ISBN 0-315-56046-0

A COMPARATIVE STUDY OF ADVANCED SUSPENSIONS BASED ON AN IN-PLANE VEHICLE MODEL

Faisal Oueslati

A Thesis

in

The Department

of

Mechanical Engineering

Presented in Partial Fulfillment of the Requirements
for the Degree of Master of Engineering
at
Concordia University
Montreal, Canada

February 1990

ABSTRACT

A COMPARATIVE STUDY OF ADVANCED SUSPENSIONS BASED ON AN IN-PLANE VEHICLE MODEL

Faisal Oueslati

Performance of three Semi-Active (SA) suspension schemes are compared in both frequency and time domains with respect to passive and active suspensions. Based on the RMS bounce acceleration transmissibility ratio and a single D.O.F. model, the SA-1 suspension scheme is found to provide a response close to that of an active suspension especially at high frequencies. The SA-2 suspension scheme, on the other hand, approaches active suspension at high frequencies but results in significant deterioration at resonance. The SA-3 suspension scheme overcomes this problem by improving the resonance control at the expense of high frequency isolation. These schemes are, then, modified and implemented in a more realistic in-plane, 4 D.O.F. model. Active suspension is shown to provide the ultimate sprung mass bounce and pitch control, at the expense of large rattle space and poor tire/ground contact force. The SA-1 suspension offers a performance (sprung mass bounce and pitch acceleration) close to that of an active system especially at high frequencies with slight loss of performance at resonance. The rattle space requirement and the tire/ground contact force are similar to those of active suspension. The SA-2 concept results in a very poor sprung

mass bounce and pitch resonance control, although at high frequencies, response is close to that of an active suspension. The SA-2 suspension scheme, however, improves the tire/ground contact force and requires a rattle space even larger than that required by an active suspension. The SA-3 suspension scheme improves the sprung mass bounce and pitch resonance, tire/ground contact force and rattle space, but results in some deterioration of the RMS transmissibility at high frequencies when compared to the SA-2 scheme. Finally, an attempt to reduce the complexity and cost of advanced suspensions is undertaken. This is achieved by using advanced suspensions, active or SA, in the front axle while the rear suspension is kept passive. In general, the performance of the combined suspensions lies between the two extremes, namely fully passive and fully active or SA. However, in the case of the SA-2 And SA-3 suspension schemes, the combined suspension results in an improvement of the sprung mass bounce and pitch resonance control when compared to a fully SA-2 or SA-3 type suspension.

ACKNOWLEDGEMENTS

The author expresses his gratitude to his thesis supervisors, Dr. S. Sankar and Dr. R. B. Bhat for their valuable guidance and encouragement, and the help extended during the preparation of this thesis. I would like also to thank all CONCAVE Research Center personnel for their technical support. The financial assistance from CONCAVE Research Center is gratefully acknowledged.

Montreal, Canada
February 1990 *F. Oueslati*

TABLE OF CONTENTS

Page

NOMENCLATURE ix

LIST OF FIGURES xi

LIST OF TABLES xvii

CHAPTER

1. INTRODUCTION 1

1.1. General 1

1.2. Literature Overview 2

1.3. Scope of the Thesis 5

2. DEMONSTRATION OF VARIOUS SUSPENSIONS FOR A 1 D.O.F. MODEL 7

2.1. General 7

2.2. Passive Suspension 7

2.3. Active Suspension 11

2.4. Semi Active Suspensions 17

2.4.1. SA Type 1 (SA-1) 18

2.4.2. SA Type 2 (SA-2) 25

2.4.3. SA Type 3 (SA-3) 32

2.5. Summary 39

3. AN IN-PLANE VEHICLE MODEL USED FOR THE ANALYSIS 40

3.1. Model Description 40

3.2. Summary 46

4. PASSIVE AND ACTIVE SUSPENSIONS FOR A 4 D.O.F. MODEL 47

4.1. General 47

4.2. Passive Suspension for a 4 D.O.F.
Vehicle Model 47
4.3. Active Suspension for a 4 D.O.F.
Vehicle Model 56
4.3.1. Choosing the
Performance Index 60
4.4. Summary 72
5. SEMI-ACTIVE (SA) SUSPENSIONS FOR A 4 D.O.F.
VEHICLE MODEL 73
5.1. General 73
5.2. SA-1 Suspension for a 4 D.O.F.
Vehicle Model 73
5.3. SA-2 Suspension for a 4 D.O.F.
Vehicle Model 82
5.4. SA-3 Suspension for a 4 D.O.F.
Vehicle Model 89
5.5. Summary 97
6. COMBINED SUSPENSIONS 98
6.1. General 98
6.2. 'Front Active / Rear Passive' Suspension 98
6.3. 'Front SA-1 / Rear Passive' Suspension . 113
6.4. 'Front SA-2 / Rear Passive' Suspension . 118
6.5. 'Front SA-3 / Rear Passive' Suspension . 122
6.6. Summary 126
7. A COMPARATIVE STUDY OF ADVANCED SUSPENSIONS . . 127
8. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER
STUDY 131
8.1. Conclusions 131

8.2. Recommendations for Further Study . . . 133
REFERENCES 134

NOMENCLATURE

A	:	state matrix
α	:	damper gain for type 2 and type 3 semi-active control laws
B	:	coefficient matrix
c	:	damping coefficient
C_1, C_2	:	front and rear damping coefficients for 4 D.O.F. model
F_d	:	semi-active actuator force
F_f	:	front semi-active damper force
F_r	:	rear semi-active damper force
F'_f	:	front semi-active damper lock-up force
F'_r	:	rear semi-active damper lock-up force
I	:	performance index
J	:	pitch moment of inertia
k_1, k_2	:	front and rear primary suspension stiffnesses
kk_1, kk_2	:	front and rear tire stiffnesses
l_1	:	horizontal distance from sprung mass CG to front wheels
l_2	:	horizontal distance from sprung mass CG to rear wheels
M	:	sprung mass
m_1, m_2	:	front and rear unsprung masses
$q_1, q_2...$	:	weighting parameters in the performance index

θ	:	pitch angle
RMS	:	root mean square
ρ_1, ρ_2	:	weighting parameters corresponding to front and rear actuator forces in the performance index
SA	:	semi-active
SA-1	:	type 1 semi-active scheme
SA-2	:	type 2 semi-active scheme
SA-3	:	type 3 semi-active scheme
t	:	time
U_1, U_2	:	vector of control forces for rear and front actuators
V	:	vehicle forward velocity
ω	:	angular frequency
ω_n	:	natural frequency for 1 D.O.F. system
ωn_1	:	bounce mode natural frequency
ωn_2	:	pitch mode natural frequency
ωn_3	:	front wheel natural frequency
ωn_4	:	rear wheel natural frequency
y	:	vector of input variables
ζ	:	damping ratio for 1 D.O.F. system

LIST OF FIGURES

Figure | Page

2.1. Passive Suspension Model. 9

2.2. RMS Acceleration Transmissibility Ratio Versus Frequency for 1 D.O.F. Passive Suspension . . . 10

2.3. Active Suspension Generic Model 14

2.4. Schematic of an Active Vehicle Suspension . . . 14

2.5. RMS Acceleration Transmissibility Ratio Versus Frequency for 1 D.O.F. Active Suspension . . . 15

2.6. SA-1 Control Policy 19

2.7. A Schematic Arrangement for Realization of SA-1 . 19

2.8. Steady State Response of SA-1 at $\omega = 0.5\omega_n$ and $\zeta = 1.0$ 21

2.9. Steady State Response of SA-1 at $\omega = 1.0\omega_n$ and $\zeta = 1.0$ 22

2.10. Steady State Response of SA-1 at $\omega = 2.0\ \omega_n$ and $\zeta = 1.0$ 23

2.11. RMS Acceleration Transmissibility Ratio Versus Frequency for 1 D.O.F. SA-1 Suspension 24

2.12. SA-2 Control Policy 26

2.13. Steady State Response of SA-2 at $\omega = 0.5\ \omega_n$ and $\alpha = 1.0$ 28

2.14. Steady State Response of SA-2 at $\omega = 1.0\ \omega_n$ and $\alpha = 1.0$ 29

2.15. Steady State Response of SA-2 at $\omega = 2.0\ \omega_n$ and $\alpha = 1.0$ 30

2.16. RMS Acceleration Transmissibility Ratio Versus Frequency for 1 D.O.F. SA-2 Suspension 31

2.17. SA-3 Model for a 1 D.O.F. System 34

2.18. Steady State Response of SA-3 at $\omega = 0.5\ \omega_n$ and $\alpha = 1.0$ 35

2.19. Steady State Response of SA-3 at $\omega = 1.0\ \omega_n$ and $\alpha = 1.0$ 36

2.20. Steady State Response of SA-3 at $\omega = 2.0\ \omega_n$ and $\alpha = 1.0$ 37

2.21. RMS Acceleration Transmissibility Ratio Versus Frequency for 1 D.O.F. SA-3 Suspension 38

3.1. Suggested 4 D.O.F. Half Vehicle Model 41

3.2. Heave Road Input 45

3.3. Pitch Road Input 45

4.1. RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of a Fully Passive Suspension . 50

4.2. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of a Fully Passive Suspension . 50

4.3. Maximum Suspension Deflection Versus Frequency of a Fully Passive Suspension 52

4.4. Maximum Tire Deflection Versus Frequency of a Fully Passive Suspension 52

4.5. A 2 D.O.F. Simplified Model Representation of a 4 D.O.F. Model 53

4.6. Overall Performance of a 2 D.O.F. Vehicle Model With Passive Suspension [36] 54

4.7. 'Chuck Hole' Type Road Profile. 55

4.8. Bounce and Pitch Response to a Chuck Hole Type Road Disturbance of the Sprung Mass of a Fully Passively Suspended Vehicle. 55

4.9. A 4 D.O.F. Vehicle Model Fitted With Active Suspension 57

4.10. RMS Bounce Acceleration Transmissibility Ratio vs Frequency of a Fully Active Suspension . . 66

4.11. RMS Pitch Acceleration Transmissibility Ratio vs Frequency of a Fully Active Suspension . . . 66

4.12. Maximum Front Suspension Deflection vs Frequency of a Fully Active Suspension . . . 67

4.13. Maximum Front Tire Deflection vs Frequency of a Fully Active Suspension 67

4.14. Overall Performance of a 2 D.O.F. Vehicle Model With Active Suspension[36] 68

4.15. Comparison of the RMS Bounce Acceleration Ratios of a 4 D.O.F. Model With Passive And Active Suspensions 69

4.16. Comparison of Bounce and Pitch Response to a 'Chuck Hole' Type Road Disturbance of the sprung Mass of an Passive and Active Suspension . 71

5.1. RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of an SA-1 Suspension . . 77

5.2. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of an SA-1 Suspension . . 77

5.3. Maximum Suspension Deflection Versus Frequency of an SA-1 Suspension 78

5.4. Maximum Tire Deflection Versus Frequency of an SA-1 Suspension 78

5.5. Comparison of the RMS Bounce Acceleration Transmissibility Ratios of a Passive, Active and SA-1 Suspensions 79

5.6. Comparison of Bounce and Pitch Response to a 'Chuck Hole' Type Road Disturbance of the Sprung Mass of Passive, Active and SA-1 Suspensions 81

5.7. RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of an SA-2 Suspension . . 84

5.8. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of an SA-2 Suspension . . 84

5.9. Maximum Suspension Deflection Versus Frequency of an SA-2 Suspension 85

5.10. Maximum Tire Deflection Versus Frequency of an SA-2 Suspension 85

5.11. Comparison of the RMS Bounce Acceleration Transmissibility Ratios of Passive, Active and SA-2 Suspensions 86

5.12. Comparison of Bounce and Pitch Response to a 'Chuck Hole' Type Road Disturbance of the Sprung Mass of Passive, Active and SA-2 Suspension. 88

5.13. SA-3 Suspension Schematic for a 4 D.O.F. Vehicle Model 90

5.14. RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of an SA-3 Suspension . . 92

5.15. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of an SA-3 Suspension . . 92

5.16. Maximum Suspension Deflection Versus Frequency of an SA-3 Suspension 93

5.17. Maximum Tire Deflection Versus Frequency of an SA-3 Suspension 93

5.18. Comparison of the RMS Bounce Acceleration Transmissibility Ratios of Passive, Active and SA-3 Suspensions 94

5.19. Comparison of Bounce and Pitch Response to a 'Chuck Hole' Type Road Disturbance of the Sprung Mass of Passive, Active and SA-3 Suspension 96

6.1. RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of a Combined 'Front Active / Rear Passive ' Suspension 106

6.2. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of a Combined 'Front Active/ Rear Passive ' Suspension 106

6.3. Maximum Front Suspension Deflection Versus Frequency of a Combined 'Front Active / Rear Passive ' Suspension 107

6.4. Maximum Rear Suspension Deflection Versus Frequency of a Combined 'Front Active / Rear Passive ' Suspension 107

6.5. Maximum Front Tire Deflection Versus Frequency of a Combined 'Front Active / Rear Passive ' Suspension 108

6.6. Maximum Rear Tire Deflection Versus Frequency of a Combined 'Front Active / Rear Passive ' Suspension 108

6.7. Comparison of the RMS Bounce Acceleration Transmissibility Ratios of Passive, Active and Combined 'Front Active / Rear Passive' Suspensions 109

6.8. Bounce and Pitch Velocity Ratio Versus Frequency of a 2 D.O.F. Pitch Model With Front Active / Rear Passive Suspension [18] 110

6.9. Comparison of the Time Response to a Chuck Hole Type Road Disturbance of a the Sprung Mass of Active, Passive and Combined 'Front Active /Rear Passive', Suspensions 111

6.10. Ascending and Descending Positions 112

6.11. RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of a Combined 'Front SA-1 /Rear Passive' Suspension 114

6.12. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of a Combined 'Front SA-1 /Rear Passive' Suspension 114

6.13. Comparison of the RMS Bounce Acceleration Transmissibility Ratios of Passive, Active, and Combined 'Front SA-1 Rear Passive' Suspensions . 115

6.14. Bounce and Pitch Velocity Ratio Versus Frequency of a 2 D.O.F. Pitch Model With Front SA-1 / Rear passive Suspension [18] 116

6.15. Comparison of the Time Response to a Chuck Hole Type Road Disturbance of the Sprung Mass of Active, Passive,SA-1 and Combined 'Front SA-1 / Rear passive' Suspensions 117

6.16. RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of a Combined 'Front SA-2 /Rear Passive' Suspension 119

6.17. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of a Combined 'Front SA-2 /Rear Passive' Suspension 119

6.18. Comparison of the RMS Bounce Acceleration Transmissibility Ratios of Passive, Active,SA-2 and Combined 'Front SA-2 / Rear Passive' Suspensions 120

6.19. Comparison of the Time Response to a Chuck Hole Type Road Disturbance of the Sprung Mass of Active, Passive, SA-2 and Combined 'Front SA-2 / Rear Passive' Suspensions 121

6.20. RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of a Combined 'Front SA-3 /Rear Passive' Suspension 123

6.21. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of a Combined 'Front SA-3 /Rear Passive' Suspension 123

6.22. Comparison of the RMS Bounce Acceleration Transmissibility Ratios of Passive, Active, SA-3 and Combined 'Front SA-3 / Rear Passive' Suspensions 124

6.23. Comparison of the Bounce and Pitch Response to a Chuck Hole Type Road Disturbance of the Sprung Mass of Passive, Active, SA-3 and 'Front SA-3 / Rear Passive' Suspensions 125

LIST OF TABLES

Table — Page

4.1. Front and Rear Actuators Gains For ' Front Active Rear Passive ' Suspension 63

4.2. Ratio of Active to Passive Parameters at Selected Frequencies 70

5.1. Ratio of SA-1 to Passive and SA-1 to Active Performance Parameters at Selected Frequencies. . 80

5.2. Ratio of SA-2 to Passive and SA-2 to Active Performance Parameters at Selected Frequencies. . 87

5.3. Ratio of SA-3 to Passive and SA-3 to Active Performance Parameters at Selected Frequencies. . 95

6.1. Front Actuator Gains For ' Front Active / Rear Passive ' Suspension 103

CHAPTER 1

INTRODUCTION

1.1. General

Contemporary suspension systems predominantly contain passive storing and dissipating elements, namely springs and dampers. These conventional vibration isolation devices have the advantage of being easy to manufacture, implement and maintain, but lack the ability to achieve a good compromise between conflicting design requirements, that is effective handling qualities and good ride comfort. Recently, however, advances in optimization techniques and automatic control in addition to availability of sophisticated transducers, processors and actuators, have been recognized to have an encouraging implication for vehicle suspension. Active suspension seems to have evolved as a result of these advances [1,2]*. Active suspension comprise actuators that, at least in part, replace passive elements. These actuators are able, theoretically, to generate a force of any magnitude and of any sign instantaneously according to some control law. Although it has been established [3] that ride comfort of vehicle passengers is improved by active suspension, a true implementation of the latter is hindered by their high production, implementation and maintenance cost, in addition to their complexity and hence less reliability. Having in mind simultaneous cost and

*Numbers in square brackets designate references performance constraints, a compromise solution is offered, that is Semi-Active (SA) suspension [4].

1.2. Literature Overview

The potential of active systems for use in vehicle suspensions lead to their experimental and theoretical investigation in a considerable number of studies. In [5] a sky-hook damper, a fictitious damper that responds to absolute rather than relative velocity, was used to simulate an active suspension for a 1 D.O.F. (degree-of-freedom) model. Based on position response, such a model demonstrated the improved ride that an active suspension can provide when compared to a passive suspension. In [6] an optimal active suspension was found to contain, in addition to the well known sky-hook damper, a so-called sky-hook spring. The latter is a fictitious spring that responds to absolute rather than relative displacement. Such results were based on an initial choice of the performance index that minimizes the Root Mean Square (RMS) rattle space or suspension travel, the RMS acceleration and the RMS jerk or rate of change of the acceleration. Although it has been rarely recognized as a major factor in ride quality, the inclusion of the jerk in the performance index results in a more general optimal suspension structure that approaches previous forms of optimal suspensions when the jerk weighting approaches zero. Quarter car 2 D.O.F. models were used by many investigators to study various aspects of active suspension. Based on such a model it was shown [7] that an active damping with body velocity feedback can achieve improved resonance control and isolation, yet a more complex active suspension incorporating high gain (fast) load leveling and active damping can provide considerably more flexibility in meeting the conflicting goals associated with suspension design. The improved performance and the more flexibility, however, are obtained at the expense of an increased power requirement. Others [8,9] have constructed experimental suspensions to verify results for a 2 D.O.F. model. Experimental results were promising, although problems arose because of the nonlinearity of the actuators. A 4 D.O.F. half car model that included unsprung masses and active suspension was used in [10] to study the pitch mode response. Based on optimal control cost function, active suspension for such model exhibited improved performance when compared to a passive suspension. Similar model was used [11] but incorporates load levelers. This was found to decrease system damping and exhibits stability problems if the feedback is too high relative to the suspension damping. An identical model was used in [12] that incorporates the time delay between the road disturbance at the front and rear wheels. To investigate all the aspects of a vehicle with active suspension, a 7 D.O.F. model was used and results confirmed the substantial improvement due to active suspension [13-16].

Most of the analysis of the SA concept was carried out using a 1 or 2 D.O.F. models. In [4] the concept of SA suspensions was first developed using a single D.O.F. model. The concept is based on the idea of having an SA device that absorbs energy in an identical fashion to an active suspension, yet during the other part of the cycle when, ideally, the damper should provide energy, the best the latter can do is to provide zero force since it does not dispose of an external power source. This scheme was found to exhibit a response close to that of an active suspension. Similar conclusion was reached in [17] were the brake pressure and the steering wheel velocity were used as leading indicator for dive and roll respectively. An SA scheme was also proven, based on frequency response plots, to be very effective in controlling the pitch and bounce in a 2 D.O.F. vehicle model [4]. The major advantage of the SA scheme for such a model is its ability to control the two resonance (bounce and pitch) independently and hence resulting in a response significantly close to that of an active suspension. In an other study [18] velocity alone and both velocity and position feedback signals were used in a 2 D.O.F. model with SA suspension. Both models exhibited ride improvement when compared to a model using passive suspension. Various other SA suspension schemes based on different control policies have been suggested and demonstrated for a simple 1 D.O.F. model [30]. They evolved as a result of difficulties faced in practical realization of the original SA scheme.

The models, being a simple 1 D.O.F. or a 2 D.O.F. pitch model, used to study SA schemes discussed above can only be valid for certain vehicle types where the main suspension connects quite directly to the ground, this is true for rail vehicle. For the case of pneumatic tired vehicles the secondary suspension representing tire stiffness need to be incorporated to see whether the benefits of SA suspensions can be extended.

1.3. Scope of the Thesis

The scope of this thesis is to investigate a comparative performance of advanced suspensions based on an in-plane vehicle model. Initially, a 1 D.O.F. model is used to gain a fundamental understanding of 3 selected SA suspension schemes. Frequency response plots are provided and discussed in comparison to passive and active suspensions. In addition, steady-state time responses to sinusoidal excitation are presented to demonstrate the behavior of the SA schemes. Because of the limited number of vehicle aspects that can be studied using a 1 D.O.F. model, a more comprehensive 4 D.O.F. model is suggested. A fully active suspension is developed for the model using optimal regulator theory and frequency response plots for the bounce and pitch modes and maximum suspension and tire deflections are presented. Then, three SA schemes are modified and fitted to the model. Frequency response plots are provided along with transient time response to a ' Chuck Hole ' type road disturbance. The last section of this work is devoted to analyzing the possibility of cutting down the price and complexity of active or SA suspensions. This is carried out by adopting active or SA suspension only for the front axle of the vehicle, while the rear suspension is kept passive. The performance of combined advanced-passive suspension systems is discussed based on frequency and transient responses. Both qualitative and quantitative comparisons and correlations with published results are also discussed wherever possible.

CHAPTER 2

DEMONSTRATION OF VARIOUS SUSPENSIONS FOR A 1 D.O.F. MODEL

2.1. General

To understand the basics of vehicle suspensions, a simple 1 D.O.F. vehicle model is used to demonstrate active, passive and 3 types of SA suspension schemes. Frequency response plots are given for changing pertinent parameters. In addition, steady-state time responses to a sinusoidal road excitation are shown to better visualize the behavior of 3 selected SA suspensions.

2.2. Passive Suspension

Traditionally, vibration isolation is accomplished through the insertion of a linear stiffness element and a linear damping element between the vibration source and the system requiring protection. The stiffness elements can be torsion bars, coil springs, leaf springs and the like; viscous dampers are, on the other hand, the most commonly used type of damping element. Passive systems will be considered to contain only springs and dampers whose rates can not be varied by external signals, hence, they do not require external energy. Passive isolation techniques are widely used in a variety of applications ranging from vehicle suspension to structural control (to reduce wind induced oscillations in tall buildings) [19].

In the case of a passive suspension, the stiffness and damping element characteristics, namely k and C, can not be varied once chosen. Hence it is necessary to carefully choose these components to provide the best possible performance. This choice, however, involves a number of compromises arising from the desire that a suspension must appear soft to minimize acceleration levels and one that must appear hard to control vehicle attitude changes and maintain good tire/ground contact [20].

Consider the 1 D.O.F. car model shown in Fig.2.1; the equation of motion is :

$$\ddot{x} + 2\zeta\omega_n(\dot{x} - \dot{y}) + \omega_n^2(x - y) = 0$$

where

$$\zeta = C / (2\sqrt{kM})$$

and $\omega_n = \sqrt{k / M}$

And the RMS acceleration transmissibility is given by :

$$T = \frac{\sqrt{1 + (2\zeta\omega / \omega_n)^2}}{\sqrt{(1-(\omega / \omega_n)^2)^2 + (2\zeta\omega / \omega_n)^2}}$$

This is plotted for several damping ratio ζ as shown in Fig.2.2.

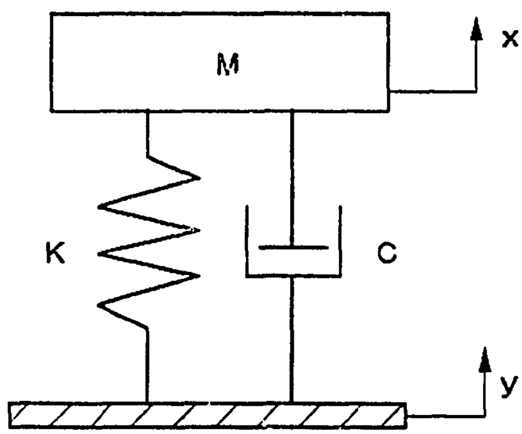


Fig.2.1. Passive Suspension Model

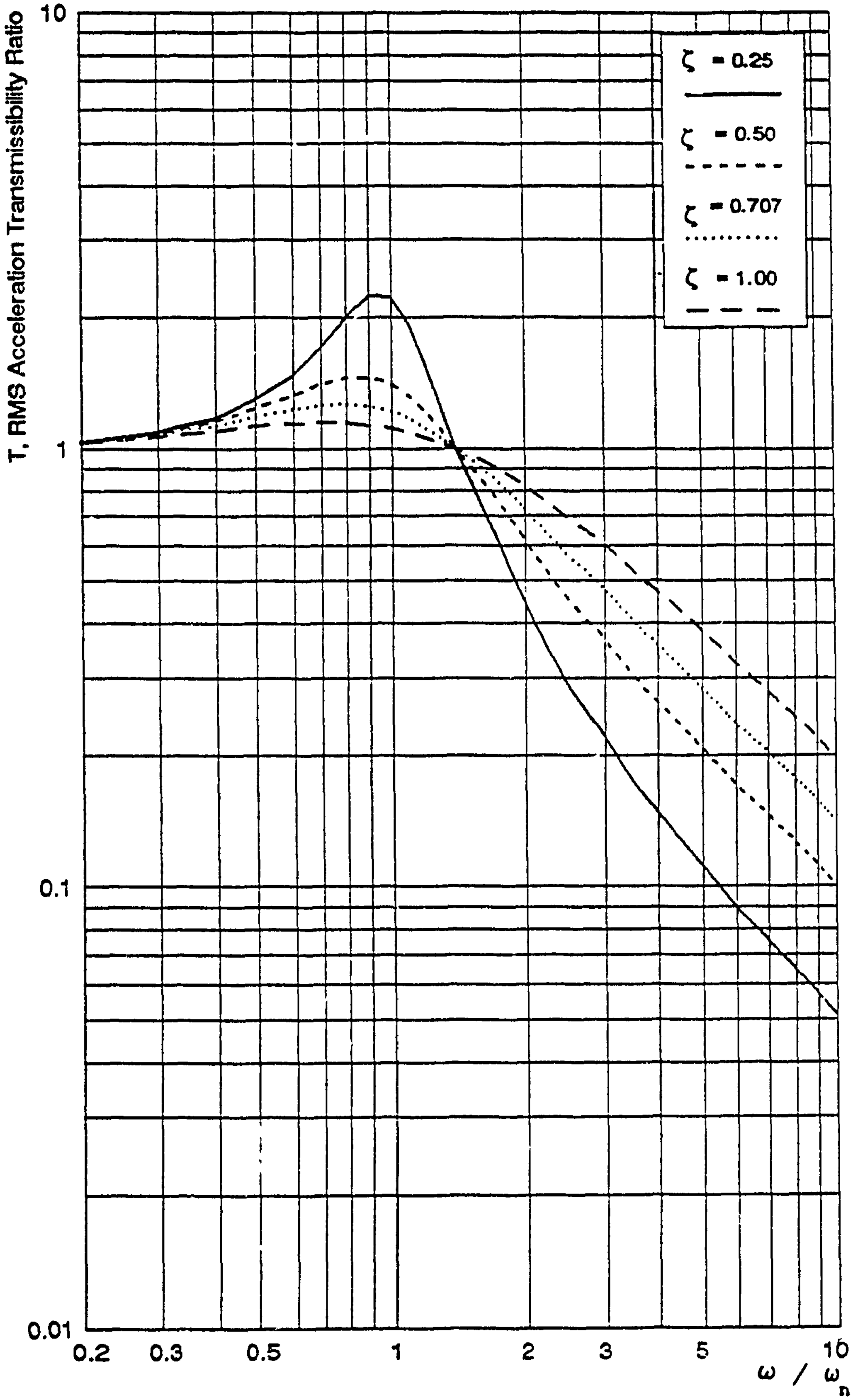


Fig.2.2. RMS Acceleration Transmissibility Ratio Versus Frequency for a 1 D.O.F. Passive Suspension

From the transmissibility plot it can be seen that as the damping ratio is increased we achieve a better response at resonance but poor isolation at higher frequencies. Similarly, as we decrease the damping ratio, we achieve better higher frequency isolation at the expense of resonance performance. A hard compromise can still be reached for this simple 1 D.O.F. system. These conflicting requirements are, however, extremely difficult to achieve if we consider actual vehicle applications where the principle dynamic modes (bounce, pitch and roll) have different natural frequencies. Additional limitation of passive elements is that they can only store (springs) or dissipate (dampers) energy. Moreover, they are restricted to generating forces in response to relative motion between attachment points of adjacent bodies.

2.3. Active Suspension

Because of the limitations of passive suspension mentioned in the previous section, an advanced suspension scheme termed active suspension was suggested. Consider the 1 D.O.F. system shown in Fig.2.3.

If the force generator is assumed to be able to generate any force, of any magnitude and of any sign instantly; the question is what that force should be in order to provide an optimum suspension ?. It is not known exactly just what F should be . This is due to the fact that the final expression for F depends primarily on what is considered an optimum suspension, i.e. how the performance criteria for the system is stated [4].

In early works, so many different methods were suggested to state the performance criteria. In [21] the weighted sum of mean square bounce acceleration and rattle space were used to determine the optimum suspension. While in [6] an optimum suspension was designed by minimizing the RMS rattle space, sprung mass acceleration and jerk. In this thesis, we are to minimize the weighted sum of the square of expected velocity and expected relative displacement of mass M. Using ' Optimal Control Theory' the expression for the force F can be shown to be given by :

$$F = b\dot{x} + k(x - y)$$

Where b and k depend on the weighing coefficients in the weighted sum [21,22].

In vehicle application, this force can be generated by a passive stiffness element and an active force generator that provides the additional $b\dot{x}$ force as shown in Fig.2.4.

The sprung mass equation of motion is therefore :

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 (x - y) = 0$$

Where ζ and ω_n are as defined earlier and the RMS acceleration transmissibility is given by :

$$T = \frac{1}{\sqrt{(1-(\omega / \omega_n)^2)^2 + (2\zeta\omega / \omega_n)^2}}$$

and is plotted for various values of ζ in Fig.2.5.

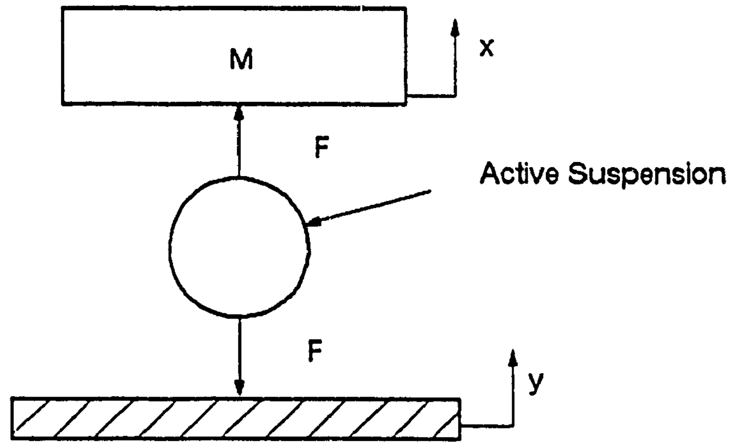


Fig.2.3. Active Suspension Generic Model

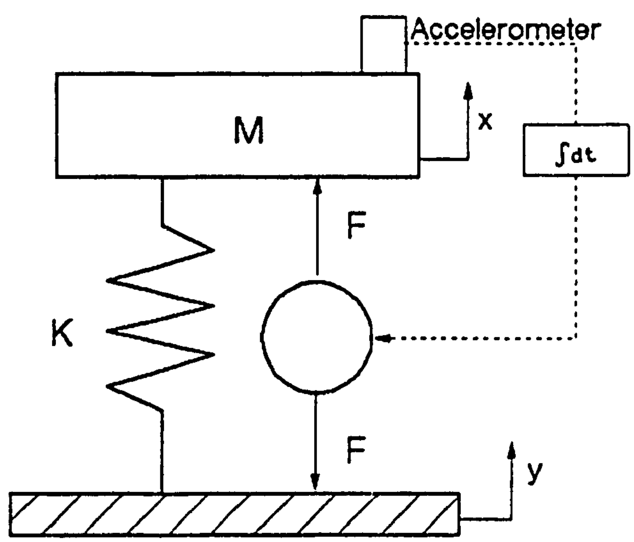


Fig.2.4. Schematic of an Active Vehicle Suspension

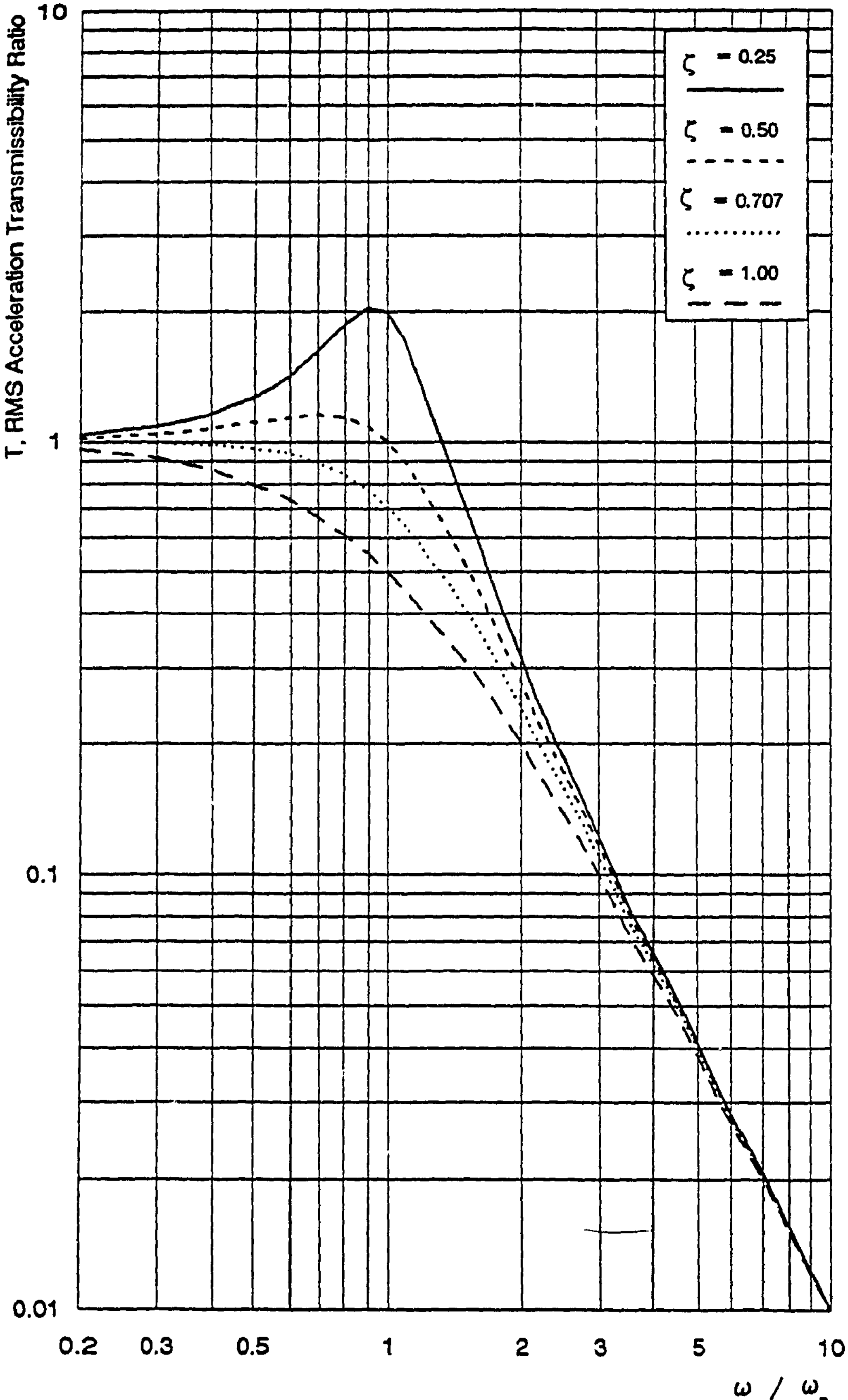


Fig.2.5. RMS Acceleration Transmissibility Ratio Versus Freequency for a 1 D.O.F. Active Suspension

Fig.2.5 illustrates the superior transmissibility characteristics of active suspension over passive suspension throughout the frequency range. Because of these significant improvements, increasing attention is being paid to the synthesis of active suspensions to control the vertical, longitudinal, and lateral motions of rail vehicles [23,24,25]. However, relatively few active suspensions have been put into service. Active suspensions are far more complex and hard to implement; they require actuators (usually hydraulic , although pneumatic, electromechanic and magnetic actuators can be used); and these actuators serve as force or torque generators. Moreover, they require sensing devices to monitor accelerations and forces of the system. But the biggest drawback of active suspensions is the large amount of external power they require for their operation, resulting in a costly and far more complex closed loop control system [26].

2.4. Semi-Active Suspensions

Passive suspensions can be easily and cheaply implemented, yet they introduce significant loss in system performance (i.e. ride quality and passenger comfort in the case of vehicle application). On the other hand, fully active suspensions improve the system performance considerably, but require, among others, significant amount of external power and complex control system implementation. To partially solve these problems, semi active (SA) suspensions were suggested. The concept of SA suspensions was first introduced in [4]. It is based on the idea of modulating the otherwise passively generated damper forces using feedback control and small amount of control power. The concept of SA suspension was shown to be a reasonable alternative suspension, with performance approaching that of an active suspension [18,27,28]. The encouraging analytical and experimental results achieved with SA control, led to their use in a variety of vehicle applications ranging from road vehicles to tracked air cushion vehicles [29].

There are so many SA suspension schemes varying in practicality, complexity and performance [30,31]. In the following sections, 3 types of SA suspensions are presented, their relative merits will be discussed based on a 1 D.O.F vehicle model.

2.4.1. SA Type 1 (SA-1)

SA type 1 (SA-1) concept was the first SA control scheme suggested [4]. It can be thought of as simplifying modification of the active suspension discussed earlier in section 2.3. An active suspension would require a control force proportional to the absolute velocity ($F = b\dot{x}$). Consequently there is a need for a servomechanism that has to either supply or absorb energy. If the servomechanism is replaced by an SA device that is capable of dissipating energy proportional to the absolute velocity, this device will, therefore, absorb energy identically to a servomechanism. Yet, during the other part of the cycle when the servomechanism supplies energy, the best the SA device can do is to produce zero force [4]. The force generated by the damper can, therefore, be expressed mathematically as follows :

$$F_d = \begin{cases} 2\zeta\omega_n\dot{x} & \dot{x}(\dot{x} - \dot{y}) > 0 \\ 0 & \dot{x}(\dot{x} - \dot{y}) < 0 \end{cases}$$

and is visualized through Fig.2.6. In Fig.2.7 a schematic arrangement for realization of this SA scheme is shown.

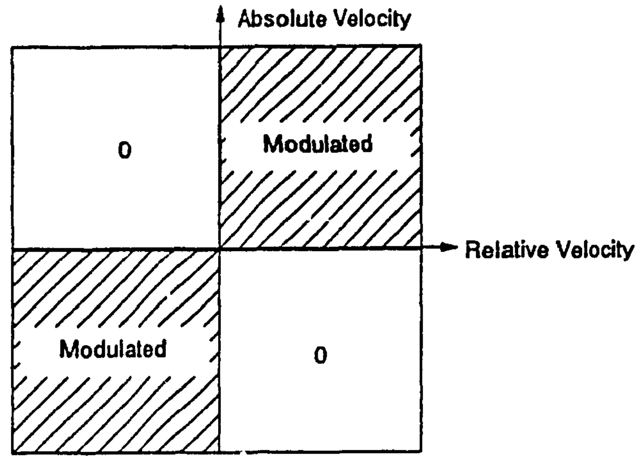


Fig.2.6. SA-1 Control Policy

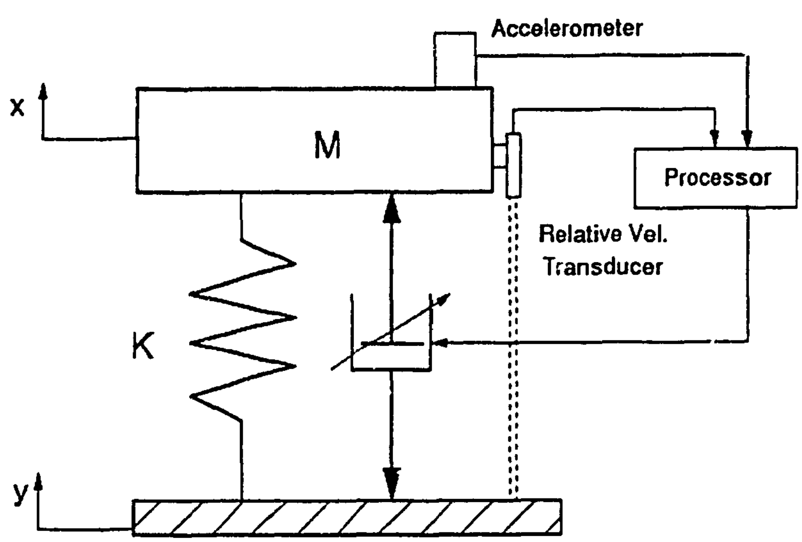


Fig.2.7. A Schematic Arrangement for Realization of an SA-1 Suspension

To better visualize the behavior of the system, a set of steady-state simulation results are shown in Figs.2.8,2.9 and 2.10, with an input frequency of 0.5,1 and 2 times the natural frequency. At 0.5 ω_n, one can easily identify the portion of the cycle when lock-up occurs ($\dot{x}=\dot{y}$ and $\ddot{x}=\ddot{y}$). At higher frequencies, however, lock-up does not occur, yet portion of the cycle when the damper force is equal to zero is visible. During this portion of the cycle, the damper is supposed to provide energy, but it does not dispose of any external power supply, consequently the best it can do is to produce no force at all. Notice that Figs.2.8,2.9 and 2.10 are not to the same scale and are simply presented to illustrate the SA-1 concept.

Fig.2.11 is the RMS acceleration transmissibility ratio of SA-1. It demonstrates the effectiveness of this concept which proves to be clearly superior to a passive suspension. At high frequencies, the RMS acceleration transmissibility ratio of SA-1 approaches that of an active suspension. This can be attributed to the fact that as the frequency is increased the amount of energy that the damper could not supply because it does not dispose of an external source of energy, is reduced and hence the loss of performance is minimized.

