CANONIC BRIDGE STRUCTURES FOR SOME TYPES OF BIQUADRATIC DRIVING POINT FUNCTIONS

by

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ABSTRACT

The thesis deals with canonic realizations of the minimum reactance biquadratic driving point impedance functions,

$$z(s) = K \frac{s^2 + A_1 S + A_0}{s^2 + B_1 S + B_0}$$

by a five element RC, RL or RLC bridge structure consisting of three resistors and two reactive elements. Realizability conditions, as well as the component values, are derived for one RC, one RL and five RLC bridge structures. These realizability conditions are also presented in the (A_1, B_1) plane, defining certain regions where $z(s)$ may be realized by one or the other of the bridge structures. It is shown that if $A_0 = B_0$, then the minimum reactance impedance $z(s)$ can always be realized by an RLC- bridge structure. It is also shown that if $A_0 \neq B_0$, then $z(s)$ can be realized only under certain coefficient conditions; but where these conditions are satisfied, there are always two realizations for $z(s)$.

CHAPTER 1

INTRODUCTION

1.1 General

One of the important problems in passive network synthesis is to obtain canonic structures for a given driving point function (DPF). It is known that such canonic structures exist for DPF's realizable by two elements kind networks (RC, RL and LC). However, a general solution to this problem does not yet exist even in the case of a second order biquadratic DPF, not realizable by two elements kind networks. Recently, some attempts have been made in this direction.

Kim[1] and Van Valkenburg[2] have shown that a minimum biquadratic function z(s) satisfying

$$z(\infty) = 4\, z(o)$$

or

$$z(o) = 4\, z(\infty) \qquad (1.1)$$

can be realized by a five-element bridge structure consisting of two resistors and three reactances, as shown in Fig. 1.1.

Subsequently, Seshu [3][4] pointed out that to realize such a minimum biquadratic DPF at least two resistors and three reactive elements are essential, and that in general, seven elements are necessary.

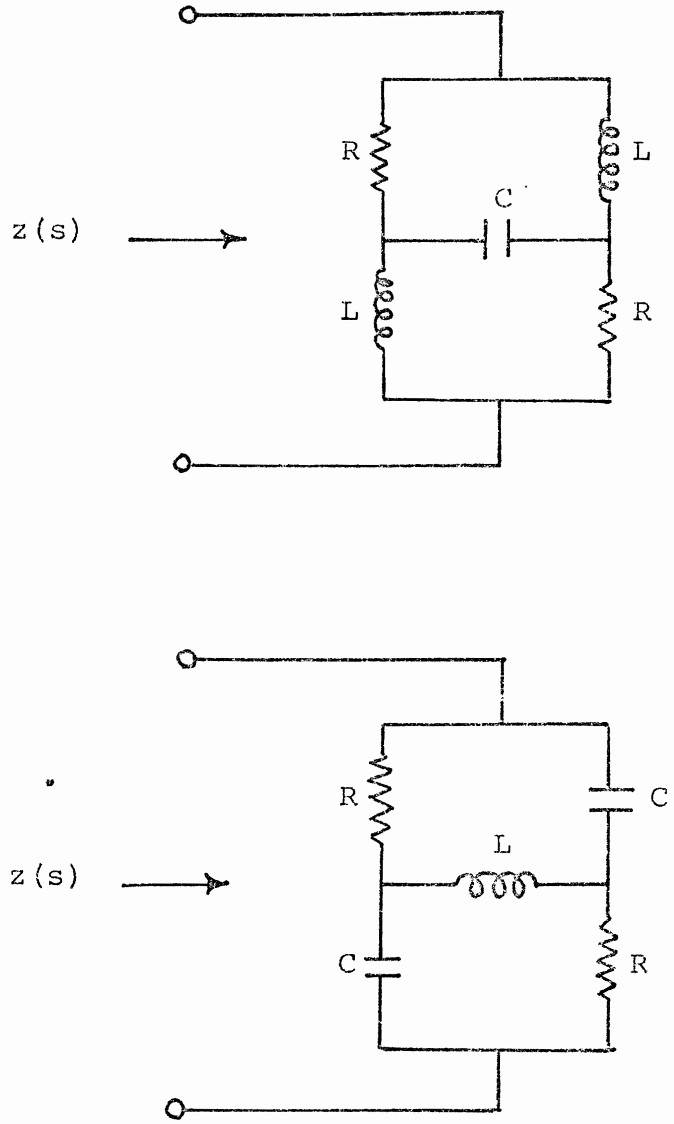


Fig. 1.1 Five element bridge Structures realizing a minimum biquadratic impedance z(s) satisfying conditions (1.1).

Foster[5] later derived conditions under which a minimum biquadratic function may be realized by a bridge structure, consisting of two resistors and three reactive elements, where the opposite arms of the bridge are of the same kind, but of unequal magnitudes, as shown in Fig. 1.2.

The realization of a non-minimum positive real biquadratic function using predistortion technique was given by Barlev[6]; however, the synthesis requires more than five elements. A five-element bridge structure containing three resistors and two reactances, as shown in Fig. 1.3, realizing a non-minimum biquadratic impedance, was given by Foster and Ladenheim[7]. Coefficient conditions for realization were derived, and a discussion of its relationship to the ladder type network was given.

1.2 Scope of the Thesis

This thesis is concerned with deriving some canonic bridge structures for minimum reactive biquadratic driving point functions, where these bridge structures consist of three resistors and two reactive elements.

There are totally thirteen possible five-element bridge structures, including dual networks, which consist of three resistors and two reactances. These structures are shown in Fig. 1.4. Structures VIII and IX have respectively a zero at the origin and at infinity, and

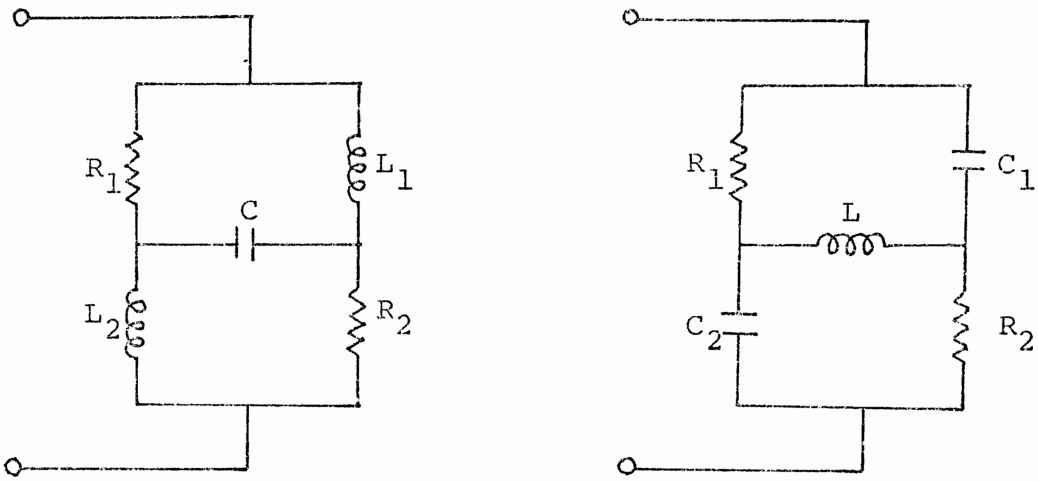


Fig. 1.2 Five element bridge Structures consisting of two resistors and three reactive elements, the opposite arms of the bridge being of the same kind but of unequal magnitudes.

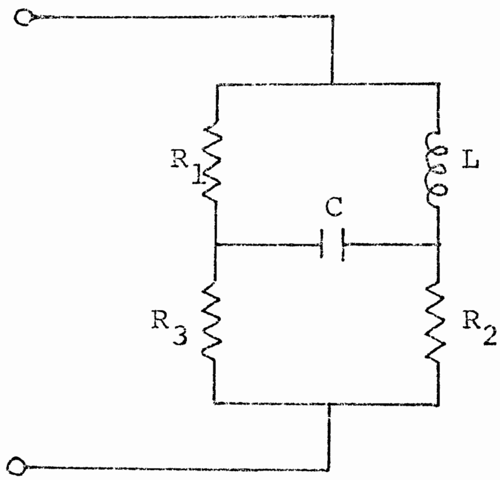


Fig. 1.3 Five element bridge Structures, with three resistors and two reactances, used by Foster and Ladenheim.

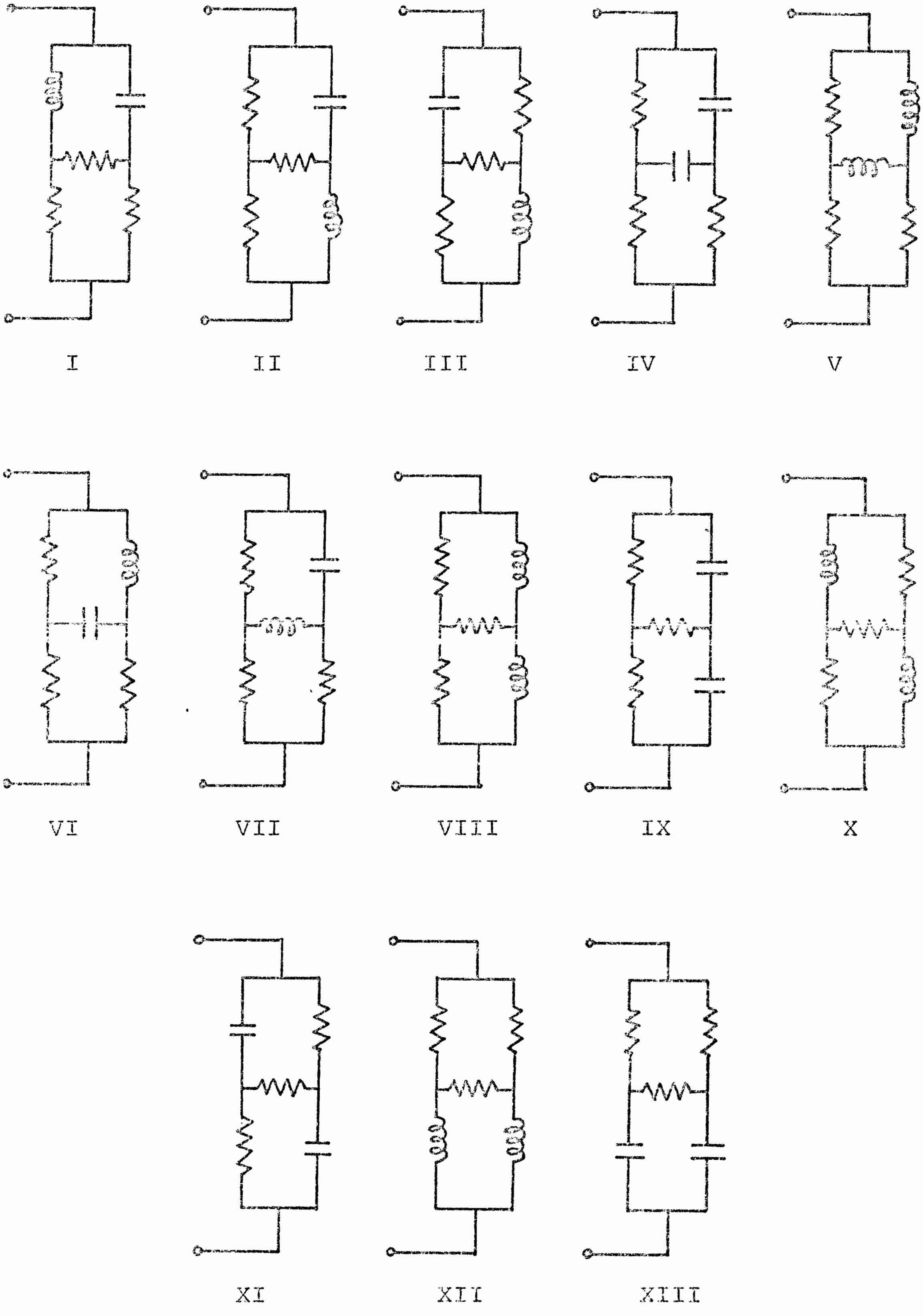


Fig. 1.4 All the possible five element bridge Structures consisting of three resistors and two reactances.

hence, will not realize a non-minimum biquadratic function.

Structures XII and XIII have respectively a pole at infinity and at the origin, and hence they also will not realize a non-minimum biquadratic function.

We are now left with Structures IV and V, VI and VII, X and XI, II, III and I, of which Foster and Ladenheim[7] have analyzed Structures VI and VII. However, for the sake of completeness, these structures are included here and the conditions for realization are derived.

In Chapter 2, we shall give general conditions for the realizability of a biquadratic positive real function by RC, RL and RLC networks. In Chapters 3, 4 and 5, we shall derive coefficient conditions under which the biquadratic function may be realized by Structures I and II, III, IV and V, respectively. Explicit formulas to calculate the element values directly from the given biquadratic function are also given. In Chapter 6, the realizability conditions for Structures VI and VII, as given by Foster and Ladenheim[7], are derived and the results are given in a more general way. Chapter 7 contains the conclusions, based on the results derived in the earlier Chapters.

CHAPTER 2

BIQUADRATIC DRIVING POINT FUNCTIONS

2.1 Reduction to a Simple Form

Let the given non-minimum biquadratic driving point function be

$$z(p) = \frac{Ap^2 + Bp + C}{Dp^2 + Ep + F} \tag{2.1}$$

where A, B, C, D, E and F are all positive. Let us scale the frequency p by

$$p = \sqrt{F/D} \cdot s \tag{2.2}$$

Then (2.1) reduces to

$$z(s) = H \frac{s^2 + a_1 s + a_0}{s^2 + b_1 s + 1} \tag{2.3}$$

where

$$\begin{aligned} a_1 &= (B/A)/\sqrt{F/D} \\ a_0 &= (C/A)/\sqrt{F/D} \\ b_1 &= (E/D)/\sqrt{F/D} \end{aligned} \tag{2.4}$$

and

$$H = A/D$$

In (2.3) we may assume, without loss of generality, $H = 1$. This corresponds to an impedance scaling. Thus, instead of considering (2.1) we may just consider the simple form of $z(s)$,

$$z(s) = \frac{s^2 + a_1 s + a_0}{s^2 + b_1 s + 1} \qquad (2.5)$$

which contains only three unknown coefficients. Once a canonic structure is given for (2.5), we may scale that network both impedance and frequency-wise and obtain a canonic realization for (2.1).

2.2 Basic Coefficient Conditions for RC, RL and RLC Networks

The impedance z(s) given by (2.5) has to be a positive real function (PRF) in order that it may be realized by passive elements. Hence,

$$a_1 b_1 \geqslant (\sqrt{a_0}-1)^2 \qquad (2.6)$$

Let us now develop z(s) as an RC-ladder by the continued fraction expansion of z(s).

$$s^2 + b_1 s + 1)\; s^2 + a_1 s + a_0 \;(1 \qquad \text{Step A}$$

$$\underline{s^2 + b_1 s + 1}$$

$$(a_1-b_1)s + a_0-1)\; s^2+b_1 s+1\;\left(\frac{1}{a_1-b_1}s \qquad \text{Step B}\right.$$

$$\underline{s^2+\frac{a_0-1}{a_1-b_1}s}$$

$$k_1 s+1)\; \overline{a_1-b_1}\,s+\overline{a_0-1}\;\left(\frac{a_1-b_1}{k_1} \qquad \text{Step C}\right.$$

$$\underline{\overline{a_1-b_1}\,s+\frac{a_1-b_1}{k_1}}$$

$$k_2)\; k_1 s+1 \;(k_1/k_2 s \qquad \text{Step D}$$

$$\underline{k_1 s}$$

$$1)\,k_2\,(k_2$$

where

$$k_1 = -\frac{a_0 - 1}{a_1 - b_1} + b_1$$

$$k_2 = (a_0 - 1) - \frac{a_1 - b_1}{k_1}$$

We see that for z(s) to be realized as an RC-ladder,

$$(a_1 - b_1) > 0 \qquad (2.7a)$$

$$(a_0 - 1) > 0 \qquad (2.7b)$$

$$k_1 > 0 \qquad (2.7c)$$

$$k_2 = (a_0 - 1)k_1 - (a_1 - b_1) > 0 \qquad (2.7d)$$

From these we see that the condition $k_1 > 0$ is contained in the other inequalities and hence is superfluous. Now from (2.7d) we have

$$(a_0 - 1)b_1 - \frac{(a_0 - 1)^2}{(a_1 - b_1)} - (a_1 - b_1) > 0$$

Since $a_1 > b_1$ we have

$$(a_0 - 1)b_1(a_1 - b_1) - (a_0 - 1)^2 - (a_1 - b_1)^2 > 0$$

or,

$$(a_0 - 1)^2 - (a_1 - b_1)(a_0 b_1 - a_1) < 0.$$

Letting

$$\alpha_0 = (a_0 - 1)^2 - (a_1 - b_1)(a_0 b_1 - a_1), \qquad (2.8)$$

we have that z(s) can be realized by an RC-network if,

$$a_0 > 1$$

$$a_1 > b_1 \qquad (2.9)$$

and $\qquad \alpha_0 < 0$

Similarly it can be shown that for $z(s)$ to be realized by an RL-network,

$$a_0 < 1 \qquad (2.10)$$

$$a_1 < b_1$$

and

$$\alpha_0 < 0$$

Thus for an RLC realization of $z(s)$,

$$\alpha_0 \nless 0 \qquad (2.11)$$

If $\alpha_0 = 0$, the continued fraction of $z(s)$ would end prematurely (at step C), showing that $z(s)$ is not biquadratic but that it is a bilinear function; in other words, there would be a linear factor common to the numerator and denominator of (2.5). That is, for a biquadratic function,

$$\alpha_0 \neq 0 \qquad (2.12)$$

Also, if $a_0 = 1$,

$$\alpha_0 = (a_1-b_1)^2 > 0 \qquad (2.13)$$

That is, the realization of $z(s)$ will have to be by an RLC - network.

Summarizing these results we have,

1. For $z(s)$ to be realizable by passive elements, it has to satisfy the positive real condition,

$$a_1 b_1 \geqslant (\sqrt{a_0} - 1)^2$$

2. For $z(s)$ to be a biquadratic function,

$$\alpha_0 \neq 0$$

3. For an RC - realization of $z(s)$,

$$\alpha_0 < 0, \; a_0 > 1, \; a_1 > b_1$$

4. For an RL realization of $z(s)$,

$$\alpha_0 < 0, \; a_0 < 1, \; a_1 < b_1$$

5. For an RLC realization of $z(s)$,

$$\alpha_0 > 0$$

6. If $a_0 = 1$, then $\alpha_0 > 0$ and hence $z(s)$ cannot be realized by either an RC or an RL network.

CHAPTER 3

CONDITIONS FOR REALIZATION OF NETWORKS I AND II.

3.1 Introduction

In this Chapter we shall derive the coefficient conditions under which a minimum reactive biquadratic positive real function

$$z(s) = \frac{s^2 + a_1 s + a_0}{s^2 + b_1 s + 1} \tag{3.1}$$

may be realized as a driving point impedance (DPI) of the structures I and II shown in Fig. 1.4.

These coefficient conditions give rise to different regions in the (a_1, b_1) plane. Explicit formulas will also be given for the element values for these structures.

3.2 Conditions for Realization by Structure I.

By straight forward analysis, the driving point impedance of structure I (Fig. 3.1) may be shown to be [8]

$$z(s) = \frac{R_2(R_1+R_3)}{\Sigma R} \frac{s^2 + \left[\frac{1}{L}\frac{R_1R_3}{R_1+R_3} + \frac{1}{C}\frac{\Sigma R}{R_2(R_1+R_3)}\right]s + \frac{R_1(R_2+R_3)}{R_2(R_1+R_3)}}{s^2 + \frac{1}{L}\frac{R_3(R_1+R_2)}{\Sigma R}s + \frac{1}{LC}} \tag{3.2}$$

Using the transformations

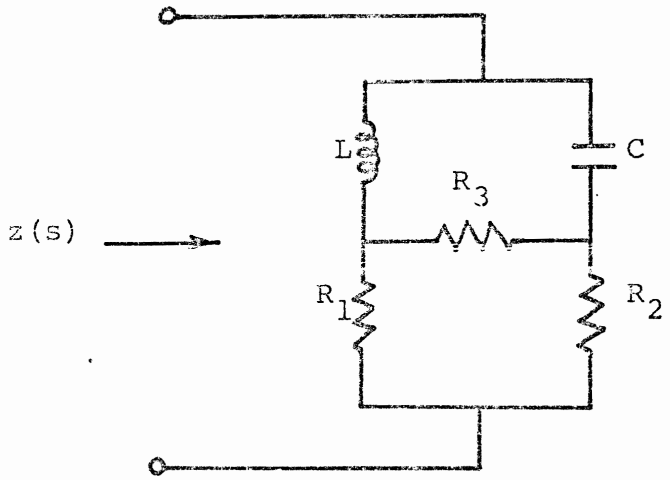


Fig. 3.1 The five element bridge Structure I.

$$r_1 = \frac{R_2 R_3}{\Sigma R_1}$$

$$r_2 = \frac{R_1 R_3}{\Sigma R_1} \qquad (3.3)$$

$$r_3 = \frac{R_1 R_2}{\Sigma R_1}$$

equation (3.2) may be written as

$$z(s) = (r_1 + r_3)\,\frac{s^2 + \left[\frac{1}{L}\frac{\Sigma r_1 r_3}{r_1 + r_3} + \frac{1}{C}\frac{1}{r_1 + r_3}\right]s + \frac{r_2 + r_3}{r_1 + r_3}}{s^2 + \frac{1}{L}(r_1 + r_2)s + \frac{1}{LC}} \qquad (3.4)$$

Equating (3.4) to (3.1) we have,

$$LC = 1$$

$$r_1 + r_3 = 1$$

$$\frac{r_2 + r_3}{r_1 + r_3} = a_0 \qquad (3.5)$$

$$\frac{1}{L}(r_1 + r_2) = b_1$$

$$\frac{1}{L}\frac{\Sigma r_1 r_3}{r_1 + r_3} + \frac{1}{C}\frac{1}{r_1 + r_3} = a_1$$

Solving these we have

$$\alpha_1 r_3^2 + 2\alpha_2 r_3 + \alpha_3 = 0 \qquad (3.6)$$

or

$$r_3 = \frac{-\alpha_2 \pm \sqrt{\alpha_2^2 - \alpha_1 \alpha_3}}{\alpha_1} \qquad (3.7a)$$

provided $\alpha_1 \neq 0$ (3.7b)

$$r_1 = 1 - r_3 \tag{3.8}$$

$$r_2 = a_0 - r_3 = (a_0-1) + r_1 \tag{3.9}$$

$$C = \frac{b_1}{r_1+r_2} \tag{3.10}$$

$$L = \frac{1}{C} \tag{3.11}$$

where

$$\alpha_1 = b_1^2 - 4 \tag{3.12}$$

$$\alpha_2 = 2(a_0+1)-a_1b_1 \tag{3.13}$$

$$\alpha_3 = a_1b_1(a_0+1)-a_0b_1^2-(a_0+1)^2 \tag{3.14}$$

It can be shown that

$$\alpha_2^2-\alpha_1\alpha_3 = b_1^2\alpha_0 \tag{3.15}$$

Also, since network I is an RLC type, we have

$$\alpha_0 > 0$$

and hence, roots of r_3 are real.

The values of R_1, R_2, R_3 may be determined by using the inverse transformation.

$$R_1 = \frac{\Sigma r_1 r_2}{r_1}$$

$$R_2 = \frac{\Sigma r_1 r_2}{r_2} \tag{3.16}$$

$$R_3 = \frac{\Sigma r_1 r_2}{r_3}$$

We shall derive now the coefficient conditions for $z(s)$ to be realized by structure I by assuming (i) $a_0 > 1$, (ii) $a_0 < 1$, and (iii) $a_0 = 1$.

3.2.1 <u>When $a_0 > 1$.</u> It may be observed from equations (3.9) to (3.11) that if r_3 and r_1 are positive, then automatically r_2 and hence L and C are positive. We will now find the conditions under which r_3 and r_1 are positive. For this purpose, we will examine all the possible sign combinations for α_1, α_2, and α_3, assuming $\alpha_1 \neq 0$, $\alpha_2 \neq 0$ and $\alpha_0 > 0$.

(i) $$\alpha_1 > 0, \ \alpha_2 > 0, \ \alpha_3 > 0.$$

Then from (3.7) there are no positive roots for r_3.

