Tracking Control of Nonholonomic Mechanical Systems Including Actuator Dynamics

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ABSTRACT

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The Control of nonholonomic systems is extremely challenging and stimulates researchers to design controllers for these highly nonlinear and complex systems. The control of these systems is important for the reason that they have numerous engineering applications such as mobile robots, space manipulators, multi-fingered robot, etc. The control techniques proposed for the nonholonomic systems can be divided into two types. The first is the kinematic control, which provides solution only at the pure kinematic level which yields driving speeds. The second is dynamic control, which takes inertia and forces into account and yields physical controls such as driving torques. But both of the above said schemes do not consider the actuator dynamics, which is an inherent part of the complete system and can affect the performance of the control system. In this thesis a strategy for designing a position tracking controller for a class of mechanical systems with nonholonomic velocity constraint is presented. Two controllers are designed at the actuator level taking into account the uncertainties in dynamics and actuator dynamics. The stability analysis for the complete system shows that the desired motion tracking is achieved by using the proposed methods. The design procedures are illustrated on a three wheeled mobile robot with simulations. Simulation results show the effectiveness of the controllers.

Dedicated to my parents and grand parents

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LIST OF ABBREVIATIONS AND SYMBOLS

q_i	Generalized co-ordinates
$\alpha(q)$	Integrating factor
o	Number of holonomic constraints
m	Number of nonholonomic constraints
k	Total number of kinematic constraints
g_i	Smooth vector fields
u_i	Control input to the kinematic subsystem
q_{di}	Desired position
I	Armature currents
λ	Lagrangian multipliers
$D(q)$	Inertial matrix
$C(q,\dot{q})\dot{q}$	Vector of centripetal and coriolis torques
$G(q)$	Gravitational torques
$B(q)$	Transformation matrix
$J(q)$	Constraint matrix
L_i	Inductances
R_i	Resistances
K_{ai}	Back EMF

ω_i	Angular velocities
ν_i	Voltage
v_i	Velocity of the robot
β_p	Unknown inertial parameter
e_i	Errors in position
z	Virtual control input
I_d	Virtual control input
ρ	Upper bound of the unknown β_p
V	Positive definite function
V_i	Positive definite function
φ	Robust controller
$\dot{\hat{L}}_i$	Adaptive control law
$\dot{\hat{R}}_i$	Adaptive control law
$\dot{\hat{K}}_{ai}$	Adaptive control law
$\dot{\hat{K}}_{Ni}$	Adaptive control law
$\dot{\hat{K}}_{NInvi}$	Adaptive control law
α_i	Controller design variables
γ_i	Positive constants
K_i	Positive constants

$\dot{\hat{\beta}}_p$ Adaptive control law

Chapter 1

Introduction

Nonholonomic mechanical systems have been known to the scientific community for more than 150 years but not until the last decade the researchers have shown interest in solving the control problems related to these systems. This is partly due to limited availability of tools to tackle these systems and partly due to lacking necessity for industrial oriented practical applications. The term "Nonholonomic systems" originates from classical mechanics and has a widely accepted meaning of "Lagrangian systems with linear constraint being non-integrable". Nonholonomic behavior can arise from either bodies rolling without slipping on top of each other or conservation of angular momentum during motion. Nonholonomic systems can be found frequently in mechanical systems such as wheeled mobile robots, car-like vehicles, the knife-edge systems, etc., which makes the study of these systems more important.

1.1 Types of Constraints in Mechanical Systems

Constraints on mechanical systems are defined as restrictions that limit the motion of the particles of a system. For instance, the motion of billiard balls are constrained to move within a plane, particles connected by a rigid rod are constrained to stay apart, etc. Constraints can be classified into two types depending on whether they are imposed on position or velocity. The constraints imposed on position are called geometric constraint and those imposed on velocity are called kinematic constraints. A geometric constraint can be expressed as

$$f(t, q) = 0 \tag{1.1}$$

where f is a vector function of class C^2, t denotes time and q is a n-dimensional vector whose components are generalized co-ordinates. Kinematic constraint can be expressed as

$$f(t, q, \dot{q}) = 0 \tag{1.2}$$

When a geometric constraint is differentiated with respect to time a differential or a kinematic constraint is always obtained. However the reverse is not always true, i.e., an existence of differential or kinematic constraint does not necessarily mean the existence of the geometric constraint. On this basis the kinematic constraints are classified as either holonomic or nonholonomic constraints.

1.1.1 Holonomic Constraint

A kinematic constraint which can be reduced to a geometric constraint or in other words a kinematic constraint which is integrable is called a holonomic constraint. Consider a kinematic constraint which is expressed as

$$J\dot{q} + b = 0 \Leftrightarrow g(t, q, \dot{q}) = 0 \tag{1.3}$$

If the matrix J and b can be expressed as $J = \frac{\partial f}{\partial q}$ and $b = \frac{\partial f}{\partial t}$, this forms necessary and sufficient condition for the integrability of the kinematic constraint to a geometric constraint of the form $f(t, q) = 0$. Each holonomic constraint reduces the number of degrees of freedom (DOF) by one. The reason being that it allows us to express one of the original generalized co-ordinates as a function of the other and deletes it from the set. For instance, two particles are connected by a massless rigid rod, so that they are constrained to move a fixed distance apart. Let the position of first particle with respect to a stationary cartesian frame be (x_1, y_1, z_1) and that of second particle be (x_2, y_2, z_2), the rigid rod constraint equation is then given by

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 = l^2 \tag{1.4}$$

This reduces the number of DOF from 6 $(x_1, x_2, y_1, y_2, z_1, z_2)$ to 5 $(x_1, y_1, z_1, \theta, \phi)$, which are taken to be position of the first particle, (x_1, y_1, z_1) and the spherical polar angles θ and ϕ. In general, the representation of a n-particle system in 3-space is given by $3n - o$, where o is the number of holonomic constraints.

1.1.2 Nonholonomic Constraint

A kinematic constraint which cannot be reduced to a position or geometric constraint ie., non-integrable kinematic constraints are called nonholonomic constraints. Nonholonomic constraints are divided into kinematic and dynamic nonholonomic constraints. Kinematic nonholonomic constraints are imposed by kinematics, such as rolling constraints, these constraints are linear in velocity. Dynamic nonholonomic constraints are constraints preserved by basic Euler-Lagrange or Hamilton equations, such as angular momentum. These constraints are not externally imposed on the system but rather a consequence of equation of motion. The focus of this thesis is only on the kinematic nonholonomic constraints and which can be represented by the Pfaffian constraint. Kinematic constraints which can be represented by

$$a(q)^T \dot{q} = 0 \tag{1.5}$$

where $a(q) \in \Re^{n \times k}$ are a k set of velocity constraints is called Pfaffian constraint. The vertical rolling disk is a basic and simple example of a system subjected to a nonholonomic constraint. Consider a homogeneous disk rolling without slipping on a horizontal plane, as shown in Fig. 1.1. It is assumed that the disk does not tilt. The configuration space for the vertical rolling disk is $Q = \Re^2 \times S^1 \times S^1$ and is

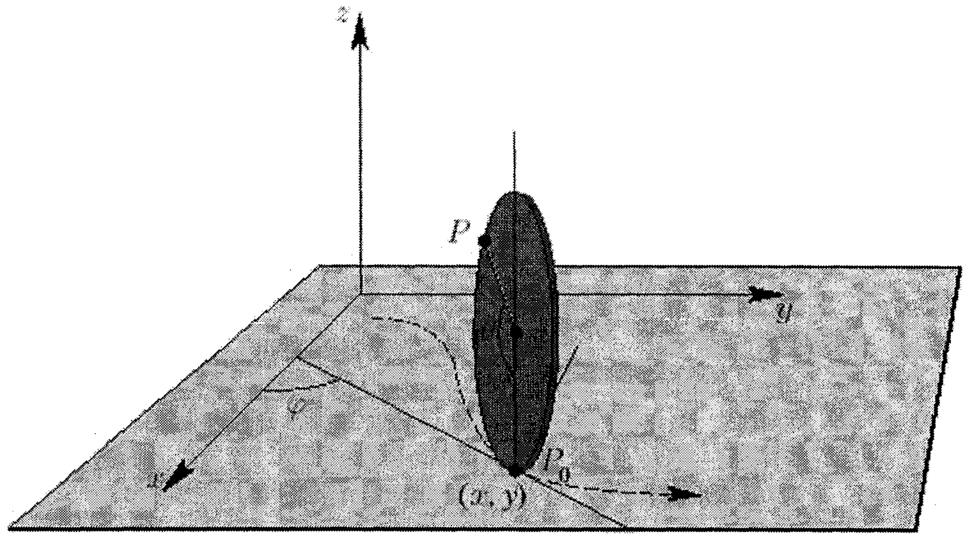

Figure 1.1: Disk rolling without slipping

parameterized by the generalized co-ordinates $q = (x, y, \theta, \varphi)$, denoting the position of the contact point in xy-plane, the rotation angle of the disk, and the orientation of the disk, respectively. The systems generalized velocities cannot assume arbitrary values but have to obey the constraint imposed by the no slip condition given by

$$\dot{x} = R\cos\varphi\dot{\theta}$$

$$\dot{y} = R\sin\varphi\dot{\theta} \tag{1.6}$$

where R is the radius of the disk. The kinematic constraints (1.6) are not integrable and as a consequence, there is no restriction on the configuration the disk attain. The above statement can be verified by the fact that disk can be driven from configuration $(x_1, y_1, \theta_1, \varphi_1)$ to a configuration $(x_2, y_2, \theta_2, \varphi_2)$ through the following sequence:

1. Roll the disk so as to bring the contact point from (x_1, y_1) to (x_2, y_2) along any curve of length $R\,(\theta_2 = \theta_1 + 2k\pi)$, where k is an arbitrary nonnegative integer.

2. Rotate the disk around the vertical axis from φ_1 to φ_2.

This proves that the constraints imposed on the motion of the vertical rolling disk are nonholonomic.

1.2 Classifications of Mechanical Systems According to the Kinematic Constraint

A singles Pfaffian constraint can be written as

$$a^T(q)\,\dot{q} = \sum_{j=1}^{n} a_j(q)\,\dot{q}_j = 0 \tag{1.7}$$

where

$$a = \begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ . \\ a_n \end{bmatrix} \qquad \dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ . \\ . \\ . \\ \dot{q}_n \end{bmatrix} \tag{1.8}$$

The constraint (1.7) is integrable if there exists a function $h : \Re^n \rightarrow \Re$ such that

$$\int a^T(q)\,\dot{q} = 0 \Longleftrightarrow h(q) = c \tag{1.9}$$

where c is the integrating constant. The constraint (1.7) becomes a holonomic if it satisfies

$$a^T(q)\,\dot{q} = \sum_{j=1}^{n} a_j^T(q)\,\dot{q}_j = 0 \Longrightarrow \sum_{j=1}^{n} \frac{\partial h}{\partial q_j}\dot{q}_j = 0 \tag{1.10}$$

This implies that there exists some function of form $\alpha(q)$, such that

$$\alpha(q)\,a_j^T(q) = \frac{\partial h}{\partial q_j}(q)\,, j = 1, ..., n \tag{1.11}$$

and $\alpha(q)$ is called the integrating factor. From the above illustration, it can be stated that a single Pfaffian constraint is holonomic if and only if there exists an integrating factor $\alpha(q)$ such that $\alpha(q)\,a^T(q)$ is the derivative of some function h. It cannot be easily verified that the constraint is integrable or not from the condition

(1.11), because it involves the unknown function $h(q)$. But using Schwarz's theorem, the integrability condition (1.11) may be replaced by

$$\frac{\partial (\alpha a_k)}{\partial (q_j)} = \frac{\partial (\alpha a_j)}{\partial (q_k)}, j, k = 1, ..., n. \tag{1.12}$$

which does not involve the unknown function $h(q)$.

Example: For the following kinematic constraint in $\Re^3$

$$\dot{q}_1 + q_1\dot{q}_2 + \dot{q}_3 = 0 \tag{1.13}$$

using the integrability condition (1.10) one can get

$$\begin{aligned} \frac{\partial \alpha}{\partial q_2} - \alpha - \frac{\partial \alpha}{\partial q_1} &= 0 \\ \frac{\partial \alpha}{\partial q_3} - \frac{\partial \alpha}{\partial q_1} &= 0 \\ \frac{\partial \alpha}{\partial q_3} q_1 - \frac{\partial \alpha}{\partial q_2} &= 0 \end{aligned} \tag{1.14}$$

Solving for $\alpha(q)$, the only possible solution is $\alpha(q) \equiv 0$. Hence, the constraint is not integrable.

The above example considered is only a single Pfaffian constraint. Finding $\alpha(q)$ becomes more difficult in the presence of multiple Pfaffian constraints of the form

$$\alpha_i(q)\, a_i^T(q)\, \dot{q} = 0 \tag{1.15}$$

where $i = 1, ..., k > n$. To check the integrability of k number of Pfaffian constraints, one has to check all the k constraints individually whether they are integrable or not and also which linear independent combinations of these constraints are integrable or not. From k number of kinematic constraints. One can often come across a system where o $(0 < o < k)$ number of constraints that are integrable and the number of constraints, $m = k - o$ are not integrable. An example of such a system is given below.

Example: The constraints on the omnidirectional symmetric three wheeled mobile robot can be represented by

$$A^T(q)\,\dot{q} = 0$$

where

$$A = \begin{bmatrix} \frac{\sqrt{3}}{2}\cos q_3 - \frac{1}{2}\sin q_3 & \sin q_3 & -\frac{1}{2}\sin q_3 - \frac{\sqrt{3}}{2}\cos q_3 \\ \frac{1}{2}\cos q_3 + \frac{\sqrt{3}}{2}\sin q_3 & -\cos q_3 & \frac{1}{2}\cos q_3 - \frac{\sqrt{3}}{2}\sin q_3 \\ l & l & l \\ r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix} \qquad \dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix}$$

in which r is the radius of the wheels, $q \in \Re^6$ and l is the distance between the center of gravity and the wheels. The three constraints cannot be individually integrated but when all the three constraints are added the constraint becomes

$$\dot{q}_3 + \frac{r}{3l}\left(\dot{q}_4 + \dot{q}_5 + \dot{q}_6\right) = 0 \tag{1.16}$$

and can be integrated as

$$q_3 = -\frac{r}{3l}\left(q_4 + q_5 + q_6\right) + c \tag{1.17}$$

On the basis of the integrability of the constraints, a mechanical system can be classified into three types that affect the reachable configuration of the system, each in an unique way.

Completely holonomic: If all the kinematic constraints are integrable i.e., $k = o$ then the system is completely holonomic. In this case, there exists o number of independent functions $h_j(q)$ such that

$$span\left\{\frac{\partial h_1}{\partial q}(q), ..., \frac{\partial h_o}{\partial q}(q)\right\} \subset span\left\{a_1^T(q), ..., a_k^T(q)\right\}, \forall q \in \Re^n. \tag{1.18}$$

From the equation (1.18), it can be inferred that the system configuration are restricted to the $(n-o)$- dimensional manifold identified by the level surfaces of the $h_j's$, i.e.,

$$\{q \in \Re^n : h_1(q) = c_1, ..., h_o(q) = c_o\} \tag{1.19}$$

on which motion has started.

Partially nonholonomic: An interesting case arises when $o < k$, i.e., there are o holonomic constraints and $m = (k-o)$ nonholonomic constraints. These kind of systems are called as partially nonholonomic systems. The reachable configurations are restricted only to the holonomic constraints o. The m nonholonomic constraints do not restrict the system configuration.

Completely Nonholonomic: If $k = m$ or $o = 0$ then all the kinematic constraints are nonholonomic. Here m constraints do not restrict the reachable configuration of the system i.e., if $q \in \Re^n$ then system can take any values in the configuration space $\Re^n$. It is to be noted that from now onwards we are only concerned with systems that are completely nonholonomic.

1.3 Thesis Layout

In Chapter 2, a brief review of the control techniques applied for the nonholonomic system at the kinematic, dynamic and actuator dynamics level is presented. In addition, the problems related to the control of nonholonomic systems, background and importance of canonical chained forms, motivation and objective of the thesis are presented.

