Analysis of a Star Local Area Network: Two Approaches, and Analysis of a 2x2 Banyan Switch

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ABSTRACT

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Jean-François Sauriol

The main objective of this thesis will be to analyze the performance of two types of networks.

First we present an analysis of a star-connected Local Area Network. This topology is seen as a valued candidate for MANs which would integrate future small scale services. The double embedding strategy used in the model's Markov chain introduces difficulties solved by a rather complicated transformation giving a large set of results according to a varying range of parameters and for variable length messages.

Then, we analyze the performance of a 2x2 Banyan Switch as part of a larger Banyan Network. These types of non-blocking networks have often been considered as the central packet switching fabric of BISDNs. The analysis follows the same approach used in the Star network but applied to the Fast Packet switching

environment of Banyan Networks. Numerous results are also obtained for different varying parameters and for variable length packets.

Finally another analysis of the same Star Network is presented. This different approach is based on a Markov chain embedded on arrival points which allows for general arrival processes. Due to limited computing facilities, numerical results for this analysis could not be obtained.

Conclusions and comparisons with previous works close this thesis.

"A mes Parents"

ACKNOWLEDGEMENTS

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I gratefully acknowledge the encouragements and constant support provided by my parents, girlfriend, brother, sister and friends.

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CHAPTER ONE

INTRODUCTION

The *Broadband Integrated Services Digital Network* (BISDN) is widely considered as the next generation telecommunications network. This is a result of a growing need for communication systems that can handle a heterogeneous and dynamically changing mix of applications.

A general overview of past and present communication networks will help in establishing the grounds for this thesis.

In the past decade or so [5], most small area and low to medium speed communications needs have been filled by privately operated *Local Area Networks* (LAN). Some familiar LANs include Ethernet, Token Ring and Polling. These architectures have been sufficiently powerful and flexible to allow each owner (whether it be Bell Headquarters or smaller scale users) to integrate data and text transmissions in their organizations. Also, when small area voice transmission were concerned, PBXs have been the favored choice.

When the case of wide area data communication networks is concerned, the larger throughputs involved created the need for various *Backbone* architectures. Common examples of wide area data networks include Arpanet (US) and Datapac (Canada). It also goes without saying that long range voice communication needs have always been filled by the ever expanding telephone network.

Fig. 1.1 summarizes the most common discrete networks outlined here, and gives the basic inter-relations between them. It becomes clear that integration of data and voice transmissions in the same network would eliminate the greater costs and lesser flexibility which stems from the present duplicity.

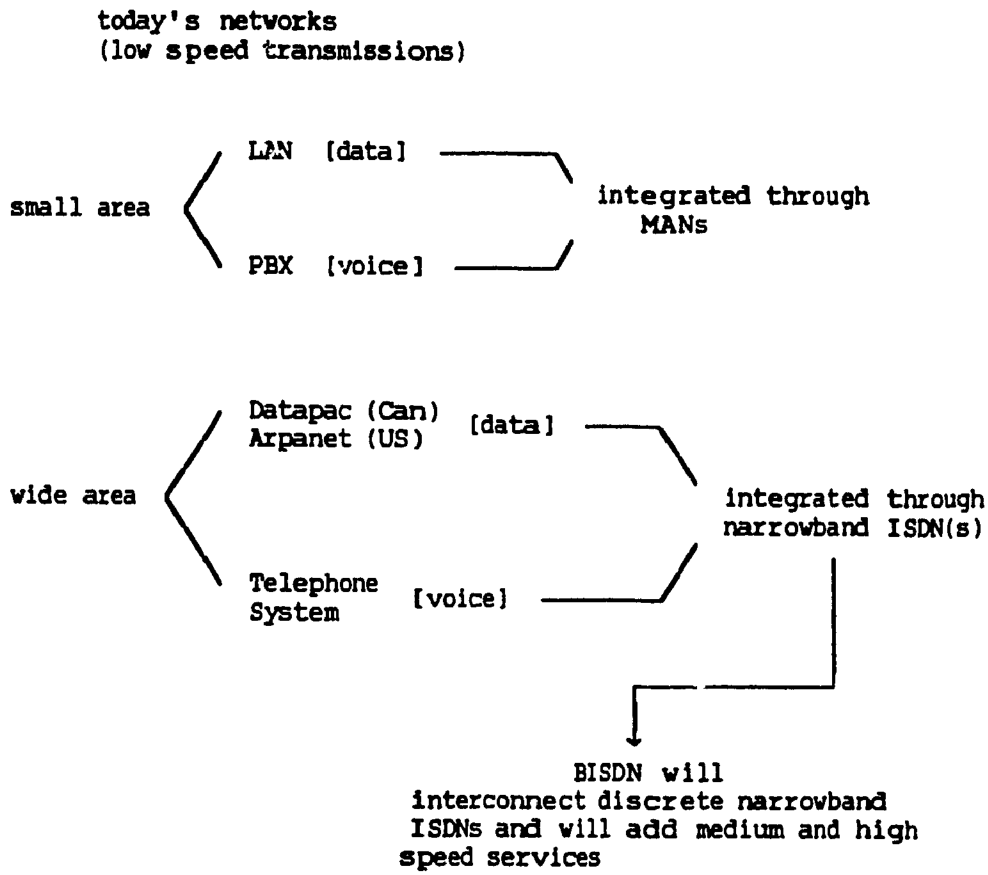


Fig. 1.1 Representation of today's most common small and wide area networks and their inter-relations.

The trend in small area networks seems to be towards *Metropolitan Area Networks* (MAN) which would integrate the owner's voice and data transmissions under the same network.

Part of this thesis will analyze the performance of a fiber-optic based, Fast Circuit Switching LAN with a Star Topology. This LAN would integrate both data and voice, and possibly video, within the same network. Two different analysis of this Star Network will be presented. Both Chapter 2 and 4, will give a performance analysis of this LAN using different approaches.

The wide area networks future trends involves a more standardized system. In fact, The "narrowband ISDN" idea of integrating data and voice wide area networks (which are rather "low speed" services), is now only a question of a few years [5], as many trials have been implemented. But when "medium speed" services (Hi-Fi sound, Videophone, quality Data/Image, ...) or "high speed" services (Conventional TV, High-Definition TV, high quality Data/Image, ...) are concerned "narrowband ISDN" falls short.

A second facet of this thesis concentrates on this larger aspect of the integration problem. It is important to realize that the standardized 64 kbit/s rate of ISDN will allow only to accommodate for the "low speed" information transfers (Data/Text, Voice, Data/Image, ...) of the future.

Services in the "medium speed" class are outside the "narrowband ISDN" capabilities. It is expected that the "broadband ISDN" will cost-effectively support switched connectivity (Fast Packet or Fast Circuit), for these low and medium speed services as well as selected services from the "high speed" class which have much higher bit rates.

Many architectures for BISDN have been proposed. In order to implement any of these, switching is needed to permit the use of multiple wavelengths, or fibers and to accommodate the bit rates and traffic variety of lower hierarchy networks. There we find the main design bottleneck : switching.

On the transmission side, related technologies have experienced an explosive growth reaching rates in the Gbit/s and still increasing. Lagging this tremendous progress in transmission, however, are the advances made in switching technology. While in the future the wideband communication networks needs for switching may be met by optical technology, at present our electronic architectures must be greatly upgraded to handle the BISDN switching needs.

Chapter 3 will then present the analysis of a *Banyan Network*. The BISDN application of Packet Switching Banyan networks have been widely analyzed in the past [15], [18], [26], [27]. The analysis will follow the same mathematical development used in Chapter 2, but will be applied to a 2x2 Banyan Switch as part of a larger Banyan Network.

It would seem that the interests of this thesis are very vast. In fact, this is only because the mathematical model used in the Star Network analysis of Chapter 2 is identical to the 2x2 Banyan Switch model derived in Chapter 3. This allows us to analyze both systems and obtain various results pertinent in each field.

Each analysis allows us to evaluate specific aspects of the respective systems. Then, the Star Network analysis of Chapter 2 allows us to generate the distribution of the number of messages in the system, the blocking probabilities, the average number of messages in the system and the mean message delay according to:

- finite buffer sizes between 2 and 10,
- variable length packets,
- symmetric or non-symmetric arrivals to the system's source queues, and
- equally or non-equally probable message destinations.

Also, these results apply to the 2x2 Banyan Switch analysis of Chapter 3. From the array of results obtained, improvements and comparisons with previous works by Jenq [15], and by Dias and Jump [18], [27] will be made and presented in Chapter 5.

Finally, the second analysis of the Star Network, presented in Chapter 4, allows us to observe the system under a general arrival process (deterministic arrivals, variable arrivals, adaptive arrivals, ...), and in symmetric traffic.

Results will be presented at the end of the respective chapters while comparisons with previous works and Conclusions will be found in Chapter 5.

CHAPTER TWO

AN ANALYSIS OF A STAR LOCAL AREA NETWORK

2.1 Introduction

This Chapter presents a performance analysis of a star-connected LAN. The central switching element of the network and its components, referred to as the *Cross-Point Switch* (CPS), provide finite buffers at the inputs and infinite output buffers. The analysis is based on a Markov chain where the points immediately following arrivals and departures to and from the CPS are chosen as the embedded points. The use of two sets of embedded points causes difficulties and requires a rather complicated transformation to determine the steady state distribution of the number of messages in the input buffers. This distribution will provide average delays and blocking probability for symmetric and non-symmetric arrivals, for different routing probabilities and for variable length messages.

2.2 System description

A star-connected LAN based on optical fiber links is currently being developed [1]. The case under consideration for this analysis is a smaller system involving two nodes connected through the CPS both being able to send messages to themselves as well as to each other.

The switching technique used in this LAN is a form of Fast Circuit switching implemented in the CPS. The details needed to substantiate the choice of the star topology for this LAN are given in [2] along with an approximate analysis of its performance.

We consider briefly the workings of this network. The star topology consists of two way links from user nodes to a central switch. (See Fig.2.1). The switch directs traffic between the links or arms connected to it. Note that this permits simultaneous connections between any pair of user nodes. (Even connection of the same user node to itself through the switch).

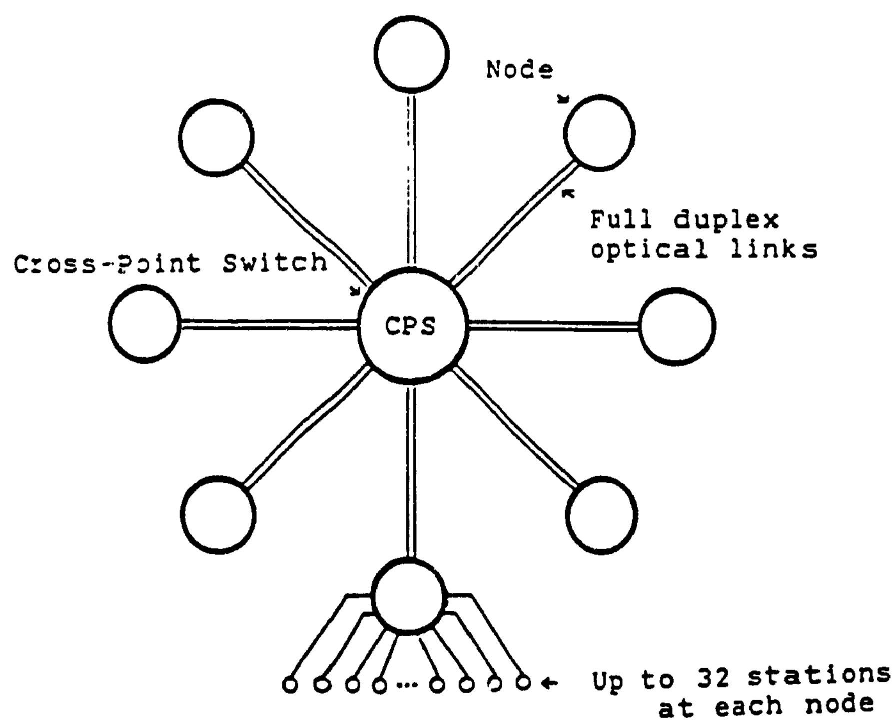


Fig. 2.1 Star Network Configuration

We differentiate between the *user nodes* and *user stations*. There may be as many as 32 user stations connected to a user node. The connection of a user node to itself allows communication of user stations located at the same node to each other.

As mentioned above the technique is a form of fast circuit switching. When a user node has information to be transmitted a *call set-up request* is sent to the central processor. The call request gives source and destination address (nodes and stations). The central switch schedules the transfer and informs the transmitting and receiving stations. The call request messages are relatively short and do not need to be processed on the fly. These factors serve to reduce the processing power that is required. When user data is transferred it is done at optical line rate with minimal processing.

2.3 The statement of the mathematical model

The communications protocol implemented in this LAN will serve messages arriving at each source node on a *First Come First Served* (FCFS) basis. This protocol has been chosen for the simplicity of operation. A representation of the more tractable two user nodes model is given in Fig.2.2.

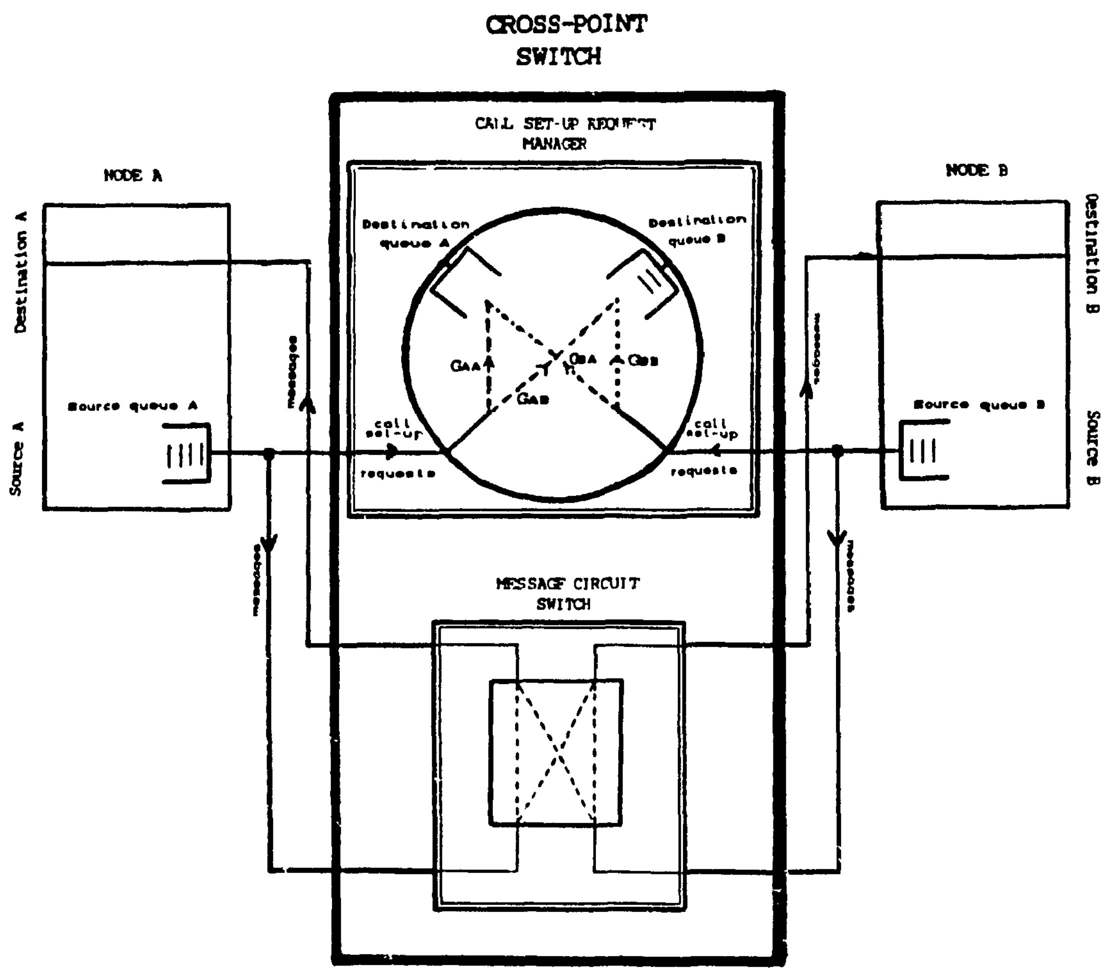


Fig. 2.2 Two User Nodes Star Network Model.