(ii) $$\alpha_1 > 0, \ \alpha_2 > 0, \ \alpha_3 < 0.$$

Then there is one positive root for r_3 namely

$$r_3 = \frac{-\alpha_2 + \sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1}$$

For $r_1 = 1 - r_3$ to be positive,

$$r_1 = 1 - \frac{-\alpha_2 + \sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1} > 0$$

Since $\alpha_1 > 0$ we have

$$\alpha_1 + \alpha_2 > \sqrt{\alpha_2^2 - \alpha_1\alpha_3}$$

or $$\alpha_1^2 + 2\alpha_1\alpha_2 + \alpha_1\alpha_3 > 0$$

i.e. $\alpha_1 + 2\alpha_2 + \alpha_3 > 0$ since $\alpha_1 > 0$

Substituting for α_1, α_2 and α_3 we have

$$(a_0 - 1)\ \alpha_4 > 0$$

where

$$\alpha_4 = b_1(a_1-b_1)-(a_0-1) > 0 \qquad (3.17)$$

Incidentally $\alpha_1 + 2\alpha_2 + \alpha_3 = (a_0-1)\alpha_4$ (3.18)

and $\alpha_1 + \alpha_2 + \alpha_4 = (a_0-1)$ (3.19)

Thus, we get the first set of conditions for realizability of structure I (when $a_0>1$) as :

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_1 &> 0 \\ \alpha_2 &> 0 \\ \alpha_3 &< 0 \\ \alpha_4 &> 0 \end{aligned} \qquad (3.20)$$

(iii) $\alpha_1 < 0,\ \alpha_2 > 0,\ \alpha_3 > 0$

In this case, there is one solution for $r_3 > 0$, namely

$$r_3 = \frac{-\alpha_2 - \sqrt{\alpha_2^2-\alpha_1\alpha_3}}{\alpha_1} \qquad (3.21)$$

$$r_1 = 1 - r_3 = \frac{(\alpha_1+\alpha_2)+\sqrt{\alpha_2^2-\alpha_1\alpha_3}}{\alpha_1}$$

Let $\alpha_1 = -p$ where $p > 0$, then for $r_1 > 0$ we should have

$$\frac{(\alpha_2-p) + \sqrt{\alpha_2^2+p\alpha_3}}{-p} > 0$$

or $(p-\alpha_2) > \sqrt{\alpha_2^2+p\alpha_3} > 0$ (3.22)

that is, $p^2-2p\alpha_2 > p\alpha_3$

Hence $\alpha_1^2 + 2\alpha_1\alpha_2 + \alpha_1\alpha_3 > 0$

Since $\alpha_1 < 0$, we have

$$\alpha_1 + 2\alpha_2 + \alpha_3 = (a_0-1)\alpha_4 < 0$$

or $\alpha_4 < 0$ since $a_0 > 1$ (3.23)

However, from (3.22), $p-\alpha_2 > 0$ or $\alpha_1 + \alpha_2 < 0$

Now from (3.19), $\alpha_4 = (a_0-1) - (\alpha_1+\alpha_2) > 0$

which contradicts (3.23).

Thus, there is no realization when $a_0 > 1$ and

$$\alpha_0 > 0,\ \alpha_1 < 0,\ \alpha_2 > 0,\ \alpha_3 > 0$$

(iv) $\alpha_1 < 0,\ \alpha_2 > 0,\ \alpha_3 < 0$

Let $-\alpha_1 = p > 0$ and $-\alpha_3 = q > 0$

Then there are two values of r_3 given by

$$r_3 = \frac{-\alpha_2 \pm \sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1} = \frac{\alpha_2 \mp \sqrt{\alpha_2^2 - pq}}{p}$$

a) Consider first the solution,

$$r_3 = \frac{\alpha_2 + \sqrt{\alpha_2^2 - pq}}{p} \qquad (3.24)$$

Then $r_1 > 0$, if

$$(p-\alpha_2) - \sqrt{\alpha_2^2 - pq} > 0$$

or $(p-\alpha_2) > \sqrt{\alpha_2^2 - pq} > 0$

Hence, $(p-\alpha_2) > 0$ or $\alpha_1 + \alpha_2 < 0$ (3.25)

and

$$p^2 - 2\alpha_2 p > - pq$$

Thus

$$\alpha_1 + 2\alpha_2 + \alpha_3 < 0$$

and hence $\alpha_4 < 0$

which implies $(\alpha_1 + \alpha_2) > 0$ for $a_0 > 1$. This is in contradiction with (3.25). Thus, there is no realization corresponding to (3.24).

b) Consider now the second solution of r_3,

$$r_3 = \frac{\alpha_2 - \sqrt{\alpha_2^2 - pq}}{p}$$

Then

$$pr_1 = (p - \alpha_2) + \sqrt{\alpha_2^2 - pq}$$

will be positive if

$$\sqrt{\alpha_2^2 - pq} > (\alpha_2 - p) \qquad (3.26)$$

Since its LHS is positive, (3.26) will automatically be satisfied if $(\alpha_2 - p) \leqslant 0$ and hence $(\alpha_1 + \alpha_2) \leqslant 0$. But since

$$\alpha_4 = (a_0 - 1) - (\alpha_1 + \alpha_2)$$

we see that $\alpha_4 > 0$.

However, if $(\alpha_2 - p) > 0$ i.e. $(\alpha_1 + \alpha_2) > 0$, then (3.26) will be satisfied if

$$\alpha_2^2 - pq > \alpha_2^2 - 2\alpha_2 p + p^2$$

or if

$$\alpha_1 + 2\alpha_2 + \alpha_3 = (a_0-1)\alpha_4 > 0, \quad \text{since } \alpha_1 < 0.$$

Thus, we see that whether $(\alpha_1+\alpha_2) \lesssim 0$ or > 0, there is always a realization provided,

$$\alpha_0 > 0,\ \alpha_1 < 0,\ \alpha_2 > 0,\ \alpha_3 < 0,\ \alpha_4 > 0$$

However, from (3.18), $\alpha_4 > 0$, $\alpha_3 < 0$ and $\alpha_1 < 0$ implies $\alpha_2 > 0$. Thus, a realization exists if

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_1 &< 0 \\ \alpha_3 &< 0 \\ \alpha_4 &> 0 \end{aligned} \tag{3.28}$$

(v) $\alpha_1 > 0,\ \alpha_2 < 0,\ \alpha_3 > 0$

Let $-\alpha_2 = q > 0$, then there are two solutions for r_3 namely

$$r_3 = \frac{q \pm \sqrt{q^2-\alpha_1\alpha_3}}{\alpha_1} \tag{3.29}$$

a) Considering the first root, we have

$$r_1 = \frac{(\alpha_1-q) - \sqrt{q^2-\alpha_1\alpha_3}}{\alpha_1}$$

Thus $r_1 > 0$ if

$$(\alpha_1-q) > \sqrt{q^2-\alpha_1\alpha_3} > 0$$

Hence for r_1 to be positive

$$\alpha_1 - q > 0 \quad \text{or} \quad \alpha_1 + \alpha_2 > 0 \tag{3.30}$$

and $\alpha_1^2 - 2\alpha_1 q + q^2 > q^2 - \alpha_1\alpha_3$

that is, $\alpha_1 + 2\alpha_2 + \alpha_3 = (a_0 - 1)\alpha_4 > 0$ (3.31)

since $(\alpha_1 + \alpha_2) > 0$ and $\alpha_2 < 0$ we conclude that $\alpha_1 > 0$ is a superfluous condition. Thus, we have a realization if

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_2 &< 0 \\ \alpha_1 + \alpha_2 &> 0 \\ \alpha_3 &> 0 \\ \alpha_4 &> 0 \end{aligned} \tag{3.32}$$

b) Now considering the second root for r_3 we have that $r_1 > 0$ if

$$(q - \alpha_1) < \sqrt{q^2 - \alpha_1\alpha_3} \tag{3.33}$$

If $(q-\alpha_1) \leqslant 0$, i.e. $(\alpha_1+\alpha_2) \geqslant 0$, the above condition is always satisfied, and no restriction is to be placed on α_4.

However, if $(q-\alpha_1) > 0$ or $(\alpha_1+\alpha_2) < 0$, then for r_1 to be positive, we should have

$$(q-\alpha_1)^2 < q^2 - \alpha_1\alpha_3$$

or $\alpha_1 + 2\alpha_2 + \alpha_3 = (a_0 - 1)\alpha_4 < 0$

This is impossible since from (3.19), $(\alpha_1+\alpha_2) < 0$ implies $\alpha_4 > 0$.

Thus a network realization exists if

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_2 &< 0 \\ \alpha_1 + \alpha_2 &> 0 \\ \alpha_3 &> 0 \end{aligned} \qquad (3.34)$$

From (3.32) and (3.34) we see that there are two realizations if

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_2 &< 0 \\ \alpha_1 + \alpha_2 &> 0 \\ \alpha_3 &> 0 \\ \alpha_4 &> 0 \end{aligned} \qquad (3.35)$$

while there is only one realization if

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_2 &< 0 \\ \alpha_1 + \alpha_2 &> 0 \\ \alpha_3 &> 0 \\ \alpha_4 &< 0 \end{aligned} \qquad (3.36)$$

It should be noted that in (3.36), $(\alpha_1+\alpha_2) > 0$ is a superfluous condition, since $\alpha_4 < 0$ implies $(\alpha_1+\alpha_2) > 0$.

Now $\alpha_4 < 0$, $\alpha_2 < 0$ implies $\alpha_1 > 0$ from (3.19).

Also $\alpha_3 > 0$, $\alpha_4 < 0$ implies $\alpha_2 < 0$, which comes from the following arguments,

$\alpha_3 > 0$ gives $a_1b_1 - 2(a_0+1) > a_0b_1^2 + a_0^2 - a_0a_1b_1 - 1$

or $$\alpha_2 < (1-a_0)^2 + a_0 b_1(a_1-b_1)$$

But $\alpha_4 < 0$ gives $b_1(a_1-b_1) < a_0-1$

$$\therefore\ \alpha_2 < (1-a_0)^2 + a_0(a_0-1) = (1-a_0) < 0$$

or $$\alpha_2 < 0$$

Hence (3.36) are equivalent to

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_3 &> 0 \\ \alpha_4 &< 0 \\ \alpha_2 &\neq 0 \end{aligned} \qquad (3.37)$$

(vi) $$\alpha_1 > 0,\ \alpha_2 < 0,\ \alpha_3 < 0$$

Then there is only one solution for r_3, namely

$$r_3 = \frac{-\alpha_2 + \sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1} \qquad (3.38)$$

Letting $-\alpha_2 = p$ and $-\alpha_3 = q$ where $p,q > 0$

we have that for $r_1 > 0$,

$$(\alpha_1 - p) > \sqrt{p^2 + \alpha_1 q}$$

Hence

$$\alpha_1 - p = \alpha_1 + \alpha_2 > 0 \qquad (3.39)$$

and

$$(\alpha_1 + \alpha_2) > \sqrt{\alpha_2^2 - \alpha_1\alpha_3}$$

or

$$\alpha_1 + 2\alpha_2 + \alpha_3 = (a_0-1)\alpha_4 > 0 \qquad (3.40)$$

The condition $(\alpha_1+\alpha_2) > 0$ is contained in $\alpha_4 > 0$

since

$$\alpha_4 = (\alpha_1+\alpha_2) + (\alpha_2+\alpha_3) \quad \text{and}$$

α_2 and α_3 are negative.

Thus we have a realization, if

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_1 &> 0 \\ \alpha_2 &< 0 \\ \alpha_3 &< 0 \\ \alpha_4 &> 0 \end{aligned} \tag{3.41}$$

(vii) $\alpha_1 < 0,\ \alpha_2 < 0,\ \alpha_3 > 0.$

In this case, there is one solution for r_3

$$r_3 = \frac{-\alpha_2 - \sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1} \tag{3.42}$$

For $r_1 > 0$ we have

$$\frac{(\alpha_1+\alpha_2) + \sqrt{\alpha_2^2-\alpha_1\alpha_3}}{\alpha_1} > 0$$

or $(\alpha_1+\alpha_2) + \sqrt{\alpha_2^2-\alpha_1\alpha_3} < 0$, since $\alpha_1 < 0$.

Letting $-\alpha_1 = p > 0$, $-\alpha_2 = q > 0$, we get

$$(p+q) > \sqrt{q^2+p\alpha_3}$$

Since both sides are positive, we have

$$(p+q)^2 > q^2+p\alpha_3$$

or,

$$\alpha_1 + 2\alpha_2 + \alpha_3 = (a_0-1)\alpha_4 < 0$$

But $\alpha_4 < 0$ implies $(\alpha_1+\alpha_2) > 0$ which contradicts $\alpha_1 < 0$ and $\alpha_2 < 0$.

Hence, there is no realization when

$$\alpha_1 < 0,\ \alpha_2 < 0 \text{ and } \alpha_3 > 0.$$

(viii) $\alpha_1 < 0, \alpha_2 < 0, \alpha_3 < 0$

There is no positive solution for r_3 and hence, no realization exists.

In the above discussion, we have assumed that $\alpha_1 \neq 0$, $\alpha_2 \neq 0$. Combining (3.20), (3.41) and (3.28), we see that there is a realization if

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_3 &< 0 \\ \alpha_4 &> 0 \\ \alpha_1 \neq 0, &\quad \alpha_2 \neq 0 \end{aligned} \tag{3.43}$$

and the value of r_3 is given by (3.38).

Also, from (3.28) we have a realization if

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_3 &> 0 \\ \alpha_4 &< 0 \\ \alpha_1 \neq 0 &\quad \alpha_2 \neq 0 \end{aligned} \tag{3.44}$$

and the value of r_3 is given by (3.21).

From (3.32) we also see that there are two realizations if

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_3 &> 0 \\ \alpha_4 &> 0 \\ \alpha_2 &< 0 \end{aligned} \tag{3.45}$$

$$\alpha_1 + \alpha_2 > 0$$
$$\alpha_1 \neq 0$$

and the values of r_3 are given by (3.7a).

We see from (3.45) that the condition $\alpha_1 \neq 0$ need not be included, since in that case, $\alpha_2 < 0$ and $\alpha_1 + \alpha_2 > 0$ cannot be simultaneously satisfied. We shall see what happens to conditions (3.43) and (3.44), if $\alpha_1 = 0$.

If $\alpha_1 = 0$, then $$r_3 = \frac{-\alpha_3}{2\alpha_2} \qquad (3.46)$$

and $$r_1 = 1 + \frac{\alpha_3}{2\alpha_2} = \frac{2\alpha_2 + \alpha_3}{2\alpha_2} \qquad (3.47)$$

In addition, if $\alpha_3 < 0$, $\alpha_4 > 0$, we see from (3.18) that $\alpha_2 > 0$ and $2\alpha_2 + \alpha_3 > 0$, and thus $r_3 > 0$, $r_1 > 0$. Hence, in conditions (3.43), $\alpha_1 \neq 0$ is not required, however, the value of r_3 is now given by (3.46). Similarly, if $\alpha_3 > 0$, $\alpha_4 < 0$, we see from (3.18) that $\alpha_2 < 0$, and $2\alpha_2 + \alpha_3 < 0$, thus showing that $r_3 > 0$, $r_1 > 0$. Hence, in conditions (3.44), the restriction $\alpha_1 \neq 0$ may be removed, but the value of r_3 is not given by (3.21), but by (3.46).

It may also be shown similarly, that the restriction $\alpha_2 \neq 0$ may be removed in (3.43) and (3.44). We do not have to worry about both α_1 and α_2 being zero simultaneously, since in that case, $\alpha_0 = 0$, which is

TABLE 3.1

Coefficient conditions for the realization of $z(s)$ by Structure I and the corresponding component values.

$(a_0 > 1,\ \alpha_0 > 0)$

Region	Coefficient Conditions	Value of r_3	Comments
1	$\alpha_3 < 0$ $\alpha_4 > 0$	$\frac{-\alpha_2+\sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1}$; $\alpha_1 \neq 0$ $\frac{-\alpha_3}{2\alpha_2}$; $\alpha_1 = 0$	One network realization.
2	$\alpha_3 > 0$ $\alpha_4 < 0$	$\frac{-\alpha_2-\sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1}$; $\alpha_1 \neq 0$ $\frac{-\alpha_3}{2\alpha_2}$; $\alpha_1 = 0$	One network realization.
3	$\alpha_3 > 0$ $\alpha_4 > 0$ $\alpha_1 + \alpha_2 > 0$ $\alpha_2 < 0$	$\frac{-\alpha_2\pm\sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1}$	Two network realization.

$r_1 = 1 - r_3;\ r_2 = a_0 - r_3;\ C = b_1/(r_1+r_2)\ ;\ L = \frac{1}{C}$

NOTE: The values of R_1, R_2, and R_3 may be determined using (3.16).

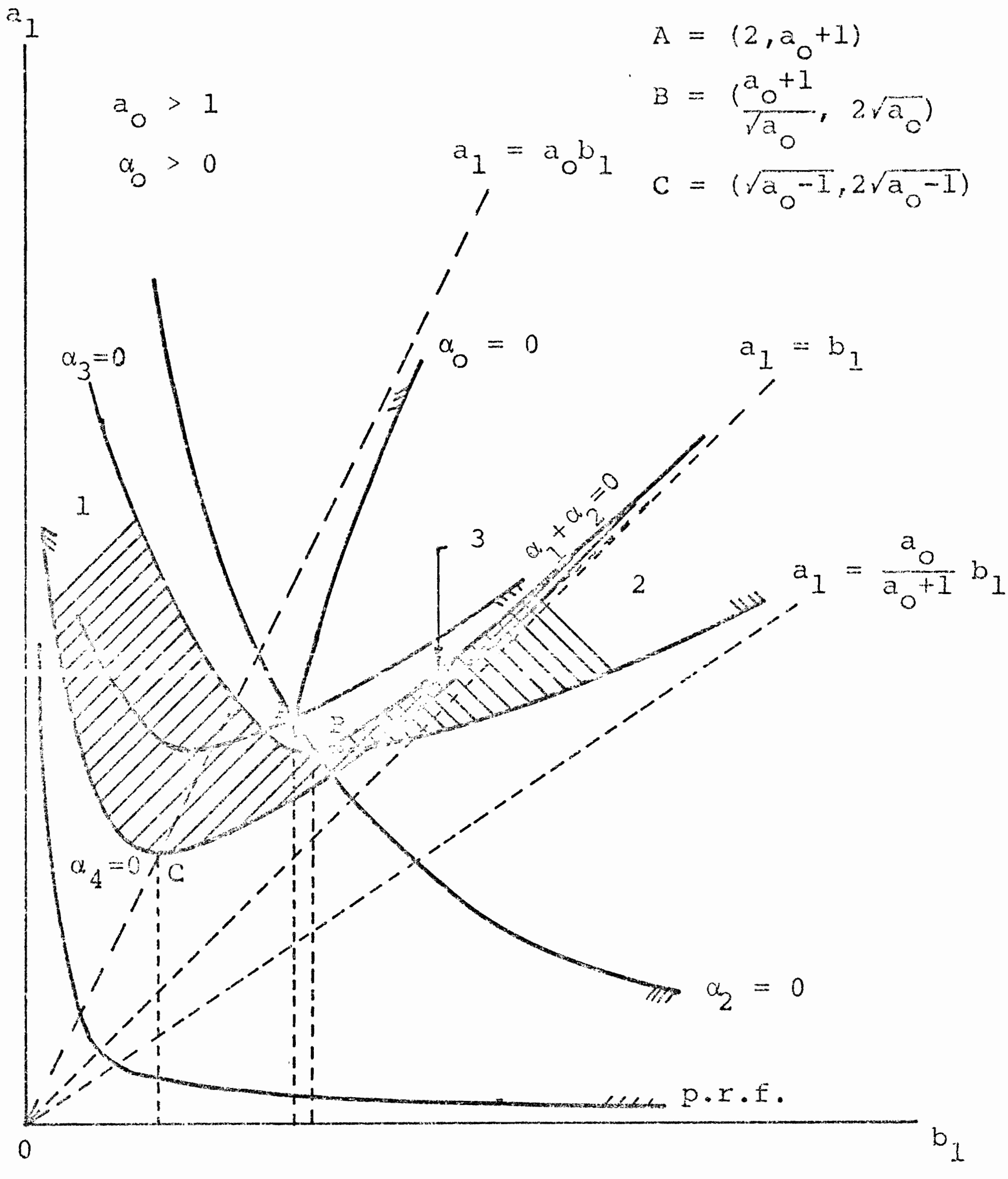


Fig. 3.2 Coefficient conditions in the (a_1, b_1) plane for the realization of z(s) by Structure I, when $a_o > 1$

contrary to our condition $\alpha_0 > 0$.

In conclusion, we see that if $a_0 > 1$, $\alpha_0 > 0$, then there is always a realization if α_3 and α_4 are of opposite signs, however, if $\alpha_3 > 0$, $\alpha_4 > 0$, then are two realizations provided, $\alpha_2 < 0$ and $\alpha_1 + \alpha_2 > 0$. All these results are tabulated in Table 3.1. These results are also presented in the (a_1, b_1) plane defining certain regions where the biquadratic function (3.1) may be realized by Structure I, when $\alpha_0 > 1$ (see Fig. 3.2).

3.2.2 <u>When $a_0 < 1$.</u> Just as in the case of $(a_0 > 1)$ we will look at all the possibilities for α_1, α_2 etc., such that r_1, r_2, r_3, L and C are positive. If $r_3 > 0$, $r_2 > 0$ then

$$r_1 = (1-a_0) + r_2$$

will automatically be positive and so also will C and L be. Thus, we have to find the conditions under which r_3 and r_2 are positive, when $a_0 < 1$ and $\alpha_0 > 0$. First, we assume that α_1 and α_2 are not zero.

(i) $$\alpha_1 > 0,\ \alpha_2 > 0,\ \alpha_3 > 0.$$

Then from (3.7) there is no positive solution for r_3.

(ii) $\alpha_1 > 0, \alpha_2 > 0, \alpha_3 < 0.$

Then there is one solution for r_3 namely

$$r_3 = \frac{-\alpha_2 + \sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1} \tag{3.48}$$

since $r_2 = a_0 - r_3$ we have

$$r_2 = \frac{(a_0\alpha_1 + \alpha_2) - \sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1}$$

Thus for $r_2 > 0$ we should have

$$(a_0\alpha_1 + \alpha_2) > \sqrt{\alpha_2^2 - \alpha_1\alpha_3}$$

Hence

$$a_0^2\alpha_1^2 + 2a_0\alpha_1\alpha_3 + \alpha_1\alpha_3 > 0$$

Since $\alpha_1 > 0$ we get

$$a_0^2\alpha_1 + 2a_0\alpha_2 + \alpha_3 > 0$$

Substituting for α_1, α_2 and α_3 we have

$$a_0^2\alpha_1 + 2a_0\alpha_2 + \alpha_3 = (1-a_0)\alpha_5 \tag{3.49}$$

where

$$\alpha_5 = a_1 b_1 - a_0 b_1^2 - (1-a_0) \tag{3.50}$$

It may also be shown that

$$a_0\alpha_1 + \alpha_2 + \alpha_5 = (1-a_0) \tag{3.51}$$

Thus $r_2 > 0$ if $\alpha_5 > 0$.

Summarizing these results we get

$$\alpha_0 > 0$$
$$\alpha_1 > 1$$
$$\alpha_2 > 0 \qquad (3.52)$$
$$\alpha_3 < 0$$
$$\alpha_5 > 0$$

(iii) $\alpha_1 < 0,\ \alpha_2 > 0,\ \alpha_3 > 0.$

In this case, there is one solution for r_3

$$r_3 = \frac{-\alpha_2 - \sqrt{\alpha_2^2-\alpha_1\alpha_3}}{\alpha_1} \qquad (3.53)$$

Hence

$$r_2 = \frac{(a_0\alpha_1+\alpha_2)+\sqrt{\alpha_2^2-\alpha_1\alpha_3}}{\alpha_1}$$

Letting $-\alpha_1 = p > 0$ we see that $r_2 > 0$ if

$$(a_0p - \alpha_2) > \sqrt{\alpha_2^2 + p\alpha_3}$$

Since the R.H.S. is positive, we have that $r_1 > 0$ if

$$(a_0p - \alpha_2) > 0 \quad \text{or} \quad (a_0\alpha_1+\alpha_2) < 0 \qquad (3.54)$$

and

$$(a_0p - \alpha_2)^2 > \alpha_2^2 + p\alpha_3$$

which may be simplified as

$$a_0^2\alpha_1+2a_0\alpha_2+\alpha_3 = (1-a_0)\alpha_5 < 0 \qquad (3.55)$$

But from (3.51) we see that $\alpha_5 < 0$ and $a_0 < 1$ imply that $(a_0\alpha_1+\alpha_2) > 0$, which contradicts (3.54). Thus there is no solution when $\alpha_1 < 0$, $\alpha_2 > 0$ and

$\alpha_3 > 0$.

(iv) $\alpha_1 < 0, \quad \alpha_2 > 0, \quad \alpha_3 < 0$.

In this case, there are two solutions for r_3 namely,

$$r_3 = \frac{-\alpha_2 \pm \sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1} = \frac{\alpha_2 \mp \sqrt{\alpha_2^2 - pq}}{p}$$

where $-\alpha_1 = p > 0$ and $-\alpha_3 = q > 0$.

a) Considering first the solution for r_3

$$r_3 = \frac{\alpha_2 + \sqrt{\alpha_2^2 - pq}}{p} \quad (3.56)$$

we see that $r_2 > 0$ if

$$(a_0 p - \alpha_2) > \sqrt{\alpha_2^2 - pq}$$

Hence we should have

$$(a_0 p - \alpha_2) > 0 \quad \text{or} \quad (a_0\alpha_1 + \alpha_2) < 0 \quad (3.57)$$

and

$$(a_0 p - \alpha_2)^2 > \alpha_2^2 - pq$$

which when simplified gives

$$a_0^2\alpha_1 + 2a_0\alpha_2 + \alpha_3 = (1 - a_0)\alpha_5 < 0 \quad (3.58)$$

But $\alpha_5 < 0$ implies $(a_0\alpha_1 + \alpha_2) > 0$, which contradicts (3.57). Hence, there is no solution corresponding to (3.56).

b) Considering now the second solution for r_3

$$r_3 = \frac{\alpha_2 - \sqrt{\alpha_2^2 - pq}}{p}$$

we have

$$pr_2 = (a_0 p - \alpha_2) + \sqrt{\alpha_2^2 - pq}$$

Thus, $r_2 > 0$ if

$$\sqrt{\alpha_2^2 - pq} > (\alpha_2 - a_0 p) \qquad (3.59)$$

This is automatically satisfied if $\alpha_2 - a_0 p \leqslant 0$ or $a_0\alpha_1 + \alpha_2 \leqslant 0$.

