

## CANADIAN THESES ON MICROFICHE

## THÈSES CANADIENNES SUR MICROFICHE



National Library of Canada  
Collections Development Branch

Canadian Theses on  
Microfiche Service

Ottawa, Canada  
K1A 0N4

Bibliothèque nationale du Canada  
Direction du développement des collections

Service des thèses canadiennes  
sur microfiche

### NOTICE

The quality of this microfiche is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us an inferior photocopy.

Previously copyrighted materials (journal articles, published tests, etc.) are not filmed.

Reproduction in full or in part of this film is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30. Please read the authorization forms which accompany this thesis.

**THIS DISSERTATION  
HAS BEEN MICROFILMED  
EXACTLY AS RECEIVED**

### AVIS

La qualité de cette microfiche dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de qualité inférieure.

Les documents qui font déjà l'objet d'un droit d'auteur (articles de revue, examens publiés, etc.) ne sont pas microfilmés.

La reproduction, même partielle, de ce microfilm est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30. Veuillez prendre connaissance des formules d'autorisation qui accompagnent cette thèse.

**LA THÈSE A ÉTÉ  
MICROFILMÉE TELLE QUE  
NOUS L'AVONS REÇUE**

**Canada**

A Study of the Active Compensation of  
Voltage Amplifiers and RC-Active Filters

Paulo Batista Lopes

A Thesis  
in  
The Department  
of  
Electrical Engineering

Presented in Partial Fulfillment of the Requirements  
for the Degree of Doctor of Philosophy at  
Concordia University  
Montréal, Québec, Canada

June 1985

© Paulo Batista Lopes, 1985

## ABSTRACT

A Study of the Active Compensation of  
Voltage Amplifiers and RC-Active Filters

Paulo Batista Lopes, Ph.D.

Concordia University, 1985

This dissertation presents the results of a systematic investigation on the generation and classification of actively compensated voltage amplifiers (ACVAs). Also, a new approach to the design of actively compensated RC-active filters is described.

A comprehensive study of ACVAs using 2 and 3 operational amplifiers (OAs) is reported in Chapters 2 and 3, respectively. In this study, ACVAs are first classified according to a combination of both practical and theoretical considerations. Each class is further divided into types. A set of realizability conditions is then associated with each type. For each type, ACVA realizations are obtained by sequential eliminations of resistors in a general model of the resistive network which imbeds the OAs in an ACVA circuit. The circuits obtained are then analyzed with respect to several performance characteristics such as relative stability, phase and magnitude errors, stability

considering the second pole of the OAs, maximum signal handling capability, etc.

In Chapter 4, a new technique for the active compensation of a single OA network is reported. The application of this technique eliminates the first-order effects of the time constants of the OAs in circuits using 2 OAs and also the second-order effects in circuits employing 3 OAs. Next, it is shown how this technique can be implemented in two important classes of active-RC filters, namely enhanced positive feedback and enhanced negative feedback networks.

Finally, in the same chapter, an optimization procedure for reducing the residual effect of the OAs in the circuits obtained through the technique above is proposed. This procedure admits easy application and is shown to be very efficient. Examples are provided in order to demonstrate the effectiveness of the results presented.

Simulations and experimental results are provided in order to demonstrate the accuracy of the theoretical analysis presented in this thesis.

V

### ACKNOWLEDGEMENTS

I would like to express my appreciation for all the guidance and assistance that has been provided to me by my supervisor, Dr. B.B. Bhattacharyya.

More than thanks are due to my wife, Maria das Graças. Her influence in my life has given me inspiration, strength and peace of mind.

My friend Jose Carlos Moreira Bermudez has helped me not only with his friendship but also with some very useful technical advice. I do appreciate having shared the office with him for almost three years.

My former teachers Luis P. Caloba and Jacques Szczupak have provided most of my technical background. Therefore, I would like to express my recognition to them.

Thanks to my family in Brazil.

Last but not least, I gratefully acknowledge the support given by CAPES (Brazilian Post-Graduate Agency) in the form of a doctoral fellowship.

To my father and the memory of my mother.

## TABLE OF CONTENTS

	Page
LIST OF TABLES .....	xi
LIST OF FIGURES .....	xiv
LIST OF IMPORTANT SYMBOLS AND ABBREVIATION .....	xvi
1. INTRODUCTION .....	1
1.1 Preliminary Considerations .....	1
1.2 The Operational Amplifier .....	2
1.3 Voltage Amplifiers and the Effects of the GB .....	6
1.4 The Effects of Actual OAs in Active-RC Networks .....	11
1.5 Special Design Techniques for OS-based Networks .....	17
1.6 Theory of Actively Compensated Voltage Amplifiers .....	21
1.7 Scope of the Thesis .....	26
2. CLASSIFICATION, GENERATION AND DESIGN OF ACTIVELY COMPENSATED VOLTAGE AMPLIFIERS USING 2OAs .....	28
2.1 Introduction .....	28
2.2 Preliminary Considerations .....	29
2.3 Classification and Generation of ACVAs .....	34
2.3.1 Infinite Input Impedance ACVAs .....	35

2.3.1.a	Class NI2A Realizations ..	38
2.3.1.b	Class NI2B Realizations ...	41
	Class II2A Realizations ..	45
2.3.2	Finite Input Impedance ACVAs .....	46
2.3.2.a	Noninverting ACVAs of the FII Group .....	52
2.3.2.b	Inverting ACVAs belonging to the FII Group .	60
2.4	Evaluation of the ACVAs .....	76
2.4.1	Design Equations and Relative Stability .....	76
2.4.2	Gain and Phase Errors ...	79
2.4.3	Effect of the Second Pole of the OAs .....	80
2.4.4	Maximum Signal Handling Capability .....	81
2.4.5	Tuning Procedure .....	83
2.5	Experimental Results .....	86
2.5.1	The ACVAs as Stand Alone Elements .....	86
2.5.2	An Application in Active-RC Filters .....	86
2.6	Conclusions	



3.	CLASSIFICATION, GENERATION AND EVALUATION OF ACTIVELY COMPENSATED VOLTAGE AMPLIFIERS USING 3OAs .....	92
3.1	Introduction .....	92
3.2	Preliminary Analysis .....	93
3.3	Performance Characteristics .....	97
3.4	Classification and Generation of 3 OA ACVAs .....	102
3.4.1	Noninverting 3 OA ACVAs .	102
3.4.2	Inverting 3 OA ACVAs ....	117
3.5	Evaluation of the ACVAs .....	138
3.5.1	Relative Stability .....	142
3.5.2	Phase and Magnitude Errors .....	145
3.5.3	Effect of the Second Pole of the OAs .....	149
3.5.4	Maximum Signal Handling Capability .....	151
3.5.5	Tuning Procedure .....	157
3.6	Experimental Results .....	160
3.6.1	The ACVAs as Stand Alone Amplifiers .....	160
3.6.2	An Active Filter Application .....	165
3.7	Conclusions .....	168

4.	DESIGN OF OPTIMUM ACTIVELY COMPENSATED NETWORKS .....	170
4.1	Introduction .....	170
4.2	Active Compensation of SABs .....	172
4.2.1	Actively Compensated Circuits Using 2 OAs ....	173
4.2.2	Actively Compensated Circuits Using 3 OAs ....	184
4.3	Optimization of Actively Compensated Circuits .....	195
4.3.1	Description of the Procedure .....	195
4.3.2	Steffen Allpass Circuit ..	198
4.3.3	The Friend Bandpass Circuit .....	210
4.4	Experimental Results .....	226
4.5	Conclusions .....	228
5.	CONCLUSIONS	
5.1	Summary of the Thesis .....	231
5.2	Suggestions for Future Research ..	235
	REFERENCES .....	237

## LIST OF TABLES

	Page
Table 2.1 : Characterization of the Classes of Infinite Input Impedance (III) ACVAs .....	39
Table 2.2 : Different Types Of ACVA Realizations of Class NI2B .....	44
Table 2.3 : Different Types of Realizations of Class II2A .....	47
Table 2.4 : ACVAs of the III Group .....	48
Table 2.5 : Properties of the ACVAs of the III Group .....	50
Table 2.6 : Types in Class NF2A .....	54
Table 2.7 : ACVAs of Class NF2A .....	55
Table 2.8 : Properties of the ACVAs of Class NF2A .....	57
Table 2.9 : Types of ACVAs of Class NF2B .....	59
Table 2.10 : ACVAs of Class NF2B .....	61
Table 2.11 : Properties of the ACVAs of Class NF2B .....	64
Table 2.12 : Types of ACVAs in Class IF2A .....	68
Table 2.13 : ACVAs of Class IF2A .....	69
Table 2.14 : Properties of the ACVAs of Class IF2A .....	70
Table 2.15 : Types of ACVAs of Class IF2B .....	72
Table 2.16 : ACVAs of Class IF2B .....	73
Table 2.17 : Properties of the ACVAs of Class IF2B .....	75
Table 2.18 : Experimental Results for the Sallen-Key Filter .....	90
Table 3.1 : Characterization of the Classes that realize Infinite Input Impedance Noninverting ACVAs .....	106

Table 3.2	: Characteristics of the Types that realize Stable and Tunable, Infinite Input Impedance, Noninverting ACVAs .....	108
Table 3.3	: Circuit Realizations for the Different Types of Noninverting ACVAs .....	114
Table 3.4	: Properties of the Tunable, Noninverting, 3 OA ACVAs .....	115
Table 3.5	: Classes of Infinite Input Impedance ACIVAs .....	120
Table 3.6	: Different Classes of Tunable ACIVAs .....	120
Table 3.7	: Characteristics of the Types that realize Stable and Tunable ACIVAs .	125
Table 3.8	: Reduced Graphs for all Types of Tunable ACIVAs .....	131
Table 3.9	: Description of the Resistive Network for Tunable ACIVAs .....	133
Table 3.10	: Properties of the ACIVAs selected as Prototypes .....	140
Table 3.11	: Design Equations that ensure a Phase Margin in the Range from $40^{\circ}$ to $50^{\circ}$ and a Gain Margin not Less than 9.5 dB .....	143
Table 3.12	: Phase and Magnitude Deviations for the ACVAs at Low Frequency ....	147
Table 3.13	: Experimental Performance for the Sallen-Key Filter using different Voltage Amplifiers .....	167
Table 4.1	: Numerator and Denominator Polynomials of the Transfer Functions of the Passive Network in the Actively Compensated Allpass Circuit	201

Table 4.2	:	Numerator and Denominator Polynomials of the Transfer Functions of the Passive Network in the Friend Bandpass Circuit .....	215
Table 4.3	:	Maximum Pole Frequency for which $ \Delta H(j2\pi f_0)  < 0.1$ .....	225
Table 4.4	:	Experimental Results for the Bandpass Circuit .....	229

## LIST OF FIGURES

	Page
Fig. 1.1 : Schematic Representation of an Operational Amplifier .....	4
Fig. 1.2 : Conventional Realizations of VAs .....	7
Fig. 1.3 : General Configuration of a 3 OA SAN ..	12
Fig. 2.1 : The General Configuration of a 2 OA ACVA .....	30
Fig. 2.2 : The General Configuration of the Resistive Network .....	36
Fig. 2.3 : The Graph for the Resistive Network for the Realization of Class NI2A after all Possible Resistor Eliminations are performed .....	42
Fig. 2.4 : Examples of Maximum Signal Handling Capability of Different VAs .....	84
Fig. 2.5 : Responses of the ACVA of type NI2B-4 (first circuit) and the conventional VA .....	87
Fig. 2.6 : The Sallen-Key Filter used for Practical Experiments .....	88
Fig. 3.1 : General Configuration of a 3 OA ACVA .	94
Fig. 3.2 : The General Configuration of the Resistive Network .....	104
Fig. 3.3 : Configuration of type NI3B-1 after the Preliminary Resistor Eliminations ....	113
Fig. 3.4 : Configuration of type NI3B-1 after all Possible Resistor Eliminations are performed .....	113
Fig. 3.5 : Minimum Value of $\theta$ for a given gain in order to assure stability .....	152

Fig. 3.6	: Maximum Signal Handling Capability .....	158
Fig. 3.7	: Theoretical and Experimental Responses for the ACIVA of type IF3AB-8 and the ACVA of type NI3C-1 .....	161
Fig. 3.8	: Results obtained for the Sallen-Key Filter .....	166
Fig. 4.1	: General Configuration of a 2 OA RC-active Network .....	174
Fig. 4.2	: General Configurations of Enhanced Positive and Negative Feedback Networks .....	180
Fig. 4.3	: General Configuration of a 3 OA Active-RC Network .....	186
Fig. 4.4	: General Configurations of Enhanced Positive and Negative Feedback Network using 3 OAs .....	190
Fig. 4.5	: Steffen Allpass Circuit .....	199
Fig. 4.6	: Plots of the Objective Function for the Actively Compensated Allpass Circuit ...	206
Fig. 4.7	: Simulated Responses for the Allpass Circuits .....	211
Fig. 4.8	: Friend Bandpass Circuit .....	213
Fig. 4.9	: Plots of the Objective Function for the Actively Compensated Bandpass Circuit ..	220
Fig. 4.10	: Modified Actively Compensated Bandpass Circuit .....	227

## LIST OF IMPORTANT SYMBOLS AND ABBREVIATIONS

	Page
OA : Operational Amplifier .....	2
$A_o$ : DC gain of the OA .....	5
$\omega_1$ : Cut-off frequency of the OA gain .....	5
GB : Gain-bandwidth product of the OA .....	5
$\tau$ : Time constant of the OA .....	5
$A(s)$ : Differential gain of the OA .....	5
$\omega_2$ : Second pole Frequency of the OA .....	5
$\theta$ : Ratio between $\omega_2$ and GB .....	5
VA : Voltage Amplifiers .....	6
$\omega$ : Angular frequency in rad/sec .....	9
SAN : Single amplifier network .....	11
$\omega_o$ : Pole-frequency in rad/sec .....	15
$f_o$ : Pole-frequency in Hz .....	224
$Q_o$ : Pole Q factor .....	15
ACVA : Actively compensated voltage amplifier .	21
ACIVA : Actively compensated inverting voltage amplifier .....	123
III : Infinite input impedance .....	35
FII : Finite input impedance .....	35
SAB : Single OA biquadratic circuit .....	171



## CHAPTER 1

### INTRODUCTION

#### 1.1 PRELIMINARY CONSIDERATIONS

The design of filtering networks has been subject of research by electrical engineers since the beginning of this century. The attempt to use transmission lines for communications over large distances was the main force behind the development of both Filter Theory and Circuit Theory [1]. In these early years, active devices such as vacuum tubes, transistors and operational amplifiers were not available yet. Therefore, circuits had to be built using resistors, capacitors, inductors and, in some cases, transformers. Eventually, these passive filters became so popular that they were widely used until the early seventies.

However, in spite of their popularity, the use of inductors in these filter circuits has always had some drawbacks. High quality inductors are bulky, expensive and the magnetic fields created by them can affect the operation of sensitive instruments that might be placed nearby.

The first experiments with active-RC filters were reported in the middle thirties. In these works, an RC network was connected in the feedback path of vacuum tube

amplifiers in order to shape their frequency characteristics [2]. However, it was only after the invention of the transistor that it was realized that the replacement of large, troublesome and expensive inductors by active devices could potentially accomplish cost and size reduction [3].

The advent of monolithic operational amplifiers (OAs) in the sixties brought a whole new perspective to the design of active networks. OAs are off-the-shelf components which are simpler to use, more reliable and can provide an improved functional performance when compared to transistors or vacuum tubes. Also, other devices such as controlled sources, GICs, FDNRs, gyrators and inductors can be readily simulated by OA-based circuits.

Numerous papers on active circuits have been published during the last three decades [4]. OAs are used in the great majority of them. It can be said that OAs now form the backbone of active circuit design and, consequently, it is worthwhile to examine their characteristics in more detail.

## 1.2 THE OPERATIONAL AMPLIFIER

Operational amplifiers have been available for almost fifty years. The first versions of OAs employed vacuum tubes and were expensive with a high power consumption. In 1951, the first discrete transistorized OA was introduced.

However, it was only after the development of integrated circuit (IC) technology that the use of OAs in circuits enjoyed a dramatic growth [5].

An ideal operational amplifier is a differential-input, single-ended-output amplifier with an infinite differential gain, an infinite input impedance and a zero output impedance. The schematic diagram of an OA is shown in Fig. 1.1.

Actual OAs do not possess the ideal characteristics mentioned above. Among many non-idealities, they present a finite frequency-dependent differential gain, a finite input impedance, a nonzero output impedance, a finite slew rate and a nonzero common mode gain. As has been pointed out [6,7], the non-idealities whose influence limit most the performance of OA-based circuits are the input impedance, the output impedance and the differential gain. The effect of both of these impedances can be made neglectable by choosing an appropriate impedance level for the network. Therefore, the differential gain remains as the most troublesome characteristic of the actual OAs.

Because of cost considerations and simplicity of use, internally compensated OAs with a single dominant pole like the Fairchild  $\mu A741$  are much more popular than externally compensated OAs. In view of this, only internally compensated OAs will be considered in this thesis.

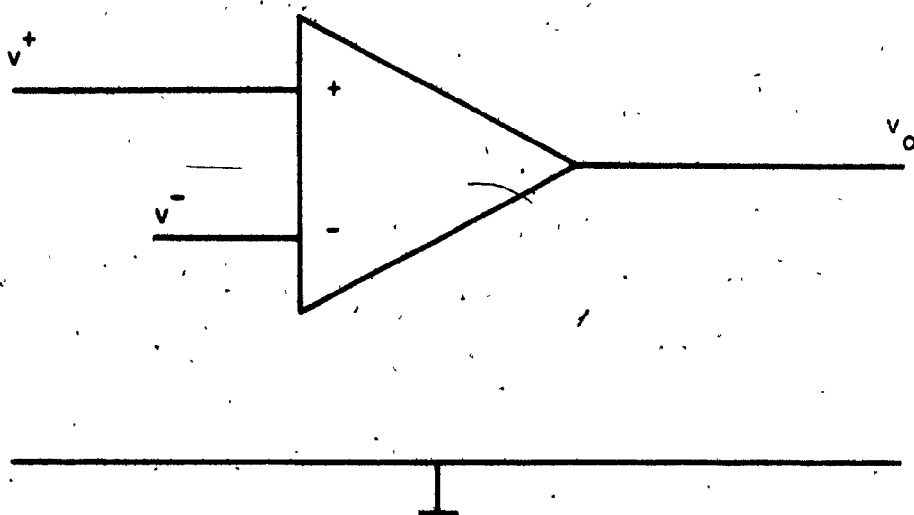


Fig. 1.1: Schematic representation of an operational amplifier

For such an OA, the differential gain can be modeled by [8]

$$A(s) = \frac{A_o}{1 + \frac{s}{\omega_1}} \quad (1.1)$$

where  $A_o$  is the DC gain,  $\omega_1$  is the cut-off frequency and  $s$  is the complex frequency variable. Usually, OAs are operated at frequencies much above  $\omega_1$  and eqn. (1.1) can be rewritten as

$$A(s) = \frac{A_o \omega_1}{s} = \frac{GB}{s} = \frac{1}{s\tau} \quad (1.2)$$

where  $GB$  and  $\tau$  are the gain-bandwidth product and the time constant of the OA, respectively.

The model in eqn. (1.2) is considered fairly accurate in the operating frequency range where OAs are commonly used. At higher frequencies, the existence of secondary poles in actual OA characteristics has to be accounted for. However, it has not been found necessary to consider more than one secondary pole. In this case, a more accurate model for the differential gain of the OAs is given by

$$A(s) = \frac{1}{s\tau(1 + \frac{s}{\omega_2})} = \frac{1}{s\tau(1 + \frac{s\tau}{\theta})} \quad (1.3)$$

where  $\omega_2$  is the second pole frequency and  $\theta$  is the ratio between the second pole frequency and the GB.

All parameters of the actual OAs such as  $A_o$ ,  $\omega_1$ ,  $\omega_2$ , and  $\theta$  vary widely with factors such as temperature, aging, supply voltages, manufacturing tolerances, etc. As a consequence of this fact, circuit designers tend to restrict the use of OAs to cases where such devices can be considered close to ideal and avoid situations where the exact knowledge of OA parameters is necessary. Exceptions to this rule are allowed only under specific laboratory conditions and rarely in a mass production environment.

Since this thesis is concerned about techniques to minimize the influence of the differential gain of the OAs in both active filters and voltage amplifiers, it is opportune to discuss such influence in both types of circuits.

### 1.3 VOLTAGE AMPLIFIERS AND THE EFFECTS OF THE GB

Voltage amplifiers (VAs) are important building blocks which find application in several areas such as active-RC filters, oscillators, instrumentation, etc [9-11]. Conventionally, VAs are realized by a single OA with two resistors providing negative feedback as shown in Fig. 1.2.

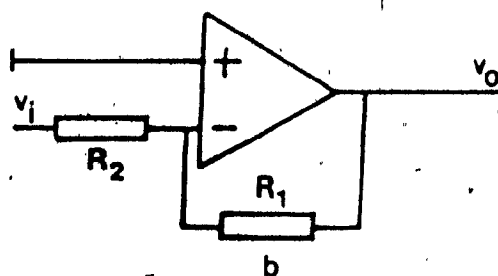
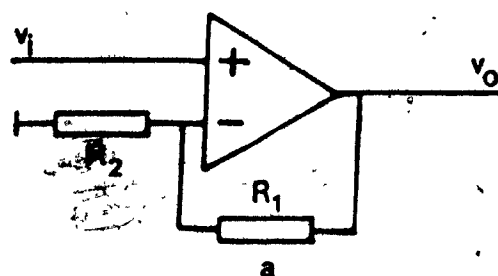


Fig. 1.2: Conventional realizations of VAs  
(a) noninverting (b) inverting

The operating frequency range of these conventional VA realizations, however, is limited up to only a few kilohertz. As it will be shown shortly, this is due to the frequency dependent differential gain of the OA which introduces severe magnitude and phase errors in the gain of the realized VA.

Let the OA in each circuit be considered ideal except for its differential gain which is modeled as in eqn. (1.2). Under this assumption, the transfer function of the positive gain or noninverting VA (Fig. 1.2a) is given by

$$H_p(s) = K_p \frac{1}{1 + s\tau K_p} \quad (1.4)$$

and the transfer function of the negative gain or inverting VA (Fig. 1.2b) is given by

$$H_n(s) = K_n \frac{1}{1 + s\tau K_p} \quad (1.5)$$

where  $K_p = 1 + R_1/R_2$  and  $K_n = -R_1/R_2$ .

If the OA were ideal,  $\tau$  would be equal to zero and the ideal values of  $H_p(s)$  and  $H_n(s)$  would be  $K_p$  and  $K_n$ , respectively. These ideal gains have magnitude and phase characteristics which are both independent of the frequency.



A look at eqns. (1.4) and (1.5) shows that the effect of the differential gain of the OA on the conventional VAs is represented by the error function

$$G(s) = \frac{1}{1 + s\tau K_p} \quad (1.6)$$

which multiplies the ideal gain values.

In order to establish the limitations of the conventional VAs let us replace  $s$  by  $j\omega$  and rewrite eqn. (1.6) in polar coordinates, i.e.

$$G(j\omega) = \frac{1}{(1 + (\omega\tau K_p)^2)^{1/2}} e^{-j\tan^{-1}(\omega\tau K_p)} \quad (1.7)$$

A little reflection on eqn. (1.7) reveals that the conventional forms of VAs will behave close to the ideal only at frequencies such that

$$(\omega\tau K_p) \ll 1 \quad (1.8)$$

and, as a much stronger restriction,

$$\tan^{-1}\omega\tau K_p \approx 0 \quad (1.9)$$

Clearly, if eqn. (1.9) is satisfied then (1.8) is also satisfied. Further, in order to satisfy (1.9),  $(\omega\tau K_p)$  must have a very small value. Therefore, eqn. (1.9) imposes a limit to the maximum operating frequency range of circuits employing such VA realizations. In practice, this limit is found to be equal to a few kilohertz if 741-type OAs with gain-bandwidth product equal to 1MHz are used.

At frequencies where these restrictions are satisfied, the normalized errors in magnitude and phase of the realized VAs are given by

$$\left| \frac{\Delta H_p(j\omega)}{K_p} \right| \approx \frac{1}{2} (K_p \omega \tau)^2 \quad (1.10)$$

$$|\Delta \phi_p(j\omega)| \approx (K_p \omega \tau) \quad (1.11)$$

for the noninverting case and

$$\left| \frac{\Delta H_n(j\omega)}{-K_n} \right| \approx \frac{1}{2} (K_p \omega \tau)^2 \quad (1.12)$$

$$|\Delta \phi_n(j\omega)| \approx (K_p \omega \tau) \quad (1.13)$$

for the inverting circuit.

Both the magnitude and phase errors will contribute to the performance degradation of the systems in which these conventional VAs are embedded. However, the latter are typically an order of magnitude greater than the former. The effect of such errors in active filters have been thoroughly examined in the literature [12-14].

#### 1.4 EFFECTS OF ACTUAL OAs IN ACTIVE-RC NETWORKS

In the last section, the effects of the OA finite gain-bandwidth on VAs were discussed. However, not all OA-based networks utilize VAs as building blocks. Therefore, it is interesting to use a more general approach to study the effect of the finite GB of the OAs in active-RC networks. In order to do so, let us consider first single amplifier networks. Later in this section, the discussion will be extended to multi-amplifier circuits.

The general configuration of a single amplifier network (SAN) is shown in Fig. 1.3. The passive RC network which embeds the OA is characterized by the following transfer functions

$$T_{1i}(s) = \frac{v_1'(s)}{v_i(s)} \Big|_{v_1(s)=0} = \frac{N_{1i}(s)}{D_{RC}(s)} \quad (1.14)$$

$$T_{11}(s) = \frac{v_1'(s)}{v_1(s)} \Big|_{v_i(s)=0} = \frac{N_{11}(s)}{D_{RC}(s)} \quad (1.15)$$

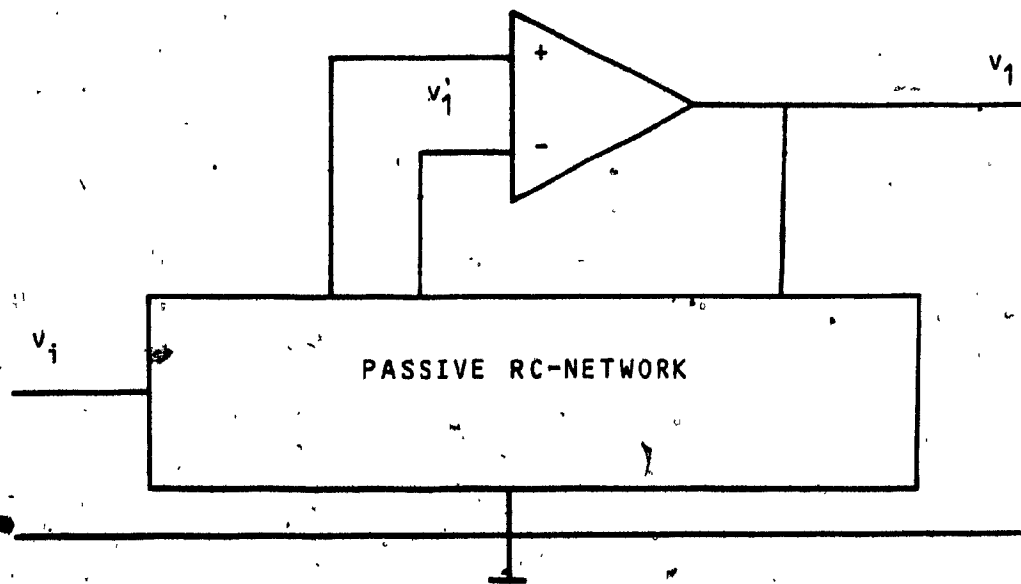


Fig. 1.3: General configuration of a SAN.

where the possible existence of private poles in either  $T_{11}(s)$  or  $T_{11}(s)$  is disregarded. It is important to note that, for second order networks as it is the case considered in this thesis,  $D_{RC}(s)$  and  $N_{11}(s)$  are second order polynomials and  $N_{11}(s)$  is a polynomial whose degree is at most equal to 2.

Under eqns. (1.14) and (1.15) and considering the OA differential gain to be equal to  $A(s)$ , the overall transfer function of the SAN is given by

$$H(s) = \frac{v_1(s)}{v_i(s)} = \frac{k_1 N_{11}(s)}{N_{11}^{1+k} \frac{D_{RC}}{A(s)}} = \frac{N(s)}{D(s)} \quad (1.16)$$

where  $k_1$  and  $k$  are positive constants.

It can be concluded from eqn. (1.16) that  $A(s)$  has no effect over the zeros of  $H(s)$ . On the other hand, the poles of  $H(s)$  are modified in two ways. First, extra poles are introduced. Second, the ideal poles of the network, i.e. the zeros of  $D(s)$  when  $A(s)$  is made infinite (which are clearly the zeros of  $N_{11}(s)$ ) are shifted to a new position.

The introduction of extra poles has small influence on the transfer function of the realized network because they are usually located at frequencies much above the

passband of the ideal characteristics. The shift on the dominant poles,  $s_0$  and  $s_0^*$  (the \* denotes complex conjugate), are of much more concern because it can dramatically change the transfer function realized by the circuit. For this reason, it is interesting to examine such departures in more details. To do so, let us consider the method proposed by Fleischer [15,16].

In order to apply the method, let us consider that the ideal poles of  $H(s)$ ,  $s_0$  and  $s_0^*$ , were shifted to the new values  $s_0 + \Delta s_0$  and  $s_0^* + \Delta s_0^*$ , respectively. Then,  $s_0 + \Delta s_0$  is a root of  $D(s)$  and, thus,

$$D(s_0 + \Delta s_0) = N_{11}(s_0 + \Delta s_0) + k \frac{D_{RC}(s_0 + \Delta s_0)}{A(s_0 + \Delta s_0)} = 0 \quad (1.17)$$

Since  $N_{11}(s)$  is the ideal value of the denominator of  $H(s)$ , it follows that

$$N_{11}(s) = (s - s_0)(s - s_0^*) \quad (1.18)$$

and

$$N_{11}(s_0 + \Delta s_0) = \Delta s_0 (s_0 - s_0^* + \Delta s_0) \quad (1.19)$$

By using eqn. (1.19) into (1.17), the following relationship can be derived

$$\Delta s_o = \frac{-k}{A(s_o + \Delta s_o)} \frac{D_{RC}(s_o + \Delta s_o)}{(s_o - s_o^* + \Delta s_o)} \quad (1.20)$$

None of the terms in the right-hand side of this equation changes rapidly. Also, for good designs,  $\Delta s_o$  should be small compared to  $s_o$ . Hence, the following approximation can be obtained by taking advantage of these two observations

$$\Delta s_o \approx \frac{-k D_{RC}(s_o)}{A(s_o)(s_o - s_o^*)} \quad (1.21)$$

If  $s_o$  is written in rectangular coordinates as

$$s_o = \omega_o \left\{ \frac{-1}{2Q_o} + j \left( 1 - \frac{1}{4Q_o^2} \right)^{1/2} \right\} \quad (1.22)$$

where  $\omega_o$  is the pole frequency and  $Q_o$  is the pole Q-factor and  $A(s)$  is modeled as in (1.2), it follows that

$$\Delta s_o = \frac{-k D_{RC}(s_o)}{(s_o - s_o^*)} \left\{ \frac{-1}{2Q_o} + j \left( 1 - \frac{1}{4Q_o^2} \right)^{1/2} \right\} \omega_o \tau \quad (1.23)$$

Eqn. (1.23) allows us to obtain a few conclusions about the effect of the OA finite gain-bandwidth on the dominant poles of SANs:

(a) There is a change in the pole frequency ( $\omega_0$ ) of the realized network.

(b) The pole Q-factor ( $Q_0$ ) is also changed.

(c) These shifts in  $\omega_0$  and  $Q_0$  are proportional to  $\omega_0 \tau$  what indicates that the ratio between the pole frequency and the OAs gain-bandwidth product must be kept small in order to realize good quality SANs.

Obviously, the last conclusion represents a limitation to the maximum operating frequency of the SANs. It is also important to note that the contributions of  $D_{RC}(s_0)$  and  $(s_0 - s_0^*)$  makes  $\Delta s_0$  to be proportional not only to  $\omega_0$  but also to  $Q_0$ . This means that the product  $Q_0 \omega_0$  must be kept small compared to the GB of the OA to enable the SAN to present a reasonably good performance.

The above conclusions for the poles of SANs can be readily extended to multiple OA networks (MANs). The main difference consists in the fact that, in general, the OAs affect both the zeros and the poles of the realized MANs. Since the derivation of the shifts of the dominant poles and zeros of MANs can be quite involved, no expressions for



such shifts are presented in this thesis. The reader is referred to [17-20] for details of such analyses.

### 1.5 SPECIAL DESIGN TECHNIQUES FOR OA-BASED NETWORKS

The analysis presented in both this section and the last one indicates that OA-based networks have their operation limited to low frequencies due to the gain-bandwidth product of the OAs used. In active filters, this limitation becomes even more severe for high Q-factors (highly selective networks). This problem can be minimized by either the use of high speed OAs or the use of special design techniques. Cost considerations make the latter preferable over the first. In this section, such special techniques are described and are briefly discussed.

Special design techniques for extending the operating frequency range of OA-based circuits can be grouped into three categories: predistortion, passive compensation and active compensation.

Predistortion [15,16] consists basically in modifying the design in order to take into account the finite GB of the OAs. For this purpose, four steps are necessary:

- (a) The network is designed as though the OAs were ideal.

- (b) The deviations of the zeros and poles of the network obtained in the first step from their ideal values are obtained.
- (c) Predistorted zeros and poles are obtained by applying shifts opposed to the ones calculated in the second step to the desired zeros and poles.
- (d) Using the predistorted zeros and poles another circuit is designed while assuming the OAs to be ideal.

The accuracy of the realized circuit will depend heavily on how precise is the determination of the poles and zeros shifts. Unfortunately, two factors contribute to the inaccuracy in this analysis. First, the calculated values of such shifts are usually obtained through approximations used to simplify the calculations. The second and most important factor is the dependence of the expressions for these shifts on the value of  $\tau$  for the different OAs on the circuit. Parameters of integrated circuits vary widely with factors such as temperature, aging, supply voltages, manufacturing tolerances, etc. Thus, the predistortion has to be tailored for each OA and a given set of environmental conditions. Even then, it will not likely remain effective if any change in any of those conditions occur or even as time goes by. Consequently, this technique works well only

under strict laboratory conditions and cannot be applied in a mass production situation.

Another possible technique for designing active circuits is accomplished by using additional RC components such that their effect neutralizes that of the differential gain of the OAs. This passive compensation scheme [21] also depends on the knowledge of the GBs of the OAs. Consequently, this compensation is tantamount to predistortion and suffers from the same limitations of that technique.

By far, the most effective design technique for OA-based networks that have been presented to date is the one that uses active compensation [22-38,51]. Active compensation relies on the use of additional OAs and resistors to form a structure in which the overall effect of the GBs of the OAs is greatly reduced. The introduction of dual and quad OA chips has made this technique cost competitive with the conventional designs.

The main characteristic of this scheme is that the correction of the GB effects depend on the matching of ratios involving resistors and the  $\tau$  of the OAs\*. Since this task can be performed by tuning the circuit and without knowledge of the values of the OA parameters, active

\*It is worthwhile noting that, in some cases, the compensation is accomplished by matching only resistor ratios.

compensation has an important advantage over the two techniques discussed before. Moreover, the close tracking among resistors implemented in IC technology and among characteristics of OAs placed on the same chip makes this scheme effective over a wide range of variations of the conditions mentioned before.

There are two approaches used to obtain actively compensated circuits. In the first one, the circuit is considered as a whole and compensation is achieved by satisfying some conditions that deal with the overall network. Another approach relies on compensating only the active building blocks used in the circuit. Let us consider each approach separately.

Compensation of the overall network was attempted in [28]. Although some very interesting theoretical considerations were obtained, the filter structures presented in this paper either have stability problems which limit the maximum realizable Q-factor and pole frequency or are wasteful on the use of capacitors. Consequently, this approach has not been popular.

Active compensation of the building blocks, on the other hand, became very popular since many already existing conventional circuits can be actively compensated by a simple replacement of a subcircuit. Such subcircuits are usually voltage amplifiers or integrators.

---

It is interesting to note that actively compensated VAs (ACVAs) find numerous applications not only because the class of VA-based active filters are largely used but also because there are many nonfiltering applications of VAs such as instrumentation, oscillators, etc.

Since a major part of this thesis deals with actively compensated voltages amplifiers, it is useful to take a look at how this compensation works.

### 1.6 THEORY OF ACTIVELY COMPENSATED VOLTAGE AMPLIFIERS

Consider a circuit employing  $m$  OAs embedded in a resistive network. Let each OA be assumed ideal except for its differential gain which is given by

$$A_i(s) = \frac{1}{s\tau_i} \quad (1.24)$$

where  $\tau_i$  is the reciprocal of the gain-bandwidth product of the  $i$ th OA. It can be shown that the transfer function realized by the circuit is given by an expression similar to

$$K(s) \approx K \frac{1 + \alpha_1 s + \alpha_2 s^2 + \dots + \alpha_{m-1} s^{m-1}}{1 + \beta_1 s + \dots + \beta_{m-1} s^{m-1} + \beta_m s^m} = K \frac{N(s)}{D(s)} \quad (1.25)$$

where  $\alpha_k$  and  $\beta_k$  ( $k=1, \dots, m$ ) are constants determined by the voltage transfer ratios and the GBs of the OAs. Further,  $\beta_m = \delta_m \gamma_m^m$  where  $\delta_m$  is a constant uniquely determined by the resistive network and  $\gamma_m^m = \prod_{i=1}^m \tau_i$ .

Let us also assume that, in the circuit, we can set  $\alpha_k = \beta_k$ ,  $k=1, \dots, m-1$ . Then, by carrying out the division of  $N(s)$  by  $D(s)$ , it follows that

$$K(s) = K \left\{ 1 - \frac{\delta_m (s \gamma_m)^m}{D(s)} \right\}$$

From this expression, using the fact that  $\gamma_m$  is the geometric mean of the  $\tau_i$ 's,  $i=1, \dots, m$ , the deviation in  $K(s)$  from the ideal value  $K$  is given by

$$\frac{\Delta K(s)}{K} = \frac{K - K(s)}{K} = \frac{\delta_m (\tau_1 \tau_2 \dots \tau_m) s^m}{D(s)} \quad (1.26)$$

A little reflection, using eqn. (1.26), shows that if  $m$  is even, the magnitude error will be the dominant one. On the other hand, if  $m$  is odd, the phase error becomes dominant.

Since, in practice,  $\tau_i \ll 1$  for all values of  $i$ , expression (1.26) seems to encourage the employment of as many OAs as feasible to minimize the deviation  $\frac{\Delta K(s)}{K}$  over the largest possible operating frequency range. However, a set of practical considerations limits the desirable number of OAs in the circuit:

- (a) With the present state of technology, chips with up to 4 OAs are readily available. Circuits containing more than that number of OAs will lose their cost effectiveness with respect to conventional designs. Further, there is no tracking among OAs placed in different chips.
- (b) The use of higher number of OAs reduces the relative stability margins in the VAs designed [29]. These margins are important for various applications of the VAs.
- (c) Since, in the derivation of the active compensation schemes, the OAs are modeled as in eqn. (1.24), it is useless to think of minimizing the error so much that the circuit will theoretically work well even at frequencies

higher than those for which the OA model is valid. In other words, designing actively compensated VAs with an excessive number of OAs may be equivalent to using a too simplistic model for the OAs.

By taking all these factors into consideration, it can be shown that ACVAs employing more than 3 OAs will be of doubtful utility.

Circuits employing 3 OAs have higher operating frequency range than circuits employing 2 OAs. On the other hand, 3 OA ACVAs are more complex and expensive than 2 OA ACVAs. By comparing the performance and complexity exhibited by these two types of circuits, it becomes clear that there is an operating frequency range at which 2 OA circuits are preferable over 3 OA realizations in spite of the smaller errors presented by the latter. This is due to the fact that, at this frequency range, the performance of 2 OA realizations is good enough to justify the rejection of the increase in complexity and costs caused by the use of 3 OA ACVAs. Of course, at frequencies above this range, the performance of 2 OA circuits deteriorates and more complex realizations must be used in order to achieve the specifications of the application at hand.

Several works have been published dealing with actively compensated VAs using 2 OAs. In the great majority



of them, however, the approach has been in an ad hoc basis, that is, circuits were selected at random and their properties examined, as opposed to a systematic and general study. Only three articles have attempted a general study of the problem, namely references [30-32].

A general study on 2 OA ACVAs was first reported in [30,31]. While interesting insights of the problem were provided in this study, only broad guidelines were provided for the classification of ACVAs. These are then used to group some previously reported circuits as well some new realizations into 6 classes. Generation procedures, however, are not at all addressed in these papers. In fact, all the ACVAs reported were presented without specifying the considerations from which they were obtained.

In [32], a more thorough study than in [30,31] was attempted. However, results are not fully satisfactory. In particular, the magnitude compensated VAs reported in this article possess phase errors that prevent their application in most applications. Further, their classification can be shown to be equivalent to the one proposed in [30,31].

On the other hand, very few articles [35-36] have been written about ACVAs using 3 OAs. Further, in all of them, the approach was on an ad hoc basis. No general study of 3 OA ACVAs has been presented so far.

## 1.7 SCOPE OF THE THESIS

The purpose of this thesis is to present the results of a systematic investigation on the generation and classification of actively compensated voltage amplifiers as well as to report a new approach to the design of actively compensated active filters.

Towards this end, a comprehensive procedure for the classification and generation of ACVAs using ~~2~~ OAs is described in Chapter 2. This classification is based on a set of both practical and theoretical considerations. Additional considerations allow each class to be further divided into types. The characterization of each type leads directly to circuit realizations of ACVAs. At the end of this chapter, the circuits obtained are evaluated with respect to performance characteristics such as magnitude and phase errors, stability considering the second pole of the OAs, tunability, etc.

In Chapter 3, the investigation initiated in Chapter 2 is extended to include ACVAs using 3 OAs. Due to the unique properties of 3 OA circuits, the procedure used in the previous chapter cannot be readily extended to this case. Therefore, the classification and generation method has to be tailored to this special situation. By doing so, some very interesting results are obtained in addition to the realizations themselves. In particular, it is proven

that it is impossible to design a tunable inverting ACVA using 3 OAs. Finally, the results of an evaluation of the circuits obtained with respect to the performance characteristics mentioned before are reported.

In Chapters 2 and 3, the design of VAs are considered regardless of the circuits in which they are used. A different approach is reported in Chapter 4. In this chapter, the overall circuit is considered and novel active compensation conditions for active-RC filters are derived. Based on these conditions, it is shown how already existing filter realizations can be modified in order to compensate them actively. Further, a simple technique to optimize the performance of such modified circuits with respect to the already reduced effect of the GBs is described. By doing so, the operating frequency range of active-RC filters can be significantly increased.

The results obtained and suggestions for future work are summarized in Chapter 5.

## CHAPTER 2

CLASSIFICATION, GENERATION AND DESIGN  
OF  
ACTIVELY COMPENSATED VOLTAGE AMPLIFIERS  
USING 2 OAs

## 2.1 INTRODUCTION

The purpose of this chapter is to provide a refined and rigorous classification as well as realization procedure for the ACVAs that use 2 OAs. A systematic synthesis methodology is described for the generation of both noninverting and inverting ACVAs possessing either finite or infinite input impedance.

First, a general model for a 2 OA ACVA is analyzed and some conditions on the resistive network which embeds the OAs are derived. Based on these conditions, ACVAs are divided into several classes. Some further considerations allow each class to be partitioned into types. Then, for each type, ACVA realizations are generated directly from the characterization of the types.

Next, the design of the circuits obtained is considered. All the circuits obtained are analyzed with respect to performance characteristics such as magnitude and phase errors, stability considering the second pole of the

OAs, etc.

Finally, the results of practical experiments using the ACVAs as stand-alone elements as well as in an active filter application are reported.

## 2.2 PRELIMINARY CONSIDERATIONS

The general configuration of a 2 OA ACVA is shown in Fig. 2.1. The OAs are considered to be ideal except for their differential gain which is expressed as in eqn. (1.24).

If  $v_1$  is taken as output, the transfer function is given by:

$$\frac{v_1}{v_i} = K \frac{1 + a_1 \tau_2 s}{1 + [b_1 \tau_2 + b_2 \tau_1] s + b_3 \tau_1 \tau_2 s^2} \quad (2.1)$$

where

$$K = \frac{f_2 F_{12} - f_1 F_{21}}{F_{11} F_{22} - F_{12} F_{21}} \quad (2.2)$$

$$a_1 = \frac{f_1}{f_2 F_{12} - f_1 F_{21}} \quad (2.3)$$

$$b_1 = \frac{-F_{11}}{F_{11} F_{22} - F_{12} F_{21}} \quad (2.4)$$

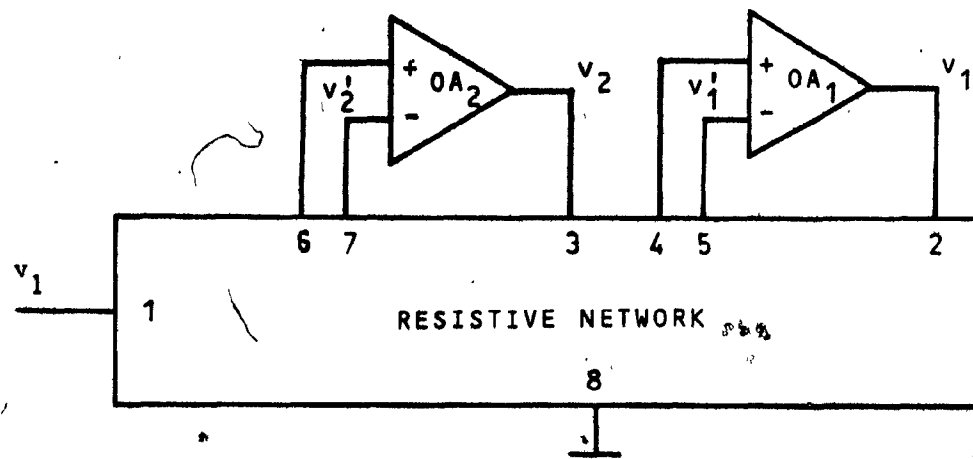


Fig. 2.1: The general configuration of a 2 OA ACVA.

$$b_2 = \frac{-F_{22}}{F_{11}F_{22} - F_{12}F_{21}} \quad (2.5)$$

$$b_3 = \frac{1}{F_{11}F_{22} - F_{12}F_{21}} \quad (2.6)$$

and

$$F_{kj} = \frac{v_k}{v_j} \Big|_{v_h \neq j=0}, h = 1, 2, 1$$

$$f_k = \frac{v_k}{v_i} \Big|_{v_1=v_2=0}$$

for  $k, j=1, 2$ .

The fact that the circuits must be stable leads to the following restrictions

$$F_{11}F_{22} - F_{12}F_{21} > 0 \quad (2.7)$$

$$F_{22} + F_{11} < 0 \quad (2.8)$$

In this case, it follows from (2.2) that

$$f_2 F_{12} - f_1 F_{21} > 0 \quad (2.9)$$

for noninverting circuits and

$$f_2 F_{12} - f_1 F_{21} < 0 \quad (2.10)$$

for inverting ACVAs.

For both cases the compensation condition is given by

$$F_{22}\tau_1 + F_{11}\tau_2 = -f_1 K_0^{-1} \tau_2 \quad (2.11)$$

and, consequently,

$$f_1 > 0 \quad (2.12)$$

for positive gain ACVAs and

$$f_1 < 0 \quad (2.13)$$

for negative gain realizations.



In order to make the stability of the circuits independent of the GBs of the OAs, eqn. (2.8) will be considered satisfied only if  $F_{11} \leq 0$  and  $F_{22} \leq 0$  (obviously  $F_{11} = F_{22} = 0$  is not acceptable). In the case  $F_{22} = 0$ , eqn. (2.11) can be satisfied regardless of the values of  $\tau_1$  and  $\tau_2$ , that is the compensation will depend only on the resistive network. If  $F_{22} \neq 0$ , then tracking OAs should be used in order to make the compensation effective when  $\tau_1$  and  $\tau_2$  vary.

Each voltage  $v_k$  ( $k=1,2$ ) can be written as

$$v_k = v_k^+ - v_k^- \quad (2.14)$$

where  $v_k^+$  and  $v_k^-$  are the voltages at the noninverting and the inverting input terminals of the  $k$ th OA, respectively, referred to ground. Hence,

$$F_{kj} = \left. \frac{v_k}{v_j} \right|_{v_h=0, h \neq j} = \left. \frac{v_k^+ - v_k^-}{v_j} \right|_{v_h=0, h \neq j} = F_{kj}^+ - F_{kj}^- \quad (2.15)$$

$$f_k = \left. \frac{v_k}{v_i} \right|_{v_1=v_2=0} = \left. \frac{v_k^+ - v_k^-}{v_i} \right|_{v_1=v_2=0} = f_k^+ - f_k^- \quad (2.16)$$

for  $k, j = 1, 2$ .

Also, because the  $F_{kj}$ 's and  $f_k$ 's are realized by a resistive network, it is easy to show that the following restrictions must hold for all  $k=1,2$

$$f_k^+ + \sum_{j=1}^2 F_{kj}^+ \leq 1 \quad (2.17)$$

$$f_k^- + \sum_{j=1}^2 F_{kj}^- \leq 1 \quad (2.18)$$

$$0 \leq F_{kj}^+ \leq 1 \quad (2.19)$$

$$0 \leq f_k^+ \leq 1 \quad (2.20)$$

## 2.3 CLASSIFICATION AND GENERATION OF ACVAS

The conventional realizations of VAs (Fig. 1.2) use only one OA and, as a consequence of this, the noninverting and the inverting circuits possess infinite and finite input impedance, respectively.

Considering the ACVAs, however, two or more OAs are employed and, therefore, it is possible to obtain realizations possessing either infinite or finite input impedance for both the noninverting and the inverting types of ACVAs. Although circuits with high input impedance are considered to be preferable in general, an ACVA which

possess finite input impedance may also present some kind of advantage like smaller magnitude and phase errors, better stability margins or even less component counts. For this reason and also because their generation is achieved in different fashions, infinite input impedance (III) and finite input impedance (FII) ACVAs are considered as two different groups of circuits. Hence, they are treated separately in the remainder of this section.

### 2.3.1 Infinite input impedance ACVAs

For the most economical realization of the ACVAs, the resistive network shown as a block in Fig. 2.1 should contain the minimum number of resistors. Clearly, then, it must contain as few nodes and branches as possible. Further, any branch between any two nodes must represent a single resistor. Hence, the most general configuration of the resistive network is as shown in Fig. 2.2 where each branch represents only one resistor.

Based on this general configuration, it is not difficult to show that the input current  $I_i$ , the current that would be drawn from a voltage source of value  $v_i$  connected between node  $i$  and ground, is given by

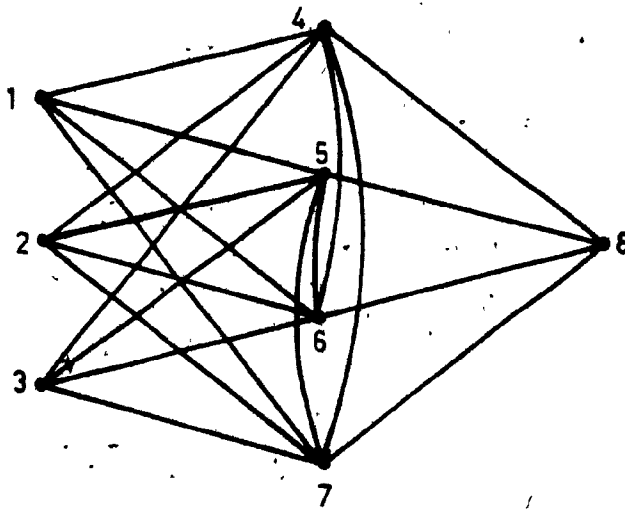


Fig. 2.2: The general configuration of the resistive network.

$$\begin{aligned}
I_i = & [G_{1,4}(1-f_1^+) + G_{1,5}(1-f_1^-) + G_{1,6}(1-f_2^+) + G_{1,7}(1-f_2^-)]v_i \\
& - [G_{1,4}F_{11}^+ + G_{1,5}F_{11}^- + G_{1,6}F_{21}^+ + G_{1,7}F_{21}^-]v_1 \\
& - [G_{1,4}F_{12}^+ + G_{1,5}F_{12}^- + G_{1,6}F_{22}^+ + G_{1,7}F_{22}^-]v_2
\end{aligned} \quad (2.21)$$

where  $G_{k,j} = 1/R_{k,j}$  and  $R_{k,j}$  is the value of the resistor connected between nodes  $k$  and  $j$ .

If the circuits are to have infinite input impedance,  $I_i$  must be equal to zero. There are two ways of accomplishing this. Either the circuit can be designed in such way that  $v_i$ ,  $v_1$  and  $v_2$  will always assume values that reduce eqn. (2.21) to zero or  $I_i$  can be made equal to zero regardless of the values of these voltages (this requires the coefficients of these voltages in eqn. (2.21) to be equal to zero). Because of sensitivity and complexity considerations, the latter is preferable over the first.

From Fig. 2.2, we note that if  $G_{1,k} = 0$ ,  $f_{(k-2)/2}^+ = 0$ , for  $k=4,6$ . Also, if  $G_{1,k} = 0$ ,  $f_{(k-3)/2}^- = 0$ , for  $k = 5,7$ . Further, it follows from eqns. (2.17) and (2.18) that if  $f_k^+ = 1$ , then  $F_{k1}^+ = F_{k2}^+ = 0$ . In view of these observations, it is easy to show that the necessary and sufficient conditions for  $I_i$  to be unconditionally equal to zero is that the  $f_k$ 's assume values equal to either zero or one. Consequently, there are 16 ways of assuring infinite input impedance for the circuits. Some of these ways, however, conflict with one or more of the eqns. (2.7) to (2.13) and (2.17) to (2.20). Only five of them satisfy all these conditions. They constitute classes\* of ACVAs of the

III group and are shown in Table 2.1. In the same table, the characterization of each class in terms of the topological connections between the input of the overall network and the input port of each OA is also indicated. Detailed analysis has shown that the realizations belonging to classes NI2C and II2B require very complex tuning procedures and, for this reason, these classes will not be considered in the remainder of this thesis.

At this point, a closer look at each of the classes is necessary in order to enable us to generate circuits for them.

#### 2.3.1.a Class NI2A realizations

For this class, we have that:

$$f_1^+ = 1 \Rightarrow F_{12}^+ = F_{11}^+ = 0$$

$$f_2^- = 1 \Rightarrow F_{21}^- = F_{22}^- = F_{22}^+ = 0$$

Thus, it follows that the DC gain and the compensation

\* It should be noted that the first letter of the name of a class in this thesis is either an N for noninverting circuits or an I for inverting ones. The second letter is an I for realizations of the III group or an F for those in the FII group. Further, the third digit is a 2 for 2 OA ACVAs or a 3 for 3 OA circuits.

TABLE 2.1: CHARACTERIZATION OF THE CLASSES OF  
INFINITE INPUT IMPEDANCE (III) ACVAs.

