

**Concatenated Coding and Space-Time Modulation:  
Transmitter and Receiver Design**

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# ABSTRACT

## Concatenated Coding and Space-Time Modulation: Transmitter and Receiver Design

Rongxin Zhang

The use of multiple transmit and receive antennas in wireless communications can significantly increase the data rate and reliability. This has spurred a great deal of research effort in space-time coding and modulation, which is an effective transmission technique for multiple-input multiple-output (MIMO) systems. In general, the design of two-dimensional space-time codes is more challenging than that of the conventional codes for single-input single-output (SISO) systems, particularly for modern multimedia wireless communications with a diversity of requirements on data rate and quality of services.

One way to simplify the design of flexible and efficient MIMO transmission schemes is to view a MIMO transmitter as a concatenated coding and modulation system, in which conventional coding is applied before space-time coding/modulation. This concatenated coding structure allows a salient choice from a large number of conventional codes for SISO systems and simplifies the design of the inner space-time modulation. The structure is also general as it subsumes many existing space-time coding schemes as special cases.

Based on the proposed general structure, a study on the effects of the inner space-time modulation on system performance is conducted. It is shown that well-known design criteria for space-time codes do not apply in a concatenated coding system. In fact, the design becomes much easier. Two MIMO transmission schemes are developed. The first

is a simple delay-diversity scheme designed for optimized performance and the second scheme is a multi-layered MIMO transmission scheme aimed at applications that requires flexible tradeoff between performance and data rate. In addition, a joint iterative receiver is developed for the proposed concatenated transmission schemes. Simulation results are provided to show the merits of the proposed schemes.

Dedicated to my family .....

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## LIST OF ACRONYMS

APP	A Posteriori Probabilty
CF	Circulant Framing
CN	Complex Noise
D-Blast	Diagonal Bell Layered Space-Time
FER	Frame Error Rate
HLST	Horizontal Layered Space-Time
LLR	Log-Likelihood Ratio
LST	Layered Space-Time
MAP	Maximum A Posterior (probability)
MIMO	Multiple-Input Multiple-Output
ML	Maximum Likelihood
PSK	Phase Shift Keying
RSC	Recursive Systematic Convolutional (code)
SISO	Single-Input Single-Output
ST	Space-Time
STBC	Space-Time Block Codes
STTC	Space-Time Trellis Codes
TCM	Trellis Coded Modulation
TLST	Threaded Layered Space-Time
V-Blast	Vertical Bell Laboratories Layered Space-Time

# Chapter 1

## Introduction

### 1.1 Research Background

In the new generation of wireless communications, we are faced with a number of challenges. These include combating fading to improve the performance and meeting the ever increasing demand for higher data rate with limited radio spectrum. Usually higher rate and better performance means wider bandwidth. As the available radio spectrum is limited, the only way for achieving both the reliability and bandwidth efficiency is designing more efficient signaling techniques. Among these techniques *multiple-input multiple-output* (MIMO) wireless transmission is an emerging cost-effective technology that promises significant improvement in these measures [1] [2].

A MIMO wireless channel is constructed with multiple antennas at two ends of the wireless link. Recent research in information theory has shown that a large gain in capacity over wireless channel can be obtained in MIMO systems [1]-[4]. It is proven in [3] that, compared with a *single-input single-output* (SISO) system in Rayleigh fading

channels, a MIMO system can improve the capacity by a factor of the minimum of the number of the transmit and receive antennas. Furthermore, recent studies have shown that the use of multiple antennas increases the diversity to combat channel fading [5] [6]: in a MIMO system with  $N_t$  transmit and  $N_r$  receive antennas, given that the path gains between individual antenna pairs are independently Rayleigh faded, the MIMO system can achieve a maximal *diversity gain* of  $N_t N_r$ .

To realize the promised theoretic capacity and diversity gain of MIMO wireless channels, an effective and practical approach, *space-time* (ST) coding has been extensively studied and applied in practices [7] [8]. Space-time coding is a joint coding and modulation technique particularly designed for MIMO transmission systems. The basic principle of space-time coding is to apply coding in both spatial and temporal domains to introduce correlations between signals transmitted on different antennas (spatial) at different time instants (temporal). The spatial-temporal correlations exploit different and often independent channel fading modes to reduce detection errors at the receiver. Two main approaches have been explored by the researchers. One is the *spatial multiplexing* approach exemplified by the so-called *layered space-time* (LST) coding proposed by Foschini [1] [3]; the other is the *diversity* approach including *space-time block codes* (STBC) [5] [9] [10] and *space-time trellis codes* (STTC) [6].

LST coding is aimed at achieving higher data rate (*multiplexing gain*) for bandwidth efficiency applications. The key component in LST coding is the spatial multiplexing that multiplexes several encoded data streams in a way such that each data stream will experience all the fading modes. At the receiver, suboptimal linear signal processing techniques can be used to separate data streams before decoding is applied [11][12]. To a

large degree, the success of an LST coding scheme depends on the multiplexing scheme at the transmitter and the signal processing techniques at the receiver. The use of suboptimal signal separation techniques makes the receiver complexity grow linearly with data rate. There are a number of LST architectures, depending on the way the modulated symbols assigned to transmit antennas (e.g., spatial multiplexing). These methods include *vertical layered space-time* (VLST) also called *vertical Bell Laboratories layered space-time* (V-BLAST) architecture [11], *horizontal layered space-time* (HLST) [12], *threaded layered space-time* (TLST) [13] and *diagonal layered space-time* (DLST or D-BLAST) [14].

In STBC, two-dimensional coding is applied on a block of input symbols, producing an output matrix whose columns represent time and rows represent antennas. The key advantage of the scheme is that it can achieve full diversity gain with orthogonal design and for certain designs has significantly lower complexity compared to STTC [9] [10]. However, the scheme suffers from smaller *coding gain* and often loss in data rate [12].

STTC was pioneered by V. Tarokh, N. Seshadri, and A. R. Calderbank [6]. It can achieve full diversity and large coding gain through a joint design of two-dimensional encoding and modulation. However, for STTC, the complexity of the optimal trellis decoding using Viterbi algorithm grows exponentially with the number of transmit antennas and data rate. The prohibitive complexity precludes its application for high data rate applications [12].

In summary, the above approaches lack the flexibility of tradeoff between bandwidth efficiency (multiplexing gain) and reliability (diversity), which is the key to the future success of wireless communications with the expectation of multi-rate and different

levels of quality of service. In [15], Zheng and Tse have shown that there exists an optimal tradeoff curve between diversity and multiplexing. It showed that D-BLAST allows the optimal tradeoff between diversity and multiplexing, but it is still not clear how to achieve the maximal diversity gain for a given multiplexing gain with structured codes instead of Gaussian random codes. This has motivated a major research thrust aimed at developing practical MIMO transmission schemes with flexible diversity-multiplexing tradeoff.

Several space-time coding schemes that provide tradeoff between data rate and reliability have been proposed, e.g., [16] and [17] for flat fading channels and [18] for both flat and frequency-selective channels. However, these schemes are based only on linear block coding which entails performance loss in terms of coding gain and excessively high complexity for optimal maximum-likelihood decoding or even for suboptimal sphere decoding. Another remarkable result is presented in [19], in which it was shown that space-time codes carved from properly constructed lattice achieve optimal multiplexing-diversity tradeoff for flat fading channels. However, the decoding complexity is still prohibitively high for a moderate number of transmit and receive antennas.

## 1.2 Problem Statements and Objectives

Despite tremendous research effort, difficulties exist in the design of flexible MIMO transmission schemes. In particular, the design criteria for space-time codes developed in [6] are very difficult to apply. In many cases, the exact coding gain and even diversity gain are difficult to be found for a given code. In addition, the need of transmission



schemes that are flexible in diversity-multiplexing tradeoff makes a combined diversity and spatial multiplexing approach necessary. A flexible MIMO transmission scheme must be able to support various data rate and, at the same time, to provide a large coding gain.

Motivated by the recent successful design of linear STBC with various data rates [16], [17], and [18], we propose a framework of MIMO transmitters consisting of two serially concatenated components as shown in Fig. 1.1. The outer codes are usually conventional one-dimensional codes to avoid the difficulties in the design of two-dimensional space-time codes; the inner code maps a group of encoded symbols to a spatial and temporal matrix and will be called space-time modulator to emphasize the “modulation” flavor of the component. An interleaver is used between them to make two parts independent.



Fig. 1.1. Block diagram of a serially concatenated MIMO transmitter.

Such a framework includes many existing space-time coding schemes such as D-Blast and some of STTC schemes. In fact, conventional one-dimensional codes are likely to be used as the outer codes in practical MIMO systems to provide large coding gain. For instance, in the 3<sup>rd</sup>-generation cellular systems [7], convolutional or turbo codes are used before Alamouti’s space-time codes. It is worthy of mentioning that although the concatenated MIMO transmission has been studied in the literature [6] and also applied in practices, the outer codes and the inner space-time codes/modulators have often been

considered separately. This is despite that well-known design criteria of space-time codes are developed for systems with only space-time codes. There are several immediate advantages to model a MIMO transmission design using the proposed general framework. First, the existence of a large amount of one-dimensional codes allows a salient choice to tailor for different needs. In addition, the design of outer code(s) needs only to be focused on coding gain and effective length [20] as in SISO fading channels while the onus of spatial diversity and multiplexing is left to the inner space-time modulator. The division of tasks between the two constituent components avoids the difficulties in the design of two-dimensional codes. Last but not the least, turbo receiver can be employed for the reception of a concatenated transmitter so that linear techniques can be used for the inner demodulation to reduce otherwise prohibitive complexity with little performance loss.

Using the above general framework, we have the following research objectives.

- Study the iterative joint receiver for concatenated MIMO transmission and develop suitable efficient space-time soft-in soft-out demodulator.
- Analyze the effects of outer coding and inner space-time modulation on the performance of the overall system and develop design criteria for the inner space-time modulator for concatenated MIMO transmission.
- Develop a space-time modulator that is suitable to be used in a concatenated MIMO transmission and is able to provide various diversity-multiplexing tradeoffs.

### 1.3 Contributions

In this thesis, we first introduce a so-called *delay-diversity* transmission under the general framework. The proposed MIMO transmission scheme can achieve full diversity given the number of antennas and data rate and provide large coding gain. Such a scheme can be regarded as a special case of STTC. We then propose an iterative joint demodulation and decoding scheme as the receiver and compare its performance with that of the optimal maximum-likelihood receiver. We show that with the presentation of the general framework, significant reduction in receive complexity is achieved by using the proposed iterative joint demodulation and decoding scheme.

We then propose a new inner space-time modulator for concatenated MIMO transmission. The overall system is a layered MIMO transmission system where the inner space-time modulator maps multiple encoded data streams into an output matrix. By analyzing the effects of the constituent components on overall performance, we develop a set of design criteria that turn out to be much easier to apply than Tarokh's criteria set. Using the design criteria, we are able to optimize important modulator parameters. The so obtained modulator maximizes the diversity gain and coding gain of those codeword pairs with minimum Euclidean distance. In addition, various multiplexing gain (data rates) can be achieved simply by choosing the number of data streams. When the number of layers (streams) is equal to or greater than the number of transmit antennas, full channel capacity can be reached.

## 1.4 Thesis Outline

The rest of the thesis is organized as follows.

Chapter 2 presents some key concepts related to this work. Firstly, MIMO channel model that will be considered in this thesis is introduced. Then diversity technique and Tarokh's space-time design criteria are presented. Diversity and multiplexing tradeoff is also discussed. Last, the general concept of turbo principle (iterative) is introduced in this chapter.

Chapter 3 provides the implementation of the proposed delay-diversity transmission scheme, that is, one-layered space-time trellis coding system. An efficient joint iterative receiver is developed.

Chapter 4 presents the multi-layered MIMO transmission system. Design criteria are established and an efficient and flexible multi-layered space-time modulator is developed.

Chapter 5 concludes the thesis by summarizing the work presented and outlining possible future works.

## 1.5 Notations

Some notations used in this thesis are list here.

<b>a</b>	A column vector in bold-face lower-case letter
<b>A</b>	A matrix in bold-face upper-case letter
<b>I<sub>i</sub></b>	An $i \times i$ identical matrix
<b>0<sub><math>i \times j</math></sub></b>	An $i \times j$ matrix with all zeros
<b>1<sub><math>i \times j</math></sub></b>	An $i \times j$ matrix with all its entries being 1
<b>A<sup>H</sup></b>	Hermitian of matrix <b>A</b>

$()^T$	Transpose of matrix or vector
$diag()$	The operator that makes a vector into a matrix with the vector elements as the matrix's diagonal entries
$E[]$	The expectation operator
$Cov()$	The covariance operator
$\Lambda()$	The a posteriori log likelihood ratio
$L()$	The a priori or extrinsic log likelihood ratio
$I$	The interleaving operator
$I^{-1}$	The de-interleaving operator

## Chapter 2

### Preliminary

In this chapter, some background knowledge, concepts, and terminology pertaining to MIMO wireless channels and the associated transmission techniques are presented to provide the basis on which the subsequent chapters will be developed. The channel model that will be considered in this thesis is described in section 2.1. Performance measures and design criteria for communications over MIMO fading channels are presented in section 2.2. In section 2.3, several popular MIMO transmission schemes are reviewed under a general MIMO transmission framework. Lastly, in the section 2.4, the receiver design based on *Turbo principles* is also introduced. The general framework will serve as the basis for the design of new MIMO transmission and reception schemes that will be presented in the later chapters.

## 2.1 Channel Model

In wireless communications, the surrounding environment such as buildings, trees and vehicles act as reflectors such that multiple reflected waves of the transmitted signals arrive at the receive antennas from different directions with different propagation delays. When these signals are collected at the receiver, they may add constructively or destructively depending on the random phases of the signals arriving at the receiver. The amplitude and phase of the combined multiple signals vary with the movement of the surrounding objects in wireless channel. The resultant amplitude fluctuation is called fading [21].

Fading, can be catalogued into flat fading also known as frequency non-selective fading and frequency selective fading. In a flat fading channel, the transmitted signal bandwidth is smaller than the coherence bandwidth of the channel. Hence, all frequency components in the transmitted signal are subjected to the same fading attenuation. In a frequency selective fading channel, the transmitted signal bandwidth is larger than the coherence bandwidth of the channel, different frequency components in the transmitted signal experience different fading attenuation. As a result, the spectrum of the received signal differs from that of the transmitted signal, this is called distortion.

Fading can also be classified as fast fading or slow fading depending on how rapidly the channel changes compared to the symbol duration. If the channel can be deemed constant over a large number of symbols, the channel is said to be a slow fading channel; otherwise it is a fast fading channel [22].

In wireless communications, the envelope of the received signal can be described by Rayleigh distribution or Ricean distribution. In a no line-of-sight propagation, Rayleigh

distribution is applied and the fading is called Rayleigh fading. While in a line-of-sight propagation, since there exists a dominant non-fading component, Ricean distribution is often used to model the envelope of the received signal. Thus it is called Ricean fading.

In this thesis, we consider applications where the channel is unknown to the transmitter but known or can be perfectly estimated at the receiver. Furthermore, our attention is limited to quasi-static, flat, Rayleigh fading channel where the channel keeps constant during a block but changes from block to block according to Rayleigh distribution. Extension to other channel conditions is possible.

Consider a MIMO communication system with  $N_t$  transmit and  $N_r$  receive antennas as shown in Fig. 2.1. The gain of the path from the  $i$ -th transmit antenna to the  $j$ -th receive antenna is denoted by  $h_{ji}$  with  $i = 1, 2, \dots, N_t$  and  $j = 1, 2, \dots, N_r$ . The path gains are assumed to be independent complex Gaussian random variables with zero mean and unit variance, that is,  $h_{ji} \sim CN(0, 1)$ . The received signal vector  $\mathbf{z}$  is written as

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (2.1)$$

where  $\mathbf{z} = [z_1, z_2, \dots, z_{N_r}]^T$  with  $z_j$  being the signal received at the  $j$ -th receive antenna,  $\mathbf{x} = [x_1, x_2, \dots, x_{N_t}]^T$  with  $x_i$  being the symbol transmitted on the  $i$ -th transmit antenna,  $\mathbf{n}$  is AWGN with  $E[\mathbf{n}\mathbf{n}^H] = N_0\mathbf{I}_{N_r}$  and  $\mathbf{H} = [h_{ji}]_{N_r \times N_t}$  is the  $N_r \times N_t$  channel matrix.

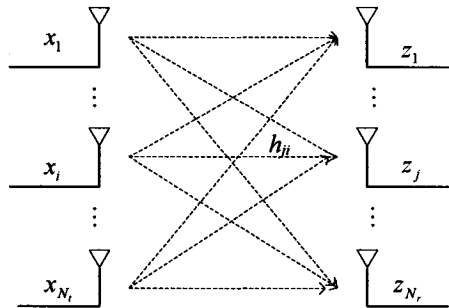


Fig. 2.1. A MIMO flat-fading channel.