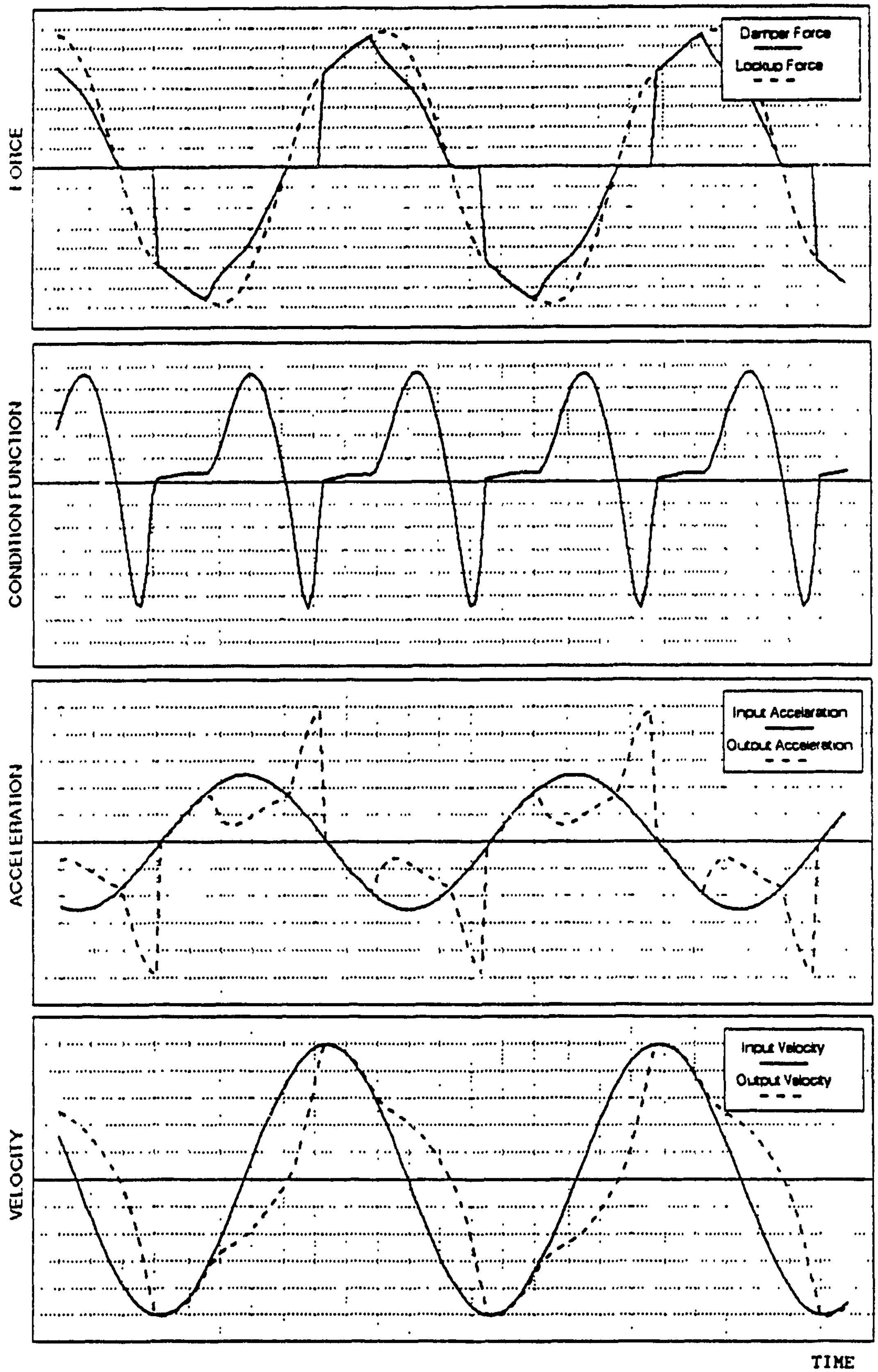


Fig.2.8. Steady State Response of SA-1 at $\omega = 0.5\ \omega_n$ and $\zeta = 1.0$

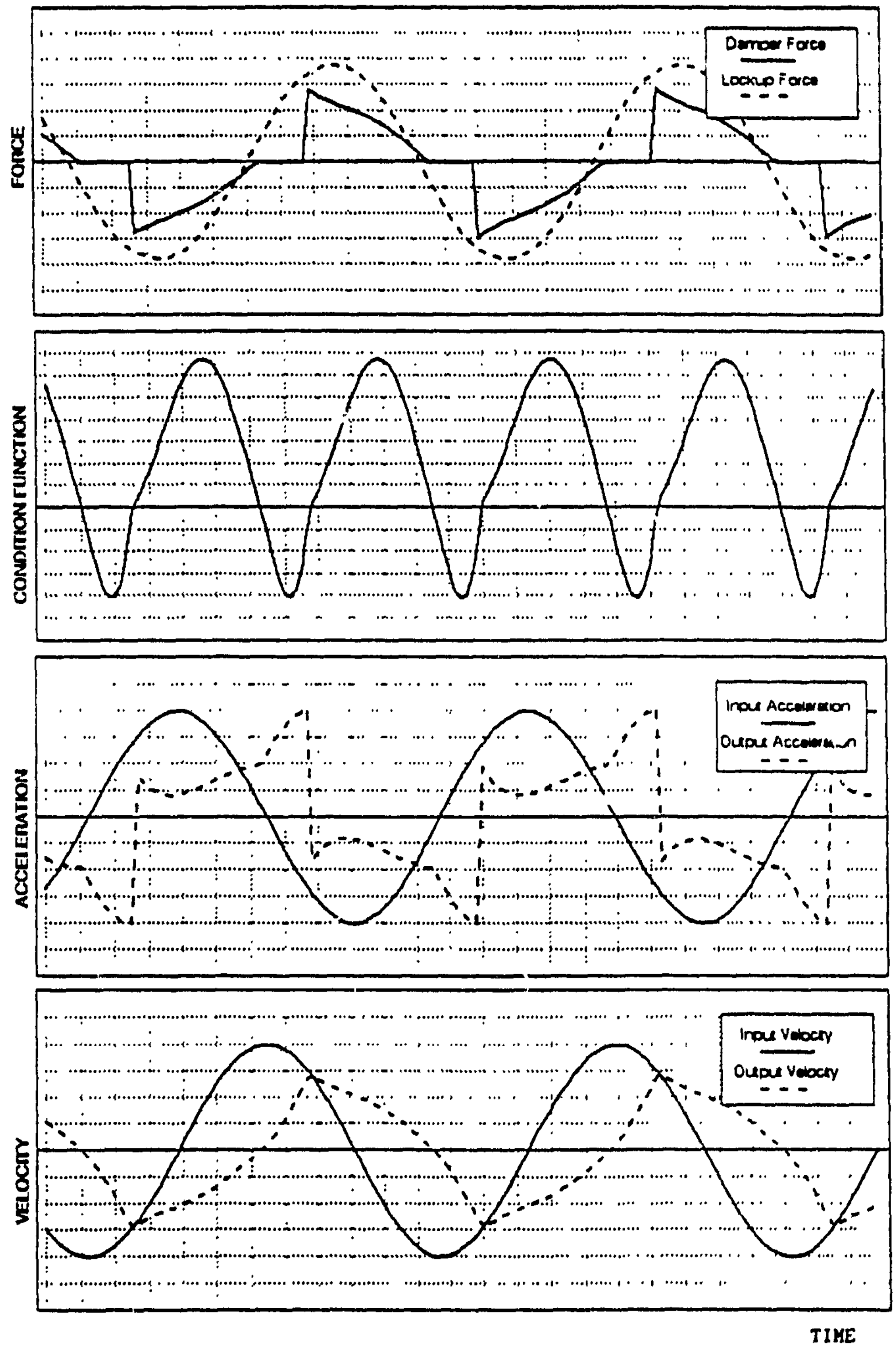


Fig.2.9. Steady State Response of SA-1 at $\omega = 1.0\ \omega_n$ and $\zeta = 1.0$

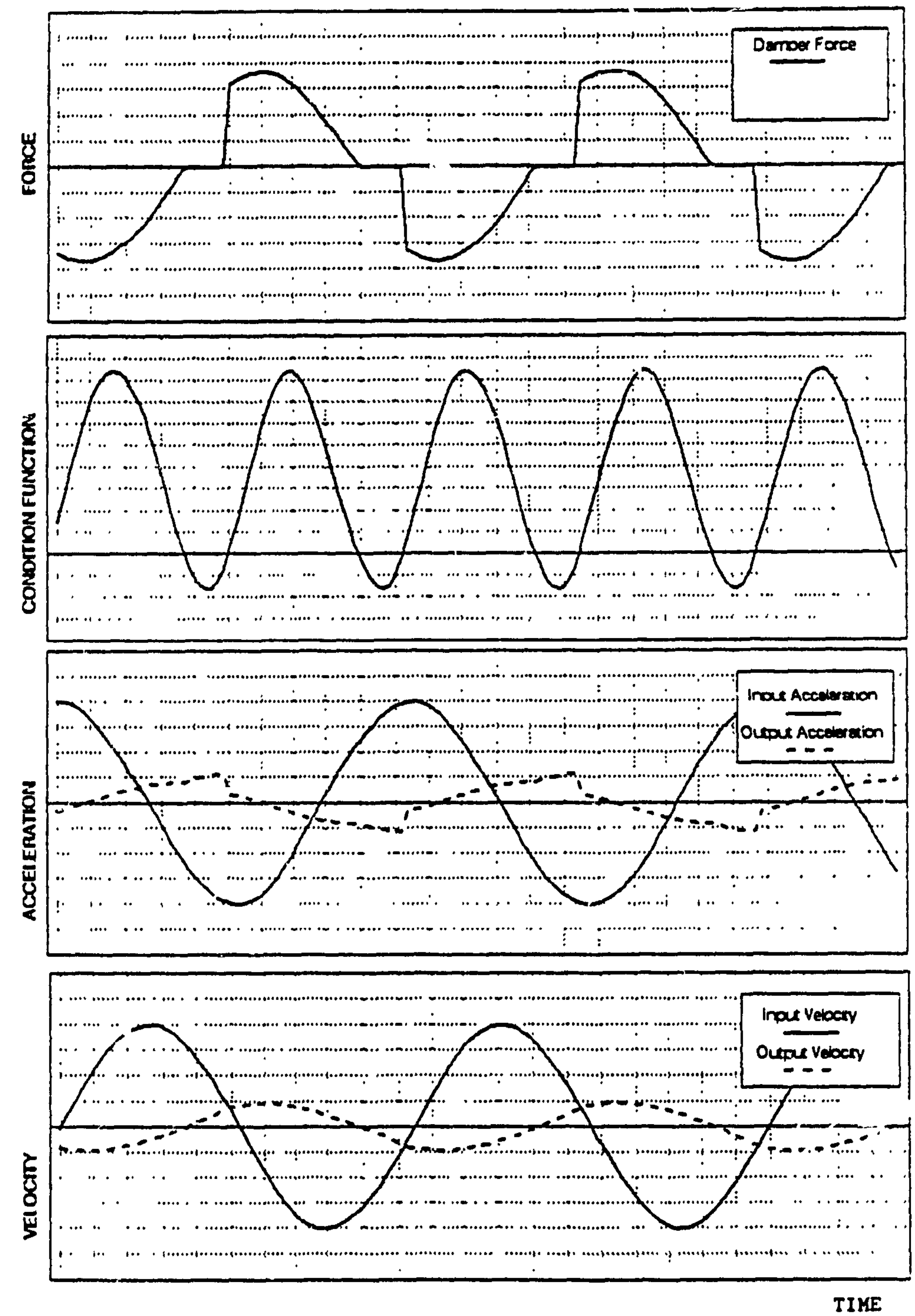


Fig.2.10. Steady State Response of SA-1 at $\omega = 2.0\ \omega_n$ and $\zeta = 1.0$

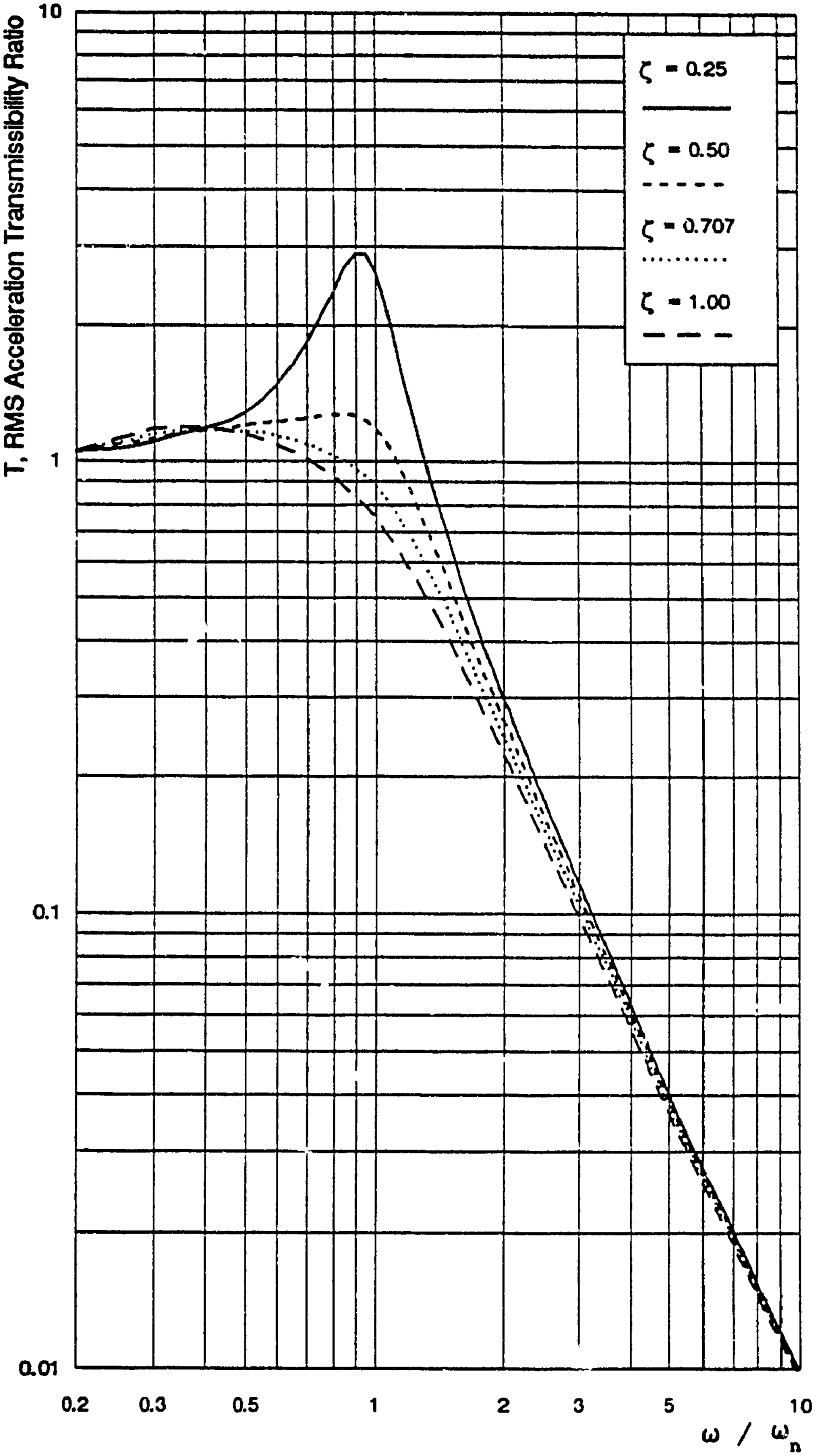


Fig.2.11. RMS Acceleration Tranmissibility Ratio Versus Frequency for a 1 D.O.F. SA-1 Suspension

2.4.2. SA Type 2 (SA-2)

This concept was originally suggested by Rakheja and Sankar [31]. They noticed that the damping force in a passive damper tends to increase the sprung mass acceleration when the spring force and damper force have the same direction i.e. $(x-y)(\dot{x}-\dot{y})$ greater than zero. A SA damper should, therefore, provide no force during this part of the cycle. When the spring force and damper force are in opposite directions, the damper generate a force having the same magnitude as the spring force but acting in the opposite direction [30], so that a net zero resulting force is obtained. Mathematically, this can be expressed as :

$$F_d = \begin{cases} -\alpha k(x - y) & (x - y)(\dot{x} - \dot{y}) < 0 \\ 0 & (x - y)(\dot{x} - \dot{y}) > 0 \end{cases}$$

Where α is the gain and k is the spring rate. F_d is visualized through Fig.2.12.The major advantage of SA-2 is that it only requires the measurement of relative quantities, which can be readily obtained even in the case of vehicle applications.

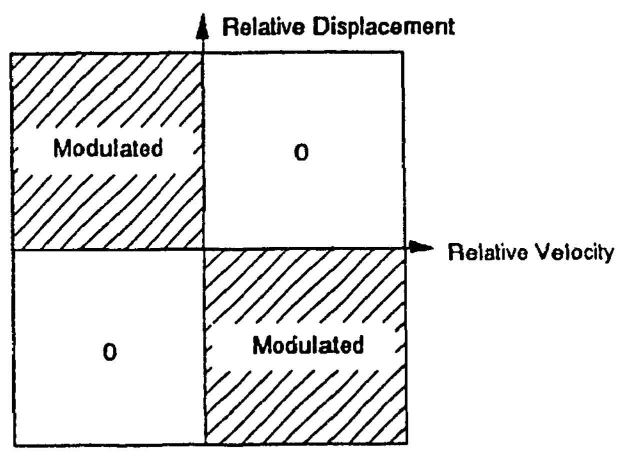


Fig.2.12. SA-2 Control Policy

Figs.2.13,2.14 and 2.15 represent the steady-state simulation results at 0.5, 1.0 and 2.0 times the natural frequency. At low frequency lock-up situation is apparent. As the frequency is increased, however, the lock-up force is visibly higher than the damper force and hence no lock-up occurs. Fig.2.16. is the RMS acceleration ratio frequency response for different values of the gain α. Beyond natural frequency the SA-2 control policy provides a far better isolation characteristics than a passive damper. At frequencies around the natural frequency, a high RMS acceleration ratio can be noticed. This is probably the major draw back of the SA-2 scheme. Increasing the gain α would not, unfortunately, solve this problem, it would indeed result in instability [30].

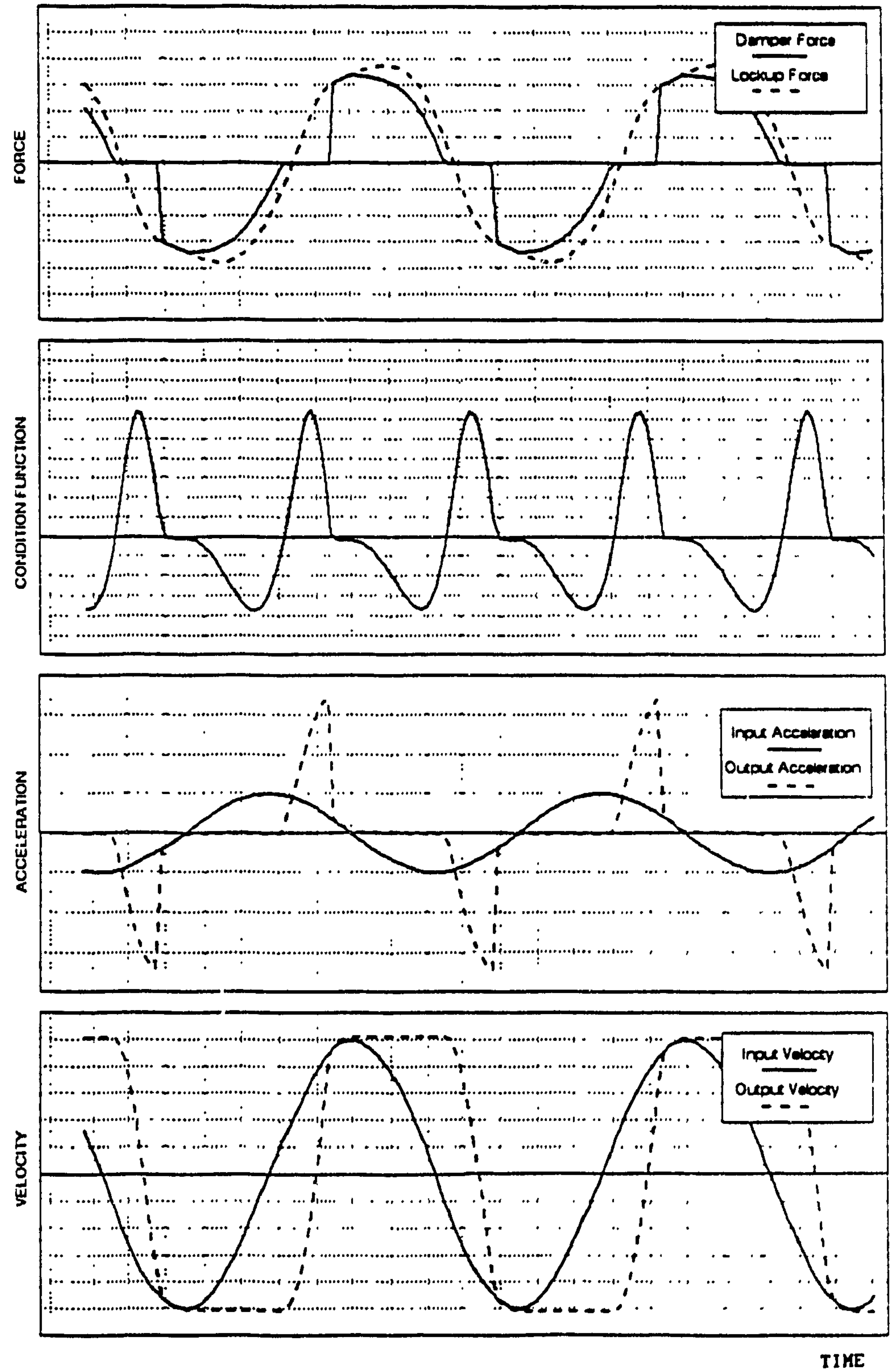


Fig.2.13. Steady State Response of SA-2 at $\omega = 0.5\ \omega_n$ and $\alpha = 1.0$

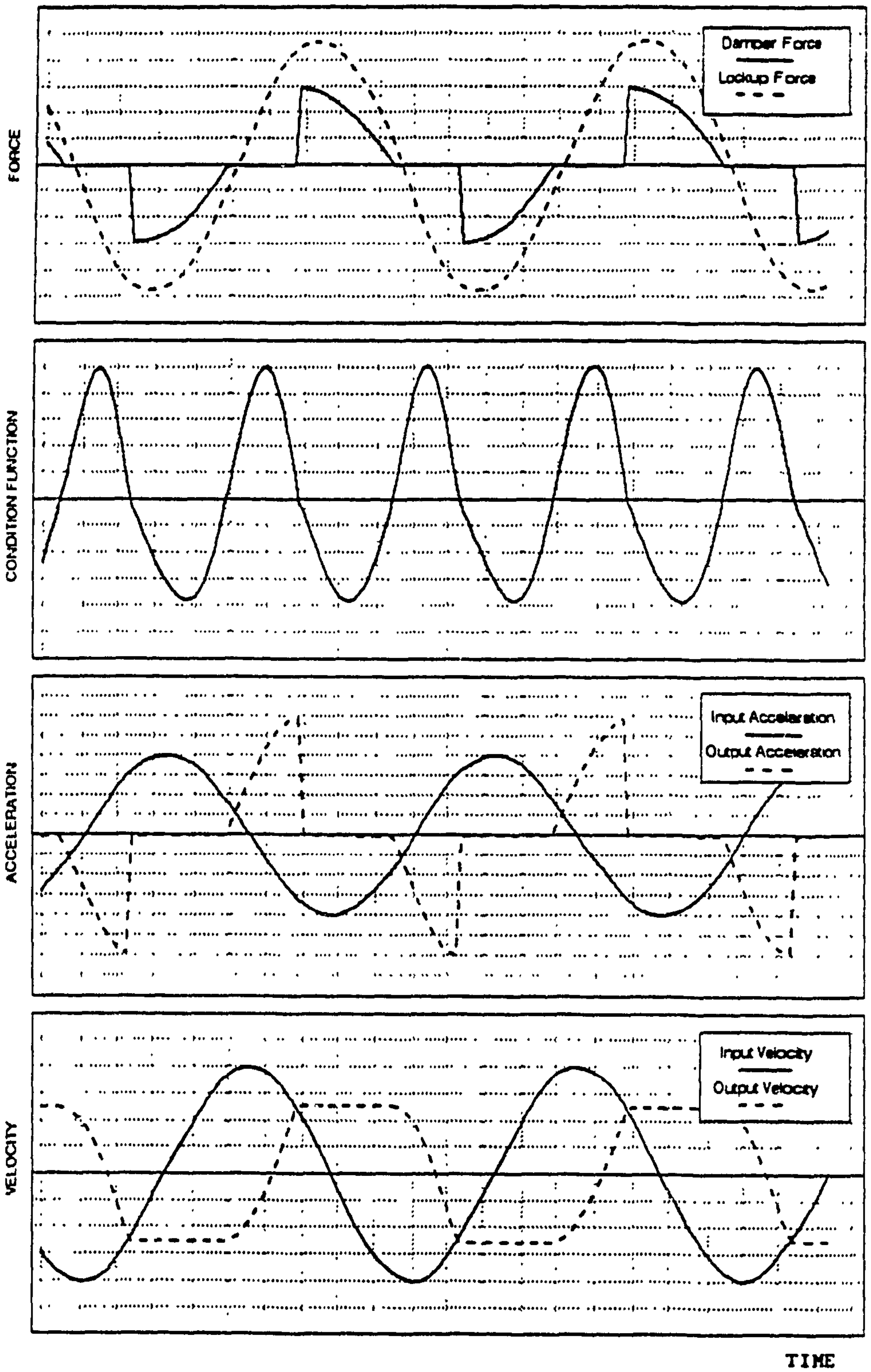


Fig.2.14. Steady State Response of SA-2 at $\omega = 1.0\ \omega_n$ and $\alpha = 1.0$

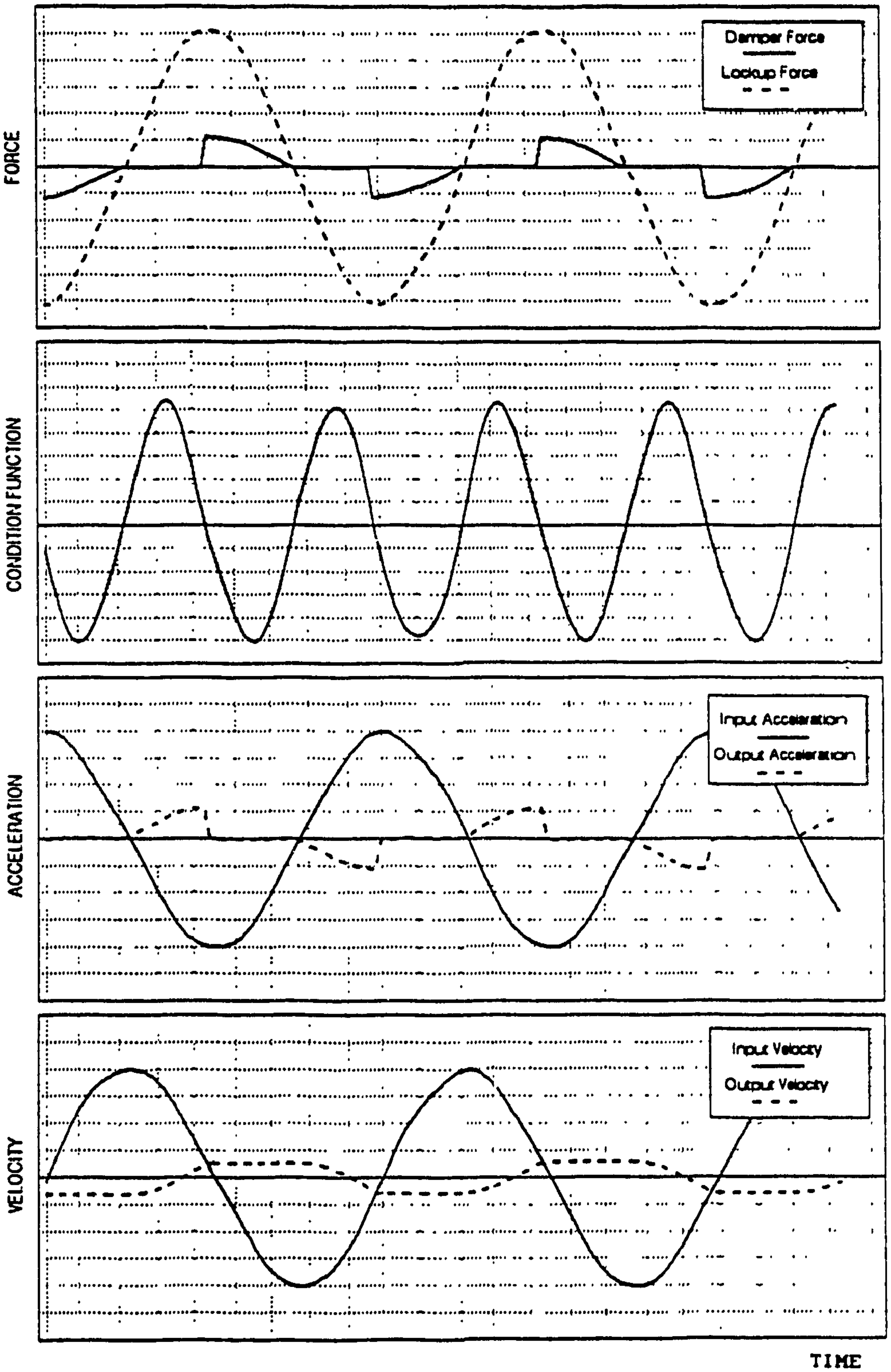


Fig.2.15. Steady State Response of SA-2 at $\omega = 2.0\ \omega_n$ and $\alpha = 1.0$

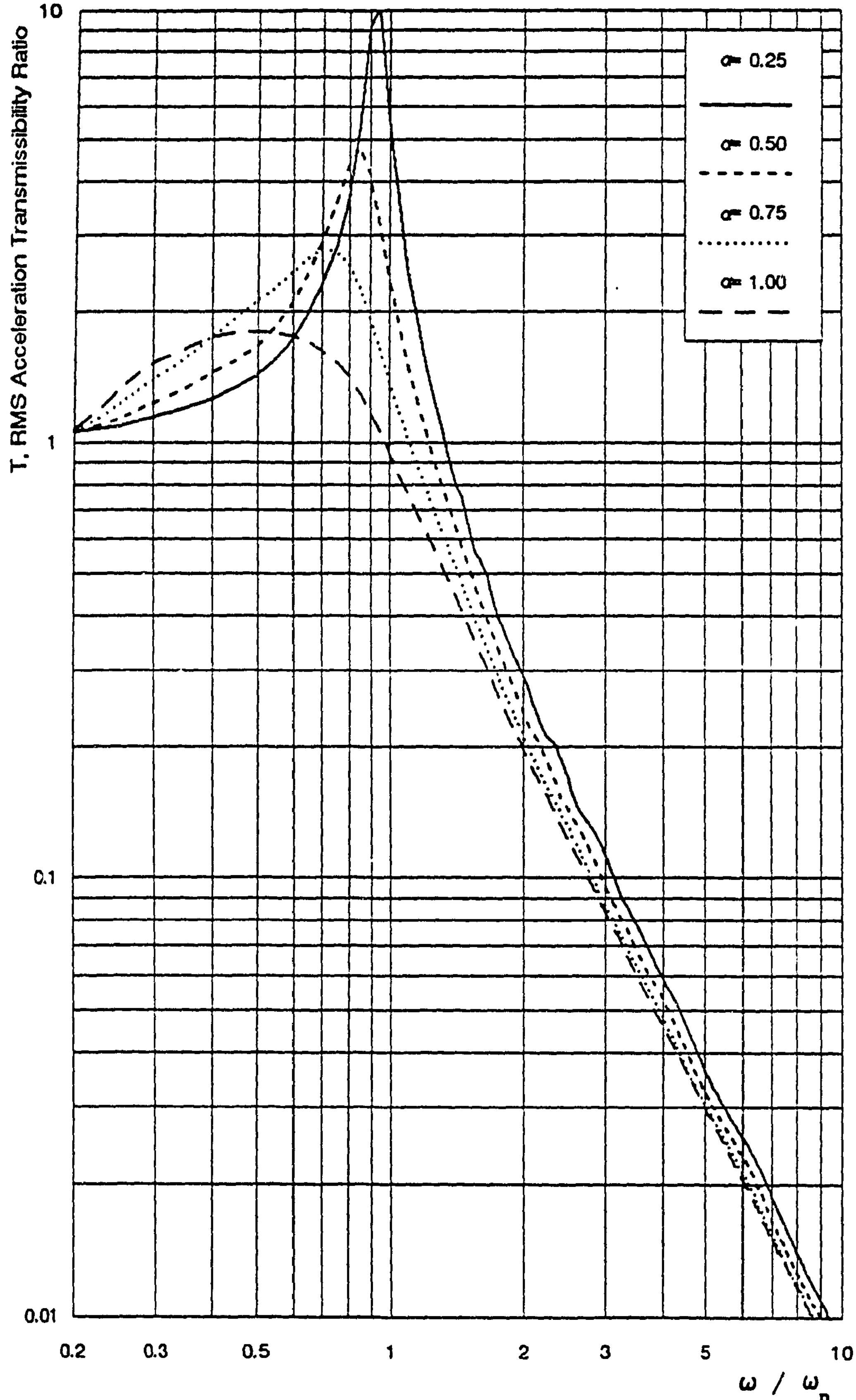


Fig.2.16. RMS Acceleration Transmissibility Ratio Versus Frequency for a 1 D.O.F. SA-2 Suspension

2.4.3. SA Type 3 (SA-3)

Because of the undesirable high RMS acceleration ratio at and around the natural frequency for SA-2, it is suggested that a passive damper is placed in parallel with the SA-2 damper and the spring as shown in Fig.2.17.

The control scheme and condition function remain, however, the same as for the SA-2 scheme. The damper gain beyond which instability occurs, can be approximated assuming a linear system behavior. The equation of motion of the system when F_d is non-zero is:

$$M\ddot{x} + c(\dot{x} - \dot{y}) + k(1 - \alpha)(x - y) = 0$$

For a stable system, the real part of the eigenvalues of the system must be within the limits of the left hand plane i.e. real part of the eigenvalues must be less or equal to zero [32]. The eigenvalues of this system can be shown to be :

$$\omega_n (-\zeta \mp \sqrt{\zeta^2 - (1 - \alpha)})$$

It can be observed that the maximum value α can take, regardless of the damping ratio ζ, is 1.0, beyond which instability occurs as discussed in section 2.4.2.

Although the above analysis is based on the linear system assumption, simulation checks seems to validate this analysis for this discontinuous SA-3 scheme.

As an example, a damping ratio of 0.25 is chosen and simulation is carried out. Figs.2.18, 2.19 and 2.20 represent the steady-state response of the system at 0.5, 1.0 and and 2 times the natural frequency. Lock-up situation is apparent at low frequencies.

Fig.2.21 is the RMS acceleration transmissibility ratio frequency response of SA-3. The insertion of a slight damping ($\zeta = 0.25$) has 2 effects (compared to SA-2) :

1. Reducing significantly the RMS acceleration ratio at and around the natural frequency.

2. Increasing the RMS acceleration ratio beyond the natural frequency.

The main advantage of SA-3 is its effectiveness in reducing the RMS acceleration ratio at and around the natural frequency. This is very important because of the sensitivity of vehicle passengers to low frequency excitation.

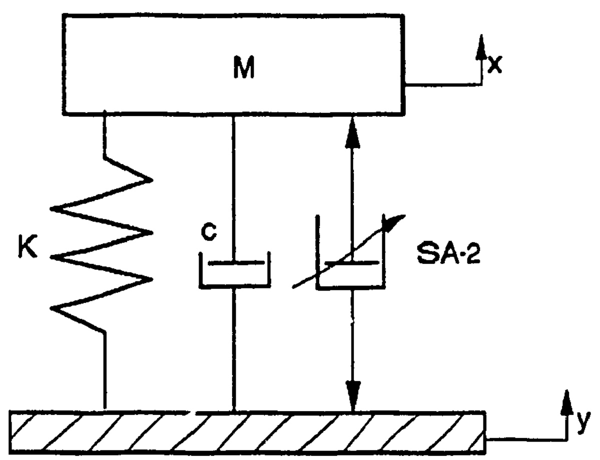


Fig.2.17. SA-3 Model for a 1 D.O.F. System

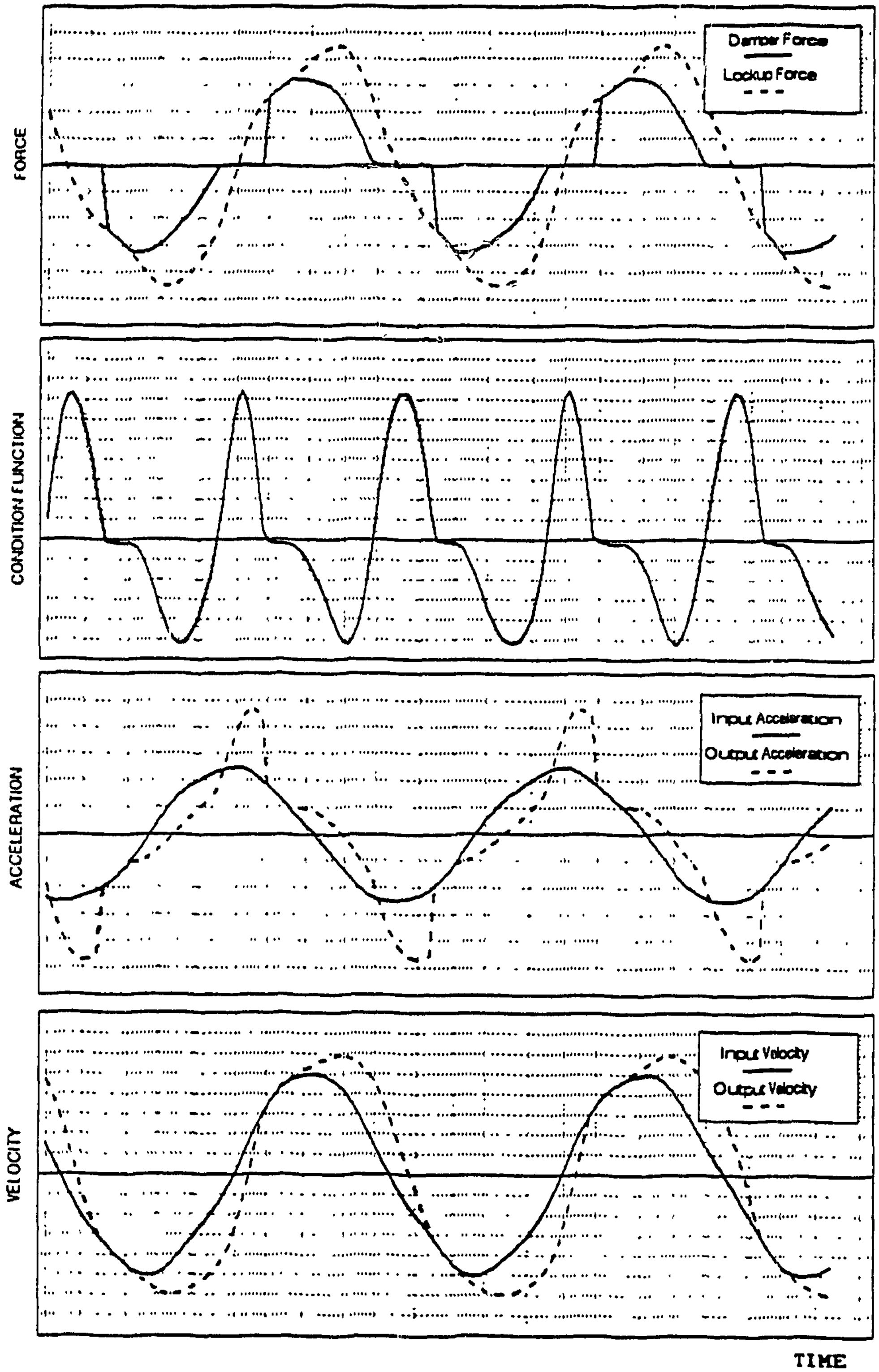


Fig.2.18. Steady State Response of SA-3 at $\omega = 0.5\ \omega_n$ and $\alpha = 1.0$

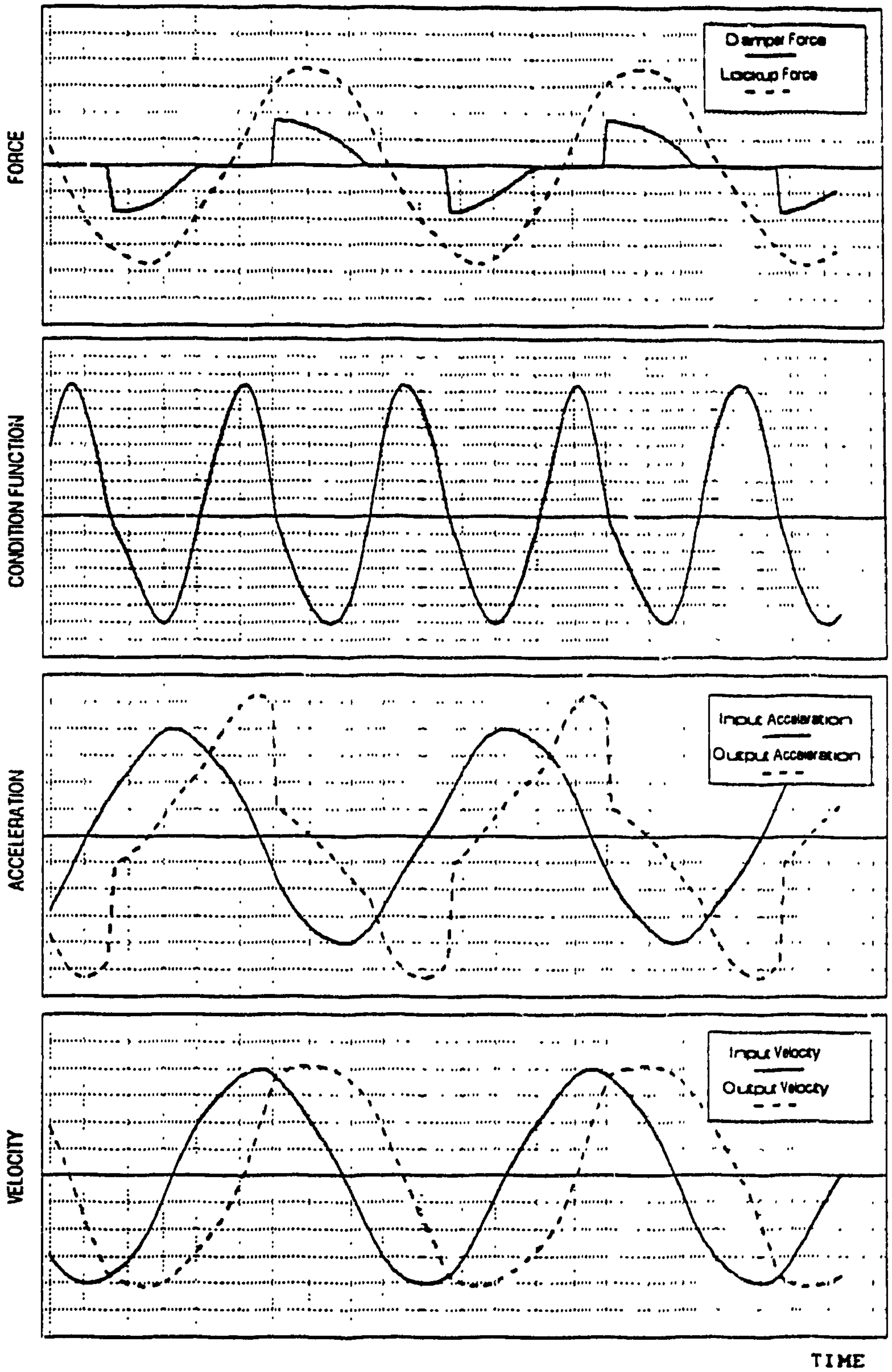


Fig.2.19. Steady State Response of SA-3 at $\omega = 1.0\ \omega_n$ and $\alpha = 1.0$

