A mesoscopic whole link continuous vehicle bunch model for multiclass

traffic flow simulation on motorway networks

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A Thesis

In

The Department

of

Concordia Institute for Information Systems Engineering (CIISE)

Presented in Partial Fulfillment of the Requirements

for the Degree of Master of Applied Science (Quality Systems Engineering)

at Concordia University

Montreal, Quebec, Canada

March 2011

CONCORDIA UNIVERSITY

School of Graduate Studies

This is to certify that the thesis prepared

By: ANATOLIJ KAREV

Entitled: *A mesoscopic whole link continuous vehicle bunch model for multiclass*

                traffic flow simulation on motorway networks

and submitted in partial fulfillment of the requirements for the degree of

Master of Applied Science (Quality Systems Engineering)

complies with the regulations of the University and meets the accepted standards with

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ABSTRACT

A mesoscopic whole link continuous vehicle bunch model for multiclass traffic flow simulation on motorway networks

Anatolij Karev

Modeling of heterogeneous driver behaviour is vital to understanding of dynamic traffic phenomenon taking place on motorway networks. In this research, we present a mesoscopic whole link continuous vehicle bunch model for multiclass traffic flow simulation on motorway networks. Two main attributes of traffic flow classification have been used are: (i) vehicle type, specifying in turn a vehicle length and, together with type of a preceding vehicle, time headway; and, (ii) desired speed, defining together with the speeds of the neighbouring vehicles, the vehicle acceleration/deceleration mode. It is assumed that vehicles in uncongested to moderate congested flow move in bunches dividing the drivers into the two main groups: (i) independent "free" drivers which usually manifest themselves as leaders of bunches; and, (ii) followers, or drivers which adapt their speed to the leader's speed and follow each other at constrained headways specified by predecessor/successor pairs. The model proposes a solution to arbitrary traffic queries involving a motion in bunches having various speed and size by assuming the rate of driver arrivals follows semi-Poisson distribution and proportion of free drivers is predefined. The solution, assuming limited overtaking possibilities for all drivers, involves formation of longer queue behind bunches moving with slower speed and transformation of some of the "leaders" into "followers" because of adjustment their speed to the speed of the preceding slow-moving bunches. The present solution considers

both stochastic and deterministic features of traffic flow and, therefore, may be easily

extended to a specific uncertainty level.

I dedicate this work

to my parents,

my spouse,

and my son Alexander.

Acknowledgements

I would like to express my deepest appreciation and gratitude to Professor Anjali Awasthi for her guidance, encouragement and advices throughout the years of my graduate study at the Concordia University, Montreal, Quebec, Canada. To have an opportunity to improve my knowledge in transportation under the professional guidance has been an invaluable experience for me.

I am also deeply indebted to many professors at CIISE, Concordia University for giving me an opportunity to take their classes, learn more about Quality Systems Engineering and get a great benefit from the respective knowledge dissemination. To mention some among them: Profs A. Ben Hamza, N. Bouguila, Z. Tian and C. Wang.

I also would like to acknowledge an indispensable support by my spouse, Elena, who patiently accepted redistribution of the duties throughout the period of my studies and by my son, Alexander, who has been giving me a powerful impetus to finish my graduate studies.

I also would like to thank most of my colleagues, discussions with whom might help me in this research study.

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List of Acronyms

CTM   -Cell Transmission Model

DTA   - Dynamical Traffic Assignment

IDM   -Intelligent Driver Model

LWR   - Lighthill, Whitham and Richards Model

MN   - Merchant Nemheuser Model

PQM   -Point Queue Model

RNG   - Random Number Generator

STA   - Static Traffic Assignment

Chapter 1:

Introduction

1.1 BACKGROUND

Traffic congestion continues to be a strong headache for transportation engineers and municipal administrations in the industrialized countries. Enormous augmentation of per capita number of personal vehicles combined with steadily growing number of transport operations during last decades multiplied by constraints in the traffic flow capacities of the already developed urban arteries produce a fairly dispirit perspective for the future of the vehicular transportation. An appropriate municipal regulation policy (McCarthy, 2001) and traffic optimisation (Peeta and Ziliaskopoulos, 2001) may help improve the current situation when frequent traffic breakdowns become integral part of the vehicular transportation. In order to overcome the problems arising from traffic congestion, two main methods of investigation have been employed in this field over the last several decades, and they will be briefly outlined below.

1.2 TWO APPROACHES IN INVESTIGATING TRAFFIC CONGESTION

1.2.1 Continuous Field Measurements

Gathering of natural traffic counts data, together with simultaneous measuring of traffic elements in order to find the statistical relationships between them, is one of the ways of investigating the transportation phenomenon called traffic congestion. The existence of already acquired and pre-processed database containing dynamic traffic attributes such as traffic counts (veh./h), vehicle speed (km/h), traffic occupancy (%) and traffic density

data (veh./km) together with traffic heterogeneity data for each specific transportation region is of major importance for designers of traffic network structures. Also, such a database is essential for validation of experimental and theoretical simulations of the traffic networks. Typical examples of this type of approach have been considered in the studies by Kerner and Rehborn (1997), Windover and Cassidy (2001) and Schönhof and Helbing (2007). The disadvantages of this approach involve: i) costs for the installation of measurement sites for a specific location; ii) permanent management requirements; and, what is the most important, iii) limitations of the objectives of a typical experimental study.

1.2.2 Construction of Mathematical /Computational Traffic Flow Models

Traffic flow modeling remains one of the best methods for investigation of the traffic congestion. It allows predicting spatio-temporal characteristics of the anticipated traffic congestion and the laws of its spreading inside the transportation network. This method involves using theoretical models which are based on the known physics of the traffic dynamics, and it has obvious advantages especially when implemented in the form of a computational code. The method, however, possesses few disadvantages as well. One of the major disadvantages is that it requires the input of traffic attributes associated with a particular traffic phenomenon, which, as mentioned earlier, can be laborious and costly to monitor *in situ*. Moreover, since the composition of traffic flow always produces specific characteristics, it is sometimes not clear which of these characteristics plays a crucial role during specific traffic phenomena. Even such a disadvantage, however, cannot detract

from the growing usefulness of this method as it continues to increase in parallel with improving computer potentials and advances in computational accuracy.

Traffic flow models can be distinguished from one another by cause effects characteristics of traffic phenomenon such as (i) the type of traffic flow phenomena modeled; (ii) methods for the solution of a physical problem; and, (iii) the role of certain factors involved in the traffic phenomenon. The same models may as well be classified according to the physical characteristics of traffic flow such as (i) level of detail (microscopic, microscopic and macroscopic); (ii) level of dynamics of the processes considered (time-independent and time-dependent); (iii) the extent to which the mathematical apparatus is applied; (iv) scale of independent variables (continuous, discrete and semi-discrete) and so forth (Hoodgendoorn and Bovy, 2001).

Traffic flow on free motorways without intersections and on-ramp merging zones has been modeled since early fifties of the last century by using macroscopic, mesoscopic or microscopic approaches. Microscopic traffic flow models consider driver's psycho-behavioural perception indicators of the traffic flow such as driver's sensitivity, perception, decision and breaking times and their relationships to the spatio-temporal characteristics of a singular transportation unity such as time gaps, headways and passage times. These models mostly rely on empirical data collected from field campaigns (Banks, 2003). Macroscopic traffic flow models consider the traffic flow as a flow of fluid throughout network without distinguishing flow participants. Therefore, such models present the traffic flow through typical traffic flow attributes such as fluid flow

rate, density and speed which follow standard fluid dynamics laws. These models may be fed by both empirical experimental data and physical flow characteristics and validated against real traffic data. Mesoscopic models consider combinations of the factors important for micro- and macroscopic models. These types of traffic models very frequently follow gas-kinetic physical laws (Hoodgendoorn and Bovy, 2001). Papageorgiou (1998) aptly argued that, even though the macroscopic flow models are closer to the developed domains in physics than the other types of models, it is in no vein to expect from them to show the same level of descriptiveness as traditional physical laws because they do not account for many traffic specificities. In other words, the macroscopic models of traffic flow are rather rough approximation of empirically obtained data by using existing physical laws in adjacent scientific fields than precise physical laws extracted and validated in the original field.

1.3 TRAFFIC HETEROGENEITY IN LWR MODELS

The first time dependent macroscopic traffic flow model involving mathematical calculation was produced in the early fifties (Lighthill and Whitham, 1955; Richards, 1956). The model which is frequently called LWR model or simply kinematic wave model has established a fundamental relationship among the main traffic attributes such as traffic flow, traffic density and vehicle speed based on fluid dynamics laws. The model assuming flow continuity proposes existence of an equilibrium state, every deflection from which produces kinematical waves.

Traffic heterogeneity is an integral part of the traffic flow, the fact that was somehow "forgotten" up to the last decade in the modeling of traffic flow at the free motorways without intersections and on-ramp merging zones. However, several important extensions to LWR model have been proposed over the course of the past decade to account for most important microscopic features and driver behavioural attitudes that allowed developing of traffic heterogeneity theory (Ossen and Hoogendoorn, 2011). In particular, Lebacque et al. (1998) considered different vehicle lengths, Hoogendoorn and Bovy (2000), Wong and Wong (2002), Zhang and Jin (2002), Jiang and Wu (2004) scrutinized different desired speeds and respective speed gradient appearance in the continuity equation, while Chanut and Buisson (2007) and Van Lint et al. (2008) considered both factors, different vehicle lengths and different desired speeds. Finally, the different speed-density relations were considered by Zhu et al. (2003), while Daganzo with coworkers and colleagues in a series of papers (Daganzo, 1997; Daganzo et al., 1997; and, Cassidy et al., 2009) showed specifics of the priority vehicles using special lanes. It was shown that considering these specifics of traffic flow, it was possible to explain various transportation phenomena observed before such as capacity drop, formation and discharge of platoons, moving bottlenecks and so forth. Moreover, Treiber and Helbing (1999) were able to explain the wide scattering of experimental data on density-flow diagram by varying a proportion of light and heavy vehicles in the flow in the course of the time. Even though the importance of traffic heterogeneity and speed variation in dissipation of the congested flow data was unambiguously underlined in their research, they admitted a predominant role of the time headways in the same experimental observations.

1.4 THESIS OBJECTIVES

A network traffic model may solve an arbitrary traffic flow query, a least-cost or a least-traffic-volume-delay optimization problem or, more generally, any optimal control problem in a different way than classical traffic flow models. Network traffic flow modeling is an indispensable tool in transportation management allowing for capturing of real-time traffic conditions, using transportation-related information effectively and choosing efficient routing strategy to timely avoid anticipated congestion. Based on real-time transportation data, the dynamic network models are capable to (i) accurately locate phase transition in the network from the free traffic flow to the congested flow; (ii) efficiently predict the main effects of the congestion propagation inside the network; (iii) consider the driver reaction on the anticipated congestion; and, (iv) propose possible network regulation strategies. A number of network traffic models have appeared during the last decades aiming on solving a broad spectrum of problems. Up to now, macroscopic traffic flow models were more successful in challenging these tasks because of their simpler nature than microscopic models which, though capturing typical driver behavioural characteristics, require greater CPU time and system memory.

Recently, however, a new class of the network traffic flow models appeared called mesoscopic continuous/discontinuous models of vehicle bunches. This type of modeling has already shown to be superior to the previous approaches because of combining greater behavioural realism and simplicity of the fluid flow representation together. Although a few specific traffic heterogeneity factors have already been considered in the macroscopic approach, to the best of authors' knowledge *no attempts have been done to*

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account for the most traffic heterogeneity factors together in continuous modeling of vehicle bunches in traffic networks. This would be the principal objective of this thesis where we are proposing a completely new continuous single link model of vehicle bunches for single link applicable to traffic networks modeling. In particular, we expect *to elucidate and evaluate the importance of the following heterogeneity factors: vehicle type and length, bunch size, vehicle-specified time headways and desired speed distribution*. The key factors among them will be revealed and quantified, accurately and systematically, by using simulation-based approach which will replicate traffic flow dynamics features involving aforementioned traffic heterogeneity factors. A further objective of this research is *to address the abovementioned traffic heterogeneity factors by combining both deterministic and stochastic traffic flow features in the complex traffic network modeling.* The examples of the deterministic features may be the experimental time headway and desired speed distributions, while an example of the stochastic feature is the way how different flow participants mix inside the entire flow. We will also try to address the issue of the spatio-temporal predictability of congestion and its dependence on the specific features such as car bunching and network loading procedures.

1.5 METHODOLOGY

After an extensive review of the literature and revealing advantages and disadvantages of the existing whole link models, an analytical study on the traffic heterogeneity factors will be performed showing the specific ranges of their importance for the parameterization. Our next step will be to propose a mesoscopic continuous vehicular bunch single link model capable of taking into account most important features of traffic

heterogeneity. Finally, a numerical approach will be developed to provide solution to the proposed parameterization.

1.6 STRUCTURE OF THE THESIS

The rest of the thesis is organised as follows. Some of the traffic features and specific phenomena important for description of congestion using macroscopic flow models and several classic dynamic traffic assignments models for traffic network will be discussed in Chapter 2. Chapter 3 presents the problem description. Chapter 4 presents the elements of the proposed mesoscopic continuous whole link vehicle bunch model applicable to network traffic modeling and considering several features of traffic heterogeneity. In Chapter 5, we will provide a numerical application of the proposed model and compare the results of this new-built model with the results obtained from previously known models and reveal some eventual improvements over them. Finally, in Chapter 6, we present the conclusions and future works. The physics of the traffic phenomena and the mathematics of its presentation will also be discussed in full detail.

Chapter 2:

Literature Review

2 EXPERIMENTAL OBSERVATIONS OF TRAFFIC PHENOMENA

Firstly, we will consider single-lane traffic flow to present the basic phenomena valid for such type of traffic flows and physical equations capable to describe them. Then, the more complex multi-lane traffic flow will be presented. Finally, the comprehensive flow involving acceleration/deceleration and multi-class traffic flow will be described.

2.1. SINGLE-LANE TRAFFIC PHENOMENA

Early studies did not recognize multi-lane traffic features and considered general traffic phenomena derived from either a single-lane traffic experimental data or two-lane averaged data. The studies also assumed that the relationships between flow attributes do not depend on type of the freeway and type of the traffic flow it serves.

2.1.1 Speed-spacing or speed-density relations

The first studies on the traffic flow were concentrated on finding maximum theoretical capacity of a single traffic lane involving traffic flow attributes such as traffic flow, q, measured in number of vehicle per hour; traffic flow density, ρ, measured in number of vehicles per km; flow speed, v, measured in km per hour; and, spacing between vehicles, s, measured in meters. The flow was considered as *time-independent*; while it was assumed that, for most of the time, an *equilibrium state* exists during which the

relationship between each two flow attributes may be defined as an algebraic linear or non-linear expression. Such relationships called flow diagrams were subjects for the early experimental investigations in the field. The two earliest studies of such kind were that by Hamlin (1927) and Johnson (1928). Both studies considered the driver behaviour as one of the factors defining the relationship between flow speed, v, and vehicle spacing, s, which was generally formulated as follows:

$$s = av^2 + bv + c \qquad (2.1)$$

where a, b and c are constants. Here, the quadratic term represented braking distance; linear term represented the driver reaction time; while the free term represented vehicle spacing defined for stationary traffic. The relationship between the theoretical maximum flow volume for a single lane, q_{max}, and flow speed, v, was defined as follows:

$$q_{max} = 1000 \cdot v / s \qquad (2.2)$$

Here, a factor *1000* appears as result of different units used for flow speed and spacing. Combining (2.1) and (2.2), the theoretical maximum capacity of the lane may be found corresponding to an optimum speed. The maximum capacity was found to be about 1970 veh/h for the optimum speed of about 35 km/h in the study by Hamlin (1927) and around 2650 veh/h for the optimum speed of about 55 km/h in the study by Johnson (1928). However, considering the sensitivity of (2.1) to many behavioural drivers' characteristics, it was concluded that the use of empirical relationships between s and v was preferable. In the study by Johnson (1928), such empirical relation was defined as follows:

$$s = 0.082v^{1.3} + 4.6 \tag{2.3}$$

Where the first term in (2.3) was called inter-vehicle clearance and second one called the average vehicle length. Combining (2.2) and (2.3), the real capacity of the lane may be found as follows:

$$q_{max} = 1000v/(0.082v^{1.3} + 4.6) \tag{2.4}$$

As mentioned in the introduction, the most important pioneering macroscopic model of traffic flow was the one developed by Lighthill and Whitham (1955) and independently by Richards (1956). The model frequently known as LWR model uses a simple physical relationship between the traffic attributes mentioned earlier and its evolution in time. The mathematical definition of this relation is given by:

$$q = \rho \cdot v \tag{2.5}$$

Thus, the density of flow, ρ, may be found by defining vehicle spacing. Even before Lighthill and Whitham (1955), Greenshields (1934) first proposed that speed and flow density are in the inverse linear proportionality to each other as follows:
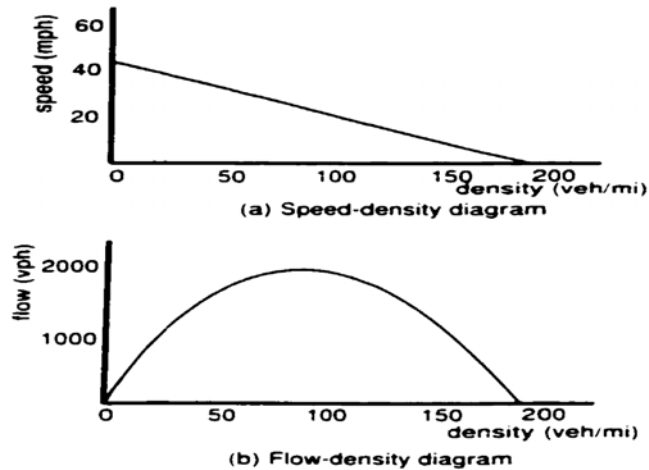
Figure 2.1 *Speed-density and flow-density relations proposed by Greenshields (1934).*

$$v = v_f \left(1 - \rho / \rho_j\right) \tag{2.6}$$

where v_f is maximum speed of traffic and ρ_j represent traffic flow density at jam, while traffic flow has symmetric parabolic shape with zero skewness (*Fig. 2.1*). Greenberg (1959) proposed logarithmic relationship between the traffic flow speed and traffic density for freeway traffic flows as follows:

$$v = v_C \ln\left(\rho_j / \rho\right) \tag{2.7}$$

where v_C is some characteristic constant, defined as a vehicle space mean speed at which maximum flow or capacity is reached. Assuming faster growth of the speed at low flow densities and slower speed drop at high flow densities, the relationship between the traffic flow and its density is an asymmetric parabola with positive skewness *(Fig.2.2)*.
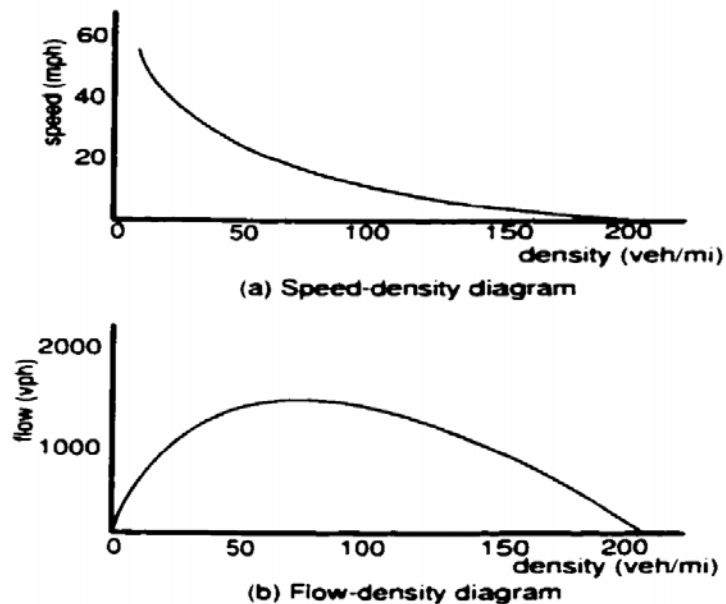
Figure 2.2 *Speed-density and flow-density relations proposed by Greenberg (1959).*

The exponential empirical speed-density relationship was proposed by Underwood (1961) for Merrit Parkway data:

$$\bar{v}_s = v_f \exp\left(-\frac{\rho}{\rho_C}\right)$$

(2.8)

where $\bar{v}_s$ is space-averaged mean speed, and ρ_C is traffic density corresponding to maximum flow rate. Another type of exponential speed-density relationship was proposed by Newell (1961):

$$v = v_f \left[1 - \exp\left\{-\frac{\lambda}{v_f}\left(\frac{1}{\rho} - \frac{1}{\rho_j}\right)\right\}\right]$$

(2.9)

where a new parameter was added λ which is the slope of spacing-speed curve at $v = 0$. A power relationship between traffic speed and traffic density was proposed by Pipes (1967) as follows:

$$v = v_f (1 - \rho/\rho_j)^n$$

(2.10)

May and Keller (1967) proposed more generalized form of power relationship for Eisenhower Expressway data as follows:

$$\bar{v}_t = v_f \left[1 - (\rho/\rho)^{1.8}\right]^5$$

(2.11)

where $\bar{v}_t$ is time-averaged mean speed. The space-averaged and time averaged speeds are related as follows:

$$\bar{v}_s = \bar{v}_t - \frac{\sigma_t^2}{\bar{v}_t}$$

(2.12)

where σ_t^2 is variance for time-averaged mean speed. Two power indexes may be further also generalized to obtain the following more complex relationship:

$$\bar{v}_t = v_f \left[1 - (\rho/\rho)^m \right]^n \tag{2.13}$$

Another complex relationship combining both exponential and power features was proposed by Drake et al. (1967) in the following shape:

$$\bar{v}_t = v_f \exp \left[-\frac{1}{2} \left(\frac{\rho}{\rho_j} \right)^2 \right] \tag{2.14}$$

More recent complex speed-density relationships were proposed by Del Castillo and Benítez (1995) and Ou et al. (2006). Recently, Leclercq (2005) and Geroliminis and Daganzo (2008) found that the non-linear complex relationship between the flow speed and traffic density is universal for both urban and freeway-traffic areas.

Equilibrium state may be defined by any combination of two out of three flow attributes together with (2.5). Combining all three plots together into one diagram and considering also the driver behaviour, one can obtain *fundamental diagram of traffic flow* (May, 1990).The importance of this diagram for understanding several standard traffic flow phenomena is beyond question, because it combines operational characteristics of traffic with operating traffic volume characteristics. Maitra et al. (1999), for example, propose the use of traffic speed/flow relationship for defining and quantifying congestion conditions, while the speed/density part which was called *equilibrium function of traffic flow* is an object of continuous studies on improvement of macroscopic models (Ou et al.,

2006). Hereafter, the parts of the fundamental diagram will be used for presenting typical

traffic flow phenomena.

2.1.2 Capacity drop: reversed-lambda or invert-V diagrams

Figure 2.3 presents several important traffic flow phenomena: a) capacity drop; b) two-

loop hysteresis formation; c) single-loop hysteresis; and, d) platoon dispersion.
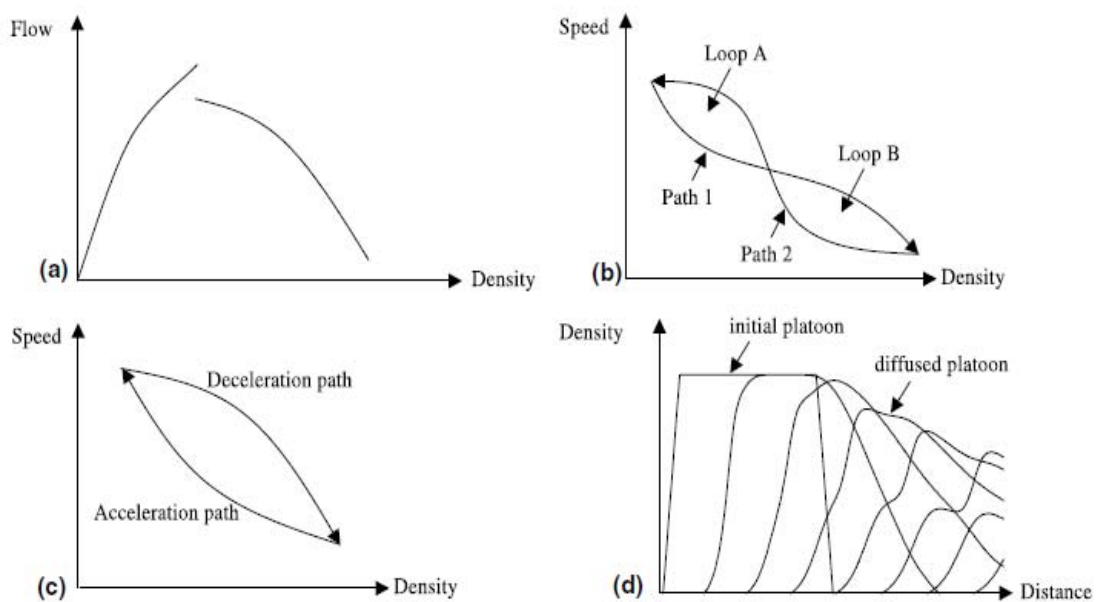
Figure 2.3 T*raffic flow phenomena: (a) two-capacity phenomenon; (b) two-loop hysteresis; (c) single loop hysteresis; (d) platoon dispersion, reproduced from Wong and Wong (2002).*

Two-capacity or capacity drop (*Fig. 2.3a*) is the most interesting phenomenon found by

Eddie (1961) from a study on Lincoln Tunnel traffic data. This phenomenon shows that

there are two different levels of traffic flow at a specific traffic flow density depending on

the direction of approaching to the maximum flow rate. Since the left and right curves in

Figure 2.3a present the operations at the free and congested flow, respectively, a discontinuity in the flow capacity may be observed when the curves are approaching to each other. In other words, it means that approaching to the peak in flow rate for the free and congested flows is accomplished in completely different ways though in both cases a deceleration in the flow rate increase has been observed. Therefore, Eddie's model defined *two-phase traffic flow*, where each phase had completely different characteristics.

The difference in the capacity at the flow-rate peak was found to be from 6 to 9 percent according to various authors. Recall that both researches, done by Greenshields (1934) and by Lighthill and Whitham (1955), respectively, have not found this phenomenon due to absence of the experimental data referring to the maximum flow rate. Lighthill and Whitham (1955), who used speed/traffic flow data for the left curve and headway/speed data for the right curve, proposed that both curves are the sides of the same parabola and matched them by interpolating both curves to the conjectural region of maximum traffic flow rate. Details of Figure 2.3(b-d) will be provided in Section 2.1.3.

