APPLICABILITY OF IRR AND NPV MODELS IN INCOME-PROPERTY ACQUISITION FEASIBILITY STUDY

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ABSTRACT

APPLICABILITY OF IRR AND NPV MODELS IN INCOME-PROPERTY ACQUISITION FEASIBILITY STUDY

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From the viewpoint of the developer-investor, the applicability of internal rate of return (IRR) and net present value (NPV) models for use in income-property acquisition feasibility studies is demonstrated by way of: 1) identifying the scope and objectives of an economic analysis and feasibility study in the realty development life cycle; 2) describing the meanings of IRR and NPV, and the decision-criteria incorporated in these models; 3) examining the ability of these models to reflect practical financial/economic realities encountered in development projects; 4) identifying investor motivations and wedding (matching) them with the performance measures incorporated in the two models; and 5) demonstrating a practical application of the IRR model through a case study of an acquisition project consisting of a downtown property and restaurant.

ACKNOWLEDGEMENTS

I wish to express my deepest gratitude and sincere appreciation to my supervisor, Dr. Alan D. Russell, for his perceptive guidance, constructive criticism, encouragement and understanding.

I would like to thank my friend Fahim Salehji for his encouragement and correcting some of the grammatical errors in my first draft.

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ADM	-	Administrative expenses.
AFCF	-	After-finance cash flows.
B_j	-	Benefits and receipts in period j.
B-ACF	-	Before-finance's minus after-finance's cash flow.
BCR	-	Discounted benefit to cost ratio.
BFCF	-	Before-finance cash flows.
$BFCR_n$	-	Before-finance net resale cash reversion in year n.
$BFICF_j$	-	Before-finance ICF_j.
BV_n	-	Book value in year n.
BV_o	-	Book value in year zero.
CCAR	-	Constant capital cost allowance rate specified by government regulations.
$CCAR_j$	-	Capital cost allowance rate in year j.
CDR	-	After-tax cost of debt rate.
CF	-	After-tax cash flows.
CGT_n	-	Capital gains tax in year n.
DCTR	-	Discount rate.
DEP_j	-	Depreciation (capital cost) allowance in year j.
DS_j	-	Debt service in year j which includes mortgage interest and mortgage principle payments.
E_o	-	Initial equity investment.

EGI_{ij} - Effective gross income of i^{th} income source in period j.

EGI_j - Effective gross income in period j.

ESLR - Constant escalation rate which is invariant with time.

$ESLR_j$ - Escalation rate in period j.

i - Mortgage interest rate.

i_B - Breakeven mortgage interest rate.

IAT - Insurance and taxes other than income tax.

ICF_j - After-tax after-finance interim cash flow from operation in year j.

INF - Inflation rate.

INT_j - Mortgage interest payment in year j.

IRR - Internal Rate of Return.

IRR_{AF} - After-tax after-finance IRR.

IRR_{BF} - After-tax before-finance IRR.

IRR_{B-A} - IRR of alternative B's minus alternative A's cash flow, OR IRR of B-ACF.

j - Number of years, value of j lies between zero and n.

k - Apparent return rate or discount rate.

LAR - Land appreciation rate.

LEV - Leverage.

m - Total number of income sources, OR number of periods that detailed modeling is justified.

MAO - Maintenance and operating expenses.

MTG - Mortgage amount: equals TC minus E_o.

n - Holding period in number of years.

NPV - Net present value.

$NRCR_n$ - Net resale cash reversion in year n.

NSP_n - Net resale price at year n.

OPE_j - Operating expense in year j.

PGI - Potential gross income.

PGI_{ij} - PGI of i^{th} income source in period j.

PP - Payback period.

RCT_n - Recapture tax in year n.

RRR - Real rate of return.

SE_n - Selling expense in year n.

SP_n - Resale price in year n.

t - Development time.

TAX_j - Income tax payable in year j.

$TAXR_j$ - Income tax rate in year j.

TBT_j - Income tax before tax shelter in year j.

TC - Total cost.

$TDEP_j$ - Tax benefits from capital cost allowance in year j, which equals tax rate times depreciation.

$TINT_j$ - Tax benefits from mortgage interest payment in year j which equals tax rate times interest payment.

TSD_j - Tax shelter benefits in year j during development period.

UM_n - Unpaid mortgage balance in year n.

UTL - Utility expenses.

V_j - Value of fluctuating variable in period j.

V_o - Initial value of fluctuating variable in period zero.

VAD - Vacancy and delinquency allowance.

VADR - Vacancy and delinquency rate.

$VADR_{ij}$ - VADR of i^{th} income source in period j.

CHAPTER ONE

INTRODUCTION

1.1 OBJECTIVE

Over the last fifteen years, considerable effort has been devoted in the literature to developing analytical models for use in the prediction of economic performance of real estate investment projects. The focus of this report is on the role of two of these models in the project development life cycle. In particular, the objectives of this report are:

1) to place in context the role of economic analysis in the project development process;
2) to assess in detail the applicability of two models used in economic analysis, namely internal rate of return (IRR) and net present value (NPV); and
3) to demonstrate the application of these models by way of a case study.

1.2 REPORT AUDIENCE

The specific audience of this report is the private developer-investor. This investor viewpoint is explored in detail in Chapter 4. The viewpoint of the private investor is similar in some respects to that of the public and non-profit institution in terms of motivations and their related goals and objectives. This can be

seen by the goals and objectives of the two organizations as defined by McGuigan and Moyer [14:18-29,519-520]*.

An economic analysis is not complete without an assessment of the risk and uncertainty associated with the project. It is beyond the scope of this report to examine various known methods and techniques for treating these issues. For an overview of risk analysis see [45], [47], [52:39-40], and [42:173-236]. Further references are given here according to the four most commonly used risk adjustment techniques as indicated by a 1972 survey by R. J. Wiley [51]: for risk adjusted discount rate approach see [24:118-120], [14:426], and [42:225]; for certainty-equivalent approach see [28:138-139], and [42:249-251]; for sensitivity analysis see [43], [49], [50], and [42:220-224]; and for probabilistic distribution model see [52], [48:23-37], and [42:271-283, 354-377].

1.3 REPORT OBJECTIVES, SUBOBJECTIVES

The management by objectives approach will be used to fulfill the objectives set for this report. Using this approach, higher level objectives are decomposed into lower level subobjectives until the subobjectives are at an easily-managable level. The objective breakdowns are summarized in the objective tree shown in Figure 1.1

The prime objective of this report is to examine the applicability of the IRR and NPV models for economic analysis in income-property acquisition feasibility studies. Economic analysis is only one of

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* This is a reference entry; number of reference is placed before the colon sign while relevant page numbers are placed after the colon.

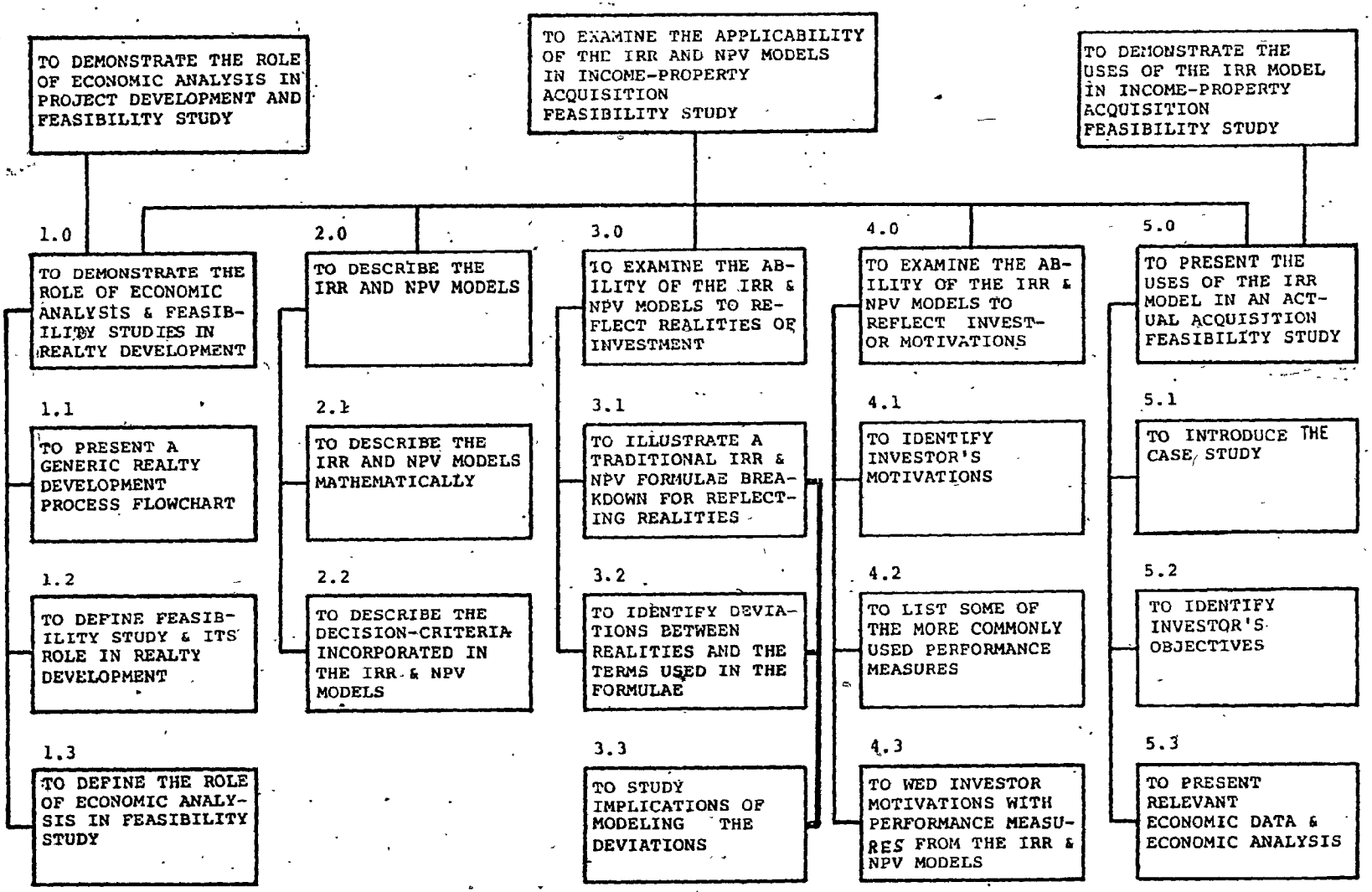


FIGURE 1.1 OBJECTIVE TREE

several major components comprising a feasibility study (see Section 2.2) which in turn is one of the several major activities in the realty development life cycle (see Section 2.1). Hence, before examining the applicability of IRR and NPV models, the question "What is the role of economic analysis in the realty development life cycle?" is addressed. The purpose and scope of economic analysis corresponds to Subobjective 1.0 in Figure 1.1.

The study of the applicability of the IRR and NPV models is treated by answering the following questions:

(1) To what extent are the IRR and NPV models able to reflect the financial and economic realities encountered in acquisition of income-property projects? (Subobjectives 2.0 and 3.0).

(2) To what extent are the IRR and NPV models able to reflect the investor's motivations and resulting goals and objectives? (Subobjective 4.0).

(3) Are IRR and NPV models practical tools for use in an income-property acquisition feasibility study? To demonstrate the practicality of the models, an actual feasibility study using the IRR model is presented. (Subobjective 5.0).

CHAPTER TWO

REALTY DEVELOPMENT LIFE CYCLE
FEASIBILITY STUDY AND ECONOMIC ANALYSIS

The purpose of this chapter is to describe the role of economic analysis in the realty development life cycle. In Section 2.2, components of an unabridged feasibility study and its role in the realty development life cycle are identified. In Section 2.3, the role of economic analysis in a feasibility study is identified. To place in perspective the roles of an economic analysis and a feasibility study, a generic realty development process flow chart is presented in Section 2.1. This flow chart also serves the purpose of providing an overview of the realty development life cycle of the developer-investor.

2.1 A GENERIC REALTY DEVELOPMENT PROCESS FLOW CHART

The generic real estate development process flow chart (Figures 2.1 to 2.11) presented herein is based on the project management concept [36]. Responsibilities of overall coordination and management are assigned to a project management unit which belongs either to an external project management company or an integrated project management department inside the developer-investor's corporation.

To provide an identical mind between the reader and the writer, attempts have been made in clarifying some of the terms used in the flow chart. Asterisks are marked on several terms which are further

explained in Appendix A: Glossary of Terms Used. Several aspects of the flow chart are described herein. Other aspects are elaborated upon in [33], [39], [35] and [36].

The flowchart is presented in the following manner:

(1) The realty development process from the viewpoint of the developer investor and project manager is divided into five phases, namely (Figure 2.1):

phase A : predevelopment phase,

phase B : development phase,

phase C : construction phase,

phase D : operation phase, and

phase E : termination phase.

(2) Except for the predevelopment and operation phases, each phase is subdivided into stages according to its degree of complexity and the scope of work involved. For instance, construction phase (phase C) is subdivided into construction stage (stage C1) and project acceptance stage (stage C2).

(3) The first and second level breakdown of the development process into phases and stages is presented in Figure 2.1.

(4) A third level breakdown of the stages into activities is presented in Figures 2.2 to 2.11. For example, Identify Problem or Opportunity is denoted as Activity A1.3 on Figure 2.2, where the activity number indicates that the activity is in stage A1, project initiation stage.

(5) Decision points throughout the flow chart are indicated by diamond-shaped squares. All decision points, except Activities B1.15, D1.10 and E2.6, indicate that approvals

should be obtained from the developer-investor or his authorized representative. The addition and deletion of decision points depends on the control system used as well as the degree of control desired (see control by decision, [37:16-19]).

(6) Most of the decision points have a feedback arrow associated with instructions such as recycle, halt or stop as required. The feedback paths are indicated by dotted lines on Figure 2.1. The exact path of feedback or backward flow depends on the nature of the problem(s) encountered at the decision point.

Two points should be noted here. Firstly, the precedence relationships depicted in the flow chart are not rigid. For example, Activity B1.14: Financial/Economic Analysis cannot be completed without first completing its precedent activity, Activity B1.13: Scheduling, but good scheduling requires feedback from financial/economic analysis; thus they form a kind of looping relationship. Melvin Silverman commented on the precedence relationship between phases in the development life cycle as follows:

> "There is no clear point in time when one phase starts and another stops...The phase-in and phase-out procedures are quite vague, and it is not unusual to find all...phases in continuous and simultaneous existence from project beginning to end." [41:14]

In fact, the need to compress the design, development and production cycle is one of the major reasons for the increased application of project management techniques in recent decades (see [41:10]). To achieve this time compression, activities and development phases must be overlapped.

The second point to be stressed here is that different activities, processes, and development activity sequences are required for different kinds of projects; (see for example the activities required for the heavy civil engineering and agriculture project in [39]). Also, different viewpoints and different companies may find that different development processes benefit them best.

In Figure 2.12, a development process flow chart used by a large Canadian development corporation is presented to illustrate the similarities between actual practice and the generic flow chart presented herein. The developer's flowchart allows each functional department's responsibilities in each development stage to be shown. However, the sequence of the activities in each stage is not shown. Perhaps the choice of this presentation format reflects the recognition that precedence relationships between some activities are not rigid, as stated earlier.

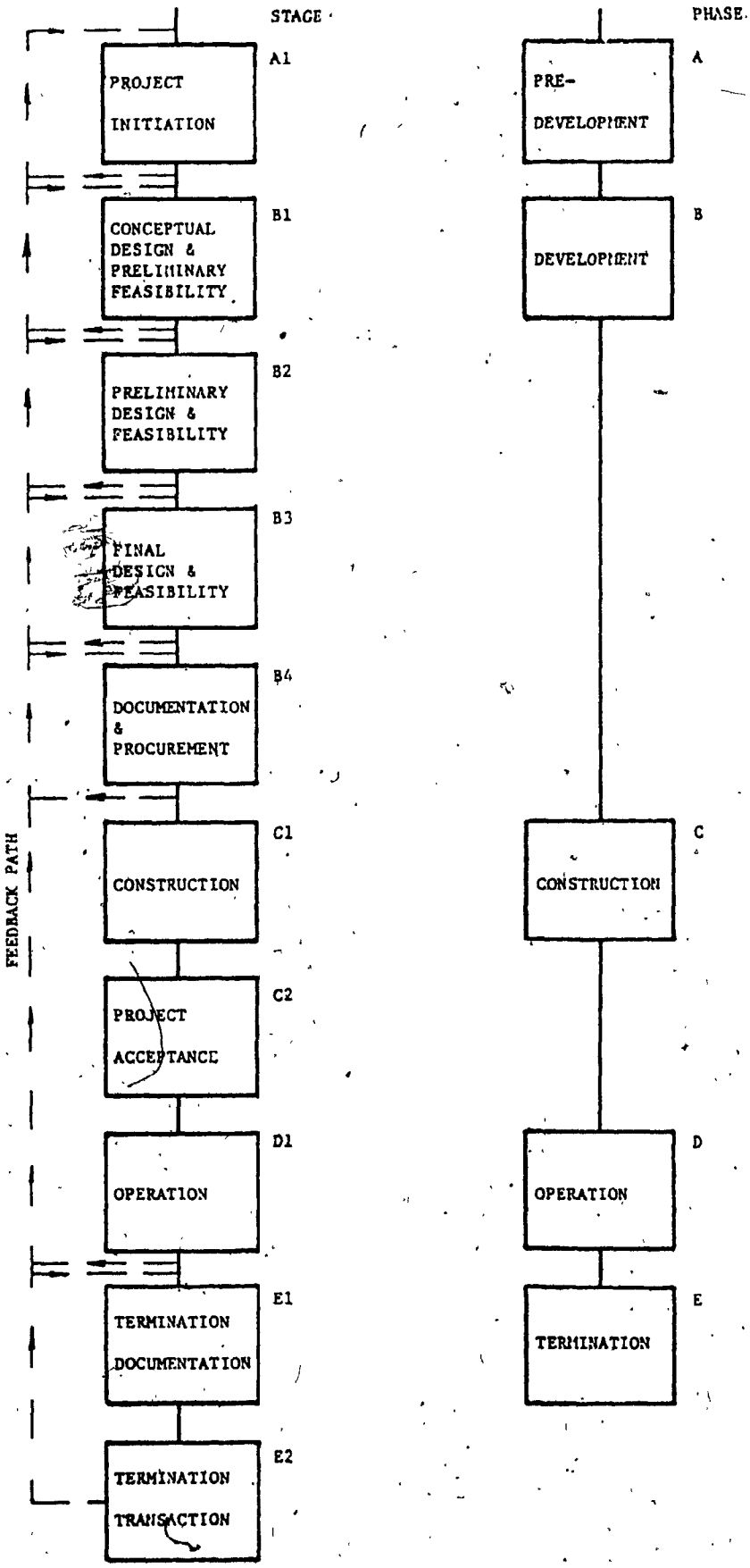


FIGURE 2.1 A GENERIC REALTY DEVELOPMENT PROCESS FLOWCHART (FIRST AND SECOND LEVEL BREAKDOWN)

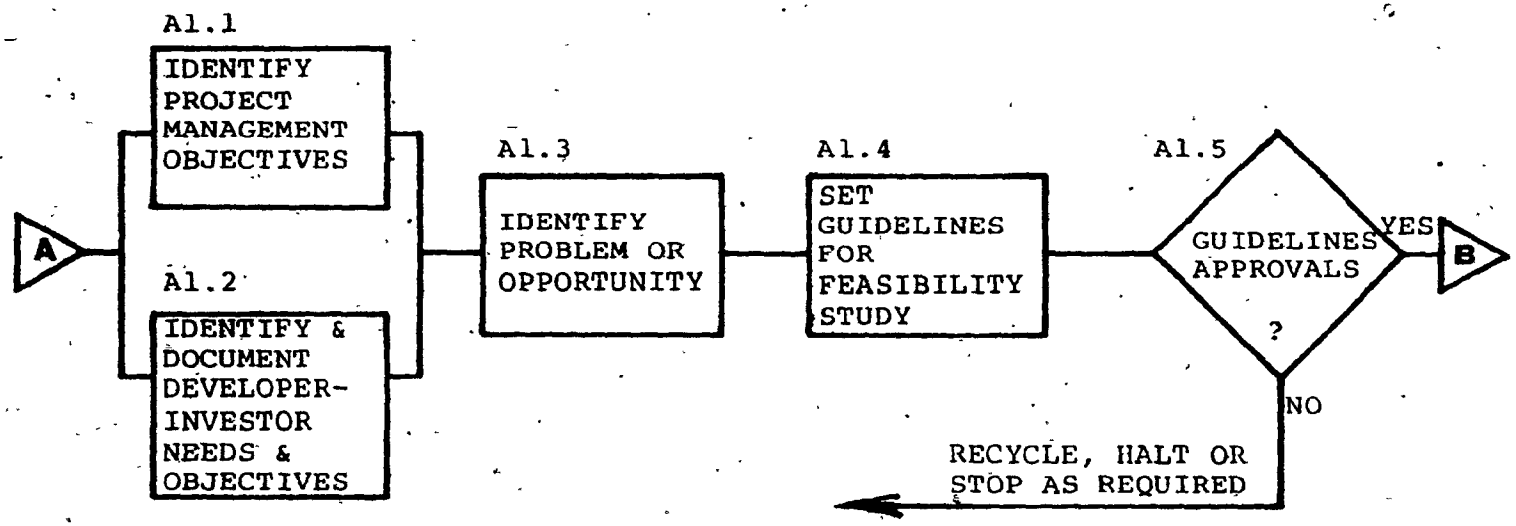


FIGURE 2.2 STAGE A1: PROJECT INITIATION

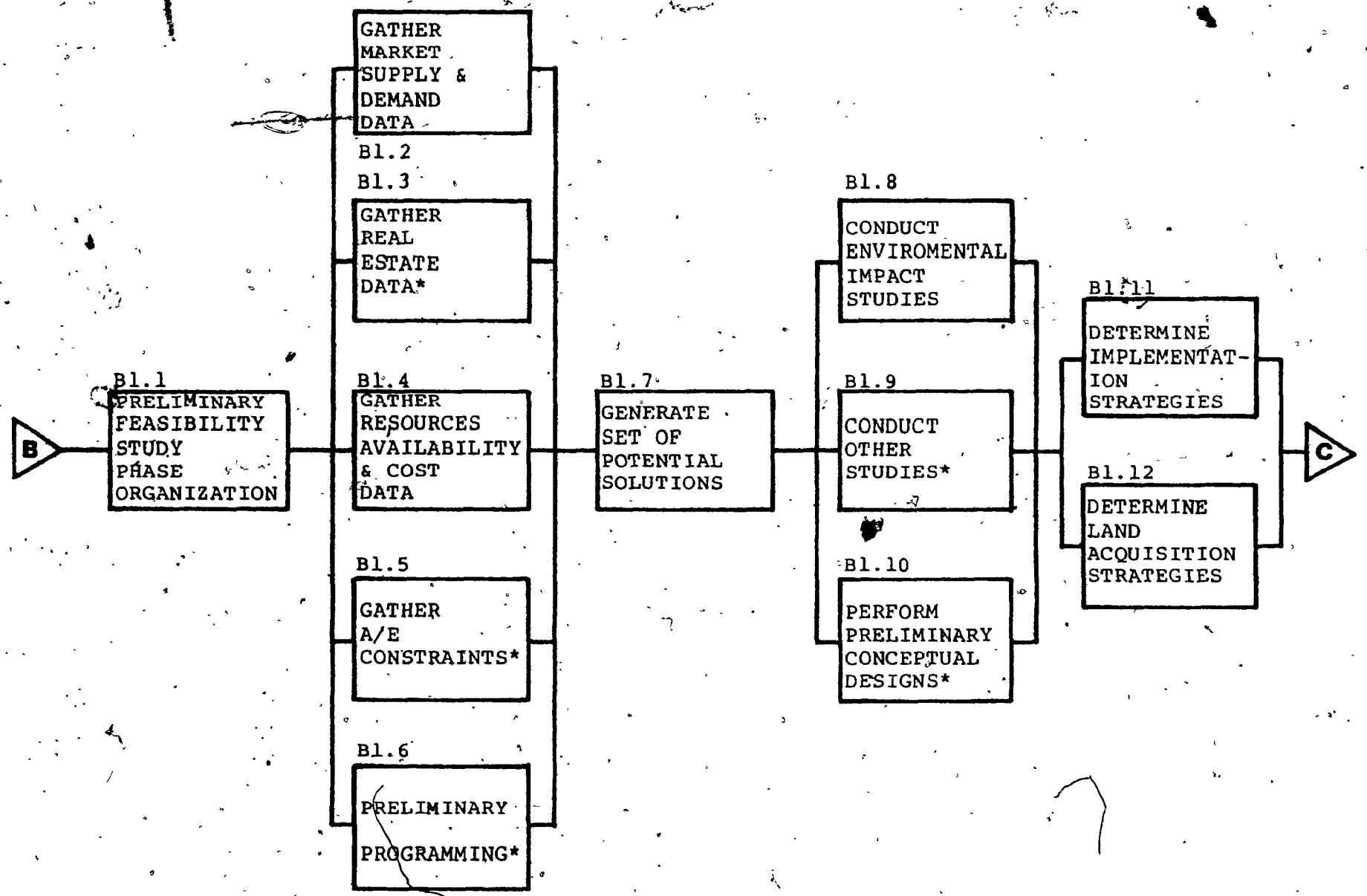


FIGURE 2.3A STAGE B1: CONCEPTUAL DESIGN & PRELIMINARY FEASIBILITY (CONT.)

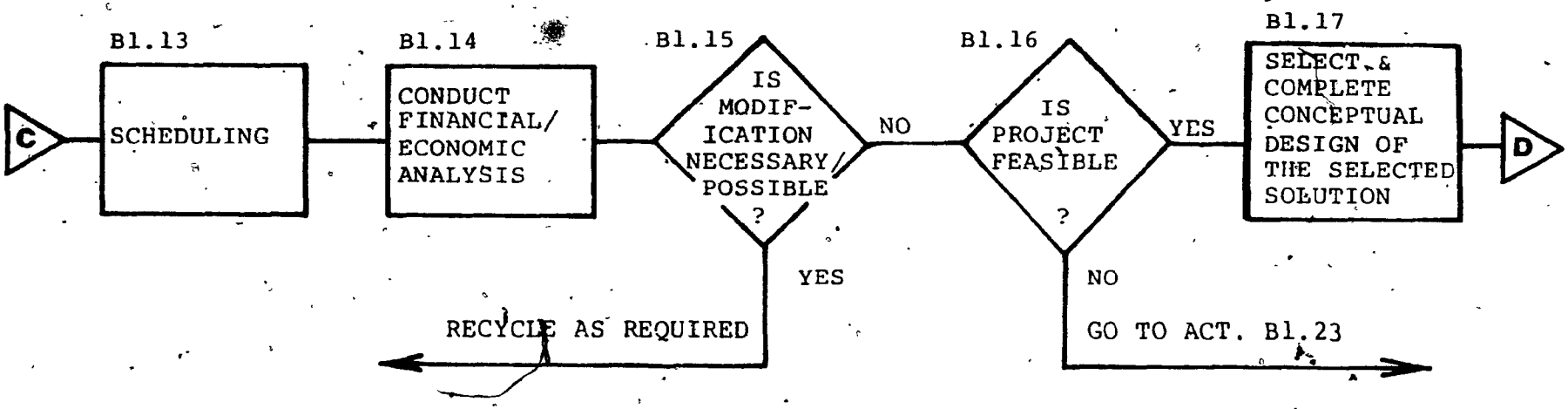


FIGURE 2.3B STAGE B1: CONCEPTUAL DESIGN & PRELIMINARY FEASIBILITY (CONT.)

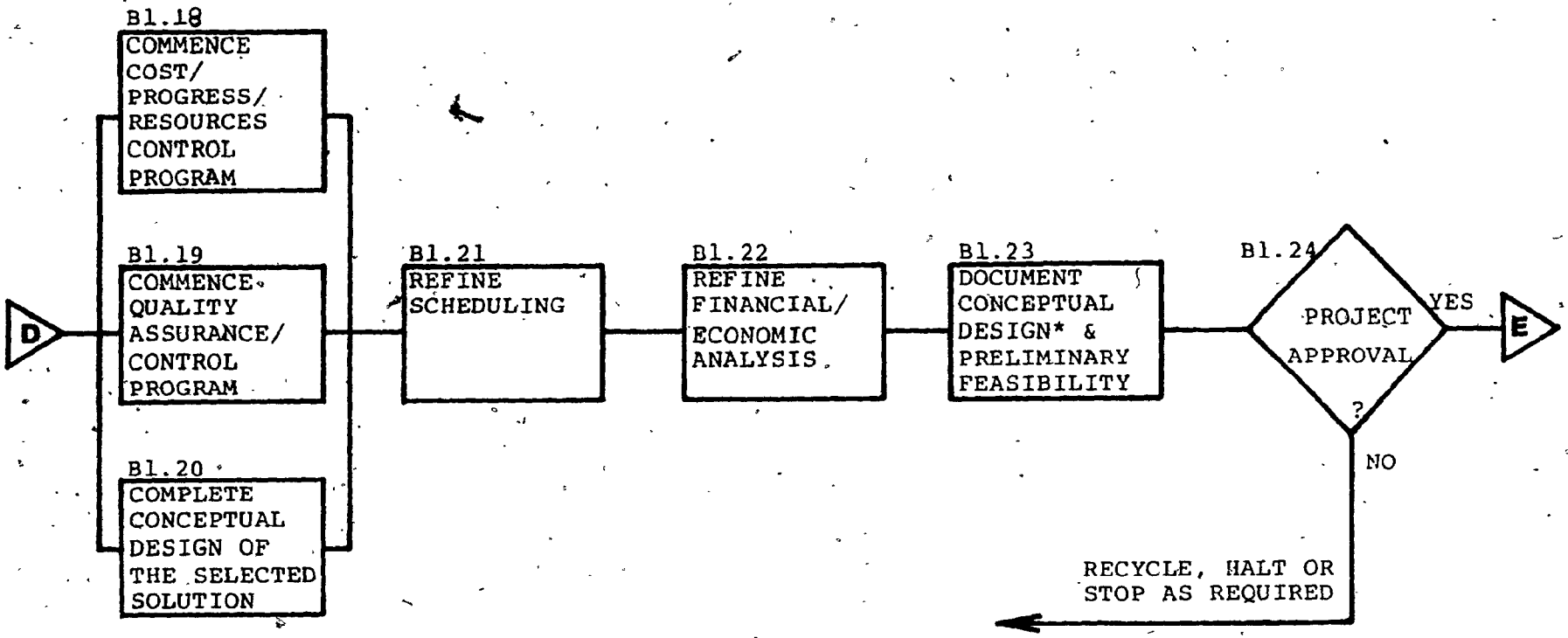


FIGURE 2.3C STAGE B1: CONCEPTUAL DESIGN & PRELIMINARY FEASIBILITY

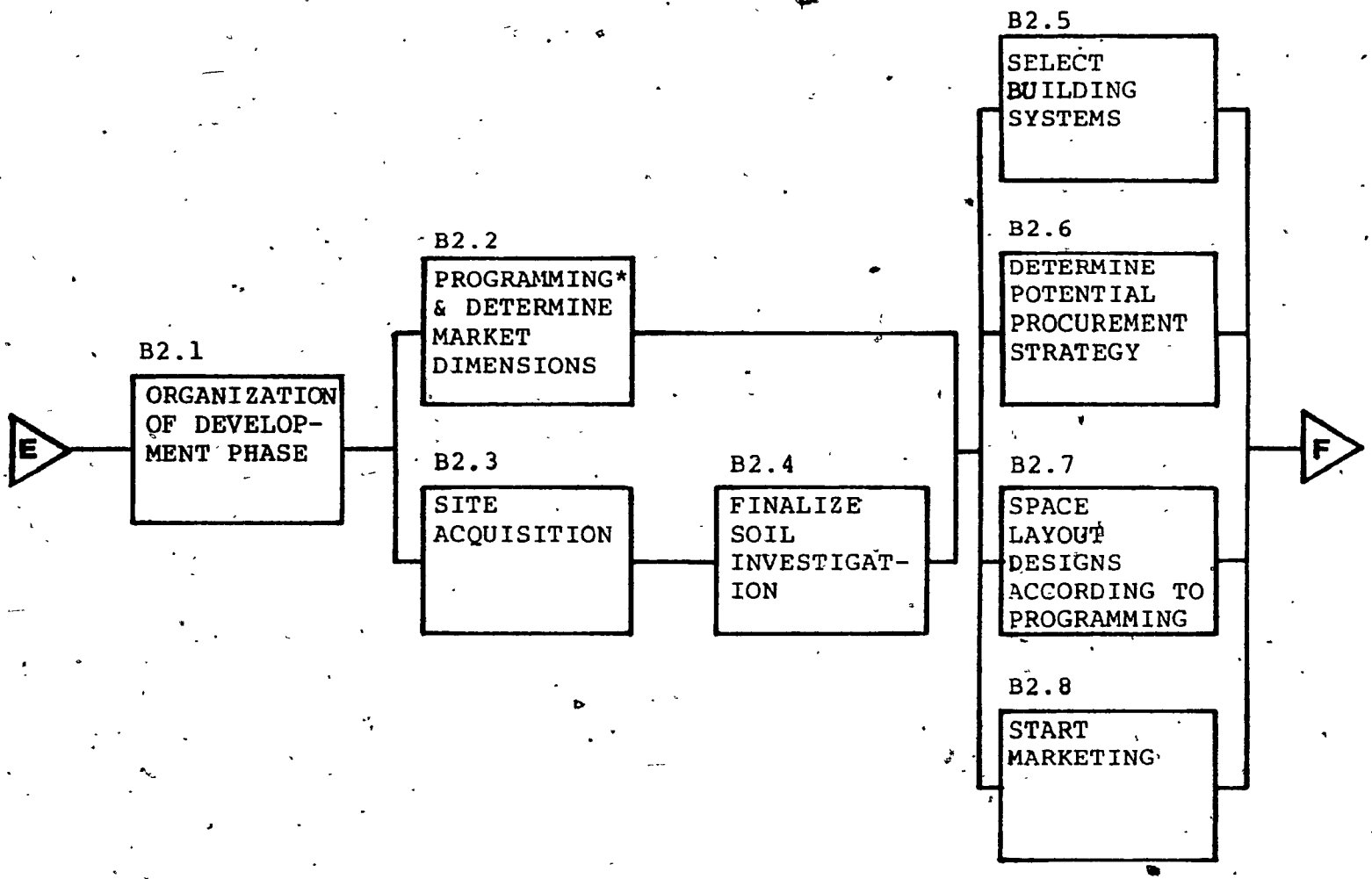


FIGURE 2.4A STAGE B2: PRELIMINARY DESIGN* & FEASIBILITY (CONT.)

