High School Students' Understanding of Percent

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ABSTRACT

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This study investigated high school students' understanding of percent. Written tests were administered to over 200 secondary I and II (grade 7 and 8) students. The test items included: questions on the meaning of percent and on the mechanical skills involved, basic percent problems and questions about fractions, decimals, ratio and proportion. Follow-up individual interviews of eighteen students were conducted.

An item analysis was carried out to determine percentage success rates and to identify types of correct and incorrect responses. The interviews were analyzed to trace students' underlying ideas of percent and its related topics.

The results indicate that high school students who had completed their formal study of percent still have a relatively poor understanding of the topic and its applications. Furthermore, mechanical skills for handling percent problems are poor, as is the ability to work with decimals, ratio and proportion.

Another finding was the presence of unanticipated patterns of misconceptions about percent which seem to have

emerged through ideas which were understood. For example the recognition that the number 100 is associated with percent often led to the misuse of this number in percent problems. The acceptance of the idea that 100% represents one whole led to the rejection of percents larger than 100. The ability to mechanically change a two digit percent to its fractional or decimal equivalent led to a generalized incorrect method to change larger percents to other forms (e.g. 115% = 0,115). The use of basic arithmetic operations to solve simple percent problems led to a tendency to randomly multiply or divide the numbers in any problem.

Suggestions for further research are given.

ACKNOWLEDGEMENTS

First and foremost, I would like to acknowldege the help of my thesis advisor, Dr. Stanley Erlwanger. His guidance, knowledge and, most importantly, his faith in me were instrumental in the undertaking and completion of this study.

I would also like to acknowledge the help of the mathematics teachers at Bialik High School and Westhill High School who administered the tests. Finally, I would like to thank the secondary I and II students who wrote the tests and made themselves available to be interviewed.

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CHAPTER I

INTRODUCTION

The mathematics curriculum encompasses many varied concepts, most of which share two common characteristics. Each topic studied is a prerequisite for another mathematical concept, yet its relevance to most students is merely limited to within the classroom. One exception is the concept of percent.

Although there are prerequisite topics leading up to its study (fractions, decimals, ratio and proportion), percent itself is not essential for the development of other mathematical concepts. Furthermore, percent is one of the few mathematical topics that has much immediate relevance outside the classroom. It appears in a variety of contexts in the daily life of both the student and the adult for whenever comparisons of amounts are to be made, these are likely to be expressed in the language of percent. Its presence is not only a part of the business world but is found in many aspects of consumer living (e.g. profit, discount, interest on loans, etc.) and recreational activities (e.g. sports). The media will, for example, contain advertisements regarding sales and conduct opinion polls using the language of percent.

In the mathematics classroom, as well as in most textbooks, percent is commonly introduced as an extension of the topic of ratio and is related to fractions and decimals. At first, much time and effort is spent on the conversion of fractions and decimals into percents and vice versa. Later, concentration is placed on the solution of basic percent problems (base × rate = percentage) which are usually divided into three 'cases' depending upon the unknown, (e.g. "What percent is 20 out of 80?" "What is 3% of 150?" and "16 is 25% of what number?"). These exercises are later accompanied with some simple word problems that require the solution of the above types of problems. Illustrative examples usually use the idea of a proportion to solve all three 'types' of problems in a similar fashion. For example, the problem "20% of what number is 15?" would be solved with the following proportion:

$$\frac{20}{100} = \frac{15}{x}$$

Finally, the discussion on percents leads to applications of percents to problems with discount, profit, interest rates, etc.

The treatment of percent in most texts and by most teachers is so similar that there appears to exist a tacit agreement on as to how the subject should be approached. The following texts demonstrate this similarity: Ebos, Robinson & Pogue, Math is 1; Dolciani, Elementary Algebra Part 2; Forbes, et al., Series M Mathematics SI Edition; MacLean, et al., Mathematics Book 7. Does this mean that the topic 'percent' is understood by students?

CHAPTER II

PURPOSE OF THE STUDY

There has been a great deal of research over the years on fractions, decimals, ratio and proportion. A combination of recent research on fractions (Chuate, 1975; Ellerbuch, 1976; Nivillis, 1976; Phillips & Kane, 1973; Stenger, 1972; Coburn, 1974; Green, 1970 and others) shows four categories of study: sequence, approach, type of algorithm and the use of materials. These categories have provided a different focus from which to investigate children's skills and led to more coordinated efforts on research with fractions (NCTM, 1980).

Decimals is another area in which much research has been conducted. Faires (1963), O'Brien (1967) and Willson (1969, 1972) studied sequencing; Fluornoy (1959) and Kuhn (1954) studied how to determine the location of the decimal point in the quotient, and Bauer (1975) studied common - fraction equivalents, expanded exponential notation and number - line approaches (NCTM, 1980).

Piaget's ideas as they relate to mathematics education have influenced research as well. His work on children's notions of ratio and proportion (1960, 1968) prompted others to study these concepts. One such study began at Laval University (Gerald Noelting). Subjects were asked to rate the strength of a pitcher of orange juice, given the number of glasses of water and orange juice within. This and other studies based on Piagetian schemes indicate that the notion of proportion is not an easy one for children to grasp.

Although the topics above are closely related to percent, research on the latter is very scarce. A survey of recent research shows that only the National Assessment of Educational Progress (NAEP) of 1973 and 1978 include some questions on percent. The results of the 1973 study were as follows:

1) "1/5 is equivalent to what percent?"

   Results: Less than 1/2 of the 13-year-olds and only 65% of the 17-year-olds answered correctly.

2) "In a school election with three candidates, Joe received 120 votes, Mary received 50 votes, and George received 30 votes. What percent of the total number of votes did Joe receive?"

   Results: Approximately 18% of the 13-year-olds while 45% and 48% of the 17-year-olds and adults respectively solved the problem correctly.

The NAEP study summarized its findings in 1973 as follows: "Consumer problems involving percents were very difficult for 13-year-olds (less than 20% correct) and generally difficult for 17-year-olds and adults. However, the adults consistently did better (approximately 60 percent correct) than 17-year-olds on all types of percentage problems (NCTM, 1978).

Five years later, the 1978 NAEP study contained more varied questions on percent (NCTM, 1981). The main findings are summarized below:

(a) Questions relating the meaning of percent to 'per 100':

1) "Express 9/100 as a percent."

Results: 36% of the 13-year-olds and 53% of the 17-year-olds responded correctly.

2) "If 37 percent of the U.S. population is under 20 years of age, what percent of the population is 20 years of age or older?"

Results: 36% of the 13-year-olds responded correctly.

3) "What percent of the circles is shaded?"

● 0 0 0

● 0 0 0

Results: 28% of the 13-year-olds and 53% of the 17-year-olds related the one-fourth of the shaded circles to 25%. Many more knew that one-fourth of the circles were shaded but failed to express the quantity in percent.

(b) Questions which attempted to assess the ability of the students to relate decimals, fractions and percents:

		% correct	
		Age 13	Age 17
1)	"Change 25% to a common fraction."	57	81
2)	"Change 125% to a decimal fraction."	27	44
3)	"Change .15 to a percent."	68	77

The results on other questions similar to questions (1), (2) and (3) show a similar success rate.

(c) Applications of percent:

		% correct	
		Age 13	Age 17
1)	"30 is what percent of 60?"	35	58
2)	"What is 4% of 75?"	8	27
3)	"12 is 15% of what number?"	4	12
4)	"What is 125% of 40?"	12	31
5)	"6 is what percent of 120?"	6	16

The results indicate a low performance for both age levels.

(d) Word problems:

1) "A store is offering a discount of 12 percent on winter coats. What is the amount a customer will save on a coat regularly priced at $30?"

Results: 10% of the 13-year-olds and 40% of the 17-year-olds responded correctly.

Altogether 20 questions were administered to the students and overall performance from the basic concepts through to applications was extremely low. The results of the findings of these exercises should give cause for concern if one expects the adolescent to exhibit a working knowledge of the concept of percent. The poor level of achievement in working with percent hints at the existence of a possibly hidden aspect in the notion of percent that is being missed in the classroom, in texts and in research.

To reiterate, percent is a direct application of previous mathematical ideas; the groundwork is laid down before the concept is approached and furthermore (unlike most other mathematical topics), percent is widely found in the student's environment. Butler and Wren (1965) made the following observation: "Since it [%] is so closely allied to the subject of decimal notation and fractions, one might conclude that its application would involve no difficulties that are not involved in the study or application of decimals as such." However, classroom experiences seem to suggest that percent is not a topic that is readily understood by students as shown by the following quote which is typical of textbooks on teaching methods: "Percentage has been and continues to be one of the most troublesome parts of arithmetic. In spite of its importance as an instrument for analysis and a vehicle for communication, many students fail to attain an assured mastery of it." (Butler & Wren, 1965, pp. 261).

Although there is little research on percent, the two NAEP studies suggest that the concept of percent contains hidden subtleties for 13 and 17 year old students as well as adults. The studies show that the areas of difficulty are the meaning of percent and the applications of percent in problems.

The purpose of this study is to explore further the nature of the difficulties secondary school students encounter in percents with respect to three aspects:

(A) The Meaning of Percent

What do students understand by the word 'percent' and the symbol '%'? How does this meaning affect their ability to solve problems involving percent?

(B) Mechanical Skills

Can a student change a percent to another equivalent number? How does a student solve the three basic percentage problems? What strategies does he use?

(C) Prerequisite Skills

What skills has the student developed in the prerequisite concepts leading up to percents? How does a student's grasp of fractions, decimals, ratio and proportion affect his understanding of the meaning of percent and his ability to handle percent problems?

CHAPTER III

METHODOLOGY

This study was intended to explore secondary school students' ideas about percent and the method employed had to be congruous with this end. Written tests and individual interviews were used. The tests would provide data about the general behaviour of students on different types of items while the interviews would shed some light on the underlying aspects behind that behaviour. It would then be possible to generalize about the group of students and also to attempt to explain the ideas which seem to influence students' responses.

TESTS

Two tests were required. These will be referred to as Test I and Test II and can be found in Appendix I. Categories of test items included the multiple choice items, true/false items, fill in the blanks, 'activity' or pictorial items, and problems requiring written solutions. The use of these various categories of items enabled the widest exploration of a student's comprehension of the topic since different objectives could best be explored by certain types of questions. The examples and discussion that follow will clarify this.

Listed below are some of the categories of test items with sample questions, some rationale for the format of the question, as well as some pertinent comments.

In the discussion below, the Roman numeral in brackets after each question refers to the test and the number refers to the item.

(a) Multiple Choice:

(i) Which one is not equivalent to the rest?

(a) 1:4 (b) 1/4 (c) 25% (d) 0,25 (e) all are equivalent (I;10).

This question was used to provide a general picture of the student's ability to deal with fractions, decimals, ratio, etc. Since the aim was not to study the strategies used to solve these types of problems, multiple choice items (or other items simply requiring an answer with no work) were quick ways to tap the student's level of performance.

(b) True/False:

(i) 100% of 80 is greater than 80. (I;18)

(ii) 172% of 80 is greater than 80. (II;9)

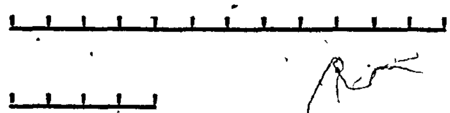
As there should be no work to solve these problems and there are only two plausible choices, they lend themselves to the true/false type of question.

(c) Activity or Pictorial:

(i) Shade in 7% of the grid below. (II;17)

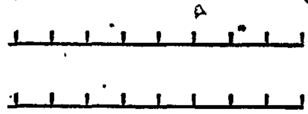
(ii) A

B



A is what percent of B? (I;7)

(iii)



A

B

A is % of B? (II;8)

These questions attempt to assess the meaning of percent in different contents. Do students have a pictorial understanding of the concept?

(d) <u>Problems Requiring Written Solutions:</u>

(i) What is 15% of 75? (I;2)

(ii) 80% of _____ = 8 (II;11)

These questions deal with the basic percentage problems. Although these could have been made into multiple choice or true/false items, more information could be obtained by having the student show his work.

Until now, the discussion of test items merely looked at the categories of questions involved. Let us turn our attention to the rationale for each question on the tests from the point of view of its relevance to the study. The objectives of the study are restated below but are broken down to more specific components. The test items that were used to investigate a particular objective are grouped together and discussed to illustrate their relevance to the investigation.

(A) Meaning of Percent:

The meaning of percent was studied through three types of items.

1. x% of 100:

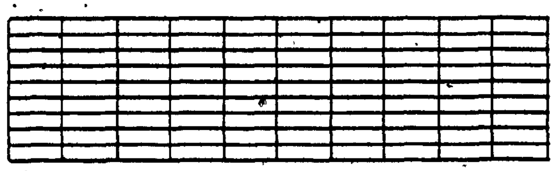
Listed below are the nine items used to investigate this particular idea.

(i) What is 17% of 100? (II;1)

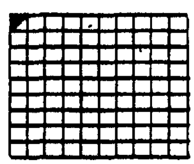
(ii) 6,2% of 100 equals? (I;27)

(iii) What is 1/2% of 100? (II;25)

(iv) Shade in 7% of the grid below. (II;17)



(v) What percent of the grid is shaded? (I;17)



(vi) 32 is ______% of 100. (I;19)

(vii) 127 = ______% of 100. (I;5)

(viii) Out of 100 apples, 40 were rotten. What percent was this? (II;3)

(ix) 70% of the children in the park like to play. If 70 children like to play, how many are in the park? (I;6)

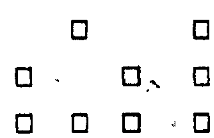
The first three items are straight forward examples of finding x% of 100 in which the number 'x' is deliberately varied - whole number, decimal and fraction. The aim was to assess whether the students would approach these questions in the same fashion or not - if not, what would the difference be?

The other questions listed test the same idea as the first three (x% of 100) but are more involved. Items (iv) and (v) are pictorial representatives of items (i) and (iii) respectively. Items (vi) and (vii) are similar to (i) except that the percent is to be found. Furthermore, item (vii) involves a percent greater than 100. Item (viii) involves the same idea as (vi) but is a word problem. Finally item (ix) assesses the basic idea that x% of 100 is always 'x'.

2. Notion of 100%:

A second idea regarding percents is that 100% represents an entire entity. The following items looked at the student's grasp of this particular notion.

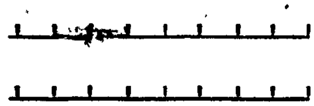
(i) Circle 100% of the squares below. (I;25)



(ii) There were 30 questions on a test. Mary got 100% of the questions correct. How many did she get correct? (I;13)

(iii) True or False?

100% of 80 is greater than 80. (I;18)

(iv)  A

B

A is % of B? (II;8)

(v) 87% of the students passed a test. What percent failed? (II;23)

The first three items are straight forward, merely requiring the students to know that 100% is the whole thing. However, these questions do differ from each other in that (i) is a pictorial representation of the idea; (ii) involves marks on a test and (iii) deals with 100% of a given number as opposed to 100% of an object.

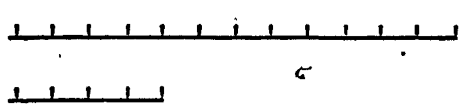
Question (iv) is again a pictorial representation of the idea under discussion; however, the student is given that two things are identical and must decide what percent one thing is of the other. Question (v) is quite different from the others in that it assesses the idea of 100% by requiring the complement.

3. <u>Percent Greater Than 100:</u>

The third idea is that a percent can be larger than 100. The items selected were designed to require little or no work.

(i) True or False?

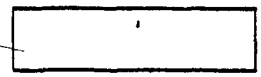
172% of 80 is greater than 80. (II;9)

(ii) A 

B

A is what % of B? (I;7)

(iii) A is 20% of B. B is what % of A? (II;16)

(iv) R is 

Draw 150% of R. (II;5)

Item (i) assesses whether the student has an intuitive feel for a percent larger than 100 - does he know that the answer must be greater than 80? In item (ii), line A is clearly three times the length of B - does the student see that A is 300% larger than B? Question (iii) is more involved than (ii) but assesses the same idea - if A is 1/5 of B, does the student see B as 500% of A. Finally, item (iv) assesses the student's ability to show pictorially 150% of a rectangular object.

The division of the first objective of the study - meaning of percent - into three subcategories is not meant to imply that this is all that is involved in understanding percent, but rather was a way to facilitate the investigation. All the items here were specifically designed to assess the student's understanding of percent in a content requiring little computation, if any. Other ideas involved in the notion of percent will be discussed later on.

(B) Mechanical Skills:

The items designed to assess the student's mechanical skills in working out percent problems involve equivalent forms and the three cases of percent.

1. <u>Equivalent Forms of a Number</u>:

The following items were used to assess the student's ability to change a number from its percent form to its decimal or fractional equivalent and vice versa.

(i) Write 128% as a fraction. (I;3)

(ii) Write 21% as a decimal. (II;7)

(iii) Write 112% as a decimal. (II;21)

(iv) Write 12,8% as a decimal. (II;26)

(v) Write 3 1/4% as a decimal. (II;24)

(vi) Which one is not equivalent to the rest?

(a) 1:4 (b) 1/4 (c) 25% (d) 0,25 (e) all are equivalent (I;10)

Item (i) involves changing a percent (larger than 100) to a fraction while items (ii) through (v) involve changing different percents to a decimal numeral. Finally item (vi) assesses the student's ability to identify equivalent numbers.

