A Performance Modeling and Analysis of Cognitive Radio Networks

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Abstract

A Performance Modeling and Analysis of Cognitive Radio Networks

Rapid growth of wireless networks has increased the demand for radio spectrum, which is a finite resource. However, the studies have shown that the licensed spectrum is not utilized most of the time. Cognitive radio is a technology that promises to bring a solution to this inefficient spectrum utilization. In cognitive radio literature, networks typically consist of two types of wireless users: primary and secondary users. While primary users have higher priority in accessing a band of spectrum, secondary users equipped with cognitive radios exploit the same band of spectrum given that their transmissions do not harm the primary users' transmissions. In this thesis, we develop performance modeling and analysis of cognitive radio networks in three different models. The first network model involves cooperation of secondary user in relaying primary user's packets. We show that the cooperation can benefit both primary and secondary users' transmissions. Secondly, we consider cognitive radio networks with multiple primary users. By having multiple primary users, the spectrum occupancy observed by the secondary transmitter and receiver may not be identical and that affects the secondary user performance. Finally, we consider cognitive radio networks with multi-antenna that enables interference cancellation and allows secondary user to transmit continuously without harmful interference to the primary user. To the best of our knowledge the above models have not been analyzed in the literature.

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List of Symbols

ρ_p	Primary traffic load
ρ_s	Secondary traffic load
$\overline{m}_p$	Mean packet service time of the primary user.
A_i	Number of arrivals during i^{th} cycle period
B_{PU1}	Busy period of PU1
B_{PU2}	Busy period of PU2
B_p	Busy period of a primary user
$B_p(s)$	Laplace transform of the busy period of the primary user
B_{sys}	Busy period of the system
D	Number of cycles of PU1 until reaching the cycle that hosts a system idle period
I_{PU1}	Idle period of PU1
I_{PU2}	Idle period of PU2
I_a	The whole idle period of PU1 or PU2
I_b	The duration from a moment that both PU1 and PU2 become idle until one of them returns active
I_p	Idle period of a primary user
$I_p(s)$	Laplace transform of the idle period of the primary user
I_{sys}	Idle period of the system
L_p	Transmission time of a primary packet

L_s	Packet transmission time of a secondary user
M_p	Service time of a primary packet
$M_p(s)$	Laplace transform of the service time of the primary user
M_s	Continuous packet service time of a secondary user
$M_s(s)$	Laplace transform of the service time of the secondary user
P_D	Probability of Detection
P_F	Probability of False Alarm
Q_p	Cycle period of a primary user
$Q_p(s)$	Laplace transform of the cycle period of the primary user
X_i	Number of packets in a secondary user queue at the end of i^{th} cycle
$X_i(z)$	Probability Generation Function of the number of packets in the secondary user queue at the end of i^{th}cycle period.
Y_i	Number of packets transmitted during the idle period of the i^{th} cycle
$\bar{d}$	Mean packet delay of a secondary user.
$f_{I_{sys}}(t)$	Probability density function of the system idle period
p_1	Probability of PR successfully receives a packet from PT
p_2	Probability of ST overhears a transmission from PT
p_3	Probability of PR successfully receives a packet from ST
p_4	Probability of SR successfully receive a packet from ST
$\bar{x}$	Mean number of packet in the secondary user system
λ	Arrival rate of a primary user
λ_1	Arrival rate of the primary user 1
λ_2	Arrival rate of the primary user 2

λ_{sys}	Arrival rate of the system
H	Probability of having a system idle period in the cycle
N	Number of transmissions until PR receives a packet successfully
$\widetilde{N}$	Number of trails for the successful transmission
T	Constant packet transmission time
V	Probability that the busy period of PU2 is going to end before the PU1 becomes active
W	Probability that PU1 becomes idle when PU2 is already idle
W_s	Packet service time of a secondary user
$\overline{W_s}$	Mean packet service time of a secondary user
r	Probability of successful transmission of a secondary packet during an idle period
α	Arrival rate of a secondary user
β	Overlap duration of primary and secondary user transmissions
σ	Probability that a slot will be idle
ω	The minimum between a continuous packet service time of the secondary user and an idle period of the primary user

Chapter 1

Introduction

The electromagnetic spectrum is considered as a natural resource managed by governmental organizations of each country. Intuitively, the more spectrum is allocated to licensees or services, the less is left for future wireless networks. It is seen that a rapid growth in usage of wireless communications within the current fixed spectrum allocation scheme could lead to the spectrum scarcity problem. However, studies have shown that large amount of allocated spectrum is not utilized in a given time and location [1]. Thus, the idea of allowing other users to make use of the available frequency spectrum, given that they do not cause harmful interference to the licensed network, is raised to deal with the spectrum scarcity problem. This may be achieved by a device known as cognitive radio which is an intelligent wireless module that is capable of sensing, learning and dynamically adjusting its physical parameters according to the radio environment. By having secondary users equipped with cognitive radios not only increases spectrum utilization but also allows more wireless users to be served.

1.1 Terminology and Classification in Cognitive Radio Networks

In cognitive radio literature, two types of wireless networks are considered; primary user and secondary user. The primary user often refers to licensed users that have been allocated a band of spectrum for exclusive use, such as cellular networks, television and radio broadcast etc. In addition, networks operating in unlicensed spectrum e.g., IEEE 802.11 WLAN, are also often considered as primary users. The secondary user is, otherwise, a wireless network equipped with cognitive radios and thus has ability to detect channel usage, analyze the channel information and make a decision whether and how to access the channel.

The first publication on cognitive radio has appeared in [2]. The authors outline the concept of cognitive radio as a smart wireless communication where devices have ability to realize, analyze the radio environment and dynamically adapt their physical transmission parameters of operation accordingly. The development of Software-Defined Radio (SDR) is seen as a possible enabler technology for cognitive radio network. The SDR is a radio technology that allows transmission parameters such as modulation schemes, power control and operating spectrum to be dynamically configurable through software in an embedding device. The development of SDR and cognitive radio have faced many challenges in fulfilling the requirement expectations [28] such as real time spectrum analysis, effective signal detection and guarantee on interference avoidance. These challenges lead to broad areas of research and studies in recent years.

The term Dynamic Spectrum Access (DSA) is often seen in cognitive radio literature and refers to an ability of accessing the spectrum dynamically, which results the spectrum being used more effectively. The DSA is an opposite approach to the current static spectrum management. The DSA can be classified in terms of network types and access strategies as shown in Figure 1.1 [3].

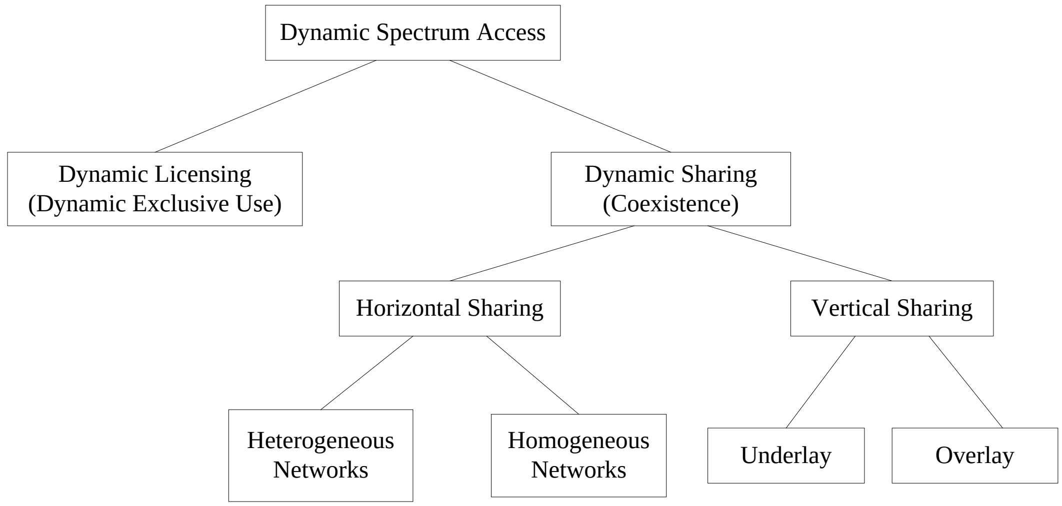

Figure 1.1 Dynamic spectrum access classifications.

The dynamic licensing basically allows licensees to freely sell part of their allocated unutilized spectrum to other network operators. In this approach, it is believed that economy will automatically drive the spectrum usage to its optimum value. Nonetheless, dynamic licensing does not require the use of cognitive radios in the network.

Dynamic sharing occurs when there is more than one network located in the same geographical area and they happen to access the same frequencies at the same time. This situation is also known as coexistence. Coexistence is divided into horizontal sharing and vertical sharing. The horizontal sharing refers to the sharing within networks that have the same regulatory status, i.e., licensed spectrum or unlicensed spectrum. The coexistence of networks operating in unlicensed spectrum, i.e., WLAN 802.11 and Bluetooth 802.15.1 is an example of heterogeneous horizontal sharing, while the coexistence among WLANs is an example of homogenous horizontal sharing.

To deal with coexistence in the horizontal sharing, many schemes have been proposed for different kinds of coexistence. Frequency hopping spread spectrum (FHSS) is used in Bluetooth PHY to cope with coexistence with WLAN 802.11. In FHSS, transmission of data moves between different channels. Thus, the harmful interference with WLAN 802.11 can be avoided. Likewise, the common spectrum coordination channel (CSCC) etiquette protocol is proposed for coexistence of WLAN 802.11 and WIMAX 802.16. This protocol requires both networks to be equipped with cognitive radios.

The vertical sharing assumes the existence of primary and secondary users in which the license of the spectrum has only been given to the primary user but the secondary user can opportunistically access the spectrum without harmful interference to the primary user. There are two approaches for secondary user to make use of the licensed spectrum while keeping the interference under the acceptable level: underlay sharing and overlay sharing. In underlay sharing, the transmission power of a secondary user is limited to just above the noise floor as shown in Figure 1.2. Ultra Wide Band communication (UWB) is an obvious example for this scenario where transmission power of secondary user is low but data is transmitted over wide spread frequencies. In this approach, noise seen by primary user, i.e., WLAN 802.11a is acceptable as it is just above the noise floor. Similarly, UWB transmission may be interfered by 802.11a but for a small portion of frequency, thus the interference is not harmful.

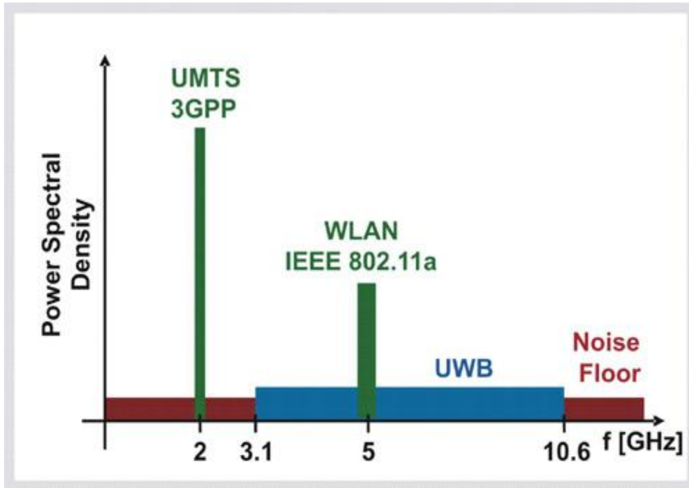

Figure 1.2 UWB as an underlay sharing

In overlay sharing, primary user owns a band of spectrum but is not using all of it at the same time. Secondary user equipped with cognitive radios can seek available spectrum, known as spectrum holes or white space, for their transmissions as shown in Figure 1.3. Once the primary user has returned to the channel, the secondary user vacates the channel immediately and may access another available channel instead, if any. This approach is also called opportunistic spectrum access (OSA).

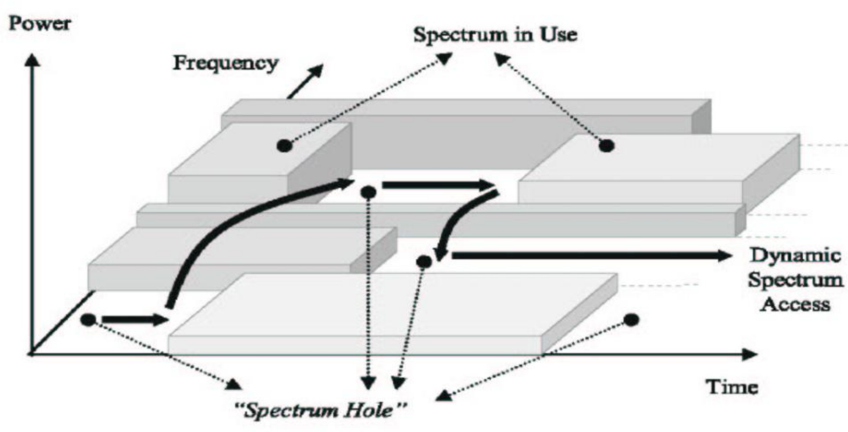

Figure 1.3 Dynamic spectrum access in overlay sharing

While keeping concepts of cognitive radio capability and dynamic spectrum access the same, some recent studies have provided a different classification of dynamic spectrum sharing. The overlay sharing is given different definition and the term interweave paradigm is introduced in [4]. Instead of using the term overlay sharing, the interweave paradigm is introduced to refer to opportunistic communication where the cognitive radio is making use of spectrum hole as we discussed earlier. The overlay sharing in [4] is otherwise described as a network that a secondary user has knowledge of a primary user's transmission and thus able to relay that transmission to the primary receiver. In addition, the knowledge of primary user's transmission can be used for interference cancellation at the secondary receiver. The secondary user is assumed to be capable of the power split in which part of its power is used to assist the primary user transmission while the remainder is used for its own transmission.

1.2 Literature Review

Development of cognitive radio technology involves extensive areas of research from physical to network layer. The studies on physical layer focus on various topics including development of Software-Defined Radio (SDR), modulation scheme based on Orthogonal Frequency-Division Multiple Access (OFDMA), spectrum sensing, channel characteristics and power control etc. In this thesis, we focus on the layer above the physical layer i.e. medium access control (MAC) protocol and other related concerns i.e. cooperative communication, resource allocation and performance evaluation. In this section, we present a survey on the topics of interest.

1.2.1 Spectrum Sensing and Cooperative Sensing

Spectrum sensing is one of the most important components in cognitive radio networks. It enables a secondary user equipped with a cognitive radio to be able to locally identify the presence of the primary user signal and thus access the spectrum properly. There are many spectrum sensing methods proposed in the literature such as energy detection, matched filter and cyclostationary detection.

Cooperative sensing refers to a method of gathering spectrum sensing information from each secondary user before making a final decision on the presence of the primary user based on the collected information. Cooperative sensing is seen as a solution for a local spectrum sensing problem that a node may incorrectly detect the presence of primary signal due to fading, noise or shadowing. The probability of miss-detection can be reduced by employing cooperative sensing. In this section, we give an overview of common spectrum sensing methods as well as cooperative sensing mechanisms.

- **Energy detection:** This is the most common spectrum sensing method in the literature due to its simplicity in implementation and computation [29]. We first refer to the conventional spectrum sensing which measures the received signal strength indicator (RSSI) and uses the following decision metric;

$$y[n] = \begin{cases} hs[n] + w[n], & H_1 : The\ primary\ user\ is\ present \\ w[n], & H_0 : The\ primary\ user\ is\ absent \end{cases}$$

Where $y[n]$ is the received signal by the secondary user, $s[n]$ is the transmitted signal by the primary user, $w[n]$ is the additive white Gaussian noise (AWGN) and h is the channel gain [30].

In energy detection, the received signal, $y[n]$, is squared and integrated over observation interval. Then the primary user is declared present or absent by comparing the calculated result to the threshold (γ) as follows;

$$\text{Accept} \quad H_0,\ if\ \ \sum_{n=1}^{N} |y[n]|^2 \leq \gamma$$

$$\text{Accept} \quad H_1, \quad otherwise$$

The main advantages of energy detection are computational simplicity and that the prior knowledge of primary user signal is not required. However, there exist some drawbacks such as the method not being able to distinguish signals from different type of networks i.e. whether the signal is from primary user or from the coexisting other secondary nodes.

- **Matched filter detection:** This is the optimum spectrum detection method when the primary user's signal, $s[n]$, is perfectly known to the secondary user. The detection mechanism involves correlating a time-reversed version of the known signal with a detected signal and comparing the result with a detection threshold. Thus, the decision is made as follows;

$$\text{Accept} \quad H_0,\ if\ \ \sum_{n=1}^{N} y[n]s[n]^* \leq \gamma$$

$$\text{Accept} \quad H_1, \quad otherwise$$

The advantage of matched filter detection is also computational simplicity and that it requires short time to achieve a certain probability of false alarm.

- **Waveform-Based Detection:** It is a simplified version of matched filter detection in terms of knowledge of primary user's signal required. In this approach, the cognitive device does not need to demodulate the primary user's signal and thus the perfect knowledge of primary user's signal is not needed. Instead, it only requires knowledge of patterns such as pilots, preamble or synchronization words. The decision metric is adopted from matched filter detection. In comparison to energy detection, the waveform-based detection achieves a better performance in terms of reliability and convergence time [29 and references herein]

- **Cyclostationary Detection:** This is another well-known spectrum sensing candidate in cognitive radio literature. A signal is seen to be cyclostationary if its statistics i.e. mean or autocorrelation is a periodic function over a certain period of time. The computational complexity in this approach is the highest among discussed approaches. The calculation starts from determining the Cyclic Autocorrelation Function (CAF) of the observed primary user's signal followed by calculating the Spectral Correlation Function (SCF) which is the discrete Fourier transform of the CAF. The decision on the presence of a primary user's signal is made by finding the peak in SCF plane.

While complex and long observation time is required, the cyclostationary detection has several advantages. Firstly, it can differentiate noise power from signal power as noise has no spectral correlation. Thus, it is also capable of distinguishing signals from different types of primary users. Furthermore, cyclostationary detection is more robust to noise uncertainty and can work with lower SNR compared to the energy detection method.

In the above, we have discussed some of the common spectrum sensing methods in cognitive radio literature. We now introduce important parameters related to spectrum sensing performance and cooperative sensing e.g. probability of detection, P_D and probability of false alarm, P_F. The probability of detection is the probability of accurately deciding the presence of the primary user's signal while the probability of false alarm refers to the probability that the secondary user incorrectly decides that the channel is idle when the primary user is actually transmitting. Let us define D_p as the detected signal power of the primary user and recall that γ is a detection threshold. Thus, P_D and P_F can be expressed as follows;

$$P_D = Pr\left(D_p > \gamma \,\middle|\, H_1\right)$$

$$P_F = Pr\left(D_p > \gamma \,\middle|\, H_0\right)$$

In wireless networks, many factors such as fading, shadowing or hidden terminals can lower the probability of detection. Cooperative sensing is essential in the sense that it allows secondary users to have more precise view of spectrum usage. For example, a secondary node that is located in the shadow area may not be able to detect the signal from a primary user even if the primary user is currently transmitting. As a result, by using only the information the node has detected, it may incorrectly identify the channel as idle. Cooperative sensing allows information exchange between the secondary nodes, the node thus will be able to use its own and others' information to decide if the channel is busy or idle more accurately.

There are several cooperative sensing techniques proposed in the literature for centralized [14], [15], [16] and decentralized networks [17], [18]. It is straight forward for centralized network to deploy cooperative sensing by letting a base station to fuse data from all cognitive devices and make a final decision of overall spectrum occupancy. The scheme is more complicated for decentralized networks as the local sensing data is needed to be exchanged among cognitive devices in the network. The cooperative sensing in decentralized networks has received relatively little attention in the studies.

Study in [14] determines the optimal number of cooperative cognitive users by considering the trade off between the discovery time (larger is the number of cooperative SUs, the shorter is the time taken) and sensing overhead (larger is the number of cooperative SUs, the higher is the overhead). The k-out-of-N data fusion rule is proposed in [15] in which the authors compare the performance with OR and AND rules. In OR rule, the primary user is declared present if at least one SUs detects it. In AND rule, the primary user is declared present only if it is detected by all SUs. The results show that the k-out-of N rule can achieve a substantial performance gain. In [16], it is shown that the optimal performance can be obtained from cooperative sensing done by a certain number of SUs that have the highest primary user's signal to noise ratio.

In decentralized networks, there is no central control point, therefore the cognitive nodes will share information among them and make their own decisions regarding the channels availability. Gossiping approach (GUESS) [17] proposes each cognitive device to sense the spectrum and make its own local decision. The decision is randomly propagated to neighbour nodes and thus converged to the point that all devices have the up-to-date average signal level. Work in [18] investigates the cooperative sensing between a pair of secondary users in which one of them is placed far away and the other one is placed close to the primary transmitter.

1.2.2 MAC Protocols

The development of MAC protocols for cognitive radio is typically based on the well-known CSMA/CA MAC protocol, deployed in WLAN networks, and the extensive studies on multi-channel wireless networks. It is important to give a review of the basic operations in CSMA/CA protocol and MAC protocols proposed for multi-channel wireless networks and cognitive radio networks.

1.2.2.1 IEEE 802.11 CSMA/CA

The MAC protocol in IEEE 802.11 is developed for single-channel network. The IEEE 802.11 MAC protocol is specified in terms of coordination functions. There are two types of coordination function namely, Distributed Coordination Function (DCF) and Point Coordination Function (PCF) [1]. DCF is implemented in Ad-hoc wireless networks and PCF is deployed in infrastructured networks where centralized control station, access point (AP), is in place. The main function of the coordination function is to specify when a station is allowed to transmit.

The DCF is based on the Carrier sensing multiple access with collision avoidance (CSMA/CA) protocol. All stations are required to contend for the channel before transmitting. A handshake (RTS/CTS) is implemented in CSMA/CA to deal with hidden-terminal problem. The basic operation of CSMA/CA is as follows; when a station has a packet to transmit, it senses the channel. If the channel is detected idle, the station will not transmit immediately but wait for a DCF interframe space (DIFS) period. This waiting time allows priority messages e.g. ACK, CTS which has short interframe space (SIFS) to first transmit. After DIFS period, if the channel remains idle, the station picks a random back off time ranged from 0 to 7. This back off time is necessarily needed to avoid collisions from the case that there is more than one station attempting to seize the channel. However, if two or more stations happen to pick the same back off time, collision will occur and the average of the next back off time will be doubled. Once the back off time expires, the winning station starts the transmission by transmitting RTS packet. Other contending stations will suspend their counters. Upon the next contention period, the contending stations resume their counters. In this way, the stations that lose the previous contention could access the channel sooner. The procedure also introduces a degree of fairness in accessing the channel.

The intended receiver responds to RTS packet with CTS packet. The RTS and CTS packets contain the duration field that refers to amount of time the transmission will consume. This allows receiver's neighbour nodes to be aware of the transmission and thus remain quiet for the whole transmission period by setting Network Allocation Vector (NAV) accordingly. Upon receiving CTS, the transmitter sends the DATA packet. The receiver then responds with ACK when the DATA is received successfully. Figure 1.8 shows the timing diagram of a successful data transmission and the contention window (CW) of the contention period [8].

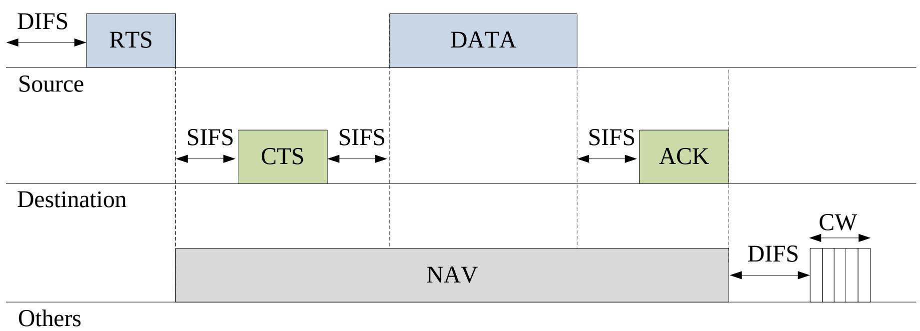

Figure 1.4 Transmission timing diagram in CSMA/CA

1.2.2.2 MAC protocols for Multi-Channel Wireless Networks

The multi-channel wireless networks are studied to improve throughput of the networks as multiple transmissions can take place simultaneously in different channels. The MAC protocol designs in this type of networks mainly rely on availability of a common control channel and a number of transceivers. While a single transceiver approach can keep the hardware as simple as WLAN 802.11, dealing with multiple channels with a single transceiver may complicate the protocol. The multi-channel MAC protocol typically needs to address two issues: channel selection and medium access. Next we describe some well-known multi-channel MAC protocols.

- **Dynamic Channel Assignment MAC (DCA-MAC):** This protocol proposed in [9] requires two transceivers at each station. One transceiver is exclusively used for control channel and other transceiver is dynamically tuned to the selected common channels for data transmission. The protocol is based on CSMA/CA. Each station maintains lists of free channels. A sender and a receiver exchange control messages (RTS/CTS) which carry channel information therefore the pair can decide which channel will be used for data transmission. The protocol does not need synchronization and be able to eliminate the hidden terminal problem as one transceiver always monitors the control channel.

- **Multi-channel MAC (MMAC):** The protocol only requires a single transceiver per station [10]. The protocol tries to eliminate the multi-channel hidden terminal problem without the need of another transceiver and dedicated control channel. The concept of Power Saving Mechanism (PSM) in IEEE 802.11 is used in this protocol. The protocol requires all stations to switch to the predefined default channel, a channel among all available channels, to exchange control messages (ATIM/ATIM-ACK and ATIM-RES). The list of channels is kept and sorted by preference level then included in the control messages. Upon agreeing on the data channel, the station tunes the transceiver to the selected data channel for data transmission. The default control channel can be used for data transmission outside the ATIM window as well. However, it is obvious that during the ATIM window, all stations are required to listen to the control messages therefore no data transmission is possible. This is the major disadvantage of the protocol.

- **Receiver-Based Channel Selection (RBCS)**: This protocol is based on CSMA/CA [11]. While similar to MAC protocol proposed in [9], the paper focuses on the method of selecting the clearest channel for data transmission. The clearest channel is defined as the channel that has the least interference sensed at the receiver. In particular, upon receiving RTS from the sender, the receiver identifies a list of free channels and selects the clearest common channel. The clearest channel is the one with the least received power based on the receiver's sensing. Simulation results in the paper shows the improvement of throughput and delay compared with the single channel IEEE 802.11 model.

1.2.2.3 MAC Protocols for Cognitive Radio Networks

MAC protocols in cognitive radio literature mainly focus on the overlay sharing; secondary user opportunistically transmits through spectrum holes. Thus, the main challenges in cognitive radio network are that the secondary user is permitted to use the channel only when the primary user is not using it. In other words, the channels are not available for secondary users to use all the time, therefore the secondary user has to sense for the idle periods and need to vacate the channels once the primary user has returned. These requirements introduce more complexities in the design of MAC protocols for cognitive radio networks.

- **Statistical Channel Allocation MAC (SCA-MAC):** The protocol in [27] is based on CSMA/CA. The channel negotiation and selection occur in dedicated control channel. The paper focuses on channel selection mechanism which considers statistics of spectrum usage to decide which channel to access. The secondary user will access the channel that has probability of successful transmission higher than a predefined threshold. It is intuitive that the proposed mechanism tends to cause relatively large delay as the sender needs to renegotiate for the channel if none of the channels meet the required success probability. The packet delay performance metric has not been considered by the paper. In addition, the protocol assumes the ability of broadband sensing and occupancy map building which are difficult to achieve in practice.

