Design Sensitivity analyses of Two-Dimensional Recursive Band-Pass and Band-Stop Digital Filters with an Application in Image Processing

Karthikeyan, Keelapandal Sundaram

A thesis

In

The Department

Of

Electrical and Computer Engineering

Presented in Partial Fulfillment of the Requirements

For the Degree of Master of Applied Science at

Concordia University

Montréal, Québec, Canada

December 2004

Your file Votre référence
ISBN: 0-494-04378-4
Our file Notre référence
ISBN: 0-494-04378-4

Abstract

Design Sensitivity analyses of Two-Dimensional Recursive Band-Pass and Band-Stop Digital Filters with an Application in Image Processing.

Two-dimensional variable recursive digital Band-pass and Band-stop filters are applied in signal processing and pro-imagining process, as well as communication systems where the frequency domain characteristics of digital filters are required to be adjustable. The main objective of this thesis has been to propose a new method of designing 2-D recursive Band-Pass and Band-Stop digital filters with variable characteristics. From the identical analog 1-D second order Butterworth Low-Pass analog ladder network, 2-D Band-Pass and Band-Stop digital filters can be obtained through the application of Low-Pass to Band-Pass / Band-Stop transformation and double generalized bilinear transformations. The denominators of these filters transfer functions are verified for VSHP. Sensitivity analyses are performed by varying the coefficients of the double generalized bilinear transformation such as k_1, k_2, a_1, a_2, b_1 and b_2 with respect to center frequency 'ω_o' and Bandwidth 'B' on the resulted 2-D Band-Pass and Band-Stop filter, which are obtained by varying the coefficients of the double generalized bilinear transformation in the specific ranges in order to maintain the stability of the filter of the Band-Pass and Band-Stop transfer function respectively.

Acknowledgements

I would like to express my deepest gratitude to my thesis supervisor, Dr. Venkat Ramachandran, for his valuable guidance and encouragement through the course of this thesis, for his extremely careful, critical, and thorough review of my work. I thank Dr. Subhash Rakheja and Dr. Rabin Raut for their helpful suggestions in the preparation of the final manuscript of the thesis. I would also thank my parents and sisters for their encouragement and supports.

Dedicated to my parents and sisters

Table of Contents

List of Tables

List of Figures

LIST OF ABBREVIATIONS AND SYMBOLS

z_1, z_2 Z-domain parameter in first and second dimensions.

s_1, s_2 Laplace domain parameter in first and second dimensions.

ω_1, ω_2 Frequencies in radians in the analog domain parameter in first and second dimensions.

H Frequency response of the filter.

1-D One-dimensional.

2-D Two-dimensional.

FIR Finite Impulse Response.

IIR Infinite Impulse Response.

VSHP Very strictly Hurwitz Polynomial.

BIBO Bounded Input Bounded Output.

PSNR Peak signal to Noise Ratio.

Chapter 1

Introduction

1.1 General

Signals arise in almost every field of science and engineering like acoustics, biomedical engineering, communictions, control systems, radar, physics, seismology and telemetry etc., Two general classes of signals can be identified, namely continuous-time and discrete-time signals. A continuous-time signal is defined at each and every instant of time. A discrete-time signal, on the other hand, is defined at discrete instants of time. A discrete-time signal, like continuous-time signal, can be represented by a unique function of frequency referred to as the frequency spectrum of the signal.

Filtering is a process by which the frequency spectrum of a signal can be modified, reshaped or manipulated according to some desired specifications. It may entail amplifying or attenuating a range of frequency components, rejecting or isolating some specific frequency components, etc. The uses of filtering are manifold such as eliminating signal contamination such as noise, to remove signal distortion brought about by an imperfect transmission channel, to separate two or more distinct signals which were purposely mixed in order to maximize channel utilization, to resolve signals into frequency components, to demodulate signals, to convert discrete-time signals into continuous-time signals, and to band limit signals.[1]

Filters are classified as analog and digital filters. Analog filters are used to process analog signals, signals which are functions of continuos-time variable. Digital filters, on the other hand, process digitized continuos waveform.

1.2 Analog and Digital filters

As stated earlier, in signal processing, the function of a filter is to remove unwanted parts of the signal, such as random noise, or to extract useful parts of the signal, such as the components lying within frequency range. The figure 1.1 illustrates the basic idea

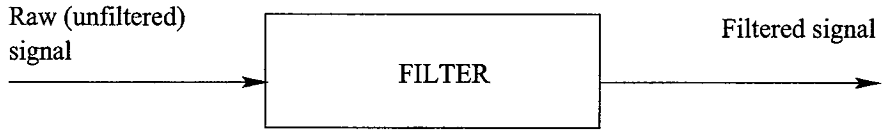

Figure 1.1: Block Diagram of a filter

An analog filter uses analog electonic circuits made up from components such as resistors, capacitors and Op Amps to produce the required filtering effect. Such filter circuits are widely used in such applications as noise reduction, video signal enchanement, graphic equalisers in hi-fi systems, and also in many other areas. There are well-established standard techniques for designing an analog filter circuit for a given requirement. At all stages, the signal being filtered is an electrical voltage or current which is the direct analogue of the physical quantity involved.