Messages arriving to the system may queue-up at either one of the *source queues*. The *destination* of each message is determined in a probabilistic manner according to a routing matrix in which the i^{th}, j^{th} element, G_{ij}, is the probability that a message from source node i will have node j as its destination.

$$G = \begin{bmatrix} G_{AA} & G_{AB} \\ G_{BA} & G_{BB} \end{bmatrix} \text{ where each row sums up to 1.}$$

Note also that the probability that a message will have its destination as its source is non-zero. This again allows connectivity of stations connected to the same node.

When a message reaches the head of its source queue a call set-up request is sent simultaneously to the central processor giving source and destination addresses (node and station). The central switch keeps a list for each destination, and each new request is placed at the end of this list. Requests are also handled on a FCFS basis. When a request reaches the head of its list, the central switch makes the necessary connection between the source and the destination pair for the transmission of the message. Notice that only the message at the head of the source queue is served even though there may be messages lower in the queue for which a line is available.

Now, each destination list may be considered as a queue with service time taken as the transmission time of a message. Clearly in the "two users" case, the maximum number of messages in a destination queue is two, corresponding to the event that both nodes have messages at the head of their queues destined for the same node (as is shown in Fig.2.2 where the head of line messages in the source queues both have node B as destination). Moreover, no more than a total of two messages may be in the destination queues at a time since only messages at the head of the source queues may send call set-up requests.

In the remainder of this chapter, we will differentiate between source and destination queues in the following manner: messages arriving into a node will queue-up at source queues A or B respectively. But call set-up requests will queue-up in destination queue A' or B' according to their routing information. This should remove any confusion between source and destination queues since destination queues are "primed".

Also, it is important to realize that messages do not queue-up in destination queues. Only call set-up requests of the head of line messages in the source queues do so. These requests remain in the destination queues until its corresponding message is fully transmitted. At that point, the Central Switch removes the respective call set-up request from its destination queue and handles the next request behind it.

It is necessary, now, to list the mathematical assumptions made in this analysis.

2.4 Basic assumptions

In the following, it is assumed that:

- finite sized input buffers are provided in the nodes.
- practically infinite output buffers are provided at the user nodes and station. Note the distinction between destination queues which are a representation of the call requests lists kept at the CPS and output buffers which are the relatively large storage facilities at the user nodes and stations which prevent blocking outside the CPS.
- all communications regarding the call set-up/take-down take place instantaneously, and do not take any of the capacity reserved for the message transmission.
- Poisson arrival process at each source queue with parameters λ_A and λ_B respectively. Note that if a source queue is full the respective arrival process is disabled (λ_A or $\lambda_B = 0$).
- Exponentially distributed message transmission times with parameters μ_A and μ_B
- Messages choose their destinations independent of each other according to the routing matrix G.

Now, for the case of exponentially distributed message transmission times, this system may be modeled by using an embedded *Markov chain*, where transition probabilities will depend on the system state defined below.

2.5 The choice of embedded points

As said earlier, this analysis is based on a Markov chain. The choice of embedded points for this Markov chain presents a problem. There are three logical choices: *departure* points, *arrival* points, arrival and departure points. We demonstrate here that the only viable choice of embedded points is both arrival and departure points by showing the problems encountered with each of the other embedding strategies.

The difficulties stem from the interdependence of one source queue on the other. Consider the following example depicted in Fig.2.3: two messages, each at the head of its source queue A and B, are sent to destination queue A'.

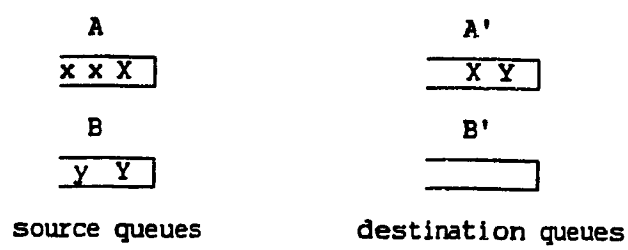


Fig. 2.3 Source Queue Interdependence example.

It is clear that the message from A, X, will not enter service until the message from B, Y, in front of it has left the destination queue A' server. When both source queues are full the behavior of the system is fairly straightforward.

Now, assume that embedded points lie immediately following departure points from the system. Since theory states that in a Markov chain the next system state depends only on the present system state it is easy to show that embedding only on departures generates difficult situations which would largely complicate the analysis.

Consider the following example: source queues A and B contain n_A and 0 messages respectively, the message at the head of source queue A is in service at destination queue A' and destination queue B' is empty (as shown in Fig.2.4).

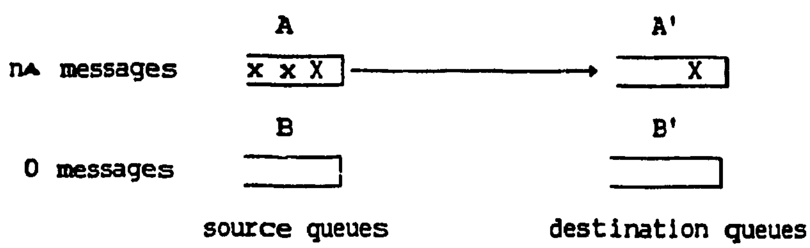


Fig. 2.4 Departure points embedding example (1)

Until the next departure from the system we may have arrivals to source queues A and B. Then if a message arrives into source queue B with B' as its destination it will enter service immediately (see Fig.2.5).

Fig. 2.5 Departure points embedding example (2)

Furthermore, this message could depart before the message already in service at destination queue A' does.

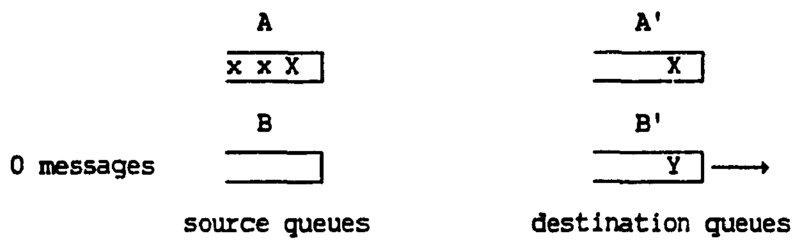


Fig. 2.6 Departure points embedding example (3)

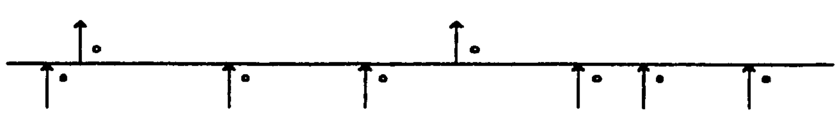
Then the next embedded point would lie immediately following this departure from queue B' seeing again 0 messages in source queue B. The departing message would have never been registered as being in source queue B from where it originated. It is possible to take this unseen arrival into account but this again, substantially complicates the analysis.

For the case where embedded points would lie immediately before arrivals to the system, please refer to Chapter 4. We simply state here that this strategy also has its drawbacks and that it did not allow us to generate numerical results.

Then, in the face of these difficulties, it was decided that the embedded points should lie immediately after both arrivals and departures to and from the system. It is clear that embedding after departures and before arrivals is impossible since this would imply anticipated knowledge of the event type (departure or arrival).

Our embedding strategy has a definite advantage. Since the embedding events are the arrivals and the departures to and from the system, the equations needed to obtain the transition probabilities resemble those in a birth-death process analysis. This in turns provides a fairly simple transition matrix to calculate. Any other embedding scheme would have generated a more complicated set of equations and a less tractable transition matrix.

We can then represent the system's Markov chain using standard notation in Fig.2.7.



-where • are the embedded points

Fig. 2.7 Representation of System's Markov chain.

Even though this embedding strategy solves one problem, it introduces another one. This difficulty lies in the steady state distribution of the number of messages in the source queues obtained from the transition matrix. The next sections will clarify the situation.

2.6 M/M/1 example

The difficulty encountered in the steady state distribution of the number of messages in the source queues arises from our embedding after arrivals and after departures to and from the system.

A more tractable example using a single M/M/1 queue will be presented here in order to explain the problems encountered as a result of our embedding strategy used in our more complicated analysis. The M/M/1 system can be represented as in Fig.2.8.

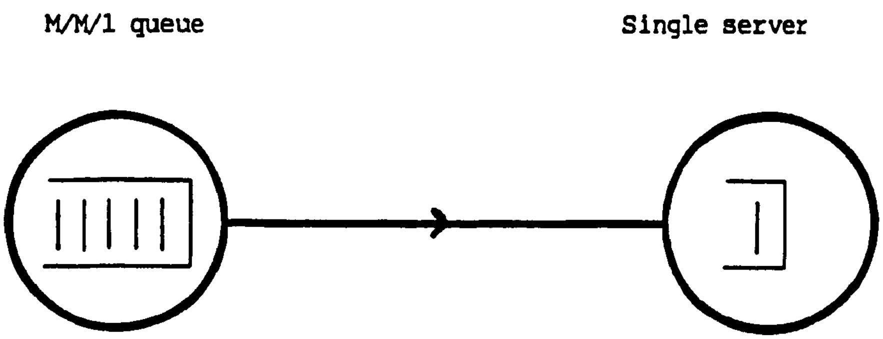


Fig. 2.8 M/M/1 system representation

It can be shown [3] that, taken separately with their respective embedding strategy, the limiting distribution of the number of messages in the M/M/1 queue immediately following a departure is equal to the same distribution immediately before an arrival for any system that changes state by unit step values (positive or negative).

Now remembering that our Markov chain is embedded *after* both arrivals and departures to and from the M/M/1 system we see that each arrival will include itself in the number of messages in the queue. In the remainder of this section, let us refer to arrivals to the M/M/1 queue and departures from the single server as simply arrivals and departures.

Let us define the probabilities of having "i" messages in the M/M/1 queue immediately following:

i) a departure as p_{di},

ii) an arrival as p_{ai}.

Again, since each arrival always includes itself in the number of messages in the queue, it becomes clear that the distribution of the number of messages in this queue immediately following an arrival (p_{ai}), is equal to the same distribution immediately following a departure (p_{di}), shifted by plus one. This is easier to visualize through the use of Fig.2.9 which presents arbitrary examples of both of these distributions.

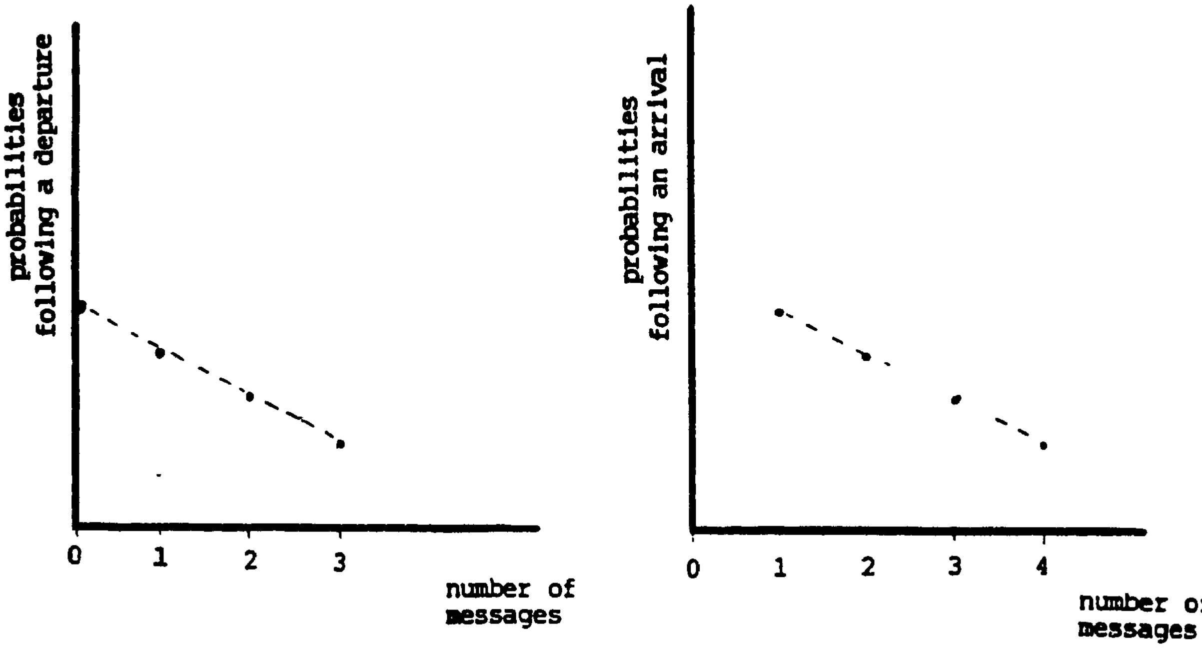


Fig. 2.9 Distributions of the number of messages in the M/M/1 queue immediately following an arrival and a departure to and from the M/M/1 system.

We notice that a translation by plus one done on the distribution following a departure results in the distribution following an arrival. Thus we can write:

$$p_{ai} = p_{d(i-1)} \qquad i > 0 \qquad -(2.1)$$

We will now determine both of these distributions. Let α_j denote the number of messages in the M/M/1 queue at the j^{th} embedded point. Assuming Poisson arrivals with parameter λ and exponentially distributed message transmission times with parameter μ, we may write:

$$\alpha_{j+1} = \begin{cases} \alpha_j - 1 & \text{with probability } \frac{\mu}{\lambda+\mu}, \\ \alpha_j + 1 & \text{with probability } \frac{\lambda}{\lambda+\mu}, \end{cases} \text{ if } \alpha_j > 0 \qquad \text{-(2.2)}$$

$$\alpha_{j+1} = 1 \qquad \text{if } \alpha_j = 0$$

Let us define $U(\alpha_j)$ as:

$$U(\alpha_j) = \begin{cases} -1 & \text{with probability } \frac{\mu}{\lambda+\mu}, \\ 1 & \text{with probability } \frac{\lambda}{\lambda+\mu}, \end{cases} \text{ if } \alpha_j > 0 \qquad \text{-(2.3)}$$

$$U(\alpha_j) = 1 \qquad \text{if } \alpha_j = 0$$

Then the above equations may be combined to give:

$$\alpha_{j+1} = \alpha_j + U(\alpha_j) \qquad \text{-(2.4)}$$

then we write:

$$z^{\alpha_{j+1}} = z^{\left(\alpha_j + U(\alpha_j)\right)} \qquad \text{-(2.5)}$$

now let

$$Q_j(z) = E\left[z^{\alpha_j} \right] = \sum_{k=0}^{\infty} z^k \, P\left[\alpha_j = k \right] \qquad \text{-(2.6)}$$

and taking the expectation on each side of Eqn 2.5 we get

$$Q_{j+1}(z) = \sum_{k=0}^{\infty} z^{k+U(k)} \; P\left[\alpha_j = k \right] \qquad -(2.7)$$

expanding using Eqn 2.3,

$$Q_{j+1}(z) = z\,P\left[\alpha_j = 0 \right] + \left[\frac{\mu z^{-1}}{\lambda+\mu} + \frac{\lambda z}{\lambda+\mu} \right] \sum_{k=1}^{\infty} z^k \, P\left[\alpha_j = k \right] \qquad -(2.8)$$

then

$$Q_{j+1}(z) = z\,P\left[\alpha_j = 0 \right] + \frac{z\rho + z^{-1}}{1+\rho} \left[Q_j(z) - P\left[\alpha_j = 0 \right] \right] \qquad -(2.9)$$

$$\text{-where } \rho = \frac{\lambda}{\mu}$$

Let us define:

$$Q(z) = \lim_{j\to\infty} Q_j(z) \;, \quad \text{and} \quad P(\alpha = k) = \lim_{j\to\infty} P(\alpha_j = k)$$

then

$$Q(z) = \frac{P(\alpha = 0) \left[z - \dfrac{\rho z + z^{-1}}{1+\rho} \right]}{1 - \dfrac{\rho z + z^{-1}}{1+\rho}} \qquad -(2.10)$$

$$Q(z) = \frac{P(\alpha = 0) \left[(1+\rho)\,z - (\rho z + z^{-1}) \right]}{(1+\rho) - (\rho z + z^{-1})} \qquad -(2.11)$$

which reduces to:

$$Q(z) = \frac{P(\alpha = 0)\left[\, z - z^{-1} \,\right]}{(1 - \rho z) + (\rho - z^{-1})} \qquad -(2.12)$$

multiplying by z at the numerator and the denominator,

$$Q(z) = \frac{P(\alpha = 0)\left[\, z^{2} - 1 \,\right]}{z\,(1 - \rho z) + (\rho z - 1)} \qquad -(2.13)$$

factoring the denominator,

$$Q(z) = \frac{P(\alpha = 0)\left[\, z^{2} - 1 \,\right]}{(1 - \rho z)\quad (z - 1)} \qquad -(2.14)$$

which reduces to:

$$Q(z) = \frac{P(\alpha = 0)\quad (z + 1)}{1 - \rho z} \qquad -(2.15)$$

and finally using $Q(1) = 1$ we get $P(\alpha = 0) = (1 - \rho)/2$. Then,

$$Q(z) = \frac{1}{2}\,\frac{1 - \rho}{1 - \rho z} + \frac{1}{2}\,\frac{(1 - \rho)\, z}{1 - \rho z} \qquad -(2.16)$$

Then we apply inverse "z" transforms and obtain:

$$P(\alpha = k) = \frac{1}{2}\,(1 - \rho)\,\rho^{k} + \frac{1}{2}\,(1 - \rho)\,\rho^{k-1} \quad k \geq 0 \qquad -(2.17)$$

-where negative powers of ρ are defined to be zero.