2. <u>Three Cases of Percent Problems</u>:

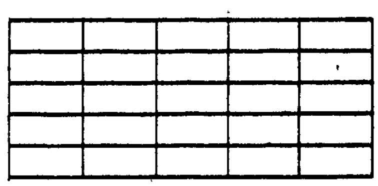
The items here assess the ability of students to deal with three cases of percent problems.

CASE I: <u>Find x% of N</u>

(i) What is 15% of 75? (I;2)

(ii) What is 120% of 60? (I;23)

(iii) Shade in 50% of the grid below: (I;21)



(iv) Circle 20% of the triangles below. (II;14)

Δ Δ Δ

Δ Δ

Δ Δ Δ Δ Δ

(v) Which is larger? 20% of 65 or 65% of 25? (II;22)

The five items test the same skill in different ways. Items (i) and (ii) are similar but item (ii) involves a percent larger than 100. Item (iii) is a pictorial representation of the other items in which the use of 50% should reveal something about the student's notion of percent. Item (iv) is another pictorial representation but involves a different percent and context. Finally item (v) is similar to (i) but the student has to compare the results.

CASE II: x is what percent of N

(i) 16 is what percent of 50? (II;4)

(ii) In a group of 40 people, 25 were on a diet. What percent was on a diet? (II;10)

Both these items test the same idea; however, the second question is a word problem, which has to be interpreted as (i) first.

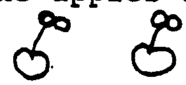
CASE III: x% of what number is N

(i) 80% of ____ = 8 (II;11)

(ii) 23% of ____ = 184 (I;11)

(iii) 80% of the books in a library were old. If 240 books were old, how many books were there altogether? (I;4)

(iv) 12% of the apples are shown below.



How many were there altogether? (I;20)

Items (i) and (ii) differ only in the level of difficulty due to the numbers involved. Item (iii) is a word problem. Item (iv) involves a pictorial representation.

In addition to mechanical skills, the above items also provide information on the student's knowldege of percent as well as fractions, decimals, ratio and proportion.

(C) Prerequisite Skills:

The test items here were designed specifically to assess the student's ability to handle fractions, decimals, ratio and proportion.

1. Fractions:

(i) 18/100 × 100 equals? (II;12)

(ii) 88/100 × 25 equals? (I;12)

The fractions chosen in the items above have denominator 100 so that if a student succeeds on these test items, his difficulty in solving a problem like 15% of 75 or 17% of 100 cannot be attributed to fractions.

2. Decimals:

(i) 0,45 × 100 equals? (I;8)

(ii) 0,12 × 55 equals? (II;6)

The rationale for these items on decimals is the same as that for fractions. Hence, if a student can change 3,8%

to a decimal, and he can also do items (i) and (ii) above, then his difficulty in solving a question like finding 3,8% of 20 is not due to difficulties with fractions and decimals.

3. Ratio and Proportion:

(i) 4:10 = x:75. Find x. (II;2)

(ii) 6,8:100 = x:4. Find x. (I;14)

(iii) 315:100 = x:20. Find x. (I;28)

(iv) 2:19 = 7:x. Find x. (II;27)

These four items involve ratio and setting up a proportion. However, the level of difficulty varies with the numbers involved. Work with the three basic percent problems discussed earlier usually involves solving such proportions. Hence, success (or failure) on these items provides a basis for analyzing the performance of students on the three cases of percent problems.

Listed below are other items that were designed to further study a student's concept of ratio and proportion.

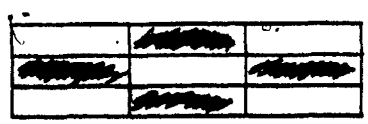
(v) John is 13 times as old as Ron. What is the ratio of John's age to Ron's? (II;13)

(vi) AAAA

BBBBBB

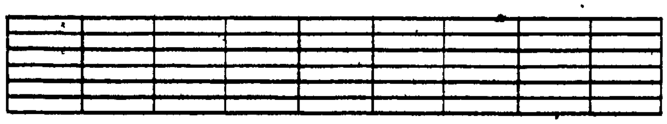
What is the ratio of A's to B's? (II;18)

(vii) What is the ratio of shaded rectangles to non-shaded in the figure below? (II;15)



(viii) If the ratio of shaded rectangles to the total number of rectangles is always as in the following

shade in the appropriate number of rectangles below. (I;26)



(ix) The ratio of students who passed a test to students who failed was 2:3. If there were 60 students in all, how many passed? (II;28)

(x) 4 out of every 5 boys likes sports. In a group of 45 boys, how many like sports? (I;22)

(xi) There is a leaky faucet in which 5 drops of water drip every minute. How many drops of water are there in one hour? (II;20)

(xii) If 2 shirts cost $35, how much would 7 shirts of the same type cost? (I;24)

(xiii) One pitcher of orange juice contains 6 glasses of water and 2 cans of orange concentrate. A second pitcher contains 4 glasses of water and 3 cans of orange concentrate. Which pitcher has the stronger orange taste? (I;16)

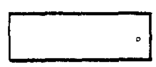
The above items were an attempt to present questions on ratio and proportion to the students in several different ways. Items (v) to (vii) should require no work. Items (viii) and (ix) are similar except that a picture is provided in (viii). Items (x) through (xii) can be solved with proportions. Item (xiii) is a 'typical' question on ratio and proportion but the numbers involved permit success on the problem even without any understanding of the concept.

Finally, the four items listed below were included in the tests but do not seem to fit into any one of the categories above.

(i) 87% + 50% equals? (II;19)

(ii) 3/4 + 20% equals? (I;15)

(iii) 4:5 = x:5 . Find x. (I;1)

(iv) This rectangle  is one-half a larger rectangle. Draw the larger rectangle. (I;9)

Items (i) and (ii) were used to see if students consider percents as individual numbers or only as x% of something. Item (iii) provided a simple example of ratio. Item (iv) was used to indicate if a student understood what was being asked in a similar item involving percent.

INDIVIDUAL INTERVIEWS

Although the written tests would provide a lot of preliminary information, it is only with the individual interviews that much of this quantitative data could be interpreted. Sometimes, what is written on paper will not give a clear picture of the student's comprehension. What may appear to be a 'silly' mistake may, in fact, be a hint of a lack of real understanding. Furthermore, even with correct answers, we can get a false picture of the student's breadth of understanding. According to Ginsburg (1981),

> "the test does not offer the kind of flexibility required for extensive exploration, for the immediate pursuit of interesting phenomena, and for the checking of intriguing hypothesis." (pp. 6)

Therefore, the personal interviews were conducted to give a more complete picture.

The clinical interview allowed for an intellectual discussion between the student and interviewer and the questions posed were dependent upon the student's responses. Hence conjectures could be tested and patterns of mathematical thought could be categorized. "When the aim is to identify structure by eliciting verbalizations, evaluating them, and checking alternative hypotheses, the clinical interview procedure (a) employs tasks which channel the subject's activity into particular areas; (b) it demands reflection; (c) the interviewer's questions are contingent on the child's response; (d) the interviewer employs basic features of the experimental method; and (e) some degree of standardization may be possible." (Ginsburg, 1981, pp. 7) Although the clinical method is not foolproof, it certainly provided a medium by which to explore the student's mathematical behaviour which otherwise would have been missed (with standardized tests only, for example). "Put simply, if you want to know what someone is thinking, ask him". (Ginsburg 1981, pp. 7)

Many of the questions in the interview were similar to the written test items but the ensuing discussions allowed for a deeper insight into the student's understanding. Items on the written tests which seemed to hint at certain general results were further investigated. Also, several items which were poorly worded on the tests were reworded and asked in the interview.

The interview itself consisted of two main parts. In the first part, each student was asked sixteen standard questions. These were presented (for the most part) to the student as if they had all been previously posed to other students. The student being interviewed was required to judge whether the answers were correct or not. This method of questioning was extremely helpful in starting the interview. Firstly, it standardized the interviews somewhat; secondly, it provided clues as to what the student seemed to grasp and what he did not grasp. Thus it was possible to formulate questions appropriately for the second part of the interview. In addition, this type of questioning seemed to calm the students down - the pressure was off them to provide answers, and they felt more comfortable criticizing or agreeing with the work of other students.

The second part of the interview consisted of, at the most, fourteen standard questions. However, here, the interview was much more flexible so that often one question would lead to a long discussion. Hence, not all fourteen questions were asked of each student; moreover the same question posed to different students often led to different discussions. When time permitted, the interviewer also asked the student some questions based on his own written tests.

Finally, a tactic used which proved useful was to ask a student to formulate his own question on a particular aspect of percent. The student's questions provided some useful information and were helpful in understanding aspects of the student's notions of percent.

The standard questions of the interview appear in Appendix II.

CHAPTER IV

PROCEDURE

This chapter describes the selection of the student sample, the administration of the tests and the interviews and the method of analysis of the data collected.

SELECTION OF STUDENTS

The sample of students was selected from two co-educational high schools in Montreal; a private school whose students come from middle and upper class homes, and the other, a typical public high school with students from a middle class background. From the private school, 85 students in 4 secondary I (grade 7) classes and 51 students in 4 secondary II (grade 8) classes were selected. The students in this school are streamed according to their mathematical ability. The top and bottom groups in each grade level were not used for the study. From the public school, 95 students in 5 secondary I classes and 21 students in 1 secondary II class were selected. In this school the students are not streamed. The classes chosen were those of the teachers willing to cooperate with this study.

ADMINISTRATION OF TESTS

The test items were organized into two tests - Test I and Test II, each consisting of 27 items. This was partly to take into account the length of a class period, the number of test items and to separate items of a similar nature. The tests were hand written and photocopies were made in order to have a set for each student.

The tests were administered during the month of February to secondary II students and during March to secondary I students. This was to ensure that the topic was covered completely in the schools before the testing began. The tests were administered by teachers in two separate sessions during the regular mathematics period of 45 and 50 minutes in the private and public school respectively. To ensure uniformity each teacher was given the same instructions. Test I was written by 231 students and test II by 239 students.

There were minor problems in the administration of the tests. A few students were present for only one of the tests. Some students came late to class or left early so that they did not complete the tests. There were also some problems with several test items. Items (I;9) and (II;5) required a ruler and were not done properly by students. Items 6, 12 and 17 on test II were cut off on many of the tests. Although teachers were provided with correction sheets, many students omitted these questions. Item (II;22) was meant to compare 20% of 65 with 65% of 20 and not with 65% of 25. Finally, item (I;16) on ratio proved to be a poor choice so that most students obtained the correct answer by chance. Where necessary, the correct versions of the test items were included in the interview.

INDIVIDUAL INTERVIEWS

Eighteen students were interviewed - twelve from the private school and six from the public school. These included ten boys and eight girls; ten of which were secondary I and eight of which were secondary II. Students from the private school were chosen in a random fashion - they were selected if they had a free period when the interviewer had a spare period as well or if they were in a class whose teacher did not mind excusing them. More organization was required for the interviews at the public school (as the researcher did not work there). Hence, the researcher provided a list of students to be interviewed to the teachers involved and students from this list who could be excused from class were chosen. The list was comprised by the researcher - students of various levels of understanding according to their written tests were interviewed.

Students did not know about the interview before hand in the public school. In the private school, sometimes a student was informed a period or two in advance and in rare instances, a day in advance (if he was to be interviewed first period in the morning).

The individual interviews began three to four weeks after the testing in order to minimize the effect of the latter and to allow the researcher an opportunity to undertake a preliminary analysis of the test results. The researcher interviewed all the students herself. The interviews were conducted in one of the spare classrooms in each school.

Each student was provided with a notebook for any rough work he wanted to do, a ruler and a pen. A taperecorder was used to tape each interview. The researcher sat next to the student and the taperecorder was placed behind them (out of sight). Each interview began with an introduction by the researcher of herself, what she intended to do, for what purpose, and that a taperecorder would be used. After any preliminary questions by the students were answered, the taperecorder was turned on and the interview began.

Each interview lasted approximately forty minutes. The students were generally communicative and willing to answer questions and solve problems. Naturally, there were some students who were nervous so time and effort had to be spent on relaxing them. Hence, they might have been presented with fewer questions than most students but this did not pose any real difficulties. A real attempt was made not to make any student feel 'stupid' - if a student was feeling this way, the interviewer moved on to a different form or aspect of questioning. In general, a relaxing atmosphere was achieved as jokes or idle 'chit-chat' were resorted to whenever the interviewer felt it was necessary to calm a student down. (Sometimes, this idle chit-chat just occurred naturally without any forethought on the part of the interviewer.)

ANALYSIS OF TESTS AND INTERVIEWS

This section describes the method used to analyze the written tests and the individual interviews.

WRITTEN TESTS

A preliminary analysis was performed on the written tests before the interviews were conducted. The written tests were carefully scrutinized for patterns of correct and incorrect items, methods of working out problems and types of errors.

A more detailed analysis of the test data was undertaken in stages: (1) calculation of the success rate on each item, (2) identification of the types of incorrect responses on each item and (3) comparison of the performance of secondary II students with the total population.

1. Calculation of Success Rates

For each item, the number of students who answered it correctly was calculated as a percentage by the formula:

$$\text{success rate} = \frac{\#\ \text{students answering correctly}}{\text{total population}} \times 100$$

The success rates of the combined population of secondary I and secondary II students on the twenty-seven items on each test are included in Appendix III.

2. Identification of Types of Incorrect Responses

Each item on the written tests was studied carefully to identify the type of common incorrect responses. The percent of students providing those incorrect responses was calculated. These results are included in Appendix IV.

3. Comparison of Secondary II Students with the Population

Separate success rates were calculated for secondary II students on each item. These were compared to the success rates of the total population (secondary I and II). The results are included in Appendix V.

INTERVIEWS

The analysis for the interviews was a qualitative one. Each interview was transcribed and then studied. The aim was to try to look for (1) evidence of correct or incorrect notions or percent, mechanical skills and prerequisite skills and (2) similarities amongst questions that students succeeded on as well as those that they had trouble with. Using these similarities as a basis for analysis, together with the general observations from the written tests, it was possible to arrive at some general conclusions together with specific representative exerpts of interviews to illustrate them.

CHAPTER V

FINDINGS AND OBSERVATIONS

In the sections that follow, the purpose is to pool together the information from the written tests and interviews to illustrate the performance of students on percent and to reveal some of the general misconceptions students seem to have about percents.

In the first part, the written tests will be examined. Each generalization about students' concepts regarding percents will be accompanied by appropriate test items that illustrate the specific observation being made.

In the second part, the same areas will be explored through the oral interviews. Selected exerpts will be provided to corroborate observations made form the written tests. In addition, new observations discovered through the interviews alone will be reported.

OBSERVATIONS FROM WRITTEN TESTS

Overall Performance

Test I and Test II were given to over 200 secondary I and secondary II students. Each test contained items on percent as well as on related aspects on fractions, decimals, ratio and proportion. The table below shows the range of success on the twenty-seven items on each test.

Percentage Range	TEST I (# of questions)	TEST II (# of questions)	TOTAL	%
76 - 100%	6	10	16	30
51 - 75%	8	9	17	31
26 - 50%	10	4	14	26
0 - 25%	3	4	7	13

This table shows that out of a possible 54 test items

i) only 16 items were answered correctly by more than 75% of the students; that is, students performed well on 30% of the items.

ii) only 33 items were answered correctly by more than 50% of the students; that is, students showed just average performance on 61% of the items. In fact, exactly half the total number of items in the two tests were answered correctly by more than 60% of the students tested.

The results thus show clearly that the overall performance of secondary school students on tests concerned with percent is quite low. This low performance becomes more significant when one considers three factors. Firstly, secondary II and in particular secondary I students had recently reviewed percent. Secondly, the test items had been deliberately designed to require little or no computation. Thirdly, the results do not reflect true understanding since a correct answer could sometimes be obtained by an incorrect method. This can be shown by comparing the success rate of similar questions as shown below.

(a) Item (II;14) | Item (I;2)

"Circle 20% of the triangles below."

Δ Δ Δ Δ Δ Δ Δ
Δ Δ Δ

success rate: 85,4%

"What is 15% of 75?"

success rate: 32,9%

Item (II;14) yielded a high success rate. However, this level of success is questionable since students obtained a low success rate on questions involving similar concepts as shown by item (I;2) which yielded a success rate of 32,9%. What could be the possible difference in the problems that could account for such a discrepancy on the respective rates of success?

Many of the incorrect responses for item (I;2) were the answers 5 arrived at by dividing 75 by 15 or 20 arrived at by incorrectly dividing 15 by 75. Further, if one divides 20 by 10 in item (II;14), the number 2, the correct answer, is obtained. This method gives a wrong answer in (I;2).

(b) Item (II;7)

"Write 21% as a decimal."

success rate: 88,7%

Item (II;21)

"Write 112% as a decimal."

success rate: 57,3%

The items assess the same idea except that (II;21) involves a percent larger than 100. The common response to the second question was 0,112. The students used a method which gave a correct answer in (II;7) but not in (II;21).

We shall now consider in some detail the performance of students on selected items involving prerequisite skills and percents in order to try and account for the overall poor performance of students.

1. Development of Prerequisite Skills:

The mere understanding of the concept of percent is not enough to be able to succeed with problems on percent. One must have the ability to work with fractions and decimals. Moreover, the ideas of ratio and proportion must be understood before one can truly have an understanding of percent. However, the written tests indicate that student performance in these skills was poor as illustrated by the examples below.

(a) Fractions:

Listed below are some of the items involving work with fractions together with their respective rates of success.