In chapter 3 the modeling of the nonholonomic mechanical system considering the kinematics, dynamics and actuator dynamics are presented.

Chapter 4 presents controller design based on the combination of robust and adaptive control techniques for the complete nonholonomic mechanical system with stability proof. Simulation results are presented to illustrates the effectiveness of the control algorithm designed.

In Chapter 5, a new adaptive controller is designed, the stability analysis and simulation results are presented.

Finally, Chapter 6 concludes with a general discussion highlighting the contributions of this research and suggestions for further research work.

Chapter 2

Control of Nonholonomic Mechanical Systems: Literature Review

In this chapter a procedure for converting a nonholonomic constraint to a nonholonomic control system, a brief review of the various control approaches and importance of including the actuator dynamics are presented.
Consider a multiple Pfaffian constraint which can be written as

$$a_i^T(q)\,\dot{q} = 0, \tag{2.1}$$

where i=*1,...,k*. The constraint (2.1) is considered as a restriction on the velocity in the direction of the matrix a_i, but the motion is not restricted on the null-space of the matrix a_i. The null-space of the matrix a_i can be computed as shown below

$$a_i^T(q)\,g_j(q) = 0, i = 1, ..., k, j = 1, ..., n - k = p \tag{2.2}$$

where, g_j is the orthogonal component of the matrix $a_i^T(q)$ and are considered to be smooth vector fields. A nonholonomic control system can be formed with some appropriate controls $u \in \Re^p$ such that the system can move without violating the nonholonomic constraint(2.1). The control system can be represented by

$$\dot{q} = g_1(q)\,u_1 + g_2(q)\,u_2 + ... + g_p(q)\,u_p \tag{2.3}$$

It implies that if $q(t)$ is a feasible trajectory for the system (2.3), then it also satisfies the constraint equation (2.1).

The basic objectives for control of the nonholonomic systems are

1. To find out whether the system is controllable i.e., a system is said to be controllable if it can be driven from one point to another point with a given class of controls and not concerned with the path taken. Although the system is controllable, if a path is prescribed, this is called path planning.

2. To analyze the stabilizability of the system i.e., finding the feedback laws. The feedback laws are particularly suited for motion control, to counteract the presence of disturbance, initial errors and modelling inaccuracies.

In order to achieve the above stated control objectives, there are few difficulties encountered for designing controller for the nonholonomic systems which are discussed in the next section.

2.1 Problems related to the Control of Nonholonomic Mechanical Systems

The system (2.3) is controllable if, for any choice of $q_1, q_2 \in \Re^n$, there exists a finite time T and an input $u : [0,T] \to U$ such that $q(T, 0, q_1, u) = q_2$ where $u \in \Re^p$ takes values in the class of piecewise-constant function U over time. The controllability can be checked from the Chow's theorem [16]. For Chow's theorem see Appendix. It is known that these kind of control systems (2.3) can be controllable but many of the standard control techniques which can be used to control the nonlinear systems cannot be directly applied to the nonholonomic systems. The difficulty in controlling the nonholonomic systems is that there are more state variables $q \in \Re^n$ to be controlled by less number of controls $u \in \Re^p$, which leads to an underactuated system.

The first thing that needs to be checked is to see whether a nonlinear system can be linearized around the equilibrium. In case of nonholonomic systems it is not possible because the linearized nonholonomic systems does not satisfy the Kalman controllability rank condition [62]. Hence the linearized nonholonomic system is uncontrollable. Feedback linearization is another approach to nonlinear control design which transforms a nonlinear systems dynamics into a linear one, so that the linear control techniques can be applied. There are two conditions discussed in [62], for the existence of input-state linearization: controllability and involutivity. It is known from the Chows theorem that nonholonomic systems are controllable. For the involutivity condition to hold, the vector fields $\{g_1, g_2, ..., g_p\}$ should be expressed as a linear combination of $\{g_1, g_2, ..., g_p\}$. But in nonholonomic systems, the vector fields are linearly independent, so the involutivity condition doesn't hold. Therefore it can be concluded that the nonholonomic systems cannot be state feedback linearizable. Another difficulty with the control of nonholonomic systems is stabilizability. Stabilization problems are concerned with obtaining feedback laws which guarantee that an equilibrium of the closed loop system is asymptotically stable. If a linear system is controllable then there exists a smooth state feedback to make the closed loop system asymptotically stable. But this does not hold for nonlinear systems.

A general theorem on necessary condition for feedback stabilization of nonlinear systems was given by Brockett [17]. It is well known that nonholonomic systems do not satisfy necessary condition of Brockett [17]. Hence nonholonomic systems with restricted mobility cannot be stabilized to a desired configuration via differentiable, or even continuous, pure-state feedback.

2.2 Control of Kinematic Nonholonomic Mechanical Systems

Taking into account the limitations stated in the previous section in designing a controller for the nonholonomic systems, the control problem can be divided into motion planning or open loop control and feedback stabilization or closed loop control.

2.2.1 Open Loop Control

The basic idea of motion planning is to obtain an open loop or feedforward controls which steer a nonholonomic control system from an initial state q_0 to a final state q_f over a given finite time interval $t = [0, T]$. The input u doesn't depend on the system state q nor on the error e. As a result, a feasible trajectory is obtained connecting q_0 and q_f without violating the nonholonomic constraint. However such a solution is not robust with respect to the disturbances, error on the initial condition or modelling inaccuracies.

There are many motion planning algorithms that have been proposed. One of the early and important work on motion planning mentioned in [21], used sinusoids at integrally related frequencies to steer a class of canonical systems called chained forms. The importance of chained forms are with an appropriate control input, the system can be driven to any configuration in $q \in \Re^n$. A brief review of chained forms is given.

From the theory of nonlinear control, the controllability of the system (2.3) can be characterized in terms of Lie algebra generated by the vector fields g_i. Lie bracket can be defined by two vector fields f, g, as

$$[f, g] = \frac{\partial g}{\partial q} f - \frac{\partial f}{\partial q} g \tag{2.4}$$

The Chows theorem states that if the system can move in every direction using Lie bracket motions (may be using higher order), then the system is controllable. This can be illustrated as follows. Consider a control system with two control inputs ($p = 2$)

$$\Sigma : \dot{q} = g_1 u_1 + g_2 u_2 \tag{2.5}$$

where $q \in \Re^n$, $n = 3$ and $u_1, u_2 \in \Re^2$, let the above control system be the result of a no slip condition imposed on the system. Since the system is completely nonholonomic according to the Chow's theorem, the system Σ is controllable and can reach any configuration in $\Re^3$. It is known that if g_1 and g_2 are two linearly independent vector fields, then the Lie bracket operation on these vector fields i.e., $[g_1, g_2]$ results in flow concatenated by g_1 and g_2 in a independent new direction given by g_3 where

$$g_3 = [g_1, g_2] \tag{2.6}$$

The vector fields g_1, g_2, g_3 span all of $\Re^3$. By using the first order Lie bracket operation the system can be made to move in a completely independent direction, which is not possible alone by g_1 and g_2. Let the nonholonomic control system Σ has $q \in \Re^n$and $u \in \Re^p$ where $n > 3$ and $p = 2$, in this cases higher order of Lie brackets are used in order to move in all the directions. From the above it is clear that by using the first or higher order of the Lie brackets, system can reach all the configurations in $\Re^n$. Brockett [17] derived the optimal controls for a set of canonical systems in which the tangent space to the configuration manifold is spanned by the input vector fields and their (first order) Lie brackets. In [21], the authors used the higher order Lie brackets to achieve controllability and had given the sufficient condition by which system of the form Σ, which is a two input nonholonomic control system can be converted to a canonical chained form. In [55] a sufficient condition for the conversion of the general multiple input system with $p > 2$ to a multiple input chained form is given. Two input chained system is broad enough to handle most of the nonholonomic systems, such as mobile robots, car like vehicle, car with trailers, etc.

The chained system, depending on the order of Lie bracket can be divided into first order systems, second order systems and so on. The general first order chained system is given by

$$\dot{q}_i = u_i \quad i = 1, ..., p$$

$$\dot{q}_{ij} = q_i u_j \quad i > j \tag{2.7}$$

In $q \in \Re^3$ with $p = 2$, the system become a two-input chained form

$$\dot{q}_i = u_1$$

$$\dot{q}_2 = u_2$$

$$\dot{q}_{21} = \dot{q}_3 = q_2 u_2 \tag{2.8}$$

Second order chained system can be obtained where first level of bracketing is not enough to span $\Re^n$, by extending the canonical form (2.7) to the next higher level of bracketing given by

$$\dot{q}_i = u_i \qquad i = 1, ..., p$$

$$\dot{q}_{ijk} = q_i u_j \qquad i > j$$

$$\dot{q}_{ijk} = q_{ij} u_k \tag{2.9}$$

where k is called the order of nilpotency. A Lie algebra is nilpotent if there exists an integer k such that all Lie products of length greater than k are zero.
The above chained form can be steered using simple sinusoidal control that may be used for generating motions affecting the i^{th} set of coordinates while leaving the previous sets of coordinates unchanged. The idea is:

1. Steer q_1 to the desired value using any input and ignoring the evolutions of the q_i's $(i > 1)$.

2. Using sinusoids at integrally related frequencies, iteratively find the inputs steering the q_i's without changing the q_j's, $j < i$.

Tilbury and Sastry in [23] used sinusoidal control to steer all coordinates at once for systems with two inputs. They also demonstrated how polynomial controls may be used to steer the two input chained form. The use of piecewise-constant inputs is another method to steer a chained form system. Under such kind of control, nonholonomic systems behave as an piecewise-linear system. Hence, forward integral of motion equation is very simple [24].

2.2.2 Feedback or Closed Loop Control

Stabilization problems are concerned with obtaining feedback laws which guarantee that an equilibrium of the closed loop system is asymptotically stable. From the Brockett's necessary condition for feedback stabilization [17], it is known that no smooth or continuous time-invariant static state feedback is possible which makes a specified equilibrium of the closed loop system locally asymptotically stable [25]. Moreover, there exists no dynamic continuous time-invariant feedback controller which makes closed loop system locally asymptotically stable [26]. Hence, a non-holonomic control system cannot be stabilized to an equilibrium using feedback linearization or any other control design approach that uses smooth time-invariable feedback.

The feedback stabilization of nonholonomic control system can be classified into three approaches using non-smooth or discontinuous time-invariant stabilization, time-varying stabilization and hybrid stabilization.

Discontinuous Time-invariant Stabilization

In [27] a discontinuous feedback law is obtained by transforming wheeled mobile robot system to a two dimensional system. Under this discontinuous feedback controller every path of the cart asymptotically approaches a particular circle which passes through the origin of the plane and is centered on the y-axis. In this way asymptotic stability is achieved.
In [28, 29, 30, 31] a nonsmooth state transformation is used to overcome the problem of stability caused by Brockett's theorem [17] and then a smooth time-invariant feedback is used to stabilize the transferred system. In the original coordinates the resulting feedback law is discontinuous.
In [32, 33] sliding mode approach is proposed to develop time-invariant feedback laws. These discontinuous feedback laws force the trajectory to eventually slide along a manifold of codimension one towards the equilibrium. The main disadvantage of sliding mode is that it may cause chattering effect.

Time-Varying Feedback Stabilization

Samson [34] proposed smooth time-varying feedback ie., feedback which explicitly depends on the time variable. Other works on the time-varying feedback control of nonholonomic systems [35, 36, 37] and its extension in robotic systems [38]. In [39, 40, 41], the method of averaging and saturation type functions are used to construct a smooth time-periodic laws for system in chained form. As demonstrated in [39] the rates of convergence provided by the smooth time-periodic feedback laws are necessarily nonexponential. A nonsmooth feedback laws with exponential convergence rate has been proposed in [42].

2.2.2.1 Tracking of Nonholonomic Mechanical Systems

Consider a nonholonomic control system in chained form is given by

$$\dot{q}_i = u_i \quad i = 1, ..., p$$

$$\dot{q}_{ij} = q_i u_j \quad i > j \tag{2.10}$$

where $q_i \in \Re^n$denotes the state of the system, $u_i \in \Re^p$ denotes the input by means of which the system can be controlled and t is the time. Further more, a feasible desired state trajectory $q_{di}(t)$ is assumed to be given for the system to track. Hence there exists a reference input $u_{di}(t)$, such that

$$\dot{q}_{di} = u_{di} \quad i = 1, ..., p$$

$$\dot{q}_{dij} = q_{di} u_{dj} \quad i > j \tag{2.11}$$

The feasible reference trajectory for the system is assumed to be given. The tracking problem can be stated as finding an appropriate control law , finding an appropriate control law

$$u_i = u_i(t, q_{di}, u_{di}, q) \tag{2.12}$$

for a given reference trajectory (q_{di}, u_{di}) such that the resulting closed loop system guarantees (2.10,2.12)

$$\lim_{t \to \infty} \|q_i(t) - q_{di}(t)\| = 0 \tag{2.13}$$

There have been many number of elegant control schemes for tracking control of nonholonomic systems [43, 48, 44, 47, 45]. Jiang and Nijmeijer in [49] proposed a backstepping based tracking control method for the kinematic and simplified dynamic model of the two wheeled mobile robot. Integrator backstepping is introduced by Kokotovic [50]. In [51, 52], integrator backstepping was successfully exploited to tackle the problem of global asymptotic stability and adaptive control of some class of nonholonomic systems.

2.3 Control of Nonholonomic Mechanical Systems including Dynamic

The dynamics of a mechanical system describes how the system responds to the actuator forces. For simplicity it is assumed that the actuator do not have dynamics of their own. The dynamic equation of mechanical system with nonholonomic constraint (2.1) using Euler-Lagrangian formulation was given by [56, 57]. The control of nonholonomic systems at the dynamic level takes inertia and forces into account and provides physical control such as driving torques. The dynamic model for the n- dimensional mechanical system with m nonholonomic constraint can be derived using Lagrange formulation. Let T be the total kinetic energy of the mechanical system

$$T = \frac{1}{2}\dot{q}^T D(q)\,\dot{q} \tag{2.14}$$

and the potential energy G associated with the mechanical system is only a function of position q

$$G = G(q)\,. \tag{2.15}$$

Lagrange $\mathcal{L}$ is defined as the difference between systems kinetic and potential energy

$$\mathcal{L}(q,\dot{q}) = T - G = \frac{1}{2}\dot{q}^T D(q)\,\dot{q} - G(q) \tag{2.16}$$

where $D(q)$ is the positive definite inertial matrix of the mechanical system. The Euler-Lagrange equation of motion are

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{q}}\right)^T - \left(\frac{\partial\mathcal{L}}{\partial q}\right)^T = Q \tag{2.17}$$

where Q is the external force applied on the system.

The above equation can be simplified and can generally be represented in the form given by

$$D(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + G(q) = J(q)^T\lambda + B(q)\,\tau \tag{2.18}$$

with nonholonomic constraint of the form

$$J(q)\dot{q} = 0 \tag{2.19}$$

where $B(q)$ is an $n \times r$ input transformation matrix with $r = n - m$, $\tau \in \Re^r$ is the applied torque on the system, $J(q)$ is a $m \times n$ constraint matrix, Q is the external force applied on the system, $\lambda \in \Re^m$ are lagrange multipliers and

$$C(q,\dot{q})\dot{q} = \dot{D}(q)\dot{q} - \frac{1}{2}\left(\frac{\partial}{\partial q}\left(\dot{q}^T D(q)\dot{q}\right)\right)^T + \left(\frac{\partial G(q)}{\partial q}\right)^T \tag{2.20}$$

In the previous section we have discussed various approaches for achieving different control objectives, but all of these take into account the kinematics of the system only. Kinematic control provides the solution only on a pure kinematic level, yielding kinematic controls such as driving speed. Control at the dynamic level takes inertia and forces into account and yields physical controls such as driving torques. Recognizing the importance of considering the system dynamics, several researchers have started to pay attention to this problem in recent years. A brief review of the control strategies considering the dynamics is presented below.