CLASS	$f_1^+$	$f_1^-$	$f_2^+$	$f_2^-$
NI2A	1	0	0	1
NI2B	1	0	0	0
NI2C	1	0	1	0
II2A	0	1	0	0
II2B	0	1	1	0

condition are given by

$$K = \frac{1}{F_{21}^+} \quad (2.22)$$

$$F_{11}^- = K^{-1} \quad (2.23)$$

The realizations for this class can be obtained by means of sequential eliminations of resistors from the general configuration shown in Fig. 2.2. These eliminations are based on the characterization of this class and lead to minimal realizations. This is shown next.

Consider, again, the general configuration shown in Fig. 2.2. As a consequence of the characterization of this class, the following resistor eliminations are performed

$$f_1^+ = 1 \Rightarrow G_{1,4} = \infty, G_{2,4} = G_{3,4} = G_{4,6} = G_{4,7} = G_{4,8} = 0$$

$$f_1^- = 0 \Rightarrow G_{1,5} = 0$$

$$f_2^+ = 0 \Rightarrow G_{1,6} = 0$$

$$f_2^- = 1 \Rightarrow G_{1,7} = \infty, G_{2,7} = G_{3,7} = G_{5,7} = G_{5,8} = 0$$



$$F_{22}^+ = 0 \Rightarrow G_{3,6} = G_{5,6} = 0$$

Further, because the value of  $F_{12}^-$  can be arbitrarily set to any non-zero quantity,  $G_{5,8}$  is not really necessary. It is interesting to note that since  $b_3$  is minimized by making  $G_{5,8} = 0$ , this choice also minimizes the magnitude and phase errors of the gain of the ACVA of this class. The final graph is shown in Fig. 2.3 and its corresponding circuit is shown in the first row of Table 2.4. The general properties of this realization are shown in Table 2.5. From this table, it is seen that this circuit does not require tracking OAs for compensation.

#### 2.3.1.b Class NI2B realizations

For this class, we have that

$$f_1^+ = 1 \Rightarrow F_{12}^+ = F_{11}^+ = 0$$

$$f_1^- = f_2^+ = f_2^- = 0$$

The DC gain and the compensation conditions are given

by

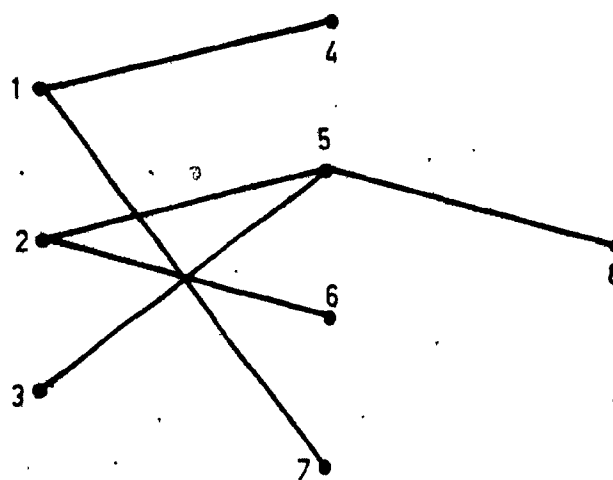


Fig. 2.3: The graph for the resistive network for the realization of class NI2A after all possible resistor eliminations are performed.

$$K = \frac{-F_{22}}{-F_{11}F_{22} - F_{12}F_{21}} \quad (2.24)$$

$$-F_{11}^T \tau_2 + F_{22}^T \tau_1 = -K^{-1} \tau_2 \quad (2.25)$$

From eqn. (2.25), it is seen that the compensation depends on the values of the GBs of the OAs.

A closer look at eqns. (2.24) and (2.25) indicates that the DC gain and the compensation condition can be adjusted independently of each other only if  $F_{11}^- = 0$ . This is a desirable property because it simplifies the tuning process. In this case, these equations can be rewritten as

$$K = \frac{F_{22}}{F_{12}F_{21}} \quad (2.26)$$

$$F_{22} \tau_1 = -K^{-1} \tau_2 \quad (2.27)$$

Different realizations can be obtained if we attempt to realize  $F_{22}^+$ ,  $F_{12}^-$  and  $F_{21}^+$  in different ways. Note that minimal realizations are obtained whenever these transfer ratios either assume values equal to 0 or 1 or, also, if the summation of transfer ratios to the same OA input node is

TABLE 2.2: DIFFERENT TYPES OF ACVA REALIZATIONS OF  
CLASS NI2B.

TYPE	$F_{12}^-$	$F_{21}^+$	$F_{21}^-$	$F_{22}^+$	$F_{22}^-$	REMARKS
NI2B-1	1	x	0	0	x	—
NI2B-2	x	1	x	0	x	$F_{21}^- + F_{22}^- = 1$
NI2B-3	x	x	0	0	x	—
NI2B-4	x	1	0	0	x	—
NI2B-5	1	x	0	x	1	—
NI2B-6	x	x	0	x	1	$F_{21}^+ + F_{22}^+ = 1$
NI2B-7	1	x	x	0	x	$F_{21}^- + F_{22}^- = 1$

NOTE: x has value between 0 and 1

equal to 1. Shown in Table 2.2 are all possible combinations of these ratios that will lead to realizations which employ 4 or less resistors. Each combination determines a different type of ACVA circuit belonging to class NI2B.

The realizations of each type can be obtained using the same elimination procedure that was used for class NI2A. To save space, the description of the steps will not be repeated. The circuits obtained are shown in Table 2.4. Note that the ACVA presented in the seventh row of this table is too complex and has no advantage over the other realizations. Therefore, it will not be considered in the remainder of this thesis. A summary of the general properties of the realizations of class NI2B is shown in Table 2.5.

### 2.3.1.c Class II2A realizations

Regarding class II2A, the DC gain and the compensation condition are now expressed, respectively, by

$$K = \frac{-F_{22}}{F_{12}F_{21}} \quad (2.28)$$

$$F_{22}\tau_1 = K^{-1}\tau_2 \quad (2.29)$$

In this class, there are three different ways for realizing  $F_{12}^+$ ,  $F_{22}^+$  and  $F_{21}^+$  with minimal number of resistors. Each of these ways determines a different type of class II2A realizations. These are shown in Table 2.3. The circuits corresponding to each of these types are obtained through the elimination procedure mentioned before and are shown in Table 2.4. . The general properties of these circuits are shown in Table 2.5. Note that the compensation condition for the realizations of this class depends on the values of the GBs of the OAs.

### 2.3.2 Finite input impedance ACVAs

For the III group of ACVAs, classes were generated by nulling the input current of the overall network. For obvious reasons, this approach can not be extended to the finite input impedance (FII) group of ACVAs. Consequently, a different methodology for generating such circuits has to be created.

In this section, it is shown how the broad classification presented in [30,31] can be modified and used to obtain a synthesis-like procedure for the generation of ACVAs of the FII group. For the sake of organization, however, this section will be divided into two subsections: one concerning noninverting structures and other considering

TABLE 2.3: DIFFERENT TYPES OF REALIZATIONS  
OF CLASS II2A

TYPE	$F_{12}^+$	$F_{12}^-$	$F_{21}^+$	$F_{21}^-$	$F_{22}^+$	$F_{22}^-$	REMARKS
II2A-1	1	0	0	x	0	x	—
II2A-2	1	0	x	x	0	x	$F_{22}^- + F_{21}^- = 1$
II2A-3	x	0	0	x	0	x	$F_{22}^- + F_{21}^- = 1$

NOTE: x has value between 0 and 1

TABLE 2.4: ACVAs OF THE III GROUP

TYPE	CIRCUIT	TRANSFER FUNCTION
NI2A [25]		$\beta^{-1} \frac{1 + \frac{1}{(1-\alpha)} \tau_2 s}{1 + \frac{\alpha}{\beta(1-\alpha)} \tau_2 s + \frac{1}{\beta(1-\alpha)} \tau_1 \tau_2 s^2}$
NI2B-1 [30]		
NI2B-2 [27]		$\beta^{-1} \frac{1 + \frac{1}{\alpha} \tau_2 s}{1 + \frac{1}{\beta} \tau_1 s + \frac{1}{\alpha\beta} \tau_1 \tau_2 s^2}$
NI2B-3 [26]		
NI2B-4		$\frac{\alpha}{\beta} \frac{1 + \frac{1}{\alpha} \tau_2 s}{1 + \frac{\alpha}{\beta} \tau_1 s + \frac{1}{\beta} \tau_1 \tau_2 s^2}$
NI2B-4 [26]		$(\beta+1) \frac{1 + \frac{(\alpha+\beta+1)}{\beta+1} \tau_2 s}{1 + (\beta+1) \tau_1 s + (\alpha+\beta+1) \tau_1 \tau_2 s^2}$
NI2B-4		TOO COMPLEX
NI2B-5 [30]		$\beta \frac{1 + \frac{\beta-1+\alpha\beta}{\alpha\beta} \tau_2 s}{1 + \beta \tau_1 s + \frac{\beta-1+\alpha\beta}{\alpha\beta} \tau_1 \tau_2 s^2}$



TABLE 2.4: (continued)

TYPE	CIRCUIT	TRANSFER FUNCTION
NI2B-6 [30]		$\beta^{-1} \frac{1 + \frac{1}{\alpha} \tau_2 s}{1 + \frac{1}{\beta} \tau_1 s + \frac{1}{\alpha\beta} \tau_1 \tau_2 s^2}$
NI2B-7		$\frac{\alpha}{\beta + \alpha - 1} \frac{1 + \frac{1}{\alpha} \tau_2 s}{1 + \frac{\alpha}{\beta + \alpha - 1} \tau_1 s + \frac{1}{\beta + \alpha - 1} \tau_1 \tau_2 s^2}$
II2A-1 [31]		$\frac{-\beta}{\alpha} \frac{1 + \frac{\alpha + \beta + \alpha\beta}{\beta} \tau_2 s}{1 + \frac{\beta}{\alpha} \tau_1 s + \frac{\alpha + \beta + \alpha\beta}{\alpha} \tau_1 \tau_2 s^2}$
II2A-2		$\frac{-\alpha}{1 - \alpha - \beta} \frac{1 + \frac{1}{\alpha} \tau_2 s}{1 + \frac{\alpha}{1 - \alpha - \beta} \tau_1 s + \frac{1}{1 - \alpha - \beta} \tau_1 \tau_2 s^2}$
II2A-3 [31]		$\frac{-\alpha}{\beta(1-\alpha)} \frac{1 + \frac{1}{\alpha} \tau_2 s}{1 + \frac{\alpha}{\beta(1-\alpha)} \tau_1 s + \frac{1}{\beta(1-\alpha)} \tau_1 \tau_2 s^2}$

TABLE 2.5: PROPERTIES OF THE ACVAs OF THE III GROUP

TYPE	DC-GAIN	COMPENSATION CONDITION	DESIGN EQUATIONS (*)	PHASE ERROR (*)	MAGNITUDE ERROR(*)	STABILITY CONDITION(**)
NI2A	$\beta^{-1}$	$\alpha = \beta$	$\beta = \alpha = K^{-1}$	$-\frac{(K\omega\tau)^3}{(K-1)^2}$	$\frac{(K\omega\tau)^2}{(K-1)}$	$\theta > 2 - \frac{2.5}{K}$
NI2B-1 NI2B-2 NI2B-3	$\beta^{-1}$	$\alpha\tau_1 = \beta\tau_2$	$\beta = \alpha = K^{-1}$	$-(K\omega\tau)^3$	$(K\omega\tau)^2$	$\theta > \frac{1.5}{K}$
NI2B-4 1st CIRCUIT	$\frac{\alpha}{\beta}$	$\alpha^2\tau_1 = \beta\tau_2$	$\alpha = K^{-1}$ $\beta = K^{-2}$	$-(K\omega\tau)^3$	$(K\omega\tau)^2$	$\theta > \frac{1.5}{K}$
NI2B-4 2nd CIRCUIT	$\beta+1$	$\alpha\tau_2 =$ $(\beta+1)^2\tau_1$ $-(\beta+1)\tau_2$	$\beta = K-1$ $\alpha = K(K-1)$	$-(K\omega\tau)^3$	$(K\omega\tau)^2$	$\theta > \frac{1.5}{K}$
NI2B-5	$\beta$	$\alpha = \frac{(\beta-1)\tau_2}{\beta(\beta\tau_1 - \tau_2)}$	$\beta = \alpha^{-1} = K$	$-(K\omega\tau)^3$	$(K\omega\tau)^2$	$\theta > \frac{1.5}{K}$
NI2B-6	$\beta^{-1}$	$\alpha\tau_1 = \beta\tau_2$	$\alpha = \beta = K^{-1}$	$-(K\omega\tau)^3$	$(K\omega\tau)^2$	$\theta > \frac{1.5}{K}$
NI2B-7	$\frac{\alpha}{\beta+\alpha-1}$	$\beta\tau_2 =$ $\alpha^2\tau_1 - (\alpha-1)\tau_2$	$\alpha = K^{-1}$ $\beta = 1 - K^{-1} + K^{-2}$	$-(K\omega\tau)^3$	$(K\omega\tau)^2$	$\theta > \frac{1.5}{K}$
II2A-1	$-\frac{\beta}{\alpha}$	$(\frac{\alpha}{\beta} + \alpha + 1)\tau_2 =$ $\frac{\beta}{\alpha}\tau_1$	$\beta = K(K+1)+1$ $\alpha = -K-1-K^{-1}$	$(K\omega\tau)^3$	$(K\omega\tau)^2$	$\theta > \frac{-1.5}{K}$

TABLE 2.5: (continued)

TYPE	DC-GAIN	COMPENSATION CONDITION	DESIGN EQUATIONS (*)	PHASE ERROR (*)	MAGNITUDE ERROR(*)	STABILITY CONDITION(**)
II2A-2	$\frac{-\alpha}{1-\alpha-\beta}$	$\alpha^2 \tau_1 + \alpha \tau_2 =$ $(1-\beta) \tau_2$	$\alpha = -K^{-1}$ $\beta = 1-K^{-2}+K^{-1}$	$(K\omega\tau)^3$	$(K\omega\tau)^2$	$\theta > \frac{-1.5}{K}$
II2A-3	$\frac{-\alpha}{\beta(1-\alpha)}$	$\beta \tau_2 =$ $\frac{\alpha^2}{(1-\alpha)} \tau_1$	$\alpha = -K^{-1}$ $\beta = \frac{1}{K(K-1)}$	$(K\omega\tau)^3$	$(K\omega\tau)^2$	$\theta > \frac{-1.5}{K}$

(\*) CONSIDERING  $A_1(s) = A_2(s) = \frac{1}{s\tau}$

(\*\*) CONSIDERING  $A_1(s) = A_2(s) = \frac{1}{s\tau(1 + \frac{s\tau}{\theta})}$

inverting circuits.

### 2.3.2.a Noninverting ACVAs of the FII group

Consider, again, the general transfer function of a 2 OA VA as expressed by eq. (2.1) to (2.6). In order to simplify the matching of the coefficients of the  $s$  term in the numerator and denominator of (2.1), either  $F_{11}$  or  $F_{22}$  (but not both at the same time) will be set to zero. By doing so, two distinct classes of realizations are derived, namely class NF2A (characterized by  $F_{22} = 0$ ) and class NF2B (characterized by  $F_{11} = 0$ ).

For class NF2A, the restriction (2.7) can be rewritten as:

$$F_{12}F_{21} < 0 \quad (2.30)$$

and because of this restriction two subclasses of realizations can be obtained:

$$\text{SUBCLASS NF2AA: } F_{12} < 0, F_{21} > 0$$

$$\text{SUBCLASS NF2AB: } F_{12} > 0, F_{21} < 0$$

Realizations for both types can be generated if we attempt to realize the  $F_{kj}^+$ 's and  $f_k^+$ 's transfer ratios using different combinations. Each combination will lead to a

different type of realizations belonging to one of the subclasses mentioned before. All the types have to agree with the restrictions implied by eqns. (2.7) to (2.9) and (2.11) to (2.12). Therefore, there are only a limited number of combinations for each subclass. The possible types are shown in Table 2.6 for both subclasses.

A look at this table reveals that ACVAs belonging to types NF22AA-2 and NFAB-2 can only realize gains at most equal to one. Hence, these cases will only lead to voltage followers.

For each of the types in Table 2.6, circuits can be obtained by applying the resistor elimination procedure mentioned before. The resulting structures are shown in Table 2.7 while the general properties of each circuit are shown in Table 2.8.

Considering class NF2B,  $f_2$  may be either negative, zero or positive. Also, for this class, the restriction (2.7) can be rewritten as in eq. (2.30). As a consequence of these two observations, the realizations of class NF2B can be grouped in one of the following subclasses:

SUBCLASS NF2BA:  $f_2 = 0, F_{12} < 0, F_{21} > 0$

SUBCLASS NF2BB:  $f_2 < 0, F_{12} < 0, F_{21} > 0$

TABLE 2.6: TYPES IN CLASS NF2A

TYPE	$F_{12}^+$	$F_{12}^-$	$F_{11}^+$	$F_{11}^-$	$f_1^+$	$f_1^-$	$F_{21}^+$	$F_{21}^-$	$f_2^+$	$f_2^-$
NF2AA-1	0	x	0	x	x	0	x	0	0	x
NF2AA-2	0	x	0	x	x	0	1	x	0	x
NF2AA-3	0	x	0	x	x	0	x	0	x	1
NF2AA-4	0	x	0	x	1	x	x	0	0	1
NF2AA-5	0	x	0	x	1	x	x	0	x	1
NF2AB-1	x	0	0	x	x	0	0	x	x	0
NF2AB-2	x	0	0	x	x	0	x	1	x	0
NF2AB-3	x	0	0	x	x	0	0	x	1	x

NOTE: x has value between 0 and 1

TABLE 2.7: ACVAs OF CLASS NF2A

TYPE	CIRCUIT	TRANSFER FUNCTION
NF2AA-1 [26]		$\beta^{-1} \frac{1 + \frac{Y}{(1-\alpha)} \tau_2 s}{1 + \frac{\alpha}{(1-\alpha)\beta} \tau_2 s^2 + \frac{1}{(1-\alpha)\beta} \tau_1 \tau_2 s^2}$
NF2AA-2		$\frac{1 + \frac{Y}{\beta(1-\alpha)} \tau_2 s}{1 + \frac{\alpha}{\beta(1-\alpha)} \tau_2 s^2 + \frac{1}{\beta(1-\alpha)} \tau_1 \tau_2 s^2}$
NF2AA-3		$\frac{a_1 + a_3}{a_1} \frac{1 + \frac{(a_1 + a_2 + a_3)}{(a_1 + a_3)(1-\alpha)} \tau_2 s}{1 + \frac{\alpha(a_1 + a_2 + a_3)}{a_1(1-\alpha)} \tau_2 s^2 + \frac{(a_1 + a_2 + a_3)}{a_1(1-\alpha)} \tau_1 \tau_2 s^2}$
NF2AA-4		$\beta^{-1} \frac{1 + \frac{a_1 + a_2}{a_2} \tau_2 s}{1 + \frac{a_1}{a_2 \beta} \tau_2 s^2 + \frac{(a_1 + a_2 + a_3)}{a_2 \beta} \tau_1 \tau_2 s^2}$
NF2AA-5		$\frac{b_1 + b_3}{b_1} \frac{1 + \frac{(a_1 + a_2)(b_1 + b_2 + b_3)}{a_2(b_1 + b_3)} \tau_2 s}{1 + \frac{a_1(b_1 + b_2 + b_3)}{a_2 b_1} \tau_2 s^2 + \frac{(a_1 + a_2 + a_3)(b_1 + b_2 + b_3)}{a_2 b_1} \tau_1 \tau_2 s^2}$
NF2AB-1 [30]		$\beta^{-1} \frac{1 + \frac{Y}{(1-\gamma)} \tau_2 s}{1 + \frac{\alpha}{(1-\gamma)\beta} \tau_2 s^2 + \frac{1}{(1-\gamma)\beta} \tau_1 \tau_2 s^2}$
NF2AB-1		$(1+\beta)^{-1} \frac{1 + \frac{Y}{(1-\gamma)} \tau_2 s}{1 + \frac{\alpha}{(1-\gamma)(\alpha+\beta)} \tau_2 s^2 + \frac{1}{(1-\gamma)(\alpha+\beta)} \tau_1 \tau_2 s^2}$
NF2AB-2		$\frac{1 + \frac{Y}{\alpha(1-\gamma)} \tau_2 s}{1 + \frac{\beta}{\alpha(1-\gamma)} \tau_2 s^2 + \frac{1}{\alpha(1-\gamma)} \tau_1 \tau_2 s^2}$

TABLE 2.7: (continued)

TYPE	CIRCUIT	TRANSFER FUNCTION
NF2AB-3		$\frac{(a_1 + a_3)}{a_1} \frac{1 + \frac{\gamma(a_1 + a_2 + a_3)}{(1-\gamma)(a_1 + a_3)} \tau_2 s}{a_1 + \frac{\beta(a_1 + a_2 + a_3)}{(1-\gamma)a_1} \tau_2 s + \frac{(a_1 + a_2 + a_3)}{(1-\gamma)a_1} \tau_1 \tau_2 s^2}$



TABLE 2.8: PROPERTIES OF THE ACVAS OF CLASS NF2A

TYPE	DC-GAIN	COMPENSATION CONDITION	DESIGN EQUATIONS (*)	PHASE ERROR (°)	MAGNITUDE ERROR (°)	STABILITY CONDITION (**)
NF2AA-1	$\beta^{-1}$	$\gamma = \alpha\beta^{-1}$	$\alpha = \frac{1+(1+4/2K)^{1/2}}{2/2K}$ $\beta = K^{-1}$ $\gamma = \frac{\alpha}{\beta}$	$\frac{-K^2\alpha}{(1-\alpha)^2} (\omega\tau)^3$	$\frac{K}{(1-\alpha)} (\omega\tau)^2$	$\theta > \frac{2(1-\alpha)}{\alpha} - \frac{\alpha}{2}$
NF2AA-2	1	$\gamma = \alpha$	$\alpha < 0.55$ $\gamma = \alpha$ $\beta = \frac{\sqrt{2}\alpha^2}{(1-\alpha)}$	$\frac{-\alpha}{\beta^2(1-\alpha)} (\omega\tau)^3$	$\frac{(\omega\tau)^2}{\beta(1-\alpha)}$	$\theta > \frac{2\beta(1-\alpha)}{\alpha} - \frac{\alpha}{2}$
NF2AA-3	$\frac{a_1+a_3}{a_1}$	$\alpha = \frac{a_1}{a_1+a_3}$	$\frac{a_2}{a_1} = \frac{K^2}{\sqrt{2}} - (1+\frac{\sqrt{2}}{2})K$ $\frac{a_3}{a_1} = K-1$ $\alpha = K^{-1}$	$\frac{-K(a_1+a_2+a_3)^2(\omega\tau)^3}{(K-1)^2 a_1^2}$	$\frac{K(a_1+a_2+a_3)(\omega\tau)^2}{(K-1)a_1}$	$\theta > \frac{2(K-1)a_1}{a_1+a_2+a_3} - \frac{1}{2K}$
NF2AA-4	$\beta^{-1}$	$\frac{a_1+a_2}{a_1} = \beta^{-1}$	$\frac{a_3}{a_1} = \frac{\sqrt{2}K}{K-1} - K$ $\frac{a_2}{a_1} = K$ $\beta = K^{-1}$	$\frac{-K^2(a_1+a_2+a_3)(\omega\tau)^3}{(K-1)a_2}$	$\frac{(a_1+a_2+a_3)K(\omega\tau)^2}{a_2}$	$\theta > \frac{2(K-1)}{K} - \frac{a_1}{(a_1+a_2+a_3)^2}$
NF2AA-5	$\frac{b_1+b_3}{b_1}$	$\frac{a_1+a_2}{a_1} = \frac{b_1+b_3}{b_1}$	$\frac{b_3}{b_1} = K-1$ $\frac{a_2}{a_1} = K-1$ $K + \frac{b_2}{b_1} = \frac{\sqrt{2}(K+\frac{a_3}{a_1})}{2}$	$\frac{-(a_1+a_2+a_3)(b_1+b_2+b_3)^2}{(K-1)a_2 b_1^2} (\omega\tau)^3$	$\frac{(a_1+a_2+a_3)(b_1+b_2+b_3)}{a_2 b_1} (\omega\tau)^2$	$\theta > \frac{2(K-1)b_1}{b_1+b_2+b_3} - \frac{a_1}{(a_1+a_2+a_3)^2}$
NF2AB-1	$\beta^{-1}$	$\gamma = \frac{\alpha}{\beta}$	$\alpha = 0.5[\frac{1}{2K^2} + 2/2]^{1/2}$ $\gamma = \frac{\sqrt{2}}{4K^2}$ $\beta = K^{-1}$ $\gamma = \frac{\alpha}{\beta}$	$\frac{-\gamma K}{(1-\gamma)^2} (\omega\tau)^2$	$\frac{K}{(1-\gamma)} (\omega\tau)^2$	$\theta > \frac{2(1-\gamma)}{K^2\gamma} - \frac{\gamma K}{2}$
NF2AB-1	$(\alpha+\beta)^{-1}$	$\gamma = \frac{\alpha}{\alpha+\beta}$	$\alpha = 0.5[0.5 + 2\frac{\sqrt{2}}{K}]^{1/2}$ $\gamma = \frac{\sqrt{2}}{4}$ $\beta = K^{-1}-\alpha$ $\gamma = \alpha K$	$\frac{-\gamma K}{(1-\gamma)^2} (\omega\tau)^3$	$\frac{K}{(1-\gamma)} (\omega\tau)^2$	$\theta > \frac{2(1-\gamma)}{\gamma} - \frac{\gamma K}{2}$
NF2AB-2	1	$\gamma = \beta$	$\frac{(1-\beta)\alpha}{\beta^2} = \sqrt{2}$	$\frac{-\gamma}{\alpha^2(1-\gamma)^2} (\omega\tau)^3$	$\frac{(\omega\tau)^2}{\alpha(1-\gamma)}$	$\theta > \frac{2\alpha(1-\gamma)}{\beta} - \frac{\beta}{2}$
NF2AB-3	$\frac{a_1+a_3}{a_1}$	$\frac{\gamma}{\beta} = \frac{a_1+a_3}{a_1}$	$\frac{a_2}{a_1} = \frac{\sqrt{2}(1-\beta K)}{\beta} - K$ $\frac{a_3}{a_1} = K-1$ $\gamma = \frac{a_1+a_3}{a_1} \beta$	$-\frac{K}{\beta} \left[ \frac{(a_1+a_2+a_3)^2}{(1-\gamma)a_1} \right] (\omega\tau)^3$	$\frac{(a_1+a_2+a_3)}{(1-\gamma)a_1} (\omega\tau)^2$	$\theta > \frac{2(1-\gamma)a_1}{\beta(a_1+a_2+a_3)} - \frac{\beta}{2}$

(\*) CONSIDERING  $A_1(s) = A_2(s) = \frac{1}{s\tau}$ (\*\*) CONSIDERING  $A_1(s) = A_2(s) = \frac{1}{s\tau(1 + \frac{s\tau}{\theta})}$

SUBCLASS NF2BC:  $f_2 > 0, F_{12} < 0, F_{21} > 0$

SUBCLASS NF2BD:  $f_2 = 0, F_{12} > 0, F_{21} < 0$

SUBCLASS NF2BE:  $f_2 < 0, F_{12} > 0, F_{21} < 0$

SUBCLASS NF2BF:  $f_2 > 0, F_{12} > 0, F_{21} < 0$

Further analysis, however, shows that the realizations of subclasses NF2BD, NF2BE and NF2BF possess very complex tuning characteristics and, for this reason, they will be disregarded in the remainder of this thesis.

Again, the generation of the circuits of class NF2B can be accomplished if the transfer ratios of the resistive network,  $F_{kj}^+$ 's and  $f_k^+$ 's, are realized using different combinations. The possible combinations are shown in Table 2.9. Some observations are in order at this stage:

- (a) Types NF2BA-10 to NF2BA-13 will only lead to circuits which can be obtained by connecting a resistive voltage divider at the input of some of the infinite input impedance circuits shown in Table 2.4. For this reason, these types will not be considered further.
- (b) Type NF2BA-3 leads only to circuits whose gain is equal to one.
- (c) The same applies for types NF2BB-3, NF2BB-5 and NF2BC-1.

TABLE 2.9: TYPES OF ACVAs OF CLASS NF2B

TYPE	$r_1^+$	$r_1^-$	$r_2^+$	$r_2^-$	$F_{12}^+$	$F_{12}^-$	$F_{21}^+$	$F_{21}^-$	$F_{22}^+$	$F_{22}^-$
NF2BA-1	x	0	0	0	x	1	x	0	0	x
NF2BA-2	x	0	0	0	x	1	x	0	x	1
NF2BA-3	x	0	0	0	x	1	1	x	0	x
NF2BA-4	x	0	0	0	x	1	x	x	0	x
NF2BA-5	1	x	0	0	0	x	x	0	0	x
NF2BA-6	1	x	0	0	0	x	x	0	x	1
NF2BA-7	1	x	0	0	0	x	1	x	0	x
NF2BA-8	1	x	0	0	x	1	1	0	0	x
NF2BA-9	1	x	0	0	x	x	x	x	0	x
NF2BA-10	x	0	0	0	x	x	x	0	0	x
NF2BA-11	x	0	0	0	x	x	x	0	x	1
NF2BA-12	x	0	0	0	x	1	1	x	0	x
NF2BA-13	x	0	0	0	x	1	1	0	0	x
NF2BA-14	x	0	0	0	x	1	x	x	0	x
NF2BB-1	x	0	0	x	0	x	x	0	0	x
NF2BB-2	x	0	0	x	0	x	1	x	0	x
NF2BB-3	1	x	0	x	0	x	x	0	0	x
NF2BB-4	1	x	0	x	0	x	1	x	0	x
NF2BB-5	x	0	0	x	x	1	x	0	0	x
NF2BB-6	x	0	0	x	x	1	1	x	0	x
NF2BC-1	x	0	x	x	0	x	x	0	0	x
NF2BC-2	x	0	x	x	x	1	x	0	0	x
NF2BC-3	1	x	x	x	0	x	x	0	0	x

NOTE: x has value between 0 and 1

- (d) Types NF2BC-2 and NF2BC-3 will only lead to circuits with complex tuning procedures. For this reason, these types will not be considered further.

For each of the types of class NF2B, circuits can be generated by applying the resistor elimination procedure mentioned before. The resulting structures are shown in Table 2.10 while the general properties of each circuit are shown in Table 2.11.

#### 2.3.2.b Inverting ACVAs belonging to the FII group

Inverting ACVAs can be generated using the same approach that was used for noninverting realizations. For this reason, the application of the procedure to inverting structures will only be described briefly.

First, two distinct classes of inverting ACVAs, classes IF2A and IF2B, are obtained by making  $F_{22} = 0$  and  $F_{11} = 0$ , respectively.

For class IF2A, restriction (2.7) can be rewritten as eq. (2.30) and, therefore, two subclasses of realizations are possible:

$$\text{SUBCLASS IF2AA: } F_{12} < 0, F_{21} > 0$$

TABLE 2.10: ACVAS OF CLASS NF2B

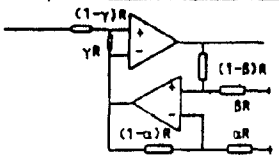
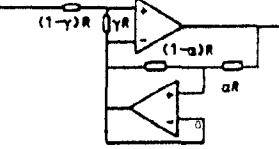
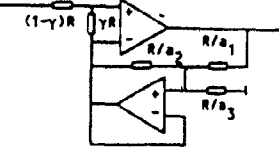
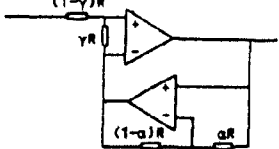
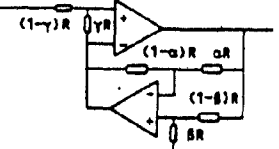
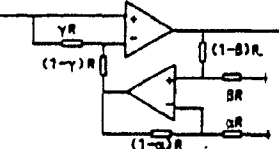
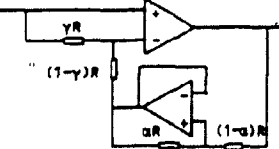
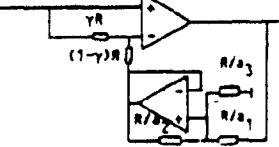
TYPE	CIRCUIT	TRANSFER FUNCTION
NF2BA-1		$\frac{\alpha}{\beta} \frac{1 + \frac{1}{\alpha} \tau_2 s}{1 + \frac{\alpha}{\gamma\beta} \tau_1 s + \frac{1}{\gamma\beta} \tau_1 \tau_2 s^2}$
NF2BA-2		$\frac{1 + \frac{1}{1-\alpha} \tau_2 s}{1 + \frac{1}{\gamma} \tau_1 s + \frac{1}{\gamma} \tau_1 \tau_2 s^2}$
NF2BA-2		$\frac{(a_1 + a_3)}{a_1} \frac{1 + \frac{(a_1 + a_2 + a_3)}{(a_1 + a_3)} \tau_2 s}{1 + \frac{(a_1 + a_3)}{\gamma a_1} \tau_1 s + \frac{(a_1 + a_2 + a_3)}{\gamma a_1} \tau_1 \tau_2 s^2}$
NF2BA-3		$\frac{1 + \frac{1}{\alpha} \tau_2 s}{1 + \frac{1}{\gamma} \tau_1 s + \frac{1}{\alpha\gamma} \tau_1 \tau_2 s^2}$
NF2BA-4		$\frac{\alpha}{\alpha + \beta - 1} \frac{1 + \frac{1}{\alpha} \tau_2 s}{1 + \frac{1}{\gamma(\alpha + \beta - 1)} \tau_1 s + \frac{1}{\gamma(\alpha + \beta - 1)} \tau_1 \tau_2 s^2}$
NF2BA-5		$\frac{\alpha}{\beta} \frac{1 + \frac{1}{\alpha} \tau_2 s}{1 + \frac{\alpha}{\gamma\beta} \tau_1 s + \frac{1}{\gamma\beta} \tau_1 \tau_2 s^2}$
NF2BA-6		$\frac{1 + \frac{1}{\alpha} \tau_2 s}{1 + \frac{1}{\gamma} \tau_1 s + \frac{1}{\alpha\gamma} \tau_1 \tau_2 s^2}$
NF2BA-6		$\frac{a_1 + a_3}{a_1} \frac{1 + \frac{a_1 + a_2 + a_3}{(a_1 + a_3)} \tau_2 s}{1 + \frac{(a_1 + a_3)}{\gamma a_1} \tau_1 s + \frac{(a_1 + a_2 + a_3)}{\gamma a_1} \tau_1 \tau_2 s^2}$

TABLE 2.10: (continued)

TYPE	CIRCUIT	TRANSFER FUNCTION
NF2BA-6		$\frac{(a_2+a_3)}{a_2} \frac{1 + \frac{1}{a} \tau_2 s}{1 + \frac{(a_1+a_2+a_3)}{a_2} \tau_1 s + \frac{(a_1+a_2+a_3)}{a_2 a} \tau_1 \tau_2 s^2}$
NF2BA-7		$\frac{1 + \frac{1}{a} \tau_2 s}{1 + \frac{1}{\gamma} \tau_1 s + \frac{1}{a\gamma} \tau_1 \tau_2 s^2}$
NF2BA-7		$\frac{(a_2+a_3)}{a_2} \frac{1 + \frac{1}{a} \tau_2 s}{1 + \frac{(a_1+a_2+a_3)}{a_2} \tau_1 s + \frac{(a_1+a_2+a_3)}{a_2 a} \tau_1 \tau_2 s^2}$
NF2BA-8		$\frac{(a_2+a_3)a}{a_2} \frac{1 + \frac{1}{a} \tau_2 s}{1 + \frac{a(a_1+a_2+a_3)}{a_2} \tau_1 s + \frac{a(a_1+a_2+a_3)}{a_2} \tau_1 \tau_2 s^2}$
NF2BA-9		$\frac{\alpha}{\alpha+\beta-1} \frac{1 + \frac{1}{a} \tau_2 s}{1 + \frac{\alpha}{\gamma(\alpha+\beta-1)} \tau_1 s + \frac{1}{\gamma(\alpha+\beta-1)} \tau_1 \tau_2 s^2}$
NF2BB-1 [30]		$\beta^{-1} \frac{1 + \tau_2 s}{1 + \frac{\gamma}{\beta} \tau_1 s + \frac{1}{\beta} \tau_1 \tau_2 s^2}$
NF2BB-2		$\beta^{-1} \frac{1 + \tau_2 s}{1 + \frac{\beta}{\gamma a} \tau_1 s + \frac{1}{a\gamma} \tau_1 \tau_2 s^2}$
NF2BB-2		$\frac{1 + s\tau_2}{1 + s\tau_1 + \frac{1}{a} \tau_1 \tau_2 s^2}$

TABLE 2.10: (continued)

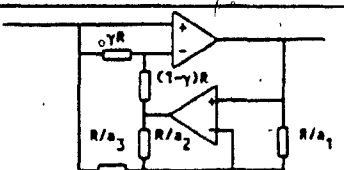
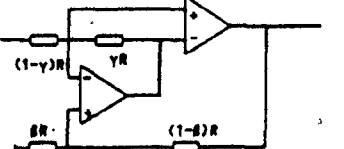
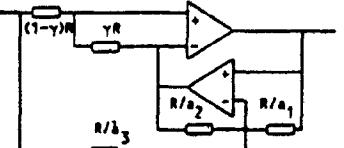
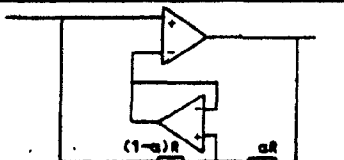
TYPE	CIRCUIT	TRANSFER FUNCTION
NF2BB-3		$\frac{1 + \frac{a_1 + a_2 + a_3}{a_2 + a_3} \tau_2 s}{1 + \frac{a_2}{\gamma(a_2 + a_3)} \tau_1 s + \frac{(a_1 + a_2 + a_3)}{\gamma(a_2 + a_3)} \tau_1 \tau_2 s^2}$
NF2BB-4 [43]		$\beta^{-1} \frac{1 + \tau_2 s}{1 + \frac{(1-\gamma)}{\gamma\beta} \tau_1 s + \frac{1}{\gamma\beta} \tau_1 \tau_2 s^2}$
NF2BB-5		$\frac{1 + \frac{a_1 + a_2 + a_3}{a_2 + a_3} \tau_2 s}{1 + \frac{a_3}{\gamma(a_2 + a_3)} \tau_1 s + \frac{(a_1 + a_2 + a_3)}{\gamma(a_2 + a_3)} \tau_1 \tau_2 s^2}$
NF2BC-1		$\frac{1 + \tau_2 s}{1 + \frac{1}{1-\alpha} \tau_1 s + \frac{1}{1-\alpha} \tau_1 \tau_2 s^2}$

TABLE 2.11: PROPERTIES OF THE ACVAS OF CLASS NF2B

TYPE	DC-GAIN	COMPENSATION CONDITION	DESIGN EQUATIONS (*)	PHASE ERROR (*)	MAGNITUDE ERROR(**)	STABILITY CONDITION(**)
NF2BA-1	$\frac{\alpha}{\beta}$	$\alpha^2 \tau_1 = \gamma \beta \tau_2$	$\alpha = \beta K$ $\gamma = \frac{\alpha^2}{\beta}$	$-\left(\frac{\omega \tau_1}{K\beta}\right)^3$	$\left(\frac{\omega \tau_1}{K\beta}\right)^2$	$\theta > \frac{3K\beta}{2}$
NF2BA-2	1	$(1-\alpha)\tau_1 = \gamma \tau_2$	$\alpha + \gamma = 1$	$-\left(\frac{\omega \tau_1}{\gamma}\right)^3$	$\left(\frac{\omega \tau_1}{\gamma}\right)^2$	$\theta > \frac{3\gamma}{2}$
NF2BA-2	$\frac{a_1+a_3}{a_1}$	$(a_1+a_3)^2 \tau_1 = \gamma a_1(a_1+a_2+a_3)\tau_2$	$\frac{a_3}{a_1} = K-1$ $\gamma = \frac{(a_1+a_3)K}{(a_1+a_2+a_3)}$	$-\left(\frac{K\omega \tau_1}{\gamma}\right)^3$	$\left(\frac{K\omega \tau_1}{\gamma}\right)^2$	$\theta > \frac{3}{2} \frac{\gamma}{K}$
NF2BA-3	1	$\alpha \tau_1 = \gamma \tau_2$	$\alpha = \gamma$	$-\left(\frac{\omega \tau_1}{\alpha}\right)^3$	$\left(\frac{\omega \tau_1}{\alpha}\right)^2$	$\theta > \frac{3}{2} \alpha$
NF2BA-4	$\frac{\alpha}{\alpha+\beta-1}$	$\alpha^2 \tau_1 = \gamma(\alpha+\beta-1)\tau_2$	$\beta = 1 - \frac{\alpha(K-1)}{K}$ $\gamma = \alpha K$	$-\left(\frac{K\omega \tau_1}{\gamma}\right)^3$	$\left(\frac{K\omega \tau_1}{\gamma}\right)^2$	$\theta > \frac{3}{2} \frac{\gamma}{K}$
NF2BA-5	$\frac{\alpha}{\beta}$	$\alpha^2 \tau_1 = \gamma \beta \tau_2$	$\alpha = \beta K$ $\gamma = \frac{\alpha^2}{\beta}$	$-\left(\frac{\omega \tau_1}{K\beta}\right)^3$	$\left(\frac{\omega \tau_1}{K\beta}\right)^2$	$\theta > \frac{3K\beta}{2}$
NF2BA-6	1	$\alpha \tau_1 = \gamma \tau_2$	$\alpha = \gamma$	$-\left(\frac{\omega \tau_1}{\alpha}\right)^3$	$\left(\frac{\omega \tau_1}{\alpha}\right)^2$	$\theta > \frac{3}{2} \alpha$
NF2BA-6	$\frac{a_1+a_3}{a_1}$	$(a_1+a_3)^2 \tau_1 = \gamma a_1(a_1+a_2+a_3)\tau_2$	$\frac{a_3}{a_1} = K-1$ $\gamma = \frac{(a_1+a_3)K}{(a_1+a_2+a_3)}$	$-\left(\frac{K\omega \tau_1}{\gamma}\right)^3$	$\left(\frac{K\omega \tau_1}{\gamma}\right)^2$	$\theta > \frac{3}{2} \frac{\gamma}{K}$



TABLE 2.11: (continued)

TYPE	DC-GAIN	COMPENSATION CONDITION	DESIGN EQUATIONS (*)	PHASE ERROR (*)	MAGNITUDE ERROR(*)	STABILITY CONDITION(**)
NF2BA-6	$\frac{a_2+a_3}{a_2}$	$\alpha\tau_1 = \frac{a_2}{a_1+a_2+a_3} \tau_2$	$\frac{a_3}{a_2} = K-1$ $\alpha = \frac{a_2}{a_1+a_2+a_3}$	$-(\frac{\omega\tau}{\alpha})^3$	$(\frac{\omega\tau}{\alpha})^2$	$\theta > \frac{3}{2}\alpha$
NF2BA-7	1	$\alpha\tau_1 = \gamma\tau_2$	$\alpha = \gamma$	$-(\frac{\omega\tau}{\alpha})^3$	$(\frac{\omega\tau}{\alpha})^2$	$\theta > \frac{3}{2}\alpha$
NF2BA-7	$\frac{a_2+a_3}{a_2}$	$\alpha\tau_1 = \frac{a_2}{a_1+a_2+a_3} \tau_2$	$\frac{a_3}{a_2} = K-1$ $\alpha = \frac{a_2}{a_1+a_2+a_3}$	$-(\frac{\omega\tau}{\alpha})^3$	$(\frac{\omega\tau}{\alpha})^2$	$\theta > \frac{3}{2}\alpha$
NF2BA-8	$\frac{(a_2+a_3)\alpha}{a_2}$	$\alpha^2\tau_1 = \frac{a_2}{a_1+a_2+a_3} \tau_2$	$\frac{a_3}{a_2} = \frac{K}{\alpha} - 1$ $\frac{a_1}{a_2} = \frac{1}{\alpha^2} - \frac{K}{\alpha}$	$-(\frac{\omega\tau}{\alpha})^3$	$(\frac{\omega\tau}{\alpha})^3$	$\theta > \frac{3}{2}\alpha$
NF2BA-9	$\frac{\alpha}{\alpha+\beta-1}$	$\alpha^2\tau_1 = \gamma(\alpha+\beta-1)\tau_2$	$\alpha = \beta K$ $\gamma = \frac{\alpha^2}{\beta}$	$-(\frac{K\omega\tau}{\gamma})^3$	$(\frac{K\omega\tau}{\gamma})^2$	$\theta > \frac{3}{2}\frac{\gamma}{K}$
NF2BB-1	$\beta^{-1}$	$\gamma\tau_1 = \beta\tau_2$	$\beta = K^{-1}$ $\gamma = \beta$	$-(K\omega\tau)^3$	$(K\omega\tau)^2$	$\theta > 2 - \frac{0.5}{K}$
NF2BB-2	$\beta^{-1}$	$\alpha\tau_1 = \gamma\beta\tau_2$	$\beta = K^{-1}$ $\alpha = [\frac{1}{2K}]^{\frac{1}{2}}$ $\gamma = \frac{\alpha}{\beta}$	$-(\alpha\omega\tau)^3$	$(\alpha\omega\tau)^2$	$\theta > 2 - \frac{\alpha}{2}$
NF2BB-2	1	$\tau_1 = \tau_2$	$\alpha = \frac{\sqrt{2}}{2}$	$-(\frac{\omega\tau}{\alpha})^3$	$(\frac{\omega\tau}{\alpha})^2$	$\theta > 2 - \frac{\alpha}{2}$

TABLE 2.11: (continued)

TYPE	DC-GAIN	COMPENSATION CONDITION	DESIGN EQUATIONS (*)	PHASE ERROR (*)	MAGNITUDE ERROR(*)	STABILITY CONDITION(**)
NF2BB-3	1	$\frac{a_2}{a_1+a_2+a_3} \tau_1 = \frac{a_2+a_3}{a_1+a_2+a_3} \tau_2$	$\frac{a_2+a_3}{a_1+a_2+a_3} = \sqrt{2} \gamma^2$	$-(\frac{a_2}{a_2+a_3})^2 (\frac{\omega\tau}{\gamma})^3$	$\frac{a_2}{(a_2+a_3)} (\frac{\omega\tau}{\gamma})^2$	$\theta > 2\gamma (\frac{a_2+a_3}{a_2})$ $-\frac{\gamma}{2}$
NF2BB-4	$\beta^{-1}$	$\frac{(1-\gamma)}{\gamma} \tau_1 = \beta \tau_2$	$\beta = K^{-1}$ $\gamma = \frac{K}{K+1}$	$-(K+1)(\omega\tau)^3$	$(K+1)(\omega\tau)^2$	$\theta > 2 - \frac{0.5}{K+1}$
NF2BB-5	1	$\frac{a_2}{a_1+a_2+a_3} \tau_1 = \frac{a_2+a_3}{a_1+a_2+a_3} \tau_2$	$\frac{a_2+a_3}{a_1+a_2+a_3} = \sqrt{2} \gamma^2$	$-(\frac{a_2}{a_2+a_3})^2 (\frac{\omega\tau}{\gamma})^3$	$\frac{a_2}{(a_2+a_3)} (\frac{\omega\tau}{\gamma})^2$	$\theta > 2\gamma (\frac{a_2+a_3}{a_2})$ $-\frac{\gamma}{2}$
NF2BC-1	1	$\alpha = 1 - \frac{\tau_1}{\tau_2}$	$\alpha = 1 - \frac{\tau_1}{\tau_2}$	$\frac{1}{(1-\alpha)^2} (\omega\tau)^3$	$\frac{1}{(1-\alpha)} (\omega\tau)^2$	$\theta > \frac{3}{2} - 2\alpha$

(\*) CONSIDERING  $A_1(s) = A_2(s) = \frac{1}{s\tau}$ (\*\*) CONSIDERING  $A_1(s) = A_2(s) = \frac{1}{s\tau(1 + \frac{s\tau}{\theta})}$

SUBCLASS IF2AB:  $F_{12} > 0, F_{21} < 0$

Realizations of each subclass are obtained if we attempt to realize the  $F_{kj}^+$ 's and  $f_k^+$ 's transfer ratios using different combinations. Each different combination corresponds to a different type. However, further analysis shows that there is only one possible combination for each subclass as shown in Table 2.12.

Realizations for each type are obtained by applying the resistor elimination procedure mentioned before. The resulting circuits are shown in Table 2.13 while their properties are shown in Table 2.14.

By applying the same considerations that were used for class NF2B, the realizations of class IF2B can be grouped in one of the following subclasses:

SUBCLASS IF2BA:  $f_2 = 0, F_{12} < 0, F_{21} > 0$

SUBCLASS IF2BB:  $f_2 = 0, F_{12} > 0, F_{21} < 0$

SUBCLASS IF2BC:  $f_2 < 0, F_{12} < 0, F_{21} > 0$

SUBCLASS IF2BD:  $f_2 < 0, F_{12} > 0, F_{21} < 0$

SUBCLASS IF2BE:  $f_2 > 0, F_{12} < 0, F_{21} > 0$

SUBCLASS IF2BF:  $f_2 > 0, F_{12} > 0, F_{21} < 0$

Once more, realizations can be obtained by using different ways of realizing the  $F_{kj}^+$ 's and  $f_k^+$ 's transfer

TABLE 2.12: TYPES OF ACVAs IN CLASS IF2A

TYPE	$f_1^+$	$f_1^-$	$F_{11}^+$	$F_{11}^-$	$F_{12}^+$	$F_{12}^-$	$f_2^+$	$f_2^-$	$F_{21}^+$	$F_{21}^-$	$F_{22}^+$	$F_{22}^-$
IF2AA-1	0	x	0	x	0	x	x	0	x	0	0	0
IF2AB-1	0	x	0	x	x	0	0	x	0	x	0	0

NOTE: x has value between 0 and 1

TABLE 2.13: ACVAs OF CLASS IF2A

TYPE	CIRCUIT	TRANSFER FUNCTION
IF2AA-1 [31]		$-\beta \frac{1 + \frac{\beta+1}{\beta} \tau_2 s}{1 + \frac{\beta+1}{\alpha} \tau_2 s + \frac{(\beta+1)(2\alpha+1)}{\alpha} \tau_1 \tau_2 s^2}$
IF2AB-1 [31]		$-\beta \frac{1 + \tau_2 s}{1 + \tau_2 s + (\alpha+1) \tau_1 \tau_2 s^2}$
IF2AB-1 [31]		$-\beta \frac{1 + \frac{1}{\alpha} \tau_2 s}{1 + \frac{1}{\alpha} \tau_2 s + \frac{(\beta+1)}{\alpha} \tau_1 \tau_2 s^2}$
IF2AB-1 [25]		$-\beta \frac{1 + \frac{\alpha(\beta+1)}{\beta(\alpha+1)} \tau_2 s}{1 + \frac{\beta+1}{\alpha+1} \tau_2 s + (\beta+1) \tau_1 \tau_2 s^2}$
IF2AB-1 [25]		$-\beta \frac{1 + \frac{\alpha(\beta+1)}{\gamma\beta(\alpha+1)} \tau_2 s}{1 + \frac{(\beta+1)}{\gamma(\alpha+1)} \tau_2 s + \frac{(\beta+1)}{\gamma} \tau_1 \tau_2 s^2}$

TABLE 2.14: PROPERTIES OF THE ACVs OF CLASS IF2A

TYPE	DC-GAIN	COMPENSATION CONDITION	DESIGN EQUATIONS (*)	PHASE ERROR (*)	MAGNITUDE ERROR(*)	STABILITY CONDITION(**)
IF2AA-1	$\beta$	$\beta = \alpha$	$\beta = -K$ $\alpha = -K$	$\frac{-(1-K)^2(1-2K)}{K^2}(\omega\tau)^3$	$\frac{(1-K)(1-2K)}{K}(\omega\tau)^2$	$\theta > \frac{-2K}{1-K} - \frac{0.5}{1-2K}$
IF2AD-1	$-\beta$	—	$\beta = -K$	$-(1-K)(\omega\tau)^3$	$(1-K)(\omega\tau)^2$	$\theta > 2 - \frac{0.5}{1-K}$
IF2AB-1	$-\beta$	—	$\beta = -K$ $\alpha = \frac{\sqrt{2}}{1-K}$	$\frac{-(1-K)}{\alpha^2}(\omega\tau)^3$	$\frac{\sqrt{2}(1-K)}{\alpha}(\omega\tau)^2$	$\theta > 2\alpha - \frac{0.5}{1-K}$
IF2AB-1	$-\beta$	$\alpha = \beta$	$\beta = -K$ $\alpha = -K$	$-(1-K)(\omega\tau)^3$	$(1-K)(\omega\tau)^2$	$\theta > 2 - \frac{0.5}{1-K}$
IF2AB-1	$-\beta$	$\alpha = \beta$	$\beta = -K$ $\alpha = -K$ $\alpha = \frac{\sqrt{2}}{1-K}$	$\frac{-(1-K)}{\gamma^2}(\omega\tau)^3$	$\frac{(1-K)}{\gamma}(\omega\tau)^2$	$\theta > 2\gamma - \frac{0.5}{1-K}$

(\*) CONSIDERING  $A_1(s) = A_2(s) = \frac{1}{s\tau}$

(\*\*) CONSIDERING  $A_1(s) = A_2(s) = \frac{1}{s\tau(1+s\frac{\tau}{\theta})}$

ratios. Each of these ways characterizes a different type of realization. The possible types for class IF2B are shown in Table 2.15. Some comments are in order at this point:

- (a) Realizations belonging to type IF2BB-3 can be obtained by connecting a resistive voltage divider at the input of the infinite input impedance ACVAs of class II2A. Because the circuits of that class have already been presented in this thesis, this type will not be considered further.
- (b) Realizations of type IF2BD-3 as well as circuits of subclasses IF2BE and IF2BF will have very complex tuning characteristics and, for this reason, they will not be considered further.

For each type, realizations can be generated by applying the previously described resistor elimination procedure. The resulting circuits are presented in Table 2.16. The general properties of these circuits are shown in Table 2.17.

TABLE 2.15: TYPES OF ACVAs OF CLASS IF2B.

