The Pricing of Stock Options and The Efficiency of the Trans Canada Options Market

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ABSTRACT

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This study is primarily concerned with pricing of options and the efficiency of the Trans Canada Options market. A procedure involving a minimization problem and the Newton-Rahpson technique for simultaneous equations is developed in order to estimate the parameters, σ^2 and r, of the Black-Scholes model. This essentially tests the functional form of the model. The results are unsatisfactory even when first order serial correlation is removed. Accordingly, the validity of the assumption that stock price changes follow a geometric Brownian motion with constant drift and intertemporally constant variance is investigated. It is found through a MINQUE-type estimator and by use of the Box-Jenkins methodology that daily variances are generally white noises with a constant mean. It is also demonstrated that the Black-Scholes price calculated by use of the MINQUE estimator of σ^2 is a better predictor than that using the historical variance.

Next, the economic and the statistical versions of the random walk hypothesis are tested for the TCO. Although substantial arbitrage profits were found through the Put-Call Parity relationship, these disappear when trading costs for Canadian markets were taken into

account. Finally, the role of the options market as an information generation mechanism is investigated. Using Pierce-Haugh causality tests, the Stiglitz-Grossman paradox is considered for possible resolution in the Options market. Also a non-parametric test of the variance shift is used to decide if the listing of stocks on the TCO is an economic signal and contains useful information.

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CHAPTER 1

INTRODUCTION

This thesis is primarily concerned with the pricing of options and the efficiency of the options market within the Canadian context. It is noted that very limited research exists in the literature in the area of option pricing in Canada. These are one unpublished paper by Mandron [1978] and three published studies by Chua and Mokelbost [1980], Mokelbost [1977] and Mandron and Perreault [1983]. Consequently, much of the research contained in this thesis is new.

This study is organized within the following framework. In Chapter 2, the Black-Scholes model is reviewed and a standard procedure for its solution is presented. Special emphasis is placed upon the inter-relationships between the Black-Scholes hedging technique (and its associated self-financing property) and the Ross riskless arbitrage approach. A review of the literature includes the several attempts made to relax one or more of the assumptions of the Black-Scholes model. Further, models in which options play the role of completing the underlying security market, are summarized.

In Chapter 3, the problems in estimating σ^2, the variance of stock price returns, are fully discussed, and the different approaches used by many authors are analysed. Following a suggestion by Cox and Ross [1976] for simultaneously estimating the parameters σ^2 and r

(the risk free interest rate) a procedure, involving a minimisation problem, is developed. This procedure uses the Newton Raphson algorithm and essentially offers a test of the functional form of the B-S model. Krausz [1979] and O'Brien and Kennedy [1982] also applied a similar technique for CBOE options. However, both these studies suffered from major drawbacks that are avoided by our methodology. Specifically, their procedure tries to find a feasible solution to $C_i^a = f_i(r,\sigma)$, $i = 1,2$, where

C_i^a = actual market prices

f_i = B-S model prices.

Such a system can have multiple solutions, so that trying to find a feasible solution is unsatisfactory. Also, the samples used in both studies were very limited and the estimation done at only one point in time, so that very limited "snapshots" of the process were obtained. Finally, if noise enters the model, through measurement errors (or some other reason), then the estimates may be entirely unreliable.

Accordingly, the procedure used herein, is to estimate $C_{it} = f_{it}(r,\sigma) + \varepsilon_{it}$ where $i = 1, \ldots. n$, and where n is the number of options on a stock which all terminate on the same date. Furthermore, a grid-search method was devised to select an appropriate initial vector (r_0,σ_0). The general finding was that the estimates of r,σ were widely dispersed and the $\hat{r}$ values were in general

quite low relative to the appropriate Treasury Bill rate. A model incorporating first order serial correlation was then formulated and estimated. Again the results (see Tables 3.9 (a) - (d)) range from very good for Northern Telecom to extremely poor $(\hat{r} < 0)$ for Husky Oil. This might signal a non-stationary variance of stock price returns. Accordingly, this problem is considered in chapter 4.

In Chapter 4, the central theme is to investigate the validity of the assumption that stock price changes follow a geometric Brownian motion with constant drift and intertemporally constant variance, and to study the implications of the results obtained for the pricing of options. The chapter is organized as follows. Firstly, the Kolmogorov-Smirnov test of non-normality is applied to the monthly data for a sample of stocks on the TSE. The general conclusion is that for a sample of 20 stocks for the period 1970-1979, the empirical distribution is approximately normal but that monthly variances of stock price changes are non-stationary in that they generally vary directly with the square of the market rate of return.

Secondly, the proposition of Mandelbrot [1963] that the distribution of daily stock price changes is not normal but belongs to the stable Paretian family with infinite variance is examined. The test is based on the work by Fama and Roll [1971] on symmetric stable distributions. The conclusion here is that for daily closing stock prices on the TSE over a sample period of 1 year, the empirical

distribution was generally found to belong to the symmetric Paretian family with the characteristic exponent α having a modal value of 1.4 . This shows that the distributions have much fatter tails than those obtained for the US (see for example, Teichmoeller [1971] and Osborne [1974] where $\hat{\alpha} \approx 1.7$). Further, the fact that in all cases a significant kurtosis coefficient was found, concides with this fat-tailed finding.

Finally, in this chapter a MINQUE-type estimator was developed. Using the Box-Jenkins methodology, suitable ARIMA models for the daily variances of stock price changes were specified and estimated. Daily variances are generally white noise with a constant mean, although an autoregressive model was the best fit for some time series $\{\sigma_t\}$. Finally, using the forecasted value, $\hat{\sigma}_R$, from the ARIMA model, the B-S price was re-calculated, and the forecasted price, $\hat{C}_R$, was a better predictor of the observed option price than the one obtained by using the historical variance.

In Chapter 5, empirical tests of the economic and statistical versions of the random walk hypothesis are conducted. The identification of the price formation mechanism for speculative markets for such assets as securities and commodities is an important problem in financial economics. Specifically, the question of whether or not such markets are efficient depends crucially on the behavior of successive price changes over time. The random walk hypothesis asserts that successive price changes are independent. This is the so-called statistical form.

For a sample of 18 options on the TCO, the hypothesis that $R_{t+1} - R_t = \omega_t$, where $R_t = \ln(C_{t+1}/C_t)$, C_t = closing option price at time t , ω_t = white noise, cannot be rejected. This is in contrast to the empirical finding by Leabo and Rogalski [1975] that warrants on the NYSE and ASE follow a restricted random walk with reflecting barriers.

The second version, or the so-called economic version, asserts that security markets are efficient in the sense that arbitrage opportunities; based on the information contained in past prices and past price changes, cannot exist, modulo transactions costs. The test of this hypothesis that was employed herein, used the Put-Call Parity Theorem. In the absence of transactions costs, a significant number of arbitrage opportunities with profits of a relatively high magnitude, was identified. In order to determine if these arbitrage opportunities could persist in the presence of transactions costs, the trading costs for options on the TCO and their associated stocks were then estimated. The estimated trading costs were considerably higher than those obtained by Phillips and Smith [1980] for the U.S., which is probably due to the relatively thin nature of the capital markets in Canada. Using these estimated trading costs for Canadian markets, the abnormal profits previously obtained were entirely eliminated.

Finally in Chapter 6, the role of the options market as an information-generation mechanism is investigated. Recently Chang [1983], obtained an option pricing formula under the assumption of an incomplete market, and then rationalized the role of the options

market as one which completes the underlying securities market. In this chapter, another possible role of the options market is investigated. This role is seen within the context of the Grossman-Stiglitz paradox. This paradox results from the argument that in a perfect market, the acquisition of information gives the investor no comparative advantage over the uninformed investor, since the process of price changes towards the new equilibrium price, will fully reveal all information to everyone. Hence there would be no incentive to acquire new information. However, if it is assumed that more uninformed investors enter the options market due to limited liability and relative small capital requirements, then changes in demand due to speculative zeal would have a sizable impact. Indeed the options market allows for more speculators and/or more speculative opportunities than in the securities market. Hence, it is hypothesized in this chapter, that investors in the securities market would acquire new information, but would enter the options market to act upon their information, since changes in volume and price in this market would be less revealing. Consequently, the following hypothesis is tested: There is causality from option volume to stock price. The testing procedure uses the Pierce-Haugh technique which involves the cross-correlograms of white noise of both time series. The overall conclusion is that for a sample of 14 pairs of estimations, the hypothesis of no lagged causality (i.e., that no contemporaneous causality is present) is accepted. Next, we test for reverse causality: there is causality from stock price to option volume. This hypothesis is based on the empirical observation that about 7 per cent of all call options on the TCO are exercised or are unprofitable by the exercise date. Consequently,

most options are traded on the secondary market, so that investors have a very short horizon with respect to option investment. This would suggest the causality stated above - from stock price to option volume. The empirical results suggest that this hypothesis is accepted with significant cross-correlation coefficients at recent lags, in several cases.

Finally in this chapter, the question as to whether the listing of stocks on the TCO is an economic signal is examined. That is, the objective is to ascertain whether information on the security variables is contained in the very act of being listed. Specifically, a non-parametric test due to Hsu [1977] is applied to a sample of 30 securities on the TCO to ascertain if there is a significant shift in the variance of stock price changes after listing. Since, it is widely accepted that the options market creates new speculative opportunities, then the above test is equivalent to finding out if the increased speculation is stabilizing. The results for this section indicated that only 25% of the securities showed a significant decrease in the stock price variance after listing and 25% underwent an increase. Hence, the likelihood that a security will undergo a significant change in σ^2 is the same as flipping a fair coin. Also, given that a significant change occurred, the probability that it is a decrease is also the same as flipping a fair coin.

Finally, in order to make sure that a significant change in σ^2 is not due to a concurrent significant change in the variance of the TSE index, the estimation procedure was conducted on the TSE for the

same time period around the listing date of each stock. Surprisingly, there is a strong correlation between changes in the variance of the stock price (before and after the listing date) and the variance of the TSE index. Hence, it is questionable whether the change in the variance in the stock price is due to being listed on the TCO. This contradicts the results by Nathan [1974] for CBOE and supports Fisher Black's [1976] comments that it is questionable whether option trading has an impact on the security variables.

CHAPTER 2

REVIEW OF THE LITERATURE

This chapter contains a review of the main developments in the search for a valuation formula for the pricing of stock options which led to the now famous Black-Scholes formula.

2.1 Definition of an Option

An option is any instrument which gives its owner the right, but not the obligation, to buy or sell an asset within a fixed period of time at a predetermined price. Specifically, a call option on the underlying asset is an option to buy (i.e., to call away) a specified number of shares from the writer for a fee, called the option premium or option price. A European call allows the security to be exercised (if rational to do so) only at the terminal date, whereas, an American call may be exercised at any time up to and including the specified date. A put option confers to its holder the right to sell the asset at a specific price for a fixed time period.

Option contracts have a long history. In Biblical times, Jacob bought an option to marry Rachel from her father for seven years labor [1].

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[1] For further details, see the Book of the Genesis, Chapter 29, King James etc.

Also, around the time of Aristotle, Thales the Milesian used options to gain superior returns in the olives market.[2] Although options have been traded in America since the late 18th century, it was not until the formation of the Chicago Board Options Exchange (CBOE) in April 1973, that trading rules and procedures were standardized. This resulted in, among other things, the standardization of exercise price and exercise date and the lowering of net transaction costs. In 1976, the Canadian Option Clearing Corporation and the Montreal Options Clearing Corporation were opened. Later these Clearing Houses merged to form the Trans-Canada options Clearing Corporation (TCO). The volume of trading on the TCO is low relative to the CBOE but, from a volume of 170,000 contracts in 1977, the TCO has grown to about 2 million contracts in 1980.

2.2 Literature Prior to the Black-Scholes Model

The literature, prior to the Black-Scholes model, contains several attempts by many economists and finance experts to find a valuation formula for the pricing of call options. An excellent summary is contained in Smith [1976]. This research followed two main lines:

(i) ad-hoc models

(ii) equilibrium models

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[2] See Aristotle's Politics, Book One, Chapter II, lowett Translation.

The first type was the result of mainly graphical techniques and their visual deductions. At the same time, many econometric models were developed. The list of contributors includes Kassouf [1977] Shelton [1967] and Gastineau [1979]. Unfortunately, these models were either computationally overwhelming and/or required continuous updating.

The second type of models were developed by Sprenkle [1964], Boness [1964] and Samuelson [1965]. There were problems with these models as well. For example, Sprenkle ignored the time value of money. The main benefit from these studies, however, is that they were a fore-runner of the Black-Scholes approach involving the creation of a riskless hedge from which a valuation formula could be derived.

2.3 Black-Scholes Formula

The seminal paper by Black-Scholes in 1973, contained the now famous option valuation formula. Their familiar model (hereafter B-S model) was based upon the following assumptions:

1) The capital markets for stocks, options and bonds are perfect.*

2) The risk-free rate, r, is constant over the life span of the option.

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* By a perfect market, it is usually meant that there are no restrictions on short sales, no transaction costs, no taxes and all securities are divisible. Hence trading can take place continuously.

3) The stock pays no dividends,

4) The stock price follows a geometric Brownian motion in continuous time with constant drift and intertemporally stationary variance of the rate of return.

Specifically, $\frac{dS_t}{S_t} = u dt + \sigma d\omega_t$.

where S_t = stock price at time t,

ω_t = white noise,

u = drift, σ = variance of stock price returns.

5) The option is a European call.

Based upon the above assumptions, Black-Scholes developed an arbitrage argument involving an optimal hedge ratio of calls and the underlying security, so that a self-financing portfolio is generated. Specifically, the model is solved as follows:

Let S_t = price of share at time t,

C_t = price of call at time t,

n_s = number of shares of stock,

n_c = number of call options.

Assume that n_s shares are bought long and n_c call options written on the shares. Then the net value of this portfolio is $R_t = n_s S_t - n_c C_t$.

Thus, the net return is

$$dR_t = d(n_s S_t) - d(n_c C_t)$$

$$= \frac{dn_s}{n_s}(n_s S_t) - \frac{dn_c}{n_c}(n_c C_t) + \frac{dS_t}{S_t}(n_s S_t) - \frac{dC_t}{C_t}(n_c C_t) \qquad (1)$$

But, by assumption (4),

$$\frac{dS_t}{S_t} = u\,dt + \sigma\,d\omega_t$$

where ω_t is a Weiner process with mean zero and variance $\sigma^2 t$. Hence since $C_t = f(S_t, t)$, by Ito's Lemma,

$$dC_t = u_c\,dt + \sigma_c\,d\omega_t \qquad (2)$$

where $u_c = \frac{\partial C}{\partial S} uS + \frac{\partial C}{\partial t} + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2$

and $\sigma_c = \frac{\partial C}{\partial S}\sigma S$

So, substituting (2) into (1), and letting $\frac{\partial C}{\partial S} = \frac{n_s}{n_c}$, (1) becomes

$$dR_t = \frac{dn_s}{n_s}(n_s S) - \frac{dn_c}{n_c}(n_c C) - n_c\left[\frac{\partial C}{\partial t} + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2\right] \qquad (3)$$

Clearly equation (3) is risk free, and in a perfect market (assumption (1)), the portfolio must earn the risk-free rate.

$$\therefore \frac{dR}{dt} = (n_s S - n_c C)r \qquad (4)$$

Comparing (3) and (4) and letting $(dn_s)S = (dn_c)C$, then[1]

$$(n_s S - n_c C)r = - n_c \left[\frac{\partial C}{\partial t} + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 \right] \tag{5}$$

Algebraically reducing (5) and observing that $\frac{\partial C}{\partial S} = \frac{n_s}{n_c}$, the following result is obtained:

$$\frac{\partial C}{\partial t} - rC + rS \frac{\partial C}{\partial S} + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 = 0 \tag{6}$$

This is the familiar deterministic partial differential equation that B-S found. Coupled with the boundary condition that

$C_{t*} = \max (S_{t*}-E,0)$, the B-S valuation formula is:

$$C_t = S_t N(d) - e^{-rt*} EN(d-\sigma\sqrt{t*})$$

where C_t = call price at time t,

S_t = stock price at time t,

$t*$ = exercise date,

E = exercise price at $t*$,

---

(1) Since $\frac{n_s}{n_c}$ is not an endogenous variable (i.e., the continuous hedging assumption that $\frac{\partial C}{\partial S} = \frac{n_s}{n_c}$ allows $\frac{n_s}{n_c}$ to be continuously determined) then $d(\frac{n_s}{n_c}) = 0$.

r = risk-free rate,

σ^2 = instantaneous variance of the stock price returns,

$$d = \frac{\ln(E/S) + (r + \frac{\sigma^2}{2})t^*}{\sigma\sqrt{t^*}}$$

Note that the only unobservable parameters in the model are σ^2 and r. Usually researchers accept the relevant Treasury Bill rate as a suitable and reliable proxy for the risk-free rate r. Hence, it would leave σ^2 as the only unobservable to be estimated. Consequently, the adequacy of the B-S model will be judged solely on how well σ^2 is estimated. (2)

2.4 Extensions of the B-S Model

Harrison and Pliska [1981] have proved that the B-S model implicitly assumes that the market is complete in the Arrow-Debreu sense. Chang [1982] then argued that it is the assumption of completeness in the B-S model which results in the Ross risk neutral valuation relationship. This relationship is based on the fact that the B-S valuation model is independent of the expected rate of return of the stock, u, and also of investor's preferences. Ross then showed that $C_t = e^{-rt^*}[\max(S_t^*-E,0]$, which gives the B-S formula.

(2) The various methods found in the literature for estimating σ^2, and the problems associated with them, are discussed fully in Chapter 3.

Other studies attempted to relax one or more of the assumptions of the B-S model. Merton [1973] assumed that the risk-free rate is stochastic and obtained a closed form solution for a simple case. Merton [1976] and Cox and Ross [1976] examined the case when the process generating the stock price returns is a jump-diffusion process, while Roll [1977] derived an analytic formula for unprotected American call options on stocks with known dividends. Finally, many numerical approaches were made to obtain pricing formulas. These include the binomial pricing (Cox, Ross and Rubinstein [1979]), Monte Carlo Simulation (Boyle [1977]), finite difference methods (Brennan-Schwartz [1978]), numerical integration (Parkinson [1977]), and the assumption of arbitrary, stochastic processes (Jarrow-Rudd [1982]).

2.5 Call Pricing in an Incomplete Market

Recently, several studies have endeavored to obtain pricing models when the market is incomplete. Kwon [1980] extended Farka's lemma to a continuous framework and showed that a pricing formula can be obtained if a "consensus" utility function is assumed. Garman [1978] has labelled this approach to be an "absolute pricing" technique. Lee-Rao-Auchmuty [1981], using Bawa's lognormal CAPM, developed a pricing formula under discrete trading. They claimed that their "new discrete trading option valuation formula is based on a larger admissable set of utility functions than is permitted in the Rubinstein-Brennan state-preference framework".

O'Brien and Schwartz [1982] provided another model using the standard CAPM. Using a numerical procedure to solve the model, they showed that it outperforms the B-S model when applied to the over-the-counter gold market. Based on the assumption that options exist for the purpose of completing the market, Chang [1982] developed a pricing model within the context of the standard CAPM. His model contains a "market-effect variable - the expected return on the underlying stock".

This completes the review of the literature on option pricing models.

CHAPTER 3

ESTIMATION OF THE UNOBSERVABLE PARAMETERS IN THE BLACK-SCHOLES MODEL

In the previous chapter, it was shown that the Black-Scholes formula gives $C_t = f(\sigma^2, r, S_t, E, t^*)$ where S_t, t^*, E are unambiguously known parameters, and σ^2 and r are unobservable parameters. Since researchers usually accept the relevant Treasury Bill rate as a suitable and reliable proxy for the unobservable risk-free rate r, only σ^2 has to be estimated. Therefore the adequacy of the Black-Scholes model will be judged solely on the degree of efficiency of the procedure used in estimating σ^2.

In this chapter, a review is made of the different estimation techniques used in the literature. As shown later, despite all the various attempts to estimate σ^2, the predicted option prices using the B-S model systematically differ from the actual values. Also, it is demonstrated in Appendix I to this chapter, that two such techniques by Latane-Rendleman [1975] and Chiras [1977] for estimating σ^2 are seriously flawed. In fact their estimates are asymptotically meaningless. Consequently, we propose an alternate approach based upon a suggestion by Cox and Ross [1976] and which essentially avoids the problems associated with the estimation of σ^2. The main feature is the Newton-Raphson method for solving non-linear simultaneous equations.

Krausz [1979] and O'Brien and Kennedy [1982] have done research on the CBOE using this approach and found that the B-S formula performed poorly. We believe our work is more extensive than in the two studies cited above in that we derive mathematically precise relations and performed cross-section studies (like O'Brien and Kennedy) but also did longitudinal studies for several pairs of options at different points in time. If the market is efficient, then our results suggest that the functional form of the B-S model is incorrect.

However, since only a limited sample was used in the longitudinal test, a full sample and simultaneous non-linear squares is then used to obtain $(\hat{\sigma},\hat{t})$ in such a manner that the efficiency of the estimates is maximized. Specifically, a two-stage non-linear estimation technique is used in which the parameter estimates $\hat{r},\hat{\sigma}$ are obtained from a minimization problem. The first stage of the procedure effectively obtains initial values (i.e., r_0,σ_0). Since the choice of the initial values can determine whether the algorithm converges rapidly to an optimum point, a grid search procedure is used which is confined to a rectangular region in the (r,σ) space. The second stage of the procedure uses the Newton-Raphson technique to obtain the optimal values of r,σ.

Based on a longitudinal test of the B-S model, the average $\hat{r}$ values are found to be low relative to the actual three-month T-Bill rate. However, before concluding on the suitability of the B-S model, the procedure used to obtain $(\hat{r},\hat{\sigma})$ is examined to determine if it is stable. More specifically, tests are conducted to ascertain if the

"too low $\hat{r}$ values" (i.e. downward bias) and "too high $\hat{\sigma}$ values" (i.e. upward bias) are the result of autocorrelation in the error terms. Significant correlation is found in the error terms, but the $\hat{r}$ estimates for the transformed model are closer to the relevant T-Bill rate in some cases and more negative in others. We attribute this to possible non-stationarity in the stock price return variances as well as the omission in the B-S model of transaction costs, margin requirements, and other features of an imperfect market. Consequently, Chapter 4 investigates the non-stationary variance assumption.

3.1 Review of the Procedures used to Estimate σ^2

(a) Historical Approach

The most popular method for estimating σ^2 was to use a time series of realized price data on a daily basis. This is seen as follows:

Let S_t = stock price at time t,

$R_t = \ln (S_t/S_{t-1})$, and

$\bar{R} = (\sum_{t=1}^{N} R_t)/N$.

Then an estimate of σ^2, using historical data, σ_H^2, is given by:

$$\sigma_H^2 = (\sum_{t=1}^{N} (R_t - \bar{R})^2)/(N-1)$$

Tests of the B-S model using σ_H^2 are contradictory. While Black and Scholes [1972] found that deep-in-the-money (out-of-the-money) calls

had predicted prices greater (less) than actual market prices, Macbeth and Merville [1979] found the opposite. Thus other procedures to estimate σ^2 were initiated.

(b) Implied Approaches

The common feature in almost all of these procedures is to let the "market do it". Trippi [1977] computed an implied volatility measure, $\hat{\sigma}_i$, for each underlying stock by numerically solving:

$$\hat{\sigma}_i = f(S_t, C_{it}, r, t^*, E_i),$$

where S_t = stock price at time t,

C_{it} = call price of option i at time t,

r = corresponding T-Bill rate,

t* = time to expiration of the option,

E_i = exercise price of the option i,

$\hat{\sigma}$ = implied standard deviation of the stock price returns.

Then, for each underlying stock, he computed

$\bar{\sigma} = \sum_{i=1}^{N} (\hat{\sigma}_i)/N$, where N is the number of options on the stock on a particular day.

Latane and Rendleman [1975] criticized this procedure on the grounds that it was "unreasonable to expect option prices for a given company to reflect the arithmetic average of implied standard deviation from all options on its stock which are traded in a particular point in time".

Therefore, they argued that a better measure for the standard deviation for a given stock was given by:

$$\bar{\sigma} = \left(\sum_{i=1}^{N} \hat{\sigma}_i^2 \, d_i^2 \right)^{1/2} / \left(\sum_{i=1}^{N} d_i \right),$$

where $d_i = \dfrac{\partial C_i}{\partial \hat{\sigma}_i}$.

However, in Appendix 3.1, the weighting formula is shown to be incorrect because it is biased towards zero when N (i.e., the number of options written on the underlying stock) becomes large.

While Chiras [1977] also observed this point, he did not supply a proof. Instead, Chiras proposed the alternate formula:

$$\bar{\sigma} = \left(\sum_{i=1}^{N} \hat{\sigma}_i \, e_i \right) / \left(\sum_{i=1}^{N} e_i \right) \text{ where}$$

$e_i = \dfrac{\partial C_i}{\partial \hat{\sigma}_i} \dfrac{\hat{\sigma}_i}{C_i}$. He found that his weighting formula was superior to those mentioned above.

Recently, Brenner and Galai [1982] computed the implied standard deviation using transactions data. They suggest that this approach gives a more reliable measure of stock price volatility since it is based on more observations rather than only the closing transaction. Although, their empirical work was only for IBM options from June 3, 1977 to October 21, 1977, the evidence presented indicates that option prices for out-of-the-money options contains errors that

"amplify the errors in the implied standard deviation. Thus, estimates which are based on a single observation are very unreliable".

(c) Simultaneous Estimation Techniques

Krausz [1979] and O'Brien and Kennedy [1982] have estimated the parameters of the B-S model by solving the system:

$$C_i^a = f_i \ (r,\sigma) \quad i = 1, 2$$

where C_i^a = actual option prices

f_i = Black-Scholes model price.

Both studies found evidence which suggests that the Black-Scholes model is invalid. However, the major problem with these studies is that the equations to be estimated could have multiple solutions, so that just trying to find a feasible solution is unsatisfactory. Also, the samples used were very limited and the estimation was done at one particular point in time. The result is that only very limited "snapshots" of the process were obtained.

(d) Other Methods

Recently, Parkinson [1980] and Garman and Klass [1980] formulated estimators of σ^2 based on the high, low, opening and closing prices, and on the transactions volume. For example, let $\{S_1, S_2, \ldots, S_n\}$ be a sequence of stock prices at equal time intervals. Let H_t = high, L_t = low for the time interval under consideration, and let $\ell_t = \ln(H_t/L_t)$. Then Parkinson showed that his estimate,

$$\hat{\sigma} = \frac{.361}{n} \sum_{i=1}^{n} \ell_i^2 \text{ , was more efficient than } \sigma_H^2 .$$

Capozza and Cornell [1979] used an autoregressive specification of the variance. They found it performed better than the implied measure of volatility. However, we will discuss this estimator in more detail in Chapter 5, where we show that Capozza and Cornell may have over-differenced their time series of weekly variances, and thereby obtained a first order moving average process.

3.2 Initial Simultaneous Estimation of the Variance and the Interest Rate.

In this section, an alternate approach of estimating the unobservable parameters of the B-S model is proposed. The approach, based upon a suggestion by Cox and Ross [1976], is essentially a test of the functional form of the model.

(a) Assumptions

For the following estimation procedure, it is assumed that:

(i) All the assumptions (given in Chapter 2) required for the derivation of the B-S formula are valid.

(ii) The B-S formula is valid in the sense that the predicted model price is the true equilibrium price.

(iii) The stock and option markets are efficient.

(iv) For the simultaneous system, $g_1(r,\sigma) = 0$ and $g_2(r,\sigma) = 0$, g_i, $i = 1,2$, are continuously differentiable in a finite bounded neighborhood of a feasible solution $(\hat{r},\hat{\sigma})$ and the

$$\text{Jacobian} = \det\begin{pmatrix} \frac{\partial g_1}{\partial r} & \frac{\partial g_1}{\partial \sigma} \\ \frac{\partial g_2}{\partial r} & \frac{\partial g_2}{\partial \sigma} \end{pmatrix} \neq 0 .$$

(b) Estimation Procedure

Consider two option series on the same stock with the same expiration date but with different exercise prices.

Let S = present stock price

C_i^a = actual call prices, $i = 1,2$

E_i = exercise prices at t^*, $i = 1,2$

C_i^{BS} = option prices derived from the B-S model

Recall $$C_i^{BS} = SN(d_i) - e^{rt^*}E_iN(d_i - \sigma\sqrt{t^*}) \qquad (3.1)$$

where $$d_i = [\ln(S/E_i) + (r+\frac{\sigma^2}{2})t^*]/(\sigma\sqrt{t^*})$$

Equation (3.1) can be written more compactly as:

$$C_i^{BS} = f_i(S,E_i,t^*,\sigma,r) \ , \ i = 1,2$$

where S, E_i, t^* are known and σ, r are unknown. But, if the B-S model is correct, then $C_i^{BS} \equiv C_i^a$, $i = 1,2$.

Hence $$f_i(\sigma,r,S,E_i,t^*) - C_i^a = 0 \qquad (3.2)$$

However, since S, E_i, t^*, C_i^a are all known then (3.2) can be written as:

$$g_i(\sigma, r, S, E_i, t^*, C_i^a) = 0 \ , \quad \text{or simply,}$$

$$g_i(\sigma, r) = 0 \ , \quad i = 1,2 \tag{3.3}$$

Hence, if the simultaneous non-linear equations are solved by the Newton - Raphson Technique, a (not necessarily unique) solution $(\hat{r}, \hat{\sigma})$ to (3.3) can be obtained. From (3.3),

$$\frac{\partial g_i}{\partial r} dr + \frac{\partial g_i}{\partial \sigma} d\sigma = 0 \tag{3.4}$$

$$\frac{d\sigma}{dr} = \frac{-\partial g_i / \partial r}{\partial g_i / \partial \sigma} \tag{3.5}$$

As shown in Appendix 3.2 to this chapter,

$$\frac{\partial g_i}{\partial r} = tEe^{-rt^*} N(d_i - \sigma\sqrt{t^*}) > 0 \ ,$$

$$\frac{\partial g_i}{\partial \sigma} = \sqrt{t}\ Ee^{-rt^*} Z(d_i - \sigma\sqrt{t^*}) > 0 \ ,$$

and, except for a minor condition, these contours are concave. (See Figure 3.1)

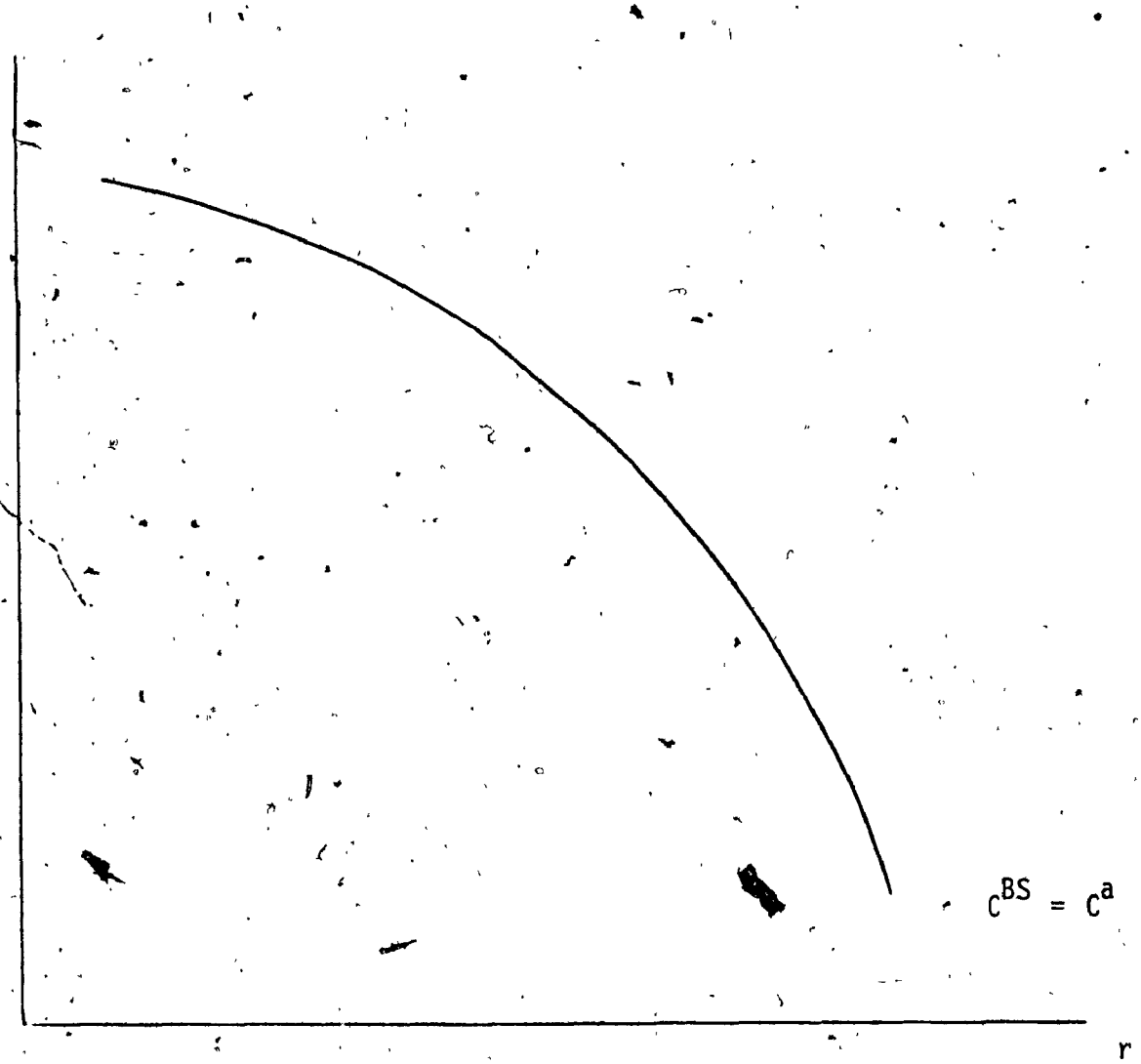

Figure 3.1 Black-Scholes Contours are concave.

