FUNDAMENTAL ISSUES IN GENERAL RELATIVITY: INERTIA, GRAVITATION AND ELECTROMAGNETIC MASS

Vesselin P. Petkov

A Thesis
in
The Department
of
Physics

Presented in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy at
Concordia University
Montreal, Quebec, Canada

December 1997

0-612-40303-3

ABSTRACT

Fundamental Issues in General Relativity:
Inertia, Gravitation and Electromagnetic Mass

Vesselin P. Petkov, Ph. D.
Concordia University, 1997

An opportunity for revealing the nature of inertia and gravitation in terms of both general relativity and the electromagnetic mass theory may have been missed in the first quarter of this century. If the entire mass of an elementary charged particle is regarded as electromagnetic in origin, a hypothesis providing a consistent explanation of inertial and gravitational phenomena emerges. Due to the anisotropy in the propagation of electromagnetic interaction in the vicinity of all (massive) objects the electric field of an electron at rest on the Earth's surface is distorted which gives rise to an electric self-force trying to force the electron to move downwards (hence the passive gravitational mass turns out to be electromagnetic); the anisotropy is compensated if the electron is falling with an acceleration g - in this case its electric field is the Coulomb field and the electron's motion is geodesic (non-resistant) in accordance with general relativity. The behaviour of an electron in an accelerated reference frame is identical (the anisotropy in the speed of light in this case is caused by the frame's accelerated motion). This hypothesis can be experimentally tested and opens up the possibility of (at least partly) controlling inertia and gravitation.

Even if one insists on the present understanding that only part of the electron mass is electromagnetic it still follows that the possibility for (partly) controlling inertia and gravitation and for an experimental test has been present since the beginning of this century when the electromagnetic mass theory was proposed. It has not been realized up to now that it immediately follows from this theory that part of the electron's active gravitational mass is electromagnetic in origin too which means that part of its gravity being caused by its charge (since part of its active gravitational mass itself is electromagnetic) is also electromagnetic in nature. And if we can control other electromagnetic phenomena nothing in principle prevents us from doing so to inertia and gravitation as well.

Acknowledgments

My first sincere thanks go to the external examiner Dr. Bernhard Haisch. Since I believe that the role of the external examiner is either to improve a thesis or to disprove it I consider myself lucky in having the ideal external examiner. Dr. Haisch has spent so much time to make sure that every point in the thesis is rigorously presented that as a result the justification of the ideas in the thesis has become stronger than ever. Responding to his constructive criticism I derived the expressions for the anisotropic speed of light and the Liénard-Wiechert-like term in the scalar and vector potentials of a charge in a non-inertial reference frame in a different, more rigorous way. These derivations will be published shortly.

I would also like to express my thanks to Professor Alfonso Rueda, the alternative external examiner, for his helpful comments and suggestions on two of my papers. I thank Professor Fritz Rohrlich as well for his comments on the same two papers.

I am grateful to Professor David Cheeke, Chair of the Department of Physics, Professor Mariana Frank, Graduate Programme Director of the Department of Physics, and Dr. Claude Bédard, Dean (then Vice-Dean) of School of Graduate Studies, for allowing me to write a Ph. D. thesis on my own research project and supervision.

I owe special thanks to Mostafa Showleh, the Physics Labs supervisor, for his open-mindedness, persistent interest and efforts in trying to set several experiments to test some of the consequences of this thesis. My thanks also go to Raymond Harris and Derek Nelligan, graduate students, for their enthusiasm and assistance.

And last, I would like to express my deep gratitude to my wonderful family for their understanding and support even through suffering. The completion of this enormous task would not have been possible without their endless love and faith in me.

Contents

1 Introduction **1**

2 A Uniformly Accelerated Charge and the Principle of Equivalence **15**

2.1 Introduction 15

2.2 A charge in a uniformly accelerated reference frame 19

2.2.1 A charge at rest in a uniformly accelerated reference frame 19

2.2.2 A charge falling in a uniformly accelerated reference frame 33

2.3 A charge in the Earth's gravitational field 35

2.3.1 A charge supported in the Earth's gravitational field 35

2.3.2 A charge falling in the Earth's gravitational field 43

2.4 Conclusions 45

3. Inertia, Gravitation and Electromagnetic Mass **48**

3.1 The electromagnetic mass of the electron 48

3.2 An electron in a uniformly accelerated reference frame 51

3.2.1 An electron at rest in a uniformly accelerated frame 51

3.2.2 An electron falling in an accelerated reference frame 56

3.3 An electron in a frame N^g at rest with respect to the Earth 59

3.3.1 An electron at rest in N^g 59

3.3.2 An electron falling in N^g 63

4. Some Insights **65**

4.1 The curvature of spacetime 65

4.2 The principle of equivalence 69

5. Has There Been an Alternative Path to General Relativity? **71**

5.1 Introduction 71

5.2 The Newtonian limit of general relativity 73

5.3 The Newtonian limit of general relativity without assumptions 78

6. **Conclusions and Contributions** **82**

6.1 Conclusions 82

6.2 Contributions 86

Appendix: Calculation of the self-force **90**

References **97**

List of Figures

2.1 Three light signals emitted at points A, B and C in an accelerating elevator arrive simultaneously at the point M' which is displaced at a distance δ from the middle point M (in a direction opposite to **a**) 20

2.2 A light signal emitted at point B in an accelerating reference frame and propagating in the direction of the frame's acceleration **a** 24

2.3 A light signal emitted at point A in an accelerating reference frame and propagating in opposite direction of the frame's acceleration **a** 26

2.4 The length l' of a charge moving towards an inertial reference frame I (as determined in I) appears larger than its actual length l 28

2.5 The length of a charge at rest in N^a appears larger in the direction of the frame's acceleration 29

2.6 A light signal emitted at B propagates upwards against the direction of the gravitational acceleration **g** 38

2.7 A light signal emitted at A propagates downwards in the direction of the gravitational acceleration **g** 39

2.8 Point B on the Earth's surface and point A above it at a distance r 40

Chapter 1

Introduction

According to the Newtonian gravitational theory a gravitational force is acting on a body when it is falling towards the Earth and when it is at rest on the Earth's surface. This situation is quite different in general relativity. A falling body is represented by a geodesic world line which means that the body is moving by inertia with no four-dimensional acceleration (in a curved spacetime) and no four-dimensional force (in a curved spacetime) forcing it to move downwards. The question whether general relativity predicts that a body on the Earth's surface is subjected to a force has been carefully avoided in the books and articles on general relativity (in fact, I am not aware of a case whether this issue has been addressed). As a result the conceptual picture of general relativity is not quite clear. On the one hand, the fact that a falling body is not subjected to a force and moves towards the Earth due to the spacetime curvature implies that the gravitational field (unlike the electromagnetic field) is rather a geometric than a force field. This means that if there is no gravitational force there is no gravitational energy either. This conclusion is reinforced by the stubborn refusal of the mathematical formalism of general relativity to produce a **tensor** of the energy and momentum of a gravitational field[1] . On the other hand, it seems that the experimental evidence leaves no doubt - there is gravitational energy. It is sufficient to mention only the tidal electric power stations converting what appears to be gravitational energy into electric energy.

If the fact that a body on the Earth's surface is subjected to a force (regarded as gravitational) is taken into account the picture represented by general relativity becomes even more confusing. The theory is telling us that a falling body is moving by inertia (despite the fact that it has a "flat space-

[1] In general relativity the energy and momentum of a gravitational field are represented by a pseudo-tensor, but a real physical quantity should be described by a tensor [1] - [3]. It was Einstein who realized that it was not possible to represent the gravitational energy and momentum by a tensor [4]. However, he kept the pseudo-tensor in order to have the global, integral laws of conservation of energy and momentum observed - without it energy and momentum of matter are not conserved globally; only the local, differential laws of conservation hold. An attempt to account for the impossibility to construct a gravitational energy and momentum tensor by assuming that the gravitational energy cannot be localized still cannot solve the problem since it does not provide a full description of the energy of gravitational field [5]. A closer look at the lack of integral laws of conservation in general relativity suggests that it appears to be a direct consequence of Noether's theorem according to which "symmetries imply conservation laws" [6]: in a curved space-time there are no symmetries (the spacetime is anisotropic) and consequently there are no global conservation laws either.

time" acceleration) which implies that the gravitational field is a geometric field with no real gravitational force (and no energy-momentum tensor of the gravitational field) and does not provide a clear answer on the nature of the force a body on the Earth's surface is subjected to. If the theory offered an unambiguous explanation of the force acting on the body this would mean that the conclusion about the geometric nature of the gravitational field within general relativity should be subjected to a severe scrutiny. Such an internal confrontation in general relativity might have produced a deeper insight into the mechanism of gravitational interaction but, unfortunately, it has never happened. At present conflicting accounts of the nature of gravitational field coexist peacefully and this does not seem to bother too much the scientists working in the field of general relativity.

Several questions must be answered in order to make the conceptual picture provided by general relativity and its mathematical formalism consistent:

1. Why is no force acting on a body falling towards the Earth's surface?

2. Why is a body on the Earth's surface subjected to a force?

3. What is the nature of that force?

4. Why does the formalism of general relativity refuse to produce a tensor of the energy and momentum of gravitational field?

5. What is the nature of gravitational field and is there a gravitational energy?

A closer look at the first two questions reveals some similarity with the motion of a body described by special relativity (i.e. a body in flat spacetime). A body falling towards the Earth's surface is described by a *geodesic* worldline and the worldline of a body on the Earth's surface is *not* geodesic. A body in flat spacetime is represented by a *geodesic* worldline if it is moving with constant velocity (i.e. moving by inertia). The worldline of an accelerating body is *not* geodesic. In both cases when the worldline is not geodesic there is a force acting on the body. We call the force on the body on the Earth's surface gravitational force and the force acting on the accelerating body - inertial force. This forces give rise to the gravitational and inertial mass of the two bodies, respectively. It is a fact that a body's inertial and gravitational mass are equivalent, but no one knows why.

We shall start with studying the connection between the appearance of force when the worldline of a body is not geodesic. This provides us with the opportunity to address both inertia and gravitation[2] . The examination of the experimental fact of equivalence of inertial and gravitational mass (i.e. the

[2] There has been another recent attempt to find a common nature of both inertia and gravitation. It has considered inertia to be generated by the vacuum electromagnetic zero-point-field [7]; gravitation has also been considered to be induced by the electromagnetic fluctuations of the vacuum [8].

equivalence of inertial and gravitational force) contains the key for answering all of the above questions[3] .

Like the inertial mass which is defined as the resistance an object offers to being accelerated (i.e. to being prevented from falling in an accelerated reference frame), the gravitational mass is similarly defined as the resistance an object offers to being prevented from falling in a gravitational field. The world path of a body falling in a non-inertial frame (either an accelerated frame N^a or a frame N^g at rest in a gravitational field) is geodesic and its motion is characterized as non-resistant (offering no resistance to falling in N^a and N^g, respectively). If, however, the body is at rest either in N^a or in N^g, an acceleration ($\mathbf{a}$ and $\mathbf{g}$ respectively) is associated with it and its world path is deformed (not geodesic). In both cases the body resists its acceleration and the deformation of its world path. This resistance to acceleration may be also regarded as a four-dimensional stress which arises in the body's deformed world path and opposes the deformation, i.e. the acceleration of the body whose world path is undergoing deformation. As the electric self-force with which each non-inertial elementary charged particle acts upon itself (on account of its own electric field) is the only known force with which a body may act upon itself, it looks natural to identify this self-force with the source of the

[3] Braginsky and Panov [56] in 1970 tested the equality of inertial and gravitational mass to 1 part in 10^{12}.

resistance which a non-inertial body offers to being accelerated, i.e. deviated from its inertial state. Stated another way, inertia and gravitation (or, more precisely, inertial and gravitational force), defined as the resistance to the accelerations **a** and **g**, respectively, may be regarded as an electromagnetic resistance originating from the interaction of a non-inertial charge with its own electromagnetic field. This approach effectively brings back the idea of the electromagnetic mass of the electron [9-13] (for a brief history of this idea see [14]).

It is now accepted that the electromagnetic mass of the electron

$$m_e = \frac{e^2}{8\pi\epsilon_0 r c^2} \tag{1.1}$$

where r is its radius, accounts for only a fraction of its mass. In fact, the issue of what part of the electron mass is electromagnetic is presently open (if not almost abandoned [15]). This issue cannot be adequately addressed from quantum-mechanical standpoint since quantum mechanics does not provide us with any model of the electron itself (and quantum objects in general), but only describes the electron's state and relates results of measurements. That is why any treatment of the electromagnetic mass of the electron remains essentially classical. There are attempts to construct relativistically invariant quantum theories of extended objects such as strings [16], solitons [17, 18], bags [19], and quantons (which despite being 4-points themselves provide a detailed model of the electron's structure by bringing the idea of atomism to its logical completion: atomism not only in space but in time as well - four-atomism [20]), but up to now no attempt has been made to apply these theories to the electromagnetic mass issue.

On the other hand, as Rohrlich [21] points out, the hope that the electron mass is of purely electromagnetic nature has been abandoned long since it was first proposed near the turn of the century and any attempt to go back to this idea will hardly look justified. If, however, it is assumed that it is the (equal) resistance to the acceleration $\mathbf{a}$ of an accelerated elementary charged particle ($\mathbf{a} = -\mathbf{g}$) and to the acceleration $\mathbf{g}$ of the same particle at rest on the Earth's surface that is responsible for the inertial and gravitational mass of that particle (leading to its entire mass being of electromagnetic nature), a consistent theory of inertial and gravitational phenomena emerges. Not only does such a theory, which for brevity will be referred to as GTMG (General Theory of Motion and Gravitation) hypothesis, account for the equivalence of inertial and gravitational phenomena (and mass), but also offers a way to be experimentally tested. The main consequences of this hypothesis are:

1. All elementary particles which possess a *rest* mass should be charged. This consequence is in agreement with the contemporary understanding that an electrically neutral particle (a neutron, for example), at the most

fundamental level, is thought to be made up of smaller constituents (quarks), which do carry electric charge. In such a way the GTMG hypothesis accounts for the mass of what we call neutral particles as well since, according to it, there are no truly electrically neutral particles.

2. Both inertial mass and gravitational mass of an elementary charged particle (an electron, for instance) are equal due to their common origin: the distorted electric field of a non-inertial electron (represented by a deformed world path) gives rise to a self-force with which the electron acts upon itself and resists any acceleration (i.e. any deformation of its world path). The fact that the instantaneous electric field $\mathbf{E}^a$ of an accelerated (with an acceleration $\mathbf{a} = -\mathbf{g}$) electron and the electric field $\mathbf{E}^g$ of an electron on the Earth's surface are *equally distorted* producing *equal* self-forces is regarded as the major justification of the GTMG hypothesis (the instantaneous field $\mathbf{E}^a$ is considered in order to obtain only its deformation due to the electron's acceleration). The distortion of the electric field of an accelerated electron ($\mathbf{a} = -\mathbf{g}$) is caused by its accelerated motion; if viewed by a non-inertial observer at rest with respect to the electron, the deformation of its field is caused by the anisotropy of the speed of electromagnetic interaction in the electron's reference frame [22]. The equally distorted electric field of an electron on the Earth's

surface is due to the greater speed of the electromagnetic interaction towards the Earth than in the opposite direction which is caused by the Earth's mass. The self-forces (resulting from the identically distorted fields of the accelerated electron and the electron on the Earth's surface) have the right forms $\mathbf{F}^a_{self} = -m^a\mathbf{a}$ ($\mathbf{F}^a_{self}$ opposes $\mathbf{a}$) and $\mathbf{F}^g_{self} = m^g\mathbf{g}$ ($\mathbf{F}^g_{self}$ has the direction of $\mathbf{g}$) and $\left|\mathbf{F}^a_{self}\right| = \left|\mathbf{F}^g_{self}\right|$ from where the equivalence of the inertial and gravitational mass, $m^a = m^g$, immediately follows.

3. According to the GTMG hypothesis there is no mass at all, but only charges which means that it is these charges (and their fields) that represent the active gravitational mass of a body. As the active gravitational mass of an object (i.e. its charges) is responsible for its gravity (i.e. the space-time curvature around it), the anisotropy of the speed of electromagnetic interaction in its vicinity turns out to be the very space-time curvature (the object's gravity) and not a consequence of it if its mass is entirely electromagnetic in nature: the larger speed of electromagnetic interaction towards the Earth than in the opposite direction is responsible for the free fall of an electron in the Earth's gravitational field (as we shall see in Chapter 3) and for the distortion of the Coulomb field of an electron (at rest on the Earth's surface) which in turn gives rise

to its passive gravitational mass. It is the consequence of the GTMG hypothesis that the anisotropy of the electromagnetic interaction in the vicinity of a body is caused by the body's charges (and their fields) that makes an experimental test possible and which may lead to controlling inertia and gravitation.

4. It follows from the equal anisotropy of the speed of electromagnetic interaction in a non-inertial (accelerated) reference frame, N^a, and in a non-inertial frame, N^g, at rest on the Earth's surface, that the principle of equivalence is a corollary of the GTMG hypothesis and not an independent postulate: the distorted electric field of an electron at rest in N^a gives rise to the electron's inertial mass; the (passive) gravitational mass of an electron on the Earth's surface also originates from its distorted electric field. The world path of the electron in both cases is equally deformed - equally deviated from its geodesic state.

