THE INCIDENCE OF TAXATION IN GROWTH MODELS

Jorge Perez

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ABSTRACT

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Jorge Perez

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Most work, theoretical and empirical, on the incidence of taxation, takes one theory of distribution, the neoclassical, as an adequate representation of economic reality. But the neoclassical theory is not the only paradigm that has been proposed to explain that reality; the works which restrict themselves to that theory are then incomplete and, more importantly, conditional on its validity. There are at least two alternative theories, which purport to explain economic reality, based on the ideas of Keynes and of Marx. Two stylized versions of those theories, presented in this thesis, have been labeled neokeynesian and neomarxian. We develop a comparative study of incidence in those three alternative frameworks, and explore their predictions regarding the balanced budget incidence of a tax on profits used to finance government

consumption. We find that, although the neomarxian theory predicts zero shifting of the tax, the other two models cannot yield precise predictions, if their defining parameters are not completely specified, either through a priori restrictions, or through empirical analysis. We carry out econometric estimations of those models, using non linear techniques, and test their capacity to represent the economic data. Our tests suggest that the neokeynesian is the best model. We use the econometric estimates of the parameters of the neokeynesian model to quantify the extent of incidence. The model suggests that a policy to increase taxation upon profits in order to finance additional government consumption results in a substantial increase in the gross rate of profit, to the extent of changing the distribution of income in favour of profits: the incidence of this policy falls on wages.

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CHAPTER 1

INTRODUCTION--INCIDENCE AND GROWTH

A) The Traditional Approach to Incidence.

With very few exceptions, most work on the incidence of taxation -- the effect of taxes on the distribution of income -- has been confined to mainstream neoclassical theory. It has been based on two more or less explicit assumptions: full employment (or a natural rate of unemployment) and ex-ante investment accomodating to ex-ante savings. This applies both to the theoretical work on incidence and to the design or, at least, the interpretation of empirical work on the subject.

It seems, indeed, as if the so called "Keynesian revolution", not to mention other dissenting theories, never reached the conventionally accepted theory of distribution and its offspring, the theory of incidence: Keynesian macro-economic theory rode high on the Kondratieff cycle; it rose from its bust in the thirties to its boom during the fifties and sixties and fell out of favour with the academic orthodoxy during the bust of the seventies and eighties, which has seen a revival of neoclassical macro theory, clothed in monetarist and rational expectations robes; but the the theory of incidence never shed its pre-Keynesian robes. Musgrave's attempt to divide the public economy in three branches -- allocative, distributive and stabilizing -- and assign to each of these a specific government function, shows this state of affairs quite clearly. While Keynesian theory would inspire the stabilizing function, the distributive function would constitute the realm of neoclassical theory.

There have been very few exceptions to this state of affairs. The most noticeable have been: the treatment of incidence in a short run Keynesian model by Kalecki (1937) the model proposed by Kaldor (1955-56) and the work of Asimakopoulos and Burbidge (1974), inspired by Kalecki (1937, 1971).

Most empirical research, both of short and long term incidence, has also been confined to a framework which can be considered mainly neoclassical (one recent exception is the thesis by Ghaeli, 1981): many past studies have dealt with short run incidence, and their findings have been interpreted in partial equilibrium terms, following a methodology first proposed by Krzyzaniak and Musgrave (1963); more current research has been based in stationary, general equilibrium, models, following a methodology proposed by Shoven and Whaley (1972).

As mentioned, neoclassical models are based on two principal assumptions: 1) The economy naturally tends, if left alone, to a state of full employment, or in more modern parlance, to a "natural rate of unemployment", and 2) Whatever is saved is invested; this is what is known as the "available funds" theory of investment. The models of growth introduced by Solow (1956) and by Swan (1956), as well as the incidence models inspired by them, such as those by Krzyzaniak (1967) and by Feldstein (1974), are of this type. Frequently, neoclassical models assume also a special type of consumption and saving behaviour of economic agents, based on life cycle (Modigliani and Brumberg, 1954) or permanent income (Friedman, 1957) theories, complemented with an assumption on the absence of socio-economic classes, such as workers and capitalists. An example of an incidence model incorporating these assumptions, besides the other two, is that proposed by Diamond (1970).

B) Beyond the Traditional Approach.

The neoclassical theory is not the only approach that has been proposed to explain the distribution of income. There are, at least, two other competing explanations. We shall label them, with S. Marglin (1984), the neokeynesian, and the neomarxian theories. Both these models drop the full employment postulate and the "available funds" theory of investment. In their place the neokeynesian models introduce an independent investment relation, which in the case of one of the tenants of this position, Joan Robinson (1962), postulates that investment is a function of expected profits. The neomarxian theory, on the other hand, proposes a theory of the determination of wages by reference to a "socially necessary" level of consumption, needed to keep the population in adequate working condition. This implies, as we shall see later, a constant share of wages in the value of the total product.

The neokeynesian and neomarxian theories postulate a heterogeneous population, in the sense that they admit the existence of distinct socio-economic classes, workers and capitalists. Also, in general, they do not define savings in terms of life cycle theories, but rather by means of propensities to save (or to consume), different among the different socio-economic classes, or alternatively, among different income types (e.g. wages and profits).

As mentioned before, the only examples that we know of application of these competing theories to the analysis of the incidence of taxes are Kaldor's (1955-56) article and the short term neokeynesian incidence model of Asimakopoulos and Burbidge (1974). No example of incidence in growth models, comparable to the neoclassical studies mentioned above, can be found.

C) Purpose of the Study and Statement of the Problem.

In this work we propose a theoretical and empirical analysis of the incidence of a profits tax in growth models. This entails not only the restatement of the theory of incidence in neoclassical growth models, but also the study of incidence in neokeynesian and neomarxian models. The empirical analysis entails additionally the construction of econometric models corresponding to the theoretical ones, the estimation and econometric testing of these models using some suitable sample of observations, the choice, if possible, of one model among those tested, as better representing the "real world" reflected in the sample, and last, the derivation of specific quantitative values for the incidence of the profits tax, using the parameters estimated in the chosen model. We should notice that a very important spin-off of this research will be the test of the alternative theories of distribution, using very recent econometric techniques.

D) Incidence Concepts and Measurement.

We have so far defined incidence simply as the effect of taxes and government expenditures on the distribution of income. It is necessary now to expand and clarify this concept. We shall do this in what follows. The discussion will not go into details and theoretical deductions, which will be presented in Chapter VI.

Bent Hansen (1967, p.93) has presented a very general definition of incidence, as "the effects on the real incomes in society of fiscal policy measures, other things being equal". By "fiscal policy measures" he means changes in the government policy variables, that is, those variables over which the government has direct control. Examples of these are the tax rates and government expenditure items, among budget policies; but the difinition is general enough to encompass changes on any other variables controlled by the state, such as those connected with the management of monetary policy, civil servants' wages, etc. We shall adopt this definition as a starting point in the discussion. As we shall immediately see, it covers the treatment of incidence traditionally followed in the field, based on full employment assumptions, as well as more general treatments not based on full employment, such as those stemming from Keynesian models.

The traditional treatment of incidence in a general, as opposed to partial, equilibrium context, utilizes concepts which are related to a neoclassical view of the economy; in this view, since full employment is automatically attained, much attention is paid to the stabilization of prices. The two traditional incidence concepts are those of balanced budget and of differential incidence.

Both these concepts are based on a balanced budget which is to be maintained so as to avoid, in the full employment system, inflationary or deflationary effects on the distribution of income (Musgrave 1959, p.212). The balanced budget incidence refers to the effect, on the distribution of income, of a change in a tax accompanied by a concomitant change in government expenditures, so as to leave the budget balanced. The fiscal policy measure is here, then a "tax - cum - expenditure" change.

The differential incidence concept refers to the effects on the distribution of income of a change in a tax accompanied by an opposite change in another tax, such that the total real tax yield is maintained as previously. This implies that the budget balance remains unchanged.

The differential incidence concept was first introduced by Wicksell, but it was not extensively used until Musgrave (1953a, 1953b) reintroduced it, because for a long period the theory of incidence was dominated by the partial equilibrium approach(1). One concept not based on a balanced budget is that called by Musgrave specific incidence. It refers to the distributional effects of changes in isolated taxes, which then entail a change in the budget balance. This, in turn, affects prices, even in

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(1) See for example the treatise by Dalton (1936)

the full employment models.

In a system where full employment is not assumed, such as the Keynesian, the differential incidence policy as defined above, will also in general result in changes in both employment and prices. This is also true of the balanced budget incidence. If other goals were desired by the government, such as to attain full employment plus price stability, the "fiscal measures" to be taken would have to specify changes in additional independent government policy variables (Hansen 1967, p.93). In this sense, the balanced budget, the differential and the specific incidence concept are in the same situation.

Incidence should always refer to complete policy packages or fiscal measures, as is specified in Hansen's definition. In certain circumstances, but not in all, some of these policy packages may be preferable to others.

What is important, when comparing the distributional consequences of alternative models, which is the problem studied in this work, is to use the same concept of incidence for them all. The models can then be compared with respect to the same "fiscal policy measure". We have chosen as this policy measure the balanced budget incidence of an increase in a tax on profits, when it is used to finance increased government expenditures on consumption goods. This particular measure has the advantage of producing relatively simple expressions for incidence, as we shall see in Chapter VI. From now on, when we shall speak of the incidence of a particular tax, we shall mean the incidence of a fiscal measure, or a fiscal policy package, built around the change in that tax.

In this work we shall only study functional incidence, that is, the effects of the fiscal policies on the distribution of income among factors of production. We shall not examine their effect on the distribution of household income (personal incidence).

For quantitative purposes we need an index, or measure, of the functional incidence of a fiscal policy action. The most natural measure of functional incidence and one that has apparently not been used in the literature, is given by the impact of the fiscal policy measure (in this case balanced budget tax versus government consumption change) on the ratio of the net of taxes profit bill to the net of taxes wage bill. We shall favour this measure. It permits one to answer questions such as: whether the change in fiscal policy alters the functional income distribution in favour of profits; or in favour of wages; or whether it is distributionally neutral, in the sense of keeping the distribution of income unchanged.

As mentioned, other measures have been used in the literature. They aim at reflecting the way the tax burden is divided between wages and profits. As Feldstein (1974) shows, in the general case, for this measure, it is necessary to consider total tax burdens, as opposed to direct tax burdens. The total tax burden can be defined as the total change in disposable real income caused by the tax change. The direct tax burden is defined as the change in the amount of revenue collected by the tax. The difference between the total and the direct tax burdens is referred to as the indirect tax burden. These quantities are, in general, negative.

The incidence measure or index proposed by Feldstein (1974), for his neoclassical (hybrid) model, which examines the balanced budget and the differential incidence of a general tax on profits, is equal to the ratio of the change in the net of tax profits to the change in the disposable income, which would result if the capital labour ratio were kept constant. He excludes the effect of changes in this latter ratio because he proves, by assuming a life cycle model of consumption and a neoclassical definition of capital, that it amounts simply to a change in the timing of consumption, and it does not change total welfare, as given by the utility level of individuals (Feldstein,1974,p 571n.; see also chapter 6, section B below).

In more general models, which do not use life cycle or permanent income theories of consumption, such as the neokeynesian and the neomarxian, this proposition would not be valid. The change in net profits and in disposable income caused by the variations of the capital labour ratio should, in this case, be included in the measure. If this is done, a more general measure is obtained, which includes Feldstein's as a particular case.

Feldstein's measure and its more general form present problems of interpretation in the (theoretically possible) case when the total variation in disposable income caused by a tax plus expenditure policy, that is, the total tax burden is positive. These problems are not present, on the other hand, with the measure we have favoured. Furthermore this latter measure presents clearer and more direct picture of the impact of fiscal policy in the distribution of income.

It will be shown in Chapter VI that, under the assumption of profit maximization, all the above measures or indices of incidence are functions of the negative of the elasticity of the gross of tax profits rate with respect to the complement of the tax, where by complement of the tax it is meant a variable equal to one minus the average profits tax rate. We shall denote this elasticity: Irg,tr.

Under the assumption of profit maximization, if the magnitude of the elasticity Irg,tr is known, as well as the average magnitude of some other parameters, the most important of which is the elasticity of substitution in production, the value of all other measures of incidence can be easily calculated. In many cases we shall deal directly with this elasticity as a measure of incidence, after having quantified its relation with the other measures. We shall call the elasticity Irg,tr the index of shifting of the profits tax, since it is related to the measures of shifting of this tax normally used in the incidence litterature. Other definitions of shifting, and the reasons to choose this one, are presented more fully in chapter VI.

E) Plan of the Study

We proceed as follows. Chapter II will present a review of the theoretical literature on the incidence of a profits tax in growth models. We shall see how this literature is limited essentially to the neoclassical model; that is to say, it assumes full employment growth and it takes ex-ante investments to be equal to ex-ante savings. As the only example of non-neoclassical incidence models is a short term model proposed by Asimakopoulos and Burbidge (1974), this one is also briefly presented.

In Chapter III we examine in turn the empirical literature on the incidence and shifting of profits taxes, most especially the corporation income tax. We discover that most work in this area deals with incidence and shifting of the tax in the short run. Only three authors (Adelman, 1957, and Lerner and Hendricksen, 1956) have dealt with incidence in the long run context, and their contributions are more than twenty years old. Although the empirical models in some instances, such as in Krzyzaniak and Musgrave's model, make some attempt to place themselves within a framework "general enough" to encompass neoclassical and neokeynesian short term theories, we shall observe that the interpretation given to the results seems to always come from a neoclassical vision of the economy. We shall also observe that the results of these works are inconclusive. The problem has not yet received a satisfactory solution.

Beginning with Chapter IV we start formulating a new approach. The first step, presented in that chapter, is to introduce three stylized models of growth and distribution -- neoclassical, neokeynesian and neomarxian. We follow here Stephen Marglin's representation of those models (Marglin, 1984).

The next chapter, V, presents modifications made to the basic model so as to introduce: a complete government sector with taxes, subsidies, consumption and savings; technical progress; non-linearities in certain consumption or saving functions; more realistic life cycles, etc.

The models can then be used to derive theoretical propositions concerning mostly the balanced budget incidence of a profits tax but also as a by-product, the incidence of some other budget policies (chapter VI). The balanced budget incidence of the profits tax is very thoroughly examined in the alternative models and a comparison is made. We discover that unless specific numerical values are given to the defining parameters of each prototype -- parameters such as elasticities of substitution, average and marginal propensities to consume, propensities to invest, etc. -- the incidence predictions of the models are, in most cases, very wide and overlapping. For the neoclassical and neokeynesian models, an increase in profits tax, in a balanced budget or differential setting can change the distribution of income in favour of wages, or in favour of profits or not at all. The neomarxian models, on the contrary, give a very definitive prediction of zero shifting of the profits tax. In general, then, the answer to the incidence question and to the problem of income distribution, requires two steps: First, choosing a theory which adequately represents the real world. Second, finding or defining a priori values for the parameters which characterize the chosen theory.

The rest of the study is dedicated to the solution of the problems presented by those two steps.

In what remains of chapter VI, we explore the a priori restrictions that are most commonly accepted by proponents of the alternative models, as well as those restrictions which result from assuming the models to be true and locally stable. We find that the stability restrictions alone are not enough to produce non overlapping incidence predictions. But the additional a priori conjectures on the value of certain parameters do result in narrower and non overlapping incidence predictions. So, for the balanced budget incidence of the profits tax versus government consumption, the neoclassical restrictions lead to a presumtion of partial shifting of the profits tax (the elasticity of the profit rate with respect to the profits tax complement, Erg,tr , is between 0.0 and 1.0). the neokeynesian restrictions lead to a prediction of overshifting (Erg,tr greater than 1.0); whereas,as mentioned,the neomarxian model predicts zero shifting (Erg,tr equal to zero) irrespectively of the restrictions on its parameters.

Turning now to the empirical determination of the parameter values, in chapter VII we present three economic models of growth and distribution corresponding to the theoretical models of Chapter V. These models are then examined for their econometric specification and estimated by three stage least squares, using a sample of pooled time series and cross country data.

In Chapter VIII we use the results from the estimated models to calculate the parameters characterising each model and examine their statistical significance. The magnitudes obtained for these parameters confirm that, in most cases, the a priori values normally conjectured in neoclassical and neokeynesian theories are of the right order of magnitude.

In Chapter IX we attempt to choose a model among the basic models examined and among their secondary variants. For this, we use econometric procedures developed recently to test for separate families of hypotheses. The results of our tests suggest that the neokeynesian model is a better representation of the empirical data. Then, the evidence gathered in this and the previous chapter is employed to quantify the extent of the balanced budget incidence of the profits tax. We find that the point estimate of the incidence measure shows that the tax increases substantially the rate of profits, to the extent of changing the income distribution in favour of profits. A statistical test shows that the relative increase in the rate of profits is significantly different from zero (at 5% level of significance).

We have then arrived at a dual conclusion. First, our data and models are consistent with a neokeynesian world. Second, the parameters and variables governing this world are of such magnitude that a tax imposed on profits changes in the long run the distribution of income in favour of profits. Wage earners pay for the tax.

CHAPTER II

THE THEORETICAL LITERATURE ON INCIDENCE IN GROWTH MODELS

A. Introduction.

The treatment of incidence in the framework of growth and indeed in the context of general equilibrium is a very recent event in modern economic writing. Although in earlier periods, classical writers such as Ricardo and neoclassicists from the Austrian and Scandinavian schools, particularly Wicksell, had dealt with incidence in models akin to the more recent growth and general equilibrium models (1), during the first half of this century

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(1) For a summary of these early theories see Musgrave(1959, pp. 388-399) and Pedone (1966). Musgrave criticizes Ricardo's theory for using the malthusian concept of subsistence wage, and Wicksell's for using the "arbitrary assumption of a fixed wage fund." He contends the marginal productivity theory is a superior alternative

the analysis of incidence was limited mainly to Marshallian partial equilibrium.

Under certain conditions laid down by Shoup (1969, p.9), partial equilibrium is adequate for the analysis of the incidence of narrowly based taxes. But for broadly based taxes such as a general tax on profits or the corporation income tax, it is necessary to utilize general stationary or growing equilibrium analysis. Early modern attempts in this direction were made by H.G. Brown(1939), Rolph(1952) and Musgrave(1953).

These works are not yet presented as completely

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to these theories. But his critiques are unfounded; marginal productivity relations of the neoclassical type are not incompatible with the Ricardian-Malthusian theory of wages; they can be integrated with the latter, although the implicit causality of the resulting system is different from the neoclassical case.

Wicksel's theory, on the other hand, is a genuine general equlibrium model in stationary state: population growth, savings and net investment are zero. Hence for this system it is correct to have total capital (the "wage fund" in this Bohm-Bawerkian system) fixed. See also Pedone's critique (1966) of Musgrave's point (Pedone's comment is somewhat vitiated by faulty mathematics: his system requires maximization of the profit rate and not of profits as he states).

defined analytical models. They assume stationarity rather than steady growth and, at least Brown and Rolph do not treat the incidence of the profits tax. Nevertheless it is important to show in a few paragraphs their way of reasoning, which opened the road to the analytical models.

Brown argued that a general tax on sales or output would not be borne by consumers (shifted forward) through price increases, as was generally believed then; but on the contrary would be shifted back to factor owners through reductions in factor prices. He separated in his analysis money and real effects. He stated that there is no connection between the tax and an "increase in the volume of circulating medium"..."Therefore, there is no basis in monetary theory for supporting the hypothesis that a general tax on all goods will make average prices permanently higher". He had argued previously that wages and other factor incomes are flexible downwards and that their supply would not be affected by the general sales tax; therefore prices would not change, and he could conclude: "If then, there is a general tax on output, the money incomes received by labourers, capitalists and landowners, must be reduced"...but, in any case, the incidence is in "practical effects" the same as if it raised all prices..."without either decrease or increase in money incomes."

Rolph (1952) further developed Brown's argument and arrived at the conclusion that, "All partial systems of excises (including those consisting of a single commodity tax) lower money incomes of owners of resources in the taxed field, of those producing products complementary to taxed items and of those whose sources are competitive in the supply side with resources in either of these two fields". (1952, p.116).

One very crucial assumption in Rolph's analysis is that an increase in taxes is not accompanied by an equal increase in government expenditure. (1952, pp. 118,119), that is, he is analyzing specific tax incidence, and his results depend on this assumption. Buchanan (1960) criticised this assumption as being illogical,since Rolph assumes as constant things which "by the nature of the analysis should vary" (Mieszkowski,1969). In fact, as discussed in the introduction, it is not a problem of logic which is involved but of what definition of incidence is being used, and of the framework within which this incidence is assumed to come about. It depends also on the time period chosen for the analysis.

Musgrave (1953a, 1953b see also, his textbook, 1959) introduced several models to discuss incidence: an "all consumption model", where there is no growth; a "capital formation model", with growth and savings in a neoclassical setting; and a "liquidity preference model", with monetary considerations. These models can be considered as attempts at a qualitative formulation of incidence in "general equilibrium terms". .

These preliminary efforts to analyse incidence in the general equilibrium framework culminated in Harberger's (1962) analytical model of the incidence of the corporation income tax. According to G. Break (1974, pp. 123-124) Harberger's model was inspired by the general equilibrium theories of international trade, in particular by the analysis of J.E. Meade (1955).

Harberger's idea was to divide the economy into two sectors: a corporate sector with a tax on profits, and a non-corporate sector with no profits tax. He also assumed fixed amounts of factors of production, capital and labour, i.e. a stationary state, as in the international trade model: full employment of factors of production and perfectly flexible prices, perfect competition in factor and product markets, profit maximization, constant returns to scale, identical consumers and no money illusion.

With these assumptions, he proved analytically that, although in the short term the tax would be borne by the corporations (in the sense that the gross of tax profit rate would not increase with the tax), in the longer term there would be movements of capital towards the non-corporate sector, such that a new equilibrium would be attained with equal net of tax rates of return in all sectors. Hence in the long term the tax could at least be partially shifted through these movements (that is the gross of tax rate of return would not fall by the total amount of the increase in the tax rate). But by assuming certain values, which he considered plausible, for the parameters in his model (income elasticities, elasticities of substitution etc.) he arrived at the conclusion that the tax was mainly borne by the corporations. In any case, his model demonstrated the theoretical, if not empirical, possibility of shifting in the neoclassical framework through intersectorial movements of capital.

In the next chapter we shall see how this theoretical possibility was also demonstrated in the context of neoclassical growth models through the iso-morphic mechanism of intertemporal "movements of capital" (savings/investments) caused by a tax on profits.

Harberger's model was extended by Mieszkowski (1967) to a very general analysis of differential tax incidence in the stationary state; by McLure (1970, 1974, 1975) to the analysis of interregional incidence, to incidence with a monetary sector, to expenditure incidence, to incidence with factor immobility, etc. The model was also applied to the analysis of unionization in the distribution of income (Johnson and Mieszkowski, (1970); the property tax (Mieszkowski, 1972); tariffs (Mieszkowski, 1966) etc. This model has been very fruitful in neoclassical incidence analysis of the stationary state. (See also more recent developments of this model by: Anderson, and Ballentine,1976; Ballentine and Eris (1975); Roskamp

(1977); Shoven and Whalley (1972), and others).

Harberger's model, as mentioned, is stationary; it ignores growth, capital formation and, in a certain sense, savings. The introduction of these factors is the subject of growth models of incidence, which will mainly occupy us in this work. In the next section we shall describe in detail the research done in this area. We shall be able to observe that the growth models of incidence presented in the literature are mainly in the neoclassical tradition. By models in the neoclassical tradition we shall understand those which assume two conditions: (1) full employment growth attained by perfect price flexibility in all markets; and (2) no independent ex-ante (planned) investment function, which implies that ex-ante investment is equal to ex-ante savings. The models in this tradition have also, in general, assumed that consumption results as the outcome of intertemporal utility maximization behaviour by households, as is the case in "life cycle" or "permanent income" models. A third characteristic of most neoclassical models, arising from this fact is, then, that consumption depends on expected wealth or permanent income, it is sensitive to the rate of interest, and it does not depend on the functional distribution of income. The basic neoclassical growth model, which we shall present later, assumes this "life cycle" savings behaviour. But we shall consider the full employment assumption as the most important characteristic of neoclassical models. If they do not have the life cycle savings assumptions, we shall still classify them as neoclassical, although of a hybrid type.

In what remains of this chapter we shall study, in section B, the literature on the incidence in neoclassical growth models. In section C, we shall study a case of incidence in a neokeynesian model.

B. <u>Incidence in Growth Models.</u>

The early application of growth models to the effects of taxation were inspired by models such as those proposed by Swan (1956) and Solow (1956). These models explicitly used production functions which allowed variable proportions as representations of technology . This device combined with the assumption of flexible wages clearing the labour market, allowed the models to reach stable full employment paths. In our terminology they showed neoclassical behaviour (1). Sometime after, R. Sato (1963) presented an analysis, inspired by these models of the time that it would take an economy to adjust to a change in an income tax (See also Kazuo Sato (1966)). Marian. Krzyzaniak presented a series of articles directly inspired by Swan's model, where he analysed the

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(1) Since for us it is given by full employment plus no independent investment function, and these models did not have one. It is important to notice here that the

incidence of a general tax on profits (Krzyzaniak 1967), and of a differential tax on profits (Krzyzaniak 1968). These two models used Cobb-Douglas production functions; which imply an elasticity of factor substitution of one. But Krzyzaniak (1970) also analysed the consequence of other substitution hypotheses by introducing CES functions. More recently, Feldstein (1974), Grieson (1975) and Gupta (1976) have used more general formulations of the production function to analyse the incidence of taxes on profits and on wages. Diamond (1970) has also used general production functions, plus an explicit "life cycle" saving function to analyse the differential incidence of a tax on interest versus a head tax. Finally, Ballentine (1978) tried to analyse the incidence of the corporation income tax in a growth model which is intended as a generalisation of Harberger's (1962) static model.

A model which is quite representative of this neoclassical growth vintage is that proposed by Feldstein

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crucial assumption of the above models is not so much the variable proportions as the fact that, either implicitly or explicitly, they assume that the marginal net product of capital is always positive, regardless of how high the capital-labour ratio rises. If this assumption is not made, even with variable proportions the growth path presents unstable behaviour of the Harrod type. This is very clearly proven by Eisner (1958).

(1974). We can use it as a basis for the exposition to follow, and mention the extensions and modifications which the other models introduce upon this basic one.

Feldstein analyses the "balanced budget" and the differential incidence of a profits tax in a growth model. For his balanced budget analysis, Feldstein assumes that the government does no investment (savings) with the tax.

Feldstein's basic model is constituted of seven relations. First, a production function, linearly homogeneous, expressing output per unit of labour, y, as a function of capital per unit of labour, k:

$$y = f (k) \quad (1a)$$

the next two are just the marginal product conditions expressing profit maximization and perfect competition in factor and product markets:

$$d f / d k = f' = rn (1 + tr) \quad (1b)$$

rn is the net of tax rate of return to capital; tr is the tax on profits;

$$y - k f' = w; \quad (2)$$

w is the wage rate. The next relation expresses savings per unit of labour as a combination of workers savings and savings on capital:

$$s = sw\ w + sp\ rn\ k \quad (3)$$

where sw is the rate of savings on wages, it is assumed to be a function of the rate of interest:

$$sw = sw (rn) \quad (4)$$

sp , the rate of saving on profits, is also a function of the rate of interest:

$$sp = sp\ (\ rn\ ) \qquad (5)$$

The last relation is the savings investment equilibrium condition for the steady-state:

$$s = n\ k \qquad (6)$$

where n is the rate of growth of the labour force. Notice that, in this equation, it is assumed that the government does no savings. The model is complete now with those seven relations in seven unknowns:

y, k, rn , w, s, sw , sp

This is a natural rate of growth model, and investment in it is equal to whatever savings happen to be: there is no independent investment function. In the sense we defined before, the model is then in the neoclassical (understood as full employment) tradition. The model is though a hybrid one, because it does not use the basic neoclassical consumption function, but rather one very similar to a consumption function proposed by Kaldor (1955-56) although with savings propensities varying with the rate of interest. Indeed, the model is very similar to Kaldor's own model, which is in fact, also a full employment model. Combining equations (1), (2), (3) and (6), it is possible to obtain an equation in terms of k and exogenous variables:

$$n\ k = sl\ [\ f\ (k) - k\ f'\ ] + sk\ k\ f'\ (\ 1 + tr\ )^{-1} \qquad (7a)$$

where

$$sl = sl\ [\ f'\ (\ 1 + tr\ )^{-1}\ ] = sl(k,tr) \qquad (7b)$$

and

$$sk = sk [f' (1 + tr)^{-1}] = sk(k,tr) \qquad (7c)$$

Feldstein uses the system given by (7a) to (7c) to study the effects of varying the tax tr. He obtains expressions for

$d k / d tr$, $d rn / d tr$, $d w / d tr$, etc.

From these he derives an expression for the balanced budget incidence of the tax, tr.

He proposes a measure of incidence of the profits tax which is given by the following expression:

$$If = k\Delta rn / [k\Delta rn + \Delta w] \qquad (8)$$

or, at the limit using the symbols d rn and d w for the differentials of rn and w:

$$If = k\, d\, rn / [k\, d\, rn + d\, w] =$$

$$[k\, d\, rn/ d\, tr] / [k\, d\, rn/ d\, tr + d\, w/ d\, tr] \qquad (9)$$

Notice that Feldstein's measure, If, does not take the total change in profits:

$$d\, rn\, k/ d\, tr = k\, d\, rn/ d\, tr + rn\, d\, k/ d\, tr \qquad (10)$$

but only that part, $k\, d\, rn/ d\, tr$, which is due to the change in the rate of return, rn . Feldstein reasons that, since $k = K / L$ changes with the tax, contrary to what happens in stationary state (or static) models, it is not adequate to use the total change on profits, since the other part of the change, $rn\, d\, k/ d\, tr$, corresponds simply to a change in the timing of consumption and brings no net gain or loss of utility. This is based on the assertion that "the net rate of return measures (to a first order of approximation) the value of the individual's time preference." With this premise, Feldstein gives a proof of the previous statement.(1) (A more detailed discussion of this can be found in chapter VI).

Having defined a measure of incidence in the budget case, Feldstein goes on to obtain an expression for this measure when the tax on profits changes from tr to tr + d tr.

He obtains:

$$If = 1 / [1 + J] \tag{11}$$

where :

$$J = \frac{(1 + tr)\ Aw\ El + (1 - Aw)\ (1 + Ek)}{Aw + Ek,l - 1} \tag{12}$$

El , Ek are the elasticities of savings out of wages and out of net profits, respectively; Ek,l is the elasticity of factor substitution; and Aw is the share of wages in total income. He asserts that, for all plausible values of the parameters, J > 0 and therefore the profits tax is at least partially shifted towards wages. The above expression is particularly sensitive to the parameter Ek,l; for example reducing Ek,l from 1 to 2/3 would double J. Feldstein analyses several numerical cases; particularly one with Ek,l = 1, fixed savings propensities (El = Ek = 0) and with Aw = 2/3 ; if then Sk = Sl capital bears two thirds of the burden; if instead Sk = 2 Sl , capital only bears half of the burden, passing the other half on to wages. Feldstein then concludes that, for realistic parameters, there is a substantial shifting of the general tax on profits; but he qualifies this conclusion, since the magnitude of the effect is in part due to the fact that the government does no investment (savings) with its tax revenue... "the transfer of funds from the private sector to the government therefore decreases national savings and reduces the capital intensity of production. This lowers the real wage and increases the gross rate of return".. (Feldstein, 1974).

Therefore if a term for savings out of government revenues were introduced in equation (6) above shifting would be less pronounced.

Feldstein (1974) also presents a differential analysis of the incidence of the profits tax which, he states, "abstracts from the problem" of government savings. His conclusions are that for the differential incidence of a profits tax against a wage tax, and for reasonable values of the parameters, the incidence is still ..."divided between capital and labour, with capital bearing the larger share but labour still bearing a significant portion".

Grieson (1975) has built a neoclassical model of growth incidence similar in most aspects to Feldstein's, but he considers explicitly the government budget equation in a very simplified way; he assumes that the government spends a constant proportion of the net national product and it has a marginal propensity to save (s), out of its tax revenues; he then arrives at a savings expression slightly more general than Feldstein's and analyses the differential incidence of a profits rather than a wages tax. He concentrates his analysis on somewhat different problems, such as that of the optimum level of taxation.

Krzyzaniak (1967) offers a model which is essentially similar to Feldstein's, just examined. Inspired by the growth model proposed by Swan (1956), he uses, instead of equation (1) above, a Cobb-Douglas production function with Hicks-Neutral technical progress, :

$$q = y = \exp\ (R'\ t)\ k^{1-b} \qquad (1c)$$

where b is the labour elasticity of production and R' is the rate of growth of Hicks-neutral technical progress. He also assumes full employment growth, all government income consumed (in his words "wasted"), hence no government savings, and a private savings function as follows (in our notation) :

$$s = [\ w + (1 - \theta)\ rn\ k\ ]\ sy + \theta\ rn\ k \quad ;\ \text{or}$$

$$s = w\ sy + [\ (1 - \theta)\ sy + \theta\ ]\ rn\ k \qquad (13)$$

where w, rn and k are as before, wages, net of tax profit rate and capital. θ is the proportion of profits retained (and saved) by enterprises; and sy is the savings propensity of households. Because of the explicit consideration of retained earnigs, this savings hypothesis is not totally similar to the basic neoclassical savings hypothesis; the expression after the second equality in (13) suggests again a certain similarity to Feldstein's essentially Kaldorian consumption function (3) above. Hence, the two models are, as stated, essentially similar to the neoclassical hybrid type.

Krzyzaniak (1970) later generalized this model to introduce the possibility of factor substitution in production, by using a CES production function rather than the Cobb-Douglas. Again this model is essentially similar to Feldstein's. One difference of form in Krzyzaniak's analysis is that he examines sizeable, rather than infinitesimal, changes in taxes and he uses a measure of incidence inspired by Dalton, which defines the tax burden as "tax induced losses to real income of persons", but these do not affect the final content and the conclusions of his analysis. The conclusions are essentially similar to those attained by Feldstein.

Gupta (1976) proposed an analysis of incidence which departs more importantly from that by Feldstein. He observed that Feldstein's model uses a Kaldorian savings function, (Kaldor, 1956; also 1957 and 1972) based on the concept of "two classes of income". Gupta replaced this function by a "generalized version" of Pasinetti's (1962) saving function based on the concept of two distinct socioeconomic classes. He then reformulated Feldstein's model with the modification:

$$n\ kw = sww\ [\ f(\ k\ ) - k\ f'(\ k\ )\ ] + spw\ rn\ kw \qquad (14)$$

$$J^n = s_{pc}\ r_n \qquad (15)$$

$$r_n\ (1 + t_r) = f'(\ k\ ) \qquad (16)$$

s_{ww} : is the workers saving propensity out of wages; and

s_{pw} : their saving propensity out of profits.(1)

s_{pc} : is the capitalists saving propensity out of profits. Equations (14) and (15) replace equation (7A) in Feldstein's model above. If workers' savings are small enough, so that $k > k_w$, the model will tend towards a Pasinetti steady state. In this case, Gupta shows ,r_n is independent of the tax t_r and the tax is shifted 100% to wages. Gupta shows that this is true even when the savings propensities are functions of r_n. But Gupta fails to show that if the workers' savings propensities out of profits and of wages are equal and if, additionally, $s_{pw} = s_{ww} > (1 - A_w)\ s_c$ the system tends towards an equilibrium where workers own all the capital. In this "Dual-Pasinetti" equilibrium (2) ,equation (15) does not hold anymore and is replaced by:

$$f(\ k_{,}\ )\ /\ k = n\ /\ s_w \qquad (15a)$$

where:

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(1) Gupta uses two savings propensities for workers, s_{pw} and s_{ww}; hence his statement that equation (14) is based on a generalized version of Pasinetti's saving function.

(2) See Samuelson and Modigliani (1966a, 1966b). They call this equilibrium "anti-Pasinetti" equilibrium .

$$sw = sww = spw \tag{17}$$

the system formed by (14), (15a), (16) and (17) has also a recursive solution. In fact equation (21a) corresponds to a model like Solow's (1956) one sector neoclassical model. Notice that equation (15a) can be solved for the equilibrium value of k as a function, ø1 of n and sw:

$$k^* = \phi 1\ (n, sw) \tag{18}$$

By marginal productivity, the wage rate w is:

$$w = f(k) - k f'(k)$$

hence in equilibrium

$$w^* = f(\phi 1) - f'(\phi 1) = \phi 2\ (n, sw) \tag{19}$$

and from (18) and (16):

$$rn^* = f'\ \phi 1 / (1 + tr) = \phi 3\ (n, sw) / (1 + tr) \tag{20}$$

Hence Feldstein's incidence measure would be:

$$If = \frac{k^*\ d\,rn^* / d\,tr}{k^*\ d\,rn^* / d\,tr + d\,wn^* / d\,tr} = \frac{-\ \phi 3\ k\ (1 + tr)^{-2}}{-\ \phi 3\ k\ (1 + tr)^{-2}}$$

$$= 1.0$$

This simply reflects the fact that the net of tax rate of return falls whereas the wage rate stays the same, therefore the tax is fully borne by profits, contrary to Gupta's Pasinetti case; paradoxically, though, the tax is also fully borne by workers who are the only capital owners in this dual economy!

Another model in the same vein of Feldstein's model, and antedating it, is given in Diamond's (1970) analysis of the incidence of a tax on interest income combined with a lump sum subsidy to savers which leaves the government budget unchanged. Diamond studies a neoclassical economy with perfectly flexible wages and profits; production represented by a constant returns to scale production function, the labour force growing at the rate 100 n% per year; investment is whatever savings permit it to be. In the above circumstances competition assures that the economy can attain, and stay on, a full employment path.

The special characteristic of Diamond's model is that it analyses explicitly savings as a process of maximization of intertemporal utility, in a sort of "life cycle" model. The model is then a neoclassical basic model in the sense defined above. By assuming, in a two period framework, that both present and future consumption are "normal goods", he is able to show that an increase in a tax on interest, accompanied by an increase in a lump sum subsidy, increases the optimal consumption of the first period and hence decreases savings (Diamond, 1970, pp. 212-215). From here on it is easy to see how Diamond's model yields conclusions similar to Feldstein's. Since savings decrease, in the steady state, capital per unit of labour will decrease; therefore since profit maximizing is assumed, the wage-gross of tax rental ratio will decrease. Hence, Diamond (1970, p.224) can conclude: "Investigation of a simple competitive model has shown that the differential incidence of an interest income rather than a lump sum tax raises the gross of tax interest rate and lowers the wage." Obviously the degree of movement in the relative wage/gross profit ratio depend on the shape of the production function, particularly on the elasticity of substitution. The model gives conclusions similar to Feldstein's, that in a growth model a tax on interest income is at least partially shifted because savings and the capital intensity of the economy are decreased. The differences of the two models are less important than this common feature arising from their basic neoclassical characteristics. So the fact that the one uses a Kaldorian savings function whereas the other uses some sort of one class life cycle savings is not an essential difference. Indeed the assumption of "normality" of present and future consumption made by Diamond implies that the elasticity of savings vis-a-vis the net rate of interest is at least positive, which is also what is implicitlly assumed by Feldstein. (1)

Another very important aspect of the incidence models treated in the literature is the analysis of the time it takes a neoclassical economy to adjust to a change in a tax on profits. This problem was treated originally by Ryuzo Sato (1963); his conclusions were later generalized

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(1) Diamond's model is mathematically more sophisticated and treats explicitly the transient states

and modified by Kazuo Sato (1966).

Ryuzo Sato uses a neoclassical model with a Cobb-Douglas production function having Hicks-Neutral technical progress, very similar to the model later used by Krzyzaniak (1967), which was presented above. Hence the production function is, in our notation, like equation (1a):

$$q = y = \exp(R' t)\ k^{1-b} \qquad (1a)$$

and by marginal productivity:

$$rn\ (1 + tr) = (1 - b)\ q\ /\ k \qquad (21a)$$

and

$$w = b\ q \qquad (21b)$$

Hence the savings equation (13) can be written:

$$s = \{ sy\ b + [(1-b)\ /\ (1 + tr)]\ (1 - \theta)\ sy + \theta \}\ q$$

$$= sa\ q \qquad (22a)$$

with

$$sa = sa(\ tr;\ b,\ \theta,\ sy\ ) = sa\ (\ tr\ )$$

Hence equation (6) can be rewritten:

$$s = \dot{s}a\ \ q = n\ k \qquad (6a)$$

Define the output capital ratio δ as:

$$\delta = Q\ /\ K = q\ /\ k$$

Then (1a) can be rewritten:

(out of the steady state) ; but its sophistication is to a certain point a drawback because it somehow "hides" the results. Also Feldstein's formulation is more flexible and can yield more empirically testable propositions .

$$\gamma = \exp(R' \quad t)\ k^{-b} \tag{23}$$

Take logarithms of (23) differentiate with respect to time and replace $d\,K / d\,t = d\,(k\,L) / d\,t$ (L = Labour) by sa.q.L, from (6a), to obtain

$$d\,\gamma / d\,t = b\,[\,(R' / b + n\,)\gamma - sa\,\gamma^2\,] \tag{24}$$

with $d\gamma / d\,t$, the time derivative of γ. This is a Bernoulli differential equation whose integration gives

$$\gamma = \frac{R' + b\,n}{b\,\{\,sa + B\exp[\,-(R' + b\,n)\,t\,]\,\}} \tag{25}$$

where B is a constant of integration.

Using formula (25), R. Sato derives an expression for the time, TR, that it would take the variable γ to get to x% of its final steady state value when the tax tr changes from tr(0) to tr(1). This expression is:

$$TR(x) = \frac{\ln\{\,1 + s(tr(0))\,x \;/\; [\,s(tr(1))\,(1 - x)\,]\,\}}{R' + b\,n} \tag{26}$$

where S(tr(0)) and S(tr(1)) are the savings proportion out of national income obtained when the tax rates are tr(0) and tr(1) respectively. R. Sato also discusses verbally the adjustment period which would result from dropping the

Cobb-Douglas assumption and using a more general production function. He arrives at the conclusion that "it is safe to say that the larger (the elasticity of technical substitution) the shorter the adjustment period" (R. Sato, 1963, p. 21, n. 2). He finally calculates the time of adjustment which would be obtained with realistic parameters: 1.3% for the rate of technical progress ; 65% for the share of labour, b; 1.5% for the rate of growth of the labour force, n; 17% for an initial proportionate tax rate on income, which, with his hypothesis on the shape of the saving function, gives s(17) = 11.66%. When the tax rate is increased to 18% the savings rate becomes s(18) = 12.54%. He thus obtains that for a 10% adjustment four years must pass; for a 50% adjustment thirty years. For a 70% adjustment fifty years and for a 90% adjustment, one hundred years. These are very long periods; they would be even longer for Ek,l <1; although for a higher rate of population growth or of technical progress they would shorten substantially.(1)

Kazuo Sato (1966) modified Ryzuo Sato's model by introducing embodied technical progress in a model where gross investment is a constant fraction of gross output;

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(1) For example the elasticity of tx with respect to n is -n / (n + b h) ; and that of h is -b h / (n + b h) ; so if n passes from 1.5% to 3.0% the times of adjustment above will be reduced by 60%;that is a 50% adjustment will be

and where capital goods differ in technical efficiency according to their vintages, and have a rate of "radioactive" decay (depreciation) of 100 % per unit of time. The rate of increase in the technical efficiency of capital goods is given by u. With these modifications the relation between the time of adjustment in Ryuzo Sato's model, TR, and that in Kazuo Sato's, TK, would be, in or notation, and assuming that 1 - b, which represents the capital share, is approximately the same when measured net and gross of depreciation:

$$TR(x) / TK(x) = (R' + b\ n) / [R' + b(n + d) + u] \quad (27)$$

We can see that the two times of adjustment differ only in the addition of a rate of depreciation d and the rate of embodied technical progress u. Indeed assuming no depreciation of capital as is done in Ryuzo Sato's model the only difference is the addition of u : technical progress behaves as if it increased neutrally by 100 (n + u) % per year in Kazuo Sato's model, whereas in Ryuzo Sato's, it increases at 100n % per year. Kazuo Sato's correction boils down to using a higher value for the technical progress parameter and taking account of the acceleration caused to the process by depreciation. If,

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obtained in 12 years and a 70% one in 20 years; similarly, if h doubles from 1.5% to 3.0%, a 50% adjustment will be obtained in 18 years and a 70% one in 30 years.

with Kazuo Sato, we accept values for the embodied technical progress of between 0 and 6%, say 3%, and for the depreciation rate of 8%, the values calculated by Ryuzo Sato with all his other parameters will be reduced in the proportion:

$$TR(x) \ / \ TK(x) = .23$$

that is to say, the times of adjustment would be one fifth of those given by Ryuzo Sato. Even assuming a zero percent rate of increase in the efficiency of embodied capital,u , but keeping the depreciation at 8%, we would obtain:

$$TR(x) \ / \ TK(x) = .32$$

a time of adjustment one third that given by Ryuzo Sato. So a more realistic calculation of the time of adjustment to 50% of final value would be: between 6 and 10 years; for 70% of final value: 10 to 17 years; and for 90% of final value: 20 to 30 years. The above figures do indeed have empirical relevance. Kazuo Sato shows that his result, concerning the rate of depreciation, hinges crucially upon his assumption that gross investment is proportional to gross output. He shows that if, instead, it is assumed that net investment is proportional to output (gross or net), that is to say, depreciation is automatically replaced, and if additionally u is assumed to be zero, the model reduces to that of Ryuzo Sato. Obviously, the assumption of automatic replacement is less realistic than the other. The shorter adjustment times are indeed, more realistic.

So far, we have examined the one sector growth models. But incidence has also been treated in two sector growth models by Krzyzaniak (1968) and by Ballentine (1978). Let us look very briefly at those models. In particular let us examine what they add to the conclusions of the one sector models. The two sector models are built with the intention of obtaining a better representation of the effects of the corporation income tax. As is well known, this tax, at least in the United States and Canada, is not "general" but "partial". That is to say, it only covers the part of the profits created in the economy which accrue to incorporated firms. Non-incorporated firms have their profit income taxed at a lower rate, and it is this differential between the two taxes which corresponds to the partial tax on corporations in Harberger's static model. In the same way, Krzyzaniak (1968) introduces in a growth model "a la Swan" two sectors, one incorporated and whose profits are taxed, and the other non-incorporated and with untaxed profits. There are now two production functions. One for the corporate sector:

$$Q1 = \exp(R1' \ t) \ K1^{a1} \ L1^{b1} \qquad (28)$$

and a second for the non-corporate:

$$Q2 = \exp(R2' \ t) \ K2^{a2} \ L2^{b2} \qquad (29)$$

where Qi , Ki , Li are respectively, total product, capital and labor and Ri′ is the rate of Harrod Neutral Technical Progress in sector i (i = 1,2); the two products of these sectors are now assumed to be different; hence the quantities produced of each cannot be added; instead Krzyzaniak aggregates them by using an index number formula:

$$Q = Q1^{m}\ Q2^{(1-m)} \qquad (30)$$

where Q is an index to represent the total product of the economy ("national product"). Next, Krzyzaniak assumes that both capital and labour are homogeneous (the same assumption is made by Harberger (1962)); they can then be added to obtain the total capital stock, K, and total labour, L. He studies what he calls "momentary equilibrium";it could also be called the "Harberger effect". This effect can be described by considering the following problem: What happens to production, profits, wages, etc., when after the imposition of the profits tax capital moves from the corporate to the unincorporated sector in such a way as to equalize the net of tax rate of return among these sectors? In other words what are the Qi Ki , Li , and the marginal products which satisfy:

$$(1 - tr)\ \partial Q / \partial K1 = \partial Q / \partial K2 \qquad (31)$$

and

$$\partial Q / \partial L1 = \partial Q / \partial L2 \qquad (32)$$

where tr is the "differential" or "partial" tax on profits (on unincorporated businesses). Using the three relations above he shows that Q can be represented by an aggregate production function:

$$Q = utr\ MO\ \exp(R'\ t)\ K^{a}\ L^{b} \qquad (33)$$

where

$$a = m\ a1 + (1-m)\ a2$$

$$b = m\ b1 + (1-m)\ b2$$

$$R' = m\ R1' + (1-m)\ R2'$$

$$utr = (1 - tr)^{m a1} / (1 - a1\ tr)^{m}$$

and MO is a constant which depends on the ai's, the bi's and on m, (i = 1,2), but not on the tax tr. The other aggregate equations can now be written in exactly the same way as in Krzyzaniak's (1967) one sector model. Hence from here on, the model reduces to the one sector model and its conclusions as to the incidence of the partial profits tax are the same as those for the general profits tax, except for the term utr; this term represents the "Harberger effects" assúmed by Krzyzaniak to occur in the "medium run". The long run effects examined in the previous model are now multiplied by (utr) [illegible].

It can then be concluded that, in Krzyzaniak's model, the effects of the differential tax in the long run are separable into two. First, the effects caused by the initial differential introduced by the tax between the corporate and the non corporate sector rate of return; second, the effects caused by the tax acting as if it were general, on the growth relations of the model. Krzyzaniak concludes in this model that for realistic values of the parameters, the "Harberger" or medium run incidence is very similar to that obtained by Harberger in his own model (this is shown by a value of utr around 1.0): the tax is borne slightly more than 100% by profits; the differences in incidence measure stem from the fact that Harberger excluded excess burden from his measure (so his value is 1.0 exactly; Harberger shows that the use of a Cobb-Douglass production function implies that the tax is exactly borne by profits). As for the long term, the conclusions of the one sector model are obtained again, but the shifting of the profits tax is somewhat greater (since it is compounded by a function of utr). In conclusion, it can be said that the consideration of two sectors in Krzyzaniak's model adds very little to the results given by the one sector growth model.

Ballentine (1978) also attacked the problem of the incidence of the tax in a two sector neoclassical growth model. Contrary to Krzyzaniak, he uses general forms for the production function; he identifies the non-corporate sector with the sector which produces only consumption goods, the corporate sector on the other hand produces a good X, which can be either invested or consumed. Another very important difference with Krzyzaniak is his assumption about the behaviour of the government. Ballentine assumes that the government saves as much as the private sector whereas Krzyzaniak assumed that the government spends all its revenues in consumption. Ballentine's model, as Harberger's indeed, avoids the bias that this causes in Krzyzaniak's model, which showed exaggerated "efficiency" effects (excess burden). . Since Ballentine also assumes that in the private sector workers and capitalists have the same saving propensities, the effects of his assumption on government behaviour is that savings are a proportion of national income, which depends only on the rate of interest but not on the distribution of income among government and private sector, nor within the private sector. These are the most important differences between Krzyzaniak's model and Ballentine's. Ballentine obtains expressions for a "momentary" and for a long run equilibrium in his model. The former is obtained by assuming instantaneous adjustments to any disturbances, such as taxes, with total factor supplies (capital and labour) given exogenously at a constant "instantaneous" level (Batra and Casas (1971)); the latter, as is known, obtains when capital is allowed to grow by the accumulation of net investment. Ballentine does not present in the article the full procedure whereby he arrives at the equilibrium expressions.

He concludes that, although in the "momentary equilibrium" (quite similar to Hargerger's expression), the tax is mostly borne by profits, for "reasonable" values of the parameters (those used by Harberger with some modifications), in the long term the tax is shifted substantially. The shifting is the greater, the smaller are the technical elasticities of substitution, and the greater is the interest elasticity of savings. Even for constant saving propensities (which is the assumpution used by Harberger) the tax is slightly shifted in the steady state growth as compared with the static Harberger's results. Again, these conclusions do not differ substantially from those obtained by Feldstein, Krzyzaniak, Gupta, Diamond and the others in their simpler one sector models.

C. The Neokeynesian Models and Incidence.

All models of incidence in the previous section are primarily neoclassical. They assume full employment growth, and do not assume independent investment behaviour as it would be represented by an ex-ante investment function. The literature does not seem to present any growth model of incidence which does not have these neoclassical characteristics. But there are several outstanding short run models which treat incidence in the non-neoclassical framework. These are the models of

Kalecki (1937) and of Asimakopoulos and Burbidge (1974) which can be considered in the neokeynesian tradition (see also Burbidge (1975, 1976) and Asimakopoulos (1975)). They introduce ex-ante investment as an independent variable and allow for the existence of unemployment.

A very simplified version of Asimakopoulos and Burbidge (1974) model (with only a profit tax) can be represented by a macro-economic system of equations as follows. Denominate Q, as total real output in the economy; L = total level of employment; a: average output per man (assume it constant); $w0$: money wage (assumed constant); m = a mark-up coefficient (average markup on prime (or variable) costs); p: factor cost price of output; C : total real private consumption; I : total real planned gross investment; G : total real government expenditures; Π : total profit bill in money terms; S: total gross money savings; tr = profits tax; $(1-\theta)$: proportion of profits paid out to rentiers (Distribution ratio); sr : ratio of savings to distributed profits for rentiers. The system is then:

A production equation

$$Q = a\,L \qquad ; \qquad L < L < Lf \qquad (33)$$

where Lf is total labor force and L is total level of employment at normal plant capacity.

A mark-up equation representing pricing:

$$p = (1 + m)\,w0 \,/\, a \qquad (34)$$

The consumption investment identity and the wage profit identity of national accounts:

$$Q = C + I + G \tag{35}$$

$$p\ Q = w0\ L + \Pi \tag{36}$$

The behavioural savings expression: Savings are a consequence of retained profits θ and a parametric savings propensity of rentiers sr (Notice a very important assumption: no savings out of wages):

$$S = (1 - tf)\ [\theta + sr(1 - \theta]\ \Pi \tag{37}$$

The savings-investment relation and the Government Budget equation (in this simplified version, a Balanced Budget):

$$S = p\ I \tag{38}$$

$$p\ G = tr.\Pi \tag{39}$$

Finally, two more relations are needed to close the model. The first is taken by Asimakopoulos and Burbidge from Kalecki's (1937) theory and constitutes a very important element in their model; they assume that in the short run gross private planned (ex-ante) investment is given and fixed.

$$I = I0 = \text{constant} \tag{40}$$

The last closure relation can be given in several ways; two of those ways are as follows. A first, competitive way, where they assume that aggregate demand is high enough to cover marginal (average variable) costs in all plants; therefore these produce at capacity and employment is fixed by the available plant; assuming additionally that full capacity of plant is just enough to give full employment, L = Lf , the models close with:

$$L = Lf \qquad (41)$$

In the second way, the non-competitive, the above full employment equation is replaced by an equation assuming that there is an exogenously given mark-up:

$$m = mo \qquad (42)$$

This procedure is justified by reference to a price leader model of oligopoly (Asimakopoulos and Burbidge, 1974, p. 271;see also Asimakopoulos, 1975); in this second case employment, L, becomes an endogenos variable and its equilibrium value will in general be different from the full employment value Lf.

Now we can deduce the incidence of the profit tax from this model. From the saving equation (38) the savings-investment equality (38) and the given investment (equation 40) follows the equation:

$$\frac{\Pi}{p} = \frac{IO}{(1 - tr)\,[\theta + (1 - \theta)\, sr]} = \frac{IO}{(1 - tr)\, sp} \qquad (43)$$

where we have renamed $[\theta + (1 - \theta)\, sr] = sp$: the average propensity to save out of profits.

This equation shows real profits Π / p as a function of the profits tax and the given propensity to save out of profits sp . It is a remarkable result because it shows that in this model the real profit bill depends only on the three relations used above to obtain equation (43). In particular, real profits do not depend on the conditions of production, nor on the pricing conditions (competitive or non-competitive), nor on the existence or not of full employment.

Notice that equation (43) implies that if the tax rate tr is increased then real profits will increase; indeed the derivative of real profits with respect to the tax rate is:

$$(\Pi / p) / \quad tr = IO / [(1 - tr)^2 sp] =$$

$$(\Pi / p) / (1 - tr) > 0$$

Notice also that profits will grow if the propensity to save out of profits sp falls. This is indeed Keynes's "widow's cruse".

This model presents a very striking contrast to the neoclassical short run one sector model. In the neoclassical model a tax on profits is fully borne by profits in the short run. In this neokeynesian model the profits tax is fully shifted.

The possibility of full short run shifting in this model is a consequence of two assumptions: The treatment of planned investment as exogenous in the short run, and the treatment of savings as dependent only on profits. These two conditions determine the short term distribution of income, by fixing the level of real profits; then "the real wage rate takes the value that is consistent with this level of profits. It is determined by the conditions in the commodity market" (Asimakopoulos and Burbidge, 1974, p. 279). In the neoclassical model, by contrast, the wage rate is determined in the labour market in such a way as to assure equilibrium of supply and demand for labour at full employment.

Asimakopoulos and Burbidge use what can be considered a classical savings function; all savings come from profits, workers do not save at all. This was the same assumption made by Kalecki in his seminal (1937) article. It is responsible for the fact that their model shows full short run shifting. If it is dropped, and their classical savings function is replaced by a Kaldorian savings function with an extra term for savings out of wages (sw w0 L), the model becomes substantially more complicated and interdependent. Profit taxes will not be fully shifted anymore; but it is possible to show that there will still be partial shifting (This can be seen for example in annex 1 to chapter III). The model is therefore still different from the neoclassical short run model which implies zero shifting when there is profit maximization.

The most important lesson to be drawn from the neokeynesian short run model examined is that total or partial shifting of a profits tax in the short run can occur, even if the economy is competitive and there is full employment. We shall come back to this point when we examine the empirical studies of incidence, which is the subject of the following chapter.

CHAPTER III

THE EMPIRICAL STUDIES OF INCIDENCE

A- Introduction

Most empirical studies of incidence have concentrated on the problem of short run shifting. Of the studies to be reviewed, only three out of sixteen deal directly with long run incidence, and all three have been written before Krzyzaniak and Musgrave's (1964) work on the short run shifting of the Corporation Income tax.

Our empirical research will concentrate on long run incidence. Strictly speaking, a review of previous work would deal with those three articles and leave the rest out. But some of the aspects touched on by the short run shifting literature are relevant to the long run, since, as we shall see, they both stem from some underlying distribution theory.

In section B, we shall briefly present the long run studies. In section C, we shall introduce the short run studies and the controversies surrounding them. We shall in particular observe that most of these studies have been presented from a microeconomic viewpoint: the theory of the firm. Macroeconomic considerations have entered the picture only indirectly if at all, as uneasy grafts onto essentially partial equilibrium models.

B) The long run studies.

These were the studies made by Lerner and Hendricksen, Adelman, and Zellner's comments on the latter, which used simple techniques to discuss the evidence available. Adelman arrived at the conclusion that the corporation income tax was borne by capital in the long term, by examining the share of profits gross of taxes on all income originated in corporations. He compared the average share in the pre-depression years 1922-29 (23%) with the average share in the post war period 1946-1955 (23.2%) and concluded on no-shifting in the face of their constancy. Zellner criticized this conclusion as "ill founded" because the framework of statistical analysis used by Adelman was defective, since he did not control for other possible factors which affected the tax in the long run. He also showed how the use of rates of return instead of income shares pointed towards long run shifting. For these reasons he deemed the evidence to be inconclusive.

In their very interesting analysis, Lerner and Hendricksen (1956) discussed both short run and long run incidence. In the short run they used a cross section of manufacturing industries classified into ten sectors. They examined the instances when a change in tax rate had been accompanied in each of the sectors by a change in the (after tax) rate of return on total investment in the same direction, or in opposite direction, or by no change at all, for each year between 1928 and 1952. They concluded that the overall evidence was not consistent with the hypothesis that a change in the corporate income tax is completely passed on in the short run; it would be at least partially absorbed by corporate capital. In the long run though, they found that, in spite of trends which brought tax rates from a level of 9% to one of 63% the trend of the rate of return after tax was constant. This can be interpreted as evidence of long run shifting. The authors try to relate the constancy of the rate of return to long term forces like technical innovations, large increases in capacity utilization during World War II and after, and illusory effects of inflation. They conclude recognizing the need to pursue the matter both at empirical and theoretical levels and with a warning against concentrating on the analysis of short run (single year) effects only.

C) The short run studies.

If the evidence of the simple long run studies is inconclusive, the situation is not any better when we examine the short run studies with their more elaborate econometric methods. Most of these studies deal with the tax on corporate profit income.

These studies can be classified according to two criteria. The first criterion is the stand they take vis-a-vis shifting. The second criterion is the method or model used.

According to the first criterion, we can divide the studies as those which claim to be consistent with no shifting of the tax; those which claim to be consistent with shifting, and those which are inconclusive. Of the thirteen short run studies we shall examine, there are six studies which claim to be consistent with short run shifting. These are Krzyzaniak and Musgrave's (1964) study, the most prominent; those inspired by their methodology: Spencer's (1969) Canadian study and Roskamp's (1965) German one; the two cross section studies of Kilpatrick (1966) and Levesque (1965); and Dusansky's (1972) study.

On the other hand, there are five studies which claim to be consistent with very little or no short run shifting. These are Hall (1964), Turek (1970), Oakland (1972) and two studies by Gordon (1967, 1968). The three studies which are not conclusive arose all as corrections to Krzyzaniak and Musgrave's study. They are those of Goode

(1966), Slitor (1966), and Cragg, Harbergr and Mieszkowski (1967). In fact, what these latter studies say is that the Kržyzaniak and Musgrave model is very sensitive to changes in specification and to the introduction of "pressure" variables to capture the economic cycle; if these variables are introduced the models do not support the shifting hypothesis. In a certain sense then, they can be interpreted as inconsistent with shifting.

According to the second criterion--the method or model they use -- the studies can be classified into three broad categories. First, those which use a regression with the profit rate as the endogenous variable to be explained, the tax rate or some other unit tax indicator as the principal explanatory variable and some other explanatory variables as controls. These models do not explicitly assume profit maximization (i.e. marginal productivity). In this category are the models by Krzyzaniak and Musgrave, Spencer, Roskamp, Goode, Dusansky, Gordon, and Slitor, Cragg, Harberger and Mieszkowski. Krzyzaniak and Musgrave, and related models, also claim to be reduced forms of "very general" macroeconomic models.

A second category is constituted by those models which, in contrast to Krzyzaniak and Musgrave (1964), are explicitly based on an assumption of the long run validity of profit maximization. These studies do not explicitly specify a complete model of the economy. Therefore they do not test directly the validity of the neoclassical theory of distribtuion (that is, they do not explicitly assume a natural rate of employment), although they seem to be implicitly based on it. The models in this category are those of Hall, Turek and Oakland.

A final category is constituted by the two cross section studies of Kilpatrick and Levesque, which are based on the analysis of concentration ratios and their relation to shifting.

We shall use this second classification in the discussion to follow.

The models which use a regression of the rate of profit against the tax rate or another tax indicator as principal explanatory variable can be expressed in the following form:

$$rg = f(Z; X1, X2...Xn) \qquad (1)$$

$$rg = \Pi / (p K)$$

where rg is the real rate of profits; Π are profit in money terms; p is a price index, K is the capital stock; Z is either the tax rate:

$$Z = tr = Tr / \Pi$$

where Tr is the total tax bill,

or another tax influence indicator, such as, for example, taxes per unit of capital. The Xi (i = 1, 2...n) are other explanatory variables. The justification of this model is some behavioural hypothesis on the way the firm will react to taxes. For example Krzyzaniak and Musgrave justify one of their models (model "A") by the assumption that

"... the firm adjusts itself so as to increase the gross rate of return sufficiently as to recoup a given fraction of the negative rate of return (defined as the ratio of tax liability Tr to capital) suffered from the tax". (Krzyzaniak and Musgrave 1964, pp. 35-36).

Most models which correspond to equation (1) assume linearity. They can therefore be written in the form:

rg = a1 Z + a1 X1 + ... + ai Xi +... + u

where u is a stochastic error term.

The first and best known model of this group is that of Krzyzaniak and Musgrave (1964). They propose two models, "A" and "B". In model A, the tax variable Z would be taxes per unit of capital: Z = Tr/K (the "negative rate of return"). In their model "B", the tax variable Z is given by the tax rate: Z = tr = Tr / M.

Musgrave and krzyzaniak add other variables to their "core" models A and B, to try to control for effects other than those caused by the tax changes on the rate of return. They justify the introduction of the other variables with a very general macroeconomic model. (Krzyzaniak and Musgrave 1964, pp. 35-36) From their macro model they arrive at the conclusion that a reduced form expression for the gross of tax rate of return would contain along with the tax rate Z, other exogenous or lagged endogenous variables. These are, after dropping those which proved not to be very significant: the capital stock at the beginning of the period K(t-1); government expenditures G; the lagged change in consumption expenditures in money terms C(t-1); the lagged ratio of inventories to sales V(t-1) and other taxes (different to the corporate tax) J. Writing in linear form and normalizing the change in consumption and government expenditures by a division by the GNP, produces model B, which is a purely reduced form expression of the rate of return with no contemporaneous endogenous variables, and which can then be estimated by least squares to arrive at consistent and unbiased results.

Model "A" has a similar linear expression, except for the tax variable, which in this case is T / K(t-1); this model is then not fully reduced, because this term is endogenous, as it is equal to the product of the endogenous variable rg by the tax rate.

Krzyzaniak and Musgrave estimate model A, which is their preferred behavioral model, by the technique of instrumental variables, using as an instrument the effective tax rate $Z^* = Tr / \Pi g$ which proves to be highly correlated with T / K(t-1). In their preferred estimation of model A, they drop the government expenditure variable which is not significantly different from zero, and arrive at coefficient for T / K(t-1) of 1.34, highly significant, which they interpret as consistent with the hypothesis that the tax is overshifted in the short run. They carry on many other multiple regressions with models A and B, at the aggregate level and by some industrial sectors. In all cases they conclude that the evidence is consistent with the hypothesis of full shifting of the corporation income tax in the short run.

Krzyzaniak and Musgrave's model was also applied with very slight modification for Canadian data by Spencer (1969) (1), and for German data by Roskamp (1965). These two models arrive at conclusions similar to their parent model: the evidence is consistent with approximately full tax shifting.

It comes as no surprise that the conclusions and the methodology used by Krzyzaniak and Musgrave have been severely criticized. The essential critique is that the model is incorrectly specified. This criticism has been waged by, among others, Goode (1966), Slitor (1966), Cragg, Harberger and Mieszkowski (1967), and Gordon (1967, 1968). Goode and Slitor were the first to note that Krzyzaniak and Musgrave's model did not seem to really separate the influence of other variables related to cyclical fluctuations. The tax coefficient therefore, ..."measures whatever influence corporate tax rates may have (on rates of return in the short term) plus part of the influence of other variables with which the tax rates are associated." (Goode 1966, p.228). The introduction of a pressure

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(1) An important difference which was to be expected in an open country with such big trade with the USA is that an additional export variable was very significant in the Canadian model.

variable such as the ratio of actual to potential GNP, along with the other variables used by Krzyzaniak and Musgrave, reduced one of the shifting coefficients obtained by them from 142% to 94.2%. (Slitor 1966, p.159)

Krzyzaniak and Musgrave objected to the use of "pressure" variables, such as the ratio of actual to potential GNP or a rate of capacity utilisation, to capture cyclical movements of the economy, on the grounds that these variables, used without lags, as Slitor and Goode do, depend on the tax rates and on other exogenous variables, and are therefore endogenous. (Krzyzaniak and Musgrave 1966, page 248). In this case the technique of least squares would give biased and inconsistent estimates.

