THERMAL CONTROL SYSTEM DESIGN FOR AUTOMATED FIBER PLACEMENT PROCESS

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ABSTRACT

THERMAL CONTROL SYSTEM DESIGN FOR AUTOMATED FIBER PLACEMENT PROCESS

Salman khan

One of the important concerns about the quality of the thermoplastic composite in Automated Fiber Placement (AFP) process is the degradation of thermoplastic resin, resulting from the overheating or the lack of proper heating by the heating system used in the automated fiber placement machine head. Heat transfer between the heating system and incoming pre-impregnated tow is not easy to control and can result in energy loss or non consistent heating of pre-impregnated tow. The advanced control systems are used to control the key processing parameter of Nip Point Temperature of the Heating System. Two advanced control systems are designed by using the dynamic thermal model of the fiber placement process. One is Linear Quadratic Gaussian (LQG) controller which is implemented to achieve the optimal results for quality performance. The other is Model Predictive Controller (MPC) which is proved more efficient as the physical capacity, safety and performance constraints of the heating system are explicitly addressed in the design of the Model Predictive Controller (MPC). Poly-ether-ether-ketone (PEEK) reinforced with carbon fiber (APC-2) is used as the focused material. The performance of the two control strategies are compared and illustrated through simulations.

ACKNOWLEDGEMENTS

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Dedicated to my beloved parents and dear sisters, Salman

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LIST OF SYMBOLS, ABBREVIATION AND NOMENCLATURE

Symbol	Definition
AFP	Automated Fiber Placement
LQR	Linear Quadratic Regulator
LQE	Linear Quadratic Estimator
LQG	Linear Quadratic Gaussian
MPC	Model Predictive Control
PEEK	Poly-ether-ether-ketone
A, B, C	System matrices of linear system in state space
Q, R	Weighing Matrices
NN	Neural Networks
x	State Variable
K	Regulator Gain
L	Kalman Filter Gain
P	Transformation Matrix between states and co-states
J	Performance Index
u	Control Vector
r	Reference Input
T_m	Melting Temperature
T_g	Glass Transition Temperature
T_h	Nip-point temperature / First Component
T_1	Second Component Temperature
T_2	Third Component Temperature

T_a	Ambient Temperature
MV	Manipulated Variable
N_p	Optimization Window / Number of Samples
N_c	Control Horizon
F, Φ	Pair of matrices in the prediction equation (5.10)
K_{mpc}	Feedback control gain using MPC
ΔU	Parameter vector for control sequence
I	Identity Matrix

CHAPTER 1
INTRODUCTION

1.1 MOTIVATION

Automated Fiber Placement (AFP) is a process in which strips of unidirectional composite prepreg tape, normally called tow (which is a combination of unidirectional individual fibers, pre-impregnated with thermoset or thermoplastic resin) is collected and laid-up on the mandrel or mold with the help of automated mechanical system. In AFP process the automation starts from the delivery, followed by preheating of the composite material for smooth lay-up on the mold surface under controlled Nip-point heat and pressure as shown in Figure 1.1.

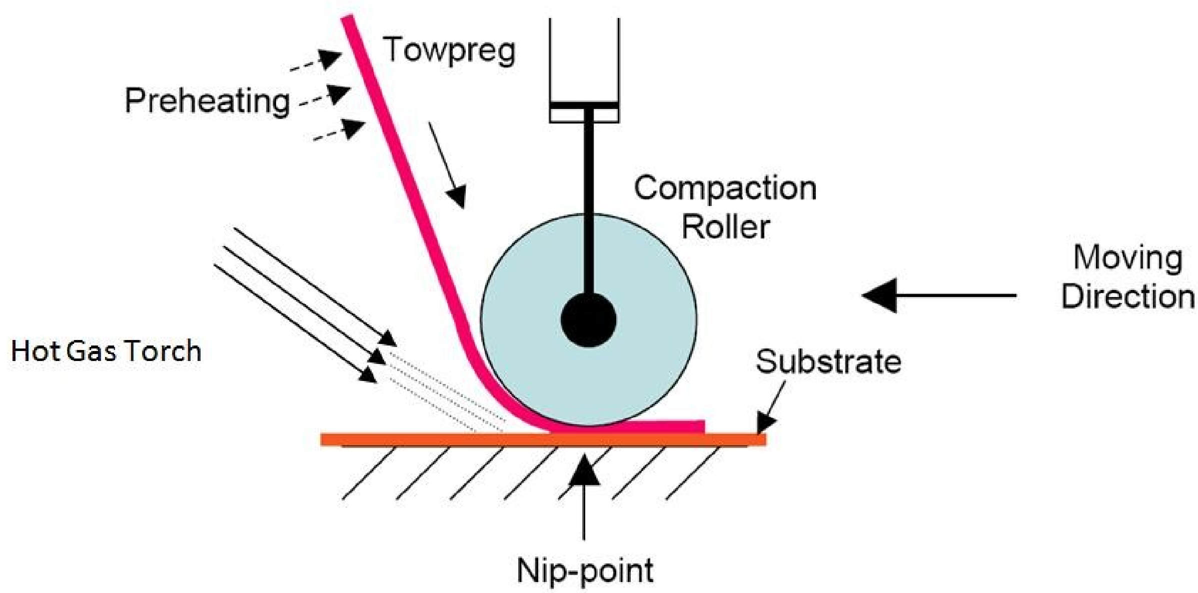

Figure 1.1: Automated Fiber Placement Process

The above process exists for both thermoset and thermoplastic composite material, however thermoplastic offers one step completion of the process, in which bonding is done simultaneously without any requirement of post processing. This process is widely known as in-situ consolidation of thermoplastic composite. Compared with thermoset, thermoplastics are more suitable for automated placement process as the combination of thermoplastics with robotics offers tremendous potential in high performance composite structures manufacturing.

AFP is considered superior to conventional methods of manufacturing composite structures, especially in terms of quality and cost. AFP is now extensively used in aerospace industry, making parts like fuselage components, empennage structure, fighter aircraft intake ducts, engine nacelle components, space launch vehicles components etc [1]. The process can be divided into four important sub-systems:

1) Delivery of composite material and trajectory planning for placement;
2) Heating of the composite material;
3) Pressure application;
4) Compaction of the material on the mold surface.

Out of the above sub-systems the most important feature of AFP machine, especially when dealing with thermoplastics is the heating system. Energy from the heater melts the surface of the material, while at the same time pressure is applied to compact the material on the mold surface. The choice of the heating system is unique and largely depends on its application. For better quality in both high speed and high temperature production, a powerful and controlled heating source has to be incorporated into the fiber placement machine head.

AFP process strongly depends on the precise control of the heating parameter and for the optimal results, the parameter is chosen according to the process heat transfer model and in some cases from the material test data.

One of the reasons for increased popularity of AFP process is cost effectiveness [2], and the selection of heating parameter during manufacturing of composite structure has a significant effect on cost and quality of the finished product.

1.2 THESIS OBJECTIVES

The selection of processing parameters has significant effect on the quality of the final product. One of the important processing parameters in automated fiber placement process is heat intensity. It should be carefully selected and controlled during the process which can be achieved by improving control system design. Thermal model based control of the heat intensity is the objective of this study.

AFP process faces some significant hurdles which need to be overcome. The main problem identified is the economic losses due to degradation of material during the process. Therefore advances continue to make the process affordable and flexible. Most of the research in the field of AFP process is focused on the process modeling and very few researchers have suggested control strategies for the heating system of AFP machine.

The objective of this study is to provide the essential knowledge about AFP process, to identify the importance of critical heating parameter of the process, and to propose thermal control strategies which help in developing an economical automated work cell for AFP process.

Automated process of fiber placement is used for composite manufacturing of aerospace structures, thus the technology is mostly used by large companies [1]. However the process has greater potential to be a part of common industrial setup as composite material is becoming a main stream material for industrial applications. This study will support the efforts of manufacturing a low cost prototype system for different commercial applications.

1.3 Thesis Organization

Chapter 2 gives the introduction of the basic components of AFP system. A brief description of the heating systems employed in AFP process is presented with a literature review on the application, thermal modeling and the control strategies previously used to control the heating system.

Chapter 3 explains the AFP process model used as a plant for proposed control strategies and gives an insight on thermoplastic composite material.

Chapter 4 proposes Linear Quadratic Gaussian (LQG) control strategy for the thermal control in AFP process. LQG is a fundamental optimal control problem, which focuses on the linear system with disturbances. Optimal control finds the control law for a system such that a specific optimality criterion is achieved. LQG controller is simply a combination of Kalman filter i.e. Linear Quadratic Estimator (LQE) with a Linear Quadratic Regulator (LQR). LQG controller shows improved performance, however limitations are observed when AFP plant is subjected to step disturbance.

Chapter 5 presents a control strategy based on Model Predictive Control (MPC). MPC is an advanced method of process control based on the iterative finite horizon optimization of the plant model. It predicts the behavior of the temperature at the Nip-point of the heating system with respect to the input. MPC not only handles the disturbance but also addresses the limitations of the heating system and the material safety requirements, when subjected to step disturbance through MPC constraint application.

Chapter 6 gives the conclusion of the thesis and recommendations for future work. The references follow Chapter 6.

1.4 Conclusion

In this chapter, AFP process is described with emphasis on the importance of thermal control. The research objectives of this thesis project are discussed and the thesis organization is outlined. Before explaining the heat transfer model and the thermal control system techniques, one should understand the basic working of Automated Fiber Placement and the concerned literature available.

CHAPTER 2
AUTOMATED FIBER PLACEMENT PROCESS AND LITERATURE REVIEW

2.1 AUTOMATED FIBER PLACEMENT PROCESS OVERVIEW

AFP process offers high production rate, better quality, efficiency and low cost manufacturing of large scale composite structures. Automation has become an important part of composite manufacturing as it provides a way to reduce the manufacturing cost, in terms of labor hour, reduction in material scrap, good processing quality and repeatability of process through use of efficient control and automation technology. AFP process is the combination of a number of processes as shown in Figure 2.1 [3].

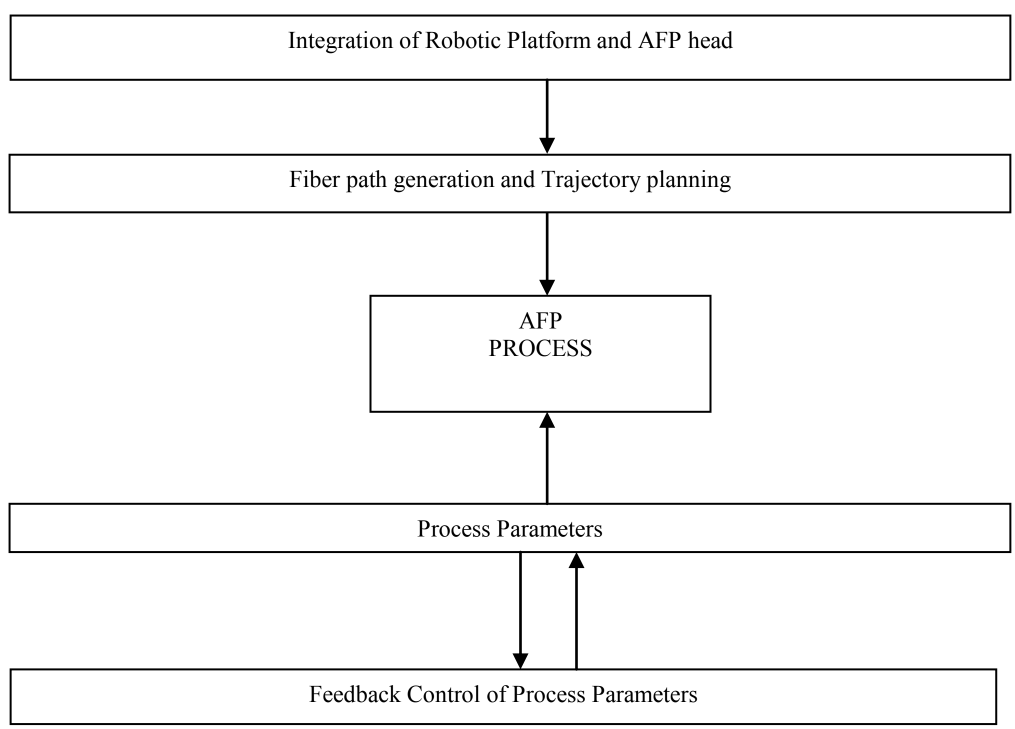

Figure 2.1: AFP process overview

Fiber placement head is mounted on robotically controlled multi-axis platform. Prototypes of different fiber placement heads have been designed and manufactured, and most of them are mounted on 6-axis gantry robotic system as shown in Figure 2.2 [4].

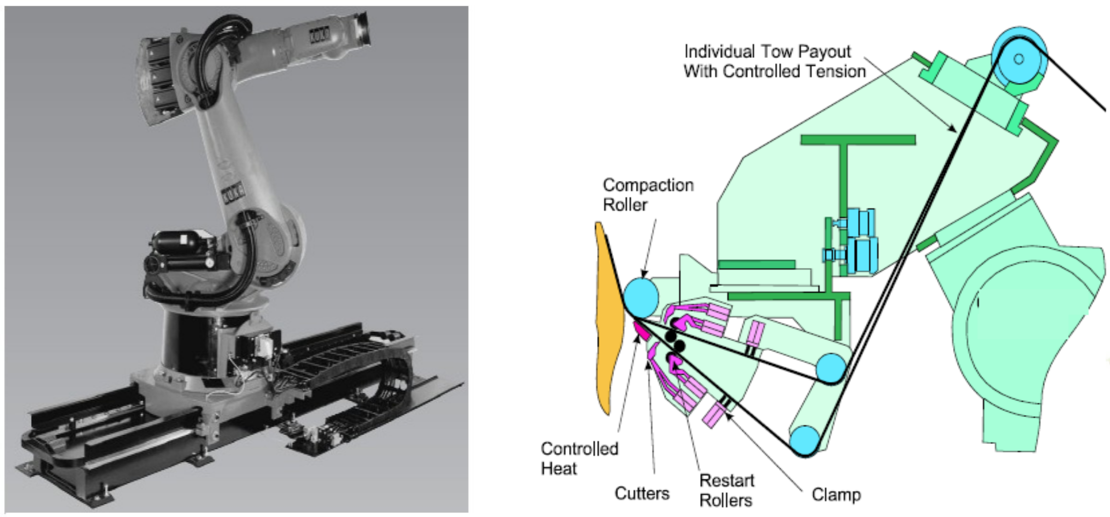

Figure 2.2: 6-axis robotic platform made by KUKA robots and the fiber placement head [4]

2.2 Fiber Placement Head

All the main functions of AFP process are executed through fiber placement head. It provides the feed, heating, consolidation, cutting, placement, and lamination of the material on the mold or tool surface. The fiber placement head is manufactured according to the application and raw material [5]. The key features of AFP head are:

1) Fiber Delivery System

2) Cut/Restart Capability

3) Compaction

4) Heating System

2.2.1 Fiber Delivery System

The delivery of fiber to the head is very important for the process flow and it should not restrict the operating zone of the AFP. In larger machines, material is delivered in the form of collimated band of tows, through the rollers located at the creel and the head. The lay out speed of individual tows is controlled by independent delivery guide system from the spool to the compaction system of the head. The system also controls the band width, during lamination in curved path.

2.2.2 Cut / Restart Capability & Trajectory Planning

AFP heads have the capacity of automatic cutting and restarting of the tows, making it possible to cut, clamp and restart without stopping the laminating procedure of the head. Cut/Restart sequence is programmed using off-line software, based upon the lay-up geometry during the path generation and simulation in CAD. Traditionally the teach pendant to jog the robot to the desired place is used, but these days trajectory planning is carried out offline [6]. Trajectory planning just provides the starting point, however the most important parameters like compaction force and heating temperature should be monitored and controlled separately.

