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# CHAPTER I

# INTRODUCTION

## PROBLEM STATEMENT

In recent years, there has been an increasing tendency to look at various systems in a systematic and scientific way. However, the use of operational research methods in educational and training systems has been limited and rarely documented.

Educational technologists have many skills, but rarely can they accurately plan how many units of each media facility are needed in a self-instructional system. Normally, intuition and common sense are used. Often, one of two undesirable outcomes occurs, either: 1) Too much money is wasted buying too many facilities; or 2) Students are unmotivated or dissatisfied because there are not enough facilities and, therefore, they must wait to gain access to the instructional facility. In either case, the outcome is less desirable and more costly (in terms of money or human inconvenience) than an optimal or near optimal solution.

This thesis argues that this type of problem, and others found in educational and training systems, can be easily solved by operational research techniques.

## PURPOSE OF THE RESEARCH

This thesis is both theoretical and operational in nature. The <u>general</u> or <u>theoretical</u> purpose of this study was to show that an operational research modeling technique -

in this case queueing theory - could be successfully applied to problems involving self-instructional facilities as an aid to problem-solving and planning resource requirements. The fundamental rationale for this assertion was that systems analytic problems repeat themselves in diverse systems and educational systems presumably are not exceptions.

As in the case of the training system being studied, one problem that has always existed in educational and training systems is the inability to optimally determine how many facilities (be they books, playground equipment, or media equipment) are needed. Consequently, one of two outcomes usually occurs, either: 1) Too much money is spent to purchase too many service facilities which will remain idle most of the time; or 2) Too little money is spent so that students must wait to gain access to the service facilities.

As mentioned, intuition and common sense - greatly influenced by monetary restrictions - have usually been used to determine how many materials are needed. Somehow, educational institutions have muddled through, but it is time to cut down on human and financial waste by applying available and applicable skills - such as operational research - to solve problems found in educational and training systems.

This inability to optimally determine resource requirements became important with the increasing problem of rising costs (Schroeder and Adams, 1976). This inability to plan resource requirements became critical when "self-pacing" or "personalized self-instruction" methods were introduced into the educational system. This teaching method is based on the theory that students would be motivated to learn more efficiently if they could work on their own, using carefully planned learning programs comprised of any combination of appropriate media - print, audio, audio-visual, etc. This learning method supported the idea that "slow" students would not be discouraged and confused by trying to keep up with the "faster" students; furthermore, the "faster" students would not be held back or become bored by a "slow" pace used to accommodate most students. This teaching method also made the teachers free to motivate all students and help where and when individual problems arose (Keller, 1968).

As long as only one type of media was used - for example, print - there seemed to be no real problem in determining facility needs: if there were 30 students, then 30 desks were needed. But when the learning programs became more complex and used various types of media simultaneously - such as audio, audio-visual, and print - and allowed students to proceed at their own pace, problems developed. Institutions tried to schedule students, but this only eliminated the advantages of self-pacing and personalized instruction.

Therefore, the crucial problem to solve was: how many units of each medium should be made available? Naturally, the easiest solution would be to purchase one of everything for each student: if classes had 30 students, then 30 units of each media (i.e. 30 multi-media units) were planned. However, it became more and more difficult to apply this extremely efficient but expensive solution to the problem of how many media facilities should be available because of rising costs and monetary restrictions.

Therefore, the general or theoretical purpose of this study was to show that the model building techniques of operational research - specifically queueing theory - could be used as an aid to determine resource requirements in such a learning system. In other words, the purpose of this thesis was to show that queueing theory could be used to control the size of (or eliminate) educational systems while simultaneously keeping purchasing costs to a minimum.

The specific or operational purpose of this study was threefold: 1) Analytical and computer simulation models were to be made of an existing media-based, self-instructional system which were then to be tested against data from existing operations for validation; 2) Analytical and computer simulation models were to be made to represent the same system with a proposed expansion of enrollment and modifications to the media pattern; and 3) Given the existing and proposed models, feasible predictions and recommendations were made for the organization of the learning resources.

The specific, self-instructional system considered was a training system at Bell Canada in Montreal. Their Business Office program was a form of PSI (Personalized System of Instruction) whereby students worked individually at their own pace. The course was comprised of 69 lessons using a simulated office and three types of self-instructional media: audio, audio-visual, and print. There was a definite lesson and media pattern to be followed, but it was still difficult to exactly identify resource requirements because students paced themselves. This caused student arrivals to and departures from the instructional facilities to be randomly distributed. This meant that it was difficult to predict how many students on the average, used each of the three media facilities simultaneously. Therefore, it was difficult to plan for such a system's media needs. The program trained about 26 students at a time, using 14 existing multi-media carrels and as many desks as were needed.

The manager of the training program intended to: 1) Expand student capacity from 26 to 45; and 2) Change one audio lesson and five print lessons to audio-visual media. An important consideration was to determine how many (new) units of each type were needed to service the students in the expanded system and keep waiting time down to about one minute (or less) per student for any lesson.

The researcher considered only the three types of media used in the instructional program: audio, audio-visual, and print (i.e. not the simulated office). The training program used many units of each media type to serve the random student arrivals at each type; thus the system had many channels. Likewise, the system had many phases because it had 69 lessons. Therefore, in operational research terminology, this self-instructional system was a multi-phase, multi-channel queueing system.

The problem statement then, was formulated as follows: "Can the model-building approach of operational research - queueing theory - be successfully applied in the planning and management of facilities for a self-instructional system?"

## THE OPERATIONAL RESEARCH APPROACH

Because operational research is new to educational theory and practice, it is important to define a few terms before going any further. Operational research (OR) is "the application of scientific methods, techniques, and tools to problems involving the operation of systems (humans, tools, materials, educational institutions, etc.) so as to provide those in control of the systems with optimum solutions to the problems that arise" (Churchman, Ackoff and Arnoff, 1957, p. 18). Furthermore, "OR is not distinguished by what it investigates, but by how it conducts its investigations" (Ackoff and Rivett, 1965, p. 61).

Stafford Beer goes further by saying that "operational research is the attack of modern science on complex problems arising in the direction and management of large systems of men, machines, materials and money in industry, business, government, and defense. Its distinctive approach is to develop a scientific model of a system, incorporating measurements of factors such as chance and risk, with which to predict and compare the outcomes of alternative decisions, strategies or controls. The purpose is to help management determine its policy and actions scientifically" (1966, p. 92).

Problems and solutions are not unique - they often repeat themselves - so the same type of situations arise in diverse ways. Consequently, Ackoff and Sasieni identify eight problem-solving (operational research) types: queueing, inventory, allocation, sequencing, routing, replacement, competing, search, and any combination of the above (1968, p. 13).

Ackoff and Rivett say that OR has three essential characteristics: 1) It is systems oriented, which means it expands to encompass the whole problem with all its interacting parts, although it does not necessarily start with the system as a whole; 2) It is inter-disciplinary because it uses methods and techniques of OR and other fields to look at complex systems. "When scientists from different disciplines study a system from different angles - particularly their field of specialization - the possible approaches to solving the problem grows" (Churchman, Ackoff, and Arnoff, 1957, pp. 8-9); and 3) It uses a scientific method, or mathematical equations (models) often used to better understand the real system (1965).

As stated, a system is a very important operational research term because a system is taken to be an organized group of components which, when interacting upon or with each other, produces a definable output when given a prescribed input (Edney, 1972, p. 66). What this means is that the system is constructed of different parts (variables) which are defined as accurately as possible. In so doing, the system is greatly simplified, is made more manageable, is better understood, and can be considered as a whole. The overall result is that the system produces a definable output or solution which might not have otherwise been defined in the complex "real world" system.

Three observations should be kept in mind when using models: 1) Models cannot replace the "real world" but, at best, reduce a complex system to manageable proportions;

2) Models are neither true nor false, their value is their contribution to our understanding the systems they represent; and 3) If models are used in parallel with the "real" systems being studied, this can stimulate more ideas for research and lead to better results (McMillan and Gonzales, 1965).

## OPERATIONAL RESEARCH INVESTIGATIONS CONCERNING INSTRUCTIONAL RESOURCES

Operational research emerged as a separate field during World War II for two reasons: 1) Tremendous changes were made in warfare technology after World War I and the military found itself using systems and equipment it knew little about; and 2) The Forces found that equipment, such as radar, often failed or acted unexpectedly in the field after being tested satisfactorily in laboratories. So an approach had to be found which would help produce more successful warfare equipment (Ackoff and Rivett, 1965).

In 1940, Blackett formed a team of experts, with little or no military knowledge, to study and solve certain military problems. His group was interdisciplinary because: 1) Scientists were scarce during the war; 2) It was believed that theories of other fields of knowledge could help solve these problems; and 3) Because it was hoped that these scientists would be more objective than "involved" military experts. The first experiments with radar worked, so other teams were set up in the British and later, the Allied Armed Forces with similar successes. Later, operational research experiments were considered such as: at what depth bombs should explode to sink enemy submarines, how guns should be set to hit planes, tanks, etc. (Ackoff and Rivett, 1965).

After the war, operational research experts moved into the government and industry. However, it was only in the 1950's with the second industrial revolution - automation - that operational research became popular with decision makers (Ackoff and Rivett, 1965).

In this study, the researcher was faced with the problem that the use of operational research methods in education and training had been limited and rarely documented (as this literature review will show). Schroeder and Adams (1976) said this could be the case because educators: 1) Lack an understanding of available operational research techniques; 2) Are unable to assess derivable benefits; and 3) Are unable to assess the interaction between the various methods and the problems at hand.

However, Mitchell (1976a) said that financial pressures are making it necessary for educational systems to use operational research techniques. He discussed and solved several single and multi-channel queueing problems and asserted that educational technologists cannot simply trust their intuition and common sense to solve instructional systems' needs. He further suggested that a new type of educational expert, an educational engineer, is needed to develop instructional systems as well as to apply operational research techniques.

Beaulieu and Dubois (1974) also argued that changes must be made in the educational system. They proposed that simulation models can assist in: 1) Planning resource requirements (human or non-human); and 2) Programming curriculum to best fulfill human and community needs. For example the models can determine how many materials and how much equipment a learning resource centre needs, what professionals (and how many) a community needs, what the courses should teach, etc. They detailed the systematic approach to be taken, considering such things as: student flow through institutions, community human resource needs, flow of community college students to higher institutions, the labor hiring rate, major factors that would vary the level of student potential, etc. They also suggested that the "real" systems are usually too complex for intuition, therefore systems analytic models - such as queueing theory - should be used. They unfortunately neglected to show the simulation model used.

Ackoff and Rivett (1965) described one of the first documented applications of operational research theory to a training problem. An American airline, wishing to minimize operational costs, asked an operational research team to determine how many air stewardesses the company should train, and how often the courses should be run. The team succeeded in optimally balancing the supply and demand for stewardesses to be trained. Then the team considered, with equal success, other problems such as where the personnel should be based, how many planes were needed and where, etc. The results made it possible for the airline to cut down on expenses yet maintain good service.

While Ackoff and Rivett (1965) were concerned with regulating student arrivals into the learning system, Dwyler (1976) considered a problem similar to the present study, i.e. that the most important problem to solve when a school plans to expand or modify its media centre is what media facilities are needed and how many? Decisions such as these must be based on fact and reliable procedures, so that the limited available resources can meet the material needs of the learners. At Thornwood High School in Illinois, students' and teachers' requests for media were used to help plan the media centre purchasing. Two things were considered - the media and material utilization and the number of students and teachers who were denied service. Queueing theory considers these two points, but Dwyler neglects to state what computer model he used to help plan his media centre requirements. He concludes by saying that his computer model was successful and that he was able to recommend what facilities were needed and how many so that neither students nor teachers were denied use of media equipment.

Redfearn (1973), on the other hand, describes his problem-solving model in great detail. He stated that a student demanding self-instructional materials resembles a customer. Learning is concerned with presenting and reinforcing knowledge, as well as motivating students; waiting to be served does not create good-will nor motivate learning. Redfearn affirmed that it is essential to consider queueing theory to optimally balance user and server idle time in a learning system. What is desired is to provide enough learning facilities so that students do not wait too long, but not too many facilities that would never or rarely be used.

Redfearn presented a simple queueing (mathematical/graphical) model - used in this research - and resolved how many serving units are needed for a certain personalized instruction course. He concluded that the model was not perfect, but useful for providing approximations. Redfearn further suggested that designers should usually consider buying cheap, simple equipment to provide many service channels and low utilization.

Mitchell (1976b) presented some problems that learning resource centre managers often face: whether to expand and decentralize a centre, how many audio-visual units should be provided to serve conference delegates, etc. One example is very similar to this research: One-User Media Centre (OUMC) set up a program for 600 students whereby the 15 hours per week of multi-media self-instruction had been assigned as follows - 1) 2 hours of TV, 2) 8 hours of slide-tape, 3) 4 hours of audio, and 4) 1 hour of film. Initially, 500 multi-media carrels were provided to meet student demand. It soon became clear that so many expensive units were unnecessary and that fewer units could keep waiting time down to one minute. Mitchell solved many such problems by using a queueing model and associated shortcuts and long formulae which he presented in the article.

Anderson (1973), by using his QUEUEING computer model, simulated: 1) A 9-hour working day; 2) A mean service time of 10 minutes; and 3) A mean arrival rate of 20 students per hour to the personalized instructional system. The simulation started off by examining the effect of 20 facilities and found that fewer were needed. By iteration, Anderson concluded that four units would satisfy service needs with a high (facility) utilization, but that five or six units should be considered so that service facilities would be freer and therefore students would probably not have to wait so long to gain access to them. He believed that his model could: 1) Indicate how many facilities were needed to keep student waiting time and purchasing costs within accepted limits; and 2) Specify how long students would probably have to wait to gain access to a learning system with a certain number of facilities.

In another area of education, Bookstein (1972) aimed to minimize library congestion at the card catalogue by determining whether the catalogue system should be arranged in alphabetical order or divided into subject-author headings. He developed a queueing model, similar to that used in this study, to look at three types of congestion: 1) The probability of drawer being used; 2) The average time needed to wait for drawer use; and 3) The average number of people attracted to the drawer at any time. Arrival and service rates were assumed to be random and drawers were randomly picked. He concluded that the alphabetic catalogue system seemed to be the better choice. While Bookstein studied the above-mentioned types of congestion, the researcher considered congestion to three types of media in a training system.

Rouse (1975) gave other examples of where queueing theory could be successfully employed to solve library problems. Some examples he mentioned dealt with books, reference and other services, balancing old and new services, etc. He believed, as does the researcher, in using queueing theory to determine the optimal "quantity" and thereby provide the best possible "quality" of library service.

McManamon (1973), in another related area, surveyed the inter-connection of existing and future cable TV for two-way transfer of audio-video and digital data signals. Because cable TV is providing more and more educational programs, it was desirable to develop an interactive system whereby the users can communicate requests (using digital telephones) and the cable network will show the desired program as soon as possible, automatically changing the users' sets to the right station. Queueing theory was used to determine how many stations will be needed to serve a minimum of 30,000 families considering first-come-first-served, an incomplete system's design, and the telephone, television, cost, technical advancement, and other projections. McManamon said a computer simulation model of multi-channel queueing theory must be used to solve this problem, because of the lack of data available and the complexity of the system.

All the above-mentioned researchers showed how the operational research method - queueing theory - could help develop and/or improve educational systems ranging from "personalized self-instruction" programs to cable television and libraries. However, only Anderson (1973), Bookstein (1972), Mitchell (1976a&b), and Redfearn (1973) provided the actual models used to plan resource requirements. The researcher used Mithcell (1976a&b) and Redfearn's (1973) queueing models to develop this study's analytical and computer models.

OBJECTIVES

This research had the following aims:

1) To show that one of the important problems an educational or training system technologist must solve when setting up or expanding a media centre is - what equipment is needed and how many of each type?

2) To show that an operational research modelling method can help solve such planning problems;

3) To use queueing theory to model and to solve a training centre's facility planning and resource allocation problem;

4) To construct both an analytical and a computer simulation model to represent an existing self-instructional system; and

5) To show how an educational technologist can engage in facility planning and resource allocation.

Therefore, the objectives of this research become the following: 1) To construct and test (a) an analytical and (b) a computer model of an existing instructional system; and 2) To conduct a systems analytic study to provide the design for a proposed new system (which consisted of a modification and expansion of the original system).

## CHAPTER II

## THE SYSTEMATIC APPROACH TO BUILDING THE MODEL OF THE QUEUEING SYSTEM

Before building the model to be used in the analytical and computer simulation models, the researcher examined the original Business Office training program of Bell Canada in more detail.

### THE SPECIFIC SYSTEMS ANALYSIS PROBLEM

The Business Office training program at Bell Canada in Montreal was a variation of the so-called Personalized System of Instruction (PSI) whereby students worked individually, at their own pace, during the 7-hour working day. The program was made up of 69 sequential lessons using a simulated office and three types of media: audio, audio-visual, and print. There was a definite lesson and media plan that students had to follow, with each lesson taught using only one of the specified media. Student self-pacing makes equipment needs difficult enough to determine, but in addition, each media sub-system, at this training centre, had a different number of lessons and their lengths varied in duration.