A completely different type of this inherent characteristic of the traffic congestion was found by other researchers *(Fig. 2.4)*. Koshi et al. (1983) investigated the data for the Tokyo expressway and found that an increase in flow rate for congested (stop-and-go) traffic flow when approaching to the peak in flow rate distribution may exhibit accelerating features instead of the decelerating ones as for developing free flow *(Fig. 2.4a)*. This type of dependence was later called as reversed-lambda diagram. Another type of the flow-density distribution called inverted-V diagram was found by Hall et al.

(1986) for Queen Elizabeth Way between Oakville and Toronto *(Fig. 2.4b)*. For this kind of distribution the discontinuity in the capacity was found to be very small or almost indiscernible.
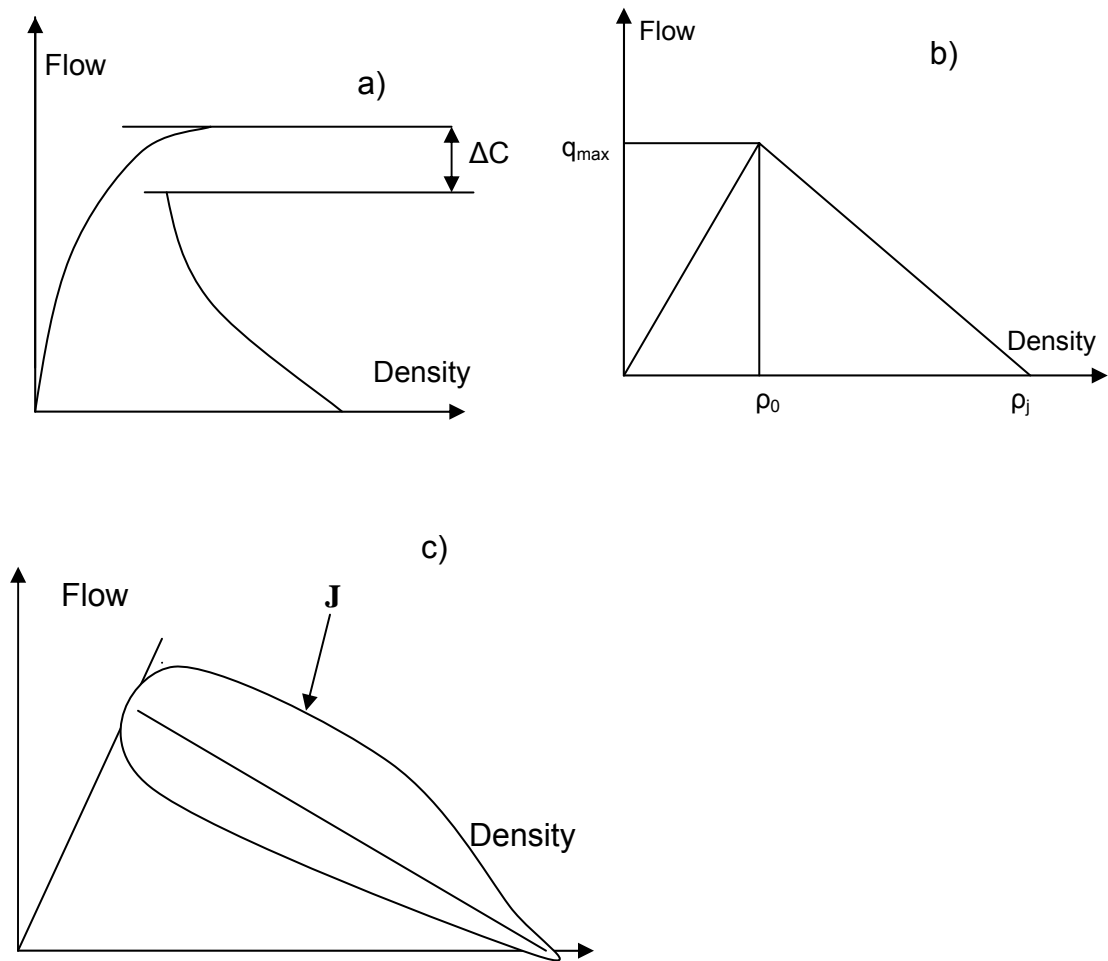
Figure 2.4 *Several variants of the capacity drop phenomenon: a) reversed-lambda law discovered by Koshi at al. (1983); b) inverted-V diagram by Hall et al. (1986); c) Kerner's relationship between free and synchronised flows (Kerner, 1998).*

A significant advance in understanding of the physics of multiphase multilane traffic flow occurred with the researches by Kerner and Rehborn (1997) and Kerner (1998). A three-phase traffic flow was proposed, including *free flow, synchronized flow and jam flow (Fig.2.4c)*. Synchronized flow was defined as a traffic flow when the driver should adjust his speed according to the surrounding flow density. Such synchronization included either overtaking including changing lane and acceleration or remaining in the same lane and adjusting the speed to the speed of the preceding vehicle. Single *jam* traffic was found to be formed from free flow only, while stop-and-go traffic flow characterized with numerous narrow jams and accomplished local phase transition from free to synchronized flow could be formed from the synchronized flow. The flow density diagram for this theory is given if Figure 2.4c. The nature of traffic flow in this theory assumes multi-lane traffic presentation and therefore will be considered later on in the subsection devoted to the multi-lane flow phenomena.

It appears, therefore, that the type of flow/density distribution cannot be unified for various highways since it depends on the type of the highway and type of the traffic flow it serves.

2.1.3. Hysteresis phenomena

The traffic hysteresis *(Fig. 2.3b and Fig. 2.3c)* is phenomenon consisting in arriving to different points of speed-density diagram, i.e. different traffic phases, when one moves from different its sides. It was firstly defined and explained by Newell (1965) and Treiterer and Myers (1974) who presented the experimental observations of the more

complex phenomenon including that explained by Newell (1965). Newell (1965) using experimental data acquired by Port of New York Authority defined that the car spacing observed during process of acceleration of flow is significantly longer as compared to the corresponding spacing observed during deceleration of flow. In the speed-spacing diagram *(Fig. 2.5)*, this phenomenon will correspond to the closed-loop observed between two different speed-acceleration relationships. Newell explained this observation by the late reaction of the succeeding car driver on the sudden acceleration or sudden deceleration by the lead car driver. Each subsequent car driver is late with his decision for acceleration or deceleration. Newell considered this assumption even in his earlier research (Newell, 1961) to explain the non-linear effects that may be produced by this phenomenon.
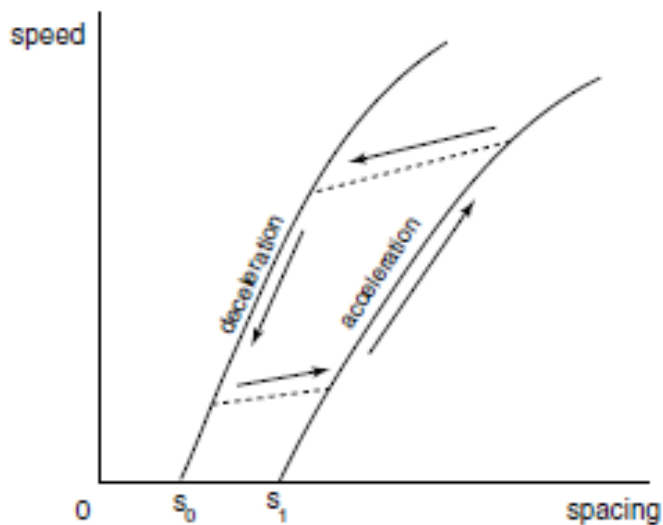
Figure 2.5. *Speed-spacing diagram by Newell (1965), reproduced.*

Disturbance, either deceleration from $v = \frac{1}{2}V$ to $v = 0$ *(Fig. 2.6)* or acceleration from

$v = 0$ to $v = \frac{1}{2}V$ *(Fig. 2.7)*, is propagating with a progressively increasing time lag

between the first lead car and all succeeding cars. That is to say, each subsequent car

reacts slower than the previous one and so on. A time lag increases rapidly as the

observer advances from the first succeeding car to the last one. The solution for the

indefinitely elongated car ($j = \infty$) corresponds to shock wave solution which

characterised by an abrupt or discontinuous change in the main medium characteristics.
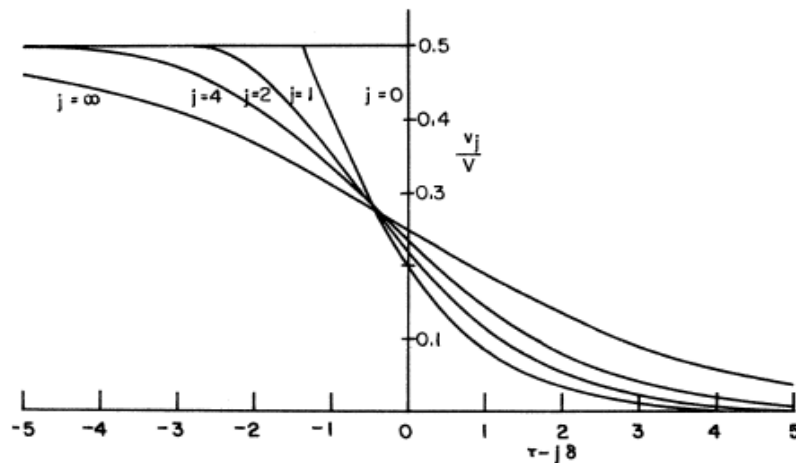
Figure 2.6 *Newell's solutions for suddenly decelerating lead car (j=0) from $v = V/2$ to*

$v = 0$ as a function of rescaled time τ/j, reproduced from Newell, (1961). Each car from

the succeeding cars (j=1 through $j = \infty$) reacts with a progressively increasing time lag

from the lead car. The solution for the last one ($j = \infty$) corresponds to shock wave
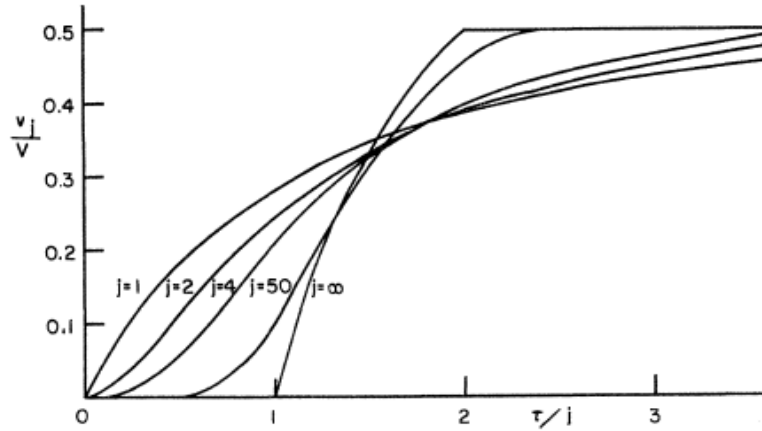
solution.

Figure 2.7 *Newell's solutions for suddenly accelerating lead car (j=0) from $v = 0$ to $v = V/2$ as a function of rescaled time τ/j, reproduced from Newell (1961). Each car from the succeeding cars (j=1 through $j = \infty$) reacts with a progressively increasing time lag to the acceleration. The first one reacts faster than the next one and so on. The solution for the last one ($j = \infty$) corresponds to shock wave solution.*

Treiterer and Myers (1974) who observed two-loop hysteresis presented in Figure 2.3b considered more than driver's behaviour during acceleration and deceleration. Accepting the driver's behaviour to be a part of the entire phenomenon *(Part 1 in Fig. 2.3b)*, they assumed that, due to unexplained reasons, beyond the late acceleration or deceleration an opposite effect should be observed *(Part 2)*. The nature of this effect which is responsible for the formation of the second loop is still unknown.

Most recent experimental observations of this phenomenon were presented by Zhang (1999). The theory allowing explanation of both capacity drop and hysteresis phenomena at the same time were presented by Zhang and Kim (2005). Zhang (1999) proposed

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different explanation for this phenomenon as compared to the explanations by both Newell (1965) and Treiterer and Myers (1974). Zhang assumed that traffic is realized by a driver through three mechanisms: anticipation, relaxation and combination of both of them. Under the non-congestion conditions, the driver can always adjust the speed according to the driving conditions ahead of him, and therefore the anticipation dominates relaxation. Under heavy congestion conditions, when driver cannot anticipate the driving conditions ahead of him, his reaction to the speed changes is late. In this case relaxation dominates the anticipation. Finally, the third state is characterized by balanced state of the traffic containing equally both mechanisms anticipation and relaxation. It is not hard to notice that anticipation and relaxation are responsible for different parts of loop A and loop B of the two-loop hysteresis in Figure 2.3b. Zhang and Kim (2005) proposed four car-following models differencing to each other by the relationship between the time-gap for the i car γ_i and inter-vehicle clearance, d_i, which is, in fact, the first term on the right-hand side in Eq. (2.3) in Subsection 2.1.1.

Zhang and Kim (2005) proposed four types of relationship between the time-gap for the j car γ_j and inter-vehicle clearance, d_j, and defined four different solutions for traffic flow/density fundamental diagram. *Model A (Fig.2.8)* which does not account for any time lag between accelerating or decelerating lead car and succeeding cars, assumes that the propagation of speed changes in the chain of cars occurs instantaneously and, therefore, may be considered as an idealization of the real world situation. The time gap is given by:

$$\gamma_j(t) = \gamma_0 + \frac{1}{v_f} d_j(t) \qquad (2.15)$$

Thus, such model does not produce either capacity drop or hysteresis, because it does not account for different traffic phases. The corresponding traffic flow/density diagram presents smooth inverted-parabola function between two flow attributes. This model gives the solution very similar to the standard fundamental diagram of traffic flow given by Greenberg (1959) which is presented in Figure 2.2 and by Del Castillo and Benitez (1995).
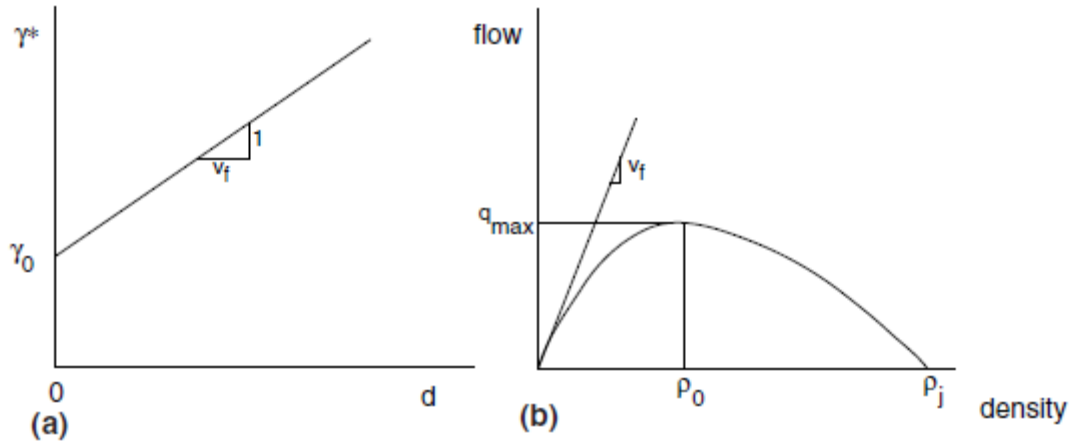
Model A

Figure 2.8 *Model A proposed by Zhang and Kim (2005), reproduced.*

Model B (Fig 2.9) assumes that a car driver behaves differently in congested flow as compared to the free traffic flow. The time gap is given by:

$$\gamma_j(t) = \frac{d_j(t)}{v_f} \quad \text{if } d_0 \le d_j(t) \text{ and} \qquad (2.16a)$$

$$\gamma_j(t) = \gamma_0 \quad \text{if } 0 \le d_j(t) \le d_0 \qquad (2.16b)$$
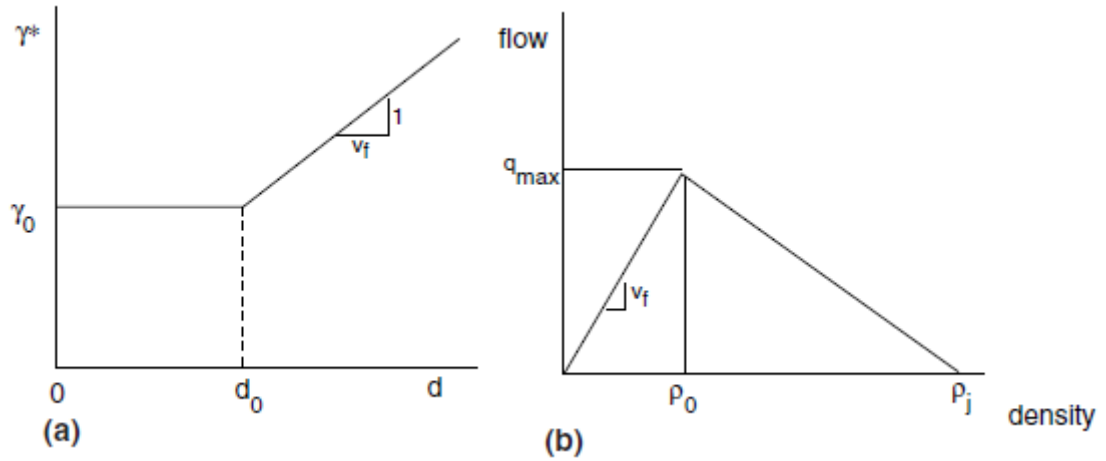
Model B
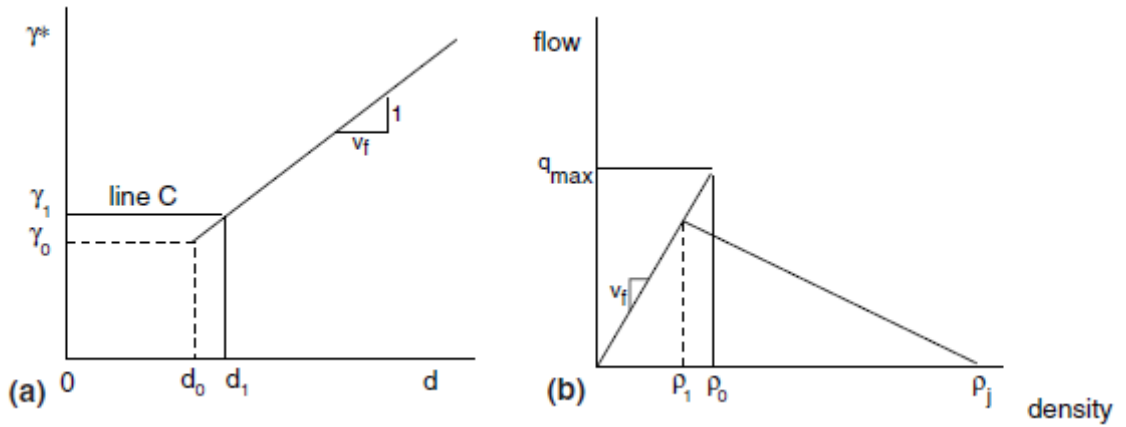
Model C
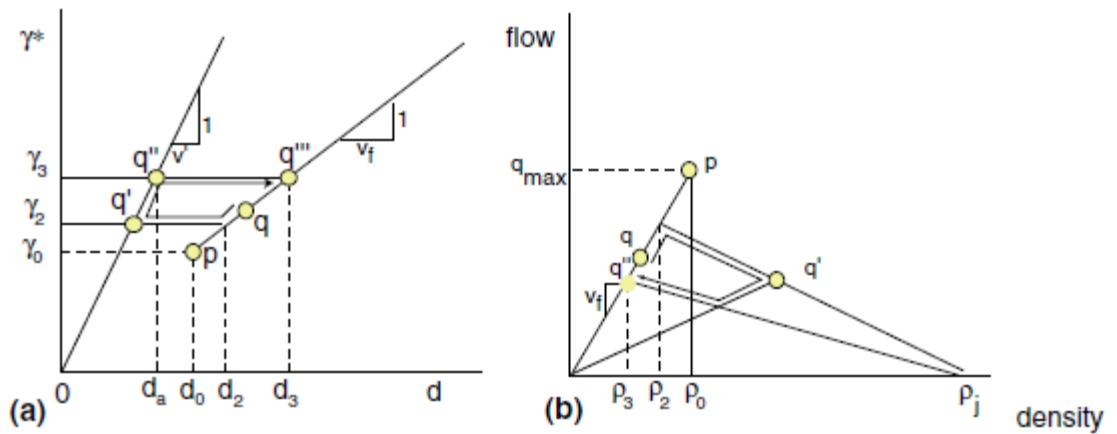
Figure 2.9 *Models B, C, and D proposed by Zhang and Kim (2005), reproduced.*

Here the congestion traffic is observed in the interval $0 \leq d_j \leq d_0$, while the free flow traffic may be observed in the interval $d_0 \leq d_j$. This model also does not produce either capacity drop or hysteresis, however, the traffic flow/density curve looses its smoothness transforming into a triangle distribution with a tip of distribution corresponding to the location of coexistence of two waves. Either of these two waves with constant wave speed is observed on the left- or on the right- hand side of the triangle tip. This model provides the traffic flow/density diagram very similar to Newell's lower order car-following model (Newell, 2002) and very similar to inverted-V diagram by Hall (1986) presented in Figure 2.4b.

Model C (Fig 2.9) assumes that the time gap may be expressed as follows:

$$\gamma_j(t) = \frac{d_j(t)}{v_f} \quad \text{if} \quad d_1 \leq d_j(t) \quad \text{and} \tag{2.17a}$$

$$\gamma_j(t) = \gamma_1 \quad \text{if} \quad 0 \leq d_j(t) \leq d_0 \tag{2.17b}$$

When $d_0 \leq d_j(t) \leq d_1$, the time gap may take the value expressed in either (2.17a) or (2.17b). This model is created on the supposition that the driver accepts very small headways in free flow but, in congested flow, he increases headways until the free flow can be attained. This model produces traffic flow/density diagram very similar to the reversed λ diagram by Koshi et al. (1983) presented in Figure 2.4a and line-J of diagram by Kerner (1998) without scattered area. The model also, with some additional transformations, may present the fundamental diagram presented by Daganzo (1999), which is shown in Figure 2.10.
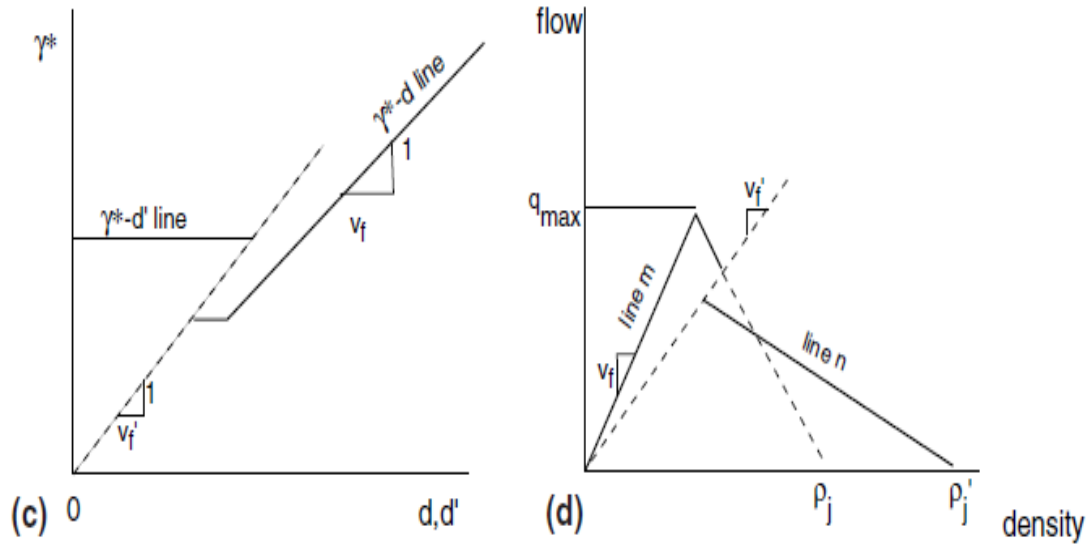
Figure 2.10 *Modified C model producing Daganzo's (2002a) fundamental diagram, reproduced from Znang and Kim, (2005).*

Model D (Fig.2.9) is the most complex case presented by Zhang and Kim (2005) when the time gap $\gamma_j(t)$ changes as a function of both gap distance $d_j(t)$ and traffic phases, i.e. *acceleration, deceleration or casting.* As can be seen from Figure 2.8, for this model, the changes of time gap may occur in horizontal and vertical direction. This model produces correctly hysteresis phenomenon (Newell, 1965), Section 2.1.3, and two capacity drop phenomena, Section 2.1.2, one at the onset of the congestion and the other at the returning to free flow. The model also correctly reproduces capacity drop, hysteresis loops and the waves obtained while transitioning from free flow to congested flow.

2.1.4 Traffic Flow in Platoon: Formation and Diffusion

Platoon phenomenon *(Fig. 2.3d)* was firstly observed and explained by Newell (1959) who considered traffic flow in a tunnel. If the traffic flow increases as in a tunnel, the vehicles tend to create platoons organized by vehicles with similar speeds. Newell considered a two-lane traffic flow with high traffic density in both lanes. This phenomenon is considered in the section for single-lane phenomena, since the passing of cars is forbidden in a tunnel and the considerations presented by Newell (1959) are very similar to those that may be applied for a single-lane flow.