In [43] a kinematic controller is designed so that the tracking error between a real robot and reference robot converges to zero and a torque controller is designed by using backstepping control technique so the velocities of a mobile robot converge to the desired velocities which are given by the kinematic controller design. Su [7] studied the dynamic tracking problem of nonholonomic system with unknown inertial parameters. In [58], a variable structure control law was proposed with which mobile robot converges to the reference trajectories with bounded errors of position. Chen [53] discussed the dynamic tracking problem with uncertainties using H_∞ techniques. In [54] the trajectory tracking control problem of dynamics nonholonomic systems was discussed when the system dynamics are not known and a robust controller were proposed. Oya, Su and Kotoh [6] proposed a position/ force tracking control of nonholonomic system with classical nonholonomic constraint with model uncertainties taking into account the system dynamics and providing a control at the dynamic level.

2.4 Dynamics of the Complete Nonholonomic Mechanical System

Actuators are used in order to produce mechanical movement in machines. There are different types of actuators that can be used for the nonholonomic mechanical systems and they can be broadly classified as hydraulic, pneumatic, piezoactuators, shape memory alloys (SMA), electrical, etc. In the present day application electrical actuators are the most popular. Electrical actuators are of two types DC motor and AC motor. AC motors are used for driving heavy loads and are commonly found in industries. DC motors are commonly used for smaller jobs. The most popular actuator is the DC motor because they are cheap and easy to control. A simple schematic diagram of a DC motor is given in Fig. 2.4.

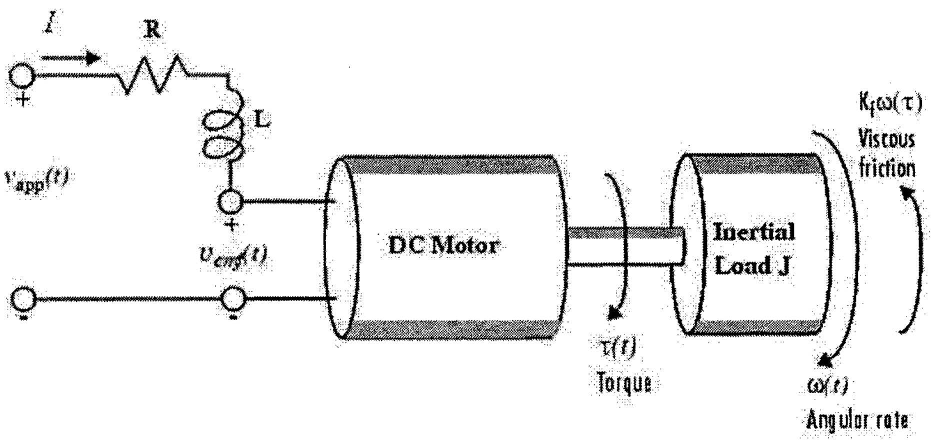

Figure 2.1: DC Motor

The torque τ seen at the shaft of the motor is proportional to the current I induced by the applied voltage,

$$\tau = K_N I \tag{2.21}$$

where K_N, the armature constant, is related to physical properties of the motor. The back electromotive force ν_{emf}, is a voltage proportional to the angular velocity ω, with K_a, the emf constant, also depends on the properties of the motor,

$$\nu_{emf} = K_a \omega. \tag{2.22}$$

The electric equation of the motor can be described by

$$\nu_{app} = L\frac{dI}{dt} + RI + K_a \omega \tag{2.23}$$

As stated in the previous section, actuator dynamics, system dynamics along with the kinematics form a complete system. Not many researchers have focused on controlling of nonholonomic system considering system kinematics, system dynamics and actuator dynamics. The justification for not considering the actuator dynamics could be that operating velocities are low. But when the systems have to perform accurately at high velocities, not including actuator dynamics can have a major impact in the overall performance of the system and in few circumstance can cause instability.

The above statement can be supported by the results obtained from the other robotic systems, such as robotic manipulators. Robotic manipulators are fixed robots with links and control problems related to these system have been very well understood and especially when actuators are not included. Experimental results [59] and theoretical results [60] indicated the detrimental effects of neglected actuator dynamics including instability and limited bandwidth. In [61] experimental tests with a Puma 560 manipulator are presented confirming the improved performance obtained when actuator dynamics is considered. After these experimental results there has been extensive research that is done and is still going on.

As wheeled mobile robots, legged mobile robots, multi-finger grasping robots, underwater robots, space robots, etc., are all subjected to nonholonomic constraint and these systems have been used extensively. The need for the error free motion control has motivated us to design a controller for a class of nonholonomic system whose kinematic system can be converted to a chained form, including system dynamics and actuator dynamics.

Chapter 3

Dynamic Modeling Including Actuator Dynamics

Consider a mechanical system with a classical nonholonomic constraint including actuator dynamics, whose dynamics are described, in local coordinates, by the following formulation

$$J(q)\dot{q} = 0 \tag{3.1}$$

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = J^T(q)\lambda + B(q)K_N I \tag{3.2}$$

$$L\frac{dI}{dt} + RI + K_a\omega = \nu \tag{3.3}$$

where q denotes a n vector of generalized coordinates; I denotes a r vector of armature current; $\lambda \in R^m$ is associated Lagrangian multipliers which expresses the contact force at the contact point between the rigid body and the environmental surface; $D(q)$ is a $(n \times n)$ symmetric, bounded, positive definite inertia matrix; $C(q,\dot{q})\dot{q}$ presents a n vector of centripetal and Coriolis torques; $G(q)$ is a n vector of gravitational torques; $B(q)$ is a $n \times r$ input transformation matrix assumed to be known because it is a function of fixed geometry of the system; K_N is a positive definite diagonal matrix which characterizes the electromechanical conversion between current and torque; $J(q)$ is $(m \times n)$ constraint matrix and the terms $L = diag[L_1, L_2, L_3, ..., L_r]$,

$R = diag\left[R_1, R_2, R_3, ..., R_r\right]$, $K_a = diag\left[K_{a1}, K_{a2}, K_{a3}, ..., K_{ar}\right]$, $\omega = \left[\omega_1, \omega_2, ..., \omega_r\right]^T$ and $\nu \in R^r$ represents the equivalent armature inductances, resistances, back emf constants, angular velocities of the driving motors and the control input voltage vector, respectively. The constraint (3.1) is assumed to be completely nonholonomic for all $q \in \Re^n$ and $t \in \Re$. To completely actuate the nonholonomic system, $B(q)$ is assumed to be a full-rank matrix and $r \geq n - m$.

Two simplifying properties can be stated regarding the dynamic structure of the system (3.2) are mentioned as follows

Property 1: There exists a so-called inertial parameter p vector β_p with components depending on the mechanical parameters (mass, moment of inertia, etc.,)[7] such that

$$D(q)\dot{v} + C(q, \dot{q})v + G(q) = \Phi(q, \dot{q}, v, \dot{v})\beta_p \tag{3.4}$$

where Φ is a $n \times l$ matrix of known functions of $q, \dot{q}, v$ and $\dot{v}$; and β_p is a l-vector of inertia parameters [11] and assumed completely unknown in this thesis.

Property 2: A suitable definition of $C(q, \dot{q})$ makes matrix $\left(\dot{D} - 2C\right)$ skew symmetric [7].

The control objective can be specified as follows. Given desired trajectories q_d and $\dot{q}_d$ which are assumed to be bounded and satisfy the constraint (3.1), with unknown inertial parameters β_p, determine a control law for ν such that q and $\dot{q}$ asymptotically converge to q_d and $\dot{q}_d$.

To solve the above tracking problem, we recall that n is the necessary and sufficient number of generalized coordinates required to describe the configuration of the systems. Likewise, the difference $p = n - m$ corresponding to unconstrained degrees of freedom is termed as velocity of degree of freedom. It is important to note that p denotes the number of generalized velocities, or of linear combinations thereof, that can be freely assigned without violating the kinematic constraint [9]. Henceforth, any set of linearly independent velocity variables will be termed as a set of independent generalized velocities.

We thus let v be a p dimensional vector of independent generalized velocities $\dot{q}$ that obeys the kinematic constraint (3.1). That is

$$\dot{q} = R(q)\, v \tag{3.5}$$

Methods of obtaining the expression of $R(q)$ can be referred in [7] and [8]. Based on (3.1) and (3.5), it is verified that

$$R^T(q)\, J^T(q) = 0 \tag{3.6}$$

Thus, $R(q)$ is an orthogonal complement of $J(q)$. Differentiating (3.5) and substituting the expression for $\ddot{q}$ in (3.2), then the dynamics (3.2), which satisfies the nonholonomic constraint (3.1), can be reformulated as

$$\dot{q} = R(q)\, v \tag{3.7}$$

$$D(q)\, R(q)\, \dot{v} + C_1(q, \dot{q})\, v + G(q) = B(q)\, K_N I + J^T(q)\, \lambda \tag{3.8}$$

where $C_1(q, \dot{q}) = D(q)\, \dot{R}(q) + \dot{C}(q, \dot{q})\, R(q)$. In the actuator dynamics (3.3) the relationship between ω and v is dependent on the type of mechanical systems and generally can be expressed as

$$\omega = \mu v \tag{3.9}$$

The structure of μ depends on the mechanical systems to be controlled. For instance, in the simulation example a type (2,0) mobile robot is used to illustrate the controller design, where μ can be derived as

$$\mu = \frac{1}{P}\begin{bmatrix} 1 & E \\ 1 & -E \end{bmatrix} \tag{3.10}$$

where P and E are shown in Fig. 3.

Eliminating ω from the actuator dynamics (3.3) by substituting (3.9), one gets

$$L\frac{dI}{dt} + RI + K_a \mu v = \nu, \tag{3.11}$$

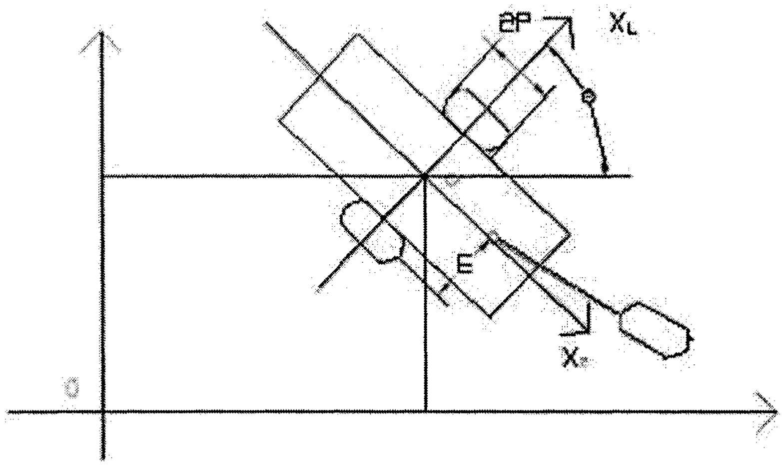

Figure 3.1: Type (2, 0) mobile robot.

Until now we have brought the kinematics (3.1), dynamics (3.2) and actuator dynamics (3.3) from the generalized coordinate system $q \in \Re^n$ to feasible independent generalized velocities $v \in \Re^p$ without violating the nonholonomic constraint (3.1).

Chapter 4

Robust Adaptive Control Algorithm with Actuator Dynamics

4.1 Controller Design

In this section a procedure for the controller design is outlined in two stages. Since the tracking control at the kinematic level is mainly for the kinematic equation which can be transferred to the so-called chained form [10], in the following development we will still follow this assumption. Furthermore, we will only take into account the case for two independent generalized velocities ($p = 2$) just for the sake of simplicity, but these results can be extended to a general case. The main goal in this thesis is to develop a motion tracking control strategy for the system subject to nonholonomic constraints with consideration of the actuator dynamics in a simple setting that reveals its essential features.

In order to design a controller it is assumed that there exist a coordinate transformation, $y = \Psi(q)$, and a state feedback, $v = \Omega_1(q)\,u$, so that the kinematic system (3.7) with $p = 2$ could be locally and globally converted to the chained form [10],[1]. The corresponding chained form equations are given by

$$\dot{y}_1 = u_1$$

$$\dot{y}_j = u_1 y_{j+1}, (2 \le j \le n-1)$$

$$\dot{y}_n = u_2 \tag{4.1}$$

Based on the transformation, $y = \Psi(q)$, the equations (3.8) and (3.11) can be modified as

$$D_2(y) R_2(y) \dot{u} + C_2(y, \dot{y}) u + G_2(y) = B_2(y) K_N I + J_2^T(y) \lambda \tag{4.2}$$

$$L\frac{dI}{dt} + RI + K_a Q(u, \mu, \Omega_1) = \nu \tag{4.3}$$

where

$$D_2(y) = D(q) \mid_{q=\Psi^{-1}(y)}$$

$$R_2(y) = R(q) \Omega_1(q) \mid_{q=\Psi^{-1}(y)}$$

$$C_2(y, \dot{y}) = [D(q) R(q) \Omega_1(q) + C_1(q, \dot{q}) \Omega_1(q)] \mid_{q=\Psi^{-1}(y)}$$

$$G_2(y) = G(q) \mid_{q=\Psi^{-1}(y)}$$

$$J_2(y) = J(q) \mid_{q=\Psi^{-1}(y)}$$

$$B_2(y) = B(q) \mid_{q=\Psi^{-1}(y)}$$

$$Q = \mu \Omega_1(q) u \mid_{q=\Psi^{-1}(y)}$$

It should be noted that the complete nonholonomic mechanical system is described by (4.1), (4.2) and (4.3). As will be clear in the later development, it is this cascade structure that makes it possible to design the robust control algorithm.

In stage 1, a strategy is developed so that the subsystem (4.1) tracks y_d with an embedded control input z, and at the same time, the output of the dynamic subsystem (4.2) is controlled to track this embedded control input by designing a virtual control input I_d. In stage 2 the actual control input ν is designed in such a way that $I \to I_d$. In turn, this allows $q(t)$ to track the desired trajectory $q_d(t)$.

4.1.1 Stage 1 (Controller Design for the Kinematic & Dynamic Subsystems)

Since the desired trajectory q_d should satisfy the constraint (3.1), therefore, there must exist a desired v_d satisfying

$$\dot{q_d} = R(q_d)\, v_d \tag{4.4}$$

Based on the fact that the kinematic system (3.1) can be converted into the chained form (4.1), there must exist a transformation $y_d = \Psi(q_d)$ and a state feedback, $v_d = \Omega(q_d)\, u_d$ such that (4.4) can be transformed as

$$\begin{aligned} \dot{y_{d1}} &= u_{d1} \\ \dot{y_{dj}} &= u_{d1} y_{dj+1}, (2 \le j \le n-1) \\ \dot{y}_{dn} &= u_{d2}. \end{aligned} \tag{4.5}$$

With the above transformations, the tracking problem considered in this thesis can be restated as seeking a strategy for specifying a control input ν for (4.1), (4.2) and (4.3), subject to the condition that parameters of the mechanical systems are not exactly known, such that $\{y, \dot{y}\} \rightarrow \{y_d, \dot{y_d}\}$.

Before proceeding further we would like to put-forward two assumptions.

Assumption (A1): The trajectories y_d and $(d^i y_{d1}/dt^i)$ $(1 \le i \le n-1)$ are bounded and $\lim_{t \to \infty} \inf |u_{d1}| > 0$.

Remark: As a matter of fact, Assumption (A1) on the boundedness of $\mathbf{y}_d$ can be relaxed to: $y_{di}(2 \le i \le n)$ are bounded, y_{d1} depending on $D_2 R_2$, C_2, and G_2. If $D_2 R_2$, C_2, and G_2 are bounded on y_1, there is no boundedness requirement for y_{d1}. This is the case in the simulation example.

Assumption (A2): The matrix $R_2^T R_2$ is nonsingular for all $y \in \Re^n$ and $t \in \Re$

Remarks: The matrix R_2 is determined by the constraint matrix $J(\mathbf{q})$ and is always a full rank matrix. Such an assumption is generally satisfied.

With the above assumptions, the following three properties can be obtained by exploiting the structure (4.2).

Property 3: The generalized inertia matrix $D_3 = R_2^T D_2(q) R_2$ is symmetric and positive definite.

Property 4: If C is defined such that *Property 2* is verified, $\left(\dot{D}_3 - 2R_2^T C_2\right)$ is a skew-symmetric matrix.