A digital filter is a device or program that performs a prescribed manipulation or algorithm on an input sequence of numbers resulting in the desired output sequence of numbers. The numbers are limited to a finite precision. A digital filter uses a digital processor to perform numerical calculations on sampled values of the signal. The processor may be general-purpose computer such as a PC, or a specialised DSP (Digital Signal Processor) chip.

The analog input signal must first be sampled and digitised using an ADC (analog to digital converter). The resulting binary numbers, representing successive sampled values of the input signal, are transferred to the processor, which carries out numerical calcualtions on them. These calculations typically involve multiplying the input values by constants and adding the products together. The results of these calculations, which now represent sampled values of the filtered signal, are fed through a DAC (digital to analog converter) to convert the signal back to analog form. The signal is represented by a sequence of numbers, rather than a voltage or current [2]. The figure 1.2 shows the basic step of such a system.

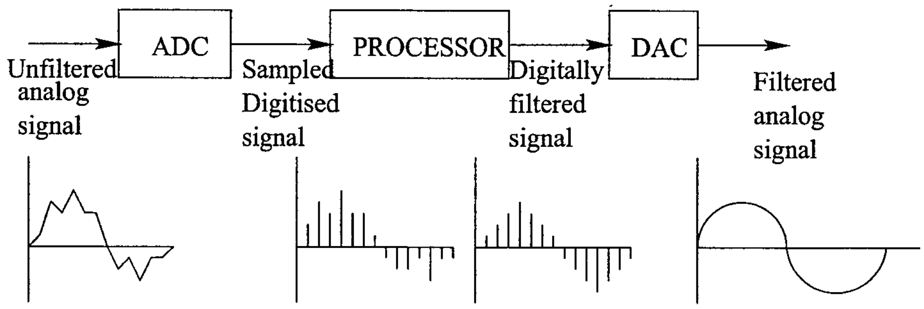

Figure 1.2: Block diagram of a digital filter

The following list gives some of the main advantages of digital filters

1. Component tolerances are uncritical.

2. Component drift and spurious environmental signals have no influence on the system performance.

3. Accuracy is high.

4. Physical size is small.

5. Reliability is high.

An important additional advantage of digital filters is the ease with which filter parameters can be changed in order to change the filter characteristics. This feature allows one to design programmable filters which can perform a multiplicity of filtering tasks. Also one can design new types of filters such as adaptive filters .

1.3 Characterization of Digital Filters

Analog filters are characterized in terms of differential equations. Digital filters, on the other hand, are characterized in terms of difference equations. Two types of digital filters can be identified, nonrecursive and recursive filters.

Nonrecursive Filters

The response of a nonrecursive filter at instant nT is of form

$$y(nT) = f\{..., x(nT - T), x(nT), x(nT + T), ...\}$$

If we assume linearity and time invariance, y(nT) can be expressed as

$$y(nT) = \sum_{i=-\infty}^{\infty} a_i x(nT - T) \tag{1.1}$$

where a_i represents constants. Now by assuming causality and then using causality criterion defined earlier, we can show that

$$a_{-1} = a_{-2} = ... = 0$$

and so

$$y(nT) = \sum_{i=0}^{\infty} a_i x(nT - T)$$

If , in addition, $x(nT)$ =0 for $n < 0$ and $a_i = 0$ for $i > N$,

$$y(nT) = \sum_{i=0}^{n} a_i x(nT - T) + \sum_{i=n+1}^{\infty} a_i x(nT - T)$$

$$= \sum_{i=0}^{N} a_i x(nT - T) + \sum_{i=N+1}^{n} a_i x(nT - T)$$

$$= \sum_{i=0}^{n} a_i x(nT - T) \tag{1.2}$$

Therefore, a linear, time-invariant, causal, nonrecursive filter can be represented by an Nth-order linear difference equation, where N is the order of the filter.

Recursive Filters

The response of a recursive filter is a function of elements in the excitation as well as the response sequence. In the case of a linear, time -invariant, causal filter

$$y(nT) = \sum_{i=0}^{N} a_i x(nT - T) - \sum_{i=0}^{M} b_i x(nT - T) \tag{1.3}$$

if the instant nT is taken to be the present, the response is a function of the present and past N values of the excitation as well as the past M values of the response. Note that the equation (1.3) simplifies to equation (1.2) if $b_i = 0$, and essentially the nonrecursive filter is a special case of the recursive one [2, 16, 30].

1.4 Different types of filters

A frequency-selective filter is one that passes signals whose frequencies are in certain ranges or bands, called the *passbands,* and blocks, or attenuates, signals whose frequencies are in other ranges, called the *stopbands.* The nature of the amplitude function $|H(j\omega)|$ or the loss function $\alpha(\omega)$ may be used to classify the various types of filters according to the location of their passbands and stopbands. An ideal filter is one which has a linear phase response in its passbands, zero loss in its passband, and infinite loss $H(j\omega)| = 0$ in its stopband.

The most often encountered types of frequency-selective filters are defined as follows:

(1) A $low - pass$ filter is one with a single passband between 0 and a cutoff frequency ω_c, with all frequencies higher than ω_c constituting the stopband. The *bandwidth* is defined as B=ω_c.

(2) A $high - pass$ filter is one with stopband $0 < \omega < \omega_c$, and pass-band $\omega > \omega_c$ where ω_c is the cutoff frequency.

(3) A $band - pass$ filter is one with a passband between two cutoff frequencies ω_c and $\omega_U > \omega_L$ and two stopbands, $0 < \omega < \omega_L$ and $\omega > \omega_L$. The *bandwidth* is defined as B=$\omega_U - \omega_L$.

