PARAMETRIC STUDY

&

ELASTIC ANALYSIS OF COUPLED SHEAR WALLS

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ABSTRACT

A PARAMETRIC STUDY & ELASTIC ANALYSIS OF COUPLED SHEAR WALLS

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The work done in this report is based on Rosman's Theory with a wider and deeper look into coupled shear walls. Besides the study of these great shear resisting structural elements, a very special effort has been made to use the results obtained for fast analysis and design of shear walls and also to give engineers a broader idea of how shear walls behave by varying certain important parameters.

Rosman's theory which is a simplified solution for coupled shear walls is used. The theory is applied to 3 different loading cases: a point load acting at the top of the wall, a uniformily distributed load (Rosman's Loading Case) and a triangularly distributed load with load concentrated at top. The theory is applied with respect to these loading cases. This is done by varying the wall parameters to include any shape or height of the wall, the purpose being to study the behaviour of the walls of different shapes under different type loading conditions and also to make the results available for use in practical problems.

Extra effort has been made to make this paper completely independent, so that the user will not have to search for other material in order to completely solve the structure for total axial loads, shear created in connecting beams, deflection at any floor or height of the wall, and moments and the stresses at the extreme fibers.

In this paper the solution is obtained by means of a computer program. The tables and curves are produced for each Loading Case, the curves make it visually easier to see the behaviour of the wall under different Loading Conditions.

A typical example is also presented for familiarizing the user with the method of calculations and use of curves.

ACKNOWLEDGEMENTS

As with any similar work, I had the help of many people. Among them, I would like to thank Dr. M.M. Douglass, my adviser, for his invaluable assistance. Mr. Danny Jensen for his support in computer work. Mr. R. Harun, eng. for his proofreading and experienced guidance. Mrs. H. McLaughlin for her diligence in typing. My wife, for her patience and understanding. My mother for her constant encouragement. I am grateful to them all.

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SECTION 6.0

1.0 NOTATION

a	=	Depth of connecting beam
A_1, A_2	=	Cross sectional area of walls
A	=	$A_1 + A_2$
b	=	Thickness of wall
c	=	Clear distance between two walls
d	=	Total width of the wall
d_0	=	Eccentricity of the opening relative to the centre line of the wall.
d_1, d_2	=	Width of walls
E	=	Modulus of elasticity of the wall
F(Q)	=	Axial force intensity factor
F(Y)	=	Deflection factors
h	=	Storey height
H	=	Total height
I	=	Moment of inertia of a centilever beam equivalent to a wall with one row of openings
I_1, I_2	=	Moment of inertia of walls
I_0	=	$I_1 + I_2$
I*	=	Moment of inertia of a connecting beam
K_1	=	$n^2 (\frac{d}{H})^4$
K_2	=	$(\frac{c}{d})$
K_3	=	$(\frac{d_0}{c})$
L	=	Distance between centroidal axes of two walls
M_x	=	Total bending moment at any point x from top of wall
M	=	Bending moments at any point from top of wall, for external loads only
M_{x1}, M_{x2}	=	Total bending moments in walls 1 & 2
n	=	Number of storeys
P	=	Point load in Load Case I
Q	=	Total axial force
(Q)	=	Total axial force factor
q	=	Shear force intensity in connecting medium

R = $\frac{\gamma H^2 L}{\beta^2}$ Non dimensional constant.

U = Integration constant

V = Integration constant

w = Uniformly distributed load in Load Case II

W = Total triangular load in Load Case III

x = Any distance from top of the wall

Y = Deflection

$\beta = \sqrt{H^2 \left(\frac{L^2}{I_0} + \frac{1}{A_1} + \frac{1}{A_2}\right) \frac{12I^*}{hc^3}}$

$\delta = \frac{12LI^*}{hc^3 I_0}$

$\eta = \frac{hc^3}{12I^*}$

ζ = Numerical coefficient

$= \frac{x}{H}$

Computer Notation

Q' same as Q

B same as β

E same as ζ

A same as U

2.0 INTRODUCTION

Since the dawn of history men built their various structures (buildings, dwellings, palaces, shrines, temples, etc.) by using shear walls.

Thousands of old buildings around us which withstood the forces of nature for hundreds of years prove that this method of construction is safe and effective.

Openings were cut in the shear walls for entrances, lighting and ventilation purposes and for different architectural and aesthetic effects. Thus shear walls were used extensively with or without openings since the beginning of civilization.

Openings in shear walls weaken the wall section. To overcome this problem, builders depended purely on past experience based on trial and error. This method was conservative in design.

In recent years the engineering profession introduced economics, and safety in the design and construction of buildings, and with the availability of new construction materials and methods a new era was born.

Since through the ages the shear wall system of construction established itself as a viable construction method, scientists and engineers started an extensive study on the amelioration and feasibility of this kind of construction.

Systematic construction first was accepted by eastern Europe then it spread to the western world because of its safety, economy and period of construction.

Since solid shear walls are very simple to analyze and published studies are amply available on the subject, this discussion will concentrate mainly on coupled shear walls.

The analysis of coupled shear walls has been the subject of numerous studies. Several researchers have put forward useful methods to assess the elastic behaviour of these major lateral load resisting structures of multi-storey buildings.

Beck (1)*, Rosman, Coull and Choudhury (4,5), simplified the analysis of coupled shear walls by replacing the connecting beams in a coupled shear wall by a continuous shear medium. Thus the high degree of indeterminancy is reduced to a simple second order differential equation by assuming that the point of contraflexure of the connecting beams occur at their mid point and the axial deformation of the beams is neglected.

The purpose of this paper is to use the available theory and prepare a parametric study for coupled shear walls.

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* Numbers in parenthesis refer to bibliography entries.

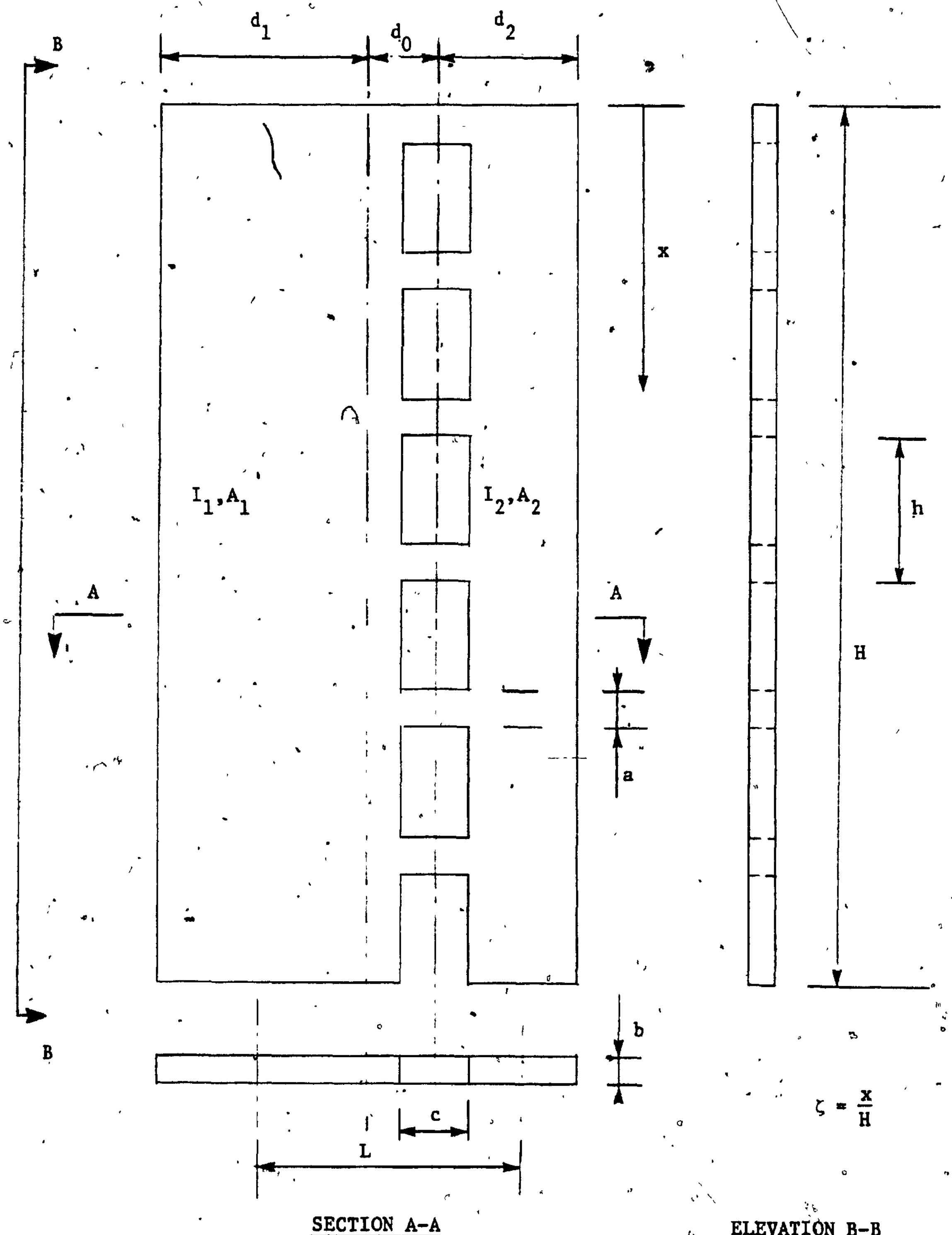

Fig. (2.1)

3.0 METHOD

3.1 INTRODUCTION

Stiffened walls behave as cantilever slabs bent in their own planes by horizontal forces. If the shear walls are solid with no openings the analysis is very simple, but in the case of walls with openings the analysis is somewhat complicated.

Perforated walls are treated sometimes as plain cantilevers with reduced cross sectional areas and moments of inertia. This is equivalent to saying that the cantilever is weakened due to the opening, with respect to a solid cantilever wall, therefore according to the above we can assume that all lines which are horizontal and coplanar before deformation takes place, will remain so after deformation.

However, this assumption is rather unrealistic because it neglects the effect of the connecting beams on the internal stresses of the wall. A better approximation is to consider the wall as a multi-storey frame, or as a cantilever consisting of several vertical bands, each of which is subjected to tangential forces representing the effects of the beams. In the above two models, the assumption that the horizontal coplanar sections remain unaffected after deformation applies only within each individual band.

The multi-storey approach is studied by A. Kacner and B. Lewicki (10)*.

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* Numbers in parenthesis refer to bibliography entries.

Many researchers have worked with the banded cantilever approach, assuming that tangential forces act on the edges of individual bands. Various solutions differ among themselves in the expression for the equivalent tangential forces replacing the beams. One author A.S. Kalmanok treats the vertical cantilever bands not as linear members, but as plates abandoning the assumption that plane sections remain planer. The resulting formulae, however, are rather complicated.

Rosman's method is recommended for practical uses. The analysis in this report is based on this method, which will be explained fully in the upcoming pages.

The differences in maximum deflection obtained by the frame and the banded cantilever methods are relatively insignificant, both with regard to the displacement and to the stresses.

Tests carried out on Perspex models show that the results obtained by calculations using the above methods give a fairly accurate picture of the behaviour of a perforated plate. It is important to verify the above solutions to be applicable to concrete structures.

At Warsaw Building Institute, concrete models were built and tested to failure by gradually increasing the horizontal loads.

The deflection of the walls tested under the action of the horizontal forces and the calculated values were within 10% of each other.

In Rosman's method the results indicate that, the shear forces and bending moments in the beams reach their maximum values at approximately $\frac{1}{3}$ H (where H is the height of the wall). This should be kept in mind in designing the reinforcement for the beams.

When stiffening walls are monolithically connected to sufficiently rigid cross walls the combined section wall should be considered as a whole for calculation purposes.

More attention should be given to the joints of walls consisting of prefabricated components because as a rule they are less rigid than joints in monolithic building system.

3.2 THEORY

Elastic analysis of a coupled shear wall has been previously done by Rosman.

To analyze the perforated shear walls using the banded cantilever method, the following assumptions are made:

a - The connecting beams of stiffness EI* are replaced by an equivalent continuous medium of stiffness EI*/H per unit height.

where:

E = modulus of elasticity of concrete

I* = the moment of inertia of a connecting beam

H = height of the wall

b - Axial deformation of connecting beams is neglected, this means that both cantilever strips located on either side of openings deflect equally due to the action of lateral loads on the wall Fig. (3.1).

c - The point of contraflexure is assumed to fall at the mid-point of the connecting beams. Fig. (3.2).

d - The whole perforated wall is assumed to behave as a cantilever.

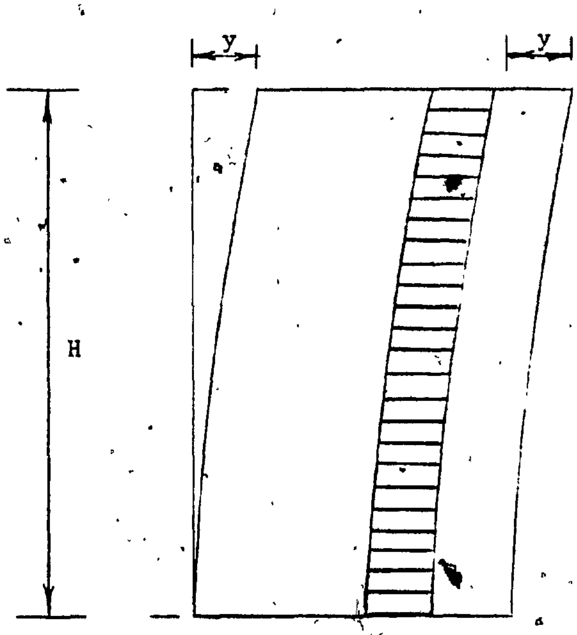

Fig. (3.1)

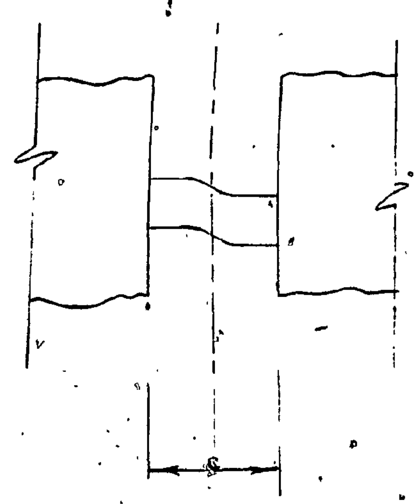

Fig. (3.2)

Since the point load action on the connecting beam of the cantilever bands is replaced by continuous, uniformly distributed tangential forces, a unit force causes a displacement δ at the middle of a connecting beam of span c Fig. (3.3).

$$\delta = 2\left(\frac{h}{3}\right)\left(\frac{c}{2}\right)\left(\frac{c}{2}\right)^2\left(\frac{1}{EI^*}\right) = \frac{hc^3}{12EI^*} \qquad (3.1)$$

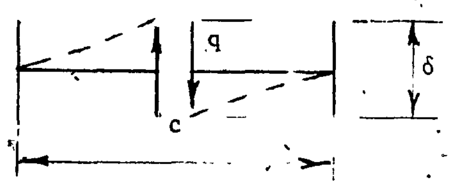

Fig. (3.3)

where c = Span of connecting beam

h = Storey height

The sum of all tangential forces at a distance x from the top of the building is expressed by the integral

$$Q = H\int_0^{\zeta} q\,d\zeta \qquad (3.2)$$

where q is a function of ζ

where Q = Sum of all the tangential forces and

$\zeta = \frac{x}{H}$

the bending moment at any Section a distance x from the top is

$$M_x = M - QL \qquad (3.3)$$

where M = Moment due to external loads only (depending on the type of load)

L = the distance between the center lines of cantilever bands

Owing to the equality of deflections of both cantilever bands, the moment M_x is divided into two components M_{x1} & M_{x2} in proportion to the moments of inertia I_1 & I_2, where I_1 and I_2 are the moments of inertia of two cantilever bands respectively.

$$M_{x1} = (M - QL)\,\frac{I_1}{I_1+I_2} \qquad (3.4)$$

$$M_{x2} = (M - QL)\,\frac{I_1}{I_1+I_2} \qquad (3.5)$$

the variation in Q as developed by Rosman (2) is given by a second order differential equation

$$Q'' - \beta^2 Q + \gamma H^2 M = 0 \qquad (3.6)$$

where $Q'' = \frac{dQ^2}{d^2\zeta}$ & M is a function of

$$\beta^2 = H^2\left(\frac{L^2}{I_0} + \frac{1}{A_1} + \frac{1}{A_2}\right)\frac{12I^*}{hc^3} \qquad (3.7)$$

$$\gamma = \frac{12LI^*}{hc^3 I_0} \qquad (3.8)$$

$$I_0 = I_1+I_2$$

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*Numbers in parethesis refer to bibliography entries

To solve the differential equation, M should be determined, therefore a loading case should be assumed.

In this paper three types of loading cases have been taken into account.

Solution of the differential equation gives values for Q, and by differentiating q is obtained i.e.,

$$q = \frac{dQ}{Hd\zeta}$$

where q is the shear per unit of height of the wall.

Knowing Q, the deflection at any point can be readily found; this is given by another second order differential equation

$$EIy'' = M_x = (M - QL) \tag{3.9}$$

where $y'' = \frac{d^2Y}{d\zeta^2}$, y being the deflection of the wall.

4.0 ANALYSIS

4.1 LOAD CASE I

POINT LOAD ACTING AT THE TOP OF THE WALL

Moment M at a distance x from the top of the wall is

$M = Px$

$M = PH$ (4.1)

Where $\zeta = \frac{x}{H}$

Substituting in Equation (3.6)

therefore the differential equation becomes:

$Q'' - \beta^2 Q + \gamma H^2 \, PH\zeta = 0$

or $Q'' - \beta^2 Q + \gamma H^3 P\zeta = 0$ (4.2)

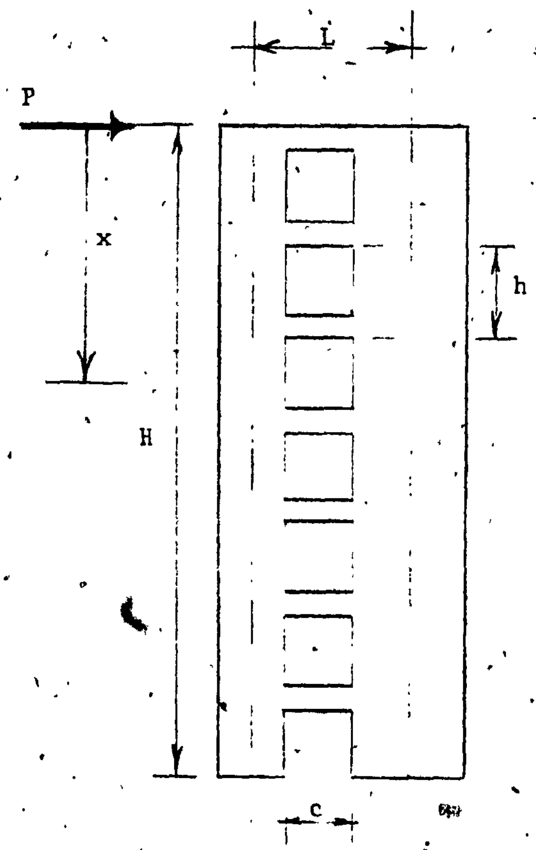


Fig. (4.1)

The solution to this differential equation is:

$$Q = H^3 p\ [C_1 \mathrm{Sinh}(\beta\zeta) + D_1 \mathrm{Cosh}(\beta\zeta) + \frac{\zeta}{\beta^2}] \qquad (4.3)$$

By differentiating:

$$\frac{dQ}{Hd\zeta} = q = \gamma H^2 p\,[\beta\ C_1 \mathrm{Cosh}(\beta\zeta) + D_1 \mathrm{Sinh}(\beta\zeta) + \frac{1}{\beta^2}] \qquad (4.4)$$

By substituting the boundary conditions in the above equations the values for C_1 & D_1 are found as follows:

Where $\beta^2 = H^2\ (\frac{L^2}{I_0} + \frac{1}{A_1} + \frac{1}{A_2})\ \frac{12I^*}{hc^3}$

The boundary conditions are

(i) when $Q = 0$, $\zeta = 0$ therefore $D = 0$

(ii) when $\frac{dQ}{Hd\zeta} = 0$, $\zeta = 1$

After differentiating the equation (4.3) and substituting the boundary conditions, C_1 is found to be

$$C_1 = -\frac{1}{\beta^3} \cdot \frac{1}{\mathrm{Cosh}\beta} \qquad (4.5)$$

therefore the equation (4.3) becomes

$$Q_1 = \gamma H^3 p\ [C_1\ \sinh\ (\beta\zeta) + \frac{\zeta}{\beta^2}] \qquad (4.6)$$

To find the deflection consider equation (3.9)

$$EI\frac{d^2y_1}{d\zeta^2} = (M_1 - Q_1L)$$

$$= H^2\ [PH\zeta - \gamma H^3 p\ (C_1 \sinh \beta\zeta + \frac{\zeta}{\beta^2})L]$$

where I is the equivalent moment of inertia of the wall. By integrating twice, y_1 becomes

$$\frac{EI}{PH^3}\ y_1 = [\frac{1}{6}\ \zeta^3 - R\ (C_1\ x \sinh \beta\zeta + \frac{1}{6}\ \zeta^3)] + U_1\zeta + V_1 \qquad (4.7)$$

Where $R = \gamma H^2 L/\beta^2$ and is a non-dimensional constant.

The boundary conditions are when $\zeta = 1$ then $\frac{dy_1}{d\zeta} = 0$

(i) when $\zeta = 1$, $\frac{dy_1}{d\zeta} = 0$

Therefore,

$$U_1 = R\ (C_1\beta\ \cosh \beta + \frac{1}{2}) - \frac{1}{2} \qquad (4.8)$$

(ii) when $\zeta = 1$, $y_1 = 0$

Therefore,

$$V_1 = [R\ (C_1\ \sinh \beta + \frac{1}{6}) - \frac{1}{6}] - U_1 \qquad (4.9)$$

From these the deflection at any point may be found.

4.2 LOAD CASE II

UNIFORMLY DISTIRBUTED LOAD ACTING ALONG THE HEIGHT OF THE WALL

Moment M at a distance x from the top of the wall is

$$M = \frac{w\zeta^2 H^2}{2} \tag{4.10}$$

Substituting M in equation (3.6)

the differential equation becomes:

$$Q'' - \beta^2 Q + \gamma H^2 \frac{w\zeta^2 H^2}{2} = 0$$

$$Q'' - \beta^2 Q + \frac{\gamma H^4 w\zeta^2}{2} = 0 \tag{4.11}$$

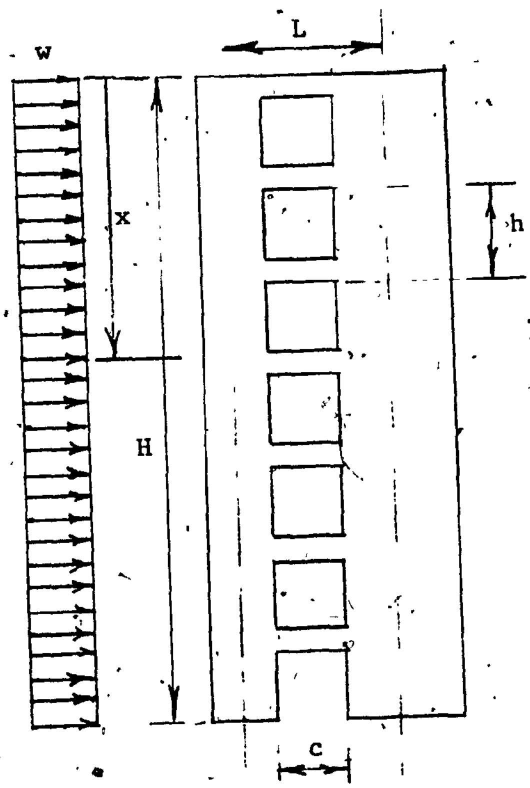


Fig: (4.2)

The solution to the above differential equation becomes:

$$Q = \gamma H^4 q \ [C_2 \sinh(\beta\zeta) + D_2 \cosh(\beta\zeta) + \frac{\zeta^2}{2\beta^2} + \frac{1}{\beta^4}] \qquad (4.12)$$

$$\frac{dQ}{Hd\zeta} = q = \gamma H^3 w \ [C_2 \ \cosh(\beta\zeta) + D_2\beta \ \sinh \beta\zeta + \frac{\zeta}{\beta^2}] \qquad (4.13)$$

after substituting the boundary conditions as in Load Case I the constants are obtained as follows:

$$D_2 = -\frac{1}{4} \qquad (4.14)$$

$$C_2 = \frac{1}{\beta^3} [\frac{1}{\beta} \mathrm{Th}(\beta) - \frac{1}{\cosh(\beta)}] \qquad (4.15)$$

Deflection is found by substituting M and Q and integrating equation (3.9) twice. This gives

$$\frac{EI}{WH^4} y_2 = [\frac{1}{24} \zeta^4 - R \ (C_2 \sinh(\beta\zeta) + D_2 \cosh(\beta\zeta) + \frac{1}{24} \zeta^4 +$$

$$\frac{1}{2\beta^2} \zeta^2)] + U_2\zeta + V_2 \qquad (4.16)$$

Applying the boundary conditions, the constants U_2 and V_2 are obtained as follows:

$$U_2 = R \ (C_2\beta \ \cosh\beta + D_2\beta \ \sinh\beta + \frac{1}{6} + \frac{1}{\beta^2}) - \frac{1}{6} \qquad (4.17)$$

$$V_2 = [R(C_2 \sinh(\beta) + D_2 \cosh(\beta) + \frac{1}{24} + \frac{1}{2\beta^2}) - \frac{1}{24}] - U_2 \qquad (4.18)$$

4.3 LOAD CASE III

UPPER TRIANGULAR LOAD PATTERN

Moment M at a distance x from the top of the wall is

$$M = WH \left(\zeta^2 - \frac{\zeta^3}{3}\right) \qquad (4.19)$$

where W = total triangular load.

