A STUDY OF ECONOMIC MACHINING AND OPTIMIZATION OF MACHINABILITY PARAMETERS

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ABSTRACT

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The problem of economic machining has been recognized as one of the decisive factors affecting the selection of tools, tool materials, cutting speeds, depths of cut and a number of other parameters. Furthermore, the problem has been attacked from varying points of view with varying degrees of success.

Two such methods are presented here. The first recognizes the large number of variables that must be handled and consequently the loss of generality that must be tolerated. However, it is faithful in its depiction of the economics of the machining operation.

The second considers maximum profit as the criterion for the selection of the optimum machining conditions rather than the conventional criteria of minimum cost or maximum production rate where the marginal revenue equals the marginal cost.

Finally, a technique is presented for the optimization of the mathematical model obtained from the above methods. The particular technique stated here is geometric programming; however, it is conceded that there are others which have proved to be equally successful.

ACKNOWLEDGEMENTS

The author wishes to thank his supervisor, Dr. M. O. M. Osman, for his encouragement and guidance throughout all stages of this report.

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NOMENCLATURE

a,b	:	indexes for feed and depth of cut in tool life equation
C	:	constant in tool life equation
C_c	:	cutting cost/piece
C_i	:	idle cost/piece
C_m	:	marginal cost
C_{min}	:	minimum unit cost, $/piece
C_o	:	cost of operating time, $/minute
C_p	:	total cost/piece
C_{pf}	:	premature failure cost/piece
C_T	:	total cost, $/minute
c_t	:	cost of tool, $/cutting edge
C_{tc}	:	tool changing cost/piece
C_{td}	:	tool depreciation cost/piece
C_{tg}	:	tool regrinding cost/piece
D	:	diameter of workpiece
d	:	depth of cut
f	:	feed rate
f_a	:	highest allowable feed, ipr
k	:	constant in extension of Taylor's equation
L	:	length of workpiece to be machined
N	:	number of tools

n	:	index of T in tool life equation
m	:	constant exponent in extension of Taylor's equation
P	:	selling price, \$/piece
Q	:	production rate, pieces/minute
q	:	index of W in premature failure equation
R_1	:	general labour and overhead costs/minute
R_2	:	constant regrinding costs
R_3	:	cost of grinding 1 inch off clearance face of tool
R_M	:	marginal revenue, \$/minute
R_T	:	tool revenue, \$/minute
R_u	:	unit revenue, \$/piece
t_c	:	tool changing time
t_f	:	final cost of tool
t_i	:	idle time per piece
t_o	:	initial cost of tool
t_e	:	$t_c + \frac{c_t}{c_o}$, min/edge
t_h	:	handling time, min/piece
t_m	:	machining time/piece
T	:	tool life, min/cutting edge
T_o	:	specific tool life for an amount of wear W
T_u	:	total time to produce a workpiece, min/piece
V	:	cutting speed, sfpm
V_{min}	:	cutting speed for min. unit cost, sfpm

V_p : cutting speed for maximum profit, sfpm

V_{max} : cutting speed for maximum production rate, sfpm

W : amount of flank wear

α : a/n

β : b/n

CHAPTER I

PURPOSE AND SCOPE OF INVESTIGATION

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PURPOSE AND SCOPE OF INVESTIGATION

1.1 Introduction

In machining any component it is first necessary to satisfy previously prescribed quality specifications such as surface finish, accuracy and surface integrity. When machining a part within the quality specifications, there usually exists a wide variety of speeds, feeds, tool materials, and other machining conditions which can be used for machining the component on a given machine. The objective of the tool engineer is then to select a set of machining parameters which first satisfy the quality specifications, and second, provide either minimum cost per piece or maximum production rate, or some combination of both.

However, the problem of economic machining is not so simple; complexities arise when faster speeds and higher feeds are required to reduce machining times and hence reduce costs; the implementation of either leads to a shorter tool life, which increases costs. It is supposed that there is some optimum set of conditions which will lead to minimum cost. The problem of economic cutting conditions has been previously investigated with varying degrees of generality and usually from rather different viewpoints.

The advent of the electronic computer has furnished another tool for the determination of optimum machining conditions and it is widely used. The main advantages are the ease with which it can handle a great range of data, and minimizing the possibility of human error in reaching a solution.

However, the fact remains that no optimization of machinability parameters is possible unless the principles of metal cutting are thoroughly understood and exploited. The first steps in this direction were taken in 1906 by Taylor when he published "The Art of Cutting Metals." He was the first to investigate the tool life and cutting speed relationship, which is still of prime importance in any discussion of machining of metals. Having said this, we are ready to proceed further and look into the interaction of tool life and cutting speed and how they affect other parameters involved.

1.2 Survey of factors affecting economic machining

There is a multitude of variables which influence total machining cost per part, such as the price of the work material, value of machine, wage rate of the operator, number of machines per operator, number of shifts and hours worked per year, tool cost and usage, production rate, tolerances and finishes required on the end product, etc. Very basically, these can all be tied to two things, feed and speed, as these two factors determine the rate at which you utilize an economic input of men, material, tooling, and equipment. "Feed" may be herein defined as the rate at which the cutting tool is pushed into the workpiece, usually expressed in inches per revolution (IPR) or inches per minute (IPM). "Speed" is defined as the differential surface velocity between the cutting tool and workpiece, regardless if either or both may be moving, expressed in surface feet per minute (SFPM).

The crux of the matter of machining economics lies in the fact that the faster you run an operation, the more efficiently you utilize the value of the machine and operator, but the more quickly you expend perishable cutting tools. Conversely, you can obtain "excellent" tool life by slowing down, but you may not then be utilizing the value of the machine and operator. Theoretically, at least, there exists a cycle for any machining operation where the "right" feeds and speeds will allow all variables acting together to produce some best economic operating position within, of course, the limitations of end product quality and reliability. This is called the "optimum" position. Generally, and in this discussion, the optimum is defined as the point where the total machining cost per part is a minimum. Some circumstances, such as war or other emergency, may demand optimum cycles set to give maximum production rate, although cycles set to maximize production are not usually the same as cycles set to minimize unit cost.

It will be noted that what is stated here is really nothing new. F. W. Taylor advanced the idea of a "twenty-minute tool life," with similar economic thinking for single point turning, back around the turn of the century. How is it then, in the age of the spaceman, that we find entire plants of machining operations with feeds and speeds set to give "good" tool life, the length of which is proudly exhibited as being a full work shift, or in some cases an entire week of shifts? It is conceded in some cases that existing feeds and speed could be optimum.

Part of the problem is that the extension from single point, "laboratory" machining economics to multi-station, multi-tool, production machining economics may be presented as a mathematically formidable one. This contributes to a second difficulty, that of communicating with practical operating personnel responsible for setting feeds and speeds and keeping machines running to "get out production." Management could make a contribution by providing appropriate accounting and performance records for engineering cost information. The common cost control system, coupled with the usual difficulties of maintaining a supply of correctly-sharpened tools, leads operating personnel to conclude that extra long perishable tool life, or perhaps buying on price alone, is the biggest contribution that can be made to reduction of total unit cost. Last, but certainly not least, there may be some lack of understanding about what physically happens in the cutting operation when feeds and speeds are changed. The net systems effect of cost versus performance features of all machining variables upon final total unit cost and product quality are prime considerations.

It is with the above thinking in mind that this work covers three areas:

1. The general effects of changing feed and speed upon physical parameters directly affecting economical and quality production.
2. A simplified derivation of the cost expression and method of application in conjunction with the Taylor plot.
3. An optimization technique to obtain the cost optimum conditions.

CHAPTER II

ON TOOL LIFE AND CUTTING SPEED

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ON TOOL LIFE AND CUTTING SPEED

2.1 Taylor equation

The work of F. W. Taylor has come to be embodied in the well-known tool life-cutting speed equation as follows:

$$T^n . V = c$$

Taylor carried out investigations to determine an optimum tool life that could be used as a standard. If the tool life is too short the tool must be reconditioned too often; on the other hand, when tool life is too long, the cutting speed is apparently too low, causing losses due to high machining time. The ratio of tool life between grinds and time for regrinding the tool should, according to Taylor's findings, be between 7 and 35. That is, the tool life should be at least 7 times the grinding time, but not more than 35 times. It must be mentioned here that grinding of Taylor's round-nosed tools took more time than that of cutting tools with straight edges.

The American Society of Mechanical Engineers recommends (Ref. 5, 19) distinction between tool life for lowest cost and tool life for largest production. As such, both will be analysed. Figure 2.1 indicates the drop and rise in tool life as caused by an increase and decrease in cutting speed. It will be seen that the exponent n has a dominating effect on the tool life-cutting speed relationship. An example is indicated by point A on the y = 0.5 line, showing that the cutting speed must be reduced by 50% when it is desired to increase tool life by 300%.

The changes will, however, be more drastic when the exponent n is smaller than 0.5, which is true in most practical cases. It will be observed that the exponent n is sometimes as small as 0.08 and often only 0.15 for high-speed steel tools and 0.3 for carbide tools. It will also be seen from Figure 2.1 that the greater the exponent n, the less the effect of a drop in cutting speed on tool life. This holds also for the converse case: the larger the exponent n, the less damaging is an increase in cutting speed for tool life.

Changes in tool life also play an important role in vibration in metal cutting, where the cutting speed varies periodically between a maximum and a minimum. Since tool life changes to a considerably greater extent than cutting speed, it is evident that even a relatively small variation in speed will cause a large variation in tool life, often exceeding the limits and causing premature breakdown of the cutting tool.

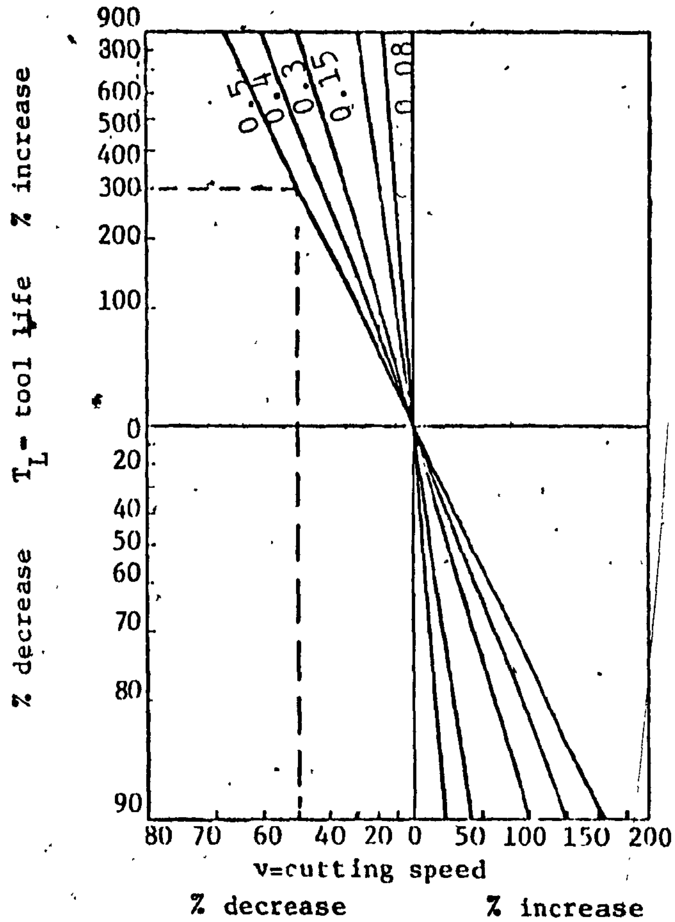

Figure 2.1. Per cent tool life change with cutting speed for tool life exponents n from 0 to 0.5.
(Ref.19)

2.2 Tool life and wear criterion

In a discussion of tool life it becomes imperative to define the wear criterion and what constitutes a tool failure or, in other words, the end of tool life. Before the advent of carbide tools it was considerably easier to determine tool life because the failure of the tool could readily be seen from the appearance of a burnished ring on the workpiece when the tool was dull. Scientifically, however, the "Schlesinger Criterion" was used in the case of high-speed steel tools. A burnished ring usually cannot be observed on a workpiece when machined with carbide tools, and hence this method is useless nowadays. The wear of carbide tools progresses, particularly at the tool flank, in a step-like fashion, as illustrated in Figure 2.2. For a certain number of cuts the wear land does not change, but increases suddenly, followed by little wear increase until another wear step occurs. This cascading effect repeats periodically until the wear land has reached a dimension w which is considered to be acceptable as a maximum.

Hence, a tool life depends on an assumed maximum of the wear of the tool and is therefore a matter of agreement and standardization. This standardization, however, is not as easy as it [illegible] appear due to the difficulties associated with measuring the wear.

It is usual practice to measure the wear land on the flank of the carbide tool (Figure 2.3) although the measuring methods need improvements. The wear land is not of uniform width but rather

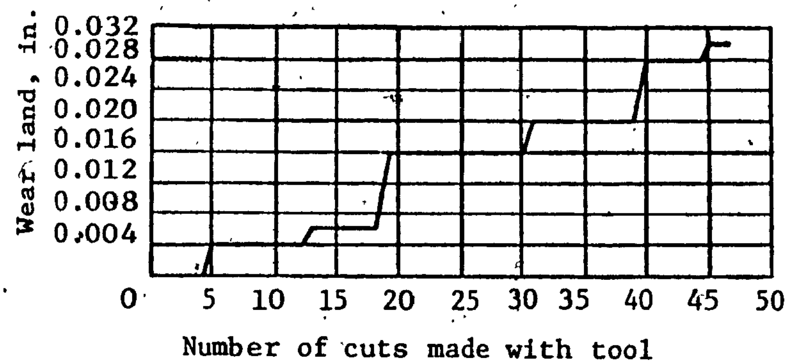

Figure 2.2. Wear progress on tool flank in steps. (Ref.19)

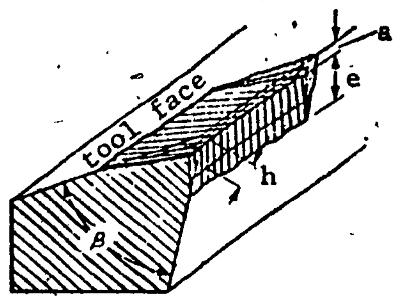

Figure 2.3. Wear land on tool flank e and displacement of cutting edge a. (Ref.19)

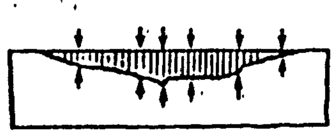

Figure 2.4. Erratic width of wear land. (Ref.19)

erratic, as sketched in Figure 2.4. Thus, the question arises as to whether the maximum of the mean of the wear land should be taken as the basis for tool life. Cratering at the tool face complicates the situation considerably, particularly when it is necessary to remove the tool from the machine for such wear measurement.

Progress has been made in measuring tool wear by various researchers. Radioactive tools have been used and the wear particles that travel with the chip or the cutting fluid have been counted with a Geiger counter.

The radioactive method gives good results, but it is difficult to use and expensive. Generally, five actions affect tool wear: 1. plastic deformation; 2. mechanical wear; 3. cracking and flaking due to pulsation of the cutting force exceeding the fatigue limit of the tool; 4. thermal shock; 5. removal of particles associated with the generation and breakdown of the built up edge.

In American practice, tool life data are usually based on a wear land of 0.03 in. at the flank of the tool in the case of carbide tools and of 0.06 in. for high-speed tools, whereby the maximum width of wear land is taken, which is easier to measure than average.

2.2 Machining variables

There are several variables involved in machining, and quite often several of these variables are ignored, but they should be considered if applicable machining data are to be derived and used.

The rate at which material can be machined is affected by 1) material to be cut, 2) cutting tool used, 3) speed, feed, and depth of cut, 4) size and type of machine, 5) power available.

