THE "VOLATILITY SMILE" OF CANADIAN INDEX OPTIONS

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ABSTRACT

The "Volatility Smile" of Canadian Index Options

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Estimating the representative agent's or the market's degree of risk aversion from securities prices has a long history. However, it is only since this century that scholars have begun using options data to do so. Options provide a particularly promising context for studying risk preferences. Embedded in the prices of traded options is rich information set relating to various aspects of the underlying asset. One piece of embedded information is the state price density. To the best of my knowledge, the implied risk aversion deriving from state price density or the "risk neutral" distribution has not been well studied in Canadian context. In this paper, the author investigates the volatility smile derived from call options on the Canadian S&P/TSX 60 Index, which is one of the most heavily traded index options in the Canadian market. Then from the option pricing function, the author recovers the risk neutral distribution by using the nonparametric approach- the regularization method proposed by Jackwerth and Rubinstein (1996). The final step is to compare the option inferred risk neutral distribution with the estimated actual distribution of index prices based on, for example, the historical price path over the same time interval. The relationship between these two distributions depends on the preference (utility) of investors about money, as they grow richer or poorer. Knowing both distributions allows us to infer what the preferences must be within an economy to be consistent with the option price and the historical returns.

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INTRODUCTION

Estimating the representative agent's or the market's degree of risk aversion from securities prices has a long history. However, it is only since this century that scholars have begun using options data to do so. Options provide a particularly promising context for studying risk preferences. Stocks are infinitely lived and so inferences must be drawn from the discounted stream of cash flows over an indefinite horizon. Usually this involves additional assumptions as to how those cash flows evolve (e.g., constant growth of dividends). Since only one value, the discounted present value of all cash flows, is known, no inferences are possible about variations in preferences over different horizons.

Options on the other hand have a fixed expiry date at which payoffs are realized. Furthermore, options contracts exist for different investment horizons. Options thus permit studying preferences over specific horizons and simultaneously over multiple horizons. Futures contracts also share this fixed-horizon characteristic. Options, however, provide a spectrum of observations for each expiry date on any given observation date - one for each quoted strike price—while futures provide only a single statistic for each expiry date/observation date pair. The multiplicity of prices for different payoffs on the same underlying asset provided by options allows us to construct a density function for the distribution of possible values of the underlying asset. In contrast, single-datum stock and futures prices allow inference only about the mean of the distribution, unless additional assumptions are made linking the observed time-series to a stochastic process or density function functional form.

Black and Scholes (1973) remains the most recognized option-pricing model in finance. Despite the model's intuitive appeal and widespread use, there is a sizeable body of academic research and a consensus among market professionals that documents significant deviations between observed option prices and those implied by the model. While the parametric assumptions of the Black-Scholes model offer parsimony, these assumptions are typically violated in practice. The result is that the Black-Scholes model is unable to capture some important features exhibited in the data. For example, as I mention in the "Implied Volatility Surface" part of literature review, there are volatility smiles and negative skewness in the distribution of the underlying stock price. Both of these features are inconsistent with the Black-Scholes assumption that the underlying stock follows geometric Brownian motion and has a lognormal distribution.

Embedded in the prices of traded options is a rich information set relating to various aspects of the underlying asset. Jackwerth (1999) outlines a range of practical applications that have been developed to use the information that can be extracted from option prices. One piece of embedded information is the state price density. In the context of option valuation, state price density can be defined implicitly as the density such that if we compute the expected payoff of any European option against this density, and discount this expectation using the risk-free rate, we will recover the true value of the option. However, the state price density is not the distribution of the underlying stock price on the option maturity date. If we were to compute the expected payoff against this "real world" distribution it would be inappropriate to discount this risky expected cash flow at the risk-free rate. For this reason, the state price density is sometimes called the "risk neutral" distribution, because this distribution produces an expected payoff that can be discounted at the risk-free rate, which would be the expected return on all assets if investors were risk neutral. The risk neutral distribution can be used in a range of applications including: 1) The pricing of exotic or illiquid derivatives, 2) Assessing the credibility of exchange rate regimes and general sentiment in foreign exchange markets (Campa, Changa, and Refalo, 1999), 3) Gauging market expectations of future changes in interest rates, (Manzano and Sanchez, 1998 and McManus, 1999) 4) Risk Management (Aäi t-Sahalia and Lo, 2000 and Shimko, 1993) and 5) Estimating investor risk preferences (Jackwerth, 2000).

To the best of my knowledge, the implied risk aversion deriving from state price density or the “risk neutral" distribution has not been well studied in Canadian context. In this paper, I investigate the volatility smile derived from call options on a Canadian index, specifically, a European call option on the Canadian S&P/TSX 60 Index, which is one of the most heavily traded index options in the Canadian market. Then from the option pricing function I recover the risk neutral distribution for Canadian index options by using the nonparametric lattice approach- the regularization method proposed by Jackwerth and Rubinstein (1996). The final step is to compare the option inferred risk neutral distribution with the estimated actual distribution of index prices based on, for example, the historical price path over the same time interval. The relationship between these first two distributions depends on the preference (utility) of investors about money, as they grow richer or poorer. Knowing both distributions allows us to infer what the preferences must be within an economy to be consistent with the option price and the historical returns.

The remainder of the paper is structured as follows. The relationship between the risk neutral distribution and the pricing of derivative securities is developed in Section 2. This section also summarizes the limitations of the Black-Scholes model and the estimation methods, both parametric and nonparametric that have been developed to either directly assume or infer the risk neutral distribution. Section 3 contains a description of the sample data and Section 4 outlines the methodology. The results are presented and interpreted in Section 5, and Section 6 concludes.

Literature Review

- **Implied Volatility Surface**

There are two important but independent features of the Black-Scholes theory. The primary feature of the theory is that it is preference-free – the values of contingent claims do not depend upon investors' risk preferences. Therefore, we can value an option as though the underlying stocks expected return is riskless. This risk-neutral valuation is allowed because we can hedge an option with stock to create an instantaneously riskless portfolio.

A secondary feature of the Black-Scholes theory is its assumption that stock prices evolve lognormally with a constant volatility at any time and market level. This stock price evolution over an infinitesimal time dt is described by the stochastic differential equation

Equation 1 $$\frac{dS}{S} = \mu dt + \sigma dZ$$

where S is the stock price, μ is its expected return and dZ is a Wiener process with a mean of zero and a variance equal to dt. The Black-Scholes formula $C_{BS}(S, \sigma, r, t, K)$ for a call with strike K and time to expiration t, when the riskless rate is r, follows from applying the general method of risk-neutral valuation to a stock whose evolution is specifically assumed to follow Equation 1.

In the Cox-Ross-Rubinstein (CRR) binomial implementation of the process in Equation 1, the stock evolves along a risk-neutral binomial tree with constant logarithmic stock price spacing, corresponding to constant volatility. The binomial tree corresponding to the risk-neutral stock evolution is the same for all options on that stock, irrespective of their strike level or time to expiration. The stock tree cannot "know" about which option we are valuing on it. Unfortunately, market prices are not exactly consistent with theoretical prices derived from the Black-Scholes formula. Nevertheless, the success of the Black-Scholes framework has led traders to quote an option's market price in terms of whatever constant volatility σ_{imp} makes the Black-Scholes formula value equal to the market price. We call σ_{imp} the Black-Scholes equivalent or implied volatility, to distinguish it from the theoretically constant local volatility σ assumed by the Black-Scholes theory. In essence, σ_{imp} is a means of quoting prices. How consistent are market option prices with the Black-Scholes formula? Kani(1995) shows the decrease of σ_{imp} with the strike level of options on the S&P 500 index with a fixed expiration of 44 days, as observed on May 5, 1993. This asymmetry is commonly called the volatility "skew." He also shows an increase of σ_{imp} with the time to expiration of at-the-money options. This variation is generally called the volatility "term structure." Jackwerth, (2004) also finds that the market implied volatilities of stock index options often have a skewed structure, as σ_{imp} falls as the strike level increases for US S&P 500, German DAX 30 and U.K. FTSE100, which means Out-of-the-money puts trade at higher implied volatilities than out-of-the-money calls. Though the exact shape and magnitude vary from day-to-day and from exchange to exchange, the asymmetry persists and belies the Black-Scholes theory, which assumes constant local (and therefore, constant implied) volatility for all options. Its persistence suggests a discrepancy between theory and the market. It may be convenient to keep quoting options prices in terms of Black-Scholes-equivalent volatilities (implied volatility), but it is probably incorrect to calculate options prices using the Black-Scholes formula.

The market implied volatilities of standard equity index options commonly vary with both option strike and option expiration. This structure has been a significant and persistent feature of index markets around the world since the 1987 crash. Figure 1 (Kani1995) shows a typical implied volatility surface illustrating the variation of S&P 500 implied volatility with strike and expiration on September 27, 1995. This surface, commonly called the "volatility smile," changes shape from day to day, but some general features persist.

The strike structure: At any fixed expiration, implied volatilities vary with the strike level. Almost always, implied volatilities increase with decreasing strike – that is, out-of-the-money puts trade at higher implied volatilities than out-of-the-money calls. This feature is often referred to as a "negative" skew.

The term structure: For any fixed strike level, implied volatilities vary with time to expiration.

In brief, there is a unique and generally different implied volatility associated with any specific strike and expiration. Each implied volatility depicted in the surface of Figure 1 is the Black-Scholes implied volatility, Σ, the volatility we have to enter into the Black-Scholes formula to have its theoretical option value match the option's market price. Σ is the conventional unit in which options market-makers often quote prices.

When options market makers quote an implied volatility Σ for an option of a given strike and expiration, they are specifying the future local volatility $\sigma = \Sigma$ that we must enter into the Black-Scholes formula to obtain the market price for the option, assuming that Σ stays constant over all times and market levels. By quoting two different implied volatilities $\Sigma 1$ and $\Sigma 2$ for two different options, say (a) a long-term option and (b) a short-term option. they are attributing two different constant local volatilities to the same underlying index. Numerous researchers find that long-term options have low implied volatilities and short-term options have high-implied volatilities. Jackwerth (2004) finds slightly U-shaped volatility smiles for European style options on the U.S. S&P 500 Index, which were flat from the start of their exchange-based trading in April 1986 until the U.S. stock market crash of October 1987. After the crash, the volatility smile becomes skewed; that is, the volatility smiles became downward sloping as the strike price increased. This pattern is not unique to the U.S. index option market. Jackwerth (2004) supports his findings by estimating the downward sloping smiles for the German DAX30, U.K. FTSE 100, as well as the Japanese Nikkei 225(his finding is actually a volatility smirk, which is a downward shape for option on index rather than a U shape of ideal volatility smile, which is quite typical for options on currency). However, to the best of my knowledge, there is no published paper that examines whether a Canadian volatility smile exists. At least, there is no paper testing Canadian implied volatility surface (tri-dimension of implied volatility, time to maturity and level of moneyness). In principle, it is possible to replace the regular binomial tree by an implied tree. Derman and Kani [1994] and, separately, Dupire [1994] have shown that, if we know standard index options prices of all strikes and expirations, then in principle we can uniquely determine the local (Canadian)volatility surface function $\sigma(S,t)$. A similar, though not identical, approach has been taken by Rubinstein [1994]. Thus, for many days of sampling, the Black-Scholes(1973) equation has to be inverted to estimate the implied volatility of Canadian S&P/TSX 60 Index options that have different strike prices and times to maturity.

- **Risk-Neutral probability density function**

I have already illustrated the deficiencies of the Black-Scholes options-pricing model. As noted earlier, volatility is the only unknown in calculating the Black-Scholes options prices. For estimating volatility, the usual practice is to take the observed options price and numerically invert the pricing formula to obtain the "implied volatility" over the term of the option. Implied volatility can also be described as the estimate of the volatility parameter that equates the observed market price of an option to the theoretical Black-Scholes price of the option.

If the assumptions underlying the Black-Scholes model were correct, then the implied volatility would be the same regardless of the exercise price or time to maturity. Instead, when the estimated implied volatilities are plotted against the exercise prices, one typically sees a "smile" or "smirk." The volatility smile shows that deep out-the-money and deep in-the-money options are priced by the market higher than theoretically forecasted by the formulas based on the lognormal distribution, that is, the estimated implied volatility is higher for options that are deep in- or deep out-of-the-money. The volatility smile is quite typical for options on currency. Often, the shape of the volatility smile for options on an index is called a "volatility smirk", because of its ascending line. The volatility smirk shows that deep in-the-money calls and deep out-the-money puts cost more than theoretically forecasted by the Black- Scholes formula, while deep out-the-money calls and deep in-the-money puts cost less. This type of shape of the volatility smile reveals that options sellers believe it is much more likely to suffer losses from selling out-the-money puts than out-the money calls. (More details are mentioned in "Implied Volatility Surface"). This empirical observation seems to refute the Black-Scholes assumption that index option prices follow a lognormal diffusion processes. For example, a smirk to one side or the other suggests a skewed distribution. This implies that extreme asset returns occur more frequently than described by the normal distribution.

If index prices do not follow normal distributions then there might be interest in finding a risk-neutral probability density function that can produce observed options prices. To illustrate how options prices and risk-neutral probability density functions are related, we can write the options prices as follows:

Equation 2 $$C(0,X) = \exp\{-r\,T\} \int_{X}^{\infty} \{\tilde{S}T - X\} q[\tilde{S}(T)] d\tilde{S}(T)$$

Equation 3 $$P(0,X) = \exp\{-r\,T\} \int_{0}^{X} \{X - \tilde{S}(T)\} q[\tilde{S}(T)] d\tilde{S}(T)$$

Here, C and P are the prices of European calls and puts at time t, respectively, with striking price X and expiring time T, r is the risk-free interest rate, and the function q [$\tilde{S}$ (T)] is the risk-neutral probability density function for the price $\tilde{S}$ (T) of the underlying asset on the maturity date of the option. It provides the probabilities attached by a risk-neutral agent to particular outcomes of the Canadian index level. Options prices thus depend on the degree to which an option is in-the-money, multiplied by the risk-neutral probability of being that deep in-the-money at expiration.

Breeden and Litzenberger (1978) were the first to derive the relationship between call options prices and the risk-neutral probability density function. They found that the risk-neutral probability density function was proportional to the second derivative of the call options price with respect to the exercise price:

Equation 4 $$\frac{\partial^2 C(0,X)}{\partial X^2} = \exp\{-rT\}q[\widetilde{S}(T)]$$

. This result, together with violations of various assumptions underlying the Black-Scholes options-pricing model, has spawned an active field of research on the extraction of risk-neutral probability density functions from observed options prices.

Until now, five main methods have been developed to obtain the risk-neutral probability density function:

1. Specifying a generalized stochastic process for the price of the underlying asset;
2. Specifying a parametric form for the risk-neutral probability density function;
3. Implying binomial tree approach constructs a binomial lattice
4. Smoothing the implied volatility smile with a quadratic approximation resulting in a smooth continuous function relating implied volatilities to exercise prices;
5. Estimates using non-parametric techniques.

In the first method, a stochastic process is specified for the index level, from which the risk-neutral probability density function can be derived and theoretical options prices calculated. Examples of stochastic processes are jump diffusions, stochastic volatility processes, and diffusions with non-constant coefficients. The parameters of the stochastic process are then estimated by minimizing a loss function defined over the pricing errors between the theoretical and actual options prices for a given set of exercise prices. This method has been used by Stein and Stein (1991) and Heston (1993). In 1991, Stein and Stein studied the stock price distributions that arise when prices follow a diffusion process with a stochastically varying volatility parameter, as described in the following two equations:

Equation 5 $dP = \mu P dt + \sigma P dz_1$

Equation 6 $d\sigma = -\delta(\sigma - \vartheta)dt + kdz_2$

Where P is the stock price, σ is the "volatility" of the stock, k, μ, δ, and θ are fixed constant, and dz_1 and dz_2 are two uncorrelated Wiener processes. Thus the model is one where volatility is governed by an arithmetic Ornstein-Uhlenbeck (orAR1) process , with a tendency to revert back to a long run average level of θ . They then use analytic techniques (related to the heat equation for the Heisenberg group) to derive a closed form solution for the distribution of stock price or option price, where volatility follows an autoregressive stochastic process. Johnson and Shanno(1987), Wiggins(1987) and Hull and White(1987) have also examined options pricing in a world where stock price dynamics are similar to those given by equation 1 and 2. The first two papers use numerical methods to determine option prices. In the third, Hull and White solve explicitly for the option price by using a Taylor expansion about k=0 (the point where volatility is nonstochastic). It is, however, not clear that such an expansion provides a good approximation to options prices when k is significant greater than zero. The Stein and Stein method allow both of k and δ to be a non-zero value. The option price F must satisfy the partial differential equation:

Equation 7

$$\frac{1}{2}\sigma^2 P^2 F_{PP} + rPF_P - rF + F_t + \frac{1}{2}k^2 F_{\sigma\sigma} + F_\sigma[-\delta(\sigma - \theta) - \phi k] = 0$$

Here ϕ denotes the market price of the stock's volatility risk and r is the riskless interest rate. The presence of ϕ in equation 7 reflect the fact that with stochastic volatility, one can't use an arbitrage argument to eliminate investor risk preferences from the options pricing problem, because the volatility σ is itself not a traded asset. Both Stein and Stein method and Hull and White(1987) assume that volatility is uncorrelated with the spot asset and use an average of Black-Scholes formula values over different paths of volatility. But since this approach assumes that volatility is uncorrelated with spot returns, it cannot capture important skewness effects that arise from such correlation. Heston(1993) offers a model of stochastic volatility that is not based on the Black-Scholes formula. It provides a solution for the price of a European call option when the spot asset is correlated with volatility. Deterministic volatility processes have been discussed by Rubinstein (1994), Derman and Kani (1994), Dupire (1994), Jackwerth and Rubinstein (1996).

The second method for characterizing the risk-neutral probability density function is to forgo discussing the diffusion process and instead directly specify a parametric form for the risk-neutral probability density function over the index level on the option's maturity date. A popular choice for the risk-neutral probability density function is that of a weighted sum of independent lognormal density functions, called a mixture of lognormal distributions (MLN). Ritchey (1990) has demonstrated that, in this case, European options prices can be written as weighted sums of Black–Scholes options prices. He assumes that the implied density function, $f(S_T)$, of the underlying asset terminal price, S_T, comprises a weighted sum of k individual lognormal density functions:

Equation 8 $$f(s_{r)}=\sum_{i=1}^{R}[\theta_i L(a_{i,}b_{i,}S_T)]$$

where $L(a_i, b_i, S_T)$ is the ith lognormal density function with parameters a_i, b_i:

Equation 9 $$L(a_i,b_i,S_T)=\frac{1}{S_t b_i\sqrt{2\pi}}e^{\{-lnS_T-a_i)^2\}/2b_i^2}$$

Equation 10 $a_i = \ln S + (\mu_i - \sigma_i^2/2)\,t$ and $b_i = \sigma_i\sqrt{t}$

In the above equations, S is the spot price of the underlying asset, t (=T-t) is the time remaining to maturity and m and s are the parameters of the normal risk neutral distribution function of the underlying returns. The weights θ_i are positive and sum to unity.