Substituting the above value in equation (3.6) we have

$$Q'' - \beta^2 Q + \gamma H^3 W \left(\zeta^2 - \frac{\zeta^3}{3}\right) = 0 \qquad (4.20)$$

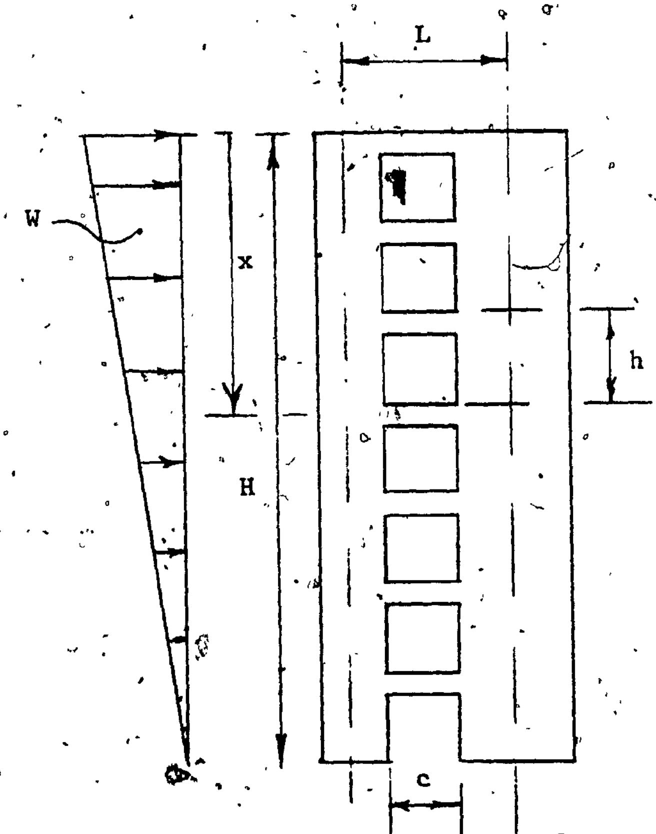


Fig. (4.3)

Solution to the differential equation (4.20) is:

$$Q = \gamma WH^3 \; [\, C_3 \,\mathrm{Sinh}\,(\beta\zeta) + D_3 \,\mathrm{Cosh}\,(\beta\zeta) - \frac{1}{\beta^2}\,[\frac{\zeta^3}{3} - \zeta^2 + \frac{2}{\beta^2}(\zeta-1)]] \qquad (4.21)$$

and

$$\frac{dQ}{Hd\zeta} = q = \gamma WH^2 \; [\beta(C_3 \,\mathrm{Cosh}\,(\beta\zeta) + D_3 \,\mathrm{Sinh}\,(\beta\zeta) - \frac{1}{\beta^2}\,(\zeta^2 - 2\zeta + \frac{2}{\beta^2})] \qquad (4.22)$$

Applying the boundary conditions the constants C_3 & D_3 are obtained as follows:

The boundary conditions are $Q = 0 \quad \zeta = 0$ or $\frac{dQ}{d\zeta} = 0, \; \zeta = 1$

(i) $Q = 0$ at $\zeta = 0$

(ii) $\frac{dQ}{d\zeta} = 0$ at $\zeta = 1$

Hence the constants D_3 & C_3 are

$$D_3 = -\frac{2}{\beta^4} \qquad (4.23)$$

$$C_3 = \frac{1}{\beta^3}\,[(\frac{2}{\beta^2} - 1) \;\; \frac{1}{\mathrm{Cosh}\beta} + \frac{2}{\beta} \;\; \mathrm{th}(\beta) \;] \qquad (4.24)$$

The deflection is found by substituting M_1 and Q_1 and integrating the equation (3.9) twice. This gives

$$EI \;\frac{d^2y}{d\zeta^2} = WH^3 \;[(\zeta^2 - \frac{\zeta^3}{3}) - \gamma H^2L \;\; C_3 \,\mathrm{sinh}\,(\beta\zeta) + D_3 \,\mathrm{Cosh}\,(\beta\zeta) -$$

$$\frac{1}{\beta^2} \;\; [\,\frac{\zeta^3}{3} - \zeta^2 + \frac{2}{\beta^2}\,(\,\zeta - 1)]] \qquad (4.25)$$

Integrating twice we have:

$$\frac{EI}{WH^3}\ \frac{dy_3}{d\zeta} = [(\frac{1}{3}\ \zeta^3 - \frac{1}{12}\ \zeta^4) - \gamma H^2 L\ [C_3 \frac{1}{\beta} \cosh (\beta\zeta) + D_3 \frac{1}{\beta} \sinh (\beta\zeta)$$

$$- \frac{1}{\beta^2} [\frac{1}{12} \zeta^4 - \frac{1}{3} \zeta^3 + \frac{2}{\beta^2} (\frac{1}{2} \zeta^2 - \zeta)]]\ ] + U_3 \qquad (4.26)$$

$$\frac{EI}{WH^3}\ y_3 = [(\frac{1}{12}\ \zeta^4 - \frac{1}{60}\ \zeta^5) - R\ [C_3 \sinh (\beta\zeta) + D_3 \cosh (\beta\zeta)$$

$$- [\frac{1}{60} \zeta^5 - \frac{1}{12} \zeta^4 + \frac{2}{\beta^2} (\frac{1}{6} \zeta^3 - \frac{1}{2} \zeta^2)]]\ ] + U_3 + V_3 \qquad (4.27)$$

Applying the boundary conditions, U_3 and V_3 are found.

Boundary conditions

(i) when $\zeta = 1$, $\frac{dy}{d\zeta} = 0$

and

(ii) when $\zeta = 1$, $y_3 = 0$

Hence the constants:

$$U_3 = R\ (C_3\beta \cosh \beta + D_3\beta \sinh \beta - \frac{1}{4} + \frac{1}{\beta^2}) - \frac{1}{4} \qquad (4.28)$$

$$V_3 = [R\ (C_3 \sinh \beta + D_3 \cosh\beta + \frac{1}{15} + \frac{2}{3\beta^2}) - \frac{1}{15}] - U_3 \qquad (4.29)$$

where β is a geometric stiffness parameter

and $R = \frac{\gamma H^2 L}{\beta^2}$

4.4 DEFINITION & LIMITS OF THE NON-DIMENSIONAL CONSTANT R

In Sections 4.1, 4.2 & 4.3 the product of $\gamma LH^2/\beta^2$ is defined as a non-dimensional constant, R. In this section R has been studied and limits to which it could vary are determined.

The values of γ and β^2 are given in previous sections and are as follows

$$\gamma = \frac{12LI^*}{hc^3 I_0}$$

$$\beta^2 = H^2\left(\frac{L^2}{I_0} + \frac{1}{A_1} + \frac{1}{A_2}\right)\frac{12I^*}{hc^3}$$

By substituting for γ and β^2, R becomes

$$R = \frac{12LI^*}{hc^3 I_0} \cdot \left[\frac{LH^2}{H^2\left(\frac{L^2}{I_0} + \frac{1}{A_1} + \frac{1}{A_2}\right)\frac{12I^*}{hc^3}}\right] \qquad (4.30)$$

where $I_0 = I_1 + I_2 = \frac{bd_1^3}{12} + \frac{bd_2^3}{12}$

$A_1 = bd_1$

and $A_2 = bd_2$

d_1 and d_2 are widths of walls 1 and 2 respectively and b is their respective thickness.

Substituting the values for I_0, A_1 and A_2 in equation (4.30) and simplifying R becomes:

$$R = \frac{12\,L^2\,d_1 d_2}{12\,L^2\,d_1 d_2 + (d_1+d_2)(d_1^3+d_2^3)} \qquad (4.31)$$

It is readily seen from equation (4.31) that

a) R tends to 0.0 when $d_1 = d_2$ tends to ∞

This shows that the walls are very large and hence stiff. Fig. (4.4)

b) On the other hand

R tends to 1.0 when $d_1 = d_2$ tends to 0.0 and L tends to ∞

This indicates that the Shear Wall system is no longer applicable. The system becomes a frame system where moment in the girders governs and shearing forces become negligible. Fig. (4.5)

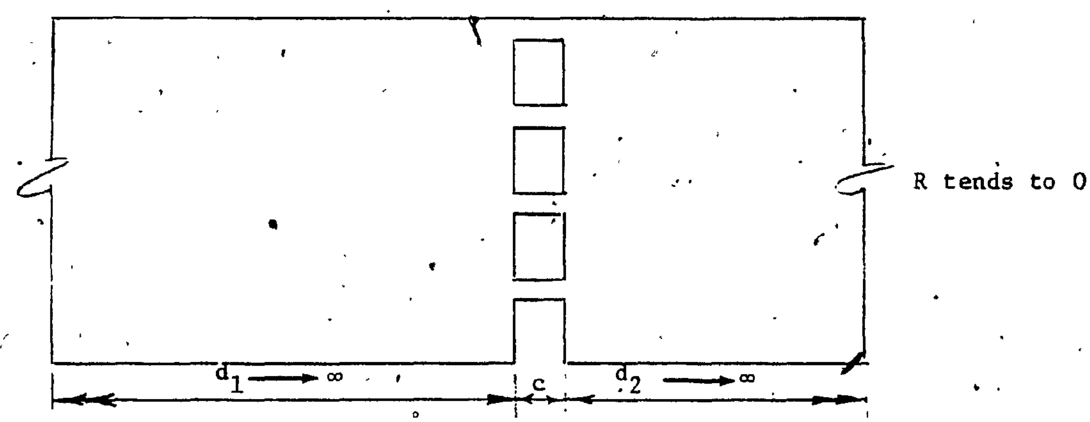


Fig. (4.4)

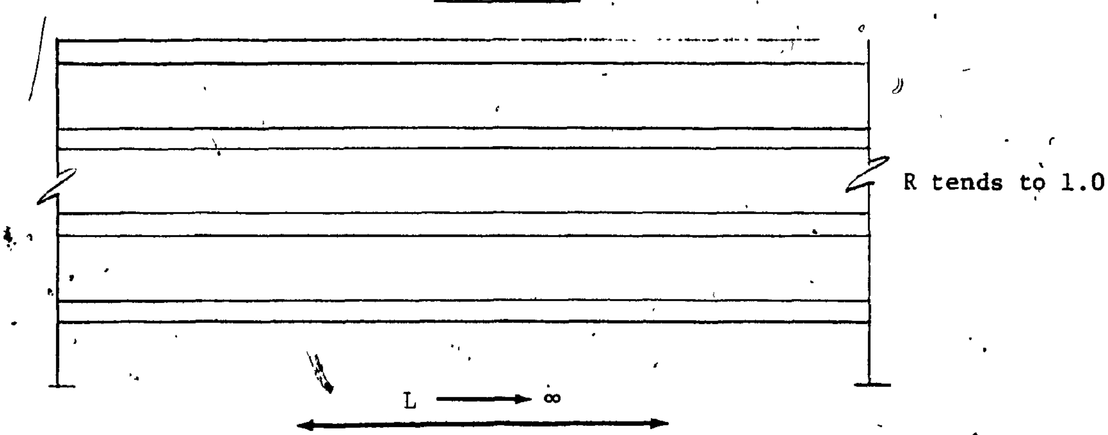


Fig. (4.5)

From above, it is seen that R can never be equal to 0.0 nor 1.0 but it can approach these values. Also it is obvious that it is more realistic for R to approach 1.0 than 0.0. The theory is applicable, even though R approaches zero, the ratio of the span to the depth of a connecting beam c/a plays a very important role in the theory used in this paper for analyzing coupled shear walls.

If the ratio c/a has a large value it indicates that the connecting beam will fail due to moment, on the other hand if this ratio is small the failure will be due to shear forces. The use of the theory is ideal in the latter case. It should be made clear that the ratio c/a of a connecting beam should be chosen in such a way that the beam is stronger in shear than in moment. Usually this ratio varies from 0.0 to 5.0 depending on lateral supports to prevent buckling.

In this particular paper, the analysis is carried out for R = 0.1 to R = 1.0.

5.0 COMPUTER PROGRAM

5.1 INTRODUCTION

The purposes of the computer program is to carry out the numerical analysis of sections 4.1, 4.2 and 4.3, so that tables and curves could be produced for use in design.

The language used in this program is Fortran. This program will be available to others. The user may modify the parameters to suit his own problem.

5.2 COMPUTING ASSUMPTIONS

5.2.1 Force Coefficients, (Q)

In the computer program the force coefficients (Q_1), (Q_2) and (Q_3) are coefficients of Q_1', Q_2', and Q_3' respectively.

Equations (4.6), (4.12) and (4.21) correspond to Q_1', Q_2' and Q_3' respectively as follows:

$$Q_1' = \gamma H^3 p \left(C_1 \sinh(\beta\zeta) + \frac{\zeta}{\beta^2}\right)$$

$$Q_2' = \gamma H^4 w \left(C_2 \sinh(\beta\zeta) + D_2 \cosh(\beta\zeta) + \frac{\zeta^2}{2\beta^2} + \frac{1}{\beta^4}\right)$$

$$Q_3' = \gamma W H^3 \left[C_3 \sinh(\beta\zeta) + D_3 \text{Cosh}(\beta\zeta) - \frac{1}{\beta^2}\left[\frac{\beta^3}{3} - \zeta^2 + \frac{2}{\beta^2}(\zeta - 1)\right]\right]$$

The above equations could be written as:

$$Q_1' = \gamma H^3 p \cdot (Q_1)$$

$$Q_2' = \gamma H^4 w \cdot (Q_2)$$

$$Q_3' = \gamma H^3 W \cdot (Q_3)$$

Where (Q_1), (Q_2) and (Q_3) are:

$$(Q_1) = C_1 \sinh(\beta\zeta) + \zeta/\beta^2 \tag{5.1}$$

$$(Q_2) = C_2 \sinh(\beta\zeta) + D_2 \cosh(\beta\zeta) + \frac{\zeta^2}{2\beta^2} + \frac{1}{\beta^4} \tag{5.2}$$

$$(Q_3) = C_3 \sinh(\beta\zeta) + D_3 \cosh(\beta\zeta) - \frac{1}{\beta^2}\left[\frac{\zeta^3}{3} - \zeta^2 + \frac{2}{\beta^2}(\zeta-1)\right] \tag{5.3}$$

Knowing the values for (Q_1), (Q_2) and (Q_3), the values for Q_1', Q_2' and Q_3' are readily found.

5.2.2 Force Coefficient (FQ)

In the computer program the force/unit height coefficients, FQ_1, FQ_2 and FQ_3 are coefficients of q_1, q_2 and q_3 respectively.

Equations (4.4), (4.13) and (4.22) corresponding to q_1, q_2 and q_3 respectively are as follows:

$$\frac{dQ'}{Hd\zeta} = q_1 = \gamma H^2 p \left(\beta C_1 \operatorname{Cosh}(\beta\zeta) + \frac{1}{\beta^2}\right) D_1 = 0$$

$$\frac{dQ'}{Hd\zeta} = q_2 = \gamma H^3 w \left(C_2\beta \operatorname{Cosh}(\beta\zeta) + D_2\beta \sinh(\beta\zeta) + \frac{\zeta}{\beta^2}\right)$$

$$\frac{dQ'}{Hd\zeta} = q_3 = \gamma H^2 W\left[C_2\beta \operatorname{Cosh}(\beta\zeta) + D_3\beta \sinh(\beta\zeta) - \frac{1}{\beta^2}\left(\zeta^2 - 2\zeta + \frac{2}{\beta^2}\right)\right]$$

The above equations could be written as

$q_1 = \gamma H^2 p \cdot (FQ_1)$

$q_2 = \gamma H^3 w \cdot (FQ_2)$

$q_3 - \gamma H^2 W \cdot (FQ_3)$

Where FQ_1, FQ_2 and FQ_3 are

$$FQ_1 = \beta C_1 \text{ Cosh } (\beta\zeta) + \frac{1}{\beta^2} \quad (5.4)$$

$$FQ_2 = C_2\beta \text{ Cosh } (\beta\zeta) + D_2 \text{ sinh } (\beta\zeta) + \frac{\zeta}{\beta^2} \quad (5.5)$$

$$FQ_3 = C_3\beta \text{ Cosh } (\beta\zeta) + D_3\beta \text{ sinh } (\beta\zeta) - \frac{1}{\beta^2} (\zeta^2 - 2\zeta + \frac{2}{\beta^2}) \quad (5.6)$$

Knowing the values for FQ_1, FQ_2 and FQ_3 the values for q_1, q_2 and q_3 are readily computed.

The above equations are evaluated by taking the stiffness parameter $\beta = 2,4, \ldots,20$ and finding the corresponding values for Q_1, Q_2, Q_3, FQ_1, FQ_2 and FQ_3 when ζ varies from 0.0 to 1.0.

Constants C_1, C_2, C_3, D_2, D_3 are given in their corresponding sections.

5.2.3 Deflection Coefficient (Fy)

In the computer program the deflection coefficients (Fy_1), (Fy_2) and (Fy_3) are coefficients of equation y_1, y_2, and y_3 respectively.

Equations (4.7), (4.16) and (4.27) correspond to y_1, y_2 and y_3 respectively

$$y_1 = \frac{PH^3}{EI} [[1/6\ \zeta^3 - R\ (C_1\ \text{Sinh}\ (\beta\zeta) + \frac{1}{6}\ \zeta^3)] + U_1\zeta + V_1]$$

$$y_2 = \frac{wH^4}{EI} [[\frac{1}{24}\ \zeta^4 - R\ (C_2\ \text{Sinh}\ (\beta\zeta) + D_2\ \text{Cosh}\ (\beta\zeta) + \frac{1}{24}\ \zeta^4 + \frac{1}{2\beta^2}\ \zeta^2)] + U_2\zeta + V_2]$$

$$y_3 = \frac{WH^3}{EI} [[(\frac{1}{12}\ \zeta^4 - \frac{1}{60}\ \zeta^5) - R\ [C_3\ \text{Sinh}\ (\beta\zeta) + D_3\ \text{Cosh}\ (\beta\zeta) - [\frac{1}{60}\ \zeta^5 - \frac{1}{12}\ \zeta^4 + \frac{2}{\beta^2} (\frac{1}{6}\ \zeta^3 - \frac{1}{2}\ \zeta^2)]]] + U_3\zeta + V_3]$$

The above equations could be written as:

$$y_1 = \frac{PH^3}{EI} \cdot (Fy_1)$$

$$y_2 = \frac{wH^4}{EI} \cdot (Fy_2)$$

$$y_3 = \frac{WH^3}{EI} \cdot (Fy_3)$$

where Fy_1, Fy_2 and Fy_3 are:

$$Fy_1 = [\frac{1}{6}\ \zeta^3 - R\ (C_1\ \text{Sinh}\ (\beta\zeta) + \frac{1}{6}\ \zeta^3)] + U_1\zeta + V_1 \tag{5.7}$$

$$Fy_2 = [\frac{1}{24}\ \zeta^4 - R\ (C_2\ \text{Sinh}\ (\beta\zeta) + D_2\ \text{Cosh}\ (\beta\zeta) + \frac{1}{24}\ \zeta^4 + \frac{1}{2\beta^2}\ \zeta^2)] + U_2\zeta + V_2 \tag{5.8}$$

$$Fy_3 = (\frac{1}{12}\ \zeta^4 - \frac{1}{60}\ \zeta^5) - R\ [C_3\ \text{Sinh}\ (\beta\zeta) + D_3\ \text{Cosh}\ (\beta\zeta) - [\frac{1}{60}\ \zeta^5 - \frac{1}{12}\ \zeta^4 + \frac{2}{\beta^2}(\frac{1}{6}\ \zeta^3 - \frac{1}{2}\ \zeta^2)]] + U_3\zeta + V_3 \qquad (5.9)$$

Knowing values for Fy_1, Fy_2 and Fy_3 the values for y_1, y_2 and y_3 are readily calculated.

Constants C_1, C_2, C_3, D_2, D_3, U_1, U_2, U_3, V_1, V_2, and V_3 are given in their respective sections, and

$$R = \frac{\gamma H^2 L}{\beta^2}$$

Fy_1, Fy_2 anf Fy_3 are evaluated by letting ζ vary from 0.0 to 1.0 while R varies from 0.1 to 1.0 and the stiffness coefficient from 2 to 20.

The procedure is as follows

a) First, starting values of 0.1 and 2 are assigned to R and β respectively while ζ is varied through its full range, and values for Fy_1, Fy_2 and Fy_3 are found in the above three cases.

b) Next R is kept at its original value and β is assigned the next value 4, 6, 8 up to 20 (full range of β)

c) Next R is assigned the next value of 0.2 and step (b) is repeated. This is continued until R reaches its final value of 1.0.

5.3 PROGRAM

5.3.1 PROGRAM TO EVALUATE (Q1),(Q2),(Q3),(FQ1),(FQ2) AND (FQ3).

```
      DO 20 IB=1,20,1
      B=IB
      PRINT 11,B
      DO 10 IE=1,21
      E=(IE-1)*0.05
10    CONTINUE
20    CONTINUE
      STOP
11    FORMAT(1H1,//,T4,*BETA=*,F6.2,//,T6,*ZETA*,T14,
     1*1000(FQ1)*,T26,*1000(FQ2)*,T38,*1000(FQ3)*
     1,T50,*1000(Q1)*,T61,*1000(Q2)*,T72,*1000(Q3)*,/,T2,80(1H-))
22    FORMAT(1H0,T5,F4.2,T14,F8.4,T26,F8.4,T38,F8.4,
     1T50,F8.4,T61,F8.4,T72,F8.4)
      END

      FUNCTION FQ1(B,E)
      B2=B*B
      B3=B**3
      BE=B*E
      ONEDB2=1.0/B2
      ONEDB3=1.0/B3
      C1=-ONEDB3*(1.0/COSH(B))
      T1=C1*B*COSH(BE)
      T2=ONEDB2
      FQ1=1000*(T1+T2)
      RETURN
      END

      FUNCTION FQ2(B,E)
      BE=B*E
      B2=B*B
      B3=B**3
      B4=B**4
      ONEDB2=1.0/B2
      ONEDB3=1.0/B3
      Z2=1.0/COSH(B)
      Z1=(1.0/B)*TANH(B)
      C2=(ONEDB3)*(Z1-Z2)
      D2=-1.0/B4
      T3=B*((C2*COSH(BE))+(D2*SINH(BE)))
      T4=E*ONEDB2
      FQ2=1000*(T3+T4)
      RETURN
      END
```

```
      FUNCTION FQ3(B,E)
      BE=B*E
      E2=E*E
      B2=B*B
      B3=B**3
      B4=B**4
      TWODB=2.0/B
      TWODB2=2.0/B2
      ONEDB2=1.0/B2
      ONEDB3=1.0/B3
      Z3=(TWODB*TANH(B)+2.0*(ONEDB2/COSH(B)))
      Z4=(1.0/COSH(B))
      C3=(ONEDB3)*(Z3-Z4)
      D3=-2.0/B4
      T5=B*((C3*COSH(BE))+(D3*SINH(BE)))
      T6=ONEDB2*((E2+TWODB2)-(2.0*E))
      FQ3=1000*(T5-T6)
      RETURN
      END

      FUNCTION Q1(B,E)
      B2=B*B
      B3=B**3
      ONEDB2=1.0/B2
      ONEDB3=1.0/B3
      BE=B*E
      C1=-ONEDB3*(1.0/COSH(B))
      T1=C1*SINH(BE)
      T2=E*ONEDB2
      Q1=1000*(T1+T2)
      RETURN
      END

      FUNCTION Q2(B,E)
      B2=B*B
      BE=B*E
      E2=E*E
      B4=B**4
      ONEDB2=1.0/B2
      B3=B**3
      ONEDB3=1.0/B3
      ONEDB4=1.0/B4
      D2=-1.0/B4
      Z1=(1.0/B)*TANH(B)
      Z2=1.0/COSH(B)
      C2=(ONEDB3)*(Z1-Z2)
      T3=(C2*SINH(BE)+D2*COSH(BE))
      T4=0.5*(E2*ONEDB2)+ONEDB4
      Q2=1000*(T3+T4)
      RETURN
      END
```

```
      FUNCTION Q3(B,E)
      B4=B**4
      B2=B*B
      E2=E*E
      E3=E**3
      BE=B*E
      TWODB=2.0/B
      B3=B**3
      ONEDB3=1.0/B3
      B2=B*B
      ONEDB2=1.0/B2
      TWODB2=2.0/B2
      D3=-2.0/B4
      Z3=(TWODB*TANH(B)+2.0*(ONEDB2/COSH(B)))
      Z4=(1.0/COSH(B))
      C3=(ONEDB3)*(Z3-Z4)
      T5=(C3*SINH(BE)+D3*COSH(BE))
      T6=(E3/3.0-E2+TWODB2*(E-1))
      Q3=1000*(T5-ONEDB2*T6)
      RETURN
      END
```

5.3.2 PROGRAM TO EVALUATE (FY1),(FY2) AND (FY3).

```
      DO 30 IR=10,100,10
      R=IR/100.0
      B=IB
      PRINT 11,R,B
      DO 10 IE=1,21
      E=(IE-1)*0.05
   10 CONTINUE
   20 CONTINUE
   30 CONTINUE
      STOP
   11 FORMAT(1H0,//,T4,*R=*,F6.2,//,T4,*BETA=*,F6.2,//T6,*ZETA*,
     1T21,*1000(FY1)*,T41,*1000(FY2)*,T61,*1000(FY3)*,/,T2
     1,70(1H-))
   22 FORMAT(1H0,T5,F4.2,T14,F14.4,T34,F14.4,T54,F14.4)
      END

      FUNCTION FY1(B,E,R)
      ONEDB1=1.0/B
      ONEDB2=1.0/B**2
      ONEDB3=1.0/B**3
      ONEDB4=1.0/B**4
      BE=B*E
      E2=E*E
      E3=E**3
      E4=E**4
      E5=E**5
      C1=-ONEDB3*(1.0/COSH(B))
      P1=C1*SINH(B)+1.0/6.0
      A1=R*(C1*B*COSH(B)+0.5)-0.5
      V1=R*P1-1.0/6.0-A1
      Y1=C1*SINH(BE)+E3/6.0
      FY1=1000*((E3/6.0-R*Y1)+A1*E+V1)
      RETURN
      END

      FUNCTION FY2(B,E,R)
      BE=B*E
      ONEDB1=1.0/B
      ONEDB2=1.0/B**2
      ONEDB3=1.0/B**3
      ONEDB4=1.0/B**4
      B2=B*B
      E2=E*E
      E4=E**4
      Z1=(1.0/B)*TANH(B)
      Z2=1.0/COSH(B)
      C2=(ONEDB3)*(Z1-Z2)
      D2=-1.0/B**4
      A2=R*(C2*B*COSH(B)+D2*B*SINH(B)+1.0/6.0+ONEDB2)-1.0/6.0
      P2=C2*SINH(B)+D2*COSH(B)+1.0/24.0+0.5*ONEDB2
      V2=R*P2-1.0/24.0-A2
      Y2=C2*SINH(BE)+D2*COSH(BE)+E4/24.0+E2/(2.0*B2)
      FY2=1000*((E4/24.0-R*Y2)+A2*E+V2)
      RETURN
      END
```

```
FUNCTION FY3(B,E,R)
ONEDB1=1.0/B
ONEDB2=1.0/B**2
ONEDB3=1.0/B**3
TWODB=2.0/B
TWODB2=2.0/B**2
BE=B*E
E2=E*E
E3=E**3
E4=E**4
E5=E**5
Z3=(TWODB*TANH(B)+2.0*(ONEDB2/COSH(B)))
Z4=(1.0/COSH(B))
C3=(ONEDB3)*(Z3-Z4)
D3=-2.0/B**4
A3=R*(C3*B*COSH(B)+D3*B*SINH(B)+0.25+ONEDB2)-0.25
P3=(C3*SINH(B)+D3*COSH(B))
P4=1.0/15.0+ONEDB2*(2.0/3.0)
V3=R*(P3+P4)-1.0/15.0-A3
Y3=(E4/12.0-E5/60.0)
Y4=C3*SINH(BE)+D3*COSH(BE)
Y5=(E5/60.0-E4/12.0)
Y6=(E3/6.0-E2/2.0)
FY3=1000*((Y3-R*(Y4-(Y5+TWODB2*Y6)))+A3*E+V3)
RETURN
END
```

WHERE A1,A2 AND A3 CORRESPOND TO U1,U2 AND U3 RESPECTIVELY IN THE TEXT.