These five general variables are further affected by

1) workpiece material type and physical characteristics
2) variations in cutting tool shape and geometry
3) cutting tool type and material
4) variations in feed and depth of cut from light to heavy and whether cut dry or with the aid of lubricant and/or co
5) variations in speed from the slowest to the fastest, depending on the machine used
6) rigidity of machine, cutting tool, and workpiece
7) specific requirements of cutting speed, tool life, surface finish, horsepower, residual stress, and heat effects.

The term machinability does not lend itself to a precise definition. In the context in which most authorities use "machinability," however, it can be described as the "ease" with which a given material can be machined in relation to a set of superimposed machining variables and the resulting cost of the operation and unit cost.

Furthermore, for a given set of machining conditions, the ease of machining varies with the workpiece material variables or those quantities that specify the properties of the workpiece material. Namely,

1) hardness

2) tensile property

3) chemical composition

4) microstructure

5) degree of coldwork

6) strain hardenability

7) configuration and dimensions of the workpiece, and

8) rigidity of the workpiece

2.4 Tool selection

In most manufacturing concerns, time is the costliest ingredient. To the cost per hour of direct labour must be added capitalized cost per hour for the machine, cost per hour for indirect labour, overhead, and other charges. The total can be a very impressive figure. If the time per piece can be reduced without excessive tool cost, all the charges are reduced in proportion. Quite often it pays to accept shorter tool life, regarding tools as expendable, in order to reduce the other charges. Time for tool change and cost of regrinding, where done, must be considered, but it is possible to balance these against speed to obtain remarkable economies.

Cutting speed is not, however, the whole consideration. By taking extremely light cuts and feeds, very high speeds with prolonged tool life can be achieved. What is needed is a high removal rate. An ideal tool material would remove the same amount of stock per cutting edge at any speed.

Magazine loaded tool holders for changing cutting edges quickly and automatic devices for loading and unloading workpieces faster will be required to take more advantage of the speed and economic resources of cemented oxides. Also, more powerful machines are needed. Machine tools built for conventional tooling cannot realize the potentials of oxides.

High speed machining is only a relative term; in cutting fully hardened (Rockwell C 60 to 65) alloy steels, it would perhaps mean an optimum speed of 600 to 700 sfpm. This is certainly within the range of machine tools used in average shops. These lower spindle speeds are often adequate to obtain minimum cost cutting speeds for cemented oxides also and would call for very little horsepower in making finishing cuts on hard alloys. Again, there is ample horsepower available on average equipment. Thus it is not only possible to use cemented oxides at low speeds, but on some materials now being machined in ever-growing volume, lower speeds are the most efficient and economical.

Cemented oxide tooling is fully reliable at low speeds. Figure 2.5 shows what happens to oxides and carbides at very low speeds. As the speed decreases below 50 sfpm, the three tool force components on both carbides and oxides increase sharply.

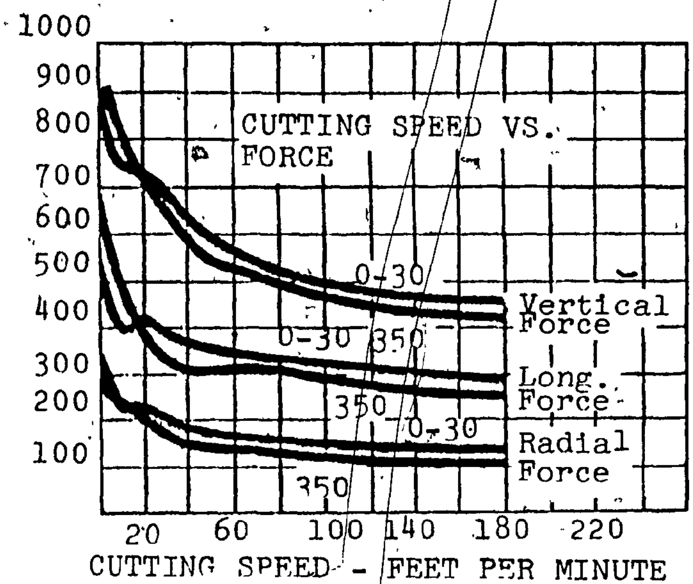

Figure 2.5. At very low speeds, forces build up rapidly on the cutting edges of both cemented oxide (0-30) and carbide tools (350). (Ref.20)

A fairly sharp dip occurs in the cemented oxide curves at about 20 sfpm. This indicates the possibility that the built up edge will increase the effective rake angle. Instability of force measurements was noted from about 40 down to 10 to 15 sfpm. This instability and the force lowering effect of the built up edge occur at a lower speed with the cemented oxide tool (0-30) than with the carbide tool (350). These facts seem to support the claim that cemented oxides are more resistant to chip welding.

The most significant difference between cemented oxides and carbides in their application is the makeup of the resulting costs to machine a workpiece. In Figure 2.6, we can see that not only has the minimum cost-cutting speed moved from around 120 to over 500 fpm, resulting in higher number of pieces produced per hour, but the total machining cost per piece dropped from about 70 cents to below 60 cents. Of basic interest is the change in the slope of the total (Ref. 5) cost curve. Differences in speed from the minimum cost-cutting speed when using carbide tools result in a rather rapid increase in the cost per piece. Using cemented oxide tools results in a total cost per piece per curve which is flatter over a large range of cutting speeds, particularly at speeds above the minimum cost-cutting speed. This means that the accuracy in determining the exact minimum cost-cutting speed when using cemented oxide tools is not nearly so critical. Also, if the calculated cutting speed cannot be maintained, only very small increases in cost per piece will result. It can be

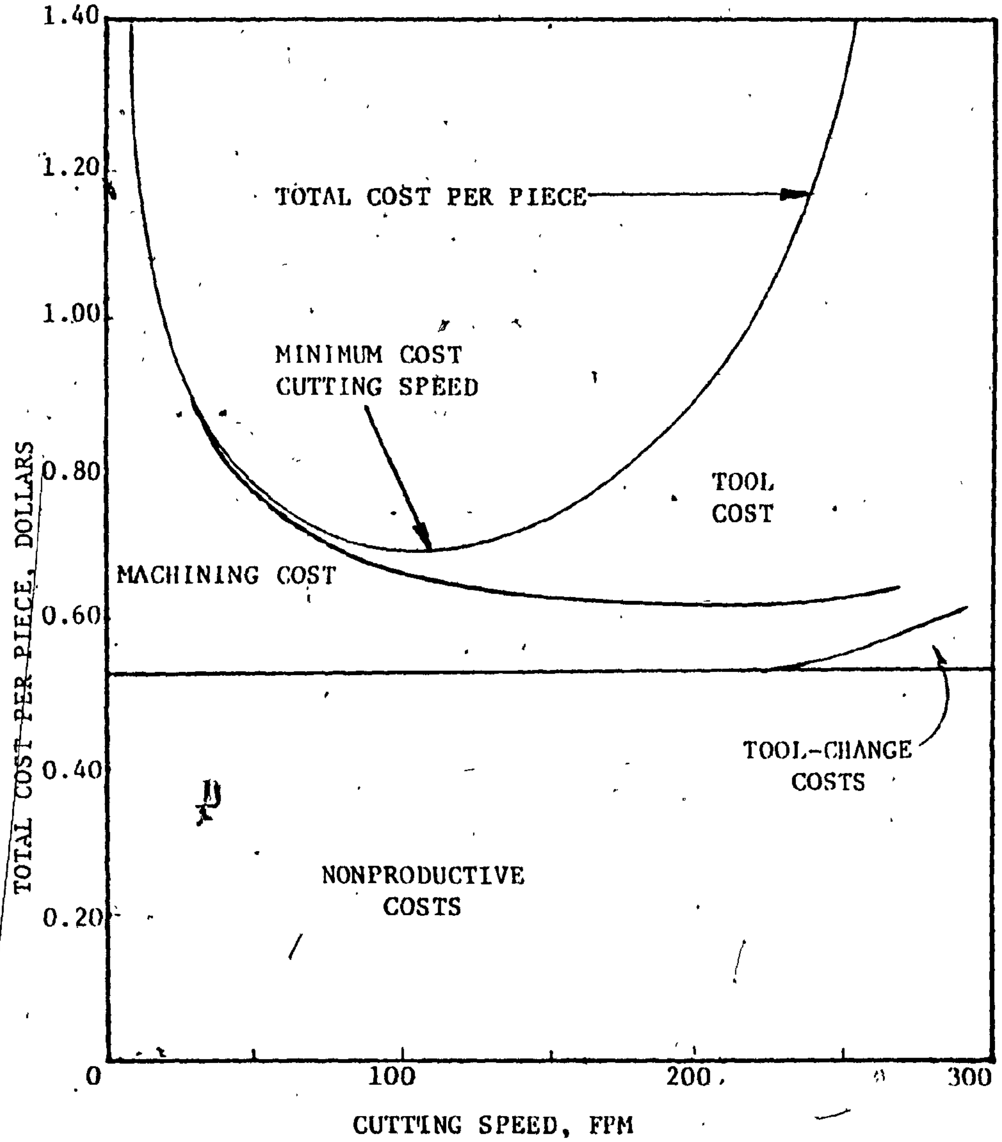

Figure 2.6. Piece costs for turning a cast iron pulley with a carbide tool.
(Ref.20)

seen that speeds well in excess of the minimum cost-cutting speed will result in only slight increase in cost and will give much higher production rates. The current interest in using cemented oxides at what appear to be excessively high cutting speeds is caused by the ability of these tool materials to save money under these conditions, and not because they must be run at high speeds only and cannot be run at low speeds.

CHAPTER III

THE ECONOMICS OF THE BASIC OPERATION

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THE ECONOMICS OF THE BASIC TURNING OPERATION

3.1 Introduction

It has been established that the tool life is most sensitive to changes in the cutting speed, less sensitive to changes in the feed rates, and least sensitive to changes in the depth of cut. Faster speeds and higher feed rates are required to reduce machining times and hence reduce costs, but the implementation of either leads to shorter tool life, which increases costs. It is supposed that there is some optimum set of conditions which will lead to a minimum cost. If this be true, there remains the further point as to whether the machine tool has sufficient power to meet the requirements of these optimum conditions.

Put qualitatively, this seems to be a simple problem; a qualitative solution, however, is beset with many difficulties, the principles of which are:

1. The mathematical manipulations involved.
2. The difficulty of obtaining sufficient reliable data (this, despite the amount of published information on metal cutting).
3. The formulation of any conclusion which could be regarded as general.

The first analyses of a more general nature in this field seem to have been given by Witthoff, who considered the effect of cutting speed and, to a less satisfactory extent, that of feed. He deduced an expression for cost in the form

$$\text{Cost} = k_1 + \frac{k_2}{fV} + \frac{k_3}{cfV(1 - \frac{1}{n})} \quad \ldots 1$$

It was not made sufficiently clear that c includes a term of the form f^a since the symbol c is derivable as follows:
The generalized tool life expression is of the form

$$VT^n f^a d^b = \text{const} \quad \ldots 2$$

which for a given depth and feed, reduces to

$$VT^n = \text{const}/(f^a d^b) = c \quad \ldots 3$$

Differentiating equation 1 with respect to V and setting the result equal to zero gives the cutting speed for minimum cost as

$$V_o = \frac{c}{[\xi(\frac{1}{n} - 1)]^n} \quad \ldots 4$$

where ξ is some constant.

Substitution of this value into the tool life expression $VT^n = C$ yielded the possibly unexpected result that the tool life for minimum cost was

$$T_m = (\frac{1}{n} - 1)\,\xi \quad \ldots 5$$

i.e., the value c had disappeared and the answer to the economic problem appeared to be independent of the feed. Witthoff suggested that the general procedure should be to determine the tool life for minimum cost from equation 5 and then to use tool life-cutting speed curves to find V_o. This is evidently where the paradox is explained, for the question immediately arises: the tool/cutting speed curve for which feed?

Whitthoff argues, with some conviction, that the feed to be selected should be the greatest consistent with the power available, but since power is a function of feed and cutting speed and, as yet, the cutting speed (for minimum cost) cannot be found until the feed is known, the problem has evidently reached the vicious circle stage. This difficulty has arisen because feed was not considered satisfactorily in the first place and a problem in two independent variables was made to look as though only one variable was effective; this cannot be so.

Independently, Atalay and Gilbert gave analyses of the problem considering only cutting speed as variable. Accordingly, they arrived at the same forms of equations as Witthoff but they gave an interpretation of the constant; viz., the ratio of the cost of changing and regrinding a tool to the cost of labour, and overheads per unit time. As the analysis did not consider feed rate, there was no need to consider methods for its selection, but the disappearance of the "constant" in the tool life equation was again noted and Gilbert states: "this means then that the tool life is independent of the loading time, the idle time, the size of part being machined, the tool shape, the size of cut, cutting fluid, etc . . ." This is true and the implication of it will be discussed later; nevertheless the reference to size of cut may have created the impression that feed does not enter into the problem.

Lickley and Chisholm considered the effect of speed and feed on cost but formulated no general expression. Their prime objective seems to have been to compare laboratory tool life tests with shop conditions. They did not consider wear as a variable nor allow for the premature failure of a certain number of tools which occurs, especially when using carbide as a cutting medium. Some of the discrepancy between their predicted and actual results may well be attributable to omission of this latter consideration.

3.2 The general cost expression

For the purposes of the present analysis the total cost/piece C will be broken down into the following parts: (Ref. 7)

1. Idle cost/piece, C_i
2. Cutting cost/piece, C_c
3. Tool changing cost/piece, C_{tc}
4. Tool regrinding cost/piece, C_{tg}
5. Tool depreciation cost/piece, C_{td}
6. Premature failure cost/piece, C_{pf}

With this definition, it follows that

$$C = C_i + C_c + C_{tc} + C_{tg} + C_{td} + C_{pf} \quad \ldots 6$$

Each of these terms will now be considered in turn.

Idle cost/piece:

If t_i be the idle time/piece, then the idle cost/piece will be

$$C_i = R_1 t_i \quad \ldots 7$$

where R_1 represents the general labour and overhead costs per minute. This will include operators' wages, maintenance power, depreciation (excluding tool depreciation), and so on.

Cutting cost/piece:

This will be directly related to the time actually taken to machine each piece, i.e.,

$$C_c = \text{(cost/minute)(cutting time per piece)}$$

$$= R_1 \frac{L \pi D}{fV} \quad \ldots 8$$

where L, D, f and V are in consistent length units.

Tool changing cost/piece:

It is usually accepted that the tool life relationship is given by

$$VT_o^n f^a d^b = \text{const} = \lambda \qquad \ldots 9$$

where T_o is the specific tool life for a given amount of wear W_o. This may be transformed to give

$$T_o = \frac{\lambda^{1/n}}{V^{1/n} f^{a/n} d^{b/n}} = \left(\frac{\lambda}{V}\right)^{1/n} \frac{1}{f^{\alpha} d^{\beta}} \qquad \ldots 10$$

where $\alpha = a/n$ and

$\beta = b/n$

The costs of regrinding are affected by the amount of wear that has taken place and, hence, if this is to be allowed for, some functional relationship must be established or assumed between the general tool life T and the general amount of wear W. The evidence available suggests that different relationships may hold for different work materials and cutting conditions. In this paper, the linear law will be considered, but evidently the same treatment will apply to other more general laws.