- **Opportunistic Spectrum Access MAC (OSA-MAC):** The protocol is proposed in [12]. It is a MAC protocol for cognitive radio networks which is based on multiple-channel MAC protocol in [10]. Additional mechanisms are added to deal with issues of cognitive radio networks. In particular, sensing phase is added into the beacon interval in order to identify the presence of a primary user. Also, the channel selection mechanism is considered. However, the protocol also inherits the drawback from [10] that is bandwidth wastage when all stations are required to switch to the default control channel for exchange of control messages. Also, the mechanism when secondary user needs to vacate the channel following return of the primary user is not presented.

- **Heterogeneous distributed MAC (HD-MAC)**: In [13], the authors develop the mechanism that allows secondary nodes to communicate in a distributed manner without the need of a dedicated control channel. Simulations have been performed to determine the probability of common channel availability and the number of commonly available channels. The results show that the neighbouring nodes have very similar spectrum availability and thus the availability of a common channel among neighbouring nodes can be assumed. With this result, they propose a group-based coordination scheme where a common channel among members in the group is assumed. The connectivity to other groups can be done by “bridge node” who shares common channels with other groups. The paper includes network setup algorithm, coordination channel selection and implementation in both legacy MAC and new proposed heterogeneous distributed MAC (HD-MAC) protocol. The HD-MAC is based on MMAC [10] with some modifications in coordination window structure to allow bridge nodes to access multiple groups, queue structure to avoid head of line blocking and data channel selection mechanism.

- **Cognitive MAC (C-MAC):** This protocol in [32] introduces the use of superframe structure in multi-channel networks where each channel maintains its own structure. Each cognitive device registers to a channel and follows its schedule. A superframe consists of Beacon Period (BP) and Data Transfer Period (DTP) in which the BP is designed to be non-overlapping allowing a node to quickly gather other channels’ information by switching channels in ascending order. The rendezvous channel is designed to facilitate the network-wide group communication, inter-channel synchronization, neighbourhood discovery and load balancing. In addition, the paper includes coexistence scenarios and adopts the Incumbent Detection Recovery Protocol (IDRP) [7] proposed for IEEE 802.22 as a method to deal with channel switching when the primary user is detected. In comparison with MMAC [10], the C-MAC also requires only single transceiver but can overcome the bandwidth wastage as a node is not required to switch to the default channel for control message exchange. However, in C-MAC a strict synchronization between channels is needed.

- **Model-Based MAC:** In [33], the paper considers the single channel IEEE 802.11 WLAN as a primary user where a dedicated control channel is assumed. The paper models the channel as a 2-state system (busy and idle). Each secondary user gathers the channel states and constructs the channel occupancy model. Hyperexponential distribution (HED) is used to model both idle and busy periods of the primary user. The model-based protocol takes a residual idle period of the primary user into account to decide the secondary user transmissions. The residual idle period is computed at both secondary transmitter and receiver and the minimum between the two is considered. If the residual idle period is higher than a predefined threshold, the secondary transmitter can transmit as many frames that it has in its queue or as many as the residual idle period can accommodate. The negotiation occurs on a dedicated control channel where the traditional three-way handshake is adopted.

1.2.3 Performance Evaluation of Cognitive Radio Networks

In [19]-[23], performance modeling of cognitive radio networks has been studied. In all these papers, the overlay sharing model where secondary users coexist with primary users and opportunistically access the unutilized spectrums have been analyzed. Primary users own a band of channels, i.e., N channels. When a secondary user detects the presence of an arriving primary packet on its current channel, it vacates that channel and moves its transmission to another available channel, if any. If all channels are busy, the secondary packet remains in a queue which might reside at secondary access point or at the node itself. The packets in the queue are served on First-Come, First-Served (FCFS) basis. The waiting secondary packet is lost when either the maximum waiting time in the queue or residence time in the service area is reached. For simplicity, it is assumed that each primary or secondary user occupies a single channel. This system model can be applied in both infrastructured and infrastructureless networks.

The above papers differ from each other in terms of system modeling and assumptions. In [19], the arrivals of primary and secondary packets are assumed to be according to Poisson processes. The channel holding time is the minimum of call holding and residence time, the time that a user resides in a service area. All the time periods mentioned are assumed to be exponentially distributed. The system model in [20] is similar to that in [19] except for the consideration of finite number of secondary users in an ad hoc wireless network in military environment. The analysis considers with two cases; the number of secondary users being less (and more) than the number of channels. The Internet traffic is often found to be bursty and correlated and the modeling of arrivals as a Poisson process not suitable. Such traffic may be better modeled as a Markovian Arrival Process (MAP). The work in [21] models the system with MAP traffic while the channel holding time is kept exponentially distributed. The traditional exponential distribution is also found inappropriate to model the service times in cellular networks [24] or Internet traffic [25]. The model in [22] uses a Markovian Arrival Process and Phase-Type (PH) distribution as the arrival process and the channel holding time, respectively. In [23], imperfect sensing is taken into account through the probabilities of false alarm and misdetection. However, arrival process and channel holding times are assumed to be Poisson and exponentially distributed, respectively.

In the above works, the systems are analyzed using multi-dimensional Markov processes. The studies determine several performance metrics of cognitive radio networks such as blocking probability of primary and secondary users, reconnection probability, channel utilization and mean number of dropped secondary packets. The blocking probability of primary user is defined as the probability that when a primary packet arrives, all the channels are occupied by other primary users while the blocking probability of secondary user is referred to the probability that all the channels are occupied by either primary or secondary users and no channel is available for a new arriving secondary packet. As expected, the blocking probability of secondary user is higher than that of the primary user under the same parameter settings. Reconnection probability is the probability that a secondary packet that waits in the queue, due to unavailability of a channel, reconnects to the system when the channel becomes available before the maximum waiting time is reached. The reconnection probability decreases as the arrival rate of the primary user increases.

In [26] the primary user is modeled as an M/G/1 queue with a general service time distribution and the secondary user queue is assumed to be saturated. The analysis is the case of a single channel with one primary and secondary user. However, the paper also provides performance results of multiple secondary users by simulation. The paper considers limiting collision probability and overlaps time between primary and secondary user in order to protect the primary user and determines throughput of the secondary user.

In [34] both primary and secondary users are modeled as M/D/1 queues with slotted time axis and error free channel. The packet transmission times are one slot and the secondary user is assumed to have perfect time synchronization and perfect sensing. The mean waiting times in the queues for both primary and secondary user have been derived.

1.3 The Standard on Cognitive Radio Networks, IEEE 802.22

The cognitive radio concept has received attention from the IEEE organization leading to development of IEEE 802.22, the first worldwide standard on cognitive radios. The IEEE 802.22, started in 2004, focuses on PHY and MAC layers of Wireless Regional Area Network (WRAN) using TV frequency bands. WRAN provides wireless broadband networking in rural areas where cable networks are not feasible. The availability of a cognitive radio standard is an important step of the technology. Here we will give an overview of the draft standard IEEE 802.22.

The TV frequency bands in consideration ranges from 54 MHz to 862 MHz, which is in VHF/UHF TV bands. There are two wireless services operating in this band; TV broadcasting and wireless microphone. The advantages of using the TV bands providing wireless network to rural areas is obvious as the signal propagation characteristic in this band is much longer compared to WLAN or WiMAX operating in above 2GHz band. This allows large coverage for a single base station to serve sparse population in the rural area. The summary of system parameters is shown in Table 1.1 [5].

Table 1.1 WRAN parameters

Parameters	**Specification**
Frequency range	54 ~ 862 MHz
Bandwidth of each channel	6, 7 and/or 8 MHz
Spectral Efficiency	0.25~3.78 b/s/Hz
Data rate	1.51 ~ 22.69 Mb/s
Payload modulation	QPSK, 16-QAM, 64-QAM
Transmit EIRP	Default 4W for CPEs
Service Coverage	33 Km at 4W CPE EIRP
FFT Mode	2048
Cyclic Prefix Modes	¼, 1/8, 1/16, 1/32
Duplex	TDD
FEC codes	LDPC, Turbo Code, and STBC

The WRAN deployment scenario is shown in Figure 1.5 [6]. It is a centralized architecture consisting of a WRAN base station (BS) serving multiple fixed-location wireless Customer Premises Equipments (CPEs). The WRAN system follows a master/slave relationship where WRAN base station is a master who controls all transmissions. CPEs transmit or sense the spectrum only if they are demanded from a WRAN base station or according to their schedules. The CPEs are supposed to be attached at a fixed outdoor location i.e. users' houses.

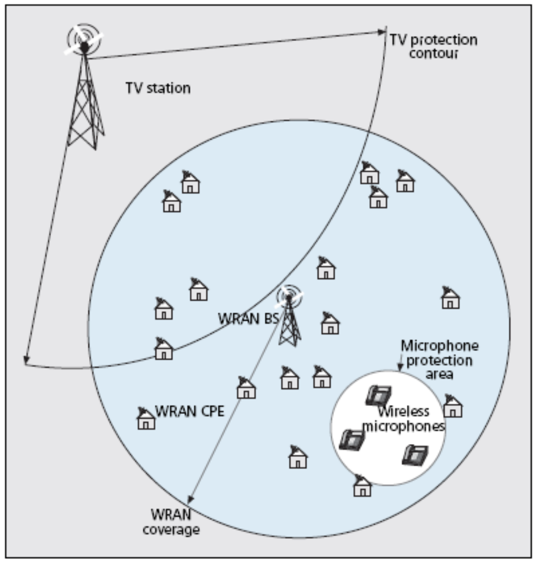

Figure 1.5 WRAN deployment scenario

The WRANs can be seen as secondary users operating in primary TV bands, it is necessary for WRANs to ensure that the interference within the protection contour (around 150 Km from TV station) is below a certain level. To achieve this, the incumbent protection parameters are set as indicated in Table 1.2 [6]

Table 1.2 Incumbent protection parameters

Parameters	**Wireless Microphones**	**TV Broadcasting**
Incumbent Detection Threshold (IDT)	-107 dBm (over 200 KHz)	-116 dBm (over 6 MHz)
Channel Detection Time (CDT)	≤ 2 sec	≤ 2 sec
Channel Move Time (CMT)	2 sec	2 sec
Channel Closing Transmission Time (CCTT)	100 msec	100 msec
Detection Probability	~ 90%	~ 90%
False Alarm Probability	~ 10%	~ 10%

In order to ensure limited interference to the primary users, WRANs shall be able to detect the incumbent signal that is above IDT with probability of detection greater than 90%. The CDT is the time that incumbent can withstand the interference, thus WRANs must detect the presence of incumbent at most within CDT. If the incumbent signal is measured above IDT in the operating channel, WRANs must vacate the channel within the CMT. The CCTT is the sum duration of WRANs transmissions during the CMT.

The 802.22 PHY is based on OFDMA for multiple access and QPSK, 16-QAM and 64-QAM modulation schemes as summarized in Table 1.1. There are two main spectrum sensing techniques adopted for the system; energy detection and feature detection. While the energy detection is only able to detect the presence of incumbent, the feature detection is able to also provide information on the type of incumbents whether it is TV broadcasting or wireless microphone.

The 802.22 MAC has followed the design of 802.16 WiMAX network but it also includes cognitive radio capabilities such as frequency management and self-coexistence management. The communication from the base station to a CPE, downstream, is based on Time Division Multiplexing (TDM) and the communication from a CPE to base station, upstream, has followed Demand Assigned TDMA [7].

In WRAN, time is divided into superframes as depicted in Figure 1.6. The structure of superframe is designed to facilitate the incumbent protection, synchronization and self-coexistence. A superframe consists of 16 frames, each of 10 msec duration. Each superframe starts with superframe preamble followed by frame preamble and Superframe

Control Header (SCH). The SCH contains control information such as the available channel, quiet period schedules and bandwidth for a CPE etc. The BS sends this information in SCH through a set of available channels. The CPE which is tuned into one of these channels is able to associate with the BS.

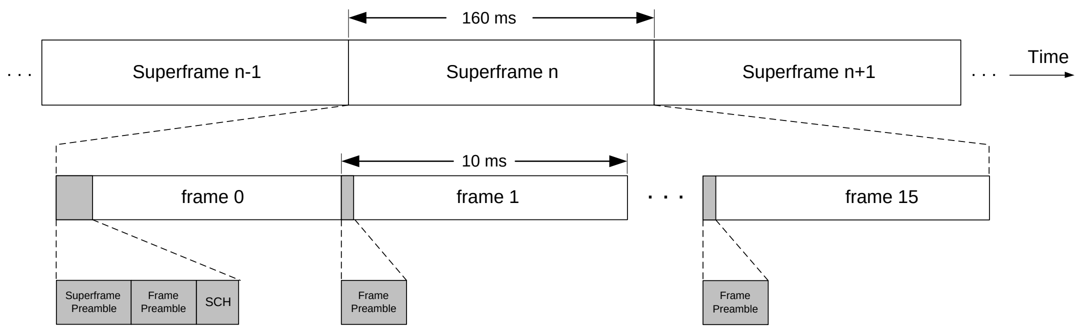

Figure 1.6 IEEE 802.22 Superframe structure

Figure 1.7 shows the detailed structure of each frame which is divided into 2 parts; downstream (DS) and upstream (US) subframe. The DS subframe is purely the DS PHY PDU while the US subframe consists of a ranging slot, bandwidth request slot, UCS notification slot followed by US PHY PDU of each CPE and SCW slot. The new MAC functions introduced specifically in 802.22 are Urgent Coexistence Situation (UCS) slot and Self-Coexistence Window (SCW) slot. As the names imply, the UCS is used by the CPEs to inform the base station about the presence of incumbent which is recently detected. Also, to deal with self-coexistence, the event when there are more than one WRAN cells located in the same geological area and operated in the overlapping bands, the Coexistence Beacon Protocol (CBP) is transmitted in the SCW slots to manage the coexistence.

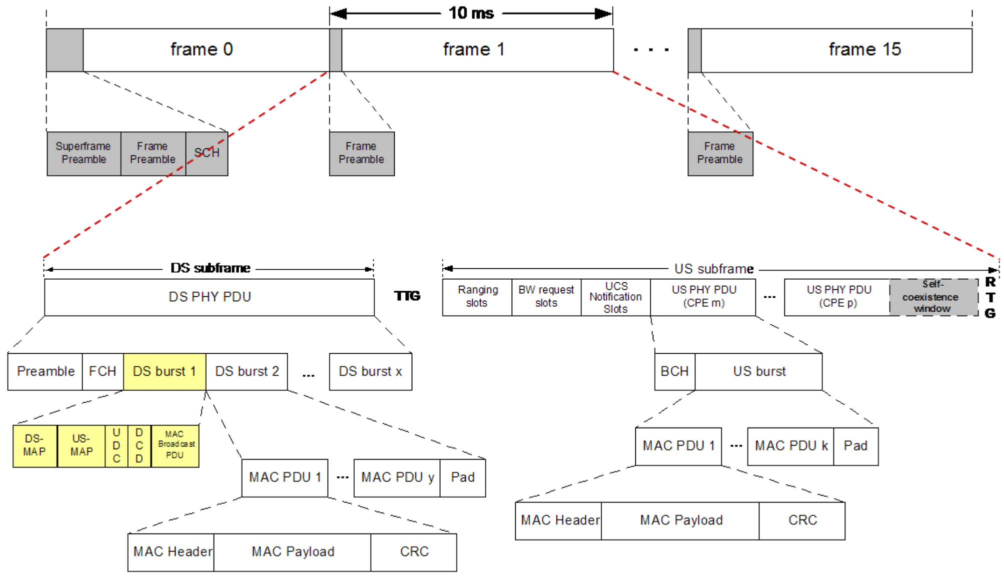

Figure 1.7 Frame structure

The overview mechanism of the 802.22 MAC operations is as follows. Base station (BS) schedules or demands CPEs to sense for available spectrum. The sensing mechanism adopted in 802.22 is called two-stage sensing which consists of fast sensing (1 msec long) and fine sensing (30 msec long) as shown in Figure 1.8. Fine sensing is performed only if the information from fast sensing is not enough to determine the presence of the incumbent. The reason for the need of two-stage sensing comes from the tradeoff between the probability of detection and throughput of WRAN system. In particular, in order to achieve high probability of detection, all WRAN transmissions in the adjacent channels need to be stopped when the sensing is performed. The stoppage of WRAN transmissions is called a quiet period. Consequently, if sensing takes a long time, the quiet period will also be long and this will affect the throughput of the WRAN. The sensing information from CPEs is reported back to the BS by the allocated US slot i.e., UCS notification slot or the measurement report messages.

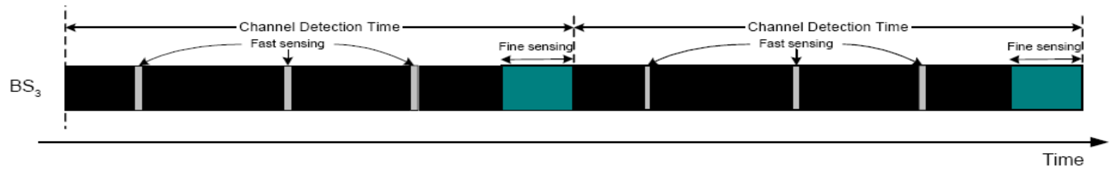

Figure 1.8 Two-stage sensing

Once the incumbent has been detected on the operating or adjacent channels, the BS demands CPEs to switch the operation to other available channels in the backup list. This channel switching should be finished within the Channel Move Time (CMT). All CPEs keep the backup channels in a priority list which is updated periodically by the BS. As a result, the communication recovery can be made very efficient as the CPEs and BS would know in advance the channel to switch into.

1.4 Major Contributions of the Thesis

In this thesis, we contribute to the performance modeling and analysis of cognitive radio networks through study of three different models.

1) In the first model, we study and analyze performance modeling of cooperative cognitive radio networks in which a secondary transmitter assists primary user's transmissions by relaying the primary user's packets to the intended receiver while transmits its own packets over spectrum holes. We also take into account the transmission errors due to wireless channel. Primary and secondary users are modeled as M/G/1 queues. We obtain Laplace transform of idle and busy periods of the primary user and the Probability Generating Function (PGF) of the number

of packets in the secondary user queue. We show that as a result of cooperation both primary and secondary users obtain capacity gain. We note that the results in [34] may be considered as a special case of this scenario since in [34] there is no secondary assistance to primary transmission and channel is assumed to be error free.

2) In the second model, we analyze cognitive radio networks where a secondary user may interfere with multiple primary users. The presence of multiple primary users degrades the secondary user performance as it decreases spectrum opportunities. We consider the scenario that secondary transmitter and receiver operate within the range of different primary users. We model primary and secondary users as M/G/1 queues. We determine Laplace transform of the system idle and busy periods and obtain the PGF of the number of packets in the secondary user queue. We show that the mean packet delay of the secondary user in the double primary users case is larger than that in the single primary user scenario.

3) In the third model, we study the use of multiple antenna in cognitive radio networks and analyze the network that employs the multiple antenna for interference cancellation. With interference cancellation, the secondary user may transmit continuously without harmful interference to the primary user. We determine the mean transmission power of the secondary user and then, obtain the PGF of the number of packets in the secondary user queue. We compare the mean packet delay of the secondary user with multiple antenna against to the case of

single antenna. The results show that multiple antenna model gives better performance over single antennna case.

To the best of our knowledge the above three models have not been analyzed before in the cognitive radio literature.

1.5 Thesis Outline

The outline of the remainder of the thesis is as follows,

Chapter 2: Performance Evaluation of Cooperative Cognitive Radio Networks. This chapter presents cooperative communication model and show the benefit of the cooperation.

Chapter 3: Performance Evaluation of Cognitive Radio Networks with Multiple Primary Users. This chapter presents the model of a secondary user within activity regions of multiple primary users. We determine the mean packet delay of the secondary user and compare the results with that in the single primary user scenario.

Chapter 4: Performance Evaluation of Cognitive Radio Networks with Multi-Antenna. We determine the mean transmission power of the secondary user and derive the mean packet delay of the secondary user.

Chapter 5: Conclusions. This chapter gives conclusion of the thesis.

Chapter 2

A Performance Evaluation of Cooperative Cognitive Radio Networks

There is a concern in the community that there may not be incentive for primary users to share spectrum with secondary users. Among different scenarios cooperative cognitive radio networks provide such an incentive. In this scenario, secondary users help in the transmission of primary user packets and that improves both primary and secondary users' performance.

We anticipate that the cooperation will increase the throughput of the primary user, therefore reducing the amount of needed channel access time. Consequently, this will result in higher channel availability for secondary user. We verify our hypothesis by modeling primary and secondary users as M/G/1 queues and determining closed-form expression of the mean packet delay of a secondary user.

2.1 System Model

We consider a system with a single primary and secondary users operating in a single channel. This configuration is shown in Figure 2.1 where PT, PR, ST and SR represent primary transmitter, primary receiver, secondary transmitter and secondary receiver respectively. Probabilities of PR successfully receiving a packet from PT and ST are denoted by p_1 and p_3 respectively. We assume that ST overhears the primary user's transmission with probability p_2. Lastly, the probability of SR successfully receiving a packet from ST is denoted by p_4.

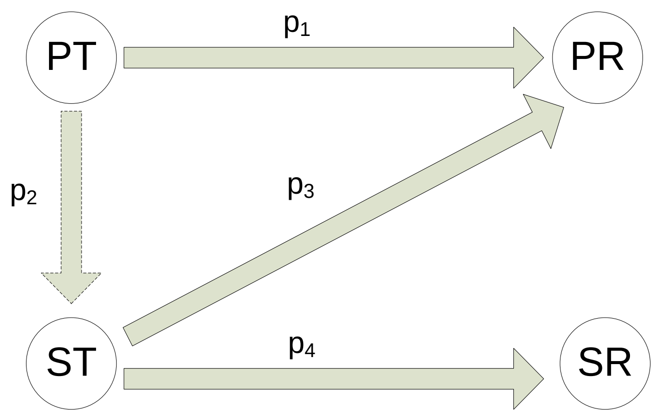

Figure 2.1 Network architecture

We assume that when PT is transmitting, ST solely acts as a relay helping the PT's transmission. When PT has a packet to transmit, it first transmits without any help from ST. If PR fails to receive the packet, PT retransmits the packet. ST may overhear the first transmission or following retransmissions, if any, of PT with probability p_2. Once ST overhears the packet, it will participate in subsequent retransmissions of that packet until the packet is received successfully by PR. ST will transmit its own packets only when PT is idle.

The primary and secondary users are modeled as M/G/1 queues where the arrival processes are Poisson and service times are generally distributed. The primary user keeps transmitting packets until it has an empty queue. Similarly, the secondary user transmits its own packets, over spectrum holes, as long as its queue is nonempty. At both primary and secondary users, we assume that a packet may be transmitted successfully or lost in each transmission due to wireless channel errors according to an independent Bernoulli process. The transmission of a packet will be repeated until it is successfully received. We assume that the service time of a packet begins with the beginning of its first transmission and ends with the completion of the successful transmission.

2.2 Modeling of the Primary User

In this section, we analyze the primary user's queue and its channel usage. The primary user has priority in accessing the channel compared to the secondary user. The channel usage seen by the secondary user alternates between busy and idle periods. During a busy period, the primary user is continuously involved in transmission of packets. A busy period terminates when the primary queue becomes empty and the next idle period begins. An idle period terminates with the arrival of a new packet to the primary user. In the following sub-section, we analyze the idle and busy periods of the primary user.

2.2.1 Idle Period of the Primary User

In an M/G/1 queue, the arrivals of packets are according to Poisson process, thus the idle period of the queue, which is equivalent to packet interarrival times, is exponentially distributed. We assume that the arrival rate is given by λ packets/sec therefore the idle period of the primary user is exponentially distributed with the same parameter. Let I_p denotes an idle period of the primary user and $I_p(s)$ represents its Laplace transform, then,

$$I_p(s) = E[e^{-sI_p}] = \int_0^\infty e^{-st} \lambda e^{-\lambda t} dt$$

$$I_p(s) = \frac{\lambda}{s+\lambda} \tag{2.1}$$

2.2.2 Busy Period of the Primary User

In our model, the primary user keeps transmitting its packets until it has an empty queue. A busy period of the primary user depends on the arrival process and the packet service time. We note that a packet may need to be transmitted multiple times because of transmission errors until it is received successfully. Therefore, the packet service time is given by the product of packet transmission time and the number of transmissions of the packet. Probability of successful transmission increases after the secondary user also joins the transmissions. We first derive the distribution of the number of transmissions until PR successfully receives the packet. Let us define the following notation,

N	$\triangleq$	number of transmissions until PR receives a packet successfully
L_p	$\triangleq$	transmission time of a primary packet
M_p	$\triangleq$	service time of a primary packet
B_p	$\triangleq$	busy period of a primary user
I_p	$\triangleq$	idle period of a primary user

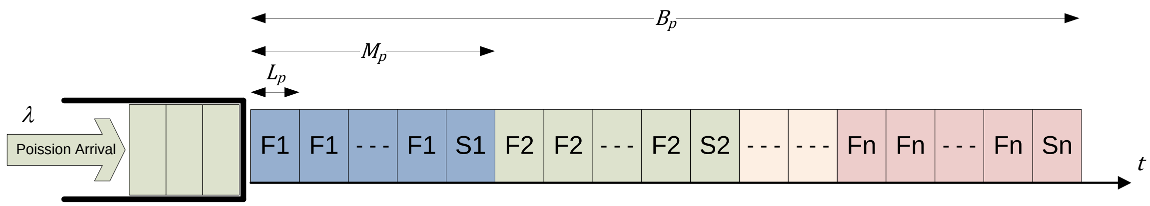

Figure 2.2 Transmissions and retransmissions of primary user's packets

Figure 2.2 illustrates transmissions and retransmissions of a primary user's packet in a channel over time and relation of a packet transmission time, packet service time and busy period of the primary user. Fi and Si, $i = 1,2,\ldots,n$ in the figure denote failure and success of i^{th} packet transmission, respectively. If PR fails to receive a packet correctly, PT retransmits the packet. During retransmissions of the packet, ST may assist PT by simultaneously relaying the packet with PT. PT continues with transmission of its packets until it has an empty queue. Now, let us determine the distribution of the number of transmissions until PR receives a packet successfully using the probability tree as depicted in Figure 2.3. The numbers in circles represent a successful transmission in the n^{th} transmission. Thus each circle is at the end of a path in the tree that ends in a successful transmission. There may be mutually exclusive multiple paths that may result in success in n^{th} transmission.