TYPE	$f_1^+$	$f_1^-$	$f_2^+$	$f_2^-$	$f_{12}^+$	$f_{12}^-$	$f_{21}^+$	$f_{21}^-$	$f_{22}^+$	$f_{22}^-$
IF2BA-1	0	x	0	0	0	x	x	0	0	x
IF2BA-2	0	x	0	0	0	x	1	x	0	x
IF2BA-3	0	x	0	0	0	x	x	0	x	1
IF2BB-1	x	1	0	0	x	0	0	x	0	x
IF2BB-2	0	x	0	0	1	x	0	x	0	x
IF2BB-3	0	x	0	0	x	0	0	x	0	x
IF2BC-1	0	x	x	x	0	x	x	0	0	x
IF2BC-2	0	x	0	x	0	x	1	0	0	x
IF2BD-1	x	1	0	x	x	0	0	x	0	x
IF2BD-2	0	x	0	x	1	x	0	x	0	x
IF2BD-3	0	x	0	x	x	0	0	x	0	x
IF2BE-1	0	x	x	0	0	x	x	0	0	x
IF2BE-2	0	x	x	0	0	x	x	0	x	1
IF2BF-1	0	x	x	0	x	0	0	x	0	x
IF2BF-2	0	x	1	x	x	0	0	x	0	x
IF2BF-3	x	1	x	0	x	0	0	x	0	x
IF2BF-4	0	x	x	0	1	x	0	x	0	x

NOTE: x has value between 0 and 1



TABLE 2:16: ACVAS OF CLASS IF2B

TYPE	CIRCUIT	TRANSFER FUNCTION
IF2BA-1		$\frac{-\alpha\gamma}{(1-\gamma)\beta} \frac{1 + \frac{1}{\alpha}\tau_2 s}{1 + \frac{\alpha}{(1-\gamma)\beta}\tau_1 s + \frac{1}{(1-\gamma)\beta}\tau_1\tau_2 s^2}$
IF2BA-2 [27]		$\frac{\gamma}{(1-\gamma)} \frac{1 + \frac{1}{\alpha}\tau_2 s}{1 + \frac{1}{(1-\gamma)}\tau_1 s + \frac{1}{\alpha(1-\gamma)}\tau_1\tau_2 s^2}$
IF2BA-3 [31]		$\frac{\gamma}{(1-\gamma)} \frac{1 + \frac{1}{\alpha}\tau_2 s}{1 + \frac{1}{(1-\gamma)}\tau_1 s + \frac{1}{\alpha(1-\gamma)}\tau_1\tau_2 s^2}$
IF2BB-1		$\frac{-(a_2+a_3)\alpha}{a_2(1-\alpha)} \frac{1 + \frac{1}{\alpha}\tau_2 s}{1 + \frac{a_2}{\gamma a_1}\tau_1 s + \frac{(a_1+a_2+a_3)}{\gamma a_1}\tau_1\tau_2 s^2}$
IF2BB-1		$\frac{-a_2}{a_1} \frac{1 + \frac{(a_1+a_2+a_3)}{a_2}\tau_2 s}{1 + \frac{a_2}{\gamma a_1}\tau_1 s + \frac{(a_1+a_2+a_3)}{\gamma a_1}\tau_1\tau_2 s^2}$
IF2BB-2		$\frac{-\alpha}{(1-\alpha)} \frac{1 + \frac{1}{\alpha}\tau_2 s}{1 + \frac{\alpha}{\gamma(1-\alpha)}\tau_1 s + \frac{1}{\gamma(1-\alpha)}\tau_1\tau_2 s^2}$
IF2BC-1		$\frac{-\alpha}{(1-\alpha)} \frac{1 + \frac{1}{\gamma}\tau_2 s}{1 + \frac{1}{(1-\alpha)}\tau_1 s + \frac{1}{\gamma(1-\alpha)}\tau_1\tau_2 s^2}$
IF2BC-2 [26]		$\frac{1 + (1 + \frac{\alpha}{\gamma} + \frac{\alpha}{\beta})\tau_2 s}{\gamma + (\gamma+1)\tau_1 s + [(\gamma+1)(1 + \frac{\alpha}{\beta}) + \alpha]\tau_1\tau_2 s^2}$

TABLE 2.16: (continued)

TYPE	CIRCUIT	TRANSFER FUNCTION
IF2BD-1		$\frac{-(a_2 + a_3)}{a_1} \frac{1 + \frac{(a_1 + a_2 + a_3)}{(a_2 + a_3)} \tau_2 s}{1 + \frac{a_2}{\gamma a_1} \tau_1 s + \frac{(a_1 + a_2 + a_3)}{\gamma a_1} \tau_1 \tau_2 s^2}$
IF2BD-2		$\frac{-(a_2 + a_3)}{a_1} \frac{1 + \frac{(a_1 + a_2 + a_3)}{(a_2 + a_3)} \tau_2 s}{1 + \frac{a_2}{\gamma a_1} \tau_1 s + \frac{(a_1 + a_2 + a_3)}{\gamma a_1} \tau_1 \tau_2 s^2}$

TABLE 2.17: PROPERTIES OF THE ACVAs OF CLASS IF2B

TYPE	DC-GAIN	COMPENSATION CONDITION	DESIGN EQUATIONS (*)	PHASE ERROR (*)	MAGNITUDE ERROR(*)	STABILITY CONDITION(**)
IF2BA-1	$\frac{-\alpha\gamma}{(1-\gamma)\beta}$	$\alpha^2\tau_1 = (1-\gamma)\beta\tau_2$	$\gamma = -K\alpha$ $\beta = \frac{\alpha^2}{(1-\gamma)}$	$-(\frac{\omega\tau}{\alpha})^3$	$(\frac{\omega\tau}{\alpha})^2$	$\theta > \frac{3}{2}\alpha$
IF2BA-2 IF2BA-3	$\frac{-\gamma}{(1-\gamma)}$	$\alpha\tau_1 = (1-\gamma)\tau_2$	$\gamma = \frac{K}{K-1}$ $\alpha = \frac{-1}{K-1}$	$[(1-K)\omega\tau]^3$	$[(1-K)\omega\tau]^2$	$\theta > \frac{3}{2(1-K)}$
IF2BB-1	$\frac{-(a_2+a_3)\alpha}{a_2(1-\alpha)}$	$\frac{\alpha}{1-\alpha}\tau_1 = \frac{a_2}{a_1+a_2+a_3}\tau_2$	$\frac{a_3}{a_2} = \frac{-K(1-\alpha)}{\alpha} - 1$ $\frac{a_1}{a_2} = (\frac{1}{\alpha} + K)(\frac{1-\alpha}{\alpha})$	$-(\frac{\omega\tau}{\alpha})^3$	$(\frac{\omega\tau}{\alpha})^2$	$\theta > \frac{3}{2}\alpha$
IF2BB-1	$-\frac{a_2}{a_1}$	$\frac{a_2^2}{a_1(a_1+a_2+a_3)}\tau_1 = \gamma\tau_2$	$\frac{a_2}{a_1} = -K$ $\gamma = \frac{-a_2K}{a_1+a_2+a_3}$	$(\frac{K\omega\tau}{\gamma})^3$	$(\frac{K\omega\tau}{\gamma})^2$	$\theta > \frac{3}{2}(\frac{-\gamma}{K})$
IF2BB-2	$\frac{1}{1-\alpha}$	$\frac{\alpha^2}{(1-\alpha)}\tau_1 = \gamma\tau_2$	$\alpha = \frac{K}{K-1}$ $\gamma = \frac{-K^2}{K-1}$	$-(\frac{(K-1)\omega\tau}{K})^3$	$(\frac{(K-1)\omega\tau}{K})^2$	$\theta > \frac{3}{2}\frac{K}{K-1}$
IF2BC-1	$\frac{-\alpha}{1-\alpha}$	$\gamma\tau_1 = (1-\alpha)\tau_2$	$\alpha = \frac{K}{K-1}$ $\gamma = \frac{-1}{K-1}$	$[(K-1)\omega\tau]^3$	$[(K-1)\omega\tau]^2$	$\theta > \frac{3}{2(1-K)}$
IF2BC-2	$-\alpha$	$\gamma\tau_1 = (\frac{\alpha}{\gamma} + \frac{\alpha}{\beta})\tau_2$	$\gamma = -K$ $\beta = 1$ $\alpha = \frac{\gamma^2}{\gamma+1}$	$[K(2-K)(1-K)+1] \times (\omega\tau)^3$	$[-K(2-K) + \frac{1}{1-K}] \times (\omega\tau)^2$	$\theta > \frac{2(1+\gamma)}{2[\gamma(\gamma+1)(\gamma+2)+1]} - \frac{(\gamma+1)^2}{2[\gamma(\gamma+1)(\gamma+2)+1]}$
IF2BD-1 IF2BD-2	$-\frac{a_2+a_3}{a_1}$	$\frac{a_2(a_2+a_3)\tau_1}{a_1(a_1+a_2+a_3)} = \gamma\tau_2$	$\frac{a_1}{a_2} = -\frac{\sqrt{2}}{K}$ $\frac{a_3}{a_1} = -K[\frac{\sqrt{2}-1}{\sqrt{2}}]$ $\gamma = \frac{-a_2K}{a_1+a_2+a_3}$	$\frac{a_2^2K}{a_1^2}(\frac{\omega\tau}{\gamma})^3$	$\frac{a_2^2K}{a_1^2}(\frac{\omega\tau}{\gamma})^2$	$\theta > \frac{2\gamma a_1}{a_2} - \frac{\gamma}{2K}$

(\*) CONSIDERING  $A_1(s) = A_2(s) = \frac{1}{s\tau}$ (\*\*) CONSIDERING  $A_1(s) = A_2(s) = \frac{1}{s\tau(1 + \frac{s\tau}{\theta})}$

## 2.4. EVALUATION OF THE ACVAs

### 2.4.1 Design equations and relative stability

All the ACVAs presented so far are stable by themselves provided the OA is modeled as in eq. (1.24). Nevertheless, it is desirable to provide design equations such that the circuits, realized using these equations, will exhibit a suitable amount of relative stability [39,40], namely, a phase margin not less than  $30^\circ$  and not greater than  $60^\circ$  (the gain margin is always infinite for any active-R 2 OA circuit when the model in eq. (1.24) is used).

However, not all the circuits presented have free design parameters to allow the control of the phase margin. Obviously, when such control is not possible, some other criterion for deriving design equations has to be sought. Fortunately, in these cases, the great majority of circuits have design equations completely specified by the gain and the compensation condition and no further consideration is necessary to design them.

Considering the cases where the control of the phase margin is possible by the use of appropriate design, one is faced with a problem. Such design equations are, in general, dependent on the values of the gain-bandwidth product of the OAs used. This is an inconvenience due to

the large variability of these parameters.

Two factors, however, reduce the effect of this dependence: first, the wide range of values for the phase margin acceptable in practice and, second, the design equations for such margin depend not on the GBs directly but on their ratio. If ACVAs are implemented with dual OAs (in which the different OAs characteristics track with each other closely), it has been found that if the design equations shown in Tables 2.5, 2.8, 2.9, 2.14 and 2.17 are used, the circuits will exhibit acceptable values of phase margin even if there is a 20% mismatch among the OAs. In practice, such mismatches are much less than 20% for dual OAs. Some comments about these design equations are in order at this stage:

- (a) All the ACVAs of the III group (Tables 2.4 and 2.5) have their design equations completely defined by the gain and the compensation condition. Hence, no control over the phase margin is possible.
- (b) All the design equations in Table 2.8 are intended to enable the circuits to exhibit a phase margin of approximately  $45^\circ$ . However, in order to use them for types NF2AA-3 and NF2AA-4, the gain has to be restricted to be greater and smaller than  $(1 + \sqrt{2})$ , respectively.

- (c) All the ACVAs in Tables 2.10 and 2.11 have phase margin of approximately  $51.8^\circ$ , except those of subclass NF2BB. The ACVAs of types NF2BB-1 and NF2BB-4 can not have their phase margin controlled. The other circuits of subclass NF2BB have design equations intended to enable them to exhibit phase margin in the range mentioned above.
- (d) The circuits in Tables 2.13 and 2.14, except the second and the fourth circuit of type IF2AB-1, have their design defined by the gain and the compensation condition. For those two exceptions, the equations presented are intended to enable them to exhibit phase margin in the range mentioned above.
- (e) Considering Table 2.18, all the ACVAs of subclasses IF2BA and IF2BB as well as the one of type IF2BC-1 have phase margin approximately equal to  $51.8^\circ$ . The circuit of type IF2BC-2 has its phase margin varying from  $54.5^\circ$  to  $51.8^\circ$  as its DC-gain is varied from 1 up to very large values. The ACVAs of subclass IF2BD have design equations intended to enable them to exhibit a phase margin equal to approximately  $45^\circ$ .

#### 2.4.2 Gain and phase errors

The use of a more complex realization for a VA instead of the conventional designs in Fig. 1.2 is justified only by the fact that the ACVAs are expected to exhibit smaller phase and magnitude errors than the latter. In order to verify such an assumption, the phase and magnitude errors of the realizations were computed and are shown in Tables 2.5, 2.8, 2.11, 2.14 and 2.17. The phase and magnitude errors of the noninverting conventional realization are given, respectively, by eqns. (1.10) and (1.11). Also, for the inverting VA realization, these errors are given, respectively, by eqns. (1.13) and (1.14). By comparing expressions (1.10) through (1.14) with the corresponding ones in Tables 2.5, 2.8, 2.11, 2.14 and 2.17, it is clear that the high frequency operation in conventional VAs is dominated by phase errors while the operation in ACVAs is restricted by magnitude errors. Since the absolute values of  $K\omega\tau$  and  $(1 - K)\omega\tau$  are much smaller than unity, magnitude errors in ACVAs are typically an order of magnitude less than the phase errors in conventional VAs. Thus, the ACVAs can be used at considerably higher operating frequencies than their conventional counterparts.

### 2.4.3 Effect of the second pole of the OAs

So far, the complex frequency-dependent gain of the OA was modeled by eq. (1.24). This model provides good accuracy for frequencies well below the GB of the OA. However, as it was explained in Chapter 1, existence of a second pole in actual OA characteristics renders this model inaccurate at high frequencies. It can even cause the ACVAs to oscillate, if not properly accounted for.

Taking this second pole into consideration, the differential gain of the  $i$ th OA can be modeled as

$$A_i(s) = \frac{1}{s\tau_i(1 + s\tau_i/\theta_i)} \quad (2.31)$$

where the second pole is given by  $-\frac{\theta_i}{\tau_i}$ .

Clearly, the study of stability, considering the second pole, can be done by making the substitution

$$s\tau_i \rightarrow s\tau_i(1 + s\tau_i/\theta_i)$$



in the denominator of the transfer function expressions in Tables 2.4, 2.6, 2.10, 2.13 and 2.16 and then checking if the resulting polynomial is Hurwitz. This is easily done by applying the Routh-Hurwitz test [39,40].

The result of the application of this test is shown in Tables 2.5, 2.8, 2.11, 2.14 and 2.17. Note that, in order to simplify the expressions, the gains of the OAs are considered equal, namely,  $\tau_i = \tau$  and  $\theta_i = 0$  for  $i = 1, 2$ .

#### 2.4.4 Maximum signal handling capability

OA based circuits may have a reduced signal handling capability at high frequencies due to basically two factors [43]. The power supply voltage level of the OAs limits the maximum signal amplitude that can be handled by the circuit. At higher frequencies, the slew rate of the OAs worsens the situation.

Regarding the power supply level, it has been shown [8] that the maximum output signal level that an OA supplied with voltages  $\pm V_{cc}$  can handle is given by

$$V_{o, \max}^{(PS)} - V_{ce}(\text{sat}) \approx V_{cc} \quad (2.32)$$

where  $V_{ce}(\text{sat})$  is the voltage between the collector and the

emitter of a transistor biased in the saturation region. Usually,  $V_{cc} \gg V_{ce}(\text{sat})$ .

For sinusoidal signals of angular frequency equal to  $\omega$ , the slew rate of the OA limits the maximum amplitude at the output to be

$$V_{o,\max}^{(SR)} = \frac{SR}{\omega} \quad (2.33)$$

where SR is the slew rate of the OA [41].

Let the voltage transfer function from the input of the ACVA to the output of the  $i$ th OA be called  $H_i(j\omega)$ ,  $i=1,2$ . From straightforward considerations, it follows that the maximum amplitude of a sinusoidal signal that the OA can handle is limited to

$$V_{i,\max} = \frac{\min\{V_{o,\max}^{(PS)}, V_{o,\max}^{(SR)}\}}{\max\{|H_1(j\omega)|, |H_2(j\omega)|\}} \quad (2.34)$$

where  $\min\{ \}$  and  $\max\{ \}$  denotes the minimum and the maximum element of the bracketed set, respectively.

By using (2.34), plots of the maximum allowable signal amplitude as a function of frequency were obtained. Fig. 2.4 shows only three examples of these plots, namely the conventional realization, the ACVA of type NI2B-4 and the ACVA of type II2A-2. For these plots, we have considered a gain equal to 2,  $V_{CC}$  equal to 15 V and a 741-type OA with SR equal to 0.5 V/ $\mu$ s and GB equal to  $2\pi \times 10^6$  rad/s. Notice that for frequencies up to 100 kHz, all the ACVAs presented here, except those of type NI2B-4, have maximum signal handling capability comparable to the one presented by the conventional realizations. For frequencies above this range, there is a slight decrease in the signal handling capability of the ACVAs compared to the conventional realization. This is expected to happen, since, for the latter, the magnitude of the transfer function decays very quickly as the frequency increases. Hence, this slight decrease in signal handling capability is a consequence of the improved response of the ACVAs.

#### 2.4.5 Tuning procedure

Regardless of the technology of implementation, the response of actual ICs departs from the nominal one due to manufacturing tolerances as well as parasitic effects. A very appropriate and economical way to correct this departure is by an after-production tuning of the circuits [9,42]. Hence, it is of practical relevance to specify a

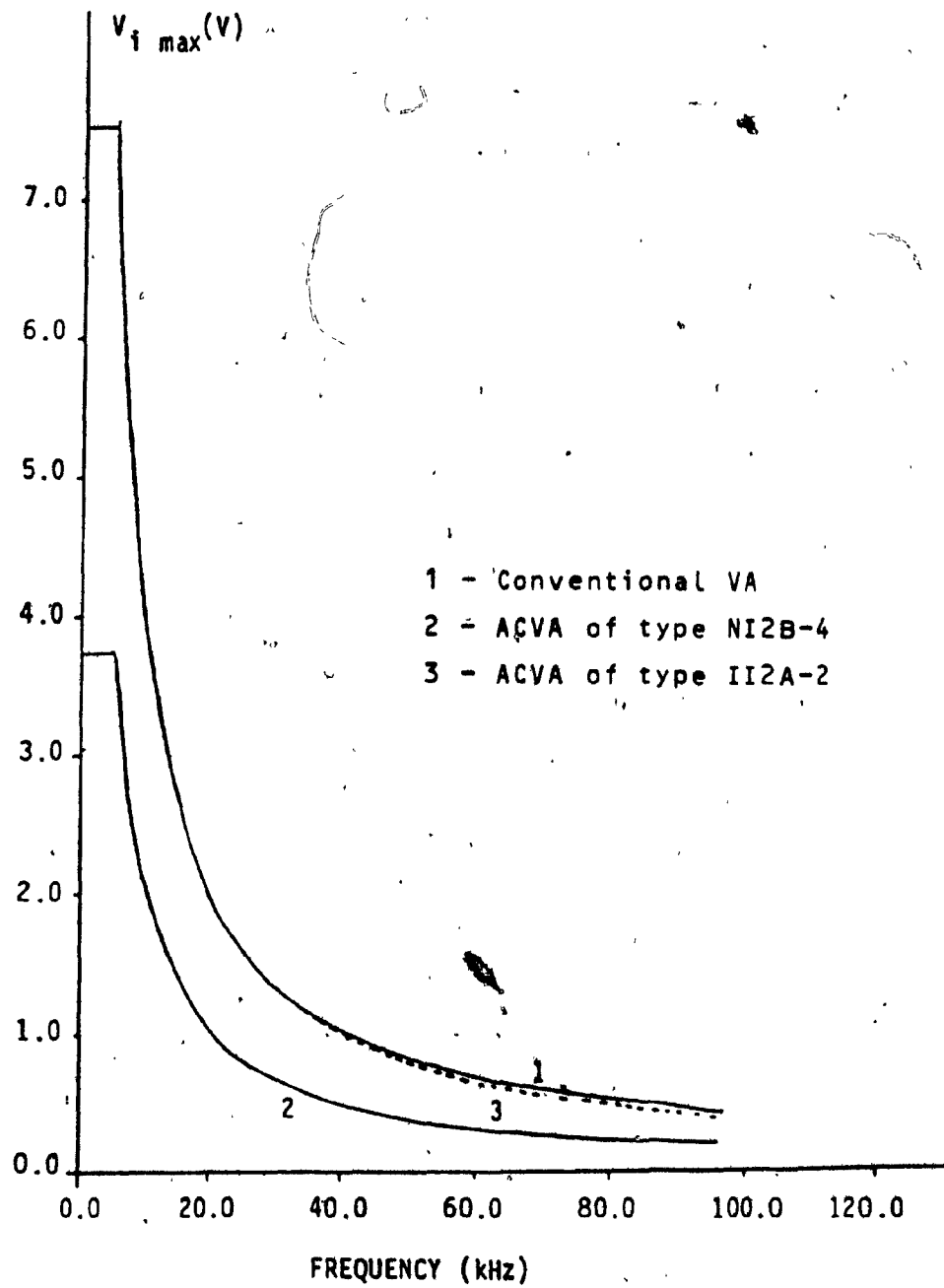


Fig. 2.4: Examples of maximum signal handling capability of different VAs.

tuning procedure for the ACVAs presented here. This procedure is accomplished through the following steps for each of the realizations:

- (a) At a frequency such that  $(K \omega \tau) \ll 1$ , adjust the DC-gain by modifying the proper resistor ratio, namely,  $\gamma, \alpha, \beta$  or  $a_i$  as the case may be.
- (b) At a frequency such that  $(K \omega \tau)^2 \ll 1$ , null the phase-shift of the circuit by modifying the appropriate resistor ratio, namely,  $\gamma, \alpha, \beta$  or  $a_i$  as the case may be.

In this tuning procedure, only resistor ratios are modified. Since resistor ratios can be increased or decreased by trimming the proper resistor, this procedure is feasible in hybrid IC technology. Further, the tracking among resistors implemented in thin or thick film technology and also among OAs placed in the same package makes the tuning effective over a wide range of variation of temperature and power supply level.

It is important to note that some of the circuits presented in this chapter are more suitable for the application of the tuning algorithm described above than the others. This is due to the fact that in these circuits the DC-gain and the compensation condition can be controlled independently of each other. In this case, the tuning is done in only one iteration in a sequential manner. For

circuits that do not possess this independence of adjustments between the gain and the compensation, the two steps mentioned must be applied iteratively until satisfactory performance is obtained.

## 2.5. EXPERIMENTAL RESULTS

### 2.5.1 The ACVAs as stand alone elements

All the ACVAs were tested in the laboratory. The GBs of the A1 and A2 amplifiers were measured to be 1063 and 1054 kHz, respectively. The resistors used were within 1% accuracy. To save space, the theoretical and the experimental responses for only the ACVA of type NI2B-4 (the first circuit) and the conventional realization are shown in Fig. 2.5. The ACVA is observed to perform well even at frequencies near 100 kHz. At higher frequencies, however, the experimental response departs significantly from the theoretical one. This is due to the effect of the second pole of the OA which was neglected in the analysis at low frequencies.

### 2.5.2 An application in active-RC filter

The Sallen-Key filter in Fig. 2.6 was designed to realize a band-pass transfer function with  $Q$  equal to 10 and

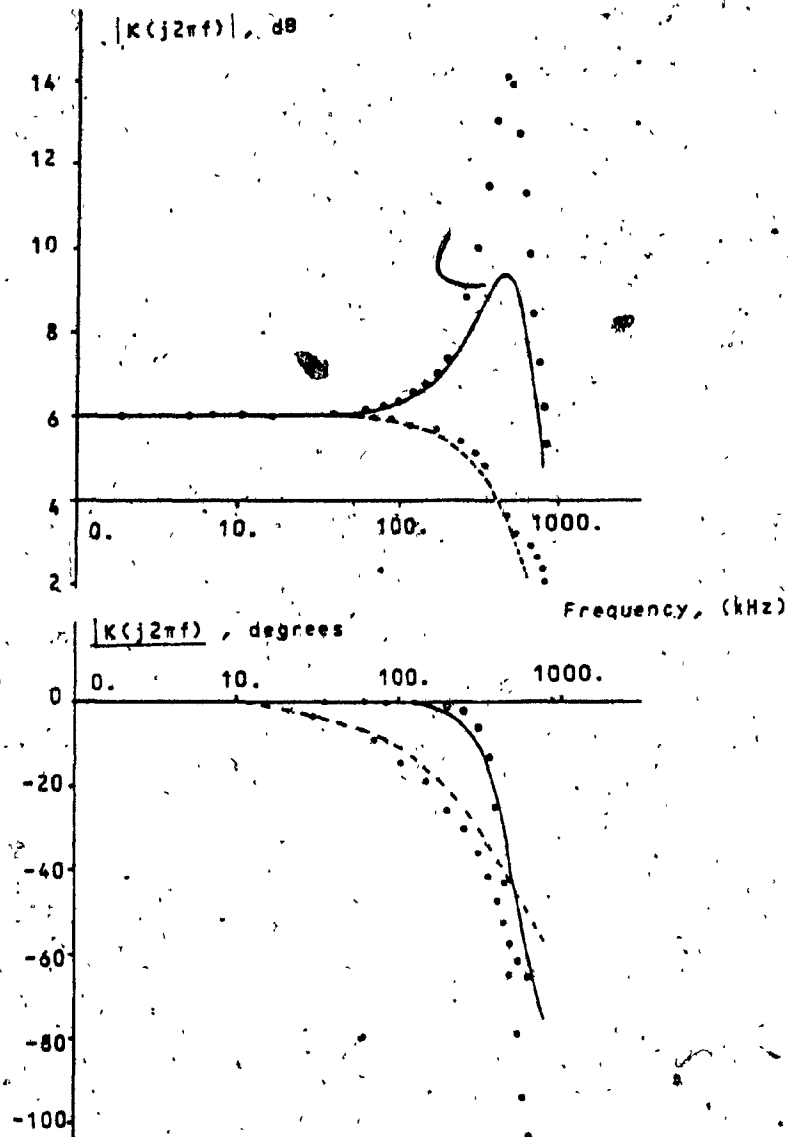


Fig. 2.5: Responses of the ACVA of type NI2B-4  
(first circuit) and the conventional VA

- theoretical, ACVA
- ○ experimental, ACVA
- - theoretical, conventional VA
- ● experimental, conventional VA

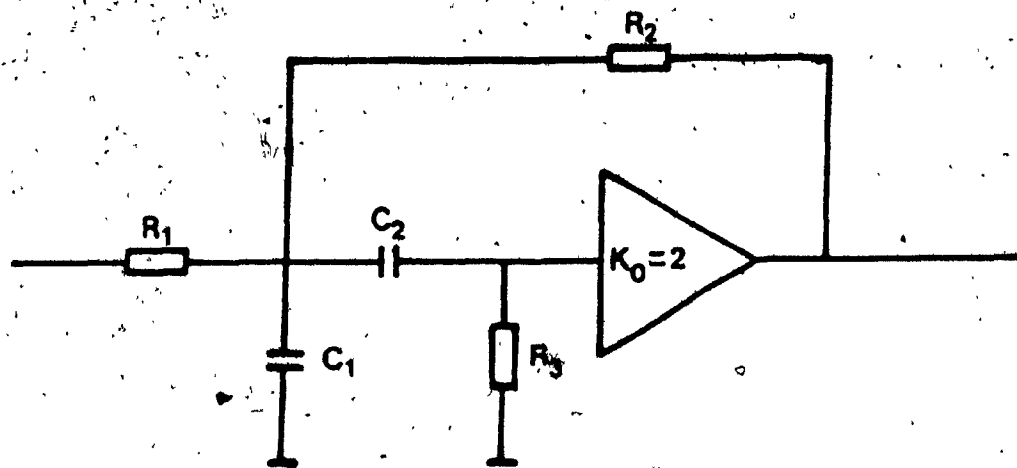


Fig. 2.6: The Sallen-Key filter used for practical experiments.



pole frequency equal to 10 kHz under the assumption of ideal OAs. A comparison between the performance of the filter when the VA was implemented using either the conventional realization or the ACVA of type NI2B-4 (the first circuit) is shown in Table 2.18.

From this table, it is clear that the filter using the ACVA significantly outperforms the one using the conventional VA.

## 2.6. CONCLUSIONS

A systematic and comprehensive synthesis procedure for generating ACVAs employing 2 OAs has been proposed. In this approach, the ACVAs are first grouped into classes using a combination of practical and theoretical considerations. Additional combinations of these considerations are employed to further subdivide each class into several types. A set of realizability conditions is then associated with each type. These conditions lead directly to the synthesis of the ACVAs using the sequential resistor elimination procedure. A set of 57 circuits has thus been obtained and classified.

An extensive study of all these ACVAs has also been made. Different performance aspects such as phase and magnitude errors, maximum signal handling capability, tunability and effect of the second pole are considered in

TABLE 2.18: EXPERIMENTAL RESULTS FOR THE  
SALLEN-KEY FILTER.

PARAMETER	THEORETICAL	CONVENTIONAL VA	ACVA
$f_0$ (kHz)	10.05	9.73	10.10
$\Delta f_0$ (%)	—	-3.15	+0.50
Q	10.55	9.73	9.85
$\Delta Q$ (%)	—	-7.86	-6.72

detail.

Finally, experimental results of the circuits as stand-alone elements as well as in an active filter application are reported. These results agree closely with the theoretical predictions.

All the circuits presented in this chapter outperform the conventional VAs in the sense that they can be used at higher frequencies than the latter. However, when the operating frequency is further increased, even 2 OA ACVAs have their performance degraded. In this case, ACVAs with an increased number of OAs should be considered. For this reason, 3 OA ACVAs are studied in the following chapter.

## CHAPTER 3

CLASSIFICATION, GENERATION AND DESIGN  
OF  
ACTIVELY COMPENSATED VOLTAGE AMPLIFIERS  
USING 3 OAs

## 3.1 INTRODUCTION

Although the operating frequency range of VAs can be extended by increasing the number of OAs in the circuits through an appropriate design, practical considerations indicate that ACVAs employing more than 3 OAs may not be useful. ACVAs using 2 OAs were the subject of the study reported in the last chapter. Clearly, then, the logical continuation of the work reported in this thesis consists in performing a similar study for 3 OA ACVAs. This is the purpose of the present chapter.

The procedures used for classifying and generating 2 OA ACVAs cannot be applied in a straightforward manner to 3 OA realizations due to the characteristics of the latter. Thus, some new criteria for classifying 3 OA ACVAs are described in this chapter. Each class is further divided into types. A set of topological conditions is derived for each type. They can be termed as realizability conditions for that type. These conditions lead directly to the different circuit realizations of 3 OA ACVAs. The circuits

are then evaluated with respect to the performance characteristics considered in Chapter 2. Experimental results are also presented.

### 3.2 PRELIMINARY ANALYSIS

The general configuration of a 3 OA amplifier is shown in Fig. 3.1. The output is taken as  $v_1$  without any loss of generality. The voltage gain can be expressed as

$$\frac{v_1}{v_i} = K \frac{1 + s[a_1\tau_2 + a_2\tau_3] + a_3\tau_2\tau_3 s^2}{1 + s[b_1\tau_1 + b_2\tau_2 + b_3\tau_3] + s^2[b_4\tau_2\tau_3 + b_5\tau_1\tau_3 + b_6\tau_1\tau_2] + b_7\tau_1\tau_2\tau_3 s^3} \quad (3.1)$$

where it is assumed that the  $i$ th OA is modeled as in eqn. (1.24), for  $i=1,2,3$ .

Further, if the resistive network is characterized by

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} v_i \quad (3.2)$$

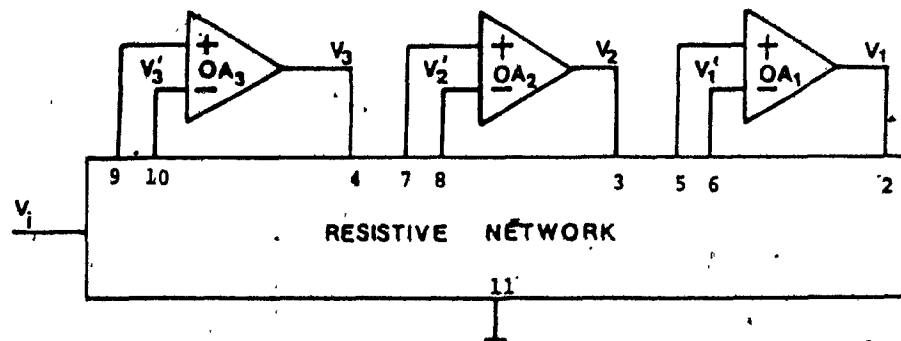


Fig. 3.1: General configuration of a 3 OA ACVA.

where

$$F_{kj} = \frac{v_k}{v_j} \Big|_{v_h=0, v_i=0, h \neq j}; \quad f_k = \frac{v_k}{v_i} \Big|_{v_h=0}; \quad h, j, k = 1, 2, 3$$

the coefficients in (3.1) are given by

$$K = \frac{f_2 F_{12} F_{33} + f_1 F_{23} F_{32} + f_3 F_{13} F_{22} - f_1 F_{22} F_{33} - f_2 F_{13} F_{32} - f_3 F_{12} F_{23}}{F_{11} F_{22} F_{33} + F_{12} F_{23} F_{31} + F_{13} F_{21} F_{32} - F_{13} F_{31} F_{22} - F_{11} F_{23} F_{32} - F_{12} F_{21} F_{33}} \quad (3.3)$$

$$a_1 = \frac{f_1 F_{33} - f_3 F_{13}}{f_2 F_{12} F_{33} + f_1 F_{23} F_{32} + f_3 F_{13} F_{22} - f_1 F_{22} F_{33} - f_2 F_{13} F_{32} - f_3 F_{12} F_{23}} \quad (3.4)$$

$$a_2 = \frac{f_1 F_{22} - f_2 F_{12}}{f_2 F_{12} F_{33} + f_1 F_{23} F_{32} + f_3 F_{13} F_{22} - f_1 F_{22} F_{33} - f_2 F_{13} F_{32} - f_3 F_{12} F_{23}} \quad (3.5)$$

$$a_3 = \frac{-f_1}{f_2 F_{12} F_{33} + f_1 F_{23} F_{32} + f_3 F_{13} F_{22} - f_1 F_{22} F_{33} - f_2 F_{13} F_{32} - f_3 F_{12} F_{23}} \quad (3.6)$$

$$b_1 = \frac{F_{23} F_{32} - F_{22} F_{33}}{F_{11} F_{22} F_{33} + F_{12} F_{23} F_{31} + F_{13} F_{21} F_{32} - F_{13} F_{31} F_{22} - F_{11} F_{23} F_{32} - F_{12} F_{21} F_{33}} \quad (3.7)$$

$$b_2 = \frac{F_{13} F_{31} - F_{11} F_{33}}{F_{11} F_{22} F_{33} + F_{12} F_{23} F_{31} + F_{13} F_{21} F_{32} - F_{13} F_{31} F_{22} - F_{11} F_{23} F_{32} - F_{12} F_{21} F_{33}} \quad (3.8)$$

$$b_3 = \frac{F_{12} F_{21} - F_{11} F_{22}}{F_{11} F_{22} F_{33} + F_{12} F_{23} F_{31} + F_{13} F_{21} F_{32} - F_{13} F_{31} F_{22} - F_{11} F_{23} F_{32} - F_{12} F_{21} F_{33}} \quad (3.9)$$

$$b_4 = \frac{F_{11}}{F_{11}F_{22}F_{33} + F_{12}F_{23}F_{31} + F_{13}F_{21}F_{32} - F_{13}F_{31}F_{22} - F_{23}F_{32}F_{11} - F_{12}F_{21}F_{33}} \quad (3.10)$$

$$b_5 = \frac{F_{22}}{F_{11}F_{22}F_{33} + F_{12}F_{23}F_{31} + F_{13}F_{21}F_{32} - F_{13}F_{31}F_{22} - F_{23}F_{32}F_{11} - F_{12}F_{21}F_{33}} \quad (3.11)$$

$$b_6 = \frac{F_{33}}{F_{11}F_{22}F_{33} + F_{12}F_{23}F_{31} + F_{13}F_{21}F_{32} - F_{13}F_{31}F_{22} - F_{23}F_{32}F_{11} - F_{12}F_{21}F_{33}} \quad (3.12)$$

$$b_7 = \frac{-1}{F_{11}F_{22}F_{33} + F_{12}F_{23}F_{31} + F_{13}F_{21}F_{32} - F_{13}F_{31}F_{22} - F_{23}F_{32}F_{11} - F_{12}F_{21}F_{33}} \quad (3.13)$$

In order to compensate the circuits against first-order (s-compensation) and second-order effects (s<sup>2</sup>-compensation), the coefficients of the s and s<sup>2</sup> terms in the numerator of (3.1) must be equal to their corresponding coefficient in the denominator, namely,

$$a_1\tau_2 + a_2\tau_3 = b_1\tau_1 + b_2\tau_2 + b_3\tau_3 \quad (\text{s-compensation}) \quad (3.14)$$

$$a_3\tau_2\tau_3 = b_4\tau_2\tau_3 + b_5\tau_1\tau_3 + b_6\tau_1\tau_2 \quad (\text{s}^2\text{-compensation}) \quad (3.15)$$

If eqns. (3.14) and (3.15) are to hold independently of the OAs used, then the following must be met



$$b_1 = 0, a_1 = b_2, a_2 = b_3 \quad (3.16)$$

for the s-compensation and

$$a_3 = b_4, b_5 = b_6 = 0 \quad (3.17)$$

for the  $s^2$ -compensation.

In case (3.16) and (3.17) are not satisfied, the validity of (3.14) and (3.15) will depend on the values of either  $\tau_1$ ,  $\tau_2$  or  $\tau_3$ . Further, for the compensation to remain effective when the  $\tau_i$ 's are varying, variations in one OA should be tracked by variations in the others. Thus, tracking OAs will be required.

### 3.3 PERFORMANCE CHARACTERISTICS

The only possible disadvantage of using of 3 OA ACVAs might be the complexity of such circuits. Therefore, in order to alleviate this problem and also to limit the study to useful and practical structures, some guidelines should be followed. We shall use the following:

- (a) Noninverting circuits must possess an infinite input impedance. This feature allows the

replacement of the noninverting conventional realization by an ACVA in all applications without additional considerations.

- (b) All circuits must be stable by themselves, i.e. as stand-alone amplifiers.
- (c) All ACVAs must be orthogonally tunable [9,42], i.e. the DC gain, the  $s$ -compensation and the  $s^2$ -compensation conditions must be adjustable independently of each other. This feature not only makes the circuits practical but also provides them with the convenience of being either functionally or deterministically tuned at low cost according to the volume of production.

In this context, it is noted that these guidelines are much stricter than those used for the 2 OA case. This is due to the fact that they are intended to counterbalance the increased complexity of 3 OA ACVAs by providing them with the convenience of being used in a very simple way. It is also noted that an ACVA that satisfies the above requirements is preferable over one that does not.

If we replace eqns. (3.3) to (3.13) in (3.1) and use routine stability tests, the following conditions are obtained

$$F_{11}F_{22}F_{33} + F_{12}F_{23}F_{31} + F_{13}F_{21}F_{32} - F_{13}F_{22}F_{31} - \\ F_{11}F_{23}F_{33} - F_{12}F_{21}F_{33} < 0 \quad (3.18)$$

$$(F_{23}F_{32} - F_{22}F_{33})\tau_1 + (F_{13}F_{31} - F_{11}F_{33})\tau_2 + \\ (F_{12}F_{21} - F_{11}F_{22})\tau_3 < 0 \quad (3.19)$$

$$F_{11}\tau_2\tau_3 + F_{22}\tau_2\tau_3 + F_{33}\tau_1\tau_2 < 0 \quad (3.20)$$

$$(F_{12}F_{23}F_{31} + F_{13}F_{21}F_{32} - 2F_{11}F_{22}F_{33})\tau_1\tau_2\tau_3 + \\ F_{11}(F_{13}F_{31} - F_{11}F_{33})\tau_2^2\tau_3 + F_{11}(F_{12}F_{21} - F_{11}F_{22})\tau_2\tau_3^2 + \\ F_{22}(F_{23}F_{32} - F_{22}F_{33})\tau_1^2\tau_3 + F_{22}(F_{12}F_{21} - F_{11}F_{22})\tau_1\tau_3^2 + \\ F_{33}(F_{13}F_{31} - F_{11}F_{33})\tau_1\tau_2^2 + F_{33}(F_{23}F_{32} - F_{22}F_{33})\tau_1^2\tau_2 > 0 \quad (3.21)$$

Also, from eqn. (3.18), it follows that

$$f_2F_{12}F_{33} + f_1F_{23}F_{32} + f_3F_{22}F_{13} - f_1F_{22}F_{33} - f_2F_{13}F_{32} \\ - f_3F_{12}F_{23} < 0 \quad (3.22)$$

for noninverting circuits and

$$f_2F_{12}F_{33} + f_1F_{23}F_{32} + f_3F_{22}F_{13} - f_1F_{22}F_{33} - f_2F_{13}F_{32}$$

$$-f_3 F_{12} F_{23} > 0 \quad (3.23)$$

for inverting realizations.

The s-compensation and  $s^2$ -compensation conditions can be expressed as

$$\begin{aligned} (F_{23}F_{32} - F_{22}F_{33})\tau_1 + (F_{13}F_{31} - F_{11}F_{33})\tau_2 + \\ (F_{12}F_{21} - F_{11}F_{22})\tau_3 = \{ (f_1F_{33} - f_3F_{13})\tau_2 + \\ (f_1F_{22} - f_2F_{12})\tau_3 \} K^{-1} \end{aligned} \quad (3.24)$$

$$F_{11}\tau_2\tau_3 + F_{22}\tau_1\tau_3 + F_{33}\tau_1\tau_2 = -f_1\tau_2\tau_3 K^{-1} \quad (3.25)$$

Further, in view of eqns. (3.18) to (3.25), the following must also hold

$$(f_1F_{33} - f_3F_{13})\tau_2 + (f_1F_{22} - f_2F_{12})\tau_3 < 0 \quad (3.26)$$

$$f_1 > 0 \quad (3.27)$$

for noninverting realizations and

$$(f_1F_{33} - f_3F_{13})\tau_2 + (f_1F_{22} - f_2F_{12})\tau_3 > 0 \quad (3.28)$$

$$f_1 < 0$$

(3.29)

for inverting ACVAs.

Each voltage  $v_k$  can be written as

$$v_k = v_k^+ - v_k^- \quad (3.30)$$

where  $v_k^+$  and  $v_k^-$  are the voltages at the noninverting and the inverting input terminals of the  $k$ th OA, respectively, referred to ground. Hence,

$$F_{kj} = \left. \frac{v_k}{v_j} \right|_{\substack{v_h=0 \\ h \neq 0}} = \left. \frac{v_k^+ - v_k^-}{v_j} \right|_{\substack{v_h=0 \\ h \neq 0}} = F_{kj}^+ - F_{kj}^- \quad (3.31)$$

$$f_k = \left. \frac{v_k}{v_i} \right|_{\substack{v_h=0 \\ h \neq 0}} = \left. \frac{v_k^+ - v_k^-}{v_j} \right|_{\substack{v_h=0 \\ h \neq 0}} = f_k^+ - f_k^- \quad (3.32)$$

Also, because the  $F_{kj}$ 's and the  $f_k$ 's are realized by a resistive network, it is easy to show that the following restrictions must hold for all  $k = 1, 2, 3$ .

$$f_k^+ + \sum_{j=1}^3 F_{kj}^+ \leq 0 \quad (3.33)$$

$$f_k^- + \sum_{j=1}^3 F_{kj}^- \leq 0 \quad (3.34)$$

$$0 \leq F_{kj}^{\pm} \leq 1 \quad (3.35)$$

$$0 \leq f_k^{\pm} \leq 1 \quad (3.36)$$

### 3.4 CLASSIFICATION AND GENERATION OF 3 OA ACVAs

ACVAs can be broadly classified into two categories: noninverting and inverting circuits. In order to simplify the design of noninverting 3 OA ACVAs, the study of this kind of VA is limited here to circuits possessing infinite input impedance. For the inverting ACVAs, both infinite and finite input impedance are acceptable provided the guidelines referred to in the previous section are observed. For this reason, noninverting and inverting ACVAs are going to be treated separately in the remainder of this section.

#### 3.4.1 Noninverting 3 OA ACVAs

The network must contain the minimum number of resistors. Clearly, then, the resistive network must contain as few nodes and branches as possible.

Additionally, any branch between any two nodes must represent a single resistor. Obviously, the minimum number of nodes for the resistive network in Fig. 3.1 is 11. Therefore, the most general configuration for the resistive network is as shown in Fig. 3.2 where each branch represents one resistor.

Based on this general configuration, it is not difficult to show that the input current  $I_i$ , the current that would be drawn from a voltage source of value  $v_i$  connected between node 1 and ground, is given by

$$\begin{aligned}
 I_i = & [G_{1,5}(1-f_1^+) + G_{1,6}(1-f_1^-) + G_{1,7}(1-f_2^+) + G_{1,8}(1-f_2^-) + G_{1,9}(1-f_3^+) \\
 & + G_{1,10}(1-f_3^-)]v_i - [G_{1,5}F_{11}^+ + G_{1,6}F_{11}^- + G_{1,7}F_{21}^+ + G_{1,8}F_{21}^- \\
 & + G_{1,9}F_{31}^+ + G_{1,10}F_{31}^-]v_1 - [G_{1,5}F_{12}^+ + G_{1,6}F_{12}^- + G_{1,7}F_{22}^+ + G_{1,8}F_{22}^- \\
 & + G_{1,9}F_{32}^+ + G_{1,10}F_{32}^-]v_2 - [G_{1,5}F_{13}^+ + G_{1,6}F_{13}^- + G_{1,7}F_{23}^+ + G_{1,8}F_{23}^- \\
 & + G_{1,9}F_{33}^+ + G_{1,10}F_{33}^-]v_3
 \end{aligned} \tag{3.37}$$

where  $G_{i,j} = 1 / R_{i,j}$  and  $R_{i,j}$  is the value of the resistor connected between nodes  $i$  and  $j$ .

If the circuits are to have infinite input impedance,  $I_i$  must be zero regardless of the values of  $v_i$ ,  $v_1$ ,  $v_2$  and  $v_3$ . This requires the coefficients of these voltages in (3.37) to be equal to zero. From Fig. 3.2, we note that if  $G_{1,i} = 0$ , then  $f_{(i-3)/2}^+ = 0$ , for  $i = 5, 7, 9$ . Also, if  $G_{1,i} =$

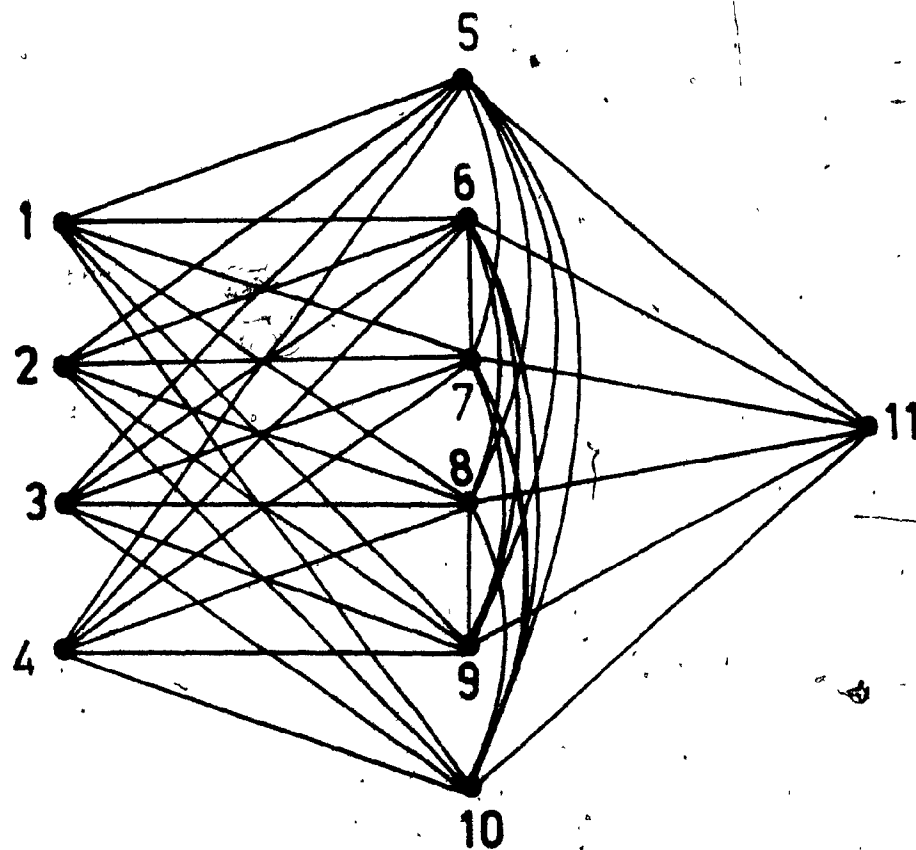


Fig. 3.2: The general configuration of the resistive network.



0,  $f_{(i-4)/2}^- = 0$ , for  $i = 6, 8, 10$ . Further, it follows from eqns. (3.33) and (3.34) that if  $f_k^+ = 1$ , then  $F_{k1}^+ = F_{k2}^+ = F_{k3}^+ = 0$ ,  $i = 1, 2, 3$ . In view of these observations, it is easy to show that the necessary and sufficient conditions for  $I_i$  to be unconditionally equal to 0 is that  $f_k^+$ ,  $k = 1, 2, 3$ , assume values equal to either 0 or 1. Hence, there are 64 ways of assuring infinite input impedance for the circuits. Some of these conditions, however, conflict with one or more of the eqns. (3.18) to (3.36). Also, some solutions are found to be similar to others, except for an interchange in two subscripts. This is equivalent to interchanging two OAs in the circuit. Consequently, after carrying out all possible simplifications, it can be shown that all infinite input impedance noninverting ACVAS employing 3 OAs belong to one of the five classes shown in Table 3.1. In the same table, the characterization of each class in terms of topological connections between the input of the overall network and the input of each OA is also indicated. It is important to mention that, it may be necessary to renumber the OAs in order to include a given circuit in one of these five classes.

We shall now impose the requirement of tunability, that is of independent control of the DC gain, the  $s$ - and the  $s^2$ -compensation on the realizations. A rational way of incorporating the tunability requirement systematically into the realizations is by subdividing each class into a number

TABLE 3.1: CHARACTERIZATION OF THE CLASSES THAT  
REALIZE INFINITE INPUT IMPEDANCE  
NONINVERTING ACVAs.

CLASS	$f_1^+$	$f_1^-$	$f_2^+$	$f_2^-$	$f_3^+$	$f_3^-$
NI3A	1	0	0	0	0	0
NI3B	1	0	0	0	0	1
NI3C	1	0	0	1	0	1
NI3D	1	0	0	0	1	0
NI3E	1	0	0	1	1	0

of types. Each type may then be investigated to ensure that it consists of structures which possess the above mentioned tunability property.

Realizations in each class have to satisfy conditions (3.18) to (3.27). Consequently, any of these conditions can be used to generate types of realizations in a given class. These types, if they satisfy the other conditions as well, will correspond to valid realizations. The condition represented by eqn. (3.24) is a convenient one for generating types. Considering that equation, there are three terms in the left hand side and two in the right hand side of the equality sign. Types can be generated if we attempt to satisfy that equation with 1, 2 or 3 and 1 or 2 terms in the left hand and right hand sides, respectively. In this manner, a maximum of 18 types for each of the 5 classes can be derived. A thorough investigation reveals that only a few of those types will lead to tunable noninverting ACVAs possessing infinite input impedance. These types are shown in Table 3.2.

Table 3.2 also shows that only the realizations belonging to types NI3B-1, NI3C-1 and NI3C-2 will not require tracking OAs. Type NI3B-2 requires tracking OAs only for the  $s^2$ -compensation.

It is also interesting to note that the types NI3A-1 and NI3B-3 are the only ones to realize voltage followers.

TABLE 3.2: CHARACTERISTICS OF THE TYPES THAT REALIZE STABLE AND TUNABLE, INFINITE INPUT IMPEDANCE, NONINVERTING ACVAS.

TYPE	CHARACTERIZATION	DC-GAIN (K)	S-COMPENSATION	S <sup>2</sup> -COMPENSATION	STABILITY CONDITION
NF3A-1	$F_{11}^+ = F_{31}^+ = F_{12}^+ = F_{13}^+ = 0$ $F_{12}^- = F_{22}^- = F_{32}^-$ $F_{13}^- = F_{23}^- = F_{33}^-$	$\frac{1}{F_{21}^+}$	$F_{32}^+ = \frac{\tau_2 K^{-1}}{\tau_1}$	$\tau_3 F_{22}^- + \tau_1 F_{33}^- = K^{-1} \tau_3$	$-(F_{13}^+ F_{21}^+ F_{32}^-) \tau_2 \tau_3 + (F_{22}^- F_{23}^+ F_{32}^-) \tau_1 \tau_3$ $+ (F_{22}^- F_{12}^+ F_{21}^-) \tau_2^2 + (F_{33}^- F_{23}^+ F_{32}^-) \tau_1 \tau_2 > 0$
NF3B-1	$F_{22}^- = F_{32}^- = F_{21}^- = F_{11}^+ = F_{12}^+ = 0$ $F_{13}^+ = F_{31}^- = F_{33}^- = 0$	$\frac{1}{F_{31}^+}$	$F_{31}^+ = K^{-1}$	$F_{11}^+ = K^{-1}$	$(F_{13}^+ F_{11}^+ F_{31}^-) \tau_2 + (F_{12}^- F_{23}^+ F_{31}^-) \tau_1 > 0$
NF3B-2	$F_{13}^- = F_{32}^- = F_{11}^- = F_{12}^- = F_{13}^- = F_{33}^- = 0$ $F_{12}^- = F_{22}^-$	$\frac{1}{F_{31}^+}$	$F_{21}^+ = K^{-1}$	$F_{22}^- = \frac{\tau_2 K^{-1}}{\tau_1}$	$(F_{22}^- F_{12}^+ F_{21}^-) \tau_3 - (F_{12}^- F_{23}^+ F_{31}^-) \tau_2 > 0$
NF3B-3	$F_{31}^+ = F_{11}^- = F_{21}^- = F_{12}^- = F_{13}^- = F_{32}^- = F_{33}^- = 0$ $F_{12}^- = F_{22}^-$ $F_{13}^- = F_{23}^-$	$\frac{1}{F_{21}^+}$	$F_{32}^+ = \frac{\tau_2 K^{-1}}{\tau_1}$	$F_{22}^- = \frac{\tau_2 K^{-1}}{\tau_1}$	$(F_{22}^- F_{32}^+ F_{23}^-) \tau_1 + (F_{22}^- F_{21}^+ F_{12}^-) \tau_3$ $-(F_{13}^+ F_{21}^+ F_{32}^-) \tau_2 > 0$
NF3C-1	$F_{11}^+ = F_{12}^+ = F_{13}^- = F_{21}^- = F_{22}^- = F_{23}^- = F_{31}^- = 0$ $F_{32}^- = F_{33}^- = F_{12}^- = 0$	$\frac{1}{F_{21}^+}$	$F_{31}^+ = K^{-1}$	$F_{11}^+ = K^{-1}$	$(F_{11}^+ F_{13}^+ F_{31}^-) \tau_2 - (F_{13}^+ F_{21}^+ F_{32}^-) \tau_1 > 0$
NF3C-2	$F_{11}^+ = F_{12}^+ = F_{13}^- = F_{21}^- = F_{22}^- = F_{23}^- = 0$ $F_{31}^- = F_{32}^- = F_{33}^- = F_{12}^- = 0$	$\frac{1}{F_{31}^+}$	$F_{21}^+ = K^{-1}$	$F_{11}^+ = K^{-1}$	$(F_{11}^+ F_{13}^+ F_{31}^-) \tau_2 + (F_{11}^+ F_{12}^- F_{21}^+) \tau_3$ $-(F_{12}^- F_{23}^+ F_{31}^-) \tau_1 > 0$

However, the latter will not realize voltage followers if the OAs have identical GBs. Furthermore, if the values of the GBs are close to each other, even while being different, a large resistance spread will be necessary in both types to realize a voltage follower.