Accordingly, a $(\hat{r},\hat{\sigma})$ solution to the system of equations, (3.3), will be such that the contours intersect. (See Figure 3.2)

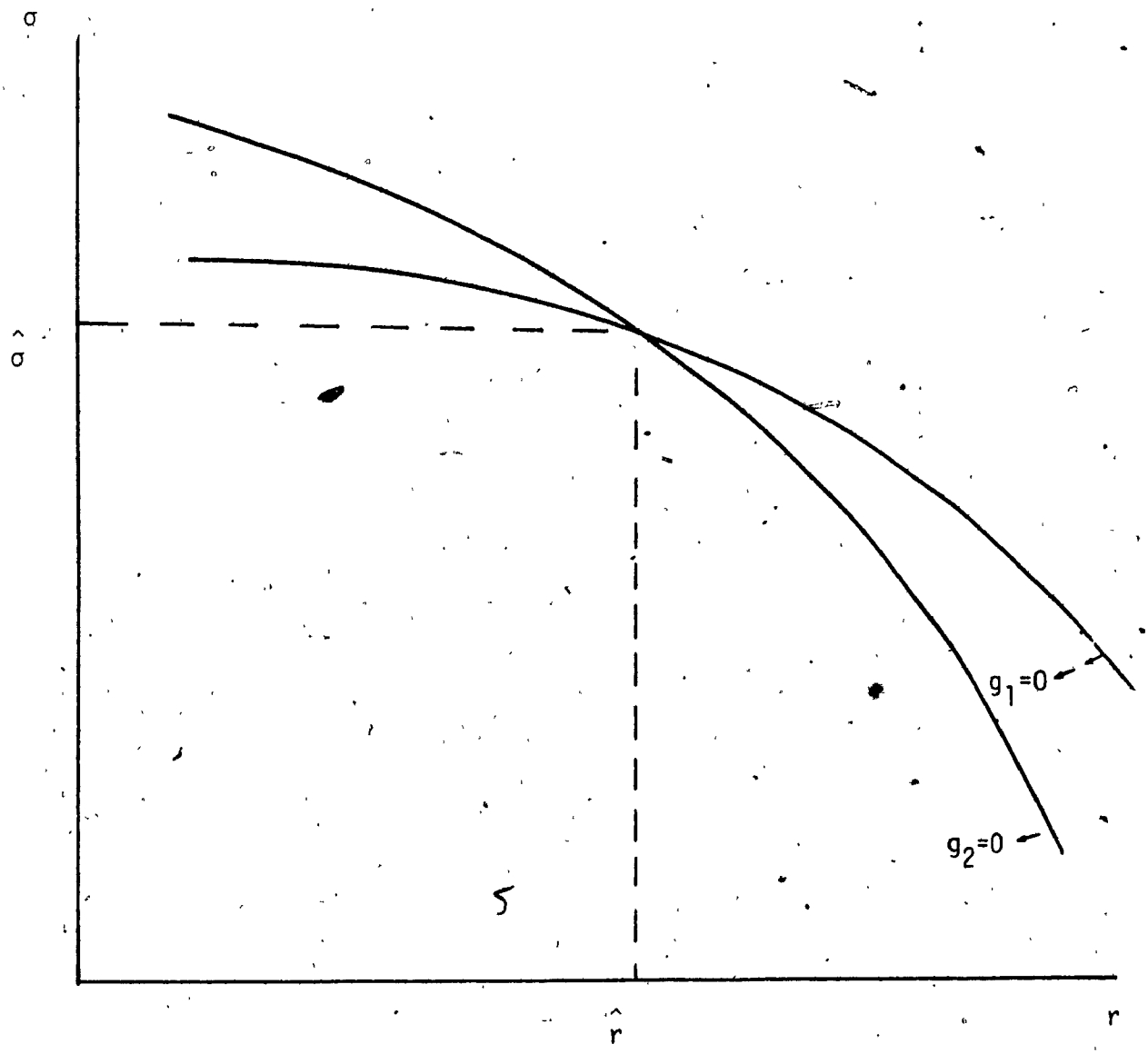

Figure 3.2 $(\hat{r},\hat{\sigma})$ is a solution to 3.3.

However, the concavity of the contours $g_i(r,\sigma) = 0$ does not rule out a multiplicity of solutions.

(c) Newton-Raphson Technique

The geometric significance of the Newton - Raphson method for solving one equation in one unknown will first be illustrated. The basic problem is to numerically solve an equation of the type $f(x) = 0$, where f has a continuous first derivative. The essential analysis (depicted in Figure 3.3) can be described as follows: Let x_0 be a first approximation of x^*, where $f(x^*) = 0$. Then $f(x)$ is approximated by its tangent line at $P = (x_0, f(x_0))$.

Hence, $f(x)$ is replaced by

$$f(x) = f(x_0) = f'(x_0)(x-x_0)$$

A better approximation of x^* is now given by x_1, that is, the solution to $f(x) = 0$ where $f(x_1) = 0$. Since $f(x_0) + f'(x_0)(x_1-x_0) = 0$,

then it follows that

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} .$$

Repeating the same procedure for Q yields

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

This is the familiar Newton-Raphson iterative procedure to solve $f(x) = 0$.

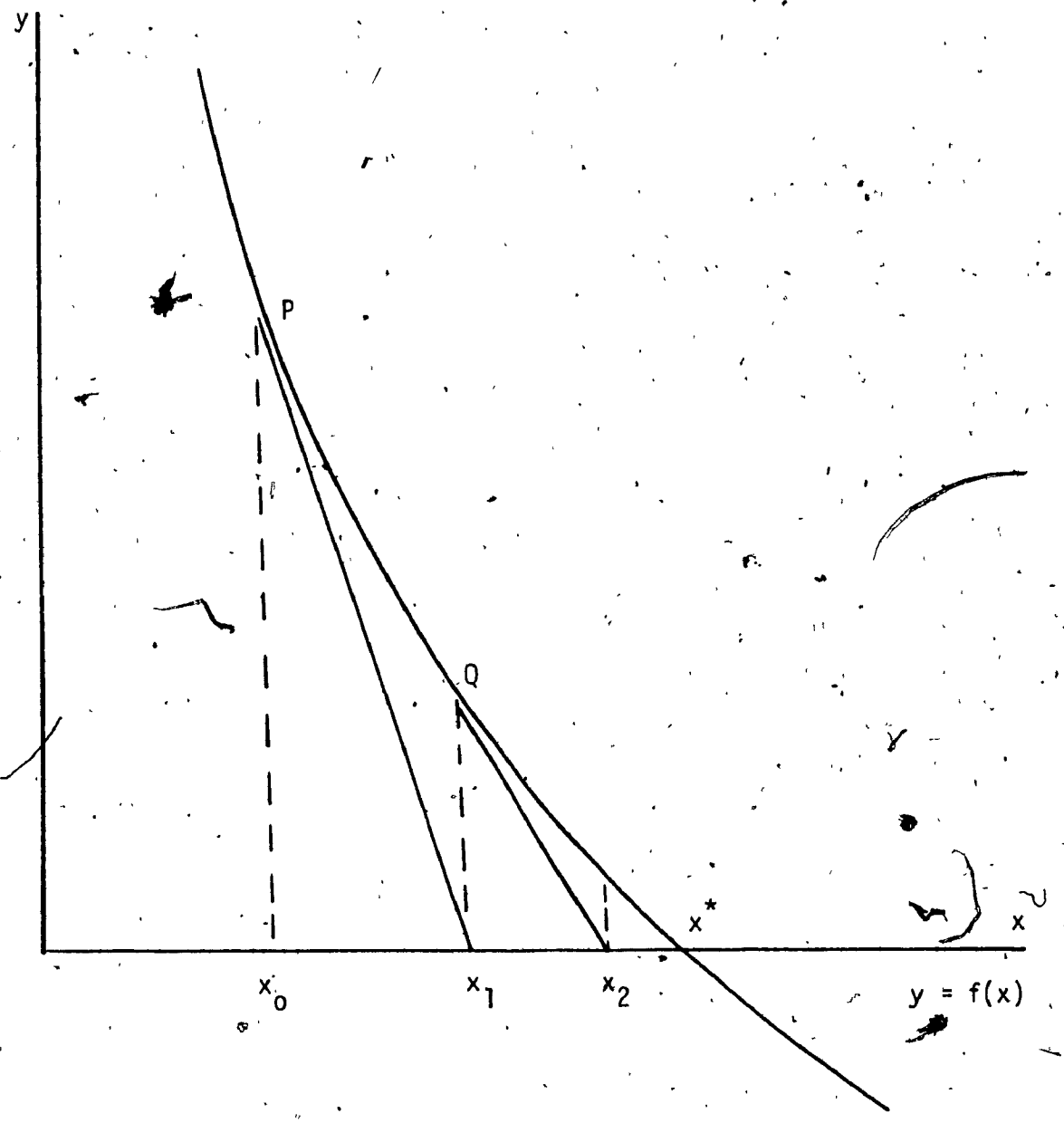

Figure 3.3 Newton-Raphson Iterative Procedure.

Furthermore, if:

(i) f'' exists and is continuous on $[x_0, x^*]$,

(ii) $f'(x) \neq 0$ and $f''(x) \neq 0$ for all $x \in [x_0, x^*]$,

and

(iii) $f(x_0)\, f''(x_0) > 0$, then $\lim_{k\to\infty} x_k = x^*$.

The Newton-Raphson method can also be easily obtained by the use of a Taylor Series as follows.

Let $f(x+h) = f(x) + h\, f'(x) + \ldots\ldots\ldots\ldots$

Ignoring all terms in h^r, $r \geq 2$, then $h = \dfrac{-f(x)}{f'(x)}$ is the appropriate correction term to apply to x to give $f(x+h) = f(x) - \dfrac{f(x)}{f'(x)} \approx 0$.

This procedure can be used to derive the Newton-Raphson procedure for simultaneous non-linear equations as follows. First consider the simultaneous equations:

$$g_1(r,\sigma) = 0 \quad \text{and}$$

$$g_2(r,\sigma) = 0,$$

where g_i, $i = 1,2$, are assumed to be continuous and differentiable. Then, the two dimensional Taylor Series is:

$$g_i(r,\sigma) = g(r_0,\sigma_0) + \frac{\partial g_i}{\partial r}(r-r_0) + \frac{\partial g_i}{\partial \sigma}(\sigma-\sigma_0) + \ldots \quad i = 1,2 .$$

Converting to a discrete iteration form, and letting $r_{j+1} = r_j + h$, $\sigma_{j+1} = \sigma_j + k$ yields:

$$\frac{\partial g_i}{\partial r} h + \frac{\partial g_i}{\partial \sigma} k = -g_i(r_j, \sigma_j)$$

Assume that $J = \det \begin{pmatrix} \frac{\partial g_1}{\partial r} & \frac{\partial g_1}{\partial \sigma} \\ \frac{\partial g_2}{\partial r} & \frac{\partial g_2}{\partial \sigma} \end{pmatrix} \neq 0$

Then, using Cramer's rule:

$$h = \det \begin{pmatrix} -g_1(r_j, \sigma_j) & \frac{\partial g_1}{\partial \sigma} \\ -g_2(r_j, \sigma_j) & \frac{\partial g_2}{\partial \sigma} \end{pmatrix} \Bigg| (J)$$

$$k = \det \begin{pmatrix} \frac{\partial g_1}{\partial r} & -g_1(r_j, \sigma_j) \\ \frac{\partial g_2}{\partial r} & -g_2(r_j, \sigma_j) \end{pmatrix} \Bigg| (J)$$

The iterative procedure is then repeated and $(\hat{r}, \hat{\sigma})$ is obtained.

(d) <u>The Data</u>

This study uses data on the stock and options of the following nine firms which are listed on the Toronto Stock Exchange (TSE): Bank of Montreal, Bell Canada, Canadian Pacific, Dome Petroleum, Gulf Canada, Inco, Noranda, Royal Bank and Stelco. For each stock, the prices of

all call options on the Trans Canada options Market (TCO) which terminated on the same date during 1981 were recorded. There are usually 4 different exercise cycles per stock each year, and a sample of two option series was obtained for each cycle. However, only frequently traded options are considered. A detailed account of the number of traded options with a common exercise date, for each stock, is given in Table 3.1. There is a statistical problem if the stock and call prices are not synchronous. For example, use of daily closing prices would inevitably lead to a problem of misalignment since published daily prices are usually end of day prices. This may be seen as follows: Assume an option contract is bought for $\$C_t$ when the stock is $\$S_t$. If the stock price closed at $\$(S+\Delta S)_{t+\Delta t}$, and no other option trades were made, then using daily closing prices could give misleading results.

Consequently, the following sampling procedure is used. On day t, and for each stock i, the traded option price is recorded as well as the time (t_i^c) of the trade. Such information is available in the daily Transactions Report at the Montreal Stock Exchange. The stock price and time (t_j^s) of the trade closest to the time of the option trade are recorded. So for day t, we select the pair of prices which correspond to (t_j^s, t_i^c) such that

$$|t_i^c - t_j^s| = \min_{(k,\ell)} \{|t_k^c - t_\ell^s|\}$$

This procedure minimizes the non-simultaneity problem of stock and option prices.

TABLE 3.1

Number of different call options for firm with a common exercise date during 1981

FIRM	Exercise Date			
	15 Feb.	16 May	15 Aug.	21 Nov.
BM	3	4	5	5
BELL	4	3	3	3
C.P.	5	6	5	5
GULF	14	11	12	9
INCO	4	8	8	8
NOR	7	10	8	6
STELCO	3	5	5	6
	18 Jan.	18 Apr.	18 Jul.	17 Oct.
DOME	10	8	8	9
ROYAL	4	4	4	5

SOURCE: Montreal Stock Exchange Daily Report.

Secondly, the stock prices are adjusted for stock splits. Also, since the B-S model is based on an assumption of no dividends, the present value of all dividends paid before the maturity date is subtracted from the stock price. But if the dividend payment is stochastic, the B-S model price is biased*. Finally, we ignore the possibility of premature exercise**.

(e) Initial Results

(i) Longitudinal Test

The numerical technique described in (c) was applied to the data described in (d) by the use of a program in FORTRAN 5. To illustrate the overall nature of the results, the detailed results obtained for Gulf Canada will first be presented. For Gulf-Canada, ten different option price series with a common exercise date on 15 August 1980 were selected and estimated at two different, ramdomly selected, points in time, i.e., 6 May 1980 and 30 May 1980. The characteristics of these options are summarized in Table 3.2 below. The sample resulted in 45 different pairs of equations at each of the two points in time, where each and every pair of equations would completely define the value of $(\hat{r},\hat{\sigma})$. However, since σ^2 is the variance of the underlying stock price returns then $\hat{\sigma}_i = \hat{\sigma}_o$ at a particular point in time. Furthermore, since investors must inpute the same risk-free rate of interest, then, it is expected that $\hat{r}_i = \hat{r}_o$ at a particular point in time. Hence, the following null hypothesis is postulated.

---

* This point is noted by Brenner and Galai [1982].

** Beckers [1981] has shown that early exercise is rarely optimal.

$H_0 : (\hat{r}_i, \hat{\sigma}_i)$, $i = 1, \ldots 45$,

are not significantly different at each point in time.

The results are recorded in Tables 3.2 and 3.4 for the 6 May 1980 and the 30 May 1980, respectively. Instances in which no intersection point exists are indicated by an "*" in Tables 3.3 and 3.4. The reason for this is that the Newton-Raphson method requires that the Jacobian be non-singular at each iteration. A test for non-singularity was inserted in the computer program so that the computation would stop when the Jacobian was zero. Thus, convergence was never achieved because of an improper choice of the initial (σ, r) values.

DISCUSSION

As is evident from Tables 3.3 and 3.4, the inputed values of $\hat{r}_j$ and $\hat{\sigma}_j$ are widely dispersed. More specifically, the values for May 1980 range from -388.9% to 54.03% and average 7.3%. However, the 3-month Treasury Bill rate for 6 May 1980, on an annualized basis, was 13.92% Worse still, the average inputed $\hat{r}_j$ value on 30 May 1980 of approximately 60% was much greater than the T-Bill rate of 11.58%.

The $\hat{\sigma}_j$ values for both dates are relatively large, especially when compared to the historically estimated standard deviation for the relevant time period. For example, on 6 May 1980, the $\hat{\sigma}_j$ range from .2605 to 1.222, whereas the standard deviation of stock price returns

from 6 May to the terminal date of 15 Aug. 1980 was .5154 . Similarly, the $\hat{\sigma}_j$ on 30 May 1980 range from .2268 to .9991 whereas the historical $\hat{\sigma}$ to the terminal date is .4467. The obvious conclusion for the longitudinal tests is that the null hypothesis is rejected. Further testing of this hypothesis, however, will be presented in section 3.3 (f).

(ii) Cross-Section Tests

In this section, different stocks with a given pair of option price series all terminating on the same date were examined for 4 consecutive days. Such a cross-sectional study has also been undertaken by O'Brien and Kennedy [1982] for one particular date. Four consecutive days were used here because interest rates are not expected to vary across different securities. So, if inputed interest rates are found to significantly vary across different stocks, and such differences persist over a few days, then arbitrage opportunities would exist. However, market forces should quickly eliminate any potential excess profits and thus the inputed risk-free rate should not be significantly different. The estimated $\hat{\sigma}, \hat{r}$ values for ramdomly selected pairs of options beginning 6 Jan. 1981 and ending 9 Jan. 1981 are shown in Table 3.5. The underlying securities are Canadian Pacific, Noranda and the Bank of Montreal and all options terminate on 15 May 1981.

TABLE 3.2

Characteristics of Gulf Canada Options

Date: 6 May 1980										S = 31.00
i	1	2	3	4	5	6	7	8	9	10
C_i	13.25	10.75	9.50	7.50	6.63	5.25	4.25	3.00	2.75	2.00
E_i	18	22	24	26	28	30	32	34	36	38
Date: 30 May 1980										S = 31.50
C_i	15.75	12.75	10.50	9.25	7.25	5.38	3.75	2.50	1.75	1.00

TABLE 3.3

Intersection Points $(\hat{r}_j, \hat{\sigma}_j)$ for $g_i(r,\sigma) = 0$, $i = 1,2$. for 6 May 1980

i	2	3	4	5	6	7	8	9	10
1	-.4192 1.222	-.3096 1.119	-.0498 .7765	-.0779 .8281	-.0261 .7263	-.0130 .7263	.0169 .6011	-.0015 .6630	-.0123 .6183
2		-.0545 .9321	.2510 .4622	.1854 .6302	.2213 .5540	.2225 .5510	.2477 .4752	.2203 .5564	.2329 .5231
3			*	.3023 .5150	.3232 .4650	.3155 .4849	.3381 .4189	.3031 .5132	.3155 .4849
4				-.5095 1.078	.1057 .6398	-.2167 .8977	.2634 .5032	.1536 .5897	[illegible]962 [illegible]5396
5					.3956 .3876	.3471 .4605	.4024 .3750	.3046 .5125	.3956 .4754
6						.2380 .5405	.4096 .3698	.2146 .5593	.2878 .4979
7							.5403 .2605	.1952 .5691	.3154 .4849
8								-3.889 1.893	-.1705 .6917
9									.5539 .3629

* Represents division by zero in the estimation, which is due to zero Jacobian. This in turn implies that the tangent planes are parallel; hence no intersection point exists.

$\hat{r}_j$ represents the annual risk-free interest rate and $\hat{\sigma}_j$ is the instantaneous variance of stock price returns on an annualized basis.

TABLE 3.4

Intersection Points $(\hat{r}_j, \hat{\sigma}_j)$ for $g_i(r,\sigma) = 0$, $i = 1,2$. for 30 May 1980

i	1	2	3	4	5	6	7	8	9	10
1		.5277 .9991	*	.6189 .6340	.6248 .4212	.6247 .2783	.6248 .2420	.6245 .2592	.6248 .3014	.6248 .2911
2			*	*	*	*	*	*	.7480 .2268	.7480 .2375
3				.2858 .9733	.6142 .4420	.6246 .2785	.6246 .2420	.6247 .2590	.6244 .3017	.6245 .2912
4					*	*	*	.7288 .1369	.7287 .2397	.7286 .2464
5						*	*	.6721 .2153	.6675 .2773	.6680 .2729
6							*	.6337 .2514	.6116 .3086	.6185 .2937
7								.5966 .2815	.5329 .3493	.5684 .3140
8									.3605 .4293	.5295 .3292
9										.6930 .2622
10										

* Since the Jacobian is zero, no solution exists. This result persists after the starting values are changed.

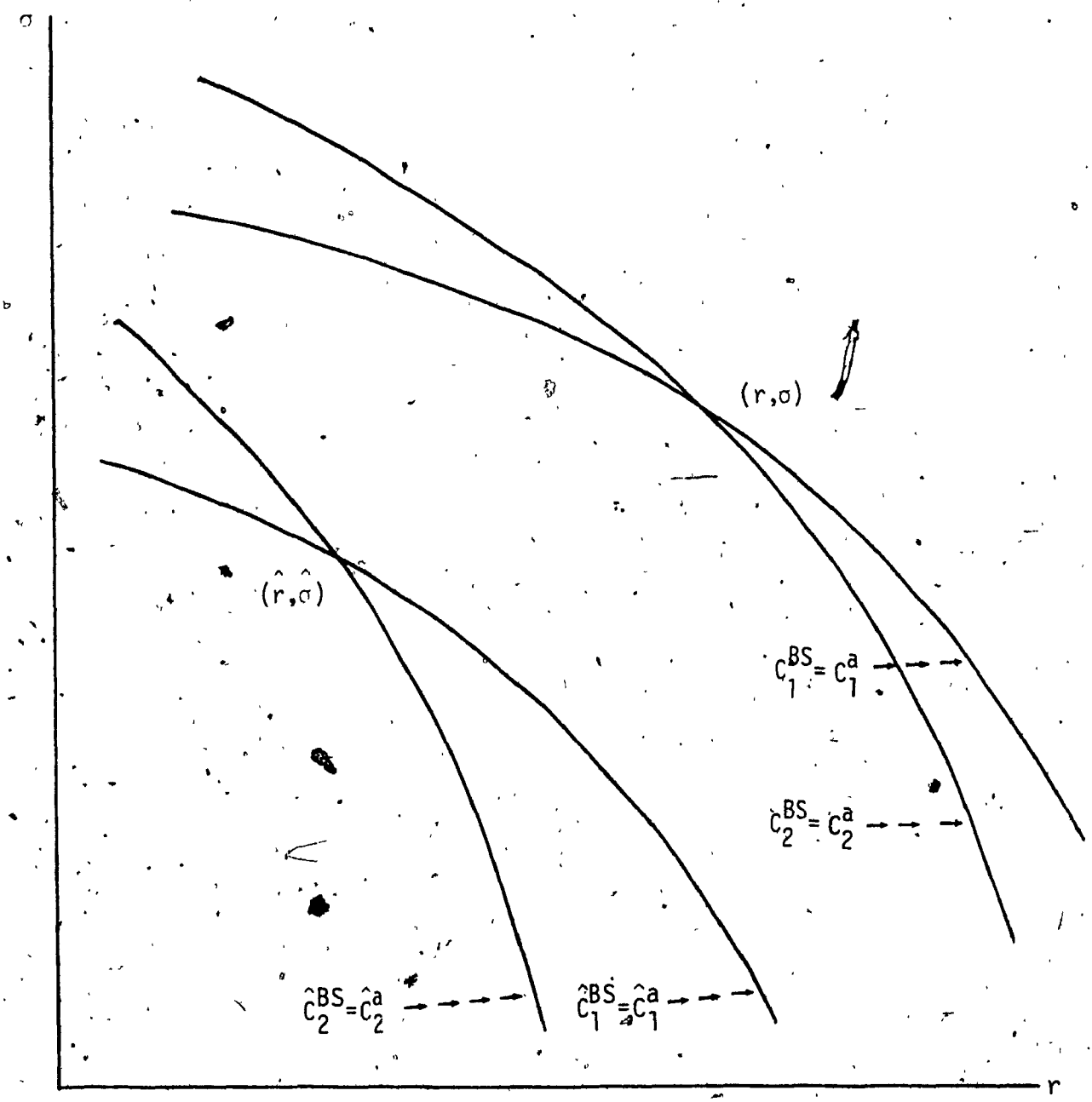

Figure 3.6 Graphical Illustration of Mispricing, $C^{BS} > C^a$.

DISCUSSION

As expected, the $\hat{\sigma}_j$ vary across the companies. However, the imputed risk-free interest rates, $\hat{r}_j$, are also dramatically different. For example, $\hat{r}_j$ on 6 Jan. 1981 was .2239 for C.P. options and -0.3609 for Noranda options. Furthermore, the average implied risk-free rate, $\bar{r}$, ranges from -.0567 to .1796 over the time period. However, the $\bar{r}$ on the fourth day compares relatively well with the 3-month Treasury Bill rate on 7 Jan. 1981 of .1675.

(iii) Summary

Based on the results of the longitudinal and cross-sectional tests presented above, the B-S model was not empirically supported. The intersection points for different pairs of options on a single stock, obtained using the Newton-Raphson technique, differed widely. This was unexpected since different exercise prices should not produce significantly different variances because σ^2 depends on S and not on E. Also, substantially different imputed risk-free rates persisted across different stocks even for several consecutive days. Hence, if the B-S model is correct, as was assumed, then arbritrage opportunities would exist for a relatively long period of time (i.e., the market appears to be inefficient). However, if it is assumed that the market is efficient, then this would lead to the conclusion that the functional form of the model is incorrect. As shown in the next section, this might be too hasty of a conclusion.

TABLE 3.5

Intersection Points $(\hat{r}_j, \hat{\sigma}_j)$ for $g_i(r,\sigma) = 0$ across stocks

from 6th Jan. 1981 to 9th Jan. 1981

NAME									
C.P.	40	45	43.63	7.25	4.00	.2239	.2830		
B.M.	27.50	32.50	33.38	6.00	2.63	-.0332	.3025	-.0567	Jan 6/81
NOR	27.50	30.00	30.63	4.38	3.00	-.3608	.4184		
C.P.	40	45	42.13	6.12	3.00	.2599	.2484		
B.M.	27.50	32.50	32.63	5.12	2.50	-.1641	.4204	.0667	Jan 7/81
NOR	27.50	30.00	29.63	3.63	2.50	.1043	.3813		
C.P.	40	45	41.25	5.25	2.25	.2661	.2205		
B.M.	27.50	32.50	32.63	5.50	2.00	.0158	.2412	-.0388	Jan 8/81
NOR	27.50	30.25	29.25	3.25	2.38	-.3984	.6069		
C.P.	40	45	41.63	6.00	2.62	.3579	.1942		
B.M.	27.50	32.50	32.00	5.13	2.00	.0180	.2815	.1796	Jan 9/81
NOR	27.50	30.00	30.00	3.63	2.50	.1629	.4569		

Common Exercise Date: 15 May 1981

- Three Month Treasury Bill Rate = .1675, annualized

3.3 Simultaneous Estimation of the Variance and Risk-Free Rate Allowing for Measurement Error

(a) Procedure

In this section of the thesis, the following two simultaneous equations in two parameters were estimated:

$$g_1(r,\sigma) = 0 \quad \text{and} \quad g_2(r,\sigma) = 0$$

However, suppose for reasons such as measurement errors and imperfect alignment of data, that there is noise in the observations, so that the true model is:

$$\begin{aligned} g_{1t}(r,\sigma) + \varepsilon_{1t} &= 0 \\ g_{2t}(r,\sigma) + \varepsilon_{2t} &= 0 \end{aligned} \qquad (1)$$

where, for $i = 1,2$, $E(\varepsilon_{it}) = 0$, $\sigma^2(\varepsilon_{it}) = \sigma^2$,

$$E(\varepsilon_{it}\,\varepsilon_{is}) = E(\varepsilon_{it})\,E(\varepsilon_{is}) \quad \text{for } t \neq s.$$

and $$E(\varepsilon_{it}\,\varepsilon_{2t}) = E(\varepsilon_{1t})\,E(\varepsilon_{2t})$$

If this is the case, then the efficiency of the $\hat{\sigma},\hat{r}$ estimates is maximized if the estimates are obtained by simultaneous non-linear least squares using a full sample rather than two points in time (as was done in the longitudinal test above).

The estimation procedure to be used is now described. Note that $g_i(\sigma,r) \equiv f_i(\sigma,r)-C_i$. Hence, the model to be estimated is:

$$C_{1t} = f_{1t}(r,\sigma) + \varepsilon_{1t} \qquad (3)$$

$$C_{2t} = f_{2t}(r,\sigma) + \varepsilon_{2t} \qquad (4)$$

The objective is to find estimates $\hat{r},\hat{\sigma}$ which minimize the residual sum of squares (SS) given by $SS = \varepsilon_1'\varepsilon_1 + \varepsilon_2'\varepsilon_2$. That is, the estimates are those that solve:

$$\min_{r,\sigma} SS = \{\sum_{t=1}^{T} (C_{1t} - f_{1t}(r,\sigma))^2 + \sum_{t=1}^{T} (C_{2t} - f_{2t}(r,\sigma))^2\}$$

Then:
$$\begin{pmatrix} \hat{r} \\ \hat{\sigma} \end{pmatrix} = \begin{pmatrix} r_o \\ \sigma_o \end{pmatrix} - \begin{bmatrix} SS_{rr} & SS_{r\sigma} \\ SS_{\sigma r} & SS_{\sigma\sigma} \end{bmatrix}^{-1} \begin{bmatrix} SS_r \\ SS_\sigma \end{bmatrix} \qquad (5)$$

(See Figure 3.4)

Initial test runs, using the Newton-Raphson technique on (5) and arbitrarily chosen values of (r_o,σ_o), gave results which were divergent. Outside some range of the local minimum, the derivatives were not consistent for a minimization problem. More specifically, SS_{rr} can be expressed as follows:

$$SS_{rr} = -2\sum(\hat{\varepsilon}_1 f_{1rr} - f_{1r}^2) - 2\sum(\hat{\varepsilon}_2 f_{2rr} - f_{2r}^2)$$

Hence, $SS_{rr} > 0$ (a condition for minimization) if $\hat{\varepsilon}_i f_{irr} < f_{ir}^2$, $i = 1,2$. Based on Lemma 2 of Appendix 3.2, $f_{rr} = \frac{t}{\sigma}[f_\sigma - \sigma f_r]$.

