FOSTER FORM REALIZATIONS OF A CLASS OF TWO-VARIABLE REACTANCE FUNCTIONS

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ABSTRACT

This report considers a class of two-variable reactance functions which can be realized in a form similar to Foster's form for single-variable functions. Each section in the above realization consists of two reactive elements in each variable. Further, it is shown that starting from such structures, the corresponding ladder structures are not synthesizable.

CHAPTER I

INTRODUCTION

1.1. LLFPB Networks

The driving point immittance functions for networks consisting of lumped, linear, finite, passive, bilateral elements may be expressed as rational functions of the complex frequency variable s, that is they may be written as :

$$Z(s) = \frac{p(s)}{q(s)} = \frac{a_o S^n + a_1 S^{n-1} + \ldots\ldots\ldots + a_{n-1} S + a_n}{b_o S^m + b_1 S^{m-1} + \ldots\ldots\ldots + b_{m-1} + b_m} \quad \ldots\ldots(1.1)$$

and can be put in the form:

$$Z(s) = \frac{a_o}{b_o} \frac{(S-z_1)(s-z_2)\ldots\ldots\ldots(s-z_n)}{(s-p_1)(s-p_2)\ldots\ldots\ldots(s-p_m)} ; \quad |m-n| \leq 1 \quad \ldots\ldots(1.2)$$

where the z's are the zeros , the p's are the poles of Z(s) and the coefficients a_i's and b_i's (i=o,1,2,...) are positive real constants. It is also noted that both p(s) and q(s) are Hurwitz polynomials i.e. they contain roots only in the left half of the s plane.

Conversely, it has been established that only those rational functions which are positive real may be realized as the driving point immittance of an LLFPB network.

A function F(s) is said to be a positive real function (p.r.f.) if and only if:

(i) F(s) is real,when s is real and

(ii) Re $F(s) \geq o$ for Re $S \geq o$

1.2. Mixed Lumped Distributed Networks

However, if the network consists of distributed elements as well as lumped elements, its driving point immittance need not be a rational function of s .This point is illustrated by the following example:

The network of Fig. 1.1 consists of resistors,capacitors,and commensurate lossless transmission lines (unit elements).

The chain matrix of a single unit element is of the form:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{ue} = \begin{bmatrix} \text{Cosh } st & Z_o \text{Sinh } st \\ \frac{1}{Z_o} \text{Sinh } st & \text{Cosh } st \end{bmatrix} \quad \ldots\ldots\ldots\ldots (1.3)$$

where t and Z_o are constants.

Defining :

$$p_1 = s \quad \text{and } p_2 = \tanh st \quad \ldots\ldots\ldots\ldots (1.4)$$

we have :

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{ue} = \frac{1}{\sqrt{1-p_2^2}} \begin{bmatrix} 1 & Z_o p_2 \\ \frac{p_2}{Z_0} & 1 \end{bmatrix} \quad \ldots\ldots\ldots\ldots (1.5)$$

Then the overall ABCD parameters of the network becomes :

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{over-all} = \begin{bmatrix} 1 & R_1 \\ 0 & 1 \end{bmatrix} \frac{1}{\sqrt{1-p_2^2}} \begin{bmatrix} 1 & Z_o p_2 \\ \frac{p_2}{Z_o} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ sc_1 & 1 \end{bmatrix} \frac{1}{\sqrt{1-p_2^2}}$$

$$\begin{bmatrix} 1 & Z_o p_2 \\ \frac{p_2}{Z_o} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ sc_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{R_2} & 1 \end{bmatrix}$$

The input impedance at points x-x is :

$$Z_{in.} = \frac{A}{C} = \frac{1}{1-p_2^2} \cdot \frac{X-R_1 Z}{Z}$$

where:

$$X = 1 + p_1^2 p_2^2 Z_o^2 C_1^2 + 3Z_o C_1 p_1 p_2 + \frac{2}{R_2} Z_o p_2 + \frac{C_1 Z_o^2}{R_2} p_2^2 + p_2^2 + 1$$

$$Z = Z_o C_1^2 p_2 p_1^2 + C_1 p_2^2 p_1 + 2C_1 p_1 + \frac{Z_0 C_1}{R_2} p_1 p_2 + \frac{2}{Z_o} p_2 + \frac{1}{R_2} p_2^2 + \frac{1}{R_2}$$

We note that it has been possible to express the input impedance as a rational function of two variables p_1 and p_2 . This approach may be extended to interconnections of non-commensurate transmission lines also.

Common examples of circuits containing both lumped and distributed elements are networkscontaining semiconductor elements and transmission lines or wave guides , or indeed networks of transmission lines alone where lumped discontinuities invitably occur.

1.3. Advantages Of Mixed Lumped-Distributed Structures

The following are some of the advantages of mixed lumped-distributed networks:

(i) In the case of mixed lumped-distributed structures, allowance may be made for the parasitics in the terminating impadance , i.e. the terminations need not be purely resistive.

(ii) A conventional quarter-wave transformer gives small or no attenuation at the higher harmonics. On the other hand , the mixed distributed impedance transformer can be designed to have the properties of an impedance transformer and a low pass filter. This property is useful where combined filtering and impedance transformation is desirable.

(iii) In the case of Comb-Line filter , lumped capacitive coupling at the input and output reduces the filter size by eliminating the transmission line matching section.

(iv) In the case of cascaded unit element filters,the number of UE'S can be reduced from (2n+1) to n , if the cascaded lines are separated by (n+1) lumped capcitors.

1.4. Realization Of MixedLumped Distributed Structures

The realization techniques available in the lumped network theory are not directly applicable in mixed-distributed cases due to the transcendental nature of the network functions.

Two different approaches are followed to solve the realization problem of these mixed lumped-distributed networks. One of them directly deals with transcendental functions which are termed as the single-variable approach.

In the other approach, the transcendental functions of s are converted into polynomial functions of several variables p_i. This is called the multivariable approach.

Some accepted definitions of multivariable network functions are given below;

Definition 1.1

A rational function $F(p_1,p_2,\ldots\ldots p_n)$ of n-complex variables $p_1,p_2,\ldots\ldots p_n$ is called a Multivariable Positive Real Function (MPRF) when the following conditions are satisfied:

(i) $F(p_1,p_2,\ldots\ldots,p_n)$ is a real function of $p_1,p_2,\ldots\ldots,p_n$, and

(ii) $\mathrm{Re}\ F(p_1,p_2,\ldots\ldots,p_n) \geq 0$ in the polydomain $\mathrm{Re}\ p_i \geq 0$, where $i=1,2,\ldots,n$.

Definition 1.2

A rational function $F(p_1,p_2,\ldots\ldots\ldots,p_n)$ is called a Multivariable Reactance Function (MRF) when the following conditions are satisfied:

(i) $F(p_1,p_2,\ldots\ldots p_n)$ is an MPRF, and

(ii) $F(p_1,p_2,\ldots\ldots,p_n) = -F(-p_1,-p_2,\ldots\ldots,-p_n)$.

Definition 1.3

A polynomial of n-complex variables $p_1,p_2,\ldots\ldots,p_n$ is called a Multivariable Hurwitz Polynomial in Narrow Sense (MHPN) if it has no zeros in the regions: $\mathrm{Re}\ p_1 > 0,\ldots\ldots,\ \mathrm{Re}\ p_{i-1} > 0$, $\mathrm{Re}\ p_i \geq 0$, $\mathrm{Re}\ p_{i+1} > 0,\ldots\ldots,\ \mathrm{Re}\ p_n > 0$, for all $i\ (1 \leq i \leq n)$.

Definition 1.4

A polynomial of n-complex variables $p_1,p_2,\ldots\ldots,p_n$, is called a Multivariable Hurwitz Polynomial in Broad Sense (MHPB) if it has no zeros in the open polydomain $\mathrm{Re}\ p_i > 0$, and if those zeros for $\mathrm{Re}\ p_i=0$ are simple.

It can be seen that these definitions are analogous to the definitions of similar functions in the single variable case. Hence, these definitions can be considerd as logical extensions of those in the single variable case.

It is known that a reactance function of a single variable (SRF) can always be synthesized using a minimum number of elements. This number is equal to the order of the function. These different canonic structures are due to Foster (1), Couer (1), Lee (2,3), Kida (4,5), Ramachandran and Swamy (6). This however, may not be true in the case of two-variable reactance functions (TRF).

In general , the two variable reactance function require either ideal transformers or passive ideal gyrators for their realization (7). However some work has been done regarding the realizations of some classes of TRF'S without transformers or gyrators (8,9,10)
It is shown that there exist a class of TRF'S which can be decomposed into a sum of two single variable reactance functions(11,12). Specifically , conditions have been obtained to decompose $Z(p_1,p_2)$ as the sum of $Z_1(p_1)$ and $Z_2(p_2)$, where $Z_1(p_1)$ is a single variable reactance function in p_1 and $Z_2(p_2)$ is a single variable reactance function in p_2. Each function can be realized by the known methods of a single variable reactance functions.

Also necessary and sufficient conditions have been obtained for the realization of TRF'S in a form similar to the Foster form consisting of one element of variable p_1 (an inductor or a capacitor), and the other element of variable p_2 (a capacitor or inductor), (8), and a typical section is as shown in Fig.1.2. For this type of realization it is shown that if the Foster form exists , the other forms similar to the single variable canonic structures exists.

1.5. Scope Of The Report

This report establishes that in the case of two-variable reactance functions, it may or may not be possible to ~~derive~~ one canonic from starting from another canonic form.

Chapter II discusses Single Variable Canonic Structures with particular emphasis on the Foster and Cauer forms. Other Canonic forms are discussed briefly for the sake of completeness.

ChapterIII discusses two-variable canonic structures which are(i)similar to Foster forms consisting of one element in each variable (Fig.1.2) ; and (ii) Ladder structures where the series and the shunt arms consists of one element in each variable.

Chapter IV gives the author's contribution. Specifically Foster forms are considered where each section consists of interconnection of four elements, two in each variable. Conditions are derived for the realization of such networks. It is further proved that such networks do not possess ladder realizations.

Chapter V discusses the conclusions arissing out of this study .

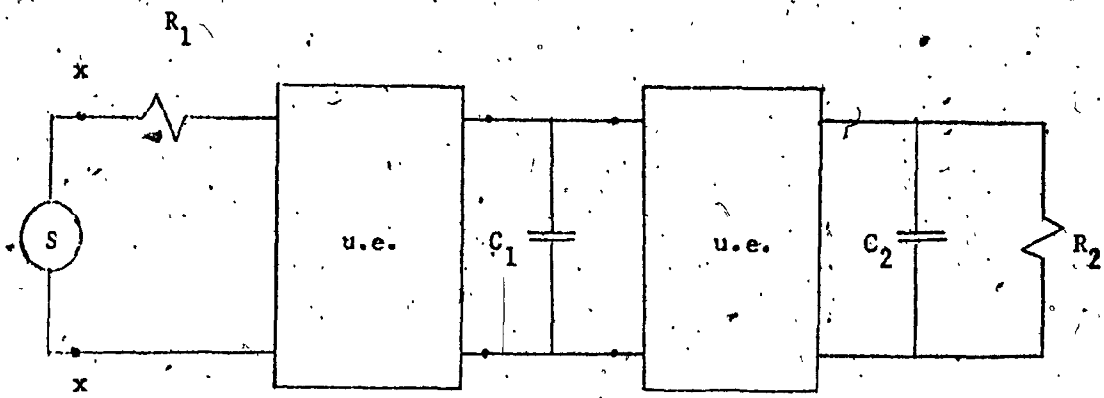


Fig.1.1

Network consists of resistor, capacitor and commensurate lossless transmission lines

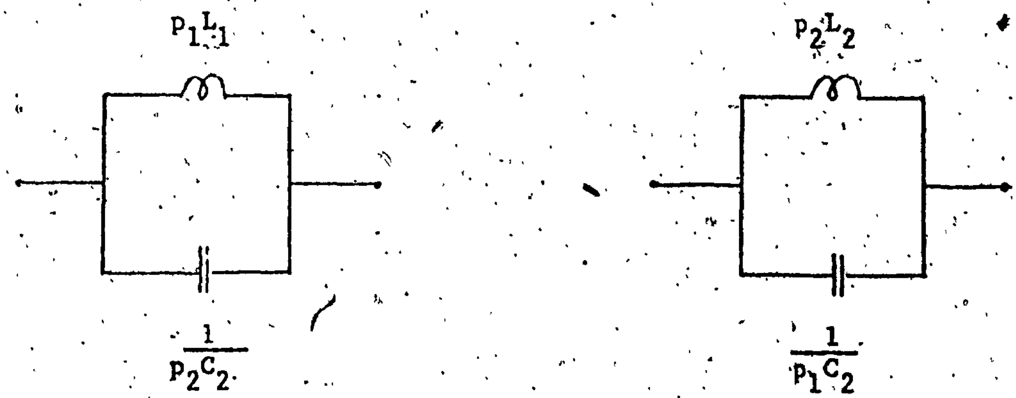


Fig.1.2

Typical sections for two variable reactance function

CHAPTER II

CANONIC REALIZATIONS OF SINGLE VARIABLE REACTANCE FUNCTIONS

2.1. Introduction

In this chapter, we shall discuss the different Canonic realizations of SRF's, with particular emphasis on the Foster and Cauer forms. These two structures will be discussed in detail as the corresponding TRF structures considered in the forthcoming chapters are analogous to these two forms. Mention will be made of the other SRF canonic structures for the sake of completeness.

2.2. Reactance Function in a Single Variable (1)

Consider an impedance function Z(s), which is positive real and has the property that Re Z(jw) = 0 for all w. If Z(s) is written in the form:

$$Z(s) = \frac{m_1(s) + n_1(s)}{m_2(s) + n_2(s)}$$

then,

$$Re\ Z(jw) = \frac{m_1(jw)m_2(jw) - n_1(jw)n_2(jw)}{m_2^2(jw) - n_2^2(jw)} \quad \text{.......(2.1)}$$

To insure that Re Z(jw) is always equal to zero, there are three nontrivial possibilities :

(i) When $m_1 = 0$ and $n_2 = 0$

(ii) " $m_2 = 0$ " $n_1 = 0$

(iii) " $m_1 m_2 - n_1 n_2 = 0$

The first possibility leads to Z(s) in the form :

$$Z(s) = \frac{n_1}{m_2} \quad \text{..........(2.2a)}$$

The second leads to :

$$Z(s) = \frac{m_1}{n_2} \quad \text{..........(2.2b)}$$

And the third leads to :

$$Z(s) = \frac{n_1}{m_2} \qquad \text{..........(2.2c)}$$

So the only two forms in which Z(s) may appear are :

$$Z(s) = \frac{m(s)}{n(s)} \qquad \text{or} \qquad Z(s) = \frac{n(s)}{m(s)}$$

And Z(s) is always the quotient of even to odd or odd to even polynomials and is a PRF.

2.3. Properties Of Reactance Functions In One Variable

(a) Let H(s) be a polynomial :

$$H(s) = m(s) + n(s)$$

where m(s) and n(s) are its even and odd parts respectively . Then the ratio m(s)/n(s) or n(s)/m(s) is a reactance function if and only if H(s) is a Hurwitz polynomial.

(b) A real rational function is a reactance function if and only if its zeros and poles are simple and alternate along the imaginary axis of the s plane.

(c) Let Z(s) be a reactance function , then the residues at the imaginary axis poles are real and positive , that is Z(s) can be written as :

$$Z(s) = Hs + \frac{K_0}{s} + \sum_{r=1}^{n} \frac{2K_{pr}\, s}{s^2 + w_{pr}^2} \qquad \text{.........(2.3)}$$

or

$$Y(s) = Hs + \frac{K_0}{s} + \sum_{r=1}^{n} \frac{2K_r\, s}{s^2 + w_r^2} \qquad \text{..........(2.4)}$$

with all poles and zeros of Z(s) on the imaginary axis.

Also :

$$Z(jw) = \text{Re}\, Z(jw) + \text{Im}\, Z(jw)$$

and ,

$$\text{Re } Z(jw) = 0 ,$$

we have :

$$Z(jw) = j\text{Im } Z(jw) = jX(w)$$

where X(w) is an odd function .

(d) There are four possibilities depending upon wheather the function has a pole at the origin or infinity or both.

	Origin	Infinity
Class I	Pole	Pole
II	Zero	Pole
III	Pole	Zero
IV	Zero	Zero

These crtical frequencies can be removed by either an inductance or a capacitance.

(e) The slope of the reactance with respect to the frequency is always positive:

let

$$X(w) = -\frac{K_0}{w} + \sum_{r=1}^{n} \frac{2K_{pr}\, w}{-w^2 + w_r^2} + Hw \qquad \text{..........(2.5)}$$

and hence

$$\frac{dX}{dw} = \frac{K_0}{w^2} + \sum_{r=1}^{n} \frac{2K_{pr}(w^2 + w_r^2)}{(-w^2 + w_r^2)^2} + H \qquad \text{..........(2.6)}$$

Now every factor in this equation is positive for all values of w.

Therefore,

$$\frac{dX}{dw} \geqslant 0 \qquad \text{for} \quad -\infty < w < \infty .$$

and the slope of the reactance versus frequency curve is always non-negative.

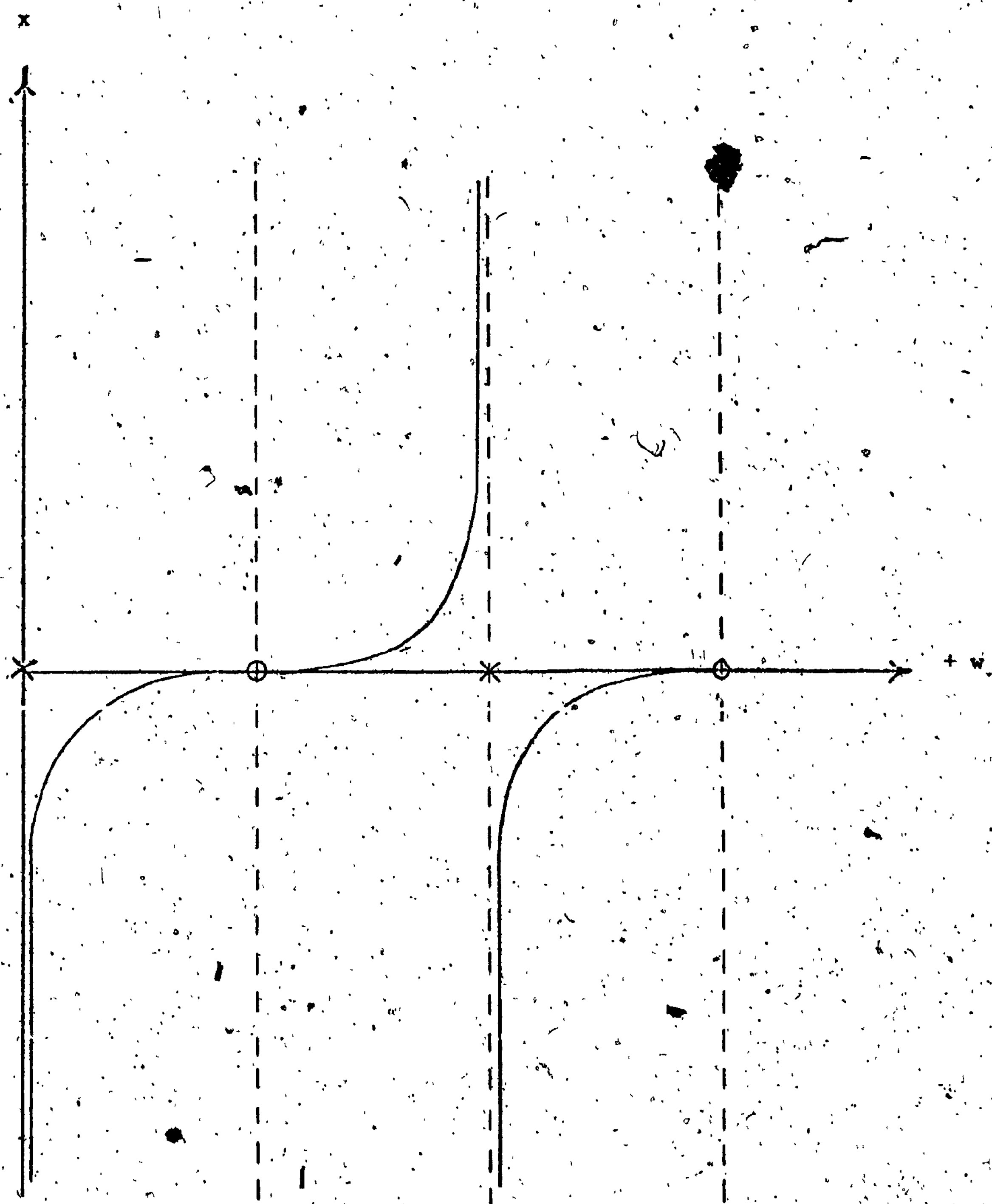


Fig. 2.1
Slope of SRF when it contains a Pole at origin.