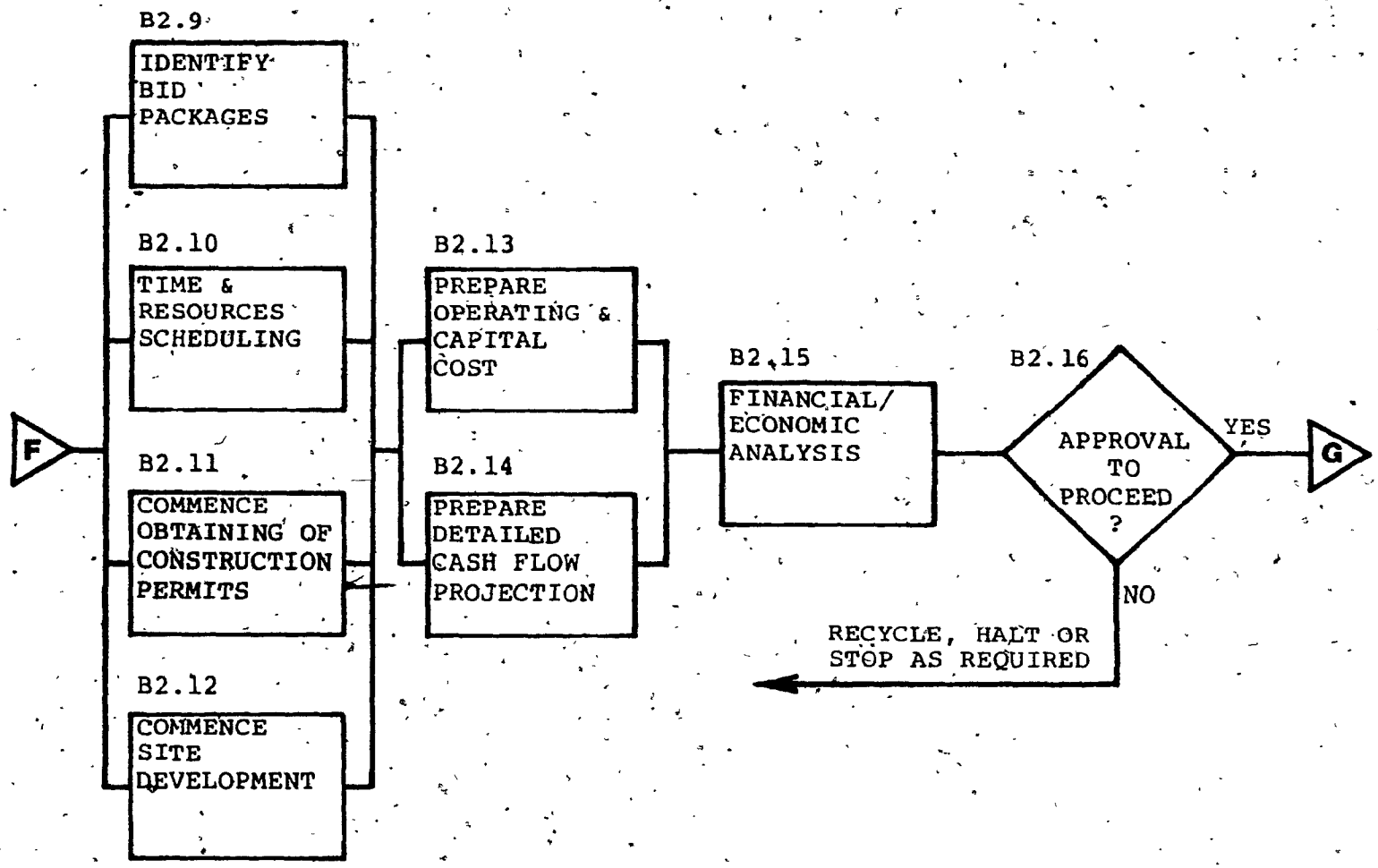


FIGURE 2.4B · STAGE B2: PRELIMINARY DESIGN & FEASIBILITY

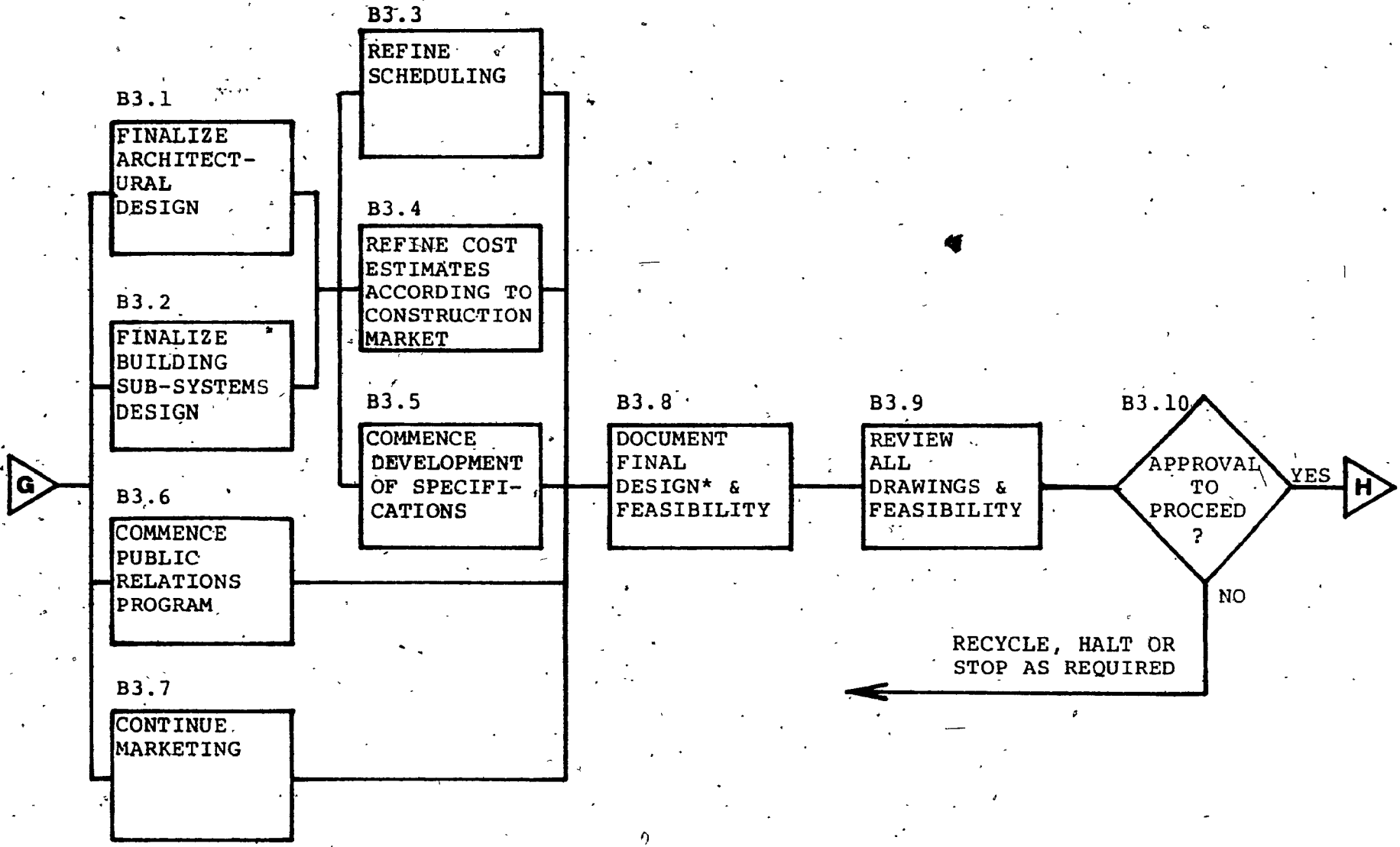


FIGURE 2.5 STAGE B3: FINAL DESIGN* & FEASIBILITY

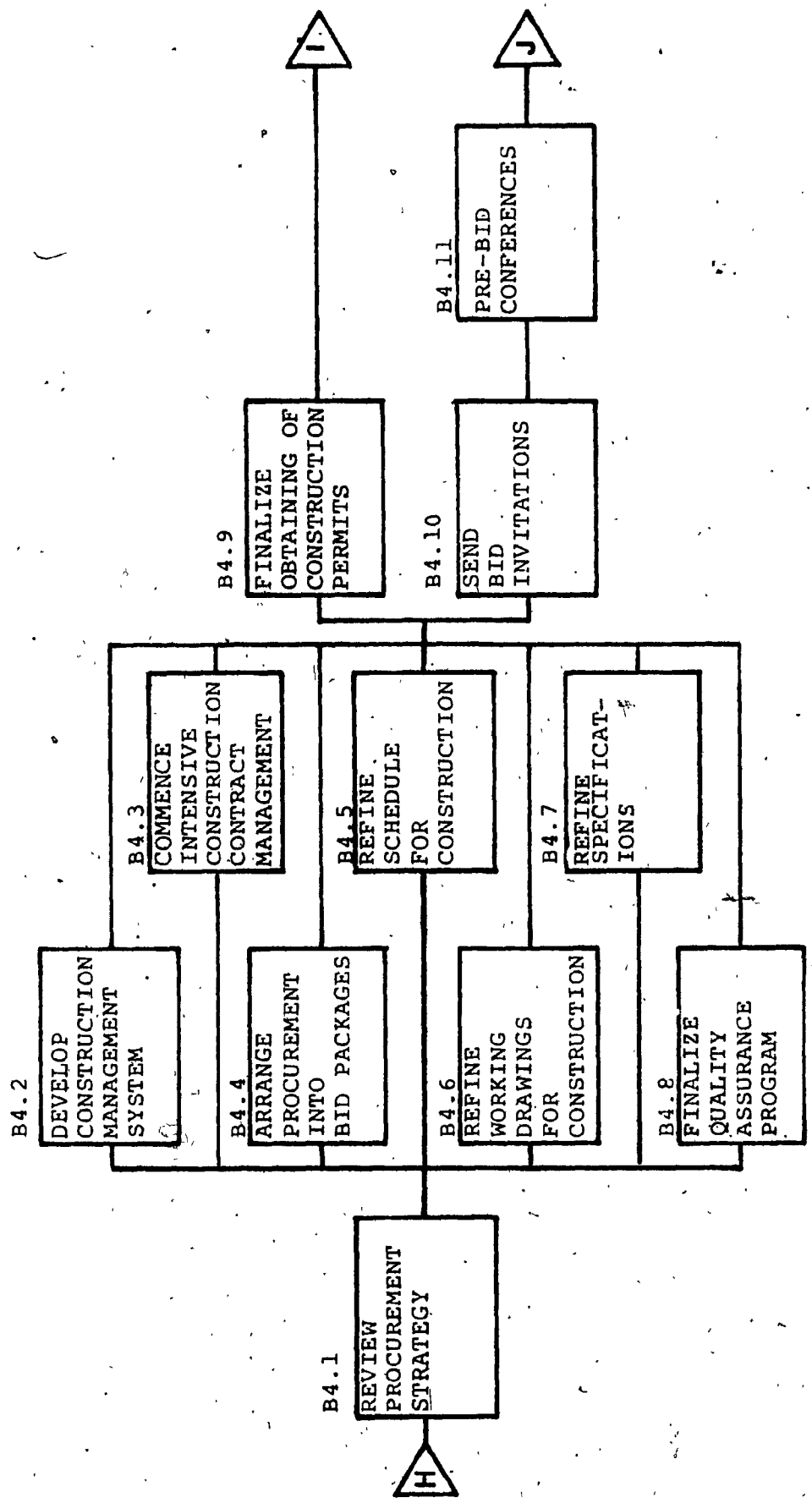


FIGURE 2.6A STAGE B4: DOCUMENTATION & PROCUREMENT

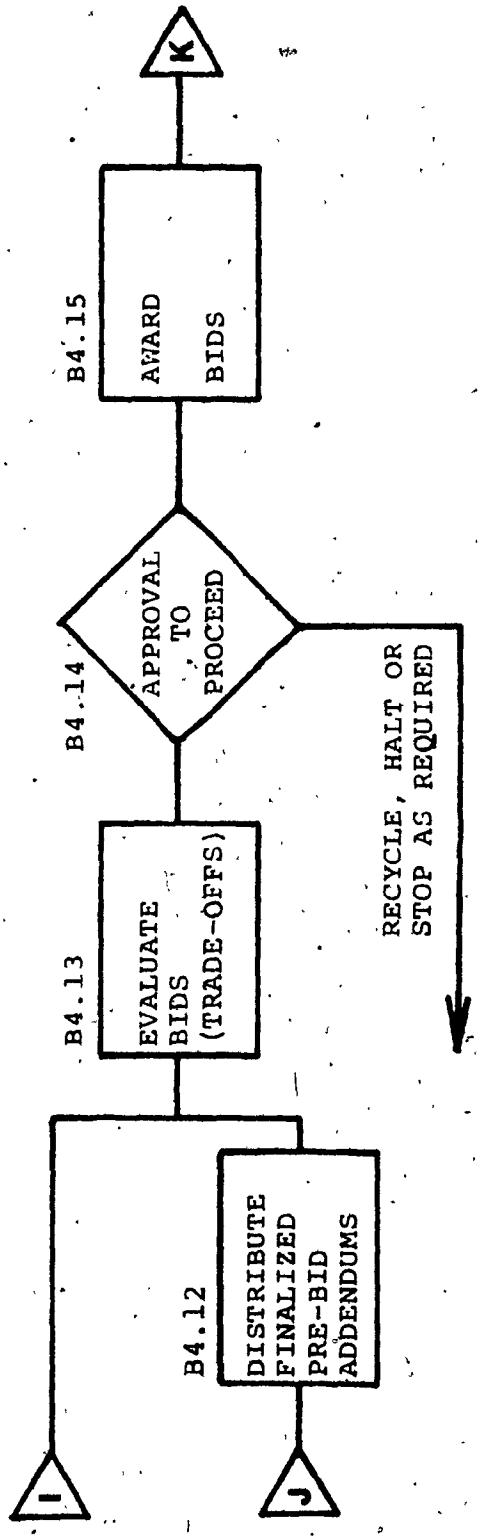
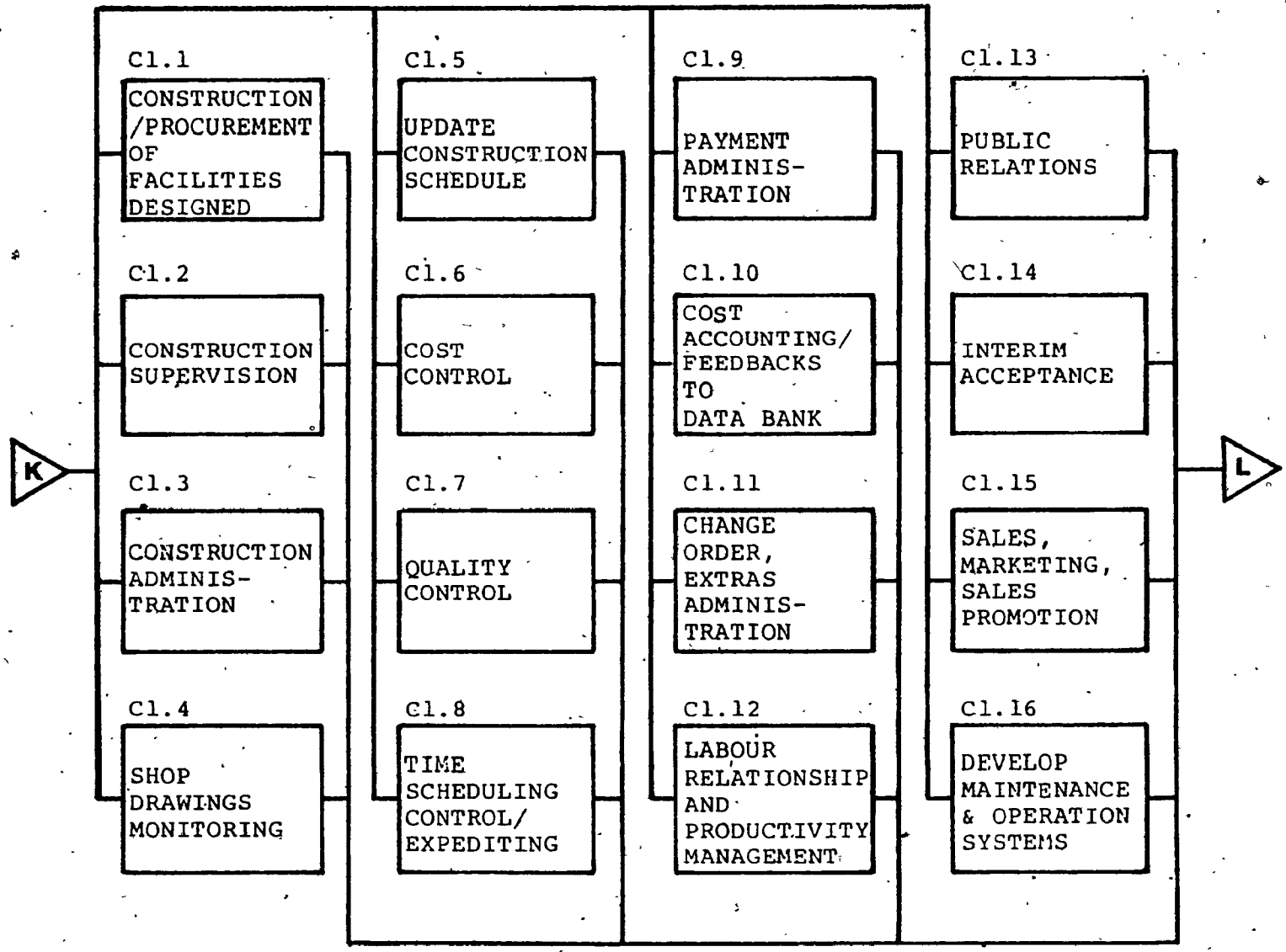


FIGURE 2.6B STAGE B4: DOCUMENTATION & PROCUREMENT



Activities in this stage are simultaneous.

FIGURE 2.7 STAGE C1: CONSTRUCTION

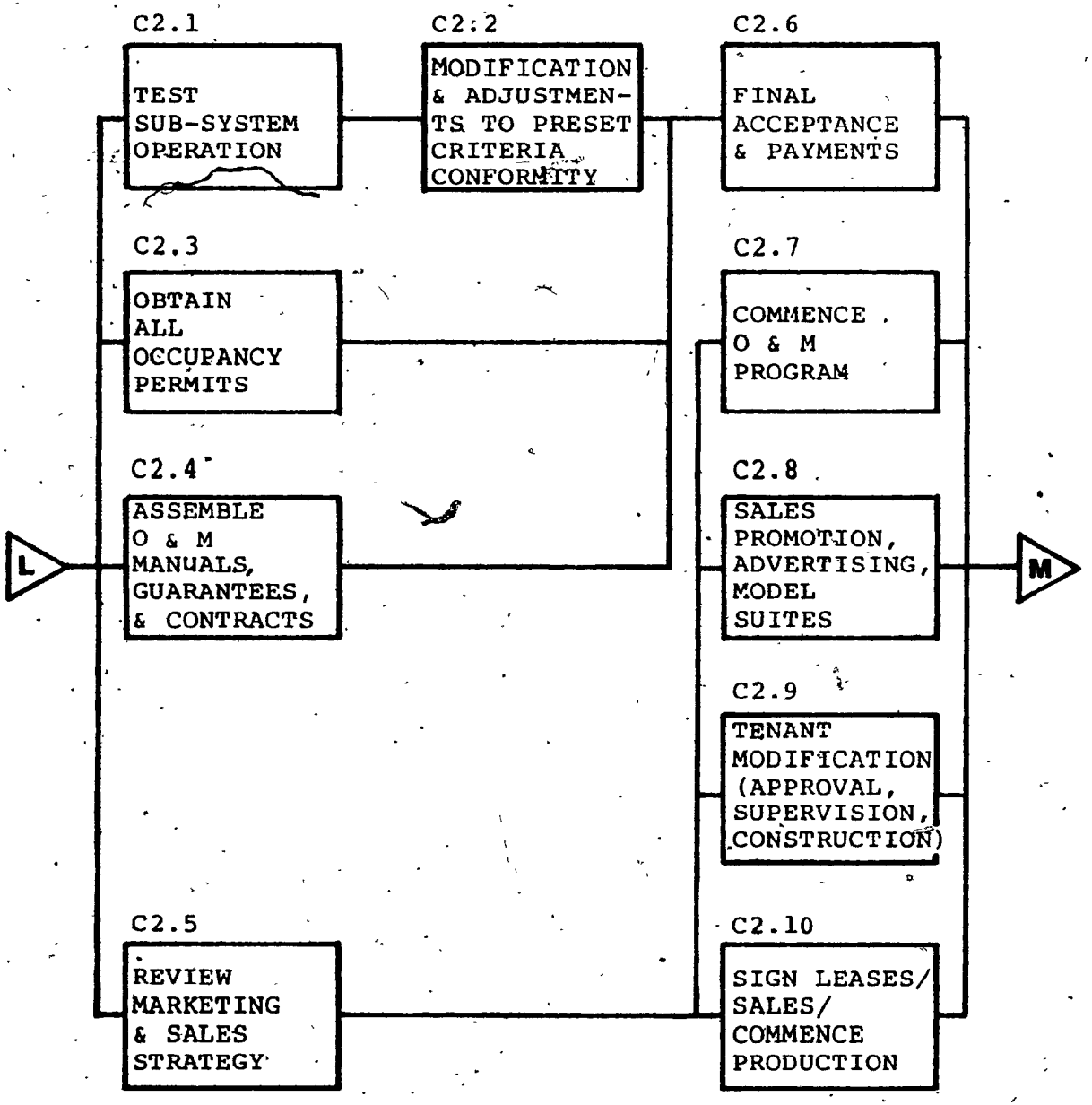


FIGURE 2.8 STAGE C2: PROJECT ACCEPTANCE

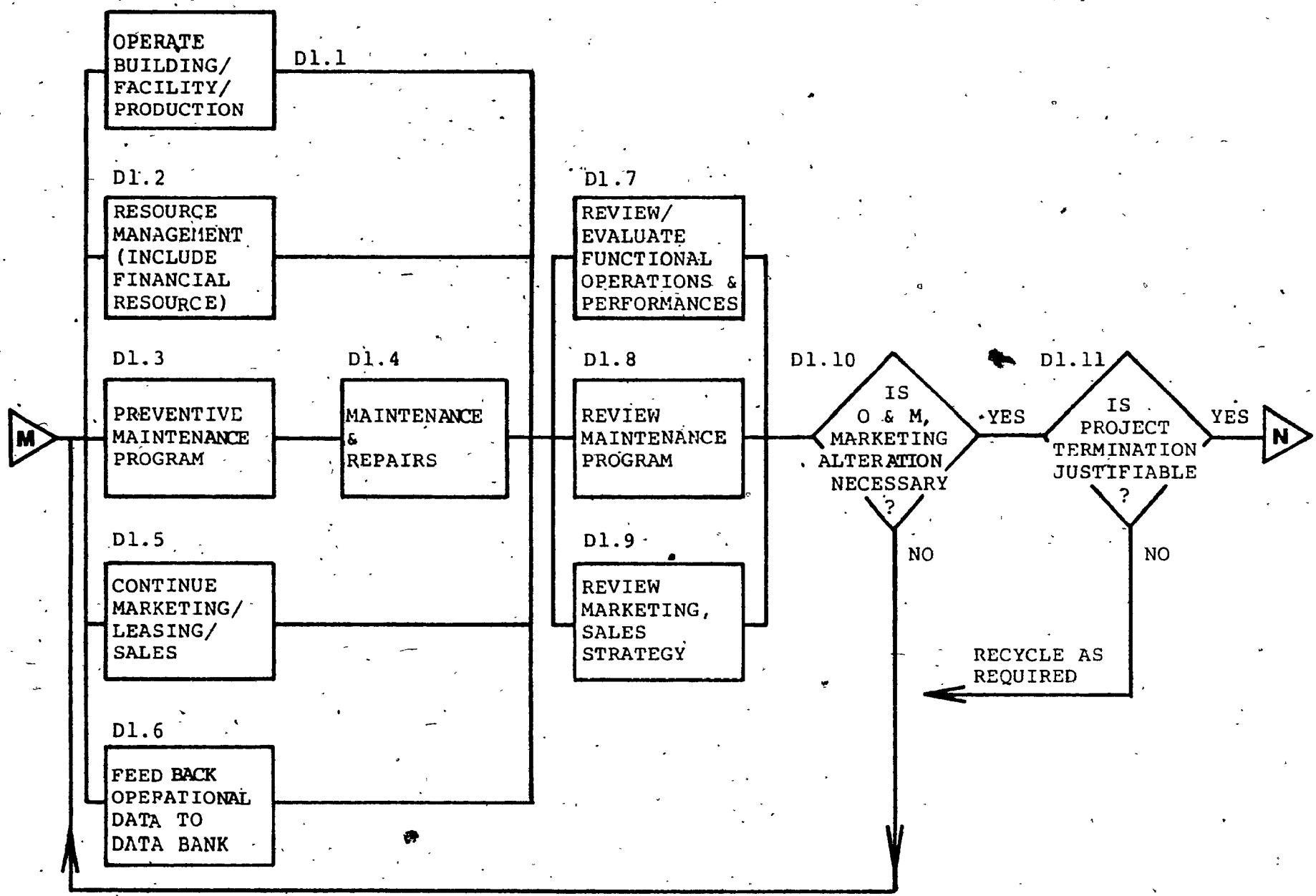


FIGURE 2.9 STAGE D1: OPERATION

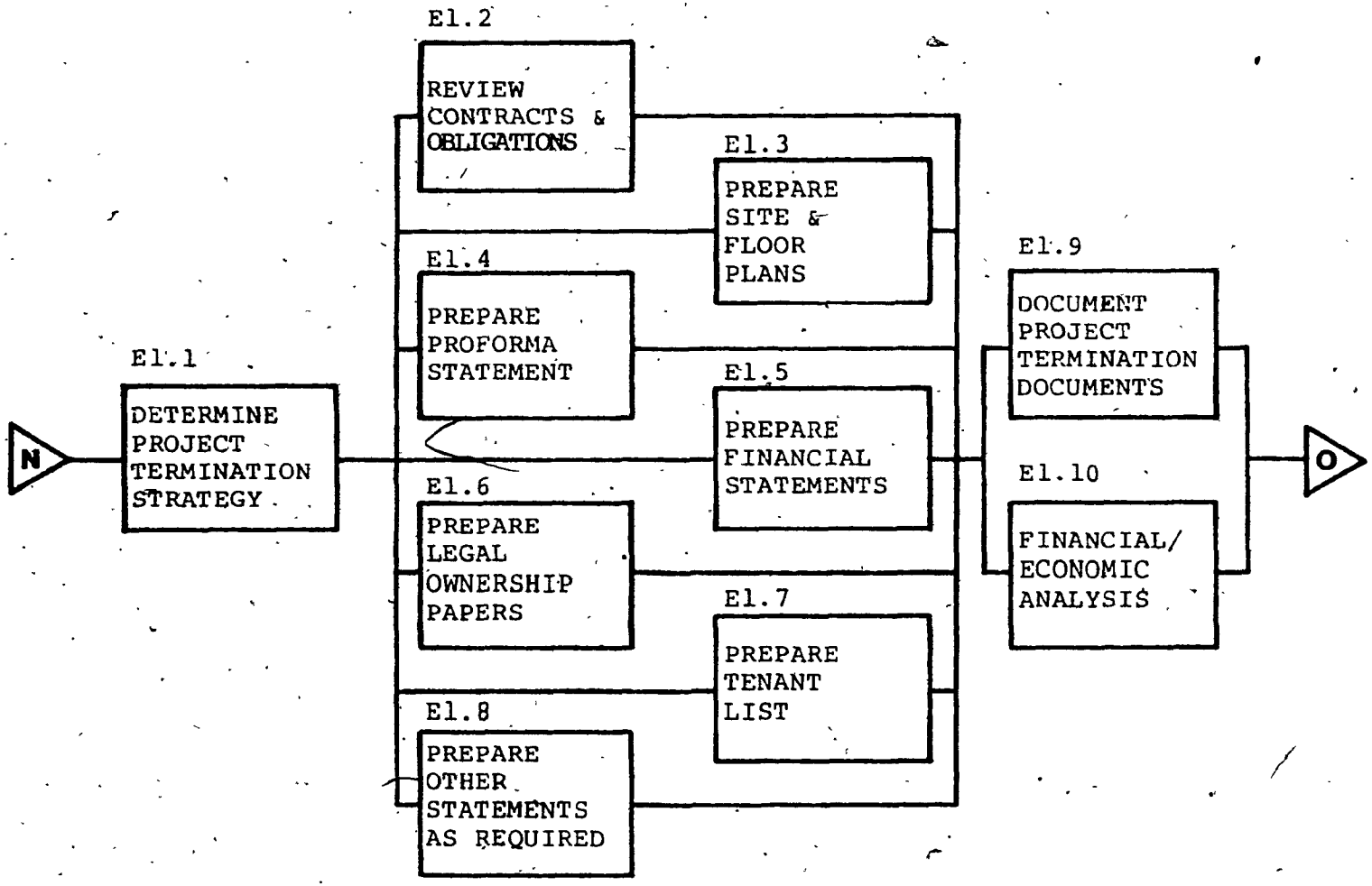


FIGURE 2.10 STAGE E1: TERMINATION DOCUMENTATION

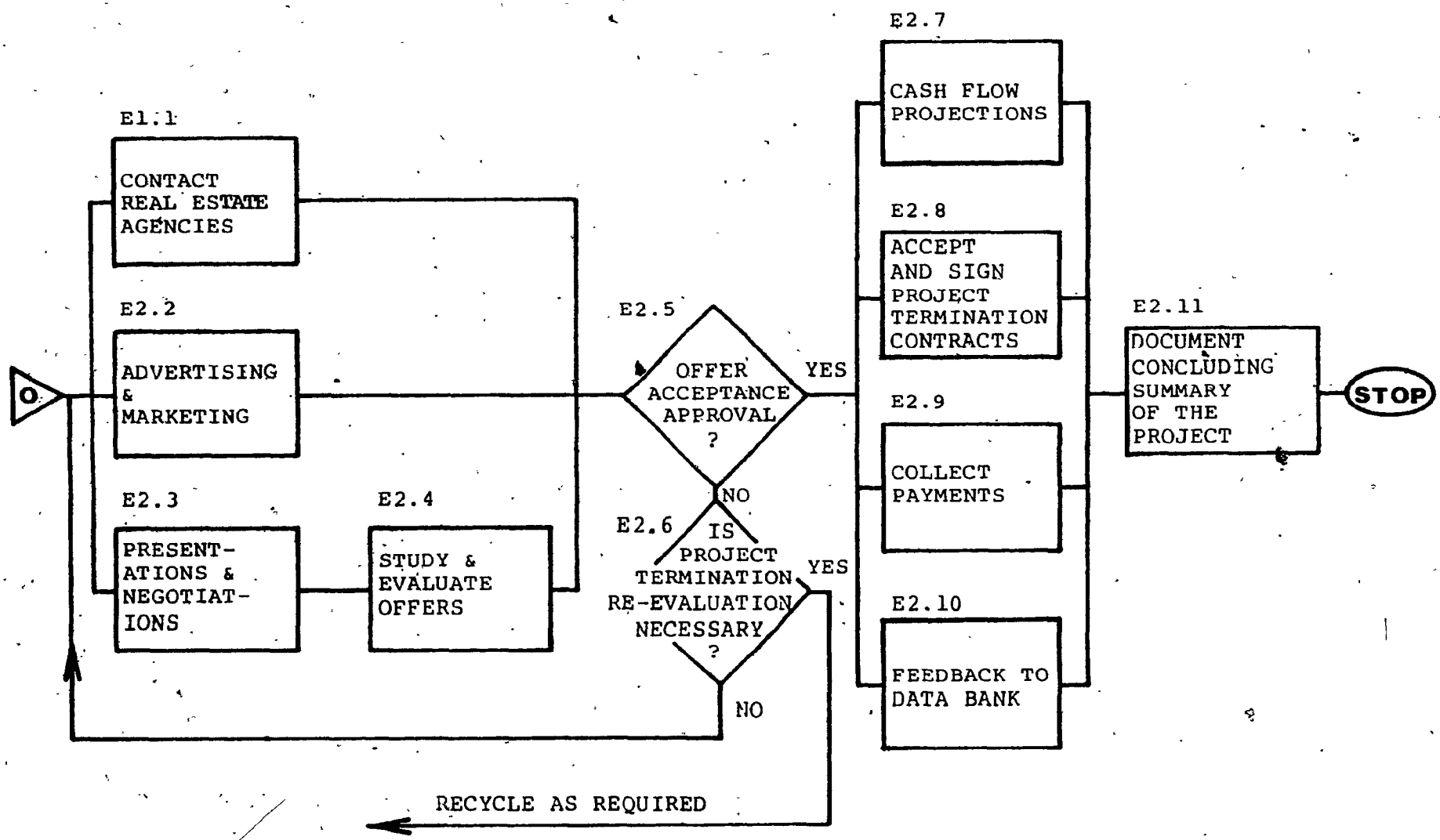


FIGURE 2.11 STAGE E2: TERMINATION TRANSACTION

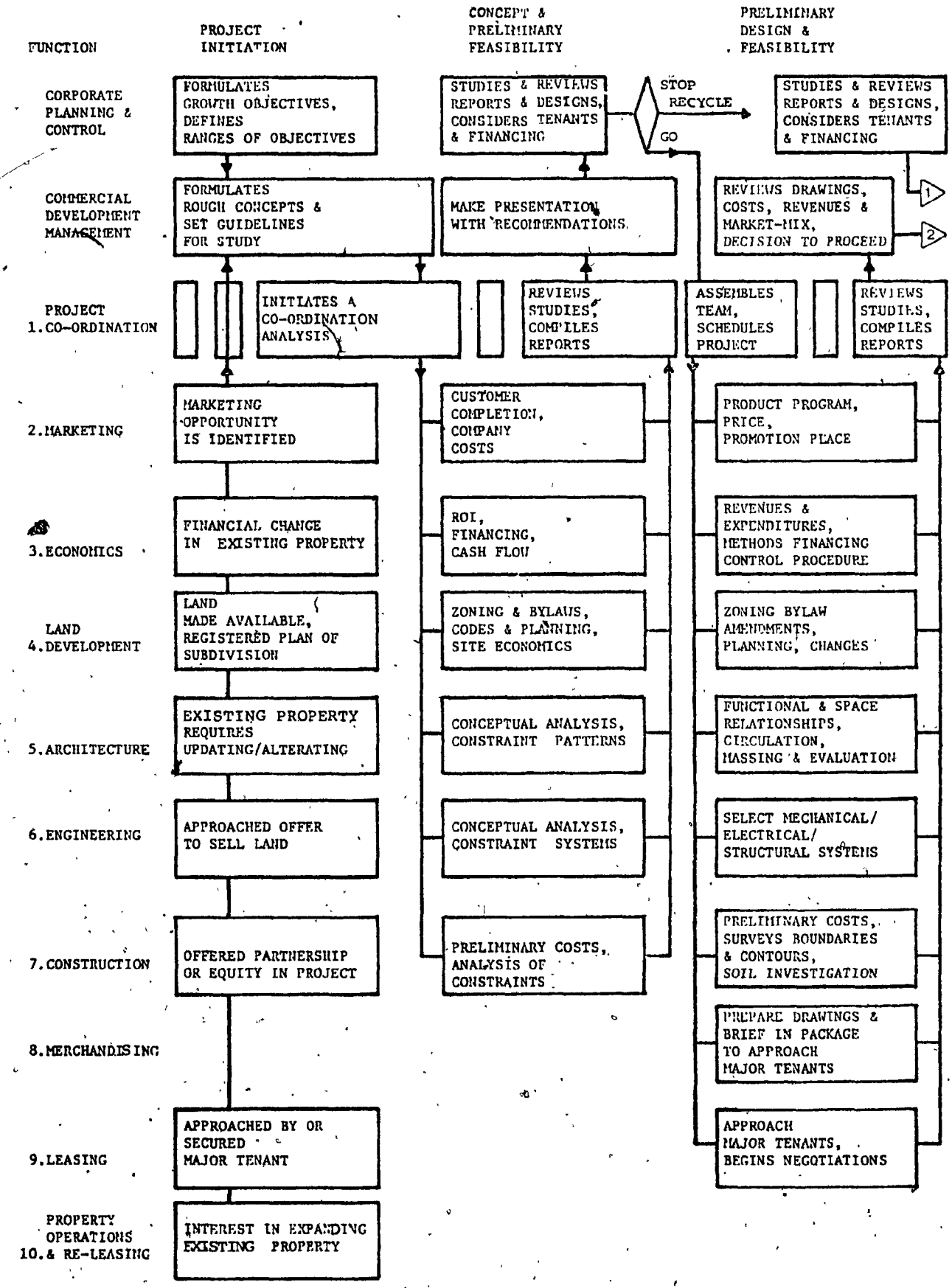


FIGURE 2.12A DEVELOPMENT PROCESS FLOWCHART OF A CANADIAN DEVELOPER (CONT.)