This is a rather unexpected result as one would have thought that this distribution of the number of messages in the M/M/1 queue would have been geometric. This alone, justifies the investigation of our embedding strategy which generates this unexpected result.

Then, except for the $\frac{1}{2}$ factor, the first and second terms of Eqn 2.17 correspond to the distributions of the number of messages in the source queue following a departure and an arrival respectively, we then write:

$$p_{di} = (1 - \rho)\,\rho^{i}\ , \quad \text{and} \quad p_{ai} = (1 - \rho)\,\rho^{i-1} \qquad -(2.18)$$

We see that the distribution following a departure is geometric, while the distribution following an arrival is shifted by one to account for the newly arrived message.

As for the $\frac{1}{2}$ factor, it corresponds to the weighting factor. This value is also in agreement with the observation that arrivals and departures are equally likely over a long interval (each arriving message will eventually depart if queue size is not to grow to infinity).

It is clear that the steady state probabilities of the number of messages in the M/M/1 queue are not valid in this distribution (Eqn 2.17). In fact, the classical approach to solving an M/M/1 system gives a geometric distribution for the number of messages in the queue at steady state. We must then apply a transformation to our results to obtain the correct distribution.

Since the problem is caused by the distribution following an arrival, p_{a_i} $0 < i < N$, which is not geometric, we will apply our transformation in order to obtain the distribution of the number of messages in the M/M/1 queue which is seen by the embedded points following only a departure, p_{d_i} $0 < i < N$. In fact, this distribution is already geometric.

Our more complicated analysis of the 2x2 Switch will also generate, from the solution of the transition matrix, a weighted combination of the distributions of the number of messages in both source queues after arrivals and departures. We will then require a transformation to obtain the distribution following a departure.

We will now tackle the analysis of the Star Network itself.

2.7 The analysis

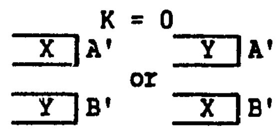
In this analysis each destination queue is to be modeled as a server of exponentially distributed messages. The object will be to obtain all the transition probabilities and then solve the resulting probability transition matrix for its steady state solution. This will give a preliminary distribution of the number of messages in the system after an embedded point. Following a transformation the distribution after a departure will be determined as explained above. Applying Little's result will then yield the average message delay.

Next, we are going to identify variables required to determine completely the state of the system for Markov chain analysis. Clearly, the number of messages in each source queue is needed, but also one has to know the status of the destination queues. Not only do we need the number of messages in the destination queues but their respective source as well.

The system state may be represented by a vector of three state variables: $\left[n_A, n_B, K \right]$, where n_A and n_B are the number of messages in source queues A and B respectively and where K determines the state of the destination queues. The transition probabilities will then be of the form $P(n_{A1},n_{B1},K_1;n_{A2},n_{B2},K_2)$.

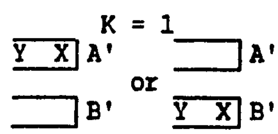
We must examine the cases which define the use of the state variable K used to handle the interdependence between the two queues. Let us define X and Y as the transmission times (length) of messages from source queues A and B respectively. Then, at any embedding point there is a maximum of 2 messages in the destination queues which are the call requests corresponding to the head of line messages from source queues A and B. Depending on their destination there are 3 possible cases depicted in Fig.2.10:

- No conflict:

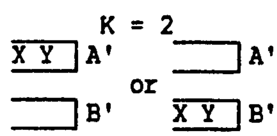
1- 

both messages are in different destination queues

- Conflicts:

2- 

both messages are in the same destination queue and the message from source queue A is in front of the one from source queue B

3- 

both messages are in the same destination queue and the message from source queue B is in front of the one from source queue A

Fig. 2.10 Possible values of state variable K.

In case 1 messages do not interfere with each other but in cases 2 and 3 the latter message must wait for the first to be transmitted until it can be transmitted itself. Note that if one or both of the destination queues are empty, K will be defined as zero.

Now since embedding is at both arrival and departure points we can only have a single arrival or departure making this analysis analogous to a birth-death process analysis. As was mentioned earlier, the simplicity of our state transition equations provides a fairly straightforward probability transition matrix. Let us then derive these state transition probabilities.

Obviously, many separate situations must be described. First, we must realize that depending on the present system state, the next embedding point may correspond to only one of four possible events, namely: arrival at A, arrival at B, departure from A', or departure from B'. In some states, some of these events will be impossible. Then, we will define the total arrival and departure rates with respect to the present system state.

We recall that if source queue A or B is full, the respective arrival process is disabled, meaning that λ_A or λ_B is set to zero. Then the total arrival rate, noted λ_T, depends on the state of the source queues, and is given by:

$$\lambda_T = \begin{cases} 0 & \text{if both source queues are full,} \\ \lambda_A & \text{if source queue B only is full,} \\ \lambda_B & \text{if source queue A only is full, or} \\ \lambda_A + \lambda_B & \text{if both source queues are non-full.} \end{cases}$$

-(2.19)

As for the total departure rate, noted μ_T, we must use state variable K, describing the state of the destination queues, to properly handle every case. We then write:

$$\mu_T = \begin{cases} 0 & \text{if } K_1=0 \text{ and both source queues are empty,} \\ \mu_A & \text{if } \begin{cases} K_1=0 \text{ and source queue A is non-empty,} \\ \text{or} \\ K_1 = 1, \end{cases} \\ \mu_B & \text{if } \begin{cases} K_1=0 \text{ and source queue B is non-empty,} \\ \text{or} \\ K_1 = 2, \end{cases} \\ \mu_A+\mu_B & \text{if } K1=0 \text{ and both source queues are non-empty.} \end{cases}$$

-(2.20)

These last two equations allow us to obtain the state transition probabilities. First defining N as the maximum source queue size we can get the probabilities that the next event will be an arrival at A or B from:

$$P\left[\text{arrival at A} \mid n_{A1} < N \right] = \frac{\lambda_A}{\lambda_T + \mu_T} \qquad -(2.21)$$

and

$$P\left[\text{arrival at B} \mid n_{B1} < N \right] = \frac{\lambda_B}{\lambda_T + \mu_T} \qquad -(2.22)$$

where λ_T and μ_T are given by Eqns 2.19 and 2.20

Similarly, we obtain the probabilities that the next event will be a departure from A' or B'. These are written:

$$P\left[\text{departure from A'} \mid (K_1=1) \text{ or } (K_1=0 \text{ and } n_{A1}>0) \right] = \frac{\mu_A}{\lambda_T + \mu_T}$$

-(2.23)

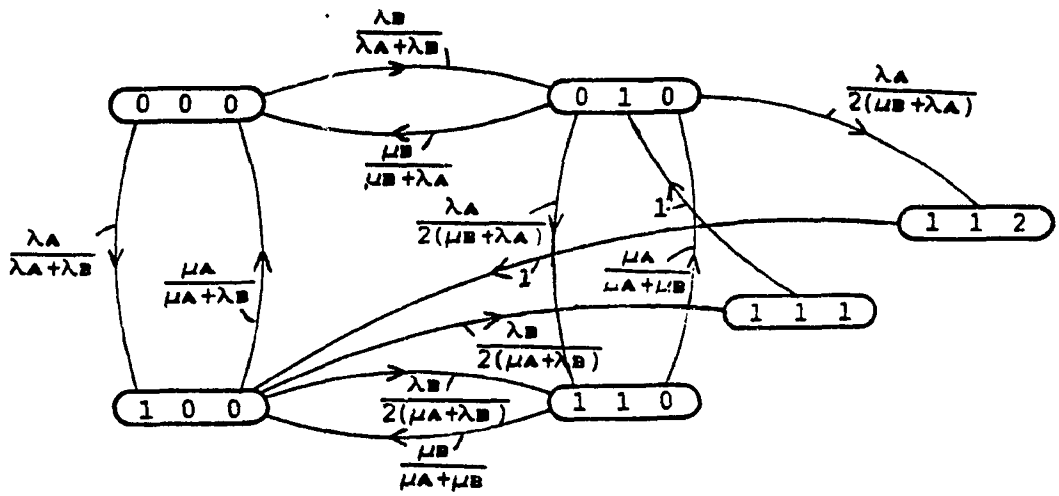
and

$$P\left[\text{departure from B'} \mid (K_1=2) \text{ or } (K_1=0 \text{ and } n_{B1}>0) \right] = \frac{\mu_B}{\lambda_T + \mu_T}$$

-(2.24)

The following example, which represents the first non-trivial case involving source queues of size N = 1, shows the simplicity of the probability transition matrix generated by the above equations.

The state transition diagram, where again each state is defined as $\left[n_A, n_B, K \right]$ is shown in Fig.2.11.



where (nA nB K) represent each possible state.

Fig. 2.11 State transition diagram of the first non-trivial case with N = 1.

From the transition probabilities between each state we obtain the probability transition matrix $\mathbb{P}$, for our example, in Eqn 2.26. In order to represent on paper the six-dimensional transition matrix, possible states are renumbered using the relation in Eqn 2.25:

$$j = 2Nn_A + Nn_B + K) \qquad -(2.25)$$

$$\mathbb{P} = \begin{array}{c} \\ \text{From} \\ 000 \\ 010 \\ 100 \\ 110 \\ 111 \\ 112 \end{array} \begin{array}{c} \begin{array}{cccccc} \text{To } 000 & 010 & 100 & 110 & 111 & 112 \end{array} \\ \begin{bmatrix} 0 & \frac{\lambda_B}{\lambda_A+\lambda_B} & \frac{\lambda_A}{\lambda_A+\lambda_B} & 0 & 0 & 0 \\ \frac{\mu_B}{\mu_B+\lambda_A} & 0 & 0 & \frac{\lambda_A}{2(\mu_B+\lambda_A)} & 0 & \frac{\lambda_A}{2(\mu_B+\lambda_A)} \\ \frac{\mu_A}{\mu_A+\lambda_B} & 0 & 0 & \frac{\lambda_B}{2(\mu_A+\lambda_B)} & \frac{\lambda_B}{2(\mu_A+\lambda_B)} & 0 \\ 0 & \frac{\mu_A}{\mu_A+\mu_B} & \frac{\mu_B}{\mu_A+\mu_B} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad -(2.26)$$

The inherent simplicity of the probability transition matrix is again obvious. We then solve the equilibrium equation in Eqn 2.27:

$$q' = q' \, \mathbb{P} \qquad -(2.27)$$

where $q' = \left[q'_0 , \; q'_1 , \; \ldots , \; q'_M \right]$

and M+1 is the number of possible states (in our example M+1 = 6)

This gives the steady state probabilities of being in each state. These probabilities are then returned to their three dimensional states using the reverse of Eqn 2.25. Removing the dependency on state variable K by taking the summation in Eqn 2.28 yields the joint probabilities of having n_A and n_B messages in each source queue.

$$q'(n_A, n_B) = \sum_{K=0}^{2} q'(n_A, n_B, K) \qquad -(2.28)$$

Finally, the marginal probabilities of having i messages in source queue A, noted q'_{Ai}, is given by:

$$q'_{Ai} = \sum_{n_B=0}^{N} q'(i, n_B) \quad , \qquad 0 \le i \le N \qquad -(2.29a)$$

and similarly for source queue B probabilities, noted q'_{Bi}:

$$q'_{Bi} = \sum_{n_A=0}^{N} q'(n_A, i) \quad , \qquad 0 \le i \le N \qquad -(2.29b)$$

It is important to remember that the probabilities obtained in Eqns 2.29a and 2.29b are weighted combinations of the four distributions seen by each set of embedded points (following arrivals at A, following arrivals at B, following departures from A', and following departures from B'). The transformation developed in the next section will be needed to obtain the distributions following departures from the respective queues.

Now, the mathematical development of this transformation uses infinite source queues as a starting point. We must then provide a relation between our finite buffer probabilities (q'_{Ai} and q'_{Bi}), and the infinite buffer probabilities, noted q_{Ai} and q_{Bi}.

In section 2.6, where we analyzed a simple M/M/1 queue, this relation involves a one-dimensional truncation. Fig. 2.12 shows an example when an infinite M/M/1 queue is truncated to N=2.

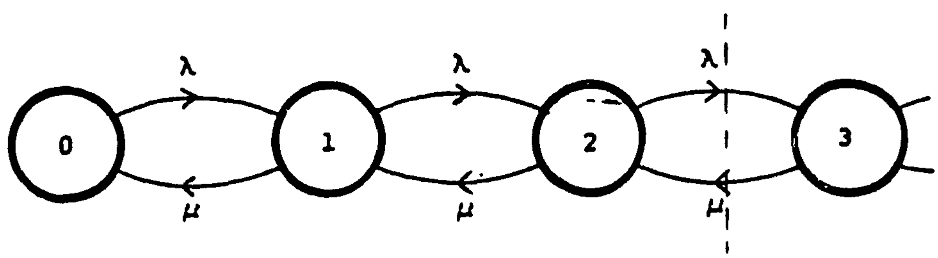


Fig. 2.12 Infinite M/M/1 queue truncated to N=2.

It is easily shown that these finite queue probabilities (q'_i) and infinite queue probabilities (q_i) would be exactly related by:

$$q'_i = \frac{q_i}{\sum_{j=0}^{N} q_j} \qquad -(2.30)$$

In our case here, where both source queues interfere with each other, this is not as straightforward. An extension of the state transition diagram of Fig.2.11 would clearly show the intricate pattern of possible transitions that would intersect a truncation to finite buffer size. The development of the exact relation between finite and infinite buffer probabilities would be quite involved. We will then approximate this relation using Eqn 2.30.

$$q'_{Ai} = \frac{q_{Ai}}{\sum_{j=0}^{N} q_{Aj}} \quad \text{and} \quad q'_{Bi} = \frac{q_{Bi}}{\sum_{j=0}^{N} q_{Bj}}, \text{ for } 0 \le i \le N. \qquad -(2.31)$$

where N is the maximum buffer size.

Note that results have shown that as N grows, this approximation becomes more and more valid. This is due to boundary effects, which are amplified when N is too small.

The next section will transform the weighted distributions obtained in Eqns 2.29a and 2.29b to provide the needed distribution of the number of messages in the source queues following departures.

2.8 The steady state distribution of the number of messages in each queue following departure points

In order to develop the needed transformation, we must establish a few relations between the different distributions arising from the four sets of embedding points in our strategy. Those four distributions stem from the embedding points immediately following:

i) a departure from A',

ii) an arrival to A,

iii) a departure from B',

iv) an arrival to B.

We repeat again that the need for a transformation arises from our embedding after both arrivals and after departures to and from the system.

We have stated before that it can be shown [3] that, taken separately with their respective embedding strategy, the limiting distribution of the number of messages in the system immediately following a departure is equal to the same distribution immediately before an arrival for any system that changes state by unit step values (positive or negative). But also this distribution is equal to the distribution seen by any random observer of the system.

We remind the reader that, in this analysis we refer to arrivals to source queues A or B as arrivals to A or B and departures from destination queues A' or B' as departures from A' or B'.

Now, it would be possible for us to obtain the joint distribution of the number of messages in both source queues following departures using our transformation, but this would make the development of this transformation quite difficult.

We shall then develop a transformation acting only on the marginal distributions q'_{Ai} and q'_{Bi} $0 < i < N$, obtained in Section 2.7. Since the system is symmetrical in operation, we will only examine the behavior of source queue A and extend the results to source queue B. Then let us define the probabilities of having "i" messages in source queue A immediately following:

i) a departure from A' as p_{di},

ii) an arrival to A as p_{ai},

iii) a departure from B' as $\tilde{p}_{di}$,

iv) an arrival to B as $\tilde{p}_{ai}$.

