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THE NONPARAMETRIC TESTS FOR EXPONENTIALITY WITH CENSORED SAMPLES

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ABSTRACT

The Nonparametric Tests for Exponentiality with Censored Samples

Li Yu Fu

Suppose that $T_1, T_2, \ldots, T_n$ are independent identically distributed random variables with continuous distribution function F. Let $Z_i = (S_i/S_{i+1})^i$, $i = 1, 2, \ldots, n-1$, where $S_i = \sum_{j=1}^{i} T_j$, $i = 1, 2, \ldots, n$. $Z_1, Z_2, \ldots, Z_{n-1}$ are independent and identical random variables distributed uniformly over (0,1) if and only if F is exponential distribution (Wang and Chang 1977). This result is applied to nonparametric tests of exponential distribution with unknown parameter when the sample is type I or type II or randomly censored respectively. And the W-test, C-test are modified to test exponentiality with the censored sample too. When failure time has an exponential distribution, we introduce a procedure to test standard random censorship model (Efron 1967) and the estimator of the parameter θ of exponential distribution and censoring parameter β. Using the Monte Carlo simulation method, the powers of several nonparametric tests of exponentiality and the Cramér-von Mises test when the alternative distribution is Weibull or gamma for type I, type II and randomly censored sample are examined and the results show that our test is better.

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I would like to express my deepest gratitude to Dr. M. Belinsky for his invaluable guidance and encouragement.

DEDICATION

I would like to dedicate this thesis to my husband Mr. Qin-Jian Guo and my mother Mrs. Xiu-Di Wu.

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

1.1 Problem and Review

One of the several important lifetime distributions in life testing is the exponential distribution with distribution function

$$F_0(t) = 1 - \exp(-t/\theta), \quad t \geq 0.$$

Suppose that $T_1, T_2, \ldots, T_n$ are i.i.d. with distribution function F. Testing the null hypothesis

$$H_0 : \ F = F_0, \tag{1.1}$$

when θ is known, has been thoroughly investigated. For instance, One method of testing $H_0 : F = F_0$ is to define statistics which consists of forming the empirical process $Y_n(t) = n^{1/2}[F_n(t) - F_0(t)]$, where $F_n(.)$ is the empirical distribution function of the sample, then use a functional on $Y_n(.)$, such as the Kolmogorov - Smirnov statistic D_n, the Aderson - Darling statistic $A_n{}^2$, Kupier statistics V_n and the Cramér - von Mises statistic $W_n{}^2$ for the test criteria. Such tests are known as EDF tests. However, the much more important situation is that in which $F_0(t)$ depends on unknown parameters. In this case, the EDF tests can be modified by inserting estimates of parameters in, but the distributions of the test statistics also depend on $F_0(t)$, and therefore the tests are no longer distribution free. There is generally no satisfactory way to get percentage points for small n except by simulation though a few special results are available. But for the exponential distribution, the percentage points have been obtained by Monte Carlo methods.

In survival studies, however, the goodness-of-fit problem is more complicated because the survival data may be censored.

1.1.1 Censoring models

Censoring distinguishes survival analysis from the other fields of statistics. A censored sample contains only partial information about the random variable of the interest. Suppose $T_1, T_2, \ldots, T_n$ is a random censored sample of failure time with distribution function F(t). Generally, there are three types of censoring.

(1) Type I censoring model:

Let $L_1, L_2, \ldots, L_n$ be some (preassigned) fixed numbers which we call the fixed censoring times and each of T_i is subjected to limited periods L_i, i= 1, 2, ...n. Instead of observing $T_1, T_2, \ldots, T_n$, we can only observe $Y_1, Y_2, \ldots, Y_n$ where $Y_i = min(T_i, L_i)$, $i = 1, 2, \ldots, n$. This is so-called Type I censored sample. If we let $L_1 = L_2 = \ldots = L_n$ = L, then L becomes the common censoring time for all observations. This special case is also a time-truncated sampling scheme, where the failure times can be ordered as $T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(r)} \leq L$, $(r \leq n)$. In this paper, we only consider this special case.

(2). Type II censoring model:

Let $r \leq n$ be fixed, we have a Type II censored sample if only the r smallest observations in the random sample $T_1, T_2, \ldots, T_n$ are observed. In a Type II censored sample, we will get an order statistics $T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(r)}$ of the T_1, T_2, ..., T_n from the life distribution.

In a Type II censored sample, r is the number of the smallest observations, which is constant and is decided upon before the data are collected. However in Type I censored model, the number of failure times, also denoted by r, is a random variable. If $f(t)$ is the probability density function of the failure time T, then $r \sim binomial(n,p)$ where $p = \int_0^l f(t)dt$. This means that using our Type I and Type II censoring models we can simply take $T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(r)}$ to be r order statistics from a Type I or Type II censored sample $T_1, T_2, \ldots, T_n$. Therefore, we may have the same formulas for both types of censoring but with different meaning of r.

(3). Random Censoring Model and the Standard Random Censorship Model

Suppose $T_1, T_2, \ldots, T_n$ is a random sample of failure times and $L_1, L_2, \ldots, L_n$ is a random sample of censoring times, with L_i associated with T_i. Suppose T_i's and L_i's are independent continuous random variables with distribution functions F(t) and H(t) respectively. We can only observe $(Y_1, \delta_1), (Y_2, \delta_2), \ldots, (Y_n, \delta_n)$ where $Y_i = min(T_i, L_i)$ and $\delta_i = 1$ when $Y_i = T_i$ and $\delta_i = 0$ when $Y_i = L_i$. This is a random censored sample. Notice that $Y_1, Y_2, \ldots, Y_n$ are i.i.d. random sample with some distribution function G. Also $\delta_1, \delta_2, \ldots, \delta_n$ are i.i.d random sample containing the censoring information.

The standard random censorship model is given by Efron in 1967. He assumed that the censoring time distribution function H(t) is related to failure time distribution function F(t) by $(1 - H(t)) = (1 - F(t))^{\beta}$, where parameter β is called the censoring parameter. This is a necessary assumption for the asymptotic distribution theory pertaining to Cramér-von Mises statistic ψ_2 obtained by Koziol and Green (1976), when F(t) is completely specified.

1.1.2 Tests for goodness-of fit with censored samples

As we know, with censoring, full knowledge of the empirical distribution is unavailable. When F_0 is completely specified, with the Type II and singly Type I (only right or only left censored) censored samples, simple modifications can be made to the EDF goodness of fit statistics and the distribution theory becomes slightly more complicated than in the corresponding uncensored situations. Barr and Davidson (1973), Koziol and Byar (1975) and Dufour and Maag (1978) considered the Kolmogorov-Smirnov test for Type I and Type II censoring. Dufour and Maag present tables of percentage points for both the Type I and Type II censored case for samples of sizes up to 25. Koziol and Bya determine the common asymptotic distribution of D_n in the case of singly Type I and Type II censoring, where with Type II censoring, r and n go to infinity, with r/n=p fixed. The generalization of ${W_n}^2$ and ${A_n}^2$ have been discussed by Pettit and Stephens (1976), who determine their asymptotic distributions and give a table of percentage points. Smith and Bain (1976) provide some small-sample percentage points for ${W_n}^2$ that are obtained by Monte Carlo methods. When F_0 contains unknown parameters the asymptotic distribution theory for ${W_n}^2$ and ${A_n}^2$ testing exponentiality, normality, and extreme value for Type I censoring has been considered by Pettit (1976). Mete and Ibrahim (1982) modified Cramér-von Mises statistic for testing exponentiality with type I censored samples, in the presence of an unknown parameter θ. They established the asymptotic distribution of Cramér-von Mises statistic when θ is estimated by its maximum likelihood estimator. Percentiles of the asymptotic distrib ... n are obtained for various levels of censoring. There is no satisfactory approach to EDF goodness of fit tests when data are arbitrarily censored. When $F_0(t)$ is fully specified, an obvious modification to Kolmogorov - Smirnov, Cramér - von Mises tests is to replace $F_n(t)$ with the product-limit estimate of $F_0(t)$. Distribution theory is difficult, however, because of the fact that the distributions of test statistics will depend on both censoring process and distribution function. Koziol and Green (1976) generalize the Cramér - von Mises statistic, denoted by ψ_2, in this way and obtain its asymptotic distribution under the assumption that censoring time are random variables independent of lifetimes, with survival function of the form $[1 - F_0(t)]^{\beta}$. They give asymptotic percentage points for a few values of β. Since both β and $F_0(t)$ are assumed to be known, these tests are not particularly useful.

There is another important method to test (1.1) which is called nonparametric method, such as Shapiro - Wilk test, Gini test and so on. Here we discuss two of them which can be modified to test exponentiality with censored sample under certain models.

First, we look at the Shapiro-Wilk test i.e. W - test. Suppose $T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(n)}$ are the order statistics of a random sample of size n, the W-test involves the computation of the statistic

$$W_E(n) = n(\overline{T} - T_{(1)})^2/(n-1)S^2 \qquad (1.2)$$

where $\overline{T} = \frac{1}{n}\sum_{i=1}^{n} T_{(i)}$ and $S^2 = \sum_{i=1}^{n}\left(T_{(i)} - \overline{T}\right)^2$. Let $U_i = (n-i+1)[T_{(i)} - T_{(i-1)}]$, $i = 1, 2, \ldots, n$, where $T_{(0)} = 0$, then $W_E(\text{n})$ can be written in terms of the U_i as

$$W_E = \frac{\sum_{i=2}^{n} {U_i}^2}{(n-1)\sum_{i=2}^{n}\sum_{j=2}^{n} {a_{ij}}^{(n)} T_i T_j} \tag{1.3}$$

where ${a_{ij}}^{(n)} = (j-1)/(n-j+1) \quad (n \geq i \geq j \geq 2)$ and ${a_{ij}}^{(n)} = {a_{ji}}^{(n)} \quad (i, j = 2, 3, \ldots, n)$. Although the exact null distribution of $W_E(\text{n})$ is not known, it does not depend on θ. Shapiro-Wilk presented tables of the upper and lower percentage points of $W_E(\text{n})$. The tables were obtained by simulation, for n from 3 to 100 for different alternatives. Since either low or high values of $W_E(\text{n})$ can occur so that this is a two-tailed test. In 1988, Samanta and Schwarz modified the Shapiro-Wilk test for testing exponentiality based on censored data. They proposed the W_1 statistic as

$$W_1 = \frac{\left(\sum_{i=2}^{n-r_1-r_2} U_{r_1+i}\right)^2}{(n-r_1-r_2-1)\sum_{i=2}^{n-r_1-r_2}\sum_{j=2}^{n-r_1-r_2} {a_{ij}}^{n-r_1-r_2} U_{r_1+i} U_{r_1+j}}, \tag{1.4}$$

where ${a_{ij}}^{(n)}$ is the same as above and r_1 of the smallest and r_2 of the largest observations are censored. when r_1=0, this is the right censored case, when r_2=0, this is the left censored case and when both r_1 and r_2 not equal to zero, this is the double censored case. For convenient computation, W_1 can be rewritten as

$$W_1 = \frac{\left(\sum_{i=2}^{m} U_{r_1+i}\right)^2}{(m-1)\left[\sum_{i=2}^{m} \frac{i-1}{m-i+1} U_{r_1+i}\left(U_{r_1+i} + 2\sum_{j=i+1}^{m} U_{r_1+i}\right)\right]}, \tag{1.5}$$

where $m = n - r_1 - r_2$. They proved that the null distribution of W_1 is the same as $W_E(m)$, where $m = n - r_1 - r_2$ is the number of available observations. The power of W_1 was compared with the Brain and Shapiro (1983) result by Samanta and Schwarz (1988) and they concluded that W_1 has good powers for complete, left censored, right censored and doubly censored samples.