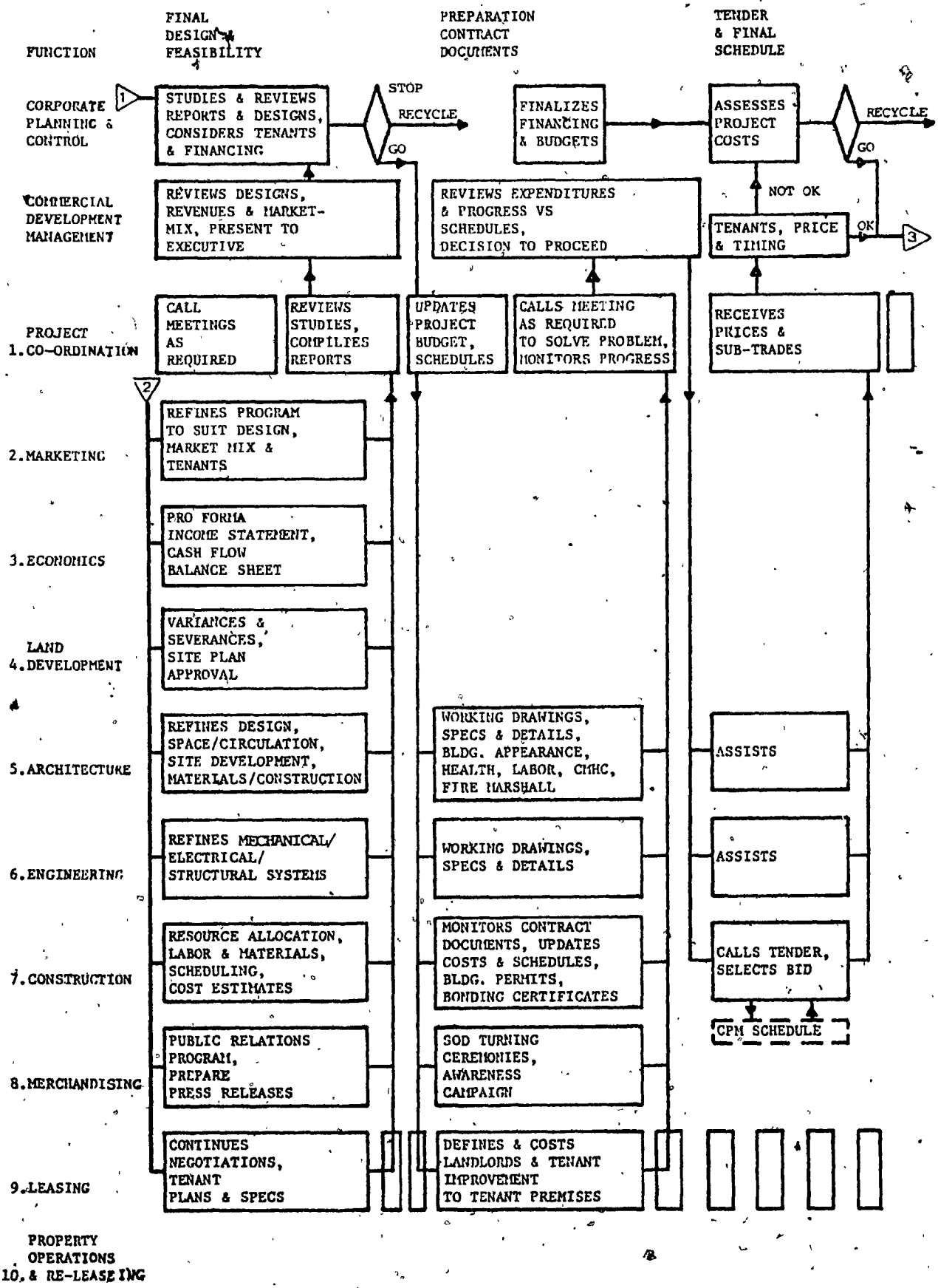


FIGURE 2.12B DEVELOPMENT PROCESS FLOWCHART OF A CANADIAN DEVELOPER (CONT.)

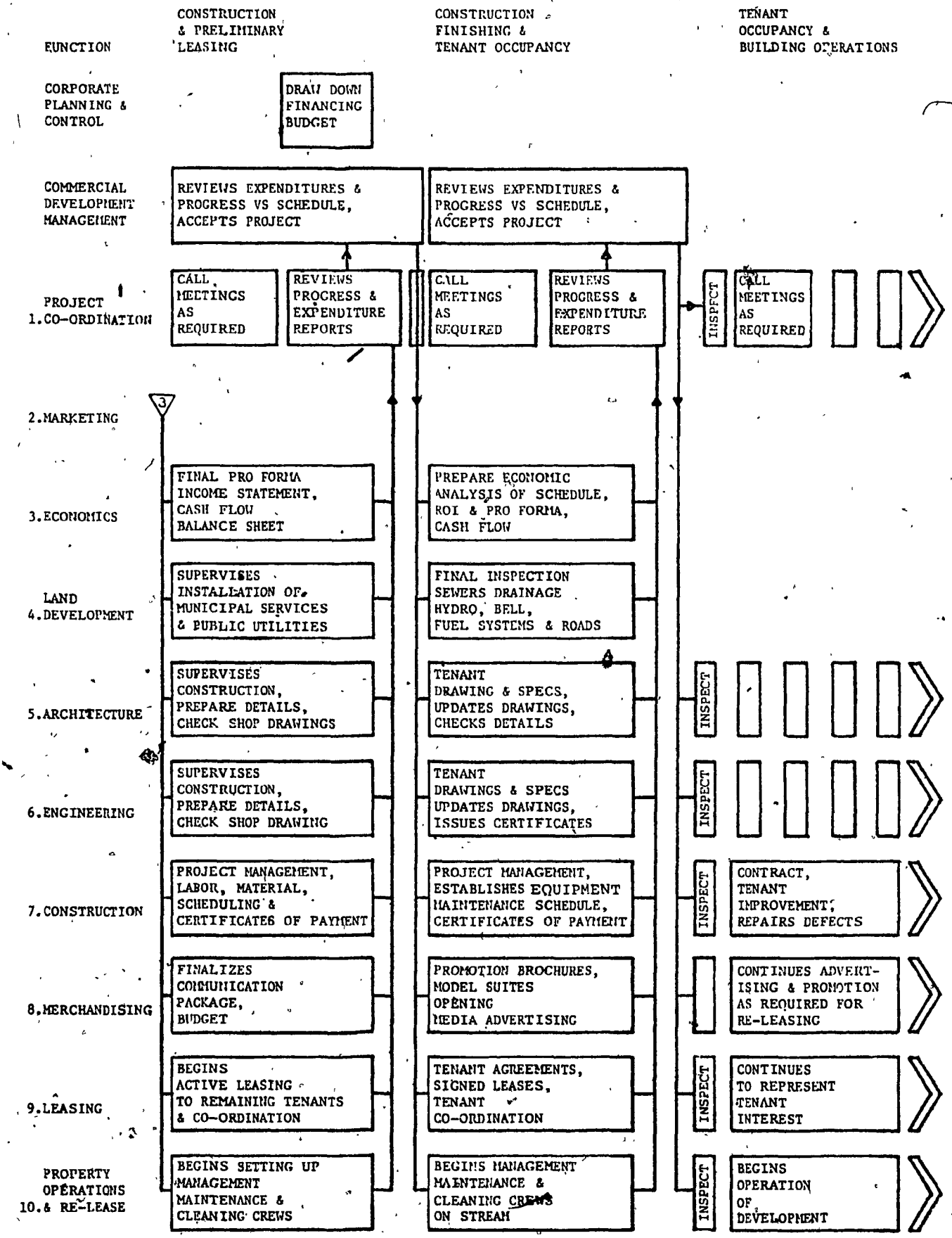


FIGURE 2.12C DEVELOPMENT PROCESS FLOWCHART OF A CANADIAN DEVELOPER

2.2 CONTENT OF FEASIBILITY STUDY AND ITS ROLE IN REALTY DEVELOPMENT

In general, decisions at an earlier phase will have a larger impact on a project's productivity than those at a later phase. The feasibility study, which guides most of the decisions at the very front end, spans over Phase A: Pre-Development Phase and Phase B: Development Phase of a project's development. The contents of an unabridged feasibility study can be classified into four categories; namely, objective identification, market analysis, constraint analysis and financial/economic analysis. These four components of a feasibility study and their role in realty development are described below.

2.2.1 OBJECTIVE IDENTIFICATION

Objective Identification refers to the identification and documentation of objectives and needs of the developer-investor and project user. Objective Identification starts at Stage A1: Project Initiation Stage. First, Activity (Act.) A1.2 is conducted to identify and document the developer-investor's objectives and needs. These are then blended with outputs from Act. A1.1 and A1.3 to form guidelines for the feasibility study in Act. A1.4. As more information becomes available, the key objectives of the developer-investor which are identified in the Project Initiation Stage can be refined in later stages for purposes of detailed design, making tradeoffs and resolving conflicts between objectives. These refinements can be made in Act. B1.6: Preliminary Programming and Act. B2.2: Programming.

2.2.2 MARKET ANALYSIS

Market analysis can be subdivided into macro-market and micro-market analysis. Macro-market analysis deals with statistical

surveys, market trends analysis and demand and supply projections as they relate to the objectives of the developer. Micro-market analysis may include market price and market rent analysis, competition analysis, market-mix determination, absorption rate studies and programming for one or more alternative sites and building concepts.

Market analyses are performed differently in each of the following three stages. The first stage of market analysis starts after guidelines for a feasibility study are set (Act. A1.4). Market data gathering (Act. B1.2) and preliminary programming (Act. B1.6) are performed to generate potential project solutions (Act. B1.7). Usually the market analyses at this stage are mostly macro-market analyses and are quite general in nature.

The second stage of market analysis focuses on the alternatives generated in the first stage. Act. B1.2 and B1.6 continue with Act. B1.3: Gather Real Estate Data to provide information for assessing project feasibility (Act. B1.16) and selecting the most appropriate project solution (Act. B1.17).

After the field of study is narrowed down to the selected project solution, the third stage of market analysis commences. The three data gathering activities, Act. B1.2, B1.3 and B1.6 carry on, and are documented in Act. B1.23 before a conceptual design is presented for approval (Act. B1.24). But market analysis does not stop here. Its role continues in other phases of the whole development life cycle. For example, it can be used to assist the determination of market dimensions (Act. B2.2) and marketing and sales strategy (Act. D1.9).

2.2.3 CONSTRAINT ANALYSIS

Constraint Analysis can be subdivided into financial constraint analysis and A/E constraint analysis. Financial constraint analysis may include analysis of lender-investor's objectives and information gathering and preliminary negotiation on such financial issues as interest rate, financial terms and money availability. A/E constraint analysis includes:
(a) legal-political and aesthetic-ethical constraint analysis which may include studies of legal opinions, public opinion, municipal by-laws, environmental impact, and socio-economic impact; and
(b) physical and technical constraint analysis which may include soil investigation, engineering studies, architectural schematics, site suitability studies, alternative building system studies, traffic analysis, survey on utilities and facilities, and surveys on existing and surrounding structures.

The degree of detail of constraint analysis increases as the field of study is narrowed by market analysis. Constraint analysis starts its data gathering in Act. B1.3, B1.4, B1.5, and B1.6. Then the findings of the analysis are used to set up the scope of studies to be performed in Act. B1.11 and B1.12, as well as the scope of designs to be determined at different development stages.

Conclusions from a Constraint Analysis can impact tremendously on both scheduling and strategy determination throughout the whole development phase. For example, they influence:

(1) scheduling at

(a) Act. B1.13 and B1.21 in Stage B1,

(b) Act. B2.10 in Stage B2, and

(c) Act. B3.3 in Stage B3; and

(2) strategy determination at

(a) Act. B1.11 for implementation strategy,

(b) Act. B1.12 for land acquisition strategy,

(c) Act. B2.6 for procurement strategy,

(d) Act. B2.8 and B3.7 for marketing strategy, and

(e) Act. B3.6 for public relations strategy.

2.2.4 FINANCIAL/ECONOMIC ANALYSIS

Financial Economic Analysis may include all kinds of cost analyses, cash flow forecasts, income tax impact analysis, risk analysis, and assessment of project performance (e.g. return on equity). The purpose of these analyses is to provide sufficient financial and economic information to permit key decision to be taken in different development stages. To achieve this goal, inputs from Objective Identification, Market Analysis, Constraint Analysis, scheduling and A/E designs are required.

The first financial/economic analysis is performed in Act. B1.14 and provides financial and economic information for the testing of project feasibility (Act. B1.16) and the selection of project solution (Act. B1.17). After conceptual design is completed, the second financial/economic analysis (Act. B1.22) is performed and documented (Act. B1.23) for conceptual design and preliminary feasibility study approval (Act. B1.24). The third financial/economic analysis (Act. B2.15) is performed for preliminary design and feasibility

approval (Act. B2.16), after detailed cash flow, and capital cost projections are made available (Act. B2.14 and B2.13). The analysis is then updated for final design and feasibility approval (Act. B3.10).

Financial/economic analysis does not stop here, however. The analysis should be updated as more information becomes available and/or when one or more inputs or assumptions change substantially. For example, at bid evaluation (Act. B4.13), the impact on project productivity of a change in capital cost should be fully assessed before a decision is made to proceed with development (Act. B4.14).

2.3 ROLE OF ECONOMIC ANALYSIS IN FEASIBILITY STUDY

The major purposes of economic analysis ina feasibility study are:

(1) to analyze the conclusions obtained from the activities, Objectives Identification and Market and Constraint Analysis in terms of their impact on project performance and project feasibility (e.g. Act. B1.14, B1.15 and B1.16);

(2) to choose an optimal feasible proposal (Act. B1.17); and

(3) to provide economic data to facilitate tradeoffs in the design, planning and procurement of subsystems.

James Graaskamp, clarified the concept of project feasibility in which economic analysis plays a key role as follows:

> "A real estate project is feasible when the real estate analyst determines that there is a reasonable likelihood of satisfying explicit objectives when a selected course of action is tested for fit to a context of specific constraints and limited resources." [34:51]

By carefully examining the sentence "there is a reasonable

likelihood of satisfying explicit objectives", the context and the role of economic analysis can be explored.

Graaskamp further clarified the concept of likelihood used in his definition as follows:

> "Likelihood requires explicit recognition that economic forecasting must treat risk explicitly and therefore does not produce a single point conclusion but rather involves a statement of reasonably foreseeable alternative outcomes, a range of possible results." (op. cit.)

This clarification underscores the importance of conducting a risk analysis as part of the economic analysis. The IRR and NPV models examined in this report, are usually considered as deterministic models. They can be considered as stochastic models, however, which permits their use in formally quantifying the risks inherent in a project. Such an advanced treatment is not considered herein. Instead, use is made of sensitivity analysis as illustrated in the case study in Chapter 5.

Explicit objectives have to be satisfied, if a project is to be considered feasible as stated in the concept of feasibility. These objectives should encompass both the quantitative and non-quantitative objectives and needs of all the users involved, especially those of the developer. (For a listing of users, see "Users" in Appendix A). Two approaches can be used to achieve this objective of "satisfying explicit objectives"; namely, single objective constrained optimization and multi-objective optimization.

In practice, especially in private business practice, the first approach is used. External objectives other than those of the developer-investor have to be satisfied, and the developer's

objectives should be maximized. These objectives can be of many types, depending on the investor's motivations as examined in Chapter 4. Among the set of objectives of the private developer-investor, the most important ones are usually financial and economic.

To maximize the performance of the financial/economic objectives of the developer, the impact on these objectives of the strategy selected for meeting external objectives has to be studied. The feasibility study provides the set of constraints which bound the choice of designs and strategies so as to fulfill these objectives. This strategy determination process is shown in Figure 2.3: strategies are first determined at Act. B1.11 and B1.12. Scheduling and economic analysis are then performed in Act. B1.13 and B1.14. Finally, the strategies are tested for modification in Act. B1.15. If modifications are justified, the process may loop back to Act. B1.11 and B1.12 for a reassessment of strategies.

To maximize the performance of the financial/economic objectives of the developer, external objectives can be input as variables or constraint equations into the maximization model(s) chosen. For example the relationship between floor area and rent which is an external objective can be input as a constraint equation into a maximization model; rental income can be treated as a variable in, say, the NPV equation which can then be maximized.

CHAPTER THREE

THE NPV AND IRR MODELS

Subobjectives 2.0 and 3.0, dealing with the mathematical descriptions of the NPV and IRR models respectively, are treated in the first two sections of this Chapter. Decision criteria embodied in the two models are discussed in the third section. In the fourth section, the basic NPV and IRR formulae are decomposed to reflect the income and expense items which describe an investment project. Finally, the relevance of the decomposed formulae in terms of modeling realities encountered in income-property acquisition projects are examined in the last section.

3.1 MATHEMATICAL DESCRIPTION OF THE NPV MODEL

Mathematically speaking, net present value (NPV) can be defined as the sum of discounted future cash flows minus discounted equity investment, where DCTR is a prespecified discount rate. In mathematical terms:

$$NPV = -E_o + \sum_{j=1}^{n} \frac{CF_j}{(1+DCTR)^j} \qquad (3.1)$$

where E_o = initial equity investment,

n = holding period,

CF_j = after-tax cash flows in period j.

To demonstrate the application of Equation 3.1, consider an equity investment, which requires an initial equity layout of $100,000, has a five-year holding period and has projected cash flows for year 1 to year 5 of $10,000, $15,000, $16,000, $17,000 and $110,000 respectively. The discount rate to be used is 13%. Thus, NPV

$$NPV = -\$100,000 + \frac{\$10,000}{(1+.13)^1} + \frac{\$15,000}{(1+.13)^2} + \frac{\$16,000}{(1+.13)^3} + \frac{\$17,000}{(1+.13)^4} + \frac{\$110,000}{(1+.13)^5}$$

$$= -\$100,000 + \$\ 8,850 + \$11,747 + \$11,089 + \$10,426 + \$\ 59,704$$
$$= \$\ 1,816$$

The concepts of time value of money and of equivalence provide the basis for the NPV model. The concept of time value of money reflects the observation that money has productivity over time if invested such that a dollar today should be worth more than a dollar in the future (see also [28:15-19]). The concept of equivalence states that two amounts of money or series of monies at different points in time are equivalent if they are equal to each other at some point in time at a specified interest rate. Thus, NPV can simply be defined as:

> the sum of present value equivalence of future benefits minus the present value equivalence of initial investment at a given interest rate.

To reflect risk and ultimately to measure risk explicitly, the NPV model can be used in association with many risk adjustment techniques such as the four most commonly-used techniques as identified in the 1972 survey on real estate investment analysis by Robert J. Wiley [51]. Application of these techniques has been described by McGuigan and

Moyer who illustrated the use of the NPV model with the risk-adjusted discount rate approach and certainty-equivalent approach in [14:426-427]; Canada who demonstrated the application of the NPV model with a sensitivity analysis approach in [42:337-339]; and Young who illustrated with examples the use of the NPV model with a probability distribution approach in [52].

3.2 MATHEMATICAL DESCRIPTION OF THE IRR MODEL

Mathematically speaking, internal rate of return (IRR) can be defined as the discount rate which equates the present value of future benefits to the present value of equity investment. In symbolic form:

$$0 = NPV = -E_0 + \sum_{j=o}^{n} \frac{CF_j}{(1+IRR)^j} \tag{3.2}$$

IRR may be determined by linear interpolation as follows:

$$IRR \doteq DCTR_1 + (DCTR_2 - DCTR_1) \times \frac{NPV_1}{NPV_1 - NPV_2} \tag{3.3}$$

where $DCTR_1$ and $DCTR_2$ are two trial discount rates such that the NPV from $DCTR_1$, NPV_1, is larger than zero and the NPV from $DCTR_2$, NPV_2, is smaller than zero. For example, by using different trial discount rates on the cash flows of the NVP example used previously, NPV_1 and NPV_2 are obtained as shown in Table 3.1. IRR is then computed, using Equation 3.3,

$$IRR \doteq 13\% + (14\%-13\%) \times \frac{\$1816}{\$1816-(-1690)} = 13.518\%$$

It should be noted that use of Equation 3.3 results in a slight overestimate of the true value of IRR. Other techniques such as one-dimensional search techniques (see [22:145-146]) can be used to determine a more accurate IRR value. The advantage of using the linear interpolation method lies in the simplicity of calculation involved. The error in the result is not significant if the difference between the trial discounts rates (used in Equation 3.3) is not big, say one percentage point. (The actual IRR for the short example should be 13.512%.)

YEAR	CF	$DCTR_1$=13%	$DCTR_2$=14%
0	$(100,000)	$(100,000)	$(100,000)
1	10,000	8,850	8,772
2	15,000	11,747	11,542
3	16,000	11,089	10,800
4	17,000	10,426	10,065
5	110,000	59,704	57,131
		NPV_1 = 1,816	NPV_2 =(1,690)

TABLE 3.1 IRR CALCULATIONS BY LINEAR INTERPOLATION METHOD

The basis for the IRR model is identical to that of the NPV model. Whereas the output of the NPV model is the present worth of revenues minus expenses, the output from the IRR model is an interest or productivity rate on the unrecovered balance (see [17:407-408]). So, IRR can be defined as:

> the annual rate of interest earned on the unrecovered balance of the initial equity after interest payments and receipts have been added and deducted, respectively, up to the point in time being considered.

This definition of IRR is useful for measuring the productivity of an individual project, which is the prime topic of concern in this report. Other applications such as the influence of an individual project on a portfolio's productivity are beyond the scope of this paper; see [17] for portfolio application.

The ability to treat risk quantitatively with the IRR model is similar to that of the NPV model. Four of the most commonly used risk adjustment techniques with the IRR model have been illustrated by many authors and will not be repeated here: Van Horne commented on the application of both the NPV and IRR models with the risk-adjusted discount rate approach and certainty-equivalent approach in [28:137-139]; Cooper and Pyhrr illustrated with a case study the application of the IRR model and the sensitivity analysis approach in [43]; Thorne demonstrated by way of an example the use of the IRR model with probability distribution approach in [48:28-30].

3.3 DECISION-CRITERIA OF THE IRR AND NPV MODELS

Decision-criteria of the IRR and NPV model for a typical project are described in this section. A typical project is defined in terms of its cash flow pattern which is characterized by outflows followed by inflows such that the total inflows exceed total outflows (see also [29:89]). Further, the project's NPV decreases with increasing discount rate and only one positive internal rate of return exists. Decisions on accepting and rejecting projects when total (non-discounted) outflows exceed total inflows or when no sign change in flows occurs (cash flows consist of either all receipts or all disbursements) are obvious. Exceptional cases not discussed in this paper are the

multiple rate of return case and the combined borrowing, investment case; discussions on them can be found in [10:503-515] and [29:108-110].

The basic decision-criterion of the IRR model for a single project is defined as:

> a project should be accepted if its IRR is larger than the required rate of return.

For the NPV model it is defined as:

> a project should be accepted if its NPV is positive based on the minimum attractive rate of return.

It should be noted that although the required rate of return is not needed for the IRR calculation, yet, ultimately it is required for decision making. The required rate of return is a prerequisite for the NPV calculations, however. In this report, it is assumed that the required rate of return is pre-determined. For discussions on how to determine the required rate of return and what it should be, see [28:101-125, 188-211], [20:303-306], and [44].

Conclusions on accepting or rejecting a project based on the decision-criteria of the IRR and NPV models will be the same. To illustrate, based on the data set forth in the short example used previously in this chapter, which is a typical project, the relationship between NPV and discount rate is plotted in Figure 3.1. According to the decision-criterion of IRR, the project's "feasible region" (that a project should be accepted) lies on the left hand side of the project's IRR since the project's IRR is larger than the required rate of return in this region. This is also the feasible region for the decision-criterion of NPV, since for any discount rate less than IRR of 13.1%, NPV is positive. Thus, application of the IRR and NPV models to a typical project will lead to the same decision

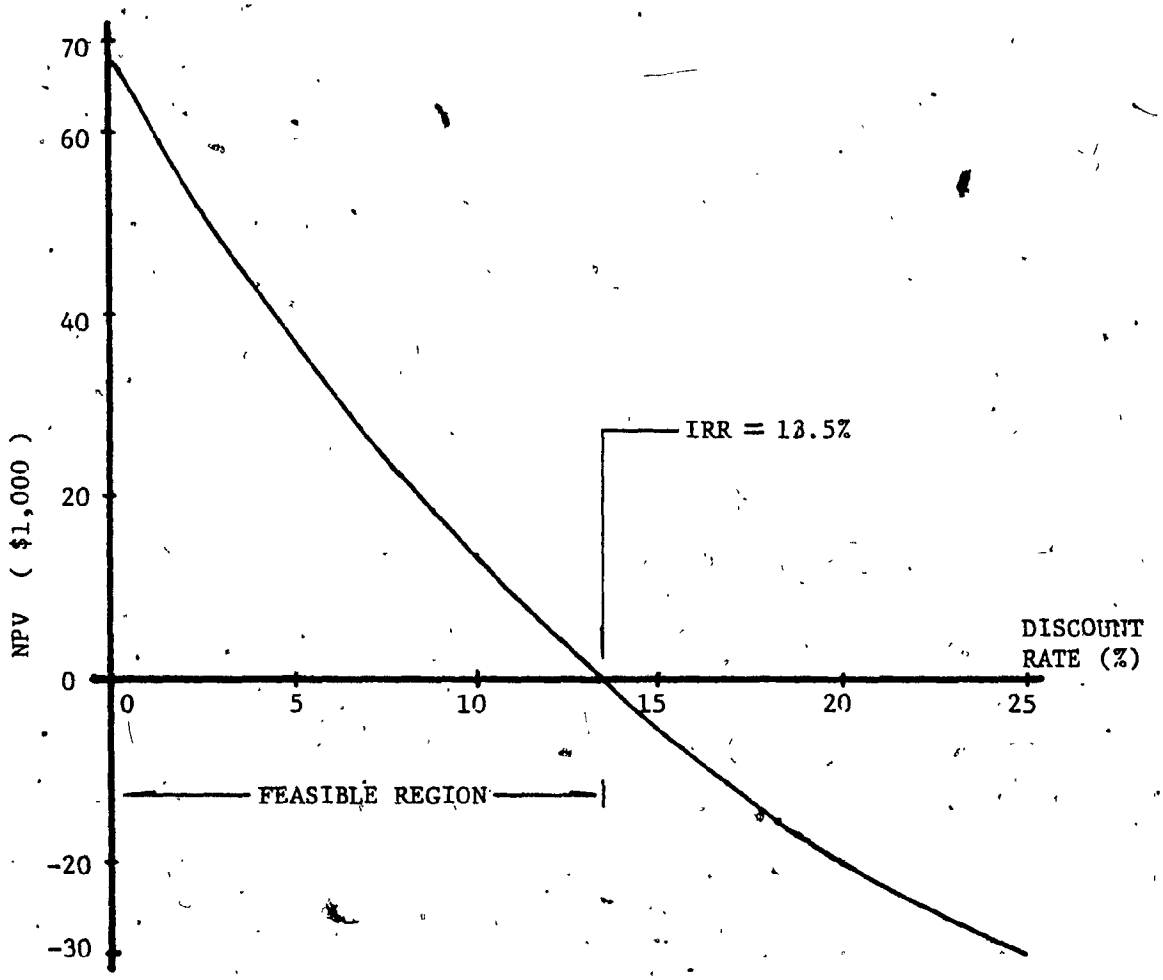


FIGURE 3.1 RELATIONSHIP BETWEEN NPV OF A PROJECT & DISCOUNT RATE

regarding acceptance or rejection.

For the ranking of mutually exclusive alternatives, attention must be directed at assessing the productivity of each additional dollar of investment. Thus, differences in equity investment and cash flow must be considered. Given two mutually exclusive alternatives A and B, they can be rewritten as:

A and A + "B-A"

where A requires the smaller equity investment. Alternative B is clearly economically superior to A, if B consists of a cash flow which is the sum of a cash flow identical to A and an economically desirable cash flow "B-A".

So, the basic decision-criterion of IRR can be restated as:

> For two mutually exclusive alternatives A and B, B is considered economically superior to A, if the IRR of "B-A" is equal to or larger than the required rate of return.

The basic NPV decision-criterion can be restated as:

> For two mutually exclusive alternatives A and B, B is considered economically superior to A, if the NPV of "B-A" is positive for the discount rate used.