Number	Item	Success Rate
(II;12)	18/100 x 100 equals?	59,4
(I;12)	88/100 x 25 equals?	52,8
(I;15)	3/4 + 20% equals?	42,9

The success rates for items (II;12) and (I;12) are low for such seemingly easy problems. Appendix III also shows the wide range of answers provided for each item. Item (I;15) may have puzzled students. One incorrect answer was 23/14 obtained by adding 3/4 + 20/10; a second wrong answer was 23/104 obtained by adding 3/4 + 20/100. In both cases, students merely added numerators and denominators. Therefore, these three examples indicate hat secondary I and II students had difficulties with fractions.

(Parenthetically, it should be added that the intent of item (I;15) was not to test arithmetic skills, but rather to see how a student would deal with a percent as a number and not as a part of something.)

(b) Decimals:

The following table contains some of the questions that involved work with decimal numerals as well as their respective success rates.

Number	Item	Success Rate
(II;6)	0,12 × 55 equals?	65,7
(I;8)	0,045 × 100 equals?	60,0
(I;14)	6,8:100 = x:4 Find x	12,6

The success rates are rather low for secondary school students. It seems that students have difficulty with operations involving decimal numerals - they do not seem to know where to place the decimal point. For example, item (I;8) yielded answers of 4,5; 45; 0,45; 0,045; 4500 and 450. All of these have the same (correct) digits but placing the decimal point caused difficulties. Similarly, on item (I;14) the decimal point in the answer was placed in various positions: 0,272; 2,72; 27,2; etc.

Students also seem uncomfortable performing any operations that do not yield whole numbers and they have difficulty providing a decimal numeral as an answer. For example, on item (I;2), they had to find 15% of 75. The success rate was 32%. However, 8% of the students were able to provide some sort of answer between 11 and 12 - quite close to the correct answer of 11,25. Hence, they performed correct steps toward solving the problem but their arithmetic and working with decimals led to difficulties. This can also be seen on item (II;27) where approximately 7% of the students provided answers such as 66;66,4; 66,6; 67; etc. while the correct response was 66,5.

It seems that a general error in division with decimals arrives with the "remainder". For example, on item (II;10), students basically had to transfer 25/40 to a percent. The long division appeared as follows.

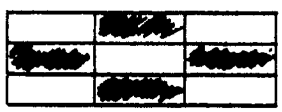
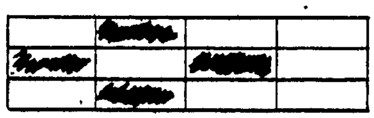
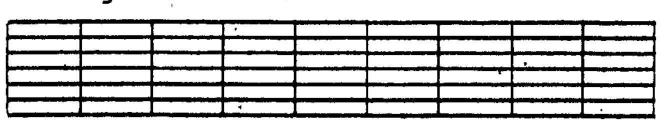
```
        .62
  40 )25.00
      24 0
       1 00
         80
         20
```

It is at this stage that some students have difficulty continuing - the problem is what does one do with the remainder of 20. Approximately 4% of the students then provided the answer 62.2%.

It should also be added that it is not always clear whether the incorrect response was due to carelessness or to poor skills. In item (II;6), the student was required to calculate 0,12 × 55. Some answers were 5,6 and 7,6 while the correct answer is 6,6.

(c) Ratio and Proportion

The notion of ratio seems to be poorly understood as illustrated by the examples that follow (see page 38) and their respective success rates.

Number	Item	Success Rate
(II;18)	AAAA BBBBBB What is the ratio of A's to B's?	85,4
(I;16)	One pitcher of orange juice contains 6 glasses of water and 2 cans of orange concentrate. A second pitcher contains 4 glasses of water and 3 cans of orange concentrate. Which pitcher has the stronger orange taste?	84,0
(II;20)	There is a leaky faucet in which 5 drops of water drip every minute. How many drops of water are there in one hour?	83,3
(I;22)	4 out of every 5 boys likes sports. In a group of 45 boys, how many like sports?	63,6
(II;15)	What is the ratio of shaded rectangles to non-shaded rectangles in the figure below. 	52,3
(I;24)	If 2 shirts cost $35, how much would 7 shirts of the same type cost?	47,2
(II;13)	John is 13 times as old as Ron. What is the ratio of John's age to Ron's?	39,7
(I;26)	If the ratio of shaded rectangles to the total number of rectangles is always as in the following  shade in the appropriate number of rectangles below. 	28,6
(II;28)	The ratio of students who passed a test to those who failed was 2:3. If there were 60 students in all, how many passed?	5,4

The success rates on the nine items range from 5,4 to 85,4.

(i) Items (II;18), (I;16) and (II;20) had the highest success rates. All three were also fairly obvious.

(ii) Items (I;26) and (II;28) appeared to be the most difficult.

(iii) The remaining items ranged from fair (39,7) to average (63,6).

It is difficult, however, to know in some cases whether the incorrect responses were due to carelessness and/or to poor arithmetic rather than to a lack of understanding on as to how to work with ratio. For example, in item (II;27), students were asked to find x given that 2:19 = 7:x. This question had a success rate of 22,6; however, one student had an incorrect response of 68,5. His work showed the possible understanding of ratio as he was calculating 19 × 3,5 and arrived at 68,5.

Percent problems are often categorized into three typical cases: (i) x% of N = ____ (ii) x = ____% of N (iii) x% of ____ = N. The test results show that many students try to solve percent problems by setting up a proportion. However, they do not seem to know where to place the numbers. Therefore, sometimes they will arrive at the correct response while at other times they will not. If one takes any one of these problems and places the given numbers in any of the four positions of a proportion, one can obtain each of the students' various responses. Presented below are several examples of 'typical' percent problems with the various answers and the student's work that led to these responses.

(i) "What is 120% of 60?" (I;23)

Responses Given	Work	Comment
7,2	$\frac{x}{60} = \frac{120}{1000}$	(note the 1000)
36	$\frac{120}{200} = \frac{}{60}$	(note the 200)
24	$\frac{x}{60} = \frac{120}{100}$	
50	$\frac{60}{120} = \frac{x}{100}$	(note the use of $\frac{x}{100}$)

(ii) "23% of _____ = 184" (I;11)

Response Given	Work	Comment
42,32	$\frac{23}{100} = \frac{x}{184}$	(some students answer-ed 4232)
800	$\frac{23}{184} = \frac{100}{x}$	

(iii) "16 is what percent of 50?" (II;4)

Response Given	Work
32	$\frac{16}{50} = \frac{x}{100}$
8	$\frac{16}{100} = \frac{x}{50}$

(iv) "What is 15% of 75?" (I;2)

Response Given	Work
20	$\frac{15}{75} = \frac{x}{100}$ or $\frac{15}{x} = \frac{75}{100}$
11,25	$\frac{15}{100} = \frac{x}{75}$

Once a student has constructed the proportion (correctly or not), his work solving it showed difficulties with other areas. Mulitplication and division with fractions usually involves cancelling. Some students solve a proportion of the form a/b = c/d by cancelling 'a' with 'd' for example, indicating trouble with fractions.

Work on other examples also demonstrated a further lack of understanding of ratio and proportion. For example, when students were asked to solve for x given that 2:19 = 7:x, one incorrect response was 22,5 arrived at by adding 19 + 3,5.. Another response was 24, obtained by adding 19 + 5. (Note: 5 = 7 - 2). In short, the examples listed above suggest strongly that the notions of ratio and proportion are not well understood by most students.

2. Changing Percents to Fractions and Decimals (and vice versa):

The examples listed below suggest that students have difficulties in changing percents to decimal numerals.

Number	Item	Success Rate	Incorrect Responses
(II;7)	Write 21% as a decimal	88,7	
(II;21)	Write 112% as a decimal	57,3	0,112

There is a large discrepancy in the success rate on these similar items. The students tend to change a percent to a decimal by the method of moving the decimal point to the left of the digits. This method need not be based on any real understanding but it does yield a correct answer for any two digit percent. However, the method fails in other cases - for example, for a percent larger than 100 like 112. In fact, it leads to 0,112 which was the common incorrect response for item (II;21).

Changing percents involving fractions and decimals to decimal numerals is even more difficult for students. The two examples below, together with their respective success rates and common incorrect responses illlustrate this.

Number	Item	Success Rate	Incorrect Responses
(II;24)	Write 3 1/4% as a decimal	16,7	3,25; 0,325; ,03 1/4
(II;26)	Write 12,8% as a decimal	28,6	12,8; 1,28

Students will correctly change a two digit percent to a fraction simply by placing the number 100 under the digits: e.g. 28% becomes 28/100. However, it seems that they are under the misconception that one is to place 100, 1000, 10000 etc. depending on the number of digits. That is, one places two, three or more zeroes depending on the number of digits in the given percent. For example, 128% = 128/1000 while 28% = 28/100. Therefore, once again, arriving at the correct response in the second example does not necessarily imply understanding or even correct mastery of the skill.

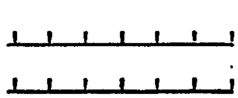
It is worth noting that some students will write 128% as 128/200, using 200 probably because 128 is larger than 100.

3. Use of the Number 100 in Solving Percent Problems

The number 100 is implicitly a part of all percentage problems and it seems that students are well aware of this fact. They will use the number 100 when working with percents even if the number is not explicitly given in the problem. Unfortunately, although they incorporate the number 100 in their work, it is often used incorrectly. For example, in item (II;14), students were presented with ten triangles and asked to shade 20% of the triangles. This question, along with its highly suspect success rate, has previously been discussed. However, approximately 8% of the students provided the answer of 5, arrived by dividing 100 by 20. Another illustration of this phenomenon can be seen in item (I;20) which states: "12% of the apples are shown below.  How many were there altogether?" Approximately 6% of the students provided the answers 8; 8,4 and 8,3 which are arrived at by dividing 100 by 12.

4. <u>Concept of 100% as a Whole</u>:

One concept that seems to be understood is the idea that 100% represents a whole. Examples of items testing the understanding of this idea, together with their respective success rates are listed below. These success rates are very high indicating that even the poorest students seem to have grasped this concept.

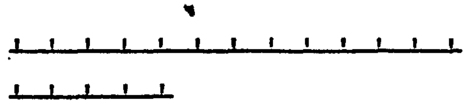
Number	Item	Success Rate
(I;25)	Circle 100% of the squares below. □ □ □ □ □ □ □ □ □	92,6%
(I;13)	There were 30 questions on a test. Mary got 100% of the questions correct. How many did she get correct?	90,0%
(II;23)	87% of the students passed a test. What % failed?	86,2%
(I;18)	True or false? 100% of 80 is greater than 80.	83,1%
(II;8)	 A B A is ____% of B	82,0%

Although it seems safe to say that most students understand this notion of 100%, in other instances this conclusion is questionable. For example, in item (II;16) ("A is 20% of B; B is what percent of A?"), approximately 6% of the students gave 100 as the answer even though some of these students probably succeeded on the above questions.

5. Percents Larger than 100:

It appears that the general acceptance of the idea that 100% is a whole leads to difficulties with a percent larger than 100%. This should not be surprising. What possible meaning could 110% of something have, for example, for a student who firmly believes that 100% is the total, the whole object?

Items portraying the difficulty with this concept are discussed below.

Number	Item	Success Rate	Incorrect Responses
(I;23)	What is 120% of 60?	31,2	72; 90; 50; ½ ;50%
(I;5)	127 = ____% of 100	46,8	27.
(I;7)	A B  A is what percent of B	13,4	300; $\frac{1}{3}$; $33\frac{1}{3}$; 33,3; 260, etc.

The success rates above are low regardless of the way the question is asked, the content or the numbers that are used. The range of incorrect responses suggests further that the students do not even have an intuitive feel for a percent larger than 100 - these responses in fact are often smaller than 100% or smaller than the original number.

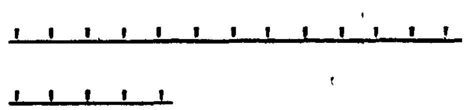
6. Use of Basic Arithmetic Operations in Solving Percent Problems:

The most unexpected observation made was the rampant use of basic arithmetic operations in solving percent problems. Although all of the observations discussed above were generally quite common, none was as widespread as the use of written arithmetic computation in the problems.

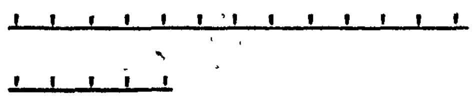
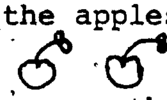
The following three specific observations emerged from a careful scrutiny of the test papers: (1) Some students will use one operation consistently for any percent problem while others will use the arithmetic operation under centain conditions. For example, some students will consistently perform the operation of division for a particular type of problem while others will perform the division only if the numbers divide evenly. Otherwise, they will resort to using other means to solve the problem. (2) The students, including the best students, will resort to written multiplication or division to solve a problem if they have difficulty with it. The tendency to resort to written computation tends to increase with the items students have difficulty with and also with the presence of numbers that divide evenly. (3) Although division was the most common type of arithmetic operation resorted to, followed by multiplication, some students added and subtracted the given numbers in a problem. Many of the incorrect responses provided by students were indeed obtained by performing some arithmetic operation with the given numbers in the problems. The computations were often correct but of course invalid for the problems at hand. The examples below best illustrate these observations.

1. Use of a Single Operation:

The items listed below are all from one student's work. They show that this student uses the operation of division when working out percent problems. (The student's work was clearly shown.)

Number	Item	Answer	How Obtained
(I;2)	What is 15% of 75?	5	75 ÷ 15
(I;7)	A B  A is what % of B?	23	13 ÷ 5
(I;11)	23% of ____ = 184	62	184 ÷ 23

The items below are all from one student's work as well. They show that this student uses the operation of subtraction.

Number	Item	Answer	How Obtained
(I;2)	What is 15% of 75?	60	75 - 15
(I;7)	A  B A is what % of B?	8%	13 - 5
(I;11)	23% of _____ = 184	16	184 - 23
(I;19)	32 is _____ % of 100	68	100 - 32
(I;20)	12% of the apples are shown below.  How many were there altogether?	88	100 - 12

2. <u>Tendency to Compute</u> (multiplication and/or division):

Number	Item	Success Rate	Some Incorrect Responses	How Obtained
(I;2)	What is 15% of 75?	32,9	5;5%	75 ÷ 15
(II;1)	What is 17% of 100?	87,0	5;5,8	100 ÷ 17
(I;20)	12% of the apples are shown below.  How many were there altogether?	15,2	24 8;8,3;8,4 6	12 × 2 100 ÷ 12 12 ÷ 2
(II;11)	80% of _____ = 8	69,0	10 640;64;6,4	80 ÷ 8 8 × 80

On item (I;2), 20% of the students provided the answer of 5, obtained by division of the numbers in the problem. It is interesting of note that although item (II;1) had a high success rate, the single most common incorrect response was a number between 5 and 6, obtained by dividing 100 by 17.

Items (I;20) and (II;11) show the use of multiplication as one of the operations in addition to division. (In item (II;11) several incorrect responses have the same digits of 6 and 4 although they differ somewhat. It seems that these answers were arrived at by some multiplication of 8 and 80.)

3. Use of Different Operations:

Number	Item	Success Rate	Some Incorrect Responses	How Obtained
(II;1)	What is 17% of 100?	87,0	83	100 - 17
(II;16)	A is 20% of B B is what % of A?	9,6	80 120	100 - 20 100 + 20
(I;6)	70% of the children in the park like to play. If 70 children like to play, how many are in the park?	54,5	30 49;490;4900	100 - 70 70 × 7

The items listed above show subtraction and addition being used in addition to multiplication and division.

An excellent example of the rampant use of random arithmetic operations is provided by item (I;4) cited below. "80% of the books in a library were old. If 240 books were old, how many were there altogether?" Incorrect responses, together with the ways of arriving at these answers are included in the table below.

Response	Arithmetic Performed
320	240 + 80
480	240 × 20 (Note: 20 = 100 - 80)
460	2 × 240 - 20
192	240 × 0,8
270	240 + 30 (30 is obtained by incorrectly dividing 240 by 80)
432	240 + 192
252	240 + 12 (12 = 240 ÷ 20)
243	240 + 3 (3 = 240 ÷ 80)

As this problem was extremely difficult for the students (a success rate of 35,5%), they resorted to the seemingly comfortable use of arithmetic. This problem shows the students using all operations and some were using a combination of several types of operations.

7. Questions with a High Success Rate:

There were items on which the students did very well. The success rate was greater than 75% on 16 items and between 60 and 74% on another 11 items. Some reference has already been made to several of these items. The purpose here is to consider them separately as a group.

a) <u>Equivalent Forms</u>:

Number	Item	Success Rate
(I;10)	Which one is not equivalent to the rest? (a) 1:4 (b) 1/4 (c) 25% (d) 0,25 (e) all are equivalent	70,0
(I;3)	Write 128% as a fraction	69,7
(II;7)	Write 21% as a decimal	88,7

Item (I;10) suggests that many students were at least able to compare a simple fraction, decimal, ratio and percent. The item is suspect since it is possible that some students might have guessed. However, the high success rate on item (I;3) and a reasonable success rate on item (II;7) seem to suggest that many students are at least able to obtain equivalent forms. However, the high success rate on item (II;7) is suspect as was discussed in section 2 above.

b) <u>Ratio and Proportion</u>:

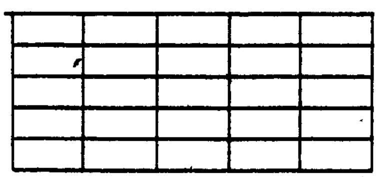
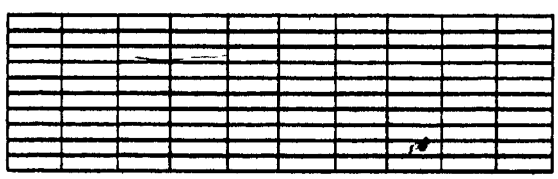
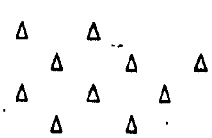
Number	Item	Success Rate
(I;1)	4:5 = x:5 Find x	84,4
(II;18)	AAAA BBBBBB What is the ratio of A's to B's?	85,4
(II;2)	4:100 = x:75 Find x	67,0

The first two items show a high success rate above 80% which suggests that students have some knowledge of ratio in contents involving small numbers and pictorial representations. The third item with a success rate of 67% also suggests the view that more than half the students have some elementary knowledge of ratio.