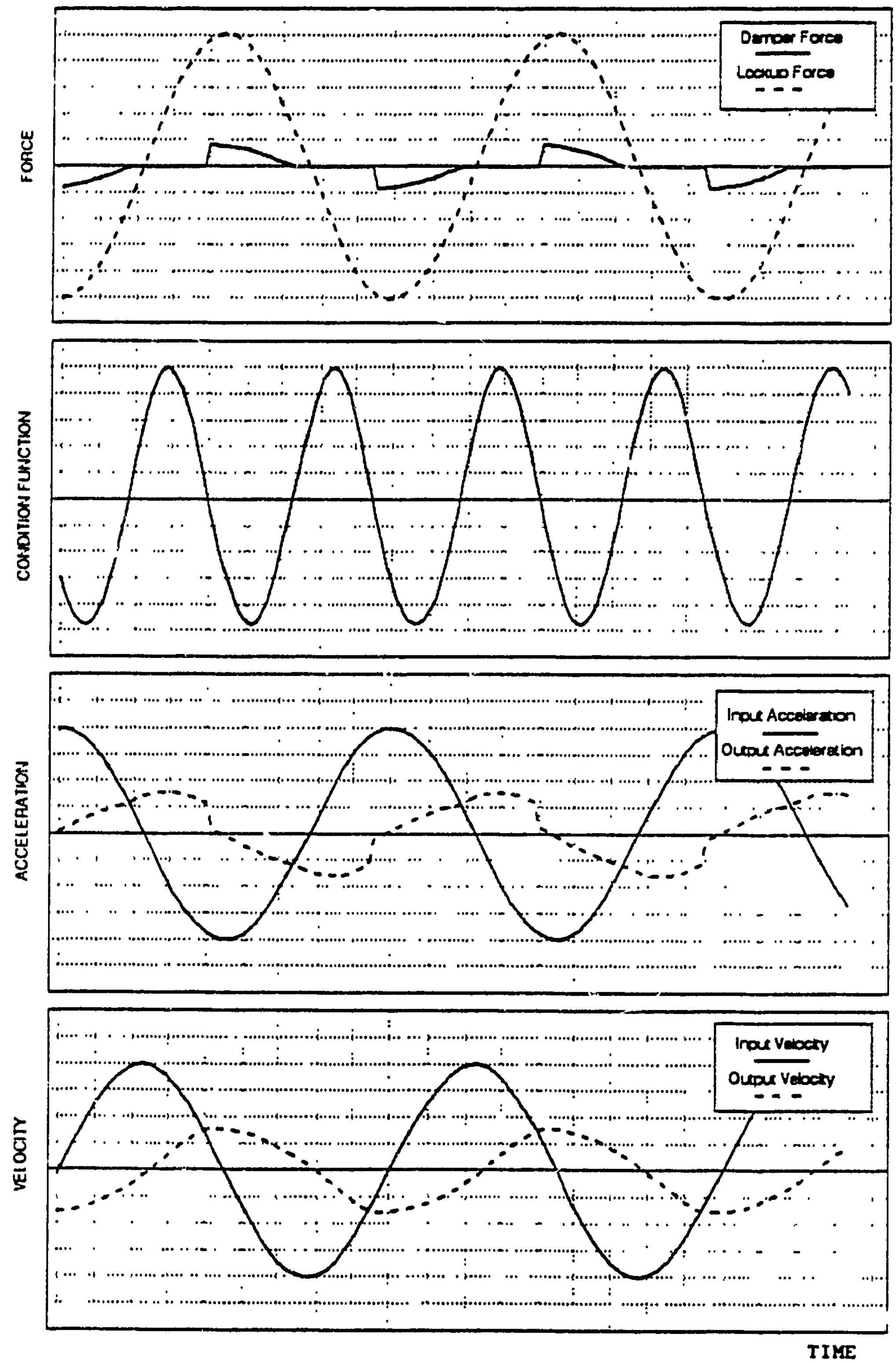


Fig.2.20. Steady State Response of SA-3 at $\omega = 2.0\ \omega_n$ and $\alpha = 1.0$

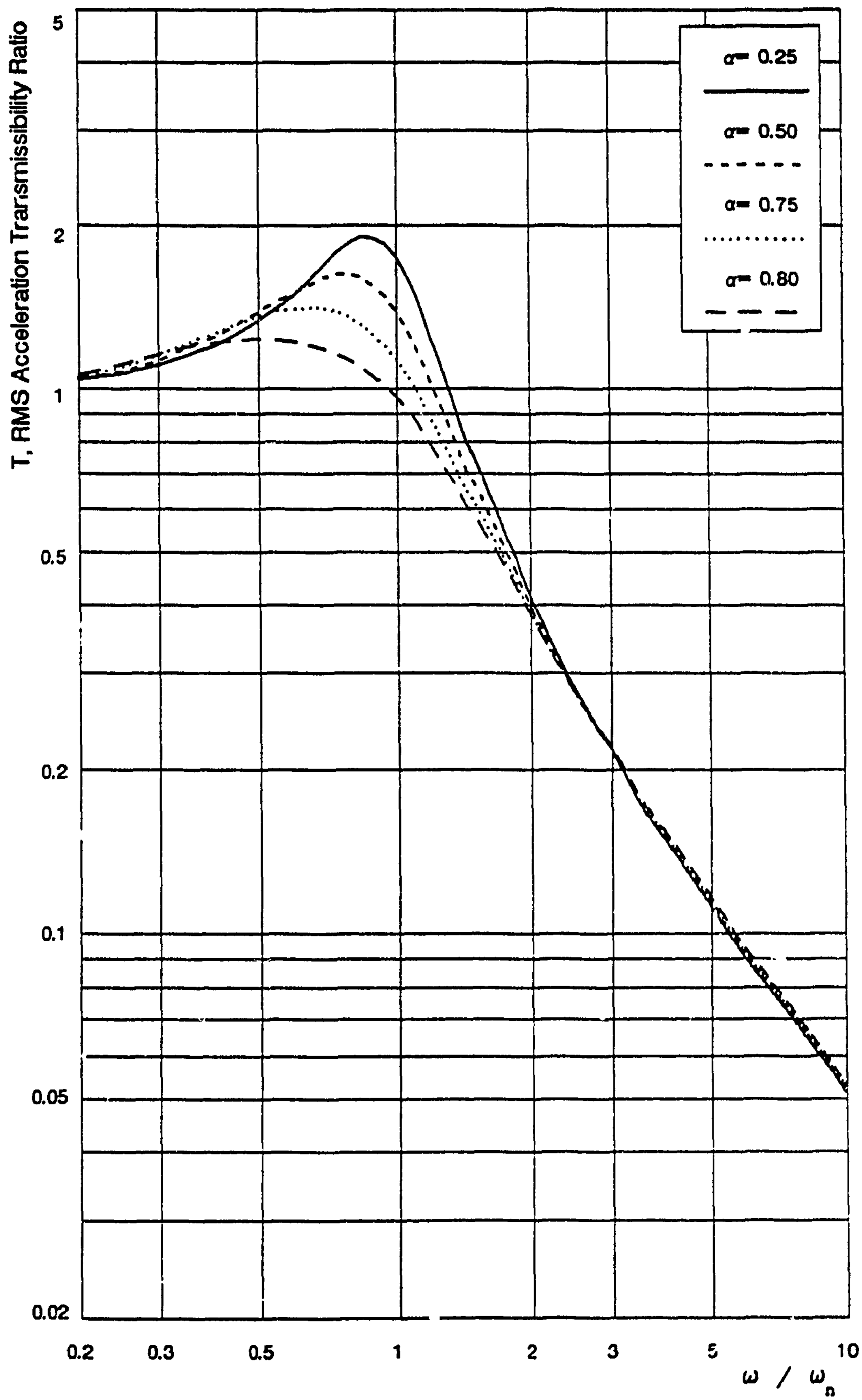


Fig.2.21. RMS Acceleration Transmissibility Ratio Versus Frequency for a 1 D.O.F. SA-3 Suspension

2.5. Summary

A basic understanding necessary to investigate advanced suspensions is gained through the study of various suspension schemes for a simple 1 D.O.F. model. Active suspension was shown to provide the ultimate performance, yet they are costly and complex. On the other hand, passive suspension has the advantage of being simple and far less costly. Unfortunately they fail to achieve a performance close to that of an active suspension. All the 3 SA suspension schemes, although differing slightly from one another, seem to provide a compromising solution offering a performance close to that of an active suspension in some instances and resulting in an important reduction of complexity and cost.

CHAPTER 3

AN IN-PLANE VEHICLE MODEL USED FOR THE ANALYSIS

3.1. Model Description

Depending on the type of study, a vehicle model with required complexity is often suggested. A 1 D.O.F. model is simple to analyze and is used mainly to gain a basic understanding of the suspension concept. Such a simplified model can sometimes be sufficient to predict the behaviour of the system. It was shown, for example, that the general performance of an active suspension compared to a passive suspension for a 7 D.O.F. model can be predicted by studying a 1 D.O.F. model [33,34]. A vehicle can, in general, be represented by a 7 D.O.F model. Such model will allow for the study of all the basic vehicle modes i.e. roll, pitch and bounce.

All the suspension schemes to be studied in this thesis have been demonstrated thoroughly for a 1 D.O.F. model in the previous chapter 2. Analytical investigations of these suspension schemes, however, need to be extended to evaluate their relative performance in the case of complex vehicle models such as an in-plane 4 D.O.F. half vehicle model that takes into account both pitch and bounce modes of the vehicle. Fig.3.1 illustrates schematically such a model.

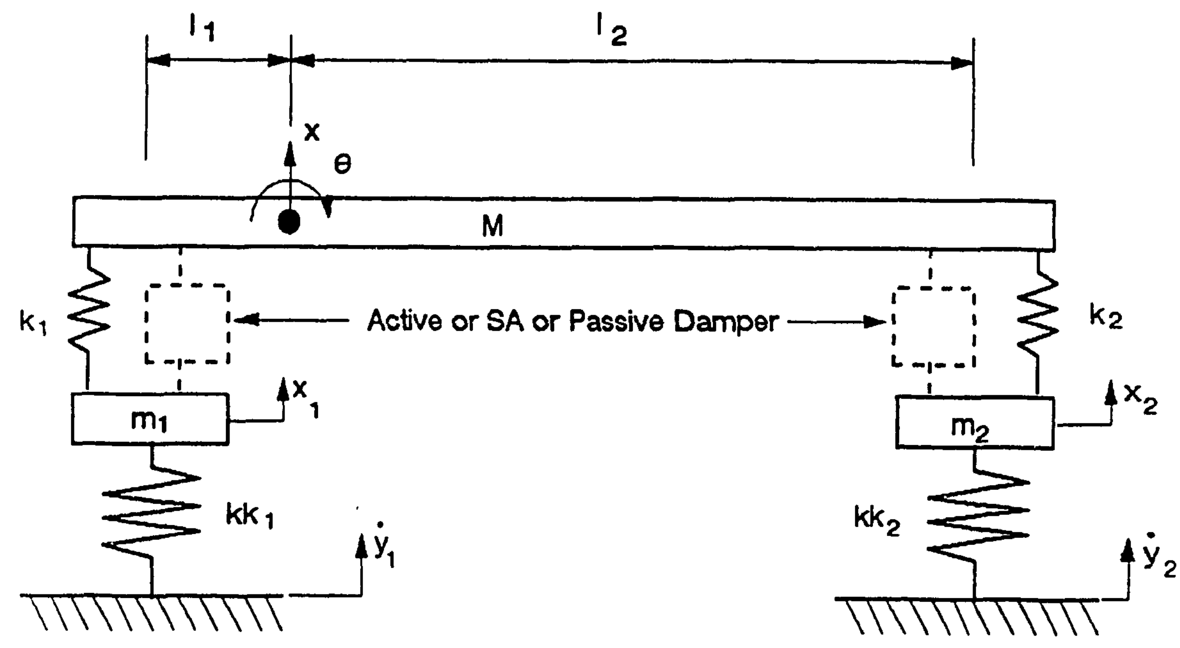


Fig.3.1. Suggested 4 D.O.F. Half Vehicle Model

The model consists of a mass M representing the vehicle body and two unsprung masses m_1 and m_2 representing the front and rear wheels and axles. As in the actual case, the vehicle body and the front and rear wheel are coupled by two compliance elements k_1 and k_2. In addition to the two springs k_1 and k_2, 2 actuators, semi-active or passive dampers are installed in parallel. Finally the model incorporates two stiffness elements kk_1 and kk_2; these are the primary suspension stiffness and are due to rubber tires, steel wheels, magnetic levitation or air cushion. In this case, the stiffness kk_1 and kk_2 are chosen to simulate the effects of inflation pressure and carcass elasticity. The energy dissipated by carcass deformation is negligible and hence no damping effect is represented in the model at that point. We assume also that the tire contacts the ground through a point follower. The follower is assumed to always stay in contact with the ground, i.e. the wheel does not hop and the road excitation is identical to the road profile. This model will allow for the study of the bounce and pitch modes of the vehicle. The system parameters listed below are chosen as representative of a small passenger car.

M	=	700.0 Kg.	Vehicle body mass
J	=	1200.0 $Kg.m^2$	Sprung mass pitch moment of inertia
m_1	=	25.0 Kg	Front wheel and axle assembly mass
m_2	=	25.0 Kg	Rear wheel and axle assembly mass
k_1	=	10000.0 N/m	Front suspension stiffness
k_2	=	10000.0 N/m	Rear suspension stiffness

kk_1 =	178000.0 N/m	Primary front suspension stiffness
kk_2 =	178000.0 N/m	Primary rear suspension stiffness
c_1 =	1250.0 N.s/m	Nominal front damping for passive system
c_2 =	1250.0 N.s/m	Nominal rear damping for passive system
l_1 =	1.0 m	Distance from Vehicle Body CG to front suspension
l_2 =	1.5 m	Distance from Vehicle Body CG to rear suspension

Based on the above system parameters the undamped natural frequencies can be calculated to be:

ωn_1 = 4.725 rad/s Bounce mode natural frequency

ωn_2 = 5.646 rad/s Pitch mode natural frequency

ωn_3 = 86.716 rad/s Front wheel natural frequency

ωn_4 = 86.727 rad/s Rear wheel natural frequency

Basically there are three measures of performance that are to be considered in this work. First, the RMS bounce and pitch accelerations which determine the ride quality i.e. how well the passenger is protected from road excitations. The second measure of performance is the maximum suspension deflection which is a design limitation and is an indication of the occurrence of the suspension hitting the stops. Finally, the maximum tire deflection which is directly proportional to the tire contact force. The latter is necessary to provide good tire/road holding characteristics. The tire deflection and accordingly contact force, however, must be kept within limits as it can not exceed design limitations.

The RMS bounce acceleration transmissibility ratio and the maximum suspension and tire deflections are evaluated for a heave road input characterized by a sinusoidal velocity excitation of 0.5 m/s magnitude as shown in Fig.3.2. The RMS pitch acceleration transmissibility ratio is obtained for a sinusoidal pitch road input generated from sinusoidal out-of-phase velocity excitation of 0.5 m/s at each wheel.

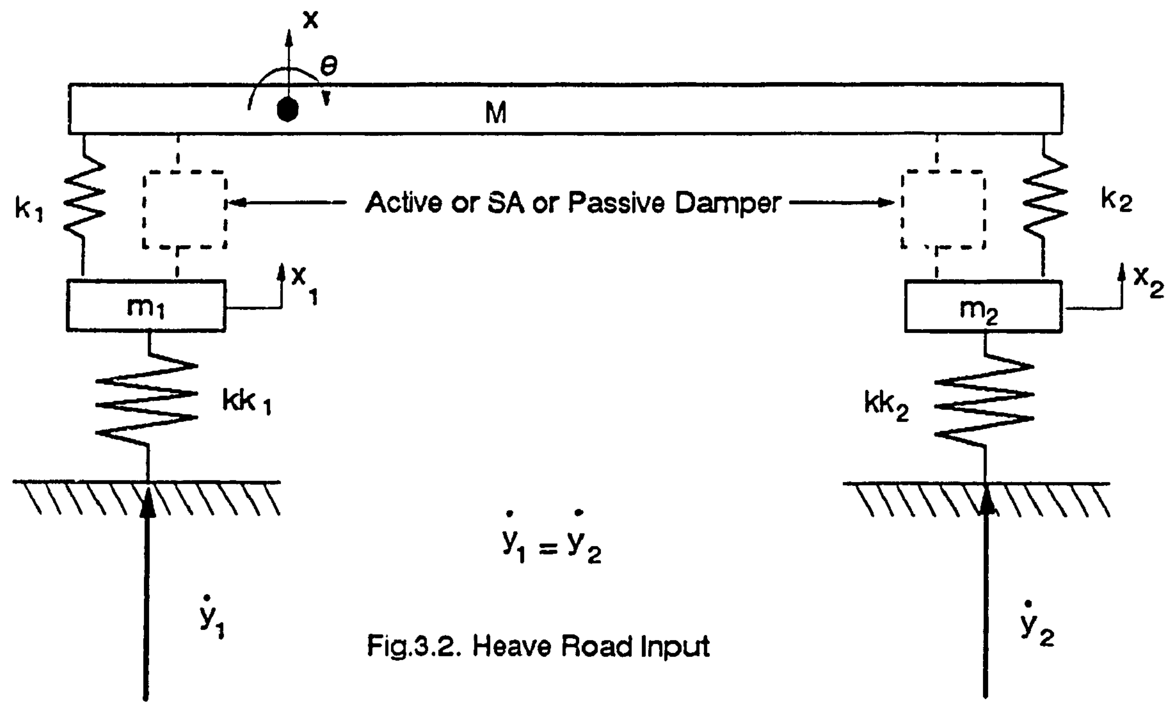


Fig.3.2. Heave Road Input

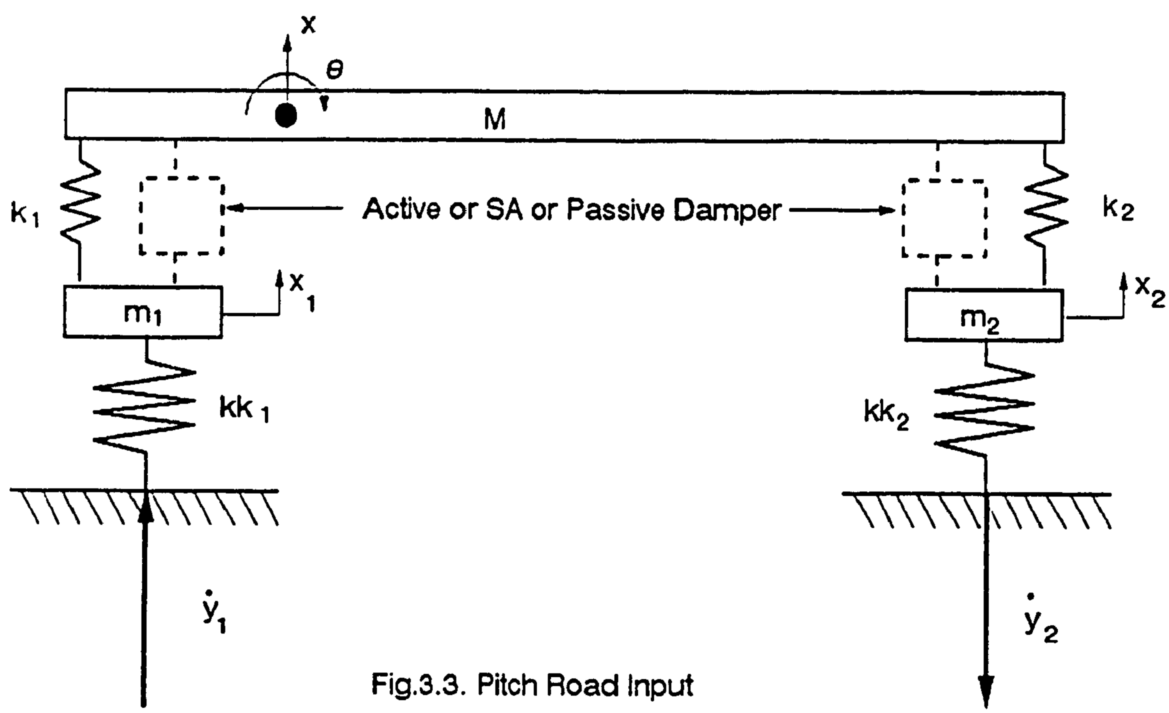


Fig.3.3. Pitch Road Input

3.2. Summary

The 4 D.O.F. half vehicle model to be used in the analysis of the different advanced suspension schemes is described. The choice of such a model arises from the desire to have a model that would allow for the study of both the heave and pitch modes along with the tires and suspensions deflection. All these quantities are of great importance in the design of vehicle suspensions. The chosen 4 D.O.F. half vehicle model is also relatively simple to analyze when compared to a 7 D.O.F. full vehicle model, yet can accurately predict the response of the latter.

CHAPTER 4

PASSIVE AND ACTIVE SUSPENSIONS FOR A 4 D.O.F. VEHICLE MODEL

4.1. General

To evaluate different suspension schemes, it is important to first study the two extreme cases, namely passive and active suspensions. In the following section, the active feedback actuator gains are determined using optimal regulator theory. Frequency response plots are provided for both passive and active suspensions for varying actuator size and varying damping coefficients, respectively. Finally, transient time response to 'chuck hole' type road disturbance is presented.

4.2. Passive Suspension for a 4 D.O.F. Vehicle model

Based on the model suggested in the previous section, assuming linear behavior for all elements and small angular displacement, the equations of motion are :

$$
\begin{aligned}
\ddot{x} &= (-k_1(x + l_1\theta - x_1) - k_2(x - l_2\theta - x_2) \\
&\quad -C_1(\dot{x} + l_1\dot{\theta} - \dot{x}_1) - C_2(\dot{x} - l_2\dot{\theta} - \dot{x}_2))/M \\
\dot{\theta} &= (-l_1k_1(x + l_1\theta - x_1) + l_2k_2(x - l_2\theta - x_2) \\
&\quad -l_1C_1(\dot{x} + l_1\dot{\theta} - \dot{x}_1) + l_2C_2(\dot{x} - l_2\dot{\theta} - \dot{x}_2))/J \\
\ddot{x}_1 &= (k_1(x + l_1\theta - x_1) - kk_1(x_1 - y_1) \\
&\quad +C_1(\dot{x} + l_1\dot{\theta} - \dot{x}_1))/m_1 \\
\ddot{x}_2 &= (k_2(x - l_2\theta - x_2) - kk_2(x_2 - y_2) \\
&\quad +C_2(\dot{x} - l_2\dot{\theta} - \dot{x}_2))/m_2
\end{aligned}
$$

Now, if we define x_3, x_4, x_5 and x_6 such that :

$$x_3 = dx/dt$$
$$x_4 = d\theta/dt$$
$$x_5 = dx_1/dt$$
$$x_6 = dx_2/dt$$

We obtain the set of first order differential equations that can be solved using the Runga-Kutta method [35].

$$\dot{x} = x_3$$
$$\dot{\theta} = x_4$$
$$\dot{x}_1 = x_5$$
$$\dot{x}_2 = x_6$$
$$\dot{x}_3 = (-k_1(x + l_1\theta - x_1) - k_2(x - l_2\theta - x_2) -C_1(\dot{x} + l_1\dot{\theta} - \dot{x}_1) - C_2(\dot{x} - l_2\dot{\theta} - \dot{x}_2))/M$$
$$\dot{x}_4 = (-l_1k_1(x + l_1\theta - x_1) + l_2k_2(x - l_2\theta - x_2) -l_1C_1(\dot{x} + l_1\dot{\theta} - \dot{x}_1) + l_2C_2(\dot{x} - l_2\dot{\theta} - \dot{x}_2))/J$$
$$\dot{x}_5 = (k_1(x + l_1\theta - x_1) - kk_1(x_1 - y_1) +C_1(\dot{x} + l_1\dot{\theta} - \dot{x}_1))/m_1$$
$$\dot{x}_6 = (k_2(x - l_2\theta - x_2) - kk_2(x_2 - y_2) +C_2(\dot{x} - l_2\dot{\theta} - \dot{x}_2))/m_2$$

Numerical simulation was carried out and a set of frequency response plots were obtained as shown in Figs.4.1, 4.2, 4.3 and 4.4 for varying damping coefficients for C_1 and C_2 ($C_1 = C_2$). Fig.4.1. is the RMS bounce acceleration transmissibility ratio frequency response. This is obtained by assuming identical road inputs to the front and rear wheels. Fig.4.1 reveals system response behaviour similar to that of a simpler 1 D.O.F. model. As the damping coefficient is increased, the RMS bounce acceleration transmissibility ratio decreases around the pitch and bounce modes natural frequencies. Beyond these frequencies, however, an increase in damper coefficient lead to an unwanted increase in the RMS ratio. This can be explained by the fact that at high frequencies an increased damping stiffens the connection of the wheel with the main body and hence more road input is transferred to the latter. The two peaks at around 5 rad/s correspond, obviously, to the bounce and pitch modes of vibration. Changing the damping value results in insignificant change in the bounce response at the wheels natural frequencies. Fig.4.2 is the RMS pitch acceleration transmissibility ratio. This is obtained for a pitch road input. Although we have a pure pitch road input and identical front and rear suspensions, two peaks at around 5 rad/s can be noticed, corresponding to the bounce and pitch modes of vibration. This is due to the fact that the vehicle body center of gravity is not right in the middle between the front and rear axles. Hence, a pure pitch input excites as well the bounce vibration mode.

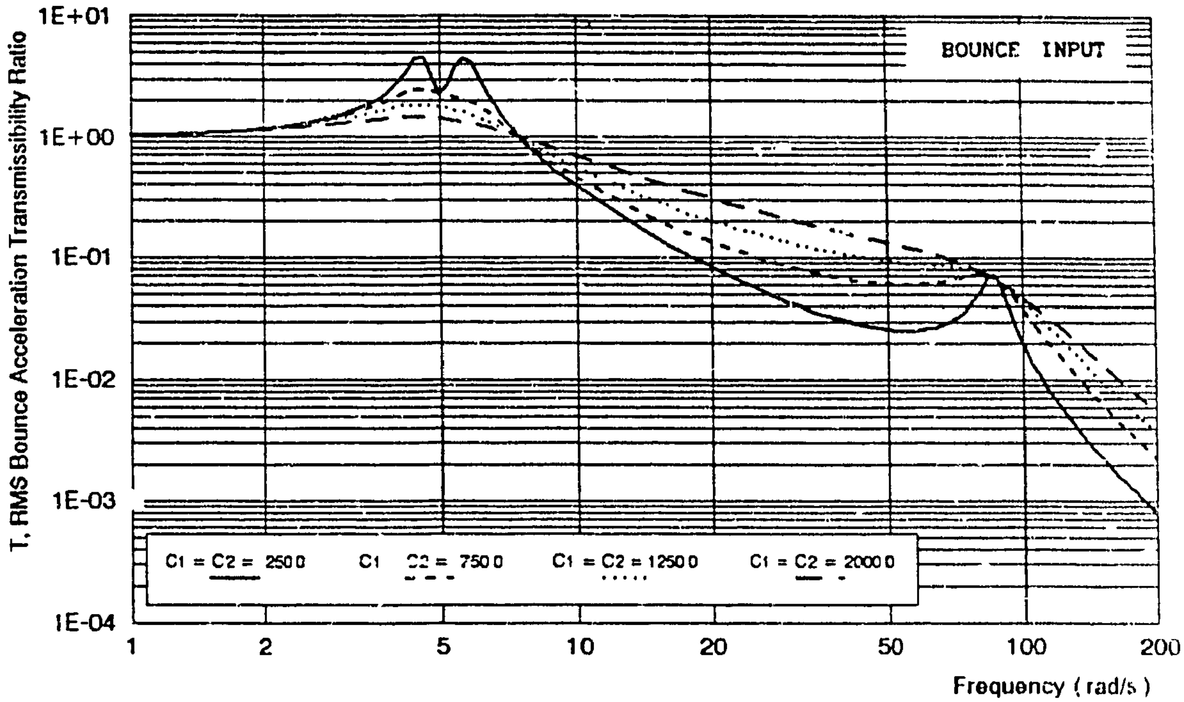


Fig.4.1. RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of a Fully Passive Suspension

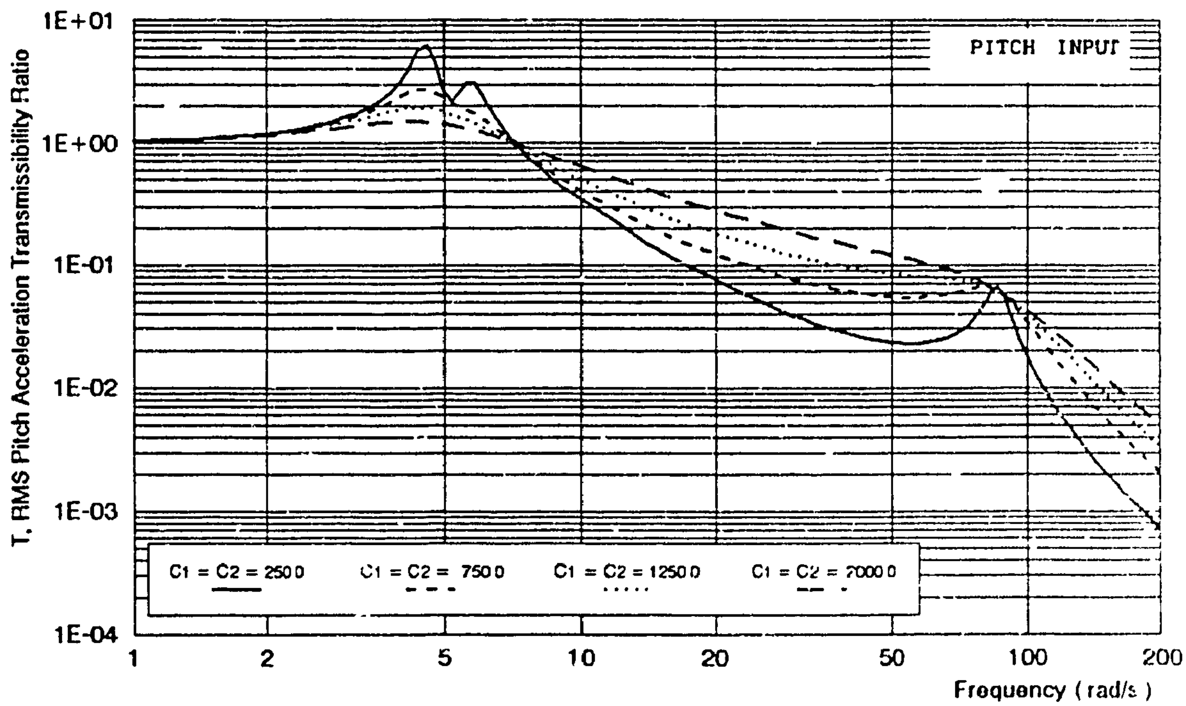


Fig.4.2. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of a Fully Passive Suspension

Fig.4.3 is the front suspension deflection frequency response for bounce input. An increase in the damper coefficient reduces suspension deflection throughout the frequency range. Suppression of the suspension deflection is especially visible at the natural frequencies.

Finally, Fig.4.4 is the front tire deflection frequency response for bounce input. Increasing the damping coefficient reduces the maximum tire deflection around the natural frequencies, yet results in an increase at frequencies between 7 and 60 rad/s.

If we are not to consider the pitch response, the 4 D.O.F. model can be simplified into a 2 D.O.F. model as shown in Fig.4.5. Such a model was analyzed for variation in the damping coefficient [37] and results are shown in Fig.4.6. The similarities between the results for a 4 D.O.F. model and these published results reveal the efficiency of a smaller and easier to analyze model in predicting the behaviour of a complex vehicle model. Such a simpler model, however, does not allow for the analysis of certain other vehicle modes such as the pitch response as in this study.

The model is also tested for a discrete road excitation. A 'Chuck Hole' type road profile is used as shown in Fig.4.7. The bounce and pitch response to such disturbance is shown in Fig.4.8. The time delay between the input to the front and rear wheels is appropriately incorporated in the simulation.

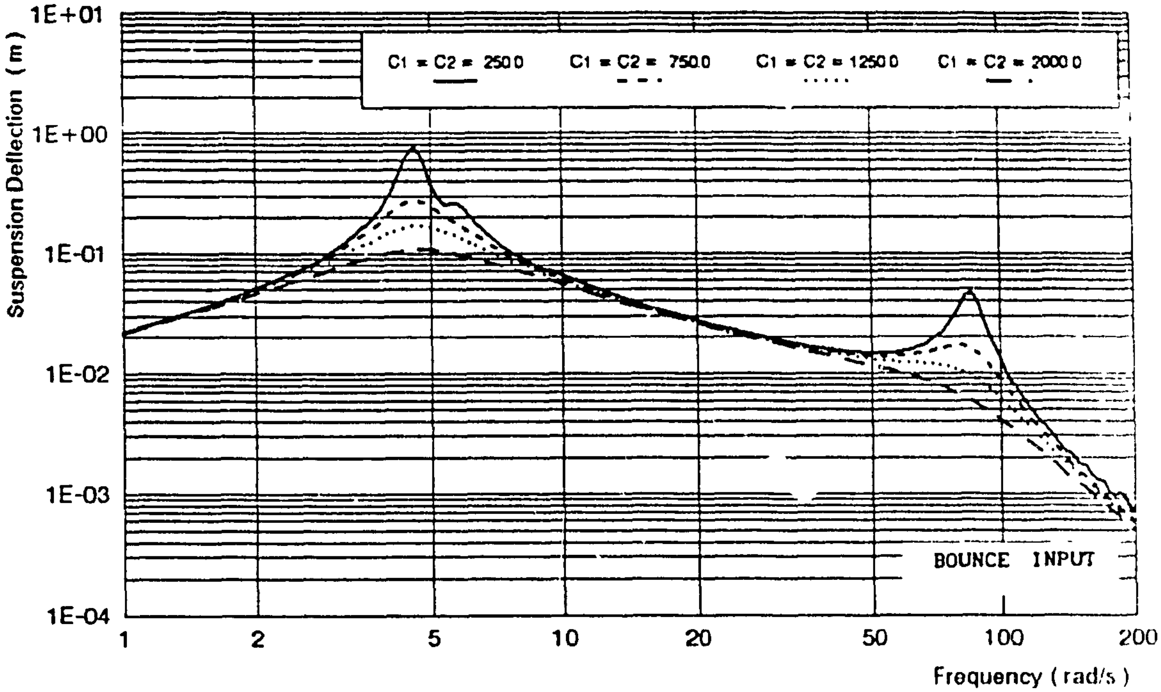


Fig.4.3. Maximum Suspension Deflection Versus Frequency of a Fully Passive Suspension

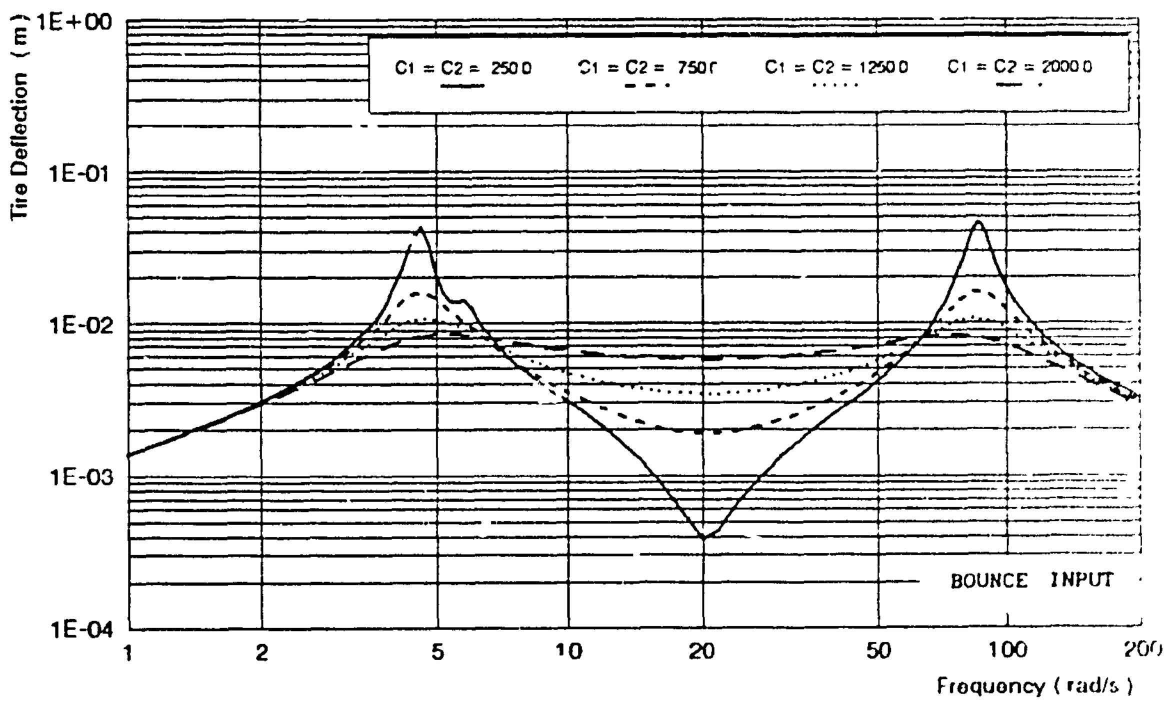


Fig.4.4. Maximum Tire Deflection Versus Frequency of a Fully Passive Suspension

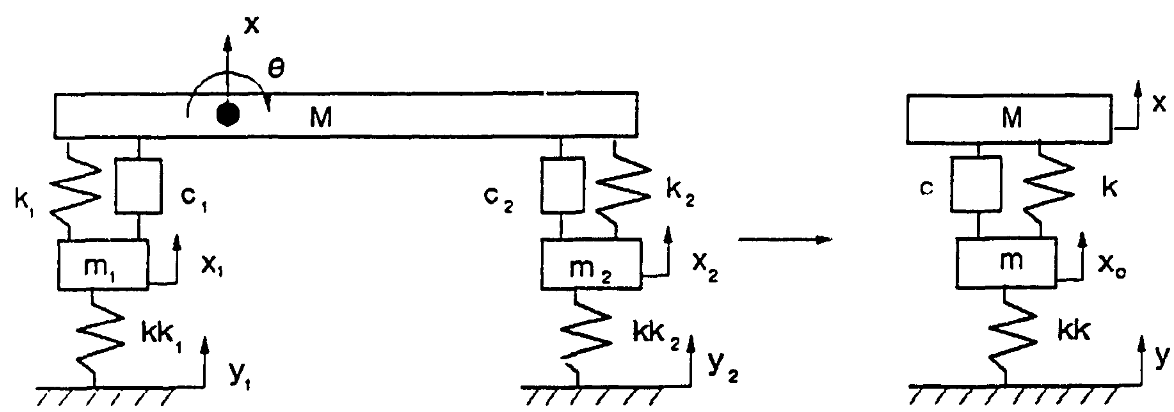
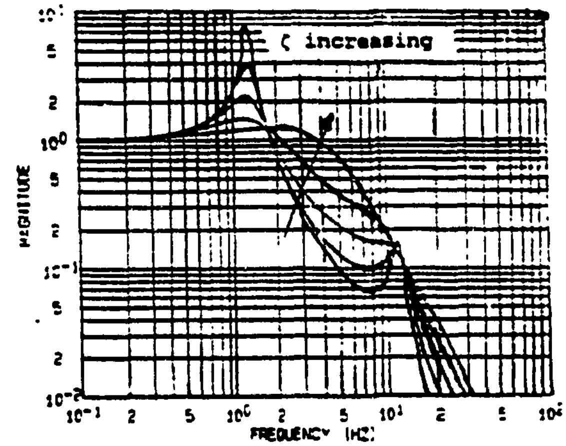
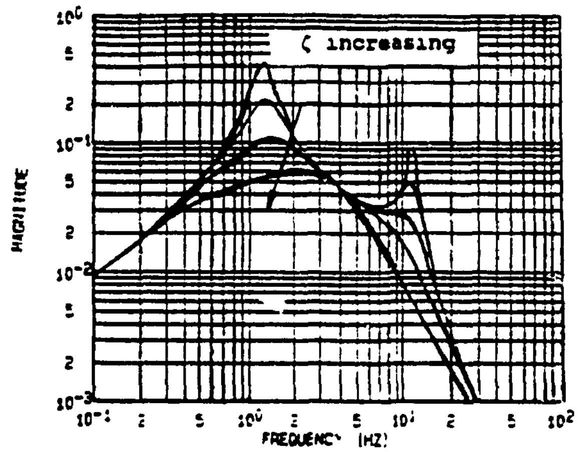


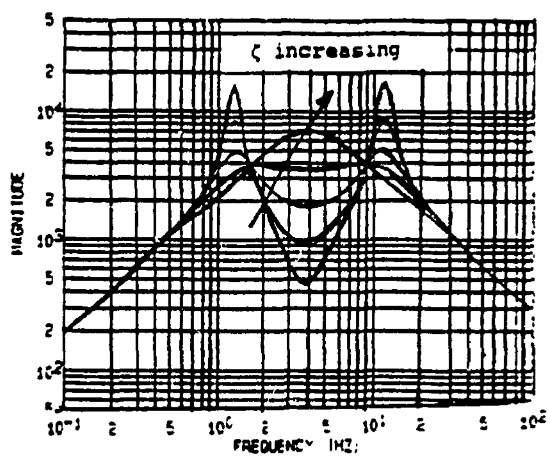
Fig.4.5. A [illegible] D.O.F. Simplified Model Representation of a 4 D.O.F. Model



Sprung Mass Velocity Frequency Response



Suspension Travel (X) Frequency Response



Tire Contact Force (F) Frequency Response

Fig.4.6. Overall Performance of a 2 D.O.F. Vehicle Model With Passive Suspension [36]

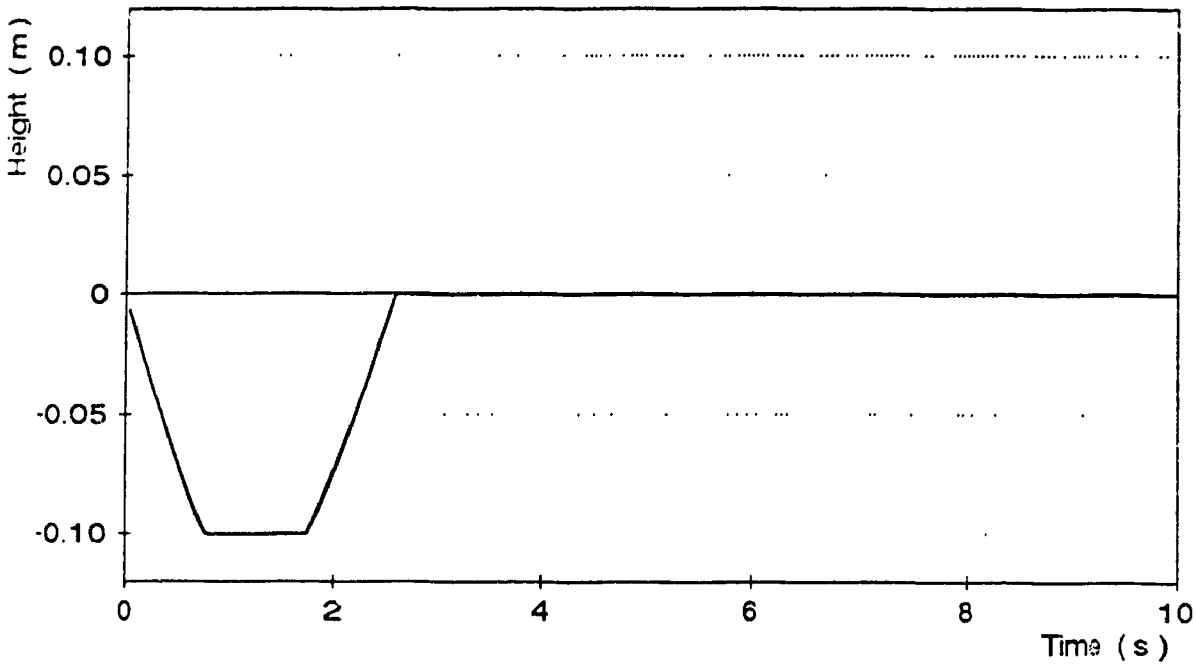


Fig.4.7. 'Chuck Hole' Type Road Profile

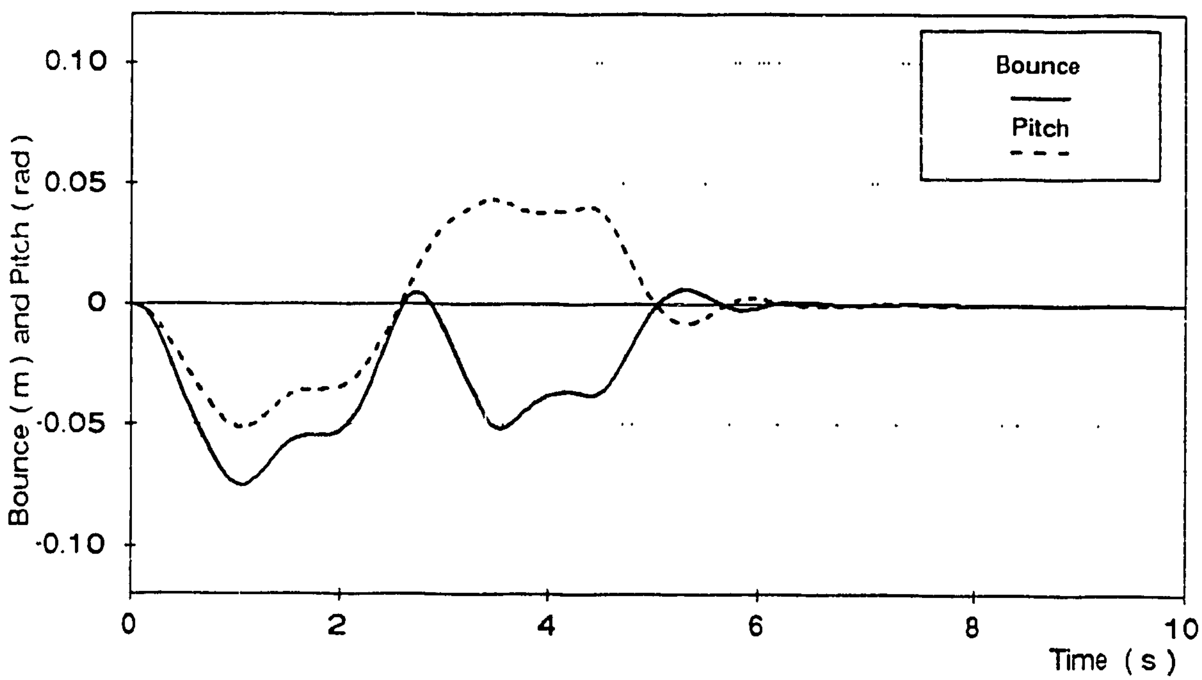


Fig.4.8. Pitch and Bounce Responce to a 'Chuck Hole' Type Road Disturbance of the Sprung Mass of a Fully Passively Suspended Vehicle

4.3. Active Suspension for a 4 D.O.F. Vehicle Model

The following section is devoted to developing a fully active suspension for the 4 D.O.F. model. The equations of motion are first developed, then the performance index is stated and the control force is determined using ' *Linear Optimal Regulator Theory* '. Frequency responses are finally provided and analyzed in comparison with the passive suspension frequency responses given earlier.