This method has been used by Gordon Gemmill and Apostolos Saflekos (1999) on FTSE-100 index options. Since their data on FTSE-100 options cover a limited range of exercise prices for each maturity, they use two lognormals, which require only five parameters: the mean of each lognormal, α1, α2, the standard deviation of each lognormal, b1, b2 and the weighting coefficient, θ. They first let the terminal pay-off on a European call maturing at time T be max(S_T - X, 0), given terminal asset price S_T and exercise price X. Assuming that the risk-free interest rate r is constant, the life of the option is t and the asset price is S, then the price of the call is the discounted expected payoff (conditional upon finishing in the money) times the probability of finishing in the money:

Equation 11 $$C(X,\tau)=e^{-rt}\int_{x}^{\infty}f(S_\tau)(S_\tau-X)dS_\tau$$

Where $f(S_T)$ is the risk-neutral probability density function of the terminal asset price at time T. Similarly, the terminal payoff on a European put is max(X - S_T, 0) and its current price is:

Equation 12 $P(X, \tau) = e^{-r\tau} \int_0^X f(S_\tau)(X - S_\tau)\, dS_\tau$

Under the assumption that the probability density function is a mixture of two lognormals (with weights θ and (1-θ)), the above equations for call and put prices can be rewritten as:

Equation 13 $C(X, \tau) = e^{-r\tau} \int_x^\infty [\theta L(a_1, b_1, S_\tau) + (1-\theta) L(a_2, b_2, S_\tau)](S_\tau - X) dS_\tau$

Equation 14 $P(X, \tau) = e^{-r\tau} \int_0^x [\theta L(a_1, b_1, S_\tau) + (1-\theta) L(a_2, b_2, S_\tau)](X - S_\tau) dS_\tau$

These equations can be used iteratively to minimize the deviation of estimated prices from observed prices, with a search being made over the five parameters. They use both puts and calls across five exercise prices and minimize the total sum of squared errors for the ten options:

Equation 15 $\sum_{i=1}^{n} [c(X_i, \tau) - \tilde{c}_i]^2 + \sum_{i=1}^{n} [p(X_i, \tau) - \hat{p}_i]^2$

where b1, b2 > 0, subscript i denotes an observation and ^ denotes an estimate. Bahra (1997) shows that equations 13 and 14 have the following closed-form solutions, which are the weighted sum of two Black-Scholes expression in equation 13 & 14 with the stock price normalized to one:

Equation 16

$$c(X, \tau) = e^{-r\tau} \{\theta [e^{a_1 + b_1^{\tau/2}} N(d_\tau) - XN(d_2)] + (1-\theta)[e^{a_2 + b_2^{\tau/2}} N(d_3) - XN(d_4)]\}$$

Equation 17

$$p(X,\tau) = e^{-r\tau}\{\theta[-e^{a_1+b_1^{\tau/2}{}_1} N(-d_\tau)-XN(-d_2)]+(1-\theta)[e^{a_2+b_2^{\tau/2}} N(-d_3)-XN(-d_4)]\}$$

where

Equation 18

$$d_1 = \frac{-\ln X + a_1 + b_1^2}{b_1}, \quad d_2 = d_1 - b_1 \quad \text{and} \quad d_3 = \frac{-\ln X + a_2 + b_2^2}{b_2}, \quad d_4 = d_3 - b_2$$

Computation of a probability density function usually occurs in the context of a more general fundamental economic question, often relating to a possible change in regime, or other phenomena that would affect expectations before showing up in time series data. In Melick and Thomas (1997), the probability density function of future oil prices reveals the effects of the Gulf war in 1991 on the expected price of oil. Leahy and Thomas (1996) derive the probability density function of the Canadian dollar-U.S. dollar exchange rate during the October 1995 referendum on Quebec independence. In these two papers, the probability density function is sometimes characterized by two modes corresponding to two political outcomes—war vs. peace, or independence vs. national unity. Campa, Chang, and Reider (1998) compute probability density function on key cross rates within the “Exchange Rate Mechanism” of the European Monetary System in order to determine the size of ERM bandwidths consistent with market expectations of exchange rate convergence/divergence. Campa, Chang, and Reider (1998) derive a number of exchange rate probability density function in order to study the relation between skewness and spot, with implications for whether exchange rates follow implicit target zones.

The third method of the implied binomial tree approach constructs a binomial lattice for the value of the underlying asset subject to the tree correctly pricing a single cross-section of option prices at a point in time. It was developed by Rubinstein (1994). The researcher specifies a set of prior risk-neutral probabilities, usually chosen to be consistent with a lognormal state price density. To correctly fit the cross section of observed option prices, these prior risk-neutral probabilities are adjusted to the "implied" risk-neutral probabilities. The restriction that the binomial tree correctly prices a set of option prices amounts to an optimization problem minimizing the sum of squared deviations between the implied and prior risk-neutral probabilities, subject to the restriction that the implied probabilities correctly price the set of options and the underlying asset. The resulting implied binomial tree represents a set of risk-neutral probabilities that are consistent with option prices at a single point in time only. It also requires reliable option prices over a range of strike prices. This method is developed by Jackwerth and Rubinsein(1996), which is the focus of this paper. For more detailed development, I describe it more in the following section.

The fourth method for characterizing the risk-neutral probability density function, one of the earliest approaches, the so called Implied Volatility Function, was developed by Shimko (1993) as a way to smooth the volatility smile with a quadratic approximation. The result was a smooth continuous function relating implied volatilities to exercise prices. These implied volatilities were then substituted into the Black–Scholes options pricing formulae, making it possible to recover options prices for a continuous series of exercise prices. This permitted the use of the Breeden and Litzenberger' (1978) results, that the state price density can be inferred from a continuum of European options written on the same asset with the same time-to-expiration when the strike prices span the set of values from zero to infinity. The state price density is inferred by calculating the second derivative of the option pricing function with respect to its strike price. Shimko interpolates implied volatilities from observed option prices by using the Black-Scholes formula. A smoothing function is then fitted to these implied volatilities and the associated strike prices that form the X-axis. The implied volatility function (the smile) is then inverted using the Black-Scholes model to produce a continuum of option prices. The Black-Scholes model is only used here to transform one measurement space to another, and the model's validity need not hold. Taking the second derivative of the continuous call pricing function yields the state price density between the lowest and the highest strike options. Malz (1997) uses the call option delta rather than the strike price as the X-axis when fitting the smoothing function to the implied volatility smile. A problem arises with smoothing the volatility smile, however. The approximation is applicable only within the range of available exercise prices, since the curve will explode outside that range. Therefore, these methods typically attach ad hoc "tails" to describe the behavior beyond the range of available exercise prices.

The fifth method for characterizing the risk-neutral probability density function follows Aït-Sahalia and Lo (1998), who refer to the state-price density (SPD) rather than the risk-neutral density. They use non-parametric options-pricing formulae, and kernel estimation techniques to construct a smooth options pricing function, to which the

Breeden and Litzenberger result may be applied, and from which the state-price density or risk-neutral probability density function may be obtained. The state-price density can also be called the pricing kernel, since once it has been obtained one can price any asset at the current time, with payoff at that date. Though this method can be quite flexible, it also tends to be very data-intensive. Though many of these approaches differ in the specification and/or estimation of the risk-neutral probability density function, they typically give similar results. For example, Campa, Chang, and Reider (1998) find that smoothing the volatility smile gives results quite similar to the MLN method, while Dumas, Fleming, and Whaley (1998) find that smoothing the volatility smile gives results very similar to deterministic volatility functions. Dumas, Fleming and Whaley (1998) find the performance of the deterministic local volatility function model of Derman and Kani (1994) and Rubinstein (1994) disappointing in the sense that it is forecasts the price of European options less accurately than the "ad hoc" Black-Scholes. They propose a stimulating empirical strategy to test the consistency of implied binomial trees method. The intuition of their test is the following: At time t they use the Rubinstein approach to compute an implied binomial tree, using the information contained in the current stock price and in the prices of options maturing at time T. Then, at time t+1 (in fact one week later), they replicate the same computations, using again the implied state prices corresponding to time T. If the Rubinstein model is well specified, the implied probabilities starting from the node reached at time should be the same, irrespective of whether they are computed at time t or at time t+1. In fact, they reject this hypothesis. They trace back the cause of this rejection to the overfitting nature of the Rubinstein method. They also show that the pricing error generated by the implied binomial tree approach exceeds that stemming from a simple extension of the Black-Scholes formula.

- Parametric or Nonparametric?

Option price data have characteristics, which are both nonparametric and parametric in nature. The economic theory of option pricing predicts that the price of a call option should be a monotone decreasing convex function of the strike price. It also predicts that the state price density, which is proportional to the second derivative of the call function, is a valid density function over future values of the underlying asset price, and hence must be non-negative and integrate to one. Except in a few polar cases, the theory does not prescribe specific functional forms. (Indeed the volatility smile is an example of a clear violation of the lognormal parametric specification implied by Black-Scholes.) All this points to a nonparametric approach to estimation of the call function and its derivatives.

On the other hand, multiple transactions are typically observed at a finite vector of strike prices. Thus, one could argue that the model for the option price – as a function of the strike price (other variables held constant) - is intrinsically parametric. Indeed given sufficient data, one can obtain a good estimate of the call function by simply taking the mean transactions price at each strike price. Unfortunately, even with large data sets, accurate estimation of the call function at a finite number of points does not assure good estimates of its first and second derivatives, should they exist. To incorporate smoothness and curvature properties, one can select a parametric family which is differentiable in the strike price, and impose constraints on coefficients. Such an approach, however, risks specification failures.

Contrasting the highly structured assumed distributional forms of parametric models, nonparametric estimation methods seek to let the data speak for itself. These methods are extremely flexible, but require large amounts of data. In this section, I have already described a range of methods, but will leave out the nonparametric technique- the regularization method proposed by Jackwerth and Rubinstein(1996), which is the focus of this paper, for more detailed development in the following section.

- **Implied Risk Aversion**

In pricing securities, we discount the expected payoff of the security when we use risk-neutral probabilities, which will normally differ from the actual probabilities. The reason is that investors are typically risk averse. That is, promising a dollar payoff to a Canadian investor in a state of the world where the investor is already wealthy is worth less to the investor than in a state where the investor is poor. The investor will, therefore, lower his or her actual probabilities to arrive at risk-neutral probabilities for pricing purposes. Similarly, the investor will raise the actual probability to arrive at a higher risk-neutral probability for dollars obtained in states of the world when he or she is poor. Once we accept that the risk-neutral distribution is not the same as the actual distribution, we can try to estimate the two distributions and divide them into each other to obtain empirical functions of scaled marginal utility (Jackwerth 2000).

A stochastic discount factor that is uniformly decreasing in states of the world is central to asset pricing models. Most pricing models are more restrictive. For example, the CAPM implies that the pricing kernel is linearly decreasing in market return. Several studies (Bansal and Viswanathan 1993 and Dittmar 2002) document that the pricing kernel is not linear, thus rejecting the implications of the traditional APT and CAPM. To compute the marginal utility function, most of studies using options data use a time series of prices to calibrate an option pricing function, which is then used to estimate stochastic discount factors. This can be done in three ways (Sophies Shive 2003). The simplest method of estimation is to assume a parametric form for the option pricing function, which incorporates the underlying price, its volatility, the risk-free interest rate, the strike price, dividend yield, and time to maturity. The second is to avoid postulating a functional form by estimating the option pricing function nonparametrically. This can be done by using a six-dimensional kernel regression with one dimension accounting for each of the factors. A third way is to blend the two methods by using a semi-parametric estimate. One way to do this is to specify a relation between certain parameters. For example, Jackwerth (2000) assumes the Black-Scholes model holds and models implied volatility nonparametrically as a function of the strike price. This assumes that the other parameters do not affect implied volatility and enter into the pricing function as specified by the Black-Scholes formula. Since nonparametric methods are data intensive, the researcher is faced with a trade off between data requirements and possible misspecification if a semi-parametric or parametric model is used. Two prior studies are closely related to this paper. Jackwerth (2000) uses a semipara- metric method to estimate risk aversion functions implied by option prices and returns on the S&P 500 index. His sample consists of S&P index options from 1986 to 1995. He finds that the risk aversion functions are well behaved (positive and monotonically downward sloping with respect to terminal wealth) until the 1987 crash. After the crash, the functions sometimes become negative around the center and can be increasing. Ait-Sahalia and Lo(2000), in the context of a study adjusting Value at Risk for risk aversion, compute an implied marginal utility function for a 1993 sample of S&P 500 data, and plot it against the possible future prices of the underlying asset. The plot shows a `bump' in the stochastic discount factor which, while not mentioned in their study, appears to be statistically significant given their confidence bounds.

- Theoretical background

The fundamental theorem of asset pricing states that the absence of arbitrage is equivalent to the existence of a positive linear pricing rule (Duffie 2001). In continuous time, a manifestation of this pricing rule is scaled stochastic discount factor. It is often written as $\frac{\partial Q}{\partial P}$, where P is the actual probability measure over states of the world, and Q is the risk neutral probability measure. This measure is useful because under Q, security prices are simply expected payoffs discounted at the risk-free rate. Some asset pricing models (e.g. consumption-based models) interpret the stochastic discount factor as the discounted marginal rate of substitution between an extra unit of consumption today and one at terminal time, and we will use this interpretation here. An agent is defined by a strictly increasing, state-independent utility function *U.* The agent is not necessarily a representative agent, but one who determines the prices of the index and of the options. This is a milder assumption than that of a representative agent whose utility function prices all assets. The agent wants to maximize utility subject to a budget constraint. researchers assume that there is an interior solution to this maximization problem, and thus that there is no arbitrage (Duffie 2001).

If Q is the risk neutral probability across states and P is the actual probability, and U is the utility function that determines the price of the asset, then the agent wants to maximize the following expression over terminal wealth W, given initial wealth I:

Equation 19 $$\max_{w} \int P(W)U(W)dW - \lambda[\frac{1}{r^t}\int Q(W)WdW - I]$$

where λ , is the shadow price of relaxing the budget constraint by one unit, and r^t is a discount factor for t periods. Previous researchers solve this for every level of terminal wealth W, and since S is the return on the index across states and the investors who determine prices must hold the entire index, we can replace W by S in what follows. Assuming there is an interior optimum, the necessary condition is:

Equation 20 $$U'(S) = \frac{\lambda Q(S)}{r^t P(S)}$$

From this equation, we can see that U' is a scaled stochastic discount factor. The parameter λ is difficult to estimate, so we cannot estimate the level of U' using this equation. Using the relation $E[U'(S)] = \frac{1}{r_f}$

Where r_f is the risk free rate (Duffie 2001) one could scale the estimate of the stochastic discount factor. Equation shows that the marginal utility function is a ratio of the risk-adjusted probability distribution to the subjective probability distribution. The numerator is a conditional estimate, because it is based on the stock prices at the date where the estimate is made. To be consistent, the denominator must also be a conditional estimate.

Bartunek and Chowdhury(1997) were the first to use the method, but they restricted the utility function to be of the power utility type. They fitted a lognormal distribution to S&P 100 index returns and estimated risk aversion coefficients between 0.2 and 0.6 to fitting S&P 100 option prices. These values appear to be low when compared with surveys that suggest coefficients of 2-5. Bliss and Panigirtzoglor(2002)used the same parameterization and added exponential utility functions. Using data for the British FTSE 100 Index returns and options and S&P 500 returns and options, they found rather moderate risk-aversion estimated—between 2 and 7—whereas the equity premium literature suggests much higher values (e.g., 55 from Mehra and Prescott 1985). A problem with these approaches is that they parameterize the utility function rather than estimating it as a nonparamentric function from the data (Jackwerth 2000)

As mentioned in "risk-neutral distribution", five main methods are used to estimate the risk-neutral distribution. The actual distribution can be derived from fitting a time-series model to the index returns. We then divide the two probability distributions to arrive at nonparametric empirical functions of scaled marginal utility. Using data from the S&P 500 options market, Ait-Sahalia and Lo(2000) used kernel methods for both distributions. Rosenberg and Engle fitted a curve to the implied volatility smile to obtain the risk –neutral distribution and fitted a GARCH model to the index returns. Coutant(2000)used Hermite polynomials for the risk-neutral distribution and fitted an ARCH-type model to the French CAC 40 index. Also using the CAC 40, Perignon and

Villa (2002) used kernel methods for fitting both the implied volatility smile and the historical return distribution. Hayes and Shin (2002) used a cubic spline to fit the implied volatilities across deltas in combination with a GARCH model for the S&P 500 returns (Jackwerth 2000).

Jackwerth(2004) uses the implied risk-aversion functions, which are closely related to marginal utility functions (they are the negatives of the derivative of log marginal utility)and documented a so-called pricing kernel puzzle. He found that the implied marginal utility function is monotonically decreasing across wealth in a precrash period-1987, as economic theory would suggest. That is, people grow less appreciative of additional wealth as they become wealthier. This picture changes during the postecrash period, however, when the implied marginal utility function is locally increasing in wealth for wealth levels close to the starting wealth level. The studies of Rosenberg and Engle (2002) and of Perignon and Villa confirmed these findings. Ait-Sahalis and Lo (2000) and Bliss and Panigirtzoglou(2002), however, did not find such an effect. Ait-Sahalia and Lo used a whole year of option price data and averaged the information, but these authors may have smoothed their kernel estimates too much. Bliss and Panigirtzoglou used a parametric form of the utility function, which precluded finding a potential pricing kernel puzzle a priori.

In order to provide an understanding of the pricing kernel puzzle, Jackwerth(2002) depicts empirical estimates of the risk-neutral and the actual distributions for four international indexes. The actual distributions appear to be somewhat normally distributed, whereas the risk-neutral distributions are (except for Japan) left skewed and leptokurtic; that is, they have fat left tails and are more peaked than a normal distribution.

He finds that it is consistent with the underlying volatility smiles compared with out-of-the –money calls. This pattern can best be viewed as "crash-o-phobia"(a term coined by Rubinstein 1994) investors who are concerned about market crashes insured themselves by buying out-of –the –money puts, which put a floor on the maximum losses the investors can sustain. The prices of these puts are then pushed up because only a few investment banks and hedge funds are willing to supply such insurance in large quantity. Still, the relationship that marginal utility is proportional to the ratio of risk-neutral probability and actual probability is based on this assumption. This is the puzzle, and a number of research papers have examined it. In conclusion, this discussion has shown that a triangular relationship exists between risk-neutral probabilities, actual probability distribution and the scaled marginal utility (pricing kernel). Given any two quantities, I can infer the third. Historically, the path taken was to assume a model for the representative agent (or a dynamically complete market, thereby the utility function). Then, we derive the risk-neutral probabilities and price all the securities. Based on this section, this view has been turned upside down; I can estimate the risk-neutral and actual probability distributions to estimate the implied utility functions, which are notoriously difficult to observe. (Jackwerth 2004)

METHODOLOGY

- **Canadian volatility smile**

The first objective of this paper is to determine if a Canadian volatility smile exists. Thus, for many days of the sample, the Black and Scholes(1973) equation has to be inverted to estimate the implied volatility of options that have different strike prices. Specifically, in this paper, the implied volatility is estimated for all call options on the TSE-60 index in the January 2,2003 to June 30,2004 period. In total, it gives 13877 options and hence the same amount of implied volatilities. This should be sufficient to examine the possibility of the existence of a Canadian smile. The implied volatilities are classified on the basis of the moneyness (ratio of the underlying spot price to strike price) and days to maturity of their respective options. With this classification, it is possible to examine if the average implied volatility changes for different values of moneyness. Then, comparisons of the smile for different times to maturity are feasible. The summary of implied volatilities for the S&P/TSX 60 Index (January 2,2003 to June 30,2004 period) is presented in Table 1.

From Table 1 and Figure 2 , we can find that when the time-to- maturity is less than 60 days, the average implied volatility decreases when the moneyness (X/S) increases up to the point when the options are at-the-money. When the moneyness is higher than 1.00, the volatility increases with increases in moneyness. This trend represent a typical "volatility smile". However, it should be noted that this U-shape, observed for the aggregate results for maturities less than 60 days, is not a good representation of the shape of the smile for all categories of maturities. The same result cannot be observed for all other categories with the time to maturity greater than 60 days.