We will now express probabilities in ii), iii), and iv) in terms of those in i). Then using similar arguments as in the previous section we can relate arrivals to A (ii) and departures from A' (i) using Eqn 2.1 giving:

$$p_{ai} = p_{d(i-1)} \qquad i > 0 \qquad -(2.32)$$

Now, because the arrival processes are independent of the departure processes, the arrivals to B act as random observers of source queue A allowing us to relate these arrivals to B (iv) to the departures from A' (i) with:

$$\tilde{p}_{ai} = p_{di} \qquad i > 0 \qquad -(2.33)$$

But because of the interference between the two destination queues, departures from B' are not quite random with respect to departures from A'. In order to simplify the analysis we will assume that departures from B' do act as random observers of source queue A and we then relate these departures from B' to departures from A' using:

$$\tilde{p}_{di} = p_{di} \qquad i > 0 \qquad -(2.34)$$

Then in summary,

$$p_{ai} = p_{d(i-1)} \qquad i > 0 \qquad -(2.35a)$$

and,

$$p_{di} = \tilde{p}_{di} = \tilde{p}_{ai} \qquad i > 0 \qquad -(2.35b)$$

We see here that the distribution after an arrival to A is the distribution after a departure from A' shifted by plus one. We also note that the other two distributions are equal to this distribution after a departure from A'. The transformation to be developed here will then provide the distribution of the number of messages in queue A following departures from queue A'.

First we shall assume infinite buffer (infinite source queue size). Let us observe the system for a long interval and make the following definitions:

n_{aA}, n_{dA} total number of arrivals and departures to and from source queue A in this interval,

n_{aB}, n_{dB} total number of arrivals and departures to and from source queue B in this interval,

clearly:

$$n_{dA} = n_{aA} \quad \text{and} \quad n_{dB} = n_{aB} \qquad -(2.36)$$

-whenever the queues are stable.

n_{ai}, n_{di} number of times i messages have been observed in source queue A following an arrival and a departure to and from queue A respectively in this interval,

p_{ai}, p_{di} probabilities of the above events,

Then, from the definition of probabilities,

$$p_{ai} = \lim_{n_{aA}\to\infty} \frac{n_{ai}}{n_{aA}}, \quad p_{di} = \lim_{n_{dA}\to\infty} \frac{n_{di}}{n_{dA}}, \quad i = 0,\ldots\infty \qquad -(2.37)$$

$\tilde{n}_{ai}$, $\tilde{n}_{di}$ number of times i messages have been observed in source queue A following an arrival and a departure to and from queue B respectively in this interval,

$\tilde{p}_{ai}$, $\tilde{p}_{di}$ probabilities of the above events,

And again,

$$\tilde{p}_{ai} = \lim_{n_{aB}\to\infty} \frac{\tilde{n}_{ai}}{n_{aB}}, \quad \tilde{p}_{di} = \lim_{n_{dB}\to\infty} \frac{\tilde{n}_{di}}{n_{dB}}, \quad i = 0,\ldots\infty \qquad -(2.38)$$

We can relate the number of departures from each queue to their respective mean arrival rates with:

$$\frac{n_{dB}}{n_{dA}} = \frac{\lambda_B T}{\lambda_A T}$$

where T is the interval length under observation

then

$$n_{dB} = \frac{\lambda_B}{\lambda_A} n_{dA} \qquad -(2.39)$$

Now let us relate the probability distributions p_{ai}, p_{di}, $\tilde{p}_{ai}$ and $\tilde{p}_{di}$ to the distribution derived in section 2.7. From the definition of probabilities, the infinite buffer probability of having i messages in source queue A following an embedded point is given by:

$$q_{Ai} = \lim_{\text{total number of observations} \to \infty} \frac{\text{number of observations of i messages in source queue A}}{\text{total number of observations}} \qquad -(2.40)$$

then,

$$q_{Ai} = \lim_{n_{dA}, n_{dB}, n_{aA}, n_{aB} \to \infty} \frac{n_{ai} + n_{di} + \tilde{n}_{ai} + \tilde{n}_{di}}{n_{dA} + n_{dB} + n_{aA} + n_{aB}} \qquad i > 0 \qquad -(2.41)$$

where the numerator in Eqn 2.41 is the total of the number of times an arrival to queue A saw i messages in queue A (n_{ai}), the number of times a departure from queue A' saw i messages in queue A (n_{di}) and the number of times an arrival to queue B and a departure from queue B' saw i messages in queue A ($\tilde{n}_{ai}$ + $\tilde{n}_{di}$). The denominator is simply the sum of all observations of the four types of events.

And from Eqn 2.36:

$$q_{Ai} = \lim_{n_{dA}, n_{dB} \to \infty} \frac{n_{ai} + n_{di} + \tilde{n}_{ai} + \tilde{n}_{di}}{2(n_{dA} + n_{dB})} \qquad i > 0 \qquad -(2.42)$$

Then,

$$q_{Ai} = \lim_{n_{dA}, n_{dB} \to \infty} \left[\frac{n_{ai} + n_{di}}{2(\ n_{dA} + n_{dB}\ )} + \frac{\tilde{n}_{ai} + \tilde{n}_{di}}{2(\ n_{dA} + n_{dB}\ )} \right] \quad i > 0$$

-(2.43)

Factoring n_{dA} and n_{dB} respectively in the denominators,

$$q_{Ai} = \lim_{n_{dA}, n_{dB} \to \infty} \left[\frac{n_{ai} + n_{di}}{2n_{dA}\left(1 + \frac{n_{dB}}{n_{dA}} \right)} + \frac{\tilde{n}_{ai} + \tilde{n}_{di}}{2n_{dB}\left(\frac{n_{dA}}{n_{dB}} + 1 \right)} \right] \quad i > 0$$

-(2.44)

taking the limits using Eqns 2.37 and 2.38,

$$q_{Ai} = \frac{p_{ai} + p_{di}}{2\left(1 + \frac{n_{dB}}{n_{dA}} \right)} + \frac{\tilde{p}_{ai} + \tilde{p}_{di}}{2\left(\frac{n_{dA}}{n_{dB}} + 1 \right)} \quad i > 0$$

-(2.45)

using Eqns 2.35a and 2.35b to get the right side in terms of "p_{di}", we get:

$$q_{Ai} = \frac{p_{d(i-1)} + p_{di}}{2\left(1 + \frac{n_{dB}}{n_{dA}} \right)} + \frac{2p_{di}}{2\left(\frac{n_{dA}}{n_{dB}} + 1 \right)} \quad i > 0$$

and from Eqn 2.39,

$$q_{Ai} = \frac{p_{d(i-1)} + p_{di}}{2\left(1 + \frac{\lambda_B}{\lambda_A}\right)} + \frac{2\frac{\lambda_B}{\lambda_A} p_{di}}{2\left(\frac{\lambda_B}{\lambda_A} + 1\right)} \qquad i > 0$$

Finally, replacing the unneeded "d" subscript from the "p_{di}" with an "A" subscript to show that these are the sought probabilities of having i messages <u>in queue A</u>, we get:

$$q_{Ai} = \frac{p_{A(i-1)} + \left(1 + 2\frac{\lambda_B}{\lambda_A}\right)p_{Ai}}{2\left(1 + \frac{\lambda_B}{\lambda_A}\right)} \qquad i > 0 \qquad -(2.46)$$

Eqn 2.46 relates the distribution of the number of messages in queue A with an infinite buffer following an embedding point (q_{Ai}) to the same distribution following a departure from A' (p_{di} or now p_{Ai}).

Now, using Eqn 2.31 to return to finite buffer results, we get:

$$q'_{Ai} = \frac{p_{A(i-1)} + \left(1 + 2\frac{\lambda_B}{\lambda_A}\right)p_{Ai}}{2\left(1 + \frac{\lambda_B}{\lambda_A}\right) \sum_{j=0}^{N} q_{Aj}} \qquad 1 \le i \le N \qquad -(2.47)$$

Clearly, the boundary cases (i=0 and i=N) must be handled separately. As mentioned earlier, an arrival to queue A will never see queue A as empty meaning that in Eqn 2.41 $n_{ao} = 0$. Following the same steps used to arrive at the general case, we get:

$$q'_{AO} = \frac{\left(1 + 2\frac{\lambda_B}{\lambda_A}\right) p_{AO}}{2\left(1 + \frac{\lambda_B}{\lambda_A}\right) \sum_{j=0}^{N} q_{Aj}} \qquad -(2.48)$$

Also, when queue is full (i=N) a departure from this queue will never see a full queue, meaning that in Eqn 2.41 $n_{dN}=0$. Again following the same steps as in the general case, we obtain:

$$q'_{AN} = \frac{p_{A(N-1)} + 2\frac{\lambda_B}{\lambda_A} p_{AN}}{2\left(1 + \frac{\lambda_B}{\lambda_A}\right) \sum_{j=0}^{N} q_{Aj}} \qquad -(2.49)$$

Then, in summary, the transformation relating the distribution found in Section 2.7 (distribution of the number of messages in finite queue A following an embedded point), to the distribution of the number of messages in finite queue A following a departure from queue A' is:

$$q'_{A0} = \frac{\left[1 + 2\frac{\lambda_B}{\lambda_A}\right] p_{A0}}{2\left[1 + \frac{\lambda_B}{\lambda_A}\right] \sum_{j=0}^{N} q_{Aj}} \qquad -(2.50a)$$

$$q'_{Ai} = \frac{p_{A(i-1)} + \left[1 + 2\frac{\lambda_B}{\lambda_A}\right] p_{Ai}}{2\left[1 + \frac{\lambda_B}{\lambda_A}\right] \sum_{j=0}^{N} q_{Aj}} \qquad 1 \le i \le N \qquad -(2.50b)$$

$$q'_{AN} = \frac{p_{A(N-1)} + 2\frac{\lambda_B}{\lambda_A} p_{AN}}{2\left[1 + \frac{\lambda_B}{\lambda_A}\right] \sum_{j=0}^{N} q_{Aj}} \qquad -(2.50c)$$

-where $\sum_{j=0}^{N} q_{Aj}$ is also a function of p_{Ai} from Eqn 2.46.

-and where q_{Ai} $0 < i < N$ are known

-- and conversely for queue B probabilities.

Eqns 2.50a, 2.50b and 2.50c form a set of N+1 linear equations (where unknowns are "p_{Ai}"), which, when solved give the probability distribution of the number of messages in finite queue A. The queue B distribution is obtained using exactly the same approach.

It is important to realize here that the distributions obtained, p_{Ai} $0 \leq i \leq N$, and p_{Bi} $0 \leq i \leq N$, are those seen immediately following departures from A' and B' respectively and for finite buffers. Infinite buffer probabilities were used only as a means towards this objective.

Taking the expectation from 0 to N, using Eqn 2.51 gives the average number of messages in both source queues.

$$\overline{n_A} = \sum_{i=0}^{N} i \, p_{Ai} \text{ , and } \overline{n_B} = \sum_{i=0}^{N} i \, p_{Bi} \qquad -(2.51)$$

Then using Little's result adapted to finite queues of Eqn 2.52 gives the average delay.

$$\overline{D} = \frac{\overline{n}}{\lambda \, (1 - P_b)} \qquad -(2.52)$$

- where P_b is the blocking probability equal to the probability of having a full source queue (p_{AN} or p_{BN} respectively),
- and where $\overline{n}$ and λ are $\overline{n_A}$ and λ_A or $\overline{n_B}$ and λ_B respectively for each queue.

2.9 Results

Note that for results presented here, mean transmission (service) rate is 2.5 messages/msec for a mean message transmission time of 0.4 ms.

The first five plots will give results for the case where both destinations are equally likely ($G_{AA} = G_{AB}$, $G_{BA} = G_{BB}$) for each message. On the other hand, the last plot will examine the case where messages will choose destination A with a greater probability than destination B ($G_{AA} > G_{AB}$, $G_{BA} > G_{BB}$).

We shall present steady state probability distribution of the number of messages in the input buffers, average delay and blocking probability for symmetric and non-symmetric arrivals and variable length messages for both cases.

Then the first plot represents mean delay with respect to total mean arrival rate for symmetric traffic and is shown in Fig.2.13. This graph is somewhat misleading. Looking at λ_T = 5 mess/msec we could believe that delay grows as N grows, which is not plausible. In fact, this graph does not account for a few particularities of the system.

First, we recall that for this finite buffer analysis, messages arriving to a full buffer are lost, meaning that average delay may increase from N = 2 to N = 10 but, for the same total mean arrival rate, many more messages are lost with a smaller queue size.

Second, finite buffer analysis are really truncations of infinite buffer analysis. Then let us assume that the system is not overloaded or that the utilization factor (which is hard to visualize here because of the interfering queues), is less than 1. Then increasing maximum queue size (N), would allow the average delay to eventually taper-off and stabilize at a finite value (with minimal number of lost messages).

What must be seen from this graph is simply that delay grows with mean arrival rate and maximum buffer size. This demonstrates the dependence of the two input queues which interfere with each other's transmissions.

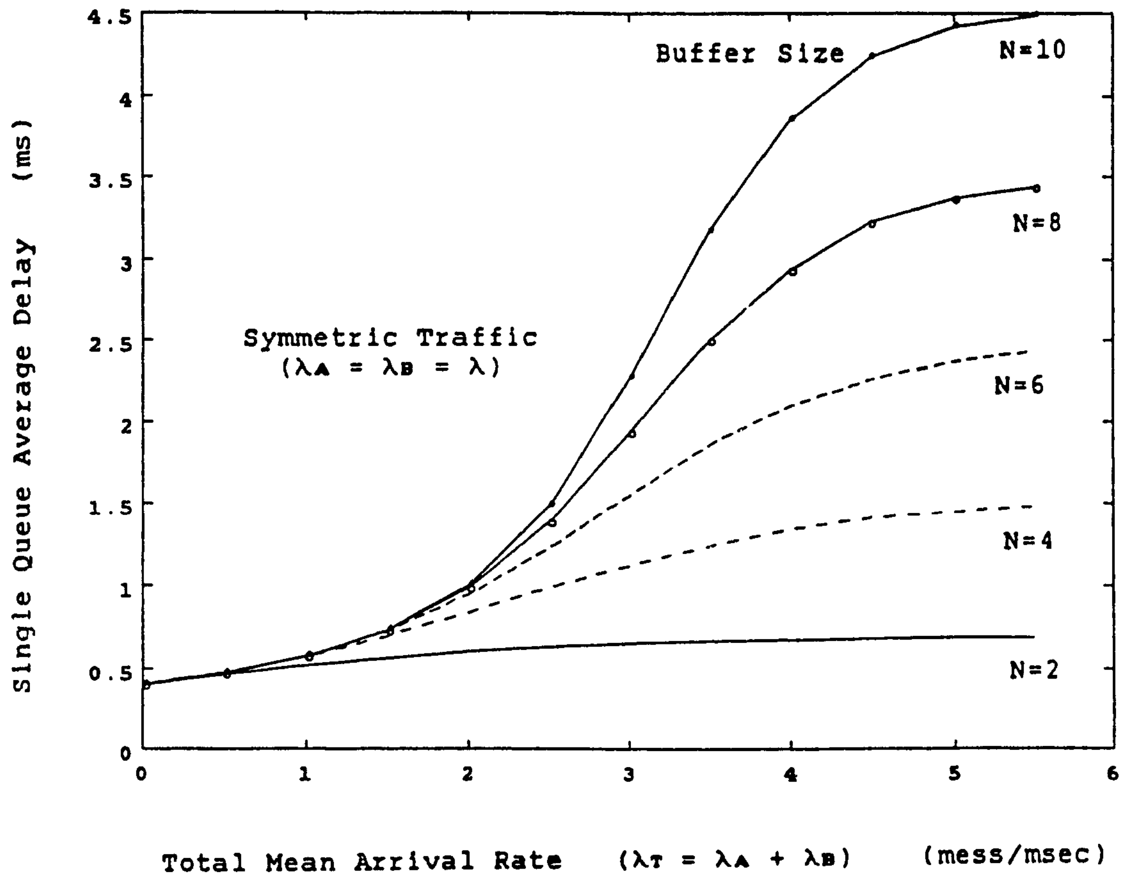


Fig. 2.13 Mean message delay against total mean arrival rate for symmetric traffic and mean message duration of 400 μs.