2.2.3 Compaction

Compaction is the pressing system installed in the fiber placement head. The function is to press the tow onto the part or mold, so that entrapped air and inner band gaps are removed. In most situations, it is necessary to introduce heat into the compactor to decrease the resin viscosity of the tows. The process is referred to as tack enhancement. Increased tack enables the incoming fibers to adhere more quickly and remain in place on the mold or substrate. Uniform compaction produces a higher quality parts, eliminates debulking, processes concave surfaces and provides greater flexibility in fiber steering. Different types of hydraulic and pneumatic compaction system have been used in AFP head for application of compaction force. For the better quality results the consolidation process has been investigated by many researchers, and still under investigation. To investigate the effect of the compaction force variations in the consolidation process, knowledge of compaction model is required, which is beyond the scope of this study.

2.2.4 Heating System

In selection of the heating system, the important factors are the size, flexibility and price. Several types of heating system are used in the AFP head [7], and the most common systems include: laser heater, hot roller/shoe heater and hot gas heating.

2.2.4.1 Laser Heater

High frequency waves can heat the material by causing the molecule in the thermoplastics to oscillate. This method is used in thermoplastics containing polar molecules. The laser heat generation is a difficult and expensive process. Lasers offer high efficiency and the better response time, which makes the system suitable for nip-point heating in moving fiber placement head; however the industry is reluctant to adopt the technique due to higher equipment cost and safety requirements.

2.2.4.2 Hot Rollers / Shoe Heater

Heating through the rigid system of roller or shoe is less expensive and regarded as a safest option. Open flame and acetylene gas torch provides a high density of energy. Usually they are so hot that they may degrade the polymer. Efficiency of rigid heating system is more than the hot gas and less than laser, researchers have closely monitored the performance of heating system for AFP of thermoplastic and simulation results have shown that rigid heating system is the most promising and outstanding system [5]. Most attractive attributes of rigid heating system are the cost and size which add to the flexibility requirements of AFP head, making the system the better option. The requirements of the systems are the precise control of heating filament and to prevent the heated composite to stick with the heat source.

2.2.4.3 Hot Gas Heating

Nitrogen gas is normally stored in high pressure vessel and the liquid form is heated electrically by a torch and then blasted through a nozzle, specially designed to provide a uniform convective heat source. The two important reasons for the common use of hot gas heating system for AFP process involving thermoplastics are, the minimum equipment cost and the traditional use of hot gas heating system for welding of thermoplastic. Hot gas heating system is capable of heating the nitrogen up to 1200˚C. Optimum processing conditions for on-line consolidation of thermoplastic using hot gas has been investigated in details [7], and now the hot gas system is used extensively.

2.3 Applications

Automated fiber placement process is widely used in aerospace industry. Since 1980s the technology has developed into a highly dependable process. It offers lower material scrap, reduced lay-up time, improved safety and quality standards, process repeatability and complex parts manufacturing. The scrap rate in automated process is about 3%-8%, which is much less than 20%-50% scrap rate in manual processes [2]. The reduced scarp rate is a major contribution to the cost reduction offered by automated process. Automated placement process is now part of larger operations of well known companies, and is extensively employed in aircraft manufacturing. It accounts for 25% of the airframe of aircraft like C-17 AirLifter, F-18 Super Hornet, F22 Raptor Joint Strike Fighter [2]. Commercially available automated tape laying machines offered by Cincinnati Milacron and Ingersoll are widely used for the manufacturing of wing skins and control surfaces of B2 aircraft from Boeing and stabilizer skins for Airbus Industries [8].

As the applications of the AFP are becoming numerous, special attention has been given to the manufacturing of complex aerospace structures. Automated Dynamics Corporation has succeeded in manufacturing of complex structures like, Helicopter Drive Shafts for Bell Helicopters, Monocoque Tail boom, V22 main rotor grip element of helicopter [9]. Automated Fiber Placement facility developed by NASA Marsch Space Flight Center uses the facility for manufacturing complex 3-D aerospace structures. NASA Langley Research Center has proved a valuable asset for obtaining data, experience and insight in automated fabrication of high performance composites [10].

Robotic Platform (6-axis robots) provided by KUKA robotics and ABB are normally used for the automated fiber placement machines [3]. User interface software like Labview are used as the interacting control software, CADFiber and CATIA CPD V5 R18 softwares are used for laminate design, simulation and programming [11]. Robotic Fiber Placement facility at Robotics and Mechatronics Research Laboratory in Monash University is widely used for the research on the trajectory planning and control of automated fiber placement process [12]. Companies like Composite Systems provide the composite tooling for the automated machines, and develop the placement head designs

[13]. Cytec Engineering Material Company is using 12 tow in–situ consolidation head for automated tape laying machine for commercial industrial applications [14].

2.4 MATERIAL SYSTEM

Compared to thermosets, advanced thermoplastics are relatively new, and are now extensively used in commercial applications. Although the market is dominated by composites process with thermoset matrix resin, a range of commercial composites based on advanced thermoplastic matrix resins have emerged for high temperature applications. Thermoplastics in general have no shelf life, low moisture absorption, thermal stability, high toughness/damage tolerance, short processing cycles, potential for manufacturing cost reduction, ability to be remolded and reprocessed and repaired by applying heat and pressure [15]. However the limited commercialization of thermoplastic products is due to its high cost.

Thermoplastic polymers are usually linear molecules with no chemical linkage between the molecules. The molecules are held together by weak secondary forces, such as van der waals or hydrogen bonding and are readily deformed by the application of heat and pressure. During the consolidation process the changes are substantially physical [16].

The term "Advanced Composite" normally implies to the high volume fraction reinforcing agent of order 60wt%. Resins for advanced composites can be classified according to their chemistry. Thermoplastics based on aromatic polymers like Poly-ether ether-ketone (PEEK), have better mechanical properties, high temperature capability, and are considered equivalent or even better than the thermosets. In its chemical structure the dominant characteristic is the polymer backbone as shown in the Figure 2.3.

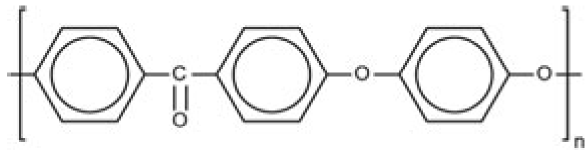

Figure 2.3: Poly-ether-ether-ketone (PEEK)

In this study, Poly-ether-ether-ketone (PEEK) matrix, reinforced with 60% graphite fiber is used as a material system for simulation study. PEEK was introduced by Imperial Chemical Industries (ICI) in 1981 with the trade name of Victrex PEEK [17]. PEEK has been the focused material for many researchers working on the automation of thermoplastic manufacturing process [18-28]. PEEK resins have melting and processing temperatures that are well suited to the technology available at the present time. Thermal stability of PEEK is very reliable within the processing temperatures, making it an ideal material for high temperature applications and replacement for metallic aircraft components. PEEK AS-4/APC-2 composite is of a high quality pure grade that contains no special additives. The results of numerous tests conducted on PEEK AS4/APC-2, indicate that this material demonstrates satisfactory behavior to aerospace environments and is insensitive to moisture [29]. In automated process the potential problem with auto consolidation is lack of consolidation due to insufficient diffusion time. Effect of heating rate and short melt times on the required processing conditions of PEEK thermoplastics are discussed in detail by Muzumdar [30]. The detail understanding of the material and the heat transfer model of the process is very important for the efficient thermal control.

2.5 THERMAL MODELING AND HEATING SYSTEMS

A few important thermal models describing the temperature distribution in automated process are discussed. In these models different heating sources have been employed like nitrogen torches, laser scanning and rigid contact. Ku Shih Lu [31] employed nitrogen gas heating system and develop a simplified model to simulate the temperature distribution. The proposed model consists of the roller, tail compactor, heating source (nozzle), and composite and the study discusses the roller contribution in heat distribution.

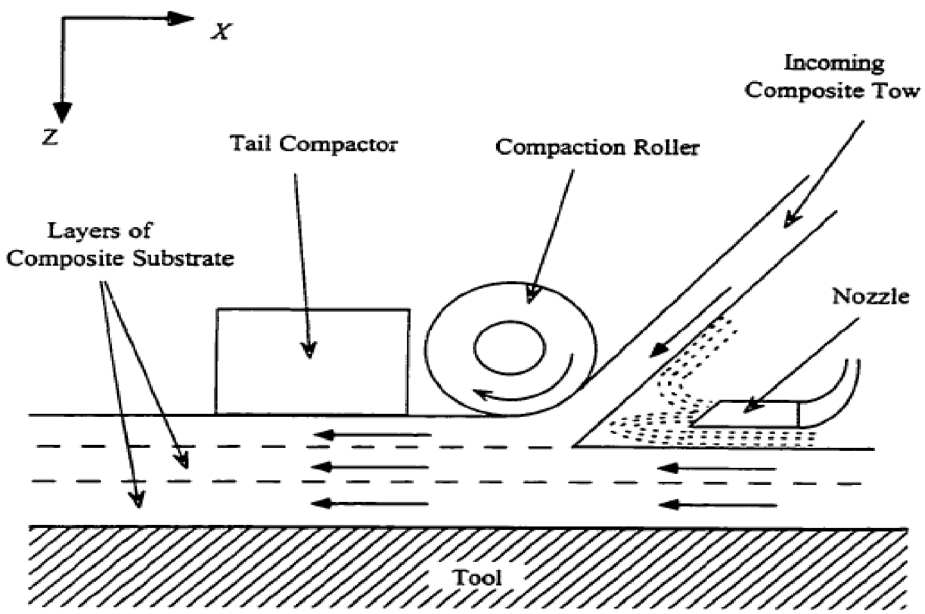

Figure 2.4: Components of thermal model developed by Ku Shih Lu [31]

Nitrogen source is a convective heat source. Eulerian frame of references is adopted to arrive at steady state temperature distribution. Roller and tail compactor need detailed knowledge of consolidation mechanics which is beyond the scope of the current study and the focus of this thesis remains on the temperature control. Muzumdar [30] developed a similar model by considering the laser assisted heating, to determine the temperature, heating rate, cooling rate, melt time and consolidation force as a function of time and position for a given heat intensity, tape speed and consolidation force. The information on temperature history is very important to predict and control the processing parameter, which needs a suitable heat transfer model. A process model which relates the heat intensity, tape speed and consolidation force to the temperature history is presented for online tape consolidation process.

Po jen Shih [32] employed the Nitrogen and Laser Heating sources to create the molten zone and adequate pressure is applied to achieve the consolidation for PEEK APC-2 material automated system. A focused heat source was aimed at the interface between incoming towpreg and substrate to create a molten zone. The intensity of the heat source must be properly adjusted to ensure the melting on both mating surfaces. In this study it was identified that when the feed speed was too high, the single point heating was not enough, therefore, preheating stage was required to elevate the temperature of the incoming tape.

Andersen and Colton [33] discussed the importance of pre-heating stage. Pre-heating is required for two reasons, i.e. The heat capacity (C_c) of the thermoplastic composite is too high and the transverse thermal conductivity (K_z) is fairly low, which makes it very difficult to quickly heat the pre-preg tape from room temperature to the processing temperature. In addition the accurate control of pre-preg temperature is highly desirable. Therefore the material is maintained at the elevated temperature, close to but less than the processing temperature, and finally pre-preg passes into the final heating stage where it is brought to the processing temperature. Wordermann and Friedrich [34] used the experimental setup to study the consolidation process and recorded the temperature profile to verify the heat transfer model.

2.6 Thermal Control System Strategies

Some control system methodologies used by the researchers to control the heating system mounted on the automated fiber placement head are discussed and an insight is given on the importance of the temperature control in automated process. Temperature and pressure at the nip point are key process variables to be controlled to ensure successful in-situ consolidation.

Dirk Heider and J Gellespie [35] used the PID and NN (Neural Networks) based control on CMAC (Cerebellar Model Arithmetic Computer) to develop an adaptive temperature control system for thermoplastic based automated fiber placement. The composite material used in this study was PEEK (APC-2), the nitrogen gas heating system was employed with AGEMA thermal camera as a feedback sensor. The objective was to maintain the temperature under the nip point according to the set point to achieve highquality. The CMAC was trained both online and offline to lower the heat transfer behavior of the system and the desired heat profile, at the nip point the heating was compared with the model output to find the suitable process input. The conventional PID was observed to be very noisy especially in the presence of disturbance, mainly because of slower response to the observed changes in the profile. Increasing the response times leads to unstable control system which reacts more on the noise. At the set point temperature of 440°C, an approximate deviation of 15°C was observed.

PF Lichterwainer [36] developed a NN based Controller to control the heating system for the automated fiber placement machine. The objective was to improve the temperature tracking performance. Inverse model of the AFP process was used for the online training of the neuro-controller to improve the performance. In this study the laser heating system was used, which provided better response time as compared to nitrogen gas system. A focused IR camera was used as a feedback sensor.

Wei-Ching Sun and W Lee [24] developed a process thermal model for automated planned of thermoplastic and developed a LQR based control methodology to control the thermal heating system for achieving high quality thermoplastic composites laminate. PEEK (APC-2) material system was used in the study and a low cost moving rigid contact nip point heater with nitrogen gas pre-heating was used as the process components. The goal was to control the bond quality of thermoplastic laminates during the manufacturing by effective control of heating system.

M Lee [37] used PID controller to control the heating system for automated fiber placement process, the study was focused on the selection of heating system and it was proved that Rigid Contact heating system with PID controller was more efficient than nitrogen gas and was more economical compared to the laser systems.

Artificial Neural Network (ANN) was also used in open loop optimization for the automated thermoplastic composite tow placement system [38]. Neural network (NN) based process model was developed to predict material quality as a function of process set points. The set points are based on experiments and experience. ANN based model facilitate the in-situ optimization of the important process parameters, however the focus of the study was the bond quality and heating and the laminates contact model were used to predict the ideal conditions.

2.7 Conclusion

In this section an overview of AFP process is presented and the components of AFP machines head are discussed. An insight is given on available heating systems mounted on the AFP head. The literature review is presented on the automated placement process, the material system, the thermal model of the process and some control system methodologies used by researchers to control the heating system, especially for thermoplastic material. Before designing the control strategies, the thermal model of the process needs to be understood in detail; therefore the next chapter is devoted to the thermal model of the AFP process.

CHAPTER 3
HEAT TRANSFER ANALYSIS IN THERMOPLASTIC COMPOSITE FIBER PLACEMENT PROCESS

In any thermoplastic polymer the thermodynamics associated with temperature change plays an important role in predicting the process parameters. Thermo physical effects at higher processing temperatures are very significant. Not only the temperature level reached but also the time spent on reaching that temperature is important [39]. The time required to reach the consolidation temperature is a function of the heating method and consolidation temperature values of specific thermoplastics. However for semi-crystalline material like PEEK AS4/APC-2, it should be above melting temperature (T_m). The consolidation time is usually long for the amorphous thermoplastics, since they do not melt and generally maintain higher viscosities at the processing temperature [40]. As a rule of thumb the glass transition temperature (T_g) is approximately *2/3Tm* (temperature in *K*). The consolidation of melt fusible thermoplastic consists of heating, consolidation and cooling as shown in Figure 3.1 [41].

