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DOUBLE PERIODIC NON-HOMOGENEOUS POISSON MODELS FOR HURRICANES DATA

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Double Periodic Non–Homogeneous Poisson Models for Hurricanes Data

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Abstract

Non–homogenous Poisson processes with periodic claim intensity rate have been proposed as claim counts in risk theory. Here a doubly periodic Poisson model with short and long–term trends is studied. Beta–type intensity functions are presented as illustrations. The likelihood function and the maximum likelihood estimates of the model parameters are derived.

Double periodic Poisson models are appropriate when the seasonality does not repeat the exact same short–term pattern every year, but has a peak intensity that varies over a longer period. This reflects periodic environments like those forming hurricanes, in alternating El Niño/La Niña years. An application of the model to the dataset of Atlantic Hurricanes Affecting the United States (1899-2000) is discussed in detail.

Keywords: Non–homogeneous Poisson process, Claim intensity function, Periodicity, Double periodic Poisson model, Maximum likelihood estimation, Hurricanes, El Niño/La Niña.

1 Introduction

Non–homogeneous Poisson (NHP) processes are considered a more realistic alternative than the classical Poisson process to model the frequency of claims in risk

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theory. The NHP time–dependent intensity function is appropriate to describe the fluctuations of risks, subject to seasonality in their claims intensity.

Beard et al. (1984) and Daykin et al. (1994) claim that the risk process is often subject to continual changes in risk propensity. This is true for both, the long–term, systematic, slow–changing trends, as well as the short–term random variations that affect the number of claims. The model to be employed must then suitably define a time–dependent function or a stochastic process $\{\lambda(t)\}_{t\geq 0}$, instead of the constant Poisson parameter λ.

Berg and Haberman (1994) use a non–homogeneous Markov birth process, of which the NHP is a special case, to predict trends such as the time to the next claim or the expected total number of claims in a year in life insurance claim occurences.

In practice, natural phenomena evolving in a periodic environment, or under seasonal conditions, affect insurance claims. For example, weather factors are known to affect automobile or fire insurance claims, while seasonal snow storms in the north and hurricanes or floods in the south affect property–casualty insurance. A periodic time–dependent intensity rate is a reasonable model for the claim frequency in such situations.

Chukova et al. (1993) shows that a random variable X with almost-lack-of-memory

$$P\{X > x + c \mid X > c\} = P\{X > x\} \;, \qquad \text{for some } c,$$

has a periodic hazard rate (intensity) function of period c, $h_X(t) = \frac{f_X(t)}{\bar{F}_X(t)}$, for $t > 0$. Obvious applications in risk theory are to model random phenomena with seasonal effects; car accidents, hurricanes. Some characterization properties of the NHP process with periodic failure rate are derived in Chukova et al. (1993) and Dimitrov et al. (1997).

A compound NHP process with periodic claim intensity rate case, called periodic risk model, is considered and the related ruin problems in these models are discussed by Dassios and Embrechts (1989) and Asmussen and Rolski (1991, 1994). These use the theory of piecewise–deterministic Markov processes, together with some standard martingale techniques and a corresponding average arrival rate risk model, respectively.

Garrido et al. (1996) exploit the corresponding properties in a risk model, where the claim intensity rates are modeled by a NHP process with (single) periodic intensity. Some properties of such processes, illustrated by a beta–shape periodic intensity function, are discussed. Morales (2004) further explores the single periodic NHP model by defining a Gaussian intensity with which he considers the problem of ruin through a simulation study.

Furthermore, Garrido and Lu (2004) consider a model with a double periodic intensity rate, where periodicity does not repeat the exact same pattern in each short–term period, rather its peak intensity varies over a longer period. This model reflects periodic environments like those forming hurricanes, in alternating El Niño/La Niña years. Parametric forms of the doubly periodic intensity function, like the double–beta and the sine–beta, are proposed. These parametric forms are fitted here to hurricanes data, emphasizing the inferential aspects.

Tropical storms and hurricanes periodically affect every coastal US state along the Atlantic and the Gulf of Mexico, from Texas to Maine, year after year. According to Cole and Pfaff (1997), much speculation exists regarding the significance of the El Niño effect. This is a phenomenon generating abnormally warm surface water temperatures off the coasts of Ecuador and Peru, affecting global climate in the short-term, including weather patterns across North America. Particular attention has been directed toward the potential effects of the El Niño phenomenon on hurricane frequency and the strength attained by tropical cyclones during El Niño years, in comparison to non–El Niño years (called La Niña). These can be seen as long–term climatological and periodical effects on North American weather.

Parisi and Lund (2000) study the annual arrival cycle and return period properties of landfalling Atlantic Basin hurricanes. A NHP process with a periodic intensity function is used to model the annual cycle of hurricane arrival times. The data used in their study contains all Atlantic Basin hurricanes that have made a landfall in the contiguous United States during the years 1935–98, inclusive. Kernel methods are used to estimate the intensity function and the standard normal kernel function is selected.

In this paper, apart from considering the seasonal effects on the hurricane arrival times, we also consider global climatological and periodical effects and try to model the occurrence times of Atlantic hurricanes using a double periodic NHP process. A double beta–type intensity function is used in this parametric model and the Atlantic hurricanes affecting the United States 1899–2000 dataset [Neumann et al. (1993) and Landreneau (2001)] is used to estimate the parameters in the model. By contrast to the method proposed by Parisi and Lund (2000), a parametric statistical inference approach is used here to estimate the intensity function. Maximum likelihood estimators of model parameters for this dataset are obtained.

A brief description of the hurricane dataset is given in Section 2. NHP models with single or double periodic intensity are introduced. The statistical inference of the model parameters is presented in Section 3. Finally, in Section 4 we discuss the fit of different models to the hurricane data and give some comments. The appendix contains some tables and remarks used in the goodness–of–fit assessment.

2 The hurricane dataset and proposed models

The data used for our study comes from Neumann et al. (1993), which reports 155 hurricanes that crossed or passed immediately adjacent to the Unites States coastline (Texas to Maine), 1899 through 1992. Landreneau (2001) contains 12 additional hurricanes for the years 1993 through 2000 and is obtained from the National Hurricane Center Web site. Henceforth we call this combined dataset "the hurricanes data". Thus, over the 102–year period 1899 though 2000, a total of 167 category 1 though 5 hurricanes crossed the Atlantic United States coastline at one or more points.

The average annual number is 1.64 over the whole period, which means an average of one to two hurricane landfalls per year. The years with a maximum number of 6 hurricanes were 1916 and 1985, while 19 out of the 102 years had no hurricanes. It can be observed that the hurricane season starts in June and ends in November over those years. Furthermore, the hurricane season peak period lasts from mid-August through October, with September having had the most major hurricanes (38.9% of all hurricanes). Figure 1 shows the annual distribution of those 167 Atlantic hurricanes, while Table 1 gives their monthly distribution. (see Appendix A.1 for a discussion on the fit of the double periodic model to this dataset).

Month	Number of occurrences	Proportion (%)
June	11	6.6
July	17	10.2
August	44	26.3
September	65	38.9
October	26	15.6
November	4	2.4
	167	100.0

Table 1: Monthly distribution of the hurricanes data

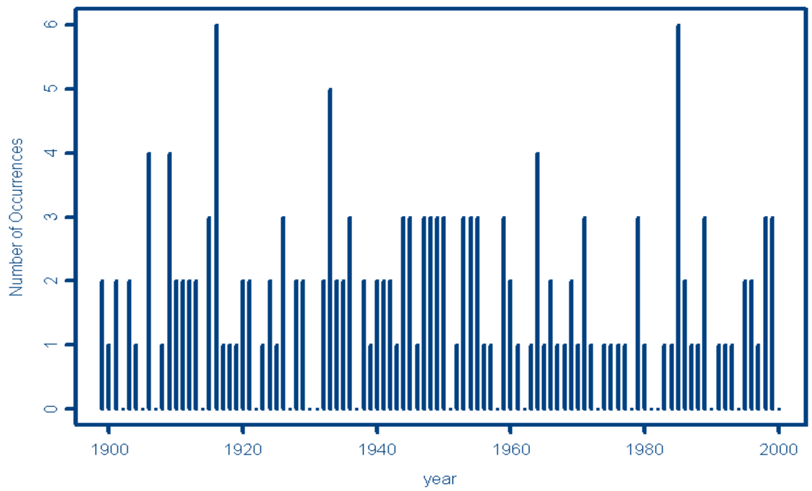

Figure 1: US Atlantic Hurricanes (1899-2000) Annual Counts

Let N_t, the number of events occurring in an interval of the form $[0,\ t)$, be a NHP process with intensity function $\lambda(t)$ for $t \geq 0$. By definition, the probability of n claims occurring in a time interval $[0,\ t)$ is given by

$$P\{N_t = n\} = \frac{e^{-\Lambda(t)}[\Lambda(t)]^n}{n!}\ , \quad n \in \mathbb{N}\ , \tag{1}$$

where Λ, called the cumulative hazard function or the cumulative intensity function of the process, is defined by $\Lambda(t) = \int_0^t \lambda(v)\, dv$ for $t \geq 0$. That is, for a NHP process with intensity function λ, N_t has a Poisson distribution with mean $\Lambda(t)$.

When its intensity function does not depend on time, i.e. $\lambda(t) = \lambda$, for all $t \geq 0$, the corresponding NHP process is the classical homogeneous Poisson process, where $\Lambda(t) = \lambda t$ is linear. From Table 1 we see that here the maximum likelihood estimator (MLE) of λ would be $\hat{\lambda} = 1.64$, hurricanes per year, for the homogeneous Poisson model. See Appendix A.1 for the goodness–of–fit analysis of this model.

Now, consider the case where the risk process evolves in a periodic environment, as when the claim arrival rate depends on the seasons. Then the intensity function of a NHP process is a periodic function, say with a period of $c > 0$ years. Consequently $t - \lfloor \frac{t}{c} \rfloor c \in [0, c)$, for $t \geq 0$, is the time of the season, where $\lfloor t \rfloor$ is the integer part of t.

Models with single and double periodicity are introduced in the following section, where they are illustrated by beta–type functions.