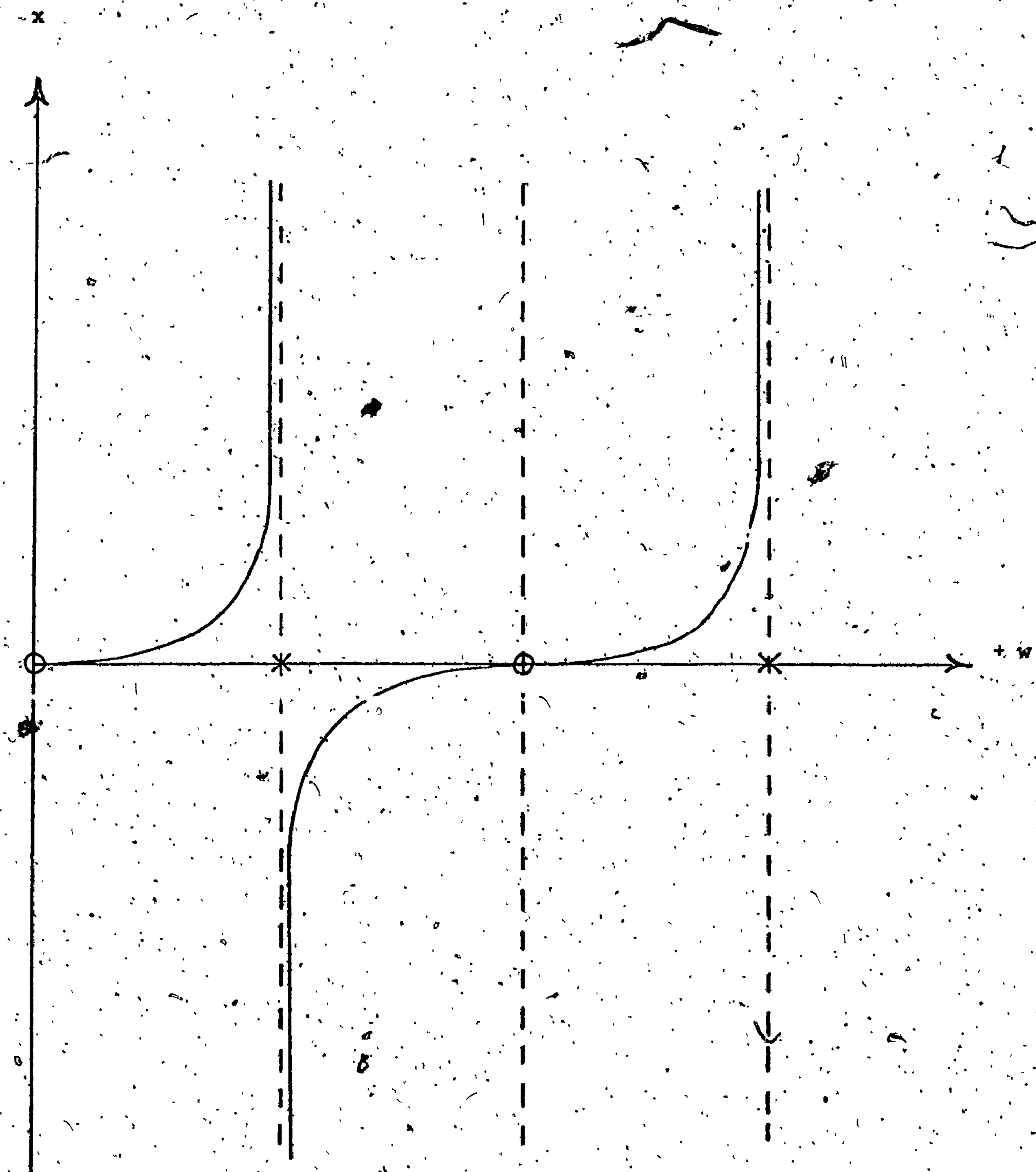


Fig.2.2

Slope of SRF when it contains a Zero at origin .

There is always either a pole or a zero at s=0 for Z(s) and the reactance curve must have the form shown in Figs. 2.1 and 2.2 respectively.
Inspection of these two curves shows that since the slope must always be positive, it is necessary that the poles and zeros separate each other and they alternate along the imaginary axis.

(f) From the foregoing discussion, we know that any SRF corresponding to Class I can be written as :

$$Z(s) = \frac{H\,(s^2 + w_1^2)(s^2 + w_3^2)\,\ldots\ldots\ldots\,(s^2 + w_{n+1}^2)}{s\,(s^2 + w_2^2)(s^2 + w_4^2)\ldots\ldots\ldots\,(s^2 + w_n^2)} \quad \ldots\ldots\ldots(2.7)$$

where $w_1, w_2, w_3, \ldots\ldots w_{n+1}$ are the internal critical frequencies. If these are known, then only one additional information must be given in order to completely specify Z(s). This fact must be the value of H or equivalent information, such as the value of the reactance at some non-critical frequency or the slope of the reactance curve at some frequency which is not a pole.
The same conclusions hold for the other Classes also.

2.4. Foster Form Realization Of SRF

The Foster form, obtained by the partial fraction expansion of the SRF is a canonic structure containing minimum number of elements equal to the number of internal critical frequencies plus one.

2.4.1. Foster's First Form

In general, the partial fraction expansion of Z(s) can be written as the summation of a number of terms as shown in Equ.(2.3). Each term can be identified as a parallel combination of an inductor and a capacitor. Thus the complete network will be a series combination of a capacitor, a number of parallel LC networks

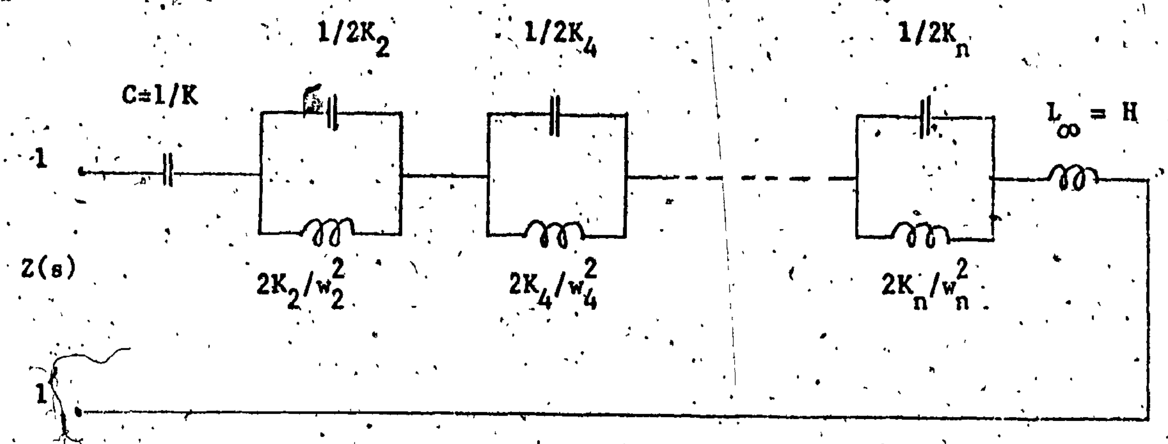


Fig.2.3

Foster's First Form

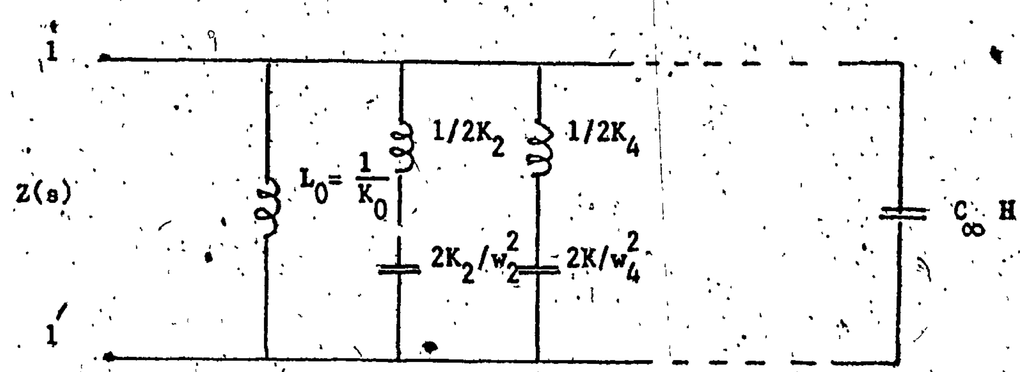


Fig.2.4

Foster's Second Form

and an inductor as shown in Fig. 2.3.

2.4.2. Foster's Second Form

The partial fraction expansion of Y(s) can be written as the summation of a number of terms as given in Equ.(2.4). Each term can be identified as a series combination of an inductor and a capacitor. Thus the complete network will be a parallel combination of an inductor , a number of series LC networks and a capacitor as shown in Fig.2.4.

Since there are four Classes of reactance functions , the different Foster forms are as shown in Table 2.1.

2.5. Cauer's Realization Of SRF

The second form of realization is called Cauer's form , and can be obtained by successive removal of poles from the reactance function and the final expansion will be in the form of a continued fraction expansion .

2.5.1 Cauer's First Form

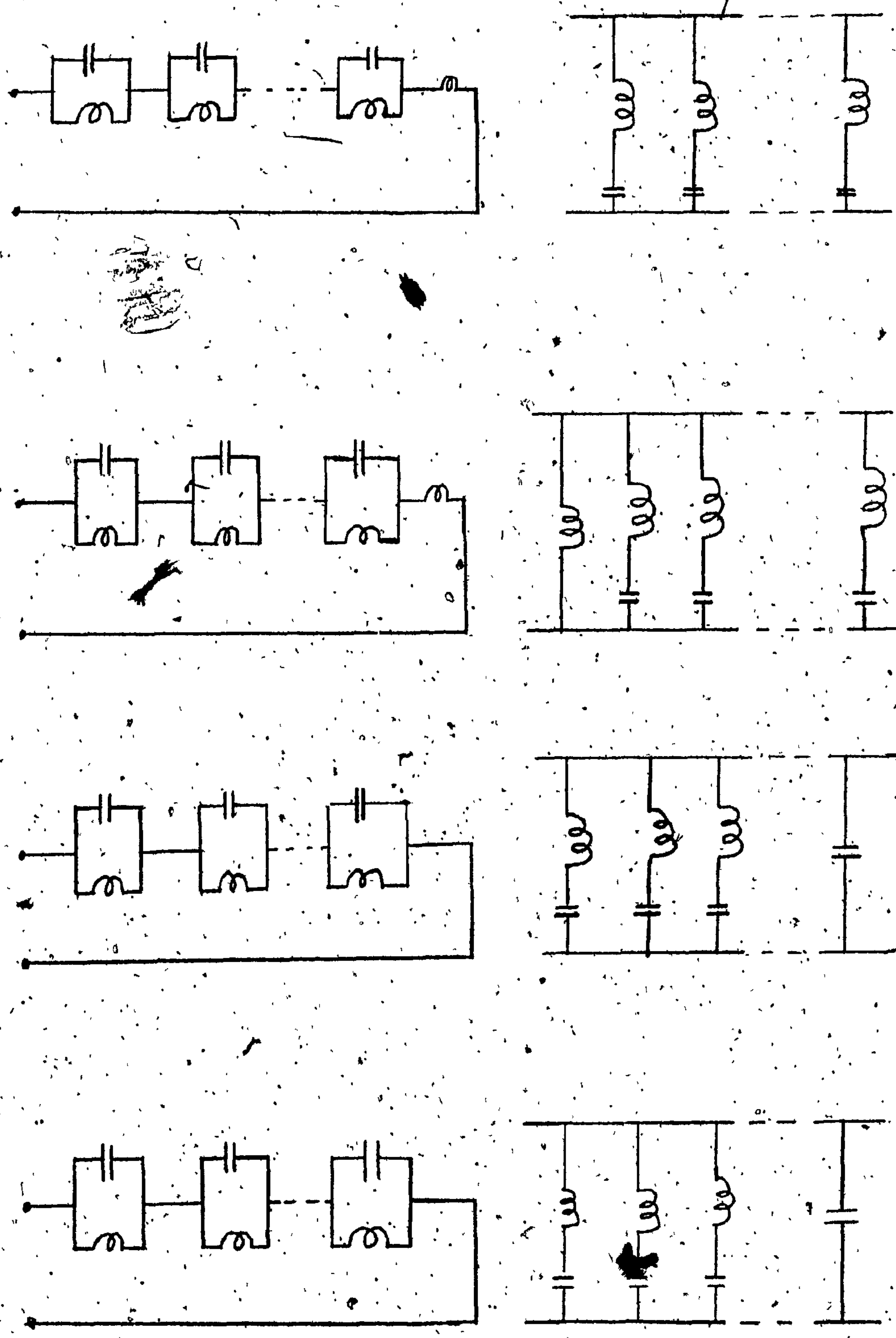
This form can be obtained by successive removal of poles at infinity of the reactance function. Then the resulting function is inverted to create a pole at infinity which is removed , and this process is continued until the expansion is complete.

The general structure of the network is as shown in Fig.2.5. where the inductors are in the series arms and the capacitor are in the shunt arms.

2.5.2. Cauer's Second Form

This form can be obtained by successive removal of poles at the origin of the reactance function . The resulting function is then inverted to create a pole at the origin which is removed , and this process is continued until the expansion is complete.

Class	Origin	Infinity
I	Pole	Pole
II	Zero	Pole
III	Pole	Zero
IV	Zero	Zero



Foster's First Form   Foster's Second Form

Table 2.1

Foster's Forms corresponding to the four Classes of SRFs

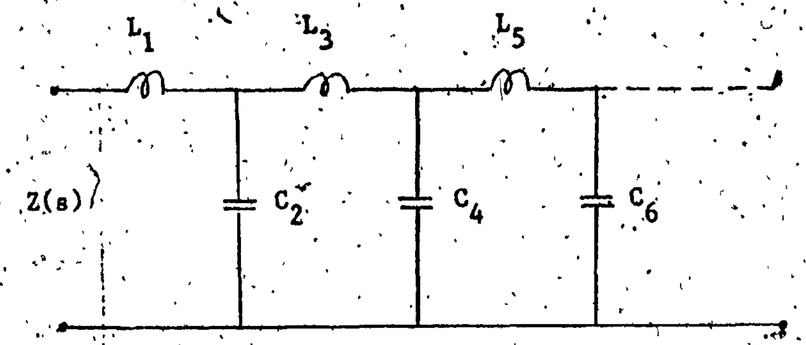


Fig.2.5

Cauer's First Form

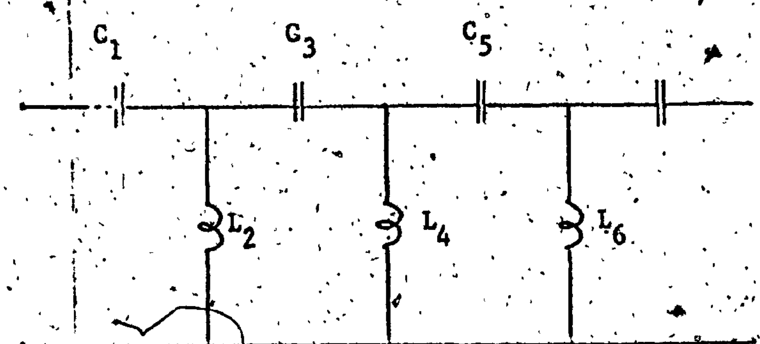
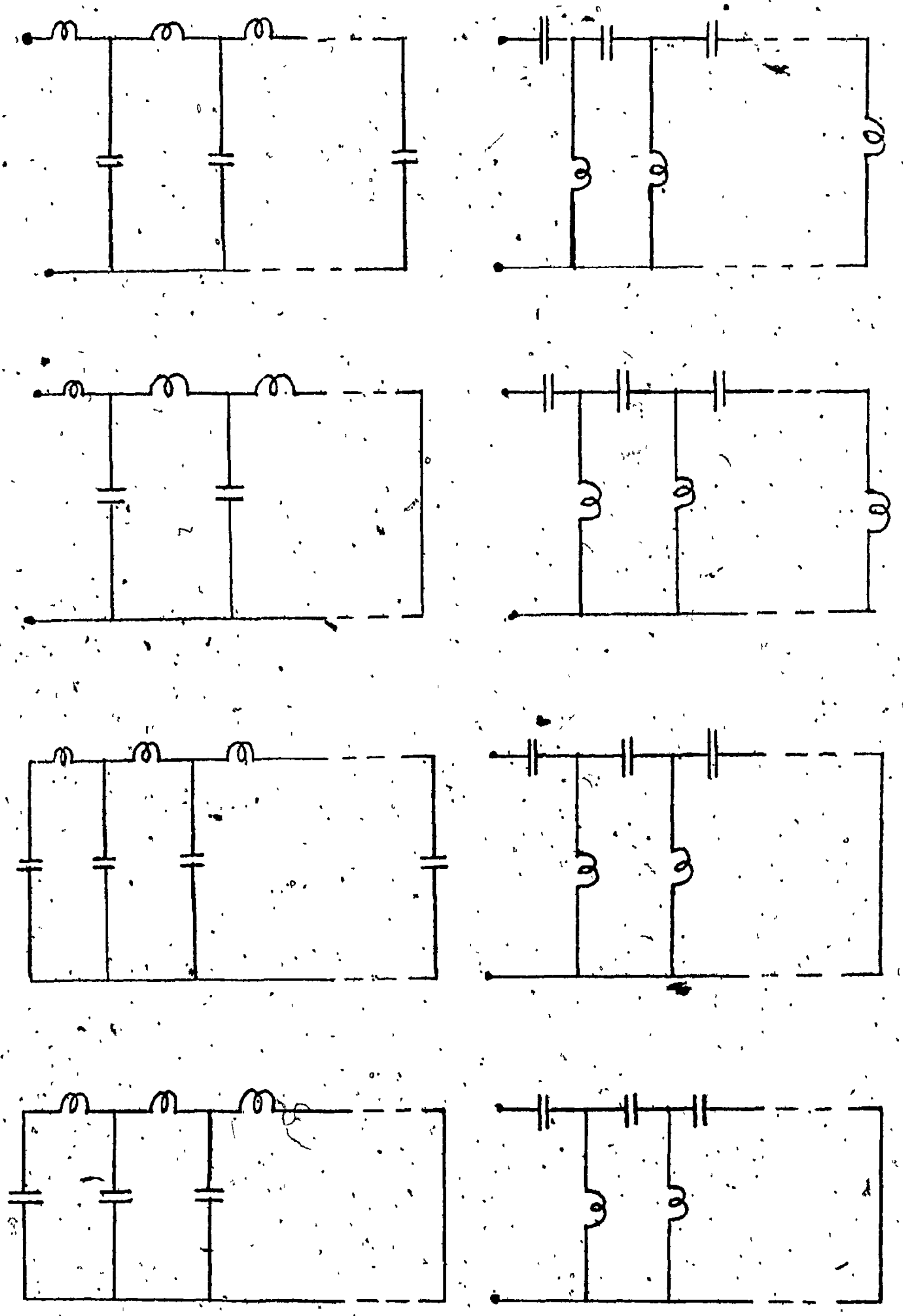


Fig.2.6

Cauer's Second Form

Class	Origin	Infinity
I	Pole	Pole
II	Zero	Pole
III	Pole	Zero
IV	Zero	Zero



Cauer's First Form   Cauer's Second Form

Table 2.2

Cauer's Forms Corresponding to the four Classes of SRFS

The general structure of the network is shown in Fig.2.6, where the capacitors are in the series arms and the inductors are in the shunt arms.