When the channel is unknown to the transmitter, the ergodic capacity of such a channel is given by [12]

$$C = E_h \left[ \log_2 \det \left( \mathbf{I}_{N_t} + \frac{P}{N_t N_0} \mathbf{H}^H \mathbf{H} \right) \right] \quad (2.2)$$

where  $P$  is the total transmitted power over the  $N_t$  transmit antennas.

## 2.2 Performance Measures and Their Tradeoff

As can be checked from (2.2), a Rayleigh fading channel can offer the capacity of  $\min(N_r, N_t)$  times of a single-antenna channel. In addition, multiple antennas can also be used to improve the communication reliability. A “good” MIMO transmission scheme must exploit as much of both resources as possible and provide a flexible tradeoff between the reliability and data rate.

### 2.2.1 Diversity and Multiplexing Gain

The increase in data rate of a MIMO transmission scheme can be characterized by multiplexing gain. A MIMO transmission scheme is said to achieve a multiplexing gain  $r$  if, at high signal-to-noise ratio (SNR) region, its data rate  $R = r \log_2 \text{SNR}$ , which is  $r$  times of the capacity of a SISO channel with the same SNR [15].

The improvement in reliability of a MIMO transmission scheme is often characterized by diversity gain. There exist two definitions for diversity gain. One is defined as the exponent of the error performance of a scheme when the data rate is fixed against the channel capacity [15]. Another more widely-used definition is the exponent of

the error performance of a scheme when the data rate is constant [6]. The first definition is useful when discussing the tradeoff between data rate and reliability; whereas the second definition is useful when discussing the error performance of a scheme with a fixed data rate. We will use both definitions in this thesis and refer to the first definition as Zheng's diversity gain and the second as Tarokh's diversity gain.

It was shown in [15] that there exists a fundamental tradeoff between the multiplexing gain and Zheng's diversity gain. Specifically, a MIMO system with  $N_t$  transmit and  $N_r$  receive antennas can provide the maximum diversity gain  $d_{max} = N_t N_r$  or the maximum spatial multiplexing gain  $r_{max} = \min(N_r, N_t)$  but not both at the same time. Any type of gain comes at the price of another [23]. Therefore, in designing a MIMO transmission scheme, flexible tradeoff is important particularly for multi-rate wireless communications.

### 2.2.2 Performance and Design Criteria

Pairwise error probability is often used to analyze the performance of a MIMO transmission scheme. The pairwise error probability between a codeword pair  $(\mathbf{X}, \hat{\mathbf{X}})$  is the probability that a decoder decides in favour of  $\hat{\mathbf{X}}$  when in fact  $\mathbf{X}$  is transmitted [12].

Let  $x_i(n)$  denote as the symbol to be transmitted at time  $n$  over transmit antenna  $i$ , and collect a block of transmitted symbols as

$$\mathbf{X} = \begin{bmatrix} x_1(1) & x_1(2) & \cdots & x_1(L) \\ x_2(1) & x_2(2) & \cdots & x_2(L) \\ \vdots & \vdots & \ddots & \vdots \\ x_{N_t}(1) & x_{N_t}(2) & \cdots & x_{N_t}(L) \end{bmatrix} \quad (2.3)$$

Then with maximum likelihood (ML) decoding, the pairwise error probability of  $(\mathbf{X}, \hat{\mathbf{X}})$  at high SNR is given by

$$P(\mathbf{X}, \hat{\mathbf{X}}) \leq \left( \prod_{i=1}^{Nr} \frac{1}{1 + \frac{E_s}{4N_o} \lambda_i} \right)^{Nr} \quad (2.4)$$

where  $\lambda_i$  is the  $i$ -th eigenvalue of matrix  $\mathbf{A} = (\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^H$  and  $r$  is the rank of matrix

**A.** The above equation can be simplified as

$$P(\mathbf{X}, \hat{\mathbf{X}}) \leq \left( \prod_{i=1}^r \lambda_i \right)^{-Nr} \left( \frac{E_s}{4N_o} \right)^{-rNr} \quad (2.5)$$

We can see from (2.5) that the pairwise error probability decreases exponentially as SNR increases. The exponent in the error probability in (2.5) determines the slope of the error probability curve as SNR increases and is called the diversity gain (Tarokh's). It can

also be observed that a coding gain of  $\left( \prod_{i=1}^r \lambda_i \right)^{\frac{1}{r}}$  is achieved.

Based on the above observations, two design criteria were obtained by Tarokh et al. in [6]:

- **The Rank Criterion:** In order to achieve the maximum diversity  $N_t N_r$ , the difference matrix  $(\mathbf{X} - \hat{\mathbf{X}})$  has to be full rank for all pairs of  $\mathbf{X}$  and  $\hat{\mathbf{X}}$ . If  $(\mathbf{X} - \hat{\mathbf{X}})$  has minimum rank  $r$  over the set of two pairs of distinct codewords, then a diversity of  $rN_r$  is achieved.
- **The Determinant Criterion:** Suppose that a diversity gain of  $rN_r$  is our target, then the design goal is making the minimum product  $\prod_{i=1}^r \lambda_i$  as large as possible over all pairs of distinct codewords  $\mathbf{X}$  and  $\hat{\mathbf{X}}$ . If a diversity gain of  $N_t N_r$  is the

design target, then the minimum product  $\prod_{i=1}^r \lambda_i$  must be maximized over all pairs of distinct codewords. Note that the coding gain  $\prod_{i=1}^r \lambda_i$  is the absolute value of the sum of determinants of all  $r \times r$  principal cofactors of  $\mathbf{A}$  taken over all pairs of codewords  $\mathbf{X}$  and  $\hat{\mathbf{X}}$ .

## 2.3 Popular MIMO Transmission Approaches and a General Framework

MIMO communication systems use antenna arrays at both the transmitter and the receiver to offer improved bandwidth efficiency and reliability. Two popular approaches in designing MIMO transmission systems have been pursued in the literature. They are the spatial multiplexing approach aimed at improved bandwidth efficiency [3] and diversity approach aimed at improved reliability [5] [6] [9]. In practice, a combined multiplexing and diversity approach is desirable to provide diversity-multiplexing tradeoff based on service requirement and link condition. In this section, we will introduce a general framework that includes both multiplexing and diversity approaches as special cases. Under this general framework, several popular schemes will be introduced.

### 2.3.1 A General Serially Concatenated Framework

Observing different schemes from the above two MIMO system approaches, we have found that most of these schemes can be modeled as serially concatenated

encoder/modulator as in Fig. 2.2. In this framework, the outer encoders (one or more depending on the multiplexing gain) are conventional one-dimensional codes such as convolutional code, trellis coded modulation (TCM), etc. The inner is a space-time modulator/multiplexer which usually works block by block with a block length much smaller than that of the outer encoder. An interleaver is often employed at the output of each outer encoder.

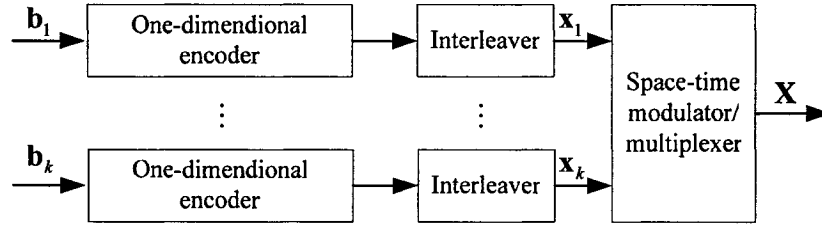


Fig. 2.2. A general framework.

Many of the existing schemes can be modeled by the general framework. For instance, space-time block codes are the inner space-time modulator/multiplexer without outer code in the general framework; while if the inner modulator/multiplexer multiplexes the  $N_t$  symbols from the  $N_t$  outer encoders in a diagonal fashion, it is D-BLAST. In what follows, we will describe the delay-diversity scheme (a special case of trellis space-time coding) and D-BLAST under the general framework.

### 2.3.2 Delay-Diversity Transmission

In transmit diversity schemes, redundancy is introduced in the information streams by applying multiple-dimensional coding across both the spatial and temporal domains to maximize reliability (diversity and coding gain) [24]-[26]. Among the diversity

approaches, the delay-diversity is a simple but effective scheme [27]. In this scheme, the interleaved symbols from the encoder and mapper are transmitted via multiple antennas with delays as shown in Fig. 2.3, where  $nD$  denotes  $n$ -symbol delay. As a result, the receiver will observe multipath-like symbol sequences, from which a diversity advantage is extracted [26]. Note that in this scheme the encoder and mapper are viewed as the outer encoder while forming the transmitted symbols matrix is viewed as a space-time modulator, i.e., the inner modulator. It is also interesting to note that the whole process including the one-dimensional encoder and the space-time modulator (repeat and delay) can be treated as a space-time encoder. Furthermore, if the outer encoder and mapper is a TCM encoder, the overall processing is a space-time trellis encoder.

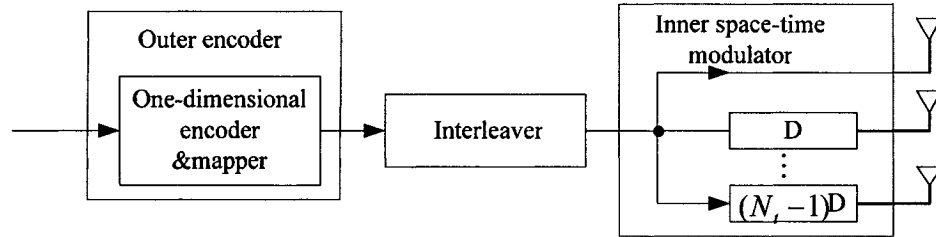


Fig. 2.3. A transmitter with delay diversity.

It can be easily checked that the diversity gain of this simple delay-diversity scheme is  $N_t N_r$ , that is, full diversity has been achieved.

In fact, delay diversity code is a special case of STTC. For example, a STTC with a trellis structure as shown in the left of Fig. 2.4 can be described by a corresponding transmission delay diversity scheme in the right part of Fig. 2.4. Surprisingly, this simple delay-diversity was conjectured to be the optimal in [6]. We will see in the chapter 3 that in the delay diversity scheme, the decoding computation is greatly saved by using turbo

receiver instead of Viterbi decoder based on the space-time trellis. Additionally, in chapter 4, we will see that this scheme combined with appropriate multiplexing, forms a new multi-layered MIMO transmission scheme which allows a flexible tradeoff between the rate and the link reliability by adapting the number of the layers.

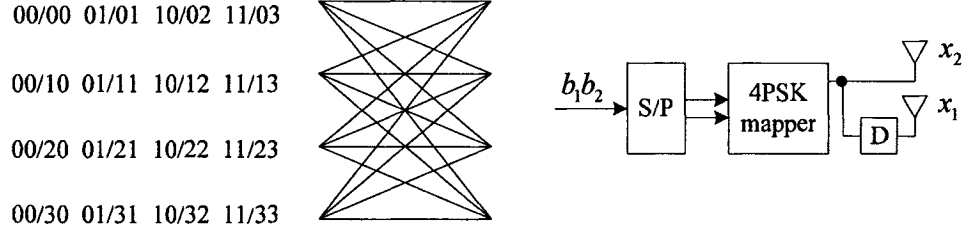


Fig. 2.4. A 4-state space-time trellis code with 4PSK and 2 transmit antennas and its corresponding delay-diversity structure.

### 2.3.3 D-BLAST

Layered space-time processing is a spatial multiplexing scheme. Its core idea is to transmit multiple independent data streams. The receiver first uses receive array to separate and detect the transmitted signals and subsequently performs one-dimensional decoding on the detected symbols. This approach achieves a high spectral efficiency (high data rate), but keeps a lower decoding complexity compared with the diversity approach like STTC, in which the complexity of the optimal maximum likelihood decoder grows exponentially with the number of bits per symbol [3] [28]. Among the existing LST schemes, D-BLAST is attractive in that it provides the optimal diversity-multiplexing tradeoff if Gaussian random codes are used [15].

In D-BLAST, the information bits first are de-multiplexed into  $N_t$  data streams. Each data stream is encoded, mapped to symbols respectively and then put along the diagonal

of the transmission matrix by a space-time modulator as shown in Fig. 2.5. For an example of  $N_t = 3$ , the encoded symbols sequences from the three data streams are respectively  $x_1(1) x_1(2) \dots$ ,  $x_2(1) x_2(2) \dots$  and  $x_3(1) x_3(2) \dots$ , where  $x_i(n)$  is the  $n$ -th symbol of the  $i$ -th data stream. After space-time modulation, a transmission array is formed as

$$\mathbf{X} = \begin{bmatrix} x_1(1) & x_2(1) & x_3(1) & x_4(1) & x_5(1) & x_6(1) & \dots \\ 0 & x_1(2) & x_2(2) & x_3(2) & x_4(2) & x_5(2) & \dots \\ 0 & 0 & x_1(3) & x_2(3) & x_3(3) & x_4(3) & \dots \end{bmatrix} \quad (2.6)$$

Obviously, the  $N_t$  data streams occupy different diagonals of the space-time grid in an interleaved fashion. Since  $N_t$  symbols are transmitted at each use of the channel, it has the potential of a multiplexing gain of  $N_t$ .

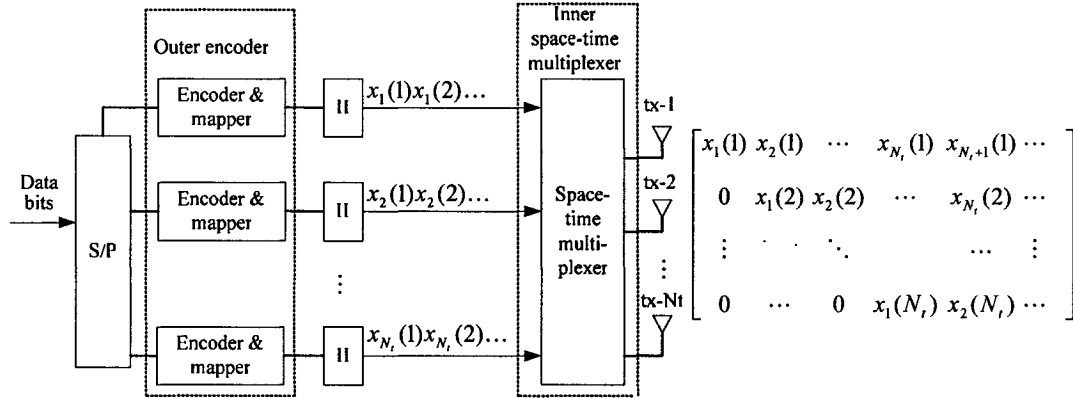


Fig. 2.5. The transmitter for D-BLAST.

D-BLAST can also be modeled by the general framework with encoders and mappers as outer encoder and space-time multiplexer as the inner space-time multiplexer. Hence, iterative Turbo receiver can be used to provide better performance than the conventional



D-BLAST receiver such as the MMSE detector followed by the decoder and the combined cancellation and linear suppression followed by the decoder [12].

One important drawback of D-BLAST is the design of the outer code for maximum diversity is difficult owing to its suboptimal spatial multiplexer.

## 2.4 Turbo Principles and Iterative Receiver

The optimal receiver for space-time coding based on trellis decoding has a high complexity especially at a high data rate [12] [29]. A joint iterative receiver is a suboptimal scheme which applies *turbo principles* in the joint detection/decoding to get a performance near that of the optimal scheme but with a lower complexity [30]-[36]. It is also a natural choice for the general framework described in the last section.

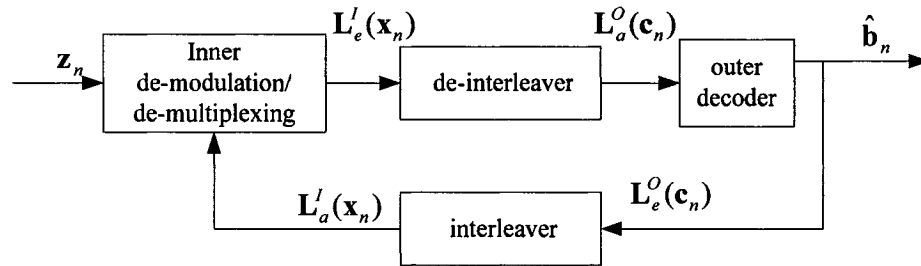


Fig. 2.6. The turbo receiver for a serially concatenated system.

Turbo principles were originally introduced in iterative turbo decoding but now have been successfully applied in joint detection/decoding and other areas.