Property 5: The dynamic structure (4.2) is linear, in terms of the same suitable selected set of inertia parameters as used in *Property 1*, one can get

$$D_2(y) R_2(y) \dot{u} + C_2(y, \dot{y}) u + G_2(y) = \Phi_1(y, \dot{y}, u, \dot{u}) \beta_p \tag{4.6}$$

For the development of the embedded control z so that the subsystem (4.1) tracks y_d, we define the following terms $e = [e_1, e_2, ..., e_n]^T = y - y_d$, $s = e - \alpha$, and $\alpha = [\alpha_1, \alpha_2, ..., \alpha_n]^T$ with

$$\alpha_1 = 0$$

$$\alpha_2 = 0$$

$$\alpha_3 = -k_2 e_2 u_{d1}^{2l-1}$$

$$\alpha_4 = -(e_2 - \alpha_2) - k_3 (e_3 - \alpha_3) u_{d1}^{2l-1} + \frac{1}{u_{d1}} \sum_{i=0}^{0} \frac{\partial \alpha_3}{\partial u_{d1}^{[i]}} u_{d1}^{[i+1]} + \sum \frac{\partial \alpha_3}{\partial e_i} e_{i+1}$$

$$\vdots$$

$$\alpha_n = -(e_{n-2} - \alpha_{n-2}) - k_{n-1} (e_{n-1} - \alpha_{n-1}) u_{d1}^{2l-1}$$

$$+\frac{1}{u_{d1}}\sum_{i=0}^{n-4}\frac{\partial\alpha_{n-1}}{\partial u_{d1}^{[i]}}u_{d1}^{[i+1]}+\sum_{i=2}^{n-2}\frac{\partial\alpha_{n-1}}{\partial e_i}e_{i+1} \tag{4.7}$$

where $l = n-2$, $u_{d1}^{[i]}$, is the i-th derivative of u_{d1}, and k_i are the positive constants. Let z be of the form

$$z=\begin{bmatrix} u_{d1}+\eta \\ u_{d2}-s_{n-1}u_{d1}-k_n s_n+\sum_{i=0}^{n-3}\frac{\partial\alpha_n}{\partial\alpha u_{d1}^{[i]}}u_{d1}^{[i+1]} \\ +\sum_{i=2}^{n-1}\sum_{i=2}^{n-1}\frac{\partial\alpha_n}{\partial e_i}e_{i+1}\end{bmatrix} \tag{4.8}$$

$$\dot{\eta}=-k_0\eta-k_1s_1-\sum_{i=2}^{n-1}s_iy_{i+1}+\sum_{j=3}^{n}s_j\sum_{i=2}^{j-1}\frac{\partial\alpha_j}{\partial e_i}y_{i+1} \tag{4.9}$$

Considering the parameter vector β_p to be uncertain, a virtual control input I_d has to be designed in such a way that the dynamic system (4.2) tracks the z and the controller design at the dynamic level is achieved. Defining $\tilde{u} = u - z$ the control objective at the dynamic level is to synthesis I_d to make $\tilde{u} \to 0$ so that $u \to z$, which is defined as

$$I_d = \hat{K}_{NInv}\tau_{md} \tag{4.10}$$

where

$$\begin{aligned}\tau_{md} &= \left[B_2^TB_2\right]^{-1}\left[B_2\right]^T\left[\Phi_1\left(y,\dot{y},z,\dot{z}\right)\varphi-K_eR_2\left(u-z\right)\right] \\ &\quad-\left[B_2^TB_2\right]^{-1}\left[B_2\right]^T\left[R_2\left(R_2^TR_2\right)^{-1}\Lambda\right]\end{aligned} \tag{4.11}$$

with Λ and φ as follows

$$\Lambda=\begin{bmatrix} k_1s_1+\sum_{i=2}^{n-1}s_iy_{i+1}-\sum_{j=3}^{n}s_j\sum_{i=2}^{j-1}\frac{\partial\alpha_j}{\partial e_i}y_{i+1} \\ s_n\end{bmatrix} \tag{4.12}$$

$$\varphi=-\rho\frac{\Phi_1^TR_2\left(u-z\right)}{\left\|\Phi_1^TR_2\left(u-z\right)\right\|} \tag{4.13}$$

where ρ is the upper bound of the unknown inertia parameter β_p, i.e., $\|\beta_p\| \leq \rho$, which is assumed to be known. In the design of the control law, K_N is considered as an unknown parameter and $\hat{K}_{NInv}$ is an estimation of the parameter K_N^{-1} and is given as below

$$\dot{\hat{K}}_{NInv} = diag\left[\dot{\hat{K}}_{NInv1}, \dot{\hat{K}}_{NInv2}, ..., \dot{\hat{K}}_{NInvr}\right]$$
$$\dot{\hat{K}}_{NInvi} = -f_i \tau_{mdi}, \quad i = 1, 2, ..., r \tag{4.14}$$

where $\tilde{u}^T R_2^T B_2 = [f_1, f_2, ..., f_r]$.

4.1.2 Stage2 (Controller Design at the Actuator Level)

Till now in this chapter we have designed a virtual controller I_d and embedded controller z for dynamic and kinematic systems respectively but u tends to z can be guaranteed, if the actual input control signal of the dynamic system I be of the form I_d which can be realized from the actuator dynamics by the design of the actual control input ν. On the basis of the above statements we can conclude that if ν is designed in such a way that I tends to I_d then $||e|| \to 0$ and $||\tilde{u}|| \to 0$ and $||\dot{\tilde{u}}|| \to 0$. Define $I = e_I + I_d$ and substituting in (4.3) one gets

$$L\dot{e}_I + RI + K_a Q = -L\dot{I}_d + \nu$$

Adding $K_I e_I$ on both sides in the above equation, one gets

$$L\dot{e}_I + RI + K_a Q + K_I e_I = K_I e_I - L\dot{I}_d + \nu \tag{4.15}$$

The actuator parameters L, R and K_a are considered unknown and are estimated online using the adaptation laws. The control input ν is designed in form given below

$$\nu = \hat{L}\dot{I}_d + \hat{R}I - K_I e_I + \hat{K}_a Q + H \tag{4.16}$$

where $\hat{L} = diag\left[\hat{L}_1, \hat{L}_2, ..., \hat{L}_r\right]$, $\hat{R} = diag\left[\hat{R}_1, \hat{R}_2, ..., \hat{R}_r\right]$ and $\hat{K}_a = diag\left[\hat{K}_{a1}, \hat{K}_{a2}, ..., \hat{K}_{ar}\right]$ are the estimates of the unknown parameters L, R and K_a respectively, satisfying

$$\dot{\hat{L}}_i = -\gamma_6 \dot{I}_{di} e_{Ii}, i = 1, 2, ..., r \tag{4.17}$$

$$\dot{\hat{R}}_i = -\gamma_7 I_i e_{Ii}, i = 1, 2, ..., r \tag{4.18}$$

$$\dot{\hat{K}}_{ai} = -\gamma_8 Q_i e_{Ii}, i = 1, 2, ..., r \tag{4.19}$$

$$H_i = -\hat{K}_{Ni} f_i, i = 1, 2, ...r \tag{4.20}$$

where $\hat{K}_N$ is an estimate of the unknown parameter K_N

$$\dot{\hat{K}}_N = diag\left[\dot{\hat{K}}_{N1}, \dot{\hat{K}}_{N2}, ..., \dot{\hat{K}}_{Ni}\right], i = 1, 2, ..., r \tag{4.21}$$

$$\dot{\hat{K}}_{Ni} = \gamma_{10} e_{Ii} f_i, i = 1, 2, ..., r \tag{4.22}$$

where γ_6, γ_7, γ_8 and γ_{10} are known constants, which determine rates of the adaptations.

4.2 Stability Analysis for the Complete System

So far we have individually designed virtual controllers z and I_d for kinematic and dynamic systems, respectively. Then, an actual control input ν is developed. In the present section, a complete stability analysis for the whole system dynamics based on the combination of the above stages is given.

Consider a positive definite function for the whole system is given by

$$V = V_1 + V_2 + V_3 + \frac{1}{2}\sum_{i=1}^{r} \gamma_{10}^{-1} \tilde{K}_{Ni}^2 \tag{4.23}$$

where

$$V_1(t) = \frac{1}{2}\left(\sum_{i=2}^{n} s_i^2 + k_1 s_1^2 + \eta^2\right) \tag{4.24}$$

$$V_2(t) = \frac{1}{2}\tilde{u}^T D_3 \tilde{u} + \frac{1}{2}\sum_{i=1}^{r} K_{Ni} \tilde{K}_{NInvi}^2 \tag{4.25}$$

$$V_3 = \frac{1}{2} e_I^T L e_I + \frac{1}{2}\sum_{i=1}^{r}\left(\gamma_6^{-1}\tilde{L}_i^2 + \gamma_7^{-1}\tilde{R}_i^2 + \gamma_8^{-1}\tilde{K}_{ai}^2\right) \tag{4.26}$$

with $\tilde{K}_N = \hat{K}_N - K_N$, $\tilde{K}_{NInv} = \hat{K}_{NInv} - K_N^{-1}$, and $\tilde{L} = \hat{L} - L$, $\tilde{R} = \hat{R} - R$, and $\tilde{K}_a = \hat{K}_a - K_a$ represent the parameter estimation errors.

4.2.1 Procedure for Obtaining the Time Derivative of V_1:

Substituting the control laws (4.8) and (4.10) along with the adaptation controllers (4.9), (4.14) and robust controller (4.13), the closed loop error equation for the kinematic and dynamic systems are given as follows

$$\dot{s_1} = \eta + \tilde{u_1}$$

$$\dot{s_2} = s_3 u_{d1} - k_2 s_2 u_{d1}^{2l} + (\eta + \tilde{u_1})\, y_3$$

$$\dot{s_3} = s_4 u_{d1} - s_2 u_{d1} - k_3 s_3 u_{d1}^{2l} + (\eta + \tilde{u_1}) \left(y_4 - \frac{\partial \alpha_3}{\partial e_2} y_3 \right)$$

$$\vdots$$

$$\dot{s}_{n-1} = s_n u_{d1} - s_{n-2} u_{d1} - k_{n-1} s_{n-1} u_{d1}^{2l} + (\eta + \tilde{u}_1) \,.\, \left(y_n - \sum_{i=2}^{n-2} \frac{\partial \alpha_{n-1}}{\partial e_i} y_{i+1} \right)$$

$$\dot{s}_n = -k_n s_n - s_{n-1} u_{d1} + \tilde{u}_2 - (\eta + \tilde{u}_1) \,.\, \sum_{i=2}^{n-2} \frac{\partial \alpha_n}{\partial e_i} y_{i+1} \tag{4.27}$$

$$\dot{\eta} = -k_0 \eta - \Lambda_1 \tag{4.28}$$

$$D_2(y)\, R_2(y)\, \dot{u} + C_2(y, \dot{y})\, u + G_2(y) = B_2(y)\, K_N e_I + B_2(y)\, K_N I_d + J_2^T(y)\, \lambda \tag{4.29}$$

The time derivative of $V_1(t)$ along the solution of (4.27) - (4.28) is

$$\dot{V}_1 = \sum_{i=2}^{n} s_i \dot{s}_i + k_1 s_1 \dot{s}_1 + \eta \dot{\eta}$$

$$\dot{V}_1 = -\sum_{i=2}^{n-1} k_i s_i^2 u_{d1}^{2l} - k_n s_n^2 - k_0 \eta^2 + \tilde{u}^T \Lambda \tag{4.30}$$

4.2.2 Procedure for Obtaining the Time Derivative of V_2:

Substituting the error $\tilde{u} = u - z$ in (4.29), we have

$$D_2R_2\dot{\tilde{u}} + C_2\tilde{u} + D_2R_2\dot{z} + C_2z + G_2 = B_2K_Ne_I + B_2K_NI_d + J_2^T\lambda \tag{4.31}$$

Using the property 5 given in the section 4.1.1, (4.31) can be rewritten as

$$D_2R_2\dot{\tilde{u}} + C_2\tilde{u} + \Phi_1\beta_p = B_2K_Ne_I + B_2K_NI_d + J_2^T\lambda \tag{4.32}$$

By rearranging the terms, one gets

$$D_2R_2\dot{\tilde{u}} = -\Phi_1\beta_p - C_2\tilde{u} + B_2K_Ne_I + B_2K_NI_d + J_2^T\lambda \tag{4.33}$$

Introducing the control law (4.10) into the above equation

$$D_2R_2\dot{\tilde{u}} = -\Phi_1\beta_p - C_2\tilde{u} + B_2K_Ne_I + B_2K_N\tilde{K}_{NInv}\tau_{md} + B_2\tau_{md} + J_2^T\lambda$$

$$D_2R_2\dot{\tilde{u}} = \Phi_1(\varphi - \beta_p) - K_eR_2\tilde{u} - C_2\tilde{u} + B_2K_N\tilde{K}_{NInv}\tau_{md} - R_2\left(R_2^TR_2\right)^{-1}\Lambda$$
$$+B_2^TK_Ne_I + J_2^T\lambda \tag{4.34}$$

Multiplying R_2^T on both sides to the above equation one gets

$$D_3\dot{\tilde{u}} = R_2^T\Phi_1(\varphi - \beta_p) - R_2^TK_eR_2\tilde{u} - R_2^TC_2\tilde{u} + R_2^TB_2K_N\tilde{K}_{NInv}\tau_{md} - \Lambda$$
$$+R_2^TB_2^TK_Ne_I \tag{4.35}$$

The time derivative of $V_2(t)$ along the solution trajectory of (4.34) is

$$\dot{V}_2 = \tilde{u}^TD_3\dot{\tilde{u}} + \frac{1}{2}\tilde{u}^T\dot{D}_3\tilde{u} + \sum_{i=1}^{r}K_{Ni}\tilde{K}_{NInvi}\dot{\tilde{K}}_{NInvi}$$

$$\dot{V}_2 = \tilde{u}^TR_2^T\Phi_1(\varphi - \beta_p) - \tilde{u}^TR_2^TK_eR_2\tilde{u} + \tilde{u}^TR_2^TB_2K_N\tilde{K}_{NInv}\tau_{md} - \tilde{u}^T\Lambda$$
$$+\tilde{u}^T\left(\frac{1}{2}\dot{D}_3 - R_2^TC_2\right)\tilde{u} + \tilde{u}^TR_2^TB_2K_Ne_I - \sum_{i=1}^{r}K_{Ni}\tilde{K}_{NInvi}f_i\tau_{mdi} \tag{4.36}$$

Since $\left(\frac{1}{2}\dot{D}_3 - R_2^T C_2\right) = 0$ and $\tilde{u}^T R_2^T B_2 K_N \tilde{K}_{NInv}\tau_{md} - \sum_{i=1}^{r} K_{Ni}\tilde{K}_{NInvi} f_i \tau_{mdi} = 0$ we have

$$\dot{V}_2 = \tilde{u}^T R_2^T \Phi_1 (\varphi - \beta_p) - \tilde{u}^T R_2^T K_e R_2 \tilde{u} - \tilde{u}^T \Lambda + \tilde{u}^T R_2^T B_2 K_N e_I \tag{4.37}$$

The first term in (4.37) can be shown that

$$\left(\Phi_1^T R_2 \tilde{u}\right)^T (\varphi - \beta_p) = \left(\Phi_1^T R_2 \tilde{u}\right)^T \left(-\beta_p - \rho \frac{\Phi_1^T R_2 \tilde{u}}{\|\Phi_1^T R_2 \tilde{u}\|}\right)$$

$$\left(\Phi_1^T R_2 \tilde{u}\right)^T (\varphi - \beta_p) \leq \left\|\Phi_1^T R_2 \tilde{u}\right\| (\|\beta_p\| - \rho) \leq 0 \tag{4.38}$$

4.2.3 Procedure for Obtaining the Time Derivative of V_3:

Introducing the control law (4.16) into (4.15) one gets

$$L\dot{e}_I + K_I e_I = \tilde{R} I + \tilde{L}\dot{I}_d + \tilde{K}_a Q + H \tag{4.39}$$