In Fig.2.14 the increasing delay for the more heavily loaded queue, queue A, is demonstrated in non-symmetric traffic. We then plot mean delay with respect to queue A mean arrival rate for different load ratios between queue A and queue B and with a maximum buffer size of 8 messages. As the arrival rate of queue B decreases we see that message delay for queue A also decreases due to less interference. When $\lambda_B = \lambda_A/100$ we notice that queue A behaves like an M/M/1 queue since transmission interferences between both queues are practically eliminated by the near zero load on queue B.

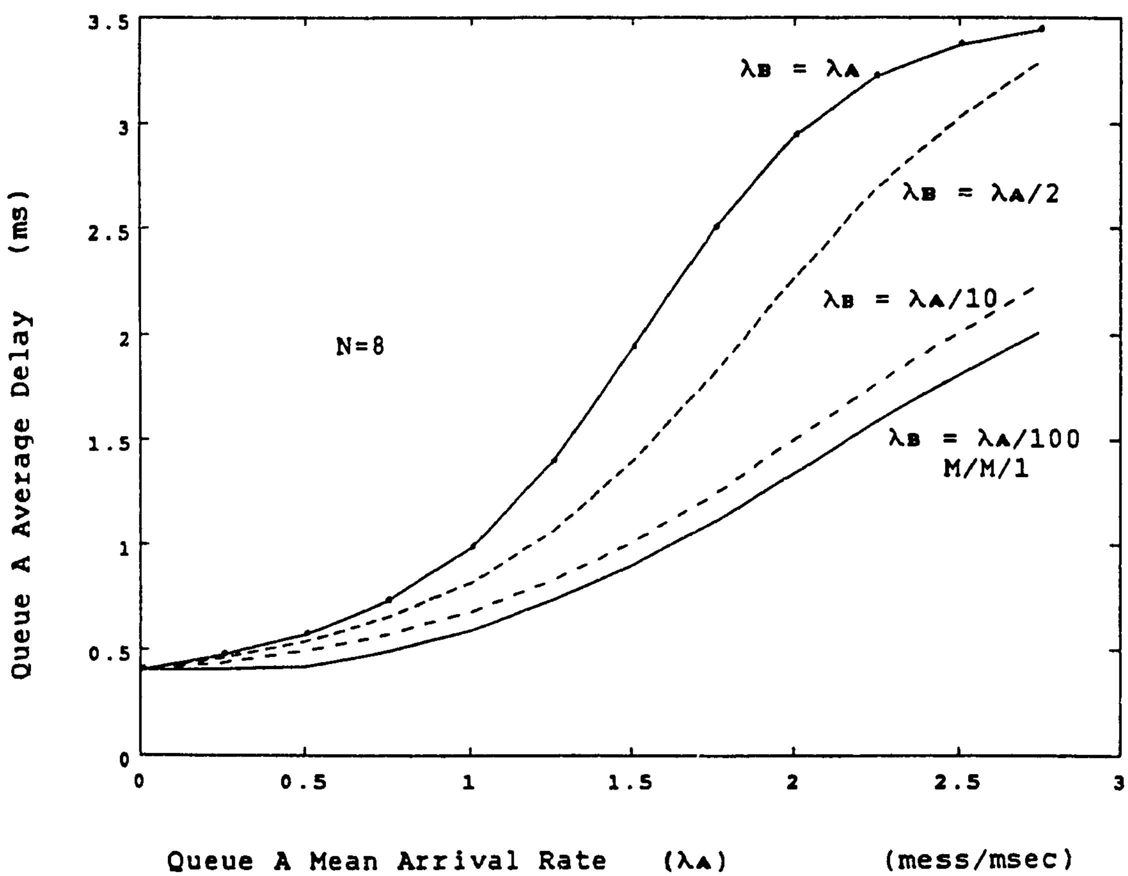


Fig. 2.14 Mean message delay of more heavily loaded queue A against queue A mean arrival rate in symmetric and non-symmetric traffic.

We can also examine Fig.2.15 which represents the average number of messages in queue A for the same load ratios between both queues and maximum buffer size of 8 messages. It still indicates that as the load on queue B increases the frequency of the transmission interferences between both queues is multiplied which makes the buffer occupancy of queue A grow accordingly.

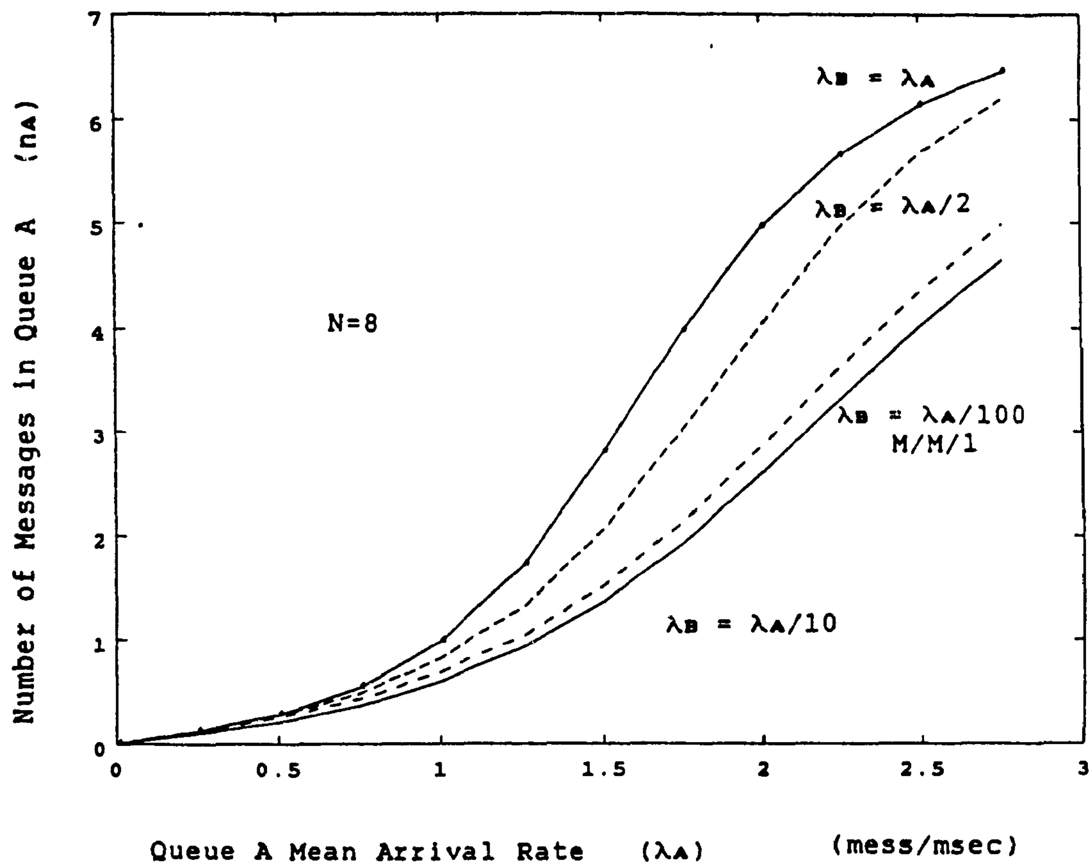


Fig. 2.15 Average number of messages in more heavily loaded queue A against queue A mean arrival rate in symmetric and non-symmetric traffic.

Fig.2.16 represents the probability distribution of the number of messages in both queues in symmetric traffic, for maximum buffer size of 10 messages, and for two different total mean arrival rates (λ_A and λ_B being equal). We clearly see the low queue occupancy at λ_T = .5 mess/msec. When the total mean arrival rate is increased to λ_T = 4.0 mess/msec, the distribution changes showing greater probabilities for the higher number of messages in the queues.

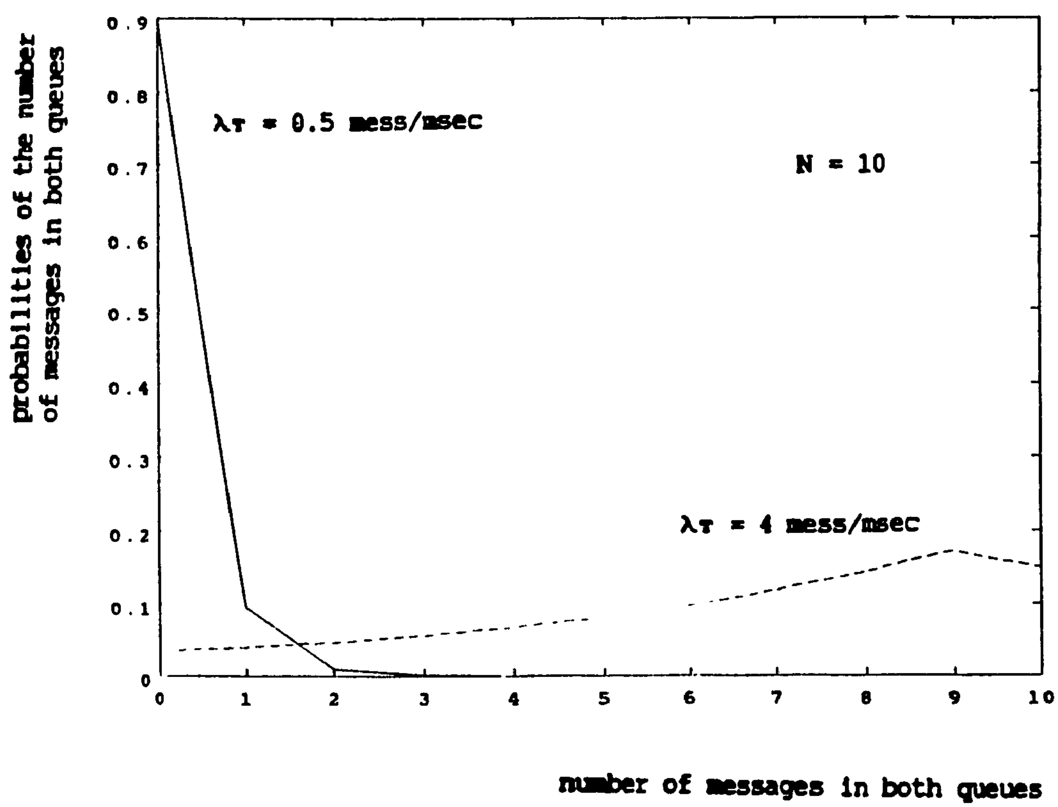


Fig. 2.16 Distribution of the number of messages in both queues in symmetric traffic, for two different values of total mean arrival rate.

We have plotted in Fig.2.17 the blocking probability in symmetric traffic with respect to single queue mean arrival rate for different maximum buffer sizes. This graph clearly shows that as maximum buffer size increases the blocking probability decreases.

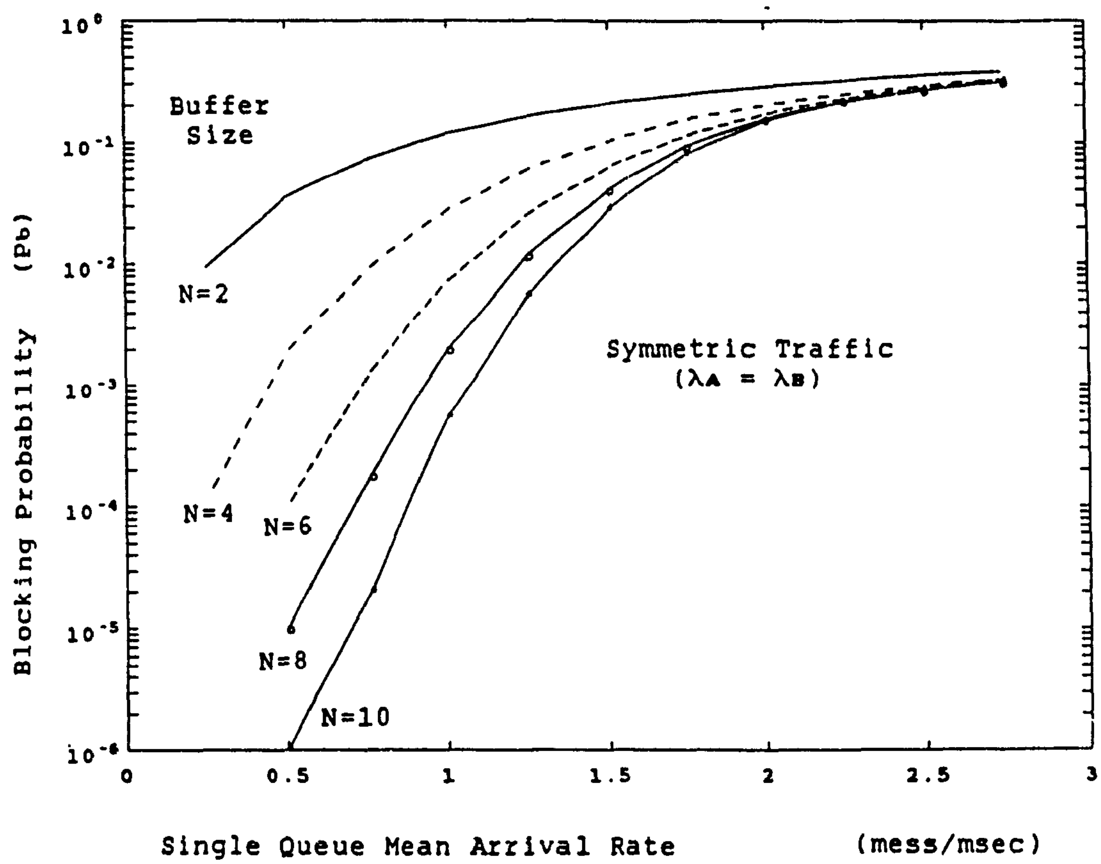


Fig. 2.17 Blocking probability against single queue mean arrival rate in symmetric traffic.

Finally we examine the cases where messages choose destination A with a probability that is twice, five times and ten times as large as destination B. We compare these with the equally likely destinations case in symmetric traffic ($\lambda_A = \lambda_B$) for maximum buffer size of 10 messages.

Then, Fig.2.18 shows that as messages favor destination A with a growing probability, the transmission delay of messages from both source queues also grows because more and more messages are sent to the same destination, queue A, increasing the frequency of interferences between the two source queues.

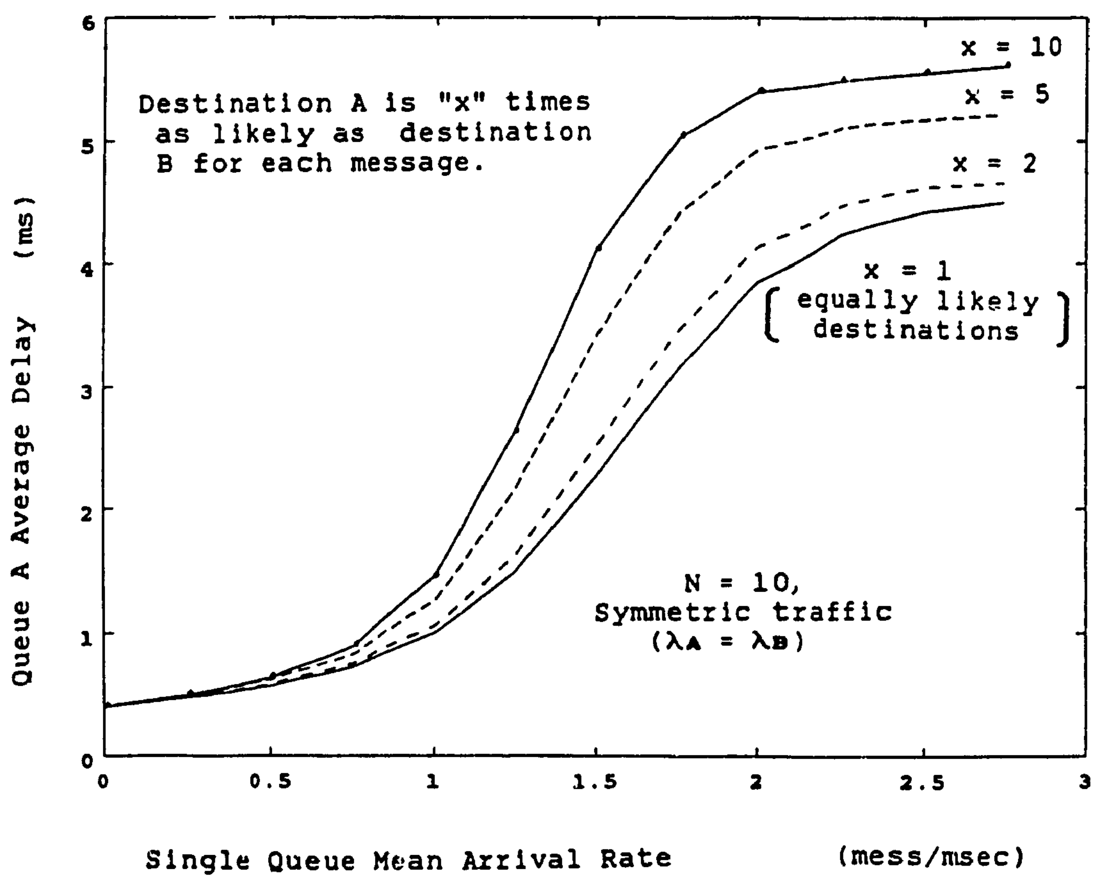


Fig. 2.18 Mean message delay of more probable destination queue A against single queue mean arrival rate in symmetric traffic.