C) <u>Concept of 100% as a Whole:</u>

It has already been shown earlier in section 4 that the concept of 100% is well understood by students.

d) <u>The Case: x% of N:</u>

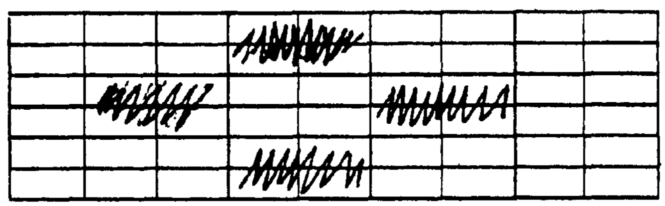
Number	Item	Success Rate
(I;2)	What is 15% of 75?	87,0
(I;21)	Shade in 50% of the grid below. 	90,9
(II;17)	Shade in 7% of the grid below. 	80,8
(II;14)	Circle 20% of the triangles below. 	85,4

The high success rates on the above items involving different contents indicate that the students understand the concept of x% of N where 'x' is a one or two digit number.

Although the performance by the students on the above items demonstrates some knowledge, it is worth noting that it can sometimes be a source of trouble. This has already been alluded to in the previous discussion in sections 1 through 6 and so we will only consider it briefly here.

i) It was shown in section 2 that students have trouble with some equivalent forms: e.g. changing percents to decimal equivalent forms: 112% and 3 1/4%. That is, their success is limited to one and two digit percents.

ii) Although students had a high success rate on item (I;10), it seems that idea that 1:4 and 1/4 are equivalent could pose a problem. There is a difference in the idea of 1/4 and the idea of a ratio 1:4. It seems that students do not differentiate between the possible interpretations of a ratio - the ratio a:b could have 'a' as a part of 'b', or both 'a' and 'b' as two distinct parts of one entity. Specifically, in item(II;28) the low success rate of 5,4% arises because students seem to treat the problem as if it states 2/3 and arrived at the incorrect answer of 40. Item (I;26), although involving a similar concept to item (II;28), had a slightly higher success rate. This could possibly be due to the fact that the students had a concrete figure to work with. However, 14% of the students gave the answer of 16. The diagram below shows how the 16 rectangles were shaded. It seems that students merely tried to reproduce the given pattern.

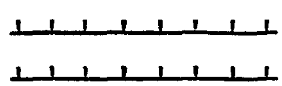


iii) It was shown in sections 3 and 4 that students seem to know that the number 100 is involved in percent problems and that 100% represents a whole. Again, both these ideas lead to trouble. It was shown that students tried to divide 100 by other numbers in the problem - especially when the number divided evenly. Finally, the notion of 100% tends to hinder students from accepting percents greater than 100.

In other words, percent, like other mathematics topics, contains hidden subtleties for the unwary student. Mathematical knowledge that is narrow and based on a shaky foundation is deceptive since it often leads to spurious generalizations.

8. Comparison of Secondary II Students with Total Population:

Appendix V lists the success rates for each item for secondary II students with the corresponding success rates for the total population (secondary I and II). Items with a difference of more than 8% will presently be looked at and discussed.

Number	Item	Success Rates Total Population	 Secondary II
(II;8)	 A B A is ___ % of B?	82	91
(II;14)	Circle 20% of the triangles below. Δ Δ Δ Δ Δ Δ Δ Δ Δ Δ	85	95
(II;18)	AAAA BBBBBB What is the ratio of A'S to B's?	85	97
(II;20)	There is a leaky faucet in which 5 drops of water drip every minute. How many drops of water are there in one hour?	83	94
(II;23)	87% of the students passed a test. What percent failed?	86	97

The above items have already been discussed in section 7 as items with a high success rate. The results show that secondary II students performed approximately 10% higher than the general population on these items.

Listed below are another group of items in which the secondary II students performed about 10% higher than the total population. However, these items are poorly done by most students.

Number	Item	Success Rate Total Population	Secondary II
(II;25)	What is 1/2% of 100?	30	43
(I;27)	6,2% of 100 equals?	33	45
(II;10)	In a group of 40 people, 25 were on a diet. What percent was on a diet?	36	51

The table below lists items which had fair results by the total population (average: 52%) yet were performed considerably better by the secondary II students (average: 64%).

Number	Item	Success Rate Total Population	Success Rate Secondary II
(I;5)	127 = _____% of 100	47	59
(I;15)	3/4 + 20% equals?	43	52
(I;8)	0,045 × 100 equals?	60	70
(I;28)	315:100 = x:20 Find x	49	64
(I;22)	4 out of every 5 boys likes sports. In a group of 45 boys, how many like sports?	64	78
(I;24)	If 2 shirts cost $35, how much would 7 shirts of the same type cost?	47	61

The probable cause of improvement in performance on items (I;8) and (I;28) is that the students are required to work out questions of these types in secondary II science and hence get more practice with them.

Items (I;22) and (I;24) also showed a higher success rate by the secondary II students. Is it possible that the concept of ratio is beginning to be grasped at some level by secondary II students?

Listed below are the items on which the secondary II students performed worse than the general population.

Number	Item	Success Rate Total Population	Success Rate Secondary II
(I;2)	What is 15% of 75?	33	23
(I;3)	Write 128% as a fraction.	70	59
(II;21)	Write 112% as a decimal	57	42

Since students seem to perform these type of questions mechanically without any real understanding, it would seem that secondary II students would be at a disadvantage. They forget the 'method' as a longer period of time has passed since they have studied percent (compared to secondary I students).

Two other items which showed a considerable difference between secondary II students and the total population were items (II;6) and (II;12). As was mentioned earlier, these questions were cut off on some papers and therefore many students left them blank. None of the secondary II papers had these questions missing and hence very few were left blank. Therefore, little information can be obtained in terms of a comparison.

A SHORT SUMMARY

A population of over 200 students was required to answer 54 questions on items involving percent and its related topics of fractions, decimals, ratio and proportion. The variety of responses and the wide range of success rates encompasses a wealth of information. Some of the major findings have been discussed above and are now summarized below.

A. The overall performance of students is low indicating a poor understanding of percent and the topics associated with it.

i) Although students seem to associate the word 'percent' and the symbol '%' with the number 100, what exactly that relationship entails is not clear to them.

ii) The concept of 100% as an entire entity seems to be accepted and understood by a majority of students.

iii) Percents larger than 100 are poorly understood and the students do not seem to have any intuitive feel for these numbers at all.

iv) Students will resort to basic arithmetic operations when solving percent problems. This tendency seems to increase with the level of difficulty of the problem. They may be consistent in the operation being used and in the instances these operations are employed or they may approach similar problems differently depending upon the particular numbers in the given problem.

B. Mechanical skills necessary to work with percents are poorly developed.

i) Students can only perform conversions between fractions, decimals, ratio and percent when simple numbers are involved. However, the general facility to change any number from one form to another is lacking.

ii) Students can perform the basic problem of finding x% of N if one or two digit numbers are involved.

C. The prerequisite skills necessary for success with percent problems - facility with fractions, decimals, ratio and proportion - are underdeveloped. The lack of these skills hinders many students in their work with percents. For example, the poor understanding of the concepts of ratio and proportion leads to difficulty in solving the three basic types of percent problems.

One disadvantage of attempting to assess a student's understanding from his test paper is that the written responses in themselves tell us very little. It becomes necessary to look for patterns from which to make inferences about the thought process used by the student. Such inferences are not always correct. For example, on the ratio item (II;27) students were asked to find x given that 2:19 = 7:x. The success rate was 22,6%. One student had an incorrect response of 68,5, although his calculation was

19 × 3,5. Now, 19 × 3,5 = 66,5 which is the correct response. Was the incorrect response here due to carelessness, poor arithmetic skills or just a lack of understanding of ratio? Secondly, it is often difficult to explain from written tests only why there is a discrepancy between the results of two similar items. For example, item (I;26) involved ratio using rectangles while item (II;28) was a word problem on ratio. The success rate for the former item was 28,6 and for the latter 5,4. How can one account for the discrepancy between these results?

Hence, the findings from the two written tests only allow us to generalize that the overall performance of the students was low and probably reflects a poor understanding of percent and related topics.

OBSERVATIONS FROM INTERVIEWS:

The individual interviews of eighteen secondary I and II students involved two basic techniques. Certain questions were merely presented to the students and they were asked to make judgments on other students' (hypothetical) responses; at other times, the students being interviewed were required to solve the problems themselves. The interview questions involved decimals, ratios, proportions and percents.

The discussion below will deal with three aspects: (1) prerequisite skills, (2) mechanical skills and (3) percent. In each case, the discussion will be based on exerpts from the interviews. Partly because of space limitations, only a few exerpts selected from individual interviews will be used. These exerpts have been chosen for two reasons - they clearly reflect the aspect being discussed and they are fairly representative of the responses in the other interviews. In using the exerpts, an attempt has been made in various ways to recreate relevant features of the interviews, and in particular the student's responses: significant portions of the student's responses are underlined; a long pause by the student is indicated by either (pause...) or just ... (e.g. well...); comments on the student's gestures are occasionally used for emphasis as are the signs '?' and '!' (e.g. that's wrong!); square brackets are used to indicate what the interviewer or the student wrote down (e.g. [1/4 = 0,25]), or to indicate what is being referred to (e.g. a student says: "This here is 1/2."

[points at 50%]). Unless otherwise indicated, numbers are used rather than the corresponding word equivalent (e.g. "I think 50% of 78 is 39.") In addition, it should be noted that the exerpts are exact quotes and not summaries of the discussions.

1. Development of Prerequistie Skills:

It was shown in the written tests that the performance of students on fractions, decimals, ratio and proportion was quite poor. The interviews provide further evidence of this and also illustrate some of the incorrect notions students have.

It was observed in the written tests that students did not use fractions in working out percent problems. To save time, the interview did not focus specifically on a student's ability to handle fractions. It was felt that a student's use of fractions would arise in the discussion on decimals, ratio, proportion and percent.

(a) Decimals

The written tests indicated that when students divide a number by 100 or 1000, they have trouble with the position of the decimal point. A similar difficulty arose in multiplication. It was surprising to see that some students would insert rows of zeroes when performing such multiplication. For example, 1,63 × 100 would be done as follows:

```
  1,63
  100
  000
 000
163
163,00
```

Difficulty in multiplying decimal numerals by 100 is illustrated below in the interviews with Diane (D), Lainie (L) and Nancy (N).

(i) Diane

Diane seems to believe that one merely adds two zeroes to the end of a number.

I: OK. Another question. Let's turn the page. 1,63 × 100 . The answer I got was 163. Was that right?

D: No.

I: What would it be?

D: 1,63...sixty-three thousand... 1,63 double zero [1,6300] .

I: OK. What if the question said: 0,023 × 100?

D: um...0,023 double zero [0,02300]

I: So what method are you using to do these?

D: Multiplying

I: How do you get the answer without doing it?

D: You just add two zeroes.

(ii) Lainie

Lainie seems to suspect that one is supposed to move a decimal point but she doesn't know how many places to move.

I: Let's say you had to do 1,63 × 100 and the answer

I got was 163. Do you agree?

L: No! [with certainty]

I: OK. What would the answer be?

L: 16,3

I: 16,3. How did you get that?

L: Because when it's 100, you move the decimal one place. [1,63 to 16,3]

I: What if it was 0,023 × 100?

L: OK. It's 0,23.

Diane and Lainie illustrate responses that were quite common among the interviews. In a few cases, students demonstrated individual methods which showed little knowledge and some guessing. Nancy's interview below is a typical example.

(iii) Nancy

Nancy seems to believe, apparently without much conviction, that 0,023 × 100, for example, is just 123.

I: Here's another one I asked them to do: 1,63 × 100 and they gave me the answer 163. Do you agree?

N: ...Ya.

I: OK. What if the question was 0,023 × 100?

N: What would the answer be? um...123

I: Where is the one coming from?

N: From here. [pointing at the one in 100]

I: OK. 123. What if it was 3,4 × 100?

N: um...13,4.

I: OK. So what is the method you are using in order to get your answer? Is it the same method for each example?

N: Ya

I: OK. So what is it that you do?

N: I just add this on like I just ...um...123 like this is 100 so there are 3 places to the 100 so I add the 23 here and it goes and this here I don't know...I just guessed.

(b) Ratio and Proportion

The interviews confirmed the finding in the written tests that students as a whole had a poor understanding of ratio and proportion. Even the brightest student has trouble with problems involving ratio and while he may perform better on ratio problems than most students do, when probed enough in an interview, he will usually show some lack of understanding.

In the discussion below, exerpts are used to illustrate three general types of difficulties encountered by students: Type I - basic idea of ratio; Type II - ratio as a fraction; Type III - solving a proportion.

TYPE I: Basic Idea of Ratio

The interviews with David and Richard indicate that students sometimes think that one cannot determine a ratio of two things if they are both unknown quantities even if one knows the relationship between the two amounts.

(i) David

Even when several examples led to the same response, David claimed that it was mere coincidence and that, in general, one cannot determine the ratio without knowing at least one of the two quantities.

I: Michael's age is two-fifths [2/5] of his mother's age. I asked them to write the ratio of Michael's age to his mother's age and some students said it couldn't be done because you didn't know their ages.

D: That's right. You would have to know one of their ages to figure it out. If you told me Michael's mother's age is 25, you would be able to figure out that Michael is 10.

I: Then what would the ratio of Michael's age to his mother's age be?

D: The ratio would be 2:5. I'm sorry 10:25.

I: Why did you say 2:5?

D: Well, reduce it, right? Hopefully? No, you could reduce it but, um...

I: What's 10:25 reduce to?

D: Reduce it...10:25. Do it this way...2:5...ya

I: OK. The ratio would be 2:5.

D: 2:5.

I: OK.

D: So I was right the first time!

I: Right, that's the same thing. I told you that Michael's age equals 2/5 his mother's age, right, and the ratio you got at the end was 2:5.

D: 2:5. But that's only if you know one of their ages. I just chose 25 because it was easier to do it with. But if you would give me a different age, it would be a different answer.

I: What about 50? Let's say the mother is 50.

D: He would be 20 which would be the same - same thing reduced.

I: Right.

D: It's not working.

I: Are we just picking numbers that make it work out always to 2:5?

D: Ya, well...

I: Because if I said 47..

D: Ya, it would be different.

(ii) Richard

Richard was asked the same question and he too had difficulty with it as is shown in his exerpt below.

I: They said that you couldn't do it because you don't know their ages.

R: No, not using x ?

I: I don't know. I mean can you write the ratio at all?

R: Uh...no, you can't. Michael's age is 2/5 of his mother's age so you could possibly say, Michael's age is 2/5 to, if you knew his mother's age, then maybe you would be able to write 2/5...uh, dot dot [:] to his mother's age. But if you don't know his mother's age, you can't.

I: What if...

R: Nothing to compare

I: What if you were able to use x, then what would you do?

R: 2/5 compared to x

TYPE II: Ratio and Fractions

The interviews with Barbara, Richard, Brian and Lee suggest that the concept of ratio of two things is seen more in terms of the familiar idea of a fraction. In other words, the ratio of a:b is seen as a/b even if a and b are distinct parts of a total quantity.

(i) Barbara

Barbara illustrates the view that if the ratio of girls to boys is 3:7, than 3/7 are girls and 7/3 are boys.

I: The ratio of girls to boys in a school is 3:7.
If I have 210 children, how many would be girls?

B: 90

I: How did you get 90?

B: 7 into 210 is 30 and I just times 30 by 3

I: What fraction of the total are girls?

B: What fraction? 3/7.

I: What fraction are boys?

B: 7/3

I: What does the ratio 3:7 girls to boys mean to you?

B: For every 7 boys, there are 3 girls.

I: OK. What is the minimum number of people you see in front of you to see that ratio?

B: All you need is 3 girls and 7 boys - 10 people.

I: What fraction are girls?

B: 3/7

I: Of 10 people, you mean

B: I don't know... Ya, 3/7

(ii) Richard

Richard responds in a similar fashion as Barbara although he is less confident about his answers.

R: 3:7 girls to boys. So that was 3 girls to 7 boys

I: Ah ha

R: How many were girls?

I: Ya

R: OK. ..uh..Let me work this out. 3 over 7 is what of 100? Uh...How do I work this out? 3...3 into ... [work: $\frac{3}{7} = \frac{}{100}$] . So I would keep on going ... [long pause]
Uh. I don't know how I would do this. I may say it would be 160 girls.

I: 160 girls. OK. How do you get 160?

R: Uh...mainly through a guess, but...

I: OK. Well, let me ask you a question. What was the basis of your guess?

R: The basis of my guess. Well, I knew there would be more boys than girls and I took it that ...that for every 3 girls there were 4 more boys

I: OK

R: So, 7 into 210 is 30. No, I would say there were 90 girls.