Consider the vehicle model fitted with an active suspension shown in Fig.4.9.

U_1 and U_2 represent the control forces generated by the front and rear actuators, respectively.

Using Newton's Second Law, the equations of motion can be shown to be:

$$\ddot{x} = (-k_1(x + l_1\theta - x_1) - k_2(x - l_2\theta - x_2) + U_1 + U_2)/M$$

$$\ddot{\theta} = (-l_1k_1(x + l_1\theta - x_1) + l_2k_2(x - l_2\theta - x_2) + l_1U_1 - l_2U_2)/J$$

$$\ddot{x}_1 = (k_1(x + l_1\theta - x_1) - kk_1(x_1 - y_1) - U_1)/m_1$$

$$\ddot{x}_2 = (k_2(x - l_2\theta - x_2) - kk_2(x_2 - y_2) - U_2)/m_2$$

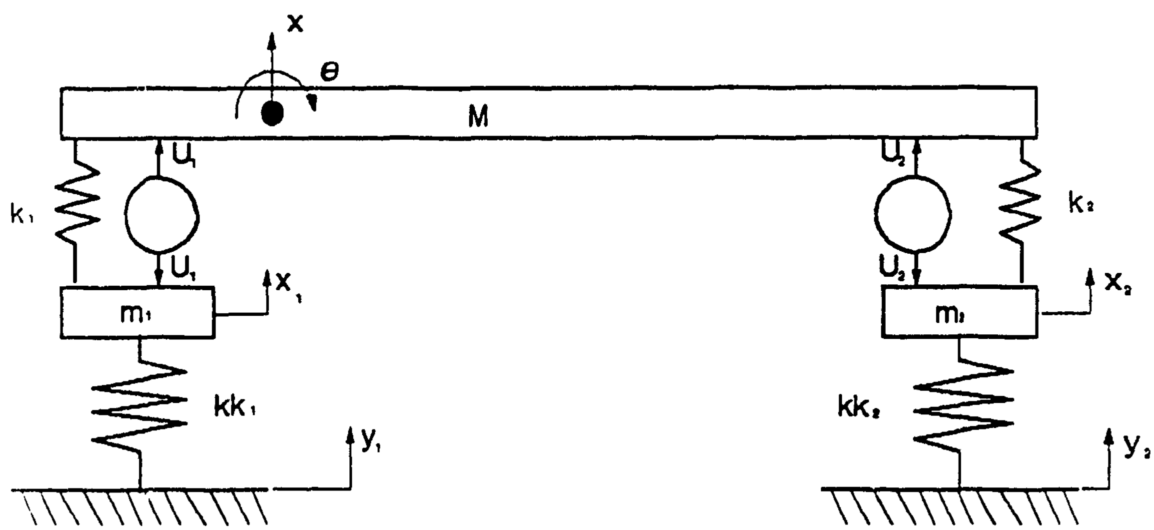


Fig.4.9. A 4 D.O.F. Vehicle Model Fitted With Actiive Suspension

Which can be reduced to the following set of first order differential equations :

$$\dot{\hat{x}} = \hat{x}_3 + l_1\hat{x}_4 - \hat{x}_5$$
$$\dot{\hat{\theta}} = \hat{x}_3 - l_2\hat{x}_4 - \hat{x}_6$$
$$\dot{\hat{x}}_1 = \hat{x}_5 - \dot{y}_1$$
$$\dot{\hat{x}}_2 = \hat{x}_6 - \dot{y}_2$$
$$\dot{\hat{x}}_3 = (-k_1\hat{x} - k_2\hat{\theta} + U_1 + U_2)/M$$
$$\dot{\hat{x}}_4 = (-l_1k_1\hat{x} + l_2k_2\hat{\theta} + l_1U_1 - l_2U_2)/J$$
$$\dot{\hat{x}}_5 = (k_1\hat{x} - kk_1\hat{x}_1 - U_1)/m_1$$
$$\dot{\hat{x}}_6 = (k_2\hat{\theta} - kk_2\hat{x}_2 - U_2)/m_2$$

Where the variables $\hat{x},\hat{\theta},\hat{x}_1,\hat{x}_2,\hat{x}_3,\hat{x}_4,\hat{x}_5$ and $\hat{x}_6$ are defined as follows :

$$\hat{x} = x + l_1\theta - x_1$$
$$\hat{\theta} = x - l_2\theta - x_2$$
$$\hat{x}_1 = x_1 - y_1$$
$$\hat{x}_2 = x_2 - y_2$$
$$\hat{x}_3 = \dot{x}$$
$$\hat{x}_4 = \dot{\theta}$$
$$\hat{x}_5 = \dot{x}_1$$
$$\hat{x}_6 = \dot{x}_2$$

The differential equations can, therefore, be rearranged in the following state space form :

$$
\dot{\hat{\left(x\right)}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & l_1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -l_2 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -k_1/M & -k_2/M & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-k_1 l_1}{J} & \frac{k_2 l_2}{J} & 0 & 0 & 0 & 0 & 0 & 0 \\ k_1/m_1 & 0 & \frac{-kk_1}{m_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_2/m_2 & 0 & \frac{-kk_2}{m_2} & 0 & 0 & 0 & 0 \end{pmatrix} \hat{\left(x\right)} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{M} & \frac{1}{M} \\ \frac{l_1}{J} & \frac{-l_2}{J} \\ -\frac{1}{m_1} & 0 \\ 0 & -\frac{1}{m_2} \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix}
$$

or

$$
\dot{\hat{\left(x\right)}} = A\hat{\left(x\right)} + B\left(U\right) + C\left(y\right)
$$

where:

$$
\hat{\left(x\right)}^T = \left(\hat{x} \quad \hat{\theta} \quad \hat{x}_1 \quad \hat{x}_2 \quad \hat{x}_3 \quad \hat{x}_4 \quad \hat{x}_5 \quad \hat{x}_6 \right) = \text{State Variables}
$$

$$
\left(U\right)^T = \left(U_1 \quad U_2 \right) = \text{Control Variables}
$$

$$
\left(y\right) = \left(\dot{y}_1 \quad \dot{y}_2 \right) = \text{Input Variables}
$$

The objective is to find an expression for the quantities U_1 and U_2 that will minimize a given cost function. Determination of these quantities will be achieved through the use of the ' Linear Optimal Regulator Theory '. The choice of the performance index is outlined in the following section.

4.3.1. Choosing the Performance Index

The choice of the performance index is dependent on the quantities we would like to cptimize and the restricted factors (maximum force that the actuators can produce). Once these quantities are chosen, the associated weighting factors are decided upon.

For this study, we select the following penalized variables :

$\hat{x} = x + l_1\theta - x_1$.....;Front suspension deflection

$\hat{\theta} = x - l_2\theta - x_2$.....;Rear suspension deflection

$\hat{x}_1 = x_1 - y_1$ ....;Front tire deflection

$\hat{x}_2 = x_2 - y_2$ ....;Rear tire deflection

$\hat{x}_3 = \dot{x}$ ....;Vehicle body bounce velocity

$\hat{x}_4 = \dot{\theta}$ ....;Vehicle body pitch velocity

We are also restricted by the maximum forces that the actuators can provide. Accordingly, we set the performance index as follows:

$$I = \frac{1}{2}\int_0^\infty (\rho_1 U_1^2 + \rho_2 U_2^2 + q_1\hat{x}^2 + q_2\hat{\theta}^2 + q_3\hat{x}_1^2 + q_4\hat{x}_2^2 + q_5\hat{x}_3^2 + q_6\hat{x}_4^2)dt$$

The weighting factors in the above expression (ρ_1, ρ_2, q_1, q_2, q_3, q_4, q_5 and q_6), are the inverse squared of the maximum value that the corresponding penalized variable is not to exceed. If, for instance, the maximum force that the front actuator can provide is 100 N, the associated weighting factor is $\rho_1 = (1/100)^2 = 0.0001$. The choice of the penalized variables and the associated weighting factors is dictated by personal judgment and hardware limitations.

The performance index I, which is a dimensionless quantity, is written in the following standard format:

$$I = \frac{1}{2}\int_0^{\infty} (U^T R\, U + \hat{x}^T Q\, \hat{x})\, dt$$

Where :

$$R = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix}$$

$$Q = \begin{pmatrix} q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & q_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & q_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & q_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Since we have been able to write the performance index in the standard form, the optimal value of the forces U_1 and U_2 is given by [37]:

$$\begin{pmatrix} U_1 \\ U_2 \end{pmatrix}_{opt} = -R^{-1}B^{T}P\,\hat{x}(t) = k^{T}\,\hat{x}(t)$$

Where the matrices R and B are as defined earlier and

$$k^{T} = \begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & k_{17} & k_{18} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & k_{27} & k_{28} \end{pmatrix}$$

The 8x8 matrix P is the symmetric positive definite steady-state solution of the algebraic *Riccati equation :*

$$PA + A^{T}P - PBR^{-1}B^{T}P + Q = 0$$

For the analysis, the following weighting factors are chosen :

$$q_1 = (\ 1/(0.1m)^2) \qquad = 100.0\ (\ m^{-1})^2$$

$$q_2 = (\ 1/(0.1m)^2) \qquad = 100.0\ (\ m^{-1})^2$$

$$q_3 = (\ 1/(0.05m)^2) \qquad = 400.0\ (\ m^{-1})^2$$

$$q_4 = (\ 1/(0.05m)^2) \qquad = 400.0\ (\ m^{-1})^2$$

$$q_5 = (\ 1/(0.1m/sec)^2) \quad = 100.0\ (\ sec.m^{-1})^2$$

$$q_6 = (\ 1/(0.1m/sec)^2) \quad = 100.0\ (\ sec.m^{-1})^2$$

And the coefficients of the matrix k are given in Table 4.1. for varying actuator size: $\rho_1=\rho_1=\rho$.

Table 4.1. Front and Rear Actuators Gains For an Active Suspension

Gains \ ρ	1E-6	5E-6	1E-5	5E-5
K11	-4137.754	-951.286	-485.199	-97.369
K12	347.962	262.838	246.846	208.453
K13	1087.843	128.848	57.133	11.372
K14	869.719	376.438	303.721	219.397
K15	-7867.014	-3526.766	-2491.147	-1101.323
K16	-6765.384	-3049.010	-2162.299	-980.164
K17	241.212	110.518	78.403	35.163
K18	-5.298	-1.792	-1.198	-0.467
K21	-347.948	-262.823	-246.848	-208.446
K22	-4137.845	-951.317	-485.178	-97.358
K23	169.617	-151.760	-191.944	-199.145
K24	1583.900	238.926	113.562	23.254
K25	-6634.913	-2973.577	-2100.445	-929.114
K26	8032.801	3624.098	2572.916	1174.740
K27	-5.363	-1.816	-1.211	-0.469
K28	236.127	108.745	77.190	34.606

Figs.4.10 and 4.11 are the RMS pitch and bounce acceleration transmissibility ratio frequency responses for different actuator sizes. As the size of the actuator is increased, a remarkable reduction in the RMS acceleration ratio is achieved at frequencies around the heave and pitch mode natural frequencies, accompanied by a negligible compromise of isolation at higher frequencies. Although the suspension performance is enhanced by an increase in actuator size, practical limitations are, however, the major drawback. Figs.4.12 and 4.13 show the maximum suspension and tire deflection frequency responses. The suspension deflection frequency response reveals an other disadvantage of increased actuator size. Although a more powerful actuator generally improves the overall system performance, it, however, results in an increase of the suspension deflection at low frequencies. Thus requiring larger rattle space. The increase in the suspension travel is caused by the larger force that the more powerful actuator applies on the wheel at low frequencies. This could be partially corrected if a larger weighting factor is assigned to the suspension deflection in the performance index I defined earlier. But this would, probably, lead to a deterioration in other system parameters because a larger portion of the actuator energy will be directed to minimizing the suspension deflection. Similarly the general performance of a simpler 2 D.O.F. model for varying damper gains is given in Fig.4.14. Clear similarity between these published results and results developed for the 4 D.O,F. model can be noticed.

Fig.4.15 demonstrates the superior RMS bounce acceleration transmissibility ratio frequency response of active suspension when compared to a passive one. This is true no matter what the damping coefficient used for the passive suspension.

Table 4.2 is the ratio of various performance parameters (RMS bounce acceleration, RMS pitch acceleration, suspension and tire deflections transmissibility ratio) of active to passive suspensions at selected frequencies. While requiring larger rattle space and resulting in lower tire contact force, active suspension offers lower RMS bounce and pitch transmissibility ratios throughout the frequency range, except at the wheels natural frequencies, when compared to passive suspension.

Finally, a better suppression of the vehicle body vibration (bounce and pitch) caused by a Chuck Hole road disturbance, is achieved through active suspension as can be seen in Fig.4.16.

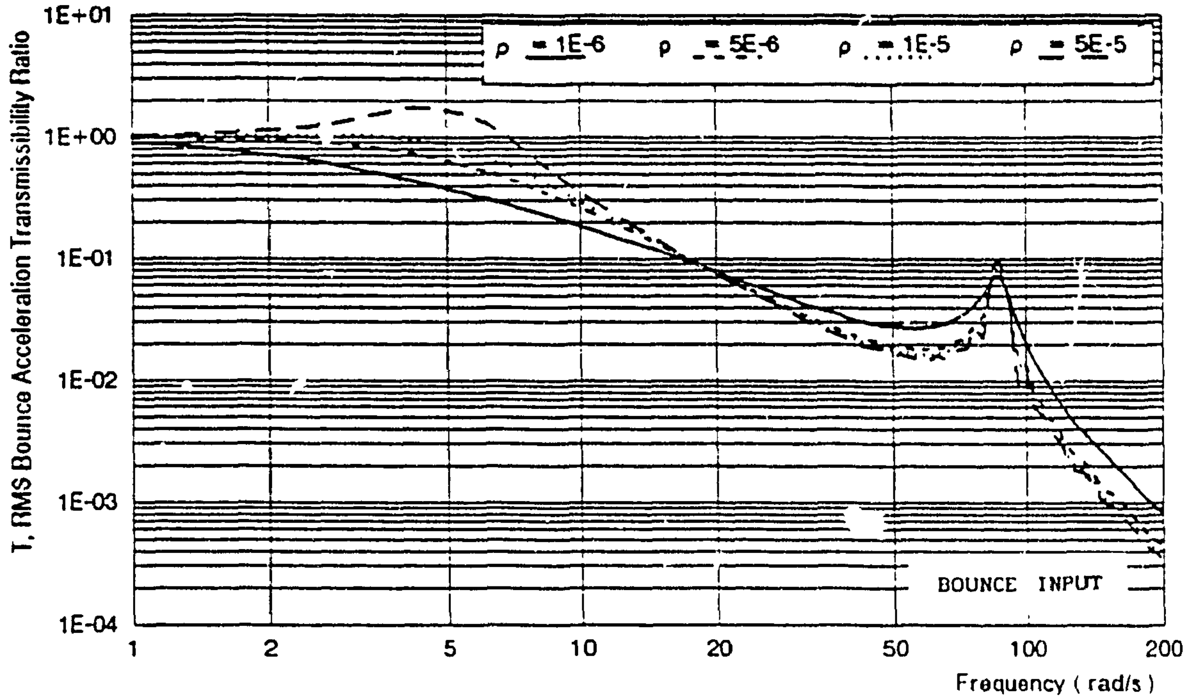


Fig.4.10 RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of a Fully Active Suspension

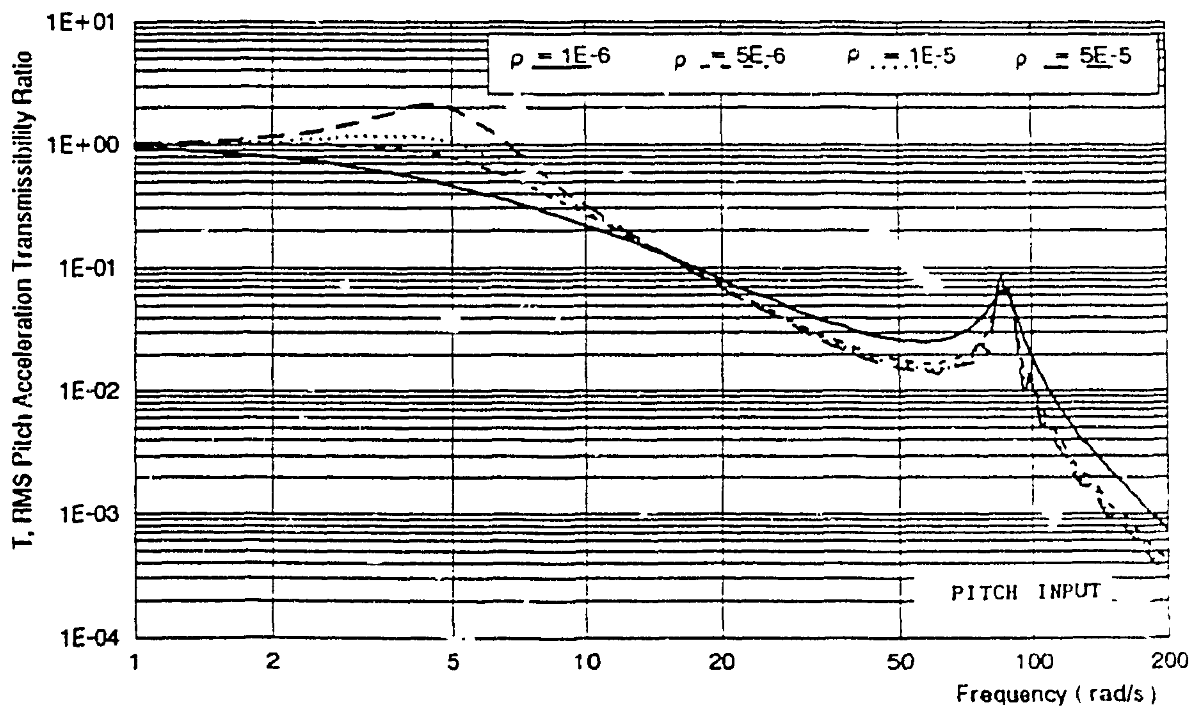


Fig.4.11. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of a Fully Active Suspension

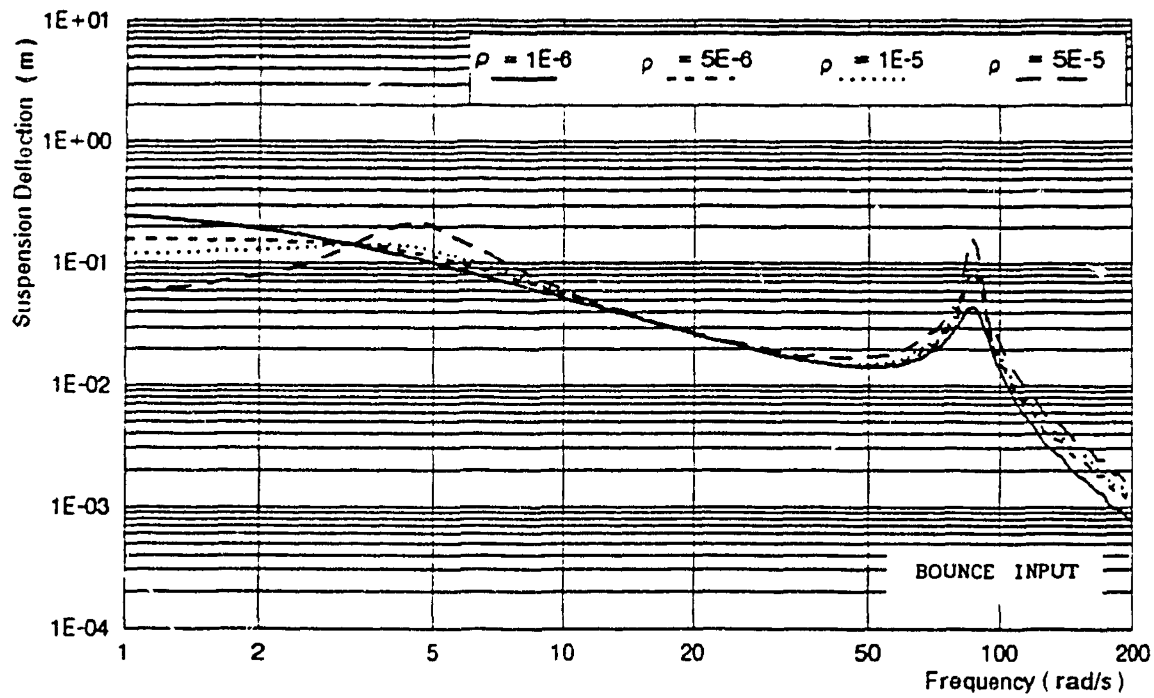


Fig.4.12. Maximum Suspension Deflection Versus Frequency of a Fully Active Suspension

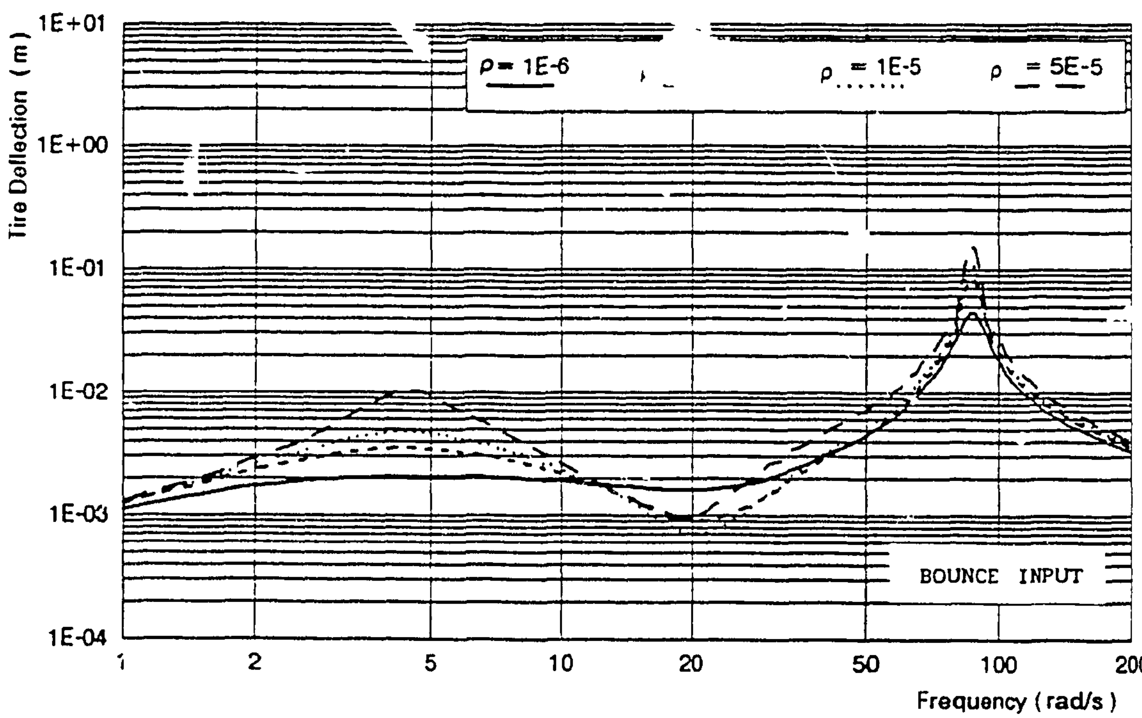
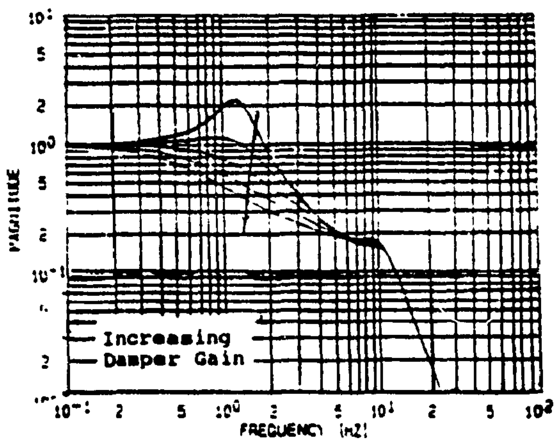
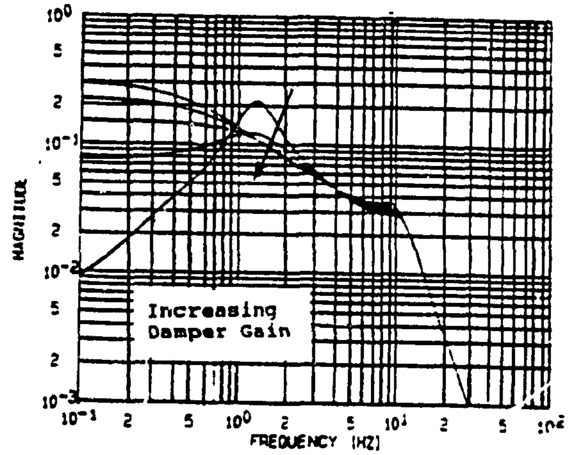


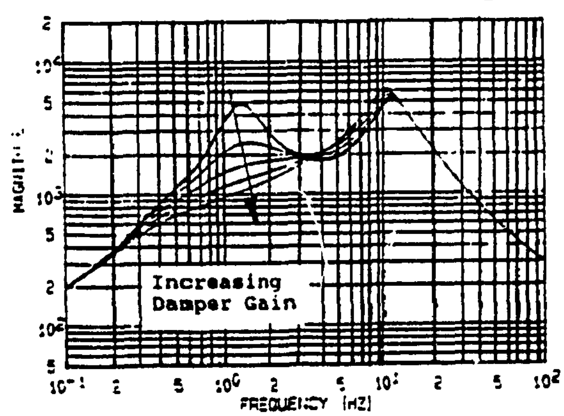
Fig 4.13. Maximum Tire Deflection Versus Frequency of a Fully Active Suspension



Sprung Mass Velocity Frequency Response



Suspension Travel (X) Frequency Response



Tire Contact Force (F) Frequency Response

Fig.4.14. Overall Performance of a 2 D.O.F. Vehicle Model With Active Suspension [36]

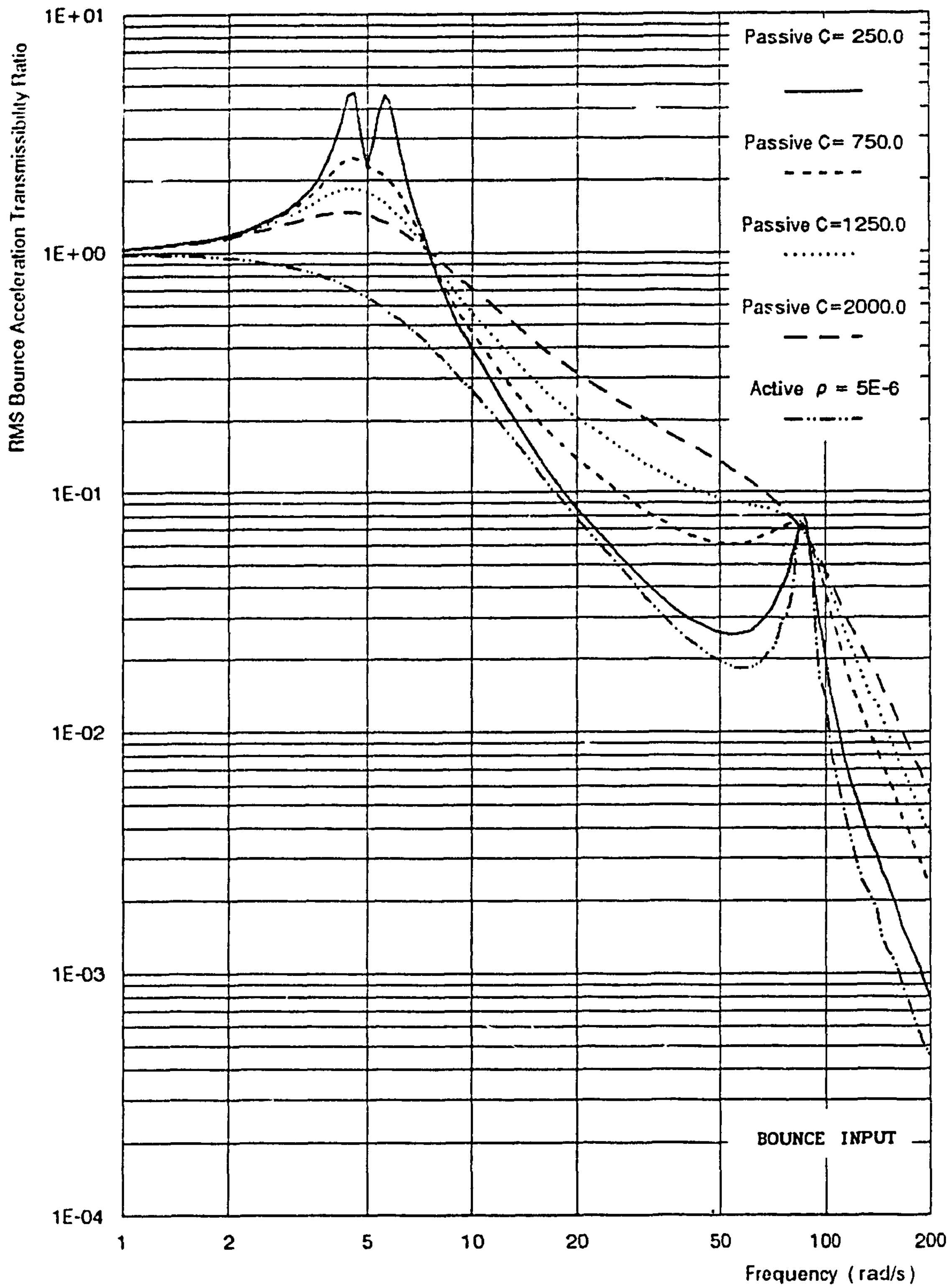


Fig.4.15. Comparison of The RMS Bounce Acceleration Transmissibility Ratios of Passive and Active Suspensions

Table 4.2. Ratio of Active to Passive Performance Parameters at Selected Frequencies: $\rho = 5E\text{-}6$; C = 1250 N.s/m

Freq. (rad/s)	RMS Bounce	RMS Pitch	Sus. Def.	Tire Def.
2.0	0.811 [T]	0.854	3.173	0.781
4.6 *	0.376	0.440	0.710	0.325
20.0	0.377	0.391	0.999	0.266
86.0 **	1.177	1.179	9.080	8.012
200.0	0.122	0.123	2.032	1.227

* Neighbourhood of bounce and pitch natural frequencies

** Neighbourhood of wheels natural frequencies

[T] Value less than unity (except for tire deflection) indicates superior performance of active suspension compared to passive suspension

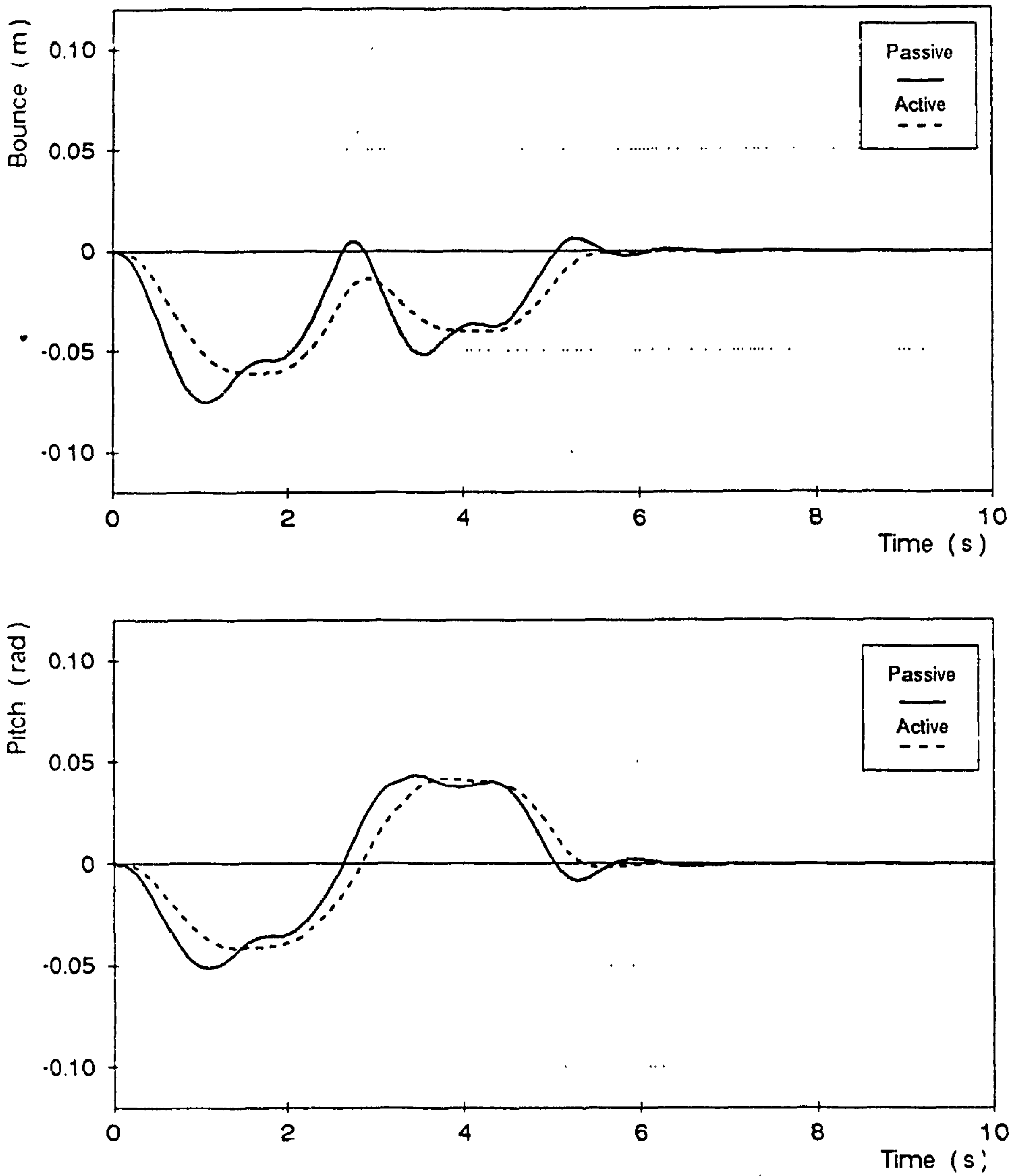


Fig.4.16. Comparison of Bounce and Pitch Responce to a 'Chuck Hole' Type Road Disturbance of the Sprung Mass of an Active and Passive Suspension

4.4. Summary

Transmissibility response plots for sprung mass bounce and pitch accelerations, and suspension and tire deflections are presented for both passive and active suspensions in a 4 D.O.F. vehicle model. Optimal gains for the active suspension system are obtained based on a quadratic performance index and linear optimal regulator control theory. The above analysis confirmed the superiority of active suspension. This is especially visible from the frequency response plots. The cost and complexity, however, are the main drawbacks of active suspension. The advantage of a passive suspension, in addition to its simplicity, is that it requires smaller rattle space and offers better road holding characteristics. Published results, although based on a quarter car model (2 D.O.F.), reveals similar conclusions to those found using a 4 D.O.F. half vehicle model.

CHAPTER 5

SEMI-ACTIVE (SA) SUSPENSIONS FOR A 4 D.O.F VEHICLE MODEL

5.1. General

As demonstrated in the previous chapter, the active suspension provides an overall superior performance when compared to a passive suspension. Unfortunately, active suspensions are far more complex and costly. Having in mind simultaneous cost and performance constraints, SA suspensions are offered. In the following section, the 3 SA suspension schemes presented in chapter 2 are modified and implemented in the 4 D.O.F. model. Frequency and transient time response plots are provided and compared to fully active and fully passive suspensions.

5.2. SA-1 Suspension for a 4 D.O.F. Vehicle Model

This scheme is based on the idea of having an SA device that can be modulated to dissipate energy in a fashion identical to an active device. During the other part of the vibration cycle when the damper should provide energy to the system, the damper is modulated to output a zero force. Therefore, this SA device does not require an external source of energy, but simply requires a continuous modulation of the damper orifice during part of the cycle. The forces generated by the front and rear dampers can, therefore, be stated as follow :

$$F_f = \begin{cases} U_1 & (\dot{x} + l_1\dot{\theta})(\dot{x} + l_1\dot{\theta} - \dot{x}_1) > 0 \\ 0 & (\dot{x} + l_1\dot{\theta})(\dot{x} + l_1\dot{\theta} - \dot{x}_1) < 0 \end{cases}$$

and

$$F_r = \begin{cases} U_2 & (\dot{x} - l_2\dot{\theta})(\dot{x} - l_2\dot{\theta} - \dot{x}_2) > 0 \\ 0 & (\dot{x} - l_2\dot{\theta})(\dot{x} - l_2\dot{\theta} - \dot{x}_2) < 0 \end{cases}$$

Where F_f and F_r are the front and rear damper forces respectively. U_1 and U_2 are the magnitude of the damper forces as defined for the active suspension in chapter 4. When $(\dot{x} + l_1\dot{\theta})(\dot{x} + l_1\dot{\theta} - \dot{x}_1) = 0$, two special cases arise. When $(\dot{x} + l_1\dot{\theta}) = 0$, we desire $F_f = 0$. In the case when $(\dot{x} + l_1\dot{\theta}) \neq 0$, but $(\dot{x} + l_1\dot{\theta} - \dot{x}_1) = 0$, the system will lock-up, if the desired force F_f is greater than the lock-up force F_f'. Similar analysis can be done for the rear suspension.

From the bounce and pitch equations of motion we can write :

$$\begin{aligned} F_f' + F_r' &= M\ddot{x} + k_1(x + l_1\theta - x_1) \\ &\quad + k_2(x - l_2\theta - x_2) \\ l_1F_f' - l_2F_r' &= J\ddot{\theta} + l_1k_1(x + l_1\theta - x_1) \\ &\quad - l_2k_2(x - l_2\theta - x_2) \end{aligned}$$

Noting that when the front SA device locks up we have $\dot{x}_1 = \dot{x} + l_1\dot{\theta}$ and similarly, when the rear suspension locks up we have $\dot{x}_2 = \dot{x} - l_2\dot{\theta}$, the above two equations can be rearranged to give the front and rear lock-up forces as :

$$F_f' = (l_2 M(\ddot{x}_1 - l_1\ddot{\theta}) + J\ddot{\theta})/(l_1 + l_2)$$
$$+ k_1(x + l_1\theta - x_1)$$
$$F_r' = (l_1 M(\ddot{x}_2 + l_2\ddot{\theta}) - J\ddot{\theta})/(l_1 + l_2)$$
$$+ k_2(x - l_2\theta - x_2)$$

Figs.5.1 and 5.2 are the RMS bounce and pitch acceleration transmissibility ratio frequency responses for various damper sizes (maximum force generated). It can be seen that increasing the damper size, while resulting in a decrease of the RMS bounce and pitch acceleration transmissibility ratios around the bounce and pitch mode natural frequencies, however, results in little change of the RMS ratio beyond 10 rad/s . A further increase in the damper size will, indeed, result in a very little improvement of the overall system behaviour as can be seen from Fig.5.1. This conclusion is significant and can be helpful in choosing the right damper size in actual application. Fig.5.3 shows that the major affect of increasing the damper size is visible at around the bounce and pitch mode natural frequencies, where increased damper size reduces the maximum suspension deflection. Fig.5.4 illustrates the tire deflection performance.

Referring to Fig.5.5, when compared to active suspension, the SA-1 suspension proves very efficient especially at frequencies higher than 10 rad/s. At low frequencies, however, the SA-1 device increase the RMS bounce and pitch acceleration ratios when compared to an active suspension, but remains highly superior to a passive one. Table 5.1 is the ratio of SA-1 to active and SA-1 to passive transmissibility performance at selected frequencies. While not as effective as active suspension, the SA-1 scheme controls adequately the RMS bounce and pitch transmissibility ratios throughout the frequency range. In addition to being easier and cheaper to implement, this scheme increases contact force. The rattle space requirements are similar to that of an active suspension except at low frequencies where the SA-1 scheme requires less rattle space. Similarly, the SA-1 suspension time response to a Chuck Hole road disturbance is very close to that of an active suspension.