Since there are four Classes of reactance functions , the different Cauer's forms are as shown in Table 2.2.

Example :

$$\text{Let : } Z(s) = \frac{s(s^2 + 2)}{(s^2 + 1)(s^2 + 4)} \quad \ldots\ldots\ldots (2.8)$$

(a) Foster's First form of realization :

$$Z(s) = \frac{As}{(s^2 + 1)} + \frac{Bs}{(s^2 + 4)}$$

$$Z(s) = \frac{\frac{1}{3}s}{(s^2 + 1)} + \frac{\frac{2}{3}s}{(s^2 + 4)}$$

from which the realization will be as shown in Fig. 2.7(a)

(b) Foster's Second form of realization :

$$Y(s) = \frac{(s^2 + 1)(s^2 + 4)}{s(s^2 + 2)}$$

$$Y(s) = s + \frac{2}{s} + \frac{1}{s+2/s}$$

of which the realization will be as shown in Fig.2.7(b).

(c) Cauer's First form of realization :

$$Y(s) = s + \frac{3s^2 + 4}{s^3 + 2s}$$

then $$Z_1(s) = \frac{s^3 + 2s}{3s^2 + 4}$$

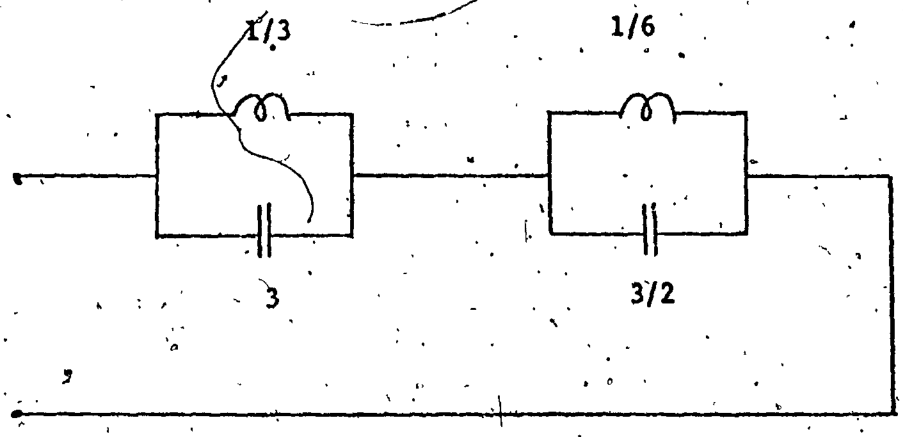


Fig.2.7(a)

Foster's First form realization of Z(s) given in equ.(2.8)

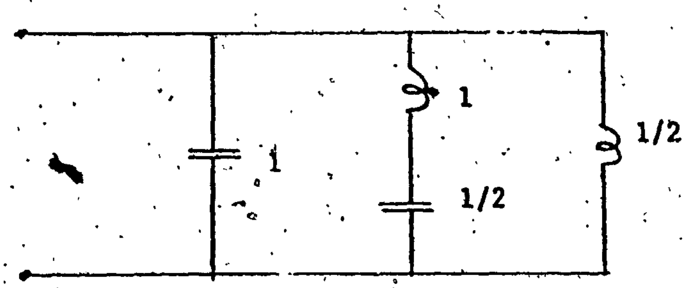


Fig.2.7(b)

Foster's Second form realization of Z(s) given in equ.(2.8)

$$3s^2+4\,)\;s^3+2s\;(\;\frac{s}{3}$$
$$\frac{s^3+\frac{4s}{3}}{\frac{2s}{3}}$$

then $$Z_1 = \frac{s}{3} + \frac{\frac{2}{3}s}{3s^2+4}$$

and $$Y_2 = \frac{3s^2+4}{\frac{2}{3}s}$$

for which the realization will be as shown in Fig.2.7(c).

(d) Cauer's Second Form of realization :

$$Z(s) = \frac{s(s^2+2)}{(s^2+1)(s^2+4)}$$

which has a zero at the origin :

then Y(s) has a pole at the origin = $\frac{2}{s}$

$$Y(s) = \frac{(s^2+1)(s^2+4)}{s(s^2+1)}$$

removing this pole ,

then $$Y_1(s) = \frac{s(s^2+3)}{(s^2+2)}$$

and $$Z_1(s) = \frac{s^2+2}{s(s^2+3)}$$

which has a pole at the origin = $\frac{2}{3s}$

then $$Z_2(s) = \frac{s}{3(s^2+3)}$$

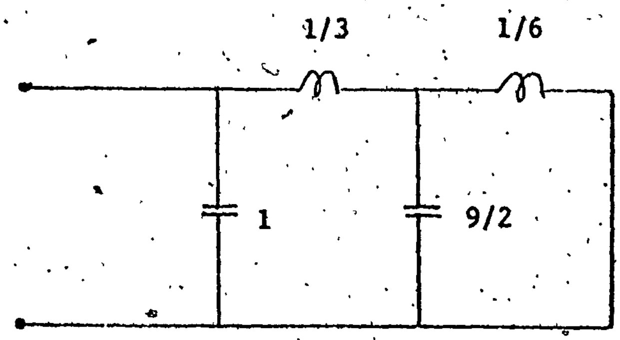


Fig.2.7(c)

Cauer's Frist form of realization of Z(s) given in equ.(2.8)

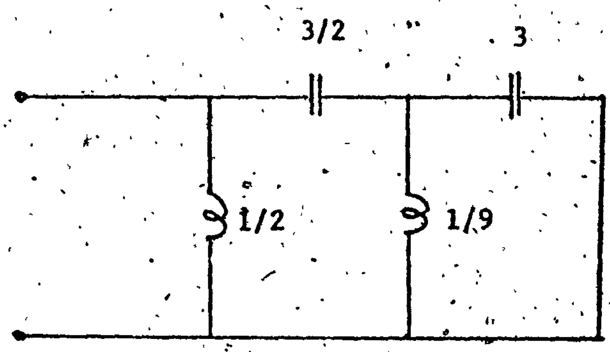


Fig.2.7(d)

Cauer's Second form of realization of Z(s) given in equ.(2.8)

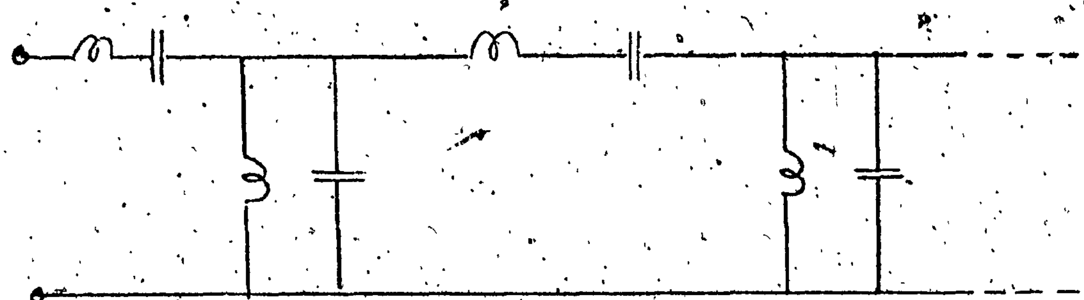


Fig.2.8

Cauer's Third Form

then $Y_2(s) = \frac{3(s^2+3)}{s}$

which has a pole at the origin $= \frac{9}{s}$

then $Y_3(s) = \frac{3(s^2+2)}{s} - \frac{9}{s} = 3s$

and $Z_3(s) = \frac{1}{3s}$

for which the realization is as shown in Fig.2.7(d).

2.6. Cauer's Third Form

This structure is as shown in Fig.2.8. This is the mixed form which is obtained by the continued fraction expansion of Z(s) alternating about origin and infinity.

2.7. Lee's Canonic Forms (2,3).

The Foster and Cauer realizations can be regarded as resulting from Canonic cycles of order two. By this, we mean that when two elements namely an inductor and a capacitor are removed from the given Z(s) or Y(s) of order n , the remaining $Z_1(s)$ or $Y_1(s)$ is of order (n-2) and is of the same form as that of Z(s) or Y(s).

In Lee's structures , the canonic cycle is of order four , that is , from the given Z(s) or Y(s) , two inductors and two capacitors are extracted so that the remaining reactance function is of order (n-4) and is of the same form as that of Z(s) or Y(s).

Lee's structure are as shown in Figs.2.9,2.10 and 2.11. As can be seen , Figs. 2.9 and 2.10 are non-symmetrical lattice structures where the series elements are of opposite kinds (and hence the shunt elements are also of opposite kinds) , that is one series arm consists of an inductor , the other series arm consists of a capacitor.

The structure shown in Fig.2.11 is a non-symmetrical Bridged - T Structure

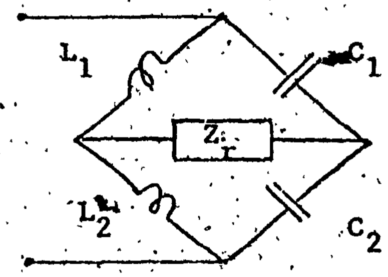


Fig.2.9

Lee's Non-Symmetrical Canonic Form I

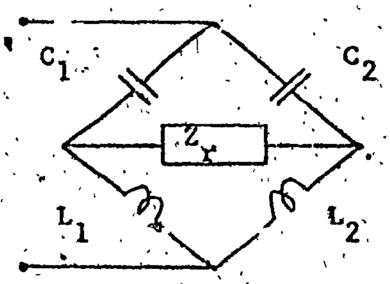


Fig.2.10

Lee's Non-Symmetrical Canonic Form II

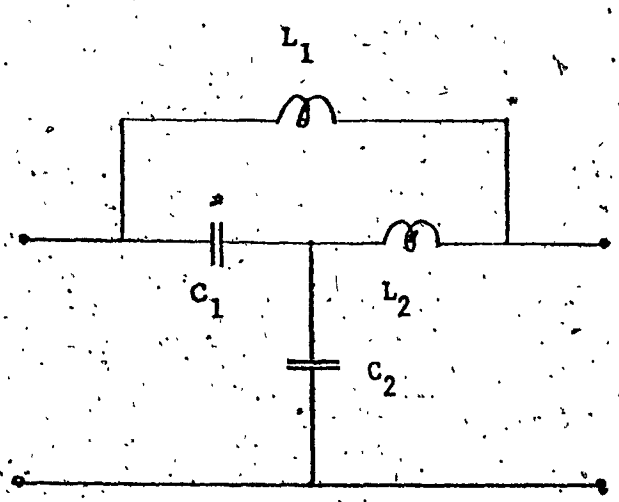


Fig.2.11

Lee's Bridged-T Canonic Structure

and variations of this are also possible.

2.8. Kida's Non-Symmetrical Lattice Structures (4,5).

The non-symmetrical lattice structure considered is as shown in Fig.2.12. It is seen that the series arms contain elements of the same kind. Yarlagadda and Tokad (4) found the conditions under which such structure can be extracted from a given reactance function. Beckoff (13,14) gave alternative proof for the conditions. However Kida (5) showed that a given reactance function can always be made to satisfy the above conditions thereby establishing that this non-symmetrical lattice constitutes a canonic cycle of order four.

2.9. Ramachandran and Swamy Structure (6)

Ramachandran and Swamy considered a Twin T-Structure as shown in Fig.2.13. They established that it is always possible to extract this structure (consisting of three inductors and three capacitors), from a given reactance function thereby consistituting a canonic cycle of order six.

They also showed that there are variations of this canonic cycle.

There exist two different realizations of the form shown in Fig.2.13. Another form of realization is shown in Fig.2.14.

Thus Ramachandran and Swamy established that there exist four different realizations of a sixth order canonic cycle of a SRF.

Discussion

Thus we see that, there exist only six basic forms of Canonic structures in the literature. Mixed Canonic realizations are posible. However, they cannot be considered as basic Canonic cycles, because they are only variations of the existing Canonic realizations.

These Canonic cycles are applicable for two-element kind functions which include resistances by similar transformations.

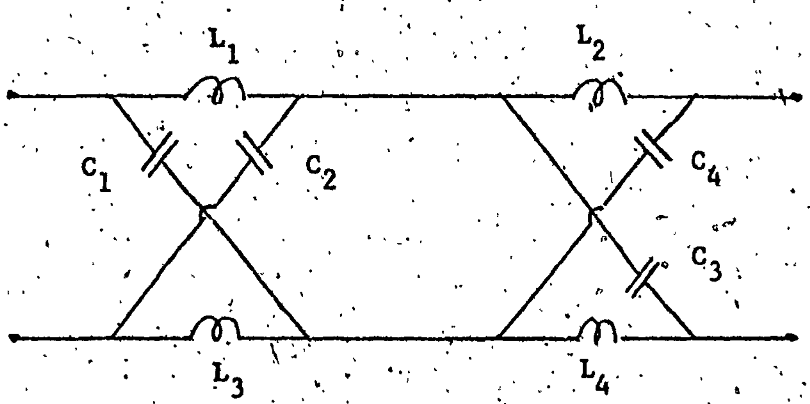


Fig.2.12

Kida's Non-Symmetrical Lattice Structure

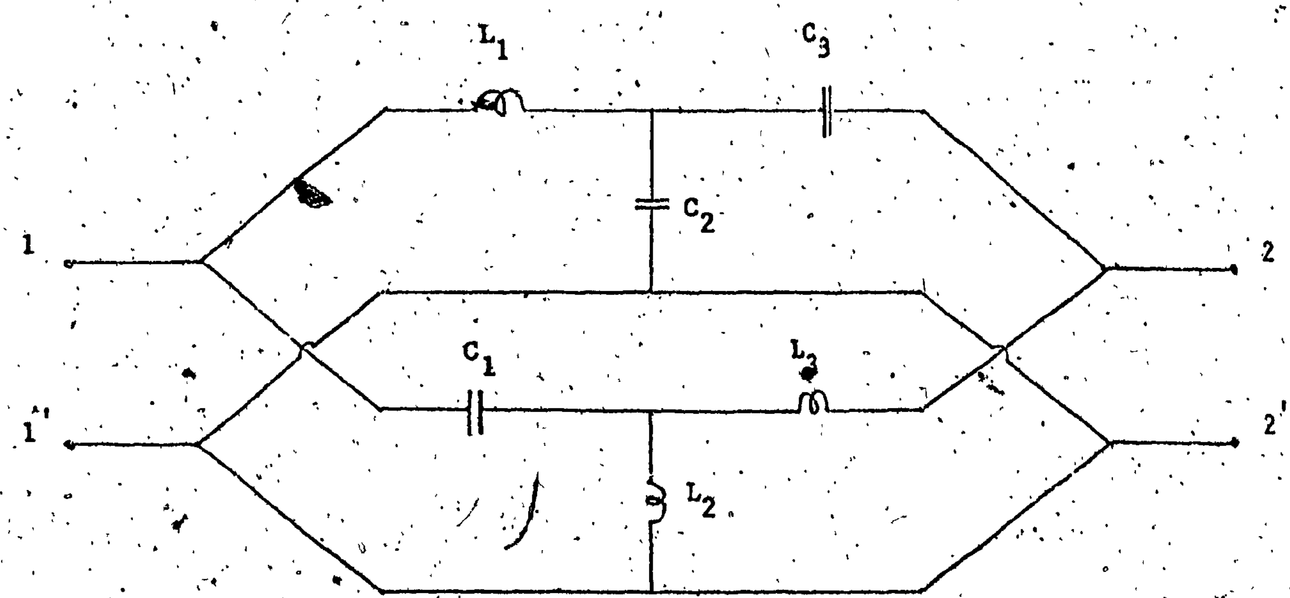


Fig.2.13

Ramachandran and Swamy Twin-T Canonic Structure

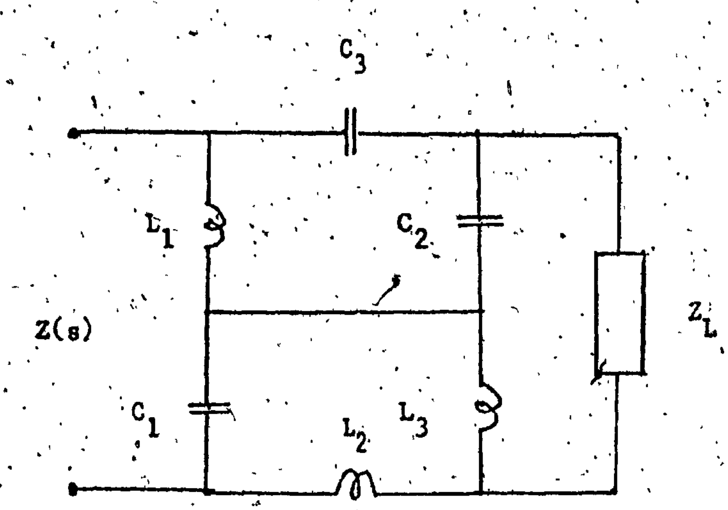


Fig.2.14

Alternate Form of Ramachandran And Swamy Twin-T Canonic Structure

CHAPTER III

CANONIC REALIZATIONS OF TWO VARIABLE REACTANCE FUNCTIONS WITH ONE ELEMENT IN EACH VARIABLE

3.1. Introduction

In this chapter, we shall discuss the two different realizations of two-variable reactance functions, which are :

(i) In the form similar to the single variable Foster form where any section consists of one element in each variable, and

(ii) In the form similar to the single-variable Cauer form, where the series and the shunt arms consist of one element in each variable.

3.2. Foster Form Realization (8)

In this form we consider two different Classes of structures:

Class A : Inductors having reactances p_1L and capacitors having reactances $1/p_2C$.

Class B : Inductors having reactances p_2L and capacitors having reactances $1/p_1C$

Tables 3.1 and 3.2 give the two Canonic structures similar to the single variable Foster form for these two classes.

The necessary and sufficient conditions for the realizability of the above two Classes of structures are given in the following two Theorems:

Theorem 3.1

The necessary and sufficient conditions for the two variable reactance function :

$$Z(p_1,p_2) = \frac{N(p_1,p_2)}{D(p_1,p_2)}$$

to be realizable by the Class A structures are :

(i) $Z(p_1,1)$ is an RL-impedance function of the variable p_1.

(ii) $Z(1,p_2)$ is an RC-impedance function of the variable p_2.

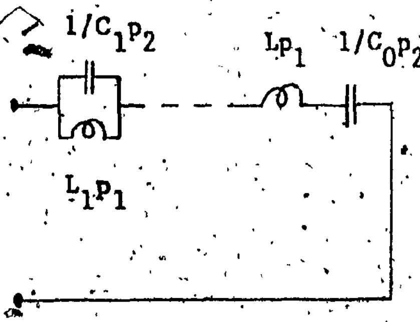
(iii) $Z(p_1,1)$ and $Z(1,p_2)$ posses the same internal critical frequencies.