Cragg, Harberger and Mieszkowski (1967) showed that the bias so caused could be in the direction of an overestimation of shifting rather than an underestimation. They started stressing the fact, already noticed by Goode and by Slitor, that the corporation income tax rose always in periods when the general economic environment was also very favorable to raising profits, such as the period before the U.S. entered the second world war and the Korean war period, and were relatively low in periods in which the economic envirohment was also very unfavorable to profits, such as the depression years between 1935 and 1939. (The Krzyazaniak and Musgrave data cover the two periods 1935-42 and 1948-59).

In the light of this evidence of a "spurious correlation" between the corporation tax rate and the gross of tax rate of return of corporate capital in manufacturing they conclude that other forces besides the tax were "...clearly at work influencing profit rates..." which may, as in the cases of World War II and the Korean war years "...have led to statutory tax rates being high under circumstances when profits were also high." (Cragg, Harberger, Mieszkowski 1967, pp, 812-813).

If Cragg, Harberger and Mieszkowski are right, then Krzyzaniak and Musgrave left out of their model a variable or some variables, which were positively correlated with both, the before tax rate of return Yg, and the tax variable. It is well known that this makes the coefficient estimated with the mispecified equation upward biased. (Johnston 1972, pp. 168-169; Maddala 1977, pp. 155-157). Hence Krzyzaniak and Musgrave's shifting coefficient would be overestimated. Cragg, Harberger and Miezkowski show that even when the technique of instrumental variables is used in the estimation, the bias of the estimated tax coefficient is still positive if their hypothesis about the cyclical variables omitted by Krzyzaniak and Musgrave's estimations are correct. They also prove that to add an endogenous pressure variable, (as Goode and Slitor did) as a proxy for their postulated truly exogenous cyclical variables results indeed, as Krzyzaniak and Musgrave stated, in a biased estimated coefficient for the tax influence. But the bias is positive if, as Cragg, Harberger and Mieszkowski (1967, PP. 813 and 816) conjecture, the correlation between the omitted exogenous variable and the rate of return is positive; its correlation with the endogenous pressure variable used as its proxy is positive; if the endogenous pressure variable is only a function of the truly exogenous cyclical variable and the tax rate; and if the influence of the latter on it is negative. These conditions, together with the fact that certain statistical moments calculated on the sample data have the right signs, assure that the bias is positive. Hence the new coefficient would be overestimated.

Cragg, Harberger and Mieszkowski recalculate Krzyzaniak and Musgrave's regression number 2 , for Model A, introducing first, as a pressure endogenous variable, the rate of employment and second, another additional dummy variable (deemed exogenous), to account for the second world war and the korean war years. The shifting coefficient is reduced in this way from 151% to 102% and to 60% respectively. In the latter case with both pressure and dummy war variables, it also becomes statistically insignificantly different from zero. And these coefficients overestimate the true shifting coefficient. Hence, they conclude, the empirical data do not support Krzyzaniak and Musgrave's hypothesis of more than full shifting.

Krzyzaniak and Musgrave in their response (1967) object to the introduction of the war dummy as arbitrary. Instead, they recalculate their equations by suppressing completely the war and mobilization years. The calculated shifting coefficient is still greater than one (115%), even with the additional pressure variable, but it is barely significant (at a 5% level of significance) and it still remains upward biased. Indeed, the Krzyzaniak and Musgrave (1967, p. 788) argument, that the direction of the bias of introducing the pressure variable is not necessarily positive, is not pertinent to the logic of the Cragg, Harberger and Mieszkowski (1967, p. 776) arguments. As the latter mention in their rejoinder, the crucial point of all their discussion is to show how sensitive are Krzyzaniak and Musgrave's results to even "small and quite plausible changes in specification".

Gordon (1970, p. 377) also notices another important problem in Krzyzaniak and Musgrave's analysis: its failure to control adequately the long term influence of increased productivity of capital on the rate of profit. Krzyzaniak and Musgrave's shifting measure is affected by the changes in the capital output ratio or its inverse, the average productivity of capital, which reflect long run responses. According to Gordon, this explains why Krzyzaniak and Musgrave's shifting measure using rates of return, differs so much from their measure using factor shares. (1)

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(1) They use this measure in chapter 2 of their book,

Gordon (1968, p. 1365) also examines the predictive power of Krzyzaniak and Musgrave's equation and finds it very low. For example their equations fail completely to explain the collapse of profits in 1929 and 1932 and the boom on profits in 1963-65 when fitted to appropriately extended time series. Krzyzaniak and Musgraves's model compares, vis-a-vis predictive capacity, very unfavourably with Gordon's own shifting model.

Spencer's application of the Krzyzaniak and Musgrave model to Canada can also be submitted to most of the above criticism, in view of the close relation between the Canadian and American economies, and in particular of their business cycles and long term trends.

Roskamp's (1965) similar study on Germany has two additional problems. First, it is not strictly comparable to Krzyzaniak and Musgrave's study because the tax coefficient in the German case included, besides the corporation income tax, a host of other taxes, such as business property taxes, real estate taxes, inheritance taxes, and other levies (Roskamp, 1965, p. 249). Second, Roskamp used, in his estimated equation, the unlagged ratio of changes in private consumption expenditure to gross national product. Contrary to the corresponding variable used by Krzyzaniak and Musgrave, this is an unlagged

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see also page 65 particularly note 4. The above criticism is also mentioned by Oakland (1972).

endogenous variable. Hence Roskamp estimates are biased and inconsistent. Gordon's criticisms are also applicable to Roskamp's results.

Another study which arrived at results consistent with shifting was Dusansky's (1972) model. The equation that he estimates, by two stage least squares is of the linear form of equation (2) above. Apart from the tax variable Tr / K, his other explanatory variables are taken from two microeconomic theories of the firm: profit maximization (variable: labour-capital ratio); and behavioural theories of the firm (variables: real wages, real price of materials, ratio of potential to actual GNP, etc.). The estimated equation is part of a large system of 16 equations in 16 endogenous and 13 exogenous variables. Some of these equations specify a macro-economic model.

Dusansky estimates his principal equation by two stage least squares and obtains a shifting coefficient of 102%. He states that his results are consistent because he used two stage least squares. This is true if the model is well specified; but his model is not well specified since the macro economic equations in it lack the corporate tax and other tax variables. Hence, his results have a specification bias.

A model which can be cast in terms of equation (1) above, but which reaches conclusions contrary to Krzyzaniak and Musgrave's model is that formulated by Gordon (1967).

Gordon starts with the formulation of a micro-economic model of mark-up pricing in a non-tax world, consistent with a theory of "satisficing". As Gordon well stresses, shifting may not automatically occur in the mark-up setting. It occurs only if firms are not at the profit maximizing level when the tax is imposed. From this basic hypothesis Gordon arrives at an equation expressing the price, p, charged by the firm as the product of a mark-up fraction, m, and the average cost at capacity output, C* (1). From this price he obtains total revenue and total "cash flow" (total revenue minus total labour and material costs) as a function of the mark-up and the different elements of costs, wages, prices of materials and labour and material output ratios (or their inverses, average productivities). Unfortunately, the equation he obtains cannot be statistically estimated because there is not, in most of the period of Gordon's data, information on wage cost (fixed and/or variable) and material cost data. Gordon is obliged to make the additional hypothesis that labour costs are linear functions of the average productivities of (fixed and variable) labour and the general price level, and that material costs are proportional to the price level. In this way, he can

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(1) This average cost contains an element of fixed cost given by part of the labour force: the non-production workers, considered fixed; but it does not include depreciation, depletion and interest paid.

obtain a basic preliminary equation in the absence of taxes. He then introduces taxes in his model by assuming that firms are able to shift part (a %) of their profits tax liability. By subtracting an allowance D for depreciation, depletion and interest paid; and normalizing the equations by a division by total capital K, he then obtains what can be considered his essential model. To this one he still introduces a pair of additional terms. The first is to take account of the fact that he is approximating costs by prices and that these could diverge in the short term, since they have different short run cyclical patterns. Hence profits would vary positively with the change in output (1 / Q)(d Q / d t). A second correction is to take account of the fact that inventory valuation profits will vary positively with price changes in the sector: (1 /p)(d p / d t).

The equation he obtains is non-linear in its coefficients. Gordon estimates it by least squares, using an iterative method to solve the non-linearity problem. He estimates it at the aggregate level for all the manufacturing corporations, and also for twenty sectoral groups. He concludes that the evidence is consistent with the hypothesis that the tax was not shifted globally: the point estimate of the shifting coefficient is 14.3% for a period similar to that used by Krzyzaniak and Musgrave (1935-41 plus 1948-59) and 11.0% if the period is extended backwards and forward (1925-41 plus 1946-62). In both cases the coefficient is not significantly different from zero (Gordon 1967, Table 1).

Sectorially, Gordon obtains that eight out of twenty manufacturing sectors were able to shift between 42% and 48% of the tax significantly, and the industries which behaved this way were the most concentrated. On the other hand, he found that the less concentrated industries showed negative shifting (that is, a negative relation between the rate of profit and the tax). Gordon's results are in direct contrast with the Krzyzaniak and Musgrave results (see also Gordon 1968).

We now come to the second group of studies, those based directly on an assumption of the long run validity of the marginal productivity relations (1). They are the studies by Hall (1964), Turek (1970) and Oakland (1972). The three are based again on hypotheses of the microeconomic behaviour of firms. To these basic hypotheses are grafted some extra variables which are supposed to control for macroeconomic factors.

Hall's approach to the problem was quite different from

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(1) That is, of profit maximization; notice that assuming the long run validity of profit maximization has little to do with the neoclassical theory of distribution; the latter, in our definition (Chapter 1), has as crucial assumption the existence of a "natural" rate of unemployment.

that of Gordon. He started with Solow's work on production functions and technical change (Solow,1957). Hall reasoned as follows. Let the output, Q, at current prices, p, in the economy be: $pQ = p(W + \Pi n + Tp + D)$

Where: W is the wage bill, Πn is the net of tax profit bill, Tp is total profit taxes in the economy and D is total depreciation. Then the observed capital share, Ar,o is: $Ar,o = (\Pi n + Tp + D) / (p Q)$

The tradional view is that profits represent a charge against capital. In that case, the output share attributable to capital is: $Ar,g = (\Pi n + Tp + D) / (p Q)$ which is equal to the observed share Ar,o. But if the tax is totally shifted, say to wages, the output share attributable to capital is, rather: $A'r,g = (\Pi n + D) / (p Q)$ which is less than the observed share Ar,o; that is, the latter "exceeds the productivity of capital by at least the amount of shifted tax" (Hall,1964, p. 260). Hall then asked whether an assumption of shifting different to the traditional one would give a better fit to the aggregate production function, after correcting the data for technical progress. Hall assumed that technical progress has been Hicks neutral and constructed three capital share series, corresponding respectively to the following assumptions: 1) that the tax is not shifted; 2) that the tax is fully shifted to wages; 3) that it is taken as a sales levy on output.He then regressed these series against capital per unit of labour, using Cobb-Douglass functions and chose as his preferred shifting hypothesis that of the three which resulted in a better fit. He used as a criterion of fit the R-squared values. The zero shifting assumption had an R-squared of 0.9722, the shifting to wages had 0.9360 and the "sale levy" shifting had 0.9455. Hall then interpreted his results as consistent with the zero shifting hypothesis conditioned to the assumption that technical progress was neutral. Musgrave criticised Hall's results; he alleged that the small difference between them was not enough to support Hall's conclusion. But as Mieszkowski (1969) has observed, $(1-R^2)$ is a better indicator of goodness of fit than R-squared, and there was a substantial difference vis-a-vis the latter indicators.

Turek (1970) generalizes Hall's model, but unlike Hall, she estimates technical change simultaneously with the tax effect, in regressions explaining the ratios of relative factors shares. She also introduces the tax rate as one of the independent variables explaining directly the ratio of relative factor shares. (Turek, 1970, p. 130).

She uses a CES production function in the formulation of the problem because this function admits the possibility of non-neutral technical progress. Her basic assumption is that firms minimize costs so that the observed ratio of factor inputs will be proportional to factor prices. She assumes that monopoly rents, if they exist, are constant through time. She then obtains an equation expressing that the ratio of marginal productivities of factors is proportional to factor prices. Introducing a CES production function, and assuming that labor and capital efficiencies increase at constant (but different) exponential rates by the action of technical progress, she obtains an expression which indicates that the logarithm of the ratio of factor shares, wg / (k rg), is a linear function of the logarithm of the capital labour ratio, K PK(O) / [L PL(O)], and of time. Expressed this way the equations assume that observed factor shares reflect the marginal productivity of factors. Hence it assumes that the tax on capital is not shifted. If the tax were completely shifted, the gross of tax profits would be affected by the tax rate. By this sort of reasoning, she can introduce the tax rate or rather its complement (1-tr) as an explanatory variable along with the capital labour ratio and time.

Turek introduces three more variables; two macroeconomic variables, which are expected to account for changes in the business cycle: the rate of unemployment, U and its rate of change plus one, $1 + \Delta U / U$; and a dummy, to account for a change in classification in the Income and Product accounts of the United States in 1947. The regression which she actually estimates is then:

$$\ln (wg / k\, rg) = \ln ao + a1 \ln (1 - tr) + a2 \ln[K\, PK(O) / (L\, PL(O)] + a2 (RK' - RL') - a3 \ln u + a4 \ln (1 + \Delta U / U)$$

the coefficients of this function are: a2 = Ek,l / (1 -

Ek,l), where Ek,l is the elasticity of factor substitution; RK' and RL' are the rates of increase in capital and labour efficiencies, which are given by the symbols PK(t) and PL(t) ;and a1 is a coefficient which is directly related to the degree of tax shifting in the short term. Her estimation of this equation by least squares gives shifting coefficients which are not significantly different from zero (at 95% level of confidence), with point values between 0.12 when statutory tax rates are used, and 0.35 when effective tax rates are used. Furthermore, using a likelihood ratio test, Turek concludes that the odds are two to one that the degree of shifting is near zero rather than 50% (and much higher odds that shifting is low rather than 100%). Hence, she concludes that her results are consistent with the hypothesis of no shifting or very little shifting. As a by-product of her regression, Turek also estimates that the elasticity of substitution of labour for capital ranges from 0.32 to 0.45, and that technical progress is more labour augmenting than capital augmenting, but the absence of "t" values does not permit to conclude if in fact technical progress is significantly non-neutral. Turek's results do not depend, as do Hall's, on the hypothesis that technical progress is Hicks-neutral.

Oakland (1972) is the last author we shall discuss who presents results consistent with the zero shifting hypothesis. His model is essentially developed around the result that, in the long run, profits are determined by the marginal productivity of capital. In the short run, this level of "normal profits" is affected by "demand conditions" in the economy, which are reflected for example in the level of capacity utilization. So, his fundamental equation expresses simply that the rate of profits on capital (Π / p k) is a function of the labour capital ratio, technical progress and actual lagged capacity utilization. If it is further assumed that there can be shifting in the short run, profits gross of tax will additionally be affected by the tax rate tr, (or its complement the retention rate, 1-tr). Finally he assumes a functional form linear in the logarithms.

Oakland estimates several versions of his basic equation by simple least squares. He finds that the retention ratio (1-tr) adds nothing to the explanatory power of the estimating equation; its coefficient is small in absolute value and statistically insignificant. He concludes that his results "strongly support the hypothesis that manufacturing firms do not engage in short run shifting". (Oakland, 1972, p. 241).

The final group of studies which we shall review correspond to the cross section analyses of Kilpatrick (1965), for the USA, and Levesque (1967) for Canada. These two studies use the same methodology. They test the hypothesis that, since shifting at the microeconomic level has to be accompanied by some monopoly power, higher changes in profits arising from tax changes are positively related to market power, as measured by concentration ratios. They choose two years separated by a substantial tax change, and relate the change in profits between these two years to the concentration ratio of approximately one hundred industrial sectors. They add to this basic relation other variables to take account of cyclical variations, special trends, disequilibrium profits, etc. From this basic multiple regression, they obtain a coefficient of change in profits per unit of the concentration measure. By assuming additionally that zero concentration corresponds to zero shifting, hence to zero change in gross of tax profits, they finally obtain a measure of the degree of shifting. Both authors estimate this degree to be around 100%.

Kilpatrick's and Levesque's works are consistent with two hypotheses. First, that profit rates are related to the concentration ratio. Second, that there is full shifting of the corporation income tax. But this second proposition depends on the crucial assumption that zero concentration ratio implies zero shifting. In fact this assumption has been contradicted by Gordon's work. Gordon (1967, p. 751-753) found that zero concentration corresponds to negative shifting of the corporation tax (1). Gordon shows that, this being the case, the average

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(1) We shall see that for a situation of excess burden

shifting coefficient of manufacturing corporations is not different from zero, even if some very concentrated industries present shifting coefficients of up to 95%.

As Mieszkowski (1969) notes, Kilpatrick and Levesque were not very successful either in isolating the non tax influence in their model. In Levesque's equation , profits seem to depend mainly on concentration, whereas Kilpatrick's equations show a positive relation between profits and concentration, even for years for which there is no change in the tax.

D. Conclusions

The only clear conclusion which can be drawn from the review of the econometric models of short run incidence is that the question has not yet been answered in a satisfactory way by these empirical works. The evidence they present is conflicting and inconclusive.

The evidence is conflicting since, among the studies reviewed here, six are consistent with shifting; three can be interpreted as inconsistent with shifting or at least inconclusive (those criticizing Krzyzaniak and

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(initial taxes different from zero), this is possible, under certain conditions, in neoclassical and neokeynesian models.

Musgrave's model); and five are consistent with no shifting or very little shifting. But all fourteen models can be found to have econometric or methodological problems which make them inconclusive. Let us examine these problems.

Krzyzaniak and Musgrave's model presents both problems of econometric specification and of methodological inconsistency. This is also true of the models which follow a similar methodology, those by Spencer and Roskamp.

Let us first discuss the inconsistency, which was first noticed by Burbidge (1976). Krzyzaniak and Musgrave propose, as a justification of their equations, a macroeconomic model which in their words is "written in a sufficiently general form to avoid any dogmatism regarding the choice between Keynesian vs Neoclassical or competitive vs imperfectly competitive systems." (1964, p. 33) But in the interpretation of their results they implicity reject the validity of any macro model other than the neoclassical.

This is clearly seen in the conclusion they draw from the results showing full shifting of the tax. They affirm that their results show the traditional theory of the profit maximizing firm is wrong. This is true only in a partial equilibrium model of the firm or in a neoclassical full employment model, where the real wage rate is totally determined by full employment plus marginal productivity (i.e. profit maximization; this is proven in a more precise form in annex 1 to this chapter). But in a "sufficiently general model", as that proposed by Krzyzaniak and Musgrave, their conclusion does not follow. In effect, as we saw in the last section, a neokeynesian model indicates that there is no incompatibility, at the macro level, between profit maximization (marginal productivity) and shifting of the tax.

Therefore, from Krzyzaniak and Musgrave's analysis, it does not follow at all that entrepreneurs are not maximizing profits or are not demanding labour according to marginal productivity. Yet this is the conclusion that Krzyzaniak and Musgrave draw from their results. The conclusion that should follow is, rather, that the results are more consistent with the neokeynesian theory than with the neoclassical theory, both of which are loosely contained in Krzyzaniak and Musgrave's macro-model and reduced form. In sum, the first problem is that the conclusion drawn from their model is a "non-sequitor".

A second problem with Krzyzaniak and Musgrave and related models is their failure to control for the change in technical progress. Krzyzaniak and Musgrave's series spans many years (1935-1942, and 1948-1959). In such a long period the change in the productivity of factors of production due to technical progress can be substantial. As Lerner and Hendricksen (1956, p. 201-202) note in their analysis, output per unit of capital (the turnover ratio) increased greatly in the American economy from the 1920's and 1930's to the 1950's, and the main cause of this increase was the high level of technological innovations in the period. The increase in the turnover ratio compensated for the increase in profit taxes, leaving the after tax rate of return unchanged in the period.

The argument made by Lerner and Hendricksen can be put in more precise terms as follows. For a production function which admits general technical progress it is shown in Appendix 2 to this chapter that, under the assumption of profit maximization, the rate of profit is positively related to a parameter p representing technical progress of any sort:

$$\partial rg / \partial p > 0$$

if technical progress is Harrod neutral or Hicks neutral, that is, in cases where the labor augmenting tendencies are at least as great as the capital augmenting tendencies. In the other case, that of pure capital augmenting technical progress (otherwise known as Solow Neutral), the rates of profit and of technical progress will still be positively related, provided that the elasticity of substitution in production Ek,l is greater than the share of wages in the product:

If $Ek,l > wgl* / Q$ then $\partial rg / \partial P > 0$

in all cases of technical progress.

Since the wage share Aw is around .65 and the elasticity of substitution is frequently considered not far from 1.0 (1),

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(1) We shall see that our empirical results give,

then, in general, technical progress and the rate of profit are positively related.

On the other hand, for the period of observation of Krzyzaniak and Musgrave's model, as technical progress was on the rise, the corporation income tax was also increasing. Then, in the period, the correlation between these two variables was positive (2). Now, Krzyzaniak and Musgrave did not control for technical progress in their model. Hence, as is shown in Theil (1971; see also Johnson 1972, PP 168-169, or Maddala 1977, pp. 155-157) the tax variable in their regression ought to have an upward bias since a variable positively related with both the endogenous variable (rg) and the tax variable was missing.

As mentioned previously,the models by Spencer and Roskamp which use the same methodology present the same problems as Krzyzaniak and Musgrave's model. Dusansky's model also presents at least one problem in common with Krzyzaniak and Musgrave's model. It does not control for technical progress; then given the length of Dusansky's period of analysis, the upward bias due to this factor is bound to be substantial. In addition, Dusansky's macro-

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approximately, Aw = .65, Ek,1 = .73 .

(2) Notice that unlike the previous relation, this one is not a necessity; the tax could have fallen as technical progress kept growing.

economic model doesn't specify a proper government budget constraint and doesn't control for taxes other than the profit tax; this is another element of misspecification for this model.

The last two models consistent with the shifting hypothesis - the cross section analyses of Kilpatrick and Levesque - have a methodology very different to the previous ones. Their conclusions of shifting of the tax at the global level hinge upon a very important assumption: that zero concentration ratio is related to zero shifting of the tax. As we saw, Gordon's studies show that this hypothesis may not be true; rather, at low concentration there may be negative shifting. Hence, here again, it is not possible to conclude from these models that the tax is fully shifted. What these models do seem to show is that, at the microeconomic level, there is a positive relation between concentration and shifting.

In sum, all models which are claimed to be consistent with the full or almost full shifting hypothesis present econometric and methodological problems which nullify their shifting conclusion.

The models consistent with the hypothesis of low or zero shifting also present econometric problems.

The modified versions of Krzyzaniak and Musgrave's models presented by Goode, Slitor, and Cragg, Harberger and Mieszkowski introduce "pressure variables" such as the unemployment rate and the ratio of actual to potential GNP to control for cyclical variations of the economy. This would be adequate for a neoclassical model, since then unemployment would be a disequilibrium variable exogeous to the model.(1) But Krzyzaniak and Musgrave's model is general enough to encompass a neokeynesian model. Now, in a neokeynesian model the "pressure variables" above are endogenous: they are explained by the model. Therefore what is being introduced from the perspective of a general model are endogenous variables. Hence, from this perspective the modified models estimated by ordinary least squares produce results with a simultaneous equation bias and are therefore inconclusive.

Turek and Oakland also introduce "pressure variables" to correct for cyclical or disequilibrium situations. But these two models start explicitly from a neoclassical framework: factor prices will be determined by marginal productivity, and unemployment cycles are states of disequilibrium. In their case, then, the use of the unemployment rate or the capacity ratio as "pressure variable" is correct and does not introduce bias, since they are exogenous to the neoclassical model. Additionally, these models control for the growth of technical progress. But they both use a structural

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(1) The reasoning in this and the next paragraph is spelled out in great detail in the appendix 1 to this chapter.

equation, the marginal productivity equation, as basis for estimation; and they estimate by ordinary least squares. Since capital per unit of labour is an endogenous variable, this estimation also has simultaneous equation bias. Hence, the conclusion of these models, for low shifting, is not firm. The same comment can be applied to Gordon's model.

In sum, it has to be concluded that the problem of the incidence of the corporation income tax in short run models remains still unsolved; or at least unsatisfactorily solved.

CHAPTER IV.

THREE MODELS OF GROWTH AND DISTRIBUTION

In this chapter we shall describe three main models of growth and distribution and three variants arising from the main models. These models are in our opinion stylized representations of how the capitalist economies work, as viewed at present by different schools. We shall follow Stephen Marglin (1984) in the stylized representation of the models and on their labeling as neoclassical, neokeynesian, neomarxian and hybrid models. We should add that the simplified representations utilized cannot cover all the aspects of the very rich thought of these schools but we hope that they have captured the essential aspects.

We shall start the presentation of the models by establishing two accounting relations or identities, familiar in the national accounts, concerning the distribution of the national product between consumption and investment on one side and wages and profits on the other side. Table 1 spells out the notation and meaning of the variables and parameters entering these relations.

CH IV

TABLE 1

NOTATION USED IN BASIC EQUATIONS OF GROWTH MODELS

Stocks:

L(t): Labour employed at the beginning of period t

K(t): Stock of "corn" capital employed at the beginning of period t

Flows:

w(t): Money wage rate per worker per unit of time during period t. w(t): real wage, in terms of corn.

Π(t): Profits per unit of time obtained during period t on corn stock K(t). $\bar{\Pi}$(t): total profit in real (corn) units.

Q(t): Corn produced per unit of time during period t.

C(t): Consumption of corn per unit of time during period t.

D(t): Total depreciation, per unit of time, of the stock of corn capital at time t.

Ratios:

g(t): Δ Q(t) / Q(t)=[Q(t+1) − Q(t)]/Q(t) : rate of growth of corn production during period t.

r(t): rate of profit net of depreciation on corn capital K(t) in corn terms.

d(t): Rate of depreciation of corn capital; d(t) = D(t) / K(t) .

p(t): Price of corn at the beginning of period t

a1: capital-output ratio in corn terms: a1 = K(t) / Q(t) Δ t]]

a0: labour-output ratio: a0 = L(t) / Q(t)

Taking account of that notation, the physical balance of production, or consumption investment relation, can be written:

$$Q(t) \ t = L(t) \Delta t + K(t+1) \qquad (1)$$

The distribution of the product between wages and profits on the other hand, can be written:

$$Q(t) \Delta t = w(t) \Delta t \ L(t) + \Pi(t) \Delta t \qquad (2)$$

We can now introduce the following definitional equations.

Capital output relation:

$$a1 = K(t+1) \ / \ [Q(t+1) \Delta t] \qquad (3)$$

Rate of growth of production:

$$g(t) = [Q(t+1) - Q(t)] \ / \ [Q(t) \Delta t] \qquad (4)$$

Rate of profit in corn (real) terms, net of depreciation:

$$r(t) = [\Pi(t) - D(t)] \ / \ K(t) =$$

$$[\Pi(t) - p(t) \ K(t) \ d(t)] \ / \ [p(t) \ K(t)] =$$

$$\Pi(t) \ / \ [p(t) \ K(t)] - d(t) \qquad (5)$$

Labour-output relation in period t:

$$L(t) = a0\ Q(t) \qquad (6)$$

Relation between money and real quantities:

$$M(t) = p(t)\ \bar{M}(t)\ ;\ w(t) = p(t)\ \bar{w}(t) \qquad (7)$$

Then, combining equation (1) with (3) and (4), we can transform the consumption-investment relation to the following form:

$$Q(t) = C(t) + a1\ [d(t) + g(t)]\ Q(t). \qquad (8)$$

Combining equation (2) with (3), (5) and (6) we can in turn transform the wage-profit relation to the following form:

$$Q(t) = a0\ w(t)\ Q(t) + [d(t) + r(t)]\ a1\ Q(t) \qquad (9)$$

Which also can be written by using (7):

$$pt = 1 = a0\ w(t) + [d(t) + r(t)]\ a1 \qquad (9a)$$

Equations (8) and (9) are the common equations or the "core" of the models of growth which we are going to present. Notice that both are definitional identities: of the distribution of the product between consumption and capital formation or investment (equation (8)); and of its distribution between the wage bill and the profit bill

(Equation 9 or 9a). Equation (8) can be further simplified by picking up a point in time when the total labour employed L(t) is defined as the unit of labour:

$$L(t) = ao\ Q(t) = 1 \qquad (10)$$

Hence, multiplying equation (8) by ao we obtain:

$$[a0\ Q(t) =]\ \ 1 = a0\ C(t) + a1\ [d(t) + g(t)] \qquad (8a)$$

The common equations are now constituted by the second equalities in (8a) and (9a). Notice that, presented in this way, the two equations have a remarkable symmetry: the line representing equation (9a) in the space(w(t), r(t)) is the same representing (8a) in the space(C(t), g(t)) ; they have the same graph.

For our empirical work, it will prove convenient to have a second representation of equations (8) and (9) which does not use the labour output (a0) and capital output (a1) ratio; but rather capital per unit of labour, k(t), and output per unit of labour, q(t). Notice that capital per unit of labour is:

$$k(t) = K(t)\ /\ L(t) = a1\ Q(t)\ \Delta t\ /\ [a0\ Q(t)] =$$
$$a1\ \Delta t\ /\ a0 = a1\ /\ a0$$

if the interval Δt is taken as the unit of time. Output per unit of labour is:

$$q(t) = Q(t) / L(t) = Q(t) / [a0\ Q(t)] = 1 / a0$$

using these two transformations from (k(t), q(t)) to (a0, a1)
we can write equation (8) or (8a) as:

$$q(t) = C(t) + k(t)\ [d(t) + g(t)] \qquad (8b)$$

and (9) as:

$$q(t) = w(t) / p(t) + k(t)\ [d(t) + r(t)] \qquad (9b)$$

Notice that the meaning of C(t) is now that of consumption per unit of labour (not to be confused with consumption per capita, which includes both workers and non workers, that is, total population).

Turning now to the possible ways of completing or closing the models, notice that the two equations (8a) and (9a) have the following unknowns:

$$C(t),\ g(t),\ w(t),\ r(t),\ ao,\ a1, d(t)$$

That is: seven unknowns for two equations. A model capable of explaining the behavior of those variables would need five more independent relations. A first relation is normally given by the assumption of a constant and known rate of depreciation $d(t) = d$ = const; in the case of the simple corn model (Marglin, 1984), this rate is 1.0.

There are then four more independent relations to be determined. Two of these relations would determine the conditions of the production technology; if, for example, this one were of the Leontief type, then a0 and a1 would be constant and two more relations would complete, or close, the models. If, on the other hand, the conditions of production could be better represented by a production function with variable proportions, and some optimizing behaviour in production, such as profit maximization or cost minimization, were assumed, this would give again two more common equations, and the models would close with two additional relations.

It is the diverse ways of obtaining the additional two relations which result in different theories of growth and distribution, different ways of explaining the real world.

Presently, there are three main competing explanations of that "world", three main theories of growth and distribution. The mainline theory is that of Neoclassical economics, which Joan Robinson has also labeled the "orthodox" theory, and which stresses full employment growth. A second is sustained by the Cambridge or Neokeynesian school, inspired by Keynes' departure from "orthodoxy"; in particular, with regards to his positions on the forces determining investment behavior and the role of investment in growth, wich is indeed completely disregarded in the Neoclassical growth theories, since, as we shall see shortly, if full employment is assumed, there is very little room left for an independent role for investment behaviour. A third theory, the Neomarxian, which claims Marx and Ricardo as its forerunners, lays stress upon the determination of the real wage rate, by factors such as conventionally accepted levels of consumption and the 'struggle of workers to maintain and better those levels (class struggle), factors which are disregarded by the other two theories.

The main theories also specify relations determining consumption or savings, which are different for the Neoclassical and the other two. In this way they present seven equations or relations to determine the seven unknown variables.

Apart from the main models, other hybrid models can be found, which combine different elements of the above theories. Since there are only two consumption functions, three mixed models can be constructed.

Among the three main theories mentioned there are many variations. Any systematic presentation of these models of growth corresponding to the three schools has to be, per force, very stylized, picking up only the most outstanding elements of those theories. The "stylized" models which we are going to present roughly follow those of Marglin (1984) and hence pick up the same outstanding elements.

We are fully aware of the risk which the forced simplification entails. Some, belonging to diverse schools, will probably feel that their theories cannot be found in the stylized models. Yet the contrary attempt, to encompass all the subtleties of the diverse theories, would have made any attempt at systematizing and comparing futile. At these moments of crisis in conventional economics, when the search for new paradigms is urgent, we think the risk is fully worth taking.

In what follows we shall present the closure relations for the Neoclassical, the Neokeynesian and the Neomarxian models; finally we shall present the mixed models.

A) Neoclassical System

In the neoclassical system the following assumptions are made about the economic agents:

a. All capital stock (all the corn for production) is owned by retired workers, who live as "rentiers".

b. Workers live two "public" periods, starting when they enter active life; during the first period they work, during the second they live retired, become rentiers and invest their savings.

c. Before they enter the first period of their "public" life, people's consumption is just part of their parents' consumption. (Samuelson, 1958)

d. There is no separate class of entrepreneurs; entrepreneurs are retired "rentiers". Accordingly all profits accrue to rentiers in the form of:

1) interest on their savings as workers and,

2) return of principal.

e. In any period corn capital is financed by the funds invested by rentiers.

f. The rate of growth of employment is equal to the rate of growth of the active population; therefore, the rate of employment is kept constant at a level that can be called the "natural" level.

g. for the simple neoclassical model of this section we shall also take the depreciation rate, $d(t) = d$ to be equal to unity.

As can be seen, the first three assumptions constitute those of a very simple model of "life cycle" savings.

The budget constraint of the typical head of household can be represented for periods 0 and 1, by:

$$C0 + C1 / (1 + r) = w / p = \bar{w} \qquad (11)$$

The utility function of the household has the arguments C0 and C1; the household maximizes:

$$\text{Max} \quad U(C0, C1) \tag{12.1}$$

subject to:

$$C0 + C1 / (1 + r) = w / p \tag{12.2}$$

The solution to this opitimization problem would give the optimum amounts of Co and C1 as a function of w and r

$$Co^* = Co\ (w/p,\ r) \tag{13}$$

$$C1^* = C1\ (w/p,\ r) \tag{14}$$

Now, if the hypothesis that when the household income increases the increase will be shown by CO and C1 so as to leave their shares unchanged is made, the utility function is restricted thereby to be linearly homogeneous. This hypothesis is currently made in "life cycle" and "permanent income" models (Friedman,1967; Modigliani and Brumberg, 1951; Ando and Modigliani,1963).

Given this homogenity restriction, which seems quite adequate, the functions (13) and (14) can be written:

$$C0^* = \bar{w}\ \phi\ (r) \tag{13.a}$$

$$C1^* = \bar{w}\ \psi\ (r) \tag{14.a}$$

where ϕ (r) and ψ (r) stand for functions of r.

Now let us see what the total consumption and the consumption per worker are at any period t: There will be a certain amount of workers L(t) and of "retirees" R(t). Assuming full employment, the total number of households will be L(t) + R(t). Total consumption, in turn, will be:

$$TC(t) = R(t)\ C1 + L(t)\ CO$$

Since retirees are nothing other than retired workers, they consume what our household consumed in his second life period, given by equation (14). By the same token workers' households consume, as our household in the first period, given by equation (13). Hence consumption per worker (or per unit of labour) will be:

$$C(t) = TC(t)\ /\ L(t) =$$

$$C1[\bar{w}(t),r(t)]\ R(t)\ /\ L(t) + Co[\bar{w}(t),r(t)]$$

where the Ci[ ], i= 0,1 indicate that those variables are functions of the arguments within the brackets.

Now, the people who are retired at t were the workers of t-1 or, in steady state:

$$R(t) = L(t-1) = L(t)\ /\ [1 + g(t)] \qquad (15a)$$

Hence consumption per worker can be written, dropping the time indicator t on the right side:

$$C(t) = C1(\bar{w},r)\ /\ (1 + g) + CO(\bar{w},r) = C(\bar{w},r) \qquad (16)$$

Or also, taking account of equation (11):

$$C(t) = w + \frac{C1(\bar{w},r)\ (r - g)}{(1 + r)\ (1 + g)} \qquad (17)$$

On the other hand, savings per worker in period t would be equal to income per worker in period t minus consumption per worker. Income per worker, gross of depreciation, in period t is (1):

$$q(t) = \bar{w} + (d + r)\, k$$

as given in equation (9-B).

Hence, savings per worker in period t, gross of depreciation, are given by:

$$GS(t) = q - C = \bar{w} + (d + r)\, k - C \qquad (18)$$

Replacing equation (17) in equation (18), and taking d = 1:

$$GS(t) = (1 + r)\, k - \frac{C1(\bar{w},r)\,(r - g)}{(1 + r)\,(1 + g)} \qquad (19)$$

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(1) In order to simplify the exposition we shall from now on drop the time subscript, t, from all variables, when this does not cause confusion. A lagged variable, such as x(t-1), will be designated x(-1); for a lead, x(t+i), we shall write x(i).

On the other hand, using equation (8-B) it is possible to see that gross savings are also equal to:

$$GS(t) = q(t) - C(t) = k\ (d + g) = k\ (1 + g) \qquad (20)$$

Hence combining (18) and (19) and defining net savings S(t) as:

$$S(t) = GS(t) - d\ k(t) \qquad (21)$$

and taking account of the fact that corn is the numeraire(p(t) = 1.0 ; $\bar{w}(t) = w(t)$), we get:

$$S(t) = r\ k - \frac{C1(w,r)\ (r - g)}{(1 + r)\ (1 + g)} = g\ k \qquad (22)$$

The last equality comes from combining equations (20) and (21) and the first from (19) and (21). Equation (22) can be simplified even further to the following expression:

$$(r - g) \left\{ k - \frac{C1(w,r)}{(1 + r)\ (1 + g)} \right\} = 0 \quad , \text{or}$$

$$C1(w,r) / (1 + r) = (1 + g) k \qquad (23)$$

Equation (22) corresponds to the familiar equilibrium condition: net ex-ante savings, S(t), equal net ex-ante investment, I(t) = gk. Equation (23) on the other hand, does not seem to have at first view a familiar interpretation. But a little manipulation reveals its meaning. Recall that in this neoclassical model all capital is owned by retired household heads. These saved in their working period a quantity of corn equal to w - Co, and with these savings they invested in such a way as to own all capital. Hence their total savings per worker,

$$R(t) (w - C0) / L(t)$$

should be equal to the total capital per worker k(t) = a1 / a0, or:

$$k(t) = (w - C0) R(t) / L(t) = (w - C0) / (1 + g) = C1 / [(1 + r) (1 + g)] \qquad (24)$$

The second equality in (24) comes from equation (15-a) and the third takes account of the budget constraint, equation (11). Hence condition (23) is nothing other than an expression of the fact that, in this model, retired households own all the capital, and this is equivalent in turn to the ex-ante equality of net savings and net investments of equation (24).(1)

As seen, equation (23) can also be written:

$$w - CO(w,r) = (1 + g)\ a1 / a0 \qquad (23a)$$

and if the utility function is assumed linearly homogeneous (see equation (13-a)), as:

$$w\,[1 - \phi\,(r)] = (1 + g)\ a1 / a0 \qquad (23b)$$

Equation 23 (or 23a, or 23b) is hence one more condition in the closure of the neoclassical model.

A second condition is the "natural rate of unemployment" condition. Calling the rate of growth of the labour force, n, this condition simply says that the common rate of growth of employment, capital and production (common because we are additionally assuming the steady state) is equal to the rate of growth of the labour force; this last has been given exogenously. That is:

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(1) There is a third way to deduce equation (23) by obtaining savings directly and then taking them equal to investments.

$g(t) = g = n$ (24)

The first equality indicates the steady-state condition; the second, the neoclassical assumption of a constant rate of unemployment.

To complete the closure of the neoclassical model it is now necessary to specify two additional relations, stemming from the conditions of production. These relations will indeed not be specific to the neoclassical model but, together with the two accounting identities (8) and (9), will be common to all models.

The simplest possible specification of the production technology is that of assuming a Leontief production function or constant factor proportions. This is expressed simply by:

$a0 = const$ (25)

$a1 = const$ (26)

In this way the neoclassical model would be complete, constituted of equations (8a), (9a), (23), (25), (25a) and (26a), which would simultaneously determine the variables: c,g,w,r,ao and a1.

More generally, it is possible to allow for the possibility of varying the factors of production, and introduce a production function relating output to capital and labour inputs.

In general the production function would be of the form:

$Q = F(K,L)$ (27)

where, recall, Q = Q(t) and L = L(t) and we impose the normal constraints on the function F, that is, it is such that the marginal products of capital and labour are positive but decreasing:

$$\partial F / \partial z > 0 \qquad \partial^2 F / \partial z^2 < 0 \,, \; z = K, L$$

We shall also assume that the economy presents, on the average, constant returns to scale. Hence the production function (27) is linearly homogeneous. It can therefore be written:

$$Q / L = F(K / L \,,\, 1) = f(K / L) \qquad (28)$$

or by the definition of q and k:

$$q = F(k,1) = f(k) \qquad (29a)$$

or, alternatively:

$$1 / a0 = F(a1 / a0 \,,\, 1) = f(a1 / a0) \qquad (29b)$$

Equation (29a) (or 29b) gives one more condition to close the models. A second condition is obtained by assuming profit maximization. This implies that the values of the marginal products have to be equal to the factor payments (1), or:

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(1) Recall that this has nothing to do with the neoclassical "Marginal Productivity Theory" of distribution. The neokeynesian and the neomarxian models are also consistent with this assumption. The crucial assumption in the neoclassical model is that of the existence of a "natural rate of unemployment".

$$FK = \partial F / \partial K = (d + r)\, p / p = d + r \qquad (30)$$

$$FL = \partial F / \partial L = w / p = \bar{w} = w \qquad (31)$$

Since the production function is assumed linearly homogeneous, this can also be written:

$$d\, f(k) / d\, k = f'(k) = f'(a1 / a0) = (1 + r) \qquad (30a)$$

$$f(k) - k\, f'(k) = f(a1 / a0) - (a1 / a0)\, f'(a1 / a0)$$

$$= w / p = \bar{w} = w \qquad (31a)$$

The neoclassical model now seems overdetermined with three additional equations instead of two. But equations (9) (a or b), (29) (a or b), (30a) and (31a) are linearly dependent. This is nothing other than a consequence of the assumption of constant returns to scale. One of the equations can be eliminated from the system with no problem. Since it is convenient to keep equation (9) in the model, according to our needs, we shall drop one of the other three equations (29), (30) or (31).

Then, in a more general way the neoclassical model can be completed by replacing the fixed proportions conditions (25) and (26) by two of the three production conditions (29), (30) or (31). As mentioned these conditions also constitute part of the other models which we shall examine.

B) Neokeynesian System

Unlike the neoclassical models, the neokeynesian as well as the neomarxian models, make a neat distinction between social classes: workers and capitalists. Capitalists in turn are frequently divided into two groups: rentiers, who own claims to capital, such as bonds and shares, and entrepreneurs who run the enterprises. The role of rentier and entrepreneur can be combined in the same person, as was typical of the 19th century entrepreneurs, but this need not be so any more in modern enterprises, where control of the decisions of the enterprise may rest in the hands of the higher management (Joan Robinson, 1956, ch 1 and ch 7 pp. 68-69).

1) Consumption and saving behaviour

Saving behavior in the neokeynesian system does not stem from an analysis of the utility maximizing individual as in the neoclassical model, rather it is based on postulates of different propensities to save among different classes or among different income categories, all more or less based on Keynesian considerations on psychological propensities to consume. Two different savings equations in the neokeynesian stream are of particular interest. One, proposed by Kaldor ,is based on propensities to save which differ according to income categories; a second, proposed by Pasinetti, is based on saving propensities which differ among classes.

Kaldor's saving equation (1) assumes simply different propensities to save out of wages, sw', and out of profits, sp', and such that:

$$sp' > sw'$$

with a strict inequality sign. In the simplest possible case of proportional saving from those income categories, the equations are:

$$TS = sw\ W + sp\ \Pi \qquad (32)$$

where W and Π are the total wage bill and profit bill in the economy, and sw and sp the average saving propensities out of wages and out of profits respectively (in this case equal to the marginal propensities). Hence, savings per employed worker would be, in corn or real terms:

$$S = sw\ w + sp\ r\ k \qquad (33a)$$

or equivalently:

$$S = sw\ w + sp\ r\ a1\ /\ a0 \qquad (33b)$$

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(1) See Kaldor (1955-56, p. 95): "Income may be divided in two broad categories, Wages and Profits...(where profits comprise)...the income of property owners generally, and not only of entrepreneurs.The important difference between them being in the marginal propensity to consume (or save), wage earners marginal savings being small in relation to those of capitalists."

where we keep our convention that quantities without time indicator refer to the present period or time t. We shall call equations (33a) and (33b) the savings functions by income categories.

Kaldor's theory, which uses savings propensities by income categories, and not by social classes, was considered inconsistent by Pasinetti, because it means that workers have different propensities to save out of their wage income and out of their capital income. Pasinetti (1962, p. 270) proposed, instead, the following savings function:

$$TS = sl(W + \Pi l) + sc\, \Pi c \qquad (34)$$

where sl and sc are savings propensities of workers and capitalists respectively, with

$$sl < sc,$$

Πl and Πc are profits net of depreciation earned by the workers and the capitalists respectively, which are proportional to the capital they own:

$$\Pi l = r\ Kl\ p$$

$$\Pi c = r\ Kc\ p$$

$$\Pi l + \Pi c = \Pi = r\ (Kl + Kc)p = r\ K\ p$$

W is the total wage bill:

$$W = w\ p\ L$$

p: is the price of the capital good.

Savings per worker in corn or real terms would then be:

$$S = TS / (p\ L) = sl[w / (p\ L) + r\ Kl / L] +$$

$$sc\ r\ Kc\ /\ L$$

hence:

$$S = sl\ (w + r\ kl) + sc\ r\ kc \qquad (35a)$$

$$kl = Kl\ /\ L;\ kc = Kc\ /\ L;\ kl + kc = k$$

or

$$S = sl[w + (1-z)\ r\ al\ /\ a0\ ] + sc\ z\ r\ al\ /\ a0 \qquad (35b)$$

$$z = Kc\ /\ K;\ 1 - z = Kl\ /\ K\ ;\quad 0 < z < 1$$

z is the proportion of capital owned by the capitalist class. ki (i = l,c) is simply total capital owned by class i divided by total workers.

Pasinetti's savings function (35a) or (35b) results in a very interesting asymptotic behaviour for the neokeynesian model as we shall soon see. But first, let us consider a slightly more general formulation of that savings function, which explicitly shows retained earnings of corporations. It is, in per worker and real terms:

$$S = sl[w + kl\ r\ (1 - \theta)] + sc\ r\ kc\ (1 - \theta) + se\ r\theta(kl + kc) \qquad (36a)$$

$$kl + kc = k$$

θ represents the proportion of total profits, r (kl + kc), retained by the corporation (or by the "entrepreneurs"); hence (1 − θ) is the proportion distributed; se is the propensity to "save" of entrepreneurs, i.e., 1−se represents their propensity to acquire consumption goods-

goods which do not contribute to increase productive capacity. One example of this type of good could be the company jet. If it is assumed that all goods acquired by corporations are investment goods, then se = 1.0.
The equation can also be written as:

$$S = sl\ [w + (1 - z)\ (1 - \theta)\ r\ a1\ /\ a0] + sc\ z\ (1 - \theta)\ r\ a1\ /\ a0 + se\ \theta\ r\ a1\ /\ a0 \qquad (36b)$$

where z is as in the equation (35a)

We can now formulate the savings investment relation in the neokeynesian models. We have to link the savings expressions of equations (34), (35) or (36) to the other variables of the models. This is done by explicitly writing the definition of the savings, net of depreciation, as non consumed income after depreciation d k:

$$S = w + (1 + r)\ k - d\ k - C \qquad (37)$$

In identity (37) savings are already expressed in per worker terms and in real or corn terms. So far we have taken the duration of our capital, corn, to be one period. Hence

$$d = 1$$

from equation (8b) and (9b) it is possible to obtain by eliminating q:

$$w + (1 + r)\ k - C = k\ (d + g) \qquad (38)$$

Replacing this in (37) and taking account of the value of the depreciation term, we get:

$$S = k\ g \quad \text{or} \quad S = (a1 \ / \ a0)\ g \qquad (39)$$

Notice that the right hand side of (39) is net investment per worker:

$$k\ g = (1 \ / \ L)\ d\ K \ / \ d\ t \ .$$

Expression (39) is nothing other than the known relation between savings and investment. Notice also that (37) together with (32), (35) or (36) implicitly define the consumption behaviour in the neoclassical model. We could indeed have formulated this model in terms of consumption rather than savings. This is what we shall do in the econometric estimation since, given the nature of the statistical data (savings is calculated as a residual in the national accounts), it is then the more adequate procedure.

Equation (39) together with one of the equations (33), (35) or (36) furnish an extra condition for the closure of the neokeynesian model. Notice that the three conditions formed are different. That obtained using (39) and (33) was the original condition used by Kaldor (1955-56).

The condition obtained by using (39) and (35a) (or 35b), on the other hand, is the starting point of the Pasinetti theorem and of its dual, proposed respectively by

Pasinetti (1962) and by Samuelson and Modigliani (1966a), which describe the asymptotic behaviour of the savings investment relation in steady state. Finally, the condition obtained by combining (39) and (36a) (or 36b) is another instance of the Pasinetti-dual theorems.

We shall examine briefly these saving-investment relations without commenting on their empirical relevance, which will be discussed later.

The Kaldorian relation, obtained by combining (39) and (33) is:

$$sw\ w + sp\ r\ k = k\ g \qquad (40a)$$

or

$$sw\ w + sp\ r\ a1 / a0 = g\ a1 / a0 \qquad (40b)$$

and replacing w from equation (9a) we get another form:

$$sw\ (1 - d\ a1)/a1 + (sp - sw)\ r = g \qquad (40c)$$

In steady state these relations should not change.

On the other hand, the savings-investment relations obtained by using Pasinetti's equation (35) combined with equation (39) do change with the assumption of steady state. Take the combination of (39) and (35a):

$$sl\ (w + r\ kl) + sc\ r\ kc = kg \qquad (41a)$$

which can also be written:

$$sl\ y + (sc - sl)\ r\ kc = gk \qquad (42)$$

where y is the net income or the product net of depreciation in real and per worker terms:

$$y(t) = q(t) - d\ k(t) = (1 - d\ a1) / a0 \quad (43)$$

Now in steady state, capital owned by each class, which is equal to accumulated savings, will be proportional to the savings of that class, hence:

$$kl / [sl\ (w + r\ kl)] = k / (g\ k) \quad (44)$$

where we have replaced total savings by its equivalent total investment: g k. The above expression can also be written, by replacing kl by k − kc, as:

$$(k - kc) / k = sl\ [w + r\ (k - kc)] / (g\ k)$$

from which we can obtain kc in terms of the other variables as:

$$kc = (g\ k - sl\ y) / (g - sl\ r) \quad (45)$$

which shows the relation between kc and the other variables, especially k, resulting from the steady state assumption. Replacing kc in the Pasinetti's savings-investment relation (42) we obtain, after some algebraic simplifications:

$$sc\ sl\ y\ r - sc\ g\ k\ r - sl\ g\ y + k\ g^2 = 0 \quad (46)$$

This is a quadratic equation in g (1). Its solution yields:

$$g = sc\ r \tag{47}$$

and

$$g = sl\ y / k = sl\ r\ (1 / Ar) \tag{48}$$

where Ar, the profit share in income net of depreciation, has been defined as:

$$Ar = r\ k / y \tag{49}$$

Relation (47) constitutes Pasinetti's (1962) theorem. Relation (48) constitutes its dual discovered by Samuelson and Modigliani (1966) and by Sato (1966). Notice that the dual solution (48) implies, by taking account of (42), that kc = 0. Conversely if kc = 0 (42) implies the dual solution (48). Therefore, in the dual state, workers eventually own all but a vanishing share of the capital :

$$kc = 0; \quad kl = k$$

Pasinetti's solution (47) is therefore restricted to values of kc strictly greater than zero:

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(1) Baranzini (1975) was the first to notice that Passinetti's problem could be expressed as the solution of a quadratic equation; the equation he obtains is in r and his deduction follows a different path. We think ours has more intuitive appeal. For other different treatments of theis problem, see Sato (1966) as well as Samuelson and Modigliani (1966). The seminal article is of course Pasinetti (1962).

$kc > 0$,

since a negative kc would have no economic meaning. To see what limits are imposed by this restriction let us replace, in equation (45), which gives the value of kc in steady state, g by its value in terms of the Pasinetti root (equation (47)), to obtain:

$$kc = (sl\ y - sc\ kr) / [r\ (sl - sc)]$$

and let us replace y by its equivalent $y = r \cdot k / Ar$, to finally obtain:

$$kc = (sl / Ar - sc) / (sl - sc) \qquad (50)$$

Now, for a meaningful Pasinetti solution kc, given by (50), should be strictly greater than zero:

$$(sl / Ar - sc) / (sl - sc) > 0 \qquad (51)$$

This implies either that $sl < Ar\ sc$ and $sl < sc$ or that $sl > Ar\ sc$ and $sl > sc$; in fact since the profit share Ar should be:

$$0 < Ar < 1$$

for (51) to obtain it is simply necessary that either:

$$sl < Ar\ sc \qquad (52a)$$

or

$$sl > sc \qquad (52b)$$

But assuming this second condition to be true leads to a contradiction, for the following reason. In steady state, as we saw in equation (44), the capital owned by each class has to be proportional to the savings of that class or, to take workers and capitalists!

$$\frac{kl}{sl\ (w + r\ kl)} = \frac{kc}{sc\ r\ kc}$$

which implies, for Pasinetti states (kc different from 0):

$$sl\ w + r\ kl\ (sl - sc) = 0 \qquad (53a)$$

or

$$kl = - sl\ w / [r\ (sl - sc)] \qquad (53b)$$

if w, r and kl are not less than zero (for economically meaningful solutions), equation (53b) can hold true only if

$$sl < sc$$

which contradicts condition (52b).

In consequence we cannot have sl > sc, and the Pasinetti solution is restricted to condition (52a), that is, to values of sl and sc such that:

$$sl < sc \quad \text{or} \quad sl < sc\ r\ k / y$$

Notice also that if the economy is in Pasinetti's regime,

$$sc\ r\ k / y = g\ k / y$$

and the criterion of validity of Pasinetti's root can also be written:

$$sl < g\ k / y$$

which is Pasinetti's (1962) original form.

The above relations and limits can also be obtained by examining the differential equations of k and of kc and their steady state paths, such as was done by Samuelson and Modigliani (1966a) and by Sato (1966). An analysis of the Pasinetti process resulting from the combination of equations (39) with the more general Pasinetti savings function (36a), which permits corporate retention of profits, would show that the addition of corporate retentions does not change the Pasinetti and dual theorems fundamentally; it simply gives them a slightly more complicated form.

In conclusion, there are two distinct forms of the savings investment relation in the neokeynesian models. One is the Kaldorian relation, which uses propensities to save expressed in terms of income categories. The second one is constituted by the Pasinetti/dual theorems, with and without corporate retentions, where there is explicit consideration of propensities to save in terms of social classes, and this leads to the special steady state results which we have examined.

The question now arises: which one of these two cases should we choose? Pasinetti in his original article (1962) seems to have suggested that the choice had to go to the Pasinetti/dual equation for logical reasons. This is only true if Kaldor's formulation is seen in terms of social classes rather than in terms of macro-categories of income. Take the starting point of Pasinetti's relation given, for example, by combining equations (39) and (36a), and replacing kc by z k , kl by (1 - z) k, and kc + kl by k:

$$sl\,[w + rk\,(1 - \theta)\,(1 - z)] + sc\,r\,k\,z\,(1 - \theta) + \theta\,se\,r\,k = g\,k \qquad (54a)$$

As written above the savings of each social class are explicit in (54a). On the other hand, the equation could be rewritten in terms of income categories as:

$$sl\ w + [sl\ (1-\theta)\ (1-z) + sc\ z\ (1-\theta) + se\ \theta]\ r\ k = g\ k \quad (54b)$$

which is formally similar to Kaldor's equation:

$$sw\ w + sp\ r\ k = g\ k$$

where sp can be defined as a weighted average of propensities to consume from capital income of different classes, and of enterprises:

$$sp = [sl\ (1-\theta)\ (1-z) + sc\ z\ (1-\theta) + se\ \theta]\ ,\quad \text{and}$$

$$sl = sw.$$

Now the question can be posed: Why, or rather how, would (54a) result in the Pasinetti/dual process rather than in some sort of macroeconomic average equation such as (54b)?

The Pasinetti/dual results would obtain only if two conditions were met. The first condition is that in the long term there be perfect arbitrage between the interest and the profit rate. In other words that, indeed, in steady state the interest rate obtained in the money market by rentiers be equal to the profit rate, any difference between the two provoking the transformation of rentiers into entrepreneurs and vice versa, so as to wipe out the difference. If this were not the case the rate of profit r would have to be replaced in equation (44) by the rate of interest. Therefore expressions (45) to (49) would not follow. Perfect arbitrage is an assumption made in the neoclassical models, but it is not clear why it should be necessary in the neokeynesian world, even in the long term since, in this model, markets do not always have to clear. The labour market in these models, for example, does not have to clear.

The second condition, or assumption, necessary for Pasinetti's processes, is that there be a social class whose only income is from capital and, more importantly, that this class be stable forever. (1) In other words changes of class or, equivalently, transfers between classes, as a consequence of inheritances going to a worker from a capitalist, or as a consequence of capitalists going bankrupt, etc. are not allowed. Even if these are allowed it would be necessary to conceive of some mechanism whereby these transfers would cancel out. For if they do not cancel out, the propensities to save appearing in equations (44) and the variable kl would not be stable and this equation could not be written.

In conclusion, the question of the choice between the Pasinetti form and the Kaldor form of the savings investment equation cannot be posed as a purely logical or theoretical one.

The choice between these two forms should be based on

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(1) This was noted by Samuelson and Modigliani (1966) and by K. Sato (1966b).

empirical studies. Unfortunately we do not have data to examine directly the savings function by income categories (Equations 33a or 33b). In our empirical analysis, we shall then be able to estimate only the Kaldorian savings function. Because of this, we shall not discuss in this work the Pasinetti-Dual version of the Neo-keynesian model.

2) The "animal spirits" function

The second condition of closure for the neokeynesian system relates the rate of growth to the rate of profits in the following way, given by Joan Robinson, (1962, pp.44-51). For a given "state of expectations" or of "animal spirits" of the entrepreneurs, these decide to invest more or less according to the profits they expect to obtain, investing in general more when profits expectations are high. In general though, in the short term, expectations are volatile due to the fact that the "real world" is ridden with uncertainty. This uncertainty is of an irreducible type, different to risk, and is furthermore not amenable to any sort of probability calculus, in spite of all neoclassical attempts to do away with it. In Keynes' own words:

> "Even apart from the instability due to speculation, there is the instability due to the characteristic of

human nature that a large proportion of our positive activities depend on spontaneous optimism rather than on a mathematic expectation, whether moral or hedonistic or economic. Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as a result of animal spirits -- of a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities. Enterprise only pretends to itself to be mainly actuated by the statements in its own prospectus, however candid and sincere. Only a little more than an expedition to the South Pole, is it based on an exact calculation of benefits to come. Thus if the animal spirits are dimmed and the spontaneous optimism falters, leaving us to depend on nothing but a mathematical expectation, enterprise will fade and die; though fears of loss may have a basis no more reasonable than hopes of profit had before...

...We should not conclude from this that everything depends on waves of irrational psychology. On the contrary, the state of long-term expectation is often steady, and even when it is not, the other factors exert their compensating effects." (Keynes, 1936, Chapter 12. See also Keynes, 1937)

It is because of this that capitalists are moved in their actions not just by calculations of expected profits, but by something else -- their "animal spirits" -- a "spontaneous urge to action" which leads them to keep accumulating. This is for Joan Robinson a crucial element of the Keynesian models. These ..."are designed to project into the long period the central thesis of the general theory, that firms are free, within wide limits, to accumulate as they please, and that the rate of savings of the economy as a whole accommodates itself to the rate of investment that they decree" (1).

In her model therefore, "the inducement to invest is conceived in terms of a desired rate of growth... the actual trend of growth is generated from within by the propensity to accumulate inherent to the system. It is steady or fluctuating according to whether it operates in tranquil conditions which generate inertia, or in a changing world, where uncertainty makes expectations volatile." (Joan Robinson 1962, p. 83).

In the steady-state then, when expectations have set in a self fulfilling steady pattern, the animal spirits

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(1) Joan Robinson, 1962, p.83. She goes too far in her assertion that firms are free to accumulate as they please. The point is rather that the rate of growth results as a consequence of independent investment behaviour of the entrepreneurs and the saving behaviour of the diverse classes in the economy.

function can be represented by the following equation:

$$g = i(r); \quad i'(r) = di / dr > 0 \qquad (55)$$

where g is the desired (and in steady-state also the actual) rate of accumulation (growth) of the economy; r the rate of profit, and i stands for the functional form of what Marglin (1984) calls the "animal spirits" function.

The neokeynesian model is, then, constituted of: equations (8a or b), (9a or b), (29) or (29a), (30a), which are common with the other models; equation (40a) (Kaldor) as the saving investment relations; and finally equation (55) the animal spirits function. Six equations for the six unknowns: c, g, w, r, k (or a1 / a0) and q (or 1 / a0). The system is therefore closed.

There are other variants of the neokeynesian model, all characterized by having a function representing the "inducement to invest" of capitalists, such as those of Harrod, Kalecki (short term) and Kaldor.(1) They introduce mechanisms different to the "animal spirits" function, in their independent investment equations. (Joan Robinson 1966, pp. 82-87). The important feature of these models though is not so much the particular mechanism used

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(1) Kaldor's model is rather of a hybrid variety, since it assumes full employment of labour.

as the existence of an independent investment (or accumulation) function.

C) <u>The Neomarxian System</u>

In order to close the neomarxian model we have to introduce an additional relation to equations (8), (9), (29), (30a), and (40a). Marx, partially following the classical economists, closed his model by postulating conditions that can be shown, in the absence of technical progress, to be equivalent to assuming a constant wage in terms of corn (1). Without elaborating too much, it can be said that this real wage is "socially determined" to allow a level of consumption sufficient to restore the labour power which the worker sells to the entrepreneur. This level of consumption is not a physical subsistence level of the malthusian type, and the related wage is not, therefore , as in the classical theories, a biological subsistence wage. It is related to the mores and customs of society. It is culturally, rather than biologically determined. It is, then, more appropriate to talk of a

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(1) To see how this wage concept can be related, in the corn model, to Marx's concept of labour value, see Marglin(1984).

conventional rather than a subsistence wage. In this setting, the level of the conventional wage is influenced by the pressures that the workers can exert upon the capitalists, in order to better their living standards. In other words, the conventional wage is influenced by the relative strength of workers and capitalists in the "class struggle". When a balance of power between workers and capitalists is maintained, the level of the conventional wage is also maintained constant, insofar as we make abstraction of technical progress.

If technical progress is introduced into the picture, the mechanism of a constant conventional wage cannot be maintained anymore. In the Marxian system, technical progress is, to use Sweezy's (1970) expression, "a necessary condition for the existence of capitalist production". If, for example as a consequence of worker's bargaining, wages grow beyond the conventional level, entrepreneurs react by cost saving measures in the form of labour saving technological innovations. Unemployment and productivity per worker increase. The increase in unemployment (in Marx's terms "the reserve army of the unemployed") exerts a downward pressure on wages. But on the other hand, the increased productivity affects the perception of the conventional wage level. Workers will fight back to attain a new level of wages coresponding to the higher level of productivity. The final outcome is not clear. Assuming as before that class power is balanced between workers and capitalists, wages will rise, that is, they will capture at least part of the productivity gains. There is still some indeterminacy, since they could grow at a slower or a faster rate than profits. One possible hypothesis is that wages grow at a rate equal to the rate of technical progress:

$$w(t) = wo\ Exp(R'\ t); \quad wo = cont. \qquad (56a)$$

where R' is the rate of growth of Harrod neutral technical progress. this hypothesis can also be expressed as a constant share of wages in the product of the economy:

$$w(t) / q(t) = a0\ w(t) = wo\ Exp(R'\ t) / [qo\ Exp(R'\ t)]$$
$$= wo / qo = const. \qquad (56)$$

This is the hypothesis that we shall adopt. But we should be aware that it implies very strong assumptions on the bargaining behaviour of workers. It implies that in capitalistic economies bargaining is done not only over the wage rate but also over its rate of growth; and furthermore that the end result of the individual bargaining processes, in a country with more than one group of organized workers, is a constant share of the product. This is easier to conceive in an economy with a highly centralized labour movement and wage bargaining process, like Sweden, than in countries such as Canada or the United States.

Finally it also implies that the relative strength of workers and capitalists in the class struggle does not change. Because of this, equations (56) and (56a) shold be modified, in order to introduce explicitly a term reflecting the relative strength of workers; that is, a term reflecting the class struggle. We shall come back to this point in chapter V, section D.

D) Hybrid Systems

So far we have presented three basic stylized models with at least four relations in common and at most two relations different among the six variables defining the system: c, g, w, r, k (or a1 / a0) and q (or 1 / a0).

Of the two different relations in the basic system one referred to the consumption or saving behaviour and the resulting saving investment equilibrium condition. The neoclassical savings equation, with its concept of life cycle saving for retirement, through lifetime utility maximization, differed from the common savings equations of the neokeynesian and neomarxian models, based on psychological propensities to save among clearly defined socioeconomic classes, resulting in either the Pasinetti/Dual equations or the Kaldorian macroeconomic savings functions.

The second different relation stipulated: for the neoclassical model, that the system had a natural rate of unemployment; for the neomarxian model, that the system had a socially necessary given wage share; and for the nekeynesian model, that the system had an independent investment relation arising from the "animal spirits" of the entrepreneurs and their urge to accumulate.

The two differing relations can be combined in other ways which give rise to three other models, which we shall call "hybrid" to distinguish them from the basic ones presented. In this way, the natural rate of unemployment relation (24) can be combined with one of the neokeynesian savings-investment relations to obtain a model much like some of Kaldor's models. We shall call this hybrid model the CK model.

On the other hand, the neokeynesian animal spirits relation (55) can be combined with the neoclassical savings investment relation (23) to obtain a second hybrid model which we shall simply designate as the KC model.

Finally, the neomarxian socially necessary wage equation (56) can be combined with the neoclassical savings investment relation (23) to obtain the third hybrid model, which we shall simply call the MC model.