The main purpose of this thesis is to show that due to the questions of the nature of gravitational field in general relativity and of the electromagnetic mass of the electron being presently open, the GTMG hypothesis is worth considering and testing because it gives a consistent and common explanation of

1. the nature of mass, inertia, and gravitational attraction;
2. the identical behaviour of a body in what we traditionally call a gravitational field with intensity $\mathbf{g}$ and in an uniformly accelerated ($\mathbf{a} = -\mathbf{g}$) reference frame (i.e. of the principle of equivalence):
3. the nature of space-time curvature;
4. why the speed of gravitational waves is equal to the speed of light, and
5. the relativistic increase of the mass.

It is also intended to imply that the GTMG hypothesis sheds some light on the meaning of the following facts as well:

1. the tensor nature of the mass;
2. the energy-mass equivalence;
3. the mass defect;
4. the effective mass of the electron in solids.

It also provides the basis for further study of the Aharonov-Bohm and Aharonov-Casher effects predicting that a similar effect should be observed when laser beams are employed instead of electron and neutron beams (such an experiment can be carried out to test the electromagnetic mass hypothesis). Since this hypothesis considers the anisotropy in the propagation of electromagnetic interaction towards and away from the Earth (and the resulting self-force acting on an electron at rest on the Earth's surface) to account fully for the gravitational effects the electron is subjected to (in accordance with general relativity's result that there is no gravitational force, but only space-time curvature), it follows that the very existence of the gravitational force (interaction) should be reexamined.

The GTMG hypothesis is worth considering even if one insists on the present understanding that only a part of the electron mass is electromagnetic. This hypothesis reveals that the possibility for controlling inertia and gravitation and for an experimental test has been, in fact, present since the beginning of this century when it was discovered that part of the electron mass is electromagnetic (which implies that the nature of an electron's inertia is partially electromagnetic in origin). It has not been noticed up to now that it immediately follows from this discovery that part of the electron's active gravitational mass is electromagnetic in origin too which means that part of its gravity being caused by its charge is also of electromagnetic nature. And if we can control other electromagnetic phenomena nothing in principle prevents us from doing so to inertia and gravitation as well.

In order to demonstrate that inertial and gravitational mass are in fact electromagnetic mass it will be necessary to prove that due to the absolute property of a worldline to be geodesic or not an association of a Coulomb field with an inertial charge (represented by a geodesic worldline) and an association of a distorted electric field with a non-inertial charge (whose worldline is not geodesic) should be observerindependent (the same for an inertial and a non-inertial observer). In other words, it should be shown that (i) for both an inertial observer, I, and a non-inertial (uniformly accelerated or at rest in a uniform gravitational field) observer, N, the electric field of an inertial charge (falling in N) is the Coulomb field, and (ii) both I and N detect the same distorted electric field of a non-inertial charge (at rest with respect to N). This will be done in Chapter 2 and the obtained result will be also used to resolve the controversial issue of whether or not a charge falling in a gravitational field radiates and whether it violates the principle of equivalence.

In Chapter 3 the idea of the electromagnetic mass of the electron and one of its consequences (the relativistic increase of mass) are briefly discussed and the electric field of an electron in an accelerated frame and the resulting self-force are calculated; it is also concerned with the calculation of the electric field of an electron in a reference frame at rest on the Earth's surface and the self-force acting on the electron. Chapter 4 discusses the meaning of spacetime curvature and the principle of equivalence. Chapter 5 considers whether there has been an alternative path to general relativity.

Chapter 2

A Uniformly Accelerated Charge and the Principle of Equivalence

2.1 Introduction

The questions whether a uniformly accelerated charge radiates and if so whether there is a contradiction with the principle of equivalence have been a matter of controversy for a long time. On the one hand different authors come to different conclusions regarding the radiation from a uniformly accelerated charge. Pauli [1] and von Laue [23] found that there should be no radiation, while Schott [24], Milner [25], Synge [26], Fulton and Rohrlich [27], Coleman [28], Kovetz and Tauber [29], Boulware [30], and Ren and Weinberg [31] drew the opposite conclusion. On the other hand, a radiating uniformly accelerated charge gives rise to a paradox: by the principle of equivalence a static charge in a uniform gravitational field (which, being static, does not radiate) should radiate if a uniformly accelerated charge radiates. The attempts to resolve this paradox have been concentrated on exploring the idea that radiation is not a covariant concept. Fulton and Rohrlich [27], Rohrlich [32], Boulware [30], and Ren and Weinberg [31] argue that the paradox is removed if it is shown that not all reference frames register radiation from the accelerated charge.

The purpose of this paper is to show that an approach to the problem of radiation from an accelerated charge, explicitly taking into consideration the inertial state of the charge, naturally agrees with the principle of equivalence and leads to no paradox and to no need for regarding the radiation from an accelerated charge as non-covariant.

The property of a worldline to be geodesic (or not) is an absolute one which, apart from being an absolute geometric characteristic, is, physically, a direct manifestation of the absolute distinction between inertial motion (represented by a geodesic worldline) and non-inertial motion (represented by a worldline which is curved, not geodesic). The worldlines of a charge which is falling in an accelerated frame, N^a, and a charge, falling in a reference frame, N^g, at rest in a uniform gravitational field, are geodesic. Both charges are inertial particles: the first moves by inertia in a flat space-time whereas the motion of the second one is also inertial (force-free) but in a curved space-time. For this reason both an inertial observer, I, (falling with the charge) and a non-inertial observer (at rest in N^a and N^g) associate a Coulomb field with them.

The worldlines of a charge at rest in an accelerated frame N^a and a charge at rest in a uniform gravitational field (i.e. in N^g) are not geodesic. As a result the electric fields of the two non-inertial charges are distorted for the observers at rest in N^a and N^g as well as for an inertial observer (the non-inertial charges have equally distorted fields, as the principle of equivalence requires, if their accelerations satisfy the relation $\mathbf{a} = -\mathbf{g}$).

In such a way, as the shape of the electric field of a charge is fully determined by its state of motion (inertial or non-inertial), i.e. by its worldline being geodesic or not, it is observerindependent - both an inertial and a non-inertial observer detect the same shape of the electric field. The relativistic distortion of the electric field of a charge (due to the relativistic contraction), which is observerdependent, depends on the observers' relative velocity. The distortion of the electric field of a charge due to its acceleration, however, is clearly distinguishable from the distortion due to the relativistic contraction. In order not to confuse the study of the dependence of the electric field shape on the charge's inertial state with its relativistic contraction, this paper avoids the relativistic deformation of the field by the usual procedure of considering the accelerated charge instantaneously at rest [33, 34].

The assumption that the shape of the electric field of a charge is determined by the charge's worldline being geodesic or not removes both the difficulty with the principle of equivalence and the need for the radiation to be considered non-covariant: the electric field of a non-inertial charge (a charge at rest either in an accelerated frame or in a uniform gravitational field) will be distorted for both an inertial observer, I, and an non-inertial observer, N, at rest with respect to the charge. As only a distorted electric field may give rise to radiation and as both I and N detect the same distorted electric field of the charge its radiation behaviour should be the same for I and N.

In Section 2 the electric fields of a charge at rest in an accelerated reference frame, N^a, and of a charge falling in N^a are calculated and is shown that an observer in N^a detects a distorted field of the charge at rest in N^a and a Coulomb field of the falling (inertial) charge. Section 3 deals with the shape of the electric field of a charge at rest in a reference frame N^g (which is static in a uniform gravitational field) and of a charge falling in N^g, and finds similarly that an observer at rest in N^g detects a distorted field of the charge at rest in N^g and a Coulomb field of the falling (inertial) charge.

2.2 A charge in a uniformly accelerated reference frame

2.2.1 A charge at rest in a uniformly accelerated reference frame

Unlike an inertial observer I, a non-inertial (accelerated) observer at rest in an accelerated reference frame N^a can determine from within his reference frame that it is an accelerated frame. All effects observed in N^a and distinguishing a non-inertial reference frame from an inertial one are attributed to its accelerated motion (there is no need for the principle of equivalence to be invoked to explain the effects). One of these effects is the anisotropy of the speed of light in N^a which leads to another effect - a distortion of the electric field of a charge at rest in N^a.

In order to find the expression for the anisotropic speed of light in N^a let us consider the following thought experiment (Figure 2.1): two light signals are emitted simultaneously in N^a (an accelerating elevator) from two points A and B separated by a distance $2r$. The signal emitted at B is propagating in a direction parallel to the acceleration of N^a and the other one (emitted at point A) in the opposite direction. A third light signal is emitted at point C and propagates in a direction perpendicular to the frame's acceleration. If N^a were an inertial frame the three light signals would arrive simultaneously

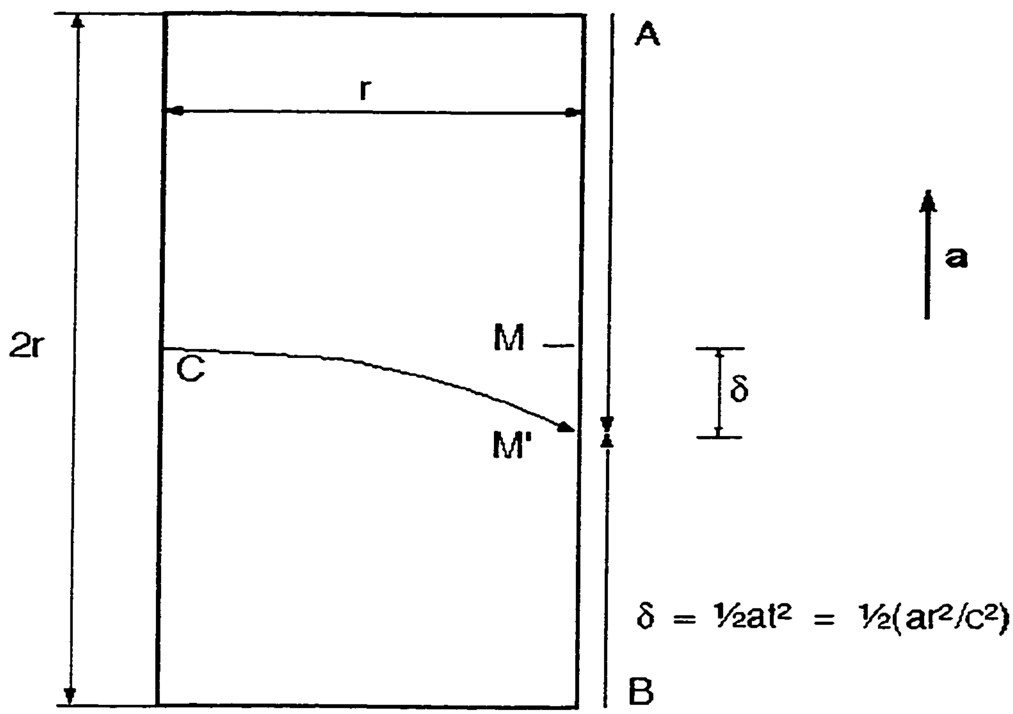

Figure 2.1: Three light signals emitted at points A, B and C in an accelerating elevator arrive simultaneously at the point M' which is displaced at a distance δ from the middle point M (in a direction opposite to $\mathbf{a}$).

at the middle point M in a time $t = r/c$. N^a, however, is an accelerating reference frame and the signals in the *same* time t will simultaneously reach not point M but point M' (why it will take the same time for the signals to converge to point M' is evident from the trajectory of the signal emitted at C - to an order of c^{-2} the light still travels the distance r and the speed of light is not affected since it is perpendicular to the frame's acceleration). An observer at rest in N^a concludes that the three light signals arrive simultaneously at M' because during the time t the light signals travel toward the middle point, N^a will move at a distance $\delta = \frac{1}{2}at^2$ (N^a is moving with an acceleration and this is an experiment allowing an observer in N^a to establish from within N^a that N^a is not an inertial reference frame). In such a way an observer at rest in the accelerating reference frame N^a will find that the light signals will simultaneously reach point M' which is displaced from the middle point M (in a direction opposite to the frame's acceleration) by

$$\delta = \frac{1}{2}at^2 = \frac{ar^2}{2c^2}$$

where $t = r/c$ is the time it takes the three light signals emitted at A, B and C to travel the distances $r + \delta$, $r - \delta$ and r, respectively as measured in the accelerating reference frame. When the light signals meet, the observer in N^a will determine that the light signal propagating downwards (in a direction opposite to $\mathbf{a}$) has travelled a distance $r + \delta$ while for the *same* time the light signal propagating upwards (parallel to $\mathbf{a}$) has covered a smaller distance $r-\delta$. This shows that in N^a the speed of light in the direction of $\mathbf{a}$ is smaller than in the opposite direction and its average value is

$$c^a_{\uparrow} = \frac{r-\delta}{t} = \frac{r}{t} - \frac{at}{2} = c - \frac{ar}{2c} = c\left(1 - \frac{ar}{2c^2}\right).$$

The average anisotropic speed of light downwards (in a direction opposite to $\mathbf{a}$) is

$$c^a_{\downarrow} = \frac{r+\delta}{t} = \frac{r}{t} + \frac{at}{2} = c + \frac{ar}{2c} = c\left(1 + \frac{ar}{2c^2}\right).$$

In vector form we have

$$c^a = c\left(1 - \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2}\right), \tag{2.1}$$

where c^a is the anisotropic speed of light as determined in N^a, and $\mathbf{r}$ is a vector with its origin at the point where a light (or more generally an electromagnetic) signal is emitted and its end at the point where the signal is measured.

Up to now (as far as I am aware) an explicit[1] expression for the anisotropic speed if light in a non-inertial frame has not been derived [2] . An average

[1] An implicit expression for the speed of light contains in Rindler's calculation of the gravitational red shift [40].

[2] I am grateful to Dr. Bernhard Haisch who drew my attention to a paper by L. I. Schiff published in 1960 [55] in which the following formula for the anisotropic speed of light

$$c' = c\left(1 - \frac{GMx^2}{c^2r^3} - \frac{GM}{c^2r}\right)$$

is derived (for the case of deflection of light rays passing close to the Sun). Dr. Haisch showed that this formula may be written as

$$c'(\phi) = c\left(1 - \frac{\mathbf{g}\cdot\mathbf{r}}{c^2}\left(1 + \cos^2\phi\right)\right)$$

anisotropic speed of light has not been defined and used either. Without it, however, the electric potential of a charge in a non-inertial reference frame cannot be calculated. Here it will be necessary to demonstrate that the expression for the anisotropic speed of light is in agreement through the principle of equivalence with the general relativistic expression for c in a curved spacetime [35]:

$$c' = c\left(1 + \frac{gr}{c^2}\right). \tag{2.2}$$

This formula gives the speed of light at a given spacetime point - it is not an average velocity (and does not contain the factor of 1/2 present in the second term of 2.1).

Consider the propagation of light in an accelerating reference frame N^a (Figure 2.2). A light signal is emitted at B and propagates upwards to point

---

where ϕ is the angle which the vector $\mathbf{r}$ (beginning at the point a light signal is emitted and ending at the point where the signal is measured) forms with the gravitational acceleration $\mathbf{g}$. $c'(\phi)$ is the speed of light at a given spacetime point. This formula is a coordinate velocity and cannot be applied for the case when the speed of light at a fixed point is viewed from another fixed point.

The average speed of light between two points, separated by a distance r, is

$$c'_{av}(\phi) = c\left(1 - \frac{\mathbf{g}\cdot\mathbf{r}}{2c^2}\left(1 + \cos^2\phi\right)\right).$$

This formula, however, cannot be used in a case in which both the source and the observation points are fixed. This can be seen from the considered experiment with the accelerating elevator - using $c'_{av}(\phi)$ to calculate δ gives the value $\delta = ar^2/c^2$ and not the correct value $\delta = ar^2/2c^2$. Considering the elevator at rest on the Earth's surface and using $c'_{av}(\phi)$ we again get a wrong value $\delta = gr^2/c^2$.

The expression for the electric potential of a charge in a non-inertial reference frame derived when $c'_{av}(\phi)$ is used is not the correct one - it does not yield the correct electric field as calculated in the non-inertial frame. $c'_{av}(\phi)$ also leads to a wrong expression for the self-force originating in the interaction of a charge in a non-inertial reference frame and its own distorted electric field..

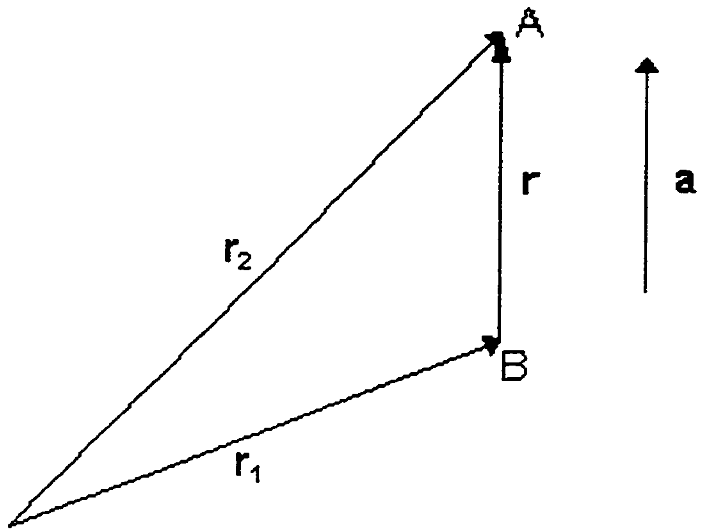

Figure 2.2: A light signal emitted at point B in an accelerating reference frame and propagating in the direction of the frame's acceleration **a**.