Figure 2.11 presents the main results obtained in the study by Newell (1959). The simulations done by Newell's models may be interpreted as follows. The average car speed inside the tunnel increases with the distance from the entrance, assuming that the traffic flow at the other side of the tunnel is free and without any additional disturbances *(Fig. 2.11a)*. The spacing is also changeable inside the tunnel. Achieving some maximum at a certain distance from the entrance, the spacing starts decreasing as the exit from the tunnel is anticipated. The platoon, a compact unity of cars forming long special interval with constant flow density inside the tunnel, is therefore formed *(Fig. 2.11b)*. However, as experimental investigations have shown, the platoons do not remain compact for long time but rather tend to diffuse in space after the specific time interval. Dispersion of the platoon is related to decreasing the height of the shoulder of platoon and decreasing its side slopes. The platoon dispersion is very often studied in the connection with traffic signal control.
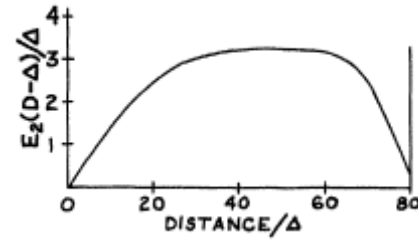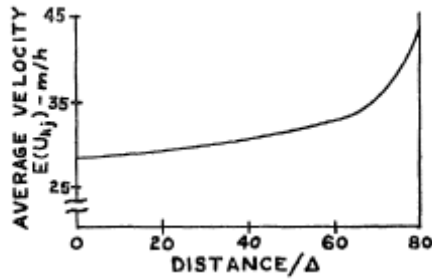
a)   b)

Figure 2.11 *Main results of the platoon formation modeling by Newell (1959): a) the average car speed as function of the scaled distance from the entrance of tunnel; b) average amount by which spacing exceeds Δ. Δ is a distance beyond the entrance to the tunnel, reproduced from Newell (1959).*

To model platoon dispersion, Grace and Potts (1964) assumed that the speeds of the cars in the platoon are distributed according to normal law. They supposed that both parameters of this distribution are related to the diffusion constant that measures the spreading of the platoon. Figure 2.12 presents main Grace and Potts's results. The main problem in the presentation of the mechanism by Grace and Potts (1964) is symmetric presentation of front and rear parts of the platoon. According to the experimental observations, the leaders of the platoon move more quickly than the rest of the group, while the cars in the rear of the platoon very soon become stragglers from the rest of the group. It is known now that the phenomenon is better modeled by multi-class traffic model, such as that presented by Wong and Wong (2002). The shapes of the platoon after several subsequent time intervals are presented in Figure 2.13. The initial trapezoidal

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shape of the platoon transforms into positive-skewed distribution with lower value of peak.
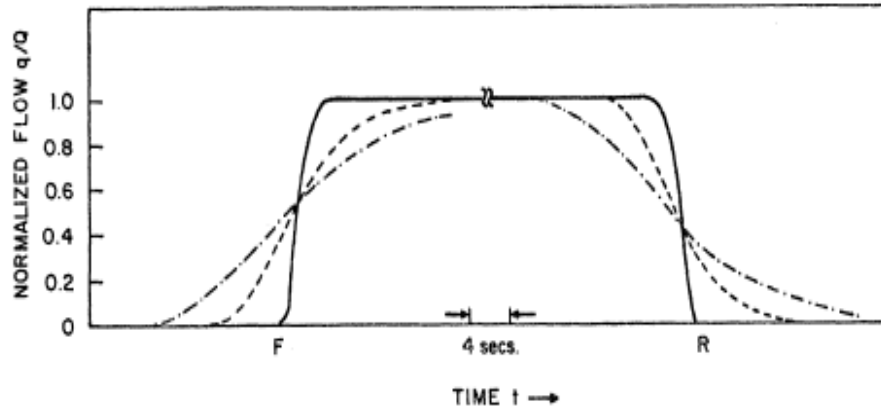
Figure 2.12 *The normalized flow of a platoon at three successive points on the highway. F and R indicate instances when the front and rear of the platoon reached the points with no diffusion, reproduced from Grace and Potts (1964).*
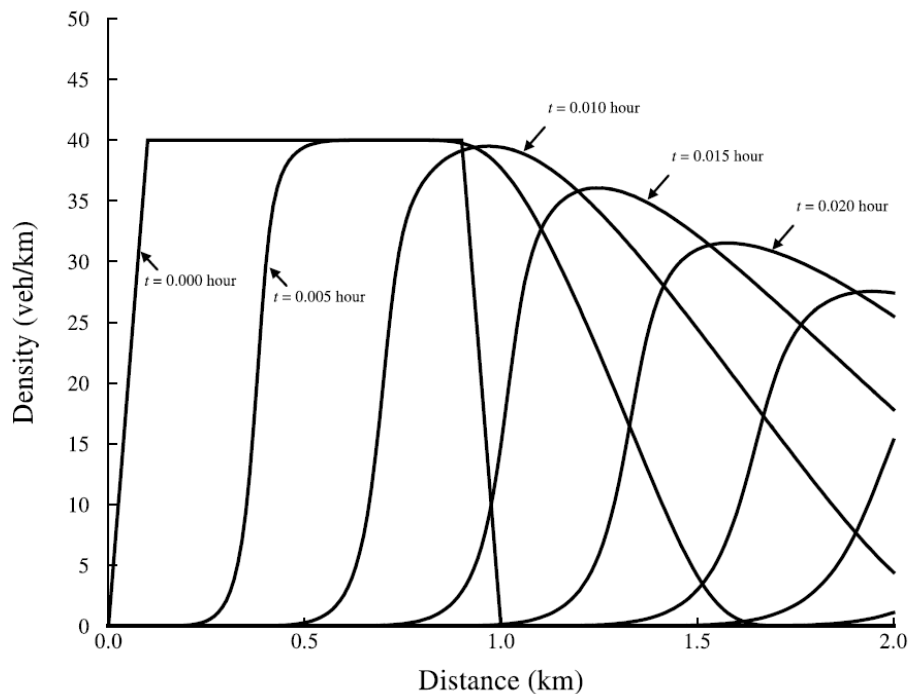
Figure 2.13 *Platoon dispersion modeled by multi-class traffic flow model by Wong and Wong (2002), reproduced.*

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2.2 MULTI-LANE TRAFFIC PHENOMENA

The two main causes for the formation of congestion which have been heavily scrutinized in recent literature are on-ramp vehicle merging into an expressway at the close-to-maximum capacity (Daganzo, 2002b) and a "phantom traffic jam" formation due to nonlinear cluster effect (Kerner and Konhäuser, 1993). Since both of them consider the multi-lane traffic flow, they will be considered in a separate section and complemented by the description of other multi-lane congestion phenomenon called bottleneck.

2.2.1 On-Ramp Vehicle Merging

Merging vehicles to the freeway performing at the limits of its capacity may gradually increase the traffic density in space and, at some distance beyond the on-ramp, bring it to a critical value enough for the transition from free flow to congested flow. The theory for the transition and establishment of congestion was developed by Daganzo (2002b). This simplified theory recognizes only two types of the drivers, the fast ones, called *rabbits* moving with speed V_f and the slow ones, called *slugs* moving with speed v_f. Two-lane traffic flow is considered *(Fig. 2.14)*.

The shoulder lane is the slower lane in which on-ramp merging of vehicles occurs, while median or passing lane is the faster lane next to the shoulder lane. In the shoulder lane, both slugs and rabbits drive with speed v_f some distance beyond on-ramp entrance due to awareness, while, in the passing lane, only rabbits drive with speed V_f. For simplification, it is assumed that slugs do not change the lane and remain all the time in the shoulder lane. On the contrary, at some distance beyond the on ramp, the rabbits start changing the

lane from the slower one to the faster one. The theory recognizes three types of stationary

states of traffic flow according to the speed of the rabbits, V:
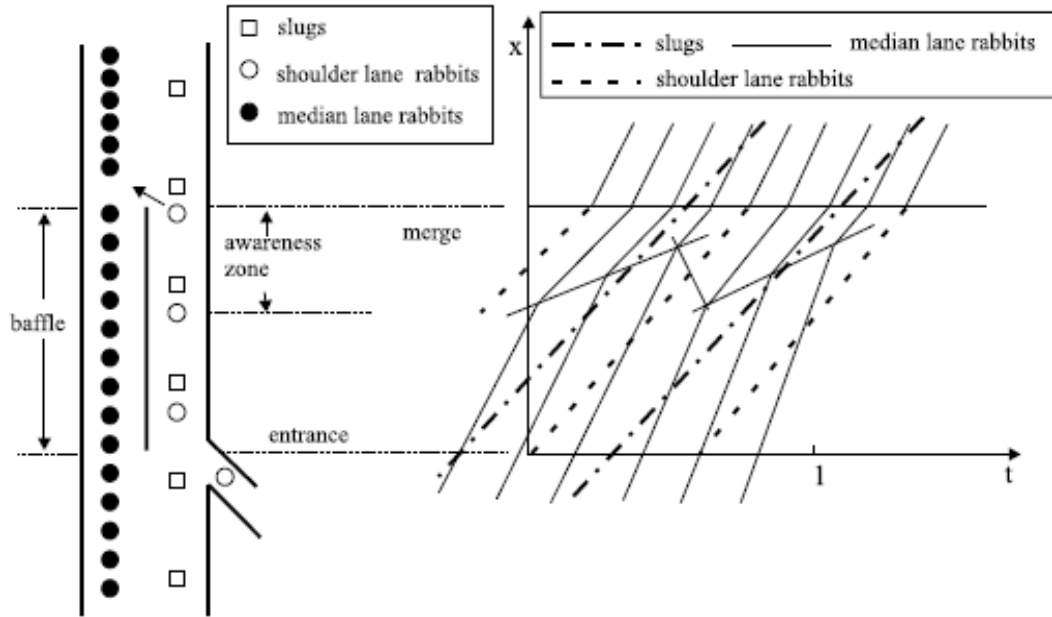
Figure 2.14 *Pumping effect at on-ramp merging, reproduced from Daganzo (2002b)*

i) free/uncongested flow when $V = V_f$; ii) semi-congested when $v_f < V < V_f$; and,

iii) congested/queued when $V \leq v_f$. Therefore, the congested flow was called 1-pipe

flow, while uncongested and semi-congested flows were called 2-pipe flow. Over-

pumping at on-ramp has as a result the following subsequent events occurring

downstream of the merge point: i) initial speed reduction in passing lane after saturation

with merging rabbits; ii) speed reduction in passing line below the slugs' speed; and, iii)

collapse of 2-pipe flow into 1-pipe flow. The first two stages are associated with

maximum flow in the passing lane. Semi-congested state may either propagate *upstream

of the merge point* into infinity or collapse into the 1-pipe regime if an additional flow

increase is observed. In the last case, if the total arrival rate which is equal to two-lane

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flow before the on-ramp plus on-ramp flow is greater than the total capacity of two lanes beyond the on-ramp, an upstream-propagating queue will be formed at the on-ramp. This on-ramp congestion formation is very similar to moving bottleneck which will be presented in Section 2.2.3.

2.2.2 Phantom Traffic Jams

Cluster of cars is a term developed in early nineties of the previous century to represent a local conglomeration of cars increasing significantly local traffic density and moving together downstream or upstream the traffic flow. In the former case, the speed of the cluster is considered to be positive, while in the latter case the speed of the cluster is negative. Formation of clusters of vehicles in the initially homogeneous traffic flow begins with a birth of a small perturbation in local traffic density bringing the traffic flow to a metastable state (Kerner and Konhäuser, 1993). The real cause of the first initial perturbation is still unknown. Such metastable state cannot remain indifferent to any changes beyond this point and, finally, transits to the more stable state characterized either by lower or higher level of serviceability. In the former case, either the speed decreases or traffic density increases or both are changed, and a cluster of cars is formed; in latter case the system returns to the initial state, and perturbation disappears.

Figure 2.15 presents five regions of stable and metastable flow in traffic flow/density diagram. Two ranges, (i) characterized by traffic density lower than some minimal value $\rho \leq \rho_{min}^m$ and (v) characterized by traffic density greater than some maximal value $\rho \geq \rho_{max}^m$, are *the regions of stable traffic flow*, where a fluctuation in the density

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immediately extinguishes after initial appearance. The next two ranges, (ii) characterized by a traffic density within the following interval $\rho_{min}^{m} \leq \rho \leq \rho_{c1}$ and (iv) characterized by traffic density within the interval $\rho_{max}^{m} \geq \rho \geq \rho_{c2}$, are *the regions of metastable traffic flow*, where only those nonhomogeneous local fluctuations in the density grow whose amplitude exceeds some critical density . Finally, the range (iii) confined between two critical values of flow density $\rho_{c1} \leq \rho \leq \rho_{c2}$ called a *region of unstable state* where infinitesimal nonhomogeneous long-wave perturbations grow without any pre-conditioning.
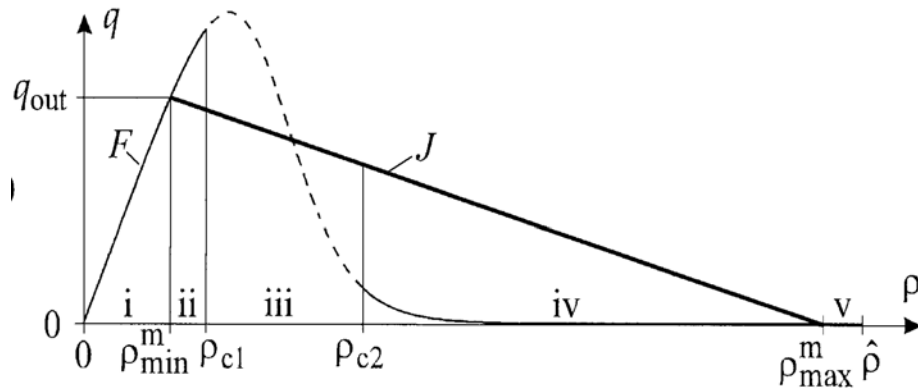
Figure 2.15 *Traffic flow/density diagram with five regions of stable (i and v), unstable (ii and iv) and metastable (iii) traffic flow, reproduced from Herrmann and Kerner (1998). F and J stand for free traffic flow and jam, respectively.*

Figure 2.16 presents the shape of the perturbation in traffic density having relative value $\Delta\rho / \hat{\rho}$ and respective variation in traffic flow, q, and space-averaged speed of vehicles, v. Here $\Delta\rho$ is a local absolute variation in traffic density, while $\hat{\rho}$ is maximum density

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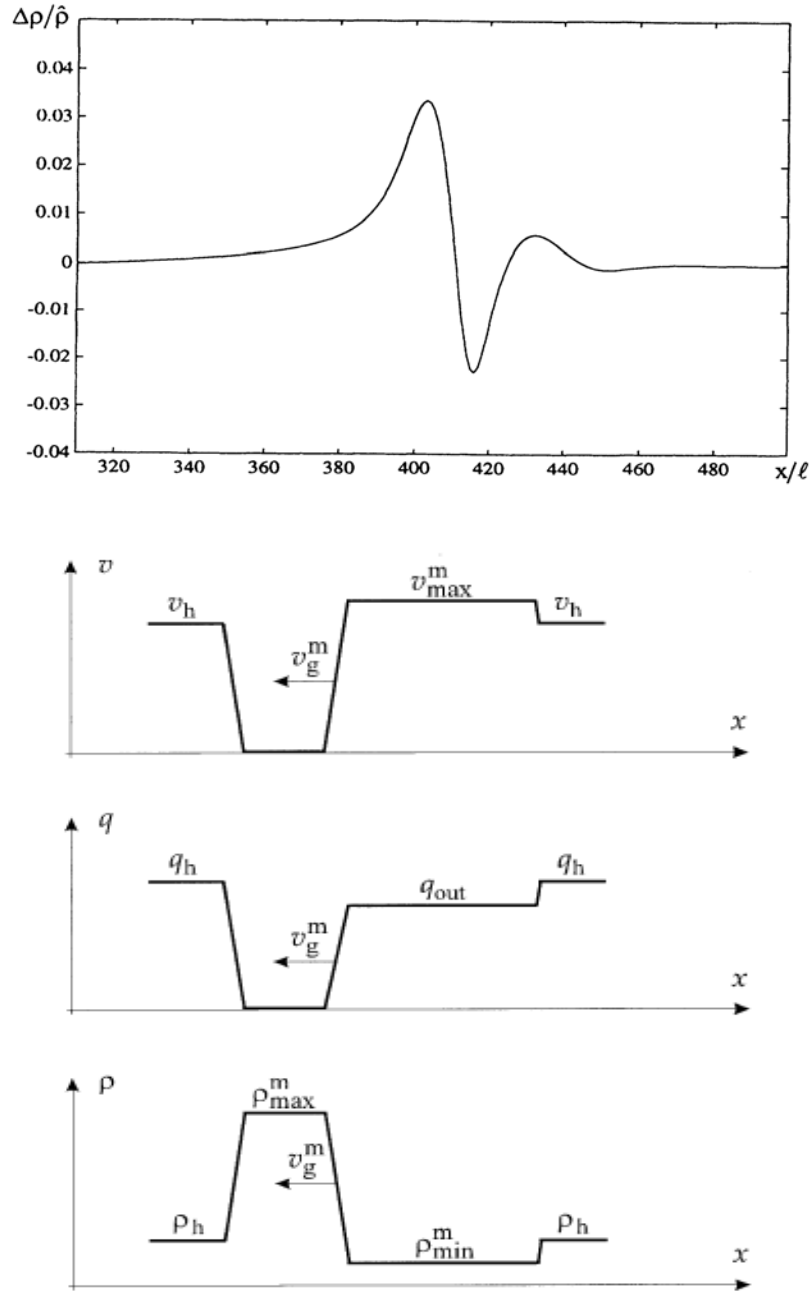
Figure 2.16. *Real (top) and modeled (bottom) shape of the perturbation in traffic density and respective variations in traffic flow, q, and space-averaged speed of vehicles, v, reproduced from Herrmann and Kerner (1998). $\Delta\rho$ is a local absolute variation in traffic density, while $\hat{\rho}$ is maximum density for n-lane road, $\hat{\rho} = n/l$, l is average length of the vehicles.*

for n-lane road, $\hat{\rho} = n/l$, l is average length of the vehicles. It may be seen from the lower diagram of Figure 2.16 that the traffic density perturbation consists of several important regions. The first of them, in fact, presents the traffic jam itself characterized by high traffic density, ρ_{max}. This zone moves upstream, in other words, opposite to the direction of traffic flow. The second region is region with homogeneous traffic flow downstream to the jam with the local traffic density, ρ_{min}, which is lower than the initial density of the incoming undisturbed traffic flow, ρ_h. The third region of the local cluster of vehicles is a transition layer between the new and initial homogeneous traffic flow which moves downstream. Since the jam region moves upstream, whereas the transition layer moves downstream, the whole width of the local cluster of vehicles monotonously increases in time.

Figure 2.17 which is reproduced from Herrmann and Kernerr (1998) presents the location of the maximum of absolute perturbation in traffic density relatively to the location of maximum of traffic flow. It may be seen from the figure that as the traffic density grows from ρ_{min} to ρ_{cr} and conditions for jam denoted by letter J are already reached, the undisturbed free traffic flow denoted by letter F still have chances to develop inside the zone of the metastable traffic flow. This happens due to capacity drop effect conditioning the achievement of metastable state only from free traffic state and never from jam flow state. Metastable state may very simply develop into the first zone of cluster of cars.
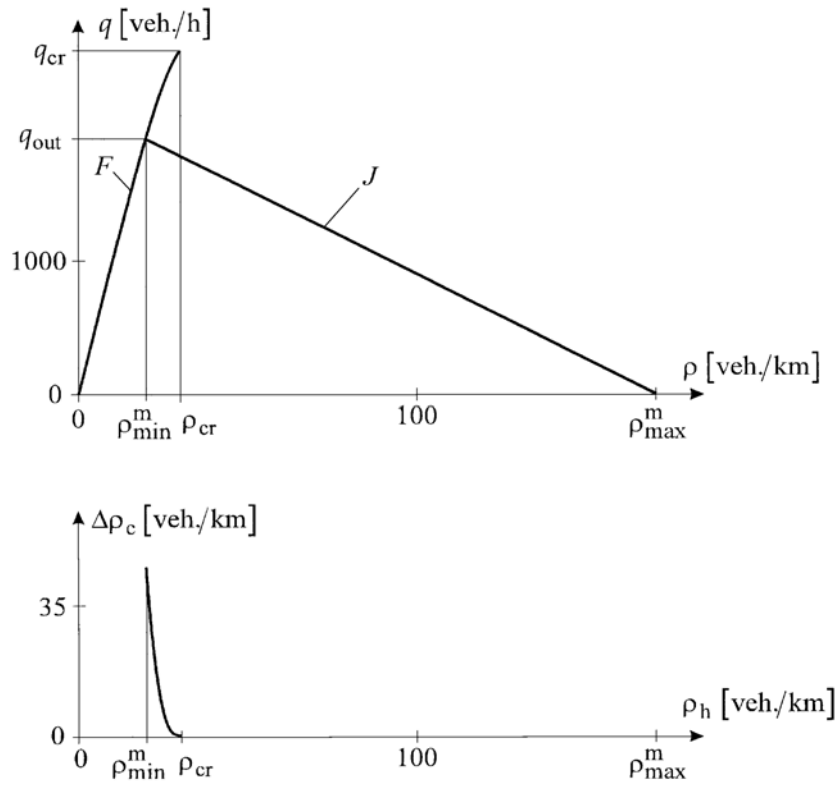
Figure 2.17. Location *of the maximum of absolute perturbation in traffic density (lower graph) relatively to the locations of free flow and jam at the traffic flow/density diagram(upper graph), reproduced from Herrmann and Kerner (1998).*

Figure 2.18 presents three dissimilar results of the traffic cluster formation and development. As it has been previously noticed, in all cases, the perturbation is widening in time expanding in both upstream and downstream directions. While in the first two cases the density perturbation propagates upstream with almost constant downstream minimum (empty zone), in the third case the perturbation created by collision of the two initial perturbations moving in dissimilar fashion propagates downstream in considerably different manner. The well-expressed secondary maximums may be observed downstream in this case.

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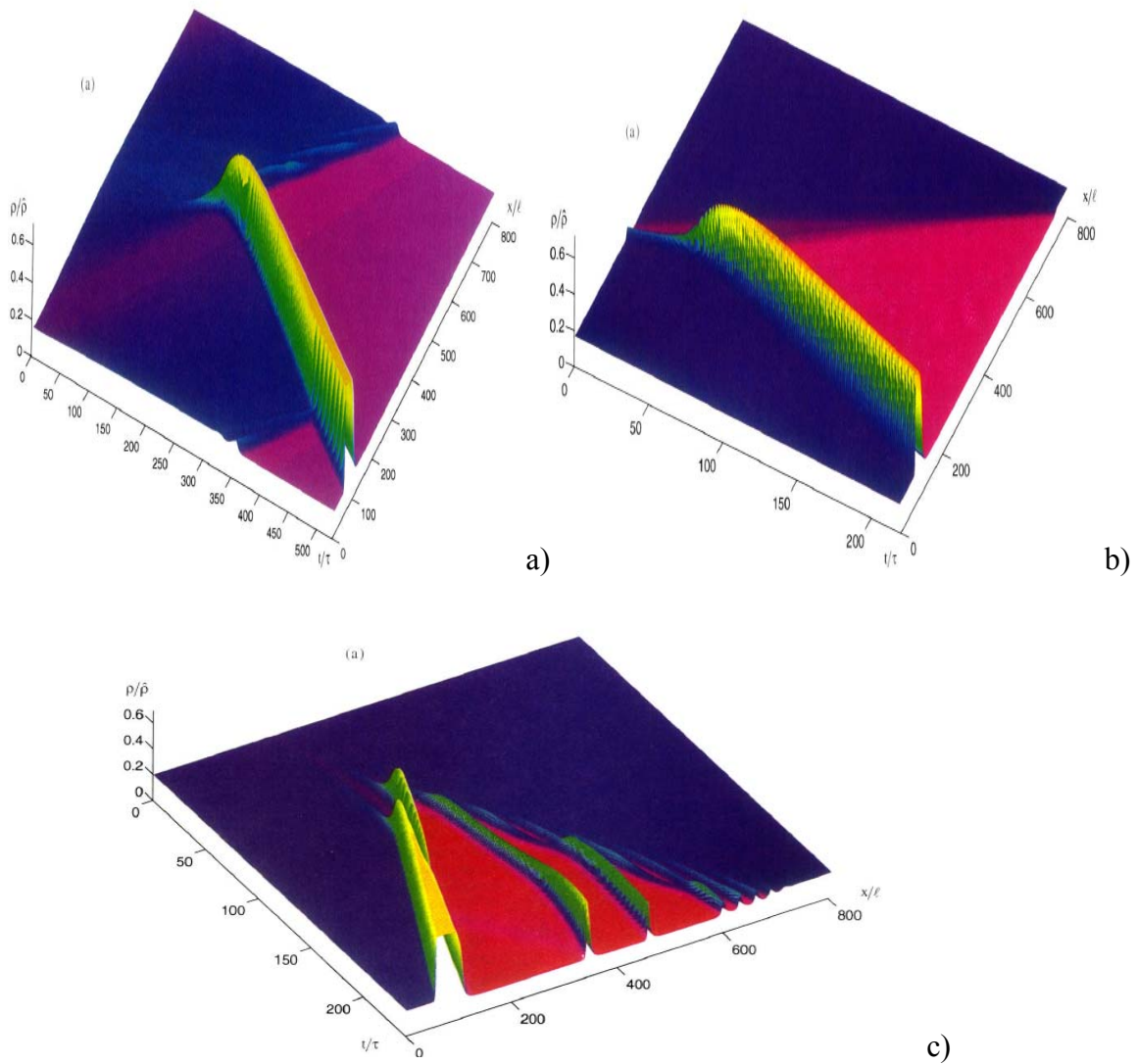
Figure 2.18 *Three possible outputs of the cluster formation: a) fast upstream motion of the perturbation; b) moderate upstream motion of the perturbation; c) slow or moderate upstream motion combined with propagating downstream secondary maximums damped over the distance from the initial location, reproduced from Kerner and Konhäuser (1994).*

The secondary maximums damp over the distance remaining almost unchangeable as time progresses. Applying the results of this theory and processing recently obtained

empirical observations in the field, Kerner and H. Rehborn (1997) and Kerner (1998, 1999) were able to show that the accepted before two-phase flow conception involving only free and congested flows fails to explain the real-life data and should be altered by the three-phase flow theory. Such a theory developed later by Kerner (2004) includes three different traffic flow phases *free flow, synchronised flow and jams formation*. All traffic models using the standard traffic flow/density diagram for equilibrium state were questioned.
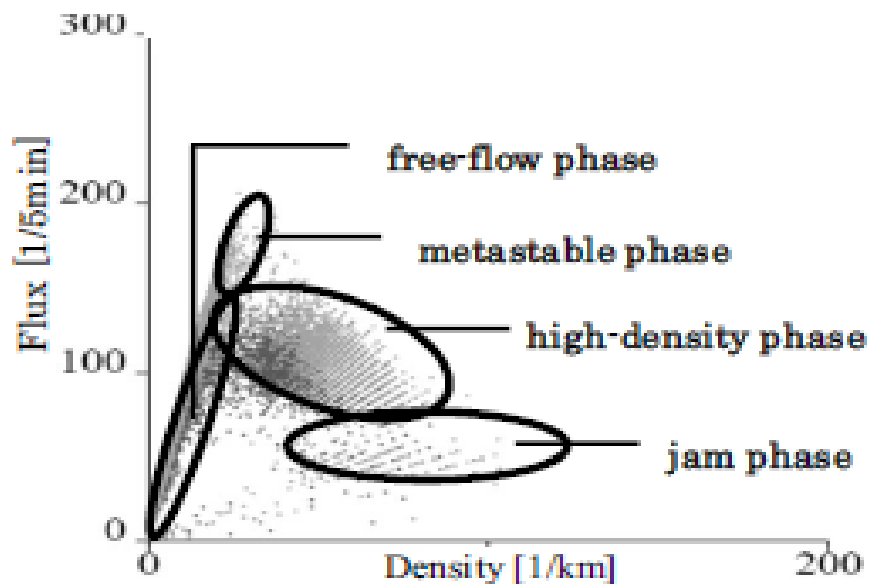
Figure 2.19 *Typical traffic flow phases accepted in the conception of synchronized flow by Kerner (2004).*

Figure 2.19 presents various flow phases involved in the proposed new concept. Due to high dispersion of experimental data from the descending part of flow/density diagram, a wide zone of synchronized flow was defined, such as presented in Figure 2.4c. Both high density and jam phases were called together as synchronised flow. Conception of

synchronised flow was seriously criticised by Schönhof and Helbing (2007) and some explanations for the questioned "out-of-flow-diagram" data were presented.