I: OK. Is that a guess or is that more educated?

R: That's a more educated guess.

I: How would you get 90?

R: Because 7 goes into 210 thirty times. So I would say 30 × 3 would be 90

I: OK.

R: I wouldn't be 100% sure. I wouldn't bet my life on that.

I: I'm not asking you to bet your life on that. There aren't many things that you'd bet your life on. What fraction of the group would be girls?

R: Fraction of the group?

I: Ya, like I don't know, 1/2, 1/4, 3/9

R: 3/7

I: 3/7 would be girls and what fraction of the group would be boys?

R: Uh, would be boys

I: Ya

R: 7/3

(iii) Brian

Brian performed very well on his tests and showed a good understanding of the basic ideas in his interview as well. On item (II;28), Brian was one of the few students to provide a correct response. The question was: "The ratio of students who passed a test to students who failed was 2:3. If there were 60 students in all, how many passed?" However, the concept of ratio still posed difficulties for him. For instance, he was shown his answer to item (II;28) and the interview continued as follows:

I: Can you explain how you got 24?

B: [pause]

I: What does it mean if the ratio of students who passed to students who failed was 2:3?

B: 2 people passed and 3 people failed

I: OK. So out of a total of 60, how many passed. How would you work that out? By the way, just because I'm asking, I'm not saying it is wrong. There's no work there [on test paper] so I just want to know how you got that answer.

B: [pause]

I: If I asked you to do it now, would you get the same answer?

B: [long pause] I don't know.

I: What fraction of the total would have passed?

B: 2/3

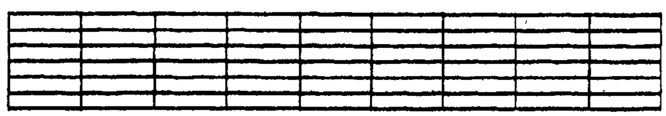
I: And what fraction would have failed?

B: <u>um...3/2?</u>

For whatever reason, Brian seemed unable to handle the question. In fact, his responses here reflect the common incorrect response of 40 in the written test obtained by calculating 2/3 of 60!

Another item similar to (II;28) but based on a diagram was (I;26): "If the ratio of shaded rectangles to the total number of rectangles is always as in the following,

shade in the appropriate number of rectangles below.



The success rates of students on (I;26) and (II;28) were 30% and 8% respectively. Although both success rates are low, students did perform somewhat better on the first question. However when item (I;26) was asked in the interview, it was found that many students who succeeded with this question did not use ratio. Through various methods and the aid of the diagram, they were at least able to obtain the correct answer.

(iv) <u>Lee</u>

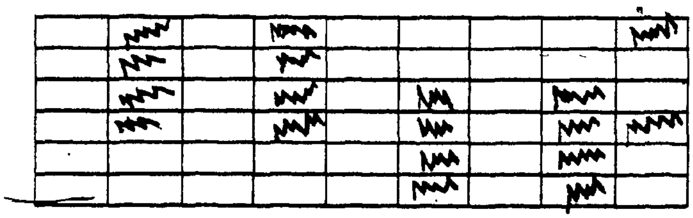
Lee's exerpt is but one example of the above. He fills in the rectangles by counting them - for every 8 not shaded, he shades 4. When he is left with 3 rectangles, he shades 2 and omits 1.

I: How many did you shade?

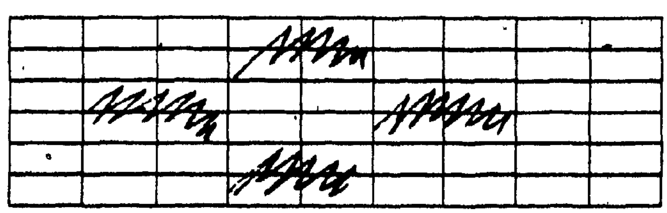
L: 18

I: How did you get that?

L: OK. I didn't know that it wasn't going to be even so I thought it would be like this: so 8:4 - here's 8 and here's 4 [leaves 8 out and shades in 4 and keeps on doing this process]; here's 8 and here's 4; here's 8 and here's 4 and then that's equal to 2:1 and since I only had enough room [only 3 left and not 12] here's 2:1 [leaves out 2 and shades 1] .



Lee's work is also more interesting in light of the way that many students arrived at the common incorrect response of 16. These students merely tried to fit the given pattern into the empty grid as shown below.



Furthermore, students who attempted to work this example out arithmetically, used the fraction 4/8 and not 4/12 just as they would use 3/7 and not 3/10 in the ratio of girls to boys discussed above.

One might be led to think that Lee's method above indicates an understanding of the concept of ratio. However, when presented with the same question as Barbara, Richard and Brian above, his performance was identical to theirs. His interview is shown below:

I: What fraction of the group is girls if the ratio is 3:7?

L: [pause]

I: Like are 1/2 of them girls? 1/4 of them? What fraction are girls?

L: <u>3/7</u>

I: What fraction would be boys?

L: <u>7/3</u>

<u>TYPE III</u>: <u>Solving a Proportion</u>

One aspect of ratio which students appear not to have grasped is the multiplicative concept involved in a proportion. This is illustrated clearly by the responses of Debbie and Karen to the following question: "Two pitchers of orange juice are made with orange concentrate. One pitcher has 3 cans of orange concentrate and 2 cans of water. The other has 7 cans of the orange and 5 cans of the water. Which of these two is going to taste more orangy?"

(i) <u>Debbie R.</u>

D: This would taste more orangy - 7 cans of orange juice.

I: OK. Why?

D: Because there's still 2 cans more orange juice than there is water while here there's only one more can [pointing to the 3 cans of orange]

ii) Karen

K: This one [indicating the second pitcher].

I: Why?

K: Because when you figure it out, this one [first pitcher] only has one more can of orange juice tnan water from these two [pitchers]

I: OK. What if I had this: 9 cans of orange and 8 cans of water?

K: That would be the same consistance [as the 3:2]

I: These two would be the same [9:8 and 3:2]. And this one [7:5] would still be stronger than those two [3:2 and 9:8]. Correct?

K: Ya.

Debbie and Karen illustrate the typical responses of most students. They focus on the number of cans of orange juice more than water, and use addition.

The exerpts below from the interviews with Nancy, Lorne and Diane illustrate further the tendency of students to use addition to solve numerical problems on proportion.

(iii) Diane

I: What about this question: 2 over 19 equals 7 over x [$\frac{2}{19} = \frac{7}{x}$] ?

D: Um...24

I: OK. Why 24? How did you get 24?

D: Because $2 + 5 = 7$ and $19 + 5 = 24$.

(iv) Lorne

I: What if you had this: 3 to 8 is 15 to x [3:8 = 15:x] ?

L: ...20

I: Why 20?

L: I dunno. I'm just thinking in the way that 5 is separating both.

(v) Nancy

I: What if I gave you this one: 3 to 8 equals 15 to x? [3:8 = 15:x]

N: 20

I: OK. How did you get 20?

N: Cause there's a 5 difference here and there's a 5 difference here.

A second difficulty students have is associated with their underdeveloped idea of a proportion. It was shown in the written tests, for instance, that by placing the given numbers in a problem anywhere in the four parts of the proportion, one could arrive at the various answers obtained by students.

The interview exerpts below illustrate typical difficulties that students have when solving percent problems using proportions.

(vi) Richard

Richard is having difficulty figuring out the following problem: "If you have 75 questions on a test and you get 25% of them correct, how many did you get correct?" After playing with the numbers, the following discussion ensued:

I: What are you trying to do here? [pointing at his work]

R: [Work: $\frac{25}{100} = \frac{x}{75}$]

Well, what I do wrong a lot is I know it's 25 over 100 equals something - I don't even know if it's x over 75 or 75 over x because I haven't worked on this in a long time.

(vii) Lainie

In the exerpt below, Lainie obtains the correct answer by using a proportion. However, she has an equation with no unknown, performs some arithmetic and arrives at the answer.

I: Look at this: 3 is 12% of what?

L: [working it out: $\frac{3}{I} = \frac{12}{100}$; $12\overline{)300}$ with quotient 25, 24, 60] 25%

I: OK. How did you get 25?

L: Cause I did $\frac{3}{I} = \frac{12}{100}$ and I cross multiplied and 12 into 300

It is doubtful from this exerpt that Lainie has much understanding of the 'proportion' she used.

2. Mechanical Skills:

Students seem to have difficulties in the conversion of a percent to its fractional and decimal equivalent forms and vice versa. In addition, in changing a percent to a fraction, for instance, the students not only showed a lack of understanding of equivalent forms, but they also revealed misconceptions about the meaning of percent and its application.

The interviews with Jerry, Karen, David and Richard are representative of the general misconceptions students seem to have in this area.

(i) Jerry

According to Jerry, it seems that if a number is greater than 100%, one changes it to a fraction by putting it over 1000.

I: Here's another question. I don't want the answer. I want you to explain to me what happened. I asked them [other students] to figure out 115% of 70. OK? One student said the answer was 20. Now, several students looked at this answer and said..

J: I'd have to draw a chart.

I: What do you mean by a chart?

J: [draws the following chart

P	W	%
	70	$\frac{115}{1000}$]

And this is parts, whole and percent. Percent is 115 over 1000 and of 70. 70 is the whole so you have to figure out the part. So you go x over 70 equals 115 over 1000.
[$\frac{x}{70} = \frac{115}{1000}$]. Is it 1000 or 100?

I: OK. Why are you putting 1000?

J: It's bigger than 100 and the next thing is 1000.

I: And why would you put 100?

J: I don't know...it would be 1000

Jerry is aware that 70 is the whole. His difficulty is that he seems to think percent means part so 115 is over 1000 since 115 is greater than 100. This leads to

$$\frac{x}{70} = \frac{115}{1000}$$

(ii) Karen

Karen seems to think there is a choice - one can put the number over 100 or over 1000.

I: Write 115% as a decimal.

K: Well, if it's out of 100, you put 1,15

I: You said if it's out of 100. What else could it be?

K: out of 1000.

I: Then the answer would be?

K: 0,115

I: OK. Now, there's something I don't understand. What does it - what do you mean? You have the choice when you see percent - it could mean either out of 100 or out of 1000?

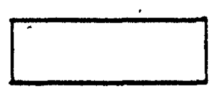
K: Well, I was told that it could be out of 100 or 1000.

I: How do you know when it is out of 100 or out of 1000? Do you have to be told in the example that it is out of 100 or 1000?

K: Ya

Later on in the interview:

I: Can you draw for me 150% of that rectangle?



K: Well what is this out of?

I: I don't know what you mean - "what is this out of"?

K: Well this could be out of 100 and then I'd have to add on more.

I: OK. What else could it be out of?

K: 1000 and then I'd have to draw 3/4 of that.

Karen has developed a theory of her own through which she can do both abstract and concrete problems using either 100 or 1000.

(iii) Debbie W.

Debbie verbalizes the notion that percent means over 100. However, she cannot use this to solve any percent problem.

I: What does 12% mean to you?

D: 12 over 100

I: OK. 12 over 100. So, if you have 100 toys and I ask you to pick up 12% of them...

D: So then you go uh...hold on...uh...if you have how many toys?

I: 100 toys and I ask you to pick up 12%

D: uh...You go...uh...I don't know

Debbie illustrates the case when a student verbalizes an apparent correct idea without knowing to apply it.

(iv) Nancy

The discussion with Nancy concerned the question: "Which is larger - 65% of 20 or 20% of 65?" It appears Nancy does not understand the problem. Indeed, Nancy does not even know that 50% means 50 out of 100.

I: Do these numbers [the 20 and the 65] affect is at all? Does it matter what these numbers are?

N: Yes

I: OK. How does it affect it?

N: Cause if this is 20 and ...um... of 10 and this was 65%..um..yes it does..um...because if this was 20%. OK. <u>If you get...um...50 out of 100 on a test, it is not very good but if you get 50% of 50 on a test, it is excellent</u>.

Clearly, Nancy's view that 50 out of 100 is not very good while 50% of 50 is excellent indicates a misconception of what percent means.

Changing percents to decimal numerals also showed the students having developed their own ideas and methods. The basic idea was to move the decimal place to the left of the digits involved in the number.

(v) <u>David</u>

David's exerpt portrays this method.

I: I asked them [other students] to switch 33% to a decimal and the answer I got was 0,33.

D: That's right.

I: OK. How do you do it?

D: Well, you have this example - <u>if it's a two number percent like 33 or 44 or something, you write a decimal in front of the two numbers and if it's only one number like 8%, you would write 0,08</u>.

I: What if there were three numbers like 115%?

D: You would go <u>0,115</u>.

Thus, David's method yields a correct answer for a one or two digit decimal and an incorrect answer for a three or more digit percent.

However, percents involving decimals and fractions themselves are treated differently. Most students merely drop the percent sign (%) and seem to believe that by doing so, they have solved the problem correctly. Hence, 5,2% is written as 5,2, and 3 1/4% as 3 1/4. Other students do have their own method for dealing with these problems. Richard's interview below illustrates one such method.

(vi) Richard

I: I asked them [other students] to write 33% as a decimal and they wrote 0,33. Is that correct?

R: Agreed

I: OK. What would 115% as a decimal be?

R: 0,115

I: What would 5,2% be?

R: 5,02

I: OK. You want to tell me how you figured them out? This one [115%] you said was...

R: This one I said was 0,115

I: OK...

R: And uh... I figured that one out, cause I know two spots after decimals it means over 100

I: OK...

R: and 115 would go into 1000 so it's three times - 3 decimal places

I: And here: 5,2

R: I would do 5 decimal...uh...what are you asking me to figure out?

I: Change this to a decimal - like get rid of the percent sign

R: 5,02 cause it's the five is before the decimal so I would forget about it and put two in two percent of 100 so it's 02

I: Ah ha. So if I asked you to just change 2% to a decimal you would write...

R: ,02

I: OK. What about 3 1/4%?

R: 3 1/4%? I would do 3 decimal...uh...3,25

I: OK. How did you work that out? How do you get 3,25?

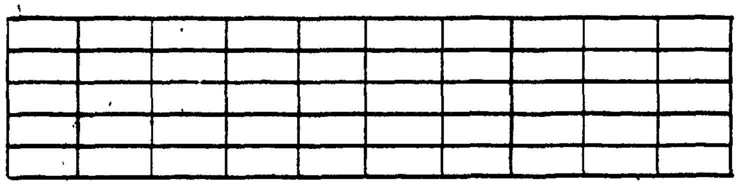
R: cause 4 times 25 is into 100

Richard has a method for each type of percent problem. For a whole number percent, regardless of the number of digits, Richard places the decimal point to the left of the digits, ensuring at least two decimal places. Therefore, for example, 2% becomes 0,02, 33% becomes 0,33 and 115% becomes 0,115. For a percent with a decimal, one seems to ignore the whole number and change the decimal part as in the method above. Therefore, 5,2% becomes 5,02. For a percent with a fraction, he used a different method and 3 1/4% became 3,25.

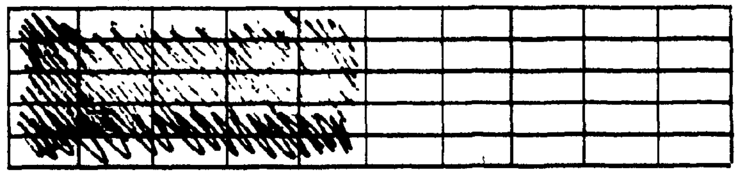
Like Richard, other students also have difficulties with such problems. When the percent was less than 1%, it seems that they merely disregard the percent symbol. For example, 1/2% of 50 is 25. When students are questioned as to a possible difference between 1/2% and 50%, their responses suggest that they believe they are equivalent. This is illustrated by the interview exerpts of Karen and Nancy.

(vii) Karen

I: Could you shade in 1/2% of this grid?



K: You shade in all that.



I: OK. What percent are you shading in?

K: 50%

I: OK. What did I ask you to shade in?

K: 1/2 of that [grid]

I: Right. I said 1/2%. Do 1/2% and 50% mean the same thing?

K: Ya.

Later on in the interview,

I: What would 1/4% of 800 be?

K: You would divide 25 into 800

I: 25 because...

K: 1/4 is 25%

I: OK. I have a question for you. What is the smallest percent?

K: zero

I: Dumb question; besides zero

K: one

I: OK. So is there nothing smaller than 1%?

K: No.

I: OK. When we say 1/4% or 1/2%, which of these two

I: numbers is bigger - 1/4% or 1/2%?

K: 1/2

I: OK. A lot bigger?

K: 25 more bigger

I: How do you get 25?

K: Cause like 1/4 is 25...4 quarters

I: and 1/2 is...

K: 2 quarters

(viii) Nancy

I: What would you do for this: 1/4% of 800?

N: 1/4% of 800 would be...250

I: OK. How do you get that?

N: Cause I figure this [1/4%] is like 25/100 and then 25 from this [800] is 250

I: Wait, you are going to subtract 25 from 800?

N: No. Um...250...wait...I...I...[long pause] 25 from 100...I'd say it would be 200.

I: What does this mean to you: 1/4%?

N: It means like if someone said to me they gave me a quarter of the pie, I'd say they gave me 25% of the pie and like 4 quarters make a dollar so...

I: Is there any difference between 1/4% and 25%?

N: No.

The two exerpts above show that Karen and Nancy persistently ignore the percent sign in a fraction (e.g. 1/4%). This leads to the idea that 1/4% is 1/4 and 1/4 is 25/100 which is 25%.

3. The Concept Percent:

The written tests have shown that many students encountered a variety of difficulties with percent problems. The interviews not only corroborated this, but also provide some insight into the students' thinking.