(4) A $band - reject$ filter is one with a stopband $\omega_L < \omega < \omega_U$ and two passbands, $0 < \omega < \omega_L$ and $\omega > \omega_U$.

(5) An $all-pass$ filter is one which passes all frequencies equally well. That is ,$|H(j\omega)|$ is constant for all frequencies, with the phase $\phi(\omega)$ generally a function of frequency[2].

1.5 Design of digital filter from analog filters

The design of filters involves the following stages

(a) The specification of the desired properties of the system,

(b) The approximation of the specifications using a causal discrete-time system, and

(c) The realization of the system

In a practical setting, the desired filter is generally implemented with digital computation and used to filter a signal that is derived from a continuous-time signal by means of periodic sampling followed by analog-to-digital conversion.

1.5.1 Design of Digital IIR fi lters from analog fi lters

The traditional approach to the design of the discrete-time IIR filters involves the transformation of a continuous-time filter into a discrete-time filter meeting prescribed specifications. This is a reasonable approach for several reasons:

(1) The art of continuous -time filter IIR filter design is highly advanced, and useful results can be achieved, it is advantageous to use the design procedures already developed for continuous-time filters.

(2) Many useful continuous-time IIR design methods have relatively simple closed form design formulas. Therefore, discrete-time IIR filter design methods based on such standard continuous-time design formulas are rather simple to carry out.

(3) The standard approximation methods that work well for continuous-time IIR filters

do not lead to simple closed-form design formulas when these methods are applied directly to the discrete -time IIR case[20, 24].

In designing a discrete-time filter by transforming a prototype continuous-time filter, the specifications for the continuous-time filter are obtained by a transformation of the specifications for the desired discrete-time filter. The system function $H_c(s)$ or impulse response $h_c(t)$ of the continuous-time filter is obtained through one of the approximations of the continuous-time filter. The system function $H(z)$ or impulse response $h[n]$ for the discrete -time filter is obtained by applying to $H_c(s)$ or $h_c(t)$ a transformation of the type discussed in this section. In such transformations, we generally require that the essential properties of the continuous-time frequency response be preserved in the frequency response of the resulting discrete-time filter. Specifically, this implies that the imaginary axis of the s-plane is mapped onto the unit circle of the z-plane. A second condition is that a stable continuous-time filter should be transformed to a stable discrete-time filter. There are two methods of digital filter design. The first method is Impulse Invariance method and second method is Bilinear transformation[1, 3, 16].

1.5.1.1 Filter Design by Impulse Invariance

In the Impulse invariance design procedure for transforming continuous-time filters into discrete-time filters, the impulse response of the discrete-time filter is chosen proportional to equally spaced samples of the impulse response of the continuous-time filter,

$$h[n] = T_d h_c(nT_d)$$

where T_d represents a sampling interval. $h_c(nT_d)$ is the impulse response of the continuous-time filter.

In the impulse invariance design procedure, the relationship between continuous-time and discrete-time frequecy is linear. Consequently, except aliasing, the shape of the frequency response is preserved. Thus the impulse invariance technique is appropriate only for bandlimited filters.

1.5.1.2 Filter Design by Bilinear Transformation[8]

The problem of aliasing is avoided by using the Generalized Bilinear Transformation instead of impulse invariance method. Bilinear Transformation is an algebraic transformation between the variable s and that maps the entire $j\Omega - axis$ in the $s - plane$ to the unit circle in the $z - plane$. Since $-\infty \leq \Omega \leq \infty$ maps onto $-\pi \leq \omega \leq \pi$, the transformation between the continuous-time and discrete-time frequency variables must be non-linear. Therefore, the use of this techniques is restricted to situations in which the corresponding warping of the frequency axis is acceptable.

With $H_c(s)$ denoting the continuous-time system function and $H(z)$ the discrete-time system function, the bilinear transformation corresponds to replacing s by

$$s = k_i \frac{z_i - a_i}{z_i + b_i} \qquad i = 1, 2$$

where k_i, a_i and b_i are coefficeints of the generalized bilnear transformation. By replacing one for the coefficients for 1-D, we get

$$s = \frac{z - 1}{z + 1}$$

that is

$$H(z) = H_c \left[\frac{z - 1}{z + 1}\right]$$

1.5.2 Design of Digital FIR filters

Nonrecursive filters are also called Finite Impulse Response (FIR) filters, which have their transfer function resulting from a finite input sequence. The output of nonrecursive filter at any point can be computed as a linear combination of a finite number of input samples. The main properties for nonrecursive filter are its inherent stability and linear phase feature. There are many methods, such as Windows methods, frequency transformation, and linear programming and these are generally used in a nonrecursive filters design[1, 23].

1.6 Two-Dimensional Digital filters

The topic of Multidimensional system (MDS) analysis and design has attracted considerable attention during recent years and is still receiving increased attention by theorists and practitioners. Specifically, interest has been directed by researchers into the area of two-dimensional (2-D) digital systems due to several reasons : high efficiency due to high-speed computations; permitting better image processing and analysis; great application flexibility and adaptivity; decreasing cost of software or hardware implementations due to the large expansion and evolution of standard computers, microcomputers, microprocessors, and high-integration digital circuits. In general, two-dimensional digital filters are designed by combining two one -dimensional digital filters. Similar to one-dimensional digital filters, two-dimensional digital filters can be classified into two main groups. The first group comprises a finite sequence transfer function and so the filters in this group are called Finite Impulse Response (FIR) filters. The second group comprises an infinite sequence transfer function and so the filters in this group are called Infinite Impulse Response (IIR) filters[23, 24, 31].