Graphical demos are presented in figure 3, figure 4, figure 5 and figure 6. The volatility appears to be relatively stable and does not fluctuate with the change in moneyness. It seems that volatility is constant for changing moneyness for these four categories of maturity. In other word, my finding for these four categories can support the volatility assumption of the Black-Scholes model. One potential reason, which seems to be reasonable, is for the time to maturity greater than 240 days. The available data in this category is too little to represent a "true" result. The observations are not well spread across the 18 months of the sample. However, this is not the only category that has the stable volatility, in the other three categories, i.e. with time to maturity within 60 to 120 days, 120 days to 180 days and 180 to 240 days as well, the observations with sufficient and well spread data also exhibit a stable volatility trend, which I cannot explain. The result of my less than 60 days maturity case is particularly sharp, which the ratio of highest /lowest implied volatility close to 3. This result is very similar as in Jackwerth's(2004) findings for C.U.K FTSE 100. When maturity is greater than 60 days, my findings of all other four categories are very similar as to those of the Japanese Nikkei 225 in Jackwerth's finding (2004).

However, if we analyze the data more closely, it is possible to see that the shape of individual smiles is more often than not a decrease from low moneyness to high moneyness (downward sloping). By plotting the implied volatilities against the moneyness (defined as X/S) of individual options, it is possible to see the shape of the volatility smirk for each option series. [1]. For maturities of 30 days and 90 days, I have 31 option series for a total 178 options available. These series of options are also used to derive the risk neutral probability distribution, for which I give more details in the next

[1] A series of options is defined as all the options that have the same maturity date, but have different strikes

section. With 31 option series, I will have 31 estimated "smile". A summary of the option series of 30-days maturity and 90-days maturity is presented in Table II. Graphic presentations are shown in Figure 7 a to Figure 7 h.

For option series of 30-days maturity, I have a typical U-shape smile for all categories of implied volatility. The figures including the highest implied volatility, lowest implied volatility, Median Implied volatility as well as Mean implied volatility all exhibits a very classical U-shape "volatility smile". However, as shown in Figure 7 e to Figure 7 h, I have somewhat upward slopes for option series of 90-days maturity. These kinds of curve are not common. One potential reason for these upward slopes can be found in the last column of Table II. For option series of 90-days maturity, when moneyness is greater than 1.1, there are only 2 available options. It is too small a sample to be truly representative. It cannot be used as representative of the typical implied volatility curve for the period January 2, 2003 to June 30, 2004. As a result, I remove the last line and re-graph them in Figure 7 i to Figure 7 l. Then I get a "volatility smirk" rather than a "volatility smile" for all categories. These findings are very similar as to the German Dax 30's volatility curve (Jackwerth's 2004). From table II, I also find that long-term options have a lower implied volatility than short-term options. This trend can be found in all categories of moneyness and in every presentation method of implied volatility. A possible explanation for the presence of a smirk is based on the violation of one of the basic assumptions of the Black-Scholes(1973) equation, namely the constant volatility assumption (Rubinstein 1994). Therefore, it would be wrong to assume that the risk-neutral probability distribution of the underlying asset is lognormal.

- Potential Explanations

Many solutions have been offered to explain the downward sloping and, to a lesser extent, the U-shaped volatility smiles. Black (1976) and Christie (1982) first mentioned the leverage effect. When the price of its stock falls, the corporation's debt-to equity ratio increases because debt stays constant while equity is being reduced. If a shock of the same size happens to asset prices before and after the fall in asset prices, the impact of the shock on equity is larger after the fall in asset values, which will cause a high debt-to equity ratio, and hence, a higher risk and volatility. Thus, volatility is higher for low strike prices. Toft and Prucyk (1997) argued, however, that the leverage effect is a rather minor effect that can explain only about half of the already relatively flat smiles of individual U.S. stock options. Thus, it would explain even less of the steep index option smiles.

Guidolin and Timmermann (2000), and Romer(1993) suggested solving the enigma of the volatility smile by using models with information aggregation. In these models investors learn about the true value of the underlying asset through trading, and prices adjust rapidly once learning takes place. Unfortunately, according to these models, decreases in asset prices are as likely as increases in asset prices, whereas the downward-sloping volatility smirk suggests that decreases in asset prices are more likely than increases. The smirk is thus more in tune with our understanding that markets sometimes melt down but rarely ever "melt up". On a downward- sloping volatility smirk, the out of the money puts are relatively expensive. Those put options essentially provide portfolio insurance; that is, they pay off when the market crashes. The options are thus priced in such a way that they incorporate some investors' fear that market crashes are rather likely. All these assumptions generate only rather moderately sloped volatility smiles, however, and do not explain the steep volatility skews in the indexes.

There have been many papers studying the departures from normality of asset return distributions. The relation between implied volatility and strike price is termed the "implied volatility smile" and this is in effect a "moneyness" bias. Figure 7 shows the "volatility smile" for call options on the S&P/TSX 60 Index. If the Black-Scholes formula were entirely valid, the plot of implied volatility against strike price of options on the same asset with identical maturity would be a straight horizontal line. Instead, there is clearly a "volatility smirk" in the case of the S&P S&P/TSX 60 Index. For call options, increasing values along the x-axis represent higher strike prices and thus, options further "out of the money". The volatility decreases as the strike price increases. The Black-Scholes formula therefore, would be expected to over-price "in the money" calls and "out of the money" puts[2] . This is because implied volatility is highest for low strike prices corresponding to "in the money" calls and "out of the money" puts, and this is an indication that the Black-Scholes formula under-prices "out of the money" calls and "in the money" puts. The graphs in Figures 7 concur with this conclusion. The implied volatility is highest for "in the money" calls and "out of the money" puts. This implies that the Black-Scholes formula would over-price such options. The opposite situation arises for "out of the money" calls and "in the money" puts. High strike prices correspond to lower implied volatilities. It is clear from figure 7 i, j, k and l that the implied volatility does decrease as strike price increases, that there exists a "volatility smirk" for stock

[2] Though I did not analyze puts, theoretically, the meaning of "in the money" calls and "out of the money" puts can be interchanged based on Put-Call Parity theorem.

index options. This "volatility smirk" arises because of the violation of fundamental Black-Scholes assumptions of constant volatility and lognormal price distribution. In order for option prices to correctly follow the Black-Scholes formula, the volatility has to be constant. Volatility is not constant over the life of most financial assets however. In reality, non-constant volatility and probability of "jumps" in the underlying asset price exist. The "volatility smirk" arises because the fundamental assumptions of the Black-Scholes formula do not hold in reality. In the following sections, the reasons for the "volatility smirk" are discussed in greater depth.

- **Implied Distribution. Actual Distribution. Normal Distribution**

The risk-free rate turned out to be a convenient choice, because it represents how a risk-neutral investor would discount. Thus the terms "risk-neutral pricing" and "risk-neutral distribution" are introduced. The implied distribution is risk neutral by construction. I can also obtain the actual distribution from fitting a time-series model to the index returns. The probability is derived based on the same return range as what I get from the risk neutral distribution. I present more details about the way of deriving the risk-neutral distribution and the actual distribution in the next section.

The implied distribution for stock index options is not lognormal. In fact, due to the downward slope of the "implied volatility smile", the risk-neutral distribution implied from option prices, which is determined by the implied volatility smile and vice versa, will be left skewed and leptokurtic. That is, there is a "fat left tail" and a "thin right tail" relative to the (log) normal distribution, which means a higher probability of extreme loss. .i.e. market crashes and they will be more peaked than a (log) normal distribution.

This indicates an increase in risk. .i.e. a greater percentage of extremely large deviations from the mean.

The evidence in the form of the volatility smirk is indicative of implicit stock return distributions that are negatively skewed and with a higher kurtosis than allowable in a normal distribution. The underlying implied distribution generally creates the "volatility smile". According to the implied distribution, downward movements in prices are more likely than the normal distribution would predict. Similarly, upward movements in prices are less likely than the normal distribution would predict. The assumptions of a normal distribution do not therefore hold realistically. If there was a distribution with "fat tails" at both ends, the market would assume that movements in both directions are more likely than that predicted by the lognormal distribution. Such a distribution would give rise to a more U-shaped "volatility smile".

Theoretically, at low strike prices, "out of the money" puts will be overpriced under the implied distribution relative to the lognormal distribution. This is because the option pays off only if the stock index value moves below a particularly low strike price. The market predicts that a downward movement is more likely than the lognormal distribution would predict. This is evident from the "fatter" left tail and more "Peak" under the implied distribution. The market thus assigns a greater value to the "out of the money" put than under the normal distribution. This means the "out of the money" puts are over-priced and have greater volatility. This is also evident from the downward "volatility smirk". This analysis holds for "in the money" calls because "out of the money" puts are essentially the same as "in the money" calls. Therefore, "in the money" calls are also over-priced by the Black-Scholes formula assuming (log) normal distribution.

At high strike prices, calls are "out of the money" and will be under-priced by the implied distribution. Such options pay off only if the stock index value moves above a particularly high strike price. According to the implied distribution's "thin" right tail at an high strike prices, the market predicts that upward movement is less likely than the lognormal distribution would predict. The market thus assigns a smaller value to an "out of the money" call because there is less likelihood of the stock index value exceeding a given high strike price. Again, the downward "volatility smirk" also proves it. Similar to the argument above, an "in the money" put is the same as an "out of the money" calls. Therefore, "in the money" puts are also under-priced. In reality, stochastic volatility leads to a greater likelihood of a negative outcome relative to the normal distribution. The implied distribution of the asset price has a greater kurtosis than the normal distribution. For stock index options, this translates to the implied distribution having the same mean and standard deviation as the lognormal distribution but a "fat left tail" and a "thin right tail". A possible reason for this is that the implied distribution accounts for a potential market crash.

I ensures surrounding the corresponding actual distribution over the same horizon, with the same return interval as the implied distribution. Jackwerth(2004) finds that the actual distribution and the normal distribution do not match well too and that "the actual distribution exhibits fat tails and is too peaked to be normal; also at times, the return distribution is skewed" Jackwerth(2004,p29). He also gives two reasons for the causes. "First, the return distribution may not be lognormal but leptokurtic. Second, a return distribution may be nonstationary over long periods. Returns are likely to be nonstationary if the economy fundamentally changes. Nonstationarity introduces leptokurtosis even if the true underlying distribution is lognormal but with time-varying parameters".

The Black-Scholes model is considered most inaccurate for options that are not "at the money". Nonetheless, any alternative to the Black-Scholes model should accurately price options that are not "at the money". In most research in this field, these are the types of options that are considered most inaccurately priced. The Black-Scholes formula is a no arbitrage model, which assumes away market frictions. This is not true in reality and may perhaps account for the pricing errors in the Black-Scholes formula.

- **Lattice approach**

Parametric methods for recovering the risk neutral probability distribution specify a priori the form of the distribution. In contrast, non-parametric methods are more flexible and capture more information on the perception of market participants because any type of probability distribution is a possible solution. The method used in this paper to recover the risk neutral probability distribution of the underlying asset is a non-parametric lattice approach- the Jackwerth-Rubinstein(1996) method. The regularization method proposed by Jackwerth and Rubinstein (1996) revisits the implied binomial tree framework. While the implied binomial tree method is primarily concerned with constructing a risk neutral density that is consistent with observed option prices, the regularization method seeks an optimal trade-off between goodness of fit and the `smoothness' of the solution. Practically, this trade-off amounts to relaxing the requirement that the tree provides an exact fit to observed option prices. It is motivated by the fact that (1) the implied binomial tree approach results in functions that are insufficiently smooth to have plausible economic interpretations and (2) risk-neutral probabilities derived from S&P 500 index options are remarkably stable in the periods prior and subsequent to the 1987 stock market crash, inferring a stable risk-neutral density. These observations suggest that a stable risk-neutral density does exist and that the estimation procedure should reflect that. A description of the variables used in those models follows.

In the computation of a (n-1)-step binomial tree, the current underlying asset price (S_0) has to be deflated by the dividend yield (g) reflecting the market participants' expectations of dividend payments throughout the life of the option. The deflator is applied because the option holders do not get the dividends. The dividend yield is assumed to be constant and known by investors. Thus, S_0 denotes the current asset price that excludes dividends, Sj (j=1,.....n) denotes the possible terminal asset prices (dividends excluded) at the maturity date of the option. Let t be the time-to-maturity of the option in years. Then So is calculated as follows: $S=S_0 e^{-gt}$. The other parameters of the binomial tree are the time interval between two successive levels of the tree $\Delta t=\frac{t}{n-1}$; the implied volatility, σ; the up move factor, $u=e^{\sigma\Delta t}$; and the down move factor, $d=\frac{1}{u.}$. The annual riskless interest rate is denoted by r.

Observed option prices are required for the Jackwerth-Rubinstein(1996) models to recover the risk neutral probability distribution(s). The objective of this method is to estimate the probabilities attached to each of the n possible terminal underlying asset prices of the (n-1)-step binomial tree. These probabilities must be consistent with the observed option prices. In this paper, the observed option prices consist of a series of call options that have the same maturity, but different strike prices. Let C_{ib} and C_{ia} be the bid and the ask prices of the options observed with corresponding strike prices X_i for I=1,....,m and let C_i be the average of bid-ask prices of these options.

For every observed option in the series, the n possible terminal option prices (payoffs) of the binomial tree are calculated. Thus, c_{ij}=Max(0, S_j-X_i), (i=1,....m and j=1,....,n) denote the terminal option prices at the maturity date of the option. From this cross-section of option prices, the lattice framework, and the observed option prices, the n risk neutral probability values of the binomial tree can be estimated with the Jackwerth-Rubinstein (1996) approach. For these models, the risk neutral probability distribution is the same for all the options in the series because it is the risk-neutral distribution of the common underlying asset. With the n probabilities, it is possible to trace the shape of the empirical risk neutral probability distribution. This approach estimates the distribution by solving an optimization problem where the goal is to find the risk neutral probability distribution that fits the observed option prices. The optimization problem in Jackwerth and Rubinstein (1996) recovers the risk-neutral probability distributions from a series of option prices that expire on the same date. This optimization problem does not assume a prior distribution. Moreover, it selects the implied distribution with the maximum smoothness. Smoothness is important because the probabilities attached to similar states of mature, as described by the returns, should be close to each other. The purpose of their objective function is to: "find the smoothest distribution in the sense of minimizing the square of the second derivative of P_j with respect to the underlying asset level, thereby minimizing the curvature exhibited in the implied probability distribution" (Jackwerth and Rubinstein, 1996, p.14):

In particular, they consider:

$$\min \sum_j (P_{j-1} \text{ -2P}_j + \text{P}_{j+1})^2$$

where

P_j =Risk neutral probability that the underlying asset reaches the ending node j of the binomial tree. Jackwerth and Rubinstein (1996) emphasize that the objective of this step does not require a prior. "Its sole purpose is to find the smoothest distribution in the sense of minimizing the second derivative of P_j with respect to the underlying asset level, thereby minimizing the curvature exhibited in the implied probability distribution. Each term corresponds to the value of a butterfly option spread, which is the finite difference approximation of the second derivative $\partial^2 P_j / \partial S_j^2$ since if the S_j are equally spaced:

$$\frac{(P_{j+1} - P_j)/(S_{j+1} - S_j) - (P_j - P_{j-1})/(S_j - S_{j-1})}{1/2(S_{j+1} + S_j) - 1/2(S_j + S_{j-1})} = \frac{\Delta[\frac{\Delta p}{\Delta s}]}{\Delta s}$$

$$= \text{constant } (\text{P}_{j-1} \text{-2P+P}_{j+1})$$

while we can omit the constant term, we need to square each individual contribution to the curvature, since the sum would otherwise be degenerate and, in any event, we are interest in a measure of absolute curvature" Jackwerth and Rubinstein (1996).

Jackwerth and Rubinstein (1996) also state that it is more appropriate to use the midpoint of the bid-ask quotes instead of the bid and ask quotes to fit the distribution.

This is to avoid dealing with an implied volatility smile that is more convex than the ones obtained with either the bid or the ask prices. However, an optimization problem, based solely on the midpoint of the bid-ask quote, is less flexible than an approach allowing for the value of the options to be in between the bid and the ask. Thus, to avoid data overfitting, the authors include constraints in the objective function. The constraint functions are added to the objective function with a penalty parameter, α, so that the value of the objective function increases each time a constraint is violated. The larger the value of α, the larger the penalty applied to the function when constraints are not respected. After experimenting with different values of the penalty parameter, the most appropriate value is determined to be α =1000. The experimental values used in the optimization problem are α =10, 100, 1000, 10000 and 100000. I have selected the highest value of α that allows the problem to converge. Jackwerth and Rubinstein (1996) also state that the typical value for the penalty parameter is 1000 for a quadratic objective function. Hence, the α of 1000 should give accurate results without overfitting the data , and I use the same penalty for all constraints.

The first two constraints that are considered in the optimization problem are innocuous and stipulate that

1) The probabilities should not be negative and
2) The sum of the probabilities must be equal to one.

The third constraint insures the presence of the risk neutrality condition. That is, the present value of the underlying asset computed with the P_j must be equal to the spot price of the underlying excluding dividends.

The last constraint requires that the value of each option with the estimated distribution be equal to its respective bid-ask quote midpoint.

The complete version of the objective function to optimize (including the penalties) the following:

Equation 21

$$\min\sum_j (P_{j-1} \text{ -2P } + P_{J+1})^2 + \alpha \;\{\; \sum_j [\max[0 - P_j]]^2 + [\sum_j P_j - 1]^2 +$$

$$[((\sum_j P_j S_j)/e^{rt}) - S_0]^2 + \sum_i [((\sum_j P_j \max[0, S_j - X_i])/e^{rt}) - C_i]^2 \;\}$$

where

P_j =Risk neutral probability that the underlying asset reaches the node j of the binomial tree

α = Penalty parameter

g = Annual dividend yield

t = Time-to-maturity of an option (in years)

r =Riskless rate

S_j = Assumed terminal index values at option expiration induced from a (n-1) period binomial tree

S_0=The current underlying price, excluding dividends

X_i=Exercise price of option I

C_i=Midpoint between the bid and the ask of the option with the exercise price X_i

Before proceeding with the above nonlinear optimization problem, the option prices that violate the general no-arbitrage conditions must be removed from the data set in order to find an admissible distribution. As explained in the next section, I verify the presence of quotes that do not respect the Merton Bound and offer the possibility of implementing profitable Vertical and Butterfly spreads.

The "what's best software" (produced by Lindo Systems Inc.) is used to solve the nonlinear optimization problem. Because of the complexity of the objective function, most optimization software packages take time to converge to a local minimum. For this model, a 51 steps binomial tree is used. This gives 52 final node probabilities to trace out the shape of the risk neutral probability distribution. The small number of 52 final nodes will not be a problem and there will not be any negative effects on my results for the following two reasons. First, the focus of this paper is on the Canadian index, rather than on a particular stock. Though, theoretically, the index return can be any amount in a particular period, in the real world, the index is much more stable than an individual stock. Though it is very common to find that an particular stock increases or decreases drastically within a very short term period, it is not usual for the Canadian index to fluctuate drastically in a very short term period, i.e. one or three months. With the 52 final nodes, the return on the underlying index falls in the range of -55% to 55% for three months and -25% to 25% for one month. This range should be sufficient to cover the real world's return of the Canadian index within one and three months. Second, the actual test results also support my assumption: Bouchard (2002) finds that her results fall entirely within the range of -55% to 55% for three month returns, even though she has 199 final nodes. I get the same results based on the 52 final nodes, all risk neutral probabilities generated from the software fall in the estimated range for both one-month returns and three-month returns. Further more, the actual one-month returns and three-month returns, which are presented in Figure 9 and Figure 10 respectively, also support my assumption. Therefore, the results from the 52 final nodes are sufficient for the purpose of this paper.