The second one is G-test. The G-test is based on the so-called Gini statistic and discussed by Gail and Gastwirth (1978) and others. Consider a random sample $T_1, T_2, \ldots, T_n$ of size n, the G-test statistic which was proposed by Gail and Gastwirth (1978) is given by

$$G_n = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} |T_i - T_j|}{2n(n-1)\overline{T}}. \tag{1.6}$$

If $T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(n)}$ are the order statistics of $T_1, T_2, \ldots, T_n$, it is easy to show that

$$G_n = \frac{\sum_{i=1}^{n-1} iU_{i+1}}{(n-1)\sum_{i=1}^{n} U_i} \tag{1.7}$$

where $U_i = (n - i + 1)[T_{(i)} - T_{(i-1)}]$ i=1,2,..., n and $T_{(0)} = 0$. This expression is useful for computation and for generalization to the case of Type II censored samples. G_n takes on values between 0 and 1 with values near 0 or 1 providing evidence against exponentiality. Under the null hypothesis, the distribution of G_n has been obtained and tabulated for n=3,..., 20 by Gail and Gastwirth (1978). For n larger than 20, the approximation

$$[12(n-1)]^{1/2}(G_n - 0.5) \sim N(0,1)$$

is very useful.

The G test is easily modified to handle Type II censored data. If only first r observations $T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(r)}$ are observed in a random sample of size n, we consider

$$G_{r,n} = \frac{\sum_{i=1}^{r-1} iU_{i+1}}{(r-1)\sum_{i=1}^{r} U_i}. \tag{1.8}$$

Since U_i/θ (i=1,2,..., r) are independent and have standard exponential distribution, it is obvious that the distribution of $G_{r,n}$ is exactly the same as that of G_r. Gail and Gastwirth examine the power of the test against certain alternative models. They show that the G test has good power against IFR or DFR alternatives. But for the randomly censored sample, there is no any nonparametric test can be used.

1.2 Plan of the Thesis

The main purpose of this paper is to investigate the tests of fit for exponentiality with censored data with unknown parameters. We mainly consider nonparametric approaches to this problem and suggest a new powerful test procedure for testing (1.1) with the type I or type II or random censoring. The censoring models have been given in chapter 1. In chapter 2, we will consider some of the mathematical properties of the exponential distribution which will be useful in understanding and developing the statistical results. In chapter 3, the non-parametric tests are discussed for Type I and Type II censoring respectively, and for each case the powers of the statistics are compared when the Weibull and gamma are alternatives to exponentiality. In chapter 4, we study the non-parametric tests for random censoring under the standard random censorship model (Efron, 1967) and compare the powers of statistics when the Weibull is the alternative to exponentiality. The summary and discussions are presented in chapter 5.

CHAPTER 2

THE EXPONENTIAL DISTRIBUTION

Before discussing goodness of fit tests for exponentiality with a censored sample, we first briefly review some properties of the exponential distribution and give some important theorems which are related to the goodness of fit problem.

2.1 Properties of the Exponential distribution

The two-parameter exponential distribution is given by

$$f(x;\theta,c) = \frac{1}{\theta}\exp\{-\frac{x-c}{\theta}\}, \quad \text{where } x \geq c \quad \text{and} \quad \theta \geq 0.$$

The hazard function of exponential distribution is

$$h(x) = \frac{f(x)}{1-F(x)} = \frac{1}{\theta}.$$

This distribution will be denoted by $X \sim \exp(\theta, c)$ or $X \sim \exp(\theta)$ if c=0. First note that c is a location parameter and θ is a scalar parameter. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from an exponential distribution. If c is assumed known, say $c = c_0$, then $X_i - c_0 \sim \exp(\theta)$. If c is unknown, then $X_{(i)} - X_{(1)} \sim \exp(\theta)$, where $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$ is an order statistic of $X_1, X_2, \ldots, X_n$. Therefore separate goodness of fit procedures will not be considered for the $\exp(\theta, c)$ model. Similarly, if the scalar parameter θ is assumed known, say $\theta = \theta_0$, then $X/\theta_0 \sim \exp(1, c/\theta_0)$. Thus, in this case the model with $\theta = 1$ can be used without loss of generality by considering the scaled data X_i/θ_0.

Theorem 1. *Suppose X_i, i=1,2,...,n, are independent exponential variables with $X_i \sim \exp(\theta)$, then*

1. $Y = \sum_{i=1}^{n} X_i \sim GAM(\theta, n)$.
2. $2\sum_{i=1}^{n} \frac{X_i}{\theta} \sim \chi^2(2n)$.

Theorem 2. *Suppose $X_1, X_2, \ldots, X_n$ be a random sample and $X_i \sim \exp(\theta)$. $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$ be a order statistic from the sample. Let*

$$U_i = (n - i + 1)(X_{(i)} - X_{(i-1)}) \quad i = 1, 2, \ldots, n$$

where $X_{(0)} = 0$, then $U_1, U_2, \ldots, U_n$ are mutually independent and $U_i \sim exp(\theta)$.

Corollary 1. *Suppose $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(r)}$ $r \leq n$ is a partial order statistics from $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$. Let*

$$U_i = (n - i + 1)(X_{(i)} - X_{(i-1)}), \quad i = 1, 2, \ldots, r,$$

where $X_{(0)} = 0$, then $U_1, U_2, \ldots, U_r$ are mutually independent and $U_i \sim \exp(\theta)$.

The next theorem was given by Wang and Chang (1977).

Theorem 3. *Suppose $X_1, X_2, \ldots, X_n$ is a random sample with continuous distribution function F. Let $Z_i = (\frac{S_i}{S_{i+1}})^i$ i=1,2, ..., n-1, where $S_i = \sum_{j=1}^{i} X_j$. Then $Z_1, Z_2, \ldots, Z_{n-1}$ are independent identical random variables distributed uniformly over (0,1) if and only if F is exponential distribution.*

Theorem 4. *If $X_1, X_2, \ldots, X_n$ are a random sample with uniform distribution over (0,1), then $C_n = -2\sum_{i=1}^{n} \ln(X_i)$ has χ-square distribution with 2n degrees of freedom.*

2.2 Statistical Inference for Exponential distribution

The exponential distribution was the first lifetime model, for which statistical methods were extensively developed. Here we will cover some basic inference procedures for the exponential distribution which relate to our topic with separate treatments for the Type I, Type II and random censoring. We also give the maximum likelihood estimator of censoring parameter β.

Suppose $T_1, T_2, \ldots, T_n$ is a random censored sample from an exponential distribution.

2.2.1 Complete sample

With complete (i.e. uncensored) samples, inference procedures are simple and well known. Such as the maximum likelihood estimator of θ is $\hat{\theta} = \sum_{i=1}^{n} T_i/n$ and T_i/θ's are independent standard exponential variates and $2T/\theta \sim \chi^2(2n)$.

2.2.2 Type II censored sample

Suppose that only the first r observations $T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(r)}$ are available in a total sample of size n, the likelihood function of $T_{(1)}, T_{(2)}, \ldots, T_{(n)}$ is

$$L(\theta) = \frac{n!}{(n-r)!}\left(\prod_{i=1}^{r} \frac{1}{\theta} \exp(-\frac{T_{(i)}}{\theta})\right)\left(\exp(-\frac{T_{(r)}}{\theta})\right)^{n-r}$$

$$= \frac{n!}{(n-r)!} \frac{1}{\theta^r} \exp[-(\frac{\sum_{i=1}^{r} T_{(i)} + (n-r)T_{(r)}}{\theta})], \quad 0 \leq \theta.$$

The natural logarithm of $L(\theta)$ is

$$\ln L(\theta) = \ln \frac{n!}{(n-r)!} - r\ln(\theta) - \frac{\sum_{i=1}^{r} T_{(i)} + (n-r)T_{(r)}}{\theta}, \quad 0 \leq \theta.$$

Thus

$$\frac{d[\ln L(\theta)]}{d\theta} = -\frac{r}{\theta} + \frac{\sum_{i=1}^{r} T_{(i)} + (n-r)T_{(r)}}{\theta^2} = 0.$$

The solution of this equation for θ is

$$\hat{\theta} = \frac{T}{r} \quad \text{where} \quad T = \sum_{i=1}^{r} T_{(i)} + (n-r)T_{(r)}$$

and $2T/\theta \sim \chi^2(2r)$.

2.2.3 Type I censored sample

Using our Type I censoring model, we observed T_i only if $T_i \leq L$ and the data therefore consist of pairs

$$(Y_i, \delta_i), \quad i = 1, 2, \ldots, n,$$

where $Y_i = min(T_i, L)$ and $\delta_i = 1$ if $Y_i = T_i$ otherwise $\delta_i = 0$. For (Y_i, δ_i)'s, the likelihood function is

$$\begin{aligned} L(\theta) &= \prod_{i=1}^{n} \{\frac{1}{\theta}\exp(-\frac{T_i}{\theta})\}^{\delta_i} \{\exp(-\frac{L}{\theta})\}^{1-\delta_i} \\ &= \frac{1}{\theta^r}\exp[-\frac{1}{\theta}(\sum_{i=1}^{n} T_i - (n-r)L)] \end{aligned} \tag{2.1}$$

where $r = \sum \delta_i$ is the observed number of failure times. Note that

$$T = \sum_{i=1}^{r} T_{(i)} + (n-r)L$$

where $T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(r)} \leq L$ are the first r observations. The maximum likelihood estimator for θ, found by maximizing (2.1), is

$$\hat{\theta} = \frac{T}{r}, \quad \text{if} \quad r > 0$$

and $2T/\theta \sim \chi^2(2r)$.

2.2.4 Randomly censored sample

By the definition of standard random censorship model, $Y_i = min(T_i, L_i)$ is observed, where T_i is failure time distributed as exponential $F(t) = 1 - \exp(-t/\theta)$ and the censoring time L_i is distributed $H(t) = 1 - \exp(-t\beta/\theta)$, which is also exponential. Since the joint p.d.f. of (Y, δ) is

$$\begin{aligned} f_0(y, \delta) =& [f(y)]^{\delta}[1 - F(y)]^{1-\delta}[h(y)]^{1-\delta}[1 - H(y)]^{\delta} \\ =& \frac{\beta^{1-\delta}}{\theta} \exp[-\frac{(1+\beta)y}{\theta}]. \end{aligned}$$

The likelihood function is

$$\begin{aligned} L(\beta, \theta) =& \prod_{i=1}^{n} \frac{\beta^{1-\delta_i}}{\theta} \exp[-\frac{(1+\beta)y_i}{\theta}] \\ =& \frac{\beta^{n-r}}{\theta^n} \exp[-\frac{(1+\beta)}{\theta} \sum_{i=1}^{n} y_i], \end{aligned}$$

where $r = \sum_{i=1}^{n} \delta_i$ is the observed number of failure times. So we have

$$\ln L(\beta, \theta) = (n - r)\ln(\beta) - n\ln(\theta) - \frac{1+\beta}{\theta} \sum_{i=1}^{n} y_i,$$

$$\frac{\partial}{\partial\theta} \ln L = -\frac{n}{\theta} + \frac{1+\beta}{\theta^2} \sum_{i=1}^{n} y_i = 0, \tag{2.2}$$

$$\frac{\partial}{\partial\beta} \ln L = \frac{n-r}{\beta} - \frac{1}{\theta} \sum_{i=1}^{n} y_i = 0. \tag{2.3}$$

Solving equations (2.2) and (2.3), we get the maximum likelihood estimators for β and θ as following

$$\hat{\theta} = \frac{\sum_{i=1}^{n} y_i}{r}. \tag{2.4}$$

$$\hat{\beta} = \frac{n-r}{r}, \quad \text{where } r \neq 0. \tag{2.5}$$

From (2.5) we can see that when $r = n$, i.e. there is no censoring, we get $\hat{\beta} = 0$, when $r = n/2$, i.e. 50% censoring, $\hat{\beta} = 1$, when $r = 2n/3$, i.e. 33% censoring, $\hat{\beta} = 0.5$, and so on. So the parameter β can be interpreted as the "censoring parameter".

CHAPTER 3

TESTS FOR EXPONENTIALITY WITH TYPE I AND II CENSORED SAMPLE

In this section we consider the tests of fit for exponentiality with Type I and Type II censored data. We discuss nonparametric approaches to this problem and suggest the new test procedure for testing (1.1) for the Type I and Type II censoring model respectively.