Either conception involves metastable phase which may be easily achieved from free-flow phase, but not from congestion phase. Another feature is queue formation which is a part of the so-called stop-and-go phenomenon. The next subsection will consider development and persistence of a queue as a result of bottleneck formation.

2.2.3 Moving and Stationary Bottlenecks

Bottleneck is a traffic phenomenon produced by sudden drop in the capacity of a freeway and resulting in slower motion of the vehicles through the location and queue formation upstream of the considered location. *Stationary bottleneck* is a drop of capacity of a multi-lane freeway resulting from decreasing the total number of lanes available for traffic flow. *Moving bottleneck* may appear as a result of presence of a slow-moving vehicle in one of the lanes of multi-lane highways.

Figure 2.20 presents contemporaneous conception of bottleneck formation. A semi-infinite bottleneck on a two-lane freeway has been created at x_0 position by complete closing one of the lanes for traffic flow (left hand side of the figure). Formation of the bottleneck is reflected in transfer from outer flow/density curve representing two-lane traffic to inner flow/density curve representing one-lane traffic (right hand side of the figure).
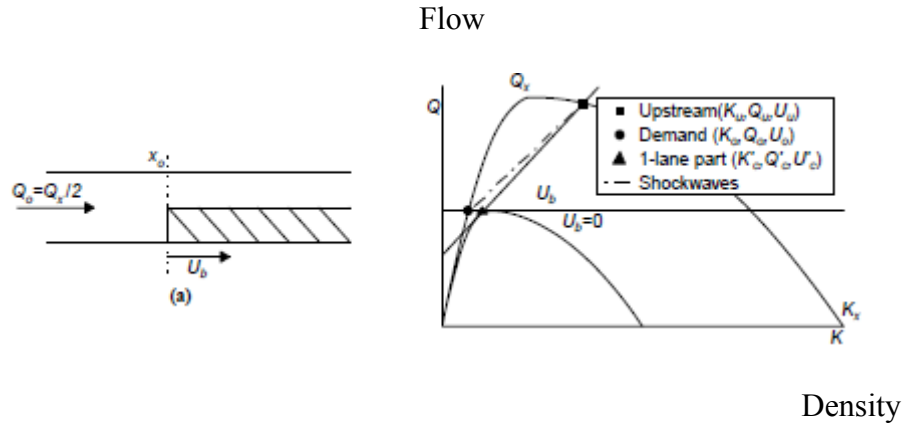
Flow

Density

Figure 2.20. *Typical bottleneck formed by decreasing the number of lanes available for traffic from two to one (on the left hand side) and respective flow/density diagram (on the right hand side) presenting flow drop (from square on the outer curve to triangle on the inner line) for a stationary bottleneck. For moving bottleneck (diamond), the flow drop is created together with discontinuity and queue upstream bottleneck.*

It is hypothesized that the demand is equal to half of the initial flow. As a result, no queue is formed upstream of the bottleneck, which may start moving slowly if the bottleneck moves with some constant speed U_b. This type of traffic phenomena requires at least two-lane highway with a possibility for overtaking. Gazis and Herman (1992) developed a theory to explain queue formation upstream a slow-moving vehicle on the two-lane highway during the periods of heavy and moderate traffic flow. Total flow is considered as a sum of two traffic flows. The first flow component, q_1, is a pack of the vehicles in both blocked and unblocked lanes, which moves with the speed of slow vehicle, v_1, while the second flow component, q_2, is a pack of the vehicles in the unblocked lanes moving with speed v_2. The total bottleneck capacity may be determined as follows:

$$q = 2\rho_1 v_1 + (v_2 - v_1)\rho_2 \qquad (2.18)$$

A newer type of theory was recently developed by Muñoz and Daganzo (2002) and Juran et al. (2009).

All traffic flow phenomena considered here involve various values of speed for different flow parts. As was seen, the speed difference between different flow parts is a 'motor' of the formation, development and persistence of the phenomena such as bottleneck, platoon diffusion and so forth. Only several LWR extension models attempted to introduce the traffic heterogeneity such as speed difference or speed gradient in existing macroscopic models. No one of the existing network link travel models, however, manages with traffic heterogeneity directly when using various macroscopic models as the tools. Two solutions are possible to improve network traffic models. One of them is to alter macroscopic models used in network traffic modeling by LWR extension models. However, every LWR extension model deals with only one specific factor and changes LWR equations accordingly. In other words, each LWR extension model presents a local solution, which may not be applied for considering other extension factors. Another approach, which is more cardinal, is *to use some microscopic features such as length and speed local variations to present traffic heterogeneity*. It is required, therefore, to understand how the 'macroscopic' travel time is calculated in the existing network traffic models. Next Section will be dealing with this specific task.

2.3 NETWORK TRAFFIC MODELING

2.3.1 Background

As mentioned earlier, the purpose, design and solution of a network traffic model are completely different when compared to simple macroscopic traffic flow models for free

motorways without intersections and on-ramp merging zones. They may solve a free traffic flow query, a least-cost or a least-traffic-volume-delay optimization problem or, more generally, any optimal control problem. The least-traffic-volume-delay optimization problems considering unvarying flow (e.g. Nguyen and Florian, 1976) and called static traffic assignment (STA) problems already considered origin-destination (O/D) pairs in searching of equilibrium flow. Yet, such solutions have not considered within-day traffic demand variations which always follow several standard variation patterns. The first least-cost optimisation model considering both O/D pairs and varying within-day traffic flow (Merchant and Nemheuser, 1978) pioneered a new generation of equilibrium flow models known as the dynamic traffic assignment (DTA) models. Contemporaneous approach to equilibrium network flow considers a complex efficient routing strategy which is not only a reckoning of shortest or cheapest path dealing with the network geometry or cost minimization problems, but a master plan considering the congestion nests across the network and proposing an efficient strategy for their avoidance (Yan et al., 2006).

To facilitate the traffic flow modeling in a complex network, the latter is represented by a family of separate freeways called links or arcs united by a family of nodes (see Astarita, 2002, for exhausting review). The incoming traffic flow to a singular node may be either merging or constant (ordinary), while the exiting one may be either diverging or constant (ordinary). Because rapidity in the reaction seems to be one of the most important features of network traffic management, and because network equilibrium may not overlap with an equilibrium flow along an individual link, it is clear that the required

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solution to the network traffic model should be faster and simpler than the respective solution to the LWR model, but converge to it under appropriate conditions and account for all microscopic and driver behavioural specificities just mentioned. As for the simplicity and rapidity of the solution as well its convergence to the LWR solution is concerned, a number of "whole-link" models have been proposed. The most frequently cited of them will be shortly reviewed in this chapter.

2.3.2 Types of Whole-Link Models

2.3.2.1 Exit Flow Functions

Merchant and Nemheuser (1978) proposed discretization of the number of vehicles on the link N in the following shape:

$$N_{i+1} = N_i + u_i - g(N_i)$$
(2.19)

where index i is time interval counter, u_i is inflow at the link, $g(N_i)$ is a link exit-flow function representing outflow at the link. $g(N_i)$ should be non-decreasing and concave. Index a showing appurtenance to a specific link is dropped in order to avoid unnecessary redundancy. Several authors (Carey, 1987; Friesz et al., 1989; Wie et al., 1990, and, Wie et al., 1994) followed the same approach called now Merchant Nemheuser model (MN), in which the shape of an a priori given function of the current number of vehicles in the link could define the current outflow. Then, if the inflow is known, the number of vehicles in the link at the next time interval may be estimated accordingly to (2.19). Finally, the link time travel could be defined. In particular, Merchant and Nemheuser (1978) proposed $g(N_i)$ functions to be linearly increasing for traffic flow lower than a link capacity, C_a, and constant, i.e. constrained, for traffic flow equal to link capacity,

while Wie et al. (1994) proposed exponential function tending asymptotically to the link capacity. Recall, that the link flow capacity, C_a, is defined as maximum traffic flow volume that is feasible for given link. Nie and Zhang (2005) tested such model for several inflow profiles and found that it does not perform well for both empty charging and totally discharging links. In the former case, any change in the inflow produces an instant reaction of the entire flow along the link, and as a result, the outflow starts at the same time interval as the inflow, i.e. travel time for the empty link is zero. In the latter case, because the travel time of the last vehicles tends to infinity, the vehicles may indefinitely remain in the link if there is no new inflow. Both drawbacks may produce significant distortion in calculating travel time, particularly, for rapidly varying inflows.

2.3.2.2 Link Travel Time Models

Friesz et al. (1993) proposed to calculate link travel time, $\tau(t)$, directly from the number of vehicle in the link by using the following linear function:

$$\tau(t) = a + bN(t) \qquad (2.20)$$

where the constants a and b, having the physical meaning of free flow travel time, τ_0, and reciprocated value to link flow capacity, C_a, respectively, may be found from the link calibration. It is clear, that no traffic flow is possible at the link travel time lower than τ_0.

Testing of more complex functions such as parabolic one (Wu et al., 1998; and Xu et al., 1999) having the following shape:

$$\tau(t) = a + bN(t) + cN^2(t) \tag{2.21}$$

has not showed any improvement in the final result since the first-in-first-out (FIFO) rule

was found to be violated for such travel time shapes. Some authors have tried to use the

Bureau of Public Roads (BPR) formula for computing travel time in the link which looks

as follows:

$$\tau(t) = \tau^0(t)\left[1 + 0.15\left(\frac{N_a}{C_a}\right)^4\right] \tag{2.22}$$

for producing a new modified BPR volume delay function (e.g. Fernandez and Cia,

1994). Both the original and modified BPR formulas, however, were found to not respect

FIFO rules and violated the traffic flow capacity of the subsequent link.

The main problem of both exit flow function and link travel time models is in confusing

the instant flow attributes of the entire link with the averaged over the time local flow

attributes. Namely, when one considers the travel time of the specific vehicle, only a part

of the flow volume surrounding the vehicle participates in the influence on the value of

travel time. In that way, when the considered vehicle enters the link, it is mainly affected

by the inflow characteristics and local number of vehicles near the link entrance, while

when the vehicle exits the link, it is mainly affected by the outflow characteristics and

local number of the vehicles near the link exit. Some improvement in this direction has

been done in a series of papers by Carey et al. (2003), Carey and Ge (2004), and Carey

and Ge (2007) proposing a new formulation of link travel time model. In fact, instead of

looking for the relationship between the travel time and the number of vehicles in the

entire link, Carey and coworkers proposed that the travel time might be a function of a

weighted average of (i) the inflow measured at the enter time interval, and (ii) the outflow measured at the exit time interval as follows:

$$\tau(t) = f\{\beta u(t) + (1-\beta)w[t+\tau(t)]\} \qquad (2.23)$$

where β is a weighting constant $0 \le \beta \le 1$. Rewriting the outflow as follows:

$$w(t+\tau(t)) = \frac{u(t)}{1+\tau'(t)} \qquad (2.24)$$

travel time may be presented with the first order ordinary differential equation:

$$\tau'(t) = -1 + \frac{(1-\beta)u(t)}{f^{-1}(\tau) - \beta u(t)} \qquad (2.25)$$

It was shown that such model satisfies FIFO if function $f(\ )$ is chosen to be nondecreasing. The main drawback of this model is in the necessity to know exact value of either outflow after link travel time, $w[t+\tau(t)]$, or $f^{-1}(\tau)$ for time interval $\tau(t)$. In both cases the approximate iterative value of travel time together with weighting constant should be given before calculation, which is very often impracticable.

A link travel time model based on an iterative procedure involving the input and output link flow capacities, jam density, flow-adjusting speed and travel time was proposed by Awasthi et al. (2006). No bounding procedure, however, was proposed for the speeds both increasing and decreasing in time.

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2.3.2.3 Cell-Transmission-Based Models

The main point of the cell-transmission model (CTM) proposed by Daganzo (1994) for approximate solving of LWR model and then extended by the same author with coworkers (Daganzo, 1995; Cayford et al., 1997) for network traffic modeling is in dividing the link lengthwise into equally-long space intervals called cells and, then, defining the number of the vehicles in each of them at any time interval by the following system of equations:

$$n_{i+1,j} = n_{i,j} + u_{i,j} - u_{i,j+1} \tag{2.26}$$

$$u_{i,j} = \min\left\{n_{i,j-1}; Q_{i,j}; \frac{w}{v}(n_{i,j}^{\max} - n_{i,j})\right\} \tag{2.27}$$

where index i, a time interval counter, is supplemented by index j which is a space interval counter having value of zero for the first cell and increasing in the direction from the link entrance to the link exit; $n_{i,j}$ stands for the number of vehicles in the cell j at the time interval i; $Q_{i,j}$ is capacity flow of the cell j, or the maximum number of vehicles that can enter into cell j; $n_{i,j}^{\max}$ is maximum number of the vehicles that can be in cell j at the time interval i; v is free flow speed; w backward wave speed, i.e. speed of disturbance propagation in congested traffic. In the system of equations (2.26) and (2.27), (2.26) is a simplified rewriting of conservation equation, while (2.27) defines that the inflow to the cell j at the time interval i cannot be greater than any of these values: (i) the instant number of the vehicles in the neighboring preceding cell; (ii) the capacity flow of the considered cell; or, (iii) amount of empty space in the considered cell, and should be chosen as a minimum value between them. The expression for inflow ensues from the

flow-density diagram having non-isosceles trapezoidal shape and side slopes v and w, while also constrained by $Q_{i,j}$ (*Fig.2.21*).
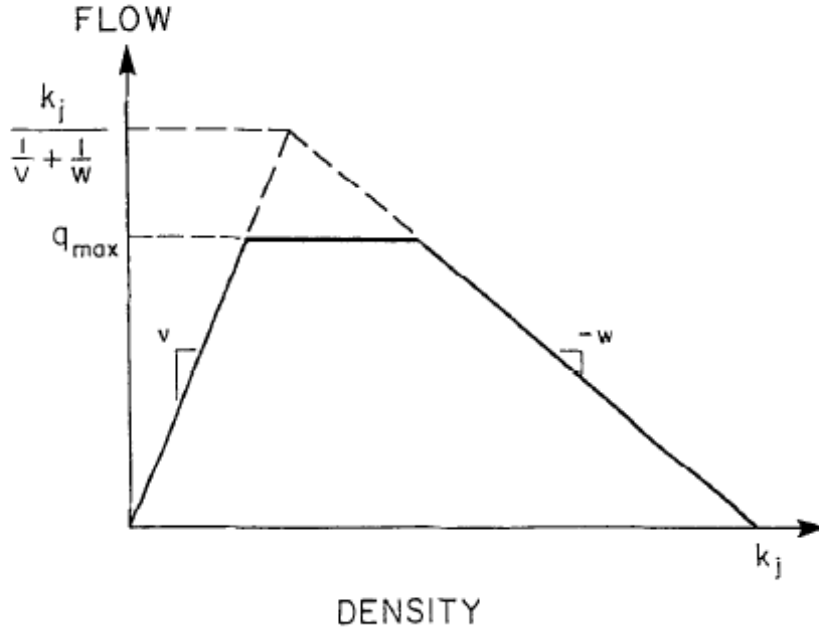
Figure 2.21 *Flow density diagram used for Daganzo cell model, reproduced from Daganzo(1995).*

Assuming that all traffic flow participants have the same spatial (i.e. cell and vehicle length) and spatio-temporal (i.e. vehicle speed) characteristics, each cell will have the same capacity flow, $Q_{i,j}$, defined by spatio-temporal flow characteristics, and the same maximum number of the vehicles in the cell, $n_{i,j}^{max}$, giving jam density when divided with cell length and, therefore, defined by spatial flow characteristics only. Under these conditions, the CTM very realistically presents real world link travel time uncongested and moderately-congested traffic flows, while, under heavily-congested flow, it aptly presents domination of queuing in the flow dynamics (Nie and Zhang, 2005). Recently, it

was shown that CTM can be derived from the exit flow function model, when the exit-flow function is properly selected and discretized (Nie, 2010). The complexity appears when the flow participants with various length and desired speed take part in the traffic dynamics. In that case, both $Q_{i,j}$ and $n_{i,j}^{\max}$ become the variables dependable on the facts such as (i) how many vehicles with specific length have been found in the considered cell; and, (ii) what is the distribution of the desired speed for these vehicles. Clearly, the existence of dissimilar $Q_{i,j}$ and $n_{i,j}^{\max}$ values for every single link in the network and their subsequent time dependence will introduce an additional complexity to the solution by requiring both excessive CPU time and increased system memory.

2.3.2.4 Point-Queue Models

Drissi-Kaïtouni and Hameda-Benchekroun (1992) proposed a model for network traffic flow in which link travel time is considered as a sum of travel time in free flow traffic along the link plus the time interval spent in a zero-long queue at the link exit (*Fig.2.22*). Such decomposition of the space into the link-long interval in which space and time have been interrelated and a single point in which space and time have been independent allowed extending the original space network into a new modified space-time network having additional space-time links. Thus, a complex DTA problem could be redefined as a relatively simpler STA problem expanded in time and resolved by manipulating *n* times with STA solutions. The main simplification of this model is in disregarding the length of the vehicles and, therefore, the length of queue forming at the link exit in congested traffic. Because of attaching physically nonexistent queue to one single point at the link exit, the model has been called point-queue model (PQM). Second simplification is in

assuming free flow traffic everywhere along the link while the vehicle passes from the link entrance to the link exit. Several authors (Kuwahara and Akamatsu, 1997; and, Li et al., 2000) followed similar approach.

Departure point   **Waiting point**

1   L_a 2

$$\tau = L_a / v_a + \tau_{wait,2}$$
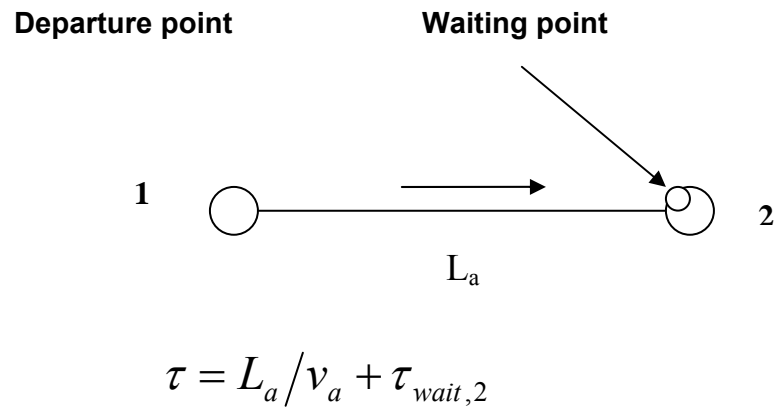
Figure 2.22 *Sketch of point-queue model*

Both CTM and PQM represent relatively well moderately-congested traffic dynamics, while the dominant role of the queue is emphasized under heavily-congested traffic. As the queue develops, the greater part of the link is occupied by the vehicles in queue decreasing the effective capacity flow and maximum number of vehicles that can be in the link. Again, *neither PQM nor CTM are capable of representing traffic flow dynamics consisting of vehicles having various lengths and various desired speeds.*

2.3.2.5 Performance Models

Astarita (1996) first proposed the link performance model by considering link travel time as dependable on the instant number of vehicles in the link per unit of length, i.e. flow

density, averaged spatially. Instead of discretizing the link in space and solve the discretized parts in the time, Gentile et al. (2005) proposed, in their performance model, to model every link by three unequally long parts: (i) an infinitesimally long input bottleneck having the capacity equal to flow capacity; (ii) a running link which is approximately link-length long; and, (iii) an infinitesimally long output bottleneck having capacity below the flow capacity or equal to it (*Fig. 2.23*).
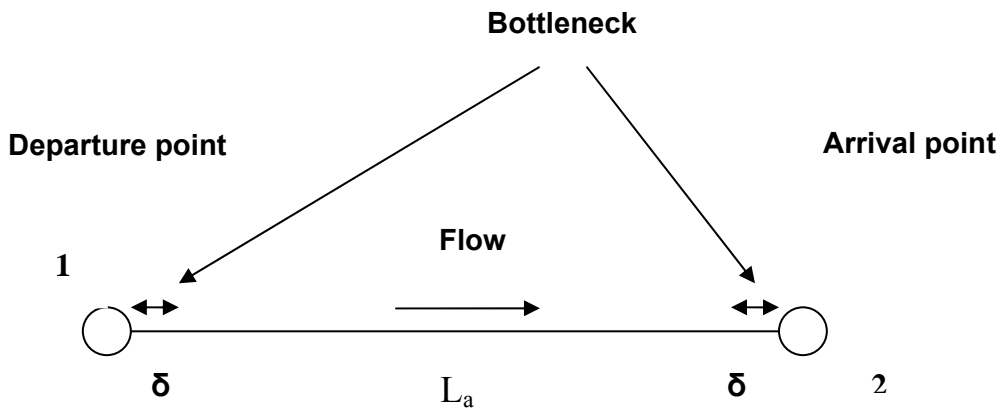
Figure 2.23 *Sketch of performance model*

Both bottlenecks have been purposely designed to model the respective queues forming at the edges of the links. The input bottleneck keeps the inflow to the running link to be bounded below the flow capacity and, thus, helps avoiding heavy-congested flow in the second part of the link. The output bottleneck might be thought to model the queue appearing at the link exit because of nonconformity between the output flow capacity of the preceding link and the input flow capacity of the succeeding link. Finally, the running link models the consistent moderately-congested flow disturbed from time to time by the

back-propagating kinematic waves initiated by the vehicle interactions. Thus, the variability of the flow states along the link has been seemingly presented. Under such a formulation, the link exit time has been a function of both the output bottleneck capacity and inflow profile in time.

The major simplification here is in assigning a particular flow phenomenon to the specific location on the link, while completely excluding other phenomena locally. For example, assuming the bottlenecks to be infinitesimally long, queuing at congested flow is forced to be modeled by vertically propagating virtual queue. Another, by no means, non-irrelevant drawback is in neglecting an interaction between the locally modeled flow phenomena. For example, assuming the running link is unaffected by the output bottleneck and so forth. Despite the presented shortcomings, the model presents a noticeable advance by trying to unite the vehicle transportation together with kinematic wave theory. The further simplifications such as applying simplified kinematic wave theory or presenting inflow as piece-wise constant function in time may be counted as irrelevant as compared to the objectives and achievements in this unification.

2.3.2.6 Vehicle-Discrete-Packet Models

The next progress in representing the vehicle traffic dynamics on a link in traffic network while solving the optimization or free traffic query problems may be considered by the research done by Di Gangi (1992) and Dell'Orco (2006). Di Gangi modeled flow propagation by considering continuous packets of vehicles with an idea to apply car-following modeling which from the time of its first appearance has been counted as

effective but not efficient method requiring high amount of CPU time. In his research, De Gangi (1992) defined several important drawbacks in modeling continuous vehicular packet as a single car, which were related to various flow attributes. Trying to overcome the difficulties encountered during modeling of traffic flow by vehicle-continuous packets, Dell'Orco (2006) proposed a new model, called vehicle-discrete packet model, in which all vehicles in the packet were attached to one single point in the head of the packet (*Fig.2.24*).
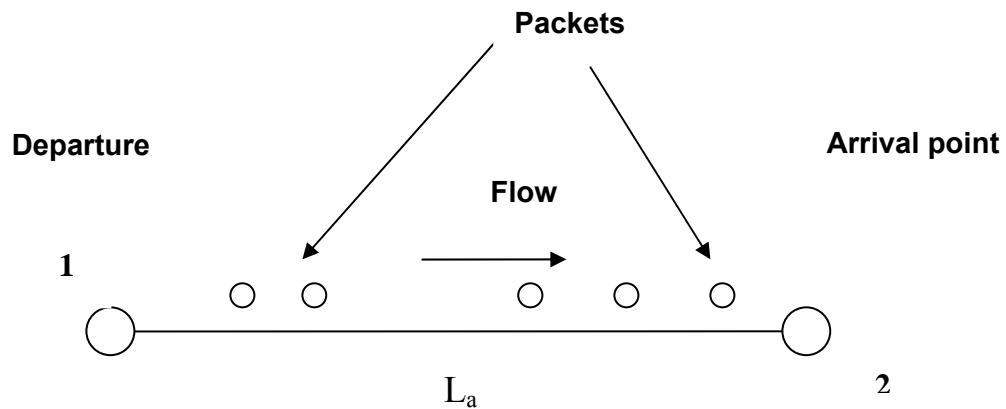
Figure 2.24 *Sketch of discrete packet model*

The positions of packets represented by points moving at the same initial speeds were resolved at each time interval by the method of final differences. In fact, a space-discrete approach already had have been applied even before development of vehicle-discrete-packet model by Dell'Orco (2006), but in a different context. In particular, most of the researches proposed exit flow function applications such as ones done by Merchant and Nemheuser (1978) and by Wie et al. (1994) were length-discrete models. Applying length-discrete approach to the vehicle packets modeling (Dell'Orco, 2006), the problem

of the time headway distributions inside each packet, which is a central one for car-following modeling, has, therefore, been avoided. Recall, that the time headway is the time interval measured between the passings of the front bumpers of the two vehicles following one after another. The models for headway distributions since long time have been built on the assumption a distinction between the leader and follower (Cowan, 1976) and, more lately, on assigning completely different distributions to different traffic volumes α. However, even such complex representation was found to be unsatisfactory for the entire spectra of the headway distributions observed in meanwhile. Therefore, on the one hand, avoiding application of the time headways between the vehicles in the packet is, obviously, a step forward in the link traffic flow modeling, because packet presentation of flow would significantly save CPU time and decrease system memory requirements. On the other hand, the headway distributions between the packets and the number of the vehicle in each packet yet need to be resolved. Moreover, because of miscount of space taken by each packet, the vehicle-discrete-packet approach complicates precise determination of both the flow capacity and jam density. Also, the equal initial, though the uniformly accelerated in time, speed for all packets, as it was modeled by Dell'Orco (2006), has been a significant simplification of real world of heterogeneous traffic flow.