CHAPTER V

THE GROWTH MODELS WITH GOVERNMENT AND TECHNICAL PROGRESS

In Chapter IV the models of growth and distribution were presented in a very simple and stylized way, which permits one to analyse with clarity their essential structure and working. For a detailed analysis along the same lines, the reader is referred to Stephen Marglin (1984). The purpose of the present work is different. It is to study the theoretical and empirical incidence of a tax on profits in the context of growth. For this purpose it is necessary to present more general forms of these models.

Two aspects are particularly important in the broadening of the models. One is the introduction of taxation and government expenditures. Without these, there is of course no incidence to speak of. The second aspect is the introduction of technical progress. This generalization is particularly important for the empirical estimation of the models, to be carried out in future chapters. Some equations, such as the neoclassical "natural rate of unemployment" relation, (equation (24) of Chapter IV), would be greatly misspecified without considerations of technical progress. Additionally, we saw in Chapter III and its appendix 2, that to ignore technical progress can cause great biases in the estimation of incidence and shifting.

Other modifications will also be implemented, such as: the generalization of the neoclassical consumption problem to the multiperiod case with unequal working and retirement spans; the subsequent transformation of the resulting neoclassical consumption function to give observable indicators of its unobservable arguments (expected wealth or permanent income), and to render it empirically estimable, etc. In general the modifications will have as a goal the development of observable and estimable forms of all equations in the model, which additionally will permit us to enunciate incidence propositions in terms of observed, or estimable, parameters.

The plan of the chapter is as follows. In Section A we shall introduce the government budget constraint and all the related variables. In Section B, we shall examine the modification to the equations common to all models, as a consequence of government actions and of technical progress. In Section C, we shall examine the corresponding modifications to the consumption functions and the saving-investment relations. In Section D, we shall analyse modifications to the other equations. Finally in Section E, we shall tie up the modified form of each model.

In view of what was discussed in Chapter IV, Pasinetti and dual processes will not be treated in this work.

A) <u>A new relation: the government budget constraint</u>

The simple models introduced in Chapter IV, with no government, were constituted of systems of six equations. The introduction of the government adds a new relation, expressing the balance between government receipts and government expenditures.

The government receipts are mostly constituted of tax yields and profits from property owned. We shall regroup the taxes collected by the government in our model, expressed in ad-valorem (proportional) terms ,as follows:

1- taxes on wages, designated by the symbol tw

2- taxes on profits of enterprises, designated by tcr, and applied to profits after depreciation, but before they are distributed

3- taxes on profits distributed by enterprises to households; they will be designated by tyr

4- taxes on sales of consumption or investment goods, designated by tx; these will include direct subsidies to production, expressed in ad-valorem terms as a negative component of tx.

The government will carry out the following types of expenses:

1- It will expend in consumption goods, i.e. in goods which do not increase productive capacity; recall that in Chapter IV we considered, as a possible good example of consumption goods for the case of corporations, the executive airplanes (although some corporate executives will find the example unconvincing). Government consumption will be expressed as GC or, in per worker terms, as gc.

2- It will act as a source of funds to enterprises, by buying shares in them, or lending them capital (directly or indirectly), i.e. it will "save", autonomously, part of its receipts; these autonomous savings will be designated as GAS, or in per worker terms as gas (1).

3- It will effect transfers of funds to households, such as unemployment insurance or retirement payments. They will be designated by SU, or in per worker terms by su.

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(1) The government also "saves" by simply depositing its money in a bank account, if this one is lent to enterprises; or if it incurs in an excess of expenses (in capital goods) over receipts (government deficit: gds). This variable (gas) is intended to show the autonomous part of the government saving activity as opposed to that, unintentional, or compensatory, which is an automatic consequence of government deficits.

The government can also incur an excess of receipts over expenses, or vice versa, i.e. a budget surplus or deficit. This will be designated by GDS, or in per worker terms gds.

There is another element in the government budget equation; since the government owns some capital through the accumulation of its savings, it will receive distributed profits in proportion to the total capital it owns. We shall designate the proportion of total capital owned by the government as zg.

We shall also have to distinguish, from now on, between pre-tax or gross of tax prices of factors and goods and after tax or net of tax prices. For, say, price "x" we shall designate it as gross of tax by a second qualifier,"g", that is as "xg";and, when relevant, we shall designate it as net of tax by a qualifier,"n",that is as: "xn". For example, wages, profit rates and prices of goods gross of tax will be, respectively, wg, rg and pg; and net of tax: wn, rn, pn.

Let us now examine in more detail the structure of the government receipts. The tax base for the tax on profits (tcr) of enterprises is total profits or:

$$\Pi g = K\ rg\ pg \qquad (1)$$

where K is total capital, rg, as we saw, the gross of tax profit rate, pg is the price gross of sales tax and Πg total gross of tax profits.

We shall take as the tax base for tyr the distributed profits net of the tax tcr, or:

$$(1 - \theta)\ (\Pi g - tcr\ \Pi g) = (1 - \theta)\ (1 - tcr)\ K\ rg\ pg$$

where θ is the retention ratio of enterprises; therefore (1- θ) is the proportion of profits which is distributed.

The tax base for tw will be the gross of tax wage bill, wg L, and the tax base for sales of consumption or investment goods will be total production value of these, Q pg.

Then the government will receive total tax receipts, Tg, of:

$$Tg = K\ rg\ tcr + K\ rg\ (1 - \theta)\ (1 - tcr)\ tyr + wg\ L\ tw + Q\ pg\ tx \qquad (2)$$

it will also receive its participation on distributed profits, which we shall assume it will treat, for tax purposes, as other distributed profits; they will then be:

$$\Pi n,g = K\ rg\ pg\ (1 - \theta)\ (1 - tcr)\ (1 - tyr)\ zg \qquad (3)$$

Hence total government receipts Yg, will be:

$$Yg = Tg + \Pi n,g = Wg\ L\ tw + K\ rg\ pg\ [\ tcr + (1 - \theta)\ (1 - tcr)\ tyr + (1 - \theta)\ (1 - tcr)\ (1 - tyr)\ zg\ ] + Q\ pg\ tx \qquad (4)$$

By defining the average ad-valorem government participation in profits, tr, as:

$$tr = tcr + (1 - \theta)\ (1 - tcr)\ tyr + (1 - \theta)\ (1 - tcr)\ (1 - tyr)\ zg \qquad (5)$$

we can also write equation (4) as:

$$Yg = Tg + \Pi n,g = wg\ L\ tw + K\ rg\ pg\ tr + Q\ pg\ tx \qquad (6)$$

The expenditure side of the government constraint was presented above. Designating total government expenditures as GE, this side would have the following form:

$$GE = GC + SU + GAS \qquad (7)$$

In other words, the government expends its receipts in consumption goods (GC), subsidies or transfers to households (SU), and it saves part of them through participation in the ownership of enterpriese (GAS).

The balance (or imbalance) in the government budget can now be expressed as:

$$Yg - GE = GDS \tag{8}$$

The excess of income (Yg) over expenditures (GE) is constituted by a government surplus, (GDS > 0), or deficit (GDS < 0). From (6), (7) and (8) we obtain:

$$wg\ tw\ L + K\ rg\ pg\ tr + Q\ pg\ tx = GC + SU + GAS + GDS \tag{9}$$

in per capita terms and taking the product as numeraire (pg = 1) this becomes:

$$wg\ tw + k\ rg\ tr + q\ tx = gc + su + gas + gds \tag{10a}$$

or in alternative notation:

$$wg\ tw + (a1\ /\ a0)\ rg\ tr + tx\ /\ a0 = gc + su + gas + gds \tag{10b}$$

tr is given by equation (5).

In the neoclassical case, the term expressing subsidies per worker, su, is an average of subsidies given to households during their working periods, su0, and those given to them during their retirement periods, su1. At any moment of time, in a two period neoclassical model, the average subsidy expressed in per worker terms will then be, in view of equation (15a) of Chapter IV:

$$su = (su0\ L + su1\ R) \ / \ L = su0 + su1 \ / \ (1 + g) = su0 + su1 \ / \ (1 + n) \qquad (11)$$

For the neokeynesian model in a two period situation and in steady state, equation (11) would also apply; but for that model it is superfluous, since life cycle savings are not assumed. Another consequence of the introduction of the government in the model, is the added number of variables representing government actions:

tw, tcr, tyr, gc, gas, su (su0 , su1), gds

Since there is one extra equation, all of these variables but one should be taken as exogenous. Notice finally that equation (10a) or (19b) is common to all models we are studying.

B) <u>The other common equations</u>

We shall examine, first, the two identities describing the distribution of the product between consumption and investment and the distribution of national income between wages and profits. Thereafter, we shall examine the two equations describing production and factor pricing.

1) The consumption-investment and wage-profit identities.

The consumption investment identity had been initially written as follows (see equation (8) of Chapter IV):

$$Q(t) = C(t) + [d + g(t)]\ X(t)\ a1 \qquad (12)$$

This equation is affected because we are assuming now that the government consumes part of the product. If the symbol C(t) is kept for private consumption and the symbol GC(t) is introduced for government consumption the equation above becomes:

$$Q(t) = C(t) + GC(t) + a1\ [d + g(t)]\ X(t) \qquad (13)$$

Part of the product now goes to the consumption by the government GC(t).

We can divide equation (13) by total labour employed and drop the time indicators to rewrite:

$$1 / a0 = c + gc + (a1/a0)\ (d + g) \qquad (14)$$

or

$$1 = a0\ (c + gc) + a1\ (d + g) \qquad (15a)$$

Recalling also that we have defined the product per unit of labour employed q, and capital per unit of labour employed k, respectively as:

$$q(t) = Q(t) / L(t) = Q(t) / [a0\ Q(t)] = 1 / a0 \qquad (16)$$

and

$$k(t) = K(t) / L(t) = (a1\ Q(t)\ \ t] / [a0\ Q(t)] = a1 / a0 \qquad (17)$$

We can also obtain from (14) in our alternative notation:

$$q = c + gc + (d + g)\ k \qquad (15b)$$

Equations (15a) or (15b) constitute the consumption investment identity when there is a government.

The second common equation is the wage-profit identity which was written, without government as (see equations (9) and (7) of Chapter IV):

$$Q(t) = a0\ w(t) / p(t) + [d + r(t)]\ a1\ Q(t) \qquad (18a)$$

or in alternative notation (see equation (9-B) of Chapter IV):

$$q(t) = w(t) / p(t) + k(t)\ [d + r(t)] \qquad (18b)$$

Now with government taxation, the wage profit distribution identity should say that the product net of general sales taxes should be equal to the factor incomes gross of their direct taxes, or dropping time variables:

$$Q\ pg(1 - tx) = wg\ ao\ Q + a1'Q\ (d + rg)$$

or

$$pn = wg\ a0 + a1\ (d + rg)\ pg \tag{19}$$

with:

$$pn = pg\ (1 - tx) = pg\ ?x \tag{20}$$

where ?x is the tax complement: $?x = 1 - tx$;
taking the price level $pg = 1$:

$$?x = 1 - tx = a0\ wg + a1\ (d + rg) \tag{21a}$$

in a similar way, (18-B) would become:

$$q\ ?x = q.\ (1 - tx) = wg + k\ (d + rg) \tag{21b}$$

Recall that tx is the net rate of sales taxes and sales subsidies. If they were going to be written explicitly separated we should replace in (21a) or (21b) tx with:

$$t'x - sur = tx$$

where sur is the ad-valorem pure subsidy rate and tx the ad-valorem pure general sales tax rate. In the empirical analysis they will be separated in this manner.

Since the product is measured, in equations (15a) or (15b) and (21a) or (21b), in normal physical units, these equations are not affected by the introduction of technical progress.

2. Production and Marginal Productivity

Let us now examine the introduction of taxes and technical progress in the production and marginal productivity equations. We shall examine, first, the modifications caused by taxes and, second, those caused by introducing technical progress.

The production function and the marginal productivity equations, before taxes, were written (see equations (29) and (30) of Chapter IV):

$$1 / a0 \equiv q = f(k) = f(a1 / a0) \qquad (22)$$

and

$$f'(k) = f'(a1 / a0) = d + r \qquad (23)$$

The production function equation (22) is not affected by the introduction of government actions; the technological relation between capital, labour and production stays the same. But the marginal productivity condition is affected as follows. The sales tax tx introduces, as all taxes do, a wedge between the income paid by the purchaser of the product and the price received by the supplier. The entrepreneur will not receive the full price of the product p = pg , but rather the after tax price pn = pg (1 − tx). On the other hand, the wages that the entrepreneur will pay will be gross of wage taxes, even if the worker will receive them net of wage taxes; the same reasoning applies to profits or, in neoclassical terms, cost of capital. In light of this the marginal productivity conditions will become:

$$f'(k)\ pn = pg\ (d + rg) \tag{24}$$

and

$$q - f'(k) = f(k) - k\ f'(k) = wg\ /\ pn \tag{25}$$

The express consideration of technical progress will introduce one more modification to those equations. Let us now examine this.

Technical progress will be treated as disembodied and exogenous. In the more general case, this type of progress is represented by the effect that it has on the factors of production: it seems to augment their productive power. For a general production function, relating total (not per capita) production (Q) to total labour (L) and total capital (K), this effect is represented by factors affecting those two variables and growing in time. The general production function can then be written:

$$Q = F[K\ PK(t), L\ PL(t)] \tag{26}$$

where PK(t) and PL(t) are two factors augmenting the productive capacity of capital and labour respectively. Assuming these factors to have an exponential form, equation (26) could also be written:

$$X = F[\ K \exp(Rk'.t), L \exp(Rl'.t)\ ] \qquad (27)$$

where Rk' and Rl' are the rates of growth of PK(t) and PL(t)respectively.

According to the size of Rk' and Rl' (or PK(t) and PL(t)) technical progress can be classified as: pure labour augmenting, or Harrod neutral, if:

$Rl' = Rh' > 0$ and $Rk' = 0$

pure capital augmenting, or Solow neutral, if:

$Rl' = 0$ and $Rk' = Rs' > 0$

and Hicks neutral, if:

$Rl' = Rk'$

Of these concepts, that of Harrod neutral technical progress is the easiest to incorporate in empirical analysis. It can be shown that if technical progress is pure labour augmenting, and there are constant returns to scale, then at a constant capital output ratio k / q (or a1) the marginal product of capital, and hence, in competition, the profit rate r will remain constant over time, and if the profit rate is constant, then the capital output raio will remain constant over time.(See Allen 1967, p. 242)

In consequence, Harrod neutral technical progress is the only one to admit a general steady state, where the capital output ratio stays constant. Since we are analyzing steady states we shall introduce only Harrod neutral technical progress.

Now, from the assumptions of constant returns to scale, i.e. of a linearly homogeneous production function, equation (27) can be rewritten in our usual notation:

$$q \exp(- Rl' t) = f\{ k \exp[-(Rl' - Rk') t] \} \qquad (28)$$

which, for Harrod neutral technical progress, becomes:

$$q = \exp(R' t) f[k \exp(-R' t)] \qquad (29a)$$

or in alternative notation:

$$1 / a0 = \exp(R' t) f[a1 \exp(-R' t) / a0] \qquad (29b)$$

where we have written:

Rl ' = Rh' = R' = rate of growth of Harrod neutral technical progress.

Equation (29a) (or 29b) is the final form of the production function, which we shall utilize in our models. Taking derivatives of (29a) the new marginal productivity conditions become:

$$d q / d k = f'[k \exp(-R' t)] = (d + rg) / (1- tx) \qquad (30)$$

and

$$\exp(R' t) f[k \exp(-R' t)] - k f'[k \exp(-R' t)] = wg / pn$$
$$= wg / (1- tx) \qquad (31)$$

Again, since the assumption of constant returns to scale is retained, equations (21a) (or 21b), (30) and (31) are linearly dependent. We shall then, in general, drop equation (31). But notice that, by taking time derivatives, this equation implies that the wage rate grows with a geometric rate of growth equal to that of Harrod neutral technical progress (See Allen 1967, page 244):

$$(1 / wg) \, d \, wg / d \, t = R' \qquad (32)$$

C) <u>Consumption functions and saving-investment relations</u>

We shall examine, first, the neoclassical consumption function and saving-investment relation; and second, the neokeynesian consumption function and saving-investment relations.

1. The neoclassical consumption function and saving-investment relation.

In chapter IV the problem of the determination of consumption by the household was posed in terms of the following two expressions (see expressions (12.1) and (12.2) of Chapter IV):

$$\text{Max } U(C0, C1) \qquad (33a)$$

s.t.

$$C0 + C1 / (1 + r) = w / p \qquad (33b)$$

We are now going to generalize these expressions by introducing a government sector, multiple periods, unequal working and retirement spans, and technical progress.

Let us first examine the introduction of the government. This changes the household budget condition for several reasons. First, the consumer will not receive a wage wg but rather a wage net of tax wn :

$$wn = wg\ (1 - tw) = wg\ 7w \qquad (34)$$

7w is the tax complement 7w = 1-tw (34a)

Second, it will still pay for consumption goods the full price of these, p = pg . Third, it will receive transfers (or "subsidies") su0 or su1 , from the government, to complement his labour income or to help during his periods of transition from job to job (unemployment or, more in the neoclassical vein, "job search") or to complement his retirement income. Fourth, the profits he receives will be net of government taxes and other transfers. Call them rn :

$$rn = rg\ (1 - tr) = rg\ 7r \qquad (35)$$

where tr is defined by equation (5) above, and 7r is simply the tax complement:

$$\gamma r = (1 - tr) \qquad (35a)$$

In the neoclassical model the retention ratio Θ should be considered as determined completely by rentier-households' decisions; controlled by these, and not by the workers-managers of the enterprises.

If we finally introduce unequal working and retiring periods, assuming that households work Tr years and live retired Tl − Tr years, the household consumption problem can now be posed as:

$$\text{Max } U[C(0), C(1), C(2) \ldots C(Tr), C(Tr + 1) \ldots C(Tl - 1)] \qquad (36a)$$

subject to:

$$\sum_{t=0}^{Tl-1} C(t)\ (1 + rn)^{-t} = A \qquad (36b)$$

where A is total expected wealth, the present value of all its expected future earnings, plus an initial capital endowment, k0 :

$$A = k0 + \sum_{t=0}^{Tr-1} wn(t)\ (1 + rn)^{-t} + \sum_{t=0}^{Tl-1} su(t)\ (1 + rn)^{-t} \qquad (37)$$

In a competitive economy the wage grows at the same rate as Harrod neutral technical progress: R' (See equation (32) above). In steady state, with perfectly realized expectations, the household will expect his wage to grow in this manner:

$$wg(t) = wg\ (1 + R')^{t}$$

therefore

$$wn(t) = wg(t)\ (1 - tw) = wn\ (1 + R')^{t} \tag{38}$$

On the other hand, in steady state, transfers will grow at the same rate as the national product; therefore transfers per worker will grow with technical progress (Allen 1967, pp. 244, 245); hence transfers to working households will be:

$$su(t) = su0\ (1 + R')^{t} \qquad t = 0, 1, ..Tr-1$$

and to retired households:

$$su(t) = su1\ (1 + R')^{t} \qquad t = Tr, Tr + 1....T1 - 1$$

The expected wealth can then be written:

$$A = k0 + (wn + su0) \sum_{t=0}^{Tr-1} (1 + rn)^{-t}(1 + R')^{t} + su1 \sum_{t=Tr}^{T1-1} (1 + rn)^{-t}(1 + R')^{t} \tag{39}$$

Then the maximizing household would arrive at the following consumption plan which would be realized in steady state:(1)

$$C^{\sim}(t) = C\ (t, A, rn) \quad ; \quad t = 0, 1...Tr...T1-1 \tag{40}$$

or assuming a homothetic utility function:

$$C^{\sim}(t) = A\ h(t, rn) \quad ; \quad t = 0, 1...Tr...T1-1 \tag{41}$$

In order to arrive at manageable expressions for the savings investment relation let us further assume that:

$$h(t,rn) = h0(rn) \qquad \text{for } t = 0, 1...Tr-1$$

---

(1) Recall that in steady state expectations are perfectly realized.

$h(t,rn) = h1(rn)$ for $t = Tr, Tr+1 \ldots T1-1$

Notice that this assumption is simply a slight variation of Modigliani and Brumberg's (1954) assumption that $h(t,rn) =$ const. for all t.

We can correspondingly write:

$$C^{\sim}(t) = A\ h0(rn) = C0(A,rn) \quad ;t = 0 \ldots Tr-1 \qquad (42a)$$

$$C^{\sim}(t) = A\ h1(rn) = C1(A,rn) \quad ;t = Tr \ldots T1-1 \qquad (42b)$$

Where C0 is a typical worker consumption and C1 is a typical retired person consumption. Total private consumption per worker will then be:

$$C = (C0 + C1\ R) / L = C0 + C1\ R / L \qquad (43)$$

In steady state, with the employed population growing, in the neoclassical model, at the geometric rate n (because of the assumption of constant unemployment), people retiring in year t entered the labor force t − Tr years ago; then:

$$R(t) = L(t - Tr) = L(t) / (1 + n)^{Tr} \qquad (44)$$

Combining (43) and (44) and dropping the time variable t for present quantities, we obtain present consumption in per worker terms, C:

$$c = C0(A,rn) + C1(A,rn) / (1 + rn)^{Tr} = C(A,rn;n,Tr) \qquad (45)$$

or for homothetic functions:

$$c = A\ [\ h1(rn) + h2(rn) / (1 + rn)^{Tr}\ ] =$$

$$A\ h(rn;n,Tr) \qquad (46)$$

Equations (45) or (46) are theoretical expressions of consumption in the neoclassical, life-cycle model. They say that consumption is a function of: expected life-time wealth, A; the opportunity cost of consumption, rn ; and the structure of the working and retired population, indicated by Tr and the rate of growth of population, n.

In order to make those equations applicable to the observed world, it is still necessary to relate expected wealth, A, to empirically observable magnitudes. We shall not work directly with A, but with a more convenient concept, that of the Hicksian permanent income yp , related to wealth A by:

$$yp = A\ rn \qquad (47)$$

The Hicksian permanent income, yp , is income which can be consumed in each period without affecting total wealth A (in other words yp is the interest yield of A; the principal stays untouched).

Permanent income is still a theoretical, not directly observable, concept. Some authors, such as Milton Friedman (1957), have proposed several observable representations of yp. For the present model we shall arrive at an observable expression for permanent income in two steps.

The first is based on our assumption of steady state. This assumption, coupled with the assumption of perfect knowledge, implies that, at any moment, expectations are perfectly realized. A perfect steady state also implies the absence of random shocks, since these would take the economy, at least temporarily, out of its steady state path. In those circumstances, each household in the economy has in fact a disposable income which is equal to its permanent income. There are no deviations between the two.

Now introduce random shocks, but assume a stable, full employment (neoclassical) economy. The random shocks will take the economy temporarily out of steady state; there will be fluctuations. But the underlying trend of growth of disposable income is stable; any deviations from it being caused by the random shocks. The (rational) households will then take that trend as an indicator of their permanent income. Additionally, for the neoclassical model, an average over the length of the business cycle of the actual disposable income should be equal to the average trended income, and therefore should be an indicator of the average permanent income. This is a consequence of the fact that, in the neoclassical model, the cycle is a purely temporary disequilibrium phenomenon. We showed this in Appendix 1 to Chapter III.

In sum, we can represent permanent income, in our theoretical model, by average disposable income in steady state. In our empirical estimations we shall represent permanent income by an average over several periods of annual disposable income, in an attempt to smooth out the business cycle. This procedure is not very different from the one proposed by Milton Friedman (1957) for the same purpose.

In the theoretical deduction of incidence propositions, to be done in the next chapter, the neoclassical permanent income will then be taken as:

$$yp = yd = wn + k\ rn + su \qquad (48)$$

where su, average subsidies in per worker terms is:

$$su = (su0\ L + su1\ R) / L = su0 + su1 / (1 + n)^{Tr} \qquad (49)$$

Equation (49) replaces equation (11) in the multiperiod case. In the empirical chapter, permanent income will also be given by equation (48), but yd will be a simple average of income per worker over several years; ideally, over the business cycle.

The neoclassical consumption equation (45) can then be represented by:

$$c = C(A,rn;n,Tr) \qquad (50)$$

and, for n and Tr constant (in the steady state) (1):

$$c = C(A,rn) = C(yd,rn) \qquad (51a)$$

or for homothetic utility functions:

$$c = yd\ h(rn) \qquad (51b)$$

yd is given by equation (48) in the manner explained before. Notice though that equation 51a (and 51b) applies

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(1) Notice that, in this particular formulation, this equation is not very different, algebraically, from the neokeynesian Kaldorian equation (58a), but the concepts behind the two are very different.

only for the steady state path, i.e. in the long term. Then with respect to incidence, only permanent tax changes can be studied with such equations. All other cases should be examined using equation (50).

We shall use equation (51a) directly in the econometric model of Chapter VII. For the deduction of incidence propositions, to be done in Chapter VI, it is more convenient to work with the savings-investment relations. To obtain the savings-investment relation, take the wage profit relation given by equation (21b), and the consumption investment relation given by equation (15b), and combine them through partial elimination of q, to obtain:

$$wg + k\ (d + rg) + q\ tx = c + gc + k\ (d + g) \qquad (52)$$

In (52), replace c from equation (51a) and gc from the government budget constraint (equation (10a)):

$$gc = wg\ tw + rg\ k\ tr + q\ tx - su - gas - gds \qquad (49a)$$

where su is now, for the multiperiod case:

$$su = su0 + su1\ (1 + n)^{Tr} \qquad (49b)$$

to obtain:

$$gas + gds + wn + k\ rn + su - C(yd,rn) = gk \qquad (53)$$

and using the definition of yd from (48):

$$gas + gds + yd - C(yd,rn) = gk = g\ a1\ /\ a0 \qquad (54)$$

Equation (54) is the neoclassical saving-investment relation.

An alternative form of that equation will also prove convenient. By definition, savings in the private sector would be equal to disposable income minus consumption. Calling these savings, in per worker units, S:

$$S = yd - C$$

Then from (51a):

$$S = yd - C(yd,rn) = S(yd,rn)$$

replacing this equation in (54) we obtain the alternative neoclassical saving-investment relation:

$$gas + gds + S(yd,rn) = g\ k = g\ a1\ /\ a0 \qquad (54a)$$

2) The neokeynesian saving-investment relation and consumption function

Our next step is to investigate the changes introduced by taxes and other government actions in the neokeynesian saving-investment relation. As explained in Chapter IV, we shall concentrate, in the main text, on the savings function by income categories, proposed by Kaldor.

In Chapter IV, the Kaldorian savings function by income categories had the following form (see equation (33a) of that chapter):

$$s = sw\ w + sp\ r\ k \qquad (55)$$

where s are savings per employed worker. With government taxation, savings are done on disposable factor incomes, hence, equation (55) becomes:

$$s = sw\ [wg(1 - tw) + su] + sp\ rg\ (1 - tr)\ k$$

$$= sw\ (wn + su) + sp\ rn\ k \qquad (56)$$

Recall that the average profit tax tr is defined by equation (5):

$$tr = tcr + (1 - \theta)(1 - tcr)\, tyr_p + (1 - \theta)(1 - tcr)(1 - tyr) zg \quad (5)$$

Equation (56) is the new Kaldorian savings function. Since consumption and savings of households are related by the following definitional identity, expressing that savings is equal to disposable income minus consumption:

$$s = wn + su + k\, rn - c$$

we can obtain the Kaldorian private consumption function from (56) and (57) as:

$$c = (1 - sw)\,(wn + su) + (1 - sp)\, k\, rn \quad (58)$$

Then the neokeynesian saving-investment relation follows by replacing c, from the consumption function (58) and gc from the government budget constraint (10a), in equation (52), and simplifying:

$$sw\,(wn + su) + sp\, rn\, k + gas + gds = g\, k \quad (59a)$$

or in alternative notation:

$$sw\,(wn + su) + sp\, rn\, a1 / a0 + gas + gds = g\, a1 / a0 \quad (59b)$$

Recall that, in the general case, sw and sp, the average propensities to consume out of wages and out of profits, need not be constant; rather they can be taken as functions of those income categories:

$$sw = sw(wn + su,\, rn\, k) \quad (60)$$

and

$$sp = sp(wn + su,\, rn\, k) \quad (61)$$

In this case it may be more convenient to write the

Kaldorian savings function as:

$$s = S(wn + su, rn\ k) \quad (56a)$$

and the Kaldorian consumption function as:

$$c = wn + su + k\ rn - S(wn + su, rn\ k)$$

$$= yd - S(wn + su, rn\ k) = C(wn + su, rn\ k) \quad (58a)$$

From equations (56), (56a) and (59a) the Kaldorian saving-investment relation can also be written:

$$S(wn + su, k\ rn) + gas + gds = gk \quad (59c)$$

D) <u>The other equations</u>

Besides the consumption functions or savings-investment relations, the models introduced in Chapter IV had second distinct closure conditions. These were, for the neoclassical model, the hypothesis of full employment, or a natural rate of unemployment, which could be written:

$$g = n \quad (60)$$

For the neokeynesian model, the hypothesis that the rate of investment was positively related to the rate of profits (the "animal spirits" function):

$$g = i(r) \quad (61)$$

and for the neomarxian model the hypothesis that the wage rate is related to a socially necessary level of consumption; and ultimately, it may be expressed as a constant proportion of the product per worker (ch IV, section C) :

$$w / q = const \tag{62}$$

The introduction of the government and of technical progress, affects the form and meaning of all of these conditions. Let us start with the neoclassical equation (60). This one is not affected by the introduction of taxes but it is by technical progress. Taking logarithms of the production function (29a), then taking time derivatives, it can be seen that, for Harrod-Neutral technical progress and in steady state, the rate of growth of the product per worker, $\dot{q} / q$, is equal to the rate of growth of technical progress, R': (Allen 1967, pp.244, 245).

$$\dot{q} / q = \dot{Q} / Q - \dot{L} / L = g - \dot{L} / L = \dot{k} / k = R' \tag{63}$$

In the neoclassical model, the natural rate of employment hypothesis means that the rate of growth of employed labour $\dot{L} / L$ is equal to the rate of growth of the labour force or, for a constant participation rate, the rate of growth of the population, n:

$$\dot{L} / L = n$$

Hence the neoclassical equation (60) becomes:

$$g = R' + n \tag{64}$$

The neokeynesian equation (61) is also affected by the introduction of taxes. The rate of profit argument in that function should now reflect the average profitability of investment for the private sector, therefore it should be net of taxes. Hence the neokeynesian "animal spirits" function should now be written:

$$g = i(rn) = i[rg\ (1 - tr)] = i(rg\ \gamma r) \qquad (65)$$

where γr is the tax complement $1 - tr$, and tr is given by equation (5).

The neomarxian equation (62), postulating a constant wage share as an outcome of the process of wage determination in the neomarxian model, is also affected by the introduction of taxes. The share to be conserved now is the net of tax share. The neomarxian wage equation becomes, then:

$$wn / q = a0\ wn = a0\ wg\ (1 - tw) = a0\ wg\ \gamma w = const \quad (66)$$

Recall that γw is the tax complement $\gamma w = 1 - tw$.

Finally, as mentioned in chapter IV, it is important to introduce in equation (66) a term which explicitly reflects the relative strength of workers in the class struggle. Theoretically, the introduction of a class struggle variable in that equation presents no problem. But for econometric estimation, we need a quantitative indicator of this variable. This is problematic. There are no readily available "class struggle" indicators. A possible one, which we shall use in this work, is the level of strike activity, as measured by an index of days lost in strikes, idls (1).

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(1) A possible objection to this indicator is that more strikes do not necessarily mean a stronger working class. Despite this objection, we consider that, as a first order of approximation, strikes are a good measurement of class power.

The neomarxian equation (66) with the variable idls would become:

$$wg\ a0 = wg / q = h'(idls) / (1 - tw) \qquad (67a)$$

In Chapter VII, a slight variation of this equation will be used. The product per worker, q, grows as the rate of Harrod neutral technical progress, R':

$$q = q0 \exp(R'\ t) \qquad (68)$$

where t is the time variable. Replacing this in (67a) and taking:

$$q0\ h'(idls) = h(idls)$$

we get,

$$wg = \exp(R'\ t)\ h(idls) / (1 - tw) \qquad (67b)$$

This equation, with a linear form for the function h( ), will be used for the econometric estimation of Chapter VII. Notice that it is equivalent to equation (67a).

E) Recapitulation

We have now completed the description of the models of growth with a government sector and with technical progress. The models are now constituted by systems of seven equations, five of which are common to them all, and two which differentiate them.

The five common equations are: the government budget constraint (10a) or (10b); the consumption-investment identity (15a) or (15b); the wage-profit identity (21a) or (21b); the production function (29a) or (29b); the marginal productivity condition (39).

The neoclassical model is completed with the neoclassical savings-investment relation (54a), and with the natural rate of unemployment condition (64). Instead of the savings-investment relation the neoclassical model could use directly the consumption function (51a) (or (51b)).

The neokeynesian model is completed with the Kaldorian savings-investment relation (59c), and the "animal spirits" function (65). Alternatively, the Kaldorian consumption function (58a) could be used instead of (59c).

The neomarxian model is completed with the kaldorian savings-investment relation (59c) or consumption function (58a), and with the marxian wage equation (66) or (67a) (or (67b)).

The hybrid models are completed as follows. The neoclassical model with a Kaldorian consumption function, or CK model, is completed with the neoclassical natural rate of unemployment condition (64) plus the Kaldorian savings-investment relation (59c) or consumption function (58a). The neokeynesian model with neoclassical consumption, or KC model, is completed with the neokeynesian animal spirits function (65), plus the neoclassical savings-investment relation (54a) or the consumption function (51a) (or (51b)). The neomarxian model with neoclassical consumption function, or MC model, is completed with the neomarxian wage equation (66) or (67a) (or (67b)) plus the neoclassical savings-investment relation (54a) or the consumption function (51a) (or (51b)).

The endogenous variables of these models are

wg, rg, c, g, a0, a1 (or alternatively to a0, a1: k, q), plus one of the government variables (tr, tx, tw, su, gas, gds or gc).

In the form presented above the models can be used to obtain theoretical propositions on the incidence of taxes or, better, the incidence of alternative government budget policies. This will be treated in the next chapter, VI.

The forms presented above are also the basis for the empirical models which will be estimated in Chapter VII by econometric methods. There, it is only necessary to choose specific functional expressions for the equations presented. We shall pursue this matter in Chapter VII.

CHAPTER 6

THEORETICAL INCIDENCE OF TAXATION IN GROWTH MODELS

In the previous two chapters we have presented several alternative models of growth and the modifications to them caused by the introduction of government actions, technical progress and other factors. The ground is now ready to derive from these models the theoretical effects which a change in the government budget would have on the distribution of income between wages and profits. In other words, we can now study the theoretical incidence of taxation in the growth models.

We proceed as follows. In Section A, we do some preliminary work, which consists essentially in a presentation of the three basic models developed in chapter V in a more ordered and condensed fashion. We also present the general procedure whereby we can obtain from the basic models total differentials of the factor prices as a function of the total differentials of the endogenous variables; the derivation of these differentials is a precondition to the development of incidence and shifting.

In section B, we present a detailed discussion of the concepts and measurements of incidence and shifting which we shall employ, and compare them to measurements proposed by other authors. Finally, in section C, we study the balanced budget incidence of a tax on profits, tr, used to finance government consumption, gc. Other cases of incidence are presented in Annex 4 to this chapter.

A) Preliminary Work

The best way to derive propositions on the incidence of taxes from multiple equation models, such as those studied here, is to transform the systems of equations defining each model into a system of equations in the differentials of the exogenous variables. This has been the procedure followed in many recent works on incidence. Harberger (1962), Mieszkowski (1967), McLure (1969, 1970, 1975) to name a few, have followed this method in their exposition of general equilibrium models of incidence. The procedure permits a very clear deduction of propositions on any type of incidence setting; also, it is intuitively appealing, because it relates directly to the different concepts of incidence proposed in the literature as we shall see in section B.

The systems of equations defining the growth and distribution models which we are studying, are scattered throughout Chapter V. As a preliminary step, it is convenient to present them here once again in a more ordered fashion. Throughout the rest of the chapter, and for the systems of total differentials which we shall develop, we have chosen the notation of those equations which employs the capital output ratio a1, and the labour output ratio a0, rather than capital per worker, k, and output per worker, q. But recall that it is an easy matter to pass from one formulation to the other by using the identities:

$$k = a1 / a0 \quad \text{and} \quad q = 1 / a0$$

To facilitate the interpretation of the equations, the definitions of all symbols used have been regrouped in Table 1. In its final form in Chapter V, each model was constituted of five common and two specific equations. They are as follows:

Common Equations

-- Consumption-investment identity, presented as equation (15a) or (15b) of Chapter V:(1)

$$1 = a0\ (c + gc) + a1\ (d + g) \tag{1}$$

-- Wage-profit identity, presented as equation (21a) or (21b) of Chapter V:

$$1 - tx = 7x = a0\ wg + a1\ (d + rg) \tag{2}$$

-- Production function, presented as equation (29a) or (29b) of Chapter V:

$$1 / a0 = \exp(R' t) \, f[\exp(-R' t) \, a1 / a0] \qquad (3)$$

-- Marginal productivity condition, presented as equation (30) of Chapter V:

$$f'[\exp(-R' t) \, a1 / a0] = (d + rg) / (1 - tx) \qquad (4)$$

Recall that f' is the derivative of f.

and

-- Government budget constraint, presented as equation (10a) or (10b) of Chapter V:

$$a0 \, wg \, tw + a1 \, rg \, tr + tx = a0 \, (gc + su + gas + gds) \qquad (5)$$

Recall that tr is the implicit tax rate on profits, defined by equation (5) in Chapter V, and repeated as equation (8) below; it represents the total government participation in the profits, through taxes and rents, in ad-valorem terms.

Specific equations

Neoclassical model

-- neoclassical saving-investment relation, presented as equation (54a) of Chapter V:

$$gas + gds + S(yd, rn; n) = g \, a1 / a0 \qquad (6c)$$

and

-- natural rate of unemployment hypothesis presented as equation (64) of Chapter V:

$$g = n + R' \qquad (7c)$$

Neokeynesian model

-- Kaldorian saving investment relation, presented as equation (59c) of Chapter V:

$$S(wn + su , rn\ a1 / a0) + gas + gds = g\ a1 / a0 \qquad (6k)$$

and

-- Animal spirits investment hypothesis, given by equation (65) of Chapter V:

$$g = i(rn) ; \quad i' > 0 \qquad (7k)$$

neomarxian model

-- Kaldorian saving investment relation, shared with the neokeynesian model, written as equation (6k) above.

-- Neomarxian socially necessary wage hypothesis, given by any of the equations (66), (67a) or (67b) in Chapter V. We shall take (67a) in this chapter:

$$wg\ a0 = h1(idls) / (1 - tw) \qquad (7m)$$

There are additionally certain auxiliary definitional equations which are employed in order to simplify the equations presented above. They are as follows.

The definition of the implicit tax rate on profits:

$$tr = tcr + (1 - \theta)\ (1 - tcr)\ tyr +$$

$$(1 - \theta)\ (1 - tcr)\ (1 - tyr)\ zg \qquad (8)$$

The definitions of net of tax factor payments:

$$rn = rg\ (1 - tr) = rg\ ?r \qquad (9)$$

$$wn = wg\ (1 - tw) = wg\ ?w \qquad (10)$$

The definition of disposable income of the households:

$$yd = wn + su + rn\ (a1\ /\ a0) \qquad (11)$$

and finally, for the neoclassical model, the decomposition of transfers to households su, in transfers to working households, su0 and to retired ones, su1

$$su = su0 + su1\ /\ (1 + n)^{TR} \qquad (12)$$

We can now proceed to obtain the differential forms of the above three models. Since the derivation of these forms is mostly a matter of tedious algebra, it has been confined to appendix 1 to this chapter. There, we have obtained, for each model, a system of seven equations in the differentials of the endogenous variables:

d rg, d wg, d a0, d a1, d g and d c,

plus one of the endogenous variables (one of d ?w, d ?r, d gas, d gds, d su, d gc). For the analysis of incidence and shifting, we have solved those equations for the total differential of rg; the total differential of wg, which is also needed, is shown there to be related to d rg by the very simple expression:

$$d\ wg = d\ ?x\ /\ a0 - (a1\ /\ a0)\ d\ rg$$

The total differentials of d rg obtained in the appendix for the three models are, then, as follows.

For the neoclassical model,

$$d\,rg = (1 / Jc) \{ - a0 \,(d\,gas + d\,gds + S1\,d\,su) - Bc,x\,d\,?x - a0\,wg\,S1\,d\,?w - [\,a1\,S1 + a0\,S2\,]\,rg\,d\,?r \} \quad (13a)$$

where Jc is the Jacobian determinant of the neoclassical system:

$$Jc = ?r \left\{ - \frac{a1\,rg\,?x\,Ek,l}{a0\,wg\,(d + rg)} \,(S1 - g / rn) - a1\,(?w / ?r)\,S1 + [\,a1\,S1 + a0\,S2\,] \right\} \quad (13b)$$

and Bc,x is:

$$Bc,x = \frac{a1\,rg\,?r\,Ek,l}{a0\,wg} \,(S1 - g / rn) + ?w\,S1 \quad (13c)$$

For the neokeynesian model the total differential of rg is:

$$d\,rg = (1 / Jk) \{ - a0 \,(d\,gas + d\,gds + sw'\,d\,su) - Bk,x\,d\,?x - a0\,wg\,d\,?w - a1\,(sp' - i')\,rg\,d\,?r \} \quad (14a)$$

where, Jk is the Jacobian determinant of the neokeynesian system:

$$Jk = ?r \ \{ - \frac{a1 \ rg \ ?x \ Ek,l}{a0 \ wg \ (d + rg)} \ (sp' - g / rn) -$$

$$a1 \ (?w \ / \ ?r) \ sw' + a1 \ (sp' - i') \ \} \qquad (14b)$$

and Bk,x is:

$$Bk,x = \frac{a1 \ rg \ ?r \ Ek,l}{a0 \ wg} \ (sp' - g / rn) + ?w \ sw' \qquad (14c)$$

Finally, for the neomarxian model, the total differential of rg is:

$$d \ rg = \frac{(Ar,g \ Ek,l - 1) \ ?w \ d \ ?x - a0 \ wg \ d \ ?w}{a1 \ (Ek,l - 1) \ ?w} \qquad (15)$$

All variables and parameters used in these expressions are defined in table 1.

From expressions (13a) to (15), representing the total differentials of the rate of profit in the models, we shall be able to derive in future sections expressions for various types of tax incidence. First we need to define more precisely a measure of incidence and compare it with other measures which have been proposed. This is done in Section B, which follows.

B) Concepts and Measurement of Incidence.

In Chapter I, we adopted the definition of incidence proposed by Bent Hansen (1967, p.93), as: "The effects on the real incomes in society of fiscal policy measures, other things being equal."

We saw that the most common "fiscal policy measures" traditionally used in incidence are those involved in the concepts of balanced budget, differential and specific incidence. In Balanced budget incidence the "fiscal policy measure" is the change in a tax rate, accompanied by a simultaneous change in real government expenditures, so as to keep the budget balance unaltered. In differential incidence the fiscal policy measure is the increase or decrease in a tax rate, accompanied by a simultaneous decrease or increase in another tax rate so as to maintain the total real tax yield, and therefore the budget balance, unaltered. In specific incidence the fiscal policy measure is simply the change in one tax; since there are no compensating changes in other taxes or government expenditures, the budget balance is in this case altered.

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TABLE 1

SYMBOLS USED IN THE MODELS OF GROWTH AND DISTRIBUTION

a0: labour output ratio, a0 = L / Q; it is equal to the inverse of the output per unit of labour, q.

q: output per unit of labour, q = Q / L

a1: capital otput ratio, a1 = K / Q; also equal to the quotient of capital per unit of labour, k, and output per unit of labour, q, a1 = k / q

k: capital per unit of labour, k = K / L

c: private consumption per worker (or, better, per unit of labour), c = C / L

gc: government consumption per worker; that is, government use of non productive goods, gc = GC / L

d: depreciation rate.

g: rate of growth of total (not per worker) output, Q, per period; g = (1 / Q) (d Q / d t). In steady state , it is also equal to the rate of growth of total (not per worker) capital, K, per period; g = (1 / K) (d K / d t).

wg: gross of tax wage rate.

wn: net of tax wage rate; related to the gross of tax wage rate by wn = wg (1 - tw)

tw: ad valorem tax rate on wages.

7w: complement of the tax rate, tw ; 7w =1-tw

rg: gross of tax and net of depreciation rate of profit.

rn: net of tax and of depreciation rate of profit. It is related to rg by: rn = rg (1 - tr) = rg 7r;

tr: implicit tax rate on profits; tr represents total government participation in profits, in ad valorem terms , obtained through taxation and, in his quality as a capital owner, through rents (distributed profits). It is given by the expression, obtained in chapter V (equation 5),

$$tr=tcr+(1-\theta)(1-tcr)tyr+(1-\theta)(1-tcr)(1-tyr)zg$$

tcr: is the ad valorem tax rate on profits of enterprises.

tyr: is the ad valorem tax rate on profits distributed to households.

θ: is the retention ratio of profits of enterprises; hence $(1-\theta)$ is the distribution ratio.

zg: is the share of total capital owned by the government.

?r: is the complement of tr , defined as ?r = (1-tr)

tx: is the ad valorem tax rate on the product, net of direct subsidies to production (with these expressed in ad valorem terms.

?x: is the tax complement of tx defined as ?x = 1-tx

R': is the rate of growth of Harrod-neutral technical progress.

su: is the expenditure of the government in transfers (subsidies) to the households, per worker. In the neoclassical model it is additionally given by:

$$su = su0 + su1 / (1 + n)^{TR}$$

TR: is the average working life of hoseholds.

n: is the rate of growth of the labour force (or active population); for a constant participation rate it is equal to the rate of growth of total population.

su0: are transfers to working households, per worker.

su1: are transfers to retired hoseholds, per worker.

gas: autonomous savings of the government; funds provided to enterprises to invest, per worker.

gds: government budget surplus (if positive) or deficit (if negative), in per worker terms.

yd: disposable income of hoseholds, in per worker terms, defined as,

$$yd = wn + su + (a1 / a0)\ rn = wn + su + k\ rn$$

y: is the national income, defined as,

$$y = wg + k\ rg$$

Ar: is the share of profits gross of tax, but net of depreciation, in the national income, ; 1 - Ar , is the wage share:

$$Ar = k\ rg / y = k\ rg / (wg + k\ rg)$$

$$1 - Ar = wg / y$$

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TABLE 1

Ar,g: is the share of profits, gross of taxes and gross of depreciation, in the national product at producer prices, defined as,

$$Ar,g = k\ (d + rg) / (?x\ q) = a1\ (d + rg) / ?x$$
$$= a1\ (d + rg) / [a0\ wg + a1\ (d + rg)]$$

Ar,d: is the share of profits net of depreciation and net of profits tax, in disposable income, defined as,

$$Ar,d = k\ rn / yd = k\ rn / (wn + su + k\ rn)$$

Bn: is the ratio of the net of tax profit bill to the net of tax wage bill, defined as,

$$Bn = K\ rn / [\ L\ (wn / pg)] = k\ rn / (wn\ pg)$$

Bg: is the ratio of the gross of tax profit bill to the gross of tax wage bill,

$$Bg = k\ rg / wg = a1\ rg / (a0\ wg) = Ar / (1 - Ar)$$

S1: is the partial derivative of the neoclassical savings function, S(yd, rn), with respect to its first argument, that is, the neoclassical marginal propensity to save out of income. It will also be written as, ms,y.

ms,y = S1

S2: is the partial derivative of the neoclassical savings function, S(yd, rn), with respect to its second argument. Notice that this implies keeping yd constant when obtaining S2.

Es,r: is the neoclassical elasticity of savings with respect to the rate of profit, defined as,

$$Es,r = rn\ S2 / S(yd, rn)$$

sw': is the partial derivative of the neokeynesian (and neomarxian) savings function, S(wn + su, k rn), with respect to its first argument, that is, the neokeynesian marginal propensity to save out of wages. It will also be written as, ms,w.

$$sw' = \partial S / \partial (wn + su)$$

ms,w = sw'

sp': is the partial derivative of the neokeynesian (and neomarxian) savings function, S(wn + su, k rn), with respect to its second argument, that is, the neokeynesian marginal propensity to save out of profits. It will also be written as ms,p.

$$sp' = \partial S / \partial (k\ rn)$$

ms,p = sp'

Sy: is the average propensity to save out of income in the

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TABLE 1

neoclassical model, defined as,

$$Sy = S / yd = (yd - c) / yd$$

mi,p: is the derivative of the animal spirits function with respect to the net rate of profit (in the neokeynesian short term mi,p is equal to the marginal propensity to invest out of profits),

$$mi,p = d\ i(rn) / d\ rn = i'(rn) = d\ g / d\ rn$$

The introduction of a budget constraint in our models simplifies greatly the application of those concepts. For a given model, the differential incidence of replacing tax t_i by tax t_j becomes simply the study of the partial derivatives $\partial rg / \partial ti$ or $\partial wg / \partial ti$ when t_j is taken as an endogenous variable in the model (so that it can adjust to produce a new budget balance). The balanced budget incidence of ti becomes the study of the partial derivatives $\partial rg / \partial ti$ or $\partial wg / \partial ti$ when one of the government expenditure variables, gc, gas, or su, is taken as endogenous. The specific incidence concept entails to run budget deficits or surpluses i.e. to study $\partial rg / \partial ti$ or $\partial wg / \partial ti$ with gds as the endogenous government variable. As mentioned in the introduction, we shall examine here the balanced budget incidence more extensively than the differential or specific incidences.

In studying balanced budget incidence we still have to decide which government expenditure variable to take as endogenous: gc, gas or su (su0, su1). The incidence results will be different in each case. If, for example, gc is taken as endogenous, the tax change, d ti, will be reflected in a change in gc. This means that total savings in the economy will fall, as households would have previously saved part of the by-gone income. If, on the contrary, gas is taken as endogenous, total savings will increase, as households would have consumed part of the income taxed away. If it is subsidies which are taken as endogenous, the government, much in the way of Harberger's original model, is redistributing back the tax taken, and the net effect on total savings will probably be smaller in absolute value than in the other two cases.

Krzyzaniak (1967) and Feldstein (1974) used the balanced budget concept with gc endogenous. But Feldstein was well aware that this hypothesis decreases national savings reducing, in his model, the capital intensity of production and the real wage .

Since we are not studying the incidence of taxes in one model, but rather comparing incidence in alternative models, it matters less which incidence concept is used than that we use the same concept for all the models. We have thus decided to fully develop balanced budget incidence with government consumption, gc, endogenous, since this one results in simpler incidence expressions; but some remarks will be made about the other cases (see Appendix 4 to this chapter)..

This leads us to our problem: the measurement of incidence. As mentioned in Chapter I a natural measure would be given by the impact of the fiscal policy change -- in this case a balanced budget change in a tax and in government consumption -- on the ratio of the net of tax profit bill to the net of tax wage bill. This ratio -- call it Bn -- is then:

$$Bn = (K\ rn) / [L(wn / pg)]$$

$$= (K\ rg\ \gamma r) / [L(wg\ \gamma w / pg)] \qquad (16a)$$

where, recall that K, L and pg are, respectively, total capital stock, total employed labour and price level. Since the latter is taken as numeraire in the growth models examined, equation (16a) can also be written:

$$Bn = k\ rn / wn = a1\ rn / (a0\ wn)$$
$$= Ar\ 7r / [(1 - Ar)\ 7w] \qquad (16b)$$

The impact of the fiscal policy measure on the distribution of income is best expressed in relative terms, by taking logarithms and differentiating equation (16a):

$$d\ Bn / Bn = d\ K / K + d\ rn / rn - d\ L / L - d\ wn / wn + d\ pg / pg$$
$$= d\ K / K + d\ rg / rg - d\ wg / wg - d\ L / L + d\ pg / pg - d\ tr / 7r - d\ tw / 7w \qquad (17a)$$

or (16b):

$$d\ Bn / Bn = d\ k / k + d\ rn / rn - d\ wn / wn$$
$$= d\ k / k + d\ rg / rg - d\ wg / wg - d\ tr / 7r + d\ tw / 7w \qquad (17b)$$

Equation (17a) shows how the relative change in the distribution of net of tax income depends on the relative change in: the capital stock, d K / K; employment, d L / L; gross of tax profits, d rg / rg , and wages, d wg / wg ; the price level, d pg / pg ; and taxes on profits d tr / 7r , and on wages, d tw / 7w . Equation (17b) presents another view of the change in the distribution of income, more directly related to the models we are examining, as a function of changes in: gross of tax wages and profits, $d\,wg / wg$ and $d\,rg / rg$; the capital labour ratio (or capital per unit of labor) $d\,k / k$; and taxes, $d\,tr / \tau r$ and $d\,tw / \tau w$.

In a general model a "fiscal policy measure" will affect most, or all, of the elements in the right hand side of equations (17a) and (17b). The end results of all these changes will constitute the impact of the measure in the net of tax distribution of income: $d\,Bn / Bn$.

Equation (17b) can also be rewritten in terms of elasticities with respect to the tax whose incidence is being studied; in the case of the incidence of a profits tax, tr, for example, equation (17b) can be expressed as:

$$E(Bn,tr) = Ek,tr + Erg,tr - Ewg,tr - 1 + Etw,tr \qquad (18c)$$

where the above elasticities are defined as:

$$Ex,tr = (\tau r / x)\ \partial x / \partial tr \text{ for } x = k,\ rg,\ wg \qquad (18b)$$

for tw as:

$$Etw,tr = (\tau r / \tau w)\ \partial tw / \partial tr \qquad (18c)$$

and for E(Bn,tr) as:

$$E(Bn,tr) = (\tau r / Bn)\ \partial Bn / \partial tr \qquad (18d)$$

Notice that, for the balanced budget incidence of a tax on profits, tr, the terms $d\,tw / \tau w$, in equation (17b), and Etw,tr in (18a), are zero.

The impact on the distribution of income, or incidence of a certain fiscal policy involving a change in tr, can then be ascertained and quantified using equations (17b) or (18a). From the definition of Bn given by equations (16a) and (16b) it can be seen that if d Bn / Bn or E(Bn,tr) is zero the distribution of income has not changed as a consequence of the balanced budget tax change. If d Bn / Bn or E(Bn,tr) is less than zero, the distribution of income has changed in favour of wages; and if d Bn / Bn or E(Bn,tr) is greater than zero the distribution of income has changed in favour of profits. Hence, as we stated, the impact of the budget change in Bn is a natural way of measuring incidence.

To the best of our knowledge, the above measure has not previously been proposed in the literature. Most other measures try to relate the change in profits, and in wages, caused by the budget change (or fiscal policy measure), to the direct or the the total tax burden; that is, they try to examine the way the tax burden is divided between wages and profits.(1) The total tax burden, d BT ,is defined as the total change in disposable real income ,d yn (in general refered as total loss of real income), caused by the tax changes:

$$\begin{aligned} d\ BT &= d\ yn = d\ (k\ rn + wn) = k\ d\ rn + rn\ d\ k + d\ wn \\ &= k\ 7r\ d\ rg - k\ rg\ d\ tr + rg\ 7r\ d\ k + 7w\ d\ wg \\ &\quad - wg\ d\ tw \end{aligned} \tag{19}$$

---

(1) See, for example, Feldstein (1974) and Krzyzaniak (1967). Krzyzaniak defines these concepts for discrete changes in taxes; Feldstein, for differential changes.

The direct tax burden, d BD, is defined as the negative of the increased revenue collected by the tax (ignoring second and third order differentials):

$$d\,BD = -\,k\,rg\,d\,tr - wg\,d\,tw - q\,d\,tx \qquad (20)$$

The difference between the total and the direct tax burden is the indirect tax burden, d BI:

$$d\,BI = d\,BT - d\,BD = k\,\gamma r\,d\,rg + rg\,\gamma r\,dk + \gamma w\,d\,wg + q\,d\,tx \qquad (21)$$

Notice that for $\gamma w = \gamma r$ the indirect tax burden becomes, if profits are maximized (equation 2', Appendix 1 to this chapter):

$$d\,BI = \gamma r\,(k\,d\,rg + d\,wg) + rg\,\gamma r\,d\,k = rg\,\gamma r\,dk \qquad (22)$$

This is true, in particular, for new taxes ($\gamma r = \gamma w = 1$) If, additionally, there is full employment and a constant capital stock, dk = 0, we obtain:

$$d\,BI = 0 \qquad (23a)$$

and

$$d\,yn = d\,BD \qquad (23b)$$

That is, the total tax burden is equal to the direct tax burden.

As mentioned in Chapter II, Feldstein (1974) argues that the terms in d k should not make part of the burden of the tax because they do not affect total welfare. He reasons, by using a life cycle model of savings, that the, net rate of return rn measures, to a first order of approximation, the value of households' time preference, and that d k represents simply a change in the timing of consumption (savings of the working period to be spent on the retiremet period). More precisely, in a two period model of life cycle savings, the budget constraint can be written w - c1 = c2 / (1 + rn). In this model capital is equal to savings in period 1: k = w - c1, then d k = - d c1 = d c2 / (1 + rn) (taking w and rn as constant). Hence, the total change in utility d U(c1,c2), resulting from d k is d U = U1 d c1 + U2 d c2 = [-U1 + U2 (1 + rn)] d k = 0, by the first order utility maximization conditions (Feldstein, 1974, p. 571 n. ; U1 and U2 denote partial derivatives of U with respect to c1 and c2 respectively). He concludes, then, that the total change in households' utility as a consequence of the capital change, d k, is zero.

For Feldstein, then, the total tax burden is given by:

d BTf = k d rn + d wn

= k ?r d rg - k rg d tr + ?w d wg - wg d tw (24a)

and the indirect tax burden by:

d BIf = k ?r d rg + ?w d wg (24b)

The direct tax burden is still given by (20).

For new taxes, even if the capital stock is not constant, in Feldstein's view, we obtain:

d BIf = 0

that is, the direct tax burden is equal to the total tax burden.

A measure of incidence of a profits tax tr (with $d\ tw = 0$), used sometimes in the literature, involving the direct tax burden is:

$$Id,r = (- k\ d\ rn) / (k\ rg\ d\ tr) \qquad (25a)$$

for profits, and

$$Id,w = (- d\ wn) / (k\ rg\ dtr) \qquad (25b)$$

for wages.

For new taxes ($\gamma r = \gamma w = 0$), under Feldstein's assumptions, these two measures sum to 1.0, since the direct and the total tax burden are equal ($d\ Bi = 0$). For more general taxes this is not true. Because of this, Feldstein calls that measure ambiguous and proposes, instead, a measure given by the following ratio:

$$If,r = k\ d\ rn / (k\ d\ rn + d\ wn)$$

$$= \frac{k\ \gamma r\ d\ rg - k\ rg\ d\ tr}{k\ \gamma r\ d\ rg - k\ rg\ dtr + \gamma w\ d\ wg - wg\ d\ tw} \qquad (26a)$$

For wages, this measure would be:

$$If,w = d\ wn / (k\ d\ rn + d\ wn) \qquad (26b)$$

where If,r and If,w sum to 1.0. Feldstein considers a profit tax, tr, totally borne by profits if $If,r = 1.0$; he says that there is partial shifting if $If,r < 1.0$; by extension, the tax would be totally shifted, in Feldstein's sense, if $If,r = 0.0$. These definitions assume that there is a "burden" to be borne or shifted;

that is to say, that k d rn + d wn is always less than zero. We shall see later that, theoretically, k d rn + d wn can be greater than zero, both in neoclassical and non-neoclassical models. Given this possibility, Feldstein's measure can also be ambiguous.

Furthermore, in models which do not assume a life cycle theory of savings, such as the neokeynesian, Feldstein's justification for ignoring the term rn d k in the burden and incidence expressions is not valid. a more general measure which, following Krzyzaniak (1967), can be called Daltonian, can be defined as:

$$Ida,r = (k\ d\ rn + rn\ d\ k) / (k\ d\ rn + rn\ dk + d\ wn)$$

$$= \frac{(k\ \tau r\ d\ rg - k\ rg\ d\ tr + rg\ \tau r\ d\ k)}{(k\ \tau r\ d\ rg - k\ rg\ d\ tr + rg\ \tau r\ d\ k + \tau w\ d\ wg - wg\ d\ tw)} \qquad (27)$$

This measure is more general than Feldstein's, but it has the same disadvantage: the "burden" term, in the denominator, need not in theory be positive. Then, the measure is also ambiguous. By extension of Feldstein's measure if Ida,r = 1.0 we can say that the burden is totally borne by profits; if Ida,r = 0.0 the burden is totally shifted; and in between, if Ida,r < 1.0, the burden is partially shifted.

Another disadvantage of both If,r and Ida,r as incidence measures is that it is not easy to infer from them whether, as a result of the budget policy change the distribution of income changed in favour of profits or in favour of wages, or stayed as before. We saw how easily this is inferred from E(Bn,tr) (equation 18a), which is the measure we favour.

All these measures, E(Bn,tr), If,r and Ida,r, can be expressed as functions of the elasticity of the gross of tax profit rate, rg, with respect to the profit tax rate, defined as:

$$Erg,tr = (?r \ / \ rg) \ \partial \ rg \ / \ \partial \ tr$$

$$= - \ (?r \ / \ rg) \ \partial \ rg \ / \ \partial \ ?r \qquad (28)$$

when profit maximization is assumed.

We shall derive those relations for the balanced budget incidence of the profits tax. Some preliminary relationships have to be established before this is done. Notice that E(Bn,tr), If,r and Ida,r depend on the differentials d k, d rn and d wn, or, alternatively, on the differentials d k, d rg, d wg, d tr and d tw ; d wg and d k can be expressed in terms of the other differentials. From equation (28) in Appendix 1 to this chapter, d wg can be written (See the definition of all variables in Table1):

$$d \ wg = - \ (q \ dtx + k \ d \ rg)$$

$$= - wg \; \{d\,tx \,/\, [(1 - Ar,g)\, \tau x] + Bg\; d\,rg \,/\, rg\} \qquad (29a)$$

dk can be written:

$$d\,k = k\; d\,k \,/\, k = k\; d\,(a1 \,/\, a0) \,/\, (a1 \,/\, a0)$$

$$= k\,(d\,a1 \,/\, a1 - d\,a0 \,/\, a0)$$

and from equation (4')in the same appendix:

$$d\,k = k\; Ek,l\; [d\,wg \,/\, wg - d\,rg \,/\, (\delta + rg)] \qquad (29b)$$

wg in that expression can in turn be replaced by its value in terms of d rg, given by equation(29a) to obtain :

$$d\,k = k\; Ek,l\; [-q\; d\,tx \,/\, wg - k\; d\,rg \,/\, wg$$

$$- d\,rg \,/\, (\delta + rg)]$$

$$= Ek,l\; k\; [-q\,(\delta+rg)\; d\,tx - \tau x\; d\,rg] \,/\, [wg\,(\delta+rg)]$$

$$= - [Ek,l\; k \,/\, (1 - Ar,g)]\; (d\,tx \,/\, \tau x)$$

$$- (Ek,l\; k\; Bg \,/\, Ar,g)\; (d\,rg \,/\, rg) \qquad (30)$$

We can now obtain d Bn / Bn in terms of d rg by replacing in equation (17b) d k / k by its value in terms of (30) and d wg by its value in terms of (29a) and reorganizing :

$$d\,Bn \,/\, Bn = [-Bg\; Ek,l \,/\, Ar,g + Bg + 1]\; (d\,rg \,/\, rg)$$

$$- [1 \,/\, (1 - Ar,g)](d\,tx \,/\, \tau x)$$

$$- d\,tr \,/\, \tau r + d\,tw \,/\, \tau w \qquad (31)$$

In the same way, replacing in expression (26a) d wg by its value in terms of d rg from equation (29a) ,Feldstein's incidence measure If,r can be rewritten :

$$If,r = \frac{k\ ?r\ d\ rg - k\ rg\ dtr}{?r\ k\ (1 - ?w\ /\ ?r)\ d\ rg - k\ rg\ dtr - wg\ dtw - ?w\ q\ dtx} \qquad (32)$$

and replacing in expresion (27) d wg from equation (29a), and d k from (30), we can obtain Ida,r in terms of d rg as:

$$\begin{aligned} Ida,r = \{ & k\ ?r\ [1 - Bg\ Ek,l\ /\ Ar,g]\ d\ rg - k\ rg\ d\ tr \\ & - (d\ tx)\ k\ rg\ ?r\ Ek,l\ /\ ((1 - Ar,g)\ ?x)\ \} \\ / \{ & k\ ?r\ [1 - Bg\ Ek,l\ /\ Ar,g - ?w\ /\ ?r]\ d\ rg \\ & - k\ rg\ d\ tr - wg\ d\ tw \\ & - (d\ tx)\ [\ ?q + k\ rg\ ?r\ Ek,l\ /((1 - Ar,g)?x)]\ \} \end{aligned} \qquad (33)$$

Notice also that the denominator of equation (33) is the total burden of the fiscal policy involving changes in taxes :

$$\begin{aligned} d\ BT &= d\ yn \\ &= k\ ?r\ [1 - Bg\ Ek,l\ /\ Ar,g - ?w\ /\ ?r]\ d\ rg \\ &\quad - k\ rg\ d\ tr - wg\ d\ tw \\ &\quad - (d\ tx)\ \{\ ?q + k\ rg\ ?r\ Ek,l\ /[(1 - Ar,g)?x]\} \end{aligned} \qquad (34)$$

Then the indirect tax burden would be, from (34) and (20):

$$\begin{aligned} d\ BI &= d\ BT - d\ BI \\ &= k\ ?r\ [1 - Bg\ Ek,l\ /\ Ar,g - ?w\ /\ ?r]\ d\ rg \\ &\quad - \{\ ?q + k\ rg\ ?r\ Ek,l\ /[(1 - Ar,g)?x]\}\ d\ tx \end{aligned} \qquad (35)$$

Equations (31) to (35) are general expressions valid for any "fiscal policy measure" ; as can be seen they present the incidence measures and the burden concepts as functions of the differential of the profit rate, d rg , and of the tax differentials .