Thus, for any other wear W other than the standard wear W, we may write

$$\frac{\text{Time for wear } W}{\text{Time for wear } W_o} = \frac{T}{T_o} = \frac{W}{W_o} \qquad \ldots 11$$

$$\text{or } T_o = \frac{W_o T}{W} \qquad \ldots 12$$

Substituting into equation 10 we have a general expression relating the machining parameters to the amount of wear, viz.

$$\frac{W_o T}{W} = \left(\frac{\lambda}{V}\right)^{1/n} \frac{1}{f^\alpha d^\beta}$$

or

$$T = \left(\frac{\lambda/W_o}{V}\right)^{1/n} \frac{W}{f^\alpha d^\beta} \qquad \ldots 13$$

If we now assume that the tool-changing time t_c is independent of the amount of wear (which seems reasonable) we may write down the tool-changing cost/piece as

$$C_{ct} = \text{(cost/tool change)(number of tool changes/piece)}$$

$$= R_1 t_c \frac{L \pi D}{fV} \left(\frac{V}{\lambda/W_o^n}\right)^{1/n} \frac{f^\alpha d^\beta}{W} \qquad \ldots 14$$

There are certain costs associated with regrinding which are quite independent of the amount of wear allowed, e.g., the general handling associated with removing the tool, inspection, lapping where necessary, and so on. Let these costs be denoted by R_2 dollars/regrind.

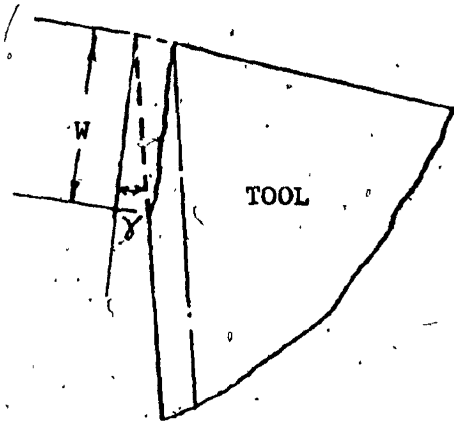

Figure 3.1. Schematic representation of a tool with flank wear of amount W.

Figure 3.1 shows the profile of a tool with clearance-face wear of amount W inches. Theoretically, the amount to be ground off in order to produce a satisfactory clearance face is AB = sin γ but in practice an amount (W sin γ + A) must be ground off as the worn face is not a defined line. Then, if R_3 be the cost of grinding per inch (measured in the direction of AB), the total cost per regrind is

$$R_2 + R_3 (W \sin \gamma + A) = (R_2 + R_3 A) + R_3 W \sin \gamma \qquad \ldots 14a$$

Hence, regrinding cost per piece (cost/regrind)(regrinds/piece) =

$$\{(R_2 + R_3 A) + R_3 W \sin \gamma\}\{\frac{L \pi D}{fV}\left(\frac{V}{\lambda / W_o^n}\right)^{1/n} \frac{f^{\alpha} d^{\beta}}{W}\} \qquad \ldots 15$$

Tool depreciation cost per piece:

Let the initial cost of the tool be t_o and, in general, we may assume that the tool has a final value t_f when it is of no further use for the particular operation considered; in particular, t_f may often be zero. If the tool is reground r times before it is of no use for the particular job, then the loss in value of the tool $(t_o - t_f)$ is spread over $(1 + r)$ usages. The value of r is determined from the amount ground off each time $(W \sin \gamma + A)$ and the limit imposed when further regrinding becomes impossible (Δ), i.e.,

$$r = \frac{\Delta}{W \sin \gamma + A}$$

$$C_{td} = \left\{\frac{t_o - t_f}{1 + \frac{\Delta}{W \sin \gamma + A}}\right\}\left\{\frac{L \pi D}{fV} \left(\frac{V}{\lambda / W_o^n}\right)^{1/n} \frac{f^{\alpha} d^{\beta}}{W}\right\} \quad \ldots 16$$

Premature failure cost per piece:

At this stage it would be pertinent to note that for brittle tool material there is no guarantee that each tool will last the time predicted by tool life tests. Although this phenomenon is not completely explicable it seems reasonable to assume that if a tool is allowed to wear more before being reground, its chances of reaching the expected tool life will be reduced.

There are thus two alternatives:

(a) To accept the sum of the costs 1-5 as the total cost, realizing that for brittle materials especially, it is unlikely to give a true picture except at small values of wear.

(b) To attempt to incorporate the notion of premature failures into the general theory, accepting that it must be based on certain assumptions which are either only partially justified or whose justifications may not be verifiable directly.

The first alternative leads to the expression for cost per piece as

$$C_p = R_1 t_i + R_1 \frac{L\,\pi\,D}{fV} + \frac{L\,\pi\,D}{fV} \left(\frac{V}{\lambda/W_o^n}\right)^{1/n} \frac{f^{\alpha} d^{\beta}}{W} \; x$$

$$\left\{R_1 t_c + R_2 + R_3 A + R_3 W \sin\gamma + \frac{t_o - t_f}{1 + \frac{\Delta}{W \sin\gamma + A}}\right\} \quad \ldots 17$$

The second alternative will now be discussed. It is evident from the foregoing discussion that, due to premature failure, the cost will increase as the wear permitted increases (provided that this is taken over a considerable number of pieces). This concept may be incorporated into the cost equation if we assume that this cost may be divided by the number of pieces so as to give an average cost per piece. This average will be greater than that for tools exhibiting no premature tool failure. A further assumption which must be made, and about which there is no experimental evidence at all, is that premature failure is independent of the number of times a tool has been reground, which appears probable.

Initially there are N tools and if 100 u (per cent) of the tools fail between each regrind, then at the first regrind there are only N(1-u) tools which have not failed; the uN tools which have failed are regarded as now useless for the present operation. At the second regrind there will be only $N(1-u)^2$ tools and, in general, at the rth regrind there will be $N(1-u)^r$ tools. After the rth regrind, the tools remaining cannot be reground and therefore the number of pieces machined which do not fail

$$= N[(1-u) + (1-u)^2 + (1-u)^3 + \ldots + (1-u)^{r+1}]\ \ell$$

$$= N\ell\ (1-u)\left[\frac{1-(1-u)^r}{1-(1-u)}\right] \qquad \ldots 18$$

where ℓ = number of pieces machined by any one tool between regrinds.

Estimating the number of pieces produced by the tools which fail prematurely is more difficult since the number will vary from tool to tool. This difficulty was overcome in an indirect manner. It was concluded that, on the average, each tool produced, before failure, 80 per cent of the components produced, between regrinds, by a tool which did not fail, i.e., 0.8 ℓ.

With these data we may proceed as follows: In the period before the first regrind Nu tools fail, producing 0.8 ℓ Nu pieces before failure. Between the first and second regrinds, there are initially N(1-u) tools, of which (100 u) per cent fail prematurely, i.e., uN(1-u); these produce 0.8 ℓ Nu(1-u) pieces before failure.

Proceeding in this manner, we may again sum a geometric series to give the total number of pieces machined by the tool which fail

$$= 0.8\,N u \ell\,[1 + (1-u) + (1-u)^2 + \ldots + (1-u)^r]$$

$$= 0.8\,N u \ell\,\left[\frac{1 - (1-u)^r}{1 - (1-u)}\right] \qquad \ldots 19$$

Hence, adding equations 18 and 19, the total number of pieces machined by N tools

$$= N\ell\,\left[\frac{1 - (1-u)^r}{1 - (1-u)}\right][(1-u) + 0.8u]$$

$$= N\ell\,\frac{1 - (1-u)^r}{1 - (1-u)}\,(1 - 0.2u) \qquad \ldots 20$$

and the number of pieces machined per tool

$$= \ell\,(1 - 0.2u)\left[\frac{1 - (1-u)^r}{1 - (1-u)}\right] \qquad \ldots 21$$

This may be regarded as increasing the tool depreciation cost in the ratio

$$\sigma = \frac{\text{total no. of pieces machined with no premature failures}}{\text{total no. of pieces actually machined}}$$

$$= \frac{\ell(r+1)}{\ell(1-0.2u)\left[\frac{1-(1-u)^r}{u}\right]}$$

$$= \frac{u(r + 1)}{(1-0.2u)[1-(1-u)^r]} \quad \ldots 22$$

Hence

$$C_{td} + C_{pf} = C_{td}\sigma \quad \ldots 23$$

Adding the various costs we have for the total cost/piece, we get

$$C_p = R_1 t_1 + R_1 \frac{L \pi D}{fV} + \frac{L \pi D}{fV} \left(\frac{V}{\chi / W_o^n}\right)^{1/n} \frac{f^\alpha d^\beta}{W} \times$$

$$\{R_1 t_c + R_2 + R_3 A + R_3 W \sin \gamma + \frac{t_o - t_f}{(1 + \frac{\Delta}{W \sin \gamma + A})}\} \times$$

$$\frac{u(r + 1)}{(1-0.2u)[1-(1-u)^r]} \quad \ldots 24$$

3.3 Factors affecting total cost/piece

It is seen from equation 23 that the cost per piece is a function of four variables, viz., W, d, f, and V. The effect of these variables on C_p will now be discussed.

The effect of W:

It has been determined that the incidence of premature failures is related to the amount of wear permitted before regrinding, by the relationship $u = mW^q$. Substituting for u in equation 24 yields a functional relationship between C_p and W which is too complex to permit a general mathematical treatment. However, for use in the present analysis, a specific set-up is considered, details of which are as follows.

Consider a turning operation for a nickel chrome steel 3 in. diam., 6 in. long, tool material, cemented carbide. Tool life relationship is

$$T = \left(\frac{163.8}{V}\right)^8 \frac{W}{d^{3.06} f^{5.6}} \quad \text{(V in in. per min.)}$$

Cost ratios: R_1 = 0.05 \$/min

R_2 = 0.20 \$/regrind

R_3 = 16.00 \$/in.

1)	Loading, min.	0.63
2)	Approach, etc., min.	0.05
3)	Engage feed, min.	0.02
4)	Remainder reduced to a time/piece, min.	0.30
	Total idle time, min.	1.00

Tool costs: t_o = 5\$, T_f = 0

Tool grinding data: A = 0.008 in.

Premature failure data: u = 1600 $W^{2.78}$

For given values of feed, depth of cut, and cutting speed, the variation of C_p is governed by the third term in equation 24; as W is increased, the quantity in braces increases, slowly at first and then more rapidly while the other variable part, 1/W, evidently decreases. The result is a curve, shown in Figure 3.2, exhibiting a minimum cost at approximately 0.03 in. wear. Evidently, alteration of feed, depth, and speed would affect the magnitudes throughout the range of wear but the general shape of the curve would not be affected since it is controlled by the two quantities stated previously, both of which are independent of f and V.

The effect of f and V:

The foregoing paragraphs have indicated that there is an optimum amount of wear to be permitted before regrinding. Assuming that this value has been chosen, equation 24 may be simplified to give

$$C_f = R_1 t_1 + R_1 \frac{L \pi D}{fV} + \frac{L \pi D}{fV} \left(\frac{V}{\lambda / W_o^n}\right)^{1/n} f^{\alpha} d^{\beta} B \qquad \ldots 25$$

where B is a constant equal to the term in braces in equation 24 divided by W.

Differentiating equation 25 with respect to f and V in turn and equating to zero, we may establish the conditions (if any) for minimum cost.

$$\frac{\partial C_f}{\partial V} = -R_1 L \pi D/fV^2 + \frac{L \pi D f^{\alpha-1} d^{\beta} B W_o}{\lambda^{1/n}} V^{(1/n - 2)} (1/n + 1) = 0$$

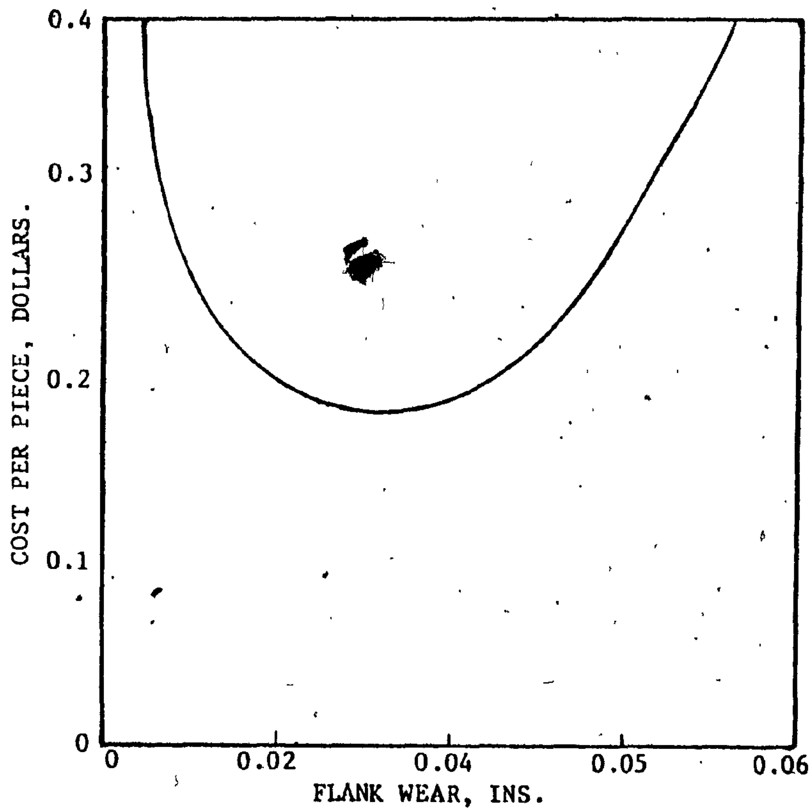

Figure 3.2 Variation of cost/piece with amount of flant wear permitted before regrinding. (Ref.5)

i.e.

$$V^{1/n} f^{\alpha} = \frac{R_1 \lambda^{1/n} n}{d^{\beta} B W_o (1-n)} \qquad \ldots 26$$

$$\frac{\partial C_p}{\partial f} = -R_1 \frac{L \pi D}{f^2 V} + \frac{L \pi D}{V} \left(\frac{V}{\lambda / W_o^n}\right)^{1/n} f^{\alpha-2} d^{\beta} B(\alpha-1) = 0$$

i.e.

$$V^{1/n} f^{\alpha} = \frac{R_1 \lambda^{1/n}}{d^{\beta} B W_o (\alpha-1)} \qquad \ldots 27$$

It should be emphasized that both of these equations are independent of the physical dimensions of the workpiece. Since equations 26 and 27 cannot be simultaneously true, it is evident that there is no absolute minimum value of C_p considered as a function of f and V. It is known, however, that for a given value of f, there is a value of V which will give minimum C_p and for a given value of V, there is a value of f which will give minimum C_p. It is also seen that as f is increased, the value of V for minimum cost decreases as would be expected.