Basically, if the transmitter of a communication system can be modelled as the combination of several components (arranged in serial or parallel fashion) such as the delay diversity in Fig. 2.4 and D-BLAST in Fig. 2.5, turbo principles are applicable in its iterative receiver. Consider a serially concatenated transmission scheme as in Fig. 2.2,

where the two components are separated by an interleaver. At the receiver, soft information in the form of confidence levels (extrinsic information) about the transmitted symbols is transmitted between the soft-in soft-out inner demodulator/de-multiplexer and the soft-in soft-out outer decoder. By exchanging soft information between the two parts, demodulation/de-multiplexing and decoding are repeated on the same set of received data until a termination criterion stops the iterative process.

Here we briefly introduce the turbo algorithm. For the inner code, we assume an  $N$ -length observation block of the received signal  $\mathbf{z} = [z_1, z_2, \dots, z_N]^T$ , which is the  $M$ -length encoded data  $\mathbf{x} = [x_1, x_2, \dots, x_M]^T$  going through a slow Rayleigh fading channel  $\mathbf{H}$ ,  $\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{n}$ . The inner decoder determines pseudo-APPs for each transmitted symbol  $x_n$  [12] [35]

$$\Lambda_1(x_n = s_i) = \ln \frac{\Pr(x_n = s_i | \mathbf{z})}{\Pr(x_n = s_0 | \mathbf{z})} \quad (2.7)$$

where  $s_i$  is from the set  $\{s_i | i = 0, 1, 2, \dots, 2^M-1\}$  if a  $2^M$ -ary modulation is assumed.

Using Bayes' rule, (2.7) can be written as

$$\begin{aligned} \Lambda_1(x_n = s_i) &= \ln \frac{\Pr(x_n = s_i)}{\Pr(x_n = s_0)} + \ln \frac{p(\mathbf{z} | x_n = s_i)}{p(\mathbf{z} | x_n = s_0)} \\ &= L_a^I(x_n = s_i) + L_e^I(x_n = s_i) \end{aligned} \quad (2.8)$$

where the first term  $L_a^I(x_n = s_i)$  represents the a priori probability of  $x_n$ , which is computed by the outer decoder in the previous iteration, interleaved and then fed back to the inner decoder. For the first iteration, one may assume equally likely code symbols, i.e., no priori information available,  $L_a^I(x_n = s_i) = 0$ . The second term  $L_e^I(x_n = s_i)$  represents the extrinsic information which is computed based on the received signal  $\mathbf{z}$  and

the a priori information about symbols transmitted at other time instants  $t$  ( $t \neq n$ ), which is also delivered by outer decoder and interleaved and fed back to the inner decoder.

For the outer code, the a posteriori probability  $\Lambda_2$  is computed based on de-interleaved the prior information delivered by the inner code and the trellis structure of the outer code.

$$\Lambda_2(x_n = s_i) = \ln \frac{\Pr(x_n = s_i | L_a^O, \text{trellis})}{\Pr(x_n = s_0 | L_a^O, \text{trellis})} \quad (2.9)$$

$\Lambda_2(x_n = s_i)$  can also be viewed as the sum of two parts

$$\begin{aligned} \Lambda_2(x_n = s_i) &= \ln \frac{\Pr(x_n = s_i)}{\Pr(x_n = s_0)} + \ln \frac{p(L_a^O, \text{trellis} | x_n = s_i)}{p(L_a^O, \text{trellis} | x_n = s_0)} \\ &= L_a^O(x_n = s_i) + L_e^O(x_n = s_i) \end{aligned} \quad (2.10)$$

where  $L_a^O(x_n = s_i)$  is the prior information of  $x_n$ , and  $L_e^O(x_n = s_i)$  is the extrinsic information given by the outer decoder. This extrinsic information is the information about the codeword gleaned from the prior information about the codewords at other time instants ( $t \neq n$ ), based on the trellis constraint of the code. After interleaving, the extrinsic information delivered by the outer decoders is then fed back to the inner decoder, as the prior information about the codeword in the next iteration.

As we mentioned before, the advantage of the general framework is lowering the receiver complexity with the use of iterative receiver, especially when a linear MMSE filter is used for the detection or cancellation. For example, for a space-time trellis code with  $s$  states and  $N_t$  transmit antennas, an MMSE iterative receiver with  $k$  iterations will reduce the complexity to  $k/s^{(N_t-1)}$  of that of the ML receiver.

## Chapter 3

# Delay-Diversity MIMO Transmission with Joint Turbo (Iterative) Receiver

In this chapter, we describe a simplified receiver for a class of space-time trellis codes, namely, *delay-diversity codes*. By viewing the delay operation as a form of modulation scheme, i.e., the inner space-time modulation in the general framework presented in the last chapter, we can employ a turbo receiver where the outer code is a one-dimensional trellis code. Since linear receiver is used as the inner demodulator, the overall complexity is lower than the standard Viterbi decoder for space-time trellis code.

The chapter is organized as follows. We begin with the description of the overall system. Then the derivation of the joint turbo interference cancellation algorithm and MAP decoding algorithm will be given in details. Finally, we will present the simulation results of the simplified iterative receiver compared to optimal receiver using Viterbi algorithm for the delay-diversity codes.

### 3.1 Description of the System

Transmitting the same data stream over multiple antennas with different delays has been recognized as an effective way of improving the reliability of wireless communications [6]. Interestingly, a more systematic space-time trellis code design has shown that these delay diversity schemes are often optimal [6]. Motivated by this, we represent this delay-based diversity coding scheme using our general transmission framework as shown in Fig. 3.1.

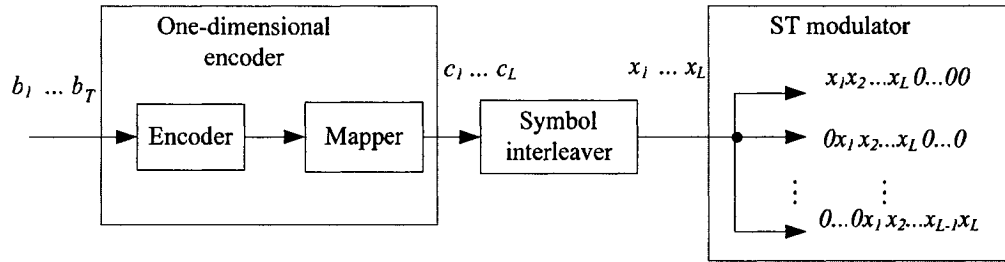


Fig. 3.1. The transmitter for delay-diversity code.

In such a transmitter, the information bits,  $b_1, b_2, \dots, b_T$ , which are assumed to be independent and equally likely taking on values of 0 or 1, first enter an one-dimensional encoder. After encoded and mapped, the symbol sequence,  $c_1, c_2, \dots, c_L$  are interleaved. The outputs of the interleaver,  $x_1, x_2, \dots, x_L$ , are then passed through the ST modulator and transmitted over the  $N_t$  transmit antennas. The ST modulator repeats each input symbol on all the  $N_t$  antennas with different delays as shown in Fig. 3.1. Often, no delay is introduced for the first transmit antenna.

With the above delay-diversity scheme in Fig. 3.1, it can be readily checked that as long as different delays are used for different antennas, a diversity gain  $N_t$  can be

achieved. Furthermore, the overall system including the one-dimensional encoder and delay network can be represented by a two-dimensional space-time encoder. If the outer code is a trellis code, then the corresponding ST code is a space-time trellis code but with much larger number of states. For instance, a one-dimensional trellis encoder with  $S$  states with a delay of  $i-1$  symbols ( $i = 1, 2, \dots, N_t$ ) for the  $i$ -th transmit antenna is equivalent to a STTC with  $S^{N_t}$  states. Apparently, the number of the states and, thus, the computational complexity of a Viterbi receiver grow exponentially with the number of the transmit antennas. Fortunately, by viewing the delay network as a form of space-time modulation, an iterative (turbo) receiver can be employed for reduced complexity.

Since the collection of receive diversity is straightforward, here we consider a system with  $N_t$  transmit antennas and one receive antenna to simplify the analysis of the receiver. The gain of the path from transmit antenna  $i$  to the receiver is denoted by  $h_i$  with  $i = 1, 2, \dots, N_t$  and is assumed to be Gaussian independent complex variables with zero mean and variance  $1/N_t$ , i.e.,

$$E\left[\sum_{i=1}^{N_t} |h_i|^2\right] = 1 \quad (3.1)$$

We also assume that the perfect channel information is available at the receiver.

With a transmitter structure shown as in Fig. 3.1, we can write the received symbol  $z_n$  as

$$z_n = \sum_{i=1}^{N_t} h_i x_{n-i} + n_n \quad (3.2)$$

where  $n_n$  is complex Gaussian noise with zero mean and variance  $N_0$ .

An iterative receiver is illustrated in Fig. 3.2. At the receiver, the received signal is first passed through a soft-in soft-out interference canceller, that is, the inner demodulator. After interference cancellation, we obtain the soft estimates of the transmitted symbols and we can further compute the extrinsic information from these estimates. Using the de-interleaved extrinsic information as the a priori information, the soft-in soft-out MAP decoder performs one-dimensional decoding to get the a posteriori probability and the updated extrinsic information of the transmitted symbol. The extrinsic information is then fed back to the canceller as the a priori information in the next iteration and the a posteriori probability is used to recover the original information bits when convergence is reached.

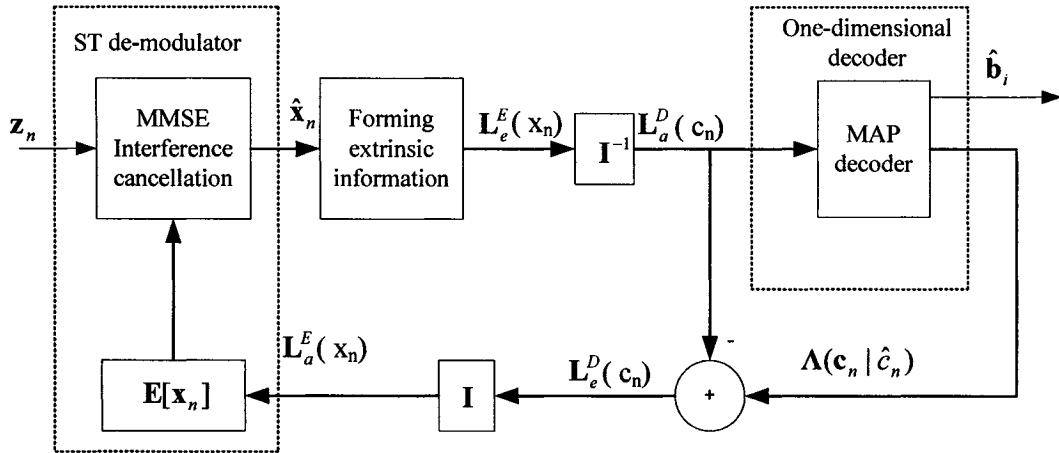


Fig. 3.2. An iterative (turbo) receiver.

In our iterative receiver, we use a MMSE linear filter as the interference canceller to lower the complexity and only the low-complexity scheme based on [32] [33] is considered. BCJR algorithm based on [12] [31] [37] is employed in the Log-MAP

decoder. In the following sections, we will use an 8PSK trellis code [20] as the outer code to introduce the implementation of our iterative receiver.

### 3.2 MMSE Interference Cancellation

In the proposed delay diversity code, multiple antennas are used at the transmitter to increase the diversity order. But at the same time the interference is introduced due to the multipath-like transmission. Thus at the receiver, a linear MMSE interference canceller is employed before decoding.

Consider an MMSE interference canceller described in Fig. 3.2. The input to the canceller is the received signal  $z_n$  and the a priori information  $L_a^E(x_n)$ . The output of the canceller is the estimate of the transmitted symbol  $x_n$  denoted by  $\hat{x}_n$ . Based on symbol estimates, the extrinsic probability  $L_e^E(x_n)$  can be computed and will be used as the a priori information in the later decoding stage.

Assume that the length of the MMSE linear filter is  $N = N_1 + N_2 + 1$  and denote the time-varying coefficient vector of the linear filter by  $\mathbf{f}_n = [f_{-N_1,n}^* \ f_{-N_1+1,n}^* \ \cdots \ f_{N_2,n}^*]^T$ . We further define the vector of the transmitted signal as  $\mathbf{x}_n = [x_{n+N_1} \ x_{n+N_1-1} \ \cdots \ x_{n-N_2-N_1+1}]^T$ , the vector of the received signal  $\mathbf{z}_n = [z_{n+N_1} \ z_{n+N_1-1} \ \cdots \ z_{n-N_2}]^T$ , the vector of noise at the receiver as  $\mathbf{n}_n = [n_{n+N_1} \ n_{n+N_1-1} \ \cdots \ n_{n-N_2}]^T$ . Then from (3.2), the received signal  $\mathbf{z}_n$  can be written as

$$\mathbf{z}_n = \mathbf{H} \mathbf{x}_n + \mathbf{n}_n \quad (3.3)$$

where  $\mathbf{H}$  is the channel matrix of size  $N \times (N + N_1 - 1)$  and is given by



$$\mathbf{H} = \begin{bmatrix} h_1 & \cdots & h_{N_t} & 0 & \cdots & 0 \\ 0 & h_1 & \cdots & h_{N_t} & \cdots & 0 \\ & & \ddots & & & \\ 0 & \cdots & 0 & h_1 & \cdots & h_{N_t} \end{bmatrix}$$

The interference canceller estimates symbol  $x_n$  as

$$\hat{x}_n = E[x_n] + \mathbf{f}_n^H (\mathbf{z}_n - \bar{\mathbf{z}}_n) \quad (3.4)$$

where  $\bar{\mathbf{z}}_n = E[\mathbf{z}_n]$ .

Let  $\bar{x}_n = E[x_n]$  and  $v_n = \text{Cov}(x_n, x_n)$ . Using the LLR outputs from the decoder, they can be found as

$$\begin{aligned} \bar{x}_n &= \sum_{\alpha_i \in \mathcal{S}} \alpha_i * P(x_n = \alpha_i) \\ v_n &= \sum_{\alpha_i \in \mathcal{S}} |\alpha_i|^2 P(x_n = \alpha_i) - |\bar{x}_n|^2 \end{aligned} \quad (3.5)$$

Assuming that the transmitted signal are mutually independent and using the statistics derived from the decoder for all the symbols, the coefficient vector of the interference canceller that minimizes  $E[|x_n - \hat{x}_n|^2]$  can be found as

$$\mathbf{f}_n = v_n (\mathbf{H} \mathbf{D}_n \mathbf{H}^H + N_0 \mathbf{I}_N)^{-1} \mathbf{s} \quad (3.6)$$

where  $\mathbf{D}_n = \text{diag}[v_{n+N_t} \cdots v_n \cdots v_{n-N_t+1}]$  and  $\mathbf{s} = \mathbf{H} [\mathbf{0}_{1 \times (N_2 + N_t - 1)} \ 1 \ \mathbf{0}_{1 \times N_t}]^T$  is the  $(N_2 + N_t)$ -th column of  $\mathbf{H}$ .

Substituting (3.6) into (3.4) we have

$$\hat{x}_n = \bar{x}_n + \mathbf{f}_n^H (\mathbf{z}_n - \mathbf{H} \bar{\mathbf{x}}_n) \quad (3.7)$$

with  $\bar{\mathbf{x}}_n = [\bar{x}_{n+N_t} \cdots \bar{x}_n \cdots \bar{x}_{n-N_t+1}]^T$

Note that in the above derivation, we have used the output LLRs from the decoder for all the symbols. However, to decouple the interference cancellation and decoding, one

shall not assume prior knowledge for the symbol to be estimated [32]-[34]. That is, the expectation and variance of the symbol to be estimated shall be  $\bar{x}_n = 0$  and  $v_n = 1$ . Therefore, the actual coefficient vector of the MMSE interference canceller can be obtained by modifying (3.6) as

$$\begin{aligned}\mathbf{f}'_n &= \mathbf{f}_n |_{v_n=1} = (\mathbf{H}\mathbf{D}_n\mathbf{H}^H + N_0\mathbf{I}_N + (1-v_n)\mathbf{s}\mathbf{s}^H)^{-1}\mathbf{s} \\ &= (\mathbf{\Phi}_n + (1-v_n)\mathbf{s}\mathbf{s}^H)^{-1}\mathbf{s}\end{aligned}\quad (3.8)$$

$\mathbf{f}'_n$  can be further written in a scaled version of  $\mathbf{f}_n$  by using matrix inversion lemma

$$\mathbf{f}'_n = (1 + (1-v_n)\mathbf{f}_n^H\mathbf{s})^{-1}\mathbf{f}_n \quad (3.9)$$

Correspondingly, (3.7) becomes

$$\begin{aligned}\hat{x}_n &= \mathbf{f}'_n{}^H (\mathbf{z}_n - \mathbf{H}\bar{\mathbf{x}}_n + \bar{x}_n\mathbf{s}) \\ &= K_n \mathbf{f}_n^H (\mathbf{z}_n - \mathbf{H}\bar{\mathbf{x}}_n + \bar{x}_n\mathbf{s})\end{aligned}\quad (3.10)$$

where  $K_n = (1 + (1-v_n)\mathbf{f}_n^H\mathbf{s})^{-1}$ .