3.1 Nonparametric tests for exponentiality with Type I or Type II censored sample

Suppose that $T_1, T_2, \ldots, T_n$ is a random sample with the exponential distribution. If this is a Type I censored random sample, then we can get r order statistics $T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(r)} \leq L$ where L is common censoring time. If this is a Type II censored random sample, then we can get r order statistics $T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(r)}$ where $r \leq n$ and is fixed in advance. Let $U_i = (n - i + 1)(T_{(i)} - T_{(i-1)})$, where $i = 1, 2, \ldots, r$ and $T_{(0)} = 0$. By the Corollary 1, $U_1, U_2, \ldots, U_r$ are mutually independent and have exponential distribution. Therefore, the problem of test (1.1) with Type I or Type II censored sample reduces to test exponentiality for an uncensored sample $U_1, U_2, \ldots, U_r$. Now the W-test and G-test can be simply used to solve this problem.

The W-statistic is

$$W_{r,n} = \frac{\left(\sum_{i=2}^{r} U_i\right)^2}{(r-1)\left[\sum_{i=2}^{r} \frac{i-1}{r-i+1} U_i\left(U_i + 2\sum_{j=i+1}^{r} U_j\right)\right]} \tag{3.1}$$

for Type I censored sample $T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(r)} \leq L$, or for Type II censored sample $T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(r)}$. Formula (3.1) is special case of (1.5) when $r_1 = 0$ and $n - r_1 - r_2 = r$. And the distribution of $W_{r,n}$ is the same as $W_E(r)$.

The G test is

$$G_{r,n} = \frac{\sum_{i=1}^{r-1} iU_{i+1}}{(r-1)\sum_{i=1}^{r} U_i} \tag{3.2}$$

for both Type I and Type II censored sample. Since U_i/θ (i=1,2,..., r) are independent and have standard exponential distribution, it is obvious that the distribution of $G_{r,n}$ is exactly the same as that of G_r.

Let $S_i = \sum_{j=1}^{i} U_j$, $i = 1, 2, \ldots, r$, be the partial sums of the random sample $U_1, U_2, \ldots, U_r$. Let $Z_i = (S_i/S_{i+1})^i$, $i = 1, 2, \ldots, r-1$. By theorem 3, $Z_1, Z_2, \ldots, Z_{r-1}$ are independent uniformly distributed on (0,1), which enables us to reduce the problem of goodness-of-fit of exponentiality based on the random sample $T_1, T_2, \ldots, T_n$ with Type I or Type II censoring to the problem of testing that the distribution of $Z_1, Z_2, \ldots, Z_{r-1}$ is U(0,1). There are many approaches to test uniformity. In this paper, we choose one of them. We define the statistic

$$C_{r,n} = -2\sum_{i=1}^{r-1} \ln(Z_i).$$

Since $Z_i \sim U(0,1)$, by theorem 4, the distribution of $C_{r,n}$ is χ^2 distribution with 2(r-1) degrees of freedom.

If we use the Cramér-von Mises statistic ti testing (1.1) with Type I or TYpe II censoring and with unknown parameter θ, the appropriate test statistics, given by Pettit (1977), are

$$W^2_{r,n} = \sum_{i=1}^{r} (\hat{F}_0(T_{(i)}) - \frac{i-0.5}{n})^2 + \frac{r}{12n^2} - \frac{n}{3}(\frac{r}{n} - \hat{F}_0(T_{(r)}))^3$$

for a Type II censored sample $T_{(1)} \leq T_{(2)} \leq \cdots \leq T_{(r)}$ $(r \leq n)$, and

$$W^2_{L,n} = \sum_{i=1}^{r} (\hat{F}_0(T_{(i)}) - \frac{i-0.5}{n})^2 + \frac{r}{12n^2} - \frac{n}{3}(\frac{r}{n} - \hat{F}_0(L))^3$$

for a Type I censored sample.

3.2 Mont Carlo Simulation

3.2.1 Simulation

For both Type I and Type II case, we simulate the $C_{r,n}$ statistic, $G_{r,n}$ statistic, $W_{r,n}$ statistic and Cramér-von Mises statistic when the alternate is Weibull distribution or gamma distribution. We assume θ is unknown. Without loss of the generality we only consider the standard exponential distribution

$$f(t) = \exp(-t).$$

the one parameter Weibull distribution

$$f(t) = \theta t^{\theta-1} \exp(-t^{\theta})$$

and the one parameter gamma distribution

$$f(t) = \frac{1}{\Gamma(\theta)} t^{\theta-1} \exp(-t).$$

The estimate of power is based on 1,000 random samples and significant level is α=0.05. The results are shown in Table (1.1), (1.2), (2.1) and (2.2). In Table (1.1) and (1.2), we give the powers of Weibull alternatives to exponential and gamma alternatives to exponential when the sample size are 20 and 50 with Type I censoring respectively. The common censoring time L is taken based on 10%, 20%, 30%, and 40% expected censored observations in a Type I censored sample. In Table (2.1) and (2.2), we list the powers of test for Weibull and gamma alternative respectively when n is only 20 and 50. In order to compare the power with the Cramér-von Mises statistic, the number of smallest observations r is taken 20, 18, 16, 14 and 12 when n=20 and r is taken 50, 45, 40, 35 and 30 when n=50. These r corresponds to 0%, 10%, 20%, 30% and 40% censorship in a Type II censored sample.

3.2.2 Notations

In Table (1.1), (1.2), (2.1) and (2.2), we use following notations.

θ --- parameter of the exponential distribution.

L ---- common censoring time.

r --- the number of smallest observation.

$C_{L,n}$ $C_{r,n}$ statistic with Type I censoring.

$C_{r,n}$ - $C_{r,n}$ statistic with Type II censoring.

$G_{L,n}$ $G_{r,n}$ statistic with Type I censoring.

$G_{r,n}$ - $G_{r,n}$ statistic with Type II censoring.

$W_{L,n}$ – $W_{r,n}$ statistic with Type I censoring.

$W_{r,n}$ - $W_{r,n}$ statistic with Type II censoring.

$W^2{}_{L,n}$ Cramér-von Mises statistic with Type I censoring.

$W^2{}_{r,n}$ - Cramér-von Mises statistic with Type II censoring.

3.2.3 Conclusions

The following conclusions may be drawn from the Tables (1.1), (1.2), (2.1) and (2.2).

(1) Botn $C_{L,n}$ and $C_{r,n}$ statistics do much better than other three tests in any case and the W-test is the weakest one.

(2) In Type I censoring case, when $\theta \geq 1.0$ the Cramér-von Mises test is slightly weaker than G-test. But when $0 < \theta < 1.0$, the Cramér-von Mises test is slightly better than G-test.

(3) In Type II censoring case, the G-test is better than Cramér-von Mises test except when $\theta \geq 1.0$ and $r \leq 12$ if n=20 or $r \leq 30$ if n=50.

(4) For all of them, the power increases with sample size, but decrease as the amount of censorship increases.

(5) When the alternate is Weibull distribution, the powers of all these tests are better than when the alternate is the gamma distribution.

CHAPTER 4

TESTS FOR EXPONENTIALITY WITH RANDOMLY CENSORED SAMPLE

In this section, we studied the tests of fit for exponentiality with randomly censored samples for composite hypotheses. We mainly discuss nonparametric approaches to this problem for random censoring under the standard random censorship model (Efron, 1967).

4.1 Tests of Exponentiality with Randomly Censored Sample

Suppose $T_1, T_2, \ldots, T_n$ is a random sample of failure times and $L_1, L_2, \ldots, L_n$ is a random sample of censoring times and L_i is associated with T_i. T_i and L_i are independent continuous random variables with distribution functions F(t) and H(t), respectively. The observation is Y_i=min(T_i, L_i). Let $\delta_i = 1$ when $Y_i = T_i$ and $\delta_i = 0$ when $Y_i = L_i$. If F(t)=1-exp(-t/θ), under the standard random censorship model, then H(t)=1-exp(-βt/θ). The joint p.d.f. of (Y,δ) is

$$f_0(y,\delta) = [f(y)]^{\delta}[1-F(y)]^{1-\delta}[h(y)]^{1-\delta}[1-H(y)]^{\delta}$$
$$= \frac{\beta^{1-\delta}}{\theta}\exp[-\frac{(1+\beta)y}{\theta}].$$

So the marginal density of δ is

$$k(\delta) = \frac{\beta^{1-\delta}}{1+\beta}, \quad (\delta = 0, 1). \tag{4.1}$$

This is a Bernoulli distribution. The conditional density of y when $\delta = 1$ is

$$f_1(y|\delta = 1) = \frac{1+\beta}{\theta}\exp[-\frac{(1+\beta)y}{\theta}]$$

i.e. exponential distribution with parameter $(1+\beta)/\theta$. Since δ has Bernoulli distribution with $p = 1/(1+\beta)$, the number of uncensored observations, denoted as m, has a Binomial distribution b(n,p). The mean value of m is $np = n/(1+\beta)$. This means that β=1 corresponds to 50 % expected censorship, β=0.5 corresponds to 33% expected censorship, and β=0 corresponds to no censoring and the expected proportion of censored observations increases with β. Here we get the same results as in Chapter 2. When the distribution of the failure time T is Weibull or scale shift exponential, we can prove that δ has the same distribution as the distribution of T is exponential.

Suppose $Y_{i_1}, Y_{i_2}, \ldots, Y_{i_m}$ is a random sample from $Y_1, Y_2, \ldots, Y_n$ where $\delta_{i_j} = 1$, j=1,2,...,m. As seen above under the standard random censorship model, the distribution of uncensored observation $Y_{i_1}, Y_{i_2}, \ldots, Y_{i_m}$ is exponential with a parameter $(\theta/(1+\beta))$. Similarly, it may be obtained that when F(t) is Weibull distribution with parameters (α, θ), the distribution of uncensored observation is Weibull with parameters $(\alpha, \theta/(1+\beta)^{(1/\alpha)})$.

Consider a test of $H_0 : T \sim exp(\theta)$ against $H_A : T \sim Weibull(\alpha,\theta)$ with randomly censored sample. Under the standard random censorship model, it is equivalent to test

$$
\begin{aligned}
H_0 &: Y_{i_j} \sim exp(\frac{\theta}{(1+\beta)}) \\
H_A &: Y_{i_j} \sim Weibull(\alpha, \frac{\theta}{(1+\beta)^{(1/\alpha)}})
\end{aligned}
\tag{4.2}
$$

with the complete sample $Y_{i_1}, Y_{i_2}, \ldots, Y_{i_m}$, but the sample size $m \sim b(n, 1/(1+\beta))$. Koziol and Green (1967) generalize the Cramér-von Mises statistic in this way and obtain its asymptotic distribution when both β and F(t) are completely specified.

They give asymptotic percentage point for a few values of β. Now we assume both θ and β are unknown. Let $Y_{(i_1)} \leq Y_{(i_2)} \leq \cdots \leq Y_{(i_m)}$ be the order statistics of $Y_{i_1}, Y_{i_2}, \ldots, Y_{i_m}$. Let $U_j = (m - i + 1)(Y_{(i_j)} - Y_{(i_{j-1})})$, j=1,2,..., m. Using Corollary 1, $U_1, U_2, \ldots, U_m$ are independent and the distribution of U_j's are exponential. Now we can simply apply W-test for $Y_{i_1}, Y_{i_2}, \ldots, Y_{i_m}$ and apply both G-test and C-statistic for $U_1, U_2, \ldots, U_m$ as follow.

1. W-test:

$$W_{m,n} = \frac{m(\overline{Y} - Y_{(i_1)})^2}{(m-1)S^2} \sim W_E(m) \tag{4.3}$$

   where $\overline{Y} = \frac{1}{m}\sum_{j=1}^{m} Y_{(i_j)}$ and $S^2 = \sum_{j=1}^{m}(Y_{(i_j)} - \overline{Y})$.