6.0 COMPUTER OUTPUT

6.1 DEFINITIONS

The computer output is the evaluation of axial load factors 1000(Q), axial load/unit height of the wall factors 1000(FQ) and deflection factors 1000(fy)

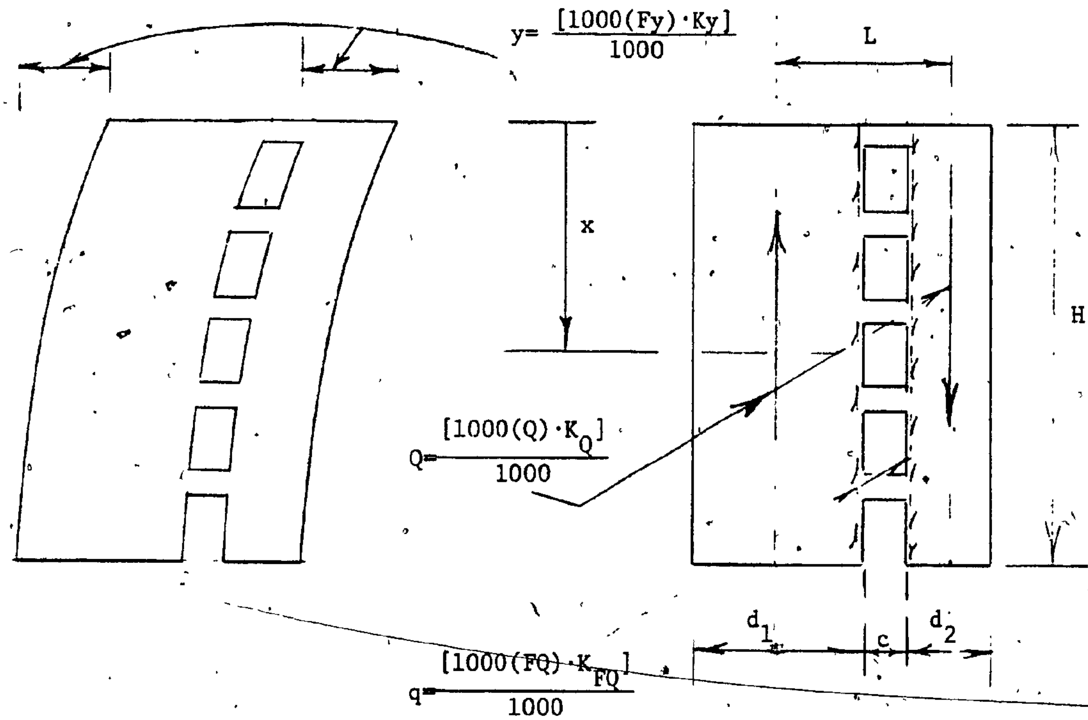


Fig. (6.1)
deflection diagram

Fig. (6.2)
force diagram

Where K_y, k_Q & K_{FQ} are multiplication factors. In the computer output, FQ_1, FQ_2, FQ_3, Q_1, Q_2, Q_3 etc. correspond to load cases 1,2 & 3, respectively Fig. (6.3):

Where $K_{Q_1} = \gamma H^3 p, \quad K_{Q_2} = \gamma H^4 w, \quad K_{Q_3} = \gamma W H^3$

$K_{FQ_1} = \gamma H^2 p, \quad K_{FQ_2} = \gamma H^3 w, \quad K_{FQ_3} = \gamma H^2 W.$

$K_{y_1} = \frac{PH^3}{EI}, \quad K_{y_2} = \frac{wH^4}{EI}, \quad K_{y_3} = \frac{WH^3}{EI}$

$\gamma = \frac{12LI^*}{hc^3 I_o}$ as given in section 3.0 (3.8)

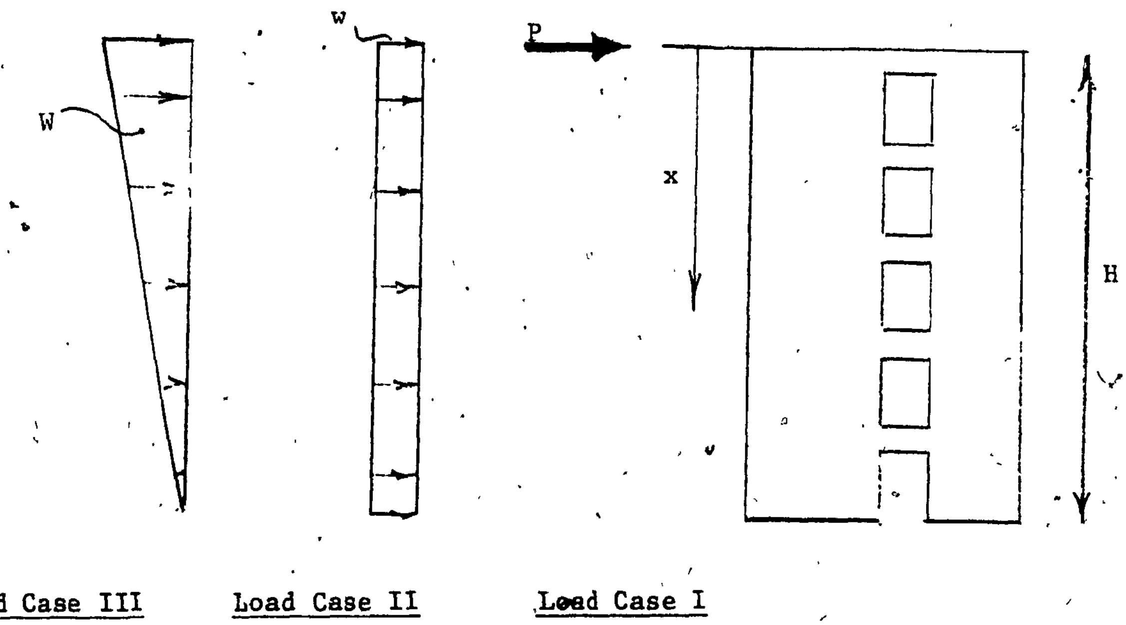


Fig. (6.3)

I = the equivalent moment of inertia of a coupled shear wall

I^* = moment of inertia of connecting beam

I_1 = moment of inertia of wall 1

I_2 = moment of inertia of wall 2

$I_o = I_1 + I_2$

6.2 EQUIVALENT MOMENT OF INERTIA

The moment of inertia I of a wall with one row of openings as in Fig. (6.1) is

$$I = \frac{1}{\eta}\ \frac{bd^3}{12} \qquad (6.1)$$

Where η is the numerical coefficient, b and d are the thickness and width of the wall respectively.

Coefficients η are treated as functions of ratios

$K_1 = \eta^2 \left(\frac{d}{H}\right)^4$, $K_2 = \frac{c}{d}$ & $K_3 = \frac{d_0}{d}$

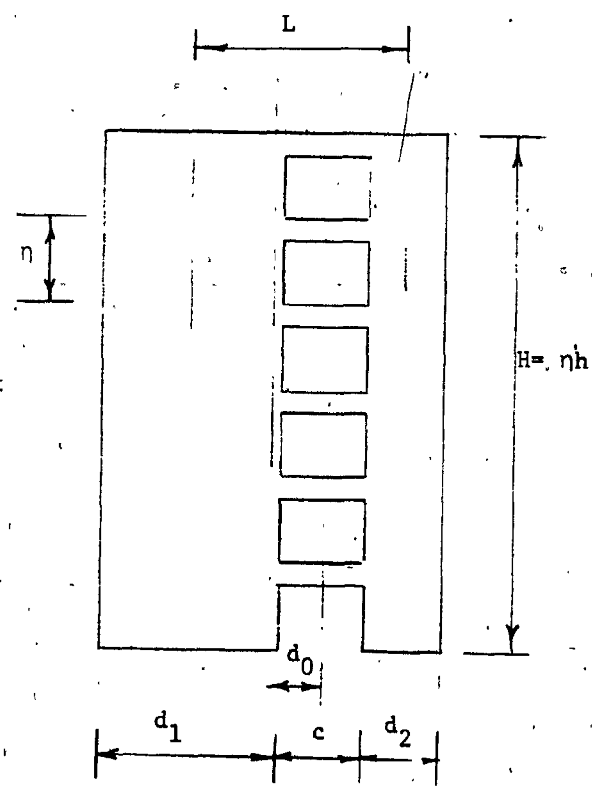


Fig. (6.4)

Where n is the number of storeys, d the width of the wall, H the height of the wall, c the width of the opening and d_0 the eccentricity of the opening relative to the center line of the wall

Coefficients η for $K_1 = 2$ and $K_1 = 8$ are given graphically in Fig. (6.2) and Fig. (6.3).

The intermediate value for η should be obtained by linear interpolation.

For a given wall, k_1, k_2 & k_3 are evaluated and η is found from the graph, and substituted in equation (6.1) to find I.

For a typical coupled shear wall, k_1, k_2 and k_3 should fall within the limits shown on the graphs, otherwise the following guidelines are recommended.

1) k_2 should not be greater than 0.35
2) If k_1 is smaller than 2, use $k_2 = 2$. This will give a more conservative value for I but less conservative than if I_0 is used, where $I_0 = I_1+I_2$.
3) In the case where k_1 is greater than 8, I should be taken equal to I_0.

The above recommendations for calculating the equivalent moment of inertia, I, represents a more accurate technique than by using the sum of inertias of the two walls.

$K_1 = 2.0$

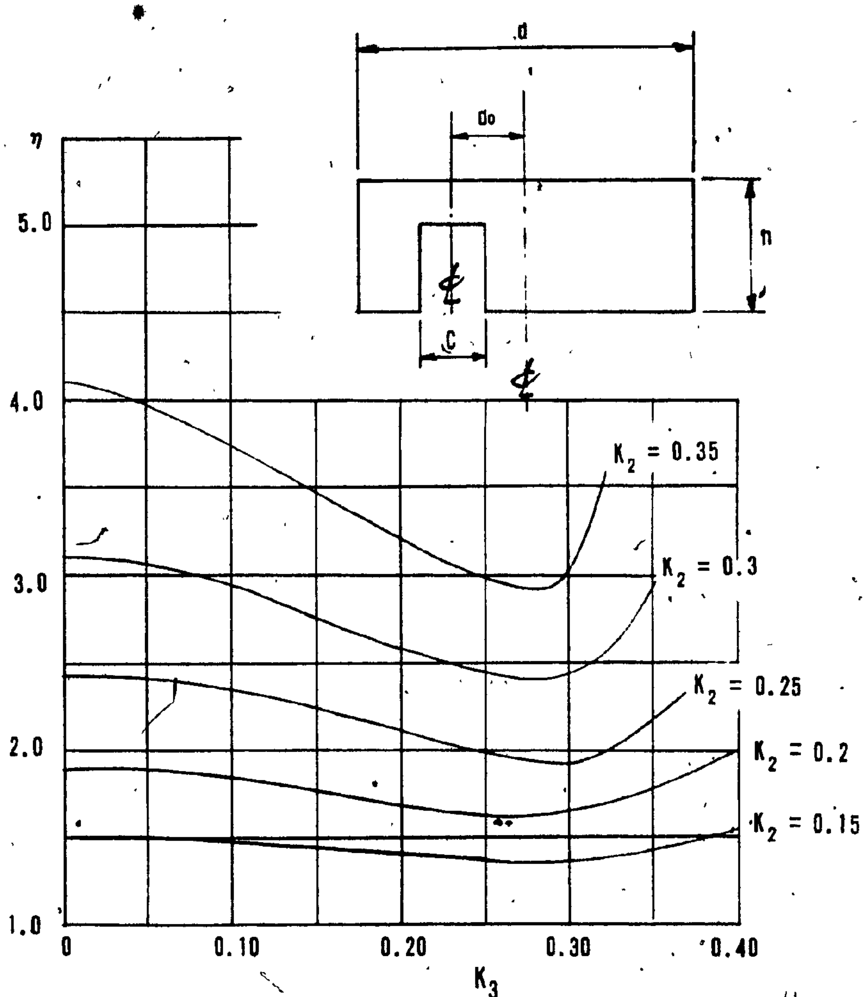


FIG. (6.5)

$$I = \frac{1}{\eta} \cdot \frac{bd^3}{12}$$

$K_1 = n^2 \left(\frac{d}{H}\right)^4$

$K_2 = \frac{c}{d}$

$K_3 = \frac{do}{d}$

Note: Graphs of Fig. (6.5) and Fig. (6.6) are by Rosman.

$K_1 = 8.0$

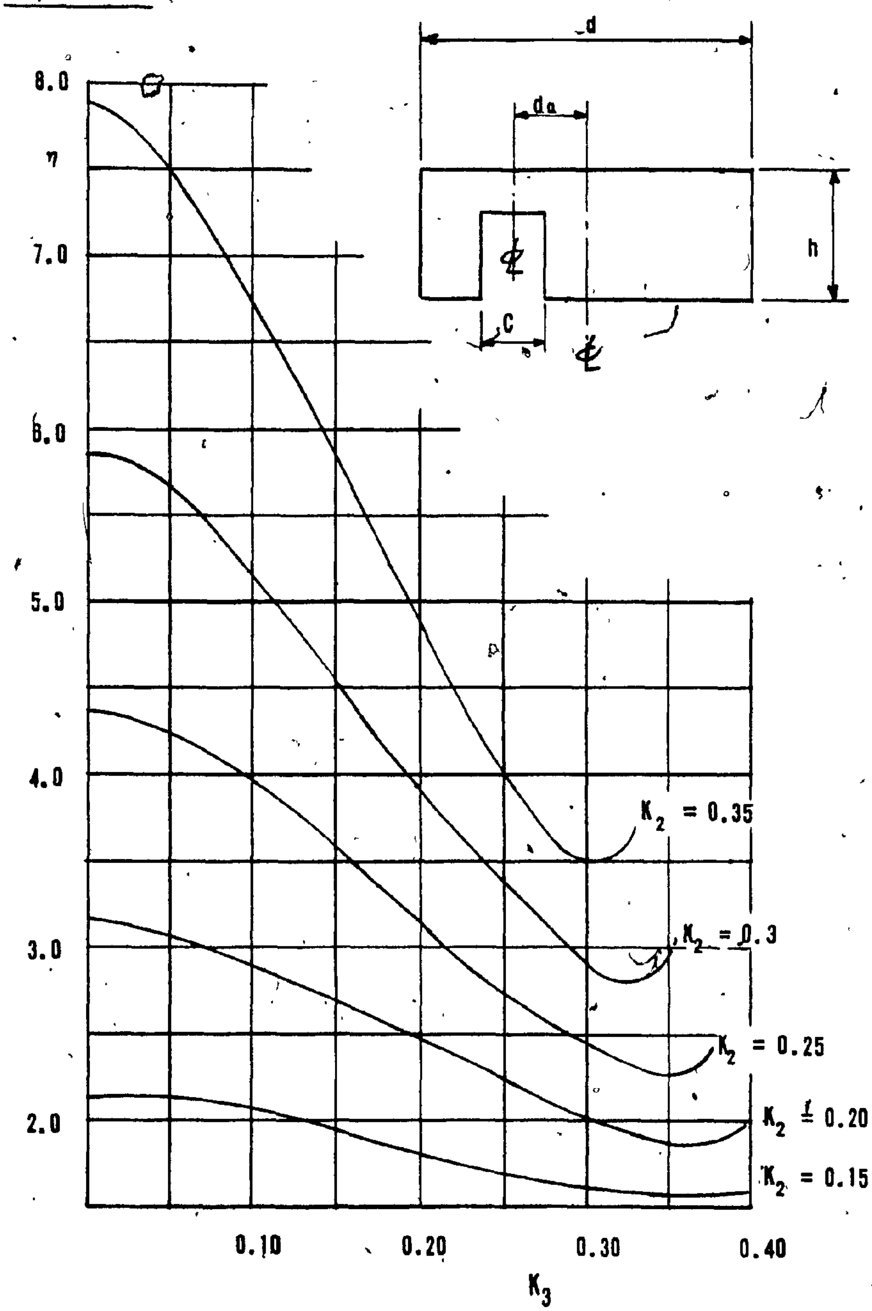


FIG. (6.6)

$K_1 = \eta^2 \left(\frac{d}{H}\right)^4$

$K_2 = \frac{c}{d}$

$K_3 = \frac{do}{d}$

$I = \frac{1}{\eta} \frac{bd^3}{12}$

BETA= 1.00

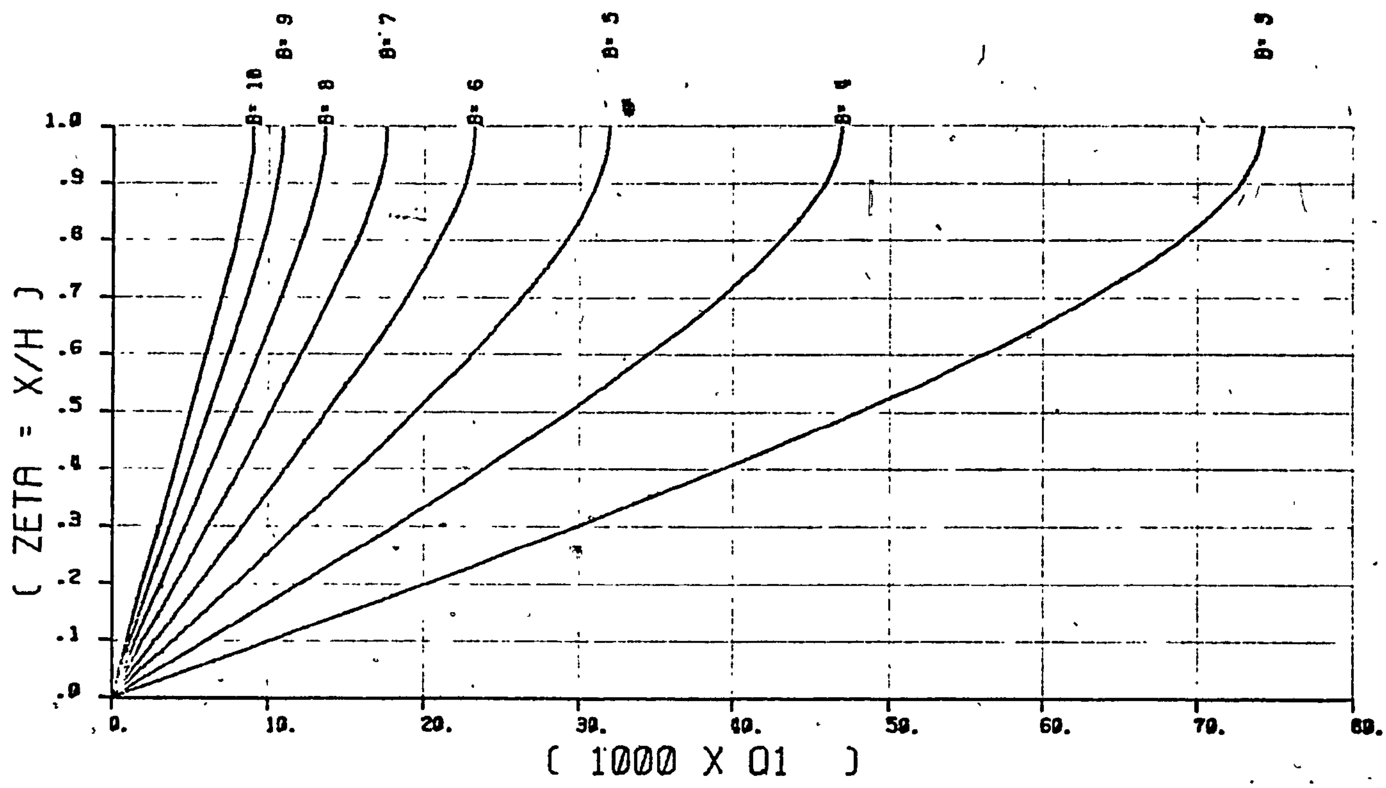
ZETA	1000(FQ1)	1000(FQ2)	1000(FQ3)	1000(Q1)	1000(Q2)	1000(Q3)
0.00	351.9457	113.5399	171.2426	0.0000	0.0000	0.0000
.05	351.1355	113.6610	171.4155	17.5838	5.6791	8.5652
.10	348.7028	113.9413	171.7743	35.0865	11.3688	17.1446
.15	344.6414	114.2565	172.0886	52.4269	17.0738	25.7419
.20	338.9414	114.4823	172.1404	69.5233	22.7929	34.3491
.25	331.5883	114.4942	171.7234	86.2935	28.5185	42.9481
.30	322.5639	114.1673	170.6429	102.6543	34.2367	51.5104
.35	311.8455	113.3758	168.7149	118.5217	39.9275	59.9982
.40	299.4064	111.9925	165.7657	133.8102	45.5644	68.3649
.45	285.2155	109.8891	161.6318	148.4331	51.1147	76.5551
.50	269.2372	106.9352	156.1589	162.3020	56.5391	84.5057
.55	251.4316	102.9985	149.2022	175.3264	61.7919	92.1462
.60	231.7542	97.9440	140.6254	187.4139	66.8204	99.3989
.65	210.1558	91.6341	130.3009	198.4698	71.5653	106.1796
.70	186.5824	83.9280	118.1090	208.3966	75.9605	112.3979
.75	160.9750	74.6815	103.9380	217.0941	79.9325	117.9575
.80	133.2696	63.7462	87.6837	224.4591	83.4005	122.7570
.85	103.3969	50.9700	69.2493	230.3850	86.2764	126.6896
.90	71.2822	36.1957	48.5447	234.7614	88.4642	129.6441
.95	36.8453	19.2615	25.4871	237.4745	89.8600	131.5048
1.00	0.0000	0.0000	0.0000	238.4058	90.3516	132.1523

Table (6.1)

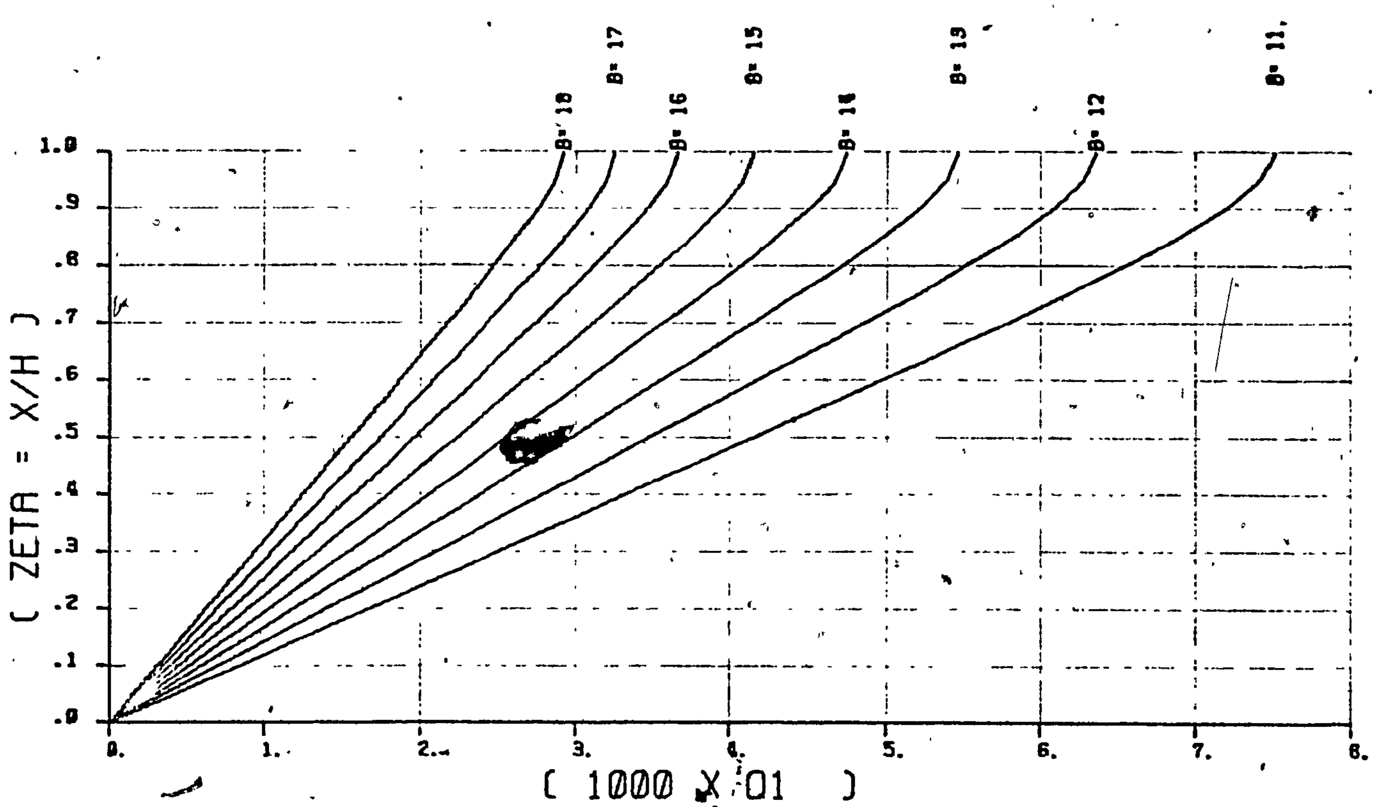
BETA= 2.00

ZETA	1000(FQ1)	1000(FQ2)	1000(FQ3)	1000(Q1)	1000(Q2)	1000(Q3)
0.00	183.5494	54.0529	82.7816	0.0000	0.0000	0.0000
.05	183.2169	54.3025	83.1547	9.1719	2.7069	4.1455
.10	182.2160	54.9706	84.1171	18.3106	5.4372	8.3253
.15	180.5367	55.9385	85.4471	27.3822	8.2089	12.5633
.20	178.1621	57.0910	86.9389	36.3526	11.0342	16.8727
.25	175.0686	58.3145	88.4011	45.1865	13.9192	21.2567
.30	171.2252	59.4961	89.6545	53.8470	16.8649	25.7093
.35	166.5933	60.5225	90.5301	62.2958	19.8663	30.2158
.40	161.1267	61.2790	90.8679	70.4924	22.9127	34.7533
.45	154.7706	61.6479	90.5148	78.3937	25.9877	39.2911
.50	147.4614	61.5078	89.3235	85.9536	29.0690	43.7909
.55	139.1260	60.7323	87.1506	93.1227	32.1279	48.2071
.60	129.6809	59.1885	83.8556	99.8477	35.1294	52.4872
.65	119.0317	56.7358	79.2991	106.0708	38.0316	56.5716
.70	107.0716	53.2246	73.3418	111.7291	40.7853	60.3938
.75	93.6811	48.4946	65.8426	116.7541	43.3338	63.8801
.80	78.7260	42.3734	56.6577	121.0711	45.6116	66.9499
.85	62.0569	34.6746	45.6390	124.5981	47.5448	69.5153
.90	43.5067	25.1961	32.6322	127.2454	49.0494	71.4807
.95	22.8898	13.7179	17.4761	128.9144	50.0311	72.7427
1.00	0.0000	0.0000	0.0000	129.4966	50.3839	73.1897

Table (6.2)

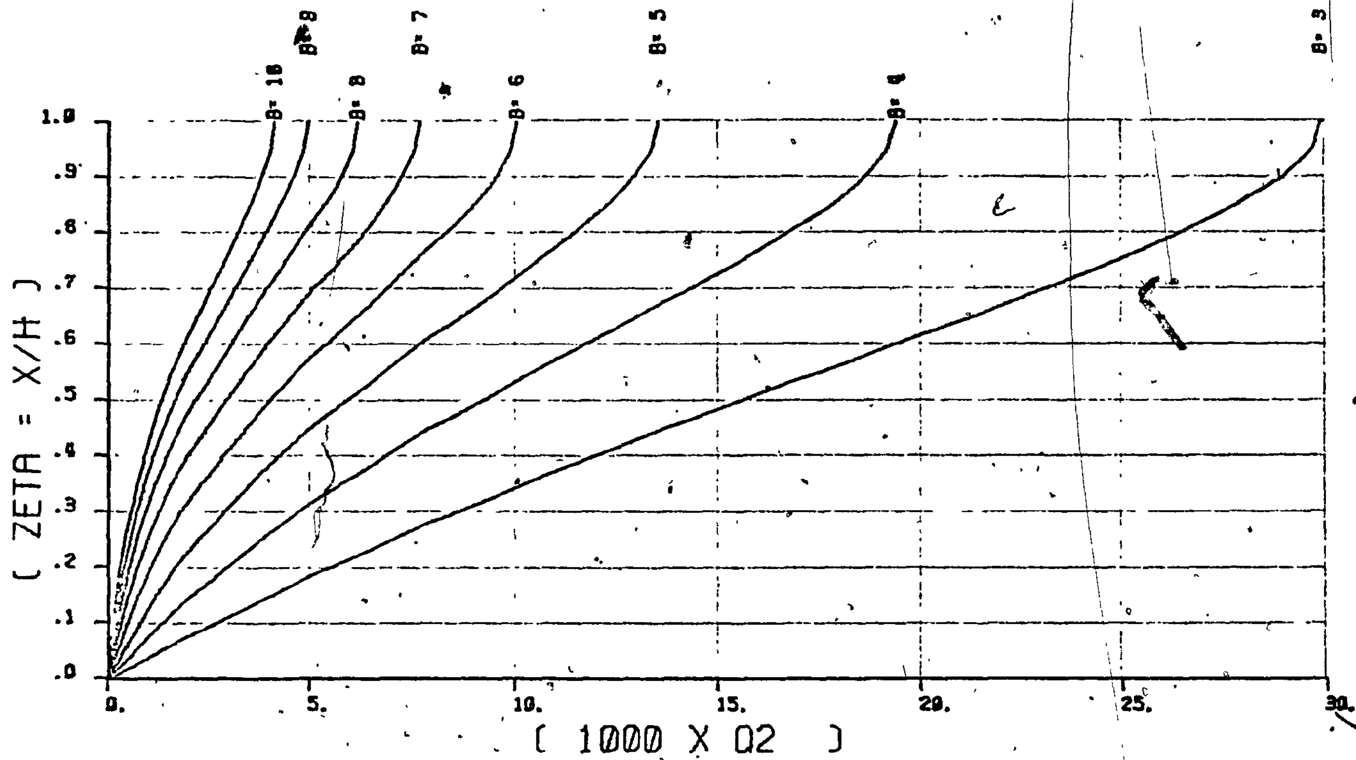


(a) TOTAL AXIAL LOAD FACTOR - LOAD CASE I

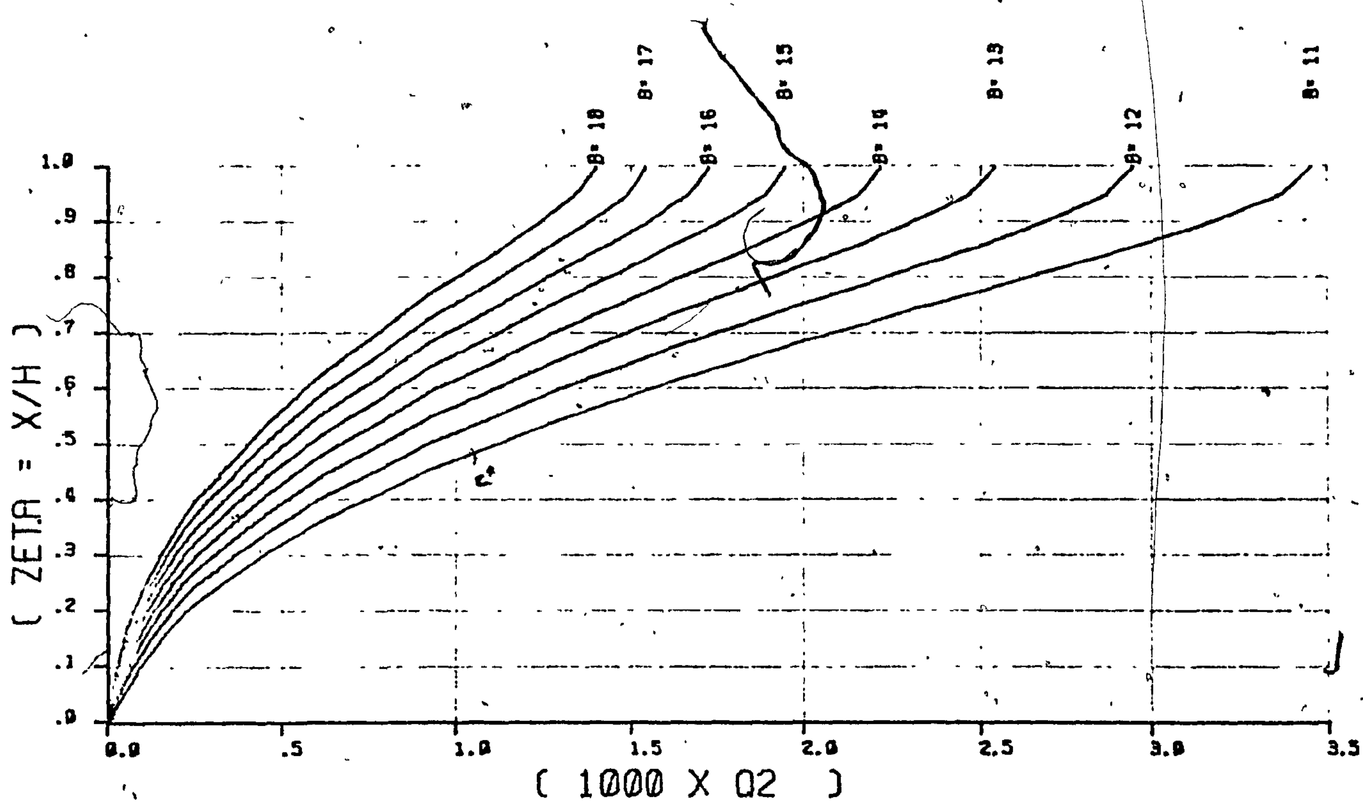


(b) TOTAL AXIAL LOAD FACTOR - LOAD CASE I

Fig. (6.7)