In Figure 2.3, let $q_j = 1 - p_j,\ j = 1, 2, 3, 4$. Consider when $Pr(N = 1), Pr(N = 2)$ and so on, we have

$$Pr(N = 1) = p_1$$

$$Pr(N = 2) = q_1 q_2 p_1 + q_1(1 - q_1 q_3)p_2$$

$$Pr(N=3) = q_1{}^2 q_2{}^2 p_1 + (q_1{}^2 q_3 + q_1{}^2 q_2)(1 - q_1 q_3) p_2$$

$$Pr(N=4) = q_1{}^3 q_2{}^3 p_1 + (q_1{}^3 q_3{}^2 + q_1{}^3 q_2 q_3 + q_1{}^3 q_2{}^2)(1 - q_1 q_3) p_2$$

$$Pr(N=5) = q_1{}^4 q_2{}^4 p_1 + (q_1{}^4 q_3{}^3 + q_1{}^4 q_2 q_3{}^2 + q_1{}^4 q_2{}^2 q_3 + q_1{}^4 q_2{}^3)(1 - q_1 q_3) p_2$$

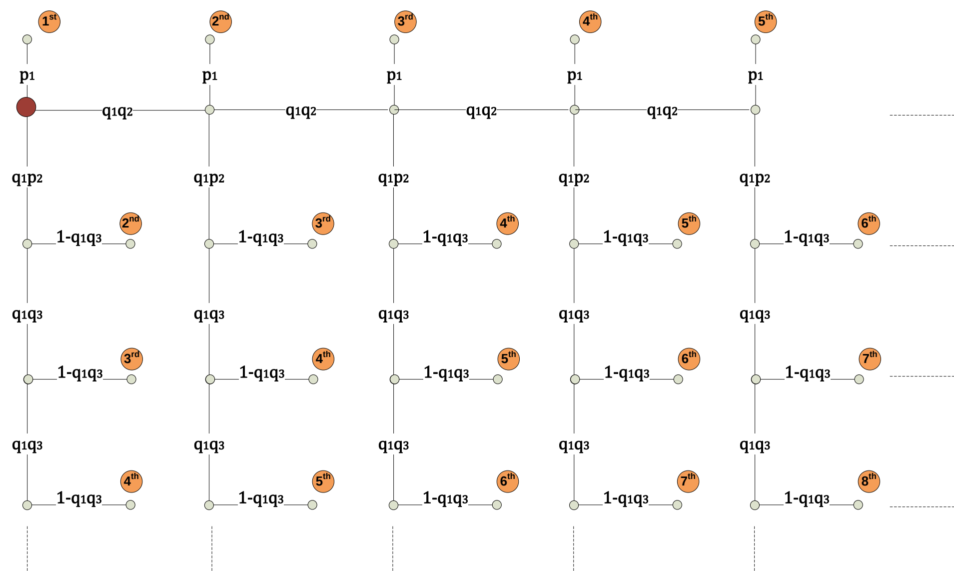

Figure 2.3 Probability tree of a number of transmissions until a packet is successfully received by PR

Thus, we can determine the distribution of N as,

$$Pr(N=n) = (q_1 q_2)^{n-1} p_1 + U(n)(1 - q_1 q_3)\left(q_1^{n-1} \sum_{i+j=n-2} \sum q_2^i q_3^j\right) p_2 \qquad (2.2)$$

where, $U(n) = \begin{cases} 0, & n = 1 \\ 1, & n > 1 \end{cases}$

During retransmissions, there are two mutually exclusive events; only primary user is transmitting and the other both primary and secondary users are transmitting. In (2.2), the first term corresponds to the success probability on the n^{th} trial when only the primary user is transmitting and the second term when both primary and secondary users are transmitting. In the second term, secondary user receives primary's transmission successfully on the $(i+1)^{st}$ transmission. During the next j transmissions, secondary user also transmits primary user packet but unsuccessfully. Finally, in the n^{th} transmission, primary's or secondary's transmission succeeds.

Let us now determine the Laplace transform of the packet service time of the primary user, $M_p(s)$, by conditioning on the packet transmission time l_p and the number of transmissions n. Let $f_{L_p}(t)$ denote the probability density function (pdf) of the packet transmission time and $L_p(s)$ its Laplace transform.

$$M_p(s) = E[e^{-sM_p}]$$

$$M_p(s|N=n) = \int_0^\infty M_p(s|L_p = l_p, N = n)\, f_{L_p}(l_p)dl_p$$

$$= \int_0^\infty e^{-snl_p} f_{L_p}(l_p)dl_p$$

By definition, the Laplace transform of a packet transmission time, $L_p(s) = E[e^{-sL_p}] = \int_0^\infty e^{-sl_p} f_{L_p}(l_p)dl_p$. Comparing the definition with $M_p(s|n)$ expression above, we get

$$M_p(s|N=n) = L_p(ns)$$

Next, $M_p(s|N=n)$ is unconditioned with respect to n to obtain $M_p(s)$,

$$M_p(s) = \sum_{n=1}^{\infty} M_p(s|N = n)Pr(N = n)$$

$$= \sum_{n=1}^{\infty} L_p(ns)Pr(N = n) \qquad (2.3)$$

where $Pr(N = n)$ is obtained from (2.2). Substituting (2.2) into (2.3), we get the following expression for $M_p(s)$.

$$M_p(s) = \sum_{n=1}^{\infty} L_p(ns)\left[(q_1q_2)^{n-1}p_1 + U(n)(1 - q_1q_3)\left(q_1^{n-1}\sum_{i+j=n-2}\sum q_2^i q_3^j\right)p_2\right] \qquad (2.4)$$

where, $U(n) = \begin{cases} 0, & n = 1 \\ 1, & n > 1 \end{cases}$

Finally, the Laplace transform of the busy period, $B_p(s)$, is given by that of M/G/1 queue. From [35] the relation between busy period and packet service time is as follows,

$$B_p(s) = M_p\left[s + \lambda - \lambda B_p(s)\right] \qquad (2.5)$$

Again from [35], the mean busy period and its second moment are given by,

$$\bar{B}_p = \frac{\bar{m}_p}{1 - \rho_p}, \qquad \rho_p = \lambda \bar{m}_p \qquad (2.6a)$$

$$\overline{B_p^2} = B_p''(0) = \frac{\overline{m_p^2}}{(1-\rho_p)^3} \qquad (2.6b)$$

where $\overline{m}_p$ and $\overline{m_p^2}$ refer to mean service time and the second moment of the service time of a packet respectively. Denote ρ_p as traffic load of the primary user.

2.2.3 Case Studies for Different Service Time Distributions

In the above analysis, we obtained Laplace transform of the busy period of the primary user, $B_p(s)$, as a function of Laplace transform of the packet service time, $M_p(s)$, in (2.5). In an M/G/1 queue, the service time distribution is general thus in this section, we present two case studies where the packet transmission time is constant and exponentially distributed.

2.2.3.1 Constant packet transmission times, $L_p = T$

In this subsection, we determine the Laplace transform of packet service time for constant packet service times. We first obtain $L_p(ns)$,

$$L_p(ns) = E[e^{-nsT}] = e^{-nsT}$$

Substituting the above $L_p(ns)$ in (2.4) then we obtain $M_p(s)$ as follows,

$$M_p(s) = \sum_{n=1}^{\infty} e^{-nsT} \left[(q_1 q_2)^{n-1} p_1 + U(n)(1 - q_1 q_3) \left(q_1^{n-1} \sum_{i+j=n-2} \sum q_2^i q_3^j \right) p_2 \right]$$

$$= \frac{p_1}{q_1 q_2} \sum_{n=1}^{\infty} (q_1 q_2 e^{-sT})^n + (1 - q_1 q_3) p_2 \sum_{n=1}^{\infty} U(n) e^{-nsT} \left(q_1^{n-1} \sum_{i+j=n-2} \sum q_2^i q_3^j \right)$$

Recall that $U(n) = \begin{cases} 0, & n = 1 \\ 1, & n > 1 \end{cases}$. Thus, the summation in the latter term can be evaluated from $n = 2$. Therefore,

$$M_p(s) = \frac{p_1 e^{-sT}}{1 - q_1 q_2 e^{-sT}} + \frac{(1 - q_1 q_3) p_2}{q_1} \sum_{n=2}^{\infty} \left((q_1 e^{-sT})^n \sum_{i+j=n-2} \sum q_2^i q_3^j \right)$$

We can reduce the double summation to a single summation by substituting $j = n - 2 - i$ and evaluating it from $i = 0$ to $n - 2$.

$$M_p(s) = \frac{p_1 e^{-sT}}{1 - q_1 q_2 e^{-sT}} + \frac{(1 - q_1 q_3) p_2}{q_1} \sum_{n=2}^{\infty} \left((q_1 e^{-sT})^n \sum_{i=0}^{n-2} q_2^i \, q_3^{n-2-i} \right)$$

$$= \frac{p_1 e^{-sT}}{1 - q_1 q_2 e^{-sT}} + \frac{(1 - q_1 q_3) p_2}{q_1 {q_3}^2} \sum_{n=2}^{\infty} \left((q_1 q_3 e^{-sT})^n \sum_{i=0}^{n-2} \left(\frac{q_2}{q_3} \right)^i \right)$$

We can obtain the closed forms of the above summations from the following result;

$$\sum_{i=a}^{b} x^i = \frac{x^a - x^{b+1}}{1 - x} \quad and \quad \sum_{i=2}^{\infty} x^i = \frac{x^2}{1 - x}$$

Thus,

$$M_p(s) = \frac{p_1 e^{-sT}}{1 - q_1 q_2 e^{-sT}} + \frac{(1 - q_1 q_3) p_2}{q_1 {q_3}^2} \sum_{n=2}^{\infty} (q_1 q_3 e^{-sT})^n \left(\frac{1 - \left(\frac{q_2}{q_3} \right)^{n-1}}{1 - \left(\frac{q_2}{q_3} \right)} \right)$$

$$= \frac{p_1 e^{-sT}}{1 - q_1 q_2 e^{-sT}} + \frac{(1 - q_1 q_3) p_2}{q_1 {q_3}^2 \left(1 - \left(\frac{q_2}{q_3}\right)\right)} \sum_{n=2}^{\infty} (q_1 q_3 e^{-sT})^n \left(1 - \left(\frac{q_2}{q_3}\right)^{n-1}\right)$$

$$= \frac{p_1 e^{-sT}}{1 - q_1 q_2 e^{-sT}} + \frac{(1 - q_1 q_3) p_2 q_3}{q_1 {q_3}^2 (q_3 - q_2)} \left(\sum_{n=2}^{\infty} (q_1 q_3 e^{-sT})^n - \frac{q_3}{q_2} \sum_{n=2}^{\infty} \left(\frac{q_1 q_2 q_3 e^{-sT}}{q_3}\right)^n\right)$$

$$= \frac{p_1 e^{-sT}}{1 - q_1 q_2 e^{-sT}} + \frac{(1 - q_1 q_3) p_2 q_3}{q_1 {q_3}^2 (q_3 - q_2)} \left(\frac{(q_1 q_3 e^{-sT})^2}{1 - q_1 q_3 e^{-sT}} - \frac{q_3 (q_1 q_2 e^{-sT})^2}{q_2 (1 - q_1 q_2 e^{-sT})}\right)$$

$$= \frac{p_1 e^{-sT}}{1 - q_1 q_2 e^{-sT}} + \frac{(1 - q_1 q_3) p_2 {q_1}^2 {q_3}^2 e^{-2sT}}{q_1 {q_3}^2 (q_3 - q_2)} \left(\frac{q_3}{1 - q_1 q_3 e^{-sT}} - \frac{q_2}{1 - q_1 q_2 e^{-sT}}\right)$$

Finally, the closed form expression of $M_p(s)$ when packet service time is constant, T, is given by,

$$M_p(s) = \frac{p_1 e^{-sT}}{1 - q_1 q_2 e^{-sT}} + \frac{(1 - q_1 q_3) p_2 q_1 e^{-2sT}}{(q_3 - q_2)} \left(\frac{q_3}{1 - q_1 q_3 e^{-sT}} - \frac{q_2}{1 - q_1 q_2 e^{-sT}}\right) \tag{2.7}$$

Substitution of (2.7) in (2.5) gives a relation of the Laplace transform of the busy period when packet transmission time is constant.

2.2.3.2 Exponentially distributed packet transmission times

In this subsection, we determine the Laplace transform of the packet service time for exponentially distributed packet transmission time with parameter μ. Laplace transform of packet transmission time is given by $L_p(s) = \frac{\mu}{s+\mu}$. Thus, $L_p(ns)$ is given by,

$$L_p(ns) = \frac{\mu}{ns + \mu}$$

Substituting the above $L_p(ns)$ in (2.4) gives $M_p(s)$ as follows,

$$M_p(s) = \sum_{n=1}^{\infty} \frac{\mu}{ns + \mu}\left[(q_1q_2)^{n-1}p_1 + U(n)(1 - q_1q_3)\left(q_1^{n-1} \sum_{i+j=n-2}\sum q_2^i q_3^j\right)p_2\right]$$

$$M_p(s) = \frac{\mu p_1}{q_1q_2}\sum_{n=1}^{\infty} \frac{(q_1q_2)^n}{ns + \mu} + \frac{\mu(1 - q_1q_3)p_2}{q_1}\sum_{n=2}^{\infty}\left(\frac{q_1^n}{ns + \mu}\sum_{i+j=n-2}\sum q_2^i q_3^j\right)$$

(2.8)

Again a relationship for the busy period is obtained by substituting (2.8) in (2.5).

2.3 Modeling of the Secondary User

In this section, we present a model of the secondary user and derive its mean packet delay. We model the secondary user as an M/G/1 queue with arrival rate of α packets/sec. The secondary user transmits as many packets as possible during the primary user's idle period. As in the case of primary user, each transmission may be successful or lost due to wireless channel errors according to an independent Bernoulli process. We first determine the mean number of packets in the secondary user queue and then apply the Little's result to obtain the mean packet delay of the secondary user.

2.3.1 The PGF of the Secondary User Queue Length

As shown in Figure 2.4, we will refer to a consecutive idle and busy periods of the primary user as a cycle period. We embed a Markov chain at the end of each cycle period to determine the number of packets at the embedding points. Let us define the following random variables.

X_i $\triangleq$ number of packets in secondary user system at the end of i^{th} cycle

Y_i $\triangleq$ number of secondary packets transmitted during the idle period of the i^{th} cycle

A_i $\triangleq$ number of secondary user packet arrivals during i^{th} cycle

M_s $\triangleq$ continuous packet service time of the secondary user

L_s $\triangleq$ packet transmission time of the secondary user

Q_p $\triangleq$ duration of a primary user cycle

r $\triangleq$ probability that service time of a secondary packet may be completed during an idle period

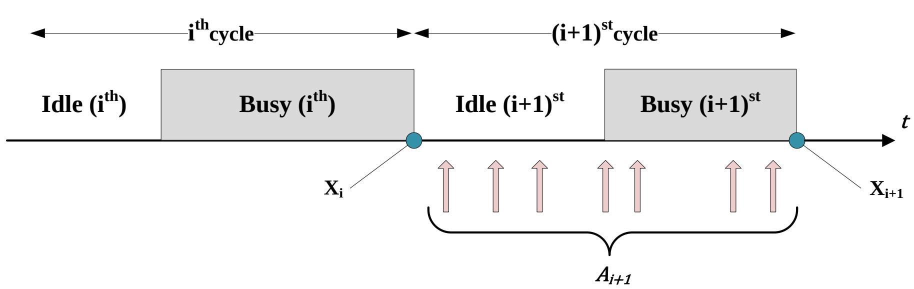

Figure 2.4 Activities in the channel and the embedded Markov points

Figure 2.4 shows activities in a channel which includes idle and busy periods of the primary user and arrivals of the secondary user packets. We make a simplifying assumption that secondary packets cannot be transmitted during the idle period that they have arrived. In particular, packets arriving during i^{th} cycle period will be transmitted at the earliest during the idle period of the $(i+1)^{st}$ cycle. Thus, at the end of $(i+1)^{st}$ cycle, the number of packets in the secondary user queue can be expressed as follows;

$$X_{i+1} = X_i - Y_{i+1} + A_{i+1}, \qquad 0 < Y_{i+1} \leq X_i \tag{2.9}$$

We note that the service time of each packet may be interrupted by the termination of primary user idle period. Since idle periods are exponentially distributed, from the memoryless property of this distribution, the service interruptions of the packets are i.i.d. according to a Bernoulli process with success probability of r defined earlier on. As a result, the probability that y packets will be transmitted successfully in the $(i+1)^{st}$ idle period given that there are x packets in the secondary user at the end of i^{th} cycle period, is given by,

$$Prob(Y_{i+1} = y | X_i = x) = \begin{cases} (1-r)r^y, & y < x \\ r^x, & y = x \end{cases} \tag{2.10}$$

Let us define $X_i(z)$ as the Probability Generation Function (PGF) of the number of packets in the secondary user queue at the end of i^{th}cycle period. Then,

$$X_{i+1}(z) = E[z^{X_{i+1}}]$$

$$X_{i+1}(z) = E[z^{X_i - Y_{i+1} + A_{i+1}}]$$

The number of secondary packet arrivals at the $(i+1)^{st}$ cycle, A_{i+1}, is independent of the number of packets in the secondary user queue at the end of i^{th}cycle. Thus subscript $(i+1)$ on the number of arrivals can be dropped as it does not depend on the cycle number. Defining $A(z)$ as the PGF of the number of secondary packet arrivals during a cycle, we have,

$$X_{i+1}(z) = E[z^{X_i - Y_{i+1}}]\, E[z^{A_{i+1}}]$$

$$= E[z^{X_i - Y_{i+1}}]\, A(z) \tag{2.11}$$

Next, we determine $E[z^{X_i - Y_{i+1}}]$ to substitute later into (2.11),

$$E[z^{X_i - Y_{i+1}}] = \sum_{x=0}^{\infty} \sum_{y=0}^{x} z^{x-y}\, Pr(X_i = x, Y_{i+1} = y)$$

From the Bayes' rule,

$$= \sum_{x=0}^{\infty} \sum_{y=0}^{x} z^{x-y}\, Pr(Y_{i+1} = y | X_i = x)\; Pr(X_i = x)$$

Next, we split the summation into two parts where $x = 0$ and $x \neq 0$ and then the latter part is further split into two subparts as $y = x$ and $y < x$.

$$
\begin{aligned}
E[z^{X_i-Y_{i+1}}] \;=\;& Pr(X_i = 0) \\
&+ \sum_{x=1}^{\infty}\left[\sum_{y=0}^{x-1} z^{x-y}\, Pr(Y_{i+1} = y|X_i = x)Pr(X_i = x)\right. \\
&\left. + z^{x-x}Pr(Y_{i+1} = x|X_i = x)Pr(X_i = x)\right]
\end{aligned}
$$

Substituting for the conditional probability from (2.10) into the above, we get

$$
E[z^{X_i-Y_{i+1}}] \;= Pr(X_i = 0) + \sum_{x=1}^{\infty}\sum_{y=0}^{x-1} z^{x-y}\,(1-r)r^y\, Pr(X_i = x) + \sum_{x=1}^{\infty} r^x Pr(X_i = x)
$$

Note that $X_i(z) = \sum_{x=0}^{\infty} \mathrm{z}^x Pr(X_i = x)$ thus the first and last terms can be combined as $X_i(z)|_{z=r}$. Therefore,

$$
E[z^{X_i-Y_{i+1}}] \;= X_i(z)|_{z=r} + (1-r)\sum_{x=1}^{\infty} z^x Pr(X_i = x)\sum_{y=0}^{x-1}\left(\frac{r}{z}\right)^y
$$

The closed form of the summation $\sum_{y=0}^{x-1}\left(\frac{r}{z}\right)^y$ is given by,

$$
\sum_{y=0}^{x-1}\left(\frac{r}{z}\right)^y = \frac{1-\left(\frac{r}{z}\right)^x}{1-\frac{r}{z}}
$$

Then we have,

$$E[z^{X_i - Y_{i+1}}] = X_i(z)|_{z=r} + \left(\frac{1-r}{1-\frac{r}{z}}\right) \sum_{x=1}^{\infty} z^x Pr(X_i = x) \ \left(1 - \left(\frac{r}{z}\right)^x\right)$$

$$= X_i(z)|_{z=r} + \left(\frac{1-r}{1-\frac{r}{z}}\right) \left[\sum_{x=1}^{\infty} z^x Pr(X_i = x) - \sum_{x=1}^{\infty} r^x Pr(X_i = x)\right]$$

$$= X_i(z)|_{z=r} + \frac{z(1-r)}{z-r} \ \{X_i(z) - Pr(X_i = 0) - [X_i(z)|_{z=r} - Pr(X_i = 0)]\}$$

Thus,

$$E[z^{X_i - Y_{i+1}}] = X_i(z)|_{z=r} + \frac{z(1-r)}{z-r} \ [\, X_i(z) - X_i(z)|_{z=r}] \tag{2.12}$$

Substituting the above in (2.11). we have,

$$X_{i+1}(z) = \left\{X_i(r) + \frac{z(1-r)}{z-r} \ [X_i(\mathrm{z}) - X_i(r)]\right\} A(z)$$

At the steady-state we have $X_i(z) = X_{i+1}(z) = X(z)$. Thus, we can drop the subscript $(i+1)$ in the above,

$$X(z) = \left\{\frac{(z-r)X(r) + z(1-r) \ [X(z) - X(r)]}{z-r}\right\} A(z)$$

Solving the above equation for $X(z)$, which determines the PGF of the secondary queue length at the embedded points, we get

$$X(z) = \frac{r(z-1)X(r)A(z)}{(z-r) - z(1-r)A(z)} \tag{2.13}$$

Let us determine the unknowns in (2.13) which are $X(r)$, $A(z)$ and r in the following subsections.

- **Derivation of $X(r)$**

Firstly, we can find $X(r)$ from the normalization condition by applying L'Hôpital's rule to (2.13) and evaluating it at $z = 1$. Solving the equation for $X(r)$, we get,

$$X(r) = \frac{r - (1 - r)A'(1)}{r} \tag{2.14}$$

- **Derivation of $A(z)$**

$A(z)$ is the PGF of the number of Poisson packet arrivals during a primary cycle period, Q_p where $Q_p = B_p + I_p$. Let us denote $Q_p(s)$ as the Laplace transform of a cycle period, $Q_p(s) = E[e^{-sQ_p}]$. From the independence of busy and idle periods, we have, $Q_p(s) = B_p(s)I_p(s)$. Note that $B_p(s)$ and $I_p(s)$ are obtained from (2.5) and (2.1), respectively. From the relation between the PGF of the number of Poisson arrivals during a random period and the Laplace transform of that random period, we have $A(z) = Q_p(s)|_{s=\alpha(1-z)}$. Thus,

$$A(z) = B_p[\alpha(1 - z)]I_p[\alpha(1 - z)]$$

Substituting for the Laplace transform of the primary user idle period from (2.1), we have,

$$A(z) = \frac{\lambda\, B_p[\alpha(1-z)]}{\alpha(1-z)+\lambda} \tag{2.15}$$

- **Derivation of r**

Finally, the last unknown in (2.13) is r. Recall that r is defined as the probability that a continuous service time of a secondary user packet is completed during an idle period. This is the probability of the event that the primary idle period lasts longer than the service time of the secondary user's packet. Let us define $f_{M_s}(t)$ as the pdf of the continuous service time of the secondary user packets.

$$r = Pr\left(I_p > M_s\right)$$

$$Pr\left(I_p > M_s \middle| M_s = m_s\right) = \int_{m_s}^{\infty} \lambda e^{-\lambda t}\, dt = e^{-m_s \lambda}$$

unconditioning the above with respect to M_s, we have

$$r = \int_{0}^{\infty} e^{-m_s \lambda}\, f_{M_s}(m_s) dm_s$$

$$r = \left. M_s(s)\right|_{s=\lambda} \tag{2.16}$$

The $M_s(s)$ can be obtained by the same approach as the $M_p(s)$ in (2.4) by dropping p_2 and replacing p_1 by the probability of successful transmission between secondary transmitter and receiver, p_4. For constant packet transmission times, T, Laplace transform of the packet service time of the secondary user, $M_s(s)$, is given by,

$$M_s(s) = \frac{p_4 e^{-sT}}{1 - q_4 e^{-sT}} \tag{2.17}$$

2.3.2 The Mean Packet Delay of the Secondary User

Now, let us determine the mean number of packets in the secondary user queue and the mean delay of a secondary packet. Let us define $\bar{x}$ as the expected value of the number of packets in the secondary user queue and $\bar{d}$ as the expected delay of a secondary packet, then,

$$\bar{x} = X'(z)|_{z=1}$$

By taking derivative of $X(z)$ in (2.13) and evaluating at $z = 1$ gives the mean number of secondary packets in the system as,

$$\bar{x} = \frac{2rA'(1)X(r) + 2(1-r)A'(1) + (1-r)A''(1)}{2[r - (1-r)A'(1)]} \tag{2.18}$$

Applying the well-known Little's result, we finally obtain the mean packet delay of a secondary user as follows

$$\bar{d} = \frac{\bar{x}}{\alpha} = \frac{2rA'(1)X(r) + 2(1-r)A'(1) + (1-r)A''(1)}{2\alpha[r - (1-r)A'(1)]} \tag{2.19a}$$

The unknowns in (2.19a), $A'(1)$ and $A''(1)$, can be determined by taking derivative of $A(z)$ in (2.15) and evaluating at $z = 1$. As a result,

$$A'(1) = \frac{\alpha(1+\lambda\bar{B}_p)}{\lambda} \tag{2.19b}$$

$$A''(1) = \alpha^2 B_p''(0) + 2\frac{\alpha}{\lambda}A'(1) \tag{2.19c}$$

2.3.3 The Secondary User Traffic Load

The system load is defined as the probability of having non-empty queue which can be written as

$$\rho_s = 1 - X(0) \tag{2.20}$$

The $X(0)$ can be obtained by substituting $z = 0$ in (2.13) which gives,

$$X(0) = X(r)A(0).$$

Similarly, we can determine $A(0)$ by substitute $z = 0$ in (2.15) then we get,

$$A(0) = \frac{\lambda B_p(\alpha)}{\alpha + \lambda}$$

We can determine $B_p(\alpha)$ from (2.5) by substituting $s = \alpha$, we get

$$B_p(\alpha) = M_p\left[\alpha + \lambda - \lambda\, B_p(\alpha)\right]$$

This equation can be solved numerically through an iterative process. First, we set up the iterative equation as follows,

$$B_{p_{n+1}}(\alpha) = M_p\left[\alpha + \lambda - \lambda\, B_{p_n}(\alpha)\right]$$

For calculation of the $B_{p_{n+1}}(\alpha)$, we initialize the iteration by choosing a value for $B_{p_0}(\alpha)$ where $0 < B_{p_0}(\alpha) < 1$ and calculate for $B_{p_{n+1}}(\alpha)$ recursively until $B_{p_{n+1}}(\alpha)$ converges. The above concludes the calculation of the secondary user traffic load.