Therefore, $\hat{\varepsilon} f_{rr} - f_r^2 < 0$, if $f_r^2 > \hat{\varepsilon}[\frac{tf_\sigma}{\sigma} - tf_r]$

$$\therefore\ f_r^2 + \hat{\varepsilon}t\, f_r - \frac{\hat{\varepsilon}t\, f_\sigma}{\sigma} > 0, \text{ and } (\hat{\varepsilon}t)^2 + \frac{4\hat{\varepsilon}\, tf_\sigma}{\sigma} < 0 .$$

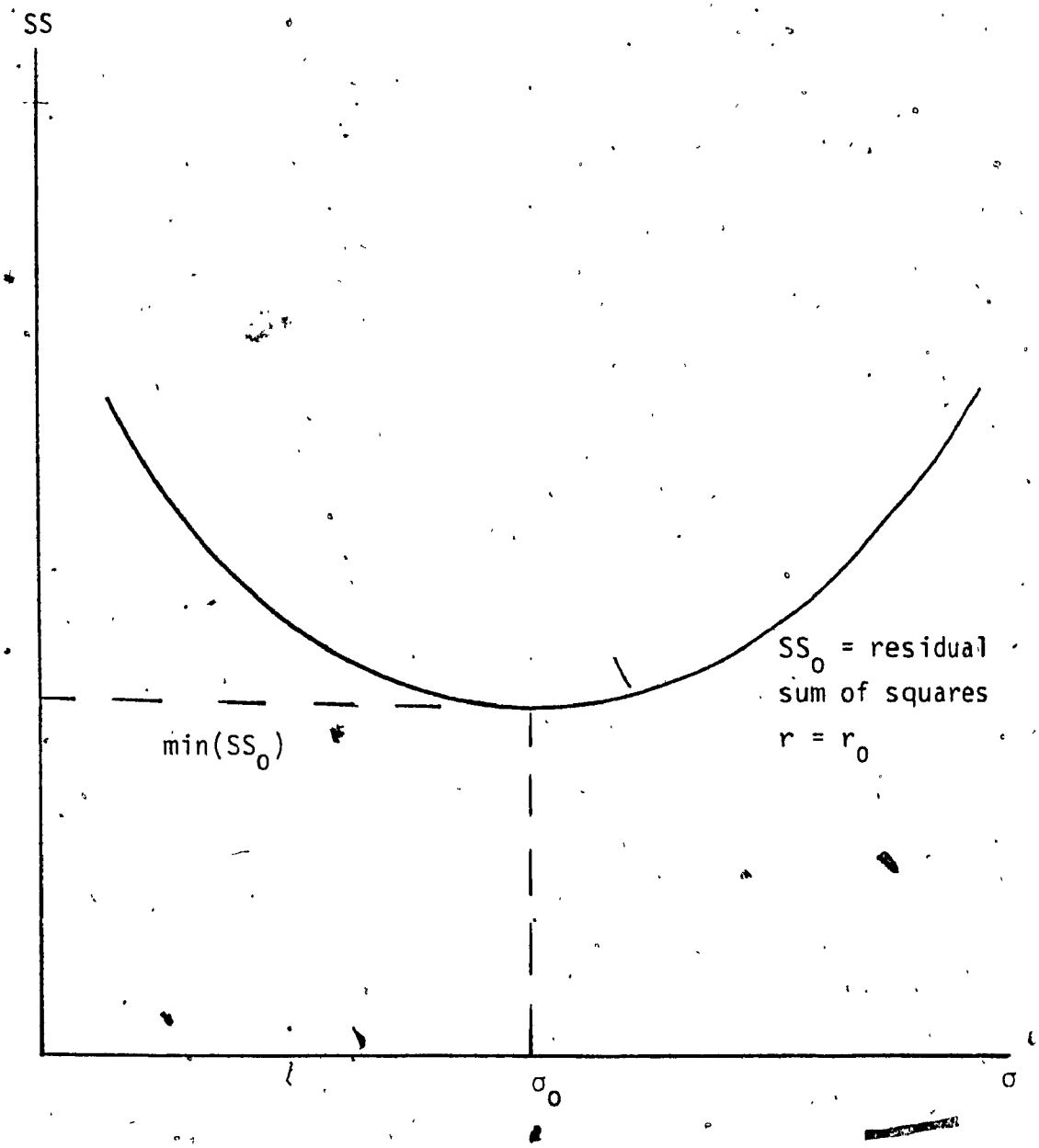

Figure 3.4. σ_0 which yields min (SS_0) for a given $r = r_0$.

Noting that $ax^2 + bx + c > 0$, if $a > 0$ and $b^2 - 4ac < 0$, then

$$\hat{\varepsilon}t + \frac{4\hat{\varepsilon}\, f_\sigma}{\sigma} < 0 \qquad (6)$$

Clearly, since $f_\sigma > 0$, condition (4) is satisfied if $\hat{\varepsilon} < 0$; in which case, $C_i - f_i\,(r,\sigma) < 0$, $i = 1,2$. That is, the B-S model overvalues the call option. If $\hat{\varepsilon} \geq 0$, condition (6) is not satisfied and saddle points can exist. Therefore, in order to alleviate the problems associated with being trapped in a saddle point, the following procedure is adopted.

$SS_{rr} > 0$, $SS_{\sigma\sigma} > 0$ and the

$$\text{Jacobian} = \det \begin{bmatrix} SS_{rr} & SS_{\sigma r} \\ SS_{r\sigma} & SS_{\sigma\sigma} \end{bmatrix}$$

$$= SS_{rr}\, SS_{\sigma\sigma} - (SS_{\sigma r})^2 > 0 .$$

Since the choice of b for this iteration is purely arbitrary, a value of b equal to .6 was initially chosen.

If for the range, $-.30 \leq r \leq .30$, no appropriate σ_0 is found within $[0,b]$ (that is, the derivatives and/or the Jacobian is of the wrong sign), then the upper bound for σ is increased to c, and the procedure is repeated for $b \leq \sigma \leq c$. Even if an appropriate σ_0 is found in $0 < \sigma \leq b$, the region $b \leq \sigma \leq c$ was considered, since a lower SS might be found within the bounds of the latter region. Thus, during stage 1, an initial value (r_0,σ_0) is obtained for the Newton-Raphson technique. (See Figure 3.5)

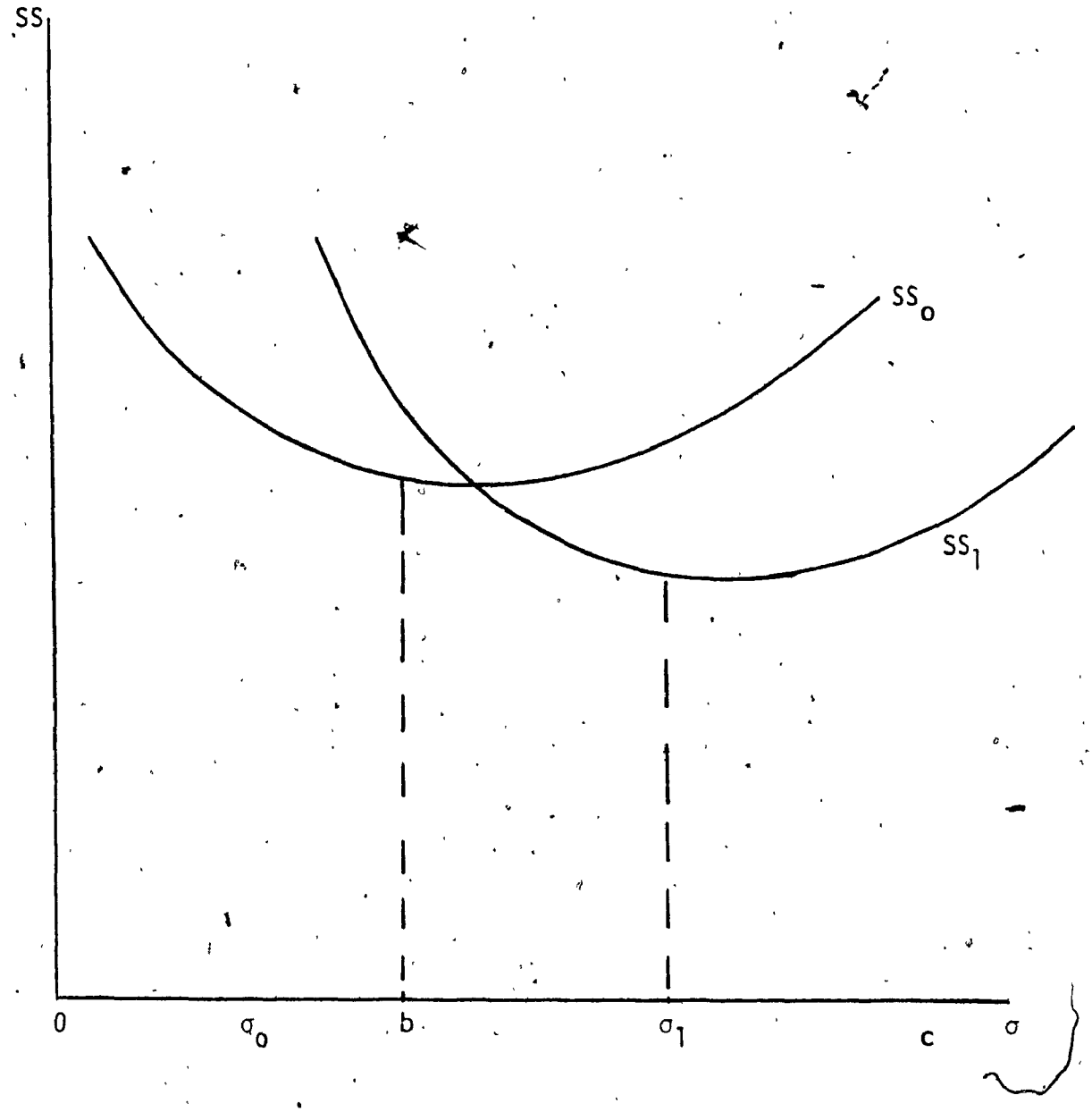

Figure 3.5 Graphical demonstration that σ_1 is preferred to σ_0, since min (SS_1) < new (SS_0).

Stage 2:

Using the (r_o, σ_o) solution from stage 1 as initial values, equation (5) above is solved to obtain:

$$\begin{pmatrix} \hat{r} \\ \hat{\sigma} \end{pmatrix} = \begin{pmatrix} r_o \\ \sigma_o \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix} \quad \text{where}$$

$$\begin{bmatrix} -A \\ -B \end{bmatrix} = \begin{bmatrix} SS_{rr} & SS_{r\sigma} \\ SS_{r\sigma} & SS_{\sigma\sigma} \end{bmatrix}^{-1} \begin{bmatrix} SS_{rr} \\ SS_{\sigma\sigma} \end{bmatrix} \tag{7}$$

and A,B represent the direction vectors of the iteration process. The convergence criteria are set at $\lambda = 10^{-6}$ and δ_1 and δ_2 are defined as:

$$\delta_1 = \frac{\hat{r}-r_o}{r_o} \quad \text{and} \quad \delta_2 = \frac{\hat{\sigma}-\sigma_o}{\sigma_o}.$$

If $|\delta_i| \leq \lambda$ for $i = 1,2$, then the algorithm stops. This is equivalent to having $|A| \leq 10^{-6}$ and $|B| \leq 10^{-6}$ for the process to stop and convergence to be achieved. If $|\delta_i| \geq \lambda$, (7) is resolved when $\begin{pmatrix} \hat{r} \\ \hat{\sigma} \end{pmatrix}$ are used as the initial values.

The estimation procedure will now be illustrated. Consider the case of two call options on Gulf Canada Stock, which expire 15 Aug. 1980, and have exercise prices $E_1 = 22.00$ and $E_2 = 27.50$, respectively. The daily data on the stock price, S, option prices C_1 and C_2, the relevant weekly Treasury Bill Rate from 5 May, 1980 to 15 Aug. 1980, are then collected. Only data for which there are

simultaneous trading for both options are considered. The results of the step-wise iteration, for r fixed to the range from $-.30$ through $.30$ and σ for the range of 0 through $.90$, are summarized in Table 3.6. Since the Jacobian is negative, the results from the first iteration are rejected. The process is then directed to a relative maximum. The other three sets of results are acceptable since $SS_{rr} > 0$, $SS_{\sigma\sigma} > 0$ and the Jacobian, $J > 0$. However, the last set for which $\hat{r} = -.30$ and $\hat{\sigma} = .81$ is preferred since the residual sum of squares, SS, is the least. However, since $A = -.01$ and $B = .05$ for this case,

$$\begin{pmatrix} \hat{r} \\ \hat{\sigma} \end{pmatrix} = \begin{pmatrix} -.30 \\ .81 \end{pmatrix} + \begin{pmatrix} -A \\ -B \end{pmatrix} = \begin{pmatrix} -.02 \\ .76 \end{pmatrix}$$

was chosen as the initial values for stage 2 of the procedure. Further, examination of Table 3.7 reveals that the final estimate of r and σ are $-.0168$ and $.7424$, respectively, for the case $E_1 = \$22.00$ and $E_2 = \$27.50$. According to the procedure in Stage 1, the initial values chosen were quite close to the final solution. This allows for the rapid convergence of the iterative procedure and it prevents it from being trapped by the numerous saddle points (when $J = 0$) or being directed towards local maxima (when $J < 0$).

TABLE 3.6

Results of the step-wise iteration for $-.30 \leq r \leq .30$ and $0 < \sigma \leq .90$

SS_{rr}	$SS_{\sigma\sigma}$	J	A	B	SS	$\hat{r}$	$\hat{\sigma}$
741.7	7696.1	-405622	-.18	.26	221.6	-.27	.44
852.5	2226.9	85795	.65	-.34	30.97	-.27	.90
855.9	2244.3	92701	.59	-.32	29.5	-.26	.90
873.0	1928.4	73402	-.01	.05	28.26	-.03	.81

(b) The Data

For each security in the data base described earlier [See 3.3 (d)], all of their frequently traded calls with an expiration date of 15 Aug. 1980 were chosen. The sampling procedure to minimize non-alignment has been described in Section 3.3 (d). A listing of the number of pairs of options selected for each security is given in Table 3.1. Therefore 147 pairs of equations had to be estimated.*

(c) Empirical Findings

The results of the longitudinal test on Gulf for 7 different option price series from 5 May to 15 Aug. 1980, are summarized in Table 3.5. The options have exercise prices varying from \$22.00 to \$34.00 with intervals of \$2.00. The following notation is used in Table 3.7.

$$E_i = 22 + 2(i-1), \quad i = 1, \ldots 7 ;$$

$$t^* = 15 \text{ Aug. } 1980 ;$$

* For an n option series on a particular stock, there are $\frac{n(n-1)}{2}$ equations. For example, for the longitudinal tests, seven option series on Gulf Canada with different exercise prices but all terminating on 15 Aug. 1980 were considered. This resulted in 21 [7(6)/(2)] different sets of simultaneous equations.

(E_i, E_j), $i \neq j$ represents the pair of equations for which the exercise prices are E_i, E_j, respectively,**

SS = sum of squared residuals;

$$SS_{rr} = \frac{\partial^2(SS)}{\partial r^2};$$

$$SS_{\sigma\sigma} = \frac{\partial^2(SS)}{\partial \sigma^2};$$

$J \equiv$ Jacobian $= S_{rr}S_{\sigma\sigma} - (S_{\sigma r})^2$; and

A,B = direction vectors of r, σ, respectively.

Two important observations can be drawn from Table 3.7. Firstly, the estimated values of the risk-free rate, $\hat{r}$, range from -.1811 to .1756. Furthermore, there are several instances of negative or quite low values of $\hat{r}$. If, for the moment, all values of $\hat{r} \leq .0500$ are discarded, then the average of the remaining $\hat{r}$ values is $\bar{\hat{r}} = .1263$. Since the three-month Bank of Canada T-Bill rate on 7 May 1980 was $r = .1392$, the average computed $\hat{r}$ value is low relative to the T-Bill rate, even when negative or low values of $\hat{r}$ have been ignored. There is a possible explanation for this result. More specifically, Merton [1976] has stated

** For example, (E_1, E_2) refers to the pair of equations for which the exercise prices are $E_1 = 22$ and $E_2 = 24$, respectively.

that finance practitioners have observed that $C^{BS} < C^{a}$ for both deep-in-the-money and deep-out-of-the-money options. In other words, the Black-Scholes price tends to understate the actual market price(*) of these type of options. The implications of this mispricing are depicted in Figure 3.6. If the B-S price corresponded exactly with the actual market price, then the (r,σ) shown in Table 3.7 would be the correct value of the interest rate and the standard deviation, respectively. However if $C^{BS} < C^{a}$ in general, then our procedure simultaneously estimates:

$$\hat{C}^{BS} = C_1^a \quad \text{and} \quad \hat{C}_2^{BS} = \hat{C}_2^a \qquad \text{instead of}$$

$$\hat{C}_1^{BS} = C_1^a \quad \text{and} \quad C_2^{BS} = C_2^a$$

This would lead to $(\hat{r},\hat{\sigma})$ where $\hat{r} < r$ and $\hat{\sigma} > \sigma$. Consequently, results which understate the interest rate and over-estimate the standard deviation would be obtained as in Table 3.7.

Secondly, the results in Table 3.7 show that the pairs of equations, $(i,7)$, $i = 1,2,3,4,5$ yield intersection points which are in the

---

* Note that the direction of the bias in the B-S price is still a matter of debate. Black-Scholes [1972] found that for deep-in-the-money options, $C^{BS} > C^{a}$, and for deep-out-of-the-money options, $C^{BS} < C^{a}$; whereas MacBeth and Merville [1979] found ~~the~~ exact opposite.

neighborhood of the T-Bill rate of $r = .1392$. The equations for which $i = 1,2, \ldots 5$, are for deep-in-the-money options and equation 7 is for a deep-out-of-the-money option. The average $\hat{r}$ value for these five equations is $\bar{\hat{r}} = .1420$ which compares very well with the T-Bill rate of .1392. Consequently, it is apparent, that the solution pair $(\hat{r},\hat{\sigma})$ to a system of equations involving one deep-in-the-money option and another deep-out-of-the-money, compares quite well to the pair $(\hat{r}_B,\hat{\sigma}_a)$ where $\hat{r}_B$ is the relevant T-Bill rate and $\hat{\sigma}_a$ is the actual standard deviation. Indeed, there is a more general tendency in the results.

Consider two pairs of equations (i,j) and (i,k) where (i,j) represents the equations for which C_i had exercise price E_i and C_j has exercise price E_j. Assume that (i,j) intersect at $(\hat{r}_1,\hat{\sigma}_1)$. If $E_k > E_j$, then $\hat{r}_2 > \hat{r}_1$ and $\hat{\sigma}_2 < \hat{\sigma}_1$. For example in Table 3.7, (1,4) represents the equations for which C_1 has exercise price $E_1 = \$22$ and C_4 has exercise price $E_4 = \$28$, and (1,5) represents the equations for which C_1 has exercise price $E_1 = \$22$ and C_5 has exercise price $E_5 = 30$. As predicted (1,4) has $\hat{r} = .0811$ and $\hat{\sigma} = .6948$ and (1,5) has $\hat{r} = .1286$ and $\hat{\sigma} = .6195$. There is a mathematical reason for this. Consider a typical contour, $C^{BS} = C^*$; that is, $f(r,\sigma) = 0$. As shown earlier, the slope of this contour is:

$$\frac{d\sigma}{dr} = -g_r/g_\sigma$$

$$= \frac{\sqrt{t}\,N(d^*)}{Z(d^*)}, \quad \text{where } d^* = d - \sigma\sqrt{t} \quad \text{and}$$

$$d = [\ln(S/E) + (r + \frac{\sigma^2}{2})t]/(\sigma\sqrt{t})$$

But $\lim_{d^* \to \infty} \frac{N(d^*)}{Z(d^*)} = \infty$ and $\lim_{d^* \to \infty} \frac{N(d^*)}{Z(d^*)} = 0$

by L'Hopital's rule. Hence, as d^* increases positively, the contour becomes steeper; whereas as d^* increases negatively, the contour flattens out. Furthermore, if E increases with S, r, σ and t fixed, $d = [\ln(S/E) + (r + \frac{\sigma^2}{2})t]/\sigma\sqrt{t}$ decreases and so the contour becomes less steep than before. Consequently, if C_1 is fixed and C_4 intersects C_1 at $\hat{r} = .0811$ and $\hat{\sigma} = .6948$, then C_4 (which is flatter because $E_4 = \$28$ and $E_5 = \$30$) will intersect C_1 at a higher $\hat{r}$ and lower $\hat{\sigma}$ (See Table 3.7).

TABLE 3.7

Results of the Non-Linear Simultaneous Estimations for Gulf Can.

$E_{i,j}$	S_{rr}	$S_{\sigma\sigma}$	J	A	B	SS	$\hat{r}$*	$\hat{\sigma}$
(1,2)	879.1	119.7	11[illegible]985	1×10^{-8}	-4×10^{-7}	15.62	-.0493	.8435
(1,3)	820.9	1503.8	119839	-1×10^{-8}	5.5×10^{-7}	17.40	-.0355	.8319
(1,4)	1577.9	3682.2	1043710	-1×10^{-8}	9.7×10^{-7}	14.08	.0811	.6948
(1,5)	1611.9	5174.2	2316964	-4×10^{-8}	5.2×10^{-7}	14.52	.1286	.6195
(1,6)	1532.7	6794.6	3985377	-4×10^{-8}	4.9×10^{-7}	16.04	.1595	.5695
(1,7)	1380.0	7259.2	4584607	-5×10^{-8}	5.5×10^{-7}	18.44	.1499	.5653
(2,3)	786.3	1678.6	112114	-4×10^{-8}	1.5×10^{-7}	21.08	-.0721	.7567
(2,4)	815.2	2186.7	175204	-8×10^{-8}	3×10^{-8}	23.56	-.0942	.7623
(2,5)	886.9	3112.3	463979	1×10^{-8}	-5.1×10^{-7}	15.99	.0279	.6845
(2,6)	894.6	4230.3	1017648	2×10^{-8}	-3.5×10^{-7}	16.74	.0905	.5986
(2,7)	843.6	4912.5	1537472	3×10^{-8}	-5×10^{-7}	19.32	.1325	.5634
(3,4)	1142.2	3929.8	161275	-3×10^{-8}	2.7×10^{-7}	12.15	-.1064	.8145
(3,5)	1182.6	5240.2	429525	-1×10^{-8}	-2.0×10^{-7}	6.68	.0380	.6445
(3,6)	1197.6	6561.7	1043480	-1×10^{-8}	-4.2×10^{-7}	8.56	.1214	.6086
(3,7)	1140.9	7299.0	1671466	-1×10^{-8}	4.4×10^{-7}	9.40	.1610	.5735
(4,5)	1144.9	6056.0	174253	2×10^{-8}	1.8×10^{-7}	6.46	-.0835	.7535
(4,6)	1226.1	8241.1	662091	4×10^{-8}	6.2×10^{-7}	7.46	.0983	.6235
(4,7)	1213.0	9749.7	1469830	0	3.9×10^{-7}	8.38	.1756	.5651
(5,6)	922.7	7501.6	128471	1×10^{-8}	3.5×10^{-7}	5.16	-.1020	.7141
(5,7)	988.3	10007.2	500734	0	4.5×10^{-7}	6.75	.0912	.6039
(6,7)	660.4	7834.4	70714	1.1×10^{-7}	-2×10^{-8}	5.48	-.1811	.7079

*The Bank of Canada three month T-Bill rate on 7 May, 1980 was r = .1392. The overall average interest rate, $\hat{r}$ was .0711.

3.4 Simultaneous Estimation of the Variance and Risk-Free Rate - Full Model

(a) Procedure

It is not sufficient to consider the computed vector $(\hat{r},\hat{\sigma})$ as being reliable estimates of (r,σ) by only estimating different pairs of equations. Indeed such a procedure does not necessarily yield a solution satisfying the full set of equations. It is evident that n equations in r unknown variables $(n > r)$ may not have a solution, although smaller subsets of m equations $(m < n)$ may have a solution. Accordingly, if there are n options on a security which all terminate simultaneously, then such a case will present n equations in two unknowns r and σ. That is, the following system is obtained.

$$C_j^a = f_j(r,\sigma) + \varepsilon_j \quad , \quad j = 1,\ldots.n \; .$$

The objective is to obtain estimated values $\hat{r}$ and $\hat{\sigma}$ such that $\sum_{j=1}^{n} \varepsilon_j' \varepsilon_j$ is minimized. Observe that in this procedure, maximum likelihood estimates of r and σ are obtained if the variance of the error terms ε_j, $j = 1,\ldots n$ are equal; i.e., $\sigma^2(\varepsilon_j) = k \;\; \forall_j = 1,\ldots n$. This is easy to see since maximizing the logarithm of the likelihood function,

$$\log \; L = \frac{1}{\text{constant}} - \sum_{j=1}^{n} \frac{n}{2} \log \sigma_j^2 - \frac{1}{2} \sum_{j=1}^{n} \left(\frac{\varepsilon_j' \varepsilon_j}{\sigma^2(\varepsilon_j)}\right) ,$$

is equivalent to minimizing the loss function, $F = \sum_{j=1}^{n} \frac{\varepsilon_j' \varepsilon_j}{\sigma^2(\varepsilon_j)}$.

But, if $\sigma^2(\varepsilon_j) = k$ for all $j = 1, \ldots n$, then minimizing F is equivalent to minimizing $\sum_{j=1}^{n} \varepsilon_j' \varepsilon_j$. Hence, for the case in which $\sigma^2(\varepsilon_j) = k$ for $j = 1, \ldots n$, maximum likelihood estimates of $\hat{r}$, $\hat{\sigma}$ are obtained. The estimation procedure is the same as was done in the previous section in that the Newton-Raphson technique was applied to find $\hat{r}, \hat{\sigma}$ which minimized $\sum_{j=1}^{n} \varepsilon_j' \varepsilon_j$.

(b) Data and Results

The data set and results are shown in Table 3.8. All options ending on July 1983 were considered for eight securities listed on the TCO. The ticker symbols for these securities are shown in Table 3.8. The number of options, range from 3 on RGO, HYO and DEN through 8 for ASM. The estimation period considered is the month of June 1983. The Newtown Raphson method was applied to the full system of equations for each security. The respective exercise prices for each set of options, as well as the values of $\hat{r}$ and $\hat{\sigma}$ obtained, are shown in Table 3.8. For comparison the 3-month Treasury Bill rate on June 1, 1983 was .0928.

It is interesting to note that $\hat{r}$ for NTL was .0986, which compares favorably with r_B = :0928. The other estimated values are, in general, quite low with respect to r_B, with a model value of about

six per cent.

On the surface, the results obtained above are quite poor relative to r_B. But the validity of the B-S model cannot be judged solely on the reliability of the estimates, $\hat{r}, \hat{\sigma}$. Thus, pronouncements concerning the adequacy of the model must be made after analysing and interpreting the behaviour of the residuals. The model is, therefore, reformulated so that the error terms are assumed to be first order serially correlated.

(c) Model with serially correlated errors

Assuming that the error terms are autocorrelated, the original model can be reformulated as follows:

$$C^a_{1,t} = f_{1t}(r,\sigma) + \varepsilon_{1t} \quad \text{and}$$

$$C^a_{2,t} = f_{2t}(r,\sigma) + \varepsilon_{2,t}$$

Now assume that $\varepsilon_{i,t} = \rho_i \, \varepsilon_{i,t-1} + u_{i,t}$, $i = 1,2$;

that is, the errors are first order serially correlated.

Also, for $i = 1,2$, assume that :

$$E(u_{i,t}) = 0 ;$$

$$E(u_{i,t} \, i_{i,s}) = E(u_{i,s}) \, E(u_{i,t}) \quad s \neq t \quad ;$$

$$E(u_{1,t}\, u_{2,t}) = E(u_{1,t})\, E(u_{2,t}) \quad ;$$

$$\sigma^2(u_{i,t}) = \sigma^2 .$$

Then $C_{i,t} - \rho_i C_{i,t-1} = f_{i,t}(r,\sigma) - \rho_i f_{i,t-1}(r,\sigma) + u_{i,t}$ for $i = 1,2$.

The transformed model now consists of a pair of simultaneous non-linear equations, corrected for autocorrelation. The procedure to estimate the vector $(\hat{r},\hat{\sigma})$ for the transformed model is as follows.

For each $i = 1,2$, estimate ρ_i under the assumption

$$\varepsilon_{i,t} = \rho_i\, \varepsilon_{i,t-1} + u_{i,t} \quad ; \text{ and}$$

get $$\hat{\rho}_i = \left[\sum_{t=1}^{T} \hat{\varepsilon}_{i,t}\, \hat{\varepsilon}_{i,t-1} \right] / \sum_{t=1}^{T} \hat{\varepsilon}_{i,t}^{\,2} ,$$

where $\varepsilon_{i,t} = C^a_{i,t} - f_{i,t}(\hat{r},\hat{\sigma})$. Thus obtain $\hat{r},\hat{\sigma}$, as in the previous section by solving the minimization problem. If the residual sum of squares for the transformed model is $TSS = u_1' u_1 + u_2' u_2$, then the objective is to minimize TSS over r,σ . Formally, the problem, is:

$$\min_{(r,\sigma)} TSS = \left\{ \sum_{t=1}^{T} \sum_{i=1}^{2} \left[(C^a_{i,t} - \hat{\rho}_i C^a_{i,t-1}) - (f_{i,t} - \hat{\rho}_i f_{i,t-1}) \right]^2 \right.$$

where $f_{i,t} \equiv f_{i,t}(r,\sigma)$.

(d) Empirical Results

The results from this estimation are recorded in Tables 3.9(a), 3.9(b), 3.9(c), 3.9(d) . The data set is the same as that recorded in Table 3.5. First, examine the case of Northern Telecom (NTL). The exercise prices considered are E_i = 30, 31.63, 33.38 36.63, 40, and 45. Recall that the estimated values for $\hat{r}, \hat{\sigma}$ for the full set of equations are .0986 and .4026, respectively. Now, each pair of the possible 15 equations is considered separately. But, first, for each option, the values of ρ_i, i = 1, ... 5 are computed. These are shown in Table 3.9(a) and options for which E = 30,40 and 45 are seen to be serially correlated. For the estimation procedure, these values of $\hat{\rho}_i$ are used as starting values for ρ_i. Also statistics for R^2, the mean of the residuals, $\mu(\hat{\varepsilon}_i)$ and the variance of the residuals, $\sigma^2(\hat{\varepsilon}_i)$ are given for each equation. The results obtained for each pair of equations for NTL are extremely good relative to r_B = .0928 . In fact, each pair of equations yield a value of $\hat{r}$ and $\hat{\sigma}$ which are quite close to the values obtained for the full system. In fact, the average of the $\hat{r}_i$ values, i = 1, ... 15 is $\bar{r}$ = .0986 and the 3-month Treasury Bill rate is r_B = .0928. It is interesting to note that equations 1 and 2 for which E_1 = 30 and E_2 = 31.63 have indicated $\sigma^2(\hat{\varepsilon}_i)$ = .040 . Hence, using the results obtained before, $\hat{r}$ = .0838 and $\hat{\sigma}$ = .4209 are maximum likelihood estimates. However $\hat{\rho}_1$ = .4250 is significant for α = 5% level of significance. When this serial correlation is accounted for and the transformed

model estimated, $\hat{r} = .0913$ and $\hat{\sigma} = .4102$ are obtained. This compares well with $r_B = .0928$. Turning now to Genstar, Table 3.9(b) shows that $r_F = .0690$ and $\hat{\sigma}_F = .4087$. It is also seen that equation 1 has a significant serial correlation coefficient of $\hat{\rho}_1 = \underset{(2.195)}{.4594}$. When equations 1 and 2 are estimated, but the autocorrelation in equation 1 unaccounted for, $\hat{r} = .1000$ and $\hat{\sigma} = .4310$ are obtained. However, when the autocorrelation in equation 1 is considered, and $\hat{r} = .1000$, $\hat{\sigma} = .4310$, $\hat{\rho}_1 = .4594$ and $\hat{\rho}_2 = .0959$ used as the vector of starting values, the iteration technique converged, yielding

$$\begin{bmatrix} \hat{r} \\ \hat{\sigma} \\ \hat{\rho}_1 \\ \hat{\rho}_2 \end{bmatrix} = \begin{bmatrix} .0871 \\ .4469 \\ -.0333 \\ -.0081 \end{bmatrix}$$

The results for Ranger Oil (RGO) are also interesting. For the third option (E_3 = 12 1/2), there is significant correlation ($\hat{\rho}_3 = .4727$). Furthermore, $\sigma^2(\hat{\varepsilon}_i) = .007$ for both options 1 and 3. Yet, the simultaneous estimation of equations 1 and 3 yielded very poor results: $\hat{r} = .0109$ and $\hat{\sigma} = .4912$. Even, when $\hat{\rho}$ is considered in the estimation the transformed model gave $\hat{r} = .0212$ and $\hat{\sigma} = .4692$. The results in Table 3.9(c) show that no more correlation exists at lag 1 and so the values of $\hat{r}$ and $\hat{\sigma}$ cannot be improved by further application of the present technique.

TABLE 3.8

Results of Simultaneous Estimation of Full Set of Options ending July 1983

Ticker Symbol	No. of Options	Exercise Prices	$\hat{r}_F$	$\hat{\sigma}_F$
RGO	3	$7\frac{1}{2}$, 10, $12\frac{1}{2}$	.0538	.4690
GST	4	25, $27\frac{1}{2}$, 30, $32\frac{1}{2}$	.0690	.4087
NTL	7	30, $31\frac{5}{8}$, $33\frac{3}{8}$, 35, $36\frac{5}{8}$, 40, 45	.0986	.4026
HYO	3	$7\frac{1}{2}$, 10, $12\frac{1}{2}$	.0151	.4377
BVI	4	15, $17\frac{1}{2}$, 20, $22\frac{1}{2}$	.0555	.3918
DEN	3	$32\frac{1}{2}$, 35, 40	.0519	.2665
DM	4	$17\frac{1}{2}$, 20, $22\frac{1}{2}$, 25	.0612	.5141
ASM	8	$17\frac{1}{2}$, 20, $22\frac{1}{2}$, 25, $27\frac{1}{2}$, 30, $32\frac{1}{2}$, 35	.0332	.5193

REMARKS

(1) The 3-month Treasury Bill rate on June 1, 1983 was $r_B = .0928$.

(2) The estimation period was June 1 to June 30, 1983.