A. Its average anisotropic velocity between the two points is given by (2.1). It is seen that c^a is a function of the distance between the point B and A. The speed of light at B is $c^a_B = c$ (since $r = 0$ at B). As c^a is defined as

$$c^a = \frac{1}{2}\left(c^a_B + c^a_A\right),$$

where c^a_A is the speed of light at point A, we can determine c^a_A from here and taking into account that $c^a_B = c$:

$$c^a_A = 2c^a - c^a_B = 2c\left(1 - \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2}\right) - c = c\left(1 - \frac{\mathbf{a}\cdot\mathbf{r}}{c^2}\right)$$

in agreement with (2.2). In the general case the speed of light at an arbitrary point M (as viewed from the point where $r = 0$) is

$$c_M^a = c\left(1 - \frac{\mathbf{a}\cdot\mathbf{r}}{c^2}\right) \tag{2.3}$$

The speed of light at A is:

$$c_A^a = c\left(1 - \frac{\mathbf{a}\cdot\mathbf{r}}{c^2}\right) = c\left(1 - \frac{ar}{c^2}\right),$$

since the light signal is propagating in the direction of a. This means that $c_A^a < c_B^a$, i.e. c_A^a is smaller than c which should not be interpreted to mean that c_A^a is smaller than c in an *absolute* sense. This only means that the speed of light at A is smaller than c_B^a *as viewed from point B*. Since c_M^a is a function of the distance between two points equation (2.3) determines the speed of light at one point (where the light signal is measured) *relative* to the point where the light signal has been emitted (at this point $\mathbf{r} = 0$). In the case of Figure 2.2 the light signal is emitted at point B and at this point the speed of light is c (since $\mathbf{r} = 0$). Let us, however, consider a different situation in the accelerating reference frame N^a - a light signal emitted at point A and propagating towards B, i.e. in a direction opposite to the frame's acceleration (Figure 2.3).

In this case the speed of light at A is $c_A^a = c$ (since at A $\mathbf{r} = 0$). The speed of light at B is

$$c_B^a = c\left(1 - \frac{\mathbf{a}\cdot\mathbf{r}}{c^2}\right) = c\left(1 + \frac{ar}{c^2}\right),$$

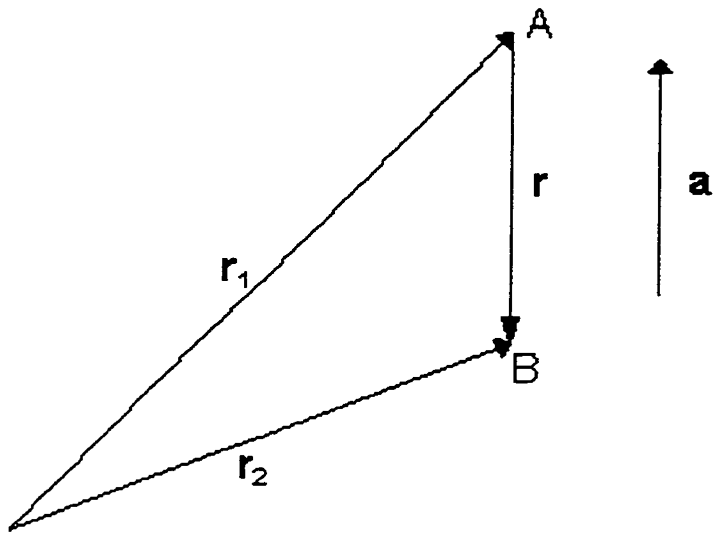

Figure 2.3: A light signal emitted at point A in an accelerating reference frame and propagating in opposite direction of the frame's acceleration **a**.

since the direction of **r** (i.e. the direction of propagation of the light signal) is opposite to **a** and $\mathbf{a}\cdot\mathbf{r} = -ar$. In other words, the light speed at B *with respect to* A is larger than c_A^a, i.e. larger than c.

The speed of light at the point of the source is always c (provided that the present convention that **r** *begins at the source point holds). If the point of detection of the signal is displaced in a direction parallel to* **a***, the speed of light at that point (relative to the source) is smaller than c; if the point of observation is situated in a direction opposite to* **a***, the light speed at that point (relative to the source) is larger than c.* Locally *the speed of light is always c.*

Now we can proceed with the calculation of the electric potential and the electric field of a charge at rest in an accelerating reference frame N^a. To do this we will rather need the average anisotropic speed of light c^a (2.1). Using c^a leads to two changes in the electric potential (2.4) of an inertial charge.

$$\varphi(r,t) = \frac{e}{4\pi\epsilon_o r}, \tag{2.4}$$

where r is the magnitude of the vector originating at the point where the charge is located and ending at the point where the potential is determined.

First, r, determined as $r = ct$ (where t is the time it takes for an electromagnetic signal to travel from the charge to the point at which the potential is determined), will have the form:

$$r^a = c^a t = ct\left(1 - \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2}\right) = r\left(1 - \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2}\right)$$

in N^a. Assuming $\mathbf{a}\cdot\mathbf{r}/2c^2 \ll 1$ we have:

$$(r^a)^{-1} \approx r^{-1}\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2}\right).$$

The second change in (2.4) due to the anisotropic speed of light in N^a is analogous to the change in (2.4) when a moving charge is described by the Liénard-Wiechert potentials in an inertial frame I (Figure 2.4) - in that case they result from an apparently larger dimension of the charge (in the direction of its motion) as viewed by I (see for instance [33], [36], [37]).

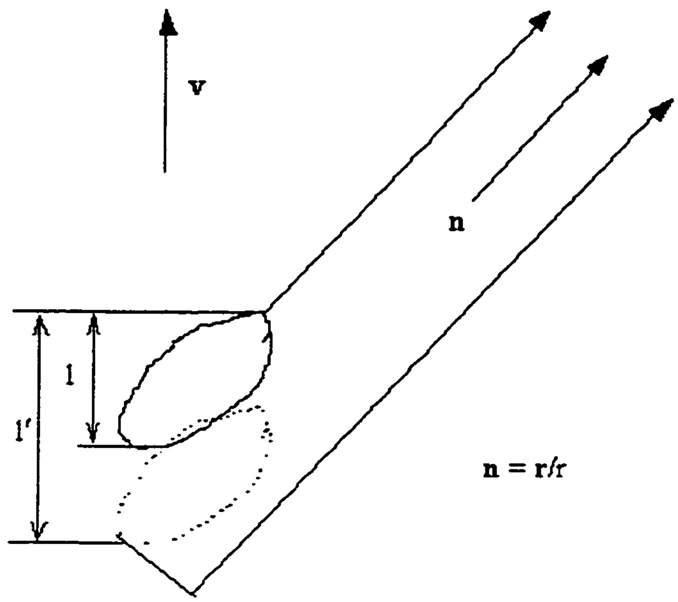

Figure 2.4: The length l' of a charge moving towards an inertial reference frame I (as determined in I) appears larger than its actual length l.

In N^a the dimension of the charge (in the direction of the acceleration of N^a) also appears larger despite the fact that it is at rest in N^a (Figure 2.5).

The reason is that the speed of the electromagnetic signal originating from the rear end of the charge (with respect to the observation point) is smaller than the speed of a signal originating from the front end (for $\mathbf{r}$ parallel to $\mathbf{a}$). The difference between the two speeds is

$$\Delta c = c\left[1 - \frac{\mathbf{a}\cdot(\mathbf{r} - \mathbf{l}/2)}{2c^2}\right] - c\left[1 - \frac{\mathbf{a}\cdot(\mathbf{r} + \mathbf{l}/2)}{2c^2}\right] = c\frac{\mathbf{a}\cdot\mathbf{l}}{2c^2},$$

where $\mathbf{l}$ is a vector parallel to $\mathbf{r}$, whose magnitude equals the dimension of the charge l. The apparent extra length Δl due to Δc which "accumulates"

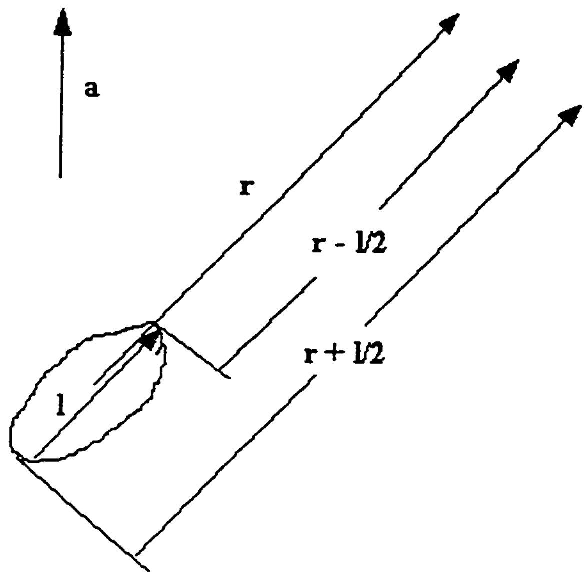

Figure 2.5: The length of a charge at rest in N^a appears larger in the direction of the frame's acceleration.

during the time $t = r/c$ (the first-approximation of the time it takes the electromagnetic signals to travel the distances $r - l/2$ and $r + l/2$, where $l \ll r$) is:

$$\Delta l = \Delta ct = \frac{\mathbf{a} \cdot \mathbf{l}}{2c^2} r = l \frac{\mathbf{a} \cdot \mathbf{r}}{2c^2}.$$

The apparent length l', is then

$$l' = l + \Delta l = l \left(1 + \frac{\mathbf{a} \cdot \mathbf{r}}{2c^2}\right).$$

It leads to an apparently larger (for $\mathbf{r}$ parallel to $\mathbf{a}$) volume element in the case of anisotropic speed of light in N^a

$$dV^a = \left(1 + \frac{\mathbf{a} \cdot \mathbf{r}}{2c^2}\right) dV \tag{2.5}$$

which leads to an apparent magnitude[3] of the charge, e^a, as determined in N^a (which is *not* the total charge e):

$$e^a = \int \rho dV^a = \int \rho \left(1 + \frac{\mathbf{a} \cdot \mathbf{r}}{2c^2}\right) dV$$

$$= e\left(1 + \frac{\mathbf{a} \cdot \mathbf{r}}{2c^2}\right),$$

where e is the magnitude of an inertial charge determined in its rest frame, i.e. e is the *total* charge; e^a does *not* represent the total charge (this apparent change of the magnitude of the charge in N^a is analogous to the change of e in the case of Liénard-Wiechert potentials when a moving charge is described in an inertial frame [33]).

Up to now no attention has been paid to the apparent change of the volume element which originates from the anisotropic speed of light in a non-inertial frame. This explains why the famous factor of 4/3 in the electromagnetic mass of the electron (derived from the expression of the self-force) still remains unaccounted for. Taking the correct volume element into consideration naturally removes the 4/3-factor without resorting to the Poincaré stresses (as will be

[3] At first glance it appears that this contradicts the experimentaly established invariance of the charge - its magnitude is the same in all reference frames. This situation is similar to the calculation of retarded potentials: " $\int [\rho] dV'$ does not in general represent the total charge of the system. The reason is that the various contributions to the integrand $[\rho]dV'$ are evaluated at different times..." ([48], p. 342). In the case of anisotropic speed of light e^a does not represent the total charge due the different speeds of electromagnetic signals originating from different parts of the charge.

shown in the next chapter).

Now we can write the electric potential of the charge in N^a:

$$\varphi^a(r,t) = \frac{e}{4\pi\epsilon_o r}\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2}\right)^2.$$

Keeping only the terms proportional to c^{-2} we obtain the final expression for φ^a:

$$\varphi^a(r,t) = \frac{e}{4\pi\epsilon_o r}\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{c^2}\right). \tag{2.6}$$

It is interesting to notice that the scalar Liénard-Wiechert potential (detected by an inertial observer)

$$\begin{aligned}\varphi(r,t) &= \frac{e}{4\pi\epsilon_o}\frac{1}{r - \mathbf{v}\cdot\mathbf{r}/c} \\ &= \frac{e}{4\pi\epsilon_o r}\frac{1}{(1 - \mathbf{v}\cdot\mathbf{n}/c)}\end{aligned}$$

where $\mathbf{n} = \mathbf{r}/r$ obtains the form of (2.6) for uniform acceleration of a charge $\mathbf{v} = \mathbf{a}t = \mathbf{a}r/c$ and $\mathbf{a}\cdot\mathbf{r}/c^2 \ll 1$:

$$\begin{aligned}\varphi(r,t) &= \frac{e}{4\pi\epsilon_o r}\frac{1}{(1 - (\mathbf{a}r/c)\cdot\mathbf{n}/c)} \\ &= \frac{e}{4\pi\epsilon_o r}\frac{1}{(1 - \mathbf{a}\cdot\mathbf{r}/c^2)} \\ &\approx \frac{e}{4\pi\epsilon_o r}\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{c^2}\right).\end{aligned}$$

This result shows that due to the absolute property of a worldline of an accelerated charge not to be geodesic, the charge's potential is identically distorted for a non-inertial observer at rest in the charge's reference frame and for an inertial observer.

The electric field of a charge at rest in N^a can be directly calculated from (2.6):

$$\mathbf{E}^a = -\nabla\varphi^a = \frac{e}{4\pi\epsilon_o}\left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{a}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a}\right). \tag{2.7}$$

The electric field (2.7) of the charge detected by an observer at rest in N^a coincides with the electric field of an accelerated charge [33, 38] determined in an inertial reference frame (in which the charge is instantaneously at rest); in such a way the electric field of the non-inertial charge is equally distorted for an inertial observer I and for an observer in N^a. The distorted field (2.7) is regarded to give rise to radiation since it contains the radiation field terms proportional to r^{-1} [38]. This, in light of the just obtained result, means that if an inertial observer detects radiation, so does an observer in N^a (since both I and N^a detect the same distorted electric field) . In fact, whether or not I and N^a register radiation is a secondary question; the most important result is that an inertial observer and an observer at rest in N^a see the same distortion of the electric field of a static charge in N^a.

Whether or not the distorted electric field of a non-inertial charge, as viewed by an observer in N^a, gives rise to radiation with respect to N^a, will perhaps remain controversial for some time, but in the end it will be recognized, I believe, that an unambiguous criterion for whether or not a charge moving with an acceleration **a** radiates is the presence of the radiation reaction force [33, 38]

$$\mathbf{F}_{rad} = \frac{1}{4\pi\epsilon_0}\frac{2e^2}{3c^3}\dot{\mathbf{a}}, \tag{2.8}$$

which results from the distorted electric field (2.7). This criterion is a direct consequence of the conservation of energy: the radiation reaction force must be present to account for the energy loss due to radiation. That is why there is radiation only if $\mathbf{F}_{rad}$ is different from zero. If the acceleration **a** is not constant, the charge radiates. If, however, the charge is in uniform (hyperbolic) motion ($\mathbf{a} = const$), $\dot{\mathbf{a}} = 0$ and therefore there is no radiation. Hence, both I and N^a will agree that a charge at rest in N^a does not radiate.

2.2.2 A charge falling in a uniformly accelerated reference frame

A charge falling in a uniformly accelerated reference frame, N^a, as viewed by an inertial observer, I, (falling with the charge or moving with constant velocity outside the accelerated reference frame) moves by inertia - its electric field is not distorted and there is no radiation. With respect to a non-inertial observer at rest in the accelerated reference frame N^a, however, the charge moves with an apparent constant acceleration $\mathbf{a}^* = -\mathbf{a}$. Using the Liénard-Wiechert potentials

$$\varphi(r,t) = \frac{e}{4\pi\epsilon_o}\frac{1}{r - \mathbf{v}\cdot\mathbf{r}/c} \tag{2.9}$$

$$\mathbf{A}(r,t) = \frac{e}{4\pi\epsilon_o c^2}\frac{\mathbf{v}}{r - \mathbf{v}\cdot\mathbf{r}/c} \tag{2.10}$$

it appears that its electric field should be distorted, having the form (2.7):

$$\mathbf{E} = \frac{e}{4\pi\epsilon_o}\left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{a}^*\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a}^*\right),$$

which suggests that one may expect radiation since the radiation reaction force (2.8) can be obtained form the distorted electric field. Such a conclusion, however, is wrong for two reasons: (i) the acceleration of the falling charge is constant ($\mathbf{F}_{rad} = 0$), and (ii) the expressions for the potentials (2.9), (2.10) hold only in an inertial frame. In order to use the Liénard-Wiechert potentials in N^a they must include the small correction due to the anisotropic speed of light (2.1); taking it into account yields the correct potentials:

$$\varphi^a(r,t) = \frac{e}{4\pi\epsilon_o}\frac{1}{r - \mathbf{v}\cdot\mathbf{r}/c}\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{c^2}\right) \tag{2.11}$$

$$\mathbf{A}^a(r,t) = \frac{e}{4\pi\epsilon_o c^2}\frac{\mathbf{v}}{r - \mathbf{v}\cdot\mathbf{r}/c}\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{c^2}\right). \tag{2.12}$$

The electric field then becomes:

$$\mathbf{E} = \frac{e}{4\pi\epsilon_o}\left\{\left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{a}^*\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a}^*\right) + \left(\frac{\mathbf{a}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a}\right)\right\}.$$

Noting that $\mathbf{a}^* = -\mathbf{a}$, we obtain the electric field of the falling charge in the accelerated reference frame N^a, which is identical with the field of an inertial charge determined in its rest frame:

$$\mathbf{E} = \frac{e}{4\pi\epsilon_o}\frac{\mathbf{n}}{r^2}.$$

This result is expected if we notice that for uniform acceleration of a charge $\mathrm{v} = \mathbf{a}^* t = \mathbf{a}^* r/c$ ($\mathbf{a}^* = -\mathbf{a}$) and considering the charge instantaneously at rest, (2.11) becomes the Coulomb potential and (2.12) - negligibly small since it is of order c^{-3}.