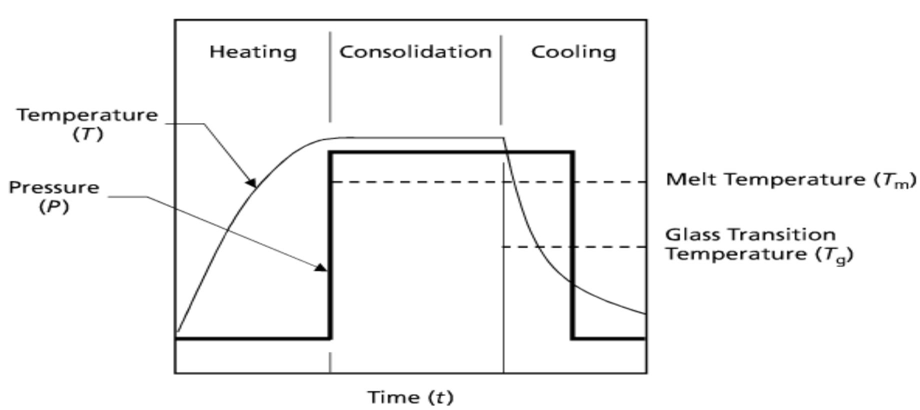

Figure 3.1: Thermoplastic Processing Cycle [41]

Appropriate selection and arrangement of the heating sources are the most important factors for the successful on-line consolidation of system. However it is very difficult to physically measure the change in temperature in the deformation zone, therefore a model consisting of all the mechanisms involved in consolidation is required for the vital temperature information. To develop a thermal model, the physics involved in consolidation must be carefully analyzed. The entire structure of the process is very complicated therefore the focus of this study is on the thermal analysis and control.

3.1 THERMAL MODEL FOR AUTOMATED FIBER PLACEMENT PROCESS

The thermal model captures all the major thermal effects so that an accurate prediction of temperature distribution of the composite can be made. Although the entire structure of the in-situ consolidation process is complicated, a simplified thermal model of in-situ thermoplastic composite tape laying process is considered [24]. Figure 3.2 gives all the major components of the thermal model.

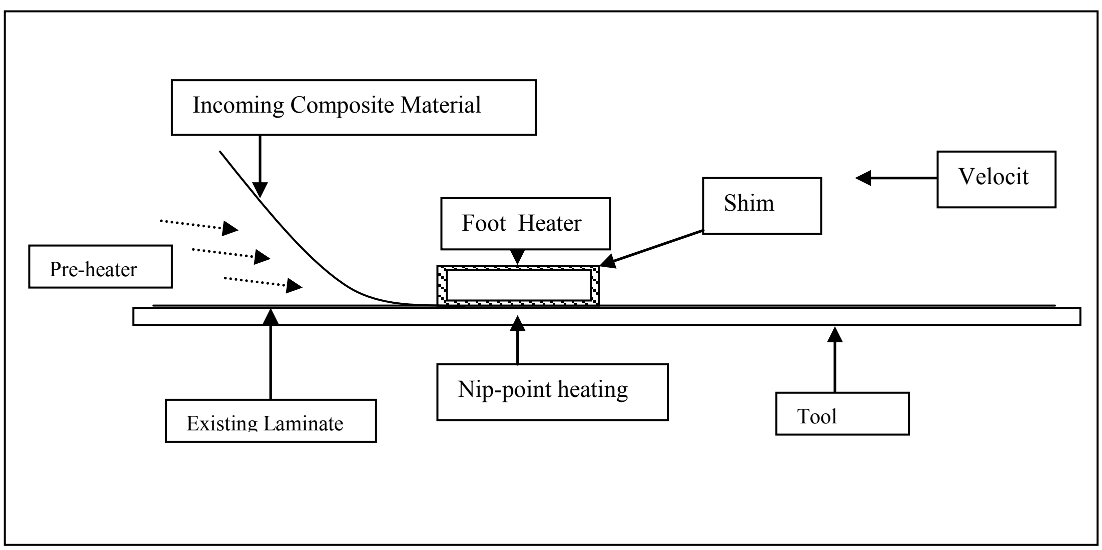

Figure 3.2: Components of thermal model

In Figure 3.2 the incoming composite material is assumed as thermoplastic composite PEEK (APC-2) with 60% of graphite fiber, the nominal processing temp of PEEK is approximately 387˚C and its manufacturing process window is assumed to be from 375˚C to 400˚C. For the nip point heating a rigid heating source of foot heater is used, as the rigid heating sources are the most economical as compared to other heating sources [5]. The source is assumed to be an equipment of power 80Watts, however the only requirement of this type of system is to prevent the heated composite to stick with the heat source, therefore a shim is used to address the problem as shown in Figure 3.2. A ceramic tool plate is selected for the model and the heater velocity (V_x) as shown in Figure 3.3 is fixed at 25mm/s, with appropriate temperature control a good quality laminate is achieved at this velocity [42]. Using the arrangement, the temperature at the critical locations in the process is predicted. Considering the critical inter-laminar bonding, the temperature at the Nip-point and at the interface between the top ply and the ply previously laid (substrate) is very important for achieving the quality product. The process is considered as a two dimensional heat conduction process with moving heat source as shown in the Figure 3.3.

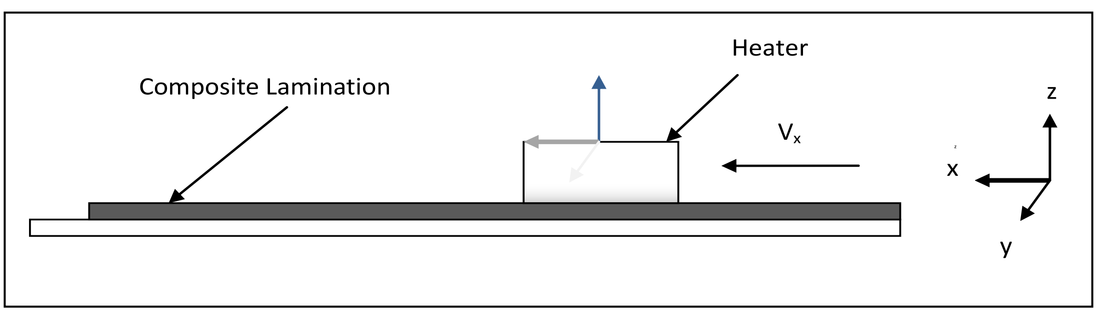

Figure 3.3: Heat conduction with moving heat source

To achieve the process model, the process dynamics shown in Figure 3.2 and 3.3 are divided into three components, based on the temperature of the critical locations in the process as shown in Figure 3.4 and 3.5.

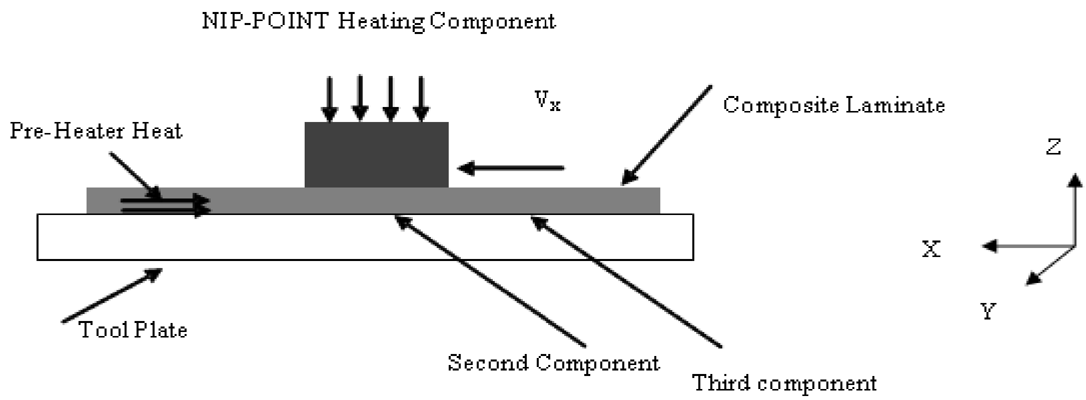

Figure 3.4: Three component model

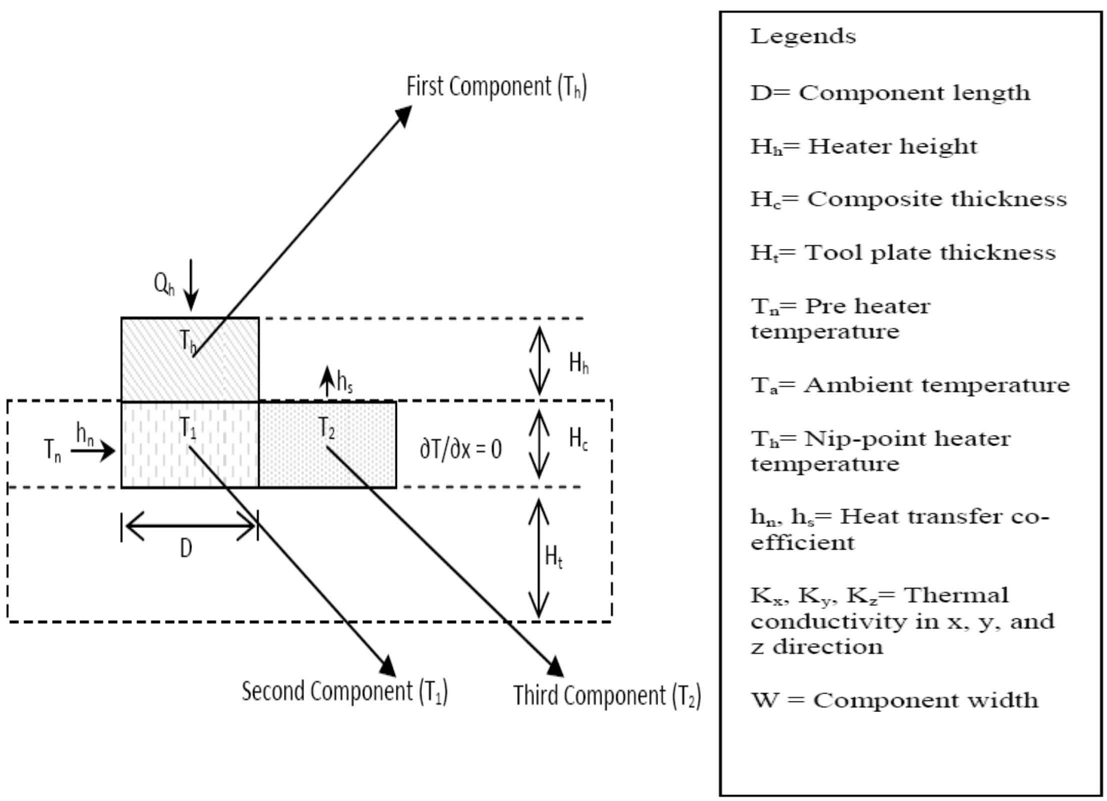

Figure 3.5: Three-component geometry and boundary conditions

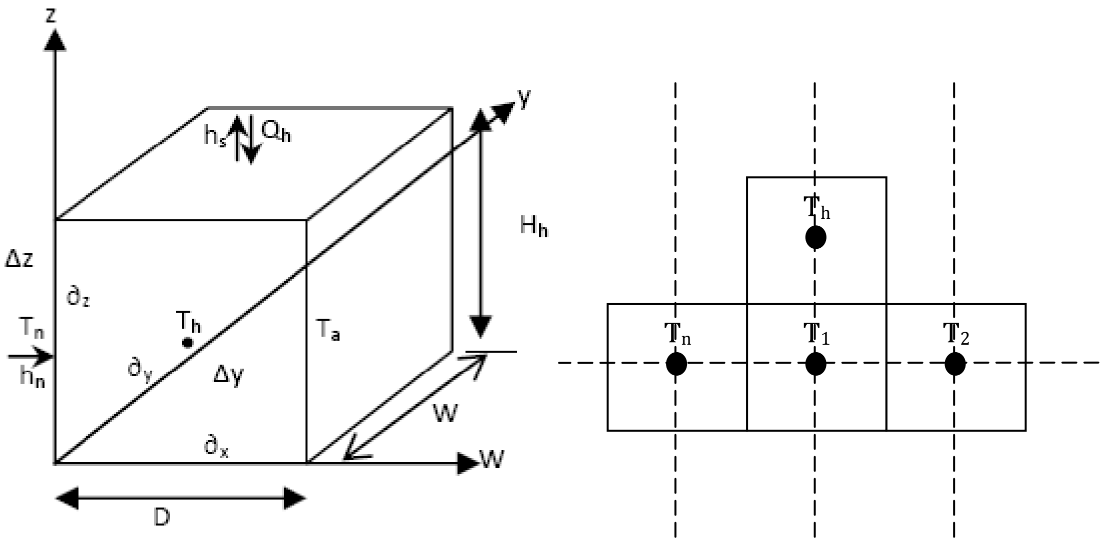

Figure 3.6: The first component as volume element

Consider the first component T_h in Figure 3.5 as the volume element shown in Figure 3.6 for the determination of the heat conduction equation. The rate of heat transfer in *'z'* direction is given by general heat transfer equation (3.1) [51].

$$\rho C \frac{\partial T}{\partial t} = K\nabla^2 T + Q \tag{3.1}$$

in *'z'* direction equation (3.1) is written as:

$$\rho C \frac{\partial T}{\partial t} = K_z \frac{\partial^2 T}{\partial z^2} + Q \tag{3.2}$$

where

Q= Total heat flow in the block.

K_z= Thermal conductivity in z direction

There are many analytical solutions of the equation (3.1) for a wide variety of initial and boundary conditions; here equation (3.2) is solved using the finite difference method using the boundary conditions shown in Figure 3.5 [52]. The finite difference method begins with the discretization of the space and time such that there is an integer number of points (n) in space and time at which we calculate the field variables, in this case just the temperature. For simplicity consider the volume element in Figure (3.6), using the boundary conditions shown in Figure 3.5 and the finite difference the energy balance for equation (3.2) is presented as in equation below.

$$\frac{\partial T}{\partial t} = DH_h W[\frac{(T_h^{n+1} - T_h^n)}{\Delta t}]$$

$$K_z \frac{\partial^2 T}{\partial z^2} = K_{hz} DW \left[\frac{2(\, T_1^{n+1} - \, T_h^{n+1})}{H_h + H_c}\right]$$

$$Q = h_n H_h W(T_n^{n+1} - T_h^{n+1}) + h_s H_h W(T_a - T_h^{n+1}) + h_s DW(T_a - T_h^{n+1}) + Q_h^n$$

Here Q_h represents the input heat from the nip point heater. The first component shown in Figure 3.5 contains the Nip-point foot heater and the concerned temperature is the Nip-point temperature T_h. This is the actual lay-up stage and the precise control of T_h is the primary objective. Therefore the total heat in Nip-point heater component is given by equation (3.3).

$$h_n H_h W(T_n^{n+1} - T_h^{n+1}) + h_s H_h W(T_a - T_h^{n+1}) + h_s DW(T_a - T_h^{n+1}) + Q_h^n +$$

$$K_{hz} DW \left[\frac{2(\, T_1^{n+1} - \, T_h^{n+1})}{H_h + H_c}\right] = \rho_h C_h D H_h W[\frac{(T_h^{n+1} - T_h^n)}{\Delta t}] \quad (3.3)$$

where

K_{hz}= K_{sz} = Thermal conductivity of heater/shim component in z direction

Δt= Time step =0.001seconds

The concerned temperature in the second component shown in Figure 3.5 is T_1. It represents the highest bonding temperature between the layers and is an important indication of the bond quality. Using the equation (3.2) for the second component in Figure 3.5, considering tool temperature is equal to ambient, and balancing the energy, the linear heat equation (3.3) is achieved.

$$K_{cx}H_cW\left[\frac{(T_n^{n+1}-T_1^{n+1})}{D}\right]+K_{cx}H_cW\left[\frac{(T_2^{n+1}-T_1^{n+1})}{D}\right]+K_{hcz}DW\left[\frac{2(T_h^{n+1}-T_1^{n+1})}{H_c+H_h}\right]+$$

$$K_{ctz}DW\left[\frac{2(T_a-T_1^{n+1})}{H_c+H_t}\right]=\rho_cC_cDH_cW\left[\frac{(T_1^{n+1}-T_1^{n})}{\Delta t}\right]-\rho_cC_cDH_cV_xW\left[\frac{(T_n^{n+1}-T_1^{n+1})}{D}\right] \quad (3.4)$$

where

K_{cx}= Thermal conductivity of composite in x direction

K_{hcx}= Resultant of thermal conductivities in x direction

K_{ctz}= Resultant of thermal conductivities in z direction

In the third component shown in Figure 3.5, T_2 represents the temperature when the foot pressure is removed. T_2 is used to estimate the spatial variation of the bond temperature and it affects the rate of crystallization [39]. By balancing the energy in the second component the equation (3.5) is achieved.

$$K_{cx}H_cW\left[\frac{(T_1^{n+1}-T_2^{n+1})}{D}\right]+K_{cx}H_cW\left[\frac{(T_a-T_2^{n+1})}{D}\right]+h_sDW(T_a-T_2^{n+1})+$$

$$K_{ctz}DW\left[\frac{2(\,T_a-T_2^{n+1})}{H_c+H_t}\right]=\rho_cC_cDH_cW\left[\frac{(T_2^{n+1}-T_2^{n})}{\Delta t}\right]-\rho_cC_cDH_cV_xW\left[\frac{(T_1^{n+1}-T_2^{n+1})}{D}\right] \qquad (3.5)$$

The values of all parameters and material properties are defined in Table 3.1.