For a particular fiscal policy measure they can be simplified. Let us examine the case of the balanced budget incidence of the profits tax tr versus government consumption gc. In this case the differentials d tw and d tx are zero; then, taking account of equations (28) and (18b) we can obtain elasticity expressions from formulas (31) to (35). Equation (31) can be rewritten :

$$E(B_n,t_r) = (\gamma_r / B_n)\, d B_n / d t_r =$$
$$[- B_g E_{k,l} / A_{r,g} + B_g + 1] E_{rg,tr} - 1 \qquad (31a)$$

For the special case of capital goods of infinite duration, that is, for d = 0, formula (31a) can be simplified as follows:

$$E(B_n,t_r) = (\gamma_r / B_n)\, d B_n / d t_r =$$
$$[(1 - E_{k,l}) / (1 - A_r)] E_{rg,tr} - 1 \qquad (31b)$$

The Feldstein and Daltonian measures (formulas (32) and (33)) can be rewritten:

$$I_{f,r} = [E_{rg,tr} - 1] / [(1 - \gamma_w / \gamma_r) E_{rg,tr} - 1] \qquad (32a)$$

$$Ida,r = \frac{[1 - Bg\ Ek,l / Ar,g]\ Erg,tr - 1}{[1 - Bg\ Ek,l / Ar,g - 7w / 7r]\ Erg,tr - 1} \qquad (33a)$$

The total burden equations (34) can be rewritten (recall, d BT = d yn):

$$(1 / k\ rg)\ d\ BT / d\ tr = [1 - Bg\ Ek,l / Ar,g - 7w / 7r]\ Erg,tr - 1 \qquad (34a)$$

and the indirect burden equation (35) can be rewritten :

$$(1 / k\ rg)\ d\ BI / d\ tr = [1 - Bg\ Ek,l / Ar,g - 7w / 7r]\ Erg,tr \qquad (35a)$$

Finally, from equation (29a) we can see that for a balanced budget change in tr, the resulting percentage change in the wage rate can be expressed as:

$$(7r / wg)\ d\ wg / d\ tr = Ewg,tr = Ewn,tr = -Bg\ Erg,tr \qquad (36)$$

Differentiating logarithmically rn = rg 7r, we obtain the net of tax profit rate as a function of Erg,tr:

$$(7r / rn)\ d\ rn / d\ tr = Ern,tr = Erg,tr - 1 \qquad (37)$$

and from (30) we obtain Ek,tr:

$$Ek,tr = (7r / k)\ d\ k / d\ tr = -Ek,l\ Bg / Ar,g \qquad (38)$$

Then, under the assumption of perfect competition, all the indicators of incidence, shifting and changes in factors payments can be expressed as functions of the elasticity of the gross of tax profits rate with respect to the profits tax, Erg,tr, and other given parameters, such as the gross of tax income distribution parameters, Bg and Ar,g, the average tax complements, ?w and ?r, and, finally, the elasticity of substitution in production, Ek,l. Bg, Ar,g , ?r and ?w can be obtained by direct observation in a given economy; Ek,l has to be obtained indirectly by estimation of a production function of the economy, within the constraints of a chosen model of behaviour of that economy.

For a sample of 11 developed countries, which we shall study, the average values of the directly observed parameters are presented in table 3 below; they are Ar,g =.4230 and Bg =.5411. In chapter VII we shall obtain econometric estimates of Ek,l for that sample of countries. We shall see that the average value of Ek,l is around .73. With these values, we have constructed table 2, which presents the relation between the incidence indicators and Erg,tr. This table will be very usefull later, for our quantification of the incidence of the profits tax.

More general representations of equations (31a) to (34a) appear in graphs 1 and 2. Graph 1(a) presents the incidence measure E(Bn,tr) as a function of the elasticity of substitution Ek,l (Eq. 31a). As can be seen, for a wide

TABLE 2

RELATION BETWEEN Erg,tr AND MEASUREMENTS OF INCIDENCE AND FACTOR PAYMENT CHANGES FOR BALANCED BUDGET INCIDENCE OF A PROFITS TAX(*)

Erg,tr	E n,tr	If,r	Id,r	dBT/(krgdtr)	dB1/(krgdtr)	Ewn,tr	Ern,tr	Ek,tr
-oo	-oo	-10.0	-.08	+oo	+oo	+oo	+oo	+oo
-1.20	-1.73	1.96		0	1.00	0.65	-2.20	1.12
-1.00	-1.61	1.82	6.29	-0.17	0.83	0.54	-2.00	0.93
0	-1.00	1.00	1.00	-1.0	0	0	-1.00	0
0.50	-0.70	.53	0.68	-1.42	-0.42	-0.27	-0.50	-0.47
1.00	-0.39	0	0.51	-1.83	-0.83	-0.54	0	-0.93
1.646	0	-0.77	0.48	-2.37	-1.37	-0.89	0.65	-1.54
2.00	0.21	-1.25	0.32	-2.67	-1.67	-1.08	1.00	-1.87
10.00	5.07		0.04	-9.34	-8.34	-5.41	9.00	-9.34
15.24	8.26	27.18	0	-13.71	-12.71	-8.24	14.24	-14.24
+oo	+oo	10.00	-.08	-oo	-oo	-oo	-oo	-oo

(*)With values of Ek,l =0.73 Bg=0.5411 and Ar,g =0.4230 ;see table 3 of this chapter

CHAPTER VI

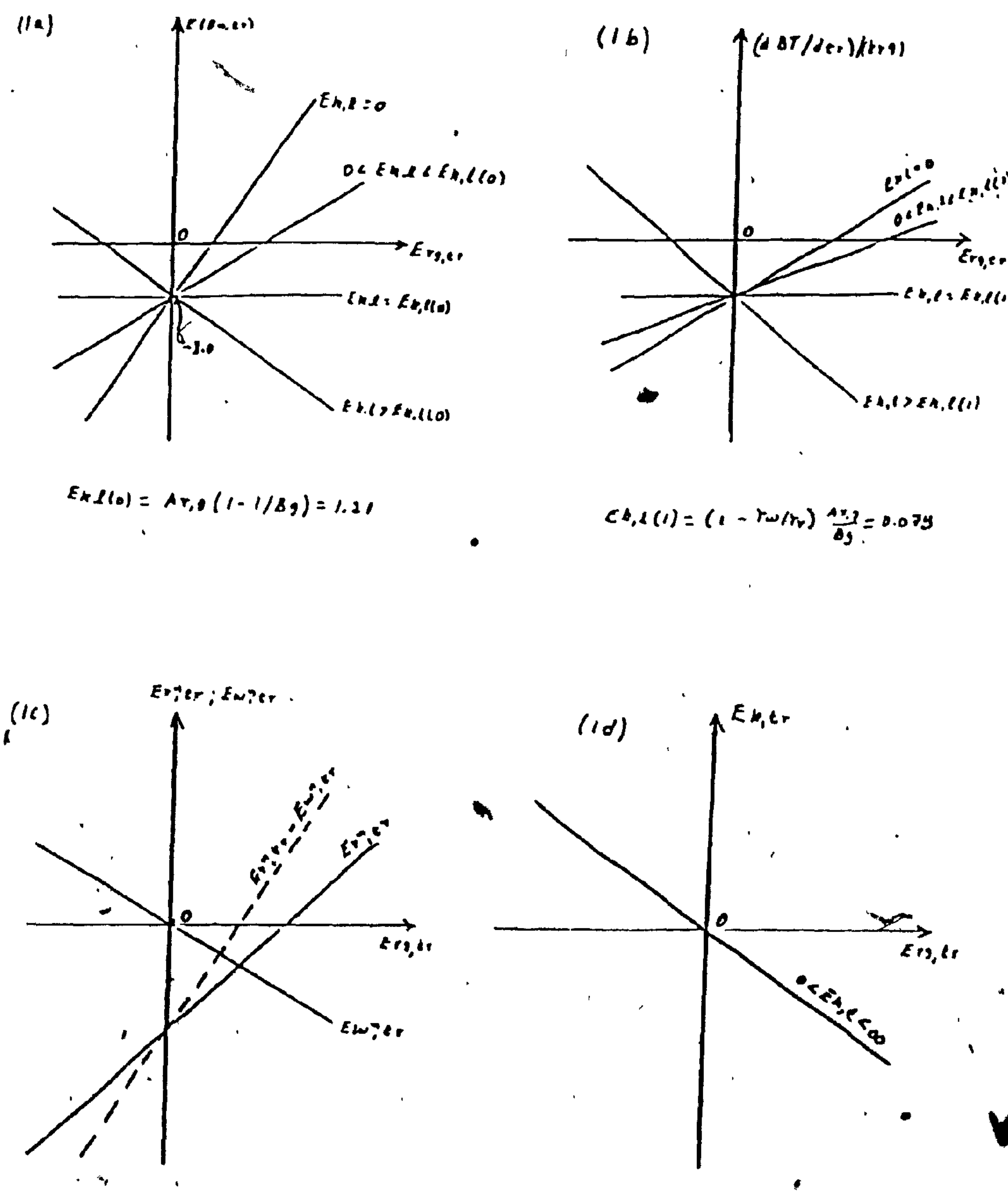
TABLE 3

AVERAGE VALUES OF DIRECTLY MEASURABLE VARIABLES FROM A SAMPLE OF 11 DEVELOPED COUNTRIES FOR THE PERIOD 1965 TO 1977

Variable	Mean Value	Variable	Mean value
rg	.1709	q = (1/a0)	.1309
a0	7.639	c′	.0810
k = (a1/a0)	.2205	a1	1.684
d	.06056	g	.05767
wg	.06963	su	.01666
tx	.1118	γx	.8882
ty	.1405	γy	.8595
tw	.2645	γw	.7355
tr	.1825	γr	.8175
tcr	.2324	γcr	.7676
gc	.0281	gas	.004157
n	.00380	yd	.09802
rn	.1397		..05121
Ar,g	.4230	Ar	.3511
Ar,d	.3143	sy = (s / yd)	.1736
g/rn	.4121	Bg	.5411
γw / γr	.8997		

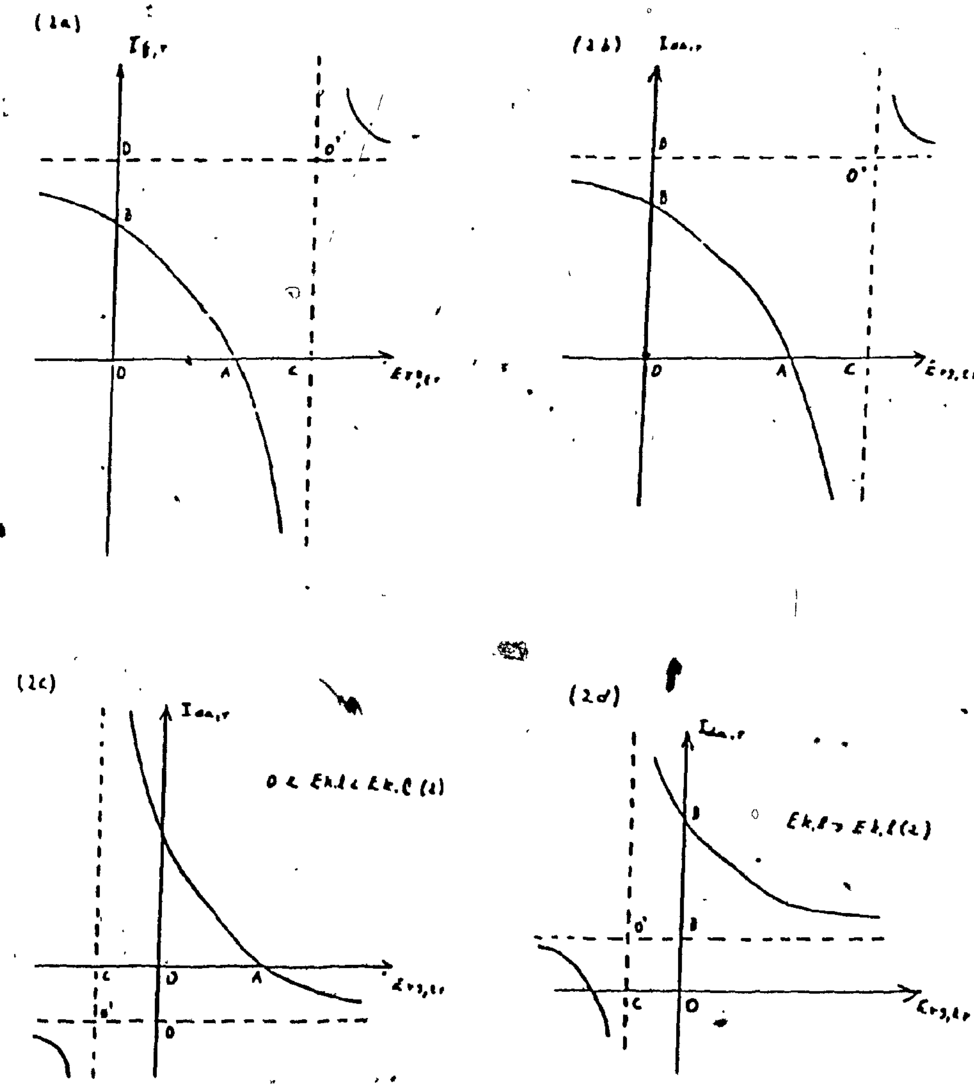
GRAPH 1

E(Bn,tr) d BT / d tr / (k rg) Erg,tr Ewn,tr and Ek,tr as functions of Erg,tr.



GRAPH 2

$I_{f,r}$ and $I_{da,r}$ as functions of $E_{rg,tr}$



Definitions:

Intercepts (a)

$OA = [1 - B_3 E_{k,l}/A_{r,3}]^{-1}$

$= [1 - 1.28 E_{k,l}]^{-1}$

$OB = 1.0$

(a) For $I_{f,r}$ take $E_{k,l} = 0$

Asymptotes (a):

$OC = [1 - B_3 E_{k,l}/A_{r,3} - \gamma_w/\gamma_o]^{-1}$

$= [0.10 - 1.28 E_{k,l}]^{-1}$

$OD = \dfrac{1 - B_3 E_{k,l}}{1 - B_3 E_{k,l}/A_{r,3} - \gamma_w/\gamma_o} = \dfrac{1 - 1.28 E_{k,l}}{0.10 - 1.28 E_{k,l}}$

$E_{k,l}(1) = (A_{r,3}/B_3)(1 - \frac{\gamma_w}{\gamma_o}) = 0.07$

$E_{k,l}(2) = A_{r,3}/B_3 = 0.78$

range of values Ek,l (in the sample of 11 develloped countries, for values of Ek,l between 0 and 1.21) there is a positive linear relationship between E(Bn,tr) and Erg,tr. If Ek,l is great enough (in our sample greater than 1.21) the relationship is inverse linear (1). In either case, if Erg,tr and Ek,l are known, the effect of the budget, or fiscal policy change, on the income ratio Bn, that is, on the distribution of income, can be found. Notice that for Ek,l =.73, our empirical estimate, the relation of E(Bn,tr) and Erg,tr is direct, as is also shown in table 2.

Graphs 1(c) and 1(d) complement the picture. As can be seen by equation (17b), E(Bn,tr) is equal to the sum:

$$E(Bn,tr) = Ern,tr + Ek,tr - Ewn,tr \qquad (39)$$

Hence graphs 1(c) and 1(d) present the decomposition of E(Bn,tr) (graph 1a) into elementary changes in net of tax factor payments and capital. As can be seen, only the capital change Ek,tr varies with the elasticity of substitution. The broken line in graph (1c) represents (Erg,tr - Ewn,tr) and it moves directly with Erg,tr; thus, it can be seen that, for small values of Ek,l this movement will dominate Ek,tr (graph 1d), and the resulting effect will be, as shown in grab 1a, a direct relation between

---

(1) Notice that for capital goods of infinite duration (equation 31b), the cut-off point is Ek,l = 1.0, rather than 1.21.

E(Bn,tr) and Erg,tr.

Graph (1b) shows the total burden ratio, (d BT / d tr) / (k rg), as a function of Erg,tr. What is important here is the sign of this expression; it is normally expected that the total burden d BT and, hence, the ratio d BT / (k rg d tr) is negative. But, as can be observed in graph (1b), the total "burden" can be positive for certain values of Erg,tr, Ek,l and $\gamma w / \gamma r$. More precisely, for positive values of Erg,tr (there is a symmetric relation for Erg,tr < 0), the burden is positive if:

$$Erg,tr > 1 / [1 - Bg\ Ek,l / Ar,g - \gamma w / \gamma r] > 0$$

and

$$1 - Bg\ Ek,l - \gamma w / \gamma r > 0$$

Notice that this can only happen if both,

$$\gamma w < \gamma r \quad \text{and} \quad Ek,l < Bg / Ar,g$$

For the values in our sample where, indeed, $\gamma w < \gamma r$, the burden is positive if

$$Erg,tr > 1 / (0.10 - 1.128Ek,l) \text{ and } Ek,l < 0.078.$$ In this case, the tax change does not produce a total (direct plus indirect) "loss" but, rather, a total "gain" in after tax income.

As we shall see in the next section, theoretically, in the neokeynesian and the neoclassical models, it is possible to encounter situations where there is a low elasticity of substitution together with a high and positive Erg,tr (see graphs 3 and 4 (case A), where Erg,tr is designated by I). Then, it is possible to find theoretical situations where a balanced budget increase in the income tax rate and in government consumption results in an increase rather than a decrease in the total net of tax income. In such cases it makes little sense to talk of the "burden" of the tax.

Graph 2 presents Feldstein's incidence measure If,r (fig 2a) and the Daltonian incidence measure Ida,r (figs 2b to 2d); these measures are defined, respectively, by formulas (32a) and (33a). As the formulas show, these measures are a ratio of the total profits loss to the total tax burden defined, in one case, If,r, with exclusion of capital changes, and in the other case, Ida,r, with inclusion of these changes.

Feldstein interprets his measure as follows (Graph 2a):A value of If,r equal to 1.0 (Erg,tr = 0) means that the burden of the tax is totally borne by profits ; a value of If,r equal to zero (Erg,tr = 1.0) means that the burden is totally shifted ; and a value in between implies partial shifting of the tax. Can we talk by extension of values of If,r less than zero as overshifting ; and greater than one as bearing more than the full burden? The answer for If,r greater than one is no. Values of If,r substantially greater than one are related to values of Erg,tr greater than the magnitud represented by line OC in graph 2a; but, as formulas (32a) and (34a) (with Ek,l =0) and graphs (2a) and (1b) indicate, OC is the point at which the burden dBT / dtr changes sign. In other words, beyond OC the "burden" is positive -- it is a gain. Then, it makes little sense to talk of profits more than fully bearing this gain.

The same observation can be applied, with some modifications, to the daltonian measures Ida,r , presented in graphs (2b) to (2d). For values of Ida,r in the range OB it is possible to talk of profits bearing the burden (Ida,r = OB, Erg,tr = O) or shifting it totally (Ida,r = 0, Erg,tr = OA) or partially. But, beyond those limits, if Ida,r is too large (Ida,r >> 1) or too small (Ida,r << 0), the tax "burden" may become a gain and the measure Ida,r becomes equivocal.

On account of these problems, we shall not use the measures If,r and Ida,r in our incidence discussion. We shall use, instead, E(Bn,tr) and Erg,tr. In the section that follows we shall use a simpler notation for these measures ; Erg,tr will simply be designated as:

Erg,tr = I

and E(Bn,tr) as:

E(Bn,tr) = I(B)

Finally, the term shifting, as used for If,r and Ida,r is equivocal. There are several ways to avoid this problem. One way, which we shall adopt, is to define shifting of a profits tax in terms of changes only in the rate of profits. In this way we say that there is shifting of a profits tax if the gross of tax rate of profits, rg, increases with a tax increase; and there is negative shifting if it decreases with a tax increase. That is there is shifting of the profits tax if:

$$d\ rg\ /\ d\ tr > 0 \quad \text{or} \quad E_{rg,tr} > 0$$

this definition coincides with the implicit meaning given to the word shifting frequently in the literature, as can be seen in chapter II and III above.

There is a second meaning that could be given to the word shifting, suggested by Musgrave (1959a). It refers to a discrepancy between the economic and legal incidence of a tax. The economic incidence is the final distribution of income after the tax or budget change. The legal or statutory incidence of a profits tax is that which would result if profits gross of tax remained unchanged after the tax (or better, budget) change; that is, if net of tax profits fell exactly by the amount of the tax change, whereas net of tax wages remained unchanged. Then, the legal incidence is characterized by:

$$d\ rg\ /\ d\ tr = 0 \quad \text{or} \quad E_{rg,tr} = 0$$

which, as we have seen (graphs 1 and related equations), implies:

$$d\ wg\ /\ d\ tr = d\ wn\ /\ d\ tr = E_{wn,tr} = 0$$

and $d\ k\ /\ d\ tr = E_{k,tr} = 0$

Therefore, it also implies (graph 1a and 1b):

$$E(Bn,tr) = E_{wn,tr} = -1.0$$

and $(1\ /\ k\ rg) \cdot d\ BT\ /\ d\ tr = -1.0$

That is, the legal, balanced budget, incidence of a profits tax is such that $E(Bn,tr) = -1$. There is shifting of the tax in this second sense if the economic incidence, or resulting distribution of income, is more favourable to profits than the legal incidence, that is, if:

$$E(Bn,tr) > -1.0$$

and there is negative shifting if:

$$E(Bn,tr) < -1.0$$

From formula (31a) we can see that there is a very simple connection between the two definitions of shifting presented. If the elasticity of substitution, Ek,l, is less than Ar,g + Ar,g/ Bg (in our sample, less than 1.21) the two definitions of shifting coincide. We shall see that our empirical investigation shows this condition to be amply satisfied.

We have now clarified the concepts of incidence employed, defined an adequate measure for them, and explored the relation of this measurement with other variables related to the after tax distribution of income. We are then ready to analyze the balanced budget incidence of a profits tax.

C) Balanced Budget Incidence of a Tax on Profits

The total differential expressions (13a) to (15) can be used to derive the incidence of many government fiscal policy measures. In this section, we shall concentrate on the examination of only one case: the Balanced Budget

Incidence of a tax on profits, tr, when government consumption, gc, is taken as the government endogenous variable. For completeness, the incidence of some other interesting government fiscal measures is examined in Appendix 4 to this chapter.

1) General Expressions

To derive the balanced budget incidence of tr vs. gc, we have to take all differentials in expressions (13a) to (15), except d tr and d gc, equal to zero. Then, using the differential form of the government budget constraint (Equation (5') of appendix 1 to this chapter):

(wg tw − gc − gas − gds − su) d a0 + rg tr d a1

+ a0 tw d wg + a1 tr d rg

= a0 (d gc + d gas + d gds + d su) + a1 rg 7r

+ a0 wg d 7w + d 7x (40)

replace d gc by its value in terms of d tr (1). In this way, we obtain from equations (13a) to (13c) the following shifting expression for the neoclassical case:

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(1) For the present case, balanced budget incidence, this is trivial, since d gc does not appear in equations 30a to 32. But the general procedure is as described.

$$Ic = Erg,tr = \partial r \, [\, a1 \quad S1 + a0 \quad S2 \,] \,/\, Jc \qquad (41a)$$

recall that Jc is the Jacobian of the neoclassical model ;replacing it by its value in equation (13b), we obtain:

$$Ic = \frac{[\, a1 \quad S1 + a0 \quad S2 \,]}{\{-\,(a1\ rg\ \partial x\ Ek,l)\ (S1 - g/rn) \,/\, [ao\ wg\ (d+rg)] - a1\ (\partial w \,/\, \partial r)\ S1 + a1\ S1 + a0\ S2 \,\}} \qquad (41b)$$

Similarly, from equations (14a) to (14c) we obtain, for the neokeynesian case:

$$Ik = Erg,tr = [\partial r\ a1\ (sp' - i')] \,/\, Jk \qquad (42a)$$

or replacing Jk by its value in equation (14b):

$$Ik = \frac{a1\ (sp' - i')}{\{\, -\,(a1\ rg\ \partial x\ Ek,l)\ (sp' - g/rn) \,/\, [a0\ wg\ (d+rg)] - a1\ sw'\ \partial w \,/\, \partial r + a1\ (sp' - i') \,\}} \qquad (42b)$$

Finally, for the neomarxian model we obtain from equation (15):

$$Im = Erg,tr = 0 \qquad (43)$$

Notice that, from auxiliary equation (8), the differential of d tr has the following general expression:

$$d\,tr = \{1 - (1 - \theta)\,[tyr + (1 - tyr)\,zg]\}\,d\,tcr$$
$$+ (1 - \theta)\,(1 - tcr)\,(1 - zg)\,d\,tyr \qquad (44a)$$

which can also be written:

$$d\,tr = [\theta + (1 - \theta)\,(1 - zg)\,\tau yr]\,d\,tcr$$
$$+ \tau cr\,(1 - \theta)\,(1 - zg)\,d\,tyr \qquad (44b)$$

where: $\tau i = 1 - ti$, $i = cr, yr$

The profits tax tcr applies to all profits, whereas the tax tyr applies only to profits distributed by enterprises. The general tax on profits is then tcr. Because of this generality, the shifting measure for tcr is the same as for tr :

$$Erg,tcr = Erg,tr:$$

This is easy to see. In effect, the tax complement τr can be written, in view of auxiliary equation (8) defining tr, as:

$$\tau r = 1 - tr = \tau cr\,\{1 - (1 - \theta)\,[tyr + (1 - tyr)\,zg]\}$$
$$= \tau cr\,[\,\theta + (1 - \theta)\,(1 - zg)\,\tau yr\,] \qquad (45)$$

Replace equation (44a) with d tyr = 0 in (13a), then replace τr by its value in expression (45) and reorganize, to finally obtain the elasticity Erg,tcr:

$$Erg,tcr = (\tau cr\ /\ rg)\quad rg\ /\quad tcr$$
$$= (\tau r\ /\ Jc\,)\ (a1\quad S1 + a0\ S2) \qquad (46)$$

Then from (46) and (41-A) we obtain:

$$E\,rg,tcr = Erg,tr \qquad (47)$$

expression (47) has been proven using the neoclassical model; but a quick look at the pertinent relations shows that it also follows for the neokeynesian model (and a fortiori for the neomarxian).

The shifting measure for the partial tax on profits tyr, on the other hand, is not equal to Erg,tr but only to a fraction of Erg,tr. In fact, replace again equation (44b), this time with d tcr = 0, in (30a), then replace ?r by its value in expression (45) and reorganize to obtain:

$$\begin{aligned} Erg,tyr &= Erg,tr\ ?yr\ ?cr\ (1-\theta)\ (1-zg)\ /\ ?r \\ &= Erg,tr\ [\ (1-\theta)\ (1-zg)\ ?yr\ ]\ /\ [\theta + (1-\theta)\ (1-zg)\ ?yr] \qquad (48) \end{aligned}$$

hence the shifting of the partial tax tyr, as measured by Erg,tr , is only part of the shifting of the general tax tcr, as measured by Erg,tcr , a result which is intuitively reasonable. What is important is that the study of Erg,tr is enough to learn about the shifting (and the incidence) of partial or general taxes on profits: once Erg,tr is known for a certain equilibrium, both Erg,tcr and Erg,tyr follow in fixed relations to the former, given by equations (47) and (48). Hence, in this chapter we shall analyze only Erg,tr.

We can now pass on to the analysis of equations (41a) to (43). The general conclusion which arises from an examination of those equations, is that, without further knowledge of the value of the quantities involved or further a priori restrictions on these values, the shifting measure "Ij" (j = c , k, m) can take any value. There is only one general exception: that of the neomarxian model, where it is clear that the tax is always fully borne by profits.

In the neoclassical and the neokeynesian case, the coefficients "Ij" (j = c , k) can take any values, if there are no restrictions, because the expressions for "Ij" represent in those cases hyperbolic surfaces in the space of parameters. As such, they can span all values of "I" from -00 to +00.

2) Restricting the Shifting Measures.

Several questions arise immediately. Can additional theoretical restrictions substantially narrow the range of values for "Ij" in each model? Can they narrow those values enough that the models would attribute non-overlapping ranges to the "Ij"? Given that some of the parameters appearing in the incidence expressions are directly measureable (i.e. average wage, income, rate of growth, tax rates, etc.), whereas others are not (i.e. Ek,l, i', sp', sw', S1, S2) if average values are obtained for the former, would this narrow the ranges of the incidence enough to distinguish models or to give clear policy implications? Briefly, how much information is needed to arrive at definite incidence implications? and further, to distinguish the models?

Let us explore the first two questions. The restrictions can be given as a priori statements constituting part of the theory, or they may be necessary to make the theory consistent, or finally they may be needed to obtain desirable general properties such as stability.

A priori statements which are part of the theory are, for example, the Kaldorian statement that the marginal propensity to save out of wages sw' is smaller than the marginal propensity to save out of profits sp' ; or the neoclassical statement that consumption in any period is a normal good and, therefore, the marginal propensities to consume (and to save) out of permanent income (∂ C / ∂ yd and S1) are greater than zero and less than one. Those are practically the only restrictions which seem to be normally given to the parameters in these theories. There are also certain presumptions which are not necessarily part of the theory such as the neoclassical presumption that S2 > 0, and that Ek,l is strictly greater than zero (1), and not far from unity; or the Kaldorian presumption that sw' is not far from zero.

By far, the most common source of restrictions seems to be the assumption of stability of the models, within a

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(1) For some, Ek,l > 0 is, more than a presumption, one of the assumptions of the neoclassical model.

certain postulated process of dynamic adjustment. This is related to Samuelson's correspondence principle. Of course, this assumption is conditional on the validity of the relevant model.

Under certain conditions, which we examine in appendix 2 to this chapter, the assumption of stability of the above models would permit to give a sign to the Jacobians Jc and Jk of equations (41a) and (42a). The conclusions obtained in the appendix can be spelled out as follows.

For the neoclassical model, an adjustment process is postulated, whose most important characteristics are the following: Prices of goods and factors are assumed to be perfectly flexible. Then if there is a situation of excess demand in the economy, that is if:

$$gas + gds + S(yd, rn) < a1\ g\ /\ a0$$

the price of the product (corn) will rise; hence the real wage rate in terms of the numeraire (corn) will fall. From this hypothesized behaviour, plus some other secondary assumptions, it is proven in appendix 2 to this chapter that local stability of the neoclassical model requires the following condition:

$$Jc > 0 \qquad (49)$$

That is, stability requires the denominator of the neoclassical shifting expressions (41a) or (41b) to be positive.

For the neokeynesian model, the most important characteristics of the adjustment process postulated are as follows. If effective demand in the economy is higher than anticipated by entrepreneurs, that is if:

gas + gds + S(wn + su, a1 rn / a0) < a1 g / a0

the entrepreneurs will revise their profits expectations upwards, and will increase investment; if effective demand is low, by the same process, they will decrease investment. From this main mechanism plus some other secondary assumptions spelled out in the appendix, we prove there that local stability of the neokeynesian model requires:

Jk > 0 (50)

that is, stability requires the denominator of the neokeynesian shifting expressions (42a) or (42b) to be positive (1).

It is important to stress that the above results depend totally on the dynamic adjustment processes postulated, which are fully discussed in the appendix. Indeed, Patinkin (1965, pp.479-499) has shown that, when very general adjustment mechanisms are postulated, it is

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(1) For another adjustment mechanism stressing prices see Marglin (1984). The mechanism presented here and in the appendix stresses, rather, quantity responses. Models with this type of response have been presented by Malinvaud (1977), Barro and Grossman (1971), and Morishima (1976), among others.

not possible to say anything about the sign of the Jacobian determinant of comparative static expressions, such as (41a) (or 41b) and (42a) (or 42b) (1).

As we shall soon see, restrictions (49) and (50) reduce the domain of variation of the parameters space in the models, but they do not reduce the range of variation of the shifting coefficients, I. The next question to explore is whether the range of I would be reduced by replacing the directly measurable parameters and variables in expressions (41b) and (42b) by their measured values.

By directly measurable parameters and variables, we mean those which do not have to be obtained by means of some postulated regression model. They can rather be obtained by direct measures using, for example, a census of the population or sampling techniques; examples of these are the data which constitute the National Accounts. We have presented, in table 3, average values of the directly observable quantities employed in the shifting expressions. Those values were obtained from a sample of eleven developed countries, which will be presented in more detail in the next chapter. Notice that taking the means of those values involves the hypothesis that those means are good measures of the steady state or equilibrium values of the systems of equations (1) to (7c), (7k) or (7m).

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(1) This implies that Samuelson's correspondence principle is of very limited usefulness.

The general analysis of equations (41b) and (42b), as well as the study of replacing the directly measurable parameters by observed values, is better done by recasting those equations in a different form, as follows.

The neoclassical shifting expression (41b) can be rewritten:

$$I_c = \frac{m_{s,y} + (\bar{s}_y / A_{r,d})\, E_{s,r}}{\{ -A_r\, E_{k,1}\, (m_{s,y} - g / r_n)\, [(1 - a_r)\, A_{r,g}] + (1 - \tau_w / \tau_r)\, m_{s,y} + (s_y / A_{r,d})\, E_{s,r} \}} \qquad (51a)$$

where A_r and $A_{r,g}$ have been defined previously as two different measures of profit share (see table 1).

$A_{r,d}$ is another measure, eequal to the share of profits, net of profits taxes, on disposable income:

$$A_{r,d} = k\, r_n / y_d = k\, r_n / [w_n + k\, r_n + s_u]$$

Notice that, for a situation of zero initial taxes, $A_{r,d} = A_r = A_{r,g}$.

$\bar{s}_y$: denotes the average propensity to consume out of income:

$$\bar{s}_y = s / y_d = (y_d - C) / y_d$$

$m_{s,y}$ denotes the neoclassical marginal propensity to save out of income:

$$m_{s,y} = \partial S / \partial y_d = S_1$$

$E_{s,r}$ is the elasticity of savings with respect to the rate of profit:

$$E_{s,r} = (r_n / S) \quad S_2$$

Notice that this elastictity represents only the price response of savings: the change in savings that would be obtained if rn changed but yd were kept constant (for example, by compensatory changes in wn or su). Recall also that, in the neoclassical growth model we are studying, there is perfect arbitrage between the rate of profit and the real rate of interest. Hence $E_{s,r}$ is also the interest elasticity of savings.

$E_{k,l}$ is, as before, the elasticity of substitution in production.

g / rn can also be written g k / (rn k), or the "investment to net profit ratio". Notice that it can equally be written:

$$g / rn = (g\,k / yd) / (yd / rn\,k) = (g\,k / yd) / A_{r,d}$$

that is, as the ratio of the share of investment in disposable income to the share of net profits in disposable income.

Finally, we have given the shifting index, "I", a qualifier c, to denote that it, "Ic", refers to the locus of values that "I" would take under the neoclassical model.

In the same way, the shifting expression for the neokeynesian model, (42b), can be rewritten:

$$I_k = \frac{m_{s,p} - m_{i,p}}{\{ - A_r\, E_{k,l}\, (m_{s,p} - g / rn) / [(1 - a_r)\, A_{r,g}] - (\gamma_w / \gamma_r)\, m_{s,w} + m_{s,p} - m_{i,p} \}} \qquad (52a)$$

whre Ik is the shifting index, $I = Erg,tr$, for the neokeynesian model. ms,p and ms,w denote the marginal propensities to save out of profits and out of wages:

$$ms,p = \partial S / \partial (k\ rn) = sp'$$

$$ms,w = \partial S / \partial (wn + su) = sw'$$

Finally mi,p is the marginal propensity to invest out of profits by the entrepreneurs:

$$mi,p = i'(rn) = dg / d\ rn$$

Notice that in the keynesian short term, when capital K is taken as given, $K = K0$, mi,p can be written, with Iv denoting total investment, and Πn total net profits:

$$mi,p = (K0\ d\ g) / (K0\ d\ rn) = d\ (K0\ g) / d\ (K0\ rn)$$
$$= d\ Iv / d\ \Pi$$

which justifies calling mi,p marginal propensity to invest out of profits.

The directly measurable parameters are now Ar , Ar,g Ar,d , δy , g / rn , $?w$ and $?r$. Those which can only be known through postulating and estimating an economic model are: ms,y , Es,r , and Ek,l for the neoclassical model; and ms,p , ms,w , mi,p and Ek,l for the neokeynesian model. Notice that the elasticity of substitution, Ek,l , is common to both models.

Using table 3, we can replace the directly measurable parameter by their value in that table to obtain, for the neoclassical model:

$$I_c = \frac{(m_{s,y} + .55\, E_{s,r})}{-1.28\,(m_{s,y} - .41)\, E_{k,l} + .10\, m_{s,y} + .55\, E_{s,r}} \qquad (51b)$$

and for the neokeynesian model:

$$I_k = \frac{(m_{s,p} - m_{i,p})}{-1.28\,(m_{s,y} - .41)\, E_{k,l} - .90\, m_{s,w} + m_{s,p} - m_{i,p}} \qquad (52b)$$

We have now restricted the incidence expressions, by theory (stability assumptions) and direct observation, as much as it is possible to do. We can attempt to answer the questions posed previously: Do these restrictions, above, narrow the range of variation of the shifting coefficients, Iz (z= c, k), enough to produce different and distinct predictions for different theories? The answer to that question has been developed in Appendix 3 to this chapter. As mentioned before, shifting formulas (51a) (or 51b) and (52a) (or 52b) correspond to families of hyperbolas in the parameter space. The restrictions arising from the stability assumptions and from observation of the directly measurable parameters permit us to eliminate certain members of those families and certain areas of the parameter space. By a systematic analysis along these lines, we have constructed in the appendix two graphs, reproduced here as graphs 3 and 4, which contain the restricted families of hyperbolas representing equation (51a and b) and (52a and b).

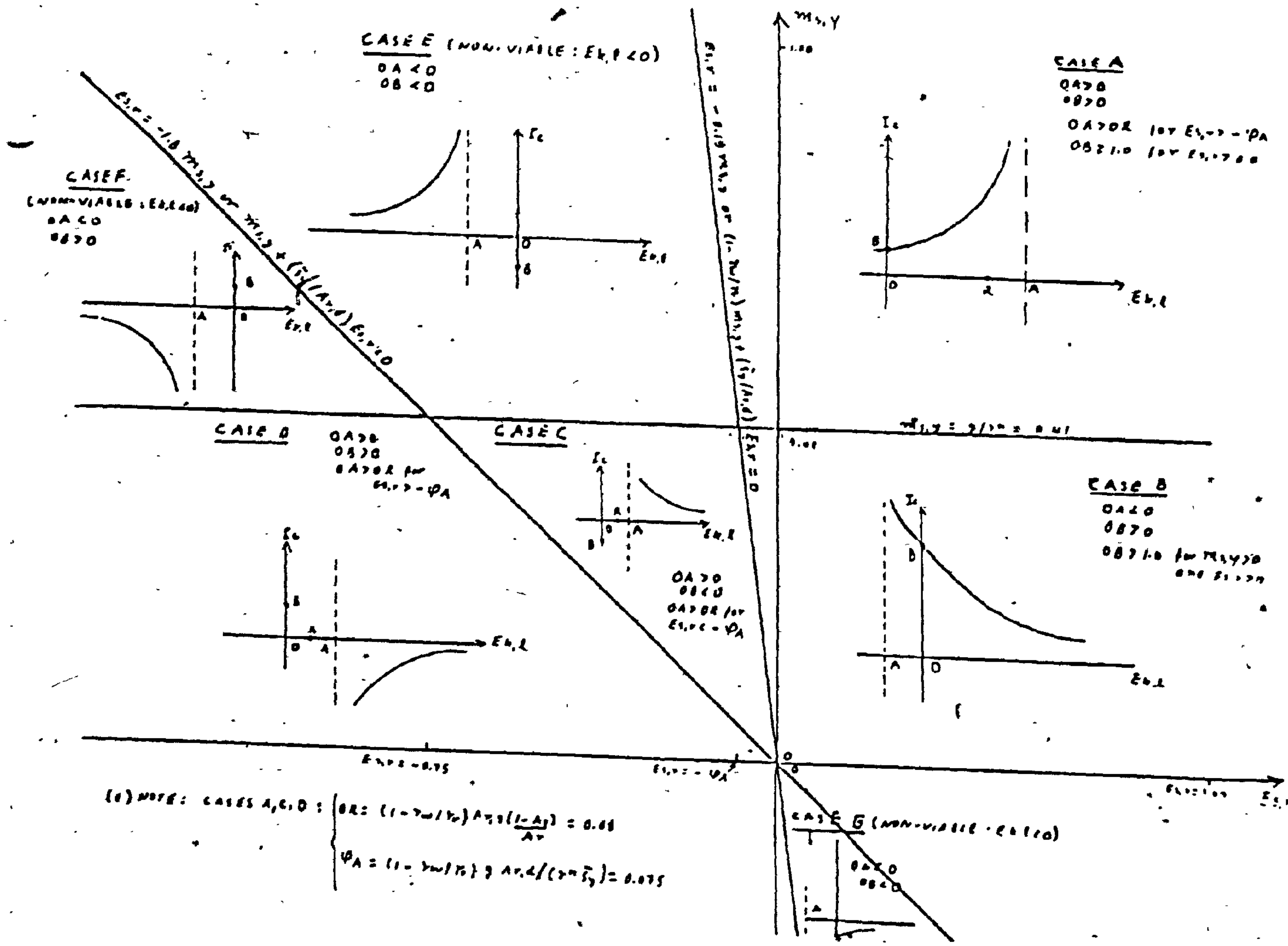
An examination of graphs 3 and 4 reveals that the question above has a negative answer: if further a priori restrictions are not imposed on the parameters which are not directly observable (ms,y, Es,r and Ek,l for the neoclassical model and mi,p, ms,p, ms,w and Ek,l for the neokeynesian) the shifting expresssions Ic, for the neoclassical model, and Ik, for the neokeynesian, can still vary from $-\infty$ to $+\infty$; therefore, the predictions of the three theories still overlap.

In order to obtain different and distinct predictions for each theory we have to assign values to the parameters which are not directly observable. This can be done by econometric estimation of the models or by direct a priori conjectures which should be integrated as part of the theories.

In the following chapters, we are going to estimate the models econometrically, and will then be able to assign values to all parameters, and obtain distinct incidence propositions. We shall also test the models and attempt to choose one as better representing the empirical data. A priori restrictions would also permit us to obtain distinct incidence propositions. In fact, the analysis which we have just made, and which is summarized in graphs 3 and 4,

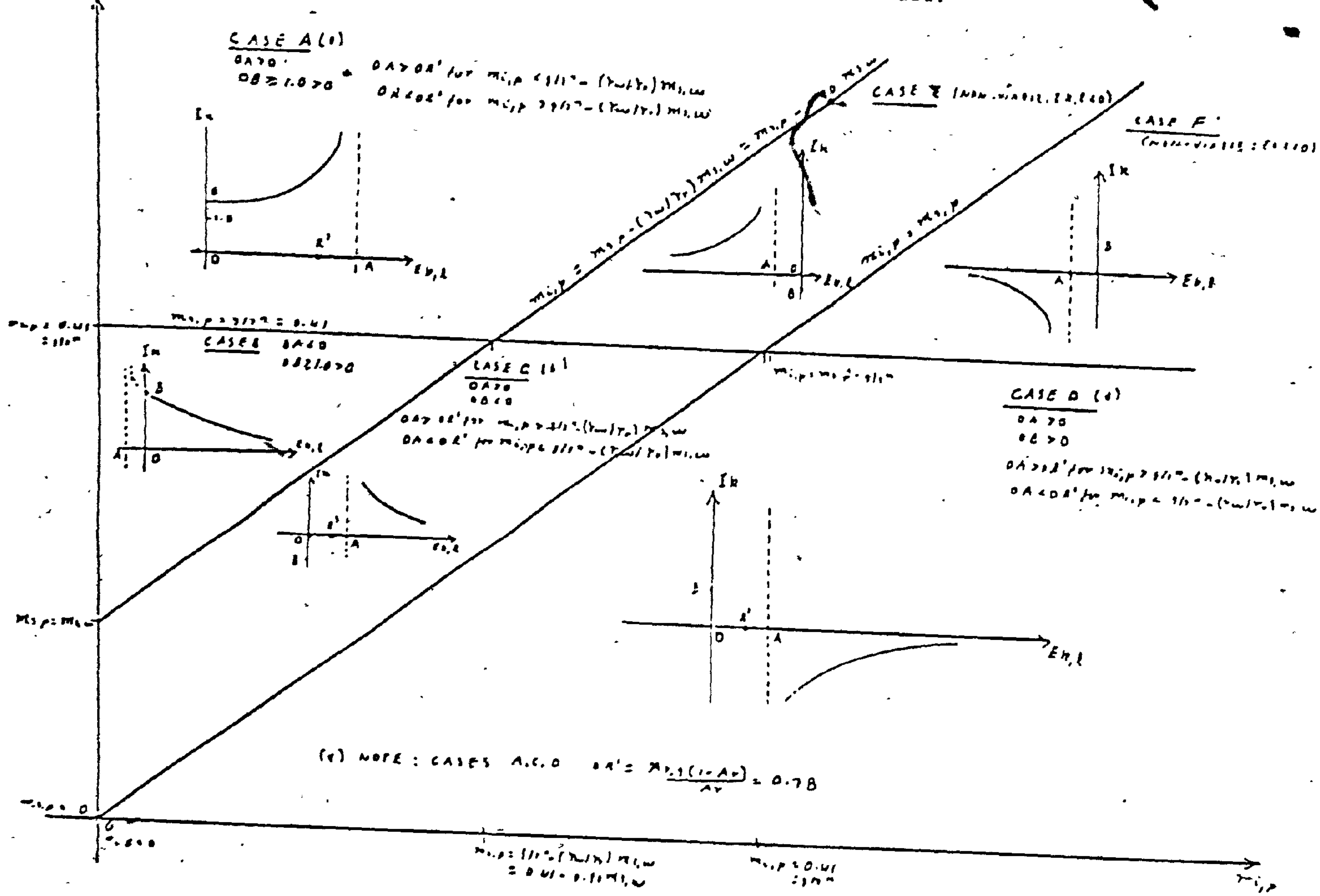
GRAPH 3

Incidence regions in the neoclassical model



GRAPH 4

Incidence-regions in the neokeynesian model



indicates what types of a priori restrictions would result in distinct incidence propositions. In what follows we shall formulate some of these propositions.

3) Some Predictions

We shall concentrate on the neoclassical and neokeynesian balanced budget incidence of a tax on profits, when the accommodating (endogenous) budget variable is government consumption. The shfting measures for these two models are summarized in Graphs 3 and 4. The <u>neomarxian</u> model, as we observed previously, is trivial in this case; it simply predicts that profits taxes are exactly borne by profits, that is: Im = 0.0.

For the <u>neoclassical</u> model, on the other hand, it is possible to read from Graph 3 the following propositions, which assume stability.

1) If the marginal propensity to save out of income, ms,y, is not negative, and the interest (or profit) elasticity of savings (E s,r) is greater than:

$$-(1 - ?w / ?r)\ Ar,d\ ms,y\ /Sy$$

that is, for the sample figures presented in table 3, if Es,r is greater than:

$$-1.8\ ms,y,$$

then the tax is at least partially shifted. (Ic > 0) (cases A and B in Graph 3). If, furthermore ms,y is greater than the investment to net profit ratio, g/r (.41 for our sample), the tax is necessarily overshifted (case A in Graph 3 with Es,r > 0).

2) If the interest elasticity of savings, Es,r , is negative and less than -Ar,d /Sy(-1.8 in the sample; Table 3) times the marginal propensity to save out of income, ms,y; and if the latter is in turn less than the investment to net profits ratio, g / rn (.41), then the tax is more than fully borne by profits (case D in Graph 3).

3) If there are no production substitution possibilities in the economy, that is if the substitution elasticity, Ek,l , is zero, the tax is necessarily overshifted (because we would have to be in case A or B in Figure 3).

4) If the wage tax complement τw is less than τr, if the elasticity of substitution in production, Ek,l, is less than:

(1 - τw / τr) Ar,g (1 - Ar) / Ar

or, from table 3 data, less than 0.08,

and the interest elasticity of savings, Es,r , is less than:

-(1 - τw / τr) (Ar,d / Sy) (g / rn)

or, from table 3 data, less than -0.075,

the tax cannot be more than fully borne by profits, that is, it is at least partially shifted (because these conditions make case D, with OA > (1 - τw/τr)

Ar,g (1 − Ar) / Ar ,or OA > 0.08, non-viable).

5) As the elasticity of substitution, Ek,l , grows to very large values, the tax tends to be exactly borne by profits (because the Ek,l axis is an asymptote to the hyperbola in Graph 3 in all cases, particularly those with viable large Ek,l.

Let us now examine the predictions made by the <u>neokeynesian</u> model. From Graph 4, for a stable model, we can observe the following:

(1) If the marginal propensity to invest, mi,p, is smaller than the marginal propensity to save out of profits, ms,p, the tax is at least partially shifted, and can be overshifted (cases A, B and C in Graph 4, which have Ik > or = 0.).

(2) If the marginal propensity to save out of profits, ms,p , is greater than the investment to net profit ratio g / rn (.41), the tax is overshifted (since for ms,p > g / rn, case A is the only one viable; then Ik > 1.0). It also follows that mi,p has to be:

$$mi,p < ms,p - (\gamma w / \gamma r)\ ms,w.$$

(3) If the marginal propensity to invest, mi,p , is higher than the marginal propensity to save out of profits, ms,p (1), then the tax is negatively shifted: Ik < 0 (since this condition delimits cases D and F in

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(1) In Marglin's (1984) model, though, mi,p > ms,p is

Graph 4, and F is not viable). It also follows that ms,p has to fulfill:

ms,p < g / rn

(Since ms,p > g / r corresponds to the non viable case F).

(4) If there is no substitution among factors of production, that is if Ek,l = 0, then the tax is necessarily overshifted or fully shifted: Ik > or = 0 (cases A and B, with a positive and relevant OB, have OB > 1.0).

(5) If the elasticity of substitution Ek,l , is less than:

OR′ (= Ar,g (1 - Ar) / Ar =(1 +*d*/rg)Ar or .81)

and the marginal propensity to invest out of profits, mi,p, is greater than:

g / rn - (?w / ?r) ms,w

then, the tax cannot be negatively shifted, that is, it has to be at least partially shifted (because these conditions make case D with OA > OR′ = .81 non viable);

(6) For a very large elasticity of substitution the tax will tend to be exactly borne by profits: Ik = 0 (because the Ek,l axis is an asymptote of the hyperbolas in Graph 4).

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a sufficient condition for instability, which would rule out this case.

4) Two Special Cases

The propositions stated on the previous paragraphs are based on the general shifting expressions (41b) and (42b), and their graphical translation, in Graphs 3 and 4. They result from the very general assumptions, concerning government actions and consumption (or savings) functions, which we have made so far. Replacing those assumptions by simpler, though still quite general ones, results in great simplifications in the shifting expressions and allows us to observe more clearly the differences in the models.

The simplified assumptions are as follows. For both neoclassical and neokeynesian models we shall assume that the government does not save, $gas = 0$; it has always a perfectly balanced budget, $gds = 0$; and it does not effect transfers to households, su [illegible] Notice that all these are government policy variables; these assumptions are then equivalent to taking a government with a more restricted fiscal policy.

Additionally, for the neoclassical model, we shall assume that the marginal propensity to save out of income, ms,y , is equal to the average propensity to save out of income, sy. For the neokeynesian model, we shall assume that the marginal propensities to save out of profits, ms,p and of wages, ms,w are constant, and that the savings function can be represented by a linear equation.

Let us now examine the form that the neoclassical shifting expression (41b) takes with these simplifying assumptions, which can be written:

$$gas = gds = su = 0 \qquad (53)$$

and

$$ms,y = \quad S \,/ \quad yd = \bar{s}y = S \,/\, yd \qquad (54)$$

From assumption (62), the savings investment relation (6C') becomes:

$$S(yd,\ rn) = g\ k$$

or

$$S(yd,\ rn)\ /\ (k\ rn) = g\ /\ rn$$

which can be written:

$$g\ /\ rn = [S(yd,\ rn)\ /\ yd]\ [yd\ /\ (k\ rn)] \qquad (55)$$

which by assumption (63), becomes

$$g\ /\ rn = ms,y\ yd\ /\ (k\ rn) \qquad (56)$$

By the use of (56) we can then obtain:

$$ms,y - g\ /\ rn = ms,y - ms,y\ yd\ /\ (k\ rn) =$$

$$ms,y\ (k\ rn - yd)\ /\ (k\ rn) = - ms,y\ wn\ /\ (k\ rn) =$$

$$\bar{s}y[- wn\ /\ (k\ rn)] \qquad (57)$$

since we have assumed $su = 0$ (58)

Then, replacing equations (54) and (57) in the neoclassical expression (41b) and simplifying we obtain:

$$Ic = \frac{1 + Es,r\ yd\ /\ (k\ rn)}{\{\ \partial x\ Ek,l\ (\partial w\ /\ \partial r)\ /\ [a1\ (d+rg)] - \partial w\ /\ \partial r}$$

$$+ 1 + E_{s,r}\ yd / (k\ rn) \}$$

and replacing $7x / [a1(d + g)]$ and $yd / (k\ rn)$ by their values in terms of $A_{r,g}$ and $A_{r,d}$ respectively:

$$I_c = \frac{1 + E_{s,r} / A_{r,d}}{(7w / 7r)\ (E_{k,l} / A_{r,g} - 1) + 1 + E_{s,r} / A_{r,d}} \qquad (59)$$

The neokeynesian simplifying assumptions were given by equations (53) and by:

$$S(wn + su\ ,\ k\ rn) = sw\ (wn + su) + sp\ k\ rn \qquad (60a)$$

with:

$$sw = \text{const.} \qquad (60b)$$

$$sp = \text{const.} \qquad (60c)$$

and therefore:

$$m_{s,w} = S / (wn + su) = sw = \text{const.} \qquad (60d)$$

and

$$m_{s,r} = S / (k\ rn) = sp = \text{const.} \qquad (60e)$$

Then by (53) the neokeynesian savings investment relation (6k) can be written as:

$$sw\ (wn + su) + sp\ k\ rn = g\ k \qquad (61)$$

also by (60e) we can write:

$$m_{s,p} - g / rn = sp - g / rn = (sp\ rn\ k - g\ k) / (k\ rn)$$

which in view of (61) and since we have assumed $su = 0$ becomes:

$$m_{s,p} - g / rn = - sw\ wn / (k\ rn) \qquad (62)$$

Replacing equations (60d), (60e) and (62) in the shifting expression (42b) we finally obtain:

$$I_k = \frac{s_p - m_{i,p}}{\{ s_w (\gamma_w / \gamma_r)\, \gamma_x\, E_{k,l} / [(d+r_g)\, a_1] - s_w (\gamma_w / \gamma_r) + s_p - m_{i,p} \}}$$

or replacing $\gamma_x / [(d+r_g)\, a_1] = 1 / A_{r,g}$:

$$I_k = \frac{s_p - m_{i,p}}{s_w (\gamma_w / \gamma_r)\, (E_{k,l} / A_{r,g} - 1) + s_p - m_{i,p}} \qquad (63)$$

The shifting expressions (59), for the neoclassical model, and (63) for the neokeynesian, are much simpler now than the original expressions (41b) and (42b). Taking account of the fact that stability requires the denominators of both, expression (59) and expression (63), to be positive, it is quite a simple matter to analyse them. Doing so, we obtain general propositions for shifting in these special cases.

The propositions for the neoclassical model can be stated as follows. In order to simplify the writing of the propositions below, define:

$$1 - (\gamma_r / \gamma_w)\,(1 + E_{s,r} / A_{r,d}) = D_s$$

Notice that:

$D_s < 1$ for $E_{s,r} > -A_{r,d}$ and

$D_s > 1$ for $E_{s,r} < -A_{r,d} < 0$

Recall also that the share of net profits in disposable income, $A_{r,d}$, has been defined as:

$$A_{r,d} = rn\ k / yd$$

and the share of gross profits in the gross product, $A_{r,g}$, has been defined as:

$$A_{r,g} = k\ (d + rg) / (q\ ?x) = a1\ (d + rg) / ?x$$

Recall that in all cases we are assuming

$$gas = gds = su = 0 \quad \text{and} \quad ms,y = Sy.$$

Then, in the <u>neoclassical</u> model:

(1) If the interest elasticity of savings, $E_{s,r}$, is greater than $-A_{r,d}$, and the elasticity of substitution in production, $E_{k,l}$ is greater than $A_{r,g}$, the tax is partially shifted: $0 < Ic < 1.0$.

(2) If $E_{s,r}$ is greater than $-A_{r,d}$ and $E_{k,l}$ is equal to $A_{r,g}$, the tax is exactly shifted: $I = 1.0$;

(3) If $E_{s,r}$ is greater than $-A_{r,d}$ and $E_{k,l}$ is smaller than $A_{r,g}$ but greater than $A_{r,g}\ Ds$ (that is $A_{r,g} > E_{k,l} > A_{r,g}\ Ds$) the tax is overshifted: $I > 1.0$;

(4) If $E_{s,r}$ is greater than $-A_{r,d}$ and $E_{k,l}$ is less than $- A_{r,g}\ Ds$ the neoclassical model is unstable.

Now let us see the cases, in the neoclassical model, when $E_{s,r}$ is less than $-A_{r,d}$; recall that in this case $Ds > 1.0$.

5) If $E_{s,r}$ is less than $-A_{r,d}$ and $E_{k,l}$ is greater than $A_{r,g}\ Ds$ the tax is negatively shifted that is: Ic

< 0 .

(6) If Es,r is less than -Ar,d and Ek,l is less than Ar,g Ds the neoclassical model is unstable.

Let us now examine the neokeynesian case. Again, to simplify, define :

$$1 - (sp - mi,p)\ ?r\ /\ (sw\ ?w) = Ds,i$$

notice that:

$Ds,i < 1$ for $sp > mi,p$ and

$Ds,i > 1$ for $sp < mi,p$

Finally, for all propositions we are assuming the special cases:

$gas = gds = su = 0$ and

a savings function with constant coefficients defined:

$$S(wn + su,\ k\ rn) = sw\ wn + sp\ k\ rn$$

We can then state, for the neokeynesian model:

(7) If the average propensity to save out of profits, sp, is greater than the marginal propensity to invest mi,p and if the elasticity of substitution, Ek,l , is greater than Ar,g , then the tax is partially shifted: 0 < Ik < 1.0.

(8) If sp is greater than mi,p and Ek,l is equal to Ar,g the tax is exactly shifted: Ik = 1.0.

(9) If sp is greater than mi,p, and Ek,l is smaller than Ar,g but greater than Ar,g Ds,i then the tax is overshifted Ik > 1.0.

(10) If sp is greater than mi,p and the average propensity to save out of wage income is zero, sw = 0,

that is if the savings function is of the classical type, the tax is exactly shifted Ik = 1.0.

(11) If sp is greater than mi,p, but Ek,l < Ar,g Ds,i (therefore necessarily less than 1.0), the model is unstable.

Now the cases, in the neokeynesian model, when sp is smaller than mi,p.

(12) If sp is less than mi,p and Ek,l is greater than Ar,g Ds,i (then, necessarily, Ek,l > 1.0), the tax is negatively shifted: Ik < 0.0.

(13) If sp is less than mi,p and the elasticity of substitution, Ek,l , is less than or equal to Ar,g Bs,i, then the neokeynesian model is unstable.

There is finally a very important neokeynesian proposition:

(14) If Ek,l is less than Ar,g and the neokeynesian model is stable then the tax is overshifted: I >1.0.

This proposition follows from the fact that if the model is stable and sp < mi,p we are in the case described by proposition (12) above. Then Ek,l > Bs,i. But for sp < mi,p , Ds,i > 1.0, therefore Ek,l > Ar,g, which contradicts the assumption that Ek,l < Ar,g. In consequence, if Ek,l < Ar,g and the model is stable it follows that sp > mi,p. But these two conditions are fulfilled only by the case described in proposition (9) above, and this proposition says that Ik > 1.0. In other words, proposition (14) and proposition (9) are equivalent.

Armed with the predictions of the special models described by the propositions above, we can now go one step further and state a-priori conjectures on the magnitude and shape of certain of the non-observable parameters, (or rather, non directly measurable parameters).

For the neoclassical model we shall give conjectures on the size of Ek,l and Es,r and on the shape of the consumption function. In general, neoclassical theory assumes a certain amount of substitutability among the factors of production; frequently Ek,l is considered not far from 1.0 in these models. Let us make these considerations more precise: We shall make the conjecture that El,l is greater than the gross share Ar,g: Ek,l > Ar,g (this share is, in most developed economies, around .3 to .4; in Table 3 it is .432).

A second frequent hypothesis made in neoclassical models, more precisely life cycle or permanent income models, is that the utility function is homothetic. Friedman (1957) as well as Modigliani and Brumberg (1953) explicitly state this in their models. It follows from this hypothesis that consumption can be represented by a function as:

C = yp h (rn)

In steady state growth models, with no short term cycles, like those treated in this work, we showed in chapter V that, then, savings can be represented by a function such as:

$S = yd\ (1 - h(rn))$

Therefore:

$$\partial S / \partial\ yd = S / yd = 1 - h(rn)$$

That is, the marginal propensity to save is equal to the average propensity to save, which is one of the hypotheses of our special neoclassical model.

Friedman has shown in his work on permanent income (1957) that empirical data are consistent with this formulation.

Finally, neoclassical theory tends to assume that savings in the overall economy respond favorably to increases in the rate of interest. This can also be taken as an assumption implicit in many policy recommendations which advise increasing the rate of interest in order to decrease consumption and increase savings. This conjecture can be formulated, simply as:

"The elasticity of savings with respect to the rate of interest is positive, or at least not negative":

$$Es,r \geq 0$$

We are now ready to formulate the neoclassical most favored case for shifting. It is as follows:

(15) In a capitalist economy, where government doesn't buy participations in capital (gas=0) where, in the long term, the budget is balanced (gds=0) and where there are no transfers to households (su=0), the most probable consequence of increasing a profits tax is that this one will be partially, but not totally,

shifted. This consequence depends on the assumption of homothetic utility functions, i.e. proportional savings (for rn = const.), and on the following conjectures:

$Es,r \geq 0$ and $Ek,l > Ar,g$

that is, the elasticity of savings with respect to the rate of interest (profits) is equal to or greater than zero, and the elasticity of substitution in production is high enough, greater than the share of gross profits in gross national revenue.

This proposition follows from proposition (1) above.

For the neokeynesian model the conjectures are less clearly stated. As concerns the savings equation, it is frequently formulated in this model as a linear function such as that assumed in the special case (60a) (see Kaldor, 1957). Frequently, it is also assumed that sw is zero, because, among other reasons, most workers' "savings" are used up in the financing of non-productive capital, such as dwellings, that is, they are consumed (Kaldor and Mirrless, 1962, see also J. Robinson 1962). Hence, frequently the conjecture is that the savings function is of the classical type:

$S = S(k\ rn) = sp\ k\ rn$

Also, as regards the substitution elasticity, it seems more consistent with the neokeynesian viewpoint to assume it to be low, in any case probably less than unity.

Then, if in the neokeynesian model we assume:

gas = gds = su = 0

and a linear savings function such as (69a), with

Sw = 0

the profits tax is exactly shifted, if the economy is stable, by proposition (10) above.

If, on the other hand, we keep all those assumptions, but replace Sw = 0 by Sw > 0, and take as plausible a value of Ek,l < Ar,g, then the neokeynesian model predicts overshifting of the profits tax, by proposition (14) above. In conclusion, in the neokeynesian model there is a strong conjecture for at least full shifting of the profits tax. This can be formulated more precisely in the following terms for the neokeynesian most favored case for shifting:

(16) In a capitalist economy, where government doesn't buy participations in capital (gas=0) where, in the long term, the budget is balanced (gds=0) and where there are no transfers to households (su=0), the most probable consequence of increasing a profits tax is that it will be at least totally shifted. This consequence depends on the assumption of a macro consumption function which is linear in terms of payments to factors of production, and on either one of the following conjectures:

sw = 0 or Ek,l < Ar,g

that is ,either savings out of wages are negligible or the elasticity of substitution is low, less than the gross profit share.

This propossition follows from propositions (10) and (14) above.

In sum, while the neoclassical most favored conjecture on the value of the non-observable (or non-directly measureable) parameters point towards partial shifting of the profits tax, in a neoclassical world; the neokeynesian most favored conjectures, although less clear, point towards at least full shifting of the tax, in a neokeynesian world.

CHAPTER VII

ESTIMATION OF THE STRUCTURAL MODELS USING CROSS SECTION AND TIME SERIES DATA

In the previous chapter we presented several alternative theoretical models of growth and incidence, and derived theoretical results for the incidence of a tax on profits, arising from those models.

Our task in the present chapter is to undertake an econometric estimation of those models, where we shall try to capture the long term phenomena involved. We shall proceed as follows. In Section A, we shall present the algebraic structure of the econometric models. In Section B, we shall present the considerations which entered in the choice of specific functional forms, as well as the specification of the random errors; that is, the statistical structure of the models. In Section C, we shall briefly present the data and the procedudre used to smooth out trade cycle effects. In Section D, we shall say some words on the identification of the models. Finally in Section E, we shall discuss the estimation method used and present the results.

A) The Algebraic Structure of the Empirical Models.

The equations of the models of growth and distribution were presented in Chapter V with general functional forms; for the econometric estimation of this chapter, specific functional forms have to be given .

It is also necessary to replace the savings/investment relations by algebraically equivalent consumption functions. This change is dictated by the nature of the data we use in the estimation. The data on savings from national accounts are calculated as residual items; as such, they will be distorted by errors and omissions incurred in the data from which they are residuals. It is then better to use directly obtained data, such as those on consumption, because they have a smaller measurement error.

Finally, we shall draw a distinction, between econometric identities and behavioural equations. The identities exactly fit all data points and do not have parameters to be estimated. The behavioural equations fit the data with an error assumed to have certain random characteristics, and usually, but not necessarily, have parameters to be estimated. (More precisely, their residual variance always is unknown.).

In what follows, we shall limit the discussion to the presentation of the specific equations of each model, and their relations with the systems presented in Chapters V and VI. The choice of the error terms for the behavioral equations, and the process whereby certain functional terms, rather than others, were chosen will be discussed in the next section. We shall, first, present the five equations common to all models; second, the specific equations; third, we shall discuss the auxiliary equations. Finally, we shall introduce a general model which will be needed later for tests of the theories. Most variables in the empirical model conserve the notation used previously; additional explanations on the meaning of these variables in the empirical models and their units are presented in Table 1. The table also presents the new variables needed in the empirical models. More details, especially on the sources of data and the construction of the capital series, are presented in the Appendix to this chapter.

1) The Common Equations.

As seen in Chapter V, there are five common equations.

The consumption-investment identity was presented as equation (15a) or equation (15b) of Chapter V. This equation is also an econometric identity. Here we shall take the form it has in equation (15b) of that chapter,

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TABLE 1

DESCRIPTION OF VARIABLES IN THE EMPIRICAL MODELS

Endogenous variables

Principal

Note--per worker quantities: in what follows, worker refers to a labourer adjusted for time worked per year; that is, a worker measures a man-year of work; per worker quantities are then here quantities per man-year of work. Quantities are also expressed, when relevant, in units of 100000 "constant" international dollars of 1975. "International" here refers to corrections to the data by means of international purchasing power parity indices, calculated by Kravis et al. (1979). "Constant" refers to corrections for inflation using price indices with 1975 as a base. All quantities per worker below are then expressed in units of 100000 constant dollars of 1975 per adjusted worker.

q: Total product per worker.

k: Total net productive capital stock per worker." Net" refers to net of accumulated depreciation. "Productive" indicates that it excludes "non-productive capital", such as dwellings. See annex to this chapter for a more detailed discussion of these data.

c: Total private consumption, per year, per worker. It includes gross demand of dwellings, that is total net demand plus dwelling depreciation.

wg: total wage bill , per worker, per year.

rg: all inclusive rate of profit, net of depreciation. It equals the ratio of all net national income other than wages to net productive capital.

g: rate of growth of total net productive capital per year. Notice that g is not the rate of growth of capital per worker.

gc: government consumption per worker, per year.