We have obtained the result we have expected - as the falling charge being an inertial particle is represented by a geodesic worldline, both an inertial and a non-inertial observer detect a Coulomb field. In other words, the results of this and the preceding section show that the shape of the electric field of a charge is determined by the state of motion of the charge (inertial or non-inertial) and is observerindependent.

2.3 A charge in the Earth's gravitational field

2.3.1 A charge supported in the Earth's gravitational field

First E. Fermi [39] in 1921 calculated the electromagnetic field of a charge supported in the Earth's gravitational field. Unfortunately his work remained unnoticed. Fermi did not obtain the correct formula for the potential and came to the wrong conclusion that the electric field of a charge supported in the Earth's gravitational field coincides with the electric field of a charge moving with an acceleration $\mathbf{a} = -\mathbf{g}/2$ (evidently, by the principle of equivalence $\mathbf{a} = -\mathbf{g}$).

The electric potential of a charge supported in the Earth's gravitational field as determined by an observer at rest on the Earth's surface can be obtained by taking into consideration the fact that the speed of light is also anisotropic in this case due to the presence of a gravitational field as follows from the principle of equivalence

$$c^g = c\left(1 + \frac{\mathbf{g}\cdot\mathbf{r}}{2c^2}\right) \tag{2.13}$$

where c^g is the average speed of the electromagnetic signal between two points separated by a distance r in a (weak) gravitational field. This formula is obtained by substituting $\mathbf{a} = -\mathbf{g}$ in (2.1). As seen from (2.13) c^g (determined in a non-inertial reference frame N^g at rest on the Earth's surface) is also direction dependent (as was the case in an accelerating reference frame). Like in the case of the anisotropic light speed in an accelerating reference frame here too the speed of light at a given spacetime point can be obtained from its average velocity (2.13). In Figure 2.6 a light signal is emitted at point B and propagates upwards towards point A, i.e. its direction is opposite to the gravitational acceleration $\mathbf{g}$. At B $c_B^g = c$ (since $\mathbf{r} = 0$). The light speed at A is then

$$c_A^g = 2c^g - c_B^g = 2c^g = c\left(1 + \frac{\mathbf{g}\cdot\mathbf{r}}{2c^2}\right) - c$$

or

$$c_A^g = c\left(1 + \frac{\mathbf{g}\cdot\mathbf{r}}{c^2}\right). \tag{2.14}$$

Since the propagation of the light signal is opposite to $\mathbf{g}$ $\mathbf{g}\cdot\mathbf{r} = -gr$ and

$$c_A^g = c\left(1 - \frac{gr}{c^2}\right),$$

which shows that $c_A^g < c_B^g$, i.e. smaller than c. Again this is *not* an *absolute* result; it only means that c_A^g is smaller than c as viewed from B.

As seen in Figure 2.7 when the light source is at A and the light signal propagates downwards (parallel to $\mathbf{g}$) the speed of light at A this time is $c_A^g = c$ (since $\mathbf{r} = 0$). The speed of light at B is

$$c_B^g = c\left(1 + \frac{\mathbf{g}\cdot\mathbf{r}}{c^2}\right) = c\left(1 + \frac{gr}{c^2}\right),$$

which means that $c_B^g > c_A^g$, i.e. larger than c. *The speed of light at the point of the source is always c (provided that the accepted convention according to which the beginning of the vector $\mathbf{r}$ is at the source point and its end at the observation point holds; otherwise the speed of light at the point of*

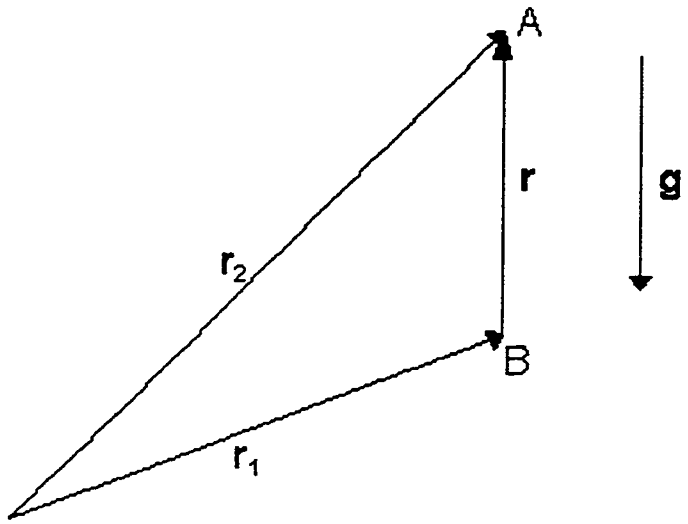

Figure 2.6: A light signal emitted at B propagates upwards against the direction of the gravitational acceleration **g**.

observation will be always c). If the point of detection of the signal is displaced in a direction parallel to **g**, *the speed of light at that point (relative to the source) is larger than c; if the point of observation is situated in a direction opposite to* **g**, *the light speed at that point (relative to the source) is smaller than c.* Locally *in a gravitational field (i.e. in curved spacetime) the speed of light is always c.* This conclusion is quite natural - in terms of spacetime curvature a curved spacetime can locally be well approximated by the tangent flat spacetime and as we know the flat spacetime physics is governed by special relativity where the speed of light is constant and is equal to c.

In an implicit form (2.14) has been used by Rindler [40]. It can be also

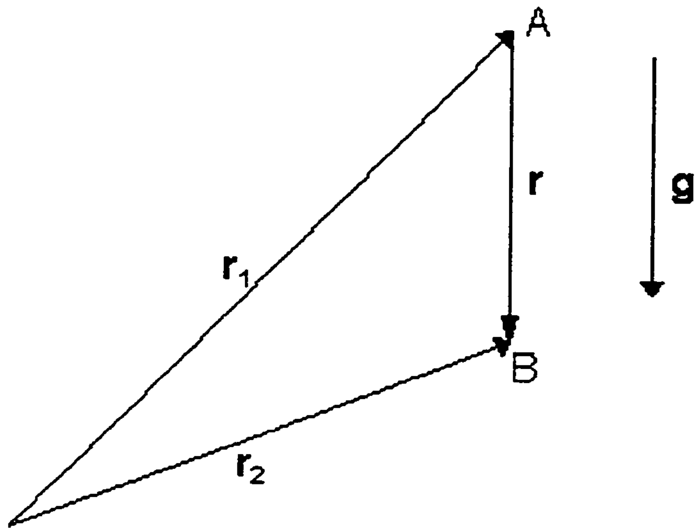

Figure 2.7: A light signal emitted at A propagates downwards in the direction of the gravitational acceleration **g**.

obtained from (2.2):

$$c' = c\left(1 + \frac{gr}{c^2}\right) = c\left(1 + \frac{\Delta\Phi}{c^2}\right),$$

where $\Delta\Phi$ is the difference in the gravitational potential. Let us consider a light signal emitted at point B on the Earth's surface (Figure 2.8). The light signal propagates upwards towards point A situated at a distance $R_A = R_B + r$ from the Earth's centre, where R_B is the Earth's radius (and the distance from the Earth's centre at which point B is situated) and r is the distance between A and B. At A the gravitational potential is $\Phi_A = -\dfrac{GM}{R_A}$; at B it is $\Phi_B = -\dfrac{GM}{R_B}$. Now we can see one more time how the direction dependence emerges.

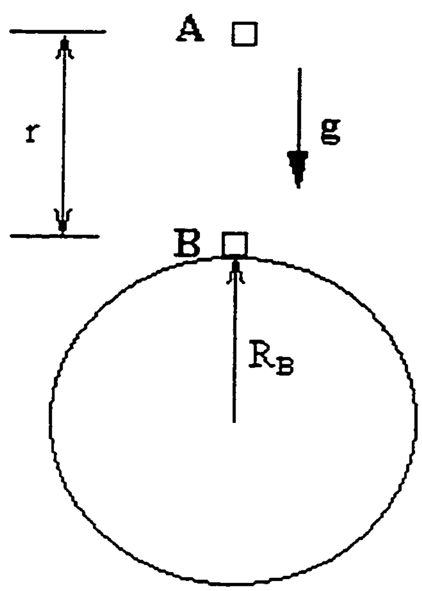

Figure 2.8: Point B on the Earth's surface and point A above it at a distance r.

As the source is at B we want to see how the potential changes upwards (from point B to point A):

$$\Delta\Phi_{BA} = -\frac{GM}{R_B} - \left(-\frac{GM}{R_A}\right) = -\frac{GM}{R_B} + \frac{GM}{R_B + r}$$

$$= -\frac{GM}{R_B} + \frac{GM}{R_B\,(1 + r/R_B)}$$

$$\approx -\frac{GM}{R_B} + \frac{GM}{R_B}\left(1 - \frac{r}{R_B}\right)$$

$$= -\frac{GM}{R_B^2} r$$

$$= -gr.$$

This means that as we go upwards the speed of light (with respect to the source) decreases:

$$c' = c\left(1 + \frac{\Delta\Phi_{BA}}{c^2}\right) = c\left(1 - \frac{gr}{c^2}\right).$$

If the light source is at A and the signal propagates towards point B, we want to see how the potential changes from A to B (i.e. downwards):

$$\begin{aligned}
\Delta\Phi_{AB} &= -\frac{GM}{R_A} - \left(-\frac{GM}{R_B}\right) = -\frac{GM}{R_B + r} + \frac{GM}{R_B} \\
&= -\frac{GM}{R_B\left(1 + r/R_B\right)} + \frac{GM}{R_B} \\
&\approx -\frac{GM}{R_B}\left(1 - \frac{r}{R_B}\right) + \frac{GM}{R_B} \\
&= \frac{GM}{R_B^2} r \\
&= gr.
\end{aligned}$$

In such a way, as we go downwards from point A to point B the speed of light increases:

$$c' = c\left(1 + \frac{\Delta\Phi_{AB}}{c^2}\right) = c\left(1 + \frac{gr}{c^2}\right).$$

Finally, in vector form we have:

$$c' = c\left(1 + \frac{\mathbf{g}\cdot\mathbf{r}}{c^2}\right), \tag{2.15}$$

which coincides with (2.14). When light is propagating opposite to $\mathbf{g}$ its speed is smaller than that when light is propagating in the same direction as $\mathbf{g}$.

As we have seen the direction dependent light velocity (2.14) directly follows from (2.2). It is also seen that the average direction dependent velocity of light between A and B can be calculated from (2.15). If the light source is at A (then $\mathbf{r} = 0$) $c' = c$ and at B: $c' = c(1 + \mathbf{g}\cdot\mathbf{r}/c^2)$. Then the average velocity is:

$$c'_{av} = \frac{1}{2}\left[c + c\left(1 + \frac{\mathbf{g}\cdot\mathbf{r}}{c^2}\right)\right] = c\left(1 + \frac{\mathbf{g}\cdot\mathbf{r}}{2c^2}\right),$$

which coincides with (2.13) confirming its correctness.

Now we can proceed with determining the electric potential of a charge in a non-inertial frame N^g, at rest in the Earth's gravitational field. Following the same reasoning as in the case of an accelerated reference frame N^a (or applying the principle of equivalence and substituting $\mathbf{a} = -\mathbf{g}$ in (2.6)) we can write the expression for the scalar potential:

$$\varphi^g(r,t) = \frac{e}{4\pi\epsilon_o r}\left(1 - \frac{\mathbf{g}\cdot\mathbf{r}}{c^2}\right), \tag{2.16}$$

where φ^g is the charge's potential in a gravitational field. The electric field of the charge $\mathbf{E}^g = -\nabla\varphi^g$ is directly obtained from (2.16):

$$\mathbf{E}^g = \frac{e}{4\pi\epsilon_o}\left(\frac{\mathbf{n}}{r^2} - \frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n} + \frac{1}{c^2 r}\mathbf{g}\right). \tag{2.17}$$

A comparison of the electric field of an accelerated charge (2.7), determined in the frame N^a, with the electric field of a charge supported in the Earth's gravitational field (2.17), determined in N^g, indicates that the electric fields of a charge moving with an acceleration $\mathbf{a} = -\mathbf{g}$ and a charge at rest on the Earth's surface are equally distorted as required by the principle of equivalence. One may expect the distorted electric field (2.17) to give rise to the radiation reaction force (2.8), but it is zero since $\mathbf{g} = const$. This means that a charge supported in the Earth's gravitational field does not radiate which agrees with the result that a charge at rest in an accelerated frame does not radiate either.

2.3.2 A charge falling in the Earth's gravitational field

All gravitational effects in general relativity are manifestation of space-time curvature due to the presence of matter. The force-free motion of a charge falling in a gravitational field is represented by the geodesic worldline of the charge. In other words, the falling charge moves by inertia and its electric field should not be distorted which means that there should not exist any radiation.

The charge's electric field is not distorted as viewed by an inertial observer, I, falling with the charge. However, if a non-inertial observer in N^g (at rest on the Earth's surface) directly uses the Liénard-Wiechert potentials (2.9), (2.10) to obtain the electric field of an accelerated charge, falling in the Earth's gravitational field ($\mathbf{a} = \mathbf{g}$), the expression obtained (considering the charge instantaneously at rest in N^g) is:

$$\mathbf{E} = \frac{e}{4\pi\epsilon_o}\left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{g}\right). \tag{2.18}$$

From the distorted field (2.18) the radiation reaction force (2.8) can be obtained which implies that the falling charge may radiate. As we have seen in the case of a charge falling in an accelerating reference frame such a conclusion is wrong for the same two reasons: (i) the acceleration of the falling charge is constant (so $\mathbf{F}_{rad} = 0$), and (ii) the expressions for the potentials (2.9), (2.10) should include the correction due to the anisotropic speed of light in N^g (i.e. in the Earth's gravitational field) in order to be valid in N^g. Taking this into account, the correct potentials are:

$$\varphi^g(r,t) = \frac{e}{4\pi\epsilon_o}\frac{1}{r - \mathbf{v}\cdot\mathbf{r}/c}\left(1 - \frac{\mathbf{g}\cdot\mathbf{r}}{c^2}\right) \tag{2.19}$$

$$\mathbf{A}^g(r,t) = \frac{e}{4\pi\epsilon_o c^2}\frac{\mathbf{v}}{r - \mathbf{v}\cdot\mathbf{r}/c}\left(1 - \frac{\mathbf{g}\cdot\mathbf{r}}{c^2}\right) \tag{2.20}$$

The electric field then becomes:

$$\mathbf{E} = \frac{e}{4\pi\epsilon_o}\left\{\left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{g}\right) + \left(-\frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n} + \frac{1}{c^2 r}\mathbf{g}\right)\right\}$$

which is the field of an inertial charge:

$$\mathbf{E} = \frac{e}{4\pi\epsilon_o}\frac{\mathbf{n}}{r^2}. \tag{2.21}$$

As in the case of a charge falling in an accelerated frame, this result is expected if we notice that for a uniformly accelerated charge $\mathbf{v} = \mathbf{g}t = \mathbf{g}r/c$ and considering the charge instantaneously at rest, (2.19) becomes the Coulomb potential and (2.20) is neglected since it is proportional to c^{-3}.

The result (2.21) (i) confirms that a Coulomb field is associated with an inertial (falling) charge by both I and N^g, and (ii) is in perfect agreement with the principle of equivalence: both I and N^a detect a Coulomb potential of a charge falling in N^a (in the case of an accelerated charge) and for both I and N^g the electric field of a charge falling in N^g is the Coulomb field (in the case of a charge supported in the Earth's gravitational field).

2.4 Conclusions

It has been shown in this chapter that the shape of the electric field of a charge is observerindependent since it is determined by the inertial state of the charge: a Coulomb field is associated with an inertial charge (represented by a geodesic worldline) by both an inertial observer, I, and a non-inertial (uniformly accelerated or at rest in a uniform gravitational field) observer N;

both I an N detect the same distorted electric field of a non-inertial charge (at rest with respect to N), whose worldline is not geodesic.