It should be noted from (3.51) that if $a_0\alpha_1 + \alpha_2 \leqslant 0$, it then automatically follows that

$$\alpha_5 > 0$$

However, if $(\alpha_2 - a_0 p) > 0$ or $(a_0\alpha_1 + \alpha_2) > 0$, then for $r_2 > 0$, we should have

$$\alpha_2^2 - pq > (\alpha_2 - a_0 p)^2$$

or

$$a_0^2\alpha_1 + 2a_0\alpha_2 + \alpha_3 = (a_0 - 1)\alpha_5 > 0$$

Thus we see that whether $(a_0\alpha_1 + \alpha_2) \leqslant 0$ or > 0, there is always a realization provided

$$\alpha_0 > 0,\ \alpha_1 < 0,\ \alpha_2 > 0,\ \alpha_3 < 0,\ \alpha_5 > 0$$

However, from (3.49), $\alpha_5 > 0$, $\alpha_3 < 0$, and $\alpha_1 < 0$ imply $\alpha_2 > 0$. Thus, a realization exists if

$$\alpha_0 > 0$$
$$\alpha_1 < 0 \qquad (3.60)$$
$$\alpha_3 < 0$$
$$\alpha_5 > 0$$

(v) $\alpha_1 > 0,\ \alpha_2 < 0,\ \alpha_3 > 0.$

Then there are two solutions for r_3 namely

$$r_3 = \frac{-\alpha_2 \pm \sqrt{\alpha_2^2-\alpha_1\alpha_3}}{\alpha_1} = \frac{q \pm\sqrt{q^2 -\alpha_1\alpha_3}}{\alpha_1} \qquad (3.61)$$

where $-\alpha_2 = q > 0$

a) Considering the first root,

$$r_3 = \frac{q + \sqrt{q^2-\alpha_1\alpha_3}}{\alpha_1} \qquad (3.62)$$

we have that $r_2 > 0$ if

$$(a_0\alpha_1 - q) > \sqrt{q^2-\alpha_1\alpha_3} > 0$$

This will be satisfied if

$$(a_0\alpha_1 - q) > 0 \text{ or } a_0\alpha_1 + \alpha_2 > 0 \qquad (3.63)$$

and

$$(a_0\alpha_1 - q)^2 > q^2 - \alpha_1\alpha_3$$

which may be simplified as

$$a_0^2\alpha_1 + 2a_0\alpha_2 + \alpha_3 = (1-a_0)\alpha_5 > 0 \qquad (3.64)$$

Since $(a_0\alpha_1 + \alpha_2) > 0$ and $\alpha_2 < 0$, we conclude that $\alpha_1 > 0$ is a superfluous condition.

Thus we have a realization if

$$\alpha_0 > 0$$

$$\alpha_2 < 0$$

$$a_0\alpha_1 + \alpha_2 > 0 \qquad (3.65)$$

$$\alpha_3 > 0$$

$$\alpha_5 > 0$$

b) Now considering the second root for r_3 we have that $r_2 > 0$ provided,

$$(q - a_0\alpha_1) < \sqrt{q^2 - \alpha_1\alpha_3}$$

If $(q - a_0\alpha_1) \leq 0$, that is if $a_0\alpha_1 + \alpha_2 \geq 0$ the above condition is always satisfied and no restriction is to be placed on α_5.

But, if $(q - a_0\alpha_1) > 0$ or $(a_0\alpha_1 + \alpha_2) < 0$, then for $r_2 > 0$, we should have the condition,

$$(q - a_0\alpha_1)^2 < q^2 - \alpha_1\alpha_3$$

which may be simplified as

$$a_0^2\alpha_1 + 2a_0\alpha_2 + \alpha_3 = (1-a_0)\alpha_5 < 0$$

This is impossible, since from (3.51), $(a_0\alpha_1 + \alpha_2) < 0$ implies $\alpha_5 > 0$.

Thus, a network realization exists if

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_2 &< 0 \\ (a_0\alpha_1 + \alpha_2) &> 0 \\ \alpha_3 &> 0 \end{aligned} \tag{3.66}$$

From (3.65) and (3.66) we see that there are two realizations if

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_2 &< 0 \\ a_0\alpha_1 + \alpha_2 &> 0 \\ \alpha_3 &> 0 \\ \alpha_5 &> 0 \end{aligned} \tag{3.67}$$

While there is only one realization if

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_2 &< 0 \\ (a_0\alpha_1 + \alpha_2) &> 0 \\ \alpha_3 &> 0 \\ \alpha_5 &< 0 \end{aligned} \tag{3.68}$$

It should be pointed out that in (3.68), $(a_0\alpha_1+\alpha_2) > 0$ is a superfluous condition, since $\alpha_5 < 0$ implies $(a_0\alpha_1 + \alpha_2) > 0$.

Now $\alpha_5 < 0$, $\alpha_2 < 0$ imply $\alpha_1 > 0$ from (3.51). It may also be shown that $\alpha_3 > 0$ and $\alpha_5 < 0$ imply $\alpha_2 < 0$. Hence, inequalities (3.68) are equivalent to

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_3 &> 0 \\ \alpha_5 &< 0 \\ \alpha_2 &\neq 0 \end{aligned} \qquad (3.69)$$

(vi) $\alpha_1 > 0, \alpha_2 < 0, \alpha_3 < 0.$

Then there is only one solution for r_3

$$r_3 = \frac{-\alpha_2 + \sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1}$$

Letting $-\alpha_2 = p > 0$ and $-\alpha_3 = q > 0$, we have that for $r_2 > 0$, we should have

$$(a_0\alpha_1 - p) > \sqrt{p^2 + \alpha_1 q} > 0$$

Thus, $r_2 > 0$ provided

$$(a_0\alpha_1 - p) > 0 \text{ or } a_0\alpha_1 + \alpha_2 > 0 \qquad (3.70)$$

and

$$(a_0\alpha_1 - p)^2 > (p^2 + \alpha_1 q)$$

which simplifies to

$$a_0^2\alpha_1 + 2a_0\alpha_2 + \alpha_3 = (1-a_0)\alpha_5 > 0 \qquad (3.71)$$

The condition $(a_0\alpha_1 + \alpha_2) > 0$ is contained in $\alpha_5 > 0$ since

$$\alpha_5 = a_0(a_0\alpha_1 + \alpha_2) + (a_0\alpha_2 + \alpha_3)$$

and α_2 and α_3 are negative.

Thus, we have a realization if

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_1 &> 0 \\ \alpha_2 &< 0 \\ \alpha_3 &< 0 \\ \alpha_5 &> 0 \end{aligned} \tag{3.72}$$

(vii) $\alpha_1 < 0, \alpha_2 < 0, \alpha_3 > 0$

In this case, there is one solution for r_3

$$r_3 = \frac{-\alpha_2 - \sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1} > 0 \tag{3.73}$$

For $r_2 > 0$ we should have

$$(a_0\alpha_1 + \alpha_2) + \sqrt{\alpha_2^2 - \alpha_1\alpha_3} < 0, \text{ since } \alpha_1 < 0.$$

Letting $-\alpha_1 = p > 0$, $-\alpha_2 = q > 0$ we get

$$(a_0 p + q) > \sqrt{q^2 + p\alpha_3}$$

since both sides are positive, the above inequality reduces to

$$(a_0 p + q)^2 > q^2 + p\alpha_3$$

or

$$a_0^2\alpha_1 + 2a_0\alpha_2 + \alpha_3 = (1-a_0)\alpha_5 < 0$$

But $\alpha_5 < 0$ implies $(a_0\alpha_1 + \alpha_2) > 0$ which contradicts $\alpha_1 < 0$ and $\alpha_2 < 0$.

Hence there is no realization when

$$\alpha_1 < 0, \ \alpha_2 < 0 \ \text{and} \ \alpha_3 > 0.$$

(viii) $\alpha_1 < 0, \alpha_2 < 0, \alpha_3 < 0$

There is no positive solution for r_3 and hence no realization exists.

In the above discussion, we have assumed that $\alpha_1 \neq 0$, $\alpha_2 \neq 0$. Combining (3.52), (3.72) and (3.60), we see that there is a realization if

$$\begin{array}{c} \alpha_0 > 0 \\ \alpha_3 < 0 \\ \alpha_5 > 0 \\ \alpha_1 \neq 0 \; , \; \alpha_2 \neq 0 \end{array} \qquad (3.74)$$

and the value of r_3 is given by (3.48).

Also, from (3.69) we have a realization if

$$\begin{array}{c} \alpha_0 > 0 \\ \alpha_3 > 0 \\ \alpha_5 < 0 \\ \alpha_1 \neq 0 \; , \; \alpha_2 \neq 0 \end{array} \qquad (3.75)$$

and the value of r_3 is given by (3.53).

From (3.67) we also see that there are two realizations, if

$$\begin{array}{c} \alpha_0 > 0 \\ \alpha_3 > 0 \\ \alpha_5 > 0 \\ \alpha_2 < 0 \\ a_0\alpha_1 + \alpha_2 > 0 \end{array} \qquad (3.76)$$

$$\alpha_1 \neq 0$$

and the value of r_3 is given by (3.61).

We see from (3.76) that the condition $\alpha_1 \neq 0$ need not be included, since in that case, $\alpha_2 < 0$ and $a_0\alpha_1 + \alpha_2 > 0$ cannot be simultaneously satisfied. Following the same procedure as was used for the case $a_0 > 1$, we may show that in conditions (3.74) and (3.75), the restrictions $\alpha_1 \neq 0$, $\alpha_2 \neq 0$ may be removed. However, when $\alpha_1 = 0$ the value of r_3 is no longer given by (3.48) or (3.53) but by

$$r_3 = \frac{-\alpha_3}{2\alpha_2} \qquad (3.77)$$

The condition $\alpha_1 = \alpha_2 = 0$ need not be considered since in that case $\alpha_0 = 0$, which contradicts the conditions $\alpha_0 > 0$.

In conclusion, we see that if $a_0 < 1$, $\alpha_0 > 0$, then there is always a realization if α_3 and α_5 are of opposite signs; however, if $\alpha_3 > 0$, $\alpha_5 > 0$, then there are two realizations provided, $\alpha_2 < 0$ and $a_0\alpha_1 + \alpha_2 > 0$. All these results are tabulated in Table 3.2. These results are also presented in the (a_1, b_1) plane defining certain regions where the biquadratic function (3.1) may be realized by Structure I, when $a_0 < 1$ (see Fig. 3.3).

TABLE 3.2

Coefficient conditions for the realization of $z(s)$ by Structure I and the corresponding component values.

$(\alpha_0 < 1, \alpha_o > 0)$

Region	Coefficient Conditions	Value of r_3	Comments
1	$\alpha_3 < 0$ $\alpha_5 > 0$	$\frac{-\alpha_2+\sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1}$; $\alpha_1 \neq 0$ $\frac{-\alpha_3}{2\alpha_2}$; $\alpha_1 = 0$	One network realization.
2	$\alpha_3 > 0$ $\alpha_5 < 0$	$\frac{-\alpha_2-\sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1}$; $\alpha_1 \neq 0$ $\frac{-\alpha_3}{2\alpha_2}$; $\alpha_1 = 0$	One network realization.
3	$\alpha_3 > 0$ $\alpha_5 > 0$ $a_0\alpha_1 + \alpha_2 > 0$ $\alpha_2 < 0$	$\frac{-\alpha_2\pm\sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1}$	Two network realizations.

$r_1 = 1 - r_3; r_2 = a_0 - r_3; C = b_1/(r_1+r_2) ; L = \frac{1}{C}$

NOTE: The values of R_1, R_2 and R_3 may be determined using (3.16).

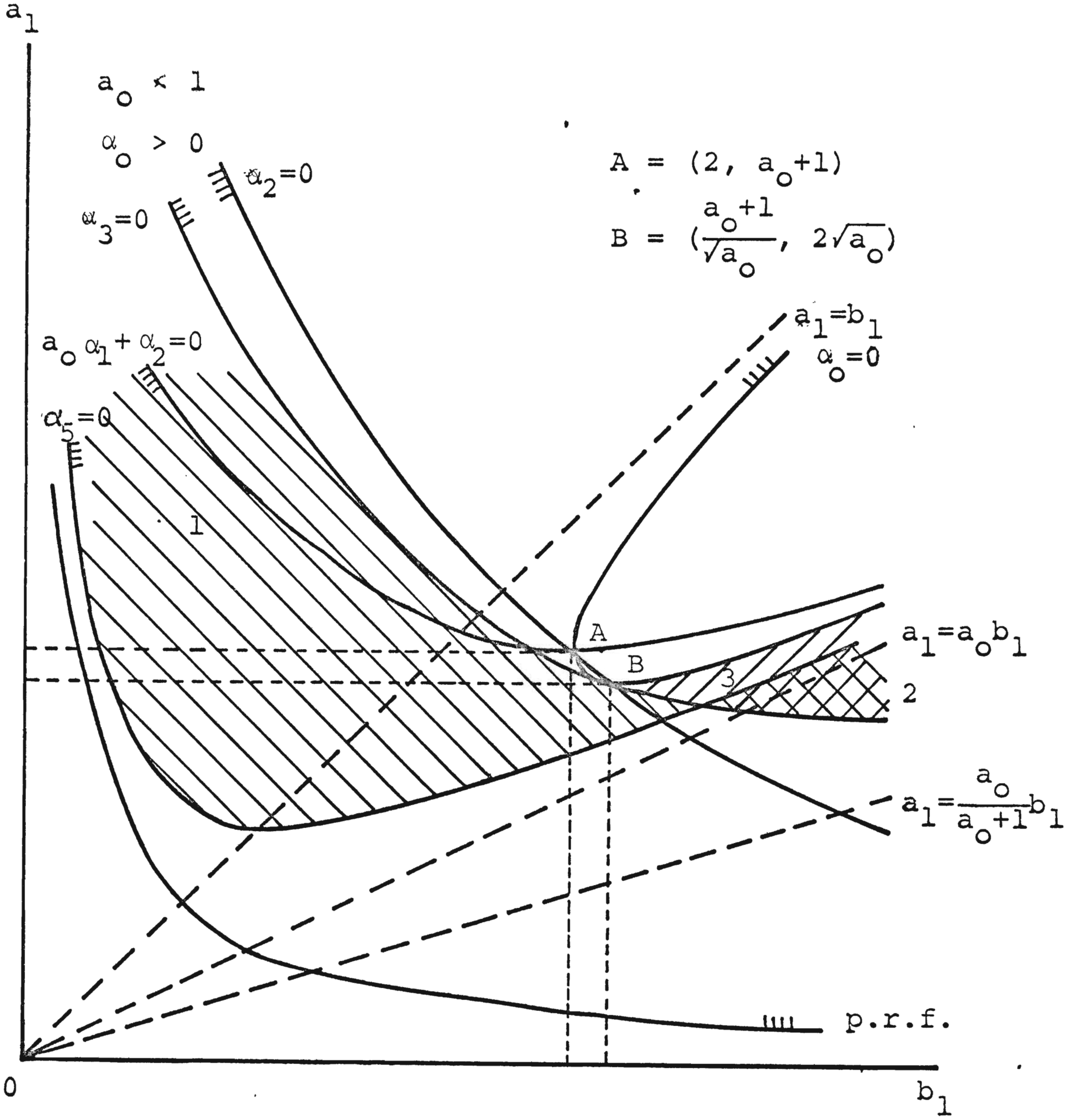


Fig. 3.3 Coefficient conditions in the (a_1, b_1) plane for the realization of z(s) by Structure I, when $a_o < 1$.

3.2.3. <u>When $a_0 = 1$.</u> When $a_0 = 1$, we see from (2.6) that the positive real condition reduces to $a_1 b_1 \geqslant 0$ and since $a_1 \neq 0$, $b_1 \neq 0$, the condition becomes

$$a_1 b_1 > 0$$

Also

$$\alpha_0 = (a_1 - b_1)^2 > 0 \tag{3.78}$$

As a result, the impedance z (s) given by (3.1) can only be realized by an RLC structure. Furthermore, the different α's reduce to

$$\alpha_1 = b_1^2 - 4$$
$$\alpha_2 = 4 - a_1 b_1$$
$$\alpha_3 = 2a_1 b_1 - b_1^2 - 4$$
$$\alpha_4 = \alpha_5 = b_1 (a_1 - b_1)$$

and

$$\alpha_2^2 - \alpha_1\alpha_3 = b_1^2 (a_1 - b_1)^2 > 0$$

From previous derrivation we have

$$r_3 = \frac{-\alpha_2 \pm \sqrt{\alpha_2^2 - \alpha_1\alpha_3}}{\alpha_1}$$
$$= \frac{(a_1 b_1 - 4) \pm b_1 (a_1 - b_1)}{\alpha_1}$$
$$= 1, \quad \frac{2a_1 b_1 - b_1^2 - 4}{b_1^2 - 4}$$

The first root will result in $r_1 = r_2 = 0$ and hence, $b_1 = 0$ from (3.10). In this case z(s) is not a proper biquadratic function. Hence, $r_3 = 1$ is not a valid solution. Taking

$$r_3 = \frac{2a_1 b_1 - b_1^2 - 4}{b_1^2 - 4} = \frac{\alpha_3}{\alpha_1}$$

we have

$$r_1 = r_2 = \frac{-2\alpha_4}{\alpha_1}$$

$$\frac{1}{L} = C = \frac{b_1}{2r_1}$$

Thus, for r_1, r_2 and r_3 to be positive, either

(i) $\alpha_1 > 0, \alpha_3 > 0, \alpha_4 < 0$

or

(ii) $\alpha_1 < 0, \alpha_3 < 0, \alpha_4 > 0$ (3.79)

Now, when $a_0 = 1$ from (3.18) and (3.19) we have

$$-\alpha_1 = 2\alpha_4 - \alpha_3 \quad (3.80)$$

Hence, if $\alpha_4 > 0$ and $\alpha_3 < 0$ then $\alpha_1 < 0$ while if $\alpha_4 < 0$ and $\alpha_3 > 0$ then $\alpha_1 > 0$. Thus (3.79) may be reduced to

(i) $\alpha_3 > 0, \alpha_4 < 0$

and

(ii) $\alpha_3 < 0, \alpha_4 > 0$

Hence, when $a_0 = 1$, z(s) can always be realized by Structure I provided

$$\alpha_3 \alpha_4 < 0 \quad (3.81)$$

that is, if α_3 and $\alpha_4=\alpha_5$ are of opposite signs, and the element values are given by

$$r_1 = r_2 = \frac{-2\alpha_4}{\alpha_1}$$

$$r_3 = \frac{\alpha_3}{\alpha_1} \qquad (3.82)$$

$$C = \frac{b_1}{2r_1},$$

$$L = \frac{1}{C}$$

The regions defined by (3.81) are shown in Fig. 3.4. in the (a_1, b_1) plane.

3.3. Conditions for Realization by Structure II.

By direct analysis, the D.P.I. of Structure II, (Fig. 3.5a) may be shown to be

$$z(s) = \frac{\Sigma R_1 R_3}{R_1+R_3} \frac{s^2 + \dfrac{R_1+R_2}{C\Sigma R_1 R_3} s + \dfrac{1}{LC}}{s^2 + \dfrac{L+C\Sigma R_1 R_3}{R_1+R_3} s + \dfrac{R_2+R_3}{LC(R_1+R_3)}} \qquad (3.83)$$

Using the transformations

$$g_1 = \frac{R_1}{\Sigma R_1 R_3} = \frac{G_2 G_3}{\Sigma G}$$

$$g_2 = \frac{R_2}{\Sigma R_1 R_3} = \frac{G_1 G_3}{\Sigma G} \qquad (3.84)$$

$$g_3 = \frac{R_3}{\Sigma R_1 R_3} = \frac{G_1 G_2}{\Sigma G}$$

or equivalently

$$\frac{1}{R_1} = G_1 = \frac{\Sigma g_1 g_3}{g_1}$$

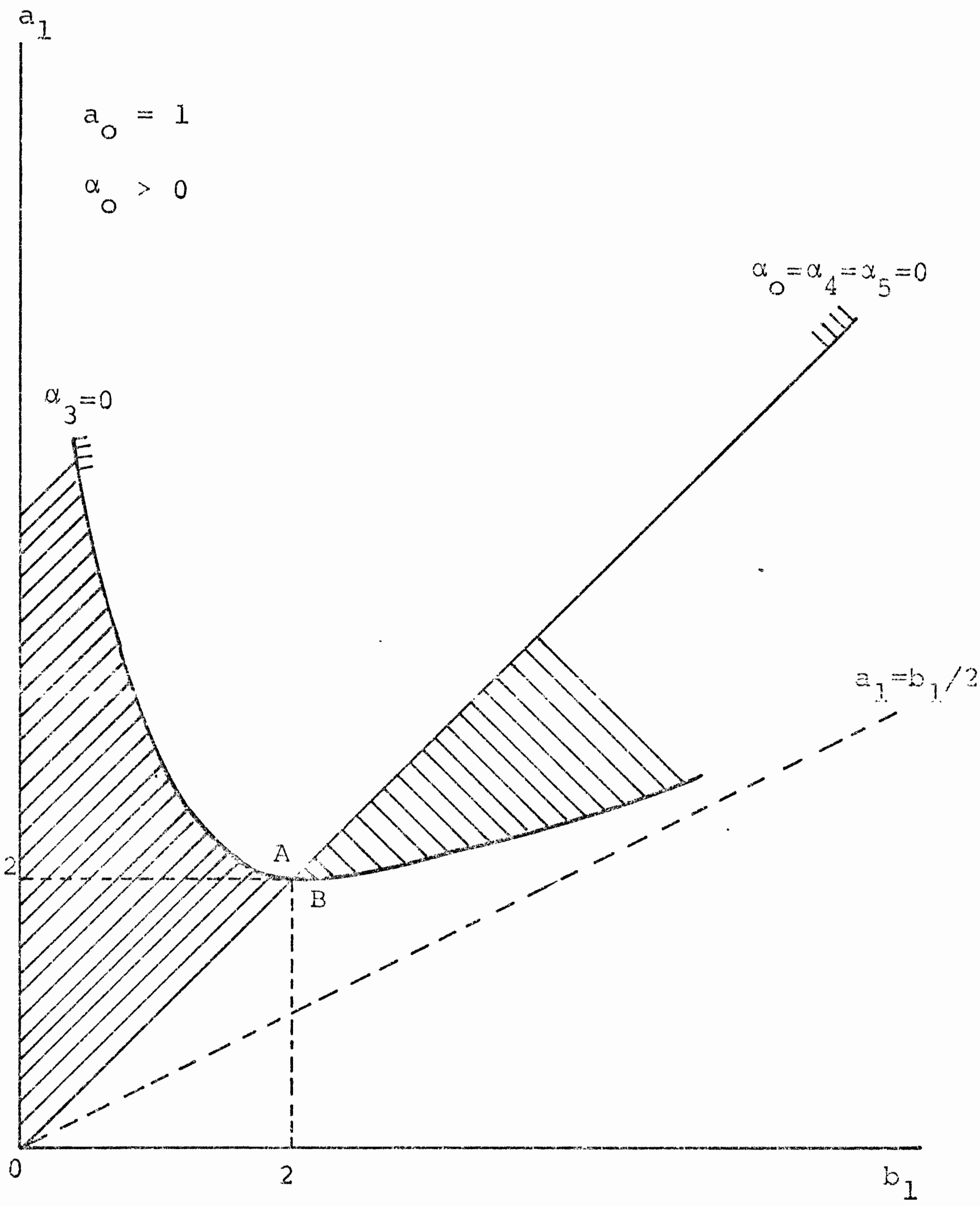
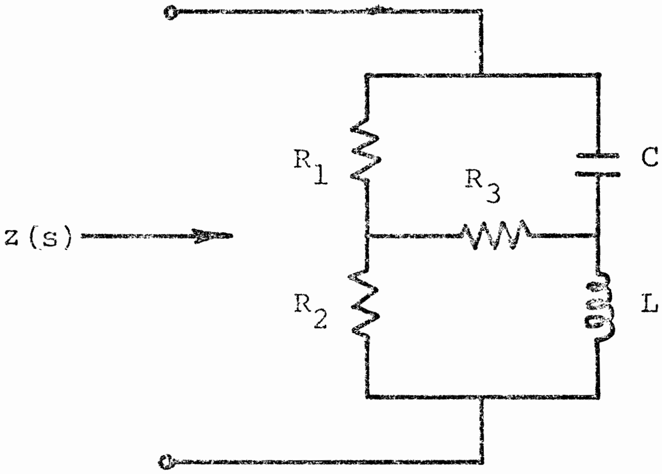
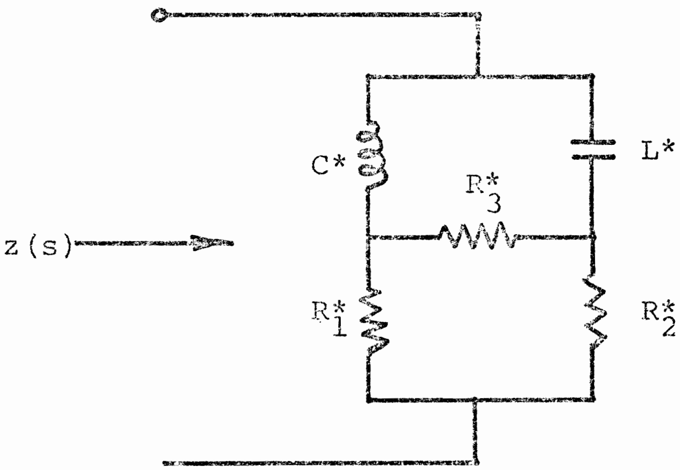


Fig. 3.4 Coefficient conditions in the (a_1, b_1) plane for the realization of z(s) by Structure I, when $a_o = 1$.