Table 3.1: Specifications and Thermal Properties of Composite used in Simulations [43]

D= Component length= 0.05(m)	K_{cx}= Composite conductivity in X direction : 6.0 (W/m˚C)
H_t= Tool plate thickness: 0.012 (m)	K_{cz}= Composite conductivity in Z direction: 0.72 (W/m˚C)
H_h= Heater height: 0.00625 (m)	(K_{sx}=K_{sz})= Shim heat conductivity: 20.9 (W/m˚C)
H_c= Composite thickness : 0.000178 (m)*ply#	(K_{tx}=K_{tz})= Ceramic heat conductivity: 1.96 (W/m˚C)
h_n= Heat transfer coefficient hn: 60 (w/km^2)	C_c= Composite Specific heat : 1425 (J/Kg˚C)
h_s= Heater transfer coefficient hs: 960 (W/km^2)	C_h=C_s= Shim/Heater specific heat: 502.4 (J/Kg˚C)
W= Component width: 0.00625 (m)	C_t = Ceramic specific heat: 1.21 (J/Kg˚C)
ρ_c= Composite density : 1560 (kg/m^3)	ρ_h=ρ_s= Shim/Heater density: 7750 (kg/m^3)
T_a= Ambient temperature=Tool Temperature=0˚C	Ceramic density (ρ_t)= 1331 (kg/m^3)
K_{hcz}= K_{hcx}= 0.695(W/m˚C)	K_{ctz}= 0.526(W/m˚C)

Applying the values of parameters given in Table 3.1 and rearranging equations (3.3), (3.4) and (3.5).

$$T_h^{n+1} = (0.97)T_h^n + (0.024)T_1^n + (1.92 \times 10^{-6})T_2^n + (0.00123)T_n^{n+1} + (0.013)Q_h^n \tag{3.6}$$

$$T_1^{n+1} = (0.047)T_h^n + (0.912)T_1^n + (7.41 \times 10^{-5})T_2^n + (0.045)T_n^{n+1} + (6.38 \times 10^{-4})Q_h^n \tag{3.7}$$

$$T_2^{n+1} = (0.508)T_h^n + (0.037)T_1^n + (0.76)T_2^n + (0.0017)T_n^{n+1} + (2.42 \times 10^{-5})Q_h^n \tag{3.8}$$

Express the equations (3.6), (3.7) and (3.8) as a linear matrix equations in the form of

$$x(n+1) = Ax(n) + Bu(n)$$

$$x(n+1) = \begin{bmatrix} T_h^{n+1} \\ T_1^{n+1} \\ T_2^{n+1} \end{bmatrix} \qquad x(n) = \begin{bmatrix} T_h^n \\ T_1^n \\ T_2^n \end{bmatrix} \qquad u(n) = \begin{bmatrix} Q_h^n \\ T_n^n \end{bmatrix}$$

where *x(n)* is a state vector, *u(n)* is the input and.

$$\begin{bmatrix} T_h^{n+1} \\ T_1^{n+1} \\ T_2^{n+1} \end{bmatrix} = \begin{bmatrix} 0.97 & 0.024 & 1.9 \times 10^{-6} \\ 0.047 & 0.912 & 7.4 \times 10^{-5} \\ 0.508 & 0.037 & 0.76 \end{bmatrix} \begin{bmatrix} T_h^n \\ T_1^n \\ T_2^n \end{bmatrix} + \begin{bmatrix} 0.013 & 0.00123 \\ 6.38 \times 10^{-4} & 0.045 \\ 2.42 \times 10^{-5} & 0.0017 \end{bmatrix} \begin{bmatrix} Q_h^n \\ T_n^n \end{bmatrix}$$

and

$$\mathrm{A} = \begin{bmatrix} 0.97 & 0.024 & 1.92 \times 10^{-6} \\ 0.047 & 0.912 & 7.41 \times 10^{-5} \\ 0.508 & 0.037 & 0.76 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.013 & 0.00123 \\ 6.38 \times 10^{-4} & 0.045 \\ 2.42 \times 10^{-5} & 0.0017 \end{bmatrix}$$

The matrices A and B are functions of material properties, boundary conditions, element size, and velocity. Setting ambient temperature to zero, a discrete time state space model with two inputs and three outputs is developed as shown in equation (3.9).

$$x(n+1) = Ax(n) + Bu(n) \tag{3.9}$$

$$y(n) = Cx(n)$$

For the above linear system, the control objective is to control the three temperatures T_h, T_1 and T_2 as represented by vector $x(n)$, to the pre-defined desired values by using the heater power and pre-heater inputs represented by vector $u(n)$, $y(n)$ represents the system output and C is the identity matrix assuming all the state variables are measurable. Two kinds of optimal control strategies are developed to fulfill the tasks in Chapters 4 and 5. The idea is to control the temperature effectively during the process by using the thermal prediction of the model, but the variations in thermal properties and disturbances are not predicted and therefore a closed-loop control system is required as shown in Figure 3.7, for the online temperature regulation to keep the bonding temperature within the manufacturing process window and to reject the undesired disturbances. The temperature at the important locations are monitored and controlled throughout the process, and the simulation results show the effectiveness of the control methods.

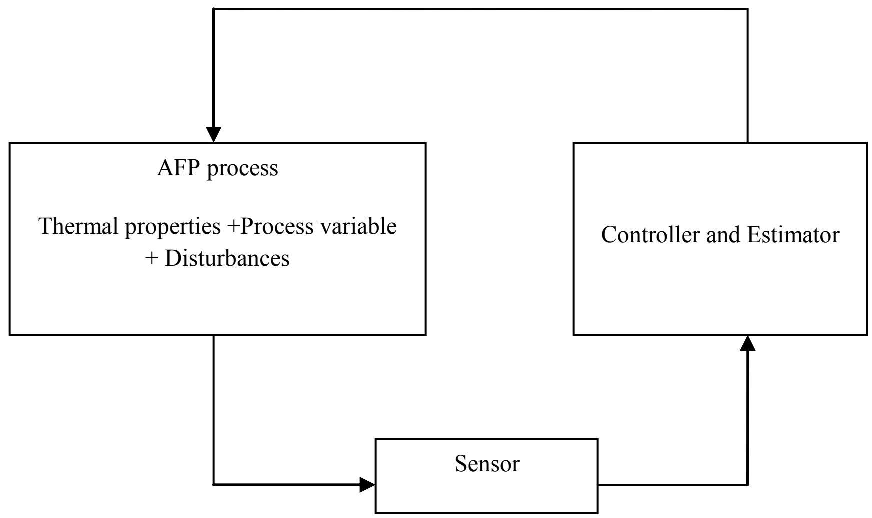

Figure 3.7: Process and Control

3.2 CONCLUSION

The chapter provides some knowledge about the material used in the AFP manufacturing and the thermal model of AFP process. PEEK thermoplastic material is used and the heat transfer during the process is explained. Thermal history of the process has a significant effect on the mechanical properties and the final part quality. The process model is developed using the basic heat conduction equation as the heat transfer plays an important role in the lay-up and consolidation of thermoplastic material. This thermal model of automated placement process is used for the control of the critical temperature parameter.

CHAPTER 4

QUADRATIC OPTIMAL CONTROL

One of the mathematical optimization methods used to design the control system is the optimal control. Optimal control problems are very common in control area. The main idea in optimal control system is to find a control law for a given system by optimizing a specific function which is called the cost function. The cost function is the function of state and control variables, and this function is chosen as the performance index. Optimal control is achieved by solving Hamilton-Jacobi-Bellman equation.

The value chosen for performance index indicates how well the measured value of the system matches the reference signal. Therefore the performance of the system is optimized by selecting the control vector $u(n)$ in such a way that performance index is minimized. The selection of performance index or the cost function is very important and is a difficult process for complex systems because it determines the nature of control and is derived according to the requirement of the process. It is possible that a control system optimized under one index is not optimal in terms of others [44]. Optimal system design is to minimize the performance index which determines the configuration of the system.

Analytical solution of optimal problem provides the complete information about optimal structures. The optimal control design for a linear system is usually based on quadratic performance index, which has been widely used in practical control system design as a measure of system performance. Quadratic performance index has been used in our optimal control system design for Automated Fiber Placement thermal process, which is modeled as a linear system. Let us consider the AFP system model described earlier in equation (3.10). In the quadratic optimal control problem we desire to determine a law for the control vector $u(n)$, as shown in equation (4.1), such that a the quadratic performance index (4.2) [47] is minimized.

$$u(n) = -K(n)x(n) \tag{4.1}$$

$$J = {}^{1}\!/_{2} \sum_{n=0}^{N-1} [\, x^T(n) Q \, x(n) + \, u^T(n) R \, u(n)] \tag{4.2}$$

Q and R are positive definite Hermitian matrices. In equation (4.2) the first part in summation brackets accounts for the relative importance of error during the control process and the second term accounts for the expenditure of the energy of the control signal [44]. If $N=\infty$ the optimal control gives a steady state solution and the gain matrix $K(n)$ in equation (4.1) becomes a constant matrix. The focus remains on determination of matrix $K(n)$ and the performance index written as in equations (4.1) and (4.2).

4.1 LINEAR QUADRATIC REGULATOR (LQR)

A specific type of linear quadratic optimal control problem is that of a linear Quadratic Regulator (LQR), in which all the matrices (i.e. A, B, Q and R) are constant. The LQR tries to regulate the $[\, r(n) - x\,(n)]$ for the reference signal tracking, and the classical optimal control theory represent the LQR in feedback form as shown in the Figure 4.1.

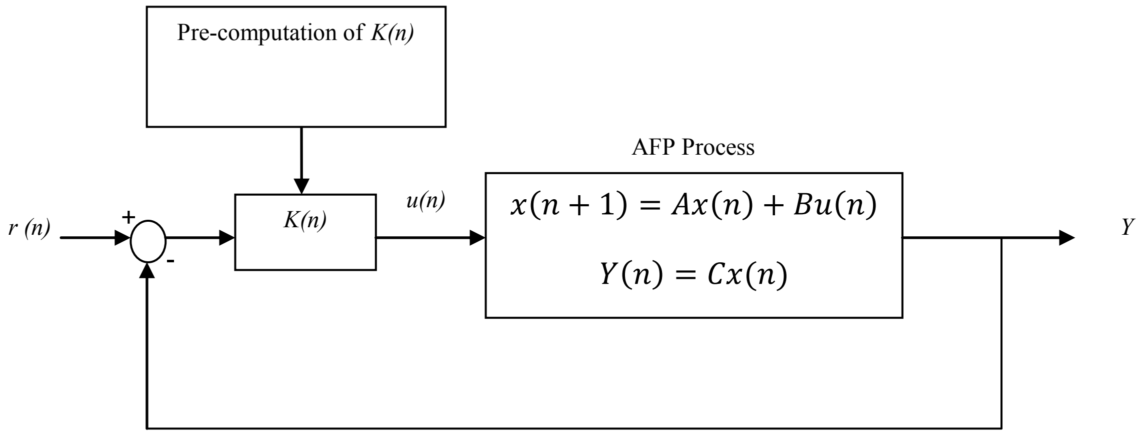

Figure 4.1: LQR assuming measureable / available states.

The weighting matrices Q and R, weighs the relative importance of the performance caused by the state vector $x(n)$ and control vector $u(n)$. Different approaches have been used by researchers to select Q and R, where selection is made either by trial and error or by using practical, iterative procedures [46]. The next step is to find the regulator gain K in control law equation (4.1), where the steady state gain K is represented as in equation (4.3) [47].

$$K = (R + B^T P B)^{-1} B^T P A \tag{4.3}$$

where P represents the transformation matrix between states. The solution of P is achieved by solving steady-state Riccati equation [47] given by equation (4.4).

$$P = Q + A^T P A - A^T P B (R + B^T P B)^{-1} B^T P A \tag{4.4}$$

In the above optimal control design shown in Figure 4.1, it is assumed that all the states are available and the outputs are measurable. Considering the actual situation, the Nip point temperature T_h and the first component temperature T_1 as shown in Figure 3.4, can be readily measured by using high temperature pyrometer which is normally used in advanced Automated Fiber Placement machines, and provides a non-contacting procedure of measuring thermal radiations. However the third component temperature T_2 is hard to measure during AFP manufacturing process, therefore a Linear Quadratic Estimator (LQE) or Kalman estimator is designed to estimate this temperature.

4.2 LINEAR QUADRATIC ESTIMATOR (LQE)

In control theory Kalman filter is most commonly represented as linear quadratic estimator (LQE). Kalman Filter provides the estimates of the measurements by prediction. It estimates the uncertainties in the prediction and finally computes the weighted average of the prediction and measured values. The weighing matrix helps having the estimates close to the true values. Kalman filter is designed based on the system model. It provides the estimations of the immeasurable state variables by using the inputs to the system and partial measurable state variables (output measurements). The design of Kalman filter takes into account the noise and uncertainties. It also averages the predictions with new measurements using weighing matrices. This process is iterated in every sampling interval.

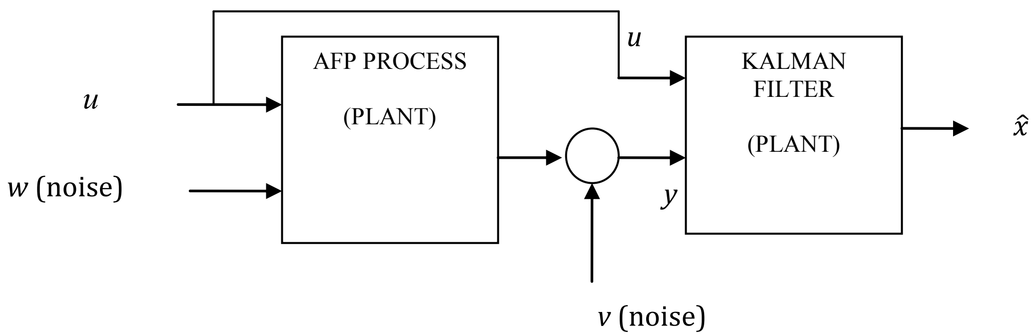

Figure 4.2: Kalman Filter

Consider a discrete time system subject to Gaussian noises w and v, in equation (4.5).

$$x(n+1) = Ax(n) + Bu(n) + Gw(n) \tag{4.5}$$

$$y(n) = Cx(n) + v(n)$$

The noise covariance represented in equation (4.6):

$$E(w[n]w[n]^T) = Q \tag{4.6}$$

$$E(v[n]v[n]^T) = R$$

The estimator is given in equation (4.7) [47], where $\hat{x}$ represents the estimated states.

$$\hat{x}[n+1 \mid n] = A\hat{x}[n \mid n-1] + Bu[n] + L(y[n] - C\hat{x}[n \mid n-1] - Du[n]) \tag{4.7}$$

The Kalman filter gain L in above equation is obtained by solving Riccati equation (4.8).

$$L = (APC^T + \bar{N})(CPC^T + \bar{R})^{-1} \tag{4.8}$$

$$\bar{R} = R + BN + N^T B^T + BQB^T$$

$$\bar{N} = A(QB^T + N)$$

The error covariance matrix P is given by:

$$P = A\,[P - P^T C\,(N + CPC^T) - CP]A^T + Q \qquad (4.9)$$

4.3 Linear Quadratic Gaussian (LQG) Control

Linear Quadratic Gaussian (LQG) controller is the combination of Linear Quadratic Estimator (LQE) and Linear Quadratic Regulator (LQR). It concerns uncertain linear system disturbed by additive white Gaussian noise, in condition where the state variables are not available for feedback control. Figure 4.3 shows the application of white noise as process and sensor disturbance and implementation of LQG Control.