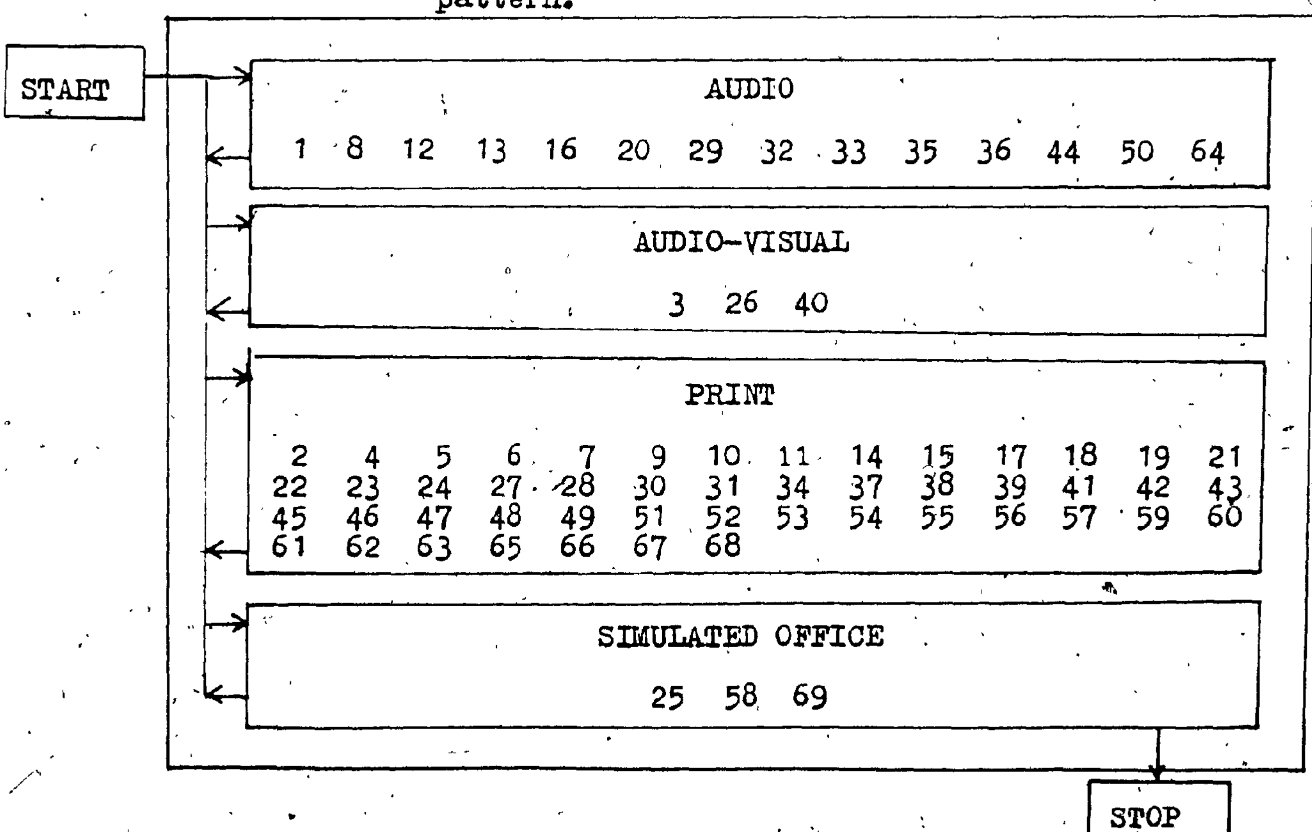
The program was made up of about 180 hours, depending on the individual student's speed. The working day was 7 hours long, excluding lunch and coffee breaks. By dividing the total service time of 180 hours, derived from the Bell documentation, by the 7-hour working day, the researcher deduced that the mean course duration was about 26 days.

Therefore, considering the arrival rate of about one student per day, it could be assumed that there was student capacity of about 26 in the existing training program at Bell Canada.

The researcher decided to study the Business Office training program by considering each media sub-system rather than the lesson sequence. To follow students through the 69 lessons would only complicate matters unnecessarily. Considering the training program as three media sub-systems (each with prescribed lessons, service mechanisms, arrival patterns, etc.) would provide a reliable estimation of media needs.

Before continuing, it is essential to visualize the system by using a diagram. Figure 2.1 indicates the lessons belonging to each media and the flow within the program.

Figure 2.1: The original training system indicating the lessons belonging to each medium and the flow pattern.



Considering Figure 2.1, we see that students using a media sub-system for any assigned lesson wait for service, i.e. there is no jumping ahead of others. This supports the assumption of first-in-first-out (FIFO) queue discipline.

It was hypothesized that the training program could be considered a queueing problem. Students arrived into the training program and started the first lesson, using audio. Then the students would leave lesson one and enter lesson two, and use print. Throughout the sequence of lessons, the trainee might re-enter the same media sub-system or move to another. This process continued until the students finished the 69 lessons in the training program.

Since this training program could be considered a queueing problem, the researcher will first discuss a few basic queueing concepts.

## BASIC QUEUEING CONCEPTS

Queueing theory is the operational research model that observes a system and its queues - it can provide general information about the number of self-instructional service facilities needed to keep costs, the number of students lined up waiting to use the facilities and the student waiting time within accepted limits. By knowing how many people are being taught and how long it usually takes students to learn, the queueing model can predict how many learning facilities (carrels, books, etc.) are needed. The model can also predict how long students wait when any specified number of facilities are available. This last prediction is extremely useful because decision-makers can then see if additional facilities and cost will significantly alter the amount of time students wait. Queueing theory is applicable to a wide variety of situations where there exists an imperfect matching between people requiring service and service facilities available.

Churchman, Ackoff and Arnoff (1957), Panico (1969), Plane and Kochenberger (1972), and many other well-known operational researchers say that queueing systems have the following three parameters: 1) An arrival pattern which is determined by the average rate of student arrivals to the learning location within a specified time span and the statistical distribution of their inter-arrivals; 2) A service mechanism which determines how many student arrivals can be served simultaneously and how long they take to complete self-instruction; and 3) The queue discipline which is usually first-come-first-served in a learning environment.

Mitchell (1976a) describes and illustrates (Figure 2.2) the four basic queue systems:

1) The simplest form a queue can take is called single-phase, single-channel queue. This simple system is made up of only one service step (phase) and only one service facility (channel). This system is a queue because people arriving, who may be students, must line up to receive attention. There are many types of simple queues such as this, and an example would be students lining up to deposit their tray of dirty dishes on the conveyor, i.e. students who must line up (and wait) before they get their chance to

get rid of their trays and depart. The length of the queue and the amount of time students spend waiting for a chance at the conveyor depends on how slowly the conveyor moves, how fast the conveyor is cleared by kitchen staff, and how many students want to deposit trays.

2) Another type of system is called the single-phase, multi-channel queue. This system is comprised of only one service step (phase) but has more than one service facility (channel) to attend to student needs. An example would be a number of librarians (channels) checking out students' books (phase). The students undergo one step, and the number of librarians serving speeds up the queue turnover or student departures.

3) A slighly more complex system is the multi-phase, single-channel queue. This queue system is made up of a couple of service steps (phases) but only one service facility (channel) is available. An example of this would be a computer course made up of three lessons with only one computer terminal available for students to use. A student could sit down and complete one or all lessons before vacating the terminal. If another student should arrive while the terminal is occupied, then he/she would have to wait until the terminal is free before commencing the course work.

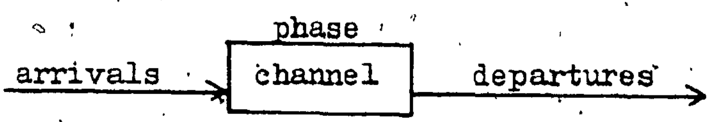
4) The final type of queue is called the multi-phase, multi-channel queue. This latter system has various stages (phases) and various service facilities (channels). An example could be a slight variation of the previous example: a three-lesson computer course with two terminals for students

to use. This latter queue system could serve about twice as many students than the same system with only one terminal. Clearly, student turnover would be greater and students would probably not have to wait in line so long.

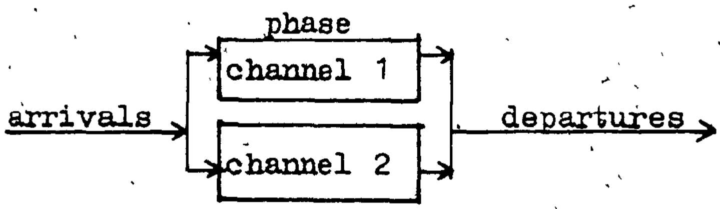
Figure 2.2 illustrates the four basic queue systems (Mitchell, 1976a, pp. 23-26).

Figure 2.2: Illustrations of the four basic queue systems.

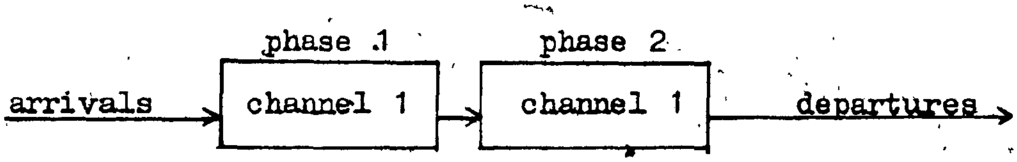
1) Single-phase, single channel:



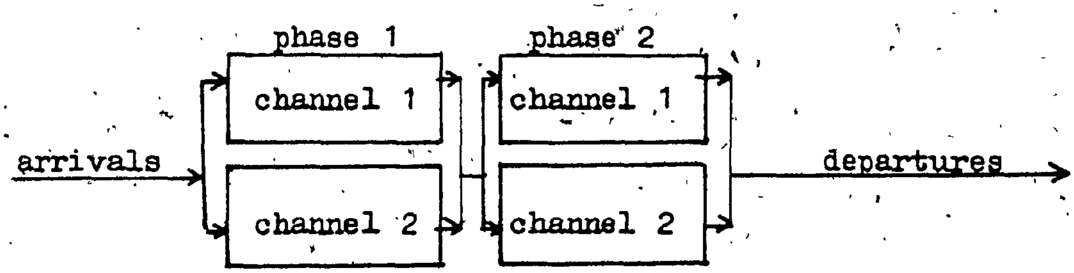

2) Single-phase, multi-channel:




3) Multi-phase, single-channel:




4) Multi-phase, multi-channel:



## THEORETICAL DEFINITIONS

Seven theoretical definitions should be clarified before going any further:

AR = Average number of students who arrive in the program every hour for service, e.g. to use audio cassettes

AT = Average time, in hours, between student arrivals for service to the program, e.g. the number of hours between people coming for audio tapes

S = Number of parallel service units of carrels available for students who take the program to use

SR = Average number of students who can be served (e.g. have access to audio recorders and tapes) every hour

ST = Average amount of time, in hours, that students take to be served

U = Utilization factor of the service facility which means the percentage of time the service facility is busy

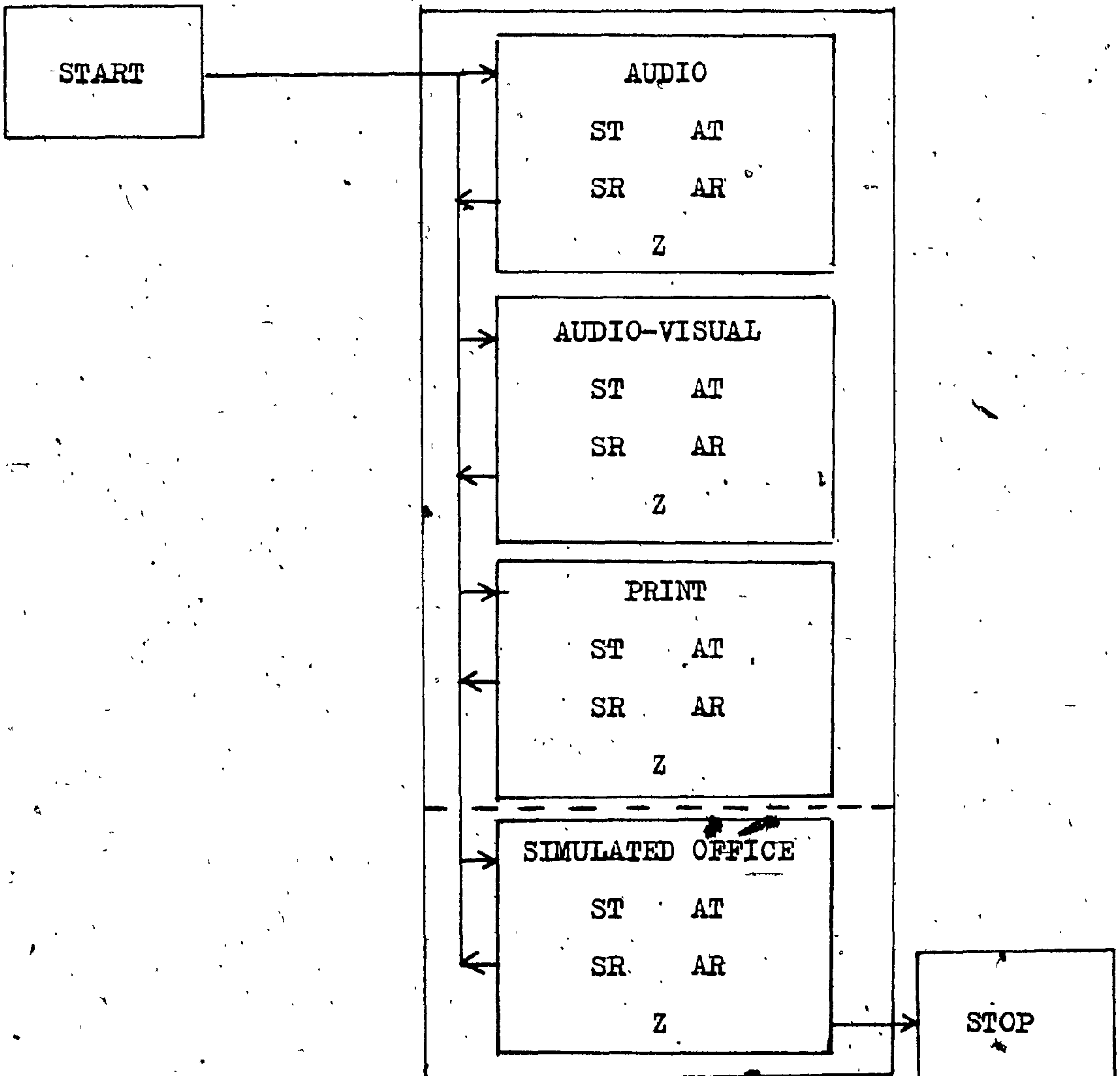
WT = Average time students wait in queue before being served

## MODEL OF THE SPECIFIC QUEUE SYSTEM

The training system was further simplified. For each media sub-system, we estimated: 1) The mean inter-arrival time (AT) and the mean arrival rate (AR) which represented the entire sub-system, not each lesson; 2) The mean of all service times (ST) and the mean service rate (SR) which also represented the entire sub-system; and 3) The number of lessons (Z) using each sub-system or medium. Figure 2.3

portrays this information for each sub-system or medium.

Figure 2.3: The simplified, aggregated model of the original queue system, indicating for each media sub-system: mean service time (ST), mean service rate (SR), mean inter-arrival time (AT), mean arrival rate (AR), and the number of lessons using each medium (Z).




The model of the original queue system was now simplified to a three-single-phase, multi-channel queueing model, each media sub-system modelled as a single-phase, single-channel system. The researcher believed that such a simplified model would be adequate to determine approximate solutions. Therefore, data from this simplified single-phase, multi-channel queueing model would be used to provide a rough approximation - a first step to determine resource requirements for the Business Office training program at Bell Canada.

It should be pointed out that the number of carrels generated by this simplified queue model will be slightly higher than results produced by a multi-phase, multi-channel queue model. This simplified model assumes that students complete all lessons using one media type before going to the next media type. Clearly, the probability of students having to wait to gain access to the multi-phase, multi-channel queue system is smaller because of the greater turn-over and smaller amount of time needed to complete a lesson and media type.

Therefore, a student arriving at an occupied medium would have to wait for another student to finish - which could take anywhere up to the full amount of time allotted to complete all the lessons using that medium. In reality, the first student in line would only wait for a student to complete a lesson.

The researcher also simplified the facilities-planning problem by considering only media needs. There also existed in the Bell training program a simulated office and two human elements - instructors and GAF (groupe d'abonnés fictifs). The instructors could affect the student flow by delays in setting up the audio-visual material, assisting when needed, and testing the students at the end of the lesson. Similarly, the GAF could cause student delays. This group of people assisted learners by calling up and pretending to be customers. Students were not marked on these exercises, they just signalled the GAF when they were ready and waited to be phoned back. These two human elements were kept in mind but not considered because the main objective was to plan media facilities. Furthermore, it was pointed out by the manager of the learning resource centre that staffing was not normally a problem. If necessary, at a later date, this operational research study could be expanded to consider a more complex system.

COMPONENTS OF THE ORIGINAL INSTRUCTIONAL SYSTEM

As in any analysis of an operational system, the researcher had to observe the various components of an existing system and collect the necessary data to build the analytical and computer-simulation models. The researcher collected information on the following: research sample, service mechanism, arrival pattern, course duration, and student capacity.

Research Sample

The research sample was drawn from the Business Office training program at Bell Canada. It was important to devise a selection process which would best represent the existing learning situation. The researcher collected data on all students who entered the program between June 1 and November 30, 1976. This six month period was selected because it was the longest and best documented period found in the files.