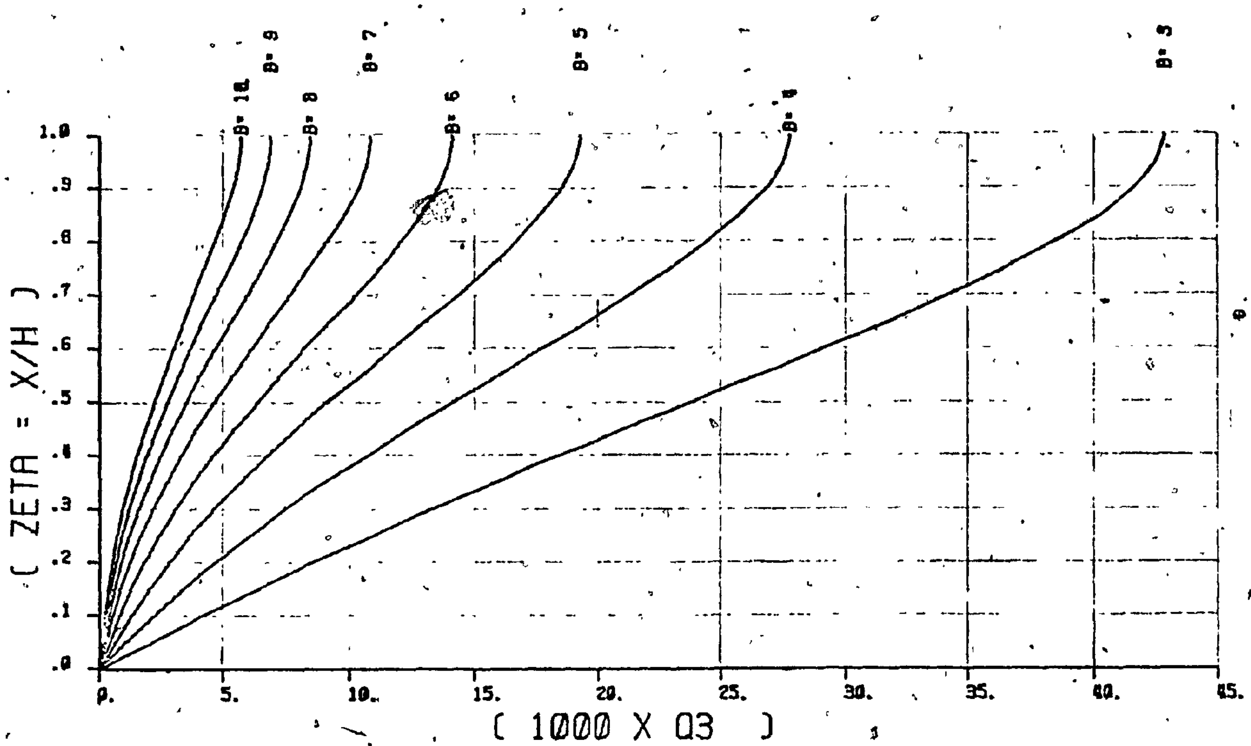


(a) TOTAL AXIAL LOAD FACTOR - LOAD CASE II

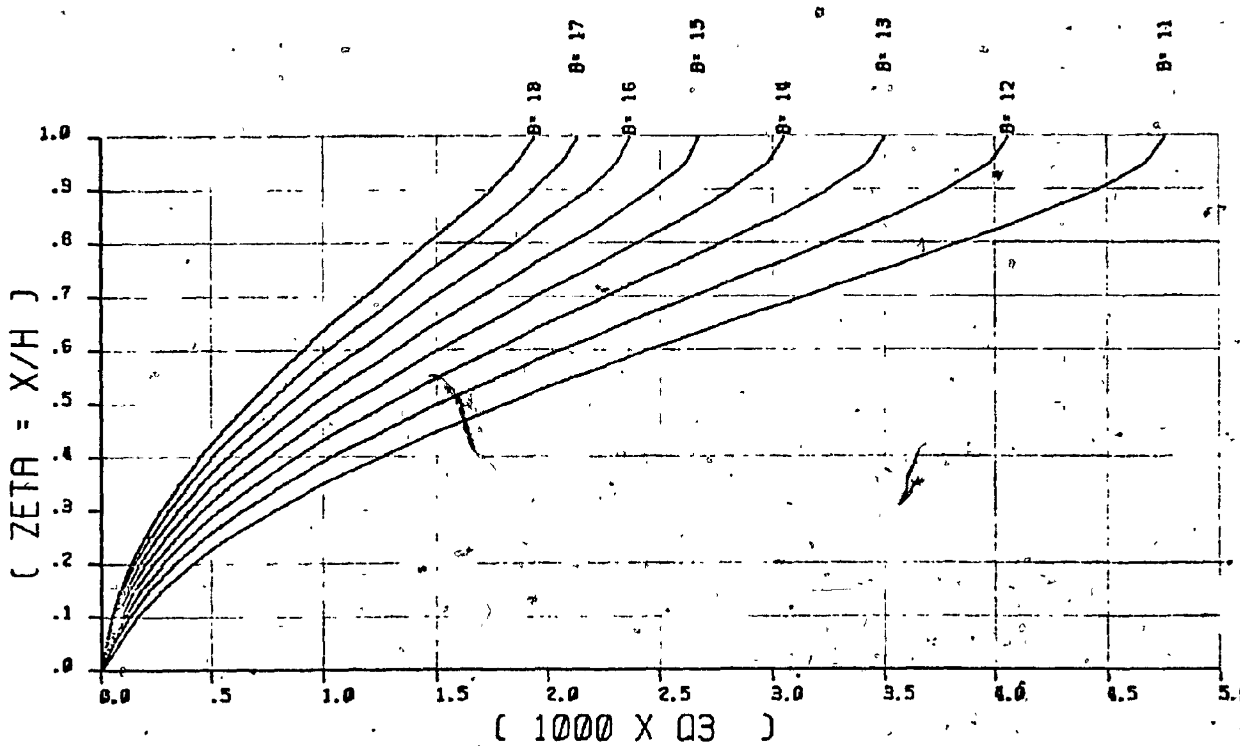


(b) TOTAL AXIAL LOAD FACTOR - LOAD CASE II

Fig. (6.8)

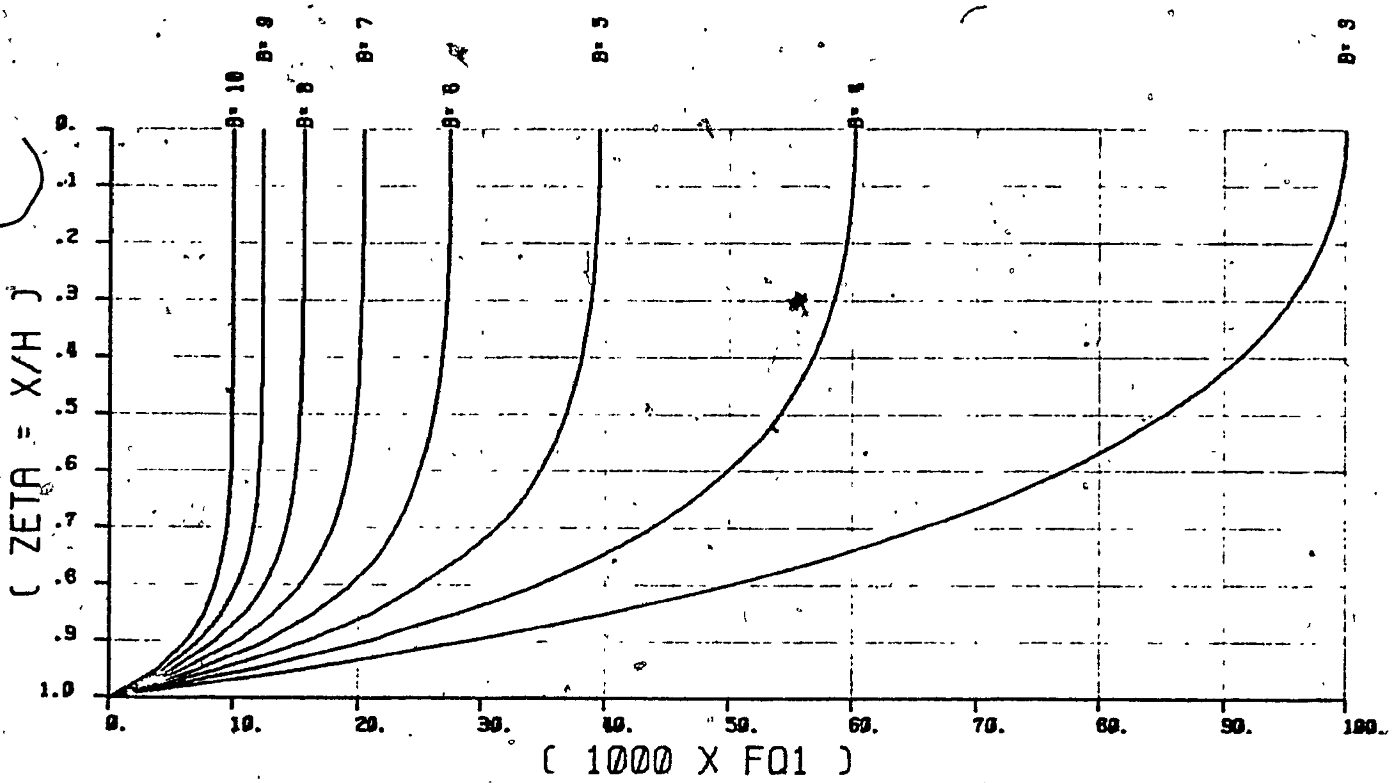


(a) TOTAL AXIAL LOAD FACTOR - LOAD CASE III

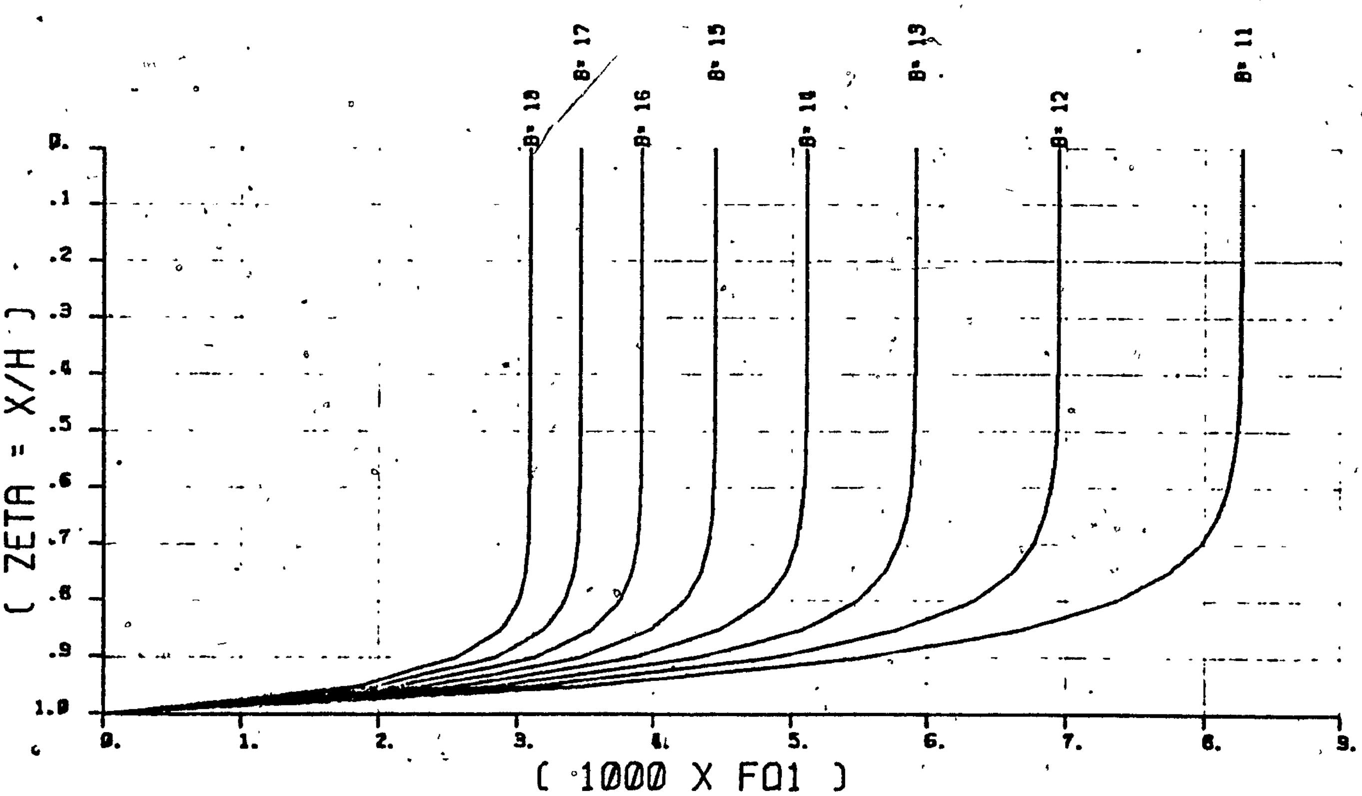


(b) TOTAL AXIAL LOAD FACTOR - LOAD CASE III

Fig. (6.9)

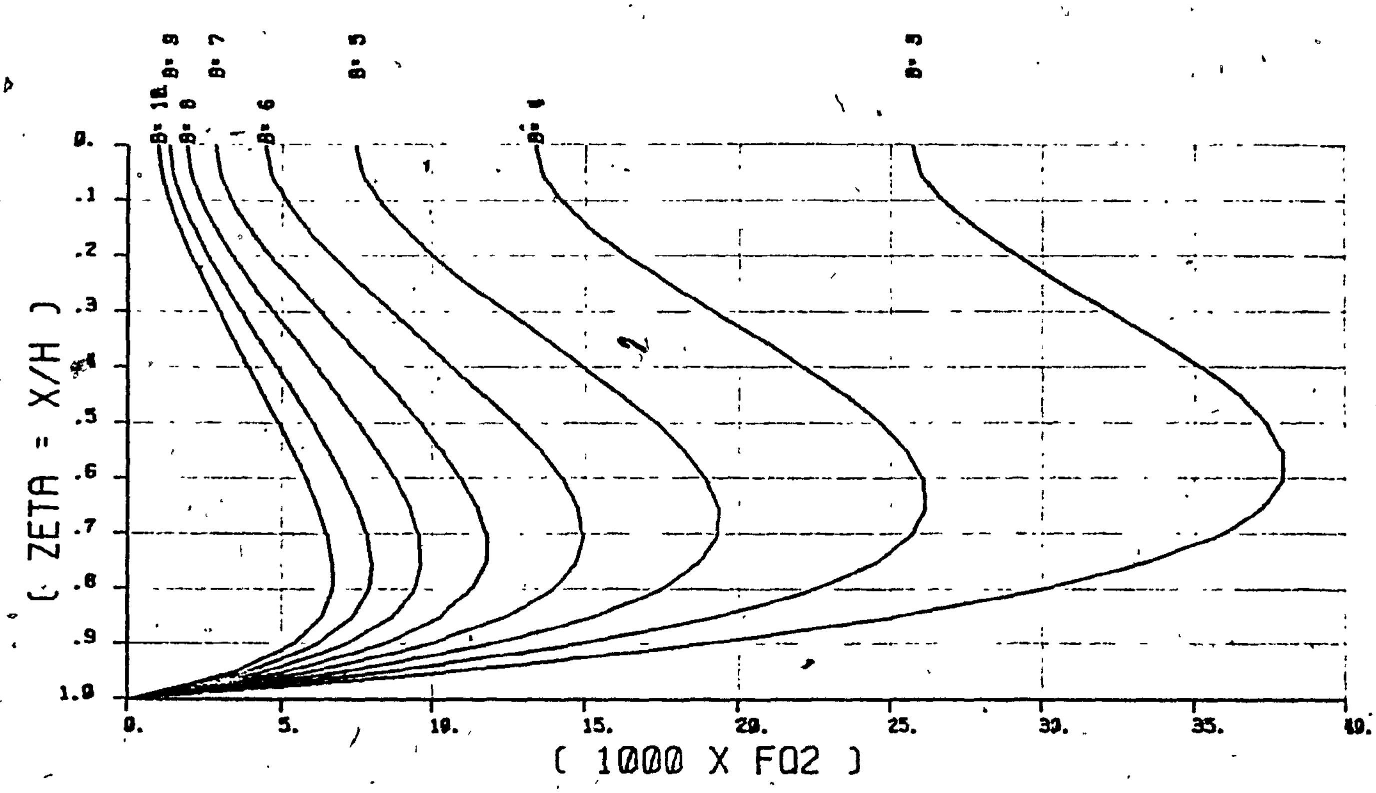


(a) AXIAL LOAD/UNIT HEIGHT FACTOR - LOAD CASE I

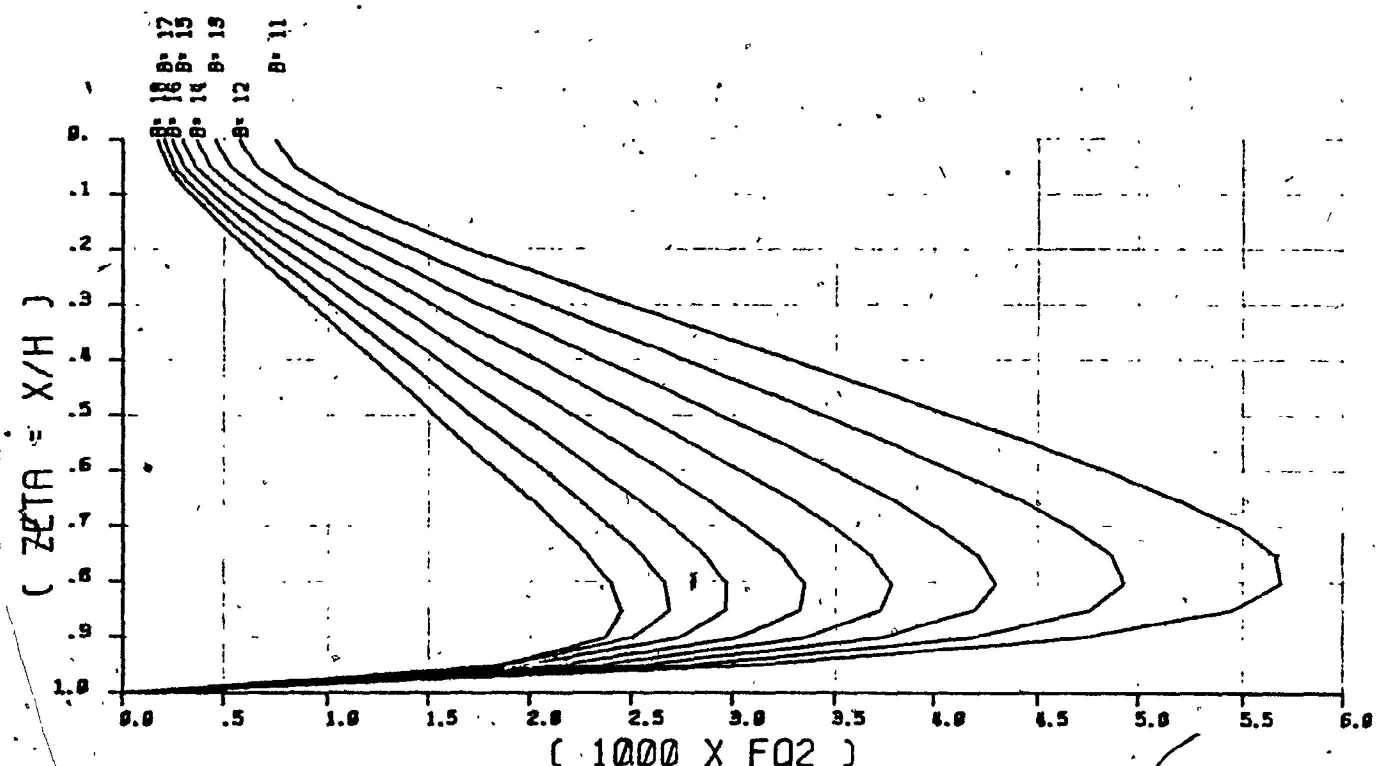


(b) AXIAL LOAD/UNIT HEIGHT FACTOR - LOAD CASE I

Fig. (6.10)

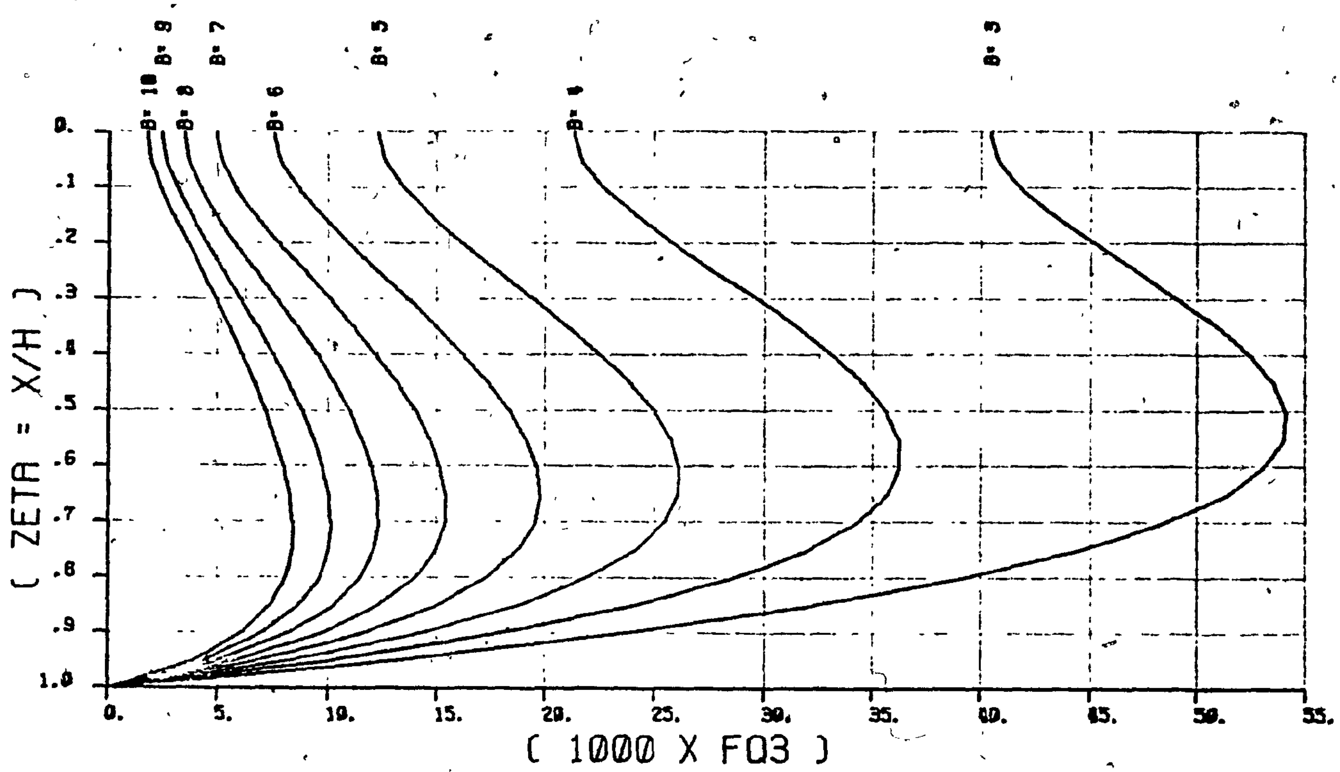


(a) AXIAL LOAD/UNIT HEIGHT FACTOR - LOAD CASE II

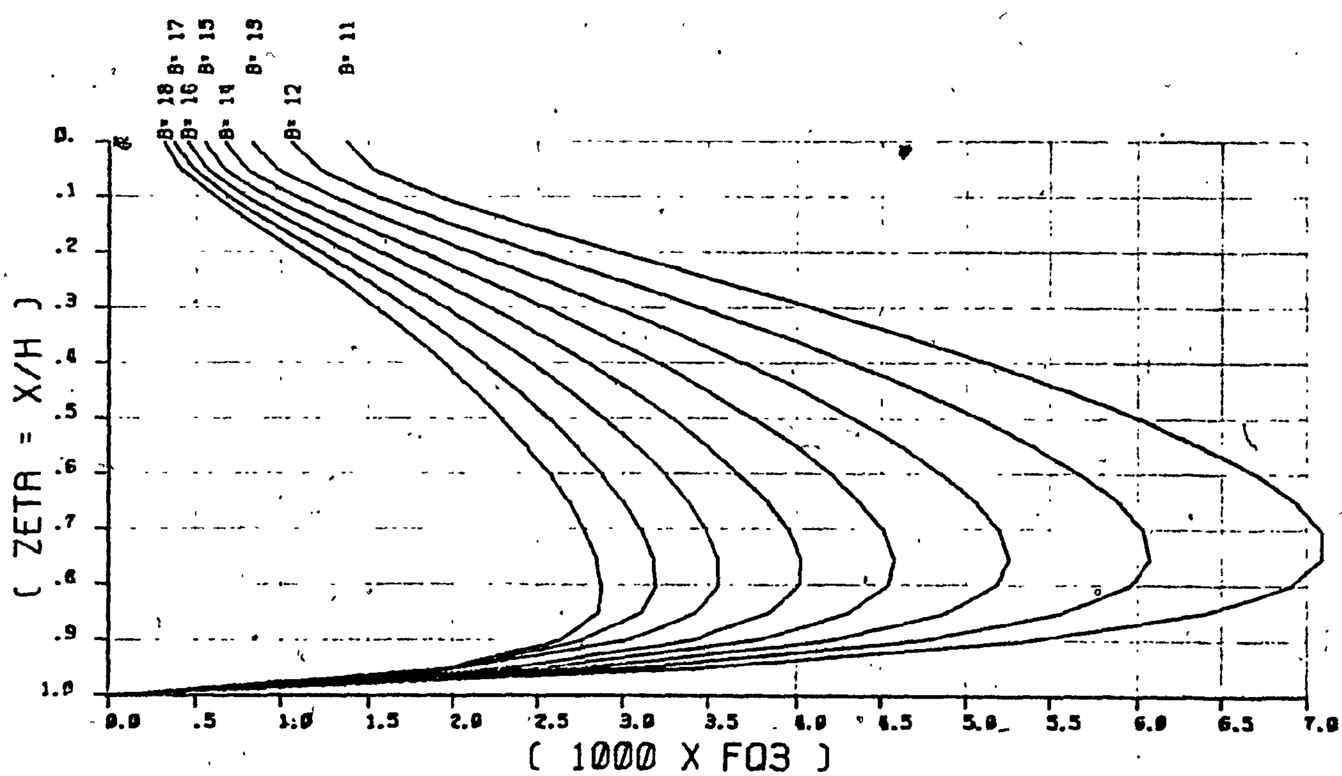


(b) AXIAL LOAD/UNIT HEIGHT FACTOR - LOAD CASE II

Fig. (6.11)