2.1 A single periodic intensity model

Assume that the short–term period is 1 (year). Let λ_1 be a beta–type function, with parameters $p_1, q_1 \geq 1$, defined on $[0,\ 1]$, such that $\lambda_1(t_1^*) = 1$, where $t_1^* \in [0, 1]$ is the mode of the function. That is

$$\lambda_1(t) = \begin{cases} \frac{\left(\frac{t-m_1}{D}\right)^{p_1-1}\left(1-\frac{t-m_1}{D}\right)^{q_1-1}}{\alpha_1^*}, & 0 \leq m_1 \leq t \leq m_2 \leq 1 \\ 0, & \text{otherwise} \end{cases}, \tag{2}$$

where $D = m_2 - m_1$ and

$$\alpha_1^* = \left(\frac{t_1^* - m_1}{D}\right)^{p_1-1} \left(1 - \frac{t_1^* - m_1}{D}\right)^{q_1-1}, \tag{3}$$

is a scale factor, while

$$t_1^* = m_1 + D\,\frac{p_1 - 1}{p_1 + q_1 - 2}, \tag{4}$$

is the mode of λ_1, so that at the mode $\lambda_1(t_1^*) = 1$ is the peak level.

Then the single periodic beta intensity function is given by

$$\lambda(t) = \lambda_0^*\, \lambda_1\big(t - \lfloor t \rfloor\big), \qquad \text{for } t \geq 0, \tag{5}$$

where $\lambda_0^* > 0$ is the (constant) peak level for this intensity and λ_1 is given in (2).

The corresponding cumulative intensity function $\Lambda(t)$ is

$$\Lambda(t) = \frac{\lambda_0^* D}{\alpha_1^*}\left[\lfloor t \rfloor B(p_1, q_1) + B\left(p_1, q_1; \frac{t - \lfloor t \rfloor - m_1}{D}\right)\right], \qquad t \geq 0, \tag{6}$$

where

$$B(p,\ q) = \int_0^1 v^{p-1}\,(1-v)^{q-1}\,dv = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

is the beta function at $p,\ q > 0$, while

$$B(p,\ q;\ t) = \begin{cases} 0\,, & \text{if } t \leq 0 \\ \int_0^t v^{p-1}\,(1-v)^{q-1}\,dv\,, & \text{if } t \in (0,\ 1) \\ B(p,\ q)\,, & \text{if } t \geq 1 \end{cases},$$

is the usual incomplete beta function.

Following the results of Garrido and Lu (2004), the NHP process $\{N_t\}_{t\geq 0}$ with intensity function given in (5) can be decomposed as

$$N_t = M_1 + M_2 + \cdots + M_{\lfloor t \rfloor} + N_{t-\lfloor t \rfloor} \ , \qquad t > 0 \ ,$$

where $\{M_i\}_{i\geq 1}$ are i.i.d. Poisson random variables distributed as N_1, with mean $\Lambda(1)$, representing counts for complete years. These M_i are independent of $N_{t-\lfloor t \rfloor}$, the latter being a Poisson r.v. with mean $\Lambda(t-\lfloor t \rfloor)$, for $t-\lfloor t \rfloor \in [0,\ 1)$, representing the count in the final incomplete year. Here $\Lambda(1)$ and $\Lambda(t-\lfloor t \rfloor)$ can be derived from (6), respectively.

An alternative simple form for λ_1, which can result in a better fit with real data, is the generalized 3–parameter beta function [denoted $G3B(p_1, q_1, \epsilon)$, see Johnson et al. (1995), Chapter 25], given by

$$\lambda_1(t) = \begin{cases} \frac{\left(\frac{t-m_1}{D}\right)^{p_1-1}\left(1-\frac{t-m_1}{D}\right)^{q_1-1}}{\alpha_1^*\left[1-(1-\epsilon)\left(\frac{t-m_1}{D}\right)\right]^{p_1+q_1}}, & 0 \leq m_1 \leq t \leq m_2 \leq 1 \\ 0, & \text{otherwise} \end{cases} , \tag{7}$$

where $p_1, q_1 \geq 1$, $\epsilon > 0$, $D = m_2 - m_1$ and

$$\alpha_1^* = \frac{\left(\frac{t_1^*-m_1}{D}\right)^{p_1-1}\left(1-\frac{t_1^*-m_1}{D}\right)^{q_1-1}}{\left[1-(1-\epsilon)\frac{t_1^*-m_1}{D}\right]^{p_1+q_1}} , \tag{8}$$

is again a scale factor, while

$$t_1^* = m_1 + D\, \frac{3 - p_1 - (1+q_1)\epsilon + \sqrt{[1+p_1+(1+q_1)\epsilon]^2 - 8(p_1+q_1)\epsilon}}{4(1-\epsilon)} , \tag{9}$$

is the mode of function λ_1, given by (7), such that $\lambda_1(t_1^*) = 1$. Note that as $\epsilon \to 1$, then (9) tends to (4).

Then for the intensity function, given by (5), the corresponding cumulative intensity function Λ is derived as

$$\Lambda(t) = \frac{\lambda_0^* D}{\alpha_1^* \epsilon^{p_1}} \left[\lfloor t \rfloor B(p_1, q_1) + B\left(p_1, q_1; \frac{\epsilon\left\{\frac{t-\lfloor t \rfloor - m_1}{D}\right\}}{1-(1-\epsilon)\left\{\frac{t-\lfloor t \rfloor - m_1}{D}\right\}} \right) \right] , \qquad t \geq 0 \ . \tag{10}$$

2.2 A double periodic intensity model

Assume that the peak values or the levels of the short–term intensity function vary periodically with period c (an integer number of years). If, as above, the short–term intensity is the beta–shape function given in (2), then the double periodic beta intensity function is given by:

$$\lambda(t) = \begin{cases} \lambda_0^* \lambda_1\big(t - \lfloor t \rfloor\big) & \text{if } 0 \le t - \lfloor \frac{t}{c} \rfloor c < 1 \\ \lambda_1^* \lambda_1\big(t - \lfloor t \rfloor\big) & \text{if } 1 \le t - \lfloor \frac{t}{c} \rfloor c < 2 \\ \vdots & \vdots \\ \lambda_{c-1}^* \lambda_1\big(t - \lfloor t \rfloor\big) & \text{if } c-1 \le t - \lfloor \frac{t}{c} \rfloor c < c \end{cases}, \tag{11}$$

where $\lambda_0^*, \ldots, \lambda_{c-1}^*$ are all positive levels. The resulting cumulative intensity function $\Lambda(t)$ is given by:

$$\begin{aligned} \Lambda(t) \quad &= \quad \lfloor \frac{t}{c} \rfloor D\, B(p_1, q_1) \sum_{j=0}^{c-1} \frac{\lambda_j^*}{\alpha_1^*} + D\, B(p_1, q_1) \sum_{j=0}^{\lfloor t - \lfloor \frac{t}{c} \rfloor c \rfloor - 1} \frac{\lambda_j^*}{\alpha_1^*} \\ &\quad + D\, B\left(p_1, q_1; \frac{t - \lfloor t \rfloor - m_1}{D}\right) \frac{\lambda^*_{\lfloor t - \lfloor \frac{t}{c} \rfloor c \rfloor}}{\alpha_1^*}, \qquad t > m_1, \end{aligned} \tag{12}$$

and $\Lambda(t) = 0$ for $0 \le t \le m_1$.

The corresponding NHP process $\{N_t\}_{t \ge 0}$ with double periodic intensity can be decomposed as

$$N_t = M_1 + \cdots + M_{\lfloor \frac{t}{c} \rfloor} + N^*_{\frac{t - \lfloor t \rfloor - m_1}{D}}, \qquad t \ge 0, \tag{13}$$

where

$$N^*_{\frac{t - \lfloor t \rfloor - m_1}{D}} = \sum_{j=0}^{\lfloor t - \lfloor \frac{t}{c} \rfloor c \rfloor - 1} N_c^{(j)} + N^{(\lfloor t - \lfloor \frac{t}{c} \rfloor c \rfloor)}_{\frac{t - \lfloor t \rfloor - m_1}{D}}, \tag{14}$$

and the $\{M_i\}_{i \ge 1}$ are i.i.d. Poisson distributed with mean $D\, B(p_1, q_1) \sum_{j=0}^{c-1} \frac{\lambda_j^*}{\alpha_1^*}$, while $N_c^{(j)}$ is Poisson with mean $D\, B(p_1, q_1) \frac{\lambda_j^*}{\alpha_1^*}$, for $j = 0, 1, \ldots, \lfloor t - \lfloor \frac{t}{c} \rfloor c \rfloor - 1$, respectively, and $N^{(\lfloor t - \lfloor \frac{t}{c} \rfloor c \rfloor)}_{\frac{t - \lfloor t \rfloor - m_1}{D}}$ is also Poisson with mean $D\, B\big(p_1, q_1; \frac{t - \lfloor t \rfloor - m_1}{D}\big) \frac{\lambda^*_{\lfloor t - \lfloor \frac{t}{c} \rfloor c \rfloor}}{\alpha_1^*}$. All these random variables are mutually independent.

2.3 A double–beta periodic intensity model

One way to reduce the number of free parameters, in the previous model in (11), is to assume a parametric form also for the long–term intensity. Here this is reasonable if it can be assumed that the short–term peak intensity values are affected periodically by some smoothly varying conditions, like the surface water temperatures in El Niño/La Niña phenomenon.

More specifically, here we assume that the peak beta values, $\lambda_0^*, \ldots, \lambda_{c-1}^*$ in the short–term intensities, follow another continuous function of period c (an integer number of years), called the long–term intensity function. For instance, a beta function $\lambda_c(t)$, is also proposed for the long–term intensity:

$$\begin{aligned}\lambda_c(t) &= a + \frac{b-a}{\alpha_c^*}\left(\frac{t-m_c}{c} - \lfloor\frac{t-m_c}{c}\rfloor\right)^{p_c-1} \\ &\quad \left[1 - \left(\frac{t-m_c}{c} - \lfloor\frac{t-m_c}{c}\rfloor\right)\right]^{q_c-1}, \qquad t > 0, \end{aligned} \tag{15}$$

where

$$\alpha_c^* = \left(\frac{t_c^* - m_c}{c}\right)^{p_c-1}\left(1 - \frac{t_c^* - m_c}{c}\right)^{q_c-1}, \tag{16}$$

is again a scale factor, so that a and b are, respectively, the minimum and maximum amplitude of the peak values. Here m_c is the starting point of the complete cycle of the long–term beta function and

$$t_c^* = m_c + c\left(\frac{p_c - 1}{p_c + q_c - 2}\right) \tag{17}$$

denotes the mode of λ_c.