It was considered unnecessary for the researcher to personally observe a large sample of students progress through the program, because the researcher had access to the available data collection mechanism - the instructors of Bell Canada. The instructors normally assisted students when necessary, set up audio-visual materials, tested students at the end of each lesson, and recorded the amount of time individual students spent on each lesson. The latter provided the main data source for the investigation.

For the six-month period, a total of 129 students was documented. From this, a random sample of 70 trainees was selected. The research sample was picked by shuffling the 129 student documentation and taking the first 70 subjects.

Mean Service Times and Mean Service Rates

The data collected in the research sample was sufficient to determine the service mechanism - mean service times and mean service rates. It was imperative to: 1) Determine the average time spent (a) in each lesson and (b) using each medium (mean service times); and 2) Determine how many students each media facility actually served per hour (mean service rates).

The researcher created a computer-based data file which contained the time taken to complete each of the 69 lessons, one trainee at a time. The researcher then used a Statistical Package for Social Sciences (SPSS) program called "condescriptive". This program provided the following information for each of the 69 lessons (variables) and three types of media: mean, standard deviation, skewness, variance, minimum, maximum, and sum. The first result was the most important because the means were the average service times for each lesson and medium. The other six results were used by the researcher to check for mistakes in the computer data file.

Table 2.1 shows the average service time (in hours) for each lesson arranged by the medium used and total service time students spent in each medium. This table was prepared from results given in the SPSS "condescriptive" program.

Table 2.1: Average service time, in hours, for each lesson, arranged by the simulated office media used for the period June 1-November 30, 1976. Audio (A), audio-visual (A-V), print (P), simulated office (SO).

| No. | A | A-V | P | SO | No. | A | A-V | P | SO |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 3.75 | | | | 36 | 7.00 | | | |
| *2 | | | 3.75 | | 37 | | | 1.00 | |
| 3 | | .75 | | | 38 | | | .75 | |
| *4 | | | 2.50 | | 39 | | | .75 | |
| 5 | | | 3.00 | | 40 | | 5.25 | | |
| 6 | | | 1.50 | | 41 | | | 1.50 | |
| 7 | | | .50 | | 42 | | | 2.75 | |
| *8 | 3.50 | | | | 43 | | | 2.25 | |
| *9 | | | 1.75 | | 44 | 1.75 | | | |
| 10 | | | 4.25 | | 45 | | | 1.25 | |
| 11 | | | 1.00 | | 46 | | | 5.25 | |
| 12 | 6.25 | | | | 47 | | | 3.75 | |
| 13 | 5.25 | | | | 48 | | | 3.00 | |
| 14 | | | 2.75 | | 49 | | | 1.75 | |
| 15 | | | .75 | | 50 | 2.00 | | | |
| 16 | 4.75 | | | | 51 | | | 1.25 | |
| 17 | | | 3.50 | | 52 | | | 3.00 | |
| 18 | | | 1.50 | | 53 | | | 3.75 | |
| 19 | | | .75 | | *54 | | | 1.25 | |
| 20 | 3.00 | | | | 55 | | | 1.50 | |
| 21 | | | 1.00 | | 56 | | | 2.50 | |
| 22 | | | 1.00 | | 57 | | | 3.75 | |
| 23 | | | 2.00 | | 58 | | | | 3.50 |
| 24 | | | .75 | | 59 | | | 2.25 | |
| 25 | | | | 4.00 | 60 | | | 2.25 | |
| 26 | | 1.00 | | | 61 | | | 1.25 | |
| 27 | | | 2.75 | | 62 | | | 1.00 | |
| 28 | | | 1.50 | | 63 | | | 2.25 | |
| 29 | 16.00 | | | | 64 | 2.50 | | | |
| *30 | | | 1.50 | | 65 | | | 1.00 | |
| 31 | | | 2.50 | | 66 | | | 1.50 | |
| 32 | 2.75 | | | | 67 | | | 1.75 | |
| 33 | 4.25 | | | | 68 | | | .75 | |
| 34 | | | .75 | | 69 | | | | 9.50 |
| 35 | 4.25 | | | | Total | 65.25 | 6.75 | 91.50 | 16.75 |

*Six media changes from audio & print to audio-visual to be considered in the expanded system

Having obtained the mean service time for each lesson and the total service time for the three media, in addition to knowing the number of lessons using each media ($Z_m$) where m refers to the media type, the researcher determined the mean service time (ST) and the mean service rate (SR) for one lesson in each medium. The following formulae were used:

$$ST = \frac{\text{total service time}}{Z_m} \qquad SR = \frac{1}{ST}$$

Although the instructors rounded student study time to the nearest 15 minutes, the researcher decided not to do the same for the calculations. The researcher wanted the model's output to limit waiting time to approximately 1 minute if possible. It was lamentable that timings were rounded off, but it would not be practical for the instructors to be more specific - it would be far too confusing and time-consuming.

It was assumed that measurement errors introduced by rounding off were not systematic but balanced, and that these errors - introduced into a model that was already a crude approximation - could justifiably be accepted and that calculations be made using them. This also applies to almost all tables to follow.

By referring back to Table 2.1 and using the above-mentioned formulae, the following mean service times (ST) and mean service rates (SR) as shown in Table 2.2, were derived for the different media in the original Business Office program.

Table 2.2: Mean service times (ST) and mean service rates (SR) for each medium in the training system.

| | ST (hours) | SR (hours) |
|---|---|---|
| audio-visual | 2.250 | .444 |
| audio | 4.661 | .215 |
| print | 1.867 | .536 |

## Exponential Distribution

Queueing theory assumes an exponential distribution of service times. By referring back to the computer data file, the researcher determined the frequency of the service timings (per hour). Table 2.3 shows the frequency distribution for each medium.

Table 2.3: The frequency distribution for the service timings, for each of the media, in the training system.

| DURATION (hours) | AUDIO-VISUAL | AUDIO | PRINT |
|---|---|---|---|
| 0 | 93 | 5 | 856 |
| 1 | 46 | 115 | 1203 |
| 2 | 5 | 198 | 713 |
| 3 | 6 | 196 | 385 |
| 4 | 22 | 170 | 167 |
| 5 | 23 | 92 | 65 |
| 6 | 7 | 67 | 20 |
| 7 | 3 | 38 | 15 |
| 8 | 5 | 26 | 9 |
| 9 | | 5 | 4 |
| 10 | | 4 | 1 |
| 11 | | 3 | 1 |
| 12 | | 4 | 1 |
| 13 | | 1 | |
| 14 | | 4 | |
| 15 | | 22 | |
| 16 | | 4 | |
| 17 | | 5 | |
| 18 | | 8 | |
| 19 | | 5 | |
| 20 | | 1 | |
| 21 | | 3 | |
| 22 | | 2 | |
| 23 | | 2 | |
| | 210 | 980 | 3430 |

These frequency distributions were then plotted for each of the media as shown in Figure 2.4 (audio-visual), Figure 2.5 (audio), and Figure 2.6 (print).

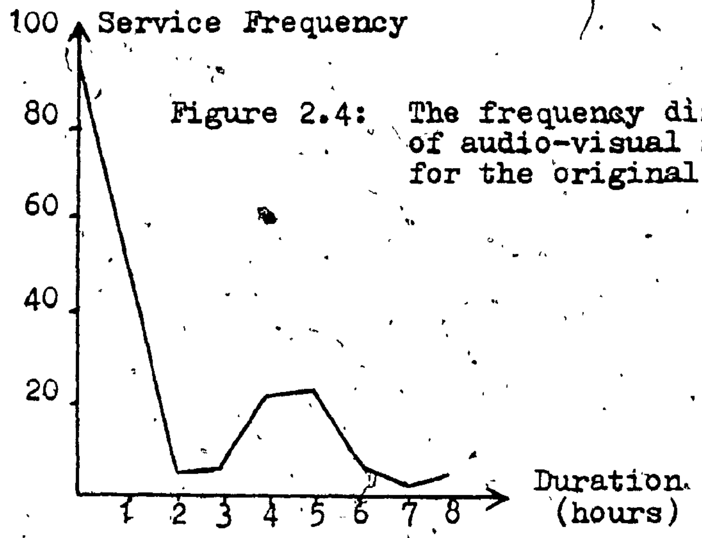



Figure 2.4: The frequency distribution of audio-visual service timings, for the original training system.

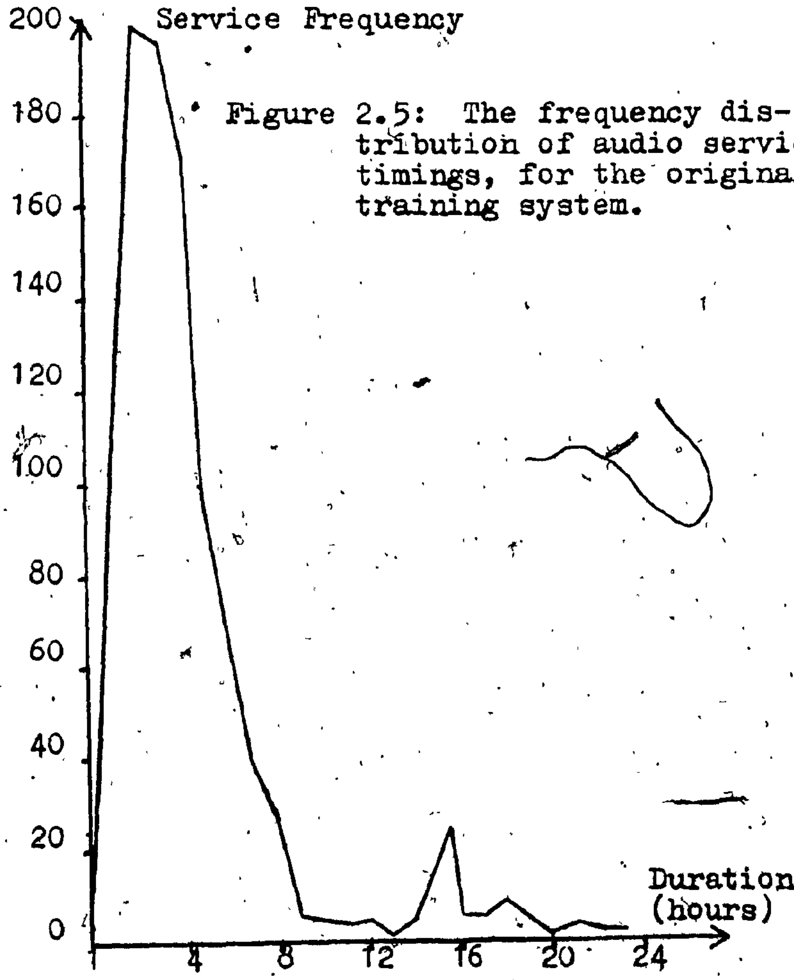



Figure 2.5: The frequency distribution of audio service timings, for the original training system.

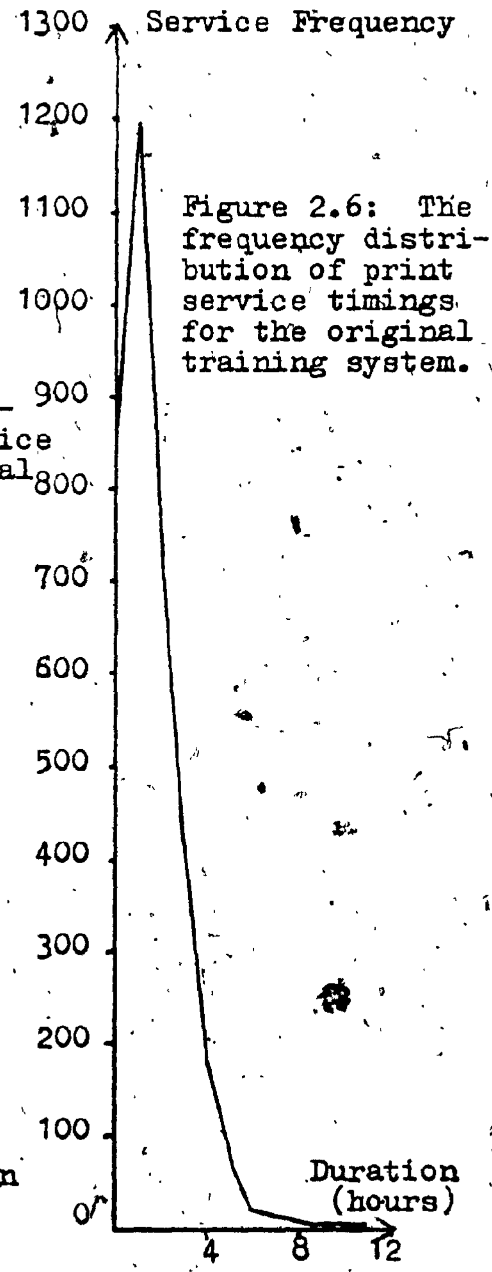



Figure 2.6: The frequency distribution of print service timings for the original training system.

The diagrams show that the service timings were approximately exponentially distributed, with a few bumps here and there. The graphs also show that there was a higher frequency of short timings than long timings. Panico (1969) states that real world data rarely follows this (exponential) service distribution. He also states that the process to determine the distribution (of plotting each timing) is laborious and rather useless because the researcher will undertake the study regardless of the service distribution.

Insofar as the three figures did approximate an exponential distribution, it could therefore be assumed that queueing theory could be used.

<u>Mean Inter-arrival Times and Mean Arrival Rates</u>

The next major component to be considered was the arrival pattern: mean inter-arrival times and mean arrival rates. It was necessary to determine when new students arrived each day at the training program (mean inter-arrival times) and estimate how many students arrived at each media per hour (mean arrival rates).

Table 2.4 depicts the student arrivals at the Bell Canada Business Office training program during the observed six-month period.

Table 2.4: The number of student arrivals per day to the training program during the 6-month period of June 1 - Nov. 30, 1976, from which the sample was taken.

| Months | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | Working Days | Arrivals |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| June | 1 | | | 1 | – | – | 2 | 1 | 2 | 2 | | – | – | 2 | 1 | 2 | 1 | | – | – | 2 | | 1 | 1 | 1 | – | – | 4 | | | x | 22 | 24 |
| July | 1 | – | – | 2 | | | | 3 | | – | – | | | 1 | | 1 | – | – | | 4 | 1 | 2 | 2 | – | – | | | 1 | 2 | 1 | – | 22 | 21 |
| Aug. | – | 3 | | 1 | 2 | | – | – | | 3 | | | | – | – | 1 | 1 | 2 | | 1 | – | – | 1 | 1 | 1 | 3 | 1 | – | – | | | 22 | 21 |
| Sept. | | 1 | 1 | – | – | | 1 | 1 | | 1 | – | – | | 2 | 2 | 1 | 1 | – | – | | 2 | | 2 | 1 | – | – | 1 | | | 2 | x | 22 | 19 |
| Oct. | 1 | – | – | | | 3 | 1 | 1 | – | – | | 4 | | | 1 | – | – | | | 2 | | | – | – | 1 | 1 | | 2 | 1 | – | – | 21 | 18 |
| Nov. | 2 | 1 | 1 | 3 | | – | – | 3 | | 2 | 2 | 1 | – | – | 1 | | 3 | | 1 | – | – | 3 | | 1 | 1 | | – | – | | 1 | x | 22 | 26 |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | Total | | 131 | 129 |

— = Weekends  x = No day

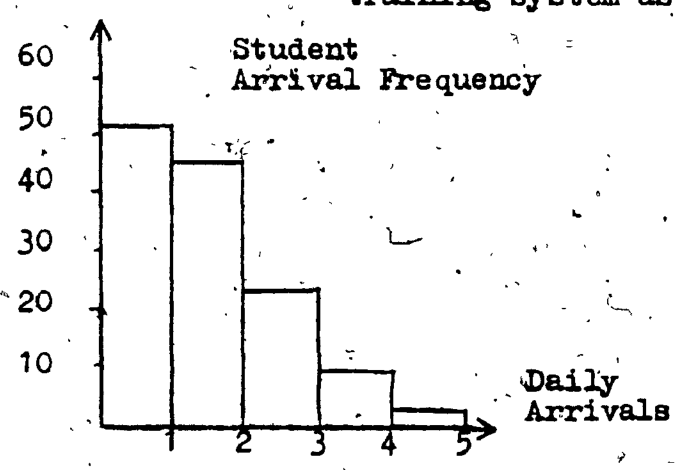
Since arrivals were not consistently one per day, Panico's (1969) method was used to obtain the daily arrival rate. By following his model, the researcher first specified the arrival frequency, as shown in Table 2.5. The mean arrival rate of one student per day was then found by dividing 129 by 131.

Table 2.5: Student arrival frequency to the training program during the 6-month period of June - November, 1976.

| Daily Arrivals | Number of Days | Total Number of Student Arrivals |
|---|---|---|
| 0 | 51 | 0 |
| 1 | 46 | 46 |
| 2 | 22 | 44 |
| 3 | 9 | 27 |
| 4 | 3 | 12 |
| | 131 | 129 |

The arrival frequency distribution depicted in Table 2.5 is shown in Figure 2.7 below.

Figure 2.7: The arrival frequency to the whole training system as shown in Table 2.5.



Having ascertained the student arrival pattern at Bell's training program, the researcher then determined the mean inter-arrival times (AT) and the mean arrival rates (AR) for each media sub-system, $Z_m$ being the number of lessons using each medium. The following formulae were used:

$$AT = \frac{\text{inter-arrival time}}{Z_m} \qquad AR = \frac{1}{AT}$$

Keeping in mind that, on the average, one student arrived daily (i.e. every 7 hours), the researcher derived the following mean inter-arrival times (AT) and mean arrival rates (AR) as shown in Table 2.6 for each media sub-system in the Business Office training program.