(a)



(b)

Fig. 3.5 (a) The five element bridge Structure II.
(b) The dual of Structure II.

$$\frac{1}{R_2} = G_2 = \frac{\Sigma g_1 g_3}{g_2} \tag{3.85}$$

$$\frac{1}{R_3} = G_3 = \frac{\Sigma g_1 g_3}{g_3}$$

and comparing (3.83) with (3.1), and simplifying, we get

$$\beta_1 g_3^2 + 2\beta_2 g_3 + \beta_3 = 0 \tag{3.86}$$

$$g_1 = 1 - g_3 \tag{3.87}$$

$$g_2 = \frac{1}{a_0} - g_3 \tag{3.88}$$

$$L = \frac{a_1}{a_0} \frac{1}{g_1 + g_2} \tag{3.89}$$

$$C = \frac{1}{L} \tag{3.90}$$

where

$$\beta_1 = (a_1^2 - 4a_0)\, a_0 \tag{3.91}$$

$$\beta_2 = [2(a_0+1) - a_1 b_1]\, a_0 \tag{3.92}$$

$$\beta_3 = a_1 b_1 (a_0+1) - a_1^2 - (a_0+1)^2 \tag{3.93}$$

and

$$\beta_2^2 - \beta_1 \beta_3 = a_0 a_1^2 \alpha_0 \tag{3.94}$$

It may be seen that equations (3.86) to (3.90) are similar to equations (3.6) to (3.11), as also equations (3.91) to (3.94) and (3.12) to (3.15). Thus, we may derive the coefficient conditions for the realization of z(s) by Structure II, by adopting procedures similar to the ones used in Sec. 3.2 for Structure I.

However, we shall derive these conditions in an entirely different way, by observing that the dual of Structure II is Structure I (Fig. 3.1) with

$$
\begin{aligned}
R_1^* &= G_1 \Omega \\
R_2^* &= G_2 \Omega \\
R_3^* &= G_3 \Omega \qquad (3.95) \\
L^* &= C \text{ Henry} \\
C^* &= L \text{ Farad}
\end{aligned}
$$

where R_1^*, R_2^*, R_3^*, L^* and C^* correspond to the different elements of the Structure I, which is the dual of Structure II. This is shown in Fig. 3.5b. If we now define r_1^*, r_2^* and r_3^* by

$$
\begin{aligned}
r_1^* &= \frac{R_2^* R_3^*}{\Sigma R^*} \\
r_2^* &= \frac{R_1^* R_3^*}{\Sigma R^*} \qquad (3.96) \\
r_3^* &= \frac{R_1^* R_2^*}{\Sigma R^*}
\end{aligned}
$$

We see from (3.95), (3.84) and (3.96) that

$$
\begin{aligned}
r_1^* &= g_1 \Omega \\
r_2^* &= g_2 \Omega \\
r_3^* &= g_3 \Omega \qquad (3.97) \\
L^* &= C \text{ Henry} \\
C^* &= L \text{ Farad}
\end{aligned}
$$

Since structures of Fig. 3.5(a) and (b) are duals of each other, we know that if the network of Fig. 3.5(a) realized the impedance (3.1), then the impedance $z^*(s)$ of the network of Fig. 3.5(b) is

$$z^*(s) = \frac{s^2 + b_1 s + 1}{s^2 + a_1 s + a_0} \tag{3.98}$$

Scaling the complex frequency s by

$$p = \frac{s}{\sqrt{a_0}} \tag{3.99}$$

we get

$$z^*(p) = \frac{p^2 + a_1^* p + a_0^*}{p^2 + b_1^* p + 1} \tag{3.100}$$

where

$$a_0^* = \frac{1}{a_0}$$
$$a_1^* = \frac{b_1}{\sqrt{a_0}} \tag{3.101}$$
$$b_1^* = \frac{a_1}{\sqrt{a_0}}$$

In Section 3.2, we have already derived conditions as well as component values for Structure I, to realize a D.P.F. of the form (3.100) for $a_0^* \gtrless 1$. These conditions may directly be used to obtain corresponding conditions for $z(s)$ to be realized by Structure II for $a_0 \lessgtr 1$. Also, since the component values of Structure I may be found in the p-plane using the results of Sec. 3.2, they may be suitably frequency scaled to obtain r_1^*, r_2^*, r_3^*, C^* and L^*, and hence, g_1, g_2, g_3, L and C, using (3.97) and (3.95).

In order to find the coefficient conditions for the realization of Structure II by this method, we need to find $\alpha_1^*, \alpha_2^*, \alpha_3^*$, corresponding to $\alpha_1, \alpha_2, \alpha_3$ used in Sec. 3.2. Now from (2.8)

$$\alpha_0^* = (a_0^*-1)^2 - (a_1^*-b_1^*)(a_0^* b^* - a_1^*)$$

or

$$\alpha_0^* = \frac{1}{a_0^2} \alpha_0 \tag{3.102}$$

$$\alpha_1^* = b_1^{*2} - 4$$

or

$$\alpha_1^* = \frac{1}{a_0^2} \beta_1 \tag{3.103}$$

$$\alpha_2^* = 2(a_0^* + 1) - a_1^* b_1^*$$

or

$$\alpha_2^* = \frac{1}{a_0^2} \beta_2 \tag{3.104}$$

$$\alpha_3^* = (a_0^*+1)a_1^* b_1^* - a_0^* b_1^{*2} - (a_0^*+1)$$

or

$$\alpha_3^* = \frac{1}{a_0^2} \beta_3 \tag{3.105}$$

$$\alpha_4^* = b_1^*(a_1^*-b_1^*) - (a_0^*-1)$$

or

$$\alpha_4^* = \frac{1}{a_0} \beta_4 \tag{3.106}$$

$$\alpha_5^* = a_1^* b_1^* - a_0^* b_1^{*2} - (1-a_0^*)$$

or

$$\alpha_5^* = \frac{1}{a_0^2} \beta_5 \tag{3.107}$$

where $\beta_1, \beta_2, \beta_3$ are given by (3.91), (3.92), (3.93) while β_4 and β_5 are given by

$$\beta_4 = a_1(b_1-a_1)-(1-a_0) \tag{3.108}$$

$$\beta_5 = a_0 a_1 b_1 - a_1^2 - a_0(a_0-1) \tag{3.109}$$

It may also be shown using (3.15), (3.18), (3.19),(3.49), (3.51) and (3.101) that the following relations hold

$$\beta_2^2 - \beta_1\beta_3 = a_1^2 a_0 \alpha_0 \qquad (3.110)$$

$$\beta_1 + 2\beta_2 + \beta_3 = (1-a_0)\beta_4 \qquad (3.111)$$

$$\beta_1 + \beta_2 + a_0\beta_4 = a_0(1-a_0) \qquad (3.112)$$

$$\beta_1 + 2a_0\beta_2 + a_0^2\beta_3 = a_0(a_0-1)\beta_5 \qquad (3.113)$$

$$\beta_1 + a_0\beta_2 + a_0\beta_5 = a_0^2(a_0-1) \qquad (3.114)$$

We shall now derive the conditions for the case $a_0 < 1$. In a similar way, conditions for $a_0 > 1$, $a_0 = 1$ may be derived.

$a_0 < 1$ corresponds to $a_0^* > 1$ for Structure I, the dual of Structure II. We know from Table 3.1 that if

$$\begin{aligned} \alpha_0^* &> 0 \\ a_0^* &> 1 \\ \alpha_3^* &< 0 \\ \alpha_4^* &> 0 \end{aligned} \qquad (3.115)$$

Structure I exists. Hence, using (3.102), (3.105) and (3.106), we see that Structure II may be realized if

$$\begin{aligned} \alpha_0 &> 0 \\ a_0 &< 1 \\ \beta_3 &< 0 \\ \beta_4 &> 0 \end{aligned} \qquad (3.116)$$

Also, from Table 3.1, we see that in the p-plane, the component values of the network in Fig. 3.5(b) corresponding to the coefficient (3.115) are,

$$r_3^* = \frac{-\alpha_2^* + \sqrt{\alpha_2^{*2} - \alpha_1^* \alpha_3^*}}{\alpha_1^*} = \frac{-\beta_2 + \sqrt{\beta_2^2 - \beta_1 \beta_3}}{\beta_1}$$

$$r_1^* = 1 - r_3^*$$

$$r_2^* = a_0^* - r_3^* = \frac{1}{a_0} - r_3^*$$

$$C^* = \frac{b_1^*}{r_1^* + r_2^*} = \frac{a_1}{\sqrt{a_0}} \frac{1}{r_1^* + r_2^*}$$

$$L^* = \frac{1}{C^*}$$

Since $p = \frac{s}{\sqrt{a_0}}$, the corresponding values in the s-plane are

$$r_3^* = \frac{-\beta_2 + \sqrt{\beta_2^2 - \beta_1 \beta_3}}{\beta_1}$$

$$r_1^* = 1 - r_3^*$$

$$r_2^* = \frac{1}{a_0} - r_3^*$$

$$C^* = \frac{a_1}{a_0} \frac{1}{r_1^* + r_2^*}$$

$$L^* = \frac{1}{a_0} \frac{1}{C^*}$$

Hence, using (3.97) we get the component values of Structure II corresponding to the coefficient conditions (3.116) to be:

$$g_3 = \frac{-\beta_2 + \sqrt{\beta_2^2 - \beta_1\beta_3}}{\beta_1}$$

$$g_2 = \frac{1}{a_0} - g_3$$

$$g_1 = 1 - g_3 \qquad (3.117)$$

$$L = \frac{a_1}{a_0}\,\frac{1}{g_1 + g_2}$$

$$C = \frac{1}{a_0}\,\frac{1}{L}$$

Once g_1, g_2 and g_3 are known, R_1, R_2 and R_3 may be determined, using (3.85).

Similarly, the other coefficient conditions under which Structure II will realize (3.1) when $a_0 < 1$ may be found using the results in Table 3.1 for the case of Structure I, when $a_0 > 1$. These results are presented in Table 3.3.

The coefficient conditions to be satisfied in order that Structure II may realize (3.1) when $a_0 > 1$ and $a_0 = 1$ may also be derived using the corresponding results of Structure I for the cases of $a_0 < 1$ and $a_0 = 1$, respectively, by using the above technique. These results are presented in Tables 3.4 and 3.5.

Figures 3.6, 3.7 and 3.8 present these results in the (a_1, b_1) plane, concerning the coefficient conditions for the realization of (3.1) by Structure II. It should be pointed out that there are certain regions, just as in the case of Structure I, when two realizations may be found.

TABLE 3.3

Coefficient conditions for the realization of $z(s)$ by Structure II and the corresponding component values.

$(a_0 < 1, \alpha_0 > 0)$

Region	Coefficient Conditions	Value of g_3	Comments
1	$\beta_3 < 0$ $\beta_4 > 0$	$\frac{-\beta_2+\sqrt{\beta_2^2 - \beta_1\beta_3}}{\beta_1}$; $\beta_1 \neq 0$ $\frac{-\beta_3}{2\beta_2}$; $\beta_1 = 0$	One network realization.
2	$\beta_3 > 0$ $\beta_4 < 0$	$\frac{-\beta_2-\sqrt{\beta_2^2 - \beta_1\beta_3}}{\beta_1}$; $\beta_1 \neq 0$ $\frac{-\beta_3}{2\beta_1}$; $\beta_1 = 0$	One network realization.
3	$\beta_3 > 0$ $\beta_4 > 0$ $\beta_1 + \beta_2 > 0$ $\beta_2 < 0$	$\frac{-\beta_2\pm\sqrt{\beta_2^2 - \beta_1\beta_3}}{\beta_1}$	Two network realizations.

$$g_1 = 1 - g_3 \;;\; g_2 = \frac{1}{a_0} - g_3 ; \quad C = \frac{1}{a_0}\frac{1}{L} \;;\; L = \frac{a_1}{a_0} \quad \frac{1}{g_1+g_2}$$

<u>NOTE:</u> The values of R_1, R_2, R_3 may be determined using (3.85)

TABLE 3.4

Coefficient conditions for the realization of $z(s)$ by Structure II and the corresponding component values.

$(a_0 > 1,\ \alpha_2 > 0)$

Region	Coefficient Conditions	Value of g_3	Comments
1	$\beta_3 < 0$ $\beta_5 > 0$	$\frac{-\beta_2+\sqrt{\beta_2^2 - \beta_1\beta_3}}{\beta_1}$; $\beta_1 \neq 0$ $\frac{-\beta_3}{2\beta_2}$; $\beta_1 = 0$	One network realization.
2	$\beta_3 > 0$ $\beta_5 < 0$	$\frac{-\beta_2-\sqrt{\beta_2^2 - \beta_1\beta_3}}{\beta_1}$; $\beta_1 \neq 0$ $\frac{-\beta_3}{2\beta_2}$; $\beta_1 = 0$	One network realization.
3	$\beta_3 > 0$ $\beta_5 > 0$ $\beta_1 + \beta_2 a_0 > 0$ $\beta_2 < 0$	$\frac{-\beta_2 \pm \sqrt{\beta_2^2 - \beta_1\beta_3}}{\beta_1}$	Two network realizations.

$g_1 = 1 - g_3;\ g_2 = \frac{1}{a_0} - g_3;\ C = \frac{1}{L}\ ;\ L = \frac{a_1}{a_0}\ \frac{1}{g_1+g_2}$

NOTE: The values of R_1, R_2 and R_3 may be determined using (3.85)

TABLE 3.5

Coefficient conditions for the realization of z(s) by Structure II, and the corresponding component values

$a_0 = 1, \alpha_0 > 0$

Region	Coefficient Conditions	Comments	Element Values
1	$\beta_3 > 0$ $\beta_4 < 0$	One network realization.	$g_1 = g_2 = \frac{-2\beta_4}{\beta_1}$ $g_3 = \frac{\beta_3}{\beta_1}$ $C = \frac{1}{L} = \frac{a_1}{2g_1}$
2	$\beta_3 < 0$ $\beta_4 > 0$	One network realization.	

NOTE: The values of R_1, R_2, R_3 may be determined using (3.85)

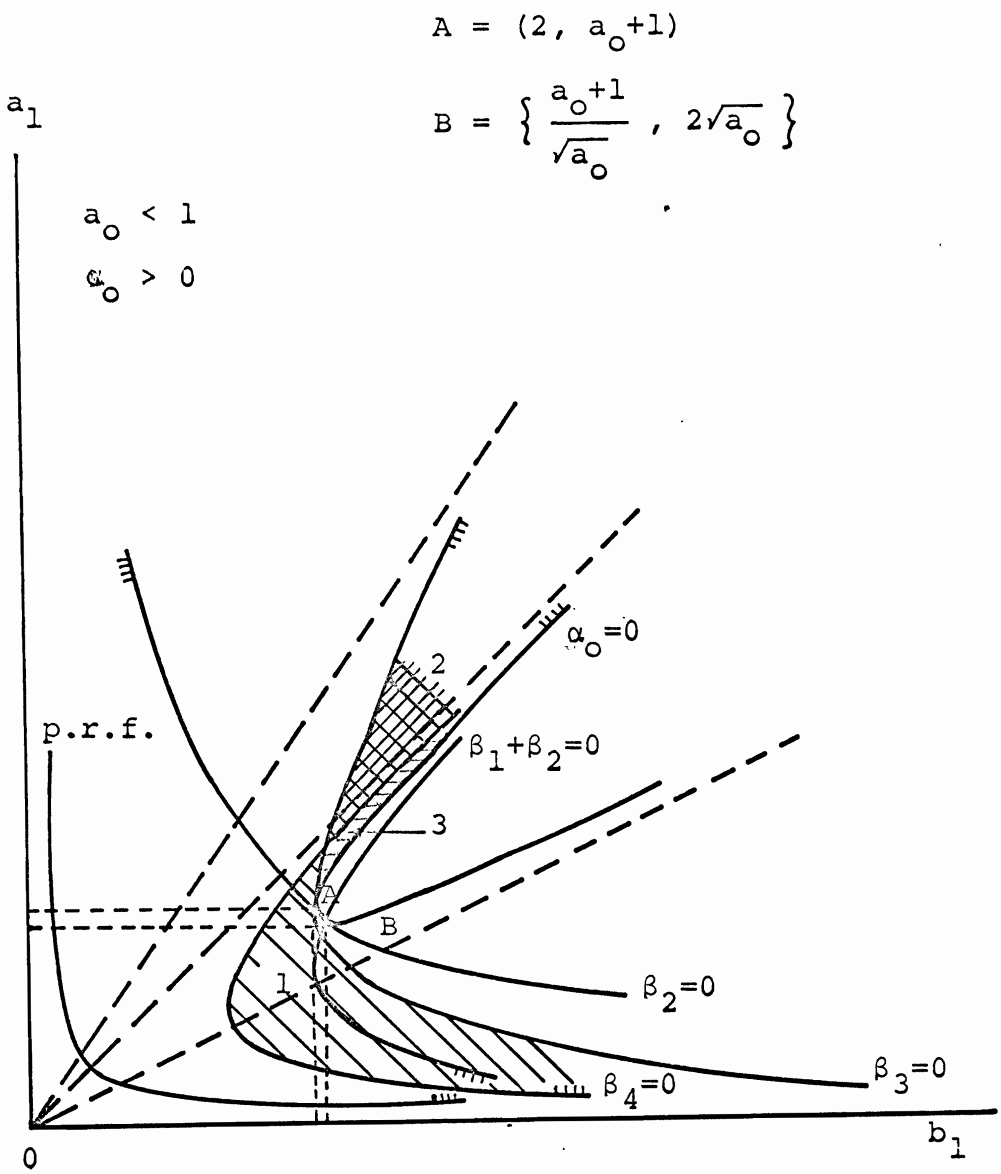


Fig. 3.6 Coefficient conditions in the (a_1, b_1) plane for the realization of z(s) by Structure II, when $a_0 < 1$.

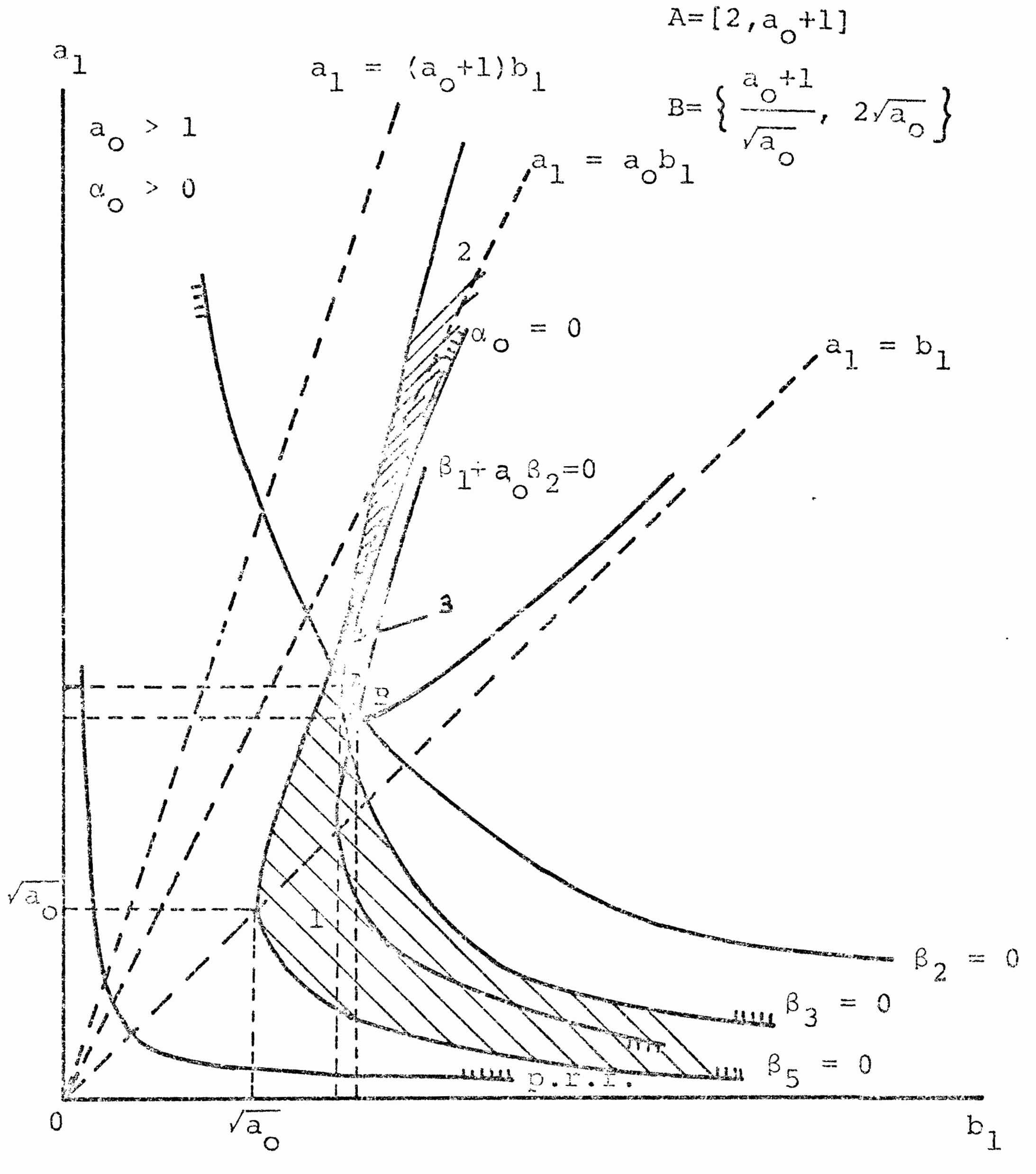


Fig. 3.7 Coefficient conditions in the (a_1, b_1) plane for the realization of z(s) by Structure II, when $a_o > 1$.

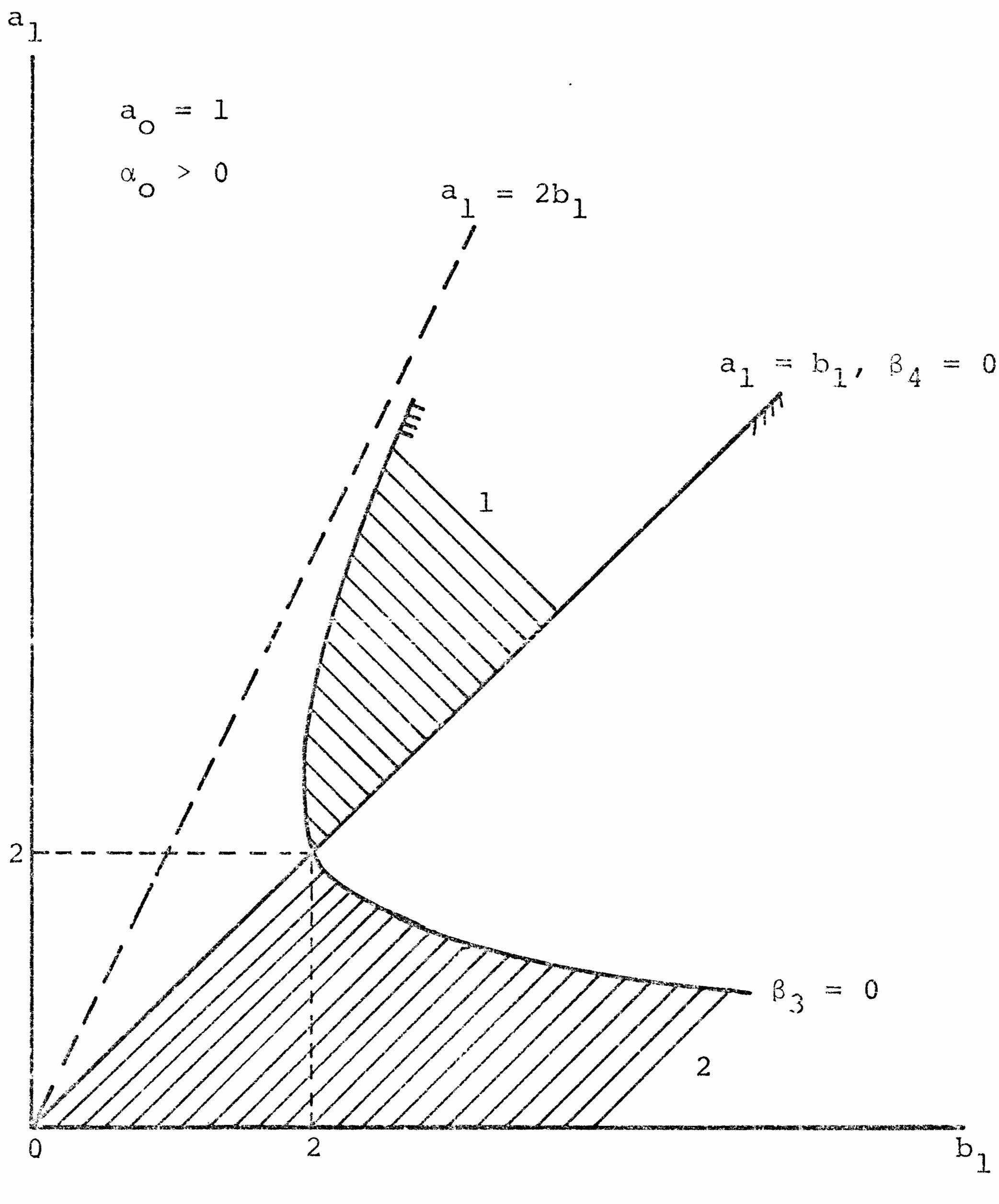


Fig. 3.8 Coefficient conditions in the (a_1, b_1) plane for the realization of $z(s)$ by Structure II, when $a_o = 1$.