It is evident from (3.9) that the coefficient vector of the MMSE interference canceller changes from symbol to symbol. This implies a large amount of computational complexity.

To reduce the complexity, we use a constant filter coefficient vector given by

$$\mathbf{f}' = (\mathbf{H}\mathbf{H}^H + N_0\mathbf{I}_N)^{-1}\mathbf{s} \quad (3.11)$$

to replace  $\mathbf{f}'_n$  such that we do not have to compute the coefficient for each time instant during a frame. Actually  $\mathbf{f}'$  is the coefficient in the first iteration with no a priori information available. In such a case,  $v_n=1$ ,  $K_n = 1$  and the coefficient of the filter is the same for  $n=1, 2, \dots, L$ , where  $L$  is the frame length.

Then  $\hat{x}_n$  in (3.10) becomes

$$\hat{x}_n = \mathbf{f}'^H (\mathbf{z}_n - \mathbf{H} \bar{\mathbf{x}}_n + \bar{\mathbf{x}}_n \mathbf{s}) \quad (3.12)$$

Now from the estimate  $\hat{x}_n$ , we compute the extrinsic information  $L_e^E(x_n = \alpha_i)$ , which is the additional information about  $x_n$  gleaned from the interference canceller.

We note the PDFs  $p(\hat{x}_n | x_n = \alpha_i)$ , are Gaussian distribution with mean  $\mu_{n,i}$  and variance  $\sigma_{n,i}^2$  given by

$$\begin{aligned} \mu_{n,i} &= E(\hat{x}_n | x_n = \alpha_i) \\ &= \mathbf{f}'^H (E(\mathbf{z}_n | x_n = \alpha_i) - \mathbf{H} \bar{\mathbf{x}}_n + \bar{\mathbf{x}}_n \mathbf{s}) = \alpha_i \mathbf{f}'^H \mathbf{s} \end{aligned} \quad (3.13)$$

$$\begin{aligned} \sigma_{n,i}^2 &= \text{Cov}(\hat{x}_n, \hat{x}_n | x_n = \alpha_i) \\ &= \mathbf{f}'^H \text{Cov}(\mathbf{z}_n, \mathbf{z}_n | x_n = \alpha_i) \mathbf{f}' \\ &= \mathbf{f}'^H (\Phi_n - v_n \mathbf{s} \mathbf{s}^H) \mathbf{f}' = \sigma_n^2 \end{aligned} \quad (3.14)$$

Replace  $\sigma_{n,i}^2$  with its time-average  $\bar{\sigma}^2 = 1/L (\sum_i \sigma_n^2)$  to simplify the computation, then we get

$$p(\hat{x}_n | x_n = \alpha_i) = \frac{1}{\pi \bar{\sigma}^2} \exp\left(-\frac{|\hat{x}_n - \mu_{n,i}|^2}{\bar{\sigma}^2}\right) \quad (3.15)$$

Thus the extrinsic information is obtained by

$$\begin{aligned} L_e^E(x_n = \alpha_i) &= \ln \frac{p(\hat{x}_n | x_n = \alpha_i)}{p(\hat{x}_n | x_n = \alpha_0)} \\ &= -\frac{|\mu_{n,i} - \hat{x}_n|^2}{\bar{\sigma}^2} + \frac{|\mu_{n,0} - \hat{x}_n|^2}{\bar{\sigma}^2} \end{aligned} \quad (3.16)$$

### 3.3 The Outer Encoding and Decoding

The outer code in the proposed transmission system is investigated in this section. As can be seen in Fig. 3.1 and Fig. 3.2, the outer code is a conventional one-dimensional encoder. At the transmitter, the information bits are encoded and mapped to symbols entering the interleaver; at the receiver, the decoder receives the soft information, i.e., the extrinsic information about the coded symbols from the MMSE canceller to compute the updated extrinsic LLR and recover the information bit.

#### 3.3.1 Encoder Structure

Assume a rate  $k_0/n_0$  code with memory length  $v$ . The input to encoder is a block of information bits,  $b_1, b_2, \dots, b_T$ , the corresponding output is  $c_1, c_2, \dots, c_L$  with  $v$  tail symbols. Note that full diversity is guaranteed regardless of the outer code used. Hence, the main design consideration for the outer code is to increase coding gain. For simplicity, the padding bits are chosen to make the encoder end in state  $\mathbf{0}$ .

Since the implementation of binary convolutional code and BPSK mapping have been addressed extensively in literatures [31] [34] as well as the bit-wise MAP algorithm [35] [38], here we will not discuss them. We choose a bandwidth efficient code, a rate 2/3 8PSK TCM (Trellis Coded Modulation) code as an example to develop a symbol-wise MAP algorithm. It has been showed that in [20], this code can achieve a performance in the fading channel compared to the same complexity code in the AWGN channel.

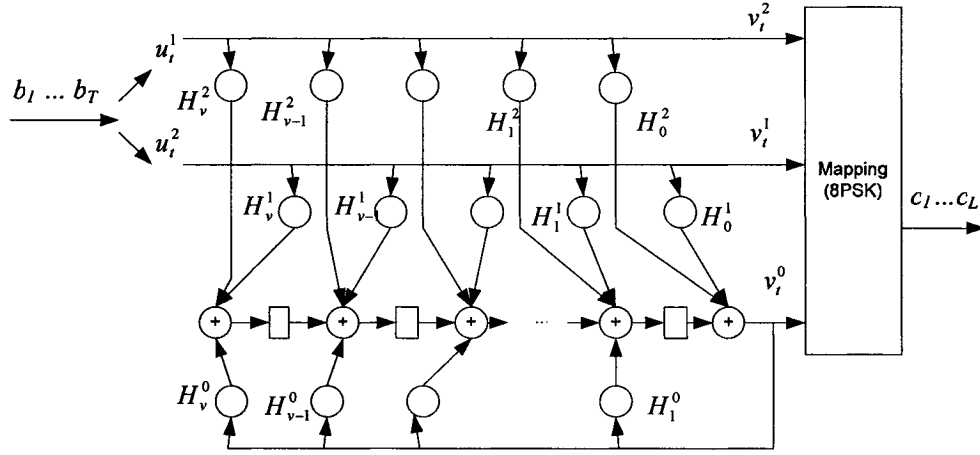


Fig. 3.3. Encoder of rate 2/3 8PSK TCM code.

Our TCM code is generated by a binary recursive systematic code encoder (RSC) followed by a mapper. The structure of the rate 2/3 8PSK encoder is shown in Fig. 3.3. Binary information bits  $b_1, b_2, \dots, b_T$  are first divided into two bit streams  $u_1^1, u_2^1, \dots, u_{T/2}^1$  and  $u_1^2, u_2^2, \dots, u_{T/2}^2$ . The two bit streams can be expressed as the polynomials of delay operator  $D$  as

$$\begin{aligned} u^1(D) &= u_0^1 + u_1^1 D + u_2^1 D^2 + \dots \\ u^2(D) &= u_0^2 + u_1^2 D + u_2^2 D^2 + \dots \end{aligned} \quad (3.17)$$

The generator polynomials of the encoder are given as

$$\begin{aligned} H^0(D) &= H_0^0 + H_1^0 D + \dots + H_v^0 D^v \\ H^1(D) &= H_0^1 + H_1^1 D + \dots + H_v^1 D^v \\ H^2(D) &= H_0^2 + H_1^2 D + \dots + H_v^2 D^v \end{aligned} \quad (3.18)$$

where  $H_0^0 = 1$  and  $v$  is the memory length of the encoder. Usually we write  $H^0(D)$ ,  $H^1(D)$  and  $H^2(D)$  in their octal forms, that is,  $H^0(D) = D^4 + D + 1 = (010011) = 23$ ,  $H^1(D) = D^2 = (000100) = 04$ ,  $H^2(D) = D^3 + D^2 + D = (001110) = 16$ .

The information bits and the output of the RSC encoder form a sequence of 3-tuples  $v_1^2 v_1^1 v_1^0, v_2^2 v_2^1 v_2^0, \dots, v_L^2 v_L^1 v_L^0$ , then these 3-tuples are mapped into 8PSK symbols sequence  $c_1, c_2, \dots, c_L$ , i.e., the output of the TCM encoder. The alphabet of 8PSK is given in table 3.1.

Table 3.1 8PSK constellation

	0	1	2	3	4	5	6	7
$v^2 v^1 v^0$	000	001	010	011	100	101	110	111
$s_i$	1	$(1+i)/\sqrt{2}$	i	$(-1+i)/\sqrt{2}$	-1	$-(1+i)/\sqrt{2}$	-i	$(1-i)/\sqrt{2}$

Denote the input to the encoder at time  $n$  by  $u_n^2 u_n^1$ , corresponding output of the TCM encoder by  $c_n$ , and the state of the encoder at time  $n$  by  $S_n$ . In the following subsections, we will use  $c_n(S', S)$  to denote the output symbol at time  $n$  caused by state transition from  $S_{n-1} = S'$  to  $S_n = S$ . To simplify the design and derivations, we assume the encoder begin with state 0 and end in state 0.

### 3.3.2 MAP Decoder

Maximum the a posteriori (MAP) algorithm was first presented in 1974 by Bahl, Cocke, Jelinik and Raviv [37], and it is also called as the BCJR algorithm in the memory of its discoverers. With reference to decoding of noisy coded sequences, the MAP decoder computes the a posteriori probability of each data bit or symbol with the a priori probability of the data, and makes decision by choosing the data bit or symbol that corresponds to the maximum the a posteriori probability.

As discussed in the above, the decoder takes the de-interleaved extrinsic information from the MMSE canceller as input to compute the a posteriori probability and extrinsic information. The extrinsic information is interleaved and fed back to the SISO canceller and the a posteriori LLR is used to compute the estimate  $\hat{b}_i$  of the information bit  $b_i$ .

Let  $S^i$  denote the set of state pairs  $(S', S)$  such that the output code symbol  $c_n$  equals to the constellation point (symbol)  $s_i$ , and  $S^0$  denote the set of state pairs  $(S', S)$  such that the output code symbol  $c_n$  equals to the constellation symbol  $s_0$ . Denote the a priori information in one frame that the decoder receives by  $\mathbf{L}_a^D$ . The a posteriori LLR of the symbol being  $s_i$  at time  $n$  is given by [12]

$$\begin{aligned} \Lambda(c_n = s_i) &= \ln \frac{\Pr(c_n = s_i | \mathbf{L}_a^D)}{\Pr(c_n = s_0 | \mathbf{L}_a^D)} \\ &= \ln \frac{\sum_{S^i} \alpha_{n-1}(S') \beta_n(S) \gamma_n(S', S)}{\sum_{S^0} \alpha_{n-1}(S') \beta_n(S) \gamma_n(S', S)} \end{aligned} \quad (3.19)$$

where  $\alpha_n(S)$  and  $\beta_n(S)$  are the forward recursive variables and the backward recursive variables defined as

$$\alpha_n(S) = \sum_{s'} \alpha_{n-1}(S') \gamma_n(S', S), \quad n=1, 2, \dots, L \quad (3.20)$$

$$\beta_n(S) = \sum_{s'} \beta_{n+1}(S') \gamma_{n+1}(S, S'), \quad n=L-1, L-2, \dots, 0 \quad (3.21)$$

with initialized value  $\alpha_0(\mathbf{0}) = 1$ ,  $\alpha_0(S \neq \mathbf{0}) = 0$ ;  $\beta_L(\mathbf{0}) = 1$ ,  $\beta_L(S \neq \mathbf{0}) = 0$ . In the above recursive equations,  $\gamma_n(S', S)$  is the probability that at time  $n$ , the output is  $c_n = s_i$  when state transition  $(S', S)$  occurs. The  $\gamma_n(S', S)$  is calculated as

$$\gamma_n(S', S) = \begin{cases} \frac{\Pr(c_n = s_i)}{\Pr(c_n = s_0)} \exp\left(-\frac{|\hat{c}_n - s_i|^2}{\bar{\sigma}^2}\right) & (S', S) \in S^i \\ 0 & \text{otherwise} \end{cases} \quad (3.22)$$

where  $\hat{c}_n$  is de-interleaved  $\hat{x}_n$ . Note that in (3.20) and (3.21) the summation is over all states  $S'$  where the transition  $(S', S)$  exists

From above equations, we see that  $\alpha_n(S)$  and  $\beta_n(S)$  drop toward zero exponentially. In order to obtain a numerically stable algorithm, we apply scaling on these parameters as the computation proceeds [34].

Let  $\tilde{\alpha}_n(s)$  denote scaled version of  $\alpha_n(s)$ . Initially,  $\alpha_1(S)$  is computed according to equation (3.20), and we set  $\hat{\alpha}_1(S) = \alpha_1(S)$ ,  $\tilde{\alpha}_1(S) = q_1 \hat{\alpha}_1(S)$  and  $q_1 = 1/\sum_s \hat{\alpha}_1(S)$ .

For  $n \geq 2$ ,  $\tilde{\alpha}_n(S)$  is computed as

$$\begin{aligned} \hat{\alpha}_n(S) &= \sum_{s'} \tilde{\alpha}_{n-1}(S') \gamma_n(S', S) \\ \tilde{\alpha}_n(S) &= q_n \hat{\alpha}_n(S), \text{ with } q_n = 1/\sum_s \hat{\alpha}_n(S) \end{aligned} \quad (3.23)$$

Similarly, let  $\tilde{\beta}_n(S)$  denote the scaled version of  $\beta_n(S)$ . Initially,  $\beta_{L-1}(S)$  is computed according to equation (3.21), and we set



$$\hat{\beta}_{L-1}(S) = \beta_{L-1}(S), \quad \tilde{\beta}_{L-1}(S) = q_{L-1} \hat{\beta}_{L-1}(S) \quad (3.24)$$

For  $n \leq L-2$ ,  $\tilde{\beta}_n(S)$  is computed as

$$\begin{aligned} \hat{\beta}_n(S) &= \sum_{s'} \tilde{\beta}_{n+1}(S') \gamma_{n+1}(S, S') \\ \tilde{\beta}_n(S) &= q_n \hat{\beta}_n(S) \end{aligned} \quad (3.25)$$

From  $L_a^D(c_n = s_i) = \ln \frac{\Pr(c_n = s_i)}{\Pr(c_n = s_0)}$ , we have  $\frac{\Pr(c_n = s_i)}{\Pr(c_n = s_0)} = \exp(L_a^D(c_n = s_i))$ , then

(3.22) can be written as

$$\gamma_n(S', S) = \begin{cases} \exp\{L_a^D(c_n = s_i) - \frac{|\hat{c}_n - s_i|^2}{\bar{\sigma}^2}\} & (S', S) \in S^i \\ 0 & \text{otherwise} \end{cases} \quad (3.26)$$

We note that the a posteriori LLR  $\wedge(c_n = s_i)$  can be expressed as the summation of two parts as

$$\wedge(c_n = s_i) = L_e^D(c_n = s_i) + L_a^D(c_n = s_i) \quad (3.27)$$

where  $L_a^D(c_n = s_i)$  is the a priori information delivered by inner code, the SISO canceller and  $L_e^D(c_n = s_i)$  is the extrinsic information obtained from the decoder which does not depend on the a priori information.

Then from (3.27) we get the extrinsic information as

$$L_e^D(c_n = s_i) = \wedge(c_n = s_i) - L_a^D(c_n = s_i) \quad (3.28)$$

### 3.4 Simulation Results

In the above, we have introduced the delay diversity transmission scheme and the MMSE turbo receiver. In this subsection, the performance of the joint cancellation/decoding receiver is presented and compared to the receiver with the optimal trellis decoding. The delay diversity transmitter and the turbo receiver are shown as in Fig. 3.1 and Fig. 3.2, respectively. The 8PSK TCM encoder illustrated in Fig.3.3 is used as the outer code. The 8PSK constellation is shown in table 3.1 with unit energy.

The MMSE linear filter length  $N$  is set to be equal to the number of the transmit antennas,  $N_t$ . A random interleaver is used. The code rate  $R$  is  $2/3$  and the block size of the information bits  $L$  is 64. The maximum number of iterations is 6 for all simulations.

First we consider the systems with encoder memory  $\nu = 3$  where the encoder polynomials are  $H^0(D)=11$ ,  $H^1(D)=02$  and  $H^2(D)=04$ . The number of antennas  $N_t$  is set to be 2, 3 and 4. Then we change the encoder memory to  $\nu = 4$  with the encoder polynomials being  $H^0(D)=23$ ,  $H^1(D)=04$  and  $H^2(D)=16$ .

In Figs. 3.4 - 3.9, we present the simulation results with various simulation settings. All the frame error rates (FER) curves are plotted against  $E_b/N_0$  where  $E_b$  is the average energy per information bit at the receiver and  $N_0$  is the PSD of Gaussian noise.