Table 2.6: The mean inter-arrival times (AT) and the mean arrival rates (AR) for each media sub-system in the system.

| | AT (hours) | AR (hours) |
|---|---|---|
| audio-visual | 2.333 | .429 |
| audio | .500 | 2.000 |
| print | .143 | 7.000 |

It was assumed that the inter-arrival media were randomly distributed even though the initial arrivals into the whole training program were somewhat regulated, i.e. about one student arrived per day. Furthermore, the system was always a bit unsteady during the starting-up periods (in the morning and after breaks), which was to be expected. On the whole, however, the system appeared to be stable with randomly distributed arrivals to the different media sub-systems beginning after the first lesson.

Poisson Distribution

While queueing models assume that the service frequency has an exponential distribution, they also assume a poisson distribution for the arrival frequency. Therefore, the arrival frequency had to fulfill the following conditions: 1) Arrivals could never occur simultaneously, there was always some measurable interval between them; 2) There existed a possibility of zero arrivals during the time segment; 3) It was more frequent for more than one student to arrive per day to the entire training program; 4) There was a single peak in the arrival distribution; and 5) The arrival distribution could not be symmetrical about the mean (Panico, 19690.

To determine the individual poisson probabilities, the researcher referred to Schlaifer's (1959) "Cumulative Poisson and Gamma Distribution Chart" (p. 712), reproduced in Figure 2.8.

Referring to Schlaifer's chart and given that the mean arrival rate was one per day, the poisson probabilities for the arrival frequency were read from the chart by using the relation: P(x) = Px' - Px (Schlaifer, 1959, p. 213). Therefore, the probabilities of the different arrival frequencies were calculated as:

P(zero) = .9999 - .6300 = .3600

P(one) = .6300 - .2600 = .3700

P(two) = .2600 - .0540 = .2060

P(three)= .0540 - .0140 = .0400

P(four) = .0140 - .0030 = .0110

Figure 2.8: Schlaifer's Cumulative Poisson and Gamma Distribution Chart (1959, p. 712)

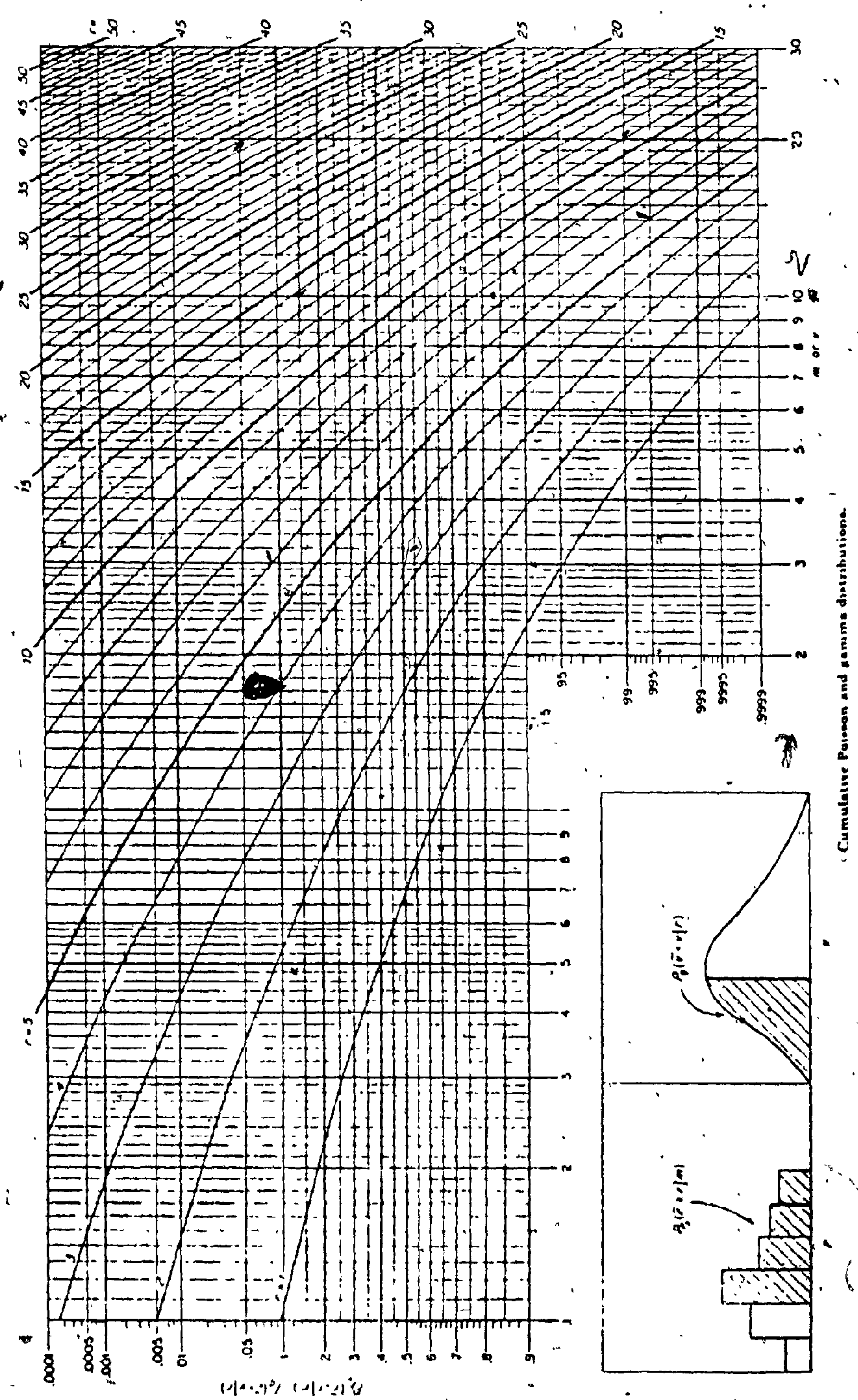
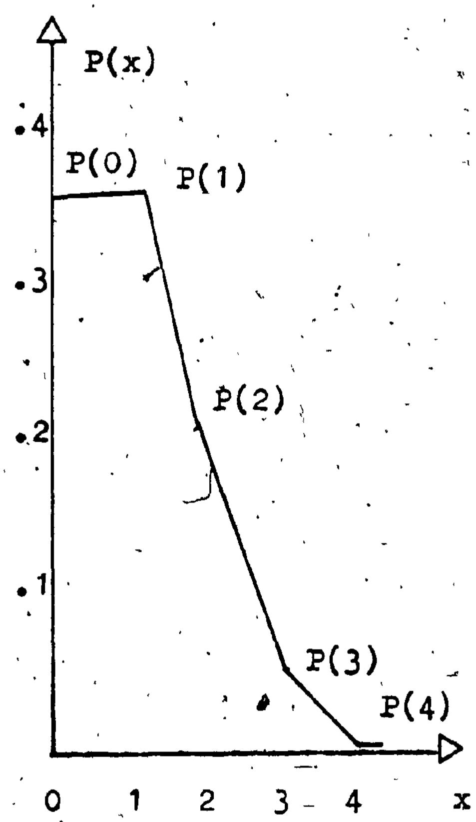


Figure 2.9 shows the probabilities of the different arrival frequencies, plotted from the previous calculations. There was only one peak and all the other conditions for a poisson distribution were fulfilled, therefore it was concluded that this arrival frequency had a poisson distribution.

Figure 2.9: The (Poisson) distribution of the arrival frequency to the original training program.



Queue Discipline

The queue discipline that existed within this training program and therefore, the model, was to deal with students as they arrived to use any one of the media on a first-in-first-out (FIFO) basis. This was the fairest and simplest discipline on the whole, because it served the first students to arrive and ruled out the confusion that might be caused by priorities.

# CHAPTER III

# ANALYTICAL SOLUTIONS TO THE MODEL OF THE ORIGINAL TRAINING SYSTEM

There are various approaches to solving queueing models. The two approaches used in this research were analytical and computer-simulation. Mathematical or analytical approaches are useful for simple systems or for acquiring quick estimations. Computer-simulations are useful for more complex problems or for acquiring more detailed estimations (Tersine and Altimus, 1974).

## ANALYTICAL MODEL

The analytical model was borrowed from Redfearn (1973). The researcher was able to approximate the number of service facilities needed in the Business Office program by performing the following three steps:

1) Determine the percent utilization (U) of the service facilities and the expected student waiting time (ST) by using the following formulae: $U = AR/(S \times SR)$ (Mitchell, 1976b, p. 32) and $WT = QT \times ST$ (Redfearn, 1973, p. 228) where QT stood for the mean queueing time;

2) Refer to Redfearn's "Mean Queueing Time" graph (Redfearn, 1973, p. 228), reproduced in Figure 3.1;

3) Use the appropriate AT, AR, ST, and SR for each medium as presented in Table 3.1 (to follow). Since the answers derived from the use of the above formulae were in "hours", they were multiplied by 60 to determine estimated

waiting time in "minutes".

Figure 3.1: Redfearn's Mean Queueing Time graph.

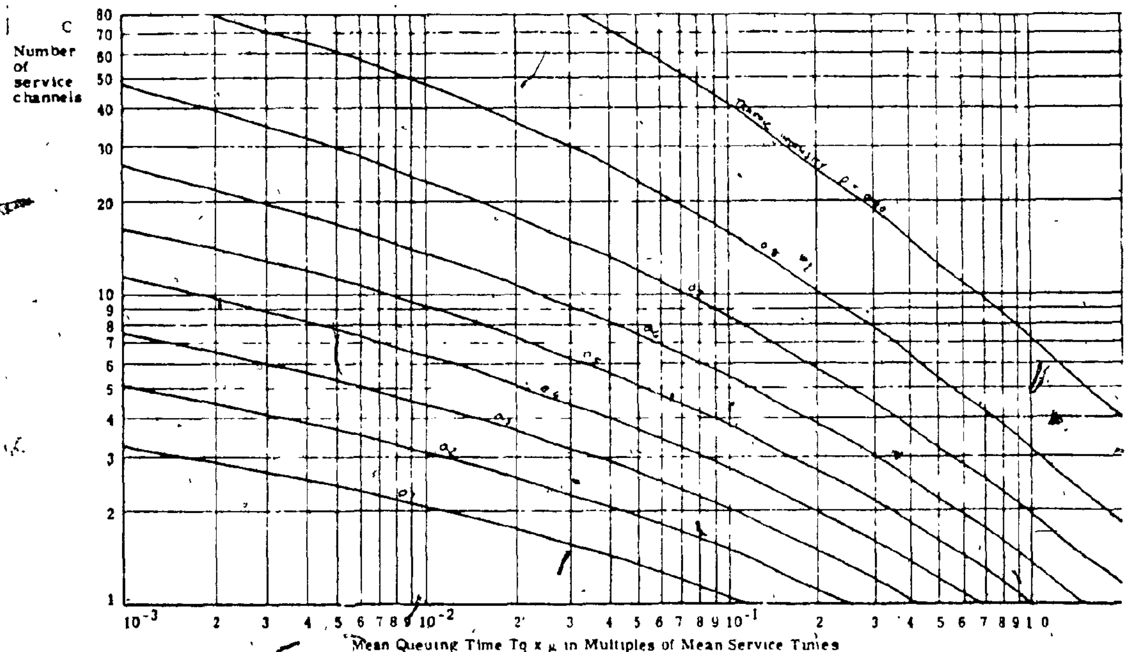



Table 3.1: AT, AR, ST, and SR for each medium in the training system, given in hours.

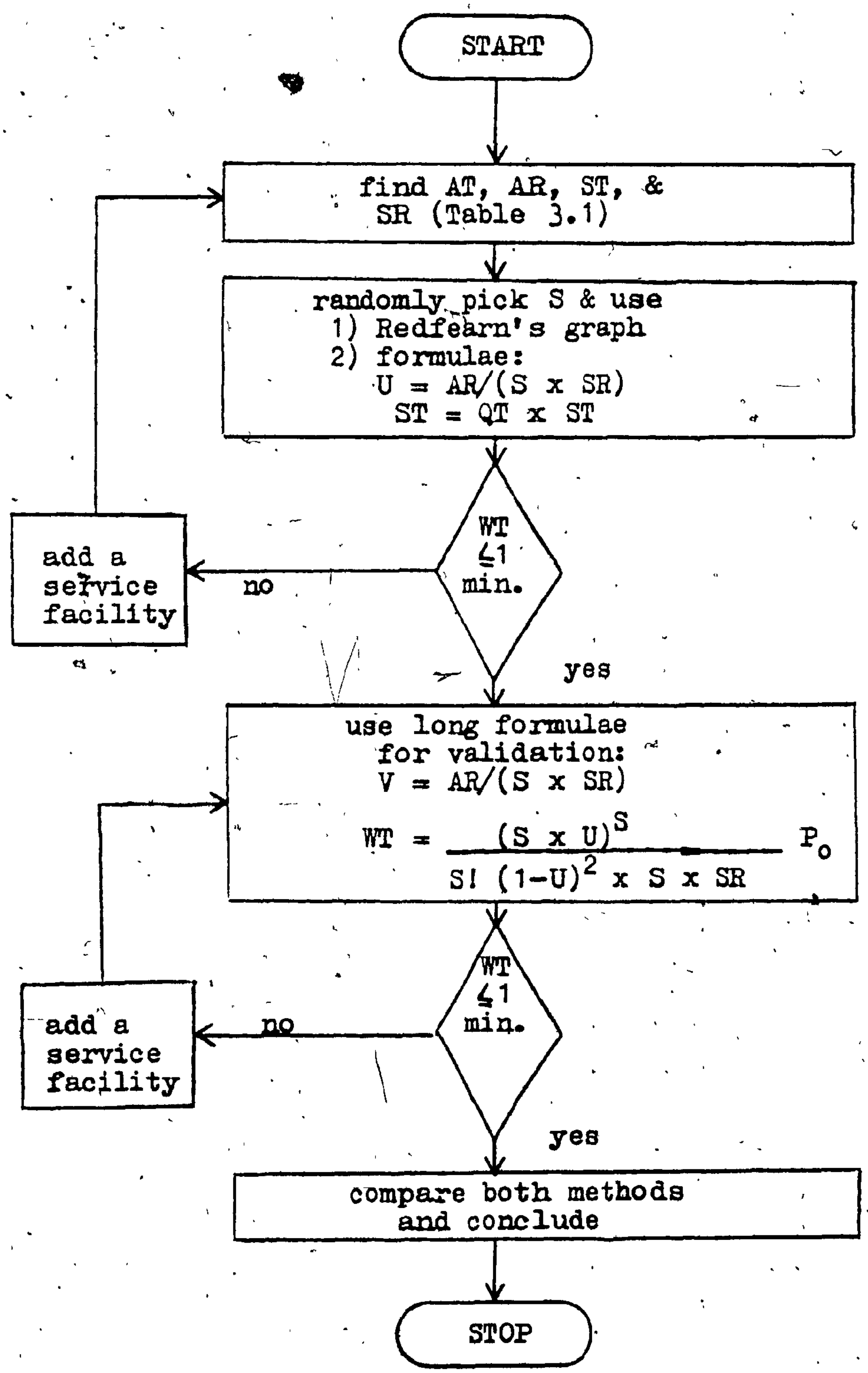
| | AT | AR | ST | SR |
|---|---|---|---|---|
| audio-visual | 2.333 | .429 | 2.250 | .444 |
| audio | .500 | 2.000 | 4.661 | .215 |
| print | .143 | 7.000 | 1.867 | .536 |

For validation, the researcher used Plane and Kochenberger's $P_0$ chart (1972, p. 196) reproduced in Table 3.2 and two of Mitchell's formulae (1976b, pp. 32-33):

$$U = AR \,/\, (S \times SR) \qquad WT = \frac{(S \times U)^S}{S!\ (1-U)^2 \times S \times SR}\ P_0$$

The following flow chart, depicted in Figure 3.2, shows the steps the researcher took to obtain the analytical solutions for each medium in the queueing model. This procedure was repeated three times to consider audio, audio-visual and print media needs.

Figure 3.2: Flow chart used to analytically determine media needs in the training system.



ANALYTICAL CALCULATIONS

Following the steps in the flow chart, Figure 3.2, the following calculations were made for the training centre's media needs. The "*" denotes the accepted number of facilities (S) to keep waiting time down to about one minute.

FORMULAE: $U = \frac{AR}{S \times SR}$ $WT = QT \times ST$

Audio-Visual Sub-System

If S = 3 and U = .32, then WT = $\begin{array}{r} .045 \\ \times\ 2.250 \\ \hline .10125 \end{array}$ hour x 60 = 6 min.

*if S = 4 and U = .24, then WT = $\begin{array}{r} .0075 \\ \times\ 2.2500 \\ \hline .016875 \end{array}$ hour x 60 = 1 min.

if S = 5 and U = .2, then WT = $\begin{array}{r} .0001 \\ \times\ 2.2500 \\ \hline .000225 \end{array}$ hour x 60 = .01 min.