CHAPTER 4

REALIZABILITY CONDITIONS FOR STRUCTURE III.

4.1 Introduction

Since Structure III is an RLC bridge network, (see Fig. 4.1), conditions derived in Sec.2.2 regarding α_0, namely

$$\alpha_0 > 0$$

have to be satisfied. We shall derive in this Chapter additional coefficient conditions to be satisfied in order that z(s) may be realized by Structure III.

4.2 Conditions for Realization by Structure III

It may be shown that for Structure III,

$$z(s) = R_2 \frac{s^2 + \frac{R_1 R_2 R_3 C + (\Sigma R) L}{(R_1+R_3) R_2 LC} s + \frac{R_1 (R_2 + R_3)}{(R_1+R_3) R_2 LC}}{s^2 + \frac{L + (\Sigma R_1 R_3) C}{(R_1 + R_3) LC} s + \frac{R_2 + R_3}{(R_1 + R_3) LC}} \tag{4.1}$$

Again comparing (3.1) and (4.1) and simplifying, we have

$$R_1 = a_0 \tag{4.2}$$

$$R_2 = 1 \tag{4.3}$$

$$\gamma_1 R_3^2 + 2\gamma_2 R_3 + \gamma_3 = 0 \tag{4.4}$$

$$C = \frac{(a_0 + 1 + R_3) b_1 - a_1}{(a_0 + R_3)(a_0 + 1)} \tag{4.5}$$

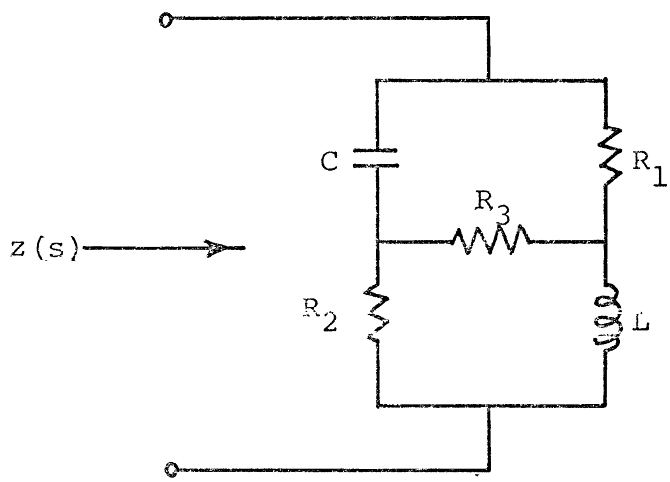


Fig. 4.1 The five element bridge Structure III.

$$LC = \frac{1 + R_3}{a_0 + R_3} \qquad (4.6)$$

where

$$\gamma_1 = b_1[a_1(a_0+1)-a_0b_1] - (a_0+1)^2 \qquad (4.7)$$

$$2\gamma_2 = a_0a_1b_1+[(a_0+1)b_1-a_1][(a_0+1)a_1-a_0b_1]-(a_0+1)^3$$

$$= 2a_0a_1b_1-(a_0+1)(\alpha_0+4a_0) \qquad (4.8)$$

$$\gamma_3 = a_0[\{(a_0+1)b_1-a_1\}a_1-(a_0+1)^2]$$

$$= a_0[a_0b_1^2-(\alpha_0+4a_0)] \qquad (4.9)$$

It may be shown that

$$\gamma_2^2 - \gamma_1\gamma_3 = \tfrac{1}{4}\alpha_0(a_0+1)^2(\alpha_0+4a_0) \qquad (4.10)$$

$$\gamma_3 - \gamma_1 = a_0b_1^2 - a_1^2 \qquad (4.11)$$

$$\gamma_1 - 2\gamma_2 + \gamma_3 = (a_1-a_0b_1)^2 \qquad (4.12)$$

and

$$a_0^2\gamma_1 - 2a_0\gamma_2 + \gamma_3 = a_0^2(a_1-b_1)^2 \qquad (4.13)$$

Now, solving (4.4) we have

$$R_3 = \frac{-\gamma_2 \pm \sqrt{\gamma_2^2-\gamma_1\gamma_3}}{\gamma_1} \qquad (4.14a)$$

with $\qquad \gamma_1 \neq 0 \qquad$ (4.14b)

Since it has been shown in Sec. 2.2 that for a minimum reactance biquadratic $z(s)$, $\alpha_0 \neq 0$ and for an RLC structure $\alpha_0 > 0$,

$$\gamma_2^2 - \gamma_1\gamma_3 > 0 \qquad (4.15)$$

and thus R_3 is real.

If γ_1 and γ_3 are of opposite signs, then

$$\sqrt{\gamma_2^2-\gamma_1\gamma_3} > |\gamma_2| \qquad (4.16)$$

and hence, R_3 has only one positive solution, whether γ_2 is $\geqslant 0$ or < 0. But if γ_1 and γ_3 are of the same signs and γ_2 of opposite sign, then there are two solutions for R_3. Let us now consider the sign combinations for γ_1, γ_2 and γ_3; with the condition $\alpha_0 > 0$. Also, we assume, first $\gamma_1 \neq 0$.

(i) $\gamma_1 < 0$, $\gamma_3 > 0$.

Then

$$R_3 = \frac{-\gamma_2-\sqrt{\gamma_2^2-\gamma_1\gamma_3}}{\gamma_1} \qquad (4.17)$$

From (4.11), $(a_0b_1^2 - a_1^2) > 0$

or $$a_0b_1^2 > a_1^2 \qquad (4.18)$$

If $a_0 \geqslant 1$, then $a_0^2b_1^2 \geqslant a_0b_1^2 > a_1^2$

or $$a_0b_1 > a_1 \qquad (4.19)$$

While, if $a_0 < 1$,

$$b_1^2 > a_0b_1^2 > a_1^2$$

or $$b_1 > a_1 \qquad (4.20)$$

Hence

$$(a_0+1)b_1 - a_1 + R_3b_1 > 0 \qquad (4.21)$$

whether $a_0 \geqslant$ or < 1

Thus for all values of a_0, from (4.5) and (4.21)

$$C > 0$$

and hence $L > 0$ from (4.6).

Hence, a network realization exists if

$$\begin{aligned} \alpha_0 &> 0 \\ \gamma_3 &> 0 \\ \gamma_1 &< 0 \end{aligned} \qquad (4.22)$$

(ii) $\gamma_1 > 0,\ \gamma_3 > 0,\ \gamma_2 < 0.$

Then, there are two roots for R_3.

Since $\gamma_3 > 0$, from (4.9)

$$2a_0a_1b_1 < (a_0+1)(\alpha_0+ 4a_0)$$

and since $\gamma_2 < 0$, from (4.8)

$$a_0b_1^2 > (\alpha_0+4a_0)$$

Thus

$$(a_0+1)b_1 > a_1$$

and hence $C > 0$.

Thus, there are two realizations, if

$$\begin{aligned} \alpha_0 &> 0 \\ \gamma_1 &> 0 \\ \gamma_3 &> 0 \\ \gamma_2 &< 0 \end{aligned} \qquad (4.23)$$

(iii) $\gamma_1 < 0,\ \gamma_3 < 0,\ \gamma_2 > 0$

Since $\gamma_1 - 2\gamma_2 + \gamma_3 = (a_1 - a_0b_1)^2$

$\gamma_1 < 0$ and $\gamma_3 < 0$ implies $\gamma_2 < 0$ which contradicts the assumption. Hence, there is no realization.

(iv) $$\gamma_1 > 0,\ \gamma_3 < 0$$

Consider the two cases, namely, $\gamma_2 \leqslant 0$ or $\gamma_2 > 0$.

Case (a) $\gamma_2 \leqslant 0$:

In this case, R_3 has one solution,

$$R_3 = \frac{-\gamma_2 + \sqrt{\gamma_2^2 - \gamma_1\gamma_3}}{\gamma_1} \tag{4.24}$$

For $C > 0$, we should have

$$b_1[(a_0+1)\gamma_1 - \gamma_2 + \sqrt{\gamma_2^2-\gamma_1\gamma_3}] > a_1\gamma_1$$

or

$$b_1\sqrt{\gamma_2^2-\gamma_1\gamma_3} > [a_1 - (a_0+1)b_1]\gamma_1 + \gamma_2 b_1 = \gamma_5 \tag{4.25}$$

where

$$\gamma_5 = [a_1 - (a_0+1)b_1]\gamma_1 + b_1 \tag{4.26}$$

Let $$\gamma_4 = (a_1-b_1)(a_0b_1-a_1) \tag{4.27}$$

If $\gamma_5 \leqslant 0$, then (4.25) always holds. If now $\gamma_5 > 0$, then from (4.25), we have by squaring

$$b_1^2(\gamma_2^2 - \gamma_1\gamma_3) > \gamma_5^2 \tag{4.28}$$

which simplifies to

$$\gamma_4 < 0 \tag{4.29}$$

Now, from (4.26), since $\gamma_1 > 0, \gamma_2 \leqslant 0$ for $\gamma_5 > 0$, we should have

$$[a_1 - (a_0+1)b_1] > 0$$

That is

$$a_1 - b_1 > 0$$

and

$$a_1 - a_0 b_1 > 0$$

and hence

$$\gamma_4 < 0$$

Thus, the condition $\gamma_4 < 0$ is automatically satisfied if $\gamma_5 > 0$. Hence, whether $\gamma_5 \leqslant 0$ or > 0, there is always a realization, if

$$\begin{aligned} \alpha_0 &> 0 \\ \gamma_1 &> 0 \\ \gamma_3 &< 0 \\ \gamma_2 &\leqslant 0 \end{aligned} \qquad (4.30)$$

Case (b) $\gamma_1 > 0,\ \gamma_3 < 0,\ \gamma_2 > 0$:

Again, R_3 is given by (4.24). For $c > 0$, we should have from (4.25)

$$b_1\sqrt{\gamma_2^2 - \gamma_1\gamma_3} > \gamma_5 \qquad (4.31)$$

If $\gamma_5 \leqslant 0$, then this condition is automatically satisfied. However, if $\gamma_5 > 0$, we get as in Case (a), that

$$\gamma_4 < 0 \qquad (4.32)$$

We shall now show that $\gamma_5 > 0$ implies (4.32) and hence that (4.32) is superfluous.

It can be shown that

$$2\gamma_2 = 2a_0a_1b_1 - a_0(a_0+1)b_1^2 + \frac{(a_0+1)}{a_0}\gamma_3$$

Since $\gamma_2 > 0$, $\gamma_3 < 0$, we have

$$2a_0a_1b_1 - a_0(a_0+1)b_1^2 > 0$$

or $$(a_0+1)b_1 < 2a_1 \qquad (4.33)$$

Also since $\gamma_5 > 0$ we have from (4.9) and (4.33) that

$$b_1(a_1-a_0b_1)(a_1-b_1) > 0$$

or $$\gamma_4 < 0$$

thus whether $\gamma_5 \leqslant 0$ or > 0, we have a realization if

$$\alpha_0 > 0$$
$$\gamma_1 > 0$$
$$\gamma_3 < 0 \qquad (4.34)$$
$$\gamma_2 > 0$$

Combining (4.30) and (4.34) we see that there is always one realization if

$$\alpha_0 > 0$$
$$\gamma_1 > 0 \qquad (4.35)$$
$$\gamma_3 < 0$$

and the corresponding R_3 being given by (4.24).

In the above discussion we have assumed $\gamma_1 \neq 0$, $\gamma_3 \neq 0$. Let us first consider when $\gamma_1 = 0$, $\gamma_3 \neq 0$. In this case,

$$R_3 = -\frac{\gamma_3}{2\gamma_2} \tag{4.36}$$

If $\gamma_3 < 0$, for $R_3 > 0$, $\gamma_2 > 0$. But from (4.12), we see that this is impossible, since $\gamma_1 = 0$, $\gamma_3 < 0$ imply $\gamma_2 < 0$. Thus, there is no realization of $\gamma_3 < 0$, $\gamma_1 = 0$. However, if $\gamma_3 > 0$ then for $R_3 > 0$, $\gamma_2 < 0$. In order that $C > 0$, we should have

$$\{(a_0+1) - \frac{\gamma_3}{2\gamma_2}\}b_1 - a_1 > 0$$

or

$$\{(a_0+1)2\gamma_2 - \gamma_3\}b_1 < 2\gamma_2 a_1 < 0$$

which is true, since $\gamma_2 < 0$, $\gamma_3 > 0$. Thus, there is a realization if

$$\begin{aligned} \gamma_1 &= 0 \\ \gamma_3 &> 0 \\ \gamma_2 &< 0 \end{aligned} \tag{4.37}$$

and the value of R_3 is given (4.36).

Let us now assume $\gamma_1 \neq 0$, $\gamma_3 = 0$. Then the value of R_3 is

$$R_3 = -\frac{2\gamma_2}{\gamma_1} \tag{4.38}$$

Let us assume first that $\gamma_1 < 0$ - then from (4.12) $\gamma_2 < 0$, which makes $R_3 < 0$. Thus, there is no realization if $\gamma_1 < 0$, $\gamma_3 = 0$.

But if $\gamma_1 > 0$, then for R_3 to be positive, $\gamma_2 < 0$. It can be said that if $\gamma_2 < 0$, $\gamma_1 > 0$, and $\gamma_3 = 0$, then automatically $C > 0$. Thus, there is a realization if

$$\gamma_3 = 0$$
$$\gamma_1 > 0 \qquad (4.39)$$
$$\gamma_2 < 0$$

and the value of R_3 is given by (4.38).

All the results are tabulated in Table 4.1 for $a_0 \neq 1$ and $a_0 = 1$, and are also presented (Figs. 4.2 and 4.3) in the (a_1, b_1) plane, defining certain regions where the impedance (3.1) may be realized by Structure III. Note that the region defined by Case 3 of Table 4.1 corresponds to one or two realizations, when $a_0 \neq 1$. Also when $a_0 = 1$, the points A, B and C in Fig. 4.2 all coincide as shown in Fig. 4.3 and as a result, there is no realization for Case 3 when $a_0 = 1$.

4.3 Conclusions

Thus we have shown that whenever γ_1 and γ_3 are of opposite signs $z(s)$ can always be realized by Structure III, if $\alpha_0 > 0$, while two structures exist if $\gamma_1 > 0$, $\gamma_3 > 0$, $\gamma_2 < 0$, $\alpha_0 > 0$, and $a_0 \neq 1$. Also, there is one realization if $a_0 \neq 1$, $\gamma_2 < 0$, with one of γ_1, γ_3 being zero, the other being positive. It should be noted that in deriving the coefficient conditions, it was not necessary to consider the different cases corresponding to $a_0 \geqslant 1$ or $a_0 \leqslant 1$, and this is due to the fact that Structure III is the dual of itself.

TABLE 4.1

Coefficient conditions for the realization of $z(s)$ by Structure III and the corresponding component values.

$(\alpha_0 > 0)$

Region	Coefficient Conditions	Value of R_3	Comments
1	$\gamma_3 < 0$ $\gamma_1 > 0$	$\frac{-\gamma_2 + \sqrt{\gamma_2^2 - \gamma_1\gamma_3}}{\gamma_1}$	One network realization.
2	$\gamma_3 > 0$ $\gamma_1 < 0$	$\frac{-\gamma_2 - \sqrt{\gamma_2^2 - \gamma_1\gamma_3}}{\gamma_1}$	One network realization.
3	$\gamma_3 \geq 0$	$\frac{-\gamma_2 \pm \sqrt{\gamma_2^2 - \gamma_1\gamma_3}}{\gamma_1}$; $\gamma_1 > 0, \quad \gamma_3 > 0$	Two realizations if $a_0 \neq 1$ No realization of $a_0 = 1$
	$\gamma_1 \geq 0$	$\frac{-\gamma_3}{2\gamma_2}$; $\gamma_1 = 0, \quad \gamma_3 > 0$	One realization if $a_0 \neq 1$ No realization if $a_0 = 1$
	$\gamma_2 < 0$	$\frac{-2\gamma_2}{\gamma_1}$; $\gamma_1 > 0, \quad \gamma_3 = 0$	One realization if $a_0 \neq 1$ No realization if $a_0 = 1$

$$R_1 = a_0, \; R_2 = 1, \; C = \frac{(a_0+1+R_3)b_1 - a_1}{(a_0+R_3)(a_0+1)}, \quad L = \frac{(1+R_3)}{C(a_0+R_3)}$$

$A = [2,\ a_o+1]$

$B = \left\{ \frac{a_o+1}{\sqrt{a_o}},\ 2\sqrt{a_o} \right\}$

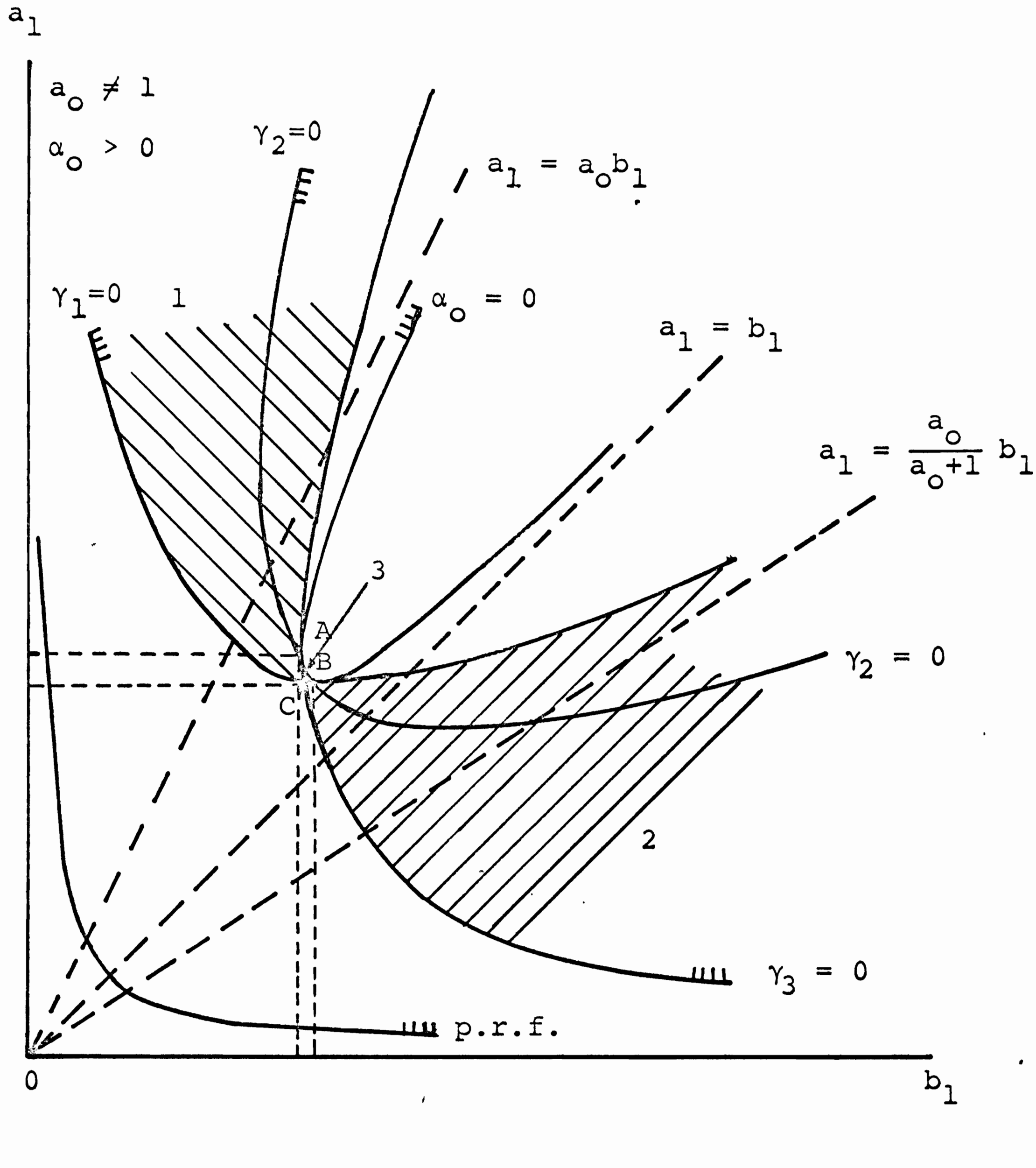


Fig. 4.2 Coefficient conditions in the (a_1, b_1) plane for the realization of z(s) by Structure III, when $a_o \neq 1$.

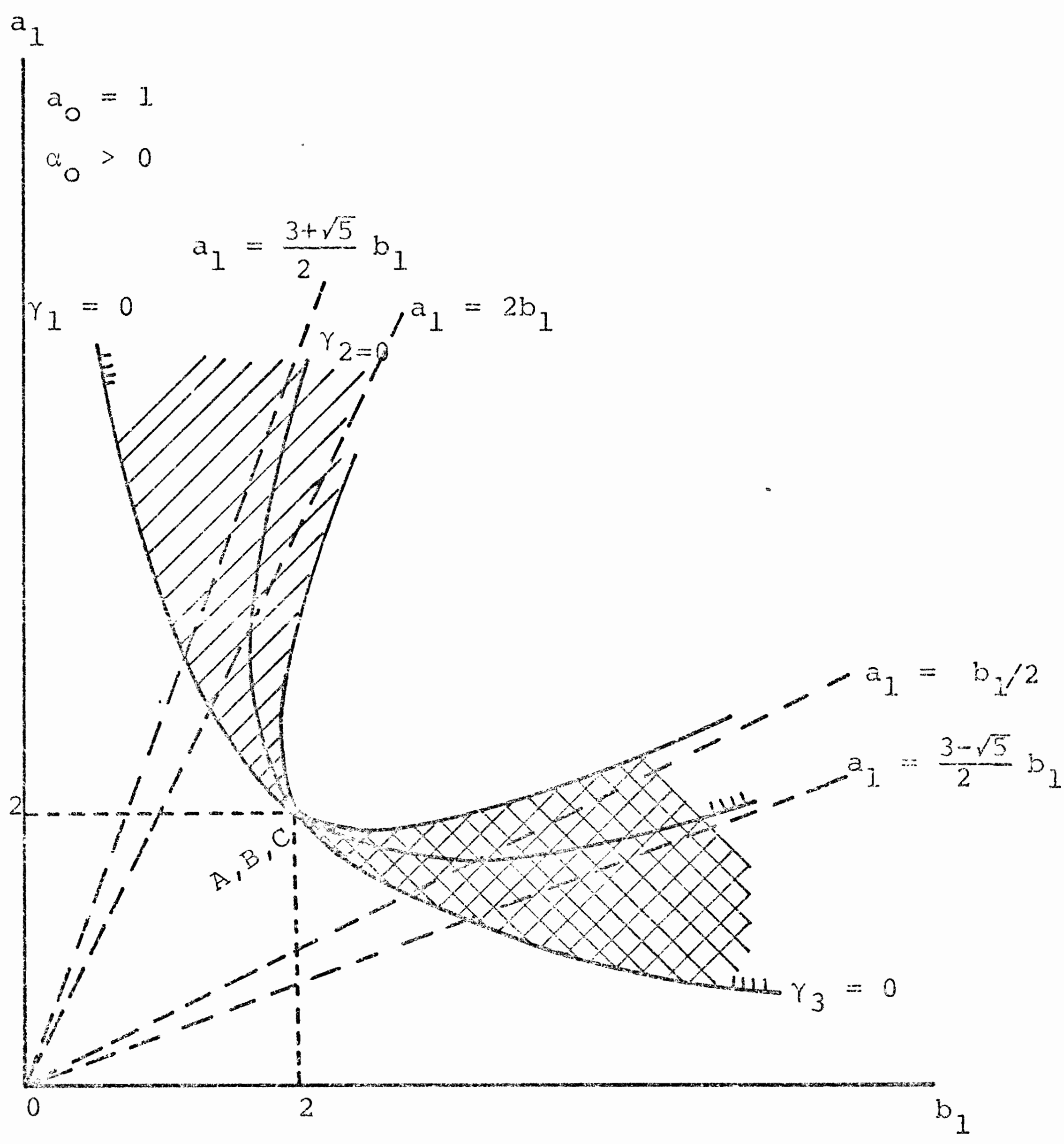


Fig. 4.3 Coefficient conditions in the (a_1, b_1) plane for the realization of $z(s)$ by Structure III, when $a_o = 1$.