Then the double–beta intensity function is given by

$$\lambda(t) = \lambda_c\big(\lfloor t\rfloor - \lfloor\frac{t}{c}\rfloor c + t_1^*\big)\lambda_1\big(t - \lfloor t\rfloor\big), \qquad \text{for } t \geq 0, \tag{18}$$

where λ_1 and λ_c are given in (2) and (15), respectively.

The solid line in Figure 2 illustrates the shape of $\lambda(t)$ in (18), when $p_1 = 3$, $q_1 = 2$, $m_1 = \frac{5}{12}$, $D = \frac{6}{12}$, $c = 5$, $p_c = 2$, $q_c = 1\frac{2}{3}$, $m_c = 3.75$, $a = 3$ and $b = 7$. The peak values of the short–term beta λ_1 fall on the dotted line, plotting the long–term beta λ_c. It serves to explain the fluctuations in the peak values of λ_1, the short–term beta periodicity.

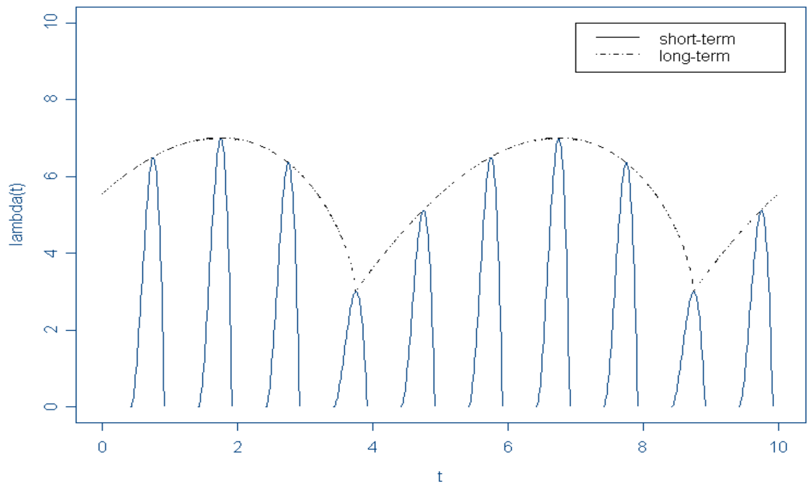

Figure 2: Double–beta intensity function $\lambda(t)$

If the intensity function λ is given by (18), then the corresponding cumulative intensity function Λ has the form

$$\Lambda(t) = \lfloor \frac{t}{c} \rfloor D\, B(p_1, q_1) \sum_{j=0}^{c-1} \frac{\lambda_c(j + t_1^*)}{\alpha_1^*} + D\, B(p_1, q_1) \sum_{j=0}^{\lfloor t - \lfloor \frac{t}{c} \rfloor c \rfloor - 1} \frac{\lambda_c(j + t_1^*)}{\alpha_1^*}$$
$$+ D\, B\left(p_1, q_1; \frac{t - \lfloor t \rfloor - m_1}{D}\right) \frac{\lambda_c(\lfloor t - \lfloor \frac{t}{c} \rfloor c \rfloor + t_1^*)}{\alpha_1^*}\,, \qquad t \geq m_1\,, \quad (19)$$

where $\lambda_c(t)$ is given by (15).

For any $t \geq 0$, the random variable N_t admits the same decomposition as in (13), where the $\{M_i\}_{i \geq 1}$ are i.i.d. Poisson, here with mean $DB(p_1, q_1) \sum_{j=0}^{c-1} \frac{\lambda_c(j+t_1^*)}{\alpha_1^*}$, they are independent of $N_c^{(j)}$, for $j = 0, 1, \ldots, \lfloor t - \lfloor \frac{t}{c} \rfloor c \rfloor - 1$, and of $N^{(\lfloor t - \lfloor \frac{t}{c} \rfloor c \rfloor)}_{\frac{t - \lfloor t \rfloor - m_1}{D}}$, which are also Poisson, here with means $DB(p_1, q_1) \frac{\lambda_c(j+t_1^*)}{\alpha_1^*}$, for $j = 0, 1, \ldots, \lfloor t - \lfloor \frac{t}{c} \rfloor c \rfloor - 1$ and $DB\left(p_1, q_1; \frac{t - \lfloor t \rfloor - m_1}{D}\right) \frac{\lambda_c(\lfloor t - \lfloor \frac{t}{c} \rfloor c \rfloor + t_1^*)}{\alpha_1^*}$, respectively.

This decomposition property of periodic NHP models is particularly useful for statistical inference, as seen in the following section.

3 Statistical inference

For the double–beta periodic intensity model in (18), the intensity is a parametric function with parameters p_1, q_1, p_c, q_c, a and b. It is possible to estimate these parameters from data using maximum likelihood estimation. Note that other model parameters as m_1, m_2, m_c and c can usually be set at values observed from the dataset.

Let d be the time scale in each short–term cycle; here $d = \frac{1}{12}$, a month in each year. Then, for the short–term intensity function in (2), denote by m_1 and m_2 two integer–multiples of d; here m_1 and m_2 correspond to two specific months in the year, marking the beginning and end of the hurricane season. Furthermore define J as

$$J = \frac{m_2 - m_1}{d} = \frac{D}{d} \ ,$$

that is the total number of months in each year over which the intensity function is positive. This gives a convenient partition of each year cycle $[0, m_1)$, $[m_1, t_1)$, $[t_1, t_2)$, ..., $[t_J, m_2)$, $[m_2, 1]$, where

$$t_j = m_1 + j\,d \ , \qquad \text{for } j = 0, \ldots, J \ . \tag{20}$$

Under the double–beta intensity function given in (18), the contribution to the likelihood for the first year of the first cycle is:

$$\begin{aligned} L_{1,1} &= e^{-\int_0^{m_1} \lambda(v)\,dv} \prod_{j=1}^{J} \left[e^{-\int_{t_{j-1}}^{t_j} \lambda(v)\,dv} \left(\int_{t_{j-1}}^{t_j} \lambda(v)\,dv \right)^{n_{j,1}^{(1)}} \right] e^{-\int_{m_2}^{1} \lambda(v)\,dv} \qquad (21) \\ &= e^{-\int_0^{1} \lambda(v)\,dv} \prod_{j=1}^{J} \left(\int_{t_{j-1}}^{t_j} \lambda(v)\,dv \right)^{n_{j,1}^{(1)}} \ , \end{aligned}$$

where $n_{j,1}^{(1)}$ is the number of events which occured within the j-th month $[t_{j-1}, t_j)$ of the first year of the first cycle, for $j = 1, \ldots, J$. The first and the last term in (21) represent the likelihood of having no hurricanes outside the time interval $[m_1, m_2]$.

In general, the contribution to the likelihood by the k-th year of the i-th cycle is similarly given by

$$L_{k,i} = e^{-\int_{k-1}^{k} \lambda(v)\,dv} \prod_{j=1}^{J} \left(\int_{(k-1)+t_{j-1}}^{(k-1)+t_j} \lambda(v)\,dv \right)^{n_{j,k}^{(i)}} \ , \qquad k = 1, \ldots, c, \ i = 1, \ldots, \lfloor \frac{t}{c} \rfloor \ ,$$

where $n_{j,k}^{(i)}$ is the number of hurricanes within the j-th month of the k-th year of the i-th cycle.

Hence for the i-th cycle, $i = 1, \ldots, \lfloor \frac{t}{c} \rfloor$, the total contribution to the likelihood is given by

$$L_i = \prod_{k=1}^{c} L_{k,i} = e^{-\int_0^c \lambda(u)\,dv} \prod_{k=1}^{c} \prod_{j=1}^{J} \left(\int_{(k-1)+t_{j-1}}^{(k-1)+t_j} \lambda(v)\,dv \right)^{n_{j,k}^{(i)}} ,$$

while the likelihood function for all $\lfloor \frac{t}{c} \rfloor$ complete cycles is

$$L_{comp} = e^{-\lfloor \frac{t}{c} \rfloor \int_0^c \lambda(v)\,dv} \prod_{k=1}^{c} \prod_{j=1}^{J} \left(\int_{(k-1)+t_{j-1}}^{(k-1)+t_j} \lambda(v)\,dv \right)^{\sum_{i=1}^{\lfloor \frac{t}{c} \rfloor} n_{j,k}^{(i)}} . \qquad (22)$$

Finally, the contribution to the likelihood from the last incomplete cycle is composed of the contributions by complete years in the last cycle, the complete months in the last incomplete year and the last incomplete month. For simplicity, set $\tau_c = \lfloor t - \lfloor \frac{t}{c} \rfloor c \rfloor$ to be the number of years in the last incomplete cycle, we have

$$\begin{aligned} L_{incomp} = \prod_{k=1}^{\tau_c} & \left[e^{-\int_{k-1}^{k} \lambda(v)\,dv} \prod_{j=1}^{J} \left(\int_{(k-1)+t_{j-1}}^{(k-1)+t_j} \lambda(v)\,dv \right)^{n_{j,k}^{(\lfloor \frac{t}{c} \rfloor + 1)}} \right] \\ & e^{-\int_{\lfloor \frac{t}{c} \rfloor c + \tau_c}^{\lfloor \frac{t}{c} \rfloor c + \tau_c + m_1} \lambda(v)\,dv} \prod_{j=1}^{J^*} \left[e^{-\int_{\tau_c + t_{j-1}}^{\tau_c + t_j} \lambda(v)\,dv} \left(\int_{\tau_c + t_{j-1}}^{\tau_c + t_j} \lambda(v)\,dv \right)^{n_{j,\tau_c+1}^{(\lfloor \frac{t}{c} \rfloor + 1)}} \right] \\ & e^{-\int_{\lfloor \frac{t}{c} \rfloor c + \tau_c + t_{J^*}}^{t} \lambda(v)\,dv} \left(\int_{\tau_c + t_{J^*}}^{t - \lfloor \frac{t}{c} \rfloor c} \lambda(v)\,dv \right)^{n_{J^*+1,\tau_c+1}^{(\lfloor \frac{t}{c} \rfloor + 1)}} , \end{aligned} \qquad (23)$$

where

$$J^* = \left\lfloor \frac{t - \lfloor \frac{t}{c} \rfloor c - \lfloor t - \lfloor \frac{t}{c} \rfloor c \rfloor - m_1}{d} \right\rfloor = \left\lfloor \frac{t - \lfloor \frac{t}{c} \rfloor c - \tau_c - m_1}{d} \right\rfloor$$

is the number of months in the last incomplete year (set to 0 when J^* is a negative integer). Note here that the last two lines reduce to

$$e^{-\int_{\lfloor \frac{t}{c} \rfloor c + \tau_c}^{\lfloor \frac{t}{c} \rfloor c + \tau_c + t - \lfloor t \rfloor} \lambda(v)\,dv} = e^{-\int_{\lfloor \frac{t}{c} \rfloor c + \tau_c}^{t} \lambda(v)\,dv} , \qquad \text{for } t - \lfloor t \rfloor \leq m_1 .$$