Using the data given in the example and d = 0.05 in., equation 25 takes the numerical form

$$C_p = 0.05 + \frac{2.83}{fV} + \frac{f^{4.6}}{522} \left(\frac{V}{163.8}\right)^7 \qquad \ldots 28$$

(C_p in dollars per piece). Figure 3.3 shows curves of C_p against V for various values of feed from which it is seen that, as f increases,

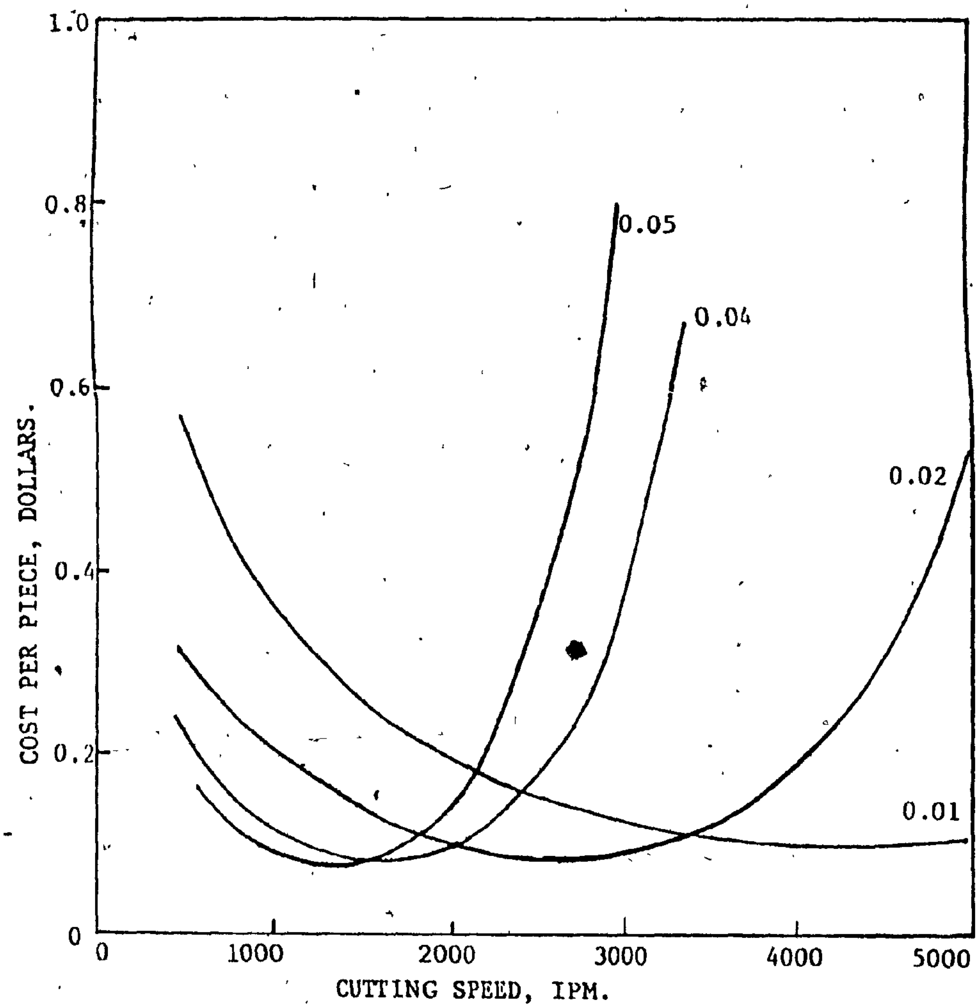

Figure 3.3 Variation of cost/piece with speed. Numbers against curves refer to feed rates in inches/revolution. (Ref.5)

not only does the value of V for minimum cost decrease, but also the cost itself decreases, although not spectacularly.

Figure 3.3 is useful for interpreting equation 28 but is not the best representation of the effect of varying feed and cutting speed since it gives no facile impression of how much change in C_p is effected by any given change in f or V. In Figure 3.4 is shown the locus of the points of minimum cost; thus for any given feed the cutting speed for minimum cost can be read off easily. As has been pointed out, there is no absolute minimum of C_p as a function of f and V, but in the range covered by Figure 3.4 the lowest cost is at f = 0.05 ipr and V = 1400 ipm. If we call this C'_p, then all combinations of f and V which will give costs, some given percentage above C'_p, must satisfy an equation of the form

$$0.005 + \frac{2.83}{fV} + \frac{f^{4.6}}{522}\left(\frac{V}{163.8}\right)^7 = \mu C'_p, (\mu>1) \quad \ldots 29$$

In Figure 3.4 are shown constant cost loci for μ = 1.1, 1.2, 1.4, 1.6, 1.8, and 2.0 (i.e., 10, 20, 40, 60, 80 and 100 per cent above C'_p). These do give some indications of how critical, or otherwise, is the selection of speeds and feeds and incidentally show how the "valley" of the minimum cost rises toward the lower feeds.

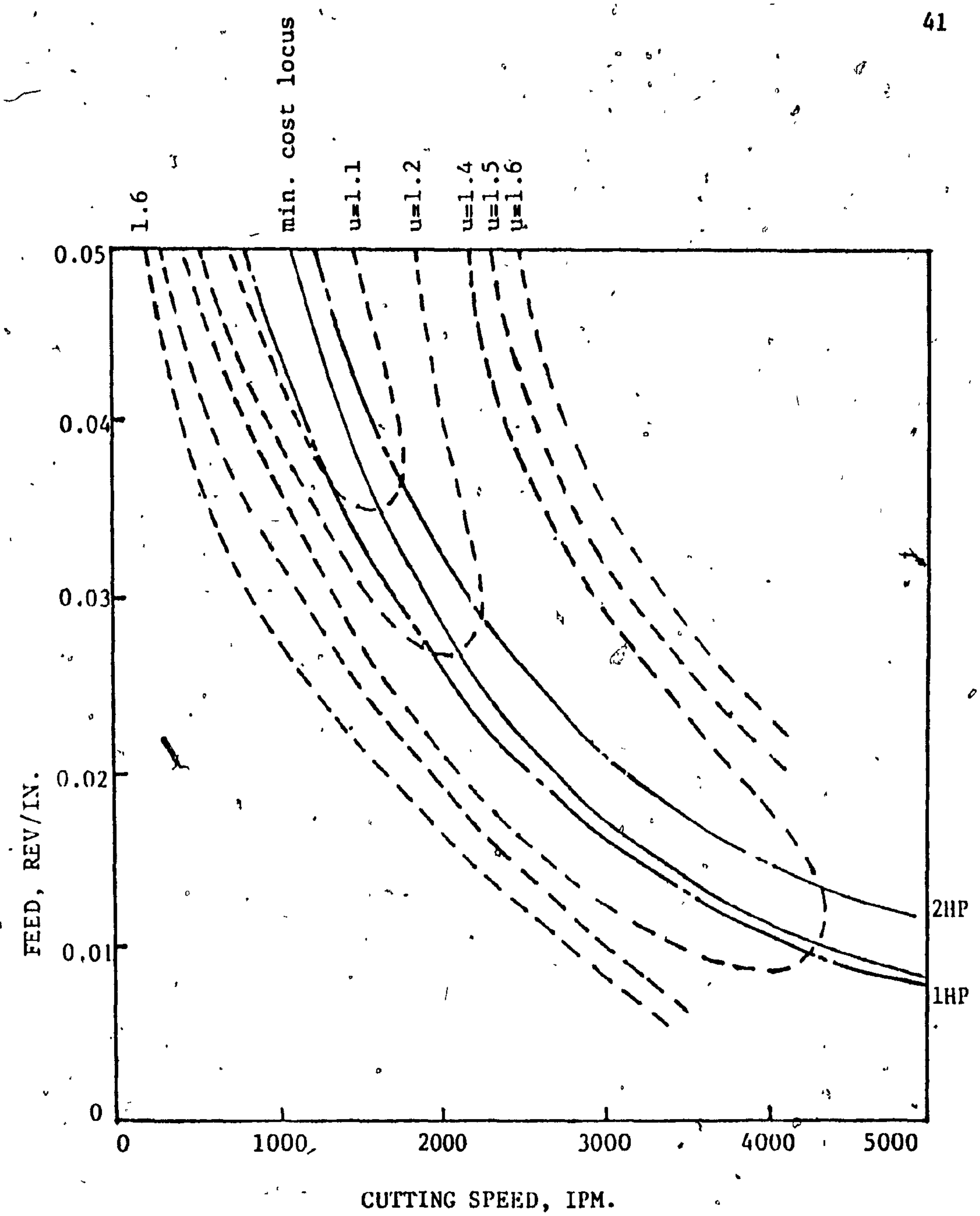

Figure 3.4 Relationship between cost/piece, feed, and cutting speed. Broken lines are loci of equal cost and chain lines are loci of constant power.
(Ref.21)

Effect of d:

The effect of depth of cut may be considered as follows. Two questions need answering: (a) Whether to take the depth in one cut or more; (b) for more than one cut whether to take the cuts in equal amounts or not. Considering the first question we have, using equation 25, the cost/piece using only one cut

$$C_{p1} = R_1 t_i + R_1 \frac{L \pi D}{fV} + \frac{L \pi D}{fV} \left(\frac{V}{\lambda/W_o^n}\right)^{1/n} f^{\alpha} d^{\beta} B \quad \ldots 30$$

Remembering that, with two cuts, the time taken per piece is doubled, the cost per piece for two cuts is

$$C_{p2} = R_1 t_i + 2R_1 \frac{L \pi D}{fV} + \frac{2L \pi D}{fV} \left(\frac{V}{\lambda/W_o^n}\right)^{1/n} f^{\alpha} \left(\frac{d}{2}\right)^{\beta} \quad \ldots 31$$

The boundary dividing the zones where these two desiderata represent the minimum cost is given by equating the right hand sides of equations 30 and 31, i.e.

$$R_1 = \left(\frac{V}{\lambda/W_o^n}\right)^{1/n} f^{\alpha} d^{\beta} (1 - 2^{1-\beta}) \beta \quad \ldots 32$$

For the data of the example this reduces to

$$\left(\frac{V}{163.8}\right)^{8} f^{5.6} = 545 \quad \ldots 33$$

At this point it should be recognized that Figures 3.3 and 3.4 have given some insight into the complex cost relationships but would involve a great deal of very tedious calculation work and graph plotting. Thus, if some alteration which will reduce the amount of preparation is not forthcoming, the analysis remains of interest only from the viewpoint of giving an insight into the problem.

However, any curve can be replotted as a straight line if the axes are subdivided in the correct manner and in the present case it is fortunate that most important loci are power laws, which means that commercially available biologarithmic paper can be used. Figure 3.5 shows much of the information of Figure 3.4 replotted on logarithmic co-ordinates. The locus of minimum cost and the loci of constant horsepower are now straight lines; the trajectories of constant cost, however, are still curved, since equation 29 is not a pure power law, but as the curves recede from their turning points they become effectively straight and of slope -1.

Returning now to the discussion on the depth of cut and the effect on the overall cost, we find that if we were to plot equation 31 on the biologarithmic graph it would appear as line AB in the figure. From this it can be seen that the region covered by the previous discussion is well inside the zone where one cut is more economical. Thus the second question posed in the foregoing need not be considered here; the use, in practice, of roughing and finishing cuts is largely a matter of securing dimensional accuracy.

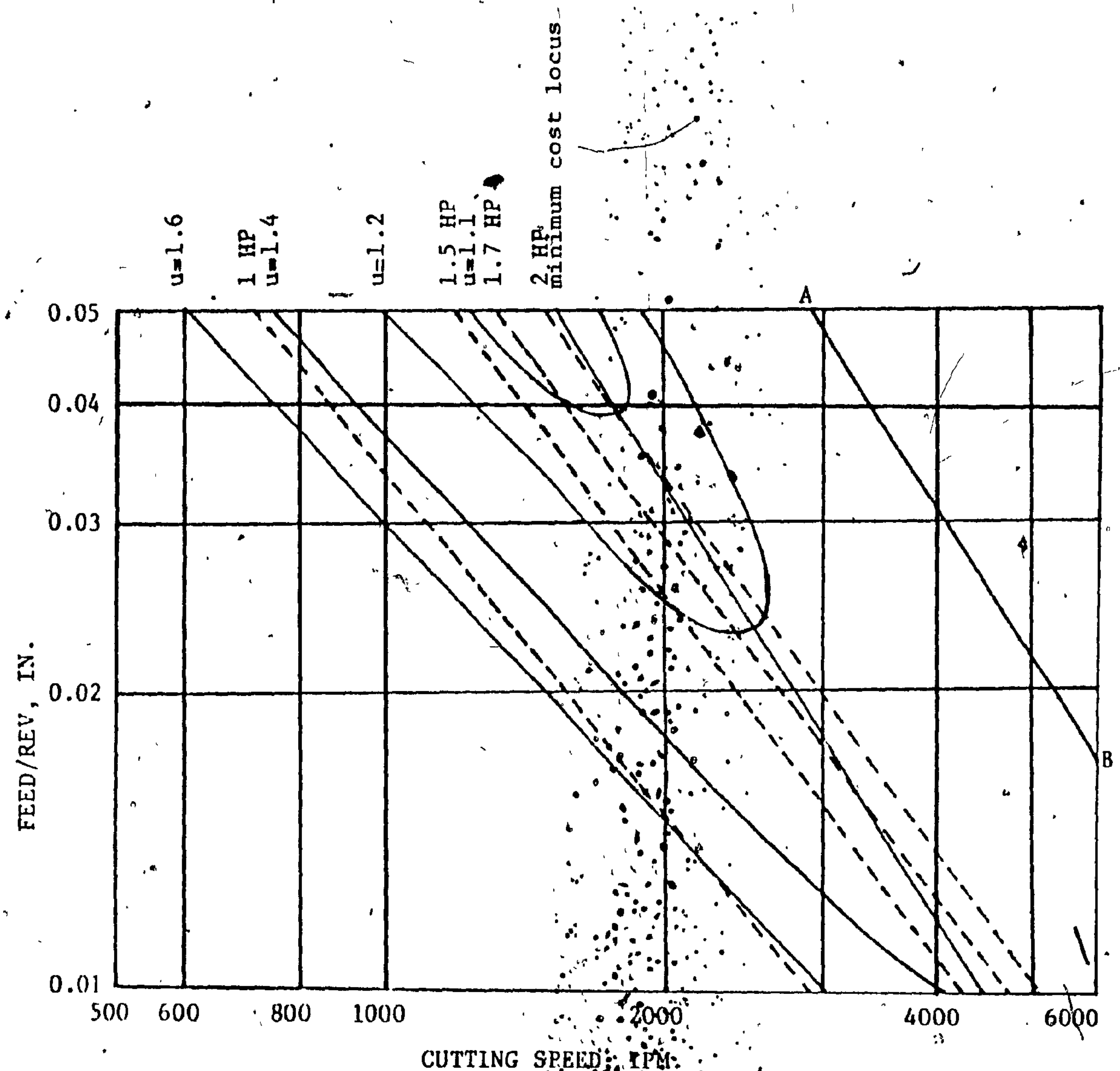

Figure 3.5 Biologarithmic plot of relationship between cost/piece, feed, and cutting speed.
(Ref.21)

CHAPTER IV

MAXIMUM PROFIT AS THE CRITERION IN THE DETERMINATION OF THE OPTIMUM CUTTING CONDITIONS

CHAPTER IV

MAXIMUM PROFIT AS THE CRITERION IN THE DETERMINATION OF THE OPTIMUM CUTTING CONDITIONS

4.1 Introduction

The problem of economic cutting conditions has been attacked from different viewpoints. Yet another approach has been forwarded by S. M. Wu and D. S. Ermer. They consider that the maximum profit is an appropriate criterion for the selection of the optimum machining conditions rather than the conventional criteria of minimum cost or maximum production rate. Thus, another dimension has been added to the problem, namely, that of maximum profit.

The basic mathematical model, which has been used in the analysis of machining economics, is a unit-cost model, or an analogous unit-time model if costs are neglected. In conjunction with these models two criteria have been used in the determination of the optimum cutting conditions - one is minimum cost and the other is maximum production rate.

If the operation is a "bottleneck" in a production sequence, it might be necessary to operate at the cutting conditions for maximum production rate. However, this is generally not the normal situation and cutting conditions are usually selected from the viewpoint of minimizing costs, under the assumption that operating at the minimum-cost conditions will tend to increase profits in the long run. It has also been recognized that between these two criteria there is a range of cutting conditions from which an optimum point could also be selected, but it has not been indicated exactly how this optimum position should be chosen.

A natural criterion for the selection of these optimum cutting conditions is maximum profit, which is in reality the major goal of industry. It was not until very recently that Okushima and Hitomi presented an analysis of the maximum-profit cutting speed (V_p). Unfortunately, their analysis is based on a simple linear break-even chart which inherently limits the necessary development of the maximum-profit concept. In addition, their explicit derivation of V_p is considered ineffectual and unnecessary by many researchers due to the fact that not only is their determination of V_p based on particular values of Taylor's exponent n, but also that they neglect the importance of exploiting the neighborhood of the theoretical maximum-profit cutting speed.

This viewpoint employs the same basic model as in prior analyses, but the maximum-profit cutting speed is determined by application of a fundamental economic principle that maximum profit occurs at the production rate where marginal revenue equals marginal cost. This marginal principle is based on the primary economic relationships between a company's production rate, costs, revenues, and profits. Introduction of these basic economic concepts and the use of maximum profit as the optimizing criterion broadens the analysis from the partially-restrictive machining cost point of view to a more general economic point of view.