(a) AXIAL LOAD/UNIT HEIGHT FACTOR - LOAD CASE III



(b) AXIAL LOAD/UNIT HEIGHT FACTOR - LOAD CASE III

Fig. (6.12)

BETA= 19.00

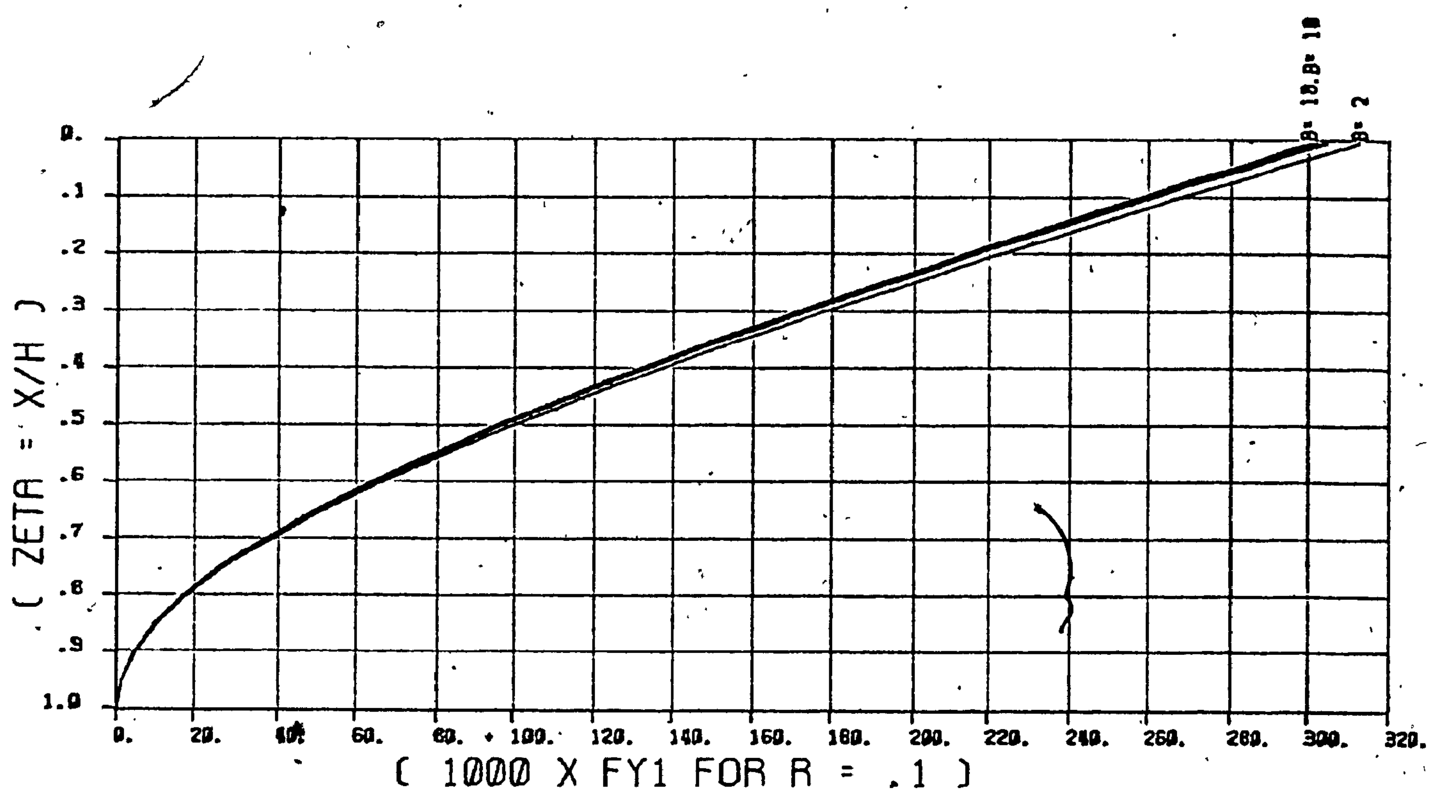
ZETA	1000(FQ1)	1000(FQ2)	1000(FQ3)	1000(Q1)	1000(Q2)	1000(Q3)
0.00	2.7701	.1458	.2762	0.0000	0.0000	-.0000
.05	2.7701	.1949	.3675	.1385	.0082	.0155
.10	2.7701	.2988	.5546	.2770	.0204	.0383
.15	2.7701	.4239	.7702	.4155	.0384	.0714
.20	2.7701	.5573	.9884	.5540	.0629	.1154
.25	2.7701	.6938	1.1991	.6925	.0942	.1701
.30	2.7701	.8315	1.3984	.8310	.1323	.2351
.35	2.7701	.9697	1.5847	.9695	.1773	.3097
.40	2.7701	1.1081	1.7576	1.1080	.2293	.3933
.45	2.7700	1.2465	1.9168	1.2465	.2881	.4852
.50	2.7699	1.3848	2.0620	1.3850	.3539	.5848
.55	2.7695	1.5230	2.1933	1.5235	.4266	.6912
.60	2.7687	1.6607	2.3101	1.6620	.5062	.8038
.65	2.7665	1.7970	2.4118	1.8004	.5927	.9220
.70	2.7608	1.9298	2.4962	1.9386	.6859	1.0447
.75	2.7461	2.0536	2.5578	2.0763	.7855	1.1712
.80	2.7081	2.1541	2.5823	2.2128	.8908	1.2999
.85	2.6098	2.1943	2.5331	2.3461	.9999	1.4282
.90	2.3558	2.0788	2.3150	2.4713	1.1078	1.5505
.95	1.6988	1.5603	1.6824	2.5752	1.2013	1.6530
1.00	0.0000	0.0000	0.0000	2.6243	1.2469	1.7017

Table (6.3)

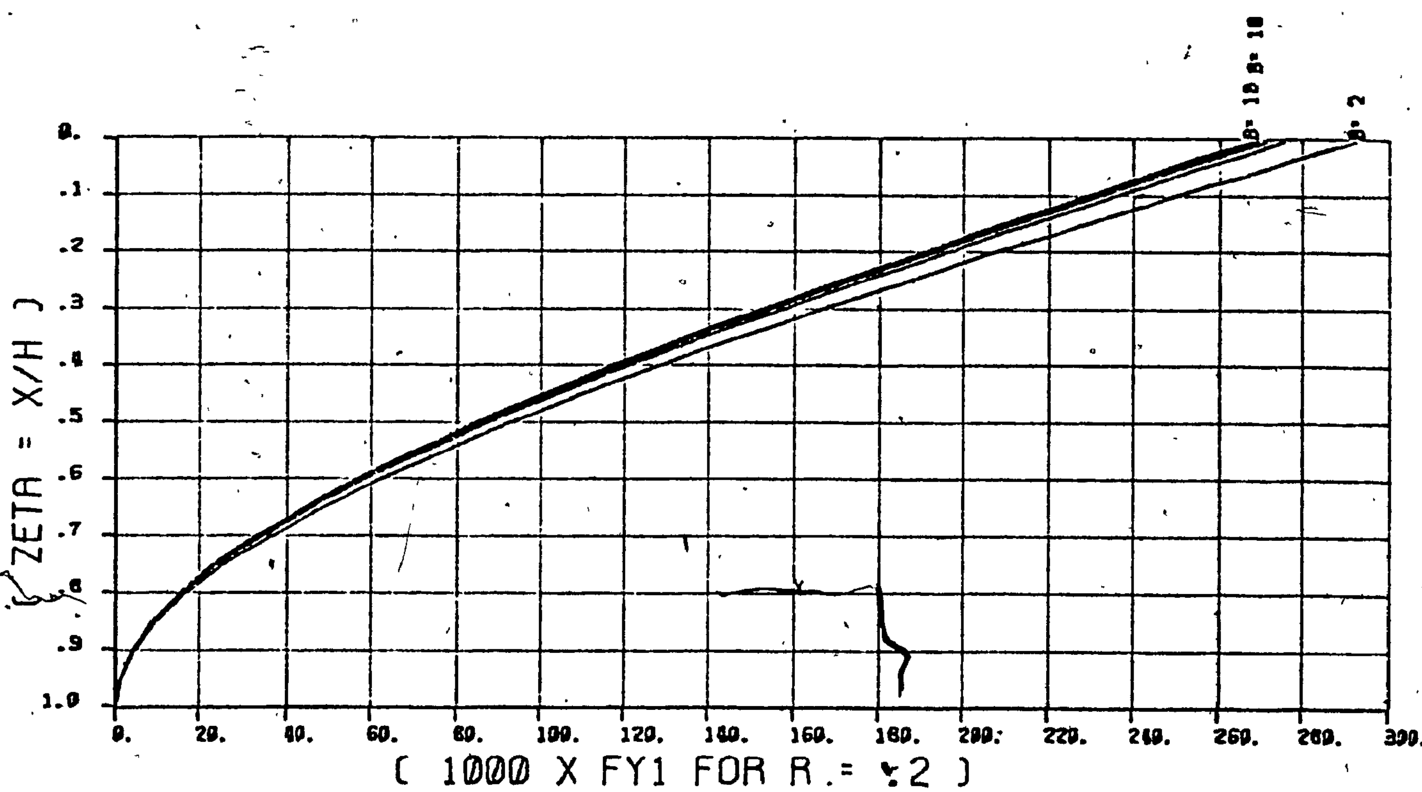
BETA= 20.00

ZETA	1000(FQ1)	1000(FQ2)	1000(FQ3)	1000(Q1)	1000(Q2)	1000(Q3)
0.00	2.5000	.1250	.2375	0.0000	0.0000	-.0000
.05	2.5000	.1710	.3232	.1250	.0071	.0134
.10	2.5000	.2669	.4963	.2500	.0179	.0337
.15	2.5000	.3812	.6937	.3750	.0341	.0634
.20	2.5000	.5023	.8921	.5000	.0561	.1031
.25	2.5000	.6258	1.0829	.6250	.0843	.1525
.30	2.5000	.7503	1.2631	.7500	.1187	.2112
.35	2.5000	.8751	1.4315	.8750	.1594	.2786
.40	2.5000	1.0000	1.5876	1.0000	.2062	.3542
.45	2.5000	1.1250	1.7312	1.1250	.2594	.4372
.50	2.4999	1.2499	1.8624	1.2500	.3187	.5271
.55	2.4997	1.3747	1.9809	1.3750	.3844	.6232
.60	2.4992	1.4992	2.0867	1.5000	.4562	.7250
.65	2.4977	1.6227	2.1790	1.6249	.5343	.8317
.70	2.4938	1.7438	2.2563	1.7497	.6184	.9426
.75	2.4832	1.8582	2.3145	1.8742	.7085	1.0570
.80	2.4542	1.9542	2.3419	1.9977	.8040	1.1736
.85	2.3755	2.0005	2.3074	2.1188	.9032	1.2902
.90	2.1617	1.9117	2.1259	2.2331	1.0018	1.4019
.95	1.5803	1.4553	1.5661	2.3290	1.0884	1.4966
1.00	0.0000	0.0000	0.0000	2.3750	1.1312	1.5423

Table (6.4)

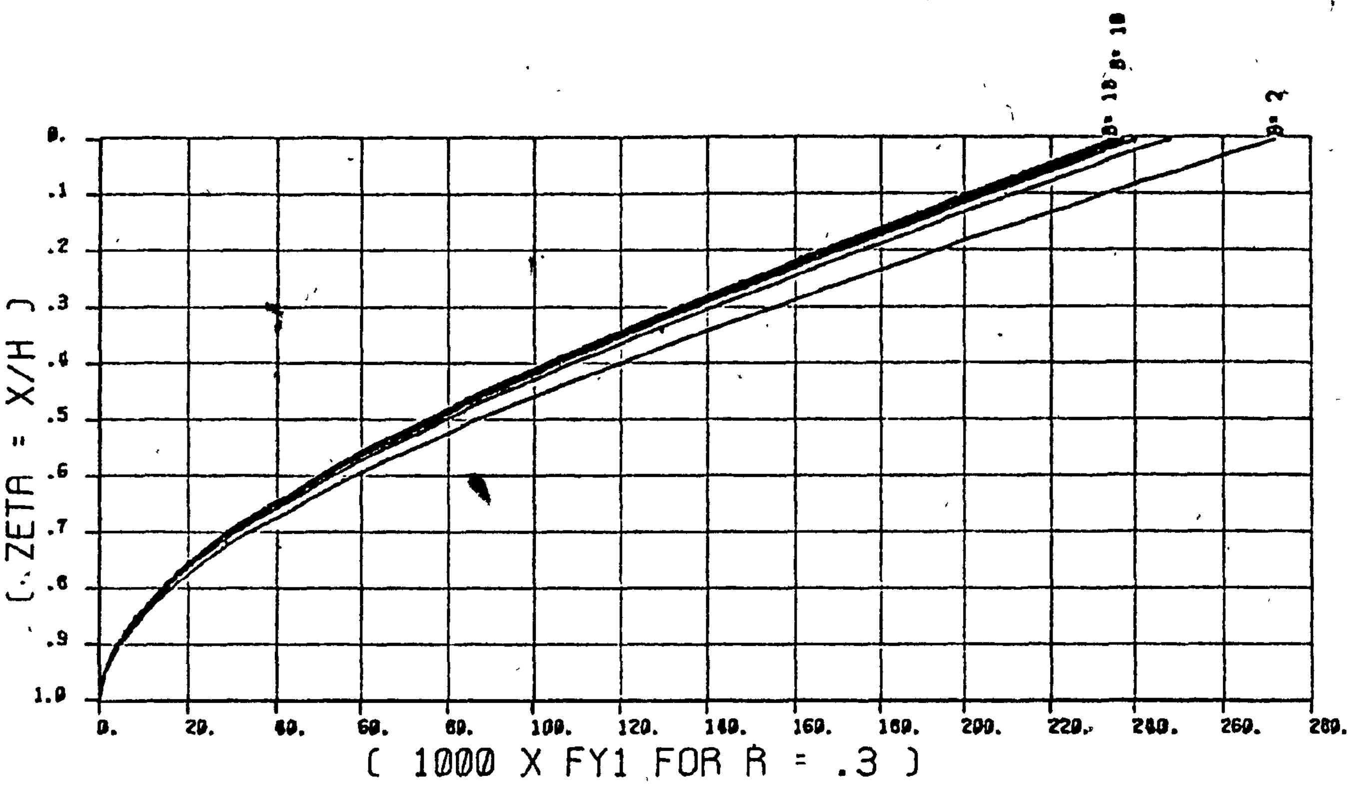


(a) DEFLECTION FACTOR - LOAD CASE I

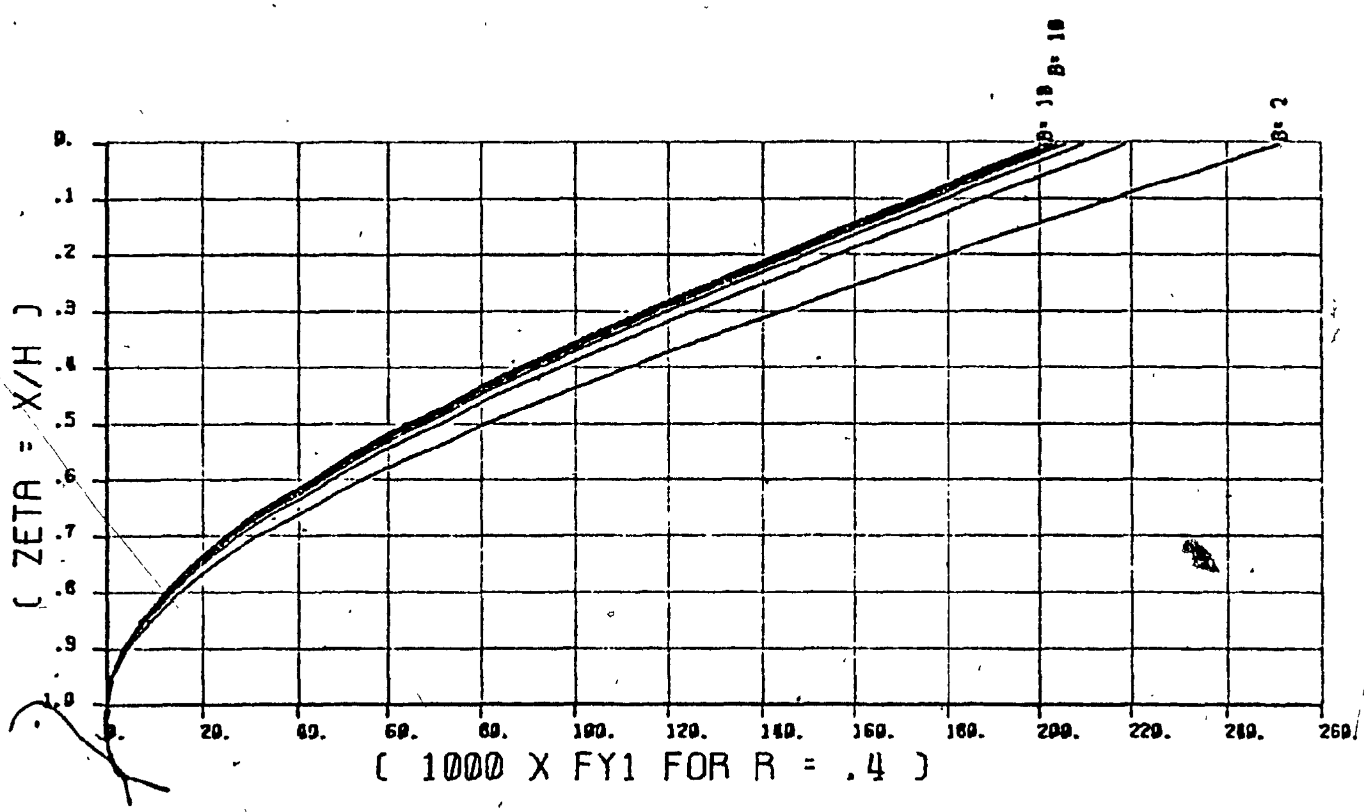


(b) DEFLECTION FACTOR - LOAD CASE I

Fig. (6.13)

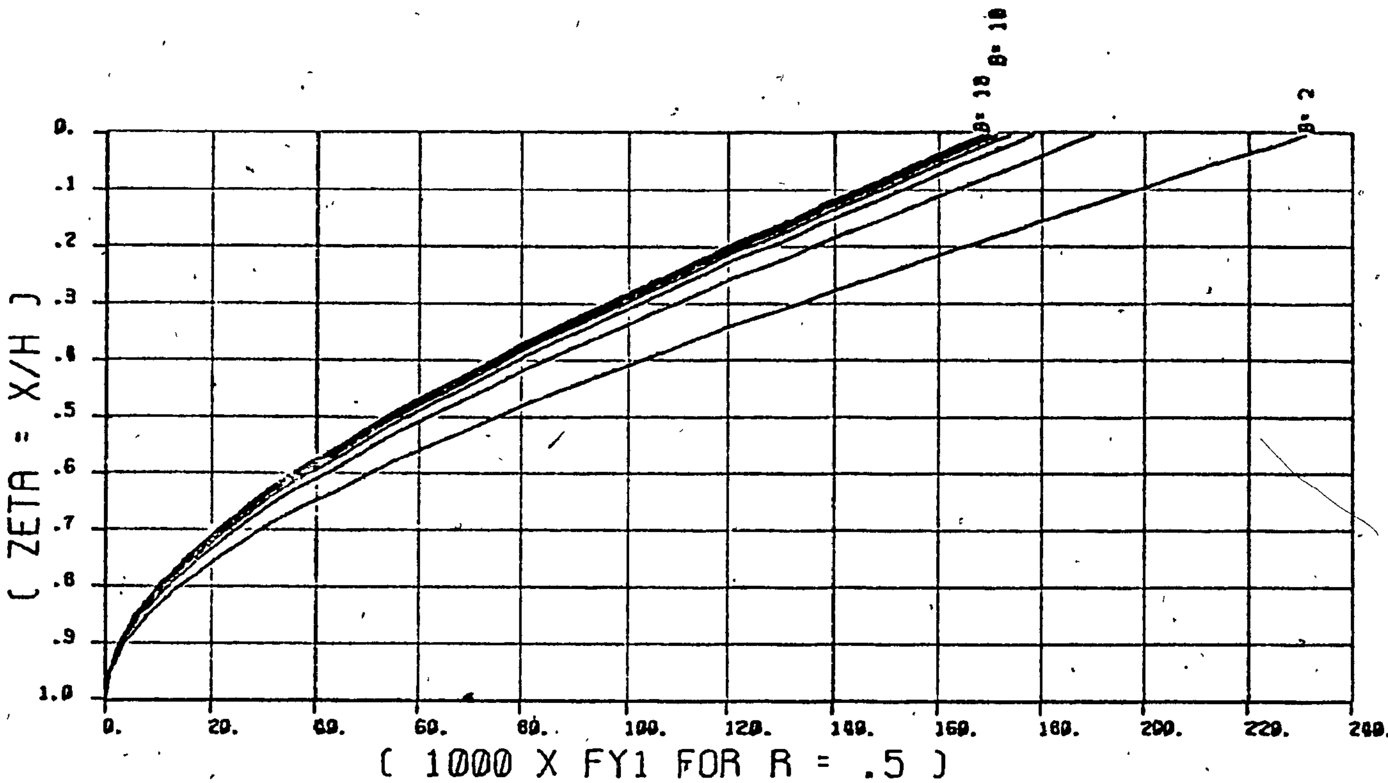


(a) DEFLECTION FACTOR - LOAD CASE 1

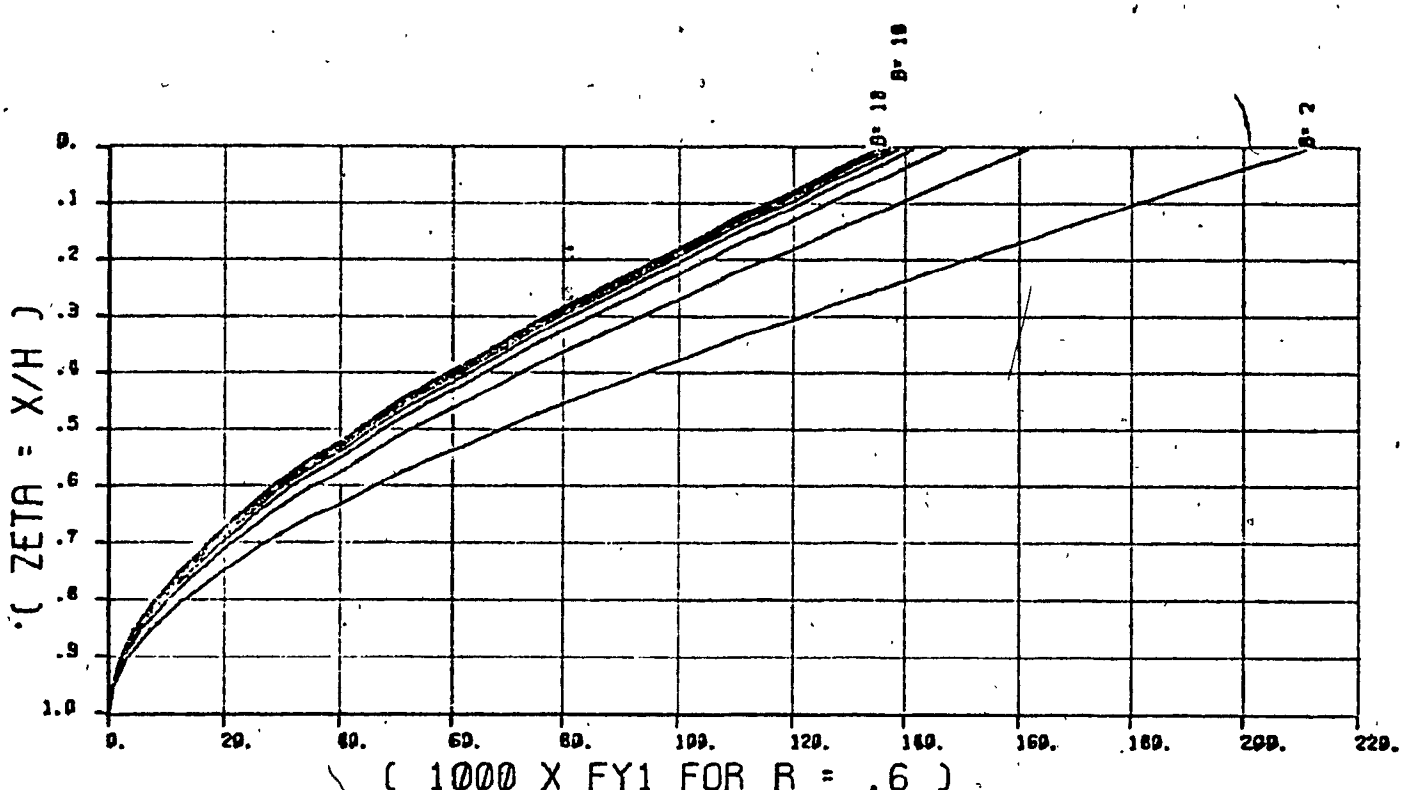


(b) DEFLECTION FACTOR - LOAD CASE I

Fig. (6.14)