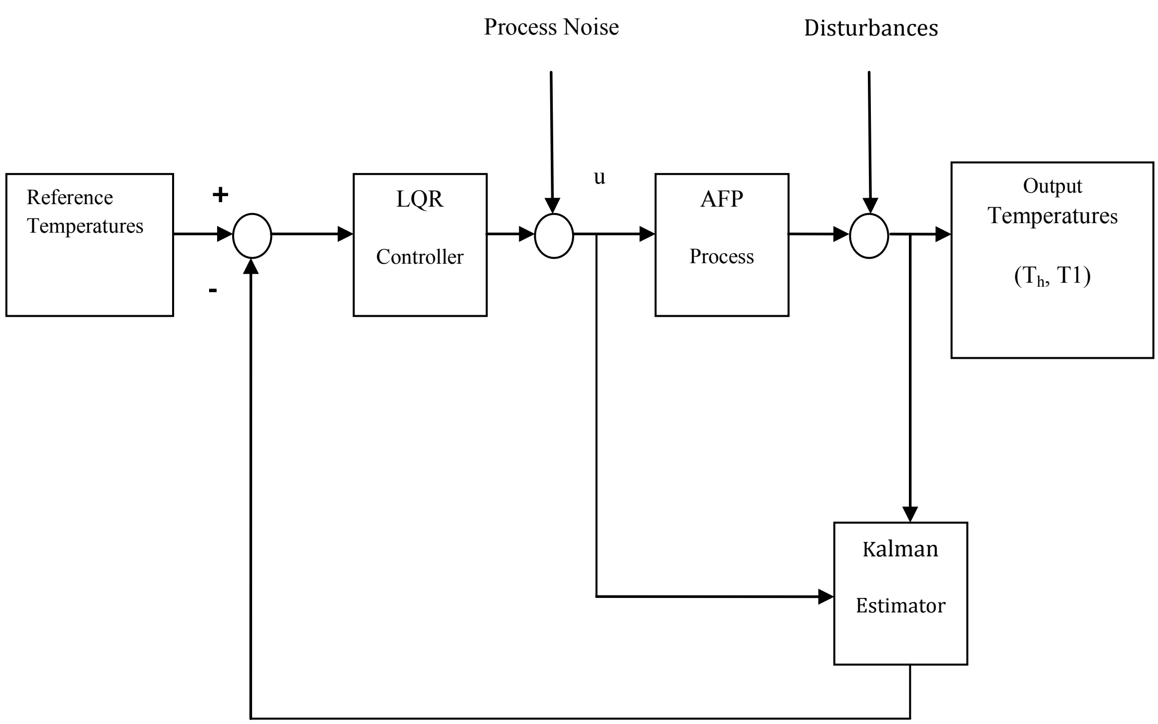

Figure 4.3: LQG Controller

4.4 Simulation and Results

We use Matlab/Simulink to carry out the simulation of LQG optimal controller as shown in Figure 4.3. The simulations are done using fixed time-step size of 1ms and the discrete time solver, with simulation time of 30 seconds. Using the optimal control theory and equations (4.3), (4.4), (4.8) and (4.9), the regulator gain *K*, the weighing matrices *Q* and *R*, the Kalman filter estimator gain L, and the error covariance matrix *P* are calculated as shown in Table 4.1.

Table 4.1: LQG parameters

Regulator Gain '*K*'	$K = \begin{bmatrix} 11.2764 & 0.8543 & 1.3066 \\ 0.9753 & 9.2449 & 1.1691 \end{bmatrix}$
Weighing matrix '*Q*'	$Q = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 100 \end{bmatrix}$
Weighing matrix '*R*'	$R = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$
Kalman filter Gain '*L*'	$L = \begin{bmatrix} 0.6097 & 0.0076 & 0.0292 \\ 0.0127 & 0.5927 & 0.0079 \\ 0.0883 & 0.0107 & 0.0378 \end{bmatrix}$
Error covariance matrix '*P*'	$P = \begin{bmatrix} 0.2662 & 0.1599 & 0.0276 \\ 0.1599 & 5.9474 & 0.2981 \\ 0.0276 & 0.2981 & 0.0493 \end{bmatrix}$

According to the conditions defined in Chapter 3 for thermoplastic composite PEEK (APC-2) ,where the manufacturing process window is from 375˚C to 400˚C [43] and the parameters defined in Table 3.1, the simulation is carried out for the system in Figure 4.3.The reference temperature for Nip-point heating T_h is set as 400˚C. Figure 4.4 shows the output temperatures T_h, T_1 and the estimated temperature T_2.

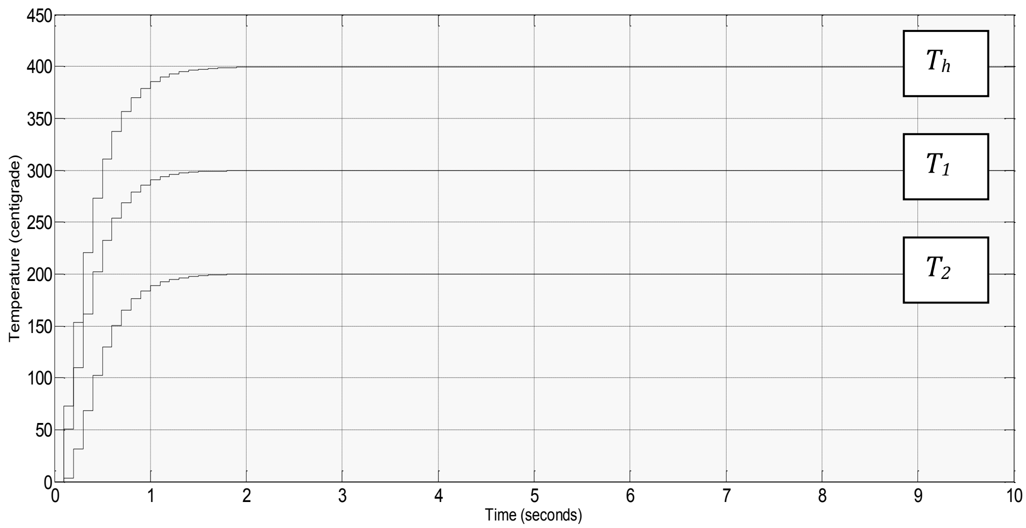

Figure 4.4: LQG Temperature Outputs T_h, T_1 and the estimated temperature T_2.

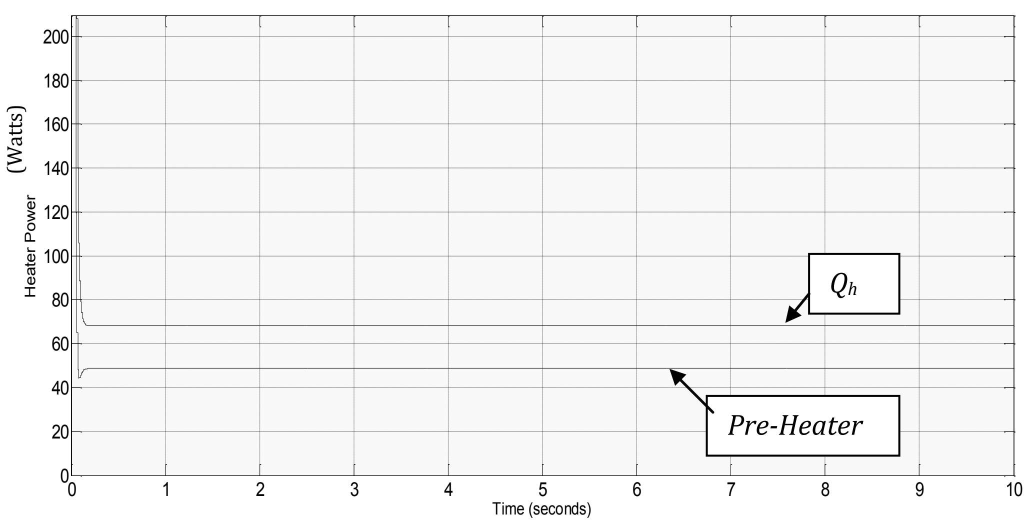

Figure 4.5: Nip- point and Pre heater Input Powers

Figure 4.6 and 4.7 shows the response curves of the Nip-point temperature T_h and the first component T_1.

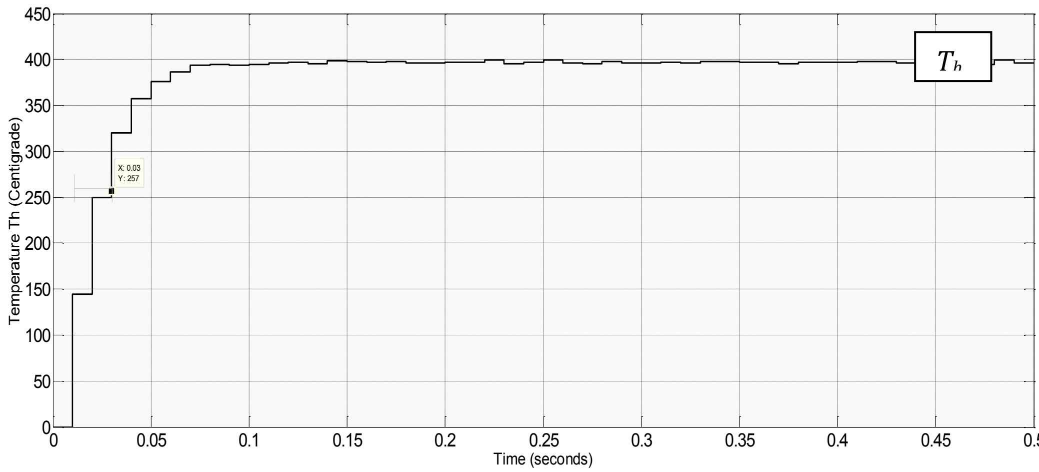

Figure 4.6: Nip Point Temperature T_h (Steady State Error = 0.62%, Rise Time = 0.03 sec, Settling Time = 0.1 sec)

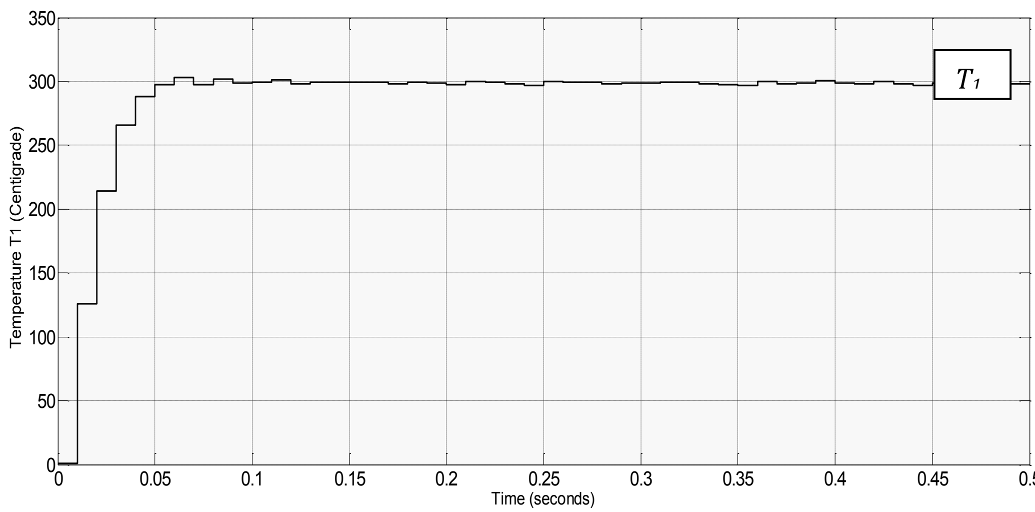

Figure 4.7: Second Component Temperature T_1 (Steady State Error = 0.44%, Rise Time = 0.03 sec, Settling Time = 0.08 sec)

4.5 Disturbance Rejection

The disturbance rejection ability of the LQG design is tested by introducing a step disturbance to the plant at t=15 second. The disturbance is modeled as shown in Figure 4.8.

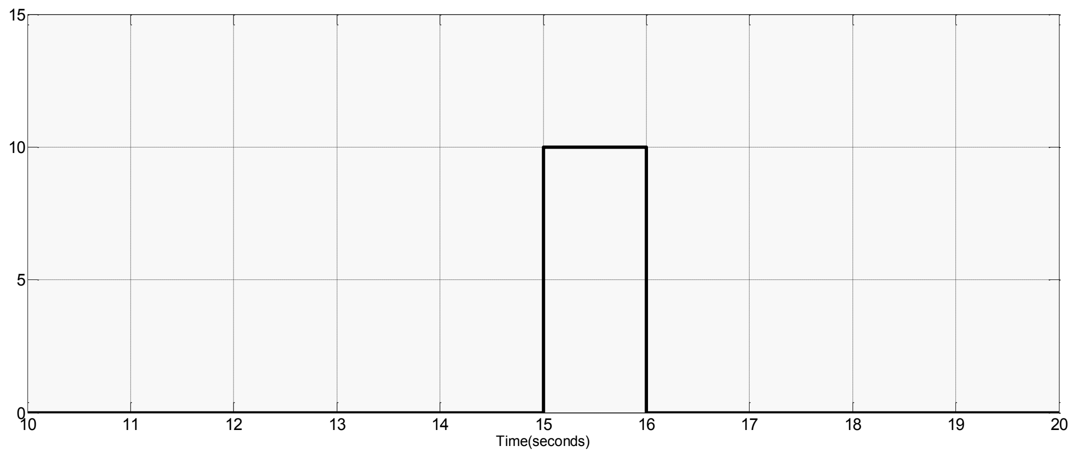

Figure 4.8: 10˚C step disturbance

The response of the Nip-point temperature T_h under step disturbance is shown Figure 4.9, where it clearly violates the reference input of 400˚C.