2. G-test:

$$G_{m,n} = \frac{\sum_{i=1}^{m} iU_{i+1}}{(m-1)\sum_{i=1}^{m} U_i} \sim G(m) \tag{4.4}$$

3. C-statistic: Let

$$S_k = \sum_{j=1}^{k} U_j, \quad k = 1, 2, \ldots, m$$

   and let

$$Z_k = \left(\frac{S_k}{S_{k+1}}\right)^k, \quad k = 1, 2, \ldots, m-1.$$

   By Theorem 3, $Z_1, Z_2, \ldots, Z_{m-1}$ are independent uniformly distributed on U(0,1). Define $C_{n,n}$ statistic by

$$C_{m,n} = -2\sum_{i=1}^{m-1} \ln(Z_i). \tag{4.5}$$

   According to theorem 4, under the H_0, $C_m \sim \chi^2(2(m-1))$.

   Also we can use EDF tests such as Cramér-von Mises statistic, kolmogrov-Simrnov statistic and so on with F(t) replaced by $\hat{F(t)} = 1 - \exp(-t/\hat{\theta})$. Here we only compare the powers of above three methods and Cramér-von Mises statistic which given by Stephens (1974).

4.2 Mont Carlo Simulation

4.2.1 Simulation

We investigated the $G_{m,n}$, $W_{m,n}$, $C_{m,n}$ and Cramér-von Mises statistic to test exponential against Weibull distribution when both θ and β are unknown, when the sample is randomly censored. Without loss of generality, we only consider the standard exponential distribution f(t)=exp(-t) and the one parameter Weibull distribution $f(t) = \theta t^{\theta-1}\exp(-t^{\theta})$. We compared the powers of the four tests and the results are shown in Table (3.1) where the sample sizes are 20 and 50 respectively. The censoring parameter β was taken as 1.00, 0.75, 0.50, 0.25 and 0.00, which corresponds to 50%, 43%, 33%, 20% and 0.0% expected censorship.

4.2.2 Notations

In Table (3.1), following notations are used.

β ——— the censoring parameter.

θ ——— the parameter of the exponential distribution.

$C_{m,n}$ — $C_{m,n}$-statistic with random censoring.

$G_{m,n}$ — $G_{m,n}$-statistic with random censoring.

$W_{m,n}$ — $W_{m,n}$-statistic with random censoring.

$W^2{}_{m,n}$ - Cramér-von statistic with random censoring.

4.2.3 Conclusions

From results of simulation, the following conclusions may be obtained.

(1) $C_{m,n}$ statistic is the more powerful test compared with the other three tests and the W-test is the weakest one.

(2) The $C_{m,n}$, the $G_{m,n}$ and the Cramér-von Mises test are comparable with the no censoring case.

(3) When $\theta \leq 0.5$ or $\theta \geq 2.5$, the power of the $C_{m,n}$, the $G_{m,n}$ and the Cramér-von Mises test are very close.

(4) For all of them, the power increases with the sample size but decrease as the amount of censorship increases.

4.3 Test for the Standard Random Censoring Model

Further, suppose that we know F(t) is exponential distribution, we now consider to test the standard random censoring model , i.e. test

$$H_0 : (1 - H) = (1 - F)^{\beta}$$
$$H_A : (1 - H) \neq (1 - F)^{\beta} \tag{4.6}$$

where both β and θ are unknown. Under the null hypothesis, $H = 1 - \exp(-\beta t/\theta)$, therefore testing (4.6) is equivalent to test

$$H_0{}' : H \sim \exp(\theta_0)$$
$$H_A{}' : H \nsim \exp(\theta_0) \tag{4.7}$$

where $\theta_0 = \theta/\beta$. Since the joint p.d.f. of (Y, δ) is

$$f_0(y, \delta) = \frac{\beta^{(1-\delta)}}{\theta} \exp\{\frac{-(1+\beta)y}{\theta}\},$$

and the marginal distribution of δ is

$$k(\delta) = \beta^{(1-\delta)}/(1+\beta), \quad \delta = 0, 1.$$

We get the conditional distribution of $(y|\delta = 0)$ is

$$g(y|\delta = 0) = \frac{(1+\beta)}{\theta} \exp(-\frac{(1+\beta)y}{\theta}),$$

which is the same as $g(y|\delta = 1)$.

Suppose $Y_{j_1}, Y_{j_2}, \ldots, Y_{j_r}$ is a random sample from $Y_1, Y_2, \ldots, Y_n$ for which δ_{j_i}=0, (i=1,2,...,r, r+m=n) and which consist of all censored observations. Under the standard random censoring model, they have the same distribution as the uncensored observations. Then the tests which test exponentiality can be applied to test whether either the censored observations $Y_{j_1}, Y_{j_2}, \ldots, Y_{j_r}$ or the uncensored observations $Y_{i_1}, Y_{i_2}, \ldots, Y_{i_m}$ has exponential distribution. If both of them have exponential distributions, then the standard random censoring model is a correct model.

CHAPTER 5

SUMMARY AND DISCUSSION

This paper has extended goodness of fit test techniques for a censored sample, special for the standard random censorship model. As shown above, amcng these methods the C-statistic is the best test for either Type I or Type II or randomly censored sample when the alternate is Weibull or gamma (for Type I and Type II censoring only) no matter what the value of θ and β take on. The G-test is slightly weaker for each case but still has good power. In table (1.1), (1.2), (2.1) and (2.2), we only give the power of each tests corresponding to 0%, 10%, 20%, 30% and 40% censorship. In fact, in Type I censoring case, using $G_{r,n}$ and $W_{r,n}$, the number of uncensored data r can be any positive integer between 0 and n, using $C_{r,n}$ r must be any number between 3 and n. But when we use Crea'nér-von Mises test, we only have asymptotic distribution when r and n go to infinity with $r/n = p$ fixed. In practice, this condition is very difficult to check. In Type I censoring case, common censoring time L also can be any positive number if we use $C_{L,n}$, $G_{L,n}$ and $W_{L,n}$.

The C-test, W-test and G-test only can be use to test exponentiality, they can not decide the what kind of exponential distribution. Because if $T_i \sim \exp(\theta)$, then $T_i/\theta \sim \exp(1)$, standard exponential distribution, and the W-statistic can be written as

$$W_E = \frac{n(\overline{T} - T_{(1)})^2}{(n-1)S^2} = \frac{n(\overline{T}/\theta - T_{(1)}/\theta)^2}{(n-1)S^2/\theta^2}$$

and G-statistics can be written as

$$G_n = \frac{\sum_{i=1}^{n} iT_i}{(n-1)\sum_{i=1}^{n} T_i} = \frac{\sum_{i=1}^{n} iT_i/\theta}{(n-1)\sum_{i=1}^{n} T_i/\theta}.$$

Also for C-statistics, for example $C_{m,n}$, we have

$$\begin{aligned} C_{m,n} &= -2\sum_{i=1}^{m-1} \log(Z_i) \\ &= -2\sum_{i=1}^{m-1} i(\log(S_i) - \log(S_{i+1})) \\ &= -2\sum_{i=1}^{m-1} i(\log(\sum_{J=1}^{i} T_{(j)}) - \log(\sum_{j=1}^{i+1} T_{(j)})) \\ &= -2\sum_{i=1}^{n-1} i(\log(\sum_{j=1}^{i} T_{(j)}/\theta) - \log(\sum_{j=1}^{i+1} T_{(j)}/\theta)). \end{aligned}$$

Therefore, these three tests are not dependent on the parameter θ. This means even if we don't known θ, we may consider the simulation of exp(1) still without loss of generality. Usually it is not possible to completely specify a model without making use of sample information. That is, it may be assumed only that the general form of the model is exponential, with the parameter θ being an unknown parameter to be estimated on the basis of sample data. Thus, if the exponential model is correct, estimating θ from the data is equivalent to estimating the mean of the sampled population. In Chapter 2, there are some theorems about inference of θ and β.

We know for the two parameters exponential distribution $F(t) = 1 - \exp(-(t - c)/\theta)$, where $t > c$ and $\theta > 0$. If c is known, the $T_i - c$ is exponential with only one parameter θ. If c is unknown, then $T_{(i)} - T_{(1)}$, i=2,3,..., n, is exponential with one parameter θ. When $T_i \sim \exp(1)$ then $\theta T_i \sim \exp(\theta)$. Also for the Weibull distribution, if $T \sim Weibull(\theta, \alpha)$ then $T/\alpha \sim Weibull(\theta, 1)$. The same result holds for the gamma distribution. That is why we only study the results of the Monte Carlo simulation for standard exponential, one parameter Weibull and one parameter gamma distribution.

The simulation programs were written in FORTRAN 77 and run on an VAX-6510 computer. The pseudorandom numbers were generated using the International Mathematical and Statistical Libraries (1984) package.

The standard random censorship model is very useful if the distribution of failure time is the one of exponential, scale shift exponential and Weibull distribution. Although we only discuss the test of the standard random censorship model and some inference of θ and β under this model when the distribution of the failure time is exponential, the idea can be extended to the scale shift exponential and Weibull distribution cases. Because if the distribution of failure time is one of them, under the standard random censorship model the censoring time has the same distribution as the failure time but with different parameter.

Table 1.1 Powers

Weibull alternatives to exponentiality

(sample size is 20 with Type I censoring)

θ	L	$C_{L,n}$	$G_{L,n}$	$W_{L,n}$	$W^2_{L,n}$
5.00	1.1815	1.000	1.000	0.894	1.000
	1.0999	1.000	1.000	0.819	0.999
	1.0378	1.000	1.000	0.684	0.986
	0.9827	0.996	0.996	0.553	0.961
4.50	1.2036	1.000	1.000	0.876	1.000
	1.1115	1.000	1.000	0.791	0.999
	1.0421	0.999	0.999	0.678	0.980
	0.9808	1.000	0.998	0.531	0.950
4.00	1.2318	1.000	1.000	0.870	0.999
	1.1263	1.000	1.000	0.747	0.991
	1.0475	1.000	0.999	0.645	0.972
	0.9784	0.995	0.990	0.522	0.920
3.50	1.2691	1.000	1.000	0.827	0.999
	1.1456	0.999	0.999	0.701	0.987
	1.0545	0.993	0.991	0.627	0.955
	0.9753	0.980	0.979	0.489	0.893
3.00	1.3205	0.999	0.999	0.771	0.991
	1.1719	0.993	0.993	0.680	0.965
	1.0638	0.982	0.973	0.552	0.902
	0.9713	0.926	0.922	0.413	0.824
2.50	1.3960	0.990	0.982	0.706	0.952
	1.2097	0.962	0.955	0.564	0.898
	1.0771	0.916	0.906	0.494	0.807
	0.9656	0.856	0.833	0.377	0.725
2.00	1.5174	0.896	0.895	0.574	0.818
	1.2686	0.843	0.816	0.431	0.719
	1.0973	0.741	0.720	0.347	0.600
	0.9572	0.618	0.590	0.250	0.511
1.80	1.5894	0.762	0.739	0.438	0.631
	1.3026	0.686	0.682	0.397	0.571
	1.1086	0.595	0.574	0.301	0.489
	0.9526	0.505	0.474	0.225	0.419
1.60	1.6842	0.599	0.610	0.357	0.503
	1.3464	0.500	0.481	0.278	0.401
	1.1230	0.457	0.418	0.213	0.346
	0.9468	0.334	0.343	0.183	0.304

Table 1.1 (continued)

