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The Logic of Global Conventionalism

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ABSTRACT

The Logic of Global Conventionalism

Michael John Assels

Global conventionalism is characterized as the doctrine which holds that two scientific theories may explain just the same observational facts without being translations of one another. This characterization is borrowed, in substance, from Shaw, but his criterion of scientificality is rejected as being controversial and unnecessary to the refutation of global conventionalism.

A refutation of GC is undertaken which proceeds from a semantic rule implicit in GC itself: The two theories under discussion must share a common interpretation with respect to their observation language.

Six lemmas are proved concerning the sets of interpretations of the theories' languages. The sixth lemma reveals that disagreement in the interpretation of the observation language is a necessary condition of the non-translatability of theories. From this lemma, a theorem is proved to the effect that GC is inconsistent.

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The errors which remain are, of course, my own.

TABLE OF CONTENTS

Chapter

CHAPTER I

PRELIMINARIES

Introduction

In what follows, we shall be making certain claims about a doctrine which Steven Shaw calls "global conventionalism" (GC).[1] This is the doctrine which holds that two distinct scientific theories may have the same observational consequences, and that any scientific extension of one theory which increases the observational import of the theory will be matched by a scientific extension of the second theory which has just the same observational consequences.

GC is philosophically interesting because if it is *false*, then any two scientific theories which explain and systematize the actual observable facts must have the same models. To know--*per impossibile*--all the observable facts would be to know the non-observable facts which serve to explain and systematize the observable. The situation would never arise in which one had to choose between two scientific theories which had equal claims to acceptance, but which made different assertions about the structure of the world.

[1] Steven G. Shaw, "An Examination of Global Conventionalism" (M.A. thesis, Concordia University, 1984) *passim*. Shaw attributes the words, but not the doctrine to Paul Horwich, "How to Choose between Empirically Indistinguishable Theories", *Journal of Philosophy*, vol. 79, no. 2 (Feb. 1982), pp. 62-77.

If, on the other hand, GC is true, we must swallow hard and become agnostics about the unobservable world, for we can conduct no experiment which might help us decide between the two theories in question.

Not surprisingly, then, there has been a good deal of discussion of conventionalism in the literature,[2] although it has not always centered on *global* conventionalism as we have briefly characterized it; a number of distinct but related doctrines have gone under the name 'conventionalism'.[3]

No one, however, has undertaken a reasonably rigorous treatment of the logical relations which hold--or would hold--among conventional alternatives. This is rather surprising, since GC is obviously a logical doctrine.

Shaw has made a start, however, by proposing a set of conditions which must be fulfilled if any two theories are to count as instances of GC. Informally, these conditions are as follows:

(i) The theories must share a common observation language, and must have just the same theorems in that language.

(ii) The theories, when interpreted, must agree in what they assign to the symbols of their shared observation language.

(iii) The theories must not be intertranslatable.

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[2]For a bibliography, see Shaw, pp. 122-26.

[3]For a thorough discussion of the various views which have been called "conventionalism", see Shaw, Ch. I, pp. 1-30.

(iv) Each theory must have as much, and only as much, theoretical structure as is necessary to entail its observational theorems.

and (v) It must be possible, for any theory which adds observational import to one of the theories while continuing to fulfill condition (iv), to find an extension of the other theory such that the two new theories fulfill the first four conditions.[4]

Conditions (i), (ii), (iii) and (v) serve to sharpen the notion of 'genuine conventional alternatives', and condition (iv) is a necessary formal condition of scientificality. Theory pairs which violate condition (i) are not observationally equivalent. It will therefore be possible to choose between them on the basis of experimental evidence. Pairs which violate condition (ii) are not concerned with the same observational facts. They are therefore not to be considered as alternatives in the relevant sense. Pairs which violate condition (iii) are synonymous. Their differences are only differences of symbolism; they describe the same objects. Pairs which violate condition (iv) are unscientific, and therefore irrelevant to the doctrine of GC. Finally, pairs which violate condition (v) are conventional alternatives only for the moment. With the development of science, it will eventually become evident that one of the theories admits of successful scientific growth while the other does not.

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[4] Shaw, pp. 70–71. We do not quote directly for two reasons: First, Shaw's formal conditions are not easy reading. Secondly, we do not wish to provoke any confusion which might arise on account of differences between Shaw's symbolism and our own. Shaw confirms, in private communication, that our rendering of his conditions is correct.