Then, by differentiating V_3 along the dynamics (4.16) - (4.19), one gets

$$\dot{V}_3 = -e_I^T K_I + e_I^T \tilde{R} I + e_I^T \tilde{L}\dot{I}_d + e_I^T \tilde{K}_a Q + e_I^T H +$$

$$\sum_{i=1}^{r} \left(\gamma_6^{-1} \tilde{L}_i \dot{\hat{L}}_i + \gamma_7^{-1} \tilde{R}_i \dot{\hat{R}}_i + \gamma_8^{-1} \tilde{K}_{ai} \dot{\hat{K}}_{ai}\right)$$

$$\dot{V}_3 = -e_I^T K_I e_I + e_I H \tag{4.40}$$

4.2.4 Time Derivative of V:

With $\dot{V}_1$ in (4.30), $\dot{V}_2$ in (4.37), and $\dot{V}_3$ in (4.40), $\dot{V}$ can be obtained as

$$\dot{V} = -\sum_{i=2}^{n-1} k_i s_i^2 u_{d1}^{2l} - k_n s_n^2 - k_0 \eta^2 + \tilde{u}^T \Lambda + \tilde{u}^T R_2^T \Phi_1 (\varphi - \beta_p) - \tilde{u}^T R_2^T K_e R_2 \tilde{u}$$

$$-\tilde{u}^T \Lambda + \tilde{u}^T R_2^T B_2 K_N e_I - e_I^T K_I e_I + e_I H + \sum_{i=1}^{r} \gamma_{10}^{-1} \tilde{K}_{Ni} \dot{\hat{K}}_{Ni} \tag{4.41}$$

$$\dot{V} \leq -\sum_{i=2}^{n-1} k_i s_i^2 u_{d1}^{2l} - k_n s_n^2 - k_0\eta^2 + \tilde{u}^T R_2^T \Phi_1 (\varphi - \beta_p) - \tilde{u}^T R_2^T K_e R_2 \tilde{u} - e_I^T K_I e_I \quad (4.42)$$

By observing (4.38), it can be concluded that the considered positive function $V(t)$ is thus non-increasing. This in turn implies that η, $\mathbf{s}$, $\tilde{\mathbf{u}}$, $\tilde{K}_{NInvi}$, e_I, $\tilde{K}_{Ni}$ as well as $\tilde{R}$, $\tilde{L}$, $\tilde{K}_a$ are bounded, and $V(t)$ converges to limit value V_{lim}. By the definition of $\mathbf{s}$, it concludes that $\mathbf{e}$ is bounded. Using the assumption (A1) , it follows that $\mathbf{y}$ is bounded. In view of (4.27)-(4.29), $\dot{\mathbf{s}}$, $\dot{\tilde{\mathbf{u}}}$, $\dot{\eta}$ are bounded. Thus, η, $\mathbf{s}$, and $\tilde{\mathbf{u}}$ are uniformly bounded. We should mention that if D_2R_2, C_2, and G_2 are bounded on y_1, there is no boundedness requirement for y_{d1}. Next, we prove that $\mathbf{s}$, $\dot{\mathbf{s}}$, and η tend to zero. Since $\ddot{V}$ is bounded and $\dot{V}$ is uniformly continuous, it can be shown that $\dot{V}$ tends to zero. Therefore, η, $s_i u_{d1} (2 \leq i \leq n-1)$, s_n, and $\tilde{\mathbf{u}}$ tend to zero. By the assumption that $\lim_{t\to\infty} inf |u_{d1}| > 0$, one concludes that $s_i (2 \leq i \leq n)$ tends to zero.

Differentiating $u_{d1}^l \eta$ yields

$$\frac{d}{dt} u_{d1}^l \eta = -k_1 u_{d1}^l s_1 + l u_{d1}^{l-1} \dot{u}_{d1}^l \eta - k_0 u_{d1}^l \eta - u_{d1}^l \left(\sum_{i=2}^{n-1} s_i y_{i+1} - \sum_{j=3}^{n} s_j \sum_{i=2}^{j-1} \frac{\partial \alpha_j}{\partial e_i} y_{i+1}\right)$$

where the first term is uniformly continuous and the other terms tend to zero. Thus it can be conclude that $\frac{d}{dt}(u_{d1}^l \eta)$ converges to zero, which in turn implies that $u_{d1}^l s_1$ and s_1 tend to zero. Therefore, $\mathbf{s}$ and $\dot{\mathbf{s}}$ tend to zero. To complete the proof and establish asymptotic convergence of the tracking error, it is necessary to show that $\{\mathbf{y}, \dot{\mathbf{y}}\} \to \{\mathbf{y_d}, \dot{\mathbf{y}}_\mathbf{d}\}$ as $t \to \infty$. This is accomplished by the following arguments.

Based on the definition of α given in (4.7) and the relationship $\mathbf{s} = \mathbf{e} - \alpha$, it is obvious that $s_i = 0 (i = 1, 2)$ yields $\lim_{t\to\infty} y_i = y_{di}$ and $\lim_{t\to\infty} \dot{y}_i = \dot{y}_{di} (i = 1, 2)$ because of $\alpha_1 = \alpha_2 = 0$. From the boundedness of u_{d1}, one obtains that α_3 and $\dot{\alpha}_3$ converge to zero, which results in $\lim_{t\to\infty} y_3 = y_{d3}$ and $\lim_{t\to\infty} \dot{y}_3 = \dot{y}_{d3}$. The convergence of α_3 and $\dot{\alpha}_3$ lead to the conclusion that α_4 and $\dot{\alpha}_4$ converge to zero, thus, $\lim_{t\to\infty} y_4 = y_{d4}$ and $\lim_{t\to\infty} \dot{y}_4 = \dot{y}_{d4}$. similarly, we can prove that $\lim_{t\to\infty} y_i = y_{di}$ and $\lim_{t\to\infty} \dot{y}_i = \dot{y}_{di} (5 \leq i \leq n)$. In summary, we have proved that $\{\mathbf{y}, \dot{\mathbf{y}}\} \to \{\mathbf{y_d}, \dot{\mathbf{y}}_\mathbf{d}\}$ as $t \to \infty$.

So it is proved that all the signals in the closed loop system remain bounded, and $||e|| \to 0$ and $||\dot{e}|| \to 0$.

4.3 Simulation Results

To verify the validity of the control approach outlined in this chapter, we consider a simplified model of a mobile wheeled robot [2] as shown in Fig. 1. This robot is of the Type (2,0), constituted by a rigid trolley equipped with non-deformable wheels and one motor for each rear wheel. The dynamic model of the mobile robot and actuator given respectively

$$
\begin{aligned}
m\ddot{x} &= \lambda \cos\theta - \frac{1}{P}\left(K_{N1}I_1 + K_{N2}I_2\right)\sin\theta \\
m\ddot{y} &= \lambda \sin\theta + \frac{1}{P}\left(K_{N1}I_1 + K_{N2}I_2\right)\cos\theta \\
I_0\ddot{\theta} &= \frac{E}{P}\left(K_{N1}I_1 - K_{N2}I_2\right)
\end{aligned}
\tag{4.43}
$$

$$
\begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix}\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} + \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}
+ \begin{bmatrix} K_{a1} & 0 \\ 0 & K_{a2} \end{bmatrix}\begin{bmatrix} Q_1(u,\mu,\Omega_1) \\ Q_2(u,\mu,\Omega_1) \end{bmatrix} = \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \tag{4.44}
$$

where

$$
\begin{bmatrix} Q_1(u,\mu,\Omega_1) \\ Q_2(u,\mu,\Omega_1) \end{bmatrix} = \begin{bmatrix} \frac{1}{P}\left(u_1\left(x\cos\theta + y\sin\theta\right) + u_2\left(1+E\right)\right) \\ \frac{1}{P}\left(u_1\left(x\cos\theta + y\sin\theta\right) + u_2\left(1-E\right)\right) \end{bmatrix}
$$

where x, y are the coordinates of the reference point O in the inertial frame, θ is the orientation of the basis with respect to the inertial frame m is the mass of the robot, and I_0 is its inertia moment around the vertical axis at point O, P is the radius of the wheels and $2E$ the length of the axis of the front wheels, and ν_1 and ν_2 are the

voltage provided to the motors which inturn provide τ_1 and τ_2 to the wheels.
The nonholonomic constraint is written as

$$\dot{x}\cos\theta + \dot{y}\sin\theta = 0 \tag{4.45}$$

Given the desired trajectory $q_d = [2\cos t, 2\sin t, t]^T$, which is circular path on the plane, the control objective is to determine a feedback control ν_1 and ν_2 so that the trajectory $q = [x, y, \theta]^T$ follows q_d.
The matrix $J(q)$ is, therefore, defined as $J(q) = [\cos\theta, \sin\theta, 0]$. The matrix $R(q)$ defined in (3.6) is chosen as

$$R = \begin{bmatrix} -\sin\theta & 0 \\ \cos\theta & 0 \\ 0 & 1 \end{bmatrix}$$

Therefore the kinematic system (4.45) can be written as

$$\begin{aligned} \dot{x} &= -v_1 \sin\theta \\ \dot{y} &= v_1 \cos\theta \\ \dot{\theta} &= v_2 \end{aligned} \tag{4.46}$$

Using a diffeomorphism transformation, $y = \Psi(q)$, and a state feedback, $v = \Omega(q)\,u$, which are defined as

$$\begin{aligned} y_1 &= \theta \\ y_2 &= x\cos\theta + y\sin\theta \\ y_3 &= -x\sin\theta + y\cos\theta \\ u_1 &= v_2 \\ u_2 &= v_1 - (x\cos\theta + y\sin\theta)\,v_2 \end{aligned} \tag{4.47}$$

the above kinematic system can be converted into the *chained* form

$$\dot{y}_1 = u_1$$

$$\dot{y}_2 = y_3 u_1$$

$$\dot{y}_3 = u_2 \tag{4.48}$$

The corresponding robot dynamic model (4.43) can be converted into

$$\begin{bmatrix} -my_2 \sin y_1 & -m \sin y_1 \\ my_2 \cos y_1 & m \cos y_1 \\ I_0 & 0 \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} - \begin{bmatrix} \cos y_1 \\ \sin y_1 \\ 0 \end{bmatrix} \lambda$$

$$+ \begin{bmatrix} -my_2\dot{y}_1 \cos y_1 - m\dot{y}_2 \sin y_1 & -m\dot{y}_1 \cos y_1 \\ -my_2\dot{y}_1 \sin y_1 + m\dot{y}_2 \cos y_1 & -m\dot{y}_1 \sin y_1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$= \frac{1}{P} \begin{bmatrix} -\sin y_1 & -\sin y_1 \\ \cos y_1 & \cos y_1 \\ E & -E \end{bmatrix} \begin{bmatrix} K_{N1} & 0 \\ 0 & K_{N2} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{4.49}$$

with

$$R_2 = \begin{bmatrix} -y_2 \sin y_1 & -\sin y_1 \\ y_2 \cos y_1 & \cos y_1 \\ 1 & 0 \end{bmatrix}$$

For the given $R(q)$, the desired trajectory $q_d = [2\cos t, 2\sin t, t]^T$satisfies $\dot{q}_d = R(q_d)\, v_d$ with $v_{d1} = 2$ and $v_{d2} = 1$. Using the above diffeomorphism transformation, the desired kinematic system $\dot{q}_d = R(q_d)\, v_d$ can be expressed as

$$y_{d1} = t$$

$$y_{d2} = 2$$

$$y_{d3} = 0 \tag{4.50}$$

with $u_{d1} = 1$ and $u_{d2} = 0$.

Introducing the actual control input $\nu = \begin{bmatrix} \nu_1 & \nu_2 \end{bmatrix}^T$ of the form given below

$$\nu = \begin{bmatrix} \hat{L}_1 & 0 \\ 0 & \hat{L}_2 \end{bmatrix} \begin{bmatrix} \dot{I}_{d1} \\ \dot{I}_{d2} \end{bmatrix} + \begin{bmatrix} \hat{R}_1 & 0 \\ 0 & \hat{R}_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}$$
$$- \begin{bmatrix} \hat{K}_{I1} & 0 \\ 0 & \hat{K}_{I2} \end{bmatrix} \begin{bmatrix} e_{I1} \\ e_{I2} \end{bmatrix} + \begin{bmatrix} \hat{K}_{a1} & 0 \\ 0 & \hat{K}_{a2} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \tag{4.51}$$

where $H_1 = -\hat{K}_{N1} f_1$ and $H_2 = -\hat{K}_{N2} f_2$

For the dynamic subsystems, the unknown parameters β_P are chosen as $\beta_P = \begin{bmatrix} m & I_0 \end{bmatrix}$ and are estimated by φ in (4.13) where Φ_1 is given by

$$\Phi_1(y, \dot{y}, z, \dot{z}) = \begin{bmatrix} -y_2\dot{z}_1 \sin y_1 - \dot{z}_2 \sin y_1 - y_2\dot{y}_1 z_1 \cos y_1 - \dot{y}_2 z_1 \sin y_1 - \dot{y}_1 z_2 \cos y_1 & 0 \\ y_2\dot{z}_1 \cos y_1 + \dot{z}_2 \cos y_1 - y_2\dot{y}_1 z_1 \sin y_1 + \dot{y}_2 z_1 \cos y_1 - \dot{y}_1 z_2 \sin y_1 & 0 \\ 0 & \dot{z}_1 \end{bmatrix} \tag{4.52}$$

Other unknown parameters K_N^{-1}, K_N, L, R and K_a are estimated using the terms $\hat{K}_{NInv}$, $\hat{K}_N$, $\hat{L}$, $\hat{R}$ and $\hat{K}_a$ with the following adaptive laws, respectively

$$\dot{\hat{K}}_{NInv1} = -f_1 \tau_{md1} \tag{4.53}$$

$$\dot{\hat{K}}_{NInv2} = -f_2 \tau_{md2} \tag{4.54}$$

where

$$f_1 = -\left(-\tilde{u}_1 y_2 \sin y_1 - \tilde{u}_2 \sin y_1\right) \frac{\sin y_1}{P}$$
$$+ \left(\tilde{u}_1 y_2 \cos y_1 + \tilde{u}_2 \cos y_1\right) \frac{\cos y_1}{P} + \frac{\tilde{u}_1 E}{P}$$
$$f_2 = -\left(-\tilde{u}_1 y_2 \sin y_1 - \tilde{u}_2 \sin y_1\right) \frac{\sin y_1}{P}$$
$$+ \left(\tilde{u}_1 y_2 \cos y_1 + \tilde{u}_2 \cos y_1\right) \frac{\cos y_1}{P} - \frac{\tilde{u}_1 E}{P}$$

$$\dot{\hat{K}}_{N1} = \gamma_{10} e_{I1} f_1$$

$$\dot{\hat{K}}_{N2} = \gamma_{10} e_{I2} f_2$$

$$\dot{\hat{L}}_1 = -\gamma_6 \dot{I}_{d1} e_{I1}$$

$$\dot{\hat{L}}_2 = -\gamma_6 \dot{I}_{d2} e_{I2}$$

$$\dot{\hat{R}}_1 = -\gamma_7 I_1 e_{I1}$$

$$\dot{\hat{R}}_2 = -\gamma_7 I_2 e_{I2}$$

$$\dot{\hat{K}}_{a1} = -\gamma_8 Q_1 e_{I1}, Q_1 = \left(\frac{y_2}{R} + \frac{D}{R}\right) u_1 + \frac{u_2}{R}$$

$$\dot{\hat{K}}_{a2} = -\gamma_8 Q_2 e_{I2}, Q_2 = \left(\frac{y_2}{R} - \frac{D}{R}\right) u_1 + \frac{u_2}{R}$$

The physical values for the simulation are taken as $m = 0.5, I_0 = 0.5$, $E = P = 1$ and $L_1 = L_2 = 2.03, R_1 = R_2 = 2, K_{N1} = K_{N2} = 1.5, K_{a1} = K_{a2} = 1$. The design parameters have an influence on the rate at which the tracking errors tend to zero. For this simulation the design parameters are set at $\gamma_6 = \gamma_{10} = \gamma_7 = \gamma_8 = 1$, $K_e = diag\,(5, 5)$, $k_0 = k_1 = k_2 = k_3 = 2$, $\rho = 1$ and $\eta\,(0) = 0$.