Audio Sub-System

If S = 16 and U = .58, then WT = $\begin{array}{r} .0055 \\ \times\ 4.6610 \\ \hline .0256355 \end{array}$ hour x 60 = 1.54 minutes

*if S = 17 and U = .55, then WT = $\begin{array}{r} .0028 \\ \times\ 4.6610 \\ \hline .0130508 \end{array}$ hour x 60 = .8 minute

Print Sub-System

If S = 19 and U = .69, then WT = $\begin{array}{r} .015 \\ \times\ 1.867 \\ \hline .028005 \end{array}$ hour x 60 = 1.7 minutes

*if S = 20 and U = .65, then WT = $\begin{array}{r} .0075 \\ \times\ 1.8670 \\ \hline .0140025 \end{array}$ hour x 60 = .8 minute

# Table 3.2: Plane and Kochenberger's $P_o$ Chart.

Multi-Server Poisson/Exponential Queuing System. Probability that the System Is Idle, $p_0$

| $\lambda/s\mu$ | Number of Channels, $s$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 10 | 15 |
| 02 | 9608 | 9418 | 9231 | 9048 | 8869 | 8694 | 85214 | 81873 | 74082 |
| 04 | 9231 | 8869 | 8521 | 8187 | 7866 | 7558 | 72615 | 67032 | 54881 |
| 06 | 8868 | 8353 | 7866 | 7408 | 6977 | 6570 | 61878 | 54881 | 40657 |
| 08 | 8519 | 7866 | 7261 | 6703 | 6188 | 5712 | 52729 | 44933 | 30119 |
| 10 | 8182 | 7407 | 6703 | 6065 | 5488 | 4966 | 44933 | 36788 | 22313 |
| 12 | 7857 | 6975 | 6188 | 5488 | 4868 | 4317 | 38289 | 30119 | 16530 |
| 14 | 7544 | 6568 | 5712 | 4966 | 4317 | 3753 | 32628 | 24660 | 12246 |
| 16 | 7241 | 6184 | 5272 | 4493 | 3829 | 3263 | 27804 | 20190 | 09072 |
| 18 | 6949 | 5821 | 4866 | 4065 | 3396 | 2837 | 23693 | 16530 | 06721 |
| 20 | 6667 | 5479 | 4491 | 3678 | 3012 | 2466 | 20189 | 13534 | 04979 |
| 22 | 6393 | 5157 | 4145 | 3328 | 2671 | 2144 | 17204 | 11080 | 03688 |
| 24 | 6129 | 4852 | 3824 | 3011 | 2369 | 1864 | 14660 | 09072 | 02732 |
| 26 | 5873 | 4564 | 3528 | 2723 | 2101 | 1620 | 12492 | 07427 | 02024 |
| 28 | 5625 | 4292 | 3255 | 2463 | 1863 | 1408 | 10645 | 06081 | 01500 |
| 30 | 5385 | 4035 | 3002 | 2228 | 1652 | 1224 | 09070 | 04978 | 01111 |
| 32 | 5152 | 3791 | 2768 | 2014 | 1464 | 1064 | 07728 | 04076 | 00823 |
| 34 | 4925 | 3561 | 2551 | 1821 | 1298 | 0925 | 06584 | 03337 | 00610 |
| 36 | 4706 | 3343 | 2351 | 1646 | 1151 | 0804 | 05609 | 02732 | 00452 |
| 38 | 4493 | 3137 | 2165 | 1487 | 1020 | 0698 | 04778 | 02236 | 00335 |
| 40 | 4286 | 2941 | 1993 | 1343 | 0903 | 0606 | 04069 | 01830 | 00248 |
| 42 | 4085 | 2756 | 1834 | 1213 | 0800 | 0527 | 03465 | 01498 | 00184 |
| 44 | 3889 | 2580 | 1686 | 1094 | 0708 | 0457 | 02950 | 01226 | 00136 |
| 46 | 3699 | 2414 | 1549 | 0987 | 0626 | 0397 | 02511 | 01003 | 00101 |
| 48 | 3514 | 2255 | 1422 | 0889 | 0554 | 0344 | 02136 | 00820 | 00075 |
| 50 | 3333 | 2105 | 1304 | 0801 | 049 | 0298 | 01816 | 00671 | 00055 |
| 52 | 3158 | 1963 | 1195 | 0721 | 0432 | 0259 | 01544 | 00548 | 00041 |
| 54 | 2987 | 1827 | 1094 | 0648 | 0381 | 0224 | 01311 | 00448 | 00030 |
| 56 | 2821 | 1699 | 0999 | 0581 | 0336 | 0194 | 01113 | 00366 | 00022 |
| 58 | 2658 | 1576 | 0912 | 0521 | 0296 | 0167 | 00943 | 00298 | 00017 |
| 60 | 2500 | 1460 | 0831 | 0466 | 0260 | 0144 | 00799 | 00243 | 00012 |
| 62 | 2346 | 1349 | 0755 | 0417 | 0228 | 0124 | 00675 | 00198 | 00009 |
| 64 | 2195 | 1244 | 0685 | 0372 | 0200 | 0107 | 00570 | 00161 | 00007 |
| 66 | 2048 | 1143 | 0619 | 0330 | 0175 | 0092 | 00480 | 00131 | 00005 |
| 68 | 1905 | 1048 | 0559 | 0293 | 0152 | 0079 | 00404 | 00106 | 00004 |
| 70 | 1765 | 0957 | 0502 | 0259 | 0132 | 0067 | 00338 | 00085 | 00003 |
| 72 | 1628 | 0870 | 0450 | 0228 | 0114 | 0057 | 00283 | 00069 | 00002 |
| 74 | 1494 | 0788 | 0401 | 0200 | 0099 | 0048 | 00235 | 00055 | 00001 |
| 76 | 1364 | 0709 | 0355 | 0174 | 0085 | 0041 | 00195 | 00044 | |
| 78 | 1236 | 0634 | 0313 | 0151 | 0072 | 0034 | 00160 | 00035 | |
| 80 | 1111 | 0562 | 0273 | 013 | 0061 | 0028 | 00131 | 00028 | |
| 82 | 0989 | 0493 | 0236 | 0111 | 0051 | 0023 | 00106 | 00022 | |
| 84 | 0870 | 0428 | 0202 | 0093 | 0042 | 0019 | 00085 | 00017 | |
| 86 | 0753 | 0366 | 0170 | 0077 | 0035 | 0015 | 00067 | 00013 | |
| 88 | 0638 | 0306 | 0140 | 0063 | 0028 | 0012 | 00052 | 00010 | |
| 90 | 0526 | 0249 | 0113 | 0050 | 0021 | 0009 | 00039 | 00007 | |
| 92 | 0417 | 0195 | 0087 | 0038 | 0016 | 0007 | 00028 | 00005 | |
| 94 | 0309 | 0143 | 0063 | 0027 | 0011 | 0005 | 00019 | 00003 | |
| 96 | 0204 | 0093 | 0040 | 0017 | 0007 | 0003 | 00012 | 00002 | |
| 98 | 0101 | 0045 | 0019 | 0008 | 0003 | 0001 | 00005 | 00001 | |

VALIDATION - COMPARISON OF THE MODELS

To ascertain whether Redfearn's analytical model had been satisfactorily applied to the problem, the researcher re-computed for audio-visual needs by referring to Plane and Kochenberger's probability that no customers are in the system chart (1972, p. 196) and two of Mitchell's formulae (1976b, pp. 32-33):

$$U = \frac{AR}{S \times SR} \qquad WT = \frac{(S \times U)^S}{S!\,(1-U)^2 \times S \times SR} \times P_o$$

(It should again be pointed out that results for the above formulae were in "hours", so the researcher multiplied them by 60 to determine waiting time in "minutes".)

Therefore, if S = 4 and U = .24, then audio-visual needs were calculated as follows:

$$WT = \frac{(4 \times .24)^4}{4!\,(1-.24)^2 \times 4 \times .444} \times P_o =$$

$$\frac{.8493465}{24.6196220} \times .3824 = .0131923 \text{ hour} \times 60 = .8 \text{ minute.}$$

CONCLUSION

The calculation above agreed with the previous analytical solution of four carrels being needed for audio-visual use, with a slight variation in WT which was to be expected. So it was assumed that the analytical estimations to the model of the original training system were valid (i.e. feasible). The analytical solutions, briefly shown in Table 3.3, estimated the total system's needs to be 41 carrels (S) with 4 assigned to audio-visual, 17 to audio, and 20 to print.

Table 3.3: Analytical solutions to the training system's media needs.

| | S | WT (minutes) |
|---|---|---|
| audio-visual | 4 | 1 |
| audio | 17 | .8 |
| print | 20 | .8 |
| Total | 41 | |

# CHAPTER IV

## COMPUTER-SIMULATION SOLUTIONS TO MODEL OF THE ORIGINAL TRAINING SYSTEM

Having completed the mathematical analysis of the original training system, the researcher then proceeded to simulate the same queueing model by using a more complex approach - a computer-simulation program. The researcher adapted McMillan and Gonzales' single-channel queue system simulation (1965, p. 264). First of all, the program had to be adapted to Concordia's computer system then modified to approximate a single-phase, multi-channel system.

The researcher decided to determine the queueing behavior in the training system by simulating the arrival and departure of 1,000 students. It was felt that by "observing" at least this many students use the training program, solutions would be reliable. Likewise, the researcher ran the program 10 times, with a different random number, and averaged the results. It was felt this would also contribute to the reliability of the results.

### OPERATIONAL DEFINITIONS

Before going any further, the following abbreviations and terms used in the computer-simulation models should be defined. Their values varied within each media sub-system:

AHRNQ = Average student-hours spent in queues

AMIQ = Average student-minutes spent in queues

AR = Mean arrival rate in an hour time span

AT = Mean inter-arrival time, i.e. the time between consecutive arrivals

AVTIS = Average student-hours spent in the system

CUMQUE = Variable which held the cumulative record of hours during which there had been a student queue of various lengths (0 to 18)

CUMUTL = Cumulative utilization, i.e. the total number of hours that the service facility was busy

CURUN = Variable that kept track of which media sub-system was being considered

CUSERV = Cumulative number of customers who had been served by and had departed from the service facility

HRSNQ = Total student-hours spent waiting in queue to use the service facility

PCUTIL = Percent utilization of the service facility

$P_0$ = Probability that no customers were in the system

QT = Mean student queueing time given in "hours"

QUE = Queue length of student waiting to use the medium being considered

S = Number of parallel service units or channels

SR = Mean service rate or the number of students that are served every hour

ST = Mean service time or how long service took to complete, given in hours

STATUS = State of the service facility: when status was 0.0, it was empty, and when it was 1.0, the facility was occupied

TIME = A sub-program built into the computer system to generate a random number function, called TIME because it uses the computer clock

TYME = Hour value of the most recent change in the state or status of the system

TNARV = Time of the next arrival

TNDPR = Time of the next departure

U = Percent utilization of the service facility

WT = Expected student waiting time in queue, given in minutes

Z = Number of lessons in the media sub-system being simulated

COMPUTER-SIMULATION MODEL

The following questions were answered for each media sub-system during the simulation: 1) How many service facilities were needed (S); 2) What was the expected average student-minutes spent in queues (AMIQ); 3) What was the percent utilization of each service facility (PUPC); 4) What was the percent utilization of the service facilities (PCUTIL); 5) What was the average or expected time, given in hours, in the system per student; and 6) What were the probabilities of queues of varying lengths forming before the service facilities (CUMQUE). In this study, the researcher was mostly interested in the first two problems mentioned above.

Figure 4.1 is a flow diagram for the Fortran (FTN) program used to simulate the Bell Canada training system. Figure 4.2 is a macro flow diagram that summarizes Figure 4.1 in an over-simplistic manner.

Figure 4.1: Flow diagram of the computer-simulated model.

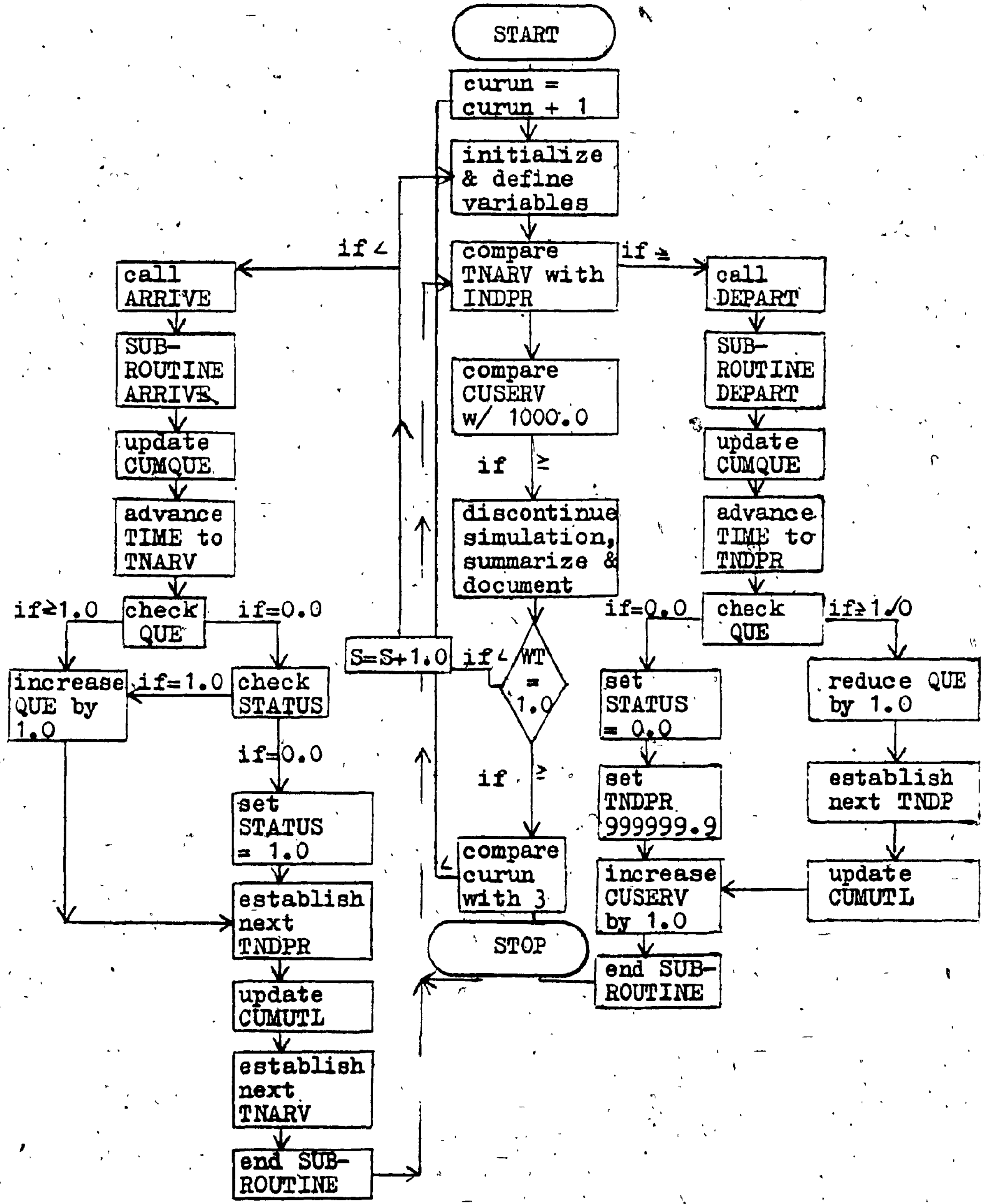
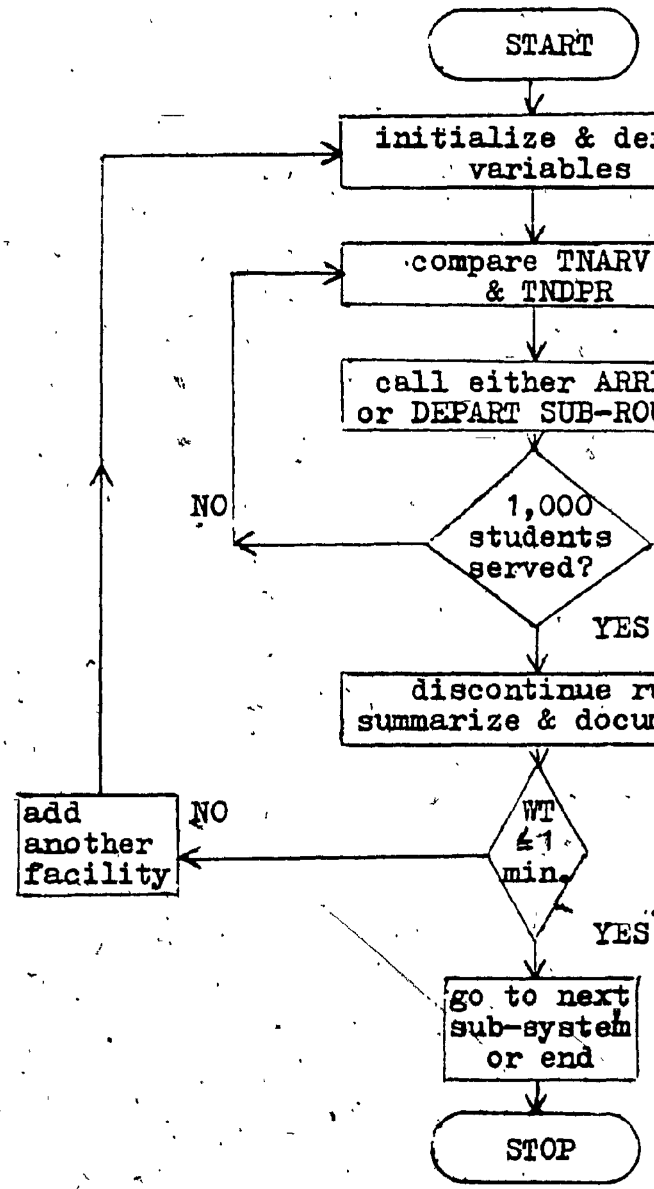


Figure 4.2: Macro flow diagram of the computer-simulation model.