These two-dimensional digital systems are being used increasingly to replace analog systems in important areas such as television, facsimile, radar, bio-medicine, remote sensing, underwater acoustics, moving-objects recognition, robotics and so on. Important operations that can be performed by two-dimensional digital systems include the following: two-dimensional digital filtering, two-dimensional digital transformations, local space processing, data compression, and pattern recongnition. Digital filtering, digital transformations and local space operations play important roles in preprocessing of images, performing smoothing, enhancement, noise reduction, extracting boundaries and edges before pattern recognition, data compression operations permitting the reduction of large number of data representing the images in digital form and solving or minimizing transmission and storage problems. Pattern recognition operations permit the extraction of significant information and configurations from the images for final interpretation and utilization.

Over the past decade, researchers have shown particular interest in two-dimensional filters, both recursive and non-recursive. These two-dimensional filters find increasing applications in many fields, such as image processing and seismic signal processing. Also two dimensional filters find increasing applications in image restoration and enhancement. As an example, two dimensional highpass filtering removes the unwanted background noise from an image so that details contained in the higher spatial frequencies are easier to perceive [4, 15, 29].

1.7 Stability of the filters

As mentioned before, 2-D filters can be classified into two main categories namely the Finite Impulse Response Filters(FIR) and the Infinite Impulse Response Filters(IIR). The Finite Impulse Response Filters have transfer functions resulting from a finite sequence and the Infinte Impulse Response Filters have the transfer functions resulting from an infinite sequence.

One important issue concerning both the above types of filters is the stability of the filter. Now it is known that Finite Impulse Response Filters are inherently stable. Infinite Impulse Response Filters may or may not be stable depending upon the transfer function.

The most commonly used definition for stability is based on the bounded-input bounded-output(BIBO) criterion. This criterion states that a filter is stable if its response to a bounded input is also bounded. Mathematically, for causal linear shift-invariant systems, this corresponds to the condition that

$$\sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} |h(n_1, n_2)| < \infty \tag{1.4}$$

where $h(n_1, n_2)$ is the impulse response of the filter.

The above definition points out an important observation that the stability criterion is always verified if the number of terms of the impulse response is finite, which is the case with FIR Filters. However, the above condition does not prove feasible to the test of stability for IIR filters. In the 1-D case, it is possible to relate the BIBO stability condition to the positions of the z-transfer function poles which have to be within the unit circle and it is possible to test the stability by determining the zeros of the denominator polynomial. Similarly, in the 2-D case, a theorem establishing the relationship between the stability of the filter and the zeros of the denominator polynomial, can be formulated. This theorem states that [8], for causal quadrant filters, if $B(z_1, z_2)$ is a polynomial in z_1 and z_2, the expansion of $1/B(z_1, z_2)$ in the negative powers of z_1 and z_2 converges absolutely if and only if

$$B(z_1, z_2) \neq 0 \ for \ \{|z_1| \geq 1, |z_2| \geq 1\} \tag{1.5}$$

The above theorem has the same form as in the 1-D case, i.e., it relates the stability of the filter to the singularities of the z-transform. However, in the 2-D case such a formulation for stability condition does not produce an efficient method for stability test, as in 1-D, due to the lack of appropriate factorization theorem of algebra. Therefore, it is necessary in principle, to use an infinite number of steps to test the stability. Also, even if it is possible to find methods to test conditions equivalent to eqn.(1.5) in a finite number of steps [25], computationally it is not easy to incorporate them in a design method and there is a problem of stabilizing the filters which may become unstable.

From the point of view of stability tests, there can be two different approaches that can be considered, in designing an IIR filter. One method is to carry out the stability test in every stage of the filter design so that the eventual filter is stable. In the second method, stability is not considered as a part of the design and a magnitude-squared transfer function is first designed. Then a stable filter is obtained, by choosing the poles in the stability region. Such an approach is convenient, because squared magnitude functions can be in a simple form and it is easy to find the poles of the filter.

However, in the 2-D case, poles in the stability region cannot be substituted for poles in the instability regions. This is because, unlike in the 1-D case, it is not possible to substitute the 2-D pole-pair combination by taking the inverse pole-pair transformation. Therefore different methods have to be used in arriving at a solution. One possible solution can be obtained by considering this as a deconvolution problem. In the quadrant filter case, a filter $H(z_1, z_2)$ can be divided as a product of four filter $^{++}H(z_1, z_2),^{+-} H(z_1, z_2),^{--} H(z_1, z_2),^{-+} H(z_1, z_2)$ each of which correspondingly represents their transfer function in the first, second, third and fourth quadrant, respectively, and each of which is stable, if computed through suitable sequence of computation. In view of the property that multiplication in the z-domain correponds to a convolution in the space domain, the problem corresponds to the reconstruction of the four sequences. Similar procedures may be applied for unsymmetrical half-plane filters as well. In this case, the coefficient matrix, corresponding to the squared magnitude function, can be considered as the convolution of twounsymmetrical half-plane sequences. Thus it is possible to decompose the sequence into two half-plane filters. However in both the above cases there are problems which can arise, as the cepstra obtained using the above procedure are not finite extent and some amount of truncation is necessary. This again modifies the transfer function and also the minimum phase property cannot be guaranteed after truncation. It is possible to observe that half-plane filters constitute a more general class than the quadrant ones since completely general transfer functions with real impulse response can be generated. The decomposition of a squared magnitude function in four different quadrant filters or two half-plane filters, each being stable if a suitable sequence of computation is chosen, can give a direct method of obtaining linear phase filtering with IIR implementation. This is especially important when visual images have to be processed, because the shape of the object is related primarily to the phase information[10].