The relationship between NPV and discount rate for two mutually exclusive alternatives is shown in Figure 3.2. The cash flow of alternative B is the same as that of the simplified example previously used in this chapter. The cash flow for alternative A and for the difference "B-A" are shown in Table 3.2. (The cash flows of alternatives A and B are arranged such that alternative A can be considered as a realty acquisition proposal which requires an initial equity investment of $60,000, while alternative B can be considered as

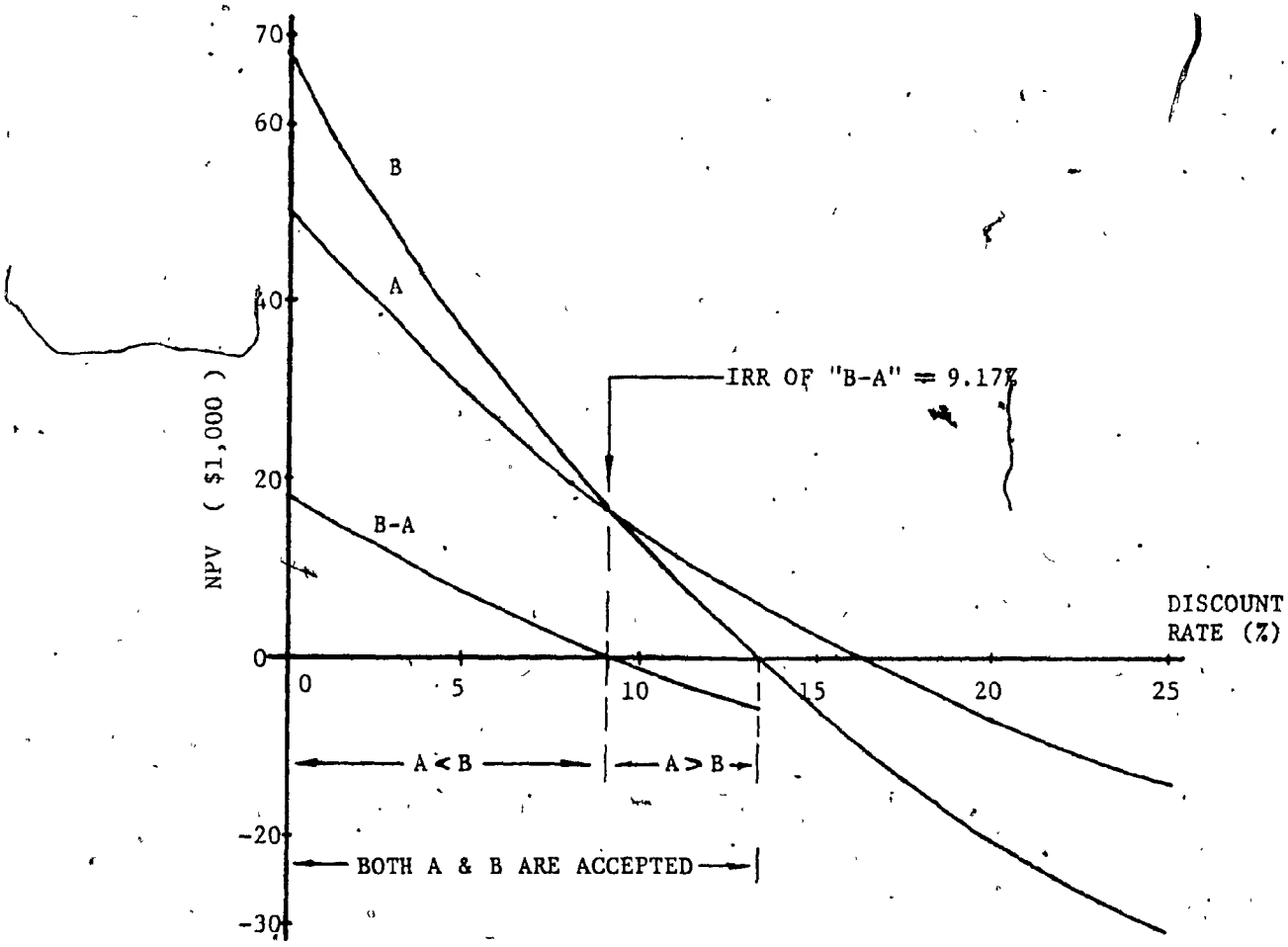


FIGURE 3.2 RELATIONSHIP BETWEEN DISCOUNT RATE & NPV OF MUTUALLY EXCLUSIVE PROJECTS

	CASH FLOW		
YEAR	ALTERNATIVE B	ALTERNATIVE A	B LESS A
0	$(100,000)	$(60,000)	$(40,000)
1	10,000	9,000	1,000
2	15,000	9,000	6,000
3	16,000	10,000	6,000
4	17,000	10,000	7,000
5	110,000	72,000	38,000
IRR	13.51%	16.33%	9.17%

TABLE 3.2 COMPARISON OF MUTUAL EXCULSIVE PROJECTS

a proposal to buy and renovate the same property such that both the initial investment and cash flows increase substantially).

The NPV functions for alternative proposals can either intersect or not intersect in the feasible region. For those which do not intersect in the feasible region, the alternative with the higher curve dominates the alternative with the lower curve. For the case where the curves intersect as shown in Figure 3.2, the preference for one alternative over another changes at the point of intersection. This point also corresponds to the IRR for the cash flow denoted by B-A which is 9.17% in this case. The IRR of the cash flow difference of the alternatives is the prime controlling factor for ranking mutually exclusive alternatives. It leads us back to the same situation and conclusion as for decision making for a single proposal if we compare the cash flow difference curve "B-A" in Figure 3.2 with the single proposal curve in Figure 3.1. The feasible region of "B-A", for which alternative B is superior to A, according to the decision-criterion of NPV coincides with that of the IRR. Thus, the decision-criteria of the IRR and NPV models lead to the same acceptance or rejection decision for a single proposal decision and for ranking mutually exclusive proposals.

3.4 DECOMPOSING THE BASIC NPV AND IRR FORMULAE

A traditional breakdown of the basic NPV and IRR formulae, Equations 3.1 and 3.2, based on an examination of the literature is documented in this section (see [50:38-39], [9:349-351], [46], [20] and [6]). The purpose of the breakdown is to reflect financial and economic realities inherent in various phases of income property acquisition projects. The breakdown of the basic NPV and IRR formulae

will be used in the next section for examining the capability of these models to reflect practical investment realities.

Recall the basic NPV and IRR formulae:

$$NPV = -E_o + \sum_{j=o}^{n} \frac{CF_j}{(1+DCTR)^j} \tag{3.1}$$

$$0 = NPV = -E_o + \sum_{j=o}^{n} \frac{CF_j}{(1+IRR)^j} \tag{3.2}$$

The cash flow term CF_j, in these equations can be further broken down into three parts, such that the basic formulae can be rewritten as:

$$NPV = -E_o + \sum_{j=o}^{t} \frac{TSD_j}{(1+DCTR)^j} + \sum_{j=t+1}^{n} \frac{ICF_j}{(1+DCTR)^j} + \frac{NRCR_n}{(1+DCTR)^n} \tag{3.4}$$

$$0 = NPV = -E_o + \sum_{j=o}^{t} \frac{TSD_j}{(1+IRR)^j} + \sum_{j=t+1}^{n} \frac{ICF_j}{(1+IRR)^j} + \frac{NRCR_n}{(1+IRR)^n} \tag{3.5}$$

where

t = development time,

TSD_j = tax shelter benefits (if any) in year j during the development period,

ICF_j = (after-tax) interim cash flows from operation in year j,

$NRCR_n$ = net resale cash reversion in year n.

The last term of Equations 3.4 and 3.5, $NRCR_n$, can be further broken down and expressed as:

$$NRCR_n = SP_n - SE_n - CGT_n - RCT_n - UM_n \tag{3.6}$$

where

SP_n = resale price in year n,

SE_n = selling expense in year n,

CGT_n = capital gains tax in year n (see also Equation 4.7),

RCT_n = recapture tax in year n (see also Equations 4.6 and 4.8),

UM_n = unpaid mortgage balance in year n.

The ICF_j term in Equations 3.4 and 3.5 could be further divided to account for the absorption phase of the project and its full operating phase. This refinement is not treated here. The key element in ICF_j can be written as:

$$ICF_j = EGI_j - OPE_j - TAX_j - DS_j \qquad (3.7)$$

where

EGI_j = effective gross income in year j,

OPE_j = operating expense in year j,

TAX_j = income tax payable in year j (which can be negative),

DS_j = debt service in year j which includes mortgage interest and mortgage principle payments.

The first three right hand side terms of Equation 3.7 can be further broken down to reflect the manner in which expenses are categorized for accounting purposes. In this chapter, the terms EGI and OPE are decomposed according to the categories of income and expense as used by the Institute of Real Estate Management in its Income/Expense Analysis (see [6:21-22]).

EGI, the first right hand side term of Equation 3.7, can be expressed as

$$EGI = PGI - VAD = PGI \times (1 - VADR) \qquad (3.8)$$

where

PGI = potential gross income,

VAD = vacancy and delinquency allowance,

VADR = vacancy and delinquency rate.

It should be noted that if different income sources exist with substantial differences in character, EGI and its related inputs can be subdivided to allow these income sources to be input and analyzed separately. Examples of different income sources are office space and 2 and 3 bedroom apartments.

OPE, the second right hand side term of Equation 3.7, can be decomposed into four main groups of expenses or as many as fifteen as per the Income/Expense Analysis. For purposes of simplicity, OPE will be treated herein as the sum of four groups of expenses. i.e.

$$OPE = MAO + UTL + ADM + IAT \qquad (3.9)$$

where

MAO = maintenance and operating expenses which include total payroll, supplies, painting and decorating, maintenance and repairs, services and miscellaneous operating expenses;

UTL = utility expenses including electricity, water, gas and heating fuel;

ADM = administration expenses which include management and leasing fees;

IAT = insurance and taxes other than income tax.

TAX_j, the third right hand side term of Equation 3.7, can be expressed as:

$$TAX_j = (EGI_j - OPE_j - DEP_j - INT_j) \times TAXR_j \qquad (3.10)$$

where

DEP_j = capital cost allowance in year j,

INT_j = mortgage interest payment in year j,

$TAXR_j$ = income tax rate in year j.

Thus, the cash flow in the j^{th} year, ICF_j may be written as:

$$ICF_j = PGI_j \times (1 - VADR_j) - (MAO_j + UTL_j + ADM_j + IAT_j) - (EGI_j - OPE_j - DEP_j - INT_j) \times TAXR_j - DS_j \quad (3.11)$$

3.5 REFLECTING FINANCIAL AND ECONOMIC REALITIES OF INCOME PROPERTY ACQUISITION PROJECTS BY THE IRR AND NPV MODELS

The ability to model financial and economic realities encountered in various phases of income-property acquisition projects in the terms encompassed in the NPV and IRR formulae is examined herein. To do this, major realities of each phase are identified, discussed and examined as to their inclusion in the NPV and IRR models. The potential of modeling omissions is then discussed. Clearly, a trade-off exists between the effort required to model accurately the nuances involved in the development process and the benefits in terms of prediction accuracy and additional information obtained. In many cases, however, the difficulty involved in obtaining the data required to model these nuances and its questionable accuracy make the effort involved unwarranted.

3.5.1 REFLECTING REALITIES IN THE PREDEVELOPMENT, DEVELOPMENT AND CONSTRUCTION PHASES

Financial and economic realities in the Predevelopment, Development and Construction Phases are:

(1) development cost payments,

(2) initial equity investment disbursements,

(3) tax shelter during development period, and

(4) capital budget ceiling constraint.

The first right hand side term of the general NPV and IRR formulae (Equations 3.4 and 3.5), E_0, is used to model the realities of development cost and disbursement. These disbursements can be modeled straightforwardly into the general formulae, as a synchronized payment if magnitude and time dispersion of the cash payments are small as in most cases of acquisition projects without renovations. Otherwise, the treatments of the development cost and/or equity disposition have to be modified such that they are subject to discounting.

Tax shelter during development period is represented adequately by the second right hand side term of the general NPV and IRR formulae.

A capital budget ceiling should be treated explicitly in the feasibility study by comparing the capital budget ceiling with the equity required to ensure that the proposed project will be feasible within the constraint budgeting problems.

3.5.2 Modeling Realities in the Operation Phase

The third right-hand side term of the general NPV and IRR formulae and its related Equation 3.11 are used to model financial and economic realities in the Operation Phase. These realities include:

(1) rent-up or absorption period during which the project cash flow differs substantially from the steady state operation phase;

(2) ICF consists of many individual components whose variations with time may substantially differ from each other. These variations can be classified into three categories (see Figure 3.3), which are:

(a) constant or changing at fixed rate(s),

(b) trend fluctuating, and

(c) cyclic fluctuating; and

(3) the timing of the components of ICF. Usually flows occur on a monthly basis or some multiple of months.

One reality which is beyond the scope of this paper is the case of refinancing.

Occurrence of non-typical ICF during absorption period, the first reality, is usually applied only to those acquisition projects which require renovations. However, it is possible to have an absorption period, even when renovations are not done. For example, a new owner of a building which has a high vacancy rate may launch an intensive sales campaign to lower the vacancy rate to that of his projected typical year. To handle the cash flow during the absorption period, time varying vacancy rates can be applied to rental income and time varying utilization rates can be used for expense items.

Differences in patterns of and rates of change of the ICF components, the second reality, requires that a range of models be used to represent the various income and expense items. If multiple income

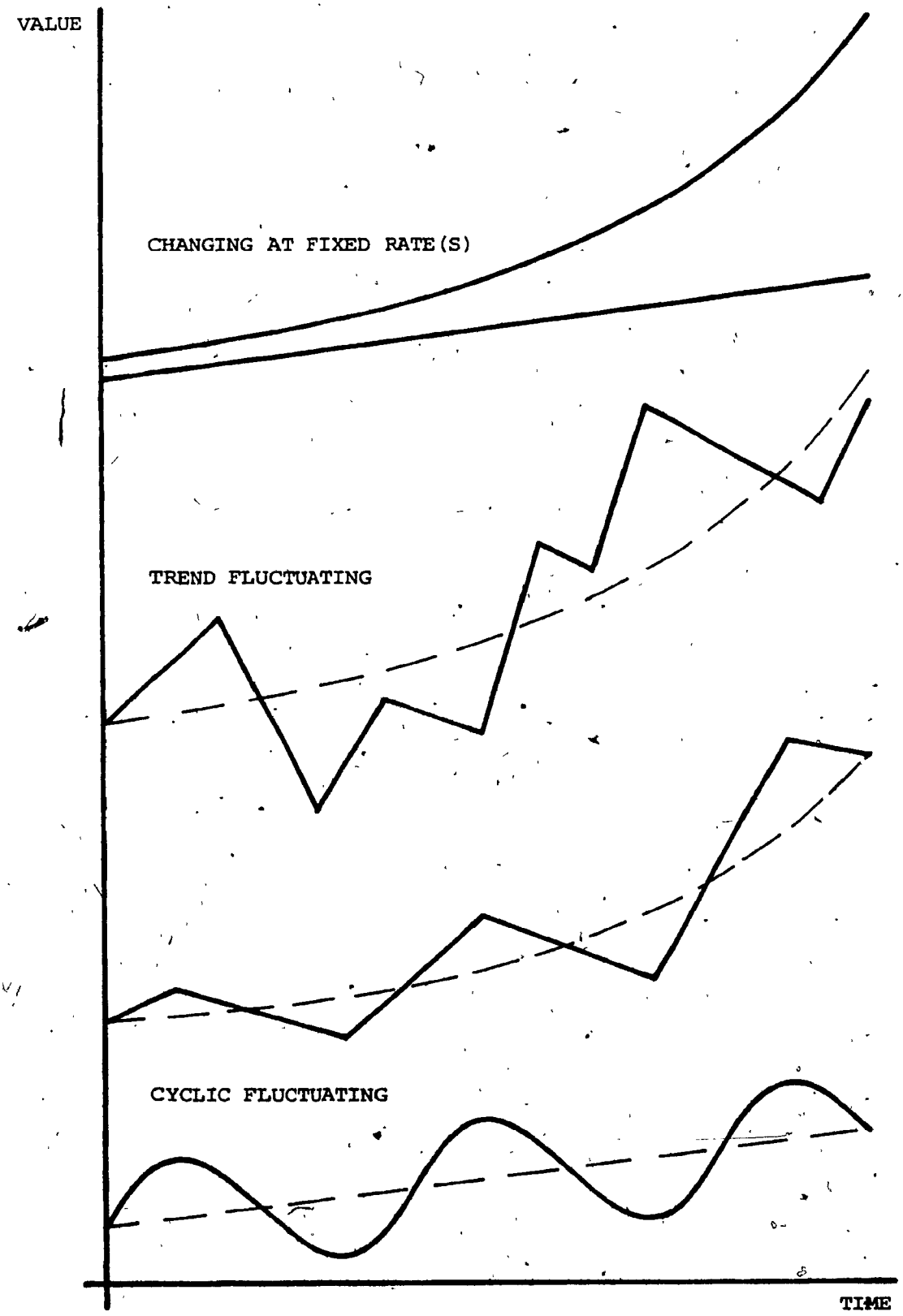


FIGURE 3.3 PATTERNS OF RATES OF CHANGE OF INTERIM CASH FLOW COMPONENTS

sources exist, the income term in Equation 3.11 can be modified as follows:

$$EGI_j = \sum_{i=1}^{m} EGI_{ij} = \sum_{i=1}^{m} PGI_{ij} \times (1-VADR_{ij}) \qquad (3.12)$$

where

m = total number of income sources;

EGI_{ij} = EGI of i^{th} income source in period j;

PGI_{ij} = PGI of i^{th} income source in period j;

$VADR_{ij}$ = VADR of i^{th} income source in period j.

If a more detailed breakdown of the four groups of operating expense items (MAO, UTL, ADM, and IAT) in Equation 3.11 is required, it can be done in a manner similar to that of the income terms.

The second reality of the Operation Phase also deals with the three categories of patterns of change in time of the ICF components previously identified. The first category consists of variables whose values either are constants or are changing at fixed rate(s). The relationship of change in value of these variables can be modeled readily in the general formulae. For example, the capital cost allowance is modeled by the following simple equation:

$$DEP_j = DEP_1 (1 - CCAR)^{j-1} \qquad (j \geq 1) \qquad (3.13)$$

where

DEP_j = depreciation allowance in period j;

DEP_1 = depreciation allowance in the first period which is equal to the initial depreciable capital times CCAR;

CCAR = constant capital cost allowance rate, specified by government tax regulations.

Trend fluctuation, the second category of pattern of change refers to changes which in the long run yield positive, negative, or zero growth rates. Trends in operating expenses for residential elevator buildings in Canada and U.S.A. from 1968 to 1975 are presented in Figure 3.4. All of the four operating expense groups have experienced positive growth rates over the seven years examined, although the yearly escalation of growth rates vary . As shown in Table 3.3, insurance and taxes other than income taxes (IAT) had a year to year escalation rate ranging from a high of 23% to a low of -18% occuring in the years 1971-1972 and 1973-1974, respectively.

If the time variation of escalation rates can be predicted with significant accuracy and if the effort of assessing and modeling them is warranted, they can be modeled by the following general formula:

$$V_j = V_0 \prod_{i=1}^{j} (1+ESLR_i) \qquad (3.14)$$

where

V_j = value of the fluctuating variable (such as IAT) in period j;

V_0 = initial value of the fluctuating variable in period zero;

$ESLR_i$ = escalation rate for the fluctuating variable in period i.

Due to our inability to accurately predict the future as well as the nature of real estate investment, the sophistication of Equation 3.13 is not often warranted, particularly for income and expense items which are small compared to total income and expenses. For such cases,

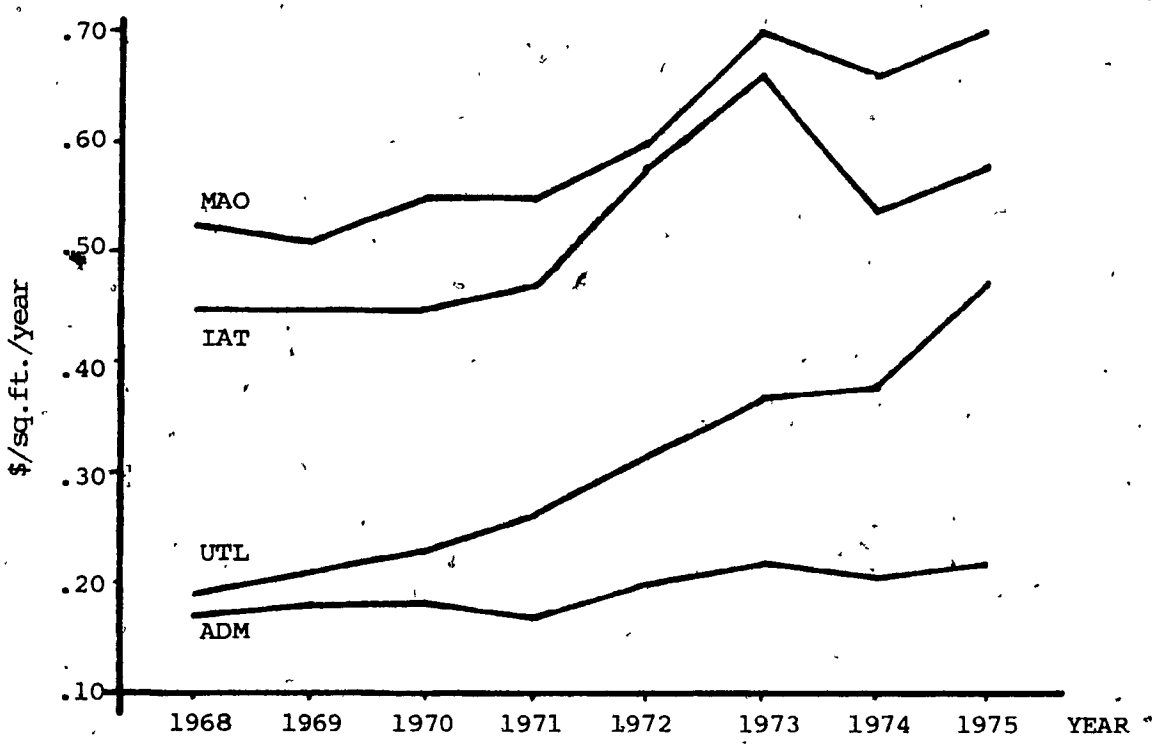


FIGURE 3.4 TRENDS IN OPERATING EXPENSES OF RESIDENTIAL ELEVATOR BUILDING IN CANADA AND U.S.A. FROM 1968 TO 1975*

	YEAR							
	1968	1969	1970	1971	1972	1973	1974	1975
$/SQ.FT./YEAR								
MAO	.52	.51	.55	.55	.60	.70	.66	.70
UTL	.19	.21	.23	.26	.32	.37	.38	.47
ADM	.17	.18	.18	.17	.20	.22	.21	.22
IAT	.45	.45	.45	.47	.58	.66	.54	.58
ESCALATION RATE								
MAO	-.02	.08	.00	.09	.17	-.06	.06	
UTL	.11	.10	.13	.23	.16	.03	.24	
ADM	.06	.00	-.06	.18	.10	-.05	.05	
IAT	.00	.00	.04	.23	.14	-.18	.07	

TABLE 3.3 TRENDS IN OPERATING EXPENSES OF RESIDENTIAL ELEVATOR BUILDING IN CANADA AND U.S.A. FROM 1968 TO 1975*

* Data used are based on IREM's Income/Expense Analysis ([6:21-23] and [7:21-23]). For detailed explanation of terms used, see Appendix of [6]. The size and composition of the data sample varies from year to year, which may account for the irregular patterns observed.

the following model is sufficient:

$$V_j = V_0(1 + ESLR)^j \qquad (3.15)$$

where ESLR is invariant with time.

Again, if detailed modeling efforts are warranted, escalation rates for those variables sensitive to project performance can either be modeled by Equation 3.14 or a combination of Equations 3.14 and 3.15:

$$V_j = V_0 (1 + ESLR)^{j-m} \prod_{i=1}^{m} (1 + ESLR_i) \quad (j > m) \qquad (3.16)$$

where 'm' equals to the number of periods that detailed modeling is justified because of confidence in prediction ability. Equation 3.16 has the advantage of minimizing the efforts of assessing the much less certain values for escalation rates for the later years. With normal economic development predictions of project performance become increasingly less sensitive to errors in estimating future escalation rates as the year(s) for which the estimate(s) is made extends further and further into the future, because of the time value of money principle.

Cyclical fluctuation, the last category of pattern of change refers to the fluctuation which occurs at a fixed or semi-fixed time period cycle. For example, heating fuel expenses will be high in winter months and low in summer months. Part of the painting and decoration expenses may occur at a long yet semi-fixed time cycle, say five years. Changes in rental income (PGI) may only be realized at the start of a new term of lease. If detailed modeling efforts are warranted, cyclical fluctuations in income and expense items can be modeled into the general IRR and NPV formulae by many different techniques.

The third reality deals with the timing of the components of interim cash flow. Most of the income and expense items of real estate investment occur on a monthly basis or some multiple of months. A discrete cash flow model using time periods of 1 month can be used directly to model this reality. If more detailed modeling is required, continuous cash flow models can be used. However, their application is most often confined to research work, as most developers will not border with continuous compounding formulation.

3.5.3 Modeling Realities in Termination Phase

Basically, there are two methods of disposition of property; namely, resale and non-taxable exchange. The case of resale and its realities will be discussed herein. Non-taxable exchanges (or like-kind exchanges) deal with postponing tax payments and reinvesting in another property. The exceptional case of exchanges is beyond the scope of this report (see [12], [18], [19] and [26] for exchanges).

For the case of resale, the realities of the Termination Phase are accompanied with a high degree of uncertainty, and are concerned with:

(1) forecasting of holding period (or year of disposition);
(2) forecasting of resale price and the related land and building escalation rates;
(3) estimating resale expenses;
(4) estimating recapture and capital gains taxes payable; and
(5) settling of unpaid mortgage balance.

The holding period can be regarded as either an endogenous variable (a function of variables explicitly defined in the model) or an

exogenous variable (a function of variables outside of the model). For the IRR and NPV models, the choice really depends on the decision-criteria used in determining the length of the holding period. (For further discussion see [2:140,156].) As an exogenous variable, holding period is not determined directly by the performance measure(s) selected for the project; other external factors such as return rate of overall portfolio guide the decision. Because of this, further discussions on holding period as an exogenous variable are excluded from the scope of this report.

As an endogenous variable, holding period is determined directly by project performance. For example, if the planned holding period is to be equal to the 'optimal holding period', i.e. that holding period for which IRR is highest, it can be determined by observing the functional relationship between holding period and the related decision criteria. (See [43:327-330] for the relationship between holding period and some performance measures).

It should be noted that, in most cases, only planned or most-likely holding period can be established and the actual date of disposition may deviate somewhat from the forecast date. The actual sale date depends on many external factors, such as market trend at time of disposition. In most cases, prediction of the actual date of disposition is almost impossible. In general, a small deviation (say plus or minus a few months) in holding period is not crucial to overall project performance (consider the findings of the case study by Cooper and Phyrr [43:327-330]).

The second reality concerns forecasting the resale price and related escalation rates. Overall project performance is extremely sensitive to such forecasts. For example, a case study by Whisler [50:43-44] indicates that a 10% change in resale price induces an equal percentage change

in IRR.

Resale price and related escalation rates are subject to trend fluctuations. Statistics on prices of three groups of real estate studied by Miller [14:23] — farms, non-residential and single-family residential — indicate that they all experienced trend fluctuations over the study period of 1969-1977. The magnitude of fluctuation, on top of a growth trend of market value of the non-residential group was enormous while the fluctuations of the remaining two groups were not significant. Thus, some property types require more detailed modeling than others.

Similar to the treatment of trend fluctuations in the previous subsection, the reality can be modeled either through Equation 3.14, or 3.15, or 3.16 in the general formulae and its related Equation 3.6. It has been demonstrated that escalation rates of resale price can have significant effects on overall project performance. Thus, in general, detailed modeling is required. Yet, the ability to accurately model this reality is highly limited by:

(i) our inability to make accurate long-term forecasts because of the large number of uncertainties involved; and

(ii) the little useful data available for preparing such forecasts.

Resale expenses, the third reality, have to be reflected in the IRR and NPV models, otherwise overall project performance will be overestimated; for example, optimum holding period will be shortened. Resale expense is modeled into the general formulae through Equation 3.6 and is usually estimated as some specified fraction of the sale price.

The fourth reality deals with the recapture and capital gains taxes payable at time of disposition. Based on present tax provisions, the taxes payable can be estimated by Equations 4.6, 4.7 and 4.8, and modeled through Equation 3.6 into the general formulae. These tax payments are subject to changes in future tax rates and tax provisions.

The last reality to be discussed here is settling of the unpaid mortgage balance. It can be modeled in the general formulae through Equation 3.6 by subtracting the cumulative sum of the principal payments from the initial mortgage amount.

CHAPTER FOUR

REFLECTING INVESTOR MOTIVATIONS BY PERFORMANCE MEASURES FROM THE IRR AND NPV MODELS

For a feasibility study and its related economic analysis to be meaningful to the investor, performance measures obtained from the model used in the analysis must reflect the investor's motivations which form the basis for his investment objectives.

The ability of IRR and NPV models to reflect investor motivations are examined in this chapter. The first section of this chapter is addressed to exploring and answering the question: "What are investor motivations?" The second section will describe some of the more commonly used performance measures. In the last section, these measures are set against investor motivations in order to determine which measures reflect the various motivations. Finally, the ability of the performance measures embodied in the IRR and NPV models to reflect investor motivations is examined.

4.1 INVESTOR MOTIVATIONS

Investor motivations are highly-varied and are a function of the investor's psychological make-up, age, financial position, attitude toward risk, etc. Investor motivations are multi-attributed; they are not always explicitly known or stated and are often contradictory.

Nine of the most common real estate investor motivations documented in the literature are described herein. An attempt is also made to demonstrate the significance of each motivation for real estate investment by means of available statistical data, where possible.

(1) HEDGE AGAINST INFLATION

High inflation rates have been and continue to be a fact of life in this decade. The U.S. Department of Commerce's consumer price index rose from 100 to 172.6 between 1967 to 1976, equivalent to a 6.2% compounded yearly inflation rate [9:348].

Wendt, in his article "Inflation and the Real Estate Investor" [30], points out that the effects of inflation on real estate investments are very complex. The attributes of real estate investment as an inflation hedge are quite different for various types of investors and properties as well as for the lease and mortgage terms and financial and economic positions of a project. Taking an income property acquisition project (which is the subject of study of this report) as an example, the economic impacts of inflation can be analyzed from the viewpoint of cash flow which consists of interim flows and net resale cash reversion.* Interim cash flow in the j^{th} year (ICF_j) can be expressed in simplified terms** for illustrative purposes as:

$$ICF_j = f\,(EGI_j^{***}, OPE_j, TAXR_j, CCAR_j) \qquad (4.1)$$

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* For a detailed description of interim cash flows and net resale cash reversion, see Section 3.2.

** It is assumed that financial position, mortgage terms, and mortgage interest rate remain unchanged during the holding period.

*** It is convenient to assume that vacancy and deliquency rates are constant over the long run, such that EGI can be expressed as a fraction of potential gross income.

where

EGI_j = effective gross income in j^{th} year,

OPE_j = operating expense in j^{th} year,

$TAXR_j$ = income tax rate in j^{th} year, and

$CCAR_j$ = capital cost allowance rate in j^{th} year.

Net operating income in j^{th} year, NOI_j, is equal to $EGI_j - OPE_j$. If there is no inflation, usually NOI (of buildings) will decrease over the long run because of aging, such that rent will decrease (assuming ceteris paribus for conditions such as surrounding environments, and technology impacts on OPE). However, in an extended inflation period, NOI in general might increase rather than decrease. For example, based on the Institute of Real Estate Management's (IREM) income and expense data on residential "elevator" buildings in the U.S.A. and Canada ([6] and [7]); NOI increased from \$1.38 per rentable sq. ft. to \$1.58 from 1968 to 1975, i.e. an equivalent 1.9% yearly compound rate of increase. At the same time, EGI increased from \$2.74 per rentable sq. ft. to \$3.58, i.e. an equivalent 3.9% yearly compound increase rate; and OPE increased from \$1.36 per rentable sq. ft. to \$2.01, i.e. an equivalent 5.7% yearly compound rate of increase. Thus, the data supports the previous argument that NOI in general might increase rather than decrease during long inflation periods, even though the percentage increase in OPE is greater than that in EGI (see also [30:344]).

The effect of inflation on net resale cash reversion, NRCR, is very complicated. Inflation has tremendous and complex impacts on resale price*. The net effect of inflation historically has caused the resale price of good quality and well located properties to rise as pointed out by both Wendt [30:353-355] and White [31:46]. A study by Wendt [30:343] further supports the above argument by statistics which

indicate that the average price of existing homes financed through savings and loan institutions rose from $23,100 to $43,300 between 1967 to 1976 which is equivalent to a 7.2% yearly compound growth rate.

So far, the effects of inflation are expressed in current dollar terms only, but investors are also interested in measurements in real dollar terms. Thus, benefits and receipts from all periods (B_j) have to be discounted by both an inflation rate (INF) and a real rate of return (RRR). This discounting operation can be presented as follows [29:259-262]:

$$\frac{B_j}{(1+INF)^j\ (1+RRR)^j} = \frac{B_j}{(1+INF+RRR+INF\times RRR)^j} = \frac{B_j}{(1+k)^j} \qquad (4.2)$$

where "k" is the apparent return rate or discount rate. Thus, it is clear that the component of an investor's discount rate should have consideration on both INF and RRR. RRR includes consideration of a risk free rate, risk, non-liquidity and a financial management fee (see also [1:350-352]).

Miller [15:24] and Sirmans and Webb [21:40] point out that there is a lack of empirical historical data on real estate investment yields. So, what has been the real rate of return of real estate investment for the past inflationary decade is very difficult to determine. Nonetheless, it has been demonstrated that real estate is historically a good inflation hedge, as inflation pushes up NOI and NRCR higher than otherwise.

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* Inflation pushes up development and construction costs which in turn pushes up the resale price (of which replacement can be a function) higher than otherwise. However, historically, during inflationary periods, interest rates rise and cause investor's target rate of return to increase which lowers the discounted investment value. But the simultaneous rise of costs, interest rates, and target rate of return have dramatic effects on lowering the supply of new buildings because fewer projects are feasible.

(2) QUALITY OF CASH FLOW

Investors are motivated by not only the absolute productivity of money but also by the quality of their receipts: cash flows. In general, the quality of cash flow can be described in one or more of the following terms: certainty, negativity, uniformity, type, and timing of cash flows. Certainty deals with the probability of obtaining the projected cash flows. Negativity deals with both the magnitude and the frequency of occurrence of negative cash flows. Uniformity refers to the stability or smoothness of the interim cash flow pattern. Type of cash flows deals with the components of real estate return which can be partitioned into (i) tax shelter for sheltering the investor's taxable income from other sources, (ii) after-tax spendable interim cash flow from operations (i.e. ICF minus the aforesaid tax shelter), and (iii) net resale cash reversion from capital appreciation (see also [27:74-75] and later discussion on capital appreciation potential). Timing deals with when the cash flows are received.