The individual interviews confirm further the finding from the written tests that students associate the number 100 with percent and that they regard 100% as a whole so that they have trouble with percents greater than 100.

a) Use of the Number 100

The exerpts from the interviews with Lainie, Richard and Michael illustrate some of the typical ways students involve the 100 to solve percent problems.

(i) Lainie

I: Let's say I had 100 toys in a room and I asked someone to pick up 12% of them.

L: Uh huh

I: So they [other students] went ahead and said they'd pick up 12 toys. Is that correct?

L: Nope

I: OK. How many toys should they pick up?

L: In the room...if you asked them to pick up 100%?

I: 12%

L: Oh, 12%! 100 over 12

I: OK. Alright. What if there were 50 toys in the room and I asked you to pick up 12%?

L: 2 toys

I: 2 toys

L: Oh no. That's half...oh no...2 toys

I: OK. Why 2?

L: Because 10...uh...<u>100 over 50 is 2</u>

Later on in the interview,

I: OK. There are 28 students in a class. 16% play sports. Can you tell me how many students play sports?

N: 9,48 [working it out as follows:

$$\frac{16}{100} = \frac{?}{28} \qquad \begin{array}{r} 28 \\ \underline{16} \\ 168 \\ \underline{78} \\ 948 \end{array}$$

]

I: OK. How did you figure that out?

N: <u>16 × 28 is 948; 100 into 948</u>

We see that Lainie uses the number 100 either as a multiplier in 100 over 12 and 100 over 50, or as a divisor in 100 into 948.

(ii) <u>Richard</u>

I: OK. Here's another question without working it out. This one is to compare these two numbers: 20% of 65 with 65% of 20. I asked them [other students] which one was larger and several students said that the second one was larger since 65% is bigger than 20%.

R: <u>I would agree that it's larger because 20% of 65 would be a lower number because there's only 20% out of 100 which is 20 out of 100, of 65. And then you have 65 out of 100 is 20 and 65 is greater out of 100 than 20 is out of 100</u>.

I: Do these numbers [65 and 20] really make a difference?

R: No. Because you're dealing with percent of this one [the number and not the percent] and it's how much out of 100 these two numbers [the percents] are. <u>This one [65%] is 65 out of 100</u>

... which is only leaving 35 and this [20%] is 20 out of 100 leaving 80 numbers so like there is more - there is not as high a number.

Richard seems to know that percent means "how much out of 100". He uses the number 100 to make the comparison: "65 is greater out of 100 than 20 is out of 100", so that the actual numbers in the problem become irrelevant.

(iii) Michael

I: Let's say we have the ratio of girls to boys in a school is 3:7. Let's say that altogether there are 210 children. Can you tell me how many of them would be girls?

M: That's a 7?

I: Yes

M: 135?

I: OK. How do you get 135?

M: Un, well, um, 3 × 25 = 75 and then just the rest is the girls.

I: 3 × 25 is 75

N: the boys

I: OK. the rest are the boys. Why did you multiply by 25?

N: Because that's what would give you 100

I: OK.

N: Like 4 × 25 will give you 100

I: Right

N: So 3 will give you 75 and then the rest...

I: OK and then the rest would have to be boys. Right. Oh. Alright. What if I asked you 20 is equal to what percent of 50?

N: It's 40.

I: Right. Why is it 40?

N: Because 20 × 2

I: Why multiply by 2; why not go 20 × 3?

N: Because 50 × 2 will give you 100

Michael's interview is puzzling because he gets the second problem correct but not the first one. Does he really understand percent? Or does he merely try to fit the number into 100: e.g. "4 × 25 will give you 100" and "50 × 2 will give you 100"?

(b) Concept of 100% as a Whole

It was shown earlier that the success rates in the written tests on items involving 100% were very high, indicating that students grasped the idea of 100% as the whole. The interviews also confirm this as illustrated by the two interview exerpts below on drawing 100% of a given rectangle.

(i) Debbie

I: What I wanted them [other students] to do was to take this rectangle and draw over here for me 100% of that rectangle.

D: The same thing.

I: They should draw the same thing? So what they did was they took a ruler, they measured it exactly, and copied it over. Would that be the right thing to do?

D: Ya

I: OK. Why should you draw the same thing?

D: Because 100% of the rectangle is the whole rectangle.

(ii) David

I: Another question I asked them was to draw for me 100% of that rectangle. They took a ruler, measured it and copied it for me exactly.

D: That's right.

I: OK. Why would one copy it exactly if you were asked for 100%?

D: Because when it's 100%, it's the total amount - it's the exact, total amount.

The exerpts show that both students respond confidently and verbalize clearly in their last statements that 100% of the rectangle is the "whole rectangle" and "the exact, total amount".

(c) Percents Larger than 100%

It was shown that students obtained low success rates on problems involving percents greater than 100. The interviews confirm that students as a whole have no grasp of this notion, even in situations involving pictorial representations. The interviews with Esther, Lorne and Lee illustrate this.

(i) Esther

Three exerpts are provided. The first and last ones deal with finding the percent of a number. The second deals with drawing 150% of a rectangle.

I: I asked the students to figure out 120% of 100 and the answer I got was 120.

E: How can you have 120%?

I: I don't know, I'm asking you?

E: No, but how can you have it; it's over the

...percentage thing.

I: OK. What about 115% of 70?

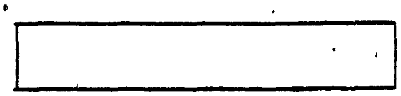
E: I don't know why they do that; there is no such thing as over 100.

I: What's the problem with over 100?

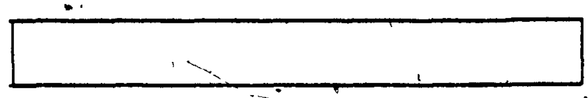
E: Cause 100 is the whole thing.

Later on in the interview,

I: I want you to draw 150% of that rectangle



E: [draws the following]



I: OK. Now I'm going to ask you what you did. What does this measure over here? [length of original rectangle]

E: 2

I: And what did you make it measure?

E: 3

I: OK. What does this measure down here? [width of original rectangle]

E: 2 ... Oh, I have to put it...

I: No, you don't have to. This is 1/2 and this one is left 1/2. [widths of both rectangles]

E: No well the whole is here and I just added 1/2. [1/2 of the original length ∴ 3 inches]

I: Five minutes ago you said you can't have a percent more than 100. Wasn't it you who said you can't have a percent more than 100?

E: Ya. It was me.

I: OK. I didn't know if I was speaking to someone else.

E: Ya, it was me. Well, you told me to do it so I had no choice.

I: Well, does it make any sense?

E: Well, it's just a rectangle if you don't count any percent.

I: Well, but I said to you: draw 150% of that rectangle.

E: I would count it as 1 1/2.

I: Well, does it make sense to ask you to do a problem like this: 150% of 20?

E: No. It doesn't make sense, but 1/2...like... I don't know. It makes sense but I can still work it out... It doesn't make sense cause it's over the whole thing but if you put it in geometry terms then I can do it.

I: OK. So if it's a picture, it can be done but if it's a number, it can't be done.

E: It can be done but it just doesn't make sense.

Still later on in the interview,

I: Can you figure out what 150% of 20 is?

E: ...7 1/2?

I: OK. Why 7 1/2?

E: 20 into 150.

I: I can see the reasoning for that - if the percent is larger than 100% it doesn't make too much sense to you, but if there's a picture or something we can understand a little bit - could you have 150% of a pie?

E: Not if you don't add another pie.

I: Right, you need a second pie.

E: 1/2 of a pie.

The three exerpts clearly suggest that to Esther "there is no such thing (percent) as over 100(%) because "100% is the whole thing". Even when she constructs 150% of a rectangle, she distinguishes between "it is just a rectangle if you don't count any percent" and "count it as 1 1/2" in "geometrical terms" i.e. a rectangle and half a rectangle.

(ii) <u>Lorne</u>

Lorne states that a question like 120% of 100 is a "killer" since "I just don't get it cause it's over 100(%)".

I: Another question was 120% of 100

L: <u>That one's a killer</u>

I: Why is that one a killer?

L: I don't know; I just don't get it. Um...120%...

I: If you had to make up a percent question what numbers would you want to have there that wouldn't be a killer?

L: Well 80% of 100.

I: OK. What's 80% of 100?

L: <u>80% of 100 is 80%</u>

I: OK.

L: But um...

I: Make up another example that wouldn't be a killer.

L: Wouldn't or would?

I: Wouldn't.

L: Well if you ask what percent of 75 is 100

I: Ya

L: It's 75%

I: What makes this problem [original one] a 'killer'?

L: 'cuz this is over 100 [points to %]

I: So as long as I make it under 100 it's no problem. What about 52% of 100?

L: 52% of 100 is 52%

Although Lorne is able to give incorrect examples of percents less than 100 such as 80% of 100 is 80%, he cannot even extend this simply to 120% of 100 is 120%.

(c) Use of Basic Arithmetic Operations in Solving Percent Problems

The individual interviews also confirmed the rampant use of arithmetic operations to solve any type of problem as observed earlier on the written tests. This aspect has been apparent in the interviews we have already discussed. In the exerpts that follow, we simply illustrate the continuing use of arithmetic operations even in very simple problems.

(i) Karen

Karen seems to divide when the numbers are "easy"; otherwise, she has to "figure it out".

I: 3 is equal to 12% of what number?

K: 4

I: OK. How did you get 4?

K: 3 times 4 is 12.

I: Alright. How come you didn't start this way? [pointing to a previous example in which Karen was asked to fill in the blank: 15 = ____ % of 20 and she proceeded as follows: $\frac{15}{20} = \frac{x}{100}$ and found what x was]

K: Cause this one's easy.

I: OK. So let's say instead of 20 over here [15 = ____ % of 20], let's say I wrote 15 is what percent of 45?

K: I'd still have to figure it out.

I: OK. So what do you mean when you say this one [3 = 12% of ____] is easy? What makes this one easy?

K: 3 × 4 = 12

I: Well, 15 × 3 = 45.

K: Well, I don't know my 15 times table.

I: OK. Let's say you knew that fact.

K: It'll make it easy.

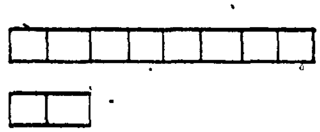
I: What would the answer be?

K: 3

(ii) Diane

Diane resorts to division although she admits she does not remember percent.

I: OK. What percent is B of A?



A

B

D: Two eighths [2/8]

I: OK, which is what percent?

D: [mumbles and pauses]

I: You don't have to do it in your head all the time.

D: I know that...4%

I: How are you getting 4%?

D: Divide 2 by 8. I can't remember how to do percent.

...I'm having trouble with it.

(iii) Esther

Esther guesses the answer by attempting both multiplication and division.

I: 3 is 12% of what number?

E: ...4? No...36?

I: Why 36?

E: I don't know. I guessed 3 × 12?

(iv) Michael

Michael resorts primarily to division.

I: Let's say there were 75 questions on a test and somebody got 25% of them correct. How many questions did he get right?

M: 30

I: OK. How did you get 30?

M: Well, 75 divided by 25 is 3.

I: OK.

M: I just added a zero.

I: Why did you add a zero?

M: To make it a tens number.

Later on in the interview,

I: Let's say I have a class of 28 students and from that class 16% of them play sports.

[Brief discussion about sports ensued.]

I: OK. So 16% of them play sports. Can you tell me how many of them play sports?

M: 4

I: How did you get 4?

M: Well, I went 4 goes into both of them.

(v) Nancy

Nancy simply uses addition.

I: Alright. 12% of my students didn't do their homework. This meant that 3 students did not do their homework. Can you tell me how many students are in my class?

N: 15

I: OK. How did you get 15?

N: 12 + 3

To summarize, the interview exerpts above confirm further a general tendency among students to resort to arithmetic operations to solve percent problems, even in those cases where they do not really understand percent.

(d) Unanticipated Phenomena Associated with Percent

Several unanticipated phenomena emerged from the interviews. We have already illustrated the restricted concept of a percent as a whole number between 1 and 100. We saw that a percent less than one, for example 1/4%, is equivalent to 25%, and also, the notion that percents greater than 100, say 120%, do not make sense.

Another more general phenomenon that emerged during the interviews involves two main ideas associated with basic percent problems. When students are asked to find x% of N, where x and N are numbers, they seem to believe that the answer is simply x, regardless of what number N is. These students also fall into two subgroups: one subgroup will not accept percents greater than 100 under any circumstances, an aspect we have already discussed. The second subgroup accepts or rejects percents greater than 100 depending on the size of the number N in relation to the percent x . For example, they accept 150% of 200 but not 150% of 20; that is, a percent greater than 100 is acceptable only if it is less than the number N . This phenomenon was not apparent in the written tests, nor was its existence anticipated. Consequently, it emerged slowly in the interviews and was only appreciated during the analysis. Hence, it appears in scattered portions of the individual interview, often quite by accident as illustrated in the seven exerpts below from the interview with Nancy and one exerpt from Michael's interview.

(i) Michael

Michael's exerpt demonstrates that x% of N is always x , regardless of N . It should be noted that Michael did not consistently use this idea but even for simple problems this misconception led to incorrect responses.

I: I told them that I had 100 toys and I asked them to pick up 12% of the toys and they said that they would pick up 12 toys. Is that correct?

M: [long pause] Yeah.

I: OK. How do you get that answer?

M: Well, becasue there's a...you have 100 toys

I: Right

M: and if you pick up 12 of them, pick up 12, because...

I: OK what if there were 50 toys

M: The answer is still 12

(ii) Nancy

The exerpts illustrate three characteristics of the above phenomenon in her ideas: x% of N is x, one can do a problem if x is less than N, and one cannot do the problem when x is greater than N. Nancy uses these ideas quite consistently.

(a) x% of N is x, where x is less than N

(i)

I: Let's say I had 100 toys in a room and I asked somebody to pick up 12% of the toys. They [other students] said that they would pick up 12 toys. Do you think that's right?

N: Um, yes

I: OK. Why?

N: Because I'd say...um...12% OK. You have 100% and you take away and then...uh...you would have 88 left, so 88 toys left; 88% plus 12% equals 100%

I: OK. What if there were 50 toys and I asked you to pick up 12%. Then how many would you pick up?

N: Um...Same thing. I would pick up 12 toys.

I: You'd pick up 12 toys. So it doesn't matter if there are 100 toys or 50 toys because you're picking up 12%. You would pick up the 12 toys.

N: Yes, but I don't know; maybe I'd pick up 6. I don't know cause half of 100...I don't know.

(ii)

I: OK. Alright. What if you have 75 questions on a test and you get 25% of them correct. How many did you get correct?

N: ...You'd get one third correct which is...what do you mean: "how many would I get correct?"

I: Well, I mean, there are 75 questions. Did you get 10 right; did you get 12 1/2 right?

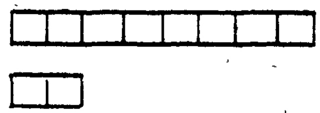
N: Oh! 25

I: OK. 25. How do you get 25 again?

N: Cause you have 75 questions and you have 25% of them which are correct, so you just take off the percent and you get 25.

(iii)

I: OK. If I asked you what percent is B of A, can you tell me what percent it is?



A

B

N: um...2%

I: How did you get 2?

N: Cause there's 2 here [pointing to the 2 squares in B]

The above exerpts show that if x is less than N, then x% of N is x, regardless of what number N is.

(b) x% of N, where x is larger than N

(iv)

I: How would you figure this out: 17% of 81?

N: Alright. You have to go 17 into 8100 or something like that and then you have to figure it out how many times.

I: Why 8100?

N: I don't know; that's how somebody told me to figure it out.

I: Well, let's say I asked you 52% of 16. Then what would you do?

N: You can't ask me that. I don't think its possible.

I: Why not? How do you determine why this one [17% of 81] is possible and this one [52% of 16] is not? What makes these examples different?

N: I think it's because 17 can go into 81 because 81 is higher than 17 but 52 can't go into 16.

I: Would this example be possible? [writing 3% of 96]

N: Ya.

I: this one? [96% of 3]

N: ...No

I: What about this one? [writes 120% of 300]

N: Um...ya.

I: How would you figure this answer? [120% of 300]

N: I'd go 120 into 300 I guess.

(v)

I: 20 is what percent of 50?

N: 20

(vi)

I: Let's say I have a group of 28 students and of these 28 students, 16% play sports. That would mean how many play sports?

N: 16

(vii)

I: Can you make up an example with 112% in it?

N: What do you mean?

I: Can you make up an example? Say you are making up a test for your class with some words and they want you to use the number 112% in the example. Can you think of something?

N: 112%...plus 50%. I don't know. Something like that or do you want me to use words?

I: Words - kids hate word problems.

N: Alright. 112% of the birds flew east. How many birds flew east?

I: Let's say you started out with 50 birds. What would be 112% of the birds or could you do that? I don't know.

N: <u>No, you can't</u>.

I: Let's say you started with 200 birds. What would 112% of the 200 birds be?

N: <u>112 birds</u>

I: But if I gave you, for example, 100 birds?

N: <u>I wouldn't be able to do that</u>.

I: Alright. So this number [the number of birds] has to be what in relation to this [the %]?

N: <u>Greater than</u>.

Here Nancy argues that x% of N can be done only if x is less than N. For instance, in exerpt (iv), she says 17% of 81 can be done but not 52% of 16 "because 17 can into 81 because 81 is higher than 17 but 52 can't go into 16"!

SUMMARY

The major points outlined in the observations above are summarized below.