θ	L	$C_{L,n}$	$G_{L,n}$	$W_{L,n}$	${W^2}_{L,n}$
1.40	1.8144	0.337	0.338	0.217	0.282
	1.4048	0.309	0.288	0.176	0.240
	1.1418	0.252	0.264	0.161	0.241
	0.9395	0.218	0.214	0.142	0.204
1.20	2.0038	0.158	0.168	0.124	0.139
	1.4867	0.132	0.135	0.113	0.119
	1.1673	0.099	0.099	0.091	0.103
	0.9297	0.098	0.095	0.077	0.101
1.00	2.3026	0.044	0.037	0.040	0.035
	1.6094	0.045	0.053	0.052	0.053
	1.2040	0.059	0.055	0.045	0.052
	0.9163	0.040	0.050	0.041	0.051
0.90	2.5262	0.087	0.079	0.070	0.092
	1.6968	0.078	0.071	0.066	0.074
	1.2291	0.074	0.072	0.048	0.071
	0.9074	0.069	0.051	0.044	0.069
0.80	2.8364	0.200	0.137	0.093	0.181
	1.8128	0.158	0.108	0.061	0.140
	1.2612	0.157	0.101	0.058	0.126
	0.8965	0.126	0.104	0.071	0.116
0.70	3.2919	0.440	0.323	0.158	0.367
	1.9736	0.377	0.284	0.138	0.307
	1.3037	0.317	0.221	0.120	0.262
	0.8826	0.298	0.186	0.084	0.218
0.60	4.0151	0.678	0.532	0.305	0.599
	2.2103	0.634	0.489	0.270	0.555
	1.3626	0.567	0.406	0.206	0.464
	0.8644	0.490	0.332	0.166	0.378
0.50	5.3019	0.931	0.821	0.545	0.888
	2.5903	0.870	0.717	0.447	0.780
	1.4495	0.809	0.626	0.363	0.676
	0.8396	0.730	0.578	0.324	0.623
0.40	8.0452	0.996	0.947	0.776	0.975
	3.2861	0.973	0.905	0.685	0.945
	1.5905	0.954	0.843	0.571	0.890
	0.8037	0.911	0.759	0.482	0.789

Table 1.1 (continued)

Weibull alternatives to exponentiality
(Sample size is 50 and with Type I censoring)

θ	L	$C_{L,n}$	$G_{L,n}$	$W_{L,n}$	$W^2_{L,n}$
5.00	1.1815	1.000	1.000	1.000	1.000
	1.0999	1.000	1.000	0.999	1.000
	1.0378	1.000	1.000	0.994	1.000
	0.9827	1.000	1.000	0.973	1.000
4.50	1.2036	1.000	1.000	1.000	1.000
	1.1115	1.000	1.000	0.999	1.000
	1.0421	1.000	1.000	0.986	1.000
	0.9808	1.000	1.000	0.966	1.000
4.00	1.2318	1.000	1.000	1.000	1.000
	1.1263	1.000	1.000	0.996	1.000
	1.0475	1.000	1.000	0.986	1.000
	0.9784	1.000	1.000	0.969	1.000
3.50	1.2691	1.000	1.000	1.000	1.000
	1.1456	1.000	1.000	0.989	1.000
	1.0545	1.000	1.000	0.972	1.000
	0.9753	1.000	1.000	0.942	1.000
3.00	1.3205	1.000	1.000	0.995	1.000
	1.1719	1.000	1.000	0.990	1.000
	1.0638	1.000	1.000	0.969	1.000
	0.9713	1.000	1.000	0.917	1.000
2.50	1.3960	1.000	1.000	0.995	1.000
	1.2097	1.000	1.000	0.967	1.000
	1.0771	1.000	0.999	0.909	0.996
	0.9656	1.000	0.997	0.839	0.983
2.00	1.5174	1.000	1.000	0.958	0.999
	1.2686	0.999	0.995	0.874	0.985
	1.0973	0.992	0.986	0.805	0.959
	0.9572	0.976	0.954	0.677	0.899
1.80	1.5894	0.995	0.992	0.905	0.978
	1.3026	0.983	0.974	0.820	0.948
	1.1086	0.962	0.944	0.716	0.886
	0.9526	0.926	0.901	0.624	0.822
1.60	1.6842	0.953	0.938	0.777	0.895
	1.3464	0.893	0.876	0.654	0.787
	1.1230	0.837	0.806	0.556	0.722
	0.9468	0.741	0.711	0.500	0.645

Table 1.1 (continued)

θ	L	$C_{L,n}$	$G_{L,n}$	$W_{L,n}$	$W^2{}_{L,n}$
1.40	1.8144	0.756	0.732	0.556	0.662
	1.4048	0.666	0.606	0.437	0.531
	1.1418	0.601	0.556	0.364	0.473
	0.9395	0.488	0.459	0.315	0.398
1.20	2.0038	0.277	0.279	0.216	0.227
	1.4867	0.285	0.259	0.196	0.218
	1.1673	0.216	0.187	0.151	0.167
	0.9297	0.184	0.168	0.135	0.152
1.00	2.3026	0.059	0.063	0.050	0.056
	1.6094	0.060	0.059	0.059	0.043
	1.2040	0.045	0.049	0.055	0.039
	0.9163	0.060	0.055	0.055	0.062
0.90	2.5262	0.119	0.096	0.049	0.109
	1.6968	0.104	0.079	0.048	0.087
	1.2291	0.110	0.085	0.062	0.094
	0.9074	0.096	0.081	0.061	0.095
0.80	2.8364	0.424	0.333	0.138	0.339
	1.8128	0.355	0.276	0.153	0.284
	1.2612	0.325	0.251	0.122	0.261
	0.8965	0.282	0.211	0.119	0.237
0.70	3.2919	0.791	0.683	0.340	0.711
	1.9736	0.729	0.566	0.306	0.600
	1.3037	0.636	0.510	0.279	0.552
	0.8826	0.561	0.411	0.227	0.435
0.60	4.0151	0.983	0.942	0.699	0.954
	2.2103	0.960	0.898	0.595	0.921
	1.3626	0.930	0.812	0.543	0.835
	0.8644	0.879	0.753	0.429	0.760
0.50	5.3019	0.999	0.994	0.926	0.999
	2.5903	0.998	0.982	0.869	0.992
	1.4495	0.992	0.966	0.765	0.969
	0.8396	0.975	0.915	0.697	0.924
0.40	8.0452	1.000	1.000	0.994	1.000
	3.2861	1.000	0.999	0.983	0.999
	1.5905	1.000	0.997	0.950	0.999
	0.8037	1.000	0.992	0.903	0.990

Table 1.1

Table 1.2 Powers
Gamma alternatives to exponentiality
(Sample size is 20 with Type I censoring)

θ	L	$C_{L,n}$	$G_{L,n}$	$W_{L,n}$	$W^2_{L,n}$
5.00	7.9936	0.998	0.991	0.543	0.972
	6.7210	0.991	0.979	0.482	0.934
	5.8904	0.978	0.964	0.373	0.872
	5.2366	0.936	0.903	0.278	0.802
4.50	7.3418	0.997	0.988	0.522	0.963
	6.1211	0.986	0.962	0.444	0.915
	5.3282	0.969	0.942	0.383	0.856
	4.7068	0.926	0.901	0.295	0.771
4.00	6.6809	0.990	0.978	0.482	0.946
	5.5151	0.971	0.946	0.404	0.864
	4.7622	0.933	0.900	0.342	0.806
	4.1753	0.883	0.835	0.278	0.723
3.50	6.0085	0.969	0.935	0.436	0.885
	4.9016	0.927	0.884	0.372	0.791
	4.1917	0.874	0.813	0.303	0.681
	3.6416	0.800	0.749	0.244	0.640
3.00	5.3223	0.908	0.865	0.370	0.782
	4.2790	0.868	0.809	0.348	0.730
	3.6156	0.812	0.757	0.270	0.630
	3.1054	0.711	0.647	0.239	0.564
2.50	4.6182	0.784	0.714	0.342	0.651
	3.6446	0.699	0.646	0.236	0.546
	3.0322	0.623	0.577	0.224	0.477
	2.5659	0.574	0.518	0.181	0.447
2.00	3.8897	0.564	0.503	0.222	0.415
	2.9943	0.458	0.410	0.188	0.331
	2.4392	0.447	0.389	0.163	0.335
	2.0223	0.382	0.335	0.143	0.298
1.80	3.5893	0.428	0.376	0.174	0.323
	2.7283	0.360	0.320	0.163	0.277
	2.1985	0.352	0.328	0.181	0.269
	1.8034	0.296	0.274	0.133	0.252
1.60	3.2822	0.283	0.255	0.153	0.221
	2.4579	0.260	0.245	0.128	0.206
	1.9552	0.252	0.237	0.113	0.200
	1.5835	0.221	0.198	0.102	0.182

Table 1.2 (continued)

θ	L	$C_{L,n}$	$G_{L,n}$	$W_{L,n}$	${W^2}_{L,n}$
1.40	2.9669	0.188	0.172	0.126	0.145
	2.1823	0.142	0.142	0.088	0.126
	1.7088	0.137	0.142	0.086	0.113
	1.3624	0.131	0.137	0.089	0.123
1.20	2.6415	0.100	0.098	0.092	0.090
	1.9001	0.081	0.088	0.075	0.075
	1.4588	0.080	0.076	0.074	0.070
	1.1400	0.079	0.085	0.073	0.079
1.00	2.3026	0.053	0.045	0.050	0.042
	1.6094	0.053	0.060	0.060	0.059
	1.2040	0.044	0.052	0.059	0.050
	0.9163	0.061	0.059	0.053	0.060
0.90	2.1267	0.064	0.050	0.054	0.052
	1.4601	0.066	0.044	0.045	0.047
	1.0744	0.072	0.064	0.056	0.064
	0.8039	0.062	0.053	0.040	0.057
0.80	1.9453	0.116	0.084	0.049	0.105
	1.3074	0.127	0.083	0.050	0.089
	0.9432	0.111	0.098	0.056	0.080
	0.6913	0.095	0.069	0.051	0.080
0.70	1.7571	0.239	0.145	0.089	0.186
	1.1506	0.234	0.150	0.070	0.165
	0.8101	0.195	0.121	0.070	0.136
	0.5785	0.192	0.136	0.077	0.143
0.60	1.5605	0.470	0.268	0.120	0.320
	0.9890	0.404	0.244	0.112	0.261
	0.6748	0.379	0.234	0.130	0.258
	0.4659	0.357	0.219	0.108	0.244
0.50	1.3528	0.698	0.453	0.226	0.518
	0.8212	0.628	0.428	0.186	0.454
	0.5371	0.581	0.370	0.173	0.405
	0.3542	0.550	0.336	0.170	0.394
0.40	1.1298	0.880	0.657	0.351	0.718
	0.6456	0.856	0.643	0.314	0.718
	0.3973	0.842	0.606	0.307	0.678
	0.2448	0.785	0.557	0.273	0.593

Table 1.2 (continued)

Gamma alternatives to exponentiality
(Sample size is 50 with Type 1 censoring)

θ	L	$C_{L,n}$	$G_{L,n}$	$W_{L,n}$	$W^2_{L,n}$
5.00	7.9936	1.000	1.000	0.942	1.000
	6.7210	1.000	1.000	0.884	1.000
	5.8904	1.000	1.000	0.849	1.000
	5.2366	1.000	0.999	0.781	0.992
4.50	7.3418	1.000	1.000	0.926	1.000
	6.1211	1.000	1.000	0.902	1.000
	5.3282	1.000	1.000	0.800	0.999
	4.7068	1.000	0.999	0.756	0.994
4.00	6.6808	1.000	1.000	0.909	1.000
	5.5151	1.000	1.000	0.848	1.000
	4.7622	1.000	0.999	0.780	0.996
	4.1753	1.000	0.998	0.722	0.996
3.50	6.0085	1.000	1.000	0.881	1.000
	4.9016	1.000	1.000	0.815	0.996
	4.1917	1.000	0.999	0.764	0.984
	3.6416	0.999	0.993	0.662	0.974
3.00	5.3223	1.000	0.997	0.839	0.996
	4.2790	0.999	0.996	0.753	0.987
	3.6156	0.998	0.994	0.695	0.976
	3.1054	0.996	0.982	0.591	0.945
2.50	4.6182	0.995	0.980	0.748	0.972
	3.6446	0.985	0.967	0.677	0.933
	3.0322	0.979	0.941	0.598	0.897
	2.5659	0.952	0.910	0.546	0.846
2.00	3.8897	0.927	0.851	0.559	0.801
	2.9943	0.918	0.821	0.503	0.770
	2.4392	0.869	0.774	0.432	0.684
	2.0223	0.797	0.704	0.379	0.624
1.80	3.5893	0.835	0.723	0.448	0.671
	2.7283	0.764	0.668	0.401	0.602
	2.1985	0.731	0.640	0.356	0.564
	1.8034	0.673	0.583	0.324	0.507
1.60	3.2822	0.668	0.568	0.360	0.496
	2.4579	0.605	0.513	0.313	0.474
	1.9552	0.551	0.467	0.261	0.404
	1.5835	0.467	0.400	0.247	0.346