2.3.4 The Secondary User Packet Service Time

In this section, we will derive the service time of a secondary user packet for packets with constant transmission time. The service time of a secondary user packet begins when the transmission of the packet starts during the idle period of the primary user and ends when the packet successfully transmitted. The packet service time may take several idle periods separated by busy periods of the primary user. We assume that the secondary user packets are served in First-Come-First-Serve basis. A secondary user packet that its service has been interrupted by the primary user's busy period will be the first packet to be transmitted during the next idle period, as shown in Figure 2.5. Let us define the following notation;

$\widetilde{N}$ $\triangleq$ number of trials for successful transmission where consecutive trials are separated by a primary busy period

ω_i $\triangleq$ $min(M_s, I_p)$

$W_s \triangleq$ packet service time of the secondary user

$B_{p_i} \triangleq$ the ith busy period of the primary user

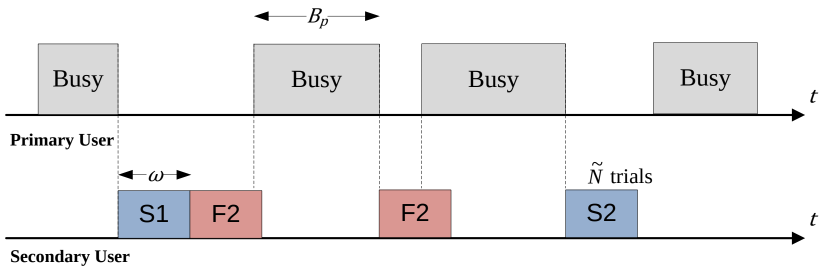

Figure 2.5 A packet service time of the secondary user

In Figure 2.5, the first packet was successfully transmitted in a single idle period whereas the second packet has failed on its first attempt and succeeds on the $\tilde{N}^{th}$ trial. Therefore,

If $\tilde{N} = 1$, $\quad W_s = \omega_1$

If $\tilde{N} = 2$, $\quad W_s = \omega_1 + B_{p_1} + \omega_2$

If $\tilde{N} = 3$, $\quad W_s = \omega_1 + B_{p_1} + \omega_2 + B_{p_2} + \omega_3$

Thus, if $\tilde{N} = k$, $\quad W_s = \sum_{i=1}^{k} \omega_i + \sum_{j=1}^{k-1} B_{p_i}$

Denote $W_s(s)$ and $\omega(s)$ as Laplace transforms of the corresponding random variables, we have,

$$W_s\left(s \middle| \tilde{N} = k\right) = [\omega(s)]^k \left[B_p(s)\right]^{k-1}$$

Unconditioning the above with respect to $\tilde{N}$,

$$W_s(s) = \sum_{k=1}^{\infty} [\omega(s)]^k \left[B_p(s)\right]^{k-1} \; Pr\left(\widetilde{N} = k\right)$$

where, $Pr\left(\widetilde{N} = k\right) = r(1-r)^{k-1}$ then we have,

$$W_s(s) = \frac{r}{(1-r)B_p(s)} \sum_{k=1}^{\infty} \left[(1-r)\omega(s)B_p(s)\right]^k$$

$$W_s(s) = \frac{r\omega(s)}{1-(1-r)\omega(s)B_p(s)}$$

(2.21)

The mean service time of the secondary user, denoted as $\overline{W_s}$, is then given by

$$\overline{W_s} = -W_s{}'(s)|_{s=0}$$

$$\overline{W_s} = \frac{(1-r)\left(\overline{B_p} + \overline{\omega}\right) - r\overline{\omega}}{r}$$

(2.22)

Let us now derive the $\omega(s)$ and its expected value, $\overline{\omega}$. For simplicity, we will assume that the idle periods are slotted with the slot duration of T second. Then, idle periods will be geometrically distributed as follows,

$$Pr\left(I_p = k\ slots\right) = (1-\sigma)\sigma^{k-1}$$

$$Pr\left(I_p \leq k\ slots\right) = 1-\sigma^k$$

Compare the above with the probability distribution of the idle period, $Pr\left(I_p < t\right) = 1 - e^{-\lambda t}$, thus $\sigma = e^{-\lambda T}$. For consistency, let us define $\gamma = 1 - p_4$ therefore, the continuous service time of a packet is given by,

$$Pr(M_s = j\ slots) = (1-\gamma)\gamma^{j-1}$$

$$Pr(M_s \le j\ slots) = 1 - \gamma^{j}$$

Refer to [40] for the distribution of ω, we have

$$\omega_k = Pr(\omega = k\ slots) = Pr\left(I_p = k\right)Pr(M_s > k-1) + Pr(M_s = k)Pr(I_p > k)$$

$$\omega_k = (1-\sigma)\sigma^{k-1}\gamma^{k-1} + (1-\gamma)\gamma^{k-1}\sigma^{k}$$

$$\omega(z) = E[z^{\omega_k}] = \sum_{k=1}^{\infty} z^k \omega_k$$

$$\omega(z) = \frac{(1-\sigma\gamma)z}{1-\sigma\gamma z}$$

Therefore the Laplace transform of ω will be given by

$$\omega(s) = E[e^{-s\omega T}] = E[z^{\omega}]|_{z=e^{-sT}}$$

$$\omega(s) = \frac{(1-\sigma\gamma)e^{-sT}}{1-\sigma\gamma e^{-sT}}$$

$$\overline{\omega} = -\omega'(s)|_{s=0} = \frac{T}{1-\sigma\gamma}$$

(2.23)

By substituting (2.23) into (2.22) we can obtain the mean service time of the secondary user with constant packet transmission times.

2.4 The Interference to the Primary User

In our model, the secondary user may interfere with the primary user if the primary user's next busy period begins when the secondary user is still transmitting in the channel. In this section, we determine the probability of interference to the primary user.

If $X_i - Y_{i+1} = 0$, this means that during the $(i+1)^{st}$ idle period, the secondary user can successfully transmit the eligible packets in its queue and no interference occurs to the primary user. On the other hand, if $X_i - Y_{i+1} > 0$, this means that the secondary user cannot transmit all the packets in its queue during the idle period and the transmission will interfere with the primary transmission. Therefore, the probability of interference to the primary user is given by the probability of $X_i - Y_{i+1} > 0$.

Let $G_i = X_i - Y_{i+1}$. Therefore,

$$Pr(interference) = Pr(G_i > 0) = 1 - Pr(G_i = 0)$$

The PGF of G_i, denoted as $G_i(z)$, has been determined in the preceding analysis, from (2.12),

$$G_i(z) = E[z^{G_i}] = E[z^{X_i - Y_{i+1}}]$$

$$= \sum_{x=0}^{\infty} \sum_{y=0}^{x} z^{x-y} \, Pr(X_i = x, Y_i = y)$$

$$= X_i(r) + \frac{z(1-r)}{z-r} \, \{X_i(z) - X_i(r)\}$$

Next, assuming steady-state by letting $i \to \infty$, we have $G_i(z) = G(z)$. From the above expression, we get,

$$G(z) = \frac{zX(z)(1-r) - rX(r)(1-z)}{z-r} \tag{2.24}$$

We can find $Pr(G = 0)$ by substituting $z = 0$ in the (2.24).

$$Pr(G = 0) = G(0) = X(r)$$

Thus,

$$Pr(interference) = 1 - X(r) \tag{2.25}$$

where $X(r)$ is previously obtained in (2.14).

By considering only the probability of interference may be misleading on the effect of secondary user over the primary user. In fact, the interference may occur only at the beginning of the primary user busy period and the primary user may be able to tolerate the interference. To verify this hypothesis, we determine the mean and percentage of the interference duration during a busy period of the primary user.

Let β denote the interference duration between the primary and secondary user transmissions. As we consider constant packet transmission times, T for the primary user, the interference duration will be uniformly distributed from 0 to T with expected value of $T/2$. Thus, we have

$$E\ [\ \beta |\ there\ is\ an\ interfernce\ ] = \frac{T}{2}$$

$$E[\beta] = \frac{T}{2}\ Pr(interference)$$

Substituting from (2.25), the mean duration of interference is given by, $E[\beta] = \frac{1}{2}T[1 - X(r)]$. Thus, the proportion of interference duration in the busy period of the primary user is $\frac{E[\beta]}{E[B_p]}$, where $E\left[B_p\right]$ is the mean busy period of the primary user.

2.5 Numerical Results

In this section, we present numerical results regarding the analysis developed in the chapter. We plot the probability of interference as well as the mean packet delay of primary and secondary users in various cases to show benefits of the cooperation.

2.5.1 Mean Packet Delay of the Secondary User

To show benefits of the cooperation, we plot the mean packet delay of a secondary user with and without cooperation. The packet transmission times of both primary and secondary users are assumed to be constant and equal to 0.1 ($L_p = L_s = T$).

The probability of successful transmission from PT to PR (p_1) as well as the probability of successful transmission from ST to SR (p_4) are both set to 0.7. Next, we consider the two scenarios,

2.5.1.1 No cooperation ($p_2 = 0$)

In this scenario, secondary user does not provide help in primary user transmissions. As $p_2 = 0$, the Laplace transform of the primary user packet service time in (2.7) reduces to

$$M_p(s) = \frac{p_1 e^{-sT}}{1-q_1 e^{-sT}} \tag{2.26a}$$

The mean service time of the primary packet ($\overline{m}_p$) is given by the negative value of a derivative of $M_p(s)$ evaluated at $s = 0$,

$$\overline{m}_p = -M_p'(0) = \frac{T}{p_1} \tag{2.26b}$$

The second moment of the primary packet service time is given by

$$\overline{m_p^2} = M_p''(0) = \frac{T^2(2-p_1)}{(p_1)^2} \tag{2.26c}$$

By substituting $\overline{m}_p$ and $\overline{m_p{}^2}$ in (2.6a) and (2.6b), respectively, to obtain the first two moments of the busy period of the primary user and other related parameters, we then can obtain the mean packet delay of the secondary user from (2.19a) . In Figure 2.6, we plot the mean packet delay of the secondary user as a function of its traffic load (ρ_s)for different values of primary traffic load (ρ_p).

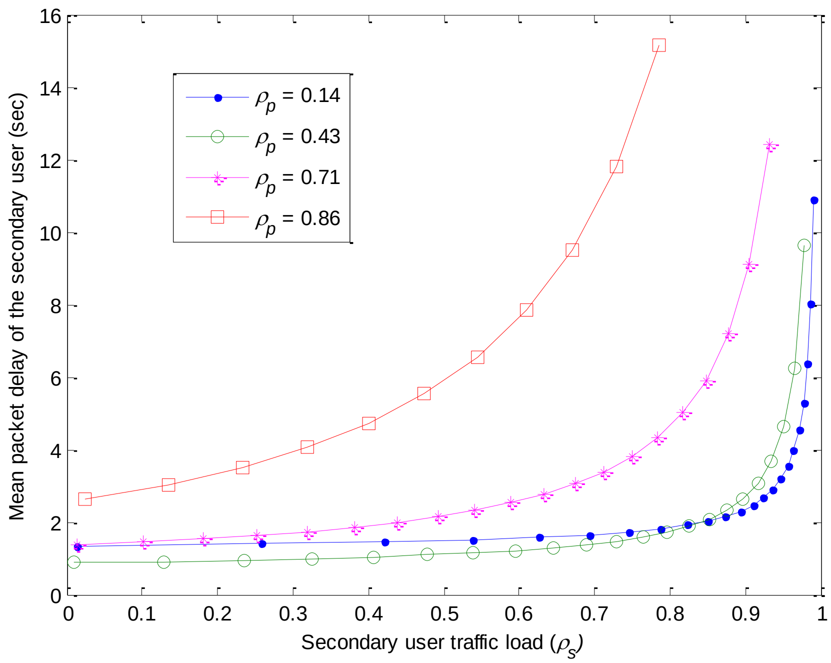

Figure 2.6 Mean packet delay of the secondary user

We observe that under light primary user load condition, the mean packet delay of the secondary user decreases as the primary load increases. For example, at the primary user load of 0.14, the mean packet delay of the secondary user is generally higher than that at the primary user load of 0.43. On the other hand, at higher primary user traffic loads, the mean packet delay of the secondary user increases as the primary user load increases. These results may be explained through our model assumptions. In our model, the secondary packets will not be transmitted in the cycle that they have arrived and their earliest transmission may occur in the idle period of the next cycle. Therefore, lower primary traffic load means longer idle periods. As a result, the packets will have to wait for the long idle period to end before they have a chance for transmission in the idle period of the next cycle. This is the reason why at low primary traffic loads, increasing the primary load results a reduction in the secondary user packet delays. Next, we consider the scenario that secondary user provides help in primary user transmissions.

2.5.1.2 With cooperation ($p_2 > 0$)

We set the probability of successful transmission from ST to PR (p_3) to 0.7. From (2.7), the Laplace transform of the primary user packet service time is given by

$$M_p(s) = \frac{p_1 e^{-sT}}{1 - q_1 q_2 e^{-sT}} + \frac{(1 - q_1 q_3) p_2 q_1 e^{-2sT}}{(q_3 - q_2)} \left(\frac{q_3}{1 - q_1 q_3 e^{-sT}} - \frac{q_2}{1 - q_1 q_2 e^{-sT}} \right)$$

Let us rewrite $M_p(s)$ as

$$M_p(s) = \frac{p_1 e^{-sT}}{1 - q_1 q_2 e^{-sT}} + \frac{C q_3 e^{-2sT}}{1 - q_1 q_3 e^{-sT}} - \frac{C q_2 e^{-2sT}}{1 - q_1 q_2 e^{-sT}} \tag{2.27a}$$

where,

$$C = \frac{(1 - q_1 q_3) p_2 q_1}{(q_3 - q_2)}$$

By taking derivative of each term in the above and evaluating at $s = 0$, we get the mean service time and its second moment as follows,

$$\bar{m}_p = -M_p'(0) = \frac{T p_1}{(1 - q_1 q_2)^2} - \frac{C T q_3 \, (q_1 q_3 - 2)}{(1 - q_1 q_3)^2} + \frac{C T q_2 \, (q_1 q_2 - 2)}{(1 - q_1 q_2)^2} \tag{2.27b}$$

$$\overline{m_p^2} = \frac{T^2 p_1 (1-q_1{}^2 q_2{}^2)}{(1-q_1 q_2)^4} + \frac{CT^2 q_3 (1-q_1 q_3)(q_1 q_3 (q_1 q_3 - 3) + 4)}{(1-q_1 q_3)^4} - \frac{CT^2 q_2 (1-q_1 q_2)(q_1 q_2 (q_1 q_2 - 3) + 4)}{(1-q_1 q_2)^4} \qquad (2.27c)$$

In the following, we plot the mean packet delay of the secondary user versus its traffic load, for different fixed values of arrival rates to the primary user, with the success probability of secondary receiving primary packet (p_2) as a parameter. The lower delay of the secondary user packet can be achieved with increasing p_2 and the delay reduction is more significant at higher primary user traffic loads.

- **Primary user packet arrival rate, $\lambda = 3$ packets/sec**

Figure 2.7 shows the plot of the mean packet delay of the secondary user as a function of its traffic load with arrival rate to the primary user set to 3 packets/sec. It may be seen that the mean packet delay of secondary user slightly decreases with increasing p_2. The decrease in mean packet delay is more significant when the secondary user traffic load is high.

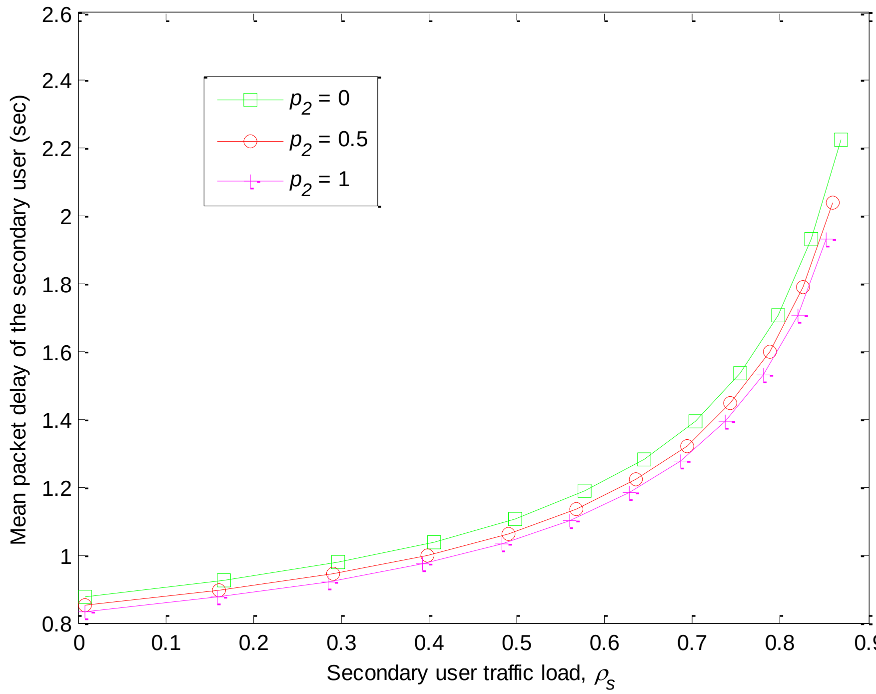

Figure 2.7 Mean packet delay of the secondary user at the primary user arrival rate of $\lambda = 3$ packets/sec

Table 2.1 shows the detailed values of the mean packet delay of the secondary user. The delay is compared between the case of no cooperation $(p_2 = 0)$ and with cooperation $(p_2 = 1)$. The results show that the mean packet delay reduction of the secondary user is around 5% - 10% in the cooperation case.

Table 2.1 Delay reduction at primary user arrival rate of 3 packets/sec

p2 / ρs	0	1	reduction in delay	% delay reduction
0.108	0.904	0.860	0.045	4.92%
0.152	0.919	0.873	0.046	5.02%
0.200	0.936	0.888	0.048	5.14%
0.285	0.971	0.921	0.050	5.13%
0.372	1.014	0.962	0.052	5.14%
0.454	1.070	1.011	0.059	5.48%
0.560	1.174	1.106	0.068	5.77%
0.672	1.331	1.253	0.078	5.86%
0.744	1.505	1.411	0.094	6.23%
0.826	1.871	1.747	0.123	6.60%
0.877	2.336	2.164	0.173	7.39%
0.900	2.690	2.486	0.203	7.56%
0.925	3.330	3.062	0.268	8.06%
0.950	4.545	4.198	0.347	7.63%
0.985	13.612	12.202	1.410	10.36%

- **Primary user arrival rate, $\lambda = 5$ packets/sec**

Under moderate primary loads, deeper reductions in secondary packet delay can be achieved. Figure 2.8 presents mean packet delay of a secondary user versus its traffic load for $\lambda = 5$. Again the Table 2.2 shows the detailed information and the percentage of reduced delay between the cases of p_2 = 1 and p_2 = 0.

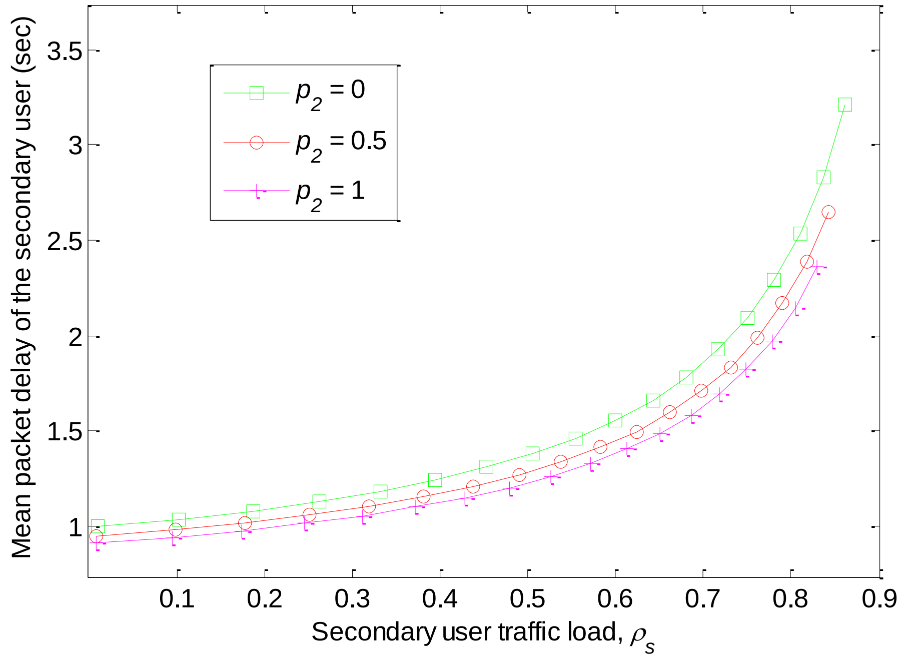

Figure 2.8 Mean packet delay of the secondary user at primary user arrival rate of 5 packets/sec

Table 2.2 Delay reduction at primary user arrival rate of 5 packets/sec

ρs \ p2	0	1	reduction in delay	% delay reduction
0.147	1.489	1.251	0.238	15.96%
0.223	1.582	1.317	0.265	16.74%
0.355	1.801	1.472	0.330	18.29%
0.422	1.951	1.575	0.376	19.27%
0.521	2.244	1.781	0.463	20.64%
0.614	2.660	2.068	0.592	22.27%
0.723	3.501	2.647	0.854	24.40%
0.852	6.066	4.421	1.644	27.11%
0.913	9.870	7.038	2.832	28.69%
0.946	15.544	10.976	4.569	29.39%
0.980	41.330	27.891	13.439	32.52%

- **Primary user arrival rate, $\lambda = 6$ packet/sec**

Under higher primary loads, we can observe that in cooperation scenario, the mean packet delay of the secondary user is considerably reduced. We take a closer look in this case, for example, when p_2 increases from 0 to 0.3, the mean secondary packet delay in cooperation case is reduced up to 24%.

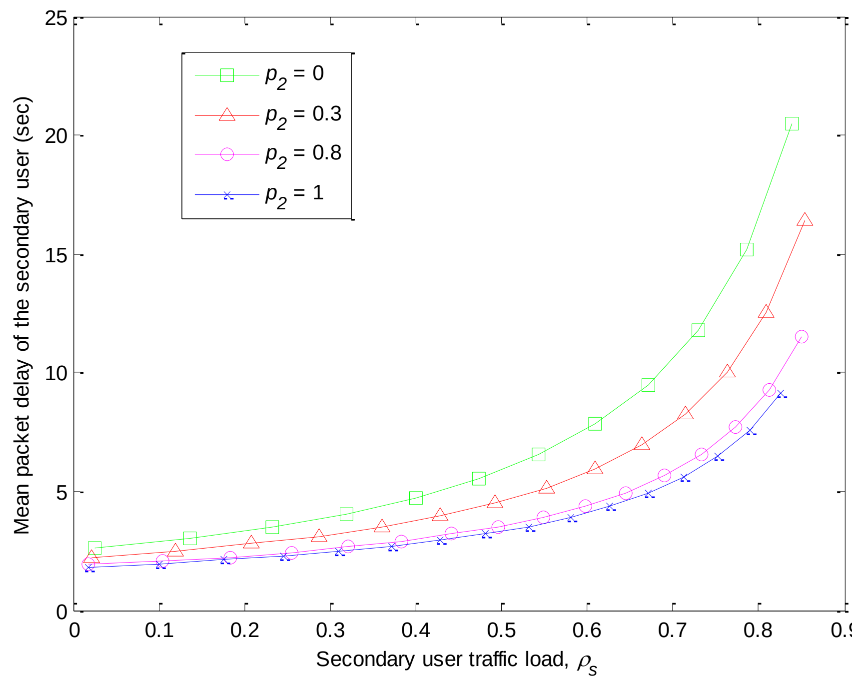

Figure 2.9 Mean packet delay of the secondary user at the primary user arrival rate of 6 packets/sec

Table 2.3 shows the detailed calculation results at $\lambda = 6$. As can be seen, in cooperation scenario the mean delay of the secondary user is significantly reduced compared to that in no cooperation case.

Table 2.3 Delay reduction at primary user arrival rate of 6 packets/sec

p2 / ρs	0	1	reduction in	% delay reduction
0.115	2.937	1.984	0.952	32.43%
0.285	3.827	2.388	1.440	37.61%
0.335	4.188	2.549	1.639	39.13%
0.430	5.039	2.925	2.113	41.94%
0.532	6.332	3.531	2.800	44.23%
0.635	8.443	4.471	3.972	47.05%
0.719	11.280	5.759	5.521	48.95%
0.806	16.957	8.158	8.799	51.89%
0.945	63.430	28.504	34.926	55.06%

2.5.2 Mean Packet Delay of the Primary User

In Figure 2.10, we plot the mean packet delay of primary user as a function of its traffic load with p_2 as a parameter. The plot shows that the mean delay of the primary user's packet consistently decreases with increasing p_2.

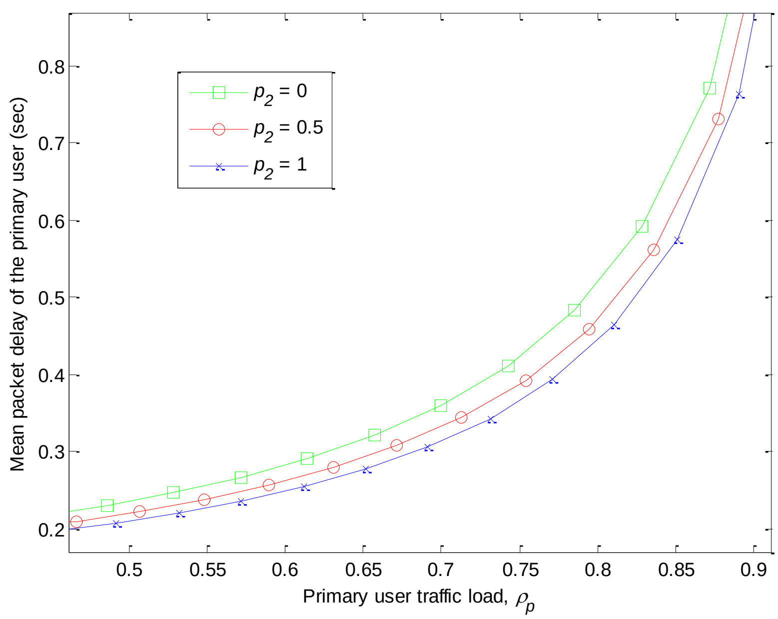

Figure 2.10 Mean packet delay of the primary user

2.5.3 Probability of Interference to the Primary User

In Figure 2.11, we plot the probability of interference seen by the primary user as a function of secondary traffic load with primary load as a parameter and the value of $p_2 = 0.8$. Probability of interference is measure of the number of primary user busy periods affected by the secondary user transmissions. The results show that the greater is the primary user traffic load, the higher is the probability of interference at any given secondary traffic load.

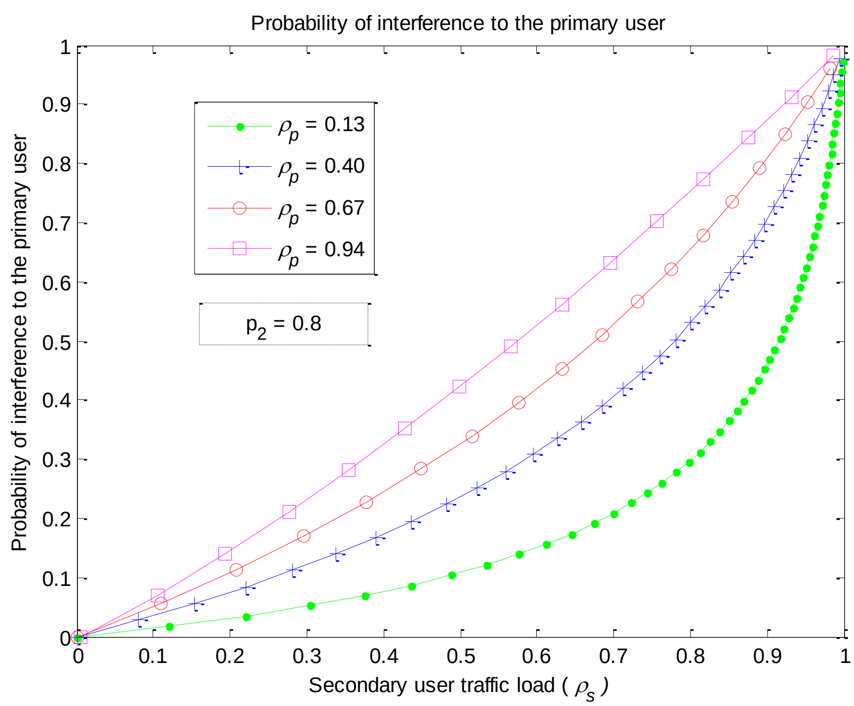

Figure 2.11 Probability of interference at different primary user loads

2.5.4 Percentage of Interference during a Primary User Busy Period

We plot the proportion of mean interference duration in the mean busy period of the primary user against the probability of interference. As can be seen, when the probability of interference is close to 1, the primary user transmissions may be interfered by only 20% of its busy period.