The next step is to obtain the amplifier circuits for each type. This task can be accomplished by means of sequential resistor eliminations in the general configuration shown in Fig. 3.2. These eliminations are similar to those performed in the previous chapter and are based on the characterization of each type. Let us illustrate the procedure for one of the types.

Consider type NI3B-1. As a consequence of its characterization and of eqns. (3.18) to (3.36), it follows that

$$f_1^+ = 1 \Rightarrow F_{11}^+ = F_{12}^+ = F_{13}^+ = 0$$

$$f_3^- = 1 \Rightarrow F_{31}^- = F_{32}^- = F_{33}^- = F_{33}^+ = 0$$

$$f_1^- = f_2^+ = f_2^- = f_3^+ = 0$$

$$F_{22}^+ = F_{22}^- = F_{32}^+ = F_{21}^+ = F_{21}^- = 0$$

These equations lead to the following resistor eliminations in the graph shown in Fig. 3.2

$$f_1^+ = 1 \Rightarrow G_{1,5} \rightarrow \infty, G_{2,5} = G_{3,5} = G_{4,5} = G_{5,7} = G_{5,8} = G_{5,9} = G_{5,10} = G_{5,11} = 0$$

$$f_1^- = 0 \Rightarrow G_{1,6} = 0$$

$$f_2^+ = 0 \Rightarrow G_{1,7} = 0$$

$$f_2^- = 0 \Rightarrow G_{1,8} = 0$$

$$f_3^+ = 0 \Rightarrow G_{1,9} = 0$$

$$f_3^- = 1 \Rightarrow G_{1,10} \rightarrow \infty, G_{2,10} = G_{3,10} = G_{4,10} = G_{6,10} = G_{7,10} = G_{8,10} = G_{10,11} = 0$$

$$f_{22}^+ = 0 \Rightarrow G_{3,7} = 0$$

$$f_{22}^- = 0 \Rightarrow G_{3,8} = 0$$

$$f_{32}^+ = 0 \Rightarrow G_{3,9} = 0$$

$$F_{21}^{+} = 0 \Rightarrow G_{2,7} = 0$$

$$F_{21}^{-} = 0 \Rightarrow G_{2,8} = 0$$

$$F_{33}^{+} = 0 \Rightarrow G_{4,9} = 0$$

After all these eliminations, the graph of the circuits of this type is shown in Fig. 3.3.

Under close scrutiny, more eliminations are possible as follows

$$F_{23}^{+} \neq 0 \text{ and } F_{33}^{+} = 0 \Rightarrow G_{7,9} = 0$$

$$F_{23}^{-} \neq 0 \text{ and } F_{33}^{+} = 0 \Rightarrow G_{8,9} = 0$$

$$F_{32}^{+} = 0, F_{12}^{-} \neq 0 \text{ and } F_{31}^{+} \neq 0 \Rightarrow G_{6,9} = 0$$

$$F_{22} = 0 \text{ and } F_{12}^{-} \neq 0 \Rightarrow G_{6,7} = G_{6,8} = 0$$

Because the values of  $F_{13}^{-}$  and  $F_{12}^{-}$  can be arbitrary (provided they are different from zero),  $G_{6,11}$  is not really

necessary. The final graph is shown in Fig. 3.4.

$F_{23}$  can assume any arbitrary positive value. By attempting to realize this voltage ratio with a minimal number of resistors, the realizations shown in Table 3.3 are obtained.

Following a similar procedure for other types, the circuits shown in Table 3.3 can be obtained. Some observations are in order at this stage:

- (a) In all types, there is a realization that employs the minimum number of resistors and other, more general, employing more components. These general configurations often offer a certain advantage at the expense of a slight increase in cost and complexity. For instance, they can be designed to possess given phase and gain margin while their minimal counterparts cannot.
- (b) The first two ACVAs of type NI3B-1 are completely equivalent. They are presented as different circuits only for the sake of completeness.

A summary of the general properties of each circuit is shown in Table 3.4.



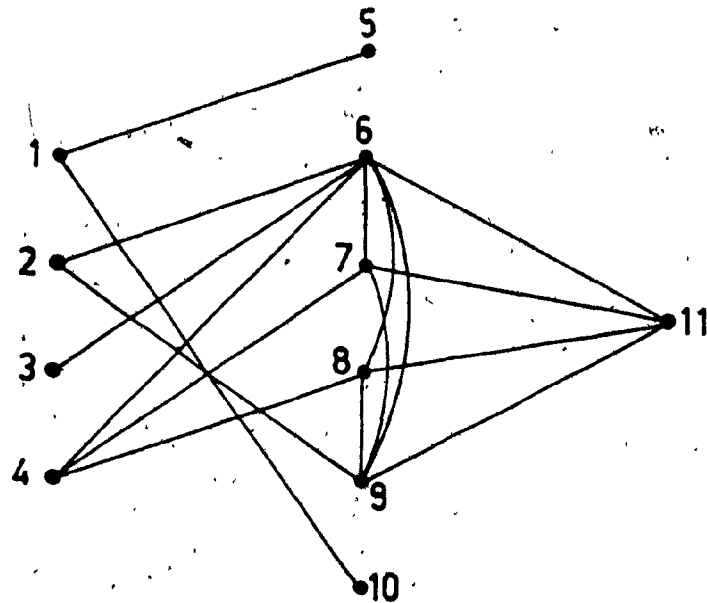


Fig. 3.3: Configuration of type NI3B-1 after the preliminary resistor eliminations.

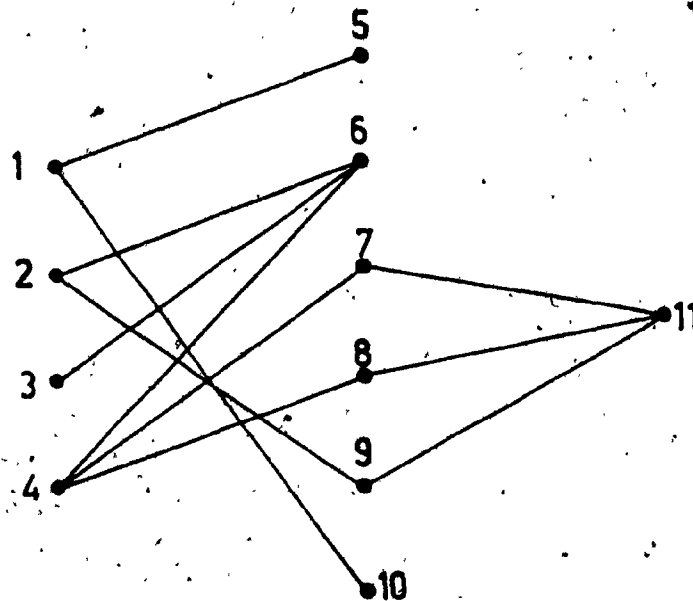


Fig. 3.4: Configuration of type NI3B-1 after all possible resistor eliminations are performed.

TABLE 3.3: CIRCUIT REALIZATIONS FOR THE DIFFERENT TYPES OF NONINVERTING ACVAS

<p>NI3A [35]</p>	<p>NI3B-1 [36]</p>	<p>NI3C-1</p>
<p>NI3B-2 [36]</p>	<p>NI3B-3</p>	<p>NI3C-2 [36]</p>
<p>NI3B-1 [36]</p>	<p>NI3B-2 [36]</p>	<p>NI3C-1 [36]</p>
<p>NI3B-1 [36]</p>	<p>NI3B-2 [36]</p>	<p>NI3C-1 [36]</p>

TABLE 3.4: PROPERTIES OF THE TUNABLE, NONINVERTING, 3 OA ACVAS.

AMPLIFIER TYPE	TRANSFER FUNCTION	DC-GAIN (K)	S-COMPENSATION	S <sup>2</sup> -COMPENSATION	STABILITY CONDITION
NI3A	$\frac{1}{\beta} \frac{1 + s \left[ \frac{t_2}{1} + \frac{a_1 t_3}{a_2} \right] + s^2 \frac{(a_1 + a_2 + a_3)}{t_1 a_2}}{1 + s \left[ \frac{t_1}{\beta} + \frac{a_1 t_3}{t_2} \right] + s^2 \left[ \frac{a_1 t_1 t_3}{t_1 a_2} + \frac{t_1 t_2}{t_1 a_2} + s^2 \frac{(a_1 + a_2 + a_3)}{t_1 a_2} \right]}$	$\beta^{-1}$	$\gamma = \frac{\beta t_2}{t_1}$	$a_1 t_1^2 + a_2 t_1^2 + \beta(a_1 + a_2 + a_3) t_2^2$	ALWAYS STABLE
NI3B-1 (a), (b)	$\frac{1}{\beta} \frac{1 + s \frac{a_3 t_2}{a_2} + s^2 \frac{(a_1 + a_2 + a_3)}{a_2}}{1 + s \frac{a_3 t_2}{a_2} + s^2 \frac{a_1}{a_2} + s^2 \frac{(a_1 + a_2 + a_3)}{a_2}}$	$\beta^{-1}$	—	$\beta(a_1 + a_2 + a_3) = a_1$	$\frac{a_2 t_1}{\beta a_3 t_2} < 1$
NI3B-1 (c)	$\frac{1}{\beta} \frac{1 + s \frac{a_3}{a_2} + s^2 \frac{(a_1 + a_2 + a_3)}{a_2}}{1 + s \frac{a_3}{a_2} + s^2 \frac{a_1}{\beta a_2} + s^2 \frac{(a_1 + a_2 + a_3)}{\beta a_2}}$	$\beta^{-1}$	—	$\beta(a_1 + a_2 + a_3) = a_1$	$\frac{a_2 t_1}{\beta a_3 t_2} < 1$
NI3B-2 (a)	$\frac{1}{\beta} \frac{1 + s \frac{(b_1 + b_2 + b_3)}{b_3} + s^2 \frac{(b_1 + b_2 + b_3)}{a_2 b_3}}{1 + s \frac{b_1}{b_3} + s^2 \frac{(b_1 + b_2 + b_3)}{\beta b_3} + s^2 \frac{(b_1 + b_2 + b_3)}{a_2 b_3}}$	$\beta^{-1}$	$\beta(b_1 + b_2 + b_3) = b_1$	$a_1 = \beta t_2$	$\frac{b_1 t_3}{\beta t_1} < 1$
NI3B-2 (b)	$\frac{1}{\beta} \frac{1 + s \frac{t_3}{\gamma} + s^2 \frac{t_3}{\alpha \gamma}}{1 + s \frac{t_3}{\gamma} + s^2 \frac{t_3}{\beta \gamma} + s^2 \frac{t_3}{\alpha \beta \gamma}}$	$\beta^{-1}$	$\gamma = \beta$	$a_1 = \beta t_2$	$\frac{t_3}{\gamma} < 1 + \frac{t_3}{t_1}$

TABLE 3.4: (continued)

AMPLIFIER TYPE	TRANSFER FUNCTION	DC-GAIN (K)	s-COMPENSATION	s <sup>2</sup> -COMPENSATION	STABILITY CONDITION
NI3B-3 (a)	$\frac{1}{\beta} \frac{1 + s \left[ \frac{2}{\gamma} + \frac{a_2 \tau_3}{a_3 \gamma} \right] + s^2 \tau_1 \tau_3 \frac{(a_1 + a_2 + a_3)}{\gamma a_3}}{1 + s \left[ \frac{\tau_1}{\beta} + \frac{a_2 \tau_3}{\gamma a_3} \right] + s^2 \tau_1 \tau_3 \frac{a_2}{\gamma a_3} + s^3 \tau_1 \tau_2 \tau_3 \frac{(a_1 + a_2 + a_3)}{\gamma a_3}}$	$\beta^{-1}$	$\gamma \tau_1 = \beta \tau_2$	$(a_1 + a_2 + a_3) \tau_1 = \frac{a_2 \tau_2}{\beta}$	ALWAYS STABLE
NI3B-3 (b)	$\frac{1}{\beta} \frac{1 + s \left[ \frac{\tau_2}{\gamma} + \frac{a \tau_3}{\gamma(1-a)} \right] + s^2 \tau_1 \tau_3 \frac{1}{\gamma \beta(1-a)}}{1 + s \left[ \tau_1 + \frac{a \tau_3}{\gamma(1-a)} \right] + s^2 \tau_1 \tau_3 \frac{a}{\gamma \beta(1-a)} + s^3 \tau_1 \tau_2 \tau_3 \frac{1}{\gamma \beta(1-a)}}$	$\beta^{-1}$	$\gamma \tau_1 = \beta \tau_2$	$a \tau_1 = \beta \tau_2$	ALWAYS STABLE
NI3C-1	$\frac{1}{\beta} \frac{1 + s \tau_2 \frac{(b_1 + b_2 + b_3)}{b_2} + s^2 \tau_2 \tau_3 \frac{(b_1 + b_2 + b_3)}{b_2(1-a)}}{1 + s \tau_2 \frac{b_1}{\beta b_2} + s^2 \tau_2 \tau_3 \frac{(b_1 + b_2 + b_3)a}{\beta b_2(1-a)} + s^3 \tau_1 \tau_2 \tau_3 \frac{(b_1 + b_2 + b_3)}{\beta b_2(1-a)}}$	$\beta^{-1}$	$\beta(b_1 + b_2 + b_3) = b_1$	$a = \beta$	$b_1 \tau_2 > b_2 \tau_1$
NI3C-2 (a)	$\frac{1}{\beta} \frac{1 + s \left[ \frac{a_3 \tau_2}{a_2(1-\gamma)} + \frac{\tau_3}{(1-\gamma)} \right] + s^2 \tau_2 \tau_3 \frac{(a_1 + a_2 + a_3)}{a_2(1-\gamma)}}{1 + s \left[ \frac{a_3 \tau_2}{a_2(1-\gamma)} + \frac{\tau_3}{\beta(1-\gamma)} \right] + s^2 \tau_2 \tau_3 \frac{a_1}{a_2(1-\gamma)\beta} + s^3 \tau_1 \tau_2 \tau_3 \frac{(a_1 + a_2 + a_3)}{a_2(1-\gamma)\beta}}$	$\beta^{-1}$	$\gamma = \beta$	$\beta(a_1 + a_2 + a_3) = a_1$	$\frac{a_3}{a_2} > \frac{\tau_1}{\tau_2} (K-1) - \frac{\tau_3}{\tau_2}$
NI3C-2 (b)	$\frac{1}{\beta} \frac{1 + s \left[ \frac{a_3(b_1 + b_2 + b_3)\tau_2}{a_2 b_3} + \frac{(b_1 + b_2 + b_3)\tau_3}{b_3} \right] + s^2 \tau_2 \tau_3 \frac{(a_1 + a_2 + a_3)(b_1 + b_2 + b_3)}{a_2 b_3}}{1 + s \left[ \frac{a_3(b_1 + b_2 + b_3)\tau_2}{a_2 b_3} + \frac{b_1 \tau_3}{\beta b_3} \right] + s^2 \tau_2 \tau_3 \frac{a_1(b_1 + b_2 + b_3)}{\beta a_2 b_3} + s^3 \tau_1 \tau_2 \tau_3 \frac{(a_1 + a_2 + a_3)(b_1 + b_2 + b_3)}{\beta a_2 b_3}}$	$\beta^{-1}$	$\beta(b_1 + b_2 + b_3) = b_1$	$\beta(a_1 + a_2 + a_3) = a_1$	$\frac{a_3}{a_2} > \frac{b_1 \tau_1}{b_1 \tau_2} - \frac{\tau_3}{\tau_2}$

### 3.4.2 Inverting 3 OA ACVAs

As it was already mentioned, inverting ACVAs can possess either finite or infinite input impedance. However, for reasons that have been mentioned before, the study of 3 OA inverting ACVAs is restricted to orthogonally tunable circuits. At this point it is worth considering the following theorem:

#### Theorem 3.1

It is impossible to realize an orthogonally tunable, inverting, finite gain, 3 OA ACVA possessing infinite input impedance.

#### Proof:

The input current,  $I_i$ , flowing into an ACVA is given by eqn. (3.37). By using the same considerations that were used for noninverting circuits, it can be shown that the necessary and sufficient conditions for  $I_i$  to be zero regardless of the values of  $v_1$ ,  $v_1$ ,  $v_2$  and  $v_3$  is that  $f_k^+$ ,  $i = 1, 2, 3$ , assume values equal to 0 or 1. Again, there are 64 ways of assuring infinite input impedance for the circuits. Nevertheless, if these solutions are examined under the

constraints imposed by eqns. (3.18) to (3.36), only three of them lead to stable, infinite input impedance, inverting ACVAs. They are shown in Table 3.5 as classes of circuits. At this point, let us examine each class individually.

### Class II3A

For this class, the  $s$ - and  $s^2$ -compensation conditions are given, respectively, by

$$(F_{23}F_{32} - F_{22}F_{33})\tau_1 + F_{13}^+F_{31}\tau_2 + F_{12}^+F_{21}\tau_3 = -F_{33}K^{-1}\tau_2 - F_{22}K^{-1}\tau_3. \quad (3.38)$$

and

$$F_{22}\tau_1\tau_3 + F_{33}\tau_1\tau_2 = K^{-1}\tau_2\tau_3 \quad (3.39)$$

This class will lead to tunable realizations if and only if  $F_{22}$  and  $F_{33}$  can be varied without affecting (3.38). For this purpose, the terms in (3.38) must occur in such a way that  $F_{22}$  and  $F_{33}$  are factors of appropriate terms in both the left and the right hand sides of the equality sign. In this case, these transfer ratios do not affect the  $s$ -compensation. Below are shown all the possible

arrangements that yield this independency between the two compensation conditions.

Arrangement a:  $F_{22} = F_{12}^+ F_{21} = 0, F_{33} = \pm F_{23} = -F_{13}^+$

Arrangement b:  $F_{22} = F_{12}^+ F_{21} = F_{13}^+ F_{31} = 0, F_{33} = \pm F_{23}$

Arrangement c:  $F_{22} = F_{12}^+ F_{21} = F_{23} F_{32}, F_{33} = -F_{13}^+$

Arrangement d:  $F_{33} = F_{13}^+ F_{31} = F_{23} F_{32} = 0, F_{22} = -F_{12}^+$

The third arrangement leads to a DC gain equal to zero. It can be seen by inspection that the other 3 arrangements lead to an interdependency between the DC gain expression and the s-compensation condition. Therefore, there is no orthogonally tunable, inverting ACVA in class II3A.

### Class II3B

For this class, the s- and the  $s^2$ -compensation conditions are given, respectively, by

$$(F_{22} F_{33} - F_{23} F_{32}) \tau_1 - F_{13}^+ F_{31} \tau_2 + F_{12}^+ F_{21} \tau_3 = (F_{33} - F_{13}^+) K^{-1} \tau_2 - F_{22} K^{-1} \tau_3 \quad (3.40)$$

and

TABLE 3.5: CLASSES OF INFINITE INPUT IMPEDANCE ACIVAs

CLASS	$f_1^+$	$f_1^-$	$f_2^+$	$f_2^-$	$f_3^+$	$f_3^-$
II3A	0	1	0	0	0	0
II3B	0	1	0	0	1	0
II3C	0	1	1	0	1	1

TABLE 3.6: DIFFERENT CLASSES OF TUNABLE ACIVAs.

CLASS	CHARACTERIZATION		
IF3A	$F_{11} \neq 0$	$F_{22} = F_{33} = 0$	
IF3B	$F_{22} \neq 0$	$F_{11} = F_{33} = 0$	
IF3C	$F_{11} \neq 0$	$F_{22} \neq 0$	$F_{33} = 0$
IF3D	$F_{22} \neq 0$	$F_{33} \neq 0$	$F_{11} = 0$



$$F_{22} \tau_1 \tau_3 - F_{33} \tau_1 \tau_2 = K^{-1} \tau_2 \tau_3 \quad (3.41)$$

As it was the case for class II3A, in order to be able to adjust these two equations independently of each other, the terms in (3.40) must be arranged in such a way that  $F_{22}$  and  $F_{33}$  are factors of appropriate terms in both sides of the equality sign. The possible arrangements are shown below

Arrangement a:  $F_{22} = F_{12}^+ F_{21} = F_{13}^+ = 0, F_{33} = F_{23}$

Arrangement b:  $F_{33} = 0, F_{22} = -F_{12}^+ = -F_{32}^-$

Arrangement c:  $F_{33} = F_{23} F_{32}^- = 0, F_{22} = -F_{12}^+$

Arrangement d:  $F_{33} = F_{12}^+ F_{21} = 0, F_{22} = -F_{32}^-$

Since it can be seen by inspection that all these arrangements lead to an interdependency among the DC gain expression, the  $s$ - and the  $s^2$ -compensation conditions, it follows that it is impossible to realize a tunable, inverting ACVA of class II3B.

#### Class II3C

For this class, the  $s$ - and the  $s^2$ -compensation conditions are given, respectively, by

$$(F_{23}^- F_{32}^- - F_{22}^- F_{33}^-) \tau_1 - F_{13}^+ F_{31}^- \tau_2 - F_{12}^+ F_{21}^- \tau_3 = (F_{33}^- - F_{13}^+) K^{-1} \tau_2 + (F_{22}^- - F_{12}^+) K^{-1} \tau_3 \quad (3.42)$$

and

$$-F_{22}^- \tau_1 \tau_3 - F_{33}^- \tau_1 \tau_2 = K^{-1} \tau_2 \tau_3 \quad (3.43)$$

The only arrangement by which (3.42) can be made independent of (3.43) is

$$F_{22}^- = F_{13}^+ = 0, \quad F_{33}^- = F_{23}^-$$

Unfortunately, this arrangement contradicts eqn. (3.18). Therefore, it is not possible to realize a tunable, finite gain, inverting ACVA of class II3C.

Since classes II3A, II3B and II3C are the only ways of assuring infinite input impedance while satisfying eqns. (3.18) to (3.36) and none of these classes lead to tunable, finite gain, inverting ACVAs, it follows that it is impossible to realize such kind of circuits with an infinite input impedance as stated in theorem 3.1.

Because of this theorem, the remainder of this section deals exclusively with the classification and generation of actively compensated inverting voltage amplifiers (ACIVAs) possessing finite input impedance. If we consider that the ACIVAs are supposed to replace their conventional counterpart (Fig. 1.2b) which also possesses finite input impedance, the nonexistence of tunable ACIVAs with infinite input impedance is not a major problem for the analog circuit designer.

In order to satisfy the tunability requirement, eqns. (3.3), (3.24), and (3.25) must be adjusted independently of each other. A look at these equations, however, reveals that this is not possible if  $F_{11}$ ,  $F_{22}$ , and  $F_{33}$  are all different from zero. Therefore, one or two of these three transfer ratios must be equal to zero in the ACIVAs. Consequently, it follows that all tunable ACIVAs can be included in one of the classes shown in Table 3.6 according to the local feedback around each OA. Since there is no clear distinction between OA-2 and OA-3 in the context discussed so far, it may be necessary to renumber the OAs in order to include a given circuit in one of these four classes.

This classification by itself, however, is not enough to assure the tunability of the circuits. A rational way of incorporating the tunability requirement systematically into the realizations to be obtained is by subdividing each class

into a number of subclasses. Each subclass may then be investigated to ensure that it comprises structures possessing the above mentioned tunability property.

Realizations in each of the classes have to satisfy eqns. (3.18) to (3.36). Consequently, any of those conditions can be used to generate subclasses of ACIVAs. The condition represented by eqn. (3.24) is a convenient one for this purpose. Considering that equation, there are three terms in the left-hand side and two in the right-hand side of the equality sign. Subclasses can be generated if we attempt to satisfy that equation with one, two or three and one or two terms in the left-hand and right-hand side, respectively. In this manner, a maximum of 18 subclasses for each of the four classes can be derived. A thorough investigation reveals that only a few of these subclasses will lead to tunable and stable ACIVAs. Also, in some subclasses, there are more than one way of realizing tunable and stable circuits. Each of these ways corresponds to a different type of circuit in a given subclass. All possible types are shown in Table 3.7 along with their individual characteristics.

It is interesting to note from Table 3.7 that subclasses IF3AB and IF3AD are the only ones whose realizations do not require tracking OAs.

TABLE 3.7: CHARACTERISTICS OF THE TYPES THAT REALIZE STABLE AND TUNABLE ACIVAS.

SUBCLASS	TYPE	CHARACTERISTICS	DC-GAIN	S-COMPENSATION	S <sup>2</sup> -COMPENSATION	STABILITY CONDITION
IF3AD	1	$F_{23}=0$ $f_1>0, F_{11}>0, f_2>0, F_{21}>0, f_3>0, F_{31}>0$ $F_{12}>0, F_{13}>0, F_{32}>0$	$-\frac{f_2}{F_{21}}$	$F_{31} - f_3 K^{-1}$	$F_{11} - f_1 K^{-1}$	$F_{13} F_{21} F_{32} \tau_1 + F_{11} F_{13} F_{31} \tau_2$ $+ F_{11} F_{12} F_{21} \tau_3 > 0$
	2	$F_{23}=0$ $f_1>0, F_{11}>0, f_2>0, F_{21}>0, f_3>0$ $F_{31}>0, F_{12}>0, F_{13}>0, F_{32}>0$				
	3	$F_{23}=0$ $f_1>0, F_{11}>0, f_2>0, F_{21}>0, f_3>0$ $F_{31}>0, F_{12}>0, F_{13}>0, F_{32}>0$				
	4	$F_{23}=0$ $f_1>0, F_{11}>0, f_2>0, F_{21}>0, f_3>0$ $F_{31}>0, F_{12}>0, F_{13}>0, F_{32}>0$				
IF3BA	1	$f_2 - F_{13} F_{21} - f_3 = 0$ $F_{31} - F_{32}$	$-\frac{f_1}{F_{12}}$	$F_{23} F_{32} \tau_1 =$ $f_1 F_{22} K^{-1} \tau_3$	$F_{22} \tau_1 - f_1 K^{-1} \tau_2$	$F_{12} F_{31} \tau_2 + F_{22} F_{32} \tau_2 > 0$
IF3BB	1	$F_{21}=0, F_{32}=0$ $f_1 = f_2; F_{12} = F_{22}; F_{13} = F_{23}$	$-\frac{f_3}{F_{31}}$	-	$F_{22} \tau_1 - f_1 K^{-1} \tau_2$	$F_{12} F_{23} F_{31} > 0$
IF3BC	1	$F_{13}=0$ $f_1 = f_2 = f_3; F_{12} = F_{22} = F_{32}$ $f_1 = 0, F_{12} = 0, F_{32} = 0$	$-\frac{f_3}{F_{31}}$	$F_{23} \tau_1 + F_{21} \tau_3 =$ $-f_2 K^{-1} \tau_3$	$F_{22} \tau_1 - f_1 K^{-1} \tau_2$	$F_{23} F_{33} \tau_2 - F_{22} F_{23} \tau_1$ $- F_{22} F_{21} \tau_3 > 0$
IF3BD	1	$F_{23} F_{32} = 0$ $f_1 = f_2; F_{12} = F_{22}$	$-\frac{f_3}{F_{31}}$	$F_{21} - K^{-1} f_2$	$F_{22} \tau_1 - f_1 K^{-1} \tau_2$	$F_{22} F_{12} F_{21} > 0$
	2	$F_{32}=0$ $f_1 = f_2; F_{12} = F_{22}$				$F_{23} F_{31} \tau_2 + F_{22} F_{21} \tau_3 > 0$
IF3BE	1	$F_{31}=0$ $f_1 = f_2; F_{12} = F_{22}; F_{13} = F_{23}$	$\frac{f_2}{F_{21}}$	$F_{32} \tau_1 - f_3 K^{-1} \tau_2$	$F_{22} \tau_1 - f_1 K^{-1} \tau_2$	$F_{13} F_{21} F_{32} \tau_2 + F_{22} F_{32} F_{23} \tau_1$ $+ F_{22} F_{12} F_{21} \tau_3 > 0$
IF3CA	1	$F_{13}=0$ $f_1 = f_2 = f_3; F_{11} = F_{21} = F_{31}$ $F_{12} = F_{22} = F_{32}$	$-\frac{f_3}{F_{31}}$	$F_{23} \tau_1 - f_2 K^{-1} \tau_3$	$F_{11} \tau_2 + F_{22} \tau_1 =$ $-f_1 K^{-1} \tau_2$	$F_{31} \tau_2 + F_{32} \tau_1 > 0$
IF3CB	1	$F_{23} F_{32} = 0$ $f_1 = f_2; F_{11} = F_{21}; F_{12} = F_{22}$	$-\frac{f_3}{F_{31}}$	-	$F_{11} \tau_2 + F_{22} \tau_1 =$ $-f_1 K^{-1} \tau_2$	$F_{11} F_{13} F_{31} > 0$
	2	$F_{32}=0$ $f_1 = f_2; F_{11} = F_{21}; F_{12} = F_{22}$ $F_{13} = F_{23}$				$F_{12} F_{23} \tau_1 + F_{11} F_{13} \tau_2 > 0$
IF3CC	1	$F_{13} F_{32} = 0$ $f_1 = f_2; F_{11} = F_{21}; F_{12} = F_{22}$	$-\frac{f_3}{F_{31}}$	$F_{21} - f_2 K^{-1}$	$F_{11} \tau_2 + F_{22} \tau_1 =$ $-f_1 K^{-1} \tau_2$	$F_{12} F_{23} F_{31} > 0$

TABLE 3.7: (continued)

SUBCLASS	TYPE	CHARACTERISTICS	DC-GAIN	S-COMPENSATION	S <sup>2</sup> -COMPENSATION	STABILITY CONDITION
IF3AA	1	$f_{31} = f_{12} = 0$	$\frac{-f_1}{f_{11}}$	$f_{23}f_{32}\tau_1 = -f_3f_{13}K^{-1}\tau_2$	-	$f_{13}f_{21}f_{32}\tau_1\tau_2\tau_3 > 0$
IF3AB	1	$f_2 = f_{32} = f_{21} = 0$ $f_3 > 0, f_{31} > 0, f_{13} > 0, f_{12} > 0, f_{32} > 0$	$\frac{-f_3}{f_{31}}$	-	$f_{11} = f_1 K^{-1}$	$f_{12}f_{23}\tau_1 + f_{11}f_{13}\tau_2 > 0$
	2	$f_2 = f_{32} = f_{21} = 0$ $f_3 > 0, f_{31} > 0, f_{13} > 0, f_{12} > 0, f_{32} > 0$				
	3	$f_2 = f_{32} = f_{21} = 0$ $f_3 > 0, f_{31} > 0, f_{13} > 0, f_{12} > 0, f_{23} > 0$				
	4	$f_2 = f_{32} = f_{21} = 0$ $f_3 > 0, f_{31} > 0, f_{13} > 0, f_{12} > 0, f_{23} > 0$				
	5	$f_{12} = f_{23} = 0$ $f_2 > 0, f_{21} > 0, f_{13} > 0, f_{31} > 0, f_{32} > 0, f_3 > 0$	$\frac{-f_2}{f_{21}}$	$f_{31} = f_3 K^{-1}$	$f_{11} = f_1 K^{-1}$	$f_{21}f_{32}\tau_1 + f_{11}f_{31}\tau_2 > 0$
	6	$f_{12} = f_{23} = 0$ $f_2 > 0, f_{21} > 0, f_{13} > 0, f_{31} > 0, f_{32} > 0, f_3 > 0$				
	7	$f_{12} = f_{23} = 0$ $f_2 > 0, f_{21} > 0, f_{13} > 0, f_{31} > 0, f_{32} > 0, f_3 > 0$				
	8	$f_{12} = f_{23} = 0$ $f_2 > 0, f_{21} > 0, f_{13} > 0, f_{31} > 0, f_{32} > 0$ $f_3 > 0$				
IF3AC	1	$f_3 = f_{13} = f_{31} = 0$ $f_1 > 0, f_{11} > 0, f_2 > 0, f_{12} > 0, f_{21} > 0$ $f_{23} > 0, f_{32} > 0$	$\frac{-f_1}{f_{11}}$	$f_{23}f_{32}\tau_1 + f_{12}f_{21}\tau_3$ $= f_2 f_{21} f_{31}^{-1} \tau_3$	-	$f_{11}f_{12}f_{21} > 0$
	2	$f_3 = f_{13} = f_{31} = 0$ $f_1 > 0, f_{11} > 0, f_2 > 0, f_{12} > 0, f_{21} > 0$ $f_{23} > 0, f_{32} > 0$				
	3	$f_3 = f_{13} = f_{31} = 0$ $f_1 > 0, f_{11} > 0, f_2 > 0, f_{12} > 0, f_{21} > 0$ $f_{23} > 0, f_{32} > 0$				
	4	$f_3 = f_{13} = f_{31} = 0$ $f_1 > 0, f_{11} > 0, f_2 > 0, f_{12} > 0, f_{21} > 0$ $f_{23} > 0, f_{32} > 0$				

TABLE 3.7: (continued)

SUBCLASS	TYPE	CHARACTERISTICS	DC-GAIN	S-COMPENSATION	S <sup>2</sup> -COMPENSATION	STABILITY CONDITION
IF3CD	1	$F_{23}^{-} = F_{32}^{-} = 0$ $f_1^{-} = f_2^{-}; F_{11}^{-} = F_{21}^{-}; F_{12}^{-} = F_{22}^{-}$ $F_{12}^{+} = 0$ $F_{32}^{-} = 0$	$\frac{-f_3}{F_{31}}$	$F_{21}^{+} = -f_2^{+} K^{-1}$	$F_{11}^{-} \tau_2 + F_{22}^{-} \tau_1 = -f_1^{-} K^{-1} \tau_2$	$F_{11}^{-} F_{13}^{-} F_{31}^{-} \tau_2^2 + F_{11}^{-} F_{12}^{-} F_{21}^{-} \tau_2 \tau_3$ $+ F_{22}^{-} F_{12}^{-} F_{21}^{-} \tau_1 \tau_3 > 0$
	2	$f_1^{-} = f_2^{-}; F_{12}^{-} = F_{22}^{-}; F_{11}^{-} = F_{21}^{-}$ $F_{13}^{-} = F_{23}^{-}; f_2^{+} = 0$				$F_{12}^{-} F_{23}^{-} F_{31}^{-} \tau_1 \tau_2 + F_{11}^{-} F_{13}^{-} F_{31}^{-} \tau_2^2$ $+ F_{11}^{-} F_{12}^{-} F_{21}^{-} \tau_2 \tau_3 + F_{22}^{-} F_{12}^{-} F_{21}^{-} \tau_1 \tau_3 > 0$
IF3DA	1	$F_{31}^{-} = 0$ $f_2^{-} = f_3^{-}; F_{22}^{-} = F_{32}^{-}$ $F_{23}^{-} = F_{33}^{-}$	TOO COMPLEX!			

Having identified the types that lead to tunable ACIVAs, the amplifier circuits can now be obtained. This task is readily accomplished by using the characterization of each type, the general configuration of the resistive network as in Fig. 3.2 and the resistor elimination procedure mentioned before.

Since the procedure is similar to the one used in the previous chapter for 2 OA ACVAs and in this chapter for noninverting 3 OA ACVAs, the elimination procedure is illustrated for only one of the possible types.

Consider type IF3AB-1. As a consequence of its characterization, the following resistor eliminations are possible

$$F_{22}=0 \Rightarrow G_{3,7}=G_{3,8}=0$$

$$F_{33}=0 \Rightarrow G_{4,9}=G_{4,10}=0$$

$$f_2=0 \Rightarrow G_{1,7}=G_{1,8}=0$$

$$F_{32}=0 \Rightarrow G_{3,9}=G_{3,10}=0$$

$$F_{21}=0 \Rightarrow G_{2,7}=G_{2,8}=0$$



$$f_1^+ = 0 \Rightarrow G_{1,5} = 0$$

$$f_2 = 0 \text{ and } f_1^- \neq 0 \Rightarrow G_{6,7} = G_{6,8} = 0$$

$$F_{11}^+ = 0 \Rightarrow G_{2,5} = 0$$

$$f_3^- = 0 \Rightarrow G_{1,10} = 0$$

$$F_{31}^- = 0 \Rightarrow G_{2,10} = 0$$

$$F_{13}^+ = 0 \Rightarrow G_{4,5} = 0$$

$$F_{12}^+ = 0 \Rightarrow G_{3,5} = 0$$

$$f_3^- = 0 \text{ and } f_1^- \neq 0 \Rightarrow G_{6,10} = 0$$

$$F_{11}^+ = F_{12}^+ = f_1^+ = F_{13}^+ = 0 \Rightarrow G_{5,7} = G_{5,8} = G_{5,9} = G_{5,10} = 0$$

$$F_{12}^- \neq 0 \text{ and } F_{32}^+ = 0 \Rightarrow G_{6,9} = 0$$

$$F_{23}^+ \neq 0 \text{ and } F_{33} = 0 \Rightarrow G_{7,9} = G_{7,10} = 0$$

$$f_3^+ \neq 0 \text{ and } f_2^- = 0 \Rightarrow G_{8,9} = 0$$

$$F_{33}^- = 0 \text{ and } F_{23}^- \neq 0 \Rightarrow G_{8,10} = 0$$

By performing these resistor eliminations, the general graph shown in Fig. 3.2 can be significantly reduced. A closer look at this reduced graph reveals that  $G_{9,11}$  is not really necessary because only the ratio between  $f_3^+$  and  $F_{31}^+$  is relevant and not their absolute value. The same reasoning applies to  $F_{11}^-$  and  $f_1^-$  and  $G_{6,11}$  can also be eliminated.

This resistor elimination procedure can be applied to all types in Table 3.7 in a similar way and, for each type, a reduced graph can be obtained. These reduced graphs are shown in Table 3.8. Now, the graphs in this table are simple enough to allow the circuits to be obtained by attempting, for each type, to realize the  $F_{ij}$ s and  $f_i$ s in different fashions. Because a very large number of circuits (150) were obtained by the application of the procedure just described, the realizations obtained are described in Table 3.9 by indicating the elements of the resistive network embedding the OAs.

TABLE 3.8: REDUCED GRAPHS FOR ALL TYPES OF TUNABLE ACIVAs.

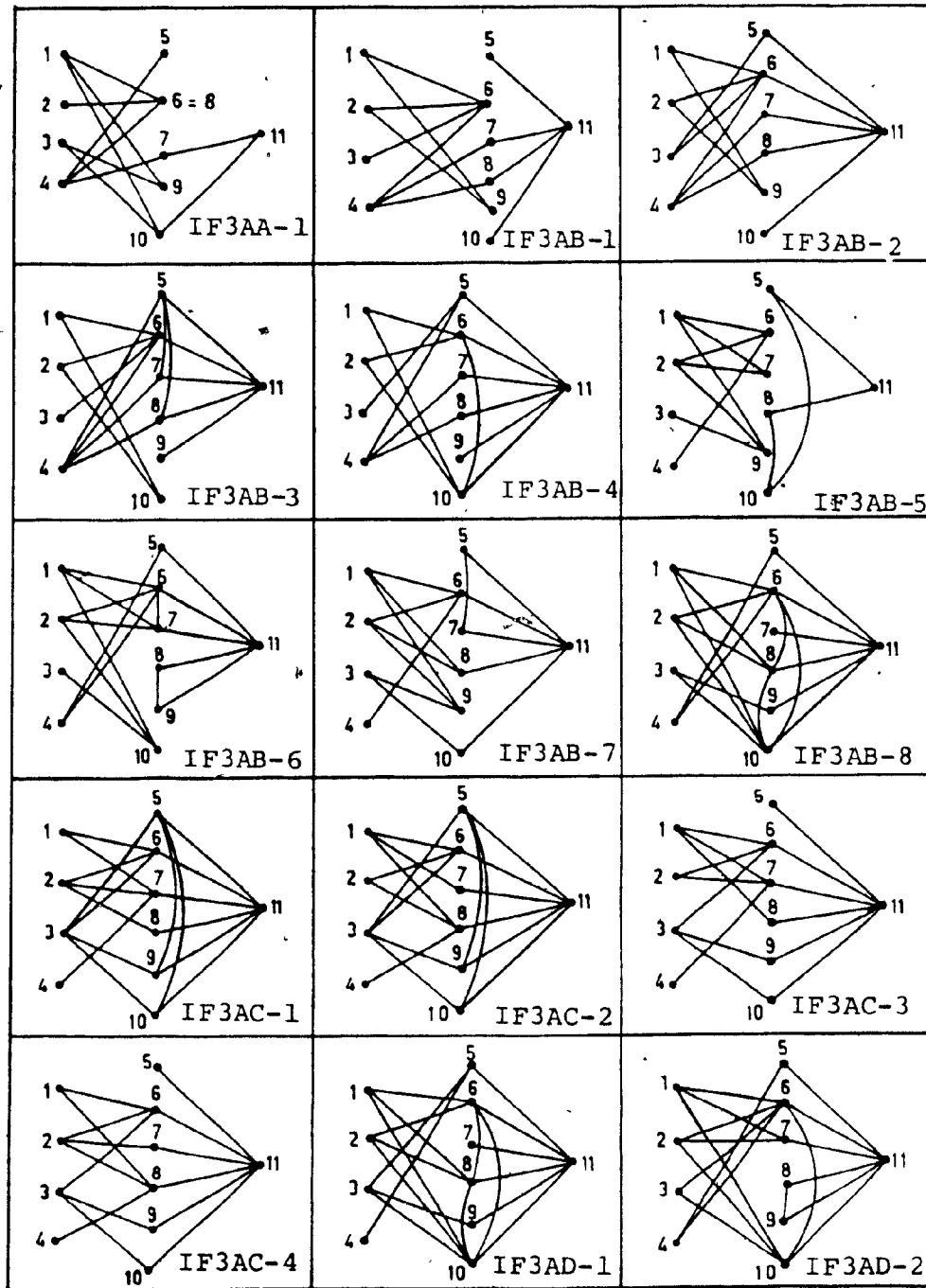


TABLE 3.8: (continued)

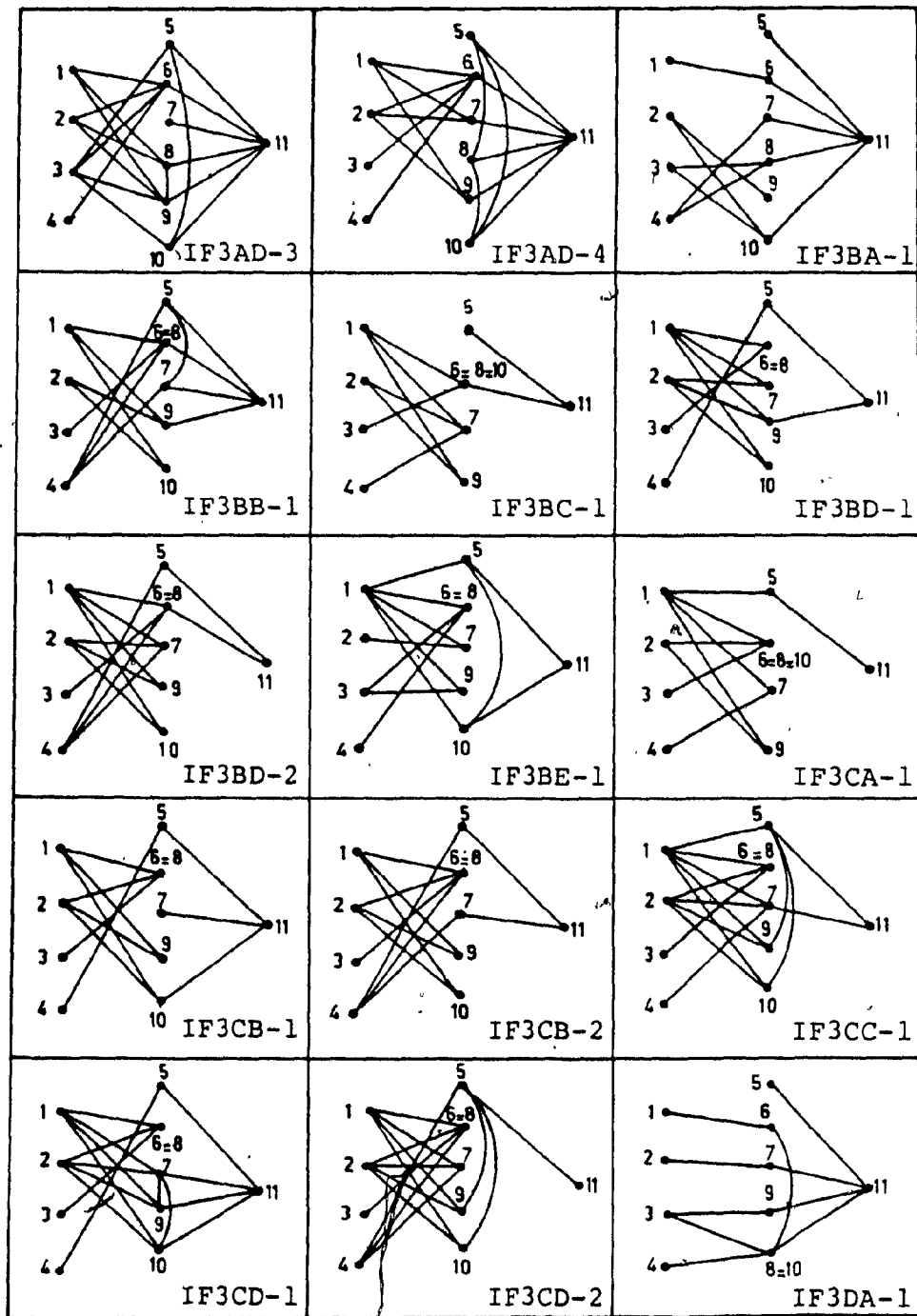


TABLE 3.9: DESCRIPTION OF THE RESISTIVE NETWORK FOR  
TUNABLE ACIVAS.

AMPLIFIER TYPE	DESCRIPTION
IF3AA-1	$R_{1,6}-R/a_2; R_{1,9}-R(1-\alpha); R_{2,8}-R/a_1; R_{3,9}-\alpha R; R_{4,5}-0; R_{4,6}-R/a_3; R_{5,8}-R_{7,11}-R_{10,11}-0$
	$R_{1,6}-R/a_2; R_{1,10}-R(1-\alpha); R_{2,8}-R/a_1; R_{3,10}-\alpha R; R_{4,7}-0; R_{4,8}-R/a_3; R_{5,8}-R_{5,11}-R_{9,11}-0$
IF3AB-1 *	$R_{1,6}-R/a_4; R_{1,9}-R(1-\beta); R_{2,6}-R/a_1; R_{2,9}-\beta R; R_{3,6}-R/a_2; R_{4,6}-R/a_3; R_{4,7}-R_{5,11}-R_{8,11}-R_{10,11}-0$
	$R_{1,6}-R/a_4; R_{1,9}-R(1-\beta); R_{2,6}-R/a_1; R_{2,9}-\beta R; R_{3,6}-R/a_2; R_{4,6}-R/a_3; R_{4,7}-R(1-\alpha); R_{7,11}-\alpha R; R_{5,11}-R_{8,11}-R_{10,11}-0$
	$R_{1,6}-R/a_3; R_{1,9}-R(1-\beta); R_{2,6}-R/a_1; R_{2,9}-\beta R; R_{3,6}-R/a_2; R_{4,7}-R/a_3; R_{4,8}-\alpha R; R_{8,11}-R(1-\alpha); R_{5,11}-R_{10,11}-0$
IF3AB-2	$R_{1,6}-R/a_2; R_{1,9}-R(1-\beta); R_{2,9}-\beta R; R_{2,6}-R/a_1; R_{4,6}-R/a_3; R_{3,5}-R_{4,8}-R_{7,11}-R_{10,11}-0$
	$R_{1,6}-R/a_4; R_{1,9}-R(1-\beta); R_{2,6}-R/a_1; R_{2,9}-\beta R; R_{3,6}-R/a_2; R_{4,6}-R/a_3; R_{3,5}-R_{4,8}-R_{7,11}-R_{10,11}-0$
	$R_{1,6}-R/a_2; R_{1,9}-R(1-\beta); R_{2,6}-R/a_1; R_{2,9}-\beta R; R_{3,5}-R(1-\gamma); R_{4,6}-R/a_3; R_{5,11}-\gamma R; R_{4,8}-R_{9,11}-R_{10,11}-0$
	$R_{1,6}-R/a_2; R_{1,9}-R(1-\beta); R_{2,6}-R/a_1; R_{2,9}-\beta R; R_{4,6}-R/a_3; R_{4,8}-R(1-\alpha); R_{8,11}-\alpha R; R_{3,5}-R_{7,11}-R_{10,11}-0$
	$R_{1,6}-R/a_4; R_{1,9}-R(1-\beta); R_{2,6}-R/a_1; R_{2,9}-\beta R; R_{3,6}-R/a_2; R_{4,6}-R/a_3; R_{4,8}-R(1-\alpha); R_{8,11}-\alpha R; R_{3,5}-R_{7,11}-R_{10,11}-0$
	$R_{1,6}-R/a_2; R_{1,9}-R(1-\beta); R_{2,6}-R/a_1; R_{2,9}-\beta R; R_{3,5}-R(1-\gamma); R_{4,8}-R(1-\alpha); R_{4,6}-R/a_3; R_{5,11}-\gamma R; R_{8,11}-\alpha R; R_{7,11}-R_{10,11}-0$
	$R_{1,6}-R/a_2; R_{1,9}-R(1-\beta); R_{2,6}-R/a_1; R_{2,9}-\beta R; R_{4,6}-R/a_3; R_{4,7}-R(1-\alpha); R_{7,11}-\alpha R; R_{3,5}-R_{4,8}-R_{10,11}-0$
	$R_{1,6}-R/a_4; R_{1,9}-R(1-\beta); R_{2,6}-R/a_1; R_{2,9}-\beta R; R_{3,6}-R/a_2; R_{4,6}-R/a_3; R_{4,7}-R(1-\alpha); R_{7,11}-\alpha R; R_{3,5}-R_{4,8}-R_{10,11}-0$
IF3AB-3	$R_{1,6}-R/a_2; R_{1,9}-R(1-\beta); R_{2,6}-R/a_1; R_{2,9}-\beta R; R_{3,5}-R(1-\gamma); R_{4,6}-R/a_3; R_{4,7}-R(1-\alpha); R_{5,11}-\gamma R; R_{7,11}-\alpha R; R_{4,8}-R_{10,11}-0$
	$R_{1,6}-R/a_3; R_{1,10}-R(1-\beta); R_{2,6}-R/a_1; R_{2,10}-\beta R; R_{3,6}-R/a_2; R_{4,8}-R_{5,8}-R_{7,11}-R_{9,11}-0$
	$R_{1,6}-R/a_3; R_{1,10}-R(1-\beta); R_{2,6}-R/a_1; R_{2,10}-\beta R; R_{3,6}-R/a_2; R_{4,7}-\alpha R; R_{7,11}-R(1-\alpha); R_{4,8}-R_{5,8}-R_{9,11}-0$
	$R_{1,6}-R/a_3; R_{1,10}-R(1-\beta); R_{2,6}-R/a_1; R_{2,10}-\beta R; R_{3,6}-R/a_2; R_{4,8}-R(1-\alpha); R_{8,11}-\alpha R; R_{4,5}-R_{7,11}-R_{9,11}-0$
	$R_{1,6}-R/a_4; R_{1,10}-R(1-\beta); R_{2,6}-R/a_1; R_{2,10}-\beta R; R_{3,6}-R/a_2; R_{4,6}-R/a_3; R_{4,8}-R_{5,8}-R_{7,11}-R_{9,11}-0$
	$R_{1,6}-R/a_4; R_{1,10}-R(1-\beta); R_{2,6}-R/a_1; R_{2,10}-\beta R; R_{3,6}-R/a_2; R_{4,6}-R/a_3; R_{4,8}-R(1-\alpha); R_{8,11}-\alpha R; R_{4,5}-R_{7,11}-R_{9,11}-0$
	$R_{1,6}-R/a_3; R_{1,10}-R(1-\beta); R_{2,6}-R/a_1; R_{2,10}-\beta R; R_{3,6}-R/a_2; R_{4,5}-R(1-\gamma); R_{3,11}-\gamma R; R_{4,8}-R_{7,11}-R_{9,11}-0$
	$R_{1,6}-R/a_3; R_{1,10}-R(1-\beta); R_{2,6}-R/a_1; R_{2,10}-\beta R; R_{3,6}-R/a_2; R_{4,7}-\alpha R; R_{7,11}-R(1-\alpha); R_{5,7}-R_{4,8}-R_{9,11}-0$
IF3AB-4	$R_{1,6}-R(1-\beta); R_{2,6}-\beta R; R_{4,5}-R(1-\gamma); R_{3,5}-\gamma R; R_{4,7}-R_{6,10}-R_{8,11}-R_{9,11}-0$
	$R_{1,6}-R(1-\beta); R_{2,6}-\beta R; R_{4,5}-R(1-\gamma); R_{3,5}-\gamma R; R_{4,8}-\alpha R; R_{8,11}-R(1-\alpha); R_{4,7}-R_{9,11}-R_{6,10}-0$
	$R_{1,6}-R(1-\beta); R_{2,6}-\beta R; R_{4,5}-R(1-\gamma); R_{3,5}-\gamma R; R_{4,7}-R(1-\alpha); R_{7,11}-\alpha R; R_{6,10}-R_{8,11}-R_{9,11}-0$
IF3AB-5 *	$R_{1,6}-R/a_2; R_{1,7}-R(1-\beta); R_{1,9}-R/b_3; R_{2,6}-R/a_1; R_{2,7}-\beta R; R_{2,9}-R/b_1; R_{3,9}-R/b_2; R_{5,11}-R_{8,11}-R_{10,11}-0$
IF3AB-6 *	$R_{1,6}-R(1-\beta); R_{1,10}-R/a_3; R_{2,6}-\beta R; R_{2,10}-R/a_1; R_{3,10}-R/a_2; R_{4,5}-R_{6,7}-R_{8,11}-R_{9,11}-0$
	$R_{1,6}-R/b_2; R_{1,7}-R(1-\alpha); R_{1,10}-R/a_3; R_{2,6}-R/b_1; R_{2,7}-\alpha R; R_{2,10}-R/a_1; R_{3,10}-R/a_2; R_{4,6}-R/b_3; R_{4,5}-R_{7,11}-R_{9,11}-0$

NOTE: RESISTORS NOT MENTIONED IN THIS TABLE ARE TO BE CONSIDERED AN OPEN-CIRCUIT.