I also obtain the actual distribution from fitting a time-series model to the index returns. The S&P/TSX 60 Index option was first introduced in December 1998. I collected all monthly returns since December 1998, and I have a total of 70 monthly returns to September 2004. Then I derive 68 three-month returns from these monthly returns. The probability is then derived based on the same return range that I get from the risk neutral distribution derived from the 52 final nodes. By plotting the actual probability based on the selected return interval, I obtain the actual distribution.

Once I have the "risk-neutral distribution" and the "actual distribution" I can divide one probability distribution by the other to arrive at nonparametric empirical functions of the scaled marginal utility. Jackwerth(2004) has proved that a triangular relationship exists between the risk-neutral probabilities, the actual probability distribution and the scaled marginal utility (the pricing kernel). Given any two quantities, we can infer the third. Jackwerth(2004) also develops an easy way to estimate the pricing kernel. He starts by calculating the value, C, of any security with payoff X_i at time T by calculating the discounted expectation under the risk –neutral probabilities (or simply by using the state prices right away):

Equation 22

$$C=\frac{[\sum_{i} P_i X_i]}{r^T}$$

$$=\sum_i \pi_i X_i$$

Two observations are important at this point. First, any security with payoff at T in our economy can be priced once we know the risk-neutral distribution. Second, the ratio of state prices to actual probabilities is also called the "pricing kernel" or "stochastic discount factor". The pricing kernel, m, tells us about the marginal utility of the representative investor in a particular future state of the economy: the poorer the investor, the higher the ratio; the wealthier the investor, the lower the ratio.

Using the definition of m$=\frac{P}{Q,}$ where Q is the actual probability and P is risk neutral distribution, Jackwerth(2004) rewrites the pricing equation. With the help of the pricing kernel, he values securities as expected scaled payoffs under the actual probabilities. In this valuation, the pricing kernel provides for the scaling:

Equation 23

$$C=\frac{[\sum_i P_i X_i]}{r^T}$$

$$=\sum_i \frac{P_i}{P_i} Q_i X_i$$

$$=\sum_i Q_i m_i X_i$$

following Jackwerth(2000)'s method, I can get the pricing kernel for the state of index that satisfy the above equation.

DATA SELECTION

- **Description of the data**

The call options on the S&P/TSX 60 Index are collected from the web site of the Montreal Exchange (www.me.org). TSE 60 options are European and they expire on the third Friday of the maturity month. The ticker symbol is SXO. The sample of call option prices considered in this paper covers the period January 2, 2003 to June 30, 2004. The expirations of the options are measured as the number of calendar days between the spot price trading date and their expiration date. In total, there are 13877 available options. A series of options is defined as all the options that have the same maturity date, but have different strike prices. Only maturities of 30 days and 90 days are considered, I have 31 option series and corresponding 31 estimated "smiles" for a total 178 available options[3].

The data on the riskless rate of interest are collected from http://www.bankofcanada.ca. The 1-month, 3-month and 6-month maturity T-Bill rates are available. When the maturity of an option, t, is between two T-Bill maturities, the appropriate riskless rate is estimated by interpolation. TSE60 Index prices are derived from a website http://finance.yahoo.com/q/hp?s=^SPTSE, its ticker symbol is ^SPTSE.

The Black-Scholes(1973) call option formula is used to find the implied volatilities required to compute the binomial tree values. The equation has to be solved numerically to estimate the implied volatility. The call option formula is the following:

Equation 24 $\text{Call}=S_0 N(d_1)\text{-}Xe^{-rt} N(d_2)$

Where

$$d_1=\frac{In(S_0/X)+(r+\sigma^2/2)/t}{\sigma\sqrt{t}}$$

[3] The total 31 option series includes 18 one-month series and 13 three-month series

$d_2 = d_1 - \sigma \sqrt{t}$

where $N(d_i)$ $i=1,2$ is the cumulative probability distribution function of a standardized, normally distributed, random variable. It provides the probability that the random variable is less than or equal to d; X is the exercise price of the option; r is the riskless rate; t is the time-to-maturity of the option; and σ is the implied volatility that needs to be estimated.

Because the spot prices of the index are often in between the available strike prices, the implied volatilities of the at-the-money options are estimated with interpolation for each series.

- **Arbitrage violations**

To find an admissible distribution, the data used for the method of Jackwerth and Rubinstein(1996) must be cleaned to remove observations that violate the no-arbitrage conditions. These authors suggest that such data be deleted from the sample or adjusted to respect the no-arbitrage conditions. The arbitrage relations that are verified are violations of the Merton lower bound and the presence of profitable Butterfly and Vertical Spreads. The put-call party relation is not verified because the put prices are not required in the recovery of the risk neutral probability distribution. The no-arbitrage conditions are verified on a daily basis for the period of January 2,2003 to June 30,2004. All the series of options are examined. Even if the risk neutral probability distributions of specific times to maturity are estimated in this paper, a broad verification that includes every day and every series of options available is highly recommended. In fact, the presence of important profitable arbitrage opportunities can be an indication of a market that is not liquid enough to support the methodology presented in the previous section. A liquid market is essential to recover the risk neutral probability distribution with a binomial tree approach because available information on the market must be captured simultaneously by the Spot and Option markets.

The Merton lower bound stipulates that if there is no dividend on the underlying asset prior to option expiration (the spot without dividend S_0 is used here), the value of the call ask price should be higher or equal to the Spot bid price minus the present value of the exercise price.

Equation 25 Merton lower bound : $C_a \geq S_{obid} - Xe^{-rt}$

The convexity relation verifies the convexity of three option prices having the same maturity, but different strike prices. If the strike prices are equidistant , the option with the middle strike price should have a bid price which is lower than half of the sum of the ask prices of the two adjacent options. If this is not the case, there is an arbitrage opportunity and a Butterfly Spread would generate risk free profits.

Equation 26

Convexity relation (strikes equidistant): $C_{2bid} \leq \frac{C_{1ask} + C_{3ask}}{2}$

If the strikes are not equidistant, a weighted sum of the option prices, based on the distances between the strike prices must be taken into account in the convexity relation if there are three options, namely C_1, C_2 and C_3 with strike prices X_1, X_2, and X_3 respectively, the following convexity relation should hold, otherwise a Butterfly Spread strategy could generate risk free profits.

Equation 27

Convexity relation (strikes non-equidistant):

$$\left(\frac{X2-X1}{X3-X1}*C_{3ask}\right)+\left(\frac{X3-X2}{X3-X1}*C_{1ask}\right)\geq C_{2bid}$$

Table III presents the arbitrage opportunities for the options on the TSE-60 index. All these observation are removed from my sample[4]. In this way, the data used for the method of Jackwerth and Rubinstein (1996) has been cleaned.

[4] There are total of 102 observations, which are removed from my sample. Comparing with the total of 1877 options, this amount is insignificant

RESULTS

Figures 8 present some typical risk neutral probability distributions obtained with the Jackwerth-Rubinstein (1996) method. Figure 9 depicts the actual distribution for the S&P/TSX 60 Index three-month returns. The actual distributions appear to be somewhat normally distributed; in comparing with the actual distribution, the risk-neutral distributions are left skewed and leptokurtic; that is, they have fat left tails. This is evident from Figure 11 b and Figure 11 c. Sometimes, the risk neutral distributions are also more peaked than the actual distribution. This is evident from the Figure 11 a. This is consistent with the shape of the underlying volatility smiles, which are depicted in Figure 7 i. j. k. and l.

The slope of the volatility smiles are possibly reflect the overpricing of out-of-the–money puts. This pattern can best be viewed as "crash-o-phobia"(a term coined by Rubinstein 1994), From Figure 11 b and c, we can see that the implied distributions become more left-skewed and change from platykurtic to leptokurtic than the actual distribution does in Figure 11 a. Specifically, the distribution is such that there is a "fat left tail" and a "thin right tail" relative to the actual distribution. Figure 11 depicts the theoretical difference in the implied risk neutral distribution observed for stock index options and the actual distribution with the same return interval as the risk neutral distribution.

The evidence in the form of the volatility smirk is indicative of implied stock return distributions that are negatively skewed and with a higher kurtosis than in an actual distribution. The underlying implied distribution generally creates the "volatility smile". Based on my finding, according to the implied distribution, downward movements in prices are viewed by risk neutral investors as more likely or more serious than the actual distribution represents. Similarly, upward movements in prices are less likely than the actual distribution describes. Theoretically, at low strike prices, an "out of the money" put will be overpriced by risk neutral investors under the implied distribution relative to the actual distribution. This is because the option pays off only if the stock index value moves below a particularly low strike price. The downward movement is less frequent than risk neutral distribution would suggest. This is evident from the "fatter" left tail under the implied distribution, which indicates a higher possibility or care of extreme loss. The negatively skewed distribution has a disproportionately large amount of outliers/extreme loss that fall within its lower (left) tail. Investors thus assign a greater value to the "out the money" put than under the actual distribution. This is also evident from the downward "implied volatility smirk". This means the "out of the money" puts are over-priced and have greater volatility under the risk neutral distribution. In other word, investors are more likely than not to be risk averse, because s/he does not like exposure to the down state at all and is willing to pay more to avoid it. They are more careful about market crashs. This analysis holds for "in the money" calls because "in the money" calls are essentially the same as "out of the money" puts. Therefore, "in the money" calls are also "over-priced" by the implied distribution.

At high strike prices, calls are "out of the money" and will be under-priced by the implied distribution. Such options pay off only if the stock index value moves above a particularly high strike price. According to the implied distribution's "thinner right tail", the negatively skewed distribution has a disproportionately small amount of outliers/extreme gain that fall within its higher (right) tail. The market predicts that upward movements are less likely than the actual distribution presents. The market thus assigns a smaller value to an "out of the money" call because it is perceived that there is less likelihood of the stock index value exceeding a given high strike price. Upward movements in prices are viewed by risk neutral investors as less likely or less serious than the actual distribution represents. They are relatively neglectful of the upside potential. In other word, again, investors are more likely than not to be risk averse rather than risk seeking, because they care little for the upside potential and will not pay more to take advantage of it. All they worry is downside risk rather than upside potential. This analysis holds for "in the money" puts because "in the money" puts are essentially the same as "out of the money" calls. Therefore, "in the money" puts are also "under-priced" by the implied distribution.

For risk averse investors, the risk neutral probability is much higher than the actual/objective probability at low asset returns. This difference decreases as the investors' wealth increases. Hence, the pricing kernel is decreasing in wealth and will have a downward slope.

The primary reason for this is that the implied distribution accounts for a potential market crash. Specifically, investors who are concerned about market downside insure themselves by buying out of the money puts, which put a floor on the maximum losses the investors can sustain. The prices of these puts are then pushed up because only a few investment banks and hedge funds are willing to supply such insurance in large quantity.

However, some researchers consider some other ways to represent the actual distribution. One of the alternative ways is to take the distribution of the abnormal return or excess over the riskless rate as a whole, then at each time of point, add back the observed riskless interest rate plus some other parameters in order to produce the alternative actual distribution. For reasons of comparison, I plot this type of actual distribution without introducing additional parameter. I have total of 70 monthly returns (December 1998 to September 2004), I derive 68 three-month returns from these monthly returns. Interest rates for each month and quarter are also available. Then I can derive the distribution of excess return over riskless rate with the same horizon as the "previous" actual distribution. The observed interest rate for each time point is then added back to have the "new" actual distribution. The comparison of these two type actual distributions is shown in Figure 10. The "new" actual distribution is more negatively skewed than the "previous" one. In other word, it has a long lower tail and is skewed left. This means it has a disproportionary large amount of outliers/extreme loss that fall within its left tail.

By dividing the two probability distributions into each other at the same return interval, we can obtain the empirical pricing kernels. I put the actual distribution, risk neutral distribution and pricing kernel in one graphic and summarize all of them in Figures12 to Figure 14 and in Figure 16(actual distribution with riskless rate add back). The individual graphs of the pricing kernel are shown in Figure 15 and Figure 17(actual distribution with riskless rate add back). Notice that the empirical pricing kernels, which are presented in Figures 12, 13, 14 and are summarized in 15, are as expected, broadly downward sloping as wealth increases. This pattern is an indication of risk-averse investors in the Canadian economy. This result is also consistent with Jackwerth(2000)' finding for American S&P 500, German DAX30, U.K. FTSE 100, as well as the Japanese Nikkei 225. Moreover, I get apricing kernel as 0 to 2.7 for the S&P/TSX 60 Index. These amount appear to be good when compare with surveys, which suggest 2-5 for S&P 500 index options (Selahattin ÿImrohoroglu, 2003)

- **The pricing kernel puzzle**

The pricing kernel puzzle is that, theoretically, the pricing kernel should be monotonically decreasing but, at times it increases for wealth levels close to some intermediate wealth level. A typical estimate of m, say m*, appears in Figure 15 and Figure 17 as a function of the return on the S&P/TSX 60 Index. The value of zero at the center of the horizontal axis of Figure 15 represents an ending level of the index (i.e., at the time of option expiration) that is equal to the current level (at the beginning of the 90-day interval). Globally, m* is a decreasing function of the ending level, as in Figure 15. However, for the range from approximately-0.05 to 0.026 for Figure 15, m* is increasing. This occurs because the proportional difference between the estimates, say Q and P, is increasing over this range, as equation 20 implies. Traditional asset pricing theory, e.g. Rubinstein (1976) and Lucas (1978), assumes that a representative investor exists. It also is common in tests of asset pricing theories to assume that a market index such as the S&P/TSX 60 Index represents the aggregate wealth held by this investor. If we make these assumptions, then the part of empirical m* of Figure 15 suggests that the representative investor is locally risk seeking. Over some range, the marginal utility is increasing in wealth, the utility function is convex, and the investor will pay to acquire fair gambles in wealth. Taking this thought experiment to the limit, however, the interpretation of m* in equations 21 is derived from the optimality conditions of an investor with concave utility. Finding of Figure 15 is inconsistent with these conditions. Hence I arrive at a puzzle. My estimate of m* is inconsistent with standard approaches in asset pricing. To estimate Q, Jackwerth and Rubinstein (1996) only require that the prices do not offer any arbitrage trading opportunities, while the sample estimator of Q requires rational expectations to be valid. Therefore, we interpret m somewhat differently. Ziegler (2003) considered three hypotheses to explain M: a. mis-estimation of the statistical distribution, b.mis-specification of investor preferences, c. heterogeneous investor beliefs and d. transaction cost as well. He believes that errors in estimating the actual distribution would need to be oddly shaped to explain the pricing kernel puzzle. With respect to the misspecification of preferences, he argues that all the typical utility functions used in finance and economics (power utility, log utility, and negative exponential utility) yield similar transformations form the actual to the risk-neutral distribution and cannot, therefore, be the cause of the pricing kernel puzzle. Finally, he suggested that an economy with heterogeneous investors who belong to either of two about equally large groups-namely; optimists and pessimists-may explain the pricing kernel puzzle.

Brown and Jackwerth(2003) use an another explanation by mean of the volatility on the wealth. Its potential impact on the pricing kernel is depicted in Figure 15 and Figure 17(actual distribution with riskless rate add back). Here, the empirical pricing kernel is composed of two reference-pricing kernels. The state 1(high-volatility) pricing kernel dominates in the tails, and the state 2(low-volatility) pricing kernel dominates in the center. As wealth increase, the likelihood of being in state 2 first increases and then decreases again. Taking the expectation over the volatility dimension yields the desired empirical pricing kernel, m. In combination, they can explain the pricing kernel puzzle because volatility tends to be low in the center, where aggregate wealth does not change much. If wealth either increases or decreases rapidly, however, volatility shoots up and the result is a U-shaped profile of volatility as a function of wealth. This pattern can be found in Figure 15 and Figure 17(actual distribution with riskless rate add back). Comparing these two figures, we can find that the pricing kernel derived from the actual distribution with riskless rate add back is more leptokurtic or more peaked than "general" actual distribution without considering the riskless rate. The phenomenon of pricing kernel is more significant, if we consider the riskless rate and back them to each point of distribution. The hump-shape of Figure 15 and Figure 17 is also statistically significant in Jackwerth (2000).

Another explanation can also be obtained from Sophie Shive(2003). She points out that if people exhibit risk aversion, the stochastic discount factor $\frac{\partial Q}{\partial P}$ (where P is the subjective probability measure over states of the world, and Q is the risk neutral probability measure) should be strictly decreasing with increasing prices of the security. This is because it corresponds to the derivative of the utility function of the agent who determines the prices. For this function to be concave, it must have a decreasing first derivative or a negative second derivative at every point. A violation of this implies that people are not risk averse over that region of outcomes. To understand this better, we consider an equation from:

Equation 28 $E_t(R_{t+1}M_{t+1}) = 1$

where M_{t+1} is the realization of the stochastic discount factor, or marginal utility of consumption tomorrow over consumption today, at time $t + 1$ (at expiration). R_{t+1} is the gross return at time $t + 1$. If we pay one dollar today, R_{t+1} is how many dollars we get tomorrow. E_t is the expectation taken over all of the possible payoffs with the time t information set. The equation shows that when the security is likely to pay off in the region of small gains, people pay `too much' for that security, and prices are pushed up to where there is negative compensation for risk. This implies that the shape as well as the mean and variance of the distribution of asset returns affect the price.

A potential further explanation for the existence of the smile is the illiquidity of out-the-money options. Options traded in a market with transaction costs or other frictions are priced within an interval determined by these costs. Further, while the bid/ask spreads in relation to the option prices are higher for away from-the-money options, the hump in the pricing kernel occurs around at-the-money, and is determined by the at-the-money options. Constantinides, Jackwerth and Perrakis (2004) also examine the restrictions on the prices of the options in the presence of transactions costs, intermediate trading over the life of the options as well as assumptions on bid-ask spread and trading fees and conclude that all these have some effects on and are proportional to index option prices and hence the utility. Again, the pricing kernel puzzle arises in the center whereas the truncation mainly affects the tails of the distribution. Also, it is not clear that market participants do indeed quote options in this particular way. Market frictions due to margin account requirements might also play a role here. The public, the clearing firms, and the market makers are all subject to margin requirements.

Biases are important especially if they are not eliminated by arbitrage. Several studies have documented that other bias may also affect prices. Coval and Shumway(2000) study Chicago Board of Trade proprietary traders. They show that traders who have morning losses are more likely to take risks in the afternoon, to avoid booking losses for the day. This suggests that they would be willing to pay more than is reasonable to avoid the risk of the morning losses. Kaustia (2000) studies IPO trading volume and shows that turnover increases for negative initial return IPOs on the day that the price first exceeds the offer price. The results of my paper also show that investors are willing to pay too much for securities whose most common payoff is slightly positive, and are likely to avoid the uncomfortable feeling of loss relative to the reference price. These results have significance for option returns as well. Coval and Shumway show generally that if the stochastic discount factor is negatively correlated with the price of a security over the entire range of the security's price, any call option on that security will have a positive expected return that is increasing in the strike price. The results in their paper suggest that this will not be the case over the whole range of strike prices. Several caveats are in order. My study assumes that the Canadian S&P/TSX 60 Index returns are functions of aggregate wealth. Also, we have assumed that the shadow price of the budget constraint is constant, and this could turn out to be false if there is a shock to the aggregate endowment. However, since the time period studied is only 2003-2004, this is no huge shock in Canadian economy. This is unlikely to be the case for investors' problems of maximize utility subject to a budget constraint or minimize expenditure subject to a utility constraint.