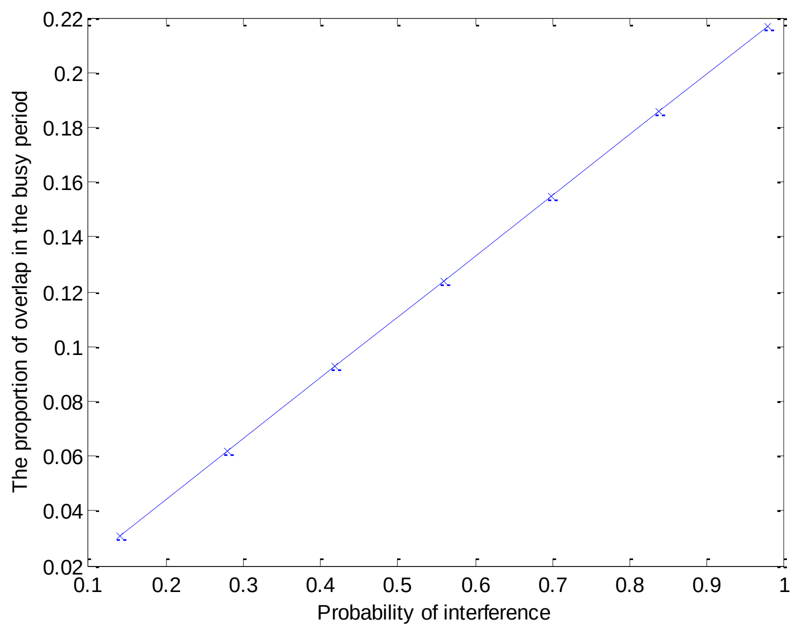

Figure 2.12 The amount of interference in the busy period

Chapter 3

A Performance Evaluation of Cognitive Radio Networks with Multiple Primary Users

In this chapter, we consider a cognitive radio network with multiple primary users using the same channel in different geological locations. If a secondary transmitter is placed within a transmission range of a primary user while a secondary receiver is located within a transmission range of another primary user, the channel usage seen by the secondary transmitter and receiver may not be identical. As a result, the probability of successful transmission of the secondary user may shrink significantly. In this chapter, we derive the mean packet delay of a secondary user for this scenario and compare the result with that in a single primary user case.

3.1 System Model

Consider a system with two primary users as shown in Figure 3.1. The system has two primary users, primary user 1 (PU1) and primary user 2 (PU2), operating in the same channel at different geological areas. A secondary transmitter (ST) and secondary receiver (SR) is within PU1's and PU2's transmission ranges, respectively. In the following, we model PU1 and PU2 as M/G/1 queues. Packets arrive at PU1 and PU2 according to Poisson process with arrival rates λ_1 and λ_2 packets/sec, respectively. The channel availability seen by ST is different from that seen by SR as they are influenced by different primary users. Consequently, the utilizable transmission opportunities for the secondary user only occur when both PU1 and PU2 are idle at the same time.

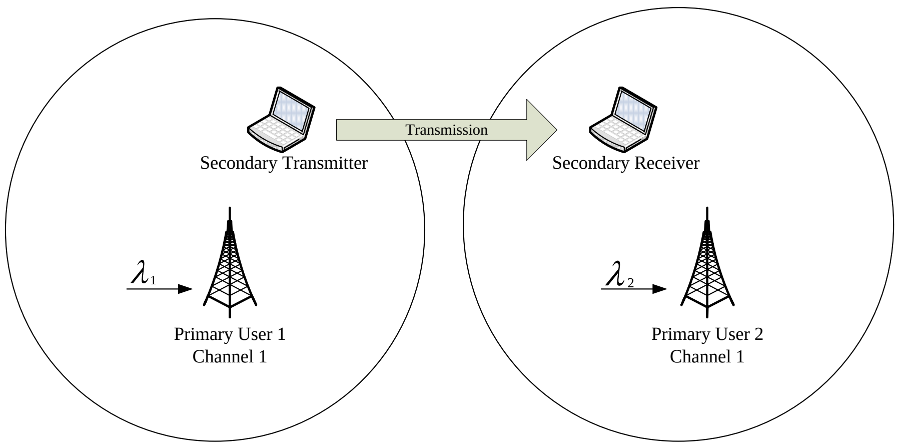

Figure 3.1 Multiple primary users network architecture

We also model the secondary user as an M/G/1 queue where the packet arrivals are according to a Poisson process with the rate of α packets/sec and service times are generally distributed. At both primary and secondary users, we assume that a packet may be transmitted successfully or lost in each transmission due to wireless channel errors according to an independent Bernoulli process. The transmission of a packet will be repeated until it is successfully received. We assume that the service time of a packet begins with the beginning of its first transmission and ends with the completion of the successful transmission. We denote the incident when both PU1 and PU2 are simultaneously idle as system idle period and when at least one primary user is active as system busy period. Next, we present the idle and busy period of a primary user.

3.2 Analysis of Idle and Busy Periods of a Primary User

The analysis of idle and busy periods of a primary user in this chapter is similar to that presented in Chapter 2. Let us refer to each primary user as PUk where $k = 1, 2$. We will use the subscript k throughout the analysis in this chapter to refer to random variables of PUk. Each primary user is modeled as M/G/1 queue. Packets arrive at PUk according to Poisson process with arrival rate of λ_k packets/sec, and service times are generally distributed. Let us define the following notation;

B_{PUk}	$\triangleq$	busy period of PUk
I_{PUk}	$\triangleq$	idle period of PUk
Q_{PUk}	$\triangleq$	cycle period of PUk which refers to consecutive busy and idle periods.
L_{PUk}	$\triangleq$	packet transmission time of PUk
M_{PUk}	$\triangleq$	packet service time of PUk

Since the arrivals are according to a Poisson process, then the idle periods of primary users are exponentially distributed and its Laplace transform, denoted as $I_{PUk}(s)$, is given by,

$$I_{PUk}(s) = \frac{\lambda_k}{s+\lambda_k} \tag{3.1}$$

The expected value of the primary user idle period and its second moment are as follows;

$$\bar{I}_{PUk} = -I'_{PUk}(s)|_{s=0} = 1/\lambda_k \tag{3.2}$$

$$\overline{I^2_{PUk}} = I''_{PUk}(0) = 2/(\lambda_k)^2 \tag{3.3}$$

Next, let us determine the Laplace transform of the packet service time of a primary user, $M_{PUk}(s)$. A primary user packet will be transmitted successfully with probability p_{PUk} then the number of transmissions until the packet is received successfully, denoted as N_k, has a geometric distribution. According to the derivation of primary user service time in (2.3), we have

$$M_{PUk}(s) \ = \ \sum_{n=1}^{\infty} L_{PUk}(ns) Pr\,(N_k = n) \tag{3.4}$$

where $Pr(N_k = n) = (1 - p_{PUk})^{n-1} p_{PUk}$. Further, we obtain the busy period of a primary user as,

$$B_{PUk}(s) = M_{PUk}[s + \lambda_k - \lambda_k B_{PUk}(s)] \tag{3.5}$$

Let $\overline{m^J_{PUk}}$ and $\overline{B^J_{PUk}}$ denote the j^{th} moments of service time and busy period of PUk, respectively. Therefore, the mean busy period of a primary user and its second moment are given by

$$\bar{B}_{PUk} = \bar{m}_{PUk}/(1 - \lambda_k \bar{m}_{PUk}) \tag{3.6}$$

$$\overline{B_{PUk}^2} = B''_{PUk}(0) = \overline{m_{PUk}^2}/(1 - \lambda_k \bar{m}_{PUk})^3 \tag{3.7}$$

The cycle period is defined as consecutive busy and idle period of a primary user, $Q_{PUk} = B_{PUk} + I_{PUk}$. Thus the first two moments of the cycle period are given by,

$$\bar{Q}_{PUk} = \bar{B}_{PUk} + \bar{I}_{PUk} \tag{3.8}$$

$$\overline{Q_{PUk}^2} = Q''_{PUk}(0) = \overline{B_{PUk}^2} + \overline{I_{PUk}^2} + 2\bar{B}_{PUk}\bar{I}_{PUk} \tag{3.9}$$

3.3 Analysis of Idle and Busy Periods of the System

In this section, we analyze idle and busy periods of the system and determine their Laplace transforms. As mentioned earlier the system idle period refers to time duration where both PU1 and PU2 are idle at the same time and the system busy period refers to time duration where at least one of the primary users is active. The channel availability seen by the secondary user changes from busy to idle according to the following two mutually exclusive cases:

Case 1: PU1 becomes idle when PU2 is already idle.

Case 2: PU2 becomes idle when PU1 is already idle.

These two cases are shown in Figures 3.2 and 3.3.

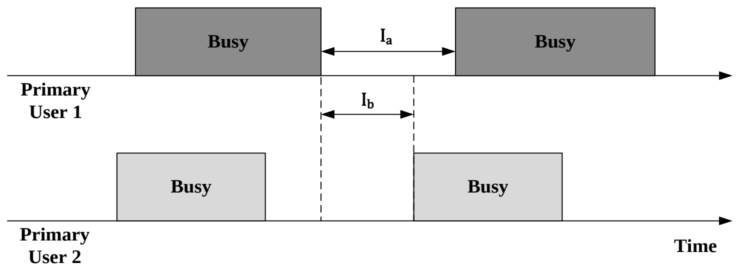

Figure 3.2 The system idle period case 1

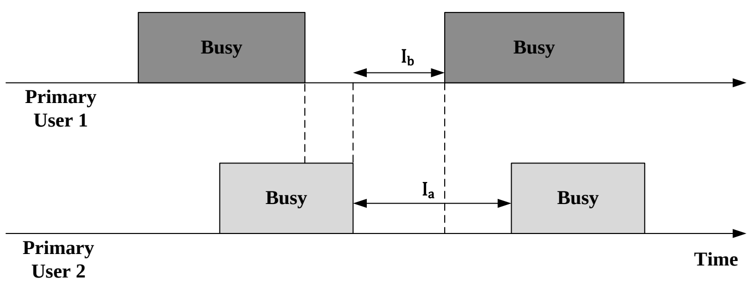

Figure 3.3 The system idle period case 2

We derive the distributions of the system idle and busy periods in the following subsections. Let us define the following notation;

$B_{sys,k}$ ≜ busy period of the system given that it had been initiated by PUk

B_{sys} ≜ busy period of the system

I_{sys} ≜ idle period of the system

3.3.1 Probability Distribution of System Idle Period

As determined before, the idle periods of PUk are exponentially distributed with parameter λ_k. From Figures 3.2 and 3.3, the system idle period is given by the minimum of a complete and a residual idle period of the two primary users. Let I_a and I_b denote a complete and a residual idle period respectively. Thus,

$$I_{sys} = min\ (I_a\ ,\ I_b)$$

From the independence of the two primary users and memoryless property of the exponential distribution, we obtain the distribution of the I_{sys} as follows,

$$\begin{aligned} Pr\left(I_{sys} > t\right) &= Pr(I_a > t, I_b > t) \\ &= [1 - Pr(I_a < t)][1 - Pr(I_b < t)] \\ &= \left[1 - \left(1 - e^{-\lambda_1 t}\right)\right]\left[1 - \left(1 - e^{-\lambda_2 t}\right)\right] \\ &= e^{-(\lambda_1 + \lambda_2)t} \end{aligned}$$

We therefore obtain the Cumulative Distribution Function (CDF) of the system idle period as follows,

$$Pr\left(I_{sys} \le t\right) = 1 - e^{-(\lambda_1 + \lambda_2)t}$$

The probability density function (pdf) of the system idle period, denoted as $f_{I_{sys}}(t)$, can be obtained by the differentiation of the above, thus,

$$f_{I_{sys}}(t) = (\lambda_1 + \lambda_2)\ e^{-(\lambda_1 + \lambda_2)t} \tag{3.10}$$

From the above, the distribution of the system idle period is exponential with parameter $\lambda_1 + \lambda_2$. Therefore, the mean of system idle period, denoted as $\bar{I}_{sys}$ is given by

$$\bar{I}_{sys} = \frac{1}{(\lambda_1+\lambda_2)} \tag{3.11}$$

3.3.2 Probability Distribution of System Busy Period

As defined earlier, during a system busy period at least one primary user is active at any time. Thus a system idle period terminates and a new system busy period begins when one of the two primary users switches from idle to a busy period. We will express the system busy period wrt the initiating primary user. Figures 3.4 and 3.5 show a system busy period initiated by the PU1 and PU2, respectively. From the figures, we will express the system busy period in terms of the cycles of the primary users. A system busy period will contain a random number of cycles of the initiating primary user with an interrupted last cycle. We will assume that each primary cycle may be the last cycle of the system busy period according to an independent Bernoulli variable. Thus the number of cycles in the system busy period will have a geometric distribution. Let D_k denote the number of cycles in a system busy period and H_k denote the probability of a cycle being the last cycle of the system busy period given that it has been initiated by the primary user k, then,

$$Pr(D_k = d) = H_k(1 - H_k)^{d-1},\ k = 1,2\ and\ d = 1, 2, 3, \ldots \tag{3.12}$$

Let the PGF of the number of cycles in a system busy period be denoted as $D_k(z)$, then, it is given by

$$D_k(z) = \frac{H_k z}{1-(1-H_k)z}, \quad k = 1,2.$$

The first two moments of the number of cycles in the system busy period are obtained by taking derivatives of the above and evaluating at $z = 1$. Therefore,

$$\overline{D}_k = D_k'(1) = \frac{1}{H_k}, \quad k = 1,2.$$

$$\overline{D_k^2} = D_k''(1) = \frac{2(1-H_k)}{{H_k}^2}, \quad k = 1,2.$$

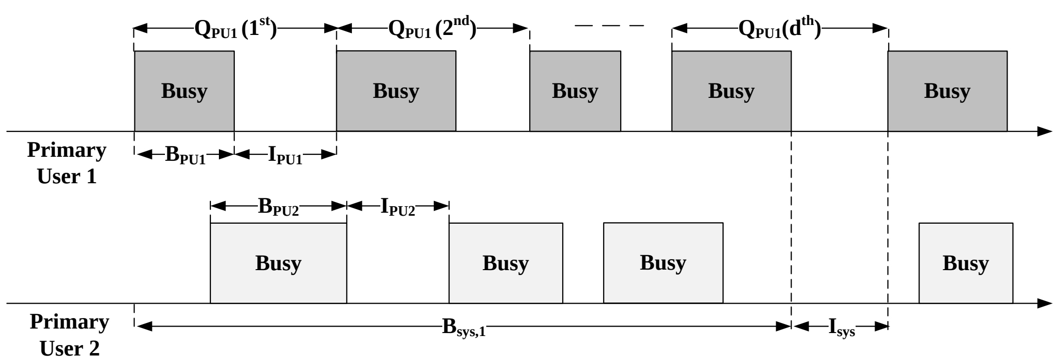

Figure 3.4 Primary user traffic in the channel where the system idle period occurs at the d^{th} cycle and the system busy period is initiated by PU1.

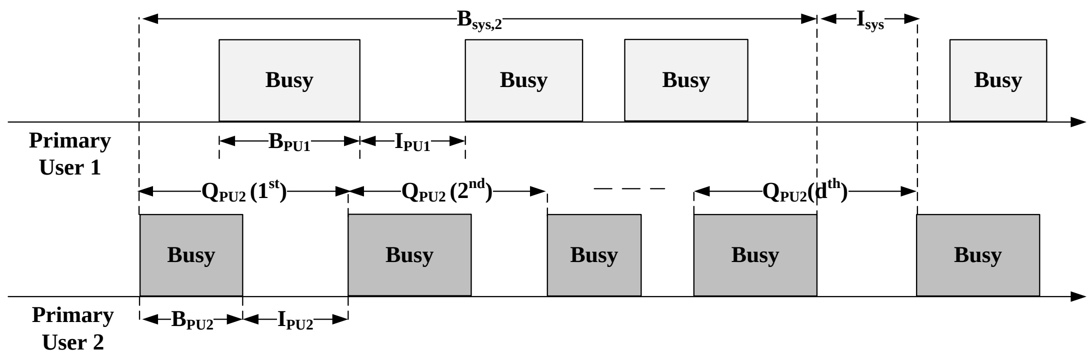

Figure 3.5 Primary user traffic in the channel where the system idle period occurs at the d^{th} cycle and the system busy period is initiated by PU2.

Let us define G_k as the probability that a system busy period will be initiated by primary user k, then,

$$G_k = \frac{\lambda_k}{\lambda_1+\lambda_2} \quad k = 1, 2. \tag{3.13}$$

Next, we will determine the duration of a system busy period given that it had been initiated by the PU1. Let $B_{sys,1}$ denote this busy period. As said above the system busy period will be expressed in terms of the PU1 cycles. Clearly, in the last cycle of system busy period, when the busy period of PU1 terminates there are only two possibilities; PU2 is either idle or busy. Based on these possibilities, the system busy period will terminate as a result of two mutually exclusive cases.

- **Case 1: PU1 becomes idle when PU2 is idle**

In this case, the system busy period terminates when the busy period of PU1 terminates as shown in Figure 3.4, thus,

$$B_{sys,1} = \sum_{i=1}^{d-1} Q_{PU1_i} + B_{PU1} \tag{3.14}$$

where Q_{PU1_i} refers to the i^{th} cycle of PU1 in the system busy period. In the above, we also assume that the summation is empty when its upper limit is smaller than its lower limit. Next we will determine the probability of this case occurring. Let us define W_1 as the probability that a busy period of PU1 will terminate in an idle period of PU2. We assume that the busy period of PU1 is equally likely to terminate at any point within a cycle of PU2, then,

$$W_1 = \bar{I}_{PU2}/(\bar{I}_{PU2} + \bar{B}_{PU2}) \tag{3.15}$$

- **Case 2: PU2 becomes idle when PU1 is idle**

In this case a system busy period terminates when the busy period of PU2 terminates as shown in Figure 3.5, thus,

$$B_{sys,1} = \sum_{i=1}^{d-1} Q_{PU1_i} + B_{PU1} + F_1 \tag{3.16}$$

where F_1 = min (R_{PU2}, I_1) with R_{PU2} being the residual busy period of the PU2 beyond the termination point of the busy period of PU1. For simplicity, we will assume that the busy period of PU2 is exponentially distributed with mean $\bar{B}_{PU2}$, as a result, $R_{PU2} = B_{PU2}$. Then, the pdf of F_1 may be determined as in (3.10) for I_{sys},

$$f_{F_1}(t) = \left(\lambda_1 + \frac{1}{\bar{B}_{PU2}}\right) e^{-\left(\lambda_1 + \frac{1}{\bar{B}_{PU2}}\right)t} \quad (3.17)$$

Denote $F_1(s)$ as the Laplace transform of the above, then it is given by,

$$F_1(s) = \frac{\left(\lambda_1 + \frac{1}{\bar{B}_{PU2}}\right)}{s + \left(\lambda_1 + \frac{1}{\bar{B}_{PU2}}\right)}$$

The first two moments of $F_1(s)$ are then given by,

$$\bar{F}_1 = -F_1'(0) = \frac{1}{\left(\lambda_1 + \frac{1}{\bar{B}_{PU2}}\right)}$$

$$\overline{F_1^2} = F_1''(0) = 2/\left(\lambda_1 + \frac{1}{\bar{B}_{PU2}}\right)^2$$

Next let us determine the probability of Case 2 occurring. Defining V_1 as the probability that residual busy period of PU2 will terminate before the idle period of PU1 terminates given that the PU2 was busy when the idle period of PU1 began. From the memoryless property of the exponential distribution,

$$Pr(B_{PU2} < I_{PU1} \mid I_{PU1} = \tau) = \int_0^\tau \frac{1}{\bar{B}_{PU2}} e^{-\frac{1}{\bar{B}_{PU2}}t} dt$$

$$= 1 - e^{-\frac{1}{\bar{B}_{PU2}}\tau}$$

$$V_1 = Pr(B_{PU2} < I_{PU1}) = \int_0^{\infty} (1 - e^{-\frac{1}{\bar{B}_{PU2}}\tau}) \lambda_1 e^{-\lambda_1 \tau} d\tau$$

$$V_1 = \frac{1/\bar{B}_{PU2}}{1/\bar{B}_{PU2} + \lambda_1} \tag{3.18}$$

Therefore the probability of case 2 occurring is given by $(1 - W_1)V_1$. Next we determine the probability that a cycle of the PU1 will contain a system idle period, H_1, which was defined in (3.12). Since this event occurs as a result of two mutually exclusive cases analyzed above,

$$H_1 = W_1 + (1 - W_1)V_1 \tag{3.19}$$

Finally, we determine the Laplace transform of the system busy period initiated by PU1 by rewriting the equations (3.14) and (3.16) as follows

$$B_{sys,1} = \begin{cases} \sum_{i=1}^{d-1} Q_{PU1_i} + B_{PU1}, & with\, prob = \tilde{p}_1 \\ \sum_{i=1}^{d-1} Q_{PU1_i} + B_{PU1} + F_1, & with\, prob = 1 - \tilde{p}_1 \end{cases}$$

where, $\tilde{p}_1 = W_1/H_1$. The above equations can be combined into one as follows,

$$B_{sys,1} = \sum_{i=1}^{d-1} Q_{PU1_i} + B_{PU1} + \phi F_1$$

where,

$$\phi = \begin{cases} 0, & with\ prob = \tilde{p}_1 \\ 1, & with\ prob = 1 - \tilde{p}_1 \end{cases}$$

Define $B_{sys,1}(s)$ as the Laplace transform of the system busy period, $B_{sys,1}(s) = E[e^{-sB_{sys,1}}]$, then we obtain

$$B_{sys,1}(s) = \tilde{p}_1\, E\left[e^{-s\left(\sum_{i=1}^{d-1} Q_{PU1_i} + B_{PU1}\right)}\right] + (1 - \tilde{p}_1)\, E\left[e^{-s\left(\sum_{i=1}^{d-1} Q_{PU1_i} + B_{PU1} + F_1\right)}\right]$$

Since that Q_{PU1_i} are i.i.d. with its Laplace transform denoted as $Q_{PU1}(s)$ and it is independent of B_{PU1},

$$B_{sys,1}(s|D_1 = d) = \tilde{p}_1 B_{PU1}(s)[Q_{PU1}(s)]^{d-1} + (1 - \tilde{p}_1) B_{PU1}(s) F_1(s)[Q_{PU1}(s)]^{d-1}$$

Unconditioning the above wrt the distribution of the number of cycles in system busy period,

$$B_{sys,1}(s) = \frac{\tilde{p}_1 B_{PU1}(s)}{Q_{PU1}(s)} \sum_{d=0}^{\infty} Q_{PU1}^d(s)\, Pr(\mathrm{D}_1 = d) + \frac{(1 - \tilde{p}_1) B_{PU1}(s) I_{PU1}(s)}{Q_{PU1}(s)} \sum_{d=0}^{\infty} Q_{PU1}^d(s)\, Pr(D_1 = d)$$

Note that $\sum_{d=0}^{\infty} Q_{PU1}^d(s)\, Pr(D_1 = d) = D_1(z)|_{z=Q_{PU1}(s)}$. Thus,

$$B_{sys,1}(s) = [\tilde{p}_1 + (1 - \tilde{p}_1) F_1(s)]\, \frac{B_{PU1}(s)}{Q_{PU1}(s)} D_1(z)|_{z=Q_{PU1}(s)} \qquad (3.20)$$

Equation (3.20) is the Laplace transform of the system busy period given that it is initiated by PU1. Similarly, the Laplace transform of the system busy period given that it has been initiated by PU2 can be obtained by the same approach and it is given by,

$$B_{sys,2}(s) = [\tilde{p}_2 + (1 - \tilde{p}_2)F_2(s)]\ \frac{B_{PU2}(s)}{Q_{PU2}(s)} D_2(z)|_{z=Q_{PU2}(s)} \tag{3.21}$$

Now we obtain the Laplace transform of the system busy period as,

$$B_{sys}(s) = G_1 B_{sys,1}(s) + G_2 B_{sys,2}(s) \tag{3.22}$$

where G_k is given in (3.13).

Next, we will determine the mean of system busy period ($\bar{B}_{sys}$) given by,

$$\begin{aligned} \bar{B}_{sys} &= -B'_{sys}(s)\Big|_{s=0} \\ &= G_1 \bar{B}_{sys,1} + G_2 \bar{B}_{sys,2} \end{aligned} \tag{3.23}$$

We can determine $\bar{B}_{sys,1}$ and $\bar{B}_{sys,2}$by taking derivatives of (3.20) and (3.21), and evaluating at $s = 0$,

$$\bar{B}_{sys,1} = (1 - \tilde{p}_1)\bar{I}_{PU1} + \bar{D}_1 \bar{Q}_{PU1} + \bar{B}_{PU1} - \bar{Q}_{PU1} \tag{3.24}$$

$$\bar{B}_{sys,2} = (1 - \tilde{p}_2)\bar{I}_{PU2} + \bar{D}_2 \bar{Q}_{PU2} + \bar{B}_{PU2} - \bar{Q}_{PU2} \tag{3.25}$$

Substituting (3.24) and (3.25) in (3.23) then we obtain the mean value of the system busy period,

$$\bar{B}_{sys} = G_1[(1-\tilde{p}_1)\bar{F}_1 + (\bar{D}_1 - 1)\bar{Q}_{PU1} + \bar{B}_{PU1}] + G_2[(1-\tilde{p}_2)\bar{F}_2 + (\bar{D}_2 - 1)\bar{Q}_{PU2} + \bar{B}_{PU2}] \tag{3.26}$$

Next, we present the second moment of the system busy period,

$$\overline{B_{sys}^2} = B''_{sys}(0) = G_1 B''_{sys,1}(0) + G_2 B''_{sys,2}(0) \tag{3.27}$$

where $B''_{sys,k}(0),\ k = 1,2$ are given by

$$B''_{sys,k}(0) = (1-\tilde{p}_k)[\,F''_k(0) + 2\bar{F}_k(\bar{D}_k\bar{Q}_{PUk} + \bar{B}_{PUk})] + Q''_{PUk}(0)(\bar{D}_k - 1) + B''_{PUk}(0)$$
$$+ \bar{Q}_{PUk}\left[\bar{Q}_{PUk}\mathrm{D}''_k(1) + 2\bar{D}_k\bar{B}_{PUk} - 2\bar{B}_{sys,k}\right]$$

3.4 Modeling of Secondary User

In this section, we present the derivation of the mean packet delay of the secondary user and the secondary user traffic load.