TABLE 3.9: (continued)

AMPLIFIER TYPE	DESCRIPTION
IF3AB-7	$R_{1,6} = R/a_2; R_{1,8} = R(1-\beta); R_{2,6} = R/a_1; R_{2,8} = \beta R; R_{4,6} = R/a_3; R_{8,9} = R_3, 10^{-R_5, 11} = 0$
	$R_{1,6} = R/a_2; R_{1,8} = R(1-\beta); R_{2,6} = R/a_1; R_{2,8} = \beta R; R_{3,10} = R(1-\alpha); R_{4,6} = R/a_3; R_{10,11} = R; R_{8,9} = R_5, 11^{-R_7, 11} = 0$
	$R_{1,6} = R/a_2; R_{1,8} = R(1-\beta); R_{1,9} = R/b_3; R_{2,6} = R/a_1; R_{2,8} = \beta R; R_{2,9} = R/b_1; R_{3,9} = R/b_2; R_{4,6} = R/a_3; R_{3,10} = R_5, 11^{-R_7, 11} = 0$
IF3AB-8	$R_{1,6} = (1-\beta)R; R_{2,6} = \beta R; R_{6,8} = R_6, 10^{-R_4, 5} = R_3, 9^{-R_7, 11} = 0$
	$R_{1,6} = (1-\beta)R; R_{2,6} = \beta R; R_{4,5} = R(1-\gamma); R_{5,11} = \gamma R; R_{6,8} = R_6, 10^{-R_3, 9} = R_7, 11^{-0}$
	$R_{1,6} = R/a_2; R_{1,8} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,8} = \beta R; R_{4,6} = R/a_3; R_{3,9} = R_4, 5^{-R_8, 10} = R_7, 11^{-0}$
	$R_{1,6} = (1-\beta)R; R_{2,6} = \beta R; R_{3,9} = (1-\alpha)R; R_{9,11} = \alpha R; R_{4,5} = R_6, 8^{-R_6, 10} = R_7, 11^{-0}$
	$R_{1,6} = (1-\beta)R; R_{2,6} = \beta R; R_{3,9} = (1-\alpha)R; R_{4,5} = (1-\gamma)R; R_{9,11} = \alpha R; R_{5,11} = \gamma R; R_{6,8} = R_6, 10^{-R_7, 11} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,10} = R/a_3; R_{2,6} = \beta R; R_{2,10} = R/a_1; R_{3,10} = R/a_2; R_{3,9} = R_4, 5^{-R_6, 8} = R_7, 11^{-0}$
	$R_{1,6} = (1-\beta)R; R_{1,10} = R/a_3; R_{2,6} = \beta R; R_{2,10} = R/a_1; R_{3,10} = R/a_2; R_{4,5} = (1-\gamma)R; R_{5,11} = \gamma R; R_{3,9} = R_6, 8^{-R_7, 11} = 0$
IF3AC-1	$R_{1,6} = R/a_2; R_{1,8} = (1-\beta)R; R_{1,10} = R/b_3; R_{2,6} = R/a_1; R_{2,8} = \beta R; R_{2,10} = R/b_1; R_{3,10} = R/b_2; R_{4,6} = R/a_3; R_{3,9} = R_4, 5^{-R_7, 11} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\alpha)R; R_{2,6} = \beta R; R_{3,7} = \alpha R; R_{2,8} = R_3, 5^{-R_3, 10} = R_9, 11^{-0}$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\alpha)R; R_{2,6} = \beta R; R_{3,9} = (1-\gamma)R; R_{4,7} = \alpha R; R_{9,11} = \gamma R; R_{2,8} = R_3, 5^{-R_3, 10} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\alpha)R; R_{2,6} = \beta R; R_{3,10} = (1-\gamma)R; R_{4,7} = \alpha R; R_{10,11} = \gamma R; R_{2,8} = R_3, 5^{-R_9, 11} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = R/a_2; R_{2,6} = \beta R; R_{2,7} = R/a_1; R_{4,7} = R/a_3; R_{2,8} = R_3, 5^{-R_3, 10} = R_9, 11^{-0}$
	$R_{1,6} = (1-\beta)R; R_{1,7} = R/a_2; R_{2,6} = \beta R; R_{2,7} = R/a_1; R_{3,9} = (1-\gamma)R; R_{4,7} = R/a_3; R_{9,11} = \gamma R; R_{2,8} = R_3, 5^{-R_3, 10} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = R/a_2; R_{2,6} = \beta R; R_{2,7} = R/a_1; R_{3,10} = (1-\gamma)R; R_{4,7} = R/a_3; R_{10,11} = \gamma R; R_{2,8} = R_3, 5^{-R_9, 11} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\alpha)R; R_{2,6} = \beta R; R_{2,8} = (1-\gamma)R; R_{4,7} = \alpha R; R_{9,11} = \gamma R; R_{3,5} = R_3, 10^{-R_9, 11} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\alpha)R; R_{2,6} = \beta R; R_{2,8} = (1-\beta)R; R_{3,9} = (1-\gamma)R; R_{4,7} = \alpha R; R_{8,11} = \beta R; R_{9,11} = \gamma R; R_{3,5} = R_3, 10^{-0}$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\alpha)R; R_{2,6} = \beta R; R_{2,8} = (1-\beta)R; R_{3,10} = (1-\gamma)R; R_{4,7} = \alpha R; R_{8,11} = \beta R; R_{10,11} = \gamma R; R_{3,5} = R_9, 11^{-0}$
	$R_{1,6} = R/a_3; R_{1,7} = (1-\alpha)R; R_{2,6} = R/a_1; R_{3,6} = R/a_2; R_{4,7} = \alpha R; R_{2,8} = R_3, 5^{-R_3, 10} = R_9, 11^{-0}$
	$R_{1,6} = R/a_3; R_{1,7} = (1-\alpha)R; R_{2,6} = R/a_1; R_{3,6} = R/a_2; R_{3,9} = (1-\gamma)R; R_{4,7} = \alpha R; R_{9,11} = \gamma R; R_{2,8} = R_3, 5^{-R_3, 10} = 0$
	$R_{1,6} = R/a_3; R_{1,7} = (1-\alpha)R; R_{2,6} = \beta R; R_{3,6} = R/a_2; R_{3,10} = (1-\gamma)R; R_{4,7} = \alpha R; R_{10,11} = \gamma R; R_{2,8} = R_3, 5^{-R_9, 11} = 0$
	$R_{1,6} = R/a_3; R_{1,7} = R/b_2; R_{2,6} = R/a_1; R_{2,7} = R/b_1; R_{3,6} = R/a_2; R_{4,7} = R/b_3; R_{2,8} = R_3, 5^{-R_3, 10} = R_9, 11^{-0}$
	$R_{1,6} = R/a_3; R_{1,7} = R/b_2; R_{2,6} = R/a_1; R_{2,7} = R/b_1; R_{3,6} = R/a_2; R_{3,9} = (1-\gamma)R; R_{4,7} = R/b_3; R_{9,11} = \gamma R; R_{2,8} = R_3, 5^{-R_3, 10} = 0$
	$R_{1,6} = R/a_3; R_{1,7} = R/b_2; R_{2,6} = R/a_1; R_{2,7} = R/b_1; R_{3,6} = R/a_2; R_{3,10} = (1-\gamma)R; R_{4,7} = R/b_3; R_{10,11} = \gamma R; R_{2,8} = R_3, 5^{-R_9, 11} = 0$
	$R_{1,6} = R/a_3; R_{1,7} = (1-\alpha)R; R_{2,6} = R/a_1; R_{2,8} = (1-\gamma)R; R_{3,6} = R/a_2; R_{4,7} = \alpha R; R_{8,11} = \gamma R; R_{3,5} = R_3, 10^{-R_9, 11} = 0$
	$R_{1,6} = R/a_3; R_{1,7} = (1-\alpha)R; R_{2,6} = R/a_1; R_{2,8} = (1-\beta)R; R_{3,6} = R/a_2; R_{3,9} = (1-\gamma)R; R_{4,7} = \alpha R; R_{8,11} = \beta R; R_{9,11} = \gamma R; R_{3,5} = R_3, 10^{-0}$
	$R_{1,6} = R/a_3; R_{1,7} = (1-\alpha)R; R_{2,6} = R/a_1; R_{2,8} = (1-\beta)R; R_{3,6} = R/a_2; R_{3,10} = (1-\gamma)R; R_{4,7} = \alpha R; R_{8,11} = \beta R; R_{10,11} = \gamma R; R_{3,5} = R_9, 11^{-0}$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\alpha)R; R_{2,6} = \beta R; R_{3,5} = (1-\gamma)R; R_{4,7} = \alpha R; R_{5,11} = \gamma R; R_{2,8} = R_3, 10^{-R_9, 11} = 0$

NOTE: RESISTORS NOT MENTIONED IN THIS TABLE ARE TO BE CONSIDERED AN OPEN-CIRCUIT.

TABLE 3.9: (continued)

AMPLIFIER TYPE	DESCRIPTION
IF3AC-1	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\alpha)R; R_{2,6} = \beta R; R_{3,5} = (1-\gamma)R; R_{4,7} = \alpha R; R_{5,11} = \gamma R; R_{2,8} = R_{3,10} = R_{9,9} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\alpha)R; R_{2,6} = \beta R; R_{3,5} = (1-\gamma)R; R_{4,7} = \alpha R; R_{10,11} = \gamma R; R_{2,8} = R_{5,10} = R_{9,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = R/a_2; R_{2,6} = \beta R; R_{2,7} = R/a_1; R_{3,5} = (1-\gamma)R; R_{4,7} = R/a_3; R_{5,11} = \gamma R; R_{2,8} = R_{3,10} = R_{9,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = R/a_2; R_{2,6} = \beta R; R_{2,7} = R/a_1; R_{3,5} = (1-\gamma)R; R_{4,7} = R/a_3; R_{5,11} = \gamma R; R_{2,8} = R_{3,10} = R_{5,9} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = R/a_2; R_{2,6} = \beta R; R_{2,7} = R/a_1; R_{3,5} = (1-\gamma)R; R_{4,7} = R/a_3; R_{5,11} = \gamma R; R_{2,8} = R_{5,10} = R_{9,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\alpha)R; R_{2,6} = \beta R; R_{2,8} = (1-\theta)R; R_{3,5} = (1-\gamma)R; R_{4,7} = \alpha R; R_{8,11} = \theta R; R_{5,11} = \gamma R; R_{3,10} = R_{9,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\alpha)R; R_{2,6} = \beta R; R_{2,8} = (1-\theta)R; R_{3,5} = (1-\gamma)R; R_{4,7} = \alpha R; R_{5,11} = \gamma R; R_{8,11} = \theta R; R_{3,10} = R_{5,9} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\alpha)R; R_{2,6} = \beta R; R_{2,8} = (1-\theta)R; R_{3,5} = (1-\gamma)R; R_{4,7} = \alpha R; R_{5,11} = \gamma R; R_{8,11} = \theta R; R_{5,10} = R_{9,11} = 0$
IF3AC-2	$R_{1,6} = (1-\beta)R; R_{2,6} = \beta R; R_{2,8} = (1-\alpha)R; R_{4,8} = \alpha R; R_{1,7} = R_{3,5} = R_{3,9} = R_{10,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{2,6} = \beta R; R_{2,8} = (1-\alpha)R; R_{3,9} = (1-\gamma)R; R_{9,11} = \gamma R; R_{4,8} = \alpha R; R_{1,7} = R_{3,5} = R_{10,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{2,6} = \beta R; R_{2,8} = (1-\alpha)R; R_{3,10} = (1-\gamma)R; R_{4,8} = \alpha R; R_{10,11} = \gamma R; R_{1,7} = R_{3,5} = R_{3,9} = 0$
	$R_{1,6} = (1-\beta)R; R_{2,6} = \beta R; R_{2,8} = (1-\alpha)R; R_{3,5} = (1-\gamma)R; R_{4,8} = \alpha R; R_{5,11} = \gamma R; R_{1,7} = R_{3,9} = R_{10,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{2,6} = \beta R; R_{2,8} = (1-\alpha)R; R_{3,5} = (1-\gamma)R; R_{4,8} = \alpha R; R_{5,11} = \gamma R; R_{1,7} = R_{3,9} = R_{10,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{2,6} = \beta R; R_{2,8} = (1-\alpha)R; R_{3,5} = (1-\gamma)R; R_{4,8} = \alpha R; R_{5,11} = \gamma R; R_{1,7} = R_{3,9} = R_{10,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{2,6} = \beta R; R_{2,8} = (1-\alpha)R; R_{3,5} = (1-\gamma)R; R_{4,8} = \alpha R; R_{5,11} = \gamma R; R_{1,7} = R_{3,9} = R_{10,11} = 0$
	$R_{1,6} = R/a_3; R_{2,6} = R/a_1; R_{2,8} = (1-\alpha)R; R_{3,6} = R/a_2; R_{4,8} = \alpha R; R_{1,7} = R_{3,5} = R_{3,9} = R_{10,11} = 0$
	$R_{1,6} = R/a_3; R_{2,6} = R/a_1; R_{2,8} = (1-\alpha)R; R_{3,6} = R/a_2; R_{3,9} = (1-\gamma)R; R_{4,8} = \alpha R; R_{9,11} = \gamma R; R_{1,7} = R_{3,5} = R_{10,11} = 0$
	$R_{1,6} = R/a_3; R_{2,6} = R/a_1; R_{2,8} = (1-\alpha)R; R_{3,6} = R/a_2; R_{3,10} = (1-\gamma)R; R_{4,8} = \alpha R; R_{10,11} = \gamma R; R_{1,7} = R_{3,5} = R_{3,9} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\theta)R; R_{2,6} = \beta R; R_{2,8} = (1-\alpha)R; R_{4,8} = \alpha R; R_{7,11} = \theta R; R_{3,5} = R_{3,9} = R_{10,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\theta)R; R_{2,6} = \beta R; R_{2,8} = (1-\alpha)R; R_{3,9} = (1-\gamma)R; R_{4,8} = \alpha R; R_{7,11} = \theta R; R_{9,11} = \gamma R; R_{3,5} = R_{10,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\theta)R; R_{2,6} = \beta R; R_{2,8} = (1-\alpha)R; R_{3,10} = (1-\gamma)R; R_{3,8} = \alpha R; R_{7,10} = \theta R; R_{10,11} = \gamma R; R_{3,5} = R_{3,9} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\theta)R; R_{2,6} = \beta R; R_{2,8} = (1-\alpha)R; R_{3,5} = (1-\gamma)R; R_{4,8} = \alpha R; R_{5,11} = \gamma R; R_{7,11} = \theta R; R_{3,9} = R_{10,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\theta)R; R_{2,6} = \beta R; R_{2,8} = (1-\alpha)R; R_{3,5} = (1-\gamma)R; R_{4,8} = \alpha R; R_{5,11} = \gamma R; R_{7,11} = \theta R; R_{3,9} = R_{10,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,7} = (1-\theta)R; R_{2,6} = \beta R; R_{2,8} = (1-\alpha)R; R_{3,5} = (1-\gamma)R; R_{4,8} = \alpha R; R_{5,11} = \gamma R; R_{7,11} = \theta R; R_{3,9} = R_{10,11} = 0$
	$R_{1,6} = R/a_3; R_{1,7} = (1-\theta)R; R_{2,6} = R/a_1; R_{2,8} = (1-\alpha)R; R_{3,6} = R/a_2; R_{4,8} = \alpha R; R_{7,11} = \theta R; R_{3,5} = R_{3,9} = R_{10,11} = 0$
	$R_{1,6} = R/a_3; R_{1,7} = (1-\theta)R; R_{2,6} = R/a_1; R_{2,8} = (1-\alpha)R; R_{3,6} = R/a_2; R_{3,9} = (1-\gamma)R; R_{4,8} = \alpha R; R_{7,11} = \theta R; R_{9,11} = \gamma R; R_{3,5} = R_{10,11} = 0$
	$R_{1,6} = R/a_3; R_{1,7} = (1-\theta)R; R_{2,6} = R/a_1; R_{2,8} = (1-\alpha)R; R_{3,6} = R/a_2; R_{3,10} = (1-\gamma)R; R_{4,8} = \alpha R; R_{7,11} = \theta R; R_{10,11} = \gamma R; R_{3,5} = R_{3,9} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,8} = R/a_2; R_{2,6} = \beta R; R_{2,8} = R/a_1; R_{4,8} = R/a_3; R_{1,7} = R_{3,5} = R_{3,9} = R_{10,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,8} = R/a_2; R_{2,6} = \beta R; R_{2,8} = R/a_1; R_{3,9} = (1-\gamma)R; R_{4,8} = R/a_3; R_{5,11} = \gamma R; R_{1,7} = R_{3,5} = R_{10,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,8} = R/a_2; R_{2,6} = \beta R; R_{2,8} = R/a_1; R_{3,10} = (1-\gamma)R; R_{4,8} = R/a_3; R_{10,11} = \gamma R; R_{1,7} = R_{3,5} = R_{3,9} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,8} = R/a_2; R_{2,6} = \beta R; R_{2,8} = R/a_1; R_{3,5} = (1-\gamma)R; R_{4,8} = R/a_3; R_{5,11} = \gamma R; R_{1,7} = R_{3,9} = R_{10,11} = 0$

NOTE: RESISTORS NOT MENTIONED IN THIS TABLE ARE TO BE CONSIDERED AN OPEN-CIRCUIT.

TABLE 3.9: (continued)

AMPLIFIER TYPE	DESCRIPTION
IF3AC-2	$R_{1,6} = (1-\beta)R_1; R_{1,8} = R/a_2; R_{2,6} = \beta R; R_{2,8} = R/a_1; R_{3,5} = (1-\gamma)R; R_{4,8} = R/a_3; R_{5,11} = \gamma R; R_{1,7} = R_{5,9} = R_{10,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{1,8} = R/a_2; R_{2,6} = \beta R; R_{2,8} = R/a_1; R_{3,5} = (1-\gamma)R; R_{4,8} = R/a_3; R_{5,11} = \gamma R; R_{1,7} = R_{3,9} = R_{5,10} = 0$
	$R_{1,6} = R/a_3; R_{1,8} = R/b_2; R_{2,6} = R/a_1; R_{2,8} = R/b_1; R_{3,6} = R/a_2; R_{4,8} = R/b_3; R_{1,7} = R_{3,5} = R_{9,10,11} = 0$
	$R_{1,6} = R/a_3; R_{1,8} = R/b_2; R_{2,6} = R/a_1; R_{2,8} = R/b_1; R_{3,6} = R/a_2; R_{3,9} = (1-\gamma)R; R_{4,8} = R/b_3; R_{5,11} = \gamma R; R_{1,7} = R_{3,5} = R_{10,11} = 0$
	$R_{1,6} = R/a_3; R_{1,8} = R/b_2; R_{2,6} = R/a_1; R_{2,8} = R/b_1; R_{3,6} = R/a_2; R_{3,10} = (1-\gamma)R; R_{4,8} = R/b_3; R_{10,11} = \gamma R; R_{1,7} = R_{3,5} = R_{9,10} = 0$
IF3AC-3	$R_{1,6} = R/a_3; R_{2,6} = R/a_1; R_{2,7} = (1-\alpha)R; R_{3,6} = R/a_2; R_{4,7} = \alpha R; R_{1,8} = R_{3,10} = R_{5,11} = R_{9,11} = 0$
	$R_{1,6} = R/a_3; R_{2,6} = R/a_1; R_{2,7} = (1-\alpha)R; R_{3,6} = R/a_2; R_{3,10} = (1-\gamma)R; R_{4,7} = \alpha R; R_{10,11} = \gamma R; R_{1,8} = R_{5,11} = R_{9,11} = 0$
	$R_{1,6} = R/a_3; R_{2,6} = R/a_1; R_{2,7} = (1-\alpha)R; R_{3,6} = R/a_2; R_{3,9} = (1-\gamma)R; R_{4,7} = \alpha R; R_{9,11} = \gamma R; R_{1,8} = R_{3,10} = R_{5,11} = 0$
	$R_{1,6} = R/a_3; R_{1,8} = (1-\theta)R; R_{2,6} = R/a_1; R_{2,7} = (1-\alpha)R; R_{3,6} = R/a_2; R_{4,7} = \alpha R; R_{8,11} = \theta R; R_{3,10} = R_{5,11} = R_{9,11} = 0$
	$R_{1,6} = R/a_3; R_{1,8} = (1-\theta)R; R_{2,6} = R/a_1; R_{2,7} = (1-\alpha)R; R_{3,6} = R/a_2; R_{3,9} = (1-\gamma)R; R_{4,7} = \alpha R; R_{9,11} = \gamma R; R_{8,11} = \theta R; R_{5,11} = R_{3,10} = 0$
	$R_{1,6} = R/a_3; R_{1,7} = R/b_2; R_{2,6} = R/a_1; R_{2,7} = R/b_1; R_{3,6} = R/a_2; R_{4,7} = R/b_3; R_{1,8} = R_{3,10} = R_{5,11} = R_{9,11} = 0$
	$R_{1,6} = R/a_3; R_{1,7} = R/b_2; R_{2,6} = R/a_1; R_{2,7} = R/b_1; R_{3,6} = R/a_2; R_{3,10} = (1-\gamma)R; R_{4,7} = R/b_3; R_{10,11} = \gamma R; R_{1,8} = R_{5,11} = R_{9,11} = 0$
	$R_{1,6} = R/a_3; R_{1,7} = R/b_2; R_{2,6} = R/a_1; R_{2,7} = R/b_1; R_{3,6} = R/a_2; R_{3,9} = (1-\gamma)R; R_{4,7} = R/b_3; R_{9,11} = \gamma R; R_{1,8} = R_{3,10} = R_{5,11} = 0$
IF3AC-4	$R_{1,6} = R/a_3; R_{1,8} = (1-\alpha)R; R_{2,6} = R/a_1; R_{2,7} = (1-\theta)R; R_{3,6} = R/a_2; R_{4,8} = \alpha R; R_{2,7} = R_{3,9} = R_{5,11} = R_{10,11} = 0$
	$R_{1,6} = R/a_3; R_{1,8} = (1-\alpha)R; R_{2,6} = R/a_1; R_{2,7} = (1-\theta)R; R_{3,6} = R/a_2; R_{4,8} = \alpha R; R_{7,11} = \theta R; R_{3,9} = R_{5,11} = R_{10,11} = 0$
	$R_{1,6} = R/a_3; R_{1,8} = R/b_2; R_{2,6} = R/a_1; R_{2,8} = R/b_1; R_{3,6} = R/a_2; R_{4,8} = R/b_3; R_{2,7} = R_{3,9} = R_{5,11} = R_{10,11} = 0$
	$R_{1,6} = R/a_3; R_{1,8} = (1-\alpha)R; R_{2,6} = R/a_1; R_{2,7} = (1-\theta)R; R_{3,6} = R/a_2; R_{3,9} = (1-\gamma)R; R_{4,8} = \alpha R; R_{9,11} = \gamma R; R_{2,7} = R_{5,11} = R_{10,11} = 0$
	$R_{1,6} = R/a_3; R_{1,8} = (1-\alpha)R; R_{2,6} = R/a_1; R_{2,7} = (1-\theta)R; R_{3,6} = R/a_2; R_{3,9} = (1-\gamma)R; R_{4,8} = \alpha R; R_{7,11} = \theta R; R_{9,11} = \gamma R; R_{5,11} = R_{10,11} = 0$
	$R_{1,6} = R/a_3; R_{1,8} = (1-\alpha)R; R_{2,6} = R/a_1; R_{2,7} = (1-\theta)R; R_{3,6} = R/a_2; R_{3,10} = (1-\gamma)R; R_{4,8} = \alpha R; R_{10,11} = \gamma R; R_{2,7} = R_{3,9} = R_{5,11} = 0$
	$R_{1,6} = R/a_3; R_{1,8} = (1-\alpha)R; R_{2,6} = R/a_1; R_{2,7} = (1-\theta)R; R_{3,6} = R/a_2; R_{3,10} = (1-\gamma)R; R_{4,8} = \alpha R; R_{7,11} = \theta R; R_{10,11} = \gamma R; R_{3,9} = R_{5,11} = 0$
	$R_{1,6} = R/a_3; R_{1,8} = R/b_2; R_{2,6} = R/a_1; R_{2,8} = R/b_1; R_{3,6} = R/a_2; R_{3,10} = (1-\gamma)R; R_{4,8} = R/b_3; R_{10,11} = \gamma R; R_{2,7} = R_{3,9} = R_{5,11} = 0$
IF3AD-1	$R_{1,6} = (1-\beta)R; R_{2,6} = \beta R; R_{3,5} = (1-\gamma)R; R_{4,5} = \gamma R; R_{3,9} = R_{6,8} = R_{6,10} = R_{7,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{2,6} = \beta R; R_{3,5} = (1-\gamma)R; R_{3,9} = (1-\alpha)R; R_{4,5} = \gamma R; R_{9,11} = \alpha R; R_{6,8} = R_{6,10} = R_{7,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{2,6} = \beta R; R_{2,10} = R/a_1; R_{3,10} = R/a_2; R_{3,5} = (1-\gamma)R; R_{4,5} = \gamma R; R_{1,10} = R/a_3; R_{3,9} = R_{7,10} = R_{6,8} = 0$
IF3AD-1 *	$R_{1,6} = R/a_3; R_{1,7} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,7} = \beta R; R_{3,6} = R/a_2; R_{4,5} = R_{6,10} = R_{8,11} = R_{9,11} = 0$
	$R_{1,6} = R/a_3; R_{1,7} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,7} = \beta R; R_{3,6} = R/a_2; R_{4,5} = (1-\alpha)R; R_{5,11} = \alpha R; R_{6,10} = R_{8,11} = R_{9,11} = 0$
	$R_{1,6} = R/a_4; R_{1,7} = (1-\beta)R; R_{1,10} = R/b_3; R_{2,6} = R/a_1; R_{2,7} = \beta R; R_{2,10} = R/b_1; R_{3,6} = R/a_2; R_{3,10} = R/b_2; R_{4,6} = R/a_3; R_{4,5} = R_{8,11} = R_{9,11} = 0$

NOTE: RESISTORS NOT MENTIONED IN THIS TABLE ARE TO BE CONSIDERED AN OPEN-CIRCUIT.



TABLE 3.9: (continued)

AMPLIFIER TYPE	DESCRIPTION
IF3AD-3 *	$R_{1,6} = R/a_2; R_{1,8} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,8} = \beta R; R_{4,6} = R/a_3; R_{3,5} = R; R_{3,10} = R; R_{8,9} = R; R_{7,11} = 0$
	$R_{1,6} = R/a_2; R_{1,8} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,8} = \beta R; R_{3,5} = (1-\gamma)R; R_{4,6} = R/a_3; R_{3,11} = \gamma R; R_{3,10} = R; R_{8,9} = 0$
	$R_{1,6} = R/a_4; R_{1,8} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,8} = \beta R; R_{3,6} = R/a_2; R_{4,6} = R/a_3; R_{3,5} = R; R_{3,10} = R; R_{8,9} = R; R_{7,11} = 0$
	$R_{1,6} = R/a_2; R_{1,8} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,8} = \beta R; R_{3,10} = (1-\alpha)R; R_{4,6} = R/a_3; R_{3,11} = \alpha R; R_{3,5} = R; R_{8,9} = R; R_{7,11} = 0$
	$R_{1,6} = R/a_2; R_{1,8} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,8} = \beta R; R_{3,10} = (1-\alpha)R; R_{3,5} = (1-\gamma)R; R_{4,6} = R/a_3; R_{3,11} = \gamma R; R_{3,10,11} = \alpha R; R_{8,9} = R; R_{7,11} = 0$
	$R_{1,6} = R/a_4; R_{1,8} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,8} = \beta R; R_{3,6} = R/a_2; R_{3,10} = (1-\alpha)R; R_{4,6} = R/a_3; R_{3,11} = \alpha R; R_{3,5} = R; R_{8,9} = R; R_{7,11} = 0$
	$R_{1,6} = R/a_2; R_{1,8} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,8} = \beta R; R_{2,9} = R/b_1; R_{3,9} = R/b_2; R_{4,6} = R/a_3; R_{3,5} = R; R_{3,10} = R; R_{7,11} = 0$
	$R_{1,6} = R/a_2; R_{1,8} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,8} = \beta R; R_{2,9} = R/b_1; R_{3,5} = (1-\gamma)R; R_{3,9} = R/b_2; R_{4,6} = R/a_3; R_{3,11} = \gamma R; R_{3,10} = R; R_{7,11} = 0$
	$R_{1,6} = R/a_4; R_{1,8} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,8} = \beta R; R_{2,9} = R/b_1; R_{3,6} = R/a_2; R_{3,9} = R/b_2; R_{4,6} = R/a_3; R_{3,5} = R; R_{3,10} = R; R_{7,11} = 0$
IF3AD-4 *	$R_{1,6} = R/a_4; R_{1,7} = (1-\beta)R; R_{1,9} = R/b_3; R_{2,6} = R/a_1; R_{2,7} = \beta R; R_{2,9} = R/b_1; R_{3,6} = R/a_2; R_{3,9} = R/b_2; R_{4,6} = R/a_3; R_{3,11} = R; R_{8,11} = R; R_{10,11} = 0$
IF3BA-1	$R_{1,6} = (1-\beta)R; R_{2,10} = \gamma R; R_{3,7} = \alpha R; R_{3,10} = (1-\gamma)R; R_{4,7} = (1-\alpha)R; R_{3,6} = \beta R; R_{2,9} = R; R_{3,8} = R; R_{5,11} = 0$
	$R_{1,6} = (1-\beta)R; R_{2,10} = \gamma R; R_{3,8} = (1-\alpha)R; R_{3,10} = (1-\gamma)R; R_{3,6} = \beta R; R_{4,8} = R; R_{2,9} = R; R_{5,11} = 0$
IF3BB-1	$R_{1,6} = (1-\gamma)R; R_{1,10} = (1-\beta)R; R_{2,10} = \beta R; R_{3,6} = \gamma R; R_{4,7} = (1-\alpha)R; R_{7,11} = \alpha R; R_{6,8} = R; R_{4,5} = R; R_{9,11} = 0$
	$R_{1,6} = R/a_1; R_{1,9} = (1-\beta)R; R_{2,9} = \beta R; R_{3,6} = R/a_2; R_{4,6} = R/a_3; R_{6,8} = R; R_{7,11} = R; R_{10,11} = 0$
IF3BC-1	$R_{1,6} = (1-\gamma)R; R_{1,7} = R/a_2; R_{1,9} = (1-\beta)R; R_{2,7} = R/a_1; R_{2,9} = \beta R; R_{3,6} = \gamma R; R_{4,7} = R/a_3; R_{6,8} = R; R_{6,10} = R; R_{5,11} = 0$
IF3BD-1	$R_{1,6} = (1-\gamma)R; R_{1,7} = (1-\alpha)R; R_{1,10} = (1-\beta)R; R_{2,7} = \alpha R; R_{2,10} = \beta R; R_{3,6} = \gamma R; R_{6,8} = R; R_{4,5} = R; R_{9,11} = 0$
	$R_{1,6} = (1-\gamma)R; R_{1,7} = (1-\alpha)R; R_{1,10} = (1-\beta)R; R_{2,7} = \alpha R; R_{2,10} = \beta R; R_{3,6} = \gamma R; R_{4,5} = (1-\theta)R; R_{5,11} = \theta R; R_{6,8} = R; R_{9,11} = 0$
IF3BD-2	$R_{1,6} = R/a_1; R_{1,7} = R/b_2; R_{1,9} = (1-\beta)R; R_{2,7} = R/b_1; R_{2,9} = \beta R; R_{3,6} = R/a_2; R_{4,6} = R/a_3; R_{4,7} = R/b_3; R_{6,8} = R; R_{5,11} = R; R_{10,11} = 0$
	$R_{1,6} = R/a_1; R_{1,7} = (1-\gamma)R; R_{1,10} = (1-\beta)R; R_{2,7} = \alpha R; R_{2,10} = \beta R; R_{3,6} = R/a_2; R_{4,5} = (1-\theta)R; R_{4,6} = R/a_3; R_{5,11} = \theta R; R_{6,8} = R; R_{9,11} = 0$
IF3BE-1	$R_{1,6} = R/b_1; R_{1,7} = (1-\beta)R; R_{1,9} = (1-\alpha)R; R_{2,7} = \beta R; R_{3,6} = R/b_2; R_{3,9} = \alpha R; R_{4,6} = R/b_3; R_{6,8} = R; R_{5,11} = R; R_{10,11} = 0$
IF3CA-1	$R_{1,6} = R/a_3; R_{1,7} = (1-\alpha)R; R_{1,9} = (1-\beta)R; R_{2,9} = \beta R; R_{2,6} = R/a_1; R_{3,6} = R/a_2; R_{4,7} = \alpha R; R_{6,8} = R; R_{6,10} = R; R_{5,11} = 0$
IF3CB-1	$R_{1,6} = R/a_3; R_{1,10} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,10} = \beta R; R_{3,6} = R/a_2; R_{4,5} = R; R_{6,8} = R; R_{7,11} = R; R_{9,11} = 0$
	$R_{1,6} = R/a_3; R_{1,10} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,10} = \beta R; R_{3,6} = R/a_2; R_{4,5} = (1-\alpha)R; R_{5,11} = \alpha R; R_{6,8} = R; R_{7,11} = R; R_{9,11} = 0$
IF3CB-2	$R_{1,6} = R/a_3; R_{1,10} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,10} = \beta R; R_{3,6} = R/a_2; R_{4,7} = (1-\gamma)R; R_{7,11} = \gamma R; R_{6,8} = R; R_{4,5} = R; R_{9,11} = 0$
	$R_{1,6} = R/a_3; R_{1,10} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,10} = \beta R; R_{3,6} = R/a_2; R_{4,5} = (1-\alpha)R; R_{5,11} = \alpha R; R_{6,8} = R; R_{4,7} = R; R_{9,11} = 0$
IF3CC-1	$R_{1,6} = R/a_3; R_{1,7} = R/b_2; R_{1,10} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,7} = R/b_1; R_{2,10} = \beta R; R_{3,6} = R/a_2; R_{4,7} = R/b_3; R_{6,8} = R; R_{5,11} = R; R_{9,11} = 0$
IF3CD-1	$R_{1,6} = R/a_3; R_{1,7} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,7} = \beta R; R_{3,6} = R/a_2; R_{4,8} = R; R_{3,5} = R; R_{7,10} = R; R_{9,11} = 0$
	$R_{1,6} = R/a_3; R_{1,7} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,7} = \beta R; R_{3,6} = R/a_2; R_{4,5} = (1-\alpha)R; R_{5,11} = \alpha R; R_{6,8} = R; R_{7,10} = R; R_{9,11} = 0$
IF3CD-2	$R_{1,6} = R/a_4; R_{1,7} = R/b_2; R_{1,9} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,7} = R/b_1; R_{2,9} = \beta R; R_{3,6} = R/a_2; R_{4,6} = R/a_3; R_{4,7} = R/b_3; R_{5,11} = R; R_{6,8} = R; R_{10,11} = 0$
	$R_{1,6} = R/a_4; R_{1,7} = R/b_2; R_{1,10} = (1-\beta)R; R_{2,6} = R/a_1; R_{2,7} = R/b_1; R_{2,10} = \beta R; R_{3,6} = R/a_2; R_{4,6} = R/a_3; R_{4,7} = R/b_3; R_{6,8} = R; R_{4,5} = R; R_{9,11} = 0$
IF3DA-1	$R_{1,6} = R/a; R_{3,9} = (1-\beta)R; R_{3,10} = R/a_1; R_{4,10} = R/a_2; R_{6,10} = R/a; R_{9,11} = \beta R; R_{10,11} = R/a_3; R_{2,7} = R; R_{8,10} = R; R_{5,11} = 0$ [30]

NOTE: RESISTORS NOT MENTIONED IN THIS TABLE ARE TO BE CONSIDERED AN OPEN-CIRCUIT.

### 3.5 EVALUATION OF THE ACVAs

Due to the large number of circuits generated in section 3.4, it is not practical to attempt to report the results of a detailed analysis of all the circuits in this thesis. Therefore, we limit ourselves to only a subset of the circuits obtained. The circuits forming this subset are selected in such way that they are attractive in practical situations and also are representative of the typical properties of the ACVAs in a general sense. In the selection of this subset, it is useful to note that the applications of noninverting VAs greatly outnumber those of inverting amplifiers. For this reason, all the noninverting 3 OA ACVAs will be selected but only some of the ACIVAs presented in this chapter will be analyzed.

A look at Table 3.7 reveals that only the ACIVAs belonging to the 8 types of subclass IF3AB and to the 4 types of subclass IF3AD do not require tracking OAs to fulfill the  $s$ - and the  $s^2$ -compensation conditions. In each of these types, there is a most general realization from which the other ACIVAs in the same type can be obtained by assigning specific values to appropriate resistive transfer ratios. Since ACIVAs that do not require tracking OAs are considered preferable in general and all circuits possessing this property are particular cases of the 12 general realizations referred to above, it seems convenient to use

them as ACIVA prototypes in our study. In this context, it is worth noting the following points:

- (a) Although circuits employing nontracking OAs are preferable in general, an ACVA requiring tracking OAs may present some advantage (e.g. smaller phase and gain errors, better stability with respect to the OAs' second pole, etc.) in a given practical situation.
- (b) The 12 circuits selected as prototypes are marked by an asterisk (\*) in Table 3.9.
- (c) The results obtained for the 12 ACIVAs chosen as a sample can readily be extended to the other circuits of the same type by appropriately substituting values for  $\alpha, \beta, \gamma, a_1, a_2, a_3, b_1, b_2, b_3$  as the case may be.
- (d) The transfer function, the  $s$ - and  $s^2$ -compensation conditions and the stability condition for each of the circuits of the selected subset are shown in Table 3.10.
- (e) In order to allow extension of the results of the evaluation reported in this chapter to other ACVAs, the following sections contain not only the results of the analysis but also a brief description of how the analysis was performed.

TABLE 3.10: PROPERTIES OF THE ACIVAS SELECTED AS PROTOTYPES.

AMPLIFIER TYPE	TRANSFER FUNCTION (K(s))	DC-GAIN (K)	S-COMPENSATION	S <sup>2</sup> -COMPENSATION	STABILITY CONDITION
IF3AB-1	$\frac{-\beta}{(1-\beta)} \frac{\frac{b_1}{1 + \frac{a_2}{a_1} s} s^2 + \frac{a_4}{\beta a_2} s^2 s^3}{1 + \frac{a_3}{a_2} s^2 + \frac{a_1}{a_2(1-\beta)} s^2 s^3 + \frac{1}{a_2(1-\beta)} s^3 s^2 s^3}$	$\frac{-\beta}{(1-\beta)}$	-	$a_4 = -K a_1$	$\frac{a_1 a_3 s^2}{(a_1 + a_2 + a_3 + a_4)} s^2 > a a_2 s^2$
IF3AB-2	$\frac{-\beta}{(1-\beta)} \frac{\frac{a_3}{1 + \frac{a_2}{a_1} s} s^2 + \frac{a_4}{(a_1 + a_2 + a_3) \gamma} s^2 + \frac{a_2}{(a_1 + a_2 + a_3) \beta \gamma} s^2 s^3}{1 + \frac{a_3}{(a_1 + a_2 + a_3) \gamma} s^2 + \frac{a_1}{(a_1 + a_2 + a_3)(1-\beta) \gamma} s^2 s^3 + \frac{1}{\gamma(1-\beta)} s^3 s^2 s^3}$	$\frac{-\beta}{(1-\beta)}$	-	$a_2 = -K a_1$	$\frac{a_1 a_3}{(a_1 + a_2 + a_3)} s^2 > \gamma a s^2$
IF3AB-3	$\frac{-\beta}{(1-\beta)} \frac{\frac{1}{1 + \frac{\gamma}{(1-\gamma)} s} s^2 + \frac{1}{(1-\gamma) a} s^2 s^3}{1 + \frac{\gamma}{(1-\gamma) a} s^2 + \frac{1}{(1-\gamma) a} s^2 s^3 + \frac{1}{(1-\gamma) a(1-\beta)} s^3 s^2 s^3}$	$\frac{-\beta}{(1-\beta)}$	-	$a_3 = -K a_1$	$a_1 s^2 > a_2 s^2$
IF3AB-4	$\frac{-\beta}{(1-\beta)} \frac{\frac{1}{1 + \frac{\gamma}{(1-\gamma)} s} s^2 + \frac{1}{(1-\gamma) a} s^2 s^3}{1 + \frac{\gamma}{(1-\gamma) a} s^2 + \frac{1}{(1-\gamma) a} s^2 s^3 + \frac{1}{(1-\gamma) a(1-\beta)} s^3 s^2 s^3}$	$\frac{-\beta}{(1-\beta)}$	-	-	$(1-\beta) \gamma s^2 > (1-\gamma) a s^2$
IF3AB-5	$\frac{-\beta}{(1-\beta)} \frac{\frac{1}{1 + \frac{b_1}{(1-\beta) b_2} s} s^2 + \frac{a_1(b_1 + b_2 + b_3)}{a_3 b_2(1-\beta)} s^2 s^3 + \frac{a_2(b_1 + b_2 + b_3)}{a_3 b_2(1-\beta)} s^3 s^2 s^3}{1 + \frac{b_1}{(1-\beta) b_2} s^2 + \frac{a_1(b_1 + b_2 + b_3)}{a_3 b_2(1-\beta)} s^2 s^3 + \frac{a_2(b_1 + b_2 + b_3)}{a_3 b_2(1-\beta)} s^3 s^2 s^3}$	$\frac{-\beta}{(1-\beta)}$	$b_3 = -K b_1$	$a_2 = -K a_1$	$\frac{a_1 b_1}{(a_1 + a_2 + a_3) b_2} s^2 > (1-\beta) \gamma s^2$
IF3AB-6	$\frac{-\beta}{(1-\beta)} \frac{\frac{1}{1 + \frac{a_2}{a_1} s} s^2 + \frac{a_4}{a_2} s^2 s^3}{1 + \frac{a_2}{(1-\beta) a_1} s^2 + \frac{a_1 + a_2 + a_3}{a_2(1-\beta) a_1} s^2 s^3 + \frac{a_3 + a_2 + a_3}{a_2(1-\beta) a_1} s^3 s^2 s^3}$	$\frac{-\beta}{(1-\beta)}$	$a_3 = -K a_1$	-	$a_1 s^2 > a_2 s^2$

TABLE 3.10: (continued)

AMPLIFIER TYPE	TRANSFER FUNCTION (K(s))	DC-GAIN (K)	S-COMPENSATION	S <sup>2</sup> -COMPENSATION	STABILITY CONDITION
IF3AB-7	$\frac{-\beta}{(1-\beta)} \frac{1 + \frac{1}{\beta} s_2 + \frac{a_2}{\beta a_3} s_2^2}{1 + \frac{1}{\beta} s_2 + \frac{a_1}{\alpha(1-\beta)a_3} s_2^2 + \frac{(a_1+a_2+a_3)}{\alpha(1-\beta)a_3} s_2^3}$	$\frac{-\beta}{(1-\beta)}$	-	$a_2 = -ka_1$	$\frac{a_1}{(a_1+a_2+a_3)} s_2 > \alpha s_1$
IF3AB-8	$\frac{-\beta}{(1-\beta)} \frac{1 + \frac{1}{\beta} s_2 + \frac{1}{\gamma\alpha} s_2^2}{1 + \frac{1}{\beta} s_2 + \frac{1}{\gamma\alpha} s_2^2 + \frac{1}{\gamma\alpha(1-\beta)} s_2^3}$	$\frac{-\beta}{(1-\beta)}$	-	-	$(1-\beta)s_2 > \alpha s_1$
IF3AD-1	$\frac{-\beta}{(1-\beta)} \frac{1 + s[\frac{1}{\alpha} + \frac{s_2}{\alpha(1-\gamma)}] + \frac{1}{(1-\gamma)\alpha} s_2^2}{1 + s[\frac{1}{\alpha} + \frac{s_2}{\alpha(1-\gamma)}] + \frac{1}{(1-\gamma)\alpha} s_2^2 + \frac{1}{(1-\gamma)(1-\beta)\alpha} s_2^3}$	$\frac{-\beta}{(1-\beta)}$	-	-	$\frac{\beta}{(1-\beta)} s_2 + \frac{\gamma(1-\beta)s_3}{(1-\gamma)} > \alpha s_1$
IF3AD-2	$\frac{-\beta}{(1-\beta)} \frac{1 + s[\frac{a_3}{\beta a_2} s_2 + \frac{1}{\alpha} + \frac{a_3}{\beta a_2} s_2^2]}{1 + s[\frac{a_1}{(1-\beta)a_2} s_2 + \frac{1}{\alpha} + \frac{a_1}{\alpha(1-\beta)a_2} s_2^2 + \frac{(a_1+a_2+a_3)}{\alpha(1-\beta)a_2} s_2^3]}$	$\frac{-\beta}{(1-\beta)}$	$a_3 = -ka_1$	$a_3 = -ka_1$	$\frac{a_1}{(1-\beta)a_2} s_2 + \frac{a_1}{\alpha} s_3 > (a_1+a_2+a_3)s_1$
IF3AD-3	$\frac{-\beta}{(1-\beta)} \frac{1 + s[\frac{1}{\alpha} + \frac{s_2}{\alpha(1-\beta)a_3} + \frac{a_2}{\alpha\beta a_3} s_2^2]}{1 + s[\frac{1}{\alpha} + \frac{s_2}{\alpha(1-\beta)a_3} + \frac{a_1}{\alpha(1-\beta)a_3} s_2^2 + \frac{(a_1+a_2+a_3)}{\alpha(1-\beta)a_3} s_2^3]}$	$\frac{-\beta}{(1-\beta)}$	-	$a_2 = -ka_1$	$\frac{a_1 a_3}{(a_1+a_2+a_3)} s_2 + a_1 s_3 > a_3 s_1$
IF3AD-4	$\frac{-\beta}{(1-\beta)} \frac{1 + s[\frac{b_3 s_2}{\beta a_3} + \frac{a_2(b_1+b_2+b_3)s_1}{\beta a_3 b_2} + \frac{a_2(b_1+b_2+b_3)}{\beta a_3 b_2} s_2^2]}{1 + s[\frac{b_1 s_2}{(1-\beta)b_2} + \frac{a_2(b_1+b_2+b_3)s_1}{\beta a_3 b_2} + \frac{a_1(b_1+b_2+b_3)}{(1-\beta)a_3 b_2} s_2^2 + \frac{(a_1+a_2+a_3)(b_1+b_2+b_3)}{(1-\beta)a_3 b_2} s_2^3]}$	$\frac{-\beta}{(1-\beta)}$	$b_3 = -kb_1$	$a_2 = -ka_1$	$a_1 a_3 b_1 s_2 + a_1 a_2 (1-\beta)(b_1+b_2+b_3)s_3 > a_3 b_2 (1-\beta)(a_1+a_2+a_3)s_1$

### 3.5.1 Relative stability

Since ACVAs form a class of feedback amplifiers, one should be concerned not only with stand alone stability but also whether they exhibit or not a suitable amount of relative stability, namely a phase margin not less than  $30^\circ$  and not greater than  $60^\circ$  and a gain margin greater than 9.5 dB [39-40]. Hence, it is desirable to derive design equations that will guarantee the DC gain, the  $s$ - and the  $s^2$ -compensations and a suitable amount of relative stability.

These design equations are dependent on the values of the gain-bandwidth product of the OAs. Since these values tend to vary widely as mentioned before, this represents an inconvenience to the designer. Fortunately, the wide range of values for the gain and phase margins acceptable in practice and the fact that design equations for such margins depend on the ratios of different GBs rather than on their values contribute to alleviate this inconvenience.

If ACVAs are implemented with quad OAs (in which the different OAs characteristics track with each other closely) and the amplifiers are realized following the conditions on Table 3.11, it has been found that they will exhibit appropriate values of gain and phase margin even if there is a 20% mismatch among GBs of different OAs. In practice, such

TABLE 3.11: DESIGN EQUATIONS THAT ENSURE A PHASE MARGIN  
IN THE RANGE FROM  $40^\circ$  TO  $50^\circ$  AND A GAIN  
MARGIN NOT LESS THAN 9.5 dB.

AMPLIFIER TYPE	GAIN MARGIN CONDITION	PHASE MARGIN CONDITION
NI3A (*)	—	$\frac{a_1}{a_2} = 4$
NI3B-1 (a), (b) (T)	$\frac{a_2 Y}{a_3} = \frac{1}{3.75K}$	$\frac{a_2 Y}{a_3} = \frac{\sqrt{2}}{K} - \frac{\sqrt{2}}{K} \sqrt{a_3/a_1}$
NI3B-1 (c) (T), (**)	$\frac{a_2}{a_3} = \frac{1}{3.75K}$	$\frac{a_2}{a_3} = \frac{\sqrt{2}}{K} - \frac{\sqrt{2}}{K} \sqrt{a_3/a_1}$
NI3B-2 (a) (T)	$\frac{b_3}{b_1} = 0.4023$	$\frac{b_3}{b_1} = 0.1$
NI3B-2 (b) (**)	—	—
NI3B-3 (a) (*)	—	$\frac{a_3}{a_2} = 0.2$
NI3B-3 (b) (**)	—	—
NI3C-1 (T), (***)	$\frac{b_2}{b_1} = \frac{1}{3.75K}$	$\frac{b_2}{b_1} = \frac{1}{1.2} [ \sqrt{2} - \sqrt{2} \sqrt{1.2(K-1)} ]$
NI3C-2 (a) (T)	$\frac{a_3}{a_2} = 3.6(K-1) - \frac{2}{3}$	$\frac{a_3}{a_2} = \frac{(K-1)}{\sqrt{2} - \sqrt{2} \sqrt{(K-1)}} - 1$
NI3C-2 (b) (T)	$\frac{a_3}{a_2} = 3.6 \frac{b_3}{b_1} - \frac{2}{3}$	$\frac{a_3}{a_2} = \frac{b_3/b_1}{\sqrt{2} - \sqrt{2} \sqrt{b_3/b_1}} - 1$

(\*) Gain margin is always infinite

(\*\*) All the design variables are already fixed

(\*\*\*) These conditions can be satisfied only for some gain values

(T) Whenever the two conditions conflict, use the smallest value of the parameter

TABLE 3.11: (continued)

AMPLIFIER TYPE	GAIN MARGIN CONDITION	PHASE MARGIN CONDITION
IF3AB-1	$\frac{a_3 a_1}{(a_1 + a_2 + a_3) a_2 \alpha} = \frac{15}{4}$	$\frac{a_1 a_2 \alpha (1-K)}{a_3^2} = \frac{540\sqrt{2}}{3425 - 1296\sqrt{2}}$
IF3AB-2	$\frac{a_3 a_1}{(a_1 + a_2 + a_3) \gamma \alpha} = \frac{15}{4}$	$\left[ \frac{(a_1 + a_2 + a_3)(1-K)}{\sqrt{2} a_3} \right]^4 \left[ \frac{\alpha \gamma (a_1 + a_2 + a_3)}{\sqrt{2} a_3} - \frac{a_1}{(a_1 + a_2 + a_3)} \right] = \frac{-6\sqrt{2}}{10}$
IF3AB-3	$\frac{a_1}{a_2} = \frac{15}{4}$	$\alpha = \frac{(1-K)}{[(1-K) + \frac{4}{15}]^2 \sqrt{2}} \left[ \frac{4}{81} + \frac{5}{3} - \frac{32}{25\sqrt{2}} \right]$
IF3AB-4	$\frac{\gamma}{(1-K)(1-\gamma)\alpha} = \frac{15}{4}$	$\alpha = \frac{3\sqrt{2} \gamma}{6(1-K)(1-\gamma)} - \left[ \frac{36\sqrt{2} \gamma^3}{25(1-K)(1-\gamma)^2} \right]^4$
IF3AB-5	$\frac{b_1 a_1 (1-K)}{b_2 (a_1 + a_2 + a_3)} = \frac{15}{4}$	$\left[ \frac{(a_1 + a_2 + a_3)(b_1 + b_2 + b_3)}{\sqrt{2} a_3 b_1} \right]^4 \left[ \frac{b_2}{\sqrt{2} b_1 (1-K)} - \frac{a_1}{(a_1 + a_2 + a_3)} \right] = \frac{-6\sqrt{2}}{10}$
IF3AB-6	$\frac{a_1}{a_2} = \frac{15}{4}$	$\alpha = \frac{(\frac{19}{15} - K)}{(1-K)^2} \left[ \frac{3475\sqrt{2} - 3888}{4050} \right]$
IF3AB-7	$\frac{\gamma}{(a_1 + a_2 + a_3) \alpha} = \frac{15}{4}$	$\frac{\alpha a_1}{a_3} = \left[ \frac{540}{1175\sqrt{2} - 1296} \right] \frac{1}{1-K}$
IF3AB-8	$\alpha = \frac{4}{15(1-K)}$	$\gamma = \frac{25\alpha^2(1-K)}{36\sqrt{2}} + \frac{5\sqrt{2}}{6(1-K)} - \frac{12\alpha}{5}$
IF3AD-1	$\alpha(1-\gamma) = \frac{4}{15(1-K)}$	$\frac{125\alpha^2(1-\gamma)^2(1-K)^2}{216\sqrt{2}} - \frac{72\alpha(1-\gamma)}{25} + \frac{25\sqrt{2}}{36(1-K)} = 1$
IF3AD-2	$\frac{a_1^2(1-K)\alpha + a_1 a_2}{(a_1 + a_2 + a_3) a_2 \alpha} = \frac{15}{4}$	$\left[ \frac{(a_1 + a_2 + a_3)(1-K)}{\sqrt{2}(a a_1(1-K) + a_2)} \right]^4 \left[ \frac{6a a_2}{5\sqrt{2}(a a_1(1-K) + a_2)} - \frac{5a_1}{6(a_1 + a_2 + a_3)} \right] = \frac{-\sqrt{2}}{2}$
IF3AD-3	$\frac{a_1 \gamma (a_1 + a_2 + a_3) + a_3 a_1}{a_3 \alpha (a_1 + a_2 + a_3)} = \frac{15}{4}$	$\left[ \frac{(a_1 + a_2 + a_3)(1-K)}{\sqrt{2}\gamma(a_1 + a_2 + a_3) + \sqrt{2} a_3} \right]^4 \left[ \frac{6\alpha}{5\sqrt{2}(\gamma(a_1 + a_2 + a_3) + a_3)} - \frac{5a_1}{6(a_1 + a_2 + a_3)} \right] = \frac{-\sqrt{2}}{2}$
IF3AD-4	$\frac{a_1 a_2 (b_1 + b_2 + b_3) + a_1 a_3 b_1 (1-K)}{a_3 b_2 (a_1 + a_2 + a_3 + a_4)} = \frac{15}{4}$	$\left[ \frac{(a_1 + a_2 + a_3 + a_4)(1-K)(b_1 + b_2 + b_3)}{\sqrt{2}(a_2(b_1 + b_2 + b_3) + a_3 b_1(1-K))} \right]^4 \left[ \frac{6a_3 b_2}{5\sqrt{2}(a_2(b_1 + b_2 + b_3) + a_3 b_1(1-K))} - \frac{5a_1}{6(a_1 + a_2 + a_3 + a_4)} \right] = \frac{-\sqrt{2}}{2}$



mismatches are much less than 20% for quad OAs.

### 3.5.2 Phase and magnitude errors

In this subsection, in order to verify the improved performance of the ACVAs, the phase and magnitude errors of these circuits are calculated and compared with similar errors of conventional VAs.

The general form of the voltage transfer function of a 3 OA ACVA is given by

$$K(s) = K \frac{1 + \alpha_1 s + \alpha_2 s^2}{1 + \alpha_1 s + \alpha_2 s^2 + \alpha_3 s^3} \quad (3.44)$$

If the phase error is defined as

$$\Delta\phi = \arg(K(j\omega))$$

for noninverting VAs and

$$\Delta\phi = \arg(K(j\omega)) - \pi$$

for inverting circuits and if the magnitude error is defined by

$$\left| \frac{\Delta K(j\omega)}{K} \right| = \left| \frac{K - K(j\omega)}{K} \right|$$

then, by performing algebraic manipulations in eqn. (3.44), it follows that

$$\Delta\phi = \alpha_3 \omega^3 \quad (3.45)$$

$$\left| \frac{\Delta K(j\omega)}{K} \right| = \alpha_1 \alpha_3 \omega^3 \quad (3.46)$$

The results of this analysis are shown in Table 3.12 where the values of the GBs of the OAs are assumed to be equal. Since  $\omega\tau$  is usually much less than 1, the errors in the ACVAs are much smaller than the corresponding ones in the conventional realizations.