(A) The individual interviews substantiate further the limited understanding of the concept of percent among students. The following specific observations were made.

(i) Again students showed a belief that the number 100 is involved with percent problems but were not always sure how to incorporate the number.

(ii) Students' verbalizations about the concept of 100% as a whole corroborated the observations from the written tests that this idea is well understood.

(iii) Difficulties in grasping percents less than 1 and greater than 100 were explained - for the most part, students found the latter notion conflicting with the idea that 100% represents an entire entity.

(iv) Students verbalized notions about the use of arithmetic operations to solve percent problems. For example, to use division, some felt that the numbers had to divide evenly; otherwise, one used a different method to solve the problem. Regardless of their methods, the interviews demonstrated that students frequently resort to the basic arithmetic operations when dealing with percents.

(v) Several phenomena that were not readily apparent in the tests also emerged from the interviews. One is the restricted notion of percent as a whole number between 1 and 100. Thus a percent less than 1, say 1/4%, is interpreted as 1/4 of 100% so that 1/4% means 25%. A second phenomenon is the notion that a percent larger than 100 does not make sense. A third phenomenon is the idea that x% of N is x regardless what number N is, except that there were certain conditions necessary regarding the relationship between x and N in order to be able to solve the problem - x must be less than N.

(B) Mechanical skills in working with percents were also shown to be poor. Students verbalized incorrect methods about changing a number from its percent form to its decimal or fraction equivalent. Some stated that when changing a percent to a fraction, one puts the number over 100, 200 or 1000 depending on the specific example. When changing a percent to a decimal, the most common incorrect method stated was to move the decimal point to the left of the digits involved.

(C) Students' verbalizations revealed an underdeveloped notion of decimals, ratio and proportion. This was in turn reflected in the difficulties they encountered when dealing with the three basic cases of percent problems.

In short, the individual interviews corroborated the overall low performance on the written tests of secondary I and II students by demonstrating that their verbalizations revealed: the acquisition of a narrow concept of percent, inadequate mechanical skills for handling these topics and underdeveloped notions of decimals, ratio and proportion.

CHAPTER VI

CONCLUSION

It was the attempt of this exploratory study to investigate the high school student's understanding of the concept of percent after this topic was completed in the schools. Understanding of the main ideas leading up to percents was also explored.

To investigate a student's understanding of a particular comcept is not an easy task. What is being referred to here is not the time and effort involved in the actual testing and interviewing of a large number of students, but rather, the designing of the actual methodology for the study. To begin with, one might question the validity of using testing and interviewing to carry out this investigation. Furthermore, although a great deal of effort went into the thought and preparation of the test items, one can always question the validity of particular items - did success (or failure) on an item imply understanding (or lack of understanding) of percent? Was the construction of the items such that the students clearly understood what was required of them?

The above queries can be raised regarding the questions asked in the interviews as well. Moreover, one must keep in mind that a student's failure on any question could be due to factors other than his understanding of the concept being tested. For example, a student may have been overly nervous or may have had difficulty expressing himself verbally in the interview. Hence, keeping in mind the limitations of this study, let us now turn our attention to its main findings.

At the outset, three main objectives were outlined for which test items and interview questions were specifically designed. Therefore, after a detailed analysis of the results was performed, it was possible to formulate tentative conclusions which attempted to answer the questions originally raised.

The first objective and the main purpose of the study was to investigate the student's understanding of percent. The study showed that this understanding among secondary I and II students is poor indeed. Success rates on test items were generally quite low and ensuing discussions in the interviews showed little or no understanding in many cases. Indeed, the small number of students who did show a general understanding of percent were those who seemed to have a basic mathematical aptitude. They had acquired the prerequisite skills leading up to percent, were able to solve percent problems and the discussions in the interviews showed a general understanding of the ideas being explored.

Students did not react any differently to the word percent than they did to the symbol '%'. In fact, to most students, both the word and the symbol implied something to do with the number 100; however, they did not know exactly what to do with this number when working out problems. Furthermore, it seems that students really did not understand what is meant by x% of a number. For example, in some instances, an answer to a question of this form (find x% of N) would be given as a percent (e.g. 80% of 100 = 80% and not 80). In other cases, students verbalized the notion that one cannot take x% of a number N if x is larger than N.

The study did show a general understanding among students of the notion of 100% as a whole entity; however, this seemed to lead to difficulties in understanding percents larger than 100. Many students rejected the existence of such numbers as they probably conflicted with the notion of 100% being a whole object. Other students, who probably performed mechanically with these numbers to obtain correct answers indicated a lack of understanding in other instances. For example, they would state that x% of a number N (where x is larger than 100) is smaller than N, or when asked to formulate problems with a percent larger than 100, they misused the numbers in such a way as to indicate little comprehension.

The second main objective of the study was to investigate the student's acquisition of mechanical skills associated with work with percents. The findings showed that students can change a two digit percent to its fractional or decimal equivalent; however, their success here often seems to be a coincidence. The methods being used to change other percents showed little understanding and raised questions about the success with two digit percents. It will be recalled that to change a percent to a decimal numeral, many students used the method of moving the decimal point to the left of the digits. Hence, they would succeed with two digit percents but not with the other numbers. To change a percent to its fractional equivalent, students would place 100 under the number if it was a two digit number but would use 1000, and in some cases 200, as the denominator if it was a three digit percent. Therefore, success with the two digit percents does not necessarily imply understanding or even mere acquisition of the skill of changing a percent to another equivalent form.

A second skill involved solving the three basic percent problems. Although the students were quite successful with the first case (finding x% of something), their overall performance on the three cases was generally poor. In many instances, the students tried to set up a proportion for these problems, but, for the most part, they did not usually know where to place the given numbers. In fact, the lack of success on such problems, as well as the confusion these problems seem to cause, highlight further the lack of comprehension of percents among students.

The third objective of the study was to investigate the student's ability to deal with fractions, decimals, ratio and proportion - in other words, the development of the prerequisite skills necessary to work with percents. Success rates with simple problems dealing with fractions and decimal numerals were quite low. Simple arithmetic operations with fractions and decimals were often performed incorrectly. Furthermore, when students did succeed with some work in these areas, their interview responses showed that they did not always seem to understand what they were doing. For example, when multiplying a decimal numeral by 100, students who used the rule: 'move the decimal point two places to the right' did not know why the rule worked. Moreover, students who memorized this rule could not remember how many places to move the decimal point.

The prerequisite concepts most poorly developed were those of ratio and proportion. The success rates on these topics were very low, and any discussion in the interview that probed a student's notion of these ideas often showed a lack of understanding.

Indeed, although some students could solve for an unknown term in a proportion when the numbers were simple ones, they were troubled when the numbers were more complex and would usually resort to methods that showed little comprehension. For example, a student might try to solve 2:19 = 7:x by adding 5 to 19 even though he was able to solve 2:19 = 6:x correctly, based on the idea that two divides into six. Word problems that required setting up a proportion for their solution generally proved to be difficult tasks for most students except the very able ones.

At the outset, it was shown that percent is a peculiar topic in school mathematics in that it has a utilitarian, rather than a mathematical value. In addition, for the study of percent, many prerequisite concepts must be learned; however, percent itself is not a prerequisite for other mathematical ideas. The literature showed little research on percents as compared to the amount of research on the prerequisite topics of fractions, decimals, ratio and proportion. Research on the latter showed that students have difficulties with these ideas.

The National Assessment of Educational Progress (NAEP) studies of 1973 and 1978 corroborated previous research that students have many difficulties with the concepts of fractions, decimals, ratio and proportion. Furthermore these studies also investigated some ideas with percents and showed that students generally performed quite poorly on problems involving percents.

The findings of the present study are consistent with the NAEP results. In addition, it showed (i) a very limited understanding of percent itself, (ii) misconceptions about changing percent to its equivalent forms, (iii) serious misconceptions about percents less than 1% and larger than 100%, (iv) difficulty solving the three basic cases of percent problems and (v) the rampant use of simple arithmetic operations to solve percent problems. It was shown that the main idea students do have about the word percent and the symbol '%' is that it is somehow associated with the number 100.

Furthermore, the only concepts that they seem to have grasped is that 100% represents an entire object, and that x% of 100 is x as long as x is a one or two digit number. Beyond that, percent remains a mystery and a source of confusion.

The disturbing feature of the study is that the students involved had completed their formal study of percent. Furthermore, since the present study involved a sample of over 200 high school students in a private school and a public school, it seems likely that the findings would hold true for high school students elsewhere as well. Hence, the findings should be of concern to teachers of mathematics and researchers.

The study raises questions for further research as well. The sequencing of the relevant topics in the mathematics curriculum - fractions to decimals to ratio and proportion and finally to percent - is a mathematically sound approach. The study showed no evidence to the contrary in that no student showed understanding of one of these topics without having acquired knowledge of the prerequisite topic. However, specific research should be performed to investigate the validity of this sequencing. If indeed it can be shown to be valid, then the poor results of this study can probably be attributed to the mode of presentation of the topics and not to the order of their presentation.

The association between fractions and decimals and percent should be further investigated. What specific aspects of fractions and decimals are needed for understanding of percent? Why do students have so much difficulty in changing a percent to its fractional or decimal equivalent?

The study discussed an association between ratio and proportion and percents. It was shown that all these concepts were poorly understood. However, one should try to explore the degree to which the understanding of percents is dependent upon the understanding of ratio and proportion. Clearly one cannot perform a percent problem as a proportion if one does not comprehend the latter but can one understand percents in a different way without total comprehension of ratio and proportion?

Another area to be explored are the concepts which were well understood. One example is the notion of 100% as a whole. What is it in the teaching of this idea that ensures higher success rates than with other concepts? Is the concept a simple one or are we doing something to make it better understood?

It was shown in this study that pictorial representations of ideas do seem to be helpful in exploring some of the abstract ideas of percent. For example, whereas most students have difficulty with percents larger than 100, some do show success when a concrete object or picture is involved even though these numbers were not totally grasped on an abstract level. One should try to investigate further what other specific aspects of percent students understand on a concrete level, even if the concept is not fully understood in the abstract. Then one could possibly use this information to gain more insight into the student's thinking about percent in general.

Although the general performance by the students in this study was poor, it can be seen that the success was greater on questions involving the most 'common' percentages (e.g. 20%, 25% and 50%) as well as with questions where the given numbers divide evenly. It has been shown that success in these questions does not necessarily imply understanding as students sometimes use incorrect methods which yield correct responses under certain conditions. However, it should also be educational to study the types of problems students are given in textbooks, classroom examples and on tests. Do the problems always involve the same type of numbers (e.g. 25% and 50% as opposed to 13% or 1/3%)? Are the answers always whole numbers and do the given numbers of a problem always divide evenly? Are all the numbers in a problem used or is there sometimes extraneous information? An investigation of the types of questions commonly presented to students might provide the reasons for the success with specific types of problems.

The results of such an investigation can prove to be especially useful in education today. In our world of modern technology, calculators and even computers are becoming more accessible to the average person. Therefore, we will no longer be restricted as to the choice of numbers in problems presented to students to ensure easy calculations. Hence, the emphasis of our teaching can be focused on understanding of concept and not on computation. Perhaps then, the future will bring more success in comprehension of percent as well as of other mathematical topics.

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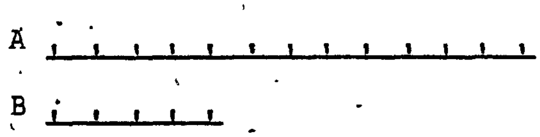
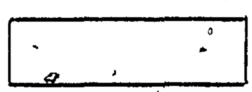
APPENDIX I

Listed below are the items from each of the two tests. The actual test papers presented to the student had the items spaced out and allowed for work space to the right of each item.

TEST I

Answer each question in the space provided, Work space is provided on the right to be used when necessary.

1. 4:5 = x:5   Find x
2. What is 15% of 75?
3. Write 128% as a fraction
4. 80% of the books in a library were old. If 240 books were old, how many books were there altogether?
5. 127 = _____% of 100
6. 70% of the children in the park like to play. If 70 children like to play, how many are in the park?
7. 

   A is what % of B?
8. 0.045 × 100 equals?
9. This rectangle  is one-half a larger rectangle. Draw the larger rectangle.

10. Which one is not equivalent to the rest?

(a) 1:4 (b) $\frac{1}{4}$ (c) 25% (d) 0,25

(e) all are equivalent.

11. 23% of _____ = 184

12. $\frac{88}{100}$ × 25 equals?

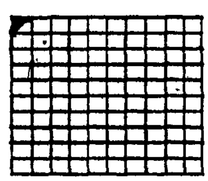
13. There were 30 questions on a test. Mary got 100% of the questions correct. How many did she get correct?

14. 6,8 : 100 = x:4 Find x

15. $\frac{3}{4}$ + 20% equals?

16. One pitcher of orange juice contains 6 glasses of water and 2 cans of orange concentrate. A second pitcher contains 4 glasses of water and 3 cans of orange concentrate. Which pitcher has the stronger orange taste?

17. What percent of the grid is shaded?

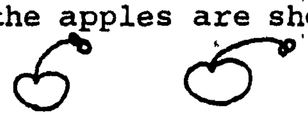


18. True or False?

100% of 80 is greater than 80.

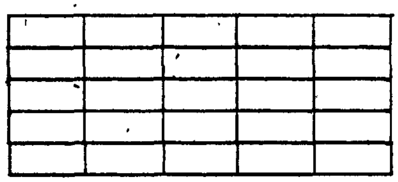
19. 32 is _____% of 100.

20. 12% of the apples are shown below.



How many were there altogether?

21. Shade in 50% of the grid below.

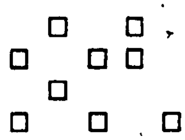


22. 4 out of every 5 boys likes sports. In a group of 45 boys, how many like sports?

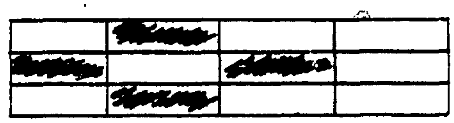
23. What is 120% of 60?

24. If 2 shirts cost $35, how much would 7 shirts of the same type cost?

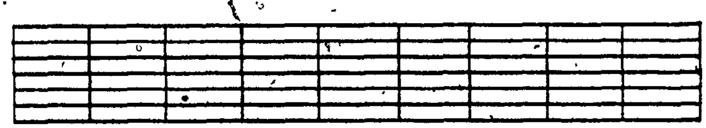
25. Circle 100% of the squares below



26. If the ratio of shaded rectangles to the total number of rectangles is always as in the following,



shade in the appropriate number of rectangles below.



27. 6,2% of 100 equals?

28. 315:100 = x:20   Find x

TEST 2

Answer each question in the space provided. Work space is provided on the right to be used when necessary.

1. What is 17% of 100?

2. 4:100 = x:75   Find x

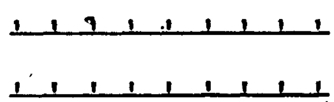
3. Out of 100 apples, 40 were rotten. What percent was this?

4. 16 is what percent of 50?

5. R is  Draw 150% of R .

6. 0,12 × 55 equals?

7. Write 21% as a decimal.

8. 
A
B

A is _____% of B?

9. True or False?
172% of 80 is greater than 80.

10. In a group of 40 people, 25 were on a diet. What percent was on a diet?

11. 80% of _____ = 8

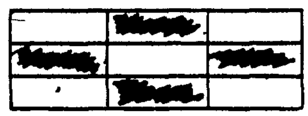
12. $\frac{18}{100} \times 100$ equals?

13. John is 13 times as old as Ron. What is the ratio of John's age to Ron?

14. Circle 20% of the triangles below.

Δ Δ
Δ Δ Δ
Δ Δ Δ
Δ Δ

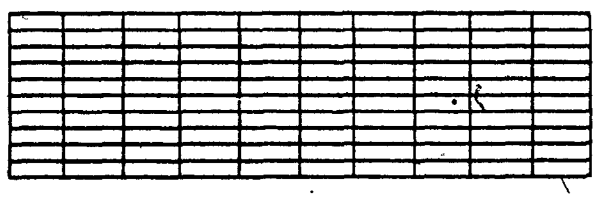
15. What is the ratio of shaded rectangles to non-shaded rectangles in the figure below?



16. A is 20% of B.

B is what % of A?

17. Shade in 7% of the grid below.



18. AAAA

BBBBBB

What is the ratio of A's to B's?

19. 87% + 50% equals?

20. There is a leaky faucet in which 5 drops of water drip every minute. How many drops of water are there in one hour?

21. Write 112% as a decimal.

22. Which is larger? 20% of 65 or 65% of 25?

23. 87% of the students passed a test. What percent failed?

24. Write $3\frac{1}{4}$ % as a decimal.

25. What is $\frac{1}{2}$ % of 100?

26. Write 12,8% as a decimal.

27. 2:19 = 7:x   Find x

28. The ratio of students who passed a test to students who failed was 2:3. If there were 60 students in all, how many passed?