TABLE 3.9(a)

Results of the Estimation Procedure for Northern Telecom (NTL)

i	1	2	3	4	5	6
E_i	30	$31\frac{5}{8}$	$33\frac{3}{8}$	$36\frac{5}{8}$	40	45
$\hat{\rho}_i$	.4250*	-.0811	.0017	.4878*	.5676*	.5114
t_i	1.99	-.345	.007	2.37	2.92	2.52
R^2	.9958	.9960	.9960	.9913	.9812	.9757
$\mu(\hat{\varepsilon}_i)$	-.007	.030	.031	.011	.030	-.059
$\sigma^2(\hat{\varepsilon}_i)$	.040	.040	.033	.076	.107	.048

$\hat{r}$
$\hat{\sigma}$

i	2	3	4	5	6
1	.0838 .4209	.0759 .4469	.0752 .4191	.0608 .4336	.0943 .3943
2		.1016 .4273	.1073 .3958	.0969 .4163	.1196 .3790
3			.1059 .3999	.0941 .4193	.1222 .3889
4				.0847 .4198	.1241 .3779
5					.1177 .3903

$\bar{r}$ = .0960; $\hat{r}_F$ = .0986 $\hat{r}_B$ = .0928

* $\hat{\rho}$ are statistically significant at α = .05

TABLE 3.9 (b)

Results of the Estimation Procedure for Genstar (GST)

i	1	2	3	4
E_i	25	27½	30	32½
$\hat{P}_i$	.4594*	.0959	.4440*	.6300*
t_i	2.19	.408	2.10	3.44
R^2	.9949	.9933	.9904	.9600
$\mu_\circ(\hat{\varepsilon}_i)$	.119	.097	−.031	−.144
$\sigma^2(\hat{\varepsilon}_i)$	.037	.042	.042	.049

Remarks

(a) For E_1, E_2, $(\hat{r}, \hat{\sigma})$

$$= (.1000, .4310)$$

For starting vector $\begin{bmatrix} .1000 \\ .4310 \\ .4594 \\ .0959 \end{bmatrix} \rightarrow \begin{bmatrix} \hat{r} \\ \hat{\sigma} \\ \hat{P}_1 \\ \hat{P}_2 \end{bmatrix} = \begin{bmatrix} .0871 \\ .4469 \\ .0333 \\ .0081 \end{bmatrix}$

(b) For the full set of equations:

$$\hat{r}_F = .0690$$

$$\hat{\sigma}_F = .4087$$

* Statistically significant for $\alpha = .05$.

TABLE 3.9 (c)

Results of the Estimation Procedure for Ranger (RGO)

i	1	2	3
E_i	7½	10	12½
$\hat{P}_i$	.2075	.1299	.4727*
t_i	.900	.559	2.28
R^2	.9902	.9692	.9970
$\mu(\hat{\varepsilon}_i)$	−.027	.047	.012
$\sigma^2(\hat{\varepsilon}_i)$	.007	.025	.007

$$\begin{pmatrix} \hat{r} \\ \hat{\sigma} \end{pmatrix}$$

i	2	3
1	.0810 .2498	.0109 .4912
2		.0938 .4621

Comments

(a) For E_1, E_3, the transformed model yields $\hat{r} = .0212$, $\hat{\sigma} = .4692$.

(b) for the full set of equations, $\hat{r}_F = .0538$, $\hat{\sigma}_F = .4690$.

* Statistically significant at $\alpha = .05$.

TABLE 3.9 (d)

Results of the Estimation Procedure for Husky Oil (HYO)

i	1	2	3
E_i	7½	10	12½
$\hat{P}_i$	.2650	.4127*	−.1298
t_i	1.16	1.92	−.555
R^2	.8224	.8821	.7352
$\mu(\hat{\varepsilon}_i)$	.017	−.014	.006
$\sigma^2(\hat{\varepsilon}_i)$	.008	.012	.003

$\hat{r}$

$\hat{\sigma}$

i	2	3
1	−.0013 .4035	−.0565 .4556
2		.0109 .4336

Comments

(a) For E_1, E_2, the transformed model yields $\hat{r} = .0121$, $\hat{\sigma} = .3918$.

(b) The full system yields $\hat{r} = -.0151$, $\hat{\sigma}_F = .0928$.

* Statistically significant at $\alpha = .05$.

Finally, the results obtained for Husky Oil (HYO) are presented in Table 3.9(d). For the 3 options considered, the full system yielded $\hat{r}_F = -.0151$ and $\hat{\sigma}_F = .0928$. In fact, the estimation of the three different pairs of options yielded no better results. Even after the serial correlation in equation 1 is removed, the improvements are marginal.

3.6 Conclusion

The major objective of this chapter was to simultaneously estimate the parameters (r,σ) of the Black-Scholes model in order to test its functional form. As a preliminary analysis, r,σ were estimated in a manner similar to Krausz [1974] and O'Brien and Kennedy [1982]. Specifically, their procedure attempts to find a feasible solution, using the familiar Newton-Raphson algorithm on the equations:

$$C_i^a \doteq f_i\,(r,\sigma) \qquad i = 1,2 .$$

where C_i = actual market prices

f_i = Black-Scholes model price .

There were three major problems with these earlier studies. First, since the equations to be estimated could have multiple solutions just trying to find one solution is unsatisfactory. Secondly, since the samples used were very limited and the estimation was only very limited, "snapshots" of the process were obtained. Finally, if noise

enters the model through measurement errors (or some other reason) then the estimates may be entirely unreliable.

Instead, the procedure used herein, was to estimate $C_{i,t} = f_{i,t}(r,\sigma) + \varepsilon_{i,t}$ and minimization of the residual sum of squares $\sum_{t=1}^{T} \sum_{i=1}^{2} \varepsilon'_{i,t}\, \varepsilon_{i,t}$. The procedure also used the Newton-Raphson algorithm to search for an optimum. Furthermore, an efficient grid-search method was devised to select an appropriate initial vector (r_0,σ_0). The general finding was that the estimates of r and σ are biased, and, in particular, that the $\hat{r}$ values were underestimated. Some possible explanations for this behaviour were provided in the chapter.

Next, a model, using the full system of equations obtained from all options on the stock ending on a particular date, was formulated. The estimates of $\hat{r},\hat{\sigma}$ were shown to be maximum-likelihood estimates when the variance of the error terms are equal. The results obtained in this case were an improvement over the 2 x 2 procedure used before, but except for NTL, the $\hat{r}$ values were still quite low compared to the relevant 3-month T-Bill rate, r_B.

A model incorporating first order serial correlation was then formulated and estimated. Again the results (see Tables 3.9(b)-(d)) range from very good for NTL to very poor for HYO. The overall average $\hat{r}$ value was still less than the T-Bill rate for the period,

and the individual values vary widely. This might signal a non-stationary variance of stock price returns. Thus, this case will be studied further in the next chapter. Furthermore, other reasons can be postulated for the widely varying estimates of r and σ. For example, the assumptions of the B-S model do not allow for real world imperfections such as transactions costs, margin requirements, differing borrowing and lending rates, etc.

APPENDIX 3.1

Latane and Rendleman [1975] used the Black-Scholes model to study the pricing of CBOE options. They obtained the implied volatility of stock price returns by numerically solving:

$$ISD^2 = f(E,S,t^*,r,C^a) \quad ,$$

where E = exercise price

S = stock price,

r = risk-free rate,

t^* = time to expiration of option,

C^a = observed option price; and

ISD^2 = implied variance.

They used the solutions to the above equation for all the n options written on the same security in the following weighted formula:

$$WISD_{it} = \frac{\left(\sum_{k=1}^{n} ISD_{kit}^2 \; d_{kit}^2\right)^{1/2}}{\sum_{k=1}^{n} d_{kit}} \qquad \ldots\ldots\ldots \quad (*)$$

where $WISD_{it}$ = the weighted average implied standard deviation for stock i at time t, and

$$d_{kit} = \frac{\partial C_k}{\partial (ISD_k)}$$

In his Ph.D. thesis, Chiras [1977] asserted that his formula was wrong based on the following simple illustration:

$$\text{Let } d_{kit} = 1 \quad ISD_{kit} = .1 \qquad n = 2 ,$$

then $WISD_{it} = .1707$. He then concluded that $WISD \to 0$ as $j \to \infty$.

Since a closed form solution for (*) has not yet been obtained, Latane-Rendleman used a numerical approach to obtain a solution. Hence, even if it is assumed that C^a follows a normal distribution, the distribution of ISD^2_{kit} would be indeterminate.

Furthermore, since $d_{kit} = \dfrac{\partial C_k}{\partial (ISD_k)}$, for $k = 1, \ldots n$, would also have a distribution that cannot be determined, the distribution of $WISD_{it}$ cannot be found. Thus, the conclusion by Chiras that $E[WISD_{it}] \to o$ as $n \to \infty$ is impossible to prove in general.

Chiras also proposed another formula for computing the implied stock price variance. This formula is stated as:

$$WISD^2 = \left(\sum_{i=1}^{N} ISD_i^2 \, e_i^2 \right)^{1/2} \Big/ \left(\sum_{i=1}^{N} e_i^2 \right) \qquad \ldots\ldots\ldots\ldots\ldots (**)$$

where $e_i = \dfrac{\partial C_i}{\partial (ISD)_i} \cdot \dfrac{ISD_i}{C_i}$.

It is noted that Latané-Rendleman's formula, (LR), given in (*) above, is not a true weighted formula since the sum of the weights is not unity. Hence, for comparison with Chiras' formula in (**), the following <u>modified</u> LR formula is proposed:

$$WISD^2 = \left(\sum_{i=1}^{n} ISD_i^2 \, d_i^2 \right) \Big/ \left(\sum_{i=1}^{n} d_i^2 \right) \qquad \ldots\ldots\ldots\ldots\ldots (***)$$

It is now demonstrated that the approach in Chapter 3 that allowed for a measurement error allow for a direct comparison between the Chiras and LR formulas. Recall the the following computation formula:

$$ISD_a^2 = f(E, S, t^*, r, C^a) .$$

Assume that the true relationship is

$$ISD_{BS}^2 = f(E, S, t^*, r, C^{BS}) .$$

Further assume that the actual price is the true BS price with an

additional measurement error:

$$C^a = C^{BS} + \varepsilon$$

Expanding f( ) as a function of the single variable C in a standard Taylor series, we obtain,

$$ISD_a^2 = ISD_{BS}^2 + \frac{\partial f}{\partial C} \varepsilon .$$

Rewriting, we get $y = \alpha + v$, the classical problem of estimating the mean of a distribution, where v is an error term which is, possibly, heteroscedastic, depending on the properties of ε . We have a simple preliminary result.

<u>Lemma 1</u> If $E(\varepsilon) = 0$ and $\sum w_i = 1$, then any estimator of the form $\sum w_i y_i$ is unbiased.

<u>Proof</u> $E[\sum w_i y_i] = \sum w_i E(y_i) = 1\alpha = \alpha$

The following lemma demonstrates the both the modified LR and Chiras formulas are closely related and are obtained by appropriate specifications of ε .

<u>Lemma 2</u> a) If $Var(\varepsilon) = \sigma^2 I$ then the modified LR formula is obtained with $w_i = \frac{d_i^2}{\sum d_i^2}$. These are in fact Gauss-Markov weights.

b) If $Var(\varepsilon) = \left(\frac{C^a\sigma}{ISD_a}\right)^2 I$, then the Chiras formula is obtained with

$$w_i = \frac{e_i^2}{\sum e_i^2}.$$

<u>Proof</u> a) Since $y_i = \alpha + v_i$ where $v = \frac{\partial f}{\partial C}\varepsilon$, then $\frac{\partial C}{\partial f} y_i = \frac{\partial C}{\partial f}\alpha + \varepsilon_i$ where $\varepsilon_i \sim N(0, \sigma^2)$. Using Ordinary Least Squares, $\hat{\alpha} = \sum w_i y_i$ where

$w_i = \frac{d_i^2}{\sum d_i^2}$, $d_i = \frac{\partial C}{\partial f}$. This gives the modified LR formula.

b) Now $y_i = \alpha + v_i$ where $v = \frac{\partial f}{\partial C}\varepsilon$ and $Var(\varepsilon) = \left(\frac{C^a\sigma}{ISD_a}\right)^2$

Then $\frac{\partial C}{\partial f} y_i = \frac{\partial C}{\partial f}\alpha + \varepsilon_i$ where $\varepsilon_i \sim N\left(0, \left(\frac{C^a\sigma}{ISD_a}\right)^2\right)$.

Using Weighted Least Squares, $\hat{\alpha} = \sum w_i y_i$ where $w_i = \frac{e_i^2}{\sum e_i^2}$, $e_i = \frac{\partial C}{\partial f} \cdot \frac{f}{C^a}$.

This gives the Chiras formula.

APPENDIX 3.2

In this appendix, the first and second partial derivatives of the Black-Scholes formula are computed. That is, if $f(r,\sigma) = S\,N(d) - Ee^{-rt}\,N(d^*)$, then, fully simplified expressions are obtained for f_r, f_{rr}, f_σ, $f_{\sigma\sigma}$, $f_{\sigma r}$. These, as was seen before, are important for the application of the Newton-Raphson algorithm. However, before these partial derivatives are obtained, some preliminary lemmas and corrollaries are required.

Given $\quad C = S\,N(d) - Ee^{-rt}\,N(d^*)$

$$\text{where} \quad d = \frac{\ln(S/E) + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$d^* = d - \sigma\sqrt{t}$$

$$N(d) = \int_{-\infty}^{d} Z(x)\,dx, \text{ where } Z(x) = \frac{1}{\sqrt{2\Pi}\,\sigma}\, e^{\frac{-x^2}{2}}$$

<u>Lemma 1</u> $\quad Z(d^*) = e^{rt}\,\frac{S}{E}\,Z(d)$

<u>Proof</u> $\quad Z(d^*) = \frac{1}{\sqrt{2\Pi}\,\sigma}\, e^{-(d^*)^2/2}$

$$= \frac{1}{\sqrt{2\Pi}\,\sigma}\, e^{-[d^2 - 2\sigma d\sqrt{t} + \sigma^2 t]/2}$$

$$= \frac{1}{\sqrt{2\Pi}\,\sigma}\, e^{-d^2/2}\, e^{d\sigma\sqrt{t} - \frac{\sigma^2 t}{2}}$$

$$= Z(d)\; e^{rt}\,\frac{S}{E}$$

Since $d\sigma\sqrt{t} = \ln(S/E) + (r + \frac{\sigma^2}{2})\,t$.

<u>Corollary 1</u> $S\,Z(d) - Ee^{-rt}Z(d^*) = 0$

<u>Proof</u> $Ee^{-rt}Z(d^*) = Ee^{-rt}(Z(d)e^{rt}\frac{S}{E})$ by Lemma

$= S\,Z(d)$

<u>Corollary 2</u> If $f(r,\sigma) = S\,N(d) - Ee^{-rt}N(d^*)$,

then $f_r = tEe^{-rt}N(d^*)$

$f_\sigma = \sqrt{t}\,Ee^{-rt}Z(d^*)$

<u>Proof</u> $f_r = S\,Z(d)\,d_r - Ee^{-rt}Z(d^*)d_r^* + tEe^{-rt}N(d^*)$

But $d_r \equiv \frac{\partial(d)}{\partial r} = \frac{\sqrt{t}}{\sigma} = d_r^*$

$\therefore f_r = d_r\,[S\,Z(d) - Ee^{-rt}Z(d^*)] + tEe^{-rt}N(d^*)$

$= tEe^{-rt}N(d^*)$ by Corollary 1

Also $f_\sigma = S\,Z(d)d_\sigma - Ee^{-rt}Z(d^*)d_\sigma^*$.

But $d_\sigma \equiv \frac{\partial(d)}{\partial\sigma} = \sqrt{t} - \frac{d}{\sigma}$ and

$d_\sigma^* = d_\sigma - \sqrt{t}$

$$\therefore \quad f_\sigma = [S\,Z(d) - Ee^{-rt}\,Z(d^*)]d_\sigma$$

$$+ Ee^{-rt}Z(d^*)\sqrt{t}$$

$$= \sqrt{t}\,Ee^{-rt}\,Z(d^*) \quad \text{by Corollary 1}$$

$$= \sqrt{t}\;\; S\,Z(d) \quad \text{by Lemma 1}$$

<u>Lemma 2</u>

$$f_{rr} = \frac{t}{\sigma}\,[f_\sigma - \sigma f_r]$$

$$f_{\sigma\sigma} = \frac{f_\sigma}{\sigma}\,[dd^* - 1]$$

$$f_{r\sigma} = \frac{-\sqrt{t}\;d\;f_\sigma}{\sigma}$$

<u>Proof</u>

The proof is quite clear by using the previous Corollaries. For example;

$$f_{rr} = -t^2Ee^{-rt}N(d^*) + tEe^{-rt}Z(d^*)d_r^*$$

$$= -t(tEe^{-rt}N(d^*)) + \sqrt{t}\,[\sqrt{t}\,Ee^{-rt}Z(d^*)]d_r^*$$

$$= -t\,f_r + \sqrt{t}\,f_\sigma\,(\frac{\sqrt{t}}{\sigma})$$

$$= t[\frac{f_\sigma}{\sigma} - f_r] = \frac{t}{\sigma}\,[f_\sigma - \sigma f_r]$$

CHAPTER 4

THE DISTRIBUTION OF STOCK PRICE CHANGES ON THE TORONTO STOCK EXCHANGE: ITS IMPLICATIONS FOR OPTION PRICING

In the previous chapter, it was shown that the estimates of σ and r (i.e., the only unobservable parameters in the Black-Scholes model) behaved poorly, even after all the significant autocorrelation was removed prior to using the Newton-Raphson minimization procedure. A central assumption of that model is that stock price changes follow a geometric Brownian motion with constant drift and intertemporally constant variance. Thus the central theme of this chapter is to investigate the validity of this assumption for the Toronto Stock Exchange (TSE), and to study the implications of the results obtained for the pricing of options. The random walk hypothesis* (and in particular, the independence assumption of stock price changes) has been extensively studied and validated (see Fama [1965] for the U.S. case and Praetz [1972] for the Australia case).** However, the nature of the distribution of stock price changes has not yet been resolved.

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* See Chapter 5 for details about this hypothesis.

** No published research seems to exist for the Canadian case.

While most researchers have shown that $\{\ln(X_t/X_{t-1})\}$ is approximately normal, Fama [1963] and Mandelbrot [1963], have shown that the symmetric stable Paretian family provides a better fit. Furthermore, Praetz [1972] for the Australia case, and Blattberg and Gonedes [1974] for the U.S. stock market, have suggested that the t-distribution is statistically more adequate. However, almost without exception, researchers have found that the estimated distribution is too "fat tailed" to be normal.

The remainder of this chapter is organized as follows. Firstly, the Kolmogorov-Smirnov test of non-normality is applied to the monthly data for a sample of stocks on the TSE. The Skewness and Kurtosis statistics are computed for each security and tests of significance are conducted. This procedure is then repeated using daily data for a sample of securities.

Secondly, a test is performed of the proposition of Mandelbrot [1963] that the distribution of daily stock price changes is not normal, but belongs to the stable Paretian family with infinite variance. This test is based on the work by Fama and Roll [1971] on symmetric stable distributions. This family is defined by

$$\ln(\phi(t)) = i\,\delta t - |ct|^{\alpha}$$

where $\phi(t)$ is the characteristic function,

$i^2 = -1$,

t = any real number,

δ = location parameter,

c = scale parameter,

α = characteristic exponent

The parameter α, $0 < \alpha \leq 2$ is very crucial in the analysis, since $\alpha = 2$ indicates a normal distribution. An α between 0 and 2 indicates an infinite-variance distribution, where the tail becomes "flatter" as α decreases to zero. Through the use of Monte-Carlo experiments, Fama and Roll [1971] have demonstrated that a method using sample fractiles may be used to compute α. Thirdly, the variance of stock price changes will be examined for non-stationarity. This examination shows that variances of monthly price returns are serially correlated and move in direct proportion to the square of the market rate of return, R_{mt}^2. This result has important implications for the CAPM since OLS estimates of this model would be inefficient in the presence of heteroscedastic errors. Further, for a sample of 20 securities on the TSE, but which are also listed on the TCO, it is empirically demonstrated, that daily variances of stock price returns are generally a white noise with a non-zero mean. This identification is made through the use of the Box-Jenkins Methodology on a time series of daily variances obtained by a MINQUE* - type estimator.

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* The MINQUE estimator was originally developed by C.R. Rao [1970].

Finally, one-period forecasts of the daily variance are made by the ARIMA model obtained above. These forecasts, along with the appropriate risk free rate, are then used to recompute call prices for the B-S model. The predicted call price is called "acceptable" if it lies between the observed bid and ask call prices.

4.1 A Test of the Normality Hypothesis

Formally, the normality hypothesis states that stock price returns follow a normal distribution with constant parameters, u, σ. Although the question of normality is not important in order to test the validity of the random walk hypothesis, the true variability, $\sigma(r_k)$ of the serial correlation coefficient, r_k, would be underestimated if the distribution is non-normal. Hence $t^* = \frac{r_k}{\sigma(r_k)}$; where r_k is the correlation coefficient of lag k, would be overstated. In turn, this would result in a larger probability of rejecting the null hypothesis of no serial correlation (i.e., a type I error), when in fact the hypothesis was true. The estimation procedure used is as follows. First, a non-parametric goodness of fit test, the Kolmogorov-Smirnov test is applied. The test is relatively weak in that it can only be used to reject the null hypothesis. That is, since it is a test of non-normality, it cannot be used for accepting H_0. In fact, tests which are applicable to a 2-decision problem (such as accepting or rejecting H_0) must be based on a characteristic property of the distribution.

(a) Kolmogorov-Smirnov Test

The Kolmogorov-Smirnov test is applied to the series $\{r_t\}_{t=1}^{N}$, where $r_t = [\ln[S_t/S_{t-1}]$. That is, for each estimation, the data consists of a random sample $\{r_1, r_2, \ldots, r_N\}$ of size N, which has some unknown continuous distribution $F(x)$. The null hypothesis is:

$$H_0: F(x) = F_0(x), \quad \text{where} \quad F_0(x)$$

is the cumulative normal distribution. The sample is then ordered so that $\{ r_1 \leq r_2 \leq \ldots \leq r_n \}$, and the test statistic is then defined as:

$$D^* = \max_i \left| F_0(r_i) - \frac{i}{N} \right|$$

Tables for determing the significance of D^* are readily available. However, for large N, $D(.05,N) = \frac{1.36}{\sqrt{N}}$, and $D(.01,N) = \frac{1.63}{\sqrt{N}}$.

The Data

The data set consists of twenty stocks selected from each sector on the Toronto Stock Exchange for the period 1 Jan. 1970 to 31 Dec. 1979. (see Table 4.1 for the full listing). End of month closing prices are recorded for each security for the 10 year period. The prices were adjusted for stock splits and cash dividends.

Empirical Findings

The results from conducting the Kolmogorov-Smirnov Test are given in Table 4.2. The most interesting finding is that the null hypothesis of normality cannot be rejected for any of the twenty securities. The critical value of D is $D^*[.05,120] = .1242$ at the 5% level, whereas the maximum value of D is .0991. However, as discussed above, the fact that the null hypothesis cannot be rejected does not mean that it can be accepted. In fact, the Kolmogorov-Smirnov test is a "weak" non-parametric test. Hence, the data is tested for significant shewness and kurtosis in the next section of the thesis.

(b) Tests of Skewness and Kurtosis

D'Agostino and Pearson [1973] stated that if there is a large incidence of ties in the ordered observations, then it is better to use a test of normality based on the measures of skewness and kurtosis. This characterizes our sample, since there is a high incidence of zeros due to thinness or constancy of prices over short intervals. (This is especially true for daily data, as is shown in a later section of this chapter.)

(i) Test for Skewness

The classical test of non-normality due to significant skewness is as follows. Define the rth moment about the mean as:

$$m_r = \sum_{i=1}^{N} (x_i - \bar{x})^r / N, \quad \text{where } x_t = \ln(S_t/S_{t-1})$$

TABLE 4.1

Full Sample of Securities on TSE

Firm	Ticker Symbol
Alcan Aluminum Ltd.	AL
Bell Canada	B
Cadillac Fairview Corp. Ltd.	CFV
Canada Packers Ltd.	CK
Campeau Corp.	CMP
Canadian Imperial Bank of Commerce	CM
Dupont Canada	DUP (A)
Gulf Canada Ltd.	GOC
Hollinger Argus Ltd.	HOC
Hudson's Bay Co.	IMO (A)
Imperial Oil Ltd.	IOC
IAC Ltd.	IAC
Inco Ltd.	N
Interprovincial Pipe Line Ltd.	IPL
MacMillian Bloedel Ltd.	MB
Placer Development Ltd.	PDL
Royal Bank of Canada	RYL (A)
Stelco Inc.	STE (A)
Union Carbide Canada Ltd.	UCC
George Weston Ltd.	WTN

Let $B_1 = m_3^2/m_2^3$. Then, for a normal distribution, the distribution of the skewness statistic, S, where $S = [(B_1)(N/6)]^{1/2}$, is approximately N(0,1). Therefore, the computed S statistic can be compared with $Z(\alpha = .05) = 1.96$. The results for the 20 securities are given in Table 4.2. Only in four cases is there significant skewness relative to the normal distribution. However, for one of these cases (Placer), the skewness can be attributed to the existence of an outlier in the sample. More specifically, the sample value of December 1979 is .3433 whereas the mean value and standard deviation for the entire period is .0017 and .1225, respectively. Significant skewness does not exist in the absence of this sample value.

It is likely that a high incidence of zero sample values would contribute to skewness since the sample mean of stock price returns is approximately zero. Thus, in order to determine if skewness is due to parameter non-stationarity, the skewness statistic was recalculated as the sample period was reduced one year at a time. In all three cases (i.e., Cadillac, Imperial oil and Stelco) significant skewness disappeared for the period 1970-1974. Therefore, it appears that parameter non-stationarity resulted in significant skewness.

(ii) Test for Kurtosis

Kurtosis is usually defined as the degree of peakedness of a distribution relative to a normal distribution. A distribution having a relatively high peak is called leptokurtic. Such a distribution

seems to describe stock price returns well since a high incidence of zero or near zero values is usually present. One measure of kurtosis is given by: $B_2 = \frac{m_4}{(m_2)^2}$.

For a normal distribution, the distribution of the kurtosis statistic, $K = (B_2-3)(\frac{N}{24})^{1/2}$, is approximately $N(0,1)$.

The Kurtosis results for the sample are given in Table 4.2 ~~of~~ the twenty stocks considered. Seven exhibited significant positive kurtosis (i.e., a clear indication of a leptokurtic distribution). In fact, except for Stelco, all of the computed B_2 values exceed 3 (the theoretical value for a normal distribution). Based on the empirical results presented in Table 4.2, the following three conclusions emerge. First, the null hypothesis of normality for monthly closing prices for a sample of 20 stocks for the period 1970-1979 cannot be rejected using the Kolmogorov-Smirnov test. However, on the basis of this test alone, it is not possible to accept the null hypothesis.

Second, the distribution of monthly stock price returns is mostly symmetric since only four instances of significant skewness were found. These appear to be caused by the existence of an outlier for Placer and to parameter non-stationarity for Cadillac, Imperial Oil and Stelco. Third, all estimations (except for Stelco) yielded Kurtosis coefficients (B_2) with values greater than three. For seven cases, these coefficients were significant at the $\alpha = .05$ level.

TABLE 4.2

RESULTS OF THE KOLMOGOROV-SMIRNOV SKEWNESS AND KURTOSIS TESTS - MONTHLY DATA (1970-1979)

STOCK	D	B_1	S	B_2	K
ALCAN	.0338	.1457	1.700	3.693	1.543
BELL	.0903	.0142	.5299	3.279	.6227
CADILLAC	.0991	.2178	2.078	6.568	7.945
CAMPEAU	.0646	.0585	1.007	3.277	.5786
CANADA PACKERS	.0694	.1480	1.713	3.858	1.911
CAN. IMP. BANK	.0496	.0039	.2803	3.678	1.511
DUPONT	.0532	.0123	.4934	3.296	.6588
GULF CAN	.0487	.0401	.8921	3.546	1.216
HOLLINGER	.0748	.0583	1.075	3.433	.9641
HUDSON BAY	.0396	.0053	.3247	4.352	3.012
IMPERIAL OIL	.0540	.3452	2.616	4.333	2.968
IAC LTD.	.0528	.1647	1.807	4.467	3.267
INCO	.0434	.0125	.4976	3.065	1.445
INTER-PROV. PIPE	.0537	.0615	1.105	4.417	3.289
MACMILLAN BLO	.0543	.0023	.2128	4.347	2.999
PLACER	.1035	2.1699	6.560	11.226	18.317
ROYAL BANK	.0525	.0000	.0029	3.256	.5713
STELCO	.0717	.2045	2.013	2.980	-.0432
UNION CARBIDE	.0789	.0054	.3278	3.352	.7855
WESTON	.0536	.0006	.1141	3.428	.9540

The critical value of D is D*[.05,120] = .1242.

The critical value of both S and K is Z = 1.96.

4.2 A Test of the Mandelbrot Hypothesis

Results similar to those reported in the previous section led Mandelbrot [1963] to propose that the distribution of stock price changes belongs to the stable Paretian class. The symmetric[1] stable Paretian family may be defined by:

$$\ln [\emptyset(t)] = i\delta t - |ct|^{\alpha} ;$$

where t = a real number,

$i^2 = -1$,

$\emptyset$ = the characteristic function,

δ = location parameter

c = scale parameter,

α = characteristic exponent, where $0 < \alpha \leq 2$.

The normal distribution is obtained when $\alpha = 2$. Since the tails of the stable Paretian class become "fatter" with a decreasing α, the normal distribution has the "thinnest tail" in the class. The interesting feature of the class for $0 < \alpha < 2$ is that the variance is infinite, in the sense that it behaves in an erratic fashion as the sample size, N, increases. Thus, the sample variance is a meaningless measure of variability when $0 < \alpha < 2$.

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* The distribution can be assumed to be symmetric since no significant skewness was found except for three securities.

If the location parameter δ is assumed to be zero, then only two parameters need to be estimated: the scale parameter, C, and the characteristic exponent, α. Fama and Roll [1968] developed an estimator, $\hat{C}$ for C, which is given by:

$$\hat{C} = \frac{\hat{x}_{.72} - \hat{x}_{.28}}{1.654}$$

where x_k is the order statistic used to estimate the kth fractile of the empirical distribution. Therefore, the data has to be ranked in some order so that x_k may be identified.(*)

The procedure for obtaining the estimator of α is more involved than that for obtaining C. Furthermore, the value of $\hat{\alpha}$ must be estimated "properly", since the type of symmetric stable distribution is determined by α. Based on the behaviour of higher fractiles, Fama and Roll [1971] showed that $\hat{\alpha}$ may be estimated as follows:

(1) For a given fractile f, compute

$$\hat{Z}_f = \frac{\hat{x}_f - \hat{x}_{1-f}}{2\hat{C}}$$

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* La Cava [1976] has suggested that since $\hat{C}$ can be used as a measure of risk for investment decisions, it can be used to rank risky assets.

(2) From a table of standardized symmetric stable fractiles (see Fama and Roll [1968]), pick the $\hat{\alpha}$ with the closest f fractile which is $\leq \hat{z}_f$. Using Monte Carlo methods, Fama and Roll found that the best results were obtained for $.95 \leq f \leq .97$.

Empirical Findings

Using the data base given in Table 4.1, $\hat{C}$, $\hat{Z}_f$ and the corresponding $\hat{\alpha}$ for each stock were computed using monthly data. Since $.95 \leq f \leq .97$, we choose $f = .96$. The results are summarized in Table 4.3. While $\hat{\alpha}$ ranges from 1.4 to 2.0, the average $\hat{\alpha}$ value is 1.8. This is quite consistent with the findings of Officer [1972] who stated that "it is appropriate to assume a stable distribution of monthly stock returns with $\hat{\alpha} \approx 1.8$ postwar".

Thus, the overall conclusion to this point in the thesis is that, for the studied sample, the distribution of stock price changes is probably leptokurtic and it belongs to a symmetric stable Paretian family with an $\hat{\alpha} \approx 1.8$.

4.3 Distribution of Daily Stock-Price Returns

In the previous section, the empirical distribution of monthly closing price changes for twenty stocks on the TSE was investigated. However, the distribution of daily price changes is more important for the pricing of options. Indeed, a central assumption of the Black-Scholes

TABLE 4.3

Estimation of the Characteristic Exponent Parameter of the Symmetric Stable Class*

STOCK	$\hat{C}$	$\hat{Z}_f$	$\hat{\alpha}$
ALCAN	.0585	2.482	2.0
BELL	.0192	2.929	1.7
CADILLAC	.0516	3.604	1.5
CAMPEAU	.0871	2.170	2.0
CANADA PACKERS	.0362	2.707	1.8
CAN.IMP.BANK	.0394	2.553	1.9
DUPONT	.0534	2.994	1.7
GULF CAN.	.0623	2.340	2.0
HOLLINGER	.0351	3.837	1.4
HUDSON BAY	.0617	2.513	1.95
IMPERIAL OIL	.0526	3.289	1.6
IAC LTD.	.0433	2.464	2.0
INCO	.0667	2.592	1.9
INTER.PROV.PIPE	.0450	2.678	1.8
MACMILLAN-BLO	.0515	2.968	1.7
PLACER	.0658	3.214	1.6
ROYAL BANK	.0331	2.986	1.7
STELCO	.0418	2.557	1.9
UNION CARBIDE	.0383	3.374	1.5
WESTON	.05036	2.880	1.7

* Based on Monthly data from the period, 1970-1979.

model is that daily stock price changes follow a Gaussian distribution with constant parameters. Thus, the procedures of Sections 4.2 and 4.3 are now applied to daily closing prices.