As a short illustrative example, consider an investor who is in a high tax bracket and who is planning to retire in a few years. Upon retirement, his tax bracket may drop substantially, and consequently he may be very motiyated to invest in a real estate project which provides immediate tax shelter and deferred earnings. Thus, the investor's motivations may encompass both type and timing of cash flows.

(3) EFFECTIVE LEVERAGE POTENTIAL

Leverage (LEV) can be defined as total cost divided by initial cash equity. The outcome of leverage financing can be described in statistical terms as follows: the expected return rate of equity involved can be increased substantially by positive leveraging, while the standard

deviation of expected return rate (i.e. upside and downside potentials) is highly magnified by leverage financing as well.

The potential of a project to have positive or negative leverage effect can be determined by the "leverage breakeven concept" which is developed in Appendix B. According to the concept, if after-tax before-finance IRR, IRR_{BF}, is larger than after-tax cost of debt rate, CDR, then positive leverage will occur. If income tax rate, TAXR, is constant for the holding period, CDR can be obtained by (see Equation B6).

$$CDR = i(1-TAXR) \tag{4.3}$$

where i is the mortgage interest rate.

Investors are motivated by the effectiveness of leverage on after-tax internal rate of return on equity, IRR_{AF}. It can be approximated by the following equation (see also Equation B10):

$$IRR_{AF} \doteq LEV\ (IRR_{BF} - CDR) + CDR \tag{4.4}$$

Based on a study by Sirmans and Webb [21:42], average mortgage loan to value ratio as reported by fifteen life insurance companies ranged from 73% to 75% between 1968 to 1976. These ratios are equivalent to a leverage factor of about 4. Another study by Miller [15:25] indicated that average non-residential mortgage interest rate for December 1977 was 9.45%. Using this interest rate, the corresponding CDR will be 4.73% (from Equation 4.3) for investors whose TAXR is 50%. For those projects which have a before-finance IRR of 7%, a leverage factor of 4 will raise the IRR about 100% to 13.83% (from Equation 4.4). The impact of leverage can even be higher for projects with a higher IRR_{BF}. For an IRR_{BF} of 9%, a leverage factor of 4 will raise the IRR 140% to 21.83%.

Leverage, however, is a two edged sword. For example, when leverage becomes higher, magnitudes of interim cash flow become smaller thus

increasing the potential for negative interim cash flows. When negative flows occur, the developer-investor will have to either (1) make up the balance from other sources, or (ii) obtain further mortgage, or (iii) sell the property to satisfy the debt. In the last case, if net resale price at that time is low, the investor may lose part or all of his equity investment.

(4) INCOME TAX SHELTER

A 1971 survey of apartment investors, by the U.S. Department of Housing and Urban Development, shows that tax shelter is the second most important motivation for individuals and is third for real estate groups and investment trusts [5]. Income tax shelter, of course, depends on income tax acts, which are complicated and are subject to change; thus, it is a source of risk as well. Some of the prime favourable income tax regulations for income property investment are:

(i) full mortgage interest payments are allowed to offset incomes, thus leverage financing becomes more attractive since large amounts of tax shelter can be generated;

(ii) the capital cost allowance is based on an accelerated depreciation method and the total capital cost (excluding land) provides the base for the calculation, regardless of the leverage factor;

(iii) capital cost allowances can be carried forward, if not fully utilized, to shelter income in later years; and

(iv) capital gains tax applies only to half of capital gains.

Tax shelter can be viewed from the viewpoint of a portfolio or individual project and are called "tax losses tax shelter" and "productiv-

ity concerned tax shelter" respectively. The tax shelter can be carried over for five years to offset incomes of like-kind. To describe them, the following simplied tax equations are used:

$$TAX_j = (EGI_j - OPE_j - DEP_j - INT_j) \times TAXR_j \qquad (4.5)$$

$$RCT_n = (NSP_n - BV_n) \times TAXR_n \qquad (\text{for } NSP_n \leq BV_o) \qquad (4.6)$$

$$CGT_n = (NSP_n - BV_o) \times 50\% \times TAXR_n \qquad (\text{for } NSP_n \geq BV_o) \qquad (4.7)$$

$$RCT_n = (BV_o - BV_n) \times TAXR_n \qquad (\text{for } NSP_n \geq BV_o) \qquad (4.8)$$

where —

TAX_j = income tax payable in year j,

EGI_j = effective gross income in year j,

OPE_j = operating expense in year j,

DEP_j = depreciation allowance in year j,

INT_j = mortgage interest payment in year j,

$TAXR_j$ = income tax rate in year j,

n = holding period in number of years,

RCT_n = recapture tax in year n,

NSP_n = net resale price in year n, equals to resale price minus selling expense,

BV_n = book value in year n,

BV_o = book value in year zero,

CGT_n = capital gains tax in year n.

"Tax losses tax shelter", which is shown in Figure 4.1, refers to the negative taxable income from a project only. The tax shelter considers the difference between income and operating expense explicitly. The negative taxable income occurs when EGI_j minus OPE_j is smaller than DEP_j plus INT_j (Equation 4.5), and/or when NSP_n is smaller than BV_n (Equation 4.6) at the end of the holding period.

"Productivity concerned tax shelter", which is shown in Figure 4.1, refers to tax exemption and tax deferment before deduction of the difference between income and operating expense. Tax exemptions are mortgage interest payments (INT_j) in operation years (Equation 4.5), and the reduced tax rate on capital gain ($NSP_n - BV_o$) at time of resale (Equation 4.7). Tax deferments consist of the depreciation allowances (DEP_j) (Equation 4.5). Depending on the net resale price (NSP_n) part or all of the depreciation claimed may have to be paid back in the form of a recapture tax (RCT_n) at the time of resale (Equations 4.6 and 4.8).

Tax shelter can be significant for two reasons. First, because mortgage interest payments can be very high for earlier years if a blended payment mortgage, where sum of principle and interest is constant, is used. Secondly, the depreciation allowance will be very high for the earlier years as well, because the accelerated depreciation method can be used on the whole capital cost, not just on the cash equity investment.

A recent study by Lorimer [13:267] showed the significance of tax shelter on real estate development: the effective tax rate was only 10%

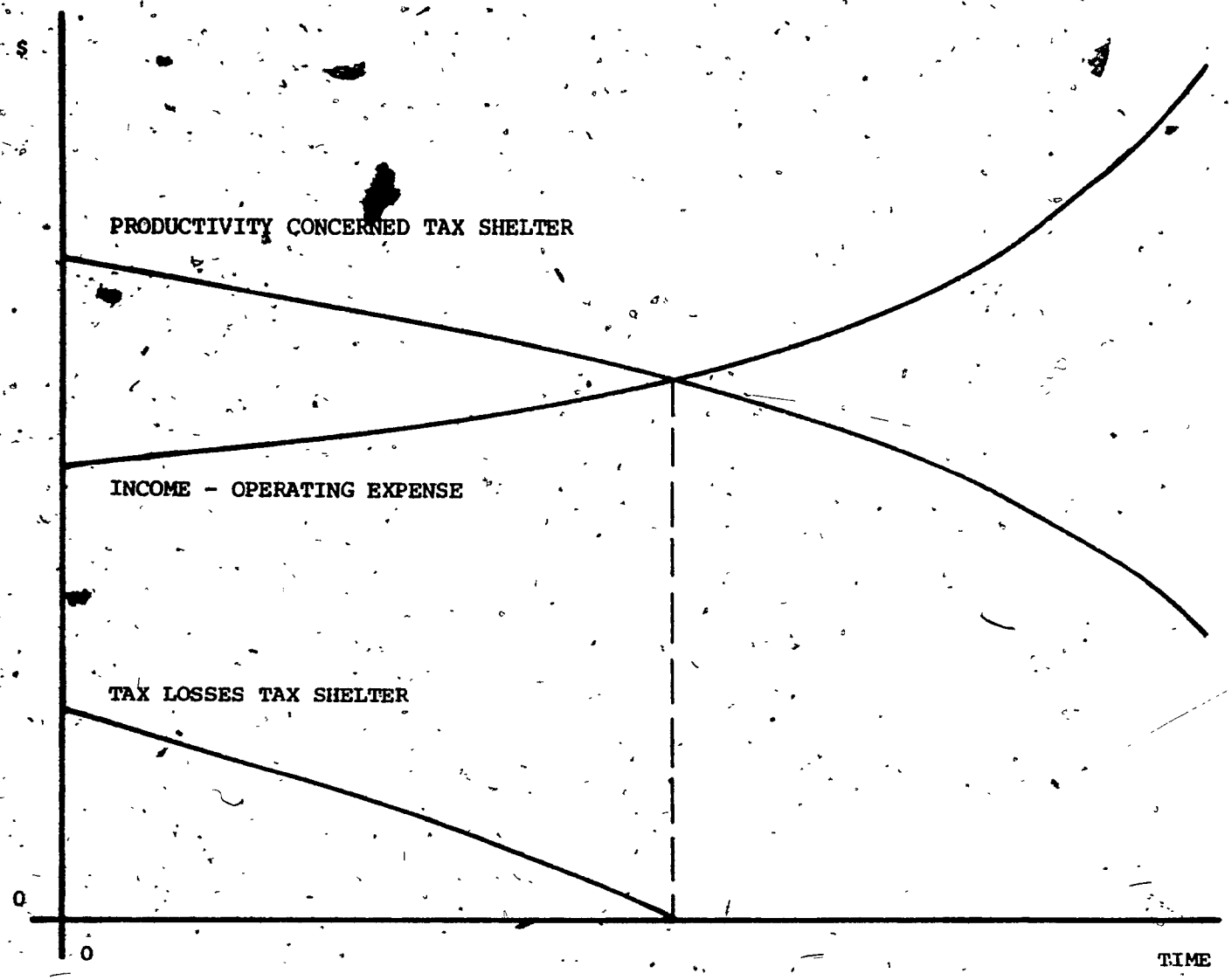

FIGURE 4.1 TAX LOSSES TAX SHELTER AND PRODUCTIVITY CONCERNED TAX SHELTER

in 197[illegible] for twenty of Canada's major public real estate corporations, (excluding Genstar and Cadillac Fairview), while the corporate tax rate was 46% in the same year.

(5) CAPITAL APPRECIATION POTENTIAL

Investors are motivated not only by interim cash flows but also by capital appreciation. Of interest to the investor is the initial equity investment he must make. This is then followed by the appreciation that this equity may undergo because of an increase in value of the property. This increase accrues wholly to the equity investor. Thus, the percentage capital appreciation can be defined as net resale cash reversion, NRCR, divided by initial equity, E_o. NRCR can be expressed by:

$$NRCR_n = SP_n - SE_n - CGT_n - RCT_n - UM_n \qquad (4.9)$$

Capital appreciation on equity can be substantial mainly because (i) a highly leveraged investment results in the capital appreciation on the total project accruing to the equity investment, and (ii) the capital gains tax is only applied to half of the capital gain. For medium to long holding periods the contribution of capital appreciation to NPV, particularly for high discount rates, can be quite small even for high appreciation rates. Nonetheless, the sensitivity of IRR to appreciation rates can vary from very high to insensitive, depending on the holding period, the appreciation rate and the discount rate. Meaningful statistical data on appreciation rates and their effect on IRR are hard to find and limited. To demonstrate the impact of capital appreciation on economic performance, typical assumptions coupled with various statistical data will be used here in conjunction with Equations 4.6, 4.7, 4.8, and 4.9.

(i) A study by Wendt [30:349] indicated that the average purchase price of existing homes financed through savings and loan institutions rose from $23,100 to $43,300 between 1967 to 1976. So, by setting total acquisition cost, TC, equal to $23,100, the resale price at the end of the 9 year holding period will be $43,300 if and only if the average property bought in 1967 is still fully competitive with the average property sold in 1976.

To relax this assumption regarding complete competitiveness, an adjustment of the resale price is necessary. This can be done using the gross income multiplier approach (i.e. assuming that resale price is directly proportional to gross possible income, GPI). Based on IREM's Income/Expense Analysis [7:15-16] GPI (per room per annum) of "garden" buildings built between 1968-1975 was $619 in 1975 and that built between 1961-1967 was $581; a difference of 6%. The rationale for choosing this age group is: a) the average age of 1968-1975 and 1961-1967 built buildings are 1972 and 1964 respectively; b) the difference between the average ages is 8 years which is similar to the 9 year holding period cited above; and c) "garden" buildings built between 1968-1975 and 1961-1967 can be regarded as representative of the average existing homes sold in 1976 and 1967 respectively. Further study on the GPI difference between the two age groups indicated that the GPI difference was 9% in 1974, 10% in 1973, and 13% in 1972. Consequently, an adjustment factor of 10% is used herein to adjust the $43,300 average purchase price to obtain a more reasonable resale price, SP_n. Thus, SP_n is assessed to be $39,000 (90% of $43,300) while TC is equal to $23,100.

(ii) Based on a study by Sirmans and Webb [21:42], the average loan to value ratio as reported by fifteen life insurance companies was 71% for the year 1967. Thus, the mortgage is assumed to be 71% of the TC; i.e. \$16,400. Then E_0 is \$6,700.

(iii) The average annual mortgage constant as reported by the aforementioned life insurance companies was 9.2% for the year 1967 [21:47]. By assuming a 25 year amortization period, the mortgage interest rate is computed to be 7.8%; (non-residential mortgage interest rate as reported by the U.S. Statistical Abstract was 8.62% for the year 1969 [15:29]). Thus, UM_n at the end of the 9 year holding period will be 83% of the \$16,400 mortgage; i.e., \$13,600.

(iv) SE_n is assumed to be 5% of SP_n; so SE_n will be \$2,000, while net resale price, NSP_n, will be \$37,000.

(v) Assuming 90% of TC is subjected to double declining depreciation for tax purposes, the book value at time of sale, BV_n, will be \$15,400.

(vi) By using a corporation tax rate of 50%, CGT_n from Equation 4.7 will be \$3,500 and RCT_n from Equation 4.8 will be \$3,800.

(vii) The U.S. Department of Commerce's Consumer Price Index rose from 100 to 172.6 from 1967 to 1976 equivalent to a 6.2% compounded yearly inflation rate [30:348].

By putting the appropriate values from above into Equation 4.9,

$$NRCR_n = \$39,000 - \$2,000 - \$3,500 - \$3,800 - \$13,600 = \$16,100.$$

So a corporate investor who had invested $6,700 in an average existing home in 1967 would receive a NRCR of $16,100 in 1976 if the property could still meet the basic criteria of modernity, location and quality. This is a 140% capital appreciation on equity over the 9 years, or an equivalence of 10.2% compounded yearly appreciation rate. By deducting the appropriate 6.2% yearly inflation rate for the same period, a real dollar yearly appreciation rate of 3.8% can be obtained. It is clear that the magnitude of capital appreciation on equity can be large.

It should be noted that real estate price increase for the period was 70% ($39,000/$23,100 - 1). And after deduction of taxes and selling expense, capital appreciation on total cost is only 30% for the same period (which can be obtained by setting UM_n of Equation 4.9 equal to zero). Thus, the capital appreciation on equity in this example (which is 140%) is approximately four times larger than that on total cost for a leverage of four.

To demonstrate the impact of capital appreciation on IRR, a uniform interim cash flow to initial equity rate (ICF Rate) of 15% is assumed. Of course if there is no capital appreciation, the project's IRR will be 15%. By considering the aforementioned 140% capital appreciation, the IRR will be 21.4%; a rise of 43%. The effect will even be higher for projects having a lower uniform ICF Rate. For the case of a 6% rate, the 140% capital appreciation will raise the IRR from 6% to 15.6%; an increase of 160%. Thus, it can be seen that capital appreciation can have a very significant impact on IRR for real estate investments having reasonable ICF Rates in the range of 6 to 15 percent.

(6) HIGH RATE OF RETURN

For two identical projects with identical risk but different return rates, investors will be motivated to invest in the project that has the higher rate of return. Since the actual receipts and benefits to the equity-investor are on an after-tax, after-finance basis, the rate of return to the equity-investor should be computed on an after-tax, after-finance basis.

Again, as mentioned earlier (see [15:24], [21:40]), statistical data on after-tax after-finance IRR are hard to find and limited. Values from the example case used in the previous discussion on the significance of capital appreciation potential will be used to demonstrate the magnitude of rate of return of real estate investment. It should be noted that the example case is based on adjusted actual sales data, and some typical tax and mortgage assumptions. If a conservative 6% uniform ICF Rate is used, the IRR will be 15.6% for the period of 1967 to 1976, which is substantially higher than the average yield rate of AAA long term utility bonds and Moody's public utilities 40 bonds for the period 1969-1976. The annual average yield of the two groups of bonds fluctuated from 7.37% to 9.88% during this period [15:29].

If we accept the concept that realty investors will not invest in a project unless its return rate is higher than the prevailing mortgage interest rate (since risk for the mortgage lender is less than for the equity-investor) then, real estate return rates have to be higher than that of bonds for the same time period. Reference [15:29] indicates that the non-residential mortgage interest rate was indeed higher than that for the two groups of bonds during the period 1969-1976.

(7) PORTFOLIO DIVERSIFICATION

A sound investment strategy should always consider the diversification of investments. Some of the prime motivations for diversifying into income-property investment are to spread business risk, to obtain tax shelter, to offset incomes from other sources, and to achieve a return rate and/or growth rate which cannot be accomplished within an investor's (or a company's) original given field of endeavour. As an example of diversification, consider the case of a portfolio comprised of fixed income-securities such as corporate bonds and real estate. When interest rates rise, the bonds decrease in value in order to yield the going rate. The real estate may maintain its value, however, because of the lower interest mortgage that a potential buyer could take over. Thus, the diversification lessens the impact of the fluctuations in interest rates.

After conducting a portfolio analysis on fifty realty properties and some common stocks, Friedman commented that:

> . . . real estate portfolios can have more return and less risk than do common stock portfolios. When the two assets are combined, the real estate assets dominate the resultant portfolios. On an after-tax basis these results are more apparent. [8:873]

(8) LIQUIDITY

Liquidity is traditionally thought to be a discouragement rather than a motivation for real estate investment. This could be a myth. Whether real estate is more liquid than others depends on the kind of investment real estate is to be compared with and on how liquidity is defined. For example, an investment in a company whose stock is not traded on the stock exchange could be as illiquid as an investment in the office building which houses the company.

Liquidity can be defined as the ability to obtain cash when needed. But how should cash be defined? Should it be initial equity in current

dollars, or initial equity in real dollars, or outstanding balance of equity involved, or market value?

If initial equity is used as a liquidity criterion, real estate may out-perform stocks and bonds especially in real dollar terms when the holding period is long enough such that taxes and selling expense from resale can be absorbed by the resale price increase and mortgage principal payments (consider Equation 4.9). Stanley F. Miller Jr's study on real estate price index and Dow Jones Industrial (D.J.I.) average from 1969 to the first quarter of 1977 [15:22] showed that D.J.I. average fluctuated and achieved a small percentage growth of 5% in terms of current dollars from 1968-1977. (D.J.I. average rose to 1040 and 1000 in 1972 and 1976, but had dropped to 600 in 1974). It is clear that the long term 5% increase of D.J.I. average was far behind the 72% increase in the consumer price index which rose from 109 to 187 during the same time period. Thus, to obtain initial equity by selling stocks and bonds in inflationary times is very difficult. Even in current dollars' terms, the resale price of stocks and bonds may fall well below the purchase price in a depressed market such as that occurred in 1974. On the other hand, the average price of a single family residence and the market value of non-residential property achieved increases of 115% and 100% respectively for the same period, which were substantially higher than the 72% increase of the consumer price index. As was demonstrated earlier in the discussion of capital appreciation potential, capital appreciation on equity for leveraged properties can be much higher than increases in the purchase price. Such high equity appreciation provides a large margin that allows resale price cutting for increasing the ability to obtain initial equity when needed even in terms of real dollars. So, liquidity of real estate can out-perform stocks and bonds in the long run.

Stanley F. Miller Jr. commented on the positive aspect of real estate liquidity as follows:

> It is believed that entry into the real estate market of an increasing number of financial institutions and pension funds (the latter tend to purchase properties for all cash) will generally increase the liquidity of real estate investment .... These observations concerning real estate liquidity are more pertinent to a discussion of income producing property as opposed to land. [15:21]

On the negative side, it is believed that, in general, real estate liquidity decreases when the uniqueness and size of a project increases since less buyers will be available. However, if revenues are well guaranteed (e.g., tenants are AAA in grade), the effect of uniqueness on liquidity may be lessened. Nonetheless, liquidity can be both a motivation and discouragement for real estate investment.

So far, real estate liquidity has been compared herein with stocks and bonds (as reflected by average Dow Jones Industrial Index) in terms of initial equity only. But should we compare it in terms of market value? Or should we compare with a specific group of shares (e.g. BAA utility shares) which have similar dividend, or return and risk profile as that of real estate? Further study and research on how to compare and how to measure real estate liquidity is required.

(9) PRIDE OF OWNERSHIP

Pride of ownership can constitute a motivation to invest in real estate. But constituents of pride of ownership are hard to define and vary from one investor to another. Quality, or height, or size of building may constitute some sense of pride to one investor but not to another. Not included in the definition of pride of ownership as used herein is the satisfaction derived by the investor because of the profitability of the project. This is already considered by other motivations. Literature on pride of ownership is scarce. Thus, no attempt is made

herein to investigate this topic in detail.

The effects of pride of ownership on the cost and designs of buildings can be significant. A study done in 1972 by Steyert [25:150-154] indicated that the total per square foot cost of a building increases with building height after the economical building height of the project is passed. For example, the economical building height was about 12 stories for a four-hundred unit apartment project. This study showed that the total per square foot cost increased from $76 to $84 in 1972 dollars when the building height increased from 12 stories to 36 stories. Thus, if an investor is motivated by pride of ownership of the tallest building in town, he may have to pay a premium for it.

4.2 PERFORMANCE MEASURES

Many performance measures are developed and used to measure the desirability of real estate investments. Although many of them are deficient in one or more ways, they are still used mainly because they can be readily and easily computed. Fourteen of the more commonly used performance measures are described herein. Most of the measures are presented in the following three step format: first, a literal definition is given; then, the measure is expressed in equation form; and finally, it is interpreted in terms of the information it provides for the economic analysis of the project for the equity-investor.

The literal definition and equation format are presented to avoid ambiguity. For example, most of the performance measures can be defined quite differently depending on whether they are before-tax or after-tax, before-finance or after-finance, and non-discounted or discounted. Unless otherwise specified, after-tax after-finance measures will be used throughout the rest of this chapter in line with the viewpoint of equity-investor

adopted.

To describe the performance measures with equations, two terms will be used from time to time. They are (after-tax after-finance) net resale cash reversion, NRCR, and interim cash flow from operation, ICF. The previously defined Equation 4.9 can be used to obtain NRCR:

$$NRCR_n = SP_n - SE_n - CGT_n - RCT_n - UM_n \qquad (4.9)$$

where CGT and RCT can be obtained from Equations 4.6, 4.7, and 4.8. ICF in year j can be obtained from:

$$ICF_j = EGI_j - OPE_j - TAX_j - DS_j \qquad (4.10)$$

where DS is debt service (mortgage interest plus mortgage principal payment), and income tax (TAX) can be obtained from the previously-defined Equation 4.5:

$$TAX_j = (EGI_j - OPE_j - DEP_j - INT_j) \times TAXR_j \qquad (4.5)$$

(1) AVERAGE ANNUAL RATE OF RETURN

It is defined as the ratio of average annual non-discounted cash flow to initial equity investment, E_o, over a specified holding period n. In equation form:

$$\text{average annual rate of return} = \sum_{j=1}^{n} ICF_j \,/\, (E_o \times n) \qquad (4.11)$$

It can be used to measure equity productivity, but the measure obtained gives no consideration to the time value of money.

(2) PAYBACK PERIOD

Payback period, PP, is the time required for cumulative non-discounted cash flow to equal the initial equity investment. To solve for PP, the following equation can be used:

$$0 = -E_0 + \sum_{j=1}^{PP} ICF_j \qquad (0 \leq PP < n) \tag{4.12}$$

$NRCR_n$ can be added to the right hand side of the equation if PP is equal to the holding period n. PP can be used to reflect how long the initial equity investment in terms of current dollars will be exposed to risk; i.e. how long it will take to recapture E_0 in terms of current dollars. Note that this definition of PP does not take into account the time value of money.

(3) RATE OF RETURN ON TOTAL COST

It can be defined as after-tax before-finance internal rate of return. To solve for rate of return on total cost, IRR_{BF}, Equation B3 derived in Appendix B can be used:

$$0 = -TC + \sum_{j=1}^{n} \frac{BFICF_j}{(1+IRR_{BF})^j} + \frac{BFCR_n}{(1+IRR_{BF})^n} \tag{4.13}$$

The notation in Equation 4.13 is defined in Appendix B.

(4) OVERALL RATE

Overall rate can be defined as total cost dividend rate which is equal to net income before recapture (typical year's EGI minus typical year's OPE) divided by total cost. (See also overall rate in [16:40]).

In equation form:

$$\text{overall rate} = \frac{\text{EGI} - \text{OPE}}{\text{TC}} \qquad (4.14)$$

It can be used to measure the productivity of the total investment in a project based on a typical year's net income, before-tax and before debt service. It gives no consideration to time value of money, cash reversion from resale, etc.

(5) EQUITY DIVIDEND RATE (BEFORE TAX)

Stehm defined equity dividend rate as the ratio of before-tax typical year's yearly cash flow to the amount of equity invested [23:23]. In equation form:

$$\text{equity dividend rate} = \frac{\text{EGI} - \text{OPE} - \text{DS}}{E_o} \qquad (4.15)$$

where EGI, OPE, and DS for a typical year. Equity dividend rate is intended to be used to measure return on equity, but it gives no consideration to recapture of initial equity, time value of money, future changes in income expenses, etc.

(6) TAX SHELTER

As described previously, there are two kinds of tax shelter measures: tax losses tax shelter and productivity concerned tax shelter. These measures were described in Section 4.1 (Equations 4.5 - 4.8). In this section, the purpose of these measures is given.

Tax losses tax shelter is usefully presented in vector form so that the magnitude of tax shelter for each operating year is known. This mode of presentation is important for financial analysis in order to pin-point both the timing and magnitude of the tax shelter that the investor can obtain

from the project to offset his taxable income from other sources. (See also [27:75].

Productivity concerned tax shelter can be used to identify the effects of tax shelter (from favourable tax regulations) on project productivity.

(7) INTERIM CASH FLOW RATES

Interim cash flow is cash flow derived from operation of the project. Of interest are interim cash flow ratios which equal non-discounted interim cash flows divided by initial equity investment, E_o. They should be presented by a vector, such that the rate for every year under consideration is shown. In equation form:

$$\text{ICF Rate for year } j = ICF_j/E_o \tag{4.16}$$

where ICF_j can be obtained from the previously defined Equation 3.10. Interim cash flow rates can be used to reflect quality of cash flows in terms of timing and relative magnitude of cash flow to equity.

(8) NET RESALE CASH REVERSION

It is non-discounted net cash reversion due to resale of the property at the end of the holding period. From Equation 3.9, net resale cash reversion, $NRCR_n$, is written as:

$$NRCR_n = SP_n - SE_n - CGT_n - RCT_n - UM_n \tag{4.9}$$

It can be used to measure capital appreciation on equity (see Section 4.1).

(9) INTERNAL RATE OF RETURN ON EQUITY (AFTER-TAX)

Mathematically, it can be defined as the discount rate that equates the expected present value of future cash flows to the expected present value of initial equity

investment given a specific holding period. To solve for the internal rate of return on equity, IRR, the following equation can be used:

$$0 = -E_o + \sum_{j=1}^{n} \frac{ICF_j}{(1 + IRR)^j} + \frac{NRCR_n}{(1 + IRR)^n} \qquad (4.17)$$

It can be used to measure productivity of equity when it is involved in the project.

(10) NET PRESENT VALUE

NPV equals the sum of the expected present value of future cash flows minus that of expected initial equity investment for a specific holding period. To discount the future dollars to present value, a prespecified discount rate DCTR which corresponds to the investor's minimum attractive rate of return is used. In equation form:

$$NPV = -E_o + \sum_{j=1}^{n} \frac{ICF_j}{(1 + DCTR)^j} + \frac{NRCR_n}{(1 + DCTR)^n} \qquad (4.18)$$

NPV can be used to compare the present value equivalence between the income and outcomes (disbursements) of a project at a prespecified discount rate.

(11) DISCOUNTED BENEFITS TO COST RATIO

It is equal to present value of expected future cash flows divided by that of initial equity investment given a specific holding period and discount rate. In symbolic form, discounted benefit to cost ratio, BCR, can be expressed as:

$$BCR = 1 + \frac{NPV}{E_o} \qquad (4.19)$$

It can be used to measure profitability of initial equity invested based on the given discount rate.

(12) LEVERAGE

Leverage, LEV, can be defined as the total cost to initial equity investment ratio. In symbolic form:

$$LEV = TC/E_0 \tag{4.20}$$

Leverage can be used to reflect the extent of the ownership and property rights controlled by the equity investment.

(13) INITIAL EQUITY

Initial Equity, E_0, can be defined as total cost minus financing, which in most cases is obtained by way of one or more mortgages, MTG. Thus,

$$E_0 = TC - MTG \tag{4.21}$$

This initial equity requirement can then be compared to available equity constraints.

(14) LOAN COVERAGE RATIO

It is computed by dividing the expected typical year's before-tax annual operating income by annual debt service. In equation form:

$$\text{loan coverage ratio} = \frac{EGI - OPE}{DS} \tag{4.22}$$

Loan coverage can be used to measure the risk of having negative before-tax cash flow and more importantly, the risk of being unable to meet the debt service payment on a before-tax basis. It gives no considerations to future changes in revenues and expenses and income tax.

4.3 SUITABILITY OF PERFORMANCE MEASURES FROM THE IRR AND NPV MODELS FOR REFLECTING INVESTOR MOTIVATIONS

Suitability of a model to reflect motivations depends not only on the performance measures made available by the model (model outputs), but also on the information inputs required by the model. In this section, the abilities of the aforementioned fourteen measures to reflect the nine motivations are examined by classifying them into three groups according to their ability to express explicitly how effectively a project will fulfill one or more investor motivations. Then, performance measures from the IRR and NPV models are identified and set against investor motivations to determine the suitability of the IRR and NPV models for reflecting them.

Montgomery commented on reflecting investor motivations as follows:

> Investment analysts face the challenge of mirroring and quantifying these concepts (of investor motivation) in each investment opportunity. If they do so effectively and consistently, they will become not only the world's best investment analysts, but also its most knowledgeable investors.*

An attempt has been made in Table 4.1 to wed (match) investor motivations with performance measures. It is the author's view that more work can be done on verifying and refining it. A measure or a set of measures are assessed as being able to reflect a motivation when there is a direct or known way to reflect the motivation by the measure(s).

Motivation reflectability can be classified into three classes: namely 'portray explicitly', 'some consideration', and 'no consideration'. Taking the motivation of capital appreciation potential for example, in

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*Montgomery, J. Thomas, "Investor Motivation and Investment Risk Analysis", in Financing Income-Producing Real Estate: A Theory and Casebook by James A. Britton Jr. and Lewis O. Kerwood, New York, New York: McGraw Hill, 1977.

Table 4.1 the measures net resale cash reversion and initial equity investment, together, are determined as being able to portray explicitly this motivation. In some cases, more than one measure are required to portray a motivation completely.

Net present value is determined as having some consideration on the same motivation in Table 4.1. In this case, NPV is influenced by capital appreciation but measures it in combination with other factors. Overall rate is determined as having no consideration on the same motivation in Table 4.1. It is clear from Equation 4.14 that capital appreciation is not incorporated in the calculation of the overall rate.