These conditions seem generally correct. However, they include, as we have noted, a partial characterization of scientific theories. Consequently, any conclusion about the truth or falsehood of GC, based on these conditions, will be open to attacks on the grounds that scientific theories are not correctly characterized by condition (iv). The formal features of scientific theories have proved notoriously elusive,[5] so that attacks will probably not be long in coming.

It will therefore be of some use to recast Shaw's conditions in such a way that they will be as neutral as possible with respect to competing notions of scientificality:

(i) The theories must share a common observation language, and must have just the same theorems in that language.

(ii) The theories, when interpreted, must agree in what they assign to the symbols of their shared observation language.

(iiia) Neither theory may be the product of translation from the other.

(iva) Each theory must be scientific.

(v) It must be possible, for any theory which adds observational import to one of the theories while continuing to fulfill condition (iva), to find an extension of the other theory such that the two new theories fulfill the first four conditions.

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[5]For an extensive review of the history of the search for formal criteria of scientificality, see Frederick Suppe, "The Search for Philosophic Understanding of Scientific Theories", in Frederick Suppe (ed.), *The Structure of Scientific Theories*, 2nd ed. (Urbana, Ill.: U. of Illinois Press, 1977), pp. 1-241.

The reason for (iva) should be obvious. The replacement of (iii) by the stronger (iiia), however, calls for some explanation. We wish, in general, to reject as an instance of GC any pair of theories such that one theory is simply an extension of the other; for in such a case, the extended theory is true whenever its extension is true. The extension is thus not an alternative but an addition to the extended theory.

Shaw eliminates this possibility with condition (iv). The extension fails to qualify as a scientific theory. Since we reject condition (iv), however, we must choose a stronger criterion of distinctness: condition (iiia).

Our new set of conditions is indeed quite neutral with respect to competing notions of scientificality, but we pay a price for this neutrality. We are now unable to establish that any pair of theories instantiates GC, since we have not specified any way of determining whether or not a theory fulfills condition (iva).

It remains possible, however, to show that *no* pair of theories instantiates GC as we have characterized it. This is what we shall do here. Specifically, we shall prove a theorem to the effect that no pair of theories can fulfill conditions (i), (ii) and (iiia). Moreover, since these conditions involve no reference to any formal requirements on scientific theories, our theorem will stand irrespective of any formal view of science.

We shall conclude that GC, as we have characterized it, is logically false, and we shall challenge the conventionalist to reformulate his doctrine in such a way as to escape the import of our theorem.

First, however, we shall need some definitions. The reader will want to know what we mean by 'translation', by 'observational equivalence', and by 'genuine conventional alternatives'.

An Elaboration of Concepts

Translation

We must begin our discussion of translation with a characterization of definitions. A sentence is a *definition* of a symbol α from symbols $\beta_1, \ldots, \beta_n$ if, and only if, it has the form

$$(x_1)\ldots(x_k) \quad (\text{--}\alpha\text{--} \leftrightarrow \_\_\_\ \beta_1, \ldots, \beta_n \ \_\_\_), \text{ where}$$

all non-logical symbols occurring in $\_\_\_\ \beta_1, \ldots, \beta_n \ \_\_\_$ belong to $\{\beta_1, \ldots \beta_n\}$;

all variables occurring free in $\_\_\_\ \beta_1, \ldots, \beta_n \ \_\_\_$ belong to $\{x_1, \ldots, x_k\}$,

and --α-- is

the formula $x_1 = \alpha$ if α is a name (in this case $k = 1$),
the sentence α if α is a sentence letter (in this case $k = 0$ and the definition is a biconditional),
the formula $\alpha x_1 \ldots x_k$ if α is a k-place predicate letter, and
the formula $x_k = \alpha(x_1, \ldots, x_{k-1})$ if α is a $(k-1)$-place function sign.[6]

A symbol α is said to be *explicitly definable* in a theory T from symbols $\beta_1, \ldots, \beta_n$ if, and only if, T entails a definition of α from $\beta_1, \ldots, \beta_n$.

A symbol α is said to be *implicitly definable* in a theory T from

[6] George Boolos and Richard Jeffrey, *Computability and Logic*, 2nd ed. (Cambridge: Cambridge U. Press, 1980), p. 246.

symbols $\beta_1, \ldots, \beta_n$ if, and only if, any two models of T with the same domain which agree in what they assign to $\beta_1, \ldots, \beta_n$ also agree in what they assign to α.

In Chapter II, we shall have occasion to make use of Beth's definability theorem: A symbol α is implicitly definable in a theory T from symbols $\beta_1, \ldots, \beta_n$ if, and only if, α is explicitly definable in T from $\beta_1, \ldots, \beta_n$.[7]

Having said what needs to be said about definitions, we are now ready to specify what we shall mean by 'translation product'.