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

In this paper, I investigate the volatility smile in a Canadian context and conduct an empirical testing on 13877 options of the S&P/TSX 60 index for period of January 2, 2003 to June 30, 2004 with different moneyness and time to maturity. I consider only maturities of 30 days and 90 days, For these I have a total of 31 option series with 178 available options and I obtain a total of 31 estimated "smiles". [5] By using these option series, I further examine the evidence for the existence of a Canadian volatility smile. I find that the classical U-shape "volatility smile" is very common for option series of 30-days maturity as moneyness change. For option series of 90-days maturity, I get some "volatility smirk" as common pattern. I also find that the implied volatility decreases as the time to maturity increases, a result that is consistent with various other studies. According to my findings, I conclude that the presence of a volatility smile in Canadian S&P/TSX 60 index, is inconsistent with the assumption of constant volatility of the Black-Scholes(1973) model. Therefore, the risk neutral probability distribution of the underlying asset may not be normal.

For this reason, the risk neutral probability distribution is recovered from option prices by a non-parametric lattice approach- the Jackwwerth-Rubinstein(1996) method. Then I compare the option inferred risk neutral distributions with the actual distribution based on the historical price path of 70 months with a total of 68 three-month returns since December 1998 over the same return interval infer as the risk neutral distributions. The relationship between these two distributions depends on the preference (utility) of investors about money, as they grow richer or poorer. Knowing both distributions allows me to infer what the preferences must be within the economy in order to be consistent with option prices and historical returns.

[5] the total 31 "smiles" include 18 one-month "smiles" and 13 three-month "smiles"

Based on my finding, the actual distributions appear to be somewhat normally distributed, whereas the risk-neutral distributions are left skewed and leptokurtic; that is, they have fat left tails and are more peaked than the actual distribution. This is consistent with the underlying volatility smirk. Downward movements in prices are viewed by risk neutral investors as more likely or serious than what the actual distribution represents. Similarly, upward movements in prices are less likely or valuable than what the actual distribution implies.

The negatively skewed distribution has a disproportionately large amount of outliers/extreme loss that fall within its lower (left) tail. The market thus assigns a greater value to the "out the money" put than under the actual distribution. Moreover, the leptokurtic distribution will have a greater percentage of extremely large deviations, which indicates an increase in risk.

This is also evident from the downward "implied volatility smirk". All these mean that the "out of the money" puts are over-priced and have greater volatility under the risk neutral distribution. In other word, investors are more likely than not to be risk averse, because s/he does not like exposure to the down state at all and is willing to pay more to avoid it.

At high strike prices, calls are "out of the money" and will be under-priced by the implied distribution. According to the implied distribution's "thin right tail", the negatively skewed distribution has a disproportionately small amount of outliers/extreme gain that fall within its higher (right) tail. The market predicts that upward movement is less likely or valuable than the actual distribution presents. The market thus assigns a smaller value to an "out of the money" call because there is less likelihood of the stock index value exceeding a given high strike price. In other word, again, investors are more likely than not to be risk averse rather than risk seeking, because they care little for upside potential and will not pay more to take advantage of it. All they worry about are downside risk rather than the upside potential.

The primary reason for this is that the implied distribution accounts for a potential market crash. Specifically, investors who are concerned about market downside ensured themselves by buying out of the money puts, which put a floor on the maximum losses the investors can sustain. The prices of these puts are then pushed up because only a few investment banks and hedge funds are willing to supply such insurance in large quantity

By dividing the risk neutral distribution by actual distribution, I obtain the empirical pricing kernels for the indexes and the empirical pricing kernels are, as expected, broadly downward sloping as wealth increases. The risk neutral probability is much higher than the actual/objective probability at low asset returns. However, this difference decreases as investors' wealth increases. This pattern is an indication of risk averse on the Canadian economy.

This result is also consistent with Jackwerth(2000)' funding for American S&P 500, German DAX30, U.K. FTSE 100, as well as the Japanese Nikkei 225.

I also find some significant increasing portions of the marginal utility function in my price kernel curve, which suggests that investors may not be risk averse in some areas of the payoff space. This phenomenon becomes more significant, if I back the observed riskless rate to each point of distribution to get an "alternative" actual distribution.

Theoretically, the pricing kernel should be monotonically decreasing at all times, but in my findings, it increases for wealth levels close to some intermediate wealth level.

Hence this is a puzzle. With my result, it implies that Canadian investors behave as if they are risk seeking (and not risk averse) with respect to the risk inherent in the index. This conclusion is inconsistent with one of the fundamental assumptions of economic theory. Another venue for future research is to understand this puzzle and reconcile index option data and Canadian economic theory.

There are several reasons to investigate this puzzle, and to investigate more generally the state price densities or risk neutral distribution for equity indices. First, the indices such as the S&P/TSX 60 Index represent a large majority of the public equity capital in their respective nations. Second, the very large market capitalization of broad equity indices has led to a long line of research on their distributional properties.

LISTS OF TABLES

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Table I S&P/TSX 60 Index Implied volatilities of individual calls for the period of January 2, 2003 to June 30, 2004

	CALL OPTION DAYS TO EXPIRATION				
X/S	<60	≥60 <120	≥120 <180	≥180 <240	≥240
<0.95	30.34(681)	14.11 (565)	15.29(439)	12.3(392)	14.24(142)
≥0.95 <0.98	14.53(703)	12.95(516)	12.84(252)	12.91(265)	12.58(125)
≥0.98 <1.01	12.96(849)	13.06(554)	13.22(273)	12.95(274)	13.27(148)
≥1.01 <1.04	12.99(816)	12.42(506)	12.98(258)	12.75(255)	12.77(132)
≥1.04 <1.07	14.49(596)	12.75(362)	13.15(207)	12.88(206)	13.59(83)
≥1.07 <1.1	18.38(235)	14.26(157)	14.14(91)	14.09(98)	15.79(34)
≥1.1	36.3(197)	17.18(106)	15.08(78)	15.59(77)	15.19(22)
	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6

Table II S&P/TSX 60 Index Implied volatilities of calls option series for the period of January 2, 2003 to June 30, 2004

	OPTION SERIES WITH 30-DAYS MATURITY					
x/s	Highest Implied Volatility	Lowest Implied Volatility	Median Implied Volatility	mean Implied volatility	Highest/Lowest Ratio	number of option
<0.95	144.8	34.72	61.25	70.714	4.170507	35
≥0. 95 <1	68.87	7.64	21.14	23.57	9.014398	25
≥1 <1.05	31.29	10.61	16.37	18.29	2.949105	32
≥1.05 <1.1	38.01	22.23	38.04	34.169	1.709852	14
≥1.1	89.35	51.46	57.14	62.81	1.7363	4
	Figure 7 a	Figure 7 b	Figure 7 c	Figure 7 d		
	OPTION SERIES WITH 90-DAYS MATURITY					
X/S	Highest Implied Volatility	Lowest Implied Volatility	Median Implied Volatility	mean Implied volatility	Highest/Lowest Ratio	number of option
<0.95	23.13	11.69	13.29	15.85	1.978614	10
≥0.95 <1	22.17	9.82	12.24	13.25	2.257637	23
≥1 <1.05	20.79	10.75	11.6	12.54	1.933953	25
≥1.05 <1.1	13.4	10.7	11.45	11.86	1.252336	8
≥1.1	19.33	18.29	18.81	18.81	1.056862	2
	Figure 7 e	Figure 7 f	Figure 7 g	Figure 7 h		

Table III Arbitrage opportunities for the options on the TSE-60 index for the period of January 2, 2003 to June 30,2004

Date	StrikePrice	Stock price	Maturity	Moneyness
2/6/03	420	0.325	0.1194444	1.1480742
2/6/03	440	0.405	0.1194444	1.2027444
2/7/03	420	0.3	0.1166667	1.1474783
2/7/03	440	0.305	0.1166667	1.2021201
2/13/03	420	0.215	0.1	1.1521369
2/13/03	440	0.225	0.1	1.2070006
2/17/03	390	0.425	0.0111111	1.0470642
2/17/03	400	0.425	0.0111111	1.073912
2/17/03	420	0.425	0.0111111	1.1276076
2/20/03	420	0.24	0.0805556	1.1296396
2/20/03	440	0.3	0.0805556	1.183432
2/24/03	420	0.18	0.0694444	1.1415525
2/24/03	440	0.25	0.0694444	1.1959122
3/4/03	420	0.175	0.0472222	1.1519789
3/4/03	440	0.225	0.0472222	1.2068351
3/6/03	400	0.225	0.0416667	1.1114507
3/6/03	420	0.2	0.0416667	1.1670233
3/6/03	440	0.25	0.0416667	1.2225958
3/7/03	420	0.125	0.0388889	1.1573755
3/7/03	440	0.5	0.0388889	1.2124886
3/10/03	420	0.125	0.0305556	1.1736433
3/10/03	440	0.3	0.0305556	1.2295311
3/11/03	420	0.125	0.0277778	1.1700468
3/11/03	440	0.275	0.0277778	1.2257633
3/13/03	420	0.125	0.0222222	1.1700468
3/13/03	440	0.325	0.0222222	1.2257633
3/18/03	420	0.125	0.0083333	1.1446013
3/18/03	440	0.15	0.0083333	1.1991061
3/19/03	420	0.125	0.0055556	1.1418318
3/19/03	440	0.475	0.0055556	1.1962048
3/20/03	400	0.15	0.0027778	1.0832476
3/20/03	420	0.15	0.0027778	1.13741
3/20/03	440	0.15	0.0027778	1.1915723
4/10/03	390	0.125	0.0194444	1.0936133
4/10/03	400	0.125	0.0194444	1.066273
4/14/03	390	0.3	0.0083333	1.0578566
4/14/03	400	0.3	0.0083333	1.0849811
4/15/03	390	0.125	0.0055556	1.0540825
4/15/03	400	0.125	0.0055556	1.0811103
4/15/03	420	0.125	0.0055556	1.1351658
4/16/03	380	0.025	0.0027778	1.0289737
4/16/03	390	0.075	0.0027778	1.056052
4/16/03	400	0.175	0.0027778	1.0831302
4/16/03	420	0.075	0.0027778	1.1372868
8/19/03	440	0	0.0861111	1.0429506
8/19/03	440	0	0.1638889	1.0429506
8/19/03	440	0	0.2611111	1.0429506
8/19/03	440	0	0.3388889	1.0429506

9/30/03	440	0.425	0.0472222	1.0562197
9/30/03	450	0.55	0.0472222	1.0802247
10/24/03	450	0.15	0.0777778	1.0533215
10/24/03	460	0.15	0.0777778	1.0767286
11/14/03	450	0.425	0.0194444	1.039285
11/14/03	460	0.425	0.0194444	1.0623802
11/17/03	450	0.15	0.0111111	1.0377511
11/17/03	460	0.15	0.0111111	1.0608122
11/18/03	440	0.125	0.0083333	1.0198405
11/18/03	450	0.05	0.0083333	1.0430187
11/18/03	460	0.2	0.0083333	1.0661969
11/19/03	450	0.1	0.0055556	1.0332476
11/19/03	460	0.175	0.0055556	1.0562087
12/15/03	450	0.175	0.0111111	1.0150223
12/15/03	460	0.175	0.0111111	1.0375784
3/11/04	500	0.025	0.0222222	1.0613233
3/11/04	510	0.025	0.0222222	1.0825497
3/15/04	490	0.025	0.0111111	1.0504652
3/15/04	500	0.025	0.0111111	1.0719033
3/15/04	510	0.025	0.0111111	1.0933413
3/16/04	490	0.025	0.0083333	1.0504652
3/16/04	500	0.025	0.0083333	1.0719033
3/16/04	510	0.025	0.0083333	1.0933413
3/17/04	490	0.025	0.0055556	1.0261995
3/17/04	500	0.025	0.0055556	1.0471423
3/17/04	510	0.025	0.0055556	1.0680852
3/18/04	490	0.025	0.0027778	1.0333193
3/18/04	500	0.025	0.0027778	1.0544074
4/14/04	490	0.125	0.0055556	1.0211099
4/14/04	500	0.125	0.0055556	1.0419489
5/12/04	490	0.15	0.025	1.076379
5/12/04	500	0.15	0.025	1.0983459
5/14/04	490	0.15	0.0194444	1.079747
5/14/04	500	0.15	0.0194444	1.1017827
5/17/04	470	0.175	0.0111111	1.0446998
5/17/04	480	0.175	0.0111111	1.0669275
5/17/04	500	0.125	0.0111111	1.1113828
5/17/04	510	0.125	0.0111111	1.1336104
5/18/04	490	0.125	0.0083333	1.0844547
5/18/04	500	0.125	0.0083333	1.1065864
5/18/04	510	0.125	0.0083333	1.1287181
5/20/04	490	0.15	0.0027778	1.0900759
5/20/04	500	0.15	0.0027778	1.1123223
5/20/04	510	0.15	0.0027778	1.1345688
6/9/04	490	0.125	0.025	1.0650322
6/9/04	500	0.125	0.025	1.0867675
6/11/04	500	0.425	0.0194444	1.079867
6/16/04	480	0.1	0.0055556	1.0273313
6/16/04	490	0.1	0.0055556	1.048734
6/16/04	500	0.1	0.0055556	1.0701368

6/17/04	480	0.125	0.0027778	1.0173587
6/17/04	490	0.125	0.0027778	1.0385537
6/17/04	500	0.125	0.0027778	1.0597486
6/17/04	510	0.02	0.0027778	1.0809436

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Figure 1. (Kani1995) The implied volatility surface for S&P 500 index options as a function of strike level and term to expiration on September 27, 1995.

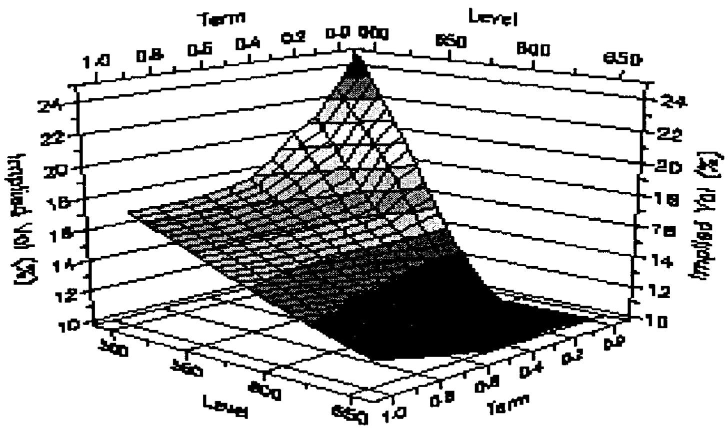


Figure 2 implied volatility for time to maturity less than 60 days

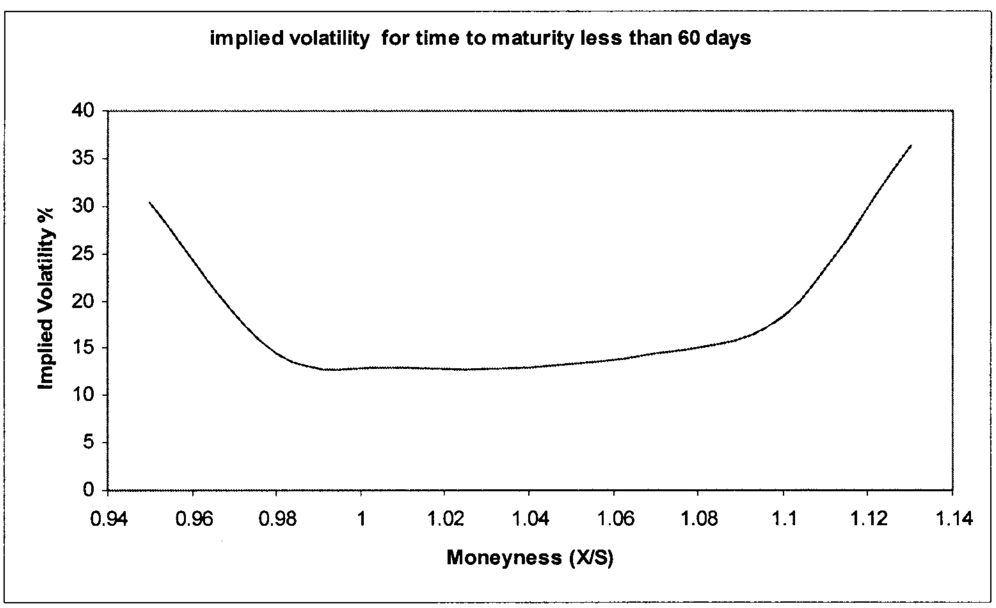


Figure 3 implied volatility for time to maturity greater than 60 days but less than 120 days

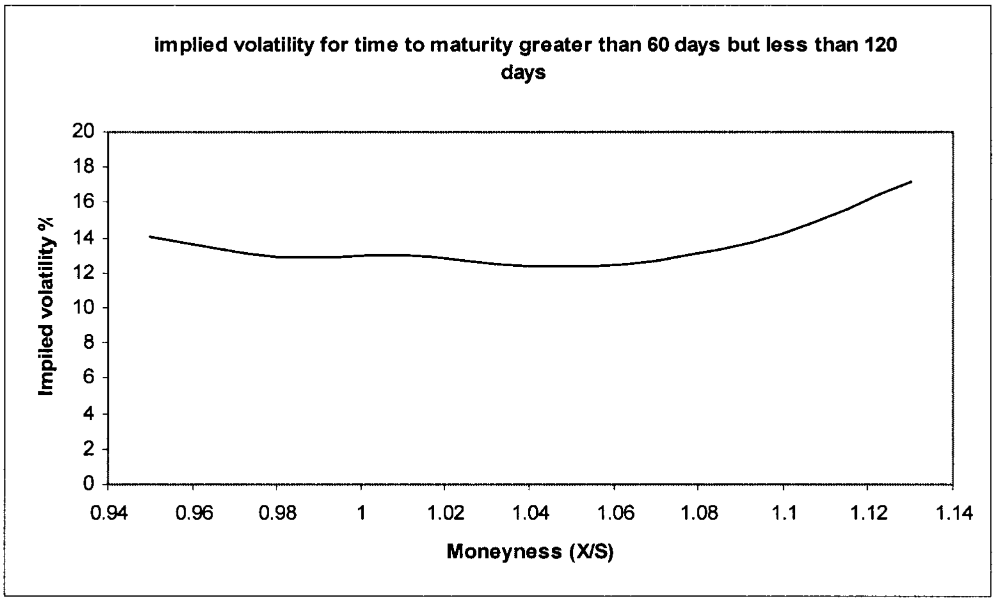


Figure 4 implied volatility for time to maturity greater than 120 days but less than 180 days