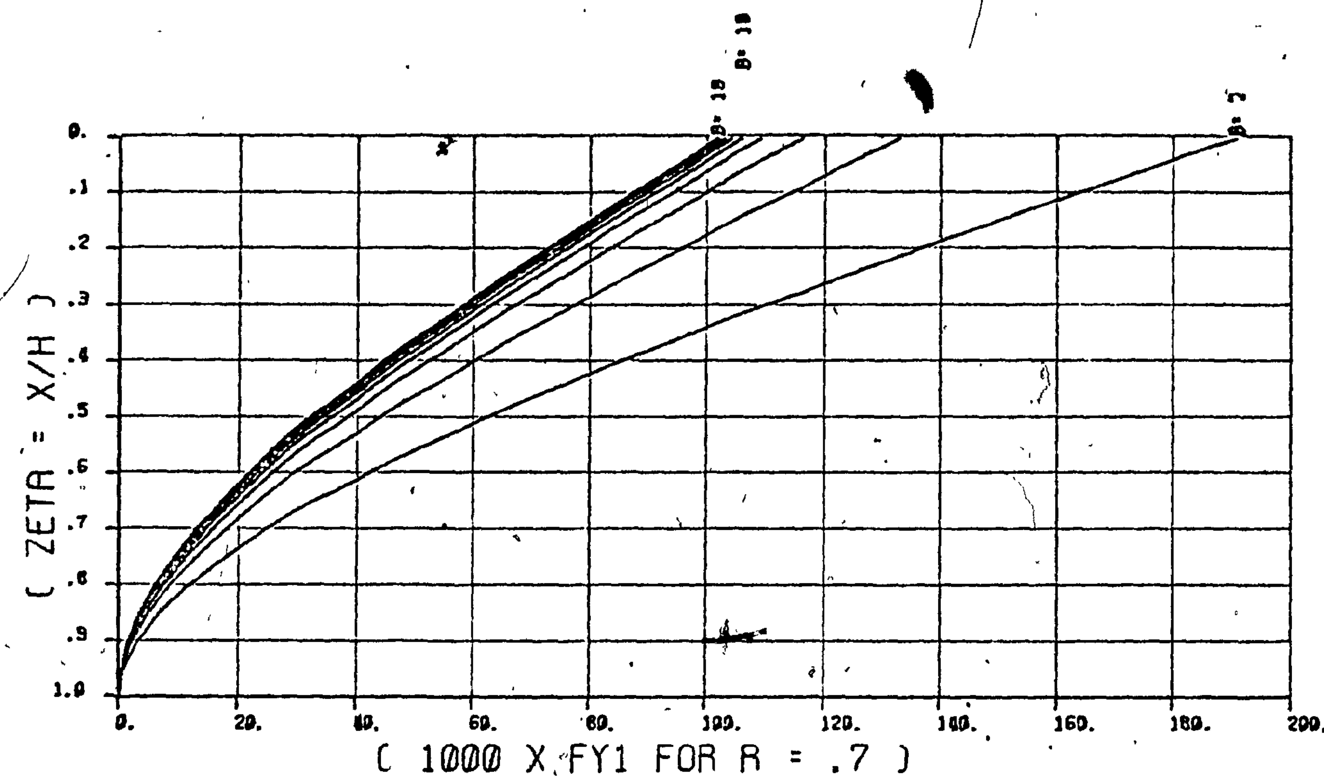


(a). DEFLECTION FACTOR - LOAD CASE I

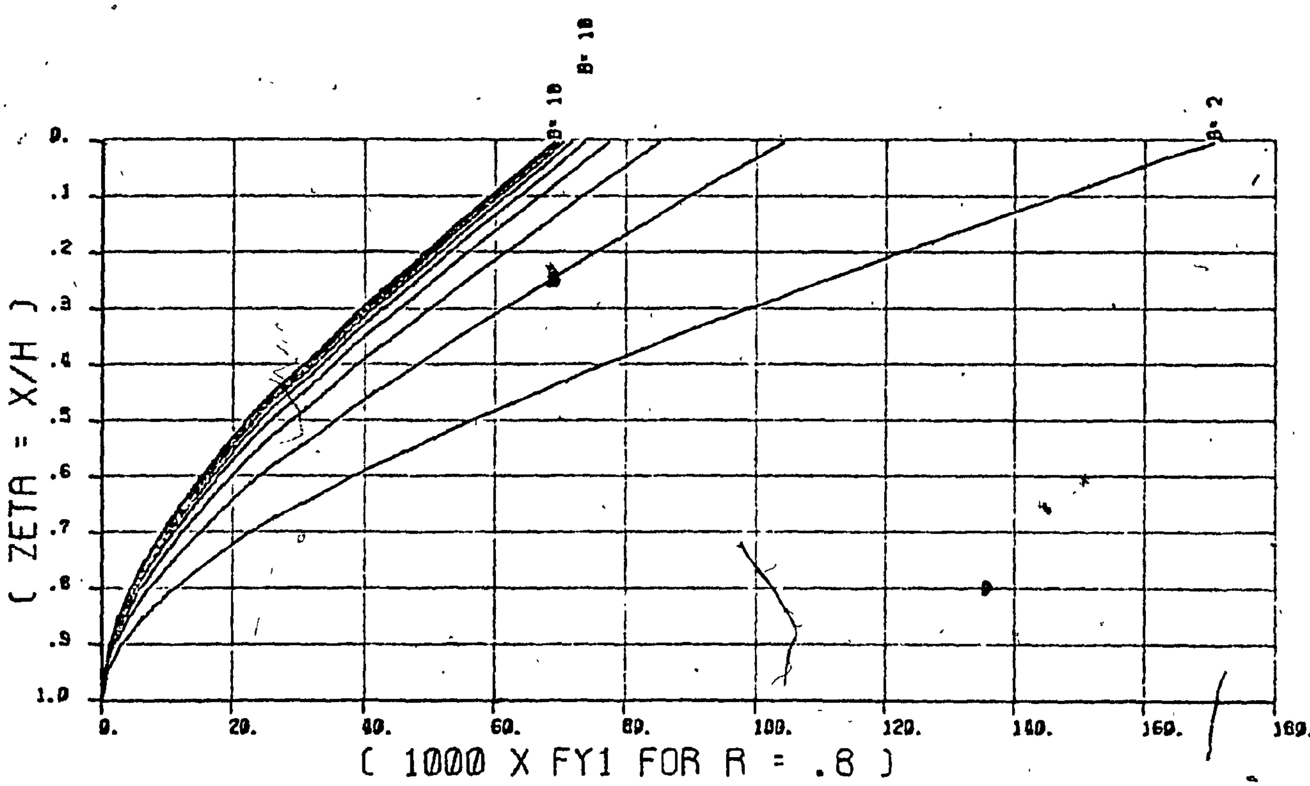


(b) DEFLECTION FACTOR - LOAD CASE I

Fig. (6.15)

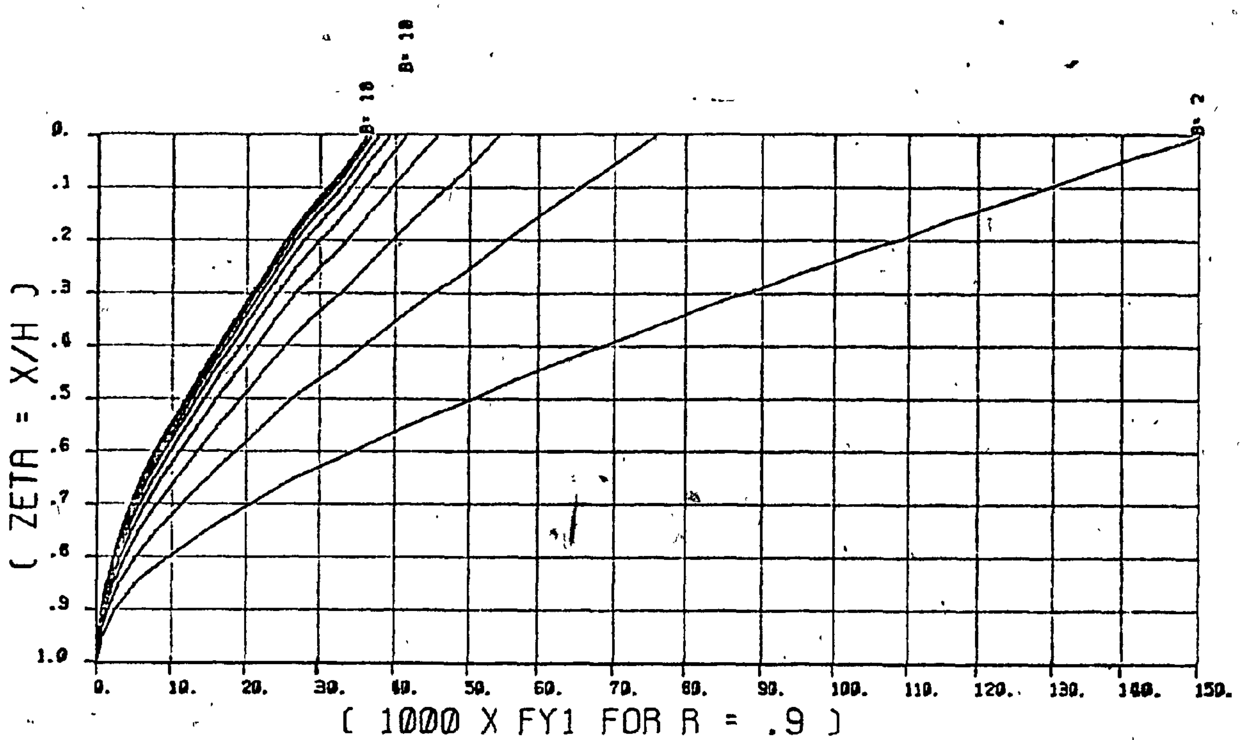


(a) DEFLECTION FACTOR - LOAD CASE I

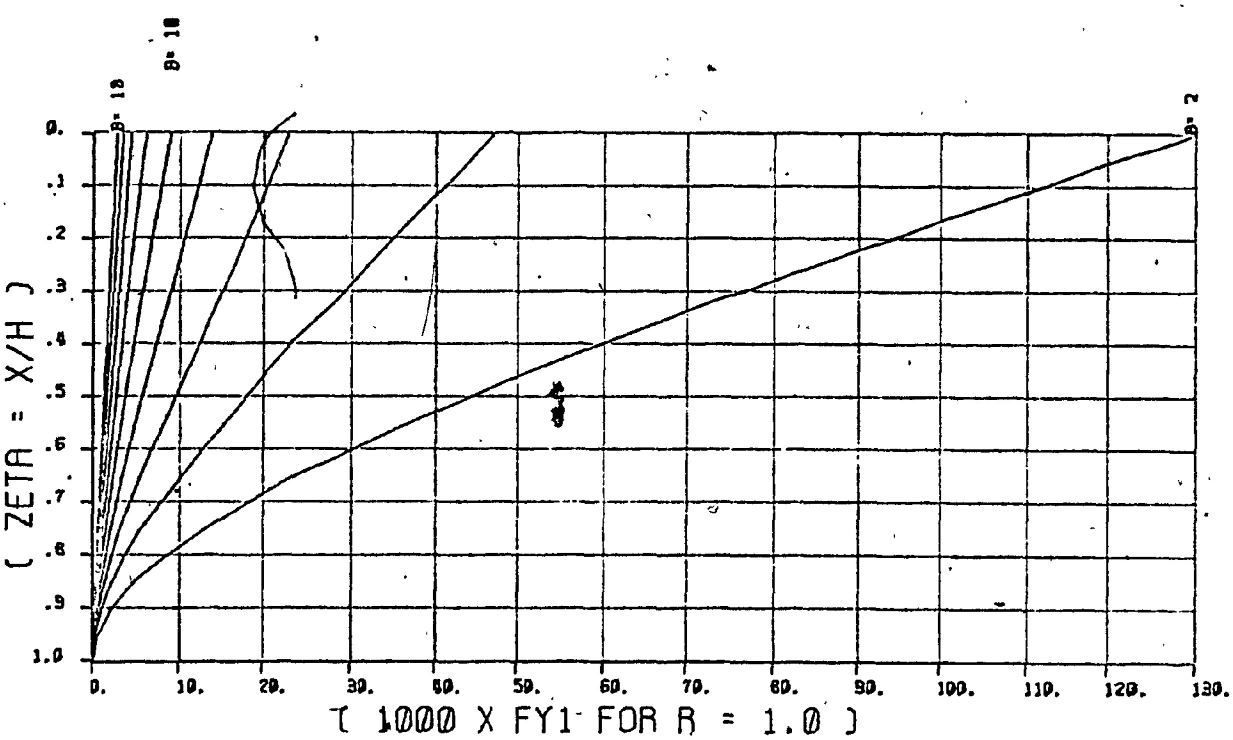


(b) DEFLECTION FACTOR - LOAD CASE I

Fig. (6.16)

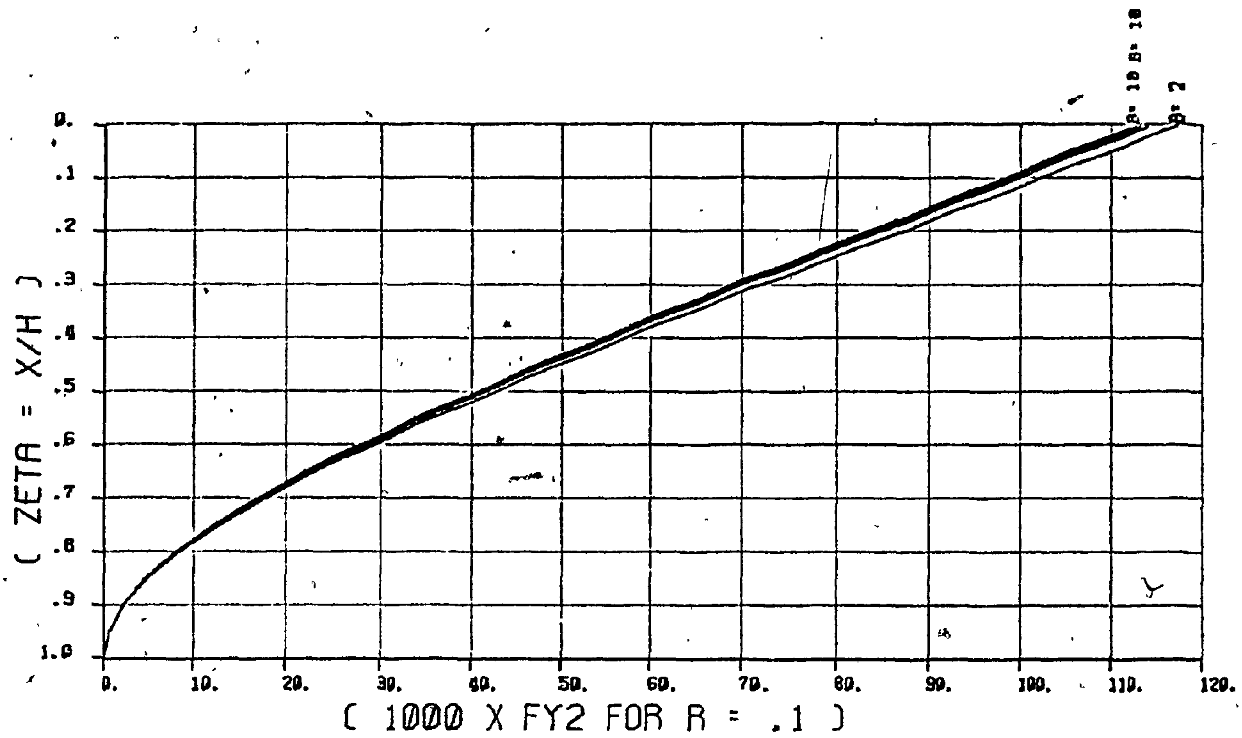


(a) DEFLECTION FACTOR - LOAD CASE I

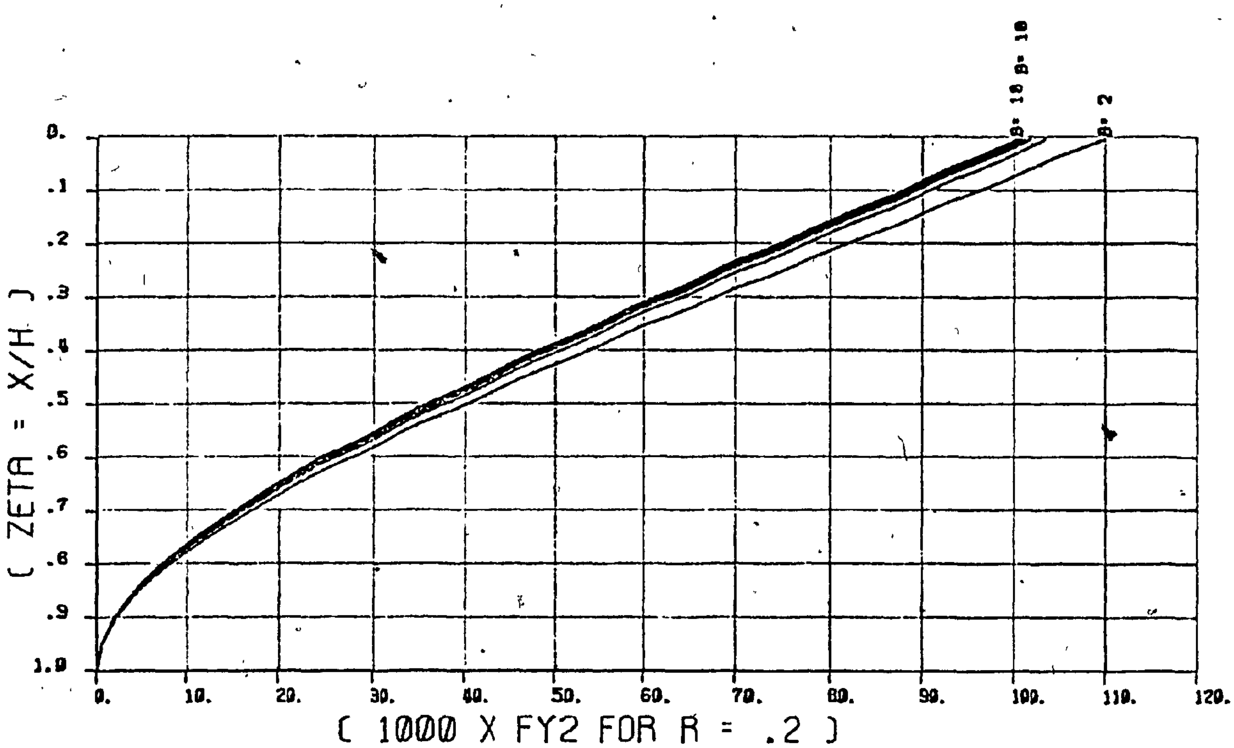


(b) DEFLECTION FACTOR - LOAD CASE I

Fig. (6.17)

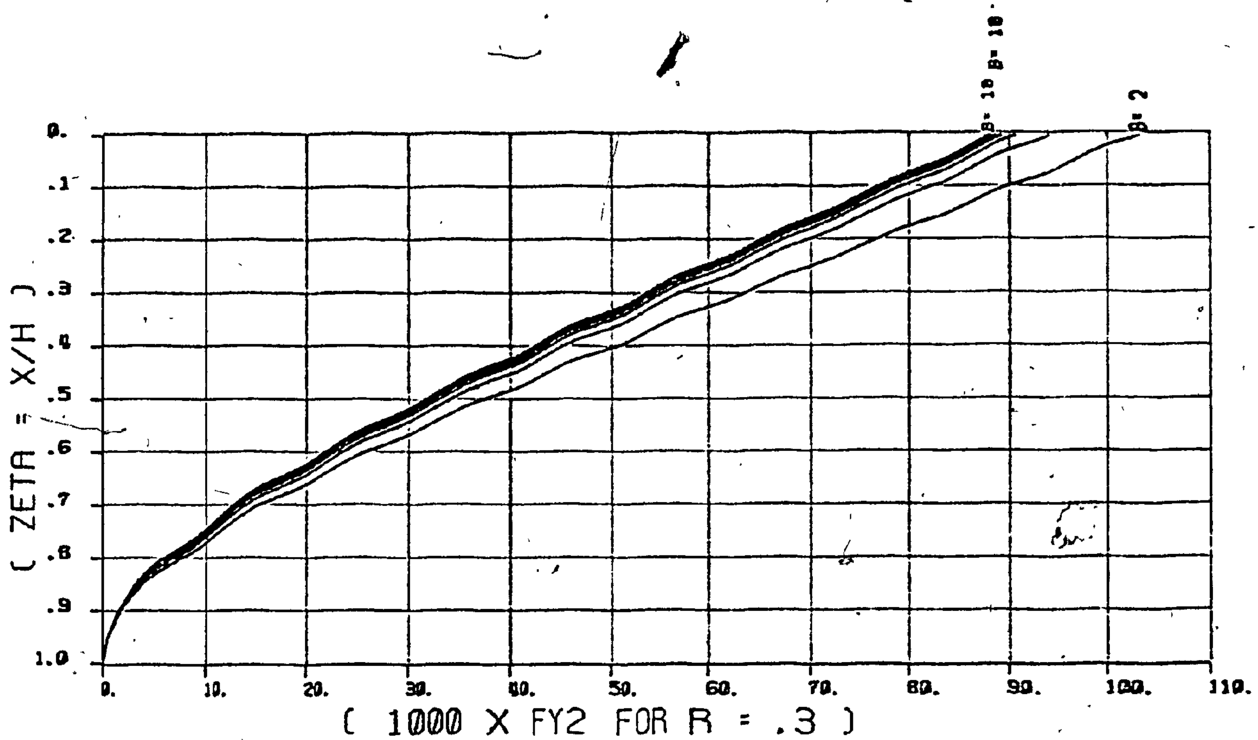


(a) DEFLECTION FACTOR - LOAD CASE II

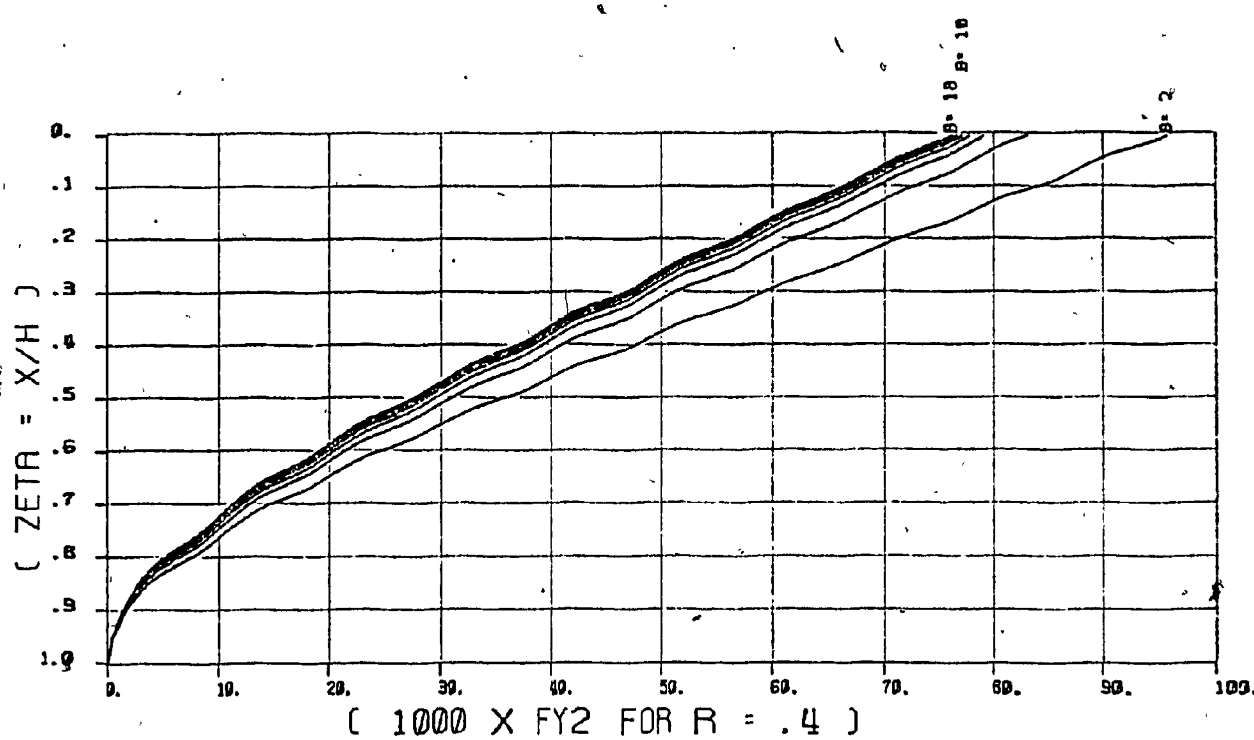


(b) DEFLECTION FACTOR - LOAD CASE II

Fig. (6.18)

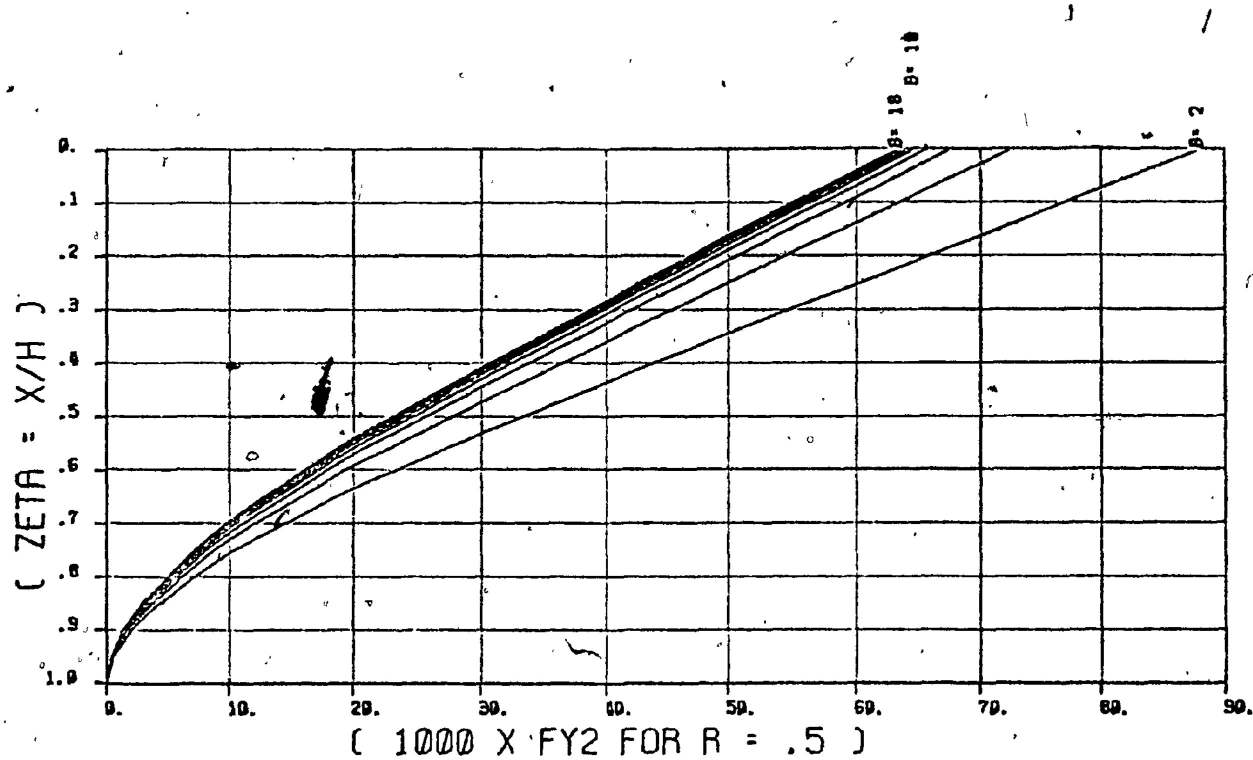


(a) DEFLECTION FACTOR - LOAD CASE II

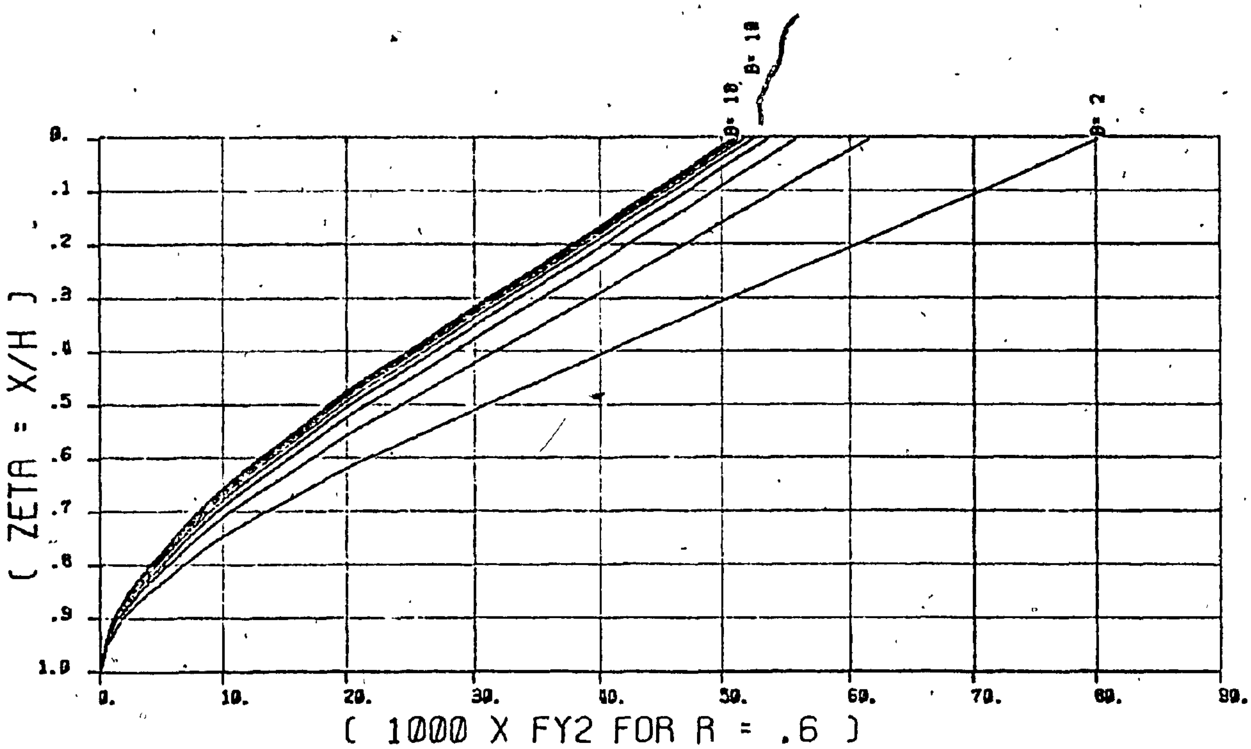


(b) DEFLECTION FACTOR - LOAD CASE II

Fig. (6.19)