3.4.1 Mean Packet Delay of the Secondary User

As before, we also model secondary user as an M/G/1 queue. We assume that packets arrive to the secondary user queue according to Poisson process with arrival rate of α packets/sec. We refer to the analysis of the secondary user in Chapter 2 where the mean number of secondary packets in the queue and the mean packet delay of the secondary user are derived and given by equation (2.18) and (2.19a) respectively. However, in the multiple primary users model, we need to take into account presence of different primary users at the secondary transmitter and secondary receiver. The Markov chain embedding points are at the end of the system periods as shown in Figure 3.6.

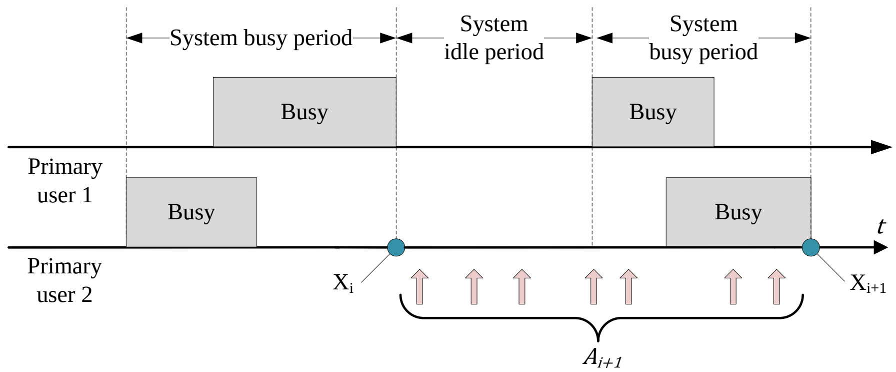

Figure 3.6 Activities in the channel and the embedded points.

The mean number of packets in the secondary queue given in equation (2.18) applies with the interpretation of primary busy and idle periods as system busy and idle periods, thus,

$$\bar{x}_{sys} = \frac{2rA'_{sys}(1)X(r_{sys}) + 2(1-r_{sys})A'_{sys}(1) + (1-r_{sys})A''_{sys}(1)}{2[r_{sys} - (1-r_{sys})A'_{sys}(1)]} \qquad (3.28)$$

where, $A_{sys}(z)$ is PGF of the number of secondary packet arrivals during the system cycle period $(Q_{sys} = I_{sys} + B_{sys})$. The unknowns, $A'_{sys}(1), A''_{sys}(1)$ and $X(r)$, can be determined in a similar approach to that in chapter 2. Thus,

$$X(r_{sys}) = \frac{r_{sys} - (1 - r_{sys})A'_{sys}(1)}{r_{sys}}$$

$$r_{sys} = P(I_{sys} > M_s)$$

$$= M_s(s)|_{s = \lambda_{sys}}$$

where λ_{sys} is the sum of the arrival rates of PU1 and PU2, $\lambda_{sys} = \lambda_1 + \lambda_2$. Next, let us determine $A'_{sys}(1)$ and $A''_{sys}(1)$. Again, we can apply the result from (2.15), thus we have,

$$A_{sys}(z) = B_{sys}[\alpha(1 - z)] \frac{\lambda_{sys}}{\alpha(1 - z) + \lambda_{sys}}$$

$$A'_{sys}(1) = \frac{\alpha(1 + \lambda_{sys}\bar{B}_{sys})}{\lambda_{sys}}$$

$$A''_{sys}(1) = \alpha^2 B''_{sys}(0) + 2\frac{\alpha}{\lambda_{sys}} A'_{sys}(1)$$

Finally, from (2.19a) the mean packet delay of the secondary user is given by

$$\bar{d} = \frac{\bar{x}_{sys}}{\alpha} = \frac{2r_{sys}A'_{sys}(1)X(r_{sys}) + 2(1-r_{sys})A'_{sys}(1) + (1-r_{sys})A''_{sys}(1)}{2\alpha[r_{sys} - (1-r_{sys})A'_{sys}(1)]} \qquad (3.29)$$

3.4.2 The Secondary Traffic Load

The secondary traffic load is defined as the probability of having non-empty queue which can be written as

$$\rho_s = 1 - X(0)$$

where, $$X(0) = X(r_{sys})A_{sys}(0)$$

$$A_{sys}(0) = \frac{\lambda_{sys}B_{sys}(\alpha)}{\alpha + \lambda_{sys}}$$

We can find $B_{sys}(\alpha)$ by substituting $s = \alpha$ in (3.22), which results in,

$$B_{sys}(\alpha) = G_1 B_{sys,1}(\alpha) + G_2 B_{sys,2}(\alpha)$$

3.5 Numerical and Simulation Results

In this section, we present the numerical and simulation results based on the analysis developed in the chapter. From calculation, we plot the mean packet delay of the secondary user versus its traffic load and compare the results with that in single primary user case. We assume constant packet transmission times for both primary and secondary users. Table 3.1 shows the values of parameters used in the calculation.

Table 3.1 Parameter setting

Primary user 1	**Primary user 2**	**Secondary user**
Arrival rate (λ_1) = 1, 2, 3 packets/sec	Arrival rate (λ_2) = 2, 4, 6 packets/sec	Arrival rate (α) = variable
Packet transmission time $(T_1) = 0.2$ sec	Packet transmission time $(T_2) = 0.1$ sec	Packet transmission time $(T_{su}) = 0.1$ sec
P(successful transmission) = 0.9	P(successful transmission) = 0.9	P(successful transmission) = 0.9

We plot the mean packet delay of the secondary user versus its traffic load (ρ_s) for different primary user traffic loads (ρ_{PU1} and ρ_{PU2}). The results show that as the primary users' loads increase, the mean packet delay of the secondary user increases as shown in Figure 3.7.

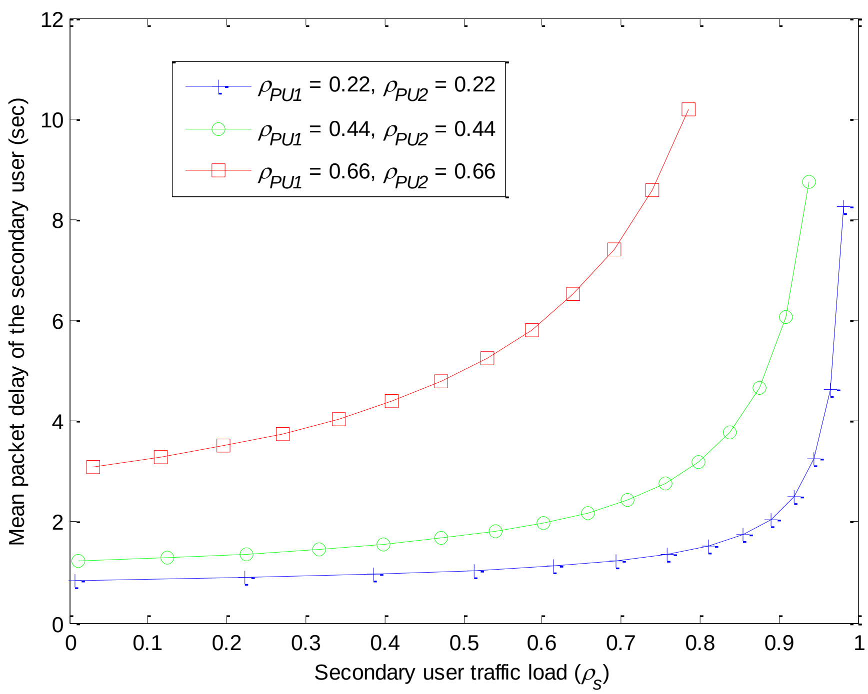

Figure 3.7 Mean packet delay of the secondary user at various primary traffic loads

An important question is how much the two-primary user model affects the mean packet delay of the secondary user compared to that in the single primary user model. In Figure 3.8, we plot the mean packet delay of the secondary user for both cases to show the differences. The traffic load in the single primary user model is set to the sum of PU1 and PU2 traffic loads in multiple primary users case. Interestingly, the mean packet delays of the secondary user of the two cases are relatively close to each other.

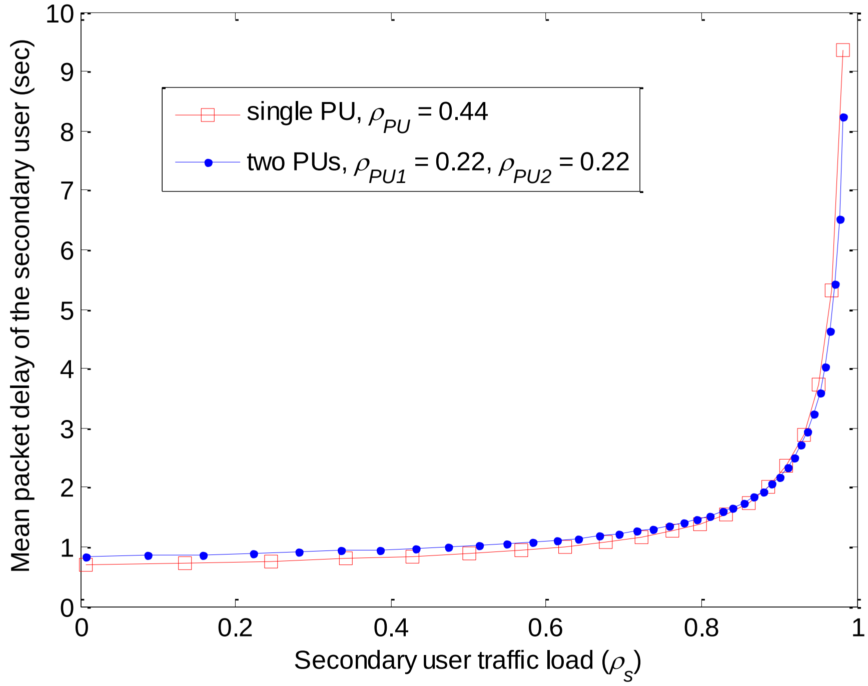

Figure 3.8 Mean packet delay of the secondary user when the primary user traffic load in the single PU case equals to the sum of two primary loads of the two primary user case.

In Figure 3.9, we also plot the mean packet delay of the secondary user when the traffic loads of PU1 and PU2 are equal to that of the single primary case. As expected, the mean packet delay of the secondary user in multiple primary users model is considerably higher than that in the single primary user model.

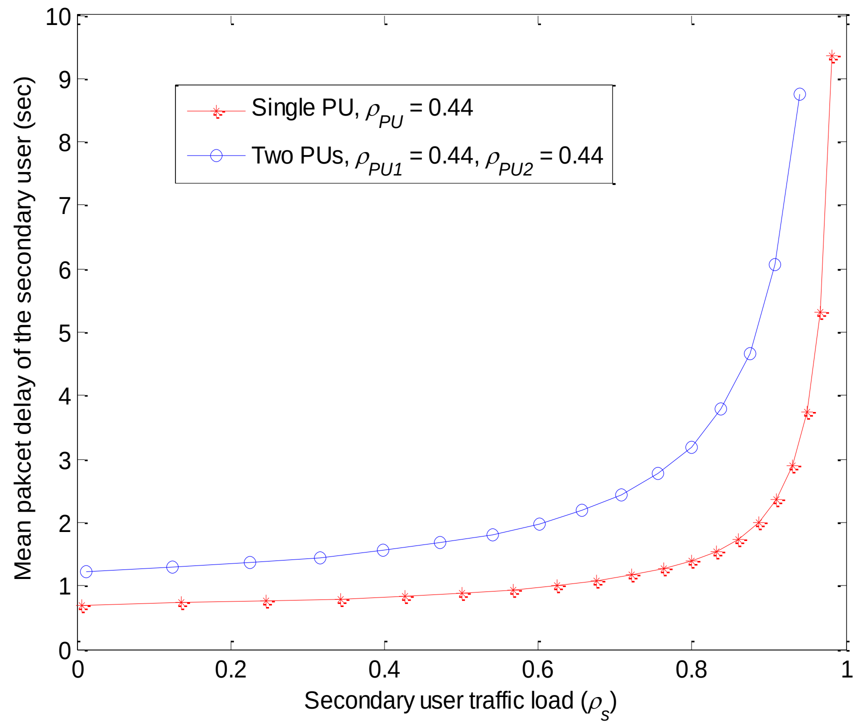

Figure 3.9 Mean packet delay of the secondary user in the single and multiple primary cases where the traffic loads of primary users are equal to each other.

Next we present some simulation results to verify the accuracy of the analysis. We developed simulation environment in Matlab® that simulates the system. Packets arrive at the primary and secondary users according to Poisson process. Packets are stored in infinite length queues and served on a FCFS basis. We record arrival and departure times of each packet in each queue. The simulation stops when the number of departures from the secondary user reaches 3000 packets. We obtain the mean packet delay of the secondary user by averaging the difference of packet departure and arrival times. The simulation results plotted in Figure 3.10 are averaged over 10 simulation runs. We set $\lambda_1 = 2$ and $\lambda_2 = 4$ while packet transmission times are, $T_1 = 0.2$ and $T_2 = 0.1$.

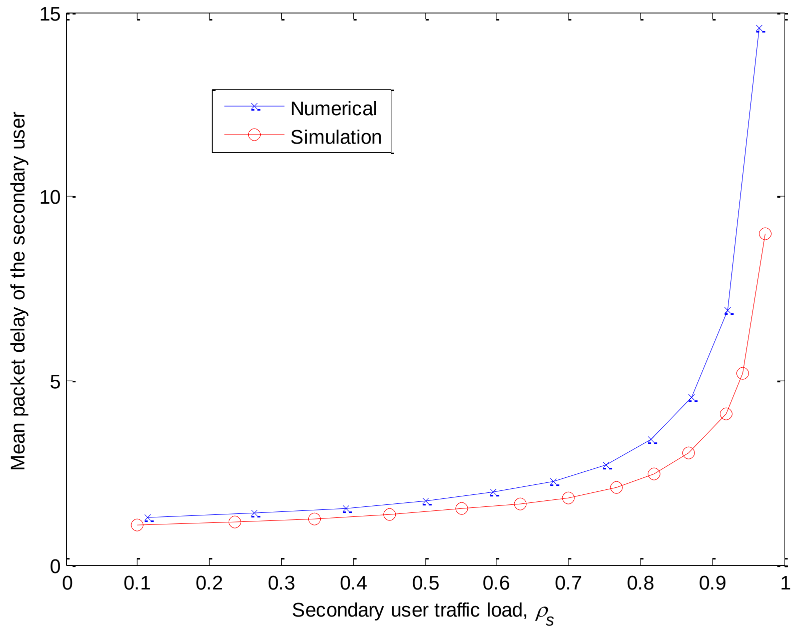

Figure 3.10 Mean packet delay of the secondary user by calculation and simulation

In Figure 3.10, we see that there is some difference between numerical and simulation results. It may be seen that the difference gets larger with the increasing secondary traffic load. However, the results are close enough and it justifies the various approximations in the analysis including the assumption of exponentially distributed primary busy period.

Chapter 4

A Performance Evaluation of Cognitive Radio Networks with Multi-Antenna

Multi-antenna techniques have been proposed in cognitive radio literature for different purposes, for example, spectrum utilization improvement, interference cancellation and spectrum sensing techniques [37]-[39]. In [37], the cyclostationarity spectrum sensing method is enhanced by employing multi-antenna at the secondary receivers. The sensing performance is improved as the multi-antenna enables spatial diversity allowing more precise view of primary user's spectrum usage. In addition, by using multi-antenna, interference cancellation can be achieved by allowing secondary user to select antenna weight that put null in the direction of the primary receiver [4], [38].

In this chapter, we focus on the use of multi-antenna in cognitive radio for interference cancellation. With this technique, the secondary user will be able to transmit continuously without harmful interference to the primary user. In particular, the secondary user transmits with two different transmission rates depending on whether the channel is idle or occupied by the primary user. We derive the mean packet delay of the secondary user in this model.

4.1 System Model

We consider a cognitive radio network that consists of a channel shared by a single primary and a secondary user. We assume a multi-antenna at the secondary transmitter and single antenna at the primary transmitter and all receivers. The primary user has priority in accessing the channel while the secondary user transmits under interference constraint to protect the primary user transmissions. The secondary user is allowed to transmit all the time; however, it transmits with full power when the channel is idle and with partial power when the channel is occupied by the primary user. In the latter case, part of the available power is used to cancel the interference to the primary user. The two different transmission power levels result in two different transmission rates of the secondary user. We note that in the following idle and busy channel will always refer to whether the primary is inactive or active in the channel.

Transmissions from the secondary transmitter may interfere with the primary receiver depending on its location, power and direction of transmission. We assume that the secondary transmitter transmits to the secondary receiver at an angle θ wrt the axis of the secondary transmitter and the primary receiver. The interference introduced to the primary receiver is a function of the secondary transmission power and the angle θ.We make similar model assumptions to the previous chapters. Thus we model the secondary user as an M/G/1 queue where the arrivals are according to a Poisson process with the rate of α packets/sec and service times are generally distributed. We assume that a packet may be transmitted successfully or lost in each transmission due to wireless channel errors according to an independent Bernoulli process. The transmission of a packet is repeated until it is successfully received. We assume that the service time of a packet begins with the beginning of its first transmission and ends with the completion of the successful transmission. The transmission rate of the secondary user will change according to the state of the primary user during the service time of a packet. For the simplicity of the analysis, we will assume that the channel will remain in its initial state for the duration of the secondary packet service time. This is reasonable under the assumption that the busy and idle periods of the primary user will be significantly longer than the service time of the secondary user packets. The validity of this assumption will be tested by simulations.

4.2 Transmission Rates and of the Secondary Transmitter

As explained earlier, the secondary user transmits at two different transmission power levels depending on the availability of the channel. If the channel is idle, the secondary transmitter fully exploits the available spectrum by transmitting at full power. On the other hand, when the channel is utilized by the primary user, the secondary transmitter needs to satisfy the interference constraint by allocating part of its available power to cancel the interference to the primary receiver while using remaining power for its own packet transmission. In this section, we relate transmission rates of the secondary user for these two situations; idle and busy channel.

Figure 4.1 shows an example location of the primary and secondary users. PR, SR, ST refer to primary receiver, secondary receiver and secondary transmitter, respectively. SR is located at an angle θ wrt the line between ST and PR. The θ is assumed to be uniformly distributed from 0 to 2π. While ST transmits to SR with transmission power $\tilde{S}$, it introduces $\tilde{S}cos\theta$ interference to PR. To cancel this interference, ST directs the same amount of power in the direction opposite to the primary receiver. Note that when the channel is idle, ST can transmit with its full power without the danger of interference to the PR. Next, let us define the following notation;

$\tilde{S}$ $\triangleq$ transmission power of the ST in the direction of SR when the channel is busy

S $\triangleq$ transmission power of the ST in the direction of SR when the channel is idle

$\tilde{C}$ $\triangleq$ transmission rate of the ST when the channel is busy

C $\triangleq$ transmission rate of the ST when the channel is idle

$\tilde{T}_s$ $\triangleq$ packet transmission time of the ST when the channel is busy

T_s $\triangleq$ packet transmission time of the ST when the channel is idle

Note that S is also the total transmission power of ST. Therefore, we obtain the relation of the transmission power during the idle and busy periods of the channel as follows,

$$S = \tilde{S} + \tilde{S}cos\theta$$

$$\tilde{S} = S/(1 + cos\theta) \tag{4.1}$$

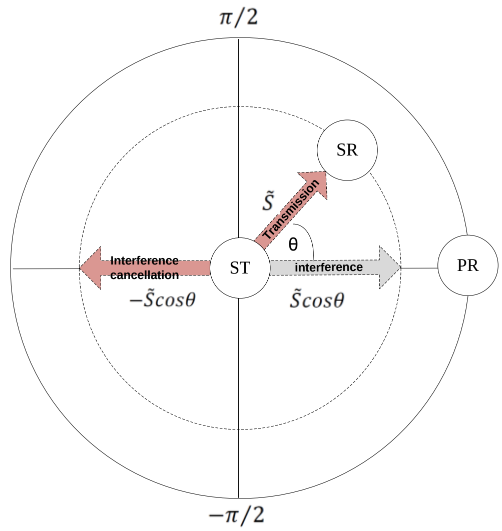

Figure 4.1 The transmission powers and directions of the secondary user

According to the well-known Shannon's channel capacity theorem, we have $C = Wlog_2(1 + SNR)$ where R is a transmission rate in bit per second, W refers to the bandwidth of the channel in hertz and SNR is signal to noise ratio of the channel in dB. The approximation of the Shannon's channel capacity theorem states,

$$C \approx 0.332\, W \left(\frac{S}{N}\right), \quad when\ \frac{S}{N} \gg 1$$

$$C \approx 1.44\, W \left(\frac{S}{N}\right), \quad when\ \frac{S}{N} \ll 1 \tag{4.2}$$

Therefore, we obtain the ratio between the transmission rates when the channel is busy and idle as follows.

$$\frac{\tilde{C}}{C} = \frac{\tilde{S}}{S}$$

Substituting for $\tilde{S}$ from (4.1)

$$\tilde{C} = C/(1 + cos\theta) \tag{4.3}$$

Since θ has been assumed to be uniformly distributed between 0 to 2π, its probability density function (pdf) is given by $1/2\pi$. From Figure 4.1 when the channel is occupied by the primary user, if ST is transmiting and SR is located on the left plane, $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$, there will be no interference directed to PR. Consequently, ST can always transmit with full transmission power, S. On the other hand, if SR is on the right plane, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, ST will need to transmit with partial power, $\tilde{S} = S/(1 + cos\theta)$as part of its power is allocated to suppress the interference. Therefore, we can determine the average transmission rate of ST when the channel is busy, denoted as $\bar{C}$, as follows.

$$\bar{C} = C \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} f(\theta) d\theta + C \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1 + cos\theta} f(\theta) d\theta$$

where $f(\theta) = 1/2\pi$,

$$\bar{C} = C \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{2\pi} d\theta + C \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{1 + cos\theta}\right) \frac{1}{2\pi} d\theta$$

$$\bar{C} = C \left(\frac{1}{2} + \frac{1}{\pi}\right) \tag{4.4}$$

In the following, we will assume that the ST always transmits with the average rate $\bar{C}$ when the channel is busy. Therefore, the relation of the packet transmission time when the channel is idle and busy is given by,

$$\tilde{T}_s = T_s / \left(\frac{1}{2} + \frac{1}{\pi}\right) \qquad (4.5)$$

4.3 Modeling of the Secondary User

The secondary user is modeled as an M/G/1 queue where arrivals are according to a Poisson process with arrival rate of α packets/sec and service times have general distribution. We will assume that the channel will be idle or busy according to an independent Bernoulli random variable at the beginnings of packet service times with probabilities p_I and p_B respectively and the state of the channel will not change during the service time of a packet. Figures 4.2 and 4.3 show the activities in the channel when the secondary packet service time begins with the idle and busy channel respectively. We choose the end of packet service times as embedded points. Let us define the following;

$M_s \triangleq$ service time of a secondary user's packet when the channel is idle

$\widetilde{M}_s \triangleq$ service time of a secondary user's packet when the channel is busy

$A_{i+1} \triangleq$ number of secondary packet arrivals during the service time of the $(i+1)^{st}$ packet if it is given by M_s

$\tilde{A}_{i+1} \triangleq$ number of secondary packet arrivals during the service time of the $(i+1)^{st}$ packet if it is given by $\widetilde{M}_s$

$X_i \triangleq$ number of packets in secondary user after the service time of i^{th} packet has been completed

$I_i \triangleq$ Bernoulli random variable that defines whether the channel is idle or busy at the beginning of the service time of i^{th} packet

We will determine mean packet delay of the secondary user in several steps

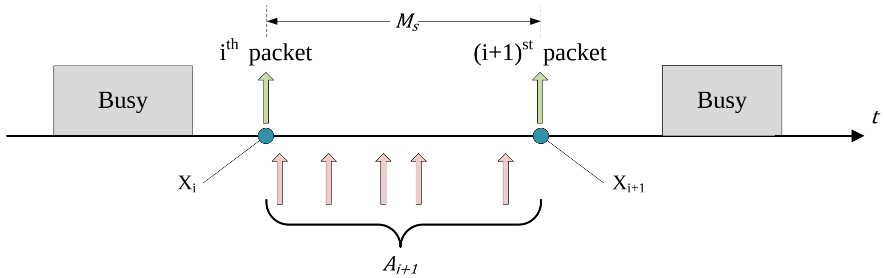

Figure 4.2 The activities in the channel in which ST transmits when the channel is idle

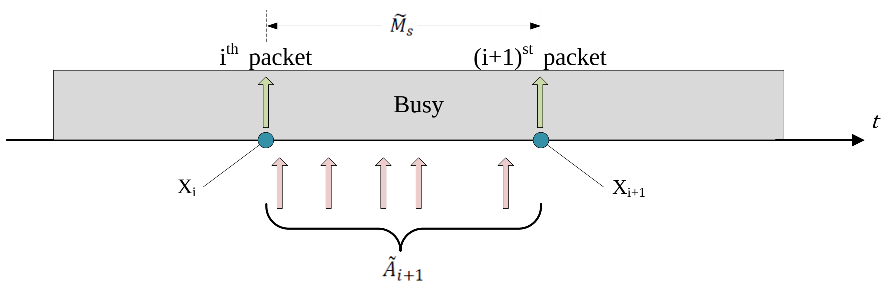

Figure 4.3 The activities in the channel in which ST transmits when the channel is busy

4.3.1 The Service Time of a Secondary User Packet

Let us first determine the Laplace transform of the service time of a secondary packet when the channel is idle or busy, denoted as $M_s(s)$ and $\widetilde{M}_s(s)$, respectively. We will assume that a packet may be successfully transmitted or lost in each transmission according to an independent Bernoulli process and the probability of successful transmission is p_s. Then the number of transmissions until a packet is received successfully denoted as N has a geometric distribution. From the Laplace transform definition, we have

$$M_s(s|N = n) = E[e^{-snT_s}], \qquad \widetilde{M}_s(s|N = n) = E[e^{-sn\widetilde{T}_s}]$$

We assume that packet transmission time T_s is constant and $\tilde{T}_s$ can be obtained from (4.5). We also have $Pr(N = n) = {q_s}^{n-1}p_s$, where $q_s = 1 - p_s$ Thus,

$$M_s(s) = \sum_{n=1}^{\infty} e^{-snT_s}\, Pr(N = n)$$

$$= \sum_{n=1}^{\infty} e^{-snT_s}\, {q_s}^{n-1}p_s$$

$$= \frac{p_s e^{-sT_s}}{1 - q_s e^{-snT_s}}$$

Therefore, we have the Laplace transform of the service time of a packet when the channel is idle and busy as follows,

$$M_s(s) = \frac{p_s e^{-sT_s}}{1-q_s e^{-snT_s}} \text{ and } \widetilde{M}_s(s) = \frac{p_s e^{-s\widetilde{T}_s}}{1-q_s e^{-sn\widetilde{T}_s}} \qquad (4.6)$$

The first two moments of the service time of a packet are obtained by taking successive derivatives of (4.6) and substituting $s = 0$, thus

$$\overline{m}_s = -M_s'(0) = \frac{T_s}{\mathrm{p}_s}, \qquad \overline{\widetilde{m}}_s = -\widetilde{M}_s'(0) = \frac{\tilde{T}_s}{p_s}$$

$$\overline{m_s^2} = M_s''(0) = \frac{{T_s}^2(2-p_s)}{{p_s}^2}, \qquad \overline{\widetilde{m}_s^2} = \widetilde{M}_s''(0) = \frac{{\tilde{T}_s}^2(2-p_s)}{{p_s}^2}$$

