Optimization Problems in Cellular Networks

Souheyl Touhami

A Thesis

in

The John Molson School of Business

Presented in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy in Administration at
Concordia University
Montreal, Quebec, Canada

July 2004

Library and
Archives Canada

Published Heritage
Branch

395 Wellington Street
Ottawa ON K1A 0N4
Canada

Bibliothèque et
Archives Canada

Direction du
Patrimoine de l'édition

395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file Votre référence
ISBN: 0-612-96968-1
Our file Notre référence
ISBN: 0-612-96968-1

The author has granted a non-exclusive license allowing the Library and Archives Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque et Archives Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou aturement reproduits sans son autorisation.

In compliance with the Canadian Privacy Act some supporting forms may have been removed from this thesis.

While these forms may be included in the document page count, their removal does not represent any loss of content from the thesis.

Conformément à la loi canadienne sur la protection de la vie privée, quelques formulaires secondaires ont été enlevés de cette thèse.

Bien que ces formulaires aient inclus dans la pagination, il n'y aura aucun contenu manquant.

Canada

Abstract

Optimization Problems in Cellular Networks

Souheyl Touhami
Doctor of Philosophy in Administration
The John Molson School of Business
Concordia University, Montreal, Canada
July 2004

Over the last decade, the wireless telephony market has experienced significant changes due to an increase in consumer demand, competition and rapid technological changes which reduced the effective life cycle of network infrastructures. Under these conditions, sustaining the growth and profitability of operations has been increasingly dependent on the efficient management of the available resources. In this research, we examine optimization problems that arise in the design and upgrade processes of the wireless component of GSM type cellular networks, namely, the antenna positioning problem (APP) and the frequency allocation problem (FAP).

APP examines the deployment of antennas throughout the service area. We propose integrated problem formulations that bring together the variety of design criteria raised both by practitioners and in the academic literature. In addition, current practices mostly use a sequential decomposition of the design process whereby the APP solution becomes a fixed input to FAP, as the simultaneous optimization would be too complex to tackle for realistic instances. In order to integrate the design process, we propose a partial incorporation of frequency allocation considerations within APP, with a limited increase in problem complexity.

Frequency management focuses on allocating a very limited radio spectrum so as to optimize the quality of service. This problem is first examined in the context of static networks, for which we develop an adaptive Tabu Search algorithm that dynamically combines several operators and search mechanisms to minimize network interference. As for Frequency Hopping networks, frequency management also entails determining the hopping sequences in addition to determining the frequency allocation. We analyze this problem for three levels of network synchronization and develop heuristic procedures to determine the parameter setting that will optimize the hopping sequences. In addition, for fully-synchronized networks, we propose to sidestep the sequence generation procedure currently used in the GSM standard, to directly generate optimized sequences.

The optimization problems considered in this work are intractable, with realistic instances being of large size and being based on estimated data. Therefore, heuristic optimization techniques are considered for solving them. We use real-life networks provided by telecom operators to test the problem formulations and the proposed optimization algorithms.

to my parents, to my sisters

Acknowledgments

I would like to express my sincere gratitude and appreciation to Dr. Jean-Marie Bourjolly. Without his guidance, understanding and support, this research would not have been completed.

I am also indebted to Dr. Gilbert Laporte for his contribution and insights throughout the course of this research.

Special thanks are also extended to Dr. Michèle Breton and Dr. Dennis Kira for agreeing to be on my committee and for providing valuable advice and support.

I would also like to thank Leslie Dejoie Oumar Diome, Ke Ding and Michel Lominy for being so generous with their time and for facilitating my introduction to the field of telecommunication.

Finally, I would like to thank my family and friends who fully shared this experience with me. This work would not have been possible without their continuous support.

Table of Contents

List of Figures

List of Tables

Notation

The Antenna Positioning Problem

- M: candidate set of antennas
- K: set of sites in the antenna candidate set
- N : discrete set of points (pixels)
- a_n : volume of traffic offered at pixel n
- p_i : Transmission power of antenna i, expressed in dBm
- p_{in}: strength of the signal transmitted by antenna i and received at pixel n, expressed in dBm
- q_{in}: degradation in signal strength from antenna i received at pixel n, expressed in dBm
- p_{min}: threshold on acceptable signal strength, expressed in dBm
- y_{in} : 1, pixel n is served by antenna i, 0 otherwise
- v_{in} : 1, antenna i is the second best server of pixel n, 0 otherwise
- ν : maximum difference in power level between first and second best server
- γ_n: carrier to interferer ratio at pixel n (γ^1, γ^2 : interference thresholds levels)
- γ_n': carrier to interferer ratio at pixel n, after incorporating effect of cell separation
- z_n^1 : 1, if $\gamma_n \leq \gamma^1$, 0 otherwise
- $z_n^{1\prime}$: 1, if $\gamma_n \leq \gamma^1$, 0 otherwise, after incorporating effect of cell separation

- z_n^2 : 1, if $\gamma^1 < \gamma_n \leq \gamma^2$, 0 otherwise
- $z_n^{2\prime}$: 1, if $\gamma^1 < \gamma_n \leq \gamma^2$, 0 otherwise, after incorporating effect of cell separation
- x_i : 1, if antenna i is active, 0 otherwise
- w_k : 1, if site k is active, 0 otherwise
- Δ_i : discrete set of possible power values p_i for antenna i.
- o_i : offered traffic at antenna i
- $o_i^\prime$: total offered traffic at antenna i after incorporating overflow traffic
- d_i : number of TRXs to be installed at antenna i
- $d_i^\prime$: number of TRXs to be installed at antenna i after incorporating overflow traffic
- b_i : blocked traffic at antenna i
- $b_i^\prime$: blocked traffic at antenna i after incorporating overflow traffic
- c_i : carried traffic at antenna i
- $c_i^\prime$: carried traffic at antenna i after incorporating overflow traffic
- cap_i : capacity of antenna i
- $cap_i^\prime$: capacity of antenna i after incorporating overflow traffic
- e_i : excess capacity at antenna i.
- $e_i^\prime$: excess capacity at antenna i after incorporating overflow traffic
- c_i^a : cost of antenna i.
- c_k^s : cost of site k.
- c^t : cost of a single TRX.
- $A(S)$: total number of active antennas in configuration S

- A^* : maximum number of antennas that can be activated
- $U(S)$: total number of active sites in configuration S
- U^* : maximum number of sites that can be opened
- $T(S)$: total number of installed TRXs in configuration S
- T^* : maximum number of TRXs that can be installed.
- $Cov(S)$: total covered area in configuration S
- Cov_* : minimum required level of coverage, expressed as a percentage of the total area in N
- $O(S)$: total network offered traffic in configuration S
- $O'(S)$: total network offered traffic in configuration S after incorporating overflow traffic
- O_*: minimum required covered traffic volume, expressed as a percentage of the total offered traffic in N
- $C(S)$: total network carried traffic in configuration S
- $C'(S)$: total network carried traffic in configuration S after incorporating overflow traffic
- C_* : minimum required carried traffic, expressed as a percentage of the total offered traffic in N
- $B(S)$: total network blocked traffic in configuration S
- $B'(S)$: total network blocked traffic in configuration S after incorporating overflow traffic
- B^*: maximum allowed blocking, expressed as a percentage of the total covered traffic in S

- $Cap(S)$: total network capacity in configuration S
- $Cap'(S)$: total network capacity in configuration S after incorporating overflow traffic
- $E(S)$: total network excess capacity in configuration S
- $E'(S)$: total network excess capacity in configuration S after incorporating overflow traffic
- E^*: maximum allowed excess capacity, expressed as a percentage of total network capacity.
- $Q(S)$: average pixel CIR in configuration S
- $Q'(S)$: average pixel CIR in configuration S, after incorporating effect of cell separation
- $TQ(S)$: traffic-weighed average pixel CIR in configuration S
- $TQ'(S)$: traffic-weighed average pixel CIR in configuration S, after incorporating effect of cell separation
- $R^1(S)$: total area with $CIR < \gamma^1$
- $R^{1\prime}(S)$: total area with $CIR < \gamma^1$, after incorporating effect of cell separation
- $TR^1(S)$: total traffic with $CIR < \gamma^1$
- $TR^{1\prime}(S)$: total traffic with $CIR < \gamma^1$, after incorporating effect of cell separation
- $R^2(S)$: total area with $\gamma^1 \leq CIR < \gamma^2$,
- $R^{2\prime}(S)$: total area with $\gamma^1 \leq CIR < \gamma^2$, after incorporating effect of cell separation
- $TR^2(S)$: total traffic with $\gamma^1 \leq CIR < \gamma^2$

- $TR^{2\prime}(S)$: total traffic with $\gamma^1 \leq CIR < \gamma^2$, after incorporating effect of cell separation
- $Cost(S)$: total network operating costs.
- $Cost'(S)$: total network excess capacity in configuration S after incorporating overflow traffic
- r : average revenue per customer whose traffic is carried
- $Revenu(S)$: total revenue generated by carried traffic
- $Cost^*$: total available budget.
- Ω : constraint set imposed by target performance requirements
- Ω' : constraint set imposed by target performance requirements after incorporating overflow traffic
- $F^A(S)$: fraction of total area with $CIR < \gamma^2$
- $F^{A\prime}(S)$: fraction of total area with $CIR < \gamma^2$, after incorporating effect of cell separation
- $F^T(S)$: fraction of total traffic with $CIR < \gamma^2$
- $F^{T\prime}(S)$: fraction of total traffic with $CIR < \gamma^2$, after incorporating effect of cell separation
- δ_{ij} : minimum frequency separation in FAP between cells i and j
- δ^1_{ij} : 1, if $\delta_{ij} = 1$, and 0, otherwise
- δ^2_{ij} : 1, if $\delta_{ij} \geq 2$, and 0, otherwise
- ρ^1 : Maximum allowed number of antenna pairs (i, j) such that $\delta^1_{ij} = 1$.
- ρ^2 : Maximum allowed number of antenna pairs (i, j) such that $\delta^2_{ij} = 1$.
- θ : adjacent-channel protection

- $Perf_c(S)$: the performance of configuration S with respect to a performance criteria c
- T_c : the target performance set by the operator
- λ_c : weight associated with performance criteria c
- $\Pi(S)$: degree of feasibility of configuration S
- $G(S)$: objective functions of configuration S
- $\varepsilon_n^c(j), \varepsilon_n^a(j)$: measure of need for co-channel and adjacent-channel separation between antennas i and j for pixel n
- $Sep^c(i,j), Sep^a(i,j)$: measure of need for co-channel and adjacent-channel separation between antennas i and j
- $move(i, p_i^k)$: elementary modification to current solution by changing the power level of antenna i to level p_i^k

The Frequency Allocation Problem

- T: set of TRXs in the network, and $N = |T|$
- C: set of cells in the network
- D_i : number of TRXs in cell i
- F_i : set of feasible frequencies available for TRX i
- F : set of all frequencies. $N_F = |F|$
- c_{ij}^1, c_{ij}^2 : Co-channel and Adjacent-channel interference between TRXs i and j
- S_{ij} : minimum required separation between frequencies allocated to TRXs i and j
- x_{if} : 1, if frequency f is allocated to TRX i, and 0 otherwise

- w_{ij}, v_{ij} : 1 if TRXs i and j use the same or adjacent frequencies, and 0 otherwise
- F_i^k : a subset labeled k of frequencies that can be feasibly allocated to cell i
- K_i : number of all such subsets F_i^k for cell i
- δ_{ij}^{kl} : 1, if it is feasible to allocate subsets F_i^k and F_j^l to cells i and j, and 0 otherwise
- I_{ij}^{kl} : interference level that results from allocating subsets F_i^k and F_j^l to cells i and j
- y_i^k : 1, if subset F_i^k is selected for cell i, and 0 otherwise
- w_{ij}^{kl} : 1, if subsets F_i^k and F_j^l are allocated to cells i and j, and 0 otherwise
- α, β : penalty coefficient associated with violating a separation constraint or a demand constraint
- $\Pi(S), \Phi(S)$: degree of feasibility of a solution S with respect to demand and separation constraints
- $\Theta(S)$: objective function for solution S
- $Swap(i : f \to g)$: elementary modification to current solution by changing the frequency assignment of TRX i from f to g
- $BS(i)^p$: Best Swap operator
- $SI(Swap)$: Swap Improvement operator
- t: current iteration counter
- $t_{\max}^D$: end iteration of the diversification phase
- $t_{\max}^I$: end iteration of the Intensification phase
- $t_{\min}^I$: starting iteration of the Intensification phase
- $t_{\max}^N$: maximum allowed number of consecutive iterations without improve-

ments in the best solution

- S: current solution
- $N(S)$: neighborhood of solution S, generated with operators $m^p(i)$
- S^*: best feasible solution
- Θ^*: current minimum objective function
- Ω : current search space: if $\Omega = \Omega^1$, then no violation of separation constraints is allowed and if $\Omega = \Omega^2$, then separation constraints are relaxed
- r : number of consecutive forced local descent phases
- π: step size increase in search intensity
- $t(c, f)$: last iteration for which the use frequency f at cell c is declared tabu
- $\eta(c, f)$: number of times frequency f has been activated in cell c

The Frequency Hopping Problem

- ΔFN : difference in frame number for any 2 TRXs
- S_{ij} : minimum frequency separation S_{ij} between a pair of cells (i, j)
- H: length of optimized sequence
- $s = (h, m)$ any sequence s results from an HSN assignment h to its cell and an MAIO assignment m
- MA_i : MA of TRX i
- $x_{ch} = 1$, if cell c is assigned HSN h, and 0 otherwise 1, if cell c is assigned HSN h
- $y_{im} = 1$, if TRX i is assigned MAIO m, and 0 otherwise

- $C^{co}_{(is,jr)}, C^{ad}_{(is,jr)}$: co-channel and adjacent-channel collisions rate between sequence $s = (h, m)$ in TRX i and sequence $r = (h', m')$ in TRX j.
- Co_{ij}, Ad_{ij} : co-channel and adjacent-channel interference level between TRX i and TRX j.
- $T_{(is,jr)}$: total collision cost associated with using sequence $s = (h, m)$ in TRX i and sequence $r = (h', m')$ in TRX j.
- θ^{co}, θ^{ad} : minimum co-channel and adjacent-channel interference threshold required for sequence separation
- $f_i(t)$: frequency used by TRX i at instant t.

Chapter 1

Overview of Wireless Cellular Networks

A wireless cellular network consists of a set of *Transmitters/Receivers (TRXs)* positioned in the service area, a set of *Mobile Units (MU)*, a set of switching and routing nodes, and connection media linking the MU with the TRXs, and these with the switching nodes. The transmission equipment allows the MUs to make wireless telephone calls in the service area. The generic call handling procedure in cellular networks can be summarized as follows:

- The MU is the hand-held set that the client uses to establish and receive telephone calls.
- Radio waves are used to carry the calls between an MU and the TRX to which it is assigned through antennas. The MUs and the TRXs and the wireless links between them form the *access network (*see Figure 1.1).
- Two MUs in the same network communicate through their assigned TRXs. Between two TRXs, the calls are routed through the switching nodes of the fixed part of the network, also known as the core network. Furthermore, this fixed network is linked to the *Public Switched Telephone Network (PSTN)* so that an MU in the network can communicate with users in other networks (see Figure 1.2)

Several TRXs are often connected to a same antenna serving an area called a *cell.* An antenna can be omnidirectional, transmitting and receiving on 360 degrees. It can also be directional, hence transmitting and receiving on a limited angle. More than one antenna can be located at the same *site*. The type and configuration of the antenna installed in each cell determine the coverage of the corresponding cell. The capacity of the cell is determined by the number of MU in its coverage area that can communicate simultaneously. As an MU moves throughout the network, its call is handed to the appropriate TRX. This is referred to as *Handoff.*

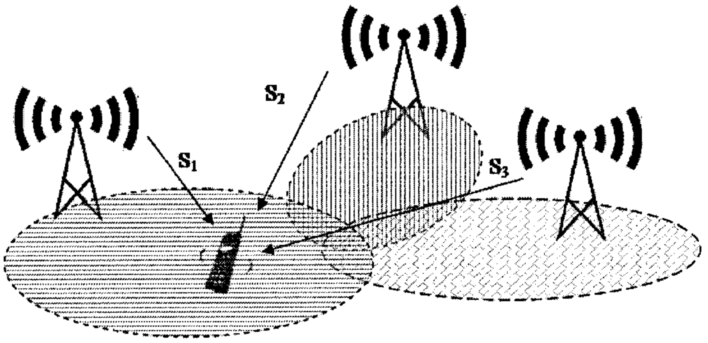

Figure 1.1: GSM network: Access component

Several resource allocation problems need to be solved for the design of wireless networks. Network routing and dimensioning problems often arise in the design of the fixed network. In this work, we examine a set of optimization problems that arise in the design of the wireless part of the network, the access network. The next sections provide a more detailed description of the resources and the trade-offs involved.

1.1 Radio Spectrum

Radio spectrum is the communication medium on the access network, providing the link between MUs and TRXs. It is a very limited resource. Only part of the overall usable radio spectrum is allocated by governmental authorities for wireless telephony applications (other applications include military communication systems, TV and radio broadcasting, industrial systems, etc.). The allocated bandwidth is further partitioned among the network operators of each region. The latter

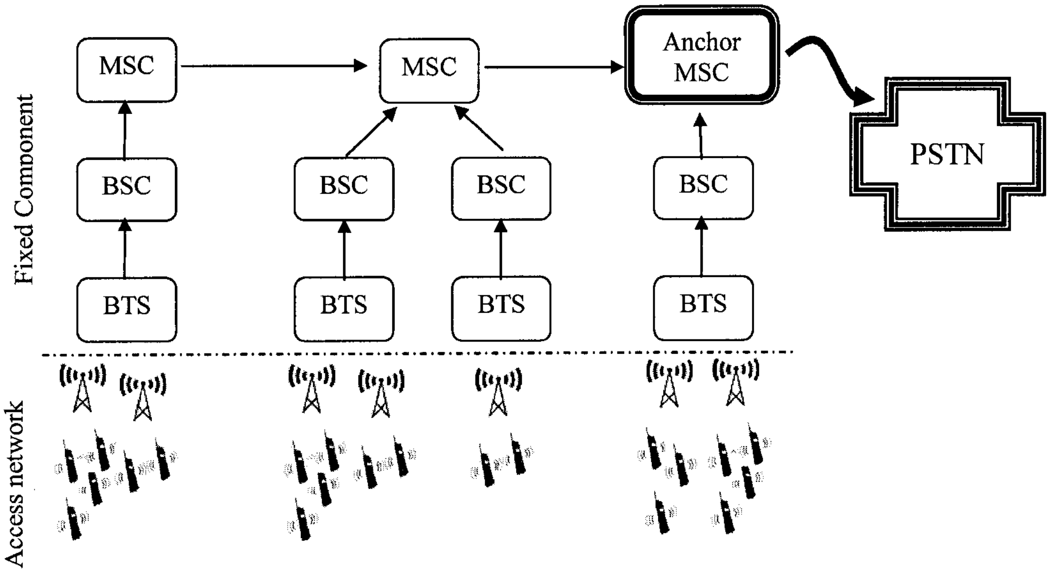

Figure 1.2: GSM network components

bid and fiercely compete for very expensive licenses. The reader can refer to [21] for a spectrum management description in the USA.

Half of the allocated band is used to carry communications from the MUs to the TRXs. This is referred to as the *Up Link.* The second half is used to carry communications in the reverse direction. This is referred to as the *Down Link.*

A network operator must accommodate a large number of users on a limited spectrum. *Multiple Access schemes* allow a network to handle multiple users while still being able to differentiate between them. In addition, sharing the radio resource among users may result in a degradation of the quality of service (QoS). It is also the role of the Multiple Access scheme to tackle this problem. For instance, a typical GSM operator would obtain about 100 radio frequencies (sometimes less) to accommodate hundreds of thousands of clients. There are three Multiple Access schemes:

- *Frequency Division Multiple Access (FDMA):*

  The available frequency band is divided into a set of frequencies, all with the same bandwidth. See Figure 1.3. Each TRX is assigned a frequency. Since for any realistic network there are more TRXs than frequencies, multiple access is achieved through reusing frequencies in different TRXs.

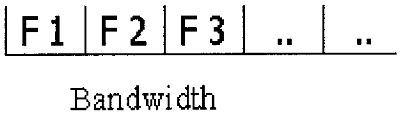

Figure 1.3: Frequency utilization - FDMA

- *Time Division Multiple Access (TDMA):*

  Under this scheme, several users share a frequency but on different time slots. Figure 1.4 illustrates this mechanism. The frequency is rotated among the assigned users, each making use of it for a short time interval, *a time slot.* The *time frame* is the length of the time interval for the rotation. The number of time slots per frame is fixed and depends on the technology being used (e.g. in GSM networks, there are 8 time slots per time frame).

- *Code Division Multiple Access (CDMA):*

  All the communications share the same frequency for the same time. However, each communication is assigned a *PN* code which allows the system to distinguish it from other calls and signals.

Since wireless networks use radio waves for signal transmission, wave propagation considera-

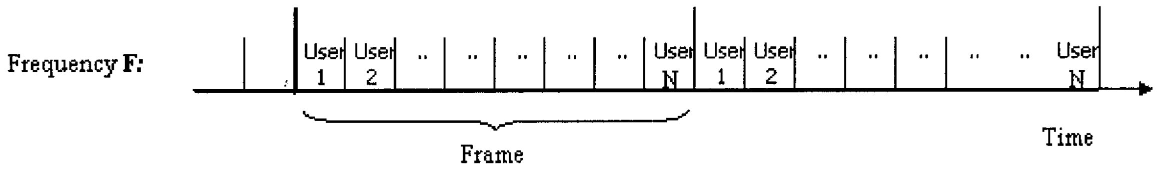

Figure 1.4: Frequency utilization - TDMA

tions come to play a major role in these systems. As a signal is transmitted (either from the MU in the up link direction or from the antenna in the down link direction), the strength of the received signal depends not only on the transmission power, transmitter height, tilt, gain and the distance between the transmitter and the receiver but also on the obstacles that lie in the path. The shapes and the type of surface of these obstacles reflect and refract the radio waves, causing a deterioration of the signal that is not uniform across the reception area.

Since several MUs will share the available radio frequencies, interference between the transmitted signals will take place. The signal propagation conditions determine interference levels across the service area and so have an impact on the quality of service at the level of the access network. In addition, under severe interference, the calls may not be established at all, hence reducing also the network capacity. Therefore, signal propagation has a dual impact both on the network quality of service and capacity.

Depending on the network technology deployed, several features may be included in wireless networks. Their role is to enhance network capacity or QoS. Features such as Power Control (transmitters dynamically adjust transmission power as a function of the interference), Discontinuous Transmission (where no signal is transmitted during silent portions of the communication) or

Frequency Hopping (changing communication frequency so as to avoid persistent interference) affect the signal transmission and as such may have to be considered during the resource allocation process.

1.2 Network Technologies

The most widely deployed type of cellular network is based on the GSM standard (General System for Mobile Communications) devised in Europe. The wireless access network in the GSM networks uses a combination of FDMA and TDMA. The available radio spectrum is divided into equally spaced frequencies (FDMA), and TDMA is applied on each frequency with 8 time slots per frame. In GSM networks, one TRX of every cell carries the *Broadcast Control Channel (BCCH)* in the first time slot. The remaining time slots of each frame are allocated as *Traffic Channels (TCH)* (See Figure.1.4). The other TRXs in the cell will carry TCH channels on all eight time slots. Each TCH can serve one customer. The BCCHs are used to establish the calls and for signaling traffic. The distinction between these two types of channels is relevant for frequency management since operators often impose higher levels of protection against interference for the BCCHs due to their critical role. The call handling procedure on the up link for GSM networks can be summarized as follows:

1. A MU uses the BCCH to request resources for establishing a call.
2. If the resources are available, 2 radio frequencies (one for the up link and one for the down link) are allocated to carry the communication between the MU and a designated TRX, located at the *Base Transceiver Station* (BTS).

3. A BTS is a set of communication equipment whose function is to relay the calls received through radio waves into the fixed part of the wireless network. A BTS includes a number of antennas, typically 3 directional antennas. Each antenna provides communication coverage for an area called a *cell*. The service capacity of a cell is defined by the number of TRXs that are linked to its antenna.

4. A call is routed through the wire network starting at its BTS, through its *Base Station Controllers* (BSC), to its *Mobile Service Switching Center* (MSC). The MSCs form the core network and are responsible for routing the call back through the network (if the party being called is within the network), or routing a call to the PSTN (if the party called is outside the network, e.g. if it is a fixed home phone). See Figure 1.2

AMPS Networks are similar to GSM networks as far as of the call handling procedure is concerned. From a resource management point of view, the main difference lies in the multiple access scheme used. AMPS networks are based on analog technology and use only FDMA with a bandwidth allocated for each frequency much smaller than in GSM. Each frequency carries only one traffic channel (as opposed to GSM which is based on digital technology where a frequency can carry up to 8 channels). TDMA systems are also very similar to GSM.

For CDMA based networks, the available spectrum is used as a single frequency. PN codes are used to distinguish between different calls. The *Universal Mobile Telecommunications System* (UMTS) standard is expected to become the prevailing standard for 4th generation networks and it employs CDMA at its core.

1.3 Network Performance

Wireless network performance is measurable by the number of users it can serve, its quality of service, and by its operating and capital cost.

The *Erlang* is the common unit used to measure the volume of communications. It is calculated as the average number of circuits busy for one hour, where a circuit is the resource (media) used to establish the link. In circuit switching networks (the earlier fixed telephone networks), each call is allocated a circuit (a physical link) reserved only for it. Thus, one Erlang traffic corresponds to having one circuit busy for one hour. Network capacity is measured by the percentage of offered traffic that is successfully carried through the network or, equivalently by the percentage of blocked traffic.

The design of the fixed part of the network involves positioning the BSCs and the MSCs, designing the topology of the core network and the capacity planning for all the links between these nodes. If there is not enough capacity to handle the total amount of offered traffic, part of it will be blocked. Therefore, the QoS in this fixed part of the network is measured by the percentage of call traffic that could not be routed to its destination. The call blocking measure may come to play at the level of access network too, since at very high interference levels or when the offered traffic is too large with respect to capacity, calls cannot be established at all and so they are blocked. For fixed networks, as home phones, a blocking rate of 0.1% is often used. For wireless networks, the target blocking rate is often set at 2%.

On the access network (the wireless part), the QoS is measured by the interference level. More formally, the *Carrier to Interferer Ratio (CIR)* is a measure often used. Other measures include the *Bit Error Rate* and the *Frame Erasure Rate.* As shown in Figure 1.1, the CIR, at any reception point, is the ratio of the strength of the desired signal (call it S_1) to the summation of the strengths of the interfering signals (S_i, $i > 1$):

$$CIR = \frac{S_1}{\sum\limits_{i>1} S_i} \tag{1.1}$$

The lower the CIR, the higher the interference and the lower the QoS (and vice-versa). Operators often define a threshold CIR^0 below which the interference is deemed to be too high. The QoS at a given cell of the access network is often measured by the percentage area of the cell, or the percentage of the call traffic for which CIR is smaller then CIR^0.

Most often, we distinguish between 2 types of interference:

- Co-Channel interference: this refers to the case where the interfering signals use the same frequency as the desired signal.
- Adjacent-Channel interference: this refers to the case where the interfering signals use a frequency $f_0 \pm 1$, while the desired signal is carried on a frequency f_0.

It is worth noting that depending on which direction of the communication one considers, the observed CIR may differ [71]. This is due to differences in propagation patterns in both directions.

Capital and operating costs constitute the third measure of network performance. For instance, increasing the number of cells in the network, allows the operator to increase the capacity and to reduce interference. Such measures, however, have associated costs that need to be considered during the resource allocation process [36].

1.4 Network Design & Management

The above paragraphs presented various concepts and design issues that one needs to consider for the design and management of cellular networks. A detailed description of these highly inter-related design factors is provided in [16]. If considered globally, the network design problem is very large and complex. It is necessary to decompose it into smaller subproblems since the size of the original problem and the frequency at which it needs to be re-solved make it next to impossible to go without decomposition. Figure 1.5 summarizes the decomposition framework used in this research. This process addresses both the design of a new network and the adjusting of an existing GSM type network:

- The data collection step involves gathering information related to the demand distribution (with variations in space and time), cost figures (equipment, real estate, etc.) and wave propagation data (terrain topography, urban architecture characteristics, etc.). This raw data provides the basis for all the design problems.
- The propagation analysis phase involves selecting appropriate propagation models for each part of the service area according to geographic, urban and demographic characteristics. These models are used to compute the print area of each candidate antenna.
- Base station and antenna configurations (positions, heights, tilts, power, etc.) are determined at the antenna management phase. The objective functions used in the literature often include maximizing coverage or minimizing costs. This is usually done indepen-

dently of the frequency allocation plan, which is considered only later. This step also involves the power management aspects where the transmission power of each antenna is determined. Knowing where the transmission units are located and their transmitter characteristics, the network planners are able to analyze the coverage area of the network through propagation analysis. Thus, they can forecast the offered traffic at each cell and estimate the potential interference levels between cells.

- The frequency allocation phase involves determining a partition of the available frequency spectrum, for the BCCHs and the TCHs, and allocating frequencies to TRXs. The considerations related to interference usually come into the picture at the frequency allocation step. However, as was discussed above, interference depends on both the frequency allocation plan and on the antenna-positioning plan. Obviously this decomposition, while simplifying the problem, decreases the overall quality of the design.
- Based on the antenna management phase, the data related to traffic captured by the access network is computed for each cell. The planning of the fixed network involves capacity and routing optimization of core network so as to carry this traffic. Determining the number of TRXs to install for each cell could be included in this phase. However, we favor performing this task within the antenna management phase as this may have a direct impact on the optimization of the deployment of antennas.
- At each step of the design process, a partial evaluation of the network performance could be carried out. The network performance can be fully evaluated only when all previous steps have been completed. The design process goes through several loops until an acceptable compromise between the conflicting design objectives and constraints is found.

The design of CDMA type networks is similar to the above process; there is no frequency allo-

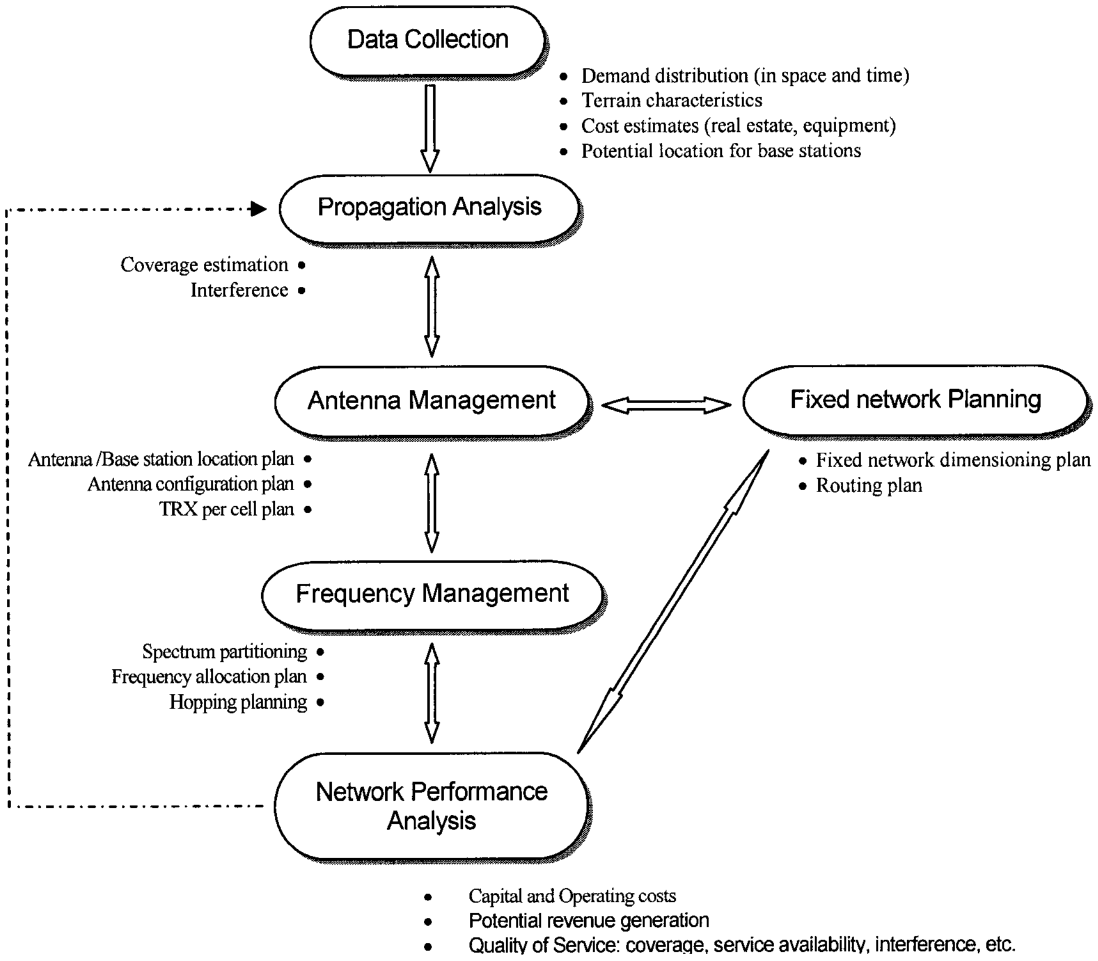

Figure 1.5: GSM network design process

cation since there is only *one* frequency, but it is replaced by PN code allocation. The infrastructure design phase becomes highly critical due to the absence of frequency differentiation between calls, and the strong coupling between teletraffic engineering issues (capacity considerations) and wave propagation (coverage considerations).

The next chapters will examine three critical optimization problems that arise in the design process of GSM networks. In each of the chapters, the relevant literature is surveyed and problem formulations are provided. Chapter 2 examines the antenna positioning problem. We propose problem formulations that combine the most important design criteria. In addition, a new problem formulation is proposed. It incorporates frequency allocation considerations within the antenna positioning problem at a reduced complexity level. For solving this problem, we propose a Tabu Search algorithm that combines several search operators. Chapter 3 examines the static frequency allocation problem where an adaptive Tabu Search-based algorithm is proposed. Frequency management in the context of a frequency hopping environment is discussed in Chapter 4. We analyze the frequency hopping problem at three levels of network synchronization and propose procedures to optimize the hopping parameter. In addition, we propose a new type of optimized hopping pattern for synchronized networks.

Chapter 2
The Antenna Positioning Problem

This chapter examines the Antenna-Positioning Problem (*APP*) for GSM cellular networks. This problem involves the selection of a subset of antennas from a larger candidate list, and determining the associated antenna parameters, subject to a set of technical constraints and multiple design criteria. As illustrated in Chapter 1, the *APP* handles a significant proportion of the network resources and is the first building block of the overall network design process. Therefore, it is essential to get it right. The proposed problem formulations attempt to bring together issues raised both in the academic literature and by practitioners, while linking the network infrastructure design phase to the subsequent frequency allocation phase. This problem is highly combinatorial in nature and realistic instances of *APP* are large in size. We propose a Tabu Search-based heuristic to solve this problem, which is then evaluated for a real life network. Results show a significant improvement over the currently implemented network configurations in the context of small and network capacity upgrade scenarios.

2.1 Introduction

The *APP* problem (also referred to as the Antenna Placement Problem) encompasses a core set of decisions that determine the deployment of physical resources of the network. The Frequency Allocation Problem (*FAP*) is the next step in the decision process. It involves the assignment of frequencies from a limited spectrum to the network radios. The quality of the assignments made while solving FAP depends on the solution approach, but it also heavily depends on the infrastructure design decisions made upstream while solving the APP. Thus, the impact of largely sub-optimal solutions to APP would also translate into sub-optimal solutions to the overall design process. The rapid growth of communication traffic and the increase in competition in the industry make the search for a better utilization of the available resources even more critical. Despite this, few tools are available to solve APP.

Generally speaking, the Antenna-Positioning Problem can be described as follows: Given a candidate set of antennas M, a discrete set of points (pixels) N representing the working area, and the radio wave propagation that link M to N, the objective is to choose a subset of antennas to activate, so as to meet the technical constraints as well as some target performance levels (these can be specified in advance, e.g., total available budget, and hence can be incorporated in the problem constraint set). A variety of criteria can be combined within the problem objective function, reflecting different priorities.

Each antenna in the network can be described by its parameters, which may include: antenna type, coordinates, height, azimuth, tilt, transmission power, cost, maximum capacity, etc. The parameters can take continuous (e.g., height) or discrete values (e.g., power level) and are limited to a specified range that depends on the equipment used. The antenna candidate set then consists of feasible combinations of these parameter settings. Since the number of such combinations may be too large, the network designers consider only the most interesting among them.

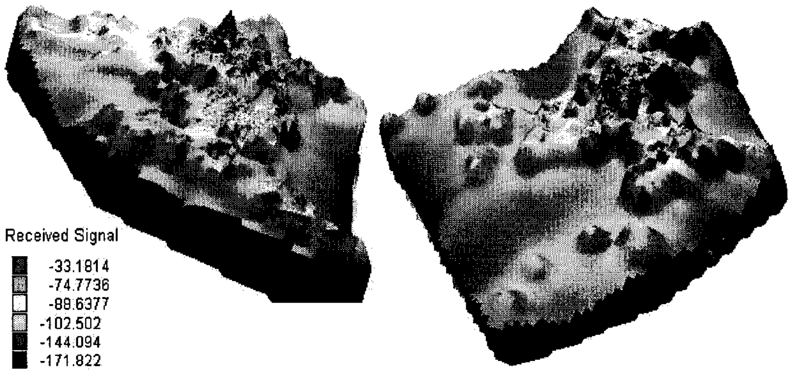

Figure 2.1: APP - Sample cell prints

The antenna print is the set of points which receive a signal from it. The cell associated with each antenna is the part of the print in which it generates the strongest received signal. By varying the antenna parameters within the allowed range, the print of the antenna will also change. Figure 2.1 shows the print of two antennas which differ only in azimuth. This figure shows the three-dimensional view of the intensity of the received signal. Note the non-uniformity of wave propagation in all dimensions as well as the significant impact of the change in azimuth. The area with received signal less than $-92dBm$ is not considered covered by the antenna as this signal is considered too weak to carry calls. Hence, in order to carry such calls the network operator must place antennas that provide a strong enough signal. This example shows how the effect of the transmission of one antenna may reach a larger area far beyond its cell and thus potentially causing significant interference. In fact a signal below $-92dBm$ though weak, can still cause interference, especially when combined with the other non-serving signal.