Similar studies have been conducted for both the U.S. and Australian cases. Osborne [1974] examined the daily returns of twelve common stocks on the Sydney Stock Exchange for a 4-year period. She found that the empirical distribution is a member of the stable Paretian class with approximately the same characteristic estimate of 1.7 for all securities. Teichmoeller [1971], using the Fama and Roll technique, also concluded that a stable Paretian distribution with an $\hat{\alpha} \approx 1.7$ was appropriate. On the other hand, Praetz [1972], using a minimum chi-squared estimator for α, could not accept the stable Paretian hypothesis for any of the stocks in his sample. He found estimates which were generally higher than 1.7. Also Fielitz and Smith [1972] found that for daily closing prices of 200 stocks listed on the NYSE, significant skewness and kurtosis made the symmetric stable distribution assumption inappropriate. If true, this would compromise the results obtained from using the technique developed by Fama and Roll.

Data

A sample of 25 securities on the TSE was used. Daily closing prices for a one-year period were recorded and adjusted for stock splits and cash dividends. For each security, the year to be examined was randomly selected from the period 1976 to 1982. This will tend to minimize the effects of co-movements in the daily prices of securities

which belong to the same sector. Thus, the sample size consisted of about 260 prices for each security. A full listing of the sample is given in Table 4.4. The Fama and Roll technique to estimate the characteristic exponent, α, under the assumption of a symmetric stable distribution was applied to each security. These results, along with the computed skewness and Kurtosis statistics, are recorded in Table 4.5.

Empirical Findings

Inconsistent with the results of Osborne [1974] and Teichmoeller [1971], the average value of $\hat{\alpha}$ is 1.4. In addition, this value is quite stable across stocks, even for different randomly selected time periods between 1976 and 1982. However, unlike Fielitz and Smith [1972], significant skewness is not evident in most cases considered. Therefore, the assumption of a symmetric stable class is not questionable. Two general conclusions emerge from these results. First, for monthly closing prices on the TSE over the period 1970-1979, the empirical distribution of stock prices is symmetric stable Paretian with the average value of $\hat{\alpha} = 1.8$. However, several instances of $\hat{\alpha} = 2.0$ were found, indicating a normal distribution.

Second, for daily stock prices on the TSE for a randomly selected one-year period between 1976 to 1982, the empirical distribution belongs to a symmetric stable Paretian class with $\hat{\alpha} = 1.4$, and the $\hat{\alpha}$ values are quite stable across securities. Furthermore, when compared to the results by Osborne and Teichmoeller, the lower $\hat{\alpha}$ values indicate that, for Canada, the distributions of daily stock price changes have "fatter" tails than in the U.S.

TABLE 4.4

Data Set Used in the Estimation of the Parameters of the Paretian Distribution

Ticker Symbol	Time Period
ASM	Jan. 1 - Dec. 31, 1977
NCN	
DM	
HYO	Jan. 1 - Dec. 31, 1978
IPL	
MB	
B	
CP	
BMO	
LBT(A)	
GOC	Jan. 1 - Dec. 31, 1979
BVI	
DEN	
TD	
RGO	
GST	July 1, 1978 - July 1, 1979
SHC	
DMT	Jan. 1 - Dec. 31, 1980
CM	
ICG	Sept. 15, 1980 - Sept. 15, 1981
BPO	July 1, 1980 - July 1, 1981
CRK	
HWR	
PDL	
AEC	Jan. 1 - Dec. 3, 1981
CLT	

TABLE 4.5

Results obtained for the estimates of the Parameters of the Paretian Distribution

Ticker Symbol	$\hat{c}$	$\hat{\alpha}$	B	B*	K	K*
ASM	.0147	1.5	.6042	5.067	5.515	8.199
NCN	.0126	1.7	.0473	1.418	1.250	3.976
DM	.0072	1.4	.2296	3.124	3.993	3.238
HYO	.0113	1.6	.0264	1.059	4.059	3.451
IPL	.0088	2.0	.0300	1.129	5.132	6.948
MB	.0083	1.8	.0719	1.749	5.495	8.132
B	.0037	1.6	.1284	2.336	6.589	11.70
CP	.0077	1.7	.0122	.7192	6.354	19.73
BMO	.0070	1.8	.0245	1.020	4.057	3.447
LBT(A)	.0070	1.8	.0077	.5725	3.598	1.950
GOC	.0121	1.5	2.015	9.255	8.369	17.50
BVI	.0130	1.4	.5250	4.725	6.522	11.48
DEN	.0110	1.4	.0001	.0745	5.880	9.388
TD	.0068	1.4	.0002	.0936	4.788	5.829
RGO	.0160	1.7	.0149	.7969	4.325	4.318
GST	.0064	1.6	.0217	.9590	8.369	17.50
SHG	.0095	1.7	.7413	5.613	5.438	7.948
CM	.0062	1.3	.0093	.2024	4.382	4.507
DMT	.0063	1.4	.0286	1.103	4.410	4.597
ICG	.0150	1.5	.0323	1.171	6.317	10.81
BPO	.0090	1.5	.0890	1.947	8.026	16.38
CRK	.0137	1.4	.0151	.8011	4.654	5.391
HWR	.0050	1.6	2.952	11.201	13.924	35.60
PDL	.0146	1.9	.0143	.7819	4.154	3.760
AEC	.0124	1.7	.0609	1.609	6.871	12.61
CLT	.0071	1.4	.1138	2.119	5.188	7.134

TABLE 4.5 (Cont'd)

Comments

(1) B = computed skewness statistic and

B* > 1.96 indicates statistical significance at the 5% level of confidence.

(2) K = computed kurtosis statistic and

K* > 1.96 indicates significance at the 5% level.

(3) $\hat{\alpha}$ has a modal value of 1.4.

(4) K* is significant in every case indicating significant leptokurtic distributions.

4.4 The Variance of Stock Price Changes

As has already been discussed in Chapter 3, the appropriate specification and the estimation of the variance of stock price changes is crucial to the pricing of options via the Black and Scholes model. However, Black [1976], Schmalensee and Trippi [1978] and Capozza and Cornell [1979] have presented evidence which suggest that the variance is correlated over time. This implies that a better valuation of the call price can be obtained using the B-S model when the "correct" specification of the process generating the variance is used. This would require a time series of variances of daily stock price changes. However, since only one sample point is available for each day, a computation of the daily variance is not possible. Capozza and Cornell [1979] computed the sample standard deviation on a weekly basis. Based on five (!) data points, they calculated

$$\hat{\sigma}_t^2 = \left[\left(\sum_{t=1}^{5} R_t^2 \right) \right] / 5$$

where $R_t = \ln (S_t / S_{t-1})$

S_t = the daily closing price at time t,

$E(R_t) =$ 0 by assumption.

A different approach, based on a well-known result in statistics, was used by Beckers [1980]. (The proof is found in, for example, Appendix C of Beckers [1980].) The result is stated as follows:

Let X be a random variable which is normally distributed with mean μ and variance σ^2. If $\mu \approx 0$, then

$E(|x|) \approx \sigma \left(\frac{2}{\pi}\right)^{1/2}$. Since, the estimated mean daily return on stocks is very close to zero in all cases considered, the assumption that $\mu = 0$ is a reasonable one. Consequently, the population variance σ_t^2, may be estimated by $\hat{\sigma}_t^2 = \frac{\pi}{2}$ ABS $\ln(S_t/S_{t-1}))$, where ABS $(x) = |x|$ and S_t is the daily stock price at time t. This gives rise to a time series $\{\hat{\sigma}_t^2\}_{t=1}^{N}$ of estimated daily variances of stock price changes. However, as is discussed more fully in Appendix 4.1 to this chapter, there is a major flaw in the estimation procedure. More specifically, since Becker's estimator is biased downward, the estimates of the parameters obtained by OLS will be understated. However, an alternative estimator which is unbiased and, like the Becker estimator, is inconsistent, is available. Thus, this section consists of the following two subsections. In the first, it is postulated that

$$\sigma_t = f(t, S_t, R_{mt})$$

where

σ_t^2 = variance of $R_t = \ln(S_t/S_{t-1})$

t = time (month)

S_t = monthly closing price

R_{mt} = monthly market rate of return.

We then obtain an appropriate specification based on R^2 and the residual sum of squares statistics is obtained.

In the second subsection, a determination is made of the extent to which the daily variance is correlated over time. The time series of daily variances is obtained by the so-called MINQUE approach, which was developed by C.R. Rao [1970]. The estimate of the autocorrelogram for each time would then suggest a suitable ARIMA model, which will be used to make a "one-step-ahead" forecast of the variance. This forecast is then used to recompute the B-S price. This price is then compared to that obtained from the use of the historical variance. Capozza and Cornell [1979] also used their series of weekly variances in a simple exponential smoothing model to forecast the next period variance. Since such a model is equivalent to a first order moving average process, their results could have arisen by over-differencing a white noise. In fact, the results given in Table 1 (a) on page 54 of Capozza and Cornell [1979] suggest this conclusion. More will be said about this point later.

(a) Test for Non-Stationarity in the Variance

The constant variance hypothesis has been studied by several researchers such as Martin and Klemkosky [1975] and Bey and Pinches [1980]. The results of these studies have had important implications for the Capital Asset Pricing Model (CAPM), since the ordinary least squares estimates of this model would be inefficient in the presence of heteroscedastic errors. Furthermore, mispricing by the Black-Scholes model would likely result if the variance is intertemporally non-stationary. For example, this might explain the contradiction ~~in the~~ empirical findings of Black and Scholes [1972] who found that deep-in-the-money

options generally have B-S prices greater than the observed market price and MacBeth and Merville [1979] who found the exact opposite result.

In order to investigate the possible non-stationarity of σ_t, a preliminary test is used to detect heteroscedasticity in monthly stock price returns. The test invokes no a priori notions about how the variance might change. An optimum distribution-free test obtained by Giaccotto and Ali [1982] is chosen because of its flexibility in coping with several alternative specifications. The test is as follows:

Let $\{R_t\}_{t=1}^{n}$ be a sequence of independent random variables. The null and alternate hypotheses are

H_o : $\sigma_1^2 = \sigma_2^2 = \ldots\ldots = \sigma_n^2$, and

H_a : σ_i^2 are not all equal, where

$\sigma_i = Var(R_i)$ Let $\sigma_t = e^{\alpha+\beta d_t}$,

where $\{d_t\}$ is a sequence of known constants. Then the hypotheses may be written as:

H_o : $\beta = 0$ and H_a : $\beta \neq 0$.

If R_t is replaced by its rank R_t^*, then the mean value of the sequence $\{R_t^*\}$ is clearly $\frac{n+1}{2}$ and the deviation is $D_t = (R_t^* - \frac{n+1}{2})$. If σ_t (the standard deviation of R_t) changes according to d_t

(as is postulated above), then a non-zero correlation can be excepted between d_t and the deviation $D_t = (R_t^* - \frac{n+1}{2})$, or some transformation of R_t^*. Based upon this argument, Ali and Giacotto derived the following statistic in order to test H_o :

$$S_n = \sum_{t=1}^{n} [(d_t - \bar{d}) \{N^{-1} (\frac{R_t^*}{n-1})\}^2],$$

where $\bar{d} = \sum_{t=1}^{n} d_t$, and $N(.)$ is the cumulative normal distribution.

Under H_o , $S_n \sim N(0,Var)$, where

$$Var = \frac{1}{n-1} [\sum_{t=1}^{n} (d_t - \bar{d}) (\sum_{t=1}^{n} (A_t - \bar{A})^2)], \text{ and}$$

$$A_t = \{N^{-1}(R_t^*/_{n-1})\}.$$

The flexibility of this approach is obtained through the choice of the sequence $\{d_t\}$.

As a first run, $d_t = t$, S_t , R_{mt}^2 , was chosen,

where t = time,

S_t = monthly closing price for each stock listed in Table 4.1, and

R_{mt} = market rate of return on a monthly basis (i.e., $R_{mt} = \ln [TSE_t/TSE_{t-1}]$, where TSE_t is the monthly TSE closing index).

The statistic $S^* = \frac{S_n}{\sqrt{Var}}$ is computed for each stock and then compared with $Z^* = 1.96$. The general pattern of the preliminary results indicates that $\sigma_t^2 = f(R_{mt}^2)$, since $S^* \geq 1.96$ most often for this case. This specification is consistent with the non-parametric test of Ali & Giacotto [1982] who, for monthly data for the Standard and Poor index from 1965 to 1977 for 384 securities, found that "the hypothesis of constant variance is unobtainable for the majority of the stocks in our sample and possibly the variance is increasing with R_{mt}^2". (p. 1256).

Thus, the variance is specified as $\sigma_t^2 = f(R_{mt}^2)$, and is estimated by ordinary least squares. However, in order to produce the "right" functional form, the familiar Box-Cox transformation is used. This transformation is useful in seeking out the best specification, and in inducing normality in the observations from skewed distributions.

Define $\sigma_t^{(\lambda)} = \frac{\sigma_t^{\lambda} - 1}{\lambda}$,

and assume that $\sigma_t^{(\lambda)} = \alpha + \beta R_{mt}^{(\lambda)}$ (1)

Thus, the estimation of (1) should yield the "best" λ (and, therefore, the best functional form).

Since $\lim_{\lambda \to \infty} \frac{x^{\lambda}-1}{\lambda} = \ln x$, the model is usually specified as: $\ln \sigma_t = \alpha + \beta \ln x_t$, when $\lambda \approx 0$.

Estimation Procedure

The estimation of model (1) above, using the data set in Table 4.1, proceeds as follows:

(i) Choose a reasonable range for λ. For each value λ_o, estimate (1) by ordinary least squares.

(ii) Calculate the Residual Sum of Squares; RSS (λ_o).

(iii) Choose $\lambda = \lambda_m$ for which the RSS (λ_m) is minimum. (This causes the concentrated likelihood function, $L(\lambda) = -\frac{\pi}{2} \ln (\text{RSS}(\lambda))$, to be at a maximum).

(iv) For $\lambda = \lambda_m$, select the estimates $\hat{\alpha}$ and $\hat{\beta}$ for the estimated model (1).

The procedure selects the best functional form as:

$$\sigma_t^{(\lambda_m)} = \hat{\alpha} + \hat{\beta} \; R_{mt}^{(\lambda_m)} .$$

However, one problem with the above approach is that it is being implicity assumed that the transformation not only gives the appropriate λ, but also produces errors which are normally distributed with mean zero and constant variance.

Empirical Findings

The above estimation procedure was applied to the sample of 20 securities given in Table 4.1. In almost all cases, $\lambda = 2$ yielded the minimum RSS. Also significant first-order serial correlation was detected in most instances. Accordingly, the following was specified:

$$\sigma_t^2 = \alpha + \beta R_{mt}^2 + \varepsilon_t \quad \text{where} \quad \varepsilon_{t+1} = \rho\varepsilon_t + V_t \text{ , and}$$

$$V_t \sim N(0, Var).$$

The following proxies are used for σ_t^2 and R_{mt} :

$$\sigma_t = ABS\ (\log\ (S_t/S_{t-1})) \quad \text{and}$$

$$R_{mt} = \log\ (TSE_t/TSE_{t-1})$$

The transformed model is then:

$$\sigma_{t+1}^2 - \rho\sigma_t^2 = \alpha(1-\rho) + \beta[R_{m\ t+1}^2 - \rho R_{mt}^2] + V_t$$

The regression results, based on a maximum Likelihood (ML) technique of the transformed model (2) are given in Table 4.6. The column labelled "Full Sample" shows the results when the regression is done for the entire 10 year period. The values of $\hat{\beta}$, their t-statistics, the R^2 values and Durbin-Watson values are all shown in the table. The most striking feature of the results is that only two securities

yield insignificant values of $\hat{\beta}_0$. These are Cadillac Fairview Ltd. and Placer Development Corp. Indeed, if the single outlier on December 1979 is removed for Placer, its $\hat{\beta}$ becomes significant. Thus, for the period and the sample chosen, the monthly variance of stock price changes varies directly with the overall market return squared. Furthermore, since the D.W. statistics are approximately equal to 2.00, the assumption of first order serial correlation in the error statistic is adequate.

Whether the β values are constant over time is unknown. One reason to suspect that β shifts over time (at least for the period considered) is based on the economic events that took place from 1970 to 1978. There was wage and price controls in the U.S. in 1971-1972, the OPEC oil embargo and the resulting recession in 1974, the recovery in 1975 and wage and price controls in Canada in 1976-1977. If a significant shift in β over time is found, this would contradict the finding of Officer [1972] that the sample standard deviation for monthly returns appears to be a well-behaved measure of dispersion. However, it would be consistent with the findings of Gultekin, Rogalski and Tinic [1982] that "average standard deviation changes markedly over time".

Assume that the shift point is at $t = 61$; that is, January 1974 (the year of the recession). Estimation of model (2) before and after the shift point is carried out for each security in the sample. That is, the following switching regression model is used:

$$\sigma_t^2 = \alpha_1 + \beta_1 R_{mt}^2 + \varepsilon_{1t} \qquad 1 \leq t \leq t_o$$

$$\sigma_t^2 = \alpha_2 + \beta_2 R_{mt}^2 + \varepsilon_{2t} \qquad t_o \leq t \leq 120$$

Technically, t_o is unknown. If the error variances are equal for both regimes, both equations should be estimated for different values of t_o. Then the value of $t_o = t_m$ for which the sum of residual sum of squares (i.e. $RSS_1 + RSS_2$) is minimized should be chosen. The value of $t_o = 61$ is appropriate according to this criterion.

The results are shown in Table 4.6 under the headings of "Pre-Shift Point" and "Post-Shift Point". In order to test if $\beta_1 = \beta_2$ for the two regimes, the following result is used: If a sample of size n_1 is independently drawn from a population having mean μ_1 and variance σ_1^2, and another sample of size n_2 is drawn from a population with mean μ_2 and variance σ_2^2, then the statistic:

$$\bar{X}_1 - \bar{X}_2 \sim N(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}) ,$$

provided that $n_1 \geq 30$, $n_2 \geq 30$. Hence, a test of the null hypothesis,

$H_o : \beta_1 = \beta_2$, against the alternative hypothesis, $H_2 : \beta_1 \neq \beta_2$, yields the following test statistic:

$$Z^* = (\hat{\beta}_1 - \hat{\beta}_2) / \sqrt{\hat{\sigma}_1^2/n_1 + \hat{\sigma}_2^2/n_2} .$$

Then Z^* is to be compared with $Z = 1.96$.

TABLE 4.6

Regression Results of $(ABS(\log(X_t/X_{t-1})))^2 = \alpha + \beta\,(\log(S_t/S_{t-1}))^2 + \Sigma_t$

Using a Maximum likelihood technique of the transformed model with

$\Sigma_{t+1} = P\Sigma_t + V_t$

STOCK	FULL-SAMPLE			PRE-SHIFT POINT			POST-SHIFT POINT		
	$\hat{\beta}$	R^2	DW	$\hat{\beta}$	R^2	DW	$\hat{\beta}$	R^2	DW
ALCAN	1.062	.1471	2.03	1.572	.2611	2.13	.5951	.0731	1.84
	(4.488)			(4.477)			(2.124)		
BELL	.0664	.0326	1.97	.0376	.0090	1.95	.1078	.1059	2.06
	(1.975)			(.7080)			(2.743)		
CADILLAC	2.231	.0933	1.99	1.983	.2283	2.00	2.498	.0725	1.99
	(3.467)			(4.202)			(2.137)		
CAN.PACKERS	.2653	.0238	1.98	.1056	.0089	1.89	.4429	.0398	2.00
	(1.689)			(.8970)			(1.565)		
CAN.IMP.BANK	.9457	.3486	1.98	.9297	.2757	2.02	.9596	.5661	1.99
	(7.920)			(4.670)			(8.536)		
DUPONT	.7838	.0566	1.98	1.402	.1471	1.96	.1565	.0025	1.99
	(2.632)			(3.117)			(.4200)		
GULF CAN.	.7716	.0549	1.98	.7584	.0656	2.10	1.149	.0992	1.96
	(2.588)			(1.963)			(2.430)		
HUDSON BAY	2.741	.4940	2.02	2.834	.4226	2.00	2.648	.6345	1.91
	(10.690)			(6.459)			(9.858)		
HOLLINGER	.7560	.1440	2.01	.4624	.1052	2.02	1.049	.1945	1.907
	(4.440)			(2.615)			(3.797)		
IMPERIAL OIL	1.963	.2883	1.98	2.391	.2797	1.98	1.651	.4927	1.98
	(6.882)			(4.691)			(7.329)		
IAC LTD.	.7090	.1159	2.00	.7236	.1833	1.99	.6947	.0839	1.98
	(3.916)			(3.577)			(2.305)		
INCO	.9921	.0721	1.96	.6109	.0201	2.02	1.256	.1878	1.72
	(3.002)			(1.046)			(3.713)		
INTER.PROV	.8844	.1324	1.99	.8180	.2560	1.98	.9513	.2015	1.99
	(4.245)			(4.266)			(4.736)		
MACMILLAN	.9756	.0685	1.88	1.005	.2459	2.02	.9386	.0469	1.74
	(2.952)			(4.268)			(1.628)		

TABLE 4.6 (continued)

STOCK	FULL-SAMPLE			PRE-SHIFT POINT			POST-SHIFT POINT		
	$\hat{\beta}$	R^2	DW	$\hat{\beta}$	R^2	DW	$\hat{\beta}$	R^2	DW
PLACER	1.879	.0184	1.96	2.622	.0193	2.00	.9406	.0324	1.348
	(1.482)			(1.056)			(1.399)		
ROYAL BANK	.6371	.2412	1.96	.4354	.1217	1.92	.8116	.3912	1.99
	(6.115)			(2.885)			(6.101)		
STELCO	.9072	.3532	1.99	1.227	.4737	1.95	.6168	.2895	1.99
	(7.993)			(7.161)			(4.798)		
UNION CARBIDE	.4714	.0369	2.04	.0305	.0004	1.97	.9004	.1063	2.00
	(2.118)			(.1110)			(2.608)		
WESTON	1.259	.1899	1.97	.3015	.0162	2.00	2.156	.4778	1.91
	(5.233)			(1.031)			(7.112)		

Comments

(1) The shift point is assumed to be at t=61 [Jan. 1974].

(2) Since $(E|X_t|)^2 \approx \hat{r}\,\sigma^2\,(\frac{2}{\pi})$, the coefficients, $\hat{\beta}$, must be multiplied by $\pi/2$. However, the signigicance of the t statistics is unchanged.

In Table 4.6, the null hypothesis of no shift in β cannot be rejected in only 6 cases. When taken along with the results presented earlier that monthly returns are approximately normal, and the present result that monthly variances are non-stationary because they generally vary with R_{mt}^2, the Rosenberg hypothesis for monthly data is now validated. Specifically, monthly stock price returns follow (approximately) a normal distribution with a non-stationary variance.

(b) <u>The Process Generating the Variance for Daily Returns</u>

The extent of the autocorrelation in the time series of the daily variances of stock price returns is now investigated. As mentioned earlier, Black [1976] and others, have suggested that such a correlation exists and Capozza and Cornell [1979] have used a simple exponential smoothing model to forecast the variance. Capozza and Cornell claimed that their forecasts yielded better B-S prices than the implied variance approach. However, since the Box-Jenkins methodology uses the dependency in the observations more efficiently than the exponential smoothing approach, it is likely to produce better forecasts. Furthermore, as is well known the Box-Jenkins methodology is well suited for time series with small sampling intervals (e.g., when using daily data). But, unlike exponential smoothing, there is no inherent procedure in the Box-Jenkins methodology to update the estimates of the parameters as new observations are obtained. However, since only one-step ahead forecasts are to be made, this is not a serious problem.

Prior Studies

Capozza and Cornell [1979] concluded that weekly stock price changes follow a first-order moving average process. They computed the sample standard deviation of stock price changes on a weekly basis as follows:

$$\hat{\sigma}_t^2 = [\sum_{t=1}^{5} R_t^2]/5 ,$$

where $R_t = \ln (S_t/S_{t-1})$;

S_t = daily closing price at time t;

and $E(R_t) = 0$ by assumption.

Using a series of weekly variances, they computed the correlograms of the first differences. Based on the finding ~~that~~ all the first autocorrelations were significant at the 95% level, they concluded that the data implies that the time series of weekly variances is a first-order moving average process.

This conclusion appears to be merely an artifact of over-differencing. Inspection of their Table 1(a) [1979,p.54] reveals that the first serial correlation coefficients of the differenced series are generally close to -.500.* In fact, the average value is -.446.

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* The statistical significance of these coefficients cannot be tested.

These results can be explained by the following theoretical argument given by Nelson [1973,p.76]:

Assume that $Z_t = \alpha + \mu_t$ is a time series which is a translated white noise; that is, it has no significant serial correlation coefficients at all lags (i.e. $\rho_k = 0, \forall k$).

Differencing of the series yields:

$\omega_t = Z_t - Z_{t-1} = u_t - u_{t-1}$, for which the correlation coefficients are:

$$\rho_j = \begin{cases} -.50 & j = 1 \\ 0 & j > 1 \end{cases}$$

Clearly, the differenced model suggests that a first-order moving average model is appropriate, even though the true model was a white noise with constant mean.

Procedure

In order to identify and estimate the appropriate model by the Box-Jenkins approach, a time series of variances of daily stock price changes is required. Since the Becker estimator is biased and inconsistent, an estimator which belongs to the MINQUE (minimum quadratic unbiased estimator) family, which was developed by C.R. Rao [1970],* is used herein. It is developed as follows.

* Rao has shown that the MINQUE estimator has the minumum average variance in the family of unbiased estimators.

Define $Y_t = \ln(S_t/S_{t-1})$, and assume that

$$Y_t = \beta_o + \mu_t \quad \text{where} \quad \mu_t \sim N(0,\sigma_t^2) \qquad (1)$$

This assumption (**) means that stock price changes follow a random walk with constant mean β .

From (1), $\hat{u}_t = Y_t - \hat{\beta}_o = u_t - [\hat{\beta}_o - \beta_o]$

$$= u_t - \bar{u}$$

$$= u_t - \frac{1}{N}\left[\sum_{t=1}^{N} u_t\right]$$

$$\therefore \hat{u}_t = (I-M_1)u_t, \text{where } M_1 = \frac{1}{N}\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 1 & 1 & \dots & 1 \\ \vdots & \vdots & \vdots & — & \vdots \\ 1 & 1 & 1 & \dots & 1 \end{bmatrix}$$

Let $I-M_1 = M$. Then $\hat{u}\hat{u}' = Muu'M'$ and $E[\hat{u}\hat{u}'] = M \Phi M'$

** Granger [1975] has shown that if the valuation formula $V_t = \sum_{i=1}^{\infty} [D^*_{t,i}]/[1+r)^i]$ holds, where $D^*_{t,i}$ is the expected dividend at time t and r is the interest rate, and if the value and price of the security are assumed equal, then the rate of return is a constant plus a white noise.

Define $\sigma = \begin{bmatrix} \sigma_1^2 \\ \vdots \\ \sigma_N^2 \end{bmatrix}$, $M = (m_{ij}^2)$ where $M = (m_{ij})$.

$$\hat{u} = \begin{bmatrix} \hat{u}_1^2 \\ \vdots \\ \hat{u}_N^2 \end{bmatrix}$$

Then $E(\hat{u}) = M\sigma$ (2)

Now let $\eta = E(\hat{u}) - \hat{u}$. Therefore, from (2), the following obtains:

$\hat{u} = M\sigma + \eta$, from which

$$\hat{\sigma} = (M^1 M)^{-1} M^1 \hat{u}$$

It follows that, $\hat{\sigma} = M^{-1} \hat{u}$ (3)

Clearly (3) gives a time series $\{\hat{\sigma}_i\}_{i=1}^{N}$ corresponding to the time series of residuals $\{\hat{u}_i^2\}_{i=1}^{N}$, which was obtained by estimating (1).

Data

A sample of 20 securities is randomly selected from the total of 33 securities listed on the TCO (see Table 4.7) Daily closing prices were recorded for each stock for the 3 month period, beginning 2 May to 28 July 1983. The correlogram of $\{\hat{\sigma}_i\}_{i=1}^{N}$ is now estimated for each of the securities for k lags, $k = 1, \ldots 10$. Under the null hypothesis that the true order of process is q, the following criterion may be used to test the significance of r_j:

$$\frac{|r_j|}{\frac{1}{\sqrt{N}}\left(1+2\sum_{i=1}^{q} r_i^2\right)^{1/2}} \geq 1.96 \quad \text{for } j > q$$

The results are recorded in Tables 4.8 and 4.9 for the series $\{\sigma_i\}_{i=1}^{N}$ and $\{\Delta\sigma_i\}_{i=2}^{N}$, respectively.

Discussion of the Results

Except for Seagrams Ltd. and Moore Corp., all the securities in Table 4.8 had insignificant correlation coefficients for lags $k = 1, \ldots 10$. In fact, the value of r_T at the 95% level of significance for $N = 60$ was -.2530. Hence, the daily variances for the remaining 18 securities probably follow a process which is a white noise with constant mean. More specifically, $\sigma_t = \alpha + u_t$, where

$$u_t \sim N(0, \sigma_u^2) \qquad (1)$$

TABLE 4.7

DATA SET OF SECURITIES

1. CAMPBELL RED LAKE
2. DENISON MINES
3. DOME MINES
4. HUSKY OIL
5. INTER-CITY GAS
6. INTERNATIONAL NICKEL
7. IMPERIAL OIL
8. MOORE CORPORATION
9. MITEL
10. NORCEN RESOURCES
11. NORTHERN TELECOM
12. RANGER OIL
13. ROYAL BANK
14. SHELL OIL
15. STEEL COMPANY OF CANADA
16. SILVER BUILLON
17. TORONTO DOMINION BANK
18. TOTAL PETROLEUM
19. TEXACO CANADA
20. SEAGRAM'S LTD.

TABLE 4:8

Autocorrelation Coefficients of $\{\sigma_t\}_{t=1}^{N}$

LAG K.

STOCK	1	2	3	4	5	6	7	8	9	10
CAMPBELL	-.1227	-.1427	-.1173	.1676	.0169	-.0329	-.1102	-.2045	.0077	.1981
DENISON	.1291	-.0330	-.0538	.1193	-.1103	-.0998	-.0577	-.0139	-.0128	.0071
DOME	-.1292	-.0784	-.1046	-.0611	.0795	-.0752	.1413	-.1019	-.0255	.0327
HUSKY	.1454	.0175	-.0279	-.0389	.2916	.0514	-.0745	-.0745	-.0689	-.1082
INTER CITY	-.1581	.0917	.0503	.0083	.0027	-.1740	.0234	-.0762	.0683	-.2897
IMPERIAL	-.0796	-.1372	.1879	.1904	-.1614	.0015	.2245	-.0626	-.1067	.0374
MOORE	-.1215	.3238#	-.1791	.1237	-.1215	.2348	-.0725	.0388	-.0917	-.0647
MITEL	-.0387	.0136	-.0295	-.0522	-.0248	-.0371	-.0043	-.0405	-.0176	-.0274
INCO	-.0479	.1805	-.1027	-.0548	.0402	-.1795	.0176	-.1565	-.0214	-.0494
NORCEN	-.1277	.0136	-.0854	-.1249	.0104	.0702	-.1418	-.1638	.0020	-.0818
NORTEL	-.0245	.0247	-.0019	.0819	.0350	-.0736	.1425	-.0558	-.0304	-.1277
RANGER	.0591	-.0411	.0716	.2338	.1016	-.0411	.1278	-.2067	-.0952	-.1209
ROYAL	-.0538	.1236	.2383	-.1038	.3069#	-.0378	-.1504	.1908	-.1549	.0844
SHELL	-.0846	-.0434	-.0131	.0200	-.0417	-.0179	.0674	-.0594	.0112	-.0088
STELCO	.0555	-.0741	.1411	-.0089	-.0119	-.0671	.0239	-.0617	-.1115	-.0301
SILVER	-.0907	-.0970	.3414#	.0294	-.0715	-.0135	-.0506	.0219	-.2404	-.1202
TOR.DOM.	.1647	-.0725	.0733	-.0037	.0122	-.0982	-.0195	.0153	.0077	-.0120
TOTAL	.0189	-.0044	-.0994	.0121	.0317	-.1215	-.0741	-.1134	-.0174	-.1035
TEXACO	-.1630	.1832	.1845	.0899	-.0175	-.0484	-.0245	-.0396	-.0976	-.1348
SEAGRAMS	.3025#	.0195	-.1512	-.0615	.1816	.0011	-.1347	-.2397	-.1996	.0540

# indicates coefficients which are significant at $\alpha = 5\%$.

TABLE 4.9

Autocorrelation Coefficients of the Differenced Series $\{\Delta \sigma_t\}$

STOCK	LAG K. 1	2	3	4	5	6	7	8	9	10
CAMPBELL	-.4311#	.0311	-.1389	.2091	-.0467	.0315	-.0113	-.1317	.0072	.1003
DENISON	-.4070#	-.0802	-.1136	.2308	-.1299	-.0066	-.0174	.0252	-.0124	-.0219
DOME	-.3673#	.0210	-.0150	-.0737	.1267	-.1635	.2356	-.1450	-.0020	-.0209
HUSKY	-.4145#	-.0488	-.0185	-.2039	.3352#	-.0805	-.0619	-.0044	.0285	-.0119
INTER-CITY	-.6045#	.1260	.0062	-.0270	.0792	-.1663	.1272	-.1050	.2169	-.2657#
IMPERIAL	-.4739#	-.1719	.1438	.1707	-.2470	-.0202	.2298	-.1089	-.0899	.0708
MOORE	-.6982#	.4242#	-.3592#	.2442	-.2619#	.2895#	-.1860	.1089	-.0718	.0394
MITEL	-.5253#	.0461	-.0102	-.0239	.0192	-.0219	.0333	-.0285	.0158	.0047
INCO	-.6106#	.2576#	-.1683	-.0251	.1548	-.1786	.1566	-.1395	.0696	-.0081
NORCEN	-.5636#	.1080	-.0175	-.0908	.0584	.1027	-.0838	-.0898	.1211	-.1368
NORTEL	-.5227#	.0323	-.0547	.0685	.0289	-.1545	.2019	-.1104	.0617	-.2444
RANGER	-.4486#	-.0974	-.0328	.1486	.0129	-.1681	.2632	-.2275	.0656	-.0365
ROYAL	-.5873#	.0324	.2193	-.3525#	.3229#	-.0447	-.2551	.3290#	-.2656	.1397
SHELL	-.5007#	-.0145	.0023	.0420	-.0403	-.0284	.0980	-.0915	.0417	.0129
STELCO	-.4199#	-.1991	.1890	-.0258	-.0589	-.0221	.0848	-.0159	-.0600	.0606
SILVER	-.4944#	-.2031	.3435#	-.0966	-.0731	.0437	-.0501	.1358	-.1405	-.0234
TOR.DOM.	-.3507#	-.1458	-.0436	.0299	.0746	-.0967	.0097	.1269	.0082	-.0904
TOTAL	-.4889#	.0384	-.1054	.0453	.0882	-.1018	.0523	-.0776	.0925	-.0559
TEXACO	-.6399#	.1390	.0553	-.0069	-.0141	-.0431	.0186	.0197	-.0105	-.0434
SEAGRAMS	-.2830#	-.0652	-.1896	-.1130	.3119	-.0323	-.0253	-.1112	-.1594	.2286

# indicates significant coefficients at $\alpha = .05$

If a regression is run on (1), and a sample of size N is taken, then the optimal forecast of σ_{N+1} would be $\hat{\alpha}$. For Seagrams, the tentative model is

$$\sigma_t = \alpha + \beta\, \sigma_{t-1} + u_t \tag{2}$$

Thus the best forecast is $\sigma_{T+1} = \dfrac{\hat{\alpha}}{1-\hat{\beta}}$. Finally, for Moore Corp, the tentative model is:

$$\sigma_t = \alpha + \beta\, \sigma_{t-2} + u_t \tag{3}$$

which is based on the observation that $r_2 = .3238$ is significant at $\alpha = .05$.