1.8 Overview of Very Strict Hurwitz Polynomial

1.8.1 Defi nition of Very Strict Hurwitz Polynomial

It is well known, a 1-D analog filter system with the transfer function

$$H_a(s) = \frac{N_a(s)}{D_a(s)}$$

is guaranteed to be stable, if the denominator of the transfer function $D_a(s)$ is a Strictly Hurwitz Polynomial (SHP), which contains all its zeros in the left-half of s-plane.

However, for 2-D analog filter system with the transfer function

$$H_a(s_1, s_2) = \frac{N_a(s_1, s_2)}{D_a(s_1, s_2)}$$

The denominator $D_a(s_1, s_2)$ is SHP cannot always guarantee the stability, as it contains non-essential sigularity of the second kind. That is, the denominator becomes zero at two points $s_1 = j\omega_{10}$ and $s_2 = j\omega_{20}$, but not in its neighbourhood [9, 26].

In fact, there are four types of 2-variable Hurwitz Polynomials, which are different from each other only in the region of analyticity[27].

Definition 1.1 A polynomial $D_a(s_1, s_2)$ is said to be a Broad Sense Hurwitz Polynomial (BHP), if $\frac{1}{D_a(s_1,s_2)}$ has no singularity in the region

$$\{(s_1, s_2)|Re(s_1) > 0, Re(s_2) > 0, |s_1| < \infty, |s_2| < \infty\}.$$

Definition 1.2 A polynomial $D_a(s_1, s_2)$ is said to be a Narrow Sense Hurwitz polynomial (NHP), if $\frac{1}{D_a(s_1,s_2)}$ has no singularities in the region

$$\{(s_1, s_2)|Re(s_1) > 0, Re(s_2) > 0, |s_1| < \infty, |s_2| < \infty\}$$

$$\cup\{(s_1, s_2)|Re(s_1) = 0, Re(s_2) > 0, |s_1| < \infty, |s_2| < \infty\}$$
$$\cup\{(s_1, s_2)|Re(s_1) > 0, Re(s_2) = 0, |s_1| < \infty, |s_2| < \infty\}.$$

Definition 1.3 A polynomial $D_a(s_1, s_2)$ is a Strictly Hurwitz Polynomial (SHP), if it has no zeros in the regions

$$\{(s_1, s_2)|Re(s_1) \geq 0, Re(s_2) > 0, |s_1| < \infty, |s_2| < \infty\}.$$

Definition 1.4 A polynomial $D_a(s_1, s_2)$ is a Very Strictly Hurwitz Polynomial (VSHP) , if the polynomial does not have zeros in the regions

$$\{(s_1, s_2)|Re(s_1) > 0, Re(s_2) > 0, |s_1| < \infty, |s_2| < \infty\}.$$

From these definitions, one can see that a VSHP is required to be necessarily a SHP. From the two-dimensional digital filter design experience, in order to get a guaranteed stable digital filter from the well-knownb bilinear transformations, the 2-D analog transfer function is required to have a 2-variable VSHP as its denominator.

1.8.2 Some Properties of VSHP[9, 27]

Our discussion for the properties of VSHP are based on the following definition of 2-D analog transfer function

$$H(s_1, s_2) = \frac{N(s_1, s_2)}{D(s_1, s_2)}$$

where

$$N(s_1, s_2) = \sum_{i=0}^{m} \sum_{j=0}^{n} B_{ij} s_1^i s_2^j$$

$$D(s_1, s_2) = \sum_{i=0}^{k} \sum_{j=0}^{l} B_{ij} s_1^i s_2^j$$

Property 1: $H(s_1, s_2)$ does not possess singularities in the closed right half of the (s_1, s_2) biplane defined as $\{(s_1, s_2)|Re(s_1) \geq 0, Re(s_2) \geq 0, |s_1| \leq \infty, |s_2| \leq \infty\}$, if and only if $D(s_1, s_2)$ is VSHP.

This property is obtained easily from the definition of VSHP.

Property 2: If $D(s_1, s_2) = D_1(s_1, s_2).D(s_1, s_2)$ is a VSHP, the necessary and sufficient condition is both $D_1(s_1, s_2)$ and $D_2(s_1, s_2)$ are VSHPs.

Proof :

Sufficient condition :

As $D_1(s_1, s_2)$ and $D_2(s_1, s_2)$ are VSHPs, $D_1(s_1, s_2)$ and $D_s(s_1, s_2)$ have no singular points in the closed half plane of $(s_1, s_2) - plane$ $\{(s_1, s_2)|Re(s_1) \geq 0, Re(s_2) \geq 0, |s_1| \leq \infty, |s_2| \leq \infty\}$. Also $D(s_1, s_2)$ is the product of $D_1(s_1, s_2)$ and $D_2(s_1, s_2)$ would not have any singular points in the closed right half of $(s_1, s_2) - plane$, i.e $D(s_1, s_2)$ is a VSHP.