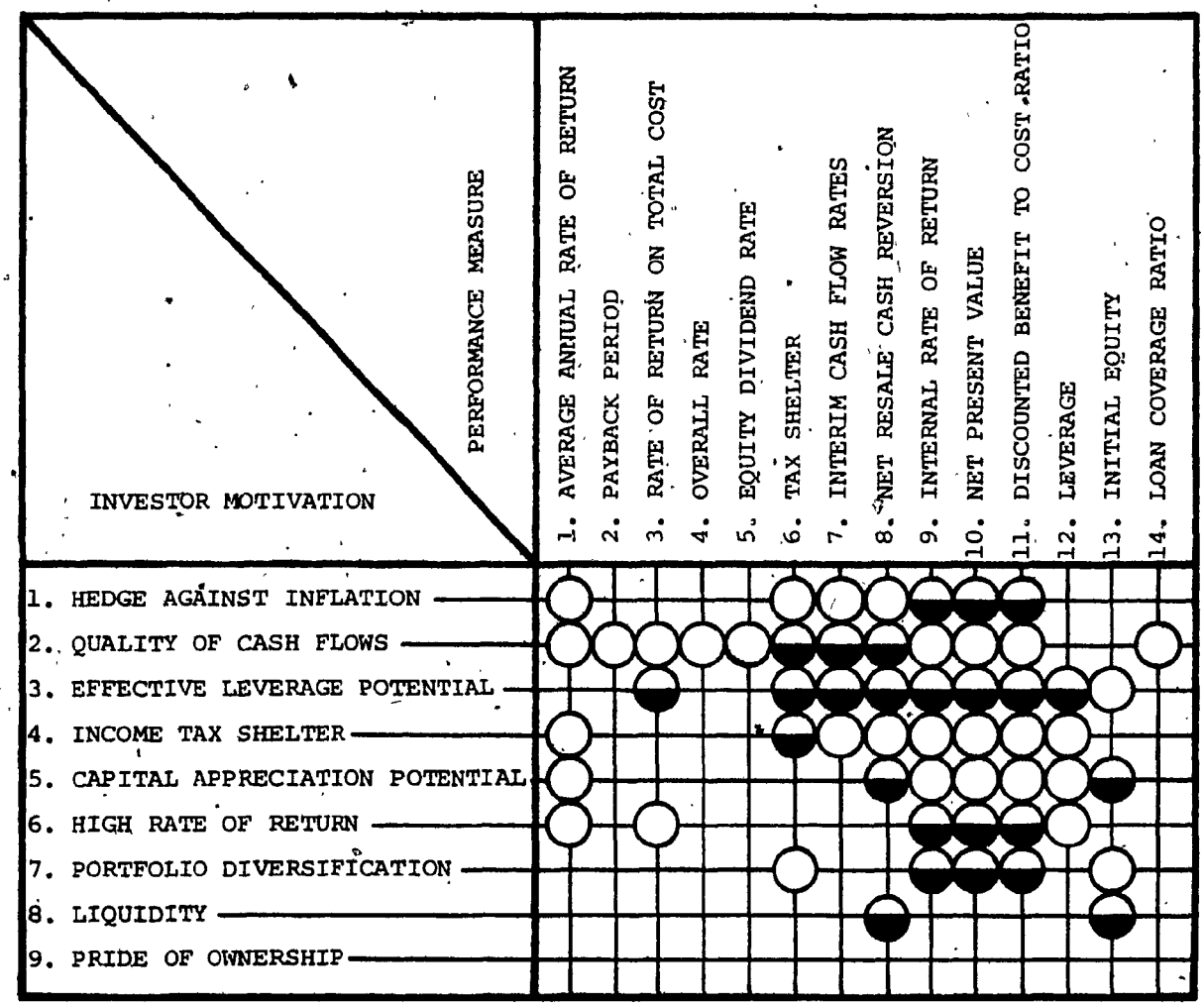
Quality of cash flows is considered unable to portray explicitly by before-tax equity dividend rate since it has no considerations on income tax and future escalation of revenues and expenditures.

Portfolio diversification can portray explicitly by, say, the internal rate of return on equity in the sense that if the risk profile and cash flows of the original portfolio are known, then they can be input with the data of the subject property under study into a probabilistic IRR model to yield the expected internal rate of return and its standard deviation for the overall portfolio. Thus, the effectiveness of spreading risk through portfolio diversification, an important investor motivation, can be measured directly.

Liquidity can portray explicitly by net resale cash reversion and initial equity if liquidity is defined as the ability to obtain initial equity when needed.

Pride of ownership cannot be reflected by the fourteen measures mainly because of its qualitative or intangible character.

Summarizing Table 4.1: Wedding Investor Motivations with Performance Measures, the following comments are made:



LEGEND: ○ THE MEASURE HAS SOME CONSIDERATION ON THE MOTIVATION.
● THE MEASURE(S) CAN PORTRAY EXPLICITLY THE MOTIVATION.

TABLE 4.1 WEDDING INVESTOR MOTIVATIONS WITH PERFORMANCE MEASURES

(i) Eight of the nine investor motivations can be reflected and portray explicitly by the fourteen performance measures, while the remaining motivation, pride of ownership, cannot, mainly due to its intangible character.

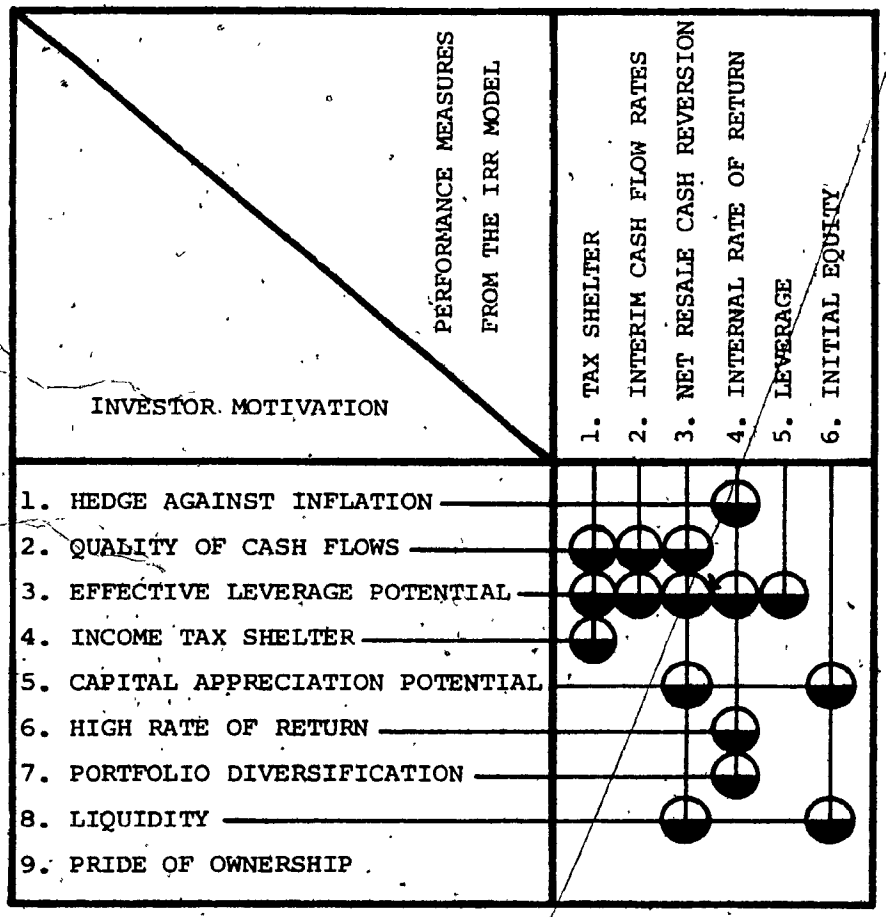
(ii) Only nine of the fourteen measures portray explicitly investor motivations; the remaining five measures do not add additional information on the motivations.

(iii) The most powerful measures are the internal rate of return on equity, net present value, and discounted BCR. Each of them can portray explicitly four motivations. As well, they give some consideration of three other investor motivations. (The NRCR measure requires other measures to portray the quality of cash flow and effective leverage potential motivations completely).

Suitability of the internal rate of return model to reflect investor motivations depends on both the measures made available by and for the model. Of the fourteen performance measures, six are made available by and for the IRR model:

i) tax shelter,
ii) interim cash flow rates,
iii) net resale cash reversion,
iv) IRR on equity,
v) leverage, and
vi) initial equity.

To examine the suitability of the IRR model to reflect investor motivations, the above six measures from the IRR model are set against the motivations in Table 4.2. The measures from the IRR model are connected by lines with the motivations which can portray explicitly by the



LEGEND: ◒ THE MEASURE(S) CAN PORTRAY EXPLICITLY THE MOTIVATION.

TABLE 4.2 SUITABILITY OF THE PERFORMANCE MEASURES FROM THE IRR MODEL FOR REFLECTING THE INVESTOR MOTIVATION

measures. For example, net resale cash reversion and initial equity measures from the IRR model are used to portray explicitly the capital appreciation potential motivation.

The six performance measures from the IRR model can portray explicitly eight of the nine investor motivations, as seen in Table 4.2; the remaining pride of ownership motivation cannot be reflected by any one of the fourteen more commonly used measures.

The remaining three measures which can portray explicitly the motivations are rate of return on total cost, NPV, and discounted BCR. Rate of return on total cost can be used with leverage to portray explicitly the effective leverage potential motivation, but they can be replaced by using IRR on equity and leverage (by means of sensitivity analysis). Presenting NPV and/or discounted BCR on top of presenting IRR on equity does not provide additional information.

The internal rate of return model is considered well suitable to portray key investor motivations. Using more performance measures on top of the six measures from the IRR model does not result in more investor motivations being treated.

Performance measures from the NPV model are similar to those from the IRR model, except the measure IRR on equity will be replaced by the measure of net present value. NPV and IRR on equity have the same motivation reflectability on the nine motivations.

CHAPTER FIVE

A CASE STUDY

INTRODUCTION

In keeping with Subobjective 5.0, a case study of a feasibility study for a small scale acquisition project is presented herein, in order to illustrate the use of the IRR model in a feasibility study and its related economic analysis.

The project examined consists of an income-producing property and a restaurant. The property and the restaurant are offered for sale as a package in the form of all the assets of China Garden Cafe Ltd. The acquisition of the property separate from the restaurant is not feasible. From an economic viewpoint, the financial and economic performances of the property and the restaurant must be analyzed separately and jointly. The individual analyses of the restaurant and the property are referred to as the Restaurant and the Property (which includes land and attached buildings) respectively. The combined studies on them are referred to as the Company.

In the first section of this chapter, the scope of the feasibility study is identified. The physical and financial description of the Company and its assets are documented in the second section. The third section examines the specific objectives of the prospective buyer. Finally, the economic analysis of the Company and its components in terms of the economic objectives of the investor are presented in the fourth section.

5.1 SCOPE OF FEASIBILITY STUDY

The objectives of the feasibility study are:

1) to identify and evaluate the economic performance of the Company and its assets; i.e. the Property and the Restaurant; and
2) to provide investment recommendations and a plan of action.

A limited time of three weeks was available for conducting the feasibility study. In addition, human and financial resources were highly constrained. The main focus of the analysis is on the Property and the Company. In conducting the analysis, the author received some help from the investor's consultants with respect to the restaurant operation aspect of the Company.

5.2 BACKGROUND INFORMATION

Financial and physical description of the Company, Property, and Restaurant, are documented in the following three subsections, respectively.

5.2.1 THE COMPANY

The Company, China Garden Cafe Ltd., has two major assets; namely the Property and the Restaurant. Combined information pertaining to the two assets, that is the Company, will be given in this subsection. Separate data on the two assets are documented in Subsections 5.2.2 and 5.2.3.

5.2.1.1 FINANCIAL STATEMENTS

Financial Statements of the Company prepared by Duval Trudeau Chartier & Associés are presented in Appendix C. The financial statements as of the year ended March 31, 1977 include the following documents:

1) results of operations;

2) change in financial position;

3) notes to financial statements (on fixed assets and long-term liabilities);

4) schedule of operating expenses; and

5) continuity of fixed assets.

5.2.1.2 OWNERSHIP

The ownership of the Company is divided into 315 shares. Nine owners hold 30 shares each, and three owners hold 15 shares each. Out of the 315 shares, only 240 shares are up for sale, or 76.2% of the Company.

5.2.1.3 OUTSTANDING MORTGAGE

A $90,000 mortgage, at an interest rate of 7 3/4% is outstanding. This mortgage is serviced by way of semi-annual installments consisting of interest on the outstanding principle plus a $5,000 payment on principle.

5.2.1.4 ACQUISITION PRICE

The sales price for the whole company has been set at $340,000, which is equivalent to $259,048 for the 240 shares which are up for sale.

5.2.1.5 INCOME/EXPENDITURE

Both unadjusted and adjusted revenues and expenditures of the Company are presented in Table 5.1. The values of the revenues/expenditures are based on the financial statements in Appendix C; unless otherwise specified below:

Item #1: Unadjusted rental income of the Property is obtained by adding the estimated $18,000 rent of the Restaurant (see explanations on Item #16 below), to the $37,939 rental income from the

	Items	Unadjusted Revenue	Unadjusted Expenses	Adjusted Revenue	Adjusted Expenses
1)	Rental income of the Property	54,939		51,480	
2)	Total sales of the Restaurant	382,225		401,336	
3)	Purchase of the Restaurant		152,962		152,962
4)	Advertising		1,395		1,395
5)	Telephone		2,708		2,708
6)	Professional fees & expenses		2,745		2,745
7)	Salaries & wages		162,741		142,740
8)	Sanitation & uniforms		15,500		15,500
9)	Crockery & dishes		319		319
10)	Light, heating & power		13,962		13,962
11)	Maintenance & repairs		4,457		4,457
12)	Property taxes		15,263		15,263
13)	Licences		3,689		3,689
14)	Insurances		11,074		11,074
15)	General expenditure		742		742
16)	Restaurant's rental expense		18,000		18,000
17)	Royalty	2,000	1,000	2,000	1,000
	Before Income Taxes & Mortgage Payments				
18)	Total	439,164	406,557	454,816	386,556
19)	Profit	32,607		68,260	
	After Income Tax & Mortgage Payments				
20)	Mortgage principle payments		10,000		10,000
21)	Mortgage interest payments		7,586		7,556
22)	Before tax shelter income tax		8,152		17,065
23)	Tax benefits from interest payments	1,889		1,889	
24)	Tax benefits from depreciation allowances	983		3,839	
25)	Total	442,036	432,265	460,544	421,177
26)	After-tax cash flow	9,771		39,367	

Table 5.1 — Unadjusted and Adjusted 1977 Income/Expenditure of the Company.

"schedule of operating expenses" in Appendix C, and deducting $1,000 from the royalty received (see Item #17). Adjusted rental income of the Property is obtained by adding the estimated $18,000 rent of the Restaurant, with observed rental income which is based on actual monthly rent received in the month of July, 1978 times twelve. Residential rent received at the month was $1,700 while the commercial rent received which excludes the Restaurant's was $1,090.

ITEM #2: Adjusted total sales of the Restaurant are obtained by increasing the unadjusted sales by five percent as recommended by the investor and his consultants on restaurant operation.

ITEM #7: Adjusted salaries and wages are obtained by first separating the unadjusted expenses for the Property and the Restaurant which are $780 and $161,961, respectively. The expenses for the Property are obtained by multiplying by twelve the actual salary paid to the Property's superintendent for the month of July, 1978. Only the expenses for the Restaurant are to be adjusted and are based on the wage rates and minimum crew size suggested by the investor's consultants on restaurant operation (6 chefs at $230/week, 5 floor-managers at $150/week, and 6 miscellaneous employees at $100/week). The adjusted expenses are $2,730/week or $141,960/year and the adjusted salaries and wages equal $142,760 ($780 + $141,960). Both unadjusted and adjusted salaries and wages do not include administration expenses of the Company.

ITEM 11: Maintenance and repairs recorded in the "schedule of operating expenses" in Appendix C are $11,829 which include a major repair on the Property that is not expected to occur every year. No

detailed breakdown of the expenses are available. Thus, the following estimates based on interviews with the vendor are used: regular yearly expenses for the Restaurant and the Property are set at $1,000 each; the remaining $9,829 is to be spread over a four-year period as distributed expenses for major repairs of the Property ($9,929 ÷ 4 = $2,457).

ITEM #12: The $15,263 allocation for property taxes is based on the 1977 real estate taxes paid as recorded in the City's tax department (see Subsection 5.2.2.1).

ITEM #13: The $3,689 expense on licenses is obtained from deducting the above $15,263 property taxes from the expenses on taxes and licenses in the "schedule of operating expenses" in Appendix C.

ITEM #16: Rent of the Restaurant is estimated by the market comparison method since income and expense records for the Restaurant and Property are kept jointly. One of the commercial tenants of the Property is a Canadian restaurant (not the Restaurant) which pays a yearly rent of $9,000 and occupies about half as much floor area as that of the Restaurant. Thus, the rent of the Restaurant is estimated to be twice as much as that of the Canadian restaurant; i.e. $18,000 (see Subsection 5.2.2.2).

ITEM #17: The Canadian restaurant pays approximately $1,000 to the Company as a royalty for operating a restaurant on the Property. It is estimated that the Restaurant should pay approximately the same amount of royalty to the Property as well. Thus, the Property has a $2,000 revenue from royalty while the Restaurant has a $1,000 royalty expenditure.

ITEM #21: Outstanding mortgage balance as of 1977 was $100,000 which is $10,000 more than the 1978 mortgage balance in Subsection 5.2.1.3. Since mortgage interest is payable on a semi annual basis, mortgage interest for the year 1977 was equal to ($100,000 x 0.0775 x 0.5) plus ($95,000 x 0.0775 x .05); i.e., $7,556.

ITEM #22: Before tax shelter income tax is obtained by multiplying the profit from Item #19 by the 25 percent applicable tax rate of the Company.

ITEM #23: Tax benefits from mortgage interest payments equal the product of the mortgage interest payments from Item #21 and the 25 percent tax rate.

ITEM #24: Unadjusted tax benefits from depreciation allowances are obtained by multiplying the depreciation allowances from "continuity of fixed assets" in Appendix C by the 25 percent tax rate. The adjusted tax benefits are obtained by multiplying the 25 percent tax rate with the book values of the following three capital cost allowance categories: i) restaurant appliances at $40,000 with 20% CCA rate; ii) goodwill of the Restaurant at $40,000 with 5% CCA rate; and iii) building at $107,000 with 5% CCA rate (the assessment of the building according to the city's taxation department was $107,000 as of 1977). The above adjustments are based on the assumption that a new company will be formed to take over the China Garden Cafe Ltd. and that the base for depreciation of the capital assets will be reassessed.

5.2.2 THE PROPERTY

The property is identified as the parcel of land located at 1240-46 Stanley Street, Montreal, P.Q., Canada (H3B 2S7) and the three buildings attached thereupon.

5.2.2.1 LEGAL AND PHYSICAL DESCRIPTIONS

The Property bears the following legal descriptions: Lot Number 1503-4 Quarter St. Antoine and Lot Number Pt. 1503 Quarter St. Antoine.

Taxation account number of the Property is 14,068,800. Assessments for tax purposes as of 1977 are $172,900 for the land and $107,100 for the building upon; a total of $280,000. Total 1977 property taxes payable was $15,263.

Size of the land is 63.4 ft x 115 ft (depth) approximately. Total land area is approximately 7,401 square feet. On top of the land are three connected old mixed commercial and residential buildings. One of them is four-stories in height while the remaining two are three-stories in height. The space within the buildings can be subdivided into three groups according to their usage:

1) ground floor is used for commercial purposes and has a gross floor area of 7,400 sq. ft.;
2) all upper floors are used for residential purposes and have a gross floor area of 16,370 sq. ft.; and
3) a basement with low clear ceiling height is used for storage and has a gross floor area of 7,400 sq. ft.

5.2.2.2 OCCUPANTS AND RENTAL INCOME

Occupants of the ground floor commercial space and their rents are (rents are based on actual rent received in the month of July, 1978 times

twelve):

1. China Garden Cafe, the Restaurant, occupies approximately 4,200 sq. ft. and its rent is estimated to be $18,000/year;
2. a Canadian restaurant occupies approximately 2,300 sq. ft. and its rent is $9,000/year; and
3. a studio occupies approximately 900 sq. ft. and its rent is $4,080/yr.

The total yearly rental income from the commercial spaces is $31,080 which yields an average commercial rate of $4.2/gross sq. ft./year ($31,080/year ÷ 7,400 sq. ft.).

Residential spaces of the upper floors are subdivided into twenty-six rooms and apartments. The yearly effective rental income is estimated to be $20,400 based on the $1,700 rent received for the month of July, 1978. Thus, the average residential rent rate is $1.25/gross sq. ft./year ($20,400/year ÷ 16,370 sq. ft).

The basement is presently used for storage and generates no income.

Vacancy rate and tenant turnover rate are expected to be low, at least for the coming few years because of the following conditions:

1) the three tenants of the commercial spaces have been leasing their spaces for more than seven years. They have heavy non-moveable investments in the buildings and have developed sound but geographic dependent businesses. Their average rent rate of $4.2/sq.ft./year is lower than that of competitive commercial spaces around; and
2) a good portion of the residential tenants are senior citizens who have been living in the building for the last decade. The past record of both residential vacancy rate and turnover rate are low. Furthermore, the $1.25/sq.ft./year residential rent rate that they are enjoying

now is substantially below that of comparable units in the downtown area.

5.2.2.3 ZONING BY-LAWS RESTRICTIONS

The following zoning by-law restrictions are based on careful interviews with the City of Montreal's Permit Department and Planning Department:

1) occupancy can be residential and/or commercial which includes entertainment centers such as bar, dancing hall, etc.;
2) maximum building height is 80 feet while that of most of the surrounding properties are about 40-45 feet only;
3) there is no set back requirement for new development; and
4) building site area for any new development is to be less than 60% of land coverage for residential development but there are no restrictions on commercial development.

5.2.2.4 PARTITION OF ACQUISITION PRICE AND MORTGAGE

Determination of the value and the amount of mortgage of the Property and the Restaurant apart from that of the Company is a prerequisite for analyzing the individual productivity of the Property and the Restaurant. This partitioning task is difficult because of the inseparable nature of the property and the restaurant, and hence the results are approximate only. For the economic analysis, the market data comparison approach will be used to assess the acquisition price of the Property. The acquisition price of the Restaurant will be assessed by deducting the acquisition price of the Property from that of the Company.

Given the character of the Property, time constraints, and availability of sales data, an abridged market data comparison analysis in conjunction with interviewing other real estate professionals was performed

in order to appraise the acquisition price of the Property. Other methods of assessing value are described in references [24:79-95] and [11]. The abridged method is outlined below:

1) assessment ratio of the sample (in Figure 5.1) is defined as the assessment value of land divided by the acquisition price;
2) acquisition price of the property being appraised is then estimated by dividing its assessment value by the mean of the assessment ratio;
3) all properties included in the data base are located in the immediate neighbourhood of the property being appraised such that the need for price adjustments due to difference in sample location are minimized; and
4) all properties included have similar characteristics to those of the property being appraised in that they are either sales of land or land with a building having little economic value, again minimizing the need for adjustments.

A sample of sales comprising two sales of land (sales number 7 and 8) and six sales of land with old building(s) having comparatively small economic values are presented in Table 5.2. The locations of the sample are shown schematically in Figure 5.1, where the numbers in the circles correspond to the sale number in Table 5.2, and the rectangular box marked "P" at the center is the location of the case study property. The mean assessment ratio from Table 5.2 is 67%. Dividing the assessment value of land of the Property, $172,900, by the mean assessment ratio yields an appraisal value of the Property equal to approximately $260,000. Thus, the acquisition price of the Restaurant is determined as $80,000 (acquisition price of the Company, $340,000, minus $260,000).

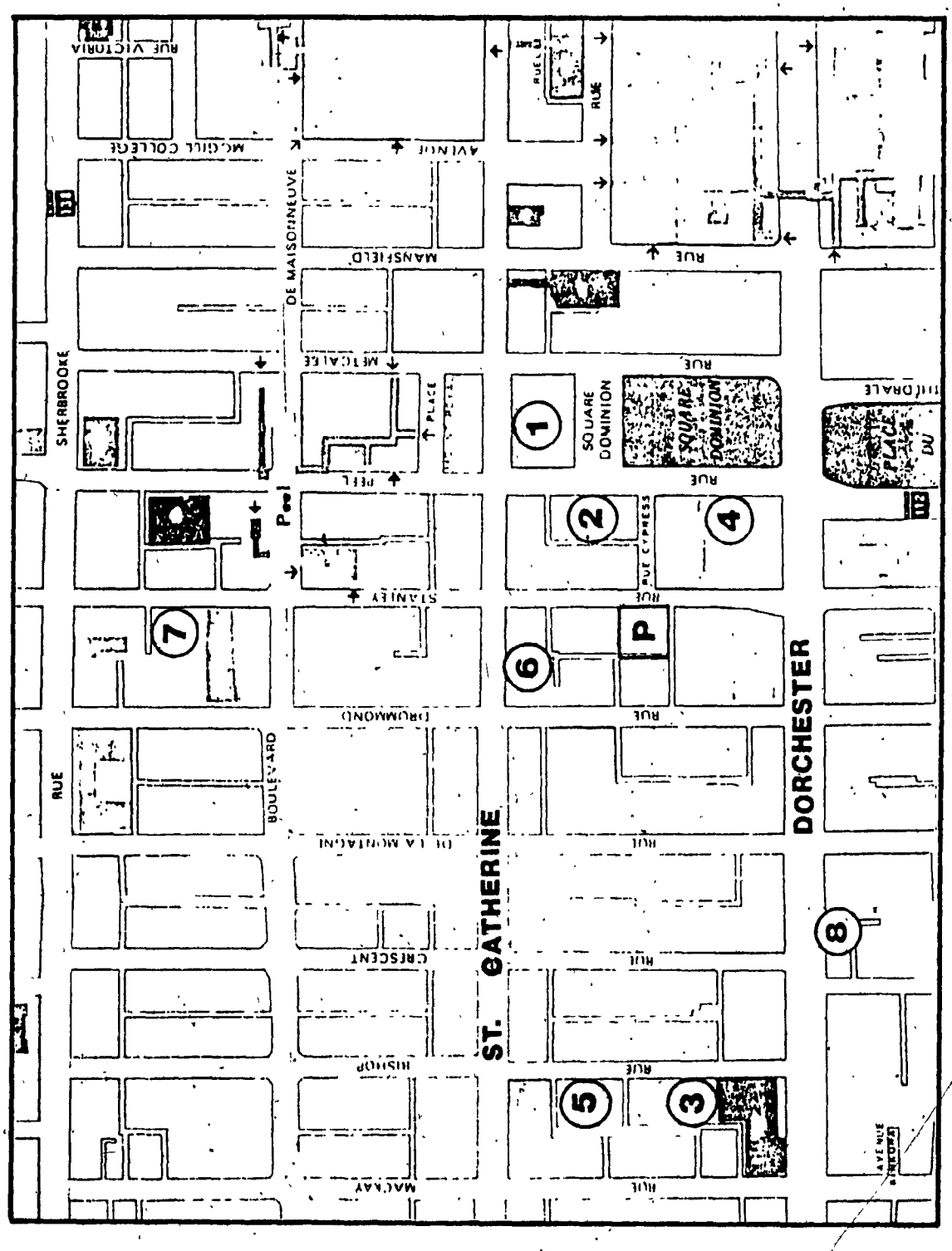


FIGURE 5.1 LOCATIONS OF SAMPLE USED IN MARKET DATA COMPARISON

Sale Number	Date For Sale	Acquisition Price	Assessment of land	Assessment Ratio
1	1975	$ 600,000	$ 303,850	51%
2	1971	245,000	167,850	69
3	1975	70,000	55,300	79
4	1975	350,000	152,550	44
5	1972	65,000	52,000	80
6	1975	500,000	212,100	42
7	1978	609,000	456,750	75
8	1978	70,000	64,650	92

The mean (average) of the assessment ratio is 67%. The standard deviation is 18%.

Table 5.2 — Sample of Sales for Market Data Comparison.

To avoid gross mistakes in the appraisal, the appraisal value of the Property was double checked with other real estate professionals. The City of Montreal's land evaluation officer for the specific area under consideration suggested that prices of adjacent land with similar utilities and traffic are approximately $29/sq. ft. The real estate agent contacted by the author suggested that it would be $31.80/sq. ft. The spread between the two opinions is less than 10% and the mean of the two rates is chosen for use herein. Multiplying the square footage of the Property (7,401 sq.ft.) by the mean yields a land value of approximately $225,000. Deducting the land value from the previous appraisal value of the Property implies a residual value of $35,000 for the old building.

Significant over-estimates of the value of the land and the old building is thought to be unlikely. Upward adjustment of the building's value will lead to a reduction in the appraised value of the Restaurant which is believed to be at its minimum. Thus, the appraised values of the Property and the Restaurant are assumed to be sufficiently accurate to form the basis of an economic analysis.

The division of the mortgage into what the Property and the Restaurant should carry respectively is based on two prime concerns of the lender when he considers approving a loan: i) the ability to pay back the mortgage principle in case of a default; and ii) the ability to generate cash to meet regular debt service payments. The ratio of the acquisition price of the Property to that of the Company is approximately three to four; however, the Restaurant is the basic mechanism for generating cash. In a very rough sense, two-thirds of the $100,000 mortgage will be assigned to the Property's responsibility. Thus, the Property and the Restaurant will carry a $66,667 and $33,333 mortgage, respectively.

5.2.2.5 INCOME/EXPENDITURE

Both unadjusted and adjusted revenues and expenditures of the Property are presented in Table 5.3. The values for the income/expenditure items in Table 5.3 are either prorated or directly transferred from the corresponding values in Table 5.1. An identical presentation format is used for Tables 5.1 and 5.3 to allow direct comparison. Consequently, some of the items used in Table 5.3 are not applicable (n.a.) to the operation of the Property. Notes on the breakdowns of the values which are not explained in Subsection 5.2.1.5 are given below:

	Items	Unadjusted Revenue	Unadjusted Expenses	Adjusted Revenue	Adjusted Expenses
1)	Rental income of the Property	54,939		51,480	
2)	Total sales of the Restaurant	n.a.		n.a.	
3)	Purchase of the Restaurant		n.a.		n.a.
4)	Advertising		n.a.		n.a.
5)	Telephone		n.a.		n.a.
6)	Professional fees & expenses		1,373		1,373
7)	Salaries & wages		780		780
8)	Sanitation & uniforms		n.a.		n.a.
9)	Crockery & dishes		n.a.		n.a.
10)	Light, heating & power		10,362		10,362
11)	Maintenance & repairs		3,457		3,457
12)	Property taxes		15,263		15,263
13)	Licences		n.a.		n.a.
14)	Insurances		5,537		5,537
15)	General expenditure		n.a.		n.a.
16)	Restaurant's rental expense		n.a.		n.a.
17)	Royalty	2,000		2,000	
	Before Income Taxes & Mortgage Payments				
18)	Total	56,939	36,772	53,480	36,772
19)	Profit	20,167		16,708	
	After Income Tax & Mortgage Payments				
20)	Mortgage principle payments		6,667		6,667
21)	Mortgage interest payments		5,037		5,037
22)	Before tax shelter income tax		5,042		4,177
23)	Tax benefits from interest payments	1,259		1,259	
24)	Tax benefits from depreciation allowances	848		1,339	
25)	Total	59,046	53,518	56,078	52,653
26)	After-tax cash flow	5,528		3,425	

Table 5.3 — Unadjusted and Adjusted 1977 Income/Expenditure of the Property.

Item 6: A detailed breakdown of the professional fees and expenses are not available from the vendor since operating records for the Property and Restaurant are not kept separately. Upon consultation with the vendor and the investor's consultant, it was decided to distribute the expenses equally among the Property and the Restaurant.

Item 10: The vendor stated that the Restaurant's 'light heating and power' expenses were approximately $3,600 for the year 1977. Deducting the expenses for the Restaurant from those for the Company ($13,962) yields the expenses for the Property, $10,362.

Item 14: Same as item #6 above.

Item 20: See Subsection 5.2.2.4.

5.2.3 THE RESTAURANT

The Restaurant is the China Garden Cafe located at 1240 Stanley Street, Montreal.

5.2.3.1 SIZE AND MARKET DIMENSION

The Restaurant occupies a gross floor area of about 4,200 sq. ft. It is a middle class Chinese restaurant whose major clients are not Chinese. The major clients during the day come from the surrounding business area, and in the evening they come from nearby discotheques and hotels.

5.2.3.2 PARTITION OF ACQUISITION PRICE AND MORTGAGE

Acquisition price of the restaurant is estimated to be $80,000, and the amount of mortgage it should carry is estimated to be $33,333; see Subsection 5.2.2.4 for the division of the acquisition price and mortgage.

5.2.3.3 INCOME/EXPENDITURE

Both unadjusted and adjusted revenues and expenditures for the Restaurant are presented in Table 5.4. The values and presentation format used in Table 5.4 are similar to Tables 5.1 and 5.3. Some of the terms used in Table 5.4 are not applicable (n.a.) to the operation of the Restaurant. Notes on the values used in Table 5.4 can be found in Subsections 5.2.1.5 and 5.2.2.5; item numbers for the entries in Table 5.4 are the same as for those in Tables 5.1 and 5.3. For further information on the financial status of the Restaurant, see the financial statement of the Company in Appendix C. It should be noted that major adjustments to the revenues and expenditures of the Restaurant are based on recommendations of the investor's consultants on restaurant operations and have been approved by the investor.

5.3 INVESTOR OBJECTIVES

Before conducting a detailed analysis of a project, the investor's objectives have to be identified (see Section 2.1) so that the analyst's efforts can be properly focused. The objectives of the investor are documented herein after a brief background description of him.

The investor is a private individual from Hong Kong who is seeking foreign investments in Canada as part of his plan for retiring in Canada. He will be retiring in a few years' time and is somewhat indifferent to immediate cash flow but is very concerned with overall project performance in the medium to long term. At the present he has no other investments in Canada and therefore he is not motivated by potential tax shelter from the Company to offset income from other sources.

Checking through each investor motivation as identified in Section 4.1 with the investor at hand, the following prime investment motivations

	Items	Unadjusted Revenue	Unadjusted Expenses	Adjusted Revenue	Adjusted Expenses
1)	Rental income of the Property	n.a.		n.a.	
2)	Total sales of the Restaurant	382,225		401,336	
3)	Purchase of the Restaurant		152,962		152,962
4)	Advertising		1,395		1,395
5)	Telephone		2,708		2,708
6)	Professional fees & expenses		1,373		1,373
7)	Salaries & wages		161,961		141,960
8)	Sanitation & uniforms		15,500		15,500
9)	Crockery & dishes		319		319
10)	Light, heating & power		3,600		3,600
11)	Maintenance & repairs		1,000		1,000
12)	Property taxes		n.a.		n.a.
13)	Licences		3,689		3,689
14)	Insurances		5,537		5,537
15)	General expenditure		742		742
16)	Restaurant's rental expense		18,000		18,000
17)	Royalty		1,000		1,000
	Before Income Taxes & Mortgage Payments				
18)	Total	382,225	369,786	401,336	349,785
19)	Profit	12,439		51,551	
	After Income Tax & Mortgage Payments				
20)	Mortgage principle payments		3,333		3,333
21)	Mortgage interest payments		2,519		2,519
22)	Before tax shelter income tax		3,110		12,888
23)	Tax benefits from interest payments	630		630	
24)	Tax benefits from depreciation allowances	135		2,500	
25)	Total	382,990	378,748	404,466	368,525
26)	After-tax cash flow	4,242		35,941	

Table 5.4 — Unadjusted and Adjusted 1977 Income/Expenditure of the Restaurant.

are identified: 1) hedge against inflation; 2) high rate of return; and 3) geographic diversification. Each of these motivations and related investor objectives are described below.