A theory T_1 in a language L_1 yields a theory T_2 (in L_2) as a *translation product* if, and only if, T_1, together with a set D_1 of definitions of theoretical symbols in L_2 from symbols in L_1, entails all theorems of T_2 and no other sentences in L_2. (The distinction between observational and theoretical symbols is, for our purposes, completely arbitrary.)

If either of two theories yields the other as a translation product, then the two theories are *translatable*; otherwise they are *non-translatable*.

Two theories are *observationally equivalent* if, and only if, they share an observation language and have exactly the same theorems in the observation language. Thus it is observational equivalence that is required by condition (i).

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[7]Evert Beth, "On Padoa's Method in the Theory of Definition", *Indagationes Mathematicae*, vol. 15 (1953), pp. 330-39. We present the theorem as it is presented in Boolos and Jeffrey, p. 246.

Genuine Conventional Alternatives

Two theories are *genuine conventional alternatives* if, and only if, they are observationally equivalent and non-translatable; that is, two theories which fulfill conditions (i) and (iiia) are genuine conventional alternatives. It should be noted that the existence of genuine conventional alternatives, as we have characterized them, does not entail GC. The alternatives may fail to fulfill any of the other three conditions.

In the next chapter, we shall argue that no pair of theories satisfies conditions (i), (ii) and (iiia). We shall be treating a theory as a set of sentences expressed, not in any natural language, but in a highly artificial symbolic language. Thus, when we speak of a theory's language, we shall be speaking of the set whose members are all the non-logical symbols (e.g., predicate letters) occurring in the theory's sentences. Because languages are sets, set theoretic predicates and operations are defined for languages. Thus we may speak of "membership in a language", "the union of two languages", etc. Of course, we shall also make extensive use of the more specific property of languages: their susceptibility of interpretation.

With these matters clarified, we may now proceed to the proof of our theorem.

CHAPTER II

PROOFS

Conventions

The proofs which we undertake here will be made a great deal shorter if we adopt a few conventions:

First, we shall assume that T_1 and T_2 are theories in the first order functional calculus which share an observation language L_0, but whose languages L_1 and L_2, respectively, are otherwise distinct (i.e., $L_1 \cap L_2 = L_0$). The language L ($= L_1 \cup L_2$) is the language which contains every symbol of each theory.

Secondly, we shall use the symbol 'D_1', ('D_2') for a set of definitions such that, for each symbol α in $L_2 \sim L_0$ ($L_1 \sim L_0$), D_1 (D_2) contains exactly one definition of α from symbols of L_1 (L_2).

Thirdly, when we speak of an 'interpretation' we shall mean an 'interpretation of L'. In fact, many different interpretations will share a common assignment to symbols of L, but for our purposes, all are equivalent. We therefore make a convenient and innocuous simplification by treating sets of equivalent interpretations--equivalent, that is, with respect to L--as though they were single interpretations.

Finally, we shall consider only some arbitrary set of interpretations which agree in what they assign to symbols of the observation language L_0. This is a crucial restriction, but it is justified in the context of this inquiry. The reader will recall that condition (ii) requires that theory pairs which instantiate GC agree, when interpreted, in what they assign to symbols of the observation language. Our restriction is thus no more than a stipulation that this condition must be fulfilled.

Before proceeding, we should pause to take note of an important consequence which follows from this last convention: that all interpretations must share a common domain. This is due to the fact that what is "assigned" to a predicate letter--we assume that any scientific theory will have at least one predicate letter--is a characteristic function. A characteristic function of an n-place predicate letter is a *total* function from the set of all n-tuples of objects in the domain to the set of truth values. Thus it is impossible for interpretations which differ in their domains to agree in what they assign to any predicate letter; the characteristic functions will inevitably differ in at least some of their arguments.

Definitions

We shall have need of some concepts having to do with interpretations and sets of interpretations:

Definition 1. Any set whose members are just the interpretations which share some common assignment to symbols of L_n ($n = 0, 1, 2$) is an L_n-*constant set*.

Definition 2. The L_n-*partition* ($n = 1, 2$) of an L_0-constant set Γ is the set of all L_n-constant sets included in Γ.

Definition 3. Every member of an L_n-partition ($n = 1, 2$) is a *part* of that partition.

Definition 4. A T_n-*modeling part* ($n = 1, 2$) of an L_n-partition is a part whose members are all models of T_n.