A discretization procedure is applied to the working area so as to obtain a set of points, or pixels.

With each such point, one can associate some information with regard to coverage requirement (whether the pixel needs to be serviced or not, and at what level of quality) or with regard to traffic density. The propagation matrix provides the link between the candidate antenna set and the pixel set. This matrix describes the wave propagation from each antenna to each pixel. This takes the form of the received signal strength or the path loss (i.e., the reduction in signal strength) at the pixel. For example, Figure 2.2 shows the print of antennas of real-life network. This figure illustrates non-uniformity in propagation patterns and how the coverage areas vary in size, shape and may not always be contiguous.

The activation of a subset of antennas from the candidate list can be evaluated with respect to a number of criteria which include network coverage, network capacity, interference levels and operating costs.

Currently, the process of locating the antennas of the network is mostly manual. The network design engineers spend a long time evaluating a number of scenarios through the use of some simulator, and making incremental adjustments until an acceptable solution is found. Due to the complexity of this problem, only a limited number of such alternatives can be evaluated manually. The process is dependent on the expertise of the engineers and it is difficult to evaluate the quality of the suggested solution. The development time of a new network configuration, or the upgrade of a network, may sometimes become a time consuming and tedious task. In addition, networks are continuously increasing in size and complexity. The “what if” planning approach is not optimal and may in fact be very far from optimality. An automatic positioning and evaluation tool can cut

Figure 2.2: APP - Site cover map

on the duration of the design process, leaving more time for other tasks.

The next section reviews most of the models presented in the literature. Section 3 presents the proposed problem formulations. The selected solution approach and experimental results are described in sections 4 and 5.

2.2 Literature Review

The number of publications dealing with the optimization aspect of the *APP* is relatively limited when compared to those dealing with FAP. Tables 2.1, 2.2, 2.3 and 2.4 below provide an overview of the various proposed models in the literature. The literature is quite recent (mostly after 1998). This reflects the pressure on cost reductions and quality improvements that the operators call for, which in turn is indicative of the increase in cellular network size and complexity. In the tables below, DL and UL refer to down link and up link, while SA, TS, EA and GA refer to Simulated Annealing, Tabu Search, Evolutionary Algorithm and Genetic Algorithm. Underlined items refer to the objective functions used in the model.

Design Issues	**Fritsch et al. 1995** [32]	**Sherali et al. 1996** [79]	**Tutschku et al. 1997** [89]	**Floriani & Mateus 1997** [29]	**Floriani & Mateus 1997** [30]	**Floriani & Mateus 1997** [31]	**Lieska et al. 1998** [55]
Demand Considerations			demand node		Bound on cell size	uniform traffic	
Interference			cell overlap		Co-channel Interf.	Co-channel Interf.	
Communication Direction	DL	DL, Max received signals	DL	DL	DL	DL	DL
Coverage Requirements	Max. coverage	100%	Max. coverage	100%	100%	Max. coverage	Max. coverage
Monetary Considerations	Fixed # ant.	Fixed # ant.	Fixed # ant.	Installation costs	Installation costs	Variable # ant.	Fixed # ant.
Antenna Characterizations	Position, height, power	Position, height	Position, height, power	Position	Position	Position, height, azimuth, power	Position
Frequency Allocation					Allocation by pattern reuse		
Methodology	Self Organizing Map in SA	Non-linear Opt	SA	IP solver	IP solver	IP solver	GA

Table 2.1: APP Literature overview - Part 1

Design Issues	**Tutschku 1998** [87]	**Reininger & Caminada 1998** [74]	**Santos et al. 1998** [76]	**Tutschku et al. 1998/99** [86][90]	**Molina et al. 1999** [63]	**Reininger et al. 1999** [75]	**Vasudevan et al. 1999** [92]
Demand Considerations	demand node		non-uniform traffic	demand node	uniform traffic		demand node
Interference				Co-channel Interf. <threshold			
Communication Direction	DL	DL+UL		DL	DL	DL	DL
Coverage Requirements	Max. coverage	100%	100%	Max. coverage	100%	100%	100%
Monetary Considerations	Variable # ant.	Min # ant.	Min # ant.	Fixed # ant.	min. # ant.	Multi-period installation costs	Installation costs
Antenna Characterizations	Position, height, power	Position		Position, height, power	Position	Position, height, azimuth, power, type	Position
Frequency Allocation					Allocation by pattern reuse		
Methodology	Greedy heuristic		Iterative Alg.	Greedy alg. + SA	Greedy alg., GA, Combination alg for total opt. (CAT)	GA	Greedy Alg

Table 2.2: APP Literature overview - Part 2

Design Issues	**Hurley** **2000** [45]	**Molina et al.** **2000** [64]	**Krishnamachar & Wicker** **2000** [54]	**Huang et al.** **2000** [44]	**Mathar& Shmeink** **2000** [60]	**Han et al.** **2001** [38]	**Vasquez & Hao** **2001** [91]
Demand Considerations	uniform traffic	non-uniform traffic		non-uniform traffic	demand node	non-uniform traffic	demand node
Interference	min overlap.				Co+ adjacent -channel Interf		min Co-channel Interf
Communication Direction	DL	DL	DL	DL	DL	DL	DL
Coverage Requirements	100%	100%	Max. coverage	coverage > target level	Max. coverage	Max. coverage	Max coverage + handover constraint
Monetary Considerations	min. # ant.	min. # ant.	Variable # ant.	variable # ant.	# ant.< K	min # ant.< K	min # ant.
Antenna Characterizations	Position, height, azimuth, power, type	Position	Position	Position, height, azimuth, power	Position, height, azimuth, power	Position	Position, height, azimuth, power, type
Frequency Allocation				Min. # of frequency			
Methodology	SA	CAT	Random walk, SA, TS, GA	Fuzzy expert system	IP-solver + SA	GA	TS with pre and post processing

Table 2.3: APP Literature overview - Part 3

Design Issues	**Mathar & Niessen** **2001** [59]	**Bose** **2001** [11]	**Zimmermann et al.** **2003** [96]	**Raisanen & Whitaker** **2003** [73]	**Mazzini et al.** **2003** [61]	**Whitaker et al.** **2004** [93]
Demand Considerations	demand node	non-uniform traffic	non-uniform traffic		non-uniform traffic	uniform traffic
Interference	Co-channel Interf	Co-channel Interf	Sum interfering signals (Co-channel)	cell overlap $<\alpha$ min. multi-coverage		
Communication Direction	DL	DL	DL		DL	
Coverage Requirements	Max. coverage	Max. coverage	100% max cell compactness	Max. coverage	100%	Max. coverage
Monetary Considerations	# ant.< K	Variable # ant.	Variable # ant.	Variable # ant.	Variable # ant. +design fixed network	Installation costs
Antenna Characterizations	Position, height, azimuth, power	Position, height, azimuth, power	Position, azimuth, power, type	Position		Position
Frequency Allocation	Allocation by pattern reuse				Separation Constraints find feasible assignment	GA
Methodology	SA	Dynamic Prog	EA	EA	Lower bounding by Lagrangian relaxationwith Branch and bound	

Table 2.4: APP Literature overview - Part 4

The decision in APP models involve selecting a combination of antennas amongst a set of potential antennas. Each potential antenna can have a set of characteristics: position, height, antenna type, tilt, angle, and transmission power. Variations in such characteristics may significantly alter the cell shape. Including all such potential variations in the model may be valuable but would add to the size and complexity of the problem. Data sets used in the reviewed articles only allow variations on a few of the characteristics (mostly location, height and power), the others being fixed at typical or average values, leaving room for fine-tuning at a later stage. In addition, although characteristics such as location may take continuous values, only some feasible discrete points that cover the range of possible values are considered.

APP modeling may involve a number of issues. For most APP models in the literature, one needs to define a model of the service area. This represents the spacial partitioning of the overall area where a grid model is applied. For instance, in [90][91][60], this partition is based on the Demand Node Concept. "A demand node represents the center of an area that contains a quantum of demand from teletraffic viewpoint, accounted in a fixed number of call requests per unit time" [86]. A partitional clustering method is used to generate such nodes. The other articles used a uniform grid: the overall area is assumed to be rectangular and is divided into pixels of equal measure. There are two difficulties associated with the use of a demand based partitioning: First, accurate data on spatial distribution of demand is very difficult to obtain. If the network is not built yet (green field scenario), the data is based on pure estimation (based on population density and population characteristics). If the network already exists (thus, the study is for the purpose of evaluation or upgrading), then the mobility of the users and the overlap between cells make accurate estimations very difficult. Second, this approach does not consider the variation in signal strength within the demand node. Nodes with low demand density will tend to cover large areas. Variations in signal strength in such extended areas may be significant. This may result in a cell with enough capacity to satisfy the demand while being unable to carry the offered traffic because of weak signals or high levels of interference. This difficulty also holds if a uniform grid partitioning is used with a low resolution.

For the models we present in this chapter, we favor the use of a uniform grid with consideration given to non-uniform traffic distributions through the pixels of the grid. This seems to be the general trend in the literature. As in [44], we recommend the use of a rough resolution at the initial stages of the design process. The resolution may be refined as the process progresses. This approach has the added advantages of allowing the designer to identify unpromising antenna candidates early in the process and hence save on computational requirements when the resolution is increased.

Demand considerations may also be included in APP models. This refers to the handling of the differences in the volume of offered traffic across the service area. Models that do not include such considerations assume that the problem of determining the capacity of the equipment is to be solved at a later stage. A limit on cell size may be imposed (such as in [30]) so as to ensure that enough capacity could be installed to serve the covered traffic. More explicit consideration may be given by computing the required capacity within the model (e.g., [91]).

The impact of interference is another aspect that has been considered in the context of APP models. In [89], interference exists if two cells overlap (i.e., if a pixel is covered by two or more antennas simultaneously). The assumption here is that all cells will be using the same frequency (worst case analysis) and that interference is unacceptable at all levels (again, worst case analysis). In this case, the objective was to maximize coverage. In [86], a pixel is considered covered only if the interference is lower than a given threshold. So in this case, consideration is included for the level of the interference. In most cases, when interference is included in APP models, it is in the form of co-channel interference as all antennas are assumed to operate on the same frequency. Only in [60] is adjacent-channel interference considered. Given the growth of wireless networks and the scarcity of the radio spectrum, incorporating interference considerations is becoming an important aspect for the efficient utilization of the available resources.

Frequency allocation is a very important problem in the design of FDMA-based wireless networks. It involves the allocation of frequencies to the network transmitters while meeting some quality protection constraints. In [30][63][59], frequency allocation and antenna positioning are considered simultaneously. Thus, these cases are not a worst-case analysis (i.e., cells are not assumed to use the same frequency). This would be the ideal case if the authors did not use the standard frequency reuse schemes for frequency allocation. These schemes assume that all cells are hexagonal in shape and wave propagation follows a uniform shape, which is unrealistic. These assumptions are limiting and may significantly outweigh the simultaneous consideration of frequency allocation and antenna positioning. In [44], minimizing the number of frequencies needed is included. However, this model does not include interference considerations, which in reality may affect the number of frequencies required. The model developed by Mazzini and Mateus [61] attempts to find a feasible frequency assignment subject to a predefined set of frequency separation constraints. Most proposed models do not include frequency allocation aspects. This is due to the significant increase in problem complexity. The second model we propose in Section 2 of this chapter attempts to develop a compromise between these two conflicting objectives, i.e., including partial consideration of the frequency management within the antenna positioning problem with a limited increase in complexity.

In a wireless network, the communication can be established only if both the down link and the up link received signals are strong enough and if the interference level is not too high. Most models reviewed in the literature consider only the down link, except in [74] where both up link and down link signals are incorporated simultaneously in the model.

The coverage achieved by a wireless network corresponds to the proportion of the service area in which communications can be established. Some APP models attempt to maximize the network coverage, while others include a constraint to force a full coverage or to ensure a minimum level of coverage. Carried traffic corresponds to traffic that can be served by the installed capacity. Most articles restrict the definition of the capacity to the number of sites and antennas and do not consider the number of TRXs to install.

As a mobile unit moves away from the coverage area of a cell, its communication must be handed over to an adjacent one. This handoff procedure requires that certain areas of the cell boundaries have multiple coverage. This means that the mobile unit can be connected to several antennas from a feasible set, allowing a smooth transition from the coverage area of one antenna to that of the next one. This type of constraint was included in [91]. To do this, for each selected antenna, part of its coverage area must also be covered by another antenna with a constraint on the maximum difference between the two received signals. The authors mention that although this is a weak form of expressing the handoff requirements, "it is sufficient to ensure good handoff when the coverage constraint is satisfied".

Monetary considerations can also be found in some *APP* models. This can take different forms. Three articles [30][29] [92] use minimizing installation costs as their objective functions. Each potential antenna position has a constant cost value associated with it. In reality, the installation cost would include a fixed portion (associated with real estate and building costs) and a variable portion associated with the capacity of the equipment installed (this in turn varies according to the volume of offered traffic and with the cell surface). The number of antennas installed in the network has also been used as a surrogate measure of the installation cost and was used as the objective function (such as in [45] and [64]). In other articles, a fixed number (such as in [55]) or a bound on the number of antennas that could be installed (such as in [59]) is included as a constraint. The idea here being to make multiple runs under various scenarios and to choose the best. In our proposed *APP* models, we use a similar approach in which we incorporate a budget constraint and limits on the number of resources that can be deployed. The model proposed in [75] incorporates capacity planning of the network over time, i.e., deciding what type of equipment to use and where, and when to install it. The addition of the time dimension increases the problem complexity, requiring a trade-off in terms of the level of model detail.

Most articles use heuristic techniques to overcome the combinatorial and nonlinear nature of the Antenna-Positioning problem. Exact solution approaches are used in some cases but validated on relatively small instances. It is difficult to fully assess the performance of the proposed algorithms because of the small size of the networks tested and the lack of benchmarks. It is worth noting that the emphasis in these articles is often on presenting the model more than on the details of the solution approach. Although most articles comment on the complexity of the proposed models, few complexity proofs are presented. Glaßer et al. [37] studied two APP models for minimizing the number of antennas subject to coverage constraints and the inverse problem. They show the NP-completeness of these two problems.

Most articles reviewed consider minimizing costs or maximizing coverage. Only a few have incorporated interference considerations within the model and even fewer within the objective function. In these few cases, interference is minimized by reducing cell overlap or by reducing the strength of the received interfering signals. Note that explicit consideration of the interference implies the ratio of the serving signal to that of the interfering signals. Only a few articles [38][96][61][73][93] propose multi-objective APP models. The fact that these articles are also the most recent highlights the general tendency towards the need to integrate all APP design aspects. Whitaker et al. [93] list the three possible strategies to combine a variety of objectives: optimizing the weighted combination of the objectives, hierarchical optimization, and generating the non-dominated solutions. For the APP formulations we propose in this work, we choose to optimize a single objective: minimizing interference. The aim here is to integrate this problem with the subsequent phases of the design process. The other design criteria are included within the constraint set of the problem. Combining the design criteria within the objective function or ranking them is difficult since this would require associating meaningful weights with each criterion. The number of design criteria could be reduced by combining the capacity, coverage and cost criteria into one single term. Coverage and capacity can be converted to dollar value as they determine the revenue generated by a network. This measure would provide an idea about the profitability of the operation when reliable cost and revenue estimates are available. Such a measure can then be used within a sensitivity analysis procedure for various levels of budget.

2.3 Proposed Problem Formulations

In this section, we propose two problem formulations in which we attempt to integrate a number of design concepts that are of concern to network operators: coverage, capacity, interference and monetary considerations. Before presenting the proposed models, we start by describing the constituting elements of the problem: the working area, the antenna set and design criteria.

2.3.1 Working area

The proposed models require a partitioning of the service area into a grid. The impact of any network configuration would then be evaluated at every pixel. These evaluations are aggregated in order to obtain an overall evaluation of the proposed configuration.

The grid resolution would be adjusted to the required level of accuracy (balanced with the computational requirements). It is possible to have all pixels on the grid either with equal size or with the same volume of offered traffic (the latter is referred to as the demand node concept [86]). We recommend that pixels correspond to areas within which wave propagation is uniform, i.e., the received signals strength at any point within the pixel is about the same. This should enable a more accurate calculation of the CIR at any pixel. As for the impact of the non-uniformity of traffic distribution across the service area, this can be handled through weights in the objective function and constraints.

Based on the selected grid, we need to define the following:

- N: Set of pixels, indexed by n. This set contains all pixels of interest. This means that all pixels that do no carry traffic or that need not be covered are dropped from consideration in order to save on memory and computational requirements.
- a_n : Volume of traffic offered at pixel n.
- p_{in}: Strength of the signal transmitted by antenna i and received at pixel n, expressed in dBm.
- q_{in}: Degradation in signal strength from antenna i received at pixel n, expressed in dBm.

- p_{min}: Threshold on acceptable signal strength, expressed in dBm. For a communication to take place at a particular pixel, we need the following condition to hold: $p_{in} > p_{min}$. This condition is also used to model the limitation on mobile unit sensitivity or minimum service requirements. For the network tested in this we use $p_{min} = -92dBm$. This value is set as to meet mobile unit sensitivity requirements and minimum requirements for a voice network both outdoors and indoors.
- $y_{in} = \begin{cases} 1, \text{if } p_{in} = \max\limits_{j}\{p_{jn}\},\ p_{in} > p_{min} \text{ and antenna } i \text{ is active} \\ 0, \text{otherwise.} \end{cases}$

  This variable is used to determine if pixel n is served by antenna i or not. This assumes that each pixel is served by its best server, i.e., the antenna providing the strongest received signal. Note that y_{in} is not defined if the signal received from antenna i, operating at its maximum allowed power level, is not strong enough. Such a condition implies that the antenna can never serve the pixel and hence the variables need not be considered in the model.
- γ_n: CIR level at pixel n.
- γ^1, γ^2 : Two interference level thresholds where $\gamma^1 < \gamma^2$. The use of these thresholds will be examined later when discussing network design criteria.
- $z_n^1 = \begin{cases} 1, \text{if } \gamma_n \leq \gamma^1 \\ 0, \text{otherwise.} \end{cases}$ and $z_n^2 = \begin{cases} 1, \text{if } \gamma^1 < \gamma_n \leq \gamma^2 \\ 0, \text{otherwise.} \end{cases}$

  These two variables are used to count the number of pixels and the volume of traffic for which the CIR level does not exceed the two defined threshold levels.

2.3.2 Candidate Antenna Set

The candidate antenna set is constituted by varying antenna parameters, which can be partitioned into 2 categories: the first set of parameters is related to the equipment while the second is dependent on the designer. Equipment manufacturers provide an array of antennas to the market. Antenna type (omni-directional versus directional antennas), propagation pattern, capacity (maximum number of radios that can be linked to the antenna) and transmission power range are the main characteristics of concern in APP. In addition, the design engineer can vary the coordinates, height, tilt and azimuth. The number of combination, that results from all possible parameters values is huge but can be reduced by eliminating infeasible or unattractive combinations (e.g. see [91]). For the models presented here, the optimization selects a subset of antennas and determining their transmission power. The candidate set includes a subset of the possible combination of antenna characteristics. We do not impose any restriction on this set.

The cell print that results from an antenna in the candidate list can be computed using some signal propagation tool either during, or prior to, the execution of the optimization algorithm. While the on-line calculations are very costly in computational time, off-line calculations may require significant memory.

The cell print can be provided in the form of the path loss matrix (i.e., reduction in signal strength from the antenna to every pixel in the network) or in the form of signal strength matrix (the received signal level at every pixel) for each possible level of transmission power at the antenna. In this work, the path loss matrix is provided as input to the optimization algorithm. We consider that the antenna transmission power is a decision variable in our models. Since the path loss matrix is valid for the range of feasible transmission power levels, one can evaluate a number of network configurations with significantly reduced memory requirements (as opposed to storing a signal strength matrix for each possible power level and for each antenna).

Each antenna is placed at a site. The latter can accommodate one omni-directional antenna or multiple directional antennas. In the proposed models we associate a cost with every opened site and a cost term with every active antenna.

If a subset of antennas is activated, the combined effect of these decisions results in an assignment of subscribers to antennas, hence identifying the cell or coverage of every active antenna. Having done this, one needs to determine the number of radio transmitters (TRXs) to be attached to every antenna.

The required number of TRXs depends on the traffic offered to the cell. The latter is equal to the summation of the pixel traffic a_n over all pixels covered by the antenna. Thus, given the estimated number of subscribers per pixel, the average traffic per subscriber and the maximum allowed blocking probability (the probability for a customer of not being served because all servers are busy), one can compute the number of TRXs needed using a conversion table. The computations are based on the Erlang B queuing model (i.e., $M/M/s$ queueing model with infinite sources and where blocked requests are lost). For any GSM antenna, the first TRX can simultaneously serve 7 communications and every subsequent TRX can handle 8 more communications (the first channel on the first TRX being reserved as the Broadcast Control Channel). The number of TRXs that an antenna can handle is limited. For the example used in this work, a maximum of 4 TRXs can be associated with every antenna. Note that there is no optimization involved in the determination

# TRXs	1	2	3	4
Traffic (Erlang)	2.93	9.00	15.76	22.82
# Subscribers	132.1	405.5	709.3	1027.3

Table 2.5: APP - Number of TRXs and Offered traffic (for 2% blocking and 0.02222 Erl per user)

of the number of TRXs. It is a function of the offered traffic which is a consequence of the optimization of the network configuration. Table 2.5 shows a sample conversion table for a 2% blocking probability and an average traffic per user of 0.02222 *Erlang* (the configuration specified by the operator, for the data set we use in this chapter). It indicates the number of subscribers, or the traffic volume, that can be served for the number of TRXs available. Equivalently, this table indicates the number of TRXs that need to be installed for each level of traffic. If the offered traffic exceeds the capacity of provided by the TRXs installed, then the excess traffic is not carried and hence is blocked. For the models presented in this chapter, we assume that all traffic related data is expressed in terms of the number of subscribers.

We need to define the following notation:

- M: Given set of antennas in the candidate list, indexed by i.
- K: Set of sites (corresponding to the antenna set in the candidate list), indexed by k.
- $x_i = \begin{cases} 1, \text{if antenna } i \text{ is active} \\ 0, \text{otherwise.} \end{cases}$
  These decision variables indicate the subset of M that needs to be activated. $X = (x_i)$ is the corresponding vector form of this variable.
- p_i : Transmission power of antenna i, expressed in dBm. Thus, for any pixel n, the received signal from antenna i is given by: $p_{in} = p_i + q_{in}$. These variables

correspond to the second set of decision variables. $P = (p_i)$ is the corresponding vector form of this variable.

- Δ_i : Discrete set of possible values of p_i for antenna i. The interval in which the power levels can be varied depends on the antenna type. For instance, for some antenna we may have $\Delta_i = \{off, 45, 46, 47, 48\}(dBm)$.
- $w_k = \begin{cases} 1, \text{if site } k \text{ is open} \\ 0, \text{otherwise.} \end{cases} = \begin{matrix} 1, \text{if } x_i = 1 \text{ for some } i \in k \\ 0, \text{otherwise.} \end{matrix}$
  This is the indicator variable associated with every site.
- o_i : Offered traffic at antenna i for a given configuration, where: $o_i = \sum_{n \in N} a_n y_{in}$.
  Offered traffic is the total traffic in the pixels covered by an antenna i.
- Using conversion table 2.5, and assuming a maximum of 4 TRXs per antenna, denote:
  - d_i : Number of TRXs to be installed at antenna i :

$$d_i = \begin{cases} 1, \text{if } o_i \leq 132.1 \\ 2, \text{if } 132.1 < o_i \leq 405.5 \\ 3, \text{if } 405.5 < o_i \leq 709.3 \\ 4, \text{otherwise.} \end{cases}$$

  - b_i : Blocked traffic at antenna i. This corresponds to the traffic offered to antenna i (i.e., antenna i provides the strongest signal) but that cannot be carried because not enough TRXs could be installed.

$$b_i = \begin{cases} 0, \ \text{if } o_i \leq 1027.3 \\ o_i - 1027.3, \text{otherwise} \end{cases}$$

  - c_i : Carried traffic at antenna i. This corresponds to the traffic served by antenna i that could be processed by the installed TRXs.

$$c_i = \begin{cases} o_i, \text{ if } o_i \leq 1027.3 \\ 1027.3, \text{ otherwise} \end{cases}$$

- cap_i : Capacity of antenna i, expressed as the total number of subscribers that can be served by the TRXs installed.

$$cap_i = \begin{cases} 132.1, \text{ if } b_i = 1 \\ 405.5, \text{ if } b_i = 2 \\ 709.3, \text{ if } b_i = 3 \\ 1027.3, \text{ if } b_i = 4 \end{cases}$$

- e_i : Excess capacity at antenna i. This corresponds to additional traffic that the currently installed number of TRXs could carry.

$$e_i = \begin{cases} 132.1 - o_i, \text{ if } o_i \leq 132.1 \\ 405.5 - o_i, \text{ if } 132.1 < o_i \leq 405.5 \\ 709.3 - o_i, \text{ if } 405.5 < o_i \leq 709.3 \\ 1027.3 - o_i, \text{if } 709.3 < o_i \leq 1027.3 \\ 0, \text{ otherwise.} \end{cases}$$

- c_i^a : Cost of antenna i.
- c_k^s : Cost of site k.
- c^t : Cost of a single TRX.

2.3.3 Network Design Criteria

A network configuration S is defined by the set of active antennas and their selected transmission power. Thus, S is defined as

$$S = (X, P) \text{ or } S = \{(x_i, p_i)\ \forall i \in M\}. \tag{1}$$

The evaluation of any network configuration, S, is based on 4 dimensions: network coverage, network capacity, network interference, and network cost and profit. Any of these criteria can be incorporated in the objective function or in the problem constraint set.

2.3.3.1 Network coverage and capacity

For a given network configuration $S = (X, P)$, the network characteristics are determined by the resources deployed (sites, antennas, TRXs). Area (traffic) coverage is the total surface (volume of traffic) that can receive a strong enough signal to establish communication. Network capacity is the maximum volume of traffic that can be carried by the installed resources. Because only limited resources are available, a configuration S may not be able to carry all offered traffic. We use the following definitions to describe capacity and coverage-related performances of any given configuration:

- $A(S)$: Total number of active antennas in configuration S: $A(S) = \sum\limits_{i \in M} x_i$.
- A^* : Maximum number of antennas that can be activated.
- $U(S)$: Total number of active sites in configuration S: $U(S) = \sum\limits_{k} w_k$.
- U^* : Maximum number of sites that can be opened.
- $T(S)$: Total number of installed TRXs in configuration S: $T(S) = \sum\limits_{i \in M} d_i$
- T^* : Maximum number of TRXs that can be installed.

- $Cov(S)$: Total covered area in configuration S: $Cov(S) = \sum\limits_{i \in M} \sum\limits_{n \in N} y_{in}$
- Cov_* : Minimum required level of coverage, expressed as a percentage of the total area in N.
- $O(S)$: Total network offered traffic in configuration S: $O(S) = \sum\limits_{i \in M} o_i$.
  This term measures the traffic coverage of configuration S and provides an estimate of the maximum revenue that can be generated by S.
- O_*: Minimum required covered traffic volume, expressed as a percentage of the total offered traffic in N.
- $C(S)$: Total network carried traffic in configuration S: $C(S) = \sum\limits_{i \in M} c_i$.
  This term corresponds to the fraction of offered traffic that can be served by the installed TRXs.
- C_* : Minimum required carried traffic, expressed as a percentage of the total offered traffic in N.
- $B(S)$: Total network blocked traffic in configuration S: $B(S) = \sum\limits_{i \in M} b_i$.
  Traffic is blocked when not enough TRXs could be installed at some antenna. This terms measures lost revenues as well as quality of service. Blocked communications may be frustrating to customers, hence the need to monitor this aspect.
- B^*: Maximum allowed blocking, expressed as a percentage of the total covered traffic in S (as opposed to measuring it against total offered traffic in N, in order to reflect the magnitude of customer dissatisfaction)
- $Cap(S)$: Total network capacity in configuration S: $Cap(S) = \sum\limits_{i \in M} cap_i$.
  This term describes the total number of customer that can be served by the proposed configuration.

- $E(S)$: Total network excess capacity in configuration S: $E(S) = \sum\limits_{i \in M} e_i$.
  This term describes the equipment efficiency of the proposed configuration and provides an estimate of the slack resources built in the proposed configuration (which could be seen as provisions for future network development).
- E^*: Maximum allowed excess capacity, expressed as a percentage of total network capacity.

2.3.3.2 Network interference

Assuming that all signals are on the same frequency (co-channel interference case), the CIR level at any pixel n is defined as the ratio of the serving signal to the combined effect of all interfering signals on the same frequency. Since all power variables are expressed in dBm, the CIR function needs to be adjusted. The relationship between power expressed in dBm and mW is as follows: $P(dBm) = 10log_{10}(\frac{P(mW)}{P_0(mW)})$, where P_0 is the reference power (1mW). Hence, the CIR at any pixel n is given by:

$$\gamma_n = \sum_{i \in M} (p_{in} y_{in}) - 10 \log_{10} \left(\sum_{\substack{j \in M \\ j \neq \arg\max (p_{in} x_i)}} 10^{\left(\frac{p_{jn} x_j}{10} \right)} \right) \tag{2.2}$$

$$where$$

$$p_{in} = p_i + q_{in} \tag{2.3}$$

$$\sum_{i \in M} y_{in} \leq 1 \tag{2.4}$$

The first term in equation 2.2 identifies the server signal of the pixel under consideration, while the second term computes the combined effect of all interfering signals. Note that since power terms are expressed in dBm, they need to be transformed back to mW and added to determine their combined effect. Equation 2.3 is used to compute the received signals at the pixels given the signal transmitted at the antennas. Equation 2.4 ensures that a pixel has no more than one serving antenna. We would like to have the CIR at all pixels as high as possible. Thus, for a given configuration, two possible interference-related design criteria would be: the average CIR and the average weighed CIR (weighed by the offered traffic), which one would like to maximize. Hence, we define:

- $Q(S)$: average pixel CIR across all covered pixels in N: $Q(S) = \frac{\sum_{n \in N} \gamma_n}{Cov(S)}$
- $TQ(S)$: traffic-weighed average pixel CIR across all covered traffic in N: $TQ(S) = \frac{\sum_{n \in N} a_n \gamma_n}{O(S)}$

Operators only have a limited radio spectrum at their disposal. For realistic size networks, increasing CIR would come at the expense of deploying more resources or reducing covered or carried traffic. In addition, at some CIR levels the communication quality is sufficiently acceptable and the benefits of a further CIR increase is imperceptible. The downside of maximizing average CIR is that the solution obtained may have pixels with unnecessarily high CIR levels while others suffer from significant interference levels. Therefore, other possible interference-related design criteria would be to minimize the area (or traffic) for which the CIR is too low.

In order to determine what would be the threshold value for which the CIR ratio is deemed too low, we need to refer the reader to the Frequency Allocation Problem (further details are presented in Chapter 3). FAP can be viewed as a generalized graph coloring problem in which the aim is to assign frequencies to TRXs so as to minimize interference. Using different frequencies on different transmitters reduces interference but because only a few frequencies are available, frequency sharing is inevitable.

When two TRXs share the same frequency, the pixels that receive their signal suffer from co-channel interference. The CIR level at any pixel would depend on the relative positions of the pixel to the interfering transmitters. For FAP to be manageable, transmitters are considered in pairs. The engineer would carry out the following steps.

- For each pixel, identify the serving antenna, i^*. This defines the cells.
- Taking in turn all other active transmitters j, compute the potential pair-wise interference, $CIR^n(i^*, j) = p_{i^*n} - p_{jn}$ value for every pixel n. This is potential interference, because the frequency allocation is not done yet and so we do not know if co-channel interference will take place or not. Also, this is pair-wise interference since we consider the interference onto the serving antenna from one transmitter at a time. The computations are carried out exactly as in equation 2.2 except that the logarithmic term contains information on one transmitter only.
- Considering all pixels covered by transmitter i and in turn consider all other transmitters j. At each step the engineer would compile the percentage area or percentage traffic for which $CIR^n(i^*, j) < \alpha$ where α is the co-channel threshold. These compiled entries constitute the *co-channel interference matrix*, which is presented as input to FAP.

Adjacent-channel interference is the second type of interference. It occurs when two

TRXs use adjacent frequencies. In GSM, each frequency has a bandwidth of 200kHz. One frequency separation does not eliminate interference but reduces its magnitude. Further frequency separations provide even more protection. It is considered that beyond adjacent-channel, the interference ceases (since the protection provided is so high).

The magnitude of the interference caused by one transmitter on an adjacent channel is calculated according to the same procedure described above for the co-channel interference case, except that one has to account for the protection provided by the separation. Let θ be the adjacent-channel protection, expressed in dBm. An interfering signal P on an adjacent channel would be equivalent to a weaker interfering signal $P' = P + \theta$ on the same frequency. The *adjacent channel interference matrix* is generated using the above procedure using the P' power levels instead of P and compiling percentage area (or traffic) using for which $CIR^n(i^*, j) < \beta$, where β is the adjacent-channel threshold. In this chapter, we use the following (typical) values for these parameters: $\theta = -18\ dBm$, $\alpha = 10$ dB, and $\beta = 14\ dB$.

Going back to APP, the other suggested design criterion is to minimize the area or traffic affected by low CIR levels. Similar to the co-channel threshold α and adjacent channel threshold β, we use two thresholds γ^1 and γ^2. In FAP, only pair-wise interference is considered while in APP the CIR at every pixel is measured as a function of the combined effect of all interferers. For this reason, we associate lower values with γ^1 and γ^2 than with α and β. For the data set tested in this research, we use $\gamma^1 = 8\ dBm$ and $\gamma^2 = 12\ dBm$. Hence, we have:

- $R^1(S)$: Total area with $CIR < \gamma^1$, computed as the number of pixels (assuming they all have the same area).

$$R^1(S) = \sum_{n \in N} z_n^1$$

- $TR^1(S)$: Total traffic with $CIR < \gamma^1$

$$TR^1(S) = \sum_{n \in N} a_n z_n^1$$

- $R^2(S)$: Total area with $\gamma^1 \leq CIR < \gamma^2$, computed as the number of pixels (assuming they all have the same area).

$$R^2(S) = \sum_{n \in N} z_n^2$$

- $TR^2(S)$: Total traffic with $\gamma^1 \leq CIR < \gamma^2$

$$TR^2(S) = \sum_{n \in N} a_n z_n^2$$

The interference related design criteria presented above hold for the case where all antennas use the same frequency. In the second model we propose in this research, we also consider adjacent channel interference. This will be discussed later when presenting the second model below in section 2.3.5.

2.3.3.3 Network costs and profit

Operating costs and profit constitute the fourth dimension of evaluation for a network configuration. We define:

- $Cost(S)$: Total network operating costs. This corresponds to the summation of site, antenna and TRXs costs:

  $$Cost(S) = \sum_{k \in K} c_k^s w_{k+} \sum_{i \in M} c_i^a x_i + \sum_{i \in M} c^t d_i$$

- $Revenu(S)$: Total revenue generated by carried traffic

  $$Revenu(S) = rC(S).$$

  where r is Average revenue per customer whose traffic is carried

- $Cost^*$: Total available budget.

Above we have listed a variety of design criteria that need to be managed simultaneously in order to obtain good APP solutions. This illustrates the need for an automatic optimization tool. For each criterion, we associate a target performance level. We can define Ω as the set of solutions achieving those target performance levels:

$$\Omega = \left\{ S:\ such\ that: \begin{array}{c} Cov(S) \geq |N|\,Cov_* \\ O(S) \geq O_* \sum\limits_{n\in N} a_n \\ C(S) \geq C_* \sum\limits_{n\in N} a_n \\ B(S) \leq B^* C(S) \\ E(S) \leq E^* Cap(S) \\ Cost(S) \leq Cost^* \\ A(S) \leq A^* \\ U(S) \leq U^* \\ T(S) \leq T^* \end{array} \right\} \tag{5}$$

Note that the above definition of Ω does not include any interference-related design criterion. We opted to include such criteria in our models within the objective function. This is because by trying to minimize interference within the objective function, we link APP to FAP, which uses a similar interference reduction objective. In addition, determining what would be an appropriate value for minimum average CIR or what should be the minimum area with low CIR is not straightforward. Such considerations are more a consequence of the available resources and the business goals of the network operator. Because of this we define two objective functions $F^A(S)$ and $F^T(S)$ such that:

$$F^A(S) = R^1(S) + R^2(S) \tag{2.6}$$

$$F^T(S) = TR^1(S) + TR^2(S) \tag{2.7}$$

$F^A(S)$ reflects the need to minimize total area with low CIR levels, and $F^T(S)$ aims at minimizing the volume of traffic affected by high interference. Moreover, any combination of the criteria used in Ω can be used instead.