The simulation results are shown in figures 4.1 - 4.4. Figs. 4.1 shows the tracking trajectory errors and Fig. 4.2 shows the geometric trajectory of x and y. As the tracking errors and trajectory followed by the mobile robot proves the validity of the proposed algorithm. In Fig. 4.3 and 4.4 shows the tracking error $I - Id = e_I$ and $\tilde{u}$ tends to zero as $t \to \infty$.

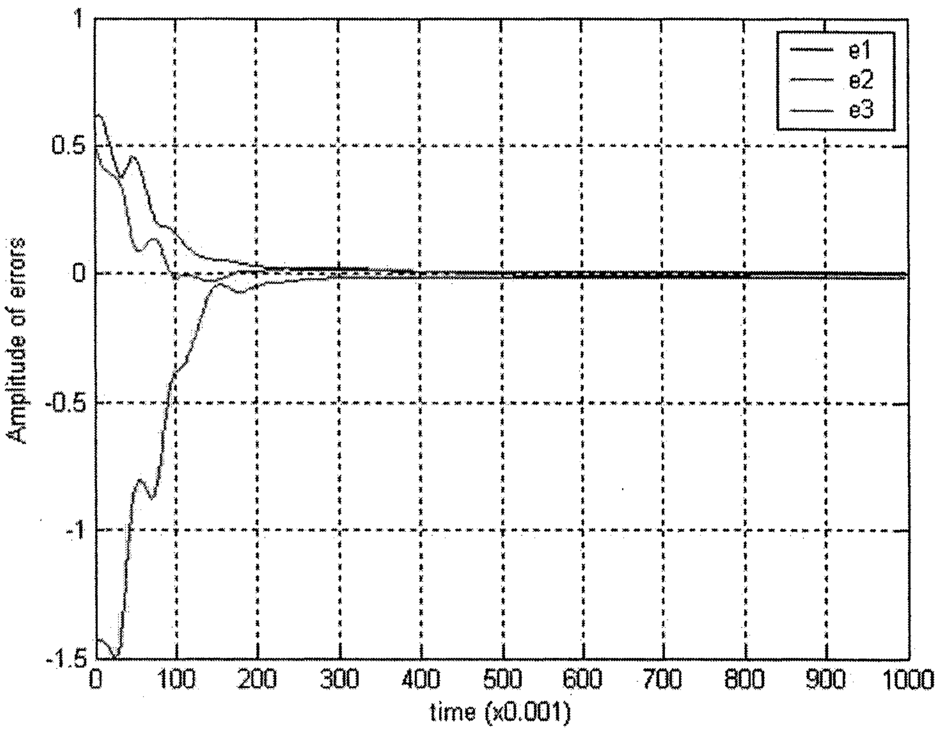

Figure 4.1: Tracking errors (e_1, e_2, e_3).

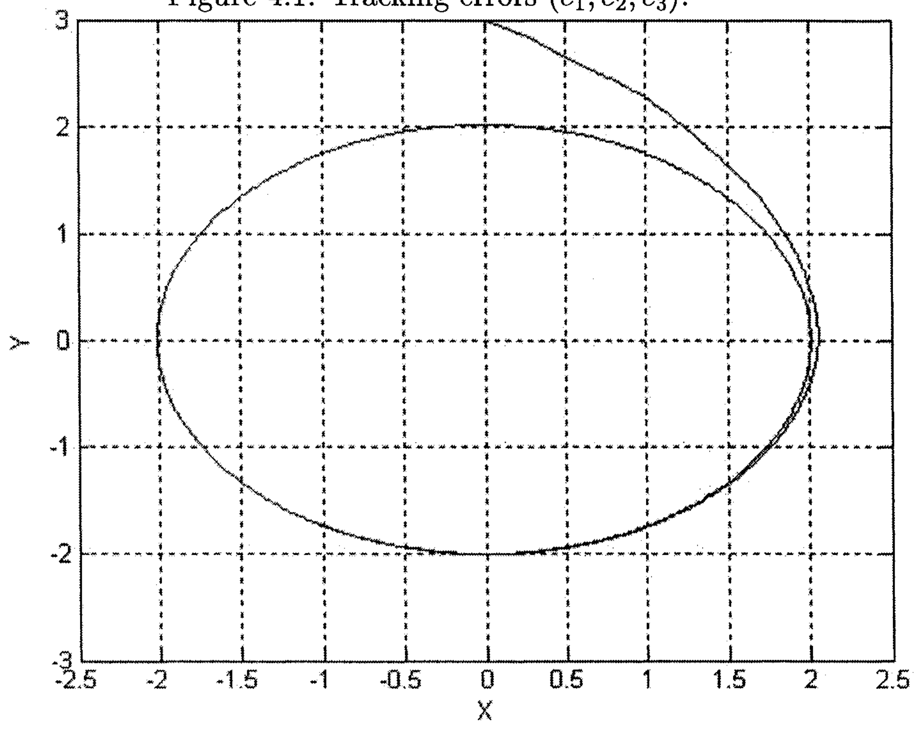

Figure 4.2: Geometric trajectory of x via y

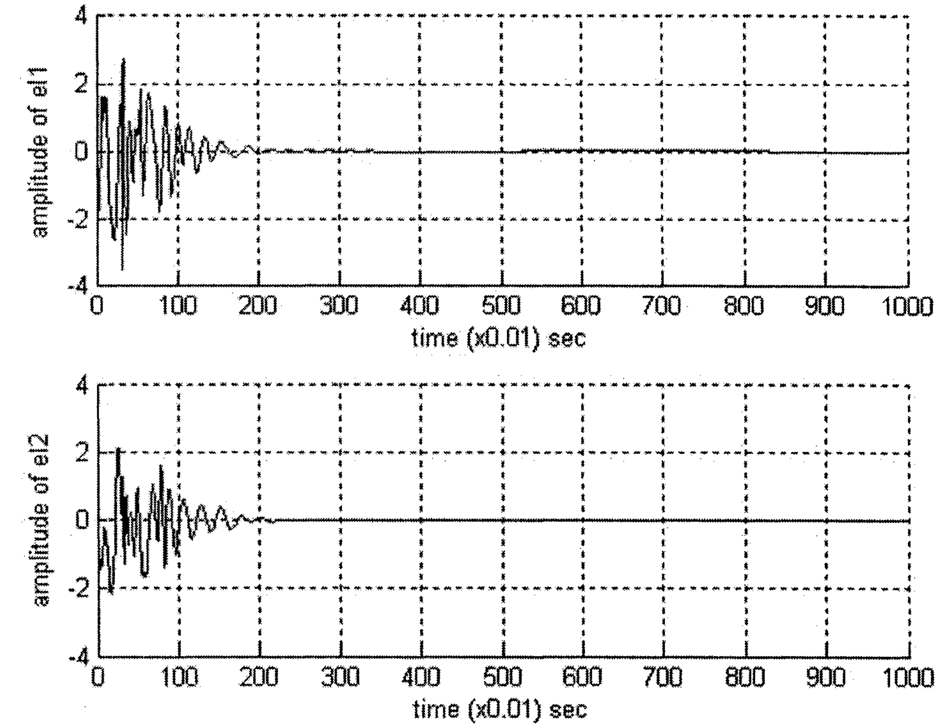

Figure 4.3: Tracking errors e_{I1} and e_{I2}

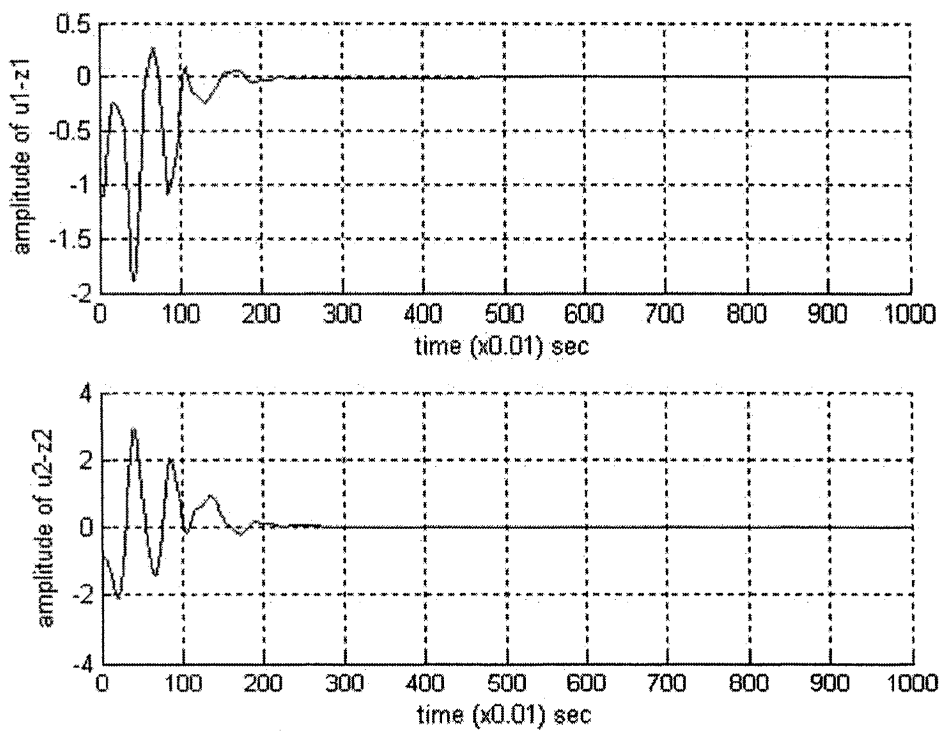

Figure 4.4: Tracking errors $\tilde{u}_1$ and $\tilde{u}_2$

Chapter 5

Adaptive Control Algorithm with Actuator Dynamics

5.1 Controller Design

In this chapter a procedure for the controller design is outlined. Since the tracking control at the kinematic level is mainly for the kinematic equation (3.7)which can be transferred to the so-called chained form [10], in the following development we will still follow this assumption. Furthermore, we will only take into account the case for two independent generalized velocities ($p = 2$) just for the sake of simplicity, but these results can be extended to a general case.

In order to design a controller it is assumed that there exist a coordinate transformation, $y = \Psi(q)$, and a state feedback, $v = \Omega_1(q)\,u$, so that the kinematic system (3.7) with $p = 2$ could be locally and globally converted to the chained form [10][1]. The corresponding chained form equations are given by

$$\begin{aligned} \dot{y}_1 &= u_1 \\ \dot{y}_j &= u_1 y_{j+1}, (2 \leq j \leq n-1) \\ \dot{y}_n &= u_2 \end{aligned} \tag{5.1}$$

Based on the transformation, $y = \Psi(q)$, the equations (3.8) and (3.11) can be modified as

$$D_2(y) R_2(y) \dot{u} + C_2(y, \dot{y}) u + G_2(y) = B_2(y) K_N I + J_2^T(y) \lambda \tag{5.2}$$

$$L\frac{dI}{dt} + RI + K_a Q(u, \mu, \Omega_1) = \nu \tag{5.3}$$

where

$$D_2(y) = D(q) \mid_{q=\Psi^{-1}(y)}$$

$$R_2(y) = R(q) \Omega_1(q) \mid_{q=\Psi^{-1}(y)}$$

$$C_2(y, \dot{y}) = [D(q) R(q) \Omega_1(q) + C_1(q, \dot{q}) \Omega_1(q)] \mid_{q=\Psi^{-1}(y)}$$

$$G_2(y) = G(q) \mid_{q=\Psi^{-1}(y)}$$

$$J_2(y) = J(q) \mid_{q=\Psi^{-1}(y)}$$

$$B_2(y) = B(q) \mid_{q=\Psi^{-1}(y)}$$

$$Q = \mu \Omega_1(q) u \mid_{q=\Psi^{-1}(y)}$$

It should be noted that the complete nonholonomic mechanical system is described by (5.1), (5.2) and (5.3). As will be clear in the later development, it is this cascade structure that makes it possible to design the adaptive control algorithm.

Since the desired trajectory q_d should satisfy the constraint (3.1), therefore, there must exist a desired v_d satisfying

$$\dot{q_d} = R(q_d) v_d \tag{5.4}$$

Based on the fact that the kinematic system (3.7) can be converted into the chained form (5.1), there must exist a transformation $y_d = \Psi(q_d)$ and a state feedback, $v_d = \Omega(q_d) u_d$ such that (5.4) can be transformed as

$$\dot{y_{d1}} = u_{d1}$$

$$\dot{y_{dj}} = u_{d1} y_{dj+1}, (2 \le j \le n-1)$$

$$\dot{y}_{dn} = u_{d2}. \qquad (5.5)$$

With the above transformations, the tracking problem considered in this chapter can be restated as seeking a strategy for specifying a control input ν for (5.1), (5.2) and (5.3), subject to the condition that parameters of the mechanical systems are not exactly known, such that $\{y, \dot{y}\} \rightarrow \{y_d, \dot{y}_d\}$.

Before proceeding further we would like to put-forward two assumptions.

Assumption (A1): The trajectories y_d and $(d^i y_{d1}/dt^i)$ $(1 \le i \le n-1)$ are bounded and $\lim_{t \to \infty} \inf |u_{d1}| > 0$.

Remark: As a matter of fact, Assumption (A1) on the boundedness of $\mathbf{y}_d$ can be relaxed to: $y_{di} (2 \le i \le n)$ are bounded, y_{d1} depending on $D_2 R_2$, C_2, and G_2. If $D_2 R_2$, C_2, and G_2 are bounded on y_1, there is no boundedness requirement for y_{d1}. This is the case in the simulation example.

Assumption (A2): The matrix $R_2^T R_2$ is nonsingular for all $y \in \Re^n$ and $t \in \Re$

Remarks: The matrix R_2 is determined by the constraint matrix $J(\mathbf{q})$ and is always a full rank matrix. Such an assumption is generally satisfied.

With the above assumptions, the following three properties can be obtained by exploiting the structure (5.2).

Property 3: The generalized inertia matrix $D_3 = R_2^T D_2 (q) R_2$ is symmetric and positive definite.

Property 4: If C is defined such that *Property 2* is verified, $\left(\dot{D}_3 - 2 R_2^T C_2\right)$ is a skew-symmetric matrix.

Property 5: The dynamic structure (5.2) is linear, in terms of the same suitable selected set of inertia parameters as used in *Property 1*, one can get

$$D_2 (y) R_2 (y) \dot{u} + C_2 (y, \dot{y}) u + G_2 (y) = \Phi_1 (y, \dot{y}, u, \dot{u}) \beta_p \qquad (5.6)$$

In the following development, two design stages will be used for the controller design.

In stage 1, a strategy is developed so that the subsystem (5.1) tracks y_d with a virtual control input z, and at the same time, the output of the dynamic subsystem (5.2) is controlled to track this virtual control input by designing another virtual control input I_d. In stage 2 the actual control input ν is designed in such a way that $I \to I_d$. In turn, this allows $q(t)$ to track the desired trajectory $q_d(t)$.

5.1.1 Stage 1 (Controller Design for the Kinematic & Dynamic Subsystems)

For the development of the virtual control z so that the subsystem (5.1) tracks y_d, we define the following terms $e = [e_1, e_2, ..., e_n]^T = y - y_d$, $s = e - \alpha$, and $\alpha = [\alpha_1, \alpha_2, ..., \alpha_n]^T$ with

$$\alpha_1 = 0$$

$$\alpha_2 = 0$$

$$\alpha_3 = -k_2 e_2 u_{d1}^{2l-1}$$

$$\alpha_4 = -(e_2 - \alpha_2) - k_3(e_3 - \alpha_3) u_{d1}^{2l-1} + \frac{1}{u_{d1}} \sum_{i=0}^{0} \frac{\partial \alpha_3}{\partial u_{d1}^{[i]}} u_{d1}^{[i+1]} + \sum \frac{\partial \alpha_3}{\partial e_i} e_{i+1}$$

.