S. M. Wu and D. S. Ermer developed their argument under the assumption that the empirical parameters in the tool-life equation are known. Brief summaries of the basic mathematical model and the conventional determination of the minimum-cost and maximum-production cutting speeds are presented, and the basic marginal principle is explained. An application of this marginal principle to determine the optimum cutting speed is illustrated by a simple example where the unit revenue is a constant. The analysis is expanded to include the case where demand is a decreasing function, and to incorporate feed as a variable in addition to cutting speed. Finally, the sensitivity of the profit response under various combinations of demand curves, feeds, and cost and time constants is considered.

4.2 Basic model

The basic model, which describes the average unit cost to produce a workpiece by means of a simple, rough turning operation, is the sum of four costs: (Ref. 13)

1. Machining cost = $c_o t_m$

where c_o is the cost of operating time, dollars per minute, and t_m is the time to machine a workpiece, minutes per piece.

2. Tool cost = $c_t \frac{t_m}{T}$

where c_t is the tool cost, dollars per cutting edge, T is the tool life, minutes per cutting edge, and t_m/T is the number of tool edges required per workpiece.

3. Tool changing cost = $c_o t_c \frac{t_m}{T}$

where t_c is the tool changing time, minutes per cutting edge.

4. Handling cost = $c_o t_h$

where t_h is the handling time, minutes per piece.

Hence, the unit cost (C_u) can be expressed as

$$C_u = c_o t_m + \frac{t_m}{T} (c_o t_c + c_t) + c_o t_n \quad \ldots 4.1$$

The machining time (t_m) is the time the tool is actually cutting. For a constant depth of cut

$$t_m = \frac{\pi DL}{12 Vf}$$

where

D is the diameter of the workpiece, in.

L is the axial length of cut, in.

V is the cutting speed, sfpm

f is the feed, ipr.

If the costs in equation 4.1 are disregarded, then the basic model for the unit time (T_u) is

$$T_u = t_m + t_c \frac{t_m}{T} + t_h \quad \ldots 4.2$$

and the reciprocal of equation 4.2 is the production rate (Q), i.e., $Q = 1/T_u$.

4.3 Determination of V_{min} and V_{max}

Using Taylor's tool life equation

$$VT^n = C$$

where C and n are empirical constants, the unit cost can be expressed as

$$C_u = \frac{c_o \pi DL}{12\ Vf} + \frac{\pi DLV^{\frac{1}{n}-1}}{12\ fC^{1/n}} (c_o t_c + c_t) + c_o t_h$$

Hence, the cutting speed for minimum cost can be derived as

$$V_{min} = \frac{C}{[(\frac{1}{n} - 1)(t_c + \frac{c_t}{c_o})]^n} = \frac{C}{[(\frac{1}{n} - 1) t_e]^n} \quad \ldots 4.3$$

where $t_e = t_c + \frac{c_t}{c_o}$

and the tool life for minimum cost is

$$T_{min} = \left(\frac{1}{n} - 1 \right) t_e \quad \ldots 4.4$$

Likewise, from equation 4.2, the cutting speed and tool life for maximum production are

$$V_{max} = \frac{C}{\left[\left(\frac{1}{n} - 1\right) tc\right]^n} \quad \ldots 4.5$$

and

$$T_{max} = \left(\frac{1}{n} - 1\right) t_c \quad \ldots 4.6$$

Since $t_e = t_c + \frac{c_t}{c_o}$ and the ratio c_t/c_o is always positive, V_{max} will always be greater than V_{min} and, consequently, there is a range of speeds defined by V_{max} and V_{min} which can be seen in Figure 4.3. These results are applicable for a given feed, and under the assumption that the Taylor equation is valid over the relevant cutting speeds.

4.4 Marginal principle and maximum profit

Suppose the relationship between the selling price of the company's product and the amount that can be sold is given by a linear demand function as shown in Figure 4.1a,

$$P = a - bW \quad \ldots 4.7$$

where

P is the selling price, dollars per minute

W is the volume sold, pieces per period

a,b are positive constants.

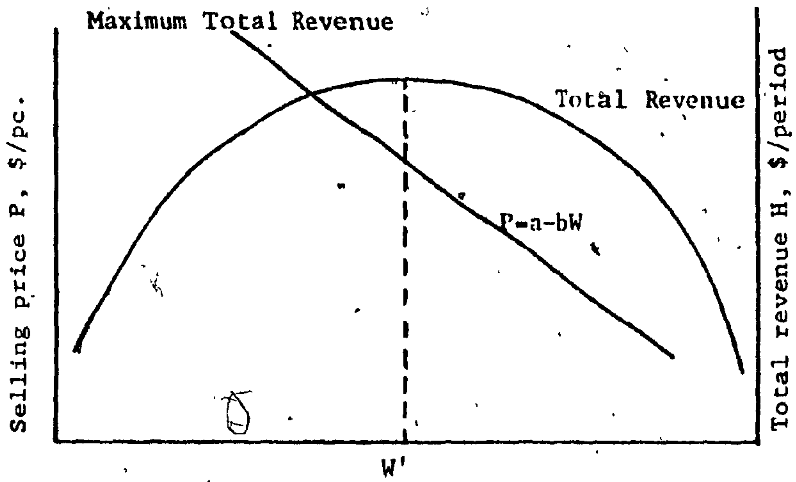

Figure 4.1a. Selling price and total revenue versus volume.
(Ref.13)

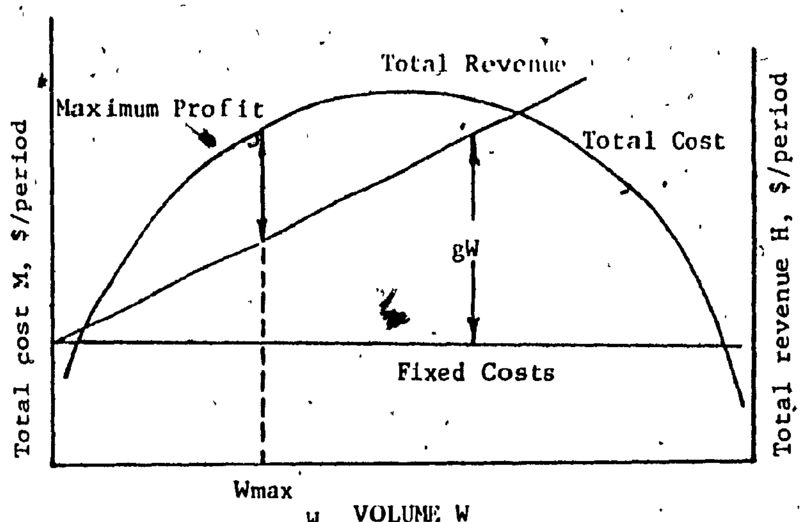

Figure 4.1b. Total cost and total revenue versus volume.
(Ref.13)

Then the total revenue resulting from any price-volume combination is the amount of product sold multiplied by the unit selling price, and using the demand function given by equation 4.7, the total revenue (H) can be expressed in terms of volume alone.

$$H = PW = (a - bW)W = aW - bW^2$$

This total revenue curve is also shown in Figure 4.1a. The volume (W') which will maximize the total revenue is:

$$\frac{dH}{dW} = a - 2\ bW = 0 \qquad \ldots 4.8$$

i.e.,

$$W' = a/2B.$$

The derivative given by equation 4.8 expresses the rate at which revenue increases with increases in volume and is called the "marginal revenue."

In general, however, the maximum total revenue does not result in maximum profit since the total cost of production must be included in the economic analysis in order to determine profit. The total cost (E) of production can be separated into two elements, fixed and variable, and for the simple case where the variable cost is a linear function of the volume, the company's total cost function is

$$E = F + gW$$

where

E is the total cost at volume W

F is the fixed cost, independent of W

G is a positive constant

Thus the profit (I) is given by

$$I = H - E = (a - g)W - bW^2 - F \quad \ldots 4.9$$

and the volume (W_{max}) which will maximize the profit under these conditions can be determined by

$$\frac{dI}{dW} = (a - g) - 2bW = 0$$

i.e.,

$$W_{max} = \frac{a - g}{2b} \quad \ldots 4.10$$

These relationships are illustrated in Figure 4.1b.

From these results it can easily be shown that at W_{max} the marginal cost is equal to the marginal revenue. It can also be shown that this is true if the variable cost is nonlinear. Therefore, to maximize profit, increase production until the marginal revenue is equal to the marginal cost.

Application of this marginal principle will indicate how the optimum cutting speed can theoretically be selected. If the revenues resulting from increases in the rate of production were known, the production rate could be increased by increasing the cutting speed from V_{min} until the incremental or marginal change in revenue is equal to the incremental change in cost. Then the cutting speed (V_p) at which this occurs will theoretically result in maximum profit and will be the speed for an optimum economic balance between cost and production rate.

4.5 Rough turning operation with a single point carbide tool

The marginal principle just stated can be best explained by using a tabular and graphical approach to determine the maximum profit cutting speed (V_p). A rough turning operation with a single point carbide tool is used to reduce the diameter of a bearing housing at a feed of 0.006 ipr. The numerical values for this example are given in Table 1. Columns (B) and (C) in Table 1 give the production rates and unit costs calculated by equations 4.2 and 4.1 for a cutting speed range of 300–650 sfpm and increments of 25 sfpm, as given in column (A). The total cost per minute (C_T) at a given cutting speed is the product of the production rate and unit cost; e.g., the total cost per minute at 450 sfpm is (Ref. 13)

$$C_T = QC_u = (0.496 \text{ pcs/min})(\$0.296/\text{pc}) = \$0.147/\text{min}$$

as is given in column (D).

If the unit revenue (R_u) was a constant at \$0.45, as indicated by the horizontal line in Figure 4.2, the marginal revenue is also

TABLE 1 Production rate, costs, revenues and profits versus cutting speed

(A) Cutting Speed V sfpm	(B) Prod. Rate Q Pces/min eq.2	(C) Unit Cost C_u \$/pc eq.1	(D) Total Cost C_T \$/min (B)x(C)	(E) Marg. Cost C_M \$/min	(F) Unit Rev. R_u \$/pc (Const.)	(G) Total Rev. P_T \$/min (B)x(F)	(H) Marg. Rev. R_M \$/min	(I) Profit \$/min (G)-(D)	(F') Unit Rev. (2) R_u \$/pc Var.	(G') Total Rev. (2) R_T \$/min (B)x(G')	(H') Marg. Rev. (2) R_M \$/min	(I') Profit \$/min (G')-(D)
300	.375	.333	.125	.001	.45	.169	.012	.044	.547	.205	.01	.080
325	.393	.318	.127	.002	.45	.179	.01	.052	.540	.215	.010	.083
350	.421	.307	.129	.002	.45	.189	.010	.051	.532	.224	.009	.095
375	.412	.299	.132	.003	.45	.198	.009	.057	.527	.233	.009	.100
400	.462	.295	.136	.004	.45	.209	.009	.070	.522	.241	.008	.105
425	.480	.294	.141	.005	.45	.215	.008	.074	.517	.246	.007	.107
450	.496	.296	.147	.006	.45	.223	.003	.075	.512	.254	.006	.107
475	.511	.302	.154	.007	.45	.230	.007	.076	.500	.260	.006	.108
500	.523	.311	.163	.006	.45	.236	.006	.073	.507	.265	.005	.102
525	.533	.324	.173	.020	.45	.240	.004	.057	.503	.263	.003	.095
550	.540	.341	.184	.011	.45	.243	.003	.054	.502	.271	.003	.037
575	.545	.362	.197	.013	.45	.245	.002	.044	.501	.273	.002	.076
600	.503	.386	.212	.015	.45	.246	.001	.034	.500	.274	.001	.061
625	.547	.416	.220	.016	.45	.246	.000	.017	.500	.274	.000	.047
650	.544	.450	.245	.017	.45	.245	.001	.000	.501	.273	-.001	.023

Costtime Parameters

c_o = \$.121/min t_c = .5 min/edge
c_t = \$.332/edge t_h = .5 min/pc.

Tool Life Parameters

.2
VT = 700

f = .006 ipr
d = .100 in

(Ref. 13)

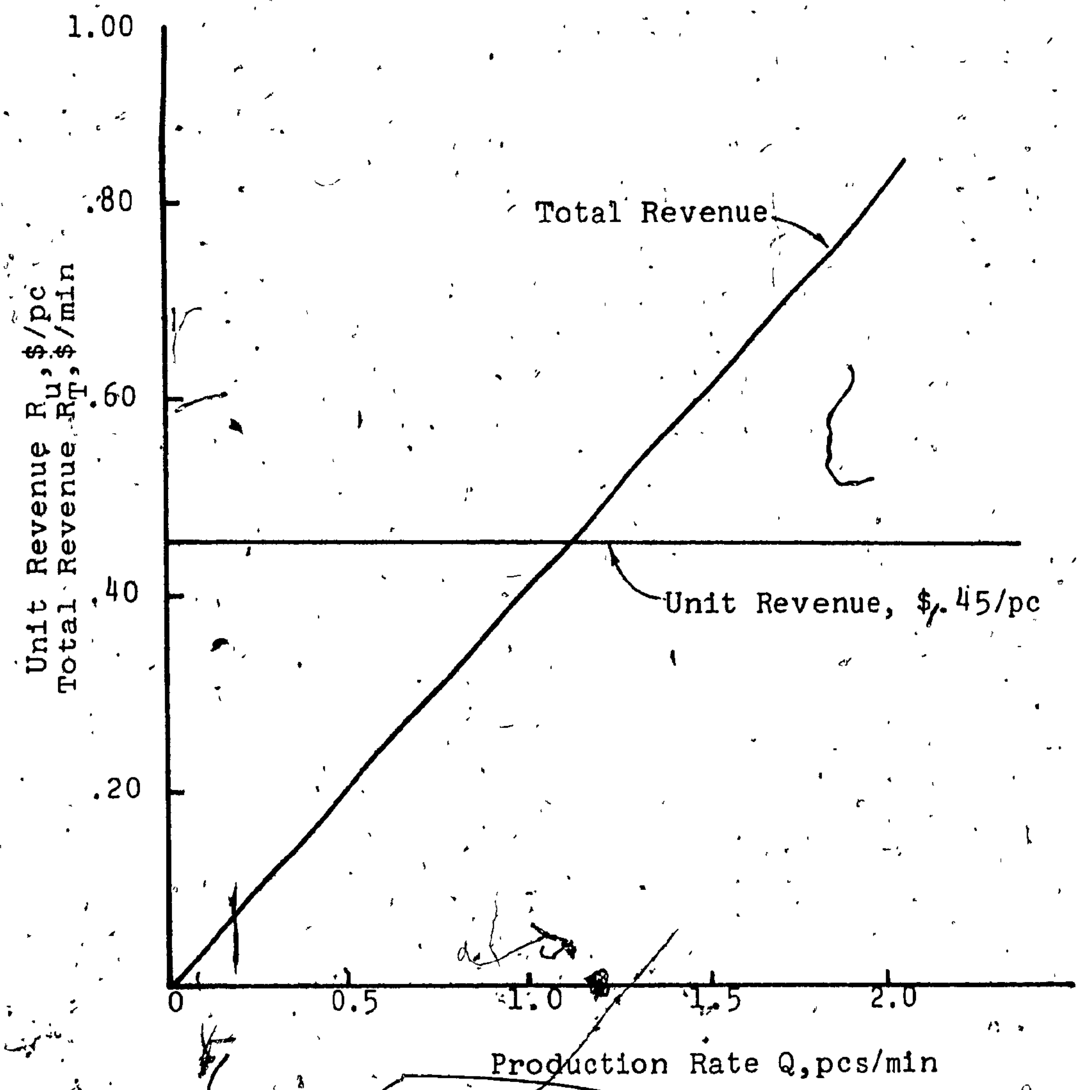

Figure 4.2. Constant unit revenue and total revenue versus production rate. (Ref.13)

\$0.45 and the total revenue (R_T) as a function of the production rate is a non-decreasing straight line, also shown in Figure 4.2.