The next sections describe the two proposed models for APP, which will be referred to as $APP1$ and $APP2$. In this first formulation ($APP1$), we model the APP under the assumption that all antennas use the same frequency. This assumption will be partially relaxed in $APP2$. Ideally one should simultaneously solve APP and FAP. However, the problem that would result would be very large and difficult to solve for realistic size networks, hence making the ideal planning scenario impractical. $APP1$ takes a conservative planning approach by assuming a worst case scenario. $APP2$ is an intermediate model: Antenna positioning is carried out while trying to partially exploit the gain from frequency allocation without actually solving FAP.

2.3.4 APP Formulation 1

In this first formulation ($APP1$), we model the APP under the assumption that all antennas operate with the same frequency.

The first proposed formulation is as follows:

$$(APP1) \quad \underset{S=\{(x_i,p_i),\forall i\in M\}}{Optimimize} \; F(S) \tag{2.8}$$

Subject to

$$p_{in} \geq x_i(p_i + q_{in}) \qquad \forall i \in M \text{ and } \forall n \in N \tag{2.9}$$

$$0 \leq (p_{in} - p_{\min})\, y_{in} \qquad \forall i \in M \text{ and } \forall n \in N \tag{2.10}$$

$$p_{in} \geq p_{jn} - L(1 - x_j) - L(1 - y_{in}) \qquad \forall i, j \in M \text{ and } \forall n \in N \tag{2.11}$$

$$\sum_{i\in M} y_{in} \leq 1 \qquad \forall n \in N \tag{2.12}$$

$$\gamma_n = \sum_{i\in M} p_{in} y_{in} - 10\log_{10}\left(\sum_{\substack{j\in M \\ j \neq \arg\max(p_{in}x_i) \\ p_{jn}\geq p_{\min}}} 10^{\left(\frac{p_{jn}x_j}{10}\right)} \right) \qquad \forall n \in N \tag{2.13}$$

$$Lz_n^1 \geq (\gamma^1 - \gamma_n) \qquad \forall n \in N \tag{2.14}$$

$$Lz_n^2 \geq (\gamma^2 - \gamma_n) - (L+1)z_n^1 \qquad \forall n \in N \tag{2.15}$$

$$w_k \geq x_i \qquad \forall i \in k \text{ and } \forall k \in K \tag{2.16}$$

$$S \in \Omega \tag{2.17}$$

$$p_i \in \Delta_i \qquad \forall i \in M \tag{2.18}$$

$$w_k, x_i, z_n^1, z_n^2 \qquad \text{Binary} \tag{2.19}$$

The objective function of $APP1$ can correspond to any of the design criteria presented previously. For the example we test our approaches on, we use $F^T(S)$. Equation 2.9 is used to compute received signals at every pixel from all active antennas (with L being a sufficiently large constant). A pixel is considered covered by the server providing a strong enough signal (inequalities 2.10) and such that it is the strongest signal among all other received signals (inequalities 2.11). Constraints 2.12 indicate that a pixel has no more than one server. The CIR level at any pixel is calculated according to equation 2.13. Inequalities 2.14 and 2.15 are used to identify pixels with CIR below $\gamma^1$and γ^2, respectively. Inequality 2.16 are used to determine the set of active sites corresponding to the set of active antennas. Condition 2.17 represents the constraint set induced by the desired network performance.

A number of other constraints could be included in the model. Handoff constraints could, for instance, stipulate that the border area of each cell receives more than one signal so as to ensure the continuity of communication when the mobile unit is travelling across cells. Vaszquez and Hao [91] defined the handoff region as the area of the cell receiving a weaker signal (than the server) but no less then a pre-specified level. The handoff constraint is expressed as a requirement for a non empty handoff region. They indicate that these constraints are weak in the sense that they are easily satisfied when coverage constraints hold. In the proposed models, we opted not to include such simple handoff constraints. A more realistic definition of the handoff region also involves identifying its location and size. The added complexity to the model is significant. Because of this, we propose to manage the handoff process at a post-optimization stage. Using a GIS-based simulation tool (available to most operators), the design engineer can visually inspect the proposed network configuration and manually manage the handoff process.

2.3.5 APP Formulation 2

The second proposed model ($APP2$) assumes a more realistic planning approach by partially incorporating the effect of frequency allocation into the model. The idea is that a good frequency allocation solution would allocate non-interfering frequencies to antennas that otherwise would suffer from high interference levels (Recall that in FAP, one would like to minimize interference between pairs of transmitters given potential co-channel and adjacent-channel interference).

From the definition of γ_n in APP1, a good solution would attempt to increase the power of the server and decrease the power of interfering antennas. Since the server of one pixel will act as interferer to some other pixels, the difficulty is then to find a good equilibrium. If one knew that a pair of antennas are guaranteed to use non-interfering frequencies, then the mutual potential interference between that pair could be dropped from consideration while solving APP. This would imply that the equilibrium solution would become independent of the relationship between that pair.

For instance, consider a pixel set A which is not covered in a particular network configuration. For this pixel set to receive a strong enough coverage, the solution has to be changed so as to increase the power of some active antenna or activate some new antenna. By doing so, one would increase the potential interference on some other pixel set B. The choice of the antenna for which to increase the power is dependent on the interference that would result. If we assume that all antennas use the same frequency then one would obtain one decision, however, if we knew the frequency allocation, the decision may change.

One can also consider the case of a network with Hierarchical Cell Structure (HCS). This refers to networks with cells operating on multiple levels. One would have a number of cells covering small areas with high traffic density and some larger cells covering several of the small cells. These large cells would capture all traffic that could not be served by the small cells. Such systems are increasingly used for high density networks. In such a context, the large and small cells will cover the same area and so the potential interference would also be very high. A good frequency allocation would not assign interfering frequencies to these cells. If one is to use a planning model that assumes a co-channel interference scenario between all antennas (most do), it is not likely to produce an HCS design.

The above argumentation for introducing information related to frequency allocation in APP dealt with interfering versus non-interfering frequencies. To be more precise, a pair of antennas can be in one of three states: co-channel interference, adjacent channel interference or no-interference. These cases correspond to the situations which are incorporated in FAP. So the question now is how to incorporate frequency-related information within APP without actually solving FAP.

FAP uses as input the co-channel and adjacent-channel interference matrices. In addition, it require a *separation matrix* (δ_{ij}). An entry in this matrix between cells i and j indicates that the assigned frequencies must be separated by at least δ_{ij}. Hence: $\delta_{ij} \geq 2$ indicates that cell i and j cannot use the same frequency or adjacent frequencies. If $\delta_{ij} = 1$ then the worst acceptable case is adjacent-channel interference, whereas $\delta_{ij} = 0$ indicates there are no restrictions imposed. Such constraints are used to protect from having high interference levels especially for pairs of antennas involved in handoffs.

In order to incorporate frequency-related information within APP without solving FAP, we propose to include δ_{ij} as decision variables within APP. This augmented model is referred to as $APP2$.

For each pair of antennas i and j, we define:

- $\delta^1_{ij} = \begin{cases} 1, \text{if } \delta_{ij} = 1 \\ 0, \text{otherwise.} \end{cases}$
- $\delta^2_{ij} = \begin{cases} 1, \text{if } \delta_{ij} \geq 2 \\ 0, \text{otherwise.} \end{cases}$

If a solution to APP2 indicates that $\delta^1_{ij} = 1$, then in a feasible FAP solution, the worst that can happen is adjacent channel interference; and if $\delta^2_{ij} = 1$ then in a feasible solution, antennas i and j will not interfere with each other. On the other hand, if $\delta^1_{ij} = 0$ and $\delta^2_{ij} = 0$, then the pair of cells are not protected and no guarantees can be given with regard to interference. Therefore, the aim is not to determine frequency assignments but rather to select which pairs to protect with the highest priority and at what level. We need to define a modified measure of the CIR at every pixel n :

$$\begin{aligned}
\gamma'_n &= \sum_{i \in M} p_{in} y_{in} \\
&\quad -10 \log_{10} \left(\sum_{\substack{j \in M \\ j \neq i^* \\ p_{jn} \geq p_{\min}}} 10^{\left(\frac{p_{jn} x_j (1 - \delta^1_{i^*j} - \delta^2_{i^*j}) + p'_{jn} x_j \delta^1_{i^*j}}{10} \right)} \right) \qquad (2.20) \\
&\quad where \\
i^* &= \underset{i \in M \text{ and } x_i = 1}{\arg\max} \{p_{in}\} \qquad (2.21) \\
p_{in} &= p_i + q_{in} \qquad (2.22) \\
p'_{in} &= p_i + q_{in} + \theta \qquad (2.23) \\
\delta^1_{ij} + \delta^2_{ij} &\leq 1 \qquad (2.24)
\end{aligned}$$

Note that if $\delta^1_{ij} = 1$, indicating that as a worst case we would have adjacent-channel interference, we need to account for the protection provided by the one channel difference. In such a situation, the interfering signal is no more p_{jn} but p'_{jn}. This is calculated in equation 2.23, where θ is the adjacent channel protection (typically set at -18 dBm). Constraint 2.24 indicates that only one type of separation constraint can be active.

Setting $\delta^2_{jj'} = 1$ for a pair of antennas j and j', would cancel out their mutual interference and hence would reduce the objective function. Naturally, any algorithm would try to set all such variables to 1. This amounts to saying that all antennas would try to use non-interfering frequencies. This is impossible for any practical size network (since a typical operator would have only about 40 frequencies to serve hundreds of antennas). The same goes for the $\delta^1_{jj'}$ variables. This would reduce interference significantly but no feasible FAP solution could meet such constraints since no operator has sufficient resources to achieve this. Therefore, we need to impose a constraint that would limit the number of pairs of antennas for which the δ variables can take value 1. This limit reflects the amount of available radio resources. To achieve this, we define:

- ρ^1 : Maximum allowed number of antenna pairs (i, j) such that $\delta^1_{ij} = 1$.
- ρ^2 : Maximum allowed number of antenna pairs (i, j) such that $\delta^2_{ij} = 1$.

This requirement can then be translated to:

$$\begin{aligned} \sum_{i,j \in M} (x_i x_j \delta^1_{ij}) &\leq \rho^1 \\ \sum_{i,j \in M} (x_i x_j \delta^2_{ij}) &\leq \rho^2 \end{aligned} \tag{2.25}$$

Now, we need to redefine all variables related to the CIR measurements so as to correspond to the modified measure γ'_n. The redefinitions are obtained by replacing γ_n by γ'_n. Thus, a new set of variables are obtained: $Q'(S)$, $TQ'(S)$, $z^{1\prime}_n$, $z^{2\prime}_n$, $R^{1\prime}(S)$, $TR^{1\prime}(S)$, $R^{2\prime}(S)$, $TR^{2\prime}(S)$, $F^{A\prime}(S)$ and $F^{T\prime}(S)$, such that:

- $z^{1\prime}_n = \begin{cases} 1, \text{if } \gamma'_n \leq \gamma^1 \\ 0, \text{otherwise.} \end{cases}$
- $z^{2\prime}_n = \begin{cases} 1, \text{if } \gamma^1 < \gamma'_n \leq \gamma^2 \\ 0, \text{otherwise.} \end{cases}$

- $Q'(S)$: average pixel CIR across all covered pixels in N: $Q'(S) = \frac{\sum_{n \in N} \gamma'_n}{Cov(S)}$
- $TQ'(S)$: traffic-weighed average pixel CIR across all covered traffic in N: $TQ'(S) = \frac{\sum_{n \in N} a_n \gamma'_n}{O(S)}$
- $R^{1\prime}(S)$: Total area with $CIR < \gamma^1$, measure in number of pixels (assuming they all have the same area):

$$R^{1\prime}(S) = \sum_{n \in N} z_n^{1\prime}$$

- $TR^{1\prime}(S)$: Total traffic with $CIR < \gamma^1$:

$$TR^{1\prime}(S) = \sum_{n \in N} a_n z_n^{1\prime}$$

- $R^{2\prime}(S)$: Total area with $\gamma^1 \leq CIR < \gamma^2$, measure in number of pixels (assuming they all have the same area):

$$R^{2\prime}(S) = \sum_{n \in N} z_n^{2\prime}$$

- $TR^{2\prime}(S)$: Total traffic with $\gamma^1 \leq CIR < \gamma^2$:

$$TR^{2\prime}(S) = \sum_{n \in N} a_n z_n^{2\prime}$$

- $F^{A\prime}(S)$: Total area with $CIR < \gamma^2$:

$$F^{A\prime}(S) = R^{1\prime}(S) + R^{2\prime}(S)$$

- $F^{T'}(S)$: Total traffic with $CIR < \gamma^2$:

$$F^{T'}(S) = TR^{1'}(S) + TR^{2'}(S)$$

Before presenting the full $APP2$ formulation, we need to introduce one more concept. Consider the situation in which one antenna covers an area where the offered traffic exceeds the maximum capacity (number of TRXs) that could be installed in it. We refer to the traffic that cannot be served by its best server as blocked traffic. Under certain conditions, blocked traffic can be served by its second best server. We refer to this as *traffic overflow*. Hence, for a particular antenna we need to distinguish between direct offered traffic and overflow offered traffic. If we ignore this second type of traffic, then the number of TRXs installed may not be enough to satisfy the total demand, leading to more overflow traffic on some other antennas and so on. In such case, one might observe a more severe traffic blocking than expected and hence a reduced quality of service. This concept may come to play an important role also in the context of HCS networks, where customers not served by lower level cells are redirected to higher level cells.

By definition, the signal from the best server is stronger than that of the second. So if both antennas use the same frequency, then the communication cannot be established on the second best server as the interference would be excessively high. Thus, for traffic overflow to take place the first and second servers need to be on different frequencies. Discussion with engineers indicate that for traffic overflow to occur at a pixel n with best server i and second server j, we need to have $\delta^1_{ij} = 1$ or $\delta^2_{ij} = 1$, indicating that, in the worst case, the best sever and the second server use adjacent frequencies. Also, for overflow to occur, we need to have $p_{in} - p_{jn} \leq \nu$, where ν is some fixed parameter. In the example we examine in this research, we have $\nu = 4dBm$. This conditions indicates that traffic overflow may occur on the cell boundaries only, which may correspond roughly to the handoff region. Although traffic overflow may occur also on the third, or fourth best server (or further), we only consider the second best server in order not to increase the problem complexity too much. In addition, design engineers indicated to us that too much traffic overflow is not recommended. Traffic blocked beyond the second server is managed by a constraint that limits the fraction of blocked traffic.

Incorporating the effect of overflow traffic implies redefining antenna offered traffic and all related variables. We define:

- $v_{in} = \begin{cases} 1, \text{if } p_{in} = \max\limits_{\{j:\, y_{jn}=0,\, p_{jn}>p_{min}\}} \{p_{jn}\} \\ 0, \text{otherwise.} \end{cases}$
  This variable identifies the second best server for pixel n.

- o'_i : Total offered traffic at antenna i, where:

$$o'_i = o_i + \sum_{j \in M} \frac{b_j}{o_j} \sum_{n \in N} a_n v_{jn}$$

  The first term computes the direct offered traffic, while the second term compiles the fraction of blocked traffic served by other antennas which overflow on antenna i as the second best server.

- Using conversion table 2.5, and assuming a maximum of 4 TRXs per antenna, define:

  - d'_i : Number of TRXs to be installed at antenna i,considering overflow traffic:

$$d'_i = \begin{cases} 1, \text{ if } o'_i \leq 132.1 \\ 2, \text{ if } 132.1 < o'_i \leq 405.5 \\ 3, \text{ if } 405.5 < o'_i \leq 709.3 \\ 4, \text{ otherwise.} \end{cases}$$

- b'_i : Blocked traffic at antenna i. This corresponds to traffic served by antenna i (i.e., antenna i provides the strongest signal) but that cannot be carried because not enough TRXs could be installed, while considering overflow traffic:

$$b'_i = \begin{cases} 0, \text{ if } o'_i \leq 1027.3 \\ o'_i - 1027.3, \text{ otherwise} \end{cases}$$

- c'_i : Carried traffic at antenna i. This corresponds to offered traffic, considering overflow traffic, served by antenna i and which could be processed by the installed TRXs.

$$c'_i = \begin{cases} o'_i, \text{ if } o'_i \leq 1027.3 \\ 1027.3, \text{ otherwise} \end{cases}$$

- cap'_i : Capacity of antenna i. This corresponds to total traffic that could be carried after adjustments have been made for the overflow traffic.

$$cap'_i = \begin{cases} 132.1, \text{ if } d'_i = 1 \\ 405.5, \text{ if } d'_i = 2 \\ 709.3, \text{ if } d'_i = 3 \\ 1027.3, \text{ if } d'_i = 4 \end{cases}$$

- e'_i : Excess capacity at antenna i. This corresponds to additional traffic that the currently installed number of TRXs could carry after considering overflow traffic:

$$e'_i = \begin{cases} 132.1 - o'_i, \text{ if } o'_i \leq 132.1 \\ 405.5 - o'_i, \text{ if } 132.1 < o'_i \leq 405.5 \\ 709.3 - o'_i, \text{ if } 405.5 < o'_i \leq 709.3 \\ 1027.3 - o'_i, \text{ if } 709.3 < o'_i \leq 1027.3 \\ 0, \text{ otherwise.} \end{cases}$$

- $O'(S)$: Total network offered traffic in configuration S, considering overflow traffic: $O'(S) = \sum_{i \in M} o'_i$.
- $C'(S)$: Total network carried traffic in configuration S, considering overflow traffic: $C'(S) = \sum_{i \in M} c'_i$.
- $B'(S)$: Total network blocked traffic in configuration S, considering overflow traffic: $B'(S) = \sum_{i \in M} b'_i$.
- $Cap'(S)$: Total network capacity in configuration S, considering overflow traffic: $Cap'(S) = \sum_{i \in M} cap'_i$.
- $E'(S)$: Total network excess capacity in configuration S, considering overflow traffic: $E'(S) = \sum_{i \in M} e'_i$.
- $Cost'(S)$: Total network excess capacity in configuration S, considering overflow traffic: $Cost'(S) = \sum_{k \in K} c^s_k w_{k+} \sum_{i \in M} c^a_i x_i + \sum_{i \in M} c^t d'_i$

For each criterion, we associated a target performance level. We can define Ω' as the set of solutions achieving those target performance levels:

$$\Omega' = \left\{ S:\ such\,that: \begin{array}{c} Cov(S) \geq |N|\,Cov_* \\ O'(S) \geq O_* \sum\limits_{n\in N} a_n \\ C'(S) \geq C_* \sum\limits_{n\in N} a_n \\ B'(S) \leq B^* C'(S) \\ E'(S) \leq E^* Cap'(S) \\ Cost'(S) \leq Cost^* \\ A(S) \leq A^* \\ U(S) \leq U^* \\ T(S) \leq T^* \end{array} \right\} \tag{26}$$

The second proposed model can be summarized as follows:

$$(APP2) \qquad \underset{S=\{(x_i,p_i),\forall i\in M\},\, \delta^1_{ij},\delta^2_{ij}}{Optimimize} \qquad F'(S) \tag{2.27}$$

Subject to

$$p_{in} \geq x_i(p_i + q_{in}) \qquad \forall i \in M \text{ and } \forall n \in N \tag{2.28}$$

$$0 \leq (p_{in} - p_{\min})\, y_{in} \qquad \forall i \in M \text{ and } \forall n \in N \tag{2.29}$$

$$p_{in} \geq p_{jn} - L(1 - x_j) - L(1 - y_{in}) \qquad \forall i, j \in M \text{ and } \forall n \in N \tag{2.30}$$

$$p_{in} \geq p_{jn} - L(1 - x_j) - Ly_{jn} - L(1 - v_{in}) \; \forall i, j \in M \text{ and } \forall n \in N \tag{2.31}$$

$$\sum_{i\in M} y_{in} \leq 1 \qquad \forall n \in N \tag{2.32}$$

$$\begin{aligned} \gamma'_n = {} & \sum_{i\in M} p_{in} y_{in} \\ & -10\log_{10} \sum_{\substack{j\in M \\ j\neq i^* \\ p_{jn}\geq p_{\min}}} 10^{\left(\frac{p_{jn}x_j(1-\delta^1_{i^*j}-\delta^2_{i^*j}) + (p_{jn}+\theta)x_j\delta^1_{i^*j}}{10}\right)} \end{aligned} \tag{2.33}$$

$$i^* = \underset{i\in M \text{ and } x_i=1}{\arg\max} \{p_{in}\} \tag{2.34}$$

$$1 \geq \delta^1_{ij} + \delta^2_{ij} \tag{2.35}$$

$$\rho^1 \geq \sum_{i,j\in M} (x_i x_j \delta^1_{ij}) \tag{2.36}$$

$$\rho^2 \geq \sum_{i,j\in M} (x_i x_j \delta^2_{ij}) \tag{2.37}$$

$$Lz^{1'}_n \geq (\gamma^1 - \gamma'_n) \qquad \forall n \in N \tag{2.38}$$

$$Lz^{2'}_n \geq (\gamma^2 - \gamma'_n) - (L+1)z^{1'}_n \qquad \forall n \in N \tag{2.39}$$

$$w_k \geq x_i \qquad \forall i \in k \text{ and } \forall k \in K \tag{2.40}$$

$$S \in \Omega' \tag{2.41}$$

$$p_i \in \Delta_i \qquad \forall i \in M \tag{2.42}$$

$$w_k, x_i, z_n^1, z_n^2 \qquad \text{Binary} \tag{2.43}$$

Equations 2.28 are used to compute the received signals. Constraints 2.30 and 2.31 identify the best server of the pixel and its second best server. The minimum signal strength constraint is described by 2.29. Inequality 2.32 ensures that each pixel has at most one best server. Constraints 2.35, 2.36 and 2.37 are used to manage the imposed antenna separation as presented above.

2.4 Proposed Heuristic Algorithm

The description of the antenna positioning problem provided in the previous section, highlights the complexity of the problem. Heuristic optimization could provide the flexibility needed to manage the variety of design criteria while handling non-linearity. We propose to use a Tabu Search-based algorithm in which the search space is adapted to the problem at hand.

Both $APP1$ and $APP2$ formulations are solved using the same procedure. A Tabu Search heuristic is used to optimize the antenna selection and the power assignment, while an embedded deterministic procedure is used to determine the antenna separations. At each iteration, the neighborhood of the current solution is generated. In solving $APP1$, the solutions in the neighborhood are evaluated and the search moves to a new solution. If solving $APP2$, the solution evaluation step also involves the application of a deterministic procedure to every solution in the neighborhood to compute antenna separations.

The frequency allocation problem is the next problem in the design process. It requires as input, the co-channel and adjacent channel interference matrices as well as the separation matrix. Thus, the full evaluation of any proposed configuration should incorporate the effect of separation constraints. By solving $APP2$, this is automatically achieved but not in $APP1$. Therefore, we apply the separation allocation procedure to the final $APP1$ solution.

2.4.1 Solution Evaluation

A network configuration S is determined by the set of active antennas, their corresponding selected transmission power, and for the cases of $APP2$, the minimum separation between pairs of antennas. So we have

$$\begin{aligned}
\text{For } APP1, \quad S &= (X, P, \delta^1 = 0, \delta^2 = 0) \\
\text{or } S &= \{(x_i, p_i, \delta_i^1 = 0, \delta_i^2 = 0) \forall i \in M\}. \\
\text{For } APP2, \quad S &= (X, P, \delta^1, \delta^2) \\
\text{or } S &= \{(x_i, p_i, \delta_i^1, \delta_i^2) \forall i \in M\}.
\end{aligned}$$

Let us denote by $Perf_c(S)$ the performance of configuration S with respect to a performance criteria c (for instance the cost associated with configuration S). We also denote by T_c, the target performance set by the operator (for instance, T_c could be the total available budget). The constraint sets Ω and Ω' contain constraints of the form $Perf_c(S) \leq T_c$, indicating that the performance of solution S with respect to design criteria c should not exceed a target level T_c. For each such constraint we associate a weight λ_c.

For each solution we have already associated an objective function $F(S)$. We also define the following function:

$$\Pi(S) = \sum_c \lambda_c \max(0, \frac{Perf_c(S) - T_c}{T_c})$$

$\Pi(S)$ describes the degree of feasibility of the solution S, with respect to the target performance indicated in Ω and Ω'. Note that for all feasible solutions, we have: $\Pi(S) = 0$.

We also define the compound objective functions:

$$G(S) = F(S) + \Pi(S)$$

$G(S)$ is used to compare solutions within the optimization process. By dynamically varying the weights λ_c during the optimization, one can direct the algorithm to seek feasibility with respect to the most violated constraints. This also can serve as a diversification mechanism.

The complete evaluation procedure of a solution S can be summarized as follows:

Procedure Evaluate (S)

```
1.   For all pixels $n$, measure the impact of $S$ (ignore the impact of separations):
     1.1.  Determine best and second servers: $i_n^1$ and $i_n^2$
     1.2.  Compute interference from all other active transmitters
     1.3.  Compute $\gamma_n$
2.   For all active antennas $j$, aggregate impact of $S$ to the level of the antenna:
     2.1.  Associate $j$ with all pixels such that $j = i_n^1$
     2.2.  Compile all variables $(o_j, d_j, b_j, c_j, cap_i, e_j)$ associated with $j$ and not related
           to $\delta_{ij}^1$ and $\delta_{ij}^2$ variables
3.   Determine separation variables $\delta_{ij}^1, \delta_{ij}^2$ using Procedure Separate $(S)$
4.   For all pixels $n$, measure the impact of $S$ (incorporate impact of separations):
     4.1.  Compute $\gamma_n'$
5.   For all active antennas $j$, compile impact of $S$ to the level of the antenna:
     5.1.  Compute offered overflow traffic
     5.2.  Compile all variables $(o_j', d_j', b_j', c_j', cap_i', e_j')$ associated with $j$ and not related
           to $\delta_{ij}^1$ and $\delta_{ij}^2$
6.   Compute $F(S)$ and $G(S)$
```

Note that while solving APP1, there is no allocation of separations, hence steps 3, 4 and 5 are skipped. The evaluation procedure requires a number of loops across pixel and antenna sets and so it is relatively time consuming.

2.4.2 Separation Allocation Procedure

The separation allocation procedure determines the values for the variables δ^1_{ij} and δ^2_{ij}, given the set of active antennas and the assigned power levels. If this procedure is called while solving $APP1$, these variables are automatically allocated a zero value (no separation). If the procedure is called while solving $APP2$ or while fully evaluating a final $APP1$ solution, then we need to determine the appropriate value of δ^1_{ij} and δ^2_{ij}.

Only a limited number of frequencies are available for use. This consideration was projected into the $APP2$ formulations by limiting the number of separations that can be imposed. These separations can be viewed as a limited resource whose impact is to be shared among all pixels in the service area. The proposed separation procedure attempts to allocate separations where they would be most profitable.

Consider a pixel n, its best server i^* and any other active antenna j, so that $p_{i^*n} > p_{jn} > p_{\min}$. We define the two variables:

- $\varepsilon^c_n(j) = \frac{1}{a_n}(p_{i^*n} - p_{jn})$
- $\varepsilon^a_n(j) = \frac{1}{a_n}(p_{i^*n} - p_{jn} - \theta)$

Both variables measure the relative impact caused by the interfering signal on the best signal, weighed by the inverse of the volume of offered traffic at the pixel. $\varepsilon^c_n(j)$ considers the case where the best and interfering signal are on the same channel, whereas $\varepsilon^a_n(j)$ handles the case of signals on adjacent frequencies.

These quantities are indicative of the need to impose a separation between antennas i^* and j : The closer the two signals are from each other, the more we need to separate them; and the heavier the traffic at a pixel, the more advantageous it is to separate them. Therefore, we would like to see that the pixels for which $\varepsilon_n^c(j)$ and $\varepsilon_n^a(j)$ are small to benefit in priority from the available number of possible separations. For each pair of active antennas i and j, we define:

$$\begin{aligned} Sep^c(i,j) &= \sum_{n \text{ such that } i = \underset{j}{argmax}(p_{jn})} \varepsilon_n^c(j) \\ Sep^a(i,j) &= \sum_{n \text{ such that } i = \underset{j}{argmax}(p_{jn})} \varepsilon_n^a(j) \end{aligned}$$

These two variables provide a surrogate measure of the need to separate each pair of antennas. We can summarize the separation allocation procedure as follows:

<u>*Procedure Separate (*S, case $APP1$*)*</u>

```
1.  For all $i, j \in M : \delta_{ij}^1 = 0, \delta_{ij}^2 = 0$
```

<u>*Procedure Separate (*S, case $APP2$*)*</u>

```
1.  For all pixels $n$ with best server $i^*$ :
    1.1.  for all antennas $j$ such that $p_{jn} > p_{min}$, Compute $\varepsilon_n^c(j)$ and $\varepsilon_n^a(j)$
2.  For all antennas $i$ and $j$ ;
    2.1.  If both $i$ and $j$ are active compute $Sep^c(i,j)$ and $Sep^a(i,j)$
```

2.2. Else, set $Sep^c(i,j)$ and $Sep^a(i,j)$ to a large number

3. Order all $Sep^c(i,j)$ in increasing order in list C
4. Order all $Sep^a(i,j)$ in increasing order in list A.
5. Starting from the antenna pair (s,r) with the smallest $Sep^c(s,r)$ value in list C, and going up, assign $\delta_{ij}^2 = 1$. Stop before condition 2.37 is violated (exceeding the maximum allowed number of this type of separations).
6. Starting from the antenna pair (s,r) with the smallest $Sep^a(s,r)$ value in list A, and going up, assign $\delta_{ij}^1 = 1$ (if $\delta_{ij}^2 = 0$). Stop before condition 2.36 is violated (exceeding the maximum allowed number of this type of separations).

2.4.3 Solution Neighborhood

The search space, $X = \{S = (X,P)\} = \{S = (\ (x_1,p_1),...,(x_{|M|},p_{|M|})\ )$ such that $x_i \in \{0,1\}$ and $p_i \in \Delta_i\}$, of APP as defined in the previous sections, consists of all possible assignments of x_i and p_i. For each such possible assignment, *Procedure Separate (S)* determines the antenna separations using a deterministic approach. Therefore, the size of the search space is given by:

$$|X| = \prod_{i \in M} |\Delta_i|.$$

For instance, if all antennas have the same set of possible power levels of size 7 (including the off level) and if the candidate set has 300 antennas, then $|X| = 7^{300}$, which is a considerably large search space.

The algorithm described here explores the search space by manipulating x_i and p_i

through the application of a sequence of operations or moves. An elementary modification to a solution would be the increase or decrease of the power level of a transmitter in the candidate list. Recall that $\Delta_i = \{p_i^0, .., p_i^{|\Delta_i|}\}$ has been defined as the discrete set of feasible power levels that an antenna i could operate on (where $p_i^0 < p_i^1 < .. < p_i^{|\Delta_i|}$, and p_i^0 corresponds to the antenna being off).

We can denote such elementary modification to a solution S by $S' = S + move(i, p_i^k)$: This indicates that configuration S' is identical to S, except for transmitter i for which the power level is changed from $p_i^l \in \Delta_i$ to $p_i^k \in \Delta_i$, where $k = l \pm 1$. Hence the size of the neighborhood of a solution S, denoted $N(S)$, is at most $2|M|$.

Because the CPU time required to evaluate a solution is very high, one cannot examine all such solutions in the neighborhood of the current solution. So one has to proceed with a reduced neighborhood. The approach used in this algorithm exploits the anticipated local effects of manipulating antenna power. Suppose we are given a configuration S. Keeping everything else unchanged, increasing the power level of an antenna i may result in:

- increasing the area and traffic covered by i.
- decreasing coverage of an adjacent antenna.
- increase TRX efficiency if the additional covered traffic does not require installing more TRXs, or increase the number of TRXs installed (and the opposite effect may occur in an adjacent antenna).
- create or increase traffic blocking, if the required number of TRXs exceeds the allowed limit.
- improve the received signal strength of the covered pixels, hence improve their

CIR level.

- increase received signal strength of pixels belonging to some adjacent antennas, hence deteriorating their CIR level (antenna i acts as an interferer for such pixels).
- open a new site, or increase the number of active antennas if i was inactive in the given configuration.

The reverse local effects may take place if the antenna power is reduced. Because these effects take place simultaneously, it is difficult to predict the overall impact on the solution, hence the need for the optimization. The proposed algorithm exploits these expected effects to direct navigation in the search space so as to favor some particular characteristic of the solution or restore feasibility of the visited solutions. For instance, suppose that for a number of iterations, all solutions visited did not achieve the required volume of carried traffic. In such a case, one may focus on neighbors of the current solution which are obtained through some power increase. The number of such neighbors is still large. One may scan the total area, select portion of it which is not covered by the current configuration and identify a small number of antennas which would be able to serve it best. The number of possible solution neighbors is then significantly reduced. One can generate similar procedures for the other system constraints or for the objective function.

Therefore, the idea is to reduce the number of solutions evaluated in the solution neighborhood by only examining solution neighbors which, we hope, would improve some aspect of the problem. This works in conjunction with the impact of the objective function (recall that problem constraints are incorporated with the objective functions by means of penalties). The combined effect of directed selection of the solution neighborhood and the penalty terms of the objective function can be expected to improve the algorithm's capacity to identify interesting solutions and, as a consequence, improve its speed.

At each iteration of the proposed algorithm, a reduced neighborhood, $N'(S)$ of the current solution S is generated. It consists of a set of solutions that could be obtained using the elementary move operator, but that promote some solution characteristic. The proposed algorithm iteratively switches between two types of search modes: *The constraint management mode* and *the system stabilization mode*. In the first mode, $N'(S)$ consists of solutions that are generated with the aim of decreasing the infeasibility of the current solution with respect to some of the violated constraints in Ω or Ω'. As for the system stabilization phase, at every iteration, $N'(S)$ consists of solutions that focus on optimizing the objective function as opposed to the constraint management phase where the focus is on restoring feasibility.

The constraint sets Ω or Ω' contain constraints for 9 design criteria. Infeasibility with respect to these constraints are managed by the following procedures. For a given input solution:

- *Procedure Improve network area coverage*

  To increase network coverage, one needs to increase the coverage area of some active antenna (by increasing power) or activating a new antenna.

6.1. Select a random portion A of the total area

6.2. For all pixels n in A which are not covered (received signal is not strong enough), for each antennas which can cover pixel n, increment a counter function.

6.3. Order antennas according to increasing value of the counter function

6.4. With a probability proportional to the counter function, select the 3 antennas i with most votes and whose status is not Tabu, assign $N'(S) := N'(S) \cup S + move(i, p_i^{k+1})$. (i.e., increase their power to the next level if they are already active, or set power at minimum level if the antenna is inactive in the current solution)

- *Procedure improve network traffic coverage*

  This procedure is similar to improving the area coverage except that the counter function is incremented by the volume of pixel traffic.

- *Procedure reduce blocked traffic*

  When traffic is blocked, indicating that offered traffic exceeds the maximum number of TRXs that could be installed, then some antenna needs to decrease its power so as to reduce the antenna coverage and increase the power of some of adjacent antennas so as to cover some of the newly uncovered traffic.

6.1. Probabilistically select some antenna i with blocked traffic (with a probability of selection being proportional to the volume of blocked traffic): $N'(S) := N'(S) \cup S + move(i, p_i^{k-1})$

6.2. Identify all antennas j that can potentially intersect with i. (Intersection occurs if a pixel receives a signal from both antennas)

6.3. With a probability proportional to the intersection area, select the 3 antennas j with largest intersection and whose status is not Tabu, assign $N'(S) := N'(S) \cup S + move(j, p_j^{k+1})$, (i.e., increase their power to the next level if they are already active, or set power at minimum level if the antenna is inactive in the current solution)

- *Procedure Reduce excess capacity*

  Excess capacity can be reduced by increasing the offered traffic at the antenna (increase its power) and by decreasing the power of some adjacent cells in order to capture more traffic.

6.1. Probabilistically select some antenna i with excess capacity (with a probability of selection being proportional to the volume of blocked traffic): $N'(S) := N'(S) \cup S + move(i, p_i^{k+1})$

6.2. Identify all antennas j that can potentially intersect with i. (intersection occurs when a pixel receives a signal from both antennas)

6.3. With a probability proportional to the intersection area, select the 3 antennas j with largest intersection and whose status is not Tabu, assign $N'(S) := N'(S) \cup S + move(j, p_j^{k-1})$. (i.e., lower power to the next level if they are already active, or set power at minimum level if the antenna is inactive in the current solution)

- *Procedure reduce excess number of sites*

  This procedure identifies a site that contains only one active antenna and adds a move to disactivate it.

6.1. From the set of all sites with only one active antenna, Probabilistically select one site with active antenna i whose status is not Tabu

6.2. Add a move to disactivate antenna i: $N'(S) := N'(S) \cup S + move(i, p_i^0)$

- *Procedure reduce excess number of antennas*

  The excess number of antennas can be reduced by disactivating some antenna.

Antennas with low volume of offered traffic are favored so that the move does not negatively impact other design criteria such as coverage or carried traffic. Probabilistically select some antenna i, with a probability of selection inversely proportional to the volume of offered traffic, and add a move to disactivate antenna i: $N'(S) := N'(S) \cup S + move(i, p_i^0)$.

- Reducing cost can be achieved by calling the procedures to reduce the number of sites or the number of active antennas, or by reducing the excess capacity.
- Increasing the volume of carried traffic is achieved by calling the procedures to reduce blocked traffic or improving traffic coverage.

The Constraint management phase combines the above described procedures to regain solution feasibility. This can be summarized as follows:

Constraint management phase (Number of iterations)

```
1.    For the specified Number of iterations
    1.1.  Probabilistically select an infeasible constraint in $\Omega$ (or $\Omega'$ if solving $APP2$)
          with a probability of selection proportional to the degree of infeasibility
          (among all constraints not satisfied by all solutions visited during the previous system stabilization phase)
    1.2.  Using the appropriate procedures described above, build $N'(S)$
    1.3.  Implement all moves in $N'(S)$
```

Notice that at each iteration of the constraints management phase, all identified moves are implemented. This is because experimental analysis indicated that in order for these moves to produce the desired effect (reduce infeasibility), the combined action of the multiple moves is often needed. In addition, only constraints that have not been satisfied by all solutions generated during the previous system stabilization phase are considered. This means that the constraint management phase only intervenes when the penalty terms in the objective function fail to restore feasibility.