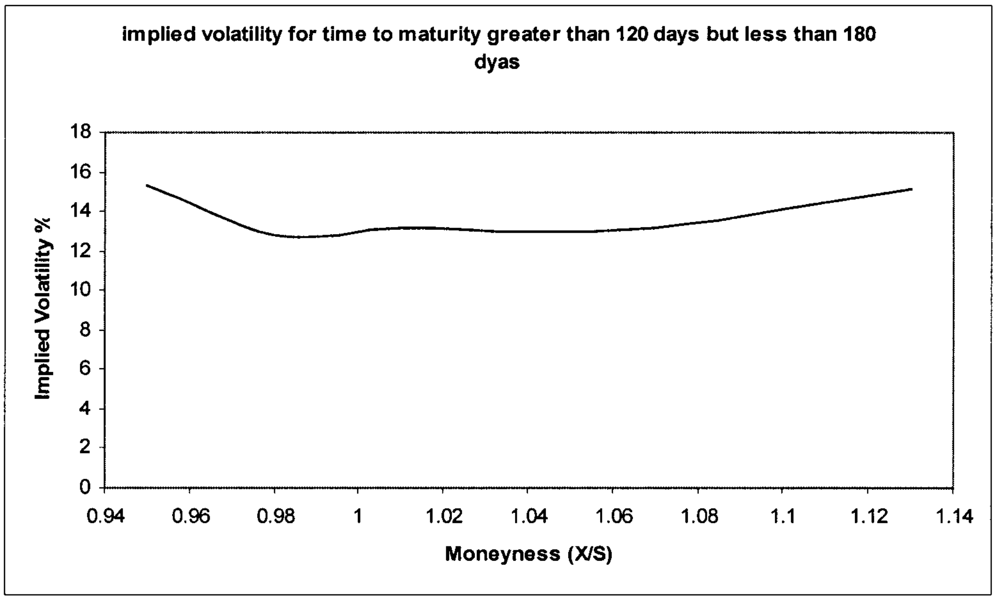


Figure 5 implied volatility for time to maturity greater than 180 days but less than 240 days

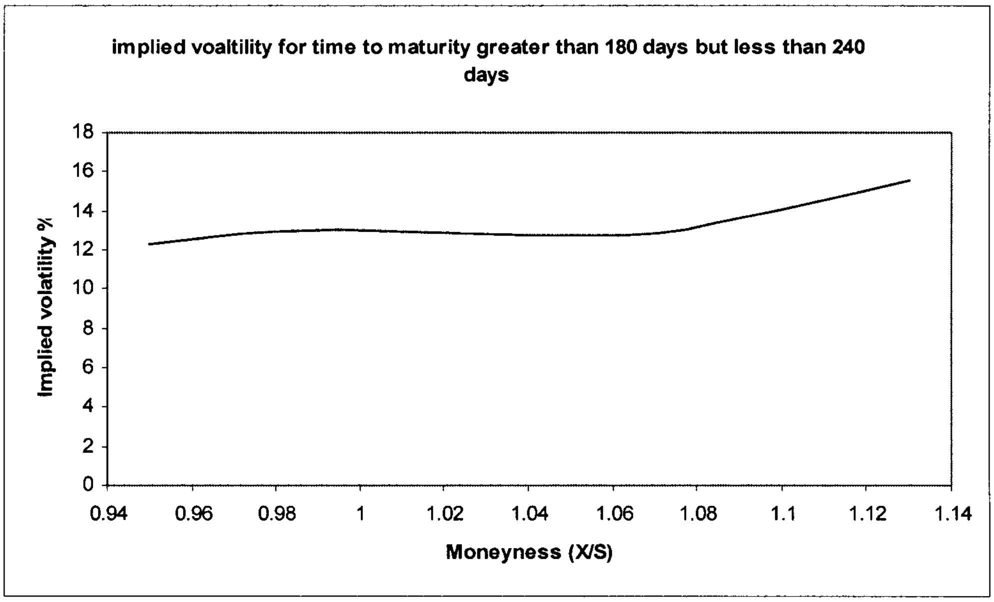


Figure 6 implied volatility for time to maturity greater than 240 days

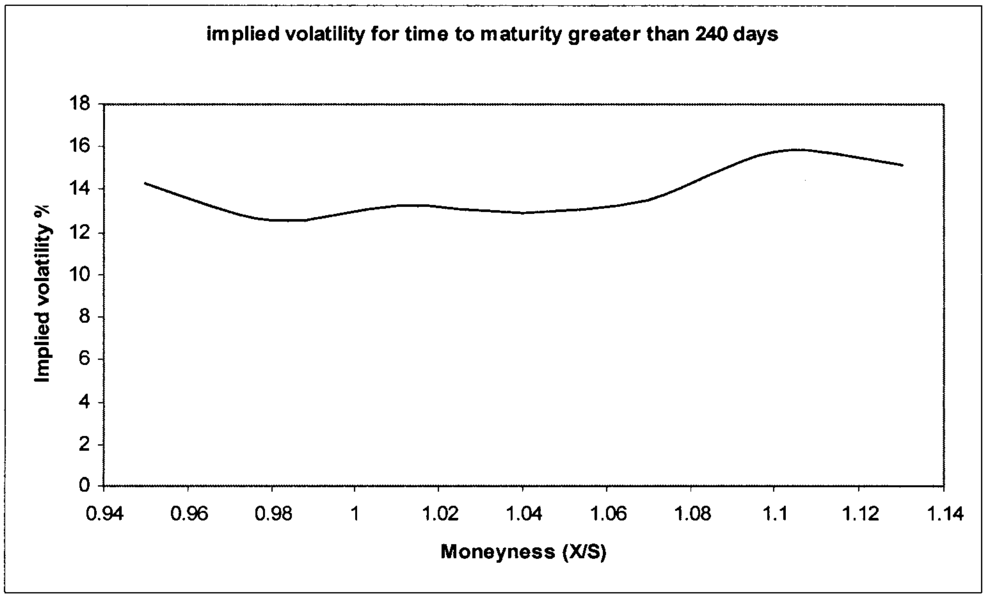
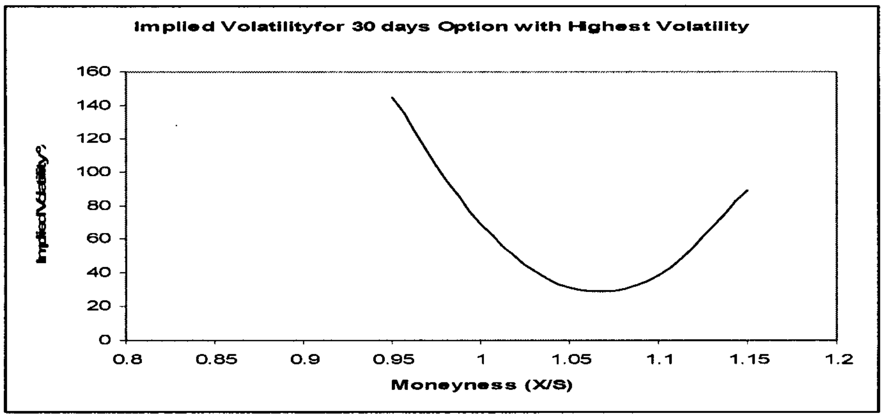
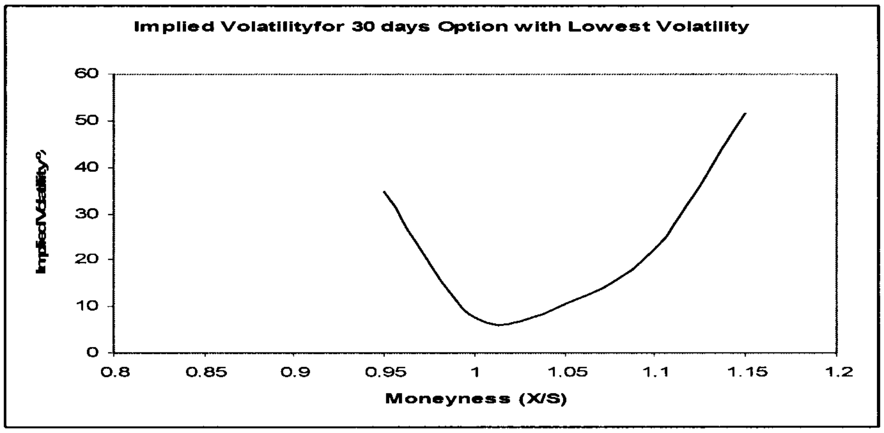


Figure 7 implied volatility for option series on the TSE-60 index for the period of January 2, 2003 to June 30, 2004

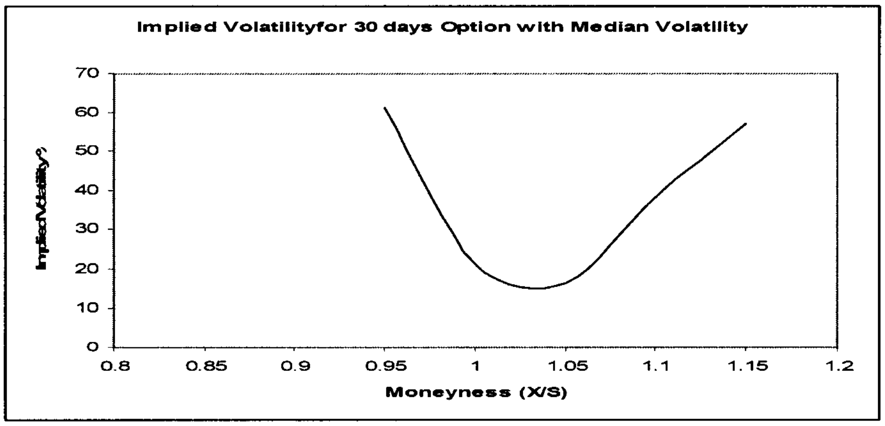
a.



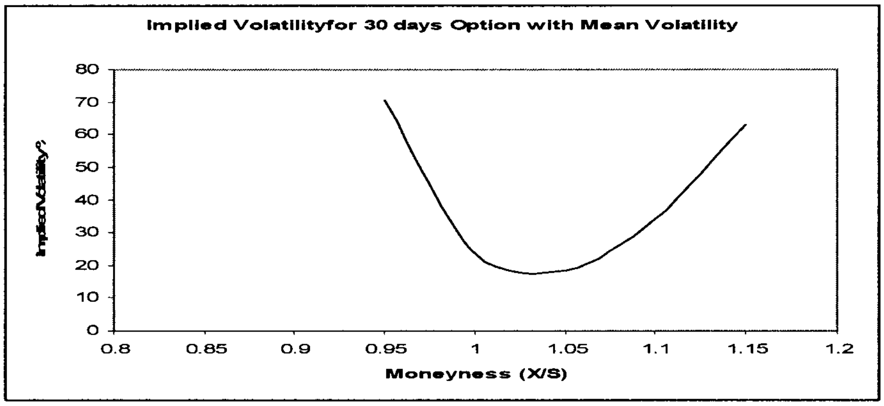
b



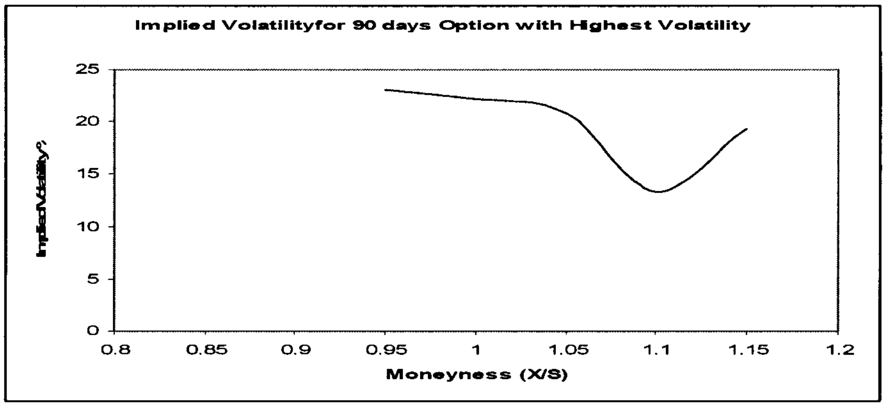
c.



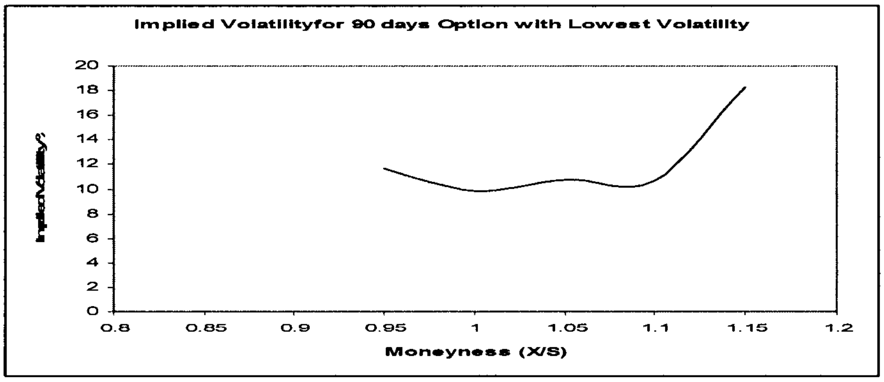
d


Implied Volatilityfor 30 days Option with Mean Volatility
80
70
60
50
40
30
20
10
0
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
Moneyness (X/S)

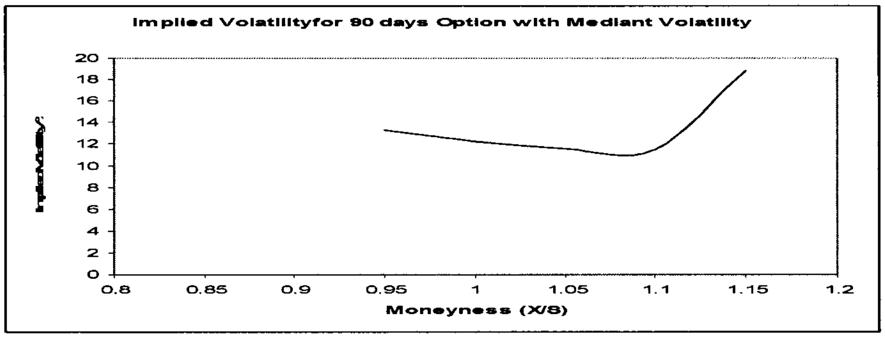
e.


Implied Volatilityfor 90 days Option with Highest Volatility
25
20
15
10
5
0
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
Moneyness (X/S)

f.

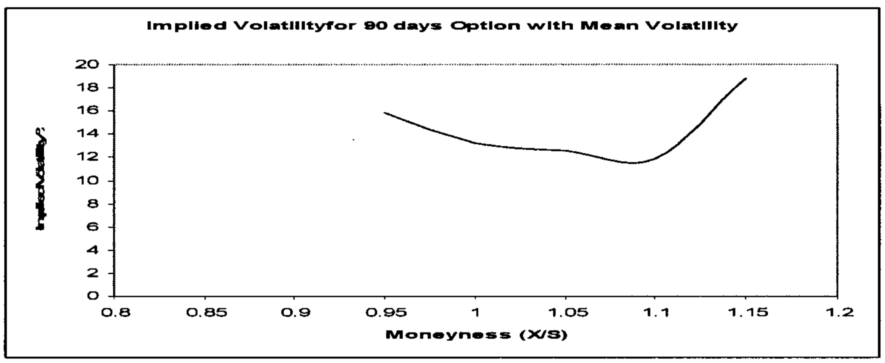

Implied Volatilityfor 90 days Option with Lowest Volatility
20
18
16
14
12
10
8
6
4
2
0
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
Moneyness (X/S)

g.



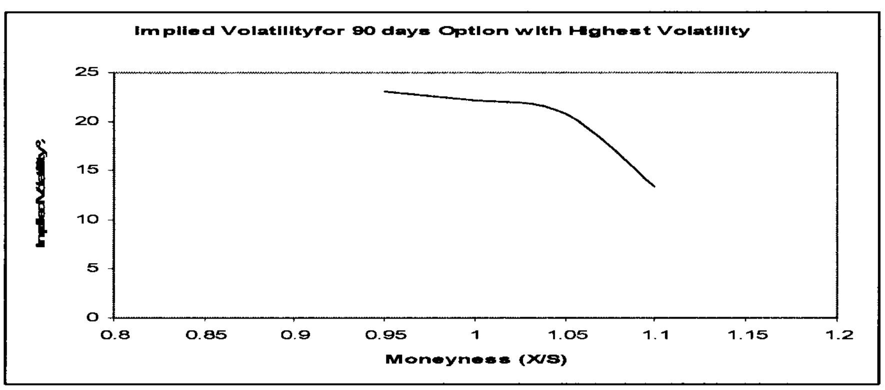
Implied Volatilityfor 90 days Option with Mediant Volatility
20
18
16
14
12
10
8
6
4
2
0
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
Moneyness (X/S)

h.



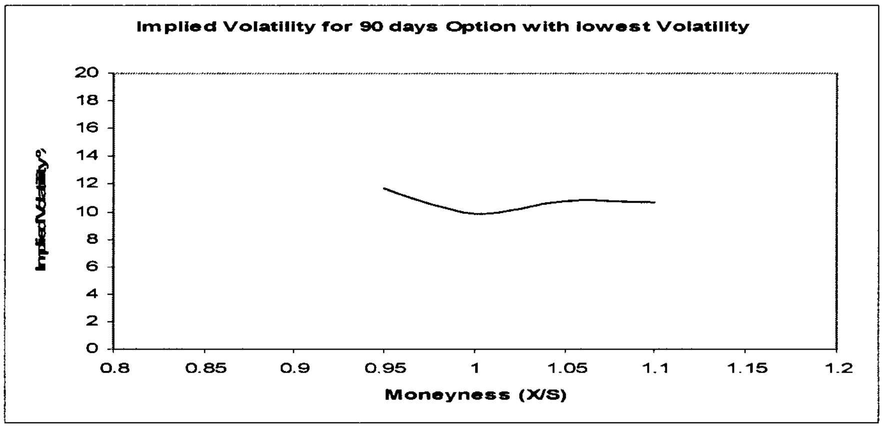
Implied Volatilityfor 90 days Option with Mean Volatility
20
18
16
14
12
10
8
6
4
2
0
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
Moneyness (X/S)

i.

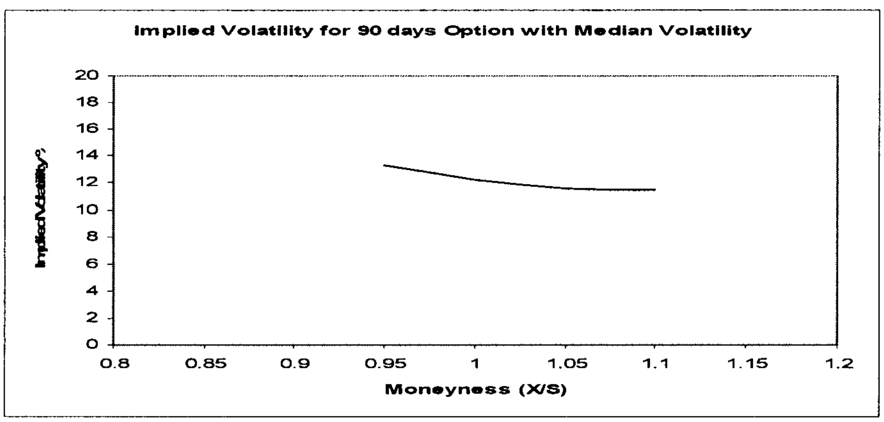


Implied Volatilityfor 90 days Option with Highest Volatility
25
20
15
10
5
0
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
Moneyness (X/S)

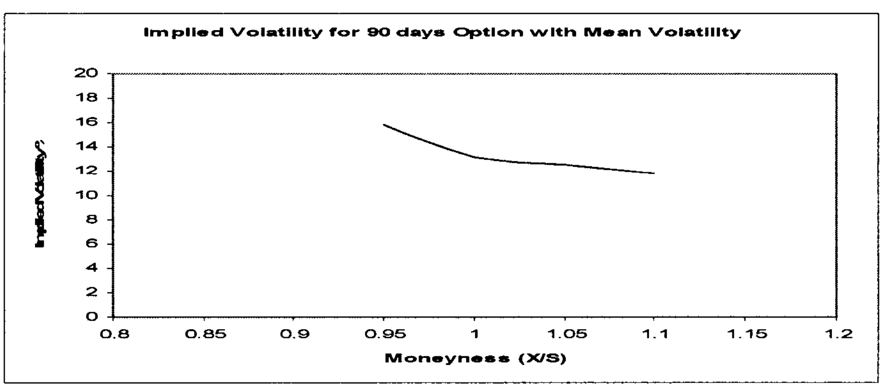
j.