2.4. CONCLUSIONS

A few conclusions should be drawn concerning the reviewed link models. Despite all mentioned shortcomings, *the packet-flow approach has been an improvement over the existing macroscopic approaches* because, in contrast to all of them, allows considering

the traffic non-homogeneities related to both the varying desired speed and the spatial inequality of traffic participants. Such an approach may combine the best features of the macroscopic and microscopic modeling in solving various optimal control problems. In particular, the improved discrete-packet-flow approach was recently used for resolving an optimization problem very similar to the DTA problem (Celikoglu and Dell'Orco, 2007). We think that *if the link flow capacity is solved integrally with time headway problem, the continuous-packet-flow approach may be even greater improvement.*

Chapter 3:

Problem Statement

3.1. ASSUMPTIONS

The assumptions used before modeling the whole link continuous bunch model are as follows:

- traffic flow is heterogeneous consisting of flow participants of different lengths moving with different desired speeds;

- driving styles of truck drivers and passenger car drivers are different;

- driving styles within a group of vehicles moving together as compact group are similar as far as their car-following behaviour is concerned;

- nonlinear effects such as shock waves are excluded in heterogeneous network traffic modeling;

- no overtaking takes place along the considered one-lane freeway.

3.2 NETWORK LINK PROBLEM FORMULATION

Let us represent the network by a graph, $\Gamma(A, P)$, which is formed by a set of arcs (links), A, and a set of nodes, P, *(Fig. 3.1)*. For each arc a, the following conservation equation is valid:

$$\frac{dN_a(t+dt)}{dt} = u(t) - w(t) \tag{3.1}$$

$$p \in P = \{1,2,3,4\} \qquad\qquad a \in A = \{a,b,c,d,e\}$$
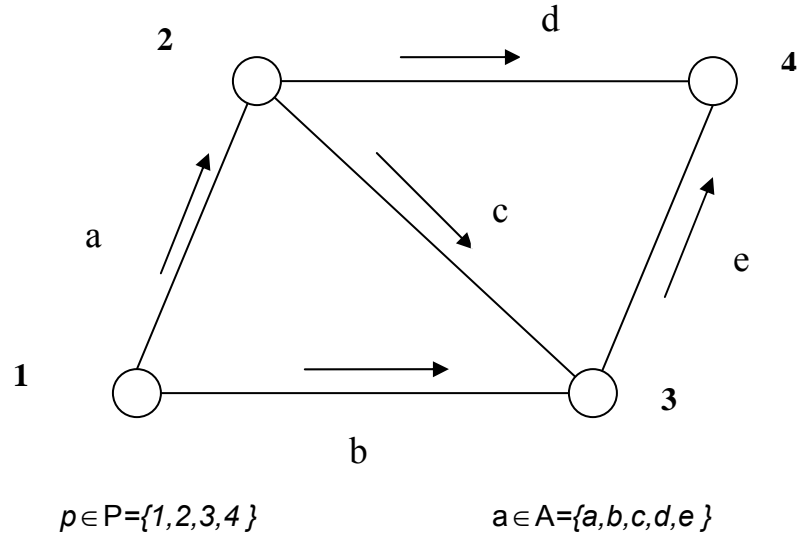
Figure 3.1 *Configuration of the traffic network*

where $N_a(t)$ is the number of vehicles in the link a at the time t, $u(t)$ is the inflow to the link a at the time t; $w(t)$ is the outflow from the link a at the time t; and, $dN_a(t)$ is a change in the number of vehicles in the link a produced during the time interval [t; t+dt]. In the macroscopic approach to the traffic flow modeling, the following formulations may be defined for the inflow and outflow, respectively:

$$u(t) = Q(t)\big|_p = \lim \frac{\Delta N_a}{\Delta t}\bigg|_{\Delta t \to 0, p} , p \in P \qquad\qquad (3.2)$$

$$w(t) = Q(t)\big|_{p+1} = \lim \frac{\Delta N_a}{\Delta t}\bigg|_{\Delta t \to 0, p+1} , p+1 \in P \qquad\qquad (3.3)$$

where the link a is located between the nodes p and $p+1$, having the traffic flow $Q(t)\big|_p$ and $Q(t)\big|_{p+1}$ across them, respectively.

Our main objective is to propose a mesoscopic continuous whole-link vehicle bunch model (considering traffic heterogeneity features) that can be conveniently applied to any kind of optimal control problem and network configuration. The main challenge in presentation of traffic heterogeneity in network traffic modeling is how to combine the singular traffic participant's car-following behavior which is pure microscopic feature with macroscopic continuous traffic flow representation generally used in whole link modeling. Three completely distinct approaches are possible.

- First one is to use LWR model as a basis for the subsequent LWR extensions considering, one by one, microscopic features of traffic heterogeneity. This type of approach, however, as was explained in previous chapter, usually presents only so-called ''local'' LWR solution extended for a specific microphysical feature and cannot be applied to the other traffic heterogeneity feature.

- Second approach is to use car-following behavior theory as a basis to introduce macroscopic features of traffic flow.

- Finally, the last approach which is called coupling approach is to develop a hybrid model in which the two traffic models, one based on microscopic car-following behavior and another based on continuum LWR presentation, can be coupled together to understand where, i.e. under what conditions and at which modeling features, one of them performs better than the other. Such comparative coupling allows progressively defining advantages and drawbacks of each modeling approach and estimating the corresponding level of uncertainty to be applied for specific modeling. For instance, the nature of most traffic phenomena is both stochastic and deterministic, while proportion of the respective representation

depends on type of phenomenon. Therefore, application of coupling approach may help classifying the phenomena according to the respective level of uncertainty in order to define the preferred tool for their modeling.

Despite the great advantages of the hybrid method, it requires great time and computational resources, which were limited in this research. Therefore, the presented approach is closer to the second approach in which we distinguish flow representation and flow participant representation. We would try to develop the dual representation of main traffic features in order to be able to vary stochastic and deterministic features as will be required by specific traffic phenomena in the future. Therefore, the mesoscopically represented results obtained in this research may be used for finding further relationships with LWR-based modeling by recalculating from them all macroscopic characteristics such as flow volume, density and macroscopic distribution of speed. This, however, may be an objective for the next investigations in the field. In this thesis, our objective is confined to initial formulation of mesoscopic continuous whole link model based on understanding of heterogeneous traffic phenomenon to be used for network traffic modeling as future works.

Chapter 4:

Solution Approach

MESOSCOPIC WHOLE LINK CONTINUOUS VEHICLE BUNCH MODEL

4.1. NOMENCLATURE

C_a link flow capacity

$f(t)$ composite time headway p.d.f., $f(t) = \theta \cdot g(t) + (1-\theta) \cdot h(t) = g_*(t) + h_*(t)$, (-);

$g(t)$ constrained component of time headway p.d.f. (-);

$g_*(t)$ nonnormalized constrained component of time headway p.d.f., $g_*(t) = \theta \cdot g(t)$, (-);

$h(t)$ free component of time headway p.d.f. (-);

$h_*(t)$ nonnormalized free component of time headway p.d.f., $h_*(t) = (1-\theta) \cdot h(t)$, (-);

K   constant in Hyperlang distribution;

l_a link length (m);

$L_j(t)$ and $L_{i,j}$ the length of the bunch j at time t and its counterpart obtained by time

discretization (m);

L_c, L_t and L_b the length of the passenger car, semi-trailer (truck) and bus respectively, (m);

L_{n_j} the length of the last car in the bunch, (m);

$N_a(t)$ and $N_{i,j}$ number of the vehicles on the link a observed at time t and its counterpart

obtained by time discretization (*veh*.);

n_j bunch size, i.e. number of the vehicles in the bunch j, which is time-independent value (*veh.*);

$p_c, p_t,$ and p_b proportions of the passenger cars, semi-trailers (trucks) and busses in the

traffic volume;

t time;

v_j^0 desired speed for the bunch j ($m \cdot s^{-1}$);

$v_c^0, v_t^0,$ and v_b^0 desired speeds for passenger car, semi-trailer (truck) and bus, respectively

(-);

$v_j(t)$ and $v_{i,j}$ real speed for bunch j and its counterpart obtained by time discretization

(m/s);

Greek symbols:

α parameter in Borel-Tunner distribution, (-);

β parameter in equation for real speed, (-)

Δ minimum headway between vehicles, (s);

δ_1 minimum free headway, (s);

δ_2 minimum constrained headway, (s);

γ_1 average headway of free vehicles, (-);

γ_2 average headway of constrained vehicles, (-);

θ^* conditional probability for a vehicle to be constrained, $\theta(t) = g_*(t)/f(t)$, (-);

ϕ proportion of the constrained vehicles, (-);

λ arrival rate of free vehicles, (s^{-1});

τ link travel time, (s);

$\tau_{1,j}$ link travel time for the first car in the bunch j, (s);

$\tau_{n_j,j}$ link travel time for the last car in the bunch j, (s);

Subscripts

1   free vehicles in the traffic volume;

2   constrained vehicles in the traffic volume;

a   arc or link;

b   bus;

c   passenger car;

t   truck

Indexes:

i   time interval counter;

j   bunch counter;

jj   counter for real speed data;

I=1,2,3….24 hour counter starting from 12:00 AM

4.2 BACKGROUND

To overcome the difficulties presented in modeling of *heterogeneous traffic flow in discrete packet approach* (Dell'Orco, 2006), a new continuous vehicle bunch model is proposed here based on the experimental time headways distributions found in recent

research study by Ye and Zhang (2009). We favour the earlier term for a group of vehicles proposed by Cowan (1976), i.e. the vehicle bunch, to the latest one proposed by Di Gangi (1992) and Dell'Orco (2006), i.e. the vehicle packet. In this chapter, the main elements of such a continuous time link model for multiclass traffic modeling using vehicle-specified time headway distributions is presented.

Two main attributes of traffic flow classification have been used: (i) a vehicle type specifying, in turn, a vehicle length and vehicle-specified time headways; and, (ii) a driver-desired vehicular speed defining, together with the speeds of the neighbouring vehicles, the vehicle acceleration/deceleration mode. Generally, three vehicle types have been used: passenger car, semi-trailer and bus. For each vehicle type, a specific normal distribution has been used for the driver-desired vehicular speed. According to the earlier experimental observations and their theoretical comprehension, it is assumed that vehicles in uncongested through moderate congested flow move in the bunches dividing drivers into the two main groups: independent, i.e. "free", drivers which usually manifest themselves as the leaders of the bunches and followers, i.e. "tracker", drivers which adapt their speed to the leader's speed and follow each other at the time headways specified by predecessor/successor pairs. It is assumed that the number of the vehicles in a bunch is approximately proportional to the traffic flow rate and, therefore, is calculated by using traffic flow-defined normal distribution. Thus, it contains both stochastic and deterministic components. A semi-Poisson mixed model has been used for calculating the time headways of both constrained and free vehicles. As the congestion continues to develop, more and more "leaders" became "followers", since the highly congested flow

obliges them to adjust their flow attributes to the attributes of the preceding bunch. To summarize, the following distinct features of continuous bunch traffic flow are assumed:

-traffic may be represented by vehicle bunches, all vehicles inside a bunch move with approximately the same speed;

-each bunch consists of one leader and several followers repeating leader's behaviour concerning the desired speed and acceleration/deceleration procedures;

-speeds of two neighbouring bunches may be completely different depending on the desired speed of the respective bunch leader;

-the bunch length is a physical value defined by the length of time headways between the drivers and the length of the vehicles in the bunch;

-no overtaking is allowed in traffic flow represented by vehicle bunches.

4.3 MODELLED TRAFFIC HETEROGENEITY ELEMENTS

4.3.1 Hourly vehicle counts decomposition

The total vehicle count per hour, denoted by I number, may be decomposed into three main components as follows:

$$N_{\Sigma,I} = N_{c,I} + N_{t,I} + N_{b,I} \qquad (4.1)$$

where $N_{\Sigma,I}$ is the total number of vehicles per hour; $N_{c,I}$ is the absolute number of passenger cars that moved on this link during considered period I; $N_{t,I}$ is absolute number of semi-trailers, i.e. articulated trucks; and, finally, $N_{b,I}$ is absolute number of buses on the link during considered periods. The proportions of the mentioned vehicle types may be defined as follows:

$$p_{c,I} = \frac{N_{c,I}}{N_{\Sigma,I}}; \quad p_{t,I} = \frac{N_{t,I}}{N_{\Sigma,I}}; \quad p_{b,I} = \frac{N_{b,I}}{N_{\Sigma,I}} \tag{4.2}$$

$$p_{c,I} + p_{t,I} + p_{b,I} = 1 \tag{4.3}$$

4.3.2 Vehicle type

It is supposed that the total number of the vehicles is distributed according the standard normal distribution, i.e. $Z \sim N(0,1)$. Defining two threshold values as $z_1 = p_{t,I}$ and $z_2 = 1 - p_{b,I}$, it is possible now to formulate the choice of the vehicle type as follows:

If $Rand \; N(0,1) \le \Pr(Z \le z_1)$ truck is chosen,

if $Rand \; N(0,1) \ge \Pr(Z \le z_2)$ bus is chosen

else if $\Pr(Z \le z_1) \le Rand \; N(0,1) \le \Pr(Z \le z_2)$ passenger car is chosen $\tag{4.4}$

4.3.3 Time headways

Cowan (1976) and Branston (1976) published the results of their experimental observations of traffic flow as the groups of vehicles moving together and called as bunches. Cowan defined several potential laws according to which the time headways between the vehicles may be distributed. Each bunch consisted of one leader imposing his own driving attitudes on several drivers that followed him and several followers each one of whom was tracking his predecessor in his anticipation of driving conditions and eventual congestion. Therefore, the free drivers usually drive at a distance called free headway from preceding vehicles, while followers drive at a distance called constrained headway from the preceding vehicles. Cowan (1976) and Branston (1976) defined a generalized queuing model accounting for the distinctions between the free drivers and

followers. Even earlier, a similar composite time headway model called semi-Poisson was proposed by Buckley (1968). The probability distribution function (p.d.f.) of such composite models may be presented as follows:

$$f(t) = \theta \cdot g(t) + (1 - \theta) \cdot h(t) \qquad (4.5)$$

where θ proportion of the constrained vehicles, (-); $g(t)$ constrained component of time headway p.d.f. (-); $h(t)$ free component of time headway p.d.f. (-). Wasielevski (1974) proposed to rewrite the probability distribution function of such composite time headway as follows:

$$f(t) = g_*(t) + h_*(t) \qquad (4.6)$$

where $g_*(t)$ is nonnormalized constrained component of time headway p.d.f., i.e. $g_*(t) = \theta \cdot g(t)$, (-), while $h_*(t)$ is nonnormalized free component of time headway p.d.f., $h_*(t) = (1 - \theta) \cdot h(t)$, (-).

After more than 30 years of extensive research in the field of the time headways (Cowan, 1976; Branston, 1976, and Ye and Zhang, 2009) it became clear that the headway distributions depend not only on division on free and follower drivers, but also on the vehicle-type-specific pair participating in interaction, the level of traffic flow and even type of the highway (rural or freeway). Hoogendoorn and Bovy (1998) proposed a technique for obtaining vehicle-type-specific headway distributions considering some of the factors mentioned. However, their technique considered only one vehicle from the interacting pair, while data were concerned to the rural area. The most extensive research

in the field considering all factors just mentioned, in our opinion, represents the study performed by Ye and Zhang (2009). In their study, however, the composite time headway model was applied for one homogeneous pair only, while, for the rest pairs, simple models were applied. An excellent flexible composite model combining shifted exponential distribution and Erlang distribution was proposed even earlier by Dawson and Chimini (1968). Archilla and Morrall, (1996) improved the model by Dawson and Chimini and presented it as cumulative distribution function (c.d.f.) as follows:

$$\Pr\left(Hw \leq t\right) = 1 - \left(1 - \theta\right)\exp\left(-\frac{t - \delta_1}{\gamma_1 - \delta_1}\right) - \theta\exp\left(-K\frac{t - \delta_2}{\gamma_2 - \delta_2}\right)\sum_{x=0}^{k-1}\frac{\left[K\frac{t - \delta_2}{\gamma_2 - \delta_2}\right]^x}{x!} \qquad (4.7)$$

where δ_1 and δ_2 are minimum free and constrained headways, respectively in (s), γ_1 and γ_2 are average headways of free and constrained vehicles in (s), respectively, while K is a parameter in Erlang distribution. Analysing the data by Ye and Zhang (2009) for various values of traffic volume flow, presenting the average values for each specific gradation of traffic flow volume, and using the composite formula by Wasielewski (1974) we were able to use the extremely low and extremely high traffic flow volumes for obtaining free and constrained components of the distributions, respectively. An example of application of such formula obtained for truck/passanger car pair for various values of θ is presented in Figure *4.1*. We used the data by Dey and Chandra (2009) for desired time gapes to calculate empty zone values, i.e. minimum constrained headways, for all homogeneous and heterogeneous pairs of vehicles.
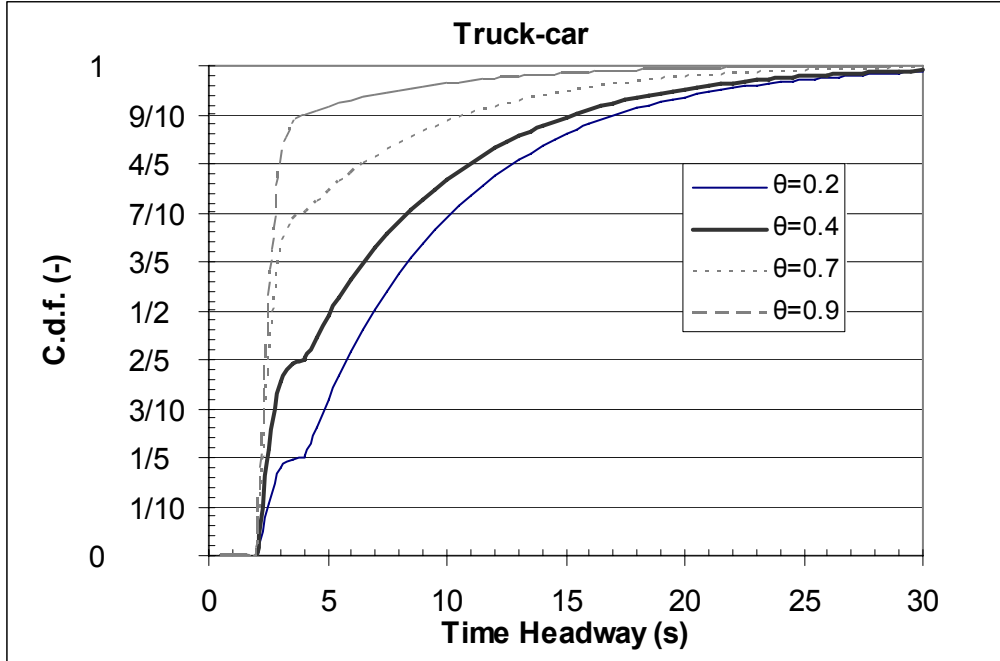
Figure 4.1 *C.d.f. of heterogeneous pair of vehicles truck/passenger car for various proportions of constrained vehicles, θ.*

Figure 4.2 shows that the difference created by various values of the proportion of constrained vehicles is more significant then the difference produced as result of various vehicle combinations.

Thus, all heterogeneous vehicle pairs, i.e. all except car-car, truck-truck and bus-bus pairs were treated according to modified Hyperlang model and original Erlang model (Dawson and Chimini, 1968; Archilla and Morrall, 1996; and Ye and Zhang, 2009) presented here by Eq.(4.7). The following vehicle type-specific headways are calculated according to this formula: Hw_{c-t}; Hw_{c-b}; Hw_{t-c}; Hw_{b-c}; H_{t-b}; and, Hw_{b-t} Values of every constants were different for various headways groups as obtained from the data by Ye and Zhang (2009).
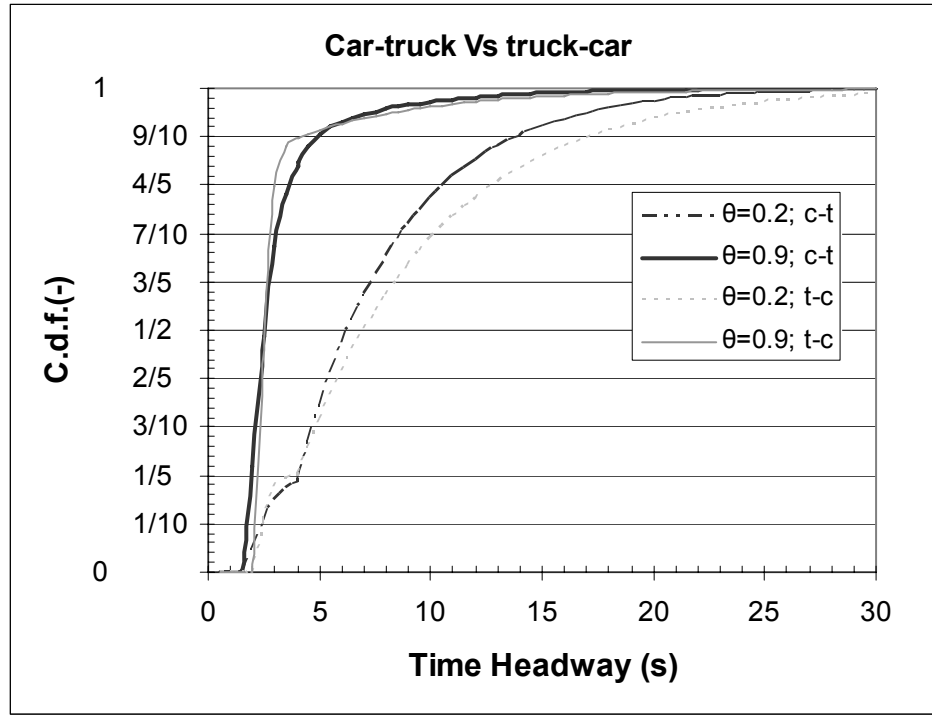
Figure 4.2. *Comparison of the C.d.f.'s of headways for two heterogeneous pairs of vehicles car/truck and truck/car for various proportions of constrained vehicles, θ.*

Recalling that the headways of homogeneous pairs would be best represented by the pair of shifted negative distribution, we have chosen Schuhl model (Schuhl, 1955) for the rest of vehicle pairs, i.e. for all homogeneous pairs such as car-car, truck-truck and bus-bus. All these groups of headways were calculated according to the following formula (Schuhl, 1955; Archilla and Morrall, 1996):

$$\Pr(Hw \le t) = 1 - (1-\theta)\exp\left(-\frac{t}{\gamma_1}\right) - \theta\exp\left(-\frac{t-\delta_2}{\gamma_2-\delta_2}\right) \tag{4.8}$$

In other words, the following vehicle-type-specific headways were calculated according to this formula: Hw_{c-c}; Hw_{t-t}; and, Hw_{b-b}.

4.3.4. Bunch size

We considered initially several probability generating functions (p.g.f.'s) for bunch size calculation such as geometric distribution, Borel-Tunner distribution (Tanner, 1961), Miller distribution (Miller, 1961) and Consul distribution (Islam and Consul, 1991). Because of absence of real bunch size database in our research, we could not compare the calculated values of bunch size to real-life data. However, comparing them to each other, we were able to realize that Borel-Tunner distribution is more stable than Miller distribution, while Consul distribution is the more general one reducing to either geometrical or Borel-Tunner distributions for some limiting cases.

The probability that the size of the bunch j is n_j (for $0 \le \alpha = m\theta \le 1$) according to Borel-Tunner distribution may be calculated as follows:

$$\Pr(n_j) = \frac{(n_j \alpha)^{n_j - 1} \exp(-\alpha n_j)}{n_j!} \qquad (4.9)$$

where $m \in N^+$, $0 < \theta < 1$, such that $1 \le m \le \theta^{-1}$, or $m < 0$, $\theta < 0$ such that $m\theta \le 1$. The Consul distribution may be calculated as follows:

$$\Pr(X = n_j) = \frac{1}{n_j}\binom{mn_j}{n_j - 1}\phi^{n_j - 1}(1 - \phi)^{mn_j - n_j + 1} \qquad n_j = 1,2,.... \qquad (4.10)$$
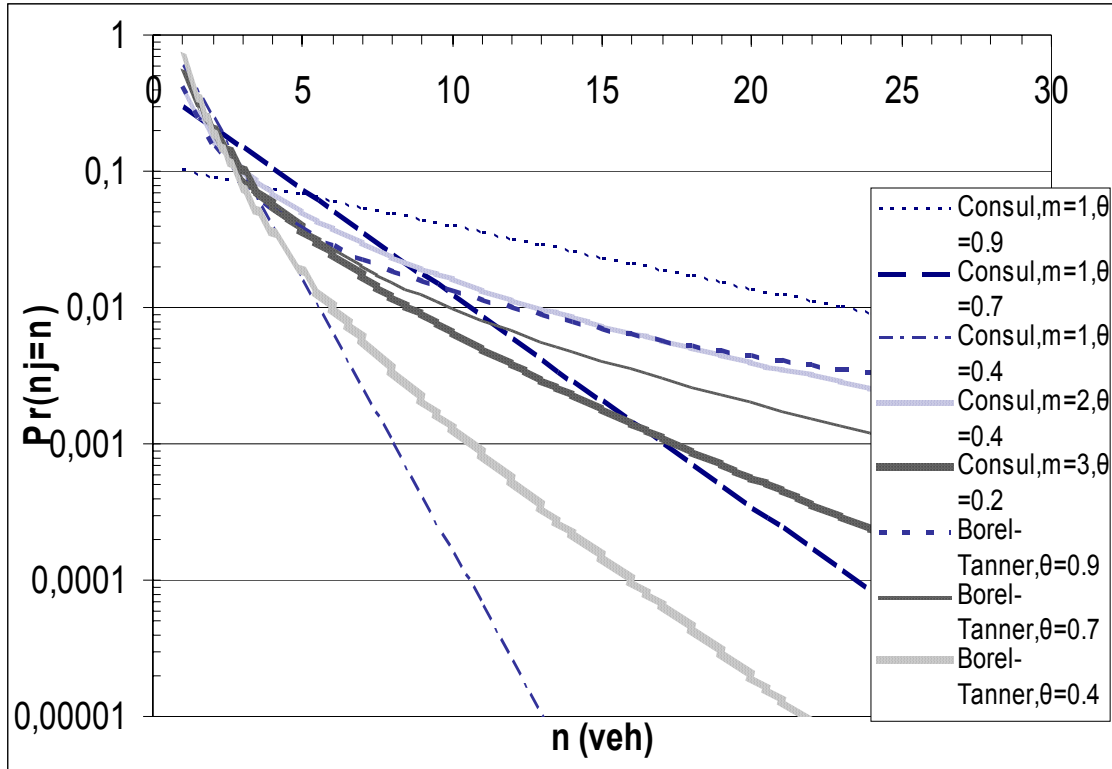
Figure 4.3 *Comparison of Consul distribution to Borel-Tanner distribution for the bunch size. Consul distribution for m=1 reduces to geometric distribution.*

As an example, Figure 4.3 presents comparison of Consul distribution to Borel-Tanner distribution. For visual demonstration only, the discrete distributions are presented by solid lines. It seems that Consul distribution is more general, though it has some limitations for vehicle constraint because of the condition $m\theta \leq 1$.