Table 1.2 (continued)

θ	L	$C_{L,n}$	$G_{L,n}$	$W_{L,n}$	$W^2{}_{L,n}$
1.40	2.9669	0.392	0.327	0.220	0.271
	2.1823	0.338	0.297	0.194	0.248
	1.7088	0.320	0.271	0.176	0.239
	1.3624	0.299	0.232	0.146	0.203
1.20	2.6415	0.145	0.134	0.110	0.114
	1.9001	0.114	0.120	0.109	0.098
	1.4588	0.153	0.146	0.112	0.129
	1.1400	0.131	0.124	0.095	0.114
1.00	2.3026	0.062	0.045	0.040	0.047
	1.6094	0.056	0.054	0.053	0.048
	1.2040	0.048	0.046	0.050	0.041
	0.9163	0.041	0.047	0.049	0.050
0.90	2.1267	0.094	0.060	0.035	0.073
	1.4601	0.092	0.073	0.057	0.079
	1.0744	0.085	0.057	0.048	0.053
	0.8039	0.076	0.065	0.060	0.079
0.80	1.9453	0.248	0.135	0.049	0.152
	1.3074	0.226	0.154	0.083	0.167
	0.9432	0.208	0.143	0.082	0.147
	0.6913	0.174	0.133	0.068	0.129
0.70	1.7571	0.517	0.309	0.113	0.348
	1.1506	0.500	0.328	0.160	0.353
	0.8101	0.461	0.292	0.148	0.313
	0.5785	0.381	0.229	0.114	0.265
0.60	1.5605	0.814	0.602	0.239	0.635
	0.9890	0.775	0.549	0.263	0.593
	0.6748	0.744	0.526	0.258	0.558
	0.4659	0.682	0.435	0.215	0.461
0.50	1.3528	0.970	0.817	0.439	0.874
	0.8212	0.954	0.806	0.433	0.839
	0.5371	0.948	0.782	0.434	0.823
	0.3542	0.906	0.714	0.399	0.758
0.40	1.1298	0.999	0.969	0.715	0.981
	0.6456	0.998	0.960	0.709	0.971
	0.3973	0.996	0.943	0.718	0.963
	0.2448	0.984	0.924	0.666	0.930

Table 1.2

Table 2.1 Powers

Weibull alternatives to exponentiality

(Sample size is 20 with Type II censoring)

θ	r	$C_{r,n}$	$G_{r,n}$	$W_{r,n}$	$W^2_{r,n}$
5.00	20	1.000	1.000	0.951	1.000
	18	1.000	1.000	0.898	1.000
	16	1.000	1.000	0.806	1.000
	14	1.000	1.000	0.668	1.000
	12	1.000	1.000	0.530	1.000
	10	1.000	1.000	0.455	1.000
4.50	20	1.000	1.000	0.954	1.000
	18	1.000	1.000	0.884	1.000
	16	1.000	1.000	0.790	1.000
	14	1.000	1.000	0.686	1.000
	12	1.000	1.000	0.536	1.000
	10	0.998	0.996	0.462	0.997
4.00	20	1.000	1.000	0.928	1.000
	18	1.000	1.000	0.843	1.000
	16	1.000	1.000	0.763	1.000
	14	1.000	1.000	0.621	1.000
	12	1.000	1.000	0.488	1.000
	10	0.992	0.992	0.407	0.995
3.50	20	1.000	1.000	0.906	1.000
	18	1.000	1.000	0.824	1.000
	16	1.000	1.000	0.694	1.000
	14	0.997	0.999	0.574	0.999
	12	0.996	0.992	0.454	0.995
	10	0.975	0.965	0.363	0.973
3.00	20	0.999	1.000	0.895	1.000
	18	1.000	0.999	0.774	0.999
	16	0.999	1.000	0.666	0.998
	14	0.993	0.988	0.510	0.986
	12	0.966	0.951	0.385	0.952
	10	0.913	0.906	0.362	0.916
2.50	20	0.997	0.997	0.812	0.997
	18	0.997	0.991	0.659	0.982
	16	0.986	0.979	0.556	0.966
	14	0.950	0.940	0.432	0.931
	12	0.906	0.873	0.309	0.879
	10	0.764	0.725	0.280	0.755

Table 2.1 (continued)

θ	r	$C_{r,n}$	$G_{r,n}$	$W_{r,n}$	$W^2{}_{r,n}$
2.00	20	0.954	0.955	0.638	0.930
	18	0.893	0.870	0.471	0.843
	16	0.847	0.819	0.412	0.783
	14	0.742	0.704	0.312	0.685
	12	0.652	0.596	0.209	0.615
	10	0.483	0.455	0.219	0.500
1.80	20	0.848	0.837	0.542	0.792
	18	0.771	0.744	0.393	0.698
	16	0.665	0.629	0.295	0.594
	14	0.541	0.512	0.248	0.505
	12	0.499	0.469	0.166	0.489
	10	0.380	0.348	0.156	0.398
1.60	20	0.611	0.595	0.374	0.559
	18	0.553	0.520	0.262	0.487
	16	0.458	0.454	0.232	0.403
	14	0.368	0.352	0.179	0.354
	12	0.325	0.308	0.126	0.320
	10	0.260	0.257	0.143	0.291
1.40	20	0.388	0.362	0.214	0.331
	18	0.309	0.289	0.160	0.261
	16	0.248	0.223	0.126	0.214
	14	0.220	0.202	0.104	0.209
	12	0.183	0.169	0.076	0.174
	10	0.132	0.114	0.062	0.138
1.20	20	0.152	0.152	0.107	0.132
	18	0.119	0.110	0.084	0.097
	16	0.089	0.082	0.072	0.073
	14	0.089	0.075	0.067	0.079
	12	0.075	0.072	0.061	0.083
	10	0.069	0.062	0.058	0.076
1.00	20	0.053	0.055	0.055	0.047
	18	0.047	0.057	0.055	0.040
	16	0.054	0.054	0.055	0.048
	14	0.043	0.044	0.055	0.051
	12	0.042	0.038	0.039	0.035
	10	0.061	0.050	0.041	0.048

Table 2.1 (continued)

θ	r	$C_{r,n}$	$G_{r,n}$	$W_{r,n}$	$W^2_{r,n}$
0.90	20	0.110	0.095	0.069	0.078
	18	0.105	0.082	0.061	0.070
	16	0.087	0.086	0.070	0.066
	14	0.080	0.066	0.061	0.056
	12	0.084	0.075	0.059	0.051
	10	0.082	0.072	0.052	0.043
0.80	20	0.242	0.214	0.119	0.179
	18	0.216	0.186	0.106	0.138
	16	0.214	0.187	0.127	0.147
	14	0.160	0.132	0.077	0.100
	12	0.151	0.117	0.088	0.089
	10	0.159	0.123	0.073	0.084
0.70	20	0.535	0.463	0.275	0.397
	18	0.502	0.421	0.256	0.342
	16	0.411	0.326	0.192	0.267
	14	0.358	0.263	0.146	0.215
	12	0.296	0.224	0.106	0.147
	10	0.263	0.174	0.097	0.116
0.60	20	0.829	0.732	0.510	0.686
	18	0.734	0.618	0.409	0.574
	16	0.671	0.548	0.324	0.499
	14	0.601	0.477	0.261	0.417
	12	0.528	0.410	0.189	0.311
	10	0.440	0.305	0.154	0.215
0.50	20	0.953	0.894	0.724	0.881
	18	0.940	0.841	0.619	0.830
	16	0.875	0.752	0.508	0.719
	14	0.849	0.692	0.449	0.657
	12	0.772	0.585	0.328	0.506
	10	0.691	0.498	0.272	0.393
0.40	20	0.998	0.9[illegible]	0.876	0.983
	18	0.991	0.967	0.821	0.960
	16	0.975	0.908	0.719	0.902
	14	0.953	0.866	0.630	0.837
	12	0.925	0.803	0.529	0.745
	10	0.874	0.700	0.427	0.612

Table 2.1

Weibull alternatives to exponentiality
(Sample size is 50 with Type II censoring)

θ	r	$C_{r,n}$	$G_{r,n}$	$W_{r,n}$	$W^2{}_{r,n}$
5.00	50	1.000	1.000	1.000	1.000
	45	1.000	1.000	1.000	1.000
	40	1.000	1.000	0.999	1.000
	35	1.000	1.000	0.995	1.000
	30	1.000	1.000	0.980	1.000
	25	1.000	1.000	0.924	1.000
4.50	50	1.000	1.000	1.000	1.000
	45	1.000	1.000	1.000	1.000
	40	1.000	1.000	0.999	1.000
	35	1.000	1.000	0.992	1.000
	30	1.000	1.000	0.973	1.000
	25	1.000	1.000	0.932	1.000
4.00	50	1.000	1.000	1.000	1.000
	45	1.000	1.000	1.000	1.000
	40	1.000	1.000	0.998	1.000
	35	1.000	1.000	0.986	1.000
	30	1.000	1.000	0.960	1.000
	25	1.000	1.000	0.894	1.000
3.50	50	1.000	1.000	1.000	1.000
	45	1.000	1.000	0.999	1.000
	40	1.000	1.000	0.999	1.000
	35	1.000	1.000	0.982	1.000
	30	1.000	1.000	0.942	1.000
	25	1.000	1.000	0.880	1.000
3.00	50	1.000	1.000	1.000	1.000
	45	1.000	1.000	0.999	1.000
	40	1.000	1.000	0.987	1.000
	35	1.000	1.000	0.960	1.000
	30	1.000	1.000	0.917	1.000
	25	1.000	1.000	0.821	1.000
2.50	50	1.000	1.000	1.000	1.000
	45	1.000	1.000	0.989	1.000
	40	1.000	1.000	0.969	1.000
	35	1.000	0.999	0.924	0.999
	30	1.000	0.999	0.851	0.997
	25	0.999	0.996	0.753	0.998

Table 2.1 (continued)

θ	r	$C_{r,n}$	$G_{r,n}$	$W_{r,n}$	$W^2_{r,n}$
2.00	50	1.000	1.000	0.993	1.000
	45	1.000	1.000	0.966	0.998
	40	0.998	0.997	0.892	0.996
	35	0.994	0.995	0.815	0.993
	30	0.983	0.966	0.684	0.963
	25	0.941	0.920	0.542	0.917
1.80	50	0.999	0.999	0.958	0.995
	45	0.996	0.998	0.891	0.994
	40	0.988	0.980	0.789	0.974
	35	0.967	0.946	0.666	0.931
	30	0.915	0.892	0.607	0.884
	25	0.843	0.791	0.430	0.798
1.60	50	0.985	0.989	0.890	0.978
	45	0.953	0.943	0.766	0.928
	40	0.903	0.882	0.618	0.850
	35	0.832	0.777	0.493	0.771
	30	0.747	0.688	0.454	0.676
	25	0.633	0.593	0.331	0.602
1.40	50	0.821	0.796	0.609	0.749
	45	0.725	0.688	0.504	0.649
	40	0.651	0.616	0.416	0.584
	35	0.554	0.520	0.333	0.496
	30	0.468	0.425	0.296	0.422
	25	0.381	0.330	0.202	0.345
1.20	50	0.302	0.306	0.228	0.266
	45	0.270	0.259	0.192	0.219
	40	0.231	0.201	0.150	0.180
	35	0.175	0.164	0.114	0.166
	30	0.176	0.165	0.124	0.168
	25	0.109	0.104	0.085	0.116
1.00	50	0.052	0.059	0.040	0.054
	45	0.059	0.059	0.040	0.056
	40	0.048	0.056	0.057	0.049
	35	0.065	0.050	0.057	0.050
	30	0.051	0.035	0.048	0.043
	25	0.043	0.056	0.057	0.047