(4.7)

4.3.2 The PGF of the Secondary User Queue Length

We assume that the Markov chain embedding points are packet departure points. From Figure 4.2 and 4.3, the number of packets in the secondary user queue at the embedded points is given by,

$$X_{i+1} = X_i - U(X_i) + A_{i+1}, \qquad with\ prob = p_I \tag{4.8a}$$

$$X_{i+1} = X_i - U(X_i) + \tilde{A}_{i+1}, \qquad with\ prob = p_B \tag{4.8b}$$

where, $U(X_i) = \begin{cases} 0, & X_i = 0 \\ 1, & X_i > 0 \end{cases}$

We can combine the equations (4.8a) and (4.8b) into one by defining,

$$I_i = \begin{cases} 0, & with\ probability\ p_I \\ 1, & with\ probability\ p_B \end{cases}$$

Thus,

$$X_{i+1} = X_i - U(X_i) + (1 - I_{i+1})A_{i+1} + I_{i+1}\tilde{A}_{i+1} \tag{4.9}$$

From (4.9) we can determine the PGF of the number of packets in the secondary user queue, $X_i(z)$. The PGF of X_{i+1} is given by,

$$\begin{aligned} X_{i+1}(z) &= E[z^{X_{i+1}}] \\ &= E[z^{X_i - U(X_i) + (1 - I_{i+1})A_{i+1} + I_{i+1}\tilde{A}_{i+1}}] \end{aligned} \tag{4.10}$$

Since the number of packets in the system and the number of new arrivals are independent of each other, (4.10) can be written as

$$X_{i+1}(z) = E\left[z^{X_i - U(X_i)}\right] E\left[z^{(1-I_{i+1})A_{i+1} + I_{i+1}\tilde{A}_{i+1}}\right] \quad (4.11)$$

Next, we give the derivations of $E\left[z^{X_i - U(X_i)}\right]$ and $E\left[z^{(1-I_{i+1})A_{i+1} + I_{i+1}\tilde{A}_{i+1}}\right]$ separately,

$$E\left[z^{X_i - U(X_i)}\right] = \sum_{k=0}^{\infty} z^{k-U(k)} Pr(X_i = k)$$

$$= z^0 Pr(X_i = 0) + \sum_{k=1}^{\infty} z^{k-1} Pr(X_i = k)$$

$$= Pr(X_i = 0) + z^{-1}\left[\sum_{k=0}^{\infty} z^k Pr(X_i = k) - z^0 Pr(X_i = 0)\right]$$

$$E\left[z^{X_i - U(X_i)}\right] = \frac{1}{z}\left[zPr(X_i = 0) + X_i(z) - Pr(X_i = 0)\right] \quad (4.12)$$

and $E\left[z^{(1-I_{i+1})A_{i+1} + I_{i+1}\tilde{A}_{i+1}}\right]$ is given by

$$E\left[z^{(1-I_{i+1})A_{i+1} + I_{i+1}\tilde{A}_{i+1}}\right] = p_I E\left[z^{A_{i+1}}\right] + p_B E\left[z^{\tilde{A}_{i+1}}\right] \quad (4.13)$$

We can drop the subscript $(i+1)$ in (4.13) as the number of arrivals during packet service times does not depend on the cycle number. Letting, $A(z) = E\left[z^{A_{i+1}}\right]$ and $\tilde{A}(z) = E\left[z^{\tilde{A}_{i+1}}\right]$. Thus,

$$E\left[z^{(1-I_{i+1})A_{i+1} + I_{i+1}\tilde{A}_{i+1}}\right] = p_I A(z) + p_B \tilde{A}(z) \quad (4.14)$$

Substituting (4.12) and (4.13) into (4.11) we obtain $X_{i+1}(z)$,

$$X_{i+1}(z) = \frac{1}{z}[zPr(X_i = 0) + X_i(z) - Pr(X_i = 0)]\left[p_I A(z) + p_B \tilde{A}(z)\right]$$

At the steady state, $X_i(z) = X_{i+1}(z) = X(z)$. Thus, we can drop the subscript $(i+1)$ in the above and solve for $X(z)$,

$$X(z) = \frac{Pr(X = 0)(z-1)\left[p_I A(z) + p_B \tilde{A}(z)\right]}{z - \left[p_I A(z) + p_B \tilde{A}(z)\right]} \tag{4.15}$$

We determine the unknowns, $Pr(X = 0)$, $A(z)$ and $\tilde{A}(z)$ in (4.15) in the following subsections,

- **Derivation of $Pr(X = 0)$**

$Pr(X = 0)$ can be obtained from the normalization condition by applying L'Hospital rule to (4.15) and substituting $z = 1$,

$$\left.\frac{Pr(X = 0)(z-1)\left[p_I A'(z) + p_B \tilde{A}'(z)\right] + \left[p_I A(z) + p_B \tilde{A}(z)\right]Pr(X_i = 0)}{1 - \left[p_I A'(z) + p_B \tilde{A}'(z)\right]}\right|_{z=1} = 1$$

Solving for $Pr(X = 0)$ and noting that $p_I + p_B = 1$, we have,

$$Pr(X = 0) = 1 - \left[p_I A'(1) + p_B \tilde{A}'(1)\right] \tag{4.16}$$

- **Derivation of $A(z)$ and $\widetilde{A}(z)$**

Secondly, $A(z)$ and $\tilde{A}(z)$ are the PGFs of the number of Poisson arrivals during the packet service times M_s and $\widetilde{M}_s$ respectively. Denote $M_s(s)$ and $\widetilde{M}_s(s)$ as Laplace transforms of M_s and $\widetilde{M}_s$ respectively. The PGF of the number of Poisson arrivals are related to the Laplace transforms of the service times as follows.

$$A(z) = M_s(s)|_{s=\alpha(1-z)} \tag{4.17a}$$

$$\tilde{A}(z) = \widetilde{M}_s(s)|_{s=\alpha(1-z)} \tag{4.17b}$$

Note that $M_s(s)$ and $\widetilde{M}_s(s)$ are given by (4.6) and α is the arrival rate of the packets to the secondary user.

4.3.3 The Mean Packet Delay of the Secondary User

From (4.15) we can determine the mean number of packets in the secondary user system by taking derivative of $X(z)$ and evaluating at $z = 1$. In (4.15), let us take the denominator onto the left hand side. Thus,

$$X(z)\left[z - \left(p_I A(z) + p_B \tilde{A}(z)\right)\right] = Pr(X = 0)(z - 1)\left[p_I A(z) + p_B \tilde{A}(z)\right] \tag{4.18}$$

Taking second derivative of the above equation gives,

$$X(z)\left\{-\left[p_I A''(z) + p_B \tilde{A}''(z)\right]\right\} + \left\{1 - \left[p_I A'(z) + p_B \tilde{A}'(z)\right]\right\}X'(z) +$$

$$\left\{z - \left[p_I A(z) + p_B \tilde{A}(z)\right]\right\}X''(z) + X'(z)\left\{1 - \left[p_I A'(z) + p_B \tilde{A}'(z)\right]\right\} =$$

$Pr(X = 0)(z - 1)\left[p_I A''(z) + p_B \tilde{A}''(z)\right] + \left[p_I A'(z) + p_B \tilde{A}'(z)\right] Pr(X = 0) +$

$\left[p_I A'(z) + p_B \tilde{A}'(z)\right] Pr(X = 0)$ (4.19)

Evaluating the above at $z = 1$, we get

$$\left\{-\left[p_I A''(1) + p_B \tilde{A}''(1)\right]\right\} + 2\left\{1 - \left[p_I A'(1) + p_B \tilde{A}'(1)\right]\right\} X'(1) = 2\left[p_I A'(1) + p_B \tilde{A}'(1)\right] Pr(X = 0)$$

From the above, we determine the mean number of packets in the secondary queue, $\bar{x}$, which is given by $\bar{x} = X'(z)|_{z=1}$. Thus,

$$\bar{x} = \frac{2Pr(X = 0)\left[p_I A'(1) + p_B \tilde{A}'(1)\right] + \left[p_I A''(1) + p_B \tilde{A}''(1)\right]}{2\left\{1 - \left[p_I A'(1) + p_B \tilde{A}'(1)\right]\right\}} \tag{4.20}$$

The unknowns, $A'(1)$, $A''(1)$, $\tilde{A}'(1)$ and $\tilde{A}''(1)$ in (4.20), can be obtained by taking derivatives in (4.17a) and (4.17b) and evaluating at $z = 1$, which are given below,

$$A'(1) = -\alpha M_s'(0), \qquad A''(1) = \alpha^2 M_s''(0) \tag{4.21a}$$

$$\tilde{A}'(1) = -\alpha \widetilde{M}_s'(0), \qquad \tilde{A}''(1) = \alpha^2 \widetilde{M}_s''(0) \tag{4.21b}$$

where the first two moments of a packet service time of a secondary user are given in (4.7). Finally, the mean packet delay of the secondary user, denoted as $\bar{d}$, is obtained by applying Little's result which states that $\bar{d} = \bar{x}/\alpha$. Thus,

$$\bar{d} = \frac{2Pr(X=0)[p_I A'(1)+p_B \tilde{A}'(1)] + [p_I A''(1)+p_B \tilde{A}''(1)]}{2\alpha\{1-[p_I A'(1)+p_B \tilde{A}'(1)]\}} \tag{4.22}$$

4.3.4 Probabilities of Idle and Busy Channel

Finally, we will determine the probabilities of idle and busy channel at the beginning of secondary user packet service time, p_I and p_B in the above expression. We will model the primary user also as an M/G/1 queue where the arrivals are according to a Poisson process with the rate of λ packets/sec and service times are generally distributed. We assume that a primary packet may be received successfully with probability p_p in a transmission and the packet will be retransmitted until it is successfully received. We assume that the service time of a secondary user packet is equally likely to occur at any point of time within a cycle of the primary user. Therefore, the probabilities of idle and busy channel, p_I and p_B, are given by the ratios of the mean idle and busy period over the mean cycle period of the primary user, respectively,

$$p_I = \frac{\bar{I}_p}{\bar{Q}_p}, \quad p_B = \frac{\bar{B}_p}{\bar{Q}_p} \tag{4.23}$$

where $\bar{Q}_p = \bar{I}_p + \bar{B}_p$. Note that in M/G/1 queue the mean idle and busy periods of the primary user are given by $\bar{I}_p = 1/\lambda$ and $\bar{B}_p = \bar{m}_p/(1-\rho_p)$ where $\bar{m}_p$ is the mean of packet service time of the primary user and ρ_p denotes the traffic load of the primary user which is given by $\rho_p = \lambda\bar{m}_p$.

4.3.5 Secondary User Traffic Load

The secondary user traffic load, denoted as ρ_s is defined as the probability of secondary user having nonempty system,

$$\rho_s = 1 - Pr(X = 0)$$

Recall that the probability of having zero packets in the secondary user system is already obtained in (4.16) and it is given by

$$Pr(X = 0) = 1 - \left[p_I A'(1) + p_B \tilde{A}'(1)\right]$$

Thus, we obtain the secondary user traffic load as follows,

$$\rho_s = p_I A'(1) + p_B \tilde{A}'(1) \tag{4.24}$$

4.4 Numerical and Simulation Results

In this section, we show the numerical results regarding the analysis developed in this chapter together with simulation results to confirm the accuracy of the analysis. In addition, we determine the probability of interference to the primary user and percentage of the duration interference in the busy period of the primary user through simulation.

4.4.1 Mean Packet Delay of the Secondary User

We plot the mean packet delay of the secondary user versus its traffic load and compare the numerical results against simulation results and to that of the single antenna case. Table 4.1 shows the values of parameters used in the analysis and simulations. From the values in Table 4.1, idle and busy channel probabilities at the beginning of a secondary user packet service time are given by, $p_I = 0.43$ and $p_B = 0.57$ which are used in the calculations and simulations.

Table 4.1 Parameter setting

	Primary user	**Secondary user**
Arrival rates	$\lambda = 4$ packets/sec	$\alpha =$ variable (multi-antenna) $\alpha =$ variable (single-antenna)
Probability of successful transmission	$p_p = 0.7$	$p_s = 0.6$
Packet transmission time	$T_p = 0.1$ sec	$T_s = 0.1$ sec $\tilde{T}_s = T_s/\left(\frac{1}{2}+\frac{1}{\pi}\right) = 0.12$ sec

Firstly, we present numerical results. Figure 4.4 shows the plots of the mean packet delay of the secondary user in the cases of multi-antenna and single-antenna. The results show that for this case, the mean packet delay of the secondary user in multi-antenna scenario is significantly reduced compare to that of the single-antenna scenario where the secondary user transmits only when the channel is idle.

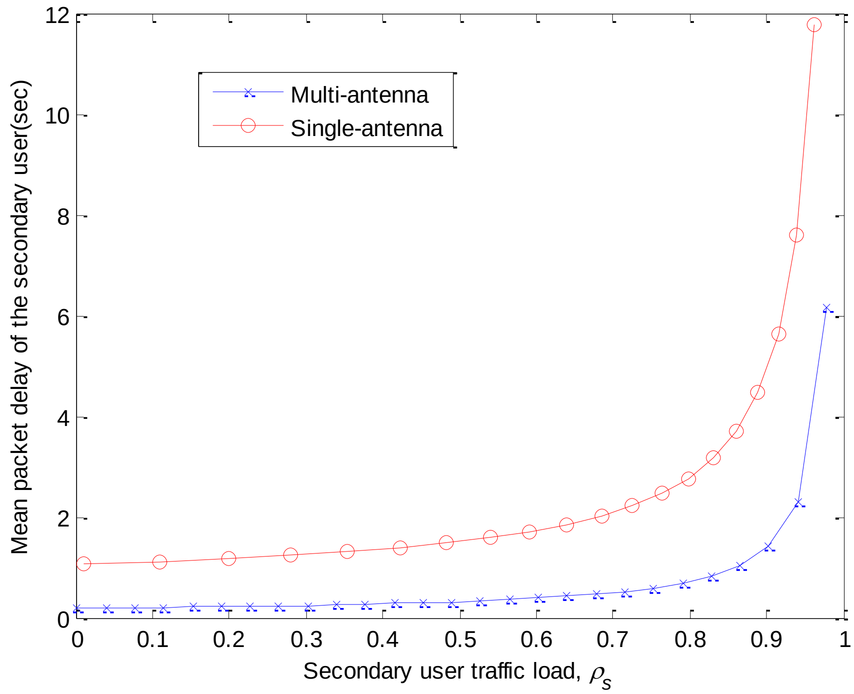

Figure 4.4 Mean packet delay of the secondary user in multi- and single-antenna cases, as a function of secondary user traffic load.

We have developed a simulation program using Matlab® which simulates the studied model. The arrivals to each of the queues are according to Poisson arrival process and the packet transmission times are constant as shown in Table 4.1. In each simulation run, we generate 5000 packet arrivals into the system and determine the mean packet delay of a secondary user. The mean packet delay results plotted in Figure 4.5 have been averaged over 10 simulation runs. Figure 4.5 also presents the corresponding numerical results and as may be seen the two agree well with each other. This confirms the accuracy of the assumption that at the beginning of secondary packet service times the channel is idle or busy according to an independent Bernoulli process.

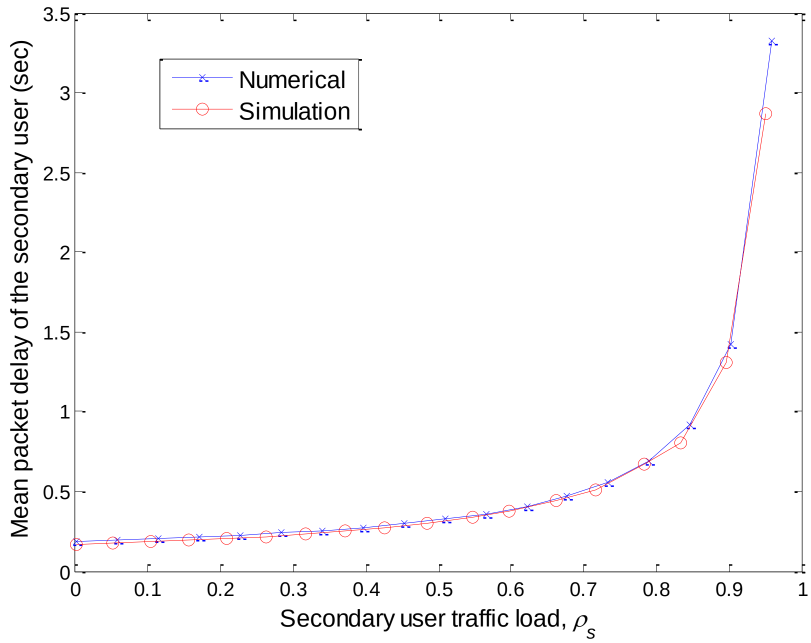

Figure 4.5 Numerical and simulation results for mean packet delay of the secondary user as a function of its traffic load

4.4.2 Probability of Interference to the Primary User

In addition to the performance of the secondary user in terms of the mean packet delay, we also determine the probability of interference to the primary user by simulation. This is the probability that the primary user becomes active during the service time of a secondary user packet that its service began when the channel was in the idle state. This means that the secondary user began service with full transmission power and continued without interference cancellation after the primary user became active. The probability of interference depends on two factors, first the channel being idle when the service time of a secondary packet begins and second the switching of the channel state from idle to busy during the service time. In simulation, we set the packet transmission time of the primary user, T_p, to 0.3 but keep the values of remaining parameters as in Table 4.1 the same.

In Figure 4.6 we plot the probability of interference as a function of the secondary user traffic load with the primary user traffic load as a parameter. As may be seen at any value of the secondary traffic load, the probability of interference decreases as the primary traffic load increases because the probability of the first factor affecting the interference decreases. This is because as the primary becomes busier the chances of secondary packet service time beginning in the idle channel state decreases. On the other hand, at any given value of the primary traffic load, the probability of interference initially increases linearly with the secondary load and then flattens out. This is because as the secondary traffic load increases the chances of service time beginning in an idle channel also increases.

As before, the probability of interference may not give the accurate measure of the impact of the secondary user on the primary user since the interference is determined from the perspective of the secondary user. A more appropriate measure may be the ratio of the mean duration of interference to the mean busy period of the primary user. Mean duration of interference is given by the product of mean packet service time and probability of interference. This measure has been plotted in Figure 4.7 and as it can be seen that the interference ratio is less than 1.6 % which is a much lower value.

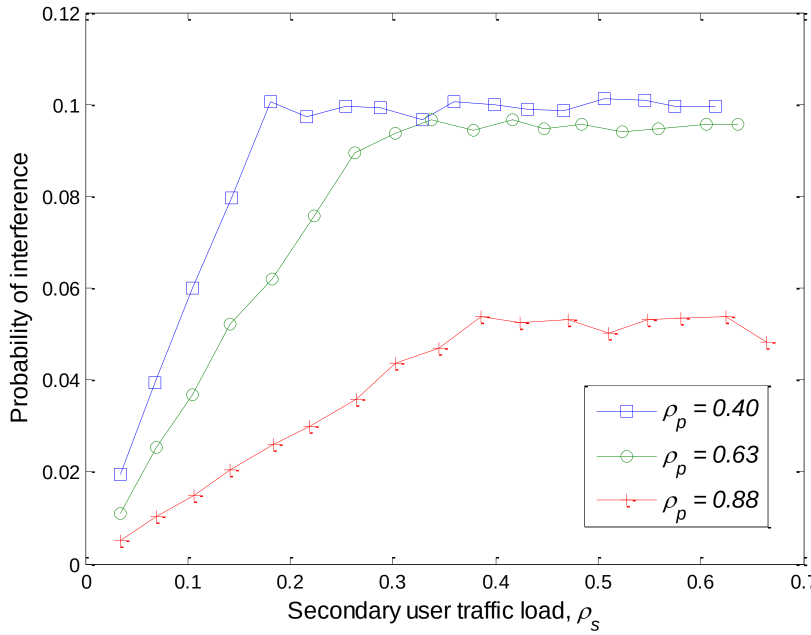

Figure 4.6 Simulation results for the probability of interference to the primary user as a function of the secondary traffic load

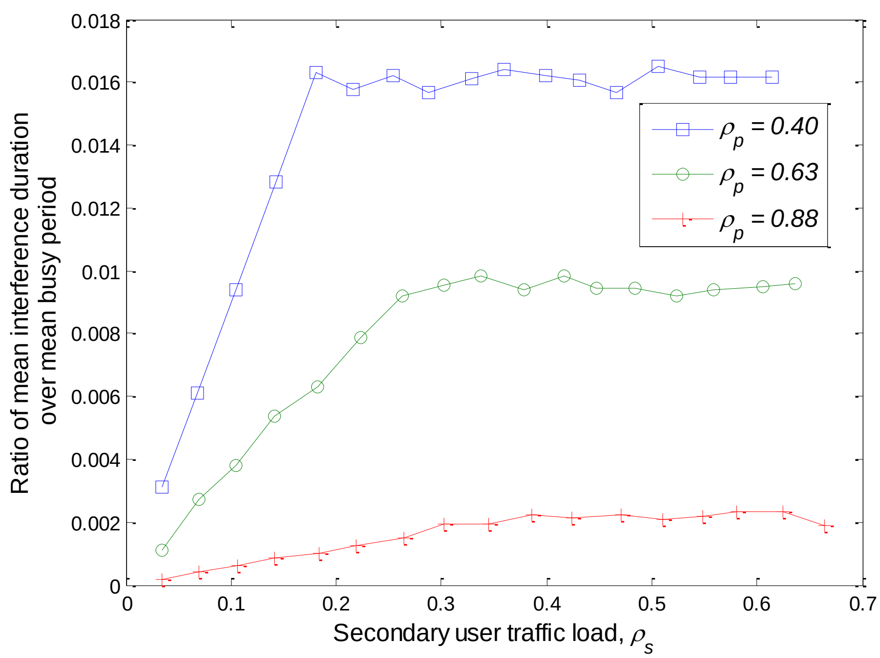

Figure 4.7 Simulation results for the ratio of mean interference duration to the mean primary busy period

Chapter 5

Conclusions and Future Work

Cognitive radio is one of the promising technologies that is said to be the "next big thing" for wireless networks. In recent years, various aspects of cognitive radio technology has been studied including spectrum sensing techniques, cooperative communication, MAC protocols, network modeling and development of IEEE 802.22 WRAN. In this thesis, we presented performance modeling of cognitive radio networks in three scenarios. We determined the performance of the secondary user and the interference experienced by the primary user.

First, we considered cooperative scenario where the secondary user assists the primary user's transmissions by relaying the primary packets to its intended destination. The secondary user utilizes the channel by transmitting its own packets when the primary user vacates the channel. We have derived the mean packet delay of the secondary user. The analytical results show that, the primary user spends less time in the channel which increases the availability of the channel to the secondary user. The mean packet delay of the secondary user in this model is significantly reduced at high primary user traffic loads.

In the second scenario, we presented performance modeling of cognitive radio network with multiple primary users. We modeled a single channel network with two primary users in which a secondary transmitter and receiver are located within transmission ranges of different primary users. In this model, secondary users may communicate if the channel is available simultaneously both to the secondary transmitter and receiver. We analyze idle and busy periods of an individual primary user and both primary users as a system and derive the mean packet delay of the secondary user. The results show that the mean packet delay of the secondary user in multiple primary user model is much higher than that in the single primary user model under equivalent loading.

In the last scenario we considered a cognitive radio network where secondary transmitters are equipped with multiple antennas. With multi-antenna, interference to the primary user can be suppressed which enables continuous availability of the channel to the secondary user. Under this scenario, the secondary user transmits with full transmission power when the channel is idle and with partial power when the channel is utilized by the primary user. We derived the mean packet delay of the secondary user and accuracy of the numerical results is verified by simulation. The results show that with multi-antenna, the mean packet delay of the secondary user may be significantly reduced. In addition, we show that the probability of the interference that may occur when the primary user returns to the channel while the secondary user is transmitting is low.

In the future, we would like to study cognitive radio networks in specific primary user environment. Clearly, the utilization of the channel depends on the characteristics of the primary user whether it is a television broadcasting, cellular communications etc.