During the system stabilization phase, the focus is on improving the system objective function. This phase employs three procedures to generate the moves so as to manage the interference-based objective function.

- *Procedure for interference management 1:*

  Managing interference involves manipulating the power levels of the active antennas. Therefore, this procedure identifies a list of antennas for which to increase power and another list for which to decrease power. This is achieved by scanning and aggregating pixel preferences.

1.1. Associate a weight φ_i with each candidate antenna i and initialize $\varphi_i := 0$

1.2. For all pixels n with current server i^*:

  1.2.1. if $\gamma_n < \gamma^1$, then $\varphi_{i^*} := \varphi_{i^*} + 1$ (if pixel n suffers from a low CIR, then it favors a power increase of its server)

  1.2.2. for all antennas $j \neq i^*$ (with transmission at power level k in the current solution, i.e., transmitting at p_j^k)

    1.2.2.1If $p_j^{k+1} + q_{jn} > p_{i^*}^k + q_{i^*n}$ (increasing the power level of antenna j, makes it a better server of pixel n), then $\varphi_j := \varphi_j + 1$

    1.2.2.2Else, if $\gamma_n < \gamma^1$, then $\varphi_j := \varphi_j - 1$ (antenna j is interfering on pixel with low CIR, so pixel favors decreasing the power level of antenna j)

1.3. Order all antennas in increasing order of φ_j

1.4. For 2 non- Tabu antennas with the highest positive φ_j, and with an arbitrary probability of 0.70, $N'(S) := N'(S) \cup S + move(i, p_i^{k+1})$ (i.e., increase the power so as to improve the received signal of the pixels they cover)

1.5. For 2 non- Tabu antennas with the lowest negative φ_j , and with an arbitrary probability of 0.70 $N'(S) := N'(S) \cup S + move(i, p_i^{k-1})$ (i.e., decrease the power so as to reduce the received signal of the pixels in adjacent cells)

- *Procedure for interference management 2:*

  This procedure takes the point of view of an active antenna. It measures pair-wise interference from adjacent active antennas and reduces the power of those causing most of the interference.

1.1. Randomly select an active antenna i.

1.2. Identify the set of active antennas j causing interference on i

1.3. Associate a weight $\varphi_i(j)$ with each candidate antenna j, and initialize $\varphi_i(j) := 0$.

1.4. For all identified antennas j, and for all pixels n with current server i:

  1.4.1.4.1.1measure pair-wise $CIR_n(i,j) = p_{in} - p_{jn}$

  1.4.1.2compute $\varphi_i(j) := \varphi_i(j) + a_n CIR_n(i,j)$

1.5. Order all antennas j in increasing order of $\varphi_i(j)$

1.6. For antenna i, if status is non- Tabu, $N'(S) := N'(S) \cup S + move(i, p_i^{k+1})$ (i.e., increase the power of the reference antenna)

1.7. For 3 non- Tabu antennas with lowest $\varphi_i(j)$, and with probability 0.7: $N'(S) := N'(S) \cup S + move(j, p_j^{k-1})$ (i.e., decrease the power so as to reduce the interference of adjacent antennas)

- *Procedure: interference management 3*

  Another approach to reduce the interference levels is to reduce the coverage of some existing antennas and add a new one. Suppose that a set of pixels covered by some antenna i and that suffer from a low CIR levels from an antenna j. The idea here to activate some new antenna h and reduce the power levels of antennas i and j. .

1.1. Randomly select an active antenna i:

1.2. Randomly select an active antenna j causing interference on i

1.3. Among all inactive antennas that intersect that can both i and j (i.e., those that can reach some of the pixels covered by antenna i and j), select antenna h, with probability of selection proportional to the intersection area. (intersection occurs if a pixel can receive the 3 signals)

1.4. Add move to decrease the power of i if non-Tabu: $N'(S) := N'(S) \cup S + move(i, p_i^{k-1})$

1.5. Add move to decrease the power of j if non-Tabu: $N'(S) := N'(S) \cup S + move(j, p_j^{k-1})$

1.6. Add move to increase the power of h if non-Tabu: $N'(S) := N'(S) \cup S + move(h, p_h^{k+1})$

The system stabilization phase uses the interference management procedures described above to identify a list of solution neighbors that may potentially lead to improving solutions. In addition, in order to avoid a significant departure from feasibility, this phase occasionally calls the procedures used within the constraint management phase.

In a traditional Tabu Search algorithm, one would scan all such neighbors and move to the best among them. Such an approach was found to be very slow due to the long time required to evaluate each solution. The proposed algorithm takes a different approach. It attempts to better exploit all the time spent for evaluations, by implementing all improving moves. The implementation goes as follows

```
1.  For all moves $m$ used to build $N'(S)$
    1.1.  $S' = S + m$
    1.2.  if $G(S') < G(S)$, then $S := S'$(an improving move is found, so the solution
          obtained becomes the current solution)
    1.3.  else if $G(S) < G(S') < 1.1 * G(S)$, then $S := S'$
```

The implementation procedure described here implements all moves that result in a solution better than the current one, or implements the best among all evaluated moves (as long as it does not significantly depart from the current solution). By exploiting all possible improvements, this approach helps cut on computational times. Since the above approach implements improving moves as soon as they are found, the sequence in which the moves are processed becomes relevant. For this reason, the moves are considered in random order, with the exception that moves which turn off active antennas are placed first in the list. This introduces a bias towards using less resources.

The system stabilization phase combines the interference management procedures so as to allow the system to reduce interference levels. It also incorporates moves to reduce constraint violations in order to depart too much from feasibility .

Procedure: System stabilization phase (Number of iterations)

1. For the specified *Number of iterations*
    1.1. With uniform probability select an interference management procedure
    1.2. Select an infeasible constraint in Ω (or Ω' if solving $APP2$) with a probability of selection proportional to the degree of infeasibility (among all constraints not satisfied by all solutions visited during the previous system stabilization phase) and select an appropriate procedure.
    1.3. Shuffle the list of moves generated
    1.4. Implement all improving moves among all proposed moves

2.4.4 Tabu List Management and Parameter Updating

The use of Tabu lists prevents the search from cycling between solutions by forbidding inverse moves. At each iteration, the proposed algorithm generates a list of moves according to varying mechanisms, making it unlikely to see moves affecting the same antennas appear in consecutive iterations. In addition, the probabilistic selection process built in the move generations procedures make cycling even more unlikely. The use of Tabu lists in the proposed algorithm aims at avoiding the remaining rare cases of cycling that may still occur.

For each antenna in the candidate set we associate a Tabu tag indicating the iteration number until which all modifications to the antenna are forbidden. The antenna status is declared Tabu for the next 20 iterations after a change in its power level has been made. The variations embedded in the move generation procedures allow us to use short Tabu durations. Initial experimental runs indicated that long Tabu durations tend to lead the algorithm into exploring a large number of solutions that are not interesting.

The proposed algorithm minimizes the area or traffic that is severely interfered (measured by $F(S)$) while simultaneously satisfying a set of design criteria (measured by the penalty terms in $\Pi(S)$), through the use of the compound objective function $G(S) = F(S) + \Pi(S)$. The weights λ_c associated with each constraint incorporated in $\Pi(S)$ are updated dynamically during the progress of the algorithm so as to restore solution feasibility. This works in conjunction with the variable move generation procedures described above. The updating takes place according to the following rule:

- After each system stabilization phase, for each constraint c in Ω or Ω' :
  - If all solutions visited during the previous system stabilization phase do not satisfy constraint c, then $\lambda_c := 2 * \lambda_c$
  - Else, $\lambda_c := 0.5 * \lambda_c$

2.4.5 Intensification and Diversification Procedures

The intensification procedure used in this algorithm attempts to see if any further gains can be exploited after a move that has generated a better solution than the current one. So for a current solution S and for solution S' obtained after one iteration of the system stabilization phase, if $G(S') < G(S)$, then an intensification procedure around antenna i is initiated. The procedure examines a small fraction of all antennas that may intersect with antenna i. For each such neighbor, the procedure evaluates all possible power level and implements the best move.

Diversification is achieved through two mechanisms: manipulation of the objective function and change of the current solution. Diversification is triggered by a failure to improve upon the best feasible solution, or a failure to visit feasible solutions for a fixed number of iterations. The procedure counts the number of consecutive iterations with no improvements in the best solution, and when the target threshold is met, a random modification is made to the current solution. This modification involves the random activation of a new site and random disactivation of an existing site.

If the number of consecutive iterations with infeasible solutions exceeds the predefined threshold, then diversification attempts to exploit any slack resources to improve solution feasibility. The procedure activates as many new antennas as possible as long as this improves the degree of feasibility of the current solution (without exceeding the allowed number of antennas, sites and TRXs). Here the focus is only to satisfy constraints.

In addition, in order to redirect the search towards new areas of the search space, the weights λ_c used for the penalty terms in the objective function, are re-initialized to random values after executing the diversification procedure

2.5 Experimental Results

2.5.1 Test data description

The proposed models and solution approach for APP are tested on a realistic data set for a medium size city. The data is provided by AirTel, Montreal. This study examines a network at two development points. The first point corresponds to the green field scenario and the second corresponds to a network upgrade scenario, where a major increase in network capacity is implemented. In both cases, we compare the configurations deployed by the operator to those produced by the proposed algorithm. The main characteristics of the network are:

- Total area: $33.742Km * 35.182Km$
- Grid resolution: $100m$
- Number of relevant pixels: 63345
- Number of pixel carrying traffic:
  - Green field scenario: 13743 pixels
  - Network upgrade scenario: 18586 pixels
- Total number of subscribers:
  - Green field scenario: 36479 subscribers
  - Network upgrade scenario: 124942 subscribers
- Minimum required signal strength: $-92dBm$
- Average traffic per subscriber: $0.02222Erlang$

The candidate antenna set is obtained by compiling a set of antennas that either have been activated or considered for activation by the design engineers. Thus, it is a filtered set of feasible antennas, of size 334 on 107 sites. We opted not to add more antennas in order to ensure that we only consider feasible cases (increasing the number of antennas would potentially lead to better solutions). Each antenna can be set to a power level from its associated discrete set Δ_i. This set is limited so that $|\Delta_i| = 7$ (including the off state). Even though the equipment has a wider operating set, this limitation would provide margin for manual fine tuning of the proposed solutions.

The path loss matrix for the candidate antenna set is compiled from field measurements (available for only a subset of antennas) and from a radio propagation tool provided by AirTel, Montreal.

Traffic data is provided in the form of estimated offered traffic for a subset of antennas operating at some given power level for each scenario. By uniformly distributing the antenna offered traffic among all its covered pixels, we obtain pixel traffic.

All computations are carried out on Pentium III PC, at 500MHz and with 576Mb of RAM. The algorithm is coded in visual C++. For each scenario, we evaluate the given configurations (i.e., the configuration that was actually implemented by the network operator) and compare them to the optimized solutions of $APP1$ and $APP2$.

The target performance levels in Ω and Ω' are determined so that feasible solutions must carry at least as much traffic as the given configuration, with no more resources (antennas, sites and TRXs) than the given configuration, and with a better level of blocked traffic or excess capacity. For instance, if the given configuration for the green field scenario uses 91 antennas, then the maximum allowed number of antennas for feasible solution in $APP1$ and $APP2$ is also 91. This implies that we require from the solution algorithm to produce solutions that are at least as good as the implemented ones. In addition, in order to be able to compare solutions, the maximum allowed number of separations is set equal to the number of separations used in the given solutions.

2.5.2 Green field network design scenario

In this scenario, we examine the capability of $APP1$ and $APP2$ to improve upon the network configuration used by the network operator and evaluate the performance of the proposed separation procedure.

Table 2.6 shows the evaluation results of the given configuration (i.e., the one implemented by the operator). In this table we compare the effect of the separations used by the operator to those produced by the separation procedure proposed in this research. Since area coverage, traffic coverage, number of antennas and number of sites are not impacted by separations, their values are the same under both columns. The traffic overflow that results from separations is indicated by the label "*overflow*" in each evaluation criteria. The label "*direct*" indicates that the criteria value is computed without incorporating traffic overflow. When signal strength is expressed in dBm, the corresponding CIR measure may take negative values (this occurs when the combined effect of interfering signals is stronger then the best signal). Averaging over such measures may be misleading. Thus, the average CIR and average weighed CIR (weighed by the volume of traffic) are computed based on signal strength expressed in Watts.

Under both columns only a small fraction of traffic is allowed to overflow and this does not require any additional TRXs. This is due to the relatively low traffic density. The proposed separations are able to slightly decrease excess capacity and blocked traffic while slightly improving the volume of carried traffic. The gain from separations is more obvious when one examines the average CIR levels before and after activating separations. The results also show that, while the introduction of separations decreases the percentage area (or traffic) with $CIR < 8dB$, there is an increase in percentage area (or traffic) with $8dB \leq CIR < 12dB$. On the overall, the fraction of interfered pixels (or traffic) decreases significantly with the introduction of separations, with a more pronounced improvement when using the procedure proposed in this research.

Table 2.7 shows the evaluations of the best feasible solutions obtained for $APP1$ and $APP2$. The objective function used is to minimize percentage traffic with $CIR < 12dB$. The solution under the $APP1$ column corresponds to the $APP1$ solution to which the separation procedure is applied. The solutions are obtained after running the algorithm for 3 hours in both cases. Because solving $APP1$ does not require determining separations nor computing their effect, more iterations are executed within the same time limit.

We see from Table 2.7 that the $APP2$ solution outperforms the $APP1$ solution on almost all measures: it uses fewer resources, achieves better coverage while carrying about the same volume of traffic. The $APP2$ solution has a slightly worse traffic blocking level but this is compensated by a much better utilization of the resources. By examining the

	Given Separations	Proposed Separations
% Coverage	45.26	45.26
% Traffic coverage	81.93	81.93
# Antennas	91	91
# Sites	34	34
# TRX (direct)	194	194
# TRX (overflow)	194	194
% Excess capacity (direct)	30.4	30.4
% Excess capacity (overflow)	30.2	30.1
% Blocked traffic (direct)	3.5	3.5
% Blocked traffic (overflow)	3.3	3.0
% Carried traffic (direct)	79.1	79.1
% Carried traffic (overflow)	79.2	79.4
# Separations	637	637
Average CIR	16.0	16.0
Average CIR (with separations)	478.6	552.1
Average weighted CIR	67.3	67.3
Average weighted CIR (with separations)	616.6	640.4
% Area with $CIR < 8dB$	30.9	30.9
% Area with $CIR < 8dB$ (with separations)	21.9	15.1
% Traffic with $CIR < 8dB$	52.3	52.3
% Traffic with $CIR < 8dB$ (with separations)	35.5	29.7
% Area with $8dB \leq CIR < 12dB$	4.4	4.4
% Area with $8dB \leq CIR < 12dB$ (with separations)	5.2	5.5
% Traffic with $8dB \leq CIR < 12dB$	10.7	10.7
% Traffic with $8dB \leq CIR < 12dB$ (with separations)	11.4	12.2

Table 2.6: APP - Green field scenario: Evaluation of given configuration

CIR-related criteria, we observe that $APP1$ achieves better performance before separations are activated, while the $APP2$ solution becomes better after allocating separations. This highlights the gain of incorporating the separation assignment within the optimization process.

It is clear from Tables 2.6 and 2.7, that the optimization produces better configurations that use fewer resources (about half the number of antennas and better than 25% reduction in the number of TRXs), achieve better equipment utilization, carry slightly more traffic and provide better quality of communications.

Having developed a network configuration by solving APP, the next step in the design process is to determine the frequency allocation by solving FAP. This latter problem is presented in detail in Chapter 3. We present in Tables 2.8 and 2.9 the combined effect of solving APP followed by solving FAP. We use the frequency allocation algorithm proposed in Chapter 3. The FAP algorithm is ran for a maximum of 1 hour. For both Table 2.8 and Table 2.9, the first pair of rows shows the percentage interfered traffic with the effect of separation accounted for prior to the resolution of FAP (hence, the data here is the same as it appears in the table describing the APP configurations in the previous tables). The next three rows describe the objective function of the solution produced by FAP, i.e., total, co-channel and adjacent channel interference respectively. As explained previously (section 2.3.3.2), modeling interference under FAP considers only the pair-wise effect. Under APP, however, we consider the combined effect of all interfering signals. In order to fully evaluate the FAP solution, the last two rows of the tables show the percentage

	APP1	APP2
% Coverage	50.48	61.32
% Traffic coverage	84.43	85.06
# Antennas	47	40
# Sites	30	31
# TRX (direct)	147	137
# TRX (overflow)	147	138
% Excess capacity (direct)	16.2	11.9
% Excess capacity (overflow)	15.9	12.2
% Blocked traffic (direct)	2.9	3.5
% Blocked traffic (overflow)	2.5	3.0
% Carried traffic (direct)	81.9	82.0
% Carried traffic (overflow)	82.3	82.5
# Separations	637	636
Average CIR	110.7	142.0
Average CIR (with separations)	493.6	934.7
Average weighted CIR	195.1	171.2
Average weighted CIR (with separations)	791.7	1435.2
% Area with $CIR < 8dB$	27.4	36.4
% Area with $CIR < 8dB$ (with separations)	4.0	1.2
% Traffic with $CIR < 8dB$	42	48.8
% Traffic with $CIR < 8dB$ (with separations)	9.8	3.3
% Area with $8dB \leq CIR < 12dB$	5.2	5.6
% Area with $8dB \leq CIR < 12dB$ (with separations)	3.5	1.8
% Traffic with $8dB \leq CIR < 12dB$	10.8	10.7
% Traffic with $8dB \leq CIR < 12dB$ (with separations)	7.1	4.2

Table 2.7: APP - Green field scenario: Evaluation of APP1 and APP2 solutions

	Given Separations	Proposed Separation
% Traffic with $CIR < 8dB$ (before FAP)	35.5	29.7
% Traffic with $8dB \leq CIR < 12dB$ (before FAP)	11.4	12.2
FAP: Total_interference	0.0	0.0
FAP: Co-channel Interference	0.0	0.0
FAP: Adjacent-channel Interference	0.0	0.0
% Traffic with $CIR < 8dB$ (After FAP)	0.431	0.495
% Traffic with $8dB \leq CIR < 12dB$ (After FAP)	0.001	0.000

Table 2.8: APP - Green field scenario: Evaluation of FAP for given configuration

of total traffic affected by significant interference as measured by the combination of all interferers after frequency assignment. The first column of Table 2.8 shows the results obtained for the network configuration and the separation plan used by the operator. The second column shows the result obtained for the network configuration used by the operator but with a separation plans computed by the algorithm proposed in this chapter. In Table 2.9, the first column shows the results obtained for the configuration obtained by solving $APP1$ while the second column shows the results corresponding to $APP2$. Note that we use the same frequency allocation algorithm (Chapter 3) in all four cases under the same parameter settings in order to maintain the same basis for comparison.

The results of Table 2.8 indicate a comparable performance whether we use the set of separation constraints provided by the operator or those generated using the proposed separation procedure. Although in both cases FAP achieves a zero objective function, there is a slight difference when it comes to incorporating the combined effect of all interferers. Both the $APP1$ and $APP2$ solutions shown in Table 2.9, achieve a worst objective function under FAP as well as a worst percentage interfered traffic (compared to the configuration

	APP1	APP2
% Traffic with $CIR < 8dB$ (before FAP)	9.8	3.3
% Traffic with $8dB \leq CIR < 12dB$ (before FAP)	7.1	4.2
FAP: Total_interference	0.294	0.834
FAP: Co-channel Interference $(10dB)$	0.294	0.834
FAP: Adjacent-channel Interference $(14dB)$	0.0	0.0
% Traffic with $CIR < 8dB$ (After FAP)	0.668	1.475
% Traffic with $8dB \leq CIR < 12dB$ (After FAP)	0.000	0.053

Table 2.9: APP - Green field scenario: Evaluation of FAP for APP1 and APP2

used by the operator). This outcome is expected since maintaining the same level of network coverage while reducing the number of active antennas requires increasing the area covered by the remaining antennas. This in turn increases the interactions between active antennas, and hence increases the total interference.

Given that both $APP1$ and $APP2$ models achieve a significant reduction in the number of antennas compared to a modest increase in interference, we can see that optimizing antenna positioning is beneficial. Note also that the 1% increase in significantly interfered traffic due the $APP1$ or $APP2$ solutions is offset by more than 5% increase in traffic as well as a reduction of about 50% in the size of the infrastructure deployed. Furthermore, we observe from Table 2.9 that the integrated consideration of interference modeled in $APP2$ produces on the overall better network design solutions than the step by step approach ($APP1$): despite the slight degradation in percentage interference traffic, the overall solution is better if we consider the increased coverage and the reduction in the number of antennas.

2.5.3 Network upgrade scenario

In this section, we examine a larger network. This involves providing service for about 125000 subscribers. For this scenario, the network operator did not impose any constraints on maintaining part of the existing infrastructure unchanged. Thus, we do not incorporate the potential costs associated with removing some of the existing sites (this feature could easily be added to the proposed algorithm). Table 2.10 shows the evaluations of the configuration implemented by the operator with the separations provided by the operator and with the separations generated with our proposed separation procedure. Table 2.11 shows the quality of the $APP1$ and $APP2$ solutions. These solutions are obtained after 5 hours of run time.

These results indicate that the separation procedure improves on the average CIR level while resulting in a decrease of the fraction of pixels or traffic with low CIR levels, as in the previous scenario. We also observe that the $APP2$ solution reduces the percentage of traffic suffering from high interference as compared to the $APP1$ solution and to the given configuration with fewer resources.

Under both scenarios, the proposed algorithm was able to produce solutions that outperform the solution generated by the operator, despite the limited size and variation of the candidate antenna set. Further solution improvements can be expected by augmenting this set to include more candidates by varying the antenna parameters. We observed that the constraints on blocked traffic and on carried traffic are the most sensitive. This is mainly due to the fact that we limit the number of TRXs per to cell to a maximum of four. Careful consideration is required while determining the target performance. Also, it may be worth noting that the algorithm visits a number of different solutions with similar objective function values. When we drop the constraints on the number of antennas, sites and TRXs and replace them by a budget constraint (through the use of some arbitrary cost figures), we observe that the number of these quasi-equivalent solutions decreases.

As in the case of the green field scenario, we would like to examine the effect of the

	Given Separations	Proposed Separations
% Coverage	96.55	96.55
% Traffic coverage	99.51	99.51
# Antennas	183	183
# Sites	66	66
# TRX (direct)	485	485
# TRX (overflow)	487	487
% Excess capacity (direct)	16.0	16.0
% Excess capacity (overflow)	15.2	14.7
% Blocked traffic (direct)	23.4	23.4
% Blocked traffic (overflow)	22.3	21.1
% Carried traffic (direct)	76.3	76.3
% Carried traffic (overflow)	77.3	78.5
# Separations	2061	2061
Average CIR	155.9	155.9
Average CIR (with separations)	178.4	481.2
Average weighted CIR	43.4	43.4
Average weighted CIR (with separations)	85.2	312.4
% Area with $CIR < 8dB$	66.1	66.1
% Area with $CIR < 8dB$ (with separations)	56.3	33.7
% Traffic with $CIR < 8dB$	71.2	71.2
% Traffic with $CIR < 8dB$ (with separations)	55.9	48.9
% Area with $8dB \leq CIR < 12dB$	9.2	9.2
% Area with $8dB \leq CIR < 12dB$ (with separations)	10.6	10.2
% Traffic with $8dB \leq CIR < 12dB$	11.6	11.6
% Traffic with $8dB \leq CIR < 12dB$ (with separations)	14.4	14.6

Table 2.10: APP - Network upgrade scenario: Evaluation of given configuration

	APP1	APP2
% Coverage	82.9	88.5
% Traffic coverage	98.1	98.4
# Antennas	149	135
# Sites	65	66
# TRX (direct)	454	440
# TRX (overflow)	463	455
% Excess capacity (direct)	11.5	12.2
% Excess capacity (overflow)	11.8	11.3
% Blocked traffic (direct)	20.9	23.1
% Blocked traffic (overflow)	19.3	21.2
% Carried traffic (direct)	77.6	75.7
% Carried traffic (overflow)	79.1	77.5
# Separations	2061	2061
Average CIR	203.4	203.0
Average CIR (with separations)	624.5	453.5
Average weighted CIR	211.7	161.6
Average weighted CIR (with separations)	343.4	364.4
% Area with $CIR < 8dB$	53.8	55.7
% Area with $CIR < 8dB$ (with separations)	22.6	18.2
% Traffic with $CIR < 8dB$	70.1	70.0
% Traffic with $CIR < 8dB$ (with separations)	43.7	33.2
% Area with $8dB \leq CIR < 12dB$	7.3	8.4
% Area with $8dB \leq CIR < 12dB$ (with separations)	8.7	8.7
% Traffic with $8dB \leq CIR < 12dB$	10.2	10.7
% Traffic with $8dB \leq CIR < 12dB$ (with separations)	12.2	13.7

Table 2.11: APP - Network upgrade scenario: Evaluation of APP1 and APP2 solutions

	Given Separations	Proposed Separation
% Traffic with $CIR < 8dB$ (before FAP)	55.9	48.9
% Traffic with $8dB \leq CIR < 12dB$ (before FAP)	14.4	14.6
FAP: Total_interference	1.437	1.232
FAP: Co-channel Interference	1.437	1.232
FAP: Adjacent-channel Interference	0.0	0.0
% Traffic with $CIR < 8dB$ (After FAP)	4.172	3.699
% Traffic with $8dB \leq CIR < 12dB$ (After FAP)	0.101	0.253

Table 2.12: APP - Network upgrade scenario: Evaluation of FAP for given configuration

APP solution in the network upgrade scenario as we proceed to the resolution of FAP. The results are presented in Tables 2.12 and 2.13 (refer to the previous section, corresponding to the green field scenario, for a description of the table fields).

As for the green field scenario, we observe that the optimized APP configurations have a higher level of interfered traffic while using less resources and carrying more traffic than the configurations used by the operator. The deterioration in interference levels is due to a reduction in the number of active antennas which needs to be compensated by increasing the area covered by each antennas. Further improvement to the optimized configuration could be expected by using a larger candidate antenna set.

2.6 Conclusion

In this chapter we propose integrated flexible models for the antenna positioning problem. The results indicate that incorporating explicit consideration of the frequency allocation problem leads to improved solutions. Antenna separation is often used to protect the hand-off procedure. By extending its use to managing interference in general and by including

	APP1	APP2
% Traffic with $CIR < 8dB$ (before FAP)	43.7	33.2
% Traffic with $8dB \leq CIR < 12dB$ (before FAP)	12.2	13.7
FAP: Total_interference	5.375	4.930
FAP: Co-channel Interference	5.375	4.930
FAP: Adjacent-channel Interference	0.000	0.000
% Traffic with $CIR < 8dB$ (After FAP)	7.771	5.814
% Traffic with $8dB \leq CIR < 12dB$ (After FAP)	0.375	0.226

Table 2.13: APP - Network upgrade scenario: Evaluation of FAP for APP1 and APP2

them within APP, one can obtain solutions that use fewer resources while achieving the same design objectives and better quality of communication. Applied to two problem instances, the proposed heuristic algorithm combined with the separation generation procedure was able to improve upon the solutions generated by the operator, highlighting the appropriateness of optimization to managing this complex problem for realistic networks.

The solution approach exploits local expected effects of elementary modifications to guide the search. This could be further enhanced by exploiting the relationship between antennas in the candidate list. In a preprocessing phase, it may be possible to optimize an area around each antenna (an area much smaller than the overall surface but larger than the antenna cell). This would form a list of partial solutions. The solution algorithm may attempt to incorporate these elements directly so as to manage the overall network.

Additional improvements can be made through a more precise cost function (e.g., a cost function that differentiates between the cost of new equipment and the cost of relocating an existing equipment) or through the introduction of a more accurate characterization of the handoff process.

Power management is a feature that is sometimes activated by network operators to further enhance network performance. This features allows antennas to dynamically adjust transmission power in order to manage interference as function of variations in traffic distribution. Similar to the separation procedure, it is possible to incorporate a power management optimization subroutine within the APP solution approach. This could be useful also to extend the proposed models to UMTS (Universal Mobile Telecommunications System), which uses CDMA as the mechanism to serve the large number of customers and so does not involve frequency allocation. The design of such systems is very dependent on the selection of the appropriate antenna configurations. The proposed models could be modified to handle such systems by changing the interference calculation formulae to account for the use of power management.

The current approach to model traffic distribution is to consider a fixed scenario (the most likely or the worst case) and then optimize APP on the basis of that scenario. The network configuration that is generated may perform very well for the selected distribution but may be sensitive to variations in traffic distribution. Consider for instance a network with significant variation in traffic between the day and evening periods. One can also consider a more complicated situation in which there are exceptional periods which may correspond to special events such as sports competitions or festivals. The distributions of traffic are combined to generate a conservative worst-case scenario. Another approach may be to formulate an objective function that simultaneously considers all traffic scenarios: the performance of any given solution is first evaluated for each scenario and the overall quality of the solution corresponds to some weighted combination of the individual scenario performance. The resulting optimization would be more robust with respect to variation in traffic distribution. For instance, if a park in the city is occasionally used to hold festivals. The traffic density outside the festival period would be low. The suggested approach may result in a configuration that involves activating sites at the boundary of the park and using directional antennas facing away from the park to cover dense traffic areas (that exist all year round). Since the sites have already been activated, it is relatively easy and inexpensive to add new antennas facing the park during the festival.

Chapter 3
The Frequency Allocation Problem

The frequency allocation problem (FAP) appears in FDMA-based networks such as GSM and AMPS networks. FAP consists of assigning a limited number of frequencies to each of the cells in the network so as to optimize some design objective (such as minimizing interference) while meeting the designer's constraints (e.g. maximum interference thresholds, frequency separations constraints, etc.). Efficient exploitation of the scarce radio spectrum resources depends heavily on good frequency allocation solutions. In addition, earlier imperfections in the design process (due to its decomposition and to the approximations that are made at each step), need to be handled within FAP: A good frequency assignment is all the more necessary since the decisions made upstream have already introduced sub-optimality. Several models have been proposed for FAP problems. Over time, these models have increased in complexity and sophistication, highlighting the ever increasing need for solutions that are better adapted to the operators' requirements.

The next section provides a review of the main models and the most commonly used solution methods. In section 2, an exact formulation for the minimum interference FAP is proposed and in section 3 a Tabu Search-based heuristic is described. Section 4 presents some experimental results.

3.1 Literature Review

A cellular network is composed of a set of cells, each of which contains a given number of transceivers (TRXs). A radio frequency needs to be assigned to each TRX so as to carry the communications. A frequency allocation solution is then a determination of what frequencies to use in each cell.

There are 3 general categories of proposed frequency allocation schemes [48]:

- Fixed channel allocation: under this scheme each TRX is assigned a frequency that does not change over time.
- Channel borrowing: each cell is assigned a pool of frequencies but a frequency can be borrowed from a selected set of neighboring cell (provided it does not cause extra interference). In order to allow this borrowing process, some form of on-line communication needs to take place between cells so as to manage interference. In addition, while the frequency is borrowed it cannot be used by the owner cell.
- Dynamic assignment: all TRXs can use any frequency from the available set. Communications need to take place between different cells so as to manage interference in an on-line fashion.

The efficiency of each of these schemes depends on a number of factors, which includes the system load. In this section, we examine the fixed frequency allocation problem since most commercially deployed systems such as GSM and AMPS networks require fixed frequency allocations.

All FAP models need to address a set of issues:

- Demand considerations:

  Often customer demand is considered as a hard constraint. The number of TRXs in each cell is given and any feasible solution needs to assign one frequency for each TRX. In some cases, however, demand has been modeled as a soft constraint, i.e. constraint violation is penalized in the objective function.

- Interference considerations

  Interference occurs as a result of frequency reuse, which must take place because of the scarcity of the radio spectrum. The magnitude of the potential interference between any pair of TRX depends on the wave propagation characteristics between this pair. This potential interference will be actualized if the pair of TRXs are assigned frequencies that are "too close" to one another.

  Interference can be included in the model in the form of constraints where pairs of TRXs may be forbidden from using frequencies that are too close to one another. These are known as the separation requirements. In some cases these constraints are considered as hard constraints, to be satisfied by any feasible solution. In some other cases, a penalty term is associated with each separation requirement, and the FAP problem becomes a constraint satisfaction problem. The penalty value may reflect the interference level that results from violating this separation requirement.

  It is also possible to handle interference considerations explicitly through the objective function. These models reflect the emphasis placed on quality of service and are especially important for operators acting within competitive markets. Such models which concentrate on interference considerations are known as *Minimum Interference* FAP. In this work, we focus on this type of model.

- Radio spectrum

  Radio spectrum is a very scarce resource. Its efficient utilization is essential in order to justify the significant monetary investments made by the operators in acquiring the license. *Minimum Span* FAP and *Minimum Order* FAP models often reflect this interest. Under minimum span FAP, the objective is to minimize the range of the frequencies used, while under minimum order FAP the objective is to minimize the number of frequencies used. The underlying motivation behind these types of models is to use as few frequencies as possible as dictated by government regulations and economic considerations.

Other radio spectrum management issues may arise for some operators. For instance, a subset of frequencies may be unavailable for some cells. Such situation may occur for example at country boundaries. We will refer to these as *locally blocked* frequencies. Also, since implementing a new frequency assignment may be a costly action, the operator may impose *locking* a prior frequency assignment for part of the network. In addition, some operators may choose to protect BCCH-carrying TRXs (which are critical for establishing calls) by reserving a subset of frequencies for BCCH use only.

Most proposed FAP models consider only the down link communications (base station to mobile station). The frequency assignment on the up link is done symmetrically on the frequency band allocated for the up link communications. However due to the differences in radio wave propagation in both direction, a good allocation on the down link may not always correspond to a good allocation for the up link. Considering both directions simultaneously increases significantly the problem size and complexity. in [71] a modification of the interference matrices is proposed. This approach combines both down link and up link considerations.

A number of variations of FAP models exist, each considering some of the issues presented above. In this work, we focus on FAP models related to civil wireless applications as opposed to military or satellite based networks. In [51] and [2], comprehensive reviews of FAP models are presented.

In its general form, FAP is modeled as a *Generalized Graph Coloring problem.* Assigning frequencies to cells is equivalent to the problem of assigning colors to a graph whose vertices represent the cells and edges represent the separation requirements. Simple forms of FAP include:

- *Vertex coloring:* a simple variation of FAP. Only co-channel separation is required and the objective is often set to obtaining the minimum span or minimum order.
- *K-coloring:* the number of frequencies to use is known and the requirement is to find a feasible assignment under co-channel separation.
- *T-coloring:* for some pairs of "close" cells, the distances between a pair a frequencies assigned to them must be larger than an integer value in the set T.
- *List coloring:* some frequencies may be locally blocked and each node in the graph must be assigned frequencies from its own pool of available frequencies under co-channel separation.
- *Set coloring:* every vertex in the graph corresponds to a cell which contains multiple TRXs and thus a vertex must be assigned a set of frequencies under co-channel separation.

Frequency allocation problems of interest to network operators are generalizations of the above *NP-Complete* problems, and thus are also *NP-Complete* [25]. A number of exact and approximate solution methods have been proposed in the literature. Exact approaches suffer from long running times, inability to tackle large size instances (as are most realistic problems), and difficulty to incorporate special constraints. However, exact approaches have been useful both in evaluating heuristic approaches (for small size problems) and in providing bounds on solutions for larger problems. The reader can refer to [66] and [25] where several IP formulations and graph theoretical results are reviewed.

Lower bounds for different FAP models have been proposed in the literature. They are useful to evaluate algorithm performance. Some proposed procedures make use of lower bounds to improve computational requirements. Based on a graph theoretical approach, Gamst [33] proposed 3 levels of lower bounds for the minimum span FAP (higher level bounds provide tighter bounds but at the expense of computational requirements). An extension of this work is later proposed by Tcha et al. [84]. The proposed procedure should provide a tighter bound with a reduction in computational complexity. Smith and Hurley [82] propose another set of bounds for the minimum span problem. Comparison of the different bounds is still to be done. For the minimum interference FAP, few lower bounds exist; they are based on linear programming relaxations [25], which do not seem to be strong enough.

Fischetti et al. [28] used a branch-and-cut approach to solve an FAP model where the objective is to minimize unsatisfied demand while satisfying a set of separation constraints. In the proposed approach, at every node of the branch-and-cut algorithm, a linear programming relaxation of the problem is used with the addition of cover and clique inequalities. The proposed approach was able to solve problems, with just over 200 cells and an average cell demand between 2 and 3 frequencies, in less than 2 hours of computational time. Maniezzo and Montemanni [57] developed a branch-and-bound algorithm to minimize the weighted cost of violating separation constraints and the cost of changing pre-assigned frequencies. Lower bounds based on the quadratic assignment problem are calculated at every node of the search tree, and dominance rules are used. Jaumard et al. [47] compare a number of column generation-based approaches based on set covering formulations.

The most common approach for solving FAP is through heuristics algorithms. We can categorize such an approach into solution construction heuristics, solution improvement heuristics and metaheuristics.

Most solution construction heuristics are based on assigning frequencies to cells according to some heuristic order. Construction heuristics have been further decomposed into *frequency exhaustive strategies* and *requirement exhaustive strategies* [85][3]. In the former case, the assignment of cells is done in such a way that a frequency is saturated before moving to the next one (i.e., no other cells can use that frequency without violating some constraint). In the latter case, the requirements of a cell are satisfied before moving to the next cell in the list (recall that a cell often requires multiple frequencies, one for each TRX).