Hence the full likelihood function is given by (22) and (23) to be

$$\begin{aligned} L &= L_{comp} \cdot L_{incomp} \\ &= e^{-\Lambda(t)} \prod_{k=1}^{\tau_c} \prod_{j=1}^{J} \left(\int_{(k-1)+t_{j-1}}^{(k-1)+t_j} \lambda(v)\, dv \right)^{\sum_{i=1}^{\lfloor \frac{t}{c} \rfloor + 1} n_{j,k}^{(i)}} \\ & \quad \prod_{k=\tau_c+1}^{c} \prod_{j=1}^{J} \left(\int_{(k-1)+t_{j-1}}^{(k-1)+t_j} \lambda(v)\, dv \right)^{\sum_{i=1}^{\lfloor \frac{t}{c} \rfloor} n_{j,k}^{(i)}} \\ & \quad \prod_{j=1}^{J^*} \left(\int_{\tau_c+t_{j-1}}^{\tau_c+t_j} \lambda(v)\, dv \right)^{n_{j,\tau_c+1}^{(\lfloor \frac{t}{c} \rfloor + 1)}} \left(\int_{\tau_c+t_{J^*}}^{t-\lfloor \frac{t}{c} \rfloor c} \lambda(v)\, dv \right)^{n_{J^*+1,\tau_c+1}^{(\lfloor \frac{t}{c} \rfloor + 1)}} . \end{aligned} \quad (24)$$

Substituting λ for the double–beta periodic intensity function in (18), then the integrals in (24) can be represented as incomplete beta functions, yielding:

$$\begin{aligned} L = e^{-\Lambda(t)} & \prod_{k=1}^{\tau_c} \prod_{j=1}^{J} \left\{ \frac{\lambda_c(k-1+t_1^*)}{\alpha_1^*} D \left[B(p_1, q_1; \tfrac{jd}{D}) - B(p_1, q_1; \tfrac{(j-1)d}{D}) \right] \right\}^{\sum_{i=1}^{\lfloor \frac{t}{c} \rfloor + 1} n_{j,k}^{(i)}} \\ & \prod_{k=\tau_c+1}^{c} \prod_{j=1}^{J} \left\{ \frac{\lambda_c(k-1+t_1^*)}{\alpha_1^*} D \left[B(p_1, q_1; \tfrac{jd}{D}) - B(p_1, q_1; \tfrac{(j-1)d}{D}) \right] \right\}^{\sum_{i=1}^{\lfloor \frac{t}{c} \rfloor} n_{j,k}^{(i)}} \\ & \prod_{j=1}^{J^*} \left\{ \frac{\lambda_c(\tau_c+t_1^*)}{\alpha_1^*} D \left[B(p_1, q_1; \tfrac{jd}{D}) - B(p_1, q_1; \tfrac{(j-1)d}{D}) \right] \right\}^{n_{j,\tau_c+1}^{(\lfloor \frac{t}{c} \rfloor + 1)}} \\ & \left\{ \frac{\lambda_c(\tau_c+t_1^*)}{\alpha_1^*} D \left[B(p_1, q_1; \tfrac{t-\tau_c-m_1}{D}) - B(p_1, q_1; \tfrac{J^* d}{D}) \right] \right\}^{n_{J^*+1,\tau_c+1}^{(\lfloor \frac{t}{c} \rfloor + 1)}} , \end{aligned} \quad (25)$$

where the function λ_c is given in (15).

Denote by $N = \sum_{i,j,k} n_{j,k}^{(i)}$ be the total number of occurrences, for $1 \le i \le \lfloor \frac{t}{c} \rfloor + 1$, $1 \le j \le J$ and $1 \le k \le c$. Further denote by

$$n_{\cdot,k}^{(\cdot)} = \sum_{j=1}^{J} \sum_{i=1}^{\lfloor \frac{t}{c} \rfloor} n_{j,k}^{(i)} , \qquad k = 1, 2, \ldots, c ,$$

the total number of occurrences in the k-th year of all complete cycles, while

$n_{\cdot,k}^{(\lfloor\frac{t}{c}\rfloor+1)}$ stands for the count in the k-th year of the last incomplete cycle. Similarly

$$n_{j,\cdot}^{(\cdot)} = \sum_{k=1}^{c} \sum_{i=1}^{\lfloor\frac{t}{c}\rfloor} n_{j,k}^{(i)} , \qquad j = 1, 2, \ldots, J ,$$

denotes the total number of occurrences in the j-th month of all complete cycles, while $n_{j,\cdot}^{(\lfloor\frac{t}{c}\rfloor+1)}$ stands for the count in the j-th complete month of the last incomplete cycle. Consequently, the log likelihood function is given by

$$\begin{aligned} l &= -\Lambda(t) + N \log \frac{D}{\alpha_1^*} + \sum_{k=1}^{c} [n_{\cdot,k}^{(\cdot)} + n_{\cdot,k}^{(\lfloor\frac{t}{c}\rfloor+1)}] \log \lambda_c(k-1+t_1^*) \\ &\quad + \sum_{j=1}^{J} [n_{j,\cdot}^{(\cdot)} + n_{j,\cdot}^{(\lfloor\frac{t}{c}\rfloor+1)}] \log \left[B(p_1, q_1; \frac{jd}{D}) - B(p_1, q_1; \frac{(j-1)d}{D}) \right] \\ &\quad + n_{J^*+1,\tau_c+1}^{(\lfloor\frac{t}{c}\rfloor+1)} \log \left[B(p_1, q_1; \frac{t-\tau_c-m_1}{D}) - B(p_1, q_1; \frac{J^* d}{D}) \right] . \end{aligned} \tag{26}$$

The maximum likelihood estimators for p_1, q_1, p_c, q_c, a and b in the double–beta intensity function are obtained by maximizing l numerically.

Similarly, the maximum likelihood estimators for parameters in the model given in Section 2.2 with a generalized 3–parameter beta short–term intensity function can be derived as follows.

To simplify expressions, let t be an integer number here. Assume that the short–term intensity function for the k-th year of a cycle is of the generalized 3–parameter beta form in (7) with parameters $p_1^{(k)}$, $q_1^{(k)}$ and $\epsilon^{(k)}$ and λ_k^* is the peak value, where $k = 1, 2, \ldots, c$. For $1 \le k \le \tau_c$, the log–likelihood function is given by

$$\begin{aligned} l_k &= -\left(\lfloor\frac{t}{c}\rfloor + 1\right) \frac{\lambda_k^* D B(p_1^{(k)}, q_1^{(k)})}{\alpha_1^{(k)} [\epsilon^{(k)}]^{p_1^{(k)}}} + [n_{\cdot,k}^{(\cdot)} + n_{\cdot,k}^{(\lfloor\frac{t}{c}\rfloor+1)}] \log \left(\frac{\lambda_k^* D}{\alpha_1^{(k)} [\epsilon^{(k)}]^{p_1^{(k)}}} \right) \\ &\quad + \sum_{j=1}^{J} [n_{j,k}^{(\cdot)} + n_{j,k}^{(\lfloor\frac{t}{c}\rfloor+1)}] \log \left[B\left(p_1^{(k)}, q_1^{(k)}; \frac{\epsilon^{(k)} \frac{jd}{D}}{1 - (1-\epsilon^{(k)}) \frac{jd}{D}}\right) \right. \\ &\qquad \left. - B\left(p_1^{(k)}, q_1^{(k)}; \frac{\epsilon^{(k)} \frac{(j-1)d}{D}}{1 - [1-\epsilon^{(k)}] \frac{(j-1)d}{D}}\right) \right] , \end{aligned} \tag{27}$$

where $n_{j,k}^{(\cdot)} = \sum_{i=1}^{\lfloor\frac{t}{c}\rfloor} n_{j,k}^{(i)}$ is the total number of occurrences in the j-th month of the k-th year of all complete cycles and $\alpha_1^{(k)}$ is the scale factor of the k-th year of each cycle. For $\tau_c < k \leq c$, we have

$$\begin{aligned} l_k \;=\; & -\lfloor \frac{t}{c} \rfloor \frac{\lambda_k^* D B(p_1^{(k)}, q_1^{(k)})}{\alpha_1^{(k)} [\epsilon^{(k)}]^{p_1^{(k)}}} + n_{\cdot,k}^{(\cdot)} \log \left(\frac{\lambda_k^* D}{\alpha_1^{(k)} [\epsilon^{(k)}]^{p_1^{(k)}}} \right) \\ & + \sum_{j=1}^{J} n_{j,k}^{(\cdot)} \log \Bigg[B\Bigg(p_1^{(k)}, q_1^{(k)}; \frac{\epsilon^{(k)} \frac{jd}{D}}{1-(1-\epsilon^{(k)})\frac{jd}{D}}\Bigg) \\ & \qquad - B\Bigg(p_1^{(k)}, q_1^{(k)}; \frac{\epsilon^{(k)} \frac{(j-1)d}{D}}{1-(1-\epsilon^{(k)})\frac{(j-1)d}{D}}\Bigg) \Bigg] . \end{aligned} \tag{28}$$

4 Discussion and remarks

As outlined in Section 2, the illustrative dataset used here comprises 167 hurricanes that made a landfall somewhere on the Atlantic United States coastline, over the 102–year period 1899 through 2000. These exhibit clear seasonal patterns. First, all hurricanes happened between the months of June to November. September generated more major hurricanes than any other month. On average, there were 1 to 2 hurricane landfalls per year over the whole period. A short–term (annual) periodic model thus seems appropriate.