CHAPTER THREE

ANALYSIS OF A 2x2 BANYAN SWITCH

3.1 Introduction

Fast Packet Switching has been proposed as a major architecture in the BISDN [14]. The implementation of this architecture requires a wideband packet switch. Among the switches under consideration for this is the Banyan Network which consists of stages of 2x2 Crossbar switches (see Fig.3.1) [11].

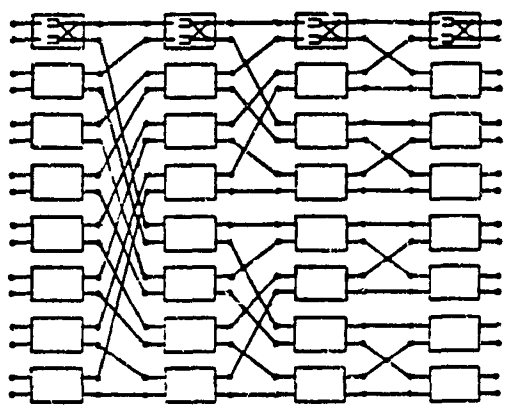


Fig. 3.1. A 4-stage Banyan network

This Chapter will present a performance analysis of a finite buffered 2x2 Banyan Switch using the same approach developed in Chapter 2. Even though much of this previous analysis would apply directly in the Fast Packet Switching environment, we will adapt it using the fundamentally different terminology and constraints found in Banyan Networks.

Then, this Chapter will introduce the Banyan Network system and the model used in its analysis. Since the assumptions made here differ somewhat from those made in the Star LAN analysis, an outline of all pertinent assumptions will also be given.

As for the analysis itself, there is no need to repeat the entire development of the equations. Sections 2.5 to 2.8 inclusively details this analysis. We will simply give an outline of the major steps followed.

Banyan networks are flexible, modular and can be extended to any number of inputs and outputs. At the inputs of each 2x2 Banyan (Crossbar) switch, buffers have been provided. A packet at the input of a Banyan switch can be moved to the next stage if there is room available in the input buffer of the next stage [15]. Then the input buffer of the next stage can be considered as an output buffer to the present Banyan switch.

In this paper we present an analysis of a 2x2 Banyan switch with finite input buffers and infinite output buffers.

The analysis is carried out using a Markov chain approach. The points immediately following an arrival and a departure to and from the switch are chosen as the embedded points. Note that the distribution of the number of messages in the buffers immediately before an arrival and immediately after a departure to and from the switch are the same [3], but that these distributions differ when embedded points are immediately following arrivals and departures as in our case. Because we are using two sets of embedded points we need a rather complicated transformation to determine the distribution of the number of messages in the buffers at steady state. Following this transformation we determine the average number of messages in the buffers, and then applying Little's result gives the average message transmission delay as a function of the load for different buffer sizes and symmetric or non-symmetric arrivals to the switch.

3.2 The Switch and its model

The basic building block of a Banyan network is the 2x2 Banyan switch. In Banyan networks, messages are forwarded to their destinations using self-routing headers. Fig.3.1 shows a 4-stage Banyan network interconnecting inputs and outputs. This paper will analyze the operation of one of the 2x2 Banyan switches as part of the total Banyan network. Fig.3.2 shows one such Switch with its finite sized input buffers.

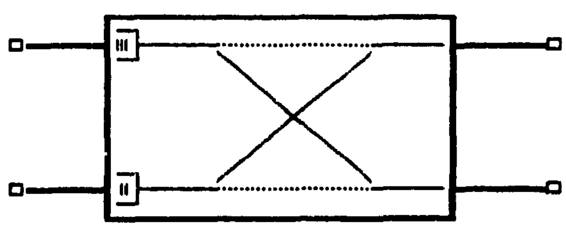


Fig. 3.2. A 2x2 Banyan Switch

The operation of the Switch is asynchronous as follows: messages queue-up in the input buffers as they arrive from the previous stage of the network. At each input, messages are served on a FCFS basis. When a message reaches the head of its input queue, transmission begins immediately if the desired output is idle. But if that output is busy serving a message from the second input queue, the message in the first input queue will wait. Following the completion of the transmission, the waiting message in the first input queue will enter service even if the next message at the head of the second input queue had the same destination. Thus outputs also serve messages coming to the head of the input queues according to the FCFS protocol.

Notice here that only the messages at the head of the input buffers can be served, even though there might be messages lower in the buffers for which an output link would be available.

We now introduce the model used in this analysis which can still be represented by Fig.3.2: Two finite sized input queues and the 2x2 interconnecting switch to both output link servers. Again if the two messages at the head of the input queues are routed to the same output server the first to arrive at the head of its queue will be transmitted while the other will wait until the server is free.

Let us introduce some assumptions made in this analysis.

3.3 Basic assumptions

In the following it is assumed that:

- Finite sized input buffers are provided in the Switch.
- Practically infinite output buffers are present outside the Switch. Clearly, this is true for the last stage of a Banyan Network and it is a good approximation for reasonably large finite input buffers in intermediate stages.
- Poisson arrival process at each input queue with parameter λ_A and λ_B respectively. Note that if an input queue is full the respective arrival process is disabled (λ_A or $\lambda_B = 0$). Also this is a good approximation for our 2x2 Switch in a large Banyan Network serving many inputs and outputs.
- Exponentially distributed message transmission times with parameters μ_A and μ_B.
- Messages choose their destinations independently of each other and both destinations are equally probable for each message.

Now, for our case in which message transmission times are exponentially distributed this system may be modeled by a Markov chain, where transition probabilities will depend on the system state.

3.4 The analysis

Sections 2.5 to 2.8 inclusively presents an analysis of a Star-connected LAN which can be applied in this Banyan Switch environment. We will simply give an outline of the major steps of that analysis for sake of continuity.

First, the system's Markov chain is embedded immediately following both arrivals and departures. The state transition probabilities are then determined in order to form the probability transition matrix. When this resulting transition matrix is solved for its steady state distribution, we obtain a preliminary distribution of the number of messages in the system.

Because of the particularities of our embedding strategy,this distribution is really a weighted combination of the distributions seen at each set of embedded points. In order to obtain the distribution of the number of messages in each input queue, a transformation is applied to the previously obtained distribution.

By varying the mean arrival rates and by analyzing the system under symmetric and non-symmetric arrivals, the obtained steady state distribution of the number of messages in the queues yields the average number of messages in the input queues and the average message delay. Note that our variable exponential message transmission times allows us to obtain results for variable length packets.

We now present the results derived from this analysis.

3.5 Results

Note that for results presented here mean transmission (service) rate is 40 messages/msec for a mean message transmission time of 25 μs. This corresponds to a mean message length of 1 Kbit at a line transmission rate of 40 Mbit/sec.

Our first plot represents mean delay with respect to total mean arrival rate for symmetric traffic and is shown in Fig.3.3. This graph is somewhat misleading. Looking at λ_T = 80 mess/msec we could believe that delay grows as N grows, which is not plausible. In fact, this graph does not account for a few particularities of the system.

First, we recall that for this finite buffer analysis, messages arriving to a full buffer are lost, meaning that average delay may increase from N = 2 to N = 10 but, for the same total mean arrival rate, many more messages are lost with a smaller queue size.

Second, finite buffer analysis are really truncations of infinite buffer analysis. Then let us assume that the system is not overloaded or that the utilization factor (which is hard to visualize here because of the interfering queues), is less than 1. Then increasing maximum queue size (N), would allow the average delay to eventually taper-off and stabilize at a finite value (with minimal number of lost messages).

What must be seen from this graph is simply that delay grows with mean arrival rate. This is in some part due to the dependence of the two input buffers which interfere with each other's transmissions.

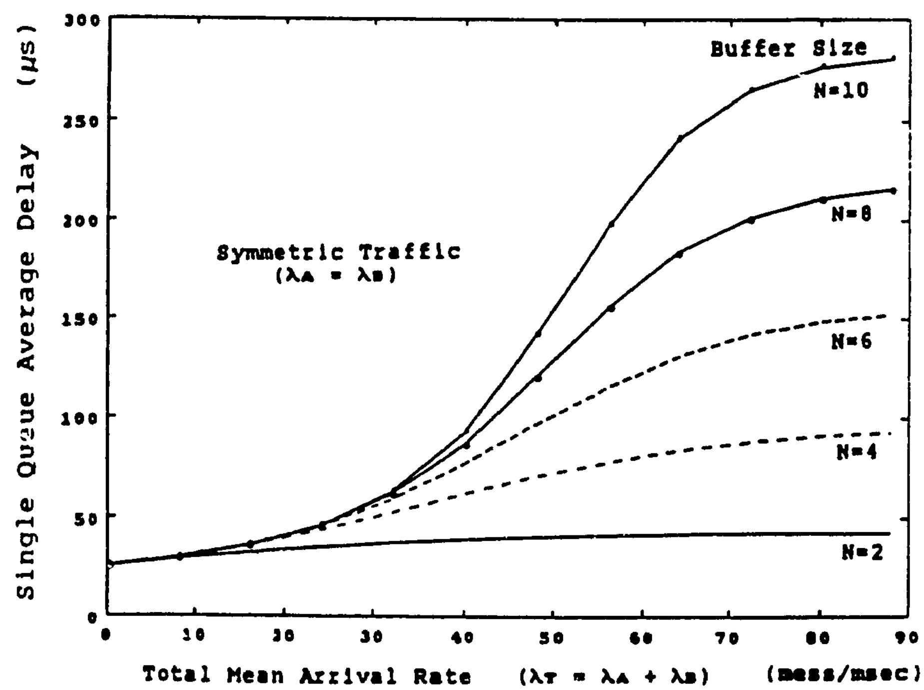


Fig. 3.3. Mean message delay against total mean arrival rate for symmetric traffic and mean message duration of 25 μs.

In Fig.3.4 we wish to demonstrate the increasing delay for the more heavily loaded queue, queue A, in non-symmetric traffic. We then plot mean delay with respect to queue A mean arrival rate for different load ratios between queue A and queue B and with a maximum buffer size of 8 messages. As the arrival rate of queue B decreases we see that transmission delay for queue A also decreases. When $\lambda_B = \lambda_A/100$ we notice that queue A behaves like an M/M/1 queue since transmission interferences between both queues are practically eliminated by the near zero load on queue B.

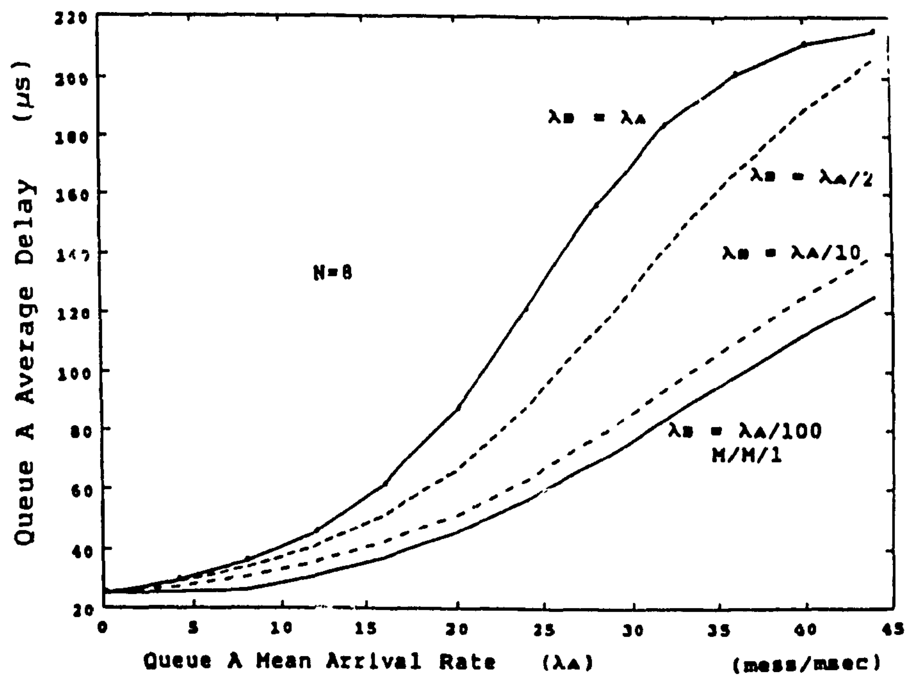


Fig. 3.4. Mean message delay of more heavily loaded queue A against queue A mean arrival rate in symmetric and non-symmetric traffic.

We can also examine Fig.3.5 which represents the average number of messages in queue A for the same load ratios between both queues and maximum buffer size of 8 messages. It still indicates that as the load on queue B increases the transmission interferences between both queues are multiplied which makes the buffer occupancy of queue A grow accordingly.

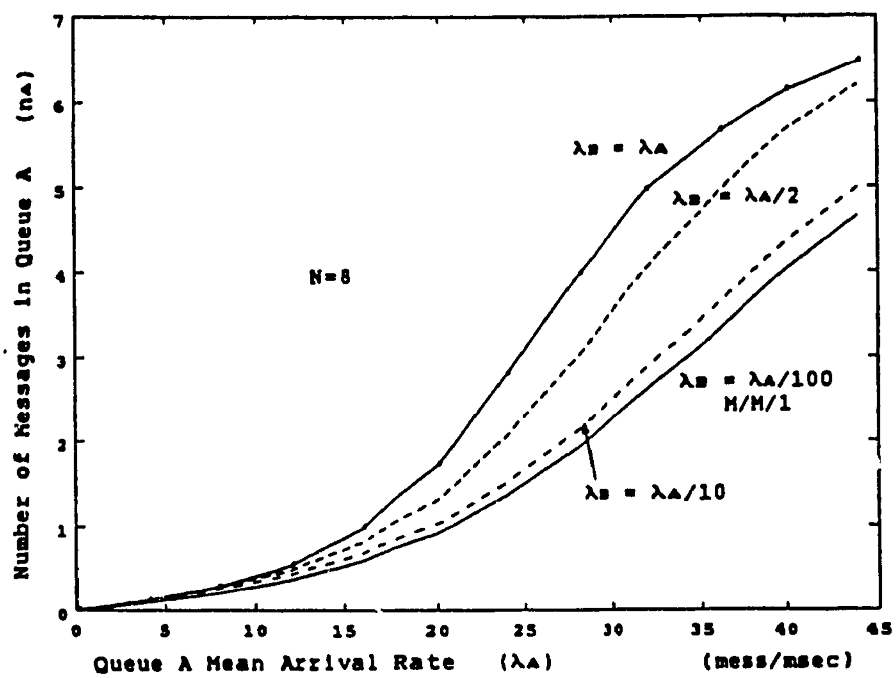


Fig. 3.5. Average number of messages in more heavily loaded queue A against queue A mean arrival rate in symmetric and non-symmetric traffic.

Fig.3.6 represents the probability distribution of the number of messages in both queues in symmetric traffic, for maximum buffer size of 10 messages, and for two different total mean arrival rates (λ_A and λ_B being equal). We clearly see the low queue occupancy at λ_T = 8.0 mess/msec. When the total mean arrival rate is increased to λ_T = 64.0 mess/msec the distribution changes showing greater probabilities for the higher number of messages in the queues.