TABLE 3.12: PHASE AND MAGNITUDE DEVIATIONS FOR THE ACVAS  
AT LOW FREQUENCIES.

AMPLIFIER TYPE	$ \Delta\phi $	$\frac{\Delta K(j\omega)}{K}$
NI3A (*)	$E(K\omega\tau)^3$	$E^2(K\omega\tau)^4$
NI3B-1 (a), (b)	$\frac{a_1}{a_2 a} K^2 (\omega\tau)^3$	$a_3 a_1 \left(\frac{K}{a_2 a}\right)^2 (\omega\tau)^4$
NI3B-1 (c)	$\frac{a_1}{a_2} K^2 (\omega\tau)^3$	$a_3 a_1 \left(\frac{K}{a_2}\right)^2 (\omega\tau)^4$
NI3B-2 (a)	$\frac{b_1}{b_3} (K\omega\tau)^3$	$\left(\frac{b_1}{b_3}\right)^2 (K\omega\tau)^4$
NI3B-2 (b)	$(K-1)^{-1} (K\omega\tau)^3$	$(K-1)^{-2} (K\omega\tau)^4$
NI3B-3 (a)	$\frac{a_2}{a_3} (K\omega\tau)^3$	$\left(1 + \frac{a_2}{a_3}\right) \frac{a_2}{a_3} (K\omega\tau)^4$
NI3B-3 (b)	$(K-1)^{-1} (K\omega\tau)^3$	$K(K-1)^{-2} (K\omega\tau)^4$
NI3C-1	$\frac{b_1}{b_2} (K-1)^{-1} (K\omega\tau)^3$	$\left(\frac{a_1}{a_2}\right)^2 (K-1)^{-1} (K\omega\tau)^4$
NI3C-2 (a)	$\frac{a_1}{a_2} (K-1)^{-1} (K\omega\tau)^3$	$\left(\frac{a_1}{a_2}\right)^2 (K-1)^{-1} (K\omega\tau)^4$
NI3C-2 (b)	$\frac{a_1}{a_2} (K-1)^{-1} (K\omega\tau)^3$	$\left(1 + \frac{a_3}{a_2}\right) \frac{a_1}{a_2} (K-1)^{-2} (K\omega\tau)^4$

(\*)  $E = 1 + \frac{a_1 \tau_3}{a_2 \tau_2}$

TABLE 3.12: (continue)

AMPLIFIER TYPE	$ \Delta\phi $	$\frac{\Delta K(j\omega)}{K}$
IF3AB-1	$\frac{(1-K)}{a_2 \alpha} (\omega\tau)^3$	$\frac{(1-K)a_1}{a_2^2 \alpha^2} (\omega\tau)^4$
IF3AB-2	$\frac{(1-K)}{\alpha \gamma} (\omega\tau)^3$	$\frac{(1-K)a_3}{(a_1+a_2+a_3)\alpha^2 \gamma^2} (\omega\tau)^4$
IF3AB-3	$\frac{(a_1+a_2+a_3)(1-K)}{\alpha a_2} (\omega\tau)^3$	$\frac{(a_1+a_2+a_3)^2(1-K)}{\alpha a_2^2} (\omega\tau)^4$
IF3AB-4	$\frac{(1-K)}{\alpha(1-\gamma)} (\omega\tau)^3$	$\frac{\gamma(1-K)}{\alpha^2(1-\gamma)^2} (\omega\tau)^4$
IF3AB-5	$\frac{(a_1+a_2+a_3)(b_1+b_2+b_3)(1-K)}{a_3 b_2} (\omega\tau)^3$	$\frac{b_1(a_1+a_2+a_3)(b_1+b_2+b_3)(1-K)^2}{a_3 b_2} (\omega\tau)^4$
IF3AB-6	$\frac{(a_1+a_2+a_3)(1-K)}{\alpha a_2} (\omega\tau)^3$	$\frac{a_1(a_1+a_2+a_3)(1-K)^2}{\alpha a_2^2} (\omega\tau)^4$
IF3AB-7	$\frac{(a_1+a_2+a_3)(1-K)}{\alpha a_3} (\omega\tau)^3$	$\frac{(a_1+a_2+a_3)(1-K)}{\alpha^2 a_3} (\omega\tau)^4$
IF3AB-8	$\frac{(1-K)}{\alpha \gamma} (\omega\tau)^3$	$\frac{(1-K)}{\alpha^2 \gamma} (\omega\tau)^4$
IF3AD-1	$\frac{(1-K)}{\alpha(1-\gamma)} (\omega\tau)^3$	$\frac{(1-K)}{\alpha^2(1-\gamma)^2} (\omega\tau)^4$
IF3AD-2	$\frac{(a_1+a_2+a_3)(1-K)}{\alpha a_2} (\omega\tau)^3$	$\frac{[a_1 \alpha(1-K)^2 + a_2(1-K)](a_1+a_2+a_3)}{\alpha^2 a_2^2} (\omega\tau)^4$
IF3AD-3	$\frac{(a_1+a_2+a_3)(1-K)}{\alpha a_3} (\omega\tau)^3$	$\frac{[(a_1+a_2+a_3)a_3 + \gamma(a_1+a_2+a_3)^2](1-K)}{\alpha^2 a_3^2} (\omega\tau)^4$
IF3AD-4	$\frac{(a_1+a_2+a_3+a_4)(b_1+b_2+b_3)(1-K)}{a_3 b_2} (\omega\tau)^3$	$\frac{b_1(1-K)^2}{a_3 b_2^2} + \frac{a_2(b_1+b_2+b_3)(1-K)}{a_3 b_2^2} \times (\omega\tau)^4$

### 3.5.3 Effect of the second pole of the OAs

As it was mentioned before, the presence of a secondary pole in actual OA characteristics can seriously compromise the stability of the ACVAs. For this reason, it is useful to analyze the stability properties of the ACVAs when a more complete model for the OAs is used.

A more accurate model for the differential gain of the OA is given by

$$A_i(s) = \frac{0}{s\tau_i(1 + \frac{s\tau_i}{\theta_i})}, \quad i = 1, 2, 3 \quad (3.47)$$

where  $-\theta_i/\tau_i$  is the second pole frequency and  $\theta_i$  is the ratio between the second pole frequency and the gain bandwidth product of the  $i$ th OA.

Hence, the study of the stability of the ACVAs when the second pole of the OAs is taken into consideration can be accomplished by making the substitution

$$s\tau_i \rightarrow s\tau_i(1 + s\tau_i/\theta_i) \quad (3.48)$$

in the denominator of the transfer function expressions presented in Table 3.4 and 3.10 and then checking the

position of the roots of the resulting polynomial. Since this polynomial has degree equal to six, the study of its roots can be very complex if attempted by traditional methods such as Routh-Hurwitz, etc.

A method that avoids this difficulty is due to Geiger [44]. To employ the method, the gains of the OAs will be considered equal, namely  $\theta_i = \theta$  and  $\tau_i = \tau$  for  $i=1,2,3$ . This method can be outlined as follows.

If the complex roots of  $D(s)$ , the denominator of the transfer function of any ACVA when eqn. (1.24) is used to model the OAs, are given by

$$p_i = -\alpha \pm j\beta$$

it can be shown that the polynomial obtained from  $D(s)$  through the transformation in eqn. (3.48) will not have roots outside the left-half of the complex plane, provided that  $\alpha$ ,  $\beta$  and  $\theta$  satisfy

$$\beta^2 < \alpha\theta$$

By using this condition, it is possible to determine the minimum value of  $\theta$  that the OAs should have in order that the ACVA remains stable for a given value of the gain. This can be done for all ACVAs. Fig. 3.5 shows some examples of such plots, namely for all the noninverting circuits and the ACIVAs of types IF3AB-4 and IF3AB-8.

#### 3.5.4 Maximum signal handling capability

The analysis of the maximum signal handling capability of 3 OA ACVAs can be performed in the same manner as described in subsection 2.4.4. The only difference is in the fact that, in the present case, there is one additional OA output to be accounted for. Thus eqn. (2.34) can be rewritten as

$$v_{i,\max} = \frac{\min\{v_{0,\max}^{(PS)}, v_{0,\max}^{(SR)}\}}{\max\{|H_1(j\omega)|, |H_2(j\omega)|, |H_3(j\omega)|\}} \quad (3.49)$$

for  $i = 1, 2, 3$  where  $v_{0,\max}^{(PS)}$  and  $v_{0,\max}^{(SR)}$  are given by eqns. (2.32) and (2.33) respectively and  $H_i(j\omega)$  is the transfer function from the input of the overall ACVA to the output of the  $i$ th OA. By using eqn. (3.49), plots of the maximum

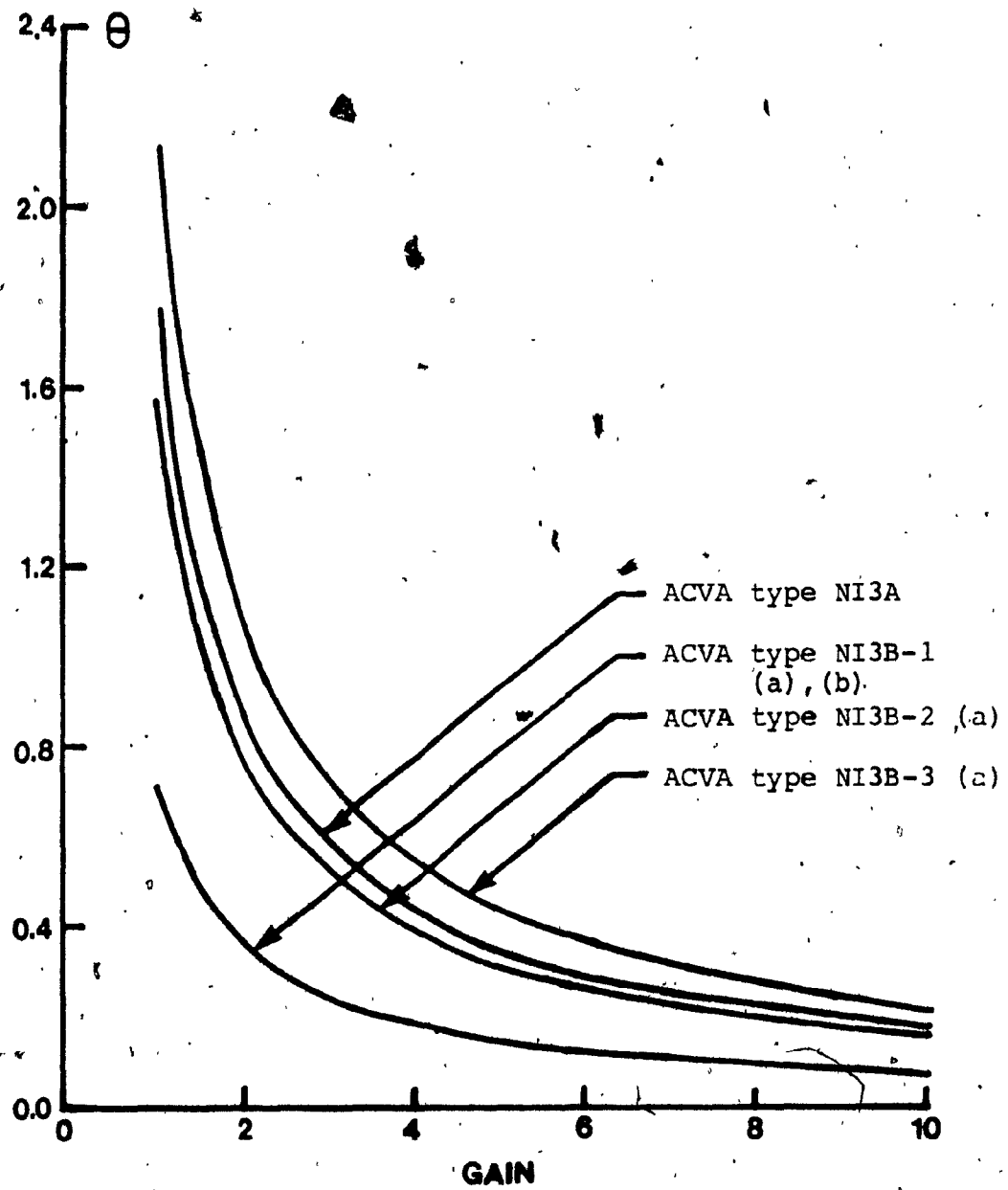


Fig. 3.5: Minimum value of  $\theta$  for a given gain in order to assure stability.



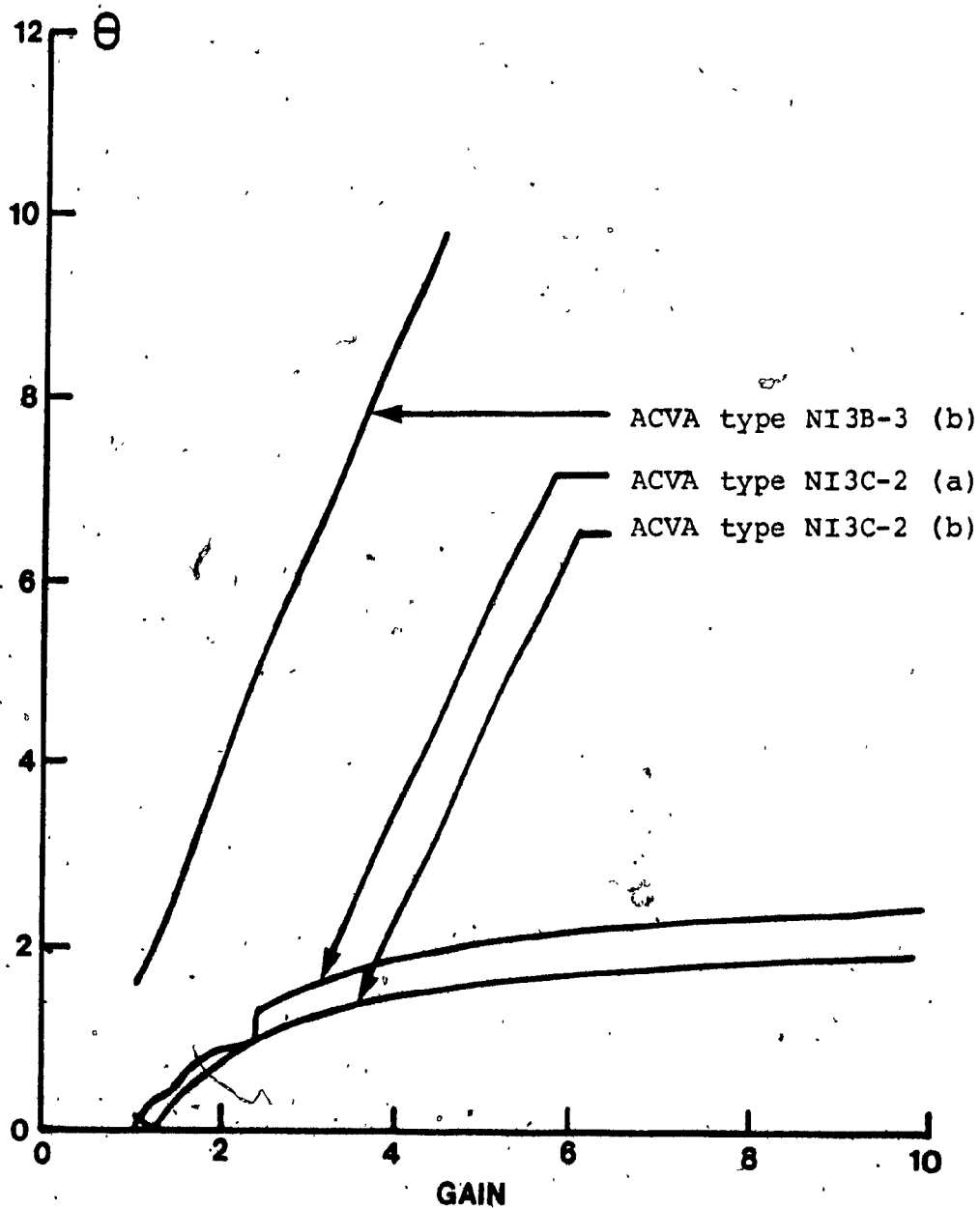


Fig. 3.5: (continued)

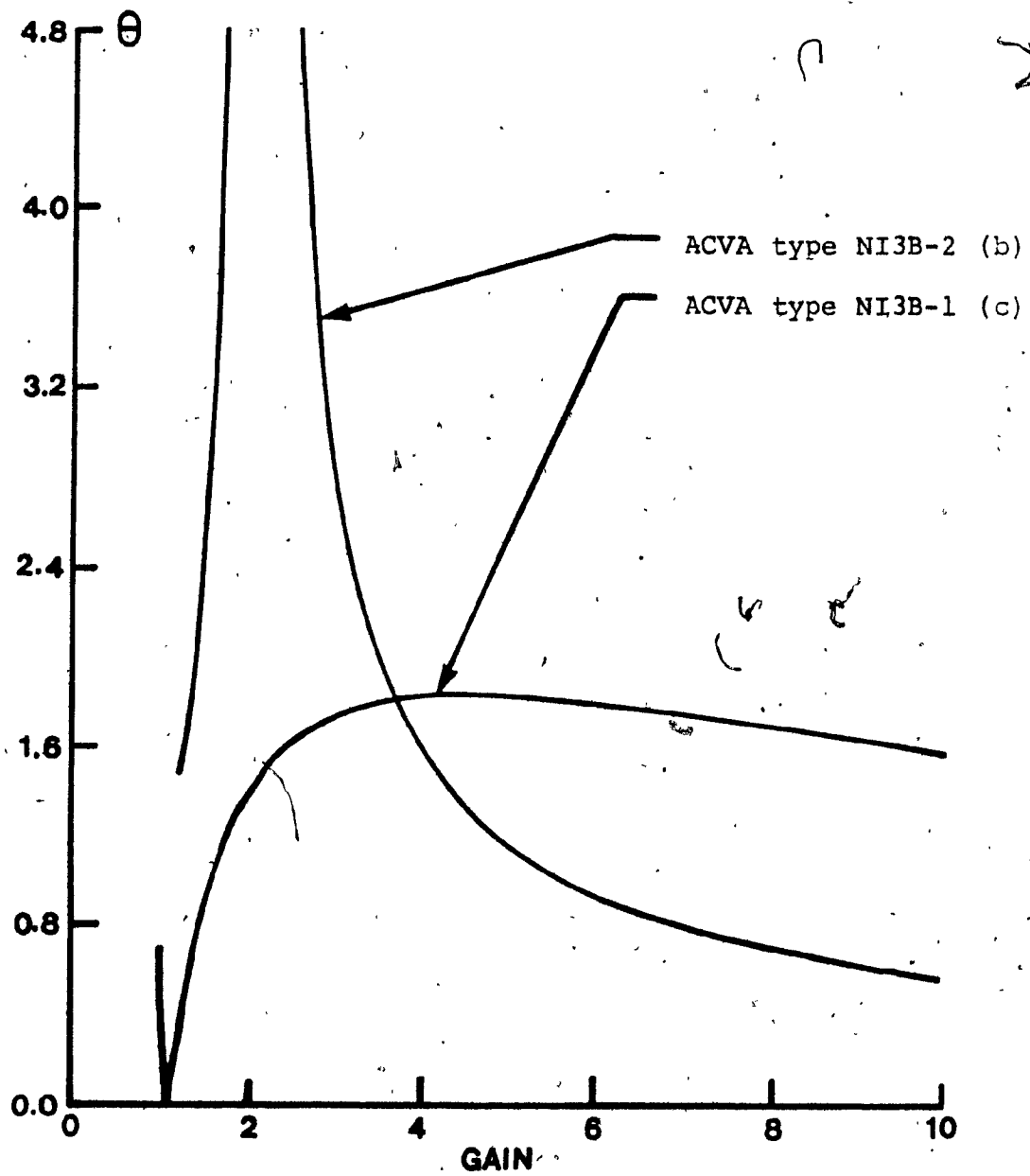


Fig. 3.5: (continued)

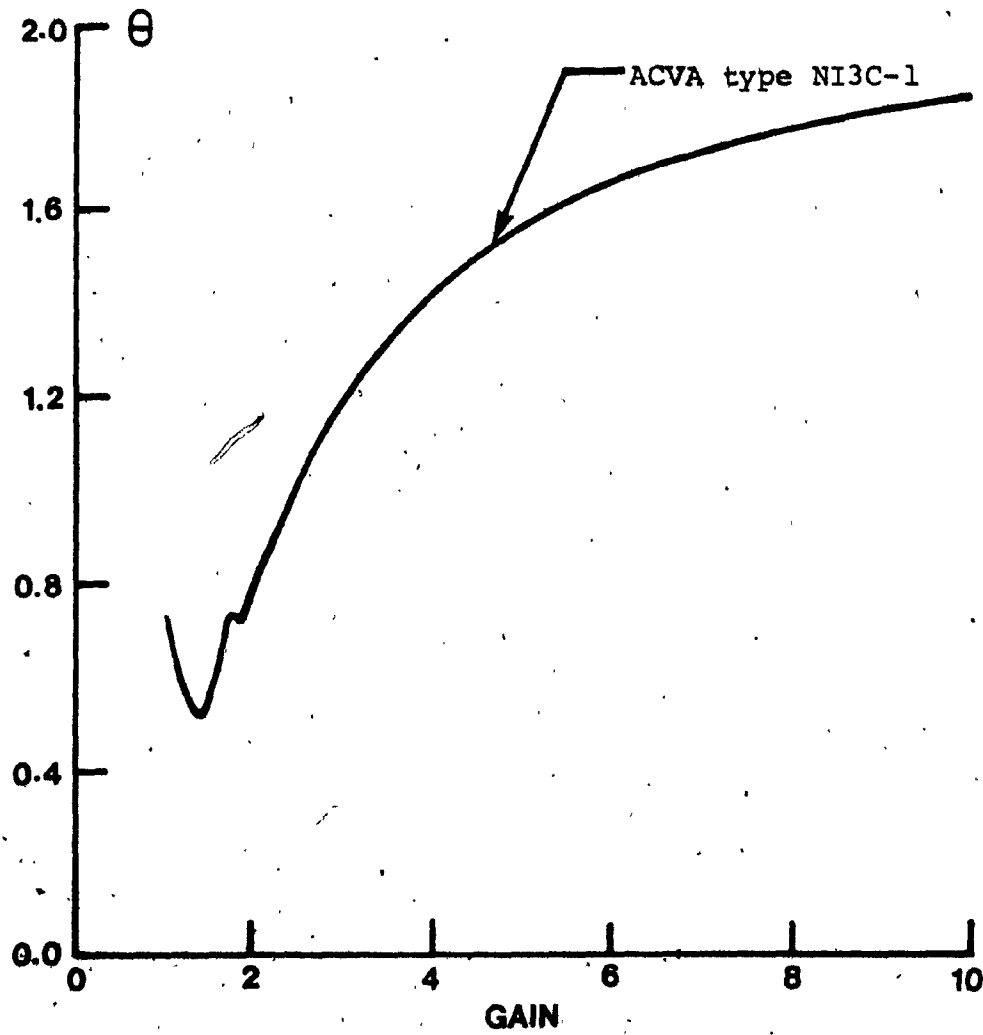


Fig. 3.5: (continued)

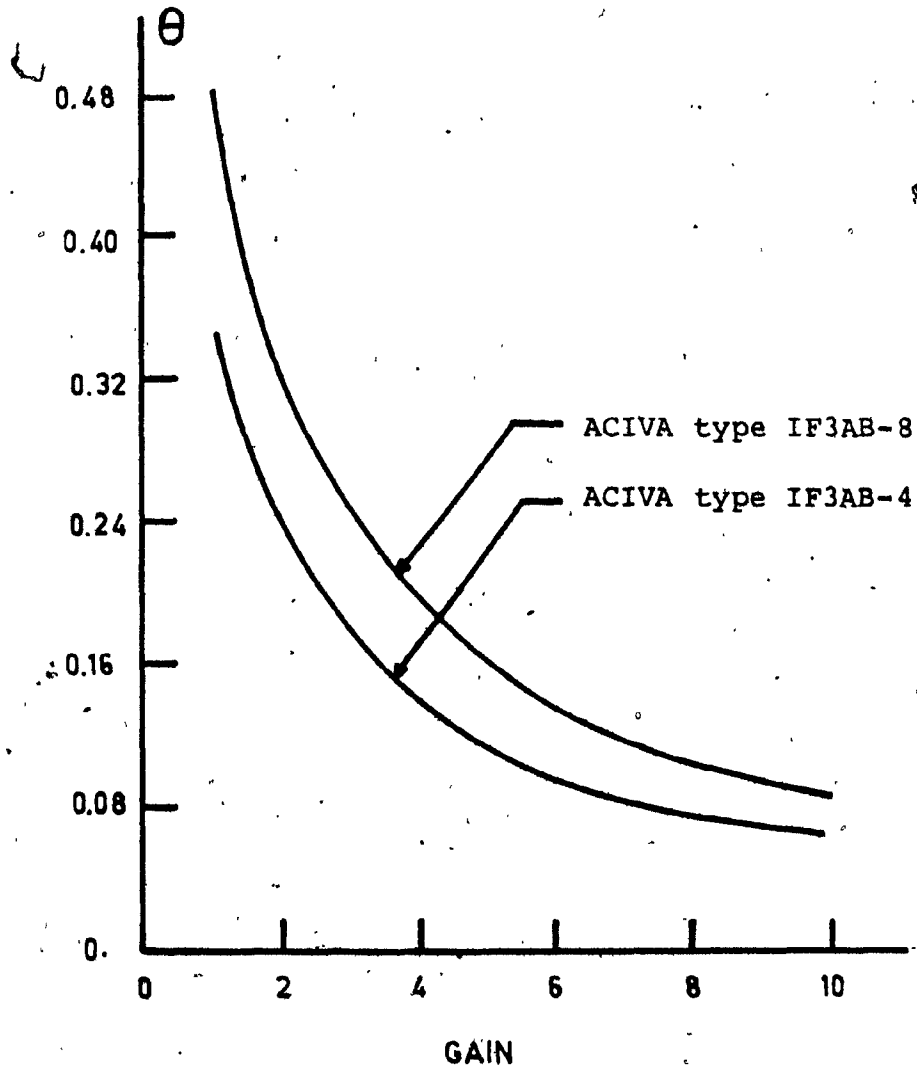


Fig. 3.5: (continued)

allowable input signal amplitude as a function of frequency can be obtained. Fig. 3.6 shows some examples of such plots. For these plots, we have considered a gain equal to 2 and -1,  $V_{CC}$  equal to 15 V and 741-type OAs with SR equal to 0.5 V/ $\mu$ s and GB equal to  $2\pi \times 10^6$  rad/s. Notice that for frequencies up to 100 kHz, the ACVAs have maximum signal handling capability comparable to the one presented by the conventional VA.

### 3.5.5 Tuning procedure

Since the response of actual hybrid integrated circuits departs from the nominal one due to manufacturing tolerances as well as due to parasitic effects, it is important to specify a tuning procedure capable of correcting such departures [9,42]. For the ACVAs presented, this procedure is carried out through the following steps:

- (a) At a frequency such that  $K\omega\tau \ll 1$ , adjust the DC gain by modifying the proper resistor ratio, namely  $\beta$ .
- (b) At a frequency such that  $(K\omega\tau)^2 \ll 1$ , null the phase shift of the circuit by modifying the appropriate resistor ratio which controls the s-compensation, namely  $\gamma$ ,  $a_1$ ,  $a_3$ ,  $b_1$ ,  $b_2$  or  $b_3$  as the case may be.

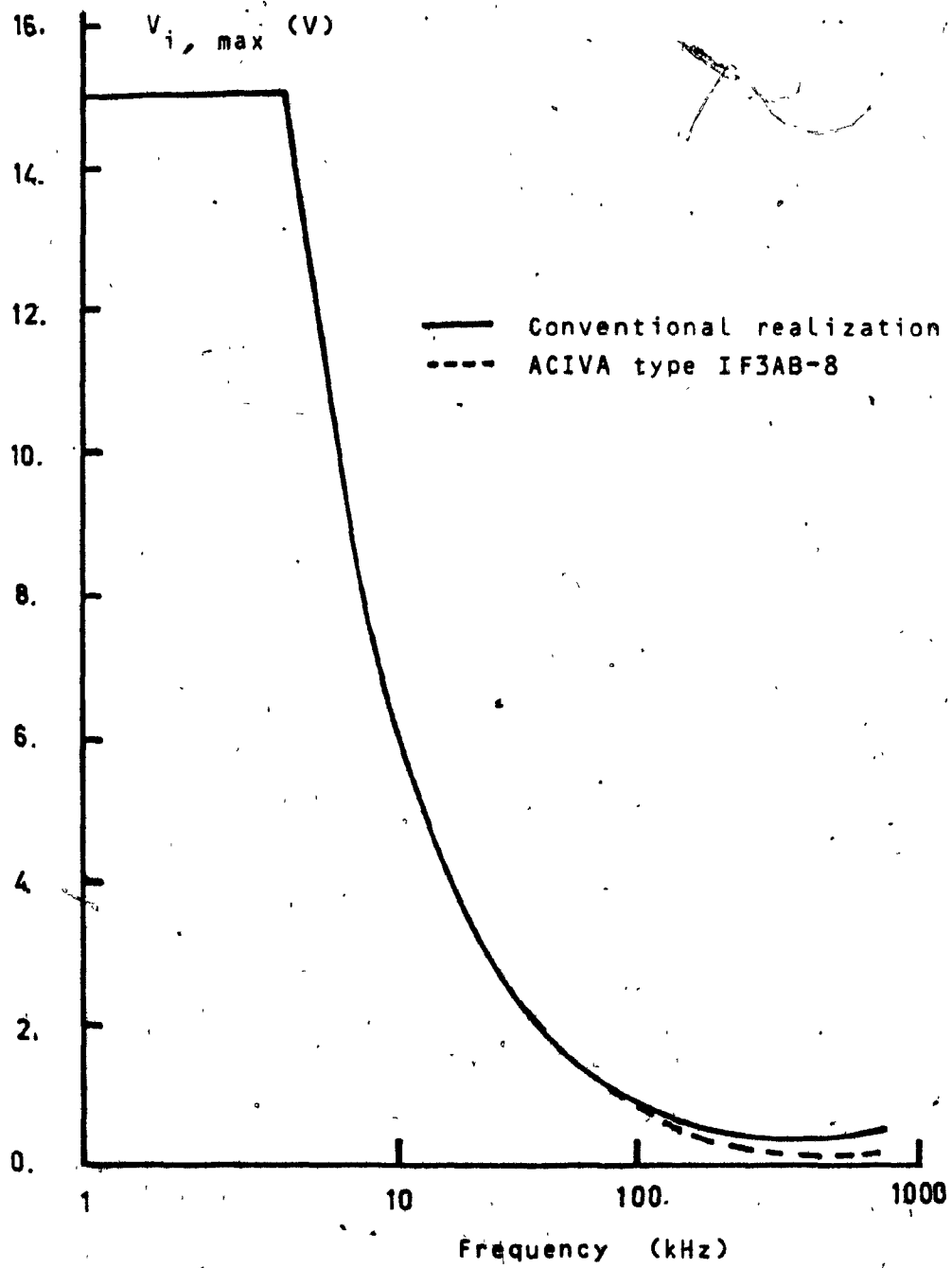


Fig. 3.6: Maximum signal handling capability.

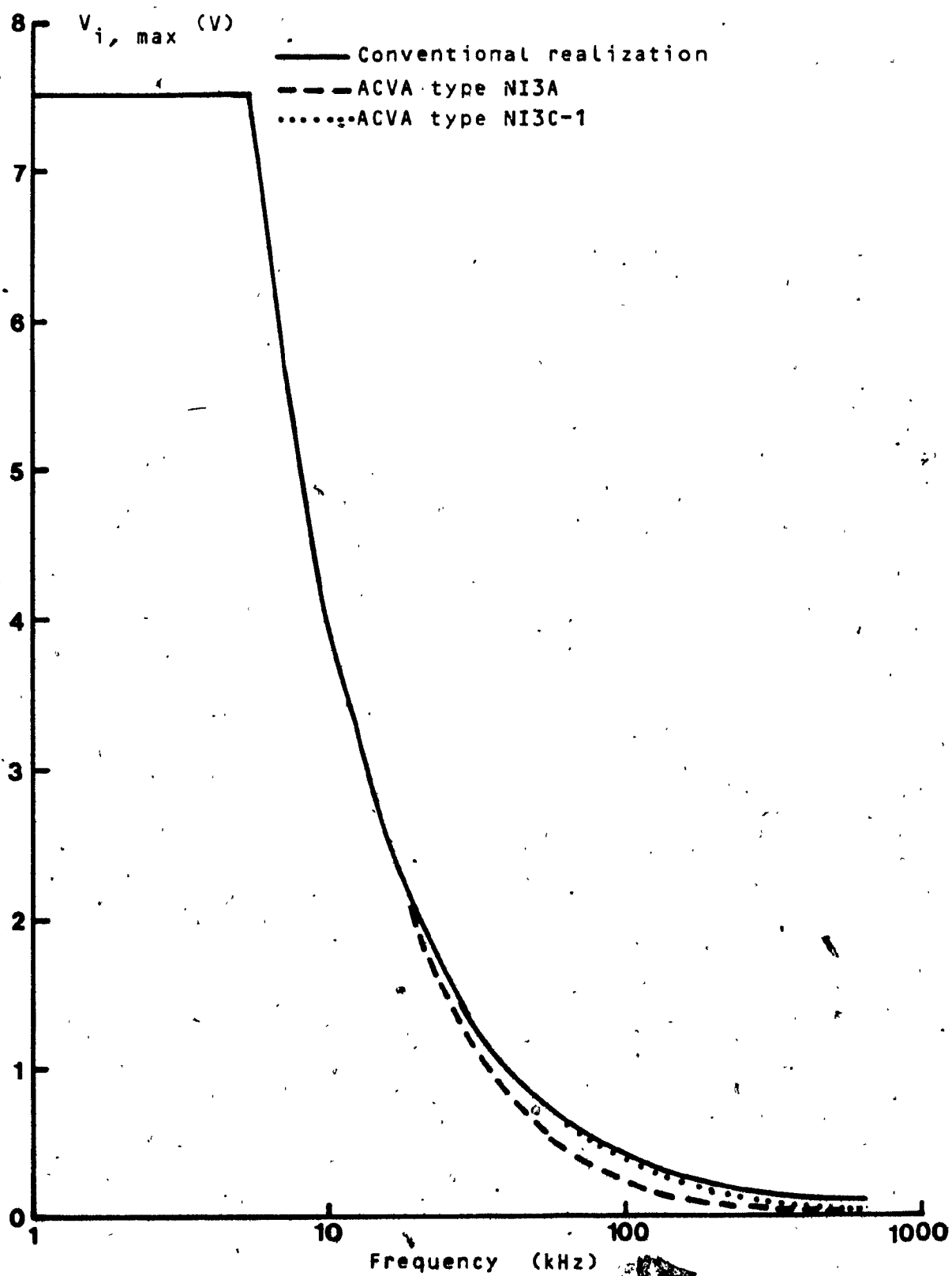


Fig. 3.6: (continued)

- (c) At a frequency such that  $(K\omega T)^3 \ll 1$ , adjust the gain of the circuit by modifying the resistor ratio which controls the  $s^2$ -compensation, namely  $\alpha$ ,  $a_1$ ,  $a_2$  or  $a_3$  as the case may be.

As in the case of the 2 OA ACVAs, the tuning procedure can be performed by modifying only resistor ratios. Therefore, as mentioned before, this procedure is feasible in IC technology and the tuning will remain effective over a wide range of temperature and power supply levels.

It is also worth mentioning that, for all the circuits presented in this chapter, ~~the~~ tuning can be performed in an orthogonal way, that is the three steps above are applied only once in a noniterative sequence.

### 3.6 EXPERIMENTAL RESULTS

#### 3.6.1 The ACVAs as stand-alone amplifiers

As it was done in Chapter 2, all the ACVAs were tested in the laboratory. The GBs of the OAs were measured to be 1084, 1068 and 1050 kHz. The resistors were within 1% accuracy. No tuning was attempted at all.

Fig. 3.7 shows the magnitude and phase response of the ACVAs of type NI3C-1 and IF3AB-8.



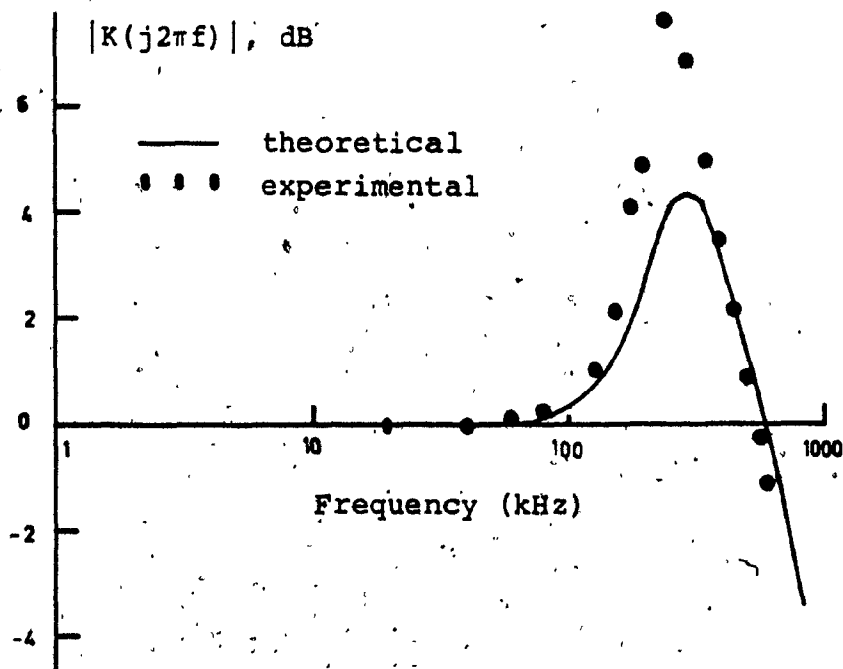


Fig. 3.7a: Theoretical and experimental magnitude responses for the ACIVA of type IF3AB-8

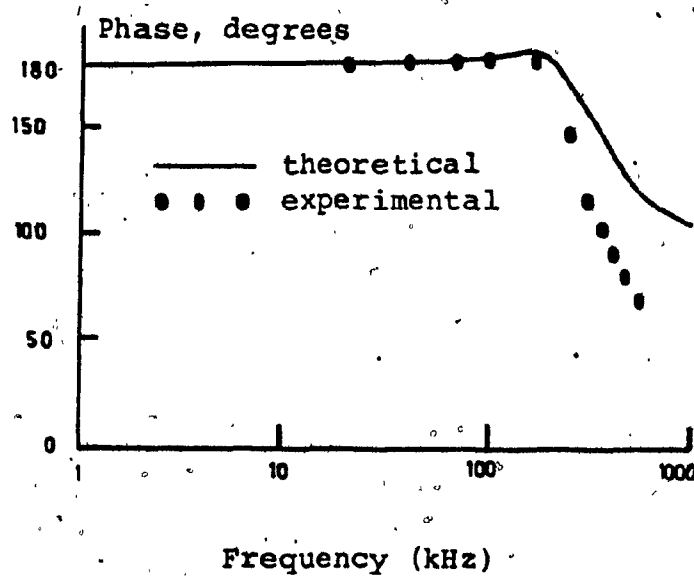


Fig. 3.7b: Theoretical and experimental phase responses for the ACIVA of type IF3AB-8\*

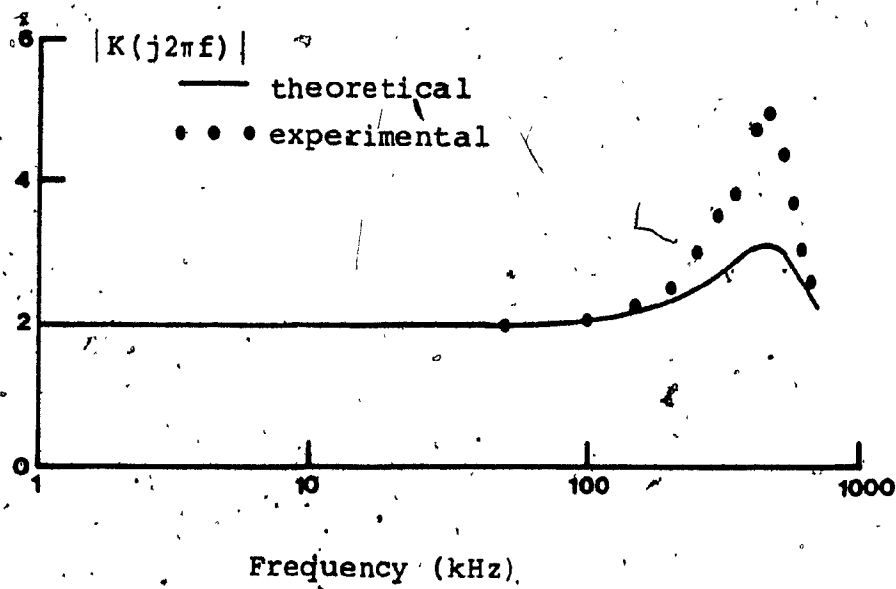


Fig. 3.7c: Theoretical and experimental magnitude responses for the ACVA of type NI3C-1

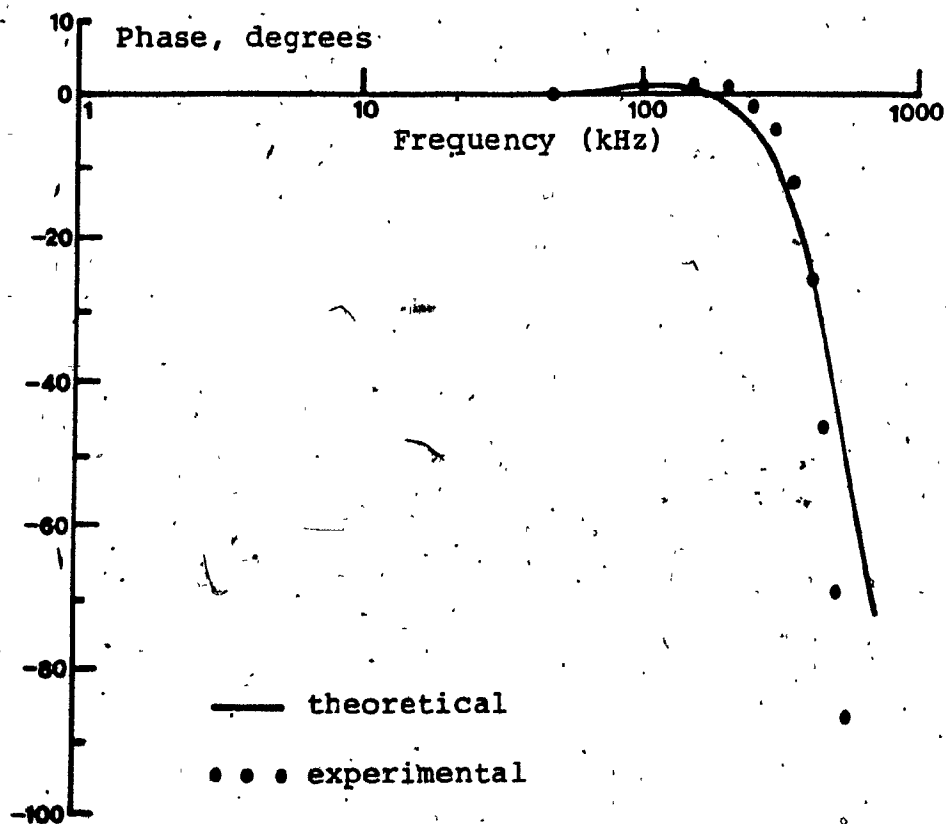


Fig. 3.7d: Theoretical and experimental phase responses for the ACVA of type NI3C-1

From this figure, it is observed that the ACVAs perform well even at frequencies near 100 kHz. At higher frequencies, the experimental responses depart from the theoretical ones owing to the effect of the second pole of the OAs which was disregarded in the analysis.

By comparing the response shown in Fig. 3.7 with those in Fig. 2.5, one can observe that 3 OA ACVAs outperform both the conventional realization and the 2 OA ACVA.

### 3.6.2 An active filter application

The Sallen-Key filter used as an example in Chapter 2 was redesigned to realize the same transfer function except for the pole frequency which was increased to 50 kHz.

Fig. 3.8 and Table 3.13 show the practical results obtained when the VA in the filter circuit was realized by the conventional realization, the 2 OA ACVA of type NI2A and the 3 OA ACVA of type NI3C-1. No tuning was attempted.

It can be clearly seen that the filter using 3 OAs presents an improved performance when compared with the filters which use the other 2 types of VAs.

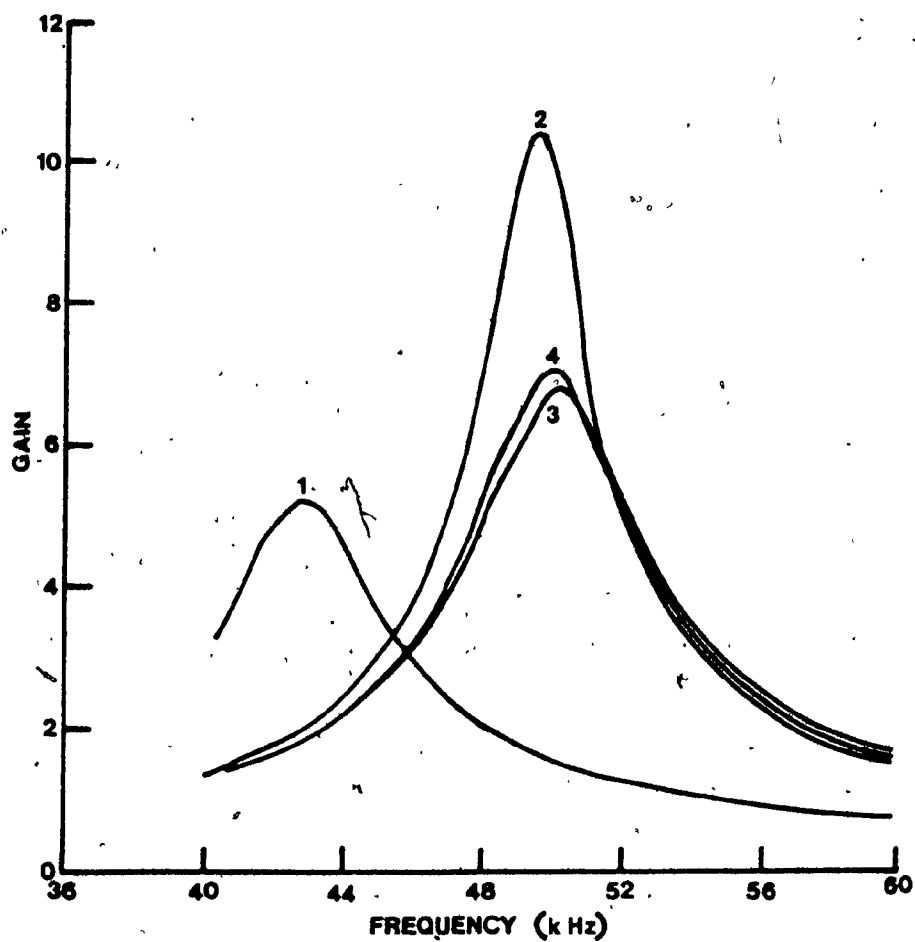


Fig. 3.8: Results obtained for the Sallen-Key filter.

- 1 - using the conventional VA
- 2 - using the ACVA of type NI2A
- 3 - using the ACVA of type NI3C-1
- 4 - ideal response

TABLE 3.13: Experimental performance for the  
 Sallen-Key filter using different  
 voltage amplifiers.

# OF OA's PARAMETER	THEORETICAL	10A	20A's	30A's
$f_0$ (kHz)	49.87	42.62	49.35	50.03
$\Delta f_0$ (%)	-	-14.54	-1.05	+0.3
$Q$	11.77	10.55	17.31	11.14
$\Delta Q$ (%)	-	-10.39	+47.07	-5.35
$ H(j\omega_0) $	7.04	5.17	10.50	6.74
$\Delta  H(j\omega_0) $ (%)	-	-26.56	+49.15	-4.26

### 3.7 CONCLUSIONS


The study presented in Chapter 2 for 2 OA ACVAs has been extended to 3 OA ACVAs in this chapter. Towards this end, a novel combination of theoretical and practical considerations (intended to be used specifically for 3 OA ACVAs) is used to classify such kind of circuit. A set of realizability conditions is developed for each of the types which were obtained by further subdividing the classes. By applying the resistor elimination procedure, the ACVA realizations can be obtained. A set of 161 circuits was obtained and classified.

Among the results presented in this chapter, it has been shown that it is impossible to realize a tunable, inverting 3<sup>rd</sup> OA ACVA possessing infinite input impedance. Therefore, the study of inverting ACVAs has been restricted to those possessing finite input impedance.

Also, an important and useful subset of the circuits obtained has been analyzed with respect to different practical aspects such as phase and magnitude errors, maximum signal handling capability, etc.

Experimental results are also reported. These results agree with the theoretical predictions.

So far, the results presented in this thesis deal with the design of voltage amplifiers. The replacement of a





conventional VA by an actively compensated one can significantly improve the operational frequency range of the systems which use VAs as building blocks. However, there are many active-RC circuits that do not rely solely on the use of finite gain amplifiers. In order to increase the operational frequency range of such circuits, a different approach has to be used. Consequently, in the following chapter, the active compensation of a general single OA active-RC circuit is considered.

## CHAPTER 4

## DESIGN

## OF

## OPTIMUM ACTIVELY COMPENSATED NETWORKS

## 4.1 INTRODUCTION

The great majority of active compensation schemes reported to date use some form of an actively compensated building block (e.g. amplifier, integrator, etc.) which is used as a direct replacement for an uncompensated block within a previously known circuit. In this approach, special emphasis has been given to the active compensation of voltage amplifiers.

The main reason for this is the existence of numerous widely used active-RC circuits which employ VAs as the active element. In order to compensate such circuits actively, the designer needs to change only the VA blocks while leaving the remainder of the network untouched. This is a useful feature since neither the design equations nor the passive sensitivity properties of the circuit are modified.

Many of the state-of-the-art VA-based single amplifier biquadratic circuits realize only lowpass, highpass and bandpass transfer functions. Other types of

transfer functions are realized by circuits which use feedforward paths from the overall input to both input terminals of the OA. Further, there are many currently used single OA biquadratic circuits (SABs) that do not rely on the use of VAs. Consequently, there are several important cases of circuits which cannot be compensated actively through the building block approach.

The main purpose of this chapter is to report a novel active compensation technique which can be applied to almost all existing SAB configurations while preserving both the design equations and the passive sensitivity properties of the uncompensated circuit. The resulting structures are, however, not SAB structures but use, in general, 2 or 3 OAs.

In order to derive this scheme, the general configurations of active-RC networks employing 2 and 3 OAs are investigated first. This analysis leads to compensation conditions concerning the whole network rather than specific blocks. The actively compensated versions of the existing SAB networks are readily obtained from these conditions.

In this context, it is interesting to note that a different set of conditions for the active compensation of a general active-RC network was presented in [28]. In this article, the authors derived conditions for eliminating the effects of the OAs on the zeros of the network and, then, further conditions were obtained to reduce the effects of

the OAs on the poles of the same circuit. Due to the former set of conditions, all circuits presented in [28] either used a cluster of OAs or doubled the number of passive RC components. Since the use of OA clusters introduces serious stability problems and the doubling the number of the RC components leads to considerable cost increase, such filters do not fully realize the potential of active compensation.

Many of the actively compensated circuits obtained through the application of the technique presented in this chapter have some free design parameters which can be used to optimize the circuit with respect to a given performance characteristic. This feature is explored in this chapter and a simple procedure is reported for minimizing the residual effects of the GBs of the OAs. This procedure is found to be very efficient despite its simplicity.

#### 4.2 ACTIVE COMPENSATION OF SABs

In this section, a technique for the active compensation of single OA biquadratic circuits (SABs) is presented. This scheme can lead to either 2 or 3 OA circuits. For comprehension purposes and because of the fact that these two types of networks are obtained in slightly different ways, 2 and 3 OA actively compensated circuits will be treated separately in this section.

#### 4.2.1 Actively compensated circuits using 2 OAs

The general configuration of a 2 OA active-RC network is shown in Fig. 4.1.

In this circuit, let us assume the OAs to be ideal except for their differential gain which is modeled as in eqn. (1.24). Further, the passive-RC subnetwork is characterized by its transfer functions which are defined as

$$T_{kj} = \frac{N_{kj}}{D_{RC}} = \left. \frac{v_k}{v_j} \right|_{v_l=0, l \neq j} \quad (4.1)$$

for  $k=1,2$  and  $j,l=1,2,i$ . Although  $T_{kj}$ ,  $N_{kj}$  and  $D_{RC}$  are all functions of the complex frequency variable  $s$ , the explicit notation for this dependence is not used in order to simplify the expressions.

Each voltage  $v_k$  can be expressed as

$$v_k = v_k^+ - v_k^-$$

where  $v_k^+$  and  $v_k^-$  are the voltages at the noninverting and the inverting input terminal of the  $k$ th OA, respectively, referred to ground. Consequently, each transfer function  $T_{kj}$  can be written as

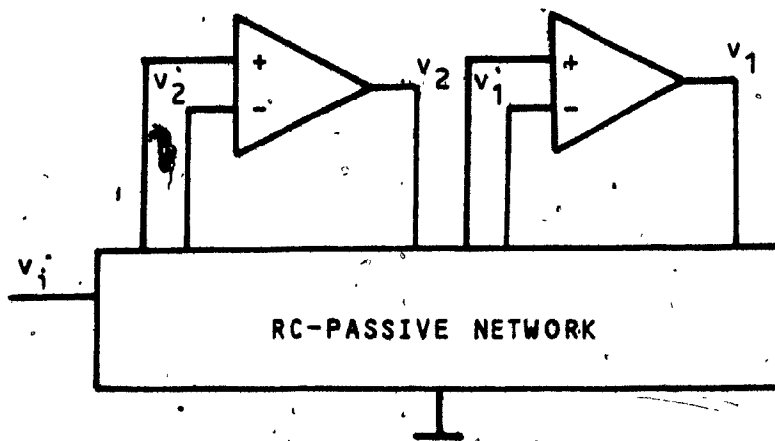


Fig. 4.1: General configuration of a 2 OA  
RC-active network

$$T_{kj} = T_{kj}^+ - T_{kj}^-$$

where

$$T_{kj}^+ = \frac{N_{kj}^+}{D_{RC}} = \frac{v_k^+}{v_j} \Big|_{v_h=0, h \neq j}$$

and

$$T_{kj}^- = \frac{N_{kj}^-}{D_{RC}} = \frac{v_k^-}{v_j} \Big|_{v_h=0, h \neq j}$$

for  $k=1,2$  and  $j,h=1,2,i$ .