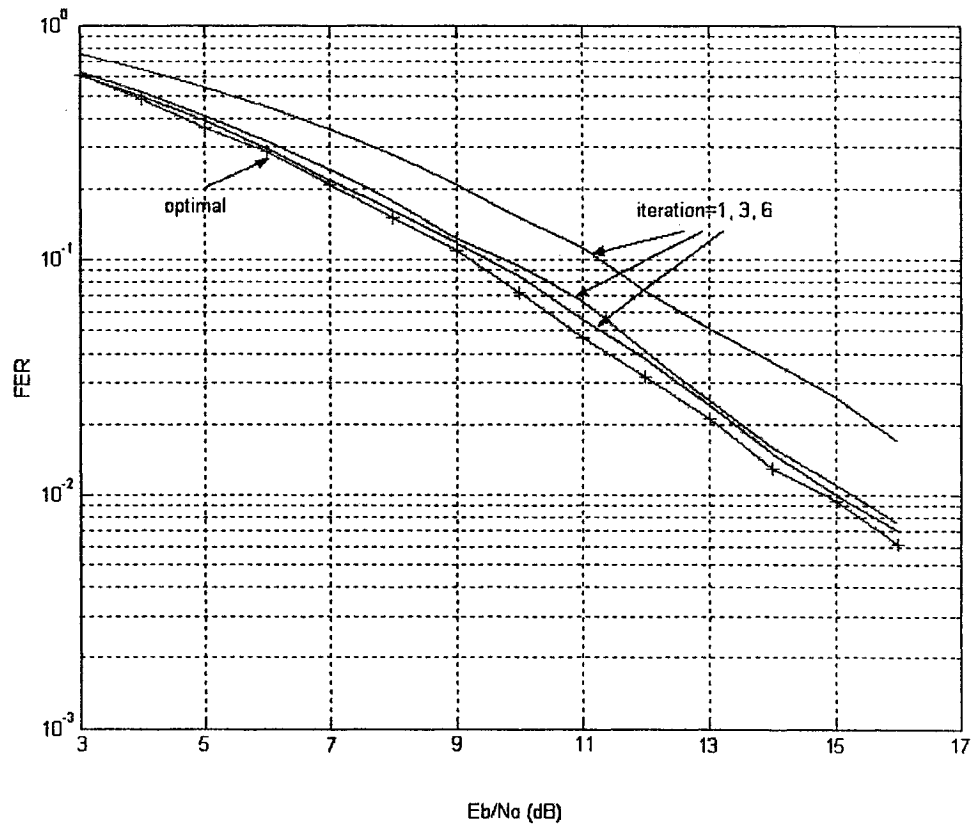


Fig. 3.4. Performance comparison of the turbo receiver and the optimal receiver in a system with 2 transmit antennas and encoder memory 3.

Fig. 3.4 presents the FERs of the joint cancellation/decoding receiver and the optimal receiver in a system with two transmit antennas and an outer encoder of memory 3. It can be seen that the FER performance has been improved greatly from  $1.7 \times 10^{-2}$  in the first iteration to  $7 \times 10^{-3}$  in the sixth iteration at  $E_b/N_0 = 16dB$ . It is also observed that when the number of the iterations is 6, the performance of the joint cancellation/decoding receiver is very close to that of the optimal trellis decoding receiver. The difference gap is less than  $0.2dB$ .

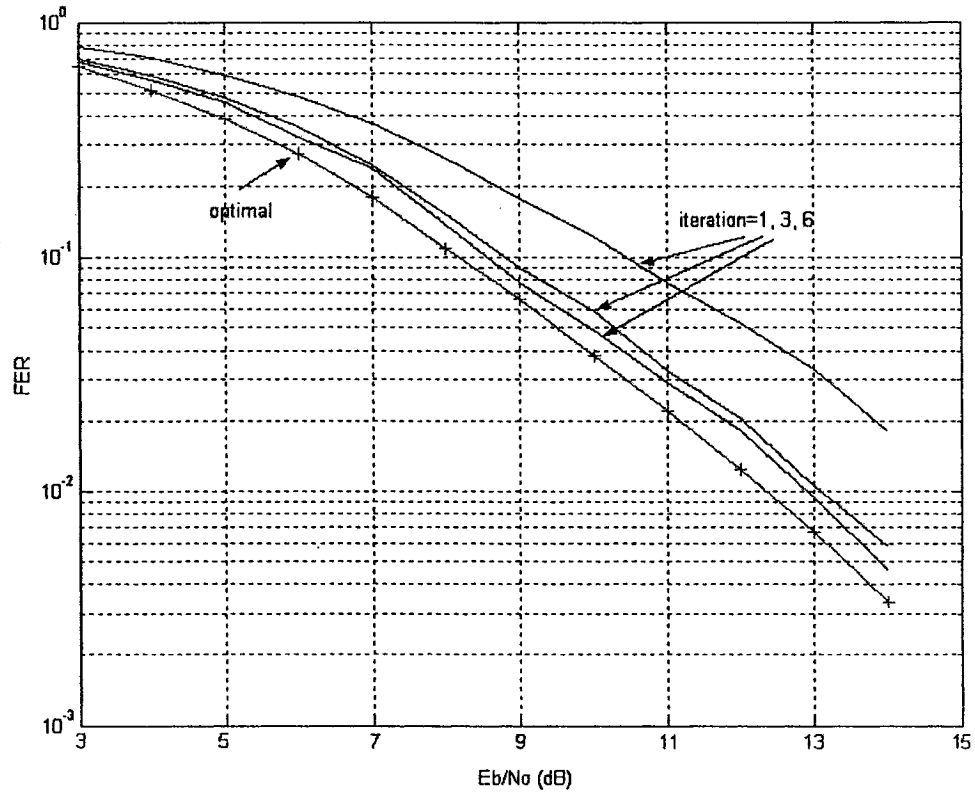


Fig. 3.5. Performance comparison of the turbo receiver and the optimal receiver in a system with 3 transmit antennas and encoder memory 3.

Fig. 3.5 illustrates the FER performance of the joint cancellation/decoding receiver and the optimal receiver in a system with three transmit antennas and an outer encoder of memory 3. We can see that there is only  $0.4dB$  difference between the joint cancellation/decoding (turbo) receiver and the optimal trellis decoding at  $FER = 6.8 \times 10^{-3}$  while the number of the iterations is 6 for the turbo receiver. However, even with 6 iterations, the joint iterative receiver has a much lower computational complexity.

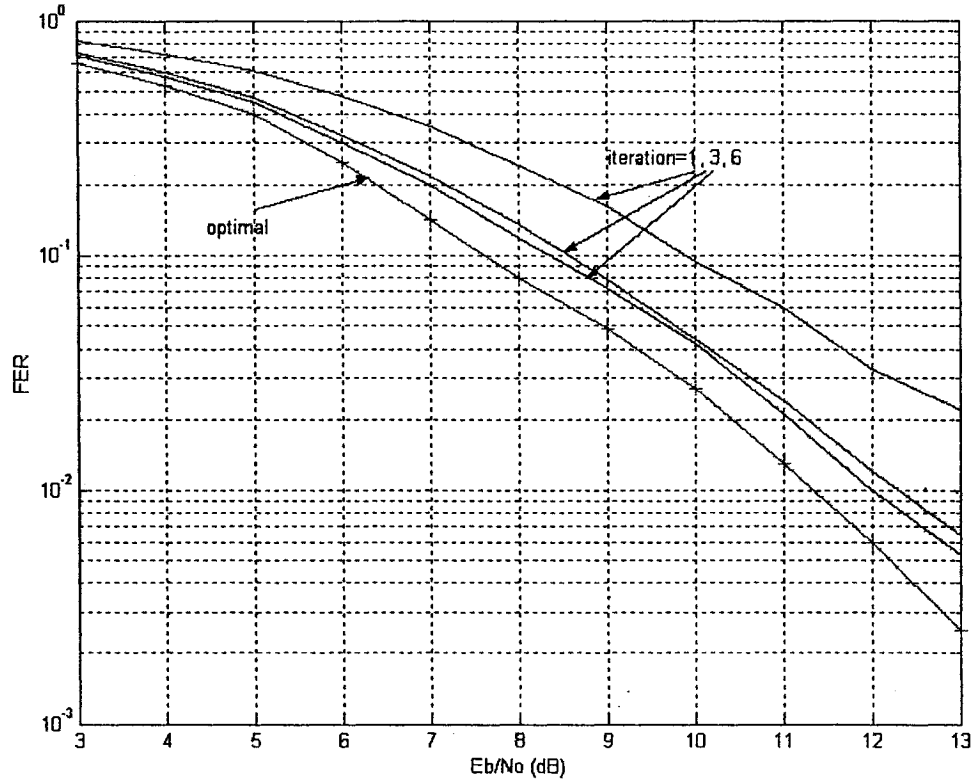


Fig. 3.6. Performance comparison of the turbo receiver and the optimal receiver in a system with 4 transmit antennas and encode memory 3.

Fig. 3.6 shows the simulation results of the joint cancellation/decoding receiver and the optimal receiver in a system with four transmit antennas and an outer encoder of memory 3. It is observed that at  $FER = 6 \times 10^{-3}$ , when the number of the iterations is 6, the performance of the joint cancellation/decoding receiver is  $0.8dB$  lower than the optimal trellis decoding receiver, which is larger than the cases when the number of transmit antennas is 2 and 3. Furthermore we note that the performance in terms of FER for 4 transmit antennas compared with 2 and 3 transmit antennas has been improved around 3.5 and  $1dB$  respectively at  $7 \times 10^{-3}$  for our proposed turbo receiver scheme.

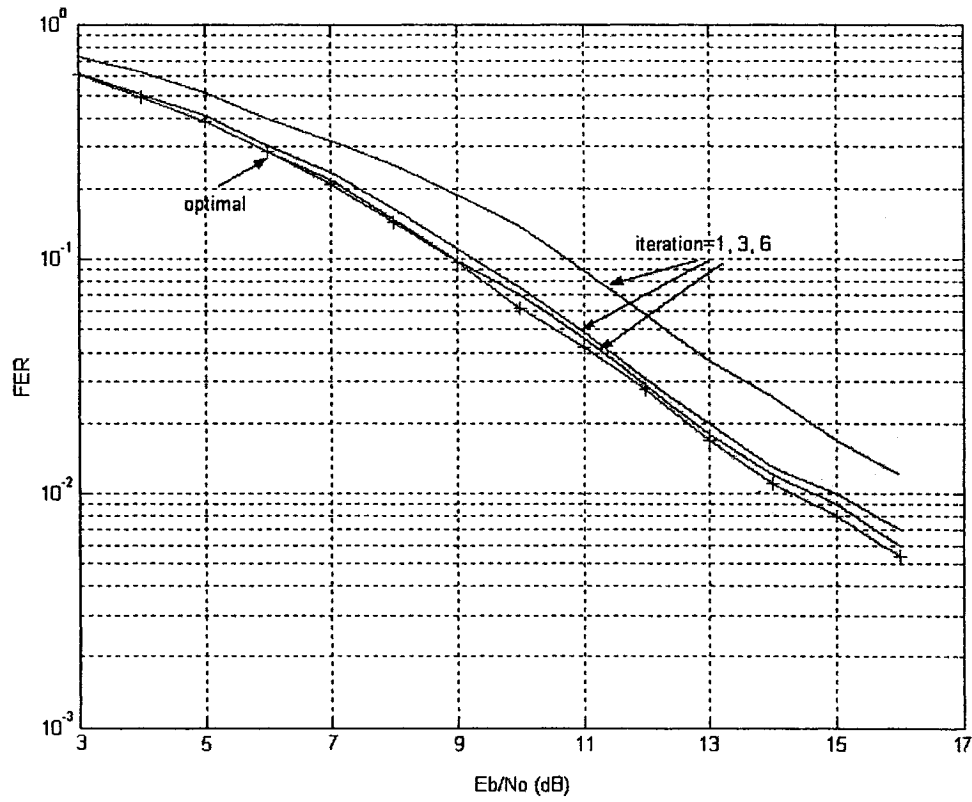


Fig. 3.7. Performance comparison of the turbo receiver and the optimal receiver in a system with 2 transmit antennas and encoder memory 4.

Fig. 3.7 shows the simulation results of the joint cancellation/decoding receiver and the optimal receiver in a system with two transmit antennas and an outer encoder of memory 4. We note that the FER performance of the turbo receiver in the sixth iteration is very near that of the optimal trellis decoding receiver: at low  $E_b/N_0$ , the curve of the turbo receiver is almost reaching the curve of the optimal trellis decoding receiver; at high  $E_b/N_0$ , the performance of the turbo receiver is only 0.1 ~ 0.2dB lower than the optimal trellis decoding receiver.

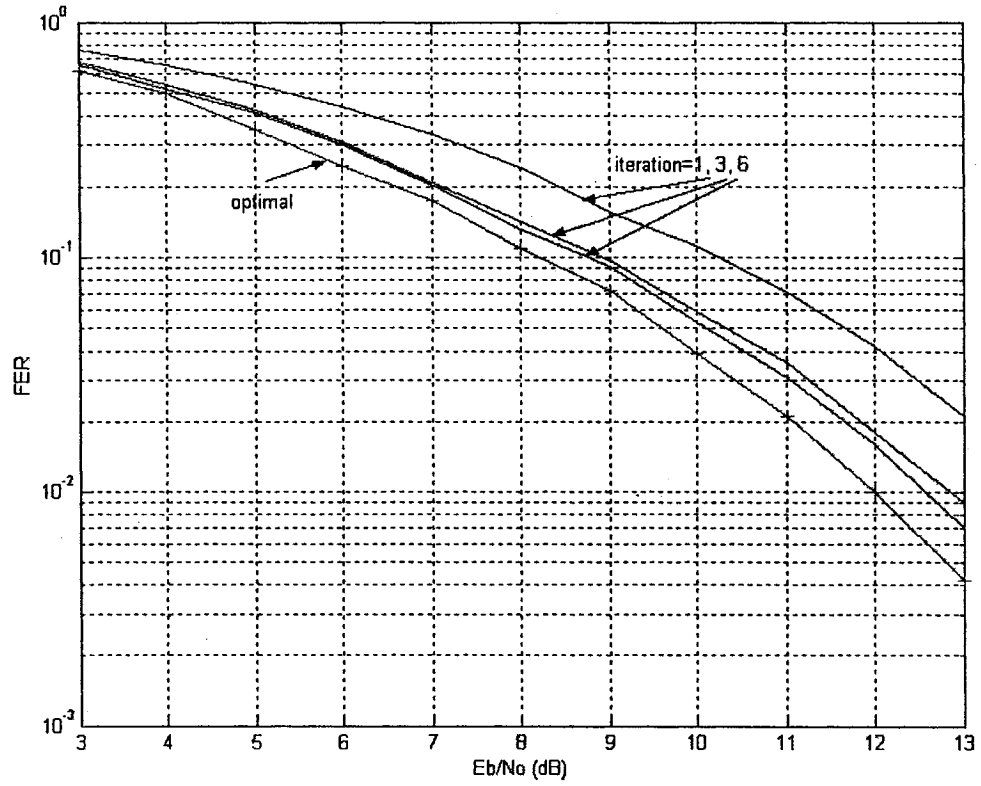


Fig. 3.8. Performance comparison of the turbo receiver and the optimal receiver in a system with 3 transmit antennas and encoder memory 4.

Fig. 3.8 illustrates the simulation results of the joint cancellation/decoding receiver and the optimal receiver in a system with three transmit antennas and an outer encoder of memory 4. It can be seen that with the number of iterations being six, when  $E_b/N_0$  is 3 ~5dB, the performance of the turbo receiver is 0.1dB lower than the optimal trellis decoding receiver; when  $E_b/N_0$  is 5 ~8dB, the performance of the turbo receiver is 0.3 ~0.5dB lower than the optimal trellis decoding receiver; at high  $E_b/N_0$ , the performance of the turbo receiver is 0.6dB lower than the optimal trellis decoding receiver.