Implied Volatility for 90 days Option with lowest Volatility
ImpliedVolatility%
20
18
16
14
12
10
8
6
4
2
0
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
Moneyness (X/S)

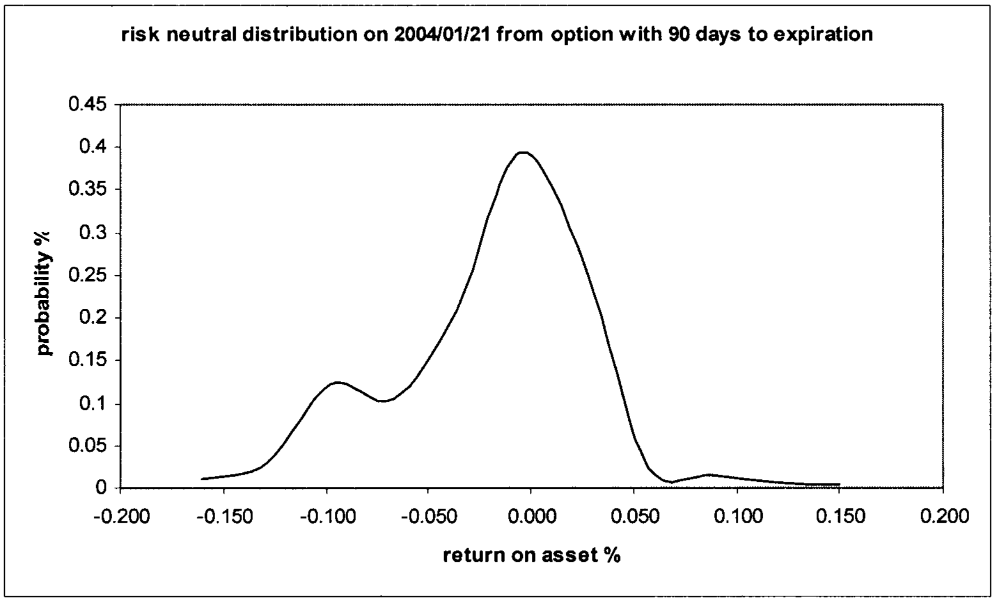
k.

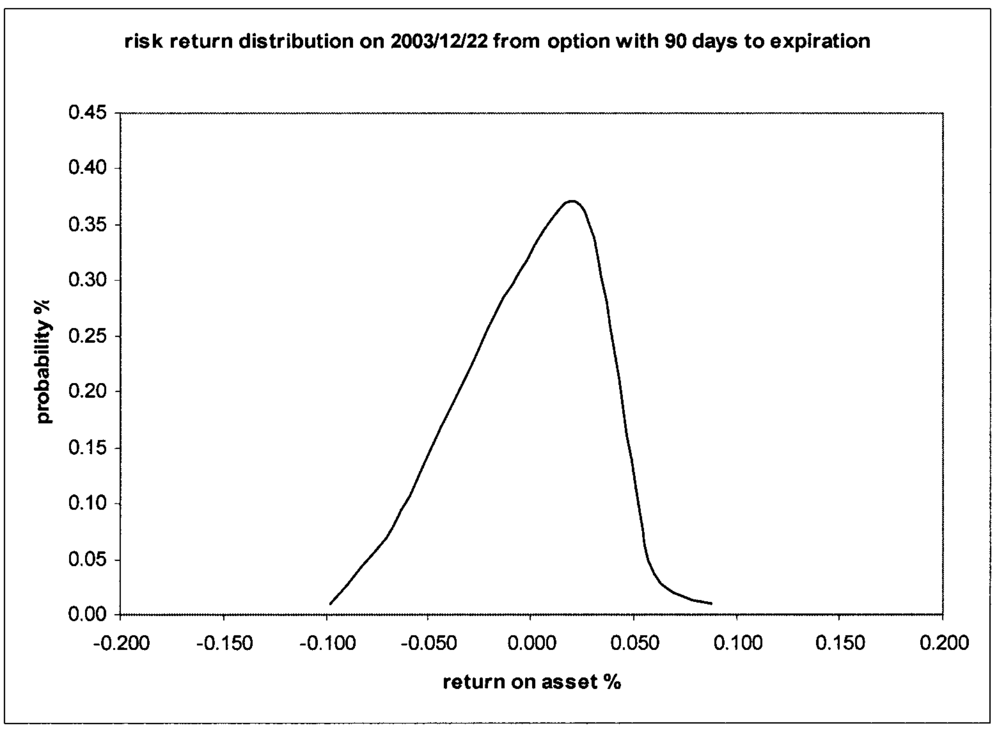

Implied Volatility for 90 days Option with Median Volatility
ImpliedVolatility%
20
18
16
14
12
10
8
6
4
2
0
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
Moneyness (X/S)

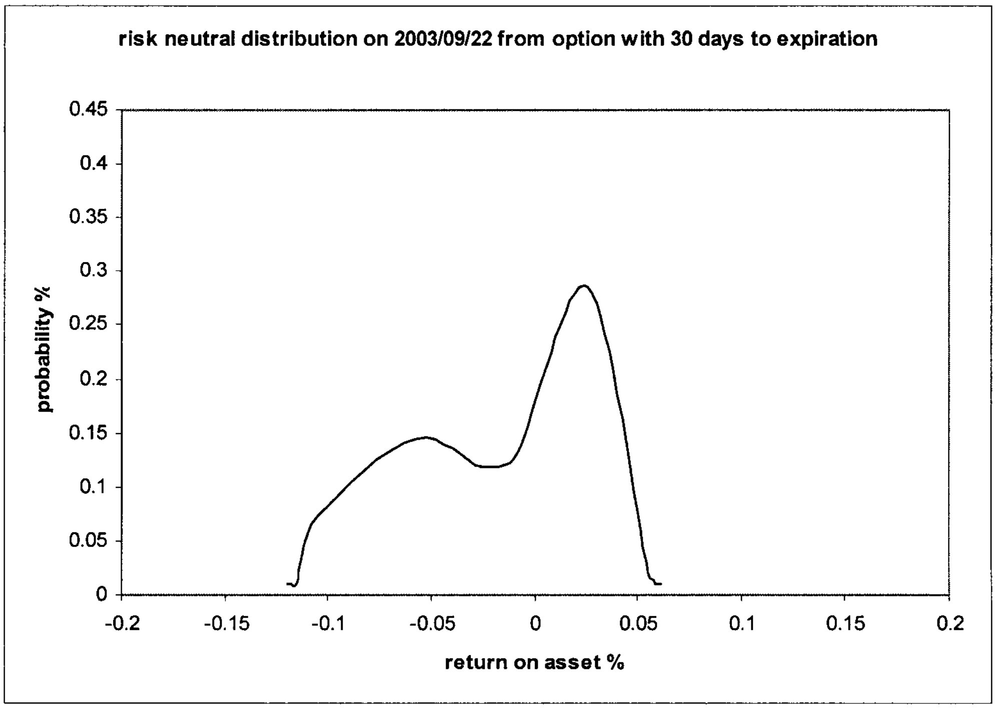
l.


Implied Volatility for 90 days Option with Mean Volatility
ImpliedVolatility%
20
18
16
14
12
10
8
6
4
2
0
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
Moneyness (X/S)

Figures 8 typical risk neutral probability distributions






risk neutral distribution on 2003/09/22 from option with 30 days to expiration
probability %
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
return on asset %

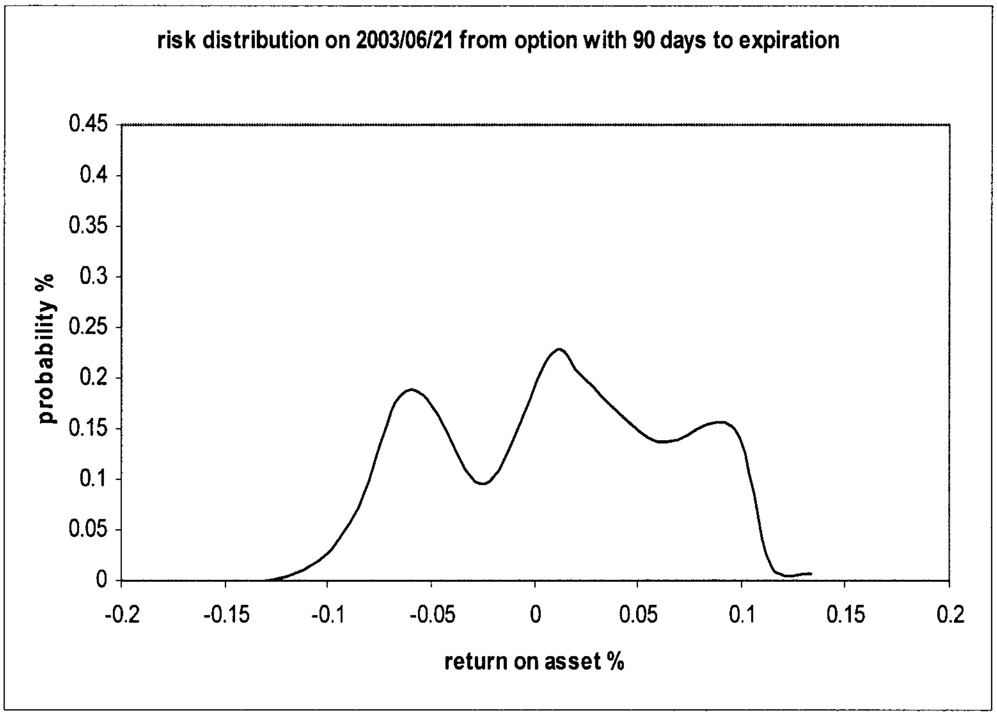

risk distribution on 2003/06/21 from option with 90 days to expiration
probability %
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
return on asset %

Figure 9 actual distribution on TSE60 index 3-months returns from January 2,1999 to June 30,2004

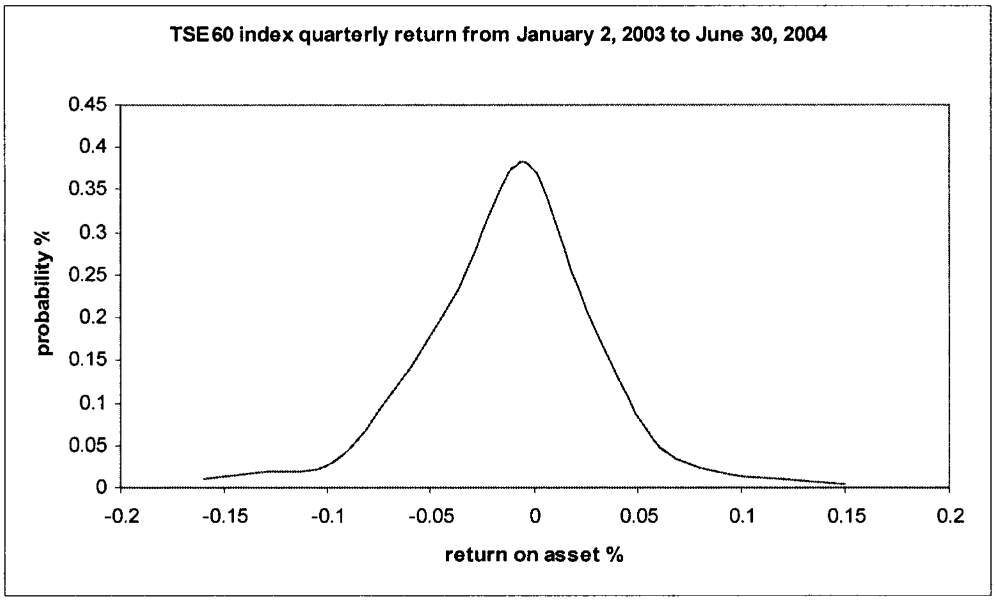


Figure 10 actual distributions of 3-month returns with and without risk free rate

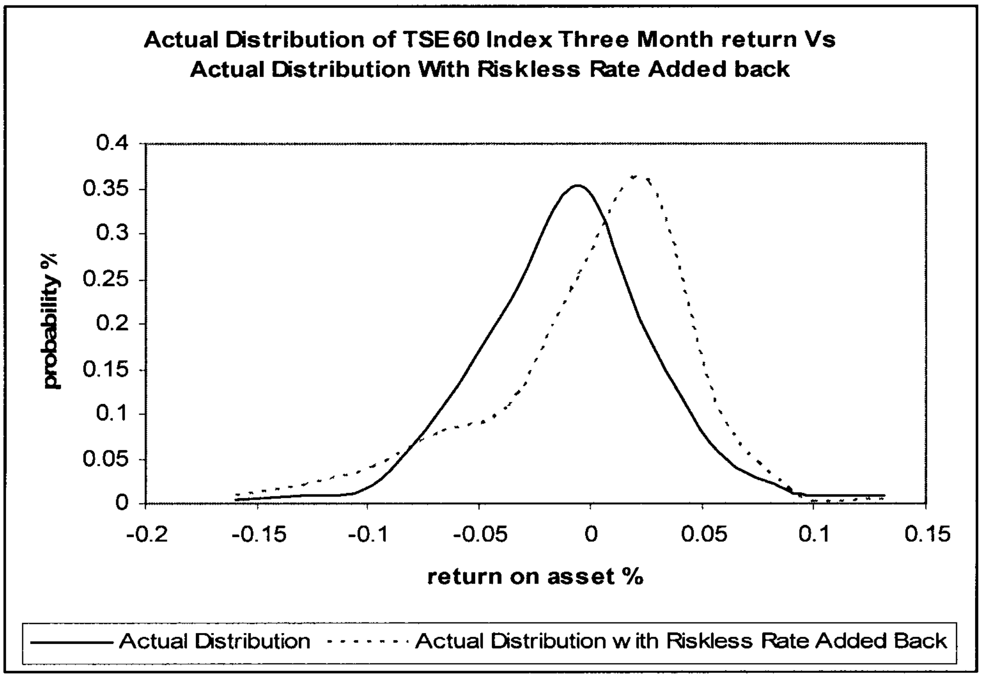
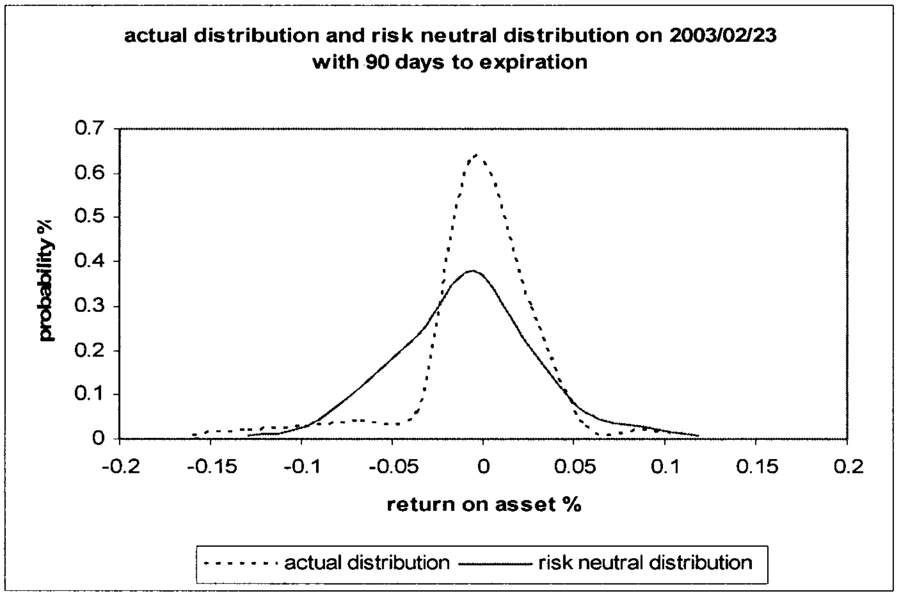
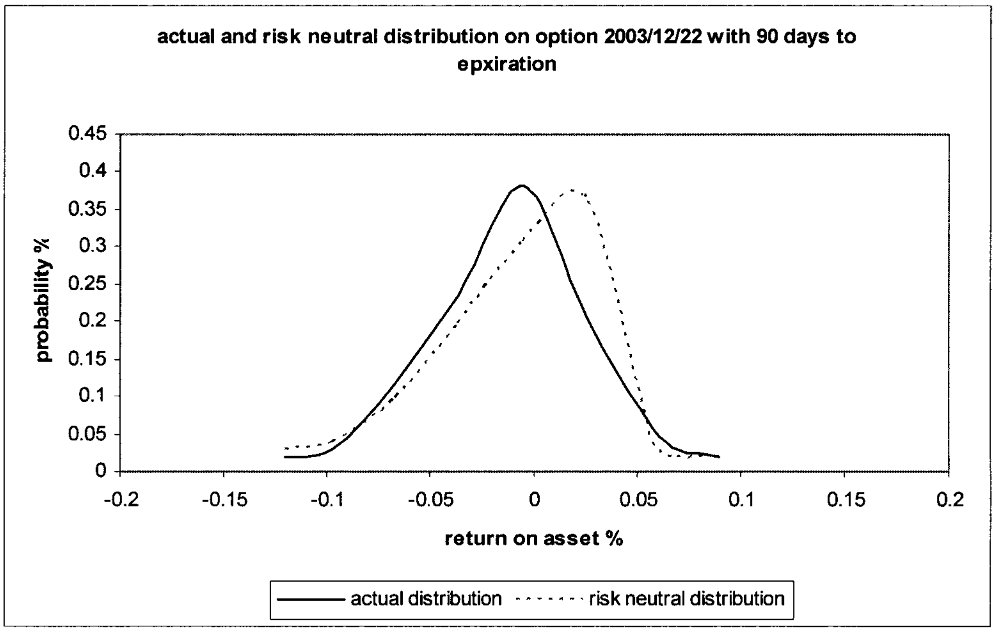


Figure 11 pricing kernel, actual distribution, risk neutral distribution for option with 90 days to expiration

a.



b.



c.