If model (1) is true for the 18 securities mentioned above, then the autocorrelogram of first differences should yield serial correlation coefficients which are not significantly different from -.500. This is exactly what is found in Table 4.9. In fact, for each security j, j = 1 ... 20, the null hypothesis, H_0: $\rho = -.500$ cannot be rejected for $\alpha = .05$*.

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* We add that the values in Table 1(a) of Capozza and Cornell [1979, p.54] have the same characteristics as the values in our Table 4.8. More specifically, the first order correlation coefficients of the different model are about -.500 and invariably cut off at lag 1 in both of the tables. Hence, the assumption by Copozza and Cornell that a first order moving average process for the different series is appropriate seems to be incorrect since it appears to be due to over-differencing.

The results, after estimating models (1), (2) and (3) are presented in Table 4.10 .

The models (1), (2) and (3) were estimated as follows. Using the estimating procedure discussed above, $\{\hat{\sigma}_t\}$ was first computed by the MINQUE estimator and then OLS was used on the models.
The results are presented in Table 4.10. Several outliers were present, where an outlier is an observed $\hat{\sigma}_t$ value which fails to confirm to the assumed model. In fact, if model (1) given by $\sigma_t = \alpha + \mu_t$ is estimated, and $\hat{\sigma}_t$ is plotted against the actual σ_t, then an outlier is represented by a significant deviation, or $E(\hat{\sigma}_t - \sigma_t) = E(\hat{\mu}_t) \neq 0$. The nature of the problem is illustrated for Mitel in Figure 4.1 . A possible explanation for the occurrence of outliers is that a time series might be subjected to an unanticipated shock due to, for example, the arrival of new information. For example, the large outlier in Figure 4.1 is due to a drop in Mitel's stock price from \$22.38 on Friday 10th June to \$17.63 on Monday 13th June 1983. This was attributed to the negative impact caused by the sudden termination of IBM's letter of understanding with Mitel.

In such cases, Weisberg [1980,p.113] states that: "one is probably justified in eliminating the case from the data set and estimating the regression model without it". Thus, a procedure given by Weisberg (see Appendix 4.2) in order to identify and significantly test for outliers was used herein. The results obtained from using this procedure can be

TABLE 4.10

Estimation of the Process Generating Daily Variances

STOCK	MODEL	Estimates of Parameters	# of outliers	$\hat{\sigma}_R$
CAMPBELL	$\sigma_t = \alpha + u_t$	$\hat{\alpha} = .000488$ [5.35]	3	.0221
DENISON	$\sigma_t = \alpha + u_t$	$\hat{\alpha} = .000159$ [4.17]	1	.0126
DOME	$\sigma_t = \alpha + u_t$	$\hat{\alpha} = .000721$ [4.73]	1	.0269
HUSKY OIL	$\sigma_{1t}=\alpha_1+u_{1t}$, $\sigma_{2t}=\alpha_2+u_{2t}$	$\hat{\alpha}_1 = .000196$ [4.02], $\hat{\alpha}_2 = .000813$ [4.63]	0	.0285
INTER CITY	$\sigma_t = \alpha+u_t$	$\hat{\alpha} = .00295$ [5.60]	1	.0172
IMPERIAL	$\sigma_t = \alpha+u_t$	$\hat{\alpha} = .000295$ [6.52]	0	.0164
MOORE	$\sigma_t = \alpha+\beta\sigma_{t-2}+u_t$	$\hat{\alpha} = .000218$ [3.19], $\hat{\beta} = .3172$ [2.47]	0	.0122
MITEL	$\sigma_{1t}=\sigma_1+u_{1t}$, $\sigma_{2t}=\sigma_2+u_{2t}$	$\hat{\alpha}_1 = .00240$ [4.51], $\hat{\alpha}_2 = .000702$ [3.61]	2	.0265
INCO	$\sigma_t=\alpha+u_t$	$\hat{\alpha} = .000317$ [4.78]	1	.0178
NORCEN	$\sigma_t=\alpha+u_t$	$\hat{\alpha} = .000174$ [5.49]	0	.0130
NORTEL	$\sigma_t=\alpha+u_t$	$\hat{\alpha} = .000491$ [5.811]	1	.0221
RANGER	$\sigma_t=\alpha+u_t$	$\hat{\alpha} = .000804$ [4.30]	2	.0286
ROYAL BK.	$\sigma_{1t}=\alpha_1+u_{1t}$, $\sigma_{2t}=\alpha_2+u_{2t}$	$\hat{\alpha}_1 = .000309$ [5.01], $\hat{\alpha}_2 = .000123$ [3.71]	0	.0111
SHELL	$\sigma_t = \alpha + u_t$	$\hat{\alpha} = .000155$ [4.82]	2	.0125
STELCO	$\sigma_t = \alpha + u_t$	$\hat{\alpha} = .000144$ [5.67]	0	.0120

TABLE 4.10 (continued)

Estimation of the Process Generating Daily Variances

STOCK	MODEL	Estimates of Parameters	# of outlines	$\hat{\sigma}_R$
SILVER	$\sigma_t=\alpha+u_t$	$\hat{\alpha}$ = .000373 [5.83]	2	.0193
TORONTO DOM	$\sigma_t=\alpha+u_t$	$\hat{\alpha}$ = .000200 [4.82]	1	.0141
TOTAL PETE	$\sigma_t=\alpha+u_t$	$\hat{\alpha}$ = .000489 [4.61]	2	.0221
TEXACO	$\sigma_t=\alpha+u_t$	$\hat{\alpha}$ = .000245 [5.03]	0	.0156
SEAGRAMS	$\sigma_t=\alpha+\beta\sigma_{t-1}+u_t$	$\hat{\alpha}$ = .0002136 [3.33]; $\hat{\beta}$ .3025 [2.447]	0	.0176

PLOT OF ACTUAL(*) AND FITTED(+) VALUES — PLOT OF RESIDUALS(O)

ID	ACTUAL	FITTED	RESIDUAL
2	.0069E-03	.1877E-02	-.117E-02
3	0.	.1931E-02	-.193E-02
4	.6681E-02	.1959E-02	.472E-02
5	0.	.1709E-02	-.170E-02
6	.5265E-03	.1959E-02	-.113E-02
7	.3507E-02	.1927E-02	.158E-02
8	.1040E-02	.1823E-02	-.783E-03
9	.4124E-03	.1918E-02	-.131E-02
10	.3609E-04	.1935E-02	-.190E-02
11	.2122E-03	.1957E-02	-.175E-02
12	.1323E-02	.1951E-02	-.252E-04
13	.4044E-03	.1884E-02	-.148E-02
14	.1509E-03	.1943E-02	-.179E-02
15	.1828E-02	.1953E-02	-.125E-03
16	.1247E-02	.1889E-02	-.641E-03
17	.2852E-04	.1910E-02	-.188E-02
18	0.	.1958E-02	-.196E-02
19	0.	.1959E-02	-.196E-02
20	.4126E-03	.1959E-02	-.155E-02
21	.2747E-04	.1943E-02	-.192E-02
22	.1617E-03	.1958E-02	-.180E-02
23	.6297E-03	.1953E-02	-.132E-02
24	.1063E-02	.1934E-02	-.872E-03
25	0.	.1918E-02	-.192E-02
26	.3274E-04	.1959E-02	-.193E-02
27	.6998E-03	.1958E-02	-.126E-02
28	.6126E-03	.1932E-02	-.132E-02
29	.5707E-01	.1943E-02	.551E-01
30	.3091E-03	-.2511E-03	.560E-03
31	.2914E-02	.1947E-02	.968E-03
32	.5548E-03	.1846E-02	-.129E-02
33	0.	.1937E-02	-.194E-02
34	.2520E-03	.1959E-02	-.171E-02
35	.7941E-04	.1949E-02	-.187E-02
36	.2628E-02	.1956E-02	.672E-03
37	.2926E-03	.1857E-02	-.160E-02
38	.1707E-02	.1949E-02	-.192E-03
39	.1713E-02	.1891E-02	-.177E-03
40	.2254E-03	.1892E-02	-.167E-02
41	.2120E-02	.1950E-02	.117E-03
42	.2074E-02	.1838E-02	.324E-02
43	.2101E-03	.1762E-02	-.155E-02
44	.2244E-02	.1951E-02	.129E-02
45	.5770E-03	.1833E-02	-.105E-02
46	.6811E-04	.1929E-02	-.186E-02
47	.1091E-02	.1956E-02	-.865E-03
48	.2889E-03	.1917E-02	-.113E-02
49	.1265E-02	.1928E-02	-.663E-03
50	.1713E-02	.1910E-02	-.196E-03
51	.5794E-04	.1892E-02	-.183E-02
52	.8015E-06	.1956E-02	-.188E-02
53	.6837E-04	.1956E-02	-.189E-02
54	.2254E-03	.1956E-02	-.173E-02
55	.5293E-03	.1950E-02	-.142E-02
56	0.	.1938E-02	-.194E-02
57	.2761E-03	.1959E-02	-.168E-02
58	.5896E-03	.1948E-02	-.958E-03
59	.9973E-04	.1959E-02	-.183E-02
60	.7133E-03	.1959E-02	-.083E-02

outlier — outlier

MIN = -.0003 MAX = .0571
RESIDUAL STD. ERROR = .7355E-02

Figure 4.1 Mitel Corp: ε_{29} is an outlier

summarized as follows (see Tables 4.10 and 4.11).

First, when no outliers and model (1) is assumed, the forecast value of σ^2_{T+1} is given by $\hat{\sigma}^2_R = \hat{\alpha}$. This value is almost identical to the value obtained by first assuming that the historical variance is constant and then calculating $\hat{\sigma}^2_H = \sum_{t=1}^{T} (\sigma_t - \bar{\sigma})^2 / T-1$. For example, Texaco has no outliers and no shifts in the variance. Thus, for Texaco, $\sigma_t = \alpha + \mu_t$ and $\hat{\sigma}_R = .0156$ while $\hat{\sigma}_H = .0155$.

Second, when a shift in the variance is detected and verified by an F-test, the best estimate of the variance is the value of $\hat{\alpha}$ obtained from the last shift. For example, the following is obtained for Husky oil.

$$\sigma_{1t} = \alpha_1 + u_{1t} \qquad t = 1, \ldots\ldots 30$$

$$\sigma_{2t} = \alpha_2 + \mu_{2t} \qquad t = 31 \ldots\ldots 60$$

This is based on an inspection of the residuals of the model $\sigma_t = \alpha + \mu_t \quad t = 1, \ldots 60$. In this case $\hat{\alpha}_1 = .00196 \neq \hat{\alpha}_2 = .000813$
(4.02) (5.63)

Thus, $\hat{\sigma}^2_{T+1}$ is forecasted as $\hat{\sigma}^2_R = \hat{\alpha}_2$; that is, as the most recent mean value. Third, outliers are eliminated based on the assumption that investors will only act on new information which causes a substantial deviation from past trend after it has been confirmed. If a new level of variances is established after the arrival, a shift has occurred. However, if the effect of the random shock was only transitory, then the

system will return to its original path. Hence, these outliers may be eliminated in the estimation procedure. This is the methodology adopted herein. Before discussing the implications of the above procedure for option pricing, it is important to note that for models (2) and (3), the OLS estimates of $\hat{\alpha}$ and $\hat{\beta}$ and the forecasted value, $\hat{\sigma}_R^2 = \frac{\hat{\alpha}}{1-\hat{\beta}}$, are all given in Table 4.10.

Black-Scholes Forecasts

Option prices for at-the-money options will now be forecasted for each of the underlying securities considered. The main reason for this selection is that past studies show that the B-S model performs very well for relatively short-term at-the-money options. The forecast date is July 29, 1983 and 3-6 month options are considered. The results are summarized in Table 4.11. Bid and Ask prices for the respective options are shown and forecasts within this range are considered acceptable. To facilitate easy interpretation of the results note that:

S_0 = closing stock price on forecast date;

$$\hat{\sigma}_H^2 = \frac{\sum (\sigma_t - \bar{\sigma})^2}{T-1} ;$$

$\hat{\sigma}_R^2$ = forecast value by a MINQUE-type estimator;

E = exercise price;

C_{Ask} = closing ask price of the option;

C_{Bid} = closing bid price of the option;

$\hat{C}_H$ = forecasted price via B-S model using $\hat{\sigma}_H$;

$\hat{C}_R$ = forecasted price via B-S model using $\hat{\sigma}_R$;

r(3), r(6) = 3 (6) month - T Bill rate of .0924 (.0959) on July 28th.

The general conclusion is that $\hat{\sigma}_R$ leads to a better forecast value than does $\hat{\sigma}_H$ in the sense that $\hat{C}_R$ lies between C_{Bid} and C_{Ask} more often than $\hat{C}_H$. Note that the implicit assumption is that observed option prices are the correct option prices. The procedure used herein generated unbiased estimates of the daily variances. Hence, since the presence of outliers could be detected, their effects could be minimized. This is not possible when computing $\hat{\sigma}_H$. Consequently, $\hat{\sigma}_H$ is generally over-estimated when outliers are present and the B-S value of $\hat{C}_H$ is too high. For example, in the case of Campbell Red Lake (CRK), the bid-ask spread was \$4.50 to \$4.63, $\hat{\sigma}_H$ value was \$5.19. However, with the elimination of the outliers, $\hat{\sigma}_R = .0221$ and $\hat{C}_R = 4.66$. A similar result occurs for INCO. When $\sigma_t = \alpha + \varepsilon_t$ is estimated with outliers present [e.g. $\varepsilon_{54} = .0035$ is a significant outlier], then $\hat{\alpha} = .0193$ and $\hat{\sigma}_R = .0193$. This compares well with the historical standard deviation

TABLE 4.11

Prediction of Option Prices for at-the-money options via the Black-Scholes model. [July 29th, 1983 is Forecast date].

STOCK	S_o	$\hat{\sigma}_H$	$\hat{\sigma}_R$	E	C_{Bid}	C_{Ask}	$\hat{C}_H$	$\hat{C}_R$
CAMPBELL	35.63	.0246	.0221	Dec 35	4.50	4.63	5.19	4.66
DENISON	47.50	.0141	.0126	Oct 50	1.25	1.50	1.85	1.57
DOME	20.63	.0268	.0269	Oct 20	2.50	2.63	2.62	2.63
HUSKY	11.50	.0258	.0285	Oct 10	2.13	2.25	2.09	2.17
INTER CITY	11.88	.0183	.0172	Dec 10	2.13	2.38	2.48	2.44
IMPERIAL	38.38	.-179	.0164	Nov 35	4.63	5.00	5.55	5.29
MOORE	54.75	.0018	.0122	Nov 55	3.50	3.75	3.41	3.50
MITEL	16.88	.0432	.0265	Dec 15	3.13	3.38	4.63	3.41
NORCEN	38.50	.0133	.0132	Nov 35	4.75	5.13	5.04	5.04
NORTEL	48.50	.0255	.0221	Oct 50	3.13	3.88	4.50	3.69
RANGER	74.25	.0314	.0286	Oct 15	1.25	1.35	1.51	1.28
ROYAL	32.63	.0136	.0111	Oct 32.50	1.90	1.95	2.12	1.74
SHELL	25.50	.0177	.0125	Nov 25	1.90	2.13	2.61	2.01
STELCO	29.00	.0121	.0120	Nov 27.50	2.88	3.00	2.92	2.92
SILVER	11.88	.0242	.0193	Dec 12	1.15	1.20	1.55	1.25
TOR. DOM.	54.38	.0164	.0141	Dec 55	4.13	4.38	4.94	4.38
TOTAL	14.75	.0266	.0221	Jan 15	1.85	2.13	2.23	1.90
INCO	19.25	.0194	.0178	Nov 20	1.35	1.60	1.50	1.36
TEXACO	38.00	.0155	.0156	Dec 35	5.25	5.63	5.35	5.35
SEAGRAMS	37.88	.0177	.0176	Oct 36.63	2.75	3.00	3.51	3.51

S_o = Stock price on Forecast date; $\hat{C}_H$ = predicted price using $\hat{\sigma}_H$

$\hat{\sigma}_H$ = Standard deviation of historical prices; $\hat{C}_R$ = predicted price using $\hat{\sigma}_R$

$\hat{\sigma}_R$ = forecast value of standard deviation from time series;

E = exercise data and price.

of $\hat{\sigma}_H = .0194$. However, when the outlier is eliminated, $\hat{\sigma}_R = .0178$ and $\hat{C}_R = \$1.36$. Although the bid-ask spread was \$1.35 to \$1.60 , $\hat{C}_R$ seems to be a good forecast since the last trade was \$1.25. Thus, the general nature of the results is that the historical variance , $\hat{\sigma}_H^2$, invariably leads to an overestimate of the option price in the presence of outliers. The estimate, $\hat{\sigma}_R$, developed herein, consistently resulted in better forecasts of the observed option price.

Conclusion

In this chapter, a number of significant results were obtained.

First, for monthly closing prices on the TSE for the period 1970 - 1979, the empirical distribution of stock price changes is approximately normal.

Second, for daily stock prices on the TSE over a sample period of 1 year, the empirical distribution was found to belong to the symmetric stable Paretian family with an average characteristic exponent of approximately 1.4 .

Third, monthly variances of stock price (for the 10-year period) are non-stationary in that they generally vary directly with the square of the market rate of return.

Fourth, daily variance [for the 3-month period 2 May to 28 July 1983] of 20 securities listed on the TCO are generally a white noise with a constant mean. However, some correlation in the time series of daily variances was found. Also the presence of outliers in $\{\sigma_t\}$ makes the historical variance, $\hat{\sigma}_H^2$, unsuitable, in the sense that investors, if they are using the B-S model, seems to discount once and for all "blips" in the variance series. A MINQUE - type estimator is developed herein, and the B-S model, using $\hat{\sigma}_R^2$, provided very good forecasts of 3-6 month at-the-money options on the Toronto Options Market.

APPENDIX 4.1

The Beckers' Estimators: A Critique

In this appendix, it will be shown that Beckers' estimator for the daily variance of stock price changes is biased and inconsistent.

Let S_t = closing stock price at time t

$X_t = \ln(S_t/S_{t-1})$

σ_t^2 = Variance of X_t

Beckers assumed that $\sigma_t = k\, S_{t-1}^{\frac{\alpha-2}{2}} \quad 0 \leq \alpha < 2$ (1)

This assumption is based upon his argument that the variance of stock price changes and the level of the stock price itself are inversely related. For $\alpha = 2$, σ_t is constant and the usual Black-Scholes assumption (i.e., $\frac{dS_t}{S_t} = \mu\, dt + \sigma\, d\,\omega_t$, where ω_t is a white noise) is obtained:

From (1), the following is obtained

$$\ln \sigma_t = \ln k + \left(\frac{\alpha-2}{2}\right) \ln S_{t-1} \qquad (2)$$

In order to estimate (2), Beckers used a familiar theorem in statistics:

<u>Theorem</u>:

Let $X \sim N(\mu,\sigma^2)$ such that $u \approx 0$. Then $E|X_t| \approx \sigma_t \left(\frac{2}{\pi}\right)^{1/2}$.

The proof of this result is rather straight forward. It is found in Beckers [1980], Appendix C.

Using this theorem, and the fact that

$$E|x_t| = |x_t| + \varepsilon_t \text{ , equation (2) becomes}$$

$$\ln\left(|X_t| + \varepsilon_t\right) = a + b \ln\left(S_{t-1}\right) \tag{3}$$

where $a = \ln\left(\frac{2k}{\pi}\right)^{1/2}$, $b = \frac{\alpha-2}{2}$.

However, $\ln\left(|X_t| + \varepsilon_t\right) = \ln\left[|X_t|\left(1 + \frac{\varepsilon_t}{|X_t|}\right)\right]$

$$= \ln|X_t| + \ln\left(1 + \frac{\varepsilon_t}{|X_t|}\right)$$

Hence (3) may be rewritten as:

$$\ln|X_t| = a + b \ln S_{t-1} + \eta_t \tag{4}$$

Beckers estimated (4) for 47 securities. He obtained significant negative estimates for 6 of these 38 securities. He concluded that this supported his hypothesis that σ_t and S_{t-1} are inversely related.

However, from (4), $\eta_t = \ln [1 + \frac{\varepsilon_t}{|X_t|}]$

where $E(\varepsilon_t) = 0$

The expansion of $\log (1+z) = z - \frac{1}{2} z^2 + \frac{1}{3} z^2 \ldots\ldots$, shows that $E(\eta_t) \neq 0$. In fact, it is easily seen that

$$E(\hat{b}) = b + E\{ \frac{\sum \ln S_{t-1}}{\sum(\ln(S_{t-1}))^2} (\frac{\varepsilon_t}{|X_t|} - \frac{1}{2} \cdot \frac{\varepsilon_t^2}{(X_t)^2} + \ldots) \}$$

$\rightarrow$ $\hat{b}$ is biased downward.

APPENDIX 4.2

Computation of the critical value of the Residual in the Outlier Test

This appendix identifies the critical value, ε^*, of the set of residuals, such that any other residual $\varepsilon_i > \varepsilon^*$ is a significant outlier. The ε^* value is calculated at the $\alpha = .01$ level of significance. The approach followed here is adapted from Weisberg [1980, p. 113].

Assume $\gamma_t = \alpha + \beta X_t + \varepsilon_t$ (1)

Estimate (1) by OLS and obtain

$$r_i = \frac{\hat{\varepsilon}_i}{\hat{\sigma}\sqrt{1-\hat{V}_{ii}}} \quad \text{where}$$

$\hat{\varepsilon}_i$ = i^{th} residual $(y_i - \hat{y}_i)$, $i = 1 \ldots\ldots N$.

$$\hat{\sigma} = \left(\frac{RSS}{N-2}\right)^{1/2}, \quad \text{where } RSS = \sum_{i=1}^{N} \hat{\varepsilon}_i^2$$

$$\hat{V}_{ii} = \frac{1}{N} + \frac{(X_i - \bar{X})^2}{\sum (X_i - \bar{X})^2}$$

As is shown in Weisberg [1980], the appropriate test statistic is

$$t = r_i \left(\frac{N-p-1}{N-p-r_i^2}\right)^{1/2} \quad \text{where}$$

p = no. of parameters to be estimated

(i.e. $p = 2$ as is shown in (1)) (2)

This value is then compared with the critical value of t given in a suitable table (for example, see Table D on page 264 of Weisberg [1980]). The critical values of ε^* are now calculated, such that if $\varepsilon_i \geq \varepsilon^*$, then ε_i is an outlier.

From (2), $r_i^2 = \dfrac{(N-p)t^2}{N-p-1+t^2}$

Hence, $$\varepsilon_i^* = (1-\hat{V}_{ii})^{1/2}\ \hat{\sigma}\sqrt{\frac{(N-p)t^2}{N-p-1+t^2}} \qquad (3)$$

So, for a given value of N and p, $\hat{\sigma}$ and $\hat{V}_{ii}$ are computed and the critical value of $t(\alpha)$ is obtained from Table D. Next, ε_i^*, is computed from (3). If $\varepsilon_i \geq \varepsilon_i^*$, then ε_i is an outlier at the $\alpha\%$ level of significance.

For example, in the case of Campbell Red Lake (CRL), $N = 60$, $p = 1$, $t = 4.03$ for $\alpha = .01$, $\hat{\sigma}^2 = .000956$, and $V_{ii} = .01566$. Then $\varepsilon^* = .00258$. Hence, any computed residual exceeding ε^* is identified as a significant outlier at the 1% level. For CRL, the following three outliers were found:

$$\hat{\varepsilon}_4 = .00285, \quad \hat{\varepsilon}_{11} = .00354, \text{ and } \hat{\varepsilon}_{44} = .00323$$

CHAPTER 5

SOME EMPIRICAL TESTS OF THE RANDOM WALK HYPOTHESIS IN CANADIAN OPTION MARKETS

The identification of the price formation mechanism for speculative markets for such assets as securities and commodities is an important problem in financial economic theory. Specifically, the question of whether such markets are efficient depends crucially on the behaviour of successive price changes over time. The most widely accepted hypothesis concerning the process of price formation is the so-called random walk hypothesis. To understand the meaning of a random walk, one can think in terms of a particle which moves in discrete jumps over time, where certain probabilities are associated with each jump. Technically, a one-dimensional random walk is a Markov chain with state $\subseteq Z$, the set of integers, in which the particle, if it is in state i, can by one transition remain in state i, or move to adjacent states $i+1$ or $i-1$. In its simplest form, one can think of the particle moving along a real line. If it is presently in state i, then, for each subsequent unit of time, it either remains in state i with probability p, moves to state $i+1$ with probability q, or to state $i-1$ with probability $1-p-q$. Note that successive movements of the particle are independent of previous movements. This is the basis of the random walk hypothesis, which states that successive price changes over time are independent and

are generated by some random process. The immediate implication is that $E(X_{n+1}/X_0, X_1, \ldots, X_n) = X_n$. That is, the expected price in the next period is just the current price, and so historical prices are of no value in the prediction of future prices. As Adam Smith [1967] succintly stated it: "Price changes have no memory and yesterday has nothing to do with tomorrow".

However, the random walk hypothesis is not inconsistent with trends in prices over time. This can be illustrated as follows. Assume that prices follow a random walk at each point in time such that:

$$\text{Prob}\ (S = S_0 + 1/S = S_0) = p \quad \text{and}$$

$$\text{Prob}\ (S = S_0 - 1/S = S_0) = q.$$ Let $p > .5$ and $q < .5$. Then, prices will follow a random walk with an upward drift. In general, the random walk hypothesis may be subdivided into two versions. The first version, or the so-called statistical form, was discussed above. It states that successive price changes are independent over time, and are generated by some random process. Cootner [1964], Osborne [1959] and Roberts [1959] using a time-domain approach, and Granger and Morgenstern [1970] using a frequency domain approach, have all empirically validated this version of the hypothesis.

The second version, or the so-called economic version, asserts that security markets are efficient in the sense that arbitrage opportunities based on the information contained in past prices and past price changes, modulo transactions cost, cannot exist. In other words, excess profits in

the presence of transactions cost, cannot be consistently earned using past prices and past price charges. Most researchers (e.g. Alexander [1961]), in order to test this hypothesis, have devised some mechanical trading rule and then compared the return associated with this rule and a simple buy and hold strategy.

The validity of the random walk hypothesis for option price changes is also an important question, because of its close association with the efficiency question. Therefore, both versions of the hypothesis for options on the T.C.O. will be tested in this chapter. To date no such empirical tests on options have been published in the literature, although Leabo and Rogalski [1975] did conduct such tests on warrants that were listed on the NYSE and ASE. Based on a serial correlation test and an exact runs test, they found that a significant negative serial correlation in warrant price series existed and that the hypothesis of an unrestricted random walk is rejected in favour of a random walk with reflecting barriers. The exact opposite result is found herein for a random sample of 18 option price series on the TCO. In only 3 cases was the hypothesis a random walk (i.e. ARIMA (0,1,0)) rejected in favor of ARIMA (0,1,1).

Next, a test of the efficiency of the TCO is conducted using the Put-Call Parity Theorem. In the absence of transactions costs, a significant number of arbitrage opportunities, with profits of a relatively high magnitude, are identified.

In order to determine if these arbitrage opportunities would persist in the presence of transactions costs, the trading costs for the options on the TCO, and their associated stocks, were then estimated. The estimated trading costs were considerably higher than those obtained by Phillips and Smith [1980] for the U.S., which is probably due to the relatively thinner nature of the capital market in Canada. Using these estimated trading costs for Canadian markets, the abnormal profits previously obtained were entirely eliminated.

5.1 Test of the Statistical Form

The statistical form of the random walk hypothesis was tested as follows.

(a) Estimation of the Autocorrelation Function

For a time series $\{x_t\}$, the autocovariance function is given by $\gamma_i = E(x_t - u)(x_{t+i} - u)$, which is the expected product of the deviations of x_t and x_{t+i} from the mean, u. Note that x_{t+i} is the realized value of the series, i periods ahead. Also observe, that if, say, two successive observations are above the mean of the process, then $\gamma_1 > 0$. However, for purposes of comparing two series, γ_i is unsatisfactory since it is susceptible to changes in the scale of measurement. Hence the autocorrelation function, $\rho_k = \frac{\gamma_k}{\gamma_0}$, standardizes the γ_k's and leads to a better means of comparing the two time series. The ρ_k's are usually estimated as follows:

Let C_k be an estimate of γ_k, and $\{O_t\}$ be a time series of option prices associated with each stock.

Define $x_t = \ln (O_t/O_{t-1})$, and $\bar{x}_t = \sum_{t=1}^{T} x_t$. Then,

$$C_k = \frac{1}{T} \left[\sum_{t=k+1}^{T} (x_t - \bar{x})(x_{t-k} - \bar{x}) \right].$$

Finally, r_k, which is an estimate of ρ_k, is given by $r_k = \frac{C_k}{C_0}$, $k = 1,2, \ldots$. For a stationary normal process, Bartlett [1946], has shown that $\sigma^2(r_k) \simeq \frac{1}{T}(1+2 \sum_{t=1}^{k-1} r_t^2)$. Thus the statistic $t^* = \frac{r_k}{\sigma(r_k)}$ is used to test for significance.

Data

A sample of 18 options were randomly selected from a sample of 7 stocks listed on the TCO. These securities are Canadian Pacific, Stelco, Bank of Montreal, Dome Pete., INCO, Noranda and Gulf of Canada. The sampling period was 1979-1980 and only options with at least 95% trading frequency were considered. When no trade in the option occurred, the average of the closing bid price, P_B, and the closing ask price, P_A, was used as a proxy for the true price, P. That is, $P = \frac{P_A+P_B}{2}$. The implicit assumption here is that the random variable,

P, is distributed uniformly over the interval $[P_B, P_A]$, so that $E(p) = (P_A + P_B)/2$ and $Var(P) = (P_A - P_B)^2/12$. When trading in the option took place, the daily closing prices were recorded. The sample of 18 options are identified in Table 5.1 .

Empirical Findings

The results of the serial correlation coefficient test, which are listed in Table 5.2, are summarized as follows. For each time series, the estimated serial coefficients were computed up to 15 lags. This totals 270 coefficients for the 18 option series. But only 10 were significant at the 5% level of significance. The implication of this result is that the null hypothesis of an unrestricted random walk cannot be rejected.

Based on this hypothesis, it is expected that the ARIMA* model (0,1,0) or $C_{t+1} - C_t = W_t$ where W_t is a white noise, would provide the best fit for the series $\{C_t\}$. Using the Box-Jenkins methodology for the identification of a tentative model and its estimation, suitable ARIMA models were obtained for each of the option series $\{C_t\}$. These fitted models and their associated Box-Pierce Q statistics (as well as the computed autocorrelograms of the residuals which indicated that they were white noises) are shown in Table 5.3.

The general conclusion of this section is that daily closing option prices for the sample selected from the TCO, are generated by a random walk process. This result is supported by a serial correlation coefficient test as well as by the Box-Jenkins procedure for identifying and estimating the proposed model. However, these results differ from those of Leabo and Rogalski [1975] who found that monthly warrant prices on the NYSE and ASE follow a restricted random walk with reflecting barriers.