$N_{kj}$ ,  $N_{kj}^+$ ,  $N_{kj}^-$  and  $D_{RC}$  are polynomials in  $s$ . Note that the definitions above exclude the possible existence of private poles in any of those transfer functions.

If  $v_1$  is taken as output signal, the transfer function of the circuit in Fig. 4.1 is given by

$$H(s, \tau_1, \tau_2) = \frac{A(s, \tau_1, \tau_2)}{B(s, \tau_1, \tau_2)} \quad (4.2)$$

where

$$A(s, \tau_1, \tau_2) = T_{12}T_{21} - T_{11}T_{22} + T_{11}\tau_2 s$$

and

$$B(s, \tau_1, \tau_2) = T_{11}T_{22} - T_{12}T_{21} + T_{11}\tau_2 s + T_{22}\tau_1 s + \tau_1\tau_2 s^2$$

The ideal transfer function is obtained by making  $\tau_1 = \tau_2 = 0$  in eqn. (4.2), namely

$$H(s, 0, 0) = \frac{T_{12}T_{21} - T_{11}T_{22}}{T_{11}T_{22} - T_{12}T_{21}} \quad (4.3)$$

$H(s, \tau_1, \tau_2)$  represents the deviation of the transfer function of the circuit from  $H(s, 0, 0)$  when  $\tau_1$  and  $\tau_2$  assume nonzero values. Therefore, the relationship between these two functions can be expressed by a Taylor series expansion [45] as

$$\begin{aligned} H(s, \tau_1, \tau_2) = & H(s, 0, 0) + \tau_1 \left. \frac{\partial H(s, \tau_1, \tau_2)}{\partial \tau_1} \right|_{\tau_1=\tau_2=0} + \\ & \tau_2 \left. \frac{\partial H(s, \tau_1, \tau_2)}{\partial \tau_2} \right|_{\tau_1=\tau_2=0} + \frac{\tau_1^2}{2} \left. \frac{\partial^2 H(s, \tau_1, \tau_2)}{\partial \tau_1^2} \right|_{\tau_1=\tau_2=0} + \\ & \frac{\tau_2^2}{2} \left. \frac{\partial^2 H(s, \tau_1, \tau_2)}{\partial \tau_2^2} \right|_{\tau_1=\tau_2=0} + \tau_1\tau_2 \left. \frac{\partial^2 H(s, \tau_1, \tau_2)}{\partial \tau_1 \partial \tau_2} \right|_{\tau_1=\tau_2=0} + \dots \end{aligned} \quad (4.4)$$



At this point, it is useful to mention that, in this expansion, terms involving derivatives of order  $n$  are much larger than terms involving derivatives of higher orders. Consequently, to make  $H(s, \tau_1, \tau_2)$  as close as possible to  $H(s, 0, 0)$ , one can try to null sequentially the first-order derivative terms, then second-order derivative terms, etc. This basically describes the idea behind the proposed active compensation.

By performing some straightforward calculations, it follows that

$$\left. \frac{\partial H(s, \tau_1, \tau_2)}{\partial \tau_1} \right|_{\tau_1=\tau_2=0} = \frac{H(s, 0, 0)}{B(s, 0, 0)} T_{22} s \quad (4.5)$$

$$\left. \frac{\partial H(s, \tau_1, \tau_2)}{\partial \tau_2} \right|_{\tau_1=\tau_2=0} = \frac{T_{1i} B(s, 0, 0) + T_{11} A(s, 0, 0)}{B^2(s, 0, 0)} \quad (4.6)$$

Hence,

$$\left. \frac{\partial H(s, \tau_1, \tau_2)}{\partial \tau_1} \right|_{\tau_1=\tau_2=0} = 0 \Leftrightarrow T_{22} = 0 \quad (4.7)$$

$$\left. \frac{\partial H(s, \tau_1, \tau_2)}{\partial \tau_2} \right|_{\tau_1=\tau_2=0} = 0 \Leftrightarrow \frac{T_{1i}}{T_{11}} = -H(s, 0, 0) \quad (4.8)$$

Eqns. (4.7) and (4.8) express the conditions to make  $H(s, \tau_1, \tau_2)$  independent of first-order effects of  $\tau_1$  and  $\tau_2$ . If these equations are satisfied, the circuit will be actively compensated against these effects. Further, if these compensation conditions hold, it follows that

$$\left. \frac{\partial^2 H(s, \tau_1, \tau_2)}{\partial \tau_1^2} \right|_{\tau_1 = \tau_2 = 0} = \left. \frac{\partial^2 H(s, \tau_1, \tau_2)}{\partial \tau_2^2} \right|_{\tau_1 = \tau_2 = 0} = 0 \quad (4.9)$$

and

$$\left. \frac{\partial^2 H(s, \tau_1, \tau_2)}{\partial \tau_1 \partial \tau_2} \right|_{\tau_1 = \tau_2 = 0} = \frac{H(s, 0, 0)}{T_{12} T_{21}} s^2 \quad (4.10)$$

From (4.10), it is seen that it is not possible to null the second-order derivatives effects in (4.4). Thus, only the terms involving first-order derivatives can be eliminated in a 2 OA circuit. Also, under the compensation, eqn. (4.4) can be rewritten as

$$H(s, \tau_1, \tau_2) = H(s, 0, 0) + \frac{H(s, 0, 0)}{T_{12}T_{21}} \tau_1 \tau_2 s^2 \quad (4.11)$$

where

$$H(s, 0, 0) = -\frac{T_{21}}{T_{12}}$$

and the higher order terms are disregarded.

After having derived the compensation conditions, let us obtain circuit realizations which satisfy them. Moreover, it is desirable to obtain a technique for compensating existing SABs actively without modifying their design equations nor their passive sensitivity properties.

Most state-of-the-art SABs are special cases of the "enhanced positive feedback" and the "enhanced negative feedback" classes of filter circuits presented in [46] and shown in Fig. 4.2a and 4.2b, respectively. These classes include the popular Sallen-Key circuits [47], the STAR network [48], the optimum SABs reported in [49], etc. Therefore, actively compensated versions of those two general configurations will find numerous applications.

Let us consider the 2 OA structures shown in Fig. 4.2c and 4.2d. Clearly, these configurations are 2 OA versions of the "enhanced positive feedback" and the "enhanced negative feedback" circuits, respectively.

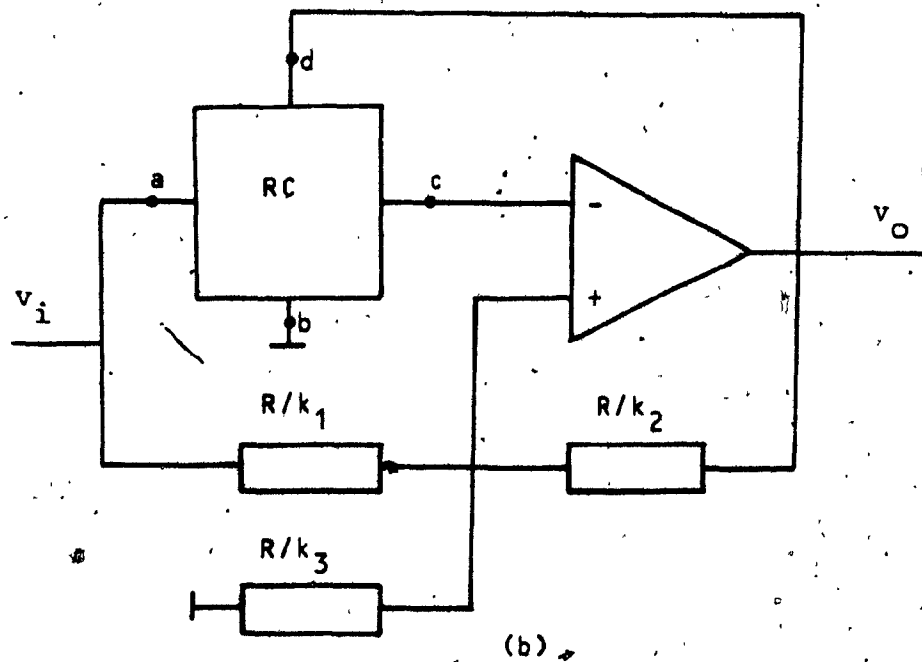
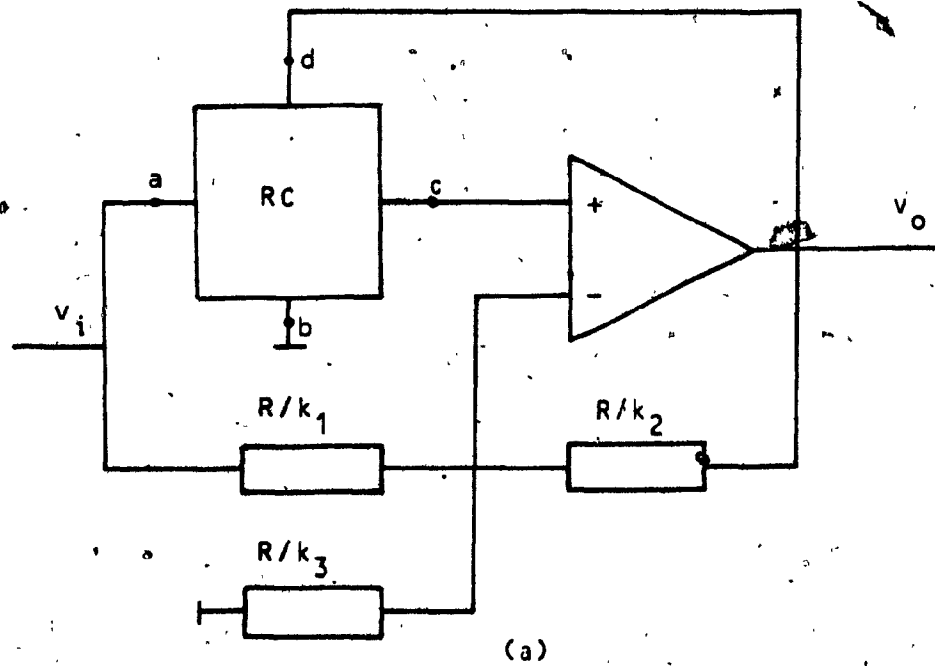


Fig. 4.2: General configuration of

- (a) conventional enhanced positive feedback network
- (b) conventional enhanced negative feedback network

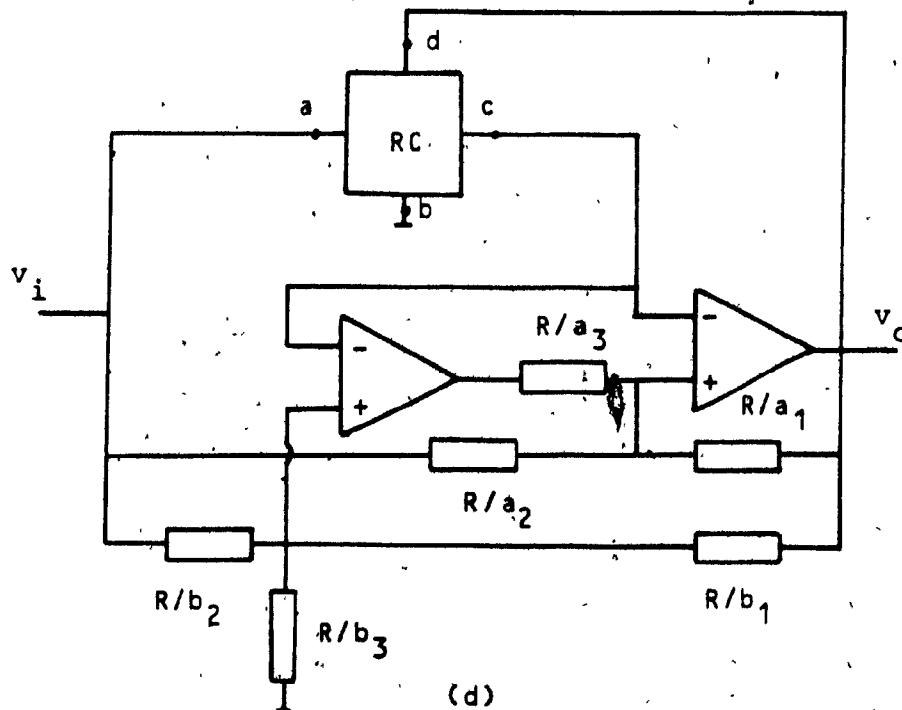
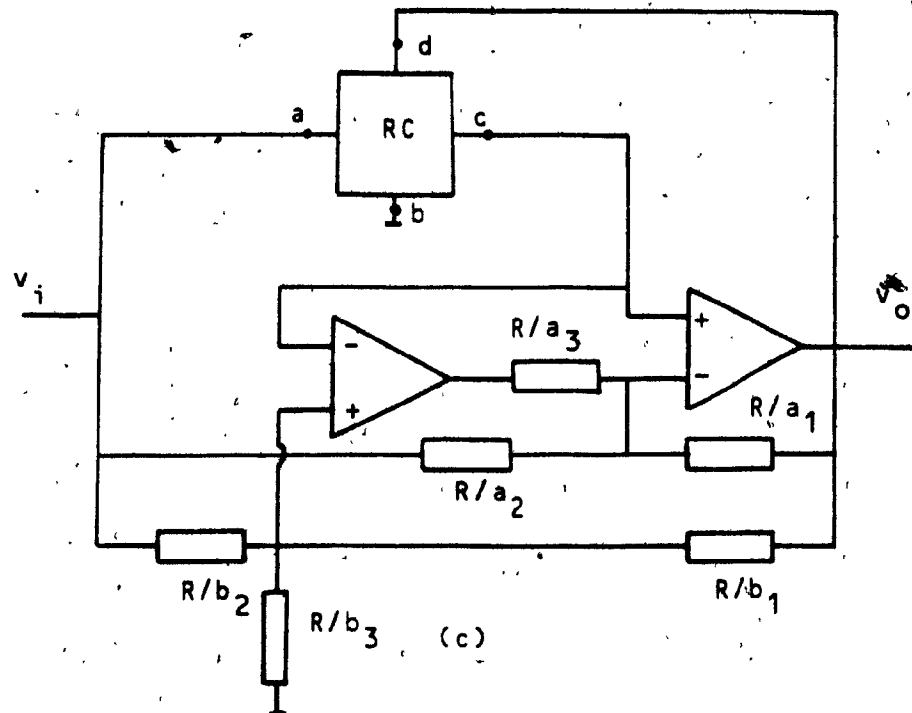


Fig. 4.2: General configuration of

(c) 2 OA enhanced positive feedback network

(d) 2 OA enhanced negative feedback network

At this point, let us assume, without any loss in generality, that  $a_1 + a_2 + a_3 = 1$  and that  $b_1 + b_2 + b_3 = 1$ . These assumptions simplify the expressions used in the analysis of these circuits.

In both cases,  $T_{22} = 0$  and condition (4.7) is satisfied.

A routine analysis of the circuit in Fig. 4.2c reveals that

$$T_{21} = \frac{b_1 D_{RC} - N_{11}^+}{D_{RC}}$$

$$T_{2i} = \frac{b_2 D_{RC} - N_{1i}^+}{D_{RC}}$$

$$T_{11} = \frac{N_{11}^+ - a_1 D_{RC}}{D_{RC}}$$

$$T_{1i} = \frac{N_{1i}^+ - a_2 D_{RC}}{D_{RC}}$$

$$T_{12} = -a_3$$

Also, for the circuit in Fig. 4.2d,

$$T_{21} = \frac{b_1 D_{RC}^{-N_{11}}}{D_{RC}}$$

$$T_{2i} = \frac{b_2 D_{RC}^{-N_{1i}}}{D_{RC}}$$

$$T_{11} = \frac{a_1 D_{RC}^{-N_{11}}}{D_{RC}}$$

$$T_{1i} = \frac{a_2 D_{RC}^{-N_{1i}}}{D_{RC}}$$

$$T_{12} = a_3$$

In order to satisfy eqn. (4.8), a circuit has to satisfy

$$\frac{T_{11}}{T_{1i}} = \frac{T_{21}}{T_{2i}}$$

For both circuits, this will be satisfied if

$$a_1 = b_1$$

(4.12a)

and

$$a_2 = b_2$$

(4.12b)

From the results reported in this subsection, it can be concluded that any circuit which is a special case of the configurations shown in Fig. 4.2a and 4.2b can be compensated actively by using the configurations in Fig. 4.2c and 4.2d, respectively, provided the conditions in eqn. (4.12) are satisfied. These conditions involve only the matching of resistors and, owing to the close tracking among resistors implemented in IC technology, the compensation will remain effective when subjected to variations in power supply voltages, temperature, humidity, etc.

#### 4.2.2 Actively compensated circuits using 3 OAs

The active compensation scheme presented in the previous subsection is effective in cancelling the first-order terms in the Taylor series expansion of  $H(s, \tau_1, \tau_2)$ . The effect of higher order terms, however, are still present in that expansion and are known to cause severe transfer function distortion when the circuit is operated at higher frequencies. Therefore, in order to further extend the operating frequency range of SABs, a scheme employing more than 2 OAs must be used.



In this context, it is interesting to note that, due to the same practical considerations that limited the number of OAs in practical ACVAs (refer to section 1.6), actively compensated RC networks employing more than 3 OAs will also be of doubtful utility.

The general configuration of a 3 OA active-RC network is shown in Fig. 4.3. Let us assume the OAs modeled as in eqn. (1.24). Also, let us extend the definitions of  $T_{kj}$ ,  $N_{kj}$ ,  $N_{kj}^+$  and  $N_{kj}^-$  and  $D_{RC}$  given in the previous subsection for  $k=1,2,3$  and  $j,h=1,2,3,i$ .

If  $v_1$  is taken as the output, the transfer function of the circuit is given by

$$H(s, \tau_1, \tau_2, \tau_3) = \frac{A(s, \tau_1, \tau_2, \tau_3)}{B(s, \tau_1, \tau_2, \tau_3)} \quad (4.13)$$

where

$$\begin{aligned} A(s, \tau_1, \tau_2, \tau_3) = & (T_{1i}T_{32}T_{23} + T_{2i}T_{12}T_{33} + T_{3i}T_{13}T_{22} - T_{1i}T_{22}T_{33} \\ & - T_{2i}T_{13}T_{32} - T_{3i}T_{12}T_{23}) + (T_{1i}T_{33} - T_{3i}T_{13})s\tau_2 + (T_{1i}T_{22} - \\ & T_{2i}T_{12})s\tau_3 - T_{1i}s^2\tau_2\tau_3 \end{aligned}$$

and

$$\begin{aligned} B(s, \tau_1, \tau_2, \tau_3) = & (T_{11}T_{22}T_{33} + T_{12}T_{23}T_{31} + T_{13}T_{21}T_{32} - T_{13}T_{31}T_{22} \\ & - T_{32}T_{23}T_{11} - T_{12}T_{21}T_{33}) + (T_{23}T_{32} - T_{22}T_{33})s\tau_1 + (T_{13}T_{31} - \end{aligned}$$

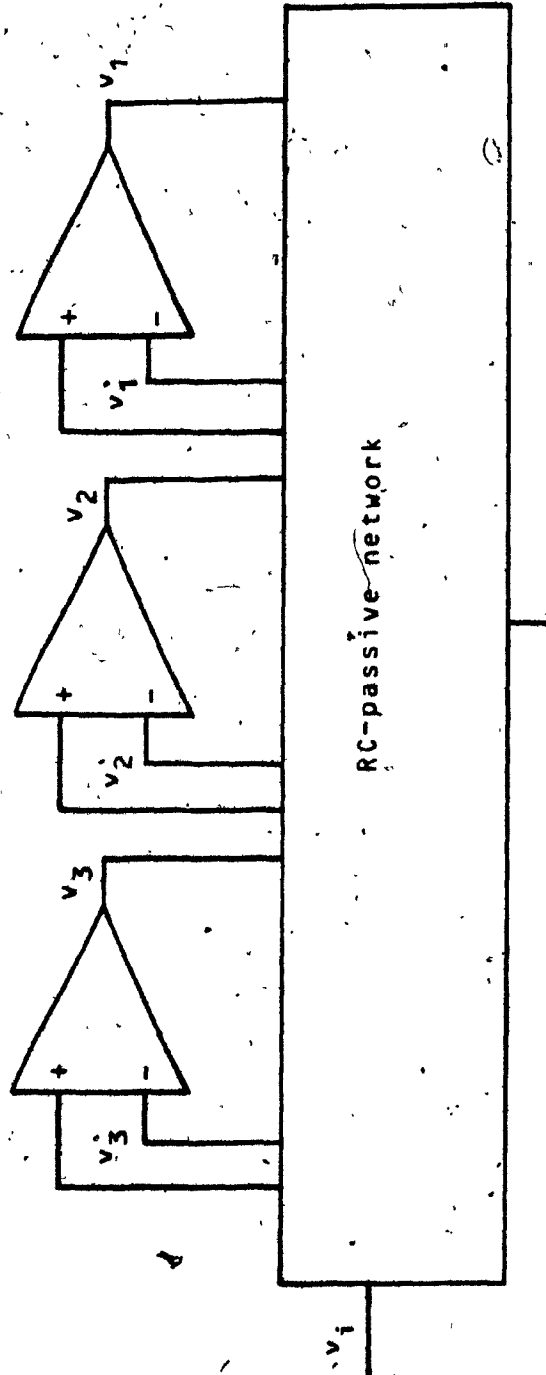


Fig. 4.3: General configuration of a 3 OA active-RC network.

$$T_{11}T_{33})s\tau_2 + (T_{12}T_{21}-T_{11}T_{22})s\tau_3 + T_{33}s^2\tau_1\tau_2 + T_{22}s^2\tau_1\tau_3 \\ + T_{11}s^2\tau_2\tau_3 - s^3\tau_1\tau_2\tau_3$$

If  $H(s, \tau_1, \tau_2, \tau_3)$  is expressed as a Taylor series, the following expansion is obtained

$$H(s, \tau_1, \tau_2, \tau_3) = H(s, 0, 0, 0) + \sum_{i=1}^3 \tau_i \frac{\partial H(s, \tau_1, \tau_2, \tau_3)}{\partial \tau_i} \Big|_{\tau_1=\tau_2=\tau_3=0} \\ + \sum_{i,j=1}^3 \frac{\tau_i \tau_j}{2} \frac{\partial^2 H(s, \tau_1, \tau_2, \tau_3)}{\partial \tau_i \partial \tau_j} \Big|_{\tau_1=\tau_2=\tau_3=0} + \\ + \sum_{i,j,h=1}^3 \frac{\tau_i \tau_j \tau_h}{6} \frac{\partial^3 H(s, \tau_1, \tau_2, \tau_3)}{\partial \tau_i \partial \tau_j \partial \tau_h} \Big|_{\tau_1=\tau_2=\tau_3=0} + \dots \quad (4.14)$$

By computing all first- and second-order derivatives in eqn. (4.14) and equating them equal to 0, it follows that

$$\frac{\partial H(s, \tau_1, \tau_2, \tau_3)}{\partial \tau_1} \Big|_{\tau_1=\tau_2=\tau_3=0} = 0 \Leftrightarrow (T_{32}T_{23} - T_{22}T_{33}) = 0 \quad (4.15)$$

$$\frac{\partial H(s, \tau_1, \tau_2, \tau_3)}{\partial \tau_2} \Big|_{\tau_1=\tau_2=\tau_3=0} = 0 \Leftrightarrow (T_{11}T_{33} - T_{31}T_{13}) = \\ H(s, 0, 0, 0) (T_{13}T_{31} - T_{11}T_{33}) \quad (4.16)$$

$$\frac{\partial H(s, \tau_1, \tau_2, \tau_3)}{\partial \tau_3} \Big|_{\tau_1=\tau_2=\tau_3=0} = 0 \Leftrightarrow (T_{11}T_{22} - T_{21}T_{12}) = \\ H(s, 0, 0, 0) (T_{12}T_{21} - T_{11}T_{22}) \quad (4.17)$$

$$\frac{\partial^2 H(s, \tau_1, \tau_2, \tau_3)}{\partial \tau_1^2} \Big|_{\tau_1=\tau_2=\tau_3=0} = 0 \Leftrightarrow (T_{32}T_{23} - T_{22}T_{33}) = 0$$

$$\frac{\partial^2 H(s, \tau_1, \tau_2, \tau_3)}{\partial \tau_2^2} \Big|_{\tau_1=\tau_2=\tau_3=0} = 0 \Leftrightarrow (T_{11}T_{33} - T_{31}T_{13}) = H(s, 0, 0, 0)(T_{13}T_{31} - T_{11}T_{33})$$

$$\frac{\partial^2 H(s, \tau_1, \tau_2, \tau_3)}{\partial \tau_3^2} \Big|_{\tau_1=\tau_2=\tau_3=0} = 0 \Leftrightarrow (T_{11}T_{22} - T_{21}T_{12}) = H(s, 0, 0, 0)(T_{12}T_{21} - T_{11}T_{22})$$

$$\frac{\partial^2 H(s, \tau_1, \tau_2, \tau_3)}{\partial \tau_1 \partial \tau_2} \Big|_{\tau_1=\tau_2=\tau_3=0} = 0 \Leftrightarrow T_{33} = 0 \quad (4.18)$$

$$\frac{\partial^2 H(s, \tau_1, \tau_2, \tau_3)}{\partial \tau_1 \partial \tau_3} \Big|_{\tau_1=\tau_2=\tau_3=0} = 0 \Leftrightarrow T_{22} = 0 \quad (4.19)$$

$$\frac{\partial^2 H(s, \tau_1, \tau_2, \tau_3)}{\partial \tau_2 \partial \tau_3} \Big|_{\tau_1=\tau_2=\tau_3=0} = 0 \Leftrightarrow -T_{11} = H(s, 0, 0, 0)T_{11} \quad (4.20)$$

Further, if eqns. (4.15) to (4.20) hold, we find

that

$$\frac{\partial^3 H(s, \tau_1, \tau_2, \tau_3)}{\partial \tau_i^3} \Big|_{\tau_1=\tau_2=\tau_3=0} = \frac{\partial^3 H(s, \tau_1, \tau_2, \tau_3)}{\partial \tau_i^2 \partial \tau_j} \Big|_{\tau_1=\tau_2=\tau_3=0} = 0 \quad (4.21)$$

and

$$\left. \frac{\partial^3 H(s, \tau_1, \tau_2, \tau_3)}{\partial \tau_1 \partial \tau_2 \partial \tau_3} \right|_{\tau_1 = \tau_2 = \tau_3 = 0} = \frac{H(s, 0, 0, 0)}{B(s, 0, 0, 0)} s^3 \quad (4.22)$$

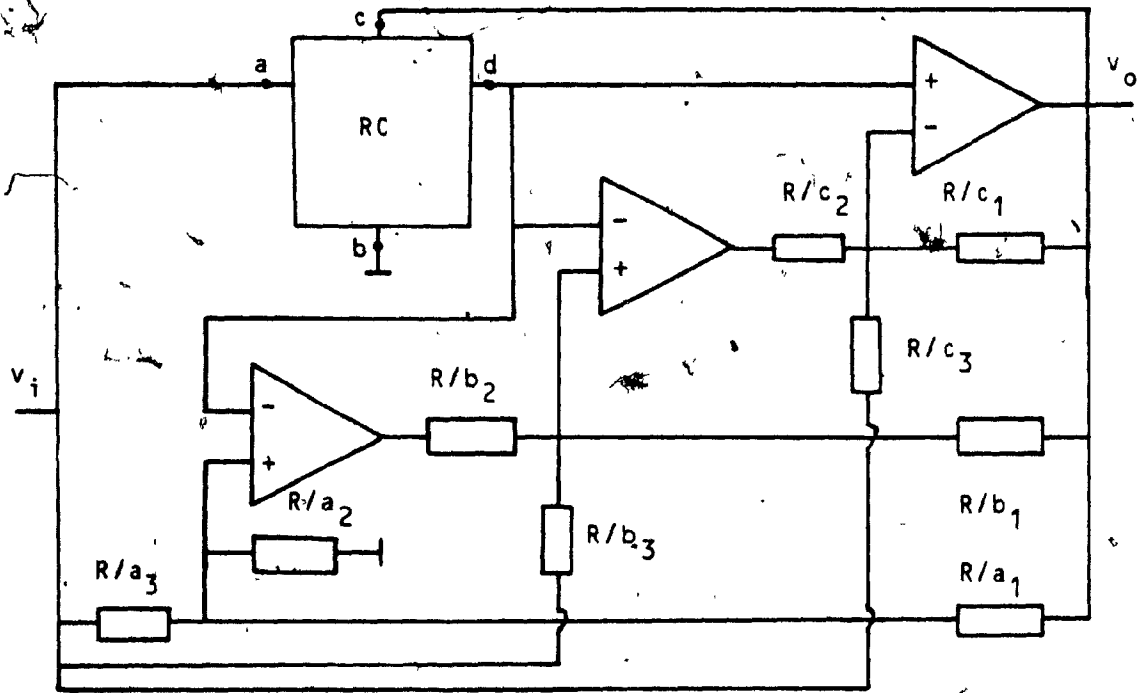
for  $i, j=1, 2, 3$ .

By taking eqns, (4.15) to (4.22) into (4.14) and neglecting higher order terms,  $H(s, \tau_1, \tau_2, \tau_3)$  is given by

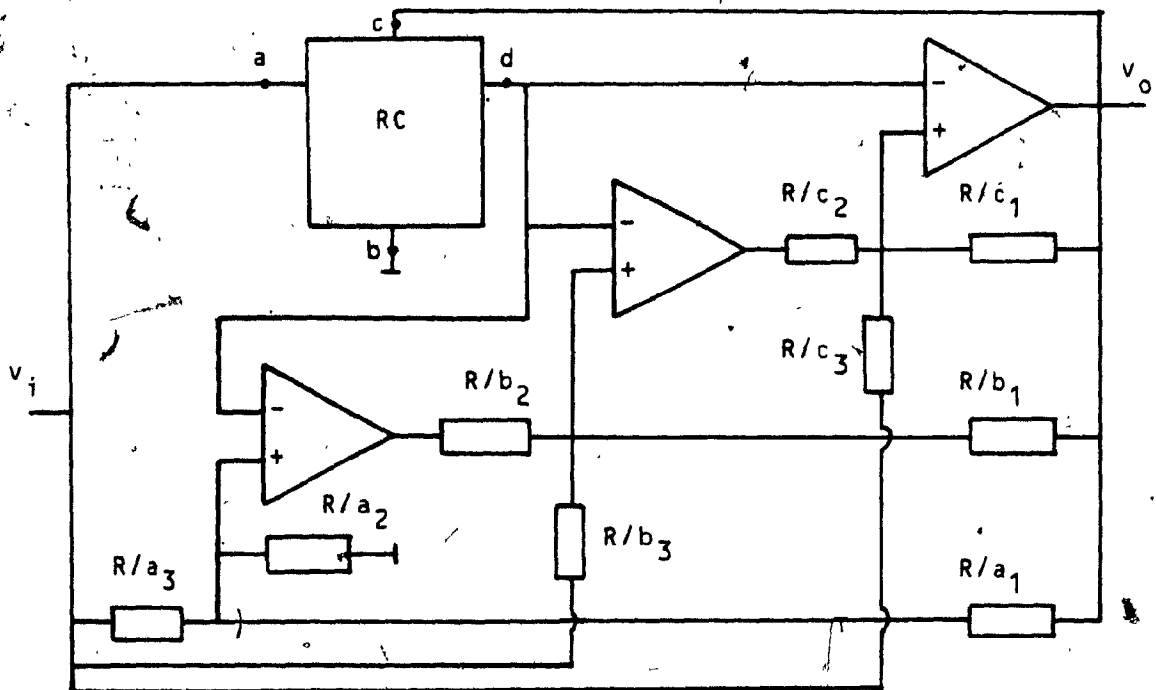
$$H(s, \tau_1, \tau_2, \tau_3) = H(s, 0, 0, 0) + \frac{H(s, 0, 0, 0)}{B(s, 0, 0, 0)} \tau_1 \tau_2 \tau_3 s^3 \quad (4.23)$$

A look at eqn. (4.23) reveals that the effect of the time constants of the OAs is much reduced if eqns. (4.15) to (4.20) are satisfied. Also, by comparing eqns. (4.23) and (4.11), it is seen that such effects are much more reduced in the scheme using 3 OAs than in the one with 2 OAs.

Two circuits, which may be designed to satisfy these conditions are shown in Fig. 4.4. The 3 OA circuits in Fig. 4.4a and 4.4b are the actively compensated versions of the structures in Fig. 4.2a and 4.2b, respectively. As in the 2 OA case, let us assume  $a_1 + a_2 + a_3 = 1$ ,  $b_1 + b_2 + b_3 = 1$  and  $c_1 + c_2 + c_3 = 1$  in order to simplify the expressions.



(a)



(b)

Fig. 4.4: General configuration of

(a) 3 OA enhanced positive feedback network

(b) 3 OA enhanced negative feedback network

For the circuit in Fig. 4.4a, a routine circuit analysis yield

$$T_{11} = \frac{N_{11}^+ - c_1 D_{RC}}{D_{RC}}$$

$$T_{1i} = \frac{N_{1i}^+ - c_3 D_{RC}}{D_{RC}}$$

$$T_{12} = 0$$

$$T_{13} = -c_2$$

$$T_{22} = 0$$

$$T_{21} = \frac{a_1 D_{RC} - N_{11}^+}{D_{RC}}$$

$$T_{23} = 0$$

$$T_{2i} = \frac{a_3 D_{RC} - N_{1i}^+}{D_{RC}}$$

$$T_{33} = 0$$

$$T_{31} = \frac{b_1 D_{RC}^{-N_{11}^+}}{D_{RC}}$$

$$T_{32} = b_2$$

$$T_{3i} = \frac{b_3 D_{RC}^{-N_{1i}^+}}{D_{RC}}$$

$$H(s, 0, 0, 0) = - \frac{T_{2i}}{T_{21}}$$

\* while, for the circuit in Fig. 4.4b,

$$T_{1i} = \frac{c_3 D_{RC}^{-N_{1i}^-}}{D_{RC}}$$

$$T_{11} = \frac{c_1 D_{RC}^{-N_{11}^-}}{D_{RC}}$$

$$T_{12} = 0$$

$$T_{13} = c_2$$

$$T_{21} = \frac{a_2 D_{RC}^{-N_{11}^-}}{D_{RC}}$$



$$T_{21} = \frac{a_1 D_{RC}^{-N_{11}}}{D_{RC}}$$

$$T_{22} = 0$$

$$T_{23} = 0$$

$$T_{31} = \frac{b_3 D_{RC}^{-N_{11}}}{D_{RC}}$$

$$T_{31} = \frac{b_1 D_{RC}^{-N_{11}}}{D_{RC}}$$

$$T_{32} = b_2$$

$$T_{33} = 0$$

$$H(s, 0, 0, 0) = -\frac{T_{21}}{T_{21}}$$

These values satisfy eqns. (4.15), (4.17), (4.18) and (4.19). In order to comply with eqn. (4.16), however, it is necessary that

$$a_3 = b_3$$

and

$$a_1 = b_1$$

Also, to satisfy eqn. (4.20), it is necessary that

$$a_3 = c_3$$

and

$$a_1 = c_1$$

Therefore, the conditions for the active compensation of the circuits in Fig. 4.4 are given by

$$a_1 = b_1 = c_1 \quad (4.24a)$$

$$a_3 = b_3 = c_3 \quad (4.24b)$$

These conditions depend only upon the matching of resistors and, consequently, as explained for 2 OA actively compensated circuits, the compensation will remain effective when the circuit is subjected to variations in power supply voltages, temperature, aging, etc.

#### 4.3 OPTIMIZATION OF ACTIVELY COMPENSATED CIRCUITS

##### 4.3.1 Description of the procedure

All the actively compensated circuits obtained through the schemes presented in section 4.2 have a larger number of design parameters than equations to be satisfied by these parameters. Hence, in all those circuits, there are some free design parameters whose values can be selected in such way that the performance of the circuit can be optimized with respect to a given characteristic. Since the operation of active-RC networks at high frequencies is mostly affected by the gain-bandwidth product of the OAs, let us use these free design parameters to optimize further the actively compensated networks presented in this chapter with respect to the GB effects.

An early work on sensitivity optimization in actively compensated circuits was presented in [50]. In this article, the optimization is accomplished by minimizing the shifts in both the pole frequency and the pole Q-factor in a

particular bandpass circuit. This simultaneous minimization is possible because the same function appears in the denominator of the expressions for such shifts. Some factors, nevertheless, contribute towards making the procedure used in that work difficult for application in other circuits:

- (a) Expressions for the shifts mentioned above are not easy to obtain for circuits employing 2 or 3 OAs.
- (b) Even if such expressions are available, it may not be easy to recognize an appropriate objective function from them.
- (c) Only the pole shift is optimized in this procedure. Circuits realizing transfer functions whose behavior depend on the zeros (e.g. allpass, notch, etc.) cannot be fully optimized.

The optimization procedure to be presented in this section avoids these disadvantages by using an easily obtainable objective function which expresses changes in the transfer function itself rather than in poles or zeros. Indeed, in practice, it is the transfer function that is of interest rather than poles or zeros.

This objective function is readily derived from eqns. (4.11) and (4.23). From these equations, it is clear that the deviations in the transfer function actually realized by the actively compensated circuits will be minimized if the effects of

$$F_2(s) = \frac{1}{T_{12}T_{21}} \quad (4.25)$$

$$F_3(s) = \frac{1}{B(s,0,0,0)} \quad (4.26)$$

are also minimized for 2 and 3 OA circuits, respectively.

Although, both  $F_2(s)$  and  $F_3(s)$  are frequency dependent, in most cases, it is seen that these functions can be expressed by a summation of lowpass and bandpass type functions whose poles are the same as the poles of the ideal transfer function. In these cases, the minimization of the objective function can be performed only at the pole frequency, namely  $s=j\omega_0$ .

The whole optimization procedure consists in applying the following steps:

Step 1: Select the free design parameters that will be used as the optimization variables.

Step 2: Using the design equations, express the other design parameters as function of the optimization variables.

Step 3: Express  $F_2(s)$  or  $F_3(s)$  (as the case may be) as a function of the optimization variables.

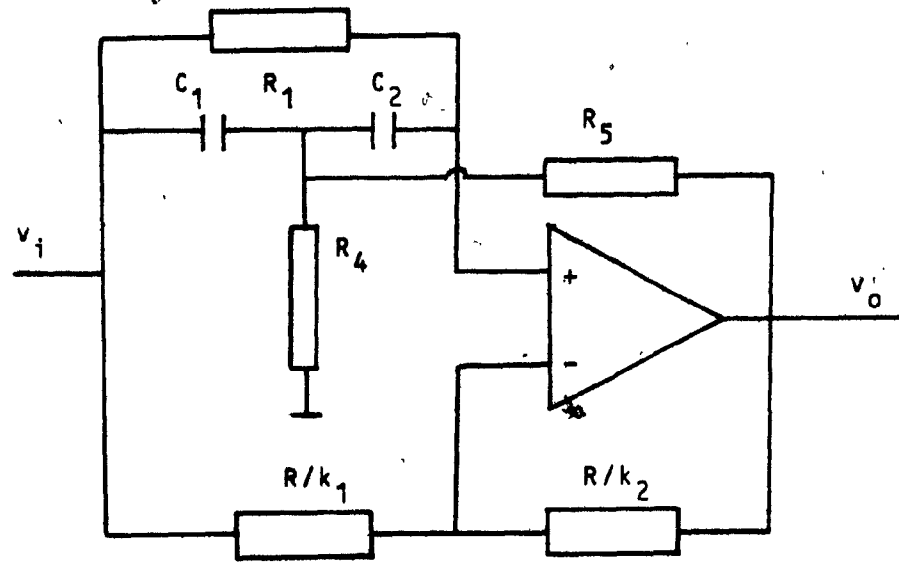
Step 4: Minimize the objective function either numerically or analytically to obtain the optimal values for the design parameters.

Some examples of the application of both the active compensation schemes and the sensitivity minimization procedure above are given next.

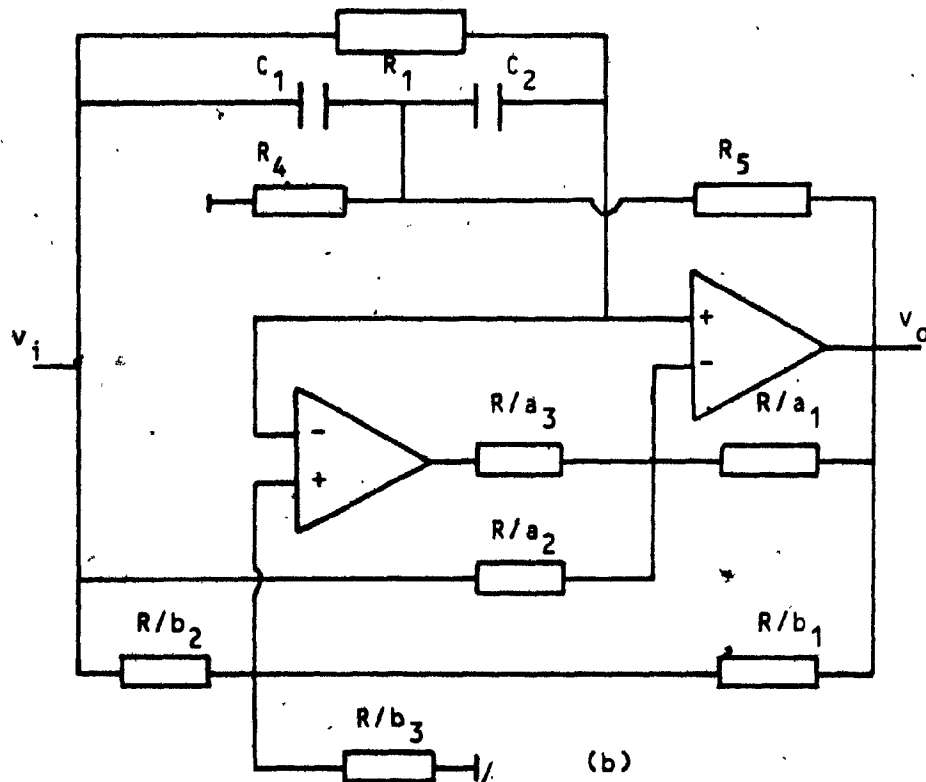
#### 4.3.2 Steffen allpass circuit

Allpass transfer function realizations cannot be optimized or even compensated by schemes which consider separately the shifts in poles and the shifts in zeros as in the technique proposed in [50]. This is due to the special characteristics of such transfer functions which require precise matching of poles and zeros. In view of the above, it is interesting to consider an application of the techniques presented in this chapter to allpass transfer functions.

The conventional structure for the allpass circuit due to Steffen [48] is shown in Fig. 4.5a. The 2 OA



(a)



(b)

Fig. 4.5: Steffen allpass circuits

(a) conventional (b) 2 OA version

actively compensated version of the same circuit is shown in Fig. 4.5b.. It is interesting to note that these two circuits are special cases of enhanced positive feedback networks. From eqn. (4.12), the compensation conditions are given by

$$a_1 = b_1 \quad (4.27)$$

and

$$a_2 = b_2 \quad (4.28)$$

The transfer functions  $T_{kj}$ 's are obtained through a simple circuit analysis and their numerator and denominator polynomials are given in Table 4.1. From this table, the ideal transfer function is

$$H(s, 0, 0) = (1 + \frac{b_3}{b_1}) \frac{s^2 + s[\frac{1}{R_1 C_1} + \frac{1}{R_1 C_2} - \frac{b_2}{(1-b_2) R_2 C_1}] + \frac{1}{R_1 R_2 C_1 C_2}}{s^2 + s[\frac{1}{R_1 C_1} + \frac{1}{R_1 C_2} + \frac{1}{R_2 C_1} - \frac{1}{b_1 R_5 C_1}] + \frac{1}{R_1 R_2 C_1 C_2}} \quad (4.29)$$



TABLE 4.1: NUMERATOR AND DENOMINATOR POLYNOMIALS OF THE  
TRANSFER FUNCTIONS OF THE PASSIVE NETWORK IN  
THE ACTIVELY COMPENSATED ALLPASS CIRCUIT

POLYNOMIAL	
$D_{RC}(s)$	$s^2 + s\left[\frac{1}{R_1 C_1} + \frac{1}{R_1 C_2} + \frac{1}{R_2 C_1}\right] + \frac{1}{R_1 R_2 C_1 C_2}$
$N_{1i}(s) = -N_{2i}(s)$	$(1-a_2)\left\{s^2 + s\left[\frac{1}{R_1 C_1} + \frac{1}{R_1 C_2} - \frac{a_2}{1-a_2} \frac{1}{R_2 C_1}\right] + \frac{1}{R_1 R_2 C_1 C_2}\right\}$
$N_{1l}(s) = -N_{2l}(s)$	$-a_1\left\{s^2 + s\left[\frac{1}{R_1 C_1} + \frac{1}{R_1 C_2} + \frac{1}{R_2 C_1} - \frac{1}{a_1 R_5 C_1}\right] + \frac{1}{R_1 R_2 C_1 C_2}\right\}$
$N_{12}(s)$	$-a_3$
$N_{22}(s)$	$0$

$$R_2 = R_4 // R_5$$

Note: It is assumed that  $a_1 = b_1$  and that  $a_2 = b_2$ .

It can be seen from eqn. (4.29) that the ideal transfer function of the circuit can be reduced to that of an allpass function whose general form is given by

$$H(s) = k \frac{s^2 - \omega_o/Q_o + \omega_o^2}{s^2 + \omega_o/Q_o + \omega_o^2}$$

provided that

$$b_2 = \frac{\beta(1+\gamma) + Q_o^{-1}\sqrt{\gamma\beta}}{1 + \beta(1+\gamma) + Q_o^{-1}\sqrt{\gamma\beta}} \quad (4.30)$$

If the optimization variables are selected as  $\gamma = C_1/C_2$  and  $\beta = R_2/R_1$ , it follows that

$$\omega_o = \frac{1}{\sqrt{\gamma\beta R_1 C_2}} \quad (4.31)$$

The parameter  $b_1$  can be used to control the overall DC gain,

K, of the transfer function realized by this circuit. By arbitrarily choosing  $K = 2$  (any other value greater than 1 can also be chosen), we have that

$$b_1 = b_3$$

and, thus,

$$b_1 = \frac{1 - b_2}{2} = \frac{1}{2[1 + \beta(1+\gamma) + Q_0^{-1}\sqrt{\gamma\beta}]} \quad (4.32)$$

By applying eqn. (4.32) into the denominator of (4.29), it can be obtained that

$$a = \frac{R_5}{R_1} = \frac{2\beta[1 + \beta(1+\gamma) + Q_0^{-1}\sqrt{\gamma\beta}]}{1 + \beta(1+\gamma) + Q_0^{-1}\sqrt{\gamma\beta}} \quad (4.33)$$

Equations (4.30) to (4.33) represent all the other design parameters as a function of  $\gamma$  and  $\beta$ .

The objective function for a 2 OA actively compensated circuit is given by

$$F_2(s) = \frac{1}{T_{12}T_{21}} = - \frac{D_{RC}}{a_3 N_{21}} \quad (4.34)$$

From Table 4.1, it is seen that

$$D_{RC} = \frac{N_{21}}{b_1} + \frac{N_{21}}{b_1 R_5 C_1} \quad (4.35)$$

Also, from the compensation conditions and the DC gain,

$$a_3 = b_3 = b_1 \quad (4.36)$$

By using (4.35), the objective function can be rewritten as

$$F_2(s) = - \frac{1}{b_1^2} \left[ 1 + \frac{s/R_5 C_1}{N_{21}} \right] \quad (4.37)$$

The frequency dependence of  $F_2(s)$  is given by the second term inside the brackets in eqn. (4.37). This term

is a bandpass type function whose poles are given by the zeros of  $N_{21}$  which are also the poles of the ideal transfer function. Consequently, the only parameter of the bandpass function which can still be modified is its center frequency gain. From this observation, it can be concluded that if  $F_2(s)$  is minimized at the pole frequency, the minimization will hold for all frequencies.

By using  $s = j\omega_0$  in eqn. (4.37) and substituting the value of  $b_1$ , the objective function becomes a real function given by

$$F = F_2(j\omega_0) = 4[1 + \beta(1 + \gamma) + Q_0^{-1}\sqrt{\gamma\beta}]^2 \left\{ 1 + \frac{Q_0\sqrt{\gamma\beta}[1 + \beta(1 + \gamma) - Q_0^{-1}\sqrt{\gamma\beta}]}{\gamma\beta[1 + \beta(1 + \gamma) + Q_0^{-1}\sqrt{\gamma\beta}]} \right\} \quad (4.38)$$

This function can be easily minimized by a numerical algorithm. Nevertheless, let us use an analytical approach in order to obtain some insight about the design of this circuit. Towards this end, Fig. 4.6 shows several plots of the objective function using different values of  $\gamma$  and  $Q_0$ . Some conclusions can be obtained directly from these plots:

- (a) For each value of  $Q_0$ , a true minimum occurs for a very large value of  $\gamma$  which is not realizable in practice.

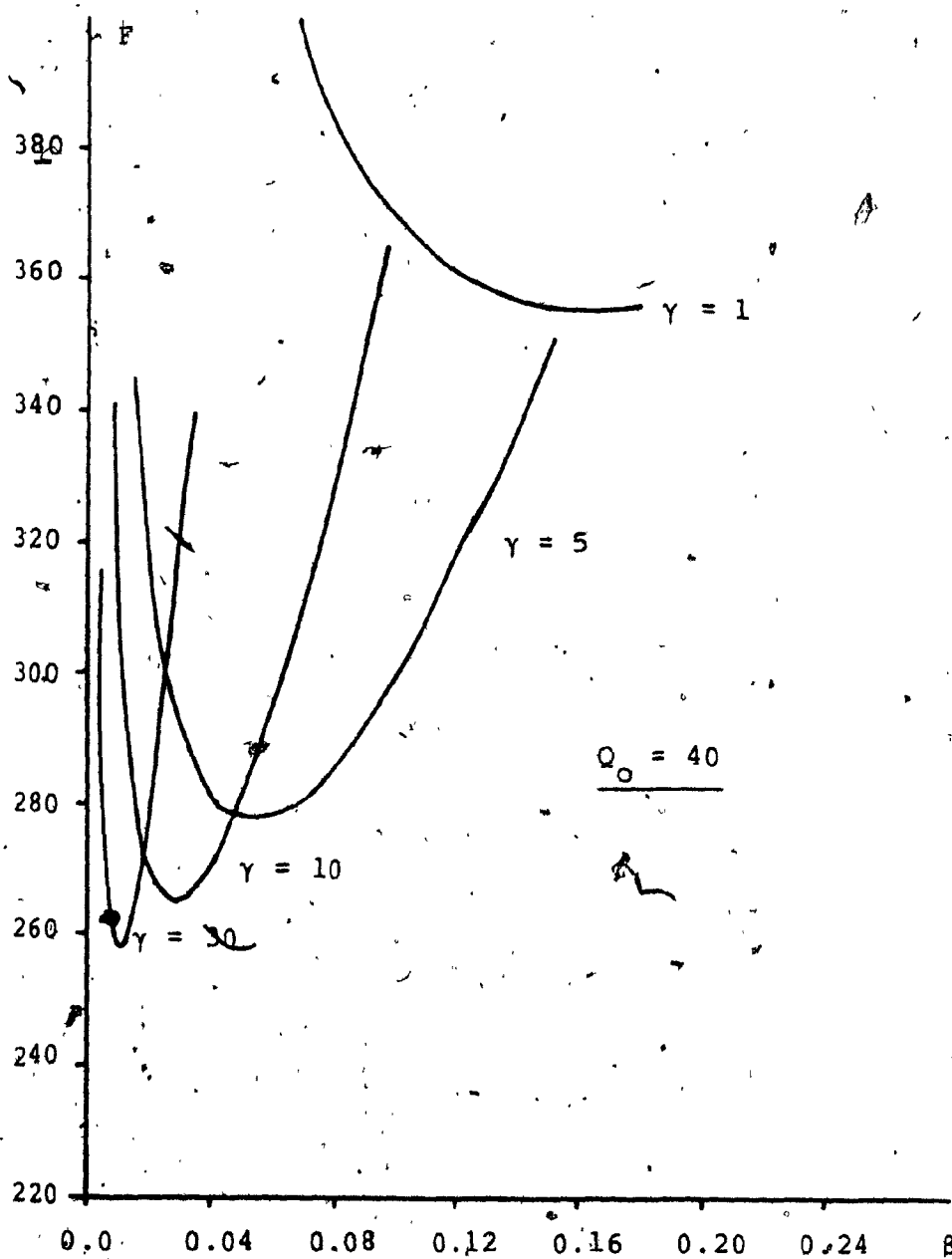


Fig. 4.6: Plots of the objective function for the actively compensated allpass circuit

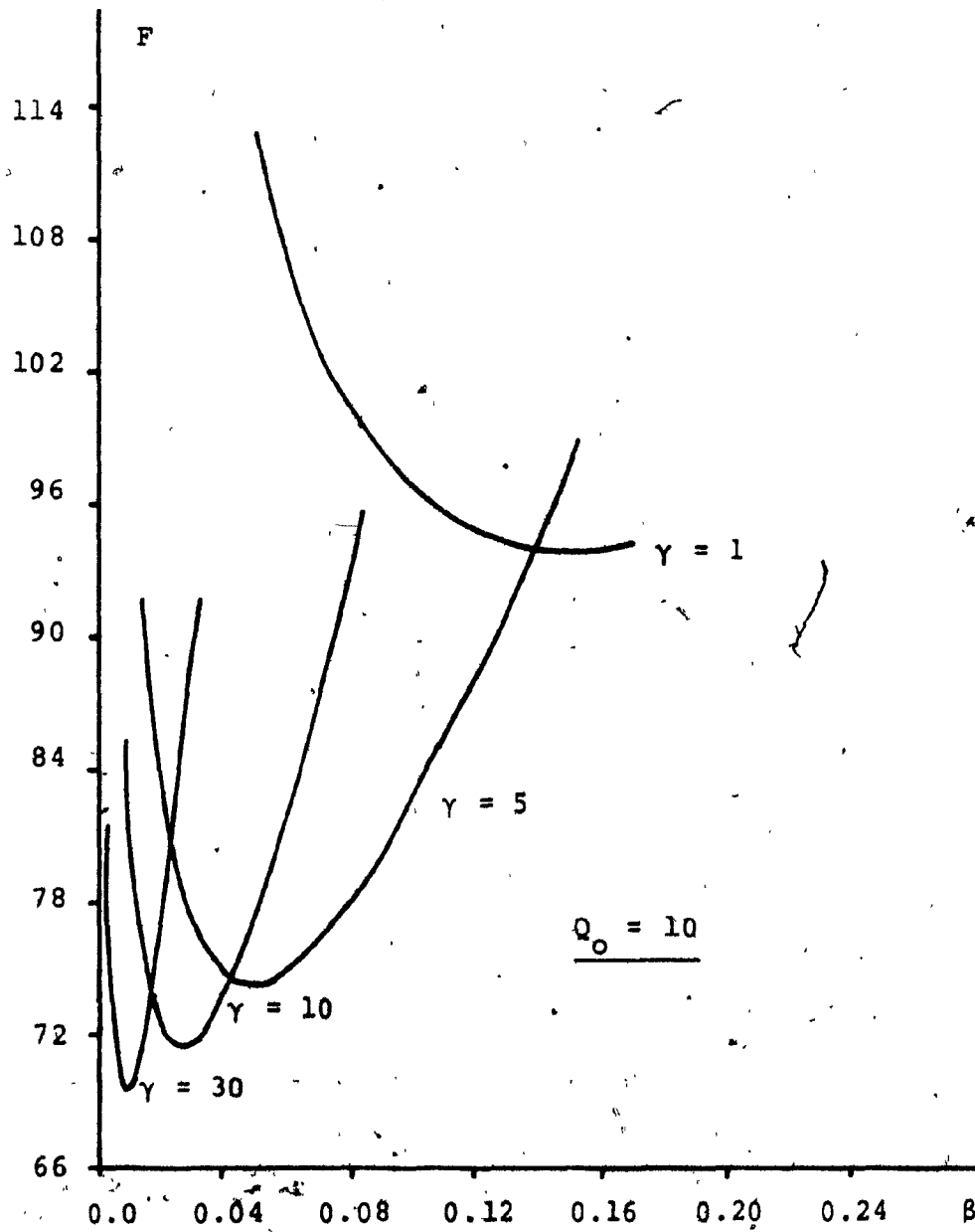


Fig. 4.6: (continued)

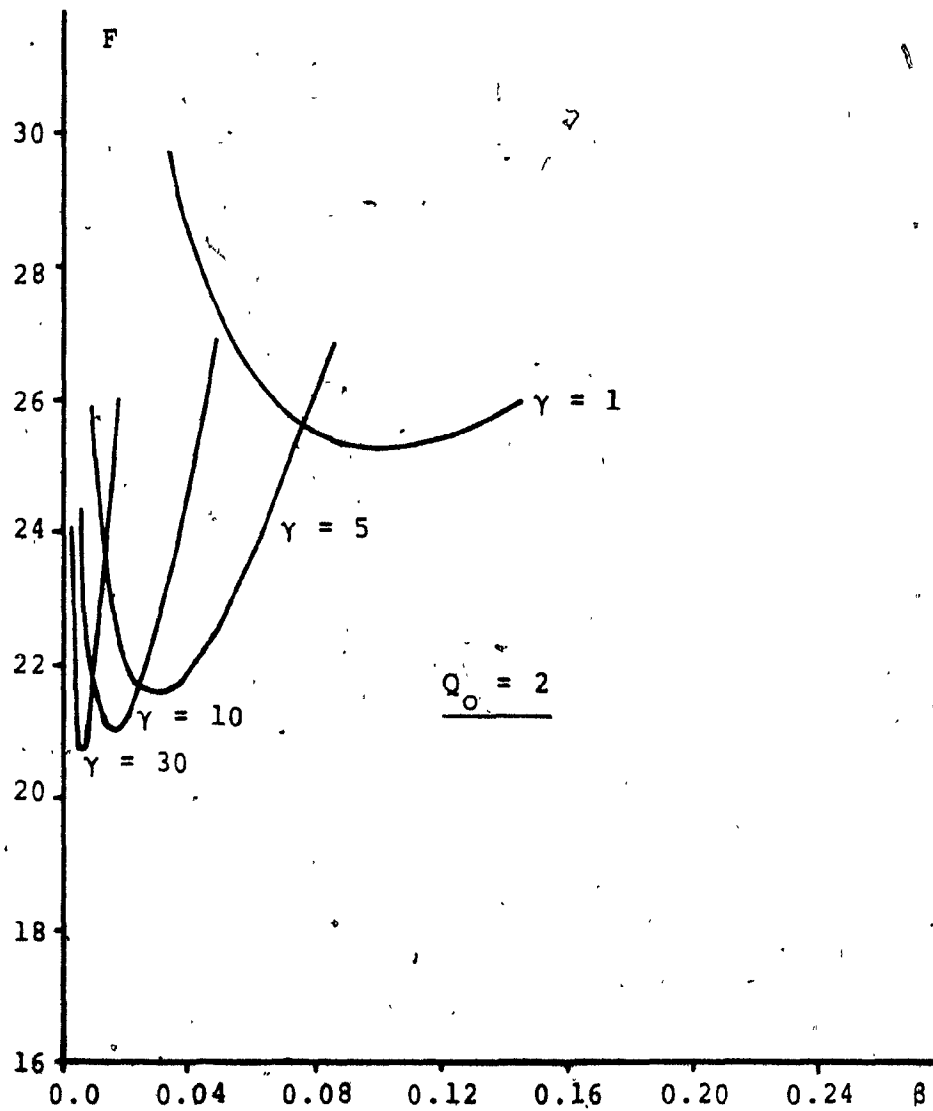


Fig. 4.6: (continued)



(b) For a given  $Q_0$ ,  $\gamma$  should be as large as possible in order to reduce most the active sensitivity. By increasing  $\gamma$  from 1 (the usual design practice) to 5, the active sensitivity can be decreased by as much as 25%. A further increase in  $\gamma$ , however, does not lead to an improvement so dramatic. Therefore,  $\gamma$  equal to 5 can be considered a suboptimal value.