The principle of equivalence is naturally observed in the proposed approach:

(i) both an observer at rest in an accelerated frame N^a and a static observer in a frame N^g (at rest in a uniform gravitational field) see the same distorted electric field of a charge at rest in N^a and N^g, respectively (provided that $\mathbf{a} = -\mathbf{g}$);

(ii) an inertial observer falling in N^a and an inertial observer falling in N^g detect the same distorted electric field of a charge at rest in N^a and N^g, respectively. The inertial observers do not register radiation because the radiation reaction force (2.8) is zero due to acceleration of the charge ($\mathbf{a}$ and $\mathbf{g}$, respectively) being constant. If one disagrees that the presence of the radiation reaction force (2.8) is a criterion whether or not there is radiation from a non-inertial charge and comes to the conclusion that the inertial observers register radiation from the non-inertial charges (static in N^a and N^g, respectively) because they see the electric field of the charges distorted (and thus containing the radiation field terms, proportional to r^{-1}), the same conclusion follows for the non-inertial observers (at rest in N^a and N^g, respectively) as well, since they see the same distortion of the fields of the non-inertial charges.

(iii) both an observer at rest in N^a and a static observer in N^g detect a

Coulomb field of a charge falling in N^a and N^g, respectively.

Chapter 3

Inertia, Gravitation and Electromagnetic Mass

3.1 The Electromagnetic Mass of the Electron

The concept of electromagnetic mass of a charged particle originated in 1881 when J. J. Thompson [9] realized that such a particle was more resistant to acceleration. After that the idea was taken up by Abraham [12] and especially Lorentz [13] who developed it further into the classical model of the electron.

The electromagnetic mass of the electron can be calculated by three independent methods [41]: (i) energy-derived electromagnetic mass $m_U = U/c^2$, where U is the field energy of the electron at rest, (ii) momentum-derived electromagnetic mass $m_p = p/v$, where p is the field momentum when the electron is moving at speed v, and (iii) self-force-derived electromagnetic mass $m_s = F_s/a$, where F_s is the self-force acting on the electron when it has an acceleration a (these are non-relativistic expressions; the relativistic ones are easily obtainable). While $m_U = m_e$, m_p has been causing problems for a number of years yielding $\frac{4}{3}m_e$. It was independently shown by several authors [42-46] that $m_p = m_e$. The self-force-derived electromagnetic mass has been the most difficult to deal with, persistently yielding the factor 4/3. By a covariant application of Hamilton's principle in 1922 Fermi [47] first removed the 4/3 factor from the self-force (described in an inertial reference frame). In 1982 Pearle [15] showed by inclusion of Lorentz contraction in the self-force calculation that $m_s = m_e$ in the case of an accelerated electron described again in an inertial reference frame. In this chapter we shall see how the factor of 4/3 is accounted for in the case of an electron at rest in an accelerated frame N^a and in a frame N^g on the Earth's surface described in N^a and N^g, respectively

The hypothesis that the whole mass of the electron is electromagnetic in origin not only explains the nature of inertia but also accounts for the relativistic increase of mass. The fact that the electromagnetic mass rises with velocity inversely as $\sqrt{1 - v^2/c^2}$ was discovered before the theory of relativity [12]. It had been shown that the calculation of the momentum $\mathbf{p}$ associated with a moving electron's electromagnetic field for an arbitrary velocity $\mathbf{v}$ yields (see [36]):

$$\mathbf{p} = \frac{m^a \mathbf{v}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where m^a is the rest electromagnetic mass of the electron; more precisely the inertial electromagnetic rest mass which characterizes the resistance an electron offers to being accelerated as determined in its rest *accelerated* frame (or in an instantaneous inertial frame). The relativistic increase of the electromagnetic mass can be also obtained from the electromagnetic energy U "attached" to a moving electron [46] (so far this effect has not been deduced from the expression for the self-force). In such a way, the relativistic increase of the electromagnetic mass (i.e. of the mass if it is entirely electromagnetic) turns out to be a direct consequence of the fact that the energy and momentum stored in the electromagnetic field of an electron are larger as determined by an observer with respect to which the electron is moving than those determined in the electron's rest frame. The relativistic increase of the electromagnetic mass also follows directly from (1.1) due to the relativistic contraction of the dimension r of the extended electron.

Here we will be concerned with the self-force-derived mass m_s, which will be denoted either by m^a (inertial electromagnetic mass) or by m^g (gravitational electromagnetic mass).

3.2 An Electron in a Uniformly Accelerated Reference Frame

3.2.1 An electron at rest in a uniformly accelerated frame

When an electron is accelerated each volume element of it experiences a force produced by the electron itself (by its own field) which gives rise to a resultant self-force acting on the electron as a whole and opposing its acceleration. The typical way of obtaining the expression for the inertial electromagnetic mass m^a is to calculate the self-force resulting from the distorted electric field of an accelerated electron. Here we shall not follow the standard approach to calculating the self-force [34, 38, 48] which describes the electron's motion in an inertial frame I. Instead, the electric field of an accelerated electron will be calculated in an accelerated reference frame N^a in which the electron is at rest. This can be done since a non-inertial (accelerated) observer at rest in N^a will attribute the self-force acting on the electron to its acceleration (due to the absolute nature of acceleration an observer in N^a can establish from within N^a that it is an accelerated frame). The advantage of calculating the electron's electric field in N^a is that it is obtained only from the scalar potential φ^a in N^a and the calculation does not involve retarded times. In order to obtain the potential φ^a one should explicitly take into account the anisotropic speed of light (and electromagnetic interaction in general) c^a in N^a [22] obtained in Chapter 2:

$$c^a = c\left(1 - \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2}\right) \tag{3.1}$$

where $\mathbf{a}$ is the acceleration of N^a and $\mathbf{r}$ is a vector with its origin at the point where an electromagnetic signal is emitted and its end at the point where the signal is determined. As we have seen in Chapter 2 the anisotropic speed of light c^a leads to two changes in the potential (3.2) of an inertial charge

$$\varphi\left(r,t\right) = \frac{e}{4\pi\epsilon_o r}, \tag{3.2}$$

where r in (3.2) and (3.1) is the same.

First, r, determined as $r = ct$ (where t is the time it takes for an electromagnetic signal to travel from the charge to the point at which the potential is determined), will have the form $r^a = c^a t$ in N^a. Assuming $\mathbf{a}\cdot\mathbf{r}/2c^2 \ll 1$ we can write:

$$\left(r^a\right)^{-1} \approx r^{-1}\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2}\right). \tag{3.3}$$

The second change in (3.2) takes into account an apparently larger dimension of the charge (in the direction of the acceleration of N^a) - it is analogous to the same apparent change of a moving charge's dimension (in the direction of its motion) as viewed by an inertial observer I. In N^a the dimension of the charge also appears larger despite the fact that it is at rest in N^a. The reason is that the speed of the electromagnetic signal originating from the rear end of the charge (with respect to the observation point) is smaller than the speed of a signal originating from the front end (for $\mathbf{r}$ parallel to $\mathbf{a}$). As we have seen in Chapter 2 this difference between the two speeds results in an apparent length l' of a volume element of the electron's charge

$$l' = l\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2}\right),$$

which leads to an apparently larger (for $\mathbf{r}$ parallel to $\mathbf{a}$) volume element in the case of anisotropic speed of light in N^a

$$dV^a = \left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2}\right)dV, \tag{3.4}$$

where dV is the actual volume element (i.e. the volume element determined when the electron is at rest in an inertial reference frame). The anisotropic volume element dV^a affects the magnitude of the charge element de^a contained in dV^a as determined in N^a:

$$de^a = \rho dV^a = \rho\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2}\right)dV, \tag{3.5}$$

where ρ is the density of the electron charge.

Now, taking into account (3.3) and (3.5), we can write the electric potential of a charge element de^a of an electron at rest in N^a(keeping only the terms proportional to c^{-2}):

$$d\varphi^a(r,t) = \frac{de^a}{4\pi\epsilon_0 r^a} \approx \frac{\rho}{4\pi\epsilon_o r}\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{c^2}\right)dV. \tag{3.6}$$

The electric field produced by de^a in N^a can be directly calculated from (3.6):

$$d\mathbf{E}^a = -\nabla d\varphi^a = \frac{1}{4\pi\epsilon_o}\left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{a}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a}\right)\rho dV,$$

where $\mathbf{n} = \mathbf{r}/r$. The electric field of the electron then is

$$\mathbf{E}^a = \frac{1}{4\pi\epsilon_o}\int\left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{a}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a}\right)\rho dV. \tag{3.7}$$

The self-force which the field of the electron exerts upon an element ρdV_1^a of the electron charge is

$$d\mathbf{F}^a_{self} = \rho dV_1^a \mathbf{E}^a = \frac{1}{4\pi\epsilon_o}\int\left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{a}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a}\right)\rho^2 dV dV_1^a.$$

The resultant self-force acting on the electron as a whole is:

$$\mathbf{F}^a_{self} = \frac{1}{4\pi\epsilon_o}\int\int\left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{a}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a}\right)\rho^2 dV dV_1^a,$$

which after taking into account (3.4) becomes

$$\mathbf{F}^a_{self} = \frac{1}{4\pi\epsilon_o}\int\int\left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{a}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a}\right)\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2}\right)\rho^2 dV dV_1. \tag{3.8}$$

Assuming a spherically symmetric distribution of the electron charge [13] and following the standard tedious procedure of calculating the self-force [34] we get (see the Appendix):

$$\mathbf{F}^a_{self} = -\frac{U}{c^2}\mathbf{a}, \tag{3.9}$$

where

$$U = \frac{1}{8\pi\epsilon_o}\int\int\frac{\rho^2}{r}dVdV_1$$

is the electron's electrostatic energy. As U/c^2 is the mass "attached" to the field of an electron, i.e. its electromagnetic mass, we can write (3.9) in the form:

$$\mathbf{F}^a_{self} = -m^a\mathbf{a}, \tag{3.10}$$

where m^a is identified with the electron's inertial mass.

The self-force $\mathbf{F}^a_{self}$ to which an electron is subjected due to its own field is directed opposite to $\mathbf{a}$ and resists its acceleration, i.e. the deformation of its world path. The famous factor of 4/3 in the electromagnetic mass of the electron does not appear in (3.10). The reason is that in (3.8) we have used the correct volume element $dV_1^a = \left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2}\right)dV_1$. This apparent change of the volume element originating from the anisotropic speed of light in a non-inertial frame has not been noticed up to now. Taking it into account naturally removes the 4/3-factor without resorting to the Poincaré stresses (designed to explain the stability of the electron).

Since its origin a century ago the electromagnetic mass theory of the electron has not been able to explain why the electron is stable - what keeps its charge together. However, this problem cannot be adequately addressed until a quantum-mechanical model of the electron's structure is obtained. On the other hand, the problem of stability of the electron (which is often used as evidence against regarding its entire mass as electromagnetic) does not interfere, as we have seen, with the derivation of the expression for the self-force (3.10) containing the electromagnetic mass. This problem can also be successfully avoided in the case of the electromagnetic mass derived from the expression for the momentum of the electron's electromagnetic field [32, 46].

3.2.2 An electron falling in an accelerated reference frame

An electron falling in a uniformly accelerated (non-inertial) reference frame, N^a, as viewed by an inertial observer, I, (falling with the electron or moving with constant velocity outside the accelerated reference frame) moves by inertia: its world path is geodesic which means that its Coulomb field is not distorted and there is no self-force acting on the electron. With respect to a non-inertial observer at rest in N^a, however, the electron moves with a constant *apparent* acceleration $\mathbf{a}^* = -\mathbf{a}$ ($\mathbf{a}$ being the acceleration of N^a) and if one uses the Liénard-Wiechert potentials

$$\varphi\left(r,t\right) = \frac{e}{4\pi\epsilon_o}\frac{1}{r - \mathbf{v}\cdot\mathbf{r}/c} \tag{3.11}$$

$$\mathbf{A}\left(r,t\right) = \frac{e}{4\pi\epsilon_o c^2}\frac{\mathbf{v}}{r - \mathbf{v}\cdot\mathbf{r}/c} \tag{3.12}$$

it appears that its electric field is distorted, having the form:

$$\mathbf{E} = -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t} = \frac{e}{4\pi\epsilon_o}\left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{a}^{*}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a}^{*}\right) \tag{3.13}$$

for the electron considered instantaneously at rest in N^a [33, 34]. This suggests that an observer in N^a may expect the distorted electric field (3.13) to give rise to a self-force which forces the electron to fall. However, the potentials (3.11) and (3.12) are valid only in an inertial reference frame; in the non-inertial frame N^a (due to the anisotropic speed of light (3.1) in N^a) they have the form [22]:

$$\varphi^a(r,t) = \frac{e}{4\pi\epsilon_o}\frac{1}{r - \mathbf{v}\cdot\mathbf{r}/c}\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{c^2}\right) \tag{3.14}$$

$$\mathbf{A}^a(r,t) = \frac{e}{4\pi\epsilon_o c^2}\frac{\mathbf{v}}{r - \mathbf{v}\cdot\mathbf{r}/c}\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{c^2}\right). \tag{3.15}$$

The electric field resulting from (3.14) and (3.15) is:

$$\mathbf{E} = \frac{e}{4\pi\epsilon_o}\left\{\left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{a}^{*}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a}^{*}\right) + \left(\frac{\mathbf{a}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a}\right)\right\}.$$

Noting that $\mathbf{a}^{*} = -\mathbf{a}$, the electric field of the falling charge in the accelerated reference frame N^a, turns out to be identical with the field of an inertial charge determined in its rest frame:

$$\mathbf{E} = \frac{e}{4\pi\epsilon_o}\frac{\mathbf{n}}{r^2}. \tag{3.16}$$

Consequently, a Coulomb field is associated with the falling electron by both an inertial observer I and an observer at rest in N^a. Comparing the electric field (3.7) of an electron at rest in N^a(determined in N^a) as calculated in the previous section and its field determined in I [33] in which the electron is instantaneously at rest shows that for both an observer in I and an observer in N^a the electron's field is equally distorted. In general, as shown in Chatter 2, due to the absolute property of a worldline to be geodesic or not, an association of a Coulomb field with an inertial electron (represented by a geodesic worldline) and a distorted electric field with a non-inertial electron (whose worldline is not geodesic) is observerindependent (the same for an inertial and a non-inertial observer).

The Coulomb field of the falling electron (3.16) implies that it is not subjected to any self-force. This result is quite natural - there is no four-dimensional stress in the electron's world path since it is geodesic which means that there is no self-force acting on the electron. If, however, the electron is prevented from falling, it starts to accelerate (with *real*, not *apparent* acceleration) and leaves its inertial state. The electron's world path deforms which results in the distortion of the electron's electric field; this in turn gives rise to a self-force that opposes the deformation of its world path.

3.3 An Electron in a Frame $\mathbf{N}^g$ at Rest With Respect to the Earth

3.3.1 An electron at rest in $\mathbf{N}^g$

In 1921 E. Fermi [39] first considered the question what is the "weight" of the electromagnetic mass of the electron, i.e. the question whether the electromagnetic mass of an electron supported in the Earth's gravitational field coincides with its electromagnetic mass when the electron is moving with an acceleration $\mathbf{a} = -\mathbf{g}$. Despite the fact that he did not obtain the correct formula for the potential and came to the wrong conclusion that the electric field of an electron supported in the Earth's gravitational field coincides with the electric field of an electron moving with an acceleration $\mathbf{a} = -\mathbf{g}/2$, Fermi did obtain the correct expression for the electromagnetic mass of an electron at rest on the Earth's surface which coincides with the electromagnetic mass of an accelerated electron. He got this result not by a direct calculation of the self-force acting on the electron, but by making a covariant application of Hamilton's principle.

Here the expression for the gravitational electromagnetic mass will be derived from the expression for the self-force acting on an electron supported in the Earth's gravitational field. Like the potential (3.6) determined in N^a, the electric potential of a charge element of an electron in a non-inertial frame N^g

at rest on the Earth's surface is distorted as a result of the anisotropic speed of light (electromagnetic interaction) c^g in N^g. The speed of light in N^g is anisotropic due to the presence of matter as shown by Einstein [35] and its average value (between two points separated by a distance r) in the Earth's gravitational field is

$$c^g = c\left(1 + \frac{\mathbf{g} \cdot \mathbf{r}}{2c^2}\right). \tag{3.17}$$

This expression can be also obtained from (3.1) by substituting $\mathbf{a} = -\mathbf{g}$ as follows from the principle of equivalence.

The general form of the electric potential of a charge element de^g of an electron at rest in N^g is:

$$d\varphi^g = \frac{de^g}{4\pi\epsilon_0 r^g}. \tag{3.18}$$

Following the same reasoning as in the case of an accelerated reference frame N^a (see Chapter 2 and the previous section) we can determine the expressions for de^g and r^g. The charge element de^g in N^g has the form

$$de^g = \rho dV^g = \rho\left(1 - \frac{\mathbf{g} \cdot \mathbf{r}}{2c^2}\right) dV, \tag{3.19}$$

where

$$dV^g = \left(1 - \frac{\mathbf{g} \cdot \mathbf{r}}{2c^2}\right) dV \tag{3.20}$$

is the anisotropic volume element in a gravitational field. Taking into account that in N^g $r^g = c^g t$ and assuming $\mathbf{g}\cdot\mathbf{r}/2c^2 \ll 1$ we can write

$$(r^g)^{-1} \approx r^{-1}\left(1 - \frac{\mathbf{g}\cdot\mathbf{r}}{2c^2}\right) \tag{3.21}$$

for r in N^g. Substituting (3.19) and (3.21) in (3.18) and keeping only the terms proportional to c^{-2} we get the final expression for the potential of the charge element de^g

$$d\varphi^g = \frac{\rho}{4\pi\epsilon_0 r}\left(1 - \frac{\mathbf{g}\cdot\mathbf{r}}{c^2}\right) dV. \tag{3.22}$$

The potential (3.22) can be also obtained by applying the principle of equivalence and substituting $\mathbf{a} = -\mathbf{g}$ in (3.6) .