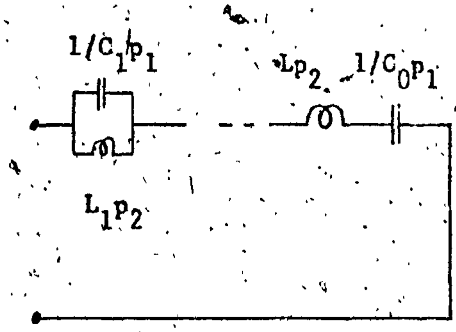
Proof :

The proof of the necessity is quite obvious from the Foster form and thus

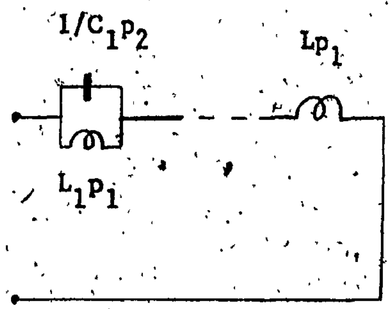
Origin Infinity

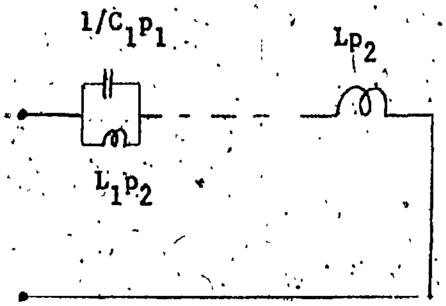
Pole Pole



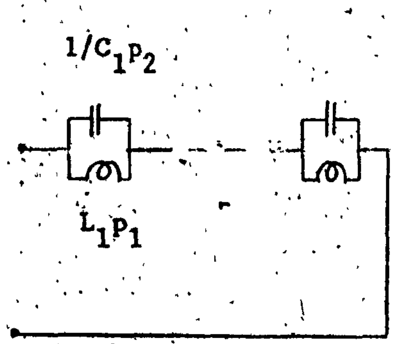


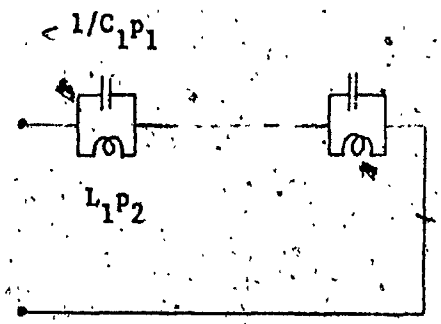
Zero Pole



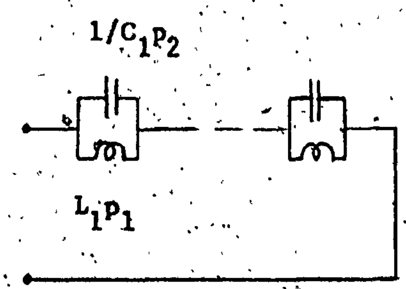


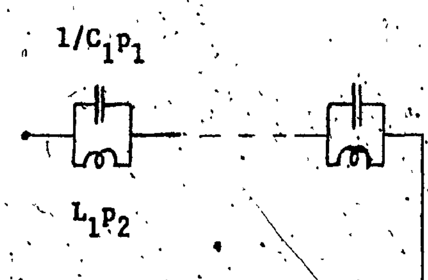
Pole Zero





Zero Zero





Class A Class B

Table 3.1

Foster First Form of TRF's with One Element in Each Variable in a section

Origin	Infinity
Pole	Pole
Zero	Pole
Pole	Zero
Zero	Zero

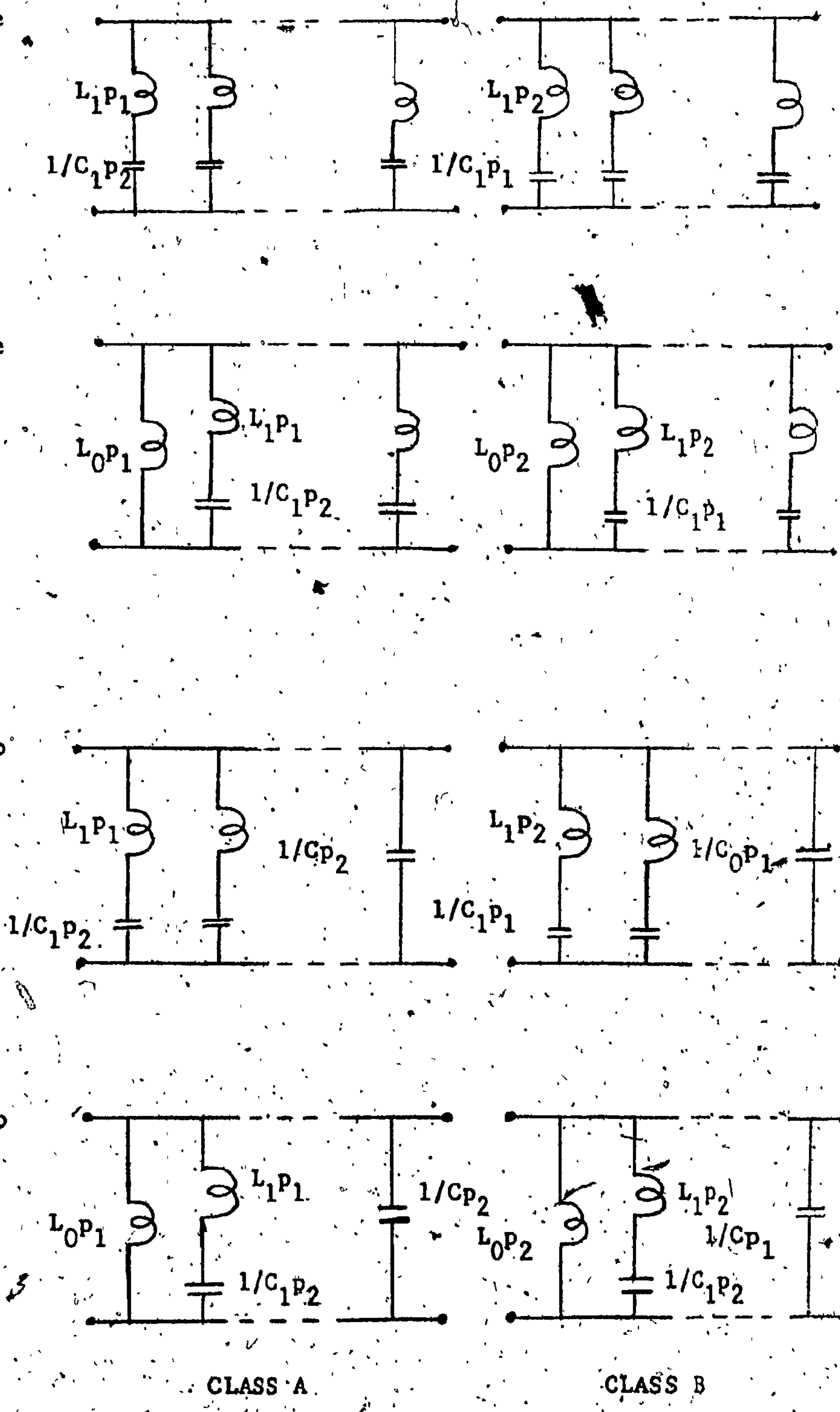


Table 3.2

Foster Second Form of TRF's with One Element in Each Variable in a section we prove the sufficiency only.

Sufficiency :

The impedance function that satisfies conditions (i) and (ii) simultaneously could be only of the form :

$$Z(p_1,p_2) = \frac{p_1\prod(p_1p_2 + Z_i)}{(p_1p_2 + S_i)} \quad \text{or} \quad \frac{\prod(p_1p_2 + A_i)}{p_2 \quad (p_1p_2 + B_i)} \qquad \text{........(3.1)}$$

where Z_i, S_i, A_i, B_i are all real, positive constants,

$$0 < S_1 < Z_1 < S_2 < Z_2 < \ldots$$

and $$Q < A_1 < B_1 < A_2 < B_2 < \ldots$$

First consider :

$$Z(p_1,p_2) = \frac{p_1\prod(p_1p_2 + Z_i)}{\prod(p_1p_2 + S_i)} \qquad \text{........(3.2)}$$

Then

$$Z(p) = \frac{Z(p_1,p_2)}{p_1} = a_0 + \frac{a_1}{p} + \sum \frac{a_i}{(p + S_i)} \qquad \text{........(3.3)}$$

where $p = p_1p_2$ and a's are positive real constants, is an RC-impedance function in p-plane. Thus we see that $Z(p_1,p_2)$ can be realized by p_1-type inductors and p_2-type capacitors.

The realizability conditions for Class B structures are given in Theorem 3.2, the proof of which is similar to Theorem 3.1, and thus it is omitted for the sake of brevity.

Theorem 3.2

The necessary and sufficient conditions for the two variable reactance function $Z(p_1,p_2)$ to be realized by the Class B structures are :

(i) $Z(p_1 1)$ is an RC-impedance function.

(ii) $Z(1,p_2)$ is an RL-impedance function.

(iii) $Z(p_1,1)$ and $Z(1,p_2)$ possess the same internal critical frequencies.

From equ.(3.3), we notice that $Z(p)$ can be realized canonically by Foster first form as RC-impedance function in the variable p. Hence it can be realized by Foster second form, and other Canonic forms discussed in chapter II.

Hence $Z(p_1,p_2)$ which is realized by Foster first form as shown in Table 3.1can be realized by other forms of canonic networks also.

<u>Example</u> :

$$\text{Let } Z(p_1,p_2) = \frac{p_1(p_1^2p_2^2 + 6p_1p_2 + 8)}{(p_1^2p_2^2 + 4p_1p_2 + 3)} \qquad \text{........(3.4)}$$

be the given reactance function.

Put $p_1 = 1$

$$\text{Then} \qquad Z(1,p_2) = \frac{p_2^2 + 6p_2 + 8}{(p_2 + 1)(p_2 + 3)} = \frac{1}{2p_2 + 3} + \frac{1}{\frac{2}{3}p_2 + \frac{2}{3}}$$

Which is an RC function as shown in Fig.3.1.

Put $p_2 = 1$

$$\text{Then} \qquad Z(p_1,1) = \frac{p_1(p_1^2 + 6p_1 + 8)}{(p_1 + 1)(p_1 + 3)}$$

is an RL function.

It is also noted that both the above functions have the same internal critical frequencies. Thus the given reactance function satisfies the condition of Theorem I and hence must be realizable by the various forms mentioned in Chapter II. The Foster form for this function is as shown in Fig.3.2.

Thus from Theorems 3.1 and 3.2, it is seen that this type of two variable reactance function is realized by the following two steps :

(i) It is first converted into the corresponding $Z(p)$, which is an RC-impedance function in one of the variables, which is realized by the different Canonic forms.

(ii) The resistance is replaced by the appropriate inductance of the variable.

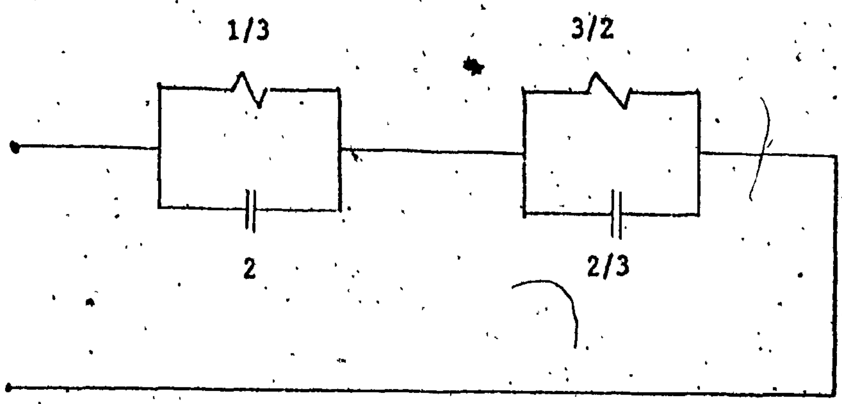


Fig.3.1

Foster's Form realization of $Z(p_1,p_2)$ given in equ.(3.4) at $p_1=1$

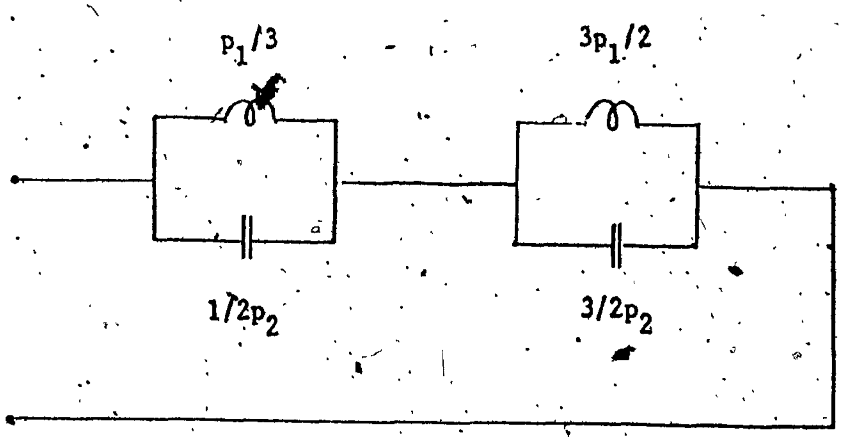


Fig.3.2

Foster's Form realization of $Z(p_1,p_2)$ given in equ.(3.4)

This means that, starting from $Z(p)$, the different single-variable Canonic RC realization can be obtained which leads to a corresponding two variable reactance function realization. Thus starting from the Foster form realization as shown in Table 3.1, the other two-variable structures similar to single-variable Canonic structures can be obtained. The typical sections of a cycle are shown in Table 3.3.

3.3. Cauer Ladder Structures With One Element Of Each Variable In The Shunt And Series Arms

In this section we shall consider ladder networks with one element of each variable in the series and the shunt arms. Before this, we shall discuss the development of two-variable array (9).

3.3.1. Two-Variable Array And Its Application :

In this part we shall discuss the two-variable array and show how the realizability conditions of the proposed ladder networks are derived from these arrays.

Let $$Z(p_1,p_2) = \frac{M(p_1,p_2)}{N(p_1,p_2)} \quad \text{or} \quad \frac{N(p_1,p_2)}{M(p_1,p_2)}$$

be the-variable reactance function of k^{th} degree in the variable p_1,p_2, and let :

$$\begin{aligned} H(p_1,p_2) &= M(p_1,p_2) + N(p_1,p_2) \\ &= (a_{k,0}\, p_1^k + a_{k-1,1}\, p_1^{k-1} p_2 + \dots\dots + a_{o,k}\, p_2^k) + \dots\dots + \\ &\quad + (a_{1,0}\, p_1 + a_{0,1}\, p_2) + a_{0,0} \end{aligned}$$

be the sum of net numerator and denominator (after cancellation of all non-constant polynomial factors common to numerator and denominator) of $Z(p_1,p_2)$. From this two-variable Hurwitz polynomial $H(p_1,p_2)$, we form the following three-rowed array:

1^{st} row : $a_{k,0}\ \ a_{0,k}\ \ a_{k-1,1} \cdots\ a_{1,k-1}\ \ a_{k-2,0}\ \ a_{k-3,1} \cdots\cdots\cdots\ a_{0,k-2} \cdots$

2^{nd} row : $a_{k-1,0}\ \ 0\ \ a_{k-2,1} \cdots\ a_{0,k-1}\ \ a_{k-3,0}\ \ a_{k-4,1} \cdots\cdots\cdots\ 0\ \cdots$

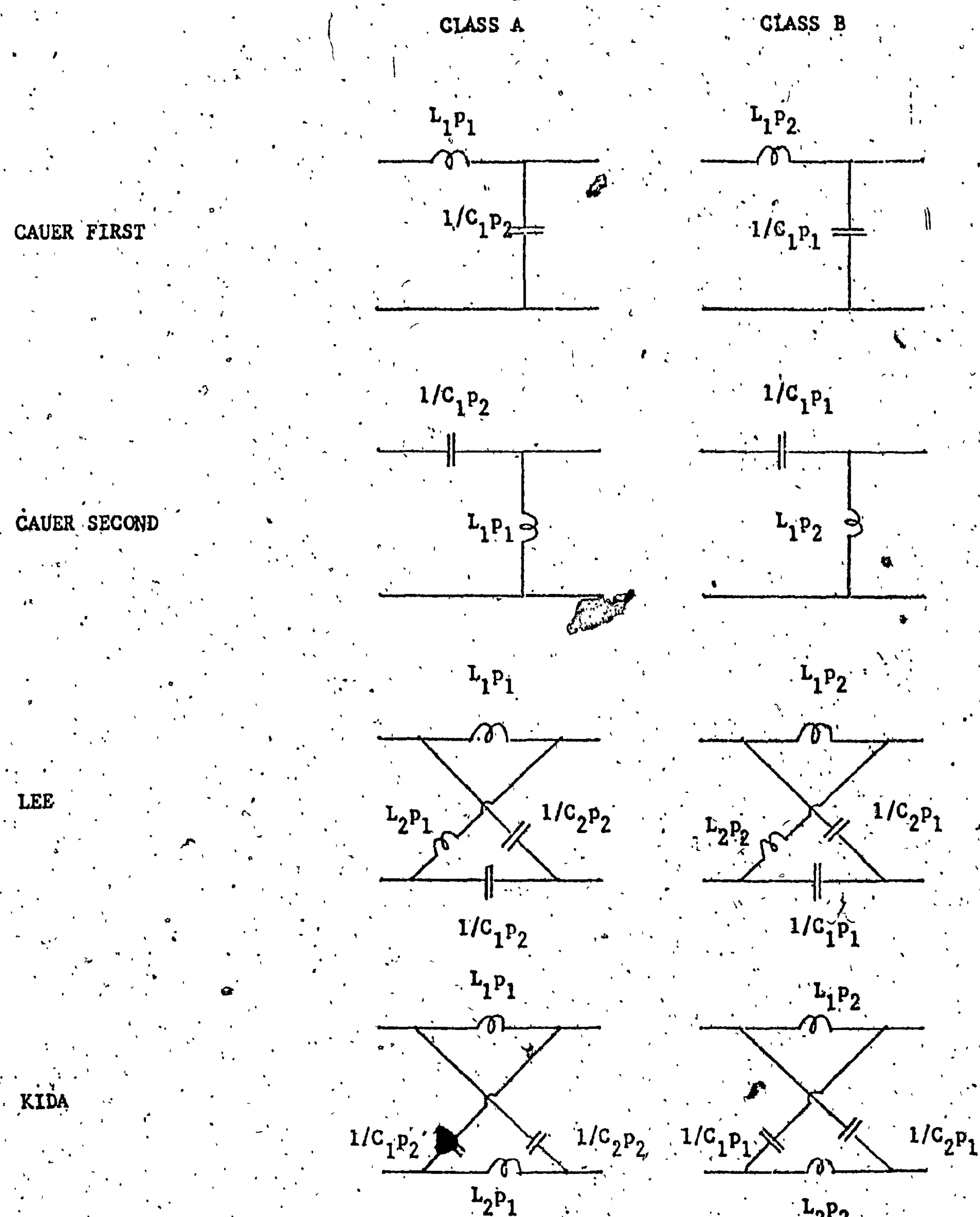


Table 3.3

Typical Sections for other Structures

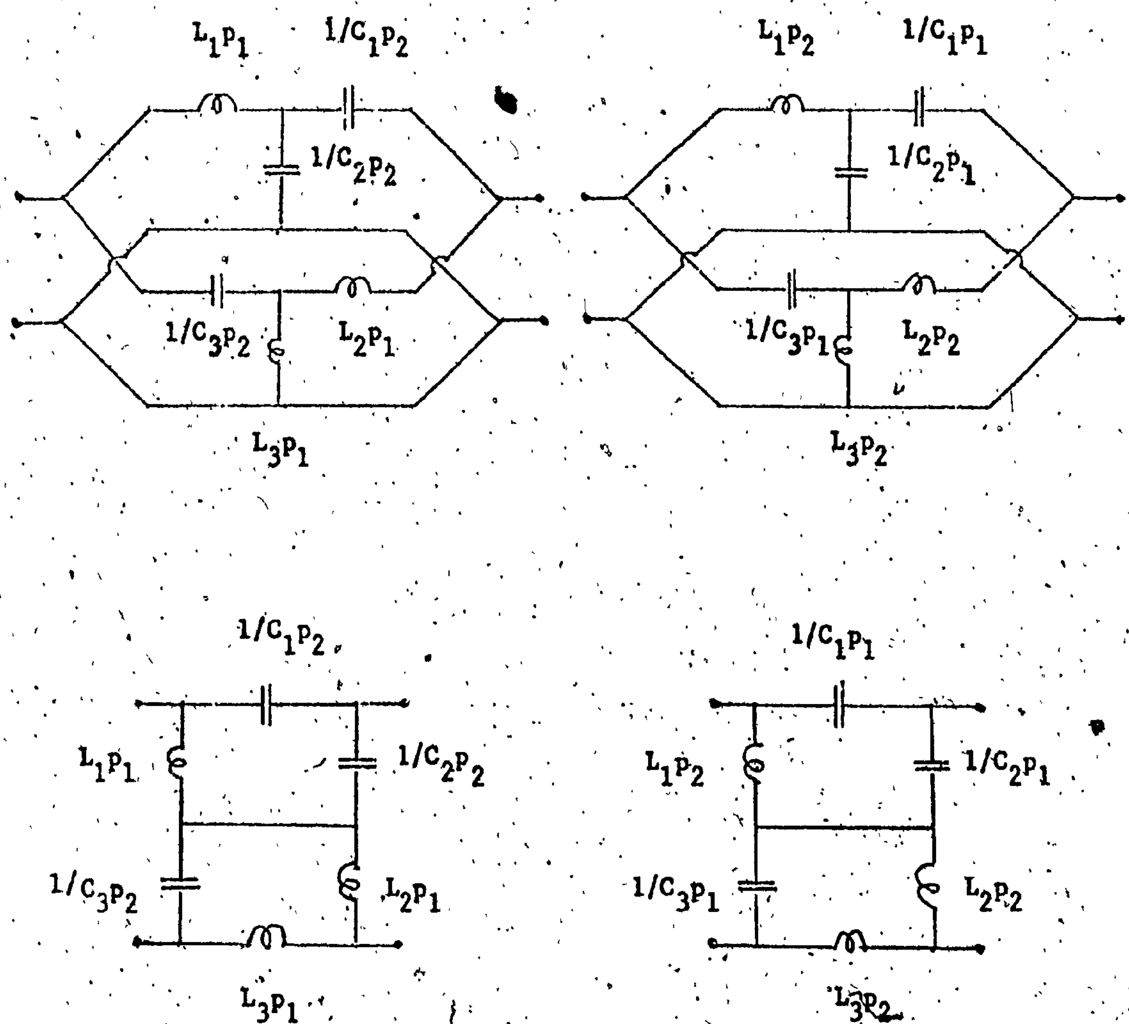


Table 3.3(cont.)