Auxiliary endogenous variables

Ar,g: The share of profits gross of depreciation in gross national income (income net of indirect taxes and subsidies) , defined as:

Ar,g = (k rg) / [q (1-tx+sur)]

This variable is used in the marginal product equation as explained in the text.

Exogenous variables

d: Depreciation rate of net productive capital , K, per year.

n′: Rate of growth of total labour force per year; equal to the rate of growth of total population minus the change in the participation of the population in the labour force.

tax rates

tx: Effective tax rate on sales of all products; indirect tax rate bill per unit of product. Equal to Tx/(qL) where:
Tx: is the total indirect tax bill.

L: is total (adjusted) workers.

tw: Total , effective, tax rate on wage income; equals

$$tw = twss + ty$$

twss: effective social security tax rate, equal to:

$$Twss / (w\ L)$$

Twss: total social security tax bill

ty: effective tax rate on income of individuals and unincorporated enterprises, defined as:

$$ty = Ty / (ynoc + w\ L + \theta h,n\ rg\ k\ L)$$

Ty: total income tax bill

ynoc: income of non corporate enterprises

θh,n: Net proportion of total profit income net of depreciation and of corporate tax, but gross of individual income tax (ty) , received by hoseholds

tcr: Effective tax rate on corporate income. Equals total tax bill on corporate income divided by total profits minus income of unincoporated enterprises. Italy includes income of unincorporated enterprises; a dummy variable, dit, has been used to take account of this. tcr is then defined as:

$$tcr = Tcr / [rg\ k\ L - ynoc\ (1 - dit)]$$

Tcr: total corporate tax bill.

tr: Total effective net participation of the government in profits, arising from taxes and government ownership of capital. It is defined as:

$$tr = tcr\ [1 - \theta noc,n\ (1-dit)] + (\theta h,n + \theta noc,n)\ ty + (1-tcr)\ \theta g,n$$

$\doteq$ tcr ($\theta h,n$ + $\theta noc,n$ dit + $\theta c,n$)
+ ty ($\theta h,n$ + $\theta noc, n$ + $\theta g,n$

$\theta g,n$: Net proportion of total profit income net of depreciation, going to government as a result only of its ownership of capital.

$\theta c,n$: : Net proportion of total profit income net of depreciation and gross of corporate income tax , staying with corporations.

$\theta noc,n$: : Net proportion of total profit income net of depreciation, going to unincorporated enterprises.

t: time variable . The data span the period 1965 to 1977 1965 coresponds to t=1 and 1967 to t=13; for the data grouped in three year periods, t takes values 2, 5, 8 and 11 for the four three year observations in each of the 11 countries in the sample.

idls: Index used as an indicator of labour unrest. It is an index of days lost due to strikes per worker.

L: total (adjusted) workers.

poaci: total civil labour force.

poac: total labour force.

ad5: an adjustment coefficient in the government accounts, per worker, per year. It is equal to ad4, defined below, minus government property income net, or:

ad5 = ad4 − $\theta g,n$ k rg (1 − tcr) where:

ad4: total government income minus income from the taxes tw, tx, ty and tcr, all in per worker per year terms; it includes, among others, items such as, the government income from ownership of capital (or government property income net: $\theta g,n$ k rg (1 − tcr)), transfers such as legacies, and any other government income not included in the variables already defined.

gas: autonomous government saving; that is government income placed autonomously in net investment. It is equal to the sum of government gross fixed capital formation (gross investment) minus depreciation, plus purchases of land net, plus purchases of intangible assets net, plus the statistical discrepancy item in the government transaction tables of the OECD new national accounts system.

gds: government surplus (if positive) or deficit, per worker per year; it is equal to the net government lending item in the new system of national accounts of OECD.

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su: transfers from government to households, per worker and per year; example: social security payments by governments.
supro: government subsidies to production per worker per year.

sur: efective rate of government production subsidy, defined as : sur=supro/q .

bt: balance of trade surplus or deficit, plus other residuals in the consumtion investment identity , per worker per year.

dit: dummy variable for italy it is equal to 1 for observations concerning italy and to 0 for observations in all other countries.

with one term added, in order to have a better approximation to the data sample. This term, designated by "bt", represents mainly the balance of trade and errors and omissions. It is calculated as a residual between the product, q, and the other terms. The consumption-investment identity is then:

$$q = c + gc + k\ (d + g) + bt \qquad (1)$$

The wage-profit identity was presented as equation (21a) or equation (21b) in Chapter V. It is also an econometric identity. We shall take here its form (21b). Recall though that the term tx stood in chapter V, for the net ad-valorem product tax minus the subsidy to production in ad-valorem terms, sur. We can then write the consumption-investment identity as:

$$q\ (1 - tx + sur) = wg + k\ (d + rg) \qquad (2)$$

The term calculated as a residual was in this case "sur".

The third equation, the production function, was presented in Chapter V as equation (29a) or (29b). This is a behavioral equation, written there in a general functional form. For the econometric estimation a simple translog form was chosen; it is written as:

$$\ln q = b0 + b1\ \ln[k\ \exp(-R'\ t)] +$$
$$0.5\ a2\ \{\ln[k\ \exp(-R'\ t)]\}^2 \qquad (3)$$

The marginal productivity condition, which is given by equation (30), in Chapter V, becomes then in its translog form:

$$(1 / q)\, dq / dt = (d + rg)\, k / [q\, (1 - tx + sur)]$$
$$= b1 + a2 \ln [k \exp (R'\, t)] \qquad (4)$$

Notice that the left hand side is the share of profits, gross of taxes, on the gross national product, at producer prices: Ar,g. The marginal productivity condition is also a behavioural equation.

The last common equation, the government budget constraint, was presented in Chapter V as equation (10a) or (10b). We have taken, for the econometric model, a special case of that equation, corresponding to a balanced budget, with gds equal to zero in the average. The econometric equation, then, is behavioural, but has no parameters to estimate. It is written:

$$gc = wg\ tw + q\ tx + k\ rg\ tr + ad5 - gas - gds - su - sur\ q \qquad (5)$$

This equation is similar to equation (10a) of Chapter (5) with an additional adjustment term, ad5, and with gds equal to zero. the adjustment term ad5 is defined in Table 1. It is essentially equal to the difference between all income and transfers received by the government and the income from the taxes tw, tx and tr. Therefore it provides an adjustment to total income received by, or transferred to the government, to take account of other taxes and various sources of income, as well as other expenditures of the government, not captured by the government variable, included explicitly in the model.

The taxes tw and tx are, respectively, the effective tax rate on wages and the effective tax rate on sales of final products. They are defined in more detail in Table 1. The term tr represents, as in Chapters V and VI the average government participation in profits as a result of taxation and net property income. It is defined by the following expression:

$$tr = ty\ (\theta n,h + \theta n,noc) + tcr\ (\theta n,h + \theta n,noc + \theta n,c + \theta n,g) + \theta n,g\ (1 - tcr) \qquad (5a)$$

This expression will be explained below (sub-section 3), short definitions of the parameters θn,h θn,noc etc., and of the tax terms, tcr and ty can be found in Table 1.

2) The Specific Equations.

Let us now examine, first, the equations specific to the neoclassical model; then those specific to the neokeynesian model; and finally, those specific to the neomarxian model. All these are behavioural equations.

The first equation, specific to the neoclassical model, is the neoclassical savings-investment relation, presented as equation (54a) of Chapter V. As we explained above, for the econometric estimation, given the nature of the data, it is better to use directly the consumption equations, presented as equations (51a) or (51b) in Chapter V:

c = C(yd, rn)

or c = yd h(rn)

Notice also the transition from the consumption function (51a) to the saving-investment relation (54a), through equations (52) and (53); in that chapter, which is perfectly reversible.

Equations (51a) or (51b) in Chapter V can now be given a specific, double logarithmic form:

ln C = b2 + b3y ln yd + b3 ln rn (6c.0)

or fully written:

ln C = b2 + b3 ln [wg (1 − tw) + su + rg (1 − tr)] +
b3 ln[rg (1 − tr)] (6c)

As is clearly seen in equations (6c.0) consumption is made a function of income yd and the real rate of interest net of taxes, or net yield of capital, given, in the neoclassical model, by rn .

As was discussed in Chapter V, the net of tax income, yd, represents, within the hypothesis of the <u>steady state</u>, a measure of permanent income. How this theoretical representation of long term phenomena is translated into an empirical measure will be treated below, in Section C.

The second, and final, specific equation of the neoclassical model is the natural rate of unemployment condition, given by equation (64) of Chapter V:

g = R' + n

For the empirical model, we shall allow for a change in the participation rate in the population; so instead of the rate of growth of total population n, we shall employ more precisely the rate of growth of the labour force, or of active population, which we shall designate with the letter "n′". If the participation rate is designated lambdn we have:

$$n' = n - (1 / \text{lambdn})\ d\ \text{lambdn} / d\ t$$

A small correction will also be done to take account of the fact that the time interval is discrete, whereas the measure of the rate of growth of technical progress, R′, in the production function, is continuous. Calling the equivalent discrete rate of change of technical progress, R", we have the following equivalence

$$R'' = \exp (R') - 1 \qquad (7c.0)$$

We shall then write the econometric form of the natural rate of unemployment condition as:

$$g = R'' + n' \qquad (7c)$$

The neokeynesian model is completed by the Kaldorian saving-investment relation, represented by equations (59c) in Chapter V, and by the "animal spirits" investment function, equation (65) of Chapter V.

We indicated in Chapter V that instead of the savings-investment relation (59c) the consumption function (58a):

$$c = C(wn + su, k\ rn)$$

could be used. This is what we shall do here for the reasons given above about the residual nature of the savings data. Taking, again, a specific double logarithmic form, the Kaldorian consumption function can be written:

$$\ln c = b2 + b3w \ln(wn + su) + b3r \ln(k\ rn) \quad (6k.0)$$

or, fully written,

$$\ln c = b2 + b3w \ln[wg\ (1-tw) + su] + b3r \ln[rg\ (1-tr)] \quad (6k)$$

For the neokeynesian animal spirits investment function, we have specified a linear functional form:

$$g = b4 + b5\ rn \quad (7k.0)$$

or fully written:

$$g = b4 + b5\ [rg\ (1 - tr)] \quad (7k)$$

Equations (6K) and (7k) complete the neokeynesian model.

As we know, the neomarxian model is completed by: the Kaldorian savings-investment relation, or alternatively the Kaldorian consumption function (6k), which it shares with the neokeynesian models; and the neomarxian "socially necessary" wage condition, which was presented in Chapter V as any of the equations (66), (67a) or (67b). The most convenient form for the econometric model is given by equation (67b), which can be written, choosing a linear form for the function h( ) as:

$$wg = \exp(R'\ t)\ (b20 + b22\ idls)\ /\ (1 - tw) \quad (7m)$$

This equation together with equation (6k) closes the neomarxian model.

3) Auxiliary Equations.

There is only one auxiliary equation which needs some additional explanation. It is equation (5a) above, defining the average government participation in profits tr. In Chapter V, we gave a theoretical definition for tr (see equation 5 of Chapter V). But, given the nature of the data, this theoretical expression cannot be directly applied to the econometric model. Instead, we have proceeded as follows. We have developed, from national accounts data, coefficients $\theta n,i$ which describe the participation of agent "i" in total profits rg k L. The agents are: the households (i = h), the government, (i = g) the corporate enterprises (i = c) and the non-corporate entrepreneurs (i = noc). The coefficients $\theta n,g$ $\theta n,h$ and $\theta n,noc$ have been obtained directly from the national accounts tables. $\theta n,c$ the corporate participation, has been obtained as a residual. Hence, these coefficients obey:

$$\theta n,h + \theta n,g + \theta n,c + \theta n,noc = 1.0$$

Notice that the participation coefficients for households, $\theta n,h$ and for government, $\theta n,g$ are net of the corporate profits tax, which is assumed to be totally "extracted" at the corporate level, as it appears in the national accounts tables. Then the composite participation of government in profits would be given by:

$$rg\ k\ L\ tr = rg\ k\ L\ tcr\ (\theta n,h + \theta n,c + \theta n,g) +$$

$$rg\ k\ L\ ty\ (\theta n,h + \theta n,noc) +$$

$$rg\ k\ L\ (\theta n,g - \theta n,g\ tcr) \qquad (8)$$

The first term in the right hand side indicates the total tax levied on corporate profits; the second term indicates that part of the income tax extracted from profits received by households and non-corporate enterprises; and the third term represents the net property income received by the government. Notice that the above equation implies that the tax base for the corporate tax, tcr, is all profits other than those going to non-corporate enterprises, and that the tax base for the income tax (ty) is, apart from wages, (which of course do not enter that expression), profits received by households ($\theta n,h$ rg k L) and profits obtained by non-corporate enterprises ($\theta n,noc$ rg k L).

The simplification of expression (8) results in auxiliary equation (5a) defining tr.

4) Two Nested Models.

One of the statistical tests which we shall present in Chapter IX will consist in obtaining a general model which will encompass the three main models examined; the neoclassical, the neokeynesian and the neomarxian models. We shall then test the significance of the variables which, when assumed equal to zero, or to some other adequate value, reduce this general model back to one of the main models. This procedure can be called "nesting", and we shall call general models so built, nested models.

The econometric and statistical characteristics of "nesting" will be discussed in Chapter IX. In this chapter we shall simply deduce the algebraic structure of the nested models.

For models represented by single linear equations, which have so far been the most common in econometrics, nesting several models is a very simple matter, as far as the algebra is concerned; though, from the statistical viewpoint, the interpretation of the resulting nested equation may not be simple at all. Algebraically in this case, nesting would consist of building a general linear equation, having as terms all those appearing in the models being nested.

For non-linear systems of equations, such as those we are studying, the procedure is not, in general, as simple and straightforward, but the general idea should be the same.

For our case, a thorough examination of three main models presented above by equations (1) to (7c), (7k) or (7m), reveals the following characteristics which should be taken into account for the "nesting":

First, as noticed many times, the models share equations (1) to (5). To arrive at the simplest nested models possible, then, those equations should be left as they are.

Second, If a term such as:

$$b3' \ln(rn)$$

is added to the Kaldorian consumption equation, the result is a general consumption equation, capable of representing, neoclassical as well as neokeynesian and neomarxian consumption functions. This general equation is:

$$\ln c = b2' + b3w' \ln(wn + su) + b3r' \ln(k\ rn) + b3' \ln(rn) \quad (9)$$

We can see that if $b3' = 0$, it becomes the neokeynesian equation (6k). On the other hand, if the parameters b3w' and b3r' are restricted to be:

$$b3w' / (\overline{wn} + \overline{su}) = b3r' / (\bar{k}\ \overline{rn})$$

where the barred values are sample averages, the equation represents, at the average sample values, the neoclassical consumption function (6c). This result is obtained by indirect inference as follows. What matters for our purpose in the consumption functions, as can be seen in Chapter VI, is the structure of the marginal propensities to consume. If this structure is the same in the consumption equation, at a point such as the mean sample values, then, for our purposes, these two equations are equivalent.

Now the neoclassical marginal propensities to consume out of wages and out of profits can be seen to be, by derivation of equation (6c):

$$\partial c / \partial (wn + su) = \partial c / \partial (k\ rn) = (\bar{c} / \overline{yd})\ b3y \quad (10)$$

Therefore in order for equation (9) to be a representation of the neoclassical equation (6c) it has to have a structure of marginal propensities to consume subject to the same restrictions, (10). Taking derivatives of equation (9) we obtain:

$$\partial c / \partial (wn + su) = b3w' \bar{c} / (\overline{wn} + \overline{su}) \qquad (11a)$$

and

$$\partial c / \partial (k\ rn) = b3r' \bar{c} / (\bar{k}\ \overline{rn}) \qquad (11b)$$

Then equations (11a) and (11b) have to have restrictions (10), which yields:

$$b3w' / (\overline{wn} + \overline{su}) = b3r' / (\bar{k}\ \overline{rn}) \qquad (12)$$

Hence if equation (9) is restricted by condition (12) it represents the neoclassical consumption function.

Consumption will then be representd in our general model by a function like:

$$\ln (c) = b2' + b3w' \ln[wg (1 - tw) + su] + b3r' \ln[k\ rg (1 - tr)] + b3' \ln [rg (1 - tr)] \qquad (6n)$$

Equation (6n) is reduced to the neokeynesian consumption function if b3' is restricted to be null; and it is reduced to the neoclassical consumption function if b3w' and b3r' are restricted by equation (12) above.

The third point to notice is that both the neoclassical and the neokeynesian models close their equation systems with relations concerning the rate of growth g as the variable to be explained: equation (7c) for the neoclassical model, and equation (7k) for the neokeynesian model. This immediately suggests that a model which would have both as particular case could have an equation of the sort:

$$g = b11 + b12\,n' + b5'\,rn \qquad (13)$$

since this equation becomes the neoclassical condition (7c) if b11 = R", b12 = 1 and b5' = 0 ; and it becomes the neokeynesian model (7k) if b12 = 0.

But the neomarxian model closes with a condition, (7m), which contains the wage rate as its right hand (explained) variable. In order to obtain a general model then, it is necessary to find an equation for the neomarxian model which is algebraically equivalent to equation (7m), but which has as a right hand side variable, not the wage rate, but the rate of growth, g. This equation could then be combined with the neoclassical and the neokeynesian expressions, to obtain an equation representing the effects of the three models. In order to do this we can make use of the identities (1) and (2), since they provide a link between wg and g. Rewrite equation (2) as follows:

$$q = wg + k\,(d + rg) + q\,(tx - sur) \qquad (14)$$

Then, subtract equation (1) from (14) and reorganize, to obtain:

$$g = wg / k + rg + (tx - sur)\,q / k - (c + gc + bt) / k \qquad (15)$$

and replace the neomarxian wage condition (7m) in (15) to obtain:

$$g = \exp(R' t)\ (b20 + b22\ idls)\ /\ [(1 - tw)\ k] +$$
$$rg + (tx - sur)\ q\ /\ k - (c + gc + bt)\ /\ k \qquad (16)$$

This equation is algebrically, and logically, equivalent to the neomarxian wage condition (7m), and it has the variable g as its right hand side explained variable. We can now combine it with the neoclassical expression (7c) and the neokeynesian expression (7k), or equivalently with equation (13) to obtain the general model. The simplest possible combination is, again, linear. It would result in :

$$g = b11 + b12\ n' + b5'\ rn +$$
$$\exp(R'\ t)\ (b20' + b22'\ idls)\ /\ [(1 - tw)\ k] +$$
$$b13\ [rg + (tx - sur)\ q\ /\ k - (c + gc + bt)\ /\ k\ ] \qquad (7n.1)$$

This is the general equation we need, or our "nested" equation. Observe that for:

$$b11 = R'',\ b12 = 1.0,\ b5' = b20' = b22' = b13 = 0$$

it is reduced to the neoclassical condition (7c) for:

$$b12 = b20' = b22' = b13 = 0$$

it is reduced to the neokeynesian condition (7m) and for

$$b11 = b12 = b5' = 0 \quad \text{and} \quad b13 = 0$$

it is reduced to the neomarxian equation (16), which is logically equivalent to (7m).

We shall observe, in Chapter IX, that in order to obtain a symmetric test of the neomarxian model versus a nested model it is better not to work with equation (7n.1), but rather with a nested equation which takes wg as its right side explained variable. This equation can be obtained by rewriting equation (15) as:

$$wg = k\ g + c + gc + bt - k\ rg - q\ (tx - sur)$$

Replacing g by its value in expression (7c) we would obtain an equivalent neoclassical condition; and replacing it by its value in (7k), an equivalent neokeynesian condition. Then taking a linear combination of these two restated conditions and of neomarxian condition (7m) we could obtain our second nested equation as:

$$\begin{aligned} wg = {} & \exp (R'\ t)\ (b20'' + b22''\ idls) \,/\, (1 - tw) + \\ & b11'\ k + b12'\ n'\ k + b5''\ k\ rg\ (1 - tr) + \\ & b13'\ [c + gc + bt - k\ rg - q\ (tx - sur)] \qquad (7n.2) \end{aligned}$$

Notice that this equation reduces to the original neomarxian equation (7m) for b11' = b12' = b5" = b13' = 0 ; to the neokeynesian equivalent model for b20" = b22" = b12' = 0 and b13' = 1.0 ; and to the neoclassical equivalent model for b20" = b22" = 0, b11' = R" and b12' = b13' = 1.0 But we shall use this second formulation to test only the neomarxian model. The other two models will be tested using equation (7n.1), to which they reduce directly.

In sum, we have built two nested models, to be used in one of the tests of the alternative theories examined, in Chapter IX. The first, which we shall designate as the nested-growth model, is given by equations (1) to (5) plus

(6n) and (7n.1). The second, which we shall designate as the nested-wage model, is given by equations (1) to (5), plus (6n) and (7n.2).

B) The Functional Forms and the Structure of the Equation Errors

In section A we presented the specific functional forms of the different main models, without elaborating on the reasons for choosing those specific forms. We did not discuss either the structure of the error terms which, in an econometric model, are part of the behavioural equations and show their random nature. Those two points will be discussed in this section. Our aim is to obtain an econometric model which is flexible, simple, workable and efficient.

The behavioural equations are: the production function, equation (3), the marginal productivity equation, (4), the government budget constraint equation, (5), the consumption functions equations, (6c), (6k) and (6n), and the final closure equations:

(7c), (7k), (7m), (7n.1) and (7n.2).

Flexibility considerations influenced the specification of the production function given by equation (3) above. Although some initial work was done with the CES production function, given the nature of the sample, it seemed more adequate to allow for at least some variation in certain characteristics of production such as the elasticity of factor substitution. This fact, as well as the greater facility to introduce dummies if adequate, led to the choice of the translog production function with an additive logarithmic error. The translog has also been shown in several empirical studies to be a better specification of the production process than the CES.

The functional form of the marginal productivity equation (4) followed immediately from the translog production function.

Simplicity considerations would have dictated to choose linear forms with additive error terms for the other equations, when not overridden by theoretical considerations. But when the three stage least squares (3SLS) method was used to estimate linear or almost linear equations with additive errors, it was discovered that the results were inefficient, with some of the equations showing very big standard errors in most coefficients. This happened even when an attempt to build the most efficient set of instrumental variables, as suggested by Amemiya (1977), was carried out. The estimations improved somewhat but not too much. This lack of efficiency was probably related to the mixed nature of the sample, partially cross-sectional.

In an attempt to reduce heteroscedasticity and increase efficiency, the consumption equations and the final closure equations were given forms linear in the logarithms of all variables, such as:

$$\ln yi = \ln b0 + \sum_{j \neq i} bj \ \ln y(j) + \sum_{k} \ln x(k) + \ln u(i) \qquad (18a)$$

where the [y(i), y(j)] are the endogenous variables (or combinations of endogenous and exogenous variables if they appear in the right hand side); the x(k) are exogenous variables; and the u(i) are the random errors of the equations. Notice that these are the forms that we finally chose for the consumption functions.

When the main three models were estimated with consumption and final closure equations of this form, the standard errors of the estimates decreased remarkably; many coefficients that were not significant for the simple linear form with additive errors, became very significant now. But the nested models could not be estimated by the TSP program with those forms. The algorithm failed to converge as the objective function became too large. These are common problems with non-linear regression procedures, related frequently to the choice of functional form (Gallant, 1977). In this case the problem seemed to be caused by the functional form of the final closure equations in the nested model (7n.1 and 7n.2). It was then decided to use for all the final closure equations (7C, 7K, 7M, 7n.1, and 7n.2) , the other efficient forms which proved to be workable in the basic and nested models. Those were the linear or almost linear forms with multiplicative errors. That is to say, the neokeynesian model was estimated using the following equation, where u stands for a random error variable:

$$g = \{b4 + b5 [rg (1 - tr)]\} u \qquad (19a)$$

or equivalently,

$$\ln g = \ln [b4 + b5 rg (1 - tr)] + \ln u \qquad (19b)$$

The neomarxian model with:

$$wg = [\exp(R' t) (b20 + b22 idls) / (1 - tw)] u \qquad (20a)$$

or equivalently,

$$\ln(wg) = \ln[(b20 + b22 idls) / (1 - tw)] + R' t + \ln u \qquad (20b)$$

the neoclassical model, using

$$g = (R'' + 1) u \qquad (21a)$$

or equivalently,

$$\ln g = \ln(R'' + 1) + \ln(u) \qquad (21b)$$

and the nested models, in the same way, using multiplicative random errors for equations (7n.1) and (7n.2).

Using these forms for the final equations (7c), (7k), (7m), (7n.1) and (7n.2); the linear in logariths forms for the consumption equations (6c), (6k) and (6n); the translog forms for the production function, equation (3), and the related marginal productivity equation (4); finally, using an additive error for the budget constraint, equation (5) all the models, estimated by Three Stage Least Squares (3SLS), showed efficient results, and the non linear estimation process was rapidly convergent. As mentioned, many other forms were either intractable or very inefficient (had very high standard errors).

Some regresions with dummy variables in the production function were carried out in order to try to capture better the intercountry variation, but were finally abandoned, as they did not produce results too different from the regressions without dummies. We should stress that all relevant variables are in per capita (or rather, per worker) terms, and this also captures the intercountry variation.

C) The Data and the Long Term Effects

As mentioned, the data used in the estimation of the econometric models appear in Table 1. Notice that they are either ratios, or quantities expressed in constant 1975 dollars per adjusted worker; that is quantities corrected to take account of inflation, of differences in purchasing power parity among countries (1), and of the hours worked per year by workers.

The original sample consisted of 143 observations,

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(1) For this we calculated indices based on the study by Kravis et al. (1979). see appendix 1 of this chapter.

pooling 13 years, 1965 to 1977, and 11 developed countries: Canada, Denmark, France, Germany, Ireland, Italy, Japan, Norway, Sweden, the United States, and the United Kingdom.

We have chosen to focus this study on the effects of taxation in the long term; that is, a period of time long enough for economic phenomena to be approximated by growth models as those presented here. Since our aim is then to capture essentially the long term effects which the models are supposed to represent, it is important that we try to smooth out the very short term and non-stationary components of the time series elements in our sample. To use a hydraulic analogy, we have to try to eliminate from our data the turbulence effects, so that we will be able to measure adequately the effects of the steady state paths.

We have attempted this "smoothing out" of short term phenomena by grouping the yearly observations. We have then taken three year simple (not moving) averages in our original sample for each of the country time series. We have in this way obtained four groups of three year simple averages for each country. The total number of observations to which the models are applied is in this way reduced to 44 for the 11 countries (1). This procedure is equivalent to taking the time interval as equivalent to three years rather than one. Since it leads to the loss

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(1) The four average values of the time variable t in each country would then be: 2, 5, 8, and 11.

of some variability in the observations, it causes some inefficiency in the estimation. In spite of this, the point estimates that we obtained had high t values and looked, in general, reasonable (1).

Simulations made by econometric models of developed countries, show that a great part of the final impact (the long term "multiplier") of a disturbance, such as a change in an exogenous variable (i.e. a tax), will have been obtained within the first three years. Hence a model, like those here, estimated on three year periods and across countries would capture essentially long term effects.

This fact is also consistent with our use of the net of tax income, yd = wn + k rn + su, as a measure of the permanent income of households, yp, for the neoclassical model, and with the theoretical justification for this procedure which was presented in Chapter V.

D) Some Words on Identification

Originally, the identification problem arose out of

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(1) Friedman and Schwartz (see Meyer 1982, p.1531.) have smoothed out the trade cycle using a procedure which is somewhat similar. So is the procedure used by Morishima and Saito (1972a), who, estimated a long term econometric model of the american economy using, as time interval, periods of five years rather than one.

the consideration of models linear in the variables and in the parameters; it deals with the conditions necessary or sufficient to obtain a one to one correspondence between the reduced form coefficients and the structural coefficients in a model. The problem appears here because in the unrestricted general linear model, this one-to-one correspondence does not exist; it is not possible to obtain a unique transformation relating the reduced form coefficients to the structural coefficients. Hence the latter have to be further restricted through exclusions, equality relations and others, in order that it be finally possible to obtain a unique relation. A very general formulation of this traditional attack has been given by F. Fisher (1966) and L. Wegge (1965). Their formulations follow the traditional lines of reasoning, but widen the approach to include, in Fisher's work, models with equations non-linear in the variables and with linear parameter restrictions; and in Wegge's work, linear models with non-linear restrictions.

A second, more recent, line of attack, applied especially in non-linear models with well defined statistical distributions of their random variables, poses a different question: whether the parameter vector alpha defining a probability distribution f(y, alpha) for the vector valued random variable y is unique. If alpha is unique then it is said to be identified. T. Rothenberg (1971) has shown that a necessary and sufficient condition for the identification so defined is that the information matrix of the model be non-singular, i.e., have a rank equal to the number of components of alpha (say m). If the parameter has restrictions, the condition is that the information matrix augmented with the Jacobian matrix of the restrictions have rank m.

As can be seen, this second approach requires the specification of the distribution function of the random variable y. Once this function is defined, the condition of identification would be fulfilled if the likelihood function presented a unique global maximum, (or, for local identification, a local maximum. (see Rothenberg (1971) p. 584). In our case, we have not specified the distribution function of the errors in the models of section A since we have estimated them by three stage least squares (3SLS).

But using 3SLS estimation implies considering that the distribution function is completely described by its first and second moments (i.e. by the parameters to be estimated and the structural error's covarience matrix). Hence, in this case also, the existence of a locally unique solution implies identification (at least locally).

The first approach, the traditional one, is less compelling in models non-linear in their variables and parameters, like those here studied. The problem can again be defined in very general terms as the analysis of the uniqueness of a (possibly) non-linear transformation from a vector of reduced form coefficients to a vector of structural coefficients with k restrictions. But this approach is not very practical.

Fisher's (1966) methods, on the other hand, combined with Wegge's (1965) can be applied with advantage to the models examined, to verify at least order conditions for identification. We shall do this in what follows.

The models in section A are not linear in the parameters. The production function (equation 3) and the neoclassical natural rate of growth (equation 7.C) contain non linearities in the parameters. Expanding the production function and introducing the following coefficients:

$$b30 = b1\ R' \qquad (22a)$$

$$b31 = a2\ R' \qquad (22b)$$

$$b32 = a2\ R'^2 \qquad (22c)$$

$$R'' = \exp(R') - 1 \qquad (22d)$$

the models can be written as linear in the new set of parameters but subject to the additional four non-linear restrictions (22). Wegge (1965) has studied the identification of models linear in the variables subject to non-linear restrictions, whereas Fisher (1965) has studied the identifications of models non-linear in the variables subject to linear restrictions. The problem posed by the equations in section A can be solved by combining these two approaches.

The first step is to write the by now linear-in-the-parameters models, following Fisher, in the general form:

$A\ q(x) = U$

Where A is the matrix of parameters, x is a vector of N basic variables (of which M are endogenous), q(x) is a vector of endogenous functions (those containing at least one endogenous variable) and exogenous functions of the basic variables, U is a vector of zeros or errors. The vector q(x) is the following for the neoclassical case, with all the endogenous functions first, and taking tr as exogenous (eliminating identity 5-A):

[q(x)]' = { q, c, gc, k (d + g), q (1 − tx + sur), wg,
k(d + rg), ln q, ln k, (ln k)², (ln k) t,
(d + rg) k / [q(1 − tx + sur)], wg tw, q tx,
k rg, [tr − θg,n (1 − tcr)°], sur q,
<u>ln[wg (1-tw) + su + k rg (1− tr)]</u>, <u>1</u>, bt, t,
t², ad5, gas, su, 1 }

A total of 26 components, 18 endogenous and 8 exogenous functions. The neo-keynesian model would replace the two underlined components by the following three:

ln[wg (1-tw) + su], ln[k rg (1 − tr)], ln[rg (1 − tr)]

and the neomarxian by:

ln[wg (1-tw) + su], ln[k rg (1 − tr)], idls / (1 − tw)

In the neokeynesian case, there would be 20 endogenous and 7 exogenous functions, and in the neomarxian case, 19 endogenous and 8 exogenous functions.

For equation i which can be written $A^i q(x)'$, Fisher's identification criterion, for linear restrictions of the type $A^i\ \phi$ = lambda is:

rank $(A^*\phi) = M^*$

where A^* is a $M^* \times M^*$ matrix obtained by augmenting A with a series of vectors h^i:

$$A^* = \begin{bmatrix} A \\ --- \\ h^{1*} \\ h^2 \\ \cdots \\ \cdots \\ h^{M^*-M} \end{bmatrix}$$

and M^* is the number of basic endogenous variables in q(x) plus the number of h^i vectors.

The vectors h have to be added to A, contrary to the linear case of identification, because, in the non-linear in the variables case, it is possible that non-linear combinations of the disturbances produce linear functions of the elements q^i, independant of those already contained in the matrix A.

Fisher has shown that those vectors h would satisfy the relation $hQ'(x)=0$; where $Q'(x)$ is the Jacobian matrix of q(x) (where the rows correspond to elements of q(x) and the columns to elements of x) (That is $Q'(x)= q(x)/ x$); Using the above relation we examined the composition of the h vectors;they were null for the three models analysed.

Hence we obtain:

$A^* = A$ and $M^* = M$

On the other hand, Wegge's analysis of non-linear and cross-equation restrictions of the form:

$\phi(aij)=0$

permits to replace the matrices ϕ by the Jacobian matrix of the ϕ, $J(\phi)$, to obtain the identification criterion applied to the overall system:

$\text{Rank } (A \quad J(\phi)) = (M^*)^2$

or in the present case,

$\text{Rank } (AJ(\phi)) = M^2$

where M is the number of "basic" endogenous variables in the system. Wegge's criterion for those equations i with no cross restrictions, reduces to:

$\text{Rank } (A \quad J\ (\phi)) = M$

or $\text{Rank } (A\ J\ (\phi)) = M$

in the present case.

With these preliminaries we can pass to the analysis of the models in section A. We shall only deal here with order conditions, that is to say, those obtained by counting restictions. The order conditions are:

$\text{Rank } (J\ (\phi)) > M$

and $\text{Rank } (J\ (\phi)) > M$

The restrictions are of several types: exclusion restrictions = ER; non-linear restrictions NLR, normalizing restrictions, NOR, and the possible other restrictions OR, such as cross-equation or restictions giving unit values to some coefficients. What the order conditions say is that: ER + NLR + NOR + OR is greater or equal than M for any equation or greater or equal than M^2 for the system overall. For the ith equation, the exclusion restrictions are equal to the total number "No" of functions in the vector q(x) minus the number of parameters NPi in that equation:

ERi =No-NPi

Hence, the order conditions can be written, for equation i as:

No - NPi + NLRi + NORi + ORi > (or =) M

NPi < (or =) No - M + NLRi + NORi + ORi

For the system as a whole, the exclusion restrictions ER are:

ER = M No - NP

hence the order conditions become:

NP < (or =) M No - M^2 + NLR + NOR + OR

Let us now examine the neoclassical system as a whole. The number of endogenous variables M is 7, the number of functions in q(x) is 26, the non-linear restrictions are 4 for the production function and one for the natural rate of growth: total: 5. The normalization restrictions are one per equation, to total 7. The other restrictions are cross-equation restrictions; there are three, equalizing some of the coefficients of the production function and the marginal productivity equations, plus another relating the rates of growth R' in the production function and the natural rate of growth equation; a total of 4 cross equation restrictions, plus 14 restrictions giving coefficients a unitary value, total other restrictions: 18. Hence

NLR +NOR + OR = 4 + 7 + 18 = 29

and

NPc < 7(26 - 7) + 29 = 162

Hence the total number of parameters in the neoclassical model should be less than 162. In fact there are only 7 in the model. The model is amply overidentified. Similar analyses for the neokeynesian model and the neomarxian models yield respectively 7(27 - 7) + 28 = 168 and 7(27 - 7) + 29 = 169 maximum number of parameters for identification. They are also amply overidentified.

A similar analysis can be done for each equation in the system, to test whether it is identified or not. Take for example the production function which has the greatest number of parameters in the model; a total of 7: a0, a1, a2, b30(b1 R'), b31(a2 R'), b32(a2 R'²), and R'. No is still 26 for the neoclassical model and 27 for the others, M=7, NOR=1, NLR=3; OR would refer to the cross equation restrictions, which are 3 (3 coefficients in the production function are equal to three coefficients in the marginal productivity equation). That would give a maximum of parameters of:

NP < (26 or 27) - 7 + 3 + 1 + OR

or PP < (23 or 24) + OR

NP < 24 or 25

Since that equation has only 7 coefficients, it is overidentified. Notice that, if all restictions but the normalization ones are ignored, a necessary condition for the identification of any equation is:

NP < (26 or 27) − 7 + 1

or NP < 20 (NC model) or 19 (NK or NM models)

Since no equation has more than 7 coefficients it can be concluded that they are all overidentified.

E) The Estimation Methods and Results

As mentioned, the estimation method finally chosen was the three stage least squares procedure for non-linear models, proposed by Hausman (1974, 1975) Jorgenson and Laffont (1974) and others (see Bernd, Hall and Haussman, 1974) and studied by Amemiya (1977). This procedure is a minimum distance method (see Malinvaud, 1980), which uses instrumental variables and consistent estimates of the variance covariance matrix of the errors of the structural equations (1).

The procedure takes account of all the restrictions across equations and within equations in the system. It is

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(1) For a more detailed description, refer to the authors quoted.

CH VII
TABLE 2

RESULTS OF THE ESTIMATION OF THE BASIC AND HYBRID MODELS BY THE 3SLS PROCEDURE

Model

Equation	Neoclassical	Neokeynesian	Neomarxian
Prod. func. and Marg. prod			
b0	-1.7288	-1.6275	-1.6889
	(.0967)	(.0999)	(.0932)
b1	.2802	.2705	.2717
	(.0924)	(.0835)	(.0881)
a2	-.0757	-.0882	-.0827
	(.0452)	(.0440)	(.0448)
R'	.0534	.0277	.0420
	(.0012)	(.0118)	(.0059)
R squared(*)	.6851,.2467	.7017,.3029	.6964,.2726
SSR(*)	1.8843,.1298	1.7851,.1201	1.8169,.1253
SER(*)	.2069,.0543	.2014,.0522	.2032,.0534
Cons. funct.			
b2	-.3154	.2657	.2657
	(.2659)	(.3092)	(.3092)
b3y	.9774		
	(.0809)	—	—
b3w	—	.7900	.7900
	—	(.0661)	(.0661)
b3r	—	.1636	.1636
	—	(.0930)	(.0930)
b3	-.0658	—	—
	(.0514)	—	—
R squared	.9668	.9616	.9616
SSR	.1621	.1877	.1877
SER	.0607	.0653	.0653
Nat rate of unempl.			
R squared	-.6511		
SSR	8.5256		
SER	.4402		
Animal spirits			
b4	—	.0255	—
	—	(.0025)	—
b5	—	.2146	—
	—	(.0205)	—
R squared		.6239	
SSR		1.942	
SER		.2100	
Wage equat.			
b20	—	—	.0369
	—	—	(.0016)
b22	—	—	-.00055
	—	—	(.00012)
R squared			.2953
SSR			5.8196
SER			.3637

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TABLE 2

RESULTS OF THE ESTIMATION OF THE BASIC AND HYBRID MODELS BY THE 3SLS PROCEDURE

	Model		
	Hybrid Neoclassical	Hybrid Neokeynesian	Hybrid Neomarxian
Equation			
Prod. func. and Marg. prod.			
b0	-1.7288	-1.6275	-1.6836
	(.0966)	(.0999)	(.0935)
b1	.2802	.2705	.2727
	(.0934)	(.0835)	(.0884)
a2	-.0757	-.0882	-.0826
	(.0452)	(.0440)	(.0448)
R'	.0534	.0277	.0410
	(.0012)	(.0118)	(.0059)
R squared(*)	.6851,.2467	.7017,.3029	.6971,.2759
SSR(*)	1.8844,.1298	1.7851,.1200	1.8125,.1247
SER(*)	.2069,.0543	.2014,.0522	.2029,.0532
Cons. funct.			
b2	.1891	-.3154	-.3154
	(.3092)	(.2660)	(.2660)
b3y	—	.9774	.9774
	—	(.0809)	(.0809)
b3w	.7171	—	—
	(.0661)	—	—
b3r	.2346	—	—
	(.0930)	—	—
b3	—	-.0658	-.0658
	—	(.0515)	(.0515)
R squared	.9654	.9668	.9668
SSR	.1692	.1621	.1621
SER	.0620	.0607	.0607
Nat rate of unempl.			
R squared	-.6511		
SSR	8.5255		
SER	.4401		
Animal spirits			
b4	—	.0254	—
	—	(.0026)	—
b5	—	.2149	—
	—	(.0207)	—
R squared		.6239	
SSR		1.9422	
SER		.2100	
Wage equat.			
b20	—	—	.0356
	—	—	(.0016)
b22(**)	—	—	.772E-5
	—	—	(.00018)
R squared			.3231
SSR			5.5896
SER			.3564

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TABLE 2

RESULTS OF THE ESTIMATION OF THE BASIC AND HYBRID MODELS BY THE 3SLS PROCEDURE

	Model	
	Nested growth	Nested wage
Equation		
Prod. func. and Marg. prod.		
b0	-1.6389	-1.6283
	(.1003)	(.0999)
b1	.2704	.2706
	(.0845)	(.0836)
a2	-.0873	-.0882
	(.0442)	(.0441)
R'	.0303	.0279
	(.0116)	(.0117)
R squared(*)	.7015,.2981	.7017,.3026
SSR	1.7864,.1209	1.7851,.1201
SER	.2015,.05242	.2014,.0522
Cons. funct.		
b2	.1886	.1886
	(.3208)	(.3208)
b3w	.7166	.7166
	(.1051)	(.1051)
b3r	.2351	.2351
	(.1224)	(.1224)
b3	-.0564	-.0564
	(.0739)	(.0739)
R squared	.9654	.9654
SSR	.1692	.1692
SER	.0620	.0620
Nested Equation		
b11	.0290	-.000129
	(.0041)	(.02407)
b12	-.0645	-.00258
	(.0916)	(.5731)
b5	.2174	.2135
	(.0363)	(.2502)
b20	.000969	.00109
	(.000609)	(.00415)
b22	-.000105	-.000107
	(.000029)	(.000168)
b13	.0306	.4878
	(.0148)	(.0572)
R squared	.7224	.9939
SSR	1.4433	.0502
SER	.1805	.0338

Government deficit	All models
R squared	.7498
SSR	.00146
SER	.00576

(*) The first figure corresponds to the prod. function the second to the marginal productivity equation.
(**) .772E-5 stands for .00000772

then quite sensitive to system influences across equations. The procedure produces, under quite general assumptions, consistent estimators and asymptotically normal errors (2). It is important to stress that the estimates produced are consistent even if the original equation errors are not normal. This is a very important property that the "Full Information Maximum Likelihood" (FIML) procedure does not possess: The FIML estimates are inconsistent if the equation errors do not conform to the assumption of normality. Given the pooled nature of the sample used here the greater robustness to the normality assumption of the 3SLS method is an important and desirable characteristic. The asymptotic normality of the errors allows chi square tests to be used. The resulting estimates of the three basic models as well as those of the nested model appear in Table 2. They are discussed in the next chapter.

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(2) These facts are proven by Amemiya (1977)

CHAPTER VIII

COMPARATIVE CHARACTERISTICS OF THE ESTIMATED MODELS

The results of the three stage least squares estimation of the basic and nested models were presented in Table 2 of Chapter VII. In this chapter we are going to calculate and analyse some some important characteristics of the models, arising from the estimated values, such as the elasticities of substitution in production, the elasticities of consumption and of investment, the propensities to consume and to invest, etc.

The values taken by these coefficients are important because the different models impose certain limits to those values. For example, in the pure neokeynesian model it is assumed that workers expend most of their disposable income on consumption, and in consequence savings come almost exclusively from capital income. Hence the propensity to consume out of wages in the neokeynesian model, should not be very different from 1.0, and in any case should not be smaller than the propensity to consume out of profits.

There are many other limits of this sort in the different models. How much the estimated values conform to these a priori limits will give a good indication of the overall adequacy of the models, of how well they represent the "real world", in so far as the latter is reflected in the sample.

In what follows we shall examine, first, the characteristics relating to production and technology; second, we shall look at the consumption (and saving) characteristics; then we shall examine the other closure equations in the models, describing the rates of growth and the wage rates. Finally, we shall briefly look at the Government Sector. We shall, in general, limit the discussion to the basic models: neoclassical, neokeynesian and neomarxian.

A) <u>Production and Technology</u>

Table 1 summarizes the principal characteristics of production and technology in the basic models arising from the coefficient estimates in Chapter VII, Table 2. It can be observed that, with the exception of the rates of technical progress there is very little variation for the values shown across models. The values shown on the tables result from suitable combinations of the joint coefficients of the translog production function and the marginal

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TABLE 1

PRODUCTION AND TECHNOLOGY CHARACTERISTICS OF GROWTH MODELS

Characteristic	Neoclass.	neokeyn.	Neomarx.	Nested G.	Nested W.
a2	-.0757	-.0882	-.0827	-.0873	-.0882
	(.0452)	(.0440)	(.0448)	(.0302)	(.0440)
El. of substitution,	.7592	.7318	.7432	.7336	.7319
Ek,l	(.0112)	(.0092)	(.0101)	(.0100)	(.0100)
Profit share, Ar	.395	.404	.397	.402	.403
	(.030)	(.026)	(.027)	(.030)	(.030)
Rate of harrod neutral	5.34%	2.77%	4.20%	3.03%	2.78%
technical progress	(.116%)	(1.17%)	(.59%)	(1.16%)	(1.18%)
Approximate rate of	3.23%	1.66%	2.52%	1.81%	1.66%
Hicks neutral tech. pr.					

Note : Standard deviations appear in parenthesis below values. All quantities not directly obtained from the estimates are calculated at middle of the sample values,except for the time variable, which is taken at t=0; the difference with the results for t average (t = 6.5) is minimal.

(*) a2 is twice the coefficient of the log of capital per worker in the translog production function.

Source: Chapter VII, table 2.

productivity conditions. In the case of the rates of technical progress, they are also constrained by the growth conditions for the neoclassical model, and the wage conditions for the neomarxian. This explains their greater variability across models.

The values shown in parentheses are approximate standard deviations of the characteristics, which in most cases have been obtained by linearizing the formulas used to calculate the characteristics around, the parameter estimates, and for values of the variables given by their sample means.

The first quantity shown is a2 : twice the coefficient of the square of the logarithm of capital per worker in the translog production function, which was written as:

$$\ln [q \exp(-R' t)] = b0 + b1 \ln [k \exp(-R' t)] + a2 \{\ln[k \exp(-R' t)]\}^2$$

with q = production per worker

k = capital stock per worker.

This coefficient gives an idea of the curvature of the translog production function in the log q vs. log k plane. A null value for that coefficient corresponds to the Cobb-Douglas production function. It is interesting to test the hypothesis of whether the production function is Cobb-Douglas, which implies important consequences for incidence.

The student t ratio for the a2 coefficient is -1.67 for the neoclassical model, -2.00 for the neokeynesian, -

1.85 for the neomarxian, −1.97 for the nested-growth model and −2.00 for the nested-wage model. Given the asymptotic normality of the coefficients, a one tailed test of the above ratios at 5% level of confidence (with a critical value of 1.645) results in the rejection of the null hypothesis for all models. The hypothesis of a Cobb-Douglas production function is rejected. But notice that the neoclassical model is almost at the limit of acceptance.

The second and third production characteristics shown are the elasticity of substitution and the share of profits on the product. The share of profits on the product is equal to the elasticity of the production per worker, q, vis-a-vis capital per worker, k, call it Eq,k:

$$E_{q,k} = (k \ / \ q) \quad q \ / \quad k = b1 + a2 \ \ln[k \ \exp(-R' \ t)] \qquad (1)$$

The elasticity of substitution Ek,l is given, in terms of Eq,k and a2 by:

$$E_{k,l} = (E_{q,k}^2 - E_{q,k}) \ / \ (E_{q,k}^2 - E_{q,k} + a2) \qquad (2)$$

It is then, as Eq,k , variable with time, t .

The values of the elasticity of substitution shown in the table were obtained for values of the exogenous time variable, t, equal to zero which corresponds to the year 1965. The standard deviations shown are approximate only, resulting from linearization around the coefficient values as explained before. As can be observed, the substitution elasticities are all very similar, in the range .73 to .76, all well below 1.0; the lowest values correspond to the nested models. Approximately 5% confidence intervals would be between .71 to .77. Hence, they are both far from fixed proportions and from Cobb-Douglas or higher values.

The share of profits Ar,g is shown with its time term apart. As can be seen, the latter is very small. For example, the neoclassical share would change from .395 at t = 0 to .42 for t = 6.5. Similar values would be obtained for the other models. The shares presented in the table are those of profits gross of depreciation to product gross of depreciation, and net of indirect taxes and subsidies, .They are all very close to .40 for t = 0 and .41 for 6.5. Approximately 5% confidence intervals for t = 0 would be .34 to .46. Hence corresponding intervals for the wage shares would be .66 to .54, with an average of around .60.

It is interesting to ask what the shares net of depreciation would be. It is possible, using some simple algebra, to see that the share, ar, of profits net of depreciation, k rg , to product net of depreciation, indirect taxes and subsidies, y = q (1 - tx + sur) - d k, can be expressed as:

$$Ar = Ar,g - (d\ k\ /\ y)\ (1 - Ar,g) \quad (3)$$

where d is the depreciation rate, k is capital stock per worker, and the other symbols are as previously defined; d, k, q are taken as usual at middle sample values.

Applying formula (3) the share of net profits on net product is .32 with an approximate 5% confidence interval of .26 to .38. Hence the share of wages on the net product would be .68 with a confidence intervals of .62 to .74.

The final two rows of Table 1 present rates of technical progress in the model. The rates of Harrod neutral, or pure labour augmenting, technical progress are those obtained from the joint estimation of the equations of the model. Notice that the terms containing those rates in the production function and the marginal productivity equation are of the form: exp(R' t), where R' is the rate of Harrod Neutral Technical Progress (heretofore HA.N.T.P). Hence the figures shown are those of instantaneous rates, expressed in percent per year. To convert them to discrete, yearly rates it is necessary to transform them by:

$$R' = \exp(R') - 1$$

where R' is the discrete yearly rate. For small values of R', as those shown, the instantaneous and the discrete yearly rates are practically equal.

As mentioned before, the neoclassical and the neomarxian rates of technical progress also enter the growth (neoclassical) and the wage (neomarxian) equations of those models. Notice that the constrained rates so obtained for these two models are higher than the unconstrained rate of the neokeynesian model (5.43% for the neoclassical and 4.20% for the neomarxian, versus 2.71% for the neokeynesian).

It should be noted that the latter models also contain rates of growth of technical progress in their nested closure equations (see equations (7n.1) and (7n.2) in Chapter VII) but since those equations impose less constraints on the values taken by the left hand side variable, their resulting rates of HA.N.T.P. are not far from the unconstrained values of the Keynesian model. All the rates are significantly different from zero; particularly the neoclassical and neomarxian models have very high Student "t" ratios.

Some will find the rates of growth of technical progress too high. It is important to keep in mind that they refer to Harrod neutral technical progress, and hence the "augmentation" of factors which they are purported to represent applies only to labour (1). It is possible to obtain an estimate of rates of Hicks Neutral Technical Progress (heretofore HI.N.T.P.) which would not be constant in time but which would express what a balanced and time variable rate of "augmentation" of both labour and capital, equivalent to the constant rate of Harrod neutral technical progress, would be. For this, take two translog production functions written, one with HA.N.T.P. and the other with HI.N.T.P. and make them identically equal, regroup and

---

(1) Recall that we chose to use Harrod neutral technical progress models, since this is the only one to admit a real steady state, where the capital output ratio stays constant. (see chapter 5, section B-2).

eliminate redundant terms to obtain finally:

$$R'hi = (1 - b1 - a2 \ln k)\, R'ha + .5\, a2\, R'ha^2\, t$$

and recall that b1 + a2 is the profit share, Ar,g , at t = 0: Ar,g(0).

Hence, simplify the above expression to:

$$R'hi = [1 - Ar,g(0)]\, R'ha + .5\, a2\, (R'ha)^2\, t \qquad (4)$$

Notice that R'hi, the rate of growth of HI.N.T.P., which would be equivalent to R'ha, the rate of growth of HA.N.T.P., could not be constant but rather variable in time. The time dependent term though is very small, for the values of our models, as compared with the other term (1), which is simply the product of the wage share by the rate of HA.N.T.P.

We have then calculated the values of the equivalent rate of Hicks neutral technical progress in the last line of Table 1, for t = 0. Notice that the values shown are approximately 60% of the values of the rates of HA.N.T.P., and they are closer to other author's estimates: 3.23% for the neoclassical; 1.67% for the neokeynesian; 2.53% for the neomarxian, etc, Notice that these values, presented in terms of HI.N.T.P. would constitute the "unexplained residual" in models like those of Denison (1967, 1970), using solely production functions and marginal productivity

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(1)For example, the neoclassical equation is:
R'hi = 3.23% - 0.01 t

conditions to explain factors of growth.

B) Consumption and Savings

The consumption equations were estimated using linear logarithmic forms, presented in Chapter VII, as follows. For the neoclassical model:

$$\ln c = b2 + b3y \ln yd + b3 \ln rn + uc \qquad (5c)$$

with c: consumption per worker

yd: equal to disposable income

rn: the net of tax profit rate and uc the random error term.

For the neokeynesian and neomarxian models, the Kaldorian consumption function:

$$\ln c = b2 + b3w \ln wn1 + b3r \ln(k\ rn) + uk \qquad (5k)$$

where wn1 is the wage net of tax plus subsidy to workers:

$$wn1 = wn + su = wg\ (1 - tw) + su$$

k rn : is the profit per worker and uk the random error term.

Finally, for the nested models:

$$\ln c = b2 + b3w' \ln wn1 + b3r \ln(k\ rn) + b3 \ln rn \qquad (5n)$$

with variables defined as previously.

Given the linear logarithmic forms of these equations, the coefficients b3y, b3w, b3r and b3, are elasticities of consumption with respect to yd, wn1, k rn and rn respectively.

Table 2 presents the values of those coefficients and their standard deviations. Those figures were used to calculate consumption and savings elasticities and marginal propensities to consume and to save, using the formulas shown in Tables 3-1, 4-1, and 7, with values of the variables taken as usual, at sample means. Confidence intervals were calculated as linear approximations using a procedure similar to the one utilized for the production characteristics.

1) Consumption Elasticities.

Table 3 shows the consumption elasticities obtained applying the formulas presented in Table 3-1. Some of these elasticities or their saving counterparts, will play an important role in the derivation of incidence values. For our present purposes the relations between consumption and profits, wages, disposable income and wealth are more interesting in terms of marginal propensities, which will follow. But the elasticity of consumption with respect to the rate of profit is interesting in itself. Recall that, in neoclassical growth models by the perfect arbitrage assumption, in the long term this is the real interest rate elasticity of consumption. But in the neokeynesian and neomarxian models, the arbitrage assumption is not made; hence, the rate of profit cannot be taken as a proxy for

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TABLE 2

ESTIMATED EQUATIONS AND STANDARD ERRORS FOR CONSUMPTION FUNCTIONS IN VARIOUS GROWTH MODELS

Coefficients	Neoclassical	Neokeynesian and neomarxian	Nested Growth
b2	-.31536	.26567	.18865
	(.26599)	(.3092)	(.32084)
b3y	.97742	--	--
	(.08093)		
b3w	--	.79001	.71660
		(.06614)	(.10513)
b3r	--	.16359	.23506
		(.09301)	(.12246)
b3	-.06584	--	-.05642
	(.05150)		(.07394)

Note:Standard deviations in parenthesis,below estimates.

TABLE 3

VALUES OF CONSUMPTION ELASTICITIES IN ESTIMATED MODELS(*)

Elasticities of consumption	Neoclassical	Neokeynesian and neomarxian	Nested growth
With respect to profits Ec,Πn Ec,k	.30719	.16359 (.09301)	.23506 (.12240)
With respect to wages Ec,wn	.67020	.79001 (.06614)	.71660 (.10513)
With respect to disposable income Ec,yd;Ec,A	.97742 (.08093)	.95360 (.08590)	.95166 (.0860)
With respect to the rate of profit Ec,rn	-.06584 (.05150)	—	-.05642 (.07394)

Note:Ec,i means consumption elasticity with respect to argument i. Argumens are: Πn:net profits(=rn k); rn:net rate of profit; k:capital stock per worker; A:wealth or assets (A=k + wn / rn); wn:net wage including subsidies(wn=wg(1-tw)+su); yd:disposable income (yd=wn+Πn=wn+k rn).

(*)Values estimated using formulas in table 3-1.Standard deviations in parenthesis.

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TABLE 3-1

SUMMARY OF RELATIONS BETWEEN ELASTICITIES AND COEFFICIENTS IN CONSUMPTION MODELS USED TO CALCULATE VALUES IN TABLE 3

Elasticities of consumption	Neoclassical	Neokeynesian and neomarxian	Nested growth
With respect to profits Ec, Πn;Ec,k	$\frac{rn\ k}{wn+rn\ k} b3y$	b3r	b3r
With respect to wages Ec,wn	$\frac{wn}{wn+rn\ k} b3y$	b3w	b3w
With respect to disposable income Ec,yd;Ec,A	b3y	(b3w+b3r)(**)	(b3w+b3r) (**)
With respect to the rate of profit Ec,rn	b3	0	b3

Notes: Ec,i means consumption elasticity with respect to argument i. Arguments are: Πn: net profits(=rn k); rn: net rate of profit; k: capital stock per worker; A: wealth or assets (A=k+wn/rn); wn: net wage including subsidies(wn=wg(1-tw)+su); yd: disposable income (yd=wn+ Πn=wn+k rn).

(**)This is obtained from the respective marginal propensity to consume in table 4, multiplied by (yd/c).

the rate of interest, and the latter simply does not appear in the consumption equations of these models.

The values taken by this interest (or profit) rate elasticity of consumption are shown in line 4 of table 3. For the neoclassical and nested equations they are respectively, −0.065 and −0.056, with the correct sign. But a Student "t" test shows that they are not ssignificantly different from zero. Since the neokeynesion equation has no term in the rate of interest its interest rate elasticity is <u>a fortiori</u> zero.

What is important to notice is that the neoclassical result is consistent with the normal conjecture made in neoclassical models, that the interest (profit) rate elasticity of consumption is negative but the evidence, although with the right sign, is not statistically significant.

2) Marginal Propensities to Consume.

Let us now look at the various marginal propensities to consume contained in the estimated models. These are shown in Table 4. They were obtained by using the formulas of Table 4-1, utilizing, when necessary, mean sample values of relevant variables. Standard deviations shown in parentheses were in general obtained, as explained before, by linear approximation.

TABLE 4

MARGINAL PROPENSITIES TO CONSUME IN ESTIMATED MODELS(*)

Marginal propensities to consume out of	Neoclassical	Neokeynesian and neomarxian	Nested growth
Profits MPC(Π)=mc,Π	.8078	.4301	.6181
	(.0669)	(.2446)	(.3218)
Wages MPC(w)=mc,w	-	.9522	.8637
		(.0797)	(.1267)
Disposable income	-	.7881	.7865
MPC(yd)=mc,yd		(.0710)	(.0711)
Capital MPC(k)=mc,k	.1128	.0601	.0656
	(.0093)	(.0342)	(.0342)
Wealth MPC(A)=mc,A	.1128	.1100	.1099
	(,0093)	(.0099)	(.0099)

Note: mc,i=MPC(i) means marginal propensity to consume out of argument i. Arguments are: Πn: net profits(=rn k); rn: net rate of profit; k: capital stock per worker; A: wealth or assets (A=k+wn/rn); wn: net wage including subsidies(wn=wg(1-tw)+su); yd: disposable income (yd=wn+Πn=wn+k rn).

(*)Values estimated using formulas in table 4-1. Standard deviations in parenthesis.

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TABLE 4-1

SUMMARY OF RELATIONS BETWEEN MARGINAL PROPENSITIES TO CONSUME AND COEFFICIENTS IN CONSUMPTION FUNCTIONS USED TO CALCULATE VALUES IN TABLE 4

Marginal propensities to consume out of	Neoclassical	Neokeynesian neomarxian and nested
Profits MPC(Π) =mc,Π	(c/yd)b3y	[c/(rn k)]b3r
Wages MPCw=mc,w	(c/yd)b3y	(c/wn)b3w
Disposable income MPCyd=mc,yd	(c/yd)b3y	(b3w+b3r)c/yd (*)
Capital MPCk=mc,k	b3y rn c/yd=rn MPCyd	b3r,c/k
Wealth MPC(A)=mc,A	b3y c /A=rn MPCyd	(b3w+b3r)c/A=rn MPCyd

Note: mc,i=MPCi means marginal propensity to consume out of argument i. Arguments are: Π n: net profits(=rn k); rn: net rate of profit; k: capital stock per worker; A: wealth, or assets (A=k+wn/rn); wn: net wage including subsidies(wn=wg(1-tw)+su); yd: disposable income (yd=wn+ Πn=wn+k rn).

(*) This expression is obtained by taking a weighted average as follows: (Πn mc,Π + wn mc,w)/(Πn + wn) = c(b3r+b3w)/yd

For the neoclassical model the marginal propensity to consume out of disposable income (mc,yd) was estimated at .81, significantly different from one at 5% level of confidence. Hence, the marginal propensity to save on the average, is around .20 (see Tables 5 and 7) in the neoclassical model, and significantly different from zero. The marginal propensity to consume out of wealth or assets, mc,A , is obtained as the product of mc,yd and the rate of profit (see Table 4-1).

For the neokeynesian model, what is needed is to test several important properties which the consumption functions should possess in these models. These properties are:

-The marginal propensity to consume out of wages, mc,wn should be significantly greater than the marginal propensity to consume out of profits mc,Πn. This corresponds to Kaldor's consumption function and plays an important role in some formulations of neokeynesian distribution theories.

-The marginal propensity to consume out of wages, mc,wn , could, additionally, not be significantly different from 1.0. If this were very definitely so, it would imply that workers' savings are very small.

-A final important characteristic to check is whether the marginal propensity to consume out of profits (or capital) is significantly different from zero. This would correspond to what could be called the "super-classical" consumption function: all wages are consumed and all profits are saved.

What do the keynesian model estimates say about the first of those three characteristics? The marginal propensity to consume out of wages does indeed exceed the marginal propensity to consume out of profits by .52, an apparently great amount. To test whether this amount is statistically significant we can calculate a linear approximation to the covariance of the two MPC's involved. Alternatively, an exact way to do the test which involves directly the values of the model coefficients, b3w and b3r, and their variance covariance matrix is as follows:

By Table 4-1:

$mc,wn - mc,\hat{r}n = (\bar{c} / \overline{wn})\ b3w - [\bar{c} / (\overline{rn}\ \bar{k})]\ b3r$.

The test is:

$$(\bar{c} / \overline{wn})\ b3w - (\bar{c} / \overline{rn}\ \bar{k})\ b3r > 0$$

Hence, the null hypothesis H0 for values of wn and rn k at their sample means, can be written:

$H0:\ z = b3w - (\overline{wn} / \overline{rn})\ b3r = 0$; against $H1:\ z > 0$

From the variance-covariance matrix for the neokeynesian model it is possible to calculate the standard deviation of z, 'z = .2405. This gives a "t" ratio of 1.8. Therefore the null hypothesis is rejected at 5% level of confidence (The critical value for one tailed tests is 1.65). Hence, the neokeynesian model values support, or rather do not contradict, the Kaldorian conjecture that the marginal propensity to consume out of wages, mc,wn is greater than the marginal propensity to consume out of profits, mc,πn . Better still, they contradict the neoclassical conjecture that those two propensities can be taken as equal.

The second conjecture, that the marginal propensity to consume out of wages, mc,wn , is equal to 1.0, or alternatively that:

$$Ec,wn = b3w = \overline{wn} / \overline{c}$$

similarly tested leads to a t ratio of .599, for the null hypothesis that b3wn = wn / c. Hence this one cannot be rejected at 5% or even higher levels of confidence. So the conjecture that the MPCW = 1.0 is not contradicted by the estimated model.

The third conjecture, that mc,πn = 0, leads to a t ratio of 1.75. Hence a one tail test at 5% level of confidence rejects that hypothesis.

Do the figures relating to the nested models confirm the above findings? The nested model shows a higher propensity to consume out of profits and a lower propensity to consume out of wages than the keynesian and marxian models. A test of whether that difference is not significant yields a "t" ratio of .58 and the hypothesis of equality cannot be rejected. Hence the test is inconclusive. The test of the hypothesis that mc,wn is 1.0 gives a "t" value of 1.07, hence it does not reject it. The one tail test that MPC = 0 yields a t value of 1.920; hence the hypothesis is rejected at 5% level of confidence.

The nested model is then less conclusive than the neokeynesian model on the subject of the difference in marginal propensities to consume out of wages and out of profits. Recall also that the nested model did not reject either the hypothesis that the profit (or equivalently in the neoclassical model, interest) elasticity of consumption is less than zero. The only clear conclusion is that the two alternative consumption functions, neoclassical and neokeynesian (Kaldorian), are not significantly different from the nested function. This will be confirmed in a later chapter, where we shall carry tests of the overall models.

Marginal propensities to save and savings elasticities, for the models are presented in Tables 5 and 6, as well as the formulas used to obtain those quantities (Table 7). A discussion of the values in those tables would simply be the mirror image of the previous discussion on consumption. The tables are presented, though, because some of the values shown will be used later to derive incidence results and because they have an intrinsic interest of their own.

To sum up, we have found the following interesting results in the analysis of the consumption functions:

(1) In the neoclassical consumption function, the effect of the rate of interest (profit) on consumption is negative, as is normally conjectured in neoclassical models, but the value of this effect is not significantly

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TABLE 5

MARGINAL PROPENSITIES TO SAVE IN ESTIMATED MODELS(*)

Marginal propensities to save out of	Neoclassical	Neokeynesian and neomarxian	Nested growth
Profits MPS(Π)=ms,Π	.1922	.5699	.3819
	(.0669)	(.2446)	(.3218)
Wages MPSw=ms,w	•	.0478	.1363
		(.0797)	(.1267)
Disposable income MPSy(d)=ms,yd	•	.2119	.2135
		(.0710)	(.0711)
Wealth MPS(A)=ms,A	.0268	.0296	.0298
	(.0093)	(.0099)	(.0099)

Note: ms,i = MPSi means marginal propensity to save out of argument i. Arguments are: Πn: net profits(=rn k); rn: net rate of profit; k: capital stock per worker; A: wealth or assets (A=k+wn/rn); wn: net wage including subsidies(wn=wg(1-tw)+su); yd: disposable income (yd=wn+Πn=wn+k rn).

(*)Values estimated using formulas in table 7. Standard deviations in parenthesis.