Lemmas

Now we prove six lemmas which bear on the concepts just defined. The first is a relatively straightforward lemma about membership in L_n-constant sets:

Lemma 1. For any L_n-constant set Γ ($n = 1, 2$) either all members of Γ are models of T_n or no members of Γ are models of T_n.

Proof

Let Γ be an L_n-constant set ($n = 1, 2$). By definition, all members of Γ agree in what they assign to symbols of L_n. So if some members of Γ are models of T_n and others are not, then T_n is not a theory in L_n. But by definition, T_n is a theory in L_n. So either all members of Γ are models of T_n, or no members of Γ are models of T_n.

Q.E.D.

It should be noted that, as a consequence of lemma 1, every part of an L_n-partition ($n = 1, 2$) which is not a T_n-modeling part contains no model of T_n.

It will be recalled that T_2 is a translation of T_1 iff T_1, together with a set D_1 of definitions of theoretical symbols in L_2 from symbols in L_1, entails all and only theorems of T_2 in L_2. It is clear enough that $T_1 \bigcup D_1$ entails *all* theorems of T_2 iff every model of $T_1 \bigcup D_1$ is a model of T_2. Our second lemma establishes the necessary and sufficient conditions for $T_1 \bigcup D_1$'s entailing *only* theorems of T_2 in L_2:

Lemma 2. $T_1 \cup D_1$ entails only theorems of T_2 in L_2 iff, for any L_2-constant set Γ which contains models of T_2, Γ contains a model of $T_1 \cup D_1$.

Proof

First the 'if':

Let $T_1 \cup D_1$ entail a sentence S in L_2 which is not a theorem of T_2. T_2 is consistent with ~S, but $T_1 \cup D_1$ is not. That is, some interpretation $\mathcal{J}$ is a model of both T_2 and ~S, but no interpretation is a model of both $T_1 \cup D_1$ and ~S. Let Γ be the L_2-constant set to which $\mathcal{J}$ belongs. ~S is a sentence of L_2, and all members of Γ agree with $\mathcal{J}$ in what they assign to symbols of L_2, so all members of Γ are models of ~S. Therefore no member of Γ is a model of $T_1 \cup D_1$, although Γ contains a model ($\mathcal{J}$) of T_2.

Now the 'only if':

Let Γ be some L_2-constant set which contains models of T_2 but no models of $T_1 \cup D_1$. Now let A be the set of sentences of L_2 whose models are just the interpretations which do not belong to Γ. Some models of T_2 (i.e., those which belong to Γ) are not models of A, so T_2 does not entail every sentence in A. Let S be a sentence in A which is not entailed by T_2. $T_1 \cup D_1$ entails every sentence in A, since the only non-models of A are the members of Γ, and $T_1 \cup D_1$ has no models in Γ. *A fortiori*, $T_1 \cup D_1$ entails S. Since S is a sentence of L_2, and S is not entailed by T_2, $T_1 \cup D_1$ entails a non-theorem of T_2 in L_2. *Q.E.D.*

Lemma 3 establishes that every interpretation corresponds to a unique pair of parts. We shall make use of this fact later, but for now we prove only that it is a fact:

Lemma 3. For any L_0-constant set Γ, each member of Γ belongs to exactly one part of Γ's L_1-partition, and to exactly one part of Γ's L_2-partition.

Proof

Let Γ be any L_0-constant set and let $\mathcal{J}$ be any member of Γ. $\mathcal{J}$ interprets L, so $\mathcal{J}$ interprets L_1 and L_2. By definition, the parts of Γ's L_1-partition are just the L_1-constant sets included in Γ. $\mathcal{J}$ must be a member of some such set, since $\mathcal{J}$ interprets L_1. So $\mathcal{J}$ is a member of some part of Γ's L_1-partition. Suppose that $\mathcal{J}$ belongs to two such parts, A and B. By definition, A is the set of all interpretations of L which share a certain assignment to symbols of L_1; and similarly for B, but with a different assignment. Since $\mathcal{J}$ belongs to A, every interpretation of L which agrees with $\mathcal{J}$ in what it assigns to symbols of L_1 belongs to A. But then every member of B is a member of A. Similarly, every member of A is a member of B. So A and B are identical. Therefore $\mathcal{J}$ belongs to exactly one part of Γ's L_1-partition.

Exactly similar reasoning establishes that $\mathcal{J}$ belongs to exactly one part of Γ's L_2-partition. *Q.E.D.*

The concept of observational equivalence was defined in Chapter I in terms of entailment of sentences in L_0. Our fourth lemma establishes that observational equivalence is also definable in terms of L_0-constant sets and the models contained in them:

Lemma 4. Two theories T_1 and T_2 are observationally equivalent iff, for every L_0-constant set Γ, Γ contains models of T_1 iff Γ contains models of T_2.