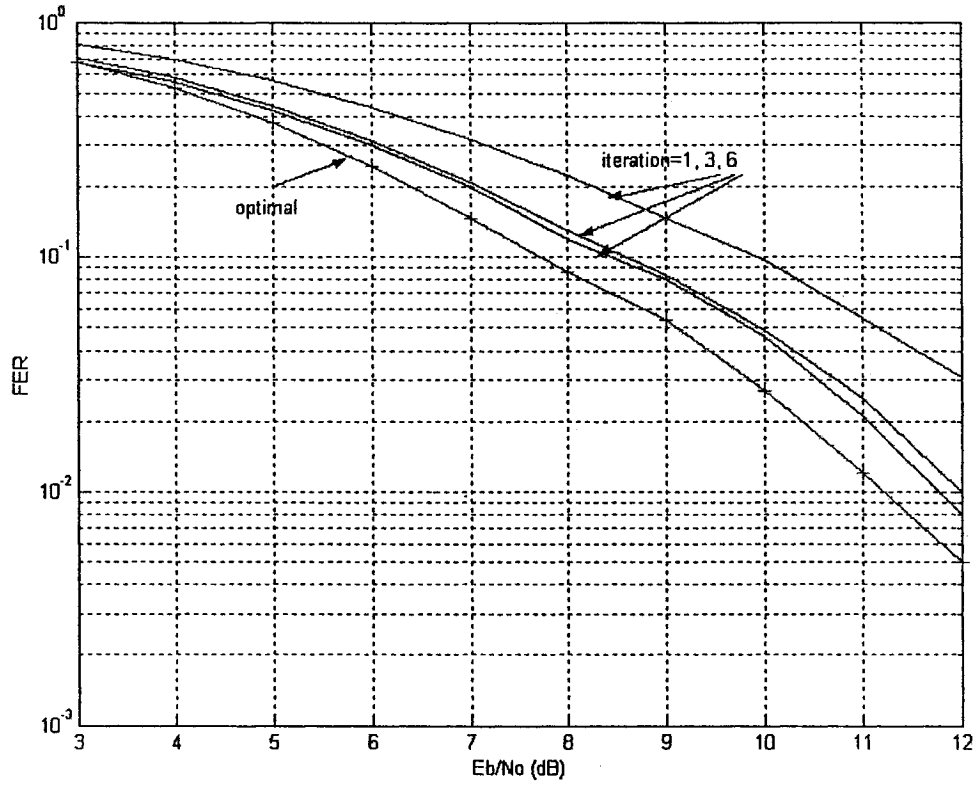


Fig. 3.9. Performance comparison of the turbo receiver and the optimal receiver in a system with 4 transmit antennas and outer code memory 4.

Fig. 3.9 illustrates the simulation results of the joint cancellation/decoding receiver and the optimal receiver in a system with four transmit antennas and an outer encoder of memory 4. It can be seen that at high  $E_b/N_0$ , when the number of iterations is six, the performance of the turbo receiver is 0.6dB lower than the optimal trellis decoding receiver. Also we observe that the performance in terms of FER for 4 transmit antennas compared with 2 and 3 transmit antennas has been improved around 3.5dB and 1dB respectively at  $7 \times 10^{-3}$  for our proposed turbo receiver scheme.

From the above simulations, we can see that in a system with two transmit antennas, the turbo receiver can achieve the performance very close to that of the optimal receiver



while the complexity is half of the latter. For the systems with three and four transmit antennas, there is  $0.4 \sim 0.8dB$  performance gap between the iterative receiver and the optimal receiver at high  $E_b/N_0$ . However, the complexity is reduced to  $6/2^{3(N_t-1)}$  of the optimal receiver when 6 iterations are used, where  $s$  is the number of states of the code. And we also note that the curve converges faster in the system with two transmission antennas than in the system with three and four transmission antennas.

### 3.5 Conclusions

In this chapter, delay-diversity MIMO transmission was investigated under the general framework. In the general framework, the repeat and delay operation was viewed as the inner space-time modulation, which was serially concatenated with a one-dimensional outer code to form the proposed delay-diversity codes. For the delay-diversity codes, a linear turbo iterative receiver consisting of MMSE interference cancellation and MAP decoding was developed.

The simulation results have shown that the iterative receiver using joint MMSE interference cancellation and MAP decoding can achieve performance close to that of the optimal trellis receiver using Viterbi decoding algorithm but with a significantly reduced complexity.

## Chapter 4

### New MIMO Transmission Design

In space-time coding, the encoding is performed in both spatial domain and temporal domain. The two-dimensional code design is much more complicated than that of the conventional one-dimensional code for SISO systems. This is mainly due to the difficulties in determining the coding gain and diversity gain of a given space-time code.

While if a MIMO transmission scheme can be modeled by a general framework as we described in chapter 3, that is, a one-dimensional encoder followed by a space-time modulator/multiplexer with an interleaver between the two components, the design will become much easier. In such a case, full diversity can be readily obtained due to the use of one-dimensional outer code. This allows the design of inner space-time modulator/multiplexer to be focused on tradeoff between data rate and performance in terms of coding gain. A space-time modulator is in fact a translator of  $k_0$  inputs to  $n_0$  symbols spreading on a two-dimensional (temporal and spatial) grid. Based on this general framework, a good space-time modulator should satisfy the following requirements:

- 1) Preserve the channel capacity and facilitate the harvest of diversity when used with an outer code. The former makes it possible to transmit at the highest data rate while the latter guarantees a good performance.
- 2) Provide a flexible tradeoff between performance (reliability) and data rate (capacity).
- 3) Allow an implementation that involves low or moderate computational complexity.

In this chapter, we develop a new multi-layered space-time modulator that is suitable to be used in conjunction with an outer SISO code. The proposed space-time modulation includes two important processes: *Circulant Framing* (CF) and spatial multiplexing. The two modulation processes together allow a full range of tradeoff between performance and data rate. In addition, it allows the use of a turbo iterative receiver to reduce the implementation complexity.

The chapter is organized as follows. First we introduce the system model including circulant framing and spatial multiplexing. Then the multi-layered MIMO transmission and its MMSE iterative receiver are studied. In the end, simulation results are given to compare the proposed multi-layered MIMO transmission scheme with D-Blast.

## 4.1 Description of the System

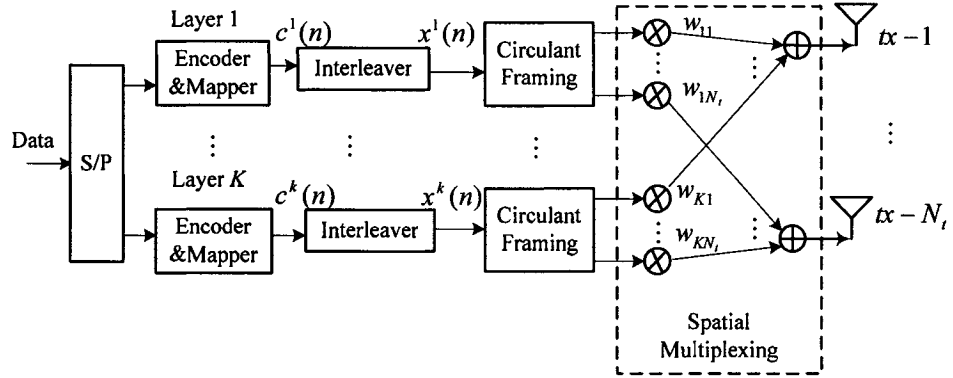


Fig. 4.1 Block diagram of a multi-layered MIMO transmitter.

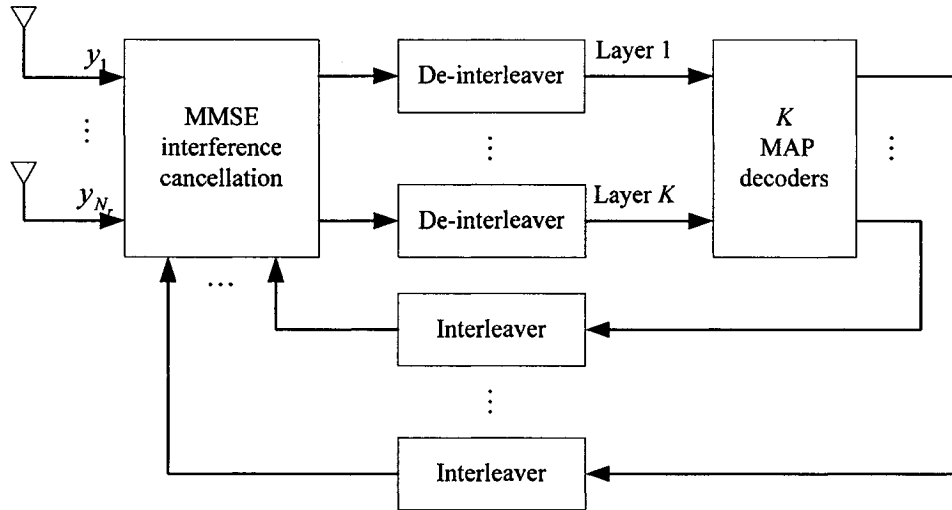


Fig. 4.2 A turbo receiver for multi-layered MIMO transmission.

The proposed space-time modulator is a multi-layered modulation scheme. Consider a system with  $N_t$  transmit and  $N_r$  receive antennas. The transmitter and the receiver are shown as in Fig. 4.1 and Fig. 4.2 respectively. At the transmitter, the data bits enter a serial/parallel converter and are de-multiplexed into  $K$  streams or layers. Each data stream

is encoded, mapped, interleaved independently and then enters  $K$  different CF modules. The output of each CF is a matrix with  $N_t$  rows corresponding to  $N_t$  antennas. The  $K$  output matrices are then linearly combined to obtain one space-time matrix and transmitted over the  $N_t$  transmission antennas. Below, we first describe the proposed mapping (CF) and then linear combining (spatial multiplexing).

#### 4.1.1 Circulant Framing

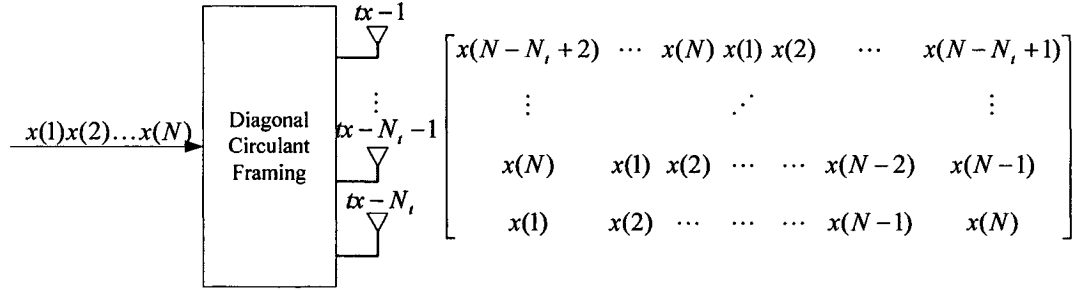


Fig. 4.3 Circulant framing.

CF is a linear space-time mapping that arranges  $N$  input data symbols on an  $N_t$ -by- $N$  space-time grid with each symbol repeating on a diagonal as shown in Fig. 4.3. If the input to the CF is  $x(1), x(2), \dots, x(N)$ , then the data stream to be transmitted on the  $N_t$ -th antenna is  $x(1), x(2), \dots, x(N)$  and the data stream on the  $i$ -th antenna is  $x(N - N_t + i + 1), x(N - N_t + i + 2), \dots, x(N - N_t + i)$ . In other word, the symbol sequence to be transmitted over the  $i$ -th antenna is the circulant shift to the right of the sequence over  $N_t$ -th antenna by  $N_t - i$ . In chapter 3 we stated that the delay-diversity transmission can be seen as a ST modulator. It is therefore interesting to compare CF

with the delay-diversity modulation scheme. In delay-diversity modulation, different delayed versions of the same data stream are transmitted over different antennas. Hence, an input of  $N$  symbols will be mapped to an  $(N_t \text{-by- } N + N_t - 1)$  space-time matrix. That is, the delay-diversity modulation is less efficient in bandwidth.

In general, CF alone does not guarantee full diversity. For instance, the rank of the difference matrix associated with two input streams to the space-time modulator  $\mathbf{x}$  and  $\hat{\mathbf{x}}$  with  $x(i) - \hat{x}(i) = x(j) - \hat{x}(j)$  for any  $i$  and  $j$  is 1. However, as discussed before, when outer code are used, the diversity gain of the inner space-time modulator alone does not actually reflect the performance of the overall system. If the interleaver ensures that any two symbols belong to one modulation block are sufficiently apart at the output of the outer code, then full diversity can still be obtained even for the above case. Even in rare cases where full diversity can not be guaranteed for a few codeword pairs, the use of outer coding and interleaving will ensure very large coding gains of the codeword pairs. To demonstrate this, suppose the effective length of the outer code is  $l$  [39] [40] and consider two codewords whose associated input to the CF modulator at one frame have a constant difference, then the two codewords must differ at least  $lN$  symbols and the Euclidean distance is at least  $lNd_{min}$  with  $d_{min}$  as the minimum distance between two symbols. In summary, if the outer code has a moderate large effective length, detection errors will mostly occur between two codewords that differ exactly at  $l$  encoded consecutive symbols. After interleaving, these  $l$  symbols will be assigned to  $l$  different modulation frames. Hence, the inner space-time modulation shall optimize the performance when two different input streams for one modulation block differ at only

one symbol. In such a case, the diversity gain with CF is  $N_t N_r$  and the overall coding gain is

$$G = \left( \sum_{t=1}^I |c(t) - \hat{c}(t)|^2 \right) / N_t \quad (4.1)$$

where  $c(t)$  and  $\hat{c}(t)$  are the  $t$ -th symbols of the output of the encoder in which the two codewords differ.

With CF, the transmission rate is one symbol per channel use. It can be readily checked that this leads to a loss in channel capacity and the loss grows as the number of transmit and/or receive antennas grows. To remedy this problem, we employ a spatial multiplexing scheme as will be described below.

#### 4.1.2 Spatial Multiplexing

To achieve spatial multiplexing gain, output matrices of the  $K$  CF modules, one for each layer, are linearly combined to form a space-time matrix of size  $N_t \times N$  to provide a transmission rate  $K$  symbols per channel use. Denote  $\mathbf{M}_k \in \mathbb{C}^{N_t \times N}$  as the CF output of layer  $k$ , then the combined matrix is

$$\mathbf{M} = \sum_{k=1}^K \mathbf{W}_k \mathbf{M}_k \quad (4.2)$$

where  $\mathbf{W}_k$  is a diagonal matrix of size  $N_t$ . Denote  $\mathbf{w}_k = \text{diag}(\mathbf{W}_k)$  as the vector collecting the diagonal entries of  $\mathbf{W}_k$ . Since the CF modulated data of layer  $k$  designated to antenna  $i$  is multiplied by the  $i$ -th entry of  $\mathbf{w}_k$  before combined with data from other layers,  $\mathbf{w}_k$  can be regarded as the spatial weight vector for layer  $k$ .

To ensure equal transmit power among layers, one can fix the  $L_2$ -norm of  $\mathbf{w}_k$  as

$$\|\mathbf{w}_k\|^2 = 1 \quad \text{for } k = 1, 2, \dots, K \quad (4.3)$$

Ideally, the weight vector shall be chosen to

- 1) maximize the coding gain of the code, and
- 2) preserve the channel capacity for a certain value of  $K$ .

Apparently, the coding gain of the overall system depends not only on the weight vector but also on the outer coder and symbol constellation. Hence, maximizing the coding gain of the overall system is rather difficult. Instead, we choose weight vectors to maximize the coding gain for codeword pairs that differ by at most one symbol per CF modulation block. This is reasonable as we discussed before. Suppose the  $l$  different symbols of the two codewords associated with layer  $k$  are  $c(1), c(2), \dots, c(l)$  and  $\hat{c}(1), \hat{c}(2), \dots, \hat{c}(l)$ , respectively. Since the  $l$  symbols will be sent to different CF modulation blocks, the coding gain of the pair is

$$G = \frac{1}{K} \left( \prod_{i=1}^{N_t} \left[ \sum_{t=1}^l w_{ki} |c(t) - \hat{c}(t)|^2 \right] \right)^{1/N_t} \quad (4.4)$$

where the factor  $1/K$  is introduced due to the fact that the transmit power of each layer is only  $1/K$  of the total transmit power.

With the constraint in (4.3), the maximizer of the (4.4) apparently satisfies

$$|w_{ki}| = \frac{1}{\sqrt{N_t}} \quad k = 1, 2, \dots, K \text{ and } i = 1, 2, \dots, N_t \quad (4.5)$$



## 4.2 Multi-layered MIMO Space-time Modulator

Now we study the channel capacity without limiting on the outer encoder and symbol constellation. As such, we need only consider one modulation block.