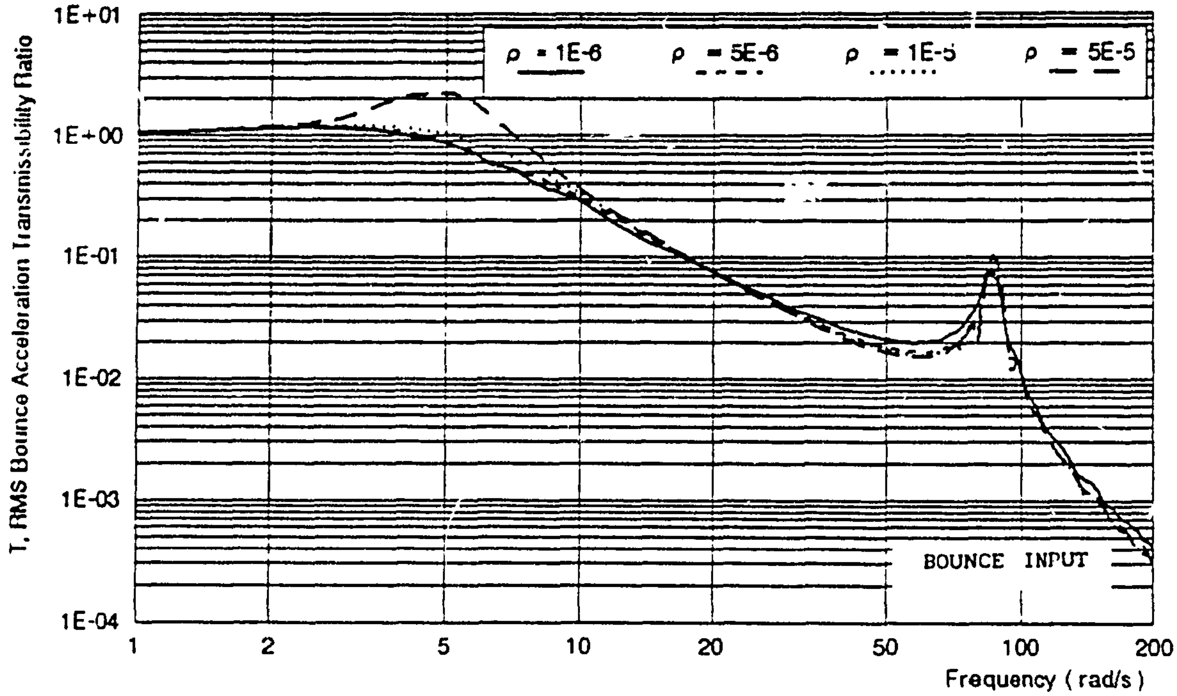


Fig.5.1. RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of an SA-1 Suspension

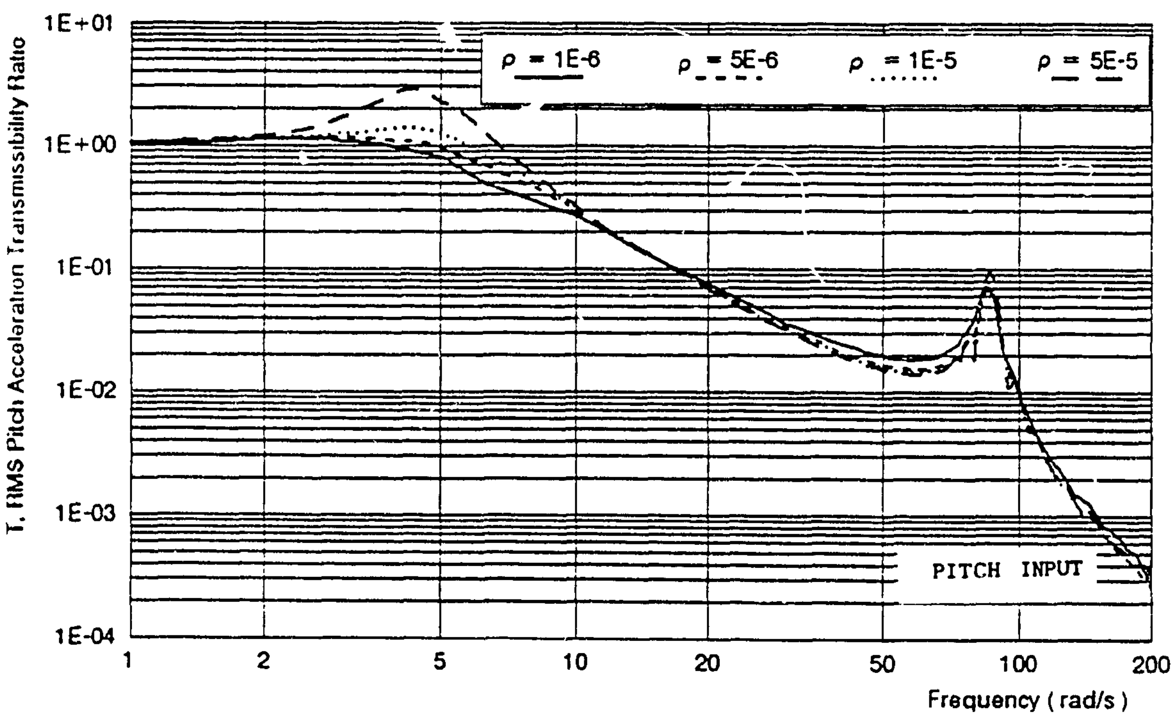


Fig.5.2. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of an SA-1 Suspension

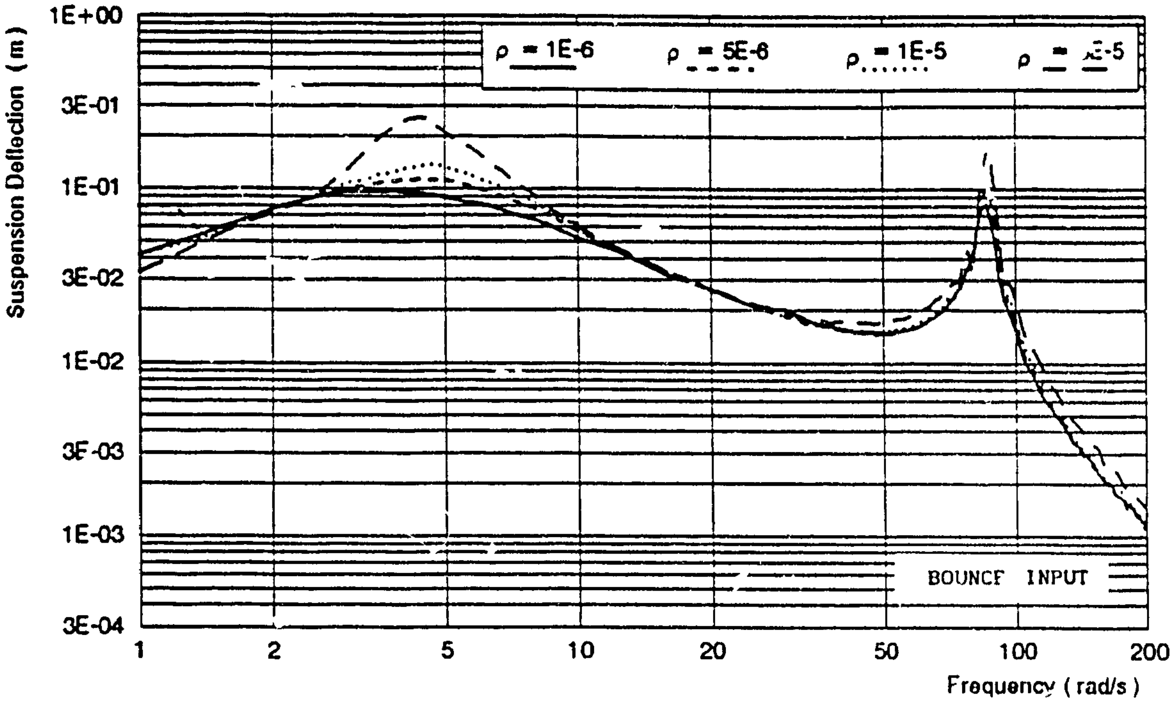


Fig.5.3. Maximum Suspension Deflection Versus Frequency
of an SA-1 Suspension

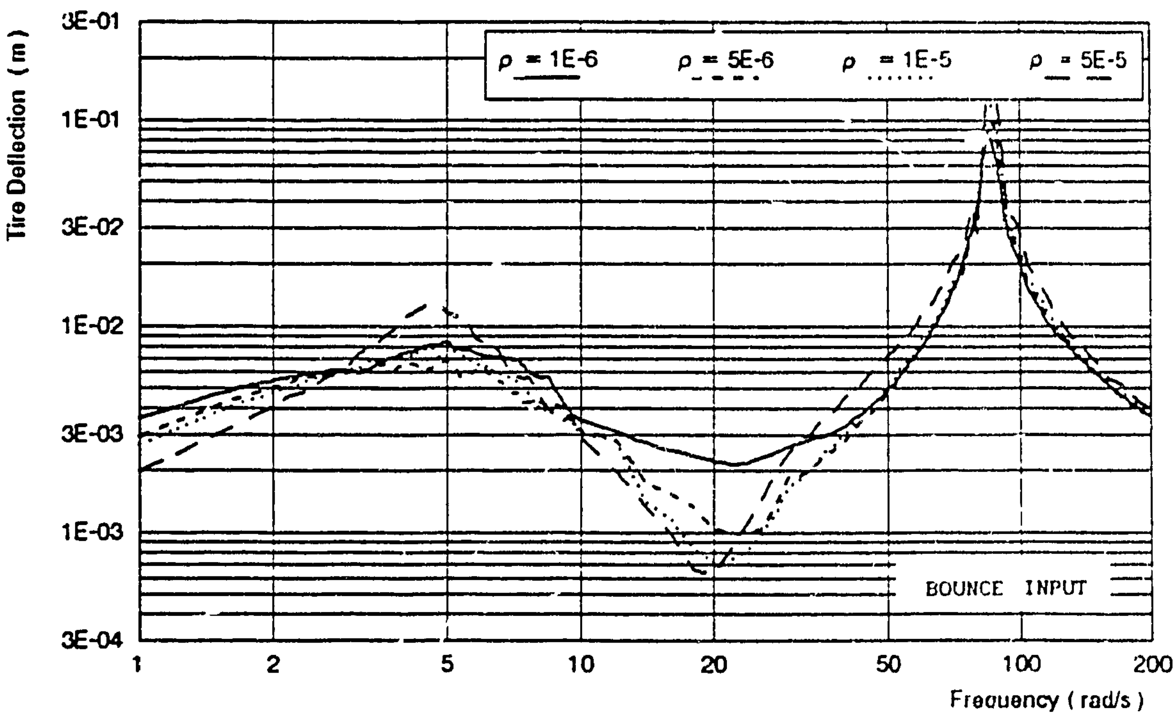


Fig.5.4. Maximum Tire Deflection Versus Frequency
of an SA-1 Suspension

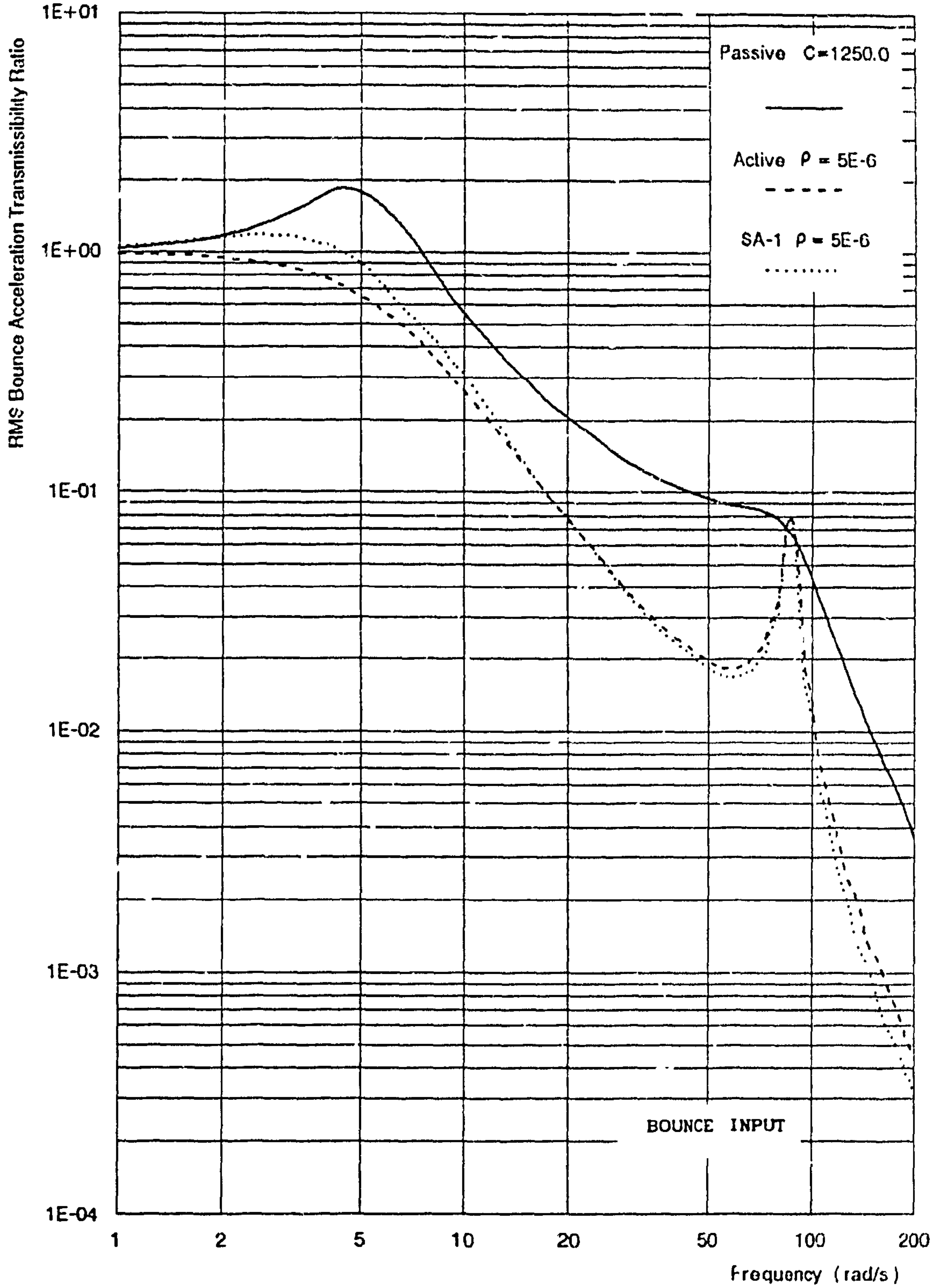


Fig.5.5. Comparison of The RMS Bounce Acceleration Transmissibility Ratios o' Passive, Active and SA-1 Suspensions

Table 5.1. Ratio of SA-1 to Passive and SA-1 to Active Performance Parameters at Selected Frequencies: ρ = 5E-5 ; C = 1250 N.s/m

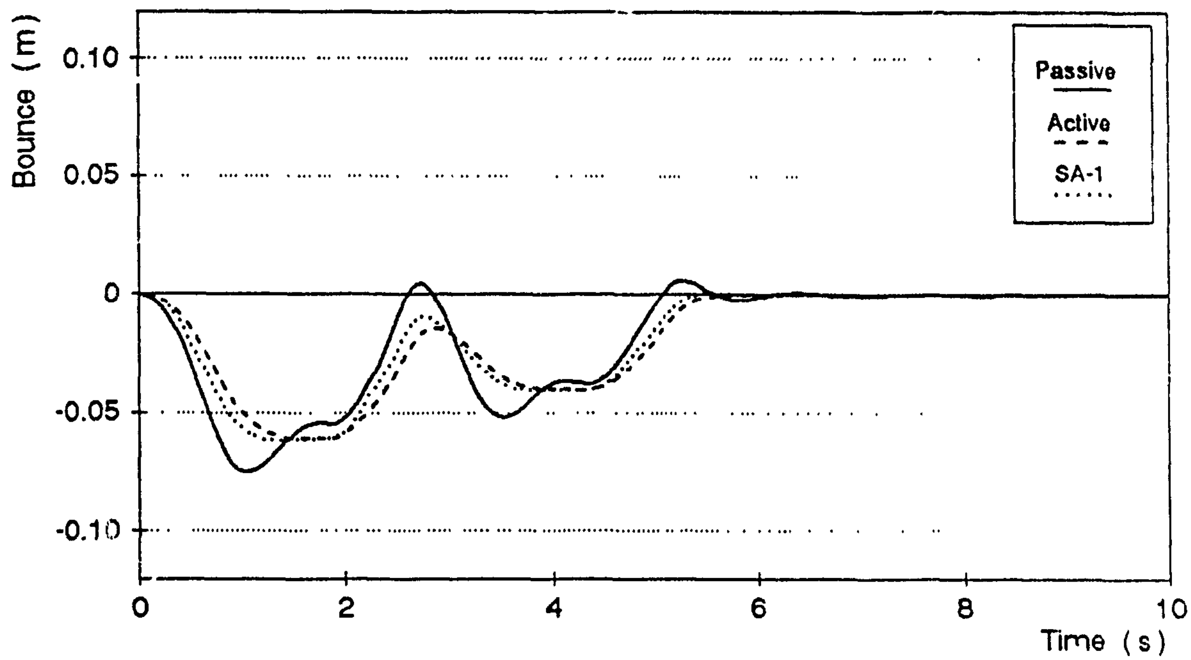
SA-1 to Passive				
Freq. (rad/s)	RMS Bounce	RMS Pitch	Sus. Def.	Tire Def.
2.0 *	0.99 T	0.97	1.56	1.69
4.6	0.52	0.55	0.66	0.57
20.0	0.38	0.39	1.00	0.30
86.0 **	1.15	1.16	10.55	9.40
200.0	0.08	0.08	2.03	1.22
SA-1 to Active				
Freq. (rad/s)	RMS Bounce	RMS Pitch	Sus. Def.	Tire Def.
2.0	1.22 TT	1.13	0.49	2.16
4.6*	1.39	1.24	0.93	1.76
20.0	1.00	0.99	1.00	1.14
86.0 **	0.98	0.98	1.16	1.17
200.0	0.67	0.66	1.00	0.99

* Neighbourhood of bounce and pitch natural frequencies

** Neighbourhood of wheels natural frequencies

T Value less than unity (except for tire deflection) indicates superior performance of SA-1 suspension compared to passive suspension

TT Value less than unity (except for tire deflection) indicates superior performance of SA-1 suspension compared to active suspension



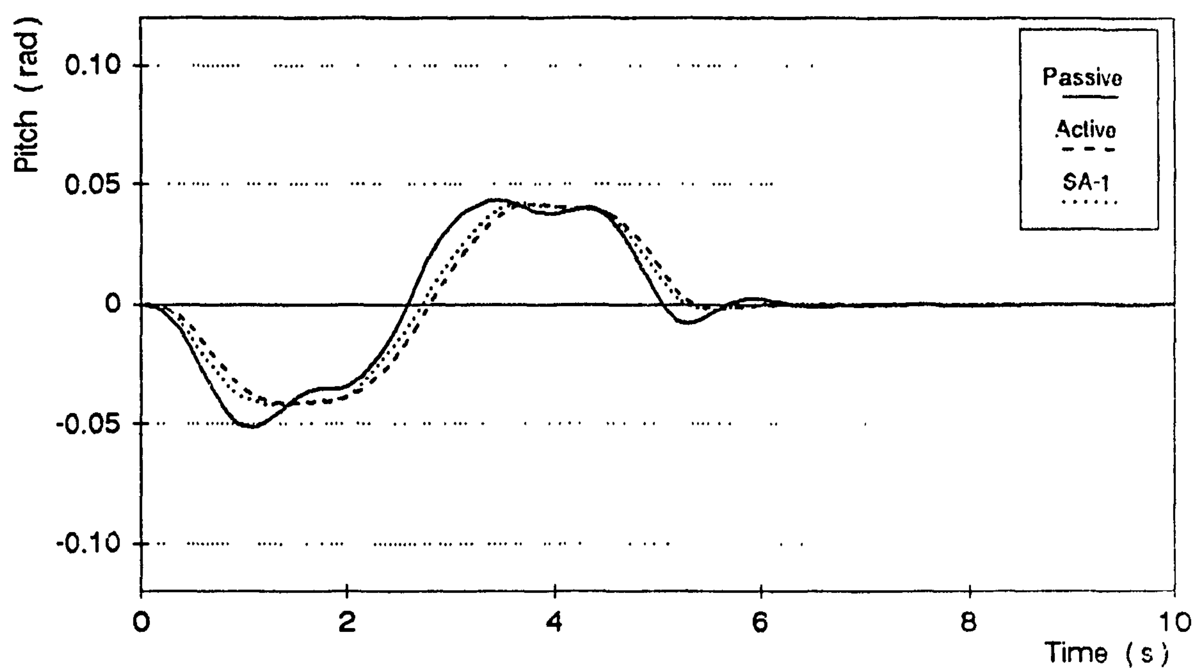


Fig.5.6. Comparison of Bounce and Pitch Responce to a 'Chuck Hole' Type Road Disturbance of the Sprung Mass of Active, Passive and SA-1 Suspensions

5.3. SA-2 Suspension for a 4 D.O.F. Vehicle Model

The concept, originally suggested by Rakheja and Sankar [31], is to be implemented in a 4 D.O.F. model. The increase of the sprung mass acceleration during part of the vibration cycle, due to the damper and spring forces being in the same direction, is to be reduced. This is achieved by using an SA device that provides zero force when the spring and damper forces are in the same direction. When the damper and the spring forces are in opposite directions , however, the SA device provides a force equal in magnitude but opposite in sign to the spring force.

$$F_f = \begin{cases} -\alpha k_1 (x + l_1\theta - x_1) & (x + l_1\theta - x_1)(\dot{x} + l_1\dot{\theta} - \dot{x}_1) < 0 \\ 0 & (x + l_1\theta - x_1)(\dot{x} + l_1\dot{\theta} - \dot{x}_1) > 0 \end{cases}$$

And

$$F_r = \begin{cases} -\alpha k_2 (x - l_2\theta - x_2) & (x - l_2\theta - x_2)(\dot{x} - l_2\dot{\theta} - \dot{x}_2) < 0 \\ 0 & (x - l_2\theta - x_2)(\dot{x} - l_2\dot{\theta} - \dot{x}_2) > 0 \end{cases}$$

Where F_f and F_r are the front and rear damper forces respectively and α is a gain. The lock-up forces and conditions are as defined earlier for the SA-1 case.

Figs.5.7 and 5.8 are the RMS bounce and pitch acceleration transmissibility ratio frequency responses for varying α gains. Increasing the feedback gain α results in a reduction of the RMS bounce and pitch acceleration transmissibility ratios at around the bounce and pitch mode natural frequencies, but increases it at lower frequencies. Increasing the gain α reduces the rattle space and tire contact force at around the bounce and pitch modes natural frequencies , and increases at lower frequencies as shown in Figs.5.9 and 5.10. A comparison of the RMS bounce acceleration transmissibility ratios of passive, active and SA-2 suspensions is presented in Fig.5.11.At low frequencies, the SA-2 suspension (with $\alpha = 0.75$) results in more than 200% increase of the RMS bounce acceleration ratio when compared to a passive suspension. At high frequencies, however, the SA-2 suspension approaches an active suspension (except at frequencies around the wheel natural frequencies). Whether compared to passive or active suspensions, the SA-2 scheme offers very poor bounce and pitch control at low frequencies. In addition it requires far more rattle space than active or passive suspensions as can be seen from Table 5.2. The only advantage of the SA-2 suspension seems to be its ability to control high frequency bounce and pitch vibration.

The time response to a Chuck Hole type road disturbance, Fig.5.12, demonstrates the inefficiency of the SA-2 suspension in suppressing the bounce and pitch vibration .

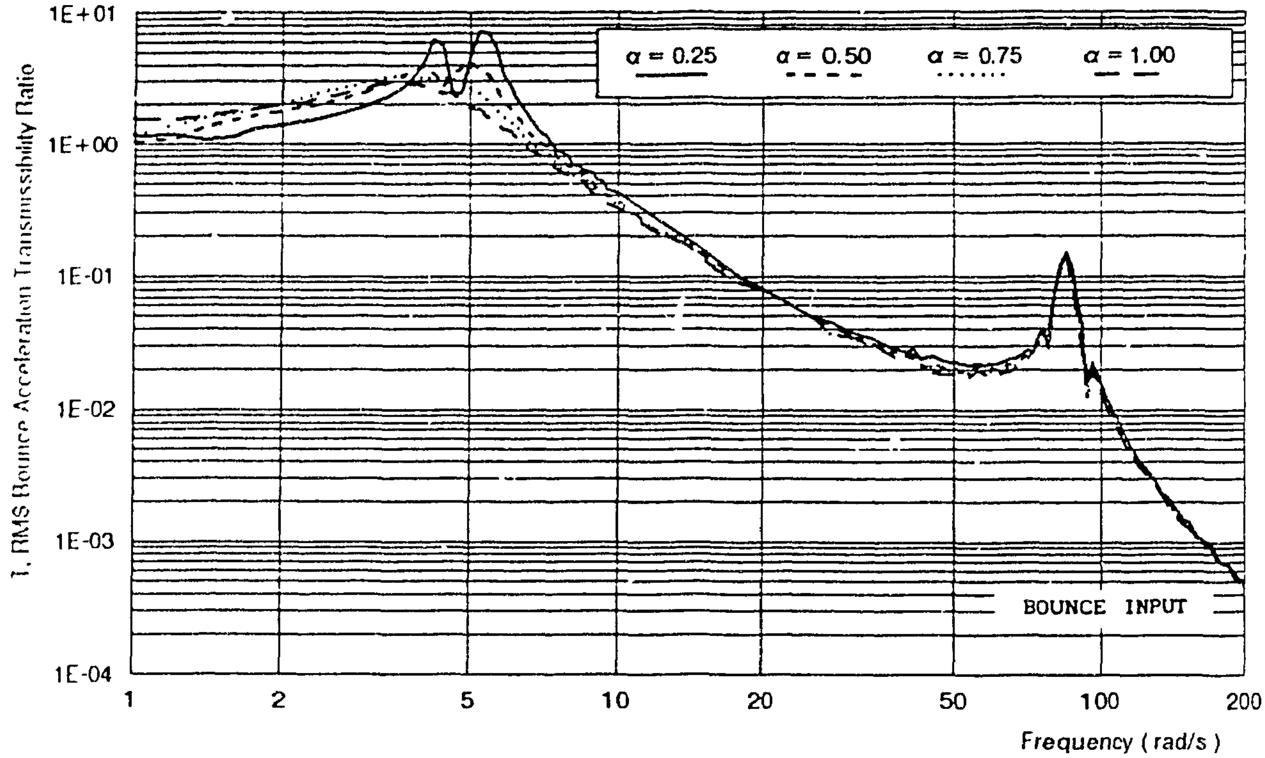


Fig.5.7. RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of an SA-2 Suspension

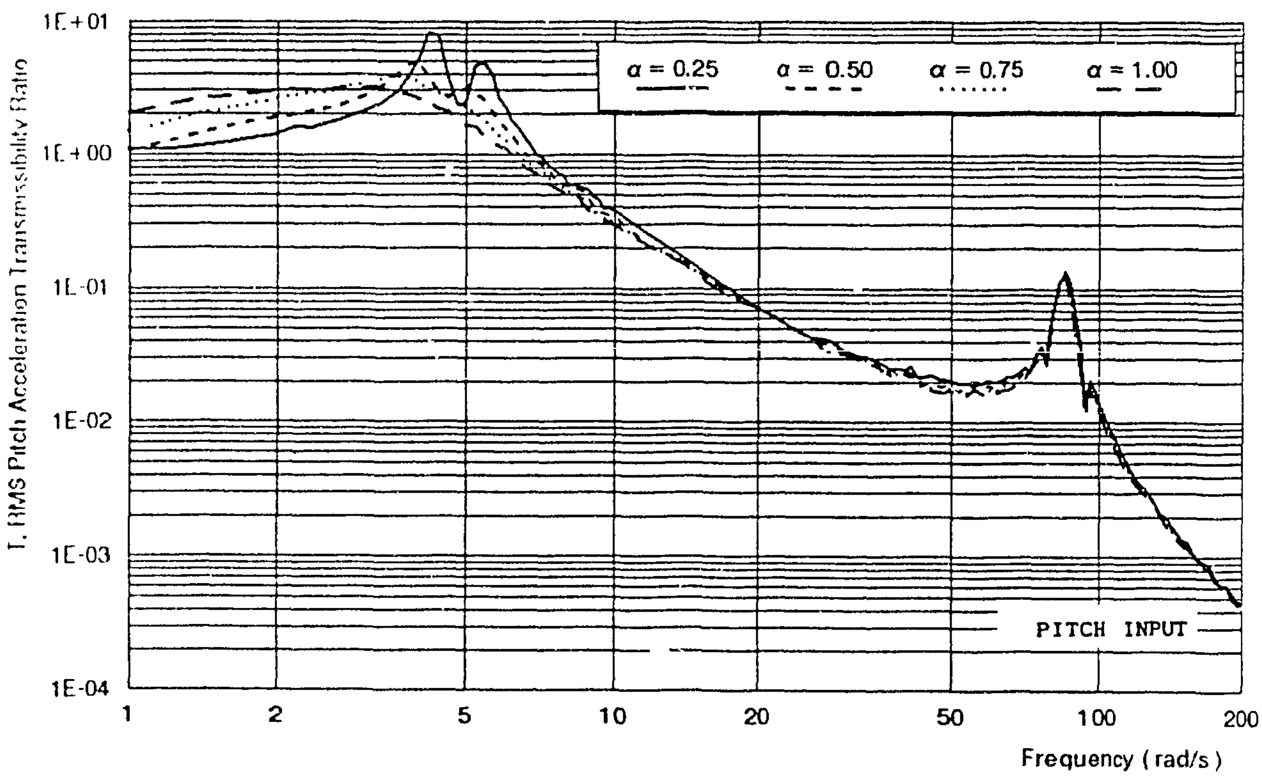


Fig.5.8. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of an SA-2 Suspension

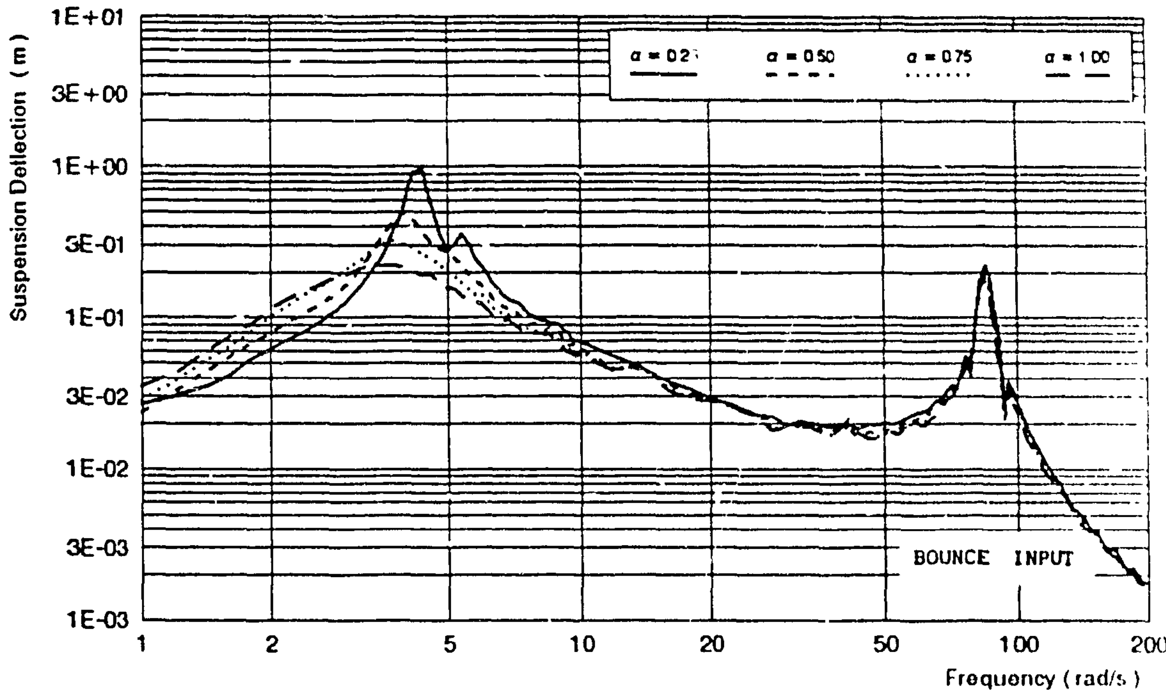


Fig.5.9. Maximum Suspension Deflection Versus Frequency
of an SA-2 Suspension

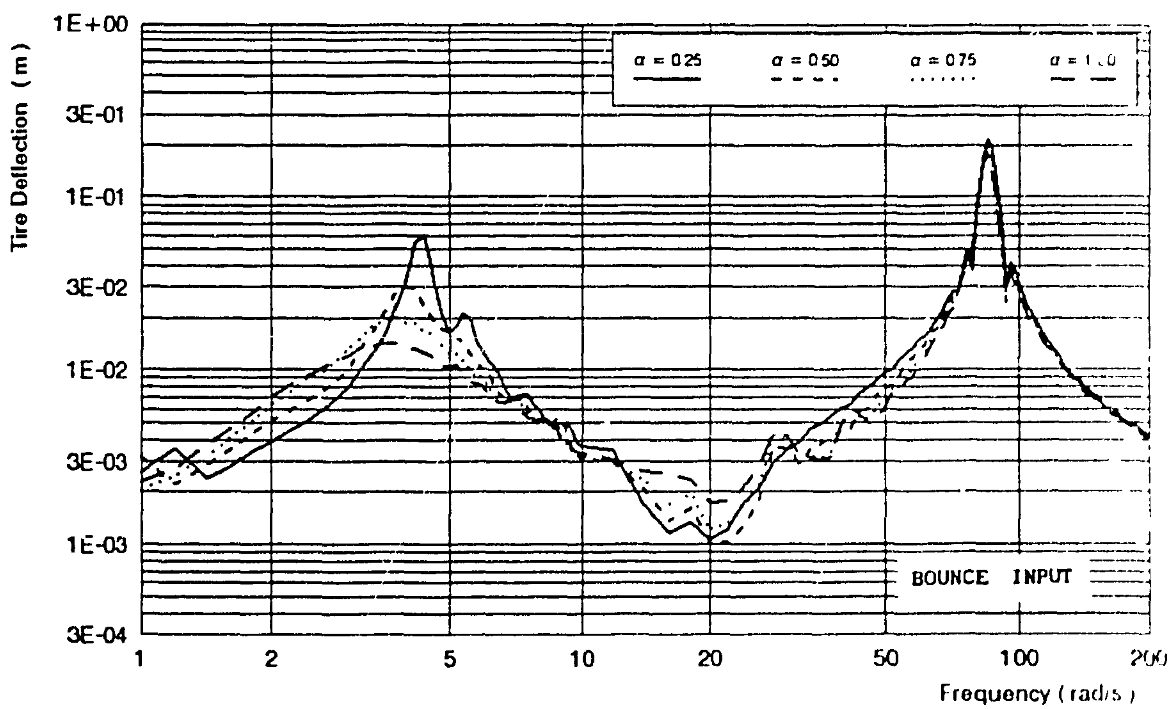


Fig.5.10. Maximum Tire Deflection Versus Frequency
of an SA-2 Suspension

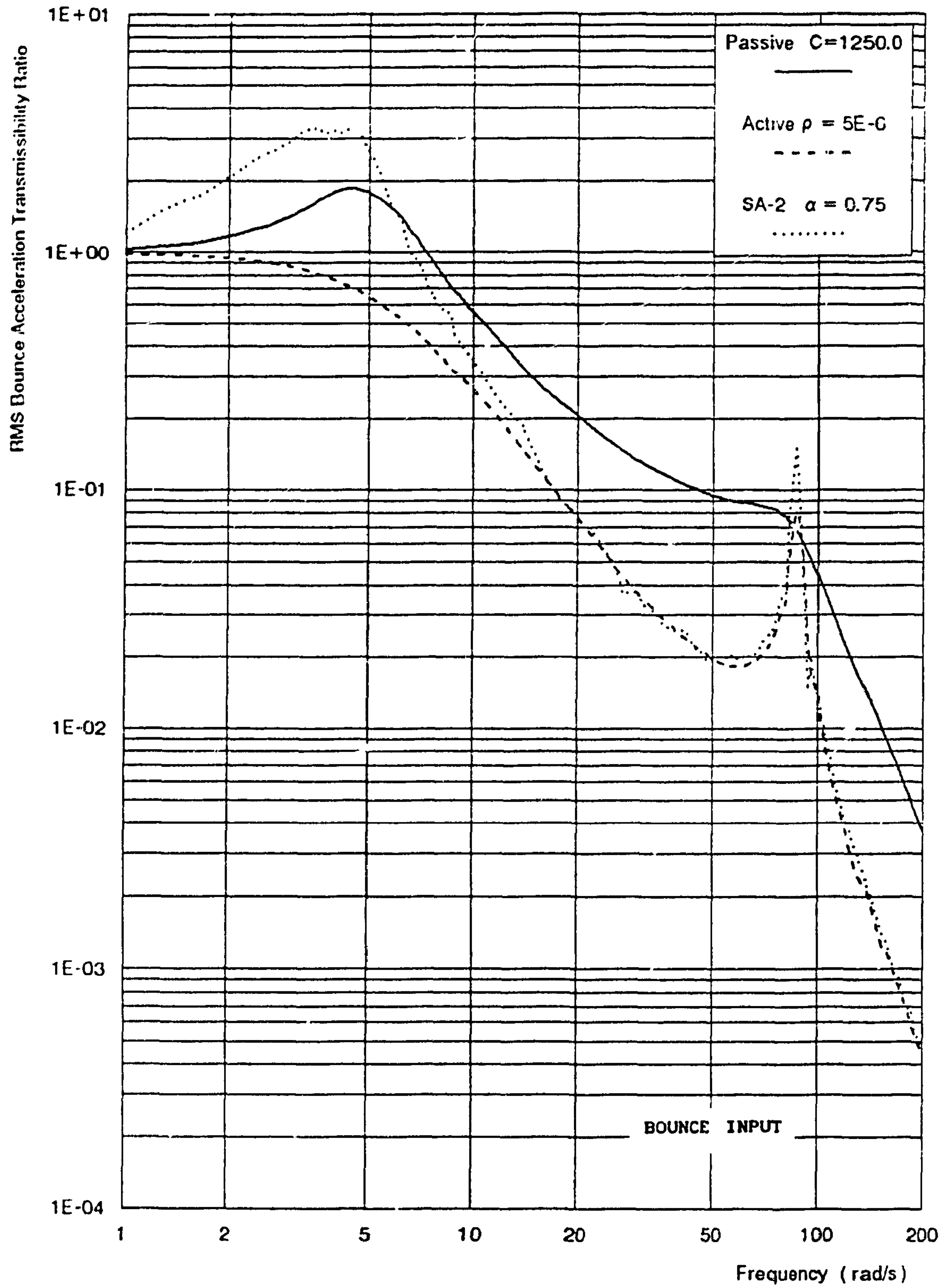


Fig.5.11. Comparison of The RMS Bounce Acceleration Transmissibility Ratios of Passive, Active and SA-2 Suspensions

Table 5.2. Ratio of SA-2 to Passive and SA-2 to Active Performance Parameters at Selected Frequencies: ρ = 5E-6 ; α = 0.75 ; C = 1250 N.s/m

SA-2 to Passive				
Freq. (rad/s)	RMS Bounce	RMS Pitch	Sus. Def.	Tire Def.
2.0	1.78 [T]	2.20	2.08	[illegible]
4.6 *	1.71	1.38	1.42	1.46
20.0	0.39	0.38	1.09	0.36
86.0 **	2.24	2.15	19.68	17.61
200.0	0.13	0.13	3.00	1.38
SA-2 to Active				
Freq. (rad/s)	RMS Bounce	RMS Pitch	Sus. Def.	Tire Def.
2.0	2.19 [TT]	2.58	0.66	2.72
4.6 *	4.54	3.14	2.01	4.49
20.0	1.03	0.98	1.09	1.33
86.0 **	1.90	1.82	2.17	2.20
200.0	1.05	1.04	1.47	1.13

* Neighbourhood of bounce and pitch natural frequencies

** Neighbourhood of wheels natural frequencies

[T] Value less than unity (except for tire deflection) indicates superior performance of SA-2 suspension compared to passive suspension

[TT] Value less than unity (except for tire deflection) indicates superior performance of SA-2 suspension compared to active suspension

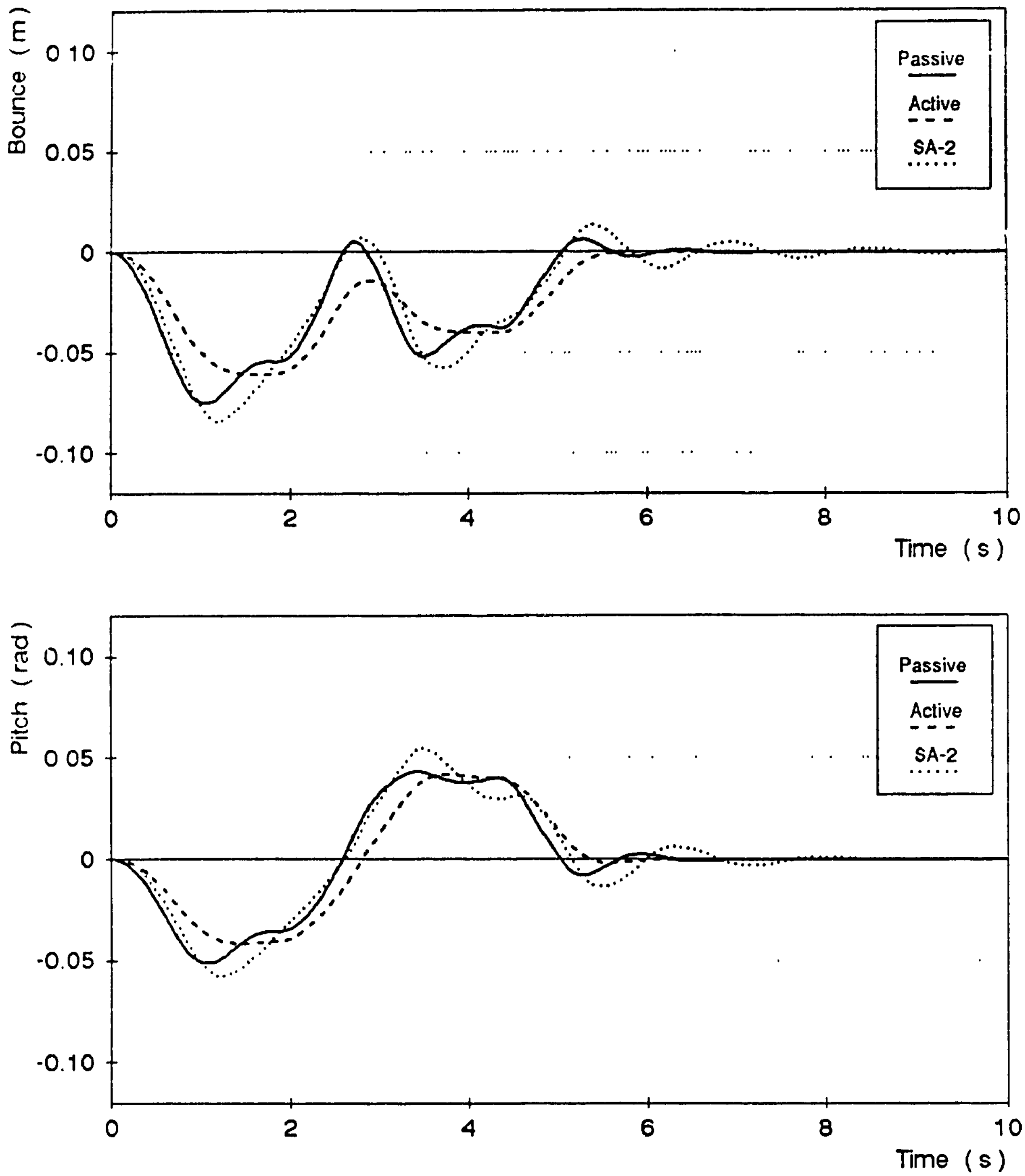


Fig.5.12. Comparison of Bounce and Pitch Responce to a 'Chuck Hole' Type Road Disturbance of the Sprung Mass of Active, Passive and SA-2 Suspensions

5.4. SA-3 Suspension for a 4 D.O.F. Vehicle Model

As in the case of a 1 D.O.F., it is suggested that a small damper be added to the front and rear suspensions in parallel with the SA-2 scheme investigated in the previous section. The additional dampers do also reflect the fact that in practice zero damping can not really be achieved. This modification results in the SA-3 suspension scheme as shown in Fig.5.13.