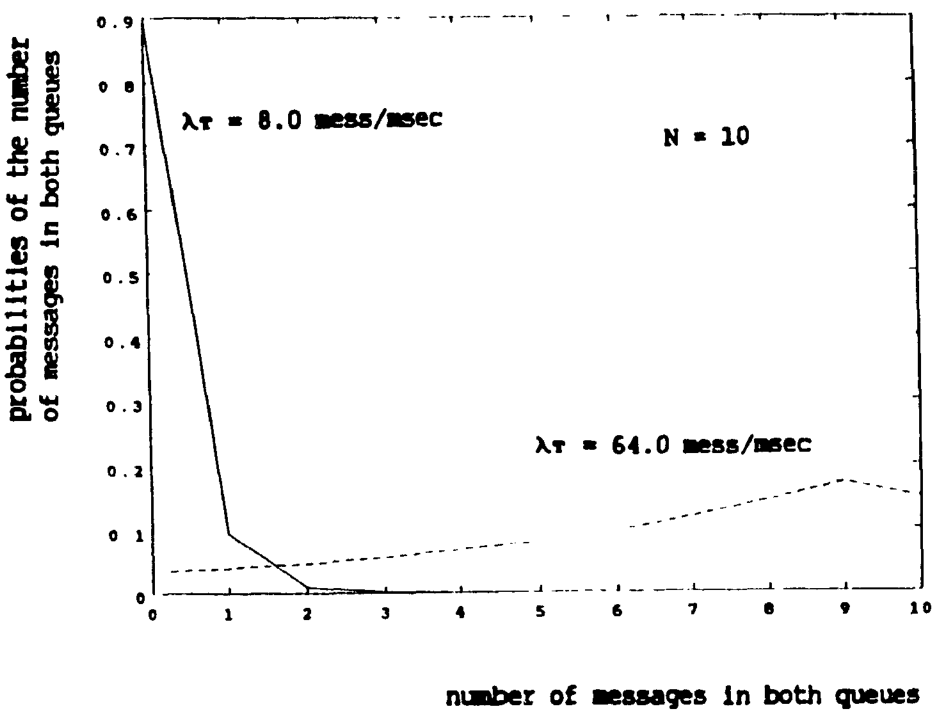


Fig. 3.6 Distribution of the number of messages in both queues in symmetric traffic, for two different values of total mean arrival rate.

Finally, we have plotted in Fig.3.7 the blocking probability in symmetric traffic with respect to single queue mean arrival rate for different maximum buffer sizes. This graph shows clearly that as maximum buffer size increases the blocking probability decreases.

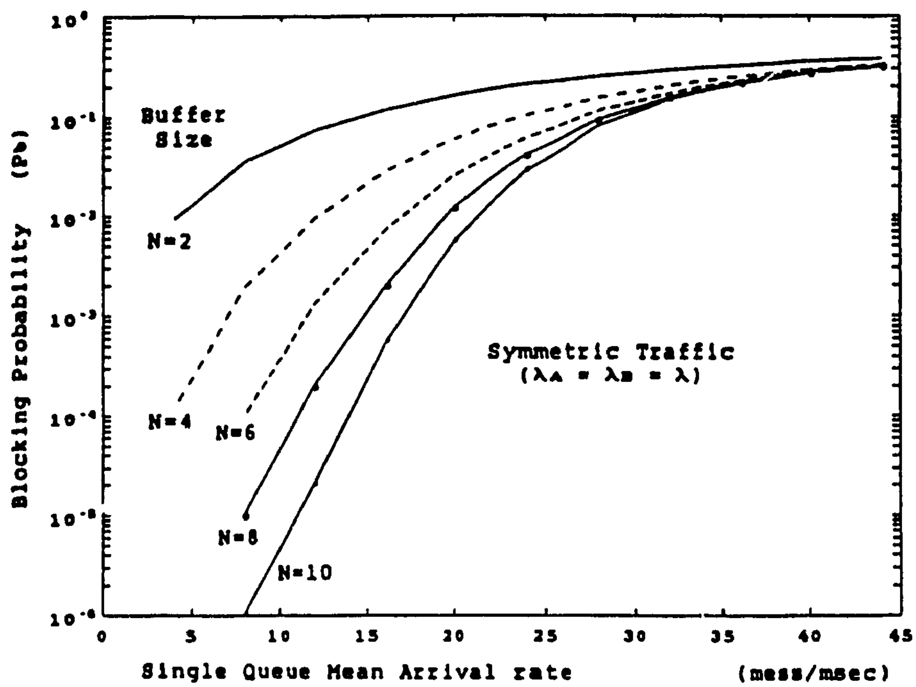


Fig. 3.7. Blocking probability against single queue mean arrival rate in symmetric traffic.

Note that in some applications where transient behavior of the switch could be a factor, the analysis can also provide results in the case where both destinations would not be equally likely for each message

CHAPTER FOUR

A MORE GENERAL ANALYSIS OF THE 2x2 SWITCH

4.1 Introduction

This Chapter will present an approximate analysis of the Star Network introduced in Chapter 2. The approach demonstrated here was originally the first one to be investigated. A major justification for this approach, even if somewhat complicated, is that we are able to let the arrival process be general. We can then examine the system's performance under various arrival processes (periodic, deterministic, exponential, ...). This analysis, again based on a Markov chain, uses the points immediately before arrivals to the system as the embedded points. Using standard notation the system's Markov chain can be represented by Fig.4.1.

-where • are the embedded points

Fig. 4.1 Representation of System's Markov chain.

As was said earlier, this embedding strategy does not allow for an exact solution to be obtained. The reason for this is: when state variable K is used with this approach (remember that state variable K was introduced in Chapter 2 to determine the state of the two destination queues), some combinations of departures between two arrivals points are impossible. It is therefore necessary to handle each one of these combinations separately as special cases. But when the maximum source queue size becomes larger than 2, these special cases multiply to a point where it is impossible to obtain closed form equations for each case.

It was then decided that state variable K was to be dropped out of this analysis making it approximate.

We point out here that it was impossible to obtain results to compare with the analysis of Chapter 2. This is due to the amount of computing involved in obtaining even one mean message delay value, which took more than two weeks on the computer facilities used (Deervax 780). The mathematical development is nevertheless valid and could be applied in other environments and solved with more powerful computing facilities.

Note that the System description and mathematical model described in Sections 2.2 and 2.3 are still valid for this approach. The assumptions made in section 2.4 remain applicable here except for the arrival process which can be general with average arrival rate λ in each source queue. As said earlier, this general arrival process allows us to analyze the system under deterministic arrivals (fixed arrival instants).

Also, message transmission times will be exponential with parameter μ for both source queues.

Let us then get right into the second analysis of this Star Network.

4.2 The analysis

In this analysis each destination queue is to be modeled as a server of exponentially distributed messages. The object will be to obtain all the transition probabilities and then solve the resulting state transition matrix for its steady state solution. This will give the distribution of the number of messages in the system from which the average message delay will be determined using Little's result. Let us make the following definitions:

X denotes the transmission time of a message from source queue A.

X' denotes the transmission time of a message from source queue B.

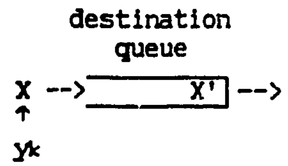
y_k denotes the service time of a message from source queue A:

$$y_k = \begin{cases} X \\ \text{or} \\ X + X' \quad \text{(details below)} \end{cases} \qquad -(4.1)$$

z_k denotes the service time of a message from source queue B:

$$z_k = \begin{cases} X' \\ \text{or} \\ X' + X \quad \text{(details below)} \end{cases} \qquad -(4.2)$$

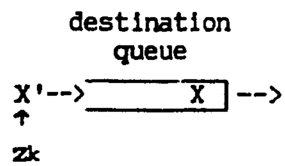
The probability of mutual interference is an important parameter in this analysis and may be evaluated from the entries in the routing matrix G, introduced in Section 2.3, assuming that messages choose their destinations independently of each other. There are two such probabilities to find: a message from source queue B may interfere with a message from source queue A and vice versa.



service time of y_k = X' + X

Prob that B interferes with A = X is sent to A and X' was in front or X is sent to B and X' was in front.
= $G_{AA}G_{BA} + G_{AB}G_{BB}$

And the opposite,



service time of z_k = X + X'

Prob that A interferes with B = X'is sent to A and X was in front or X'is sent to B and X was in front.
= $G_{BA}G_{AA} + G_{BB}G_{AB}$

We then see that both probabilities are equal. Then

p = Prob that A interferes with B = Prob that B interferes with A

$$p = G_{AA}G_{BA} + G_{AB}G_{BB}. \qquad -(4.3)$$

Remember here that a message may interfere with or may be interfered by one and only one other message!

Now, we define the system state as [n_A, n_B] where n_A and n_B are the number of messages in both source queues respectively. Our system is modeled by a Markov chain where the transition probabilities depend on the system state. Then if we let:

$$I = n_{A1} - n_{A2} \qquad -(4.4)$$

and

$$J = n_{B1} - n_{B2} \qquad -(4.5)$$

we get

$$P(\, n_{A1},\, n_{B1};\, n_{A2},\, n_{B2}\,) = P_{I,J} \qquad -(4.6)$$

where:

n_{A1}, n_{B1} denotes the number of messages in source queues A and B respectively, immediately before the current arrival.

n_{A2}, n_{B2} denotes the number of messages in source queues A and B respectively, immediately before the next arrival.

and,

$P_{I,J}$ is the probability that I messages depart from source queue A and J messages depart from source queue B between the current and next arrivals. These will be used to determine the entries in the state transition matrix.

Now, out of the I and J messages transmitted some will interfere on those in the other source queue and others will not. Then from Eqn 4.3 we have the probability '(1-p)' that a message will not interfere with the transmission of messages from the other source queue. We define:

i — as the number of messages out of the I messages from source queue A which do not interfere with the transmission of messages from source queue B. Then the number of interfering messages is (I-i).

j — as the number of messages out of the J messages from source queue B which do not interfere with the transmission of messages from source queue A. Then the number of interfering messages is (J-j).

From these definitions and from the independence of interferences, we get:

$$P(i) = \begin{pmatrix} I \\ i \end{pmatrix} (1-p)^{i} \quad p^{I-i} \qquad -(4.7)$$

and

$$P(j) = \begin{pmatrix} J \\ j \end{pmatrix} (1-p)^{j} \quad p^{J-j} \qquad -(4.8)$$

In order to get the $P_{I,J}$ wanted in Eqn 4.6 we define the total service times for the I and J messages and the total service times of the interfering and non-interfering messages as:

α_I denotes the total service time of all I messages from source queue A:

$$\alpha_I = y_1 + y_2 + \dots + y_I \qquad -(4.9)$$

β_J denotes the total service time of all J messages from source queue B:

$$\beta_J = z_1 + z_2 + \dots + z_J \qquad -(4.10)$$

and,

α_i denotes the total service time of all i non-interfering messages from source queue A.

β_i denotes the total service time of all j non-interfering messages from source queue B.

γ denotes the total service time of all (I-i) and (J-j) interfering messages from both source queues.

At this point we have all that is necessary to obtain the basic transition probabilities $P_{I,J}$ used to determine the entries of the state transition matrix. We may write:

$$P_{I,J} = \int_0^\infty P\left[\alpha_{I+1}, \beta_{J+1} \geq t \geq \alpha_I, \beta_J \mid t \right] dA(t) \qquad -(4.11)$$

where

dA(t) is the inter-arrival time distribution, and α_K and β_K are as defined above.

Next we will rewrite both total service time equations, Eqns 4.6 and 4.7, in such a way as to show explicitly the messages whose service times have been interfered with. Then:

$$\alpha_I = y_1 + \ldots + y_k^{(j+1)} + \ldots + y_{k+m}^{(J)} + \ldots + y_I \qquad -(4.12)$$

$$\beta_J = z_1 + \ldots + z_l^{(i+1)} + \ldots + z_{l+n}^{(I)} + \ldots + z_J \qquad -(4.13)$$

where

$y_k^{(j+1)}$ to $y_{k+m}^{(J)}$ represent the messages from A which were interfered by a message from B.

and,

$z_l^{(i+1)}$ to $z_{l+n}^{(I)}$ represent the messages from B which were interfered by a message from A.

In the above, the superscripts simply show the number of the messages from one source queue which interfered on the respective messages from the other source queue, but it does not show their sequence. Since messages choose their destinations independently of each other according to the routing matrix, the messages may interfere with one another in any order. This order is not important since it does not change the sums α_I and β_J needed to calculate the basic transition probabilities $P_{I,J}$. Next, we express the service times in terms of message transmission times from Eqns 4.1 and 4.2:

$$\alpha_I = X_1 + \dots + X_k + X'_{j+1} + \dots + X_{k+m} + X'_J + \dots + X_I \qquad -(4.14)$$

$$\beta_J = X'_1 + \dots + X'_l + X_{l+1} + \dots + X'_{l+n} + X_I + \dots + X'_J \qquad -(4.15)$$

Rearranging these equations to show the interfering and non-interfering parts separately, we get:

$$\alpha_I = \overbrace{X_1 + \dots + X_l}^{\alpha_1} + X_{l+1} + \dots + X_I + X'_{j+1} + \dots + X'_J \qquad -(4.16)$$

$$\beta_J = \underbrace{X'_1 + \dots + X'_j}_{\beta_1} + \underbrace{X'_{j+1} + \dots + X'_J + X_{l+1} + \dots + X_I}_{\gamma} \qquad -(4.17)$$

where α_1, β_1 and γ are as defined above.

Note that we have arranged the messages in this fashion to show that the γ part is common in both equations.

Now since we must find a closed form solution to Eqn 4.11, we have to distinguish between three mutually exclusive cases involving the behavior of the source queues:

A- both source queues remain non-empty during the inter-arrival period,

B- one of the source queues goes empty during the inter-arrival period,

C- both source queues go empty during the inter-arrival period.

These cases will be analyzed separately.

4.2.1 Both queues busy (non-empty)

The integrand in Eqn 4.11 states that exactly I and J messages from source queues A and B respectively, are served between the current and the next arrivals. We may write:

$$P\left[\alpha_{I+1}, \beta_{J+1} \geq t \geq \alpha_I, \beta_J \mid t \right] =$$

$$P\left[\text{I messages served from A, J messages served from B in t} \mid t \right] \qquad -(4.18)$$

Looking at Eqns 4.16 and 4.17 we see that we have separated the interfering messages from the non-interfering messages and that the γ term appears in both. This allows us to rework the integrand in Eqn 4.11. If we leave for the common γ part ,which includes the (I-i) and (J-j) interfering messages, t_1 seconds to be transmitted, the α_1 and the β_1 part will have $(t-t_1)$ seconds left since both α_I and β_J have t seconds until the next arrival. Then we may write:

$$P\left[I \text{ messages served from A, } J \text{ messages served from B in } t \mid t, t_1 \right] =$$

$$P\left[\begin{array}{l} i \text{ messages transmitted in } (t-t_1), \\ j \text{ messages transmitted in } (t-t_1), \\ ((J-j), (I-i)) \text{ messages transmitted in } t_1 \mid t, t_1 \end{array}\right] \qquad -(4.19)$$

Now, from Eqns 4.16 and 4.17 we see that these three events are independent. Then in each of the above events we can say that we have three independent queues of exponentially distributed messages each being served by its own server. Then during the inter-arrival time the number of messages (i, j and ((I-i)+(J-j))) served will have a Poisson distribution (in fact a pure Poisson death process) with parameter μ associated with our exponential transmission time. Then the above probability may be broken down and written as:

$$P\left[\text{ I messages served from A, J messages served from B in t } \mid t, t_1 \right] =$$

$$P\left[\text{ I messages transmitted in } (t-t_1) \mid t, t_1 \right] \times$$

$$P\left[\text{ J messages transmitted in } (t-t_1) \mid t, t_1 \right] \times$$

$$P\left[\ (J-j), (I-i)\ \text{ messages transmitted in } t_1 \mid t, t_1 \right] =$$

$$\frac{(\mu(t-t_1))^{i}\ e^{-\mu(t-t_1)}}{i!}\quad \frac{(\mu(t-t_1))^{j}\ e^{-\mu(t-t_1)}}{j!}\quad \frac{(\mu t_1)^{I+J-i-j}\ e^{-\mu t_1}}{(I+J-i-j)!}$$

-(4.20)

and we will remove the dependencies in the following order: t_1, then i and j, and finally t and get the $P_{I,J}$ wanted.