The electric field of the charge element de^g in N^g is calculated by using the scalar potential (3.22):

$$d\mathbf{E}^g = -\nabla d\varphi^g = \frac{1}{4\pi\epsilon_o}\left(\frac{\mathbf{n}}{r^2} - \frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n} + \frac{1}{c^2 r}\mathbf{g}\right)\rho dV$$

and the field of the electron is

$$\mathbf{E}^g = \frac{1}{4\pi\epsilon_o}\int\left(\frac{\mathbf{n}}{r^2} - \frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n} + \frac{1}{c^2 r}\mathbf{g}\right)\rho dV. \tag{3.23}$$

A comparison of the electric field of an accelerated charge element (3.7), determined in the frame N^a, with the electric field of a charge element supported in the Earth's gravitational field (3.23), determined in N^g, indicates that the electric fields of a charge element moving with an acceleration $\mathbf{a} = -\mathbf{g}$ and a charge element at rest on the Earth's surface are equally distorted as required by the principle of equivalence.

The self-force with which the electron field interacts with an element ρdV_1^g of the electron charge is

$$d\mathbf{F}^g_{self} = \rho dV_1^g \mathbf{E}^g = \frac{1}{4\pi\epsilon_o}\int\left(\frac{\mathbf{n}}{r^2} - \frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n} + \frac{1}{c^2 r}\mathbf{g}\right)\rho^2 dV dV_1^g.$$

The resultant self-force with which the electron acts upon itself is:

$$\mathbf{F}^g_{self} = \frac{1}{4\pi\epsilon_o}\int\int\left(\frac{\mathbf{n}}{r^2} - \frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n} + \frac{1}{c^2 r}\mathbf{g}\right)\rho^2 dV dV_1^g,$$

which after taking into account the explicit form (3.20) of dV_1^g becomes

$$\mathbf{F}^g_{self} = \frac{1}{4\pi\epsilon_o}\int\int\left(\frac{\mathbf{n}}{r^2} - \frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n} + \frac{1}{c^2 r}\mathbf{g}\right)\left(1 - \frac{\mathbf{g}\cdot\mathbf{r}}{2c^2}\right)\rho^2 dV dV_1. \qquad (3.24)$$

Assuming a spherically symmetric distribution of the electron charge and calculating the self-force as in the case of an accelerated electron we get:

$$\mathbf{F}^g_{self} = \frac{U}{c^2}\mathbf{g}, \qquad (3.25)$$

where

$$U = \frac{1}{8\pi\epsilon_o}\int\int\frac{\rho^2}{r}dV dV_1$$

is the electron's electrostatic energy. As U/c^2 is the electromagnetic mass of the electron, (3.25) obtains the form:

$$\mathbf{F}^g_{self} = m^g \mathbf{g} \qquad (3.26)$$

where m^g here is interpreted as the electron's passive gravitational mass. As in the case of the self-force acting on an accelerated electron the factor 4/3 in the electromagnetic mass does not appear in (3.26) for the same reason: the correct volume element (3.20) was used in (3.24).

The self-force $\mathbf{F}^g_{self}$ which acts upon an electron on account of its own field is directed parallel to $\mathbf{g}$ and resists its acceleration arising from the fact that the electron (at rest on the Earth's surface) is prevented from falling, i.e. from moving by inertia.

3.3.2 An electron falling in $\mathbf{N}^g$

General relativity describes an electron falling in a gravitational field by a geodesic world path. It implies that it moves by inertia and its Coulomb field should not be distorted which means that there should not exist any self-force acting on the electron. The electron's Coulomb field is not distorted as viewed by an inertial observer falling with the electron. In order to obtain the electric field of an accelerated electron in the Earth's gravitational field ($\mathbf{a} = \mathbf{g}$) with respect to a non-inertial observer (at rest in N^g) one cannot use the Liénard-Wiechert potentials (3.11) and (3.12) in N^g since N^g is not an inertial frame. In order to be used in N^g they should include the correction due to the anisotropic speed of light (3.17) in N^g. As shown in Chapter 2 in N^g the potentials (3.11) and (3.12) have the form:

$$\varphi^g(r,t) = \frac{e}{4\pi\epsilon_o}\frac{1}{r - \mathbf{v}\cdot\mathbf{r}/c}\left(1 - \frac{\mathbf{g}\cdot\mathbf{r}}{c^2}\right) \qquad (3.27)$$

$$\mathbf{A}^{g}(r,t) = \frac{e}{4\pi\epsilon_o c^2}\frac{\mathbf{v}}{r - \mathbf{v}\cdot\mathbf{r}/c}\left(1 - \frac{\mathbf{g}\cdot\mathbf{r}}{c^2}\right). \tag{3.28}$$

The electric field of the electron falling in N^g (and considered instantaneously at rest in N^g) obtained from (3.27) and (3.28) is:

$$\mathbf{E} = -\nabla\varphi^g - \frac{\partial\mathbf{A}^g}{\partial t} = \frac{e}{4\pi\epsilon_o}\left\{\left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{g}\right) + \left(-\frac{\mathbf{g}\cdot\mathbf{n}}{c^2 r}\mathbf{n} + \frac{1}{c^2 r}\mathbf{g}\right)\right\}.$$

The electric field of the falling charge in the reference frame N^g, proves to be identical with the field of an inertial charge determined in its rest frame:

$$\mathbf{E} = \frac{e}{4\pi\epsilon_o}\frac{\mathbf{n}}{r^2}. \tag{3.29}$$

As in the case of an electron falling in an accelerated frame, here too both an inertial observer (falling with the electron) and a non-inertial observer (at rest in N^g) detect a Coulomb field of the electron falling in N^g. In other words, while the electron is falling in the Earth's gravitational field its world path is geodesic (as general relativity tells us) and its electric field at any instant is the Coulomb field which means that no force is acting on the electron (i.e. there is no resistance to its accelerated motion).

Chapter 4

Some Insights

4.1 The Curvature of Space-Time

In general relativity the curvature of space-time is considered to be caused by the presence of matter. The anisotropy in the propagation of light in the vicinity of material bodies is a consequence of the space-time curvature due to the mass of the bodies. If, however, the entire mass of a body is electromagnetic (originating from the self-force acting on each elementary charged particle of the body), it follows that its active gravitational mass also originates from its charges (since according to the GTMG hypothesis there is no mass but only charges) which means that it is the charges (and their fields) that change the properties of space-time around the body in such a way that the speed of electromagnetic interaction towards the body is greater that the speed in the opposite direction. This means that the anisotropy in the speed of light around the Earth (i.e. inside a non-inertial reference frame N^g at rest on the

Earth's surface) is caused by all elementary charged particles constituting the building blocks of the Earth. Due to this anisotropic speed of light in N^g, expressed by (3.17), the electric field of an electron on the Earth's surface (determined in N^g) is distorted and the resulting self-force (3.26) pulls the electron downwards. Hence, what we have called gravitational force turns out to be, according to the GTMG hypothesis, the electric self-force acting on an elementary charged particle due to its own electric field (so we have obtained the answer to the question what is the nature of the a body on the Earth's surface is subjected to - it is *not* a gravitational force; its an electric force). As we have seen in Chapters 2 and 3 the electric field of an electron falling towards the Earth's surface (determined in N^g) at any instant is the Coulomb field. In other words, the electron is falling because it tries to keep its electric field symmetric (the Coulomb field); if the electron is prevented from falling, its electric field becomes distorted due to the greater speed of the electromagnetic interaction towards the Earth than in the opposite direction and a self-force which tends to force the electron to move downwards with an acceleration $\mathbf{g}$ arises. The anisotropy in the propagation of the electromagnetic interaction in the Earth's vicinity is compensated if a charge is moving downwards with an acceleration $\mathbf{g}$ (as shown in Chapters 2 and 3) which explains why all bodies are falling with the *same* acceleration towards the Earth and why a falling body is not subjected to any force (as the formalism of general relativity predicts).

In such a way, the anisotropy of the speed of electromagnetic interaction caused by the elementary charged particles (the Earth is built of) is responsible for the distortion of an electron's electric field which in turn accounts for the gravitational attraction the electron is subjected to. That is why the anisotropy in the electromagnetic propagation turns out to be the very curvature of space-time according to the GTMG hypothesis. The interpretation of space-time curvature in terms of anisotropy is not so unexpected since there have always been two options for understanding the meaning of Riemann curvature tensor in general relativity - it represents either a real (geometrical) curvature of space-time or some kind of anisotropy in the (physical) properties of space-time. Considering the mass to be of purely electromagnetic nature leads to the latter option. This conclusion implies that the gravitational field does not possess its own, separate existence - it is rather a change in the properties of space time which is caused by the charges of (massive) objects. The consequence of the GTMG hypothesis that the anisotropy in the propagation of electromagnetic interaction in the vicinity of a body (which we call the body's gravitational field) being caused by its charges and their fields is electromagnetic in origin also sheds light on the question why it has not been possible to construct an energy-momentum tensor of the gravitational field in general relativity: what we have called gravitational energy turns out to be electromagnetic energy.

The motion of an electron is completely determined by its field - (i) if it is the Coulomb field, there is no (resistant) self-force acting on the electron, its world path is geodesic which means that the electron is moving by inertia - both an inertial observer I and a non-inertial observer N detect a Coulomb field of the electron and agree that its motion is inertial (force-free or non-resistant) ; (ii) if the electron's field is distorted (both I and N detect the same distortion), its world path is also deformed (not geodesic) and a self-force arises and opposes the deviation of the electron from its inertial state, i.e. opposes the deformation of its world path.

In general, planets are orbiting the Sun, light is bending when passing near the Sun, and bodies are falling towards the Earth due to the greater speed of the electromagnetic interaction towards the Sun/Earth than in the opposite direction. A more descriptive definition of inertial (geodesic) motion can now be given. An electron is moving by inertia if its Coulomb field is not distorted. Such a non-resistant motion is represented by a geodesic world line. If the geodesic world lines are straight lines the space-time is isotropic with respect to the propagation of electromagnetic interaction (flat space-time); if the geodesic lines are curved the space-time is anisotropic (curved space-time).

The property of a world line to be geodesic is an absolute one determined by the structure of space-time (in accordance with general relativity).

As we can in principle control and manipulate any kind of electromagnetic phenomena and as the anisotropy of the speed of light being caused by charges is an electromagnetic phenomenon in origin as well, one day we can produce such an anisotropy and make a body move in whatever direction we want (this possibility still holds, but partly if the mass turns out to be only partly electromagnetic as presently believed).

4.2 The Principle of Equivalence

The principle of equivalence is a direct consequence of (i) the fact that the speed of light is equally anisotropic inside an accelerated reference frame N^a and a reference frame N^g at rest on the Earth's surface (as seen form (3.1) and (3.17)) and (ii) the determination of the motion of an electron by its field.

The anisotropy in the speed of light in N^a is caused by the accelerated motion of N^a; in N^g it is caused by the Earth's charges. The electric field of an electron at rest in N^a (as viewed by an observer in N^a) is distorted due to the anisotropic speed of light (3.1) in N^a giving rise to the (inertial) self-force (3.10) and the inertial mass of the electron. The electric field of an electron at rest in N^g (as viewed by an observer in N^g) is distorted due to the anisotropic speed of light (3.17) in N^g which gives rise to the (gravitational) self-force (3.26) and the gravitational mass of the electron. The inertial and gravitational mass of an electron are equivalent since they are the same thing: the electron's electromagnetic mass arising from the electromagnetic resistance an electron offers to being deviated from its geodesic (inertial) state.

An observer in N^a at any instance detects a Coulomb field of an electron falling in N^a which means that it is moving by inertia since no self-force is acting on it; if something prevents the electron from falling, however, its electric field becomes distorted and the self-force (3.10) appears opposing the change in the inertial state of the electron. An observer in N^g at any instance also detects a Coulomb field of an electron falling in N^g which implies that there is no self-force acting on the electron; in other words, the falling electron is (as in N^a) moving by inertia. If the electron is prevented from falling in N^g, its electric field deforms, the self-force (3.26) arises and resists the deviation of the electron from its geodesic (inertial) state.

Chapter 5

Has There Been an Alternative Path to General Relativity?

5.1 Introduction

An opportunity for revealing the nature of inertia and gravitation in terms of the electromagnetic mass theory may have been missed at the time (and after) Minkowski gave the four-dimensional formulation of special relativity. In 1911 Einstein [35] showed that the speed of light in an accelerated reference frame was anisotropic and realized that it should be anisotropic in a gravitational field as well. If the electromagnetic mass theory had been explored thoroughly it would have been possible to notice that it is the anisotropy in the speed of electromagnetic interaction in an accelerated frame N^a and in a frame N^g at rest in a gravitational field that is responsible for the distortion of the electric field of an electron at rest in N^a and N^g, respectively which gives rise to the electron's inertial and gravitational mass.

Substituting the anisotropic speed of light c^g in the expression for the interval yields:

$$ds^2 = (c^g)^2 dt^2 - dx^2 - dy^2 - dz^2,$$

where c^g here is the anisotropic speed of light defined at any space-time point (2.2). Substituting c^g in the interval and noting that

$$c^g = c\left(1 + \frac{gr}{c^2}\right) = c\left(1 + \frac{\Phi}{c^2}\right),$$

where Φ is the difference in the gravitational potential, the interval obtains the form

$$ds^2 = \left(1 + \frac{\Phi}{c^2}\right)^2 c^2 dt^2 - dx^2 - dy^2 - dz^2,$$

or in a more compact form

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu,$$

where

$$g_{\mu\nu} = \begin{pmatrix} \left(1 + \frac{\Phi}{c^2}\right)^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{5.1}$$

is the metric tensor in the case of anisotropic speed of electromagnetic interaction. It is immediately seen from here that the metric in this case is not the metric of a flat spacetime. This observation may have lead to an alternative path to general relativity in which the emphasis on the mechanism of gravitation may have been greater than that on its mathematical description.

To answer the Chapter's question it will be necessary to demonstrate that general relativity can be fully constructed from the metric tensor (5.1) (this is the subject of future work). It will be shown in this chapter that the Newtonian limit of general relativity follows immediately from (5.1) *without* any additional assumptions.

5.2 The Newtonian Limit of General Relativity

The standard way of obtaining the Newtonian limit of general relativity is the following [5], [49] -[54]. It is assumed that in the limit of weak and slowly-varying gravitational field (which means that the velocity of the object producing that field is smaller that c) the geodesic equation (5.2) reduces to $\frac{d^2x^i}{dt^2} = g$ and Einstein's equation (5.3) reduces to the differential form of Newton's law of gravitation (expressed by the Poisson equation $\nabla^2\Phi = 4\pi G\rho$).

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0 \tag{5.2}$$

$$R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} = \frac{8\pi G}{c^4}T_{\alpha\beta} \tag{5.3}$$

In equation (5.2) $\Gamma^{\mu}_{\alpha\beta}$ are the metric connections which can be expressed by the metric tensor $g_{\alpha\beta}$

$$\Gamma_{\mu\nu\sigma} = \frac{1}{2}(g_{\mu\nu,\sigma} + g_{\nu\sigma,\mu} - g_{\sigma\mu,\nu}) \tag{5.4}$$

and ds is the proper time.

In equation (5.3) $R_{\alpha\beta}$ is the Ricci tensor obtained by a contraction of the Riemann curvature tensor $R_{\sigma\alpha\rho\beta}$:

$$R_{\alpha\beta} = g^{\rho\sigma} R_{\sigma\alpha\rho\beta}.$$

R is the scalar (spacetime) curvature (the contracted Ricci tensor $R = g^{\alpha\beta} R_{\alpha\beta}$.), $g_{\alpha\beta}$ is the metric tensor of curved spacetime, G is the gravitational constant and $T_{\alpha\beta}$ is the stress-energy tensor (of matter).

In the classical (Newtonian) limit of general relativity the curvature of space time can be regarded as a perturbation of flat spacetime. Mathematically this can be expressed by representing the metric tensor of curved spacetime $g_{\alpha\beta}$ by the metric tensor of flat spacetime $\eta_{\mu\nu}$ and another "perturbation" tensor $h_{\mu\nu}$ whose components are much less than unity:

$$g_{\alpha\beta} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{5.5}$$

The components of $g_{\alpha\beta}$ are of the order of unity since

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

and the components of $h_{\mu\nu}$ are proportional to c^{-2} as we shall see below.

As the velocity of an object whose gravitational field is slowly varying is much less than c, we have

$$dx^0 = cdt \gg dx^i,$$

where as usual roman letters $i, k, l, ...$ take on the values 1, 2 and 3, while the Greek letters $\alpha, \beta, \gamma ...$ vary from 0 to 3. Also the proper time $ds = cd\tau$ in the non-relativistic approximation becomes equal to cdt. In this approximation $dx^0/ds \gg dx^i/ds$. When this is taken into account the geodesic equation (5.2) reduces to [54]:

$$\frac{d^2x^i}{dt^2} + c^2\Gamma^i_{00} = 0 \tag{5.6}$$

since in the Newtonian limit Γ^i_{00} are the only components of the metric connections that are not negligible.