During the simulation, students could either arrive or depart. A sub-routine was designed for both these events, and will be explained before the main program.

Arrival Sub-routine

Keeping in mind that TYME was always the hour value of the most recent change in the state or status of the system, the first thing the program routine had to update were CUMQUE (variable recording the student-hours spent in queues of varying lengths of up to 18 students) and TYME. The sub-routine took note of how many students were already in the queue and added the latest arrival. The program also determined how long the queue had that particular length. Now that CUMQUE was updated, TYME was updated as well, with the following Fortran statements:

```
M = QUE,
CUMQUE(M+1) = CUMQUE(M+1) + TNARV - TYME
TYME = TNARV
```

Next, the program checked the queue length and state of the service facility. If the QUE was empty (0.0), then the student could use the facility and the facility became occupied (STATUS = 1.0). If there was someone using the facility, then the ~~student~~ joined the queue, as seen in the following lines:

```
    IF (QUE - 1.0)1, 3, 3
1.  IF (STATUS - 1.0)2, 3, 3
2.  STATUS = 1.0
3.  QUE = QUE + 1.0
```

The third consideration in the arrival routine was to establish the time spent in the facility (T) and the time of the next departure (TNDPR). T was determined by multiplying the negative exponential service time (ST) by the random number generator. TYME was still the recent time of the arrival:

```
T = -ST*ALOG(RANF[X])
TNDPR = TYME + T
```

Knowing the T increment during which the facility was estimated to be occupied, the program could then update cumulative utilization (CUMUTL). T was multiplied by S to take into account that S number of facilities existed in the system:

```
CUMUTL = CUMUTL + (T*S)
```

Finally, the program determined the expected time of the next arrival (TNARV) by adding the product of the poisson arrival rate and the random number generator to TYME:

```
4 TNARV = -1.0/AR * ALOG(RANF[X]) + TYME
```

At this point,t he computer returned to the main program. The whole arrival sub-routine, as seen in Figure 4.3, ran as follows:

Figure 4.3: The arrival sub-routine.

```
C ARRIVE SUBROUTINE
      SUBROUTINE ARRIVE(QUE,STATUS,TNDPR,CUMUTL,TNARV,
     1CUMQUE,TYME,ST,S,AR)
      DIMENSION CUMQUE(40)
C
C UPDATE CUMULATIVE QUEUE AND RESET TYME
      M=QUE
      CUMQUE(M+1)=CUMQUE(M+1)+TNARV-TYME
      TYME=TNARV
C
C CHECK QUEUE LENGTH AND STATE OF SERVICE FACILITY
      IF(QUE-1.0)1,3,3
    1 IF(STATUS-1.0)2,3,3
    2 STATUS=1.0
C
C ESTABLISH TYME IN SERVICE FACILITY AND TYME OF NEXT DEPARTURE
      T=-ST*ALOG(RANF(X))
      TNDPR=TYME+T
```

```
C
C UPDATE CUMULATIVE UTILIZATION
      CUMUTL=CUMUTL+(T*S)
      GO TO 4
C
C INCREASE QUEUE LENGTH
    3 QUE=QUE+1.0
C
C ESTABLISH TYME OF NEXT ARRIVAL
    4 TNARV=-1.0/AR*ALOG(RANF(X))+TYME
      RETURN
      END
```

Departure Sub-routine

Because the status of the system was affected again, CUMQUE and TYME had to be reset. The procedure was similar to that used in the arrival sub-routine, but now TNDPR (the most recent status-disturbing event) was considered:

```
M = QUE,
CUMQUE(M+1) = CUMQUE(M+1) + TNDPR - TYME
TYME = TNDPR
```

Next, the sub-routine checked the queue length. To initiate the simulation, TNDPR was set at 999999.9 so that it was impossible for the next disturbance to be a TNDPR. If there was no one in the QUE, STATUS remained 0.0, but if there were, then the QUE was decreased:

```
If (QUE  - 1.0)1, 2, 2
STATUS = 0.0
TNDPR = 999999.9
GO TO 3
QUE = QUE - 1.0
```

Having done the above, it was necessary to determine how long the student would be in the facility (S) to then determine the TNDPR. This step was identical to the one found in the arrival sub-routine:

```
T = -ST * ALOG(RANF[X])
TNDPR = TYME + T
```

The last step of the routine was to update CUMUTL and indicate that another student had been served. Once again, T was multipled by S number of facilities:

CUMUTL = CUMUTL + (T*S), 3 CUSERV = CUSERV + 1.0

The entire departure sub-routine is seen in Figure 4.4, which follows.

Figure 4.4: The departure sub-routine.

```
C
C DEPARTURE SUBROUTINE
      SUBROUTINE DEPART(QUE,STATUS,TNDPR,CUMUTL,CUMQUE,
     +TYME,CUSERV,ST,S)
      DIMENSION CUMQUE(40)
C
C UPDATE CUMULATIVE QUEUE AND RESET TYME
      M=QUE
      CUMQUE(M+1)=CUMQUE(M+1)+TNDPR-TYME
      TYME=TNDPR
C
C CHECK QUEUE LENGTH
      IF(QUE-1.0)1,2,2
    1 STATUS=0.0
      TNDPR=999999.9
      GO TO 3
C
C DECREASE QUEUE LENGTH
    2 QUE=QUE-1.0
C
C ESTABLISH TYME IN SERVICE FACILITY AND TYME OF NEXT DEPARTURE
      T=-ST*ALOG(RANF(X))
      TNDPR=TYME+T
C
C UPDATE CUMULATIVE UTILIZATION AND CUMULATIVE SERVED
      CUMUTL=CUMUTL+(T*S)
    3 CUSERV=CUSERV+1.0
      RETURN
      END
```

## Main Program

The main program consisted of: 1) Initializing statements; 2) Defining variable values; 3) Summarizing and printint out the desired results; 4) Adding another service facility and beginning again if waiting time exceeds one minute; and 5) Proceeding to the next run or ending the program run.

To begin with, the main program, called TIME(x) and RANSET(x), a sub-program in Concordia's computer system, were used to determine the random number by using the clock time.

Next, the program gave values to the variables that changed with each sub-routine, or CURUN: $Z_m$, S, ST, SR, AT, and AR. S was initially set at the numbers suggested by the analytical model for each media sub-system. ST was determined for each media sub-system by dividing the total service time (found in Table 2.1) by $Z_m$. SR was found by dividing 1 by ST. For each media sub-system, AT was determined by dividing the inter-arrival time to the training system (7 hours) by $Z_m$. Finally, AR was found by dividing 1 by AT. Therefore, $Z_m$, S, ST, SR, AT, and AR were the following values for each of the media sub-systems (Note: the computer was allowed to calculate them to decrease the possibility of errors occuring):

Table 4.1: $Z_m$, S, ST, SR, AT, and AR as specified in the 3 sub-systems of the simulation model.

| | audio-visual | audio | print |
|---|---|---|---|
| $Z_m$ | 3 | 14 | 49 |
| S | 4 | 17 | 15 |
| ST | 6.75/3 | 66.25/14 | 94.5/49 |
| SR | 1/ST | 1/ST | 1/ST |
| AT | 7/3 | 7/14 | 7/49 |
| AR | 1/AT | 1/AT | 1/AT |

**Figure 4.5: The introduction to the main program.**

```
      PROGRAM QUEUE(INPUT,OUTPUT)
      DIMENSION CUMQUE(40)
      REAL AVTIS,CUMQUE,CUMUTL,CUSERV,HRSNQ,PCUTIL
      REAL QUE,STATUS,TYME,TNARV,TNDPR,AR,AT
      REAL SR,ST,S,Z,AHRNQ,AMIQ,CURUN
C
C INITIALIZE SYSTEM AT TYME ZERO AND DEFINE VARIABLES
C
      CALL TIME(X)
      CALL RANSET(X)
      CURUN=1.0
      S=5.
      PRINT 1,CURUN
 1    FORMAT(*CURUN =*,F4.1,*  --AUDIO-VISUAL--*)
 2    ST=(6.75/3.)/S
      SR=1./ST
      AT=7./3.
      AR=1./AT
      Z=3.0
      GO TO 10
C
 3    S=10.0
      PRINT 4 ,CURUN
 4    FORMAT(*CURUN =*,F4.1,*  --AUDIO--*)
 5    ST=(66.25/14.)/S
      SR=1./ST
      AT=7./14.
      AR=1./AT
      Z=14.0
      GO TO 10
C
 6    S=16.0
      PRINT 7 ,CURUN
 7    FORMAT(*CURUN =*,F4.1,*  --PRINT--*)
 8    ST=(94.5/49.)/S
      SR=1./ST
      AT=7./49.
      AR=1./AT
      Z=49.0
C
 10   CONTINUE
      PRINT 9,S
 9    FORMAT(*NOW USING *,F4.1,* CARRELS.*)
      TYME=0.0
      TNDPR=999999.9
      STATUS=0.0
      QUE=0.0
      CUMUTL=0.0
      CUSERV=0.0
      AVTIS=0.0
      PCUTIL=0.0
      HRSNQ=0.0
      AHRNQ=0.0
      AMIQ=0.0
      TNARV=0.0
      DO 11 M=1,20,1
 11   CUMQUE(M)=0.0
      TNARV=-1.0/AR*ALOG(RANF(X))
```

After the program was initialized, it was imperative to consider the calling routine. The following routine had to compare TNARV and TNDPR and call the appropriate sub-routine. Furthermore, if 1,000 students had already been served, the program then summarized and documented its finding:

```
C
C CALL UPON TYME ROUTINE
 12   IF(TNARV-TNDPR)13,14,14
 13   CALL ARRIVE(QUE,STATUS,TNDPR,CUMUTL,TNARV,CUMQUE,
     +TYME,ST,S,AR)
      GO TO 12
 14   CALL DEPART(QUE,STATUS,TNDPR,CUMUTL,CUMQUE,TYME,
     +CUSERV,ST,S)
C
C TERMINATE SIMULATION IF 1000 ARRIVALS HAVE BEEN SERVED
      IF(CUSERV-1000.0)12,15,15
```

Now the program could determine the percent utilization of the service facility (PCUTIL) and then the average utilization of each facility (PUPC). The PCUTIL was found by dividing CUMUTL by TYME and multiplying the result by 100. PUPC was found by dividing PCUTIL by the number of service facilities being used.

```
C
C DETERMINE PERCENT UTILIZATION OF THE SERVICE FACILITY
 15   PCUTIL=CUMUTL/TYME*100.0
C
C DETERMINE PERCENT UTILIZATION OF EACH SERVICE FACILITY
      PUPC=PCUTIL/S
```

Having determined CUMQUE in the sub-routines, the program could then calculate: total student-hours spent in queues (HRSNQ), average student-hours spent in queues (AHRNQ), and finally average student-minutes spent in queues (AMIQ):

```
C DETERMINE STUDENT-HOURS SPENT IN QUEUE
      HRSNQ=0.0
      DO 16 M=1,19,1
      N=M
16    HRSNQ=HRSNQ+N*CUMQUE(M+1)
C
C ADD STUDENT-HRS IN FACILITY AND DETERMINE AVE. TYME IN SYSTEM
      AVTIS=(HRSQ+CUMUTL)/1000.0
C
C DETERMINE AND PRINT AVERAGE WT (IN MIN.)
      AHRNQ=HRSNQ/1000.0
      AMIQ=(AHRNQ*60.0)/Z
```

To determine HRSNQ, the researcher had a loop which multiplied the number of students in each queue (from 0 to 18) by the amount of time students spent in these queues (CUMQUE). These 19 results were then added together. The calculations looked somewhat like this:

1 x hours queue had 1 student
+ 2 x hours queue had 2 students
+18 x hours queue had 18 students

AVTIS (the average time students spent in the system) was found by adding the total time students spent in the different queues (HRSNQ) to the total number of hours the service facilities were busy (CUMUTL). This result was then divided by the number of students that were simulated, i.e. 1,000.

To obtain AHRNQ, HRSNQ was divided by 1,000 because the program simulated 1,000 students being served. To determine AMIQ, AHRNQ was multiplied by 60 to transform the average "hours" into average "minutes". The same calculation was then divided by Z (number of lessons using the media) to estimate how long students usually waited for service in the medium. AMIQ represented the average waiting time per lesson (in minutes), not the cumulative waiting time per media.

The next task was to print out the results of these calculations, using PRINT and FORMAT statements. This was a straight-forward procedure.

```
      PRINT 17,AMIQ
 17   FORMAT(2X,*AVE. MINUTES IN QUEUE =*,F10.4)
C
C PRINT PERCENT UTILIZATION PER CARREL
      PRINT 18,PUFC
 18   FORMAT(2X,*PERCENT UTILIZATION PER FACILITY =*,F10.6)
C
C CONVERT CONTENT OF CUMQUE TO PROBABILITIES
      DO 19 M=1,20,1
 19   CUMQUE(M)=CUMQUE(M)/TYME
C
C PRINT PERCENT UTILIZATION AND AVERAGE TYME IN SYSTEM
      PRINT 20,PCUTIL,AVTIS
 20   FORMAT(2X,*PERCENT UTILIZATION OF SERVICE FACILITIES =   *,
     +F12.6,//,2X,*AND AVERAGE TIME IN SYSTEM PER CUSTOMER =*,
     +F5.2)
C
C PRINT PROBABILITIES OF QUEUES IN VARYING LENGTHS
      DO 22 M=1,19,1
      L=M-1
      PRINT 21,L,CUMQUE(M)
 21   FORMAT(2X,*PROBABILITY OF   *,I2,2X,*CUSTOMERS IN QUEUE =*,
     +F10.7)
 22   CONTINUE
```

When the program finished documenting the run, the program compared AMIQ to one minute. If AMIQ (waiting time) was more than one minute, the program added another service facility (S) and looped back to the same CURUN or media sub-system for a re-run. This portion of the program is seen in the following:

```
C
C ADD ANOTHER FACILITY AND LOOP BACK TO SAME CURUN
C
      IF(AMIQ.LE.1.0)GO TO 24
      S=S+1.0
      IF(CURUN.EQ.1.0)GO TO 2
      IF(CURUN.EQ.2.0)GO TO 5
      IF(CURUN.EQ.3.0)GO TO 8
 24   CONTINUE
```

Finally, when the waiting time (AMIQ) was less than or equal to one minute, the program then proceeded to the next CURUN or ended the program run. Unlike the previous loop, this one had to return to the line defining the value of S:

```
C
C PROCEED TO THE NEXT CURUN OR END PROGRAM RUN
      CURUN=CURUN+1.0
      IF(CURUN.EQ.2.0)GO TO 3
      IF(CURUN.EQ.3.0)GO TO 6
 25   CONTINUE
      STOP
      END
C
```

The total program appeared as shown in Figure 4.6:

Figure 4.6: The whole queue program.

```
      PROGRAM QUEUE(INPUT,OUTPUT)
      DIMENSION CUMQUE(40)
      REAL AVTIS,CUMQUE,CUMUTI,CUSERV,HRSNQ,PCUTIL
      REAL QUE,STATUS,TYME,TNARV,TNDPR,AR,AT
      REAL SR,ST,S,Z,AHRNQ,AMIQ,CURUN
C
C INITIALIZE SYSTEM AT TYME ZERO AND DEFINE VARIABLES
C
      CALL TIME(X)
      CALL RANSET(X)
      CURUN=1.0
      S=5.
      PRINT 1,CURUN
 1    FORMAT(*CURUN =*,F4.1,*  --AUDIO-VISUAL--*)
 2    ST=(6.75/3.)/S
      SR=1./ST
      AT=7./3.
      AR=1./AT
      Z=3.0
      GO TO 10
C
 3    S=18.0
      PRINT 4 ,CURUN
 4    FORMAT(*CURUN =*,F4.1,*  --AUDIO--*)
 5    ST=(66.25/14.)/S
      SR=1./ST
      AT=7./14.
      AR=1./AT
      Z=14.0
      GO TO 10
```

```
C
 6     S=16.0
       PRINT 7 ,CURUN
 7     FORMAT(*CURUN =*,F4.1,*  --PRINT= *)
 8     ST=(94.5/49.)/S
       SR=1./ST
       AT=7./49.
       AR=1./AT
       Z=49.0
C
 10    CONTINUE
       PRINT 9,S
 9     FORMAT(*NOW USING *,F4.1,* CARRELS.*)
       TYME=0.0
       TNDPR=999999.9
       STATUS=0.0
       QUE=0.0
       CUMUTL=0.0
       CUSERV=0.0
       AVTIS=0.0
       PCUTIL=0.0
       HRSNQ=0.0
       AHRNQ=0.0
       AMIQ=0.0
       TNARV=0.0
       DO 11 M=1,20,1
 11    CUMQUE(M)=0.0
       TNARV=-1.0/AR*ALOG(RANF(X))
C
C CALL UPON TYME ROUTINE
 12    IF(TNARV-TNDPR)13,14,14
 13    CALL ARRIVE(QUE,STATUS,TNDPR,CUMUTL,TNARV,CUMQUE,
      +TYME,ST,S,AR)
       GO TO 12
 14    CALL DEPART(QUE,STATUS,TNDPR,CUMUTL,CUMQUE,TYME,
      +CUSERV,ST,S)
C
C TERMINATE SIMULATION IF 1000 ARRIVALS HAVE BEEN SERVED
       IF(CUSERV-1000.0)12,15,15
C
C DETERMINE PERCENT UTILIZATION OF THE SERVICE FACILITY
 15    PCUTIL=CUMUTL/TYME*100.0
C
C DETERMINE PERCENT UTILIZATION OF EACH SERVICE FACILITY
       PUPC=PCUTIL/S
C
C DETERMINE STUDENT-HOURS SPENT IN QUEUE
       HRSNQ=0.0
       DO 16 M=1,19,1
       H=M
 16    HRSNQ=HRSNQ+H*CUMQUE(M+1)
C
C ADD STUDENT-HRS IN FACILITY AND DETERMINE AVE. TYME IN SYSTEM
       AVTIS=(HRSQ+CUMUTL)/1000.0
C
C DETERMINE AND PRINT AVERAGE WT (IN MIN.)
       AHRNQ=HRSNQ/1000.0
       AMIQ=(AHRNQ*60.0)/Z
```

```
      PRINT 17,AMIQ
 17   FORMAT(2X,*AVE. MINUTES IN QUEUE   *,F10.4)
C
C PRINT PERCENT UTILIZATION PER CARREL
      PRINT 18,PUPC
 18   FORMAT(2X,*PERCENT UTILIZATION PER FACILITY =*,F10.6)
C
C CONVERT CONTENT OF CUMQUE TO PROBABILITIES
      DO 19 M=1,20,1
 19   CUMQUE(M)=CUMQUE(M)/TYME
C
C PRINT PERCENT UTILIZATION AND AVERAGE TYME IN SYSTEM
      PRINT 20,PCUTIL,AVTIS
 20   FORMAT(2X,*PERCENT UTILIZATION OF SERVICE FACILITIES =   *,
     1F12.6,/,2X,*AND AVERAGE TIME IN SYSTEM PER CUSTOMER =*,
     +F5.2)
C
C PRINT PROBABILITIES OF QUEUES IN VARYING LENGTHS
      DO 22 M=1,19,1
      L=M-1
      PRINT 21,L,CUMQUE(M)
 21   FORMAT(2X,*PROBABILITY OF   *,I2,2X,*CUSTOMERS IN QUEUE =*,
     +F10.7)
 22   CONTINUE
C
C ADD ANOTHER FACILITY AND LOOP BACK TO SAME CURUN
C
      IF(AMIQ.LT.1.0)GO TO 24
      S=S+1.0
      IF(CURUN.EQ.1.0)GO TO 2
      IF(CURUN.EQ.2.0)GO TO 5
      IF(CURUN.EQ.3.0)GO TO 8
 24   CONTINUE
C
C PROCEED TO THE NEXT CURUN OR END PROGRAM RUN
      CURUN=CURUN+1.0
      IF(CURUN.EQ.2.0)GO TO 3
      IF(CURUN.EQ.3.0)GO TO 6
 25   CONTINUE
      STOP
      END
C
C ARRIVE SUBROUTINE
      SUBROUTINE ARRIVE(QUE,STATUS,INDER,CUMUTL,TNARV,
     1CUMQUE,TYME,SI,S,AR)
      DIMENSION CUMQUE(40)
C
C UPDATE CUMULATIVE QUEUE AND RESET TYME
      M=QUE
      CUMQUE(M+1)=CUMQUE(M+1)+TNARV-TYME
      TYME=TNARV
C
C CHECK QUEUE LENGTH AND STATE OF SERVICE FACILITY
      IF(QUE-1.0)1,3,3
    1 IF(STATUS-1.0)2,3,3
    2 STATUS=1.0
```

```
C
C ESTABLISH TYME IN SERVICE FACILITY AND TYME OF NEXT DEPARTURE
      T=-ST*ALOG(RANF(X))
      TNDPR=TYME+T
C
C UPDATE CUMULATIVE UTILIZATION
      CUMUTL=CUMUTL+(T*S)
      GO TO 4
C
C INCREASE QUEUE LENGTH
    3 QUE=QUE+1.0
C
C ESTABLISH TYME OF NEXT ARRIVAL
    4 TNARV=-1.0/AR*ALOG(RANF(X))+TYME
      RETURN
      END
C
C DEPARTURE SUBROUTINE
      SUBROUTINE DEPART(QUE,STATUS,TNDPR,CUMUTL,CUMQUE,
     +TYME,CUSERV,ST,S)
      DIMENSION CUMQUE(40)
C
C UPDATE CUMULATIVE QUEUE AND RESET TYME
      M=QUE
      CUMQUE(M+1)=CUMQUE(M+1)+TNDPR-TYME
      TYME=TNDPR
C
C CHECK QUEUE LENGTH
      IF(QUE-1.0)1,2,2
    1 STATUS=0.0
      TNDPR=999999.9
      GO TO 3
C
C DECREASE QUEUE LENGTH
    2 QUE=QUE-1.0
C
C ESTABLISH TYME IN SERVICE FACILITY AND TYME OF NEXT DEPARTURE
      T=-ST*ALOG(RANF(X))
      TNDPR=TYME+T
C
C UPDATE CUMULATIVE UTILIZATION AND CUMULATIVE SERVED
      CUMUTL=CUMUTL+(T*S)
    3 CUSERV=CUSERV+1.0
      RETURN
      END
```

Before analysing the results of the computer model, the researcher had to keep in mind two factors that could influence the results:

1) The data collected by Bell Canada instructors was rounded off to the nearest 15 minutes. Therefore, any results are gross approximations. Therefore, it was decided that computer print-out results would be submitted to the researcher's judgement. The researcher then decided to accept any results under 2.5 minutes and reconsider the results. Redfearn (1973) and many other operational researchers agree that such a decision is acceptable because models such as this computer-simulation are not perfect but useful to provide approximations;

2) Since the queue model was over-simplified to represent a single-phase, multi-channel queue system, the required number of servers would be higher than those which a multi-phase, multi-channel queue model would produce. This means that one must consider recommending fewer media service facilities than the computer-simulation model suggests.

COMPUTER PRINT-OUT RESULTS

Applying the computer-simulation program to the model of the training system and setting service facilities (S) to the numbers suggested by the analytical study, the researcher ran the program 10 times. Each run used a different random number to generate varying results which were then averaged together for a more reliable solution. Because a print-out of ten simulation runs would be too lengthy and confusing, the researcher shows only one program run, in Figure 4.7, and later summarizes all results.

Figure 4.7: Computer print-out of figures obtained by running the queue program.

```
CURUN = 1.0  --AUDIO-VISUAL--
NOW USING  5.0 CARRELS.
  AVE. MINUTES IN QUEUE =     2.2610
  PERCENT UTILIZATION PER FACILITY = 19.011141
  PERCENT UTILIZATION OF SERVICE FACILITIES =      95.055707
  AND AVERAGE TIME IN SYSTEM PER CUSTOMER = 2.30
  PROBABILITY OF   0  CUSTOMERS IN QUEUE =  .9636978
  PROBABILITY OF   1  CUSTOMERS IN QUEUE =  .0277314
  PROBABILITY OF   2  CUSTOMERS IN QUEUE =  .0069179
  PROBABILITY OF   3  CUSTOMERS IN QUEUE =  .0015203
  PROBABILITY OF   4  CUSTOMERS IN QUEUE =  .0001326
  PROBABILITY OF   5  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF   6  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF   7  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF   8  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF   9  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  10  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  11  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  12  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  13  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  14  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  15  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  16  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  17  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  18  CUSTOMERS IN QUEUE = 0.0000000
NOW USING  6.0 CARRELS.
  AVE. MINUTES IN QUEUE =     1.3234
  PERCENT UTILIZATION PER FACILITY = 15.226656
  PERCENT UTILIZATION OF SERVICE FACILITIES =      91.359934
  AND AVERAGE TIME IN SYSTEM PER CUSTOMER = 2.24
  PROBABILITY OF   0  CUSTOMERS IN QUEUE =  .9778735
  PROBABILITY OF   1  CUSTOMERS IN QUEUE =  .0178740
  PROBABILITY OF   2  CUSTOMERS IN QUEUE =  .0037194
  PROBABILITY OF   3  CUSTOMERS IN QUEUE =  .0004362
  PROBABILITY OF   4  CUSTOMERS IN QUEUE =  .0000769
  PROBABILITY OF   5  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF   6  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF   7  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF   8  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF   9  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  10  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  11  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  12  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  13  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  14  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  15  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  16  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  17  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  18  CUSTOMERS IN QUEUE = 0.0000000
```

```
NOW USING  7.0 CARRELS.
  AVE. MINUTES IN QUEUE =      1.2035
  PERCENT UTILIZATION PER FACILITY = 13.624544
  PERCENT UTILIZATION OF SERVICE FACILITIES =      95.371808
  AND AVERAGE TIME IN SYSTEM PER CUSTOMER = 2.24
  PROBABILITY OF  0  CUSTOMERS IN QUEUE =  .9778807
  PROBABILITY OF  1  CUSTOMERS IN QUEUE =  .0193300
  PROBABILITY OF  2  CUSTOMERS IN QUEUE =  .0021026
  PROBABILITY OF  3  CUSTOMERS IN QUEUE =  .0006152
  PROBABILITY OF  4  CUSTOMERS IN QUEUE =  .0000716
  PROBABILITY OF  5  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  6  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  7  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  8  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  9  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 10  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 11  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 12  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 13  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 14  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 15  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 16  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 17  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 18  CUSTOMERS IN QUEUE = 0.0000000
NOW USING  8.0 CARRELS.
  AVE. MINUTES IN QUEUE =       .6097
  PERCENT UTILIZATION PER FACILITY = 11.606371
  PERCENT UTILIZATION OF SERVICE FACILITIES =      92.850966
  AND AVERAGE TIME IN SYSTEM PER CUSTOMER = 2.21
  PROBABILITY OF  0  CUSTOMERS IN QUEUE =  .9882701
  PROBABILITY OF  1  CUSTOMERS IN QUEUE =  .0106357
  PROBABILITY OF  2  CUSTOMERS IN QUEUE =  .0010930
  PROBABILITY OF  3  CUSTOMERS IN QUEUE =  .0000012
  PROBABILITY OF  4  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  5  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  6  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  7  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  8  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF  9  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 10  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 11  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 12  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 13  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 14  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 15  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 16  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 17  CUSTOMERS IN QUEUE = 0.0000000
  PROBABILITY OF 18  CUSTOMERS IN QUEUE = 0.0000000
CURUN = 2.0   --AUDIO--
NOW USING 18.0 CARRELS.
  AVE. MINUTES IN QUEUE =      1.1908
  PERCENT UTILIZATION PER FACILITY = 52.932005
  PERCENT UTILIZATION OF SERVICE FACILITIES =     952.776087
  AND AVERAGE TIME IN SYSTEM PER CUSTOMER = 4.79
```

```
PROBABILITY OF  0  CUSTOMERS IN QUEUE = .7258229
PROBABILITY OF  1  CUSTOMERS IN QUEUE = .1328202
PROBABILITY OF  2  CUSTOMERS IN QUEUE = .0693751
PROBABILITY OF  3  CUSTOMERS IN QUEUE = .0367377
PROBABILITY OF  4  CUSTOMERS IN QUEUE = .0174354
PROBABILITY OF  5  CUSTOMERS IN QUEUE = .0074737
PROBABILITY OF  6  CUSTOMERS IN QUEUE = .0083258
PROBABILITY OF  7  CUSTOMERS IN QUEUE = .0020091
PROBABILITY OF  8  CUSTOMERS IN QUEUE = 0.0000000
PROBABILITY OF  9  CUSTOMERS IN QUEUE = 0.0000000
PROBABILITY OF 10  CUSTOMERS IN QUEUE = 0.0000000
PROBABILITY OF 11  CUSTOMERS IN QUEUE = 0.0000000
PROBABILITY OF 12  CUSTOMERS IN QUEUE = 0.0000000
PROBABILITY OF 13  CUSTOMERS IN QUEUE = 0.0000000
PROBABILITY OF 14  CUSTOMERS IN QUEUE = 0.0000000
PROBABILITY OF 15  CUSTOMERS IN QUEUE = 0.0000000
PROBABILITY OF 16  CUSTOMERS IN QUEUE = 0.0000000
PROBABILITY OF 17  CUSTOMERS IN QUEUE = 0.0000000
PROBABILITY OF 18  CUSTOMERS IN QUEUE = 0.0000000
NOW USING 19.0 CARRELS.
 AVE. MINUTES IN QUEUE =      .9346
 PERCENT UTILIZATION PER FACILITY = 47.491114
 PERCENT UTILIZATION OF SERVICE FACILITIES =    902.331169
 AND AVERAGE TIME IN SYSTEM PER CUSTOMER = 4.56
 PROBABILITY OF  0  CUSTOMERS IN QUEUE = .7642923
 PROBABILITY OF  1  CUSTOMERS IN QUEUE = .1233313
 PROBABILITY OF  2  CUSTOMERS IN QUEUE = .0667969
 PROBABILITY OF  3  CUSTOMERS IN QUEUE = .0268064
 PROBABILITY OF  4  CUSTOMERS IN QUEUE = .0108630
 PROBABILITY OF  5  CUSTOMERS IN QUEUE = .0034544
 PROBABILITY OF  6  CUSTOMERS IN QUEUE = .0009388
 PROBABILITY OF  7  CUSTOMERS IN QUEUE = .0003614
 PROBABILITY OF  8  CUSTOMERS IN QUEUE = .0028513
 PROBABILITY OF  9  CUSTOMERS IN QUEUE = .0003044
 PROBABILITY OF 10  CUSTOMERS IN QUEUE = 0.0000000
 PROBABILITY OF 11  CUSTOMERS IN QUEUE = 0.0000000
 PROBABILITY OF 12  CUSTOMERS IN QUEUE = 0.0000000
 PROBABILITY OF 13  CUSTOMERS IN QUEUE = 0.0000000
 PROBABILITY OF 14  CUSTOMERS IN QUEUE = 0.0000000
 PROBABILITY OF 15  CUSTOMERS IN QUEUE = 0.0000000
 PROBABILITY OF 16  CUSTOMERS IN QUEUE = 0.0000000
 PROBABILITY OF 17  CUSTOMERS IN QUEUE = 0.0000000
 PROBABILITY OF 18  CUSTOMERS IN QUEUE = 0.0000000
CURUN = 3.0  --PRINT--
NOW USING 16.0 CARRELS.
 AVE. MINUTES IN QUEUE =      .5236
 PERCENT UTILIZATION PER FACILITY = 84.480474
 PERCENT UTILIZATION OF SERVICE FACILITIES =   1351.687582
 AND AVERAGE TIME IN SYSTEM PER CUSTOMER = 1.85
 PROBABILITY OF  0  CUSTOMERS IN QUEUE = .2899798
 PROBABILITY OF  1  CUSTOMERS IN QUEUE = .1282576
 PROBABILITY OF  2  CUSTOMERS IN QUEUE = .1136090
 PROBABILITY OF  3  CUSTOMERS IN QUEUE = .1006152
 PROBABILITY OF  4  CUSTOMERS IN QUEUE = .0741308
 PROBABILITY OF  5  CUSTOMERS IN QUEUE = .0461389
```

```
PROBABILITY OF  6  CUSTOMERS IN QUEUE = .0456091
PROBABILITY OF  7  CUSTOMERS IN QUEUE = .0505590
PROBABILITY OF  8  CUSTOMERS IN QUEUE = .0410673
PROBABILITY OF  9  CUSTOMERS IN QUEUE = .0180847
PROBABILITY OF 10  CUSTOMERS IN QUEUE = .0164751
PROBABILITY OF 11  CUSTOMERS IN QUEUE = .0126166
PROBABILITY OF 12  CUSTOMERS IN QUEUE = .0077197
PROBABILITY OF 13  CUSTOMERS IN QUEUE = .0074055
PROBABILITY OF 14  CUSTOMERS IN QUEUE = .0036452
PROBABILITY OF 15  CUSTOMERS IN QUEUE = .0021208
PROBABILITY OF 16  CUSTOMERS IN QUEUE = .0037770
PROBABILITY OF 17  CUSTOMERS IN QUEUE = .0013969
PROBABILITY OF 18  CUSTOMERS IN QUEUE = .0034626
   2.482 CP SECONDS EXECUTION TIME
```

The computer-simulation results derived from the ten program runs for the queueing model of the training system are briefly shown in Table 4.2 (for audio-visual), Table 4.3 (for audio) and Table 4.4 (for print).