(c) For given values of  $\gamma$  and  $Q_0$ , a suboptimal value of  $\beta$  can be obtained from a plot of the objective function. This plot can be easily generated by using a programmable calculator. Further, from Fig. 4.6, it is seen that these plots are very steep near the minimum and, consequently,  $\beta$  must be carefully obtained.

In order to demonstrate the improved performance of the optimized actively compensated version of the Steffen allpass circuit, let us consider the design of an allpass transfer function with a pole frequency equal to 15 kHz and  $Q_0$  equal to 10.

If  $\gamma$  is chosen to be equal to 5, a look at Fig. 4.6b reveals that the suboptimal value of  $\beta$  will be 0.05. However, in order to check the dependence of the design on the accuracy with which is realized, let us use a value of 0.055 for this parameter (a 10% increase in the suboptimal

value).

The actively compensated Steffen allpass circuit of Fig. 4.5b was designed using these values for  $\gamma$  and  $\beta$ . Also, the conventional circuit was designed according to the design in [42]. Finally, the conventional circuit was designed using the actively compensated amplifier presented in [25].

All three circuits were simulated considering 741-like OAs with gain-bandwidth product equal to 1 MHz. The results of these simulations are shown in Fig. 4.7 along with the ideal response. Note that the magnitude of the transfer function of the circuit optimized in this subsection was normalized in Fig. 4.7a in order to perform the comparisons. From this figure, the improved performance of the circuit treated in this subsection can be clearly observed. Also, it is seen that the simple use of an ACVA in this circuit does not lead to an improved performance because such building blocks cannot upgrade circuits which use active devices with differential input. Further, it is seen that the effect of a 10% error in  $\gamma$  is well tolerated by the network.

#### 4.3.3 The FRIEND-DELIYANNIS bandpass circuit

The conventional and the 3 OA actively compensated versions of the FRIEND bandpass circuit [48] are shown in

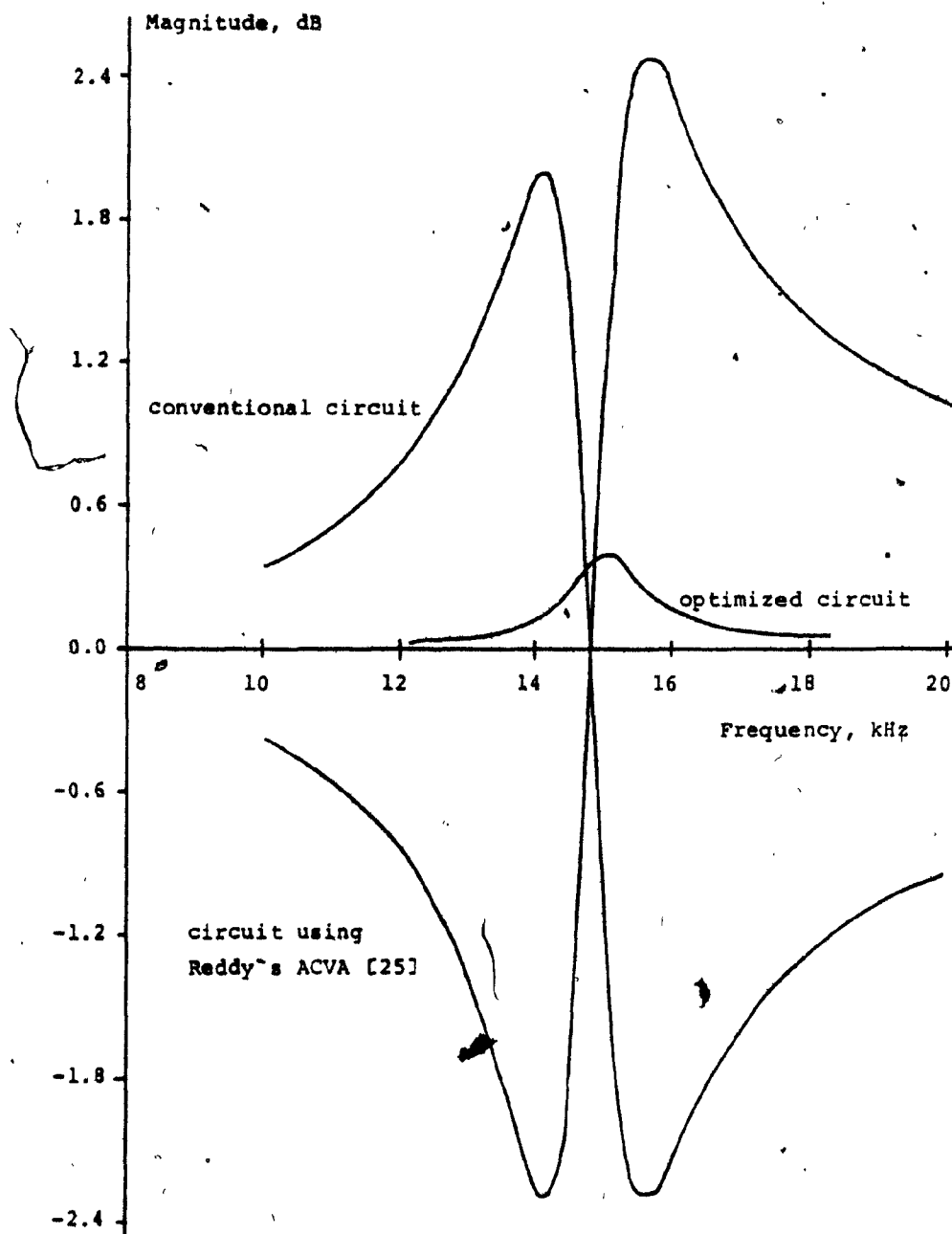


Fig. 4.7a: Magnitude responses of the different allpass circuits

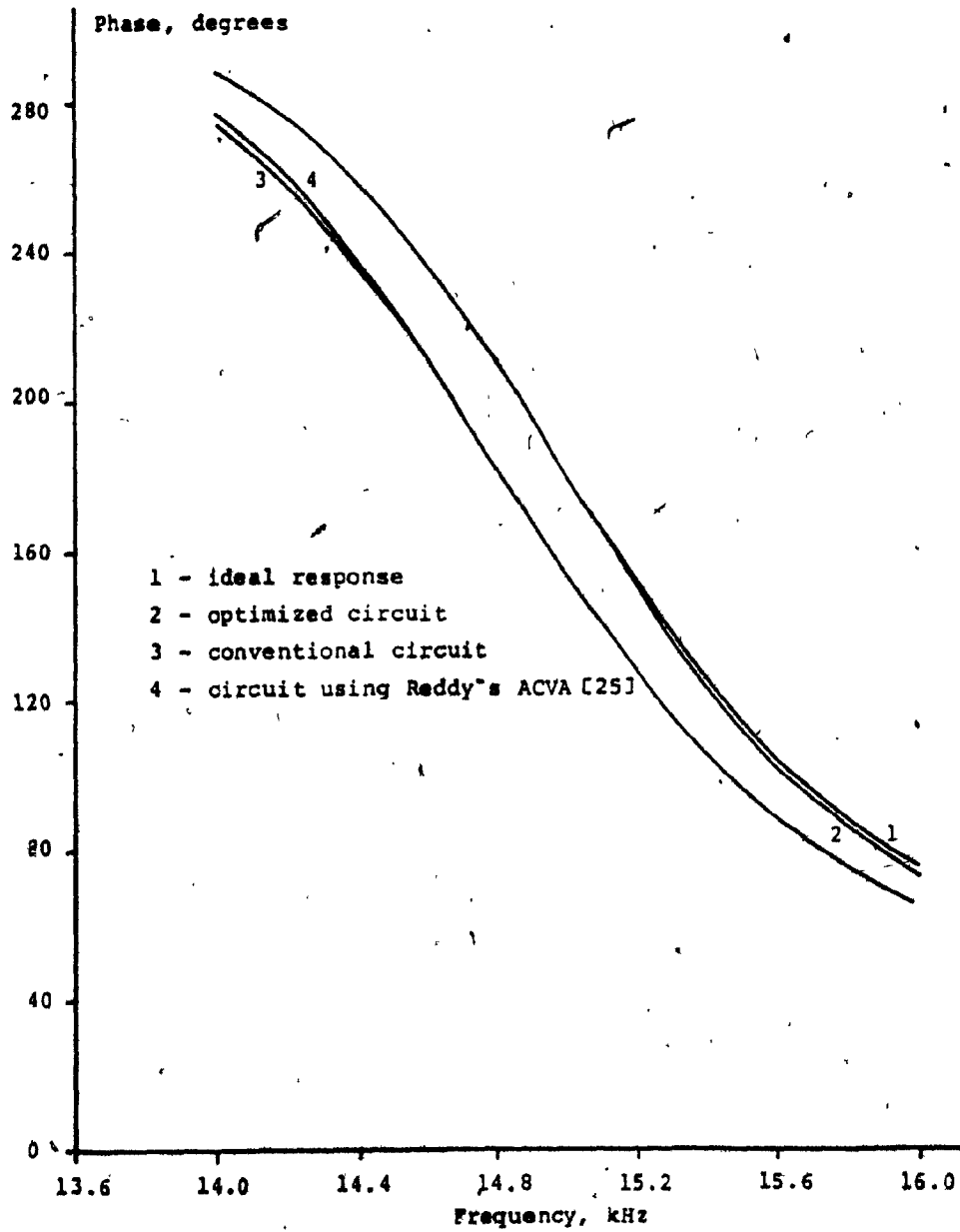
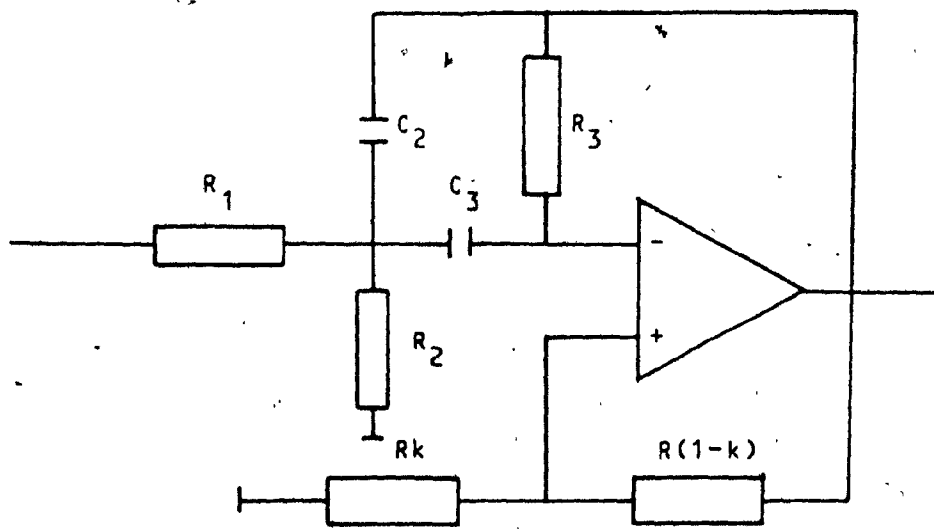
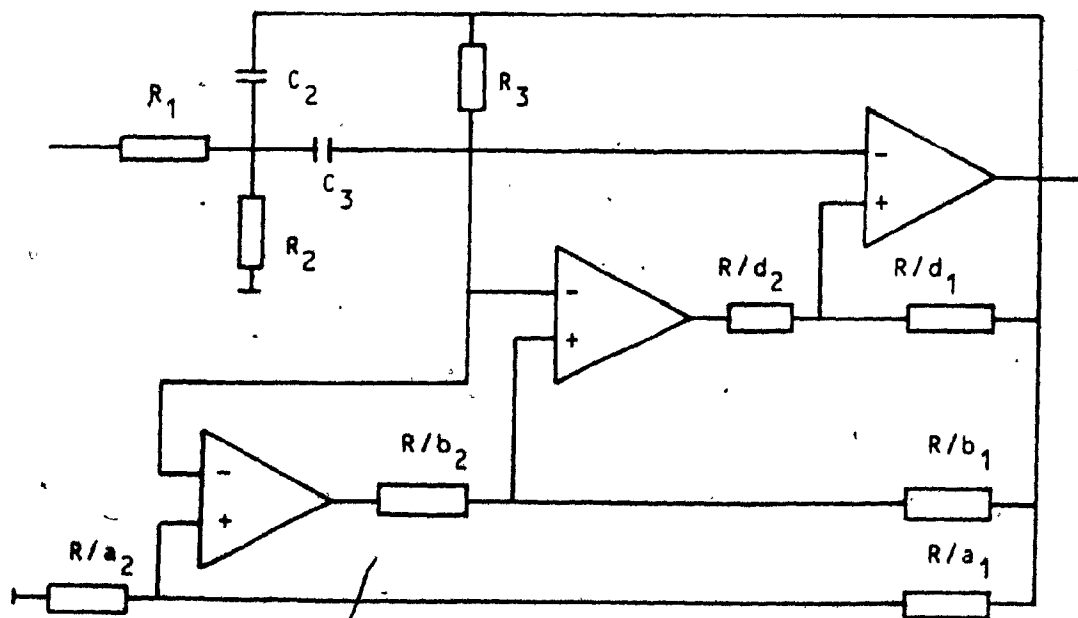


Fig. 4.7b: Phase responses of the different allpass circuits



(a)



(b)

Fig. 4.8: The FRIEND-DELIYANNIS bandpass circuit

(a) conventional circuit

(b) 3 OA actively compensated version

Fig. 4.8. These circuits can be classified as enhanced negative feedback networks. Considering the latter circuit, the compensation condition is given by

$$a_1 = b_1 = c_1 \quad (4.39)$$

which implies that

$$a_2 = b_2 = c_2 \quad (4.40)$$

The numerator and the denominator of the transfer functions  $T_{kj}$ 's are given in Table 4.2 where it is assumed that eqns. (4.39) and (4.40) are satisfied. From this table, the ideal transfer function is

$$H(s, 0, 0, 0) = \frac{s/(1-d_1)R_1C_2}{s^2 + s\left[\frac{1}{R_3C_3} + \frac{1}{R_3C_2} - \frac{d_1}{(1-d_1)R_4C_2}\right] + \frac{1}{R_3R_4C_3C_2}} \quad (4.41)$$

This is a bandpass function whose generic form is given by

TABLE 4.2: NUMERATOR AND DENOMINATOR POLYNOMIALS OF THE  
TRANSFER FUNCTIONS OF THE PASSIVE NETWORK IN  
THE FRIEND-DELIYANNIS BANDPASS CIRCUIT

POLYNOMIAL	
$N_{1i} = -N_{2i} = -N_{3i}$	$-\frac{s}{R_1 C_2}$
$N_{11} = N_{21} = N_{31}$	$(d_1 - 1) \{ s^2 + s [\frac{1}{R_3 C_3} + \frac{1}{R_3 C_2} + \frac{d_1}{(1-d_1) R_4 C_2}] + \frac{1}{R_3 R_4 C_2 C_3} \}$
$N_{13}$	$d_3$
$N_{32}$	$b_2$
$D_{RC}$	$s^2 + s [\frac{1}{R_3 C_2} + \frac{1}{R_3 C_3} + \frac{1}{R_4 C_2}] + \frac{1}{R_3 R_4 C_2 C_3}$
$N_{12} = N_{22} = N_{23} = N_{33}$	0

$$R_4 = R_1 // R_2$$

$$H(s) = \frac{k\omega_o/Q_o}{s^2 + s\omega_o/Q_o + \omega_o^2}$$

If the optimization variables are defined as

$$P = \frac{d_1}{1 - d_1}$$

and

$$\gamma = \frac{C_2}{C_3}$$

respectively, it follows from the design equations that

$$\beta = \frac{R_3}{R_4} = \frac{[2P(1+\gamma)Q_o^2 + \gamma] - [\gamma^2 + 4P(1+\gamma)\gamma Q_o^2]^{\frac{1}{2}}}{2P^2 Q_o^2} \quad (4.42)$$

$$\omega_o = 1/\sqrt{\gamma B R_4 C_3} \quad (4.43)$$

$$Q_o = [\sqrt{\gamma B} + \frac{1}{\sqrt{\gamma B}} - P\sqrt{B/\gamma}]^{-1} \quad (4.44)$$



$$\delta = \frac{R_1}{R_4} = \frac{Q_o \sqrt{\gamma \beta}}{\gamma k d_2} \quad (4.45)$$

The function to be minimized is

$$F_3(s) = \frac{1}{B(s, 0, 0, 0)} = \frac{1}{T_{13}^* T_{21} T_{32}} = \frac{D_{RC}}{N_{21} d_2 b_2} \quad (4.46)$$

Also, from Table 4.2, it is seen that

$$D_{RC} = -\frac{N_{21}}{1 - d_1} + \frac{s}{(1 - d_1) R_4 C_2} \quad (4.47)$$

By taking eqn. (4.47) into (4.46) and using the fact that  $1 - d_1$  is equal to  $d_2$ , the objective function can be rewritten as

$$F_3(s) = \frac{1}{(1 - d_1)^3} \left[ -1 + \frac{s/R_4 C_2}{N_{21}} \right] \quad (4.48)$$

The second term inside the brackets is a bandpass type function in  $s$  whose poles are the same as the poles of the ideal transfer function. Hence, they cannot be modified. Only the gain at the center frequency can be altered in this function. Since the dependence of  $F_3(s)$  upon the frequency is totally contained in this term, it can be concluded that if the objective function is minimized at  $s = j\omega_0$ , it will be minimized for all frequencies. Therefore,  $F_3(s)$  can be treated as a scalar function defined as

$$F = F_3(j\omega_0) = \frac{1}{(1-d_1)^3} \left[ -1 + \frac{Q_0}{R_4 C_2 \omega_0} \right] \quad (4.49)$$

By using eqns. (4.42) to (4.45),  $F$  can be rewritten as

$$F = (1 + P)^3 \left[ -1 + \frac{[1 + 4PQ_0^2(1 + \gamma^{-1})]^{\frac{1}{2}} - 1}{2P} \right] \quad (4.50)$$

The optimal values of  $\gamma$  and  $P$  that minimize  $F$  can be easily obtained through a numerical algorithm.

Nevertheless, as before, some interesting insight can be obtained if we plot  $F$  as a function of  $P$  for different values of  $\gamma$  and  $Q_0$ . Examples of such plots are shown in Fig. 4.9. At this point, some useful conclusions can be derived:

- (a) From both Fig. 4.9 and eqn. (4.50), it can be seen that the true minimum is reached when  $\gamma$  goes to infinity. Obviously, then, the mathematically optimal design for the circuit under consideration is not practically realizable.
- (b) The active sensitivity of this circuit is reduced when  $\gamma$  is increased. This sensitivity reduction is very pronounced when this parameter is increased from 1 to 5. Further increments in  $\gamma$ , however, do not lead to such strong improvement. Consequently, it is advisable to use a value for  $\gamma$  as high as possible but still in the range from 1 to 5.
- (c) For a given value of  $Q_0$ , there is a suboptimal value of  $P$  for which the objective function is at a minimum. This optimal  $P$  is reasonably independent of  $\gamma$ .
- (d) All curves are very shallow around the minimum point. Therefore, one can use any value of  $P$  in

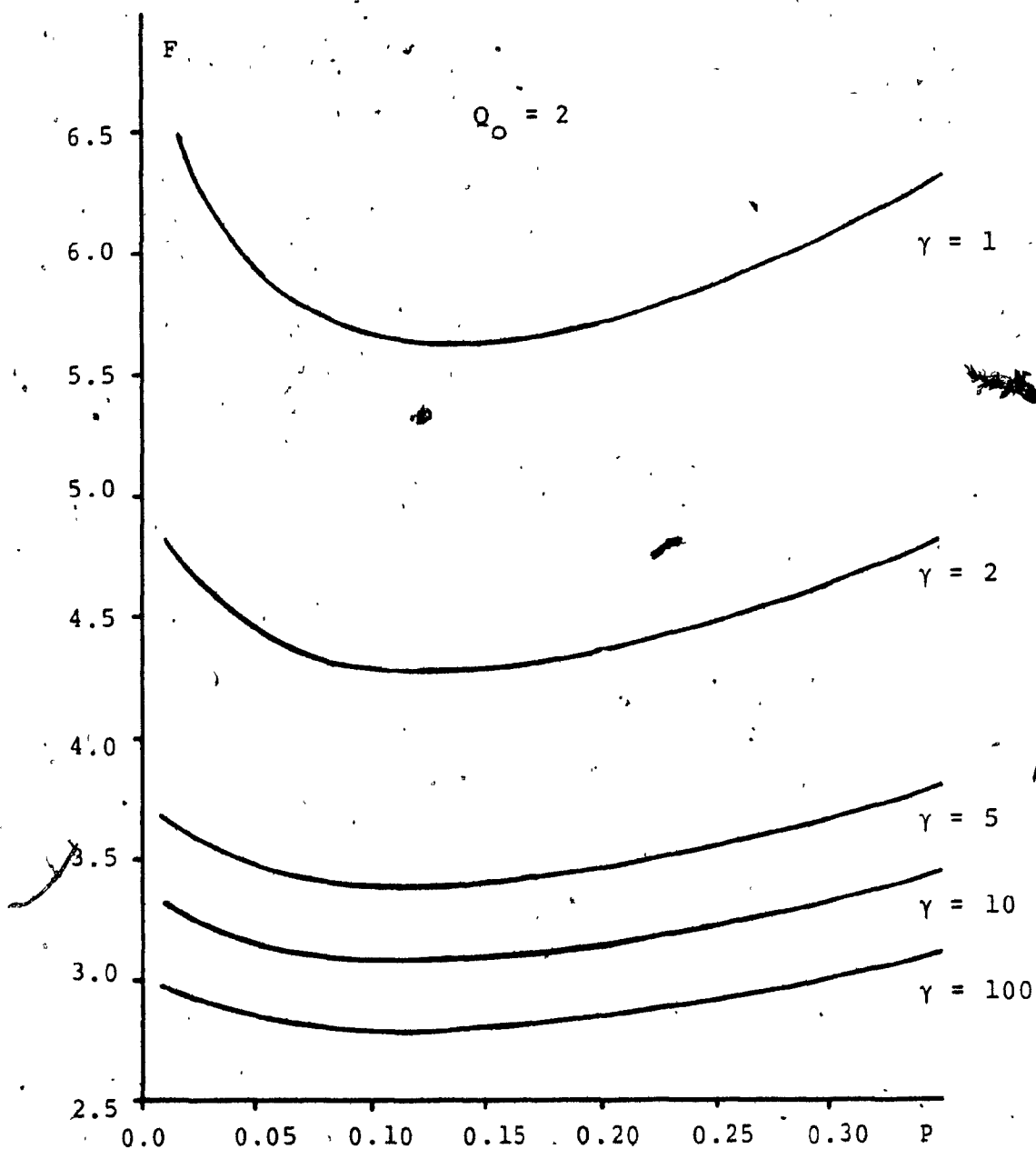


Fig. 4.9: Plots of the objective function for the actively compensated bandpass circuit.

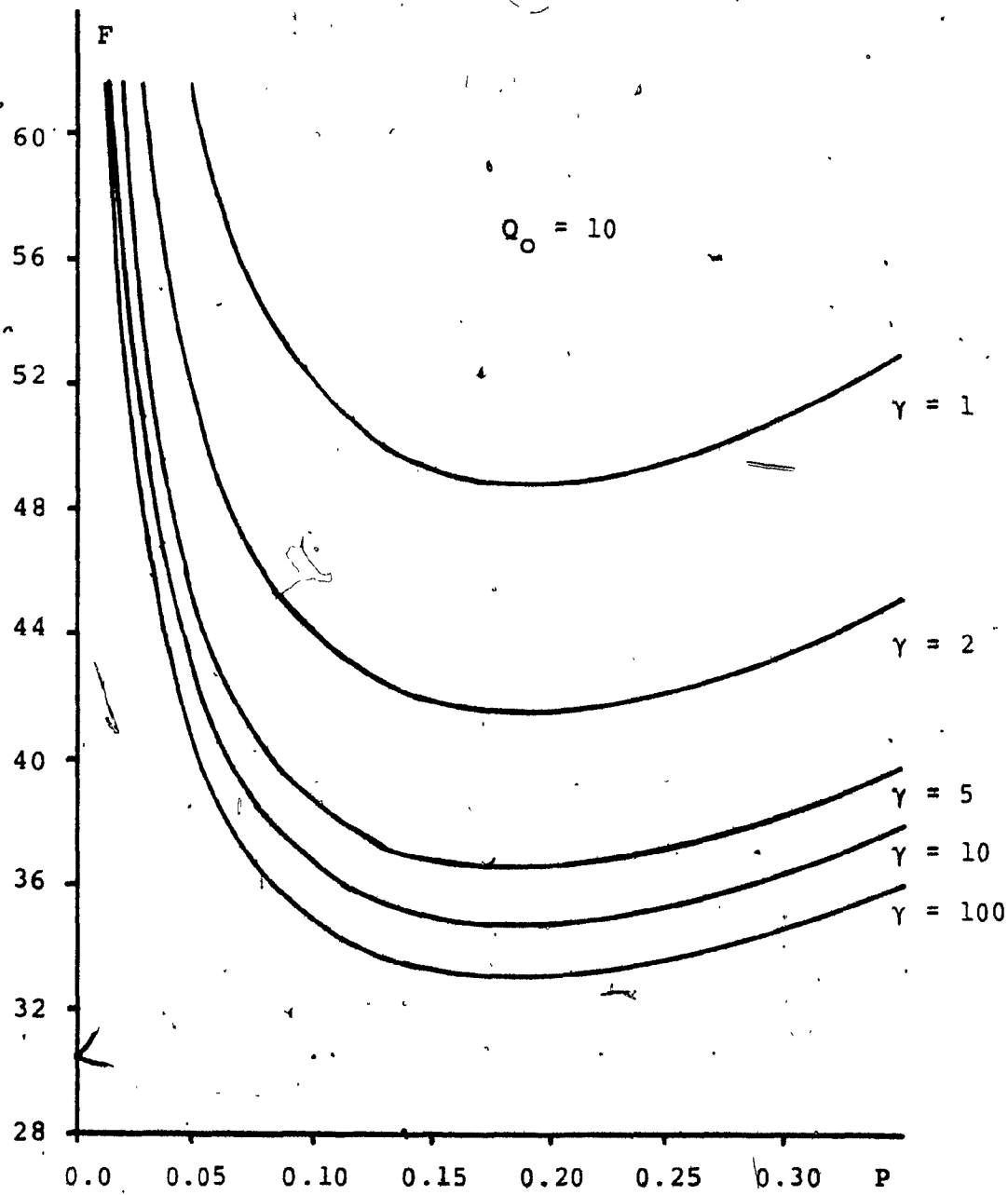


Fig. 4.9: (continued)

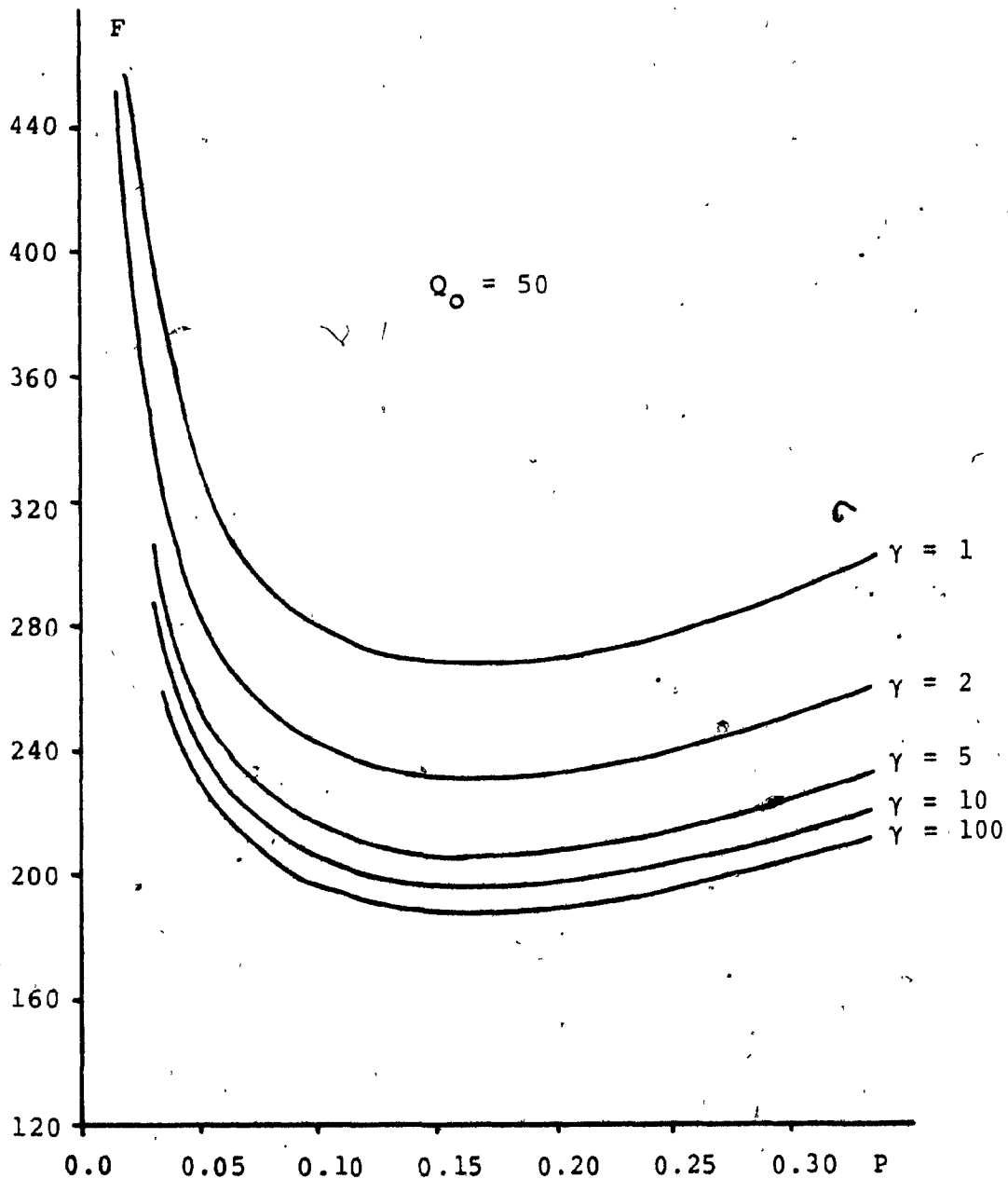


Fig. 4.9: (continued)

the range of  $0.1 < P < 0.2$  without significant increase in  $F$  for any values of  $\gamma$  and  $Q_0$ .

- (e) If the circuit is designed using  $\gamma$  in the range from 1 to 5 and  $P$  in the range from 0.1 to 0.2, practical examples show that the resistor spread will be always smaller than 50. This spread is quite tolerable in current IC technology.

In [50], an actively compensated bandpass circuit employing 2 OAs is claimed to have "better performance than any other actively compensated filters". It is interesting to compare the bandpass filter treated in this subsection (which we shall call BP3) against the one in that article (which we shall call BP2). For this purpose, let us define a suitable figure of merit.

Eqns. (4.11) and (4.23) express the deviations in the transfer function from its ideal value due to the actual OA characteristics for actively compensated circuits using 2 and 3 OAs, respectively. If the fractional transfer function deviation is defined as

$$H(s) = \frac{H(s, \tau_1, \tau_2) - H(s, 0, 0)}{H(s, 0, 0)}$$

or

$$H(s) = \frac{H(s, \tau_1, \tau_2, \tau_3) - H(s, 0, 0, 0)}{H(s, 0, 0, 0)}$$

it follows from those equations that

$$H(s) = F_2(s) \tau_1 \tau_2 s^2 \quad (4.51)$$

or

$$H(s) = F_3(s) \tau_1 \tau_2 \tau_3 s^3 \quad (4.52)$$

for 2 and 3 OA circuits, respectively.


For the two circuits under comparison,  $H(s)$  is maximum at the pole frequency, namely  $s = j2\pi f_0$ . Also,  $H(j2\pi f_0)$  increases as  $f_0$  increases. Therefore, a suitable figure of merit for comparing the two filters is the maximum  $f_0$  for which  $|H(j2\pi f_0)|$  is smaller than 10%. This maximum  $f_0$  can be easily calculated since  $F_2(s)$  and  $F_3(s)$  can be obtained by a routine circuit analysis.

The results of this analysis are shown in Table 4.3 for different values of  $Q_0$ , GB equal to 1 MHz and  $\gamma$  equal to 5. For each  $Q_0$ , the BP3 circuit used  $P$  equal to 0.18 while the BP2 circuit used  $P$  equal to 0.2 as recommended in [50].



TABLE 4.3: MAXIMUM POLE FREQUENCY FOR WHICH

$$|\Delta H(j2\pi f_o)| < 0.1$$



Filter	$Q_o = 2$	$Q_o = 10$	$Q_o = 50$
BP2	186.3 kHz	57.3 kHz	24.2 kHz
BP3	309.2 kHz	139.9 kHz	78.6 kHz

From this table, it is evident that BP3 outperforms significantly the other circuit. However, it is important to note that the values of  $f_0$  in Table 4.3 are obtained while modelling the OAs as in eqn. (1.24). When actual OAs are considered,  $f_0$  is further limited by several OA characteristics (e.g. the second pole of the OAs, the slew rate, etc.). Therefore, the values in that table serve for comparison between the two circuits but do not necessarily represent the maximum pole frequency achievable in practice.

#### 4.4 EXPERIMENTAL RESULTS

The 3 OA actively compensated bandpass filter presented in this chapter was designed for a pole frequency equal to 80 kHz and a Q-factor equal to 10. For this design,  $\gamma$  equal to 5 and  $P$  equal to 0.18 were used. The GBs of the OAs were measured to be 999, 1001 and 983 kHz. Also, the second pole of the OAs were measured to be at 1869, 1847 and 1850 kHz, respectively.

Before testing, the circuit was simulated on a computer and it was found to be unstable when considering a two-pole model for the OAs. In order to stabilize the circuit, the return ratio [39] of the network was modified by adding a resistor of value  $R/b_3$  to the circuit as shown in Fig. 4.10. The value of  $b_3$  was calculated in order to reduce  $b_2$  by a factor of 10 while leaving  $b_1$  unaffected.



Consequently, the return ratio was decreased by a factor of 10 and  $F_3(s)$ , the objective function considered in subsection 4.3.3, was increased 10 times.

The resulting circuit was built and measured in the laboratory along with the bandpass circuit presented in [50]. A comparison between the performance of both circuits is shown in Table 4.4.

This comparison demonstrate clearly the superior performance of the 3 OA circuit. This good performance at such unusually high frequency for active-RC filters was made possible by not only the use of active compensation but also by the use of the optimization procedure presented in this chapter.

#### 4.5 CONCLUSIONS

An active compensation technique was presented in this chapter for use in cases where the building block approach cannot be applied. This technique can be applied to most already existing SABs regardless of the transfer function being realized. It was shown that the application of this scheme results in nulling the first-order effects of the GBs of the OAs in circuits using 2 OAs and also the second-order effects in 3 OA circuits.

TABLE 4.4: EXPERIMENTAL RESULTS FOR THE  
BANDPASS CIRCUIT

PARAMETER	BP2	BP3
$\Delta f_o$ (%)	0.61	-0.55
$\Delta Q_o$ (%)	21.03	4.5
$\Delta  H(j2\pi f_o) $ (%)	21.35	4.8

Theoretical values:  $f_o = 80.455$  kHz

$Q_o = 8.72$

$|H(j2\pi f_o)| = 10.75$

Further, a procedure for minimizing the residual effects of the GBs of the OAs was presented. This procedure is very simple and can be readily applied for all actively compensated circuits with free design parameters. Two examples demonstrate that significant performance improvement can be achieved by combining active compensation and sensitivity minimization.

## CHAPTER 5

## CONCLUSIONS

## 5.1 SUMMARY OF THE THESIS

Active-RC filter networks using OAs have been extensively researched and used during the last decades. Conventional OA-based circuits, however, work well only at frequencies much below the gain-bandwidth product of the OAs used. This limitation is due to the effect of the frequency dependent differential gain of the OAs which introduces severe distortion in the transfer function realized by the circuit.

This thesis deals with a very effective technique for extending the operating frequency range of these OA-based networks, namely active compensation. In order to use this technique, one can either replace the active element within a given circuit by an actively compensated version of that element or consider the circuit as a whole and compensate it actively by modifying its overall structure. In the first approach, special attention has been given to the active compensation of voltage amplifiers (VAs) due to the great popularity of active filters which use such building blocks.

In this dissertation, the results of a comprehensive and systematic study on the generation and classification of actively compensated voltage amplifiers (ACVAs) are reported. Also, a new technique for the active compensation of active-RC circuits is described.

Towards this end, Chapter 2 contains an investigation of ACVAS using 2 OAs. This investigation starts with the analysis of a general 2 OA amplifier circuit. From this analysis, performance conditions are derived including the ones that lead to the active compensation. These conditions are then used to classify the ACVAS according to a set of both practical and theoretical considerations. Additional considerations allow each class to be further divided into types. The characterization of each type leads directly to the circuit realizations. A set of 52 circuits is obtained and classified. Particularly, a complete set of infinite input impedance ACVAS using 2 OAs and 4 or less resistors is obtained. Finally, all circuits presented in this chapter are analyzed with respect to performance characteristics such as magnitude and phase errors, stability considering the second pole of the OAs, etc.

The investigation of ACVAS is extended in Chapter 3 to include 3 OA circuits. For this purpose, a general model of a 3 OA ACVA is analyzed and some performance conditions are thus obtained. Some additional restrictions are used to reduce the effect of the increased complexity of 3 OA ACVAS



and to make their use simple and practical. Then, 3 OA ACVAs are classified according to a set of practical and theoretical considerations which are not the same as the ones used for classifying 2 OA ACVAs. The circuit realizations are obtained from this classification through the sequential resistor elimination procedure mentioned in Chapter 2. A set of 161 ACVA realizations are generated. It is interesting to note that, among other results, it is shown in this chapter that an inverting 3 OA ACVA possessing infinite input impedance cannot be orthogonally tunable. Also, an important subset of the circuits obtained are analyzed with respect to the several performance factors referred to above.

The replacement of a VA building block by an ACVA can upgrade the performance of an important but restricted class of circuits. However, there are numerous applications where VA-based circuits are not used. In these cases, a different active compensation technique must be used in order to extend the operating frequency range of these networks. Such a technique is presented in Chapter 4. Its use can eliminate first-order effects of the gain-bandwidth product of the OAs in circuits employing 2 OAs and also second-order effects in 3 OA networks.

In order to derive this technique, the general structures of 2 and 3 OA circuits are analyzed. According to the results of this analysis, a Taylor series expansion

is used to express the relationship between the actual transfer function of 2 and 3 OA networks and the transfer function that would be realized if the OAs were ideal. From this series, conditions for the active compensation of such networks are readily obtained. Then, practical schemes are proposed to implement these conditions in both enhanced positive feedback and enhanced negative feedback single amplifier biquads.

All circuits obtained through the application of the technique mentioned above have some free design parameters whose values can be selected in order to optimize the performance of the network with respect to a given characteristic. This feature is explored in Chapter 4 and an optimization procedure for minimizing the residual effects of the gain-bandwidth of the OAs in actively compensated circuits is presented. The objective function used in this procedure can easily be obtained by a simple analysis of the circuit to be optimized. Therefore, this procedure is readily applicable to different circuits. Examples are given to demonstrate the improved performance of the filters designed according to this procedure.

Simulations and experimental results are included at all the appropriate places in the thesis in order to demonstrate the accuracy of the theoretical conclusions derived.

## 5.2 SUGGESTIONS FOR FUTURE RESEARCH

The study of ACVAs reported in this thesis is limited to finite gain amplifiers. An immediate extension of this investigation would be the study of infinite gain amplifiers. Such amplifiers could be used as a direct replacement for the OAs themselves in several active networks.

In the derivation of the active compensation schemes for VAs, the OAs are modeled as in eqn. (1.24). Nevertheless, the second pole of the OAs are observed to affect significantly the performance of the circuits at frequencies above 100 kHz. Thus, a useful direction of research would be the attempt to obtain an active compensation scheme for the ACVAs in which a double pole model is used for the OAs. The ACVAs thus obtained could be used at considerably higher frequencies.

Integrators are very important building blocks which are used in numerous active-RC circuits. Therefore, it would be worthwhile to perform a study of this kind of circuit. Such study could be conducted in a way similar to the one used in this thesis for ACVAs.

In conclusion, the author hopes that the study and techniques presented will be useful to the analog circuit design in practice. Moreover, owing to the widespread use

of active-RC networks in several areas, the author also hopes that the results obtained will be found useful in a variety of industrial applications.

## REFERENCES

- [1] S. Darlington, "A History of Network Synthesis and Filter Theory for Circuits Composed of Resistors, Inductors and Capacitors", IEEE Trans. Circuits and Systems, Vol. CAS-31, pp. 3-13, January 1984.
- [2] H.W. Bode, "Network Analysis and Feedback Amplifier Design", Van Nostrand-Reinhold, New York, 1945.
- [3] M.S. Ghausi, "Analog Active Filters", IEEE Trans. Circuits and Systems, Vol. CAS-31, pp. 13-31, January 1984.
- [4] R. Schaumann, M. Soderstrand and K. Laker (Eds.), "Modern Active Filter Design", IEEE Press, New York, 1981.
- [5] R.J. Widlar, "Design Techniques for Monolithic Operational Amplifiers", IEEE J. Solid-State Circuits, Vol. SC-4, pp. 184-191, August 1969.
- [6] K. Ramakrishna, K. Soundararajan and V.K. Aatre, "Effect of Amplifier Imperfections on Active Networks", IEEE Trans. Circuits and Systems, Vol. CAS-26, pp. 922-931, November 1979.
- [7] H.J. Bachler and W. Guggenbuhl, "Active-Filter Pole Displacement Due to Nonideal Operational Amplifiers", IEE J. Electronics, Circuits and Systems, Vol. 2, pp. 89-91, July 1978.
- [8] P.R. Gray and R.G. Meyer, "Analysis and Design of Analog Integrated Circuits", John Wiley Sons, New York,

1977.

- [9] G.S. Moschytz, "Linear Integrated Networks: Design", Van Nostrand Reinhold, New York, 1975.
- [10] L.P. Huelsman, "Theory and Design of Active RC Circuits", McGraw-Hill, New York, 1968.
- [11] J. Millman and C.C. Halkias, "Integrated Electronics", McGraw-Hill, New York, 1972.
- [12] A. Budak and D.M. Petrela, "Frequency Limitations of Active-RC Networks Using OAs", IEEE Trans. Circuit Theory, vol. CT-19, pp. 322-329, July 1972.
- [13] G.S. Moschytz, "Gain-Sensitivity Product - A Figure of Merit for Hybrid-Integrated Filters Using Single Operational Amplifiers", IEEE J. Solid-State Circuits, vol. SC-6, pp. 103-110, June 1971.
- [14] A.S. Sedra and J.L. Espinoza, "Sensitivity and Frequency Limitations of Biquadratic Active Filters", IEEE Trans. Circuits and Systems, vol. CAS-22, pp. 122-130, February 1975.
- [15] P.E. Fleischer, "Sensitivity Minimization in a Single Amplifier Biquad Circuit", IEEE Trans. Circuits and Systems, vol. CAS-23, pp. 45-55, January 1976.
- [16] G. Daryanani, "Principles of Active Network Synthesis and Design", John Wiley and Sons, New York, 1976.
- [17] R. Tarmy and M.S. Ghausi, "Very High-Q Insensitive Active-RC Networks", IEEE Trans. Circuit Theory, vol. CT-17, pp. 358-366, August 1970.
- [18] W.B. Mikhael and B.B. Bhattacharyya, "A Practical

Design for Insensitive RC-Active Filters", IEEE Trans. Circuits and Systems, vol. CAS-22, pp. 407-415, May 1975.

- [19] Y. Sun, "Formulas for Pole-Q and Resonance Frequency Shifts In RC-Active Circuits due to Amplifier Finite Gain-Bandwidth", Proc. 23th Midwest Symp. On Circuits and Systems, pp. 201-205, Toledo, OH., 1980.
- [20] R. Moon and F.W. Stephenson, "Second-Order Q and Shifts in RC Active Filters Having NonIdeal Amplifiers", Electr. Letters, vol. 18, pp. 365-367, April 1982.
- [21] G. Wilson, "Compensation of Some OA Based RC Active Networks", IEEE Trans. Circuits and Systems, vol. CAS-23, pp. 443-446, July 1976.
- [22] P.M. Vogel, "Method for Phase Correction in Active RC Circuits Using Two Integrators", Electr. Letters, vol. 10, pp. 273-275, May 1971.
- [23] D. Akerberg and K. Mossberg, "A Versatile Active RC Building Block with Inherent Compensation for the Finite Bandwidth of the Amplifiers", IEEE Trans. Circuits and Systems, vol. CAS-21, pp. 75-78, January 1974.
- [24] P.O. Brackett and A.S. Sedra, "Active Compensation for High-Frequency Effects in Op. Amp. Circuits with Applications to Active-RC Filters", IEEE Trans. Circuits and Systems, vol. CAS-23, pp. 68-73, February 1976.

- [25] M.A. Reddy, "Operational-Amplifier Circuits with Variable Phase Shift and Their Application to High-Q Active RC-Filters and RC-Oscillators", IEEE Trans. Circuits and Systems, vol. CAS-23, pp.384-389, June 1976.
- [26] S. Natarajan and B.B. Bhattacharyya, "Design and Some Applications of Extended Bandwidth Finite Gain Amplifiers, J. Franklin Inst., vol. 305, pp. 321-341, June 1978.
- [27] A.M. Soliman and M. Ismail, "Active Compensation of Operational Amplifiers, IEEE Trans. Circuits and Systems, vol. CAS-26, pp. 112-117, February 1979.
- [28] R. Geiger and A. Budak, "Active Filters with Zero Amplifier Sensitivity", IEEE Trans. Circuits and Systems, vol. CAS-26, pp. 277-288, April 1979.
- [29] S. Natarajan, "Some Design and Optimization Techniques for Extending the Operating Frequency Range of Active-RC Filters", Ph.D. Dissertation, Concordia University, Montreal, Quebec, Canada, December 1978.
- [30] A.M. Soliman, "Classification and Generation of Active Compensated Non-Inverting VCVS Building Blocks", Int. J. Circuit Theory and Appl., vol. 8, pp. 385-405, October 1980.
- [31] A.M. Soliman, "Classification, Generation and Application of Inverting Amplifier Structures", AEU, vol. 35, pp. 311-320, July 1981.
- [32] J.L. Huertas and A. Rodriguez-Wasquez, "On the Active



Compensation of Operational Amplifier Based VCVS", IEEE Trans. Circuits and Systems, vol. CAS-29, pp. 497-506, July 1982.

[33] A. Budak, G. Wullink and R. Geiger, "Active Filters with Zero Transfer Function Sensitivity with Respect to the Time Constants of Operational Amplifiers", IEEE Trans. Circuits and Systems, vol. CAS-27, pp. 849-854, October 1980.

[34] P.B. Lopes and B.B. Bhattacharyya, "A Systematic Approach to the Realization and Design of Actively Compensated Two OA Voltage Amplifiers", to appear in the Int. J. Circuit Theory and Appl. in 1985.

[35] S. Natarajan and B.B. Bhattacharyya, "Design of Actively Compensated Finite Gain Amplifiers for High Frequency Applications", IEEE Trans. Circuits and Systems, vol. CAS-27, pp. 1133-1139, December 1980.

[36] R. Geiger and A. Budak, "Design of Active Filters Independent of First- and Second-Order Operational Amplifier Time Constant Effects", IEEE Trans. Circuits and Systems, vol. CAS-28, pp. 749-757, August 1981.

[37] P.B. Lopes and B.B. Bhattacharyya, "Classification, Generation and Evaluation of Actively Compensated Noninverting Finite Gain Amplifiers", Proc. IEE Part G, vol. 131, pp. 177-189, October 1984.

[38] P.B. Lopes and B.B. Bhattacharyya, "Generation, Classification and Evaluation of Actively Compensated Inverting Voltage Amplifiers", to be published.

- [39] S.S. Hakim, "Feedback Circuit Analysis", Iliffe Books, London, 1966.
- [40] J.K. Roberge, "Operational Amplifiers: Theory and Practice", John Wiley and Sons, New York, 1975.
- [41] G.E. Tobey, J.G. Graeme and L.P. Huelsman, "Operational Amplifiers: Design and Applications", McGraw-Hill, New York, 1971.
- [42] G.S. Moschytz and P. Horn, "Active Filter Design Handbook", John Wiley and Sons, New York, 1981.
- [43] P.S.R. Diniz, "A New High Performance Voltage Controlled Voltage source", Int. J. Electronics, vol. 53, pp. 349-352, October 1982.
- [44] R. Geiger, "Parasitic Pole Approximation Techniques for Active Filter Design", IEEE Trans. Circuits and Systems, vol. CAS-27, pp. 793-799, September 1980.
- [45] E. Kreyszig, "Advanced Engineering Mathematics", John Wiley and Sons, New York, 3rd edition, 1972.
- [46] (a) A.S. Sedra, "Generation and Classification of Single Amplifier Filters", Int. J. Circuit Theory and Appl., vol. 2, pp. 51-67, March 1974. (b) A.S. Sedra and L. Brown, "A Refined Classification of Single Amplifier Filters", Int. J. Circuit Theory and Appl., Vol. 7, pp. 127-137, March 1979.
- [47] P.R. Sallen and E.L. Key, "A Practical Method of Designing RC Active Filters", IRE Trans. Circuit Theory, vol. CT-2, pp. 74-85, March 1955.
- [48] J.J. Friend, C.A. Harris and D. Hilberman, "STAR: An

Active Biquadratic Filter Section", IEEE Trans. Circuits and Systems, vol, CAS-22, pp. 115-121, February 1975.

- [49] A.S. Sedra, M.A. Ghorab and K. Martin, "Optimum Configurations for Single-Amplifier - Biquadratic Filters", IEEE Trans. Circuits and Systems, vol. CAS-27, pp. 1155-1163, December 1980.
- [50] S. Natarajan, "Active Sensitivity Minimization in SAB's with Active Compensation and Optimization", IEEE Trans. Circuits and Systems, vol. CAS-29, pp. 239-245, April 1982.
- [51] P.B. Lopes and B.B. Bhattacharyya, "Design of Optimum Actively Compensated Networks", to be published.