Some proposed heuristics incorporate graph theory related concepts. In [50], two heuristics are proposed for the minimum order FAP with co-channel interference considerations only. The first heuristic sequentially identifies stable sets in the graph representing the network (nodes correspond to cells and edges correspond to separation constraints). At each step, after the identification of the stable set, the next available frequency is assigned to all the cells in that set. The procedures continues until all nodes (cells) are allocated the required number of frequencies. In the second proposed heuristic, a number of stable sets are identified in the graph, the problem is formulated as set covering problem of on the stable sets. Hence, an optimal solution corresponds to the smallest collection of stable sets so that all cells in the network are allocated while satisfying separation constraints. Smith et al. [80] present a heuristic, which is based on identifying a core part of the network and assigning frequencies to this core first (in this case the core is a clique). This assignment is then fixed and the remaining part of the network is assigned frequencies. If the span of the complete network and the partial network are significantly different, then the core is augmented with another node and the process is repeated (the partial assignment of frequencies can be done using any heuristic). This type of method can be expected to be useful especially for large networks since it provides an interesting problem decomposition approach. Borndörfer et al. [9] present a number of fast solution construction heuristics and solution improvement heuristics for the minimum interference problem. These heuristics try to exploit local information in each cell (such as the degree of the cell in the corresponding graph) and current characteristics of the partial solution formed so far to determine the next assignment or change in the solution.

In [85], a heuristic approach is proposed to solve FAP where the objective is to minimize changes in an existing partial assignment and to minimize the number of frequencies used. The proposed procedure iteratively identifies the most difficult nodes (cells) for the next assignment. This procedure has the added advantage of re-considering past assignments and is able to reduce some of the myopic behavior of iterative construction approaches. Among the solution construction procedures that cannot be classified within the assignment strategies described above is the use of neural networks. Kim et al. [49] propose the use of Hopfield-based neural network to solve the minimum interference FAP. The problem is represented as a network where each node corresponds to the assignment of a frequency to a cell. Separation and demand considerations are handled in the energy (objective) function to be minimized.

Metaheuristics have received a great deal of attention as solution approaches for solving FAP. ANTS based metaheuristics have been applied to FAP in [56][3]. Maniezzo and Carbonaro [56] use lower bounds to identify unpromising moves of the agents in the algorithm and reduce computational times. The lower bounds are based on the linear programming relaxation of a modification of the formulation proposed in [10]. Relaxation of the classical IP formulation is reported to produce bounds that are not tight enough. Results indicate a better performance than Simulated Annealing and Tabu Search, although the evaluation is difficult due to differences in running time. Abril et al. [3] also used an ANTS-based heuristic on FAP. For this application, lower bounds proposed by Gamst [33] on graph coloring are used. The proposed algorithm is reported to outperform Simulated Annealing in terms of solution quality at the expense of longer running times.

In [83], a generalized quadratic assignment formulation is used to model a minimum interference FAP, where the separation constraints are incorporated in the objective function and the demand requirements are considered as hard constraints. A genetic algorithm is used to solve this problem where a solution is represented by an assignment matrix and the crossover operator preserves the hard constraints. Often times, the difficultly with genetic algorithms is to find a combination of solution representation and crossover operators that are fast and that efficiently handle problem constraints. Beckman and Killat [7] propose the use of a Combined Genetic Algorithm (CGA) to solve minimum order FAP. The genetic algorithm generates an ordering of the cells. A frequency assignment strategy is applied to each of the ordered lists produced by the genetic algorithm. The proposed approach is flexible in terms of handling a variety of constraints while still making use of the solution diversification capabilities of genetic algorithms.

Tabu Search has been used to solve several FAP models. In [39] and [12], Tabu Search is used to minimize the number of separation constraint violations. In [17], a minimum interference FAP model is solved with Tabu Search. Here, a more accurate measure of interference is considered: Interference is calculated in terms of the combined effect of all interfering cells as shown by equation 1.1. On the other hand, most models in the literature consider pair-wise interference, i.e., the potential interference effect of cell j on cell i and of cell k on cell i, separately. In [18], Castelino and Stephens apply a Tabu Thresholding algorithm to minimize the number of violated separation constraints. The algorithm was able to achieve better results than a Tabu Search algorithm [20]. Later [19], the authors propose a better method to identify promising moves through considering the combined effect of separation constraints. Bourjolly et al. [13] propose a Tabu Search algorithm which incorporates an adaptive memory module for the minimum interference FAP. The proposed algorithm was shown to be competitive with other commercially available software.

Simulated Annealing has also been used for FAP. Duque-Antón et al. [23] applied Simulated Annealing to minimum interference FAP models. Demand, interference, and radio spectrum constraints are handled as soft constraints that are incorporated within the objective function. In [72], the basic Simulated Annealing is applied with a simple swap operator (a frequency assignment is swapped for another) as the neighborhood generation mechanism. This is the neighborhood operator most often used in Tabu Search- and Simulated Annealing-based algorithms. Since such neighborhoods are usually found to be very large or to include a number of non-interesting solutions, neighborhood reduction procedures were found to be beneficial.

The application of metaheuristics to FAP has often revealed that the search space is large and hence, neighborhood reduction mechanisms are needed in order to accelerate the search. In addition, the experimental results have shown that the search space is characterized by a large number of local optima, highlighting the need to implement diversification procedures. Also it is interesting to point out that there is no general agreement on how to handle problem constraints: while in some applications, constraints are included in a soft manner within the objective function, in others, they are treated as hard constraints. Metaheuristics applied to problem instances with a dense set of constraints may achieve better results when constraints are considered in a soft manner since this would allow them to faster escape from local optima and achieve a better exploration of the search space. The difficulty, in this case, is to define the appropriate objective function that balances various goals. On the other hand, a looser constraint set may be more appropriately handled with hard constraints, since this would help reduce the search space size. This difficulty of handling problem constraints is further compounded by the fact that problem instances do vary considerably in specifications, making the design of effective algorithms across the range of problem characteristics a difficult task.

An evaluation of the different proposed algorithms is difficult due to the fact that experimental results are often reported on different data sets. There is a need for a set of benchmarking problem instances that cover the ranges of problem characteristics faced by network operators. A set of links to test problems as well as an extensive bibliography can be found at the FAP web site at *http://fap.zib.de*. Also a France Telecom benchmark (*http://www.cs.cf.ac.uk/User/Steve.Hurley/freq.htm*) has been used to compare a number of algorithms and commercial tools. Good performance of the algorithm appears to depend heavily on exploiting the problem structure or on utilizing a "smart" exploration of the search space.

3.2 The Minimum Interference FAP

3.2.1 Exact Formulations

In this section, two linear integer programming models (*MIFAP1* and *MIFAP2*) for the frequency allocation problem are proposed. The problem consists of assigning a radio frequency for each TRX in the network so as to minimize the overall co-channel and adjacent-channel interference, while satisfying the following set of constraints:

- The available set of frequencies to be used is predetermined.
- Each TRX must be allocated a frequency
- BCCH-carrying TRXs and TCH-carrying TRXs can be limited to using frequencies from pre-specified subsets of frequencies. These subsets may be disjoint.
- Each TRX may be prohibited from using a specific subset of frequencies
- A given minimum separation between the frequencies allocated to each pair of TRXs needs to be satisfied at all times.

The level of study of the first model (*MIFAP1*) is the TRX as opposed to the cell which is the level of study of the second model (*MIFAP2*). *MIFAP1* is the analogous to the model presented in [25].

We need to define the following:

- T: Set of TRXs in the network, and $N = |T|$.
- F_i : Set of feasible frequencies for TRX i. A frequency is feasible if it is not blocked and if it is within the appropriate frequency set (i.e., if a TRX carries the BCCH, then the frequency belongs to the subset reserved for this type. Similarly for TCH-carrying TRXs).
- F : Set of all frequencies. $N_F = |F|$.
- c^1_{ij} : Co-channel interference between TRXs i and j, if they use the same frequency.
- c^2_{ij} : Adjacent-channel interference between TRXs i and j, if they use adjacent frequencies.
- s_{ij} : Minimum required separation between frequencies allocated to TRXs i and j.

The decision to be made in this problem is to select which frequency to use at each TRX. Thus, we define:

$$x_{if} = \begin{cases} 1, \text{ if frequency } f \text{ is allocated to TRX } i, & i = 1,..,N \text{ and } f = 1,..,N_F. \\ 0, \text{ otherwise.} \end{cases}$$

Note that if a TRX i cannot receive frequency f (because such a frequency is forbidden or is not of the appropriate type), then the variable is not defined. Also define:

$$w_{ij} = \begin{cases} 1, \text{ if TRXs } i \text{ and } j \text{ use the same frequency} \\ 0, \text{otherwise} \end{cases} \quad \forall i \neq j \in T.$$

and

$$v_{ij} = \begin{cases} 1, & \text{if TRXs } i \text{ and } j \text{ use adjacent frequencies} \\ 0, & \text{otherwise} \end{cases} \quad \forall i \neq j \in T.$$

The proposed model for the minimum interference frequency allocation problem (*MIFAP1*) at the TRX level is as follows:

$$(MIFAP1) \quad \underset{x_{if}}{Minimize} \sum_{i,j \in T} \left[(c_{ij}^1 + c_{ji}^1) w_{ij} + (c_{ij}^2 + c_{ji}^2) v_{ij} \right] \tag{3.1}$$

Subject to

$$\sum_{f \in F_i} x_{if} = 1 \quad \forall i \in T \tag{3.2}$$

$$x_{if} + x_{jg} \leq 1 \quad \begin{array}{l} \forall\, i < j \in T,\ s_{ij} > 0 \ \text{ and} \\ \forall\, f \in F_i, g \in F_j \text{ such that } |f - g| < s_{ij} \end{array} \tag{3.3}$$

$$x_{if} + x_{jf} - w_{ij} \leq 1 \quad \forall\, i < j \in T \text{ and } f \in F_i \cap F_j \tag{3.4}$$

$$x_{if} + x_{jg} - v_{ij} \leq 1 \quad \begin{array}{l} \forall\, i < j \in T \text{ and} \\ \forall\, f \in F_i, g \in F_j \text{ such that } |f - g| = 1 \end{array} \tag{3.5}$$

$$\begin{array}{ll} x_{if} \text{ binary} & \forall\, i \in T \text{ and } f \in F_i \\ w_{ij}, v_{ij} \text{ binary} & \forall\, i < j \in T \end{array} \tag{3.6}$$

Equations 3.2 ensure that every TRX in the network receives exactly one frequency from its feasible domain. Inequalities 3.3 enforce the separation constraints between TRXs. Inequalities 3.4 and 3.5 are used to identify when two TRXs use the same frequency or adjacent frequencies, respectively. The objective function 3.1 minimizes the overall co-channel and adjacent-channel interference. This model incorporates interference considerations both in the constraint set as separation requirements, and in the objective function. This allows the user to make sure that very high interference levels do not exist through the hard separation constraints, while managing the remaining interference in a flexible way.

By considering the special case of co-channel separation constraints ($s_{ij} \leq 1, \forall i, j$), constant co-channel and adjacent-channel interference and by having the same frequency set for all TRXs, the above model can be relaxed to the K-coloring problem, which is *NP-Complete* [35]. The model requires less than $N(N-1) + NN_F$ variables. Let ρ be the density of the separation matrix S. The required number of constraints is then bounded from above by $N + N(N-1)N_F\left[1 + \rho N_F\right]$. For instance, for a 20 TRXs, 5 frequencies and a separation matrix with density 0.2, the required number of variables is 480 and the required number of constraints is 3820.

We now present *MIFAP2* which is formulated with respect to cells, where each cell must be allocated a given number of frequencies. The idea here is to identify, for every cell, all possible subsets of frequencies that can be feasibly assigned to that cell. Each such subset needs to satisfy the minimum separation requirement between frequencies assigned to TRXs in the same cell. It also needs to satisfy any restrictions to BCCH- and TCH-carrying TRXs. The problem then becomes to choose a combination of subsets of frequencies to allocate to cells. This combination needs to satisfy minimum separation requirements for frequencies allocated to different cells. We define the following:

- C: set of cells in the network
- D_i : number of TRXs in cell i.
- s_{ij} : Minimum required separation between frequencies allocated to cells i and j
- F_i^k : a subset labeled k of frequencies that can be feasibly allocated to cell i. (i.e., $|F_i^k| = D_i$ and if $f, g \in F_i^k$, then $|f - g| > s_{ii}$). The number of all such subsets for cell i is K_i.
- $\delta_{ij}^{kl} = \begin{cases} 1, & \text{if allocating subset } F_i^k \text{ to cell } i \text{ and allocating subset } F_i^l \text{ to cell } j \text{ is feasible} \\ 0, & \text{otherwise} \end{cases}$

  Allocating subset F_i^k to cell i and subset F_j^l to cell j is feasible if for all feasible frequencies $f \in F_i^k$ and $g \in F_j^l$, we have $|f - g| \geq s_{ij}$. The δ_{ij}^{kl} values are computed beforehand.

- I_{ij}^{kl} = interference level that results from allocating subset F_i^k to cell i and allocating subset F_j^l to cell j .

  To compute I_{ij}^{kl}, one identifies the common and adjacent frequencies in F_i^k and F_j^l. The appropriate values of co-channel and adjacent channel interference are added.

The problem decision variables for *MIFAP2* are:

- $y_i^k = \begin{cases} 1, & \text{if subset } F_i^k \text{ is selected for cell } i. \\ 0, & \text{otherwise} \end{cases}$

  This variable identifies the frequency assignment for each cell

- $w_{ij}^{kl} = \begin{cases} 1, & \text{if subset } F_i^k \text{ is selected for cell } i \text{ and subset } F_j^l \text{ is selected for cell } j \\ 0, & \text{otherwise} \end{cases}$

  This variable is used to identify which of the subsets F_i^k and F_j^l are assigned to cells i

and j.

The proposed model for the minimum interference frequency allocation problem (*MIFAP2*) at the cell level is the following:

$$(MIFAP2) \quad \underset{y_i^k}{Minimize} \sum_{\substack{i,j \\ k,l}} [I_{ij}^{kl} w_{ij}^{kl}] \tag{3.7}$$

Subject to

$$\sum_{k=1}^{K_i} y_i^k = 1 \quad \forall\, i \in C \tag{3.8}$$

$$y_i^k + y_j^l \leq 1 \quad \begin{array}{c} \forall\, i,j \in C, \\ \forall k = 1,..,K_i, \\ \forall l = 1,..,K_j, \\ \text{such that } \delta_{ij}^{kl} = 1 \end{array} \tag{3.9}$$

$$y_i^k + y_j^l - w_{ij}^{kl} \leq 1 \quad \begin{array}{c} \forall\, i,j \in C, \\ \forall k = 1,..,K_i, \\ \forall l = 1,..,K_j, \end{array} \tag{3.10}$$

$$y_i^k, w_{ij}^{kl} \text{ binary} \quad \begin{array}{c} \forall\, i,j \in C, \\ \forall k = 1,..,K_i, \\ \forall l = 1,..,K_j, \end{array} \tag{3.11}$$

In the above *MIFAP2* formulation, equations 3.8 ensures that every cell is assigned exactly one subset of frequencies. Inequalities 3.9 enforce the separation constraints between cells (the within-cell separation requirements are already taken care of during the generation of the possible subsets F_i^k for each cell i). Inequalities 3.10 are used to identify the interference level that results from the assignment. The *MIFAP2* formulation is similar to the *FAP2* model presented in [47]. *FAP2* formulates a minimum order FAP with co-channel and adjacent channel separation constraints. *MIFAP2* can be used to model a general set of separation constraints. *MIFAP2* is a column generation formulation with an exponential number of variables.

3.2.2 On the PN Code Allocation Problem in CDMA

In CDMA systems all transmissions are on the same frequency. The overlap of multiple signals may result in the loss of the transmitted data. To avoid such collision, CDMA-based networks use spread spectrum communication techniques. The latter assign orthogonal codes to the different communications so as to distinguish between them. There are two types of CDMA systems: Frequency-hopped CDMA and Direct-sequence CDMA systems. In the first type of system, the transmitter changes its radio frequency carrier at regular intervals according to a hopping pattern. In Direct-sequence CDMA systems, a phase shift is applied to the signal on the radio frequency according to a signature sequence. With a good choice of the frequency hopping pattern or of the signature sequence, signals from different transmitters interfere very little with each other. Such hopping patterns or signature sequences are often referred to as pseudonoise (PN) sequences or codes. Some different PN code construction methods are described in [78].

Such PN codes share the fixed channel capacity and thus their number must not exceed a given bound. Therefore, the assignment of PN codes to the different communications of the CDMA-based mobile network is similar to the frequency allocation problem that arises in the FDMA-based networks. However, from a resource allocation point of view, there is one major difference between the two problems. As we saw in the description of FAP, the assumption is that the structure of the network is determined, i.e. we already know the coverage of each cell and hence it is possible to calculate the interference level that results from using interfering frequencies. In CDMA-based networks, the cell shapes change dynamically with the number of active users due to the power control feature (transmitters adjust their transmission power so as to overcome interference). One can overcome this problem by considering the shape of the cell at an average number of customers.

To model the PN code allocation problem one needs to determine the cost of sharing the available codes (In FAP, this corresponds to calculating co-channel and adjacent channel interference levels that result from using the same or adjacent frequencies). The cost of sharing the codes depends on the type of coding used. For example, the hamming correlation function [78] counts the number of collisions between two hopping sequences. Such functions could be used as measures for interference.

Hu [43] consider a packet-radio network which consists of a set of single transceiver nodes. Each node is connected to all nodes to which it is allowed to forward data packets and adjusts its transmission power according to the distance. The author discusses a number of code assignment schemes. The assignment methods presented are based on graph coloring heuristics that attempt to minimize the number of codes required. Battiti et al. [5] also use a graph coloring approach to propose a set of heuristic and bounding algorithms for a similar problem. The models considered in these papers can be compared to minimum order FAP: linked nodes cannot use the same frequency (co-channel separation requirement) and the objective is to minimize the number of codes required (which is equivalent to minimizing the number of frequencies required).

3.3 Proposed Solution Approach

The frequency assignment problem is relatively simple to state. However, the mathematical complexity of the problem, further compounded by variations in the characteristics of problem instances, make FAP a difficult problem to solve. In this work, we propose an adaptive Tabu Search algorithm for the resolution of FAP. The input provided by the user consists of the set of TRXs, the frequency band (and the related restrictions), the co-channel and adjacent-channel interference matrices as well as the separation matrix. The user also specifies the termination criteria stated as the maximum number of iterations or the total allowed run time.

The proposed solution approach uses a compound operator that first changes the frequency of some TRX, and then attempts to identify other improving moves that may results from the initial frequency change. The search is slowly intensified by increasing the size of the solution neighborhood. To further improve the convergence of the algorithm the search iterates between a normal tabu search mode and a local descent search mode. The next paragraphs provide a detailed description of the algorithm.

3.3.1 Solution Evaluation

A solution S to the FAP formulation is an assignment of frequencies to TRXs. Thus, a solution

S is defined as:

$$S = \{x_{if}, \forall i \in T \text{ and } \forall f \in F_i\}$$

where x_{if} indicates if frequency f is activated in TRX i or not.

The objective function of FAP aims at minimizing the total interference. The proposed heuristic also incorporates problem constraints within the objective function, since we allow infeasible solutions (solutions violating demand or separation constraints) to be visited during the search. We first define the following parameters:

- α : Penalty coefficient associated with violating a separation constraint.
- β : Penalty coefficient associated with violating a demand constraint, i.e., coefficient associated with having a TRX with no frequency assignment.

We define the function $\Pi(S)$ to describe the degree of feasibility of a solution S with respect to demand constraints, such that:

$$\Pi(S) = \sum_{i \in T} \left(1 - \sum_{f \in F_i} x_{if}\right)$$

Note that for all solutions that satisfy demand constraints, we have $\Pi(S) = 0$. In addition, we define the function $\Phi(S)$ to count the number of separation constraints that are violated by solution

S, where:

$$\Phi(S) = \sum_{i,j \in T} \sum_{\substack{f \in F_i, g \in F_j \\ |f-g| < s_{ij}}} x_{if} x_{jg}$$

Here also, for all feasible solutions we have $\Phi(S) = 0$.

Therefore, the full objective function $\Theta(S)$ can now be stated as:

$$\Theta(S) = \sum_{i,j \in T} \left[(c_{ij}^1 + c_{ij}^1) w_{ij} + (c_{ij}^2 + c_{ij}^2) v_{ij} \right] + \alpha \, \Pi(S) + \beta \, \Phi(S)$$

While the first term in $\Theta(S)$ describes the co-channel and adjacent-channel interference levels incurred under solution S, the second term describes the solution feasibility with respect to demand requirements and the last term describes solution feasibility with respect to separation requirements.

3.3.2 Search Operator & Solution Neighborhood

The search space of the problem at hand is very large, especially since demand requirements and separation constraints are considered as soft constraints (with penalty terms incorporated within the objective function). For a search space defined by varying the frequency of one TRX at a time, the size of the search space is $\prod_{i \in T} |F_i|$. For instance, for a network with just 100 TRXs and 40 frequencies available for both BCCH and TCH-carrying TRXs, we have 40^{100} possible solutions. The search operator is designed to identify promising moves for the algorithm.

The search space is explored by changing the frequency assignments through a sequence of operations or moves at each iteration. An elementary operation consists of changing the frequency assignment of a TRX i from frequency f to frequency g. This is denoted by $Swap(i : f \rightarrow g)$. Applying such an elementary move to a solution S, we obtain a new solution S'. This is denoted by: $S' = S + Swap(i : f \rightarrow g)$.

The operator used for this algorithms first identifies the best frequency change for a given TRX among a subset of frequencies, and then examines a set of related TRXs for further potential gains. We need to define the following operators: the Best Swap operator and the Swap Improvement operator.

- *The Best Swap operator*

  Consider a TRX i currently assigned a frequency f in a given solution S. This operator identifies the frequency change that produces a new solution with the lowest objective function in a subset of the allowed frequencies F_i'. By examining only a fraction p of the possible frequency changes in F_i', we can manipulate the search intensity of this operator. The Best Swap operator is denoted by $BS(i)^p$. So we have:

$$\Theta(S + BS(i)^p) = \Theta(S + Swap(i : f \rightarrow g^*)) = \min_{\substack{g \in F_i' \\ Rand(g) < p}} \Theta(S + Swap(i : f \rightarrow g))$$

  where $Rand(g)$ is a randomly generated number in $[0, 1]$. This operator selects the best move among the set of allowed moves F_i'. The definition of this set varies as the algorithm progresses. This is because we allow for a temporary relaxation of the separation constraints. In such a situation, some frequencies, though infeasible due to separation constraints, become available. In addition, the set of allowed moves F_i' does not include frequencies that are declared Tabu nor does it contain any locally blocked frequencies (as some cells may be prohibited to use some specific frequencies).

- *The Swap Improvement operator*

  This operator tries to improve upon the solution produced by one or more frequency changes in the current solution. Consider for instance a pair of TRXs i and j such that $c^1_{ij} > 0$ or $c^1_{ji} > 0$, and suppose that a frequency change $Swap(i : f \rightarrow g)$ is applied to the current solution. In case TRX j is assigned frequency g, then, due to co-channel interference that results from this frequency change between this pair (i, j), changing the frequency of TRX j may lead to a better solution. Considering the opposite case where TRX j is not currently assigned frequency f, then this frequency may become an attractive move as the potential co-channel interference with TRX i would not be there anymore.

  One can make similar arguments for TRX pairs related by adjacent-channel interference or by separation constraints. We need to consider all three possible TRX relations especially that in some instances the co-channel or adjacent channel interference matrices may indicate a zero value for some pairs of TRXs not because of the absence of interference but because the separation constraints (if satisfied) would guarantee that no interference would occur.

  We denote this operator by $SI(Swap)$. Let S be an initial solution and let set M be an ordered set of changes of the form $Swap^l = Swap(i : f \rightarrow g)$ that were applied to the initial solution S in order to produce a solution S'. We have $S' = S + \sum_{Swap^l \in M} Swap^l$. The set M is ordered according to the order $l = 0, .., |M|$, in which the changes $Swap^l$ are applied. This operator start with a set M such that $|M| = 1$. This neighborhood operator can take a depth first approach or breath first approach. As it is not possible to determine which would work better and in order to introduce more robustness in the search process, a random selection is made at the beginning of the call of this operator. It functions as follows:

```
1. Start with $Swap^{l=0} = Swap(i : f \rightarrow g)$
2. Randomly select the examination approach: Depth first vs. Breath first
3. FOR all TRXs $j \neq i \in T$, such that:
   $Rand(j) < \frac{c^1_{ij}}{\max_k c^1_{ik}}$ OR $Rand(j) < \frac{c^2_{ij}}{\max_k c^2_{ik}}$ OR $Rand(j) < 0.1 * S_{ij}$
   3.1 IF $x_{jg} = 1$ OR $x_{j(g\pm1)} = 1$ , THEN
         IF $\Theta(S' + BS(j)^{1.0}) < \Theta(S')$
            THEN
               3.1.1 IF Depth first,
                        THEN $S' := S' + BS(j)^{1.0} + SI(BS(j)^{1.0})$
               3.1.2 ELSE    $S' := S' + BS(j)^{1.0}$ AND
                             $M := M \cup \{BS(j)^{1.0}\}$

   3.2 IF $x_{jf} = 0$ AND $x_{jh} = 1$, THEN
         IF  frequency $f$ is allowed at TRX $j$
            AND $\Theta(S' + Swap(j : h \rightarrow f)) < \Theta(S')$
               3.2.1 IF Depth first,
                        THEN $S' := S' + Swap(j : h \rightarrow f)$
                                 $+SI(Swap(j : h \rightarrow f))$
               3.2.2 ELSE $S' := S' + Swap(j : h \rightarrow f)$ AND
                          $M := M \cup \{Swap(j : h \rightarrow f)\}$
4. IF Breath first, THEN $S' := S' + SI(Swap^{l+1})$
```

In step 3 of the $SI(M)$ operator, $Rand(j)$ is a randomly generated number in $[0, 1]$. The number of TRXs that are related to the reference TRX i is usually very large in realistic problem instances. Thus, examining all possible TRXs for possible improvement to the current solution would be time consuming. To counter this, we attempt to focus on the moves that are more likely to improve the current solution: TRXs with strong co-channel or adjacent-channel interference, or that are related by a separation constraint.

Step 3.1 of the $SI(M)$ operator focuses on TRXs using the same or adjacent frequencies as the new frequency of the reference TRX i, while step 3.2 considers such TRXs that may switch to the old frequency of the reference TRX i.

If a Breath first approach is selected, the Swap Improvement operator first attempts to identify improving moves that result from the current reference frequency change $Swap^l$ before moving to examine the next one in the list M in step 4 (as opposed to a depth-first search approach, where the effect of each change is treated immediately in steps 3.1.1 and 3.2.2).

The Best Swap and Swap Improvement operators are combined together to form a compound operator: Given a TRX i and a solution S, we define the neighborhood operator $m^p(i)$ such that: $m^p(i) = BS(i)^p + SI(BS(i)^p)$.

We denote the solution neighborhood that results from using this operator by $N^p(S)$. This operator first finds the best frequency change for the current TRX using the Best Swap operator and then attempts to identify improving moves using the Swap Improvement operator. By manipulating the parameter p, we control the number of evaluations and hence control the intensity of the $BS(i)^p$ part of the search. However, increasing the value of p does not necessarily increase the overall search intensity of the $m^p(i)$ operator, nor vice versa. In fact, for a low value of p, $BS(i)^p$

may produce a relatively bad solution but the chain improvements that result from $SI(BS(i)^p)$ may compensate for that, producing an even better solution than if a large value for p were used.

Since there is no way of determining the appropriate value for p beforehand, we opted to dynamically change its value during the search. Attempts to experimentally find the appropriate value of this parameter failed as we found that there is no clear dominant value across problem instances. For this, at each iteration the value of p is randomly selected in the interval $[0, 1]$. Therefore, the parameter p provides us with a mechanism to diversify the search.

So far in describing the neighborhood operator we referred to changing the frequency of some TRXs to a new frequency which is selected from a set of allowed frequencies F_i'. The definition of this set depends on the definition of the search space: whether or not we allow relaxation of problem constraints. Such a relaxation has the effect of increasing the search space, which in turn may increase the likelihood of producing better solution but at the expense of higher computational requirements. In the algorithm we propose in this work, separation constraints are relaxed temporarily so that $m^p(i) = BS(i)^p|_{\Omega=\Omega^2} + SI(BS(i)^p)|_{\Omega=\Omega^1}$. This means that during the computation of $BS(i)^p$, solutions that are infeasible with respect to separation constraints are considered, while no more infeasibility is allowed when computing $SI(BS(i)^p)$. Given that the number of computations involved during this latter step is much larger, a relaxed definition of the search space would result in significantly higher computational times. In addition, this would increase the proportion of infeasible solutions that are visited but discarded due to the penalty terms included in the objective function, making it a wasted effort.

By considering a relaxed search space definition only in the first portion of the neighborhood operator, we are able to take advantage of the benefits of the relaxation at limited computational costs. Furthermore, by definition of the Swap Improvement operator, it will attempt to restore any infeasibility that results from the Best swap operator. This, combined with the effect of penalty terms in the objective function, would increase the likelihood of producing feasible solutions by the $m^p(i)$ operator.

An extension to the $m^p(i)$ operator may be to consider all possible frequency changes instead of considering $BS(i)^p$. This extension would be in the form of:

$$\Theta(S + m_2^p(i)) = \min_{\substack{g \in F_i' \\ Rand(g) < p}} \Theta\left[S + Swap(i : f \rightarrow g) + SI(\{Swap(i : f \rightarrow g)\})\right]$$

This operator is more powerful. It scans all allowed moves for TRX i, and for each such possibility, it attempts to identify any improving moves using the Swap Improvement operator. Experimenting with this operator, we found that the search strength is offset by the much higher computational requirements. Overall, we found that using the $m^p(i)$ operator results in a better performance of the algorithm when different values of the parameter p are used.

The neighborhood of a solution is obtained by applying the $m^p(i)$ operator to the TRXs of the network. Because of the large number of TRXs in any realistic problem instance, the evaluation time required at every iteration would be excessive, if one were to apply the operator to all TRXs. For this reason, neighborhood sampling is used in the proposed algorithm.

The neighborhood sample is formed by randomly selected TRXs, with a uniform probability of selection. Increasing the size of the neighborhood provides a search intensification mechanism at the expense of increasing computational requirements. Since it is difficult to determine an appropriate value for the size of the neighborhood sample, we opted to dynamically increase it as the search progresses.

3.3.3 Algorithm Search Structure

The algorithm starts in normal Tabu search mode. After reaching a predefined number of consecutive iterations without any improvement in the best solution or if it improves upon the best solution, then the algorithm moves to an intensive search mode. This intense phase continues until no more improvements could be expected. The intensification phase involves switching to a local descent algorithm where only improving moves are accepted.

Before describing the algorithm in detail, we define the following:

- t: current iteration counter
- $t^I_{\max}$: end iteration of the Intensification phase
- $t^I_{\min}$: starting iteration of the Intensification phase
- $t^N_{\max}$: maximum allowed number of consecutive iterations without improvements in the best solution
- S: current solution
- $N(S)$: neighborhood of solution S generated according to the neighborhood operator
- S^*: best feasible solution
- $\widetilde{\Theta}$: current minimum objective function
- Ω : current search space: if $\Omega = \Omega^1$, then no violation of separation constraints is

allowed and if $\Omega = \Omega^2$, then separation constraints are relaxed

The proposed adaptive Tabu search algorithm (ATSA) follows the general structure of a Tabu Search algorithm with the exception that it iterates between a normal search mode and local descent mode. The algorithm can be summarized as follows:

```
1.  Initialization
    1.1.  Generate initial solution $S$
    1.2.  Set $S^* := S, \widetilde{\Theta} := \Theta(S^*), t^N := 0, t := 0$
2.  WHILE termination criterion not met
    2.1.  Build $N^p(S)$
    2.2.  Evaluate $S' = \arg\min_{S'' \in N^p(S)} \Theta(S'')$
    2.3.  IF in Tabu Search mode, THEN
                  $S := S'$
              ELSE (case Local Descent mode)
                  $S := \begin{cases} S', \text{ IF } \Theta(S') < \widetilde{\Theta} \\ S, \text{ Otherwise} \end{cases}$ AND $\widetilde{\Theta} := \Theta(S)$
    2.4.  IF $\Theta(S) < \Theta(S^*), \Pi(S) \leq \Pi(S^*)$, AND $\Phi(S) \leq \Phi(S^*)$,
              THEN $S^* := S$
    2.5.  Assign tabu tags and update parameters
```

The proposed algorithm starts in classic Tabu Search mode. This phase continues until one of two conditions is met:

1. The search reaches a solution that improves upon the best feasible solution. The algorithm switches to a local descent mode in order to converge quickly towards a local optimum.

2. There are $t^N_{\max}$ consecutive iterations without improvements in the best solution since the last forced switch to a local descent mode. In this case, the algorithm is "forced" to go into local descent mode. This is to be differentiated from the previous "un-forced" case.

   The value of $t^N_{\max}$ is set so as to use the Tabu search phase to escape from the local optimum that was explored during the last descent phase and to move to another area of the search space. For this we have $t^N_{\max} = 2(t^I_{\max} - t^I_{\min})$. This strategy allows the algorithm to be adaptive in how much effort to spend during each phase.

When the local descent mode is started, we set $t^I_{\max} = t + 200$, $t^I_{\min} = t$ and $\widetilde{\Theta} = \Theta(S)$. While in this mode, each time the algorithm encounters a solution S with $\Theta(S) \leq \widetilde{\Theta}$, the end iteration of this phase is reset to $t^I_{\max} := t + 200$ and $\widetilde{\Theta} = \Theta(S)$. The duration of the look-out window is set so that for each TRX, the expected number of evaluation of all possible moves is more than one.

Let r be the number of consecutive forced local descent phases (this corresponds to the number of unsuccessful consecutive attempts to improve upon the current best). Since the algorithm dynamically intensifies the search by increasing the neighborhood sample size, we define $\pi \in [0, 1]$ as the step size increase. Then, we have:

- The neighborhood sample is increased such that if $r > 1$ then $|N(S)| := |N(S)| + \pi|T|$ (where T is the set of TRXs in the network)
- If $r = r_{\max} = 5$, then initiate a diversification phase

- Generate a new start solution
- Set $r := 0$, and $|N(S)| := 0.01|T|$

3.3.4 Tabu List Management and Parameter Updating

The Tabu structure used in the proposed solution approach consists of placing a Tabu Tag $t(c, f)$ on the cell-frequency pair. This tag indicates the iteration number before which a particular frequency cannot be activated at the cell under consideration. The Tabu Tag is placed on the cell as opposed to being on the TRX since TRXs belonging to the same cell are similar in the sense that they share the same interference levels with other cells. The duration of the tabu tag is proportional to the occurrence of use of frequency f at cell c. This is computed as follows:

$$t(c, f) = t + \lambda \frac{\eta(c, f)}{\eta(c)}$$

where t is the current iteration counter, $\eta(c, f)$ is the number of times frequency f has been activated in cell c, $\eta(c)$ is the number of times a frequency change has occurred for cell c, and λ is constant. The value assigned to this constant λ is critical since it determines the length of the tabu status of moves. The value λ is determined experimentally at 100.

The proposed algorithm does not utilize any aspiration criterion. Initial experiments suggested that such a feature increases the number of evaluations carried out during each iteration without improving the final results. This may be due to the relatively short tabu tenure used.

Parameter updating occurs after every iteration. The algorithm updates the coefficients associated with the problem constraints: parameter α associated with demand constraints and parameter β associated with separation constraints. This takes place so as to slowly direct the search towards feasible solutions. After each iteration, for a given solution S:

- If $\Pi(S) > 0$ (i.e., solution S does not meet demand constraints), then $\alpha := \alpha(1 + 5\%)$
  else $\alpha := \alpha(1 - 5\%)$
- If $\Phi(S) > 0$ (i.e., solution S does not meet separation constraints), then $\beta := \beta(1 + 10\%)$
  else $\beta := \beta(1 - 5\%)$

During the search, the algorithm temporarily relaxes the separation constraints. In order to quickly move away from the current infeasibility, a fast change of the penalty values is implemented. Initial experimental investigation suggested that a rapid change to the parameters achieves this.

Note that for the penalty associated with violating separation constraints, the change in the coefficient β values is not symmetric. By decreasing β at a lower rate than the increase, the algorithm is biased so that it remains in the feasible regions of the search space for longer periods of time.

If the algorithm fails to reach a feasible solution, the above updating procedure will only increase the value of the penalty associated with the constraints. However, the algorithm may reach a point where it cannot improve feasibility without temporarily deteriorating it. This cannot occur if the penalty value is too high. To overcome this, one approach would be to fix a maximum value for the penalty terms. The difficulty then is to determine what should be the appropriate value. For this algorithm we consider a different approach. By design, the algorithm allows the relaxation of the separation constraints. So what needs to be done is to reduce the value of the penalty term even if the best solution is still infeasible. This takes place if the infeasibility persists for a long time. If the infeasibility is related to the demand constraints, we still reduce the penalty term associated with separation as this allows the required type of change to the solution.

3.3.5 Generation of the Initial Solution

The proposed algorithm can either start with some given solution or gradually generate its own solution. The latter can take place since we allow a relaxation of the demand constraints with a penalty term in the problem objective function. Initially, the penalty coefficient α associated with demand constraints is set to a large value, favoring the fast generation of more complete solutions. Since the algorithm favors solutions with lower objective function values, by using this approach, the procedure runs the risk of first assigning frequencies to TRXs far apart from each other (low interference between them) before assigning frequencies to the more critical TRXs (TRXs in the dense area of the network whose management is more difficult). As consequence, the initial solution quality may be very bad.