4.3.5. Desired speed

The anticipation of both the safety requirements and the current driving conditions by various drivers is usually not the same. Therefore, their choice of the speed which is most suitable for the present situation or, in other words, their choice of the desired speed is

never the same. If there is no changing the lanes, then a vehicle moving with the lower speed, by preventing the faster vehicles from going ahead, affects the speed of the vehicles following it on the same lane. Many authors considered the desired various speed distributions for specific vehicle types on completely different manner. Hoogendoorn and Bovy (2000) introduced speed dependence in LWR model, Wong and Wong (2002) proposed to write down main LWR equations p times equal to classes of the desired speed, while Jiang and Wu (2004) introduced speed gradient in the LWR model.

For the desired speed treatment, the exact values collected during experimental series may be used which should be fixed for the already existing impeding (Archilla and Morall, 1996).

$$v_c^0 = N\left(\mu_c = 27.3; \sigma_c^2 = 2.3^2\right)$$

$$v_t^0 = N\left(\mu_t = 26.6; \sigma_t^2 = 1.5^2\right)$$

$$v_b^0 = N\left(\mu_b = 26.5; \sigma_b^2 = 1.36^2\right) \qquad (4.11)$$

It should be noticed that the values measured at low traffic volume flow do not correspond to the desired values at high traffic volume. These three normal distributions are presented in Figure 4.4. Car data are more dispersed as compared to truck and bus data. The authors (Archilla and Morall, 1996) do not mention, whether the data is already fixed for the constrained part during measurements. Recall, that the unconstrained data may be obtained from censored speed data by using modified Kaplan-Meier survival non-parametric distribution as follows (Hoogendoorn, 2005; Catbagan and Nakamura, 2008):

$$\hat{S}_{\infty}\left(v^{0}\right)=\prod_{jj=1}^{M_{v^0}}\left(\frac{M-jj-1}{M-jj-\theta}\right)=1-\hat{F}_{\infty}\left(v^{0}\right) \qquad (4.12)$$

where $F_{\infty}\left(v^{0}\right)$ represents free speed cumulative distribution for specific type of vehicles, M_{v^0} is number of samples of v_{jj} that are smaller or equal to v^{0}, M is total number of headway observations.
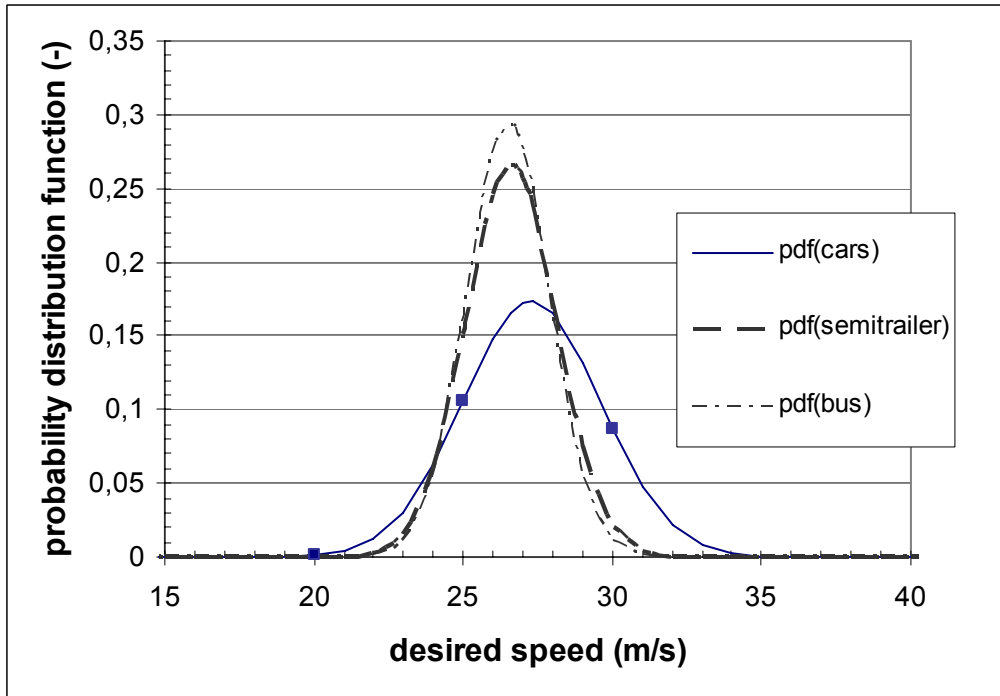
Figure 4.4 *Three standard normal distributions obtained from (4.11) for car, truck and bus.*

4.3.6. Real speed

For calculating the real speed of the current bunch, an inequality was developed in this research comparing *the free headway between the first car in the current bunch j and the last car in the preceding bunch j-1, i.e. either* $Hw_{1,c}$, $Hw_{1,t}$ *or* $Hw_{1,b}$, *on the one hand, and desired time gap* (Dey and Chandra, 2009) *between the first car in the current bunch*

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j and the last car in the preceding bunch j-1, i.e. $\Delta_{1,c}$, $\Delta_{1,t}$ *or* $\Delta_{1,b}$, on the other hand. To take a decision whether to apply deceleration of the current vehicle bunch, the following inequality was obtained in this research comparing the desired gap and free headway between *j* and *j-1* bunches moving at speeds v_j and v_{j-1}, respectively:

$$\underbrace{\Delta_{1,(c,t,b)}}_{\text{Desired gap}} + \underbrace{\left\{ \left(\sum_1^{n-1} Hw_{kk,kk+1} \right)_j + \frac{2L_F}{v_{j-1}(1-\beta)+v_j(1+\beta)} \right\} \frac{(v_{j-1}-v_j)(1+\beta)}{2}} \le \underbrace{Hw_{1,(c,t,b)} v_{j-1}}_{\text{Free headway}}$$

B*unch length calculated as a weighted value*

of the lengths at the present and next time steps

(4.13)

where β is a coefficient which is very close to the perception coefficient, and *kk* is a counter for the number of the vehicles inside each bunch. The subscripts *c, t* and *b* stand for car, truck and bus, respectively. *Recall that time gap between two vehicles,* Δ*, is a time headway between them decreased for the value of time required to cover the length of the first vehicle.* If the calculated gap between leader of the present bunch and the last driver of the preceding bunch, *i.e.* free headway minus the length of the last vehicle in previous bunch, is lower than the desired time gap for the same pair of vehicles, the deceleration should be applied.

For calculating the acceleration/deceleration, the formula of intelligent driver model (IDM) by Treiber et al. (2000) was used in the following shape:

$$\dot{v}_j = a_{max}\left(1-\left(v_j/v_j^0\right)^4-(S^*/\Delta)^2\right)^2$$

(4.14)

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where a_{max} is maximum acceleration, a_{max}=0.73 m/s^2 ; v_j^0 is desired speed for bunch j; v_j is real speed for the same bunch; Δ is desired gap between first vehicle in the current bunch j and the last vehicle in the preceding bunch j-1, while S^* is real distance between them (bumper to bumper) calculated as follows:

$$S^* = s_0 + v_j T + \frac{v_j \Delta v}{2\sqrt{a_{max} b_0}} \tag{4.15}$$

where: $s_0 = 2$ m; $T = 1.6$ s; $b_0 = 1.67$ m/s^2; Δv is speed difference between the bunches. More about acceleration/deceleration procedure, as well about the way of its application to the last follower and the related problems will be mentioned in Section 5.6.

4.3.7. Bunch length

The average length of each type of the vehicle mentioned in Subsection 4.3.1 has been chosen in accordance with existing data in literature: the average passenger car length is 4.5 m; the average length of eighteen-wheeler is up to 25 m; the length of the school bus is up to 12.5 m. As mentioned before, three solutions for presenting the vehicles with various lengths were proposed in previous research studies (Lebacque et al., 1998; Chanut and Buisson, 2007; and Van Lint et al., 2008). The first solution was defined for presenting flow dynamics of buses which generally are longer and slower in the acceleration than the rest of the traffic participants. Such approach consisted of decreasing the value of link flow capacity by the value of flow covered by the length of the bus with all ensuing consequences such as decreasing of the flow capacity and jam density. This type of the solution could not be applied directly to network traffic

modeling since both traffic flow characteristics mentioned will become dependable on the time and length of the link. Both the recent studies done by Chanut and Buisson (2007) and by Van Lint et al. (2008) used term called passenger car equivalent, obtained for car and trucks together. It expresses a new flow capacity and jam density which became weighted functions of the flow capacities and jam densities calculated for two flows consisted of car and trucks only, respectively. Again, such a solution could not be applied directly to network traffic modeling. Because the effect of time headways between the vehicles was found to be dominating over the effect of the vehicle length on both the flow capacity and jam density (Treiber and Helbing, 1999) and because the time headways were found to be defined by the vehicle type and volume of traffic flow (Ye and Zhang, 2009), we define here nine different time headway distributions depending on the predecessor/successor pair. The values of the parameters in the defined distributions are dependable on the classification of the traffic flow related to congestion development. The discrete value of bunch length calculated after each acceleration/deceleration procedure may be found as follows:

$$L_{i,j} = v_{i,j}(\sum_{kk=1}^{n_j-1} Hw_{kk,kk+1} + L_F)$$
(4.16)

where $Hw_{kk,kk+1}$ are constrained time headways, i.e. headways inside the bunch, L_F is length of the last follower in the bunch, and $v_{i,j}$ is real speed of the bunch, kk is counter inside the bunch.

4.4. CALCULATION PROCEDURE

Initially the bunch size and type of vehicles in the bunch are calculated for each bunch. For the first vehicle in the bunch, the desired speed is calculated. The procedure for

adoption of desired speed in the model will be explained in detail in the next section. Time headways are calculated for each pair of the vehicles inside the bunch according to (4.7 or 4.8). For every time step (i counter), the conditional length of the bunch is calculated, i.e. $L_{i,j}$, (4.16) and the distance it travels on the link may be found as $S = v_{i,j} \cdot \Delta \tau$. During each time step, the speed of each bunch is fixed. At the fist time step only it is equal to the desired speed, i.e. v_j^0. At every next time step, the current real speed of bunch, $v_{i,j}$, is calculated which is a function of the speed at the previous time step, $v_{i-1,j}$, and of the current speed of preceding bunch, $v_{i,j-1}$. No overtaking is allowed. Same procedure is repeated j times (number of bunches) for the entire vehicle counting during the current hour I (traffic volume per one hour).

The calculation for one hour is performed in Matlab according to flow chart presented in Figure 4.5. If new proportions of the type of vehicles are introduced, the process is repeated. The critical issue is here in how to design several consecutive i loops (time counter) and j loops (bunch counter). For each subsequent bunch, speed and current distance from the link input will be dependable on the positions of the vehicle ahead and the information should be available concerning the drivers in the preceding bunch. Current bunch moving with the slow speed may have influence on one or many following bunches depending on the dynamic conditions and the speed difference between bunches.

Minimum desired gaps for specific vehicle pair are introduced as constant in the shape of matrix. Also traffic counts per hour decomposed into the main three components are introduced at this stage in the shape of proportions.
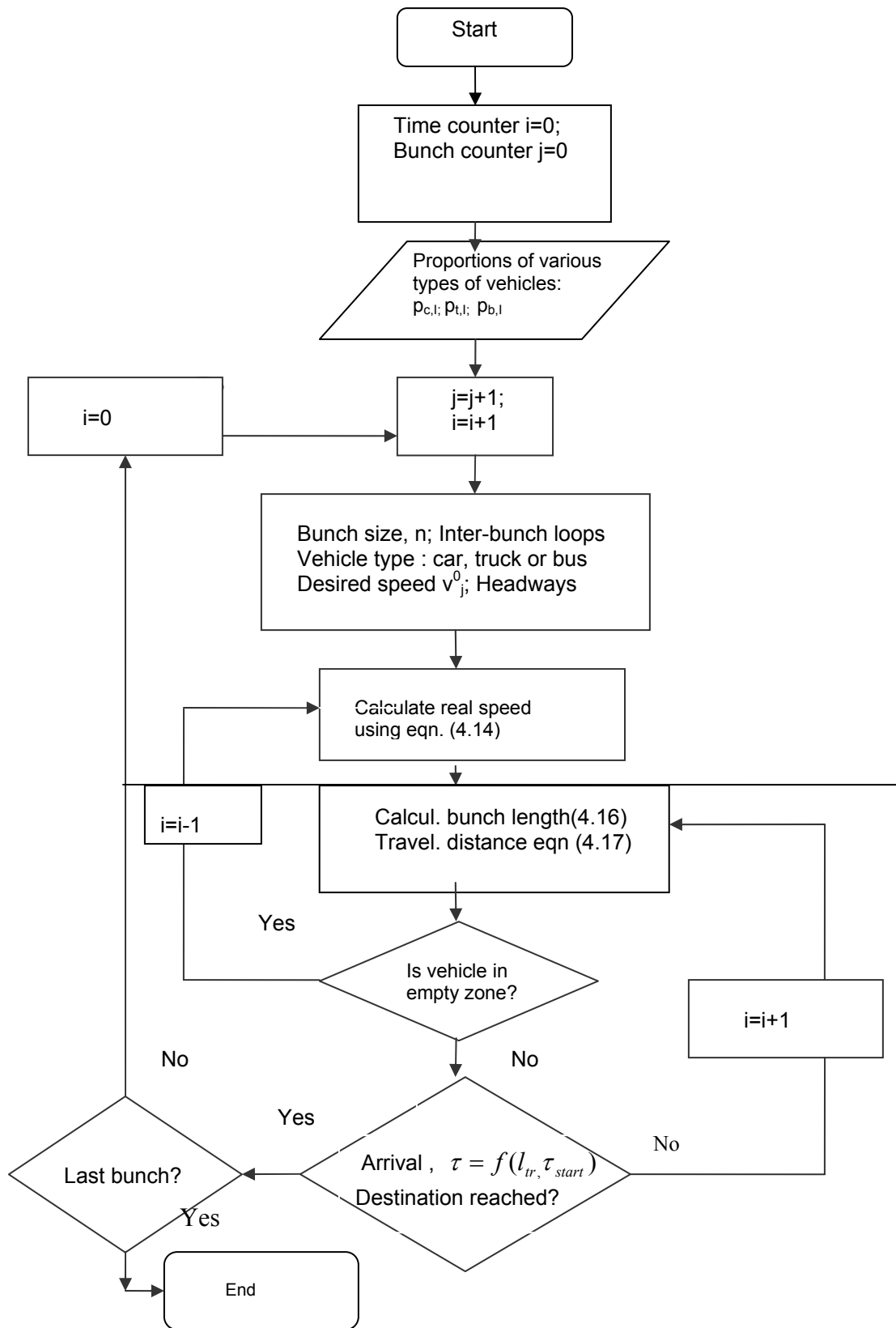
Figure 4.5. *The flow chart for model calculation of link travel time during one hour.*

Bunch counter starts, and at this stage the bunch size, the types of vehicles and both free and constrained headway are calculated. After calculating bunch desired speed and bunch traveled distance, the latter is compared to the traveled distance concerning the previous bunch at the same time step. The inter-bunch distance is calculated and it is compared to the minimum desired time gap between two specific vehicles. The info about the last driver in the preceding bunch is saved at previous bunch counter loop. If the first vehicle in the current bunch gets into empty zone, the calculation for real speed performed by using deceleration formula. After speed adjustment and new travel distance calculation, a new inter-bunch distance is calculated. Next time step, an additional speed adjustment will bring the speed of the current bunch closer to speed of preceding bunch. The number of the steps during adjustment procedure depends on the time step. In this research a time step of 10 s was used, which required only three four adjustment to reach the new speed value. After checking if arrival destination is reached, a new time step is performed and previously explained procedure for inter-bunch distance evaluation is repeated. After reaching the destination, it is checked if the current bunch was the last bunch in the hour. After the last bunch is reached, the calculation either continues for the next hour with new vehicle type proportion data or finally stops.

4.5 OUTPUT

The travelled distance is calculated at each time step $\Delta\tau = 10$ s according to the following simple formula:

$$S_{i,j} = S_{i-1,j} + v_{i,j} \cdot \Delta\tau \tag{4.17}$$

The link travel time is calculated for each bunch separately as a sum of the time steps required to cover the entire distance equal to the length of the link plus travel time for the part of last time step required to the first car finish the distance l_a as follows:

$$\tau_{L,j} = (i_L - 1) \cdot \Delta\tau + \xi_L \cdot \Delta\tau + \varsigma_L \cdot \Delta\tau$$

$$\tau_{F,j} = (i_F - 1) \cdot \Delta\tau + \xi_F \cdot \Delta\tau + \varsigma_F \cdot \Delta\tau \qquad (4.18)$$

Where i_L and i_F are numbers of the whole time steps required to cover distance l_a by the leader and last follower in the bunch, respectively, ξ_L, ζ_L ξ_F and ζ_F are respective parts of time step, when the travel starts and/or finishes inside the time step for the leader and for the last follower in the bunch, respectively. The travel time for the last car in the bunch is fixed for the length of the bunch at the beginning of travel.

As may be seen, the link travel time becomes dependable on many parameters mentioned such as initial desired speed, real speed, time headway distribution and conditional probability for a vehicle to be constrained.

Chapter 5:

Numerical Application

5.1. INPUT DATA

In this chapter, an implementation of the continuous whole link model developed in the previous chapter is presented. The heterogeneous traffic flow on the modelled link contains the vehicles of various types moving in bunches with different desired speeds. To make the features of heterogeneous traffic and certain specific issues of flow dynamics resulting from this heterogeneity more visible, we decided to modify a few flow elements used as model input in this preliminary assessment. Our concerns here have been put on the flow attributes whose variations for various flow participants usually occur in similar fashion within a confined area. Therefore, as they are, such variations cannot normally induce some conclusive evidence of their anticipated impact from a few singular observations only. Quantifying the level of heterogeneity, for example, by using delta-function, may help to distinguish the main trends in the anticipated impact and give an answer to the question if the consideration of this factor is important for the modeling at all and at what cost.

5.1.1. Desired speed

Particularly, we speak here about the flow speed and bunch size quantification. As was seen in previous chapter, the desired speed, for all three traffic flow participants considered, usually vary following normal distribution in very narrow speed interval

(Fig.4.3). However, although the expected values of some normal distributions are relatively close, such as those of bus and articulated truck, their statistical variations may be either similar or different. In particular, the standard deviation of the desired speed for passenger cars is almost twice greater than that for bus and articulated truck. Thus, we propose to represent the flow speed variations for each traffic participant type by delta function achieving the discrete values of 20, 25 and 30 m/s. The respective probability mass function is presented in Figure 5.1.
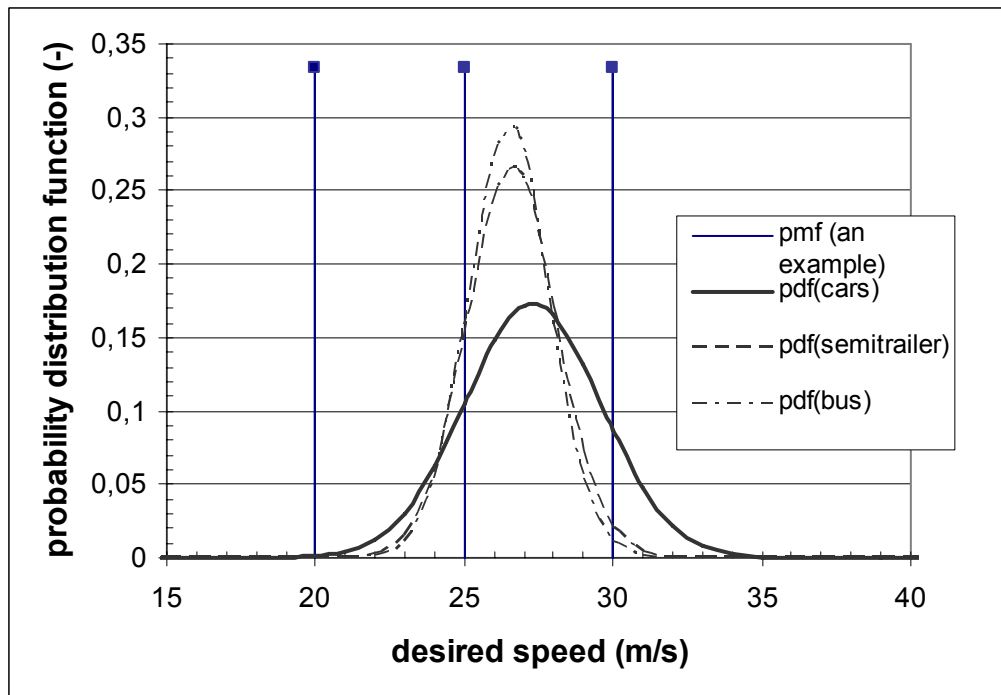
Figure 5.1. *The modeled probability mass function shaped as delta function with the values of the desired speed of 20, 25 and 30 m/s used for the model initiation. A comparison is made to the three real-life normal distributions for car, truck and bus.*

Notice, that the proposed discretization of the continuously varying input parameter is accepted to be the same for all traffic participants. In other words, considering three types

of flow participants with three discrete values of the desired speed produces totally 9 (3x3) different classes of traffic participants. Each of them may represent a specific group of the   entire driver community population. The aged cautious drivers may be represented by the delta function value at 20 m/s, while reckless and young drivers may be represented by the delta function value of 30/m and so forth. With such formalization, the main concern in future work will only be devoted to a proper definition of the nominal value of delta function for each group.
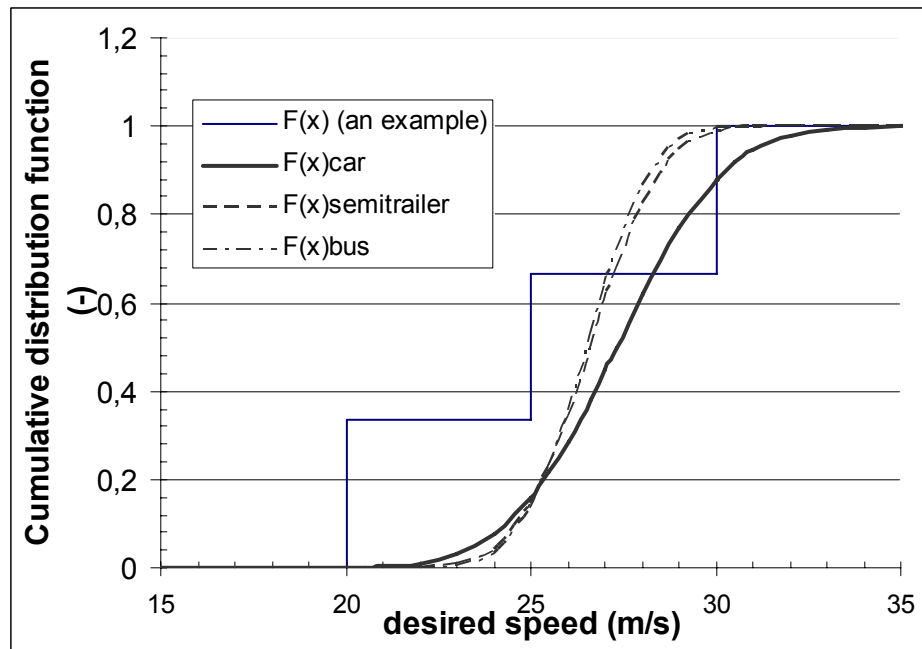
Figure 5.2. *The modeled cumulative probability mass function of a delta function with the desired speed 20, 25 and 30 m/s as compared to the three normal distributions for car, truck and bus.*

For example, Figure 5.2 presents the comparison of the cumulative distribution functions for the three vehicle types to the cumulative probability mass function for the proposed three-value discrete speed distribution. It is seen from figure 5.2 that the appropriate

decrease of the delta function value for the speed of 20 m/s and 30 m/s together with

respective augmentation of the value of the function for the speed of 25m/s (performed to

keep the total value of the cumulative probability mass function equal to a unity) will get

cumulative mass distribution function significantly closer to the experimental curves.

5.1.2. Bunch size treatment

The next input parameter, bunch number and bunch size, may be modeled by using two

different approaches. One of them is to generate the random numbers by using one of the

bunch probability mass functions defined in Section 4.3.4 with the predefined total event

number N and percentage of constrained drivers, θ; another one deals with simplifying

the modeling by assumption that the average bunch size is somehow proportional to the

traffic flow volume. In the first approach, the random numbers have been generated by

using random number generator (RNG) and compared at each step to the value of the

chosen cumulative mass distribution function unfolded from the long range tail. The

description of the entire procedure may be found in statistical literature. The outcome is

more successful, when the trials with greater amounts of random numbers are performed.