Only from (5.6) we cannot obtain the classical equation of motion. It is necessary to presuppose the classical result and to identify the terms of equation (5.6) with the corresponding terms of the classical equation.

Consider a case of a constant acceleration g in the i direction:

$$\frac{d^2x^i}{dt^2} = g. \tag{5.7}$$

A comparison of (5.6) and (5.7) gives

$$\Gamma^i_{00} = -\frac{g}{c^2}. \tag{5.8}$$

Substituting (5.8) in (5.6) yields

$$\frac{d^2x^i}{dt^2} = g.$$

To obtain the differential form of Newton's law of gravitation it is necessary to use an alternative form of Einstein's equation (5.3) [54]:

$$R_{\alpha\beta} = \frac{8\pi G}{c^4}\left(T_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}T^\mu_\mu\right). \tag{5.9}$$

In the Newtonian limit (5.9) reduces to

$$R_{00} = \frac{8\pi G}{c^4}\left(T_{00} - \frac{1}{2}g_{00}T^0_0\right) \tag{5.10}$$

since in this limit (with velocities $\ll c$) the spatial components of momenta are negligible compared to energies; that is why only the energy density T_{00} of the stress-energy tensor is kept. The corresponding approximation for the Ricci tensor is [54]:

$$R_{00} = \frac{1}{2}\eta^{\alpha\sigma}\left(h_{\sigma 0,0\alpha} - h_{00,\sigma\alpha} + h_{0\alpha,\sigma 0} - h_{\sigma\alpha,00}\right), \tag{5.11}$$

where $\eta^{\alpha\sigma}$ is the metric tensor of flat spacetime and $h_{\sigma\alpha}$ is the "perturbation" tensor of equation (5.5); as usual comma denotes differentiation. As was the case of the classical limit of the geodesic equation here too the time derivatives are negligible compared to spatial derivatives (since each time derivative is proportional to c^{-1} - $\frac{d}{cdt}$). Then equation (5.11) reduces to

$$R_{00} = -\frac{1}{2}\eta^{ij}h_{00,ij} = \frac{1}{2}h_{00,ii}. \tag{5.12}$$

Equation (5.12) by itself again does not yield a term of the classical gravitational equation. It is necessary to use the result of the gravitational red shift experiment to determine h_{00} [54]*:*

$$h_{00} = -\frac{2GM}{rc^2} = \frac{2\Phi}{c^2}.$$

Then (5.12) representing the left-hand side of equation (5.10) becomes

$$R_{00} = \frac{\Phi_{,ii}}{c^2} = \frac{\nabla^2\Phi}{c^2}. \tag{5.13}$$

If it is supposed that the rest density of matter is ρ we can write

$$T_{00} = \rho c^2$$

and then the right-hand side of (5.10) takes the form of

$$T_{00} - \frac{1}{2}g_{00}T_0^0 = \frac{\rho c^2}{2}. \tag{5.14}$$

Substituting (5.13) and (5.14) in (5.10) we get

$$\nabla^2\Phi = 4\pi G\rho,$$

which is exactly the Poisson equation representing the differential form of Newton's law of gravitation.

As we have seen general relativity does not transform naturally (by itself) into the Newtonian gravitational theory in the limit of weak fields and slow velocities of the bodies producing those fields (we have twice *identified* general relativistic quantities with the corresponding classical quantities to get the classical gravitational equations). By contrast special relativity naturally transforms into classical mechanics in the limit $v^2/c^2 \to 0$.

5.3 The Newtonian Limit of General Relativity Without Assumptions

Now we shall see that using the metric tensor for the case of anisotropic light speed naturally leads to the Newtonian gravitation laws without the unavoidable assumptions necessary to get the Newtonian limit of general relativity.

If the metric tensor is determined on the basis of the anisotropic speed of electromagnetic interaction it has the form (5.1). Only its diagonal elements are different from zero: $g_{ii} = -1$ and

$$g_{00} = \left(1 + \frac{\Phi}{c^2}\right)^2 \approx 1 + \frac{2\Phi}{c^2}. \tag{5.15}$$

A comparison of (5.5) and (5.15) shows that in this case it is not necessary to make the assumption to represent $g_{\mu\nu}$ by the metric tensor of flat spacetime $\eta_{\mu\nu}$ and a "perturbation" tensor $h_{\mu\nu}$ (as was the case in (5.5)). Here g_{00} contains two terms by itself. The "perturbation" term (equivalent to h_{00}) is proportional to c^{-2}. This demonstrates that the gravitational effects we are subjected to in our everyday life result from perturbations to $\eta_{\mu\nu}$ which are of the order of c^{-2}.

As all $g_{ii} = -1$ (i.e. constants) substituting (5.1) in the equation for the metric connections (5.4) shows that only $\Gamma^{\mu}_{00} = g^{\mu\mu}\Gamma_{\mu 00}$ is different from zero. Γ^{0}_{00} is negligible since it is proportional to c^{-3}

$$\Gamma^0_{00} = g^{00}\Gamma_{000} = \frac{1}{2}g^{00}\frac{dg_{00}}{cdt} = \frac{1}{2}g^{00}\frac{dg_{00}}{cdt} = \frac{1}{c^3}g^{00}\frac{d\Phi}{dt}.$$

This means that the geodesic equation reduces to

$$\frac{d^2x^i}{dt^2} + c^2\Gamma^i_{00} = 0. \tag{5.16}$$

The explicit form of Γ^i_{00} is

$$\Gamma^i_{00} \;=\; \frac{1}{2}g^{ii}\left(g_{i0,0} + g_{0i,0} - g_{00,i}\right)$$

$$= -\frac{1}{2}g^{ii}g_{00,i} = \frac{1}{2}g_{00,i}$$

$$= \frac{1}{2}\frac{\partial}{\partial x^i}g_{00}.$$

As $g_{00} = \left(1 + \frac{2\Phi}{c^2}\right)$ Γ^i_{00} obtains the form

$$\Gamma^i_{00} = \frac{1}{c^2}\frac{\partial\Phi}{\partial x^i}. \tag{5.17}$$

Substituting (5.17) in (5.16) yields

$$\frac{d^2x^i}{dt^2} = -\frac{\partial\Phi}{\partial x^i}. \tag{5.18}$$

As

$$-\frac{\partial\Phi}{\partial x^i} = g$$

(5.18) becomes

$$\frac{d^2x^i}{dt^2} = g$$

showing that a free body in a region in which the speed of light is anisotropic (or in traditional terms in a gravitational field) moves with an acceleration g (determined by the expression (2.13)) in the direction of larger speed of light. Thus, we have obtained this classical result without any assumptions.

Now if we substitute (5.1) in the most general expression for the Ricci tensor

$$R_{\alpha\beta} = g^{\rho\sigma}R_{\sigma\alpha\rho\beta} = g^{\rho\sigma}g_{\sigma\mu}R^{\mu}_{\alpha\rho\beta}$$

$$= g^{\rho\sigma} g_{\sigma\mu} \left(\Gamma^{\mu}_{\alpha\beta,\rho} - \Gamma^{\mu}_{\alpha\rho,\beta} + \Gamma^{\mu}_{\eta\rho}\Gamma^{\eta}_{\alpha\beta} - \Gamma^{\mu}_{\eta\beta}\Gamma^{\eta}_{\alpha\rho} \right)$$

it reduces to

$$R_{00} = -\frac{1}{2} g^{ii} g_{00,ii} = \frac{1}{2} g_{00,ii} = \frac{1}{c^2} \Phi_{,ii} = \frac{1}{c^2} \nabla^2 \Phi.$$

Here we have obtained the same result (5.13) as in the previous section but without making any assumption. Assuming $T_{00} = \rho c^2$ which leads to

$$T_{00} - \frac{1}{2} g_{00} T^0_0 = \frac{\rho c^2}{2}$$

the alternative form of Einstein's equation (5.9) reduces to the Poison equation:

$$\nabla^2 \Phi = 4\pi G \rho.$$

In such a way the GTMG which uses only the fact of anisotropic speed of electromagnetic interaction in the vicinity of (massive) objects (and the idea of electromagnetic mass) to explain the nature of gravitational interaction is consistent with general relativity. This, however, is only a necessary condition for the GTMG hypothesis to be correct; a sufficient condition would mean that general relativity can be fully constructed on the basis of the GTMG hypothesis. This is the subject of future work.

Chapter 6

Conclusions and Contributions

6.1 Conclusions

It has been shown that both inertial mass and gravitational mass may result from the self-force with which each non-inertial elementary charged particle acts upon itself through its own electric field. This hypothesis leads to the conclusion that both inertial and gravitational masses have the same origin and are of entirely electromagnetic nature.

Regarding the mass of each elementary charged particles as electromagnetic in origin provides a consistent and common explanation of inertia and gravitation. Inertia is the electromagnetic resistance a charged particle offers to being accelerated originating in the interaction of the particle's charge with its own electric field (which means that the particle's inertial mass is electromagnetic). Gravitation also turns out to be of electromagnetic nature. The anisotropy in the propagation of light (and electromagnetic interaction in general) in the Earth's vicinity is caused by its charges. As the speed of light downwards (towards the Earth's centre) is larger than the speed upwards the electric field of an electron for instance is distorted which gives rise to an electric self-force trying to force the electron to move downwards. We call this force gravitational but it is electric (meaning that the passive gravitational mass is also electromagnetic); even if the GTMG hypothesis (regarding the entire mass as electromagnetic) turns out to be wrong, this force is again of partly electric nature since it is now accepted that part of the mass of each elementary charged particle is electromagnetic in origin (but it has not been realized before). The anisotropy is compensated if the electron is falling with an acceleration g - in this case its electric field is the Coulomb field and the electrons motion is geodesic (non-resistant). In such a way the GTMG hypothesis offers answers to the open questions of general relativity as formulated in the introduction of the thesis:

1. Why is no force acting on a body falling towards the Earth's surface? *Answer:* the body is falling since each of its charged particles "tries" to compensate the anisotropy in the propagation of electromagnetic interaction in order to keep its electric field symmetric (the Coulomb field); if the body is preventing from falling the electric fields of each charged particle distorts due to the anisotropic speed of electromagnetic interac-

tion and an electric self-force appears which forces each particle of the body to move downwards.

2. Why is a body on the Earth's surface subjected to a force? *Answer:* as explained above a body on the Earth's surface is prevented from falling (which means that the body can no longer compensate the anisotropy of light speed and cannot keep its charged particles' Coulomb fields not distorted); as a result each charged particle's electric field is distorted which gives rise to an electric self-force acting on the particle.

3. What is the nature of that force? *Answer:* it is an electric force; more precisely, an electric self-force arising from the interaction of a particle's charge with its own electric field whenever the field is distorted.

4. Why does the formalism of general relativity refuse to produce a tensor of the energy and momentum of gravitational field? *Answer:* it seems there is no such thing as a gravitational field (which is different from and is causing the anisotropy in the speed of light) and the formalism of an adequate physical theory naturally refuses to represent something which is not out there.

5. What is the nature of gravitational field and is there a gravitational energy? *Answer:* (i) according to the GTMG hypothesis what we call

a gravitational field is the anisotropy in the speed of propagation of electromagnetic interaction in the vicinity of an object which is caused by the object's charges; (ii) what traditionally is called gravitational energy is in fact electromagnetic energy - the tidal electric power stations for instance are converting not gravitational energy into electric energy but electric energy into electric energy.

Even if one insists that not the whole mass of an electron but only a part of it is of electromagnetic nature, the proposed approach is worth developing because its conclusions still hold partly and it stimulates further advancement in the following areas: studying the properties of the vacuum, boosting further research in the foundations of quantum mechanics (in order to obtain a model of the quantum object), understanding the meaning of the relativistic increase of the mass, the mass defect, the energy-mass relation, and the effective mass of the electron in solid state physics. The GTMG hypothesis reveals that even if only part of the electron mass is considered to be electromagnetic (as presently believed), it follows that part of its active gravitational mass is also electromagnetic which demonstrates that a body's gravity is partly of electromagnetic origin too. Stated another way, the possibility for at least partly controlling inertia and gravitation has always been present since the beginning of this century but has not been recognized..

The consequence of the GTMG hypothesis that the space-time curvature means anisotropy in the propagation of electromagnetic interaction arising from an object's charges (and their fields) is directly testable. Such a test can determine whether the whole mass of an elementary charged particle is electromagnetic in origin or not. This hypothesis opens up the possibility of (at least partly) controlling inertia and gravitation and can be experimentally tested by a version of the Aharonov-Bohm effect setup employing laser beams instead of electron beams.

6.2 Contributions

Bellow are listed what I regard as contributions in the thesis:

1. Demonstrating that the inertial and gravitational properties of a body can find a common and consistent explanation if its entire mass is considered to be electromagnetic in origin. This approach opens up the possibility of controlling inertia and gravitation: if inertia and gravitation are of electromagnetic nature and if we can control and manipulate (at least in principle) any electromagnetic phenomena, it follows that both inertia and gravitation can in principle be controlled as well.

2. The approach followed in the thesis reveals that the possibility for partly controlling inertia and gravitation has always been present since the beginning of this century when the electromagnetic mass theory was proposed but has not been recognized. Since then it has been accepted (in fact, it is an almost forgotten fact) that part of the mass of every elementary charged particle is electromagnetic in origin, but it has not been realized that if so inertia and gravitation are partly controllable. Concerning gravitation it becomes even more evident: as the mass of an elementary charged particle is of partly electromagnetic nature it immediately follows that its active gravitational mass is also of partly electromagnetic nature which means that the particle's gravity is partly caused by its charge.

3. Deriving direction-dependent expressions for the speed of light in a non-inertial frame (accelerated or supported in a gravitational field). It is impossible to calculate the electric potential and the electric field of an electron in a non-inertial reference frame without an expression for the average direction-dependent velocity of light.

4. Showing that for both an inertial observer I and a non-inertial observer N a Coulomb field is associated with an inertial charge (represented by

a geodesic worldline) and both I and N detect a distorted field of a non-inertial charge (whose worldline is not geodesic). This result resolves the dispute whether or not a falling charge in a gravitational field radiates: as a falling charge is described by a geodesic worldline its motion is inertial and there is no radiation (the charge's field is the Coulomb field).

5. Determining the electron's electric potential in an accelerated reference frame and in a frame supported in a gravitational field (and correcting Fermi's error of 1/2 in the gravitational case). This makes possible to calculate the electric field of an electron in a non-inertial frame where the calculations are essentially facilitated since only the scalar potential is used and no retarded times are involved.

6. This is a contribution which I consider to be of essential importance for the thesis. To correct Fermi's error it was necessary to realize an effect of an apparent enlargement of the volume element dV due to the anisotropic speed of light: $dV^a = \left(1 + \frac{\mathbf{a} \cdot \mathbf{r}}{2c^2}\right) dV$. This is in an accelerated frame; in a gravitational field the sign is minus. This effect is analogous to the apparent enlargement of the volume element of a moving charge as calculated in an inertial frame; when the correct volume element is used in the expressions for the electric potentials (of a charge

moving with respect to an inertial frame) we get the retarded (Liénard-Wiechert) potentials. When the correct volume element dV^a is used in the electric potential of an electron at rest in an accelerating frame we get the expression for its distorted potential $\varphi^a(r,t) = \frac{e}{4\pi\epsilon_o}\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{c^2}\right)$. Without the realization of this effect it is not possible to calculate the electric potential, the electric field and the self-force of a charge in a non-inertial reference frame; this in turn reveals that one of the reasons why the ideas developed here have not been put forward earlier is the failure to recognize that effect.

7. Using the volume element dV^a in the calculation of the self-force in a non-inertial frame naturally removes the famous factor of 4/3 in the expression for the electromagnetic mass without resorting to the Poincaré stresses. This has not been done before either.

Appendix
Calculation of the self-force $\mathbf{F}^a_{self}$

As we have seen the anisotropic volume element is

$$dV^a = \left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2}\right) dV, \tag{6.1}$$

where dV is the actual volume element (i.e. the volume element determined when the electron is at rest in an inertial reference frame). The anisotropic volume element dV^a affects the magnitude of the charge element de^a contained in dV^a as determined in N^a:

$$de^a = \rho dV^a = \rho\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2}\right) dV, \tag{6.2}$$

where ρ is the density of the electron charge.