TABLE 6

VALUES OF SAVINGS ELASTICITIES IN ESTIMATED MODELS(*)

Elasticities of saving	Neoclassical	Neokeynesian and neomarxian	Nested growth
With respect to profits Es,Πn=Es,k	.3483	1.0321 (.4430)	.6918 (.5829)
With respect to wages Es,wn	.7594	.1888 (.3150)	.5384 (.5007)
With respect to disposable income Es,yd=Es,A	1.1075 (.3854)	1.2210 (.4091)	1.2302 (.4096)
With respect to the rate of profit Es,rn	.3135 (.2453)	—	.2687 (.3521)

Note: Es,i means savings elasticity with respect to argument i. Arguments are: Πn: net profits(=rn k); rn: net rate of profit; k: capital stock per worker; A: wealth or assets (A=k+wn/rn); wn: net wage including subsidies(wn=wg(1-tw)+su); yd: disposable income (yd=wn+ Πn=wn+k rn).

(*)Values estimated using formulas intable 7. Standard deviations in parenthesis.

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TABLE 7

SUMMARY OF RELATIONS BETWEEN ELASTICITIES OF CONSUMPTION AND OF SAVINGS VIS A VIS DIFFERENT ARGUMENTS AND BETWEEN MARGINAL PROPENSITIES TO CONSUME AND TO SAVE

Argument i	Marginal propensities $MPSi=ms,i$	Elasticities Es,i
Profits Π	$MPS\ \Pi=1-MPC\Pi$	$Es,\Pi n=\dfrac{\Pi n/c-Ec,\Pi n}{yd/c-1}$
Wages (incl subsidies) $wn+su$	$MPSwn=1-MPCwn$	$Es,wn=\dfrac{wn/c-Ec,wn}{yd/c-1}$
Disposable income yd	$MPSyd=1-MPCyd$	$Es,yd=\dfrac{yd/c-Ec,yd}{yd/c-1}$
Rate of profit rn	$\partial S/\partial rn=-\partial c/\partial rn$	$Es,rn=\dfrac{-Ec,rn}{yd/c-1}$
Capital k	$\partial S/\partial k=rn-\partial c/\partial k$	$Es,rn=\dfrac{\Pi n/c-Ec,k}{yd/c-1}=Es,\Pi n$
Wealth A	$\partial S/\partial A=rn-\partial c/\partial A$	$Es,A=\dfrac{rn\ A/c-Ec,A}{yd/c-1}=Es,yd$

Note: Es,i / Ec,i means savings/consumption elasticity with respect to argument i. Arguments are: Πn: net profits(=rn k); rn: net rate of profit; k: capital stock per worker; A: wealth or assets (A=k+wn/rn); wn: net wage including subsidies(wn=wg(1-tw)+su); yd: disposable income (yd=wn+Πn=wn+k rn).

different from zero in the present estimation.

(2) In the neokeynesian model, the marginal propensity to consume out of wages is significantly greater than that out of profits, hence Kaldor's conjecture is not rejected; furthermore the marginal propensity to consume out of wages is not significantly different from one, which means that a classical consumption function is not rejected either by the presented estimations.

(3) Finally, both the neokeynesian and the neoclassical consumption functions are not significantly different from the nested consumption function. So they both seem "not-unacceptable" representations of the real world for the present estimation.

C) Other Closure Equations in the Basic Models

We shall examine, first, the neoclassical natural rate of unemployment equations; second, the neokeynesian "animal spirits" equation and finally, the neomarxian wage determination equation.

1) The Neoclassical Natural Rate of Unemployment.

The neoclassical model is closed by imposing the condition that the rate of growth of employment, g, be equal to the rate of growth of Harrod neutral technical progress, R", plus the rate of growth of the labour force: (1)

$$g = R'' + n' = \exp(R') - 1 + n' \qquad (6)$$

Written this way, there is very little, if nothing, to examine about that relation. It will be adequately tested in another chapter using methods of choice among different models.

But now it is interesting to estimate a more general equation, nesting equation (6), and test whether both are significantly different. This equation is:

$$g = b11 + b12\ n' \qquad (7)$$

Notice that it is linear; since the parameter n′ is exogenous, ordinary least squares can be used; in any case two stage least squares (2SLS) would produce the same results if n′ were among the instruments. The estimated model is (standard deviations in parenthesis,below the coefficients):

$$g = \underset{(0.0036)}{0.0506} - \underset{(0.0262)}{0.277}\ n'; \quad R^2 = 0.0259 \qquad (7a)$$

In order to test whether the neoclassical hypothesis is adequate we should test b11 = R" and b12 = 1, where R" would be the real rate of Harrod neutral technical

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(1) Recall that R" is the rate of growth in discrete time and R′ is that rate in continuous time; hence (1 + R") = exp(R′).

progress. Unfortunately we do not know what that real rate would be. All we can see is that .0506 is not very far from the R" estimated in the neoclassical model as a whole by the 3SLS and that it is significantly different from zero and has the right sign. But we can indeed test whether b12 is significantly different from 1. The "t" value for this hypothesis is -4.87; thus b12 not only has the wrong sign but it is also significantly different from 1.0, its theoretical value. Hence the nested equation is significantly different from the neoclassical natural rate of growth. We shall see in another chapter that this partial result is confirmed when the global neoclassical model is tested against the other alternatives and against a nested model which combines them all.

2) The Neokeynesian "animal spirits" function.

As we saw in Chapter III, the animal spirits function relates to the rate of growth:

$$g = i(rn) = i[rg\ (1 - tr)] \qquad (8)$$

The estimated equation had a linear form:

$$g = b4 + b5\ rn \qquad (9)$$

The estimated parameters were:

$$g = 0.02552 + 0.2146\ rn \qquad (9a)$$

$$(0.00254)\ (0.0205)$$

One parameter of interest can be derived from this equation: the marginal propensity to invest out of profits, mi,p,

$$mi,p = d\, g \,/\, d\, rn \quad ;$$

recall that in the short term, with capital K given,

$d\, g \,/\, d\, rn = d\,(g\, K) \,/\, d\,(rn\, K)$.

The marginal propensity to invest out of profits is simply the coefficient b5, with a value of 0.21, significantly different from 0. The neokeynesian conjecture that investment is determined by the investment decisions of the entrepreneurs, given the state of their expectations, especially in what respects the rate of profit, are then patently not contradicted by our present model and data.

It can also be observed that this propensity is smaller than the marginal propensity to save out of profits, ms,p, which, by Table 5, is 0.57.

3) The Neomarxian Wage Equation.

The neomarxian wage equation, postulating essentially that the wage rate is determined outside the neoclassical labour market, and possibly influenced by the extent of workers' struggle through strikes and other actions, was given the following form:

$$wg\,(1 - tw) = (b20 + b22\, idls \,/\, 100)\, \exp\,(R'\, t) \qquad (10)$$

where: idls is an index of days lost to strike; R' is the parameter for the rate of HANTP; b20 can be considered to be the socially determined level of the subsistence wage when labour is not on strike (idls = 0).

Notice that the basic wage rate grows with the extent of technical progress. The results of estimating that equation were:

$$b20 = 0.03692 \quad b22 = -0.0005494 \quad R' = 4.203\% \quad (10a)$$
$$(0.001641) \quad (0.0000117) \quad (0.5938\%)$$

The socially determined level of the wage rate, b20, is then in the average, 3692 international dollars of 1975 per worker. From b22 we can also obtain the elasticity of the wage rate with respect to the strike rate; at mid-sample values this elasticity is: -0.0399.

All coefficients are significantly different from zero; b22 (and its corresponding elasticity) has a negative sign: There is an inverse relationship between wages and days lost in strikes. However, the causality cannot be determined by the model; it is not possible to say, without further investigation, whether more strikes lower the wage rate or whether countries with lower wages have greater labour unrest.

4) The Government Sector.

An identity representing the government sector in the models would have the following form:

$$gc = -gds + wg\ tw + q\ tx + k\ rg\ tr + ad5 - gas - su - sur\ q \quad (11)$$

where the definitions of the variables can be found in table 1 to chapter VII. gds represents the long term surplus or deficit of the government. For the long term incidence model we have conjectured that this deficit is nil.

$$gds = u \quad (12)$$

where the expectation of the discrepancy (error), u, is E(u) = 0.

The government equation in the models consists of replacing condition (12) in identity (11) above. As such, the equation has no estimable parameters.

CHAPTER IX

TESTS OF THE MODELS AND QUANTIFICATION OF INCIDENCE

A) Introduction

In Chapter VI we saw that in order to completely define the incidence of taxes in a given model, it was necessary to obtain estimates of certain parameters of that model which could not be directly observed, such as the elasticities of substitution and the consumption (or saving) parameters. In Chapter VII we estimated the models by econometric methods, and in Chapter VIII we discussed the values that result, for the non-directly observable parameters, from these econometric estimations. We can now calculate the value of the incidence coefficient in each of the alternative models studied and have general estimates of the incidence of a general profits tax, one for each model.

In order to determine in a more definite way what is the best estimate of incidence, we also have to answer one more question: which of the alternative models proposed, best "explains reality". In Section B, we shall attempt an answer to this question, by use of very new techniques to econometrically test the alternative models. Then, in Section C, we shall use the results of Chapter VII and VIII and those of Section B in this chapter, to obtain the most likely magnitude of the incidence of a general tax on profits.

B) Test of the Models

We shall explain, first, the procedures used to test the models. Then, we shall present and discuss the results.

1) Description of the Tests.

In their basic forms, the models examined, (neoclassical, neomarxian, neokeynesian and hybrid models) cannot be obtained from one another by restricting the coefficients in their equations. If this were the case, to choose among them it would suffice to test those restrictions going from the more general model, which would encompass the others, to the more specialized. The models would then be naturally nested and there would not be any difficulty in testing them by well known statistical tests of hypotheses. But the three models under examination correspond to separate families of hypotheses, for which the traditional testing procedures are not applicable directly. It is then necessary to look for other ways.

So far, these other ways of testing the separate families of hypotheses have either consisted of building a model more general than each of the basic models and which contains them, or can be related to this procedure, which we have designated as nesting. Cox (1961, 1962), Pesaran (1974), Quandt (1973), Pesaran and Deaton (1980) and Davidson and MacKinnon (1981) have all proposed tests of models which are all, in one way or another, related to this procedure. The procedure can also be directly applied to the econometric models we have estimated.

We shall use two types of tests of this nature. One of them will be inspired by, but not similar to, the test proposed by Davidson and MacKinnon (1981). The second type will simply consist of applying classical tests of hypothesis to the general or "nested" models which we proposed and estimated in Chapter VII; the "nested growth" and the "nested wage" model.

i) <u>Test Related to Davidson and MacKinnon's Procedure.</u>

The easiest way to introduce this test is to start, as Davidson and MacKinnon (1981) do, with a single equation non linear regression model. The test is as follows.

Let H0 be the null hypothesis to be tested represented by the non linear model:

$$H0: \quad y_i = f(x_i, B) + u_{o,i} \tag{1}$$

where yi is the ith observation on the dependent variable, f( ) stands for a non linear function, xi is a vector of observations on exogenos variables, B is a k vector of parameters to be estimated, and the error term is uo,i assumed to be Normally Identically Distributed with mean zero and variance v_o $[NID(0,v_o)]$.

Model H0 is to be compared with an alternative hypothesis represented by model H1 :

$$H1: \quad y_i = g(z_i, D) + u_{1,i} \tag{2}$$

where yi is as before, g( ) stands for a non linear function, zi is the vector of observations on the exogenous variables of the alternative model, D is an l vector of paramters to be estimated, and u1,i is assumed to NID$(0,v_1)$ if H1 is true. It is assumed that the two hypotheses belong to separate families, i.e. H0 is not nested in H1 and vice-versa.

Now, form the following general model:

$$y_i = (1 - \phi 5)\, f(x_i, B) + \phi 5\, \hat{g}_i + u_i \tag{3}$$

where gi = $\hat{g}(z_i, \hat{D})$ and $\hat{D}$ is the estimate of D (Maximum Likelihood estimate in Davidson and MacKinnon's article). On the null hypothesis that H0 is true, the combination parameter ø5, should have a value of zero. On the other hand...

"g is simply a function of the exogenous variables zi and the parameter estimates D. The former are independent of ui by assumption. Asymptotically, the latter are also independent of ui, because the influence of any particular error term on the estimates tends to zero as the sample size tends to infinity. Thus, asymptotically, gi will be independent of ui, so that one may validly test whether ø5 = 0 (in equation 3) by using a conventional asymptotic test, or equivalently a likelihood ratio test." (Davidson and MacKinnon, 1981, p.782).

Davidson and MacKinnon go on to propose other tests based on the same principle, and show how it is possible to apply these to the test of one hypothesis H0 against several alternatives Hi (i = 1, 2...m) by using weights (1 - ø5,j) and ø5,j for H0 and H1 respectively.

To apply this procedure to the models analysed here it is necessary to show that: (1) the procedure can be generalized to systems of non linear equations; and (2) it is valid for 3 stage least squares (3SLS) estimation.

To generalize the procedure we can represent any of the six basic or hybrid models which we are studying, say the Jth model by a system of equations of the sort:

$$yl,i = flJ(Yl,i; xi; BJ) + ulJ,i \qquad l = 1,m; \; i = 1,n \tag{4}$$

where we have:

i: index of observations;

l: index for the endogenous variables; total of variables = m

J: index for the models; J = C (neoclassical), K (neokeynesian), M(neomarxian), CK (hybrid neoclassical with Kaldor's consumption function), etc.

flJ( ): functional form of the lth structural equation of the Jth model.

ŷl,i: ith observation on the vector of endogenous variables other than yl,i ;

xi: ith observation on the vector of exogenous variables;

ul,i: ith observation on the random error of the lth structural equation

BJ: vector of parameters of Jth model;

The reduced form of that model could be written (abstracting from error terms):

$$yl,i = glJ(xi,BJ) \quad (5)$$

with:

glJ( ): functional form of the lth reduced form equation of the Jth model.

Then we can build a general model with composite structural equations such as:

$$yl,i = (1 - \emptyset l,H)\ flJ(\hat{y}l,i;xi;BJ) + \emptyset l,H\ \hat{g}lH + ui \quad (6)$$

$$; l = 1, m$$

where:

øl,H is the set of combination (or nesting) parameters

$\hat{g}lH = glH(xi, \hat{B}H)$; $l = 1, m$ (7)

J,H = C,K,M,CK,KC,MC (all basic and mixed models).

$\hat{B}H$ is the vector of 3SLS estimated parameters of model H.

This composite structural model is estimated by three stage least squares. Model J is here the null hypothesis, H0, and model H is the alternative hypothesis H1. Equation (6) combines them through the nesting parameters $\alpha l,H$ (l = 1, m). Notice that the alternative hypothesis model has been estimated and solved, that is, the values $\hat{y}l,i = \hat{g}l,H$ conditional on H1, have been obtained.

Since this model is estimated by 3SLS the parameters are consistent; therefore on the null hypthesis that H0 is true the set of parameters $\alpha l,H$ should have a joint value not significantly different from zero. Since the 3SLS estimates are asymptotically normal, chi square tests can be applied to that joint hypothesis.

Even if equation (6) and the parameters of model H shown in equation (7) were estimated by Full Information Maximum Likelihood (FIML), it would be possible to apply tests to the coefficients $\alpha l,H$. In fact, by a reasoning similar to that of the single equation case, the terms $\hat{g}lH$ in equation (6) will be asymptotically independent of ui, given that they are functions only of the vector of exogenous variables, xi, and the vector of parameter estimates, $\hat{B}H$. Hence, asymptotic likelihood ratio tests on the set of $\phi 1H$ being zero would be valid.

For the 3SLS estimation procedures, which we employed, the tests obtained are less efficient than those using FIML, but the model estimations are more robust, and more adequate, given the pooled nature of the sample; all this was amply discussed in Chapter VII.

To fix ideas, the growth equation corresponding to a composite model, such as that given by equation (6), for testing the neoclassical versus the neokeynesian model would be formed as:

$$\ln g_i = (1 - \phi_{k,5}) \ln(R''_i + l_i) + \phi_{k,5} \ln(\hat{g}_{k,i}) + u_{5,i} \qquad (8)$$

where i is the observation number, i = 1, n ; the $\hat{g}_{k,i}$ correspond to the solved values of the endogenous variable, g in the neokeynesian model, for sample values of the exogenous variables; that is it depends, as in equation (7), of the 3SLS estimated coefficients, $\hat{B}k$, of the neokeynesian model, and on the values of the exogenous variables in the sample. All other terms are as defined previously.

In the same way, the consumption equation would look like:

$$\ln c_i = (1 - \phi_{k,4}) [b2 + b3y \ln(yd,i + b3 \ln rn,i] + \phi_{k,4} \ln (\hat{c}_{k,i}) + u_{4,i} \qquad (9)$$

where the $\hat{c}_{k,i}$ are again solutions for consumption c in the neokeynesian model.

The three other behavioural equations, the government budget constraint, the marginal productivity and the production function, could be written in a similar way. Let us denote the øI,j coefficients in those equations øk,3 , øk,2 and øk,1 respectively. Then the model constituted of equations (8), (9) and the other three can be estimated by 3SLS. To test the null hypothesis that the neoclassical model is true, we have to test, jointly, that the vector øk given by:

$$øk' = (øk,1 \ , \ øk,2 \ , \ øk,3 \ , \ øk,4 \ , \ øk,5)$$

is null. That is:

$$H0: øk' = 0 \qquad \text{versus} \quad H1: øk' \neq 0$$

This test corresponds to that presented in the first line of Table 1a, for one to one tests.

Using basically the same procedure, we can test the neoclassical model versus the neomarxian model, by building a system where the neoclassical equations are combined with the neomarxian equations, in a way similar to that shown by equations (6), (8) and (9); if, in this case, we call the vector of combination, or nesting, parameters øm , given by:

$$øm = (øm,1 \quad øm,2 \quad øm,3 \quad øm,4 \quad øm,5)$$

one for each behavioural equation, the null hypothesis is now:

$$H0: \ øm' = 0 \qquad \text{versus} \qquad H1: \ øm' \neq 0$$

Repeating this procedure, we can test the other basic and hybrid models against one single alternative model.

The results of all the tests carried out in this way are shown in Table 3; as seen, they all test the nullity of a vector ϕi (i=C, K, M, CK, KC, MC) which has five components corresponding to the five nesting coefficients $\phi i,j$ one per behavioural equation in each model.

The models can also be tested, using the same procedure, against two alternative models. We shall denominate these tests: "composite".

The composite tests compare each model against two alternative models. For example, for the test of the neoclassical model versus the neokeynesian and the neomarxian taken together, the growth equation would now be:

$$\ln gi = (1 - \phi k,5 - \phi m,5) \ln[R"i + li] + \phi k,5 \ln \hat{g}k,i + \phi m,5 \ln \hat{g}m,i + u5,i \qquad (10)$$

with $\hat{g}k,i$ as before and $\hat{g}m,i$ now being the solutions of the growth rate g for the neomarxian model. Since the same would be done to the four other behavioural equations the test would now be:

$$H0: (\phi k' \ \ \phi m') = (\phi k,1 \ \ \phi k,2 \ \ \phi k,3 \ \ \phi k,4 \ \ \phi k,5 \ \ \phi m,1 \ \ \phi m,2 \ \ \phi m,3 \ \ \phi m,4 \ \ \phi m,5) = 0$$

versus:

$$H1: (\phi k' \ \ \phi m') \neq 0$$

Table 1b summarizes the composite tests carried out. These composite alternate hypotheses produce, of course,

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TABLE 1a

DAVIDSON AND MCKINNON TYPE TESTS PERFORMED
ONE TO ONE TESTS(*)

Alternative hypothesis

Null hypothesis	Neoclassical	Neokeynesian	Neomarxian
Neoclassical (C)	—	(δk1,δk2,δk3, δk4,δk5)=0	(δm1,δm2,δm3, δm4,δm5)=0
Neokeynesian (K)	(δc1,δc2,δc3 δc6,δc7)=0	—	(δm1,δm2,δm3, δm6,δm7)=0
Neomarxian (M)	(δc1,δc2,δc3 δc7,δc8)=0	(δk1,δk2,δk3, δk7,δk8)=0	—
Hybrid neo-classical (CK)	—	(δk1,δk2,δk3, δk7,δk5)=0	(δm1,δm2,δm3, δm7,δm5)=0
Hybrid neo-keynesian (KC)	(δc1,δc2,δc3 δc6,c4)=0	—	(δm1,δm2,δm3, δm6,δm4)=0
Hybrid neo-marxian (MC)	(δc1,δc2,δc3 δc4,δc8)=0	(δk1,δk2,δk3, δk4,δk8)=0	—

DAVIDSON AND MCKINNON TYPE TESTS PERFORMED
ONE TO ONE TESTS(*) (CONT.)

Alternative hypothesis

Null hypothesis	Hybrid Neoclassical	Hybrid Neokeynesian	Hybrid neomarxian
Neoclassical (C)	—	(δkc1,δkc2,δkc3, δkc4,δkc5)=0	(δmc1,δmc2,δmc3, δmc4,δmc5)=0
Neokeynesian (K)	(δck1,δck2,δck3 δck6,δck7)=0	—	(δmc1,δmc2,δmc3, δmc6,δmc7)=0
Neomarxian (M)	(δck1,δck2,δck3 δck7,δck8)=0	(δkc1,δkc2,δkc3, δkc7,δkc8)=0	—
Hybrid neo-classical (CK)	—	(δkc1,δkc2,δkc3, δkc7,δkc5)=0	(δmc1,δmc2,δmc3, δmc7,δmc5)=0
Hybrid neo-keynesian (KC)	(δck1,δck2,δck3 δck6,δck4)=0	—	(δmc1,δmc2,δmc3, δmc6,δmc4)=0
Hybrid neo-marxian (MC)	(δck1,δck2,δck3 δck4,δck8)=0	(δkc1,δkc2,δkc3, δkc4,δkc8)=0	—

Note: The first index "i" of the coefficient ij denominates the model acting as alternative hypothesis; it takes the values: i=C (neoclassical), K(neokeynesian), M(neomarxian), CK(hybrid neoclassical), KC(hybrid neokeynesian, MC(hybrid neomarxian). The index "j" in ij denominates the equation in the null hypothesis model to which it is applied; it takes the values: j=1(production function), 2(marginal productivity), 3(gov. budget constraint), 4(neoclassical consumption function), 5(neoclassical growth equation, 6(neokeynesian growth equation), 7(neokeynesian/neomarxian consumption function), 8(neomarxian wage equation).

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TABLE 1b

DAVIDSON AND MCKINNON TYRE TESTS PERFORMED
COMPOSITE TESTS(*)

Null hypothesis	test
Neoclassical (C)	(dk1,dm1,dk2,dm2,dk3,dm3,dk4,dm4,dk5,dm5)=0
Neokeynesian (K)	(dc1,dm1,dc2,dm2,dc3,dm3,dc6,dm6,dc7,dm7)=0
Neomarxian (M)	(dk1,dc1,dk2,dc2,dk3,dc3,dk7,dc7,dk8,dc8)=0
Hybrid neo-classical (CK)	(dk1,dm1,dk2,dm2,dk3,dm3,dk7,dm7,dk5,dm5)=0
Hybrid neo-keynesian (KC)	(dc1,dm1,dc2,dm2,dc3,dm3,dc6,dm6,dc4,dm4)=0
Hybrid neo-marxian (MC)	(dk1,dc1,dk2,dc2,dk3,dc3,dk4,dc4,dk8,dc8)=0

Note: The first index "i" of the coefficient ij denominates the model acting as alternative hypothesis; it takes the values: i=C (neoclassical), K(neokeynesian), M(neomarxian), CK(hybrid neoclassical), KC(hybrid neokeynesian, MC(hybrid neomarxian). The index "j" in ij denominates the equation in the null hypothesis model to which it is applied; it takes the values: j=1(production function), 2(marginal productivity), 3(gov. budget constraint), 4(neoclassical consumption function), 5(neoclassical growth equation, 6(neokeynesian growth equation), 7(neokeynesian/ neomarxian consumption function), 8(neomarxian wage equation).

much more stringent tests of the models than the simple hypothesis. The results of these tests are presented in the first column, "composite", of Table 3. All these tests were carried out taking as alternative hypotheses only the basic models. That is: both the basic and hybrid neoclassical models were tested against the basic neokeynesian plus the basic neomarxian; both the basic and hybrid neokeynesian models were tested against the basic neoclassical plus the basic neomarxian; and both the basic and hybrid neomarxian models were tested against the basic neoclassical plus the basic neokeynesian models.

Tests of one model versus three alternative ones or versus a combination of two hybrid models were not done because they would not have added any new information (1). This can be seen with an example:

A test of the neomarxian model, as null hypothesis, versus the neoclassical and neomarxian jointly, compares the neomarxian wage equation with the neokeynesian and neoclassical growth equations, and with the neokeynesian and neoclassical consumption equations. Hence all the equations which can be different among any two models are included in the comparison. Therefore, they exhaust all additional independent information which can be attributed to all possible alternatives.

---

(1) Logically, the null and the alternative hypothesis must be disjointed partitions.

ii) <u>Tests by Direct Nesting</u>. As mentioned, the second procedure applied was to obtain the simplest possible type of general models, by combining the systems of equations of the three basic models in a way that was extensively described in Chapter VII, Section A. This resulted in two general or "nested" models, which we have called the nested-growth and the nested wage-models.

Recall from Section A.4 of Chapter VII that the nested-growth model was composed of the five equations common to all models, given by relations (1) to (5) in that chapter, plus the following two "nested" relations. The nested consumption function:

$$\ln C = b2 + b3w \ln wn + b3r \ln(k\ rn) + b3 \ln rn + \ln uc \qquad (11)$$

with uc : the random error term; and the nested-growth equation:

$$\ln g = \ln \{ b11' + b12' n' + b5' rn + \exp(R' t)\ (b20' + b22' idls) / [(1 - tw)\ k] + b13' [rg + (tx - sur)\ q / k - (c + gc + bt) / k] \} + \ln ug,1 \qquad (12)$$

with ug,1 : the random error term. This equation is similar to equation (7n.1) of Chapter VII but written explicitly with a logarithmic random error, for reasons explained in Section B of that Chapter.

The nested wage model has the same consumption function as the nested growth model, given by relation (11), but it has a nested-wage equation instead of equation (12), as follows:

$$\begin{aligned} \ln wg = \ln \{ & \exp(R'\ t)\ (b20'' + B22''\ idls)\ /\ (1 - tw) + \\ & b11''\ k + b12''\ n'\ k + b5''\ rn\ k + \\ & b13''\ [c + gc + bt - rg\ k - q\ (tx - sur)] \\ & \} + \ln uw,1 \qquad (13) \end{aligned}$$

with uw,1 : the random error term. Again this equation is similar to equation (7n.2) of Chapter VII, but written with logarithmic random errors.

The nested-growth model is used to test the (basic and hybrid) neokeynesian and the neoclassical equations; since, as was explained in Chapter VII, it reduces to those models by suitable restrictions of the coefficients. These restrictions are shown in Table 2.

The neomarxian model cannot be obtained in its original structural form by restricting the nested-growth model. In fact, if we take:

$$b11' = b12' = b5' = 0 \quad \text{and} \quad b13 = 1.0$$

we obtain an equation such as:

$$\begin{aligned} \ln g = \ln \{ & \exp(R'\ t)\ (b20' + B22'\ idls)\ /\ [(1 - tw)\ k] \\ & + [rg + (tx - sur)\ q\ /\ k - (c + gc + bt)\ /\ k] \\ & \} + \ln ug,1 \qquad (14) \end{aligned}$$

Abstracting from the error term, this equation is similar to equation (16) in Chapter VII and therefore it is algebraically equivalent to the wage equation in the basic

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TABLE 2

TESTS[1] PERFORMED ON THE NESTED MODELS

Null hypothesis	Test
	1)Tests performed on the nested growth model
Neoclassical (C)	(b11',b5',b20',b22',b13',b12')=(,0,0,0,0,1) and b3w'=[(wn+su)/(krn)]b3r'=2.18 b3r'
Neokeynesian (K)	(b12,b20,b22,b13)=(0,0,0,0) and b3=0
Hybrid neo-classical (CK)	(b11',b5,b20,b22,b13,b12)=(,0,0,0,0,1) and b3'=0
Hybrid neo-keynesian (KC)	(b12',b20',b22',b13')=(0,0,0,0) and b3w'=2.18 b3r'
Neoclassical with general cons. func.	(b11',b5',b20',b22',b13',b12')=(,0,0,0,0,1)
Neokenesian with general cons. func.	(b12',b20',b22',b13')=(0,0,0,0)
	2)tests performed on the nested wage model
Neomarxian (M)	(b11',b12',b5',b13')=(0,0,0,0) and b3'=0
Hybrid neo-marxian (MC)	(b11',b12',b5',b13')=(0,0,0,0) and b3w'=2.18 b3r'
Neomarxian with genural cons. func.	(b11',b12',b5',b13')=(0,0,0,0)

neomarxian model. But because of the logarithmic error term, equation (14) cannot be transformed back to the original wage equation with a multiplicative, or equivalently additive-logarithmic, error. In fact, equation (14) can be rewritten:

$$gk - u_{g,1}\ [k\ rg + (tx - sur)\ q - k\ (c + gc + bt)] = [\exp(R'\ t)\ (b20' + b22')\ idls\ /\ (1 - tw)\ ]\ u_{g,1} \quad (15)$$

This can be written, by replacing in the left hand side $u_{g,1}$ by $(v_{g,1} + 1)$:

$$wg = [\exp(R'\ t)\ (b20' + b22'\ idls)\ /\ (1 - tw)\ ]\ (1 + v_{g,1}) + [k\ rg + (tx - sur)\ q - k\ (c + gc + bt)]\ v_{g,1} \quad (16)$$

The mathematical expectation of this equation is similar to the mathematical expectation of the basic neomarxian wage-equation:

$$wg = [\exp(R'\ t)\ (b20 + b22\ idls)\ /\ (1 - tw)\ ]\ u_{w,1} \quad (17)$$

but the structures of the errors $v_{g,1}$ and $u_{w,1}$ differ. For this reason, it is better to test the neomarxian model by restricting the nested-wage model. This latter model fully reduces to the original neomarxian structural equations and keeps the same error structure. In fact, if the following restrictions are applied to the nested wage equation (13):

$$b11'' = b12'' = b13'' = 0$$

Equation (13) reduces fully, including its random error, to the basic neomarxian wage equation (17). So, the test of the basic and of the hybrid neomarxian model will be done using the nested-wage equation. They appear in Table 2.

iii) <u>The Tests and the Asymptotic Normality of the Estimates</u>. We have mentioned, in the previous paragraphs, that the test procedures described will make use of the asymptotic normality of the 3SLS estimates. Let us describe more precisely how this will be done.

Take a null hypothesis of the general form (1):

$$B(2) = \psi[B(1)] \qquad (18)$$

where B(2) is a vector of parameters of length n and B(1) is a vector of the remaining K-n parameters in the model. The statistic, Z, corresponding to (18) in terms of the vector of estimated parameters b = [b(1) , b(2)], is:

$$Z = b(2) - \psi[b(1)] \qquad (19)$$

Z will be close to zero if the null hypothesis (18) is true. Malinvand (1981) proves that Z is asymptotically normal. It has a variance covariance matrix:

$$V(Z) = V(2,2) - V(2,1)\,(\partial\psi/\partial b(1))' - \partial\psi/\partial b(1)\,V(1,2) + \partial\psi/\partial b(1)\,V(1,1)\,(\partial\psi/\partial b(1))'$$

where V(i,j) are the conforming blocks of the asymptotic variance-covariance matrix of the estimator b. Then the quadratic form:

$$F = Z'\,[V(Z)]^{-1}\,Z \qquad (21)$$

---

(1) See Bernd, Hall and Hausmann (1974), whose exposition we follow closely. See also Malinvaud (1981 ch.9).

is distributed asymptotically as chi square under the null hypothesis with n degrees of freedom. As can be seen in Tables 1 and 2, in all tests that will be carried out the functions ø are linear; and in most cases they are simply constant. In this latter case the statistic (21) reduces to:

$$F = Z' [V(2,2)] Z \qquad (22)$$

which is straightforward to compute.

2) Test Results.

The results of the tests inspired in Davidson and MacKinnon's procedure are presented in Table 3. Those obtained using the nested model appear in Table 5.

The first noticeable feature in Table 3 is that the null basic hypothesis C and the null hybrid hypothesis CK, have almost the same chi square values for tests against the same alternatives. The same is true for the the other pairs of models: basic and hybrid neokeynesian (K and KC) and basic and hybrid neomarxian (M and MC). A more detailed look at the coefficients, reveals that the contribution to the chi-square values of the øi,j parameters corresponding to the consumption functions is very small. For example, the chi-square value contributed by the ø coefficient in the consumption function of the

TABLE 3

DAVIDSON AND MCKINNON TYPE TESTS PERFORMED
SUMMARY OF RESULTS

	Alternative hypothesis			
Null hypothesis	Composite	Neoclassical	Neokeynesian	Neomarxian
Neomarxian				
Neoclassical (C)	107.40(10)	—	19.38(5)	29.52(5)
Neokeynesian (K)	53.69(10)	49.98(5)	—	12.53(5)**
Neomarxian (M)	106.18(10)	101.67(5)	33.10(5)	—
Hybrid neo-classical (CK)	159.57(10)	—	19.37(5)*	31.42(5)
Hybrid neo-keynesian (KC)	59.95(10)	49.20(5)	—	12.60(5)**
Hybrid néo-marxian (MC)	106.73(10)	100.69(5)	33.43(5)	—

(cont.)

	Alternative hypothesis		
Null hypothesis	Hybrid Neoclassical	Hybrid Neokeynesian	Hybrid Neomarxian
Neoclassical (C)	—	46.62(5)	43.27(5)
Neokeynesian (K)	55.04(5)	—	15.00(5)**
Neomarxian (M)	125.31(5)	51.27(5)	—
Hybrid neo-classical (CK)	—	42.62(5)	42.26(5)
Hybrid neo-keynesian (KC)	54.06(5)	—	15.41(5)**
Hybrid néo-marxian (MC)	124.83(5)	50.88(5)	—

Notes:
In parenthesis degrees of freedom
(*) Null hypothesis cannot be rejected at .01% level of significance
(**) Null hypothesis cannot be rejected at 1% (and .5%) level of significance

basic neoclassical model, C, tested against the KC model is .27 (the t value of this coefficient was .076 and covariances with the ø's were zero), against a total chi-square, for all ø's, of 42.62 (see Table 3). For the other model: the hybrid neoclassical, CK, tested against the KC model, the chi-squared contributed by the coefficient in its, now Kaldorian, consumption function is .066 against a total chi-square for all ø's, of 42.62 in Table 3. Therefore, the consumption functions in these models do not add a significant difference. Since the only difference between the C and the CK models is that the first uses the neoclassical consumption function and the second uses the neokeynesian one, tests of these models against the same alternative are not very different, because the consumption functions are not statistically very different.

The second noticeable feature of Table 3, is the bad performance of the Basic (M) and Mixed (MC) neomarxian models in all tests; they are always rejected in the one to one composite tests.

The neoclassical models perform better than the neomarxian ones; the basic (C) and hybrid (CK) neoclassical models are accepted in tests against the neokeynesian model, but rejected in all other instances.

The neokeynesian models are the least rejected. They also have the lowest chi-square values, i.e., the "higher" levels of acceptance. Both the basic (K) and the hybrid

(KC) neokeynesian models are accepted in tests against the basic and hybrid neomarxian models at the 1% level of significance. Now, these are indeed different tests, even if they are performed against models whose only difference is the consumption behavior. The basic neoclassical model, for example, is accepted, at .01% level of significance, when tested against the basic Keynesian (K) model, but clearly rejected at that level when tested against the hybrid Keynesian (KC) model.(1) The neokeynesian models are then accepted more times than the others: they are also the only ones to be accepted at 1% level of significance.

The composite tests, which compare each model against the two other basic models, which have a different growth (or wage) equation, result in the rejection of all models. In conclusion no model is as good as a combination of two basic models complementing it. But again the lowest chi-square values are obtained for the neokeynesian tests; although they are not low enough to be significant, even at a very small level of significance.

For incidence, it is convenient to pick up only one model, as best representing the "real world", as it is reflected in the sample. For this it is sufficient to choose a cutoff point high enough to eliminate other cases.

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(1) The situation observed here -- where it is not possible to clasify the models neatly as winners or losers -- is not unusual in this type of extended testing.

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TABLE 4

PERFORMANCE OF MODELS IN DAVIDSON AND MCKINNON TESTS

Model	As null hypothesis the model was accepted the following times: At .01% level of significance	At 1% level of significance
Neoclassical (C)	Once	None
Neokeynesian (K)	twice	twice
Neomarxian (M)	None	None
Hybrid neo-classical (CK)	Once	None
Hybrid neo-keynesian (KC)	Twice	Twice
Hybrid neo-marxian (MC)	None	None

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TABLE 5
TESTS PERFORMED ON THE NESTED MODELS
SUMMARY OF RESULTS

Null hypothesis	Test results (Chi square values)
1) Tests performed on the nested growth model	
Neoclassical (C)	340.30
Neokeynesian (K)	24.29*
Hybrid neo-classical (CK)	339.78
Hybrid neo-keynesian (KC)	24.79*
Neoclassical with general cons. func.	339.72
Neokenesian with general cons. func.	23.79*
2) tests performed on the nested wage model	
Neomarxian (M)	172.22
Hybrid neo-marxian.(MC)	171.98
Neomarxian with general cons. func.	171.64

(*)Does not differ significantly from nested model at .01 level of significance

Here, the cutoff point can be the 1% level of significance. In this case the best model is the neokeynesian, both in its basic and hybrid versions.

A more problematic endeavour would be to establish a hierarchy of models in their capacity to represent the sample. We can do this by establishing other lower cutoff points. If this is taken as the .01% level of significance, then, as can be observed in Table 4, the following hierarchy of models can be established. First, both the nekeynesian models - basic and hybrid; second, both neoclassical models - basic and hybrid; finally, the neomarxian models, which are rejected even at the .01% level of significance.

The tests carried out using the nested model are presented in Table 5. All of the tests performed using the nested model compare each model or each combination of two models against the nested model, which is in fact a combination of the three basic models, or equivalently of the three hybrid models.

The tests of the three basic and three hybrid models confirm in general what was said previously for the Davidson and MacKinnon tests. First, the results of the basic and the corresponding hybrid models are essentially the same, for the same reasons as before: because the consumption equations do not differ too much in statistical terms. Second, the neokeynesian basic and hybrid models are accepted, whereas all the other models are rejected.

Three more simple models were added in order to check the impact of the growth or wage equations alone. Those are the neoclassical model with a general consumption function (CG) and the neokeynesian model with a general consumption function (MG). They are essentially similar to the corresponding basic and hybrid models, but with slightly lower chi square values on account of the more general consumption function used.

To summarize, from those two sets of results we can conclude that, among the tested models, the neokeynesian -- both basic and hybrid -- seem to best represent the real world, as this is reflected in our multicountry sample.

C) Quantification of the incidence of a general profits tax

From the results in the previous section we can conclude that the most likely incidence of a general profits tax is that resulting from the Neokeynesian model. This incidence can be quantified using results obtained in chapters VI to VIII. We shall do this in what follows. For comparison, we shall also quantify the incidence of the profits tax resulting from the neoclassical and the neomarxian models.

We shall use two measures of incidence which we chose in chapter VI as more interesting. One is the elasticity, $E_{rg,tr}$, of the gross of tax rate of profits with respect to the tax complement, which we defined by

$$E_{rg,tr} = (\tau r / rg)(\partial rg / \partial tr)$$

As we saw this magnitude gives the degree of shifting of the tax.

The second measure is the elasticity $E(Bn,tr)$ of the ratio of net of tax profits to wages ($Bn = k\, rn / wn$) with respect to the tax complement. We defined this magnitude by the expression

$$E(Bn,tr) = (\tau r / Bn)(\partial Bn / \partial tr)$$

as we saw, this magnitude indicates what happens to the after tax distribution of income with a budget change; in other words it indicates whether the tax changes, at the margin, the distribution of income in favour of profits or in favour of wages.

Recall from chapter VI that these two measures are related, under the models laid out in that chapter, by the following expression:

$$E(Bn,tr) = [\, - Bg\, E_{k,l} / A_{r,g} + Bg + 1 \,]\, E_{rg,tr} - 1 \qquad (23)$$

$$Bg = k\, rg / wg = 0.541 \; ; \; A_{r,g} = k\, rg / (q\, \tau x) = 0.423$$

The elasticity, $E_{rg,tr}$, on the other hand, can be calculated by the use of the shifting expressions (51b), (52b) and (43) of chapter VI. Most probable values for the parameters shown in these expressions can be obtained from tables 5 and 6 of chapter VIII. For the neoclassical model those values are the marginal propensity to save out of income, ms,y =.19, the elasticity of savings with respect to the interest rate, Es,r=.31, and the elasticity of substitution in production, Ek,l=.76. These values, replaced in formula (51b) of chapter VI, give a coefficient Erg,tr of 0.916; and from formula (23) above E(Bn,tr) is - 0.48.

The most probable parameter values for the neokeynesian model are, from tables 5 and 6 of chapter VIII: 0.73 for the elasticity of substitution in production, Ek,l; 0.048 for the marginal propensity to save out of wages, ms,w; .57 for the marginal propensity to save out of profits, ms,r; and 0.21 for the marginal propensity to invest out of profits, i′, in the animal spirits function (from section C of chapter VIII). These values replaced in equation 51b of chapter VI result in a shifting coefficient Erg,tr of 2.09;and from equation (23) above, in a coefficient E(Bn,tr) of .26.

The neomarxian model results, directly from equation (43) of chapter VI, in an exact prediction for the shifting coefficient Erg,tr of 0.0 ; and therefore a coefficient E(Bn,tr) of -1.0 (1).

Notice that the parameter values in the neoclassical model correspond to the downward sloping hyperbola in the positive quadrant of the (I, Ek,l) plane presented as case

---

(1) Notice that in this neomarxian case, the values given are not statistical estimates, but necessary consequences of the formulation of the model.

A in graph 4 of Chapter VI. The parameter values of the neokeynesian model correspond to case A in Graph 5 of Chapter VI: the upward sloping hyperbola in the positive quadrant of the (I, Ek,l) plane. Moreover, those incidence values do not contradict the conjectures on Incidence done by proponents of each of those models presented at the end of Section D in Chapter VI. The conjectures on the parameters are a fortiori also accepted.

More information can still be extracted from the estimated empirical model, concerning standard deviations. For this we can obtain shifting expressions which make fuller use of the estimates of the empirical models. To do so we need not repeat the process of derivation of incidence followed in Chapter VI. Expressions (41-B), (42-B) and (43) in that Chapter would again be obtained, but the parameters would be functions of the empirical coefficients. If we derive those functions our task will consist simply of replacing them in those expressions.

Let us start with the neoclassical model. The marginal propensity to save out of income S1, and the partial derivative of the savings function with respect to its second argument, the rate of profit, S2, can be expressed in terms of the consumption function C(yd, rn) as:

$$S1 = 1 - \partial C(yd, rn) / \partial yd$$

and

$$S2 = - \partial C(yd, rn) / \partial rn$$

Hence taking account of the log-log form of the consumption expression in the neoclassical econometric model, presented as equation (6c.0) of Chapter VII, we can obtain:

$$S1 = 1 - b3y\ C / yd$$

and

$$S2 = - b3\ C / rn$$

In a similar way from the translog production function examined in Chapter VIII, we obtain the elasticity of substitution:

$$Ek,l = 1 / (1 + a2 / D)$$

$$D = (Eq,k)^2 - Eq,k$$

$$Eq,k = (k / q)\ d q / d k = b1 + a2 \ln[k \exp(-R'\ t)]$$

Replacing all these values in the expression for neoclassical incidence; (41-B) of Chapter VI, we obtain the empirical shifting expression shown in Table 6 here.

The neokeynesian expression shown in Table 6 is obtained in a similar way, replacing in equation (42-B) sp', sw' and i' as follows:

$$sp' = ms,p = 1 - \partial C / \partial (k\ rn) = 1 - b3r\ C / (k\ rn)$$

$$sw' = ms,w = 1 - \partial C / \partial (wn + su)$$
$$= 1 - b3w\ C / (wn + su)$$

$$i' = d\ i / d\ rn = b5$$

TABLE 6

EMPIRICAL EXPRESSIONS OF SHIFTING FROM THE ECONOMETRIC MODELS

Neoclassical shifting(*)

```
              B4(1-B2 b3y)-B5 b3
Erg,tr=------------------------------------------------------------
        -B1(1-B2 b3y-B3) Ek,1-B6(1-B2 b3y)+B4(1-B2 b3y)-B5 b3
```

where(**):

B1= Ar 7x/[(1- Ar)(d+rg)]=2.07 B4=a1=k/q=1.6845

B2=c/yd=.8264 B5=c/(rn q)=4.4214

B3=g/rn=.4121 B6=k 7w/(q 7r)=1.5155

Ek,1=el. of substitution=1/(1- a2 D) D= (Eq,k)² - Eq,k

Eq,k=b1-a2 ln(k)exp(- R't)

Neokenesian shifting(*):

```
                 B4(1-B7 b3r-b5)
Frg,tr=--------------------------------------------------------
        -B1(1-B7 b3r-B3) Ek,1-B6(1-B8 b3w)+B4(1-B7 b3r-b5)
```

where(**):

B7=c/(rn k)=2.6295 B8=c/(wn+su)=1.2052

All other coefficients as above.

(*)The coefficients b3y, b3, Ek,1(b1,a2,R'), b3r, b3w and b5 are as defined in in the empirical equations of chapter VII.

(**)These coefficients are calculated at sample means using Table 3 of chapter VI.

Notice that the expressions in Table 6 are functions of the estimated parameters of the empirical model b3y, b3, Ek,l (b1, a2, R′), b3r, b3w, b5. It is then possible to obtain approximate variances for the shifting coefficients by linearizing the expressions in that table and using the variance matrices estimated for the parameters. In the case of the function I = f(b1, b2 ...bn) = f(b) for example, where the bi are estimated parameters with variances:

$$\mathrm{var}(b_i) = v_{ii}$$

$$\mathrm{cov}(b_i, b_j) = v_{ij}$$

the approximate variance of f(b) would be:

$$\mathrm{var}(b) = \sum_{i=1}^{n} (\partial f / \partial b_i)^2 v_{ii} + 2 \sum_{i,j;\, i \neq j} (\partial f / \partial b_i)(\partial f / \partial b_j) v_{ij}$$

For the neoclassical model f(b) is represented by the first formula in Table 2 and the vector b is b^c = (b1, a2, R′, b3y, b3).

For the neokeynesian model f is the second formula in that table and the vector b is b^k = (b1, a2, R′, b3w, b3r, b5).

These formulas can be applied to obtain standard deviations for the shifting measure Erg,tr and the incidence measure E n,tr. A summary of the results obtained is presented in table 7. The point estimates of Erg,tr and E(Bn,tr) in the table are, necessarily, equal to those obtained before using the data of chapter VII.

The point estimates of shifting, Erg,tr in table 7, indicate that the neoclassical model predicts partial

TABLE 7

AVERAGE ESTIMATES AND STANDARD DEVIATIONS OF SHIFTING AND INCIDENCE MEASURES IN NEOCLASSICAL AND NEOKEYNESIAN MODELS (1)

	Neoclassical model	Neokeynesian model
Shifting measure Erg,tr		
Average	.92	2.09
Standard deviation	.49	1.10
Incidence measure		
Average	-.48	.26
Standard deviation	.28	.67

(1) For a balanced budget change in a tax on profits versus government consumption. The neomarxian model does not appear because, as mentioned in the text, it predicts a shifting coefficient Erg,tr of 0.0 and an incidence coefficient E(Bn,tr) of -1.0. These two values are not statistical estimates.

shifting of the profits tax (Erg,tr = 0.92). The neokeynesian model predicts overshifting (Erg,tr = 2.09). The neomarxian model predicts no shifting.

A one tailed test of whether the shifting coefficient Erg,tr is greater than zero, yields a student t value of 1.88 for the neoclassical model and 1.90 for the neokeynesian model. Then, both neoclassical and neokeynesian models reject the null hypothesis that the tax is not shifted (Erg,tr=0) at 5% level of significance (critical t value 1.65).

The point estimates of incidence E(Bn,tr) indicate that the neoclassical model predicts that, in the average, tax cum expenditure changes will change the distribution of income in a direction unfavourable to profits per worker, k rn, and favourable to wages, wn (E(Bn,tr) = -0.48). The neokeynesian model predicts that the budget change will change the distribution of income in a direction favourable to profits per worker and unfavourable to wages (E(Bn,tr) = 0.26). As for the neomarxian model, it predicts that the profits tax will be even more unfavourable to profits per worker than the neoclassical prediction; indeed it predicts that the actual incidence of the tax will be equal to its legal, or statutory, incidence.

A one tailed test of whether the neoclassical incidence coefficient Erg,tr is less than zero, yields a t statistic of -1.71; hence the null hypothesis that the tax is not unfavourable to profits (E(Bn,tr = 0) is rejected in the neoclassical model at 5% level of significance. A similar test for the neokeynesian model, that E(Bn,tr) is greater than zero, does not permit to reject the null hypothesis at 5% level of significance. Hence we cannot reject the possibility that in the neokeynesian model the tax change may leave the distribution of income unaffected (E(Bn,tr) = 0).

We can finally answer the question : what is the balanced budget incidence of a profits tax versus government consumption ? Since the neokeynesiian model--it was concluded in the previous section-- is the best representation of the real world, the incidence is as given by that model. That is, the tax is substantially shifted, with a probable value of the shifting coefficient Erg,tr of 2.09, significantly different fron zero. Furthermore, the tax probably changes the distribution of income in favour of profits per worker (E(Bn,tr) =0.26), or at least, it leaves it unchanged. Most probably, then, workers pay for the tax on profits.

CHAPTER X

CONCLUSIONS

The evidence presented in this work supports the hypothesis that, in the long term at least, the economies of industrialized countries are better represented by a neokeynesian model.As a consequence,and given the magnitude of the parameters which characterize this model,we have concluded that an increase in a general tax on profits,used to increase government expenditures,changes the distribution of income in favor of profits and against wages.In sum, the incidence of the profits tax falls on wages.

These results go against the received economic wisdom.As we saw in chapter II, incidence has so far been confined to what could be called neoclassical ways of thinking: models assuming full employment and without independent investment functions. We could find only one exception, in the work of Asimakopoulos and Burbidge. We also saw in Chapter III that, in spite of claims to generality made by certain authors (Krzyzaniak and

Musgrave), this neoclassical way of thinking equally besets the empirical literature.

We have argued that the neoclassical way of thinking provides too narrow a focus of research for the study of incidence. We have thus widened this focus, to encompass not only neoclassical models of growth and distribution, but two other models as well: the neokeynesian and the neomarxian.

The application of neokeynesian and neomarxian growth models to the study of the incidence of taxation has not been carried out heretofore. So, our first task has been to explore what these models imply for incidence, in particular for the incidence of a tax on profits. In the process it was also necessary to recast the neoclassical model in terms comparable to the other two. What emerged from the completion of this exercise, in Chapter VI, was not very definite.

With the exception of the neomarxian model, which predicts that a tax on profits is not shifted, we observed that, in order to obtain specific incidence results, it is necessary to give specific values to all the parameters and variables of the models. Some of these parameters and variables can be easily observed and measured directly or by statistical sampling methods, but others, such as the elasticity of substitution, the various marginal propensities to consume or save, the interest elasticities of savings, and the marginal propensities to invest, can only be measured indirectly, by econometric methods, which depend on the explicit or implicit acceptance of a model as true, in order to avoid misspecifications. Then, their value is, to a certain extent, conditional on the chosen model.

We saw that the values of these indirectly observable parameters are frequently explicitly restricted in the formulation of a model. A case in point are the restrictions to the production function in neoclassical models to obtain a "well behaved" function. In other cases, stability considerations also impose restrictions in those parameters. These two types of restrictions are to a certain point necessary for the consistency of the models. Beyond pure restrictions, the models are frequently accompanied by conjectures or presumptions on the value of their parameters, such as when the elasticity of substitution is taken to be substabtially greater than zero and not far from one, in neoclassical models. These presumptions may be founded in previous results widely accepted among those sharing their belief in a model or theory (sharing a paradigm in Kuhn's sense).

By giving an explicit form to some of these conjectures on characteristic parameter values, we were able to derive in Chapter VI propositions of incidence characteristic of each model. We then concluded that for the balanced budget incidence of a profits tax versus government consumption, the neoclassical model carries a presumption of partial shifting of the profits tax; that is, this model's most accepted conjecture is that the elasticity of the rate of profit with respect to the profit tax complement, Erg,tr, takes values between zero and one. For the neokeynesian model we concluded that there is a presumption of greater shifting than for the neoclassical model, with values of Erg,tr greater than one. For the neomarxian model, as we already mentioned, there is a certain prediction that the profits tax is borne by profits, that is, that Erg,tr is zero.

The parameters we obtained from the econometric estimation of these models (Chapters VII and VIII) confirm that the values normally conjectured in neoclassical and neokeynesian theories are in the right order of magnitude, and therefore the conjectures on shifting mentioned above are basically correct. A more precise quantification of incidence in Chapter IX indicates that, for the sample of developed countries we have used, the average value of Erg,tr in the neoclassical model (for balanced budget incidence of tr versus government consumption) is .92; for the neokeynesian model, it is 2.09; for the neomarxian model it stays at 0.0.

In terms of the effect on the distribution of income which, we measured by the coefficient E(Bn,tr), the neoclassical model predicts that the profits tax affects the distribution of income against profits [E(Bn,tr) = - 0.48]; the neokeynesian model predicts that it affects the distribution of income in favour of profits [E(Bn,tr) =0.26]; and the neomarxian model that it is even more damaging to profits than the neoclassical case, indeed, it predicts that the economic incidence of the tax is equal to its legal incidence.

But all this still leaves open the question that we set out to answer: what is, in the real world, the incidence of a profits tax? In order to answer this question one last step had to be done; to choose a model, among those examined, as best explaining the real world. We endeavoured to do this in Chapter IX, using very new econometric techniques for the test of models which belong to different families of hypotheses. We concluded that the evidence favours, although weakly, the neokeynesian model. This model best explains the real world. We should then expect an increase in a profits tax, accompanied by a balanced budget increase in government consumption, to increase the gross of tax rate of profits and to change the distribution of income in favour of profits and against wages. In this sense, it is possible to say that workers pay for the profits tax.

We should also stress the other byproduct of this research, the test of the models of growth and distribution, and the conclusion, albeit weak, that neokeynesian models best explain the real world. In other words, models which assume independent investment behaviour and which do not assume full employment, or a natural rate of unemployment, best explain reality . In this sense, one hundred years after Keynes birth, and forty seven after the publication of the General Theory, his insights are still valid.

APPENDIX 1 TO CHAPTER III

THREE SHORT TERM MACRO-ECONOMIC MODELS AND THE SHIFTING ASSUMPTIONS

The macro model proposed by Krzyzaniak and Musgrave is very general. It is difficult to infer from it what should, in theory, happen to profits when the profits tax changes, i.e. to infer what is the incidence of the tax.

We shall present here three short run macro-models greatly simplified, which represent three alternative theories of the way the economy works; a neoclassical, a neokeynesian, and a neomarxian model. We will then be able to infer the short run incidence of a tax on profits, and other relevant results.

Most importantly we shall be able to observe how the econometric specification of Krzyzaniak and Musgrave's model depends on which model has been taken as point of reference, i.e. as the "true" model. When the reference model encompasses the neoclassical and the neokeynesian short run models--as Krzyzaniak's and Musgrave's model does--we shall be able to see that their result, that the tax is shifted in the short run, doesn't reject at all that general model; it rather indicates that the empirical data are consistent with the Keynesian elements of that general model. We shall also examine what happens when the marginal productivity equation, common to all models below, is replaced by several alternative hypothesis.

The basic short run models, which follow the lines of the neokeynesian model of Asimakopoulos and Burbidge presented in Chapter II can be represented by the following system of equations:

$q = f(Ko / L)$ (1)

with $d f / d (Ko / L) = f' > 0$ (1a)

and $d f' / d(Ko / L) = f'' < 0$ (1b)

$C + I + (\Pi / p)\, tr - BS = q L$ (2)

$q L = (w/p) L + \Pi / p$ (3)

$w/p = f (Ko / L) - (Ko / L) f'(KO / L)$ (4)

$I = (1 - tr)\, sp\, (\Pi / p) + (w/p) L\, sw + BS$ (5k)

or: $I = [(1 - tr) (\Pi / p) + (w/p) L] sy + BS$ (5c)

$L = ho(w/p)$; $ho' = d\, ho / d (w/p) > 0$ (6c)

or $I = Io$ (6k)

or $(w/p) = (w/p)o$ (6m)

where the notation is as follows:

q : product per unit of employed labour

L : employed labour

Ko : capital stock; in this short term model constant or exogenous

C,I : consumption and investment in real terms

Π / p : profits in real terms

p : price of production

w/p : real wage rate

BS : budget surplus in real terms; an exogenous variable (by assumption)

f( ) : stands for the production function; it is, as seen, linearly homogeneous in L and Ko.

Io :exogenously determined investment in the neokeynesian model

(w/p)o: exogenously determined real wage rate in the neomarxian model

ho( ): function representing labour supply in the neoclassical model

sp, sw, sy: parameters representing respectively the propensity to consume out of net profits, out of net wages and out of disposable income (neoclassical case).

tr : profits tax rate: only tax assumed to exist in this model

The neoclassical model is constituted of equations (1) to (4), plus (5c) and (6c). The neokeynesian of (1) to (4) plus (5k) and (6k). The neomarxian of (1) to (4) plus (5m) and (6m).

There are 6 endogenous variables in all models:

q, L, C, I, (Π / p), (w/p)

We can then, in principle, solve for those endogenous variables as functions of the exogenous variables:

tr, Ko, BS and Io or (w/p)o

and the parameters: sp and sw, or sy.

Let us do so for the variables determining the distribution of income. We shall obtain partially reduced forms for those variables. These will permit us to see the effects of changes in the profits tax.

For the neoclassical model the real wage rate can be obtained by replacing (6c) in (4):

$$w/p = f[Ko / ho(w/p)] - Ko / ho(w/p)\, f'[Ko / ho(w/p)] \qquad (9)$$

which can be solved for w/p:

$$(w/p)^{*c} = h^{w.c}(Ko) \qquad (10)$$

where the asterisk indicates a value satisfying the model, and $h^{w.c}(\ )$ stands for the functional form.

then L is obtained from (6c):

$$L^{*c} = ho[h^{w.c}(Ko)] = h^{l.c}(Ko) \qquad (11)$$

On the other hand, from (3), (1) and (4) we can obtain:

$$\Pi / (p\ L) = f\ (Ko/L) - w/p = f - [f - (Ko/L)f']$$

$$= (Ko/L)\ f'$$

or

$$\Pi / (p\ Ko) = f'(Ko/L) \qquad (12)$$

replacing (11) in (12):

$$(\Pi / p)^{*c} = Ko\ f'[Ko / h^{l.c}(Ko)] = h^{\pi.c}(Ko) \qquad (13)$$

Several implications can then be deduced from this model. First, the change in the tax rate leaves before tax profits unaffected:

$$(M / p) / \quad tr = \quad h^{n \cdot c}(K_o) / \quad tr = 0 \qquad (14)$$

Hence after tax profits, $(1 - tr)\ h^{n \cdot c}$, will fall by the full amount of the tax. As we had stated in the text,in the short term, the tax is fully borne by profits in this neoclassical model.

Second, there is full employment, in the sense that involuntary unemployment is zero: all supplied labour has been employed. The amount of unemployment is:

$$U^{*c} = h_o[(w/p)^{*c}] - L^{*c} = 0$$

by virtue of equations (11) and (10)

Third, it is clear from equations (5c) and (4) that the distribution of income is determined in the labour market, by movements of the real wage to the point where labour supply is equal to labour demand.

Fourth, notice by (5c) that once the distribution of income and employment are fixed in the labour market, investment simply becomes equal to the amount of savings at "full employment", i.e. investment acommodates to savings. Notice also that Budget deficits (negative BS) would decrease investment.

The accomodation of investment to savings, which is trivial in this very simple model is nevertheless a good representation of the neoclassical paradigm. Even in a more complicated model with a market for funds where investment would depend inversely on the cost of capital given by the interest rate, say ir, and the savings would be dependent on the interest rate too, it can be said that savings and investment accomodate to the conditions in the factor market. Replace for example (5c) by:

$$I(ir) = [(1 - tr)\ (\bar{h} / p) + (w/p)\ L]\ sy(ir) + BS \quad (5c.1)$$

where ir indicates the interest rate or cost of capital.

By (10), (11) and (13), in the short run, m/p, w/p and L depend on Ko then (5c.1) can be solved for ir as:

$$ir = \bar{h}^{i,c}(Ko, tr, BS) \quad (14)$$

and then

$$I = I(ir) = I[h^{i,c}(Ko, tr, BS)] = hI,c(Ko, tr, BS)$$

and similarly for savings. Notice that a quantity equation for money could also be introduced to determine the level of prices:

$$M = (1/v)\ p\ q\ L \quad (15)$$

where M : quantity of money (supply) and

v : velocity of money.

This changes nothing to the conclusions outlined previously.

For the neokeynesian model, we can obtain from (1) and (3)

$$w/p = f(Ko / L) - \bar{h} / (p\ L) \quad (16)$$

and replacing this as well as (6k) into 5k:

$$Io = [(1-tr)\ sp - sw]\ (\bar{h}/p) + f(Ko / L)\ sw\ L + BS \quad (17)$$

and recall from (12) that:

$$\bar{h} / (p\ Ko) = f'(Ko / L) \quad (18)$$

These two equations define the equilibrium values of $\bar{h}$/p and L; notice that these values are functions not only of Ko, but also of tr and BS:

$$(M / p)^{*k} = h^{m,k}(Ko, Io, tr, BS) \qquad (19)$$

$$L^{*k} = h^{l,k}(Ko, Io, tr, BS) \qquad (20)$$

w/p can be obtained by (16)

Then from (17) and (18) we can obtain:

$$(M/p)/\,tr = \frac{(M/p)\ sp}{(1-tr)sp - sw(f - Ko\ f'/L)L^2/(Ko^2 f") - sw}$$

$$= \frac{(M / p)\ sp}{(sp - sw) - sw\ L^2 w/(f"\ Ko^2 p) - tr\ sp} \qquad (20a)$$

then, a new tax (initial tr = 0) in this model is at least partially shifted (1). Recall that in the special case of the Asimakopoulos and Burbidge (1974) model, which uses a classical savings function (sw=0), profits are determined solely by equation (17) and the tax is then totally shifted.

So, unlike the neoclassical model, the neokeynesian admits at least partial shifting. A second difference with the neoclassical concerns employment. The employment level obtained by this system given by equation (18) would only by coincidence be equal to the employment voluntarily

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(1) Expression (20a) is positive for tr = 0 (new tax), f" < 0 and sp > sw, which are the normal assumptions of the model. Notice that the last equality in (20a) is obtained from eq. (4).

supplied by the labourers, whether this takes the form of a neoclassical function Ls = ho(w/p) or any other:

$$Ls = h_2(w/p)$$

where Ls stands for labour supply.

In general

$$Uk = Ls - L^{*k} \neq 0$$

unlike the neoclassical model where Uc = 0. Hence the neokeynesian model admits an equilibrium with involuntary unemployment.

Third, in this model, savings, in a certain way, accomodate to investment, by changes in the distribution of income, thanks to the different savings propensities of workers and profit holders. Notice also that, for this to happen, there is no need for the wage rate to change. A simple change in the employment level L, will change the distribution of income and consequently savings. In this model, unlike the neoclassical, investment plays a crucial role in the distribution of income and in the determination of the employment level.

Fourth, notice that for the special case , sw = 0, the distribution of income is determined only by equation (5k); e.g., it is determined solely in the goods market (Asimakopoulos and Burbidge, 1974).

To obtain reduced forms for the neomarxian model, replace (6m) into (4):

$$(w/p)o = f(Ko / L) - (Ko / L)\, f'(Ko / L) \qquad (21)$$

from which we can obtain L:

$$L^{*m} = h^{1 \cdot m}[Ko,(w/p)o] \quad (22)$$

Then from (12) and (22) we can obtain real profits:

$$\Pi / p = Ko\ f'\{Ko / h^{1 \cdot m}[Ko,(w/p)o]\}$$

$$= h^{\pi \cdot m}[ko,(w/p)o] \quad (23)$$

We can then observe the following. First, the distribution of income depends on the exogenously determined real wage and on the exogenous capital stock. In particular the gross of tax wage rate, (w/p), the gross of tax wage bill (w L / p), and gross of tax profits do not change when the profits tax changes. Therefore this tax is totally borne by profits; there is zero shifting.

Second, again the neomarxian model, as the neokeynesian, allows for less (or more) than full employment. If workers are willing to supply a certain amount of labour Ls;

$$Ls = h_2(w/p)$$

In general this amount supplied will be different to the equilibrium employment in the model L^{*m} and "involuntary" unemployment will be:

$$Um = Ls - L^{*m} \neq 0$$

The same would be true for the neomarxian model as well as for the neokeynesian even if full labour supply were defined simply as the labour force, Lf:

$$Lf > \text{or} = Ls$$

Third, notice that in the neomarxian model the distributiion of income is essentially determined outside both the labour and the goods markets through the exogenously given "socially necessary" wage. The demand for labour equation (4) determines then the amount of employment at that wage.

Finally, notice that in this model, as in the neoclassical, investment accomodates to savings [see equation (5k)]. But, unlike the neoclassical model, this investment doesn't correspond to full employment savings.

We can now compare the specification of the short run empirical models with the theoretical models. From our discussion above and from equations (13) we can state that under the assumption of a neoclassical world a reduced form equation for profit rate should give a coefficient for a profits tax term not singificantly different from zero. Indeed, equation (13) is the theoretical reduced form for profits in the neoclassical model. Now equation (13) implies that at each observation the model is in neoclassical equilibrium. If this is not the case, i.e. if in fact what we observe is situations of disequilibrium, i.e. out of equilibrium, then it is necessary to obtain a correct specification to control for those situations.

The explanation for the states of disequilibrium in the neoclassical model would be the existence of "random shocks", external to the system. The disequilibrium would be observed through variables such as unemployment, U. As we saw in neoclassical equilibrium, this variable is always zero; from the point of view of the neoclassical model, the non-zero "U" is a variable which cannot be expressed as a function of other variables in the system: it is exogenous. Then it is legitimate to use unemployment, U, as an exogenous control variable for disequilibrium (cyclical) deviations from the neoclassical model. Hence an acceptable reduced form to test the incidence hypothesis conditional on the assumption that the neoclassical model we presented above explains the world would be like:

$$rg = \Pi \;/\; (p\ Ko) = h^{c}(Ko, tr, U) + e \qquad (24)$$

where e is the random error and $h^{c}(\ )$ stands for the reduced functional form.