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Appendix

A. Simulation Source Code - Multiple Primary Users

```
Tmax = 100000;
Tp1 = 0.2;   % a packet transmission time of a PU1
Tp2 = 0.1;   % a packet transmission time of a PU2
T2 = 0.1;    % a packet transmission time of a SU
Tnow = 0;    % current time initially
 lamda1 = 2;  % arrival rate of a PU1
lamda2 = 3;  % arrival rate of a PU2
Pp1 = 0.9;   % Probability of successful transmission of PU1
Pp2 = 0.9;   % Probability of successful transmission of PU2
Ps = 0.9;    % Probability of successful transmission of SU
 NoArr_p1 = 0;  % no of arrivals to PU1 queue
NoArr_p2 = 0;  % no of arrivals to PU2 queue
NoArr2 = 0;    % no of arrivals to SU queue
NoDep_p1 = 0;  % no of departures from PU1 queue
NoDep_p2 = 0;  % no of departures from PU2 queue
NoDep2 = 0;    % no of departures from SU queue
 NoEvent = 0; % no of entries in B
Q_p1 = 0;    % PU1 queue length
Q_p2 = 0;    % PU2 queue length
Q2 = 0;      % SU queue length
idle = 0;    % SU idle event

 % Allocate arrival time and departure time arrays
ArrTime_p1 = zeros(10000,1); % Arrival time of PU1 packets
```

```
ArrTime_p2 = zeros(10000,1); % Arrival time of PU2 packets
ArrTime2 = zeros(10000,1);   % Arrival time of SU packets
DepTime_p1 = zeros(10000,1); % Departure time of PU1 packets
DepTime_p2 = zeros(10000,1); % Departure time of PU2 packets
DepTime2 = zeros(10000,1);   % Departure time of SU packets

% B(i,j,k) indicates future events time and type and PU1/PU2/SU
% i is event number indicating the location of the entry
% j indicates type of entry, Type 1 = arrival, Type 2  = departure
% k indicates queues, 1 = PU1, 2 = PU2, 3 = SU

% Schedule the first event as arrivals for PU1, PU2 and SU
B(1,1) = Tnow + exprnd(1/lamda1);
B(1,2) = 1;
B(1,3) = 1;

B(2,1) = Tnow + exprnd(1/lamda2);
B(2,2) = 1;
B(2,3) = 2;

B(3,1) = Tnow + exprnd(1/alpha);
B(3,2) = 1;
B(3,3) = 3;
NoEvent = NoEvent + 3; % Number of Events in the future event array (B)

% Sort the arrival times of PU1, PU2 and SU
 B = sortrows(B,1); % sort by colume 1 in ascending order

 while NoDep2 < 3000
        NoEvent = NoEvent - 1;
```

```
% *********************** ARRIVAL ******************************
  if   B(1,2) == 1
  % ********************* PU1  ARRIVALS ***********************
    if B(1,3) == 1  % The arrival is from PU1
    % Arrival procedure of PU1
       NoArr_p1 = NoArr_p1 + 1;
      Tnow = B(1,1);
      ArrTime_p1(NoArr_p1,1) = B(1,1);
      Q_p1 = Q_p1 + 1;
    % Determine the start time of system busy period
       if Q_p1 + Q_p2 == 1
          Tbusy_sys = Tnow;
       end
    % Move the next event up when the event has occurred
      B = circshift (B,-1);
   % Schedule the next arrival of PU1
      if NoArr_p1 < Tmax
         B(NoEvent+1,1) = Tnow + exprnd(1/lamda1);
         B(NoEvent+1,2) = 1;
         B(NoEvent+1,3) = 1;
         NoEvent = NoEvent + 1
     end
    % Schedule a departure when its queue = 1 (Head of Queue)
      if Q_p1 == 1
         B(NoEvent+1,1) = Tnow + Tp1;
         B(NoEvent+1,2) = 2;
         B(NoEvent+1,3) = 1;
         NoEvent = NoEvent + 1;
      end
```

```
% ******************** PU2  ARRIVALS **********************
elseif B(1,3) == 2
    % Arrival procedure of PU2
      NoArr_p2 = NoArr_p2 + 1;
      Tnow = B(1,1);
      ArrTime_p2(NoArr_p2,1) = B(1,1);
      Q_p2 = Q_p2 + 1;
    % Determine the start time of system busy period
      if Q_p1 + Q_p2 == 1
          Tbusy_sys = Tnow;
     end
      % Move the next event up when the event has occurred
      B = circshift (B,-1);
    % Schedule the next arrival of PU2
      if NoArr_p2 < Tmax
          B(NoEvent+1,1) = Tnow + exprnd(1/lamda2);
          B(NoEvent+1,2) = 1;
          B(NoEvent+1,3) = 2;
          NoEvent = NoEvent + 1;
      end
   % Schedule a departure when its queue = 1 (Head of Queue)
      if Q_p2 == 1
          B(NoEvent+1,1) = Tnow + Tp2;
          B(NoEvent+1,2) = 2;
          B(NoEvent+1,3) = 2;
          NoEvent = NoEvent + 1;
      end
% ******************** SU  ARRIVALS ************************
elseif B(1,3) == 3    % The arrival is from SU
    % Arrival procedure of SU
```

```
        NoArr2 = NoArr2 + 1;
        Tnow = B(1,1);
        ArrTime2(NoArr2,1) = B(1,1);
        Q2 = Q2 + 1;
      % The idle period of the SU ends
        if Q2 == 1 && idle > 0
          Tbusy = Tnow;
          idleperiod(idle,1) = Tbusy - Tidle;
        end
      % Move the next event up when the event has occurred
        B = circshift (B,-1);
      % Schedule the next arrival of SU
        if NoArr2 < Tmax
             B(NoEvent+1,1) = Tnow + exprnd(1/alpha);
             B(NoEvent+1,2) = 1;
             B(NoEvent+1,3) = 3;
             NoEvent = NoEvent + 1;
        end
  end
  % Sort the entries in B
  C = B(1:NoEvent,1:3);
  B = sortrows(C,1);
% ************************ DEPARTURE *************************
elseif B(1,2) == 2
  % ********************* PU1  Departure *******************
  if B(1,3) == 1 % The departure is from PU
    u = rand;
    if u < Pp1 % the packet will depart
    % Departure procedure of PU1
      Tnow = B(1,1);
```

```
    Q_p1 = Q_p1 - 1;  % decrease the queue length by one

    NoDep_p1 = NoDep_p1 + 1;

    DepTime_p1(NoDep_p1,1) = B(1,1); % Departure time

  % Move the future event up

    B = circshift (B,-1);

  % Schedule departure for SU (SU starts transmitting)

   if Q_p1 == 0 && Q_p2 == 0
      cycle = cycle + 1;
      NoQ2(cycle,1) = Q2;
     Tidle_sys = Tnow; % start T of the system idle period

        if  Q2 > 0 && ArrTime2(NoDep2+1) < Tidle_sys

           B(NoEvent+1,1) = Tnow + T2;

           B(NoEvent+1,2) = 2;

           B(NoEvent+1,3) = 3;

           NoEvent = NoEvent + 1;

        end

   end

  % Schedule departure of PU1

    if Q_p1 > 0

       B(NoEvent+1,1) = Tnow + Tp1;

       B(NoEvent+1,2) = 2;

       B(NoEvent+1,3) = 1;

       NoEvent = NoEvent + 1;

   end

    else % retransmission & reschedule the same packet

    Tnow = B(1,1);

    B(1,1) = Tnow + Tp1;

    B(1,2) = 2;

    B(1,3) = 1;

    NoEvent = NoEvent + 1;

 end

% ******************** PU2  Departure *******************
```

```
elseif B(1,3) == 2 % The departure is from PU2
    u = rand;
    if u < Pp2 % the packet will depart
   % Departure procedure of PU2
        Tnow = B(1,1);
        Q_p2 = Q_p2 - 1;  % decrease the queue length by one
        NoDep_p2 = NoDep_p2 + 1;
        DepTime_p2(NoDep_p2,1) = B(1,1); % Departure time
   % Move the future event up
        B = circshift (B,-1);

   % Schedule departure for SU (SU starts transmitting)
       if Q_p2 == 0 && Q_p1 == 0
           cycle = cycle + 1;
           NoQ2(cycle,1) = Q2;
            Tidle_sys = Tnow;
           if Q2 > 0 && ArrTime2(NoDep2+1) < Tidle_sys
               B(NoEvent+1,1) = Tnow + T2;
               B(NoEvent+1,2) = 2;
               B(NoEvent+1,3) = 3;
               NoEvent = NoEvent + 1;
           end
        end
   % Schedule departure of PU2
        if Q_p2 > 0
           B(NoEvent+1,1) = Tnow + Tp2;
           B(NoEvent+1,2) = 2;
           B(NoEvent+1,3) = 2;
           NoEvent = NoEvent + 1;
        end
    else % retransmission & reschedule the same packet
```

```
        Tnow = B(1,1);
        B(1,1) = Tnow + Tp2;
        B(1,2) = 2;
        B(1,3) = 2;
        NoEvent = NoEvent + 1;
    end
% ******************** SU  Departure *********************
elseif B(1,3) == 3
     u = rand; % generate a random number
    if Q_p1 == 0 && Q_p2 == 0 % if both PU1&PU2 are idle, the SU packet gets a chance to depart
        if u < Ps % the packet will depart
        % Departure procedure of SU
            Tnow = B(1,1);
            Q2 = Q2 - 1;  % decrease the queue length by one
            NoDep2 = NoDep2 + 1;
            DepTime2(NoDep2,1) = B(1,1); % Departure time
        % Determine mean idle period of the SU
             if Q2 == 0
                idle = idle + 1;
                Tidle = Tnow;
             end
        % Move the future event up
            B = circshift (B,-1);
        % Schedule the next departure for SU
            if Q2 > 0 && ArrTime2(NoDep2+1) < Tbusy_sys
                B(NoEvent+1,1) = Tnow + T2;
                B(NoEvent+1,2) = 2;
                B(NoEvent+1,3) = 3;
                NoEvent = NoEvent + 1;
            end
```

```
            else % Retransmission
              Tnow = B(1,1);
              B(1,1) = Tnow + T2;
              B(1,2) = 2;
              B(1,3) = 3;
              NoEvent = NoEvent + 1;
            end
        else
            Tnow = B(1,1);
            B = circshift(B,-1);
        end
    end
    % Sort the entries in B
    C = B(1:NoEvent,1:3); % To prevent unwanted values, at the
    B = sortrows(C,1);    % last row of B, from sorting
  end
end
% Delay of each packet
SUEachDelay = DepTime2 - ArrTime2;
UsedSUDelay = SUEachDelay(1:NoDep2,1);
% The mean delay obtained by averaging each delay
SUmeanDelay = mean(UsedSUDelay);
% Determine rous from simulation
if idle > 1
PrIdle = (ArrTime2(1,1)+sum(idleperiod))/Tnow; % prob(SU is having an empty queue)
meanIdleSU = mean(idleperiod);  % mean idle period of the SU
rous = 1 - PrIdle;
elseif idle == 1
   rous = 1;
end
```

B. Simulation Source Code – Multi-Antenna

```
Tmax = 5000;
T1 = 0.1;    % a packet transmission time of a PU
T2i = 0.1;   % a packet transmission time of a SU @ idle channel
T2b = 0.12;  % a packet transmission time of a SU @ busy channel
Tnow = 0;    % current time initially
 lamda = 4;   % arrival rate of a PU
Pp = 0.7;    % Probability of successful transmission of SU
Ps = 0.6;    % Probability of successful transmission of SU
 NoArr1 = 0;  % no of arrivals to PU queue
NoArr2 = 0;  % no of arrivals to SU queue
NoDep1 = 0;  % no of departures from PU queue
NoDep2 = 0;  % no of departures from SU queue
Fail = 0;
NoFail = 0;
idle = 0;    % PU idle event
idle2 = 0;   % SU idle event
 NoEvent = 0; % no of entries in B
Q1 = 0;      % PU queue length
Q2 = 0;      % SU queue length
D1 = 0;      % PU sum of delay
D2 = 0;      % SU sum of delay
 % Allocate arrival time and departure time arrays
ArrTime1 = zeros(5000,1);   % Arrival time of PU packets
ArrTime2 = zeros(5000,1);   % Arrival time of SU packets
DepTime1 = zeros(5000,1);   % Departure time of PU packets
DepTime2 = zeros(5000,1);   % Departure time of SU packets
 % Schedule the first event as arrivals for both PU and SU
B(1,1) = Tnow + exprnd(1/lamda);
```

```
B(1,2) = 1;
B(1,3) = 1;
B(2,1) = Tnow + exprnd(1/alpha);
B(2,2) = 1;
B(2,3) = 2;
NoEvent = NoEvent + 2; % Number of Event in the future event array (B)
 % Sort the arrival times of PU and SU
 B = sortrows(B,1); % sort by colume 1 in ascending order

while NoDep1 < Tmax || NoDep2 < Tmax
     NoEvent = NoEvent - 1;
   % *********************** ARRIVAL ****************************
   if   B(1,2) == 1
      % ******************** PU  ARRIVALS *********************
      if B(1,3) == 1  % The arrival is from PU
         % Arrival procedure of PU
            NoArr1 = NoArr1 + 1;
            Tnow = B(1,1);
            ArrTime1(NoArr1,1) = B(1,1);
            Q1 = Q1 + 1;
            A1(Q1,1) = B(1,1);
          % Determine the idle duration
            if Q1 == 1 && idle >= 1
               Tbusy = Tnow;
               idleperiod(idle,1) = Tbusy - Tidle;
            end
         % Move the next event up when the event has occurred
         if NoEvent >= 1
            B = circshift (B,-1);
         end
```

```
   % Schedule the next arrival of PU
   if NoArr1 < Tmax
      B(NoEvent+1,1) = Tnow + exprnd(1/lamda);
      B(NoEvent+1,2) = 1;
      B(NoEvent+1,3) = 1;
      NoEvent = NoEvent + 1;
   end
% Schedule a departure when PU Q = 1 (Head of Queue)
   if Q1 == 1
      B(NoEvent+1,1) = Tnow + T1;
      B(NoEvent+1,2) = 2;
      B(NoEvent+1,3) = 1;
      NoEvent = NoEvent + 1;
   end
% ******************** SU  ARRIVALS *************************
elseif B(1,3) == 2    % The arrival is from SU
    % Arrival procedure of SU
      NoArr2 = NoArr2 + 1;
      Tnow = B(1,1);
      ArrTime2(NoArr2,1) = B(1,1);
      Q2 = Q2 + 1;
      A2(Q2,1) = B(1,1);
      if Q2 == 1 && idle2 > 0
         Tbusy2 = Tnow;
         idleperiod2(idle2,1) = Tbusy2 - Tidle2;
      end
    % Move the next event up when the event has occurred
    if NoEvent >= 1
         B = circshift (B,-1);
    end
```

```
    % Schedule the next arrival of SU
     if NoArr2 < Tmax
         B(NoEvent+1,1) = Tnow + exprnd(1/alpha);
         B(NoEvent+1,2) = 1;
         B(NoEvent+1,3) = 2;
         NoEvent = NoEvent + 1;
     end
     % Schedule a departure when SU Q = 1 (Head of Queue
     if Q2 == 1
        if Q1 == 0 % the channel is idle
           T2 = T2i;
        else T2 = T2b;
        end
       B(NoEvent+1,1) = Tnow + T2;
       B(NoEvent+1,2) = 2;
       B(NoEvent+1,3) = 2;
       NoEvent = NoEvent + 1;
      end
  end
  % Sort the entries in B
  if NoEvent <= 3
     C = B(1:NoEvent,1:3);
     B = sortrows(C,1);
  end
  if NoEvent == 4
     B = sortrows(B,1);
  end
% *********************** DEPARTURE ***************************
elseif B(1,2) == 2
```

```
% ********************** PU  Departure *********************
if B(1,3) == 1 % The departure is from PU
    u = rand;
    if u < Pp % the packet will depart
   % Departure procedure of PU
        Tnow = B(1,1);
        Q1 = Q1 - 1;  % decrease the queue length by one
        NoDep1 = NoDep1 + 1;
        DepTime1(NoDep1,1) = B(1,1); % Departure time
        D1 = D1 + Tnow - A1(1,1);  % Add up delay
       if NoDep1 == Tmax
          Tend = Tnow;
       end
   % Determine the idle period of PU
       if Q1 == 0
          idle = idle + 1; % the idle event
          Tidle = Tnow;
       end
% Determine the service time of a packet
    if Fail > 0
        NoFail = NoFail + 1;
        m(NoFail,1) = Tnow - TFail + T1; % note the service  time of each retransmitted packets
        Fail = 0; % reset the fail event = 0
    end
   % Schedule departure of PU
        if Q1 > 0
            B(1,1) = Tnow + T1;
            B(1,2) = 2;
            B(1,3) = 1;
            NoEvent = NoEvent + 1;
```

```
      for n = 1:Q1 % Move arrival time of each packet up
          A1(n,1) = A1(n+1,1);
      end
  % Queue is empty, move the future event up
      elseif Q1 == 0
          B = circshift (B,-1);
      end
  else % retransmission & reschedule the same packet
      Tnow = B(1,1);
      Fail = Fail + 1; % the first attempt was failed
      if Fail == 1;
      TFail = B(1,1);  % time the first failed attempt event
      end
      B(1,1) = Tnow + T1;
      B(1,2) = 2;
      B(1,3) = 1;
      NoEvent = NoEvent + 1;
  end
% ********************** SU  Departure ***********************
elseif B(1,3) == 2
  % check if the channel is idle or busy,set the transmission
  % time of SU accordingly
  if Q1 == 0
      T2 = T2i;
  else
      T2 = T2b;
  end
  u = rand; % generate a random number
  if u < Ps % the packet will depart
  % Departure procedure of SU
```

```
    Tnow = B(1,1);
    Q2 = Q2 - 1;  % decrease the queue length by one
    NoDep2 = NoDep2 + 1;
    DepTime2(NoDep2,1) = B(1,1); % Departure time
    D2 = D2 + Tnow - A2(1,1);  % Add up delay
    % Determine mean idle period of the SU
    if Q2 == 0
      idle2 = idle2 + 1;
      Tidle2 = Tnow;
    end
    if NoDep2 == Tmax
      Tend2 = Tnow;
    end
% Schedule departure for SU
    if Q2 > 0
       B(1,1) = Tnow + T2;
       B(1,2) = 2;
       B(1,3) = 2;
       NoEvent = NoEvent + 1;
    for n = 1:Q2
       A2(n,1) = A2(n+1,1);
    end
% Queue is empty, move the future event up
    elseif Q2 == 0
       B = circshift (B,-1);
    end
else
    Tnow = B(1,1);
    B(1,1) = Tnow + T2;
    B(1,2) = 2;
```

```
            B(1,3) = 2;
            NoEvent = NoEvent + 1;
          end
        end
        % Sort the entries in B
        if NoEvent <= 3
          C = B(1:NoEvent,1:3); % To prevent unwanted values, at the
          B = sortrows(C,1);    % last row of B, from sorting
        end
        if NoEvent == 4
           B = sortrows(B,1);
        end
    end
end
% Delay of each packet
PUEachDelay = DepTime1 - ArrTime1;
SUEachDelay = DepTime2 - ArrTime2;
% The mean delay obtained by averaging each delay
PUmeanDelay2 = mean(PUEachDelay);
SUmeanDelay2 = mean(SUEachDelay);
% Determine rous
if idle2 > 1
PrIdle2 = sum(idleperiod2)/Tend2; % prob(SU is having an empty queue)
rous = 1 - PrIdle2;
elseif idle2 == 1
   rous = 1;
end
% number of packets in the SU
meanNoQ2 = mean(NoQ2); % mean queue length of SU
rous_sim2 = nnz(NoQ2)/cycle;
```

```
% service time of SU
serviceT_SU = DepTime2(1:2500,1)-Tms_start(1:2500,1);
ms_sim = mean(serviceT_SU);
rous_sim3 = ms_sim*alpha;
```

C. The Exact Analysis of Mean Queue Length of the Secondary User

Define the following notation;

A_{i+1} ≜ number of arrivals during the $(i+1)^{st}$ cycle

B_{i+1} ≜ number of arrivals during the busy period of the $(i+1)^{st}$ cycle

C_{i+1} ≜ number of arrivals during the idle period of the $(i+1)^{st}$ cycle

n ≜ number of slots to transmit a packet successfully during an idle period

m_j ≜ number of slots to transmit j packets successfully during an idle period

N_k ≜ probability that a packet is transmitted successfully in k slots during an idle period

m_{jk} ≜ probability that j packets are transmitted successfully in k slots during an idle period

$$A_{i+1} = B_{i+1} + C_{i+1} \tag{1}$$

Define $B(z) = B_p[\alpha(1-z)]$ then equation (2.9) in Chapter 2 can be written as,

$$X_{i+1} = X_i - Y_{i+1} + B_{i+1} + C_{i+1} \tag{2}$$

$$X_{i+1}(z) = E[z^{X_i - Y_{i+1} + C_{i+1}}]B(z) \tag{3}$$

Next, let us show the derivation of $E[z^{X_i - Y_{i+1} + C_{i+1}}]$

$$N_k = (1-\gamma)\gamma^{k-1}, \;\; k = 1,2,3, \ldots$$

$$Pr(n \leq k) = 1 - \gamma^k$$

$$n(z) = E[z^n] = \frac{(1-\gamma)z}{1-\gamma z} \tag{4}$$

From the definitions, we have

$$m_j = \sum_{i=0}^{j} n_i$$

$$m_j(z) = E[z^{m_j}] = \sum_{k=j}^{\infty} z^k m_{jk} = [n(z)]^j = \left[\frac{(1-\gamma)z}{1-\gamma z}\right]^j \tag{5}$$

Let us define the following, then we have,

$S_j(k) \triangleq$ probability that at least j packets will be successfully transmitted during k slots

$W_j(k) \triangleq$ probability that exactly j packets will be successfully transmitted during k slots

$$S_j(k) = \sum_{i=j}^{k} m_{ji} \quad\quad for\ k \geq j$$

$$S_j(k) = 0 \quad\quad\quad for\ \ k < j$$

$$W_j(k) = \begin{cases} S_j(k) - S_{j+1}(k), & j \geq 0 \\ 0, & j > k \end{cases}$$

From (3), to determine $E[z^{X_i - Y_{i+1} + C_{i+1}}]$ we need to condition on constant X_i and I_p,

$$E\left[z^{X_i - Y_{i+1} + C_{i+1}} | X_i = x, I_p = k\right] = E\left[z^{X_i - Y_{i+1}} \middle| X_i = x, I_p = k\right] E\left[z^{C_{i+1}} \middle| I_p = k\right]$$

$$= \sum_{y=0}^{x} z^{x-y} W_y(k)\, e^{-skT}|_{s=\alpha(1-z)}$$

$$= \left\{\sum_{y=0}^{x-1} z^{x-y}\left[S_y(k) - S_{y+1}(k)\right] + S_x(k)\right\} e^{-\alpha(1-z)kT}$$

We can rewrite the above in continuous form as follows

$$E\left[z^{X_i - Y_{i+1} + C_{i+1}} | X_i = x, I_p = t, kT < t < (k+1)T\right]$$

$$= \left\{\sum_{y=0}^{x-1} z^{x-y}\left[S_y(k) - S_{y+1}(k)\right] + S_x(k)\right\} e^{-\alpha(1-z)t}$$

$$E[z^{X_i - Y_{i+1} + C_{i+1}} | X_i = x] = \sum_{k=0}^{\infty} \int_{kT}^{(k+1)T} \left\{\sum_{y=0}^{x-1} z^{x-y}\left[S_y(k) - S_{y+1}(k)\right] + S_x(k)\right\} e^{-\alpha(1-z)t} f_{I_p}(t)\, dt$$

$$= \sum_{k=0}^{\infty} \int_{kT}^{(k+1)T} \left\{\sum_{y=0}^{x-1} z^{x-y}\left[S_y(k) - S_{y+1}(k)\right] + S_x(k)\right\} e^{-\alpha(1-z)t} \lambda e^{-\lambda t}\, dt$$

$$= \lambda \sum_{k=0}^{\infty} \left\{\sum_{y=0}^{x-1} z^{x-y}\left[S_y(k) - S_{y+1}(k)\right] + S_x(k)\right\} \int_{kT}^{(k+1)T} e^{-[\alpha(1-z)+\lambda]t}\, dt$$

$$= \frac{\lambda}{\alpha(1-z)+\lambda} \sum_{k=0}^{\infty} \left\{\sum_{y=0}^{x-1} z^{x-y}\left[S_y(k) - S_{y+1}(k)\right] + S_x(k)\right\} e^{-[\alpha(1-z)+\lambda]kT} (1 - e^{-[\alpha(1-z)+\lambda]T})$$

Recall that $\sigma = e^{-\lambda T}$ and let $u = \sigma e^{-\alpha(1-z)T}$, we have

$$E\left[z^{X_i - Y_{i+1} + C_{i+1}} | X_i = x\right]$$

$$= \frac{\lambda}{\alpha(1-z)+\lambda} \sum_{k=0}^{\infty} \left\{ \sum_{y=0}^{x-1} z^{x-y} \left[S_y(k) - S_{y+1}(k)\right] + S_x(k) \right\} u^k (1-u)$$

$$= \frac{\lambda(1-u)}{\alpha(1-z)+\lambda} \sum_{k=0}^{\infty} \left\{ \sum_{y=0}^{x-1} z^{x-y} \left[S_y(k) - S_{y+1}(k)\right] + S_x(k) \right\} u^k$$

Let us define the following z-transform,

$$S_y(z) = \sum_{k=y}^{\infty} z^k S_y(k) = \sum_{k=i}^{\infty} z^k \sum_{i=y}^{\infty} m_{yi}$$

$$= \sum_{i=y}^{\infty} m_{yi}\, z^i \sum_{k=i}^{\infty} z^{k-i} = \sum_{i=y}^{\infty} m_{yi}\, z^i \sum_{k-i=0}^{\infty} z^{k-i}$$

$$S_y(z) = \frac{1}{1-z} \sum_{i=y}^{\infty} m_{yi}\, z^i = \frac{m_y(z)}{1-z}$$

Thus,

$$E\left[z^{X_i - Y_{i+1} + C_{i+1}} | X_i = x\right] = \frac{\lambda(1-u)}{\alpha(1-z)+\lambda} \left\{ \sum_{y=0}^{x-1} z^{x-y} \left[S_y(u) - S_{y+1}(u)\right] + S_x(u) \right\}$$

$$= \frac{\lambda}{\alpha(1-z)+\lambda} \left\{ \sum_{y=0}^{x-1} z^{x-y} \left[m_y(u) - m_{y+1}(u)\right] + m_x(u) \right\}$$

For simplicity in writing, let $a(z) = \frac{(1-\gamma)u}{(1-\gamma u)z}$ and $b(z) = \frac{(1-\gamma)u}{(1-\gamma u)}$

$$= \frac{\lambda}{\alpha(1-z)+\lambda}\left\{\sum_{y=0}^{x-1} z^x\{[a(z)]^y - z[a(z)]^{y+1}\} + [b(z)]^x\right\}$$

$$E[z^{X_i - Y_{i+1} + C_{i+1}}|X_i = x] = \frac{\lambda}{\alpha(1-z)+\lambda}\left\{z^x \frac{1-[a(z)]^x}{1-a(z)}[1 - za(z)] + [b(z)]^x\right\}$$

Unconditioning the above with respect to X_i,

$$E[z^{X_i - Y_{i+1} + C_{i+1}}] = \frac{\lambda}{\alpha(1-z)+\lambda}\sum_{x=0}^{\infty}\left\{\frac{1-za(z)}{1-a(z)}[z^x - [za(z)]^x] + [b(z)]^x\right\}Pr(X_i = x)$$

$$= \frac{\lambda}{\alpha(1-z)+\lambda}\left\{\frac{1-za(z)}{1-a(z)}[X_i(z) - X_i(za(z)) + X_i(b(z))\right\}$$

$$X_{i+1}(z) = \frac{\lambda B(z)}{\alpha(1-z)+\lambda}\left\{\frac{1-za(z)}{1-a(z)}[X_i(z) - X_i(za(z)) + X_i(b(z))\right\}$$

At the steady state, $X_{i+1}(z) = X_i(z) = X(z)$. Thus,

$$X(z) = \frac{\lambda B(z)}{\alpha(1-z)+\lambda}\left\{\frac{1-za(z)}{1-a(z)}[X(z) - X(za(z)) + X(b(z))\right\}$$

Solve the above equation for $X(z)$ we get

$$X(z) = \frac{\lambda B(z)\{[1-za(z)]X(za(z)) - [1-a(z)]X(b(z))\}}{\lambda B(z)[1-za(z)] - [\alpha(1-z)+\lambda][1-a(z)]}$$

Apply L'Hopital rule,

$$X\left(a(1)\right) = \frac{\lambda\left\{[1 - a(1)]\alpha\overline{B_p} - a(1)\right\} - \alpha[1 - a(1)]}{\lambda a(1)}$$

$$X'\left(a(1)\right) = \frac{a(1)\left[\lambda\left(1 + \alpha\overline{B_p}\right) + \alpha\right] - \lambda\left[a(1) - \alpha\overline{B_p}\left(1 - a(1)\right)\right] - \alpha\left(1 - a(1)\right)}{\lambda[a(1)]^2}$$

Determine mean queue length by taking derivative of (34) and evaluate at z = 1

$$\bar{x} = \frac{2\alpha a'(1) - \lambda\left\{2a(1)\left[\alpha\overline{B_p}X\left(a(1)\right) + \left(a'(1) + a(1)\right)X'\left(a(1)\right)\right] + 2X\left(a(1)\right)a'(1)\right\} - \lambda\left\{\alpha^2 B_p''(0)\left(1 - a(1)\right) - 2\alpha\overline{B_p}\left(a'(1) + a(1)\right) - 2a'(1)\right\}}{2\left\{\lambda\left[\alpha\overline{B_p}\left(1 - a(1)\right) - a(1)\right] + \alpha\left(1 - a(1)\right)\right\}}$$

where

$$a'(1) = \frac{(1 - \gamma)\sigma[\alpha T - 1 + \gamma\sigma]}{(1 - \gamma\sigma)^2}$$