Proof.

First the 'only if':

Let Γ be any L_0-constant set such that Γ contains models of T_1 but no models of T_2 (or vice versa; the reasoning is the same). Let A be the set of sentences of L_0 such that the models of A are just the interpretations which do not belong to Γ. Every model of T_2 is a model of A, but, since T_1 has a model in Γ, some model of T_1 is not a model of A. That is, T_2 entails a set of sentences in L_0, but T_1 does not entail those sentences. So T_1 and T_2 are not observationally equivalent.

Now the 'if':

Let T_1 entail a sentence S in L_0 which is not entailed by T_2 (or vice versa; again, the reasoning is the same). Since T_2 does not entail S, S is not valid. So $\sim S$ has a model. Since T_1 entails S but T_2 does not, T_2 is consistent with $\sim S$ but T_1 is not. That is, some model of T_2 is a model of $\sim S$ but no model of T_1 is a model of $\sim S$. Let $\mathcal{J}$ be any interpretation which is a model of both T_2 and $\sim S$, and let Γ be the L_0-constant set to which $\mathcal{J}$ belongs. $\sim S$ is a sentence of L_0, and every member of Γ agrees with $\mathcal{J}$ in what it assigns to symbols of L_0, so every member of Γ is a model of $\sim S$. Therefore no member of Γ is a model of T_1. That is, Γ contains models of T_2 but no models of T_1. *Q.E.D.*

Lemma 5. A set of sentences in L is logically equivalent to a set of definitions of symbols of $L_2 \sim L_0$ from symbols of L_1 if, and only if, it has exactly one model in each L_1-constant set.

Proof

Let D_1 be a set of definitions of symbols in $L_2 \sim L_0$ (i.e., in the theoretical language of T_2) from symbols in L_1. By Beth's definability theorem, any two models of D_1 which agree in what they assign to L_1-symbols will agree in what they assign to $(L_2 \sim L_0)$-symbols. Let Γ be any L_1-constant set. By definition, all members of Γ agree in what they assign to L_1-symbols. So all members of Γ which are models of D_1 agree in what they assign to $(L_2 \sim L_0)$-symbols. Thus all members of Γ which are models of D_1 agree in what they assign to all symbols of L. But Γ does not contains two distinct members which agree in what they assign to symbols of L, so D_1 has at most one model in Γ.

If, for any L_1-constant set Γ, Γ contains no models of D_1, then it will not be possible to specify a model of D_1 in Γ. We now show how to specify such a model.

D_1 contains, for each symbol α in $L_2 \sim L_0$, exactly one sentence S_α of definitional form:

$$(x_1) \ldots (x_k)\ (\text{--}\alpha\text{--} \longleftrightarrow \text{\_\_\_}\beta_1, \ldots, \beta_n\text{\_\_\_}).$$

Let $\mathcal{I}$ be any member of Γ. Now let $\mathcal{J}$ be the interpretation of L which agrees with $\mathcal{I}$ in what it assigns to symbols of L_1, and which, for each α in $L_2 \sim L_0$, assigns to the k-open place formula --α-- whatever it assigns to the k-open place formula

$$\text{\_\_\_}\beta_1, \ldots, \beta_n\text{\_\_\_}\ \&\ x_1 = x_1\ \&\ \ldots\ \&\ x_k = x_k.$$

Thus, for each α, $\mathcal{J}$ assigns to S_α whatever it assigns to

$$(x_1) \ldots (x_k)\ (\text{\_\_\_}\beta_1, \ldots, \beta_n\text{\_\_\_}\ \&\ x_1 = x_1\ \&\ \ldots\ \&\ x_k = x_k \longleftrightarrow \text{\_\_\_}\beta_1, \ldots, \beta_n\text{\_\_\_}).$$

Since every such sentence is valid, all are true in $\mathcal{J}$. So every S_α is true in $\mathcal{J}$. That is, $\mathcal{J}$ is a model of D_1. Moreover, since $\mathcal{J}$ agrees with the member $\mathcal{J}$ of Γ in what it assigns to symbols in L_1, $\mathcal{J}$ is in Γ.

Thus, D_1 has at least one model, and at most one model, in every L_1-constant set. The same is true of any set of sentences logically equivalent to D_1, since logical equivalents have exactly the same models.