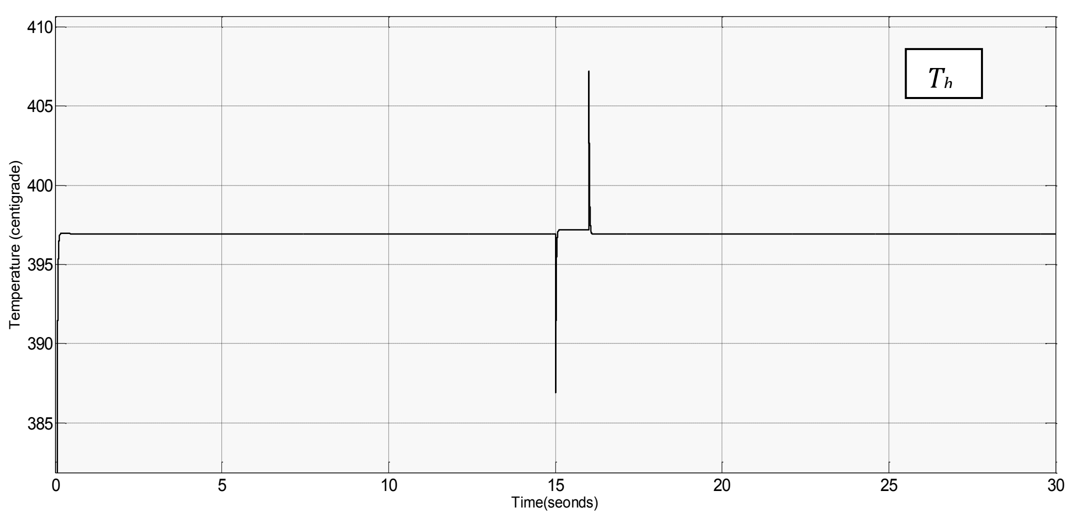

Figure 4.9: Nip-point temperature T_h with step disturbance (Steady State Error = 0.75%, Rise time = 0.03 sec, Settling Time = 0.16 sec, Spike = 2%)

The input power for Nip point heating violatese the physical constraint of the heating system of 80Watts as shown in Figure 4.10.

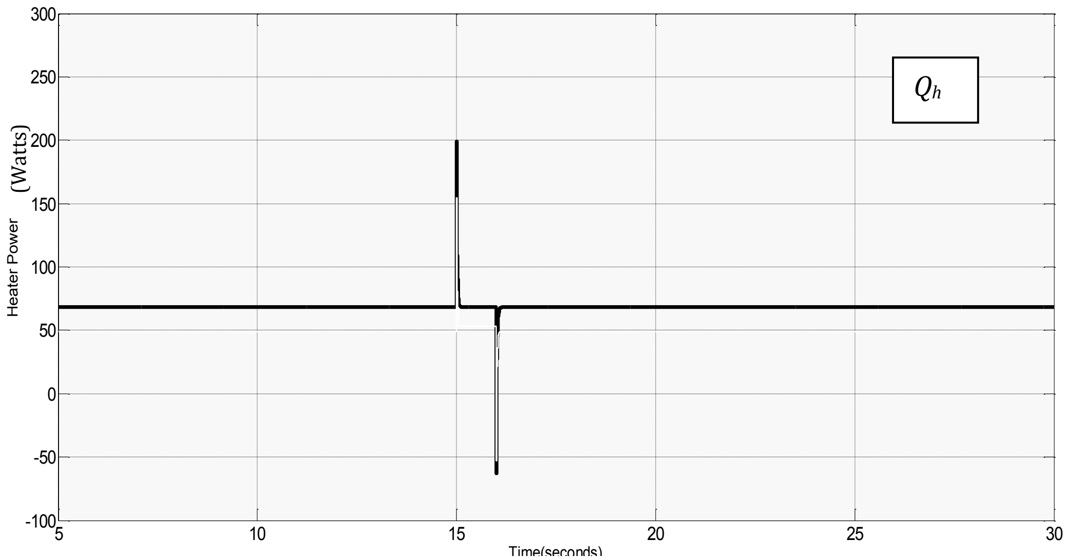

Figure 4.10: Nip-point Heater Power after step disturbance.

Figures 4.4 and 4.5 show the system output and input with LQG controller, the performance of the controller is efficient and the temperature of the Nip-point heater is kept within the range of 375-400˚C. However after the application of step disturbance the Nip-point temperature as shown in Figure 4.9 violates the temperature limit and the sharp spike is observed at the input power of the Nip-point heater as shown in Figure 4.10. Although there is a slight increase in steady state error in Figure 4.9 but the spike of 2% is not acceptable in AFP process as it may degrade the quality of the material, moreover the sharp spike in Figure 4.10 violates the physical constraint of 80 watt heating equipment.

4.6 CONCLUSION

In this chapter the basic optimal theory is discussed and the Linear Quadratic regulator (LQR) is designed to achieve the simple feedback control system. Linear Quadratic Estimator (LQE) provides the estimate of immeasurable temperature T_2, and finally with the combination of LQR and LQE the Linear Quadratic Gaussian (LQG) controller is designed. The simulation results are analyzed under disturbance and clear violation of input and output constraints were observed. Temperature and power spikes are not acceptable in AFP process. Therefore the controller must address the constraints for reliable performance.

CHAPTER 5
MODEL PREDICTIVE CONTROL

Model Predictive Control (MPC) is one of the advanced control technologies that have been in use in process industries for 25 years. The main reason of high demand for MPC is its ability to deal with equipment and safety constraints while providing optimal control performance. Initially the technology was developed for specific application in petroleum industry but today MPC can be found in variety of applications. Different companies are making MPC-based control systems for industrial process, making its application doubled in 4 years from 1995 to 1999 [45].

5.1 BASIC FORMULATION

The basic understanding of MPC can help in solving the problems related to practical implementation of control strategies in Automated Fiber Placement (AFP) processes. MPC techniques can solve the challenging problems related to temperature control in AFP process. MPC is the combination of control algorithms which incorporate process model to predict the future response of the plant. The model is used to predict the behavior of the dependent variable (outputs) of the modeled dynamics with respect to the change in the independent variables (inputs). At each sample it optimizes the future response of the plant by finding the sequence of future inputs or Manipulated Variable (MV) adjustments as shown in Figure 5.1 [48]. MPC methodology is presented in both constrained and unconstrained forms.

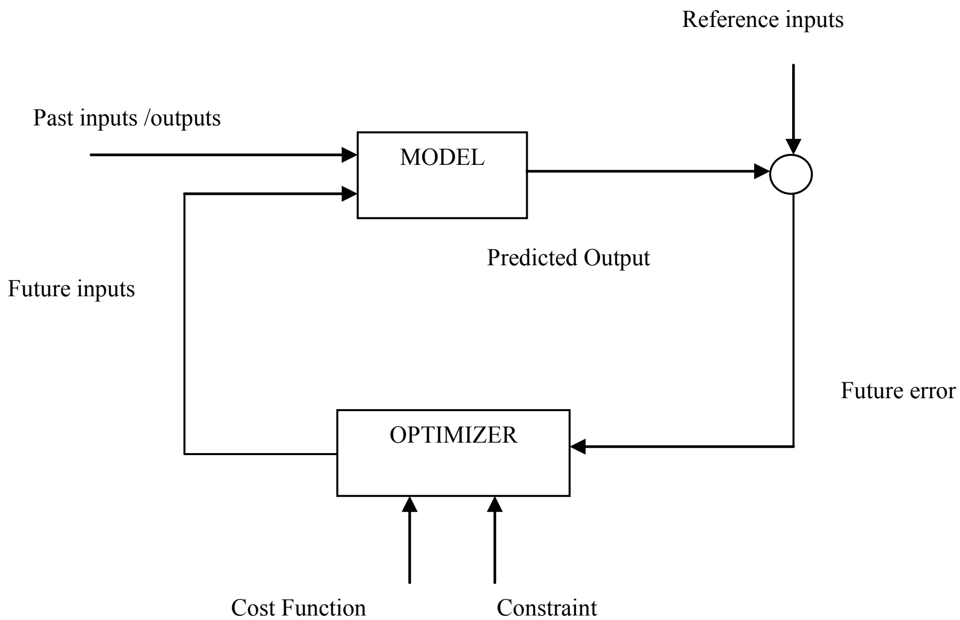

Figure 5.1: Basic Formulation of MPC control

Consider the flow chart of MPC operation in Figure 5.2.

1) At present time t_i, the process behavior over the optimization window N_p, this is also regarded as the numbers of samples.
2) Now using the process model in equation (3.9), the changes of process output according to manipulated variable is predicted.
3) The current and further moves of manipulated variables are selected by minimizing the sum of square of the future errors. Future error mentioned here is the deviation of controlled variable from the set point (reference input).
4) In later part constraints on MV (inputs) and outputs are considered within the minimization as shown in Figure 5.1.

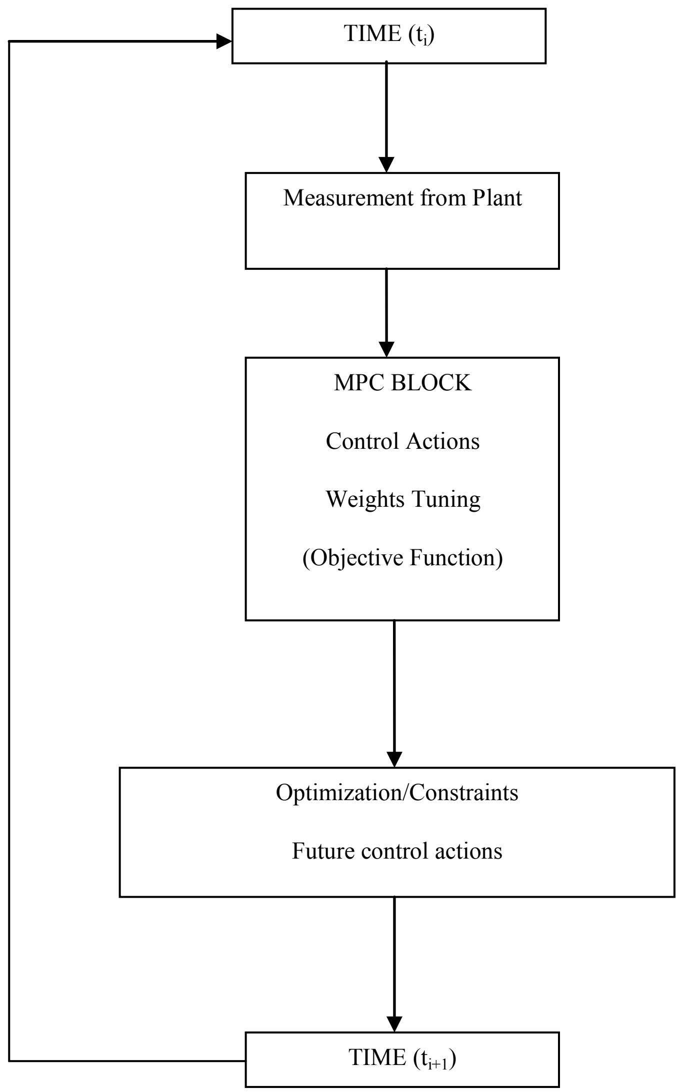

Figure 5.2: Flow chart of MPC process

Only first value of computed moves is applied to the plant. At next step t_{i+1} shown in the Figure 5.2, the horizon is shifted one step forward and the optimization is carried out again. Because of this strategy the control is referred as Moving Horizon Control.

5.2 IMPLEMENTATION

In this project, the model predictive controller is designed based on the state space model of automated fiber placement process defined in equation (3.9). In the state space model the current information is represented by the state variables which are used for future prediction. The first step is to establish the plant model that is feasible for the MPC closed-loop performance. If there exist higher frequency modes in the plant, they can pose devastating effect on the system. To compensate for this, we include a penalty on high frequency control action by augmenting the plant model with a high pass filter. Consider the AFP model as described in equation (3.10).

$$x(n+1) = Ax(n) + Bu(n) \tag{5.1}$$

$$y(n) = Cx(n)$$

Considering the moving horizon strategy, we need to change the model by introducing an integrator for MPC design purpose. Therefore the model presented in equation (5.1) is changed to suit our design purpose in which integrator is embedded. The new model is called augmented model and represented by same A_m, B_m and C_m matrices as shown in equation (5.2)

$$\overbrace{\begin{bmatrix} \Delta x(n+1) \\ y(n+1) \end{bmatrix}}^{x_m(n+1)} = \overbrace{\begin{bmatrix} A & 0_m^T \\ CA & I \end{bmatrix}}^{A_m} \overbrace{\begin{bmatrix} \Delta x(n) \\ y(n) \end{bmatrix}}^{x_m(n)} + \overbrace{\begin{bmatrix} B \\ CB \end{bmatrix}}^{B_m} \Delta u\,(n)$$

$$y_m(n) = \overbrace{\begin{bmatrix} O_m & I \end{bmatrix}}^{C_m} \begin{bmatrix} \Delta x(n) \\ y(n) \end{bmatrix} \tag{5.2}$$

where O_m $3x3$ zero matrix, $\Delta x(n+1) = x(n+1) - x(n)$ and $\Delta u\ (n) = u(n) - u(n-1)$.

$\Delta U(n)$ is the input and referred as the future control vector represented as:

$$\Delta U = [\Delta u(n_i), \Delta u(n_i + 1) \dots \dots \dots, \Delta u(n_i + N_c - 1)]^T \tag{5.3}$$

where, N_c is called the control horizon which indicates the number of parameters used to develop future control. Now the future state variables are represented as;

$$x_m(n_i + 1|n_i), x_m(n_i + 2|n_i) \dots \dots \dots \dots x_m\left(n_i + N_p \middle| n_i\right)$$

and the output is represented as

$$Y = [y_m(n_i + 1|n_i), y_m(n_i + 1|n_i) \dots \dots y_m\left(n_i + N_p \middle| n_i\right)]^T \tag{5.4}$$

where N_p is the optimization window and also regarded as number of samples.

Considering equation (5.3) and (5.4), the predictive output is represented as [49]:

$$Y = Fx_m(n_i) + \ \Delta U \tag{5.5}$$

where, ϕ represents the transition matrix.

$$F = \begin{bmatrix} C_m A_m \\ C_m {A_m}^2 \\ . \\ . \\ C_m {A_m}^{Np} \end{bmatrix} \qquad \phi = \begin{bmatrix} C_m B_m & . & . & 0 \\ C_m {B_m}^2 & . & . & . \\ . & . & . & . \\ . & . & . & . \\ C_m {B_m}^{Np-1} & . & . & C_m {B_m}^{Np-Nc} B_m \end{bmatrix}$$

5.2.1 OPTIMIZATION AND GAIN CALCULATION

MPC predicts the output, and tries to bring it as close as possible to the desired input which is regarded as set point signal and considered as constant. This is achieved by delivering the optimal control ΔU by minimizing the errors E_p between reference points and predicted outputs.

Therefore, the control signal is calculated as:

$$\Delta u(n_i) = K_{MPC}(r(n_i) - x_m(n_i))$$

Where $r\ (n_i)$ represents the reference input for i^{th} future sample and K_{MPC} is the MPC gain, which depends on output and input weights, predictions and control horizons. The output weights are adjusted to bring the corresponding output close to its reference trajectory. Now the quadratic objective function is represented as:

$$J = \sum_{i=1}^{Np} ||w^y(y(n+i\ |n) - r(n+i)||^2 + \sum_{i=1}^{Nc} ||\ w^{\Delta u}(\Delta u(n+i\ |n)||^2$$

where w^y , $w^{\Delta u}$ represents the input and output weights for i^{th} future sample in discrete time. Defining cost function in terms of Y and ΔU from equation (5.5) we get

$$J = (R_s - Y)^T\ (R_s - Y) + \ \Delta U^T R \Delta U \tag{5.6}$$

where Rs represents the reference point signal and $R \in R^{n*n}$ is a matrix of tuning parameters. The first term of above equation is linked to the objective for minimizing the error and the second term is related to minimizing the control effort. By putting the value of Y from equation (5.5) to find the optimal ΔU that minimizes cost function J, we obtain.

$$J = (R_s - Fx_m(n_i))^T\ (R_s - Fx_m(n_i)) + \ 2\Delta U^T \phi^T (R_s - Fx_m(n_i)) + \ \Delta U^T (\phi^T \phi + R)\Delta U$$

Taking the first derivative of J and putting necessary condition $\frac{\partial J}{\partial \Delta u} = 0$ we obtain,

$$\Delta U = (\phi^T \phi + R)^{-1} \phi^T (R_s - Fx_m(n_i)) \tag{5.7}$$

where $(\phi^T \phi + R)^{-1}$ is called Hessian Matrix in optimization literature [49]. The optimal value of ΔU is obtained by solving equation (5.6), the components of the equation depends on the system parameters and are constant for the system, but as mentioned earlier, according to receding horizon control principal, we only take the first element of ΔU at time t_i as shown in Figure 5.2. Therefore, one has

$$\begin{aligned} \Delta U(n_i) &= [1 \quad 0 \quad . \quad . \quad 0]\ (\phi^T \phi + R)^{-1} (\phi^T R_s r(n_i) - \ \phi^T F x_m(n_i)) \\ &= \ K_y r(n_i) - K_{mpc} x_m(n_i) \\ &= \ K_{MPC}(r(n_i) - x(n_i)) \\ &= \ K_{MPC}(E_p(n_i)) \end{aligned}$$

where K_y is the first element of $(\phi^T \phi + R)^{-1} \phi^T R_s$ and K_{mpc} is the first row of $(\phi^T \phi + R)^{-1} \phi^T F$.