Except for the added small dampers , the SA-3 and SA-2 suspensions are identical. The lock up-forces and conditions are as defined for the SA-1 suspension (at lock-up the additional dampers have no effect since relative velocities are zero).

Figs.5.14 and 5.15 are the RMS bounce and pitch acceleration transmissibility ratio frequency responses. The additional small damping resulted in an important improvement of the RMS bounce and pitch acceleration ratios at low frequencies. In addition, a remarkable reduction in the suspension deflection, hence rattle space, is achieved as can be seen from Fig.5.16.

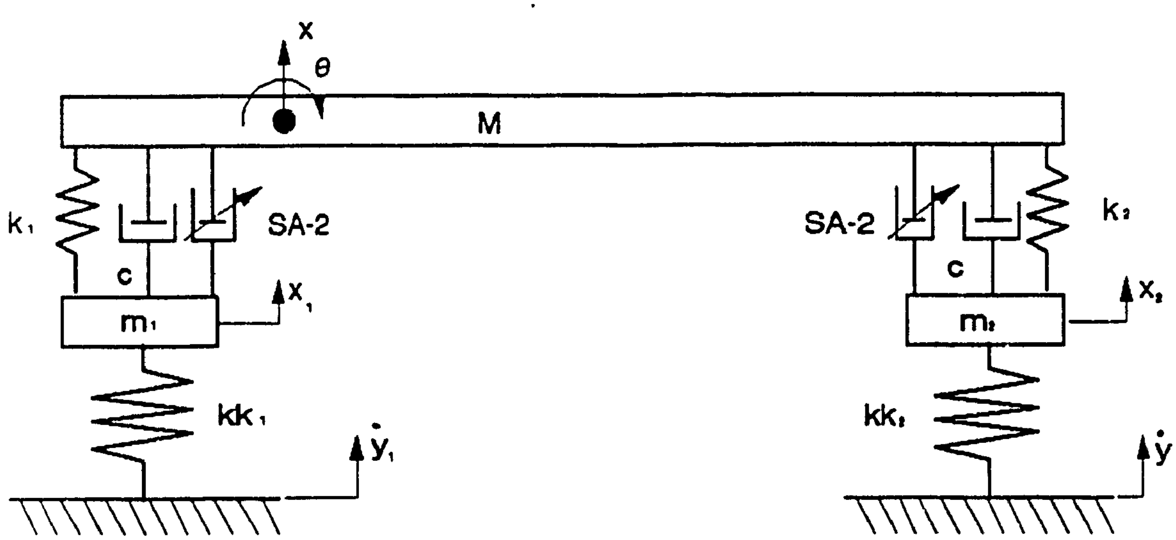


Fig.5.13. SA-3 Suspension Schematic for a 4 D.O.F. Vehicle Model

The major drawback of the inclusion of such small damping is the increase of the RMS acceleration ratio at high frequencies as can be seen form Fig.5.14. This increase is caused by the slightly stiffer connection between the wheel and the body (brought about by the additional passive dampers). A comparison of the RMS bounce acceleration transmissibility ratios of passive, active and SA-3 suspensions is presented in Fig.5.18.Table 5.3 summarizes the overall performance of the SA-3 suspension when compared to either active or passive suspension. Although the SA-3 is not as adequate as an active suspension in controlling the RMS bounce and pitch transmissibility ratios, it reduces significantly the rattle space requirements and increase the tire contact force.

The time response to a Chuck Hole road disturbance reflects the effectiveness of the SA-3 suspension in suppressing the bounce and pitch vibration when compared to an SA-2 suspension.

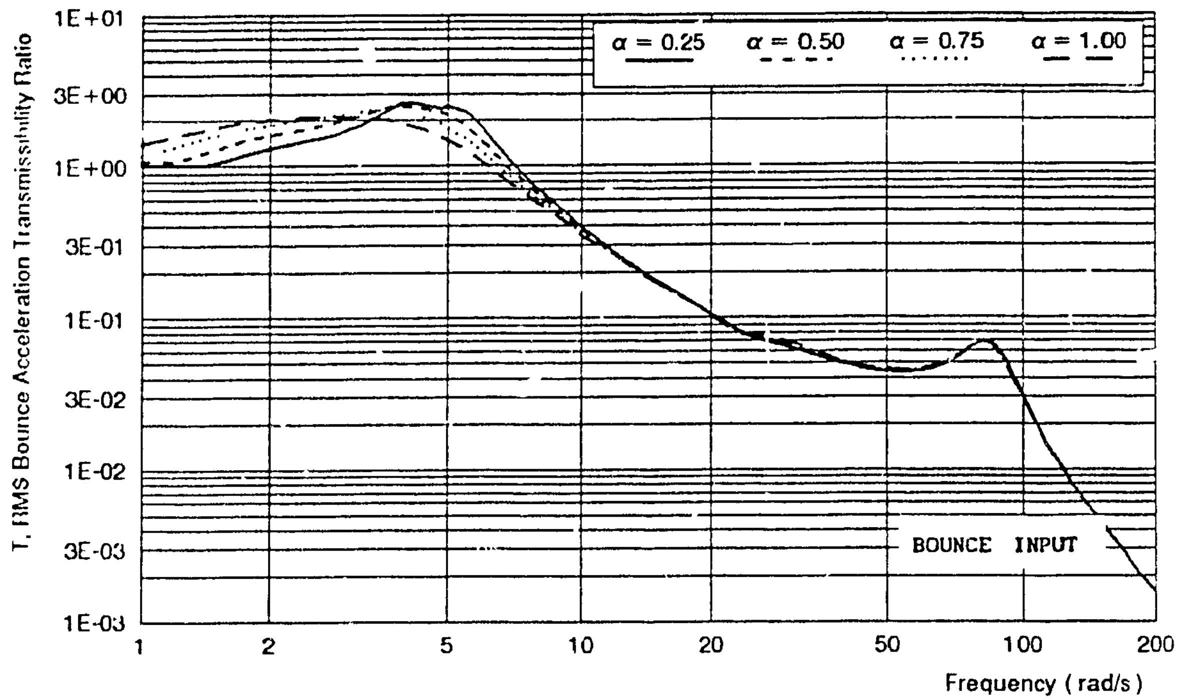


Fig.5.14. RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of an SA-3 Suspension

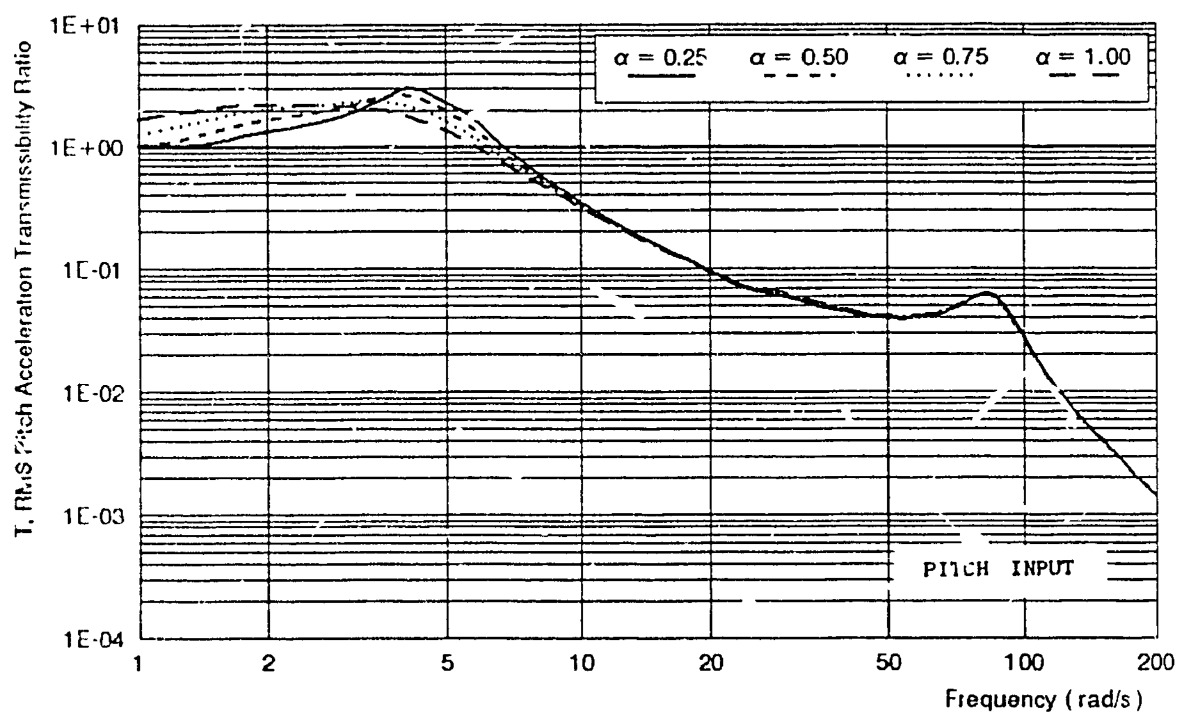


Fig.5.15. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of an SA-3 Suspension

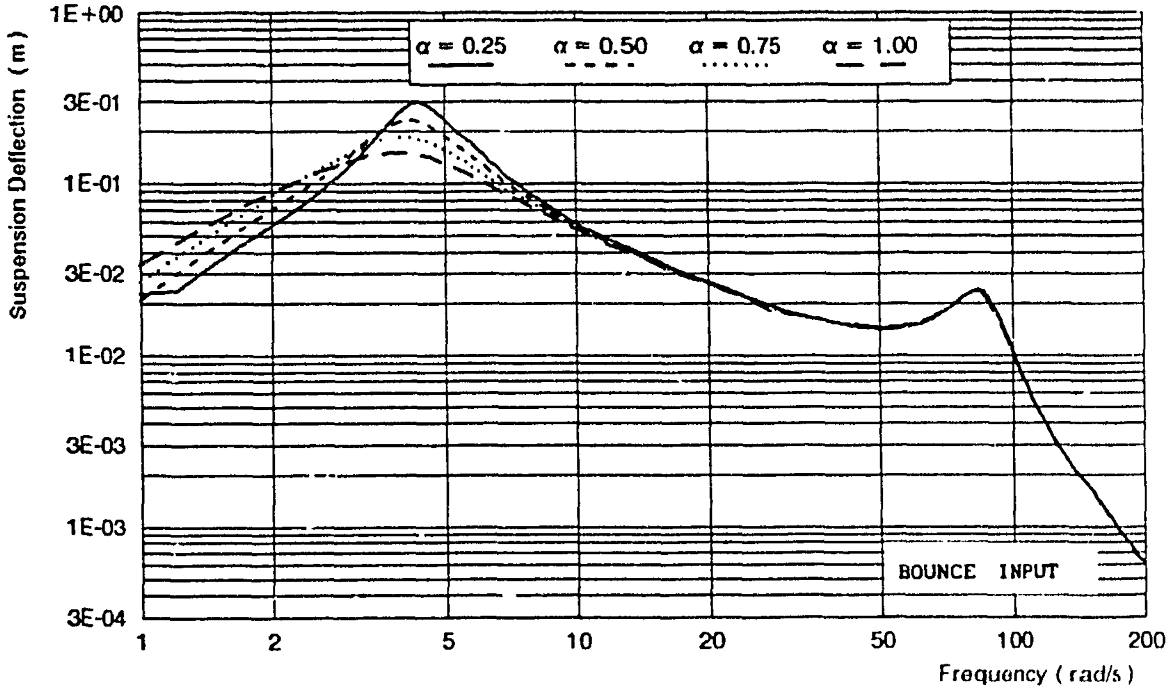


Fig.5.16. Maximum Suspension Deflection Versus Frequency of an SA-3 Suspension

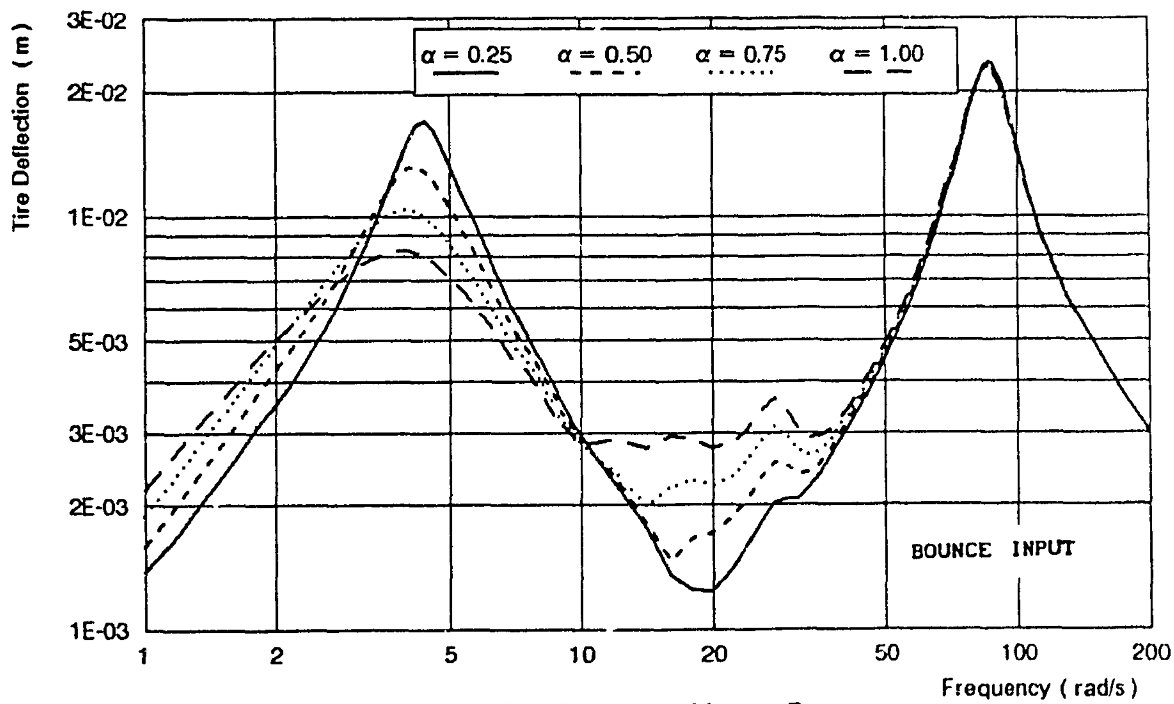


Fig.5.17. Maximum Tire Deflection Versus Frequency of an SA-3 Suspension

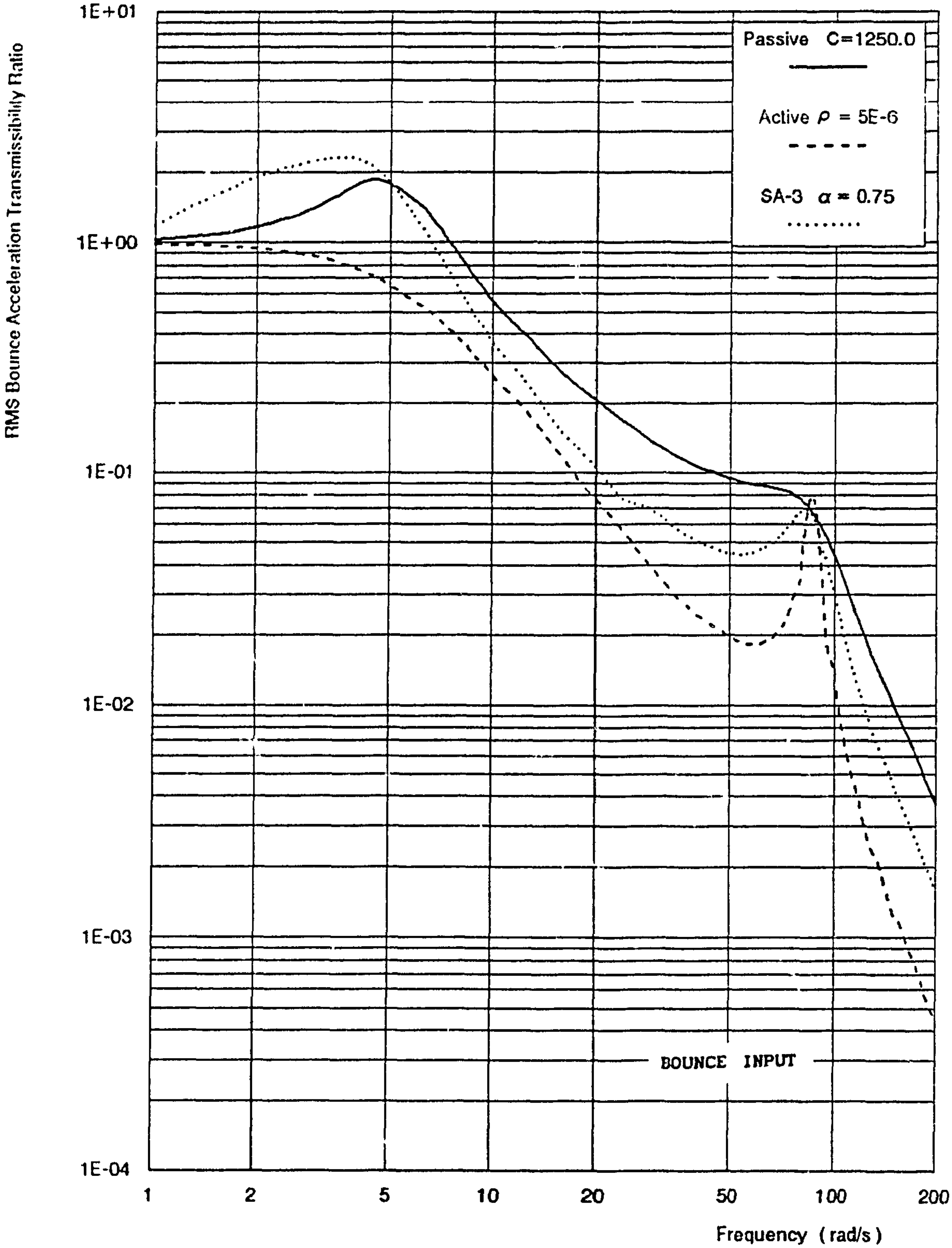


Fig.5.18. Comparison of The RMS Bounce Acceleration Transmissibility Ratios of Passive, Active and SA-3 Suspensions

Table 5.3. Ratio of SA-3 to Passive and SA-3 to Active Performance Parameters at Selected Frequencies: ρ = 5E-6 ; α = 0.75 ; C = 1250 N.s/m

SA-3 to Passive				
Freq. (rad/s)	RMS Bounce	RMS Pitch	Sus. Def.	Tire Def.
2.0	1.64 ^{T}	1.76	1.73	1.64
4.6 *	1.10	0.99	1.01	0.87
20.0	0.51	0.52	0.99	0.65
86.0 **	0.97	0.97	2.37	2.1[illegible]
200.0	0.43	0.43	1.11	1.03
SA-3 to Active				
Freq. (rad/s)	RMS Bounce	RMS Pitch	Sus. Def.	Tire Def.
2.0	2.02 ^{TT}	2.06	0.55	2.10
4.6 *	2.91	2.24	1.43	2.69
20.0	1.37	1.32	0.99	2.43
86.0 **	0.83	0.83	0.26	0.27
200.0	3.50	3.48	0.55	0.84

* Neighbourhood of bounce and pitch natural frequencies

** Neighbourhood of wheels natural frequencies

^{T} Value less than unity (except for tire deflection) indicates superior performance of SA-3 suspension compared to passive suspension

^{TT} Value less than unity (except for tire deflection) indicates superior performance of SA-3 suspension compared to active suspension

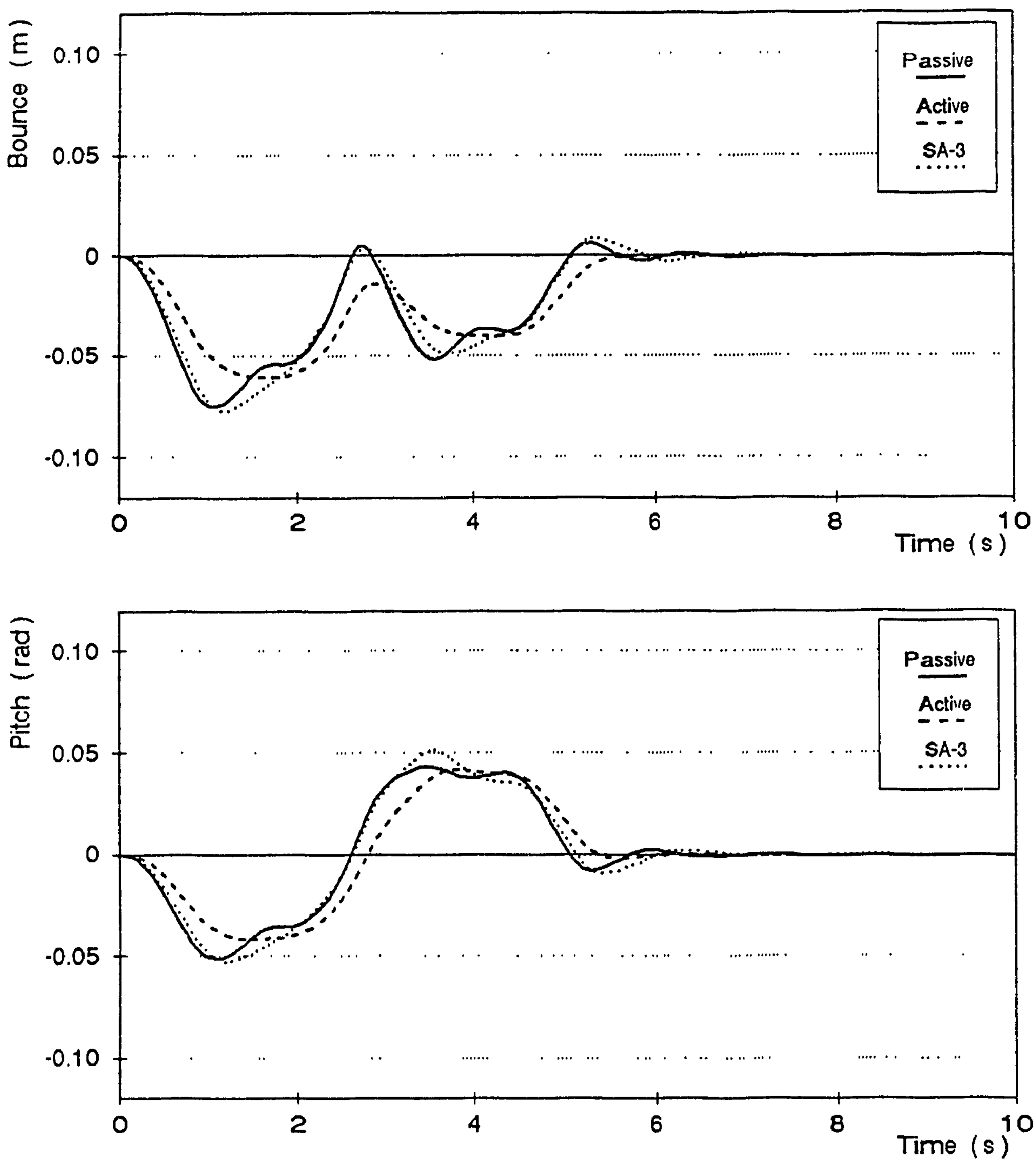


Fig.5.19. Comparison of Bounce and Pitch Responce to a 'Chuck Hole' Type Road Disturbance of the Sprung Mass of Active, Passive and SA-3 Suspensions

5.5. Summary

The SA-1 suspension scheme offers a performance considerably close to that of an active suspension. The main drawback is the number of feedback signals required, which results in extensive hardware requirements. The SA-2 scheme eliminates the need for hard to measure feedback quantities and requires only the measurement of relative displacements and velocities. This scheme, however, fails to adequately control the resonance response. The SA-3 scheme partially solves the latter problem by reducing the RMS acceleration ratio at low frequencies while resulting in slight deterioration of the high frequency response.

CHAPTER 6

COMBINED SUSPENSIONS

6.1. General

Although SA suspensions are a compromise between active and passive suspensions, their implementation is still hindered by their relatively high cost and complexity. This is especially true if they are to be fitted to the front as well as the rear axles. A further attempt to reduce the cost of these suspensions is suggested and investigated in the following. This is done by using an SA or an active suspension only in the front where it is most needed to control the vibration level and to protect the driver and the engine assembly. The rear suspension, however, is maintained passive.

6.2. 'Front Active / Rear Passive' Suspension

From Fig.3.1. and using Newton's second Law, the equations of motion of the system can be written as:

$$\ddot{x} = (-k_1(x + l_1\theta - x_1) - k_2(x - l_2\theta - x_2)$$
$$- C(\dot{x} - l_2\dot{\theta} - \dot{x}_2) + U_1)/M$$
$$\ddot{\theta} = (-l_1k_1(x + l_1\theta - x_1) + l_2k_2(x - l_2\theta - x_2)$$
$$+ l_2C(\dot{x} - l_2\dot{\theta} - \dot{x}_2) + l_1U_1)/J$$
$$\ddot{x}_1 = (k_1(x + l_1\theta - x_1) - kk_1(x_1 - y_1) - U_1)/m_1$$
$$\ddot{x}_2 = (k_2(x - l_2\theta - x_2) - kk_2(x_2 - y_2)$$
$$+ C(\dot{x} - l_2\dot{\theta} - \dot{x}_2))/m_2$$

Which can be reduced to the following set of first order differential equations :

$$\dot{\hat{x}} = \hat{x}_3 + l_1\hat{x}_4 - \hat{x}_5$$
$$\dot{\hat{\theta}} = \hat{x}_3 - l_2\hat{x}_4 - \hat{x}_6$$
$$\dot{\hat{x}}_1 = \hat{x}_5 - \dot{y}_1$$
$$\dot{\hat{x}}_2 = \hat{x}_6 - \dot{y}_2$$
$$\dot{\hat{x}}_3 = (-k_1\hat{x} - k_2\hat{\theta} - C(\hat{x}_3 - l_2\hat{x}_4 - \hat{x}_6) + U_1)/M$$
$$\dot{\hat{x}}_4 = (-l_1k_1\hat{x} + l_2k_2\hat{\theta} + l_1U_1 + l_2C(\hat{x}_3 - l_2\hat{x}_4 - \hat{x}_6))/J$$
$$\dot{\hat{x}}_5 = (k_1\hat{x} - kk_1\hat{x}_1 - U_1)/m_1$$
$$\dot{\hat{x}}_6 = (k_2\hat{\theta} - kk_2\hat{x}_2 + C(\hat{x}_3 - l_2\hat{x}_4 - \hat{x}_6))/m_2$$

Where the variables $\hat{x}$, $\hat{\theta}$, $\hat{x}_1$,$\hat{x}_2$,$\hat{x}_3$,$\hat{x}_4$,$\hat{x}_5$ and $\hat{x}_6$ are defined as follow :

$$\hat{x} = x + l_1\theta - x_1$$
$$\hat{\theta} = x - l_2\theta - x_2$$
$$\hat{x}_1 = x_1 - y_1$$
$$\hat{x}_2 = x_2 - y_2$$
$$\hat{x}_3 = \dot{x}$$
$$\hat{x}_4 = \dot{\theta}$$
$$\hat{x}_5 = \dot{x}_1$$
$$\hat{x}_6 = \dot{x}_2$$

The equations of motion can, therefore, be rearranged in the following space state equation :

$$\left(\dot{\hat{x}}\right)=\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & l_1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -l_2 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -k_1/M & -k_2/M & 0 & -C/M & Cl_2/M & 0 & 0 & C/M \\ \frac{-k_1 l_1}{J} & \frac{k_2 l_2}{J} & 0 & \frac{Cl_2}{J} & \frac{-Cl_2^2}{J} & 0 & 0 & \frac{-Cl_2}{J} \\ k_1/m_1 & 0 & \frac{-kk_1}{m_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_2/m_2 & 0 & \frac{-kk_2}{m_2} & \frac{C}{m_2} & \frac{-Cl_2}{m_2} & 0 & -\frac{C}{m_2} \end{pmatrix}\left(\hat{x}\right)+\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{M} \\ \frac{l_1}{J} \\ \frac{-1}{m_1} \\ 0 \end{pmatrix} U_1+\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix}$$

or

$$\left(\dot{\hat{x}}\right) = A\left(\hat{x}\right) + B\left(U\right) + C\left(y\right)$$

where:

$$\left(\hat{x}\right)^T = \left(\hat{x} \quad \hat{\theta} \quad \hat{x}_1 \quad \hat{x}_2 \quad \hat{x}_3 \quad \hat{x}_4 \quad \hat{x}_5 \quad \hat{x}_6\right) = \text{State Variables}$$

$$\left(U\right)^T = U_1 = \text{Control Variable}$$

$$\left(y\right) = \left(\dot{y}_1 \quad \dot{y}_2\right) = \text{Input Variables}$$

The objective is to find an expression for the quantity U_1 that will minimize a given cost function. Determination of the latter will be achieved through the use of the ' Linear Optimal Regulator theory ' as discussed in the case of a fully active suspension in chapter 4.

The performance index is as follows:

$$I = \frac{1}{2}\int_0^\infty (\rho_1 U_1^2 + q_1\hat{x}^2 + q_2\hat{\theta}^2 + q_3\hat{x}_1^2 + q_4\hat{x}_2^2 + q_5\hat{x}_3^2 + q_6\hat{x}_4^2)\,dt$$

The weighting factors in the above expression (ρ_1, ρ_2, q_1, q_2, q_3, q_4, q_5 and q_6), are as defined for a fully active suspension.

The performance index, I, is, therefore, written in the following standard format:

$$I = \frac{1}{2}\int_0^\infty (U^T \rho_1 U + \hat{x}^T Q \hat{x})\,dt$$

Where :

$$Q = \begin{pmatrix} q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & q_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & q_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & q_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

And hence the optimal value of the force U_1 is given by [37] :

$$U_{1\,opt} = -(B^T P\ \hat{x}(t))/\rho_1 = k^T\ \hat{x}(t)$$

Where the matrix B is as defined earlier and

$$k^T = \left(k_{11}\ \ k_{12}\ \ k_{13}\ \ k_{14}\ \ k_{15}\ \ k_{16}\ \ k_{17}\ \ k_{18} \right)$$

The 8x8 matrix P is the symmetric positive definite steady-state solution of the algebraic *Riccati equation :*

$$PA + A^T P - PBB^T P/\rho_1 + Q = 0$$

The coefficients of the matrix k are given in Table 6.1. for varying actuator size.

Table 6.1. Front Actuator Gains for a ' Front Active / Rear Passive' Suspension

Gains \ ρ	1E-6	5E-6	1E-5	5E-5
K11	-4142.158	-954.446	-488.077	-99.501
K12	-731.537	230.524	333.703	285.079
K13	1133.390	132.598	56.650	9.107
K14	3676.271	1960.216	1401.320	563.003
K15	-8413.236	-3651.632	-2530.884	-1070.262
K16	-6055.382	-2902.975	-2127.933	-1032.471
K17	240.862	110.429	78.358	35.166
K18	-16.178	-4.158	-2.161	-0.366

Figs.6.1 and 6.2 demonstrate the effect of increased damper size on the RMS bounce and pitch acceleration transmissibility ratios. As the actuator size is increased, a decrease in the RMS bounce and pitch ratios is achieved at low frequencies with an insignificant increase at high frequencies. As expected, varying the actuator size does not have any effect on the rear suspension and tire deflections (Figs.6.4 and 6.6). The front rattle space and tire contact force, Figs.6.3 and 6.5, are similar to that of a fully active suspension except for the dip of the tire deflection around the bounce and pitch natural frequencies for high actuator size. While offering RMS bounce and pitch acceleration ratios close to that of a fully active suspension at low frequencies, the loss of performance of the combined 'Rear Passive / Front Active' suspension occurs predominantly at higher frequencies as shown in Fig.6.7.

Similar results are published in [18] as can be seen from Fig.6.8. Although the model used is a simpler 2 D.O.F. pitch model, results at low frequencies (dominated by the bounce and pitch modes) exhibit the same overall behaviour. The discrepancies are caused by the fact that the actuator gains are not identical in both studies and the fact that while the published results are based on velocity ratio response, the results of this work are based on the RMS acceleration transmissibility ratio. Similarities with published results are mostly apparent for ρ = 1E-5. Notice the linear scale is used in Fig.6.8, while Logarithmic scale is used in Figs.6.1 and 6.2.

The response to a Chuck Hole type road disturbance Fig.6.9, is governed by whether the vehicle is in the descending or ascending position as shown in Fig.6.10. In the descending position, the response approaches that of an active suspension and in the ascending position, the response follows that of a passive suspension.

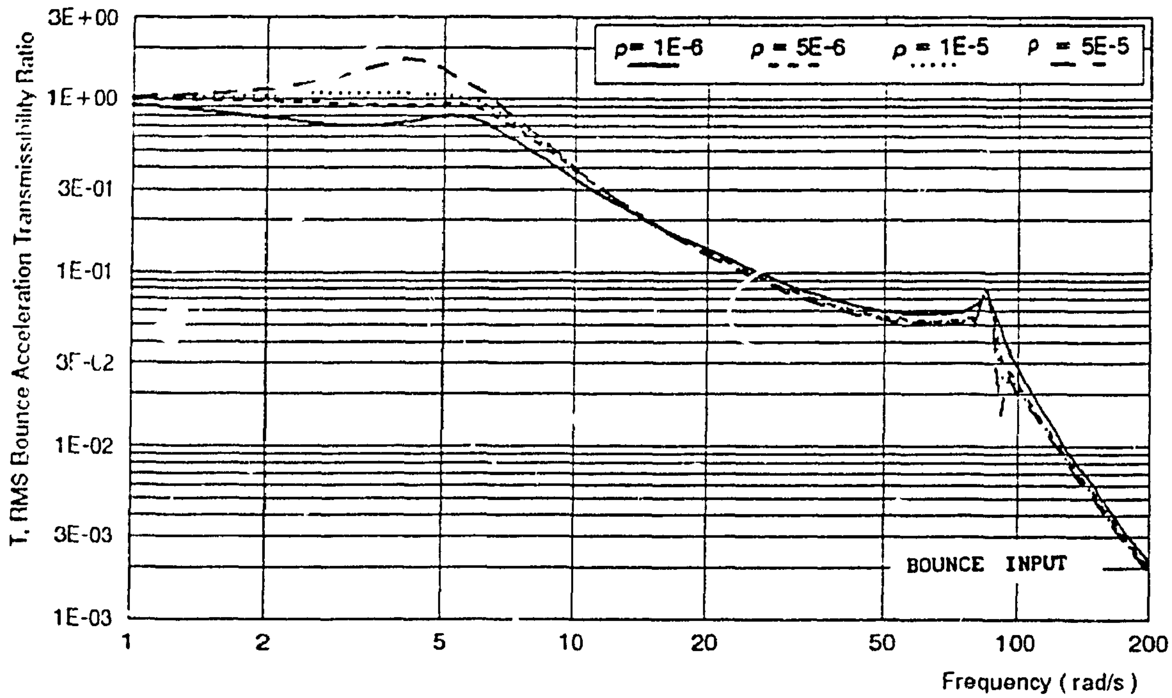


Fig.6.1. RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of a Combined ' Front Active / Rear Passive' Suspension

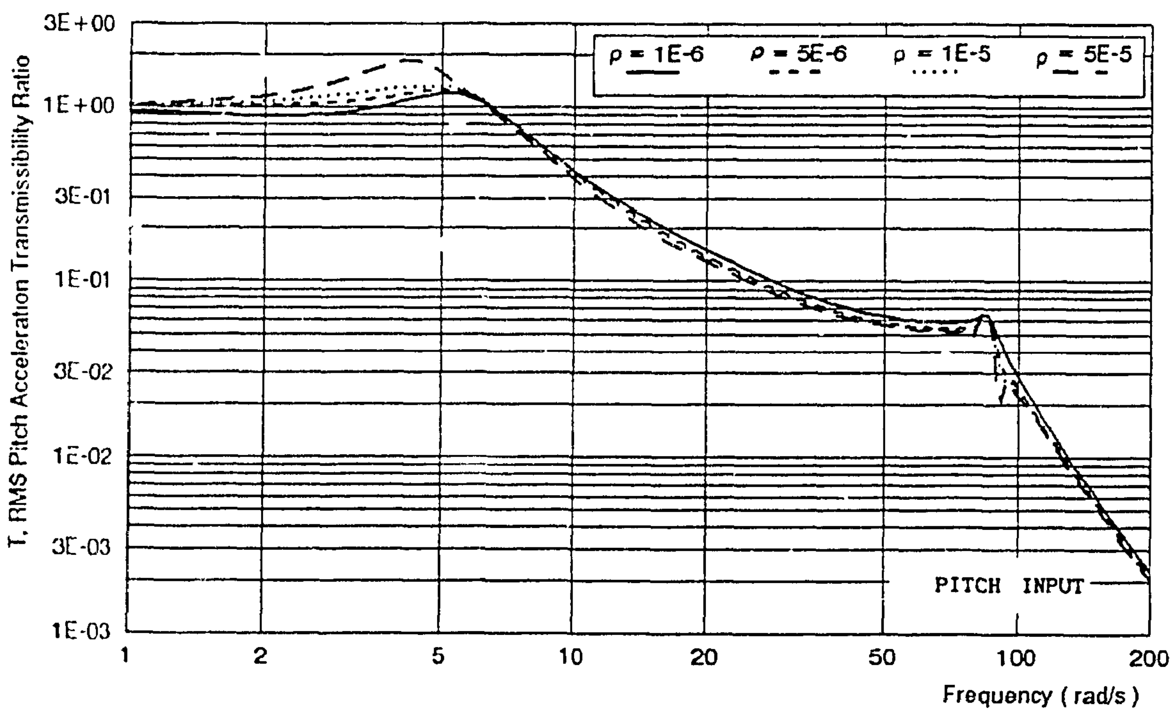


Fig.6.2. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of a Combined ' Front Active / Rear Passive' Suspension

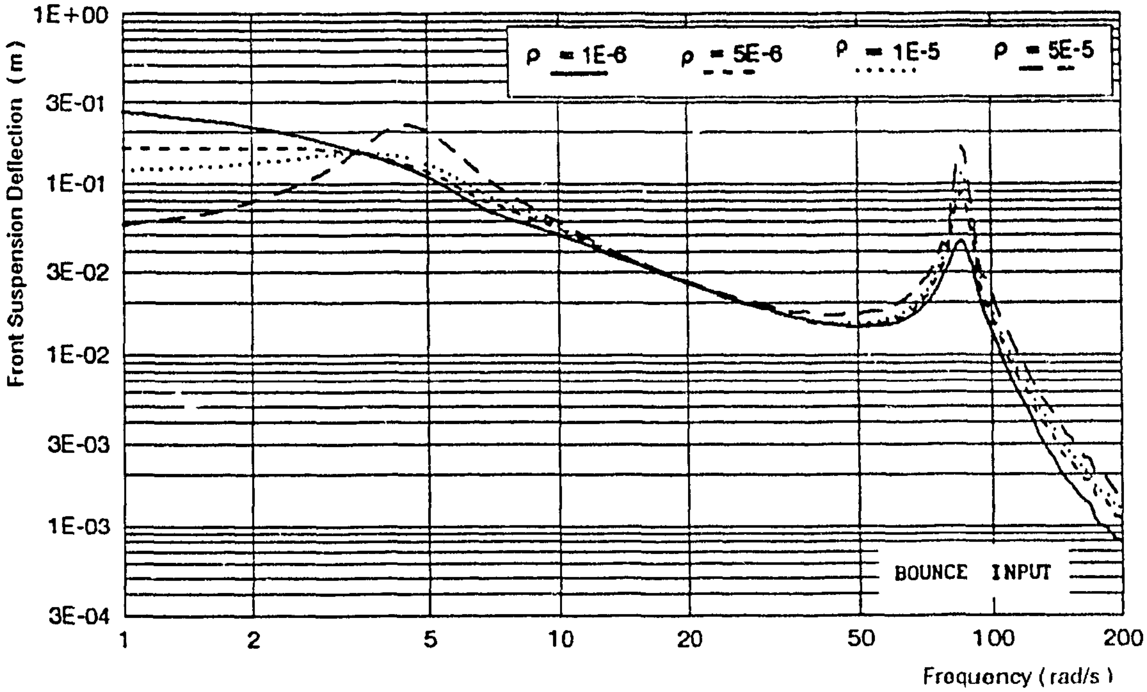


Fig.6.3. Maximum Front Suspension Deflection Versus Frequency of a Combined 'Front Active / Rear Passive' Suspension

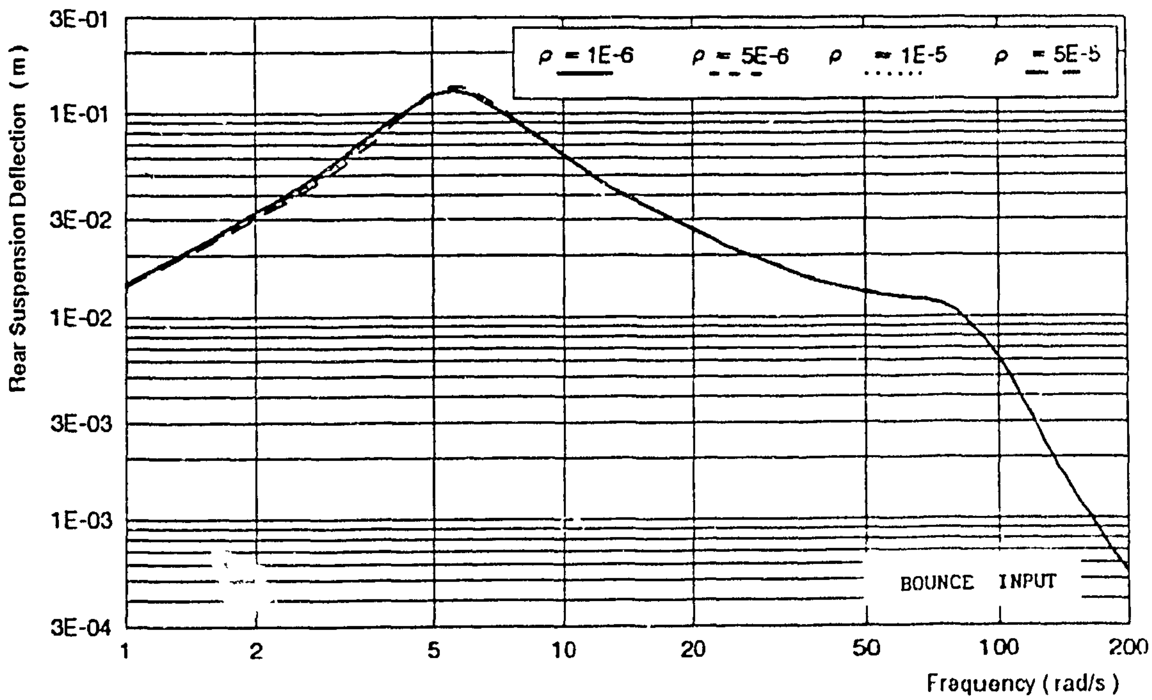


Fig.6.4. Maximum Rear Suspension Deflection Versus Frequency of a Combined 'Front Active / Rear Passive' Suspension