CHAPTER 5

REALIZABILITY CONDITIONS FOR STRUCTURES IV AND V.

5.1 Introduction

In this Chapter, we shall consider the coefficient conditions under which the impedance function z(s) given by (3.1) may be realized by Structures IV and V (Fig.5.1). We have already derived in Chapter 2, the conditions under which (3.1) may be realized by an RC or an RL structure. In fact, it is well known that when those conditions are satisfied, z(s) may always be realized by the Foster and Cauer canonic structures. We shall, in this Chapter, show that under certain additional coefficient conditions, z(s) may also be realized by the canonic bridge structures IV and V.

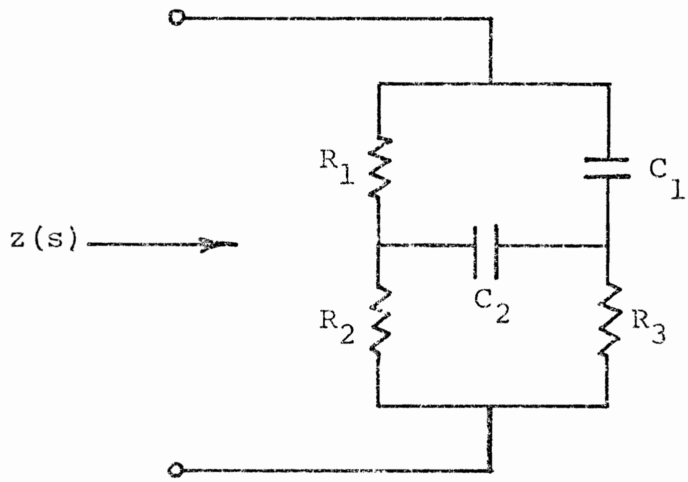
5.2 Conditions for Realization by Structure IV.

For Structure IV, z(s) may be shown to be

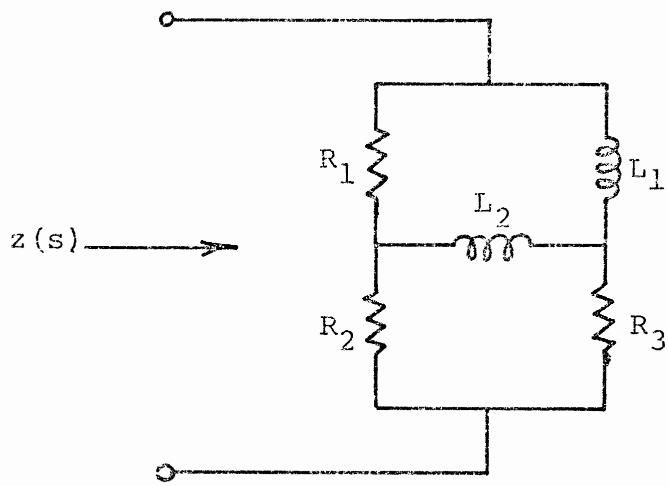
$$z(s) = \frac{R_2 R_3}{R_2 + R_3} \frac{s^2 + \frac{R_3 (R_1 + R_2) C_1 + (\Sigma R_1 R_3) C_2}{R_1 R_2 R_3 C_1 C_2} s + \frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2}}{s^2 + \frac{(\Sigma R) C_1 + (R_2 + R_3) C_2}{R_1 (R_2 + R_3) C_1 C_2} s + \frac{1}{R_1 (R_2 + R_3) C_1 C_2}} \tag{5.1}$$

Comparing (3.1) and (5.1), we get

$$R_2 R_3 = R_2 + R_3$$



(a)



(b)

Fig. 5.1 (a) The five element bridge Structure IV.
(b) The five element bridge Structure V.

$$R_3(R_1+R_2)C_1+(\Sigma R_1R_3)C_2 = a_1R_1R_2R_3C_1C_2$$

$$(R_1+R_2) = a_0R_1R_2R_3C_1C_2$$

$$(\Sigma R)C_1+(R_2+R_3)C_2 = b_1 \qquad (5.2)$$

$$R_1(R_2+R_3)C_1C_2 = 1$$

From the above equations, we may derive the following relations,

$$\delta_1R_2^2 - 2\delta_2R_2 + \delta_3 = 0 \qquad (5.3)$$

$$R_3 = \frac{R_2}{R_2-1} \qquad (5.4)$$

$$R_1 = a_0-R_2 \qquad (5.5)$$

$$C_1 = \frac{(1+a_0-R_2)b_1-a_1}{(a_0+1)R_1} \qquad (5.6)$$

and

$$R_1R_2R_3C_1C_2 = 1 \qquad (5.7)$$

where

$$\delta_1 = -b_1[a_0b_1-(a_0+1)a_1] - (a_0+1)^2 \qquad (5.8)$$

$$2\delta_2 = [a_1-(a_0+1)b_1][a_0b_1-(a_0+1)a_1]+a_0a_1b_2-(a_0+1)^3 \qquad (5.9)$$

$$\delta_3 = a_0a_1[b_1(a_0+1)-a_1] - a_0(a_0+1)^2 \qquad (5.10)$$

Recalling the results derived in Chapter 2, regarding the realization of z(s) by an RC structure, we have

$$a_0 > 1$$

$$a_1 > b_1$$

and $$\alpha_0 < 0$$

since $\alpha_0 = (a_0-1)^2 - (a_1-b_1)(a_0b_1-a_1) < 0$, and $a_1 > b_1$ we should have

$$(a_0b_1-a_1) > 0 \tag{5.11}$$

Thus, for any RC biquadratic function,

$$a_0 > 1 \tag{5.12}$$

$$a_0b_1 > a_1 > b_1 \tag{5.13}$$

and

$$\alpha_0 < 0 \tag{5.14}$$

Since $\alpha_0 < 0$ we also have,

$$\delta_1 > 0,\ \delta_2 > 0 \text{ and } \delta_3 > 0 \tag{5.15}$$

It may further be shown that

$$\delta_2 > \delta_1 \tag{5.16}$$

$$a_0\delta_1 - \delta_2 > 0 \tag{5.17}$$

$$\delta_1 - 2\delta_2 + \delta_3 = (a_0b_1-a_1)^2 > 0 \tag{5.18}$$

$$a_0^2\delta_1 - 2a_0\delta_2 + \delta_3 = a_0^2(a_1-b_1)^2 > 0 \tag{5.19}$$

and

$$\delta_2^2 - \delta_1\delta_3 = \tfrac{1}{4}(a_0+1)^2\alpha_0(\alpha_0+4a_0) \tag{5.20}$$

From (5.3) we have

$$R_2 = \frac{\delta_2 \pm\sqrt{\delta_2^2-\delta_1\delta_3}}{\delta_1}$$

Thus, if

$$\delta_2^2-\delta_1\delta_3 \geqslant 0$$

or $$\alpha_0 + 4a_0 \lessgtr 0 \qquad (5.21)$$

R_2 exists.

Case (a) : $(\alpha_0 + 4a_0) < 0$

If the condition $(\alpha_0+4a_0) < 0$ (5.22)

is satisfied, then there are two values for R_2.

Let these be R_{2a} and R_{2b}, hence

$$R_{2a} = \frac{\delta_2 + \sqrt{\delta_2^2-\delta_1\delta_3}}{\delta_1}$$

$$R_{2b} = \frac{\delta_2 - \sqrt{\delta_2^2-\delta_1\delta_3}}{\delta_1}$$

Obviously $R_{2a} > R_{2b}$.

From (5.19) we get

$$a_0^2\delta_1^2 - 2a_0\delta_1\delta_2 + \delta_1\delta_3 > 0$$

or

$$\delta_2^2 - \delta_1\delta_3 < a_0^2\delta_1^2 + \delta_2^2 - 2a_0\delta_1\delta_2 = (a_0\delta_1-\delta_2)^2$$

hence $$\sqrt{\delta_2^2-\delta_1\delta_3} < |a_0\delta_1 - \delta_2|$$

Using (5.17) we get

$$\sqrt{\delta_2^2-\delta_1\delta_3} < a_0\delta_1 - \delta_2$$

or,

$$\frac{\delta_2 + \sqrt{\delta_2^2-\delta_1\delta_3}}{\delta_1} = R_{2a} < a_0$$

Using the above inequality, and the fact that $R_{2b} < R_{2a}$,

we get $$R_{2b} < R_{2a} < a_0 \qquad (5.23)$$

similarly, we can show by using (5.18), that

$$R_{2a} > R_{2b} > 1 \tag{5.24}$$

Combining (5.23) and (5.24), we have

$$1 < R_{2b} < R_{2a} < a_0 \tag{5.25}$$

From (5.25), (5.4) and (5.5), we see that corresponding to R_{2a} and R_{2b}, we have realizable R_1 and R_3. We shall now show that whether $R_2 = R_{2a}$ or R_{2b}, C_1 is always positive.

Let $\delta_4 = [a_1 - b_1(a_0 + 1)]\delta_1 + b_1\delta_2$

Then it can be shown that

$$2\delta_4 = (a_0+1)b_1(\alpha_0+4a_0) - 2a_1(a_0+1)^2$$

Using (5.22) we conclude that

$$\delta_4 < 0 \tag{5.26}$$

Now, $C_1 > 0$ if

$$(1+a_0-R_2)b_1 > a_1 \tag{5.27}$$

Letting $R_2 = R_{2a}$, (5.27) is satisfied if

$$\sqrt{\delta_2^2 - \delta_1\delta_3} > \frac{\delta_4}{b_1}$$

which is true, since $\delta_4 < 0$.

Now, letting $R_2 = R_{2b}$, (5.27) is satisfied if

$$-\sqrt{\delta_2^2 - \delta_1\delta_3} > \frac{\delta_4}{b_1}$$

or

$$-\frac{\delta_4}{b_1} > \sqrt{\delta_2^2 - \delta_1\delta_3}$$

Since both sides are positive, the above condition becomes

$$\delta_4^2 > b_1^2(\delta_2^2 - \delta_1\delta_3)$$

Substituting for $\delta_1, \delta_2, \delta_3$ and δ_4 this reduces to

$$(a_1 - b_1)(a_0 b_1 - a_1) > 0$$

which is always true, in view of (5.13).

Hence, $C_1 > 0$ whether $R = R_{2a}$ or R_{2b}; consequently, from (5.7), C_2 is also > 0.

Therefore, there are two realizations in the form of Structure IV, if conditions (5.12) to (5.14) and (5.22) are satisfied. However, the condition (5.21), namely $(\alpha_0 + 4a_0) \leqslant 0$ implies the conditions (5.13) and (5.14).

Thus, for the biquadratic impedance z(s), two realizations may always be found, if

$$a_0 > 1$$

and (5.28)

$$\alpha_0 + 4a_0 < 0$$

Further, the component values are given by (5.21) and (5.4) to (5.7). The region in the (a_1, b_1) plane, where z(s) may be realized by Structure IV, is shown in Fig. 5.2.

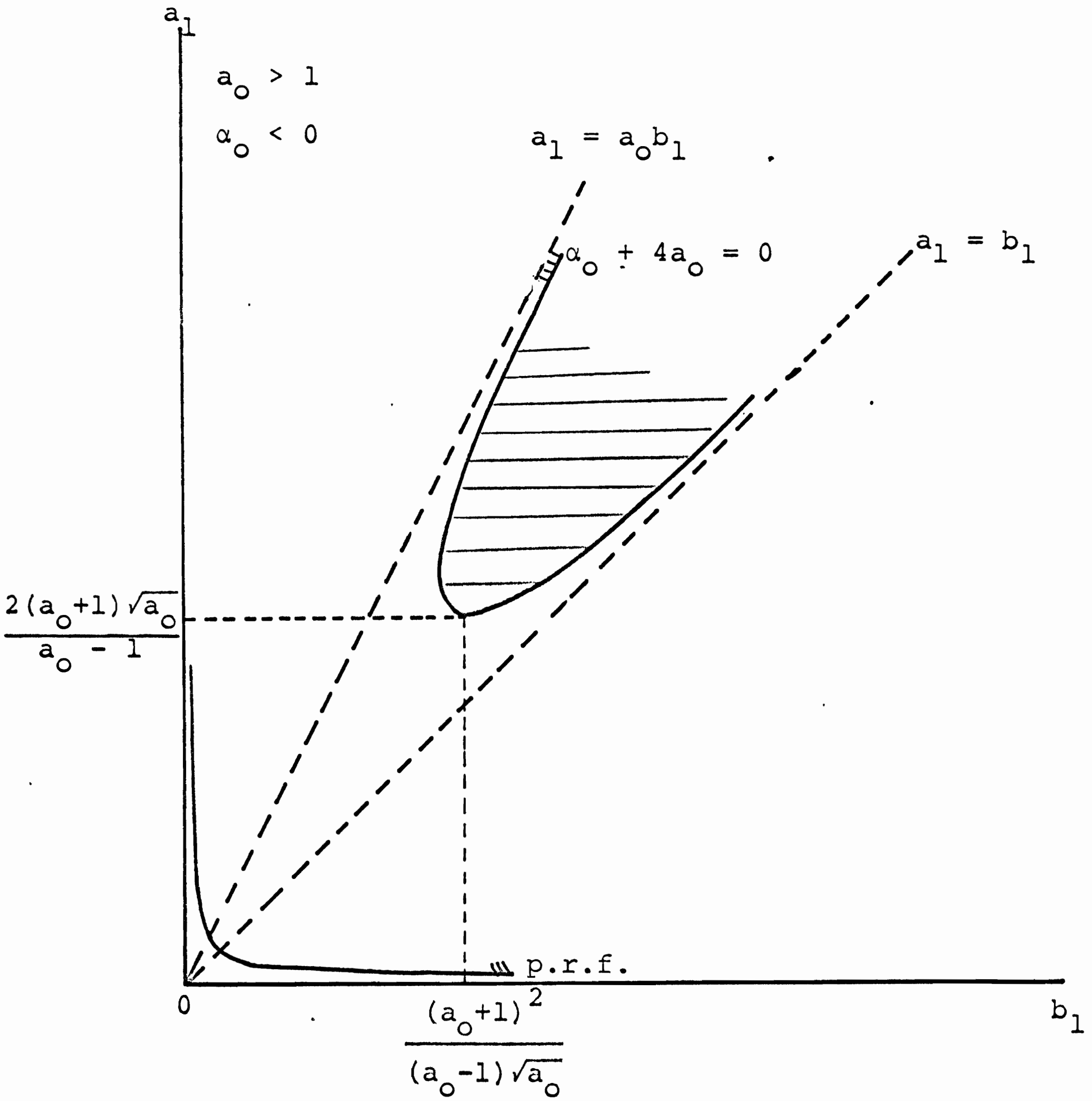


Fig. 5.2 Coefficient conditions in the (a_1, b_1) plane for the realization of z(s) by Structure IV.

Case (b) : $(\alpha_0 + 4a_0) = 0$

In this case, $\delta_2^2 - \delta_1\delta_3 = 0$ and there is only one value for R_2 and is

$$R_2 = \frac{\delta_2}{\delta_1} \qquad (5.29)$$

It may again be shown that

$$\delta_2 > \delta_1 \qquad (5.30)$$

and hence, $R_2 > 1$. We now see from (5.4) that $R_3 > 0$.

Now $$R_1 = a_0 - R_2 = \frac{a_0\delta_1 - \delta_2}{\delta_1} \qquad (5.31)$$

which may be shown to be positive. Also,

$$C_1 = \frac{[(1+a_0)\delta_1 - \delta_2]b_1 - \delta_1 a_1}{(a_0 + 1)R_1} \qquad (5.32)$$

which will be positive if

$$[(1 + a_0)\delta_1 - \delta_2]b_1 > \delta_1 a_1$$

which simplifies to $\delta_4 < 0$, which is always true from (5.26).

Hence if $$a_0 > 1 \qquad (5.33)$$

and $(\alpha_0 + 4a_0) = 0$, there is one realization of $z(s)$ in the form of Structure IV.

All the above results are presented in Table 5.1.

5.3 Conditions for Realization by Structure V.

For Structure V (Fig. 5.1), $z(s)$ may be shown

TABLE 5.1

Coefficient conditions for the realization of z(s) by Structure IV, and the corresponding component values

$(a_0 > 1,\ (\alpha_0 + 4a_0) \leqslant 0)$

Component Values	$(\alpha_0 + 4a_0) < 0$	$(\alpha_0 + 4a_0) = 0$
R_1	$a_0 - R_2$	$a_0 - R_2$
R_2	$\dfrac{\delta_2 \pm \sqrt{\delta_2^2 - \delta_1\delta_3}}{\delta_1}$	$\dfrac{\delta_2}{\delta_1}$
R_3	$\dfrac{R_2}{(R_2-1)}$	$\dfrac{R_2}{(R_2-1)}$
C_1	$\dfrac{(a_0 + 1 - R_2)b_1 - a_1}{(a_0 + 1)R_1}$	$\dfrac{(a_0 + 1 - R_2)b_1 - a_1}{(a_0 + 1)R_1}$
C_2	$\dfrac{1}{R_1R_2R_3C_1}$	$\dfrac{1}{R_1R_2R_3C_1}$
	Two network realizations	One network realization

to be

$$z(s) = (R_1+R_2)\,\frac{s^2+\dfrac{(\Sigma R_1R_3)L_1+R_3(R_1+R_2)L_2}{(R_1+R_2)L_1L_2}\,s+\dfrac{R_1R_2R_3}{(R_1+R_2)L_1L_2}}{s^2+\dfrac{(R_2+R_3)L_1+(\Sigma R)L_2}{L_1L_2}\,s+\dfrac{R_1(R_2+R_3)}{L_1L_2}} \tag{5.34}$$

By following the same procedure as in Sec. (5.2), we can derive the conditions for realization of $z(s)$ by Structure V, and also the different component values. However, by observing Structure V to be the dual of Structure IV, we may derive these results by adopting the technique discussed in Sec. 3.3. The realizability conditions are :

$$a_0 < 1 \tag{5.35}$$

$$\alpha_0 + 4a_0 < 0$$

in which case, there are two realizations, or

$$a_0 < 1$$

$$\alpha_0 + 4a_0 = 0$$

in which case, there is only one realization. The component values for these cases are given in Table 5.2.

5.4 Conclusions.

We have shown that if $(\alpha_0 + 4a_0) \leqslant 0$ and $a_0 \neq 1$, the given $z(s)$ may be realized as a canonic bridge RC or RL structure. If $a_0 > 1$, it corresponds to an RC structure (Fig. 5.1a), while $a_0 < 1$ corresponds to an RL structure (Fig. 5.1b).

It is also shown that there are two realizations of

TABLE 5.2

Coefficient conditions for the realization of $z(s)$ by Structure V, and the corresponding component values

$(a_0 < 1, \ (\alpha_0 + 4a_0) \leqslant 0)$

Component Values	$(\alpha_0 + 4a_0) < 0$	$(\alpha_0 + 4a_0) = 0$
R_1	$1 - R_2$	$1 - R_2$
R_2	$\dfrac{\delta_2 \pm \sqrt{\delta_2^2 - \delta_1\delta_3}}{\delta_1}$	$\dfrac{\delta_2}{\delta_1}$
R_3	$\dfrac{R_2 a_0}{(R_2 - a_0)}$	$\dfrac{R_2}{(R_2 - a_0)}$
L_1	$\dfrac{R_1}{(a_0+1)-R_2)b_1-a_1}$	$\dfrac{R_1}{(a_0+1 -R_2)b -a_1}$
L_2	$\dfrac{R_1(R_2+ R_3)}{L_1}$	$\dfrac{R_1(R_2+R_3)}{L_1}$
	Two network realizations	One network realization.

NOTE: δ_1, δ_2 and δ_3 are given by (5.8), (5.9) and (5.10).

these structures if $(\alpha_0 + 4a_0) < 0$, while there is only one realization if $(\alpha_0 + 4a_0) = 0$. Thus, for the region enclosed by $\alpha_0 < 0$, and $\alpha_0 + 4a_0 > 0$, the bridge canonic Structures IV or V, do not exist, even though the Foster and Cauer canonic forms exist.

CHAPTER 6

CONDITIONS FOR REALIZATION OF STRUCTURES VI AND VII.

6.1 Introduction

Foster and Ladenheim[7] have shown that, under certain coefficient conditions, $z(s)$ given by (3.1) may be realized by Structures VI and VII (Figs. 6.1a and 6.1b) respectively, if $a_0 < 1$ and $a_0 > 1$. However, they have not given explicitly the regions where more than one realization may be found for $z(s)$ in the form of Structure VI or VII. In this Chapter, we shall not only derive the coefficient conditions for the different regions where $z(s)$ may be realized by these structures, but also present these results in the (a_1, b_1) plane.

6.2 Conditions for Realization by Structure VI.

It has been shown that $z(s)$ may be realized by Structure VI, if the following conditions are met:

(i) $a_0 < 1, \alpha_0 > 0,$

(ii) $\mu_1, \mu_2, \mu_3,$ all do not have the same sign,

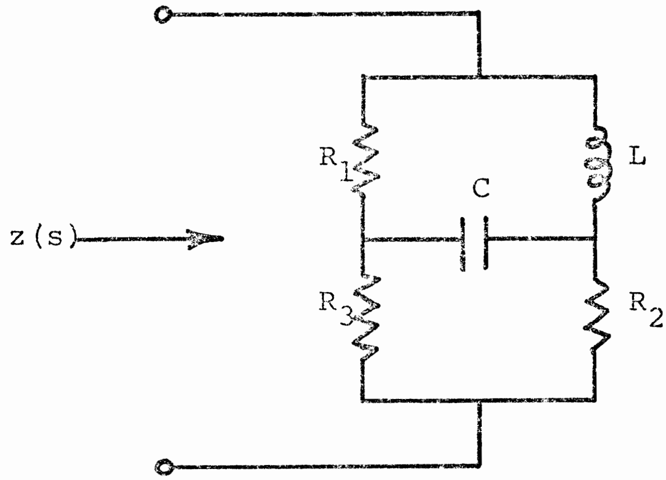
(iii) $\mu_4 \geqslant 0,$

where

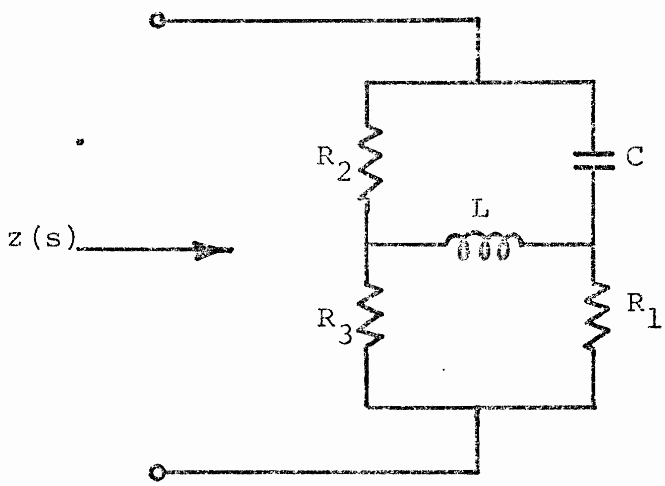
$$\mu_1 = (1-a_0) - b_1(a_1-a_0b_1) \tag{6.1}$$

$$2\mu_2 = (1-a)(1-3a_0) + (a_1-b_1)(a_1-a_0b_1) \tag{6.2}$$

$$\mu_3 = a_0[(1-a_0) + a_1(a_1-b_1)] \tag{6.3}$$



(a)



(b)

Fig. 6.1 (a) The five element bridge Structure VI.
(b) The five element bridge Structure VII.

$$\mu_4 = \mu_2^2 - \mu_1\mu_3 = \alpha_0[(1-a_0)(1-9a_0) + (a_1-b_1)(a_1-a_0 b_1) + 4a_0 a_1 b_1)] \quad (6.4)$$

They have also shown that the component values are given by

$$\mu_1 R_3^2 + 2\mu_2 R_3 + \mu_3 = 0 \quad (6.5)$$

$$(a_0+R_3)R_1^2 - (a_0+R_3)(1-R_3)R_1 - (1-a_0)R_3^2 = 0 \quad (6.6)$$

$$R_2 = \frac{a_0(R_1+R_3)}{(R_1+R_3)-a_0} = \frac{(1-R_1)R_3}{(R_1+R_3)-1} \quad (6.7)$$

$$L = \frac{R_1(R_1R_2 + R_1R_3 + R_3^2)}{(R_2+R_3)[b_1(R_1+R_3)-a_1]} \quad (6.8)$$

$$C = \frac{\Sigma R[b_1(R_1+R_3)-a_1]}{R_1(R_1R_2+R_3^2)} \quad (6.9)$$

We shall first show that this bridge structure does not exist if $a_0 \geqslant 1$.

Case (a) : $a_0 = 1$

In this case, equation (6.5) becomes

$$(1+R_3)R_1^2 - (1-R_3^2)R_1 = 0 \quad (6.10)$$

Hence, $R_1 = 0$, $R_1 + R_3 = 1$ or $R_2 = \infty$ from (6.7). Thus, if $a_0 = 1$, Structure VI is no longer a bridge network and hence, we shall not discuss this special case any further.