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* Details about the identification and estimation of an ARIMA model using the Box-Jenkins methodology were discussed in Chapter 4.

TABLE 5.1

The Sample of 18 options on the TCO which were used in the Test of the Statistical Form of the Random Walk Hypothesis

Security (Ticker Symbol)	E	t_0	t_1	N
1. DMP	35	3/27/79	10/19/79	136
2. DMP	60	1/21/80	10/17/80	158
3. DMP	80	8/31/78	1/19/79	88
4. N	22.50	2/19/79	11/16/79	160
5. N	20	5/23/78	2/16/79	175
6. N	22.50	5/22/79	2/15/80	168
7. C.P.	30	3/16/79	8/17/79	93
8. C.P.	22.50	8/ 4/78	2/16/79	106
9. C.P.	30	3/16/79	11/16/79	127
10. NOR	40	10/6/78	5/18/79	126
11. NOR	45	1/15/79	8/17/79	120
12. NOR	20	8/23/79	2/15/80	117
13. STE	35	6/21/80	11/21/80	67
14. BMO	25	8/21/78	5/18/79	185
15. BMO	25	8/18/78	2/16/79	109
16. GOC	24	12/17/79	5/16/80	93
17. GOC	110	9/21/79	2/15/80	100
18. GOC	100	9/13/79	2/15/80	105

E = exercise price; t_0 = starting date; t_1 = exercise date; N = sample size.

TABLE 5.2

Results of the Serial Correlation Coefficient Test of the Random Walk Hypothesis

LAG (K)

#	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	-.036	-.170	-.061	.029	.118	-.089	.013	-.006	-.026	.116	.037	-.045	-.006	.012	.005
2	.052	-.179*	.000	.174	-.078	-.047	-.097	.020	.076	.034	-.018	-.097	.084	.151	-.076
3	.021	-.020	.023	.115	-.014	-.042	-.146	.052	.144	.095	.061	.163	-.097	-.039	-.130
4	.064	-.046	.041	.092	.028	.035	.088	.040	-.052	-.046	-.006	-.101	.018	.010	-.026
5	-.007	-.011	-.003	-.025	.038	-.162*	.018	-.010	-.087	.014	.048	.067	.048	-.002	.094
6	-.071	.000	.176*	-.051	.055	.123	-.057	-.081	.090	.020	-.017	.110	-.014	-.086	.102
7	.075	.056	-.150	-.019	.013	-.096	.043	-.141	.028	-.079	.034	.039	-.106	-.044	-.170
8	.180	.065	.014	-.020	.006	-.082	-.212*	-.125	.009	-.043	-.134	.122	.017	.061	.053
9	.141	.034	-.084	.009	-.016	.154	-.040	-.072	-.111	-.168	-.202*	.106	.033	.127	-.149
10	.012	-.019	-.262*	.051	.006	-.017	.024	.075	.023	-.084	.048	.087	.023	-.058	-.061
11	.077	-.045	-.092	-.059	.200*	.015	.072	.145	.041	-.064	-.093	-.019	.092	-.076	-.044
12	-.135	-.137	.107	.030	.034	-.004	.122	-.085	-.062	-.062	.052	-.046	-.126	-.019	.205*
13	.070	-.168	.065	.053	-.021	.044	.005	-.038	-.126	-.186	-.106	-.086	-.017	-.011	-.068
14	.010	-.022	.079	.067	-.022	.008	.056	.050	-.007	-.011	-.009	-.008	.055	.062	.043
15	.031	.072	.016	-.140	.330*	-.001	-.031	-.033	-.059	.131	.125	-.027	-.013	-.040	-.056
16	.148	.001	.069	-.093	-.059	.006	.055	-.005	.084	.062	-.022	.016	-.115	-.118	-.043
17	-.095	.112	-.113	.061	.041	-.016	.084	.000	-.177	-.025	-.056	-.039	-.072	-.052	.165
18	.059	.008	-.008	-.003	.042	-.119	-.096	-.091	-.119	-.014	-.089	.010	-.148	.276*	.074

refers to the same ordering as in Table 5.1. For example, number 1, refers to the option on DMP with Exercise Price of $35 ending on 10/19/79.

* indicates statistical significance based on the Bartlett formula.

Table 5.3

Results of the Box-Jenkins Procedure to estimate the option series in Table 5.1

#	ARIMA (p,d,q)*	Q Statistic
1	(0,1,0)	29.50
2	(0,1,0)	32.45
3	(0,1,0)	14.67
4	(0,1,0)	22.78
5	(0,1,0)	24.78
6	(0,1,0)	29.97
7	(0,1,0)	21.43
8	(0,1,0)	23.90
9	(0,1,1)	33.10
10	(0,1,0)	28.12
11	(0,1,0)	40.04
12	(0,1,0)	24.64
13	(0,1,1)	14.29
14	(0,1,0)	21.45
15	(0,1,1)	19.74
16	(0,1,0)	19.04
17	(0,1,0)	16.64
18	(0,1,0)	14.21

refers to numerical ordering as in Table 5.1

* p = number of autoregressive coefficients;
d = degree of differencing;
q = number of moving average coefficients.

5.2 Some Tests of the Economic Form of the Random Walk Hypothesis

The economic form of the the random walk hypothesis asserts that security markets are efficient in the sense that arbitrage opportunities cannot exist when transactions costs are considered. In order to test this form of the random walk hypothesis, most researchers employ some filter rule strategy and then assess the returns generated from using the strategy with those obtained from using a simple buy and hold strategy.

A different approach to testing this hypothesis will be used herein. In particular, an approach will be used which is similar to that used by Frenkel and Levich [1977] for the foreign exchange market. They tested the interest parity (IP) theory in order to detect any unexploited profit opportunities. The IP theory states that:

$$\frac{F-S}{S} = \frac{i-i^*}{i+i^*} \quad , \quad \text{where } F \text{ and } S$$

are forward and spot rates, respectively; i is the domestic interest rate (e.g., T-Bill rate); and i^* is the foreign interest rate. Although Frenkel and Levich found that deviations from the parity line existed, they could not be exploited in the presence of transactions costs. Thus, the hypothesis of market efficiency could not be rejected.

We will also test for option market efficiency by empolying a similar parity theory. This is the so-called Put-Call Parity Theorem that was formulated by Stoll [1969] for European call and put options

on non-dividend paying stocks. Specifically, the theorem is as follows:

Theorem

Let S_t be the price of the underlying security at time t. Let C_t and P_t be the call and put price of a call and a put option, respectively, which have the same expiration date, t^*, and exercise price, E. Also, let r be the short-term Treasury Bill rate for the period $[t,t^*]$.

Then $C_t - P_t = S_t - Ee^{-r}(t^*-t)$ (1)

Proof

Create a portfolio in which 100 shares of the stock are bought; one call option contract sold; one put option is bought, and $Ee^{-(r)(t^*-t)}$ dollars are borrowed for the time period, $[t^*,t]$.

Then the initial cash flow is:-

$$C_t + Ee^{-r(t^*-t)} - S_t - P_t .$$

At t^*, either $S_{t^*} \geq E$ or $S_{t^*} < E$. If $S_{t^*} \geq E$, then the final cash flow is:

$$-(E-S_{t^*}) - E + S_{t^*} + 0 = 0 \qquad (2)$$

Equation (2) can be explained as follows. Since $S_t^* \geq E$, the call option, which was sold at time t, will be exercised and so would involve a cash outflow of $(E-S_{t^*})$. The loan will also be due at t^*

and this will involve a cash outflow of E dollars. Hence the total outflow is $(E-S_{t*}) + E$. The cash inflow will result from the stock being sold for S_{t*} at $t*$. The put option which was bought will be worthless at $t*$, since $S_{t*} \geq E$. Hence, the total cash inflow is $S_{t*} + 0$. Thus, the total net cash flow is $-[(E-S_{t*}+E]+[S_{t*}+0] = 0$.

Similarly, if $S_{t*} < E$, the call option is worthless at $t*$, and the put option is worth $E-S_{t*}$, at $t*$. Thus, the net cash flow is:

$$0 + S_{t*} + (E-S_{t*}) - E = 0 \qquad (3)$$

Thus, in all states for S_t at $t*$, the final value of the portfolio which was formed at t is zero. Hence, to prevent arbitrage the initial cash flow must also be zero. That is,

$C_t-S_t-P_t + Ee^{-r(t*-t)} = 0$ or, the so-called Put-Call Parity theorem,

$C_t = S_t+P_t - E^{e-r(t*-t)}$ must be satisfied. This result can be interpreted as follows: P_t, the price of the put, can be viewed as being the premium the investor must pay in order to insure a price of \$E for the stock at $t*$. In other words, P_t is the price of an insurance policy. Since $Ee^{-r(t-*)}$ is borrowed, then $S-Ee^{-r(t-t*)}$ is the margined value of the stock. So $C_t = P_t-S+Ee^{-r(t*-t)}$ can be interpreted in words as: A call option is equivalent to an insurance policy plus a margined stock.

Merton [1973] modified (1), by showing that a sufficient condition for a rational investor not to pre-exercise the call option before T^* is:

$$E[1-e^{-r(t_i-t)}] > d_i , \qquad (4)$$

where t_i is the ex-dividend date and d_i is the dividend payment announced for each i.

If the stock pays no dividends, condition (4) is always satisfied, and pre-mature exercise is non-optimal. Merton also argued, that, even in the case of non-dividend paying stocks, early exercise is a possibility for puts. Then, relation (1) becomes:

$$P_t + S_t - E \leq C_t \leq P_t + S_t - Ee^{-r(t^*-t)} \qquad (5)$$

There are three published empirical tests of the Put-Call Parity theorem. Stoll [1969] tested equation (1) for a sample of ten frequently reported options on the OTC market. His results, based on weekly prices over the period 1966-1967, were consistent with the theory. Unfortunately, however, unlike an established auction market like the TCO, options on the OTC are adjusted to incorporate cash dividend payments. Gould and Galai[1974] tested the right-hand side of (4) for the special case where $S_t = E$. Their results for the period 1967-1969 supported the efficiency hypothesis, although exchange members could obtain abnormal profits. Finally, Klemkosky and Resnick [1979] tested a sample of 606 options for the period 1977-1978. In their test, known dividends were

assumed and early exercise was not rational. Their results also supported the Put-Call Parity theorem.

Gould and Galai [1974] concluded their paper by stating: "assuming that the COBE and similar options markets that may come into existence in the next few years provide more information at lower costs, it will be interesting to see if greater efficiencies are achieved in option trading". Now that the TCO exists, traded option prices are available for both Canadian calls and puts. Thus, a re-examination of the Put-Call Parity theorems using different (i.e., Canadian) data is possible.

Two separate estimations will be conducted herein to test the Put-Call Parity Theorem. First, in order to be able to compare our results with those obtained by Gould and Galai [1974], inequality $C_t \leq P_t + S_t - Er^{-r(t^*-t)}$ is estimated for the case, $S_t = E$. The inequality can be rewritten:

$$C_t \leq P_t + S_t - [1 - e^{-r(t^*-t)}] \qquad (6)$$

Second, the inequality (5) is estimated for the case $S_t \neq E$. Specifically, the following is estimated:

$$\alpha \leq C \leq \beta, \qquad \text{where}$$

$$\alpha_t = P_t + S_t - E, \quad \text{and} \quad \beta_t = P_t + S_t - Ee^{-r(t^*-t)}$$

TABLE 5.4

NUMBER OF PAIRS OF PUTS AND CALLS WITH EQUAL EXERCISE PRICES AND TERMINAL DATES FOR EACH OF THE TEN STOCKS

EXERCISE DATE	GULF	STELCO	B.M.O.	C.P.	NOR	DOME	INCO	BELL	ROYAL	WALKER	TOTAL
18 May 1979							2				2
17 Aug 1979			2				3	3			8
16 Nov 1979	8		2	4	2		4	3			25
15 Feb 1980	10		3	4	3		5	4			29
18 Apr 1980						8					8
16 May 1980	17			6	8		8	3			42
18 Jul 1980						10					10
15 Aug 1980	12		5	5	8		7	3			40
17 Oct 1980						9					9
21 Nov 1980	10			5	6		7	3			31
16 Jan 1981						8					8
20 Feb 1981	7		5	5			6	3			36
20 Mar 1981						8					8
15 May 1981	8	3		4	4		6	2			27
19 Jun 1981						8				5	13
17 Jul 1981						9			3		12
21 Aug 1981	5	3	6	3			5	3			25
4 Sep 1981							6				6
18 Sep 1981						10				5	15
16 Oct 1981						9		3	4		16
20 Nov 1981	8	3	5	4	6						26
18 Dec 1981						10					10

GRAND TOTAL 413

Note that P_t = put price at time t

S_t = stock price at time t

C_t = call price at time t.

r = short term Treasury Bill rate announced for the period [t,t*], which is assumed herein to be constant on a weekly basis;

t* = expiration date of both the call and put options; and

E = the exercise price of both the call and put options at t* .

Data for the First Estimation

For the 3-year period of January 1, 1979 through December 31, 1981, all pairs of calls and puts with equal exercise prices and the same expiration date were randomly selected for the following ten stocks: Bank of Montreal, Bell, Canadian Pacific, Dome Petroleum, Gulf Canada, INCO, Noranda, Royal Bank, Stelco and Hiram Walker Resources. As indicated in Table 5.4, this resulted in 413 pairs of put and call options.

The Put-Call Parity inequality in (6) was derived under the condition that the American call was identical to an European call (i.e., early exercise is not rational). Thus, a correction must be made for dividends, since Merton [1973] has shown that call options can be optimally exercised just prior to the ex-dividend date if the dividend is sufficiently large. Thus, hedges were formed for each pair of options between the ex-dividend dates and the terminal date. The cash

TABLE 5.5

Ex-Dividend Dates and Dividends for Each of the Ten Stocks

STOCK	(1979) 1	2	3	4	(1980) 1	2	3	4	(1981) 1	2	3	4
BM	25 Jan (.31)	25 July (.34)	25 Oct (.36)	—	25 Jan (.37)	24 Apr (.38)	25 July (.39)	29 Oct (.40)	26 Jan (.42)	1 May (.44)	31 July (.46)	30 Oct (.48)
BELL	9 Mar (1.14)	11 June (.38)	10 Sept (.38)	10 Dec (.41)	10 Mar (.41)	9 June (.41)	9 Sept (.41)	9 Dec (.45)	9 Mar (.45)	9 June (.45)	9 Sept (.45)	9 Dec (.49)
C.P.	20 June (.80)	19 Dec (.90)	—	—	17 June (.90)	19 Dec (.95)	—	—	17 June (.95)	—	—	—
DOME*												
GULF	22 Feb (.35)	18 May (.35)	24 Aug (.40)	26 Nov (.40)	25 Feb (.15)	12 May (.11)	18 Aug (.11)	24 Nov (.11)	23 Feb (.11)	25 May (.11)	24 Aug (.11)	23 Nov (.11)
INCO	9 Feb (.10)	26 Apr (.10)	31 July (.10)	30 Oct (.20)	8 Feb (.15)	25 Apr (.18)	29 July (.18)	28 Oct (.18)	9 Feb (.18)	27 Apr (.18)	28 July (.18)	29 Oct (.05)
NOR	22 Feb (.50)	28 May (.60)	9 Aug (.70)	19 Nov (.25)	20 Feb (.30)	12 May (.30)	11 Aug (.30)	17 Nov (.35)	23 Feb (.35)	11 May (.35)	10 Aug (.35)	16 Nov (.35)
ROYAL	18 Jan (.50)	18 Apr (.55)	18 July (.55)	18 Oct (.58)	18 Jan (.60)	18 Apr (.60)	18 July (.64)	20 Oct (.58)	19 Jan (.72)	20 Apr (.40)	20 July (.45)	19 Oct (.49)
STELCO	27 Mar (.45)	26 June (.55)	27 Sept (.45)	27 Dec (.65)	27 Mar (.45)	25 June (.45)	29 Sept (.45)	24 Dec (.65)	26 Mar (.45)	25 June (.45)	28 Sept (.45)	24 Dec (.65)
WALKER	2 Mar (.33)	1 June (.33)	30 Aug (.33)	29 Nov (.33)	29 Feb (.33)	30 May (.33)	28 Aug (.33)	28 Nov (.33)	27 Feb (.33)	14 May (.33)	28 Aug (.33)	27 Nov (.33)

*No cash dividend paid.

TABLE 5.6

D_t Values for First Estimation (S_t = E)

.0082	.0114	.0006	-.0279	.0310
-.0084	.0119	.0121	-.0134	-.0246
.0185	.0219	-.0002	.0008	.0022
.0363	.0005	-.0298	.0040	-.0008
.0238	.0052	-.0153	.0059	-.0159
.0142	-.0015	-.0136	-.0016	-.0226
-.0068	-.0003	.0078	.0179	-.0306
-.0134	-.0865	.0126	.0168	-.0267
.0333	-.0113	.0169	.0130	-.0288
.0249	-.0154	-.0062	-.0079	-.0114
.0163	-.0244	-.0199	-.0184	.0170
-.0047	.0224	.0148	-.0042	-.0455
-.0461	.0180	.0183	-.0463	-.0105
-.0120	.0061	-.0651	-.0042	-.0324
-.0069	.0056	.0159	-.0021	-.0138
-.0039	-.0082	-.0076	-.0093	-.0635
.0099	-.0006	.0265	-.0097	-.0295
.0070	-.0150	-.0140	-.0010	-.0831
-.0133	.0007	-.0105	.0217	-.0405
.0068	.0009	.0149	.0290	-.0035

$\mu(D_t)$ = -.0051 min (D_t) = -.0865

$\sigma(D_t)$ = .0229 max (D_t) = .0363

dividends and the ex-dividend dates for each of the ten stocks over the three year period are given in Table 5.5 . To illustrate the sampling procedure, consider the case of INCO. Table 5.4 shows that there were two pairs of options for INCO that ended on 19 May 1979. However , Table 6.9 indicates that 26 April 1979 was an ex-dividend date. Hence, only the data points from 27 April to 18 May 1977 were used. Thus, since over this period, the call was identical to an European call, inequality (6) was the appropriate relationship to estimate.

However, several instances were observed in which there were no simultaneous trading; that is, the call was traded but the put was not, and vice-versa. Thus, these sample points were eliminated. Thus, for the 413 pairs of options, 143 hedges for which $S_t = E$ were formed.

Empirical Findings for the First Estimation

For each of the data points, the following was estimated: $D_t = C_t - P_t - S_t[1-e^{-r(t^*-t)}]$. If the estimated $D_t > 0$, then this would imply that the put-call relationship was violated and arbitrage opportunities would exist. The results are shown in Table 5.6 for a randomly selected sample of 100 values of D_t .

Since $D_t > 0$ for 42% of the data points, this indicates that the Parity Theory is violated fairly often.
However, the average value of D_t, for which $D_t > 0$, is only .0137.

Thus, the average arbitrage profit for these particular data points is $1.37, a negligable amount in the presence of trading costs. Thus, unlike the empirical results obtained by Gould and Galai [1974], the results presented herein reveal that the efficiency of the market is supported, since the very small arbitrage profits which are identified herein would be neutralized by very small trading costs.

Data for the Second Estimation

Since the condition $S_t = E$ no longer applies, hedges can be formed involving the underlying security and the associated put and call options. For this reason, the time period considered is January 1 through June 30, 1981. All pairs of calls and puts with equal exercise prices and the same expiration date were selected for the following 15 stocks: ALCAN, AQUITAINE, B.P. CAN, Bank of Montreal, Canadian Pacific, Dome Petroleum, C.P. Enterprises, Gulf Canada, Imperial Oil, INCO, Noranda, Shell Canada, Total Petroleum, Hiram Walker and West Coast Transmission. Since a potentially large number of hedges could be formed for each stock, a random selection of n hedges were formed for each stock where $30 \leq n \leq 50$. The actual number of hedges formed by this procedure is given in Table 6.11.

Empirical Findings

The empirical results for the second estimation for each of the 15 stocks are given in Table 6.7. The number of violations of the Parity Theorem ranges from zero for Alcan to 23 out of 30 for Total Pete. Furthermore, for the cases in which $C_t - B_t > 0$, the average value

TABLE 5.7

Results of the Second Estimation of the Put-Call Parity Theorem

STOCK	# of Hedges	# $(C_t > \beta_t)$	Mean $(C_t > \beta_t)$
ALCAN	38	0	0
AQUITAINE	38	10	.1568
B.P.CAN.	37	14	.1227
BANK MTL.	40	17	.2386
C.P.	37	2	.1135
DOME	42	18	.1844
ENT. C.P.	35	6	.6947
GULF CAN.	41	11	.1922
IMPERIAL OIL	41	10	.2111
INCO	35	12	.3000
NORANDA	48	17	.3363
SHELL	35	16	.1123
TOTAL PETE	30	23	.2672
WALKER	30	18	.0963
WEST COAST	38	16	.1028

(1) μ = \$39.63

(2) The sum of # $(C_t > \beta_t)$ = 140

ranges from .1227 for B.P. Canada to .6947 for Canadian Pacific. For the overall sample, 140 deviations, of the type $C_t - \beta_t > 0$ (or 36.8% of the total sample) were observed. The overall average deviation was $39.63! Thus, not only many profitable opportunities exist but the actual average profit (before trading costs) can be quite large. To determine if these profits still exist when trading costs are fully accounted for, it is necessary to estimate the size of trading costs for the Canadian market.

However, before proceeding to such an estimation, the case of Dome Petroleum will be studied further. The primary reason for doing so is that Dome Petroleum paid no cash dividends over the entire three year period, 1979-1981. As a result, the options written on Dome stock are equivalent to European options and so pre-exercise would be irrational. Data for 110 weekly hedges for Dome over the period are recorded in Table 5.8. The mean arbitrage profit is $83.90 per contract for those cases in which the deviation is positive. To determine why such a large average upward deviation exists, the Put-Call Parity relationship was estimated by OLS. Specifically,

$$C = P + S - E \exp(-r(t^*-t)).$$

Let $K = E \exp(-r(t^*-t))$.

Then estimate: $\frac{C}{K} = \alpha_0 + \alpha_1(P/K) + \alpha_2(S/K-1)$.

For the Put-Call Parity Theorem to hold, α_0 should equal 0.

TABLE 5.8

Results of the Put-Call Parity Estimation for Dome Petroleum

(a)

Year	# of Hedges	E	t*	$\#(C_t < \alpha_t)$	$\#(d_t \leq C_t \leq \beta_t)$	$\#(C_t > \beta_t)$
1979	13	45	19/10/79	4	0	9
1979	13	50	19/10/79	2	3	8
1980	19	50	18/07/80	0	11	8
1980	25	55	18/07/80	0	13	12
1981	21	20	16/10/81	3	8	10
1981	19	25	16/10/81	2	4	12
Σ	110			11	39	60

(1) = $(C_t > \beta_t) \equiv$ number of hedges for which $C_t > \beta_t$ ie upward violation

(2) Mean = .8390
$(C_t > \beta_t)$

(b)

$$C/K = .2017 + .9896\,(P/K) + 1.003\,(S/K-1)$$
$$(2.678)\quad(16.782)\qquad(43.959)$$

$R^2 = .9844$

D.W. = 2.079

The results, given in Table 5.8, show that:

$$C/K = .2017 + .9896\ P/K + 1.003\ (S/K-1)$$
$$\quad (2.678) \quad (16.728) \quad (43.959)$$
$$R^2 = .9844$$
$$D.W. = 2.079$$

Since $\hat{\alpha} = \underset{(2.678)}{.2017}$, the hypothesis that

$\alpha_0 = 0$ is clearly rejected. Thus, on average, calls on Dome Petroleum are over-valued by \$20.17 per contract (for 100 shares). Also, since $K = E \exp(-r(t^*-t)$, then $S/K-1 > 0$ means that $S > K$, and that the call is in-the-money. The coefficient, $\hat{\alpha}_2 = \underset{(43.959)}{1.003}$, indicates that the calls move approximately "dollar for dollar" with the stock when the calls are in the money. This could account for the over-valuation of the calls and for the violations of the Put-Call Parity Theorem.

5.3 Estimation of the Trading Costs for the TCO

Several empirical studies, including those by Galai [1978], Chiras and Manaster [1978] and Trippi [1977], have found abnormally high returns from hedges and concluded that the options market is inefficient. However, Phillips and Smith, by estimating the trading costs* involved in dealing in stocks and options on the U.S. markets, have shown that such superior returns do not exist when trading costs are considered. Hence, to conclude that the market is inefficient may be unjustified. As Jensen [1978] noted, market efficiency implies that economic profits from trading, net of costs, are zero.

Before proceeding to a discussion of the sampling procedure and estimation technique used herein, a relatively major flaw in the Phillips and Smith study need to be discussed. Although they considered four different weeks from June 1977 to February 1978, they sampled consecutive daily prices for each week. This would almost surely lead to correlation in the prices used and thus lead to biased results. Furthermore, since the U.S. market is more liquid than the Canadian market, estimates of U.S. trading costs would most likely be significantly less than those for the TCO.

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* Total transaction costs include taxes, trading costs (or the bid-ask spread) and brokers' commissions and relatively large arbitrage profits be earned on average. But, given the remark by Jensen, and the study by Phillips and Smith the effect of trading costs on the hedge profit potential must be evaluated.

The nature of trading costs can be described as follows. At any time t, two price quotations exist for any option and for its underlying security. The ask price, P_t^A, is the minimum price per share or option contract at which a market maker is willing to sell one round lot (100 shares) of the security or one option contract. The bid price, P_t^B, is the maximum price, at which a market maker is willing to buy the security. The difference, $P_t^A - P_t^B$, is the so-called bid-ask spread, and it represents a cost to the trader for immediate execution.

Data

For the period from January 1st through June 30th 1981, a random sample of 15 stocks listed on the MSE was selected (See Table 5.7). For each of the stocks, options which were relatively at the money were then randomly selected. Specifically, an option was selected if, at time t, $|S_t - Er^{-r(t^*-t)}| \leq .10$. The other words, the option was selected if the present discounted value of its exercise price differed from its underlying stock price by at most \$.10*. For each selected option, the average intra-day ask and bid prices were recorded. Specifically, if $P_{t,i}^A$, $i = 1,\ldots n$ are the n ask prices for a security or option, then the average ask price, ASK_t, is computed as

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* We also restrict $C_t \geq .50$, $P_t \geq .50$. This is because P_t, the bid-ask spread, is quite small, for low values of C_t, and so would distort the average.

$ASK_t = \sum_{i=1}^{n} P^A_{t,i} / n$. Similarly, for day t,

$$BID_t = \sum_{i=1}^{n} P^B_{t,i}/n .$$

This random selection of acceptable data points should solve the correlation problem discussed above. The final sample consisted of 515 pairs of ASK_t/BID_t prices for call options, 515 pairs for put options and 180 pairs for the underlying securities. For each bid-ask combination, and for each call, put and stock, the following were then computed:

(1) Cost ($) = $ASK_t - BID_t$

(2) Cost (%) = $(ASK_t - BID_t)/(ASK_t + BID_t)/2$

Empirical Findings

The results of the 2420 estimations are summarized in Table 5.9. The Phillips-Smith results for the U.S. markets are also shown for comparison purposes. As expected, the trading costs for the Canadian market are about twice as large as those for the U.S. market. For example, the average trading cost was $35.14 for a call contract on the TCO, whereas it was $20.46 for a call contract on the CBOE.

TABLE 5.9

Estimates of the Trading Costs for Stocks on the MSE and Options on the TCO, and similar estimates (*) for the NYSE and CBOE **

	Mean Cost ($)	σ($)	Mean Cost (%)	σ (%)
STOCK (MSE)	26.52	18.71	.86	.63
CALLS (TCO)	35.14	17.61	10.48	6.19
PUTS (TCO)	38.74	12.28	18.77	9.05
STOCK (NYSE)	20.46	—	.62	—
CALLS (CBOE)	18.23	—	4.51	—
PUTS (CBOE)	19.10	—	5.77	—

(*) These are obtained by Phillips and Smith [1980].

(**) For the period, trading costs for calls and puts are per contract, and for stocks are per round lot of 100 shares.

In order to exploit the violations from the Put-Call Parity Theorem, a portfolio must be created in which a round lot of 100 shares of the security is brought, one call contract is sold, and one put option contract is bought. Thus, in order to effect a Put-Call hedge, one must, on average, incur \$34.14 + \$38.74 + \$26.52 = \$100.40 of trading costs. This would completely eliminate all of the apparent profit opportunities shown earlier in Table 5.9. Therefore, the general conclusion is that, although very high average (pre-trading cost) profits seem to exist from violations of the Put-Call hedge on the TCO, these abnormal arbitrage profits generally do not cover the very high trading costs that exist on the TCO.

CONCLUSION

The major conclusions of this chapter are:

(a) Based on a sample of 18 options, randomly selected from a sample of 7 securities listed on the TCO, the daily closing option prices are generated by an unrestricted random walk process. This result is supported by a serial correlation coefficient test as well as a Box-Jenkins procedure for identifying and estimating the underlying model. This result is in contrast to the one by Leabo and Rogalski [1975] who found the closing monthly prices on the NYSE and ASE follow a random walk with reflecting barriers.

(b) Estimation of the Put-Call Parity Theorem shows that upward violations exist for calls, puts and stocks on the TCO. This would seem to indicate that large arbitrage profit opportunities exist on the TCO, and that the validity of the economic form of the random walk hypothesis is not supported. However, estimates of the trading costs involved in obtaining these profits on the TCO are relatively large. They are almost twice those computed by Phillips and Smith for U.S. When these estimated costs are considered, the apparent arbitrage profits are eliminated.

CHAPTER 6

THE OPTIONS MARKET AS AN INFORMATION GENERATION MECHANISM

Kreps [1979] and Harrison-Pliska [1981] have shown that the Black-Scholes model implicitly assumes that the market is complete in the Arrow-Debreau sense. Chang [1982] argued that it is the assumption of completeness in the B-S model which results in the Ross risk neutral valuation relationship (i.e., that the price of the option can be obtained without requiring the utility function of investors). In turn, this implies that the expected rate of return of the stock is irrelevant. However, in a complete market without transactions costs, the option on the underlying security would be a redundant asset and so there would be no apparent reason why there would be a market for options. To resolve this paradox, Chang developed an option pricing model under the assumption of an incomplete market. Options can then be viewed as derived assets which serve the economic role of completing the market.

Another role of the options market is postulated herein. This role is based on the Stiglitz-Grossman argument that, in a noise-free market, private information has no value. The logic is as follows: When information enters the market, the informed traders adjust their expectations about the future price. This results in the bidding up (or down) of the current price to the expected future price. Hence, the

trader obtains the benefit of the new information by observing the movements of the current price. This is only so if all costly information is perfectly transmitted and freely revealed from the informed to the uninformed, thereby offering no return for investment in information. This is the so-called Grossman-Stiglitz paradox. They state, however, that if the price is influenced by noise due to random supply etc, then the informed traders would have an advantage in acquiring new information. Clearly, this is also the case if there are different "grades" of information that enter the market.

If the current spot price reveals all information to the uninformed an option (or futures) market could provide no informational role. But, if "noise" is present in the system of information transmission then there would be an incentive to invest in costly information and informed traders could earn an positive net return on their investment. But then the current price is not a sufficient statistic for forming future price expectation. Uninformed investors will form expectations about future spot price conditional on the current spot price as well as the option price, whereas informed traders base their expectations upon their acquired information. If the options price serve to eliminate the "noise" in the market, then we are back to the previous situation in which there no incentive to invest in information. Hence, as long as "noise" persists, there will be a difference in expectations about the future spot price and therefore a positive return to investment in information. The options market will then provide an information-generation role. In fact, if "noisy" prices do

hide information, then the formation of options market should eliminate some or all such "noise", thereby reducing volatilities.

Based upon the assumption that investors only trade because of information reasons the following testable hypothesis is postulated (It is noted that trading may take place to meet consumption needs or to facilitate portfolio shifts). There is causality running from option volume to the stock price. The rationale for this hypothesis is as follows. When new information enters the market, the informed investors trade in the more noisy options market, because the new information is not fully revealed in this market. Thus, changes in volume in the option precede a change in the stock price. Hence, an unforecastable change in the option volume leads an unforecastable change in the underlying stock price. The role of the options market is then an information generation mechanism in which informed traders can earn a better return for their investment in costly information than in the stock market.

It has been empirically documented* that most traders in options do not exercise their contracts but trade their contracts on the

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* Reports by the Montreal Options Market indicate that about 7-8% of the total volume in option trades are actually exercised or expire without value.

secondary market. In other words, it appears that option traders have a very short horizon. Thus, investors would reorganize their portfolios of options when a change in the stock price variance is expected or when the expected long-run steady state price level is reached. Hence, one would expect a causality running from the underlying stock price to option volume. That is, an unforecastable change in the stock price precedes an unforecastable change in the option volume. Coupled with the previous hypothesis, the following testable is postulated: There is a feedback between option volume and the underlying stock price. This hypothesis is tested using the Pierce Haugh [1977] technique in section 6.2.