5.3 Simulation / Results of MPC without Constraint

Matlab/Simulink was used to carry out the simulation of MPC design without constraints. The optimization window N_p is set as 10 and the control horizon N_c is set as 2. However it was noted that without the presence of disturbances the performance of unconstrained MPC is similar to those in LQG control system. Figure 5.3 shows the output temperatures T_h, T_1 and T_2 and it is noticed that the performance is similar to the results shown in Figures 4.4 and 4.5.

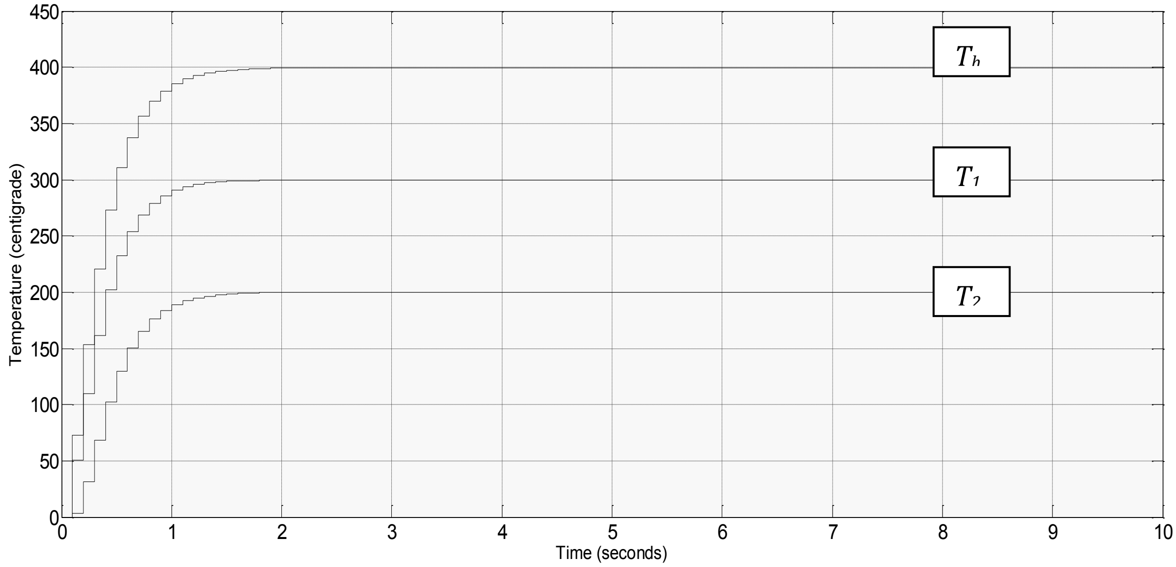

Figure 5.3: Temperature Outputs T_h, T_1 and the estimated temperature T_2 with unconstrained MPC

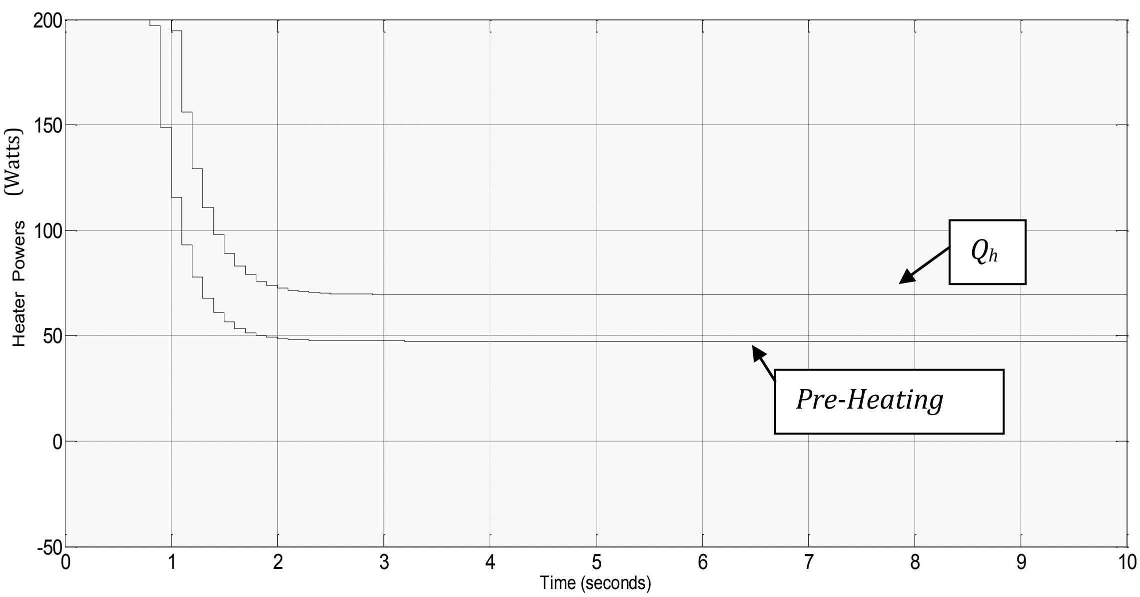

Figure 5.4: Nip-point and Pre heater Input Powers

5.4 CONSTRAINTS

One of the important reasons of taking the constraints into account is to obtain profitable operation. Such constraints are associated with direct cost, especially the energy cost. In AFP the manufacturing cost can be minimized by keeping the heat supplied within the manufacturing constraints to ensure the quality of composite. Similarly the input constraints are applied on the input equipment for heating control within its capacity.

It is noticed that the performance of LQG control system designed for the original system is deteriorated significantly when the disturbances were applied, as the system has operational constraints. In the system of Automated Fiber Placement, the constraints like the heating equipment limitations and the temperature limitations for the PEEK (AS4) material need to be addressed to achieve efficient control system. The conventional control system operates very closely to the limits and in many situations the constraint violations are observed shown in Figures 4.9 and 4.10, therefore for safety reasons the reference points must be changed. This change decreases the quantity and quality of the manufactured composite and results in profit loss.

The MPC addresses the constraints within the design procedure and provides better performance under disturbances. Figure 5.5 compares the typical response of MPC controller under constrained environment with conventional control strategy.

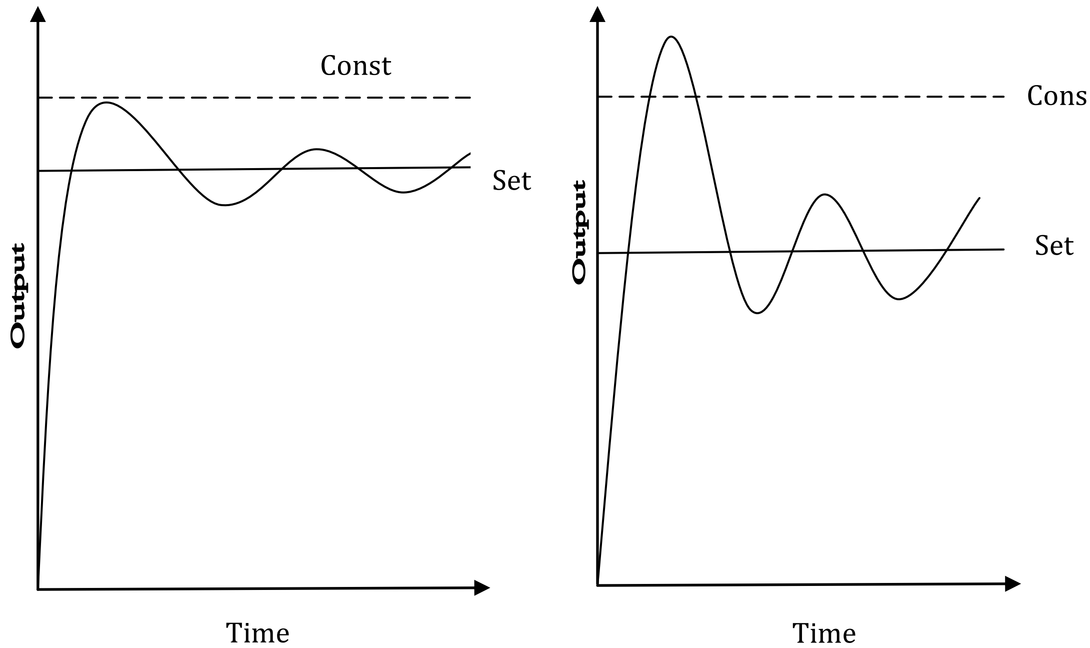

Figure 5.5: Constraints in MPC and conventional control

Three types of Constraints are normally used:

1) Constraint on rate of change of controls variable *ΔU*. If we met the upper and lower limits in *ΔU* the constraints are specified as

$$\Delta u^{min} \le \Delta u \le \Delta u^{max} \quad (5.8)$$

2) Constraint amplitude of control variable (input), they are the very common physical hard constraint on the system.

$$U^{min} \le U(k) \le U^{max} \quad (5.9)$$

3) Output Constraints

$$Y^{min} \le Y(k) \le Y^{max} \quad (5.10)$$

Output constraints are not very straight forward, as it causes large changes in both amplitude and rate of control variables. Therefore we need either to make the output constraints soft or to consider the control constraints in addition to output constraints. Constraints can be hard or soft, however, in some particular situation the violation is unavoidable, and therefore the MPC provide the tolerance band by softening the constraint, making a violation mathematically acceptable [49]. The constraints are applied on MPC as linear inequalities. Constraints are applied on both current and future moves within the predictive horizon to allow algorithm to prevent further violation. By putting the value of Y from equation (5.5) in (5.10), the output constraints are expressed as:

$$Y^{min} \leq F x_m(n_i) + \phi \, \Delta U \leq Y^{max} \tag{5.11}$$

The constraints presented in equations (5.8), (5.9) and (5.10) can be combined in equation (5.12)

$$C^u \Delta U(n) \geq C(n+1 \,|n) \tag{5.12}$$

$$C^u = \begin{bmatrix} -I \\ I \\ -I \\ I \\ -\phi \\ \phi \end{bmatrix}$$

$$C\ (n+1\ |n) = \begin{bmatrix} -U^{max} \\ U^{max} \\ -\Delta u^{max} \\ \Delta u^{max} \\ -Y^{min} + Fx_m(n_i) \\ Y^{min} - Fx_m(n_i) \end{bmatrix}$$

After forming the constraints, the next step is to include the linear constraints into quadratic cost function *J* as defined in equation (5.5). Therefore the objective is to find the optimal solution for a system (5.1) subject to the constraints (5.12). The MPC controller can be obtained by using the standard quadratic programming problem [48]. Quadratic programming is a separate field of study from this thesis. Therefore the focus in this project is on the basic quadratic programming techniques to derive the solution of the cost function with respect to the applied constraints. The cost function *J* with constraints can be arranged as:

$$\text{Minimize : } J = \frac{1}{2}\Delta U^T H \Delta U + \ \Delta U G^T \quad (5.13)$$

$$\Delta \text{U} = \text{control vector}$$

$$s.t \quad C^u \Delta U \ \geq C\ (n+1\ |n)$$

where *H* is a symmetric, positive definite Hessian Matrix. *G* is a gradient vector and is equal to $2\phi^T W_y^T W_y E_p(\ n+1\ |n)$ and $C\ (n+1\ |n)$ represents the components of constraints. The constraint MPC algorithm is implemented by initializing the predictive vector *Y* in equation (5.5), updating the state, obtaining measurement from the plant and computing the error E_p, computing gradient vector *G*, calculating constraint vector *C(n + 1 | n)* and solving the quadratic programming as shown in equation (5.13). The constraints tend to increase the computational time needed to solve the quadratic programming. The equation (5.13) is solved at each controller execution after every prediction.

5.5 Simulation and Results of Constrained MPC

We use MPC toolbox [50] to implement the constrained MPC control system. The MPC toolbox provides a user friendly Graphical User Interface (GUI) for fine tuning of inputs and output weights $w^{\Delta u}$ and w^{y} as given in (5.14). In order to test the constraint feature of the MPC controller of AFP heating system as shown in Figure 5.6, a step disturbance is introduced as shown in Figure 4.8.

$$w^{\Delta u} = \begin{bmatrix} 0.09 \\ 0.0125 \end{bmatrix} \quad w^{y} = \begin{bmatrix} 0.8 \\ 0.78 \\ 1 \end{bmatrix} \tag{5.14}$$

The output constraints are set according to the operating limits of PEEK (APC-2) material as 400˚C and the input constraints are set according to the heating equipment capacity and limitation which are assumed to be 80Watts. The focus here is the Nip-Point temperature T_h, and the input power of the heater. Figure 5.7 shows the output temperatures with step disturbance in unconstrained MPC and Figure 5.8 and 5.9 shows the violations of the input and output constraints presented in equation (5.15).