The proposed algorithm used a quick solution construction heuristic, which considers cells in random order and assigns each of their TRXs the best available frequency (using the Best Swap operator). This step does not necessarily produce a complete solution as some assignment may be infeasible. The best swap operator is applied a second time in local descent mode in order to

Time(sec)	40	150	300	900	1500	2250	4500
DOCAF	13774	13532	13297	13081	12947	12907	12810
ATSA-FAP	13958.1	13341.8	13059.2	12770.8	12696.4	12596.4	12502.0

Table 3.1: FAP - Proposed algorithm vs DOCAF

improve upon the incumbent.

3.4 Experimental Results

The initial investigations of the proposed algorithm ATSA-FAP are carried out on a real-life instance provided by AirTel, Montreal. This data set includes 337 cells with 927 TRXs and a contiguous block of 50 frequencies. For these initial investigations, we compare ATSA-FAP to DOCAF [13], a commercial tool developed by Prestige Telecom, Montreal. This software ranked in second place in the challenge organized by France Telecom [14] relative to a collection of commercial frequency planning tools. For this problem instance, we were provided with the total interference of the best solution generated by DOCAF for several points in time. Table 3.1 shows the progress of our proposed algorithm versus the progression of DOCAF. Note that consideration was made to account for the difference in clock speeds of the platforms on which the problem instance was run. The proposed algorithm shows a fast convergence rate.

We evaluate the performance of the proposed adaptive Tabu Search algorithm presented in this chapter on a set of benchmark instances available at the *FAP* web site [26]. The COST 259 benchmarks include a set of 32 realistic GSM network planning scenarios. For these problem instances, the solutions provided by several frequency planning methods are also made available.

Table 3.2 provides a description of the characteristics of these problem instances. The density of the co-channel interference matrix, the adjacent-channel interference and separation matrices are referred to as ρ^{co}, ρ^{ad} and ρ^{sep}, respectively. The frequency spectrum available may be divided into frequency blocks, which is marked in Table 3.2 with the '+' sign. A more extensive analysis of the characteristics of a subset of the COST 259 benchmarks is available in [24]. The analysis indicates that although the underlying graph is not always connected, it contains a major component while the other components include only a few cells. In Addition, Eisenblatter [24] reports that the diameter of the underlying graphs (defined as the maximum length among the shortest paths between pairs of vertices of the largest component) is relatively small: between 3 and 10. This is indication of the tightness of the relationships between the cells forming the network. The need for careful optimization is further strengthened by the fact that both the average vertex degree and the size of the maximum cliques are much larger than the available number of frequencies. The difficulty of these problem instances is further illustrated by the wide gap between the best known lower bounds and the set of available results (see Tables 3.6 and 3.8). The results achieved by the lower bounding approach proposed by Montemanni et al. [65] also show a wide gap with respect to the best known solutions.

The tightness of the relationship between network cells make the use of problem decomposition very delicate. We have attempted a decomposition approach by extracting K-club using the algorithm proposed by Bourjolly et al. [15]. However, the results achieved were not promising due to the difficulty in recombining the sub-problems. Difficulties of the same nature were encountered while experimenting with Adaptive memory type heuristics.

Scenario	# Cells	# TRXs	# Frequencies	ρ^{co}	ρ^{ad}	ρ^{sep}
Siemens 1	506	930	20 + 23	0.079	0.040	0.0142
Siemens 2	254	977	4 + 72	0.480	0.216	0.037
Siemens 3	894	1623	55	0.0799	0.031	0.017
Siemens 4	760	2785	39	0.196	0.111	0.007
K	264	267	50	0.3892	0.045	0.033
Bradford–1-eplus	1878	2947	75	0.0845	0.0050	0.0046
Bradford–1-race	1878	2947	75	0.0294	0.0032	0.0045
Bradford–1-free	1878	2947	75	0.0367	0.0030	0.0045
Bradford–2-eplus	1876	3406	75	0.0847	0.0050	0.0046
Bradford–2-race	1876	3406	75	0.0295	0.0032	0.0045
Bradford–2-free	1876	3406	75	0.0368	0.0030	0.0045
Bradford–4-eplus	1886	3996	75	0.0838	0.0049	0.0046
Bradford–4-race	1886	3996	75	0.0292	0.0031	0.0045
Bradford–4-free	1886	3996	75	0.0364	0.0030	0.0045
Bradford–10-eplus	1876	4871	75	0.0847	0.0050	0.0046
Bradford–10-race	1876	4871	75	0.0295	0.0332	0.0045
Bradford–10-free	1876	4871	75	0.0368	0.0030	0.0045
Bradford–0-eplus	1886	1886	75	0.0838	0.0049	0.0046
Bradford–0-race	1886	1886	75	0.0292	0.0032	0.0045
Bradford–0-free	1886	1886	75	0.0364	0.0030	0.0045
bradford_nt-1-eplus	1886	1971	75	0.0838	0.0049	0.0046
Bradford_nt–1-race	1886	1971	75	0.0292	0.0031	0.0045
bradford_nt-1-free	1886	1971	75	0.0036	0.0030	0.0045
bradford_nt-2-eplus	1886	2214	75	0.0838	0.0049	0.0046
Bradford_nt–2-race	1886	2214	75	0.0292	0.0031	0.0045
bradford_nt-2-free	1886	2214	75	0.0364	0.0030	0.0045
bradford_nt-4-eplus	1886	2775	75	0.0838	0.0049	0.0046
Bradford_nt–4-race	1886	2775	75	0.0292	0.0031	0.0045
bradford_nt-4-free	1886	2775	75	0.0364	0.0030	0.0045
bradford_nt-10-eplus	1886	4145	75	0.083	0.005	0.0046
Bradford_nt–10-race	1886	4145	75	0.036	0.003	0.0045
bradford_nt-10-free	1886	4145	75	0.029	0.003	0.0045

Table 3.2: FAP - Cost 259 Benchmark Characteristics

	Fixed $\lvert N(S)\rvert =$ 0.01|T|	0.05|T|	0.10|T|	0.15|T|	0.20|T|	0.25|T|
Siemens 1	4.02	3.27	3.36	3.35	3.34	3.22
Siemens 2	18.93	17.84	17.96	18.18	18.14	18.14
Siemens 3	8.99	8.89	9.01	10.04(2)	11.01	8.97
Siemens 4	103.69	113.32	110.25	109.76(2)	109.11	103.75

Table 3.3: FAP - Effect of Solution Neighborhood - Fixed Neighborhood size

We study the critical aspects of the proposed algorithm in tables 3.3, 3.4 and 3.5. These test are carried out on four of the benchmark instances. These benchmarks were selected because they provide a wide range of characteristic while still being homogenous as they come from the same source. Each case corresponds to 90 minute runs on a Pentium IV 1.8GHz PC. If the best solution achieved is not feasible, the number of violations is indicated between parenthesis.

One of the specificities of the proposed algorithm is the dynamic adjustment of the solution neighborhood size. Table 3.3 shows the performance of the algorithm when using a fixed neighborhood size. This size is set as a fraction of the number of TRXs in the network. Table 3.4 shows the case when the neighborhood size is dynamically adjusted during the optimization. This table tests the effect of π, where the neighborhood size is updated when a forced descent is triggered such that $|N(S)| := |N(S)| + \pi|T|$. For both tables 3.3 and 3.4, the parameter p of the neighborhood operator, is randomly changed in every iteration.

	Dynamic $\vert N(S)\vert$ with $\pi =$					
	0.005	0.01	0.02	0.03	0.04	0.05
Siemens 1	3.42	3.29	3.17	3.35	3.26	3.27
Siemens 2	17.79	17.56	17.45	18.04	17.69	17.52
Siemens 3	8.79	8.60	8.53	8.56	8.47	8.59
Siemens 4	102.03	103.45	103.01	103.06	103.12	99.57

Table 3.4: FAP - Effect of Solution Neighborhood - Dynamic Neighborhood size

Clearly from table 3.3, there is no dominant fixed size for the solution neighborhood. The drawback of using larger neighborhood appears mostly for very long runs: Due to the computational load of larger neighborhoods, a smaller portion of the search space could be explored. In addition, with using a larger neighborhood, the likelihood of converging to the same local optima increases, hence further reducing the exploratory capacity of the algorithm.

As illustrated by table 3.4, the dynamic increase in solution neighborhood size produces better solutions on the overall. However, the difficulty here involves determining by how much to increase the neighborhood size. The experimental results did not indicate any clear ideal level for this increase. We opt to choose a low level for π at 0.01. The aim behind this approach is to provide a slow intensification of search and hence explore larger portions of the search space, and provide a faster convergence rate for when only short runs can be made.

As discussed previously, the parameter p of the neighborhood operator $m^p(i)$ provides a mechanism to diversify the search. Table 3.5 compares the effect of using a fixed value for p (columns 2 through 6) to using a randomly changed value (column 1). These tests indicates that there is no preferred fixed value for p, but on the contrary, while a fixed value may produce good solutions for

	variable p in [0.25,1.00]	Fixed $p =$ 0.10	0.25	0.50	0.75	1.00
Siemens 1	3.29	3.20	3.28	3.16	3.25	3.19
Siemens 2	17.56	17.68	17.43	17.30	17.51	17.15
Siemens 3	8.60	8.96	8.79	8.54	8.36	11.13
Siemens 4	103.45	100.59	102.30	104.11(2)	103.70	113.21

Table 3.5: FAP - Effect Neighborhood operator parameter p

one instance, the result would not be as good on another. The use of a random value, on the other hand, seems to be more consistent across benchmarks.

In Tables 3.6, 3.7 and 3.8, we compare the results of the proposed algorithm to the ones listed in the FAP web site (Column ATSA-FAP, followed by the CPU time column expressed in seconds). We provide the results achieved by the longest available runs. These tables also provides the lower bounds (under column LB) of some test instances reproduced from [24], as well as the total interference achieved by solutions generated by several frequency planning methods (the solutions are available at the FAP web site [26]). Note that for the Siemens data sets (1 through 4) we do not obtain the same values for the total interference as the one reported in the FAP web site. The tables below show the value of the objective function as computed by our code.

We were not able to locate all the articles describing the solution methods. Both SA(TUHH) [6] and Telefonica [22] use Simulated Annealing-based Heuristic, while Tabu Accepting is used for TA(RWTH) [40] and TA(Siemens) [41]. A dynamic Tabu Search algorithm is used in DTS [81] where the tabu lists are dynamically reduced during the search process. The DC5_IM(ZIB) [9] corresponds to a construction heuristic which is run multiple times, each with a different start seed. Each run is then followed by the application of an improvement heuristic that performs frequency changes as long as they improve upon the current solution (The full procedure is referred to as DSATUR with Costs at 5% followed by Iterated 1-Opt). The best results in Tables 3.6, 3.7 and 3.8 are due to Mannino et al. (K-Thin(UR1) [58]). It is based on a Simulated Annealing procedure that incorporates a dynamic programming subroutine to compute local optima. The latter efficiently reoptimizes the frequency assignment of a subset of related TRXs each time the algorithm reaches a new best solution or after a fixed number of iterations. The results achieved by this algorithm outperform all those generated by other planning methods.

The time required to generate the solutions is not provided in the FAP web site. However, for some of the solutions, we were able to locates the time figures in the related article. The computational times appear in parenthesis in Tables 3.6, 3.7 and 3.8 are provided in hours. For the sake of comparison, we mention that the K-Thin(UR1) solutions were generated with a dual 1.5GHz Intel Xeon processor. As for our proposed algorithm, it is coded in Visual C++. The tests are carried out on Pentium IV 1.8GHz processor equipped with 384MB of memory, or on a Pentium IV 2.8GHz processor equipped with 512MB of memory (indicated with a "*").

The proposed algorithm achieves a moderate performance, ranking in second or third position in several cases. However, one must consider that our runs are relatively short (as compared to when run times are available). The proposed algorithm slowly increases the solution neighborhood size as an intensification mechanism, before restarting the search when further increases are not profitable. In many instances, the proposed algorithm did not complete the sequence of increases in the reported runs. For this we expect a better performance for longer runs.

Scenario	K-Thin (UR1) [58]	SA (TUHH) [6]	TA (RWTH) [40]	TA (Siemens) [41]	U (Siemens) [27]	DTS (Glamorgan) [81]	DC5_IM (ZIB) [9]	LB [24]	ATSA *FAP*	Time (sec)
Siemens 1	2.679 (5h)	3.158	2.987	2.795	3.791	-	-	0.1280	2.784	21654*
Siemens 2	16.364 (9h)	17.368	17.135	16.811	19.311	16.3587	-	6.9463	17.063	22270*
Siemens 3	6.338 (15h)	7.7189	6.830	6.465	9.525	6.394	-	0.4132	7.588	69951*
Siemens 4	82.479 (18h)	94.185	88.816	86.178	110.853	87.130	-	27.632	92.747	35709*
K	-	-	-	0.458	-	-	0.447	0.1887	0.452	37221*

Table 3.6: FAP - Cost 259 Benchmarks: Results - Part 1

Scenario Bradford–	K-Thin (UR1) [58]	SA (TUHH) [6]	TA (RWTH) [40]	U (Siemens) [27]	DC5_IM (ZIB) [9]	Telefonica [22]	ATSA *FAP*	Time (sec)
1-eplus	33.802 (64h)	33.995	40.52	-	56.332	-	38.337	69840 *
1-race	0.015 (26h)	0.035	0.0398	0.078	0.159	0.089	0.0357	214795 *
1-free	0.119 (30h)	0.160	0.195	0.295	1.512	-	0.261	197440 *
2-eplus	79.383 (75h)	80.029	89.961	-	123.92	113.578	86.776	108820 *
2-race	0.325 (39h)	0.425	0.422	0.799	1.449	-	0.304	169981 *
2-free	2.694 (37h)	2.948	3.691	4.238	12.934	9.292	3.972	42892.*
4-eplus	167.007 (90h)	167.7	185.416	-	253.051	-	180.998	157136 *
4-race	2.937 (36h)	3.042	3.297	-	10.245	-	3.671	26233 *
4-free	19.996 (46h)	22.091	31.024	-	60.895	-	30.796	10801 *
10-eplus	395.510 (128h)	400.003	435.276	-	553.996	-	441.882	38549
10-race	27.390 (46h)	30.219	32.604	-	62.345	-	37.2408	39790
10-free	113.690 (63h)	117.8	127.332	-	213.959	-	134.998	93375

Table 3.7: FAP - Cost 259 Benchmarks: Results - Part 2

Scenario Bradford–	K-Thin (UR1) [58]	SA (TUHH) [6]	TA (RWTH) [40]	TA (Siemens) [41]	U (Siemens) [27]	DC5_IM (ZIB) [9]	LB [24]	ATSA *FAP*	Time (sec)
0-eplus	0.596	0.796	0.906	-	1.019	1.103	0.0096	1.47238	40813
0-race	0	0	0	-	-	0	-	0	0
0-free	0	0	0	-	-	0	-	0	0
nt-1-eplus	0.856	1.039	1.101	0.859	-	1.785	0.0297	2.11648	42138
nt–1-race	0	0	0	-	-	0	-	0	0
nt-1-free	0	0	0	-	-	0	-	0	0
nt-2-eplus	3.204	3.786	4.074	3.204	-	6.234	0.475	5.77309	42018
nt–2-race	0	0	0	-	-	0	-	0	0
nt-2-free	0	0	0	-	-	0	-	0	196
nt-4-eplus	17.722	19.003	19.510	17.728	-	29.165	4.034	24.4247	41521
nt–4-race	0	0	0	-	-	0	-	0	25023
nt-4-free	0	0	0	-	-	0	-	0	29726
nt-10-eplus	144.937	148.122	158.659	146.199	-	208.138	54.099	170.585	95204
nt–10-race	1.094	1.727	1.729	1.073	-	4.392	-	2.2474	41895
nt-10-free	5.422	8.625	7.593	5.862	-	15.339	-	11.2728	40883

Table 3.8: FAP - Cost 259 Benchmarks: Results - Part 3

3.5 Conclusion

In this chapter, we develop an adaptive Tabu Search algorithm for FAP. It uses a strong neighborhood operator to define the search space in order to speed up the convergence. Local optima are quickly identified and examined by switching to a local descent type search. The relaxation of the separation constraints allows the algorithm to counter the tightness of the relationship between network cells. Currently longer runs on the test instances are being carried out to investigate the long term performance of the algorithm.

The literature reveals that a significant effort has been, and is still being, devoted to the resolution of FAP due to its practical importance as well as to the particular challenges it poses. One direction of research which has not be explored in depth is problem decomposition. The tight relations between the elements of realistic instances (as shown by the benchmarks considered in this chapter) make decomposition a tricky task. It is possible to extract subproblems in a variety of ways by looking at various graph structures such as cliques, stable sets, k-clubs, etc. However, our initial investigations suggest that the difficulty lies mostly in finding an appropriate method to recombine subproblems. We believe that the increase in the size of problem instances that are being considered in the literature is indicative of the need to explore this avenue of research.

Chapter 4
The Frequency Hopping Problem

Frequency hopping is a feature in GSM-like cellular systems in which the frequency carrying the communication rapidly changes over time. This improves the communication quality without the addition of new equipment. The frequency hopping problem involves the determination of the subset of frequencies allocated to each cell to be used for hopping, as well as determining the hopping sequence of each TRX in the network. In this chapter we investigate the potential gain from optimizing frequency management. We consider three levels of network synchronization, and for each case, we propose procedures to optimize and evaluate frequency hopping plans. In addition, we propose a novel hopping pattern for fully-synchronized network. Results obtained from real life instances indicate the quality improvement due to FH is best under fully-synchronized networks. The proposed FH management procedures are also shown to improve upon the plan selected by the operator.

4.1 Introduction

The mobile telephony market has been undergoing dramatic changes reflecting an increase in the volume of demand as well as in the quality of service requirements. A major limiting resource in this highly competitive market is the available frequency spectrum. Improving the efficiency of the exploitation of this resource would allow the operators to accommodate or attract more customers and hence generate more revenue. On the other hand, an inefficient use of the available spectrum can only be overcome through the deployment of more equipment and hence increasing operating costs. In FDMA-based networks, static frequency allocation has been the basic tool for optimizing the spectrum management. This may sometimes be insufficient for highly congested networks. *Frequency Hopping* (FH) is a feature in GSM-type networks (such as GSM, PCS1900 and DCS1800) that allows expansion of the available capacity of mobile networks or to improve the quality of service through varying the transmission frequency over time according to some predetermined pattern. The result of FH is an improved CIR level achieved by interference averaging and frequency diversity without the addition of new equipment.

An efficient use of the radio spectrum corresponds to an efficient allocation of frequencies to TRXs so as to minimize interference. The static frequency allocation problem has been modeled as a combinatorial optimization problem, and several good algorithms for solving it are available (see Chapter 3). However, the effectiveness of static frequency allocation is somewhat limited when applied to congested networks because the frequency allocation they provide is static: When two mobile subscribers are simultaneously allocated interfering frequencies for their respective communications, the interference remains as long as one of the communications is active since the frequencies on which the communications are established do not change. If, due to network characteristics, the interference levels are high, the mobile subscribers may experience bad communication quality or, even worse, one or both communications may be interrupted. One approach to solve this problem is to reduce the transmission power of one or both transmitters. This results in reducing cell coverage and hence may require the installation of a new cell. Activating FH is another approach.

FH is a feature in GSM-type networks that aims at overcoming high interference levels through the sharing of interference among users in the network. The FH concept was first introduced in 1940 by Hedy Lamarr and George Antheil and taken up in 1957 by engineers at the Sylvania Electronic Systems [42]. FH implies broadcasting a signal over a changing series of radio frequencies, quickly switching from frequency to frequency. The initial objective was to reduce the danger of detection or jamming for military radio communications. In modern cellular communication systems, FH also allows a better exploitation of the available radio spectrum.

In this chapter, we propose to analyze and optimize the radio spectrum utilization under FH. This involves a two step approach: 1) allocating frequencies to cells and 2) determining the frequency hopping sequences for each TRX. For the second step, we distinguish between three levels of network synchronization. For each level, we propose a heuristic procedure to optimize the GSM parameters that control the hopping sequences. Results obtained on real life network indicate that synchronization can lead to a better exploitation of FH and that the proposed heuristics are able to significantly improve upon solutions generated by the network engineers. In addition for fully-synchronized networks, we propose to generate the hopping sequences (as opposed to controlling the GSM sequence generation procedure).

In section 4.2 we describe frequency hopping in more detail. In Section 4.3, we describe the proposed approach for managing FH. The performances of the proposed models and algorithms are examined for real life networks in section 4.4.

4.2 The Frequency Hopping Mechanism

GSM makes use of the inherent frequency agility of the transceivers and mobile units (MU) (i.e. of their ability to transmit and receive on different frequencies) to implement slow Frequency Hopping. FH is a technique where the frequency used by a given pair composed of a cell and an MU is allowed to change over time at a prescribed rate (217 times per second). Each cell may contain one or more transceivers (TRXs). The first time slot in the first TRX of a cell is used as the Broadcast Control Channel (BCCH). The remaining time slots in that TRX and the time slots in the other TRXs are used as Traffic Channels (TCH). Because the BCCH is used for establishing calls and for signaling purposes, the TRX carrying it is not allowed to hop. The *cell allocation* (CA) is the subset of frequencies allocated to a cell. Under static allocation, the CA contains a number of frequencies that is equal to the number of TRXs (one frequency for each TRX). The *mobile allocation* (MA) is the subset of frequencies from the CA that is allocated to a particular call. Under FH, the BCCH-carrying TRX has an MA containing only one frequency (since no hopping may occur).

From an implementation point of view, there are two types of FH: *Baseband* FH and *Synthesized* FH, and this depends on the type of equipment installed at the TRXs:

- In Baseband FH, each TRX can only be tuned to one fixed frequency. As a consequence, the cell allocation contains as many frequencies as the number of TRXs in that cell. Hopping is obtained by shifting the communication from one TRX to another.
- In Synthesized FH, the TRXs are capable of, and are allowed to, retune to different fre-

quencies. Any given call goes through the same TRX, but the frequency used changes over time. This allows the number of frequencies in the CA to be larger than the number of TRXs (no more than 64 in GSM, however). Generalized FH [62] is a special case of synthesized FH where the number of allocated frequencies is equal to the total number of frequencies available to the operator.

As far as the MU is concerned, the difference between the two types of hopping technologies is only defined by the set of frequencies on which it is going to hop. Hence, as far as modeling the frequency allocation under FH goes, the hopping technology only determines the number of frequencies in the CA.

Both the transmitter and the receiver need to have the same hopping pattern so that they are always tuned to the same frequency.. In GSM, two types of frequency hopping patterns are used [1]:

- Under *Random Hopping (RH)*, the hopping sequence is randomly generated according to a deterministic pseudorandom generator.
- Under *Cyclic Hopping (CH)*, the hopping sequence cycles through the available frequencies in the MA from the lowest index to the highest index.

In GSM, the *Hopping Sequence Number* (HSN) is the parameter that determines the type of hopping pattern to be used: If HSN is set equal to 0 then the Cyclic Hopping is active. If HSN is set to an integer value in the interval $[1, 63]$, then random hopping is active and each value corresponds to a different hopping sequence. Each cell is allocated one HSN value.

The sequence generator in GSM also depends on the *Frame Number* (FN) which is a time counter. Each TDMA frame (of length $8 * 3/5200$ seconds) is numbered by the frame number. It cycles in the range $[0, FN\_MAX]$ (where $FN\_MAX = 26 * 51 * 2048 = 2715647$). By design, calls connected to the same cell are aligned, that is to say they have the same FN.

The *Mobile Allocation Index Offset* (MAIO) is another parameter involved in the sequence generator. It is used to avoid that two aligned TRXs with the same HSN (i.e., the same hopping sequence) interfere with each other. The MAIO makes sure that the two TRXs start hopping from different positions in the hopping sequence. For instance, consider the case of TRXs belonging to the same cell. Since these TRXs are part of the same Base-Station, the hopping sequences are aligned and also the interference level would be unacceptably high if they used the same hopping pattern (they would be using the same frequencies all the time). To avoid this situation, the hopping sequences for these TRXs are made orthogonal by adjusting them to different MAIOs.

Detailed description of the hopping mechanism and sequence generation procedure can be found in [1]. For a given TRX with specified MA (where $|MA|$ is denoted by N), HSN, MAIO and at a given FN, the *Mobile Allocation Index* (MAI) indicates the index (from 0 to $N - 1$) of the frequency in the MA to be used during frame FN. The MAI is the output of the hopping sequence generator.

The frequency hopping problem in GSM networks can be now be stated as consisting of:

- Determining the CA of each cell in the network.
- Assigning an HSN to each cell
- Assigning an MAIO to each TRX

The aim behind the introduction of FH is the reduction of the interference. From a signal processing point of view, frequency hopping has two types of direct effects: interference diversity and frequency diversity [46][69]:

- *Interference diversity* results from the change in the set of TRXs using the same frequency over time. Any TRX would experience high interference level for a short amount of time only. Continuous interference can cause several bursts in a row to be erroneous. The signal coding in GSM would perform poorly in such cases. FH reduces the probability of consecutive corrupted bursts and so coding mechanism can work better.
- The signal propagation occurs on multiple paths resulting in variations in the power of the received signals. This results in strong fading of the signal, which negatively impacts the signal quality. With *frequency diversity,* this effect is reduced since only some isolated bursts will be affected (which can be corrected with the error correcting code incorporated in GSM communication).

Recall that in the static frequency allocation problem, the objective is to allocate frequencies so as to minimize the sum of the interfered area (or traffic) in each cell. This is measured by first computing the pair-wise CIR at every point in the network (i.e., ratio of the signal from the server cell to that from the interfering cell). For each cell and with respect to every other cell, the network engineers would compile the fraction of the cell area or traffic with CIR level below some given threshold. The underlying idea is that points with CIR below this threshold would experience a large fraction of erroneous bursts. The interference and frequency diversity that results from FH reduce the probability of such errors. Thus, in order to reflect this gain, Nielsen and Wigard [69] and Nielsen et al. [67] compute the reduction of this threshold due to FH that would keep the same level of erroneous bursts. The determination of the magnitude of this reduction is based on the use of a simulation of the network environment. Such calculations are beyond the scope of this work. Therefore, we take the position as in [67], [69] and [8] so that frequency and interference diversity gains are incorporated in the interference matrices.

For the models presented in this work, the objective in managing FH has to do with the interference sharing aspect. For the static frequency allocation problem, the objective is to minimize the interfered area or traffic. Since the carrying frequency varies under FH, the time dimension is introduced in the objective. This is presented with more details when describing each part of the FH management problem. Note that in the experimental analysis carried out in this chapter, we use the same interference matrices as the ones used for static frequency allocation. This is due to two reasons: the first is the lack of a simulation environment that would enable us to compute the interference and frequency diversity gains. Secondly, it is desirable to highlight the interference sharing aspect which corresponds to our aim behind the optimization of resource allocation under FH.

Dynamic frequency hopping (DFH) has been proposed to improve upon the conventional frequency hopping [52]. The idea here is to use dynamic resource allocation for the purpose of interference avoidance: While in conventional FH the hopping patterns are pre-determined (given the MAs, HSNs and MAIOs assignments), with DFH, the hopping patterns are built in real-time in order to adapt to changes in the interference, which in turn depends on the set of active users.

Under static frequency allocation and conventional FH, a worst-case scenario is often considered, where all TRXs are assumed to be active. Hence, DFH is able to cope with changes in the distribution of mobile units in time and space. However, DFH requires continuous measurement of the interference at every frequency hop and the transmission of these measurements to some central controller. This implies an overhead cost on the network that may be significant. A second requirement for DFH is system wide synchronization. As indicated in [52], such a feature would be feasible in wireless standards such as EGPRS (an extension of GSM) and IS-136.

For the case where synchronization is feasible, we propose the use of optimized pre-defined hopping patterns either by optimizing the HSN-MAIO allocation or by building the actual sequences. By using optimized hopping patterns, we attempt to control the interference levels and to distribute interference overtime for all TRXs in the network. Since the proposed optimized hopping patterns are pre-defined, there is no overhead cost on the network associated with measuring, signaling and selecting frequencies as is the case in DFH.

Frequency hopping is often analyzed in light of elementary frequency re-use schemes according to which the cells are assumed to have the same nice hexagonal shape, and where traffic and wave propagation distributions are assumed to be uniform (e.g., [46][94][68]). These assumptions are not realistic, especially in view of the fact that frequency hopping is most useful for large dense networks that often have highly irregular propagation patterns and varying traffic density.

The MA allocation problem under FH is considered to be similar to the frequency allocation problem. However, little work has considered the specific aspects of FH. In [77], Simulated

Annealing was used in conjunction with simulation for the construction of the MA. The simulation system is used to evaluate frequency plans at the level of the subscriber, as opposed to the common FAP approach (i.e., using cell co-channel and adjacent interference based objective function). In [67], a heuristic algorithm that combines a search tree with dynamic constraint setting is proposed (more details can be found in [69]). The focus of this work is to demonstrate how to incorporate the effect of both interference and frequency diversity within the objective function. The resulting function is similar to static minimum interference FAP, except for the addition of weights that account for the frequency hopping effects.

Bjorklund et al. [8] consider MA allocation problem under FH as a generalization of the static frequency allocation problem. In static FAP, the number of frequencies allocated to each cell is equal to the number TRXs. The generalization allows the allocation of a number of frequencies that is larger than the number TRXs. The proposed model determines the size of the MA while considering the separation constraints of the same type as for static frequency allocation. The authors propose a Simulated Annealing algorithm with a neighborhood defined by the change of the MA of a randomly selected cell. Bourjolly et al. [15] (this model is presented in detail in section 4.3.1) consider a similar model but recommend a redefinition of the separation constraints in order to further exploit the gains from FH. The model also differs in the objective function by attempting to minimize the maximum cell interference as opposed to minimizing total interference. The argument here is that using the Min-Max approach during the MA allocation results in lower instantaneous interference, while the switching of frequencies through FH would take care of the averaging of interference over time.

The HSN-MAIO assignment problem has not received much attention either. The HSN and the MAIO parameters are used to control FH in GSM networks by creating virtually decorrelated hopping sequences. The HSN can take integer values in the interval $[0, 63]$ and for a TRX i with an MA of size N_i, the MAIO can take integer values in the interval $[0, N_i - 1]$. Gamst [34] examined the problem of managing the HSN. The problem is modeled as a coloring problem on a graph whose nodes correspond to cells and whose edges represent the existence of a common frequency in the MAs of the corresponding cells. The resulting HSN assignment problem prohibits the use of the same HSN for a pair of cells if they use at least one common frequency. Nielsen and Wigard [69] recommend a similar approach to managing the HSN allocations. The underlying assumption of this approach is that sequences with different HSN are not correlated. This is contradicted by the findings in [70].

In fact, Nyberg et al. [70] study the collision properties of random hopping sequences as a function of the MA, HSN, MAIO and FN. Here, the collision rate between 2 sequences is defined as the fraction of bursts for which the two sequences use interfering frequencies. For independent random sequences the collision rate should depend on the MA allocation only. The authors show that these sequences are in reality correlated and the level of correlation depends on the assigned HSN-MAIO values in addition to the MA allocations. This implies that the HSN, MAIO and FN values jointly determine the real level of collisions between any 2 sequences. Therefore, just assigning different values for the HSN or MAIO may not guarantee that the resulting sequences will have a low collision rate.

Nyberg et al. [70] suggest generating a correlation table for all possible parameters values. For synchronized networks, the authors suggest using this table in an on-line mode for hopping sequence selection. The on-line requirement stems from the fact that the level of correlation depends on the difference in the FN of the two sequences. Since hops occur at the same time in synchronized networks, the use of highly correlated sequences would result in systematic interference, which is exactly what FH is designed to avoid. For asynchronous networks, the authors suggest computing the spread of collision rates as a function of the MAIO and FN and minimizing such spreads. The difficulty of this approach is that for every pair of TRXs, we need to calculate the collision rate for all possible HSN-MAIO combinations over a range of FNs. This involves significant computational and memory requirements.

While radio spectrum management in a static environment has received considerable attention, little work has been done for the context of frequency hopping. In the next section, we propose to optimize the mobile allocations (MA) as well as the HSN-MAIO assignments. In addition, for synchronized networks, we propose the optimization of the hopping sequence (as opposed to optimizing the HSN-MAIO parameters for the GSM sequence generator). We take the position of optimizing FH in off-line mode so as to avoid the overhead cost associated with real-time management.

4.3 Proposed Solutions for Optimizing Frequency Hopping

The previous section introduced the frequency hopping mechanism. It also showed that the resource allocation under FH depends on the network synchronization level. In this work, we view the resource allocation under FH in two steps: first build the MA for each TRX and then determine their hopping sequences. As illustrated in Figure 4.1, we distinguish between 3 types of networks. As the name implies, asynchronous networks are those with no synchronization at all. Thus if one considers any two TRXs belonging to different cells, their corresponding frame numbers can take any value in the allowed range (0 to FN_MAX). We denote by hop-synchronized network those where the frequency switch occurs at the same time, but the frame number for any 2 TRXs could be different (ΔFN is the difference in frame number). The third type of networks corresponds to the case where the frame number is always the same for all TRXs at any time. This distinction is useful for evaluating hopping sequences.

The proposed approach to optimize FH consists of the following steps:

1. Construct the MA for each TRX
2. Determine the hopping pattern for each TRX
    2.1. For networks supporting full-synchronization, build optimized hopping sequences
    2.2. For networks that do not support full-synchronization or that cannot support optimized hopping sequences, determine the HSN and MAIO assignment.

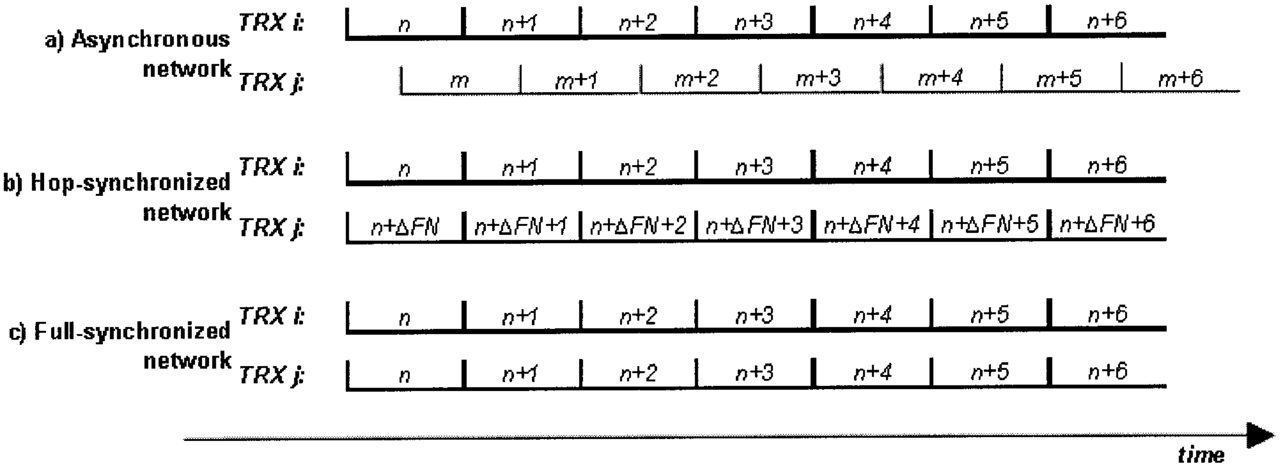

Figure 4.1: FH: Network synchronization levels

4.3.1 Optimizing MA Allocation

The first step of managing frequency hopping is to build the MA, i.e., determining the frequencies on which hopping will take place. The work described in this section is partly published in [15] . Before describing the algorithm for optimizing MA allocation, we must identify the required input. Under static frequency allocation, optimization problems require as inputs the Co-channel and Adjacent-channel interference matrices. Each entry in these matrices corresponds to the interference level that one cell exerts on the other if they use the same frequency (or if adjacent frequencies are used in the case of adjacent-channel interference). One possible interference measure is expressed in terms of the relative size of the area in which the Carrier to Interferer Ratio (CIR) is lower than a given threshold. The same data is also required for determining hopping patterns.

In addition, static frequency allocation requires the separation matrix as input. Each entry in this matrix corresponds to the minimum frequency separation S_{ij} between a pair of cells (i, j). In other words, if frequency f is allocated to cell i, then all frequencies g allocated to cell j must satisfy: $|f - g| > S_{ij}$. Minimum separations are introduced in order to ensure sufficient service quality for specific links in the network. The diagonal entries of the separation matrix (S_{ii}) represent the minimum separation between frequencies allocated to the different TRXs of a same cell.

The proposed MA construction algorithm uses a partially relaxed form of these separation constraints to build the MA. The relaxation is later dropped when building the optimized hopping sequences for synchronized networks. In this approach, we still require that the minimum within-cell separation be satisfied at all times. In addition, in every cell, one TRX is responsible for carrying the Broadcast Control Channel (BCCH), which is not allowed to hop. Since this channel is very important in establishing calls and ensuring synchronization, signaling and hand-over, we require that separation constraints between any TRX and all BCCH-carrying TRXs must be satisfied.

Under Synthesized FH, the number of frequencies allocated to a cell may be larger than the number of TRXs. If the original separation constraints are used, then the size of the MA will be bounded from above due to these constraints, and this bound is likely to be very strict. However, at any time during the hopping process, only a subset of these frequencies is active. Thus, the original separation constraints may be over-protective. Therefore, in the relaxed-separation constraints, a frequency f is a candidate to be included in the MA of a cell i if, for each remaining cell j, the MA of j contains a subset of frequencies such that:

- All frequencies in this subset satisfy separation constraints with f, and

- This subset contains a number of frequencies greater than or equal to the number of TRXs in cell j.