Necessary condition :

$D(s_1, s_2)$ is a VSHP.

Suppose that $D_1(s_1, s_2)$ is not a VSHP, then there exists some points in the closed right half plane of the $(s_1, s_2) - plane$ $\{(s_1, s_2)|Re(s_1) \geq 0, Re(s_2) \geq 0, |s_1| \leq \infty, |s_2| \leq \infty\}$, $D_1(s_1, s_2)$ will equal to zero. So $D(s_1, s_2)$ will be zero at those points too, it will contradict our assumption that $D(s_1, s_2)$ is $VSHP$, so $D_1(s_1, s_2)$ must be a VSHP.

Similarly, we can prove that $D_2(s_1, s_2)$ must be a VSHP.

Property 3:

If $D(s_1, s_2)$ is VSHP, then $\frac{\vdots \delta D(s_1,s_2)}{\delta s_1}$ and $\frac{\vdots \delta D(s_1,s_2)}{\delta s_2}$ are also VSHPs.

Proof :

We can write $D(s_1, s_2)$ in the following form

$$D(s_1, s_2) = A_m(s_2)s_1^m + A_{m-1}(s_2)s_1^{m-1} + ... + A_1(s_2)s_1 + A_0(s_2) \tag{1.6}$$

or

$$D(s_1, s_2) = B_n(s_1)s_2^n + B_{n-1}(s_1)s_2^{n-1} + ... + B_1(s_1)s_2 + B_0(s_1) \tag{1.7}$$

For any point s_2 in the open halif of $s_2 - plane$ $\{s_2|Re(s_2) \geq 0, |s_2| < \infty\}$,(1.6) is an m^{th} order SHP with respect to s_1 including $|s_1| \leq \infty$. Now we need to check the behaviour at $|s_2| = \infty$.

Differentiating (1.7) with respect s_1, we have

$$\frac{\delta D(s_1, s_2)}{\delta s_1} = B_n^{'}(s_1)s_2^n + B_{n-1}^{'}(s_1)s_2^{n-1} + ... + B_1^{'}(s_1)s_2 + B_0^{'}(s_1) \qquad (1.8)$$

Since (1.8) is known to be a SHP,

$$\frac{\delta D(s_1, s_2)}{\delta s_1} |_{s_2 \rightarrow \frac{1}{s_2}}$$

is also a SHP.

For $s_2 \rightarrow \infty$, we can get

$$\frac{\delta D(s_1, s_2)}{\delta s_1} |_{s_2 \rightarrow \infty} = \frac{B_n^{'}(s_1)}{0} \qquad (1.9)$$

As $B_n(s_1)$ is SHP of s_1, it has no zeros in the closed right-half of $s_1 - plane$ $\{s_1|Re(s_1) \geq 0, |s_1| < \infty\}$. So (1.10) will not be undeterminate. We can get the $\frac{\delta D(s_1,s_2)}{\delta s_1}$ is VSHP.

Similarly, we can prove that $\frac{\delta D(s_1,s_2)}{\delta s_2}$ is also VSHP.

Property 4:

When we express $D(s_1, s_2)$ as (1.6) or (1.7), then both $A_i(s_2)$, $i = 0, 1, 2, ...m$ and $B_j(s_1)$, $j = 0, 1, 2, ...m$ are SHPs of s_2 and s_1 respectively.

Proof :

In (1.6), since $D(s_1, s_2)$ is a VSHP, from Property 1.3, $A_0(s_2)$ is a SHP in s_2, which is obtained by setting $s_1 = 0$. Differentiating (1.6) partially with respect to $s_1$and putting $s_1 = 0$, we get $A_1(s_2)$ is a SHP in s_2, since, from property 3, we know that $\frac{\delta D(s_1,s_2)}{\delta s_1}$

is a VSHP. By continuing this process, it is established that all the polynomials $A_1(s_2)$, $i = 0, 1, 2, ..., m$ are SHPs in s_2.

Using the same method, by sucessive differentiation with respect to s_2, and then putting $s_2 = 0$, it can be proven that all the polynomials $B_j(s_1)$, $j = 0, 1, 2, ..., n$ are SHPs in s_1.

Property 5 :

If a real 2-variable VSHP can be written as $D(s_1, s_2) = \sum_{i=0}^{m} \sum_{j=0}^{n} d_{ij} s_1^i s_2^j$, then the coefficient $d_{mn} d_{ij} > 0$ for all i and j .

Proof:

The VSHP can be written in the compact form

$$D(s_1, s_2) = \sum_{i=0}^{m} A_i(s_2) s_1^i \tag{1.10}$$

and

$$D(s_1, s_2) = \sum_{j=0}^{n} B_j(s_1) s_2^j \tag{1.11}$$

From Property 4, both $A_i(s_1)$, $(i = 0, 1, ..., m)$ and $B_j(s_2)$, $(j = 0, 1, ..., n)$ are one-variable SHP's.