$$0^{\circ}C \leq Y(n) \leq 400^{\circ}C \tag{5.15}$$

$$0Watts \leq U(n) \leq 80Watts$$

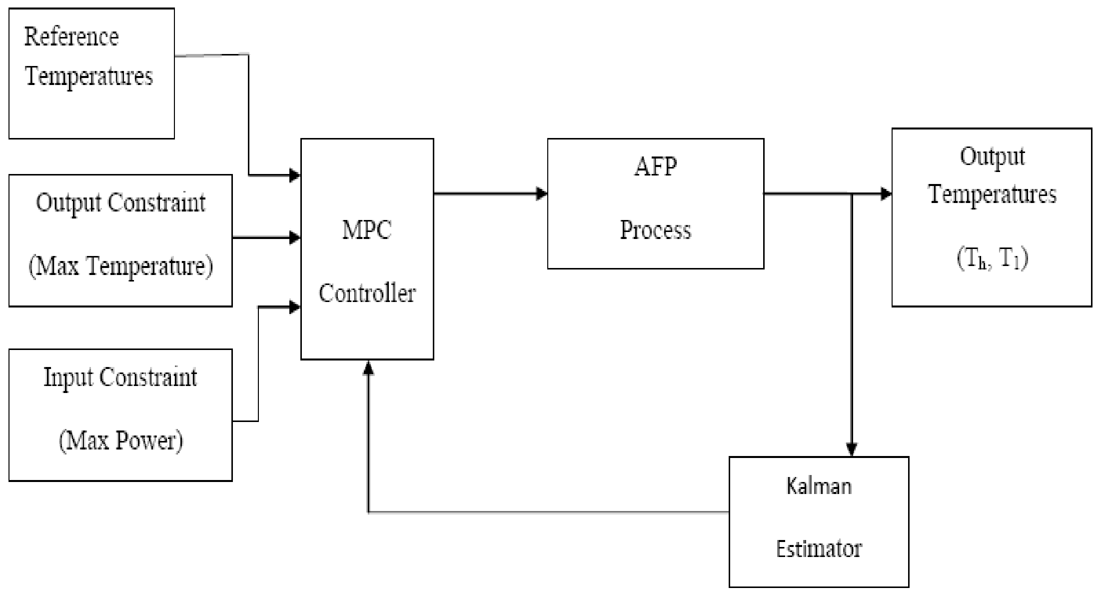

Figure 5.6: MPC in SIMULINK

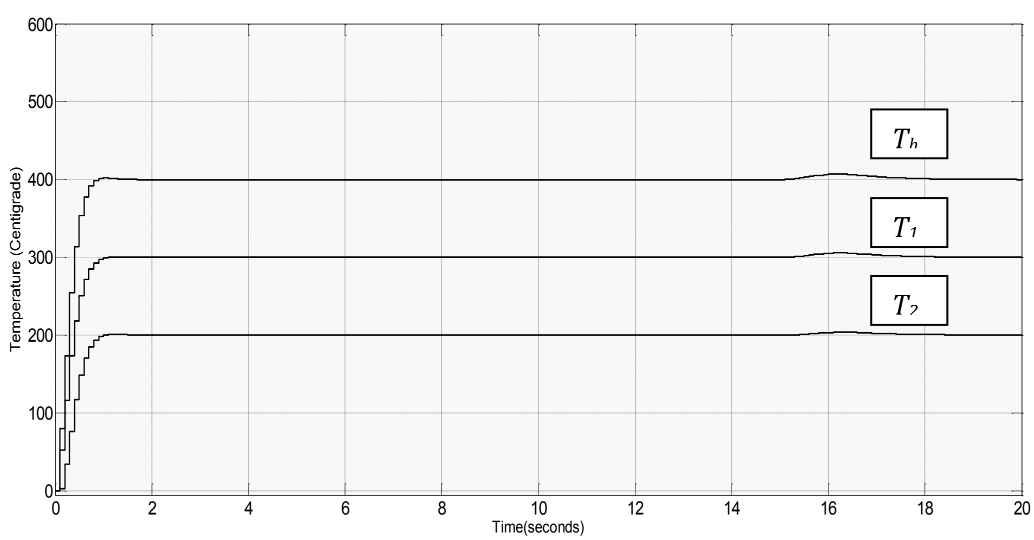

Figure 5.7: Output temperatures with disturbance

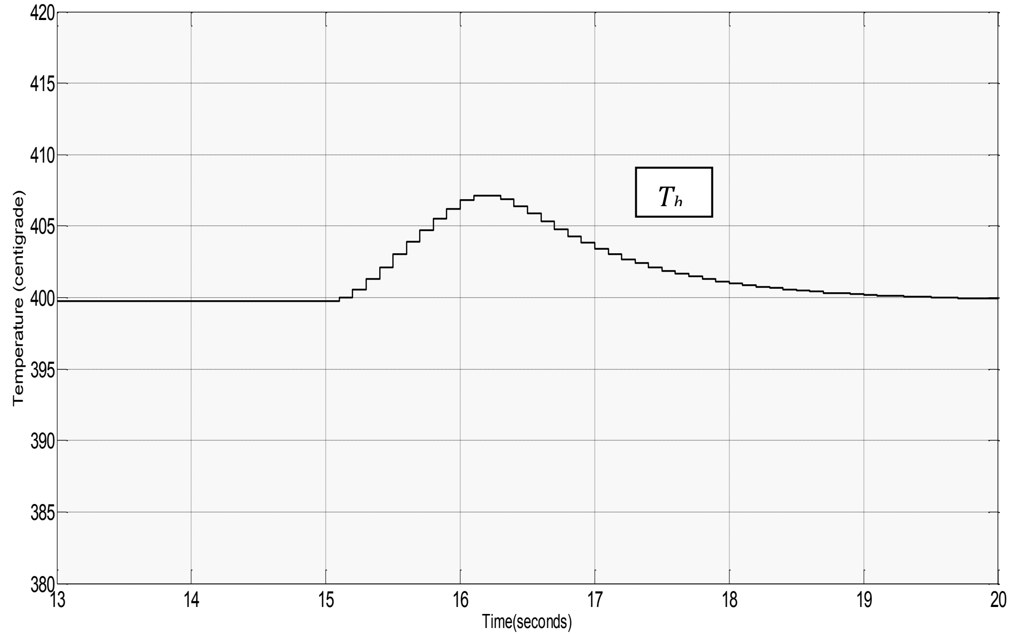

Figure 5.8: Nip-point temperature T_h with step disturbance

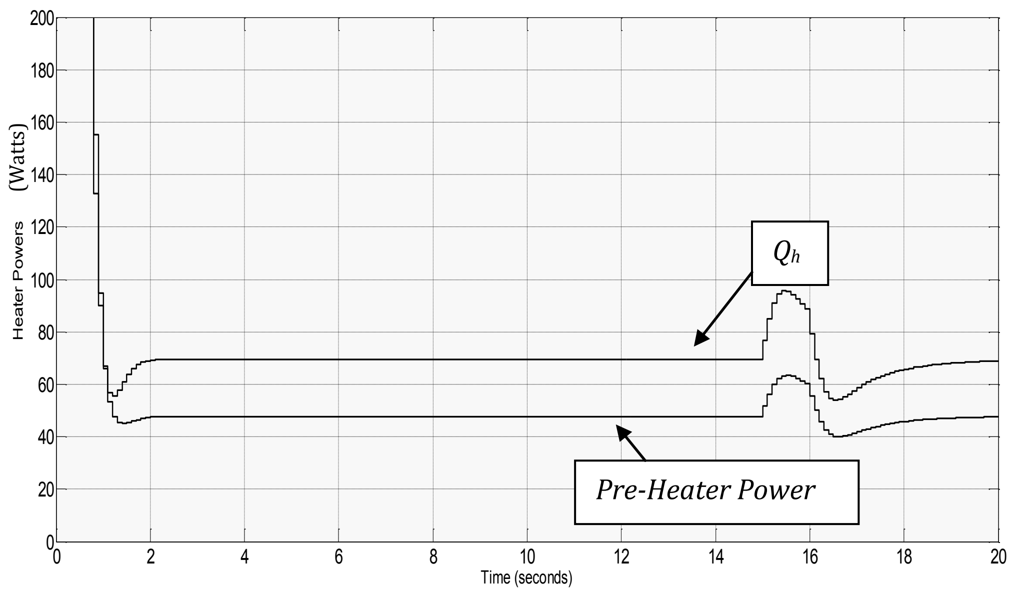

Figure 5.9: Nip-point and Pre-Heater Inputs with step disturbance

Now the constraints shown in equation (5.15) are applied on the MPC design presented in Figure 5.6. The Nip-point Temperature T_h and the input power were efficiently controlled and the violations were avoided as shown in Figure s 5.10, 5.11 and 5.12.

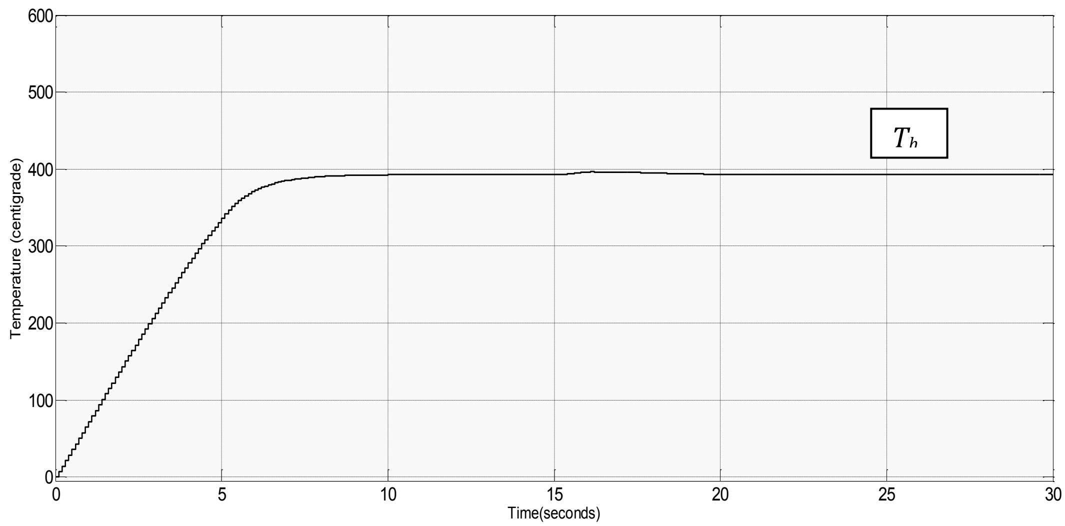

Figure 5.10: Nip-point temperature T_h with constrained MPC (steady state error=1.8%, rise time= 4.7 sec, settling time= 8.7 sec)

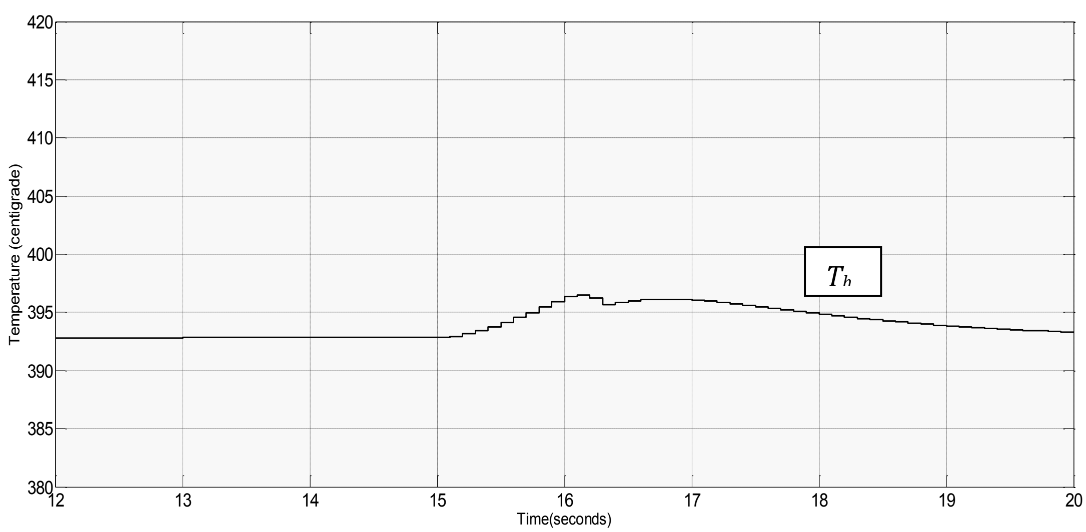

Figure 5.11: Nip-point temperature T_h with constrained MPC

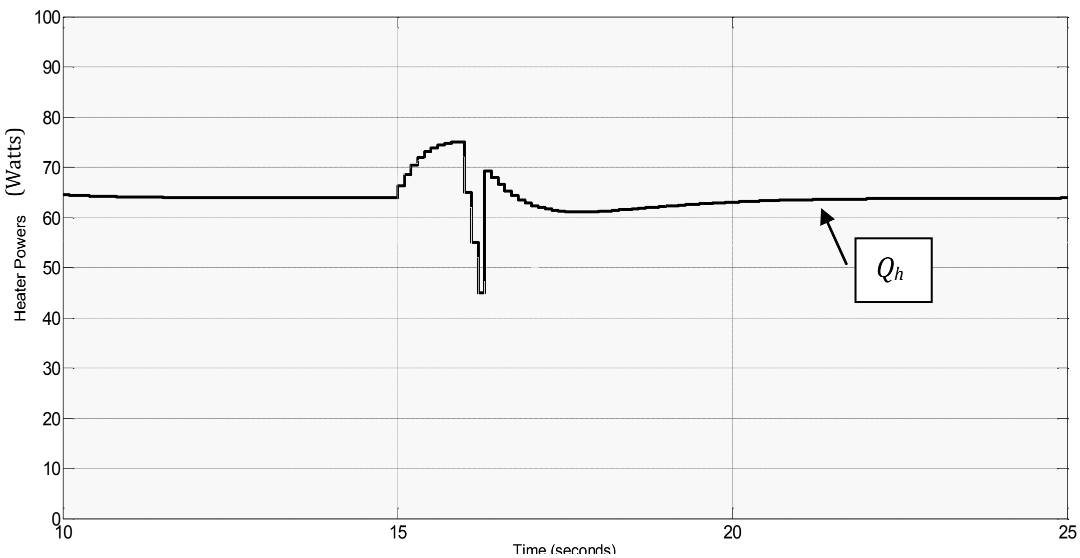

Figure 5.12: Nip-point heater power with constrained MPC

Table 5.1: Nip-point temperature response in LQG and Constrained MPC

LQG	CONSTRAINED MPC
Steady State Error = 0.75%	Steady State Error = 1.8%
Rise Time = 0.03 sec	Rise Time = 4.7 sec
Settling Time = 0.16 sec	Settling Time = 8.7 sec
Overshoot = 2%	Overshoot = NA

Table 5.1 compares the response of the Nip-point temperature T_h in LQG and constrained MPC, and it was found that despite the slight delay due to extra computations of constraints, the constrained MPC design avoids the overshoot in the output by keeping the T_h near to the operational limit of 400˚C and efficiently addresses the input constraint of 80Watts as shown in Figure 5.12.

5.6 CONCLUSION

The chapter provides the basic understanding of the MPC formulation, its applications and the features beneficial for meeting the AFP temperature control objectives. Model Predictive Controller is designed for AFP temperature control application and the results are compared with those of LQG controller deigned in Chapter 4. Besides the similar results for unconstrained application, the constrained MPC was proved more efficient than the conventional LQG controller, especially for AFP application, where the Nip-point temperature and the input power constraints are vital for material, product quality and economical composite manufacturing. Therefore the MPC provides the better design to address the physical and performance constraints of the process compared with LQG controller.

CHAPTER 6
CONCLUSION

6.1 CONTRIBUTION OF THE THESIS

The thesis focuses on the thermal control strategies employed for automated thermoplastic fiber placement process. The study gives an overview of the basic working process of AFP and a literature review on its applications. The fiber placement head carrying different types of heating systems is discussed and a literature review on the thermal modeling and control strategies is provided in Chapter 2. Chapter 3 provides the basic understanding of heat transfer and the critical parameters of AFP process, as the thermal history of the process greatly affects the mechanical properties and final part quality. In Chapters 4 and 5, two advanced control systems are designed by using the dynamic thermal model of the fiber placement process.

The main contributions of the thesis are summarized as follows:

1) Critical Nip-point heating parameters in AFP are identified and a thermal model involving rigid heating system and the thermoplastic composite (PEEK) is used to design a feedback control system for temperature control.

2) LQG-based optimal controller was designed in Chapter 4 for precise temperature control and estimation. The LQG controller showed improved performance however the limitations are found when the system is subjected to disturbances. The LQG controller under disturbances violated the temperature limitations defined for PEEK and the input signals generated are very harsh for the practical application. Although we can control the violations by changing the reference inputs but it will compromise the quality and profit.

3) To address the disturbances and deal with the violations observed in LQG controller, Model Predictive Control (MPC) based controller is designed in Chapter 5. MPC quickly adapts to the defined constraints and takes into account the process limitations through explicit implementation of constraints. It allows the operation of system close to the constraints, which leads to a profitable operation. One of the main objectives of automated fiber placement systems is the cost reduction; therefore, the constraints must be addressed.

6.2 Recommendations for future work

Expensive manufacturing equipment has always limited the role of automated fiber placement system for smaller applications. The study is a step towards developing an affordable AFP technology. In this thesis heating control strategies are proposed, however, with the help of experimentation we can develop a complete model for both compaction and heating. The thermal model used for controller design needs experimental validation. It will be very beneficial for developing a single centralized control unit, based on Model Predictive Control (MPC) to control all the critical parameters i.e. heating, compaction and feed rate. This will make AFP technology economical and affordable.

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