The total revenue per minute as a function of the cutting speed is determined from the multiplication of the unit revenue (\$/pc) and production rate (pcs/min) at each cutting speed, and the results are given in column (G). Columns (E) and (H) give the marginal costs (C_M) and the marginal revenues (R_M), respectively; i.e., the incremental changes in the total cost and total revenue as the cutting speed increases in steps of 25 sfpm. The marginal cost equals the marginal revenue ($C_M = R_M = \$0.007/min$) at 0.511 pcs/min and thus the optimum cutting speed for maximum profit is 475 sfpm, with a corresponding tool life of seven minutes.

To illustrate, the data in columns (A) to (I) in Table 1 are plotted in Figure 4.3. The cutting speeds for minimum cost and maximum production rate are 420 and 610 sfpm by equations 4.3 and 4.5, a cutting speed range of 190 sfpm. Accordingly, if this operation were performed at a cutting speed of 420 sfpm, the tool life for minimum cost by equation 4.4 would be 13 minutes, the average unit cost would be a minimum at approximately \$0.295 per housing; and the production rate would be approximately 0.475 housings per minute; whereas if V were increased to 610 sfpm, the tool life at the maximum production rate by equation 4.6 would be two minutes, the unit cost would be \$0.395 per housing, and the production rate would be a maximum at approximately 0.55 housings per minute. It is also

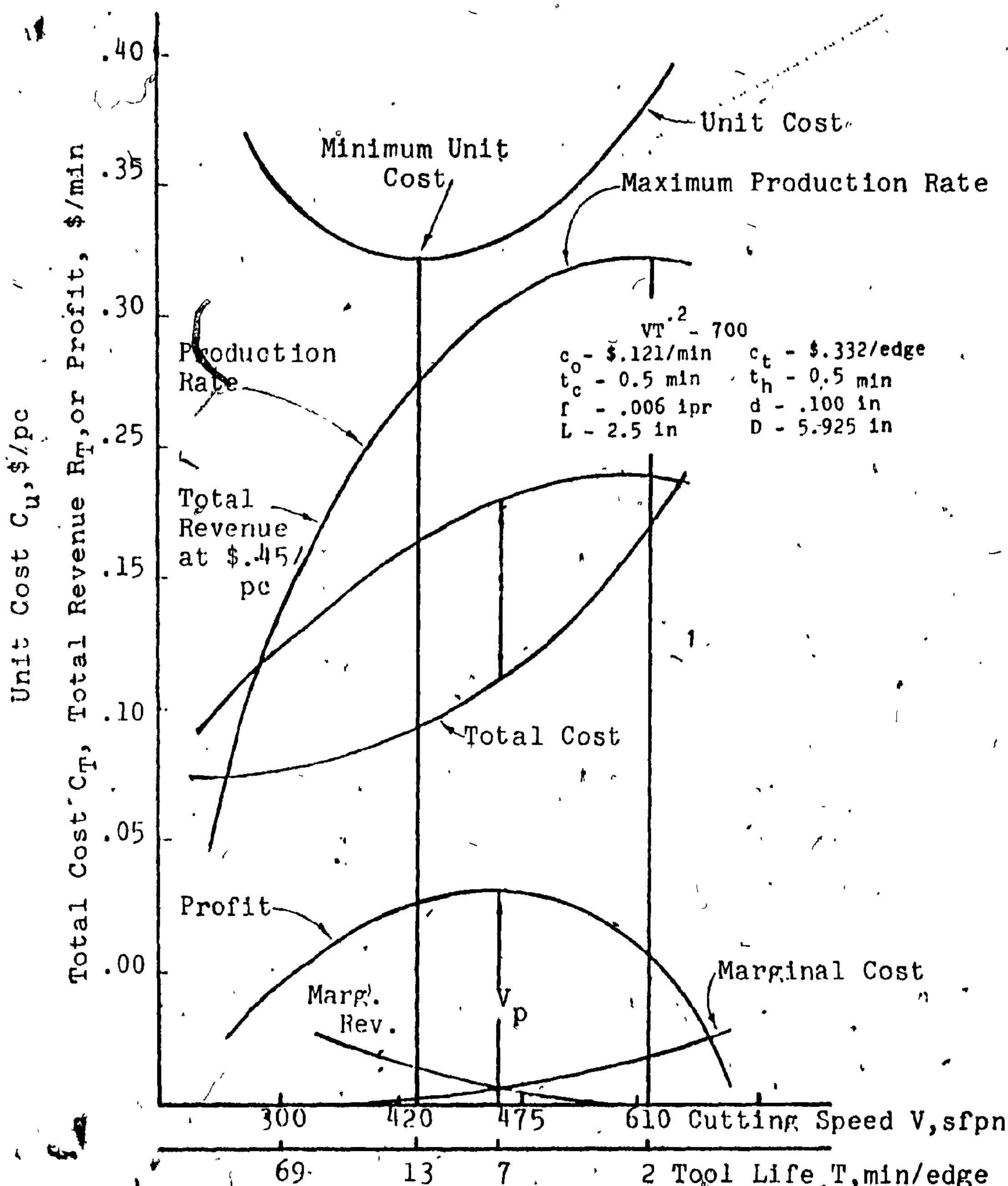

Figure 4.3. Production rate, costs, revenues, and profit versus cutting speed and tool life; and the determination of the maximum profit speed V. (Ref. 13).

shown in Figure 4.3 that the maximum profit is obtained at the cutting speed corresponding to the intersection of the marginal cost and marginal revenue curves, or equivalently at the maximum distance between the total cost and total revenue curves.

Some general trends can be observed if the unit revenue is held constant at various levels. For instance, if the unit revenue was a constant at $0.75 instead of $0.45, then the maximum profit occurs at 500 sfpm, as shown in Figure 4.4. If the unit revenue was a constant at $1.00, the V_p is 525 sfpm. In other words, as the level of the unit revenue is increased, the total cost curve does not change, but the maximum profit cutting speed increases from 475 to 500 to 525 sfpm for unit revenues at $0.45, $0.75, and $1.00 respectively; and V_p approaches V_{max} as an upper limit. Alternatively, as the level of the unit revenue decreases and approaches the minimum unit cost, as illustrated in Figure 4.4 by the total revenue curve for a unit revenue of $0.30, the cutting speed for maximum profit decreases and approaches V_{min} as a lower limit.

It is interesting to note that the range between V_{min} and V_{max} varies according to the relative magnitude of the cost ratio (c_t/c_o) as compared with the tool changing time (t_c). This is so because the only difference in calculating V_{min} and V_{max} is that V_{max} is a function of t_c, whereas V_{min} is a function of $t_e = t_c + c_t/c_o$. Therefore, if the c_t/c_o ratio is small compared with t_c, the difference between t_e and t_c will be small, and the $V_{min} - V_{max}$

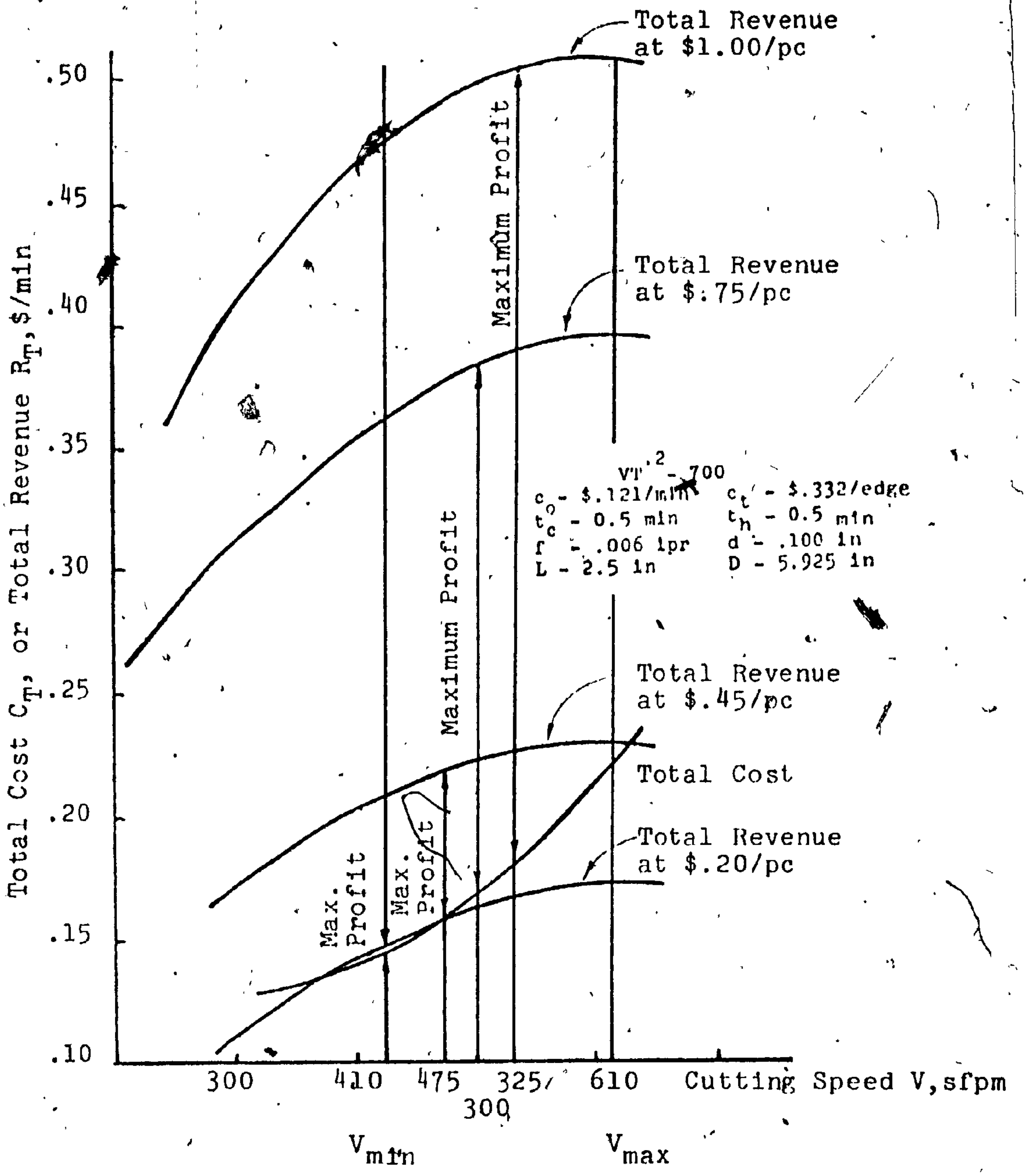

Figure 4.4) Total revenue curves for increasing constant unit revenues, and the corresponding increase in the maximum profit cutting speeds. (Ref.13)

range will be narrow. For instance, if the cost and time parameters in the first case were changed to those shown in the second case of the following, while all other conditions remain the same, the c_t/c_o ratio (0.67) is two-thirds of t_c (1.0) and the $V_{min} - V_{max}$ range is only 50 sfpm as compared with the previous range of 190 sfpm for the first case, where the c_t/c_o ratio (2.75) is over five times as large as t_c (0.5).

	Units	Case 1	Case 2
c_t	\$/edge	0.332	0.10
c_o	\$/min	0.121	0.15
c_t/c_o	min/edge	2.75	0.67
t_c	min/edge	0.5	1.0
t_e	min/edge	3.25	1.07
$V_{min} - V_{max}$	sfpm	190	50

The values of c_o, c_t, and t_c given in the summary were chosen for illustration purposes. The importance of being able to determine accurately these cost and time parameters should be explored.

The unit cost and production rate curves for the conditions of Case 2 are shown in Figure 4.5, along with the corresponding total cost curve. Also shown in Figure 4.5 are the total revenue and profit curves based on a constant unit revenue of \$0.45 and the maximum profit occurs at 500 sfpm. However, it can be seen from the profit curve that there is no significant difference in profit over this

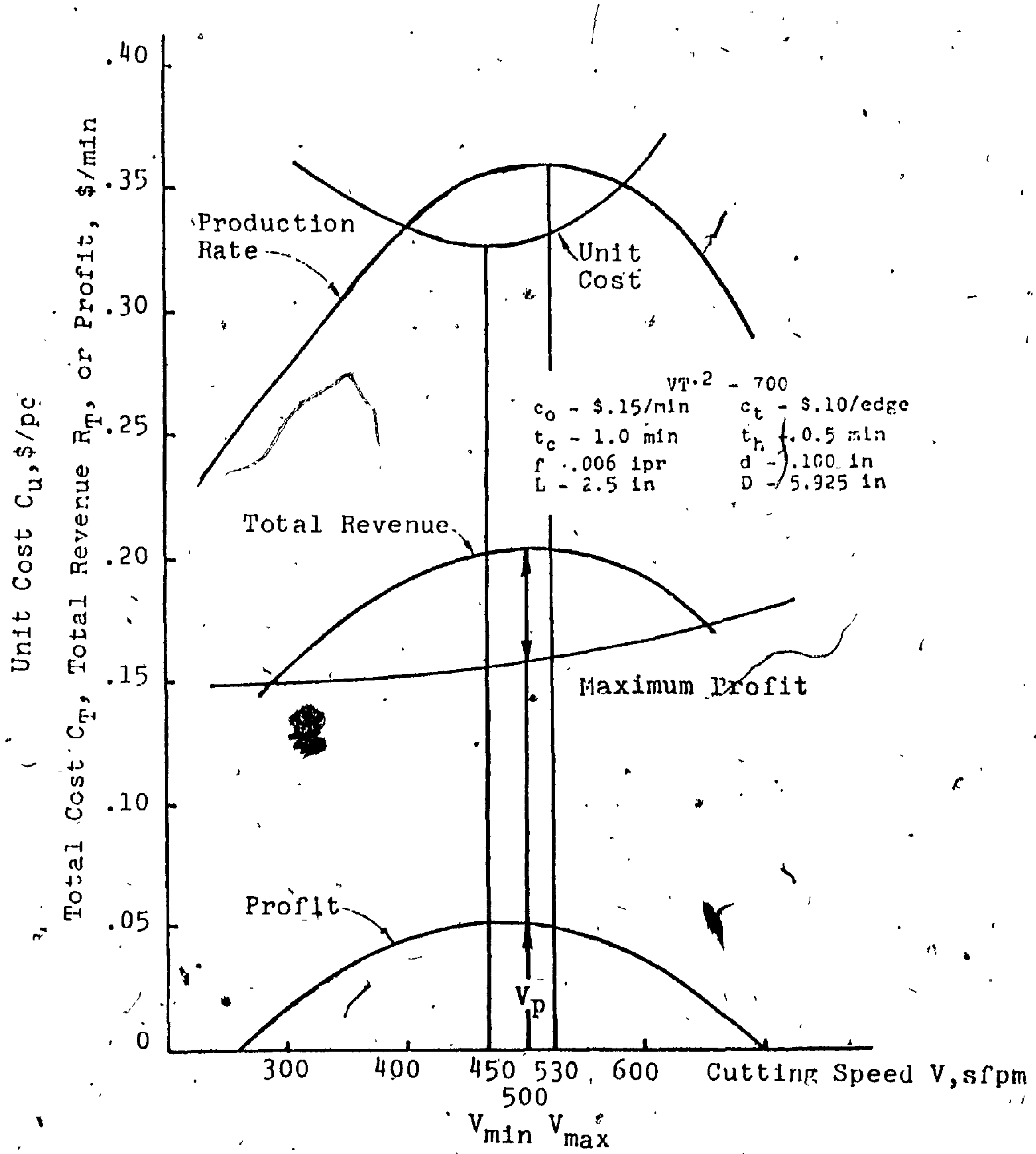

Figure 4.5. A range of maximum profit cutting speeds for the special case where the V_{min} - V_{max} is relatively narrow. (Ref.13)

relatively narrow range of 50 sfpm, and in such special cases any speed between V_{min} and V_{max} will be close to the optimum.