First consider a NHP model with single periodicity. Figure 3 gives the generalized 3–parameter beta intensity described in (7), that was fitted to these annual hurricane frequencies. The parameter MLE's here are $\hat{p}_1 = 1.9198$, $\hat{q}_1 = 11.3050$, $\hat{\epsilon} = 0.1349$ and $\hat{\lambda}_0^* = 6.5145$, obtained with the Excel solver using the method described in Section 3.

The constant intensity $\lambda_1(t) = \hat{\lambda} = 1.64$, the homogeneous Poisson process MLE, is also graphed on Figure 3 for comparison. Graphically it is clear that the classical model gives here a crude representation of hurricane frequencies (this hypothesis is tested more formally in Appendix A.1).

Climatological studies suggest that the hurricane intensity does not repeat the exact same short–term pattern every year. Rather, it slightly varies from year to year, as in alternating El Niño–La Niña cycles. For example, research on the tropical cyclones affecting the coast of Texas, during El Niño/La Niña years of 1900–1996, shows that the highest percentage of all major hurricanes which have affected the coast of Texas occurred when El Niño was present for at least part of the given year [see Cole and Pfaff (1997)]. Some actuaries also believe that El

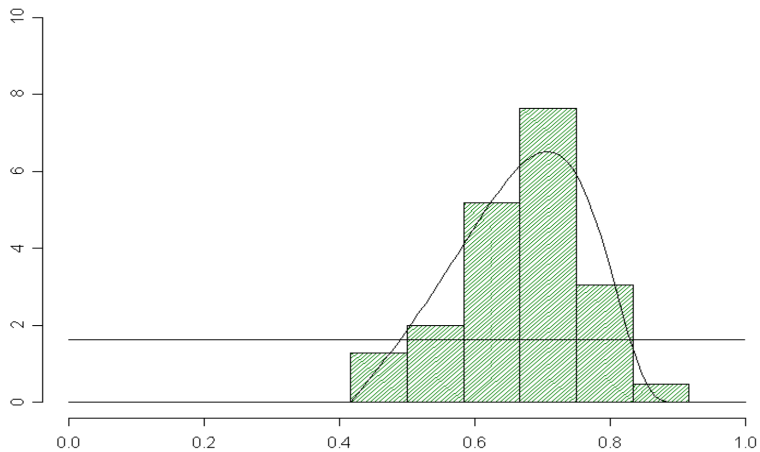

Figure 3: Histogram and fitted hurricane intensity $\lambda(t)$ for 1–year cycles

Niño/La Niña cycles in the Pacific affect tropical storm systems in the Atlantic.

Our hurricane data also exhibits some long–term periodicity, under the influence of the global El Niño/La Niña phenomenon. The 5–year cycle in Figure 4 shows how the 3rd and 4th years of the cycle have lower occurrences of hurricanes, the 4th year being the lowest. This is followed by a peak lasting for a period nearly three years long.

This motivates our assumptions of the doubly periodic NHP process presented in Section 2. Here the seasonality of the Atlantic hurricane repeats a similar short–term pattern every year meanwhile the peak intensity, affected by the El Niño phenomenon, varies over a longer periodic cycle.

Climatologists observed that the typical El Niño cycle occurs within a two–to–seven year cycle. From a graphical analysis of the dataset, we conclude that a long-term period $c = 5$ years and a short–term period of one year reasonably describe the Atlantic hurricanes.

Figure 4 compares the observed and expected monthly average number of hurricanes over the 5–year cycle for the 1899–2000 dataset. A double 2–parameter beta intensity function was used and the following MLE's were obtained: $\hat{p}_1 = 3.0145$, $\hat{q}_1 = 2.4389$, $\hat{p}_c = 1.5463$, $\hat{q}_c = 1.3642$, $\hat{a} = 3.2354$ and $\hat{b} = 6.9634$, where $\hat{q}_1$ and $\hat{q}_c$ are obtained from (4) and (17), respectively, while the estimated standard deviations for $\hat{p}_1$, $\hat{p}_c$, $\hat{a}$ and $\hat{b}$ are 0.3582, 0.7653, 0.7890 and 0.9126, respectively (see Appendix A.2 for a derivation).

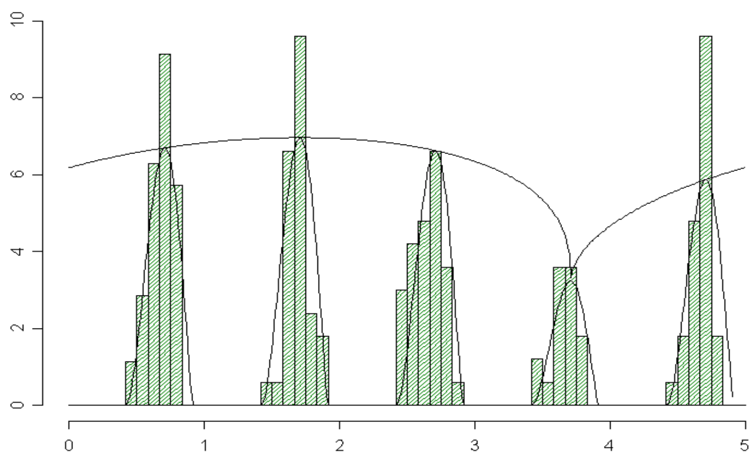

Figure 4: Hurricanes data and 5–year double–beta intensities

Climatology suggests that the levels for the long–term cycle are governed by some underlying smoothly changing function, represented by the second beta function. The fit for each short–term cycle seems quite good, supporting our periodic theory. But the model does not adequately explains the short–term peaks over the long–term cycle. The El Niño/La Niña is a global phenomenon, perhaps too complex to capture with such a simple parametric model.

Depending on the intended use of the model, the fit can be improved by the introduction of additional parameters. For instance when a generalized 3–parameter beta intensity is used for the short–term cycle, while the long–term beta function is kept at 2–parameters, the following MLE's are obtained: $\hat{p}_1 = 1.8946$, $\hat{q}_1 = 12.3899$, $\hat{\epsilon} = 0.1205$, $\hat{p}_c = 1.5639$, $\hat{q}_c = 1.3921$, $\hat{a} = 3.5868$ and $\hat{b} = 7.7307$. It is clear from Figure 5 that the fit is improved, although not perfect, at the cost of introducing only one additional parameter.

If fit is more important than simplicity of the model or smoothness, the number of parameters can be further increased by letting the short–term cycle peak values be free. Figure 6 gives the histogram and fitted beta intensities, as in (11), for monthly hurricane frequencies over a 5–year long–term cycle.

Here the generalized 3–parameter beta function in (7) was used as the short–

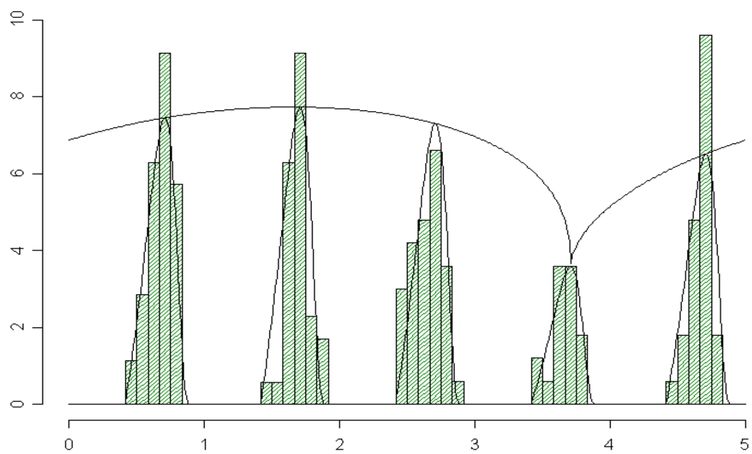

Figure 5: Hurricanes data and 5–year generalized double–beta intensities

term intensity and the MLE's, given in Table 2, were derived from (27). The fit improvement is substantial for each short–term intensity in the 5–year cycle. Yet, the model now fails to explain how hurricane intensities vary from El Niño to La Niña years. A possible remedy may be the use of random effects on certain years of the cycle. This will be the subject of further research on regime–switching double–periodic Cox processes.

Year	$p_1^{(k)}$	$q_1^{(k)}$	$\epsilon^{(k)}$	λ_k^*
1	2.0087	150.0076	0.0097	9.3381
2	4.7926	3.0123	1.3227	7.2847
3	1.1459	11.9872	0.0820	5.8373
4	2.0586	121.7060	0.0150	3.9563
5	3.0769	155.4399	0.0165	8.4431

Table 2: MLE's of 3–parameter beta intensities for the hurricanes data

In conclusion, it appears that NHP risk models are more realistic in practice than classical Poisson processes, as their intensity rate is a function of time. This is clearly the case for hurricane landfalls.

In general, NHP processes with a periodic claim intensity can be useful in modeling risk processes that evolve in a periodic environment. The proposed double–beta

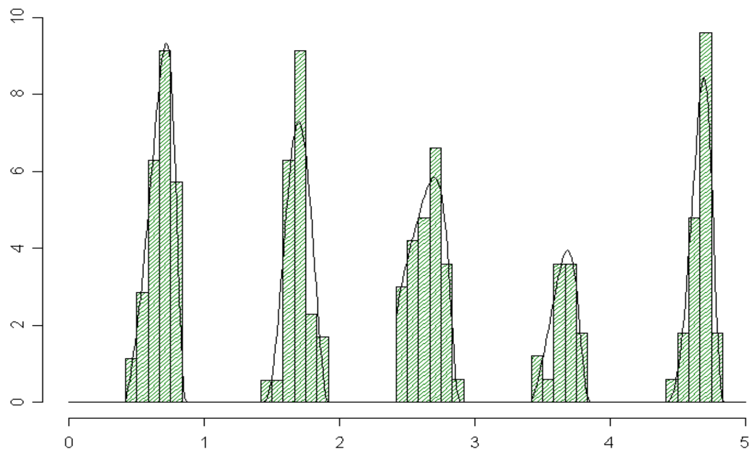

Figure 6: Hurricanes data 5-year short-term generalized beta fit

periodic claim intensity not only generalizes the classical risk model, but it can also give a more realistic representation than (single) periodic models with only short–term periodic intensity functions.

The flexibility in shape of the beta function and the explicit results obtained for the risk process, as well as the tractability of the statistical estimation of model parameters, should make these double–beta periodic models easy to use in practice. We hope that the illustration of the hurricane dataset serves to show that NHP risk models can also be tractable if properly parameterized.