This is so if the equation is to be applied to annual data which reflect the business cycle. An alternative way of proceeding would be to take observations to use in the econometric model in such a way as to smooth out the disequilibria (i.e. the business cycles) in the model. For this the period of observation for time series has to be widened. The ideal length now is the average period of the business cycle. This is what Milton Friedman and Ana Schwartz did in a recent work (1), and indeed, this is what we shall do in our own empirical models (2). In this case the correct specification of the reduced model would once again be:

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(1) See Friedman and Schwartz, quoted by Mayer (1982).

(2) Essentially, simple (not moving) averages for all variables for periods of T years, where T is approximately the length of the cycle, are used.

$$\tilde{rg} = \tilde{M} / (\tilde{p}\ \tilde{Ko}) = (1 / \tilde{Ko})\ h^{n.e}(\tilde{Ko}) + \tilde{e} \qquad (25)$$

where the tilde indicates transformed variables.

This corresponds to (13). U or other disequilibrium variables shouldn't appear since disequilibrium has been accounted for by the adequate choice or transformation of the data.

Another way to proceed, under the assumption of a neoclassical world, is to use one of the structural equations such as (9) or (12). Then again, corrections for the disequilibrium states would have to be done by introducing "U" or another pressure variable. Now the system should be estimated by two stage least squares or another method which would give consistent estimates, given the appearance of explanatory endogenous variables. In this case a test equation for shifting would look, on the lines of equation (12) like:

$$rg = M / (p\ Ko) = f'(Ko / L)\ h_3(U)\ h_4(1-tr)\ e \qquad (26)$$

where we have taken a multiplicative functional form. f' is of course the marginal product. $h_3(U)$ is a function representing disequilibrium effects through U; $h_4(1-tr)$ would represent shifting effects and e is a random error. If h_4 has, in the average, a value near 1, the model is consistent with zero shifting. Notice that this is essentially the model presented by Oakland and by Turek. But they did not use two stage least squares or any other technique to account for the endogeneity of L and therefore their results still can have a simultaneous equation bias.

In short, if a model such as (24) or (26) is built to test for shifting, conditional on the validity of the neoclassical short term model, a result for the tax term which shows it not to be significantly different from zero indicates that the empirical observations are consistent with the neoclassical world. If, on the other hand, the tax term is significantly different from zero, this indicates that the data are inconsistent with the neoclassical model such as it is formulated by equations (1) to (4), (5c) and (6c).

Let us now examine the reduced form of a model built on the premise that the neokeynesian short term model is a good representation of the world, i.e. that it is a "true model". This reduced form, for rg , would correspond to equation (19), which in turn results from (16), (17) and (18). It is (1):

$$rg = \Pi \;/\; Ko = (1 \;/\; Ko)\; h^{\Pi \cdot k}(Ko,Io,tr,BS) \qquad (27)$$

or

$$rg = h^{k}(Ko,Io,tr,BS) + e \qquad (28)$$

where e is a random error.

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(1)This reduced form follows the Asimakopoulos and Burbidge short term formulation, where investment I is taken as exogenous (equal to Io). A representation more in agreement with the long term neokeynesian model would take investment as dependant on expected profitability; call it re, I = I(re). In the short term, expected profits can be

If the observations in the model reflect a world in neokeynesian equilibrium, they should be well represented by a model such as (28). Furthermore since this model admits unemployment in its equilibrium outcomes, this variable cannot be added to (28) to control for short term departures from the neokeynesian equilibrium. The neokeynesian disequilibrium would be reflected, for example in differences between ex-ante savings, i.e. the right hand side of equations (17), and ex-ante investment, i.e.the exogenously given Io [or I (reo) see previous note]. But we do not dispose of such an indicator.

The neokeynesian model can explain the fluctuations of the data without having to interpret them as out of (neokeynesian) equilibrium. Variations in Io, due to changes in expectations of entrepreneurs (their "animal spirits") would be a way to explain the cycle, within the neokeynesian equilibrium mechanism . Indeed here again the best way to avoid the problems introduced by the cycle is to take periods of observations with a length near that of the cycle, and fit model (28) to these observations.

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taken as exogenous (Malinvaud, 1980 presents a justification for this hypothesis) : re = reo then I = I(reo) = Io and equation 27 is obtained. In the long term, re cannot be taken as exogenous and equation 27 would not be the reduced form of a long term neokeynesian model.

Furthermore, a coefficient for the tr tax variable significantly different from zero in model (28) would simply indicate that the data are consistent with the neokeynesian model. Even a coefficient near zero would not indicate inconsistency with a neokeynesian model having a Kaldorian savings function such as the right hand side of (5k); although they would indicate inconsistency with the Kaleckian model of Asimakopoulos and Burbidge.

If we start, then, from the hypothesis that the neokeynesian representation of the world is correct, shifting is consistent with this model. Furthermore, it is inadequate to add an unemployment term to equation (28) to control for disequilibrium, since unemployment is explained by the neokeynesian model (1): it is endogenous in this model (2).

Notice that the reduced form (27) corresponds approximately to Krzyzaniak and Musgrave's empirical model "B". Therefore the result they obtain, that the tax is shifted, is consistent with the short run neokeynesian model presented in this appendix. What is inconsistent, is their conclusion that the shifting result disproves

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(1) This is not to say that the explanation is a good or bad one.

(2) It is given by an extra equation: Uk = Ls - L*k, where L*k is full employment, whatever its definition.

profit maximization. Since profit maximization is part of the neokeynesian short term model presented here, they are obviously wrong.

The only way to make Krzyzaniak and Musgrave's results coherent is to assure that they are taking as their "true world" a purely micro-economic construct, or alternatively a non-neokeynesian theory, neoclassical or neomarxian. Then, indeed, in those circumstances one way to explain incidence would be to replace the marginal productivity equations (4) by some other adequate hypothesis, such as the "satisficing" model. But to presume that they are working on such an assumption would contradict their words, the justification of their model as "very general", and would make no sense of the macro- economic system they propose in their work.

Let us now examine the reduced form of a model built on the premise that the neomarxian model is a good representation of reality. A reduced form for the profit equation would be similar to equation (23), resulting from (22) and (12):

$$rg = \Pi / Ko = h^{n \cdot m}[Ko, (w/p)o] \quad (29)$$

To test for shifting we should have to add a term in (1-tr) and a random error :

$$rg = h^{n \cdot m}[Ko, (w/p)o] \; h_s(1-tr) \; e \quad (30)$$

where we have assumed a multiplicative form.

Then, if $h_s(\;)$ is not significantly different from

1.0, (30) is consistent with (29), i.e., with the neomarxian model; otherwise, it is inconsistent. Since the neomarxian model, as the neokeynesian, admits unemployment, it is again inadequate here to introduce a "pressure variable" to account for disequilibrium situations. Disequilibrium has to be handled by taking a long enough period of observation to smooth it out, but in this case a long term model is more adequate.

To sum up, if a very general model is constructed, capable of encompassing the three models represented by equations (1) to (6), and then a test of shifting of a tax on profits is effected on its reduced forms, the results indicate the following. A significat shifting result would be consistent with the neokeynesian model as formulated above, and inconsistent with the other two. A zero shifting result, on the other hand, would be consistent with all three models as formulated above.

It is important to realize that the marginal productivity equation (4) is not needed to obtain any of the results above. It can be replaced by a mark-up equation as Asimakopoulos and Burbidge do. Let that equation be:

$$w/p = q / (1 + m) \tag{31}$$

where m is the mark-up on prime (here labour) costs. First observe that written as above the equation adds nothing to the models presented previously. There is an extra equation and an extra endogenous variable m. m would then depend on the distribution of income according to either one of the alternative schemes showed, neoclassical, neokeynesian or neomarxian.

But if m is taken as exogenous another equation in the system (1) to (6) has to be dropped; then assume:

$m = mo = \text{exogenous}$

and replace equation (4) by:

$$w/p = q / (1 + mo) \qquad (32)$$

Then solving the equations as previously it can be shown that the previous results do not change.

As an example, take the neoclassical model, given now by equations (1) to (3), (32), (5c) and (6c). Then by replacing (1) and (6c) in (32):

$$(w/p) = f(Ko / L) / (1 + mo)$$

$$= f[Ko / ho(w/p)] / (1 + mo) \qquad (33)$$

therefore

$$(w/p)^{*.c.m} = h^{w.c.m}(mo,Ko) \qquad (34)$$

and

$$L^{*.c.m} = h^{l.c.m}(mo,Ko) \qquad (35)$$

and by (3):

$$(\Pi / p)^{*.c.m} = f(Ko / h^{l.c.m})\, h^{l.c.m} - h^{w.c.m}\, h^{l.c.m}$$

therefore

$$(\Pi / p)^{*.c.m} = h^{\Pi.c.m}(Ko,mo)$$

And it can be seen the gross of tax profits or the gross of tax wage bill doesn't change with the changes in the tax rate. Hence, the incidence conclusions have not changed, although there is no profit maximization. Also, labour supplied is equal to labour demanded and there is no involuntary unemployment, although the economy is not "efficient" because it is not maximizing profits.

In a similar way, the results already observed for the other models can be reproduced here. So a simple substitution of a mark-up model for the competitive one would not be sufficient to explain shifting in the neoclassical context.

But there are two ways among others, in which a neoclassical model could result in shifting. One is by replacing the profit maximizing equation (4) or the mark-up equation (32) by a simple behavioural hypothesis stating that entepreneurs simply aim at a fixed after tax profit target, $(\Pi / p)_{c.o.}$. So (4) or (32) are replaced by:

$$(1-tr)\ \Pi / p = (\Pi / p)_{c.o} \qquad (37)$$

Then, obviously, in this trivial model gross of tax profits will increase if the tax rate tr increases: the tax will be totally shifted.

A second way the neoclassical model would predict shifting is given by replacing the profit maximizing equation (4) by a fixed mark-up equation where it is assumed that entrepreneurs perceive the profits tax as a direct cost. If this were the case the mark-up equation would be:

$$p = (1 + mo)\ [w / q + tr\ \Pi e / (q\ L)] \qquad (38)$$

where Πe are expected profits, on which the tax is expected to be applied. To obtain shifting, we also need to assume that entrepreneurs have some sort of rational expectations: expected profits will be equal to actual realized profits,

$$\Pi e = \Pi$$

Then, the mark-up equation closing the neoclassical model will be:

$$1/(w/p) - (1+mo) \{1 - (\Pi/p)tr/[(w/p)L]\} / q = 0 \quad (39)$$

which can also be written, by using equation (3), as:

$$(\Pi / p) / [(w/p)L] = mo / [1 - tr (1 + mo)] \quad (40)$$

Equation (40) is enough to see that in this model the tax is at least partially shifted: if the tax rate, tr, increases, the right hand side of (40) will increase. Therefore real gross of tax profits will increase. Hence, workers will bear at least part of the tax.

Notice that equation (40) applies also to the neokeynesian and the neomarxian models. Then, in these two models the assumption embedded in equation (38) implies at least partial shifting of the tax. An interesting characteristic of the neomarxian model with equation (38) is that, since (w/p) is given by (w/p)o, the profits tax increase is shifted only through a reduction of employment, which decreases the wage bill.

Closing the neokeynesian model wih assumption (37) rather than (4) or (38) also gives 100% shifting quite trivially since profits after tax stay at the fixed level, (Π / p)o.

The same is true of the neomarxian model, an increase in the profit tax will keep after tax profits at the same level. Since at the same time the real wage rate is also given at (w/p)o, shifting will again occur through a reduction of employment, which will decrease the wage bill.

Notice, finally, that assumption (37) is incompatible with a Kaleckian model like that of Asimakopoulos and Burbidge, since with sw=0, only by chance will we obtain equality (17).

APPENDIX 2 TO CHAPTER III

TECHNICAL PROGRESS AND THE RATE OF PROFIT

Let a production function with general form of technical progress be:

$$Q = F(K\ PK,\ L\ PL) \quad (1)$$

where Q and K are output and capital in normal physical units and L is labour in normal labour units.

PK is a capital augmenting factor, representing the effect of technical progress on capital; similarly PL is a "labour augmenting" factor.

PK and PL increase with time if there is technical progress:

$$PK = PK(t), \quad d\,PK / d\,t > 0$$

$$PL = PL(t) \quad d\,PL / d\,t > 0$$

Assuming constant returns to scale, equation (1) can also be written, in terms of output and capital per unit of labour, respectively q and k as:

$$q = PL\ F(k\ PK / PL) = PL\ f(k\ PK / PL) \quad (2)$$

or further

$$q = f(k) \quad (3.1)$$

$$q = q / PL \qquad (3.2)$$

$$k = k / PK \qquad (3.3)$$

q and k are capital and output per unit of labour in "efficiency" units.

Let us further assume that PK ad PL can be expressed as parametric functions of a variable "PT" and that this function is exponential:

$$PK = PT^{BK} \qquad (4.1)$$

$$PL = PT^{BL} \qquad (4.2)$$

Then, technical progress will be purely labour augmenting or Harrod Neutral if BK = 0 (PK = 1) and BL ≠ 0; it will be Hicks neutral if BK = BL = B (PK = PL = PB) and it will be capital augmenting or Solow neutral if BL = 0 (PL = 1) and BK ≠ 0.

Then under normal profit maximization hypotheses

$$(d + rg)\ pq = PT^{BK} f'(k) \qquad (5.1)$$

$$k = k\ PT^{BK-BL} \qquad (5.2)$$

where d is the geometric depreciation rate of capital and pq the price of output.

From equation (5) we can derive an expression for

$$rg = PT^{BK} f'(k) / pq - d \qquad (6.1)$$

To simplify take d = 0 and pq = 1 (output as numeraire); this will not change our conclusions.

Then obtain

$$rg / PT = [PT^{BK} f'\ BK + PT^{BK}(BK-BL)\ k\ f''] / PT$$

$$= [BK\ rg + PT^{BK}(BK-BL)\ k\ f''] / PT \qquad (7)$$

where we have used (6.2); on the other hand the elasticity of substitution for a general production function with constant returns to scale, such as (3.1) is:

$$Ek,l = [f' (f - k f')] / [- k f f''] \quad (8)$$

Then eliminating q from (7) by use of (8) and simplifying we get.

$$k f'' = [PT^{-BK} rg (q - k PT^{-BK} rg)] / [-Ek,l q]$$

$$= [PT^{-BK} rg \; wg] / [-Ek,l q].$$

and

$$rg / \quad PT = [BK + (BL - BK)Aw / Ek,l] rg / PT \quad (9)$$

where: Aw = wg / q is the share of wages in the product. It can be seen that expression (9) is positive for all values of BK and BL such that BL > BK. In the worst case, that of Solow neutral T.P., with BL=0 we get:

$$rg / \quad PT = BK \, rg \, (Ek,l - Aw) / (Ek,l \, PT)$$

which is still positive if Ek,l >Aw; since Aw is around 2/3, if Ek,l > .65 then even for BL = 0 will d rg / d PT > 0.

Hence we can conclude: technical progress and the rate of profits are positively related in all cases where labour augmenting tendencies are at least as strong as the capital augmenting tendencies (i.e. if BL > or = BK), in particular for Harrod and Hicks neutrality. In the worst case of pure capital augmenting technical progress (BL = 0), the relation is still positive if the elasticity of substitution Ek,l is greater than the wage share Aw.

APPENDIX 1 TO CHAPTER VI

In this Appendix, we shall derive the differential forms of the equations presented in section A of chapter VI. To ease the task, let us first repeat here those equations. Recall that all symbols are defined in table 1 of chapter VI.

Common equations

$$1 = a0 \ (c + gc) + a1 \ (d + g) \qquad (1)$$

$$1 - tx = 7x = a0 \ wg + a1 \ (d + rg) \qquad (2)$$

$$1 / a0 = \exp(R' \ t) \ f[\exp(-R' \ t) \ a1 / a0] \qquad (3)$$

$$f'[\exp(-R' \ t) \ a1 / a0] = (d + rg) / (1 - tx) \qquad (4)$$

Neoclassical closure equations

$$gas + gds + S(yd, rn; n) = g \ a1 / a0 \qquad (6c)$$

$$g = n + R' \qquad (7c)$$

Neokeynesian closure equations

$$S(wn + su\ ,\ rn\ a1\ /\ a0) + gas + gds = g\ a1\ /\ a0 \qquad (6k)$$

$$g = i(rn)\ ;\quad i' > 0 \qquad (7k)$$

Neomarxian closure equations

(the first neomarxian closure equation is the same as the neokeynesian savings equation (6k))

$$wg\ a0 = h1(idls)\ /\ (1 - tw) \qquad (7m)$$

Auxiliary definitional identities

$$tr = tcr + (1 - \theta)\ (1 - tcr)\ tyr + (1 - \theta)\ (1 - tcr)\ (1 - tyr)\ zg \qquad (8)$$

$$rn = rg\ (1 - tr) = rg\ \tau r \qquad (9)$$

$$wn = wg\ (1 - tw) = wg\ \tau w \qquad (10)$$

$$yd = wn + su + rn\ (a1\ /\ a0) \qquad (11)$$

$$su = su0 + su1\ /\ (1 + n)^{TR} \qquad (12)$$

We can now derive the differential forms of these equations. For easier association with the growth models, we shall retain the numbering of the equations as above, but the corresponding equations in differentials will be primed.

The differentiation of equation (1) produces directly after some algebraic manipulations:

$$d\ c + d\ gc + (c + gc)\ d\ a0 / a0 + (a1 / a0)$$
$$(d + g)\ d\ a1 / a1 + (a1 / a0)\ d\ g = 0 \qquad (1')$$

Notice that the differential of a variable, say x, is denoted by "d x". For the differentials of a0 and a1, though, it will prove convenient to keep them in the form they have in equation (1'), as "d a0 / a0" and "d a1 / a1".

Let us now differentiate equation (2). This results in the expression:

$$d\ ?x = a0\ dwg + a1\ d\ rg + wg\ d\ a0 + (d + rg)\ d\ a1 \qquad (13)$$

This expression can be further simplified by use of a particular form of the differential of equation (3), which we shall now develop. A straight differentiation of equation (3) results in the following expression:

$$- (1 / a0)\ d\ a0 / a0 = (a1 / a0)\ f'\ (d\ a1 / a1 - d\ a0 / a0)$$

reorganizing this expression, and replacing f' by its value in terms of (d + rg) given by equation (4):

$$(d\ a0 / a0)\ [- (?x / a0) + (d + rg)\ a1 / a0] / ?x$$
$$= [(d + rg)\ (a1 / a0) / ?x]\ da1 / a1$$

Now, from equation (2) we obtain:

$$- 7x / a0 + (d + rg)\ a1 / a0 = - wg$$

which can be replaced in equation (14), to obtain, after reorganizing and simplifying:

$$wg\ d\ a0 + (d + rg)\ da1 = 0 \qquad (15a)$$

Defining the share of profits gross of depreciation in national revenue, "Ar,g" , as:

$$Ar,g = [\ (d + rg)\ a1 / a0\ ] / [wg + (d + rg)\ a1 / a0]$$
$$= a1\ (d + rg) / 7x \qquad (16)$$

Equation (15a) can also be written as:

$$Ar,g\ d\ a1 / a1 + (1 - Ar,g)\ d\ a0 / a0 = 0$$

Equation (15a) can be replaced in equation (13) to obtain the final form for the differential of equation (2):

$$d\ 7x = a0\ d\ wg + a1\ d\ rg = d\ wg + k\ d\ rg \qquad (2'')$$

The differential form of equation (3) could be represented by equations (15a) or (15b), but we shall obtain, instead, another expression more convenient for our purposes, after deriving an expression for the differential of equation (4). For this latter derivation start by taking logarithms of (4) and then differentiate, to obtain:

$$d f' / f' =$$

$$f'' \exp(-R' t)\ (a1 / a0)\ (d\ a1 / a1 - d\ a0 / a0) / f'$$

$$= d\ rg / (d + rg) - d\ 7x / 7x \qquad (17)$$

Equation (17) could be used directly as the differential expression for equation (4). But for practical purposes we would need to be able to give empirical values to the second derivative f". In most economic work, rather than f" the concept of elasticity of substitution is used. We shall therefore modify equation (17) to use this concept rather than f". In general, for a production function with Harrod neutral technical progress, expressing total production Q, in terms of total capital, K, and total labour, L:

$$Q = F[K, L \exp(R' t)] = L \exp(R' t)\ f(k) \qquad (18a)$$

where,

$$k = K / [L \exp(R' t)] = k \exp(-R' t)$$

$$= (a1 / a0) \exp(-R' t) \qquad (18b)$$

the elasticity of substitution in production, "Ek,l", is defined as:

$$Ek,l = - d \ln(K / L) / d \ln(FK / FL) =$$

$$- (d K / K - d L / L) / (d FK / FK - d FL / FL) \qquad (19)$$

where FK and FL are, respectively, the marginal products of capital and labour:

$$FK = \quad F / \quad K = d\,f / d\,k = f' = (d + rg) / ?x \qquad (20)$$

$$FL = \quad F / \quad L = \exp(R'\,t)\,f - k\,f' = wg / ?x \qquad (21)$$

The last inequalities in (20) and (21) come from the assumption of profit maximization, implicit in equation (3) of last chapter. Taking logarithms and differentiating equations (18b), (20) and (21), we get:

$$d\,K / K - d\,L / L = d\,a1 / a1 - d\,a0 / a0$$

$$d\,FK / FK = d\,rg / (d + rg) - d\,?x / ?x$$

$$d\,FL / FL = d\,wg / wg - d\,?x / ?x$$

Replacing all these expressions in equation (19) we obtain:

$$Ek,l = (d\,a1 / a1 - d\,a0 / a0) / [d\,rg / (d + rg) - d\,wg / wg] \qquad (22)$$

which can be rearranged as:

$$d\,a1 / a1 - d\,a0 / a0 = Ek,l\,[\,d\,wg / wg - d\,rg / (d + rg)\,] \qquad (4')$$

This is the expression which we shall adopt for the differential form of the marginal productivity equation (4).

Let us now return to the differential form of the production function, (3). We saw that this form could be represented by either equation (15a) or (15b). However we prefer to obtain an expression which doesn't contain the differential d a1. d a1 can be eliminated from, say, equation (15b), by the use of equation (22). Doing this we obtain:

$$d\,a0 / a0 = - Ar,g\ Ek,l\ [d\,wg / wg - d\,rg / (d + rg)] \qquad (3')$$

This is the final expression which we shall adopt for the differential form of the production function, equation (3).

The next equation is the government budget constraint, (5). A straightforward differentiation of this equation results in:

$$\begin{aligned} &(wg\ tw - gc - gas - gds - su)\ da0\ + \\ &rg\ tr\ d\,a1 + a0\ tw\ d\,wg + a1\ tr\ d\,rg \quad = \\ &a0\ (d\,gc + d\,gas + dsu) + a1\ rg\ d\,7r\ + \\ &a0\ wg\ d\,7w + d\,7x \end{aligned} \qquad (5')$$

Notice that:

$$7i = 1 - ti \text{ and } d\,7i = - d\,ti\ ;\ i = r, w, x.$$

We have completed the differential forms of the common equations. Let us now examine those equations specific to each model.

The differential of the neoclassical saving-investment relation (6c) can be obtained as follows.

First, differentiate equation (6c):

$$d\ gas + d\ gds + S1\ d\ yd + S2\ d\ rn =$$
$$(a1 / a0)\ d\ g + g\ (a1 / a0)\ (d\ a1 / a1 - d\ a0 / a0) \qquad (23)$$

where S1 denotes the partial derivative of the neoclassical savings function S(yd, rn; n) with respect to its first argument, yd, and S2 represents its partial derivative with respect to the second argument, rn.

Then replace (see auxiliary equations 9 to 11):

$$d\ rn = \tau r\ d\ rg + rg\ d\ \tau r \qquad (24)$$

and

$$d\ yd = d\ (wn + rn\ a1 / a0 + su) = \tau w\ d\ wg + wg\ d\ \tau w +$$
$$(a1 / a0)\ \tau r\ d\ rg + (a1 / a0)\ rg\ d\ \tau r +$$
$$rn\ (a1 / a0)\ (d\ a1 / a1 + d\ a0 / a0) + d\ su \qquad (25)$$

to obtain:

$$a1\ [S1\ rn - g]\ [d\ a1 / a1 - d\ a0 / a0] +$$
$$[a1\ S1 + a0\ S2\ ]\ \tau r\ d\ rg +$$
$$a0\ S1\ \tau w\ d\ wg + \ - a1\ dg =$$
$$- [a0\ S1 + a0\ S2]\ rg\ d\ \tau r$$
$$- a0\ S1\ wg\ d\ \tau w - a0\ d\ gas$$
$$- a0\ S1\ d\ su - a0\ d\ gds \qquad (6c')$$

This is the differential form of the neoclassical saving-investment relation. Finally, that of the neoclassical natural rate of unemployment hypothesis (7c) is straightforward. Differentiating (7c) we obtain:

$$d\ g = d\ (n + R') = d\ n + d\ R'$$

To derive incidence propositions the two exogenous variables, n and R', will be taken as unchanged. Hence, for this purpose, the equation can be written:

$$d\ g = d\ n + d\ R' = 0 \qquad (7c')$$

Notice also that the differential form of the neoclassical saving-investment relation (6C') can be further simplified by eliminating, in the left hand side, in view of (7c'), the term: −a1 d g.

The differential form of the Kaldorian saving-investment relation, 6K', common to the neokeynesian and the neomarxian model can be obtained as follows. First, differentiate equation (6K):

$$sw'\ [d\ wn + d\ su] + sp'\ rn\ [d\ a1\ /\ a1 - d\ a0\ /\ a0]\ a1\ /\ a0$$
$$+ sp'\ (a1\ /\ a0)\ d\ rn + d\ gas + d\ gds =$$
$$(a1\ /\ a0)\ d\ g + g\ [d\ a1\ /\ a1 - d\ a0\ /\ a0]\ a1\ /\ a0 \qquad (26a)$$

where, sw′ is the partial derivative of the neokeynesian savings function S (wn + su, k rn), with respect to its first argument, wn + su,

$$sw' = ms,w = S / (wn + su) \qquad (26b)$$

and sp′ is the derivative with respect to its second argument, k rn,

$$sp' = ms,p = S / (rn\ a1 / a0) = S / (k) \qquad (26c)$$

i.e., sw′ and sp′ are the marginal propensities to save, respectively, out of wages and out of profits. They will alternatively be written as ms,w and ms,p.

Replace in (26a) d rn by its value in (24) and dwn by (see equation 10):

$$d\ wn = 7w\ d\ wg + wg\ d\ 7w \qquad (27)$$

Then reorganize to obtain:

$$a1\ (sp'\ rn - g)\ [d\ a1 / a1 - d\ a0 / a0] + a0\ sw'\ 7w\ d\ wg$$
$$+ a1\ sp'\ 7r\ d\ rg - a1\ d\ g =$$
$$a0\ sw'\ wg\ d\ 7w - a1\ sp'\ rg\ d\ 7r - a0\ d\ gas - a0\ d\ gds$$
$$- a0\ sw'\ d\ su \qquad (6k')$$

This is the differential form of the Kaldorian saving-investment relation, common to the neokeynesian and neomarxian models. The differential form of the neokeynesian animal spirits investment hypothesis (7K), is straightforward:

$$d\ g = i'\ d\ rn$$

which can be rewritten, taking account of (24):

$$dg - i'\ 7r\ d\ rg = i'\ rg\ d\ 7r \qquad (7k')$$

where $i' = d\ i\ /\ d\ rn$

Finally, the differential form of the neomarxian socially necessary wage hypothesis, (7M), can be obtained for a constant index of strikes as follows..

Rewrite equation (7M) as:

$$wg\ a0\ 7w = h1(idls)$$

then take differentials and reorganize to obtain :

$$d\ wg + wg\ d\ a0\ /\ a0 = -\ wg\ d\ 7w\ /\ 7w \qquad (7m')$$

We have now completed the differential forms of all equations in the growth models. For each model we now have a system of seven equations, linear in the differentials of the endogenous variables:

$$d\ rg\ ,\ d\ wg\ ,\ d\ a0\ ,\ d\ a1\ ,\ d\ g\ ,\ d\ C$$

plus one of the differentials of the government variables (one of $d\ 7x$, $d\ 7w$, $d\ 7r$, $d\ gas$, $d\ gds$, $d\ su$, $d\ gc$). The coefficients are all evaluated at a solution (equilibrium) point of the systems. From these systems we could obtain the total differentials of the endogenous variables, that is, expressions giving the differentials of those variables as functions of the differentials of the exogenous variables and the equilibrium values of all variables. We shall see in Section B of chapter VI that, to study incidence, it is only necessary to examine the total differentials of rg and wg. But, from equation (2'), d wg can be directly obtained from d rg by a very simple expression:

$$d\ wg = d\ ?x\ /\ a0 - (a1\ /\ a0)\ d\ rg \qquad (28)$$

We shall then concentrate only in obtaining the total differential of rg. For this, we simply have to solve the system (1') to 7c' (or to 7k' or to 7m') for d rg. The solution is a matter of straightforward algebra; it is facilitated by the frequent appearence ofthe expression:

$$d\ a1\ /\ a1 - d\ a0\ /\ a0$$

in the equations, and by the fact that this expression can be reduced to one solely in terms of d rg and equilibrium values of variables. Indeed from (2'), (4') and (2) we get:

$$d\ a1\ /\ a1 - d\ a0\ /\ a0 = Ek,l\ [\ d\ wg\ /\ wg - d\ rg\ /\ (d + rg)\ ]$$
$$= Ek,l\ /\ wg\ \{\ d\ ?x - ?x\ d\ rg\ [\ a0\ (d + rg)\ ]\ \} \qquad (29)$$

By extensive use of relations (28) and (29) it is possible to arrive at the total differentials of rg in the models. The expressions obtained are as follows. For the neoclassical model, the total differential of rg is:

$$d\,rg = (1 / Jc)\ \{ - a0\ (d\,gas + d\,gds + S1\ d\,su) - Bc,x\ d\,7x - a0\ wg\ S1\ d\,7w - [\,a1\ S1 + a0\ S2\,]\ rg\ d\,7r\ \} \qquad (30a)$$

where Jc is the Jacobian determinant of the neoclassical system:

$$Jc = 7r\ \{ - \frac{a1\ rg\ 7x\ Ek,l}{a0\ wg\ (d + rg)}\ (\,S1 - g / rn\,) - a1\ (7w / 7r)\ S1 + [\,a1\ S1 + a0\ S2\,]\ \} \qquad (30b)$$

and Bc,x is:

$$Bc,x = \frac{a1\ rg\ 7r\ Ek,l}{a0\ wg}\ (\,S1 - g / rn\,) + 7w\ S1 \qquad (30c)$$

For the neokeynesian model the total differential of rg is:

$$d\,rg = (1 / Jk)\ \{ - a0\ (d\,gas + d\,gds + sw'\ d\,su) - Bk,x\ d\,7x - a0\ wg\ d\,7w - a1\ (sp' - i')\ rg\ d\,7r\ \} \qquad (31a)$$

where, Jk is the Jacobian determinant of the neokeynesian system:

$$Jk = ?r \left\{ - \frac{a1\ rg\ ?x\ E_{k,l}}{a0\ wg\ (d + rg)} (sp' - g / rn) - a1\ (?w / ?r)\ sw' + a1\ (sp' - i') \right\} \qquad (31b)$$

and Bk,x is:

$$B_{k,x} = \frac{a1\ rg\ ?r\ E_{k,l}}{a0\ wg} (sp' - g / rn) + ?w\ sw' \qquad (31c)$$

Finally, for the neomarxian model, the total differential of rg is:

$$d\ rg = \frac{(A_{r,g}\ E_{k,l} - 1)\ ?w\ d\ ?x - a0\ wg\ d\ ?w}{a1\ (E_{k,l} - 1)\ ?w}$$

Recall that Ar,g is the share of profits gross of depreciation in total product:

$$A_{r,g} = (d + rg)\ k / (q\ ?x)$$

Equations (30) to (32), representing the total differentials of the rate of profit in the models, can be used to derive expressions for all types of tax-cum-expenditure incidence. In chapter VI, they are used to derive the balanced budget incidence of the profits tax vs. government consumption; in appendix 4 to chapter VI, to derive other important incidence cases.

APPENDIX 2 TO CHAPTER VI

STABILITY AND THE GROWTH MODELS

As mentioned in the text, the assumption of stability under certain dynamic processes imposes restrictions in the range of variations of the parameters of the models. This idea was first developed by Samuelson (1947) under the name of "Correspondence Principle". Later Patinkin (1965) found that, if very general dynamic adjustment processes are postulated, the assumption of convergence will not impose practically any restrictions on the parameters. Hence very specific dynamic processes are necessary.

Furthermore, these specific processes have to be as much part of the models of growth as the steady state (static) relations we have studied so far. For example, in the Neoclassical model, where prices and price flexibility play a crucial role, a postulated dynamic equation representing the way the economy goes back towards equilibrium after a disturbance (shock), or after a purposeful change resulting form government policy, would not be adequate if it did not take account of these essential traits of the neoclassical model. The neoclassical adjustment processes should show prices accomodating to situations of excess demand or excess supply in accordance with Neoclassical theory. Adjustment through rigid prices and flexible output, for example would not do for this model.

In sum, for meaningful results, the adjustment processes have to have two conditions : first, they have to be very specific; second, they have to be consistent with the model they refer to.

Since our purpose is to restrict the possible values of the parameters in the incidence expressions (41a) to (43) in chapter VI, which are nothing other than comparative static expressions, we may only deal with local stability conditions. We shall proceed as follows. First, we shall present a short but general model relating stability conditions to the sign of the Jacobian determinant of the three basic models, represented by equation 1 to 7c 7k or 7m in chapter VI. Secondly, we shall present the principal dynamic adjustment hypothesis, consistent with each model of growth. Then we shall complete the specification of the dynamic adjustment with other auxiliary hypotheses, common to all models in so far as possible. Finally, we shall derive the sign of the Jacobian determinants from assuming stability in the linearized models studied.

1) Stability Conditions and Comparative Statics

Assume the following model:

$$fi(X,B) = 0 \quad i = 1,2 \ldots n \qquad (1)$$

where:

$X' = (x1, x2 \ldots xn)$ is a row vector of n endogenous variables, transpose of the column vector X. The variable X varies in time. $B' = (b1, b2 \ldots bn)$ is a row vector of m endogenous variables and parameters, transpose of B.

Equation (1) can represent any of the models we are studying shown in equation 1 to 7C, 7K or 7M or chapter VI. The model can also be represented using a vector function as

$$F(X,B) = 0 \qquad (2)$$

where

$F' = [f1, f2 \ldots fn]$ is the transpose vector of functional forms, fi, in equation (1).

Let us represent equilibrium values by hatted variables, like $\hat{X} = (\hat{x}1, \hat{x}2 \ldots \hat{x}n)$. Hence in equilibrium the models are, identically:

$$F(\hat{X},B) = 0$$

or

$$fi(\hat{X}, B) = 0 \quad i = 1, 2 \ldots n$$

If equilibrium is temporarily disturbed because of an external shock or due to a small change, d b, in the b's , relations (3.1) or (3.2) will not obtain any more, but rather: $F = F(\hat{X},B) \neq 0$

In this case we shall assume that the endogenous variables will move along dynamic paths back to equilibrium, i.e. we shall assume that the system is stable.

Very general dynamic paths of adjustment can be given by the system of differential equations (where T stands for time):

$$d\,x_i \;/\; d\,T = \dot{x}_i = \quad u_{ij}\, f_j(X,B) \qquad i = 1,2 \ldots n \qquad (4.1)$$

or in matrix notation:

$$d\,X \;/\; d\,T = \dot{X} = \Omega\, F(X,B)$$

where, Ω isthe matrix:

$$\Omega = [\omega_{ij}]\;;\; i,j = 1,\ 2 \ldots n$$

Equations (4.1) and (4.2) simply say that the rate of movement of each endogenous variable out of equilibrium depends on each equation of the system f_j in a linear way; the coefficient of this adjustment process, or the "speeds" are ω_{ij}. The signs of the ω_{ij} would be deduced from the theory underlying each model.

As Patinkin (1965) shows, this process turns out to be too general to be of any use in producing useful constraints (1). We shall choose a much more specific adjustment, by assuming that the derivative of each endogenous variable depends only on one of the equations f_i in its adjustment path. Hence the matrix Ω is diagonal;

----

(1) The characteristic determinant of the related system

call it Ψ^D:

$$\Psi = \Psi^D \equiv [\psi_i \delta_{ij}] \qquad (5)$$

$$\delta_{ij} = 0 \text{ if } i \neq j \; ; \; \delta_{ij} = 1 \text{ if } i = j$$

Which variable depends on which equation, as well as the sign of the corresponding speed of adjustment will depend on the theory underlying the model. Here is where the requirement of consistency of the adjustment process with that theory arises.

The adjustment process will now be:

$$d\, X / d\, T = \dot{X} = \Psi^D F(X,B)$$

Linearizing this equation around the equilibrium values X we obtain:

$$\dot{X} = \Psi^D [\partial F(\hat{X},B) / \partial X] [X - \hat{X}]$$

or

$$\dot{X} = \Psi^D X \; \partial F(\hat{X},B) / \partial X - \Psi^D \hat{X} \; \partial F(\hat{X},B) / \partial X = 0 \qquad (6)$$

using the differential operator D = d/dT we can write (6) as:

$$[\Psi^D \; \partial F(\hat{X},B) / \partial X - D] X - \Psi^D [\partial F(\hat{X},B) / \partial X] \hat{X} = 0 \qquad (7)$$

The characteristic equation of this system is:

$$\left| \Psi^D \; \partial F(\hat{X},B) / \partial X - \text{lam } I \right|$$

V of linearized differential equations is $|\Psi J|$ (where J is the Jacobian of F, J = $\partial F / \partial X$) and although its sign is given by the stability assumption, it is still not possible to obtain from this the sign of the Jacobian.

$$= \omega_i \left| \partial f_i(\hat{X},B) \, / \, \partial X - lam\,\delta_{ij} \right| \quad i,j = 1,2...n \quad (8)$$

An nth degree equation in lam, whose solution will give the characteristic roots lam(i) of system (7). It can be seen for example in Samuelson (1947) that in order for the linearized system (6) or (7) to be stable its characteristic matrix

$$[\omega^D \, \partial \, F(\hat{X},B) \, / \, \partial \, X - lam \; I] \qquad (9)$$

has to be negative definite. This condition can be used to impose restrictions on the parameters B, perhaps with additional hypotheses.

In particular, that condition is met if the sign of the characteristic determinant:

$$Dc = \left| \omega^D \, \partial \, F(\hat{X},B) \, / \, \partial \, X \right| = \prod_{i=1}^{n} \omega_i \left| \partial \, F \, / \, \partial \, X \right| \qquad (10)$$

is equal to:

$$sign(Dc) = sign\,[(-1)^n]$$

Therefore the condition is:

$$sign \left| \partial \, F \, / \, \partial \, X \right| = sign \; \{\prod_{i=1}^{n} \omega_i^{-1} \, Dc\} = sign[(-1)^n] \quad (12)$$

On the other hand, as is well known, comparative static results for a small change in the vector of parameters and exogenous variables B, can be obtained differentiating equation (3):

$$[\, \partial \, F(\hat{X},B) \, / \, \partial \, X] \, dx + [\, \partial \, F(\hat{X},B) \, / \, \partial \, B \,] \, d\,B = 0 \qquad (3.1)$$

then:

$$d\,X = [\, \partial \, F \, / \, \partial \, X]^{-1} \, [\, \partial \, F \, / \, \partial \, B] \, d\,B \qquad (3.2)$$

Equation (3.1) is a very compact matrix representation of the system of equations 1' to 7c', 7k' or 7m', in differentials, of chapter VI. Equation (3.2) is a matrix representation of the total differentials of all endogenous variables in the model of chapter VI; three of these are given by the total differential of rg in expression (13 a), (14 a) and (15) in chapter VI; as can be seen, the sign of the determinant of the Jacobian matrix in (3.2) can be obtained from the stability conditions (12). This is the condition we shall use.

2) Dynamic adjustment processes in the growth models

Our next task is to define the adjustment processes followed by each model when it is out of equilibrium. Essentially, for the models formed of equations 1 to 7C, 7K or 7M of chapter VI, this consists of assigning the derivative of one of the endogenous variables to one of the equations, when the systems have been written in implicit form, such as that of equations (1) or (2) of this appendix. Then, by assuming an economically meaningful adjustment mechanism, consistent with the theory at hand, to choose signs for the speeds of adjustment.

Let us describe the economic adjustment mechanisms consistent with each model.

For the Neoclassical model, the crucial and most distinguishing feature is the assumption of full employment (or equivalently of a constant or "natural" rate of employment) for the equilibrium path. This conclusion is justified in the Neoclassical model by assuming that prices of goods and of factors of production are quite flexible and respond to situations of excess demand or supply. When demand exceeds supply in a certain market, i.e. when there is excess demand, the price of the goods exchanged will tend to rise.

In the neoclassical model, the equilibrium in the goods market is represented by equation (6c) of chapter VI, the savings/investments expression. Take equation (6c) and write it:

$$gas + S(\hat{yd}, \hat{rn}, n) - \hat{g}\ \hat{a1} / \hat{a0} = 0 \quad (13.1)$$

$$\hat{yd} = \hat{wn} + su + \hat{rn}\ \hat{a1} / \hat{a0} \quad (13.2)$$

$$\hat{g} = n + R' \quad (13.3)$$

where the hats indicate equilibrium values. If there is excess demand, savings will be less than investments.

$$gas + S(yd, rn, n) - (n + R')\ a1 / a0 < 0 \quad (14)$$

Assume that this is the case, due to a shift in savings caused by an exogenous shock. Assume also that all other relations in the model initially stay in equilibrium, and only go out of equilibrium sequentially (in time) if one of their intervening variables is affected in a previous stage, in another relation.

In this case, a possible path back to equilibrium which would be consistent with neoclassical theory would be the following. Because of the situation of excess demand, the price of the product (corn) will tend to rise; since this price is the numeraire, the wage rate in terms of corn will tend to fall:

$\dot{wg} < 0$

Notice that this could make the initial situation of excess demand worse, if we assume savings to be a normal good:

$$\partial S / \partial yd > 0$$

The fall in the real wage rate would set some other relations in the model out of equilibrium. Take equation 2 of chapter VI; it would become

$$wg\ a0 + a1\ (d + rg) - (1 - tx) < 0$$

This equation would adjust through changes in the rate of profit rg; rg would then have to rise

$\dot{rg} > 0$

This would set the marginal productivity equation 4 in chapter VI out of equilibrium as:

$$f' - (d + rg) / (1 - tx) < 0$$

In order for this equation to adjust, the capital labour ratio a1 / a0 or the capital output ratio a1 would have to decrease (since f" < 0);

$\dot{a1} < 0$ or $(a1 / a0)^{\cdot} < 0$

and this indeed would happen because of the profit maximizing behaviour of the entrepreneurs and the perfect smoothness and "well behavedness" of the production function which are all assumed in the neoclassical model.

We then come back to the savings investment relation

(13.1) above. savings would tend to go upwards thanks to the increase in rg but downward thanks to the decrease in a1 / a0; but investment would also tend to go downwards thanks to the decrease in a1 / a0. For stability it would have to be assumed that the overall effect would be equilibrating, i.e. excess demand would disappear (1).

Notice that the process just described is very much like a statement of causality in the neoclassical model. It also accomplishes the two tasks necessary to establish the dynamic model; an allocation of endogenous variables in one to one correspondance with model relations (equations); and a choice of sign for the coefficients. Hence it seems that Samuelson's "correspondence principle" is very much an indirect way to introduce causality in economic models; to break the apparent simultaneity of relations in the general equilibrium models. This is why the correspondence principle only gives useful results when the dynamic processes chosen are very specific.

As stated, the neoclassical adjustment process described so far would give correspondences and ui signs as shown in equations (2c), (4) and (6c) of table 1). In a similar way, we can choose signs for the other equations consistent

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(1) Notice that a sufficient (but not necessary) conditions for the a1/a0 effect to be equilibrating is: $\partial S / \partial k < \partial (n + R') / \partial k$ where: $k = a1 / a0$; then, ms,y < g / rg.

APPENDIX 2--TABLE 1

DYNAMIC ADJUSTMENT PROCESSES FOR NEOCLASSICAL AND NEOKEYNESIAN MODELS

Equations common to both models

$$\dot{c} = -\mathring{u}1\ [a0(c+gc)+a1(d+g)-1] \qquad (1)$$

$$(1/a0)^{\bullet} = -\mathring{u}3\ \{1/a0-\exp(R't)f[\exp(-R't)a1/a0]\} \qquad (3)$$

$$\dot{a}1 = -\mathring{u}4\ [(rg+d)\ \gamma x - f'] \qquad (4)$$

$$\dot{g}c = \mathring{u}5\ [a0\ wg\ tw+a1\ rg\ tr+tx-a0(gc+gas+gds+su)] \qquad (5)$$

Equations specific to the neoclassical model

$$\dot{r}g = -\mathring{u}2\ [a0\ wg+a1(d+rg)-\gamma x] \qquad (2c)$$

$$\dot{w}g = \mathring{u}6\ [gas+gds+s(yd,rn,n)-(a1/a0)g] \qquad (6c)$$

$$\dot{g} = -\mathring{u}7\ [g-R'-n] \qquad (7c)$$

Equations specific to the neokeynesian model

$$\dot{w}g = -\mathring{u}2\ [a0\ wg+a1(d+rg)-\gamma x] \qquad (2k)$$

$$\dot{r}g = -\mathring{u}6\ [gas+gds+s(wn+su,rn\ a1/a0)-(a1/a0)g] \qquad (6k)$$

$$\dot{g} = -\mathring{u}7\ [g-i(rn)] \qquad (7k)$$

with neoclassical theory. Equation (7c) in table 1 simply translates the tendency of the rate of investment in the neoclassical system to return towards the investment consistent with the natural rate of employment, a consequence, according to neoclassical theorists, of the tendency of all markets including the labour market, to clear, thanks to price flexibility and well behaved functions at work in the economy. Equation (5) shows the process of adjustment of government consumption necessary to restore the budgeting balance. Equation (3) shows the adjustment of production to the equilibrium values. Finally equation (1) shows the adjustment of consumption.

As is shown in table A.3-1 some of the adjustment processes described for the neoclassical case are common to, and consistent with, the other models. The most crucial processes, those which characterize the neoclassical model, are those translated into equation: (6c) and (2c) and into equation (7c). Equations (6c) and (3c) reflect the crucial neoclassical assumption that excess demand in a market causes adjusting price movements; whereas equation (7c) reflects the other crucial neoclassical assumption that the labour market clears thanks to price flexibility, perfect factor movements and well behaved production functions.

For the neokeynesian model, the most important economic adjustment mechanism refers to the expectations of profits of the entrepreneurs and their effect on investment and effective demand, If, thanks for example to an external shock, effective demand is higher than the entrepreneurs expected, the entrepreneurs will revise upwards their profits expectations and will increase investment (1). This process can be translated into the two following dynamic equations, for the neokeynesian model with Kaldor savings (the numbering corresponds to that of table 1):

$$\dot{r}g = -\text{ü}6 \ [gas + S(wn, a1 / a0 \ rn) - (a1 / a0) \ g] \quad (6k)$$

$$\dot{g} = -\text{ü}7 \ [g - i(rn)] \quad (7k)$$

Equation (2k) is simply, as in the neoclassical case, a complement to the process of adjustment of factor prices.

The other equations are shared with the neoclassical model and correspond to the adjustment processes mentioned for that model. There is, though, an economic difference. The neokeynesian model does not assume full employment

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(1) Although the neokeynesian model presented here is based on Marglin (1984), the dynamic process is different to that presented by Marglin. Marglin's mechanism stresses price responses; the mechanism here stresses quantity responses in a manner related to the models by Malinvaud (1977), Barro and Grossman (1971) and Morishima (1976).

and does not need to assume perfect flexibility of wages, nor a well behaved production function; but this doesn't change the algebraic form of the common equations (1).

In conclusion the principal economic mechanisms involved in the dynamic adjustment processes characteristic of each model are quite distinct: for the neoclassical model, they postulate rapid and complete price movements which wipe out excess demand or supply and clear all markets, including that of labour; for the neokeynesian model, they postulate changing expectations of profits and corresponding movements in investment, which equilibrate excess demand or excess supply, in what essentially is quantity accomodations (through increased investment), as opposed to the neoclassical price accomodations.

Having established the dynamic adjustment equations, grouped in table 1, the next step is to obtain the linearized models and apply the stability criteria laid out in the previous section. The set of linearized differential equations is presented in table 2.

In that table, hatted values correspond to

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(1) We shall not discuss the stability of the neomarxian model; the purpose of our discussion is to find conditions which permit us to make specific statements about shifting of the profits tax, and the shifting of this tax in the neomarxian model is totally specified at zero.

APPENDIX 2--TABLE 2

DYNAMIC ADJUSTMENT PROCESSES FOR NEOCLASSICAL AND NEOKEYNESIAN MODELS -- LINEARIZED EQUATIONS

Equations common to both models

$$\dot{c} = -\mu_1 \left\{c-\hat{c}+gc-\hat{gc}+(\hat{c}+\hat{gc})\frac{a0-\hat{a}0}{\hat{a}0}+(\hat{a}1/\hat{a}0)(d+\hat{g})\frac{a1-\hat{a}1}{\hat{a}1}\right\} \quad (1)$$

$$\dot{a}0 = -\mu_3 \left\{\frac{a0-\hat{a}0}{\hat{a}0}+\hat{A}_{r,g}\,\hat{E}_{k,1}\left[\frac{wg-\hat{wg}}{\hat{wg}}-\frac{rg-\hat{rg}}{d+\hat{rg}}\right]\right\} \quad (3)$$

$$\dot{a}1 = -\mu_4 \left\{\frac{a1-\hat{a}1}{\hat{a}1}-\frac{a0-\hat{a}0}{\hat{a}0}-\hat{E}_{k,1}\left[\frac{wg-\hat{wg}}{\hat{wg}}-\frac{rg-\hat{rg}}{d+\hat{rg}}\right]\right\} \quad (4)$$

$$\dot{gc} = \mu_5 \{(\hat{wg}\,tw-\hat{gc}-gas-gds-su)(a0-\hat{a}0)+\hat{rg}\,tr(a1-\hat{a}1) + \hat{a}0\,tw(wg-\hat{wg})+\hat{a}1\,tr(rg-\hat{rg})-a0(gc-\hat{gc}) + \hat{a}1\,\hat{rg}(tr-tr_{,0})\} \quad (5)$$

Equations specific to the neoclassical model

$$\dot{rg} = -\mu_2\{\hat{a}0(wg-\hat{wg})+\hat{a}1(rg-\hat{rg})\} \quad (2c)$$

$$\dot{wg} = \mu_6 \left\{-\hat{a}1(\hat{m}_{s,y}\,\hat{rn}-\hat{g})\frac{a0-\hat{a}0}{\hat{a}0}+\hat{a}1(\hat{m}_{s,y}\,\hat{rn}-\hat{g})\frac{a1-\hat{a}1}{\hat{a}1} + \hat{a}0\,\hat{m}_{s,y}\,\eta_w(wg-\hat{wg}) + [\hat{a}1\,\hat{m}_{s,y}+\hat{a}0\,\hat{S}2][\eta_r(rg-\hat{rg})+\hat{rg}(\eta_r-\eta_{r,0})]\right\} \quad (6c)$$

$$\dot{g} = -\mu_7\,(g-R'-n) \quad (7c)$$

Equations specific to the neokeynesian model

$$\dot{wg} = -\mu_2\{\hat{a}0(wg-\hat{wg})+\hat{a}1(rg-\hat{rg})\} \quad (2k)$$

$$\dot{rg} = -\mu_6 \left\{-\hat{a}1(\hat{m}_{s,p}\,\hat{rn}-\hat{g})\frac{a0-\hat{a}0}{\hat{a}0}+\hat{a}1(\hat{m}_{s,p}\,\hat{rn}-\hat{g})\frac{a1-\hat{a}1}{\hat{a}1} + \hat{a}0\,\hat{m}_{s,w}\,\eta_w(wg-\hat{wg}) + \hat{a}1\,\hat{m}_{s,p}[\eta_{r,0}(rg-\hat{rg})+\hat{rg}(\eta_r-\eta_{r,0})]-\hat{a}1(g-\hat{g})\right\} \quad (6k)$$

$$\dot{g} = -\mu_7\,\{g-\hat{g}-\hat{i}'\,\eta_{r,0}\,(rg-\hat{rg})-\hat{i}'\,\hat{rg}(\eta_r-\eta_{r,0}) \quad (7k)$$

equilibrium values of the models from which the linear expansion is effected.

Most of the equations are straightforward derivations from the corresponding equations of table 1. Equations (2c) and (2k) and equations (3) and (4) need some explanation. Let us derive them here. To derive equation (2c) of table 2 let us start with equation (2) of the chapter VI.

$$7x \;/\; a0 - wg - (d + rg)\; a1 \;/\; a0 = 0 \qquad (17)$$

combining with the production function:

$$7x \; \exp(R't)\; f - wg - (d + rg)\; a1 \;/\; a0 = 0 \qquad (18)$$

taking derivatives:

$$f \exp(R'\,t)\; d\, 7x + 7x\, f'\, d\,(a1 \,/\, a0) - d\, wg$$

$$- (d + rg)\; d\,(a1 \,/\, a0) - (a1 \,/\, a0)\; d\, rg = 0 \qquad (18.1)$$

By profit maximization we have:

$$f' = (d + rg) \;/\; 7x \qquad (19)$$

then; equation (18-1) can be simplified to:

$$d\, 7x \;/\; a0 - d\, wg - (a1 \,/\, a0)\; d\, rg = 0$$

or

$$d\, 7x - a0\; d\, wg - a1\; d\, rg = 0 \qquad (20)$$

Therefore we can write directly equation (2c) in its linearized form as:

$$\dot{rg} = -Q\,[a0\,(wg - \hat{wg}) + a1\,(rg - \hat{rg})] \qquad (2c)$$

Equation (2k) also follows directly

Now equation (3). Take:

$$(1/a0)^{\bullet} = -ů3 \ \{1/a0 - \exp(R't) \ f[\exp(-R't) \ a1/a0]\}$$

(21)

and linearize to obtain:

$$-(1/a0^2)\ \dot{a}0 = -ů3\ [-(1/a0)^2\ d\ a0$$

$$- f'(a1/a0)\ (d\ a1\ /\ a1 - d\ a0\ /\ a0)]$$

where

$$d\ z = z - \hat{z}\ ;\ z = a1,\ a0,\ \text{etc.}$$

By profit maximization:

$$\dot{a}0 = -ů3\ [a0^2(d+rg)/\ ?x\ \ (a1/a0)(d\ a1\ /\ a1 - d\ a0\ /\ a0)$$

$$+ d\ a0] \qquad (22)$$

Then from equation 22 (or 41) of chapter VI obtained from assuming profit maximization:

$$\dot{a}0 = -ů3\ \{\ d\ a0 + a0^2(d+rg)\ /\ ?x\ \ (a1/a0)\ Ek,l$$

$$[d\ wg\ /\ wg - d\ rg\ /\ (d+rg)]\ \}$$

or

$$\dot{a}0 = -ů3\ a0\ \{\ d\ a0\ /\ a0 + ar,g\ Ek,l$$

$$[d\ wg\ /\ wg - d\ rg\ /\ (d + rg)]\ \} \qquad (23)$$

Since a0 is positive, redefining ů3 as ů3 a0 we obtain equation (3) of table 2, by replacing dz by (z − z) (z = a0, wg, rg) in equation (23).

Finally take equation (4) of table 1:

$$\dot{a}1 = -ů4\ [(d+rg)\ /\ ?x - f']$$

and linearize:

$$\dot{a}1 = -ů4\ \{ - f''\exp(-R't)(a1/a0)\ (d\ a1\ /\ a1 - d\ a0\ /\ a0)$$

$$+ d\ [(d + rg)\ /\ ?x]\ \} \qquad (25)$$

We now have to replace the term which contains f" with a term which contains Ek,l. This is easiest done by rewritting (25) as:

$$\dot{a}1 = -\mu 4 \ \{d \ [(d+rg) \ / \ \gamma x] \ - \ f' \ (d \ f' \ / \ f')\} \quad (26a)$$

where we have left the differential of f':

$$d \ f' = f'' \exp(-R't) \ d \ k$$

$$= f'' \exp(-R't) \ (a1/a0) \ (da1 \ / \ a1 - d \ a0 \ / \ a0) \quad (26b)$$

undevelopped.

Now, from equations (19), (20) and (21) in chapter VI, which define Ek,l we can write:

$$Ek,l = \frac{- [da1 \ / \ a1 - d \ a0 \ / \ a0]}{d \ f' \ / \ f' - d \ (wg \ / \ \gamma x) \ / \ (wg \ / \ \gamma x)}$$

From which we obtain d f' / f' as:

$$d \ f' \ / \ f' = (1 \ / \ Ek,l) \ (d \ a0 \ / \ a0 - d \ a1 \ / \ a1)$$

$$+ \ d \ (wg \ / \ \gamma x) \ / \ (wg \ / \ \gamma x) \quad (27)$$

Replace (27) in (26-A) to obtain:

$$\dot{a}1 = -\mu 4 \ \{ - (f' \ / \ Ek,l) \ (d \ a0 \ / \ a0 - d \ a1 \ / \ a1)$$

$$- \ f' \ d \ (wg \ / \ \gamma x) \ / \ (wg \ / \ \gamma x) + d \ [(d+rg) \ / \ \gamma x] \ \} \quad (28)$$

replacing f'by (d+rg) / γx (marginal productivity) and simplifying we finally obtain:

$$\dot{a}1 = \mu 4 \ [(d+rg) \ / \ (\gamma x \ Ek,l)] \quad \{ - d \ a0 \ / \ a0$$

$$- d\,a1 / a1 + E_{k,1} [d\,rg / (d+rg) - d\,wg / wg] \} \qquad (29)$$

since (d+rg) / (7x $E_{k,1}$) is positive we can redefine the speed of adjustment ů4 as:

ů4 -------> ů4 (d+rg) / (7x $E_{k,1}$)

and replace the differentials

$\dot{d}\,z$ (z = a0, a1, rg, wg)

by linear expressions $z - \hat{z}$, in equation (29). Doing this we obtain equation (4) in table 2.

By writing equations (2) (3) and (4) as we have done, we have created a dynamic system which is consistent with the systems in differentials given by equations 1 to 7c′ or 7k′ of chapter VI.

This is also true of the other equations in table 2. It was necessary to do this in order to be able to relate the conditions of stability of the systems in table 2 to the Jacobian of the systems of equations 1 to 7c′ or 7k′ of chapter VI. But let us keep in mind that the forms adopted were not the only possible ones.

The systems of equations in table 2 are then written in the linearized form conforming to that of equation (6) of this appendix. From them we can write the characteristic equations of the systems, obtain the characteristic determinant in the way indicated by equation (10) and apply conditions (12) of this appenddix.

By applying condition 12 we finally arrive at the conclusion that the sign of the Jacobian of the neoclassical system, $\partial F / \partial X = Jc$, given by equation (30b) of chapter VI is:

$$Jc > 0$$

In the same way we conclude that the sign of the Jacobian determinant of the neokeynesian system $\partial F / \partial X = Jk$, given by equation (31b) in chapter VI, is:

$$Jk > 0$$

APENDIX 3 TO CHAPTER VI

RESTRICTING THE MODELS

In this appendix, we explore how restrictions arising from the stability assumptions and from direct observation of some variables, discussed in chapter VI, affect the range of variation of the shifting coefficients Ic and Ik. We attempt to answer the question posed in section C of that chapter: can these restrictions narrow the range of variation of the shifting coefficients enough to produce different and distinct predictions for the different theories?

Let us start by rewritting the shifting expressions to be examined. Recall that they represent the balanced budget incidence of a profits tax (tr) used to finance government consumption (gc). All symbols used are defined in table 1 of chapter VI. The shifting expressions are as follows.

Neoclassical shifting, Ic:

$$Ic = \frac{ms,y + (Sy / Ar,d)\ Es,r}{\{ - Ar\ Ek,l\ (ms,y - g / rn) / [(1 - ar)\ Ar,g]} \quad (1a)$$

$$+ (1 - \gamma_w / \gamma_r)\, m_{s,y} + (\xi_y / A_{r,d})\, E_{s,r} \quad \}$$

neokeynesian shifting, I_k:

$$I_k = \frac{m_{s,p} - m_{i,p}}{\{ - A_r\, E_{k,l}\, (m_{s,p} - g / r_n) / [(1 - a_r)\, A_{r,g}] - (\gamma_w / \gamma_r)\, m_{s,w} + m_{s,p} - m_{i,p} \}} \qquad (2a)$$

In chapter VI equation 1a was presented as equation (51a) and equation (2a) as (52a). When the directly measurable parameters A_r, $A_{r,g}$ $A_{r,d}$, ξ_y, g / r_n, γ_w and γ_r were replaced by observed average values (table 3 of chapter VI), thoose expressions could be written as follows.

Neoclassical:

$$I_c = \frac{(m_{s,y} + .55\, E_{s,r})}{-1.28\, (m_{s,y} - .41)\, E_{k,l} + .10\, m_{s,y} + .55\, E_{s,r}} \qquad (1b)$$

Neokeynesian:

$$I_k = \frac{(m_{s,p} - m_{i,p})}{-1.28\, (m_{s,y} - .41)\, E_{k,l} - .90\, m_{s,w} + m_{s,p} - m_{i,p}} \qquad (2b)$$

In chapter VI equation (1b) was presented as (51b) and

(2b) as (52b).

We can now examine the range of variation of those expressions in the space of non-bservable parameters ($m_{s,y}$, $E_{s,r}$, $m_{s,p}$, $m_{s,w}$, $m_{i,p}$, and $E_{k,l}$). As mentioned in section C of chapter VI, those expressions correspond to a set of hyperbolas in the parameter space. If drawn in the (I, $E_{k,l}$) plain, those hyperbolas are all asymptotic to the $E_{k,l}$ axis. The expressions for the positions of that asymptote and intercept are as follows.

For the neoclassical model, the intercept, on the $E_{k,l}$ axis, of the asymptote parallel to the I axis is:

$$A_{es,c} = \frac{-\S_y E_{s,r} / A_{r,d} - (1 - \gamma_w / \gamma_r)\, m_{s,y}}{(g / r_n - m_{s,y})\, A_r / [(1-a_r)\, a_{r,g}]}$$

$$= \frac{-0.55\, E_{s,r} + 0.10\, m_{s,y}}{0.52 - 1.28\, m_{s,y}} \qquad (3)$$

where we have called that intercept: $A_{es,c}$. Notice that it is a function of the parameters $E_{s,r}$ and $m_{s,y}$, which are not directly measurable.

The intercept of the neoclassical hyperbola with the I_c axis -- call it $I_{o,c}$ -- is:

$$I_{o,c} = \frac{m_{s,y} + (\xi_y / A_{r,d})\, E_{s,r}}{\xi_y\, E_{s,r} / A_{r,d} + (1 - \tau_w / \tau_r)\, m_{s,y}}$$

$$= \frac{m_{s,y} + .55\, E_{s,r}}{.10\, m_{s,y} + .55\, E_{s,r}} \qquad (4)$$

For the neokeynesian model the intercept, on the $E_{k,l}$ axis, of the asymptote parallel to the I_k axis -- call it $A_{es,k}$ -- is:

$$A_{es,k} = \frac{(\tau_w / \tau_r)\, m_{s,w} - m_{s,p} + m_{i,p}}{(g / r_n - m_{s,p})\, A_r / [(1 - a_r)\, a_{r,g}]}$$

$$= \frac{0.90\, m_{s,w} - m_{s,p} + m_{i,p}}{0.52 - 1.28\, m_{s,p}} \qquad (5)$$

and the intercept of the neokeynesian hyperbola wth the I_k axis -- call it $I_{o,k}$ -- is:

$$I_{o,k} = \frac{m_{s,p} - m_{i,p}}{-(\tau_w / \tau_r)\, m_{s,w} + m_{s,p} - m_{i,p}}$$

$$= \frac{ms,p - mi,p}{-\ 0.90\ ms,w + ms,p - mi,p} \qquad (6)$$

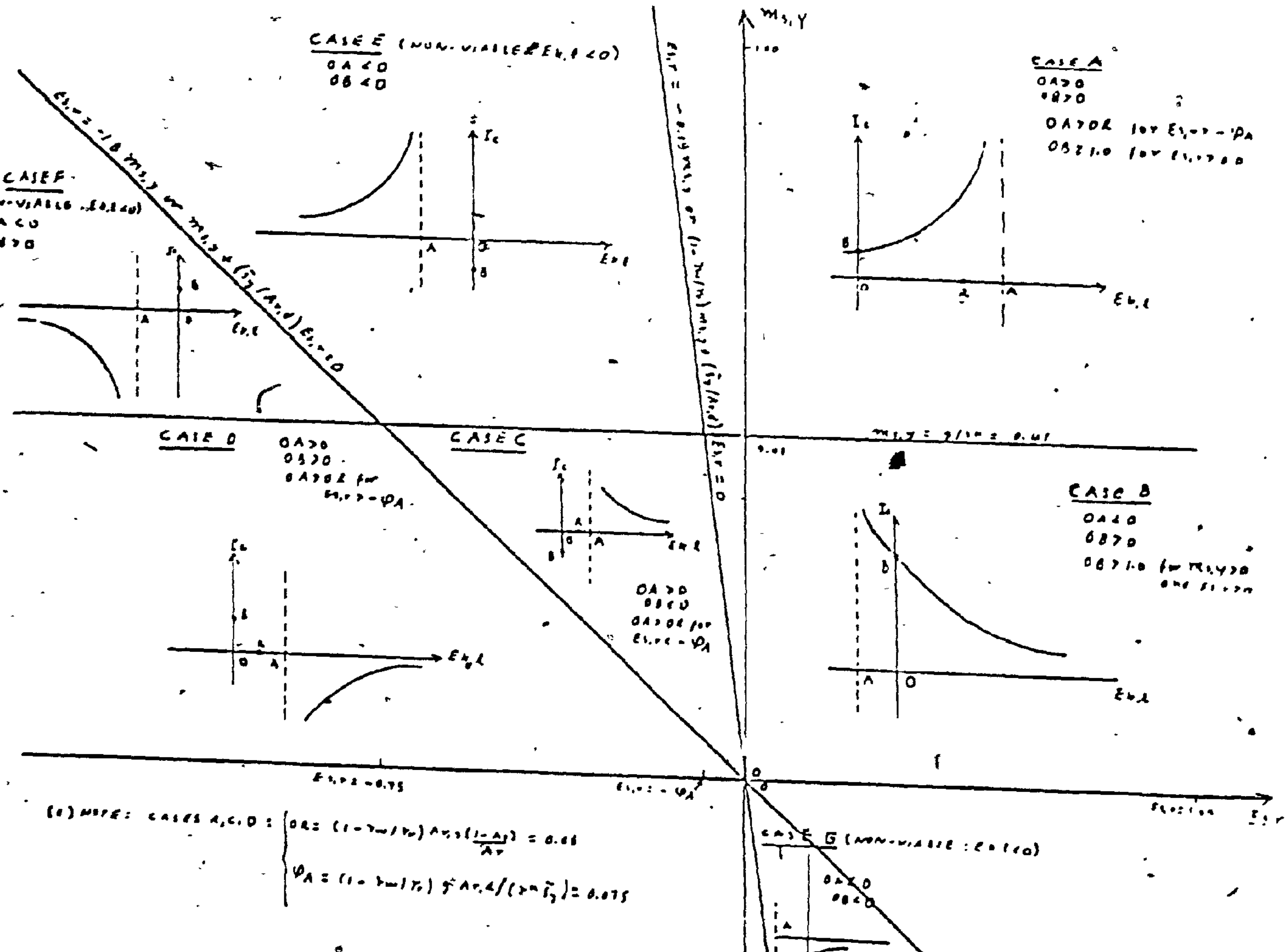
Given that, for those expressions to be positive, both their numerator and denominator have to have the same sign, and vice versa, the parameter space can be divided in regions corresponding to determined positions of the hyperbolas with respect to the (Iz, Ek,l) axes. Those regions and the corresponding hyperbolas, are drawn in Graphs 1 and 2.