For the one-variable SHP $A_m(s_2) = \sum_{j=0}^{n} d_{mj} s_2^j$, the coefficients need to be positive or $d_{mn} d_{nj} > 0$ for all $j = 0, 1, 2, ...n - 1$

And for the one-variable SHP $B_j(s_1) = \sum_{i=0}^{m} d_{ij} s_1^i$, the coefficients need to be positive or $d_{mj} d_{ij} > 0$

So, $d_{mn} d_{ij} > 0$, for all i and j. Thus, Property 5 is proved.

Property 6:

If we express $D(s_1, s_2)$ as (1.6) and (1.7), each of the functions $\frac{A_i(s_2)}{A_{i-1}(s_2)}$, $i = 1, 2, ..., m$, is a maximum reactive positive real function in s_2. Also each of the functions $\frac{B_i(s_1)}{B_{j-1}(s_1)}$, $j = 1, 2, ..., n$, is a minimum reactive positive real functions in s_1.

Proof :

Dividing both side of (1.6) by $A_m(s_2)$, we have

$$\frac{1}{A_m(s_2)}D(s_1, s_2) = s_1^m + \frac{A_{m-1}(s_2)}{A_m(s_2)}s_1^{m-1} + ... + \frac{A_1(s_2)}{A_m(s_2)}s_1 + \frac{A_0(s_2)}{A_m(s_2)} \qquad (1.12)$$

For any specified s_2 in the range of $Re(s_2) \geq 0$, (1.12) is a SHP in s_1, which means that it has all its zeros in the open left half of the $s_1 - plane$. Let δ_i $(i - 1, 2, 3, ...m)$ be its zero. Then we have

$$\frac{A_0(s_2)}{A_m(s_2)} = \prod_{i=1}^{m}(-\delta_i)$$

and

$$\frac{A_1(s_2)}{A_m(s_2)} = \sum_{i=1}^{m}\left(\prod_{j=1, j\neq i}^{m}(-\delta_i)\right)$$

So, we have

$$\frac{A_1(s_2)}{A_0(s_2)} = \sum_{i=1}^{m}(-\frac{1}{\delta_i})$$

As $Re(\delta_i) < 0$, $Re\left(\frac{A_1(s_2)}{A_0(s_2)}\right) > 0$ for all $Re(s_2) \geq 0$. So we get the conclusion that $\frac{A_0(s_2)}{A_1(s_2)}$is strict positive real function. In addition, from property 5, $A_0(s_2)$ and $A_1(s_2)$ are SHPs without any missing coefficients. So $\frac{A_0(s_2)}{A_1(s_2)}$ is also minimum reactive function. In the similar manner, we can show that $\frac{A_i(s_2)}{A_{i-1}(s_2)}$ $(i = 1, 2, ..., m)$ are minimum reactive positive real functions in s_2.

Using the similar procedure, we can prove that $\frac{B_j(s_1)}{B_{j-1}(s_1)}$ $(j = 1, 2, ..., n)$ are minimum reactive positive real functions in s_1.

1.9 Generation of VSHP

When VSHP is used in the denominator of a 2-D analog transfer function, it is guaranteed that the resulting 2-D digital bilinear transfer function obtained through the application of the well-known bilinear transformation is stable. Therefore, VSHP is highly useful in 2-D digital filter design. We can first generate a 2-variable Very Strictly Hurwitz Polynomial (VSHP) using its various properties, and assign the generated VSHP to the denominator of the 2-D analog transfer function, then obtain the digital transfer function through double bilinear transformations. Here we review some some methods, which are used to generate VSHP.

1.9.1 Using Terminated n-port Gyrator Networks[11]

For a n-port gyrator network, its ports are termined by capacitances. In much a case, the overall admittance matrix will be

$$A = \begin{bmatrix} \mu_1 & g_{12} & g_{13} & \cdots & g_{1n} \\ -g_{12} & \mu_2 & g_{23} & g_{23} & g_{2n} \\ -g_{13} & -g_{23} & \mu_3 & \cdots & g_{3n} \\ \vdots & & & \ddots & \\ -g_{1n} & -g_{2n} & -g_{3n} & \cdots & \mu \end{bmatrix} \tag{1.13}$$

The determinant of the matrix A can be expressed as

$$D_n = \sum_{1 \leq i \leq n} \mu_i \left|A_i\right| + \sum_{1 \leq i_1 < i_2 < i_3 < n} \mu_{i_1}\mu_{i_2}\mu_{i_3} \left|A_{i_1 i_2 i_3}\right| + \cdots + \mu_1\mu_2\mu_3 \cdots \mu_n \qquad (n\ is\ odd) \tag{1.14}$$

$$or \quad D_n = |A_n| + \sum_{1 \le i_1 < i_2 < n} \mu_{i_1}\mu_{i_2} |A_{i_1 i_2}| + \sum_{1 \le i_1 < i_2 < i_3 < i_4 < n} \mu_{i_1}\mu_{i_2}\mu_{i_3}\mu_{i_4} |A_{i_1 i_2 i_3}| + \cdots + \mu_1\mu_2\mu_3 \cdots \mu_n (niseve \tag{1.15}$$

where $|A_{i_1 i_2}|$ is the determinant of the sub-matrix of A obtained by deleting both i_1^{th} and i_2^{th} rows and columns, the same holds for $|A_{i_1 i_2 i_3}|$,$|A_{i_2 i_3 i_4}|$, etc .

By making some of the $\mu_i s$ equal to s_1, and some of $\mu_i s$ equal to s_2, under certain conditions, (1.14) and (1.15) will yield two-variable VSHPs.