Case (b) : $a_0 > 1$

From (6.7) for $R_2 > 0$, we should have,

$$(R_1 + R_3) > a_0 \quad (6.11)$$

For $a_0 > 1$, (6.6) becomes

$$(a_0+R_3)R_1^2 - (a_0+R_3)(1 - R_3)R_1 + (a_0 - 1)R_3^2 = 0 \quad (6.12)$$

Since the coefficients of R_1^2 and the constant terms are positive, equ. (6.12) will not have any positive real roots if $R_3 > 1$; hence we have the necessary condition

$$R_3 < 1 \quad (6.13)$$

Therefore $$(1 + R_3) < 2a_0 \quad (6.14)$$

Now, R_1 is given by

$$R_1 = \frac{(a_0 + R_3)(1 - R_3) \pm \sqrt{\Delta}}{2(a_0 + R_3)} \quad (6.15a)$$

where $$\Delta = (a_0+R_3)^2(1 - R_3)^2 - 4(a_0+R_3)(a_0 - 1)R_3^2 \quad (6.15b)$$

Therefore, to satisfy (6.11)

$$R_1 + R_3 = \frac{(a_0+R_3)(1 + R_3) \pm \sqrt{\Delta}}{2(a_0+R_3)} > a_0$$

or

$$2a_0(a_0+R_3) < (a_0+R_3)(1 + R_3) \pm \sqrt{\Delta}$$

Hence, we should have

$$(a_0+R_3)[(2a_0 - (1 + R_3)] < \pm \sqrt{\Delta} \quad (6.16)$$

Since $2a_0 > (1 + R_3)$ from (6.14), the above equation cannot be satisfied with the negative sign for the R.H.S. expression of (6.16). Taking the positive sign, for $(R_1 + R_3) > a_0$ we should have

$$(a_0+R_3)^2 [2a_0 - (1 + R_3)]^2 < \Delta \quad (6.17)$$

which simplifies to

$$4a_0^3 - 4a_0^2 < 0$$

$$\text{or} \qquad a_0 < 1$$

which contradicts our assumption that $a_0 > 1$. Thus $z(s)$ cannot be realized by Structure VI if $a_0 > 1$.

Let us now consider in detail the realization of $z(s)$ by Structure VI when $a_0 < 1$. First, we observe from (6.5) that if μ_1 and μ_3 are of the opposite signs, then

$$\mu_4 > 0$$

and

$$\sqrt{\mu_4} = \sqrt{\mu_2^2 - \mu_1\mu_3} > |\mu_2|$$

whether μ_2 is positive, zero or negative, and hence there is always a positive solution for R_3. If μ_1 and μ_3 are of the same sign, but μ_3 of the opposite sign and $\mu_4 > 0$, then there are two real roots for R_3 both of which, are either positive or negative. Let us now consider these cases, separately.

(i) $\mu_1 > 0, \mu_3 < 0$

Then $\mu_4 \geqslant 0$ is automatically satisfied and the positive solution for R_3 is

$$R_3 = \frac{-\mu_2 + \sqrt{\mu_2^2 - \mu_1\mu_3}}{\mu_1} \tag{6.18}$$

(ii) $\mu_1 < 0, \mu_3 > 0$

Here again, $\mu_4 \geqslant 0$ is automatically satisfied and

R_3 is given by

$$R_3 = \frac{-\mu_2 - \sqrt{\mu_2^2 - \mu_1\mu_3}}{\mu_1} \qquad (6.19)$$

(iii) $\mu_1 > 0, \mu_3 > 0, \mu_4 \geqslant 0$

It can be shown in this case, that

$$\mu_2 < 0 \text{ if } (\frac{1}{9} < a_0 < 1)$$

and

$$\mu_2 > 0 \text{ if } (a_0 < \frac{1}{9})$$

Thus, there is no network realization if $a_0 < \frac{1}{9}, \mu_1 > 0$, $\mu_3 > 0$ and $\mu_4 \geqslant 0$. However, there are two realizations if

$$\mu_1 > 0$$

$$\mu_3 > 0$$

$$\mu_4 > 0$$

$$\frac{1}{9} < a_0 < 1$$

The corresponding values of R_3 being given by

$$R_3 = \frac{-\mu_2 \pm \sqrt{\mu_2^2 - \mu_1\mu_3}}{\mu_1} \qquad (6.20)$$

While there is only one realization if

$$\mu_1 > 0$$

$$\mu_3 > 0$$

$$\mu_4 = 0$$

$$\frac{1}{9} < a_0 < 1$$

the corresponding R_3 being given by

$$R_3 = \frac{-\mu_2}{\mu_1} \qquad (6.21)$$

(iv) $\mu_1 < 0, \mu_3 < 0, \mu_4 \geqslant 0.$

In this case, it can be shown that

$$\mu_2 < 0 \text{ if } (\frac{1}{9} < a_0 < 1)$$

and

$$\mu_2 > 0 \text{ if } (a_0 < \frac{1}{9})$$

Thus, there is no network realization if

$$\frac{1}{9} < a_0 < 1, \mu_1 < 0, \mu_3 < 0, \mu_4 \geqslant 0.$$

However, there are two realizations if

$$\mu_1 < 0$$
$$\mu_3 < 0$$
$$\mu_4 > 0$$
$$a_0 < \frac{1}{9}$$

The values of R_3 being given by (6.20), while there is only one realization if

$$\mu_1 < 0$$
$$\mu_3 < 0$$
$$\mu_4 = 0$$
$$a_0 < \frac{1}{9}$$

the value of R_3 now being given by (6.21).

In all the above cases, once R_3 is known, the other component values of Structure VI may be found using (6.6) to (6.9). The different coefficient conditions and the component values of Structure VI are given in Table 6.1.

TABLE 6.1

Coefficient conditions for the realization of $z(s)$ by Structure VI, and the corresponding component values.

$(a_0 < 1,\ \alpha_0 > 0)$

Region	Coefficient Conditions	Value of R_3	Comments
1	$\mu_1 > 0$ $\mu_3 < 0$	$\frac{-\mu_2+\sqrt{\mu_2^2 - \mu_1\mu_3}}{\mu_1}$	One network realization.
2	$\mu_1 < 0$ $\mu_3 > 0$	$\frac{-\mu_2-\sqrt{\mu_2^2 - \mu_1\mu_3}}{\mu_1}$	One network realization.
3	$\mu_1 > 0$ $\mu_3 > 0$ $\mu_4 \geqslant 0$ $\frac{1}{9} < a_0 < 1$	$\frac{-\mu_2\pm\sqrt{\mu_2^2 - \mu_1\mu_3}}{\mu_1}$; $\mu_4 \neq 0$ $\frac{-\mu_2}{\mu_1}$; $\mu_4 = 0$	Two network realizations. One network realization.
4	$\mu_1 < 0$ $\mu_3 < 0$ $\mu_4 \geqslant 0$ $a_0 < \frac{1}{9}$	$\frac{-\mu_2\pm\sqrt{\mu_2^2 - \mu_1\mu_3}}{\mu_1}$; $\mu_4 \neq 0$ $\frac{-\mu_2}{\mu_1}$; $\mu_4 = 0$	Two network realizations. One network realization.

$$(a_0+R_3)R_1^2 - (a_0+R_3)(1-R_3)R_1 + (a_0-1)R_3 = 0;\ R_2 = \frac{a_0(R_1+R_3)}{(R_1+R_3)-a_0} = \frac{(1-R_1)R_3}{(R_1+R_3)-1}$$

$$L = \frac{R_1(R_1R_2+R_1R_3+R_3^2)}{(R_2+R_3)[b_1(R_1+R_3)-a_1]}; \qquad C = \frac{(\Sigma R)[b_1(R_1+R_3)-a_1]}{R_1(R_1R_2+R_3^2)}$$

In order to present these results in the $(a_1 b_1)$ plane, it is necessary to consider the following five intervals,

(i) $\frac{1}{3} < a_0 < 1$

(ii) $a_0 = \frac{1}{3}$

(iii) $\frac{1}{9} < a_0 < \frac{1}{3}$

(iv) $a_0 = \frac{1}{9}$

(v) $a_0 < \frac{1}{9}$

This is due to the fact that $\mu_4 = 0$ changes from an ellipse to a hyperbola as a_0 passes through the value $\frac{1}{9}$, while the shape of α_2 also changes as a_0 passes through $\frac{1}{3}$. Figures 6.2, 6.3, 6.4, 6.5 and 6.6 show different regions where the biquadratic $z(s)$ may be realized by Structure VI. It should be observed from these figures that whenever $\mu_1\mu_3 < 0$, there is always a realization, whatever be the value of a_0 (as long as $a_0 < 1$), and μ_2. It is also to be noted that except when $a_0 = \frac{1}{9}$, there is always a certain region where two realizations in the form of Structure VI may be found for $z(s)$.

6.3 Conditions for Realization by Structure VII

It may be observed that Structure VII is nothing but the dual structure of Structure VI. Hence, we may adopt the technique used in Sec. 3.3 in deriving the coefficient

$$A = \left\{ \sqrt{\frac{1-a_o}{2a_o}},\ (3/2)\sqrt{2a_o(1-a_o)} \right\}$$

$$B = \left\{ 3\sqrt{\frac{1-a_o}{2}},\ \sqrt{\frac{1-a_o}{2}} \right\}$$

$$C = \left\{ \sqrt{1+1/\sqrt{a_o}},\ \sqrt{a_o+\sqrt{a_o}} \right\}$$

$$D = \left\{ \frac{1}{a_o}\sqrt{(1-a_o)(3a_o-1)},\ 0 \right\}$$

$$E = \left\{ 0,\ \sqrt{(1-a_o)(3a_o-1)} \right\}$$

$$F = \left\{ \sqrt{\frac{(1-a_o)(9a_o-1)}{a_o}},\ 0 \right\}$$

$$G = [0, \sqrt{(1-a_o)(9a_o-1)}]$$

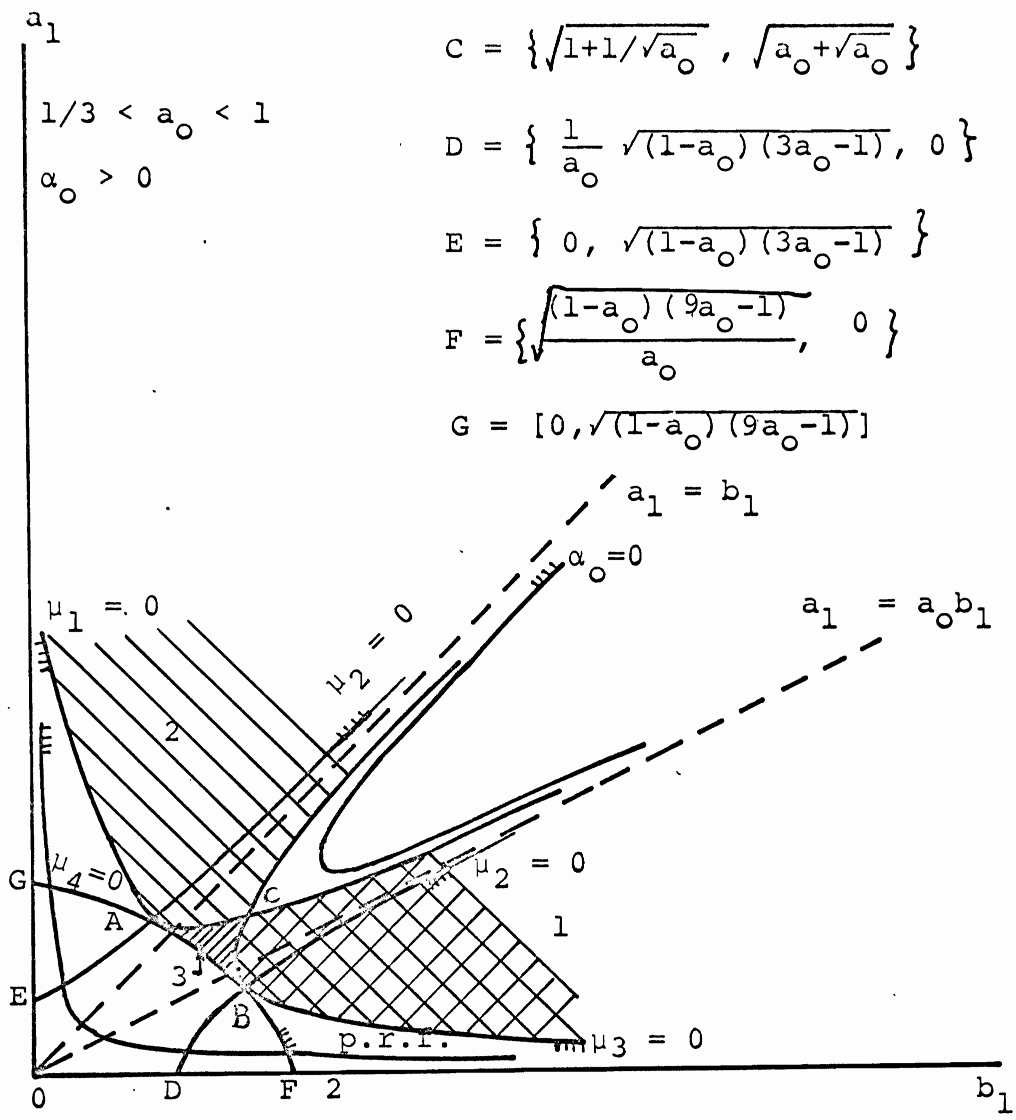


Fig. 6.2. Coefficient conditions in the (a_1,b_1) phase for the realization of z(s) by Structure VI, when $1/3 < a_o < 1$.

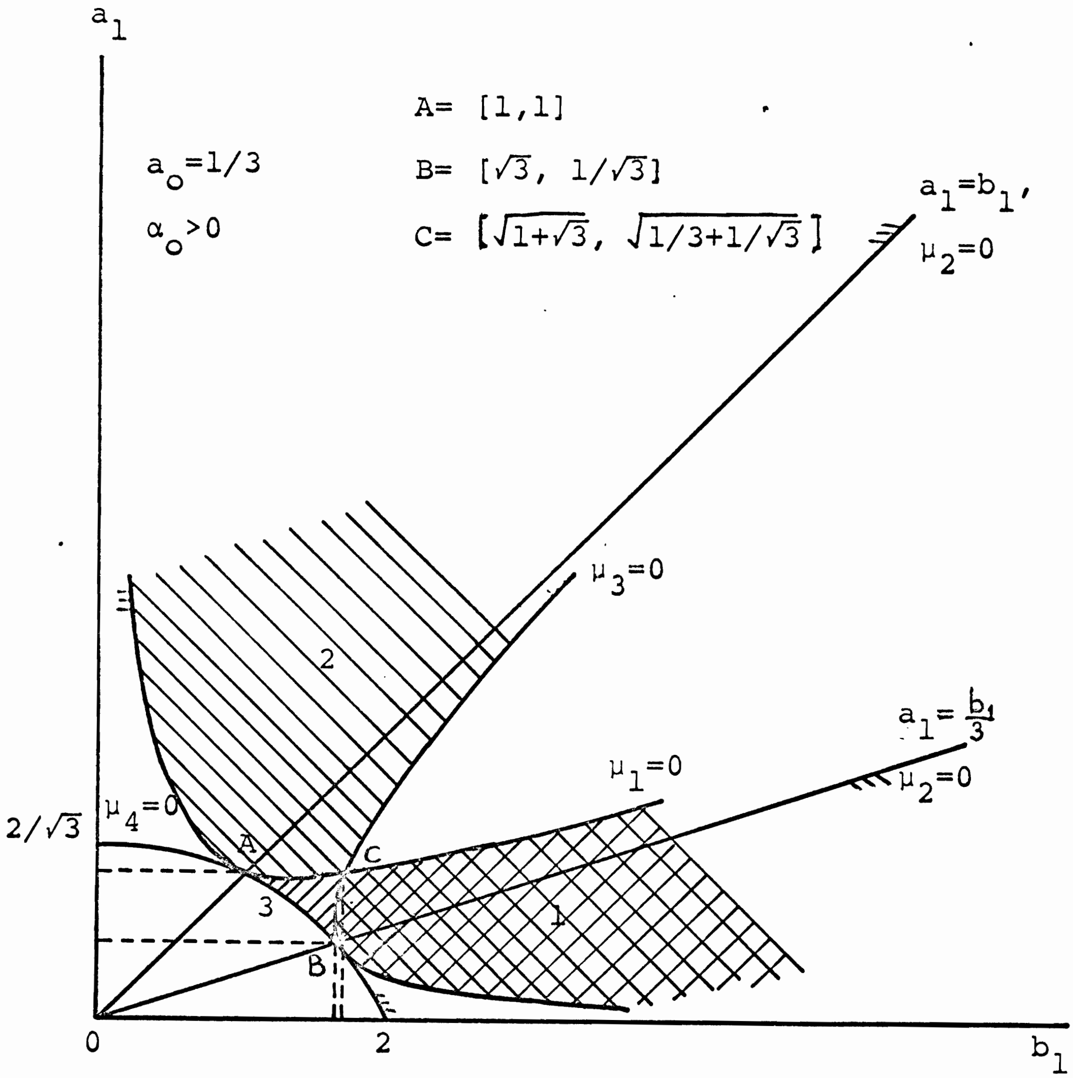


Fig. 6.3 Coefficient conditions in the (a_1, b_1) phase for the realization of z(s) by Structure VI, when $a_o = 1/3$.

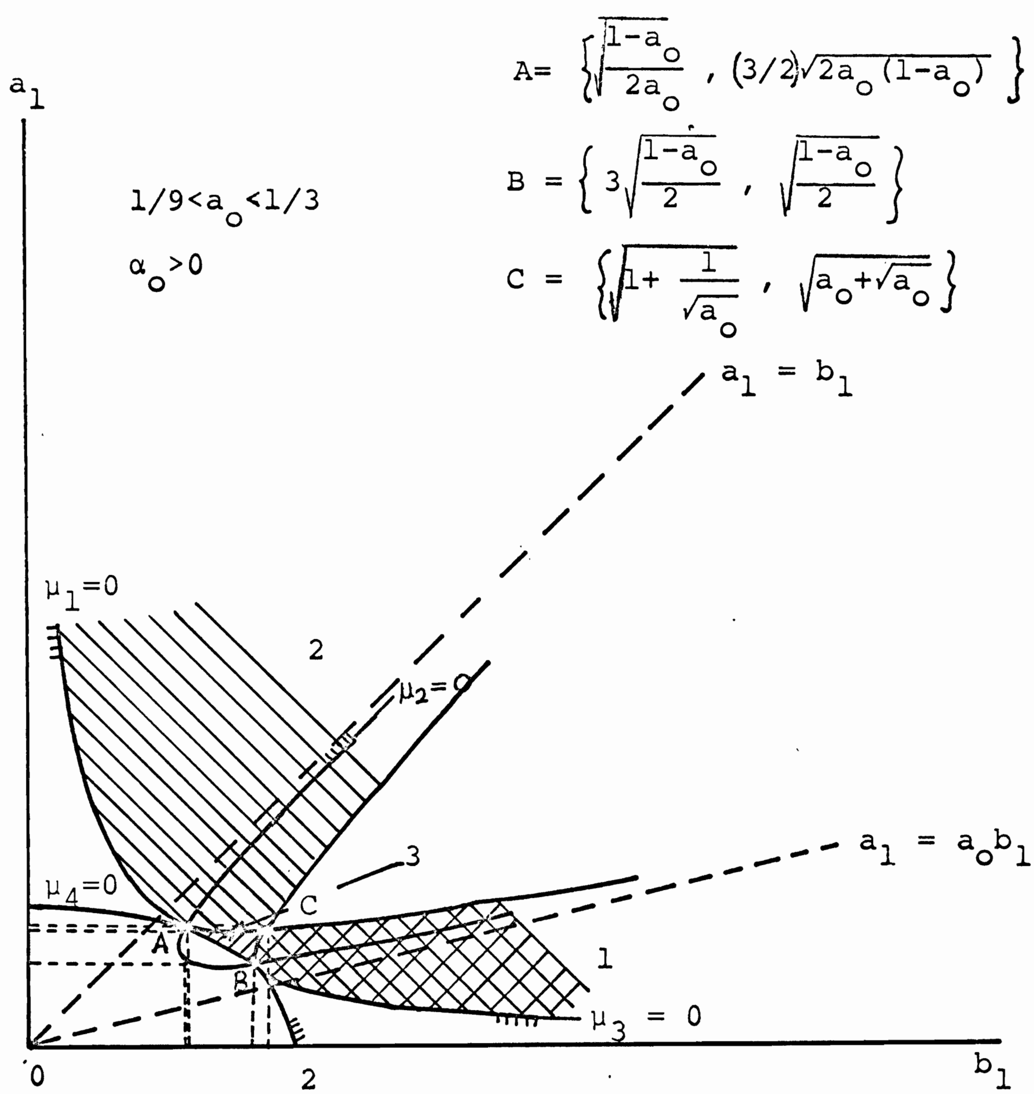


Fig. 6.4 Coefficient conditions in the (a_1, b_1) plane for the realization of z(s) by Structure VI, when $1/9 < a_o < 1/3$.

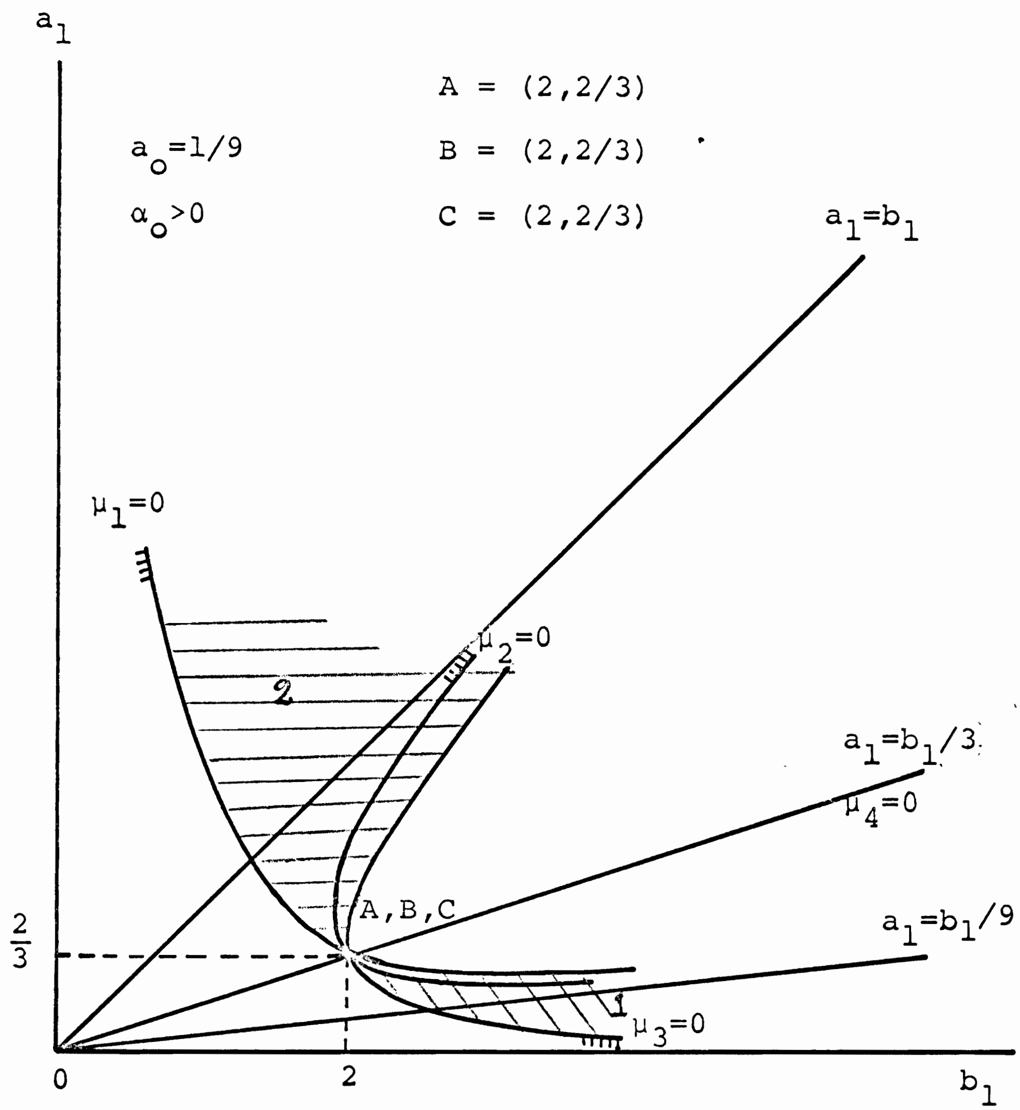


Fig. 6.5 Coefficient conditions in the (a_1, b_1) plane for the realization of z(s) by Structure VI, when $a_o = 1/9$.