Typical Sections for other Structures

3^{rd} row : 0 $a_{0,k-1}$ $a_{k-1,0}$ .... $a_{1,k-2}$ 0 $a_{k-2,0}$ .... $a_{0,k-3}$ ....

The rules for writing the array are as follows :

(i) The first row contains terms of degrees, k, (k-2), (k-4), (k-6), ...etc. For convenience, the k^{th} degree terms are arranged first followed by (k-2), (k-4), (k-6) .....etc, degree terms.

(ii) The second and third rows contain (k-1), (k-3), (k-5), ...etc., degree terms in the following manner:

If $a_{x,y}$ is a particular term in the first row, then $a_{x-1,y}$ and $a_{x,y-1}$ are the terms respectively in the same column of the second and third row. If (x-1) or (y-1) is less than zero, then the corresponding term is zero.

From the above array, we form the following 3x3 determinats :

$$\Delta_{(k,0),(0,k),(x,y)} = \begin{vmatrix} a_{k,0} & a_{o,k} & a_{x,y} \\ a_{k-1,0} & 0 & a_{x-1,y} \\ 0 & a_{0,k-1} & a_{x,y-1} \end{vmatrix}$$

where 0 x k and 0 y k

and $$b_{x,y} = \frac{\Delta_{(k,0),\ (0,k),\ (x,y)}}{a_{k-1,0} \cdot a_{0,k-1}}$$

From the calculated values of $b_{x,y}$ and the earlier second and third row terms, the following new array is formed:

1^{st}row : $a_{k-1,0}$ $a_{0,k-1}$ $a_{k-2,1}$......$a_{1,k-2}$ $a_{k-3,0}$ $a_{k-4,1}$......$a_{0,k-3}$.....

2^{rd}row : $b_{k-2,0}$ 0 $b_{k-3,1}$......$b_{0,k-2}$ $b_{k-4,0}$ $b_{k-5,1}$......0.......

3^{rd}row : 0 $b_{0,k-2}$ $b_{k-2,0}$......$b_{1,k-3}$ 0 $b_{k-4,0}$......$b_{0,k-4}$

In this array, the first row contains terms of degrees (k-1), (k-3), (k-5))... etc. and second and third rows contain terms of degrees (k-2), (k-4)...etc. The rules of forming this array are the same as those for the earlier ones. From this array we then calculate the following determinants:

$$\Delta_{(k-1,0),\ (1,k-1),\ (x_1,y_1)} = \begin{vmatrix} a_{k-1,0} & a_{0,k-1} & a_{x_1,y1} \\ b_{k-2,0} & 0 & b_{(x_1-1),\ y_1} \\ 0 & b_{0,k-2} & b_{x_1,(y_1-1)} \end{vmatrix}$$

where $0 \leqslant x_1 \leqslant (k-1)$ and $0 \leqslant y_1 \leqslant (k-1)$.

and

$$c_{x_1,y_1} = \frac{\Delta_{(k-1,0),\ (0,k-1),\ (x_1,y_1)}}{b_{k-2,0} \cdot b_{0,k-2}}$$

From the above calculated values of c_{x_1,y_1} and $b_{x,y}$ we form the new array similar to the above and calculate d_{x_2,y_2}, c_{x_3,y_3},...etc.

3.3.2 Two-Variable Low-Pas Ladder Networks (TLPL)

Using the above developed arrays, the realizability conditions for the TLPL are given by the following theorem.

Theorem 3.3

The given two-variable reactance function

$$Z(p_1,p_2) = \frac{M(p_1,p_2)}{N(p_1,p_2)} \quad \text{or} = \frac{N(p_1,p_2)}{M(p_1,p_2)}$$

can be realized by the TLPL of Fig. 3.3, if and only if

(i) $H(p_1,p_2)$, the sum of net numerator and denominator of $Z(p_1,p_2)$ is a polynomial in p_1,p_2 with no missing terms.

(ii) when $H(p_1,p_2)$ is arranged in the form of the two variable arrays, the following conditions hold:

(a) $b_{x,y} = 0$ if $x + y = k$

> 0 if $x - y < k$

(b) $c_{x_1,y_1} = 0$ if $x_1 + y_1 = (k-1)$

> 0 if $(x_1 + y_1) < (k-1)$

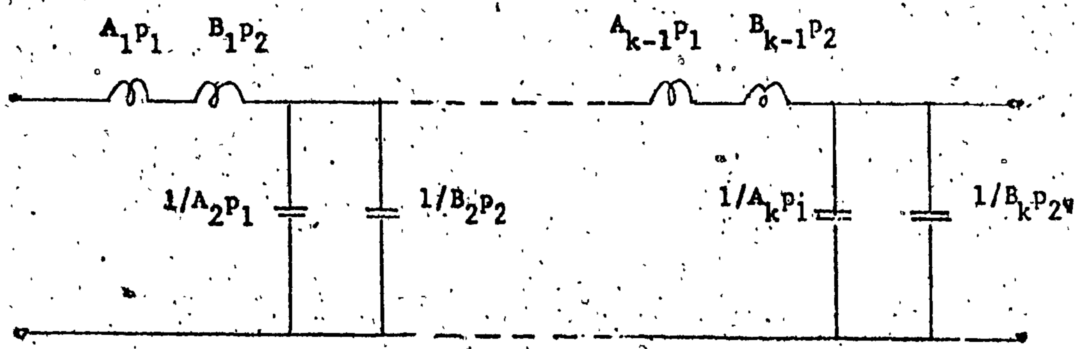


Fig.3.3

Two-variable Low-Pass Ladder Network

(c) and similar conditions hold for

$^{d}x_2$, y_2, $^{e}x_3$, y_1...etc.

Proof:

Necessity:

The transmission matrix of the two-part TLPL of Fig. 3. is the product of following matrices:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & A_1p_1 + B_1p_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ A_2p_1 + B_2p_2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ A_kp_1 + B_kp_2 & 1 \end{bmatrix}$$

By direct matrix multiplication, the necessity of condition (i) can be seen. If the given $Z(p_1, p_2)$ is realizable as TLPL, then the K^{th} degree terms of $H(p_1, p_2)$ must be divisible by the $(k-1)^{th}$ degree terms with a quotient of $(A_1p_1 + B_1p_2)$ where A_1, B_1 0 , with the remaining terms being positive.

In fact this is what condition (ii) (a) implies. Thus by repeating the same argument, the necessity of condition (ii) is proved.

Sufficiency:

For this part of the proof, we shall consider $Z(p_1, p_2) = \dfrac{N(p_1, p_2)}{M(p_1, p_2)}$

and the same arguments apply for the case $Z(p_1, p_2) = \dfrac{M(p_1, p_2)}{N(p_1, p_2)}$

condition (ii) (a) implies that the K^{th} degree terms of $H(p_1, p_2)$ are devisible by $(k-1)^{th}$ degree terms with a quotient of $(A_1p_1 + B_1p_2)$ with $A_1 > 0$ and $B_1 > 0$ and all remaining terms being positive. Thus

$$Z(p_1, p_2) = (A_1p_1 + B_1p_2) + \frac{N'(p_1p_2)}{M(p_1p_2)}$$ and N'/M is of $(k-1)^{th}$ degree in

p_1, p_2. By applying condition (ii) (b) we infer that

$$\frac{M(p_1, p_2)}{N'(p_1, p_2)} = (A_2p_1 + B_2p_2) + \frac{M'(p_1, p_2)}{N'(p_1, p_2)}$$

where $(A_2, B \quad 0)$ and M'/N' is of degree (k-2). Condition (i) implies that no determinant is trivially zero, that is, no column in the array has only zeros. Thus, by repeating the above argument, the sufficiency follows..

3.3.3 Other Types of Two-Variable Ladder Networks

By marking various transformations, to the TLPL, other types of Ladder Networks can be obtained as shown in Tables 3.4 & 3.5.

Example

Given

$$Z(p_1,p_2) = \frac{(p_1 + 3p_2)}{(2p_1^2 + 7p_1 p_2 + 3p_2^2) + 1} \qquad \text{......(3.5)}$$

Can be realized by the ladder structure shown in Fig. 3.4

As $Z(p_1,p_2)$ can be expressed as a continued fraction expansion ,

$$Z(p_1,p_2) = (p_1 + 2p_2) + \frac{1}{(2p_1 + p_2) + \frac{1}{p_1 + 3p_2}}$$

Since the numerator of $Z(p_1,p_2)$ cannot be factorized in the form required for Fosters' form expansion, this function is not realizable in the forms discussed in Table 3.1 and 3.2.

Discussion

In this chapter, we have discussed Foster's forms of some two-variable reactance functions. A typical section of the Foster form consists of one element in each variable, the element being of opposite kinds.

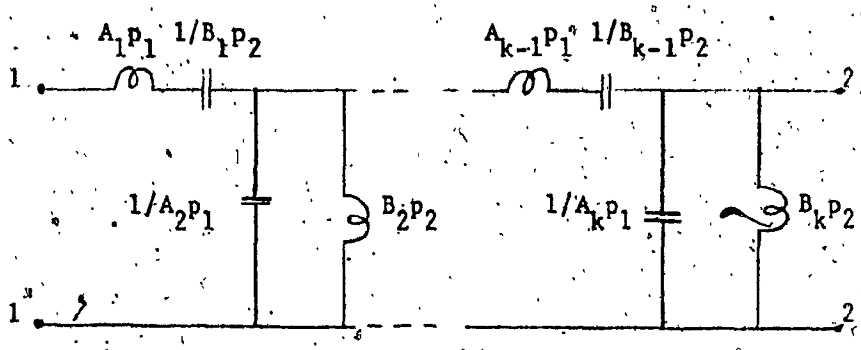
It is shown that if such a Foster form is realizable, corresponding forms similar to the single variable canonic forms are also realizable.

Also we have discussed the realizability conditions of ladder structures whose series and shunt arms consist of one element of each variable.

It may be noted that, if the two variable reactance function is realizable by TLPL or other ladder structures shown in Table3.3 it may not have the corresponding Foster form of the type discussed above, since the even and

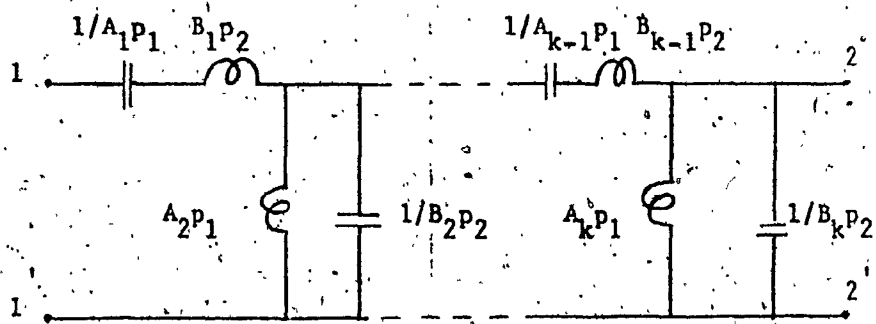
Transformations

$p_1 \longrightarrow 1/p_1$



(a)

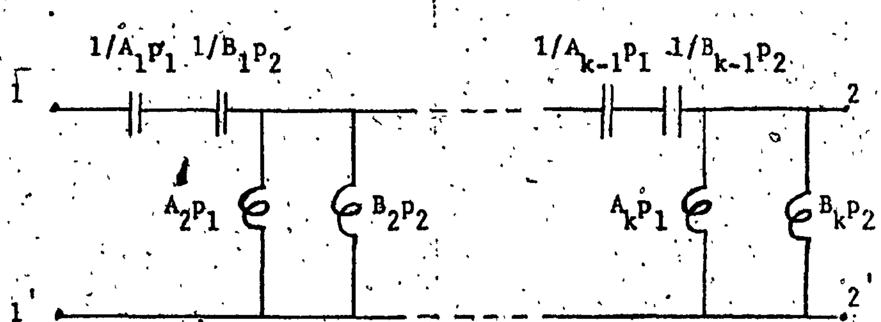
$p_2 \longrightarrow 1/p_2$



(b)

$p_1 \longrightarrow 1/p_1$

$p_2 \longrightarrow 1/p_2$



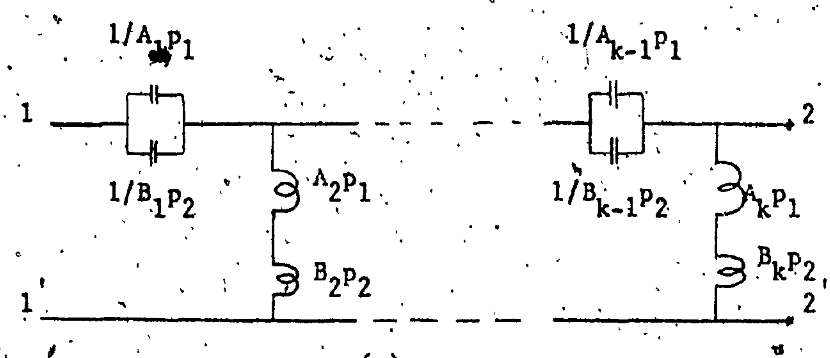
(c)

Table 3.4

Two-Variable Ladder Networks consisting of p_1 and p_2 type elements in series in the series arms and in parallel in the shunt arms

Transformations

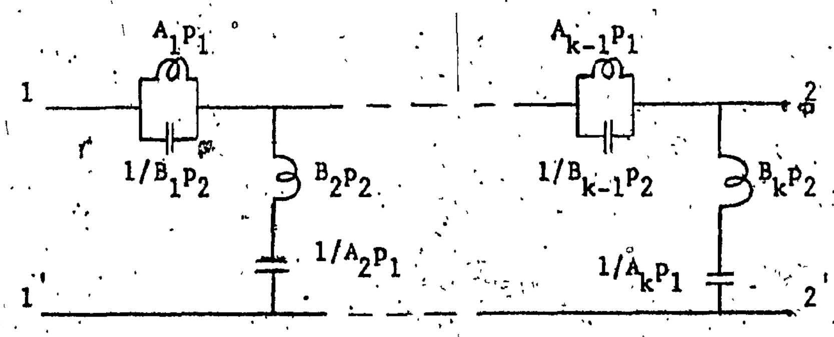
$A_1p_1 + B_1p_2 \rightarrow$ in TLPL $\frac{1}{A_1p_1+B_1p_2}$



(a)

$p_1 \rightarrow 1/p_1$

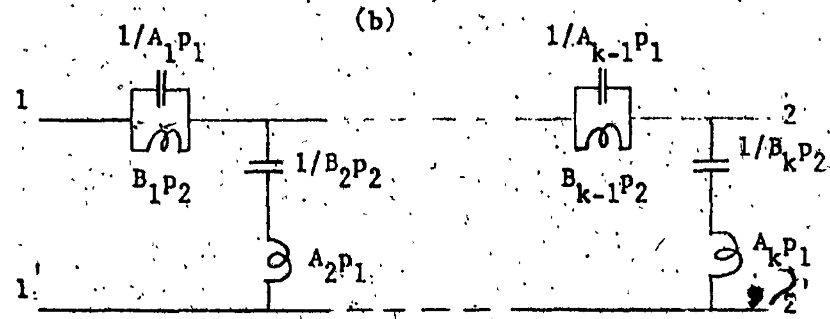
in (a)



(b)

$p_2 \rightarrow 1/p_2$

in (a)

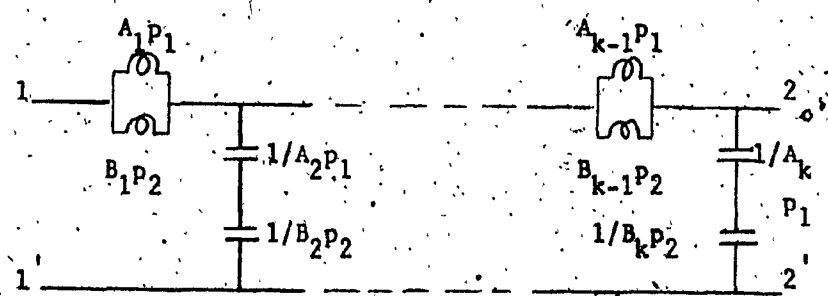


(c)

$p_1 \rightarrow 1/p_1$

$p_2 \rightarrow 1/p_2$

in (a)



(d)

Table 3.5

Two-Variable Ladder Networks consisting of p_1 and p_2 type elements in parallel in the series arms and in series in the shunt arms .

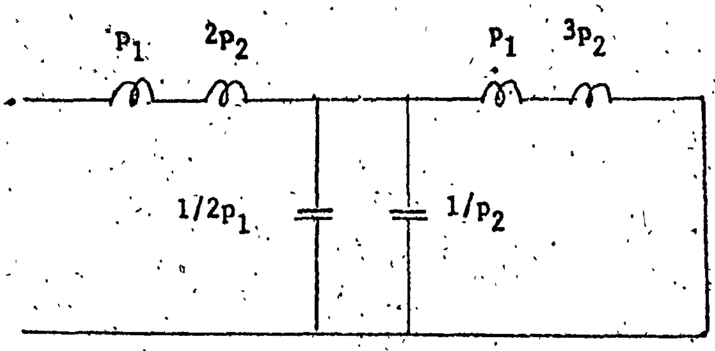


Fig.3.4

Ladder Structure Realization of $Z(p_1,p_2)$ given in equ.(3.5) odd polynomials of the TLPL functions may not necessarily be factorizable as required by the foster forms. It has to be inferred that in the case of single variable reactance structures, if a given function is realizable by the Cauer ladder form (or Foster, Lee, Kida, Ramachandran and Swamy forms), the other structures are always realizable, whereas in the two-variable case this is not necessarily true. For those two-variable functions for which all such structures are realizable are discussed only in (8).

CHAPTER IV

CANONIC REALIZATION OF TWO VARIABLE REACTANCE FUNCTION WITH TWO ELEMENTS IN EACH VARIABLE

4.1. Introduction

In this chapter, we shall discuss the realization of two variable reactance functions, using two elements in each variable, in a form similar to the single variable Foster form.

We propose the following four classes of structures in the study:

Class A: Two series inductors having reactances p_1L_1 and p_2L_2 in parallel with two parallel capacitors having reactances $\frac{1}{p_1C_1}$ and $\frac{1}{p_2C_2}$, as shown in Fig.4.1(a).

Class B: Two series capacitors having reactances : $\frac{1}{p_1C_1}$ and $\frac{1}{p_2C_2}$, in parallel with two parallel inductances having reactances p_1L_1 and p_2L_2, as shown in Fig.4.2(b).