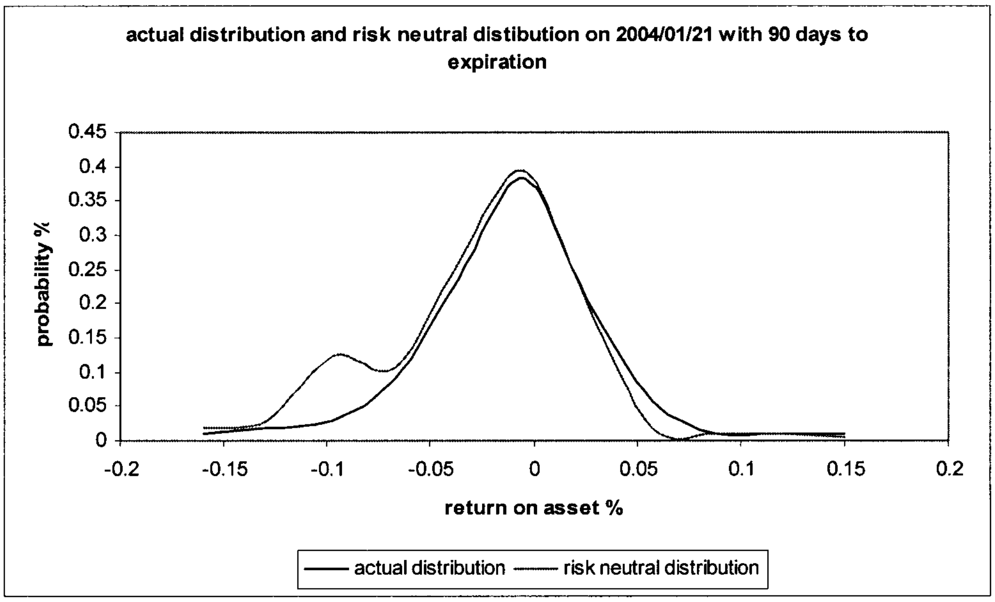


Figure 12 pricing kernel, actual distribution and risk neutral distribution on 2003/02/23 with 90 days to expiration

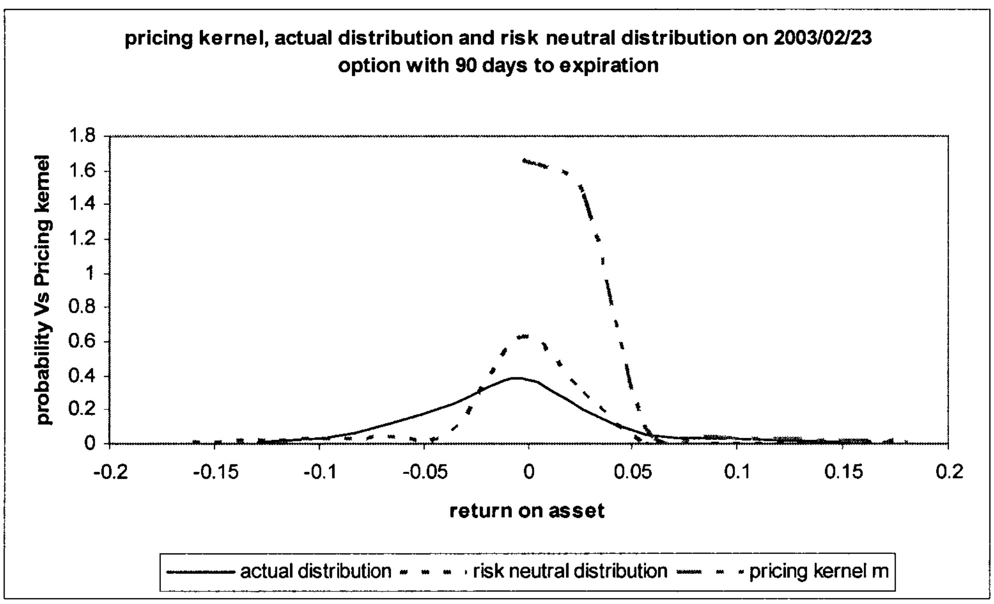


Figure 13 pricing kernel , actual distribution and risk neutral distribution on 2003/12/12 with 90 days to expiration

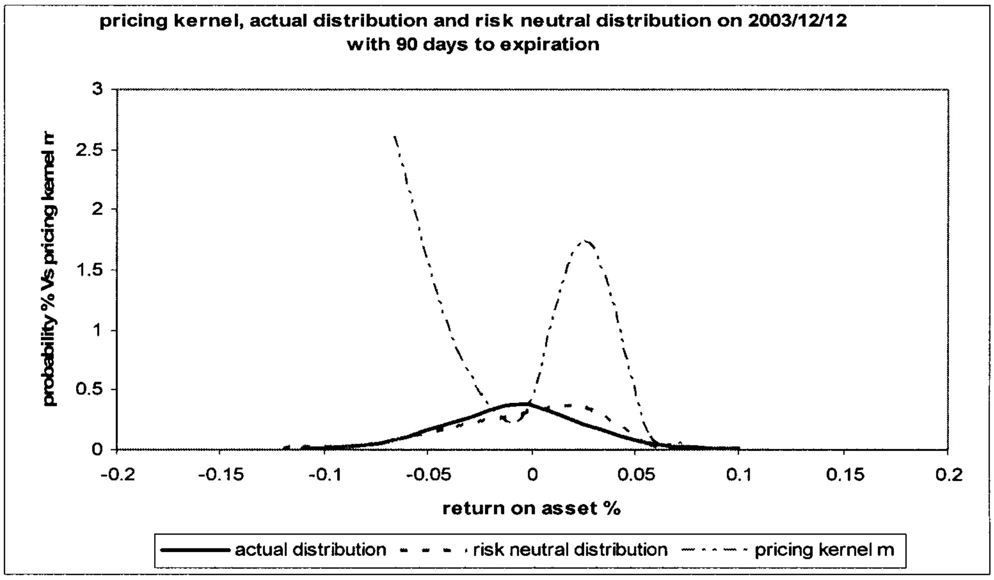


Figure 14 pricing kernel, actual distribution and risk neutral distribution on 2004/01/21 option with 90 days to expiration

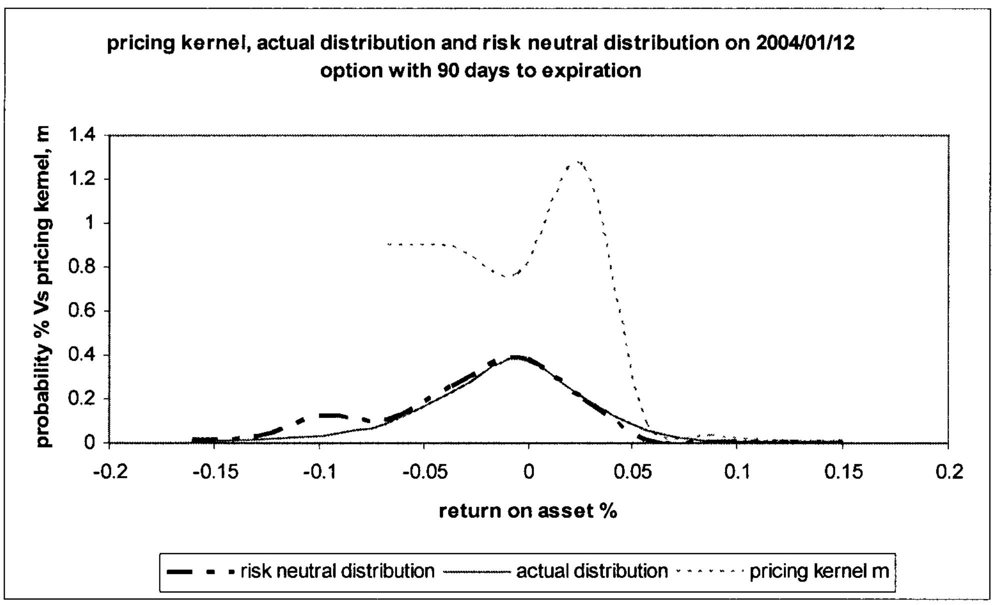
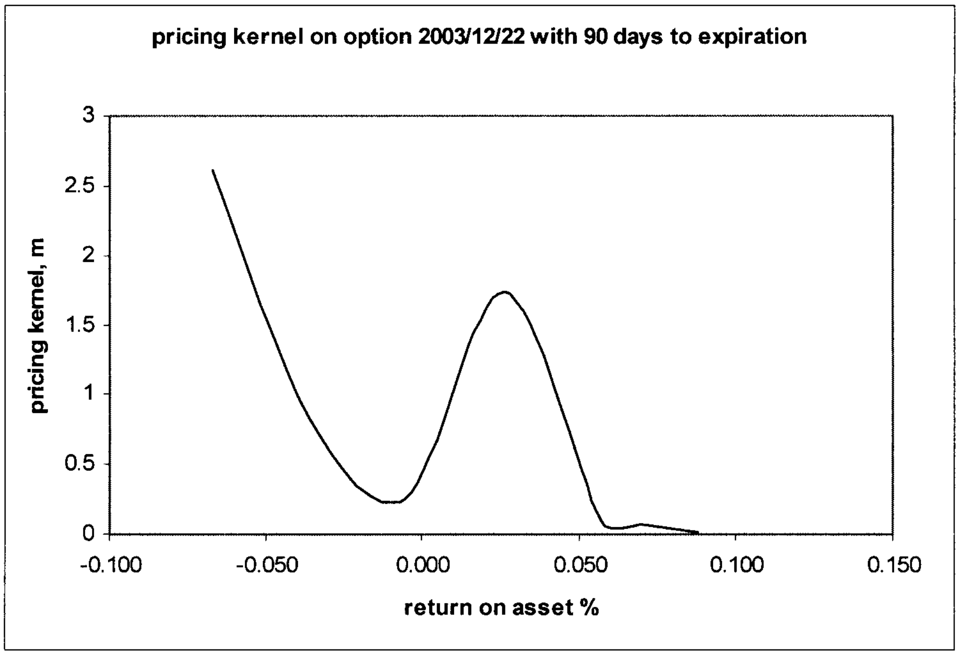


Figure 15 pricing kernel on option 2003/12/22 with 90 days to expiration



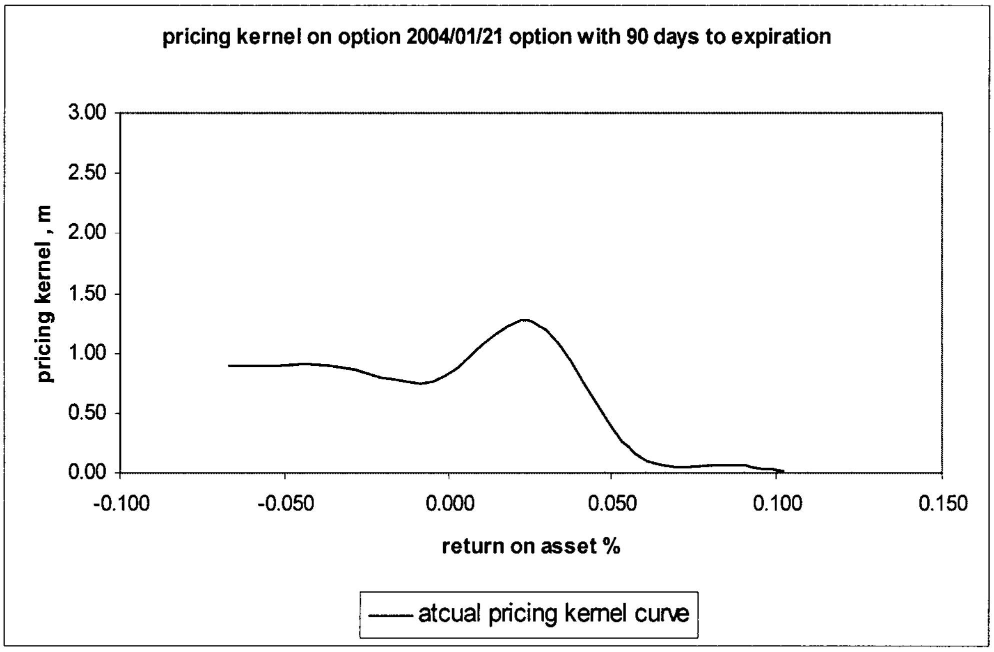
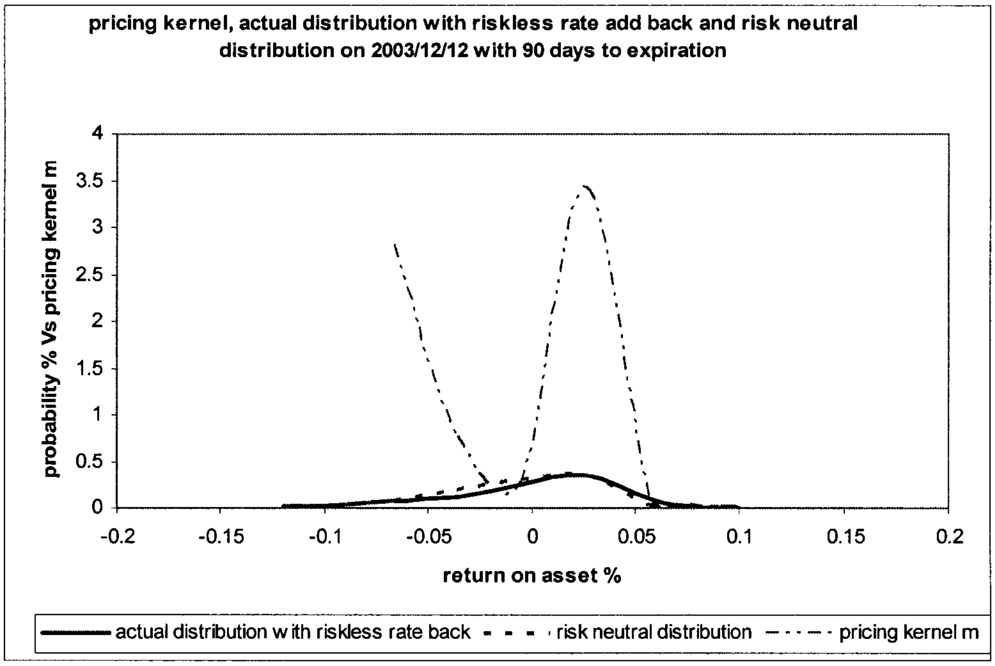


Figure 16 pricing kernel, actual distribution with riskless rate add back and risk neutral distribution



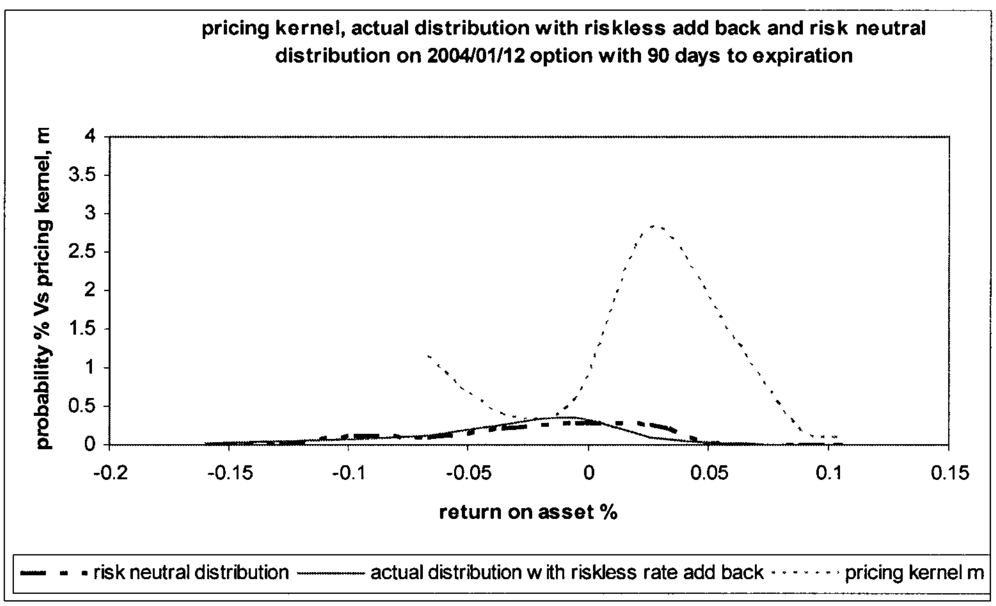
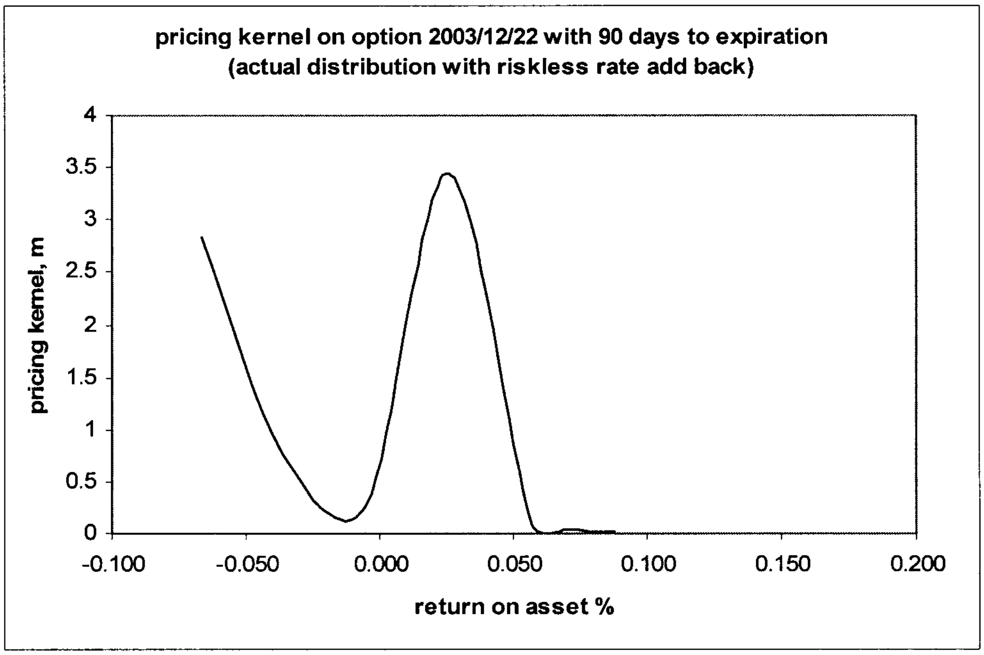
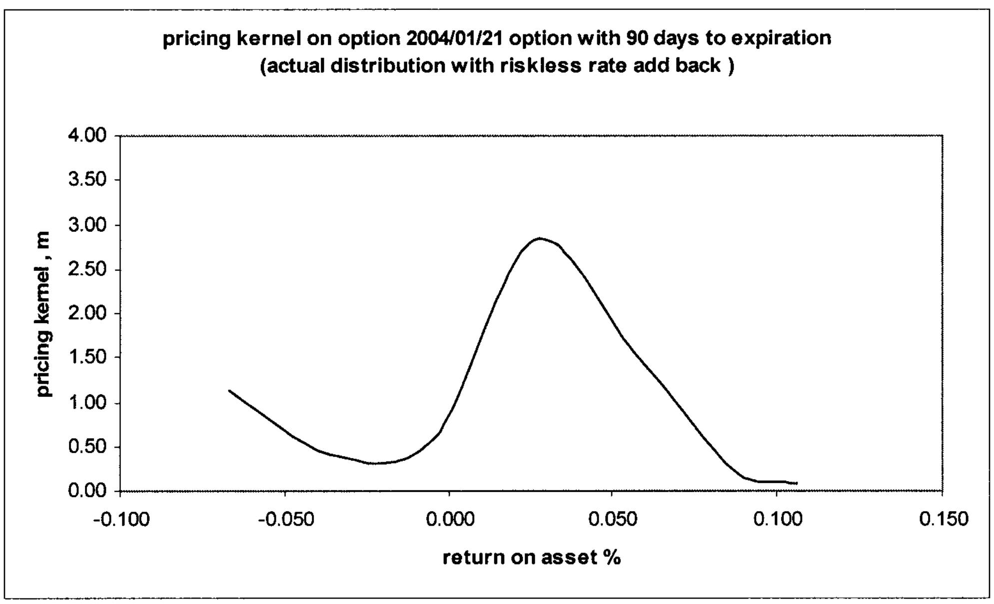


Figure 17 pricing kernel on option 2003/12/22 with 90 days to expiration (actual distribution with riskless rate adds back)





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APPENDICES

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