The first motivation, hedge against inflation, arises from the investor's concern that he maintains his purchasing power for his retirement. Upon examining with the investor the inflation rate in Canada over the past few years, an average inflation rate of 9.5%, was selected for use. This assumption has important implications for the selection of a minimum attractive rate of return.

The second investor motivation is high rate of return. Upon examining the collective and individual characteristics of the investment with the investor, the following three objectives were set:

1) minimum after-tax IRR of the Property must be greater than or equal to 9.5%;

2) minimum after-tax IRR of the Restaurant must be greater than or equal to 25%; and

3) minimum after-tax IRR of the Company must be greater than or equal to 20%.

Geographic diversification, the third motivation, is defined by the investor as diversifying geographically from Hong Kong where most of his investments are presently located. Studies pertaining to this motivation are mostly macro-market analysis and macro-economic analysis. They are not encompassed in the scope of economic analysis as defined earlier in Sections 2.2 and 2.3; but rather are performed in a macro-market analysis and Activity B1.9 - Conduct Other Studies in feasibility study (see Sections 2.1 and 2.2).

Economic analysis of project performances in terms of the investor objectives derived from the first and second motivations are documented in the next section. The third motivation is not treated.

The project analyzed is sufficiently small in capital requirements that the investor was not faced with budget or equity constraints.

5.4 ECONOMIC ANALYSIS

A computer program called CBEES was used to aid in the economic analysis for the case study. The limitations of this program are described in the first subsection below.

5.4.1 LIMITATION OF THE COMPUTER PROGRAM USED

The computer program used in the economic analysis of this case study is called CBEES; Canadian Building Economic Evaluation Study*. As far as application to the case study, the CBEES program was used for modifying the general IRR and NPV formulae and related equations as outlined in Section 3.4. The main unit of time employed by the program is a month. The major weaknesses of the program as they related to the type of acquisition project described herein deal with: 1) construction period, 2) rent-up period, and 3) mortgage financing details.

Construction and rent-up periods for an acquisition project are generally zero. However, the CBEES program requires non-zero entries. To minimize the impact on project performance from these discrepancies, minimum time periods allowed by the CBEES program were chosen for these items as follows:

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*The CBEES program is developed by the Canadian Division of the Portland Cement Association in cooperation with the Faculty of Administrative Studies, York University, Toronto, Ontario. The CBEES program used here has been reprogrammed by Concordia University.

a) construction duration equals 1 month;

b) time from start of construction until first revenue starts equals 1 month; and

c) minimum rent-up period equals 3 months.

This program requirement may cause irregular and unpredictable cash flow in the first year.

Another inaccuracy in the analysis results from the manner in which CBEES handles debt service payments and the way they are made in the actual case study. The schedule of debt service payments for the case study corresponds to a fixed amount of principle payment plus interest on outstanding mortgage balance, payable on a semi-annual basis. The CBEES model assumes equal debt service payments (principle plus interest) throughout the whole amortization period. Then differences in mode of mortgage repayment result in differences in cash flow and tax shelter from interest charges.

The significance of the above restrictions imposed by the CBEES model in terms of financial performance was not investigated. However, since the magnitude of net after-tax cash flow difference between the CBEES calculation and the exact calculation is expected to be relatively small as compared to the overall cash flow, it is assumed that the impact on economic performance will not be big.

5.4.2 ECONOMIC ANALYSIS OF THE PROPERTY

The primary objective of the economic analysis of the property is to determine if the IRR is greater than or equal to the minimum after tax return rate of 9.5%. Economic performance of the Property is first analyzed

in Subsection 5.4.2.1 based on data from Subsection 5.2.2 and some professional assumptions. After examining these results the scope of a sensitivity analysis is identified and performed with the results documented in Subsection 5.4.2.2. Finally, in Subsection 5.4.2.3, conclusions are drawn from the economic analysis.

5.4.2.1 FUNDAMENTAL ANALYSIS

Computer inputs for the fundamental analysis of the Property are summarized below (see also the inputs identified in Subsection 5.4.1):

1) The most likely holding period is 10 years. However, a 5-year period is also studied;
2) Acquisition price of the Property is $260,000;
3) Initial value of building is $35,000 and its yearly depreciation rate is 20%;
4) Initial value of the land is $225,000 and its yearly appreciation rate is 5%;
5) Selling expense is 7.5% of resale price;
6) Income tax rate is 25%;
7) Rental income of the Property is treated in 3 groups of income sources (Items #8-10). Zero vacancy and delinquency allowance rates are used since effective rental incomes are used;
8) Yearly commercial rent rate is $31,080, yearly rent escalator is 7%, and term of rent escalator is 3 yrs. (i.e. rent levels adjusted at 3 yrs. interval);
9) Yearly residential rent rate is $20,400, yearly rent escalator is 5%, and term of rent escalator is 1 year;
10) Yearly income from royalty is $2,000, its yearly escalator is 9.5%, and term of the escalator is 1 year;

11) Cash operating expenses are decomposed into 6 groups of expenses below (Item #12 - 17);

12) Professional Fees and Expenses are $1,373, and the yearly escalation is 8%;

13) Salaries and Wages are $780 and the yearly escalation is 9.5%;

14) Light, Heating and Power are $10,352 and the yearly escalation is 8%;

15) Maintenance and Repairs are $3,457 and the yearly escalation is 9%;

16) Property taxes are $15,263, and the yearly escalation is 7%;

17) Insurance is $5,537 and the yearly escalation is 9.5%;

18) Mortgage is $60,000 (two-thirds of the $90,000 outstanding balance) at a yearly interest rate of 7.75%, two equal payments per year over an amortization period of 9 years;

19) One type of asset is used to generate capital cost allowance which is building, its initial (depreciable value) book value is $107,700 (see discussion in Item #24 in Subsection 5.2.1.5). CCA rate used is 5%.

Key results of the fundamental analysis of the Property are:

1) Initial cash equity investment is $200,000;

2) After-tax interim cash flows from operation (ICF) and ICF Rates (ICF/E_0) are presented in Table 5.5 (the non-uniform pattern of ICF and ICF Rates is because of the yearly escalation clause used):

YEAR	ICF	ICF RATES
1	$ 2,568	.0128
2	4,093	.0205
3	2,445	.0122
4	4,419	.0221
5	4,200	.0210
6	2,065	.0103
7	4,368	.0218
8	3,972	.0199
9	1,219	.0061
10	8,617	.0431

Table 5.5 — ICF and ICF Rates of the Property

3) Net resale cash reversion of the property for holding periods of 5 and 10 years, respectively, are $248,609 and $336,044.
4) After-tax internal rate of return on equity for holding periods of 5 and 10 years, respectively, are 5.69% and 6.68% which is less than the investor's minimum attractive after-tax rate of return.

5.4.2.2 SENSITIVITY ANALYSIS

An assessment is made herein of the sensitivity of IRR to changes in key data inputs. After studying the inputs and outputs of the fundamental analysis, it was decided to conduct sensitivity analysis on the following four inputs:

1) holding period;
2) land appreciation rate, since it is highly uncertain, and the comparatively low ICF Rates (in comparison to IRR) indicates that capital

appreciation is a dominant component of IRR;

3) book value of building, since it has been adjusted upward 200% from $35,000 to $107,100; and

4) acquisition price, since it may be open to substantial negotiation.

Results of these four sensitivity analysis are presented in Figures 5.2 through 5.5; where 'n' denotes holding period.

Figure 5.2 indicates that the optimal holding period (where IRR is a maximum) does not lie in the 5 to 10 year domain selected for the study. However, the difference in IRR between the 10 and 15 year holding period is negligible.

IRR is extremely sensitive to land appreciation rates (LAR). Figure 5.3 shows that a one percentage point change in LAR yields an almost similar change in IRR. Thus, careful assessment of the LAR is crucial. It was performed by reviewing the market value of land with real estate agent and the City of Montreal's land evaluation officer for the specific area under study. The review led to the LAR being adjusted downward from 5% to 3%. According to the curves in Figure 5.3, the IRR's for the 5 and 10 year holding periods dropped to 3.91% and 5.15% respectively for a LAR of 3%/year. In percentage terms, the 2 point drop in LAR led to a 33% and 23% decline in IRR for the 5 and 10 year holding periods.

The sensitivity analysis for change in book value of the building versus IRR is shown in Figure 5.4. For holding periods of 5 and 10 years, the IRR will drop 0.56 and 0.31 percentage points, respectively, if the $64,400 unadjusted book value of building (from the financial statement of the Company in Appendix C) is used instead of the $107,100 taxation assess-

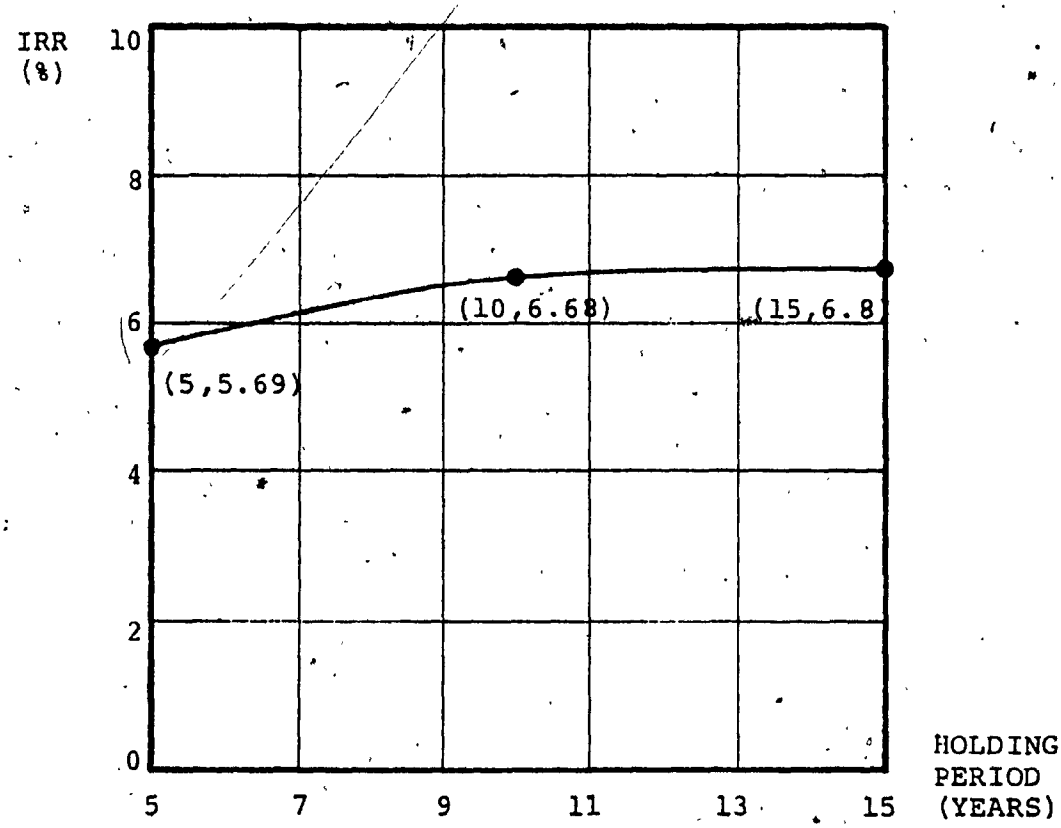

FIGURE 5.2 SENSITIVITY OF HOLDING PERIOD ON IRR OF PROPERTY

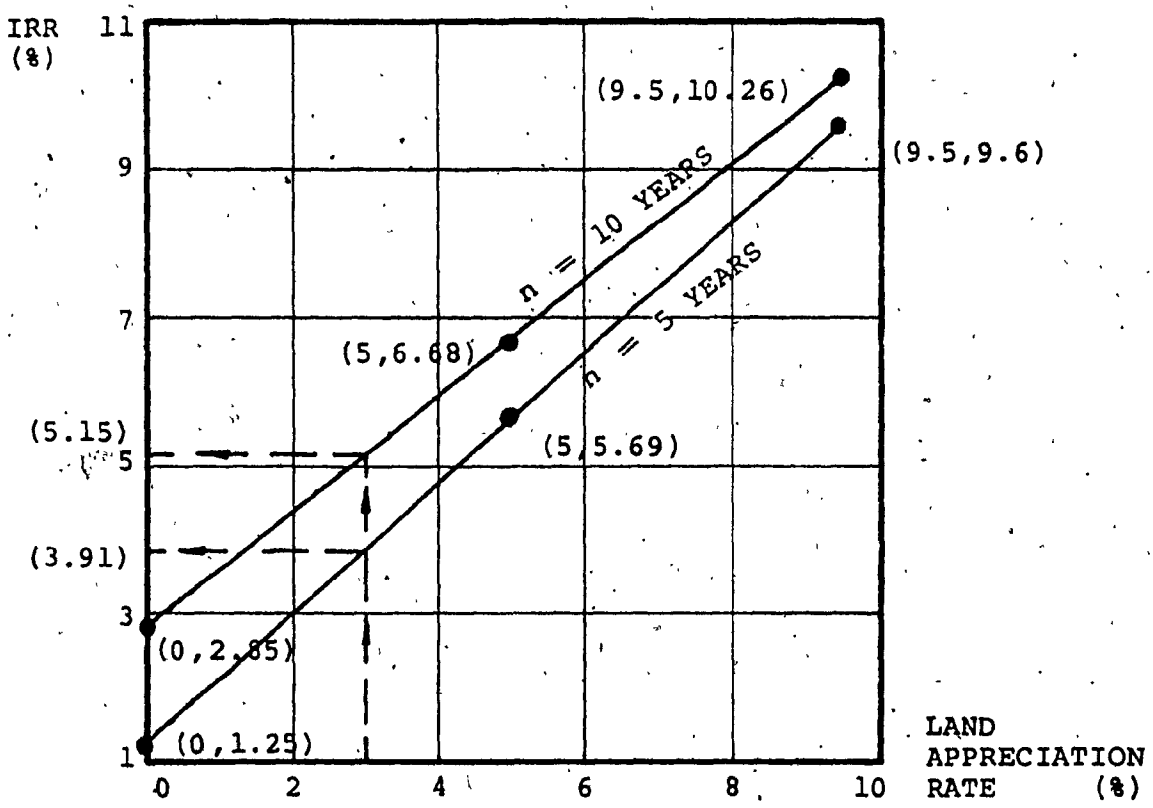

FIGURE 5.3 SENSITIVITY OF LAND APPRECIATION RATE ON IRR OF PROPERTY

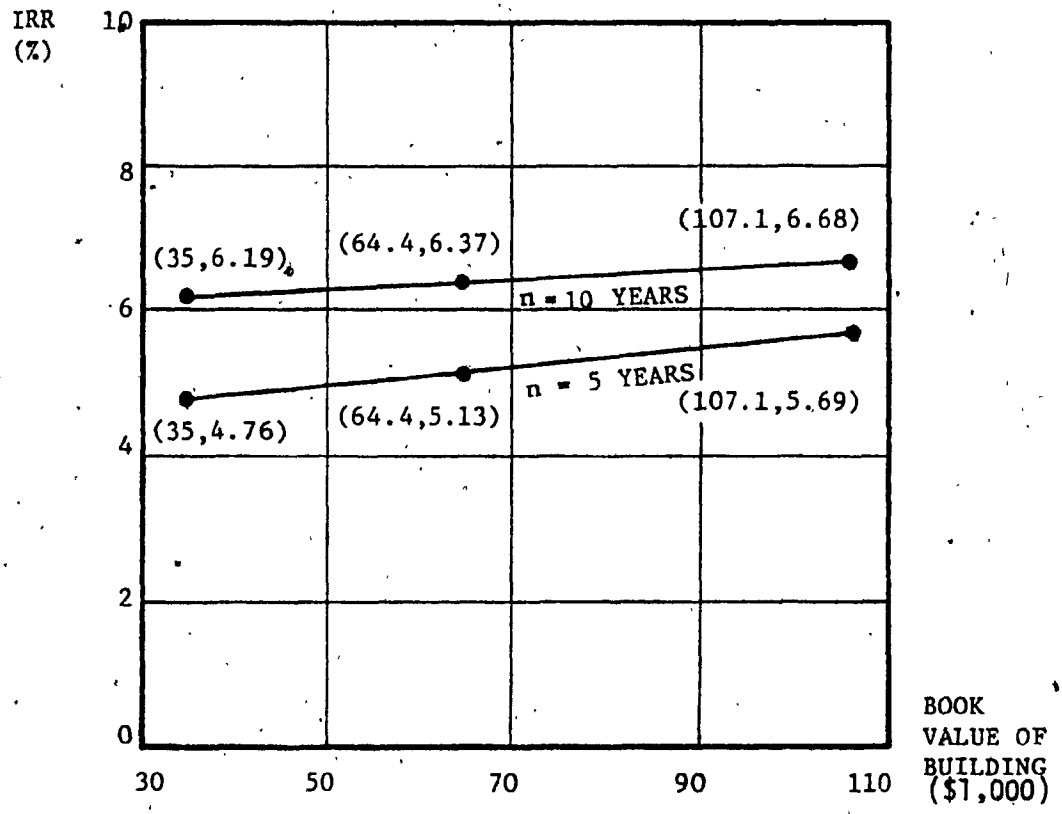

FIGURE 5.4 SENSITIVITY OF BOOK VALUE OF BUILDING ON IRR OF PROPERTY

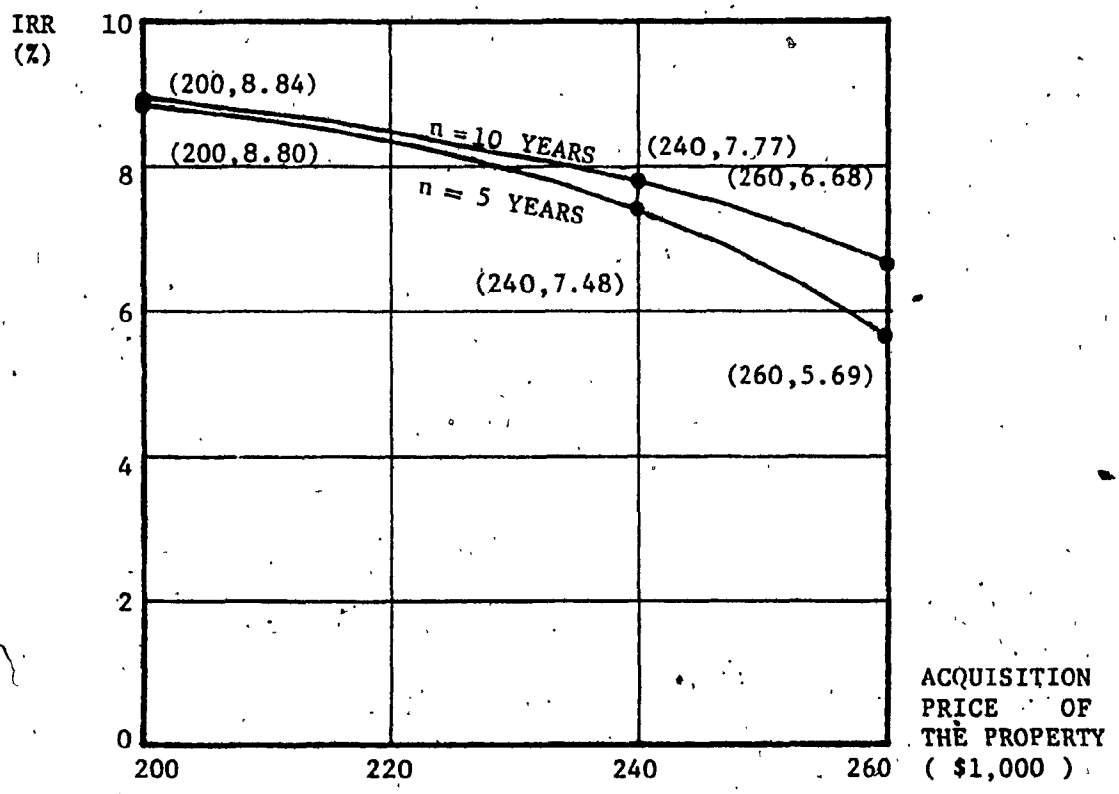

FIGURE 5.5 SENSITIVITY OF ACQUISITION PRICE ON IRR OF THE PROPERTY

ment value. Thus, economic performance is relatively insensitive to assumptions regarding book value of the Property.

The results of the sensitivity analysis of change in acquisition price of the Property versus IRR are shown in Figure 5.5. It is estimated that $240,000 is the most likely acquisition price for the property, some $20,000 below the initial estimate of $260,000. This drop in acquisition price results in IRR's of 7.48% and 7.77% for holding periods of 5 and 10 years, respectively — equivalent to a 31% and 16% increase in IRR. It is estimated that the most optimistic acquisition price would be $200,000 and the corresponding IRR's for 5 and 10 year holding periods would be 8.8% and 8.84% respectively.

5.4.2.3 ANALYSIS CONCLUSION

The sensitivity analysis suggests that a 10 year holding period is acceptable, and the IRR of the fundamental analysis of the Property should be adjusted considerably. Based on the refined LAR of 3%/year and the most likely acquisition price of $240,000 the IRR for a 10 year holding period for the Property would be approximately

$$IRR \doteq 6.68\% \times (1-23\%) \times (1 + 16\%) = 5.97\%.$$

By adjusting the IRR with the LAR of 3%/year and the most optimistic acquisition price of $200,000, the IRR is approximately equal to:

$$IRR \doteq 6.68\% \times (1 - 23\%) \times (1 + 32\%) = 6.79\%.$$

The adjusted IRR's of the Property cannot meet the investor objective of 9.5% minimum after tax rate of return for the Property. Therefore, the Property investment must be considered INFEASIBLE.

5.4.3 ECONOMIC ANALYSIS OF THE RESTAURANT

The restaurant is analyzed in order to assess its performance vis-a-vis the 25% minimum attractive rate of return criterion.

5.4.3.1 FUNDAMENTAL ANALYSIS

The computer inputs for the fundamental analysis of the Restaurant are summarized below (see also Subsections 5.2.3 and 5.4.2):

1) The most likely holding period is 10 years; a 5-year holding period is also studied;
2) The acquisition price of the Restaurant is $80,000 and the yearly depreciation rate is 20%;
3) Selling expenses are estimated as 7.5% of the resale price;
4) Income tax rate is 25%;
5) The yearly effective income of the Restaurant is $401,336 and it is assumed to increase at the rate of 9.5%/year;
6) The cash operating expenses for the restaurant are decomposed into 14 groups as described in items 7 to 20;
7) Purchases for the Restaurant are $152,962 and are assumed to increase at 9.5%/year;
8) Advertising is $1,395, and increases at 8%/year;
9) Telephone is $2,708 with a yearly escalator of 5%;
10) Professional Fees and Expenses are $1,373, and are assumed to increase at 8%/year;
11) Salaries and Wages are $141,960 with a yearly escalator of 9.5%;
12) Sanitation and Uniforms are $15,500 with a yearly escalator of 9.5%;
13) Crockery and Dishes are $319, with a yearly escalator of 9.5%;
14) Light, Heating and Power are $3,600 with a yearly escalator of 8%;
15) Maintenance and Repairs are $1,000 with a yearly escalator of 9.5%;

16) Licenses are $3,689 with a yearly escalator of 5%;

17) Insurance is $4,437, with a yearly escalator of 9.5%;

18) General Expenditure is $742, with a yearly escalator of 9.5%;

19) Restaurant's rental expense is $18,000 and is assumed to increase at 7%/year;

20) The annual royalty is $1,000, with a yearly escalator of 9.5%; and

21) Two types of assets are used to generate capital cost allowance: restaurant appliances at $40,000 with a 20% CCA rate; and goodwill of the Restaurant at $40,000 with a 5% CCA rate.

Results of the fundamental analysis of the Restaurant are as follows:

1) Initial cash equity investment is $50,000;

2) After-tax interim cash flows from operation (ICF) and ICF Rates (ICF/E_0) are presented in Table 5.6 below*;

YEAR	ICF	ICF RATES
1*	$ 48,676	0.9735
2	35,915	0.7183
3	41,041	0.8208
4	45,730	0.9146
5	51,011	1.0202
6	56,929	1.1386
7	63,535	1.2707
8	70,890	1.4178
9	79,062	1.5812
10	90,492	1.8098

Table 5.6 — ICF and ICF Rates of the Restaurant.

*The first year's ICF and ICF Rate are higher than they should be. It is caused by the three months absorption period assumption embodied in the CBEES program used.

3) Net resale cash reversion for holding periods of 5 and 10 years, respectively, are $10,900 and $10,507. (The low NRCR is based on the assumption that the old building which houses the restaurant will be demolished in 10 years' time and the restaurant will have to be relocated).

4) The after-tax internal rate of return on equity for holding periods of 5 and 10 years, respectively, are 125.14% and 125.6%.

5.4.3.2 SENSITIVITY ANALYSIS

The IRR of the Restaurant is four times higher than the 25% minimum required. Thus, a detailed sensitivity analysis does not appear to be required. However, two simplified analysis assuming a 5% gross over-estimate of effective income and a $50,000 gross under-estimate of acquisition price, respectively, were performed to check against gross errors.

The simplified method used consisted of:

1) Using a yearly discrete cash flow model instead of the monthly model used in the CBEES program; and
2) Using the figures of the interim cash flows provided by the CBEES model (Table 5.6).

The IRR of the Restaurant from the simplified analysis is 90%; a considerable under-estimation of the actual IRR.

The simplified analysis for an over-estimate of effective income by 5% is performed by deducting from ICF 5% of the $401,336 effective income used in the fundamental study. This deduction also influences the income tax paid. The deduction for the jth year is:

$$\text{Deduction}_j = \$401,336 \times 5\% \times (1\text{-}25\%) \times (1 + 9.5\%)^{j-1}.$$

Based on these deductions, the IRR for a 10 year holding period was determined to be 56%.

Combining the 5% gross over-estimate of effective income with a $50,000 gross under-estimate of acquisition price yields an IRR of 25.5% for a 10 year holding period.

5.4.3.3 ANALYSIS CONCLUSION

Thus, even considering the combined effects of pessimistic adjustments to key variables, the IRR for the Restaurant investment is still higher than the 25% minimum required. Consequently, the restaurant may be considered as a FEASIBLE investment.

5.4.4 ECONOMIC ANALYSIS OF THE COMPANY

The Prime investor objective to be considered in analyzing the Company (Property plus Restaurant) is the 20% minimum required IRR.

5.4.4.1 FUNDAMENTAL ANALYSIS

The results of the computer analysis of the Company are as follows:

1) Initial cash equity investment is $250,000.
2) After-tax interim cash flows from operation (ICF) and ICF Rates (ICF/E_0) are presented in Table 5.7.
3) Net resale cash reversion for holding periods of 5 and 10 years, respectively, are $259,509 and $346,550.
4) After-tax internal rate of return on cash equity for holding periods of 5 and 10 years, respectively, are 18.31% and 20.87%.

YEAR	ICF	ICF Rates
1	$ 51,243	0.2050
2	40,008	0.1600
3	43,486	0.1739
4	50,149	0.2006
5	55,211	0.2208
6	55,995	0.2240
7	67,902	0.2716
8	74,862	0.2974
9	80,281	0.3211
10	99,109	0.3964

Table 5.7 — ICF and ICF Rates of the Company.

5.4.4.2 SENSITIVITY ANALYSIS

A sensitivity study considering the highly uncertain land appreciation rate (LAR) and the acquisition price of the Company were considered to be essential. Results of these two analysis are presented in Figures 5.6 and 5.7 in which 'n' denotes the holding period.

After further refinement on the assessment of the LAR, a more reliable 3%/year LAR was adopted (see also Subsection 5.4.2.2). From Figure 5.6, the 5 and 10 year holding period IRRs of the Company become 17.7% and 20.3%, respectively for the 3%/year LAR.

Further study on the acquisition price of the Company indicated that a $30,000 downward adjustment of the original estimate of $310,000 is reasonable. From Figure 5.7, the 5 and 10 year holding period IRRs of the Company for the revised acquisition price become 22.6% and 23.85%, respectively.

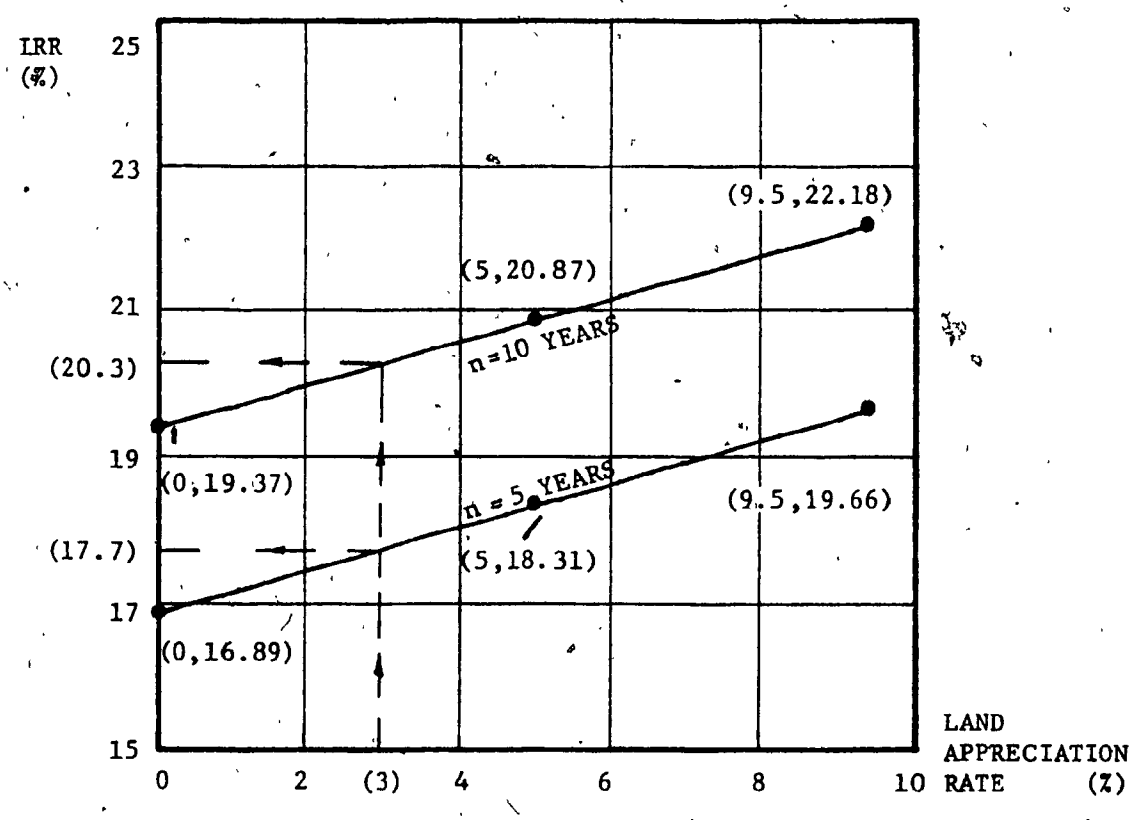

FIGURE 5.6 SENSITIVITY OF LAND APPRECIATION RATE ON IRR OF THE COMPANY

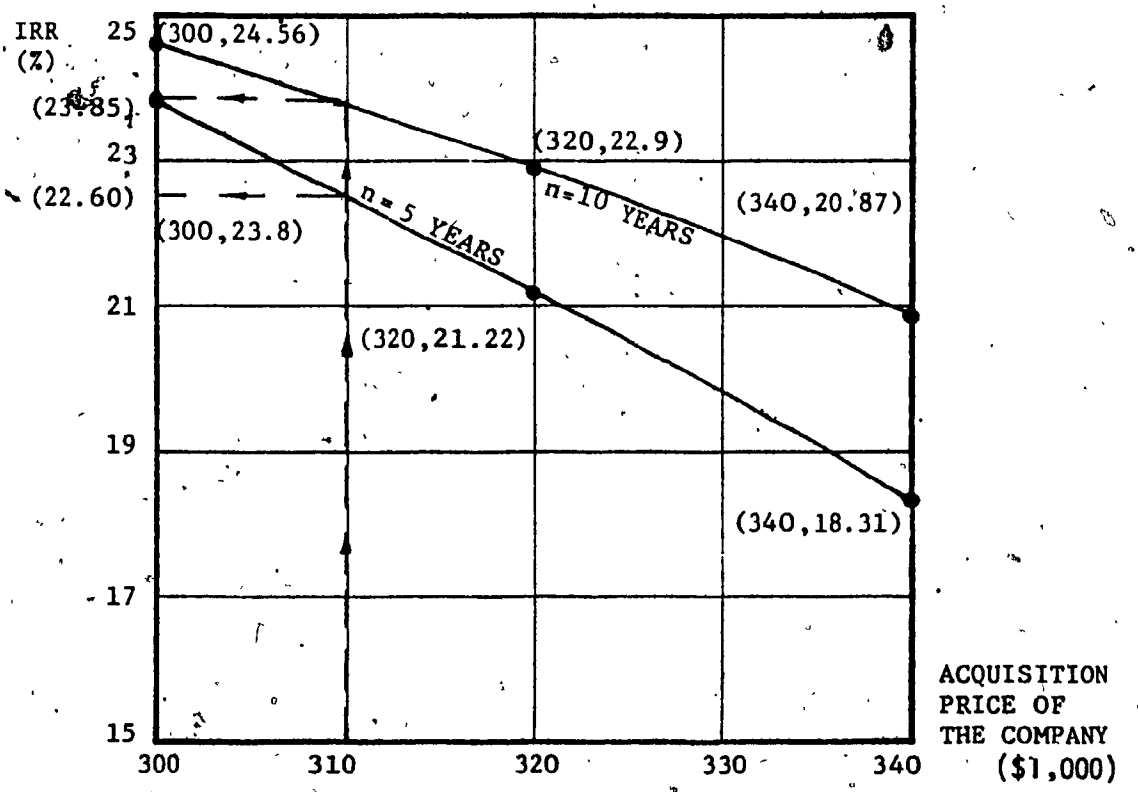

FIGURE 5.7 SENSITIVITY OF ACQUISITION PRICE ON IRR OF THE COMPANY.