Class C: Series inductor and capacitor having reactances p_1L_1 and $\frac{1}{p_2C_1}$ in parallel with series inductor and capacitor having reactances p_2L_2 and $\frac{1}{p_1C_2}$ as shown in Fig.4.1(c).

Class D: Two parallel inductors having reactances p_1L_1 and p_2L_2 in series with two parallel capacitors having reactances $\frac{1}{p_2C_1}$ and $\frac{1}{p_1C_2}$ as shown in Fig.4.1(d).

4.2. Realizability Conditions For Foster Like Realization Of Class A to D structures

The necessary and sufficient conditions for the realizability of the above four Classes of structures are given in the following Theorems:

Theorem 4.1.

The necessary and sufficient condition for the two variable reactance function:

$$Z(p_1,p_2) = \frac{N(p_1,p_2)}{D(p_1,p_2)}$$

to be realizable by a series combination of Class A structure are :

(i) $Z(1,p_2)$ and $Z(p_1,1)$ posses the same poles.

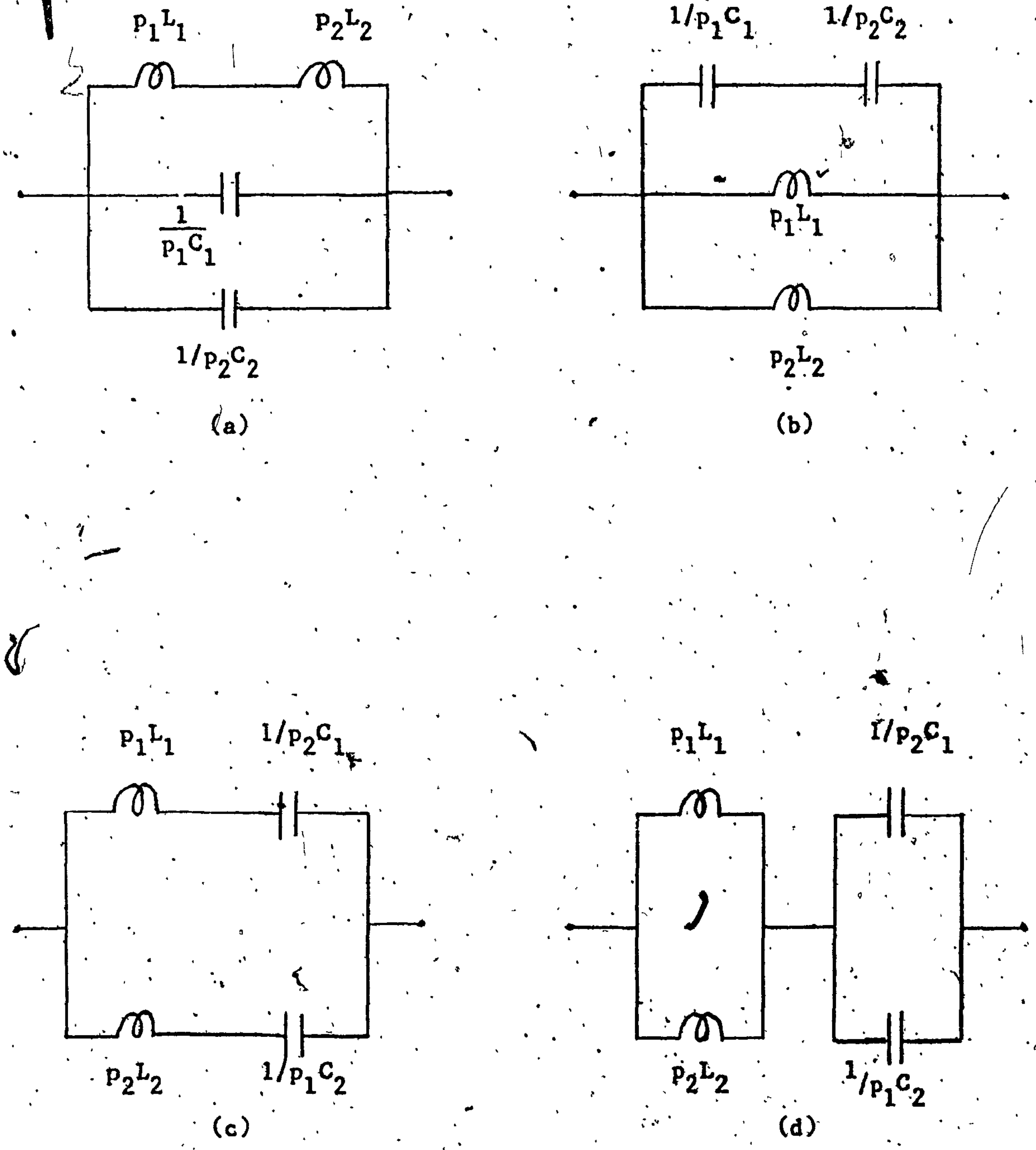


Fig.4.1

TRF's structures using two elements in each variable

(ii) $Z(1,p_2)$ shall be realizable as a series combination of several structures, each being of the form shown in Fig.4.2(a).

(iii) $Z(p_1,1)$ shall be realizable as a series combination of several structures, each being of the form shown in Fig.4.2(b).

(iv) For every section given by the impedance :

$$Z(p_1,p_2) = \frac{a_1p_1 + b_1p_2}{f_1p_1^2 + d_1p_1p_2 + e_1p_2^2 + 1}$$

the following condition should be satisfied:

$$\begin{vmatrix} f_1 & d_1 & e_1 \\ a_1 & b_1 & 0 \\ 0 & a_1 & b_1 \end{vmatrix} = 0$$

Proof:

The impedance function of the section shown in Fig.4.1(a) is :

$$Z(p_1,p_2) = \frac{p_1L_1 + p_2L_2}{p_1^2L_1C_1 + p_2^2L_2C_2 + p_1p_2(L_1C_2 + L_2C_1) + 1} \qquad \text{..........(4.1)}$$

which can be written as :

$$Z(p_1,p_2) = \frac{a_1p_1 + b_1p_2}{f_1p_1^2 + d_1p_1p_2 + e_1p_2^2 + 1}$$

where a_1,b_1,d_1,e_1,f_1, are positive real constants.

for several sections in series, the impedance function will be :

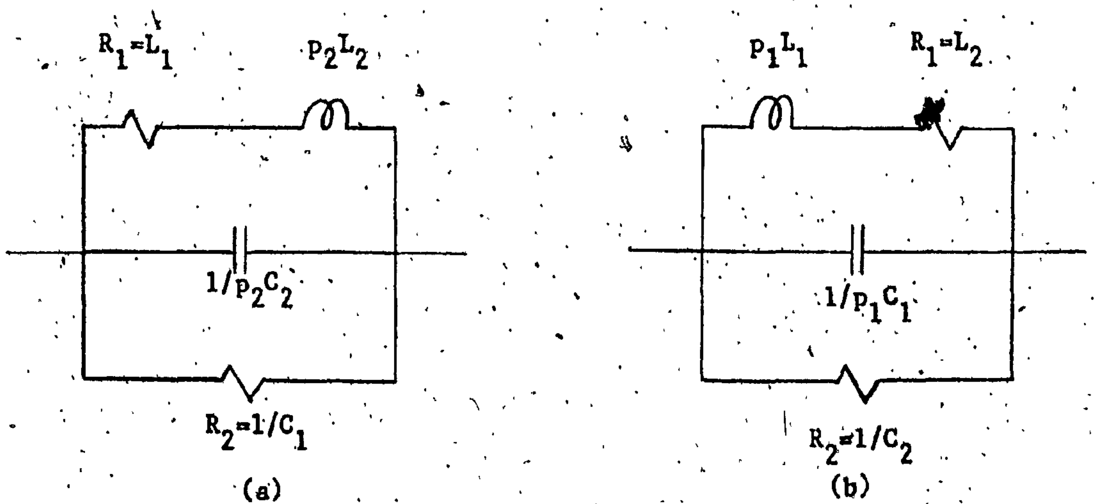


Fig.4.2

Realization of Class A Structures

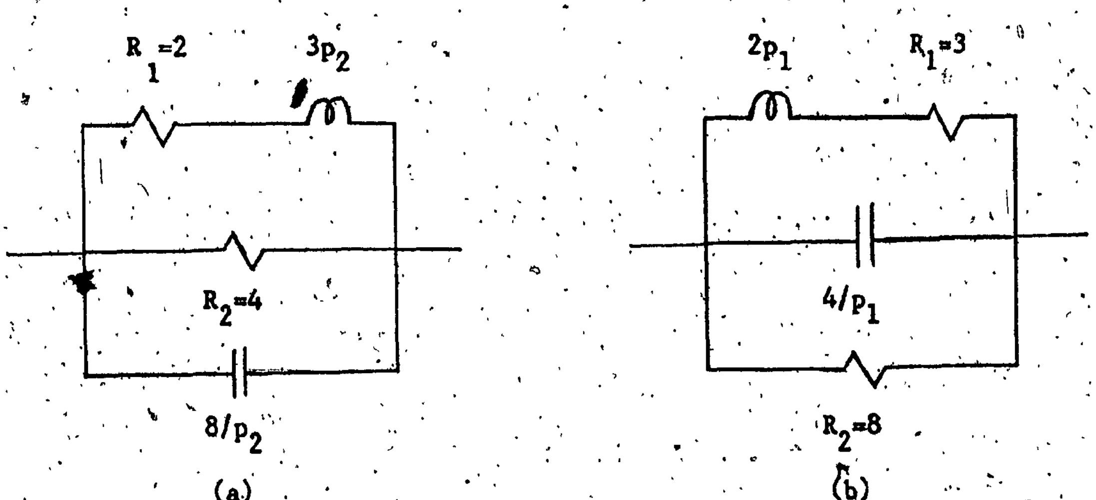


Fig.4.3

Numerical Example for the realization of Class A Structure

$$Z(p_1,p_2) = \sum \frac{a_1p_1 + b_jp_2}{f_1p_1^2 + d_1p_1p_2 + e_1p_2^2 + 1} \qquad \text{..........(4.2)}$$

For $p_1=1$, the circuit becomes as shown in Fig.4.2(a), for which:

$$Z_1(1,p_2) = \frac{p_2L_2 + L_1}{p_2^2L_2C_2 + p_2(L_1C_2 + L_2C_1) + (1 + L_1C_1)} \qquad \text{..........(4.3)}$$

The zero for $Z_1(1,p_2)$ is :

$$p_2 = -\frac{L_1}{L_2}$$

and the poles for $Z(1,p_2)$ are : ....(4.4)

$$p_2 = \frac{-(L_1C_2 + L_2C_1) \pm \sqrt{L_1^2C_2^2 + L_2^2C_1^2 - 2L_1L_2C_1C_2 - 4L_2C_2}}{2L_2C_2}$$

For $p_2=1$, the circuit becomes as shown in Fig.4.2(b).

Then :

$$Z_2(p_1,1) = \frac{p_1L_1 + L_2}{p_1^2L_1C_1 + p_1(L_1C_2 + L_2C_1) + (L_2C_2 + 1)} \qquad \text{..........(4.5)}$$

The zero for $Z(p_1,1)$ is :

$$p_1 = -\frac{L_2}{L_1}$$

The poles for $Z(p_1,1)$ are : ....(4.6)

$$p_1 = \frac{-(L_1C_2 + L_2C_1) \pm \sqrt{L_1^2C_2^2 + L_2^2C_1^2 - 2L_1L_2C_1C_2 - 4L_1C_1}}{2L_1C_1}$$

Comparing equations (4.4) and (4.6), we notice that the zeros are not the same, but that $Z(p_1,1)$ and $Z(1,p_2)$ posses the same poles, which is condition (i).

For $Z(p_1,p_2)$ to be realizable in the form required it should possible to expand $Z(1,p_2)$ and $Z(p_1,1)$ in the form given by equs.(4.3) and (4.5), where each term should be realizable by the structures shown in Figs.4.2(a) and 4.2(b) respectively. Then only the realization is possible. Thus, $Z(1,p_2)$ and $Z(p_1,1)$ is first realized as a series combination of the networks shown in Figs.4.2(a) and 4.2(b).

Now, considering any one section, its impedance should be of the form:

$$Z_1(1,p_2) = \frac{b_1p_2 + a_1}{e_1p_2^2 + d_1p_2 + (1+f_1)}$$

and

$$Z_2(p_1,1) = \frac{a_1p_1 + b_1}{f_1p_1^2 + d_1p_1 + (1+e_1)}$$

Hence

$$Z(p_1,p_2) = \frac{b_1p_2 + a_1p_1}{f_1p_1^2 + d_1p_1p_2 + e_1p_2^2 + 1} \quad \text{........(4.7)}$$

From which the required realization is carried out and then utilizing condition (iv), the elemental values are:

$$L_1 = a_1 \qquad L_2 = b_1 \qquad C_1 = \frac{f_1}{a_1} \qquad C_2 = \frac{e_1}{b_1}$$

<u>Numerical Exapmle:</u>

It is required to realize :

$$Z(p_1,p_2) = \frac{2p_1 + 3p_2}{\frac{1}{2}p_1^2 + p_1p_2 + \frac{3}{8}p_2^2 + 1}$$

This function has Class A form, then using Theorem 4.1 :

$a_1 = 2$, $b_1 = 3$, $d_1 = 1$, $f_1 = \frac{1}{2}$, $e_1 = \frac{3}{8}$

and since

$$\begin{vmatrix} \frac{1}{2} & 1 & \frac{3}{8} \\ 2 & 3 & 0 \\ 0 & 2 & 3 \end{vmatrix} = 0$$

Then $L_1 = a_1 = 2$, $L_2 = b_1 = 3$,

$C_1 = \frac{f_1}{a_1} = \frac{1}{4}$, $C_2 = \frac{e_1}{b1} = \frac{1}{8}$

at $p_1 = 1$, the realization is as shown in Fig.4.3(a).

at $p_2 = 1$, the realization is as shown in Fig.4.3(b).

Theorem 4.2.

The necessary and the sufficient conditions for the two variable reactance function $Z(p_1,p_2)$ to be realizable by a series combination of Class B structures are:

(i) $Z(1,p_2)$ and $Z(p_1,1)$ posses the same poles.

(ii) $Z(1,p_2)$ shall be realizable as a series combination of several structures, each being of the form shown in Fig.4.4(a).

(iii) $Z(p_1,1)$ shall be realizable as a series combination of several structures, each being of the form shown in Fig.4.4(b).

(iv) For every section given by the impedance :

$$Z(p_1,p_2) = \frac{a_1 p_1^2 p_2 + b_1 p_1 p_2^2}{e_1 p_1^2 p_2^2 + h_1 p_1^2 + f_1 p_2^2 + d_1 p_1 p_2}$$

the following condition should be satisfied:

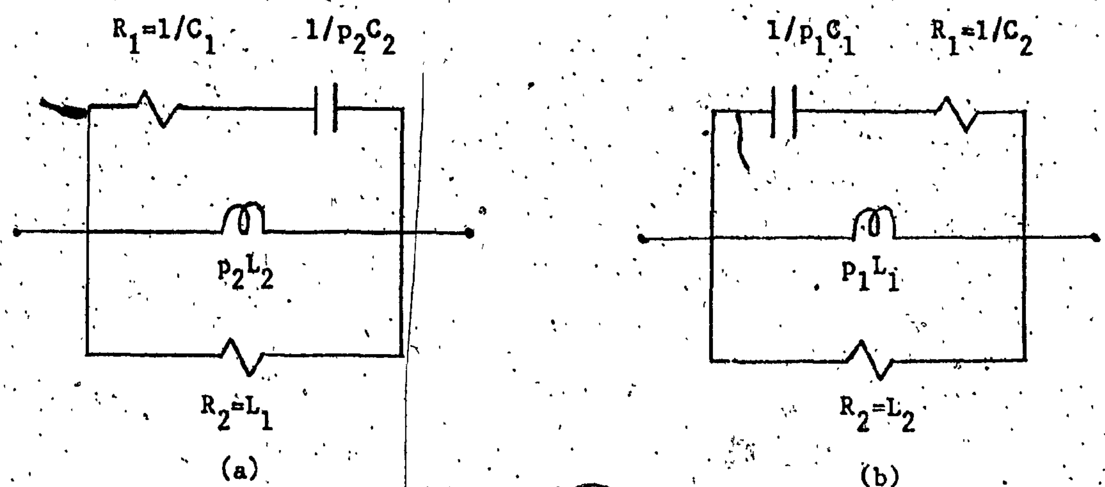


Fig.4.4

Realization of Class B Structures

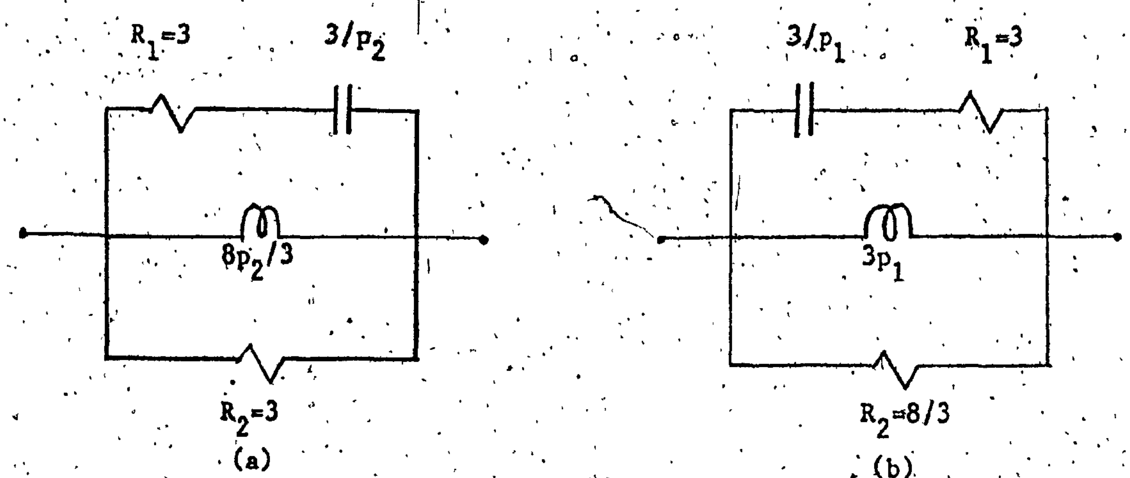


Fig.4.5

Numerical Example for the realization of Class B Structure

$$\begin{vmatrix} f_1 & d_1 & h_1 \\ a_1 & b_1 & 0 \\ 0 & a_1 & b_1 \end{vmatrix} = 0$$

Proof:

The impedance function of the section shown in Fig.4.1(b) is:

$$Z(p_1,p_2) = \frac{p_1^2p_2L_1L_2C_1 + p_1p_2^2L_1L_2C_2}{p_1^2p_2^2\, L_1L_2C_1C_2 + p_1^2L_1C_1 + p_2^2L_2C_2 + p_1p_2(L_1C_2 + L_2C_1)} \quad \text{.........(4.8)}$$

which can be written as :

$$Z(p_1,p_2) = \frac{a_1p_1^2p_2^2 + b_1p_1p_2^2}{e_1p_1^2p_2^2 + h_1p_1^2 + f_1p_2^2 + d_1p_1p_2}$$

where $a_1, b_1, d_1, e_1, h_1, f_1$, are positive real constants.

For several sections in series, the impedance function can be expressed as follows:

$$Z(p_1,p_2) = \sum \frac{a_1p_1^2p_2 + b_1p_1p_2^2}{e_1p_1^2p_2^2 + h_1p_1^2 + f_1p_2^2 + d_1p_1p_2} \quad \text{.........(4.9)}$$

For $p_1=1$, the circuit becomes as shown in Fig.4.4(a), for which:

$$Z_1(1,p_2) = \frac{p_2^2L_1L_2C_2 + p_2L_1L_2C_1}{p_2^2(L_1L_2C_1C_2 + L_2C_2) + p_2(L_1C_2 + L_2C_1) + L_1C_1} \quad \text{..... (4.10)}$$

The zeros for $Z_1(1,p_2)$ are :

$$p_2 = \frac{-L_1L_2C_1}{L_1L_2C_2} = -\frac{C_1}{C_2} \quad \text{or} \quad p_2 = 0$$

and the poles for $Z(1,p_2)$ are: ....(4.11)

$$p_2 = \frac{-(L_1C_2 + L_2C_1) \pm \sqrt{L_1^2C_2^2 + L_2^2C_1^2 - 2L_1L_2C_1C_2 - 4L_1^2L_2C_1^2C_2}}{2(L_1L_2C_1C_2 + L_2C_2)}$$

For $p_2=1$, the circuit becomes as shown in Fig.4.4(b).