Let  $x^k(n)$  be the  $n$ -th input to the CF module of the  $k$ -th layer. The contribution from the  $k$ -th layer to the  $j$ -th receive antenna at time instant  $n$  denoted by  $y_j^k(n)$  for  $n = 1, 2, \dots, N$  can be written as

$$\begin{aligned} y_j^k(n) &= \sum_{i=1}^{N_t} x^k([n - N_t + i]_N) w_{ki} h_{ji} \\ &= [\tilde{h}_{j1}^k \ \tilde{h}_{j2}^k \ \dots \ \tilde{h}_{jN_t}^k] \begin{bmatrix} x^k([n - N_t + 1]_N) \\ x^k([n - N_t + 2]_N) \\ \vdots \\ x^k([n]_N) \end{bmatrix} \end{aligned} \quad (4.6)$$

where  $\tilde{h}_{ji}^k = h_{ji} w_{ki}$  and

$$[m]_N = \begin{cases} m & \text{for } 1 \leq m \leq N \\ N + m & \text{for } m < 1 \end{cases}$$

With  $x^k(n)$  and  $y_j^k(n)$ , we define vector  $\tilde{\mathbf{x}}^k$  and  $\tilde{\mathbf{y}}_j^k$  as

$$\tilde{\mathbf{x}}^k = [x^k(N - N_t + 1) \ x^k(N - N_t + 2) \ \dots \ x^k(1) x^k(2) \ \dots x^k(N - N_t)]^T$$

$$\tilde{\mathbf{y}}_j^k = [y_j^k(1) \ y_j^k(2) \ \dots \ y_j^k(N)]^T$$

By (4.6) we can write  $\tilde{\mathbf{y}}_j^k$  as

$$\tilde{\mathbf{y}}_j^k = \tilde{\mathbf{H}}_j^k \tilde{\mathbf{x}}^k \quad (4.7)$$

where  $\tilde{\mathbf{H}}_j^k$  is circulant and is given by

$$\tilde{\mathbf{H}}_j^k = \begin{bmatrix} \tilde{h}_{j1}^k & \tilde{h}_{j2}^k & \cdots & \tilde{h}_{jN_t}^k \\ & \tilde{h}_{j1}^k & \tilde{h}_{j2}^k & \cdots & \tilde{h}_{jN_t}^k \\ & & \ddots & \ddots & \ddots & \ddots \\ & & & \tilde{h}_{j1}^k & & \tilde{h}_{jN_t}^k \\ \tilde{h}_{jN_t}^k & & & & \ddots & \vdots \\ \vdots & \ddots & & & & \ddots & \vdots \\ \tilde{h}_{j2}^k & & \tilde{h}_{jN_t}^k & & & & \tilde{h}_{j1}^k \end{bmatrix}$$

Denote  $y_j(n)$  as the actual received signal at the  $j$ -th receive antenna and collect the  $N$  received signal from a modulation block in a vector as

$$\begin{aligned} \tilde{\mathbf{y}}_j &= [y_j(1) \ y_j(2) \ \cdots \ y_j(N)]^T \\ &= \sum_{k=1}^K \tilde{\mathbf{y}}_j^k + \mathbf{n}_j \\ &= \underbrace{[\tilde{\mathbf{H}}_j^1 \ \tilde{\mathbf{H}}_j^2 \ \cdots \ \tilde{\mathbf{H}}_j^K]}_{N \times KN} \begin{bmatrix} \tilde{\mathbf{x}}^1 \\ \tilde{\mathbf{x}}^2 \\ \vdots \\ \tilde{\mathbf{x}}^K \end{bmatrix} + \mathbf{n}_j = \tilde{\mathbf{H}}_j \tilde{\mathbf{x}} + \mathbf{n}_j \end{aligned} \quad (4.8)$$

where  $\mathbf{n}_j$  is the complex Gaussian noise term. Hence, the collection of the received signals at all the  $N_r$  antennas can be written as

$$\begin{aligned} \tilde{\mathbf{y}} &= [\tilde{\mathbf{y}}_1 \ \tilde{\mathbf{y}}_2 \ \cdots \ \tilde{\mathbf{y}}_{N_r}]^T \\ &= \begin{bmatrix} \tilde{\mathbf{H}}_1 \\ \tilde{\mathbf{H}}_2 \\ \vdots \\ \tilde{\mathbf{H}}_{N_r} \end{bmatrix} \tilde{\mathbf{x}} + \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \vdots \\ \mathbf{n}_{N_r} \end{bmatrix} = \tilde{\mathbf{H}} \tilde{\mathbf{x}} + \mathbf{n} \end{aligned} \quad (4.9)$$

In order for the convenience of the later development, we take the  $K$  transmitted symbols from the  $K$  layers at time instant  $n$  to define vector  $\mathbf{x}(n)$  as  $\mathbf{x}(n) = [x^1(n) x^2(n) \cdots x^k(n) \cdots x^K(n)]^T$ . Similarly we take the  $N_r$  received symbols at time instant  $n$  from the  $N_r$  receive antennas to define vector  $\mathbf{y}(n)$  as  $\mathbf{y}(n) = [y_1(n) y_2(n) \cdots y_{N_r}(n)]^T$ . We further define  $\mathbf{x} = [\mathbf{x}(1) \mathbf{x}(2) \cdots \mathbf{x}(N)]^T$  and  $\mathbf{y} = [\mathbf{y}(1) \mathbf{y}(2) \cdots \mathbf{y}(N)]^T$ .

Since  $\mathbf{x}$  and  $\mathbf{y}$  are permutations of  $\tilde{\mathbf{x}}$  and  $\tilde{\mathbf{y}}$  respectively, we can write

$$\begin{aligned}\mathbf{x} &= [\mathbf{x}(1) \mathbf{x}(2) \cdots \mathbf{x}(N)]^T = \mathbf{P}_x \tilde{\mathbf{x}} \\ \mathbf{y} &= [\mathbf{y}(1) \mathbf{y}(2) \cdots \mathbf{y}(N)]^T = \mathbf{P}_y \tilde{\mathbf{y}}\end{aligned}\tag{4.10}$$

where  $\mathbf{P}_x$  and  $\mathbf{P}_y$  are two permutation matrices. From (4.9) and (4.10), we have

$$\begin{aligned}\mathbf{y} &= \mathbf{P}_y \tilde{\mathbf{y}} = \mathbf{P}_y \tilde{\mathbf{H}} \tilde{\mathbf{x}} + \mathbf{P}_y \mathbf{n} \\ &= \mathbf{P}_y \tilde{\mathbf{H}} \mathbf{P}_x^T \mathbf{P}_x \tilde{\mathbf{x}} + \mathbf{P}_y \mathbf{n} \\ &= \hat{\mathbf{H}} \mathbf{x} + \hat{\mathbf{n}}\end{aligned}\tag{4.11}$$

It can be verified that  $\hat{\mathbf{H}} = \mathbf{P}_y \tilde{\mathbf{H}} \mathbf{P}_x^T$  is block circulant and given by

$$\hat{\mathbf{H}} = \begin{bmatrix} \hat{\mathbf{H}}_1 & \hat{\mathbf{H}}_2 & \cdots & \hat{\mathbf{H}}_{N_t} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{H}}_1 & \hat{\mathbf{H}}_2 & \cdots & \hat{\mathbf{H}}_{N_t} & \ddots & \vdots \\ \vdots & \cdots & \ddots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \vdots & \ddots & \hat{\mathbf{H}}_1 & \ddots & \ddots & \hat{\mathbf{H}}_{N_t} \\ \hat{\mathbf{H}}_{N_t} & \ddots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots & \ddots & \ddots & \hat{\mathbf{H}}_2 \\ \hat{\mathbf{H}}_2 & \cdots & \hat{\mathbf{H}}_{N_t} & \mathbf{0} & \cdots & \mathbf{0} & \hat{\mathbf{H}}_1 \end{bmatrix}\tag{4.12}$$

with

$$\hat{\mathbf{H}}_i = \begin{bmatrix} \tilde{h}_{1i}^1 & \tilde{h}_{1i}^2 & \cdots & \tilde{h}_{1i}^K \\ \tilde{h}_{2i}^1 & \tilde{h}_{2i}^2 & \cdots & \tilde{h}_{2i}^1 \\ \vdots & \ddots & \ddots & \vdots \\ \tilde{h}_{N_i,i}^1 & \tilde{h}_{N_i,i}^2 & \cdots & \tilde{h}_{N_i,i}^K \end{bmatrix} = \begin{bmatrix} h_{1i} \\ h_{2i} \\ \vdots \\ h_{N_i,i} \end{bmatrix} [w_{1i} \ w_{2i} \ \cdots \ w_{Ki}] = \mathbf{h}_i \mathbf{g}_i$$

From (4.11), the proposed space-time modulation can be regarded as a  $KN$ -input  $N_r N$ -output system with *channel matrix*  $\hat{\mathbf{H}}$ . When the actual channel  $\mathbf{H}$  and, hence,  $\hat{\mathbf{H}}$  is unknown to the transmitter, the ergodic capacity of  $\hat{\mathbf{H}}$  is [2]

$$C_{\hat{\mathbf{H}}} = \frac{1}{N} E_h \left[ \log_2 \det \left[ \mathbf{I}_{N_r N} + \frac{P}{KN_o} \hat{\mathbf{H}} \hat{\mathbf{H}}^H \right] \right] \quad (4.13)$$

where  $P$  is the average transmit power per channel use. Then we can have the following theorem:

**Theorem 4.1:** The proposed space-time modulator including CF and spatial multiplexing preserves channel capacity when

$$\mathbf{g}_i^H \mathbf{g}_j = \begin{cases} K/N_i & i = j \\ 0 & i \neq j \end{cases} \quad (4.14)$$

where  $\mathbf{g}_i = [w_{1i} \ w_{2i} \ \cdots \ w_{Ki}]^T$ ,  $i = 1, 2, \dots, N_i$ , is the vector collecting the  $i$ -th spatial weights of all the layers.

**Proof:** Since  $\hat{\mathbf{H}}$  in (4.12) is block circulant,  $\hat{\mathbf{H}} \hat{\mathbf{H}}^H$  is banded block diagonal and can be written as

$$\hat{\mathbf{H}}\hat{\mathbf{H}}^H = \begin{bmatrix} \mathbf{R}_0 & \mathbf{R}_1 & \cdots & \mathbf{R}_{N_t-1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{R}_1^H & \mathbf{R}_0 & \mathbf{R}_1 & \cdots & \mathbf{R}_{N_t-1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{R}_{N_t-1}^H & \cdots & \mathbf{R}_1^H & \mathbf{R}_0 & \mathbf{R}_1 & \cdots & \mathbf{R}_{N_t-1} \\ \mathbf{0} & \ddots & \cdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \cdots & \ddots & \ddots & \mathbf{R}_1 \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{R}_{N_t-1}^H & \cdots & \mathbf{R}_1^H & \mathbf{R}_0 \end{bmatrix} \quad (4.15)$$

where

$$\mathbf{R}_j = \sum_{i=1}^{N_t-j} \hat{\mathbf{H}}_i \hat{\mathbf{H}}_{i+j}^H \quad \text{for } j=0, 1, \dots, N_t-1 \quad (4.16)$$

Note that  $\hat{\mathbf{H}}_i = \mathbf{h}_i \mathbf{g}_i^T$  and substitute (4.14) into (4.16), we have  $\mathbf{R}_0 = \frac{K}{N_t} \mathbf{H} \mathbf{H}^H$  and  $\mathbf{R}_j =$

$\mathbf{0}$  for any  $j > 0$ . Hence, (4.13) becomes

$$\begin{aligned} C_{\hat{H}} &= \frac{1}{N} E_h \left[ \log_2 \prod_{i=1}^N \det[\mathbf{I}_{N_r} + \frac{P}{N_t N_o} \mathbf{H} \mathbf{H}^H] \right] \\ &= E_h \left[ \log_2 \det[\mathbf{I}_{N_r} + \frac{P}{N_t N_o} \mathbf{H} \mathbf{H}^H] \right] \end{aligned} \quad (4.17)$$

which is exactly the capacity of the original physical channel  $\mathbf{H}$ . This has established the theorem.

Several immediate consequences of Theorem 4.1 are stated below.

**Corollary 4.1:** The proposed space-time modulator preserves channel capacity only when  $K \geq N_t$ .

This is obvious. Since only when  $K \geq N_t$ , there exist  $N_t$  orthogonal vectors of size  $K$ .

**Corollary 4.2:** The proposed space-time modulator preserves channel capacity if  $N_t$

layers are used and the spatial weight vectors all have unit  $L_2$ -norm and are mutually orthogonal to each other, i.e.,

$$\mathbf{w}_i^H \mathbf{w}_j = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \quad (4.18)$$

The above corollary also tells us that there is little incentive to employ more than  $N_t$  layers. Hence, from now on, we set the number of layers to be  $1 \leq K \leq N_t$ . When  $K$  increases, a higher data rate (a larger multiplexing gain) can be achieved. Now (4.12) and (4.16) together constitute a set of design rules for the spatial weight vectors. An immediate choice is to choose  $\mathbf{w}_k$  as the  $k$ -th column of the DFT matrix of size  $N_t$ . That is

$$w_{ki} = \frac{1}{\sqrt{N_t}} \exp(-j2\pi(k-1)(i-1)/N_t) \quad \text{for } 1 \leq k \leq K \text{ and } 1 \leq i \leq N_t \quad (4.19)$$

In the rest of the chapter, spatial weight given above will be assumed.

### 4.3 Receiver Design for Multi-layered MIMO Transmission

For the proposed coded, multi-layered MIMO transmission, a natural choice is a joint iterative decoding and demodulation receiver as described in chapter 3. Since decoding and message passing in the form of likelihood ratios are the same as described before, in this section, we will only describe the inner demodulator based on MMSE interference cancellation.

For layer  $k$ , we assume the time-varying coefficient vector of the linear filter  $\mathbf{f}^k(n) = [\mathbf{f}_{-N_1}^{*k}(n)^T \cdots \mathbf{f}_m^{*k}(n)^T \cdots \mathbf{f}_{N_2}^{*k}(n)^T]^T$  with  $\mathbf{f}_m^{*k}(n) = [f_{1,m}^{*k}(n) \ f_{2,m}^{*k}(n) \cdots f_{N_t,m}^{*k}(n)]^T$ .

Let  $\hat{x}^k(n)$  denote the estimation of  $x^k(n)$ , the transmitted symbol from layer  $k$  at time  $n$ , and define  $\bar{x}^k(n) = E[x^k(n)]$ ,  $\bar{\mathbf{x}} = E[\mathbf{x}]$ .

Similarly, define  $\bar{y}_j(n)$  as the estimated received signal using the decoder output, i.e.,

$$\bar{y}_j(n) = \sum_{k=1}^K \sum_{i=1}^{N_t} \bar{x}^k([n - N_t + i]_N) w_{ki} h_{ji} \text{ and collect them into a vector of size } N_r N. \text{ We}$$

denote the vector as  $\bar{\mathbf{y}} = [\bar{y}_1(1) \cdots \bar{y}_{N_r}(1) \bar{y}_1(2) \cdots \bar{y}_{N_r}(2) \bar{y}_1(N) \cdots \bar{y}_{N_r}(N)]^T$ .

The MMSE detector will apply a linear filter to the received signal less the estimated received signal as

$$\hat{x}^k(n) = \bar{x}^k(n) + [\mathbf{f}^k(n)]^H (\mathbf{y} - \bar{\mathbf{y}}) \quad (4.20)$$

According to MMSE cost criterion,  $\mathbf{f}^k(n)$  is chosen to minimize the mean square error between the transmitted symbol  $x^k(n)$  and the output of the linear filter, that is

$$\mathbf{f}^k(n) = \text{argmin} E[|x^k(n) - \hat{x}^k(n)|^2] \quad (4.21)$$

Before solving the equation to get  $\mathbf{f}^k(n)$ , we define

$$\begin{aligned} v^k(n) &= \text{Cov}(x^k(n), x^k(n)) \\ \mathbf{v}(n) &= [v^1(n) \ v^2(n) \cdots v^K(n)]^T \\ \mathbf{V}_n &= \text{diag}(v((n - N_t + 1)) \cdots v(n) \cdots v((n - N_t + N))) \\ \mathbf{u} &= [\mathbf{0}_{I \times (K(N_t - 1) + k - 1)} \ \mathbf{1} \ \mathbf{0}_{I \times (K(N - N_t) + K - k)}] \end{aligned} \quad (4.22)$$

Based on the assumption that  $\hat{x}^k(n)$  is independent from its own prior information,  $\mathbf{f}^k(n)$  can be derived as below for cases of  $N_t \geq N_r$  and  $N_t \leq N_r$ , respectively.