4.6 Effect of feed

The importance of incorporating feed as a variable in the basic model has been acknowledged in prior studies of machining economics. To include feed in the basic model the generalized tool life equation is used, and hence the total unit cost is given by

$$C_u = \frac{c_o \pi DL}{12\ Vf} + \frac{DLV^{\frac{1}{n}-1} f^{\frac{m}{n}-1}}{12\ K^{1/n}} (c_o t_c + c_t) + c_o t_h \quad \ldots 4.9$$

Based on this model, the unit cost is known to be minimized at the highest allowable feed (f_a) with the corresponding cutting speed

$$V_{min} = \frac{k}{f_a^m [(\frac{1}{n} - 1)\ t_e]^n} \quad \ldots 4.10$$

Similarly, the maximum production rate will be obtained at f_a with the corresponding cutting speed

$$V_{max} = \frac{k}{f_a^m [(\frac{1}{n} - 1)\ t_c]^n} \quad \ldots 4.11$$

It is also true that the profit will be maximized at the highest allowable feed, which will be illustrated in the two cases where the unit revenue is a constant and where it is a decreasing variable.

In addition, the effect of feed on the $V_{min} - V_{max}$ range and on the rate of change of the profit will also be considered.

If it is assumed that the highest allowable feed for the turning operation in the previous discussion is $f_a = 0.012$ ipr and that the Taylor equation for this feed is $VT^{0.2} = 500$, then it can be shown by the marginal principle that the maximum profit cutting speed is 350 sfpm, with a corresponding tool life of six minutes, and that the maximum profit is $0.145 per minute, which is almost twice that at f = 0.006 ipr. These results are shown in Figure 4.6, which indicates the increase in total revenue at $f_a = 0.012$ ipr as compared with f = 0.006 ipr in Figure 4.3. The total cost at V_{p_r} is approximately the same at both feeds, and consequently the magnitude of the maximum profit at f = 0.012 ipr is larger than at f = 0.006 ipr. This increase in profit is obtained for any level of constant unit revenue, as illustrated by a comparison of Figure 4.4, in which f = 0.006 ipr, and Figure 4.7 in which f = 0.012. For example, at $0.75 per unit the maximum profit at $f_a = 0.012$ ipr is $0.345 per minute, as compared with $0.23 per minute at f = 0.006 ipr.

4.7 Conclusions

As has been observed earlier that to consider the machining operation in isolation is unrealistic and could lead to misleading results. Most often one will be operating at neither the minimum cost speed nor at the maximum production rate. Thus it would be

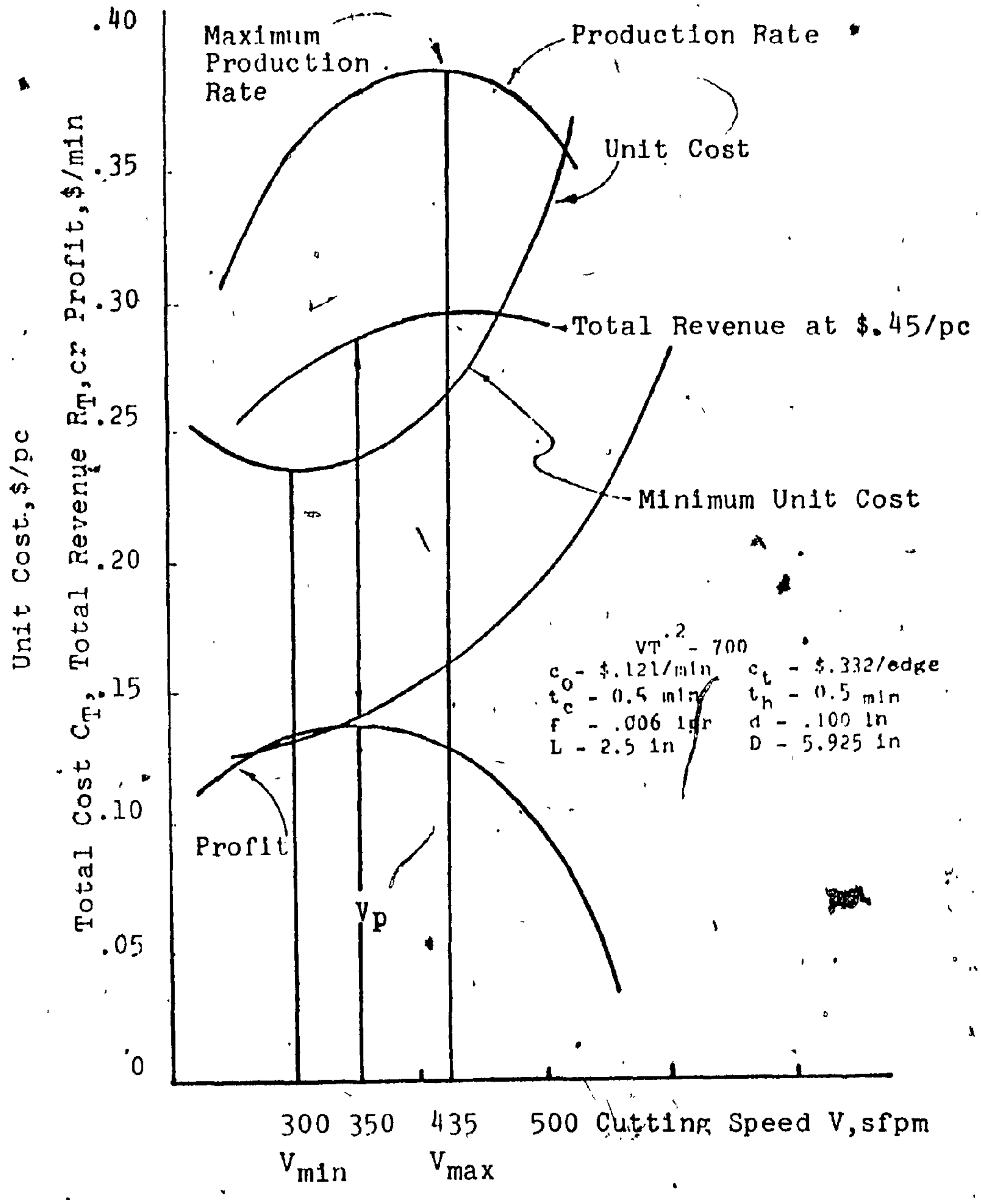

Figure 4.6. Production rate, costs, revenues, and profit versus cutting speed at the highest allowable feed, and the determination of the maximum profit cutting speed. V_p. (Ref.13)

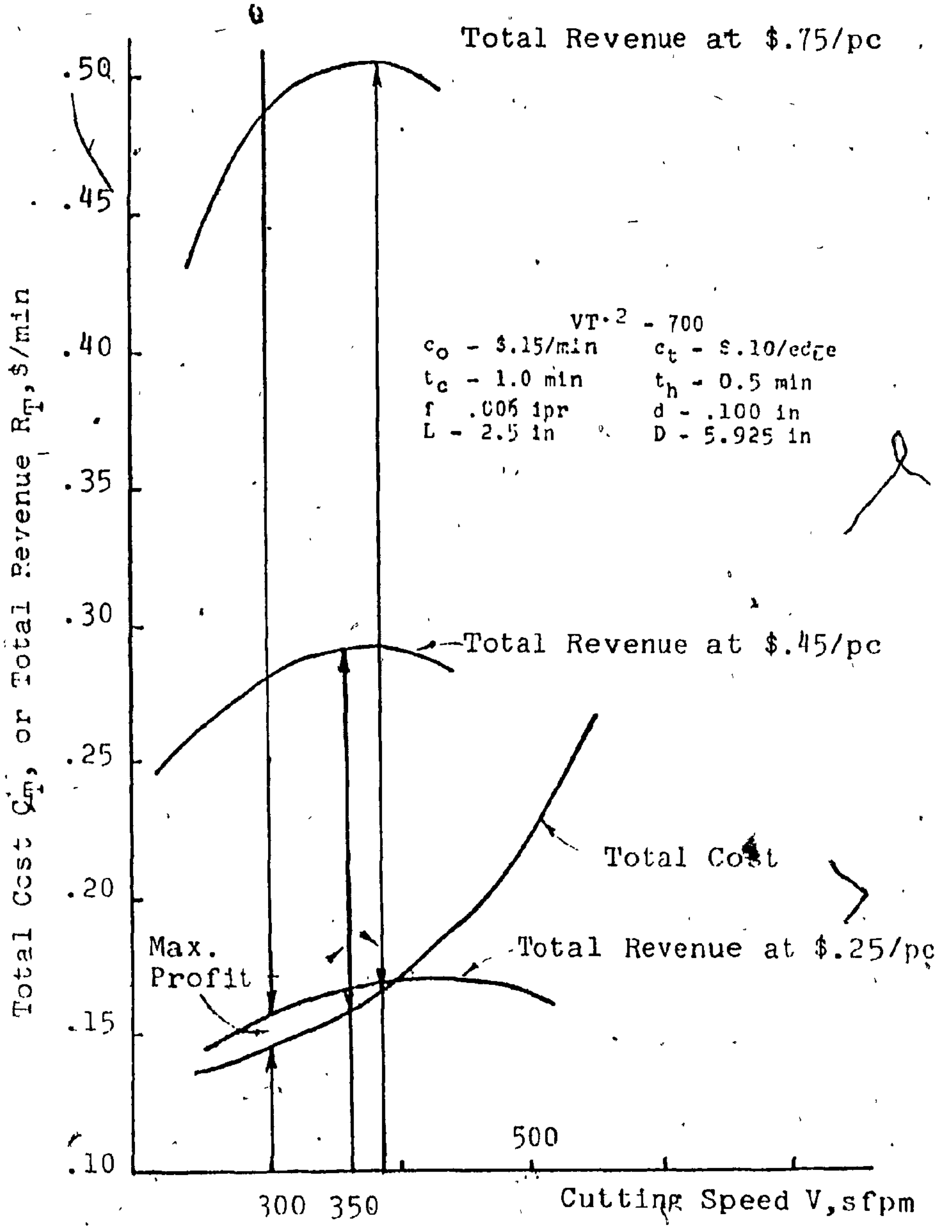

Figure 4.7. Total revenue curves for various constant revenues and corresponding cutting speeds for maximum profit, at the highest allowable feed. (Ref.13)

more pertinent to try and operate at the maximum profit conditions. This condition can be obtained by operating at the production rate where marginal revenue equals the marginal cost. The maximum profit cutting speed will be in the so-called Hi-E range, i.e., the speeds for minimum cost and maximum production rate. The highest allowable feed will be the optimum feed for maximum profit. However, an accurate determination of the cost and time parameters and the tool life parameters is important in order to determine the operating conditions.

CHAPTER V

OPTIMIZING MACHINABILITY PARAMETERS

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5.1 Introduction

So far we have analysed the turning process in detail and obtained the mathematical models that best describe it in all its perspectives. The mathematical model so obtained can now be optimized with respect to various factors to obtain optimum operating conditions. Since no optimization results will be more accurate than that allowed by the accuracy of the mathematical model, it is therefore of paramount importance to have the model depict the physical process as faithfully as possible.

However, before we go on to the mechanics of optimization, it would be instructive to note the various efforts which have been made to simplify the application of this analysis. Although metal cutting was pretty much an art instead of a science for many years, there is evidence of early concern for machinability factors and of efforts to organize systematically these factors so they could be applied scientifically. F. W. Taylor was the one who pioneered in the field of machinability, and his paper "The Art of Cutting Metals" exhibits the early efforts in the field. In the same Transaction, a slide rule, patented in 1904, for solving machinability problems, appeared for the first time. Among the variables represented on this slide rule are tool life, coolant, class of material, feed, and depth of cut.

Continuing assaults on the problem-solution relationship have been made since these early efforts. A nomograph (Figure 5.1) published in ASME's Manual on Cutting of Metals (second edition 1952) deals with the relation between cutting speed and other conditions. Another effort is illustrated in the same manual with a series of charts (V19 through V24) and the following formula. Values from the charts are used in the formula, (Ref. 6)

$$V_t = V_s \times K_h \times K_f \times K_d \times K_{nr} \times K_{ea} \times K_{ra}$$

where

V_t = desired cutting speed with subscript t representing desired tool life

V_s = standard speed from a table

K_h = factor for Brinell hardness number

K_f = factor for feed

K_d = factor for depth of cut

K_t = factor for tool life

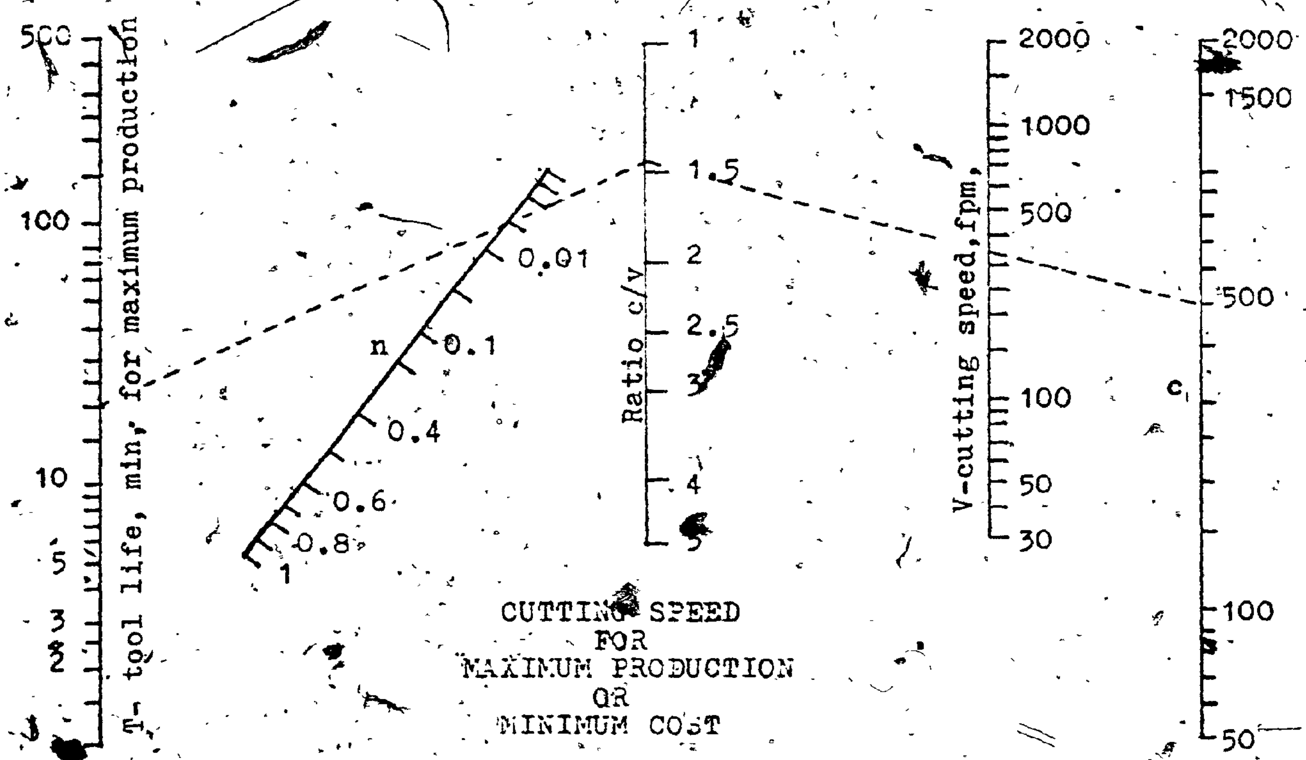

Figure 5.1. A nomograph for obtaining the optimum cutting speed.
(Ref. 6)

K_{nr} = factor for nose radius

K_{ea} = factor for entering angle

K_{ra} = factor for side rake angle

An early success by General Electric in simplifying the solution of machinability problems is the analog computer developed in the mid-1950's by Dr. W. W. Gilbert and his group at the Company's Advanced Manufacturing Engineering Services in Schenectady. Another development came in 1960 in the form of a machinability slide rule or "Hi-E" pocket calculator, developed jointly by R. G. Brierly and E. J. Weller of Metallurgical Products Department in connection with the Carboloy Hi-E (high efficiency machining) philosophy applied to production operations by T. E. Hayes. This aid is still frequently used today in determining feed, speeds and depths of cut for efficient machining.