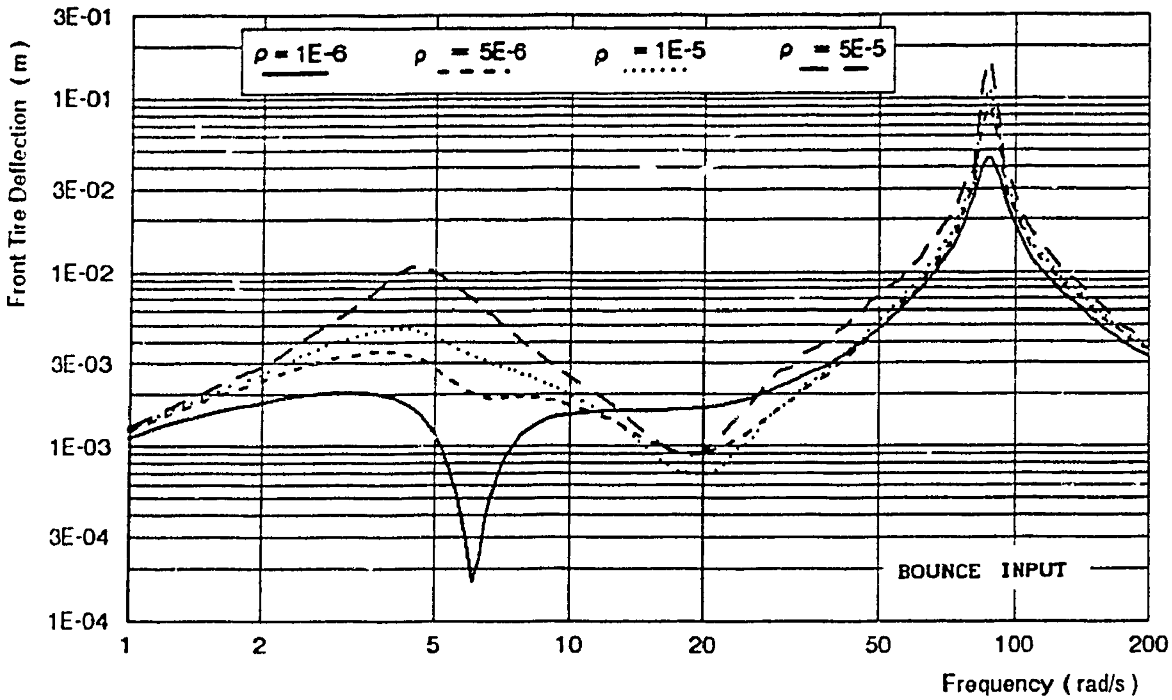


Fig.6.5. Maximum Front Tire Deflection Versus Frequency
of a Combined 'Front Active / Rear Passive' Suspension

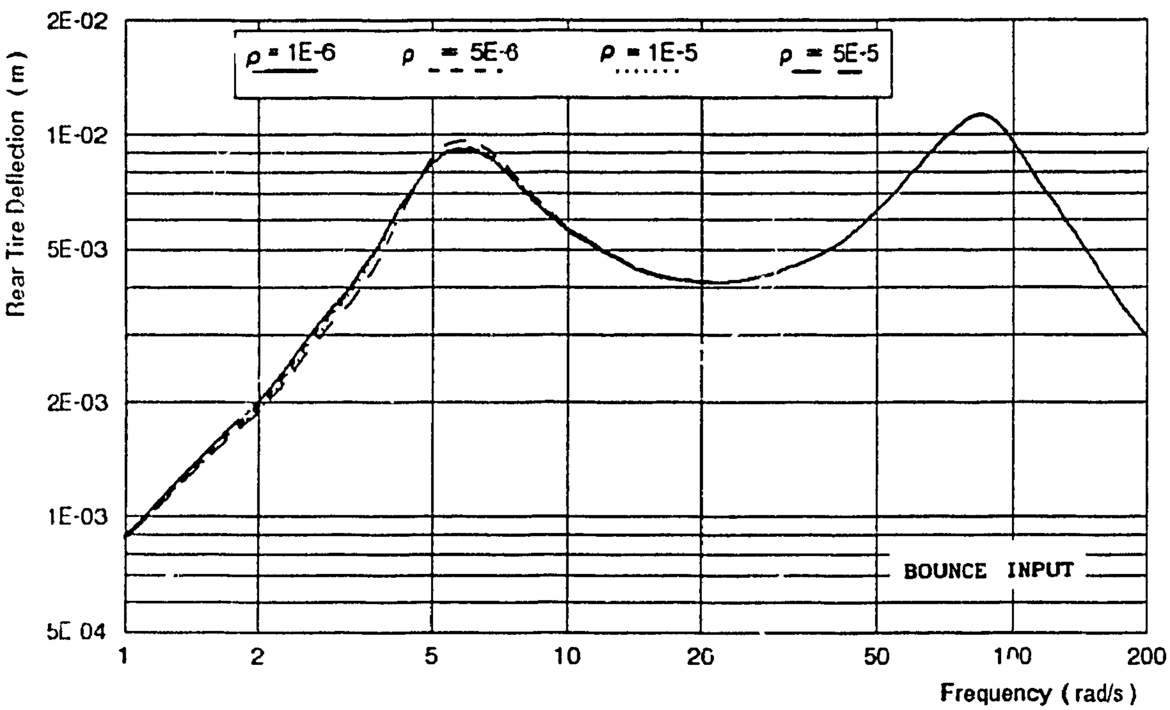


Fig.6.6. Maximum Rear Tire Deflection Versus Frequency
of a Combined 'Front Active / Rear Passive' Suspension

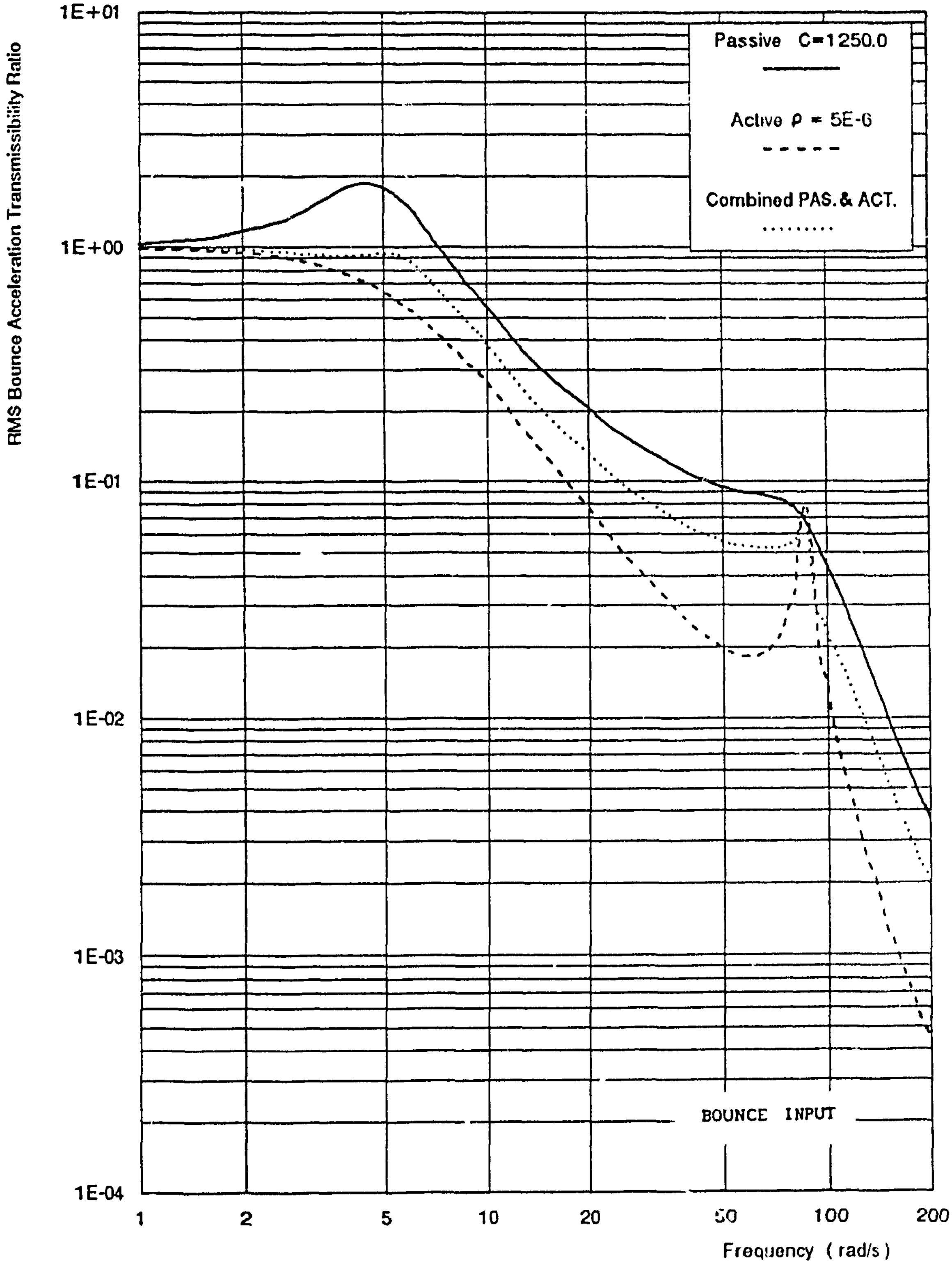


Fig.6.7. Comparison of The RMS Bounce Acceleration Transmissibility Ratios of Passive, Active and Combined 'Front Active / Rear Passive' Suspensions

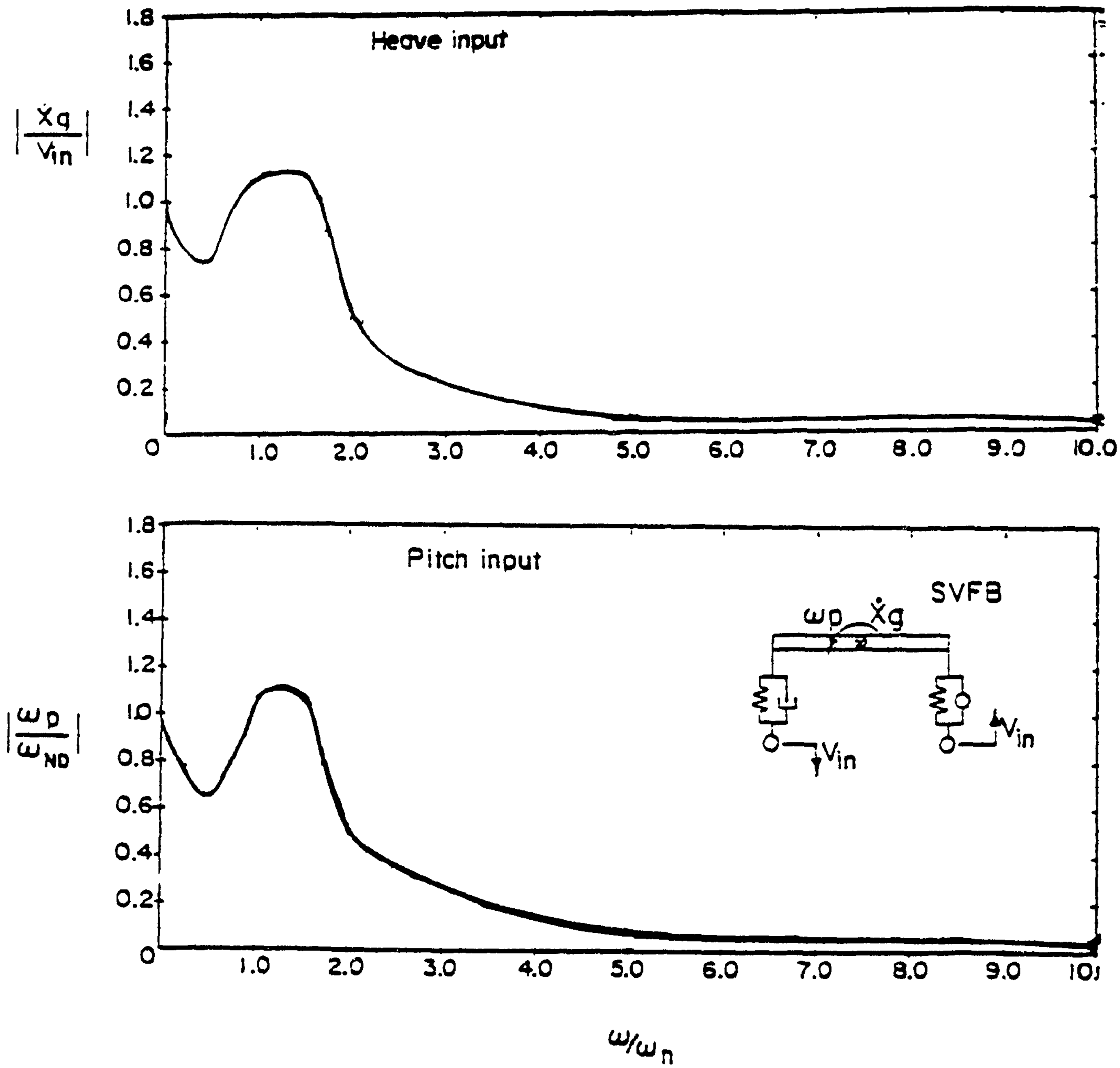
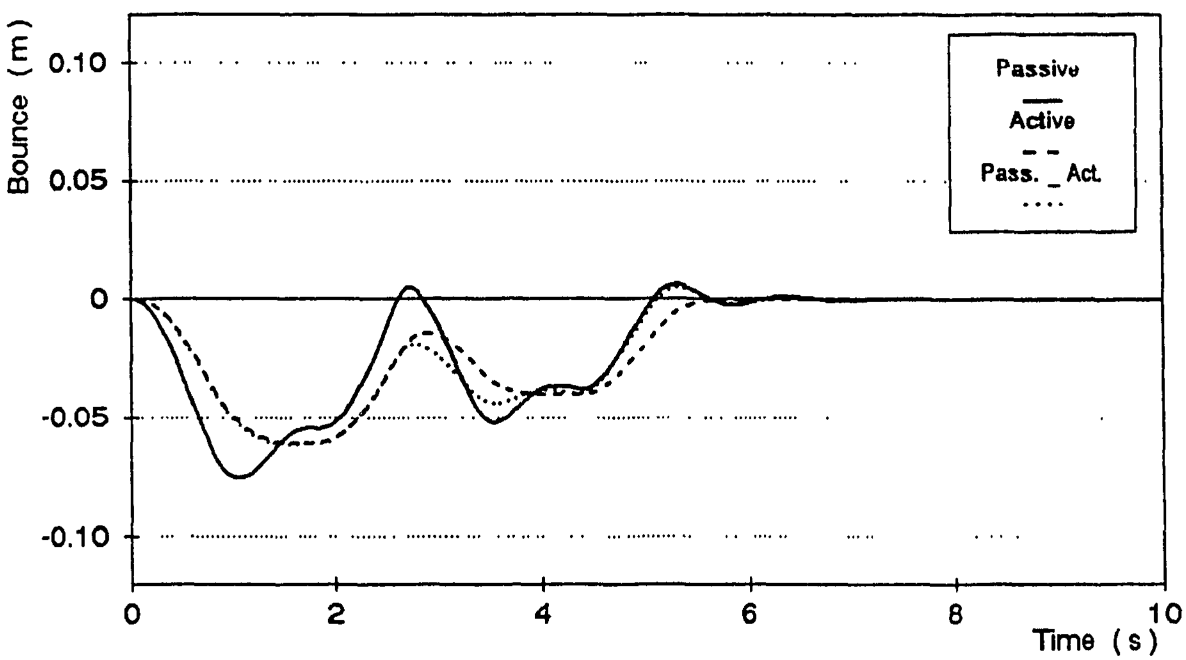


Fig.6.8. Bounce and Pitch Velocity Ratio Versus Frequency of a 2 D.O.F. Pitch Vehicle Model With Combined 'Front Active / Rear Passive' Suspension [18]



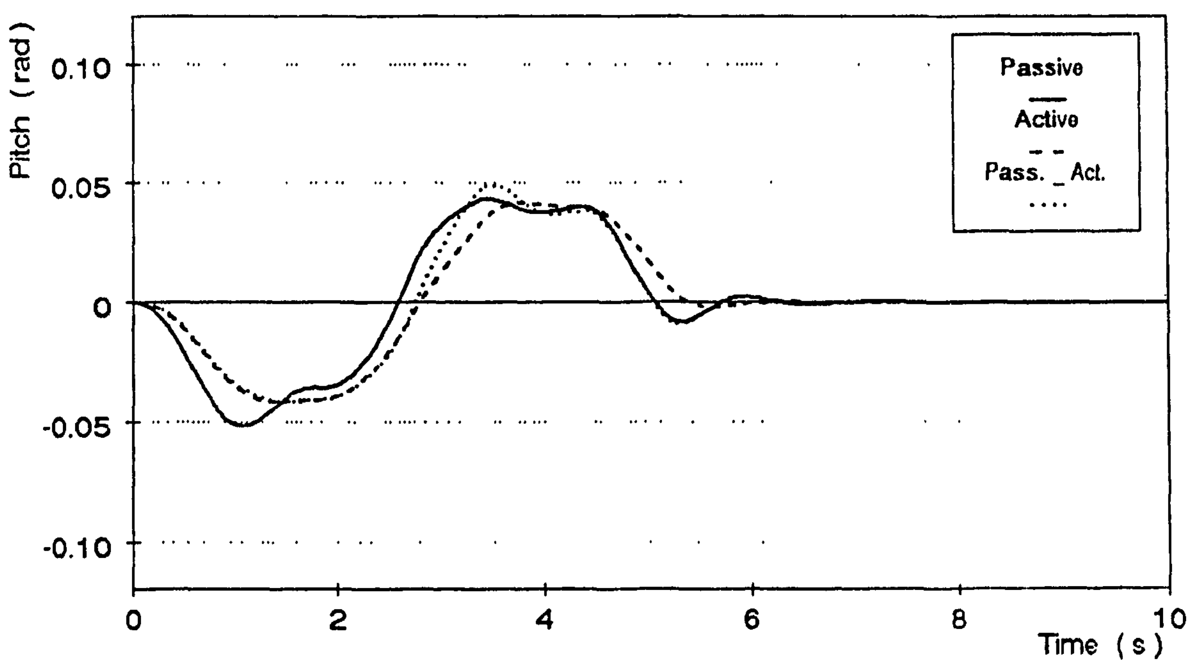
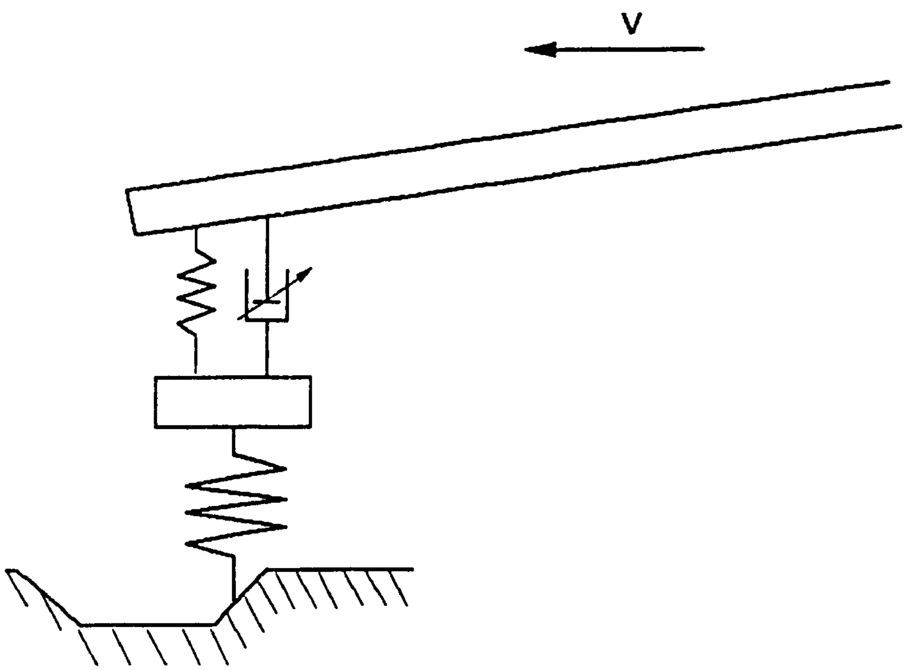
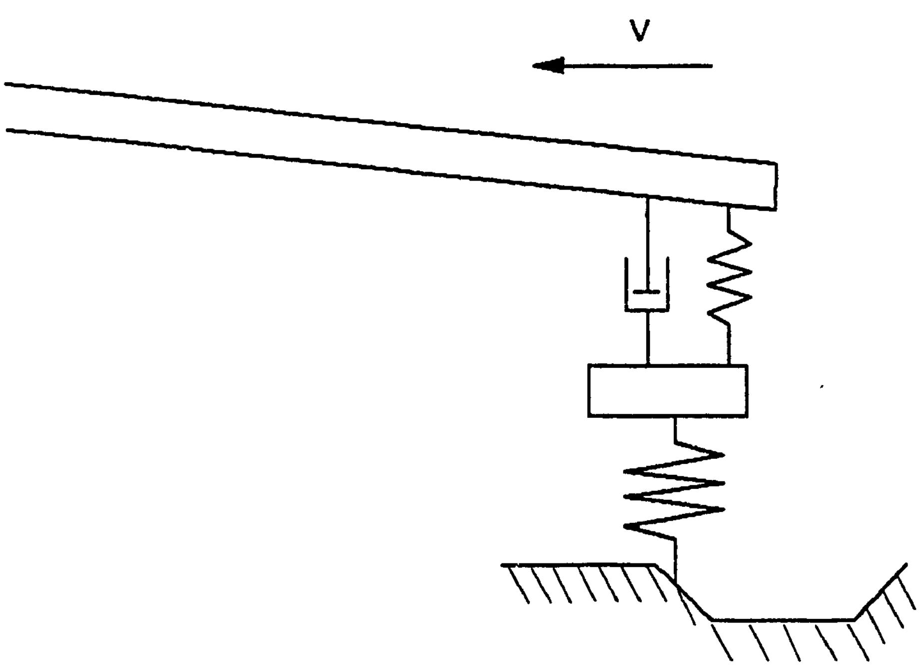


Fig.6.9. Comparison of Bounce and Pitch Responce to a 'Chuck Hole' Type Road Disturbance of the Sprung Mass of Active, Passive and Combined ' Front Active / Rear Passive ' Suspensions



Descending Position



Ascending Position

Fig.6.10. Ascending and Descending Positions

6.3. 'Front SA-1 / Rear Passive ' Suspension

This scheme is similar to the one described in section 5.1, except for the fact that the rear suspension is passive. Using the mathematical approach presented in section 5.1 and combining with the optimal damper force U, calculated in section 6.1, the vibration response of a 4 D.O.F model with front SA-1 / rear passive suspension is investigated. Figs.6.11 and 6.12 present the RMS bounce and pitch acceleration ratio frequency responses. Increasing the SA-1 damper size reduces the RMS bounce and pitch acceleration ratios at around the bounce and pitch mode natural frequencies. A further increase of the SA damper size, however, results in little or no improvement of the RMS ratio throughout the frequency range. Although not included here, the front suspension and tire deflections are not affected by changing the SA damper size. While the front suspension and tire deflections are similar to those for a fully SA-1 suspension. The overall performance of a combined 'Front SA-1 / Rear Passive' suspension lies, as expected, between that of an SA-1 suspension and that of a passive suspension Fig.6.13. Similarly published results [18] indicate similar behaviour although a simpler 2 D.O.F. model is used (Fig.6.14). The discrepancies are due to the same factors listed earlier. Slight deterioration of the time response to a 'Chuck Hole' type road disturbance resulted because of alteration of the rear SA-1 suspension with a passive suspension as shown in Fig.6.15.

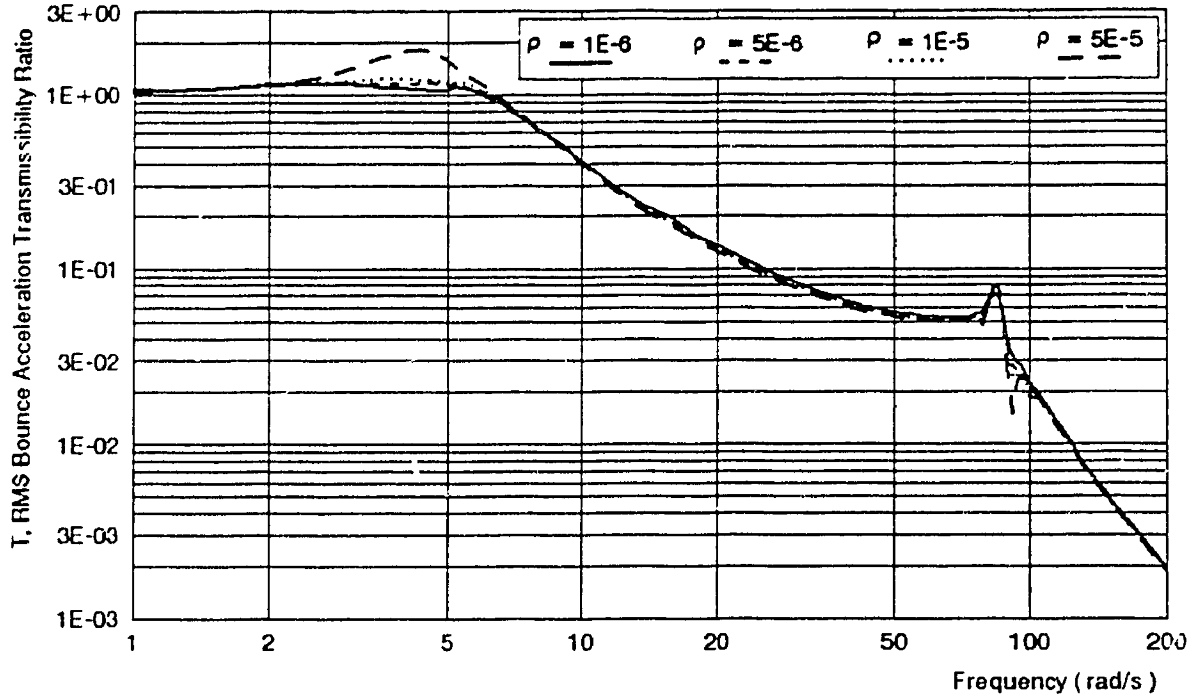


Fig.6.11. RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of a Combined ' Front SA-1 / Rear Passive' Suspension

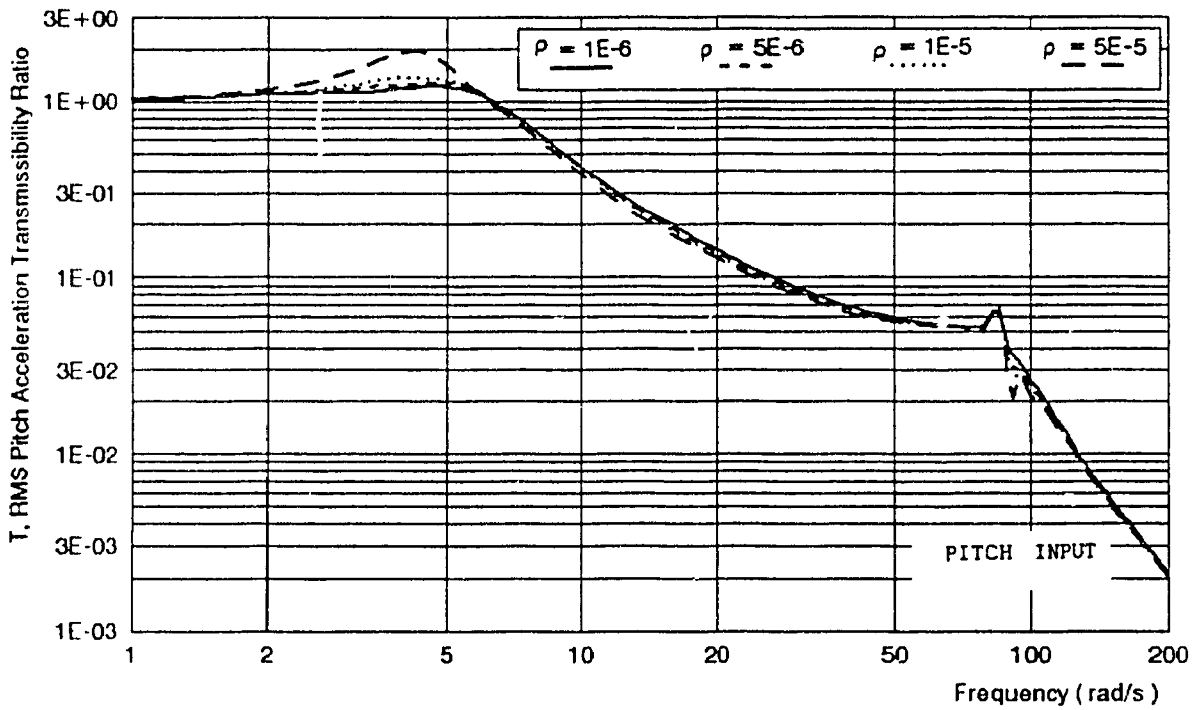


Fig 6.12. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of a Combined ' Front SA-1 / Rear Passive' Suspension

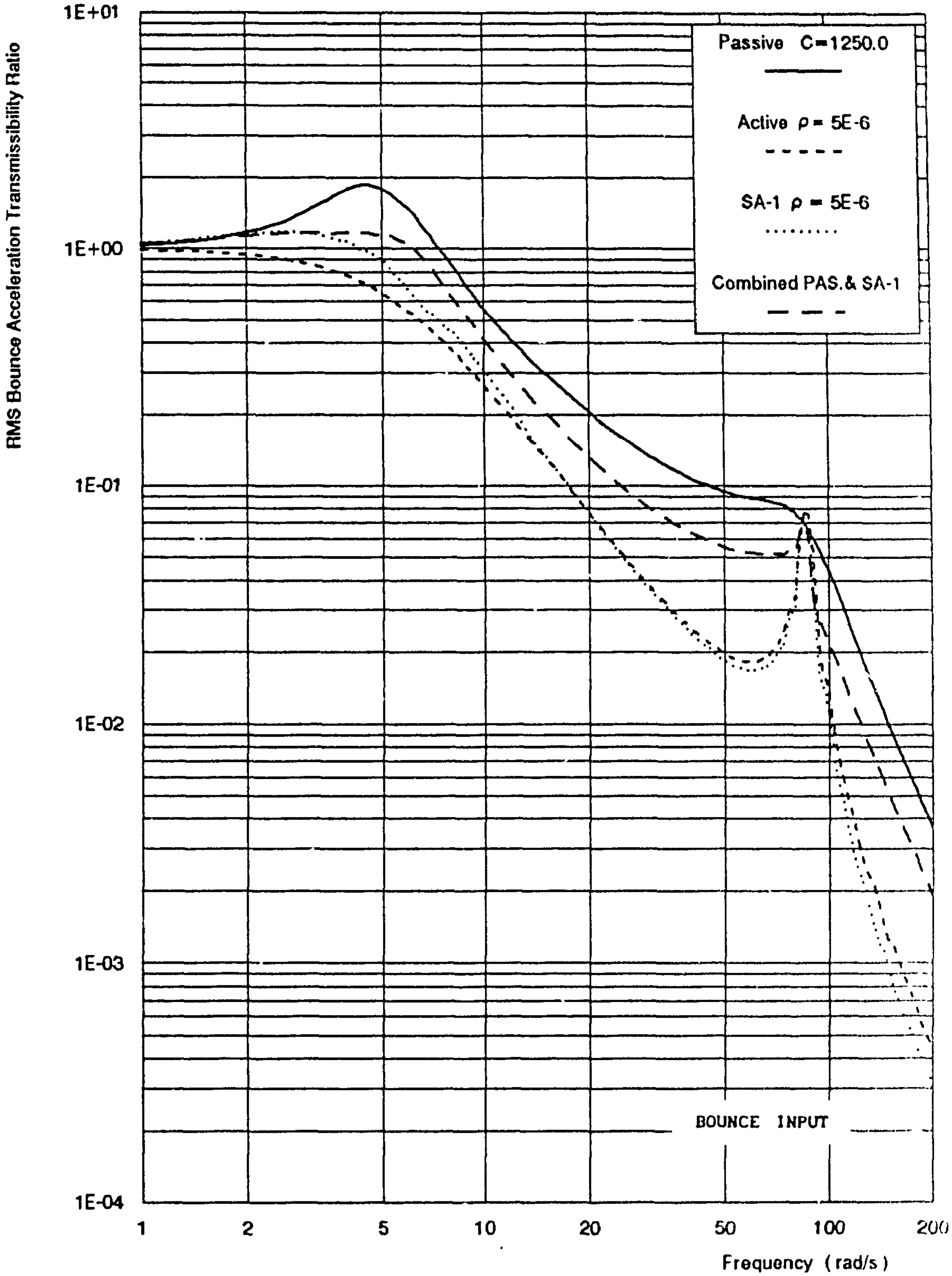


Fig.6.13. Comparison of The RMS Bounce Acceleration Transmissibility Ratios of Passive, Active, SA-1 and Combined 'Front SA-1 / Rear Passive' Suspensions

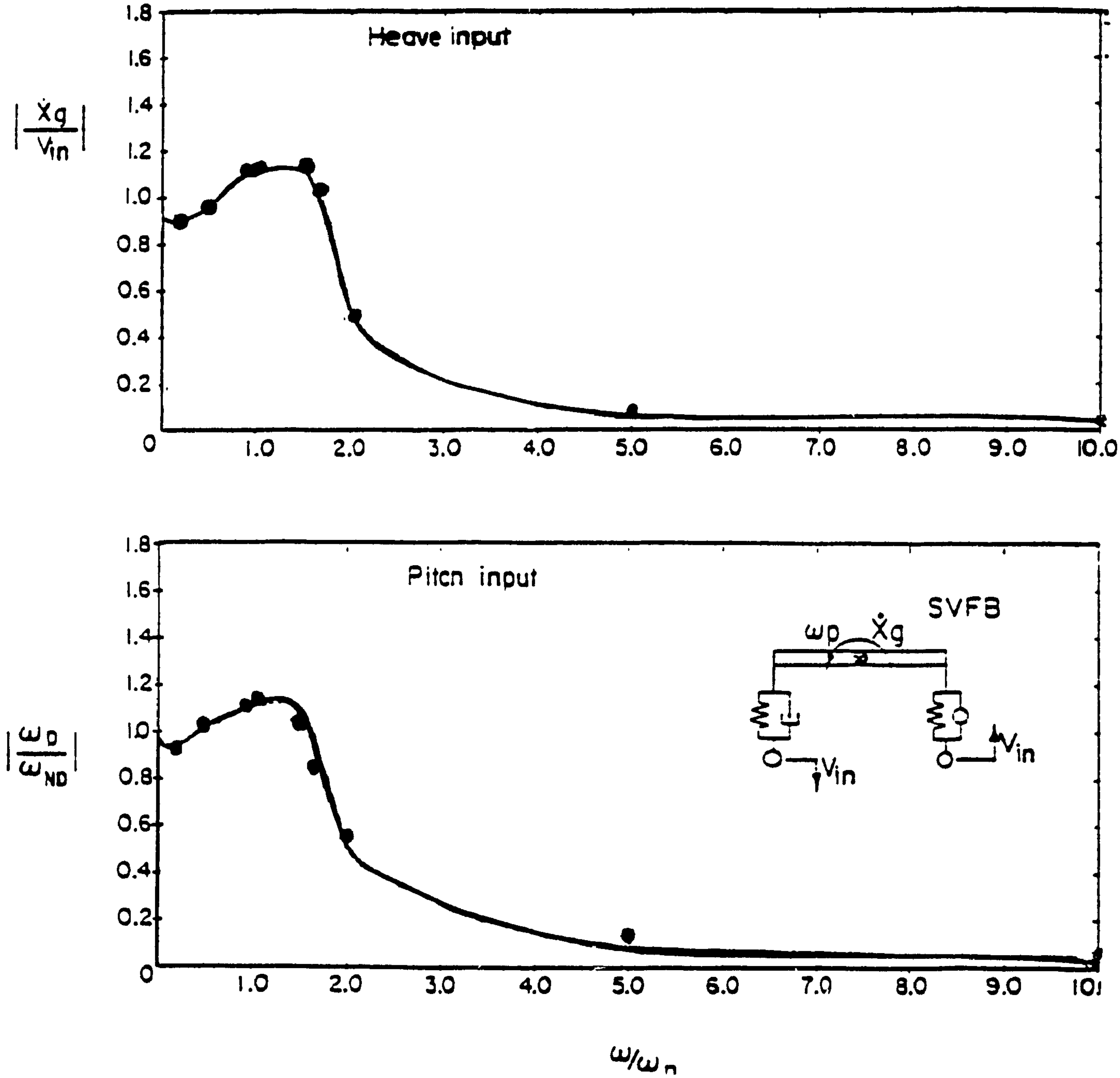


Fig.6.14. Bounce and Pitch Velocity Ratio Versus Frequency of a 2 D.O.F. Pitch Vehicle Model With Combined 'Front SA-1 / Rear Passive' Suspension [18]

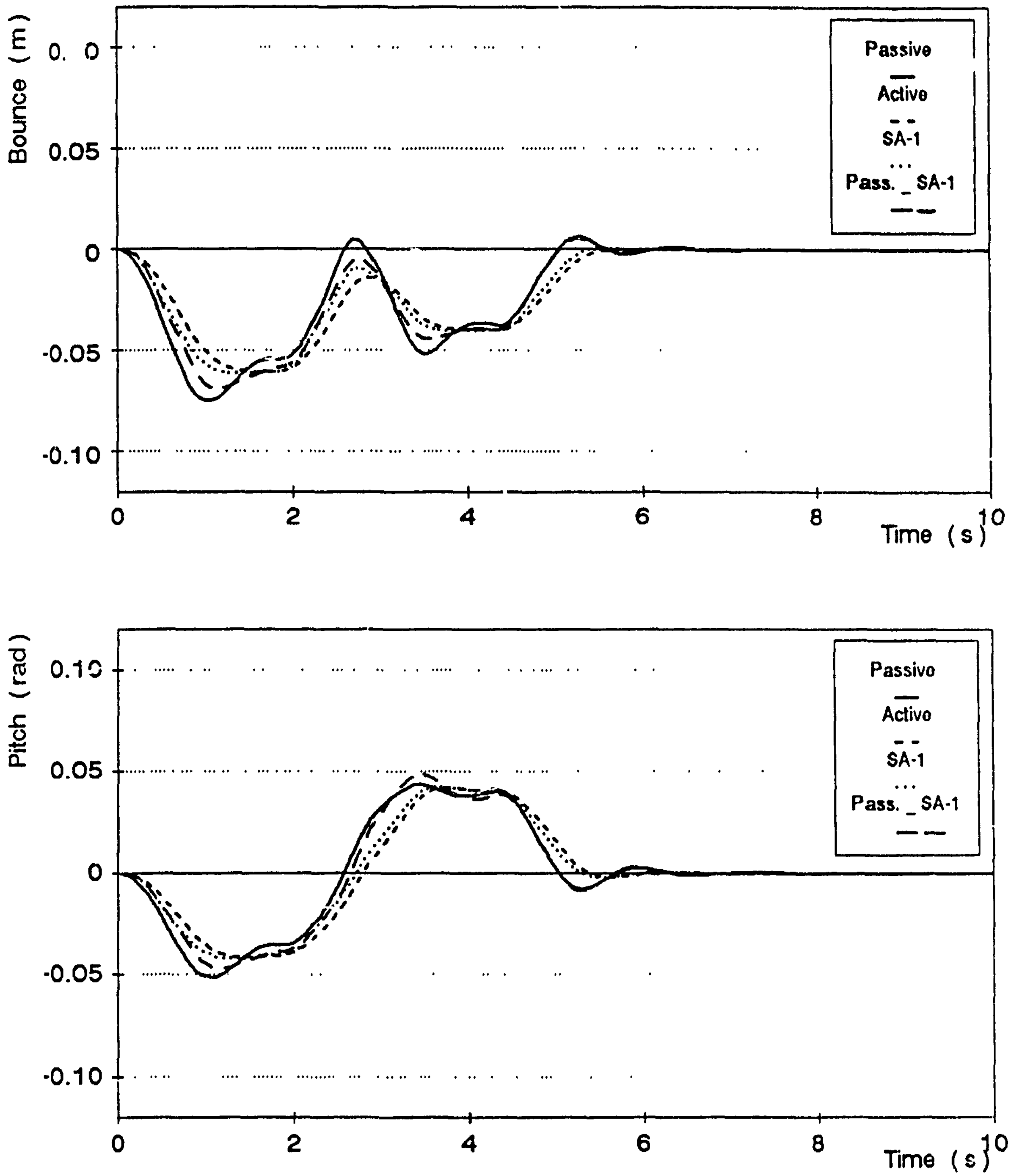


Fig.6.15. Comparison of Bounce and Pitch Responce to a 'Chuck Hole' Type Road Disturbance of the Sprung Mass of Active, Passive, SA-1 and Combined ' Front SA-1 / Rear Passive ' Suspensions

6.4. ' Front SA-2 / Rear Passive ' Suspension

This scheme is very similar to the one described in section 5.2, except for the passive rear suspension. As in the case of SA-2 suspension, the increase of the feedback gain α reduces the RMS bounce and pitch acceleration transmissibility ratios around the first and second mode natural frequencies, yet results in an unfavorable increase of the RMS ratio at lower frequencies as shown in Figs.6.16 and 6.17. At frequencies around and below the bounce and pitch mode natural frequencies, the combined ' Front SA-2 / Rear Passive ' suspension has, in fact, a better RMS bounce acceleration transmissibility ratio than that of a fully SA-2 suspension as can be seen from Fig.6.18. Hence, the less costly less complex combined suspension gives a better performance than that of a fully SA-2 suspension at low frequencies. The major loss of performance occurs, however, at high frequencies, but the combined suspension remains superior to that of a fully passive suspension.

The time response to a 'Chuck Hole' type road disturbance, Fig.6.19,shows that the combined suspension offers, indeed, better bounce and pitch vibration suppression than that of a fully SA-2 Suspension.