Table 4.2: Computer-simulation results for audio-visual needs in the training system.

| | WT for the 10 Simulation Runs (Minutes) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| S | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Mean WT |
| 4 | 2.22 | 2.10 | 2.75 | 2.32 | 2.31 | 2.44 | 2.50 | 2.20 | 2.39 | 2.00 | 2.32 |
| 5 | 1.82 | 1.85 | 2.64 | 1.65 | 2.00 | 2.08 | 2.38 | 2.00 | 2.22 | 1.74 | 2.04 |
| 6 | 1.06 | 1.07 | 1.21 | 1.37 | 1.47 | 1.13 | 1.09 | 1.37 | 1.37 | 1.01 | 1.22 |
| 7 | 1.12 | .93 | .94 | 1.25 | .93 | .99 | 1.05 | 1.00 | .86 | .85 | .99 |
| 8 | .78 | | | .74 | | | .80 | .70 | | | .76 |

Table 4.3: Computer-simulation results for audio needs in the training system.

| S | WT for the 10 Simulation Runs (Minutes) 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Mean WT |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 17 | 1.59 | 2.40 | 2.10 | 3.00 | 1.50 | 1.90 | 1.37 | 2.14 | 1.73 | 1.30 | 1.90 |
| 18 | 1.06 | 1.69 | 1.44 | 2.28 | .96 | 1.31 | .81 | 1.14 | 1.41 | 1.06 | 1.32 |
| 19 | 1.10 | .90 | .88 | 1.31 | | .95 | | 1.09 | 1.40 | .91 | 1.70 |
| 20 | .92 | | | 1.20 | | | | .73 | .87 | | .93 |
| 21 | | | | .73 | | | | | | | .73 |

Table 4.4: Computer-simulation results for print needs in the training system.

| S | WT for the 10 Simulation Runs (Minutes) 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Mean WT |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 14 | | | | | | | | | | | |
| 15 | 1.18 | 1.40 | 1.37 | 1.26 | 1.15 | 1.33 | 1.02 | 1.54 | 1.28 | 1.13 | 1.27 |
| 16 | .40 | .96 | .95 | .91 | .88 | .92 | .73 | .99 | .89 | .84 | .85 |

## CONCLUSIONS

### Audio-Visual

Running the sub-routine ten times produced varying results for the different number of service facilities available. This, of course, reflected the random number function. To keep student waiting down to one minute, the results (see Table 4.2) would indicate that seven audio-visual carrels were needed. However, keeping in mind the conservative nature of the model, as outlined on p. 64, fewer facilities should suffice. In fact, four facilities would keep student waiting time for audio-visual facilities under 2.5 minutes as stipulated. Fewer than four facilities is well over the accepted maximum waiting time.

Audio

The computer results (see Table 4.3) suggest that twenty facilities were needed to keep waiting time within the accepted limits of one minute. However, considering again the conservative nature of the model, the researcher found that 17 audio carrels should be more than adequate to keep WT under 2.5 minutes. The researcher tried running the program with fewer carrels, but then waiting time was well over three minutes.

Print

The print results (see Table 4.4) caused some confusion in the researcher's mind. The analytical model suggested that 20 facilities (i.e. desks) were needed, but the computer simulation indicated that only 16 desks were needed to keep waiting time within the desired minute. The results also show that 15 carrels would keep the waiting time down to well under 2.5 minutes. When fewer facilities were used, however, the model would not accept them and "rambled" into the exchange package. Therefore, 15 desks should be considered to satisfy print needs.

Taking into account that the model assumes an infinite queue, the researcher would recommend the above number of facilities. The more conservative number of desks (generated by the analytical model) was suggested because the print subsystem is a finite queue. The computer solutions are briefly shown in Table 4.5.

Table 4.5: Computer-simulation solutions to the training system's needs.

| | S | WT (minutes) |
|---|---|---|
| audio-visual | 4 | 2.32 |
| audio | 17 | 1.90 |
| print | 15 | 1.27 |

# CHAPTER V

# SYSTEMS DESIGN AND SOLUTIONS

# TO THE MODIFIED AND EXPANDED TRAINING SYSTEM PROBLEM

## SYSTEMS DESIGN

In the proposed new system, the management intended to: 1) Expand capacity from 26 to 45 students; and 2) Change 6 lessons, 1 audio and 5 print, to audio-visual media as shown in Table 2.1. The important consideration was to determine how many units of each media (i.e. audio-visual, audio, and print) would be needed to keep waiting time down to about one minute. This was too complex a problem to decide without a systematic analysis, therefore the queueing model was modified to describe the proposed training system.

### Service Mechanism

To be able to determine the service mechanism, the researcher had to make the media-lesson changes and assume the average service times for the lessons remained the same. The total service time and the number of lessons ($Z_m$) per media sub-system in the proposed scheme are shown in Table 5.1.

Table 5.1: Total service time and the number of lessons ($Z_m$) for each media sub-system in the proposed system.

| | $Z_m$ | Total ST (hour) |
|---|---|---|
| audio-visual | 9 | 20.0 |

| | $Z_m$ | Total ST (hour) |
|---|---|---|
| audio | 13 | 62.0 |
| print | 44 | 81.5 |

Having done this, the researcher could then solve for mean service times (ST) and mean service rates (SR) for each media sub-system. Using the same formulae (applied in Chapter II, p. 28) to determine the service mechanism of the original training system, the results set down in Table 5.2 were found. As noted before, results were given to the third decimal for computational purposes.

Table 5.2: Mean service times (ST) and mean service rates (SR) for each media in the modified and expanded training system.

| | ST (hour) | SR (hour) |
|---|---|---|
| audio-visual | 2.222 | .450 |
| audio | 4.769 | .210 |
| print | 1.852 | .540 |

Arrival Pattern

Having determined the new service mechanism, the researcher then had to figure out the new arrival pattern. Given the capacity of 45 students and the mean training duration of 26 days, the arrival ratio to the whole system became 1.73 per day, i.e. a new student arriving about every four (4.05) hours. Following the formulas (in

Chapter II, p. 34) used in the original training system, mean inter-arrival times (AT) and mean arrival rates (AR) were found for the three media, as seen in Table 5.3. Results were again given to the third decimal for computational purposes.

Table 5.3: Mean inter-arrival times (AT) and mean arrival rates (AR) for the media in the modified and expanded training system.

| | AT (hours) | AR (hours) |
|---|---|---|
| audio-visual | .450 | 2.222 |
| audio | .312 | 3.210 |
| print | .092 | 10.864 |

ANALYTICAL SOLUTIONS TO THE QUEUE MODEL OF THE EXPANDED AND MODIFIED TRAINING SYSTEM

Taking into account the new service mechanisms and arrival patterns, but keeping the other system components and queue model untouched, the researcher was then able to analytically solve the facility needs for the expanded and modified model of the training system. The analytical model was the same one explained and used in Chapter III, page 38. The following calculations were made:

Audio-visual -

if S = 9, U = .55, then WT = .019
x 2.222
.042218 hour x 60 =
2.5 minutes

* if S = 10, U = .49, then WT = .00675
x 2.22200

.0149985 hour x 60 =
.9 minute

Audio -

*if S = 24, U = .64, then WT = .00375
x 4.76900

.0178837 hour x 60 =
1 minute

if S = 25, U = .61, then WT = .0017
x 4.7690

.0081073 hour x 60 =
.5 minute

Print -

if S = 27, U = .74, then WT = .013
x 1.852

.024076 hour x 60 =
1.44 minutes

*if S = 28, U = .72, then WT = .009
x 1.852

.01668 hour x 60 =
1 minute

Therefore, the analytical estimations to the expanded and modified training system's needs are summarized in Table 5.4.

Table 5.4: Analytical solutions to the expanded and modified system's media needs.

| | ST | WT (minutes) |
|---|---|---|
| Audio-visual | 10 | .9 |
| Audio | 25 | 1.0 |
| Print | 28 | 1.0 |

## COMPUTER-SIMULATION SOLUTIONS TO THE EXPANDED AND MODIFIED TRAINING SYSTEM

Applying the already documented computer program to the expanded and modified training system was no real problem, as only a few things had to be changed: $Z_m$, S, AT, AR, ST, and SR. Figure 5.1 shows the modified portion of the program:

Figure 5.1: Modified program.

```
      PROGRAM QUEUE (INPUT,OUTPUT)
      DIMENSION CURQUE(40)
      REAL AVTIS,CUMQUE,CUMQTI,CUSERV,HRSNO,PCUTIL
      REAL QUE,STATUS,TYME,INARV,INDPR,AR,AT
      REAL SR,ST,S,Z,AHRNO,AMIQ,CURUN
C
C INITIALIZE SYSTEM AT TYME ZERO AND DEFINE VARIABLES
C
      CALL TIME(X)
      CALL RANSET(X)
      CURUN=1.0
      S=10.
      PRINT 1,CURUN
 1    FORMAT(*CURUN =*,F4.1,*  --AUDIO-VISUAL--*)
 2    ST=(20./9.)/S
      SR=1./ST
      AT=4.05/9.
      AR 1./AT
      Z=9.0
      GO TO 10
C
 3    S=24.0
      PRINT 4 ,CURUN
 4    FORMAT(*CURUN =*,F4.1,*  --AUDIO--*)
 5    ST=(62./13.)/S
      SR=1./ST
      AT=4.05/13.
      AR=1./AT
      Z=13.0
      GO TO 10
C
 6    S=24.0
      PRINT 7 ,CURUN
 7    FORMAT(*CURUN =*,F4.1,*  --PRINT--*)
 8    ST=(81.5/44.)/S
      SR=1./ST
      AT=4.05/44.
      AR=1./AT
      Z=44.0
```

```
C
10    CONTINUE
      PRINT 9,S
9     FORMAT(*NOW USING *,F4.1,* CARRELS.*)
      TYME=0.0
      TNDPR=999999.9
      STATUS=0.0
      QUE=0.0
      CUMUTL=0.0
      CUSERV=0.0
      AVTIS=0.0
      PCUTIL=0.0
      HRSNQ=0.0
      AHRNQ=0.0
      AMIQ=0.0
      TNARV=0.0
      DO 11 M=1,20,1
11    CUMQUE(M)=0.0
      TNARV=-1.0/AR*ALOG(RANF(X))
```

By running the modified version of the program 10 times, the researcher obtained the results shown in Table 5.5 (for audio-visual), Table 5.6 (for audio), and Table 5.7 (for print).

Table 5.5: Computer-simulation results for audio-visual needs in the expanded and modified training system.

| Runs | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Mean WT |
|---|---|---|---|---|---|---|---|---|---|---|---|
| S | | | | | | | | | | | |
| 9 | 1.75 | 2.26 | 2.91 | 2.33 | 3.00 | 2.26 | 2.75 | 3.50 | 2.88 | 2.21 | 2.58 |
| 10 | 1.19 | 1.49 | 1.49 | 1.49 | 1.43 | 1.60 | 1.41 | 1.79 | 1.27 | 1.22 | 1.44 |
| 11 | 1.34 | .96 | 1.11 | .93 | 1.31 | 1.00 | 1.41 | 1.12 | 1.18 | 1.23 | 1.16 |
| 12 | .77 | | .85 | | .90 | | .88 | .82 | .80 | .92 | .85 |

Table 5.6: Computer-simulation results for audio needs in the expanded and modified training system.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Mean WT |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 22 | | | | | | | | | | | |
| 23 | 3.25 | 2.80 | 2.29 | 2.12 | 3.00 | 2.75 | 2.90 | 2.59 | 2.31 | 2.26 | 2.63 |
| 24 | 1.65 | 1.18 | 1.46 | 1.13 | 1.50 | 1.75 | 1.63 | 1.15 | 1.31 | 1.70 | 1.45 |
| 25 | 1.62 | 1.41 | 1.37 | 1.19 | 1.16 | 1.37 | 1.15 | .89 | 1.23 | 1.52 | 1.29 |
| 26 | 1.60 | 1.16 | 1.40 | 1.25 | 1.03 | 1.28 | 1.02 | | 1.12 | 1.09 | 1.22 |
| 27 | 1.08 | .91 | 1.08 | 1.19 | 1.27 | 1.10 | 1.10 | | 1.07 | .80 | 1.07 |
| 28 | .89 | | .83 | 1.09 | .85 | 1.17 | .89 | | 1.26 | | .87 |
| 29 | | | | .88 | .46 | .79 | | | .79 | | .73 |

Table 5.7: Computer-simulation results for print needs in the expanded and modified training system.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Mean WT |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | | | | | | | | | | | |
| 21 | | | | | | | | | | | |
| 22 | | | | | | | | | | | |
| 23 | 1.20 | 1.31 | .87 | .72 | .69 | .83 | .99 | 1.01 | .92 | 1.71 | 1.03 |
| 24 | .80 | .94 | .39 | .50 | .46 | .55 | .56 | .62 | .76 | 1.50 | .71 |
| 25 | | | | | | | | | | | |
| 26 | | | | | | | | | | | |

## CONCLUSIONS

### Audio-Visual

The ten program runs (see Table 5.5) show that twelve facilities would keep the student waiting time under one minute. However, this computer-simulation is conservative in nature, so ten audio-visual carrels should be considered to keep the waiting time (for the computer-simulation model) down to under 2.5 minutes. Fewer carrels kept students waiting more than this amount of time.

### Audio

The simulation results (see Table 5.6) suggest that 28 facilities were needed to keep student waiting time within the limit. However, it would seem that, at most, 24 carrels should be considered to keep waiting time (for the computer-simulation of the queueing model) to within 2.5 minutes.

### Print

The simulation results (see Table 5.7) suggest that 24 desks would suffice to keep student waiting time within the desired one minute. Knowing that the model was conservative, the researcher tried fewer carrels, but the model wouldn't accept fewer carrels than 23 as it would just "ramble" into the exchange package. Therefore, 23 facilities should be considered because the researcher would not suggest fewer facilities without print-out results for back-up.

The computer solutions are briefly shown in Table 5.8 on page 80.

Table 5.8: Computer-simulation solutions to the expanded and modified training system.

| | S | WT (minutes) |
|---|---|---|
| Audio-visual | 10 | 1.44 |
| Audio | 24 | 1.45 |
| Print | 23 | 1.03 |

# CHAPTER VI

## DISCUSSION AND CONCLUSIONS OF BOTH ANALYTICAL AND COMPUTER-SIMULATION MODELS OF THE ORIGINAL AND PROPOSED TRAINING SYSTEMS

### DISCUSSION

Given the solutions to both the analytical and computer-simulation models of the original and proposed (expanded and modified) training systems, the researcher then examined the two sets of results together.

A comparison of the analytical and computer-simulation results are briefly shown in Table 6.1 (for the original training system) and in Table 6.2 (for the expanded and modified version of the training system).

Table 6.1: Analytical and computer-simulation results for the original training system.

| | Analytical | | Computer-simulation | |
|---|---|---|---|---|
| | S | WT (minutes) | S | WT (minutes) |
| Audio-visual | 4 | 1.01 | 4 | 2.32 |
| Audio | 17 | .80 | 17 | 1.90 |
| Print | 20 | .80 | 15 | 1.27 |
| Total | 41 | | 36 | |

Table 6.2: Analytical and computer-simulation results for the proposed (expanded and modified) training system.

| | Analytical | | Computer-simulation | |
|---|---|---|---|---|
| | S | WT (minutes) | S | WT (minutes) |
| Audio-visual | 10 | .9 | 10 | 1.44 |
| Audio | 24 | 1.0 | 24 | 1.45 |
| Print | 28 | 1.0 | 23 | 1.03 |
| Total | 62 | | 57 | |

When examining the results, the researcher found three things of interest:

1) The analytical and computer-simulation models were consistent;

2) The computer-simulation results tended to recommend a higher number of service facilities in the audio and audio-visual sub-systems, but the answers became identical because the researcher chose to accept WT of up to 2.5 minutes in the computer-simulation model;

3) In the print sub-system, the reverse situation occured. The computer-simulation suggested that fewer facilities were needed than the analytical model. When both models of the original and expanded training systems were compared, the computer-simulation indicated that five fewer carrels were needed.

The researcher supports the accuracy of the analytical model for two reasons. First of all, Redfearn (1973) built a multi-queueing model.. Secondly, the researcher compared the system existing at Bell Canada with the results for the original training system model (Table 6.1). The Business Office program had 14 multi-media carrels and as many desks as were needed - about 14. The analytical and computer-simulation results were close, but they could not be closer because the models and the training system were designed a bit differently (i.e. the program operated with two types of service facilities - multi-media and print - and the models considered three types - audio, audio-visual, and print).

The researcher also supports the approximations of the single-phase, multi-channel simulation model. This second model over-estimated media needs because of the way the system was designed, but by knowing this, the researcher was able to adjust the model slightly. As it has already been mentioned, the researcher decided to allow more than one minute waiting time in the simulation - deciding that 2.5 minutes would be acceptable. As it turned out, this readjustment nearly made the results for both the analytical and simulation models identical.

With the readjustments, the results for the audio and audio-visual sub-systems became identical. It would seem that by accepting a waiting time of 2.5 minutes in the simulation, it could adequately represent a multi-phase, multi-channel queueing system.

However, the results for the print sub-systems were a bit of a shock - at first, in both the original and modified models of the queueing system, the computer suggested that five fewer carrels were needed. This could have been brought about by a bug in the computer-simulation or by the fast turnover of students arriving and leaving print - there were many lessons of short duration. The researcher noted however, that Bell was working with about 14 desks - one less than the conservative simulation suggested for the original queue system. It would seem therefore, that the computer-simulation was not far wrong.

It is concluded that both models are reliable (i.e. consisten) but that they should be used under different conditions. The analytical model was clearly the faster model. Furthermore, if an educational technologist was working on a very low budget, this model would be recommended because all that would be needed would be the formulae, a calculator, Redfearn's graph, and the Schlaifer graph. On the other hand, should a more elaborate study be made to determine media needs in a learning centre, it would be recommended to use both models or only the computer model. This latter model can process more students and under differing conditions. This model can also be run a number of times and an average of several runs could be taken to produce the final solution.

## RECOMMENDATIONS

Given the results generated by both the analytical and computer-simulation models, the researcher would suggest that the manager of the Business Office program consider the following recommendation, seen in Table 6.3, for their