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A Appendix

A.1 Goodness–of–fit

Figures 1 and 3 provide graphical evidence that annual, respectively monthly, hurricane counts show a periodic behaviour.

More formally, we can test the alternate hypothesis of a constant hurricane intensity, $\lambda_1(t) = \hat{\lambda} = 1.637254902$, resulting in a Poisson number of hurricanes per year. Table 3 reports the Poisson expected and observed number of years with 0, 1, 2, 3 and 4 or more hurricanes (the last observations were grouped to be representative).

A simple Chi-square test ($X^2 = 1.81 < \chi^2_{3;\,0.05} = 7.81$) does not reject the homogeneous Poisson assumption. Still, it is clear from Table 3 that the fit is poor in the tail of the distribution.

The Poisson model with constant intensity predicts well the expected number of years with lower hurricane frequencies (e.g. $n = 0$, 1 or 2 hurricanes per year), but gives a poorer prediction of the number of years with higher frequencies ($n = 3$ and $n \geq 4$). The fit in the tail is usually very important in insurance applications.

Counts	Observed	Expected	Chi-square
0	19	19.84	0.04
1	34	32.48	0.07
2	25	26.59	0.10
3	18	14.51	0.84
4+	6	8.57	0.77
Total	102	102.00	1.81

Table 3: Chi-square goodness–of–fit test for the homogeneous Poisson model

Furthermore, the homogeneous Poisson model fails to recognize the short–term seasonal and long–term cyclical patterns that the hurricanes data exhibit in Figure 5. A more appropriate statistical inference here is to test the significance of the additional parameters in our double–beta periodic models.

Since the classical Poisson model is a special case of the double–beta periodic model with 4 parameters, in Figure 4, we can use a likelihood ratio test for the homogeneous Poisson hypothesis, against the alternative of a full 4–parameters model. The test statistic $r = 2(499.645 - 345.407) = 308.476 > \chi^2_{3;\,0.05} = 7.81$, is significant, supporting the full model hypothesis.

Similarly, in testing for the extra parameter in the complete model used for Figure 5, with a generalized 3–parameter beta function for the short–term intensity, the statistic $r = 2(345.407 - 335.936) = 18.942 > \chi^2_{1;0.05} = 3.84$ is also significant. This complete double–beta periodic model with 5 parameters explains the observed periodicity more adequately than the above reduced models.

The other assumption that should be tested is that of dependence on time. The hurricane counts observed here are not assumed to be mutually dependent (autocorrelated), but rather dependent on the time (season) of occurrence. Once a cycle completes, every 5 years, then this dependence on time gets reset. Subsequent 5 year cycles are thus independent, as in the decomposition in (13). Figure 7 shows the absence of autocorrelations, in these 5-year cycle counts.

A.2 Information matrix and estimated standard deviations

The detailed calculation of the estimated standard deviations of the MLE's is given here for the parameters in our double–beta intensity model. The variance–

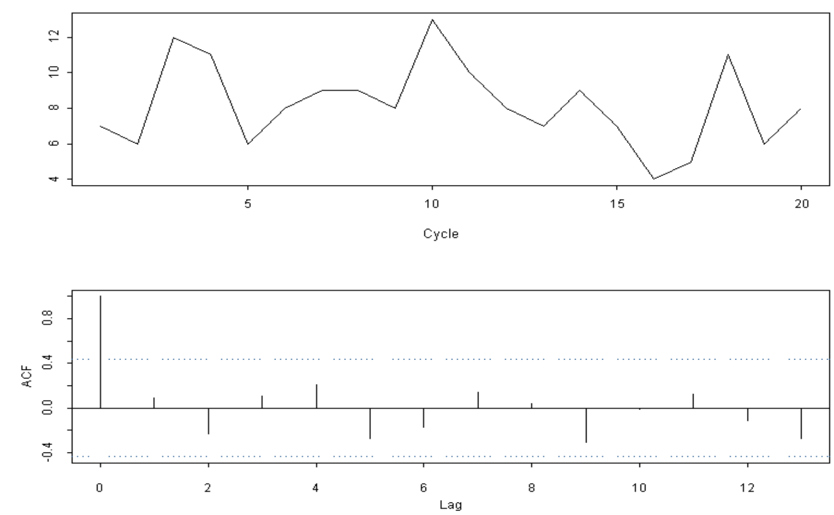

Figure 7: Hurricanes data autocorrelations, 5-year cycle counts

covariance matrix is obtained by inverting the matrix whose rs–element is

$$\mathbf{I}(\theta)_{rs} = -E\left[\frac{\partial^2}{\partial\theta_s\partial\theta_r}\, l(\theta)\right] ,$$

where $\theta = (p_1,\, p_c,\, a,\, b)^T$ is the vector of MLE's in our model and $\mathbf{I}(\theta)$ is called the information matrix.

For simplicity, we assume that t is an integer in the following derivation. In this case, the log–likelihood function is given by

$$\begin{aligned} l \;=\; & -\Lambda(t) + N \log \frac{D}{\alpha_1^*} + \sum_{k=1}^{c} n_{\cdot,k}^{(\cdot)} \log \lambda_c(k-1+t_1^*) \\ & + \sum_{j=1}^{J} n_{j,\cdot}^{(\cdot)} \log\left[B(p_1, q_1; \frac{jd}{D}) - B(p_1, q_1; \frac{(j-1)d}{D})\right] , \end{aligned} \tag{29}$$

where the cumulative intensity function Λ has the form

$$\Lambda(t) = \lfloor \frac{t}{c} \rfloor D\, B(p_1, q_1) \sum_{j=0}^{c-1} \frac{\lambda_c(j+t_1^*)}{\alpha_1^*} + D\, B(p_1, q_1) \sum_{j=0}^{\lfloor t - \lfloor \frac{t}{c} \rfloor c \rfloor - 1} \frac{\lambda_c(j+t_1^*)}{\alpha_1^*} , \tag{30}$$

and $\lambda_c(t)$ is given by (15).

The first partial derivatives of (29) w.r.t. p_1, p_c, a and b are:

$$\begin{aligned}
\frac{\partial l}{\partial p_1} &= -\frac{\partial \Lambda(t)}{\partial p_1} - N\frac{\frac{\partial \alpha_1^*}{\partial p_1}}{\alpha_1^*} + \sum_{j=1}^{J} n_{j,\cdot}^{(\cdot)} \frac{B'_{p_1}(p_1, q_1; \frac{jd}{D}) - B'_{p_1}(p_1, q_1; \frac{(j-1)d}{D})}{B(p_1, q_1; \frac{jd}{D}) - B(p_1, q_1; \frac{(j-1)d}{D})} , \\
\frac{\partial l}{\partial p_c} &= -\frac{\partial \Lambda(t)}{\partial p_c} + \sum_{k=1}^{c} n_{\cdot,k}^{(\cdot)} \frac{\frac{\partial \lambda_c(k-1+t_1^*)}{\partial p_c}}{\lambda_c(k-1+t_1^*)} , \\
\frac{\partial l}{\partial a} &= -\frac{\partial \Lambda(t)}{\partial a} + \sum_{k=1}^{c} n_{\cdot,k}^{(\cdot)} \frac{\frac{\partial \lambda_c(k-1+t_1^*)}{\partial a}}{\lambda_c(k-1+t_1^*)} , \\
\frac{\partial l}{\partial b} &= -\frac{\partial \Lambda(t)}{\partial b} + \sum_{k=1}^{c} n_{\cdot,k}^{(\cdot)} \frac{\frac{\partial \lambda_c(k-1+t_1^*)}{\partial b}}{\lambda_c(k-1+t_1^*)} ,
\end{aligned}$$

where

$$\frac{\frac{\partial \alpha_1^*}{\partial p_1}}{\alpha_1^*} = \nu * := \ln\left(\frac{t_1^* - m_1}{D}\right) + (\frac{5}{7}) \ln\left(1 - \frac{t_1^* - m_1}{D}\right) ,$$

$$B'_{p_1}(p_1, q_1; t) = \int_0^t v^{p-1}(1-v)^{q-1}\Big[\ln(v) + (\frac{5}{7})\ \ln(1-v)\Big]dv , \qquad t \in (0,\ 1) ,$$

while the first partial derivatives of (30) w.r.t. p_1, p_c, a and b are given by

$$\begin{aligned}
\frac{\partial \Lambda(t)}{\partial p_1} &= \Lambda(t)\left[\frac{B'_{p_1}(p_1, q_1)}{B(p_1, q_1)} - \nu *\right] , \\
\frac{\partial \Lambda(t)}{\partial p_c} &= \frac{D\, B(p_1, q_1)}{\alpha_1^*}\left[\lfloor\frac{t}{c}\rfloor \sum_{j=0}^{c-1} \frac{\partial \lambda_c(j+t_1^*)}{\partial p_c} + \sum_{j=0}^{\lfloor t - \lfloor \frac{t}{c}\rfloor c\rfloor - 1} \frac{\partial \lambda_c(j+t_1^*)}{\partial p_c}\right] , \\
\frac{\partial \Lambda(t)}{\partial a} &= \frac{D\, B(p_1, q_1)}{\alpha_1^*}\left[\lfloor\frac{t}{c}\rfloor \sum_{j=0}^{c-1} \frac{\partial \lambda_c(j+t_1^*)}{\partial a} + \sum_{j=0}^{\lfloor t - \lfloor \frac{t}{c}\rfloor c\rfloor - 1} \frac{\partial \lambda_c(j+t_1^*)}{\partial a}\right] , \\
\frac{\partial \Lambda(t)}{\partial b} &= \frac{D\, B(p_1, q_1)}{\alpha_1^*}\left[\lfloor\frac{t}{c}\rfloor \sum_{j=0}^{c-1} \frac{\partial \lambda_c(j+t_1^*)}{\partial b} + \sum_{j=0}^{\lfloor t - \lfloor \frac{t}{c}\rfloor c\rfloor - 1} \frac{\partial \lambda_c(j+t_1^*)}{\partial b}\right] ,
\end{aligned}$$

here $B'_{p_1}(p_1, q_1) = B'_{p_1}(p_1, q_1; 1)$ and the first partial derivatives of $\lambda_c(t)$ w.r.t. p_c,

a and b are given by

$$\begin{aligned}
\frac{\partial \lambda_c(j+t_1^*)}{\partial p_c} &= \big[\lambda_c(j+t_1^*) - a\big]\Big\{\ln\big(\frac{t-m_c}{c} - \lfloor\frac{t-m_c}{c}\rfloor\big) - \ln\big(\frac{t_c^*-m_c}{c}\big) \\
&\quad +\big(\frac{2}{3}\big)\Big[\ln\big(1-\frac{t-m_c}{c} - \lfloor\frac{t-m_c}{c}\rfloor\big) - \ln\Big(1-\frac{t_c^*-m_c}{c}\Big)\Big]\Big\} \ , \\
\frac{\partial \lambda_c(j+t_1^*)}{\partial a} &= 1 - \frac{1}{\alpha_c^*}\big(\frac{t-m_c}{c} - \lfloor\frac{t-m_c}{c}\rfloor\big)^{p_c-1}\big(1-\frac{t-m_c}{c} - \lfloor\frac{t-m_c}{c}\rfloor\big)^{q_c-1} \ , \\
\frac{\partial \lambda_c(j+t_1^*)}{\partial b} &= \frac{1}{\alpha_c^*}\big(\frac{t-m_c}{c} - \lfloor\frac{t-m_c}{c}\rfloor\big)^{p_c-1}\big(1-\frac{t-m_c}{c} - \lfloor\frac{t-m_c}{c}\rfloor\big)^{q_c-1} \ .
\end{aligned}$$