Figure 5.3 compares two samples obtained by using Consul distribution model for *m=1*

and $\theta = 0.7$.
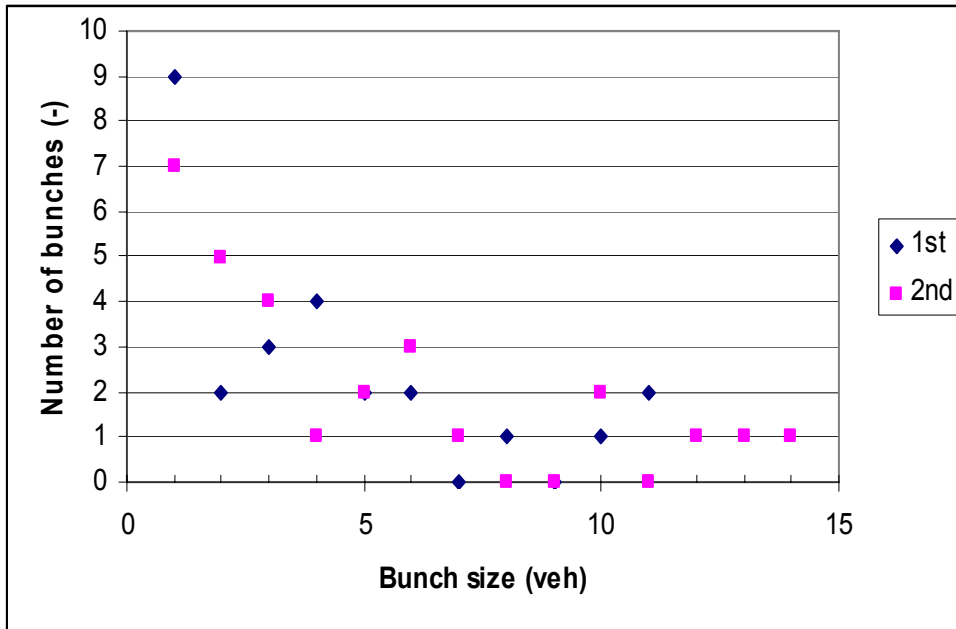
Figure 5.3. *Two sets of randomly generated bunch size distributions obtained by using Consul bunch distribution model for N=110, m=1 and θ = 0.7 (geometrical distribution with respective value of θ).*

In the second method used to generate bunch samples, we use a simple assumption that the average bunch size may be in some power law relationship with the traffic flow volume. This assumption seems to be logical if a chain of events related to the traffic flow volume increase is recollected. Namely, the greater flow volume produces greater number of the follower drivers, which in turn generally results in the increased average bunch size. The number of impeded drivers forced to be follower progressively increases, though the number of those willing to leave the bunch increases as well. Even though some of them may leave the present bunch at any lucky situation, they will be immediately forced to take their place in another bunch and so on. This dynamical birth-

and-destroy process (Islam and Consul, 1991) will continue until the substantial changes in the flow conditions occur. To account for the variations in the averaged number of the drivers impeded by the traffic flow volume and forced to take their place inside a bunch and to leave it out after a while, we propose to represent the averaged bunch by delta function with three possible values.
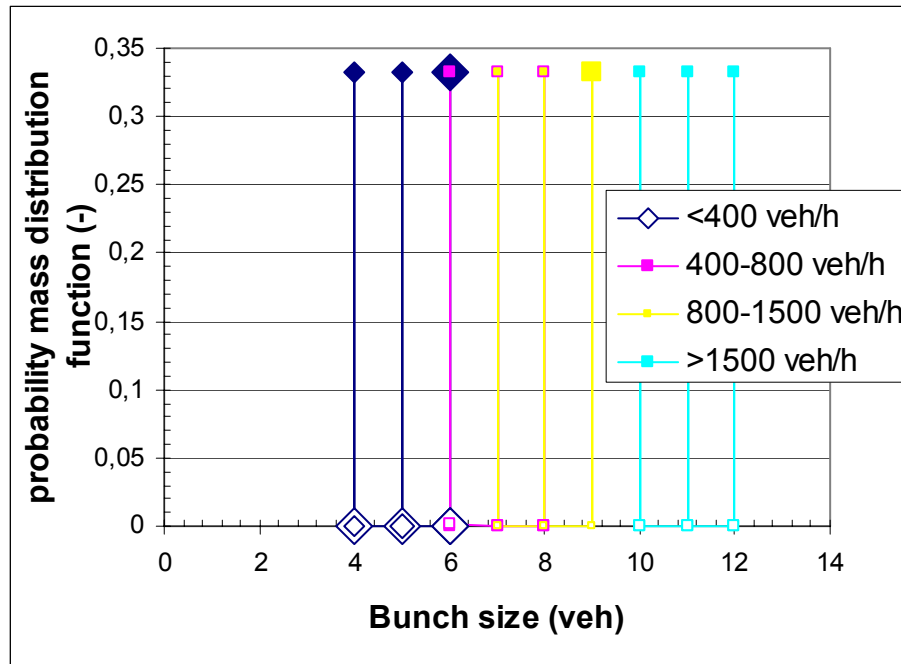
Figure 5.4. *The assumed probabilities of the average bunch size for various flow volume values: uncongested $Q \le 400$ veh/h; low congested 400 veh/h$\le Q \le 800$ veh/h; moderately congested $800 veh/hour \le Q \le 1500 veh/hour$; and, heavily congested $Q \ge 1500$ veh/h.*

Figure 5.4 presents the assumed theoretical probability mass distribution functions for the average bunch size at various levels of the traffic flow volume. Again, the nominal values

of bunch size are taken arbitrary. Such representation of the average bunch size by delta function has a number of advantages over the previous traditional approach such as saving CPU time by avoiding tedious calculations of numerous bunches with one vehicle only and by counting the anticipated queue length during congestion conditions in number of the bunches of specific size. The choice of traffic flow volume classification was according to the classification proposed by Ye and Zhang (2009). The main disadvantage of this method of bunch size modeling is, of course, absence of bunch randomness inherent with its birth-and-destroy dynamical structure. In other words, the process is closer to steady state than to any other transitional process.

5.1.3. Generating the time headways

The samples of randomly chosen constrained and free headways for various combinations of vehicle pair have been generated by using RNG and the headway distributions defined in Section 4.3.3 on the same way as it has just been explained for the bunch size. As an example, Figure 5.5 presents the time headways for heterogeneous pair truck-truck, which were randomly generated by using shifted exponential distribution with shifted value of 1.3 s (Ye and Zhang, 2009), which is specific case of the Schuhl distribution (Schuhl, 1955) presented here in Section 4.3.3. The sample size was $N = 90$. Again, as in the case with randomly generated bunches, it may be observed that the nonproportionally large values are possible even in such small samples.
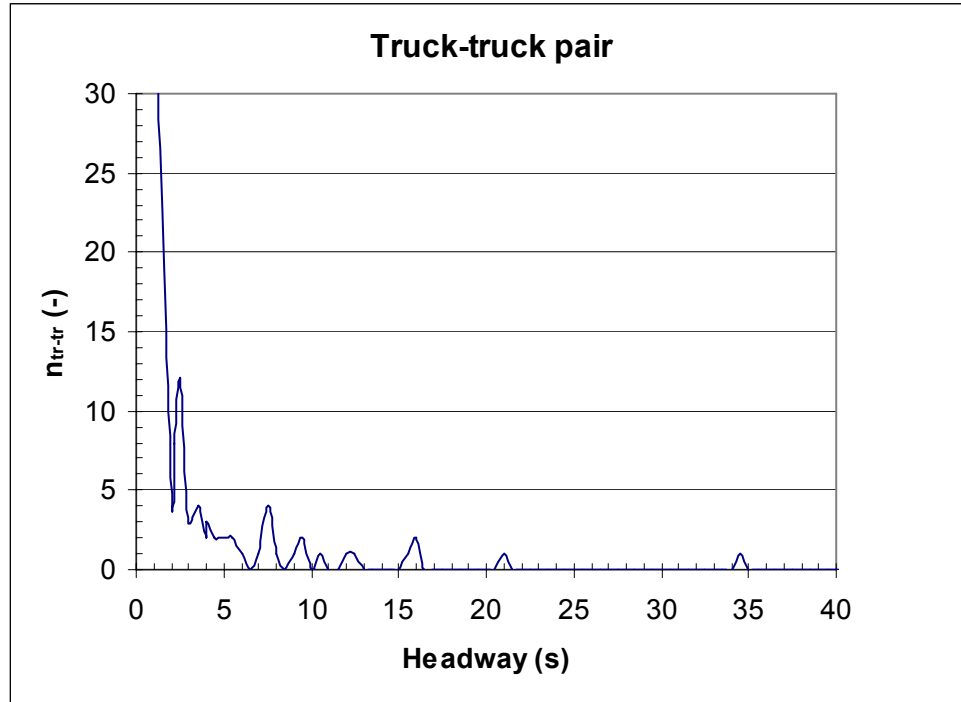
Figure 5.5. *The randomly generated time headways for the homogeneous vehicle pair truck-truck obtained by using shifted exponential distribution with shifted value of 1.3 s. Sample size is $N = 90$.*

5.1.4. Real vehicle counts and designing suitable dataset

We collected directional traffic counts available for the wide community from the Open data catalogue of the site of city Vancouver. Data, collected by automatic vehicle detectors, i.e. loop detectors, from Trans-Canada Highway, coordinates 570296 and 570297, do not contain any additional information concerning traffic heterogeneity since such data are typically not collected at an individual vehicle level, but at some short time interval such as part of the minute. Therefore we required to modify initial data generating the drivers of different classes.
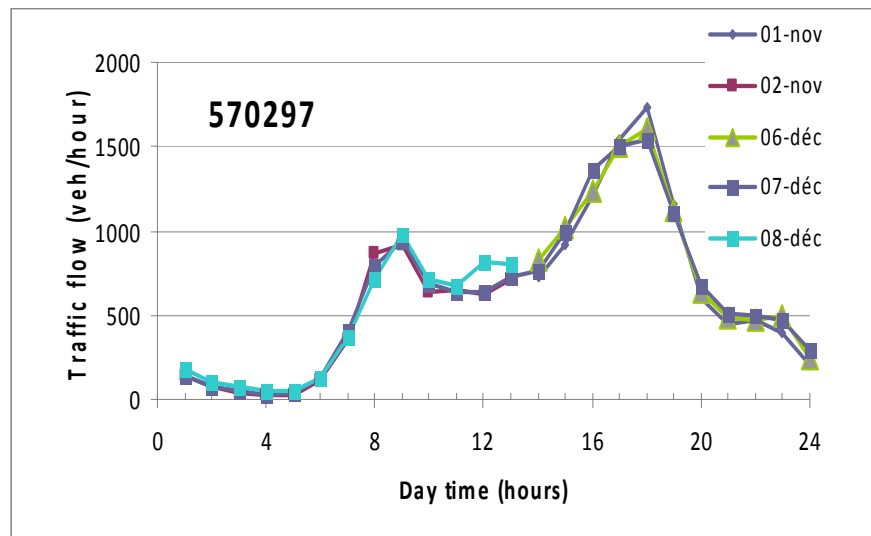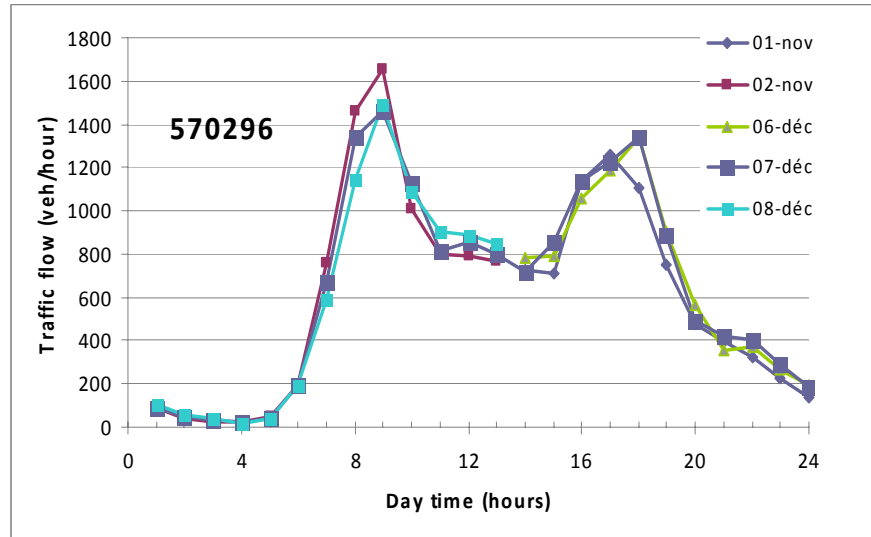
Figure 5.6. *Two typical within-day variations of traffic flow volume on Trans-Canada Highway at Vancouver with morning (afternoon) primary peak and afternoon (morning) secondary peak.*

The generated heterogeneous data in this research, of course, are subjective, since their generation was done according to our comprehension of the within-day demand for slow,

fast, light and heavy vehicles. However, having real heterogeneous data would not change the entire calculation procedure. Usually, such diversification in traffic flow follows the standard within-day traffic flow volume variations. The two typical non-resemble within-day variations on Trans-Canada Highway at Vancouver are presented in Figure 5.6. As an example of redesigning these data by taking into account (2x2) traffic heterogeneity, in Table 5.1, we present the classification of the flow volume for the main four pre-defined groups according to speed factor and vehicle type.

Hour of the day	V_1 (Slow Car)	V_2 (Fast Car)	V_3 (Slow Truck)	V_4 (Fast Truck)	Total 570296 WB
1	18	25	8	42	93
2	8	15	7	18	48
3	3	12	1	15	31
4	5	8	2	8	23
5	12	17	11	5	45
6	25	140	14	15	194
7	45	540	23	65	673
8	57	1130	30	100	1317
9	60	1275	50	150	1535
10	140	750	30	157	1077
11	280	300	71	190	841
12	350	204	65	225	844
13	400	205	48	150	803
14	302	250	70	120	742
15	225	202	110	250	787
16	190	800	65	57	1112
17	210	920	50	45	1225
18	180	939	60	80	1259
19	250	468	45	85	848
20	133	248	90	40	511
21	100	95	100	95	390
22	90	65	110	102	367
23	85	85	35	55	260
24	75	60	11	25	171

Table 5.1. *Diversification of driver classes in order to obtain (2x2) traffic heterogeneity by using within-day traffic counts.*

5.2. TRAVEL TIME COMPUTATION

Figure 5.7 presents a test link with 2 vehicle bunches containing 5 and 7 vehicles, respectively. The time headways inside the bunch, i.e. constrained headways, and the time headways between bunches, free headways, may be generated by using RNG and proposed functions after the type of each vehicle is randomly generated by using normal distribution and proposed vehicle proportions. Knowing headways and initial speed allows computing length of the bunch.

Free Headway

Constrained Headway

Bunch 2							Bunch 1				
12	11	10	v9	v8	v7	V6	v5	v4	v3	v2	v1
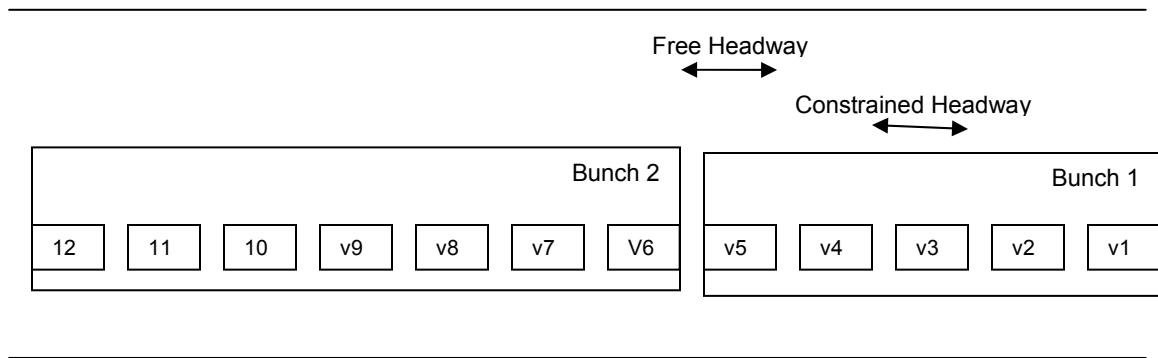
Figure 5.7 *Test link for numerical example*

We consider here a 6-km long link where the type of vehicles move in bunches in the sequence presented in Table 2. The desired speed is chosen by the leader and accepted by all followers in the bunch. It is assumed that the presence of slowly moving bunch ahead of the current one will gradually reduce the speed of some bunches behind it to the specific value equal or greater than the speed of slowly moving bunch. Although no overtaking is not allowed, we assume that impeded driver may accelerate immediately after the last slowly moving car in the bunch ahead has reached the link output. Such

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assumption is valid for the link output being a diverging node of the traffic network. In such a case, the leader of the present bunch may start acceleration procedure immediately after he has noticed the direction chosen by slowly moving bunch ahead and has anticipated to choose an alternative link.

Bunch number	Bunch size	Desired speed	Leader	Second veh.	Third veh.	Fourth veh.	Fifth veh. Condit Last	Sixth veh. Condit Last	Seventh veh. Condit Last	Note
1	5	20	Car	truck	car	bus	truck			
2	7	30	Truck	bus	truck	car	bus	car	car	
3	5	25	Truck	truck	bus	car	car			
4	7	25	Truck	car	car	truck	car	truck	car	
5	6	30	Car	bus	bus	truck	car	truck		
6	7	20	Car	bus	car	bus	truck	bus	truck	
7	5	25	Car	car	car	bus	car			
8	5	20	Truck	bus	truck	truck	bus			
9	7	30	Truck	truck	bus	bus	truck	bus	truck	
10	5	25	Car	bus	truck	truck	car			
11	7	20	Car	bus	car	bus	car	bus	car	
12	6	30	Truck	truck	car	car	car	car		
13	5	20	Car	truck	car	car	car			
14	5	25	Car	truck	bus	bus	car			
15	7	30	Truck	truck	bus	bus	car	bus	car	

Table 5.2. *First fifteen randomly generated bunches.*

The computational steps involved in calculating the travel time of the first 12 vehicles (v1-12) using the proposed model are presented as follows:

1).Time t=0,

First Bunch: The number of vehicles present in Bunch 1 is 5 (v1, v2, v3, v4 and v5). The free time headway with the preceding vehicle is 11.51 s. The initial speed of the bunch is 20 m/s and the location of the first vehicle is at the start of the test link, while the positions of the vehicles inside the bunch are defined by the respective time headways calculated for specific vehicles pairs. For instance, the headway between the leader and

the succeeding vehicle is 8.36 s, while the free headway between the last vehicle in Bunch 1 and the first vehicle in Bunch 2 is 34.5 s. The time counter starts, when first vehicle starts entering the link, so at this time interval all vehicles are outside the link.

Second Bunch: The number of vehicles in Bunch 2 is 7, and initial speed accepted for this bunch is 30 m/s. The headway of the first vehicle with the succeeding vehicle is 10.32 s.

It should be mentioned here that the positions of the vehicles in the bunch are calculated only at this time interval. After the bunch length is calculated, only the positions of front bumper of the leader and the rear bumper of the last follower in the bunch are followed in space. This simplified calculation procedure, which save a great amount of calculation time, is a crucial distinction of this model with the classical car-following models where the position of each vehicle is permanently followed in space.

2).Time t= 10 s,

The front line of Bunch 1 has entered the link at the distance 200 m. The Bunch 2 still has not entered the link.

3).Time t=40 s,

The rear line of Bunch 1 has past the distance 43 m inside the link. The Bunch 2 still has not entered the link.

4).Time t=80 s,

The front line of Bunch 1 has covered distance of 1600 m, no interference is observed since the low traffic flow volume is considered, while the bunch moves with lower value of desired speed. The front line of Bunch 2 has entered the link at the distance 152 m.

5).Time t=140 s,

The comparison of the free headway and desired time gap between the front line of Bunch 2 and rear line of Bunch 1 reveals that the Bunch 1 may get inside empty zone. This means that application of deceleration is required for Bunch 1 starting from previous time step. Calculated deceleration for t=130 s is -0.69 m/s². Therefore, the value of speed at this time interval is 23.1 m/s. For the next time interval new value of deceleration is calculated which is equal to -0.236 m/s². The decoration for the last follower is calculated in similar manner with the time lag equal to $(n-1) \cdot \tau_{sens}$, where n is a number of the driver in bunch and τ_{sens} is reaction or sensitivity time required for driver to start reacting correspondingly on acceleration or deceleration of preceding driver. It occurs that this time may have also heterogeneous nature and nay be different for various types of drivers. However, for the purposes of this research this value was taken as constant for all drivers.

This way the computation continues until all the vehicles exit the link. The travel time for each leader and last follower in all bunches are calculated accordingly to (4.15). It is clear now that the ξ_L and ξ_F are, in fact, ratios between the distance covered by either front line or rear line of bunch during time interval starting when cross the link start line and maximal distance that can be covered during time interval of $\Delta t = 10$ s with the desired speed. Respectively, ζ_L and ζ_F are the ratios between overdriven distance over $l_a = 6km$

and maximal distance that can be covered during time interval of $\Delta t = 10$ s with the real speed for respective bunch.

5.3. MODEL RESULTS

As an example, we present in Table 5.3 the final results for the leaders and last followers of first 3 bunches for different time instances until all the vehicles exit the link**.**

Time Step	Bunch–Vehicle	Event	Position on link (m)	Acceleration/ Deceleration	Travel Speed	Travel Time
T=0	B1-L	enters	0	0	20	-
	B1-F		-757	0	20	
	B2-L				30	
	B2-F				30	
	B3-L				25	
	B3-F				25	
T=140	B1-L		2800	0	20	-
	B1-F		2043	0	20	
	B2-L	decceleration	1917.5	-0.69	23.1	
	B2-F		336.5	-0.73	28.9	
	B3-L				25	
	B3-F				25	
T=300	B1-L	exit	6000	0	20	300
	B1-F		5243	0	20	
	B2-L		5140.4	0	20	
	B2-F		4063.4	0	20	
	B3-L		3906.5		25	
	B3-F		2948.5		25	
T=350	B2-L	exit	6140.4	0	20	297.1
	B2-F		5063.4	0	20	
	B3-L		4955.1	-0.08	21.6	
	B3-F		4141	-0.19	22.1	
T=400	B3-L	exit	6191.5	0.11	25.2	284.5
	B3-F		5358	-0.01	24.8	

Table 5.3 *Model results for first three bunches.*

5.4 MODEL VERIFICATION

To examine the correctness of the model, in this section some link travel time calculations will be presented under uncongested and congested conditions. We would like to reveal the advantages and disadvantages of this model and to provide a qualitative assessment of the level of its accuracy and predictive capacity as regards to the results obtained by simple time travel calculated either as time of point moving from link input to link output or as a function of the number of vehicles on the link. We would also like to clear up the role of two main attributes of traffic flow classification used: a vehicle type specifying, in turn, a vehicle length and vehicle-specified time headways; and, a driver-desired vehicular speed in the value of obtained link time travel.

5.4.1. Uncongested flow

We consider here the traffic flow with a very low flow volume as obtained for after-midnight hours in Table 5.1. Even under such conditions, the presence of slowly moving bunch, which we call here bottleneck initiator, is decisive factor in reaching or non-reaching the anticipated travel time by the present bunch. The first twelve travel curves for the six leaders and six last drivers in the first six bunches are presented in Figure 5.8. This type of model allows obtaining the same link travel time for the first bunch no matter whether there are or not the drivers on the link at the moment the flow started. Even if there are some vehicles which are significantly far away on the link from the link input, they can not disturb the traffic flow and, therefore, impact actual value of link travel time for the first bunch. Presence of bottleneck initiator (bunch 1 in this case moving with speed of 20 m/s) in front of the moving bunch is the main factor in

anticipated decreasing of travel time for two bunches following it. The presence of bottleneck initiator is anticipated by the subsequent following bunch and by the bunch following after very easily. In the first case, however, it is anticipated at earlier stage of travel of the second bunch for which the desired speed was 30 m/s and at significantly later stage for the third bunch for which the desired speed was 25 m/s. For the last bunch another important phenomenon is observed. The length of this bunch firstly decreases corresponding to deceleration anticipated through the second bunch deceleration which resulted from the presence of the bottleneck initiator and then suddenly increases as result of the acceleration of the second bunch at the link output.
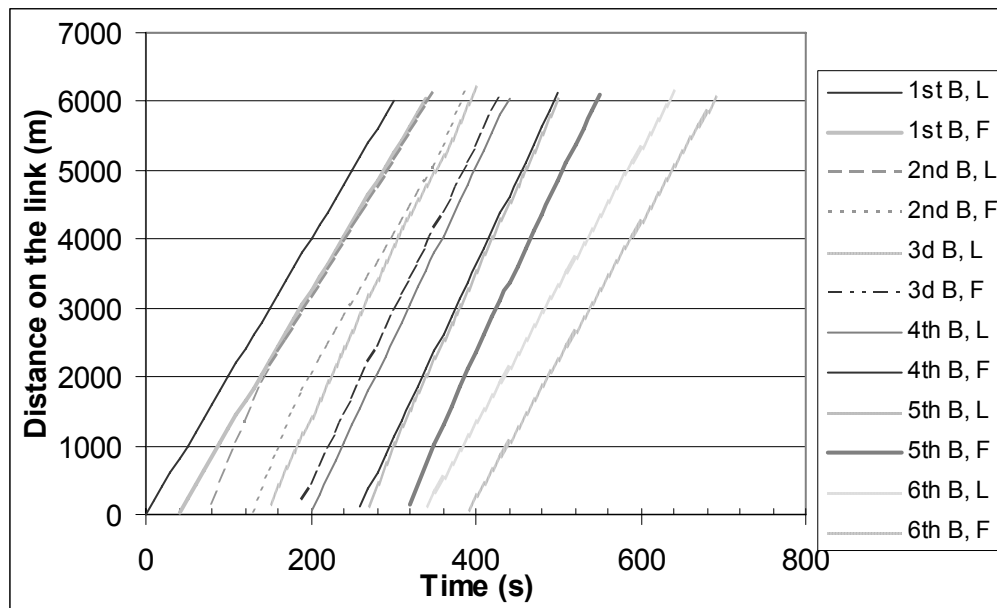
Figure 5.8 *Calculated travel trajectories for leaders and last drivers in the first six bunches under uncongested conditions.*

In Table 5.4, we present comparison of the expected and modeled arrival time for uncongested conditions. The letter L stands for the leader of the bunch, while F stands for the last follower in the bunch. It may be observed that the presence of bottleneck initiator is 'felt' by the subsequent bunches in the same way as congestion conditions would be. The drivers preferring the speed of 30 m/s as desired one are most affected as compared to the other classes of drivers. In particular, if a bunch with the drivers having the highest desired speed follows a bunch with the drivers having the lowest desired speed, a delay occurs by more than one third of the expected time.