Suppose now that a set A of sentences in L has exactly one model in each L_1-constant set. Any two models of A which agree in what they assign to symbols of L_1 agree in what they assign to symbols of $L_2 \sim L_0$. So, by Beth's theorem, A entails a set D_1 of definitions of $(L_2 \sim L_0)$-symbols from L_1-symbols. Because D_1 is such a set of definitions, it has exactly one model in each L_1-constant set. That is, A and D_1 have the same number of models. Moreover, since A entails D_1 every model of A is a model of D_1. So A and D_1 have just the same models; i.e., they are logically equivalent.

Therefore, a set of sentences in L is logically equivalent to a set of definitions of symbols of $L_2 \sim L_0$ from symbols of L_1 if, and only if, it has exactly one model in each L_1-constant set. *Q.E.D.*

Our last lemma is the longest. It sets forth, in terms of the concepts defined in the last section, the necessary and sufficient conditions for two theories' being genuine conventional alternatives. We beg the reader's forgiveness in advance, and we proceed:

Lemma 6. If T_1 and T_2 are two observationally equivalent theories, then T_1 and T_2 are non-translatable iff (i) there is an L_0-constant set Γ_1 such that Γ_1's L_1-partition contains more T_1-modeling parts than there are T_2-modeling parts in Γ_1's L_2-partition, and (ii) there is an L_0-constant set Γ_2 such that Γ_2's

L_2-partition contains more T_2-modeling parts than there are T_1-modeling parts in Γ_2's L_1-partition.

Proof

First the 'if':

Let Γ_1 be an L_0-constant set whose L_1-partition contains more T_1-modeling parts than there are T_2-modeling parts in Γ_1's L_2-partition. Let Γ_2 be an L_0-constant set whose L_2-partition contains more T_2-modeling parts than there are T_1-modeling parts in Γ_2's L_1-partition. Let T_2 be a translation product of T_1. Then, for some set D_1 of definitions, $T_1 \cup D_1$ does not entail any sentence S in L_2 unless T_2 entails S. By lemma 2, $T_1 \cup D_1$ has at least one model in each L_2-constant set which contains models of T_2. So $T_1 \cup D_1$ has at least one model in each T_2-modeling part of Γ_2's L_2-partition. That is, the number of models of $T_1 \cup D_1$ in Γ_2 is greater than or equal to the number of T_2-modeling parts in Γ_2's L_2-partition. But, by lemma 5, D_1 has exactly one model in each part of Γ_2's L_1-partition. So $T_1 \cup D_1$ has exactly one model for each T_1-modeling part in Γ_2's L_1-partition. That is, the number of models of $T_1 \cup D_1$ is exactly equal to the number of T_1-modeling parts in Γ_2's L_1-partition. So the number of T_1-modeling parts in Γ_2's L_1-partition is at least as great as the number of T_2-modeling parts in Γ_2's L_2-partition. But this contradicts our hypothesis, so T_2 is not a translation product of T_1.

Exactly similar reasoning, exchanging the '1's and '2's in the subscripts, will prove that T_1 is not a translation product of T_2. So T_1 and T_2 are non-translatable.

Now the 'only if':

Let T_1 and T_2 be two observationally equivalent theories such that, for every L_0-constant set Γ, there are at least as many T_1-modeling parts in Γ's L_1-partition as there are T_2-modeling parts in Γ's L_2-partition. We now show that T_2 is a translation product of T_1 (i.e., that there exists a D_1 such that the L_2-theorems of $T_1 \cup D_1$ are just the L_2-theorems of T_2).

Since, for each L_0-constant set Γ, there are at least as many T_1-modeling parts in Γ's L_1-partition as there are T_2-modeling parts in Γ's L_2-partition, it is possible to put the T_2-modeling parts in Γ's L_2-partition into one-one correspondence with the members of some subset of T_1-modeling parts in Γ's L_1-partition. Let γ be any total function (from T_2-modeling parts into T_1-modeling parts) which establishes such a correspondence.[8] By lemma 3, for any Γ, each member of Γ belongs to exactly one part of Γ's L_1-partition and to exactly one part of Γ's L_2-partition. So we can specify a set of A of sentences in L by specifying the models of A as follows:

For each L_0-constant set Γ,

(i) for every T_1-modeling part B in Γ's L_1-partition, if B is a value of γ, then the interpretation which belongs both to B and to the T_2-modeling part C in Γ's L_2-partition such that $\gamma(C) = B$, is a model of A, and no other member of B is a model of A;

(ii) if γ has any arguments, then for some arbitrary argument C of γ, and for every part B in Γ's L_1-partition, if B is not a value of γ, then the interpretation which belongs both to B and to C is a model of A, and no other member of B is a model of A;

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[8]We invoke the axiom of choice here.

and (iii) if γ has no arguments, then for some arbitrary part D of Γ's L_2-partition, and for every part B in Γ's L_1-partition, the interpretation which belongs both to B and to D is a model of A, and no other member of B is a model of A.