Integrating w.r.t. t_1 from 0 to t, Eqn 4.20 reduces to:

$$P\left[\text{I messages served from A, J messages served from B in t } \mid t, t_1 \right] =$$

$$\frac{1}{i!\ j!\ (I+J-i-j)!} \int_0^t (\mu t_1)^{I+J-i-j}\ (\mu(t-t_1))^{i+j}\ e^{-\mu(2t-t_1)}\ dt_1$$

-(4.21)

Rearranging the t_1 terms gives:

$$P\left[\text{I messages served from A, J messages served from B in t } \mid t, t_1 \right] =$$

$$\frac{\mu^{I+J}\ e^{-2\mu t}}{i!\ j!\ (I+J-i-j)!} \underbrace{\int_0^t t_1^{(I+J-i-j)}\ (t-t_1)^{i+j}\ e^{\mu t_1}\ dt_1}_{A}$$

-(4.22)

The integral in Eqn 4.22 which we will call A, can be simplified using Eqn 4.23:

$$(t-t_1)^{i+j} = \sum_{n=0}^{i+j} \begin{bmatrix} i+j \\ n \end{bmatrix} t^{i+j-n} \; (-t_1)^n \qquad -(4.23)$$

We get:

$$A = \int_0^t \sum_{n=0}^{i+j} \begin{bmatrix} i+j \\ n \end{bmatrix} t^{i+j-n} \; (-1)^n \; t_1^{(I+J-i-j+n)} \; e^{\mu t_1} \; dt_1 \qquad -(4.24)$$

Interchanging the order of integration and summation gives:

$$A = \sum_{n=0}^{i+j} \begin{bmatrix} i+j \\ n \end{bmatrix} t^{i+j-n} \; (-1)^n \underbrace{\int_0^t t_1^{(I+J-i-j+n)} \; e^{\mu t_1} \; dt_1}_{B} \qquad -(4.25)$$

The integral B in Eqn 4.25 may be determined to give:

$$B = \left[\frac{e^{\mu t_1}}{\mu} \sum_{m=0}^{I+J-i-j+n} \frac{(I+J-i-j+n)!}{(I+J-i-j+n-m)!} \; \frac{t_1^{(I+J-i-j+n-m)}}{(-\mu)^m} \right]_0^t \qquad -(4.26)$$

Evaluating Eqn 4.26 from 0 to t and substituting the solution first in Eqn 4.25 and then in Eqn 4.22 gives:

$$P\left[I \text{ from A, } J \text{ from B in } t \mid t, i, j \right] =$$

$$\frac{\mu^{I+J} \, e^{-2\mu t} \, (i+j)!}{i! \, j! \, (I+J-i-j)!} \left\{ \sum_{n=0}^{i+j} \frac{(-1)^n \, t^{i+j-n} \, q!}{n! \, (i+j-n)!} \times \right.$$

$$\left. \left[e^{\mu t} \sum_{m=0}^{q} \frac{(-1)^m \, t^{(q-m)}}{(q-m)! \, \mu^{(m+1)}} - \frac{(-1)^q}{\mu^{(q+1)}} \right] \right\} \qquad -(4.27)$$

where $q = I+J-i-j+n$

Now, before removing the dependency on i and j we must realize a critical aspect of the relationship between these two variables.

Clearly, if I=5 and J=10 only 5 of the 10 messages from queue B can interfere with the 5 messages from queue A since each message can interfere with only one message. Thus in each case the maximum number of interfering messages will be min(I, J). Since i and j denote the number of non-interfering messages in each queue, the maximums they can reach are:

$$\left.\begin{array}{l} i_{max} = I - \min(I, J) \\ j_{max} = J \end{array}\right\} \text{ if } \min(I, J) = J$$

or -(4.28)

$$\left.\begin{array}{l} i_{max} = I \\ j_{max} = J - \min(I, J) \end{array}\right\} \text{ if } \min(I, J) = I$$

Therefore in the removal of the dependency, the following cases are impossible:

$$P\left[\text{imp. cases}\right] = \sum_{i=0}^{K} \sum_{j=0}^{L} P\left[I \text{ from A, } J \text{ from B in } t \,\middle|\, t,i,j\right] P(i)\, P(j)$$

-(4.29)

where $K = \begin{cases} I & \text{if } \min(I,J)=I \\ I-\min(I,J) & \text{otherwise} \end{cases}$,

$L = \begin{cases} J & \text{if } \min(I,J)=J \\ J-\min(I,J) & \text{otherwise} \end{cases}$ from Eqn 4.28,

and $P(i)$, $P(j)$ and $P\left[I \text{ from A, } J \text{ from B in } t \,\middle|\, t,i,j\right]$ come from Eqns 4.7, 4.8 and 4.27 respectively.

We will then divide by (1- P[imp. cases]) to make sure that the total probability law remains valid. Putting it all together gives:

$$P_{I,J\,(busy)} =$$

$$\int_0^{\infty} \frac{\displaystyle\sum_{i=I-\min(I,J)}^{I} \; \sum_{j=J-\min(I,J)}^{J} P\left[I \text{ from A, } J \text{ from B in } t \,\middle|\, t,i,j\right] P(i) P(j)}{\left[1 - P\left[\text{ impossible cases }\right]\right]} \, dA(t)$$

-(4.30)

Eqn 4.30 gives the general case solution for when both queues remain busy between two arrival points. Many special cases come into play and must be analyzed separately. For example when I and/or J are zero the queues behave differently. These special cases will not be analyzed in detail here.

Now that we have the solution when both queues remain non-empty, we may use this result in handling the cases when one of the queues goes empty.

4.2.2 One of the queues goes empty

This case will be handled with a simple trick. We will assume that the queue which empties does so in y seconds leaving i_1 (or j_1) messages in the other queue. We then uncondition w.r.t. y and i_1 (or j_1) and get the solution. This can be illustrated using the diagram of Fig.4.2 for the case when source queue B goes empty:

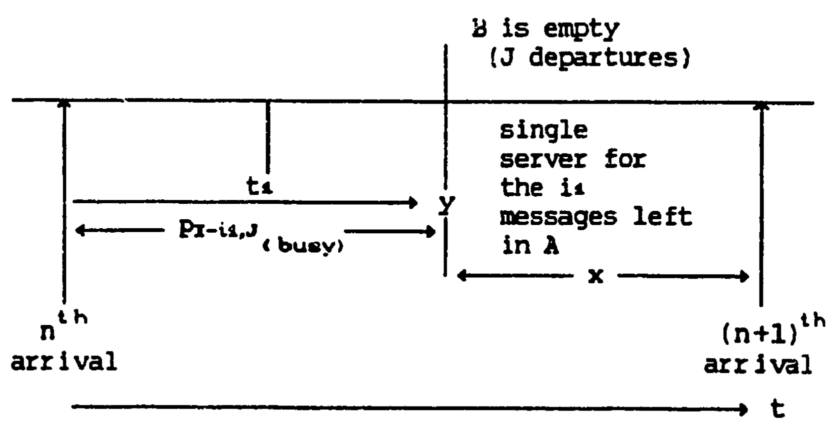


Fig. 4.2 Case where Queue B empties during the inter-arrival period.

We then uncondition w.r.t. x from y to t, w.r.t. y from 0 to t and finally, w.r.t. t from 0 to ∞.

Clearly, during the first y seconds both queues remain busy which is handled by Eqn 4.30 and during the remaining t-y seconds, only queue A will send messages, the service being modeled by a single server (Poisson process). The equation governing this case will be written:

$$P_{I,J_{(B\ empties)}} =$$

$$\int_0^{\infty} \left[\sum_{i1=0}^{I} \int_0^t \left\{ \left[P_{I-i1,J_{(busy)}} \right]_0^y \int_y^t \frac{(\mu x)^{i1}}{i1!} \, e^{-\mu x} \, dx \right\} dy \right] dA(t) \qquad -(4.31)$$

And similarly for the case when A goes empty:

$$P_{I,J_{(A\ empties)}} =$$

$$\int_0^{\infty} \left[\sum_{j1=0}^{J} \int_0^t \left\{ \left[P_{I,J-j1_{(busy)}} \right]_0^y \int_y^t \frac{(\mu x)^{j1}}{j1!} \, e^{-\mu x} \, dx \right\} dy \right] dA(t) \qquad -(4.32)$$

Eqns 4.31 and 4.32 give the general case solution for the "one queue goes empty" cases. Here again some special cases must be handled separately.

Let us then get into the last possibility which cover the case when both queues go empty between two arrival points.

4.2.3 Both queues go empty

To analyze this case we will consider two mutually exclusive events and simply add the results. Surely, if both queues go empty within the next arrival, queue A will empty either before or after queue B. We will solve both cases separately. The following diagrams will help in visualizing both events. If A empties before B we get the situation shown in Fig.4.3:

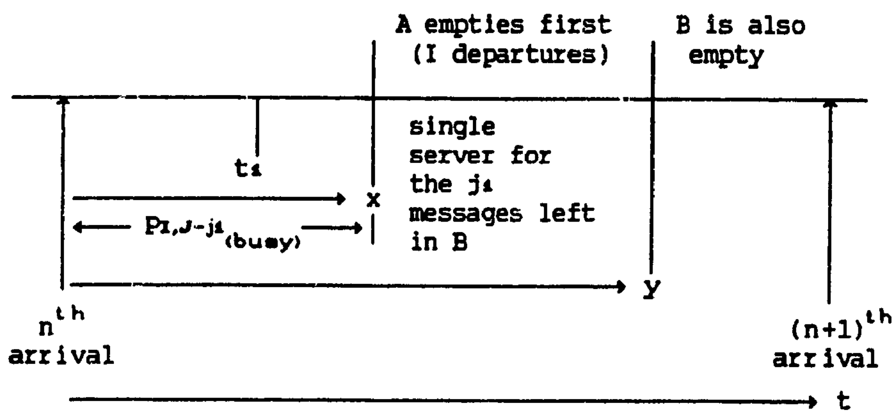


Fig. 4.3 Case where queue A empties before queue B during the inter-arrival period.

We then uncondition w.r.t. x from 0 to y, w.r.t. y from 0 to t and finally, w.r.t. t from 0 to ∞.

On the other hand B may empty before A as shown in Fig.4.4:

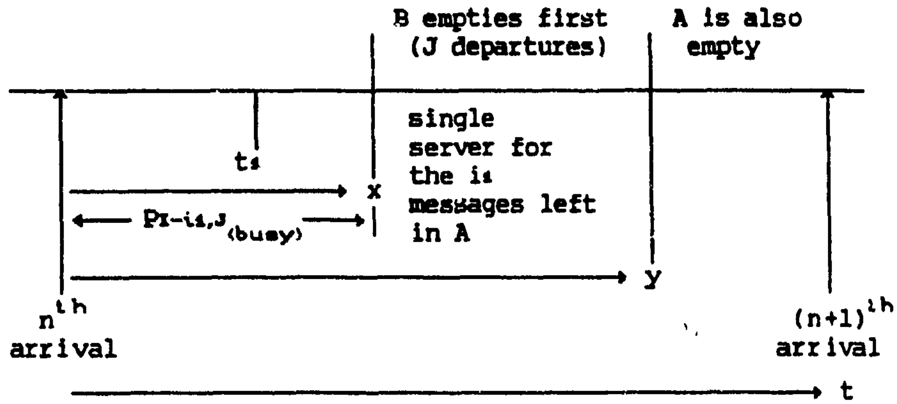


Fig. 4.4 Case where queue B empties before queue A during the inter-arrival period.

Then again we uncondition w.r.t. x from 0 to y, w.r.t. y from 0 to t and finally, w.r.t. t from 0 to ∞.

Note also that the $P_{I,J-j_1\,(busy)}$ and $P_{I-i_1,J\,(busy)}$ parts are still handled by Eqn 4.30.

Writing the equation for the sum of these two mutually exclusive events gives:

$$P_{I,J\,(A,B\ \text{empty})} =$$

$$\int_0^{\infty}\left[\sum_{j_1=0}^{J}\int_0^{t}\int_0^{y}\left\{\left[P_{I,J-j_1\,(\text{busy})}\right]\int_x^{y}\frac{(\mu z)^{j_1-1}}{(j_1-1)!}\,e^{-\mu z}\,dz\right\}dx\,dy\right]dA(t)$$

$$+$$

$$\int_0^{\infty}\left[\sum_{i_1=0}^{I}\int_0^{t}\int_0^{y}\left\{\left[P_{I-i_1,J\,(\text{busy})}\right]\int_x^{y}\frac{(\mu z)^{i_1-1}}{(i_1-1)!}\,e^{-\mu z}\,dz\right\}dx\,dy\right]dA(t)$$

-(4.33)

Now that we have closed form solutions for all three separate cases we can obtain the transition probabilities forming the transition matrix which will be solved for its steady state solution. This will then give the steady state distribution of the number of messages in both source queues. Taking the expectation from 0 to N (the maximum buffer size) yields the average number of messages in the queues and using Little's result adapted to finite queues repeated in Eqn 4.34 gives the average delay of a message.

$$\bar{D} = \frac{\bar{n}}{\lambda\,(\,1 - P_b\,)} \qquad -(4.34)$$

- where P_b is the blocking probability equal to the probability of having a full source queue.

4.3 Results

Unfortunately, the amount of computing involved in calculating the numerical solutions even for a small state space (maximum of two messages in each queue) is so large that weeks are needed to come up with data. This is due to the imbedded integrals and summations and to the limited computing facilities used (Deervax 780). We are therefore unable to present any results which could be compared with those obtained in the previous analysis of Chapter 2.

CHAPTER FIVE

CONCLUSIONS

The main objective of this thesis was to propose and analyze the performance of two different switching techniques. A first system used in an integrated LAN (MAN) environment was designed as a Star Network and was analyzed in two different ways. A second system, applicable in the BISDN environment, was composed of 2x2 Banyan Switches as part of a larger Network and was analyzed using an adaptation of the first analysis of the Star Network.

Then, in Chapter two an analysis of a two users star-connected Fast Circuit switching LAN was presented. The novel approach of this analysis provided results, namely distribution of the number of messages in the system, mean message delay and blocking probabilities, for a wide range of varying parameters:

* Symmetric or non-symmetric arrivals to the source queues,
* Different input buffer sizes,
* Variable length messages
* And equally or non-equally probable message destinations.

These results agreeably compare with the approximate results published in [2] which analyzed larger Networks (16 users) but assumed independence between the source queues and did not handle finite buffer sizes.

In Chapter four the same Star Network is analyzed using a different approach which would provide approximate results if larger computing facilities were used. With this method we could analyze the system under general arrival process (fixed, variable, adaptive, ... arrival rates), and in symmetric traffic.

Even though this LAN application, integrating many services, mainly extends towards the MAN environment, the Fast Circuit switching fabric of the CPS, and therefore the two previous analysis, can be extended to wider BISDN architectures where higher switching speeds would be used.

In Chapter three the analysis of the second chapter was applied to the Packet switching fabric of Banyan Networks.

The 2x2 Banyan Switch uses small variable length packets with equally probable destinations. The flexibility of this analysis allows for many combinations of the parameters in order to obtain a wide set of results similar to those from the Chapter two analysis.

It is interesting to realize that our analysis takes a look at Banyan networks from a microscopic point of view where we observe the behavior of a single switching element of a larger network. Other approaches by Jenq [15] and by Dias and Jump [27], [18], examine the entire network from a macroscopic point of view.

A significant aspect of the obtained results involves our capacity to obtain the distribution of the number of messages in the system, the mean message delay, the blocking probabilities and the average number of messages in the system for variable length packets. In fact, in the fast developing field of BISDN, variable length messages will be part of reality [8], [13]. This is why our use of variable length packets improves on the analysis published by Jenq [15].

Also, Jenq only analyzed Banyan Networks using single-buffered 2x2 crossbars. The results obtained here allow for buffer sizes to vary from 2 to any value of maximum buffer size (N). Note that we have computed the above results for values of N= 2, 4, 6, 8 and 10.

In other papers by Dias and Jump [27], [18], again only fixed sized packets were allowed. But also, they simplified their network model in order to avoid the complexity and large number of states in an exact Markov chain analysis. In fact, Chapter three does follow a Markov chain analysis. An approximation was used (increasing in validity as N increases), but the entire state space of the Markov process is generated here.

Furthermore, Dias and Jump used an approximate algorithm in order to obtain results for multiple buffers between the stages. Our analysis does not suffer from such an approximation since it applies to arbitrary queue sizes (any number of buffers).

In future works, the Star Network analysis of Chapter two could be extended to larger topologies where the greater number of nodes would enlarge the state space (possible values) of state variable K and where it would be necessary to obtain the different interdependence relations and the interference probabilities between the nodes.

Also, larger Banyan Networks could be analyzed by obtaining an interdependence algorithm between the stages of 2x2 Banyan Switches which make up the architecture of the Network. The single 2x2 Banyan Switch analysis of Chapter three would then be used in conjunction with the interdependence algorithm to obtain a large scale analysis.

It is believed that the double embedding strategy of the analysis in Chapters two and three, and the transformation used to obtain the steady state distribution of the number of messages in the system could be applied in other instances where a similar Markov chain analysis would be a viable solution.

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