Bunch	Positioning	Expected (s)	Modeled (s)
1	20m/s,L	300	300
	20m/s,F	300	300
2	30m/s behind 20m/s, L	200	267,1
	30m/s behind 20m/s, F	200	263,3
3	25m/s behind 30 m/s, L	240	248,6
	25m/s behind 30m/s,F	240	243,6
4	25m/s behind 25m/s,L	240	240
	25m/s behind 25m/s, F	240	240
5	30m/s behind 25 m/s, L	200	233,3
	30m/s behind 25 m/s, F	200	233,9
6	20 m/s behind 30 m/s, L	300	300
	20 m/s behind 30 m/s, F	300	300

Table 5.4 *Expected and modeled travel times for the first six bunches under uncongested conditions. L stands for leader, while F stands for last follower.*

Obviously, the absolute travel time value in this case is a function of many factors such as link length, dimension of the free headway between two interacting bunches and the

way how deceleration is performed – from long distance or after getting into the empty zone.

5.4.2. Congested flow

Figure 5.9 presents travel curves calculated for the six leaders and six last vehicles in the first six bunches under congested conditions. The dimension of bunches are approximately twice shorter than under uncongested conditions (for the same speed), while the headways are approximately 4-5 time shorter as compared to those under uncongested conditions. Therefore, the interaction between slowly moving vehicle and the one approaching it with the higher speed occurs faster. In particular, the interaction between the second and the first vehicles, which occurred under uncongested conditions at the distance approximately equal to one-thirds of the entire trajectory, now occurs almost immediately after the start. Also note that the interaction between the third and the second vehicles, which previously occurred at the distance equal to two-thirds of the entire trajectory, now occurs at the distance of 1 km from the start. Obviously, longer interactions are observed as compared to the uncongested conditions and therefore longer delays are expected than under uncongested conditions. Note that under congested conditions the bunches contain twice more vehicles the under uncongested conditions: 5, 6 or 7 vehicles in first case against 10, 11 or 12 vehicles in the second case (see Figure 5.4).
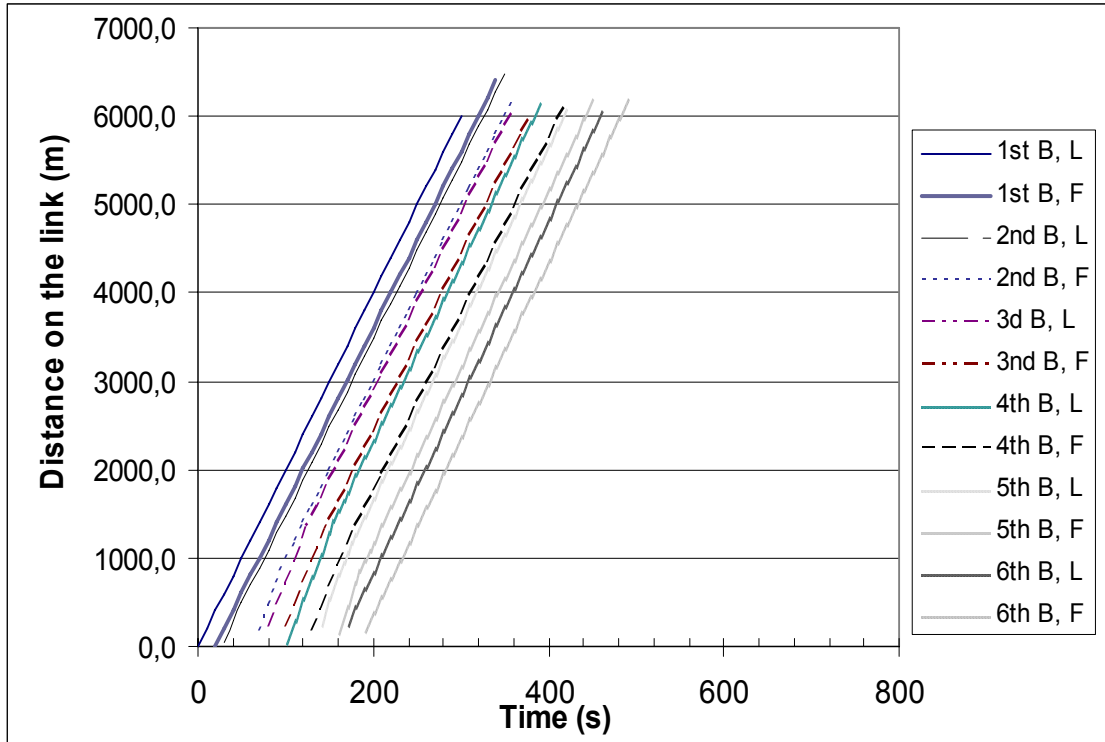
Figure 5.9 *Calculated travel trajectories for the first six leaders and six last drivers in the bunches under congested condition.*

The travel trajectories allow obtaining a lot of other important information. For instant, verification at the position of 6 km confirms that no outflow is observed during first 280 s. If there are some vehicles on the link at the inflow moment, their trajectory will start in the middle of the graph.

In Table 5.5, the expected and modeled travel times are compared for the leaders and followers of the first six bunches under moderately congested conditions. It important to mentioned that the first bunch is taken to be unaffected by the moderately congested conditions, i.e. it is assumed that near entry node the part of link required for loading the first bunch is still empty. In opposite case, the calculation of new equilibrium speed is

required according to some of the proposed relationships for equilibrium speed such as

(2.6). For maximal speed of the flow, the desired speed for the bunch is used.

Bunch	Positioning	Expected (s)	Modeled (s)
1	20m/s, L	300	300
	20m/s, F	300	300
2	30m/s behind 20m/s, L	200	297,1
	30m/s behind 20m/s, F	200	286,3
3	25m/s behind 30 m/s, L	240	284,5
	25m/s behind 30m/s,F	240	287,6
4	25m/s behind 25m/s,L	240	284,8
	25m/s behind 25m/s, F	240	289,4
5	30m/s behind 25 m/s, L	200	284,8
	30m/s behind 25 m/s, F	200	287,1
6	20 m/s behind 30 m/s, L	300	300
	20 m/s behind 30 m/s, L	300	300

Table 5.5. *Expected and modeled travel times for the first six bunches under moderately*

congested conditions. The first bunch gets into empty part of the link.

Because of very compact configuration between bunches, the delays are very high for all

types of bunch configurations. It is expected that the delays will be even greater, when

one consider the delay for the first bunch during loading procedure. Loading procedure in

such case is very complex and has not been considered in these examples.

5.5. MODEL VALIDATION

To validate the model results against the results of some of the models presented in

Chapter 3, it is necessary to present bunches through the parameters which figure in Eq.

(3.1): the input flow, number of the vehicles on link and output flow. They may be

recalculated from the trajectories of vehicles presented in Figures 5.8 and 5.9. The results

of such recalculation of data from Figure 5.8 are presented in Figure 5.10 and Figure 5.11.The former presents the inflow in the shape of the first seven bunches and outflow of the first three bunches during period of the first 450 seconds, while the later presents variations of number of the vehicles on the link during the same period. Both inflow and outflow data are presented using discrete spatial presentation, so a number of the vehicles on the link calculated with the aid of these functions, is always integer value as it may be seen from Figure 5.11. The variations in the inflow presented in Figure 5.11 are result of variations in free and constrained headways, vehicle length and bunch speed while the outflow is mostly affected by deceleration of bunches following the slowest first bunch.
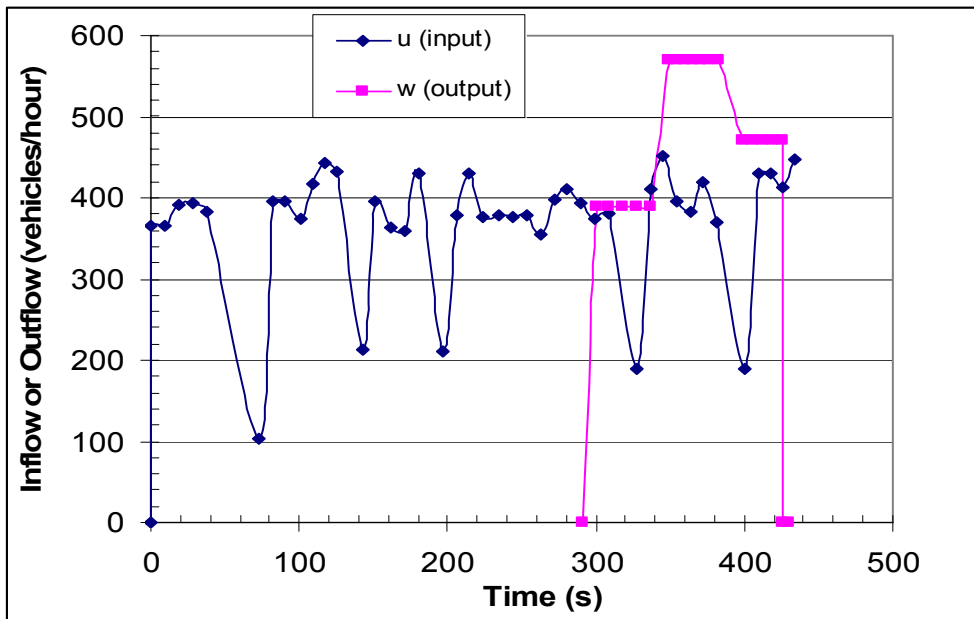
Figure 5.10 *Link inflow and outflow under uncongested conditions recalculated from input of first seven bunches and output of first three bunches*

It may be seen that the choice of $g(N)$ function, for example for exit flow model, Eq. (2.19), is not so simple, because real shape of $g(N)$ function depends on the combination of many factors. First of all, it will depend on which bunch, faster or slower one, will follow the bottleneck initiator.
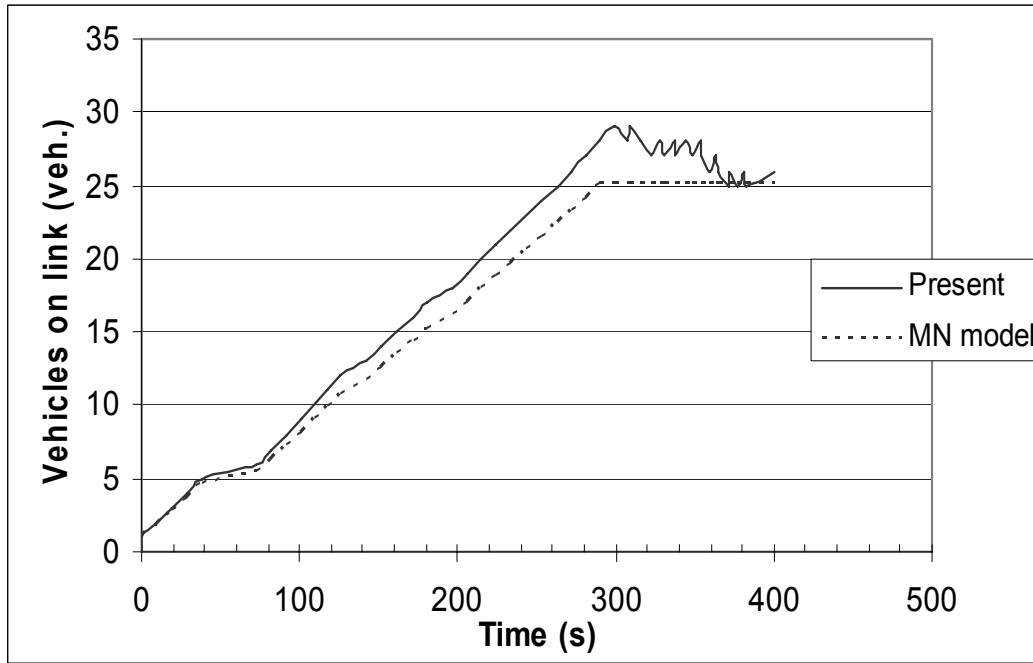
Figure 5.11 N*umber of vehicles on link calculated by using inflow and outflow functions for present model and MN exit flow function model.*

Suppose that the shape of *g(N)* is chosen accordingly to the following rule (Nie and Zhang, 2010):

$$g = \min\{\pi N, G\} \tag{5.1}$$

where, π is a slope parameter for N curve with time axis, which is smaller that 1, and G is maximum outflow that can be obtained for the interval considered. The comparison of

of number of vehicles on the link calculated by MN model and present model is presented in Figure 5.11. As may be concluded from Figure 5.10, the shape of output function may be different depending on the randomly chosen desired speeds for the interacting bunches. Although the discrepancy between the numbers of the vehicles on the link calculated by two models is not so great, as may be seen in Figure 5.11, the difference in travel times will be very significant. The main reason is completely different shapes of the assumed output function, i.e. presented by Eq (5.1), and calculated by output function (Fig. 5.10). As a result, MN model will produce travel time starting from the instant when $g(N)$ function has some finite value, i.e. starting from t=0, while the presented model will define travel time only starting from the instant t=300 s, a moment when the leader of the first vehicle bunch reaches link exit node.

5.6. CONCLUSIONS

The main advantage of this model is in its simple structure and in an opportunity to investigate various features of the traffic heterogeneity. The link travel time is calculated for the ensemble of vehicles called vehicle bunch, and it is completely independent either of the number of vehicles on the link or link outflow.

This model has been designed in order to improve link travel time calculation as applied to the traffic networks by taking into account some known features of *heterogeneous traffic flow* such as desired speed variation, vehicle length difference and car-following driver behaviour. Some of these specifics of traffic participants may be less important at low traffic flow volume, but become dramatically important under congestion conditions,

others may be important at all the time. Since both inappropriate speed and abnormal length of a traffic participant may constantly require application of acceleration and deceleration procedures from surrounding traffic participants, the question of correct calculating the acceleration/deceleration value becomes central for such model. In fact, the time added or subtracted with deceleration or acceleration, respectively, together with the subtraction term responsible for the driver's adaptation to lower speed make the main difference in travel time calculation as compare to standard link travel time. Dey and Chandra (2009) recognize even five operations related to overtaking involving deceleration and acceleration among the others, while Barton and Morrall (1998) showed that the longer truck length require almost 1.6 longer overtaking time. Obviously, no mater whether the driver will decide to overtake the flow obstacle or will try to adapt to new flow conditions, the distance between two vehicles moving with the different speeds is a crucial parameter which needs always to be compared to the desired gap distance. A simple formula proposed in previous chapter, where this distance is calculated as a weighed value of two speeds involved in interaction, has shown its total and limited legitimacy at various flow conditions. Thus, the question of how to define the value of the weighed constant remains open because it changes drastically depending of type of the flow.

In order to further improve this part of calculation, we tried to apply some other formulae for such calculation known in literature. Most of them are applicable only under specific flow conditions. After rigorous verification of dozen of formulae starting with the first car-following models, where the vehicle deceleration is proportional to the difference in

speeds of two interacting vehicles, vehicle own speed and inversely proportional to the distance between the vehicles, two formulae attracted our attention and were kept for subsequent application: optimal velocity model by Bando et al. (1995) and intelligent driver model (IDM) by Treiber et al. (2000). A comparative verification of the formulae was performed. Both formulae are very simple in adaptation and were treated at various conditions. The authors of IDM propose to use their formula in the following way: firstly to let a follower to get into the empty zone and then start calculating deceleration required to vehicle to get out of empty zone at the safe desired gap distance.
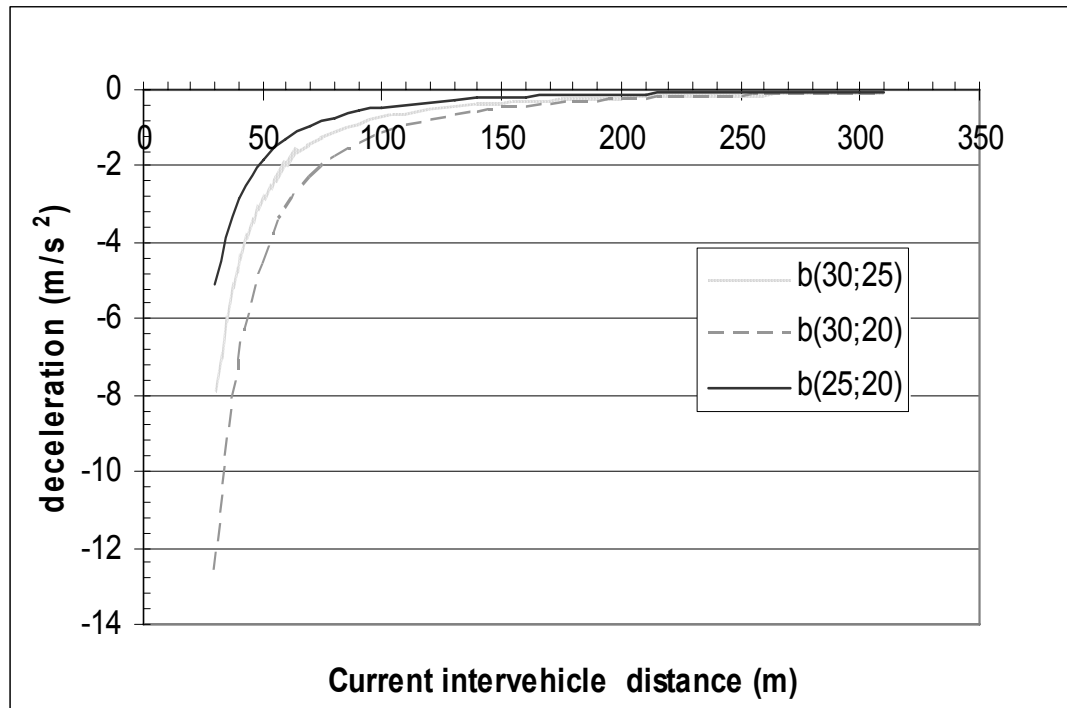
**Figure 5.12**. *Results of the calculations according to IDM deceleration formula (Treiber et al., 2000) adapted for the conditions of this research. The designations belong to the following conditions: b(30;25) vehicle at speed of 30 m/s follows vehicle at speed of 25 m/s; b(30;20) vehicle at speed of 30 m/s follows vehicle at speed of 20 m/s; and, b(25;20) vehicle at speed 25 m/s follows vehicle at speed of 20 m/s.*

Under such formulation, however, the formula gives extreme deceleration over $1.0 \cdot g = 9.8$ m/s^2, as shown in Figure *5.12*. To avoid such abnormal values, the formula was applied outside the empty zone at some specific appropriate distance close to the threshold between constrained and free headways. The value of this threshold varies in the range between 4 s and 6 s according to various authors. Hoogendoorn (2005) proposes to calculate this value by using modified non- parametric Kaplan-Meier survival function presenting the real headway data on logarithmic scale. The threshold may be obtained graphically at the point where the data start deviate from strait line. In our modeling we did not use real headway data, so value of 4 s was adopted for this research. This choice allows a tighter configuration of the vehicles on link under similar conditions. However, applying IDM formula from this distance, we never obtained very close configuration of two interacting vehicles of order of 2 s. Instead, the formula allowed keeping some specific distance of order of 3-3.5 s. Therefore, in our opinion, the question of the appropriate acceleration formula still remains open in this field of research.

Specifically to this research, a good formula for deceleration must account for the way of calculation of deceleration for the last vehicle in the bunch. Since the IDM formula as all other car-following model formulae involves the inter-vehicular distance, applying this formula directly to our model means to consider the distance to the free driver, i.e. the first driver in the bunch, which will give wrong values of deceleration corresponding to such long distances. In this research we do not follow the positions of the vehicles inside

the bunch but only two points of the bunch: a forward bumper of the first vehicle in the bunch and a rear bumper or the last vehicle. If we try to calculate the positions of all vehicles inside the bunch, all advantages of this model disappear and it will require significantly greater amount of CPU time which is not our intention.

Considering now the results obtained by presence of the bottleneck initiator on the link of network, the following conclusion may be drawn. More information is required in order to make an informed decision concerning the choice of the specific link at the diverging node. In particular, for the closest followers of the bottleneck initiator very important information would be concern the bottleneck initiator's anticipated destination and his current distance to this destination.

Chapter 6:

Conclusions and Future Works

6.1. BACKGROUND

Since the first dawn of the transportation age, it is well known that the driving styles of various groups of driver population differ substantially. The origin of this lies in distinct human anticipation of various life events and their categorization on the extreme and standard ones on the one hand and in the different technical characteristics of the vehicles they drive, on the other hand. For instance, the drivers of the trucks and buses have better observation of the driveway and surroundings from the height of their cabins which, in fact, allows them to make more informed decisions than the small-passenger-car drivers' decisions. It is also well known that the acceleration/deceleration characteristics of the heavy vehicles are significantly different when compared to the light small passenger cars. Assuming certain general driver's behaviour, the various car-following microscopic models have been developed in transportation sciences since the early sixties of the last century. Such models have been designed firstly for simple calculations of the fastest path and their simplified features were due partly to limited computer possibilities and partly to the under development of the optimization sciences at that time. Later on, as the optimization control spread in every single region of the social and engineering sciences and computer technology penetrated in every parts of our life, the appetites for predictability of the logistics and transportation sciences augmented enormously. Such ideas were heated up by new achievements in the transportation instrumentation as well

by the foundation and development of the dynamic traffic assignment (DTA) and its large-scale and real-time application. The enlargement of the scale of application considering greater traffic networks has been competing nowadays against the introduction of microscopic specifics of the singular traffic flow participant. Extensions of both macro- and microscopic limits mainly are limited by the computer possibilities and our knowledge in the field. Under such conditions, a number of new transportation models have appeared, considering, one by one, the various traffic micro-features discovered in the meantime. Despite the large influence of specifics in the driver behaviour or traffic heterogeneity on traffic flow dynamics, no attempts have been done up to now to classify them and unite them together. Only recently the first attempt has been performed in this field (Ossen and Hoogendoorn, 2011) proposing how to shape this part of transportation sciences. The two types of traffic heterogeneity were defined: between and inside the classes. The present model brings, for the first time, few traffic heterogeneity factors between the classes discovered before uniting them by known mathematico-statistical apparatus and empirical relationships in the field. The present model is not a DTA link model in classical sense, because it does not consider any type of optimization control inequality, nor it is a direct application of car-following methodology to bunch representation of traffic flow including the stability analysis for the scheme mentioned. For such deep research objectives, this study requires more time and resources. But, *it is a strong intention of this research study to point out the unresolved problem of traffic heterogeneity in traffic network modeling and to show how few heterogeneity factors may be united together to explain an arbitrary traffic query.*

The next step will be to apply such an approach to traffic heterogeneity in a real DTA problem.

It should be noted here that the traffic science to the great extent still remains empirical knowledge where the great part of the discovered relationships still remains confined within specific intervals of the traffic parameters. Therefore, the importance of investigating the interplay of the various factors in the field by using mathematical apparatus uniting them together is beyond question. In the next part, we will evaluate advantageous and disadvantageous sides of the presented model. Then we will enumerate the sequence of the processes to be taken into account for improving the quantitative predictability and diagnosis of the traffic state under given conditions. Finally, recommendations will be presented for further investigations in this field.

6.2. SPECIFICS OF THE PROPOSED MODEL

Because of the high empiricism of the great part of relationships used in the field, a typical model based on the observational data may explain only one singular traffic phenomenon. On the other hand, a mathematical model based on the theoretical relationships used in the field usually deals with only one singular phenomenon. The reason of this lies in the fact that the transportation science uses the physical laws obtained in other physical fields and adapts them for use in the present field. Suffice it to mention kinetic gas theory or Navier-Stokes equations applications in transportation. Therefore, it is very difficult to define the system of equations that may explain all traffic phenomena together, because some of the parameters used in outsourced scientific fields

may loose their sense when applied in the transportation. Combining the various empirical and physical laws may allow explaining greater number of phenomena. One of the advantages of this model is its capability to explain several completely different traffic phenomena. In particular, already at this stage it is noticeable that the model may be an excellent tool for modeling moving bottlenecks and platoons which were reviewed in Chapter 2. Both these traffic phenomena require a speed difference between traffic participants which this model affords and multi-lane motorway presentation which was not considered in this model due to the limited time frame of this research study. In fact, overtaking which was not allowed in this model and any type of two-pipe flow with or without mixing and important for ramp merging processes are very difficult to model. Therefore, their presentation in the present model was avoided. However, *the proposed model deals with the dynamics of acceleration/deceleration of singular bunches moving with various speeds* which is already good step ahead in modeling of the phenomena mentioned. Another very important point in such modeling is that a bottleneck may be formed at any point on the link and not only at the link input and/or link output as in models seen in Section 2.3.

The next advantage of this model is the possibility for the modeller to carefully chose O/D pairs in traffic network modeling because of discretization of flow on the bunches with specific value. This means that particular O/D pairs may be designed for every bunch. This will allow to properly distribute them in the network and to have a simple evidence of their real-time positions. Finally, a great advantage is that *travel time is*

calculated directly for each bunch and not for the entire flow as it was in most of DTA link travel models.

The greatest disadvantage of this model is that because of the varying speed on the various parts of the link, loading procedure is more difficult to perform at once, particularly at congested state. Some additional information is required concerning the global link speed distribution during loading (Lin et al., 2008) in order to know if the desired speed of loading bunch will match or not global link speed at the loading point. Moreover, the link capacity problem should be mentioned as jam density, input and output capacities become time dependable in the case when heterogeneity of traffic flow is considered because of: (i) vehicle length; (ii) varying vehicle-type-specified headways; and, (iii) varying desired speeds. In such a case, all time dependable parameters will be re-calculated every time when a new bunch has been uploaded or downloaded from a singular link.

6.3 FUTURE WORKS

The model as it is already showed has a lot of advancements as compared to certain link travel models. However, the real world application of this model requires the introduction of overtaking and multi-lane flow. Without such improvement, the applicability of the model is very limited.

Some next improvements capable to drastically improve applicability of this model are related to traffic attributes calculated in the model. In fact, presently, the constrained

headways are calculated only at the first loading on the link, while the free headways are recalculated during the interactions between bunches. In order to improve the stochastic level of the model, the constrained headways might be calculated at each time step. This procedure will make headway more dynamically changeable at each time step, and the bunch length will not be constant even at the constant bunch speed. Present variant of the model accounts for bunch compression and extension only at the time of the interaction between fast and slow bunch and sudden acceleration, when the highway becomes clear at the diverging point. The most unexplored consequence of this consists of speed variations inside a bunch and subsequent local bunch compressions and extensions.

Choice of the specific speed value should be more carefully designed. Firstly, the nominal value of the desired speed should be chosen according to the traffic flow volume conditions. Secondly, the second and third value of delta function may be chosen at lower and upper 95 % confidence interval of the chosen value of desired speed. 2.5% from both sides of the chosen speed value may be assumed to perturb the normal traffic flow because of significantly lower or greater value then the desired one.

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