APPENDIX II

THE CLINICAL INTERVIEW

Part I

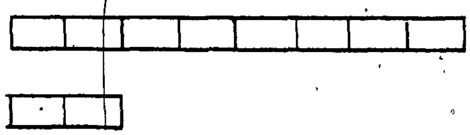
1. $\frac{3}{5} = \frac{x}{5}$ Some students wrote $x = 3$. What do you think? What if the question was $\frac{3}{4} = \frac{x}{20}$? What would the answer be?
2. There were 100 toys in a room and Michael picked up 12% of the toys. This meant that he picked up 12 toys. Is that correct? Why do you think so? What if there were 50 toys altogether and he was asked to pick up 12%? Why?
3. A lot of questions had to do with writing a percent as a decimal.
   (a) I asked for 33% as a decimal and the answer I got was 0,33. Is that correct? How do you figure it out? Why does it work out that way?
   (b) What if I asked for 115% as a decimal?
   (c) 5,2% as a decimal?
   (d) 3 1/4%?
4. Peter got 15/25 on his test. This meant he got 60%. Is that right? How do you figure it out?
5. Here's another question: $1{,}63 \times 100$. The students said the answer was 163. Do you agree? What if I asked you $0{,}023 \times 100$? What method are you using?

6. How do you read this? 2:5 = x:15? Some students said x = 12. Do you agree? Why or why not? What about 3:8 = 15:x ; what's x?

7. I have two books. The first book is 1/3 as large as the second book. Whis book is larger? How many times larger? Let's take another example with two different size items.

A

B



B is what percent of A?

8. I asked a student what 115% of 70 was and he said the answer was 20. A second student said that this was impossible and he didn't even work anything out - he just looked at the answer and said this was impossible. How do you thihk he knew this?

9. There were two questions on the test. One was to calculate 20% of 65 and the other was to calculate 65% of 20. Several students thought the second answer must be bigger since 65% is larger than 20%. Do you think this makes sense? Why or why not?

10. I asked students to write 18% as a fraction and the answer I got was 18/100. Is this correct? Why or why not?

11. Michael's age is 2/5 that of his mother's. I asked students to tell me the ratio of Michael's age to his mother's. Some said it was impossible without knowing

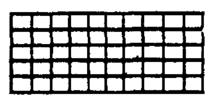
the actual ages. Do you think they are right?

Look at this card: AABABAAA

What is the ratio of A's to B's?

12. I asked students to draw 100% of the rectangle below. They drew exactly the same rectangle over again. Why do you think they did this?

13. Look at this grid. Please shade in 1/2% of the grid.



14. I asked students to calculate 120% of 100. They said the answer was 120. What do you think?

What if I asked for 1/2% of 100?

15. Read this:

$$\frac{2}{7} = \frac{12}{x}$$

What do you think x is?

16. 31 is what percent of 100? How do you get this?

3 is 12% of what number? Why?

Part II

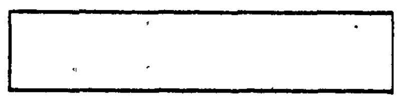
1. Calculate 17% of 81.

2. $\frac{2}{19} = \frac{7}{x}$. Find x .

3. The ratio of girls to boys in a school is 3:7. If there is a group of 210 children, how many are girls?

4. 20 = ______% of 50?

5. Draw 150% of this rectangle.

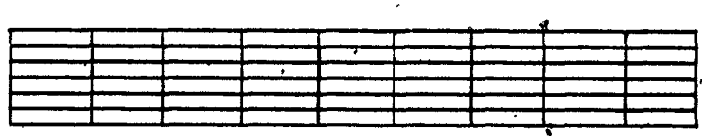


6. 12% of my students did not do their homework. If 3 students didn't do their homework, how many are in my class?

7. In a class of 28 students, 16% play sports. How many play sports?

8. If the ratio of shaded rectangles to non-shaded rectangles is always as in the following,

shade in the appropriate number of rectangles in the grid below.



9. Write 25/40 as a percent.

10. What is 1/4% of 800?

11. A is 50% of B .

    B is what percent of A ?

12. What is 3,8% of 20?

13. 148 is what percent of 60?

14. I have two pitchers of orange juice. One has 3 cans of orange concentrate and 2 cans of water while the other has 7 cans of orange concentrate and 5 cans of water. Which one tasts more orangy? Why?

APPENDIX III

OVERALL SUCCESS RATES

TEST I		TEST II	
Question	Success Rate (%)	Question	Success Rate (%)
1	84,4	1	87,0
2	32,9	2	67,0
3	69,7	3	87,4
4	35,5	4	61,9
5	46,8	6	65,7
6	54,5	7	88,7
7	13,4	8	82,0
8	60,0	9	69,0
10	70,0	10	36,4
11	29,4	11	69,0
12	52,8	12	59,4
13	90,0	13	39,7
14	12,6	14	85,4
15	42,9	15	52,3
16	84,0	16	9,6
17	55,8	17	80,8
18	83,1	18	85,4
19	66,2	19	84,9
20	15,2	20	83,3
21	90,9	21	57,3
22	63,6	22	71,1
23	31,2	23	86,2
24	47,2	24	16,7
25	92,6	25	30,5
26	28,6	26	42,7
27	33,3	27	22,6
28	48,9	28	5,4

APPENDIX IV

% BREAKDOWN OF VARIOUS ANSWERS PER QUESTION

Each of the questions from both tests is cited below. Beneath each question are a list of various responses provided by the students together with the percentage of the population providing those answers. The first response is the correct answer and hence the percent should correspond with the percent given in Appendix III. The category 'other' merely groups together miscellaneous responses provided by the students, none of which could be classified as common to the population. Finally, the symbol 'ɓ' symbolizes that the students left the question blank.

TEST I

Item 1: 4:5 = x:5 Find x

Response	Percent
4	84,4
other	4,8
ɓ	10,8

Item 2: What is 15% of 75?

Response	Percent
11,25 or 11 1/4	32,9
5 or 5%	20,3
20(20%;0,2;2)	10,8
11-12	7,8
other	19,5
ɓ	8,7

Item 3: Write 128% as a fraction.

Response	Percent
$\frac{128}{100}$; $\frac{32}{25}$; $1\frac{7}{25}$; $1\frac{28}{100}$	69,7
$\frac{128}{1000}$	10,4
other	10,8
∅	7,8

Item 4: 80% of the books in a library were old. If 240 books were old, how many were there altogether?

Response	Percent
300	35,5
192 (1920;19200)	11,3
260	2,6
270	2,6
432	1,7
320	1,7
1/3; 33 1/3	1,7
other	14,3
∅	25,6

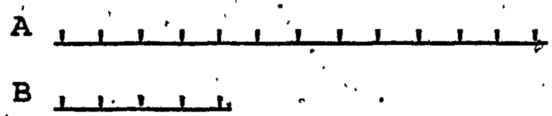
Item 5: 127 = _____% of 100

Response	Percent
127	46,8
27	4,3
other	16,9
∅	32.0

Item 6: 70% of the children in the park like to play. If 70 children like to play, how many are in the park?

Response	Percent
100	54,5
70	8,2
49;490;4900	8,2
14;140;1400	3,0
30	6,9
other	5,3
∅	13,9

Item 7:



A is what percent of B?

Response	Percent
300	13,4
1/3	4,8
33 1/3; 33;33.3	9,5
260	4,8
13/5;5/13	1,7
1/4	0,9
75	0,9
25	2,2
other	45,0
ø	16,8

Item 8: 0,045 × 100 equals?

Response	Percent
4,5	60,0
45	4,3
0,45	3,5
0,045	4,3
4500	6,9
450	1,3
other	9,0
ø	10,7

Item 10: Which one is not equivalent to the rest?

(a) 1:4 (b) 1/4 (c) 25%
(d) 0.25 (e) all are equivalent

Response	Percent
E	70,0
A	9,0
B	0,9
C	3,4
D	11,6
ø	5,1

Item 11: 23% of ______ = 184

Response	Percent
800	29,4
42,32;4232;42 8/25	8,2
8	8,7
161	1,7
other	18,6
ø	33,4

Item 12: $\frac{88}{100} \times 25$ equals?

Response	Percent
22	52,8
2200/100	2,2
2200/2500;22/25	3,0
88/2500	0,9
2200	3,0
other	25,5
ƀ	12,6

Item 13: There were 30 questions on a test. Mary got 100% of the questions correct. How many did she get correct?

Response	Percent
30	90.0
other	4,8
ƀ	5,2

Item 14: 6,8:100 = x:4

Response	Percent
0,272	12,6
2,72	3,0
27,2	1,3
272	1,3
6,8;68;0,068; $\frac{68}{100}$	2,6
other	35,9
ƀ	43,3

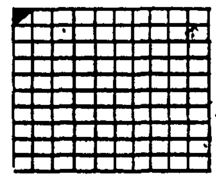
Item 15: 3/4 + 20% equals?

Response	Percent
95%;0,95;95/100; $\frac{19}{20}$	42,9
95	3,9
20 3/4;20 3/4%	4,3
23/4;23/104; 23/14	2,2
15;15%;15/100	5,2
other	26,0
ƀ	15,5

Item 16: One pitcher of orange juice contains 6 glasses of water and 2 cans of orange concentrate. A second pitcher contains 4 glasses of water and 3 cans of orange concentrate. Which pitcher has the stronger orange taste?

Response	Percent
second	84,0
first	6,1
same	0,4
b̸	9,5

Item 17: What percent of the grid is shaded?



Response	Percent
1/2%;0,5%	55,8
$\frac{1}{200}$; $\frac{\frac{1}{2}}{100}$; 0,005	12,1
50%	0,9
0,005%	0,9
99 1/2	4,8
0,05	1,7
other	13,0
b̸	10,8

Item 18: True or False? 100% of 80 is greater than 80.

Response	Percent
F	83,1
T	12,6
b̸	4,3

Item 19: 32 is _____% of 100

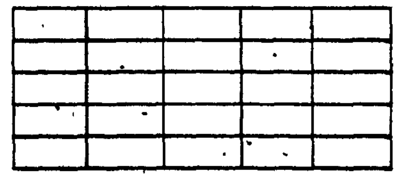
Response	Percent
32	66,2
other	16,1
b̸	17,7

Item 20: 12% of the apples are shown below.

How many were there altogether?

Response	Percent
16 2/3; 16,$\bar{6}$; 16,6	15,2
16-17	14,7
24	8,2
6	2,6
8; 8,4; 8,3	6,1
2	4,8
other	23,4
ø	25,0

Item 21: Shade in 50% of the grid below.



Response	Percent
correct	90,9
wrong	6,1
ø	3,0

Item 22: 4 out of every 5 boys like sports. In a group of 45 boys, how many like sports?

Response	Percent
36	63,6
40	3,5
44	3,5
9	5,2
other	13,0
ø	11,2

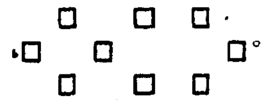
Item 23: What is 120% of 60?

Response	Percent
72	31,2
90	2,6
50	4,3
1/2; 50%	3,5
other	39,8
ø	18,6

Item 24: If 2 shirts cost $35, how much would 7 shirts of the same type cost?

Response	Percent
122.50	47,2
245,00	15,2
other	28,1
ƀ	9,5

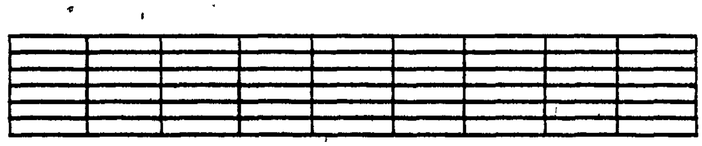
Item 25: Circle 100% of the squares below.



Response	Percent
correct	92,6
wrong	3,9
ƀ	3,5

Item 26: If the ratio of shaded rectangles to the total number of rectangles is always as in the following

shade in the appropriate number of rectangles below.



Response	Percent
18	28,6
16	13,9
27	4,3
24	2,2
9	2,2
other	25,1
ƀ	23,7

Item 27: 6,2% of 100 equals?

Response	Percent
6,2	33,3
62	8,2
620	5,2
6,2/100;0,062	3,5
62%	2,2
63	1,3
0,62	1,7
other	13,4
ø	31,2

Item 28: 315:100 = x:20
Find x .

Response	Percent
63	48,9
15,75;1575;15,15	4,8
235	0,9
other	16,0
ø	29,4

TEST II

Item 1: What is 17% of 100%

Response	Percent
17	87,0
5-6	2,5
83	2,0
17/100	1,7
other	4,3
ø	2,5

Item 2: 4:100 = x:75
Find x

Response	Percent
3	67
other	17
ø	16

Item 3: Out of 100 apples, 40 were rotten. What percent was this?

Response	Percent
40	87,4
60	6,3
other	3,8
ƀ	2,5

Item 4: 16 is what percent of 50?

Response	Percent
32	61,9
8	12,1
16	2,9
other	12,5
ƀ	10,6

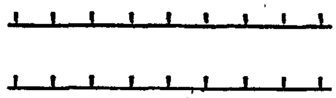
Item 6: 0,12 × 55 equals?

Response	Percent
6,6	65,7
other	18,8
ƀ	15,5

Item 7: Write 21% as a decimal.

Response	Percent
0,21	88,7
other	7,1
ƀ	4,2

Item 8:



A is what % of B?

Response	Percent
100	82,0
other	9,2
∅	8,8

Item 9: True or False? 172% of 80 is greater than 80.

Response	Percent
T	69,0
F	27,2
∅	3,8

Item 10: In a group of 40 people, 25 were on a diet. What percent was on a diet?

Response	Percent
62,5	36,4
10	6,7
15	5,0
62,2	3,8
other	36,0
∅	12,1

Item 11: 80% of _____ = 8

Response	Percent
10	69,0
100	7,5
1000	3,8
640;64;6,4	2,5
other	8,4
∅	8,8

Item 12: $\frac{18}{100} \times 100$ equals?

Response	Percent
18	59,4
other correct answers (ex: 1800/100)	2,1
1800	7,1
1800/10000	3,8
other	14,2
ø	13,4

Item 13: John is 13 times as old as Ron. What is the ratio of John's age to Ron?

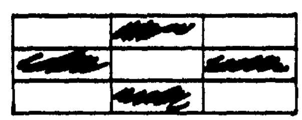
Response	Percent
13:1	39,7
13:x	7,5
x:13	3,8
13:13	2,5
1:13	5,9
13:0	1,7
other	16,3
ø	22,6

Item 14: Circle 20% of the triangles below.

Δ Δ Δ Δ
Δ Δ Δ
Δ Δ Δ

Response	Percent
2	85,4
5	7,5
other	3,8
ø	3,3

Item 15: What is the ratio of shaded rectangles to non-shaded rectangles in the figure below?



Response	Percent
4:5	52,3
4:9	27,6
5:4	7,1
other	8,8
ø	4,2

Item 16: A is 20% of B. B is what % of A?

Response	Percent
500	9,6
80	37,7
120	7,5
100	6,3
20	12,6
other	11,7
⌀	14,6

Item 17: Shade in 7% of the grid below.

Response	Percent
7 rectangles	80,8
7 columns	3,4
other	7,9
⌀	7,9

Item 18: AAAA

BBBBBB

What is the ratio of A's to B's?

Response	Percent
4:6 (2;3)	85,4
other	8,8
⌀	5,8

Item 19: 87% + 50% equals?

Response	Percent
137%	84,9
68% - 59%	1,7
other	7,5
⌀	5,9

Item 20: There is a leaky faucet in which 5 drops of water drip every minute. How many drops of water are there in 1 hour?

Response	Percent
300	83,3
other (130;12; misc.)	8,8
ɓ	7,9

Item 21: Write 112% as a decimal.

Response	Percent
1,12	57,3
0,112	29,7
other	7,1
ɓ	5,9

Item 22: Which is larger?

20% of 65 or
65% of 25?

Response	Percent
second	71,1
first	21,3
same	0,8
ɓ	6,8

Item 23: 87% of the students passed a test. What percent failed?

Response	Percent
13	86,2
other	6,3
ɓ	7,5

Item 24: Write 3 1/4% as a decimal

Response	Percent
0,0325	16,7
,03 1/4	9,6
,325	9,6
3,25	23,4
other (,3 $\frac{1}{4}$; 3,$\frac{1}{4}$; misc.)	28,5
∅	12,2

Item 25: What is 1/2% of 100?

Response	Percent
1/2;0,5	30,5
50	25,5
50%	13,8
$\frac{1}{200}$;0,005	7,5
$\frac{1}{2}$%;0,5%	4,2
99 $\frac{1}{2}$	1,7
other	9,3
∅	7,5

Item 26: Write 12,8% as a decimal.

Response	Percent
0,128	42,7
12,8	21,8
1,28	9,6
other	16,7
∅	9,2

Item 27: 2:19 = .7:x Find x .

Response	Percent
66,5	22,6
66 - 67	7,1
24	10,5
other	29,7
∅	30,1

Item 28: The ratio of students who passed a test to students who failed was 2:3. If there were 60 students in all, how many passed?

Response	Percent
24	5,4
40	37,2
20	9,6
other	29,3
ø	18,5

APPENDIX V

A Comparison of Secondary II Success Rates with General Population (Percents were rounded to the nearest whole percent)

TEST I

Question	Secondary II	General Population
1	89	84
2	23	33
3	59	70
4	39	36
5	59	47
6	52	55
7	14	13
8	70	60
10	72	70
11	28	29
12	53	53
13	98	90
14	14	13
15	52	43
16	91	84
17	62	56
18	88	83
19	70	66
20	13	15
21	98	91
22	78	64
23	23	31
24	61	47
25	100	93
26	33	29
27	45	33
28	64	49

TEST II

Question	Secondary II	General Population
1	89	87
2	75	67
3	95	87
4	65	62
6	77	66
7	91	89
8	91	82
9	77	69
10	51	36
11	66	69
12	74	59
13	37	40
14	95	85
15	49	52
16	6	10
17	88	81
18	97	85
19	88	85
20	94	83
21	42	57
22	75	71
23	97	86
24	11	17
25	43	30
26	40	43
27	29	23
28	5	5