This frequency f still needs to satisfy original minimum separation with all BCCH-carrying TRXs and with the other TRXs of cell i. This relaxation is justified because, in essence, FH works against having high interference levels for several consecutive time periods, which could result in communication dropping. Therefore, the proposed relaxation is a way of increasing communication capacity through a better use of the TCHs while preserving interference-free guarantee for the critical non-hopping service provided by the BCCHs.

For this first phase of managing frequencies under FH, the objective function used is to minimize the maximum interference level. An alternate measure would be to minimize total interference. However, this would often result in having a small number of cells with high interference levels that the frequency hopping cannot overcome. For this reason, the Min-Max approach was selected.

Given an initial partial MA, the following algorithm sequentially increments the MA of all cells until the desired size is reached. At each step, the selected frequency causes the minimum increase in the maximum cell interference level.

MA construction procedure:

1. Initialization, Generate an initial *MA* for each cell, using a static frequency allocation algorithm. DOCAF[1] is used to allocate frequencies to BCCH-carrying TRXs (which do

---

[1] DOCAF^{TM} is a commercial frequency allocation software package for GSM and AMPS networks, developed by Prestige Telecom, Canada.

not hop) and provide an initial *MA* for each cell in such a way as to satisfy all separation constraints. The size of this initial *MA* is equal to the number of TRXs in the cell.

2. Compute the amount of interference incurred by each cell: Each time the same frequency appears in the *MA* of two cells, or adjacent frequencies are in the *MA* of two cells, the interference levels of these cells is augmented by the appropriate interference value (from the co-channel and adjacent channel interference matrices).

3. Set the number of passes (Nb_pass), i.e. the target increase in *MA* size. (This is a parameter that is determined by the user). Under Baseband FH, there would be no increase, and the algorithm would stop. This parameter is the same for all cells. A potential improvement to the algorithm may be to allow this parameter to vary across cells.

4. For $pass = 1$ to Nb_pass Do

4.1.Order the cells in decreasing order of interference levels

4.2.Select the cell with the highest interference level that has not been selected in the previous pass. Call it cell **i**.

4.3.For each frequency **f** that satisfies the relaxed separation constraints with respect to cell **i** :

4.3.1.Evaluate the impact of adding **f** to the *MA* of cell **i**:

4.3.1.1.Let C_f denote the cell that would show the highest interference level as a consequence of adding **f** to the *MA* of cell **i**.

4.3.1.2.And let L_f denote the interference level of cell C_f

4.3.2.Let $\mathbf{f}^* = \underset{f}{argmin}\{L_f\}$

4.4.Add frequency **f*** to the *MA* of cell **i**.

(If a feasible **f*** is not found, disregard cell **i** in future executions of steps 4.1 and 4.2 during this pass, and go to step 4.1.)

4.5. Update the interference levels as in step 2. Go to step 4.1.

4.3.2 Optimized Hopping Sequences

Cyclic or Random hopping sequences are obtained by manipulating the HSN-MAIO parameters. We propose the use of *Optimized Hopping* (OH) as a third type of hopping pattern. This proposal requires a fully-synchronized network (case c in Figure 4.1). OH does not use the GSM sequence generation mechanism but involves the determination of the hopping sequences. The work described in this section is partly published in [15].

Optimized Hopping sequences correspond to scheduling the use of the frequencies available in the MA of each TRX over time. We consider a time horizon H. The sequence of length H is then repeated indefinitely. We can view sequence generation under Optimized Hopping as generating entries to fill a matrix whose columns corresponds to the TDMA frames in the horizon H and the rows correspond to the hopping TRXs of the network. An entry in this matrix corresponds to the index of the frequency that is to be used at a particular TRX and in a particular TDMA frame over the given horizon.

Similar to the MA allocation phase, the input data needed are the co-channel and adjacent-channel interference matrices and the original separation matrix. During the MA construction phase, the relaxed form of the separation constraints is used. For generating the optimized hopping sequences, we use the original separation constraints as described above. This approach may be over-protective: under static frequency allocation, the original separation constraints are protection against high interference levels. Since under FH the carrier frequency changes, less protection would be required. However, in order to allow comparison with static frequency allocation solutions, we still use the original constraints.

The objective typically used under static allocation is to minimize the sum or, equivalently, the average interference across all TRXs in the network. Because FH introduces the time dimension into the problem, this must be captured in the optimization objective function. In addition, the aim of frequency hopping is to achieve interference averaging across TRXs and over time. More specifically, its aim is to reduce the number of consecutive TDMA frames in which interference is high so as to allow error-recovery algorithms to function properly. Thus, the objective selected in our approach is to minimize the maximum cumulative interference incurred. Note that we consider a short horizon H so that the accumulated statistics are representative of the interference perceived by the user. In other words, for any given solution, the interference incurred of each TDMA frame is computed for each TRX. These are then summed across all frames in the horizon to produce the cumulative interference for each TRX. The quality of any solution is measured by the maximum cumulative interference. The algorithm attempts to minimize this quantity. An alternative objective could have been to compute the average cumulative interference. This type of objective may produce a solution with lower average but with some TRXs having very high interference. This may be considered as conflicting with the initial objective, i.e., interference averaging across TRXs. Moreover, using the maximum cumulative interference as an objective has the added advantage of producing solutions where high interference levels are not incurred during consecutive TDMA frames, thus avoiding call drops.

The following heuristic sequentially builds the hopping sequence of all TRXs in the network over the time horizon.

Optimized Hopping sequence generation procedure

```
1. Initialization
2. For $t = 1$ to H Do:
   2.1. Compute the interference level incurred by each TRX: Each time the same frequency is assigned to two TRXs, or adjacent frequencies are assigned to two TRXs, the cumulative interference levels of these TRXs is augmented by the appropriate interference value (from co-channel and adjacent channel interference matrices)
   2.2. Order the TRXs in decreasing order of interference levels.
   2.3. Select the TRX with the highest cumulative interference level that was not selected in the current iteration t. Call it TRX i.
   2.4. For each frequency f that satisfies the original separation constraints with respect to TRX i:
      2.4.1. Evaluate the impact of using f at TRX i at time t:
         2.4.1.1. Let $T_f$ denote the TRX of the network that would show the highest interference level as a consequence of using f at TRX i at time t.
         2.4.1.2. And let $L_f$ denote that interference level
```

2.4.2.Let $\mathbf{f}^* = \underset{f}{argmin}\{L_f\}$

2.5.Assign frequency $\mathbf{f}^*$ to be used by TRX **i** at time t.

2.6.Update the interference levels as in step 2.1. Go to step 2.2.

4.3.3 Optimizing HSN-MAIO Allocation

Our proposal for optimizing the hopping sequence requires full network synchronization. However, with current network equipment, hopping sequences are the output of the deterministic sequence generator already implemented in GSM. In this case, hopping sequences are generated by determining an HSN-MAIO plan. For all TRXs, under Random or Cyclic hopping, any sequence s results from an HSN assignment h to its cell and an MAIO assignment m. So we denote $s = (h, m)$.

We can define the following:

- $x_{ch} = \begin{cases} 1, & \text{if cell } c \text{ is assigned HSN } h \\ 0, & \text{otherwise} \end{cases}$
- $y_{im} = \begin{cases} 1, & \text{if TRX } i \text{ is assigned MAIO } m \in \{0,..,|MA_c|-1\} \text{ where } i \in c \\ 0, & \text{otherwise} \end{cases}$
- $C^{co}_{(is,jr)}$ is the co-channel collision rate that results from using sequence $s = (h, m)$ in TRX i and sequence $r = (h', m')$ in TRX j.
- $C^{ad}_{(is,jr)}$ is the adjacent-channel collision rate that results from using sequence $s = (h, m)$ in TRX i and sequence $r = (h', m')$ in TRX j.
- Co_{ij} is the co-channel interference level between TRX i and TRX j.
- Ad_{ij} is the adjacent-channel interference level between TRX i and TRX j.

- $T_{(is,jr)} = Co_{ij}C^{co}_{(is,jr)} + Ad_{ij}C^{ad}_{(is,jr)}$ is the total collision cost associated with using sequence $s = (h, m)$ in TRX i and sequence $r = (h', m')$ in TRX j.

Similar to the separation constraints in the static frequency allocation problem (see Chapter 3), one can impose constraints that would prohibit the simultaneous use of a pair of sequences at a pair of TRXs if this causes a high level of interference. For instance, such a simultaneous use of sequence s at TRX i and sequence r at TRX j could be prohibited if the following condition holds:

$$\begin{aligned} Co_{ij}C^{co}_{(is,jr)} &> \theta^{co} \\ &\text{or} \\ Ad_{ij}C^{ad}_{(is,jr)} &> \theta^{ad} \end{aligned} \tag{4.1}$$

where, θ^{co} and θ^{ad} are some prescribed threshold values. Such condition would hold if the joint effects of interference level (co-channel or adjacent channel) and collision rates were too high.

The simultaneous HSN-MAIO assignment problem can now be formulated as follows:

$$\underset{x_{ch}, y_{im}}{Minimize} \sum_{\substack{i \in c_1, j \in c_2 \\ s=(h,m), r=(h',m')}} T_{(is,jr)}(x_{c_1 h} y_{im})(x_{c_2 h'} y_{jm'}) \quad (4.2)$$

$$\begin{aligned}
\sum_h x_{ch} &= 1 \quad \forall c && (4.3)\\
\sum_m y_{im} &= 1 \quad \forall i && (4.4)\\
x_{c_1 h} y_{im} + x_{c_2 h'} y_{jm'} &\leq 1 \quad \begin{array}{l} \forall i, j, \text{ where } s = (h, m) \text{ and } r = (h', m') \\ \text{for which 4.1 holds} \end{array} && (4.5)\\
x_{ih}, y_{im} &\in \{0, 1\} && (4.6)
\end{aligned}$$

In the above formulation, the objective function 4.2 attempts to minimize the total expected interference. Constraints 4.3 insure that every cell is assigned one HSN value and constraints 4.4 insure that every hopping TRX is assigned one MAIO value. For BCCH-carrying TRXs, we have $y_{jo} = 1$ (i.e. automatically assigned to MAIO 0 since the MA contains only one frequency). Constraints 4.5 forbid the joint assignment of a pair of sequences that have an excessive collision rate. Instead of using such constraints 4.5, we can replace the objective function coefficients corresponding to the prohibited pair of sequences by a large number.

The difficulty with this formulation for the HSN-MAIO assignment problem resides in computing the $T_{(is,jr)}$ values. In order to get an idea about the number of such values that need to be computed, we denote:

- C: Set of cells

- d_c: Number TRXs in cell c.
- MA_c: Mobile allocation of hopping TRXs in cell c.

There are 64 possible HSN values for each cell and for each TRX the MAIO can have any of $|MA_c|$ values. Thus, the above model requires:

- Number of x_{ch} variables is $64|C|$
- Number of y_{ih} variables is $\sum_{c \in C}(|MA_c| - 1)(d_c - 1)$
- For each pair of cells we need to compute the collision between all possible sequences. Thus the total number of $T_{(is,jr)}$ parameters to be computed is $(64|MA_c|)^2 N^2/2$.
- For example: if $N = 100$ cells and $d_c = 3$ and $|MA_c| = 6$ for all cells, then the model involves 8200 variables and the computation of about 738 million $T_{(is,jr)}$ terms.

Given the huge number of simulations that need to be run to compute the $T_{(is,jr)}$ terms, we need to reduce the model size. A first approach is to restrict the model by looking at the possible collisions between a small subset of cells S_c for each cell c, and hence, making the implicit assumption that collisions with other cells not in S_c are not significant. For instance, we can identify for each cell c a set S_c of critical neighbors. This set may contain all cells with high potential interference with cell c.

The above problem simplification may still require a significant number of simulations that need to be run. A second approach would be to decompose the problem: First assign an HSN value to each cell as suggested in [34] and [69], and in a second step, assign an MAIO value to each TRX. In [34], the proposed model consists of assigning HSNs to cells such that no two cells whose MA share a frequency can be allocated the same HSN. Given a fixed HSN assignment, it remains now to assign the MAIOs to TRXs. This 2-step HSN-MAIO allocation procedure is based on the idea that for a given pair of cells that are sharing a frequency, the collision rates are more significant if they are assigned the same HSN. This is not necessarily true. In [70], it was shown that the collision depends on HSN, MAIO and FN. Because of this, we opt to assign both HSN and MAIO simultaneously.

So far we have not addressed the issue of calculating the $T_{(is,jr)}$ parameters. This depends on the types of network synchronization that are depicted in Figure 4.1. The next paragraphs describe the procedures for calculating these parameters and the HSN-MAIO assignment procedures corresponding to each level of synchronization. The input to the procedures are the list of cells and the number of associated TRXs, the cell CA list as well as the co-channel and adjacent channel interference matrices.

4.3.3.1 HSN-MAIO assignment in a Fully-Synchronized network

For a fully-synchronized network one can evaluate the co-channel and adjacent channel collision rates between hopping sequences given an MA, HSN and MAIO assignment for each TRX. This is done by generating all sequences and counting all instances of co-channel or adjacent channel hits. This is possible since all TRXs share the same frame number (FN).

In GSM, the complete hopping sequence has a length of FN_MAX (where $FN\_MAX = 26 * 51 * 2048$). Any algorithm designed to optimize the HSN-MAIO assignment requires the generation of several sequences for each TRX (corresponding to different HSN and MAIO choices) and computing collision rates between pairs of sequences. Thus, generating the complete sequences is very time consuming and hence it is preferable to generate just a partial sequence. In addition collision rates computed over a partial range are not likely to differ significantly from those computed over the full sequence length as long as the partial range is not too short. In our experimental investigation we use a length of one Superframe (defined in GSM as 26*51 TDMA frames).

For the HSN-MAIO assignment in a fully-synchronized network, we use a heuristic procedure: Starting from an empty solution, the heuristic sequentially builds a partial solution by considering each cell in turn. For each cell, the heuristic determines the best cell HSN and the best combination of MAIO assignments for all of its TRXs. The determination is such that it causes the minimum increase in the objective function 4.2 in the HSN-MAIO model described above. We denote by $s_i^* = (h_c^*, m_i^*)$ the assignment of HSN h_c^* to cell c and MAIO m_i^* to TRX i of cell c. The assignment procedure can be summarized as follows:

Optimized HSN-MAIO assignment: Case of Fully Synchronized networks

```
1.  Initialization
    1.1.  Let $D = \emptyset$ and $D^0 = C$
    1.2.  Randomly order the cells in $D^0$
2.  Considering each cell $c \in D^0$ in order:
    2.1.  For all possible HSN assignments, $h_c$
        2.1.1. For all possible MAIO combinations $\{m_i, \forall i \in c\}$ corresponding to all TRXs
```

```
    $i \in c$
        2.1.1.1 Compute all $T_{(is_i,jr_j^*)}$ where $j$ is a TRX belongs to cells $c' \in D$ (that have
                already been considered) and where $s_i = (h_c, m_i)$ and $r_j^* = (h_{c'}^*, m_j^*)$
    2.1.2. Let $\Delta T^* = \min\limits_{(h_c,\{m_i\})} \sum\limits_{i \in c} \sum\limits_{j \in c' \in D} T_{(is_i,jr_j^*)}$ and $(h_c^*, \{m_i^*, \forall i \in c\}) = \operatorname*{argmin}\limits_{(h_c,\{m_i\})} \sum\limits_{i \in c} \sum\limits_{j \in c' \in D} T_{(is_i,jr_j^*)}$
    2.1.3. let $D = D \cup \{c\}$ and $D^0 = D^0 \backslash \{c\}$
```

The above procedure builds a solution for the HSN-MAIO assignment problem in a fully-synchronized network. It could be further improved by running the procedure a number of times and starting each time with a differing ordering of the cells.

4.3.3.2 HSN-MAIO assignment in a Hop-Synchronized network

In a hop-synchronized network, the hopping takes place at the same time but TRXs in different cells do not have the same FN. Thus at a particular TDMA frame, if TRX i is using frequency f, then the frequency g being used at another TRX j depends on the difference in FN between the cells (for a fixed MA, HSN and MAIO assignment). For a hop-synchronized network, if at any instance, 2 TRXs use interfering frequencies, then we know that the duration of this interference lasts for exactly one frame. This is to be opposed to the asynchronous case where this does not hold.

In a hop-synchronized network, one needs to know the FN for each TRX in addition to knowing the MA, HSN and MAIO assignment in order to compute the collision rates between any two TRXs. However, the difference in FN of two TRXs is not known in advance. Therefore, we propose that the HSN-MAIO planning be done by averaging the collision rates between TRX pairs over all possible FN values. Let :

- ΔFN : difference in frame number between TRXs i and j. Note that if i and j belong to the same cell then $\Delta FN = 0$ since TRXs in the same cell are aligned. Since FN can take any value in the range 0 to FN_MAX, then ΔFN can take values in the range 0 to $FN\_MAX - 1$.
- $C^{co}_{(is,jr,\nabla FN)}$ and $C^{ad}_{(is,jr,\Delta FN)}$ are the co-channel and adjacent channel collision rate that results from using sequence $s = (h, m)$ in TRX i and sequence $r = (h', m')$ in TRX j with a frame number difference of ΔFN.
- Averaging over possible ΔFN, we obtain:

$$\begin{aligned} C^{co}_{(is,jr)} &= \frac{\sum\limits_{\Delta FN=0}^{FN\_MAX-1} C^{co}_{(is,jr,\Delta FN)}}{FN\_MAX}, \\ C^{ad}_{(is,jr)} &= \frac{\sum\limits_{\Delta FN=0}^{FN\_MAX-1} C^{ad}_{(is,jr,\Delta FN)}}{FN\_MAX} \end{aligned}$$

Similar to the fully-synchronized case, computing $C^{co}_{(is,jr,\Delta FN)}$ over all the sequence length (FN_MAX) is very time consuming. In the hop-synchronized network these computations need to be computed an even larger number of time (because of the averaging over possible values of ΔFN). Thus for the same reasons as for the fully-synchronized network, we also generate the sequences over a partial range. In our experimental investigation we use a length of one Superframe (defined in GSM as 26*51 TDMA frames).

The computational times are still too high as the collision rates need to be computed over the full range of possible ΔFN. To overcome this difficulty, we compute $C^{co}_{(is,jr)}$ and $C^{ad}_{(is,jr)}$ by sampling over the possible values of ΔFN. For our experimental investigation this sample has a size of 100 points. Initial experimentation indicated that without this sampling, just evaluating a given HSN-MAIO plan takes over 4000 seconds for a network with 605 TRX. The use of sampling reduced this to about 50 seconds. As for the accuracy of the sampling approach, obviously increasing the sample size would improve the accuracy. A sample size of 100 points appeared to produce sufficiently precise results for the networks tested.

For HSN-MAIO assignment the procedure we propose for the hop-synchronized network is similar in structure to the one used for the fully-synchronized network by incrementally building a partial solution. The main difference lies in the computation of the hopping collision rates as described above.

4.3.3.3 HSN-MAIO assignment in an Asynchronous network

In an asynchronous network, and at any instance, the time difference between any TRX pair belonging to different cells can be anywhere within the sequence length. Thus, in order to compute the collision rates between sequences one can use a Monte Carlo-based simulation procedure as the one used in the hop-synchronized network. In such a case a collision between two sequences would not occur necessarily for the full duration of a TDMA frame, but would only involve part of the frame. So the simulation would involve examining the system with a time line discretized to a very small level (a fraction of a time frame). This is time consuming. Embedding such simulation into an optimization algorithm would be extremely time consuming.

For the investigation carried in this chapter we compute the theoretical performance of the asynchronous network by computing expected collision rates based on the assumptions that 1) the pseudo-random sequence generator used in GSM network procedures un-correlated sequences and 2) the frame numbers in different sequences are independent.

We denote by $f_i(t)$ the frequency used by TRX i at instant t. Based on the random sequence assumption, the probability that a frequency f is being active in TRX i is given by:

$$Prob(f_i(t) = f) = \begin{cases} 0, \text{ if } f \notin MA_i \\ \frac{1}{|MA_i|}, \text{ if } f \in MA_i \end{cases}$$

where MA_i is the set of frequencies on which TRX i is allowed to hop. Since sequence hop times are assumed independent, we have:

$$Prob(f_i(t) = f \text{ and } f_j(t) = g) = \begin{cases} 0, \ \text{ if } f \notin MA_i \text{ or } g \notin MA_j \\ \frac{1}{|MA_i|}\frac{1}{|MA_j|}, \text{ if } f \in MA_i \text{ and } g \in MA_j \end{cases}$$

Under the assumptions of independence of the hop times and the randomness of the hopping sequences, one can then compute the joint probability distribution governing the hopping patterns of two sequences. This depends on the MAs only and is independent from any HSN-MAIO assignments. Using these computations, one can compute the expected co-channel and adjacent-channel collisions between two TRXs and so we have:

- $C^{co}_{(is,jr)} = |MA_i \cap MA_j| \frac{1}{|MA_i|}\frac{1}{|MA_j|}, \forall\ s, r$, and
- $C^{ad}_{(is,jr)} = |\{f : \text{such that } f \in MA_i \text{and } f \pm 1 \in MA_j\}| . \frac{1}{|MA_i|}\frac{1}{|MA_j|}, \quad \forall\ s, r.$

4.4 Experimental Results

In this section we examine the impact of optimizing resource allocation in the context of FH in two real-life networks. The larger data set is used to analyze FH optimization in detail in the case of a fully-synchronized network. The second data set includes an actual FH plan which has been implemented by the network operator. We use this data set to compare the proposed algorithms for all levels of synchronization.

4.4.1 Data Set 1

The first data set considered in these experiments corresponds to a real-life network with 337 cells, and a total of 927 TRXs. A cell contains a maximum of 3 TRXs. The available bandwidth consists of 50 frequencies to be used both by the BCCH-carrying TRXs and the TCH-carrying TRXs. For Optimized Hopping, we consider a hopping sequence of length 50. The separation constraints require a separation of 3 within a cell and a separation of 2 within a given site. In the experiments described below, Optimized, Random and Cyclic Hopping, and static frequency allocation under DOCAF are compared. The comparisons are based on the same MAs, which were allocated using the procedure described above. Also, the static frequency allocation corresponds to the initial MA allocation used in step 1 of the MA construction algorithm.

It is also necessary to mention that for the purpose of comparison, the interference matrices used for static and FH frequency allocation are the same. Recall that the deployment of FH in a GSM networks produces several effects which reduce the severity of the interference between cells. For instance. an entry in the co-channel interference matrix corresponds to the fraction of traffic or area with CIR level below some predefined threshold. This parameter in turn corresponds to a maximum allowed level of the frame erasure rate (FER). The use of FH improves the error recovery algorithms and the signal processing capabilities of the network. This implies that with FH, one can achieve the desired level of FER with a less severe level of CIR threshold. For these experiments the interference reduction that results from the frequency and interference diversity gains is not incorporated, and so the focus is on the gain due to interference sharing that results from the frequency allocation.

In order to investigate the impact of the MA size, we allowed it to vary from the number of TRXs of a cell to that number plus 10. Figure 4.2 and Figure 4.3 show the impact of the MA size on the TRXs' cumulative interference (averaged over time to allow comparison with static frequency allocation). In these figures, Static corresponds to static allocation. OH, CH and RH correspond to Optimized, Cyclic and Random Hopping, respectively. At this point, it is important to note that the effect of hopping can only be fully appreciated as a function of time, and that averaging over time may be misleading since a lower average over a network can hide the fact that several calls cannot be carried through because of some high interference being present at the corresponding TRXs over the whole horizon; this is particularly true when comparing FH with static allocation: for instance, Figure 4.2 shows that for the data at hand, static allocation results in an average TRX cumulative interference that is lower than what it is under frequency hopping, although it is not as good. Figure 4.3 shows that Optimized Frequency Hopping outperforms all three other methods with respect to the maximum cumulative interference. These Figures also indicate that an MA size equal to the number of TRXs plus 5 provides a good balance for OH in terms of deterioration in average and improvement in maximum cumulative interference. Beyond this value, the curves level off.

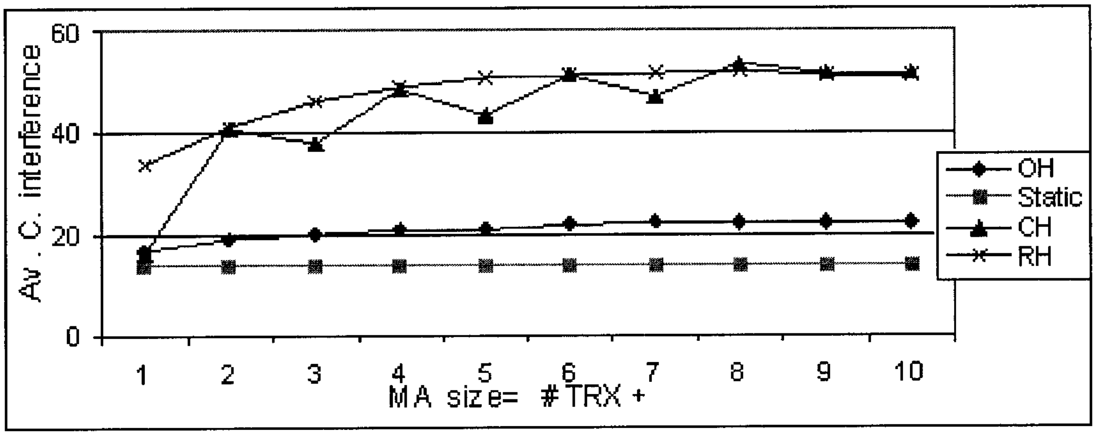

Figure 4.2: FH: Variation in average cumulative interference with MA size.

Figure 4.4 shows the index of the TRX that incurred the maximum cumulative interference up to each of the 50 TDMA frames in the horizon that we consider, under Optimized Hopping. Similarly, Figure 4.5 shows the index of the TRX that incurred the maximum interference during each TDMA frame under Optimized Hopping. Note that the index of the TRX with the highest interference changes with time in both figures. A similar pattern is also observed under Random and Cyclic Hopping. However, Optimized Hopping outperforms Cyclic and Random Hopping in terms of interference levels as shown in Figures 4.6 and 4.7.

To illustrate the impact of hopping, Figure 4.8 shows the interference incurred by a randomly

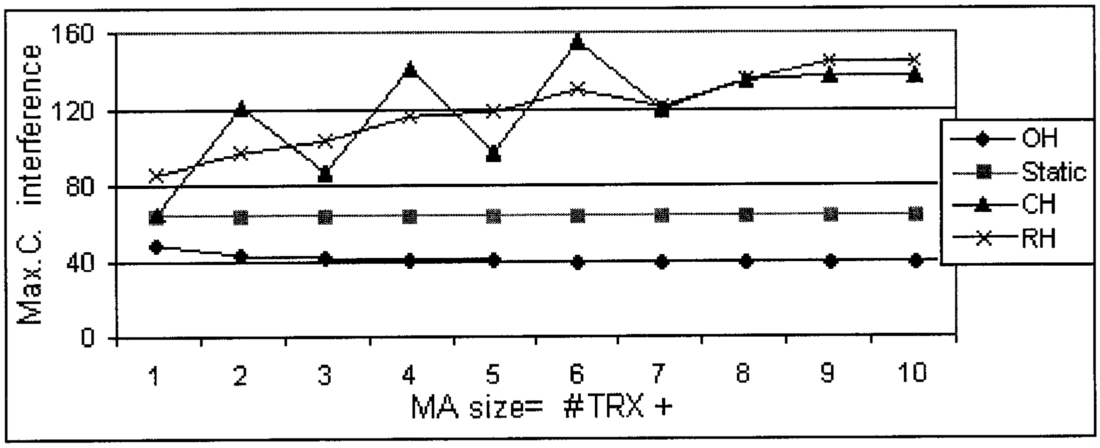

Figure 4.3: FH: Variation in maximum cumulative interference with MA size.

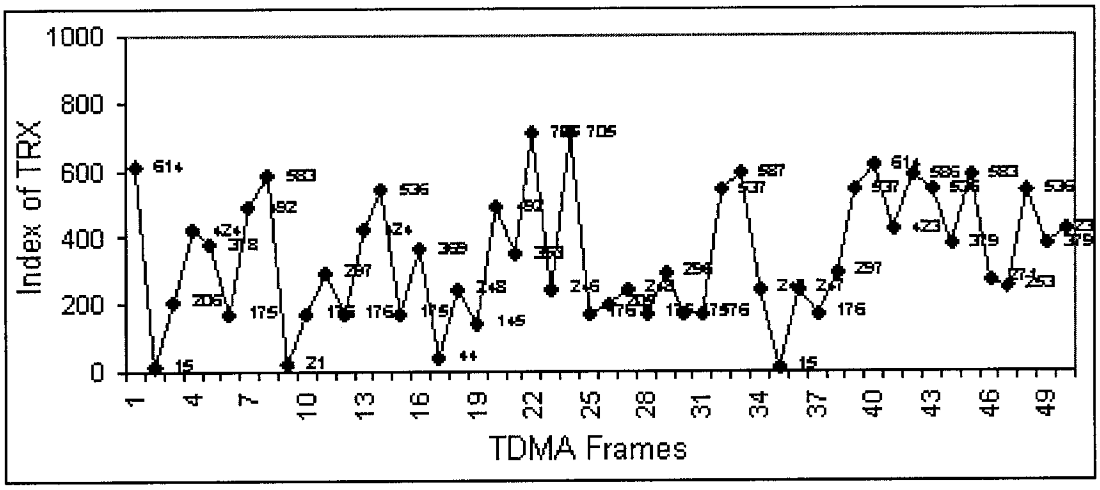

Figure 4.4: FH: Index of TRX carrying maximum cumulative interference.

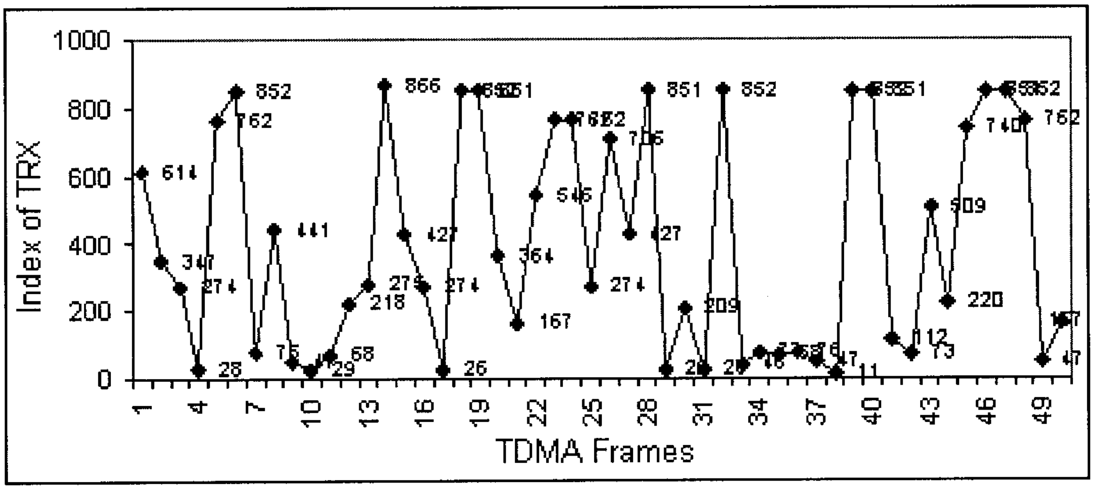

Figure 4.5: FH: Index of TRX carrying maximum interference.

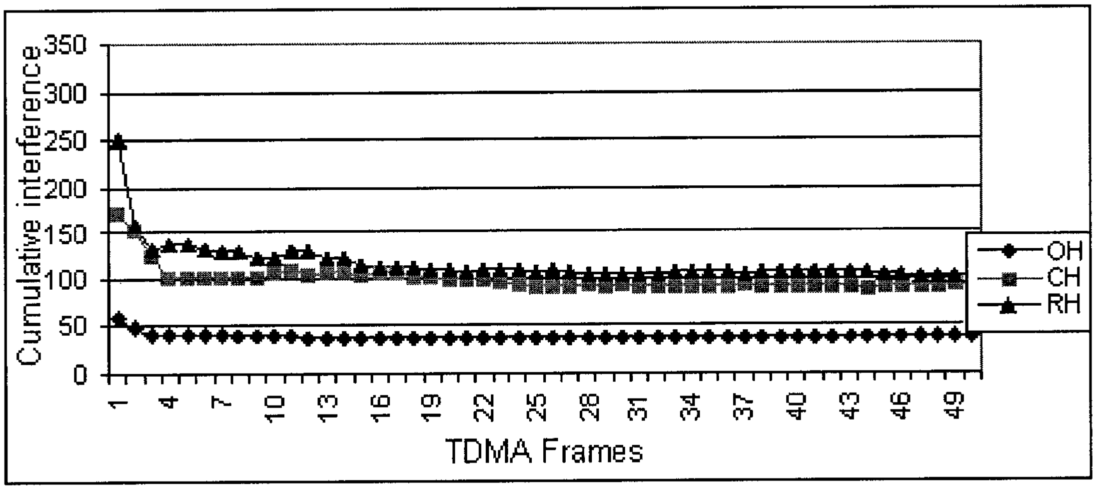

Figure 4.6: FH: Maximum cumulative interference.

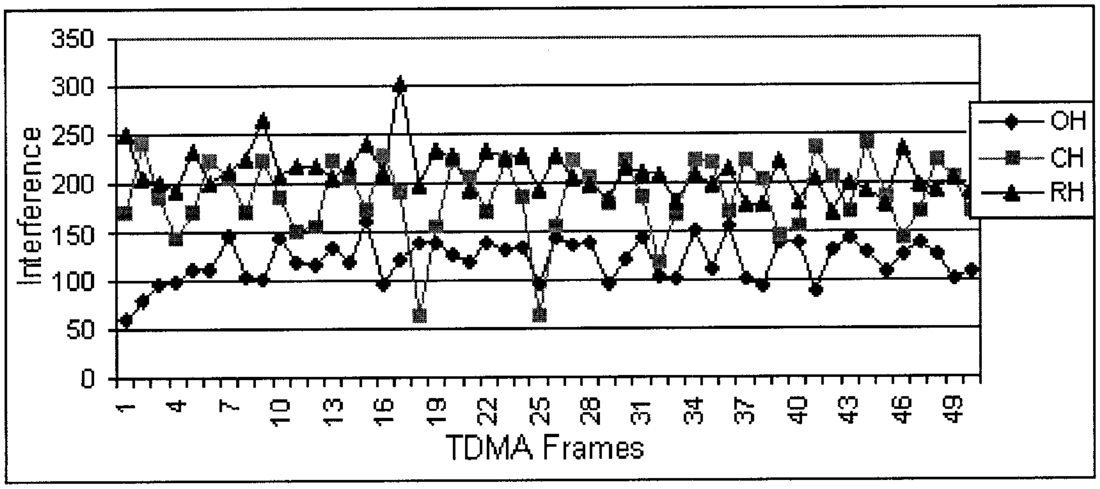

Figure 4.7: FH: Maximum interference.

selected TRX that is allowed to hop. Figure 4.9 shows the evolution of interference for a randomly selected non-hopping TRX (a BCCH-carrying TRX) that belongs to the same cell as the one shown in Figure 4.8. In both cases, Optimized Hopping shows lower interference levels than Cyclic and Random Hopping. For these specific TRXs, Static allocation often shows lower levels of interference than Optimized Hopping. The reason for this is that OH reduces interference levels for some other TRXs for which interference levels under static allocation are high.

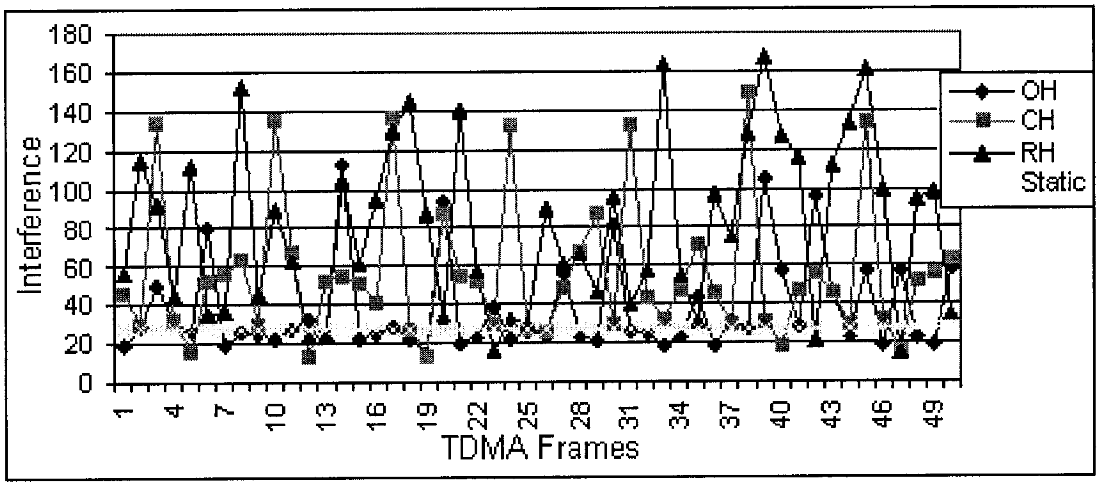

Figure 4.8: FH: Evolution of interference for a sample hopping TRX.