Table 2.1 (continued)

θ	r	$C_{r,n}$	$G_{r,n}$	$W_{r,n}$	$W^2_{r,n}$
0.90	50	0.178	0.152	0.094	0.125
	45	0.158	0.141	0.088	0.117
	40	0.134	0.119	0.076	0.097
	35	0.115	0.102	0.078	0.083
	30	0.105	0.081	0.064	0.069
	25	0.087	0.065	0.054	0.044
0.80	50	0.530	0.493	0.290	0.429
	45	0.476	0.393	0.192	0.344
	40	0.440	0.348	0.173	0.296
	35	0.386	0.309	0.163	0.259
	30	0.318	0.233	0.136	0.191
	25	0.244	0.194	0.097	0.153
0.70	50	0.888	0.824	0.594	0.796
	45	0.849	0.768	0.460	0.739
	40	0.747	0.616	0.370	0.574
	35	0.670	0.566	0.341	0.518
	30	0.600	0.477	0.273	0.401
	25	0.503	0.381	0.193	0.311
0.60	50	0.994	0.982	0.879	0.977
	45	0.978	0.940	0.722	0.936
	40	0.947	0.888	0.661	0.872
	35	0.925	0.835	0.591	0.807
	30	0.867	0.751	0.487	0.698
	25	0.795	0.644	0.338	0.590
0.50	50	1.000	1.000	0.982	1.000
	45	1.000	0.992	0.927	0.996
	40	0.997	0.982	0.876	0.985
	35	0.994	0.969	0.806	0.966
	30	0.985	0.934	0.735	0.918
	25	0.937	0.849	0.576	0.822
0.40	50	1.000	1.000	0.999	1.000
	45	1.000	1.000	0.994	1.000
	40	1.000	1.000	0.983	1.000
	35	1.000	0.998	0.961	0.997
	30	0.998	0.994	0.904	0.991
	25	1.000	0.973	0.817	0.965

Table 2.1

Table 2.2 Powers
Gamma alternatives to exponentiality
(Sample size is 20 with Type II censoring)

θ	r	$C_{r,n}$	$G_{r,n}$	$W_{r,n}$	$W^2_{r,n}$
5.00	20	1.000	0.998	0.581	0.997
	18	1.000	0.996	0.470	0.996
	16	0.997	0.991	0.380	0.991
	14	0.992	0.970	0.321	0.972
	12	0.971	0.948	0.260	0.958
	10	0.933	0.885	0.213	0.912
4.50	20	1.000	0.999	0.550	0.996
	18	0.996	0.985	0.452	0.988
	16	0.994	0.973	0.387	0.972
	14	0.976	0.944	0.315	0.948
	12	0.947	0.914	0.222	0.915
	10	0.871	0.821	0.238	0.862
4.00	20	0.997	0.984	0.513	0.982
	18	0.989	0.968	0.433	0.968
	16	0.975	0.942	0.353	0.936
	14	0.958	0.921	0.289	0.924
	12	0.920	0.855	0.201	0.864
	10	0.834	0.782	0.202	0.811
3.50	20	0.985	0.963	0.441	0.955
	18	0.968	0.926	0.348	0.919
	16	0.938	0.877	0.326	0.874
	14	0.905	0.840	0.251	0.840
	12	0.825	0.747	0.179	0.762
	10	0.734	0.640	0.192	0.703
3.00	20	0.946	0.881	0.372	0.875
	18	0.895	0.826	0.293	0.810
	16	0.858	0.768	0.251	0.754
	14	0.776	0.685	0.229	0.681
	12	0.721	0.636	0.171	0.667
	10	0.637	0.564	0.187	0.606
2.50	20	0.791	0.710	0.299	0.687
	18	0.742	0.658	0.252	0.646
	16	0.683	0.589	0.207	0.563
	14	0.624	0.546	0.180	0.543
	12	0.518	0.443	0.140	0.467
	10	0.434	0.394	0.144	0.432

Table 2.2 (continued)

θ	r	$C_{r,n}$	$G_{r,n}$	$W_{r,n}$	$W^2{}_{r,n}$
2.00	20	0.567	0.475	0.213	0.482
	18	0.513	0.431	0.172	0.412
	16	0.456	0.383	0.133	0.356
	14	0.376	0.318	0.137	0.314
	12	0.302	0.275	0.114	0.291
	10	0.240	0.210	0.092	0.246
1.80	20	0.415	0.352	0.171	0.333
	18	0.358	0.303	0.156	0.295
	16	0.333	0.289	0.116	0.268
	14	0.284	0.237	0.119	0.256
	12	0.250	0.230	0.095	0.242
	10	0.195	0.157	0.090	0.198
1.60	20	0.269	0.223	0.125	0.220
	18	0.277	0.239	0.129	0.223
	16	0.242	0.193	0.096	0.196
	14	0.218	0.181	0.084	0.183
	12	0.168	0.141	0.068	0.168
	10	0.132	0.136	0.095	0.156
1.40	20	0.141	0.125	0.086	0.117
	18	0.156	0.129	0.078	0.121
	16	0.118	0.103	0.067	0.100
	14	0.122	0.098	0.070	0.104
	12	0.107	0.094	0.065	0.109
	10	0.099	0.097	0.067	0.108
1.20	20	0.075	0.082	0.075	0.070
	18	0.070	0.066	0.057	0.069
	16	0.077	0.076	0.054	0.066
	14	0.064	0.072	0.054	0.078
	12	0.049	0.045	0.042	0.057
	10	0.085	0.078	0.072	0.091
1.00	20	0.071	0.062	0.051	0.052
	18	0.056	0.054	0.045	0.051
	16	0.041	0.045	0.056	0.039
	14	0.043	0.054	0.053	0.051
	12	0.045	0.051	0.047	0.039
	10	0.050	0.057	0.057	0.055

Table 2.2 (continued)

θ	r	$C_{r,n}$	$G_{r,n}$	$W_{r,n}$	$W^2_{r,n}$
0.90	20	0.066	0.056	0.056	0.053
	18	0.083	0.061	0.058	0.055
	16	0.071	0.066	0.051	0.055
	14	0.076	0.064	0.068	0.053
	12	0.062	0.066	0.055	0.049
	10	0.062	0.065	0.053	0.050
0.80	20	0.169	0.145	0.076	0.117
	18	0.139	0.104	0.064	0.092
	16	0.126	0.115	0.075	0.086
	14	0.134	0.101	0.057	0.066
	12	0.128	0.090	0.051	0.057
	10	0.120	0.086	0.055	0.058
0.70	20	0.303	0.199	0.116	0.166
	18	0.284	0.175	0.103	0.157
	16	0.252	0.169	0.097	0.140
	14	0.238	0.172	0.090	0.140
	12	0.216	0.132	0.083	0.104
	10	0.208	0.141	0.065	0.084
0.60	20	0.520	0.348	0.153	0.300
	18	0.470	0.309	0.153	0.269
	16	0.422	0.280	0.144	0.226
	14	0.395	0.276	0.147	0.214
	12	0.375	0.261	0.117	0.202
	10	0.363	0.224	0.104	0.149
0.50	20	0.750	0.539	0.277	0.502
	18	0.735	0.522	0.260	0.471
	16	0.660	0.474	0.247	0.423
	14	0.638	0.425	0.209	0.371
	12	0.572	0.363	0.178	0.311
	10	0.538	0.354	0.164	0.254
0.40	20	0.932	0.758	0.471	0.763
	18	0.916	0.718	0.426	0.712
	16	0.875	0.692	0.384	0.654
	14	0.850	0.664	0.378	0.629
	12	0.773	0.580	0.323	0.518
	10	0.762	0.533	0.250	0.435

Table 2.2

Gamma alternatives to exponentiality
(Sample size is 50 with Type II censoring)

θ	r	$C_{r,n}$	$G_{r,n}$	$W_{r,n}$	$W^2{}_{r,n}$
5.00	50	1.000	1.000	0.970	1.000
	45	1.000	1.000	0.931	1.000
	40	1.000	1.000	0.883	1.000
	35	1.000	1.000	0.852	1.000
	30	1.000	1.000	0.748	1.000
	25	1.000	1.000	0.660	1.000
4.50	50	1.000	1.000	0.956	1.000
	45	1.000	1.000	0.898	1.000
	40	1.000	1.000	0.864	1.000
	35	1.000	1.000	0.799	1.000
	30	1.000	1.000	0.736	1.000
	25	1.000	1.000	0.640	1.000
4.00	50	1.000	1.000	0.935	1.000
	45	1.000	1.000	0.890	1.000
	40	1.000	1.000	0.852	1.000
	35	1.000	1.000	0.772	1.000
	30	1.000	1.000	0.735	1.000
	25	1.000	0.997	0.582	0.996
3.50	50	1.000	1.000	0.906	1.000
	45	1.000	1.000	0.855	1.000
	40	1.000	1.000	0.792	1.000
	35	1.000	0.999	0.750	0.999
	30	1.000	0.996	0.637	0.996
	25	0.999	0.989	0.538	0.993
3.00	50	1.000	1.000	0.857	1.000
	45	1.000	1.000	0.805	1.000
	40	0.999	0.999	0.735	0.999
	35	0.999	0.994	0.637	0.995
	30	0.996	0.987	0.627	0.981
	25	0.987	0.957	0.475	0.969
2.50	50	1.000	0.994	0.782	0.991
	45	0.998	0.987	0.684	0.989
	40	0.994	0.971	0.617	0.973
	35	0.978	0.935	0.529	0.932
	30	0.963	0.921	0.506	0.922
	25	0.931	0.856	0.387	0.870

Table 2.2 (continued)

θ	r	$C_{r,n}$	$G_{r,n}$	$W_{r,n}$	$W^2_{r,n}$
2.00	50	0.960	0.902	0.569	0.899
	45	0.931	0.856	0.510	0.850
	40	0.921	0.814	0.446	0.810
	35	0.886	0.770	0.386	0.776
	30	0.815	0.689	0.337	0.701
	25	0.723	0.616	0.241	0.641
1.80	50	0.893	0.774	0.482	0.764
	45	0.834	0.726	0.401	0.728
	40	0.795	0.685	0.364	0.671
	35	0.726	0.621	0.316	0.610
	30	0.655	0.525	0.274	0.541
	25	0.553	0.463	0.201	0.482
1.60	50	0.716	0.594	0.345	0.563
	45	0.624	0.517	0.286	0.500
	40	0.570	0.472	0.257	0.451
	35	0.503	0.407	0.211	0.414
	30	0.464	0.378	0.219	0.383
	25	0.383	0.313	0.150	0.332
1.40	50	0.416	0.326	0.187	0.316
	45	0.372	0.292	0.171	0.283
	40	0.319	0.269	0.167	0.239
	35	0.304	0.243	0.151	0.226
	30	0.236	0.202	0.127	0.212
	25	0.241	0.197	0.108	0.218
1.20	50	0.138	0.121	0.095	0.114
	45	0.125	0.103	0.095	0.100
	40	0.123	0.114	0.091	0.109
	35	0.118	0.111	0.080	0.111
	30	0.108	0.092	0.081	0.104
	25	0.093	0.089	0.066	0.104
1.00	50	0.057	0.060	0.040	0.065
	45	0.048	0.050	0.039	0.047
	40	0.045	0.058	0.053	0.053
	35	0.051	0.059	0.047	0.050
	30	0.043	0.044	0.054	0.053
	25	0.059	0.054	0.047	0.054

Table 2.2 (continued)