Now the second partial derivatives of (29) w.r.t. p_1 is obtained as follows:

$$\frac{\partial^2 l}{\partial p_1^2} = -\frac{\partial^2 \Lambda(t)}{\partial p_1^2} + \sum_{j=1}^{J} n_{j,\cdot}^{(\cdot)} \left[\frac{B'_{p_1}(p_1,q_1;\frac{jd}{D}) - B'_{p_1}(p_1,q_1;\frac{(j-1)d}{D})}{B(p_1,q_1;\frac{jd}{D}) - B(p_1,q_1;\frac{(j-1)d}{D})}\right]'_{p_1} \ ,$$

where

$$\begin{aligned}
\left[\frac{B'_{p_1}(p_1,q_1;\frac{jd}{D}) - B'_{p_1}(p_1,q_1;\frac{(j-1)d}{D})}{B(p_1,q_1;\frac{jd}{D}) - B(p_1,q_1;\frac{(j-1)d}{D})}\right]'_{p_1} &= \frac{B''_{p_1,p_1}(p_1,q_1;\frac{jd}{D}) - B''_{p_1,p_1}(p_1,q_1;\frac{(j-1)d}{D})}{B(p_1,q_1;\frac{jd}{D}) - B(p_1,q_1;\frac{(j-1)d}{D})} \\
&\quad - \left[\frac{B'_{p_1}(p_1,q_1;\frac{jd}{D}) - B'_{p_1}(p_1,q_1;\frac{(j-1)d}{D})}{B(p_1,q_1;\frac{jd}{D}) - B(p_1,q_1;\frac{(j-1)d}{D})}\right]^2 \ ,
\end{aligned}$$

here we define for $t \in (0,\ 1)$,

$$B''_{p_1,p_1}(p_1,q_1;t) = \int_0^t v^{p-1}\,(1-v)^{q-1}\Big[\ln^2(v) + 2\big(\frac{5}{7}\big)\ln(v)\ln(1-v) + \big(\frac{5}{7}\big)^2\ln^2(1-v)\Big]dv \ ,$$

and

$$\begin{aligned}
\frac{\partial^2 \Lambda(t)}{\partial p_1^2} &= \frac{\partial \Lambda(t)}{\partial p_1}\left[\frac{B_{p_1}(p_1,q_1)}{B(p_1,q_1)} - \nu*\right] + \Lambda(t)\left[\frac{B'_{p_1}(p_1,q_1)}{B(p_1,q_1)} - \nu*\right]'_{p_1} \\
&= \frac{\partial \Lambda(t)}{\partial p_1}\left[\frac{B'_{p_1}(p_1,q_1)}{B(p_1,q_1)} - \nu*\right] + \Lambda(t)\left[\frac{B''_{p_1,p_1}(p_1,q_1)}{B(p_1,q_1)} - \left(\frac{B''_{p_1}(p_1,q_1)}{B(p_1,q_1)}\right)^2\right] \ ,
\end{aligned}$$

where $B''_{p_1,p_1}(p_1,q_1) = B''_{p_1,p_1}(p_1,q_1;1)$.

The second partial derivatives of (29) w.r.t. p_1 are:

$$\begin{aligned}
\frac{\partial^2 l}{\partial p_1\,\partial p_c} &= -\frac{\partial^2 \Lambda(t)}{\partial p_1\,\partial p_c} = -\frac{\partial \Lambda(t)}{\partial p_c}\left[\frac{B'_{p_1}(p_1,q_1)}{B(p_1,q_1)} - \nu*\right] ,\\
\frac{\partial^2 l}{\partial p_1\,\partial a} &= -\frac{\partial^2 \Lambda(t)}{\partial p_1\,\partial a} = -\frac{\partial \Lambda(t)}{\partial a}\left[\frac{B'_{p_1}(p_1,q_1)}{B(p_1,q_1)} - \nu*\right] ,\\
\frac{\partial^2 l}{\partial p_1\,\partial b} &= -\frac{\partial^2 \Lambda(t)}{\partial p_1\,\partial b} = -\frac{\partial \Lambda(t)}{\partial b}\left[\frac{B'_{p_1}(p_1,q_1)}{B(p_1,q_1)} - \nu*\right] ,
\end{aligned}$$

while the second partial derivatives of (29) w.r.t. other parameters are:

$$\begin{aligned}
\frac{\partial^2 l}{\partial p_c^2} &= -\frac{\partial^2 \Lambda(t)}{\partial p_c^2} + \sum_{k=1}^{c} n_{\cdot,k}^{(\cdot)} \frac{\partial}{\partial p_c}\left[\frac{\frac{\partial \lambda_c(k-1+t_1^*)}{\partial p_c}}{\lambda_c(k-1+t_1^*)}\right]\\
&= -\frac{\partial^2 \Lambda(t)}{\partial p_c^2} + \sum_{k=1}^{c} n_{\cdot,k}^{(\cdot)} \left\{\frac{\frac{\partial^2 \lambda_c(k-1+t_1^*)}{\partial p_c^2}}{\lambda_c(k-1+t_1^*)} - \left[\frac{\frac{\partial \lambda_c(k-1+t_1^*)}{\partial p_c}}{\lambda_c(k-1+t_1^*)}\right]^2\right\} ,\\
\frac{\partial^2 l}{\partial p_c\,\partial a} &= -\frac{\partial^2 \Lambda(t)}{\partial p_c\,\partial a} + \sum_{k=1}^{c} n_{\cdot,k}^{(\cdot)} \frac{\partial}{\partial a}\left[\frac{\frac{\partial \lambda_c(k-1+t_1^*)}{\partial p_c}}{\lambda_c(k-1+t_1^*)}\right]\\
&= -\frac{\partial^2 \Lambda(t)}{\partial p_c\,\partial a} + \sum_{k=1}^{c} n_{\cdot,k}^{(\cdot)} \left\{\frac{\frac{\partial^2 \lambda_c(k-1+t_1^*)}{\partial p_c\,\partial a}}{\lambda_c(k-1+t_1^*)} - \frac{\frac{\partial \lambda_c(k-1+t_1^*)}{\partial p_c}\,\frac{\partial \lambda_c(k-1+t_1^*)}{\partial a}}{\left[\lambda_c(k-1+t_1^*)\right]^2}\right\} ,\\
\frac{\partial^2 l}{\partial p_c\,\partial b} &= -\frac{\partial^2 \Lambda(t)}{\partial p_c\,\partial b} + \sum_{k=1}^{c} n_{\cdot,k}^{(\cdot)} \frac{\partial}{\partial b}\left[\frac{\frac{\partial \lambda_c(k-1+t_1^*)}{\partial p_c}}{\lambda_c(k-1+t_1^*)}\right]\\
&= -\frac{\partial^2 \Lambda(t)}{\partial p_c\,\partial b} + \sum_{k=1}^{c} n_{\cdot,k}^{(\cdot)} \left\{\frac{\frac{\partial^2 \lambda_c(k-1+t_1^*)}{\partial p_c\,\partial b}}{\lambda_c(k-1+t_1^*)} - \frac{\frac{\partial \lambda_c(k-1+t_1^*)}{\partial p_c}\,\frac{\partial \lambda_c(k-1+t_1^*)}{\partial b}}{\left[\lambda_c(k-1+t_1^*)\right]^2}\right\} ,\\
\frac{\partial^2 l}{\partial a^2} &= -\sum_{k=1}^{c} n_{\cdot,k}^{(\cdot)} \left[\frac{\frac{\partial \lambda_c(k-1+t_1^*)}{\partial a}}{\lambda_c(k-1+t_1^*)}\right]^2 ,\\
\frac{\partial^2 l}{\partial a\,\partial b} &= -\sum_{k=1}^{c} n_{\cdot,k}^{(\cdot)} \left[\frac{\frac{\partial \lambda_c(k-1+t_1^*)}{\partial a}\,\frac{\partial \lambda_c(k-1+t_1^*)}{\partial b}}{\lambda_c(k-1+t_1^*)}\right]^2 ,\\
\frac{\partial^2 l}{\partial b^2} &= -\sum_{k=1}^{c} n_{\cdot,k}^{(\cdot)} \left[\frac{\frac{\partial \lambda_c(k-1+t_1^*)}{\partial b}}{\lambda_c(k-1+t_1^*)}\right]^2 ,
\end{aligned}$$