.

$$\alpha_n = -(e_{n-2} - \alpha_{n-2}) - k_{n-1}(e_{n-1} - \alpha_{n-1}) u_{d1}^{2l-1} + \frac{1}{u_{d1}} \sum_{i=0}^{n-4} \frac{\partial \alpha_{n-1}}{\partial u_{d1}^{[i]}} u_{d1}^{[i+1]} + \sum_{i=2}^{n-2} \frac{\partial \alpha_{n-1}}{\partial e_i} e_{i+1} \tag{5.7}$$

where $l = n-2$, $u_{d1}^{[i]}$, is the i-th derivative of u_{d1}, and k_i are the positive constants. Let z be of the form

$$z = \begin{bmatrix} u_{d1} + \eta \\ u_{d2} - s_{n-1}u_{d1} - k_n s_n + \sum_{i=0}^{n-3} \frac{\partial \alpha_n}{\partial \alpha u_{d1}^{[i]}} u_{d1}^{[i+1]} \\ + \sum_{i=2}^{n-1} \sum_{i=2}^{n-1} \frac{\partial \alpha_n}{\partial e_i} e_{i+1} \end{bmatrix} \tag{5.8}$$

$$\dot{\eta} = -k_0\eta - k_1 s_1 - \sum_{i=2}^{n-1} s_i y_{i+1} + \sum_{j=3}^{n} s_j \sum_{i=2}^{j-1} \frac{\partial \alpha_j}{\partial e_i} y_{i+1} \tag{5.9}$$

Considering the parameter vector β_p to be uncertain, a virtual control input I_d has to be designed in such a way that the dynamic system (5.2) tracks the z and the controller design at the dynamic level is achieved. Defining $\tilde{u} = u - z$ the control objective at the dynamic level is to synthesis I_d to make $\tilde{u} \to 0$ so that $u \to z$, which is defined as

$$I_d = \hat{K}_{NInv}\tau_{md} \tag{5.10}$$

where

$$\begin{aligned} \tau_{md} = {} & \left[B_2^T B_2\right]^{-1} \left[B_2\right]^T \left[\Phi_1 \left(y, \dot{y}, z, \dot{z}\right) \hat{\beta}_p - K_e R_2 \left(u - z\right)\right] \\ & - \left[B_2^T B_2\right]^{-1} \left[B_2\right]^T \left[R_2 \left(R_2^T R_2\right)^{-1} \Lambda\right] \end{aligned} \tag{5.11}$$

with Λ and $\hat{\beta}_p$ (estimation of β_p) as follows

$$\Lambda = \begin{bmatrix} k_1 s_1 + \sum_{i=2}^{n-1} s_i y_{i+1} - \sum_{j=3}^{n} s_j \sum_{i=2}^{j-1} \frac{\partial \alpha_j}{\partial e_i} y_{i+1} \\ s_n \end{bmatrix} \tag{5.12}$$

$$\dot{\hat{\beta}}_p = -\Gamma \phi_1^T R_2 (u - z) \tag{5.13}$$

where Γ is a symmetric positive definite constant matrix. In the design of the control law, K_N and β_p are considered as an unknown parameter and $\hat{K}_{NInv}$ is an estimation of the parameter K_N^{-1} and is given as below

$$\dot{\hat{K}}_{NInv} = diag\left[\dot{\hat{K}}_{NInv1}, \dot{\hat{K}}_{NInv2}, ..., \dot{\hat{K}}_{NInvr}\right]$$

$$\dot{\hat{K}}_{NInvi} = -f_i \tau_{mdi}, \quad i = 1, 2, ..., r \tag{5.14}$$

where $\tilde{u}^T R_2^T B_2 = [f_1, f_2, ..., f_r]$.

5.1.2 Stage2 (Controller Design at the Actuator Level)

Till now we have designed a virtual controllers z and I_d for the kinematic and dynamic systems respectively. u tends to z can be guaranteed, if the actual input control signal of the dynamic system I be of the form I_d which can be realized from the actuator dynamics by the design of the actual control input ν. On the basis of the above statements we can conclude that if ν is designed in such a way that I tends to I_d then $\|e\| \to 0$ and $\|\tilde{u}\| \to 0$ and $\left\|\dot{\tilde{u}}\right\| \to 0$.
Define $I = e_I + I_d$ and substituting in (5.3) one gets

$$L\dot{e}_I + RI + K_a Q = -L\dot{I}_d + \nu$$

Adding $K_I e_I$ on both sides in the above equation, one gets

$$L\dot{e}_I + RI + K_a Q + K_I e_I = K_I e_I - L\dot{I}_d + \nu \tag{5.15}$$

The actuator parameters L, R and K_a are considered unknown and are estimated online using the adaptation laws. The control input ν is designed in form given below

$$\nu = \hat{L}\dot{I}_d + \hat{R}I - K_I e_I + \hat{K}_a Q + H \tag{5.16}$$

where $\hat{L} = diag\left[\hat{L}_1, \hat{L}_2, ..., \hat{L}_r\right]$, $\hat{R} = diag\left[\hat{R}_1, \hat{R}_2, ..., \hat{R}_r\right]$ and $\hat{K}_a = diag\left[\hat{K}_{a1}, \hat{K}_{a2}, ..., \hat{K}_{ar}\right]$ are the estimates of the unknown parameters L, R and K_a respectively, satisfying

$$\dot{\hat{L}}_i = -\gamma_6 \dot{I}_{di} e_{Ii}, i = 1, 2, ..., r \tag{5.17}$$

$$\dot{\hat{R}}_i = -\gamma_7 I_i e_{Ii}, i = 1, 2, ..., r \tag{5.18}$$

$$\dot{\hat{K}}_{ai} = -\gamma_8 Q_i e_{Ii}, i = 1, 2, ..., r \tag{5.19}$$

$$H_i = -\hat{K}_{Ni} f_i, i = 1, 2, ...r \tag{5.20}$$

where $\hat{K}_N$ is an estimate of the unknown parameter K_N

$$\dot{\hat{K}}_N = diag\left[\dot{\hat{K}}_{N1}, \dot{\hat{K}}_{N2}, ..., \dot{\hat{K}}_{Ni}\right], i = 1, 2, ..., r \tag{5.21}$$

$$\dot{\hat{K}}_{Ni} = \gamma_{10} e_{Ii} f_i, i = 1, 2, ..., r \tag{5.22}$$

where γ_6, γ_7, γ_8 and γ_{10} are known constants, which determine rates of the adaptations.

5.2 Stability Analysis for the Complete System

So far we have individually designed virtual controllers z and I_d for kinematic and dynamic systems, respectively. Then, an actual control input ν is developed. In the present section, a complete stability analysis for the whole system dynamics based on the combination of the above stages is given.

Consider a positive definite function for the whole system is given by

$$V = V_1 + V_2 + V_3 + \frac{1}{2}\sum_{i=1}^{r} \gamma_{10}^{-1} \tilde{K}_{Ni}^2 \tag{5.23}$$

where

$$V_1(t) = \frac{1}{2}\left(\sum_{i=2}^{n} s_i^2 + k_1 s_1^2 + \eta^2\right) \tag{5.24}$$

$$V_2(t) = \frac{1}{2}\tilde{u}^T D_3 \tilde{u} + \frac{1}{2}\tilde{\beta}_p^T \Gamma^{-1} \tilde{\beta}_p + \frac{1}{2}\sum_{i=1}^{r} K_{Ni} \tilde{K}_{NInvi}^2 \tag{5.25}$$

$$V_3 = \frac{1}{2} e_I^T L e_I + \frac{1}{2}\sum_{i=1}^{r}\left(\gamma_6^{-1}\tilde{L}_i^2 + \gamma_7^{-1}\tilde{R}_i^2 + \gamma_8^{-1}\tilde{K}_{ai}^2\right) \tag{5.26}$$

where $\tilde{\beta}_p = \hat{\beta}_p - \beta_p$, $\tilde{K}_N = \hat{K}_N - K_N$, $\tilde{K}_{NInv} = \hat{K}_{NInv} - K_N^{-1}$, and $\tilde{L} = \hat{L} - L$, $\tilde{R} = \hat{R} - R$, and $\tilde{K}_a = \hat{K}_a - K_a$ represent the parameter estimation errors.

5.2.1 Procedure for Obtaining the Time Derivative of V_1:

Substituting the control laws (5.8) and (5.10) along with the adaptation controllers (5.9), (5.14) and (5.13), the closed loop error equation for the kinematic and dynamic systems are given as follows

$$\dot{s_1} = \eta + \tilde{u_1}$$

$$\dot{s_2} = s_3 u_{d1} - k_2 s_2 u_{d1}^{2l} + (\eta + \tilde{u_1})\, y_3$$

$$\dot{s_3} = s_4 u_{d1} - s_2 u_{d1} - k_3 s_3 u_{d1}^{2l} + (\eta + \tilde{u_1}) \left(y_4 - \frac{\partial \alpha_3}{\partial e_2} y_3 \right)$$

$$.$$

$$.$$

$$\dot{s}_{n-1} = s_n u_{d1} - s_{n-2} u_{d1} - k_{n-1} s_{n-1} u_{d1}^{2l} + (\eta + \tilde{u}_1) . \left(y_n - \sum_{i=2}^{n-2} \frac{\partial \alpha_{n-1}}{\partial e_i} y_{i+1} \right)$$

$$\dot{s}_n = -k_n s_n - s_{n-1} u_{d1} + \tilde{u}_2 - (\eta + \tilde{u}_1) . \sum_{i=2}^{n-2} \frac{\partial \alpha_n}{\partial e_i} y_{i+1} \tag{5.27}$$

$$\dot{\eta} = -k_0 \eta - \Lambda_1 \tag{5.28}$$

$$D_2(y) R_2(y) \dot{u} + C_2(y, \dot{y}) u + G_2(y) = B_2(y) K_N e_I + B_2(y) K_N I_d + J_2^T(y) \lambda \tag{5.29}$$

The time derivative of $V_1(t)$ along the solution of (5.27) -(5.28) is

$$\dot{V}_1 = \sum_{i=2}^{n} s_i \dot{s}_i + k_1 s_1 \dot{s}_1 + \eta \dot{\eta}$$

$$\dot{V}_1 = -\sum_{i=2}^{n-1} k_i s_i^2 u_{d1}^{2l} - k_n s_n^2 - k_0 \eta^2 + \tilde{u}^T \Lambda \tag{5.30}$$

5.2.2 Procedure for Obtaining the Time Derivative of V_2:

Substituting the error $\tilde{u} = u - z$ in (5.29), we have

$$D_2 R_2 \dot{\tilde{u}} + C_2 \tilde{u} + D_2 R_2 \dot{z} + C_2 z + G_2$$
$$= B_2 K_N e_I + B_2 K_N I_d + J_2^T \lambda \quad (5.31)$$

Using the property 5 given in the section 3, equation (5.31) can be rewritten as

$$D_2 R_2 \dot{\tilde{u}} + C_2 \tilde{u} + \Phi_1 \beta_p = B_2 K_N e_I + B_2 K_N I_d + J_2^T \lambda \quad (5.32)$$

By rearranging the terms, one gets

$$D_2 R_2 \dot{\tilde{u}} = -\Phi_1 \beta_p - C_2 \tilde{u} + B_2 K_N e_I + B_2 K_N I_d + J_2^T \lambda \quad (5.33)$$

Introducing the control law (5.10) into the above equation

$$D_2 R_2 \dot{\tilde{u}} = -\Phi_1 \beta_p - C_2 \tilde{u} + B_2 K_N e_I$$
$$+ B_2 K_N \tilde{K}_{NInv} \tau_{md} + B_2 \tau_{md} + J_2^T \lambda \quad (5.34)$$

$$D_2 R_2 \dot{\tilde{u}} = \Phi_1 \left(\hat{\beta}_p - \beta_p\right) - K_e R_2 \tilde{u} - C_2 \tilde{u} + J_2^T \lambda$$
$$+ B_2 K_N \tilde{K}_{NInv} \tau_{md} - R_2 \left(R_2^T R_2\right)^{-1} \Lambda + B_2^T K_N e_I \quad (5.35)$$

Multiplying R_2^T on both sides to the above equation one gets

$$D_3 \dot{\tilde{u}} = R_2^T \Phi_1 \left(\hat{\beta}_p - \beta_p\right) - R_2^T K_e R_2 \tilde{u} - R_2^T C_2 \tilde{u}$$
$$+ R_2^T B_2 K_N \tilde{K}_{NInv} \tau_{md} - \Lambda + R_2^T B_2^T K_N e_I \quad (5.36)$$

The time derivative of $V_2(t)$ along the solution trajectory of equation (5.36) is

$$\begin{aligned}
\dot{V}_2 &= \tilde{u}^T D_3 \dot{\tilde{u}} + \frac{1}{2} \tilde{u}^T \dot{D}_3 \tilde{u} + \tilde{\beta}_p^T \Gamma^{-1} \dot{\tilde{\beta}}_p \\
&\quad + \sum_{i=1}^{r} K_{Ni} \tilde{K}_{NInvi} \dot{\tilde{K}}_{NInvi} \\
&= \tilde{u}^T R_2^T \Phi_1 \left(\hat{\beta}_p - \beta_p\right) - \tilde{u}^T R_2^T K_e R_2 \tilde{u} - \tilde{u}^T R_2^T \Phi_1 \tilde{\beta}_p^T \\
&\quad + \tilde{u}^T R_2^T B_2 K_N \tilde{K}_{NInv} \tau_{md} - \tilde{u}^T \Lambda + \tilde{u}^T R_2^T B_2 K_N e_I \\
&\quad + \tilde{u}^T \left(\frac{1}{2} \dot{D}_3 - R_2^T C_2\right) \tilde{u} - \sum_{i=1}^{r} K_{Ni} \tilde{K}_{NInvi} f_i \tau_{mdi} \qquad (5.37)
\end{aligned}$$

Since $\left(\frac{1}{2}\dot{D}_3 - R_2^T C_2\right) = 0$ and $\tilde{u}^T R_2^T B_2 K_N \tilde{K}_{NInv} \tau_{md} - \sum_{i=1}^{r} K_{Ni} \tilde{K}_{NInvi} f_i \tau_{mdi} = 0$ we have

$$\dot{V}_2 = -\tilde{u}^T R_2^T K_e R_2 \tilde{u} - \tilde{u}^T \Lambda + \tilde{u}^T R_2^T B_2 K_N e_I \tag{5.38}$$

5.2.3 Procedure for Obtaining the Time Derivative of V_3:

Introducing the control law (5.16) into (5.15) one gets

$$L\dot{e}_I + K_I e_I = \tilde{R} I + \tilde{L} \dot{I}_d + \tilde{K}_a Q + H \tag{5.39}$$

Then, by differentiating V_3 along the dynamics (5.16) - (5.19), one gets

$$\dot{V}_3 = -e_I^T K_I + e_I^T \tilde{R} I + e_I^T \tilde{L} \dot{I}_d + e_I^T \tilde{K}_a Q + e_I^T H +$$

$$\sum_{i=1}^{r} \left(\gamma_6^{-1} \tilde{L}_i \dot{\hat{L}}_i + \gamma_7^{-1} \tilde{R}_i \dot{\hat{R}}_i + \gamma_8^{-1} \tilde{K}_{ai} \dot{\hat{K}}_{ai}\right)$$

$$\dot{V}_3 = -e_I^T K_I e_I + e_I H \tag{5.40}$$

5.2.4 Time Derivative of V:

With $\dot{V}_1$ in (5.30), $\dot{V}_2$ in (5.38), and $\dot{V}_3$ in (5.40), $\dot{V}$ can be obtained as

$$\begin{aligned} \dot{V} &= -\sum_{i=2}^{n-1} k_i s_i^2 u_{d1}^{2l} + \tilde{u}^T \Lambda - \tilde{u}^T R_2^T K_e R_2 \tilde{u} \\ &\quad -k_n s_n^2 - k_0 \eta^2 - \tilde{u}^T \Lambda + \tilde{u}^T R_2^T B_2 K_N e_I \\ &\quad -e_I^T K_I e_I + e_I H + \sum_{i=1}^{r} \gamma_{10}^{-1} \tilde{K}_{Ni} \dot{\hat{K}}_{Ni} \\ &\leq -\sum_{i=2}^{n-1} k_i s_i^2 u_{d1}^{2l} - k_n s_n^2 - k_0 \eta^2 \\ &\quad -\tilde{u}^T R_2^T K_e R_2 \tilde{u} - e_I^T K_I e_I \end{aligned} \tag{5.41}$$

By observing the above, it can be concluded that the considered positive function $V(t)$ is thus non-increasing. This in turn implies that η, $\mathbf{s}$, $\tilde{\mathbf{u}}$, $\tilde{K}_{NInvi}$, e_I, $\tilde{K}_{Ni}$ as well as $\tilde{R}$, $\tilde{L}$, $\tilde{K}_a$ are bounded, and $V(t)$ converges to limit value V_{lim}. By the definition of $\mathbf{s}$, it concludes that $\mathbf{e}$ is bounded. Using the assumption (A1), it follows that $\mathbf{y}$ is bounded. In view of (5.27)-(5.29), $\dot{\mathbf{s}}$, $\dot{\tilde{\mathbf{u}}}$, $\dot{\eta}$ are bounded. Thus, η, $\mathbf{s}$, and $\tilde{\mathbf{u}}$ are uniformly bounded. We should mention that if D_2R_2, C_2, and G_2 are bounded on y_1, there is no boundedness requirement for y_{d1}. Next, we prove that $\mathbf{s}$, $\dot{\mathbf{s}}$, and η tend to zero. Since $\ddot{V}$ is bounded and $\dot{V}$ is uniformly continuous, it can be shown that $\dot{V}$ tends to zero. Therefore, η, $s_i u_{d1}(2 \leq i \leq n-1)$, s_n, and $\tilde{\mathbf{u}}$ tend to zero. By the assumption that $\lim_{t\to\infty} inf|u_{d1}| > 0$, one concludes that $s_i(2 \leq i \leq n)$ tends to zero.