In the remainder of the chapter, the impact of listing on the options market on the underlying firm's stock price is studied. Considerable research has been undertaken relating to the response of stock price returns to various factors such as dividend announcements (e.g., Petit [1972]), stock splits (e.g., Copeland [1979]), discount rate changes (e.g., Waud [1970]), and Federal Reserve changes (e.g., Grube, Joy, Parton [1979])). These factors may all be classified as stimuli. The important issue in all of these studies is whether the market is informationally efficient relative to the stimulus studied. Another relevant and important issue is whether the stimulus has informational value. As Grube, Jay and Parton [1979] put it: "is the stimulus an economic signal?"

The impact of option trading on the price, volatility and volume of the underlying security has received limited attention in the literature. In fact, there has been some debate as to whether there is any significant impact at all (e.g., Black [1976]). It is quite clear that since stocks must meet relatively stringent criteria prior to listing on the Options Market, (*) that increased public interest may result in greater demand for the shares. Furthermore, option listing makes certain hedging strategies possible, that may make ownership of the underlying stock desirable. As a result, Black [1976] does concede that "there will be some effects on stock prices and trading volume. Option trading may even have some slight effects on stock volatilities". Previous studies in this area include Nathan [1974] who found a decrease in volatility after listing; and Hayes and Tennenbaum [1979] who found that listed options increase the volume of trading in the underlying shares. It is quite clear that these conclusions have important implications for the underlying market. Increased volume after listing would be of definite interest to brokers and market makers because of their interest in the maximization of turn-over. It would also be of interest to management, since the increased liquidity of its shares would facilitate the raising of new equity capital.

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* The main criteria for listing on the TCO are:

(1) a minimum of 5 million shares outstanding
(2) a minimum of 5000 shareholders holding bound lots
(3) a trading volume of at least 500,000 shares annually in each of the previous two years; and
(4) a market price of at least $5 per share.

Also, a decrease in the volatility of a stock's price, and the resulting stability should be of interest to investors. The reason is as follows: It is easier to predict a system with less noise than one with more noise. If, as a result of listing, there is less noise in the stock market price series, then a shift in the trend of this series would be more discernible as responding to changes in fundamental factors. Therefore information would be transmitted more quickly and efficiently with less noise. However, this can be criticized. The CAPM prices assets according to the equation:

$$E(R_i) = r + \beta_i[E(R_m) - r]$$

where r = risk free rate

β_i = risk index of the ith security

= $Cov\ (R_i, R_m)/Var\ (R_m)$

R_m = rate of return on the market portfolio.

Hence, the ith security price variance plays no role in the asset equation. Hence, it is argued that, under the assumptions of a perfect market, an effective change in the variance of the security after listing on the options market is unimportant to the investor. Indeed in the CAPM, β_i plays the crucial role. However, this may be answered as follows. Assuming an imperfect market in which the kth investor holds shares of n_k companies where $n_k < n$ and n is the number of firms in the market, Levy [1978] has shown that the variance has a strong impact on the risk-return relationship.

For securities which were widely held, he asserted that β_i would provide a better explanation for price behaviour, while for most securities which are not widely held, σ_i^2 would provide a better explanation for price behaviour. The general conclusion of Levy's work is that the variance, σ_i^2, is important to investors.

Accordingly, whether or not there was a shift in the variance of stock price returns around the listing date of each security is investigated in Section 6.3. The technique used is the one used by Hsu [1977] to determine if there was a variance shift in the U.S. stock market return series during the Watergate crisis.

CAUSALITY

Let $\{y_t\}$ and $\{x_t\}$ be two realized time series of two economic variables. Granger [1969] defined two particular forms of causality as follows:

(1) x <u>simply</u> causes y if

$$\sigma^2(y_t|y_{t-1}, \ldots, x_{t-1} \ldots) < \sigma^2(y_t|y_{t-1} \ldots).$$

That is, the variance of the errors in predicting y_t, when past x values are included in the information set, is less than the variance if the x values were omitted.

(2) x <u>instantaneously</u> causes y if

$$\sigma^2(y_t|y_{t-1}, \ldots, x_t, x_{t-1} \ldots) < \sigma^2(y_t|y_{t-1} \ldots *).$$

Note that, as proved by Pierce and Haugh [1977], it is impossible to determine a unique direction of causality if x instantaneously causes y .

The Granger concept of causality is essentially based on a notation of predictability or temporal ordering. Zellner[1] [1979] criticized the Granger's procedure as merely a statistical one - a "measurement without theory" which involves "a special form of predictability but no mention of economic law." Schwert [1979] also argued that it is unclear that causality and temporal ordering should be synonymous in economic systems. He argued that "economic agents make decisions based on expectations of what state of the world will occur in the future, and the process of forming expectations about the future can change the interpretation of Granger causality." This problem is likely to occur in financial markets where arbitrage profits may exist. So, as Schwert concluded "the Granger concept of causality based on temporal ordering will not lead to sensible conclusions about directions of causation in many instances."

(1) The philosopher Kant believed that causality and predictability are one and the same.

Accordingly, the two-step Pierce-Haugh test for causality is used herein. The first step is to transform each variable so that the resulting time series is stationary*. Suitable ARMA models are fitted and estimated for each of the transformed variables x_t, y_t and the residuals or "innovations" $\{\varepsilon_{xt}\}$ and $\{\varepsilon_{yt}\}$ are retrieved. The second step requires the computation of cross-correlation coefficients between ε_{xt} and ε_{yt} at each lag $k=-\ell \ldots \ell$; that is,

$$r_{xy}(k) = \frac{C_k[\varepsilon_{xt}\ \varepsilon_{yt}]}{\sqrt{C_o(\varepsilon_{xt})}\ \sqrt{C_o(\varepsilon_{yt})}}$$

Each $r_{xy}(k)$ is computed with its asymptotic standard error** in order to test for statistical significance. Furthermore, a joint test of the null hypothesis $H_o : \rho_{xy}(k) = 0$ for all $k = -\ell, \ldots, \ell$, was developed by Haugh [1972]. This requires the computation of the test statistic, $S = \sum_{k=-\ell}^{\ell} [r_{xy}(k)]^2$. under the null hypothesis, this test statistic is asymptotically distributed as a χ^2 distribution with $2\ell + 1$ degrees of freedom.

* Zellner [1979] objected to this procedure on the grounds that the effects of filtering (whether by differencing or by a more general filter) can be very drastic and might be "throwing the baby out with the bath".

** The asymptotic standard error, under the null hypothesis that $\rho_{xy}(k) = 0$ is given by $\frac{1}{\sqrt{N}}$, where N is the length of the time series.

6.1 The Pierce-Haugh Methodology: A Test for Feedback Between Stock Price and Option Volume

Data

The sample is listed in Table 6.1. It consists of 8 securities and their associated call options. The main criterion for selecting the call options was that they traded a minimum of 90% of the time. The daily closing option volume series and the daily closing price series of the underlying securities were recorded. The second criterion for selection was to avoid call options for which there was a stock split in the underlying security during the life-time of the call. This was done because preliminary tests of the effects of a stock split on option volume indicated that there was an increased trend in the option volume after the split date. (See Appendix 6.1 for details of this test and the results obtained.)

Estimation Procedure

The Pierce-Haugh [1977] procedure is described as follows. The first step is to estimate a suitable ARIMA model by the Box-Jenkins methodology. This is done by first obtaining a tentative specification of the time series through the autocorrelogram and the partial correlogram. An iterative least squares procedure on ARIMA (p,d,q)* is done. The residuals are retrieved and the residual autocorrelogram

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* p = # of autoregressive coefficients;
q = # of moving average coefficients; and
d = degree of differencing.

is computed. The Q-statistic for ARIMA (p,d,q) is given by

$$Q = (N-d) \sum_{j=1}^{k} (\hat{\rho}_j^2)$$

where N = sample size, k = no. of lags, $\hat{\rho}_k$ = estimated serial correlation coefficient at lag k. If the fitted model is correct, $Q \sim \chi^2(K-p-q)$ asymptotically. An insignificant Q value indicates that the residuals are not serially correlated (i.e., a white noise). This first step of model identification, estimation and diagnostic checking is applied to the option volume series $\{V_t\}$, and to the underlying security rates of return, $\{R_t = \log(S_t/S_{t-1})\}$, where S_t is the closing stock price at time t. The second step is to compute the cross-correlogram of the two series, $\varepsilon(V_t)$ and $\varepsilon(R_t)$, where

$\varepsilon(V_t)$ = white noise of option volume residuals; and

$\varepsilon(R_t)$ = white noise of stock price residuals.

The significance of these coefficients is made through the theorem of Haugh [1976]. This is stated as: If $\varepsilon(V_t)$ and $\varepsilon(R_t)$ are white noises after estimation by a suitable ARIMA model, then under the null hypothesis of independence, the sample cross correlation and coefficients are asymptotically uncorrelated and normal with mean of zero and variance of $\frac{1}{N}$.

Empirical Findings

The above procedure was applied to the data set in Table 6.1. In order to demonstrate the details of the methodology, the case of BANK OF MONTREAL (BMO) is given in its entirety.

TABLE 6.1

SAMPLE OF OPTIONS AND UNDERLYING SECURITIES

SELECTED FOR THE CAUSALITY TEST

Firm	t_o	t_1	E
BMO	19/11/79	15/8/80	24.63
STE	31/7/80	21/11/80	40.00
DMP	23/7/79	18/4/80	50.00
C.P.	31/8/79	15/2/80	40.00
WTC	1/2/83	17/6/83	15.00
N	1/2/83	23/5/83	15.00
TPN	1/2/83	17/6/83	25.00
RGO	1/2/83	17/6/83	July 10.00
BVI	1/2/83	18/7/83	20.00
PDL	1/2/83	17/6/83	25.00
HY	1/2/83	17/6/83	July 10.00
B	1/2/83	20/5/83	25.00
AL	1/2/83	17/6/83	35.00
HRW	1/2/83	18/5/85	June 30.00

Comments

(1) t_o = initial date of option price series.

(2) t_1 = terminal date of option. However, if the option price series was ended before t_1, the terminal date is indicated. See, for example, RGO.

The call option on BMO has exercise price $24.63 and life-span is 19 Nov. 1979 to 15 August 1980 (N = 185).

(i) Identification of the ARIMA (p,d,q) for BMO

The autocorrelograms and partial correlograms for the original stock price series, $\{S_t\}$, the first differenced series, $\{S_t - S_{t-1}\}$, and the second differenced series, $\{S_t - 2S_{t-1} + S_{t-2}\}$, are computed. The appropriate degree of differencing is suggested by the variance of the serial correlation coefficients. The table below shows the results:

# of lags	degree of differencing	variance
30	0	4.4466
30	1	.09819
30	2	.173350

Hence, a first differenced model, $\{S_t - S_{t-1}\}$, is suggested. Furthermore, visual inspection of the autocorrelogram and partial correlogram of the differenced series, shows that none of the serial correlation coefficients are significant. Hence, as a tentative model for the stock price series for BMO, an ARIMA (0,1,0) is suggested.

The option volume series is now considered. The correlation coefficients and the partial correlation coefficients for lags up to 30 are computed for $\{V_t\}$ and $\{V_t - V_{t-1}\}$. The table below shows the results:

# of lags	degree of differencing	variance
30	0	1286.4
30	1	1741.0

So, an undifferenced model $(d = 0)$ is suggested. Furthermore, inspection of the partial correlogram and the autocorrelogram of $\{V_t\}$ suggest that a model of the form ARIMA (0,0,4) is appropriate, although ARIMA (1,0,3) and ARIMA (3,0,0) are possible.

(ii) Estimation of Tentative Models for BMO

The Box-Jenkins methodology is applied to the tentative models obtained above. For $\{S_t\}$ the estimated model is ARIMA (0,1,0) and the estimated residual correlogram, $\varepsilon(S_t)$, is computed for 30 lags (10 lags are shown below). This series is clearly a white noise. The Q-statistic is $31.80 < \chi^2_{.01}(175) = 204.5$. The first 10 coefficients are:

r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}
.0619	-.0301	-.0653	.0962	-.0297	.0698	.0422	.0185	-.0967	.1105

Also, estimation of the 3 tentative models for $\{V_t\}$ show that, in terms of R^2, the variance of the residual correlogram and Q-Statistic, the model ARIMA (3,0,0) is appropriate. The estimated model is

$$V_t = 13.669 + .2993\, V_{t-1} + .1761\, V_{t-3}$$
$$\quad (3.722) \quad (3.994) \qquad (2.349)$$

where $R^2 = .1323$. The estimated residuals $\varepsilon(V_t)$ is a white noise with Q statistic = $32.30 < \chi^2(175) = 205.5$. The first 10 serial correlation coefficients are:

r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}
-.0161	.0049	-.0347	.0501	-.0314	-.0488	.2260	.0823	-.0435	.0223

(iii) Estimation of Cross-Correlogram of $\varepsilon(S_t)$ and $\varepsilon(V_t)$.

The estimated residuals $\varepsilon(S_t)$ and $\varepsilon(V_t)$ are now cross-correlated to obtain the sample cross-correlation coefficients, $\hat{\rho}_k$, where

$$\hat{\rho}_k = \frac{C_k[\varepsilon(S_t), \varepsilon(V_t)]}{\sqrt{C_o(\varepsilon(S_t))}\,\sqrt{C_o(\varepsilon(V_t))}}$$

where $C_k\,[\varepsilon(S_t), \varepsilon(V_t)]$ is an estimate of the cross-covariance function.

In order to test for causality running from V_t to S_t, the cross-correlogram for 10 lags is computed for $\varepsilon(S_t - S_{t-1})$ on V_t. $[(S_t - S_{t-1})$ is used instead of (S_t), since it was seen above that

$(S_t - S_{t-1})$ is a white noise]. Similarly, to test for causality running from S_t to V_t, the cross-correlogram for 10 lags is computed for $\varepsilon(V_t)$ on $\varepsilon(S_t - S_{t-1})$. The results are shown in Table 6.2. This completes the detailed analysis for BMO. Similar analyses were done for the full sample and the results are shown in Tables 6.3 and 6.4.

TABLE 6.2

CROSS CORRELOGRAMS OF $\varepsilon(V_t)$ on $\varepsilon(S_t - S_{t-1})$ AND $\varepsilon(S_t - S_{t-1})$ on $\varepsilon(V_t)$ for 10 LAGS FOR BMO

	Lag										
Model	0	1	2	3	4	5	6	7	8	9	10
$\varepsilon(S_t - S_{t-1})$ on $\varepsilon(V_t)$	-.007	.133	.069	-.067	-.004	.061	-.056	-.121	.050	.094	-.081
$\varepsilon(V_t)$ on $\varepsilon(S_t - S_{t-1})$	-.007	-.045	.234	.104#	.032	.120	.002	-.080	.023	.062	.073

significant at 5% level ; $r_k^* > \frac{1.96}{\sqrt{N}} = .1445$

TABLE 6.3

Results of Box-Jenkins Methodology as Applied to Sample listed in Table 6.1

Firm	Estimated Model		Q - Statistic ($)	
	DS_t (*)	V_t	DS_t	V_t
BMO	$DS_t=W_t$	$V_t=13.7+.29V_{t-1}+.18V_{t-2}$ (3.7)(a) (4.0) (2.4)	31.8	32.3
STE	$DS_t=W_t$	$V_t=20.5+.29V_{t-5}$ (2.8) (2.5)	18.6	9.5
DMP	$DS_t=.16DS_{t-1}$ (2.41)	$V_t=15.9+.32V_{t-1}+.24V_{t-3}$ (3.9) (4.4) (3.3)	34.2	29.2
CP	$DS_t=.28DS_{t-1}-.20DS_{t-4}$ (2.8) (-1.9)	$V_t=16.9+.23V_{t-1}$ (2.8) (2.33)	9.06	19.6
WTC	$DS_t=W_t$	$V_t=8.2+.24V_{t-1}$ (2.1) (2.2)	16.7	1.93
N	$DS_t=W_t$	$V_t=42.7+.29V_{t-4}+.19V_{t-5}$ (13.8) (4.0) (2.7)	13.8	11.3
TPN	$DS_t=W_t$	$V_t=23.0+.19V_{t-3}$ (2.9) (1.7)	8.0	10.0
RGO	$DS_t=W_t$	$V_t=73.6+.26V_{t-1}+.31V_{t-2}$ (2.0) (2.3) (2.8)	21.3	15.5
BVI	$DS_t=.13-.26DS_{t-3}$ (2.1) (-2.5)	$V_t=.22V_{t-1}+.31V_{t-5}$ (2.1) (2.9)	15.8	10.5
PDL	$DS_t=W_t$	$V_t=47.9+W_t$ (5.4)	19.5	18.5
HY	$DS_t=-.25W_{t-2}$	$V_t=79.7+.39V_{t-1}$ (2.3) (3.9)	17.3	15.9
B	$DS_t=W_t$	$V_t=136.3+.46V_{t-1}-.29V_{t-2}+.26V_{t-3}$ (2.6) (3.7) (-2.2) (2.1)	12.4	2.0
AL	$DS_t=W_t$	$V_t=26.1+W_t$ (3.9)	20.7	12.0
HRW	$DS_t=W_t$	$V_t=17.4+.28V_{t-1}$ (8.5) (2.7)	11.8	2.00

(*): $DS_t=S_t=S_{t-1}$; $: all Q statistics are insignificant at 5% level

a : t statistics ; W_t; white noise error.

TABLE 6.4

Cross-Correlograms of the White Noise Series $\varepsilon(V_t)$ and $\varepsilon(DS_t)$ for 0 to 4 lags.

Firm	LAG 0	1	2	3	4
BMO	-.007	.133 -.045	.069 .234=	-.067 .104	-.004 ..... (a) .032 ..... (b)
CP	.232#	.089 .223=	-.071 -.032	-.023 -.097	.222= .118
STE	.297=	.013 -.073	.016 .200*	-.017 .068	.103 -.127
DMF	.169	-.021 .287=	.002 -.028	-.079 -.070	.045 -.022
WTC	.173=	.046 .079	.002 .150	-.087 -.052	-.153 .111
N	.278=	.023 -.098	-.014 -.056	-.027 -.027	.204* .006
TPN	.140	.136 .141	.036 .084	.072 -.096	.013 -.028
RGO	.495=	-.024 .2092=	.167 -.1303	-.072 .116	-.149 -.116
BVI	.067	.154 .152	.064 -.043	-.062 .066	.058 .020
PDL	.048	-.062 .1087	.032 -.064	-.112 .149	-.113 .136
HY	.242=	.227= .106	.033 .013	.003 .125	-.033 .129
B	.089	.170 .180	.016 -.090	-.127 .128	.132 .199*
AI	.162	-.072 .078	-.140 -.054	-.054 .123	.049 .145
HRW	.210*	.161 .053	-.042 .2131*	.006 .086	.090 -.083

Comments

a: these coefficients are for the model $\varepsilon(DS_t)$ on $\varepsilon(V_t)$

b: these coefficients are for the model $\varepsilon(V_t)$ on $\varepsilon(DS_t)$

=: significant at 95% level

*: significant at 90% level

DISCUSSION OF RESULTS

The results, given in Table 3.6, are for each of the 14 models estimated for $\{DS_t\}$ and $\{V_t\}$. The general result is that DS_t is ARMA (1,0), a result which confirms to conventional wisdom that properly anticipated prices vibrate randomly. Also, the stochastic process generating option volume is generally autoregressive of degree d, where $d \leq 4$.

The results in Table 6.4 are for the Pierce-Haugh causality test. Cross-Correlation coefficients are estimated for the white noise series $\varepsilon(DS_t)$ on $\varepsilon(V_t)$ and are given in the first row of the results for each security. The lags are from $k = 0$ through 4. Similarly, the sample cross correlation coefficients for $\varepsilon(V_t)$ on $\varepsilon(DS_t)$ are given is the second row of the results for each security. Again, the lags are from $k = 1$ through 4.

Hence, the first row of coefficients are used to test for causality from option volume to the underlying security price. Of the 14 models estimated, [i.e. $\varepsilon(DS_t)$ on $\varepsilon(V_t)$], 5 showed significant simultaneous causality at the 95% confidence level and 2 others at the 90% level. Only one model (Husky Oil) exhibited significant lagged causality. Hence, the null hypothesis of no lagged causality from option volume

to stock prices is accepted. Thus, there results do not support our argument for a resolution of the Grossman-Stiglitz paradox. However, a significant lagged causality from option volume to stock price would be difficult to obtain. This would require a united and concerted action on the part of all informed traders who, upon obtaining new information will enter the noisy options market and "play out" their expectations. Failing such a group action, the causality might be difficult to ascertain and our hypothesis harder to test.

The second row of coefficients, which are the sample cross-correlation coefficients for the model $\varepsilon(DS_t)$ on $\varepsilon(V_t)$, indicate different results. Four models have significant lagged causality for $\alpha = .05$, while two others are significant for $\alpha = .10$. While this is not overwhelming evidence of causality from stock prices to option volume, the results do suggest that investors in the option market have a very short horizon, and do tend to revise their portfolio when stock prices change in response to new information. Consequently, stock prices seem to lead option volume, thereby accounting for the empirical fact that only about 7% of all contracts are exercised at the exercise date.

6.2 A Test of a Shift in the Variance After Listing on the TCO

The economic nationale for this study has been discussed in Section 6.1. Therefore, the basis for this test and the data set used are now described.

Data

As of January 29, 1982, there were 52 calls listed on the TCO. Forty-Five of these were listed on the TSE and twenty-four on the MSE (seventeen were dual listings). A random sample of 30 securities on the TOC was selected, and their listing date were recorded (see Table 6.5). For each security, a sample of 100 daily closing stock prices around the listing date was obtained.

Estimation Procedure

The estimation technique applied herein is due to Hsu [1977]. Let $R_t = \ln(S_t/S_{t-1})$ where $\{S_t\}_{t=1}^{N}$ is a sequence of random stock prices and assume that $\{R_t\}_{t=2}^{N}$ is a sequence of independent random variables such that $E(R_t) = u$ and $Var(R_t) = \sigma_t^2$. The hypothesis to be tested can be formulated as follows:

$$H_o : \sigma_1^2 = \sigma_2^2 = \ldots\ldots = \sigma_N^2 = \sigma^2 .$$

$$H_1 : \sigma_1^2 = \sigma_2^2 = \ldots\ldots = \sigma_k^2 = \sigma^2 \quad \text{and}$$

$$\sigma_{k+1}^2 = \sigma_{k+1}^2 = \ldots\ldots = \sigma_N^2 = \sigma^2 + \varepsilon$$

where $|\varepsilon| > 0$.

Define $X_t = (R_t - \bar{R})^2$. Hsu, following an approach suggested by Chernoff and Zacks [1964], obtained a test statistic which is locally most powerful. This test statistic is given by

$$H = \{(\sum_{t=1}^{N} (t-1)X_t)/((N-1)\sum_{t=1}^{N} X_t)\}$$

Under the assumption that H_0 is true, and observing that H is a linear function of Dirichlet variables, he showed that

$$E(H) = \frac{1}{2} \quad \text{and} \quad Var(H) = \frac{N+1}{6(N-1)(N+2)}$$

Empirical Findings

The empirical findings obtained by applying the Hsu estimation technique to the sample of 30 companies listed in Table 6.5 are given in Table 6.6. Seventeen securities showed significant shifts in the variance for the 100-day period around the listing date. Of these 17, seven securities had significant negative H statistics, indicating a statistically significant decrease in the variance after the listing date. The other ten securities had a significant increase in the variance after the listing date. These results are interesting in the sense that the probability that a security, upon being listed on the TCO, will undergo a significant shift in its daily variance is about 50 per cent (i.e., the same as flipping a fair coin.) Furthermore, given that a change in the variance did take place, the probability that it is a decrease is also about 50 per cent (i.e., the same as flipping a fair coin.) The results are different from those obtained by Nathan [1974] who found an over-all significant decrease

in the variance for stocks listed on the CBOE. For the TCO, only 25 per cent of the sample (which itself represented 58 per cent of the population) showed a significant decrease in the daily variance of stock price changes.

It is sometimes argued that if markets are incomplete, the option market creates new speculative opportunities, and so a test of a shift in the variance after listing is equivalent to finding out if the increased speculation is stablizing. The results of this section show that a difinitive conclusion is not borne out. That is, the likelihood that a security will undergo a significant change in o^2 is fifty per cent.

Furthermore, in order to make sure that a significant change in o^2 is not due to a concurrent change in the variance of the TSE index, the Hsu procedure is now applied to the TSE for the same time period around the listing date of each stock which showed a significant decline in o^2. Recall, that on Jan. 5th, 1981, these six securities were listed on the TCO were ICG, BPO, HWR, WTC which showed a significant decline in o^2 at 5% and 10% level of significance.

TABLE 6.5

Sample of 30 securities listed on TCO

and their listing date

SECURITY (TICKER SYMBOL)	LISTING DATE	SECURITY (TICKER SYMBOL)	LISTING DATE
IPL	Nov. 8, 78	RGO	Jan. 4, 79
MB	Aug. 15, 78	TD	Nov. 15, 79
B	Dec. 4, 78	DM	Nov. 8, 77
CP	Sept. 25, 78	HYO	Jan. 1, 77
BMO	Nov. 8, 78	TPN	Jan. 8, 79
LBT (A)	Aug. 28, 78	GST	Jan. 29, 79
CM	Mar. 24, 80	BNS	Dec. 11, 79
DMT	Sept. 8, 80	TXC	Jan. 11, 82
PDL	Apr. 21, 81	WTC	Jan. 5, 81
AEC	Apr. 21, 81	SHC	Jan. 8, 79
CLT	Apr. 21, 81		
ASM	Apr. 4, 77		
NCN	Oct. 19, 77		
ICG	Jan. 5, 81		
BPO	Jan. 5, 81		
CRK	Jan. 5, 81		
HWR	Jan. 5, 81		
GOC	Aug. 31, 79		
BVI	Aug. 2, 79		
DEN	Feb. 19, 79		

TABLE 6.6

Test of Variance Shift After Listing Date

using Hsu's Technique

SECURITY	H	H*	SECURITY	H	H*
ICG	.3465	-3.301#	PDL	.5777	1.894
WTC	.3868	-3.050#	AEC	.5618	1.508
BNS	.5717	1.925#	CLT	.4216	-1.910
TXC	.4088	-2.153#	CRK	.4575	-1.035
DM	.6452	3.359#	BPO	.3552	-3.528#
HYO	.5492	1.198	HWR	.3509	-3.631#
ASM	.4179	-1.999#	GOC	.6028	2.507#
NCN	.6027	2.504#	BVI	.6552	3.782#
IPL	.4617	-.9340	DEN	.5344	.8372
MB	.4889	-.2478	RGO	.9610	11.236#
B	.5818	1.994#	TD	.6169	2.850
CP	.6882	4.586#	TPN	.4162	-2.042#
BMO	.4899	-.2440	SHC	.4308	-1.685
LBT	.6758	4.285#	GST	.4716	-.6913
CM	.5144	.3529	DMT	.5232	.5657

Notes: $H^* = \dfrac{H - E(H)}{(Var(H))^{1/2}}$

= significant at the 5% level.

when the Hsu procedure was applied to a sample of 100 daily closing TSE indices centered around Jan. 5th, 1981, $H = .3146$ and $H^* = 4.569$. Thus, there was a concurrent significant decrease in the variance the changes in the TSE index. In fact, the standard deviation for the 50 trading days prior to Jan. 5th was $\hat{\sigma}_1 = .0111$ and for the post period was $\hat{\sigma}_2 = .0067$. The null hypothesis of equal variances is rejected since $F^* = 2.749 > F_{.05}(49,49) = 1.690$. Similarly, for TXC, ASM, TPN and CLT, there was a similar concurrent downward shift in the variance of the changes in the TSE index.

CONCLUSION

The main conclusions of this chapter are:

(a) Based on the Grossman-Stiglitz argument that, investment in information would have a zero return, in a perfect market, it was hypothesized in this chapter that investors would enter the options market to act upon new information, since changes in volume and price in this noisy market would be less revealing. Consequently, the hypothesis that there is causality from option volume to stock price was treated. A Pierce-Haugh test failed to support the hypothesis since only contemporaneous causality was detected.

(b) There is considerable empirical evidence that option investors usually exercise their contracts prior to the exercise date. As a result, the hypothesis that there is causality from stock price to option volume was tested. The Pierce-Haugh test for causality supported the hypothesis in several cases in which lagged (but only one-period mostly) causality was found. Hence it might seem that investors quickly revise their options portfolio upon changes in the stock price, but the evidence is very weak.

(c) Finally, a non-parametric test was applied to a sample of 30 securities on the TCO to ascertain if there is a significant shift in the variance of stock price changes upon listing on the options market. These results indicated that 25% of the securities showed a decrease in σ^2 and 25% of the securities showed a decrease in σ^2 after listing. Furthermore, when a significant decrease in σ^2 occurred, there was a similar concurrent decrease in the variance of the changes in the TSE index. Hence, it is questionable whether the change in the variance in the stock price is due to being listed on the TCO. This contradicts the results by Nathan [1974] for the CBOE and supports Fischer Black's [1976] comments that is questionable whether option trading has an impact on the security's variables.

CONCLUSION

This thesis dealt with three major themes:

(a) The estimation of the parameters of the Black-Scholes model through an optimization problem involving the Newton-Raphson iterative procedure for simultaneous variables.

(b) The efficiency of the TCO

(c) The role of the Options Market as an information generating mechanism.

With respect to these three main problems, the following conclusions were found.

(a) When the Newton-Raphson was applied to a minimization problem in order to estimate σ^2 and r, the results, even when first order serial correlation was removed, were unsatisfactory, since the estimates of $\hat{r}$ differ dramatically. Accordingly, the validity of the assumption that stock price changes follow a geometric Brownian Motion with constant drift and intertemporally constant variance was studied and the implications of the results obtained for option pricing was investigated. The general conclusion is that for a sample of 20 stocks on the TSE for the period 1970-1979, the empirical distribution is approximately normal but monthly variances vary directly with the square of the market rate

of return. Also it was shown that for daily closing prices on the TSE over a sample period of one year, the empirical distribution was generally found to belong to the symmetric Paretian family with α having a modal value of 1.4. Finally, a MINQUE-type estimator for σ^2 was developed and using the Box-Jenkins methodology, suitable ARIMA models for daily variances were estimated. The overall conclusion is that daily variances are generally white noises with a constant mean. Based on these models, forecasts of Black-Scholes prices were made and they were better predictors of the observed option prices than the ones obtained by using the historical variance.

(b) With respect to the statistical test of the random walk hypothesis for the TCO, a sample of 18 options showed that the null hypothesis that successive price changes are independent cannot be rejected. Furthermore, although arbitrage opportunities with significantly large profits were identified through the Put-Call Parity relationship, these abnormal profits disappeared when trading costs for options on the TCO and those for the associated stock were taken into account. Both the statistical and the economic versions of the random walk hypothesis for options on the TCO were validated.

(c) In the last section, the role of the options market as an avenue for the optimal investment of private information and therefore for the resolution of the Grossman Stiglitz paradox is studied. The overall conclusion is that for a sample of 14 pairs of estimations, the hypothesis of no lagged causality from option volume to stock price is accepted. Finally, in an effort to determine if the listing of stocks on the TCO (an action which created new speculative opportunities) results in stabilizing speculation, the Hsu non-parametric test variance shift is applied to securities on the TCO. The general conclusion is that the probability of a significant shift, given listing, is the same as flipping a fair coin. This conclusion contradicts the results by Nathan [1974] for the COBE but supports Black's [1976] comments that it is questionable whether option trading has an impact on the security variable.

APPENDIX 6.1

A TEST FOR A SHIFT IN THE TREND OF THE OPTION-VOLUME SERIES IN THE NEIGHBORHOOD OF THE STOCK SPLIT

The familiar non-parametric Cox and Stuart Test for trend is used. This test can be described as follows. Let $\{X_t\}_{t=1}^{N}$ be a sequence of random variables. Group the random variables into pairs (where ties are removed) as follows: $\{X_1, X_{1+k}), (X_2 . X_{2+k}), \ldots \{X_{N-k}, X_N)\}$

where $k = \frac{N}{2}$ if N is even

$= \frac{N+1}{2}$ if N is odd

The assumptions are:

(1) $\{X_i\}$ are mutually independent; and

(2) The $\{X_i\}$ are either identically distributed or have a trend.

The hypothesis to be tested are:

H_o : There is no upward trend.

H_a : There is an upward trend.

The test statistic is T = no. of pairs (x,y) for which $y > x$. In fact, this is a sign test where, if $y > x \Rightarrow$ "+" and if $y < x \Rightarrow$ "-". Hence the critical T value for $\alpha = .05$ and $N \geq 20$ is $T = \frac{N}{2} + \sqrt{N}$

DATA The following data set is considered in this test.

STOCK	SPLIT FACTOR	SPLIT-DATE
Dome Pete	4:1	May 14, 1979
Gulf Canada	3:1	Apr.30, 1980
NORANDA	3:1	Aug.22, 1979
Royal Bank	2:1	Mar. 9, 1981

Results The empirical results are listed in the following table. Clearly, for the stocks considered there is an increased trend in the option volume after the split date.

STOCK	E	t^*	T^*
NORANDA	15.00	Feb. 15, 80	30 (26.4)#
Dome Pete	32.5	Oct. 19, 79	25 (19.8)
Gulf Canada	30.	Nov. 19, 79	37 (25.2)
Royal Bank	30.	Oct. 16, 81	17 (14.5)

This is the critical value of $T = n-t$ where $t \approx \frac{n}{2} - \sqrt{n}$ $n \geq 20$, $\alpha' = .05$.

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