θ	r	$C_{r,n}$	$G_{r,n}$	$W_{r,n}$	$W^2_{r,n}$
0.90	50	0.095	0.075	0.049	0.070
	45	0.098	0.077	0.041	0.068
	40	0.106	0.063	0.066	0.077
	35	0.094	0.082	0.063	0.075
	30	0.081	0.071	0.068	0.059
	25	0.085	0.079	0.065	0.075
0.80	50	0.274	0.195	0.101	0.175
	45	0.261	0.195	0.098	0.163
	40	0.213	0.143	0.089	0.137
	35	0.229	0.156	0.092	0.123
	30	0.202	0.153	0.078	0.119
	25	0.183	0.121	0.080	0.098
0.70	50	0.552	0.396	0.180	0.369
	45	0.522	0.370	0.155	0.336
	40	0.521	0.359	0.171	0.352
	35	0.465	0.343	0.180	0.298
	30	0.441	0.291	0.172	0.259
	25	0.364	0.259	0.134	0.213
0.60	50	0.861	0.690	0.343	0.677
	45	0.815	0.612	0.265	0.608
	40	0.782	0.589	0.290	0.555
	35	0.750	0.544	0.276	0.526
	30	0.721	0.521	0.274	0.472
	25	0.621	0.443	0.205	0.392
0.50	50	0.979	0.879	0.578	0.883
	45	0.976	0.856	0.525	0.882
	40	0.947	0.824	0.493	0.813
	35	0.941	0.807	0.492	0.791
	30	0.910	0.751	0.443	0.713
	25	0.851	0.680	0.364	0.629
0.40	50	0.998	0.981	0.831	0.985
	45	1.000	0.979	0.780	0.983
	40	0.998	0.974	0.765	0.972
	35	0.996	0.945	0.722	0.942
	30	0.989	0.919	0.691	0.923
	25	0.977	0.880	0.622	0.873

Table 2.2

Table 3.1 Powers

Weibull alternatives to exponentiality
(sample size is 20 with random censoring)

β	θ	$C_{m,n}$	$G_{m,n}$	$W_{m,n}$	$W^2_{m,n}$
0.00	5.00	1.000	1.000	0.951	1.000
	4.50	1.000	1.000	0.952	1.000
	4.00	1.000	1.000	0.935	1.000
	3.50	1.000	1.000	0.919	1.000
	3.00	1.000	1.000	0.867	1.000
	2.50	0.998	0.999	0.809	0.997
	2.00	0.952	0.955	0.645	0.928
	1.80	0.845	0.835	0.546	0.811
	1.60	0.652	0.646	0.366	0.608
	1.40	0.373	0.362	0.201	0.335
	1.20	0.154	0.149	0.110	0.144
	1.00	0.049	0.043	0.041	0.045
	0.90	0.107	0.104	0.069	0.095
	0.80	0.297	0.259	0.166	0.210
	0.70	0.553	0.469	0.290	0.413
	0.60	0.824	0.749	0.507	0.710
	0.50	0.967	0.911	0.747	0.902
	0.40	0.999	0.982	0.889	0.989
0.25	5.00	1.000	1.000	0.862	1.000
	4.50	1.000	1.000	0.872	1.000
	4.00	1.000	1.000	0.821	1.000
	3.50	0.999	1.000	0.810	1.000
	3.00	0.997	0.997	0.734	0.997
	2.50	0.988	0.991	0.673	0.981
	2.00	0.878	0.873	0.507	0.846
	1.80	0.728	0.709	0.362	0.693
	1.60	0.494	0.478	0.251	0.463
	1.40	0.293	0.288	0.153	0.26[illegible]
	1.20	0.101	0.102	0.073	0.104
	1.00	0.063	0.060	0.044	0.057
	0.90	0.094	0.105	0.090	0.081
	0.80	0.215	0.183	0.128	0.151
	0.70	0.453	0.380	0.221	0.334
	0.60	0.756	0.635	0.431	0.607
	0.50	0.906	0.831	0.622	0.805
	0.40	0.993	0.951	0.829	0.954

Table 3.1 (continued)

β	θ	$C_{m,n}$	$G_{m,n}$	$W^1_{m,n}$	$W^2_{m,n}$
0.50	5.00	1.000	1.000	0.746	1.000
	4.50	1.000	1.000	0.717	1.000
	4.00	1.000	1.000	0.716	1.000
	3.50	0.999	0.998	0.657	0.998
	3.00	0.987	0.985	0.617	0.985
	2.50	0.964	0.956	0.523	0.951
	2.00	0.798	0.786	0.366	0.755
	1.80	0.618	0.626	0.286	0.595
	1.60	0.414	0.421	0.208	0.400
	1.40	0.228	0.204	0.111	0.217
	1.20	0.094	0.091	0.048	0.098
	1.00	0.048	0.047	0.050	0.047
	0.90	0.069	0.070	0.059	0.066
	0.80	0.243	0.195	0.117	0.160
	0.70	0.396	0.349	0.231	0.279
	0.60	0.700	0.574	0.375	0.531
	0.50	0.865	0.776	0.554	0.734
	0.40	0.977	0.904	0.738	0.894
0.75	5.00	1.000	1.000	0.676	1.000
	4.50	0.998	0.999	0.627	0.999
	4.00	0.999	0.999	0.618	0.999
	3.50	0.986	0.993	0.573	0.990
	3.00	0.969	0.968	0.530	0.963
	2.50	0.902	0.905	0.432	0.889
	2.00	0.688	0.678	0.280	0.655
	1.80	0.529	0.517	0.219	0.525
	1.60	0.333	0.346	0.196	0.329
	1.40	0.210	0.197	0.121	0.210
	1.20	0.094	0.102	0.061	0.103
	1.00	0.048	0.052	0.054	0.058
	0.90	0.076	0.084	0.073	0.067
	0.80	0.177	0.150	0.084	0.121
	0.70	0.387	0.325	0.183	0.261
	0.60	0.606	0.519	0.327	0.452
	0.50	0.831	0.696	0.486	0.676
	0.40	0.954	0.871	0.673	0.860

Table 3.1 (continued)

β	θ	$C_{m,n}$	$G_{m,n}$	$W_{m,n}$	$W^2_{m,n}$
1.00	5.00	0.993	0.994	0.545	0.996
	4.50	0.994	0.993	0.518	0.996
	4.00	0.993	0.991	0.531	0.992
	3.50	0.973	0.976	0.492	0.976
	3.00	0.932	0.936	0.427	0.931
	2.50	0.856	0.854	0.325	0.843
	2.00	0.616	0.624	0.251	0.608
	1.80	0.458	0.449	0.193	0.446
	1.60	0.303	0.302	0.123	0.307
	1.40	0.187	0.158	0.091	0.184
	1.20	0.078	0.078	0.066	0.082
	1.00	0.051	0.048	0.046	0.039
	0.90	0.094	0.072	0.063	0.060
	0.80	0.168	0.152	0.105	0.118
	0.70	0.326	0.272	0.179	0.210
	0.60	0.549	0.454	0.274	0.388
	0.50	0.763	0.626	0.418	0.590
	0.40	0.912	0.806	0.592	0.801

Table 3.1

Weibull alternatives to exponentiality
(sample size is 50 with random censoring)

β	θ	$C_{m,n}$	$G_{m,n}$	$W_{m,n}$	$W^2{}_{m,n}$
0.00	5.00	1.000	1.000	1.000	1.000
	4.50	1.000	1.000	1.000	1.000
	4.00	1.000	1.000	1.000	1.000
	3.50	1.000	1.000	1.000	1.000
	3.00	1.000	1.000	1.000	1.000
	2.50	1.000	1.000	0.999	1.000
	2.00	1.000	1.000	0.984	1.000
	1.80	0.999	1.000	0.964	1.000
	1.60	0.975	0.978	0.867	0.967
	1.40	0.808	0.789	0.608	0.748
	1.20	0.336	0.308	0.221	0.273
	1.00	0.060	0.067	0.050	0.054
	0.90	0.171	0.170	0.105	0.121
	0.80	0.503	0.458	0.267	0.388
	0.70	0.906	0.833	0.598	0.793
	0.60	0.993	0.980	0.881	0.979
	0.50	1.000	0.998	0.974	1.000
	0.40	1.000	1.000	0.999	1.000
0.25	5.00	1.000	1.000	1.000	1.000
	4.50	1.000	1.000	1.000	1.000
	4.00	1.000	1.000	0.999	1.000
	3.50	1.000	1.000	1.000	1.000
	3.00	1.000	1.000	1.000	1.000
	2.50	1.000	1.000	0.995	1.000
	2.00	1.000	1.000	0.960	0.998
	1.80	0.993	0.994	0.899	0.987
	1.60	0.943	0.942	0.782	0.923
	1.40	0.709	0.684	0.484	0.633
	1.20	0.257	0.263	0.204	0.226
	1.00	0.053	0.057	0.058	0.042
	0.90	0.126	0.121	0.081	0.092
	0.80	0.443	0.394	0.238	0.340
	0.70	0.821	0.768	0.524	0.710
	0.60	0.984	0.949	0.791	0.936
	0.50	0.997	0.995	0.959	0.994
	0.40	1.000	1.000	0.995	1.000

Table 3.1 (continued)

β	θ	$C_{m,n}$	$G_{m,n}$	$W_{m,n}$	$W^2{}_{m,n}$
0.50	5.00	1.000	1.000	0.997	1.000
	4.50	1.000	1.000	0.998	1.000
	4.00	1.000	1.000	0.994	1.000
	3.50	1.000	1.000	0.995	1.000
	3.00	1.000	1.000	0.988	1.000
	2.50	1.000	1.000	0.975	1.000
	2.00	1.000	0.998	0.916	0.994
	1.80	0.976	0.976	0.812	0.966
	1.60	0.889	0.877	0.668	0.851
	1.40	0.597	0.577	0.388	0.537
	1.20	0.194	0.182	0.131	0.172
	1.00	0.047	0.052	0.052	0.054
	0.90	0.142	0.121	0.099	0.113
	0.80	0.365	0.338	0.211	0.288
	0.70	0.743	0.641	0.434	0.596
	0.60	0.955	0.899	0.716	0.867
	0.50	0.995	0.983	0.911	0.981
	0.40	1.000	0.999	0.983	1.000
0.75	5.00	1.000	1.000	0.993	1.000
	4.50	1.000	1.000	0.995	1.000
	4.00	1.000	1.000	0.983	1.000
	3.50	1.000	1.000	0.990	1.000
	3.00	1.000	1.000	0.972	1.000
	2.50	1.000	1.000	0.955	1.000
	2.00	0.988	0.990	0.861	0.984
	1.80	0.955	0.954	0.748	0.924
	1.60	0.824	0.806	0.585	0.770
	1.40	0.537	0.514	0.349	0.491
	1.20	0.183	0.185	0.141	0.174
	1.00	0.047	0.052	0.051	0.041
	0.90	0.120	0.113	0.083	0.106
	0.80	0.352	0.328	0.212	0.296
	0.70	0.685	0.617	0.396	0.535
	0.60	0.927	0.864	0.666	0.840
	0.50	0.990	0.963	0.850	0.962
	0.40	1.000	0.995	0.965	0.994

Table 3.1 (continued)

β	θ	$C_{m,n}$	$G_{m,n}$	$W_{m,n}$	$W^2{}_{m,n}$
1.00	5.00	1.000	1.000	0.984	1.000
	4.50	1.000	1.000	0.981	1.000
	4.00	1.000	1.000	0.970	1.000
	3.50	1.000	1.000	0.965	1.000
	3.00	1.000	1.000	0.940	1.000
	2.50	1.000	1.000	0.898	1.000
	2.00	0.985	0.981	0.777	0.968
	1.80	0.925	0.926	0.624	0.897
	1.60	0.758	0.728	0.478	0.696
	1.40	0.460	0.428	0.314	0.412
	1.20	0.170	0.151	0.130	0.162
	1.00	0.038	0.051	0.060	0.051
	0.90	0.112	0.102	0.070	0.085
	0.80	0.331	0.302	0.211	0.252
	0.70	0.642	0.561	0.366	0.500
	0.60	0.875	0.807	0.599	0.781
	0.50	0.978	0.953	0.798	0.939
	0.40	1.000	0.994	0.946	0.996

Table 3.1

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