Then :

$$Z_2(p_1,1) = \frac{p_1^2L_1L_2C_1 + p_1L_1L_2C_2}{p_1^2(L_1L_2C_1C_2 + L_1C_1) + p_1(L_1C_2 + L_2C_1) + L_2C_2} \quad \text{....(4.12)}$$

The poles for $Z(p_1,1)$ are :

$$p_1 = \frac{-(L_1C_2 + L_2C_1) \pm \sqrt{L_1^2C_2^2 + L_2^2C_1^2 - 2L_1L_2C_1C_2 - 4L_1L_2^2C_1C_2^2}}{2(L_1L_2C_1C_2 + L_2C_2)}$$

The zeros for $Z(p_1,1)$ are : ......(4.13)

$$p_1 = -\frac{C_2}{C_1} \quad \text{or} \quad p_1 = 0$$

Comparing equations (4.11) and (4.13), we notice that the zeros are not the same, but that $Z(p_1,1)$ and $Z(1,p_2)$ posses the same poles, which is condition (i).

For $Z(p_1,p_2)$ to be realizable in the form required it should be possible to expand $Z(1,p_2)$ and $Z(p_1,1)$ in the form given by equations (4.10) and (4.12), where each term should be realizable by the structures shown in Figs.4.4(a) and 4.4(b) respectively. Then only the realization is possible. Thus, $Z(1,p_2)$ and $Z(p_1,1)$ is realized as a series combination of the networks shown in Figs.4.4(a) and 4.4(b).

Now, considering any one section, its impedance should be of the form:

$$Z_1(1,p_2) = \frac{b_1 p_2^2 + a_1 p_2}{(e_1 + h_1)\, p_2^2 + d_1 p_2 + f_1}$$

and $$Z_2(p_1,1) = \frac{a_1 p_1^2 + b_1 p_1}{(e_1 + f_1)\, p_1^2 + d_1 p_1 + h_1}$$

Hence $$Z(p_1,p_2) = \frac{a_1 p_1^2 p_2 + b_1 p_2^2 p_1}{e_1 p_1^2 p_2^2 + h_1 p_1^2 + f_1 p_2^2 + d_1 p_1 p_2}$$

From which the required realization is carried out and then utilizing condition (iv), the elemental values are:

$$L_1 = \frac{b_1}{f_1} \qquad L_2 = \frac{a_1}{h_1} \qquad C_1 = \frac{e_1}{b_1} \qquad C_2 = \frac{e_1}{a_1}$$

Nnmerical Example:

It is required to realize :

$$Z(p_1,p_2) = \frac{2p_1^2 p_2 + 3p_2^2 p_1}{p_1^2 p_2^2 + \frac{3}{4} p_1^2 + p_2^2 + 2p_1 p_2}$$

This function has Class B form, then using Theorem 4.2 :

$a_1 = 2$, $b_1 = 3$, $d_1 = 2$, $e_1 = 1$, $f_1 = 1$, $h_1 = \frac{3}{4}$

and since:

$$\begin{vmatrix} 1 & 2 & \frac{3}{4} \\ 2 & 3 & 0 \\ 0 & 2 & 3 \end{vmatrix} = 0$$

Then $L_1 = \frac{b_1}{f_1} = 3$, $L_2 = \frac{a_1}{h_1} = \frac{8}{3}$

$C_1 = \frac{e_1}{b_1} = \frac{1}{3}$, $C_2 = \frac{e_1}{a_1} = \frac{1}{3}$.

at $p_1 = 1$, the realization is as shown in Fig.4.5(a).

at $p_2 = 1$, the realization is as shown in Fig.4.5(b).

Theorem 4.3.

The necessary and sufficient conditions for the two variable reactance function $Z(p_1,p_2)$ to be realizable by a series combination of Class C structures are:

(i) $Z(1,p_2)$ and $Z(p_1,1)$ shall not posses the same poles and have the partial fraction expansion of the form:

$$Z(1,p_2) = \sum \frac{(A_i p_2 + 1)(B_i p_2 + 1)}{d_1 p_2^2 + (e_1 + g_1)p_2 + h_1}$$

And

$$Z(p_1,1) = \sum \frac{(A_i p_1 + 1)(B_i p_2 + 1)}{e_1 p_1^2 + (d_1 + h_1)p_1 + g_1}$$

(ii) $Y_i(1,p_2)$ and $Y_i(p_1,1)$ shall have the form:

$$Y_i(1,p_2) = \frac{F_i}{A_i p_2 + 1} + \frac{K_i p_2}{B_i p_2 + 1}$$

and

$$Y_i(p_1,1) = \frac{F_i p_1}{A_i p_1 + 1} + \frac{K_i}{B_i p_1 + 1}$$

Proof:

The impedance function of the section shown in Fig.4.1(c) is:

$$Z(p_1,p_2) = \frac{p_1^2 p_2^2 L_1 L_2 C_1 C_2 + p_1 p_2 (L_1 C_1 + L_2 C_2) + 1}{p_1^2 p_2 L_1 C_1 C_2 + p_1 p_2^2 L_2 C_1 C_2 + p_1 C_2 + p_2 C_1} \quad \text{.......(4.13)}$$

which can be written as :

$$Z(p_1,p_2) = \frac{a_1 p_1^2 p_2^2 + b_1 p_1 p_2 + 1}{d_1 p_1 p_2^2 + h_1 p_1 + g_1 p_2 + e_1 p_1^2 p_2}$$

where $a_1, b_1, d_1, e_1, h_1, g_1$, are positive real constants.

For several sections in series, the impedance function will be :

$$Z(p_1,p_2) = \sum \frac{a_1 p_1^2 p_2^2 + b_1 p_1 p_2 + 1}{d_1 p_1 p_2^2 + h_1 p_1 + g_1 p_2 + e_1 p_1^2 p_2} \quad \text{.........(4.14)}$$

For $p_1=1$, the circuit becomes as shown in Fig.4.6(a), for which:

$$Z_1(1,p_2) = \frac{p_2^2 L_1 L_2 C_1 C_2 + p_2 (L_1 C_1 + L_2 C_2) + 1}{p_2^2 L_2 C_1 C_2 + p_2 (L_1 C_1 C_2 + C_1) + C_2} \quad \text{.........(4.15)}$$

The zeros for $Z(1,p_2)$ are :

$$p_2 = \frac{-(L_1 C_1 + L_2 C_2) \pm \sqrt{L_1^2 C_1^2 + L_2^2 C_2^2 - 2 L_1 L_2 C_1 C_2}}{2 L_1 L_2 C_1 C_2}$$

The poles for $Z(1,p_2)$ are: ....(4.16)

$$p_2 = \frac{-(L_1 C_1 C_2 + C_1) \pm \sqrt{L_1^2 C_1^2 C_2^2 + C_1^2 + 2 L_1 C_1^2 C_2 - 4 L_2 C_1^2 C_2}}{2 L_2 C_1^2}$$

For $p_2=1$, the circuit becomes as shown in Fig.4.6(b).

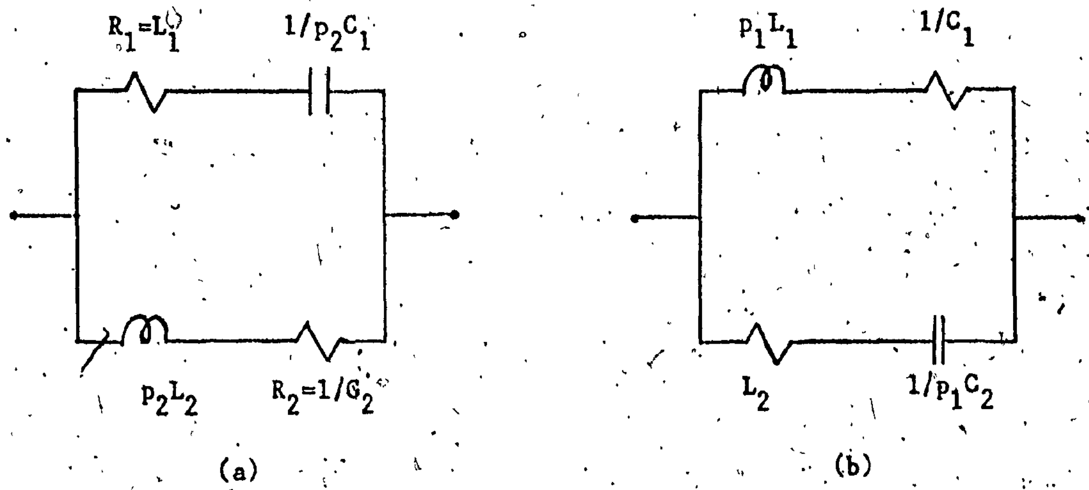


Fig.4.6

Realization of Class C structures

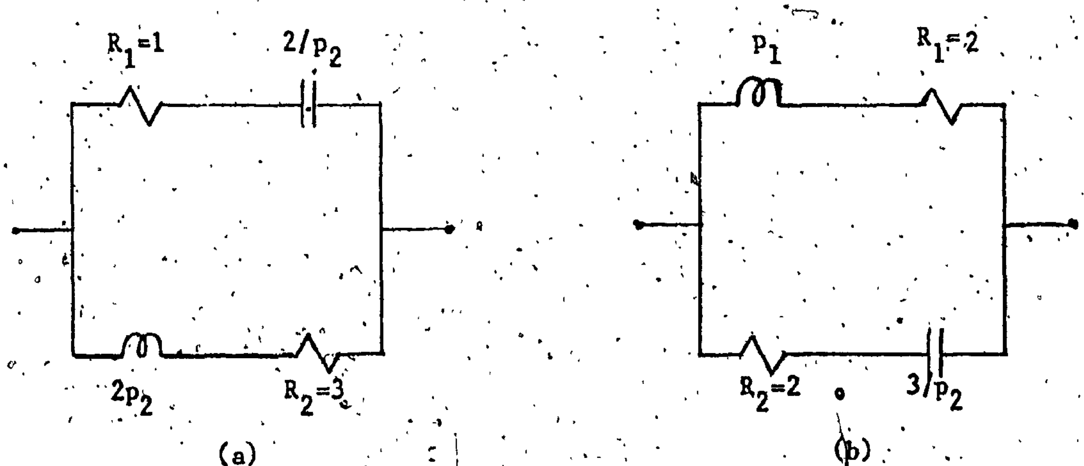


Fig.4.7

Numerical Example for the realization of Class C Structure

Then :

$$Z_2(p_1,1) = \frac{p_1^2L_1L_2C_1C_2 + p_1(L_1C_1 + L_2C_2) + 1}{p_1^2L_1C_1C_2 + p_1(L_2C_1C_2 + C_2) + C_1} \qquad \text{............(4.17)}$$

The zeros for $Z(p_1,1)$ are :

$$p_1 = \frac{-(L_1C_1 + L_2C_2) \pm \sqrt{L_1^2C_1^2 + L_2^2C_2^2 - 2L_1L_2C_1C_2}}{2L_1L_2C_1C_2}$$

The poles for $Z(p_1,1)$ are : ....(4.18)

$$p_1 = \frac{-(L_2C_1^2 + C_2) \pm \sqrt{L_2^2C_1^2C_2^2 + C_2^2 + 2L_2C_1C_2^2 - 4L_1C_1C_2^2}}{2L_1C_1C_2}$$

Comparing equations (4.16) and (4.18), we notice that the zeros are the same for any one section, but that $Z(p_1,1)$ and $Z(1,p_2)$ may not posses the same poles and zeros.

For $Z(p_1,p_2)$ to be realizable in the form required, it should be possible to expand $Z(1,p_2)$ and $Z(p_1,1)$ in the form given by equations (4.15) and (4.17), where each term should be realizable by the structures shown in Figs.4.6(a) and 4.6(b) respectively. Then only the realization is possible.

Thus, $Z(1,p_2)$ (or $Z(p_1,1)$) is first realized as a series combination of the network shown in Fig.4.6(a)(or in Fig.4.6(b)). Consider any section, its impedance should be of the form:

$$Z_i(1,p_2) = \frac{(A_ip_2 + 1)(B_ip_2 + 1)}{d_ip_2^2 + (e_i + g_i)p_2 + h_i}$$

and its admittance should be of the form:

$$Y_i(1,p_2) = \frac{F_i}{A_i p_2 + 1} + \frac{K_i p_2}{B_i p_2 + 1}$$

in order that this section be realizable by the structure given in Fig.4.6(b).

Similarly,

$$Z_i(p_1,1) = \frac{(A_i p_1 + 1)(B_i p_1 + 1)}{e_i p_1^2 + (d_i + h_i)p_1 + g_i}$$

$$Y_i(p_1,1) = \frac{F_i p_1}{A_i p_2 + 1} + \frac{K_i p_2}{B_i p_2 + 1}$$

should be of the shown form.

Combining the above two, for any one section, the impedance should be of the form:

$$Z_i(p_1,p_2) = \frac{(A_i p_1 p_2 + 1)(B_i p_1 p_2 + 1)}{d_i p_1 p_2^2 + h_i p_1 + g_i p_2 + e_i p_1^2 p_2} \qquad \text{.............(4.19)}$$

or

$$Y_i(p_1,p_2) = \frac{1}{\frac{A_i}{F_i} p_2 + \frac{1}{F_i p_1}} + \frac{1}{\frac{B_i}{K_i} p_1 + \frac{1}{K_i p_2}}$$

and the elemental values are :

$$L_1 = \frac{B_i}{K_i}, \qquad L_2 = \frac{A_i}{F_i}, \qquad C_1 = K_i, \qquad C_2 = F_i$$

<u>Numerical Example:</u>

It is required to realized :

$$Z(p_1,p_2) = \frac{2p_1^2 p_2^2 + 7p_1 p_2 + 6}{2p_1 p_2^2 + p_1^2 p_2 + 2p_1 + 3p_2}$$

This function can be put in the form:

$$Z(p_1,p_2) = \frac{(p_1p_2 + 2)(2p_1p_2 + 3)}{2p_1p_2^2 + p_1^2p_2 + 2p_1 + 3p_2}$$

This function has Class C form, then using Theorm 4.3 :

$$Y(p_1,p_2) = \frac{2p_1p_2^2 + p_1^2p_2 + 2p_1 + 3p_2}{(p_1p_2 + 2)(2p_1p_2+3)}$$

then $$Y(p_1,p_2) = \frac{1}{p_1 + \frac{2}{p_2}} + \frac{1}{2p_2 + \frac{3}{p_1}}$$

$$L_1 = \frac{B_i}{K_i} = 1 , \qquad L_2 = \frac{A_i}{F_i} = 2 , \qquad C_1 = K_i = ½ , \qquad C_2 = F_i = 1/3.$$

at $p_1 = 1$, the realization is as shown in Fig.4.7(a).
at $p_2 = 1$, the realization is as shown in Fig.4.7(b).

Theorem 4.4.

The necessary and sufficient conditions for the two variable reactance function $Z(p_1,p_2)$ to be realizable by a series combination of Class D structures are :
(i) $Z(1,p_2)$, $Z(p_1,1)$ shall not posses the same poles or zeros and have the partial fraction expansion of the form:

$$Z_i(1,p_2) = \frac{A_ip_2}{G_i + B_ip_2} + \frac{D_i}{E_i + F_ip_2}$$

$$Z_i(p_1,1) = \frac{A_ip_1}{G_ip_1 + B_i} + \frac{D_i}{E_ip_1 + F_i}$$

Proof:

The impedance function of the section shown in Fig.4.1(d) is:

$$Z(p_1,p_2) = \frac{p_1^2 p_2 L_1 L_2 C_2 + p_2^2 p_1 L_1 L_2 C_1 + p_1 L_1 + p_2 L_2}{p_1^2 L_1 C_2 + p_2^2 L_2 C_1 + p_1 p_2 (L_1 C_1 + L_2 C_2)} \qquad \text{.............(4.20)}$$

which can be written as :

$$Z(p_1,p_2) = \frac{a_1 d_1 p_1^2 p_2 + a_1 b_1 p_1 p_2^2 + f_1 p_1 + e_1 p_2}{p_1^2 k_1 + p_1 p_2 h_1 + p_2^2 g_1}$$

where $a_1, b_1, d_1, e_1, f_1, g_1, h_1, k_1$, are positive real constants.

For several sections in series, the impedance function will be :

$$Z(p_1,p_2) = \sum \frac{a_1 d_1 p_1^2 p_2 + a_1 b_1 p_1 p_2^2 + f_1 p_1 + e_1 p_2}{p_1^2 k_1 + p_1 p_2 h_1 + p_2^2 g_1} \qquad \text{.............(4.21)}$$

For $p_1=1$, the circuit becomes as shown in Fig.4.8(a), for which:

$$Z_1(1,p_2) = \frac{p_2^2 L_1 L_2 C_1 + p_2 (L_1 L_2 C_2 + L_2) + L_1}{p_2^2 L_2 C_1 + p_2 (L_1 C_1 + L_2 C_2) + L_1 C_2} \qquad \text{.............(4.22)}$$

The zeros for $Z(1,p_2)$ are :

$$p_2 = \frac{-(L_1 L_2 C_2 + L_2) \pm \sqrt{(L_1 L_2 C_2 + L_2)^2 - 4 L_1^2 L_2 C_1}}{2 L_1 L_2 C_1}$$

The poles for $Z(1,p_2)$ are: ....(4.23)

$$p_2 = \frac{-(L_1 C_1 + L_2 C_2) \pm \sqrt{L_1^2 C_1^2 + L_2^2 C_2^2 - 2 L_1 L_2 C_1 C_2}}{2 C_1 L_2}$$

For $p_2=1$, the circuit becomes as shown in Fig.4.8(b), for which:

$$Z_2(p_1,1) = \frac{p_1^2L_1L_2C_2 + p_1(L_1L_2Cl + L_1) + L_2}{p_1^2L_1C_2 + p_1(L_1C_1 + L_2C_2) + L_2C_1} \qquad \text{.........(4.24)}$$

The zeros for $Z(p_1,1)$ are:

$$p_1 = \frac{-(L_1L_2C_1 + L_1) \pm \sqrt{(L_1L_2C_1 + L_1)^2 - 4L_1L_2^2C_2}}{2L_1C_2}$$

The poles for $Z(p_1,1)$ are : ....(4.25)

$$p_1 = \frac{-(L_1C_1 + L_2C_2) \pm \sqrt{L_1^2C_1^2 + L_2^2C_2^2 - 2L_1L_2C_1C_2}}{2L_1C_2}$$

Comparing equations (4.23) and (4.25), we notice that $Z(p_1,1)$ and $Z(1,p_2)$ may not posses the same poles and zeros.

For $Z(p_1,p_2)$, to be realizable in the form required, it should be possible to expand $Z(1,p_2)$ and $Z(p_1,1)$ in the form given by equations (4.22) and (4.24), where each term should be realizable by the structures shown in Fig.4.8(a) and 4.8(b) respectively. Then only the realization is possible.