This specification of A's models is complete for it determines, for each member of each B in each Γ, whether the member is a model of A or not; and these are just the interpretations of L. Moreover, the specification is unambiguous for any choice of γ, C and D, since the three clauses apply to distinct B's. (Clauses (i) and (iii) do not "overlap"; γ has values iff it has arguments.) Finally, the specification assigns to A exactly one model in each part in each Γ's L_1-partition (i.e., in each L_1-constant set). So, by lemma 5, A is logically equivalent to the set D_1 of definitions of symbols of $L_2 \sim L_0$ from symbols of L_1.

Clause (i) of the specification ensures that each T_2-modeling part in each Γ's L_2-partition contains at least one model of $T_1 \cup A$. So, by lemma 2, and by the equivalence of A and D_1, $T_1 \cup D_1$ does not entail any sentence S in L_2 unless T_2 entails S. By hypothesis, T_1 and T_2 are observationally equivalent, so by lemma 4, for any L_0-constant set Γ, T_1 has no models in Γ iff T_2 has no models in Γ. But γ has no arguments just when T_2 has no models in Γ. So clause (iii) is applicable only when $T_1 \cup A$ can have no models in Γ. Clauses (i) and (ii) make an interpretation a model of A only if it is a member of an argument of γ (i.e., only if it is a model of T_2). So the specification ensures that every model of $T_1 \cup A$ (and hence, every model of $T_1 \cup D_1$) is a model of T_2. Thus, $T_1 \cup D_1$ entails all, and only, theorems of T_2 in L_2. That is, T_2 is a translation product of T_1.

Moreover, if the '1' and '2' be exchanged in the subscripts of the hypothesis, exactly similar reasoning proves that T_1 is a translation product of T_2.

Therefore, T_1 and T_2 are non-translatable only if (i) there is an L_0-constant set Γ_1 such that Γ_1's L_1-partition contains more T_1-modeling parts than there are T_2-modeling parts in Γ_1's L_2-partition, and (ii) there is an L_0-constant set Γ_2 such that Γ_2's L_2-partition contains more T_2-modeling parts than there are T_1-modeling parts in Γ_2's L_1-partition. *Q.E.D.*

With all our lemmas proved, we may now move on to our theorem.

Theorem

The theorem follows from lemma 6:

Theorem 1. No theory has a genuine conventional alternative.

Proof

Let T_1 be a theory with a genuine conventional alternative T_2. By definition, T_1 and T_2 are observationally equivalent and non-translatable. So by lemma 6, there is an L_0-constant set Γ_1 such that Γ_1's L_1-partition contains more T_1-modeling parts than there are T_2-modeling parts in Γ_1's L_2-partition, and there is an L_0-constant set Γ_2 such that Γ_2's L_2-partition contains more T_2-modeling parts than there are T_1-modeling parts in Γ_2's L_1-partition. Γ_1 and Γ_2 are distinct, for if $\Gamma_1 = \Gamma_2$, then the number of T_1-modeling parts in Γ_1's L_1-partition is both greater than and less than the number of T_2-modeling parts in Γ_1's L_2-partition. But by our convention, all interpretations agree in what they assign to symbols of L_0. So by definition 1, there is only one L_0-constant set. Therefore Γ_1 and Γ_2 cannot be distinct, but this is absurd. *Q.E.D.*

CHAPTER III

CONCLUSION

Theorem 1 asserts that no theory has a genuine conventional alternative, which is to say that no pair of theories satisfies conditions (i) and (iiia). Therefore, if theorem 1 is true, GC is false.

But is theorem 1 true? It is not a theorem of pure logic, because it rests upon a substantive convention about allowable interpretations: the convention which admits only such interpretations as agree in what they assign to symbols of the observation language. It is reasonable, therefore, to ask under what conditions theorem 1 holds true. The answer, expressed pedantically, is that the theorem is true in all cases where the convention is legitimately adopted. When is the convention legitimately adopted? Whenever condition (ii) is true.

Since condition (ii) is a necessary condition for the truth of GC, we now come to see just what theorem 1 is. It is a theorem of GC! Thus, GC itself entails its own falsehood, and we may assert unconditionally that GC is false. Conditions (i), (ii) and (iiia) are inconsistent.