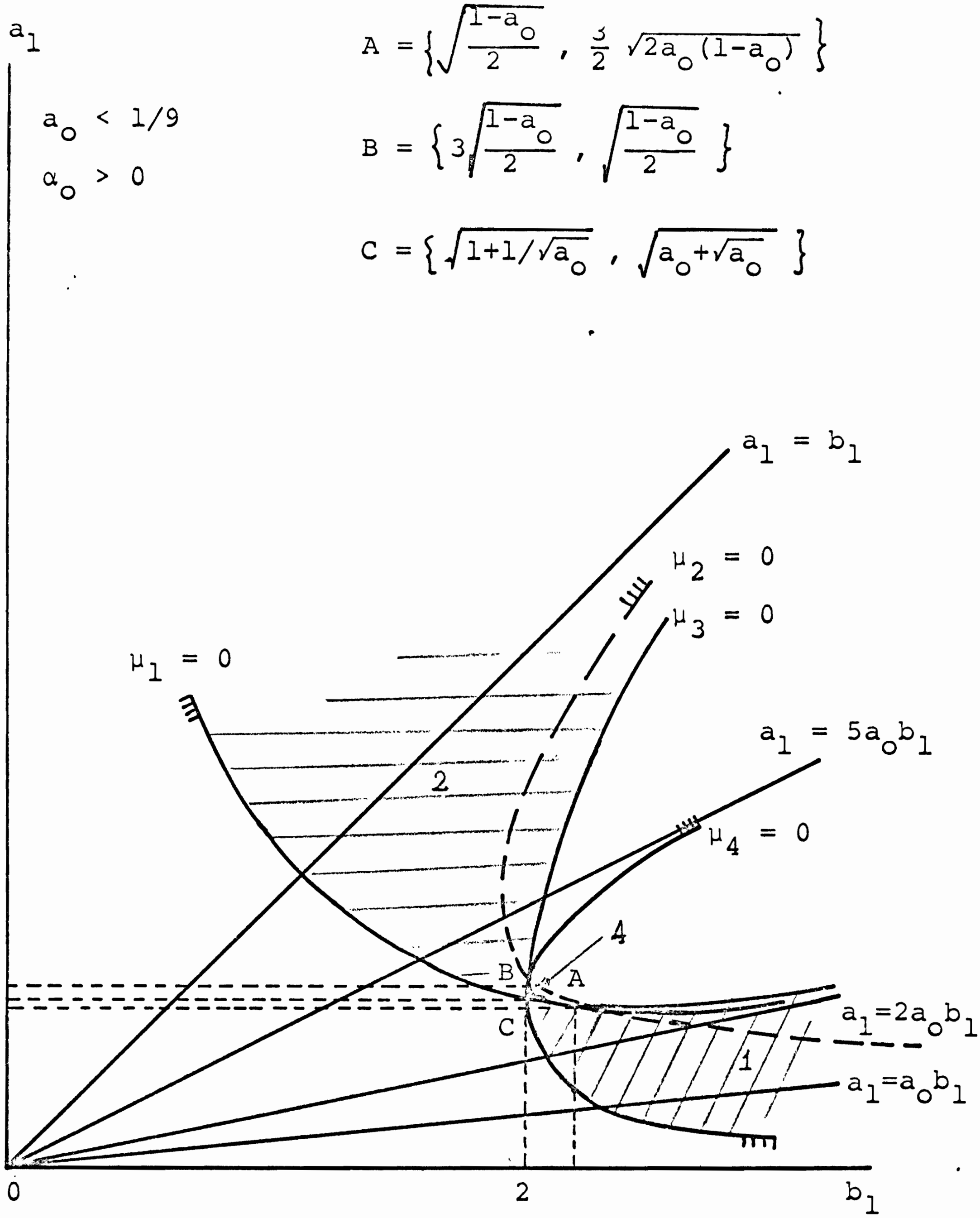


Fig. 6.6 Coefficient conditions in the (a_1, b_1) plane for the realization of z(s) by Structure VI, when $a_o < 1/9$.

conditions, as well as the component values for Structure VII from those of Structure VI. These results are tabulated in Table 6.2. The values of ν_1, ν_2, ν_3, and ν_4, used in Table 6.2 are given below.

$$\nu_1 = a_0(a_0-1) - a_1(a_0b_1-a_1)$$

$$2\nu_2 = (a_0-1)(a_0-3) - (a_1-b_1)(a_0b_1-a_1)$$

$$\nu_3 = (a_0-1) - b_1(a_1-b_1)$$

$$\nu_4 = \nu_2^2 - \nu_1\nu_3$$

TABLE 6.2

Coefficient conditions for the realization of $z(s)$ by Structure VII, and the corresponding component values.

$(a_0 > 1,\ \alpha_2 > 0)$

Region	Coefficient Conditions	Value of G_3	Comments
1	$\nu_1 > 0$ $\nu_3 < 0$	$G_3 = \frac{-\nu_2 + \sqrt{\nu_2^2 - \nu_1\nu_3}}{\nu_1}$	One network realization.
2	$\nu_1 < 0$ $\nu_3 > 0$	$G_3 = \frac{-\nu_2 - \sqrt{\nu_2^2 - \nu_1\nu_3}}{\nu_1}$	One network realization.
3	$\nu_1 > 0$ $\nu_3 > 0$ $\nu_4 \geqslant 0$ $1 < a_0 < 9$	$G_3 = \frac{-\nu_2 \pm \sqrt{\nu_2^2 - \nu_1\nu_3}}{\nu_1}$; $\nu_4 \neq 0$	Two network realizations.
		$= \frac{-\nu_2}{\nu_1}$; $\nu_4 = 0$	One network realization.
4	$\nu_1 < 0$ $\nu_3 < 0$ $\nu_4 \geqslant 0$ $a_0 > 9$	$G_3 = \frac{-\nu_2 \pm \sqrt{\nu_2^2 - \nu_1\nu_3}}{\nu_1}$; $\nu_4 \neq 0$	Two network realizations.
		$= \frac{-\nu_2}{\nu_1}$; $\nu_4 = 0$	One network realization.

$$(1+a_0G_3)G_1^2 - (1+a_0G_3)(1-G_3)G_1 - (a_0-1)G_3^2 = 0;\quad G_2 = \frac{G_1+G_3}{a_0(G_1+G_3)-1} = \frac{(1-G_1)G_3}{(G_1+G_3)-1}$$

$$C = \frac{G_1(G_1G_2+G_1G_3+G_3^2)}{(G_2+G_3)[a_1(G_1+G_3)-b_1]};\quad L = \frac{(\Sigma G)[a_1(G_1+G_3) - b_1]}{a_0G_1(G_1G_2+G_3^2)}$$

CHAPTER 7

CONCLUSIONS

In the preceding Chapters, we have derived the coefficient conditions under which the biquadratic minimum reactance z(s)

$$z(s) = \frac{s^2 + a_1 s + a_0}{s^2 + b_1 s + 1}$$

may be realized by Structures I - VII. We have already mentioned in Chapter 2, that knowing the realizability conditions in terms of a_1, b_1 and a_0, we may obtain the realizability condition for a general minimum reactance biquadratic function.

$$z(s) = K \frac{s^2 + A_1 S + A_0}{s^2 + B_1 s + B_0} \qquad (7.1)$$

by noting that

$$\frac{A_1}{\sqrt{B_0}} = a_1$$

$$\frac{B_1}{\sqrt{B_0}} = b_1 \qquad (7.2)$$

$$\frac{A_0}{B_0} = a_0$$

Substituting for a_0, a_1, b_1 in the quantities α's, β's, γ's, δ's, μ's and ν's, we may define a set of modified α's, β's, etc., in terms of A_1, B_1, A_0 and B_0; the necessary modified α's, β's, etc., are given in the Appendix.

The results obtained in the preceding Chapter, regarding the conditions under which $z(s)$ may be realized by one of the bridge structures (I-VII), are, for the sake of compactness, presented in the (A_1, B_1) plane in Figs. (7.1), (7.2) and (7.3) respectively; they are also tabulated in Tables (7.1), (7.2) and (7.3).

It is seen from Table (7.1), that if

$$\alpha_0 + 4A_0B_0 \leqslant 0$$

and (7.3)

$$A_0 > B$$

then $z(s)$ given by (7.1) may always be realized by the RC - bridge structure IV. It is to be pointed out that if

$$\alpha_0 + 4A_0B_0 < 0$$

there are two realizations of Structure IV, while if

$$\alpha_0 + 4A_0B_0 = 0$$

there is only one realization. Thus, if conditions (7.3) are satisfied, then in addition to the Cauer and Foster canonic forms, we may find Canonic RC - bridge networks in the form of Structure IV.

It is observed from Table (7.2) that if

$$\alpha_0 + 4A_0B_0 \leqslant 0$$

and (7.4)

$$A_0 < B_0$$

then $z(s)$ given by (7.1) may always be realized by the

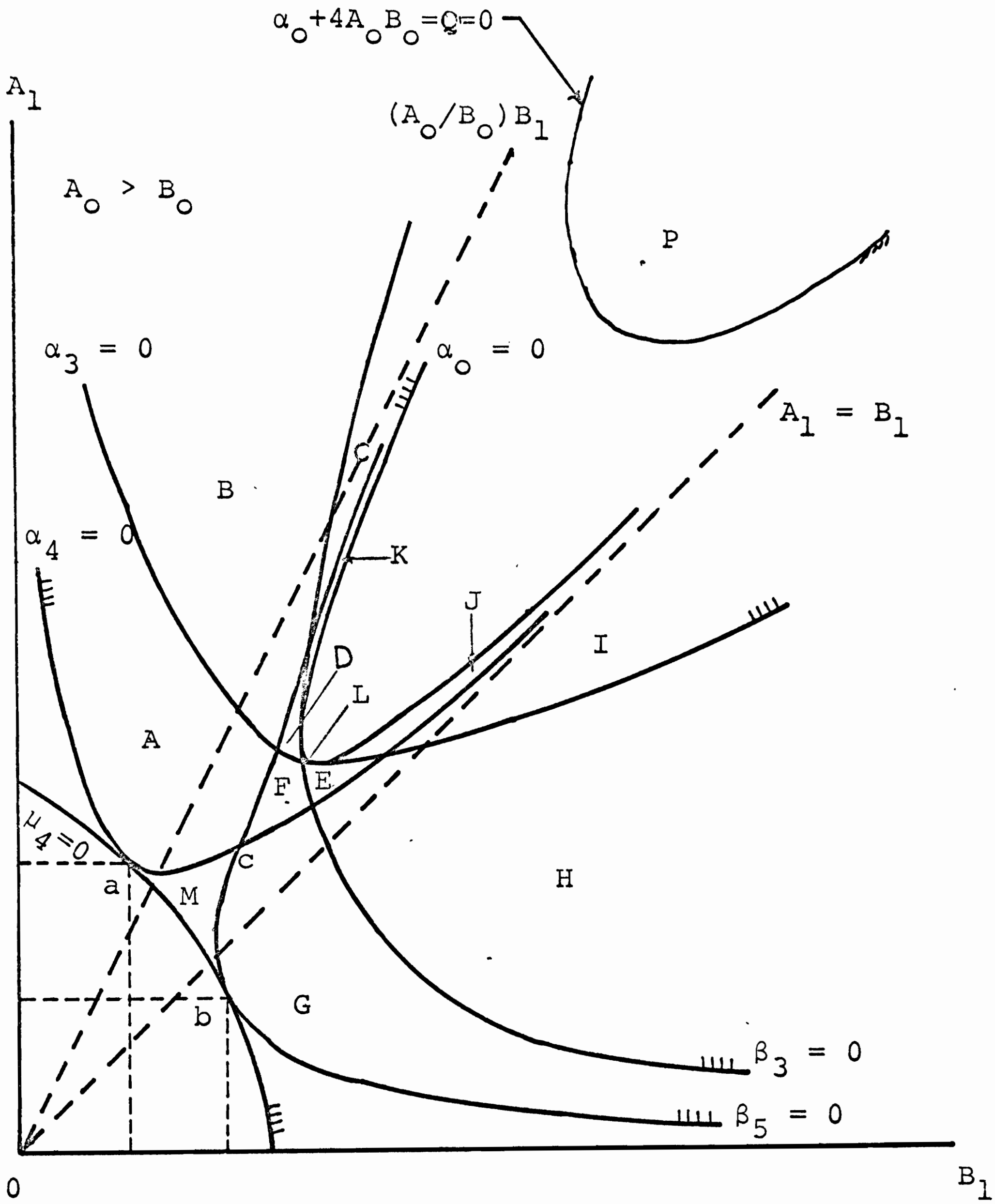


Fig. 7.1 Coefficient conditions in the (A_1, B_1) plane for the realization of z(s) by Structure I - VII, when $A_o > B_o$.

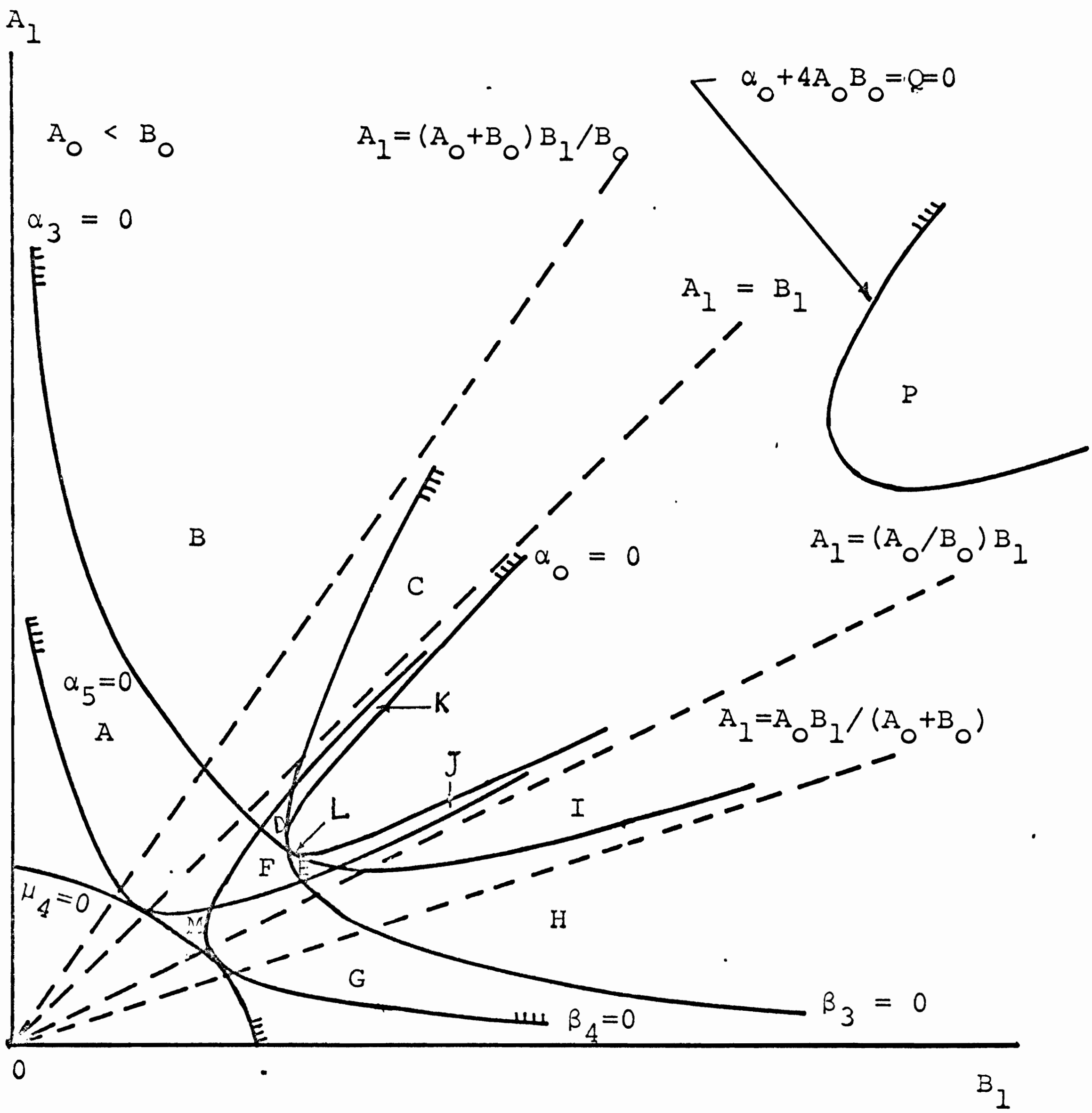


Fig. 7.2 Coefficient conditions in the (A_1,B_1) phase for the realization of z(s) by Structures I - VII, when $A_o < B_o$.

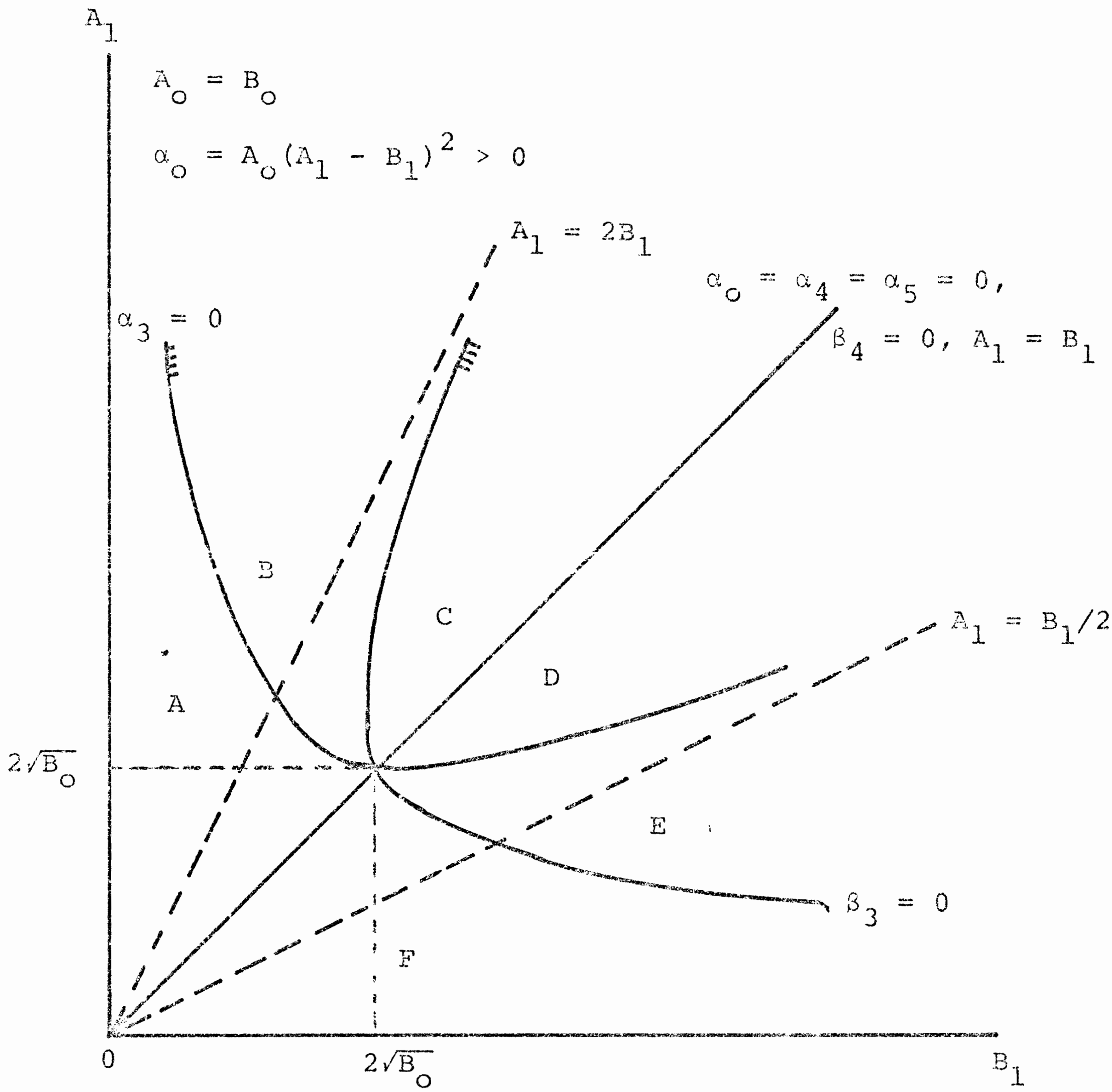


Fig. 7.3 Coefficient conditions in the (A_1, B_1) plane for the realization of z(s) by Structures I - VII, when $A_o = B_o$.

TABLE 7.1

Coefficient Conditions for the Realization of $z(s)$ by Structures I-VII, when $A_0 > B_0$.

		Regions															
		A	B	C	D	E	F	G	H	I	J	K	L	M	N	P	Q
Coefficient Conditions	α_0										+	+	+				
	α_3	−	+		+	−	−		−	+	+		+				
	α_4	+				+	+	−	−	−	+			−	+		
	β_3		−	+	−	+	−	−	+			+	+				
	β_5	−	−	−	+		+	+				+		−	+		
	α_2										−	−					
	$\alpha_1+\alpha_2$										+						
	$B_0\beta_1+A_0\beta_2$											+					
	γ_2												−				
	ν_4													+	+		
	ν_5													−	+		
	$\alpha_0+4A_0B_0$															−	0
Structure	I	1				1	1			1	2						
	II			1	1		1	1				2					
	III		1		1	1			1				2				
	IV															2	1
	V																
	VI																
	VII	1	1	1				1	1	1				2	2		

NOTE: The values of different α's, β's etc., are given in Appendix

TABLE 7.2

Coefficient Conditions for the Realization of $z(s)$ by Structures I-VII, when $A_0 < B_0$.

		Regions															
		A	B	C	D	E	F	G	H	I	J	K	L	M	N	P	Q
Coefficient Conditions	α_0										+	+	+				
	α_3	−	+		+	−	−		−	+	+		+				
	α_5	+				+	+	−	−	−	+			−	+		
	β_3		−	+	−	+	−	−	+			+	+				
	β_4	−	−	−	+		+	+				+		−	+		
	α_2										−	−					
	$\beta_1+\beta_2$											+					
	$A_0\alpha_1+\alpha_2B_0$										+						
	γ_2												−				
	μ_4													+	+		
	μ_5													−	+		
	$\alpha_0+4A_0B_0$															−	0
Structures	I	1				1	1			1	2						
	II			1	1		1	1				2					
	III		1		1	1			1				2				
	IV																
	V															2	1
	VI	1	1	1				1	1	1				2	2		
	VII																

TABLE 7.3

Coefficient Conditions for the Realization of z(s) by Structures I-VII, when $A_0 = B_0$.

		Regions					
		A	B	C	D	E	F
Coefficient Conditions	α_3	−	+		+	−	
	β_3		−	+		+	−
	α_4	+		+	−		−
Structures for Realization	I	1			1		
	II		1			1	
	III			1			1
	IV						
	V						
	VI						
	VII						

RL - bridge Structure V. It is to be pointed out that if

$$\alpha_0 + 4A_0B_0 < 0$$

there are two realizations of Structure V, while if

$$\alpha_0 + 4A_0B_0 = 0$$

there is only one realization. Thus, if conditions (7.4) are satisfied, then in addition to the Cauer and Foster canonic forms, we may find Canonic RL-bridge networks in the form of Structure V.

Finally, it is seen from Tables(7.1), (7.2) and (7.3), that if $A_0 = B_0$, z(s) may be realized by one of the RLC- bridge structures I, II or III, and that this circuit is unique. It is also seen that if $A_0 \neq B_0$, then z(s) can be realized, provided certain coefficient conditions are satisfied. Furthermore, once these conditions are satisfied, there are always two network realizations for z(s), either in the form of two different structures, or in the form of two different realizations of the same structure.

Thus, we have shown that under certain conditions, a minimum reactance biquadratic impedance z(s) may be realized by a canonic bridge structure, consisting of three resistive and two reactive elements. It would be of interest to find the coefficient conditions under which (7.1) may be realized by RLC- structures, other than the bridge networks; further it is worthwhile examining the possibilities of such canonic realizations for higher order RLC- driving point functions.

APPENDIX

Modified α's, β's, etc., in terms of the coefficients A_1, B_1, A_0 and B_0.

$$\alpha_0 = (A_0 - B_0)^2 - (A_0B_1 - A_1B_0)(A_1 - B_1)$$

$$\alpha_1 = B_1^2 - 4B_0$$

$$\alpha_2 = 2(A_0 + B_0) - A_1B_1$$

$$\alpha_3 = A_1B_1(A_0 + B_0) - A_0B_1^2 - (A_0 + B_0)^2$$

$$\alpha_4 = B_1(A_1 - B_1) - (A_0 - B_0)$$

$$\alpha_5 = B_0A_1B_1 - A_0B_1^2 - B_0(B_0 - A_0)$$

$$\beta_1 = A_1^2 - 4A_0$$

$$\beta_2 = 2(A_0 + B_0) - A_1B_1 = \alpha_2$$

$$\beta_3 = A_1B_1(A_0 + B_0) - B_0A_1^2 - (A_0 + B_0)^2$$

$$\beta_4 = A_1(B_1 - A_1) - (B_0 - A_0)$$

$$\beta_5 = A_0A_1B_1 - B_0A_1^2 - A_0(A_0 - B_0)$$

$$\gamma_2 = A_0B_0A_1B_1 + [(A_0 + B_0)B_1 - A_1B_0][(A_0 + B_0)A_1 - A_0B_1] - (A_0 + B_0)^3$$

$$\mu_4 = (B_0 - A_0)(B_0 - 9A_0) + (A_1 - B_1)(B_0A_1 - A_0B_1) + 4A_0A_1B_1$$

$$\mu_5 = B_0 - 9A_0$$

$$\nu_4 = (A_0 - B_0)(A_0 - 9B_0) + (B_1 - A_1)(A_0B_1 - B_0A_1) + 4B_0A_1B_1$$

$$\nu_5 = A_0 - 9B_0$$

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