Therefore $Z(1,p_2)$(or $Z(P_1,1)$) is first realized as a series combination of the network shown in Fig.4.8(a)(or in Fig.4.8(b)), which will be of the form:

$$Z_i(1,p_2) = \frac{A_ip_2}{G_i + B_ip_2} + \frac{D_i}{E_i + F_ip_2}$$

$$Z_i(p_1,1) = \frac{A_ip_1}{p_1G_i + B_i} + \frac{D_i}{p_1E_i + F_i}$$

Hence $$Z_i(p_1,p_2) = \frac{A_ip_1p_2}{G_ip_1 + B_ip_2} + \frac{D_i}{E_ip_1 + F_ip_2}$$

and the elemental values are:

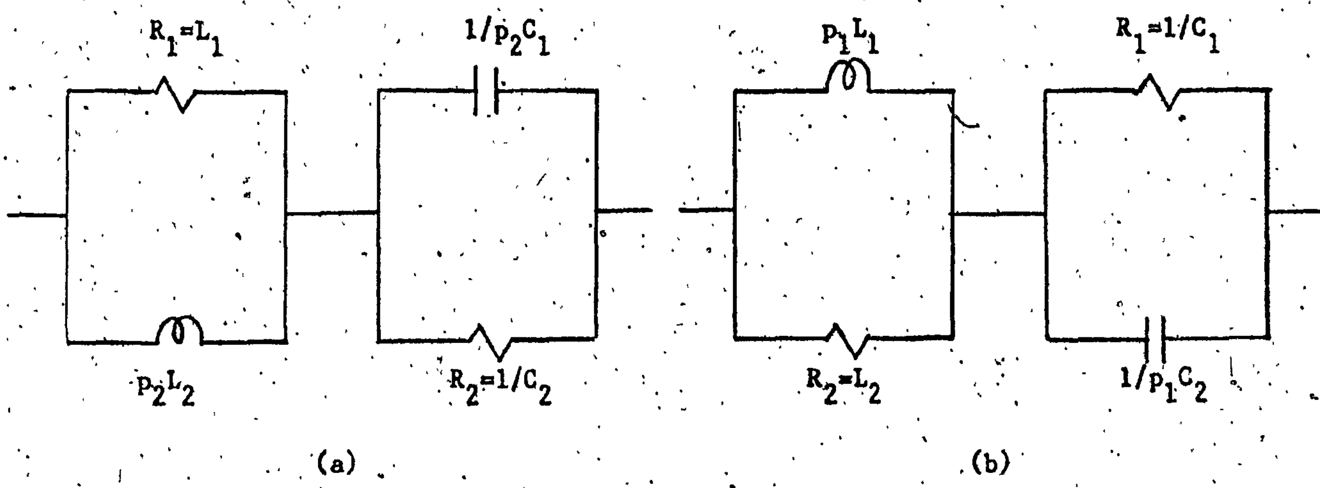


Fig.4.8

Realization of Class D Structures

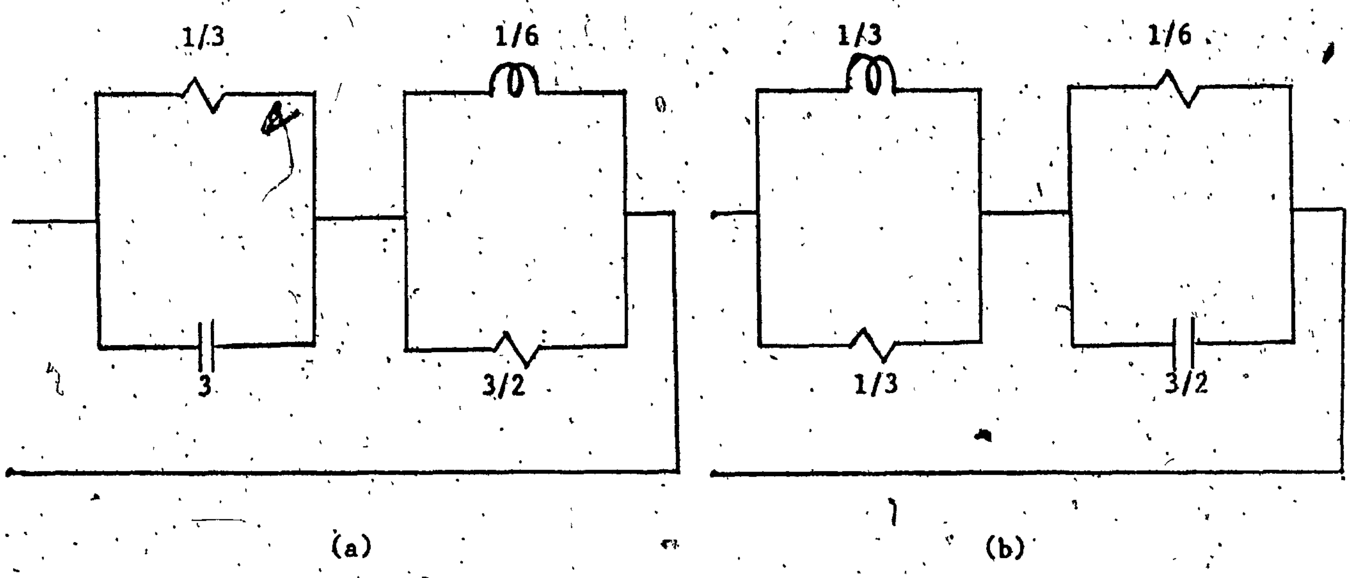
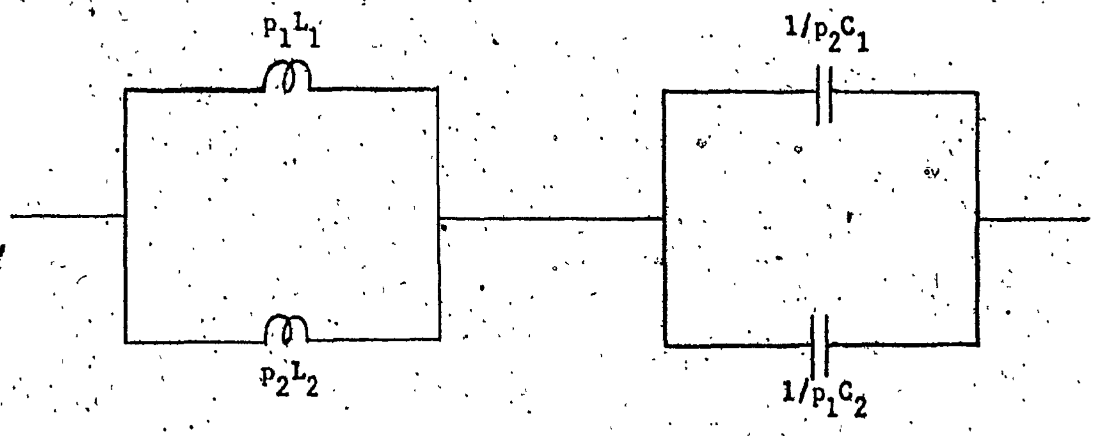


Fig.4.9

Numerical Example for the realization of Class D Structures



(c)

Fig.4.9(cont.)

$$L_1 = \frac{A_1}{B_1}\ , \qquad L_2 = \frac{A_1}{G_1}\ , \qquad C_1 = \frac{F_1}{D_1}\ , \qquad C_2 = \frac{E_1}{D_1}$$

Numerical Example:

It is required to realize :

$$Z(p_1,p_2) = \frac{\frac{1}{3}\, p_1^2 p_2 + \frac{2}{3}\, p_2^2 p_1 + \frac{4}{3}\, p_1 + \frac{2}{3}\, p_2}{p_1^2 p_2^2 + 5p_1 p_2 + 4}$$

At $p_1=1$: $$Z(1,p_2) = \frac{2/3\ p_2^2 + p_2 + 4/3}{(p_2+1)(p_2+4)}$$

Then $$Z(1,p_2) = \frac{1}{3p_2+3} + \frac{1}{3/2 + 6/p_2}$$

and the realization is shown in Fig.4.9(a).

At $p_2=1$: $$Z(p_1,1) = \frac{1/3\ p_1^2 + 2p_1 + 2/3}{(p_1+1)(p_1+4)}$$

Then $$Z(p_1,1) = \frac{1}{3 + 3/p_1} + \frac{1}{3/2p_1 + 6}$$

and the realization is shown in Fig.4.9(b).

As can be seen this function is realizable by Class D structures and the realization of $Z(p_1,p_2)$ is as shown in Fig.4.9(c).

4.3. Equivalent Ladder Structures

In this section, it will be shown, with suitable examples, that corresponding ladder realizations of these four Classes of structures do not exist in general.

(i) Series connection of Class A structures:

consider the network shown in Fig.4.10.

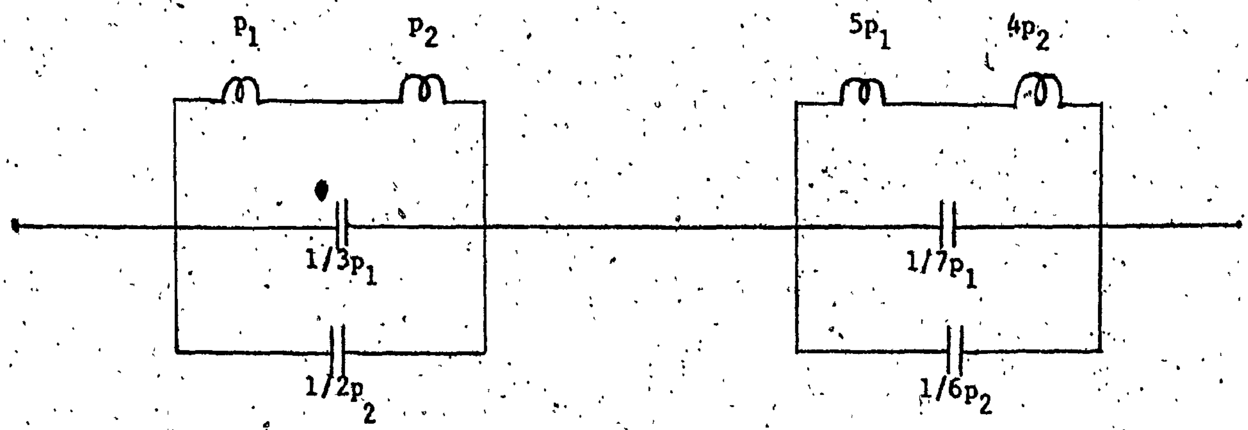


Fig.4.10

Series Connection of Class A Structures

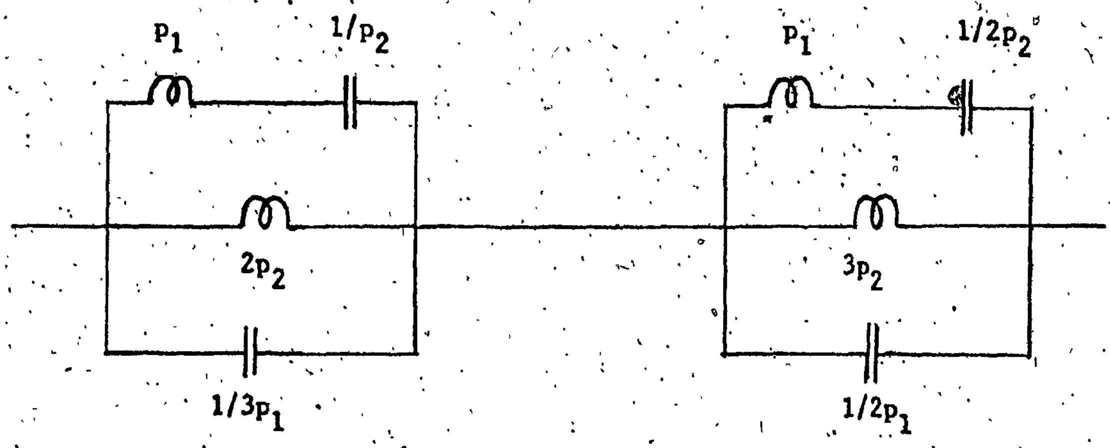


Fig.4.11

Series Connection of Class B Structures

$$Z(p_1,p_2) = \frac{50p_1^3 + 130p_1^2p_2 + 112p_1p_2^2 + 32p_2^3 + 6p_1 + 5p_2}{105p_1^4+349p_1^3p_2+432p_1^2p_2^2+236p_1p_2^3+48p_2^4+38p_1^2+63p_1p_2+26p_2^2+1}$$

Then :

$$Y(p_1,p_2) = 2.1p_1+1.5p_2+\frac{p_1^3p_2+1.8p_1^2p_2^2+0.8p_1p_2^3+25.4p_1^2+43.5p_1p_2+18.5p_2^2+1}{50p_1^3+130p_1^2p_2+112p_1p_2^2+32p_2^3+6p_1+5p_2}$$

It can be seen that the passive ladder realization is not possible, since the fourth degree terms do not vanish, which means it is not possible to carry on the continued fraction expansion.

(ii) Series connection of Class B structures

consider the network shown in Fig.4.11.

$$Z(p_1,p_2) = \frac{60p_1^3p_2^4 + 100p_1^2p_2^3 + 24p_1p_2^4 + 18p_2^3 + 45p_1p_2^2 + 5p_2}{72p_1^4p_2^4+132p_1^3p_2^3+60p_1^2p_2^4+12p_2^4+74p_1^2p_2^2+58p_1p_2^3+6p_2^2+15p_1p_2+1}$$

Then :

$$Y(p_1,p_2) = 1.2p_1 + \frac{12p_1^3p_2^3+31.2p_1^2p_2^4+12p_2^4+20p_1^2p_2^2+36.4p_1p_2^3+6p_2^2+9p_1p_2+1}{60p_1^3p_2^4 + 100p_1^2p_2^3 + 24p_1p_2^4 + 18p_2^3 + 45p_1p_2^2 + 5p_2}$$

$$= 1.2\,p_1 + \frac{D}{N} \qquad \text{(say)}$$

Further continued fraction expansion of D/N so as to result in a passive network is not possible, as $5p_2$ in N/D, cannot be removed because the resulting function contains negative numbers.

(iii) Series connection of Class C structures:

consider the network shown in Fig.4.12.

$$Z(p_1,p_2) = \frac{432p_1^4p_2^3+432p_1^3p_2^4+372p_1^3p_2^2+300p_1^2p_2^3+92p_1^2p_2+68p_1p_2^2+7p_1+5p_2}{144p_1^4p_2^2+360p_1^3p_2^3+144p_1^2p_2^4+96p_1^3p_2+150p_1^2p_2^2+60p_1p_2^3+12p_1^2+17p_1p_2+6p_2^2}$$

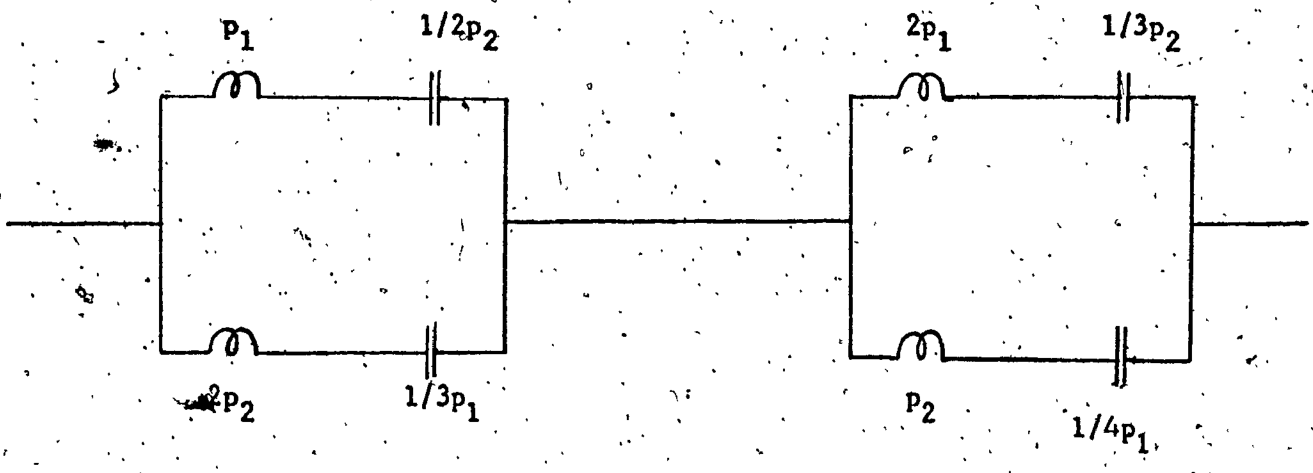


Fig.4.12

Series Connection of Class C Structures

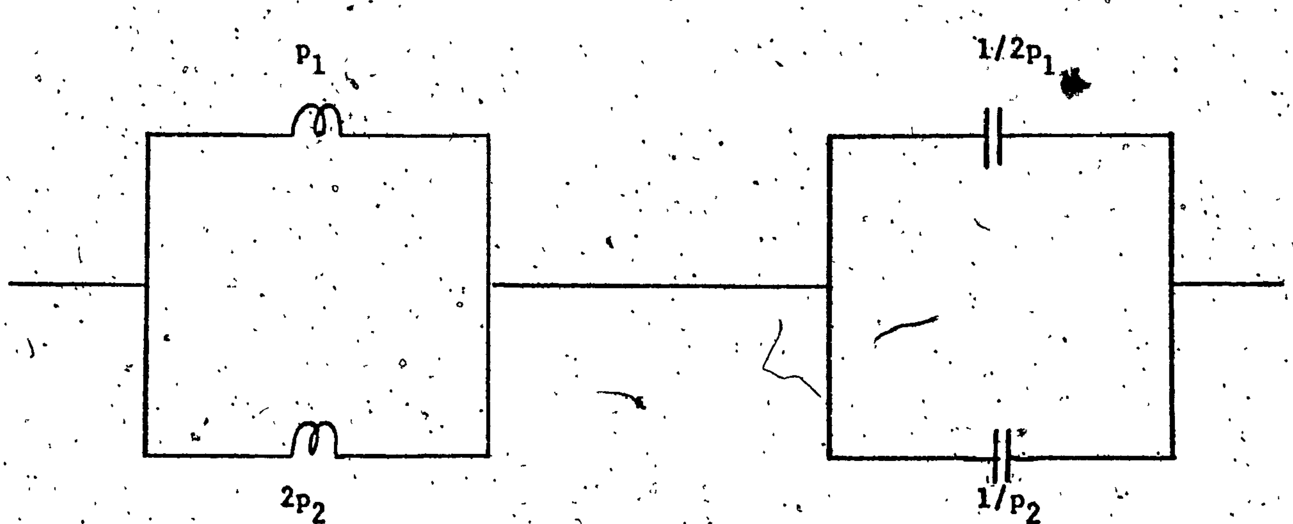


Fig.4.13

Series Connection of Class D Structures

The continued fraction expansion fails, whenever we try to remove p_2.

(iv) Series connection of Class D structures:

consider the network shown in Fig.4.13,

$$Z(p_1,p_2) = \frac{2p_1p_2^2 + 4p_1^2p_2 + p_1 + 2p_2}{2p_1^2 + 5p_1p_2 + 2p_2^2}$$

As can be seen, the continued fraction expansion fails to give a passive ladder realization.

Discussion:

In this chapter, four new structures are proposed where each section has two elements in each of the variables p_1 and p_2. Their conditions of realizations have been derived. It is also shown that, for these structures it is not possible to obtain, in general, equivalent ladder realizations.

CHAPTER V
CONCLUSIONS

This report has considered the possibility of realizing a certain class of two-variable reactance functions in a manner similar to Foster's form for single variable functions. The realizations consist of several sections in series, each section containing two elements for each variable.

To begin with, the different canonic realizations of SRF's are discussed, with particular attention being focussed on Foster and Cauer forms. For the sake of completeness, other forms, namly Lee's non-symmetrical two forms and Lee's Bridged-T structure, Kida's non-symmetrical Lattice Structures and Ramachandran and Swamy's two forms of the twin-T structure, are also breifly discussed.

Next, a class of TRF's are studied which can be realized by a form similar to Foster forms consisting of one element in each variable. Since the realizations procedures for these structures are similar to those of single-variable two-element kind network, such two variable reactance functions can be synthesized in the form similar to the single variable canonic structures. In addition it is shown that there exists another class of TRF's where, starting from a ladder structure, it is not possible to obtain corresponding Foster forms.

In this report, it is clearly established that starting from some Foster forms, corresponding ladder realizations are not possible.

Therefore, it is to be concluded that the TRF's can be broadly divided into two classes:

(i) where, starting from one structure, another structure, is realizable, and:

(ii) where, starting from one srtucture, another structure is not realizable.

In this report, only sections with two elements in each variable are considered. It should be interesting to study sections consisting of more number of elements in each variable. Also, it should be interesting to study the class of TRF's for which both Foster and Cauer forms exist.

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