Although the analog computer and Hi-E pocket calculator will continue as useful problem-solving tools for some time to come, there is no question that the best device known today for solving machinability problems is the digital computer. This equipment is capable of handling a great magnitude of data; it minimizes the possibility of human error in reaching a solution, and it is almost unbelievably fast.

5.2 Hi-E or high-efficiency machining

Hi-E is a technique for determining proper cutting speed for either maximum production or minimum production cost.

When piece cost vs speed is plotted on the same chart with pieces per unit time vs speed, the result is as shown in Figure 5.2. The Hi-E range is bounded on the left by the cutting speed at which minimum cost occurs (the lowest point on the cost curve) and on the right by the speed at which the maximum production rate occurs (the highest point on the production curve; the point beyond which increasing tool-change time outweighs cutting speed gains).

The optimum cutting speed will be found within the Hi-E range. The exact speed selected will depend on whether the emphasis is to be on cost or production rate. Any speed within the Hi-E range will be a compromise between the two factors, whereas any speed outside the Hi-E range will sacrifice both cost and production.

The basic Hi-E equations are as follows:

$$T_p = \left(\frac{1}{n} - 1 \right) \times TCT$$

$$T_c = \left(\frac{1}{n} - 1 \right) \times \left(\frac{t}{M} + TCT \right)$$

where

T_p = tool life (in the cut) for maximum production

T_c = tool life (in the cut) for minimum part cost

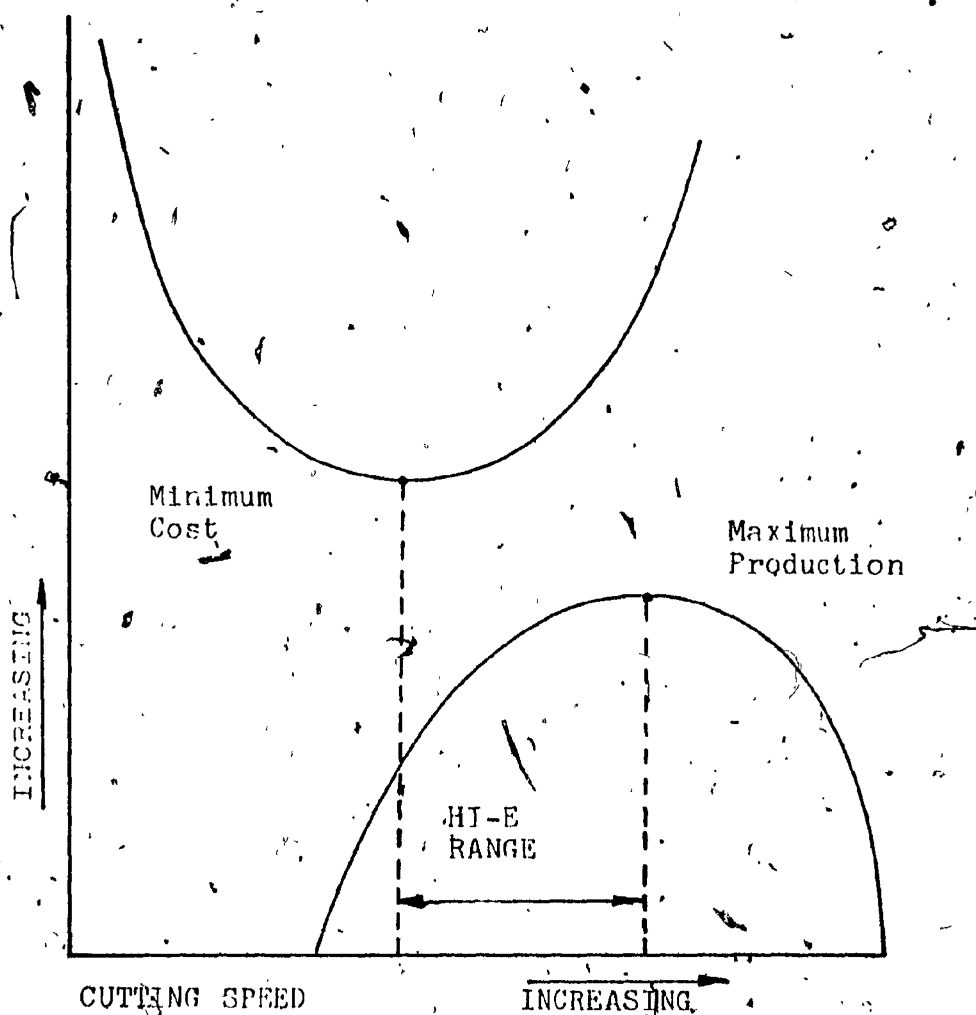

Figure 5.2. The minimum cost and maximum production rate curves with HI-E range.
(Ref.20)

n = slope of tool life line

t = total cost of cutting edge, including cost to change tool and to regrind the cutting edge, and depreciation of brazed tools or mechanical holders

M = machine labour and overhead rate

TCT = tool change time

Since n = 0.250 for carbide tools and n = 0.125 for high speed steel, then

for carbide,

$$\left(\frac{1}{n} - 1 \right) = 3$$

for high speed steel,

$$\left(\frac{1}{n} - 1 \right) = 7$$

and the equations for carbide can be written as follows:

$$T_p = 3 \times TCT$$

$$T_c = \left(3 \times \frac{t}{M}\right) + TCT$$

The logic represented by these equations is included in the computer program.

5.3 Optimization of machining variables by geometric programming

Various researchers have proposed models that they derived from varying viewpoints. However, the basic mathematical model which has been used in the analysis of the machining economics is a unit cost model or an analogous unit time or production rate model. Profit (Okushima and Hitomi 1964; Wu and Ermer 1966) and profit rate (Armarego and Russel 1966) models have also been proposed.

Optimal cutting conditions are usually determined on the basis of the following two criteria as earlier stated:

(a) the minimization of the unit cost

(b) the minimization of the unit time or the maximization of the production rate.

The actual selection of one of these two criteria is dependent upon the particular case being studied as well as upon the dominant production and market conditions.

The problem of the selection of optimal machining conditions in the basic turning operations has been analysed until recently with varying degrees of generality by many investigators.

Thus, a number of authors (Gilbert 1950; Shaw 1959; Brewer and Rueda 1963) studied the economics of machining and considered the influence of cutting speed, V, whereas other investigators (Witthoff 1947, 1948, 1952; Brewer 1958; Brown 1962, Brewer and Rueda 1964; Cook 1966) analysed the effect of both cutting speed and feed rate, s.

The aforementioned analyses were based on the minimization of unit cost.

Unit cost and production rate were also optimized by the determination of the optimal value of cutting speed for turning, milling, drilling, reaming and tapping.

Brewer (1958), Radford and Richardson (1970) considered the problem of optimizing unit cost in turning, taking into account the cutting power available from the machine tool a constraint, and Petropoulos (1971) studied the constrained problem of the selection of optimal machining conditions in face milling by using unit cost and production rate as optimizing criteria and a graphical representation of the optimization conditions and the constraints imposed in the log V - log s plane.

Basically, in the above-mentioned analyses, the optimization procedure, wherever carried out, involved partial differentiation of the unit cost or the production rate (both expressed as a function of V and s) with respect to V or s and equating zero. This is not a realistic approach to the problem because both independent variables, namely, cutting speed and feed rate, could not be treated simultaneously.

Brewer (1966) suggested the use of Langrangian multipliers for the optimization of the constrained problem of unit cost with mainly cutting power as a constraint, but did not proceed with an analytical optimization procedure.

Here the constrained problem of the selection of optimal machining conditions in turning is successfully and easily treated by the application of a relatively new non-linear programming technique, namely, geometric programming. The basic theory and formal proofs for this optimization technique can be found in any work on Operations Research.

Geometric programming deals, in general, with the problem of minimizing non-linear functions, the objective functions, subject to inequality constraints of a certain type and maximizing product functions, the dual functions, subject to certain linear constraints. The original minimization problem called primal program, is transformed to a maximization problem, termed dual program, which greatly simplifies the numerical solution of the optimization problem.

5.4 The formulation of the primal and dual programs

The basic model describing the cost to produce a workpiece by a simple turning operation has, as stated earlier, been derived by different researchers and each obtained a slightly different model. For the purpose of the following discussion the model adopted is as follows: (Ref. 22).

$$C_u = c_o t_m + \frac{t_m}{T}(c_o t_c + c_t + c_o t_h) \qquad \ldots 5.1$$

where

C_u = unit cost or cost per piece

c_o = cost of operating time

c_t = tool cost

t_m = DL/318.5 vs time to machine a workpiece

t_c = tool changing time

T = tool life

V = cutting speed

s = feed rate

L = axial length of cut

D = diameter of the workpiece

The extended Taylor's tool life equation which is used in the expression of unit cost takes the form

$$VT^m s^n = \lambda \qquad \ldots 5.2$$

where m, n, and λ are empirical constants for a given machine tool, workpiece material cutting tool material combination, tool geometry, depth of cut and environment.

It has been shown that equation 5.2 fits satisfactorily to experimental data over a fairly wide range of values of the machining rate variables. Substituting for T and t_m in equation 5.1.

We thus obtain

$$C_u = c_{o1} V^{-1} s^{-1} + c_{o2} V^{(1/m)-1} + K(\$/pc) \qquad \ldots 5.3$$

in which

$$c_{o1} = \frac{c_o DL}{318.5}$$

$$c_{o2} = \frac{DL}{318.5\ \lambda^{1/m}}\ (c_o t_c + c_t)$$

$$K = c_o t_h = \text{constant}$$

The constraints:

Practical considerations limit the possible range of cutting speed and feed rate. The main restrictions are:

(a) The maximum cutting power available

(b) The surface roughness required

(c) The maximum cutting force permitted by the rigidity of the machine tool and accuracy required

(d) The maximum feed rate and rotational speed available from the machine tool

In our investigation, we shall consider items (a) and (b) above as the dominant constraining functions.

Cutting power for a turning operation is given as

$$P_c = C_p V s^{1-z}\ (\text{KW}) \qquad \ldots 5.4$$

where

$$C_p = \frac{k_{s1.1}(\cos^{k})^{-z} d}{6120}$$

$k_{s1.1}$ and z are empirical constants (Kienzle and Victor 1957), k = side cutting edge angle, and d = depth of cut.

Bhattacharyya (1970) and Olsen (1965) have proposed the following model for the surface roughness:

$$R_a = C_R V^{-1.52} s \; (\mu m) \qquad \ldots 5.5$$

for $23 < V < 230$ m/min and $s < 0.75$ mm/rev, where

$$C_R = 2.2 \times 10^4$$

and R_a is the CLA value of surface roughness in μm for 1045 SAE steel and for a cemented carbide tool of ISO-P10 grade with nose radius of 1 mm. In formulating the surface roughness model it should be remembered that there has been ample experimental evidence that the depth of cut has little effect on surface roughness over the range from 0.25 to 3.5 mm.

The primal and dual problem:

From the above analysis regarding the objective function given by equation 5.3, the constant term K is neglected and the constraining functions expressed by equations 5.4 and 5.5, the primal program can be formulated as follows by denoting $V \equiv x_1$ and $s \equiv x_2$, the primal variables:

$$\text{Min } g_1(X) = c_{o1} x_1^{\alpha_{o11}} x_2^{\alpha_{o12}} + c_{o2} x^{\alpha_{o21}} x^{\alpha_{o22}}$$

subject to the forced constraints

$$g_1(X) = c_{11}\, x_1^{\alpha_{111}}\, x_2^{\alpha_{112}} < 1$$

$$g_2(X) = c_{21}\, x_1^{\alpha_{211}}\, x^{\alpha_{212}} < 1$$

and to the natural constraints

$x_1 < 0$ and $x_2 < 0$

where

$$c_{11} = \frac{C_R}{R_{a_m}}$$

$$c_{21} = \frac{C_p}{P_{cm}}$$

P_{cm} and R_{a_m} being the maximum value of the cutting power available from the machine tool and the maximum permissible CLA value of surface roughness, respectively. The coefficients c_{o1}, c_{o2}, c_{11} and c_{21} are positive constants and the exponents α_{o11}, α_{o12} are real numbers.

The associated dual program will take the following form

$$\text{Max } V(\delta) = \left(\frac{c_{o1}}{\delta_{o1}}\right)^{\delta_{o1}} \left(\frac{c_{o2}}{\delta_{o2}}\right)^{\delta_{o2}} (c_{11})^{\delta_{11}} (c_{21})^{\delta_{21}} \quad \ldots 5.8$$

the dual variables, δ_{o1}, δ_{o2}, δ_{11}, and δ_{21} being subject to linear constraints, viz., satisfy a normality condition

$$\delta_{o1} + \delta_{o2} = 1 \qquad \ldots 5.9$$

as well as two orthogonality conditions

$$\begin{aligned} \alpha_{o11}\,\delta_{o1} + \alpha_{o21}\,\delta_{o2} + \alpha_{111}\,\delta_{11} + \alpha_{211}\,\delta_{21} &= 0 \\ \alpha_{o12}\,\delta_{o1} + \alpha_{o22}\,\delta_{o2} + \alpha_{112}\,\delta_{11} + \alpha_{212}\,\delta_{21} &= 0 \end{aligned} \qquad \ldots 5.10$$

and the non-negativity conditions

$\delta_{o1} > 0$, $\delta_{o2} > 0$, $\delta_{11} > 0$, and $\delta_{21} > 0$

Once the problem has been formulated, the usual techniques can be applied. The major difficulty is encountered in the formulation as in all optimization problems.

The geometric programming technique elaborated here is only one of the sophisticated methods available to tackle a problem of this sort; there are several others.

CONCLUSIONS

CONCLUSIONS

The problem of economic machining is analysed from two different viewpoints in this report. However, there are other methods which have been employed to arrive at the optimum cutting conditions with varying degrees of success. Furthermore, each of those methods may be subjected to criticism on account of its inherent merits and demerits.

The two techniques presented here are in the author's opinion the most comprehensive and describe the process in the greatest depth. The optimization technique used is geometric programming, however, as has been mentioned before, other techniques could be employed to obtain equally satisfactory results. This, again, depends upon the mathematical model developed and the type of constraints.

But after all has been said and done the fact remains that the priorities of actual production at times compel one to deviate from the optimum cutting conditions. Practical realities are such that if optimum cutting conditions call for tool changes after three-quarters of a shift, to lower the speed and use one shift as the optimum tool life may result in higher overall efficiency. In highly automated production lines such as automotive production, the use of production rates which will produce the required number of parts per hour to match production line schedules would probably be the optimum machining cutting conditions. Similarly, in times of war the maximum production rate would most likely be more desirable than any other consideration.

But the everlasting search for cheaper machining methods goes on, and for this a better understanding of the tool chip interface during machining is needed. The newer tools and the throw-away carbide inserts have proved to be far better than conventional tooling and the search goes on.

However, in retrospect it can be summarised that there is an optimum cutting speed in all machining operations, and if the principles of economic machining were faithfully applied the savings in costs would be phenomenal.

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