Now, taking into account the expression for r^a

$$\frac{1}{r^a} = \frac{1}{r}\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2}\right)$$

and (6.2), we can write the electric potential of a charge element de^a of an electron at rest in N^a(keeping only the terms proportional to c^{-2}):

$$d\varphi^a(r,t) = \frac{de^a}{4\pi\epsilon_0 r^a} \approx \frac{\rho}{4\pi\epsilon_o r}\left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{c^2}\right) dV. \tag{6.3}$$

The electric field produced by de^a in N^a can be directly calculated from (6.3):

$$d\mathbf{E}^a = -\nabla d\varphi^a = \frac{1}{4\pi\epsilon_o}\left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{a}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a}\right)\rho dV,$$

where $\mathbf{n} = \mathbf{r}/r$. The electric field of the electron then is

$$\mathbf{E}^a = \frac{1}{4\pi\epsilon_o} \int \left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{a}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a} \right) \rho dV. \tag{6.4}$$

The self-force which the field of the electron exerts upon an element ρdV_1^a of the electron charge is

$$d\mathbf{F}^a_{self} = \rho dV_1^a \mathbf{E}^a = \frac{1}{4\pi\epsilon_o} \int \left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{a}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a} \right) \rho^2 dV dV_1^a.$$

The resultant self-force acting on the electron as a whole is:

$$\mathbf{F}^a_{self} = \frac{1}{4\pi\epsilon_o} \int\int \left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{a}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a} \right) \rho^2 dV dV_1^a,$$

which after taking into account (6.1) becomes

$$\mathbf{F}^a_{self} = \frac{1}{4\pi\epsilon_o} \int\int \left(\frac{\mathbf{n}}{r^2} + \frac{\mathbf{a}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a} \right) \left(1 + \frac{\mathbf{a}\cdot\mathbf{r}}{2c^2} \right) \rho^2 dV dV_1. \tag{6.5}$$

Keeping only the terms proportional to c^{-2} (6.5) becomes

$$\mathbf{F}^a_{self} = \frac{1}{4\pi\epsilon_o} \int\int \left(\frac{\mathbf{n}}{r^2} + \frac{3}{2}\frac{\mathbf{a}\cdot\mathbf{n}}{c^2 r}\mathbf{n} - \frac{1}{c^2 r}\mathbf{a} \right) \rho^2 dV dV_1. \tag{6.6}$$

We have reached this result assuming that the charge element de^a acts upon the charge element de_1^a. In this case the vector $\mathbf{r}$ begins at de^a and ends at de_1^a, i.e. $\mathbf{n}$ points from de^a to de_1^a. If we assumed that de_1^a acts upon de^a the result should be the same. As interchanging the two charge elements reverses the direction of $\mathbf{n}$ the self-force in this case will be

$$\mathbf{F}^a_{self} = \frac{1}{4\pi\epsilon_o}\int\int\left(-\frac{\mathbf{n}}{r^2}+\frac{3}{2}\frac{\mathbf{a}\cdot\mathbf{n}}{c^2 r}\mathbf{n}-\frac{1}{c^2 r}\mathbf{a}\right)\rho^2 dV dV_1. \qquad (6.7)$$

Adding equations (6.7) and (6.6) and dividing the result by 2 we get

$$\mathbf{F}^a_{self} = \frac{1}{4\pi\epsilon_o}\int\int\left(\frac{3}{2}\frac{\mathbf{a}\cdot\mathbf{n}}{c^2 r}\mathbf{n}-\frac{1}{c^2 r}\mathbf{a}\right)\rho^2 dV dV_1. \qquad (6.8)$$

In order to do the integral (6.8) let us consider the integral [34]

$$\mathbf{I} = \int\int\left(\frac{\mathbf{a}\cdot\mathbf{n}}{r}\mathbf{n}\right)dV dV_1. \qquad (6.9)$$

We can put $\mathbf{n} = \mathbf{n}_{\parallel}+\mathbf{n}_{\perp}$, where $\mathbf{n}_{\parallel}$ is parallel to $\mathbf{a}$ and $\mathbf{n}_{\perp}$ is perpendicular to $\mathbf{a}$. Then

$$\begin{aligned}
(\mathbf{a}\cdot\mathbf{n})\,\mathbf{n} &= \mathbf{a}\left(\mathbf{n}_{\parallel}+\mathbf{n}_{\perp}\right)\left(\mathbf{n}_{\parallel}+\mathbf{n}_{\perp}\right) \\
&= \left(\mathbf{a}\cdot\mathbf{n}_{\parallel}+\mathbf{a}\cdot\mathbf{n}_{\perp}\right)\left(\mathbf{n}_{\parallel}+\mathbf{n}_{\perp}\right) \\
&= \left(\mathbf{a}\cdot\mathbf{n}_{\parallel}\right)\mathbf{n}_{\parallel}+\left(\mathbf{a}\cdot\mathbf{n}_{\parallel}\right)\mathbf{n}_{\perp}+\left(\mathbf{a}\cdot\mathbf{n}_{\perp}\right)\mathbf{n}_{\parallel}+\left(\mathbf{a}\cdot\mathbf{n}_{\perp}\right)\mathbf{n}_{\perp} \\
&= \left(\mathbf{a}\cdot\mathbf{n}_{\parallel}\right)\mathbf{n}_{\parallel}+\left(\mathbf{a}\cdot\mathbf{n}_{\parallel}\right)\mathbf{n}_{\perp}
\end{aligned}$$

since $(\mathbf{a}\cdot\mathbf{n}_{\perp}) = 0$. Substituting this result in (6.9) yields

$$\mathbf{I} = \int\int\left(\frac{\mathbf{a}\cdot\mathbf{n}_{\parallel}}{r}\mathbf{n}_{\parallel}\right)dV dV_1 + \int\int\left(\frac{\mathbf{a}\cdot\mathbf{n}_{\parallel}}{r}\mathbf{n}_{\perp}\right)dV dV_1. \qquad (6.10)$$

To facilitate the calculations further let us assume that $\mathbf{r}$ is rotated 180° about an axis parallel to $\mathbf{a}$ running through the centre of the spherical charge distribution of the electron. Then the vector $\mathbf{n} = \mathbf{n}_{\parallel}+\mathbf{n}_{\perp}$ becomes $\mathbf{n}_{\parallel}-\mathbf{n}_{\perp}$. This means that in the second integral in (6.10) for every elementary contribution

$$\left(\frac{\mathbf{a}\cdot\mathbf{n}_{\parallel}}{r}\mathbf{n}_{\perp}\right)dVdV_1$$

there is also an equal and opposite contribution

$$-\left(\frac{\mathbf{a}\cdot\mathbf{n}_{\parallel}}{r}\mathbf{n}_{\perp}\right)dVdV_1$$

which shows that the second integral in (6.10) is zero and we can write

$$\mathbf{I}=\int\int\left(\frac{\mathbf{a}\cdot\mathbf{n}_{\parallel}}{r}\mathbf{n}_{\parallel}\right)dVdV_1. \tag{6.11}$$

The integral $\mathbf{I}$ is now a function only of $\mathbf{n}_{\parallel}$. In order to return to the general case of $\mathbf{n}$ (and not restrict ourselves to using $\mathbf{n}_{\parallel}$) we will express the integral in (6.11) in terms of $\mathbf{n}$ and a unit vector $\mathbf{u}$ in the direction of $\mathbf{a}$. Since $\mathbf{n}_{\parallel}$ is parallel to $\mathbf{a}$, we have $\mathbf{a}\cdot\mathbf{n}_{\parallel}=an_{\parallel}$. Then we can write

$$\begin{aligned}\left(an_{\parallel}\right)\mathbf{n}_{\parallel} &= \mathbf{a}\left(n_{\parallel}\right)^2=\mathbf{a}\left[1^2\left(n_{\parallel}\right)^2\right]= \\ &= \mathbf{a}\left(1n_{\parallel}\right)^2=\mathbf{a}\left(un_{\parallel}\right)^2=\mathbf{a}\left(un\cos\theta\right)^2 \\ &= \mathbf{a}\left(\mathbf{u}\cdot\mathbf{n}\right)^2\end{aligned}$$

where θ is the angle the vector $\mathbf{n}$ forms with the vector of the acceleration $\mathbf{a}$. Now we can write the integral (6.11) in the form

$$\mathbf{I}=\mathbf{a}\int\int\frac{\left(\mathbf{u}\cdot\mathbf{n}\right)^2}{r}dVdV_1. \tag{6.12}$$

Following Abraham [12] and Lorentz[13] we have assumed a spherically symmetric distribution of the electron charge. This shows that as all directions in space are indistinguishable the integral in (6.12) should be independent of the direction of the unit vector $\mathbf{u}$. In such a way the average of this integral over all possible directions of $\mathbf{u}$ should be equal to the integral itself:

$$\begin{aligned} \int\int \frac{(\mathbf{u}\cdot\mathbf{n})^2}{r} dV dV_1 &= \frac{1}{4\pi}\int d\Omega \int\int \frac{(\mathbf{u}\cdot\mathbf{n})^2}{r} dV dV_1 \\ &= \frac{1}{4\pi}\int\int \frac{dV dV_1}{r}\int (\mathbf{u}\cdot\mathbf{n})^2 d\Omega \end{aligned} \tag{6.13}$$

where $d\Omega$ is an element of the solid angle within which a given unit vector $\mathbf{u}$ lies. To do this integral we choose a polar coordinate system with the polar axis along $\mathbf{n}$. Then $\mathbf{u}\cdot\mathbf{n} = \cos\theta$ and $d\Omega = \sin\theta d\theta d\varphi$ and

$$\begin{aligned} \frac{1}{4\pi}\int (\mathbf{u}\cdot\mathbf{n})^2 d\Omega &= \frac{1}{4\pi}\int_0^{\pi}\cos^2\theta\sin\theta d\theta\int_0^{2\pi} d\varphi \\ &= \frac{1}{2}\int_0^{\pi}\cos^2\theta\sin\theta d\theta \\ &= \frac{1}{2}\left(-\cos^3\theta \,|_0^{\pi}\right) \\ &= \frac{1}{2}\left[-\frac{1}{3}(-1-1)\right] \\ &= \frac{1}{3}. \end{aligned}$$

Substituting this result in (6.13) yields

$$\int\int \frac{(\mathbf{u}\cdot\mathbf{n})^2}{r} dV dV_1 = \frac{1}{3}\int\int \frac{dV dV_1}{r}.$$

Thus for the integral (6.12) we have

$$\mathbf{I} = \mathbf{a}\int\int \frac{(\mathbf{u}\cdot\mathbf{n})^2}{r} dV dV_1 = \frac{\mathbf{a}}{3}\int\int \frac{dV dV_1}{r}. \tag{6.14}$$

By substituting (6.14) in (6.8) we obtain

$$\begin{aligned}\mathbf{F}^a_{self} &= \frac{1}{4\pi\epsilon_o}\int\int \left(\frac{3}{2}\frac{\mathbf{a}}{3c^2 r} - \frac{\mathbf{a}}{c^2 r}\right)\rho^2 dV dV_1 \\ &= -\frac{\mathbf{a}}{8\pi\epsilon_o c^2}\int\int \rho^2 dV dV_1\end{aligned}$$

and finally the expression for the self-force becomes

$$\mathbf{F}^a_{self} = -\frac{U}{c^2}\mathbf{a}, \tag{6.15}$$

where

$$U = \frac{1}{8\pi\epsilon_o}\int\int \frac{\rho^2}{r} dV dV_1$$

is the electron's electrostatic energy. As U/c^2 is the mass "attached" to the field of an electron, i.e. its electromagnetic mass, we can write (6.15) in the form:

$$\mathbf{F}^a_{self} = -m^a\mathbf{a}, \tag{6.16}$$

where m^a is identified with the electron's inertial mass.

The self-force $\mathbf{F}^a_{self}$ to which an electron is subjected due to its own field is directed opposite to $\mathbf{a}$ and resists its acceleration, i.e. the deformation of its world path. The famous factor of 4/3 in the electromagnetic mass of the electron does not appear in (6.16). The reason is that in (6.5) we have used the correct volume element $dV_1^a = \left(1 + \frac{\mathbf{a} \cdot \mathbf{r}}{2c^2}\right) dV_1$.

References

[1] W. Pauli, *Theory of Relativity*, (New York: Dover, 1958).

[2] E. Schrodinger, *Phys. Z.*, **19** 4 (1918).

[3] H. Bauer, *Phys. Z.*, **19** 163 (1918).

[4] A. Einstein, *Ann. Phys.*, **49** 769-822 (1916).

[5] P. A. M. Dirac, *General Theory of Relativity*, (New York: John Wiley and Sons, 1975).

[6] S. Weinberg, *The Quantum Theory of Fields*, Vol. 1, (New York: Cambridge University Press, 1995), 307.

[7] B. Haisch, A. Rueda, and H. E. Puthoff, *Phys. Rev. A* **49**, 678 (1994).

[8] H. E. Puthoff, *Phys. Rev. A* **39**, 2333 (1989).

[9] J. J. Thompson, *Phil. Mag.* **11**, 229 (1881).

[10] O. Heaviside, *The Electrician* **14**, 220 (1885).

[11] G. F. C. Searle, *Phil. Mag.* **44**, 329 (1897).

[12] M. Abraham, *The Classical Theory of Electricity and Magnetism*, 2nd ed. (London: Blackie, 1950).

[13] H. A. Lorentz, *Theory of Electrons*, 2nd ed. (New York: Dover, 1952).

[14] J. W. Butler, *Am. J. Phys.* **37**, 1258 (1969).

[15] P. Pearle, in D. Teplitz, ed. *Electromagnetism: Paths to Research* (New York: Plenum Press, 1982).

[16] J. Scherk, *Revs. Mod. Phys.* **47**, 123 (1975).

[17] G. Whitham, *Linear and Nonlinear Waves* (New York: John Wiley and Sons, 1974).

[18] S. Coleman, *Phys. Rev. D* **11**, 2088 (1975).

[19] K. Johnson, *Acta Physica Polonica B* **6**, 865 (1975).

[20] A. H. Anastassov, *Annuaire de l'Universite de Sofia St. Kliment Ohridski, Faculte de Physique* **81**, 135 (1993).

[21] F. Rohrlich, *Phys. Rev. D* **25**, 3251 (1982).

[22] V. Petkov, *J. Phys.* submitted.

[23] M. von Laue, *Relativitäts Theorie* 3rd edn Vol 1 (Braunschweig: Frederick Vieweg und Sohn, 1919).

[24] G. A. Schott, *Electromagnetic Radiation* (London: Cambridge University Press, 1912).

[25] S. M. Milner, *Phil. Mag.* **41** 405 (1921).

[26] J. L. Synge, *Relativity: The Special Theory* 2nd edn (Amsterdam: North-Holland Publ. Co., 1964).

[27] T. Fulton and F. Rohrlich, *Ann. Phys.* **9** 499 (1960).

[28] S. Coleman, in *Electromagnetism: Paths to Research* ed D Teplitz (New York: Plenum Press, 1982).

[29] A. Kovetz and G. E. Tauber, *Am. J. Phys.* **37** 382 (1969).

[30] D. G. Boulware, *Ann. Phys.* **124** 169 (1980).

[31] H. Ren and E. J. Weinberg, *Phys. Rev.* **D49** 6526 (1994).

[32] F. Rohrlich, *Classical Charged Particles* (New York: Addison-Wesley, 1990).

[33] D. J. Griffiths, *Introduction to Electrodynamics* 2nd edn (London: Prentice Hall, 1989).

[34] B. Podolsky and K. S. Kunz, *Fundamentals of Electrodynamics* (New York: Marcel Dekker, 1969).

[35] A. Einstein, *Ann. Phys.* **35**, 898 (1911).

[36] R. P. Feynman, R. B. Leighton and M. Sands, *The Feynman Lectures on Physics*, Vol 2, (New York: Addison-Wesley, 1964).

[37] M. Schwartz, *Principles of Electrodynamics* (New York: Dover Publications, 1972).

[38] J. D. Jackson, *Classical Electrodynamics* 2nd edn (New York: Wiley, 1975).

[39] E. Fermi, *Nuovo Chimento* **22** 176 (1921).

[40] W. Rindler, *Essential Relativity: Special, General and Cosmological,* 2nd ed., (New York, Heidelberg, Berlin: Springer-Verlag, 1977), pp.117-118.

[41] D. J. Griffiths and R. E. Owen, *Am. J. Phys.* **51**, 1120 (1983).

[42] H. Mandel, *Z. Physik* **39**, 40 (1926).

[43] W. Wilson, *Proc. Phys. Soc.* **48**, 736 (1936).

[44] M. H. L. Pryce, *Proc. Roy. Soc. A* **168**, 389 (1938).

[45] B. Kwal, J. *Phys. Rad.* **10**, 103 (1949).

[46] F. Rohrlich, *Am. J. Phys*. **28**, 63 (1960).

[47] E. Fermi, *Phys. Zeits.* **23**, 340 (1922).

[48] W. K. H. Panofsky and M. Phillips, *Classical Electricity and Magnetism*, 2nd ed. (Massachusetts, London: Addison-Wesley Publ. Co. Inc., 1962).

[49] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation* (San Francisco: W. H. Freeman, 1973).

[50] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (New York: John Wiley and Sons, 1972).

[51] H. Ohanian and R. Ruffini, *Gravitation and Spacetime*, 2nd ed., (New York, London: W. W. Norton, 1994).

[52] R. M. Wald, *General Relativity* (Chicago, London: The University of Chicago Press, 1984).

[53] L. P. Hughston and K. P. Tod, *An Introduction to General Relativity* (Cambridge: University Press, 1990).

[54] I. R. Kenyon, *General Relativity* (Oxford, New York, Tokyo: Oxford University Press, 1990).

[55] L. I. Schiff, *Am. J. Phys.*, **28**, 340 (1960).

[56] V. B. Braginsky and V. I. Panov, *Zh. Eksp. i Teor. Fiz.* **61**, 873 (1971); translated in *Sov. Phys. JTEF* **34**, 463 (1972).