Differentiating $u_{d1}^l \eta$ yields

$$\frac{d}{dt}u_{d1}^l\eta = -k_1 u_{d1}^l s_1 + l u_{d1}^{l-1}\dot{u}_{d1}^l\eta - k_0 u_{d1}^l\eta$$

$$-u_{d1}^l(\sum_{i=2}^{n-1} s_i y_{i+1} - \sum_{j=3}^{n} s_j \sum_{i=2}^{j-1}\frac{\partial\alpha_j}{\partial e_i}y_{i+1})$$

where the first term is uniformly continuous and the other terms tend to zero. Thus it can be conclude that $\frac{d}{dt}(u_{d1}^l\eta)$ converges to zero, which in turn implies that $u_{d1}^l s_1$ and s_1 tend to zero. Therefore, $\mathbf{s}$ and $\dot{\mathbf{s}}$ tend to zero. To complete the proof and establish asymptotic convergence of the tracking error, it is necessary to show that $\{\mathbf{y}, \dot{\mathbf{y}}\} \to \{\mathbf{y_d}, \dot{\mathbf{y}}_\mathbf{d}\}$ as $t \to \infty$. This is accomplished by the following arguments.

Based on the definition of α given in (5.7) and the relationship $\mathbf{s} = \mathbf{e} - \alpha$, it is obvious that $s_i = 0(i = 1, 2)$ yields $\lim_{t\to\infty} y_i = y_{di}$ and $\lim_{t\to\infty} \dot{y}_i = \dot{y}_{di}(i = 1, 2)$ because of $\alpha_1 = \alpha_2 = 0$. From the boundedness of u_{d1}, one obtains that α_3 and $\dot{\alpha}_3$ converge to zero, which results in $\lim_{t\to\infty} y_3 = y_{d3}$ and $\lim_{t\to\infty} \dot{y}_3 = \dot{y}_{d3}$. The convergence of α_3 and $\dot{\alpha}_3$ lead to the conclusion that α_4 and $\dot{\alpha}_4$ converge to zero, thus, $\lim_{t\to\infty} y_4 = y_{d4}$ and $\lim_{t\to\infty} \dot{y}_4 = \dot{y}_{d4}$. similarly, we can prove that $\lim_{t\to\infty} y_i = y_{di}$ and $\lim_{t\to\infty} \dot{y}_i = \dot{y}_{di}(5 \leq i \leq n)$. In summary, we have proved that $\{\mathbf{y}, \dot{\mathbf{y}}\} \to \{\mathbf{y_d}, \dot{\mathbf{y}}_\mathbf{d}\}$ as $t \to \infty$. So it is proved that all the signals in the closed loop system remain bounded, and $||e|| \to 0$ and $||\dot{e}|| \to 0$.

5.3 Simulation Results

The physical values for the simulation are taken as $m = 0.5, I_0 = 0.5$, $E = P = 1$ and $L_1 = L_2 = 2.03, R_1 = R_2 = 2, K_{N1} = K_{N2} = 1.5, K_{a1} = K_{a2} = 1$. The design parameters have an influence on the rate at which the tracking errors tend to zero. For this simulation the design parameters are set at $\gamma_6 = \gamma_{10} = \gamma_7 = \gamma_8 = 1$, $K_e = diag\,(5, 5)$, $k_0 = k_1 = k_2 = k_3 = 2$, $\hat{\beta}\,(0) = 0$, $\Gamma = 1$, and $\eta\,(0) = 0$.
Simulation results are shown below. Fig. 5.1 shows the tracking trajectory errors and Fig. 5.2 shows the geometric trajectory of x and y.In Figs. 5.3 and 5.4 shows the output of the actuator system I tracks the virtual control input I_d and u tends to z. It can be seen from the simulation results that the tracking errors and path followed by the robot proves the effectiveness of the designed controller.

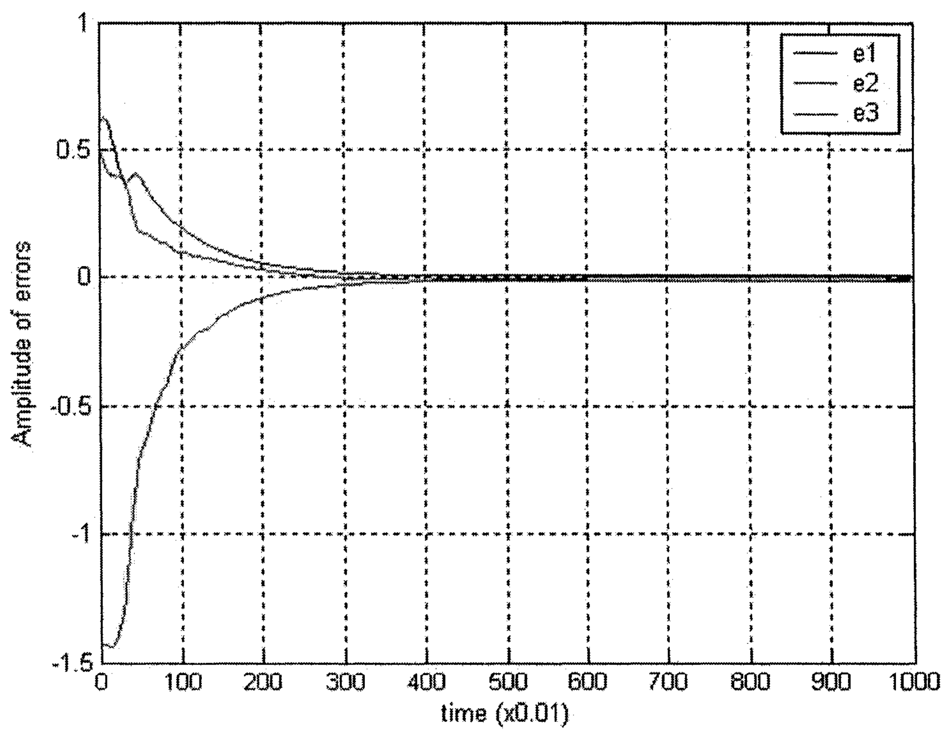

Figure 5.1: Tracking errors (e_1, e_2, e_3).

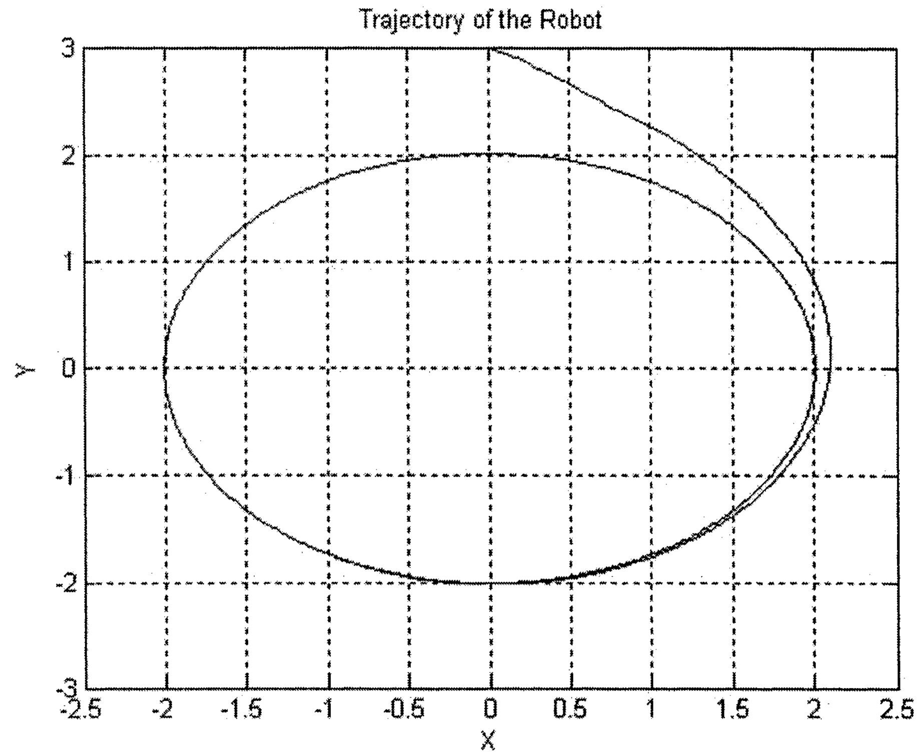

Figure 5.2: Geometric trajectory of x via y

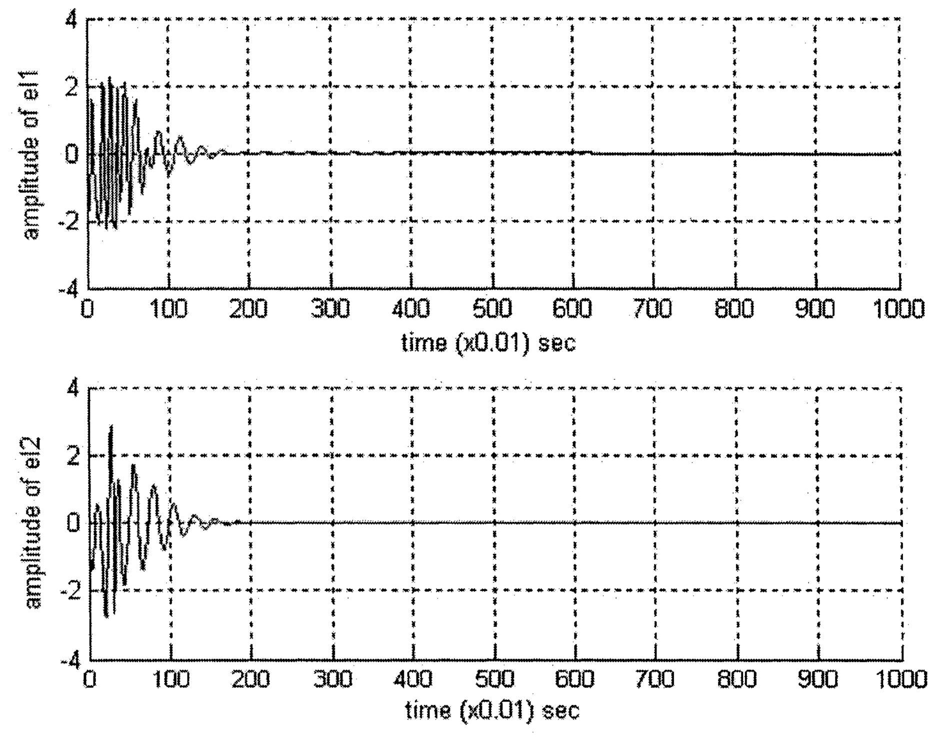

Figure 5.3: Tracking errors e_{I1} and e_{I2}

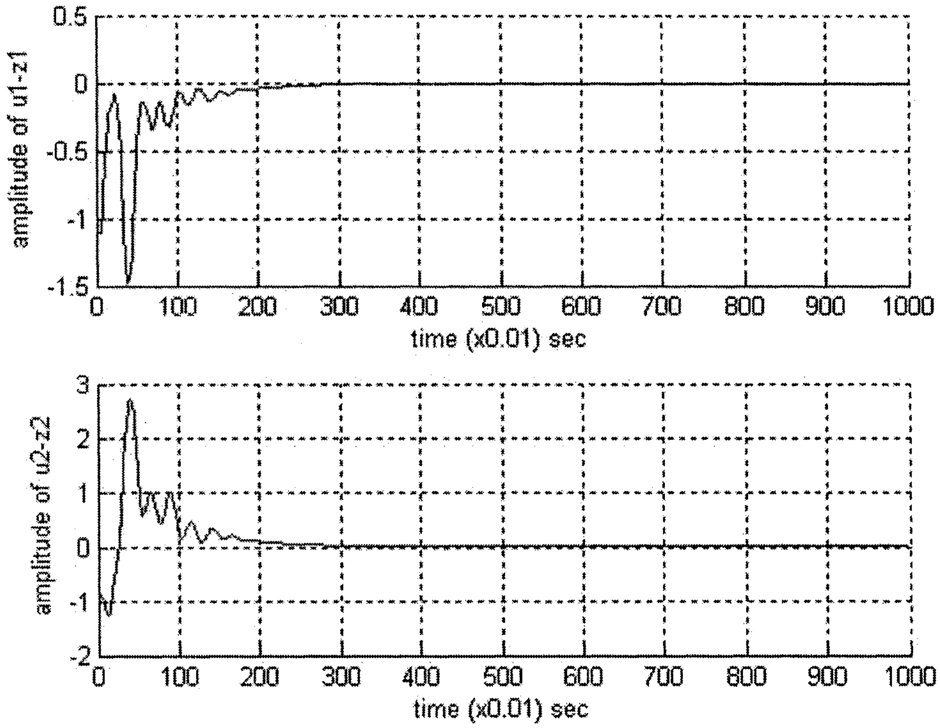

Figure 5.4: Tracking errors $\tilde{u}_1$ and $\tilde{u}_2$

Chapter 6

Conclusions and Future Work

6.1 Conclusions

In this thesis two control algorithms have been proposed. Both of them are feedback tracking controllers for a class of uncertain nonholonomic mechanical systems. The uncertain nonholonomic mechanical system forms a cascade system consisting of kinematic nonholonomic constraint, dynamics system and actuator dynamics. The control objective was to determine a feedback control law designed at the actuator level so that the trajectory q follows q_d. In order to achieve the control objective an adaptive and a robust adaptive tracking control algorithms have been designed. Both controllers have been designed to deal with the uncertainties in dynamics and actuator dynamics.

Since the actual control input is voltage. The control input has been designed in such a way that the output of the dynamic system v and actuator system I are forced to track the desired input to kinematic system z and dynamic system I_d respectively. The stability analysis proves that the position tracking control has been achieved. A simplified mobile robot of the type (2,0) was used in order to demonstrate the design procedure. The simulation results showed effectiveness of the controllers and it was observed that robust adaptive controller have faster rate of tracking than the adaptive controller.

6.2 Future Work

In this study, the design procedure and simulation results are provided. The controllers designed need an acceleration measurements. The future work should validate the controller designed through experiments and removing the need for the acceleration signal. The current control scheme is restricted only to the first order nonholonomic systems, extending it to the second order nonholonomic systems will be more challenging. In most of the nonholonomic control schemes and in this thesis also we consider only drift free nonholonomic systems. Designing a general control scheme for the nonholonomic systems with drift can also be considered.

Appendix

Chow's Theorem

If the accessibility rank condition

$$dim\Delta_c(q_0) = n$$

holds, then the control system (2.3) is locally accessible form q_0. If the accessibility rank condition holds for all $q \in \Re^n$, the system is locally accessible. Conversely, if system (2.3) is locally accessible, then $dim\Delta_c(q) = n$ holds in an open and dense subset of $\Re^n$. Where c is defined as *accessibility algebra.*

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