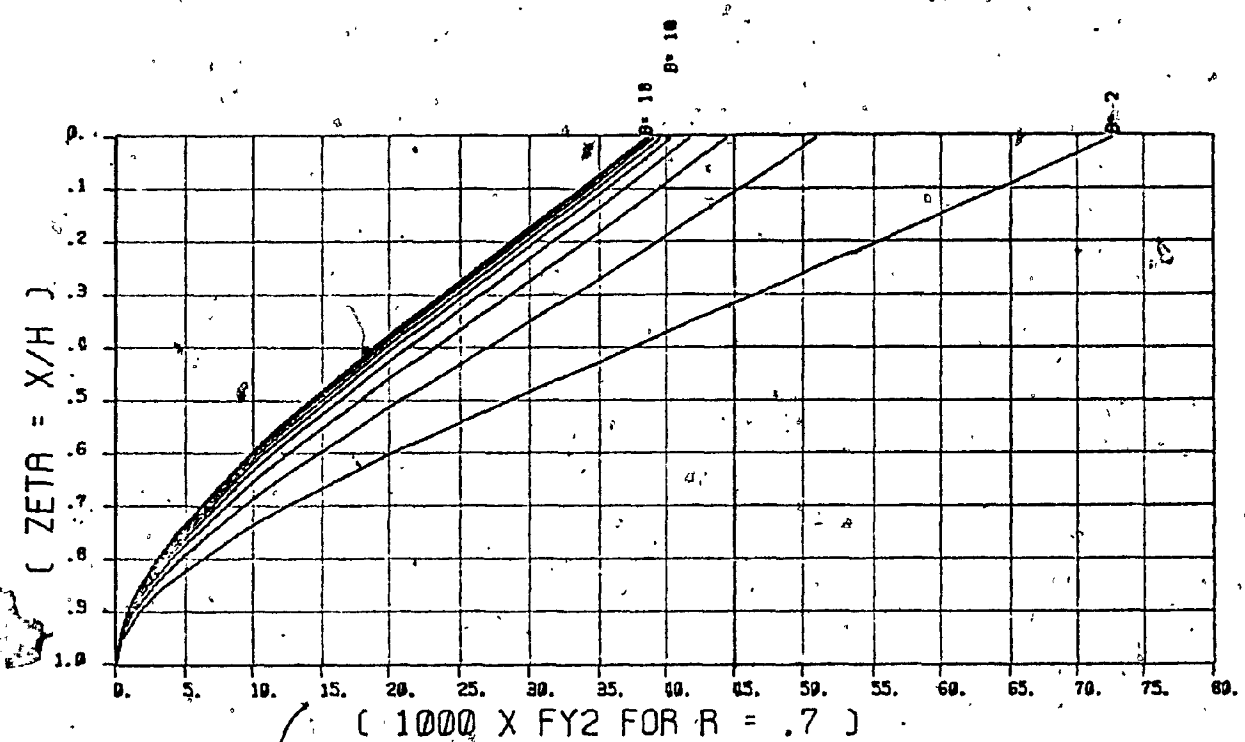


(a) DEFLECTION FACTOR - LOAD CASE II

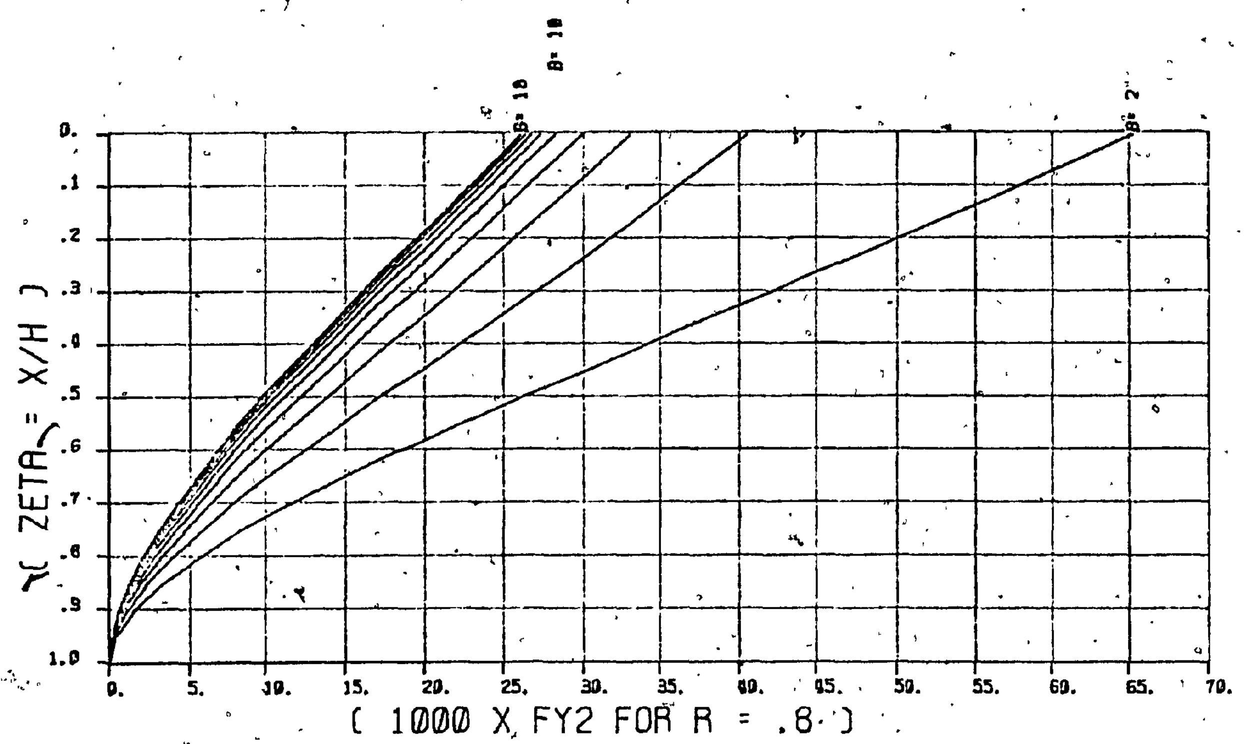


(b) DEFLECTION FACTOR - LOAD CASE II

Fig. (6.20)

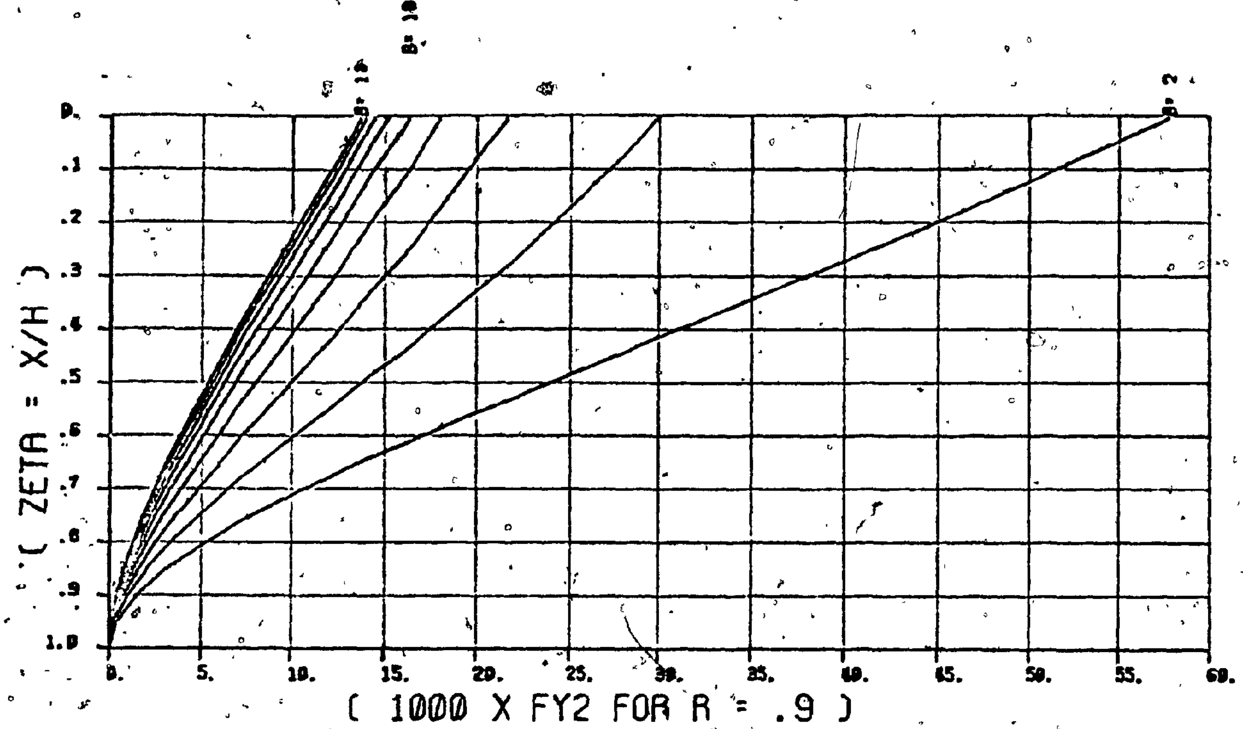


(a) DEFLECTION FACTOR - LOAD CASE II

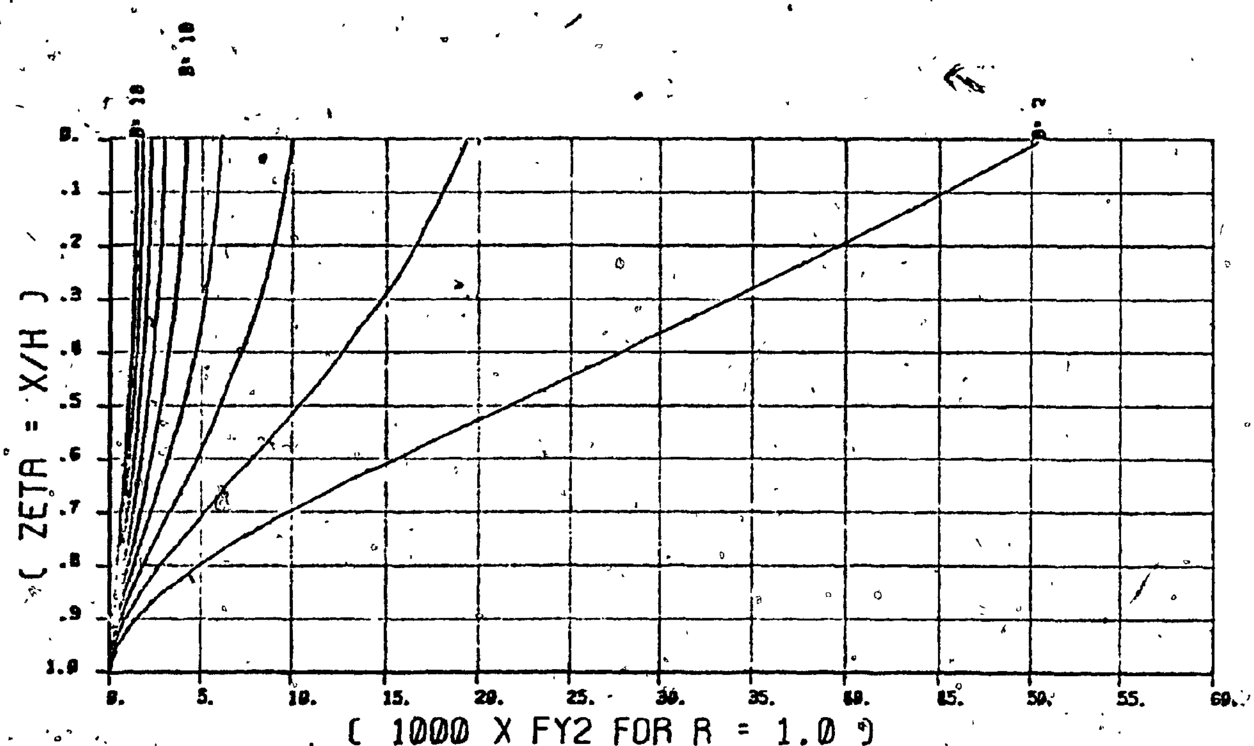


(b) DEFLECTION FACTOR - LOAD CASE II

Fig. (6.21)

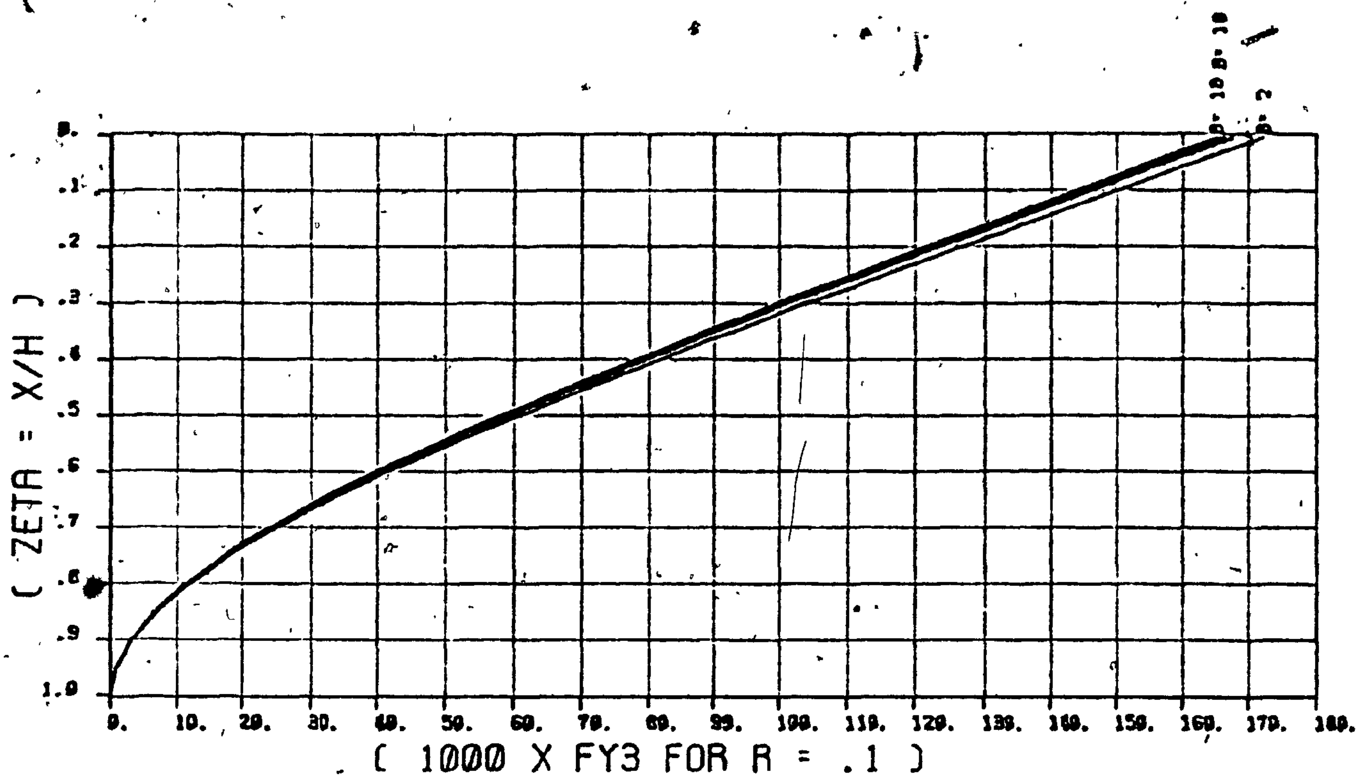


(a) DEFLECTION FACTOR - LOAD CASE II

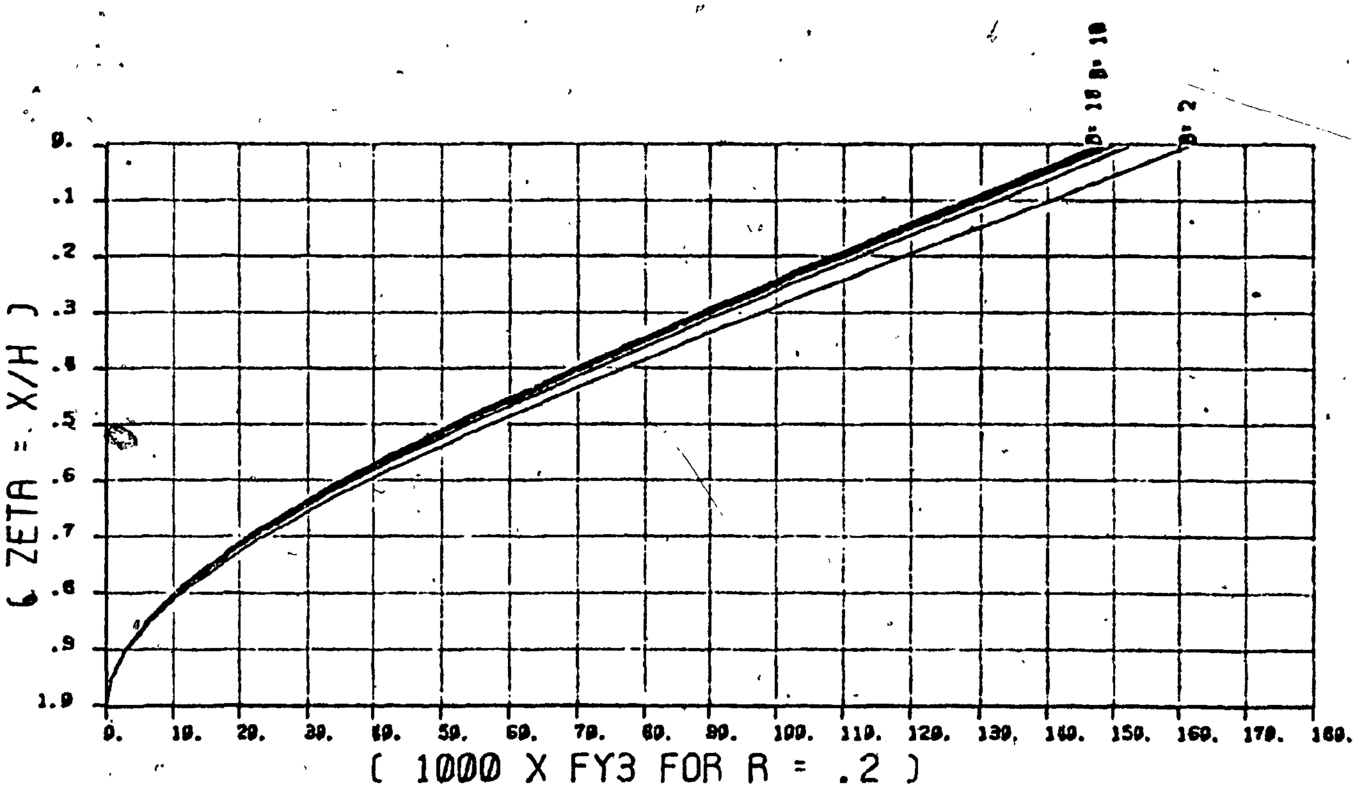


(b) DEFLECTION FACTOR - LOAD CASE II

Fig. (6.22)

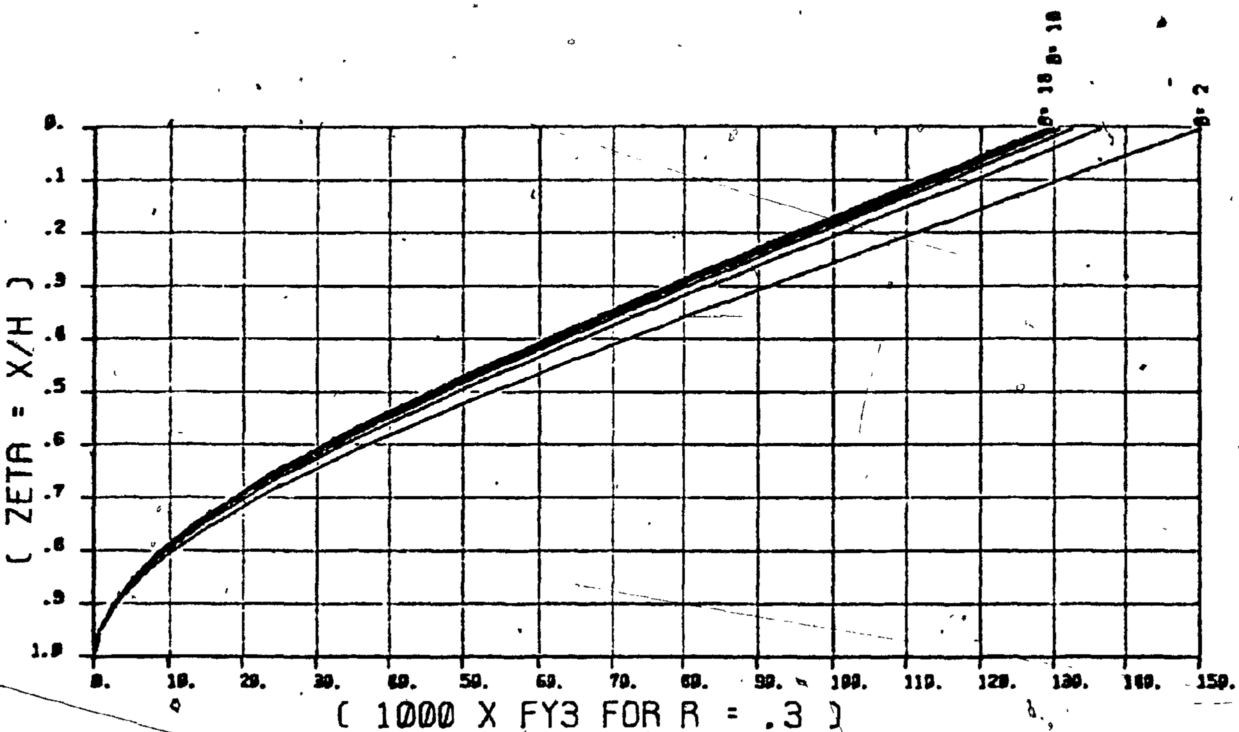


(a) DEFLECTION FACTOR - LOAD CASE III

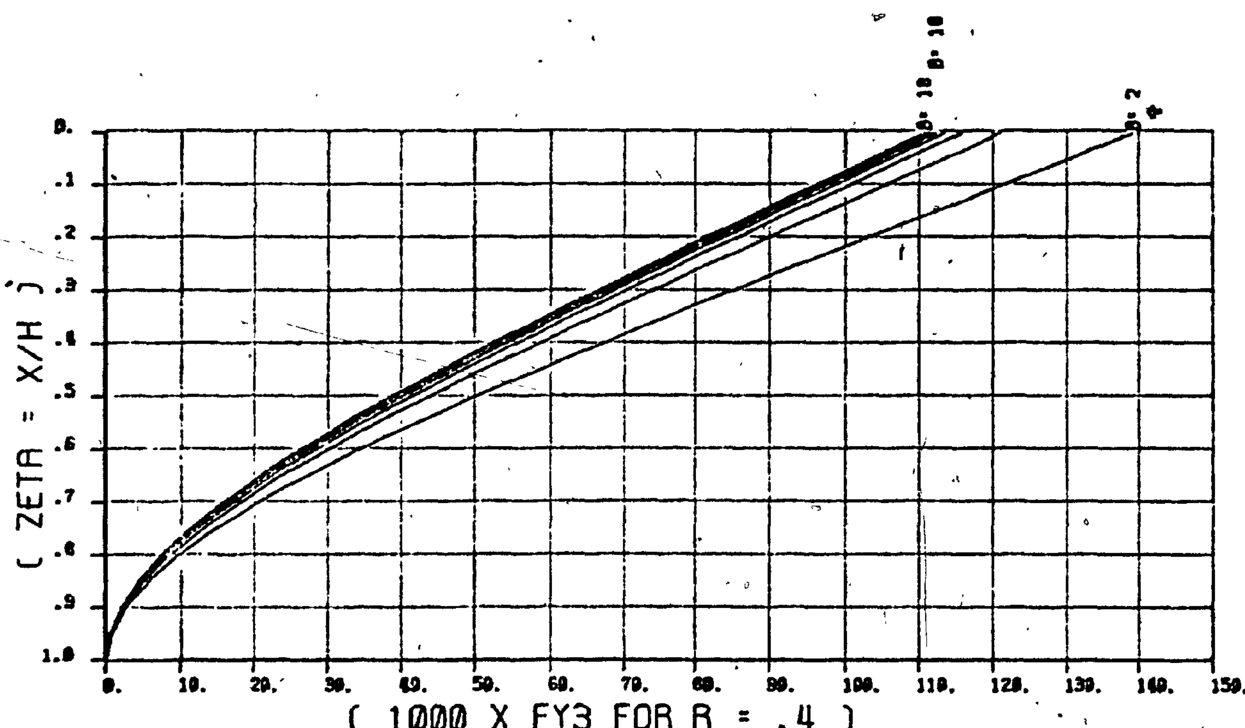


(b) DEFLECTION FACTOR - LOAD CASE III

Fig. (6.23)

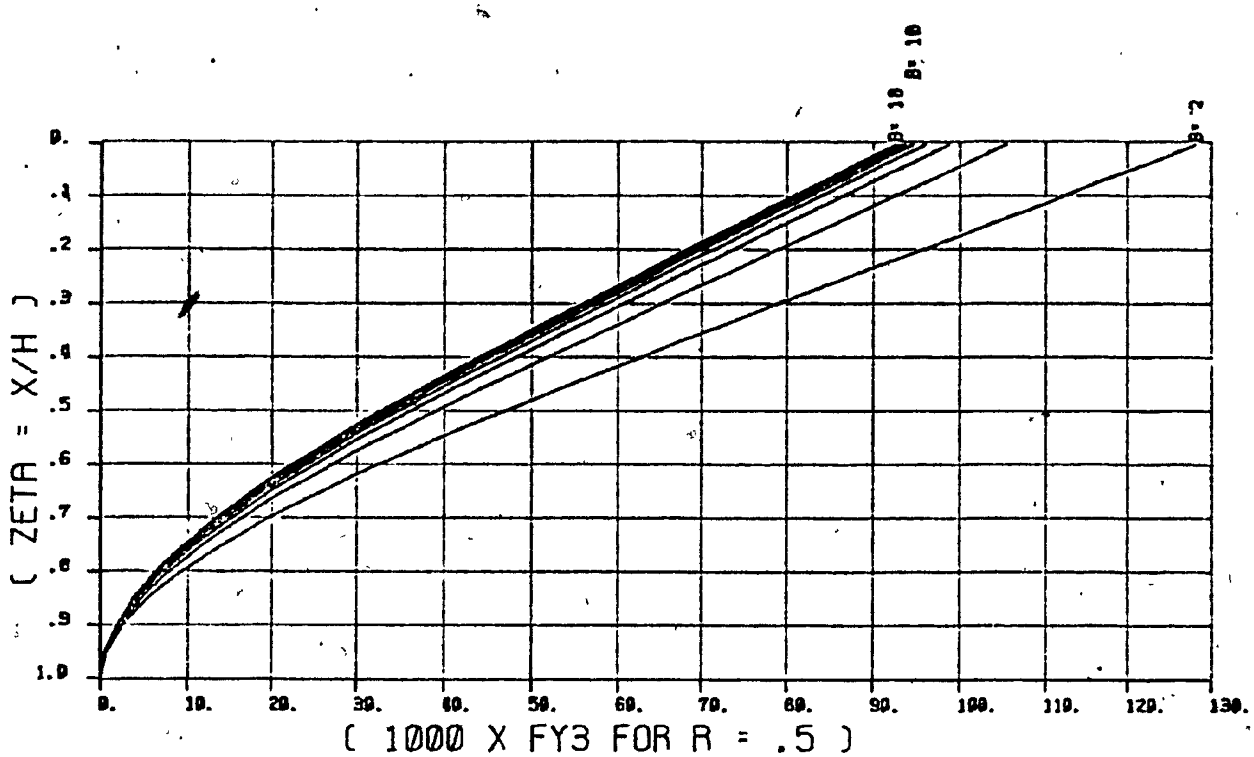


(a) DEFLECTION FACTOR - LOAD CASE III

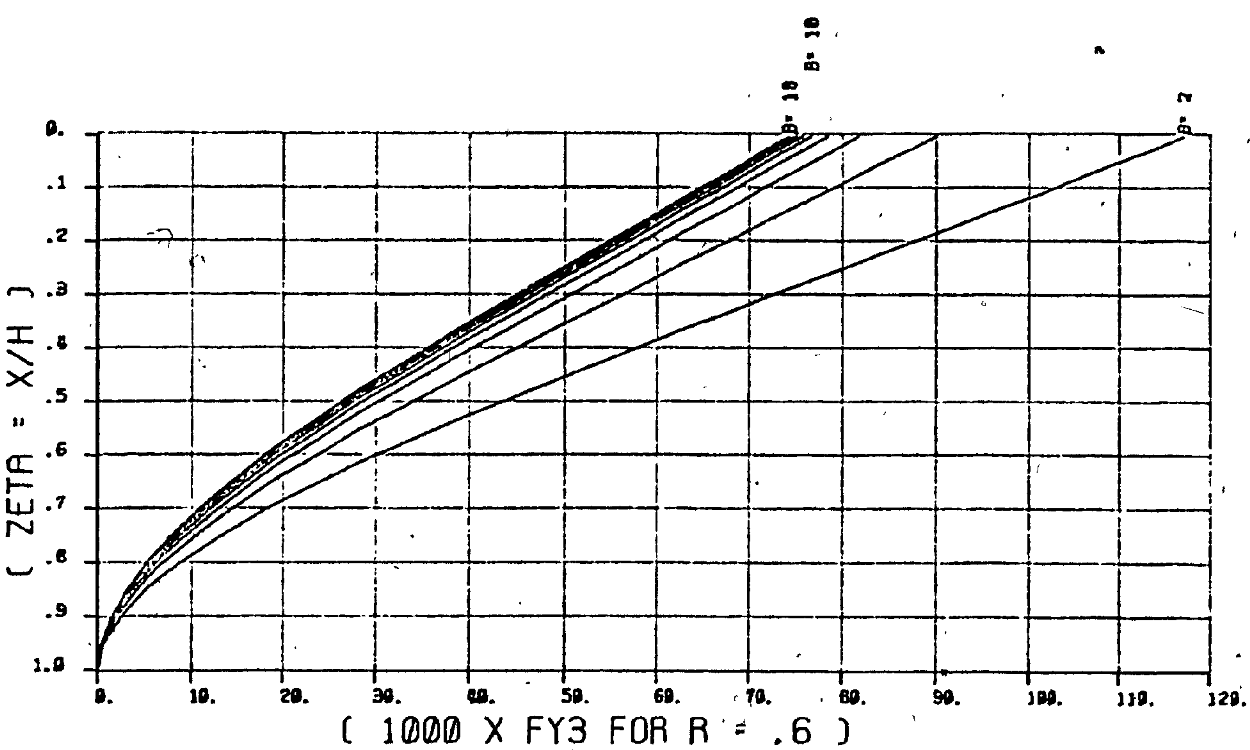


(b) DEFLECTION FACTOR - LOAD CASE III

Fig. (6.24)