In order to better capture the time dimension, the interference levels experienced by each TRX during each TDMA frame are compared to the maximum interference levels experienced over the horizon. The maximum interference level determines the interference range. This range is divided into 4 sub-ranges or categories (the first being the best and the fourth being the worst). For each TDMA frame, each TRX is put in one of the four categories. Figure 4.10 shows the average percentage of time spent by TRXs in each category. The interval shown in the legend of Figure 4.10

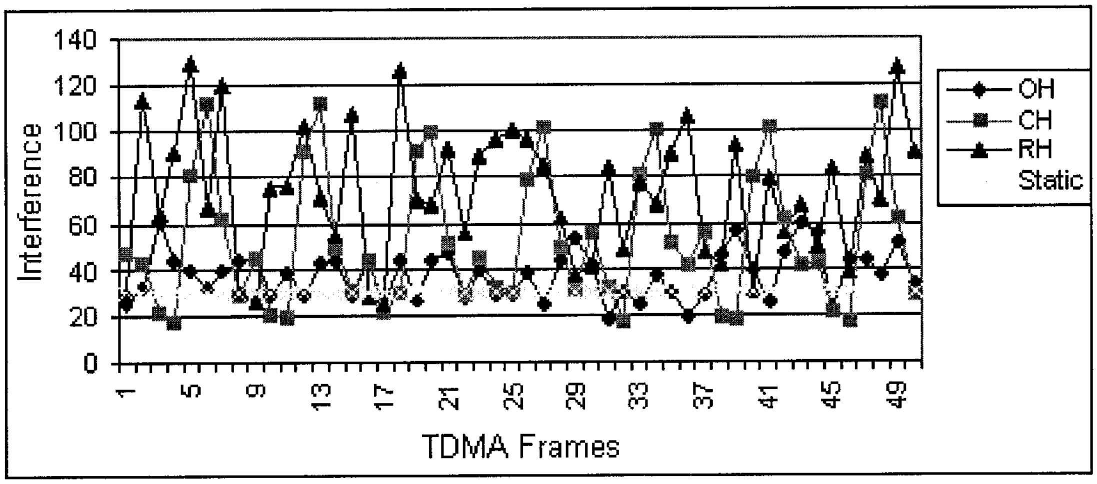

Figure 4.9: FH: Evolution of interference for a sample non-hopping TRX.

is the range covered by the four categories for each type of hopping. Although CH and RH have wider ranges, OH shows a higher percentage of time spent in category one. This indicates that OH significantly outperforms CH and RH. On the other hand, OH looks worse than Static allocation, but keep in mind that high interfernce persists in time for Static allocation while this is not the case under OH.

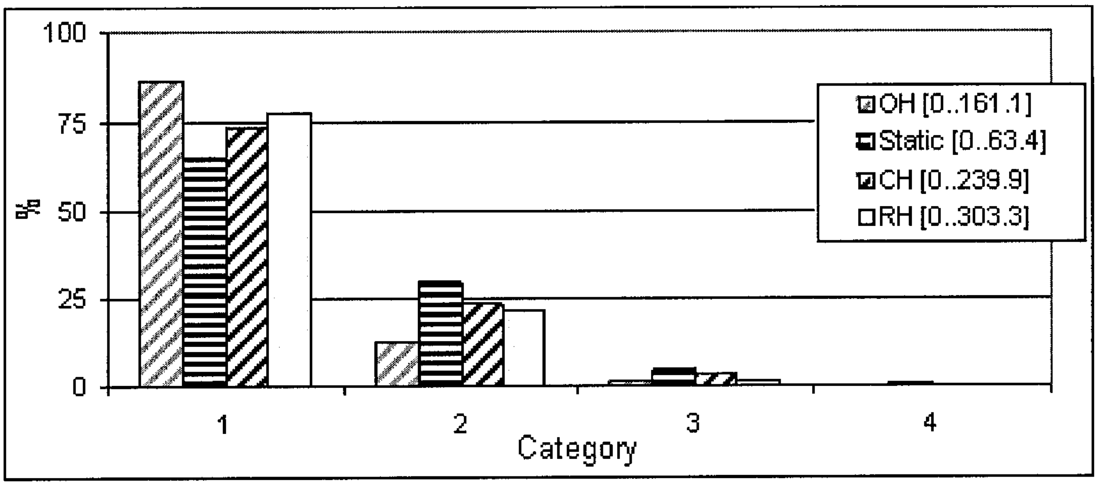

Figure 4.10: FH: Average percentage of time spent per category.

It is also interesting to get an idea about the amount of time during which a TRX would experience high interference levels. Figure 4.11 shows the number of times each TRX in the network experienced interference levels of category 3 or 4 for two consecutive TDMA frames, under Optimized Hopping. This figure indicates that high levels of interference are experienced for short periods of time. Figure 4.12 shows the number of TDMA frames during which interference was in category 3 or 4 (out of a maximum of 50, which is the length of the horizon considered), for each TRX under Optimized Hopping. On the average a TRX would spend about 0.65 TDMA frames in category 3 or 4. Although not shown, Optimized Hopping outperforms the other types of hopping patterns.

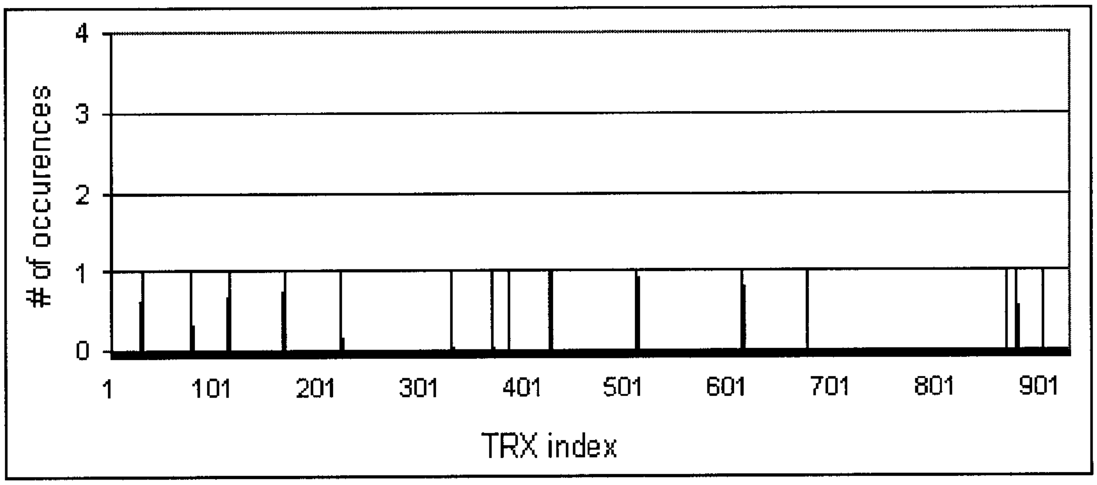

Figure 4.11: FH: Number of 2 consecutive observations of a TRX being in category 3 or 4.

4.4.2 Data Set 2

The above analysis has illustrated the gain from optimizing FH in the context of a fully-synchronized network. Optimized hopping was shown to provide a significant advantage. The next set of experiments are designed to investigate the impact of optimizing FH in asynchronous, hop-synchronized and fully-synchronized networks. The analysis is based on a network with 199 cells and a total of 605 TRXs (with a maximum of 4 TRXs per cell). The available radio bandwidth contains 40 frequencies.

Table 4.1 summarizes the results of these experiments for the three levels of synchronization considered. This table compares a frequency hopping plan that was implemented by the operator

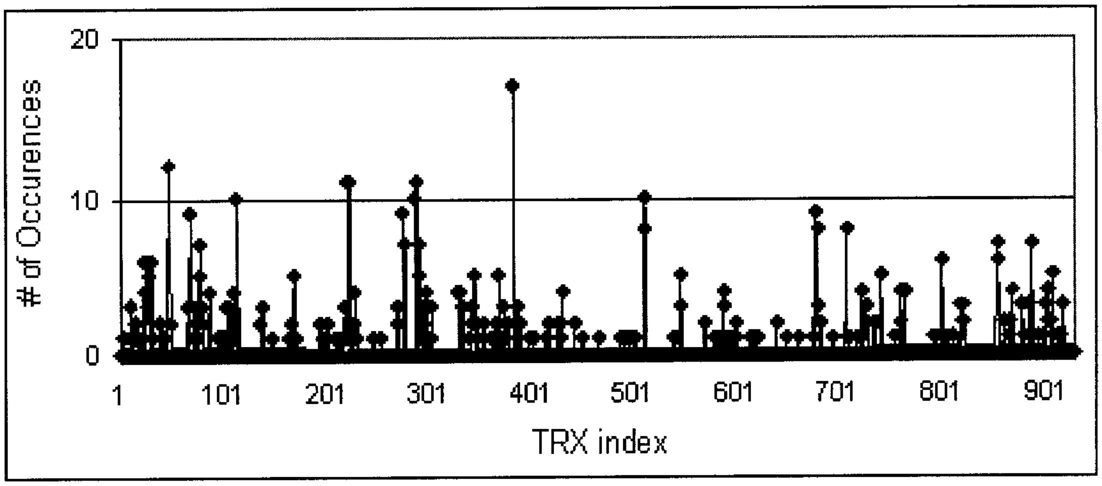

Figure 4.12: FH: Number of times in category 3 or 4.

to the optimized plans generated by the procedures proposed in this chapter (where a frequency hopping plan is composed of an MA allocation plan and an HSN-MAIO assignment plan). For each scenario, Table 4.1 provides the average, maximum and standard deviation of the interference among TRXs in the network. The MA plan provided (implemented by the operator) allocates 6 frequencies to each cell for hopping in addition to 1 frequency for the non-hopping TRX. The optimized MA plan was obtained by running the MA construction procedure several times in order to obtain the best MA size. The best compromise between minimizing average interference and minimizing maximum interference was achieved with an MA of size 6. For asynchronous networks, the procedure used to evaluate the performance of a given plan does not depend on the HSN-MAIO allocation, but depends on the MA allocation plan only. This explains why the evaluation results are the same for all HSN-MAIO plans with the same MA plan.

Table 4.1 compares several HSN-MAIO assignment plans: for the MA allocation provided by the operator, we compare a random HSN-MAIO plan, the actual plan used by the operator and optimized plans generated by the proposed heuristics. These are also compared to the use of optimized hopping sequences (for fully-synchronized network). Note that the HSN-MAIO plan provided by the operator uses Cyclic Hopping for all cells.

A first observation that could be made from Table 4.1 is that MA planning strongly determines the quality of the FH solutions. This holds for asynchronous, hop-synchronized or fully-synchronized networks. The improvement appears both as a reduction in the average as well as in the maximum interference.

		Asynchronous Network	**Hop-Synchronized Network**	**Fully-Synchronized Network**
MA plan provided Random HSN_MAIO	Avg.	31.1	19.9	19.9
	Max.	213.2	129.0	126.9
	StDev.	29.7	19.1	19.1
		(<1 sec.)	(53 secs)	(28 secs)
MA plan provided HSN_MAIO plan provided	Avg.	31.1	19.9	19.3
	Max.	213.2	133.3	147.6
	StDev.	29.7	19.2	21.4
		(<1 sec.)	(304 secs)	(28 secs)
MA plan provided Optimized HSN_MAIO	Avg.	31.1	19.6	12.8
	Max.	213.2	126.4	100.1
	StDev.	29.7	18.9	14.2
		(<1 sec.)	(39278 secs)	(7819 secs)
MA plan provided Optimized sequence	Avg.	31.1	-	9.6
	Max.	213.2	-	102.6
	StDev.	29.7	-	13.0
		(<1 sec.)		(28 secs)
Optimized MA plan Random HSN_MAIO	Avg.	26.9	15.4	15.0
	Max.	130.0	43.8	47.4
	StDev.	22.9	11.6	11.6
		(<1 sec.)	(53 secs.)	(29 secs)
Optimized MA plan Optimized HSN_MAIO	Avg.	26.9	15.1	9.0
	Max.	130.0	42.9	40.0
	StDev.	22.9	11.3	8.1
		(<1 sec.)	(38770 secs)	(7496 secs)
Optimized MA plan Optimized sequence	Avg.	26.9	-	1.9
	Max.	130.0	-	19.2
	StDev.	22.9	-	2.8
		(<1 sec.)		(29 secs)

Table 4.1: FH - HSN-MAIO management: Experimental resuts

For the same MA allocation (either the allocation provided by the operator, or the optimized one), the optimization of the HSN-MAIO assignment does not seem to produce any gain in the cases of asynchronous or hop-synchronized networks. However, in a fully-synchronized network, optimizing HSN-MAIO assignments improves the solution quality. These observations indicate that because of the randomness in asynchronous or hop-synchronized networks, the resulting hopping sequences are about equally correlated. On the other hand, in fully-synchronized networks, the absence of randomness reveals the differences in correlation between hopping sequences, which can be exploited by the HSN-MAIO assignment procedure.

For fully-synchronized networks, we propose the use of Optimized Hopping (i.e., optimizing the hopping sequence as opposed to optimizing the HSN-MAIO assignments). Results indicate that Optimized hopping provides a better approach than optimizing HSN-MAIO allocation for both MA allocation plans. The gain is more pronounced with optimized MA allocation.

This experimental analysis shows that optimization of the resource allocation under FH can significantly reduce the interference. In addition, we observe that full-synchronization allows us to take further advantage of FH. The use of the sequence generation procedure of GSM systems in the context of synchronized networks can be improved upon by actually building the hopping sequence through Optimized Hopping. The experimental analysis of the first data set shows how Optimized hopping outperforms cyclic or random hopping in the case of a fully-synchronized network.

4.5 Conclusion

In a synchronized network, Optimized Hopping is seen to perform better than Cyclic or Random Hopping, both by reducing the interference levels, and by reducing the time spent at high interference levels. The results also indicate that Optimized Hopping outperforms static frequency allocation with regard to interference sharing among TRXs and over time, and with regard maximal interference levels. It is worth mentioning here that the static frequency allocation program, DOCAF, used in this analysis is a powerful one, competitive with the best tools available on the market, while Optimized Hopping is based on a simple greedy algorithm that could be improved significantly.

The solution approaches developed in this chapter are used to investigate the effects of frequency hopping. Their performance could be further improved through the use of a metaheuristic such as Tabu search. For example, the frequency allocation algorithm proposed in Chapter 3 could be modified to accommodate MA allocation. The major differences between the static FAP and frequency hopping reside in the definition of the separation constraints and the objective function.

For hop-synchronized and asynchronous networks, the MA allocation plan was found to be the determinant factor for the solution quality. The HSN-MAIO planning is not relevant as long as the randomization is present through differentiation of the HSN-MAIO values. This indicates that synchronization can lead to a better exploitation of the gains from FH. This can be achieved through the optimization of the HSN-MAIO plan or better yet, through the use of the proposed Optimized Hopping pattern.

Bibliography

[1] *Multiplexing and Multiple Access on the Radio Path, GSM Specifications.* European Telecommunications Standards Institute, ETSI, 1999.

[2] K. I. Aardal, S. P. M. V. Hoesel, A. M. C. A. Koster, C. Mannino, and A. Sassano. Models and solutions techniques for frequency asssignment problems. Technical Report 01-40, Konrad-Zuse-Zentrum fur Informationstechnik Berlin, December 2001.

[3] J. Abril, F. Comellas, A. Cortes, J. Ozon, and M. Vaquer. A multiagent system for frequency assignment in cellular radio networks. *IEEE Transactions on Vehicular Technology*, 49(5):1558–1565, 2000.

[4] H. R. Anderson and J. P. McGeehan. Optimizing microcell base station locations using simulated annealing techniques. In *Vehicular Technology Conference, 44th*, volume 2, pages 858–862, 1994.

[5] R. Battiti, A. A. Bertossi, and M. A. Bonuccelli. Assigning codes in wireless networks: Bounds and scaling properties. *Wireless Networks*, 5:195–209, 1999.

[6] D. Beckmann and U. Killat. Frequency planning with respect to interference minimization in cellular radio networks. Technical Report TD(99) 032, COST 259, Vienna, Austria, 1999.

[7] D. Beckmann and U. Killat. A new strategy for the application of genetic algorithms to the channel-assignment problem. *IEEE Transactions on Vehicular Technology*, 48(4), July 1999.

[8] P. Björklund, P. Värbrand, and D. Yuan. Optimal frequency planning in mobile networks with frequency hopping. Technical Report LiTH-ITN-R-2002-3, linköping institute of technology, 2002.

[9] R. Borndorfer, A. Eisenblatter, M. Grotschel, and A. Martin. Frequency assignment problem in cellular phone networks. *Annals of Operations Research*, 76, 1998.

[10] R. Borndörfer, A. Eisenblatter, M. Grötschel, and A. Martin. The orientation model for frequency assignment problems. Technical Report TR 98-01, Konrad-Zuse-Zentrum fur Informationstechnik Berlin, 1998.

[11] R. Bose. A smart technique for determining base-station locaions in an urban environment. *IEEE Transactionson Vehicular Technology*, 50(1):43–47, January 2001.

[12] A. Bouju, J. F. Boyce, C. H. D. Dimitropoulos, G. V. Scheidt, and J. Taylor. Tabu search for the radio links frequency assignment problem. *Applied Decision Technologies*, 1995.

[13] J.-M. Bourjolly, L. Déjoie, K. Ding, O. Dioume, and M. Lominy. Frequency allocation in cellular phone networks: An OR success story. In *Joint Canadian Operationnal Research Society / Optimization Days Conference, Quebec City, May 2001, First Prize in the Competition on the Practice of OR*, 2001.

[14] J.-M. Bourjolly, L. Dejoie, K. Ding, O. Dioume, and M. Lominy. Canadian telecom makes the right call. *OR/MS Today*, 29(2):40–44, 2002.

[15] J.-M. Bourjolly, G. Laporte, and G. Pesantc. An exact algorithm for the maximum k-club problem in an undirected graph. *European Journal of Operational Research*, 138(1):21–28, April 2002.

[16] J. Button, K. Calderhead, I. Goetz, M. Hodges, R. Patient, R. Pilkington, R. Reeve, and P. Tattersall. Mobile network design and optimization. *BT Technol Journal*, 14(3):29–46, July 1996.

[17] A. Capone and M. Trubian. Channel assignment problem in cellular systems: A new model and a tabu search algorithm. *IEEE Transactions on Vehicular Technology*, 48(4), July 1999.

[18] D. Castelino and N. Stephens. *Tabu Thresholding for the Frequency Assignment Problem*, pages 343–360. Kluwer Academic Publishers, 1996.

[19] D. Castelino and N. Stephens. A surrogate constraint tabu thresholding implementation for the frequency assignment problem. *Annals of Operations Research*, 86:259–270, January 1999.

[20] D. J. Castelino, S. Hurley, and N. M. Stephens. A tabu search algorithm for frequency assignment. *Annals of Operations Research*, 63:301–319, 1996.

[21] D. Cohen. The spectrum management process: How it is working to support the new wireless. In J. Reed, T. Rappaport, and B. D. Woerner, editors, *Wireless Personal Communications: Advances in Coverage and Capacity*, 1997.

[22] L. J. de Urries, M. A. D. Guerra, and I. Berberana. Frequency planning using simulated annealing. Technical Report TD(00) 054, COST 259, Bergen, Norway, 2000.

[23] M. Duque-Anton, D. Kunz, and B. Ruber. Channel assignment for cellular radio using simulated annealing. *IEEE Transactions on Vehicular Technolgy*, 42(1), February 1993.

[24] A. Eisenblätter. *Frequency Assignment in GSM Networks: Models, Heuristics, and Lower Bounds*. PhD thesis, Technische Universität Berlin, 2001.

[25] A. Eisenblätter, M. Grötschel, and A. M. Koster. Frequency planning and ramifications of coloring. Technical Report 00-47, Konrad-Zuse-Zentrum fur Informationstechnik Berlin, 2000.

[26] A. Eisenblätter and A. Koster. FAP web - a website about frequency assignment problems,. *http://fap.zib.de*, 2000.

[27] R. Enders. Unpublished, siemens AG, germany.

[28] M. Fischetti, C. Lepschy, G. Minerva, G. Romanin-Jacur, and E. Toto. Frequency assignment in mobile radio systems using branch-and-cut techniques. *European Journal of Operational Research*, 123:241–255, 2000.

[29] L. Floriani and G. Mateus. An optimization model for the BST location problem in outdoor cellular and PCS systems. In V. Ramaswami and P. Wirth, editors, *Teletraffic Contributions for the Information Age, Volume 2a of Teletraffic Science and Engineering*. Elsevier, 1997.

[30] L. Floriani and G. Mateus. Optimization models for effective cell planning design. In

A. Ferreira and D. Krob, editors, *Proceedings of DIAL M for Mobility '97*, 1997.

[31] L. C. P. Floriani and G. R. Mateus. An optimization model for the BST location problem in outdoor cellular and PCS systems. In *International Teletraffic Congress 15, Washington, DC, USA*, pages 527–537, 1997.

[32] T. Fritsch, K. Tutschku, and K. Leibnitz. Field strength prediction by ray-tracing for adaptive base station positioning in mobile communication networks. In *2nd ITG Conference on Mobile Communication*, September 1995.

[33] A. Gamst. Some lower bounds for a class of frequency assignment problems. *IEEE Transactions on Vehicular Technology*, 35(1):8–14, February 1986.

[34] A. Gamst. Application of graph theoretical methods to GSM radio network planning. In *IEEE International Sympoisum on Circuits and Systems*, volume 2, pages 942–945, 1991.

[35] M. R. Garey and D. S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. Freeman, San Francisco, 1979.

[36] B. Gavish and S. Sridhar. Economic aspects of configurating cellular networks. *Wireless networks*, 1:115–128, 1995.

[37] C. Glaßer, S. Reith, and H. Vollmer. The complexity of base station positioning in cellular networks. In *Approximation and Randomized Algorithms in Communication Networks, ICALP Workshops, Carleton Scientic*, volume 8, pages 167–177, 2000.

[38] J. K. Han, B. S. Park, Y. S. Choi, and H. K. Park. Genetic approach with a new representation for base station placement in mobile communications. In *Vehicular Technology Conference, Fall*, volume 4, pages 2703 –2707, 2001.

[39] J.-K. Hao, R. Dorne, and P. Galinier. Tabu search for frequency assignment in mobile radio networks. *Journal of Heuristics*, 4, 1998.

[40] M. Hellebrandt and H. Heller. A new heuristic method for frequency assignment. Technical Report TD(00) 003, COST 259, Valencia, Spain, 2000.

[41] H. Heller. Private communication, 2000.

[42] http://www.ieee.org/organizations/history_center/lamarr.html.

[43] L. Hu. Distributed code assignment for CDMA packet radio networks. *IEEE/ACM Transactions on Networking*, 1(6):668–677, 1993.

[44] X. Huang, U. Behr, and W. Wiesbeck. Automatic base station placement and dimensioning for mobile network planning. In *Vehicular Technology Conference 52nd*, volume 4, pages 1544 –1549, 2000.

[45] S. Hurley. Automatic base station selection and configuration in mobile networks. In *Vehicular Technology Conference, 52nd*, volume 6, pages 2585 –2592, 2000.

[46] K. Ivanov, N. Metzner, G. Spring, H. Winkler, and P. Jung. Frequency hopping-spectral capacity enhancement of cellular networks. In *Proc. IEEE Fourth International Symposium on Spread Spectrum Techniques and Applications (ISSSTA'96)*, pages 1267–1272., Mainz,

1996.

[47] B. Jaumard, O. Marcotte, C. Meyer, and T. Vovor. Comparison of column generation models for channel assignment in cellular networks. *Discrete Applied Mathematics*, 112:217–240, 2001.

[48] I. Katzela and M. Naghshineh. Channel assignment schemes for cellular mobile telecommunication systems. *IEEE Personal Communications*, 3(3):10–31, June 1996.

[49] J.-S. Kim, S. H. Park, P. W. Dowd, and N. M. Nasrabadi. Cellular radio channel assignment using a modified hopfield network. *IEEE Transactions on Vehicular Technology*, 46(4), November 1997.

[50] Y. Kishi, T. Mizuike, and F. Watanabe. A unified approach for frequency assignment of cellular mobile networks. In *Third Annual International Conference on Universal Personal Communications,*, pages 563 –567, 1994.

[51] A. Koster. *Frequency Assignment; Models and Algorithms*. PhD thesis, Proefschrift Universiteit Maastricht, 1999.

[52] Z. Kostic, X. Qiu, K. Chawla, L. F. Chang, J. Chuang, and N. Sollenberger. Dynamic frequency hopping in cellular systems with network assisted resource allocation. In *IEEE 51st Vehicular Technology Conference Proceedings, Spring, Tokyo*, volume 3, pages 2459 –2463, 2000.

[53] B. Krishnamachari. Global optimization in the design of mobile communication systems. Master's thesis, Cornell university, 1999.

[54] B. Krishnamachari and S. Wicker. Experimental analysis of local search algorithms for optimal base station location. In *International Conference on Evolutionary Computing for Computer, Communication, Control and Power (ECCAP 2000), Chennai, India*, January 2000.

[55] K. Lieska, E. Laitinen, and J. Lahteenmaki. Radio coverage optimization with genetic algorithms. In *The Ninth IEEE International Symposium on Personal Indoor and Mobile Radio Communications*, volume 1, pages 318 –322, 1998.

[56] V. Maniezzo and A. Carbonaro. An ants heuristic for the frequency assignment problem. In *Proceedings of MIC'99*, 1999.

[57] V. Maniezzo and R. Montemanni. An exact algorithm for the min-interference frequency assignment problem. Technical Report WP-CO0003, Scienze dell'Informazione, University of Bologna, 2000.

[58] C. Mannino, G. Oriolo, and F. Ricci. Solving stability problems on a superclass of interval graphs. Technical Report 26-02, DIS-Università di Roma "La Sapienza", 2002.

[59] R. Mathar and T. Niessen. Optimum positioning of base stations for cellular radio networks. *Wireless Networks*, 6:421–428, 2000.

[60] R. Mathar and M. Schmeink. Integrated optimal cell site selection and frequency allocation for cellular radio networks. In *Informs Select Proceedings of the 2000 Telecommunications*

Conference, 2000.

[61] F. F. Mazzini, G. R. Mateus, and J. M. Smith. Lagrangean based methods for solving large scale cellular networks design problems. *Wirelss Networks*, 9:659–672, 2003.

[62] H. N. B. Meriem and S. Tabbane. Simulations of frequency hopping schemes with power control in a GSM network. In *IEEE 51st Vehicular Technology Conference Proceedings, Spring, Tokyo.*, volume 3, pages 2380 –2383, 2000.

[63] A. Molina, G. E. Athanasiadou, and A. R. Nix. The automatic location of base-stations for optimised cellular coverage: A new combinatorial approach. In *Vehicular Technology Conference, 49th*, volume 1, pages 606–610, 1999.

[64] A. Molina, A. R. Nix, and G. E. Athanasiadou. Cellular network capacity planning using the combination algorithm for total optimisation. In *Vehicular Technology Conference Proceedings, Tokyo, 51st*, volume 3, pages 2512 –2516, 2000.

[65] R. Montemanni, D. H. Smith, and S. M. Allen. An improved algorithm to determine lower bounds for the fixed spectrum frequency assignment problem. *European journal of Operational Research*, to appear.

[66] R. Murphey, P. Pardalos, and M. Resende. *Handbook of Combinatorial Optimisation, Suplement Volume*, chapter Frequency Assignment Problems. Number 98.16.1. Kluwer Academic Publishers, 1999.

[67] T. T. Nielsen, J. Wigard, P. H. Michaelsen, and P. Mogensen. Resource allocation in a frequency hopping PCS1900/GSM/DCS1800 type of network. In *IEEE 49th Vehicular Technology Conference*, volume 1, pages 209 –214, 1999.

[68] T. T. Nielsen, J. Wigard, and P. Mogensen. On the capacity of a GSM frequency hopping network with intelligent underlay-overlay. In *VTC'97 Phoenix, USA*, 1997.

[69] T. T. Nielsen, J. Wigard, and T. T. Nielsen. *Performance Enhancements in a Frequency Hopping GSM Network.* Kluwer Academic Publishers, 1st edition, 2000.

[70] H. Nyberg, S. Craig, S. Magnusson, and E. Edgren. Collision properties of GSM hopping sequences. In *International Symposium on Personal, Indoor and Mobile Radio Communications, PIMRC*, volume 2, pages 1004 –1008, 2000.

[71] A. D. Pasquale, N. P. Magnani, and P. Zanini. Optimizing frequency planning in the GSM system. In *International Conference on Universal Personal Communications*, volume 1, pages 293 –297, 1998.

[72] A. Quellmalz, A. Knälmann, and B. Müller. Efficient frequency assignment with simulated annealing. In *Antennas and Propagation*, volume 407, April 1995.

[73] L. Raisanen and R. M. Whitaker. Multi-objective optimization in area coverage problems for cellular communication networks: Evaluation of an elitist evolutionary strategy. In *Proceedings of the 2003 ACM Symposium on Applied Computing*, 2003.

[74] P. Reininger and A. Caminada. Connectivity management on mobile network design. In *10th Conference of the European Consortium for Mathematics in Industry*, 1998.

[75] P. Reininger, S. Iksal, A. Caminada, and J. J. Korczak. Multi-stage optimization for mobile radio network planning. In *Vehicular Technology Conference, 1999 IEEE 49th*, volume 3, pages 2034 –2038, 1999.

[76] L. Santos, L. S. Ferreira, V. Garcia, and L. M. Correia. Cellular planning optimisation for non-uniform traffic distributions. In *The Ninth IEEE International Symposium on Personal Indoor and Mobile Radio Communications*, volume 3, pages 1198–1202, 1998.

[77] E. Sarkola. Channel assignment methods in frequency hopping cellular radio networks. In *The 8th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, 1997. Waves of the Year 2000. PIMRC '97*, volume 3, pages 771 –775, 1997.

[78] D. V. Sarwate. Optimum PN sequences for CDMA systems. In *Proceedings of IEEE Third International Symposium on Spread Spectrum Techniques and Applications*, pages 27–35, 1994.

[79] H. Sherali, C. Pendyala, and T. S. Rappaport. Optimal location of transmitters for micro-cellular radio communication system design. *IEEE Journal on Selected areas in communications*, 14(4), May 1996.

[80] D. Smith, S. Hurley, and S. Thiel. Theory and methodology: Improving heuristics for the frequency assignment problem. *European Journal of Operational Research*, (107), 1998.

[81] D. H. Smith. Unpublished, university of glamorgan, UK.

[82] D. H. Smith and S. Hurley. Bounds for the frequency assignment problem. *Discret Mathematics*, 167/168:571–582, 1997.

[83] K. A. Smith. A genetic algorithm for the channel assignment problem. In IEEE, editor, *Global Telecommunications Conference, 1998.*, volume 4, pages 2013 –2018, 1998.

[84] D.-W. Tcha, Y.-J. Chung, and T.-J. Choi. A new lower bound for the frequency assignment problem. *IEEE/ACM Transactions on Networking*, 5(1), February 1997.

[85] D.-W. Tcha, J.-H. Kwon, T.-J. Choi, and S.-H. Oh. Perturbation-minimizing frequency assignment in a changing TDMA/FDMA cellular environment. *IEEE Transactions on Vehicular Technology*, 49(2):390–396, 2000.

[86] K. Tutschku. Demand based radio network planning of cellular mobile communication systems. In *INFOCOM'98, San Francisco, USA*, March 1998.

[87] K. Tutschku. Interference minimization using automatic design of cellular communication networks. In *Vehicular Technology Conference 98, Ottawa, Canada*, May 1998.

[88] K. Tutschku, N. Gerlich, and P. Tran-Gia. An integrated approach to cellular network planning. In *Networks'96, Sydney, Australia*, November 1996.

[89] K. Tutschku, K. Leibnitz, and P. Tran-Gia. ICEPT - an integrated cellular network planning tool. In *Vehicular Technology Conference 97, Phoenix, USA*, May 1997.

[90] K. Tutschku, R. Mathar, and T. Niessen. Interference minimization in wireless communication systems by optimal cell site selection. In *3rd European Personal Mobile Communications Conference, Paris, France*, March 1999.

[91] M. Vasquez and J.-K. Hao. A heuristic approach for antenna positioning in cellular networks. *Journal of Heuristics*, 7(5):443–472, September 2001.

[92] M. Vasudevan, K. Rasmussen, C. Yu, and W. Egner. An algorithmic approach to growth planning in a wireless network. In *Vehicular Technology Conference, 50th*, volume 4, pages 2029 –2033, 1999.

[93] R. M. Whitaker, L. Raisanen, and S. Hurley. A model for conflict resolution between coverage and cost in cellular wireless networks. In *Proceedings of the 37th Hawaii International Conference on System Sciences*, 2004.

[94] J. Wigard and P. Mogensen. Capacity of a GSM network with fractional loading and random frequency hopping. In *PIMRC'96, Taipei*, 1996.

[95] J. Wigard and P. Mogensen. A simple mapping from C/I to FER and BER for a GSM type of air-interface. In *PIMRC'96, Taipei*, 1996.

[96] J. Zimmermann, R. Hons, and H. Muhlenbein. ENCON: An evolutionary algorithm for the antenna placement problem. *Computers and Industrial Engineering*, 44:209–226, 2003.

Glossary

- *Antenna*
  - *Print*, page 16

    The area receiving a from the antenna.
  - *Cell*, page 16

    The area with the antenna print for which the antenna provides the strongest signal.
  - *Serving antenna*

    The antenna carrying the communication of the customer. The best server corresponds to the antenna providing the strongest signal. The second best server is the antenna with the second strongest signal that is within a preset range of the best server.
- Base station, page 7

  Set of equipment including the antenna and the TRXs.
- *Carrier to Interferer Ration, CIR*

  A measure of the quality of the signal received. Other measures may include the Bit Error Rate or the Frame Erasure Rate.
  - *Pair-wise CIR*, page 40

    Considering transmitters in pairs, this measure corresponds the ratio of the serving signal to each of the interfering signals.
  - *Combined CIR*, pages 9, 38, 50

    For this measure of interference, CIR is computed as the ratio of the serving signal to the sum of all interfering signals.

- *Cell Allocation, CA*, page 146

  Set of frequencies assigned to a cell. This corresponds to the union of the subsets of frequencies available for use at each of the TRXs in the cell.

- Erlang, page 8

  Unit to measure customer traffic. One Erlang corresponds to using the communication medium (one circuit) for one hour.

- *Frequency*

  - Spectrum, page 2

    The radio bandwidth available for use. It is divided into frequencies.

  - *Separation*, pages 49,110

    A protection on a pair of transmitters so that the assigned frequencies are distant from each other by at least the specifies level of separation.

  - *Locally locked*, page 101

    One or more frequencies that cannot be assigned to a particular transmitter.

  - *Channel*, page

    The use of a frequency at a particular transmitter is shared by multiple users, each of which is assigned a different time slot or channel.

  - *Broadcast control channel, BCCH*, page 6

    In GSM systems, the first channel of the first TRX in a base station is used to establish calls and various signaling purposes.

  - *Traffic channel, TCH channel*, page 6

    These channels are used to carry customer communications.

- *Handoff*, page 2

  Procedure by which the communication of a moving user is transferred from one base station to an adjacent one.

- *Hopping Sequence*

  In Hopping GSM systems, the frequency on which the communication is established varies in time over the subset of frequencies with the MA of the serving TRX.

  - *Hopping sequence number, HSN*, page 147

    This parameter determines the hopping sequence to use.

  - *Mobile Allocation Index Offset, MAIO*, page 148

    This parameter set the offset with respect to the beginning of the sequence. It is used to make two sequences of the same HSN orthogonal, i.e., there is no collision between the two sequences. Collision occurs when interfering frequencies are used simultaneously.

  - *Frame Number, FN*, page 147

    A discrete time counter for each antenna.

  - *Mobile Allocation Index, MAI*, page 148

    The index of the frequency from the MA to use at a specified FN. It is the output of the sequence generator as a function of HSN, MAIO and FN.

  - *Random Hopping, RH*, page 147

    The hopping sequence is generated according to a deterministic pseudo-random procedure. There are 63 possible random sequences in GSM.

  - *Cyclic Hopping, CH*, page 147

    In GSM, it is possible to select a cyclic hopping pattern where the sequence goes through the set of frequencies in the MA in increasing or of their index before restarting.

  - *Optimized Hopping, OH,* page 160

    A Proposed hopping sequence that replaces the use of the deterministic sequence generator in GSM. It involves generating a sequence that optimizes the cumulative interference while avoid consecutive burst with high interference levels.

- *Interference matrix*

  - *Co-channel*, page ??

    This matrix provides for each pair of cells a measure of the area or traffic for which the pair-wise CIR level would be below a preset threshold if the pair used the same frequency.

  - *Adjacent-channel*, page 41

    This matrix provides for each pair of cells a measure of the area or traffic for which the pair-wise CIR level would be below a preset threshold if the pair used adjacent frequencies.

- *Mobile Allocation, MA,* page 146

  The subset of frequency assigned for use in a TRX. In non-hopping GSM networks, this set contains just one frequency. In hopping GSM systems, and for TRXs that do not carry the BCCH, the subset may contain more than one frequency.

- *Mobile Unit,* MU, page *1*

  The hand held set used by the customer.

- *Multiple Access Scheme*, page 3

These allow the network to accommodate multiple user on the available frequency spectrum.

  - *Frequency division multiple access, FDMA*

    Multiple access is achieved by dividing the spectrum into frequencies and assigning different communications to different frequencies.

  - *Time division multiple access, TDMA*

    Multiple access is achieved by assigning the available band to each communication for a distinct time slot.

  - *Code division multiple access, CDMA*

    a coding mechanism is used to differentiate between different communications.

- *Traffic*, pages 34, 55

  - *Offered*

    This is the volume of communication requests

  - *Covered*

    This is the volume of offered traffic that receives a strong enough signal to establish the communication

  - *Carried*

    This is the volume of covered traffic that can be served by the number of TRXs installed

  - *Blocked*

    This is the volume of covered traffic that cannot be served by the number of TRXs installed

- *Overflow*

  This is the volume of covered traffic that cannot be served by the number of TRXs installed in the primary serving antenna but that can go through the secondary server.

- *Transmitters/Receivers,* TRX, page *1*

  It is used to relay communication between the customer and the core network.

- *Up Link - Down link, page 3*

  This link direction: from MU to TRX or the inverse.