The conventionalist cannot attack this conclusion on the grounds that it embodies some questionable assumption about the formal conditions of scientificality. We have been careful not to make any such assumptions, unless they are inherent in the notion of an observation language. But if this is so, it becomes difficult to see how the conventionalist can even state his doctrine. If there is no common ground whatever between theories, how can it mean anything to say that they are conventional alternatives? Conventional alternatives in the explanation of what?

If our conclusion is to be attacked, it must be on the grounds that we have misrepresented the conventionalist's position. This is a charge to which we are, admittedly, vulnerable. We have not reproduced the words of any conventionalist who attempted to give a rigorous characterization of the logical nature of his views. We are not aware that any such attempt has been made. We have therefore had to rely upon our own assessment of GC, which in turn relies upon Shaw's.

We have tried to offer some justification for each of the conditions which we set forth. Nevertheless, it remains possible that what has just been knocked down is nothing but a straw man. We take the risk, but we ask the reader to take note of two important points. The first is that the "fallacy" of the straw man is not a fallacy. It is a valid argument against a position which no real man has adopted. GC, as we have characterized it, is false whether or not it is the view of any conventionalist. That is worth knowing. The second point is that when the real man is hiding in the tall grass, there is no better way of getting him to stand up and show himself than to knock down a straw man with his name on it. If we have *not* refuted GC, we shall find out soon enough what GC *really* is. That will be worth knowing.

BIBLIOGRAPHY

Articles

Beth, Evert. "On Padoa's Method in the Theory of Definition", *Indagationes Mathematicae* 15 (1953), pp. 330-39.

Carnap, Rudolf. "The Methodological Character of Theoretical Concepts", in Feigl, Herbert and Scriven, Michael (Eds.), *Minnesota Studies in the Philosophy of Science*. Minneapolis: University of Minn. Press, 1956. Vol. 1, pp. 38-56.

Craig, William. "Replacement of Auxiliary Expressions". *Philosophical Review*, 65 (1956), pp. 38-55.

Hempel, Carl G. "The Theoretician's Dilemma", in Feigl, Herbert, Maxwell, (Eds.), *Minnesota Studies in the Philosophy of Science*. Minneapolis: University of Minn. Press, 1958. Vol. II, pp. 37-98.

Horwich, Paul. "How to Choose between Empirically Indistinguishable Theories", *Journal of Philosophy*, Vol. 79, no. 2 (February, 1982), pp. 62-77.

Putnam, H. "The Refutation of Conventionalism", *Nous*, 8 (1974), pp. 25-41.

________. "An Examination of Grunbaum's Philosophy of Geometry", in B. Baumrin (Ed.), *Philosophy of Science: The Delaware Seminar*, Vol. 2 (1962-63). pp. 205-255.

________. "Models and Reality", *Journal of Symbolic Logic*, Vol. 45, no. 3 (September, 1980), pp. 464-482.

Quine, W.V.O. "Two Dogmas of Empiricism", in *From a Logical Point of View*, Cambridge, Mass.: Harvard University Press, 1980. pp. 20-46.

________. "On the Reasons for Indeterminacy of Translation", *Journal of Philosophy*, (1970), pp. 178-183.

Books

Angel, R.B. *Relativity: The Theory and its Philosophy*. Oxford: Pergamon Press, 1980.

Boolos, George and Jeffrey, Richard. *Computability and Logic*, 2nd ed. Cambridge: Cambridge U. Press, 1980.

Giedymin, J. *Science and Conventionalism: Essays in Conventionalism and Twentieth Century Philosophy of Science*. Oxford: Pergamon Press, 1981.

Grünbaum, A. *Geometry and Chronometry in Philosophical Perspective*. Minneapolis: University of Minnesota Press, 1968.

Kuhn, T.S. *The Structure of Scientific Revolutions*, 2nd ed. Chicago: University of Chicago Press, 1970.

Nagel, E. *The Structure of Scientific Theories*. New York: Harcourt, Brace, 1961.

Poincaré, Henri. *Science and Hypothesis*. New York: Dover Publications, 1952.

Reichenbach, H. *The Philosophy of Space and Time*. New York: Dover, 1958.

Sklar, L. *Space, Time and Space-Time*. California: University of California Press, 1974.

Sneed, J. *The Logical Structure of Mathematical Physics*. Dordrecht: Reidel, 1971.

Suppe, Frederick, ed. *The Structure of Scientific Theories*, 2nd ed. Urbana, Ill.: University of Illinois Press, 1977.

<u>Thesis</u>

Shaw, Steven G. "An Examination of Global Conventionalism". M.A. thesis. Concordia University, 1984.