Graphs 1 and 2 exclude the unstable and indicate the non-feasible branches of the hyperbolas. The unstable branches can be obtained by observing that the stability conditions (49) and (50) in chapter VI (see also appendix 2 to chapter VI) eliminate the space situated either to the right or to the left of the asymptotes in each hyperbola. The non-feasible branches are those which are entriely contained in a subspace where some parameter doesn't exist; for example, those branches entirely situated in the negative interval of the Ek,l axis, since Ek,l has to be greater than 0. It is indeed this process of exclusion of unstable and non-feasible branches which gives the possibility of limiting somewhat the range of variation of the incidence coefficients, Iz (z = c, k).

As an example, let us examine how the neoclassical "case A" curve, on the upper right hand corner of graph 1,

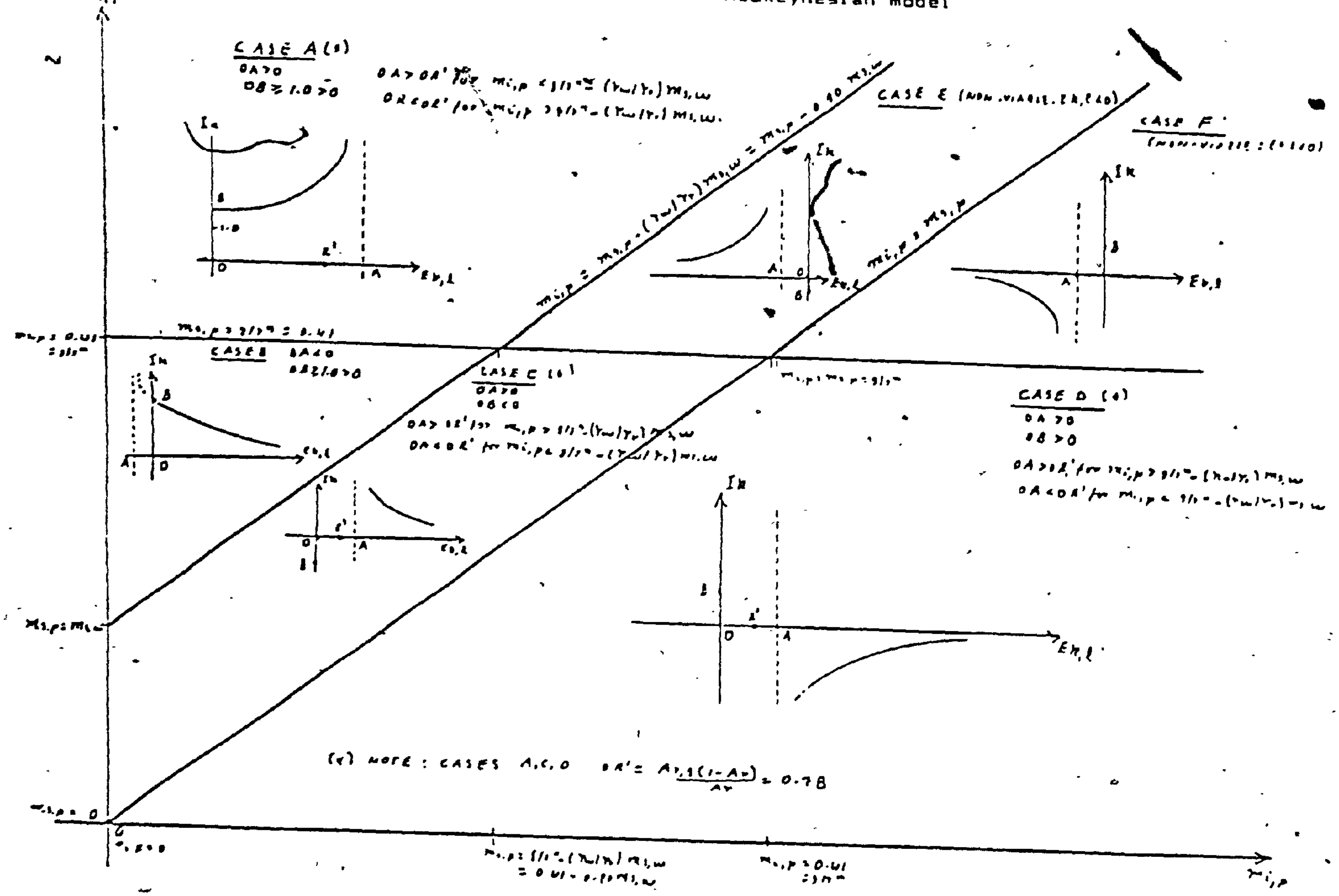
GRAPH 1

Incidence regions in the neoclassical model



GRAPH 2

Incidence regions in the neokeynesian model



was obtained. As mentioned, this hyperbola, as all those studied, is asymptotic to the $E_{k,l}$ axis; its position and shape are therefore completely determined by its intercept with the I_c axis, $I_{o,c}$ = OB, and by the intercept of the other asymptote with the $E_{k,l}$ axis, $A_{es,c}$ = OA. For case A in graph 1, both $I_{o,c}$ and $A_{es,c}$ are positive:

$$A_{es,k} = OA > 0 \qquad (7a)$$

$$I_{o,c} = OB > 0 \qquad (7b)$$

Condition (7a) implies that both numerator and denominator of equation (3) have to have the same sign; that is, either:

$$-\xi_y E_{s,r} / A_{r,d} - (1 - \gamma_w / \gamma_r) m_{s,y} > 0$$

$$\text{and} \quad (g / r_n - m_{s,y}) A_r / [(1-a_r) a_{r,g}] > 0 \qquad (8a)$$

or

$$-\xi_y E_{s,r} / A_{r,d} - (1 - \gamma_w / \gamma_r) m_{s,y} < 0$$

$$\text{and} \quad (g / r_n - m_{s,y}) A_r / [(1-a_r) a_{r,g}] < 0 \qquad (8b)$$

An examination of those inequalities shows that they limit, in graph 1, a space to the right of the straight line:

$$\xi_y E_{s,r} / A_{r,d} + (1 - \gamma_w / \gamma_r) m_{s,y} = 0$$

or equivalently : $0.55\ E_{s,r} + 0.10\ m_{s,y} = 0$

and above the line:

$$g / r_n - m_{s,y} = 0$$

or equivalently: $.52 - m_{s,y} = 0$,

plus a space to the left of:

$$\delta y\ E_{s,r} / A_{r,d} + (1 - \tau_w / \tau_r)\ m_{s,y} = 0$$

and below:

$$g / r_n - m_{s,y} = 0.$$

In other words, they limit the space comprised only of cases A, C and D in graph 1.

In the same way, condition (7b) requires the denominator and numerator of equation (4) to be both simultaneously positive, or both simultaneously negative; that is, either:

$$m_{s,y} + (\delta y / A_{r,d})\ E_{s,r} > 0$$

and $\delta y\ E_{s,r} / A_{r,d} + (1 - \tau_w / \tau_r)\ m_{s,y} > 0$
(9a)

or

$$m_{s,y} + (\delta y / A_{r,d})\ E_{s,r} < 0$$

and $\delta y\ E_{s,r} / A_{r,d} + (1 - \tau_w / \tau_r)\ m_{s,y} < 0$
(9b)

These inequalities limit, in graph 1, a space to the right of:

$$\delta y\ E_{s,r} / A_{r,d} + (1 - \tau_w / \tau_r)\ m_{s,y} = 0$$

or of $.10\ m_{sy} + .55\ E_{s,r} = 0$

and to the right of $m_{s,y} + (\delta y / d)\ E_{s,r} = 0$

or of $m_{s,y} + 0.55\ E_{s,r} = 0$

plus the space to the left of both these lines. In other words, they limit the space comprised only of cases A, B, F and D.

The common space is that comprised only of cases A and D. Therefore, conditions (7a) and (7b), or equivalently (8a) to (9b), limit the neoclassical curves with OA > 0 and OB > 0 to be located in the space comprised only of case A and of case D. Further characterisation of these two cases needs the introduction of the stability condition.

The neoclassical stability condition (49) in chapter VI says:

$$Jc > 0$$

or, since Jc is the denominator of (1a) and (1b) (ignoring division by the positive constant ?r; see equation (41a) in chapter 4):

$$- Ar\ Ek,l\ (ms,y - g / rn) / [(1-ar)\ ar,g] + (1 - ms,y\ ?w / ?r + \S y\ Es,r / Ar,d) > 0$$

If ms,y > g / rn (= .41) this inequality becomes:

$$Ek,l < \frac{-\S y\ Es,r / Ar,d - (1 - ?w / ?r)\ ms,y}{(g / rn - ms,y)\ Ar / [(1-ar)\ ar,g]} \qquad (10a)$$

If ms,y < g / rn (= .41), it becomes:

$$Ek,l > \frac{-\S y\ Es,r / Ar,d - (1 - ?w / ?r)\ ms,y}{\quad} \qquad (10b)$$

(g / rn − ms,y) Ar / [(1-ar) ar,g]

Notice that by equation (3), the right hand side of (10a) and (10b) is equal to the asymptote intercept Aes,c then by (3), (10a) and (10b), in graph 1:

Ek,1 < Aes,c (= OA) for ms,y > g / rn (= .41) (11a)

and

Ek,1 > Aes,c (=OA) for ms,y < g / rn (= .41) (11b)

Therefore in case A (ms,y > .41), by (11a), the branch to the right of AH corresponds to unstable neoclassical situations, and is assumed away. In the same way, in case D (ms,y < .41), by (11b) the branch to the left of AH is suppressed, because it is unstable.

We have then obtained case A and its corresponding parameter subspace. The last task is to examine whether there are limitations to the range of variation of the intercept OB = Io,c and of the position, OA = Aes,c , of the asymptote AH. For this, notice that equation (3), determining Aes,c , is a hyperbolic expression in terms of Aes,c and the parameters Es,r and ms,y; and equation (4), determining Io,c is a hyperbolic expression in terms of Io,c , ms,y and Es,r. What we are interested in, for case A, is to examine whether OB = Io,c attains a minimum value greater than zero or than 1.0, and if so, in what conditions; and whether OA = Aes,c attains a minimum greater than zero. Because of the nature of hyperbolas (3) and (4), with asymptotes parallel or coincident with the axis on the space: ($E_{k,l}$, $m_{s,y}$, $E_{s,r}$), there are no interior minima. Hence, the extrema either will be at some boundary in subspace A or will coincide with the asymptotes. The easiest way to analyse this is to draw the families of hyperbolas described by (3) and (4), at the ($A_{es,c}$, $m_{s,y}$) plane, with $E_{s,r}$ still as a parameter. This is shown in graph 3, A and B.

An analysis of equation (3) shows asymptotes at:

$m_{s,y} = g / rn = .41$, and at:

$A_{es,c} = [(1 - \gamma_w / \gamma_r)\ a_{r,g}] / [a_r / (1 - a_r)] = .08$,

which have been drawn in Graph 3a. Since we are studying case A of Graph 1, the branches of interest in equation (3) are those to the right of line SQ in Graph 3a, i.e. those where $m_{s,y} > .41$. Without any other restriction, it can be seen that $A_{es,c}$ can take any values from $-\infty$ to $+\infty$; but we restrict $E_{s,r}$ to be positive, or to be $E_{s,r} > -\varphi A$, or:

$E_{s,r} > [-(1 - \gamma_w/ \gamma_r)\ g / rn] / (\xi_y / A_{r,d}) = -0.075$

then the minimum value taken by $A_{es,c}$ is that of the asymptote RQ in Graph 3a, that is:

$OR = (1 - \gamma_w / \gamma_r)\ (1 - a_r)\ a_{r,g} / A_r = .08$

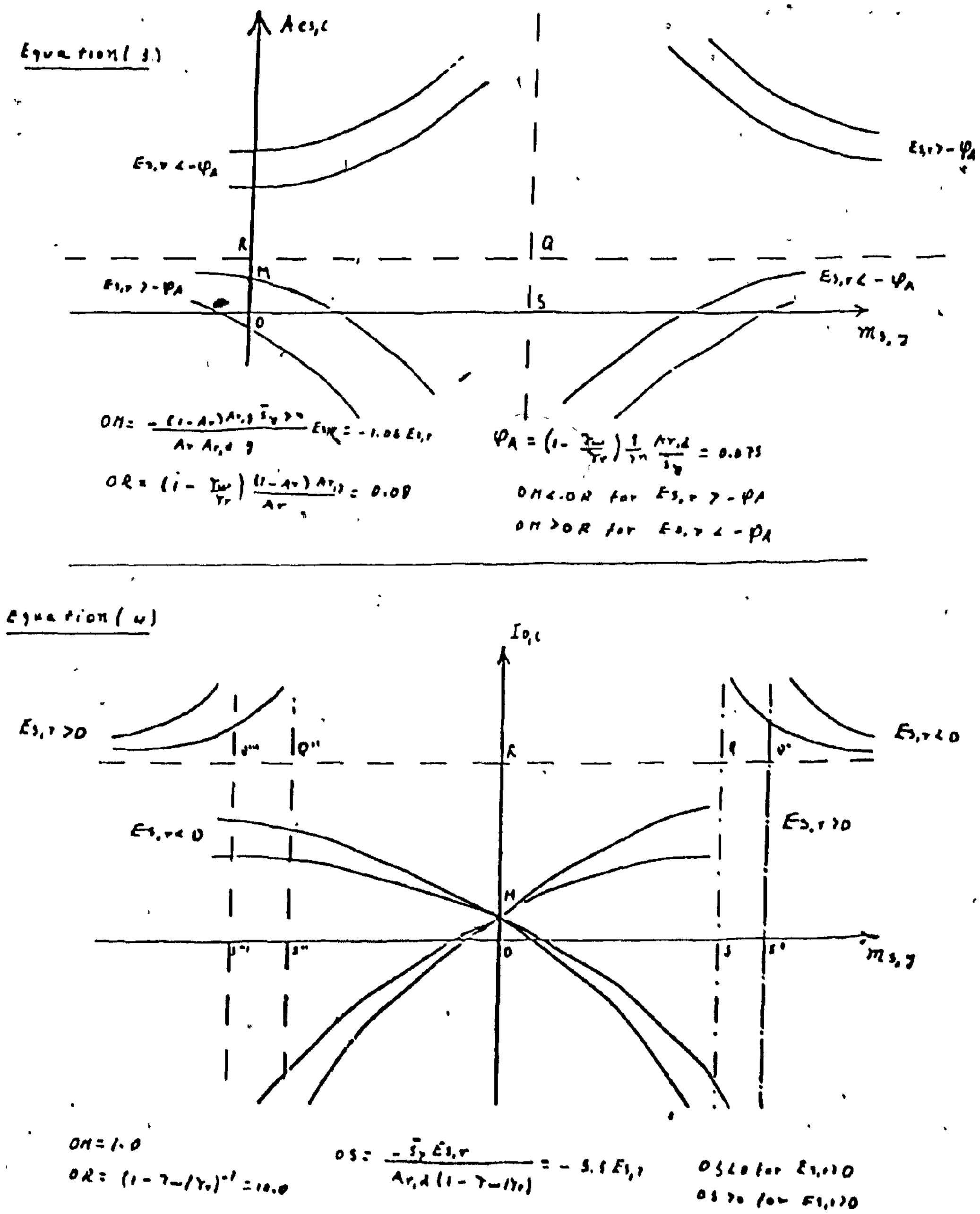
To sum up, for the neoclassical case A curve, in Graph 1, $OA = A_{es,c} > .08$ if $E_{s,r} > -.075$; this also applies, _a fortiori_, if $E_{s,r}$ is positive.

In the same way, an analysis of equation (4) shows asymptotes at:

$I_{o,c} = OR = 1 / (1 - \gamma_w / \gamma_r) = 10.0$ and at:

GRAPH 3

Neoclassical model--Families of hyperbolas given by equations (3) and (4)



$m_{s,y} = - \xi_y E_{s,r} / [A_{r,d} (1 - \gamma_w / \gamma_r)] = - 0.55 E_{s,r}$

some of which have been drawn in Graph 3b. Equation (4) also shows an intercept for all curves of the family at $I_{o,c} = OM = 1.0$, drawn also in Graph 3b. As can be seen in that graph, for values of $E_{s,r} > 0$ and for $m_{s,y} > 0$ the value taken by $I_{o,c}$ is greater than OM, since the curve's slope is positive in that region; then, *a fortiori*, for $m_{s,y}$ [illegible] $g / r_n = .41$ $I_{o,c}$ also takes values greater than $OM = 1.0$. In consequence, for case A in Graph 1, when $E_{s,r} > 0$ $OB = I_{o,c} > 1.0$.

We have then completed the analysis which shows all the restrictions which it is possible to apply to the curve case A in Graph 1. Similar analyses can be done for all other cases in graph 1 and for all the neokeynesian cases in Graph 2. The results of these analyses are summarized in those two graphs.

APPENDIX 4 TO CHAPTER VI

OTHER ASPECTS OF TAX INCIDENCE

We gave, in chapter VI, a very detailed exposition of the Balance Budget incidence of a general tax on profits, used to finance government consumption, expressed in terms of the shifting indices, Ic, Ik, and Im. The total differential expressions (13a) to (15), presented in that chapter, permit us also to obtain expressions for the incidence of any other tax under any type of government related expenditure. Indeed, those expressions can be taken as the generalized effect on profits, or the generalized shifting indices, of government fiscal policy.

It is worthwile to examine very briefly some other particular aspects of tax incidence. We have thus taken some of the most interesting tax-expenditure combinations, obtained their shifting expressions, and compared these, whenever possible, with the shifting indices of a generalized profits tax used to finance government consumption, studied in chapter VI (the indices Ic, Ik and Im).

The resulting shifting expressions are presented in Tables 1 and 2. With an example taken from Table 1, the notation employed has the following meaning:

$Irg,tr^{c,gas}$ is the elasticity of the profit rate (first sub index, rg) with respect to the profit tax complement (7r = 1 - tr; second subindex, tr) for the neoclassical model (first super index, c), when gas is taken as the endogenous government variable, i.e. when the tax increase is used to finance government savings (second super index, gas). In other words,

$$Irg,tr^{c,gas} = (rg \;/\; 7n) \quad rg \;/ \quad tr$$ (gas endogenous, neoclassical model)

Notice that in this notation the expression for the balanced budget incidence of a general tax on profits when gc is endogenous, for the neoclassical model, studied in the previous sections, is:

$$Irg,tr^{c,gc}$$

and the differential incidence of tr vs tw would be:

$$Irg,tr^{c,tw}$$

In Tables 1 and 2 we show seven cases of incidence. Two cases present the balanced budget incidence of a profits tax, first, when its increase is used to finance government autonomous savings (gas), and second, when it is used to finance subsidies or income transfers to households or employees (su). Two cases present the differential incidence of the general profits tax, one versus a tax on wages, tw, and a second one versus a

APPENDIX 4—TABLE 1

OTHER CASES OF INCIDENCE FOR THE NEOCLASSICAL MODEL

1) Shifting of a new(*) profits tax (d tr) used to spend in government savings, with a balanced budget.

$$I^{c,gas}_{rg,tr} = \frac{Nc - a1}{Dc} = I^{c,gc}_{rg,tr} - a1/Dc \qquad (1\text{-}a)$$

where: $Nc = a1\ ms,y + a0\ 62$ (1-b) ; $Dc = Jc / \partial r$

Jc: Jacobian determinant of neoclassical system (see eq. 13-b).

2) shifting of a new (*) profits tax (dtr) used to spend in government transfers (d su)

$$I^{c,su}_{rg,tr} = \frac{Nc - ms,y\ a1}{Dc} = I^{c,gc}_{rg,tr} - ms,y(a1/Dc) \qquad (2)$$

3) Differential incidence of a new (*) tax on profits (d tr) versus a wage tax (d tw)

$$I^{c,tw}_{rg,tr} = \frac{Nc - ms,y\ a1}{Dc} = I^{c,gc}_{rg,tr} - ms,y(a1/Dc) \qquad (3)$$

4) Differential incidence of of a new(*) profits tax (d tr) versus a gneral sales tax (d tx)

$$I^{c,tx}_{rg,tr} = Nc/Dc - (a1\ Nx/Dc) \gtreqless I^{c,gc}_{rg,tr} \ ; Nx = \frac{Ar(ms,y - g/rn)Ek,l + (1-Ar)ms,y}{1 - Ar} \qquad (4)$$

5) Shifting of a constant income tax (d ty) used to increase government consumption (d gc), with a balanced budget (d ty=d tw=d tyr)

$$I^{c,gc}_{rg,ty} = \frac{a0\ wg\ ms,y + Nc\ rg\ (1-0)(1-zg)}{rg\ Dc} \ \frac{(1-\theta)(1-zg) I_{rg,tr} + a0\ wg\ ms,y}{rg \cdot Dc} \qquad (5)$$

6) Shifting of a new(*) wage tax (d tw) used to spend in government transfers (d su) with a balanced budget

$$I^{c,gc}_{rg,tw} = I^{c,gc}_{wg,tw} = 0 \qquad (6)$$

7) Balanced budget impact on profits rg of a general sales tax used to incrase government consumption.

$$I^{c,gc}_{rg,tx} = \frac{\partial x[Ar(m^{g}s,y - g/rn)Ek,l + (1-Ar)(\partial w / \partial r)ms,y]}{(1-Ar)\ rg\ Dc} \qquad (7)$$

(*) These expressions obtain only for initial tr=tw=tx-gas=gc=su=0, that is for "new" government budgets.

CH VI

APPENDIX 4—TABLE 2

OTHER CASES OF INCIDENCE FOR THE NEOKEYNESIAN MODEL

1) Shifting of a new(*) profits tax (d tr) used to spend in government savings, with a balanced budget.

$$I^{k,gas}_{rg,tr}=\frac{Nk-a1}{Dc}=I^{k,gc}_{rg,tr}-a1/Dk \qquad (1-a)$$

where: $Nk=a1(ms,p-i')$; $Dk=Jk\ /\ 7r$

Jk: Jacobian determinant of neokeynesian system (see eq. 31-b).

2) shifting of a new (*) profits tax (dtr) used to spend in government transfers (su)

$$I^{k,su}_{rg,tr}=\frac{Nk-ms,w\ a1}{Dk}=I^{k,gc}_{rg,tr}-ms,w(a1/Dk) \qquad (2)$$

3) Differential incidence of a new (*) tax on profits (d tr) versus a wage tax (d tw)

$$I^{k,tw}_{rg,tr}=\frac{Nk-ms,w\ a1}{Dk}=I^{k,gc}_{rg,tr}-ms,w(a1/Dk) \qquad (3)$$

4) Differential incidence of of a new(*) profits tax (d tr) versus a general sales tax (d tx)

$$I^{k,tx}_{rg,tr}=Nk/Dk-(a1\ Nx/Dk) \ \gtrless\ I^{k,gc}_{rg,tr}-a1Nx/Dk;\quad Nx=\frac{Ar(ms,p-g/rn)Ek,1+(1-Ar)ms,w}{1-Ar}$$

5) Shifting of a constant income tax (d ty), used to increase government consumption (d gc), with a balanced budget (d ty=d tw=d tyr)

$$I^{k,gc}_{rg,ty}=\frac{a0\ wg\ ms,w+Nk\ rg\ (1-0)(1-zg)7cr}{rg\ Dk\ \ r3\ (\ 7cr/\ 7yr)}=\left[(1-0)(1-zg)I_{rg,tr}+\frac{a0\ wg\ ms,w\ 7cr}{rg\ Dk7cr}\right]\frac{}{7r3}$$

where: $7r3=1-(1-\theta)tyr-(1-\theta)(1-tyr)zg=\theta+(1-0)7yr(1-zg)$

6) Shifting of a new(*) wage tax (d tw) used to spend in government transfers (d su) with a balanced budget

$$I^{k,su}_{rg,tw}=I^{k,su}_{wg,tw}=0 \qquad (6)$$

7) Balanced budget impact on profits rg of a general sales tax used to incrase government consumption.

$$I^{k,gc}_{rg,tx}=\frac{7x[\ (ms,p-g/rn)Ek,1+(1-Ar)(7w\ /\ 7r)ms,w]}{(1-Ar)\ rg\ Dk} \qquad (7)$$

(*) These expressions obtain only for initial tr = tw = tx = gas = gc = su = 0, that is for "new" government budgets.

general sales tax (on all goods), tx. Finally, three cases present the impact, or incidence, of other taxes on factor prices: First, the balanced budget incidence of a constant income tax used to finance gc, in other words, the impact of a constant income tax on the functional distribution of income (although in most countries the income tax is not constant but increasing, in our empirical work we were obliged to use average rates). Secondly, the balanced budget incidence of a tax on wages used to finance transfers, su, to households or workers. Finally, the balanced budget incidence of a general sales tax, tx, used to finance government consumption.

The expressions in Tables 1 and 2 are obtained from the total differential expressions (13a) to (15) in Chapter VI, in the way explained at the beginning of Section C. To obtain, for example, the shifting index for the differential incidence of tr versus tw, in the neoclassical model,

$$I^{c,tw}_{rg,tr}$$

we have to proceed as follows. In the neoclassical expression (13a, Chapter VI) take all the differentials of all variables, except tw and tr equal to zero to obtain:

$$d\,rg = \frac{[a0\ wg\ ms,w\ d\,tw + (a1\ ms,y + a0\quad S2)\ rg\ dtr]}{Jc} \qquad (1)$$

Then, by the use of the differential form of the budget constraint (equation (40) in chapter VI; see also equation (5') in Appendix 1 to Chapter VI), replace dtw by its value in terms of dtr (tw is in this case taken as the government endogenous variable). We would obtain:

$$(wg\ tw - gc - gas - gds - su)\ d\ a0 + rg\ tr\ da1 + a0\ tw\ d\ wg + a1\ tr\ d\ rg = -\ a1\ rg\ dtr - a0\ wg\ d\ tw \qquad (2)$$

Notice though that the above expression contains terms in da0, da1, dwg and drg. These endogenous variables would also have to be eliminated by means of the system of equation in differentials (1') to (7C') presented in annex 1 to chapter VI.

This would result in quite elaborate shifting expressions, which would involve a great deal of simple algebraic manipulations. We can avoid this and still obtain a knowledge of the differential incidence of tr versus tw if we assume an initial situation of zero taxes and zero government expenditures. In other words, if we limit our analysis to new taxes. This simplifies expression (2) to:

$$a0\ d\ wg + a1\ d\ tr = 0$$

or

$$d\ tw = -\ (a1\ rg\ d\ tr)\ /\ (a0\ wg) \qquad (3)$$

By replacing this expression in equation (1) and reorganizing, we finally obtain Erg,tr when tw is the government endogenous variable:

$$Irg,tr^{c,\,tw} = (Nc - a1\ ms,y) \ / \ (Jc \ / \ \gamma r)$$

$$= Irg,tr - ms,y\ a1 \ / \ Dc \qquad (4a)$$

where we have defined:

$$Nc = a1\ ms,y + a0 \quad S2 \qquad (4b)$$

$$Dc = Jc \ / \ \gamma r \qquad (4c)$$

Jc is the Jacobian determinant of the neoclassical system, given by equation (13b). Expression (4a) appears as equation (2) of Table 1. All other expressions in Tables 1 and 2 are obtained in a similar form. In most cases, it was assumed that initial government taxes and expenditures were zero; the exceptions were those cases where gc is taken as endogenous since, then, that simplifying hypothesis is not needed.

In Table 2 the terms Nk and Dk stand for:

$$Nk = a1\ (ms,p - mi,p) \qquad (5a)$$

$$Dk = Jk \ / \ \gamma r \qquad (5b)$$

where Jk is the Jacobian determinant of the neokeynesian system, given by equation (14b, Chapter VI).

For the examinaation of Tables (1) and (2) it is convenient to keep in mind that Dc and Dk are positive if the models are stable because then the Jacobian Jc and

Jk have to be positive. By examining Tables 1 and 2 we can now establish the following propositions.

(1) Balanced budget incidence with gas endogenous. Using the proceedings of a new general profits tax (d tr) to finance government savings (gas), rather than government consumption (gc), would result in decreased shifting if the tax was shifted before, or in increased negative shifting if the tax was previosly negatively shifted. This is true for both the neoclassical and the neokeynesian model.

(2) Balanced budget incidence with su endogenous. Using the new profits tax proceedings to finance subsidies or transfers to households and workers, rather than government consumption, also results in a decrease in the amount of shifting if the tax was shifted, or in an increase in negative sshifting if it was not. Again, this is true for both neoclassical and neokeynesian models.

(3) Differential incidence of tr versus tw. The differential incidence of a new tax on profits rather than on wages produces, as expected, the same results as the balanced budget incidence of the profits tax used to finance subsidies to households or workers, discussed in the previous paragraph, since a subsidy is nothing other than a negative tax on hoseholds income. Again this applies to both neoclassical and neokeynesian models.

(4) Differential incidence of tr versus tx. The

differential incidence of a new profits tax rather than a generalized tax on sales, tx, may produce more or less shifting than the basic case when the increased profits tax finances government consumption, according again to both the neoclassical and neokeynesian models.

(5) Balanced budget incidence of a wages tax, tw, used to finance subsidies or transfers to households or to workers (su endogenous). In all cases shifting is zero (but notice that there is a redistribution from wage earners to those receiving social security and other government transfers).

(6) Balanced budget incidence of tx used to finance gc. In general the expressions in Tables 1 and 2 do not permit us to relate the effect to the basic case of the profits tax; the impact on profits can be greater or smaller than this case, depending on the magnitude of the different parameters in Tables 1 and 2.

Finally, Tables 1 and 2 present the balanced budget incidence of an average income tax, ty, used to increase government consumption, gc. Notice again that this tax is only an approximation to income taxation in the real world where marginal income taxes are in general increasing. The increase in the income tax is represented by:

d tr = d tw = d ty

The effective tax on wages is a composite of the income tax ty and the tax on social security, twss:

tw = ty + twss

If only the former increases, we get:

d twss = 0   d tw = d ty

On the other hand, tyr is nothing other than the effective income tax rate applied to profits:

tyr = ty   then d tyr = d ty

The resulting shifting expression for the balanced budget incidence of the income tax, ty, used to finance government consumption, gc, is presented as equations (5) in Tables 1 and 2. Again there is no simple relation with the incidence of the basic case, the general profits tax.

We can finally examine the incidence of other taxes in the neomarxian case. As discussed previously, for the neomarxian model, a profits tax increase, used to increase any item of government expenditure, is exactly borne by profits themselves; there is no shifting. But as is shown in Table 3, the balanced budget incidence of other taxes as well as the differential incidence of the profits tax is more varied. the expressions developed in Table 3 result very straightforwardly from the total differential of the neomarxian model, given by equation (15).

It can be seen in Table 3 that the elasticity of substitution Ek,l and the gross profit share Ar,g are the determinant factors in the incidence of taxes in the neomarxian framework. According to the values taken by Ek,l in all cases and by Ar,g in some, taxes can be partially shifted, or totally shifted, or negatively shifted, by the taxed factor or factors. The knowledge of

CHAPTER VI

APPENDIX 4--TABLE 3

OTHER CASES OF INCIDENCE IN THE NEOMARXIAN MODEL

1) Impact on wages of an increase in a wage tax, $(\eta/wg)\,\partial wg/\partial tw$ accompanied by a balanced budget increase on any item of government expenditure (gas, gc, or su)

$$(\eta w/wg)\,\partial wg/\partial tw = I^{m,gc}_{wg,tw} = a0/[a1(Ek,l-1)] \qquad (1)$$

2) Impact on profits of an increase in a general sales tax, tx, accompanied by a balanced budget increase on gas, gc or su

$$(\eta x/rg)\,\partial rg/\partial tx = I^{m,gc}_{rg,tx} = \frac{\eta x(Ar,g\ Ek,l - 1)}{a1\ rg(Ek,l - 1)} \qquad (2)$$

3) Differential incidence of a new (*) tax on profits (d tr) versus a wage tax (d tw)

$$I^{m,tw}_{rg,tr} = \eta r/[\eta w(1 - Ek,l)] = 1/(1 - Ek,l) \qquad (3)$$

4) Differential incidence of of a new(*) profits tax (d tr) versus a gneral sales tax (d tx)

$$I^{m,tx}_{rg,tr} = \frac{\eta r/(Ar,g\ Ek,l - 1)}{(Ek,l - 1)} = \frac{(Ar,g\ Ek,l - 1)}{(Ek,l - 1)} \qquad (4)$$

Differential incidence of a new(*) wage tax, tw, versus a general sales tax tx

$$(\eta w/wg)\,\partial wg/\partial tw = I^{m,tx}_{wg,tw}$$

$$= \eta w\ Ek,l(1 - Ar,g)/(Ek,l - 1) = \frac{(1 - Ar,g)}{Ek,l - 1}$$

(*) These expressions obtain only for initial tr = tw = tx = gas = gc = su = 0, that is for "new" government budgets.

the magnitude of those parameters is again paramount. Notice that if $E_{k,l}$ is less than one the neomarxian model would predict that the wage tax used to finance government expenditures, including social security, would be negatively shifted; the sales tax for the same purposes would on the contrary be partially shifted (since $A_{r,g} < 1$); changing a profits tax t_r by a wage tax t_w would increase gross profits and partially shift the burden to wages; but the contrary would arrive if t_r were replaced by a sales tax t_x; and finally changing a wage tax to a sales tax would decrease gross of tax wages. All this would be inverted if $E_{k,l}$ were greater than one but, in some cases, not greater than $1 / A_{r,g}$ ($1 < E_{k,l} < 1 / A_{r,g}$).

APPENDIX 1 TO CHAPTER VII

SOURCES AND PROCEDURES TO OBTAIN THE DATA

1. The Net Capital Stock Figures.

Many sources, which will be mentioned below, were examined and employed to obtain capital stock figures. For some of the countries, time series covering the period of study, 1966-1977, were available for capital stock, while for others only some years were available. Unfortunately, capital stock series from different sources can be built in such a way that they may turn out to be very heterogeneous and not adequate for cross section studies. The careful adjustments that Denison (1967) had to apply to his data are ample proof of this. Moreover, some of the sources of capital do not always clearly specify the characteristics of the stock calculated; or they do it wrongly. A case in point is Ward (1976). A table at the end, apparently presenting gross capital stock for some countries, contains in some cases net capital stock figures.

Because of all this, it was decided to use the different capital stock sources to calculate a benchmark for net capital stock for productive capital, as homogeneous as possible. Then OECD Gross investment (Gross Fixed Capital Formation) and Depreciation (consumption of fixed capital) data were used to obtain capital series for the sample countries from 1966 to 1977. OECD data, from the OECD National Accounts series, have been collected to give as much homogeneity in the procedures as possible. The OECD data excluded consumer durables but included dwellings and governmment capital. We adjusted them to eliminate dwellings in the manner explained below. Our Net Capital Stock series are, in this way, more homogeneous and better fit to estimate a production function across countries, than they would have been utilizing estimates by separate sources with little control.

Some of the original estimators used in the derivation of the benchmarks appear in Table 1. These raw data had to be transformed and completed as explained below. One of the most important steps was needed to pass from gross to net capital stock figures. We had to derive some approximate formulas to do this when no direct information of the net to gross capital ratio was available. In what follows we shall present a derivation of those formulas and their condomitant hypotheses. Then, we shall present some notes on the way the benchmarks for net capital stock and gross capital stock were obtained from the raw data. These benchmarks are shown in Table 1.

i) <u>The Ratio of Net to Gross Capital</u>. Net capital stock is simply capital stock net of accumulated depreciation. Take: GK(t) and NK(t) to indicate respectively gross and net capital stock at the beginning of year t; R(t) and D(t) as retirement and depreciation respectively at year t. Then, gross and net capital stock obey the following recursive relations:

$$GK(t) + GI(t) - R(t) = GK(t+1) \qquad (1)$$

$$NK(t) + NI(t) - D(t) = NK(t+1) \qquad (2)$$

Apply these recurrences to succesive past years; calling the first year in the past, year 0; then:

$$GK(t) = GK(0) + \quad [GI(i) - R(i)] \qquad (3)$$

$$NK(t) = NK(0) + \quad [GI(i) - D(i)] \qquad (4)$$

These two equations show the relation between (gross and net) capital l years in the past and capital now.

Subtracting (4) from (3):

$$GK(t) - NK(t) = GK(0) - NK(0) + \quad [D(i) - R(i)] \qquad (5)$$

Define the net to gross capital ratio for time j as:

$$Ak(t) = NK(t) / GK(t) \qquad (6)$$

then (5) can be written:

$$1 / Ak(t) = 1 + [\quad D(i) - R(i)] / NK(t)$$

$$+ [(1-Ak(0) NK(0)] / [Ak(0) NK(t)] \qquad (7)$$

Now, assume that the equipment has a life of L years and that it is totally retired at the end of this life. Hence, retirement at time t is equal to the Gross investment L years before:

$$R(t) = GI(t-L) \tag{8}$$

Assume also that capital accumulation has been growing smoothly enough to be represented by an average rate of growth, which we shall simply call g. In the perfectly regular growth case the economy would be in steady state and the rate of growth of capital of gross investment and of net investment would be the same. Take this to be a good representation of the "smooth growth". Then we have:

$$GK(t) = GK(0) + (1 + g)^{t} \tag{9}$$

$$NK(t) = NK(0) + (1 + g)^{t} \tag{10}$$

$$GI(t-L) = (1 + g)^{-L}\ GI(t) \tag{11}$$

Assume also a geometric depreciation rate,d, on net capital to be a good representation of the depreciation process:

$$D(j) = d\ NK(j) \tag{12}$$

Then from (10) and (12):

$$[1 / NK(t)] \quad D(i) = [d / NK(t)] \quad NK(i)$$

$$= \{ [d\ NK(0)] / [NK(0)(1 + g)^{t}] \} \quad (1 + g)^{i}$$

$$= \{d\ [(1 + g)^{t} - 1]\} / \{g\ (1 + g)^{t}\}$$

$$= (d / g)\ [1 - (1 + g)^{-t}] \tag{13}$$

In the same way from (11) and (8):

$$[1 / NK(t)] \quad R(i) = [1 / NK(t)] \quad GI(i-L)$$

$$= [1 / NK(t)] \quad GI(i)\ (1 + g)^{-L}$$

$$= [(1 + g)^{-L} / NK(t)] \quad GI(0)\ (1 + g)^{i}$$

$$= \frac{(1 + g)^{-L}[NI(0) + D]}{NK(0)\ (1 + g)^{t}} \cdot \frac{[(1 + g)^{t} - 1]}{g}$$

$$= [(g + d) / g\ ]\ (1 + g)^{-L}\ [1 - (1 + g)^{-t}] \qquad (14)$$

also

$$NK(0) / NK(t) = NK(0) / [NK(0)\ (1 + g)^{-t}]$$

$$= (1 + g)^{-t} \qquad (15)$$

Then replacing (13), (14) and (15) in (7):

$$\begin{aligned}[1 / Ak(t)] &= 1 + [1 - Ak(0)]\ (1 + g)^{-t} / Ak(0) \\ &\quad + (d / g)\ [1 - (1 + g)^{-t}] \\ &\quad - (1 + g)^{-L}\ (1 + d / g)\ [1 - (1 + g)^{-t} \\ &= 1 + [1 - Ak(0)]\ (1 + g)^{-t} / Ak(0) \\ &\quad + [1 - (1+g)^{-t}]\ [d/g - (1+g)^{-L}(1+d/g)]\end{aligned} \qquad (16)$$

If a large enough time interval has been covered $(1 + g)^{-t}$ will be small; in the limit:

$$\lim\ (1 + g)^{-t} = 0$$

Hence, formula (16) can be reduced to

$$\begin{aligned}1 / Ak(0) = 1 / Ak &= 1 + d / g - (1 + d / g)\ (1 + g)^{-L} \\ &= (1 + d / g)\ [1 - (1 + g)^{-L}]\end{aligned} \qquad (17)$$

where: d is the average geometric depreciation rate on net capital; g is the average geometric rate of growth of capital and of investment (in "smooth" situations, i.e.

which can be approximated by a steady state); and L is the average life of the capital stock.

A good estimator d^* of d from a set of empirical data is

$$d^* = D(i) \;/\; NK(i) \qquad (18)$$

Formula (12) permits then to calculate the ratio of net to gross capital stock Ak, knowing the geometric depreciation rate (applied to net capital), the rate of growth of capital stock and the average life of capital.

If instead of net capital data we dispose of gross capital data to estimate the depreciation rate, we can estimate d^*, rather than using (18), as follows:

$$d^* = D(i) \;/\; NK(i) = D(i) \;/\; [Ak \; GK(i)] \qquad (19)$$

but Ak is unknown; then call:

$$d^{1*} = D(i) \;/\; GK(i) \qquad (20)$$

therefore from (20) and (19):

$$d^* = d^{1*} \;/\; Ak \qquad (21)$$

Replace d^* by its expression (21) in equation (17) and solve again for 1 / Ak to obtain:

$$1/Ak = [1 - (1+g)^{-L}] \;/\; \{1 - (d^1/g)\; [1 - (1+g)^{-L}]\} \qquad (22)$$

where d^1 is estimated by d^{1*}, given by formula (21).

If we do not have an estimate of the average life of capital, L, we can use the following approximate relation:

$$L = 1 \;/\; d^1 = 1 \;/\; (Ak\; d) = 1 \;/\; (0.5\; d) \qquad (23)$$

Formulas (17) and (22) could have been obtained more straightforwardly by assuming a steady state. If this assumption is made it follows that capital investment, depreciation and retirements are growing at the same rate g. Taking continuous time, this can be written:

$$\dot{NK}(t) \;/\; NK(t) = \dot{GK}(t) \;/\; GK(t) = \dot{D}(t) \;/\; D(t)$$

$$= \dot{R}(t) \;/\; R(t) = g \qquad (24)$$

where we have used the Newtonian notation for time derivatives:

$$\dot{x}(t) = d\,x(t) \;/\; d\,t$$

In continuous time equations (1) and (2) can be written:

$$NK(\dot{t}) = GI(t) - D(t) \qquad (25)$$

and

$$GK(t) = GI(t) - R(t) \qquad (25)$$

then by (25) and (26):

$$NK(t) + D(t) = GK(t) + R(t) \qquad (27)$$

and dividing (27) by GK(t) = (1 / Ak) NK(t) :

$$Ak\ (g + d) = g + R(t) \;/\; GK(t) \qquad (28)$$

But we saw that in steady state:

$$Rt = GI(t - L) = (1 + g)^{-L}\ GI(t) =$$

$$= (1 + g)^{-L}\ [NK(t) + D(t)]$$

Replacing this in (28):

$$Ak\ (g+d) = g + (1+g)^{-L}\ [NK(t) + D(t)] \;/\; [NK(t) \;/\; Ak]$$

$$= g + Ak\ (g + d)\ (1 + g)^{-L} \qquad (29)$$

Solving (29) for 1 / Ak we would obtain

$$1 \;/\; Ak = (1 + d \;/\; g)\ [1 - (1 + g)^{-L}] \qquad (30)$$

equal to expression (17).

ii) <u>Obtaining Benchmarks for Gross and Net Capital Stocks</u>. As mentioned, the capital stock benchmarks were obtained taking as a starting point various incomplete estimates of capital stock which appear in table 1. What follows is a short description of how those data were combined with other information and simplifying assumptions to arrive at the capital stock Benchmarks presented in table 2.

CANADA

Urquhart and Buckley (1965) and Statistics Canada (1978) data. Urquhart's proportions of Industry Capital to Total Capital, Dwellings to total, etc. are applied to the recent figures of Statistics Canada, to obtain breakdown.

DENMARK

Denison's figures of Net Capital are used as a starting point for net capital. Then a ratio Ak^{1} of gross to net capital is obtained according to the formula (17) above, with d = D / NK = 0.0428 g = 0.053 and L, calculated as follows: L = 1 / (d / 2) ≅ 2 / d = 41 years. Then 1 / Ak is 1.64 (Ak = .61).

FRANCE.

Net capital: Malinvand's data translated to 1975 prices. A correction to Net capital in industry and government is done to exclude infrastructure from industry and include it with government. From page 141 of Malinvand (1975), the ratio between industry excluding infrastructure and

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APPENDIX 1—TABLE 1

SOME ORIGINAL ESTIMATES OF CAPITAL STOCK USED IN THE DERIVATION OF BENCHMARKS

GROSS CAPITAL STOCK

Country, year and currency	Total	Industry	Government	Dwellings	Observations and sources
Can. '65 Mill. '71C$	--	189672.4(a)	(a)	--	Stats. Can.
Can. '55 Mill. '49C$	--	38040.2	15013.1(b)	21742.1	Urquhart
Denm. '72 Bill. '60DnKr	--	86.2(a)	--	--	Groes
Fran. '65 Mill. '56Fr	1314340(d)	--	--	328170	Malinvaud et all.
Fran. '64 Bill. '56Fr	--	471.33	--	--	Denison (e)
Fran. '50 Bill. '56Fr	--	279.36	--	--	"
Germ. '60 Bill. '54DM	--	450.8X1.058= 477.3(f)	--	--	Denison;Kirner(e)
Germ. '60 Bill. '62DM	945.6(g)	--	71.9(g)	347.6	Lutzel
Ital. '62 Bill. '54Lir.	--	19661(i)	--	--	Denison, Agostinelli
Ital. '64 Bill. '54Lir.	--	22851(i)	--	--	Fua
Jap. '65 Bill. '65Yen	95542	46059(k)	24451	25032	Ohkawa et al.
Jap. '64 Bill. '65Yen	--	40634(j)	--	--	Denison,Cheng
Jap. '65 Bill. '65Yen	--	44825(k)			Denison
Swed. '68 Mill. '68SwKr	--	157979(l)	--	--	Tengblad and Westelund
U.S. '65 Bill. '72US$	--	1135.2	--	1119.6	Musgrave
U.S. '60 Bill. '54US$	--	672.0	--	--	Quotes by Hickman
U.S. '60 Bill. '58US$	--	726.7(p)	--	--	Denison
U.S. '50 Bill. '58US$	--	485.1(p)	--	--	"
U.K. '65 Bill. '70L.	146.6	33.67(q)	--	43.6	Griffin(1979)
U.K. '66 Bill. '70L	152.0	--	--	44.4	"
U.K. '64 Bill. '58L.	--	37.5	--	--	Denison
U.K. '50 Bill. '58L.	--	37.8	--	--	"

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APPENDIX 1--TABLE 1

NET CAPITAL STOCK

Country, year and currency	Total	Industry	Government	Dwellings	Observations and sources
Can. '65 Mill. '71C$	—	123359(a)	(a)	—	Stats. can.
Can. '55 Mill. '49C$	—	23041.1	9234.7(b)	12891.3	Urquhart
Denm. '64 Bill. '53DNKr	88.8	54.3	3.8	30.6	Denison(e)
Denm. '55 Bill. '55DNKr	56.6	31.9	2.4	22.2	"
Fran. '65 Mill. '56Fr	657170	385280(c)	72350(c)	199540	Malinvaud et all.(e)
Fran. '63 Mill. '56Fr	590280	261494(e)	152116(e)	176670	"
Fran. '64 Bill. '56Fr	—	247.72	—	—	Denison(e)
Fran. '50 Bill. '56Fr	—	147.62	—	—	"
Germ. '60 Bill. '54DM	—	288.9X1.058= 305.9	—	—	Denison, Kirner(e)
Germ. '60 Bill. '62DM	521.0	287.1	39.8	194.1	Lutzel
Ire. '65 Mill. '58L.	—	316.0(g')	—	—	Kennedy(1971)
Ital. '62 Bill. '54Lir.	—	20028(h)	—	—	Denison, Agostinelli
Ital. '50 Bill. '54Lir.	—	12360(h)	—	—	Denison
Jap. '64 Bill. '65Yen	—	22860	—	—	Denison, Chung(j)
Jap. '65 Bill. '65Yen	—	24806	—	—	Denison
Nor. '65 Mill. '70NrKr	235436(s)	145584	36593	53259	Stat. Arbok (1977), Nor.
Nor. '60 Mill. '55NrKr	—	60975	—	—	Denison
Nor. '50 Mill. '55NrKr	—	36140	—	—	"
Nor. '60 Mill. '50US$	—	12609	—	—	Colin Clark Econ. Record (1970)
Swed. '68 Mill. '68SwKr	600850	—	—	—	Ward
Swed. '65 Mill. '68SwKr	533423	—	—	—	Ward
U.S. '65 Bill. '72US$	645.9	—		721.4	Musgrave
U.S. '60 Bill. '54US$	—	360.6(m) or 293.0(n)	—	—	Bert Hickman (quotes)
U.S. '60 Bill. '50US$	—	387.5(p)	—	—	Denison
U.S. '50 Bill. '50US$	—	260.2(p)	—	—	"
U.K. '65 Bill. '73L.		21.25(q)	—	—	Griffin
U.K. '64 Bill. '50L.	—	34.4	—	—	Denison
U.K. '50 Bill. '50L.	—	19.5	—	—	"

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APPENDIX 1--TABLE 1

Footnotes

(a)industry plus government

(b)includes institutional capital

(c)Industry includes infrastrunture ,government excludes it

(d)estimated using Malinvaud et all.(1965) estimated ratio of net capital to gross capital of .5 ;our estimated breakdown between gov. assets and industry using Malinvaud et all.(1965) tables 5.6 and 5.7

(e) stock figures at the beginning of year

(f)correction to include Saar and West Berlin

(g)excludes roads dams and similar forms of public construction

(g')manufacturing only

(h) excludes agriculture

(i)only eqipment ; end of year figures

(j) year end figures

(k)construction work in progress excluded

(l)construction ,transport and finance, insurance and real estate and community social and personal services are missing;also, probably, part of agriculture.

(m) U.S. net stock ;straight line depreciation, bulletin F lives

(n)Net stock double rate declining balance depreciation rates, bull. F lives

(p) does not include government owned enterprises (estimated at around 4% of capital stock).

(q)manufacturing only

(s)includes land and forests and inventories industry including it is: 44.3/59.4 = .745 in 1963.

Gross Capital: also Malinvand's figures for total capital.

GERMANY

Gross capital:Lutzel's (1977) beginning of year figures (equal to Ward's (1976) end of year figures) and Lutzel's proportions with a correction for infrastrucutre (roads, dams, etc.). Proportions are also taken from Lutzel's, as well as net to gross capital ratios for 1970, which are used to obtain Net capital benchmarks.

IRELAND

Starting figure is Kieran-Nevin estimate of net capital in manufacturing. Then, using ratios of cumulated gross investment for: manufacturing / (total capital), (total capital) / industry etc., as explained in Italy's note, estimates of total capital and capital for dwellings, industry and government are obtained, which are then translated into constant '75 prices. For estimates of total gross capital, the Gross / Net capital ratio, 1 / Ak, is obtained using formula (17) above; 1 / Ak was 1/0.627. The ratio of cumulated GI in manufacturing to cumulated total gross investment for 1970-76 was 0.181. Total GK obtained for 1975 is: 5525.3 Mill '75 Pounds. (Data for Gross Investment from OECD, National Accounts, 1950-71, Vol. 1 (Publ.No. 30.81.01.3).

ITALY

Starting point is Denison (Agostinelli) figure for Net capital in industry, excluding agriculture, for 1962. From

The OECD National Accounts 1960-77, Vol. II (Publ. No. 30.79.03.3) page 141 we obtain; by sectors, accumulated figures for gross investment by summing over the years 1970-1977. This is done for total-investment, agriculture, all industry, dwellings (Residential Buildings), Government; and also for machinery and equipment including transportation. From these figures proportions between industry and other sectors are developed. The application of these proportions to net capital results in estimates of net capital for government, dwellings, industry including agriculture, and total net capital. The proportion of machinery and equipment is used to obtain a figure of net capital for these items; this figure, multiplied by the GK / NK = 1 / Ak proportion is not far from Fua's estimate for 1964 which gives a check of the procedure. The 1962 net capital is then carried on to 1965 by adding the gross investment minus the depreciation in the intermediate years. The gross capital figures are obtained by applying a gross to net capital ratio of 1 / Ak = 1.5565 (Ak = 0.64), obtained by formula (17) using data from Denison from 1950 to 1965, with g = 0.0416 and d = D / NK = 0.0501.

JAPAN

Data from Ohkawa, Shinohura and Meismer (Eds.), Table A-43, are the starting point; we take their data for total gross capital stock and translate it to 1975 prices. We also take their estimates for gross capital stock of Dwellings, Industry and government. To obtain Net capital stock, we use Denison-Chung's data for industry; for total net stock a ratio Ak (NK / GK) of .67 is estimated using formula (17). d is estimated from OECD depreciation data combined with the GK data of Ohkawa et al. (1979); rate of growth is obtained comparing GK in 1970 and 1954 from Ohkawa's series. Net capital for dwellings is obtained by using the proportions of GK Dwelling to GK total minus GK industry; since both government and dwellings capital is mostly structures this procedure seems adequate.

NORWAY

Net capital estimates for 1965, for industry, government dwellings and total, come from the Statistik Arbok (Statistical Yearbook) of 1977, Central Bureau of Statistics, Oslo, page 67. The figures exclude land, forests and inventories, as do Denison's figures, which are based on an older edition of the same source. Gross capital is calculated by multiplying net capital by the net to gross capital ratio 1 / Ak estimated to be (1/.66) = 1.51, by formula (17) with: d = 4.25% and g = 5.00%; g is estimated from the capital figures in the Statistik Arbok; d is calculated using depreciation and gross investment data from OECD and the capital stock figures of the Arbok. Colins Clark's estimate of Net capital for Norway in Dollars, when translated to Krone of 1955 is 69170 Million Krone which compares well, and confirms, the statistical Arbok figures used by Denison.

SWEDEN

Data for total gross capital from Ward were the starting point of the estimation of the benchmark. Since, in spite of Ward's assertion, it was not sure that these figures were gross capital; a check was done using Tengblad and Westelund (1976) figures of gross capital stock for 1968, which are incomplete: they do not include construction, transport and finance, insurance and real estate, community social and personal services, and government and dwellings. The OECD national accounts, 1969-77 (Vol. II, page 149) contain data for constant Krone and current Krone gross investment for the period 1960-77, at a very detailed level. The constant data were cumulated (summed up) by detailed group for the period. Then, ratios of this cumulated investment were obtained and applied to Tengblad and Westelund data. The result was an estimate of total gross capital stock of 559,000 million of 1968 Krone for 1968, quite comparable to Ward's similar figures for 1968 of 600,850 1968 Krone. Hence, Ward's figure of total gross capital stock for 1965 was taken as a starting point. Using the ratios of cumulated GI, obtained as explained before, estimates of gross capital stock for dwellings and government were obtained. Finally subtracting these from the total, an estimate of capital stock for industry was obtained. To obtain the total net capital stock benchmark, a ratio Ak of net to gross capital stock of 0.645 was applied. This ratio was obtained from formula (22) above.

The data for Sweden were: $g = 0.039$, $d^1 = D / GK = 0.0226$ and $L = 1 / d^1 = 44$ years. For dwellings and government, a ratio Ak of 0.679 was applied, implying a life of 60 years, instead of 44, for this type of capital, which is mostly composed of structures. The net capital for industry was obtained as the difference of total net capital and that for dwellings and for government.

UNITED STATES

Data for 1965 Gross and Net capital for industry and dwellings are from Musgrave "Fixed non-residential business and residential capital in the U.S. 1925-1975" Survey of Current Business, April 1976, and idem. August 1979. Musgrave's figures are translated to 1975 prices and to beginning of year. For the government capital stock, a ratio of cumulated investment in the government sector vs. cumulated total investment in industry plus dwellings was obtained from OECD National Accounts, for the period 1960-1977. This ratio was applied to industrial plus dwelling capital, to obtain an estimate of government Gross capital stock. The net capital stock for government was obtained by applying a net to gross ratio Ak of 0.648 obtained from residential structures (dwellings).

UNITED KINGDOM

Total gross capital stock for all sectors and for dwellings, for 1965, comes from Griffin (1977, 1979). Net capital for Industry in 1965 is estimated by using Denison's data for 1964 (beginning of year) and adding an estimate of Gross investment minus depreciation for 1964, from OECD national accounts. Gross capital is then obtained for industry by applying the overall gross to net ratio 1/Ak obtained from Denison's 1964 figures. Gross capital for government is finally obtained as residual. To obtain net capital for all sectors, it was assumed that the net to gross capital ratio for industry from Denison's 1964 figures would be representative of all sectors; this ratio was Ak = 0.60. Applying it to the gross capital figures, net capital figures were estimated.

iii) The Construction of the Total Net Capital and Productive Net Capital Stock Figures -- 1965 - 1977.

Once obtained, the benchmarks shown in Table 2 served as initial values to construct total net capital stock series from 1965 to 1977. For this, the depreciation and Gross investment data from the OECD national accounts (1980a,1980b) were also used. The formula used is simply:

$$NK^{to}(t+1) = NK^{to}(t) + GI^{to}(t) - D^{to}(t) \qquad (31)$$

$$t = 1965, 1966...1976$$

where: $NK^{to}(t)$ is total net capital at beginning of period t. $NK^{to}(1976)$ would be the total net capital benchmark. $D^{to}(t)$ is total depreciation expense during period t. $GI^{to}(t)$ is total gross investment during period t. All data are in constant 1975 prices.

To obtain productive capital series, the process was

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APPENDIX 1--TABLE 2

BENCHMARKS FOR NET AND GROSS CAPITAL IN 1965 (BEGINNING OF YEAR) IN 1975 PRICES AN DOMESTIC CURRENCIES

NET CAPITAL

Country and currency			Total	Industry	Government	Dwellings
Canada	Mill.	'75C$	252189	128649	51562	71978
Danemark	Mill.	'75DNKr	300900	184200	12900	103800
France	Mill.	'75Fr	1209871	527635	315435	366801
Germany	Bill.	'75DM	1638.6	784.4	362.4	491.8
Ireland	Mill.	'75L.	5525.3	4039.0	292.8	1193.5
Italy	Bill.	'75Lir.	146317	103996	9401	32920
Japan	Bill.	'75Yen	110864	42570	32188	36106
Norway	Mill.	'75NrKr.	343905	202406	57627	83872
Sweden	Mill.	'75SwKr	547331	317542	97027	132762
U.S.	Mill.	'75US$	2050800	812700	309600	928500
U.K.	Mill.	'75L.	173600	104600	17000	52000

GROSS CAPITAL

Country and currency			Total	Industry	Government	Dwellings
Canada	Mill.	'75C$	392095	199416	78702	113977
Danemark	Mill.	'75DNKr	493500	302100	21200	170200
France	Mill.	'75Fr	2416066	—	—	—
Germany	Bill.	'75DM	2452.9	1196.6	574.3	682.0
Ireland	Mill.	'75L.	—	—	—	—
Italy	Bill.	'75Lir.	227744	—	—	—
Japan	Bill.	'75Yen	164951	77966	40998	45987
Norway	Mill.	'75NrKr	520832	—	—	—
Sweden	Mill.	'75SwKr	847270	508846	142898	195526
U.S.	Mill.	'75US$	3377000	1447300	[illegible]82600	1447100
U.K.	Mill.	'75L.	289300	174300	28400	86600

APPENDIX1--TABLE 3

AGE OF MACHINES AND EQUIPMENT COMPARED TO AGE OF STRUCTURES IN SOME COUNTRIES

Country	Machines and equipment	Structures	Ratio of average ages	Ratio of greater ages
Canada	6, 35	20, 7	.43	.47
Norway	5, 12, 30	25, 60, 100	.25	.30
Sweden	20, 25	~65	.37	—
U.S.	—	65, 80	—	—

Source:table 2, page 215, OECD. The growth of output 1960-1980, Dec 1970 [(OECD) 11.70.02.1] somewhat more elaborate, because we did not dispose of separate depreciation series for productive capital and for dwellings. Productive capital is taken here as Industry plus government capital or, equally, total capital minus dwellings capital.

The problem of depreciation was solved as follows. Call depreciation of productive capital per period D^{pr}, that of Dwelling Ddw, and that of total capital D. Then we have the identity:

$$D(t) = D^{pr}(t) + D^{dw}(t) \qquad (32)$$

Now taking geometric depreciation rates di on net capital NKi(t) , we have:

$$D^{i}(t) = d^{i}\ NK^{i}(t) \qquad i = pr,w \qquad (33)$$

then: replacing in equation (2)

$$d\ NK(t) = d^{dw}\ NK^{dw}(t) + d^{pr}\ NK^{pr}(t) \qquad (34)$$

or

$$d = d^{dw}\ \theta^{dw} + d^{pr}\ \theta pr \qquad (35)$$

where θi is the share of Net capital of class i in total net capital.

$$\theta i = NK^{i}(t)\ /\ NK(t) \qquad (35a)$$

Finally (5) can also be written:

$$1 = (d^{dw}\ /\ d)\ \theta^{dw} + (d^{pr}\ /\ d)\ \theta pr \qquad (36)$$

and calling

$$d^{i} = B^{i}\ d \quad ,\ i = pr,\ dw \qquad (37)$$

$$1 = B^{dw}\ \theta^{dw} + B^{pr}\ \theta pr \qquad (38)$$

Notice also that:

$$\theta^{dw} + \theta pr = 1.0 \qquad (38a)$$

Notice that productive depreciation expense can be written:

$$D^{pr} = d^{pr}\ Kpr = B^{pr}\ d\ (Kpr\ /\ K)\ K = B^{pr}\ \theta pr\ D \qquad (39)$$

So if we know total depreciation expense D, the ratio of productive capital to total capital, θpr, and the ratio of the productive depreciation rate (d^{pr}) to the total depreciation rate (d), we can obtain depreciation expense of productive capital, D^{pr} by applying formula (9).

From the Benchmark Table 2 we can get initial values of θpr as: $\theta pr = (NK^{ind} + NK^{gov})\ /\ NK^{to}$

We also have the OECD series of total depreciation Dt. We then need estimates of Br. For this, it is easier to obtain an estimate of B^{dw} and using formula (38) to obtain B^{pr} as:

$$B^{pr} = (1 - B^{dw}\ \theta^{dw})\ /\ \theta pr \qquad (40)$$

Now B^{dw} is:

$$B^{dw} = d^{dw}\ /\ d = (1\ /\ Ldw)\ /\ (1\ /\ L) = L\ /\ Ldw \qquad (41)$$

where Ldw is the useful life of dwelling capital; L is that of total capital stock.

Some typical ages of machinery and equipment and structures are shown in Table 3. It can be seen there that the average ratio of structure ages to machinery and equipment ages is around .35, and the average age of structures is around 60 years. On the other hand, the average age of total capital used in our study, as the inverse of a linear depreciation rate, is around 30 years, i.e. it has a ratio to structures of .5. These numbers are of course very approximate; but they give an appreciation of the approximate magnitude of the ratio. We have chosen a ratio of 0.4 which seems reasonable in the light of those numbers:

$$B^{dw} = 0.4$$

Then, since for the benchmark year, 1965, θ^{dw} is in the average around .30, we have for 1965:

$$\theta^{dw} = 0.30$$

then, by eq (38a) $\theta pr = 0.70$

and

$$B^{pr} = (1 - 0.40\ \theta^{dw}) \ /\ \theta pr = 1.26$$

Then, on the average (since these values change with country and year):

$$d^{dw} = 0.40\ d \qquad \text{and} \qquad d^{pr} = 1.26\ d$$

The depreciation rate for productive capital is 26% higher than the depreciation rate of total net capital. Notice also that B^{pr} is not very sensitive to changes in B^{dw}. If, for example, B^{dw} is taken as 0.3, B^{pr} becomes 1.30 instead of 1.26, a difference of only 4%. Hence the value of B^{pr} is quite robust with respect to changes in the ratio of ages of capital.

Using formula (39) and, for each country:

$$NK^{pr}(t+1) = NK^{pr}(t) + GI^{pr}(t) - D^{pr}(t)$$

$$= NK^{pr}(t) + GI^{pr}(t) - B^{pr}\ \theta pr\ D \qquad (43)$$

t = 1965, 1966... 1976

i = pr, dw

It is possible then to calculate, for each country, a series of total productive capital, using OECD's figures

for gross investment in productive capital, total depreciation, $\theta pr = 1 - \theta^{dw} = 0.70$ and $B^{pr} = 1.26$.
If desired the process can be iterated to obtain updated values of B^{pr} from formula (42). It is a very simple matter to program this process in fortran, or even a statistical language like TSP. The exercise results in series of: Net productive capital, NK^{pr}, and depreciation expense for productive capital, D^{pr}, and for dwellings, D^{dw}.

2) Correction for international purchasing power. The purchasing power parity indices.

These indices were taken from Kravis and Summers (1978, table 1.8). In this work, indices of international purchasing power parity are given for the years 1970 and 1973 for four categories of expenditures: consumption, capital formation, government expenditures and the gross domestic product. Since for each country, we deflated all relevant variables to constant national purchasing power of 1975, the 1973 values in table 1.8 were used, on the assumption that relative international purchasing power and exchange rates did not change appreciably between 1973 and 1975.

From table 1.8, then, the following purchasing power parity indices were obtained directly (U.S=100.00):

Country	Consumption index,PIPPC	Capital formation index PIPPK	Government index PIPPG	GDP index PIPPY
France	1.07	.97	.95	1.01
Germany	1.26	1.01	1.26	1.16
Italy	.87	.85	.95	.87
Japan	.91	1.09	.85	.94
U.S.	1.00	1.00	1.00	1.00
U.K.	.85	.84	.75	.84
Denmark,Norway,Sweden	1.02	1.02	1.13	1.01

The index for the scandinavian countries was built from data in table 1.8 by assuming it to be equal to the average of the indices for Belgium and the Netherlands. The U.S. index was also assumed good for Canada (given the great similarities of price structures and their correlation). Finally the italian indices were also used for Ireland (notice that they are also very similar to the U.K.'s).

The purchasing power parity international indices can then be used to deflate the diverse variables of the model,already expressed in constant "national" dollars .The consumption index is used to correct private consumption; the capital formation index is used to correct investment and capital; the government index corrects the various government expenditures,such as the government consumption, the subsidies, other expenses, etc. Finally, the GDP index is used to correct the national product.

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