5.4.4.3 ANALYSIS CONCLUSION

Based on the refined 3%/year LAR and the adjusted $310,000 acquisition price of the Company, the 5 years' IRR of the Company should be:

$$IRR \doteq 18.31\% \times (1 - 3.3\%) \times (1 + 23\%) = 21.78\%;$$

and the 10 years' IRR of the Company should be:

$$IRR \doteq 20.87\% \times (1 - 2.7\%) \times (1 + 14\%) = 23.15\%.$$

After the above adjustments the overall IRRs of the Company can meet the investor objective of 20% minimum required IRR of the Company marginally.

Two of the three investor's economic objectives concerning the Restaurant and the Company are met. The most probable IRRs of the Property are about three percentage points lower than the 9.5% minimum required.

Thus, the acquisition of the China Garden Cafe Ltd. as a whole is concluded to be INFEASIBLE.

CHAPTER SIX

SUMMARY

Adopting the viewpoint of the developer-investor, this study has demonstrated the applicability of two analytical models, namely internal rate of return and net present value, to income-property feasibility studies. In particular, attention was directed at:

1) identifying the scope and objectives of an economic analysis and its related feasibility study in the realty development life cycle;
2) describing the internal rate of return and net present value models and the decision-criteria embodied in them;
3) examining the ability to model practical financial/economic realities in various phases of income-property acquisition projects in the terms encompassed in the IRR and NPV models;
4) identifying and describing investor motivations and wedding these motivations with economic/financial performance measures provided by the IRR and NPV models; and
5) demonstrating a practical application of the IRR model for an actual acquisition case study.

Economic analysis and feasibility study have the common goal of determining whether the projected performance of an investment alternative meets the objectives of the investor-developer. The IRR and NPV models studied in this report aid in meeting this goal as they reflect

explicitly investor motivations from which his financial/economic objectives and performance measures arise.

The IRR and NPV models are flexible enough to allow practical realities in the various phases of the project development life cycle to be modeled in various degrees of detail and accuracy. Trade-offs between modelling the nuances involved in the development process and the resulting benefits in terms of prediction accuracy and additional information obtained are essential, and are made possible by the flexibility and comprehensiveness of the two models.

The trade-off process is a joint responsibility of the economic analyst, project manager and the investor. The use of a sensitivity analysis procedure can be of considerable use in this process.

The case study presented demonstrates the practicality of the IRR model for use in analyzing a relatively straightforward investment opportunity. The use of a computerized version of the model further enhanced the application of the IRR model.

Additional contributions of this report include:

1) a generic realty development process flowchart which provides an overview of the various activities involved in the development life cycle from project initiation to project termination; and
2) a leverage breakeven point concept which provides an easy-to-use technique for determing the occurrence of positive or negative leverage for a project and for assessing sensitivity of the after-tax IRR to leverage.

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APPENDIX A

GLOSSARY OF TERMS USED

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GLOSSARY OF TERMS USED

A/E CONSTRAINTS: Architectural and engineering constraints refer to the legal, political, aesthetic, ethical, physical and technical constraints in Subsection 2.2.3.

CONCEPTUAL DESIGN: By definition, it is considered to advance design to the 10 to 15 percent level of design completion. It may encompass the following (see also [40:3-4]):

1. soil investigation (preliminary);
2. surveying and mapping;
3. A/E constraint analysis;
4. definition of type of structures;
5. general overall dimensional design of the project;
6. identifying locations, limits, spatial allowances and key dimension of major functions to be housed;
7. identifying major equipment to be housed;
8. general arrangement of mechanical and electrical systems; and
9. identifying major direct impacts of construction and operations on existing and proposed structures, systems, facilities and properties, and general methods of handling such impacts.

CONSTRUCTION CONTRACT MANAGEMENT: It deals mainly with the handling of legal correspondence arising from construction; for instance, notice for job acceleration and change orders. It also encompasses dealings with bonds, construction insurance, construction contracts and subcontracting.

FINAL DESIGN: It is completion of design from preliminary design. It includes development of final A/E designs, preparation for documentation of all bidding contracts, documents, and preparation of detailed cost estimates and cash flow projections. (See also [40:4]).

OTHER STUDIES: Any appropriate studies, investigations and evaluations which are not specially identified in the generic realty development flowchart but which may be required by the nature of the project [33:44]. An example study can be "the study on complication of foreign investment in Canada" which is required by the case study of this technical report.

PRELIMINARY CONCEPTUAL DESIGNS: Refers to preliminary designs of two or more probable project solutions selected from the set of potential solutions generated earlier in Activity B1.7. Their content depends on the nature of the project and the availability of information. The quality and detail of conceptual designs should be such that strategy determination, scheduling and financial/economic analysis can be performed for the selection of the best project solution from the set of potential solutions.

PRELIMINARY DESIGN: It is considered to extend conceptual design to the 30 to 40 percent level of design completion. It may include the following (see also [40: 4]):

1. site development;
2. detail programming;
3. rough space layout designs;
4. selection of all systems and subsystems (such as structural, HVAC, mechanical and electrical systems);
5. identifying bid packages;
6. development of outline specifications; and
7. capital cost estimates for bid packages.

PRELIMINARY PROGRAMMING: Its prime purpose is to provide essential information for analyzing the set of potential project solutions. Thus, its scope is usually centered on critical zoning by-laws, financial constraints, and fundamental quantitative and qualitative criteria of the sponsor and project users.

PROGRAMMING: Programming identifies, reconciles, and transforms needs and requirements of all users involved into design language (e.g., dimensional requirement of each functional unit to be housed). Programming establishes the criteria which the designer must fulfill (see [32] and [33:59]).

REAL ESTATE DATA: It includes property data in terms of locality, availability, community services, facilities, and cost and terms of acquisition. It should also include such additional information as may be required to formulate a land acquisition strategy. See also [33:27-28].

USERS: Cunningham classified building users into nine categories (in [32:6-8]):

1. Sponsor, e.g., promoter, developer, and owner;
2. Agent, e.g., legal adviser, realtor, financier, insurer, programmer, sponsor representative, etc.;
3. Authority, e.g., building inspector, health officer, fire department, appeals boards, national standard institute, etc.;
4. Designer, e.g., architect, engineer, consultants, etc.;
5. Constructor, e.g., contractor, subcontractors, manufacturers, suppliers, construction workers and foremen;
6. Operator, e.g., post-construction operator, maintenance, and utility service personnel;
7. Tenant, e.g., organizations and parties that occupy the building as lessees or owner;
8. Occupant, e.g., individuals, usually employees of sponsors or tenants, but they can include tenants and sponsor themselves; and
9. Public, e.g., community, neighbours, and visitors.

APPENDIX B

DETERMINING AFTER-TAX BREAKEVEN POINT OF LEVERAGE FINANCING

APPENDIX B

DETERMINING AFTER-TAX BREAKEVEN POINT OF LEVERAGE FINANCING

Leverage potential, rate of return on equity, and income tax are three prime concerns of most real estate investors. In this Appendix, a concept, called leverage breakeven concept, is developed for determining when after-tax positive or negative leverage occurs. Based on the findings of this Appendix, leverage effects on after-tax internal rate of return on equity can be approximated very easily. Income tax will be considered explicitly to reflect the tax concerns of investors. All measures used in this article are after-tax, unless otherwise specified.

Breakeven point of leverage financing is defined herein as the point at which after-tax internal rate of return on equity is not affected by mortgage financing; (risk associated with leverage financing is not considered herein). Above this breakeven point, the effects of positive leverage will occur and the after-tax internal rate of return on equity will be increased by additional mortgage financing. Below this point, the IRR will decrease with additional financing.

THE LEVERAGE BREAKEVEN CONCEPT

The economic productivity of a project can be expressed in terms of the internal rate of return, IRR, which can be obtained from the following equation:

$$0 = -E_o + \sum_{j=1}^{n} \frac{EGI_j - OPE_j - TBT_j + TDEP_j + TINT_j - DS_j}{(1 + IRR)^j} + \frac{SP_n - SE_n - RCT_n - CGT_n - UM_n}{(1 + IRR)^n} \qquad (B1)$$

where —

E_o = initial cash equity investment.

n = prespecified holding period.

EGI_j = effective gross income in year j.

OPE_j = operating expense in year j.

TBT_j = income tax before tax shelter in year j; that income tax in year $j = TBT_j - TDEP_j - TINT_j$.

$TDEP_j$ = tax benefits from capital cost allowance in year j, which equals the tax rate times depreciation.

$TINT_j$ = tax benefits from mortgage interest payment in year j which equals the tax rate times interest payment.

DS_j = debt services (mortgage interest plus mortgage principle payment).

SP_n = resale price of the property in year n.

SE_n = selling expense in year n.

RCT_n = recapture tax in year n.

CGT_n = capital gain tax in year n.

UM_n = unpaid mortgage balance in year n.

Rearranging all the variables which are not dependent on financing, a simplifed after-finance formula can be obtained:

$$0 = -(TC-MTG) + \sum_{j=1}^{n} \frac{BFICF_j - (DS_j - TINT_j)}{(1 + IRR_{AF})^j} + \frac{BFCR_n - UM_n}{(1 + IRR_{AF})^n} \qquad (B2)$$

where —

TC = total cost.

MTG = mortgage amount equals TC minus E_o.

$BFICF_j$ = before-finance interim cash flow (from operation) in year j;

= $EGI_j - OPE_j - TBT_j + TDEP_j$.

$BFCR_n$ = before-finance net resale cash reversion (from resale of the project) in year n.

= $SP_n - SE_n - RCT_n - CGT_n$.

After-finance IRR, IRR_{AF}, may be thought of as the after-tax after-finance equity productivity rate on the equity capital employed in the project under the specified mortgage arrangement. It is the after-tax internal rate of return on equity for the equity investor (not mortgage lender-investor).

In a no mortgage case — before-finance — these variables which are financing dependent (MTG, DS_j, $TINT_j$ and UM_n) will be equal to zero and Equation B2 can be modified to a before-finance formula:

$$0 = -TC + \sum_{j=1}^{n} \frac{BFICF_j}{(1 + IRR_{BF})^j} + \frac{BFCR_n}{(1 + IRR_{BF})^n} \qquad (B3)$$

Before-finance IRR, IRR_{BF}, obtained is after-tax before-finance overall productivity rate of the project.

To determine if the potential for positive or negative leverage exists, a mutually exclusive alternative comparison approach can be used; since only one of the before-finance or after-finance alternatives can be chosen. Given two multually exclusive alternatives A and B, they can be rewritten as:

A and A + "B-A"

where A requires the smaller equity investment. The corresponding decision-criterion is that B is considered economically superior to A if the IRR of "B-A" is larger than the return on the total capital employed, IRR_{BF}.

For ranking after-finance and before-finance alternatives, the case of after-finance can be considered as alternative A while before-finance is considered as alternative B. The required rate of return used in deter-

mining the breakeven point of leverage financing should be the overall productivity rate (IRR_{BF}) of the project rather than the discount rate of the firm. It is because leverage effects are concerned with changes in project performances as measured by IRR due to changes in financing. These effects are independent of the investor's minimum attractive rate of return.

The decision-criterion can be restated as the before-finance alternative and its IRR_{BF} are superior to the after-finance alternative and its IRR_{AF} if IRR of "B-A" is larger than the IRR_{BF} of the project. Negative leverage occurs when IRR_{BF} is superior to IRR_{AF}. Positive leverage occurs when IRR of "B-A" (IRR_{B-A}) is smaller than IRR_{BF}. When the after-tax mortgage financing rate equals IRR_{BF}, IRR_{B-A} equals IRR_{BF} and the investor, on the basis of after tax return is indifferent to the two financing alternatives.

IRR_{B-A} can be obtained by solving the following equation which is derived from subtracting the equity and cash flow terms of Equation B2 from those of B3:

$$0 = -MTG + \sum_{j=1}^{n} \frac{DS_j - TINT_j}{(1 + IRR_{B-A})^j} + \frac{UM_n}{(1 + IRR_{B-A})^n} \qquad (B4)$$

The IRR of before-finance's minus after-finance's cash flow, IRR_{B-A}, obtained can be interpreted as the after-tax interest rate the invester must pay for financing. Because it is tax position related, this rate does not reflect the return rate of the lender.

A NUMERICAL ILLUSTRATION OF THE LEVERAGE BREAKEVEN CONCEPT

To illustrate the leverage breakeven concept, financial/economic data of the realty income-property investment example in ref. [50:35-48], which is reproduced in Table B1, is employed. (For a detailed description of the example investment, see [3:105-109]). The schedule of income/expenses and cash flow are shown in Table B2.

Cash flow details and results of the calculation of after-finance, and before-finance, and "B-A" IRRs based on Equations B2, B3, and B4, respectively, are shown in Table B3. In particular, for Table B3:

(1) The before-finance cash flows, BFCF, are obtained from the data in Tables B1 and B2;
(2) The tax benefits from mortgage interest payments, TINT, are equal to the product of mortgage interest payments and the 50% income tax rate used;
(3) "B-A" cash flows, B-ACF, are obtained by subtracting TINT in year j from annual debt services; and
(4) after-finance cash flows, AFCF, are obtained by deducting the B-ACF from the BFCF.

The mortgage interest rate used in Table B3 is 8.5%; corresponding IRRs for different leverage levels are documented in the last three rows of the Table. Likewise, by using different mortgage interest rates of 14.9%, 15% and 20%, sensitivity of IRR_{AF} and IRR_{B-A} to changes in mortgage interest rate and leverage can be obtained. The results of these sensitivity analyses are summarized in Table B4 and Figure B1.

As stated previously, positive leverage occurs when IRR_{B-A} is smaller than IRR_{BF}. This is exactly the case in Table B3. The 4.25% IRR_{B-A} from the last row of the table) is smaller than the project's IRR_{BF} of 7.45%. The advantages of positive leverage can be observed by observing how IRR increases with increasing leverage (LEV).

Also, as noted before, leverage will not affect IRR_{AF} if IRR_{B-A} equals IRR_{BF}; i.e., the project is at its breakeven point of leverage financing. In Table B4, when mortgage interest rate (i) equals 14.9%, IRR_{B-A} is equal to the project's 7.45% IRR_{BF}. Thus, the corresponding values of IRR_{AF} (from the second last row) remain constant for all values of leverage.

Finally, negative leverage occurs when IRR_{B-A} is larger than IRR_{BF}. In Table B4, i has a small increase from 14.9% to 15% such that the corresponding IRR_{B-A} is a little bit larger than IRR_{BF}. Negative leverage effects can be observed by comparing the values of the corresponding IRR_{AF} with different leverage levels; IRR_{AF} decreases with increasing leverage. The same negative leverage effect is magnified when i is increased substantially to 20%. IRR_{AF} drops from 7.45% to 2.91% when leverage rises from 1 to 3.

The relationship between equity productivity rate (IRR_{AF}) and mortgage interest rate and leverage (for the specific TAXR) is summarized in Figure B1. The vertical line at the centre is the breakeven line on which IRR_{B-A} equals IRR_{BF} for all leverage values. The mortgage interest rate on this line may be denoted as the breakeven mortgage interest rate, i.e., positive leverage occurs at the right hand side of the breakeven line. The sufficient conditions for positive leverage to occur is "IRR_{B-A} smaller than IRR_{BF}".

A CONVENIENT WAY TO ASSESS REVERSED MORTGAGED PRODUCTIVITY RATE (IRR_{B-A})

It has been observed that the reversed mortgage productivity rate happens to be equal to after-tax "cost of debt rate" (CDR). Thus, for constant income tax rate, IRR of "B-A" (see [28:102-104]):

$$IRR_{B-A} = CDR = i(1-TAXR). \quad \text{(for constant TAXR)} \tag{B5}$$

For example, in Table B3, i is equal to 8.5% and TAXR is equal to 50%. By substituting the values of i and TAXR into Equation B5, CDR becomes 4.25% which equals the value of IRR_{B-A} in Table B3. By substituting the value of TAXR and different i from Table B4 into Equation B5, corresponding values of CDR can be obtained and are equal to IRR_{B-A}.

ASSESSING THE BREAKEVEN MORTGAGE INTEREST RATE (i_B)

If income tax rate is constant for the holding period, i_B can be derived directly from Equation B5 since the overall productivity rate must equal the after-tax cost of financing at the breakeven point:

$$IRR_{BF} = IRR_{B-A} = CDR = i_B \times (1 - TAXR) \tag{B6}$$

So,

$$i_B = IRR_{BF} \div (1 - TAXR) \tag{B7}$$

By knowing and comparing i_B with the mortgage interest rate, the potential for positive or negative leverage can be determined easily.

BEFORE-TAX AND AFTER-TAX OVERALL RATE

Some analysts define the conditions for positive leverage occurance as "positive leverage will occur if the before-tax overall rate is larger than the mortgage interest rate." The problem with this approach is that tax considerations and mode of financing are ignored. Consequently, investment opportunities offering positive leverage potential despite high mortgage rates may not be recognized.

For example, if we define before-tax overall rate as before-tax internal rate of return on total cost, (which is equivalent to setting, the terms TBT_j, $TDEP_j$, RCT_n and CGT_n of Equation B3 equal to zero), then based on the data from Table B1 and B2*, before-tax internal rate of return on total cost is equal to 11.5%.

Thus, based on the pre-tax criterion, financing rates in excess of 11.5% would be rejected as they would yield negative leverage. But the after tax criterion shows that positive leverage occurs for mortgage interest rates up to 14.9 percent.

APPROXIMATING LEVERAGE EFFECTS ON AFTER-TAX INTERNAL RATE OF RETURN ON EQUITY

Based on the cost of capital concept (see [28:102-105])

overall productivity rate	$\triangleq$	percentage of total cost financed by debt	x	cost of debt	+	percentage of total cost financed by initial equity	x	internal rate of return on equity	(B8)

By replacing the percentages with leverage (LEV), and using the notation previously defined, Equation B8 can be rewritten as:

$$IRR_{BF} \triangleq \left(1 - \frac{1}{LEV}\right) \times CDR + \left(\frac{1}{LEV}\right) \times IRR_{AF} \qquad (B9)$$

A handy equation for approximating leverage effects on IRR can be obtained by rearranging the above equation for IRR_{AF}:

---

* Before-tax cash flows for the project from year 0 to year 10, respectively, are (150,000); 16,800; 16,904; 17,008; 17,112; 17,216; 17,320; 17,424; 17,528; 17,632; and 196,336.

$$IRR_{AF} \doteq LEV \times (IRR_{BF} - CDR) + CDR \qquad (B10)$$

This equation may be used as follows: IRR_{BF} from Equation B3 and Table B3 is 7.45%, CDR from Equation B5 is 4.25%. For a leverage factor of 1.5, the after-tax internal rate of return on equity can be obtained by substituting the values into Equation B10 to yield:

$$IRR_{AF} \doteq 1.5 \times (7.45\% - 4.25\%) + 4.25\% = 9.05\%.$$

The value of the internal rate of return on equity obtained from Equation B10 is approximately equal to the actual IRR_{AF} (8.88%) found in Table B3; the approximation is 0.17 percentage points higher than the actual return rate.

For a leverage factor of 3, IRR_{AF} from Equation B10 is:

$$IRR_{AF} \doteq 3 \times (7.45\% - 4.25\%) + 4.25\% = 13.85\%.$$

The discrepancy between the actual and approximate result increases with increasing leverage.

Total property cost	$150,000
Land cost	30,000
Building cost	120,000
Investment	50,000
Loan assumed (8½% interest)	100,000
Annual payment	9,500
Depreciation (125% declining balance, 40 years)	
Vacancy factor (4% of gross income)	
Property taxes ($3,000 the first year, increasing $30 per year thereafter)	
Gross income ($30,000 the first year, increasing $400 per year thereafter)	
Expenses ($9,000 the first year, increasing $250 per year thereafter)	
Sales price at end of year 10	190,000
Income tax rate	50 %
Capital gains tax rate	25 %
Real estate commission on sale	6 %
Proposed life of the investment	10 years

TABLE B1 APARTMENT FINANCIAL INFORMATION*

	Gross Income	Vacancy Allowance	Real Estate Taxes	Expenses	Depreciation	Interest	Taxable Income	Cash Flow	Present Value Factor (13%)	Present Value of Cash Flow
Initial investment	—	—	—	—	—	—	—	−$50,000	1.000	−$50,000
Year 1	$30,000	$1,200	$3,000	$9,000	$3,750	$8,500	$4,550	5,025	0.885	4,447
2	30,400	1,216	3,030	9,250	3,633	8,415	4,856	4,976	0.783	3,896
3	30,800	1,232	3,060	9,500	3,519	8,323	5,166	4,925	0.693	3,413
4	31,200	1,248	3,090	9,750	3,409	8,223	5,480	4,872	0.613	2,987
5	31,600	1,264	3,120	10,000	3,303	8,114	5,799	4,816	0.543	2,615
6	32,000	1,280	3,150	10,250	3,200	7,996	6,124	4,758	0.480	2,284
7	32,400	1,296	3,180	10,500	3,100	7,869	6,455	4,696	0.425	1,996
8	32,800	1,312	3,210	10,750	3,003	7,730	6,793	4,630	0.376	1,741
9	33,200	1,328	3,240	11,000	2,909	7,579	7,144	4,560	0.333	1,518
10	33,600	1,344	3,270	11,250	2,818	7,416	7,502	82,081	0.295	24,214

TABLE B2 SCHEDULE OF INCOME, EXPENSES AND CASH FLOW*

* Reprinted from [50:38, 39].

LEV	1	1.5			3			5		
L/V	0	.3333			.6667			.8000		
MTG	0	50,000			100,000			120,000		
DS	0	4,750			9,500			11,400		
LIM	0	42,582			85,165			102,198		
	BFCF	TINT	B-ACF	AFCF	TINT	B-ACF	AFCF	TINT	B-ACF	AFCF
Yr. 0	(150,000)		(50,000)	(100,000)		(100,000)	(50,000)		(120,000)	(30,000)
1	10,275	2,125	2,625	7,650	4,250	5,250	5,025	5,100	6,300	3,975
2	10,269	2,104	2,646	7,623	4,208	5,292	4,976	5,049	6,351	3,918
3	10,264	2,081	2,669	7,595	4,162	5,338	4,925	4,994	6,406	3,858
4	10,261	2,056	2,694	7,567	4,112	5,338	4,872	4,933	6,466	3,785
5	10,260	2,029	2,721	7,539	4,057	5,443	4,816	4,868	6,532	3,728
6	10,260	1,999	2,751	7,509	3,998	5,502	4,758	4,798	6,602	3,658
7	10,262	1,967	2,783	7,479	3,935	5,565	4,696	4,721	6,678	3,583
8	10,266	1,932	2,818	7,448	3,865	5,635	4,630	4,638	6,762	3,504
9	10,271	1,895	2,855	7,416	3,790	5,710	4,560	4,548	6,852	3,419
10	10,277	1,854	2,896	7,381	3,708	5,792	4,485	4,450	6,950	3,329
10	162,761		42,582	120,179		85,165	77,596		102,198	60,563
IRR_{BE}	7.45%									
IRR_{AF}	7.45%			8.88%			12.7%			17.08%
IRR_{B-A}			4.25%			4.25%			4.25%	

Mortgage Constant = 9.5%; Amortization Period = 27.5961 years

LEV = leverage; L/V = loan to value ratio; MTG = Amount of mortgage.

Table B3 — Cash Flow Sheet for i = 8.5%.

	i = 8.5%			i = 14.9%			i = 15%			i = 20%
LEV	1.5	3	5	1.5	3	5	1.5	3	5	3
L/V	.333	.667	.800	.333	.667	.800	.333	.667	.800	.667
MTG	50,000	100,000	120,000	50,000	100,000	120,000	50,000	100,000	120,000	100,000
DS	4,750	9,500	11,400	7,615	15,230	18,276	7,662	15,324	18,389	20,131
IRR_{AF}	8.88%	12.70%	17.08%	7.45%	7.45%	7.45%	7.42%	7.35%	7.28%	2.91%
IRR_{B-A}	4.25%	4.25%	4.25%	7.45%	7.45%	7.45%	7.50%	7.50%	7.50%	10%

IRR_{BF} is 7.45%

Table B4 — Summary of Sensitivity of IRR to Change in Mortgage Interest Rate and Leverage.

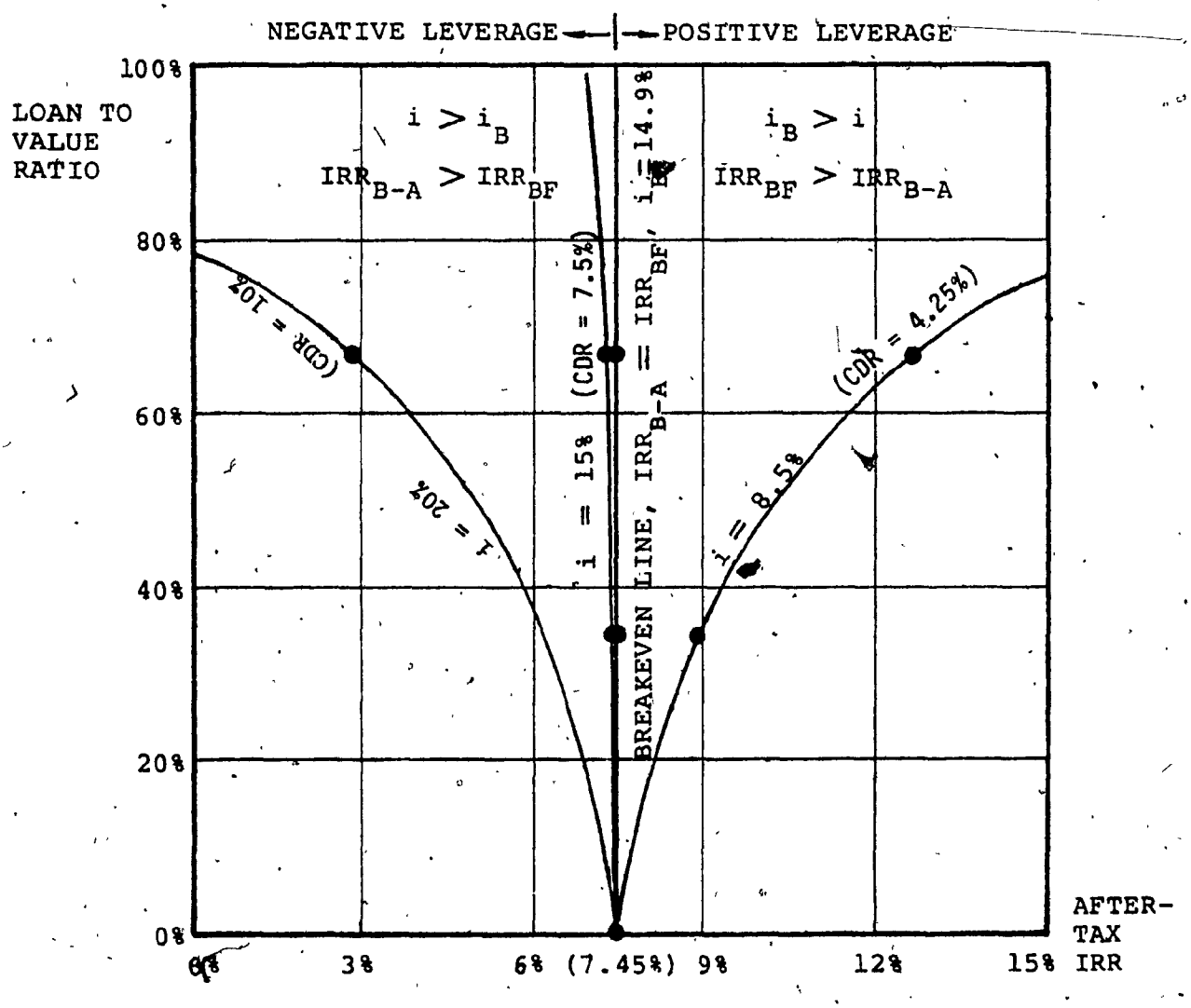

FIGURE B1 RELATIONSHIP BETWEEN IRR_{AF} AND i AND LEVERAGE

APPENDIX C

FINANCIAL STATEMENT OF THE COMPANY

CHINA GARDEN CAFE LIMITED

RESULTS OF OPERATIONS

YEAR ENDED MARCH 31, 1977

SALES			$382,224.63
COST OF SALES			
Inventories at beginning of year	$ 17,132.11		
Purchases	154,826.86		
Operating expenses (Note 1)	201,111.44	$373,070.41	
Inventories at end of year		15,067.54	358,002.87
GROSS PROFIT			24,221.76
SELLING, ADMINISTRATIVE AND FINANCIAL EXPENSES			
Advertising		1,395.15	
Telephone		2,707.68	
Professional fees and expenses		2,744.73	
Interest on long term debt		8,783.25	15,630.81
NET PROFIT BEFORE ESTIMATED INCOME TAXES AND EXTRAORDINARY ITEM			8,590.95
ESTIMATED INCOME TAXES			2,319.56
NET PROFIT BEFORE EXTRAORDINARY ITEM			6,271.39
EXTRAORDINARY ITEM			
Income taxes recovery due to application of loss sustained in prior year against current year taxable income			2,319.56
NET PROFIT FOR THE YEAR			$ 8,590.95

CHINA GARDEN CAFÈ LIMITED

CHANGES IN FINANCIAL POSITION

YEAR ENDED MARCH 31, 1977

WORKING CAPITAL AT BEGINNING OF YEAR		$ 5,113.96
VARIATION DUE TO CURRENT YEAR TRANSACTIONS		
SOURCE OF ADDITIONAL WORKING CAPITAL		
Resulting from current operations:		
Net profit for the year	$ 8,590.95	
Add: Depreciation	3,932.00	12,522.95
APPLICATION OF WORKING CAPITAL		
Decrease in long term debt		10,000.00
INCREASE IN WORKING CAPITAL FOR THE YEAR		2,522.95
WORKING CAPITAL AT END OF YEAR		$ 7,636.91

WORKING CAPITAL RECONCILIATION

AT MARCH 31, 1977 AND 1976

	1977	1976
CURRENT ASSETS	$43,944.70	$53,132.08
CURRENT LIABILITIES	36,307.79	48,018.12
WORKING CAPITAL	$ 7,636.91	$ 5,113.96
RATIO: Current assets / Current liabilities	1.21 : 1	1.11 : 1

CHINA GARDEN CAFE LIMITED

NOTES TO FINANCIAL STATEMENTS

YEAR ENDED MARCH 31, 1977

1. FIXED ASSETS

	Rate	Cost	Accumulated depreciation	Net
Land	-	$193,250.00	--	$193,250.00
Building	5%	88,868.27	$24,445.24	64,423.03
Office furniture and fixtures	20%	1,259.45	1,240.40	19.05
Restaurant furniture and fixtures	20%	55,593.53	53,447.67	2,145.86
		$338,971.25	$79,133.31	$259,837.94

The fixed assets are depreciated on the declining balance method at the above-mentionned annual rates.

The depreciation amounts to $3,932.00 for the year and is included in operating expenses in RESULTS OF OPERATIONS.

2. LONG TERM LIABILITIES

Loan payable, 7 3/4%, maturing November 1st, 1987, payable by semi-annual instalments of $5,000 in principal and secured by way of a mortgage on fixed assets having a book value of $257,673.03 at balance sheet date	$110,000.00
Loans from shareholders, without interest	139,902.80
	249,902.80
Less: Principal maturing within one year	10,000.00
	$239,902.80

The required instalments in principal for the loan payable are as follows for the five years ending after March 31, 1978

1979	$ 10,000.00
1980	10,000.00
1981	10,000.00
1982	10,000.00
1983	10,000.00
	$ 50,000.00

CHINA GARDEN CAFE LIMITED

SCHEDULE OF OPERATING EXPENSES

YEAR ENDED MARCH 31, 1977

Salaries and wages		$162,741.01
Sanitation and uniforms		15,499.99
Crockery and dishes		318.74
Light, heat and power		13,961.48
Maintenance and repairs		11,829.29
Taxes and licences		18,952.03
Insurances		11,073.44
General expenses		742.41
Rental income		(37,938.95)
Depreciation		
- Building	$3,391.00	
- Furniture and fixtures	541.00	3,932.00
		$201,111.44

CHINA GARDEN CAFE LIMITED

CONTINUITY OF FIXED ASSETS

YEAR ENDED MARCH 31, 1977

	COST				ACCUMULATED DEPRECIATION						
	at 3/31/76	Additions	(Disposals)	at 3/31/77	at 3/31/76	(on disposals)	Net	Depreciable value 3/31/77	Depreciation	Accumulated depreciation 3/31/77	Residual value 3/31/77
Land	$193,250.00	-	-	$193,250.00	-	-	-	$193,250.00	-	-	$193,250.00
Building	88,868.27	-	-	88,868.27	$21,054.24	-	$21,054.24	67,814.03	$3,391.00	$24,445.24	64,423.03
Office furniture and fixtures	1,259.45	-	-	1,259.45	1,235.40	-	1,235.40	24.05	5.00	1,240.40	19.05
Restaurant furniture and fixtures	55,593.53	-	-	55,593.53	52,911.67	-	52,911.67	2,681.86	536.00	53,447.67	2,145.86
	$339,971.25	-	-	$338,971.25	$75,201.31	-	$75,201.31	$263,769.94	$3,932.00	$79,133.31	$259,837.94