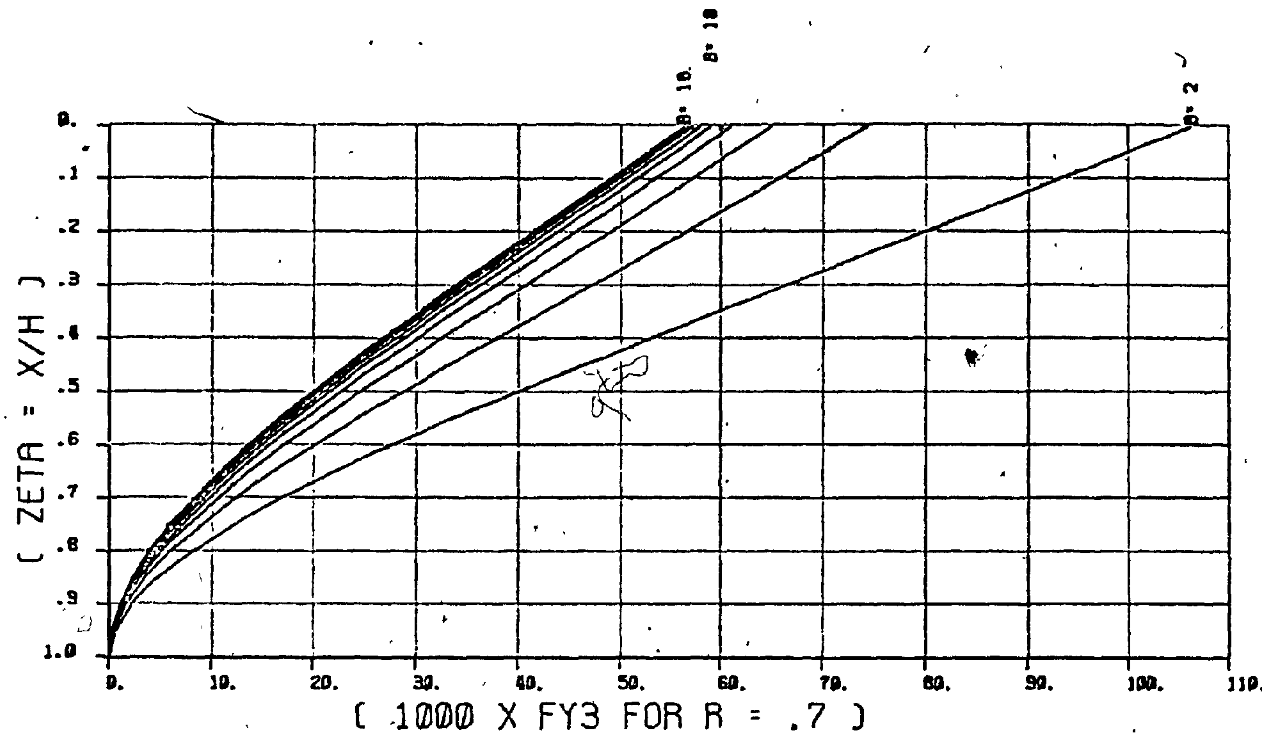


(a) DEFLECTION FACTOR - LOAD CASE III

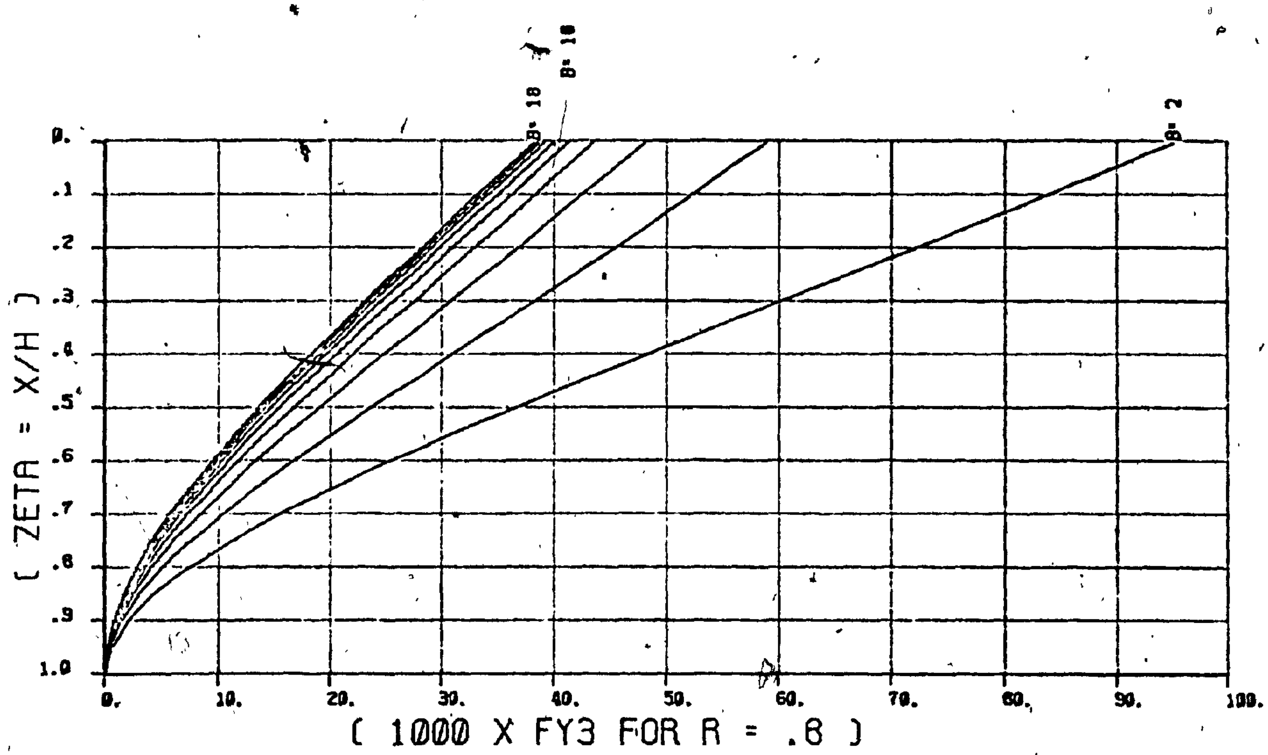


(b) DEFLECTION FACTOR - LOAD CASE III

Fig. (6.25)

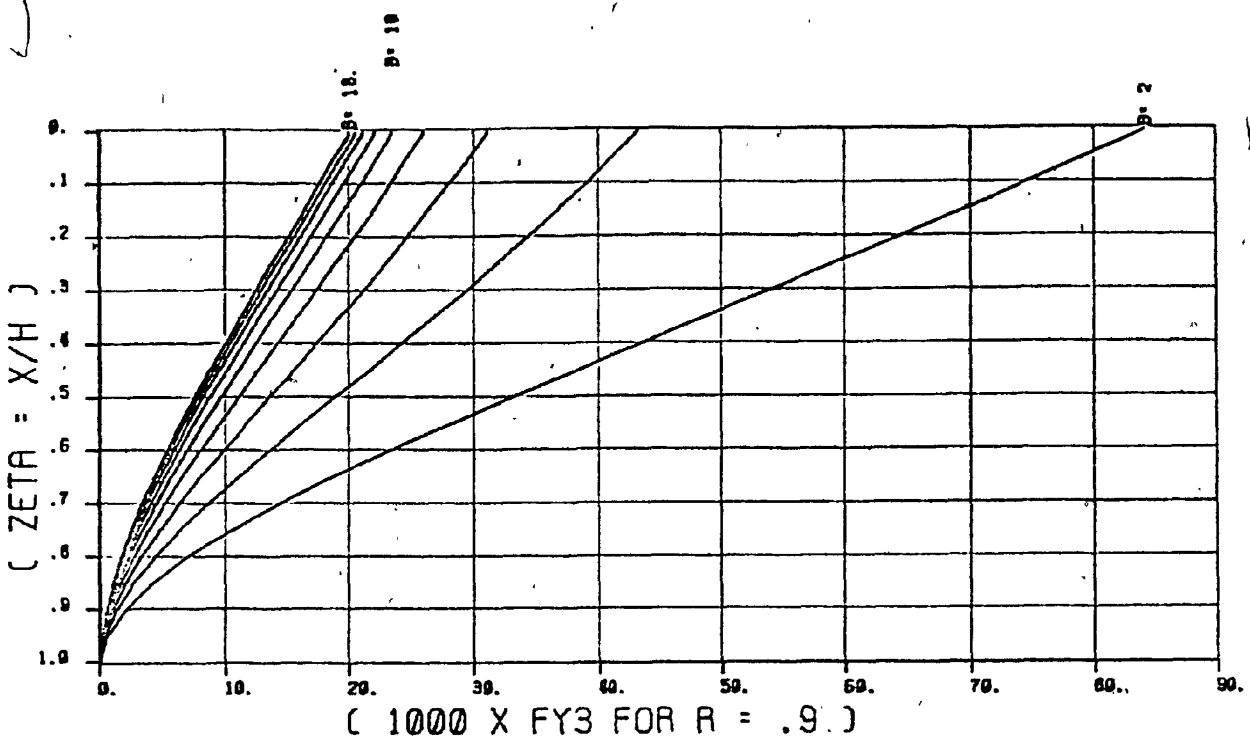


(a) DEFLECTION FACTOR - LOAD CASE III

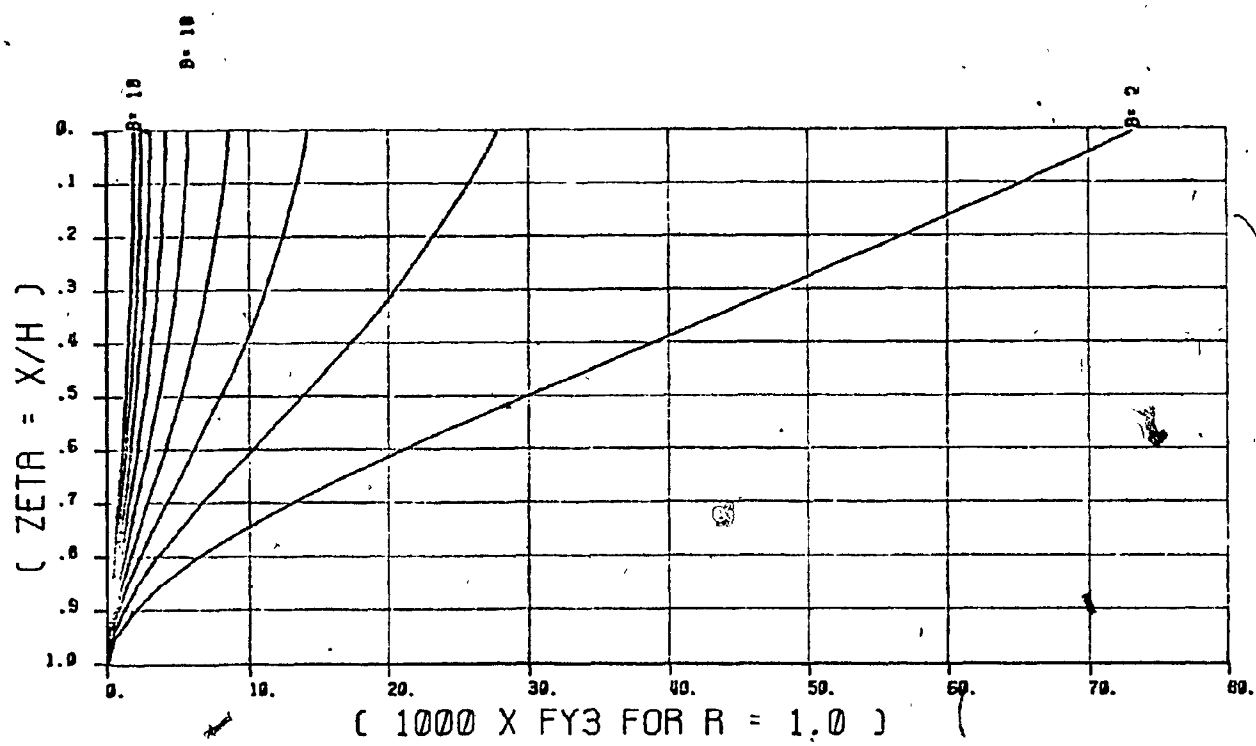


(b) DEFLECTION FACTOR - LOAD CASE III

Fig. (6.26)



(a) DEFLECTION FACTOR - LOAD CASE III



(b) DEFLECTION FACTOR - LOAD CASE III

Fig. (6.27)

7.0 DESIGN EXAMPLE

In order to illustrate the use of the design curves an example of a typical system of a coupled shear wall is shown in Fig. (7.1).

7.1 EXAMPLE

In this example it is assumed that the walls are connected by 0.4 m deep connecting beams.

Where:

$H = 24.75$ m

$h = 2.75$ m

$L = 5.75$ m

$d_1 = 4.5$ m

$d_2 = 4$ m

$c = 1.5$ m

$b = 0.2$ m

$a = 0.4$ m

$d_0 = 0.25$ m

$n = 9$

Solutions will be obtained for the following

a) Q maximum

b) q for $\zeta = 0.5, 0.6, 0.7, 0.8$ & 0.9

c) y_{max}, and relative deflection between 8th & 9th floors.

Loading will be assumed as the problem progresses.

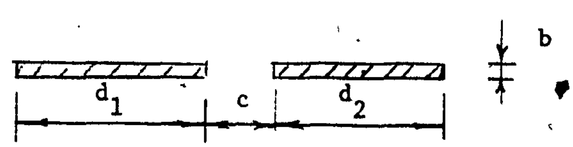


Fig. (7.1)

Solution

$I_* = \frac{0.2 \times 0.4^3}{12}$ $= 0.0011 \quad m^4$

$I_1 = bd_1^{\,3}/12 = 0.2 \times 4.5^3/12$ $= 1.52 \quad m^4$

$I_2 = bd_2^{\,3}/12 = 0.2 \times 4^4/12$ $= 1.07 \quad m^4$

$A_1 = 0.2 \times 4.5$ $= 0.9 \quad m^2$

$A_2 = 0.2 \times 4$ $= 0.8 \quad m^2$

$I_0 = I_1 + I_2 = 1.52 + 1.07$ $= 2.59 \quad m^4$

$$\beta^2 = H^2 \left(\frac{L^2}{I_0} + \frac{1}{A_1} + \frac{1}{A_2}\right) \frac{12I^*}{hc^3}$$

$$= 24.75^2 \left(\frac{5.75^2}{2.59} + \frac{1}{0.9} + \frac{1}{0.8}\right) \frac{12 \times 0.0011}{2.75 \times 1.5^3} = 13.18$$

$\beta = \sqrt{\beta^2}$ $= 3.63$

$$\gamma = \frac{12LI^*}{hc^3 I_0} = \frac{12 \times 5.75 \times 0.0011}{2.75 \times 1.5^3 \times 2.59} = 0.00316$$

$$R = \frac{\gamma L H^2}{\beta^2} = \frac{0.00316 \times 5.75 \times 24.75^2}{13.18} = 0.84$$

In accordance with Rosman's graphs for equivalent moment of inertia

$K_1 = n^2 \left(\frac{d}{H}\right)^4 = 9^2 \left(\frac{10}{24.75}\right)^4 = 2.16$

$K_2 = \frac{C}{d} = \frac{1.5}{10} = 0.15$

$K_3 = \frac{d_0}{d} = \frac{0.25}{10} = 0.025$

From these values and the use of graph in section 6.0 of this paper

$$\eta = 1.5$$

$$I = \frac{bd^3}{12} \times \frac{1}{\eta} = \frac{0.2 \times 10^3}{12} \times \frac{1}{1.5} = 11.11 \ m^4$$

From Fig. (6.7), Fig. (6.8) and Fig. (6.9), (a) for $\zeta = 1.0$ and $\beta = 3.63$

$1000\ (Q_1)$	= 56	(Q_1)	= 0.056
$1000\ (Q_2)$	= 23	(Q_2)	= 0.023
$1000\ (Q_3)$	= 34	(Q_3)	= 0.034

From Fig. (6.10), Fig. (6.11) and Fig. (6.12), (a) for $\zeta = 0.5, 0.6,$ $0.7, 0.8$ and 0.9 and $\beta = 3.63$

ζ	$1000\ F(Q_1)$	$1000\ F(Q_2)$	$1000\ F(Q_3)$	$F(Q_1)$	$F(Q_2)$	$F(Q_3)$
0.5	64	30	41	0.064	0.03	0.041
0.6	61	32	43	0.061	0.032	0.043
0.7	53	29	40	0.053	0.029	0.040
0.8	41	26	33	0.041	0.026	0.033
0.9	23	17	20	0.023	0.017	0.020

From Fig. (6.16) (b), Fig. (6.17) (a), Fig. (6.21) (a), Fig. (6.22) (b), Fig. (6.26) (a) and Fig. (6.27) (b) for $\zeta = 0$ and $\zeta = \frac{2.75}{24.75} = 0.11.$

$\beta = 3.63, \quad R = 0.84$

ζ	$1000\ F(Y_1)$	$1000\ F(Y_2)$	$1000\ F(Y_3)$	(FY_1)	(FY_2)	(FY_3)
0	107.33	42	60.67	0.1073	0.042	0.0607
0.11	95.33	37.67	53	0.0953	0.0377	0.053

Assume $P = 450$ kN

$w = 36$ kN/m

$W = 450$ kN

$KQ_1 = \gamma H^3 P = 0.00315 \times 24.75^3 \times 450 = 21558.8$ kN

$KQ_2 = \gamma H^4 w = 0.00316 \times 24.75^4 \times 36 = 42686.5$ kN

$KQ_3 = \gamma H^3 W = 0.00316 \times 24.75^3 \times 450 = 21558.8$ kN

$F(FQ_1) = \gamma H^2 P = 0.00316 \times 24.75^2 \times 450 = 871.0$ kN/m

$K(FQ_2) = \gamma H^3 w = 0.00316 \times 24.75^3 \times 36 = 1724.7$ kN/m

$K(FQ_3) = \gamma H^2 W = 0.00316 \times 24.75^2 \times 450 = 871.0$ kN/m

$$KY_1 = \frac{PH^3}{EI} = \frac{450 \times 24.75^3}{21 \times 10^6 \times 11.11} = 0.0202 \text{ m}$$

$$KY_2 = \frac{wH^4}{EI} = \frac{36 \times 24.75^4}{21 \times 10^6 \times 11.11} = 0.0579 \text{ m}$$

$$KY_3 = \frac{WH^3}{EI} = \frac{450 \times 24.75^3}{21 \times 10^6 \times 11.11} = 0.0202 \text{ m}$$

E_C = Modulus of elasticity of concrete $= 21 \times 10^6$ kN/m^2

$Q_1 = (Q_1) \cdot KQ_1 = 0.056 \times 21558.8 = 1207.3$ kN

$Q_2 = (Q_2) \cdot KQ_2 = 0.023 \times 42686.5 = 981.8$ kN

$Q_3 = (Q_3) \cdot KQ_3 = 0.034 \times 21558.8 = 733.0$ kN

ζ	$F(Q_1) \cdot K(FQ_1) = q_1$ kN/m	$F(Q_2) \cdot K(FQ_2) = q_2$ kN/m	$F(Q_3) \cdot K(FQ_3) = q_3$ kN/m
0.5	55.74	51.74	35.71
0.6	53.13	55.19	37.45
0.7	46.16	50.02	34.84
0.8	35.71	44.84	28.74
0.9	20.03	29.32	17.42

ζ	$F(Y_1) \cdot KY_1 = Y_1$ m	$F(Y_2) \cdot KY_2 = Y_2$ m	$F(Y_3) \cdot KY_3 = Y_3$ m	
0	0.003133	0.002432	0.00177	Maximum
0.11	0.002783	0.002183	0.001547	at 8th Floor
Relative Deflection	0.000350	0.000249	0.000223	

(i) when $\zeta = 0$, $y = y_{max}$.

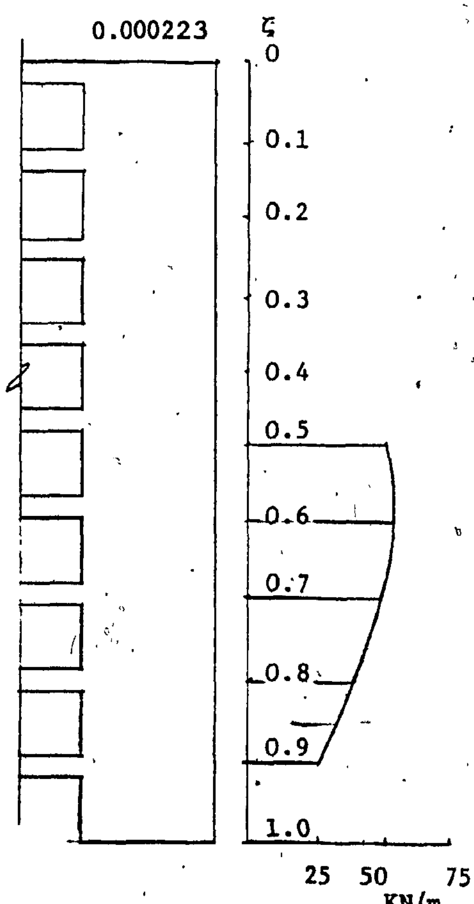


Fig. (7.2)

Relative deflection = $Y_0 - Y_{0.11}$

Fig. (7.2) shows where the maximum shear occurs by taking the average value between two consecutive floors. The value of shear on the connecting beam can be obtained by using the same methods. Similarly the shearing can be obtained for beams at any level.

The bending moment in the wall in each case is given by the following:

$$M_x = M - QL$$

M for three different loading cases is given in section 4.0.

$$M_1 = PH\zeta$$

$$M_2 = \frac{W\ \zeta^2 H^2}{2}$$

$$M_3 = WH\ (\zeta^2 - \frac{\zeta^3}{3})$$

The maximum moment occurs when $\zeta = 1.0$

$M_1 = 450 \times 24.75 \times 1.0 \quad = 11137.5$ kN.m

$M_2 = 36 \times 1.0^2 \times 24.75^2/2 = 11026$ kN.m

$M_3 = 450 \times 24.75\ (1 - \frac{1}{3}) \quad = 7425$ kN.m

By substituting for M, Q and L we have

$M_x = 11137.5 - 1206.3 \times 5.75 = 4195.5$ kN.m Load Case I

$M_x = 11026 - 981.8 \times 5.75 \quad = 5280.6$ kN.m Load Case II

$M_x = 7425 - 733 \times 5.75 \quad = 3210.2$ kN.m Load Case III

The moment in each wall is given by

$$M_{x1} = M_x \frac{I_1}{I_0}$$

$$M_{x2} = M_x \frac{I_2}{I_0}$$

$$M_{x1} = 4195.5 \frac{1.52}{2.59} = 3157.7 \text{ kN.m}$$

Load Case I

$$M_{x2} = 4195.5 \frac{1.07}{2.59} = 1733.3 \text{ kN.m}$$

$$M_{x1} = 5380.6 \frac{1.52}{2.59} = 3157.7 \text{ kN.m}$$

Load Case II

$$M_{x2} = 5380.6 \frac{1.07}{2.59} = 2223 \text{ kN.m}$$

$$M_{x1} = 3210.2 \frac{1.52}{2.59} = 1884 \text{ kN.m}$$

Load Case III

$$M_{x1} = 3210.2 \frac{1.07}{2.59} = 1326.22 \text{ kN.m}$$

The moment could be found at any height in the wall by taking the appropriate values for ζ and Q.

The above is a typical example of a coupled shear wall. It should be borne in mind that the solution will also apply to other shapes of wall sections. Fig. (7.3).

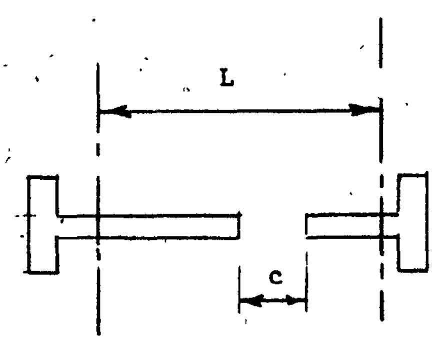


Fig. (7.3)

8.0 DISCUSSION OF RESULTS

8.1 STIFFNESS PARAMETER β

The stiffness parameter β is the wall stiffness constant and depends solely on wall dimensions.

$$\beta^2 = H^2\left(\frac{L}{I_o} + \frac{1}{A_1} + \frac{1}{A_2}\right) \frac{12I^*}{hc^3}$$

or

$$\beta^2 = H^2\left(\frac{12L^2}{b\,(d_1^3 + d_2^3)} + \frac{d_2 + d_1}{bd_1\,d_2}\right) \frac{12I^*}{hc^3}$$

By keeping b, d_1, d_2, h, c and I* constant, it is obvious the value of β increases by increasing the height of the wall. This indicates that β is large when the shear wall is less stiff. Therefore, the smaller the value of β the stiffer is the wall. β is assumed to vary from 1.0 to 20 which for practical purposes covers all possible wall systems. The major factor contributing to the above is H, the height of the wall. Variation of b, d_1, d_2, h, c and I* also contributes towards the wall stiffness but variation of β with respect to these variables is limited to a small value in comparison with H.

It is more common to vary β between 4 and 12 for practical design purposes.

8.2 PARAMETER γ

$$\gamma = \frac{12L\ I^*}{hc^3\ I_o}$$

The value of γ is independent of H, therefore it varies only when the internal shape of the wall is varied. This gives an indication of internal stiffness of the wall with respect to openings and the total width of the wall. As in the case of β the smaller the value of γ the stiffer is the wall. In the analysis, γ is not given a special status.

8.3 NON-DIMENSIONAL CONSTANT R

R is given by $R = \frac{\gamma L H^2}{\beta^2}$

From figure (4.4) in section 4.0 it is seen that when R tends to 0.0 the structure becomes increasingly stiff. R was computed for some typical walls. The values were greater than 0.7 and in the majority of the cases the values of R were greater than 0.8. This indicates that for practical purposes R would fall between 0.75 and 0.95. Any value smaller than 0.75 indicates a very stiff structure. The R curves demonstrate this point. Also, it is observed that when R is given values from 0.1 to 0.6 for different values of β, the variation in deflection is very small and when R is given values greater than 0.6 the graphs indicate a greater change in deflection with respect to the change in the stiffness parameter.

9.0 CONCLUSION

A simple procedure has been employed for the analysis of linear response of any type of coupled shear wall system to three types of lateral loading. In this paper the basic elastic solution is obtained by means of a computer program. The tables and curves are produced for each loading case. The force and deflection factors obtained from the graphs serve as design aids.

The curves serve to make it visually easier to see the behaviour of the wall under different loading conditions.

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