where

$$
\begin{aligned}
\frac{\partial^2 \Lambda(t)}{\partial p_c^2} &= \frac{D\,B(p_1,q_1)}{\alpha_1^*}\left[\lfloor\frac{t}{c}\rfloor\sum_{j=0}^{c-1}\frac{\partial^2\lambda_c(j+t_1^*)}{\partial p_c^2}+\sum_{j=0}^{\lfloor t-\lfloor\frac{t}{c}\rfloor c\rfloor-1}\frac{\partial^2\lambda_c(j+t_1^*)}{\partial p_c^2}\right],\\
\frac{\partial^2 \Lambda(t)}{\partial p_c\,\partial a} &= \frac{D\,B(p_1,q_1)}{\alpha_1^*}\left[\lfloor\frac{t}{c}\rfloor\sum_{j=0}^{c-1}\frac{\partial^2\lambda_c(j+t_1^*)}{\partial p_c\,\partial a}+\sum_{j=0}^{\lfloor t-\lfloor\frac{t}{c}\rfloor c\rfloor-1}\frac{\partial^2\lambda_c(j+t_1^*)}{\partial p_c\,\partial a}\right],\\
\frac{\partial^2 \Lambda(t)}{\partial p_c\,\partial b} &= \frac{D\,B(p_1,q_1)}{\alpha_1^*}\left[\lfloor\frac{t}{c}\rfloor\sum_{j=0}^{c-1}\frac{\partial^2\lambda_c(j+t_1^*)}{\partial p_c\,\partial b}+\sum_{j=0}^{\lfloor t-\lfloor\frac{t}{c}\rfloor c\rfloor-1}\frac{\partial^2\lambda_c(j+t_1^*)}{\partial p_c\,\partial b}\right],
\end{aligned}
$$

while

$$
\begin{aligned}
\frac{\partial^2\lambda_c(j+t_1^*)}{\partial p_c^2} &= \frac{\partial\lambda_c(j+t_1^*)}{\partial p_c}\Bigg\{\ln\big(\frac{t-m_c}{c}-\lfloor\frac{t-m_c}{c}\rfloor\big)-\ln\big(\frac{t_c^*-m_c}{c}\big)\\
&\quad+\big(\frac{2}{3}\big)\Big[\ln\big(1-\frac{t-m_c}{c}-\lfloor\frac{t-m_c}{c}\rfloor\big)-\ln\Big(1-\frac{t_c^*-m_c}{c}\Big)\Big]\Bigg\},\\
\frac{\partial^2\lambda_c(j+t_1^*)}{\partial p_c\,\partial a} &= \left[\frac{\partial\lambda_c(j+t_1^*)}{\partial a}-1\right]\Bigg\{\ln\big(\frac{t-m_c}{c}-\lfloor\frac{t-m_c}{c}\rfloor\big)-\ln\big(\frac{t_c^*-m_c}{c}\big)\\
&\quad+\big(\frac{2}{3}\big)\Big[\ln\big(1-\frac{t-m_c}{c}-\lfloor\frac{t-m_c}{c}\rfloor\big)-\ln\Big(1-\frac{t_c^*-m_c}{c}\Big)\Big]\Bigg\},\\
\frac{\partial^2\lambda_c(j+t_1^*)}{\partial p_c\partial b} &= \frac{\partial\lambda_c(j+t_1^*)}{\partial b}\Bigg\{\ln\big(\frac{t-m_c}{c}-\lfloor\frac{t-m_c}{c}\rfloor\big)-\ln\big(\frac{t_c^*-m_c}{c}\big)\\
&\quad+\big(\frac{2}{3}\big)\Big[\ln\big(1-\frac{t-m_c}{c}-\lfloor\frac{t-m_c}{c}\rfloor\big)-\ln\Big(1-\frac{t_c^*-m_c}{c}\Big)\Big]\Bigg\}.
\end{aligned}
$$

Now the elements of the information matrix $\mathbf{I}(\theta)$ can be calculated with the following formulas:

$$
\begin{aligned}
-E\left[\frac{\partial^2 l}{\partial p_1^2}\right] &= \frac{\partial^2\Lambda(t)}{\partial p_1^2}-\sum_{j=1}^{J}E\big[n_{j,.}^{(\cdot)}\big]\left[\frac{B'_{p_1}(p_1,q_1;\frac{jd}{D})-B'_{p_1}(p_1,q_1;\frac{(j-1)d}{D})}{B(p_1,q_1;\frac{jd}{D})-B(p_1,q_1;\frac{(j-1)d}{D})}\right]'_{p_1},\\
-E\left[\frac{\partial^2 l}{\partial p_1\,\partial p_c}\right] &= \frac{\partial\Lambda(t)}{\partial p_c}\left[\frac{B'_{p_1}(p_1,q_1)}{B(p_1,q_1)}-\nu*\right],\\
-E\left[\frac{\partial^2 l}{\partial p_1\,\partial a}\right] &= \frac{\partial\Lambda(t)}{\partial a}\left[\frac{B'_{p_1}(p_1,q_1)}{B(p_1,q_1)}-\nu*\right],\\
-E\left[\frac{\partial^2 l}{\partial p_1\,\partial b}\right] &= \frac{\partial\Lambda(t)}{\partial b}\left[\frac{B'_{p_1}(p_1,q_1)}{B(p_1,q_1)}-\nu*\right],
\end{aligned}
$$

$$
\begin{aligned}
-E\left[\frac{\partial^2 l}{\partial p_c^2}\right] &= \frac{\partial^2\Lambda(t)}{\partial p_c^2} - \sum_{k=1}^{c} E\big[n^{(\cdot)}_{\cdot,k}\big]\left\{\frac{\frac{\partial^2\lambda_c(k-1+t_1^*)}{\partial p_c^2}}{\lambda_c(k-1+t_1^*)} - \left[\frac{\frac{\partial\lambda_c(k-1+t_1^*)}{\partial p_c}}{\lambda_c(k-1+t_1^*)}\right]^2\right\}, \\
-E\left[\frac{\partial^2 l}{\partial p_c\,\partial a}\right] &= \frac{\partial^2\Lambda(t)}{\partial p_c\,\partial a} - \sum_{k=1}^{c} E\big[n^{(\cdot)}_{\cdot,k}\big]\left\{\frac{\frac{\partial^2\lambda_c(k-1+t_1^*)}{\partial p_c\,\partial a}}{\lambda_c(k-1+t_1^*)} - \frac{\frac{\partial\lambda_c(k-1+t_1^*)}{\partial p_c}\,\frac{\partial\lambda_c(k-1+t_1^*)}{\partial a}}{\big[\lambda_c(k-1+t_1^*)\big]^2}\right\}, \\
-E\left[\frac{\partial^2 l}{\partial p_c\,\partial b}\right] &= \frac{\partial^2\Lambda(t)}{\partial p_c\,\partial b} - \sum_{k=1}^{c} E\big[n^{(\cdot)}_{\cdot,k}\big]\left\{\frac{\frac{\partial^2\lambda_c(k-1+t_1^*)}{\partial p_c\,\partial b}}{\lambda_c(k-1+t_1^*)} - \frac{\frac{\partial\lambda_c(k-1+t_1^*)}{\partial p_c}\,\frac{\partial\lambda_c(k-1+t_1^*)}{\partial b}}{\big[\lambda_c(k-1+t_1^*)\big]^2}\right\}, \\
-E\left[\frac{\partial^2 l}{\partial a^2}\right] &= \sum_{k=1}^{c} E\big[n^{(\cdot)}_{\cdot,k}\big]\left[\frac{\frac{\partial\lambda_c(k-1+t_1^*)}{\partial a}}{\lambda_c(k-1+t_1^*)}\right]^2, \\
-E\left[\frac{\partial^2 l}{\partial a\,\partial b}\right] &= \sum_{k=1}^{c} E\big[n^{(\cdot)}_{\cdot,k}\big]\left[\frac{\frac{\partial\lambda_c(k-1+t_1^*)}{\partial a}\,\frac{\partial\lambda_c(k-1+t_1^*)}{\partial b}}{\lambda_c(k-1+t_1^*)}\right]^2, \\
-E\left[\frac{\partial^2 l}{\partial b^2}\right] &= \sum_{k=1}^{c} E\big[n^{(\cdot)}_{\cdot,k}\big]\left[\frac{\frac{\partial\lambda_c(k-1+t_1^*)}{\partial b}}{\lambda_c(k-1+t_1^*)}\right]^2,
\end{aligned}
$$

where

$$
\begin{aligned}
E\big[n^{(\cdot)}_{j,\cdot}\big] &= D\left[B(p_1,q_1;\frac{jd}{D}) - B(p_1,q_1;\frac{(j-1)d}{D})\right] \\
&\quad \left[\lfloor\frac{t}{c}\rfloor \sum_{j=0}^{c-1}\frac{\lambda_c(j+t_1^*)}{\alpha_1^*} + \sum_{j=0}^{\lfloor t-\lfloor\frac{t}{c}\rfloor c\rfloor-1}\frac{\lambda_c(j+t_1^*)}{\alpha_1^*}\right], \qquad j=1,2,\ldots,6, \\
E\big[n^{(\cdot)}_{\cdot,k}\big] &= \begin{cases} \big(\lfloor\frac{t}{c}\rfloor+1\big)\,D\,B(p_1,q_1)\,\frac{\lambda_c(k-1+t_1^*)}{\alpha_1^*}, & 1\le k\le \lfloor t-\lfloor\frac{t}{c}\rfloor c\rfloor \\ \lfloor\frac{t}{c}\rfloor\,D\,B(p_1,q_1)\,\frac{\lambda_c(k-1+t_1^*)}{\alpha_1^*}, & \lfloor t-\lfloor\frac{t}{c}\rfloor c\rfloor < k\le c \end{cases}.
\end{aligned}
$$

For the $1899-2000$ hurricanes data with the 4–parameter double–beta intensity, the MLE's obtained are $\hat{p}_1 = 3.0145$, $\hat{p}_c = 1.5463$, $\hat{a} = 3.2354$ and $\hat{b} = 6.9634$. The corresponding information matrix and its inverse are given by

$$
\mathbf{I}(\theta) = \begin{pmatrix} 12.76374 & 2.64269 & -1.39801 & -3.487777 \\ 2.64269 & 3.14166 & -0.53981 & -1.94906 \\ -1.39801 & -0.53981 & 1.80932 & 0.32309 \\ -3.48777 & -1.94906 & 0.32309 & 2.75323 \end{pmatrix},
$$

$$\mathbf{I^{-1}}(\theta) = \begin{pmatrix} 0.12831 & 0.00000 & 0.07162 & 0.15414 \\ 0.00000 & 0.58574 & 0.10286 & 0.40258 \\ 0.07162 & 0.10286 & 0.62256 & 0.09049 \\ 0.15414 & 0.40258 & 0.09049 & 0.83285 \end{pmatrix}.$$

Hence the estimated standard deviations for MLE's $\hat{p}_1$, $\hat{p}_c$, $\hat{a}$ and $\hat{b}$ are 0.3582, 0.7653, 0.7890 and 0.9126, respectively.

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