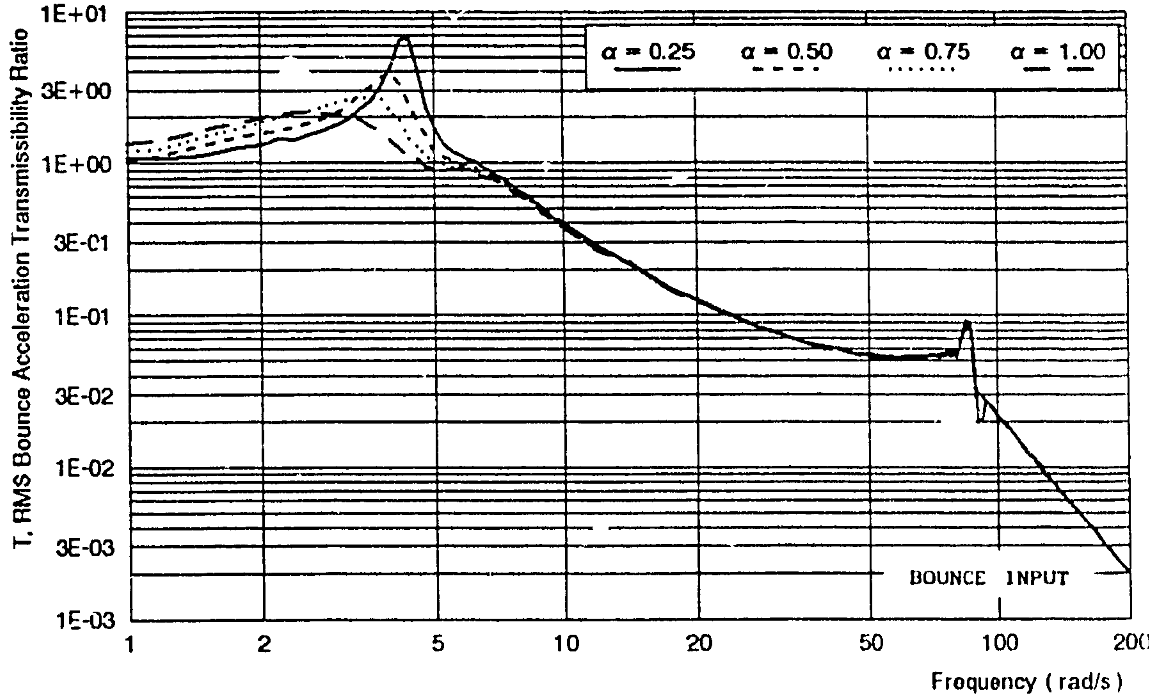


Fig.6.16. RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of a Combined ' Front SA-2 / Rear Passive' Suspension

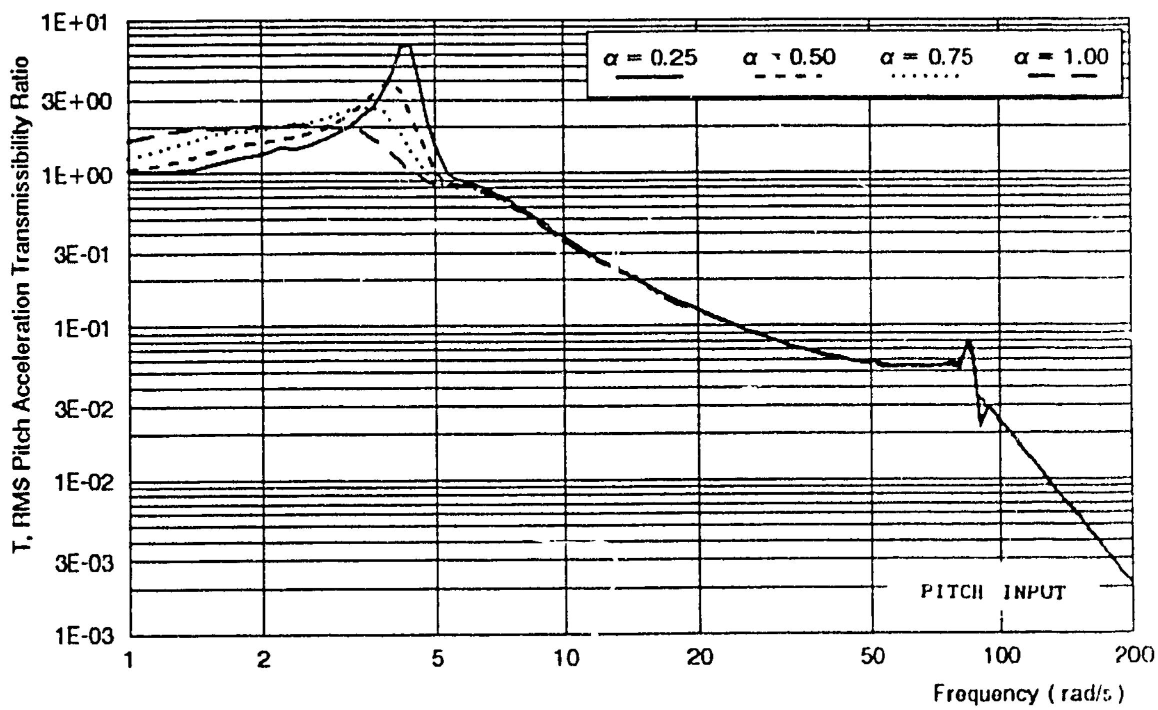


Fig.6.17. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of a Combined ' Front SA-2 / Rear Passive' Suspension

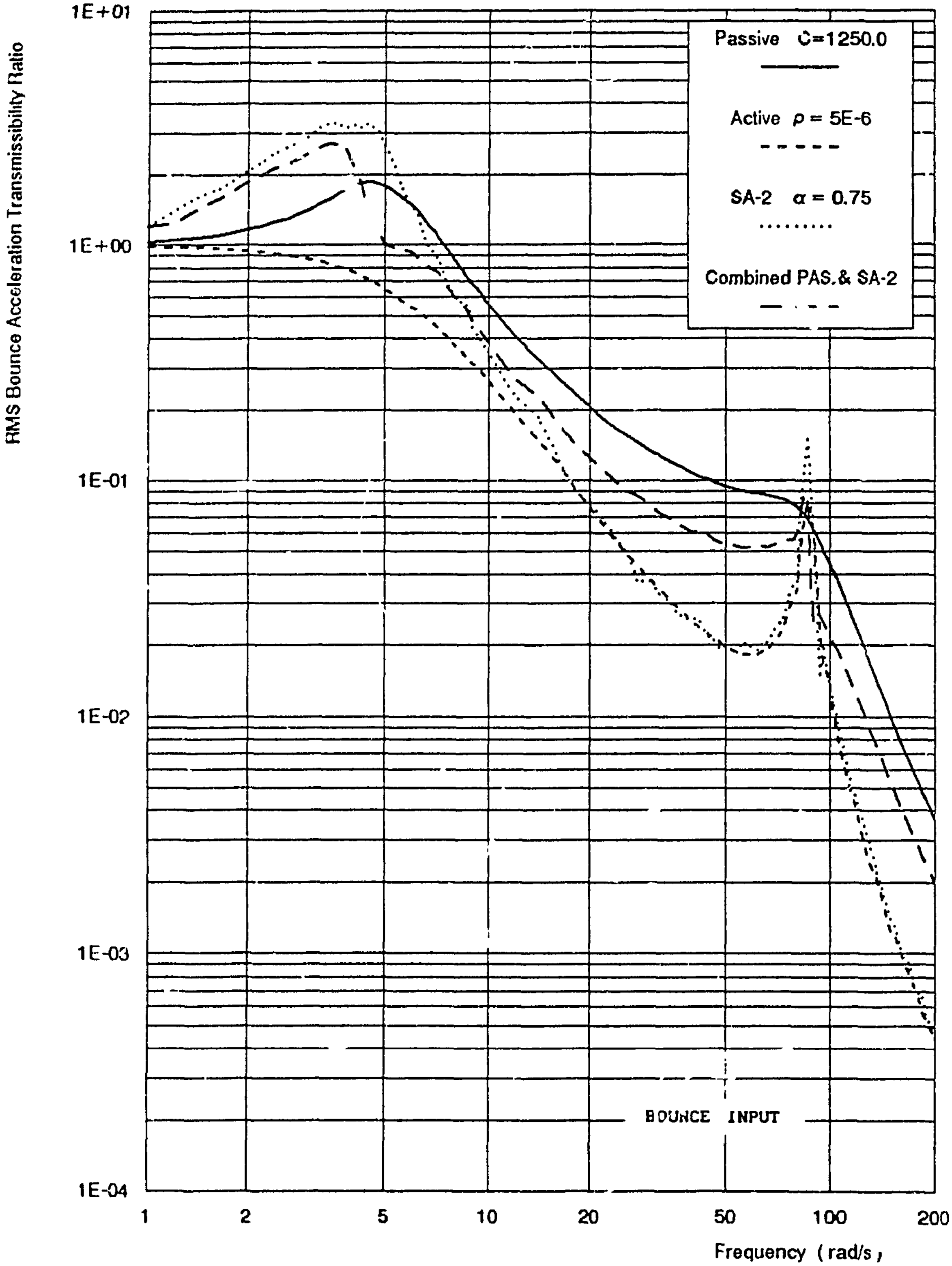


Fig.6.18. Comparison of The RMS Bounce Acceleration Transmissibility Ratios of Passive, Active, SA-2 and Combined 'Front SA-2 / Rear Passive' Suspensions

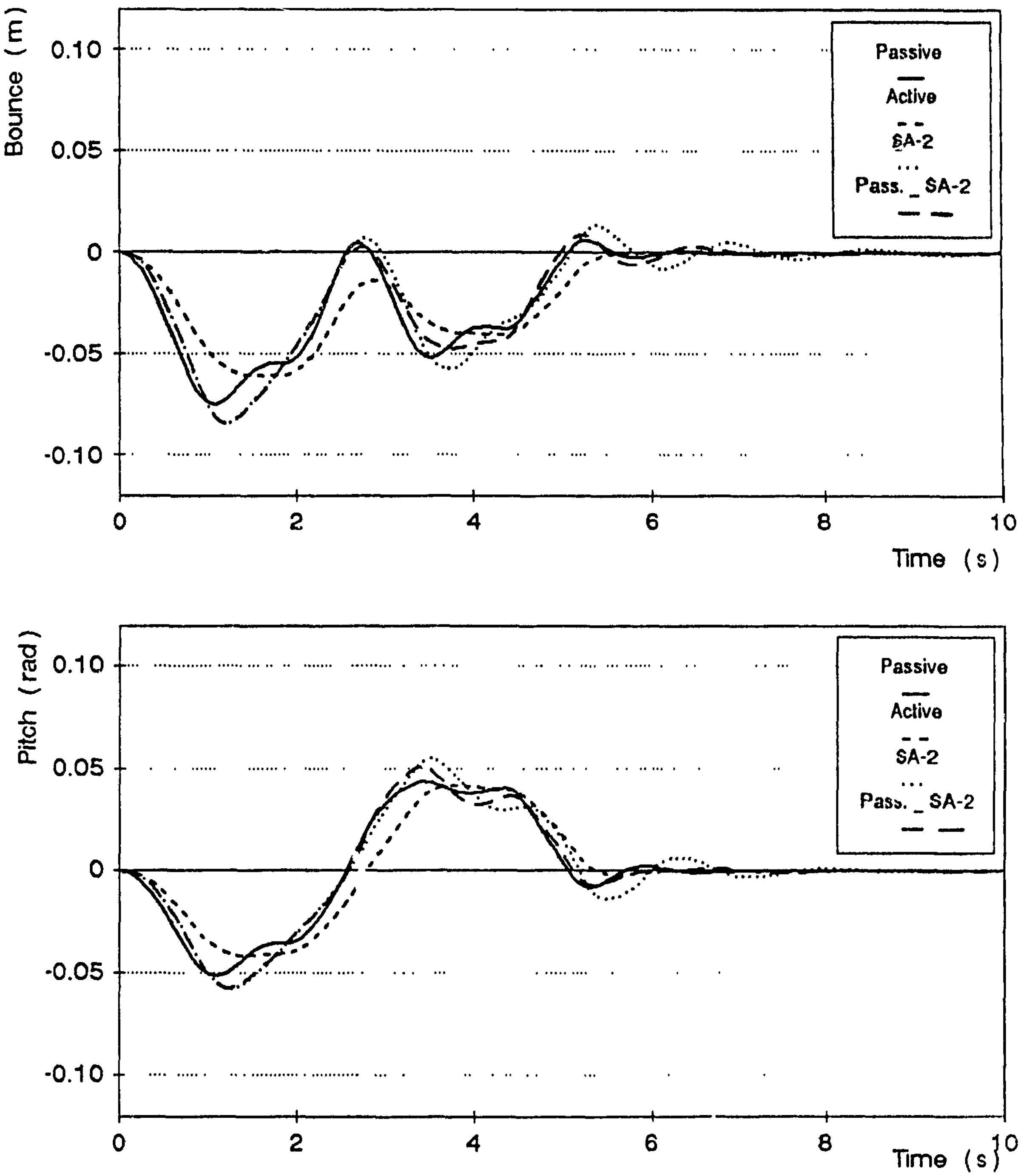


Fig.6.19. Comparison of Bounce and Pitch Responce to a 'Chuck Hole' Type Road Disturbance of the Sprung Mass of Active, Passive, SA-2 and Combined ' Front SA-2 / Rear Passive ' Suspensions

6.5. 'Front SA-3 / Rear Passive ' Suspension

this scheme is very similar to the one described in section 5.3, except for the passive rear suspension.For this type of suspension, an increase of the gain α reduces the RMS bounce and pitch acceleration ratios at frequencies around the first and second mode natural frequencies yet results in an increase of the RMS ratio at lower frequencies. At frequencies higher than 7 rad/s, the increase in the feedback gain α has, virtually, no effect on the RMS bounce and pitch frequency ratios as shown in Figs.6.20. to 6.21.

When compared to a fully SA-3 suspension, Fig.6.22, the combined ' Front SA-3 / Rear Passive ' suspension offers lower RMS bounce acceleration transmissibility ratio at low frequencies. At high frequencies, however, an increase of the latter can be noticed. This is caused by replacing the rear SA-3 suspension with a passive one.

Similarly, the response to a Chuck Hole type Road disturbance, Fig.6.23, is indeed better than that of a fully SA-3 suspension.

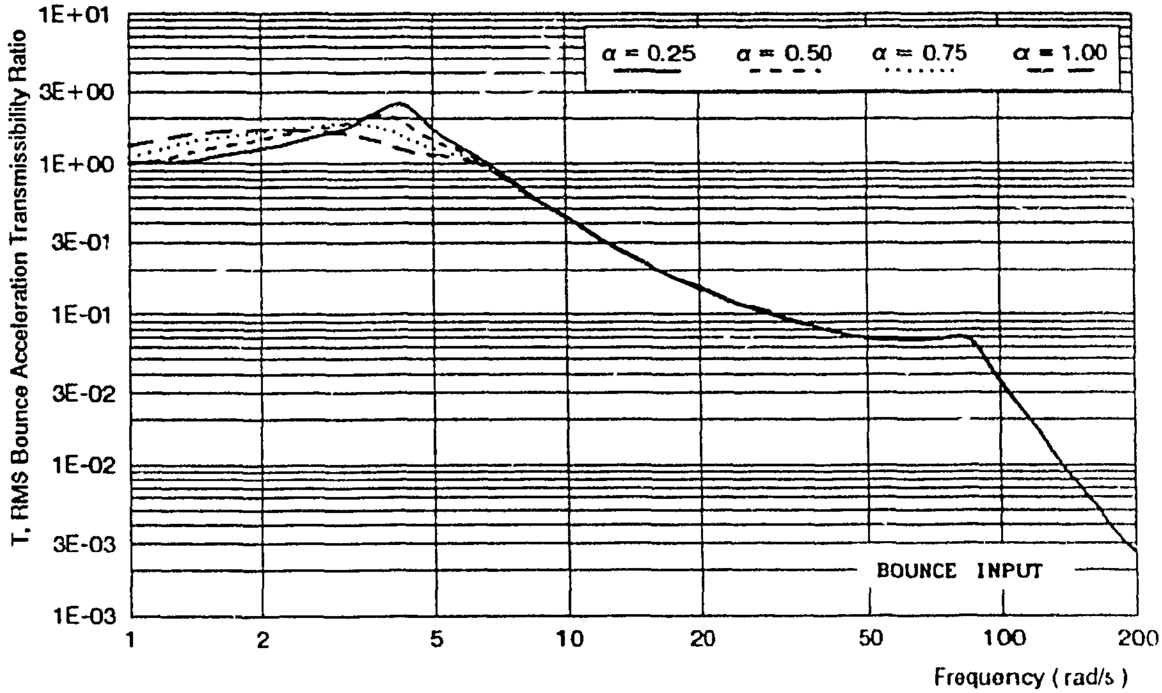


Fig.6.20. RMS Bounce Acceleration Transmissibility Ratio Versus Frequency of a Combined ' Front SA-3 / Rear Passive' Suspension

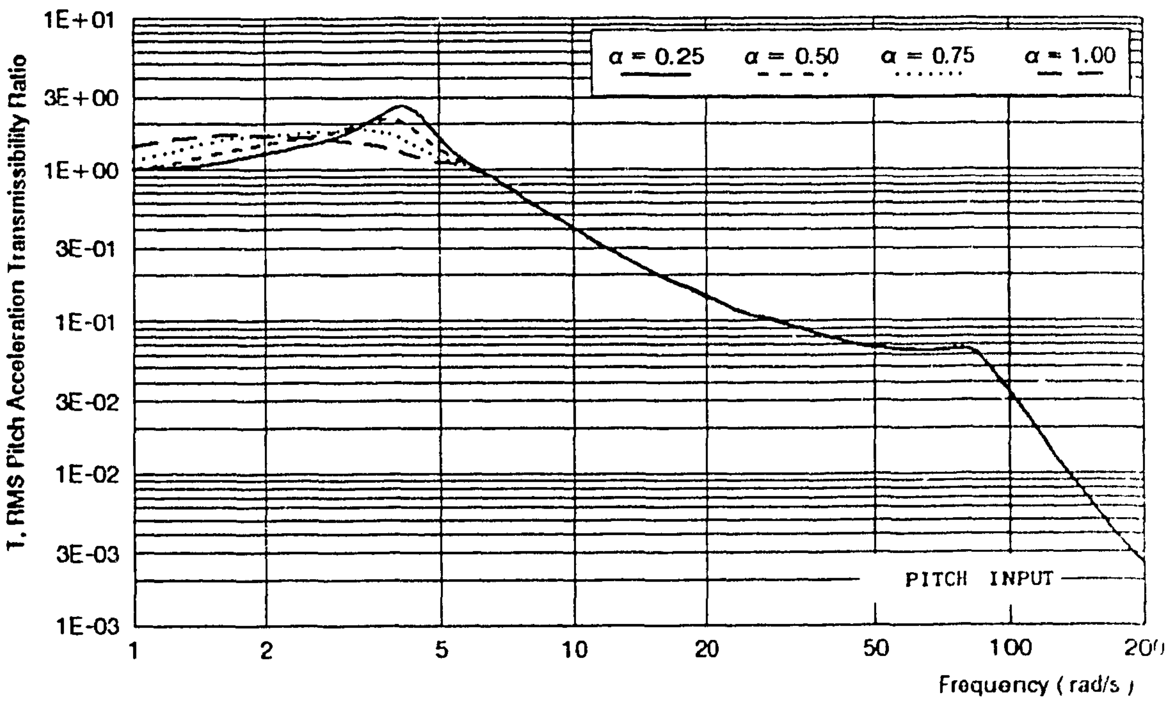


Fig.6.21. RMS Pitch Acceleration Transmissibility Ratio Versus Frequency of a Combined ' Front SA-3 / Rear Passive' Suspension

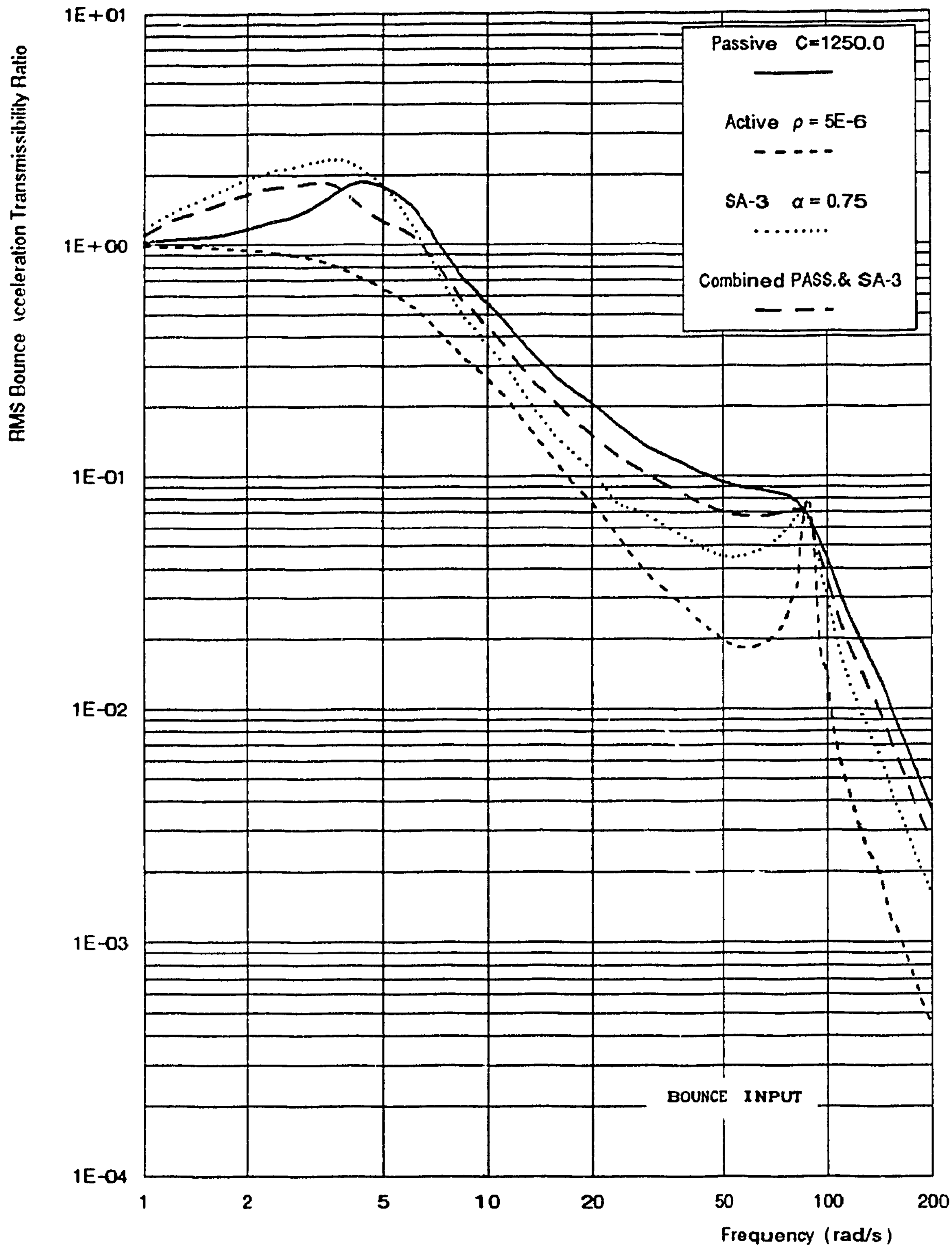
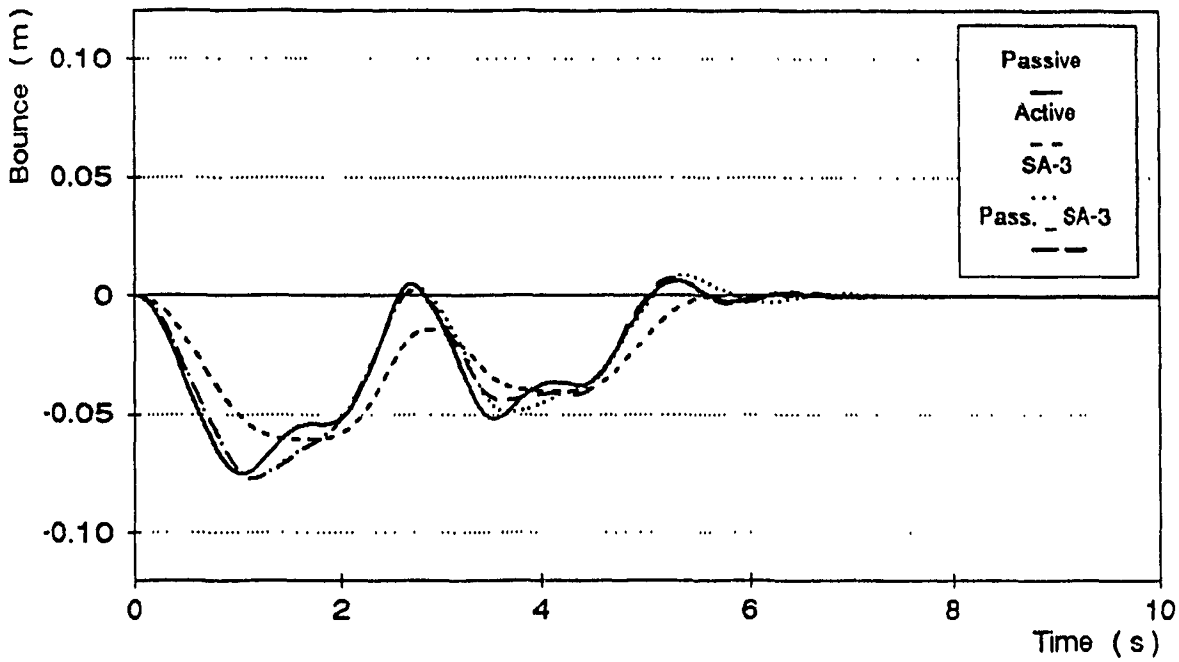


Fig.6.22. Comparison of The RMS Bounce Acceleration Transmissibility Ratios of Passive, Active, SA-3 and Combined 'Front SA-3 / Rear Passive' Suspensions



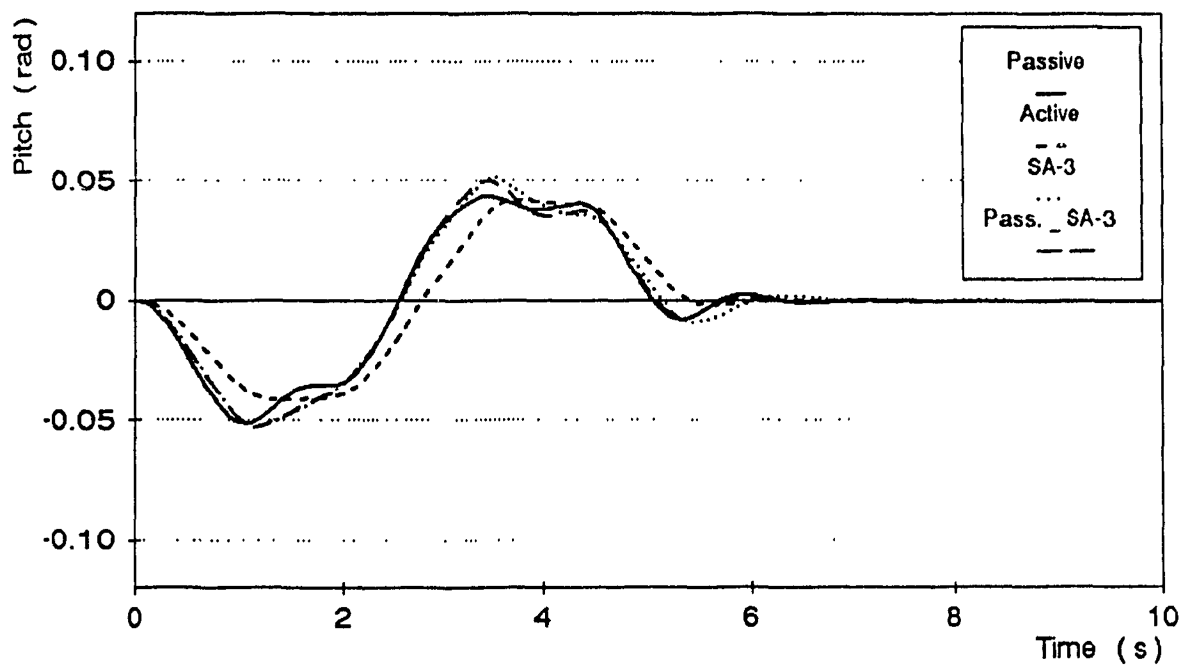


Fig.6.23. Comparison of Bounce and Pitch Responce to a 'Chuck Hole' Type Road Disturbance of the Sprung Mass of Active, Passive, SA-3 and Combined ' Front SA-3 / Rear Passive ' Suspensions

6.6. Summary

The potential of combined suspensions is well apparent in the above study. The performance of combined suspensions lies, generally, between the performance of a fully passive and a fully active (or SA) suspension. While resulting in a slight loss of performance, the potential of combined suspensions is in their reduced cost and complexity. Keeping in mind the importance of low frequency vibration isolation, the combined SA-1 offers an attractive way of achieving such goals. In addition to being simpler and cheaper, the combined SA-2 and SA-3 schemes offer a low frequency response superior even to that of a fully SA-2 and SA-3 suspensions, respectively.

CHAPTER 7

COMPARATIVE STUDY OF ADVANCED SUSPENSIONS

Performance of three Semi-Active (SA) suspension schemes are compared in both frequency and time domains. Their relative merits and drawbacks are discussed with respect to passive and active suspensions. The first semi-active suspension scheme, termed SA-1, is based on the idea of having an SA device that can be modulated to dissipate energy in a fashion identical to an active device. The SA-2 scheme is introduced in an attempt to overcome the increase in the sprung mass acceleration due to the spring and damper forces being in the same direction. During this part of the vibration cycle, the SA damper outputs zero force. However, when the spring and damper forces are in opposite directions the SA device provides a force equal in magnitude but opposite in sign to spring force. Hence, the total net resulting force acting on the sprung mass is zero. Finally, the SA-3 suspension scheme is a modified version of the SA-2 scheme with a small damper placed in parallel with the latter in order to overcome the poor resonance control and to relate to real life applications where zero damping is normally unachievable.

Based on a simple single D.O.F. vehicle model, a basic understanding of various suspension schemes is gained. The RMS bounce acceleration transmissibility ratio of an SA-1 suspension scheme is found to provide a response close to that of an active suspension especially at high frequencies. The SA-2 suspension scheme, on the other hand, approaches active suspension at high frequencies but results in significant deterioration of the resonance control. The SA-3 suspension scheme overcomes this problem by improving the resonance control at the expense of high frequency isolation. The SA dampers lock-up is most apparent through time responses to sinusoidal excitation at different frequencies.

Performance of the SA schemes are, then, investigated in a more realistic in-plane 4 D.O.F. model. Active suspension is shown to provide the ultimate sprung mass bounce and pitch control. Active control, however, requires complex hardware implementation arising from the necessity to measure all of the system states (in the case of full state feedback). In addition, large amount of external power required for the actuators, large rattle space and poor tire ground contact force are some of the drawbacks of active suspension. The SA-1 suspension scheme solves the problem of high external energy requirements of active suspension. This scheme offers a pitch and bounce RMS acceleration performance close to that of active systems especially at high frequencies with slight loss at resonance. However, it remains far more complex than passive systems since it still requires the measurement of all the system states as in the case of active suspension and similarly it requires large rattle space and results in poor tire/ground contact force.

Noting that a damper tends to increase the sprung mass acceleration during part of the vibration cycle, the SA-2 scheme is suggested to overcome this problem. This concept has the advantage of requiring the measurement of easy to obtain relative displacements and relative velocities, therefore solves the major problem associated with the SA-1 scheme. Unfortunately, it results in a very poor sprung mass bounce and pitch resonance control; although at high frequencies, response is close to that of an active suspension. The SA-2 suspension scheme, however, improves the tire/ground contact force and requires a rattle space even larger than that required by an active suspension. This unacceptable poor resonance control is partially solved by adding a small passive damper in parallel with the SA-2 damper resulting in the SA-3 scheme. The latter controls better the sprung mass bounce and pitch resonance performance but results in some deterioration of the RMS transmissibility at high frequencies. This deterioration increases with an increase in the damping coefficient of the additional passive damper.

The SA-3 suspension presents the most attractive advanced suspension scheme among the ones considered. It has the advantage of being easier to implement when compared to an active or SA-1 suspension, requiring only the measurement of relative quantities. Yet, it offers a sprung mass bounce and pitch performance superior to that of a passive suspension and a rattle space, tire/ground contact force comparable to that of passive suspension. Although the 3 SA suspension concepts offer a compromising solution to suspension problems, they remain costly and complex when compared to passive suspensions, especially if they are to be installed in all the 4 wheel locations. This can be overcome by using advanced suspensions, active or SA, in the front axle while the rear suspension is kept passive. The choice of the front suspension for the location of the active or SA suspension arises from the fact that the engine assembly and the driver are generally located in the front section of the vehicle. In addition, the passive suspension, which provides good tire/ground contact force, can be quite effective if placed in the rear section if we assume rear-wheel drive. In general, the performance of the combined suspensions lies between the two extremes, namely fully passive and fully active or SA. However, in the case of the SA-2 and SA-3 suspension schemes, the combined suspension results in an improvement of the sprung mass bounce and pitch resonance control when compared to a fully SA-2 or SA-3 type suspension. The encouraging results obtained from combined suspensions are very significant since they reveal the effectiveness of such method in reducing the complexity and cost of advanced suspensions while maintaining adequate performance.

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

8.1. Conclusions

Three Semi-Active (SA) suspension schemes are compared in both frequency and time domains with respect to passive and active suspensions based on a 1 D.O.F. model. The 1 D.O.F. model gives a very basic perception of the suspensions performance, hence, it is not sufficient to predict the overall behaviour of a suspension in real life applications. Based on such a model, a basic understanding of various suspension schemes is gained. The RMS bounce acceleration transmissibility ratio of an SA-1 suspension scheme is found to provide a response close to that of an active suspension especially at high frequencies. The SA-2 suspension scheme, on the other hand, approaches active suspension at high frequencies but results in significant deterioration of the resonance control. The SA-3 suspension scheme overcomes this problem by improving the resonance control at the expense of high frequency isolation.

Performance of the SA schemes are, then, investigated in a more realistic in-plane 4 D.O.F. model. Active suspension is shown to provide the ultimate sprung mass bounce and pitch control at the expense of high rattle space requirement and poor tire contact force. The SA-1 suspension scheme solves the problem of high external energy requirements of active suspension. This scheme offers a pitch and bounce RMS acceleration performance close to that of active systems especially at high frequencies with slight loss at resonance. similarly it requires large rattle space and results in poor tire/ground contact force.

The SA-2 suspension scheme has the advantage of requiring the measurement of easy to obtain relative displacements and relative velocities. Unfortunately, it results in a very poor sprung mass bounce and pitch resonance control; although at high frequencies, response is close to that of an active suspension. The SA-2 suspension scheme, improves the tire/ground contact force, however, it requires a rattle space even larger than that required by an active suspension. This unacceptable poor resonance control is partially solved by adding a small passive damper in parallel with the SA-2 damper resulting in the SA-3 scheme. The latter controls better the sprung mass bounce and pitch resonance performance but results in some deterioration of the RMS transmissibility at high frequencies.

In general, the performance of the combined suspensions lies between the two extremes, namely fully passive and fully active or SA. However, in the case of the SA-2 and SA-3 suspension schemes, the combined suspension results in an improvement of the sprung mass bounce and pitch resonance control when compared to a fully SA-2 or SA-3 type suspension.

8.2. Recommendations for Future Work

As recommendations for future work, it is important to consider the following:

1. The dynamics of the hardware parts (actuators, SA dampers, signal processors, and the like.,) were not included in this study. It is suggested that the dynamics associated with hardware parts be included in future work to gain an insight into their effect on suspension performance.

2. The concept of preview control, a control scheme in which an input is sensed before it reaches the wheel, can be of great use in the case of combined suspensions. In such a case, the actuator or the SA device are placed in the rear axle. The input to the front passive suspension is used to prepare the actuator or SA device for the incoming road irregularities. This will result in an improved performance by partially overcoming the problems associated with bandwidth limitation of the hardware parts.

3. This study is limited to sinusoidal and discrete obstacle type of road inputs. The investigation should be extended to study random terrain inputs to broaden the perspective of these advanced suspensions.

4. Finally, an experimental prototype set up could be built to gain a better insight into the effectiveness and complications of implementation of advanced suspensions.

REFERENCES

1. Karnopp D.C., *"Are Active Suspensions Really Necessary?"*, ASME Paper Presented at the Winter Annual Meeting, San Francisco, Calif., December 10-15, 1978.

2. Goodall R.M. and Kortum W., *"Active Control in Ground Transportation - A Review of the State of the Art and Future Potential"*, Vehicle System Dynamics, Vol. 12, 1983, pp. 225-227.

3. Sharp R.S. and Crolla D.A., *"Road Vehicle System Design - A Review"*, Vehicle System Dynamics, Vol. 16, 1987, pp. 167-192.

4. Karnopp D., Crosby M.J. and Harwood R.A., *"Vibration Control Using Semi-Active Force Generators"*, Journal of Engineering for Industry, Trans. ASME, Vol. 96, May 1974, pp. 619-626.

5. Margolis Donald, *"The Response of Active and Semi-Active Suspensions to Realistic Feedback Signals"*, Vehicle System Dynamics, Vol. 11, 1982, pp. 267-282.

6. Hrovat D. and Hubbard M., *"Optimum Vehicle Suspensions Minimizing RMS Rattlespace, Sprung-Mass Acceleration and Jerk"*, Journal of Dynamic Systems, Measurement and Control, Trans. ASME, Vol. 103, Sept. 1981, pp. 228-236.

7. Karnopp D., *"Active Damping in Road Vehicle Suspension Systems"*, Vehicle System Dynamics, Vol. 12, 1983, pp. 291-316.

8. Sutton H.B., *"The Potential for Active Suspension Systems"*, Auto.Engr.(UK), Apr/May 1979, pp. 21-24.

9. Sutton H.B., *"Synthesis and Development of an Experimental Active Suspension"*, Auto.Engr.(UK), Oct/Nov 1979, pp. 51-54.

10. Thompson A. G. and Pearce C. E. M., *"An Optimum Suspension for an Automobile on a Random Road"*, SAE Trans, Paper 790478 (1979).

11. ElRazaz and ElMadany M. M., *"Eigenvalue Sensitivities as an Effective Tool for Active Automotive Suspension Design"*, Diagnostic Vehicle Dynamics and Special Topics, DE-Vol. 18-5.

12. Louam N., Wikson D. A. and Sharp R. S., "*Optimal Control of a Vehicle Suspension Incorporating the Time Delay Between Front and Rear Wheel Inputs*", Vehicle System Dynamic, Vol 17, 1988, pp. 317-336.

13. Malek K.M. and Hedrick J.K., "*IAVSD Extensive Summaries: Decoupled Active Suspension Design for Improved Automotive Ride Quality/Handling Performance*", Vehicle System Dynamics, Vol. 14, 1985, pp. 78-81.

14. Fruhauf F., Kasper R. and Luckel J., "*IAVSD Extensive Summaries: Design of an Active Suspension for a Passenger Vehicle Model Using Input Processes With Time Delays*", Vehicle System Dynamics, Vol. 14, 1985, pp. 115-120.

15. Barek F. and Sachs H.K., "*IAVSD Extensive Summaries: On the Optimal Ride Control of a Dynamic Model for Automotive Vehicle System*", Vehicle System Dynamics, Vol. 14, 1985, pp. 196-200.

16. Abdelhady M. B. A. and Crolla D. A., "*Theoretical Analysis of Active Suspension Performance Using a Four Wheel Model*", Journal of Automobile Engineering, Vol. 203, 1989, pp. 125-135

17. Karnopp D. and Margolis D., "*Adaptive Suspension Concepts for Road Vehicles*", Vehicle System Dynamics, Vol. 113, 1984, pp. 145-160.

18. Margolis D., "*Semi-Active Heave and Pitch Control for Ground Vehicles*", Vehicle System Dynamics, Vol. 11, 1982, pp. 31-42.

19. Hrovat D., Barak P. and Rabins M., "*Semi-Active Versus Passive or Active Tuned Mass Dampers for Structural Control*", Journal of Engineering Mechanics, Vol. 109, No.3, 1983, pp. 691-705.

20. Sharp R. S. and Hassen S. A., "*The Fundamentals of Passive Automotive Suspension System Design*", Society of Environmental Engineers Conference on Dynamics in Automotive Engineering, 1984, pp. 104-115.

21. Bender E.K., Karnopp D. and Paul I.L., "*On the Optimization Of Vehicle Suspensions Using Random Process Theory*", ASME Publication 67, Vol. 12, 1967.

22. Bender E.K., "*Optimum Linear Random Vibration Isolation*", Reprints JACC, 1967, pp. 135-143.

23. Sarma G.N. and Kozin G., "*An Active Suspension System Design for a High-Speed Wheel-Rail System*", Journal of Dynamic Systems, Measurement and Control, Trans. ASME, Vol. 93, 1971, pp. 233-241.

24. Hullender D.A., Wormley D.N. and Richardson H.H., "*Active Control of Vehicle Air-Cushion Suspensions*", Journal of Dynamic Systems, Measurement and Control, Trans. ASME, Vol. 94, 1972, pp. 41-49.

25. Sinha O.K., Wormley D.N. and Hedrick J.K., "*Rail Passenger Vehicle Lateral Dynamic Performance Improvement Through Active Control*". Journal of Dynamic Systems, Measurement and Control, Trans. ASME, Vol. 100, Sept. 1978, pp. 270-283.

26. Hedrick J.K., "*Railway Vehicle Active Suspensions*", Vehicle System Dynamics, Vol. 10, 1981, pp. 267-283.

27. Kresnicki E.J., "*Comparison of Analytical and Experimental Results for a Semi-Active Isolator*", Proceedings of the 50th Shock and Vibration Symposium, Boulder Co., Oct. 1979.

28. Kresnicki E.J., "*The Experimental Performance of an ON-OFF Active Damper*", Proceedings of the 51st Shock and Vibration Symposium, San Diego, Oct. 1980.

29. Margolis D.L., Tylee J.L. and Hrovat D., "*Heave Mode Dynamics of a Tracked Air Cushion Vehicle With Semi active Airbag Secondary Suspension*", Journal of Dynamic Systems, Measurement and Control, Trans. ASME, 1975, pp. 399-407.

30. James Alanoly and Seshadri Sankar, "*A new Concept in Semi-Active Vibration Isolation*" , ASME paper No. 86-DET-28.

31. S. Rakheja and S. Sankar, "*Vibration and Shock Isolation Performance of a Semi-Active On-Off Damper*", ASME paper No. 85-DET-15, Presented at the ASME Design Engineering Technical Conference, Cincinnati, OH. Semtember 10-13, 1985.

32. John Van de Vegte, "*Feedback Control Systems*", Prentice-Hall,Inc., Englewood Cliffs,NJ. 1986.

33. Chalasani R. M., "*Ride Potential of Active Suspension Systems-Part I: Simplified Analysis Based on a Quarter-Car Model*", ASME Symposium on Simulation and Control of Ground Vehicles, AMD-Vol. 80, DSC Vol 2, 1986, pp. 187-204.

34. Chalasani R. M., "*Ride Potential of Active Suspension Systems-Part II: Comprehensive Analysis Based on a Full-Car Model*", ASME Symposium on Simulation and Control of Ground Vehicles, AMD-Vol. 80, DSC Vol 2, 1986, pp. 205-234.

35. Leon Lapidus and John H. Seinfeld, "*Numeriacl Solution of Ordinary Differential Equations*", Mathematics in Science and Engineering, Vol. 74, Academic Press, New York and London, 1971.

36. Redfield R. C. and Karnopp D. C., "*Performance Sensitivity of an Active Damped Vehicle Suspension to Feedback Variation*", Journal of Dynamic Systems, Measurement and Control, Trans. ASME, 1989, pp. 51-59.

37. Rajnikant V. Patel and Neil Munro, "*Multivariable System Theory and Design*", International Series on Systems and Control, Volume 4. 1st ed. Oxford,Eng. New York: Pergamon Press, 1982.