**Case 1** ( $N_t \geq N_r$ ):

$$\mathbf{f}^k(n) = \sum_{\mathbf{n}}^{-1} \hat{\mathbf{H}} \mathbf{u}^T \quad (4.23)$$

with  $\sum_n = \hat{\mathbf{H}} \mathbf{V}_n \hat{\mathbf{H}}^H + N_0 \mathbf{I}_{N_r N}$ . Hence, the output of the detector is

$$\hat{x}^k(n) = K_n [\mathbf{f}^k(n)]^H (\mathbf{y} - \hat{\mathbf{H}} \bar{\mathbf{x}} + \bar{x}^k(n) \hat{\mathbf{H}} \mathbf{u}^T) \quad (4.24)$$

with  $K_n = (1 + (1 - v^k(n)) [\mathbf{f}^k(n)]^H \hat{\mathbf{H}} \mathbf{u}^T)^{-1}$

**Case 2** ( $N_t < N_r$ ):

$$\mathbf{f}^k(n) = \sum_n^{-1} \mathbf{Q} \mathbf{u}^T \quad (4.25)$$

with  $\sum_n = \mathbf{Q} \mathbf{V}_n \mathbf{Q}^H + N_0 \mathbf{Q}$  where  $\mathbf{Q} = \hat{\mathbf{H}}^H \hat{\mathbf{H}}$ . Hence, the output of the detector is

$$\hat{x}^k(n) = K_n [\mathbf{f}^k(n)]^H (\hat{\mathbf{H}}^H \mathbf{y} - \mathbf{Q} \bar{\mathbf{x}} + \bar{x}^k(n) \mathbf{Q} \mathbf{u}^T) \quad (4.26)$$

with  $K_n = (1 + (1 - v^k(n)) [\mathbf{f}^k(n)]^H \mathbf{Q} \mathbf{u}^T)^{-1}$

Since the PDFs,  $p(\hat{x}^k(n) | x^k(n) = \alpha_i)$ , are approximately Gaussian distribution with mean  $\mu_{n,i}^k = E(\hat{x}^k(n) | x^k(n) = \alpha_i)$  and variance  $(\sigma_{n,i}^k)^2 = \text{Cov}(\hat{x}^k(n), \hat{x}^k(n) | x^k(n) = \alpha_i)$ , we obtain

$$\begin{aligned} \mu_{n,i}^k &= K_n \alpha_i [\mathbf{f}^k(n)]^H \hat{\mathbf{H}} \mathbf{u}^T \\ (\sigma_{n,i}^k)^2 &= K_n^2 ([\mathbf{f}^k(n)]^H \hat{\mathbf{H}} \mathbf{u}^T - v^k(n) [\mathbf{f}^k(n)]^H \hat{\mathbf{H}} \mathbf{u}^T \mathbf{u} \hat{\mathbf{H}}^H \mathbf{f}^k(n)) \\ &= (\sigma_n^k)^2 \end{aligned} \quad (4.27)$$

Hence the PDFs can be written as

$$p(\hat{x}^k(n) | x^k(n) = \alpha_i) = \frac{1}{\pi (\sigma_n^k)^2} \exp\left(-\frac{|\hat{x}^k(n) - \mu_{n,i}^k|^2}{(\sigma_n^k)^2}\right)$$

Thus the extrinsic information of  $\hat{x}^k(n) = \alpha_i$  is obtained by

$$L_e^E(\hat{x}^k(n) = \alpha_i) = \ln \frac{p(\hat{x}^k(n) | x^k(n) = \alpha_i)}{p(\hat{x}^k(n) | x^k(n) = \alpha_o)} \quad (4.28)$$



The above LLR is the output of the inner demodulator and is to be sent as the priori information to the decoder.

## 4.4 Simulation Results

We have introduced the multi-layered MIMO transmission system in the previous sections. Here we will present simulation results of the new multi-layered scheme and compare them with that of D-Blast scheme of the same data rate. The transmitter and the receiver structure for the proposed multi-layered MIMO system are shown in Fig. 4.1 and Fig. 4.2.

A rate 1/2 and memory  $\nu = 4$  convolutional code is used as outer one-dimensional code. The generator polynomials are  $H^1(D) = 04$ ,  $H^2(D) = 13$ . QPSK modulation is performed on the encoded bits. A random interleaver is used. The number of information bits in one frame  $L$  is 96. The number of iteration is 8 for all simulations. For multi-layered scheme, the number of layers equals to the number of the transmit antennas and the number of the receive antennas, i.e.  $K = N_t = N_r$ . The circulant block length  $N$  is 4 for  $N_t = 2, 3$  and 6 for  $N_t = 4$ . The MMSE linear filter length equals to the circulant block length  $N$ . For a fair comparison, for D-Blast scheme the MMSE linear filter length is also chosen as 4 for  $N_t = 2, 3$  and 6 for  $N_t = 4$ .

In Figs. 4.4 – 4.6 are the performance comparison between the multi-layered scheme and the D-Blast scheme when the number of the transmit antennas is 2, 3, and 4 respectively.

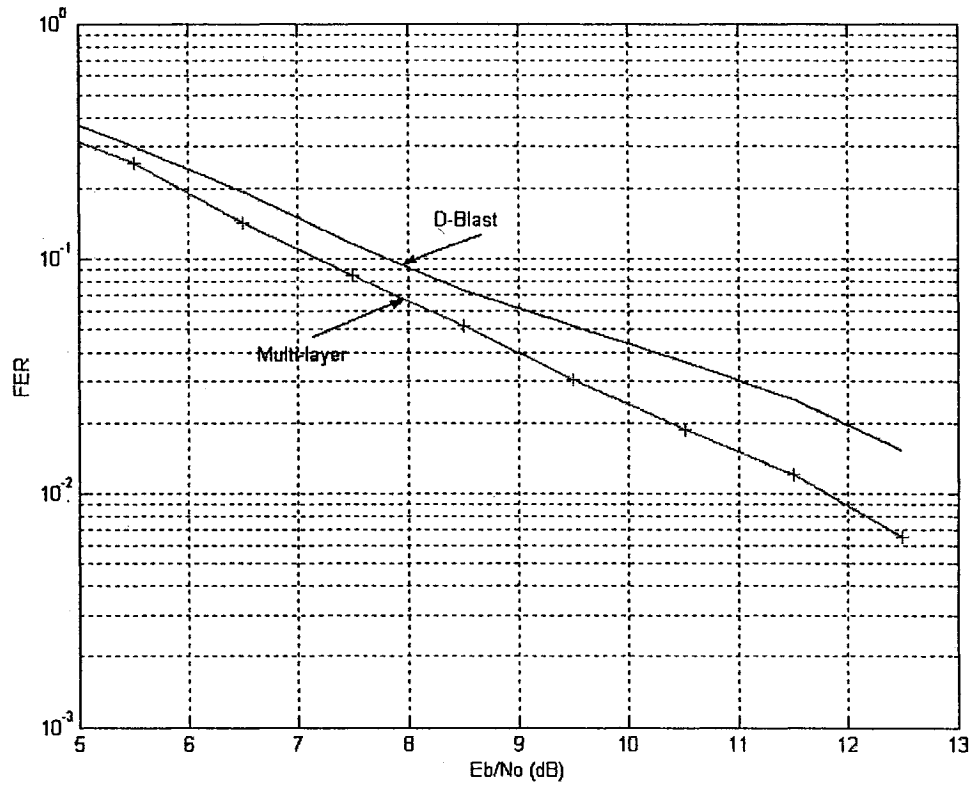


Fig. 4.4. FER of the proposed 2-layer scheme and D-Blast: 2 transmit and 2 receive antennas.

From Fig. 4.4 it can be seen that the proposed scheme with two layers provides a better performance. Specifically, the FER of the proposed scheme is  $6.5 \times 10^{-3}$  while it is  $1.6 \times 10^{-2}$  for the D-Blast scheme at  $E_b/N_0 = 12.5dB$ . It is also observed that the performance of the proposed scheme with two layers has a  $1.5dB$  gain when FER is  $1.6 \times 10^{-2}$ .

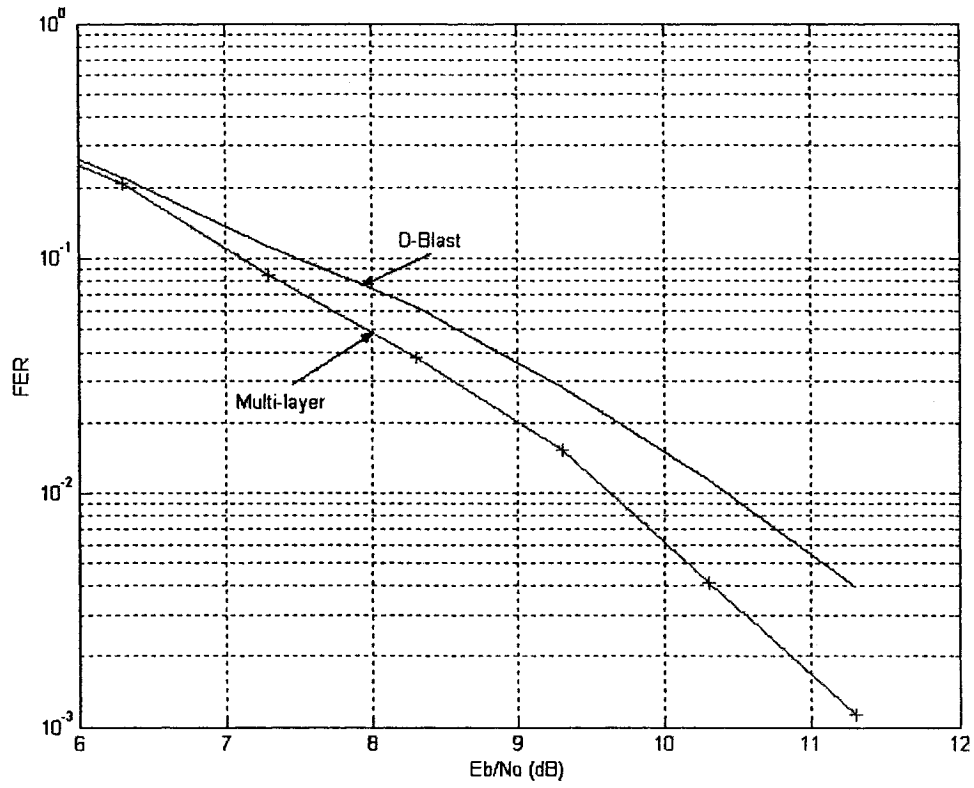


Fig. 4.5. FER of the proposed 3-layer scheme and D-Blast: 3 transmit and 3 receive antennas.

Fig. 4.5 illustrates the simulation results of the three-layer MIMO transmission scheme and the D-Blast scheme when there are three transmit and three receive antennas. One can see that at  $E_b/N_0 = 11.3dB$ , the FER of the proposed scheme with three layers is  $1.2 \times 10^{-3}$  while the FER of the D-Blast scheme is  $4.0 \times 10^{-3}$ . Also we observe that the three-layer scheme has  $1dB$  gain over the D-Blast scheme at  $4.0 \times 10^{-3}$ .

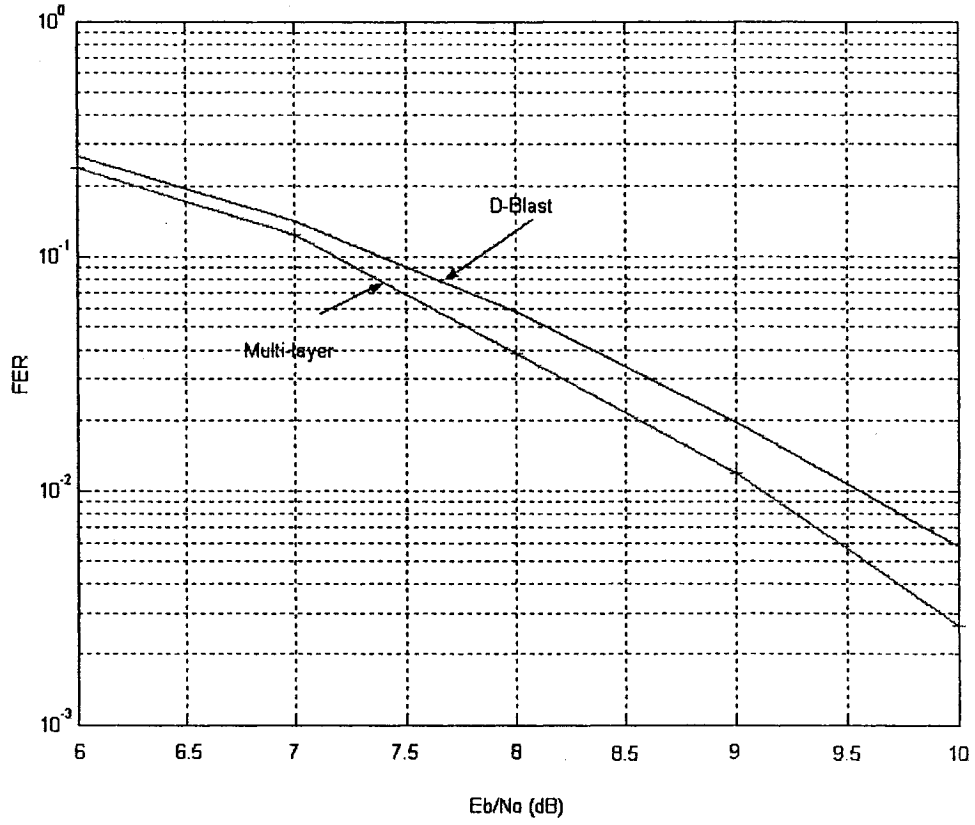


Fig. 4.6. FER of the proposed 4-layer scheme and D-Blast: 4 transmit and 4 receive antennas.

Fig. 4.6 shows the simulation results of the proposed scheme and D-Blast when there are four transmit and four receive antennas and four layers. We can see that at  $E_b/N_0 = 10dB$ , the FER of the proposed scheme is  $3.0 \times 10^{-3}$  while the FER of the D-Blast scheme is  $6.0 \times 10^{-3}$ . Also we observe that the proposed scheme has  $0.5dB$  gain over the D-Blast scheme at  $FER = 6.0 \times 10^{-3}$ .

From above three figures, one can also observe that the proposed scheme provides a larger diversity gain. This is due to the design that ensures full diversity for the codeword pairs with minimum Hamming distance.

We further note that with the increase of the number of the layers (also the number of transmit and receive antennas) from 2 to 3 and 4, not only the data rate is increased but also the FER performance is improved around  $0.8dB$  and  $2.5dB$ . Again, this is mainly due to the increase in the diversity gain for codeword pairs with minimum Hamming distance.

## 4.5 Conclusions

In this chapter, a new multi-layered MIMO transmission scheme has been proposed. In the heart of the proposed MIMO transmission scheme is a layered space-time modulator that allows various tradeoff between data rate and diversity. Different tradeoffs can be simply achieved by varying the number of the layers. It has also been shown that channel capacity can be preserved when the number of layers is equal to and greater than the number of transmit antennas.

In addition to the proposed MIMO transmission scheme, a study on the design of inner space-time modulator has been carried out. Design criteria were obtained, which turned out to be much easier to apply than Tarokh's design criteria for space-time codes. With the developed design criteria, optimal multiplexing weights were found.

Simulations have been carried out and results have demonstrated that the proposed scheme significantly outperforms D-Blast.

## Chapter 5

### Conclusions and Future Works

In this thesis, we proposed a general frame work for the design of MIMO transmission scheme. In the proposed framework, a MIMO transmission system is modeled as one or more one-dimensional outer codes serially concatenated with a space-time modulator to simplify the design of two-dimensional space-time codes and to provide additional flexibility in various tradeoff. An interleaver is used between them to make two parts independent.

Using the general framework, we first proposed the delay-diversity transmission scheme in which the repeated delay operation is viewed as the inner space-time modulation such that a turbo receiver can be employed with a one-dimensional trellis code as the outer code. It is shown that the proposed delay-diversity MIMO transmission scheme is in fact a space-time trellis coding scheme. However, with the new interpretation, the design is considerably simplified as the inner modulator guarantees full diversity regardless of the outer code used. As a result, one needs only to choose conventional codes with maximum hamming distance for a given complexity constraint.

Viewing the transmission system as a concatenated codes system, we have developed a joint iterative (turbo) receiver which has much reduced complexity compared to the optimal Viterbi receiver. Specifically, the complexity now grows linearly with the number of transmit antennas instead of exponentially. Simulation results show that performance close to that of the optimal receiver can be obtained.

Then we developed a new multi-layered MIMO transmission scheme. The new scheme employs a number of one-dimensional encoders followed by a space-time modulator. The space-time modulator allows a full range of tradeoff between performance and data rate.

To optimize the multi-layered space-time modulator, a study on the performance of the overall system was conducted. A set of design criteria for the inner space-time modulator have been established, which show that the design becomes easier due to the use of outer codes. Using the design guidelines, we have found the optimal modulation parameters. The resultant modulator preserves the channel capacity when the number of layers is equal to or greater than the number of transmit antennas.

A simplified MIMO interference canceller has been developed to be used as the inner soft-in soft-out demodulator in a joint iterative receiver. Simulation results demonstrated that the proposed scheme significantly outperforms D-Blast.

The advantages of the proposed general design framework have been demonstrated by the two successful designs. However, its full potential remains to be explored. For instance, a joint optimization on the outer encoding, constellation, and inner space-time modulation is desirable. Furthermore, joint iterative receiver with reduced complexity is still possible by utilizing the special features of the inner space-time modulation. It is also

desirable in practices to estimate the FER of a given set of transmission parameters. Since the ML receiver is too complex to be feasible, all the above design considerations shall be based on iterative (turbo) receiver. In such a case, extrinsic information transfer (EXIT) chart is a handy tool for the further optimization of the overall system. This can be an immediate framework to fully exploit the benefits of the proposed general design framework for MIMO transmission schemes.



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