1.9.2 Using the properties of positive semi-defi nite matrices [12]

In this case, we first define three $n \times n$ square matrices A, μ and G as

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{bmatrix} \tag{1.16}$$

$$\mu = \begin{bmatrix} \mu_1 & & & & 0 \\ & \mu_2 & & & \\ & & \mu_3 & & \\ & & & \ddots & \\ 0 & & & & \mu_n \end{bmatrix} \tag{1.17}$$

$$
G = \begin{bmatrix} 0 & g_{12} & g_{13} & \cdots & g_{1n} \\ -g_{12} & 0 & g_{23} & g_{23} & g_{2n} \\ -g_{13} & -g_{23} & 0 & \cdots & g_{3n} \\ \vdots & & & \ddots & \\ -g_{1n} & -g_{2n} & -g_{3n} & \cdots & 0 \end{bmatrix} \tag{1.18}
$$

where A is a general symmetrical $n \times n$ square matrix

μ is an $n \times n$ diagonal matrix

G is an $n \times n$ skew-symmetric matrix

These matrixes A, μ and G are physically realizable.

Now, we define a matrix C as:

$$
C = A\mu A^T + G \tag{1.19}
$$

The determinant of matrix C is given by

$$
M = det(C) \tag{1.20}
$$

The polynomial M_n

$$
M_n = M + \sum_{j=1}^{n} k_j \frac{\partial M}{\partial \mu_i} \tag{1.21}
$$

is a two-variable VSHP when some of the μ_i's are properly made equal to s_1 and some of the μ_i's equal to s_2 and it is made sure the requirements of $VSHP$ are met.

1.9.3 Using the properties of the derivative of even or odd parts of Hurwitz polynomial [13, 14]

From (1.16) and (1.17), one can obtain an n^{th} order polynomial M_n as

$$M_n = det[\mu I + A] \tag{1.22}$$

From, the diagonal expansion of the determinant of matrix, M_n can be written as (1.14) and (1.15). We can observe that M_n is the odd (even) part of a $n - variable$ Hurwitz polynomial when n is odd (even), So $\frac{\partial M_n / \partial \mu_i}{M_n}$ is a reactance function. Therfore,

$$M = M_n + \sum_{j=1}^{n} K_j \left(\frac{\partial M_n}{\partial \mu_i} \right) \tag{1.23}$$

is a n-variable Hurwitz polynomial.

Assigning some of $\mu_i^{'}s$ to s_1 and some to be s_2, and ensuring the conditions of two-variable VSHP, a two-variable VSHP could be generated from (1.23).

1.10 Scope and Organization of the Thesis

The objective of the thesis is to propose a new approach in the design of a 2-D digital filter, which has variable magnitude characteristics in the frequency domain by performing the sensitivity analyses. This approach is based on the generalized bilinear transformation method. To get the digital filter with variable characteristics, sensitivity analyses are performed by varying one or more of the coefficients of the digital transfer function. The generalized double bilinear transformation is one of the processes that can introduce variable coefficients into the transfer function of the resulting digital filters.

In Chapter 2, two-dimensional band-pass digital filters are designed and analyzed. First, the second order Butterworth low-pass ladder structure is taken, the values of the inductor and capacitor are calculated. Analog transfer function of a Band-Pass filter is obtained using the low-pass to band-pass transformation. The inductor and capacitor values are determined. Then, double generalized bilinear transformation is applied to the analog transfer function to get its digital transfer function. Also in this chapter, the stability for the 2-D digital transfer function are considered. Using the links between the stability conditions to the coefficients of the double generaliezed bilinear transformations, we get the conditions for each of these coefficients. The effect of each coeffiecient on the resulting 2-D band-pass digital filter's magnitude response are studied in detail.

The 2-D Band-Stop digital filters having variable magnitude characteristics are studied in Chapter 3. Starting from the same analog low-pass transfer function as in Chapter 2, low-pass to band-stop transformation is applied and the resulting analog transfer function is digitized using the same double bilinear transformation. The coefficients are constrained by the stability condition for digital filters introduced in Chapter 2. The manner in which each double generalized bilinear transformation coefficient effects the digital filters magnitude responses are studied in detail.

Chapter 4 discusses the application of the band-pass and the band-stop filter. The summary, conclusions and the directions for the future work are given in Chapter 5.

Chapter 2

Two-Dimensional Band-Pass Filters from Low-Pass Butterworth fi lters

In this chapter, the 2-D band-pass filters are analyzed. The low-pass butterworth filter is transformed to a band-pass filter, and it is transformed to a digital filter by using the generalized Bilinear transformation. Then, sensitivity analyses are performed by varying the coefficient of the generalized bilinear transformation and the effect of the coefficient are observed in the design of 2-D band-pass filter. Section 2.1 gives the brief introduction to 2-D low-pass butterworth filter. Section 2.2 describes the low-pass to band- pass transformation. The band-pass structure, its transfer function and the determination of parameter values are explained in section 2.3 and 2.4 respectively. Section 2.5 describes the generalized bilnear transformation. Section 2.6 gives the detail frequency responses of band -pass filter on varying the different bilinear transformation coefficients. Finally, section 2.7 gives the summary and discussions of the results contained in this chapter.

2.1 Butterworth Low-Pass Filters[2, 30]

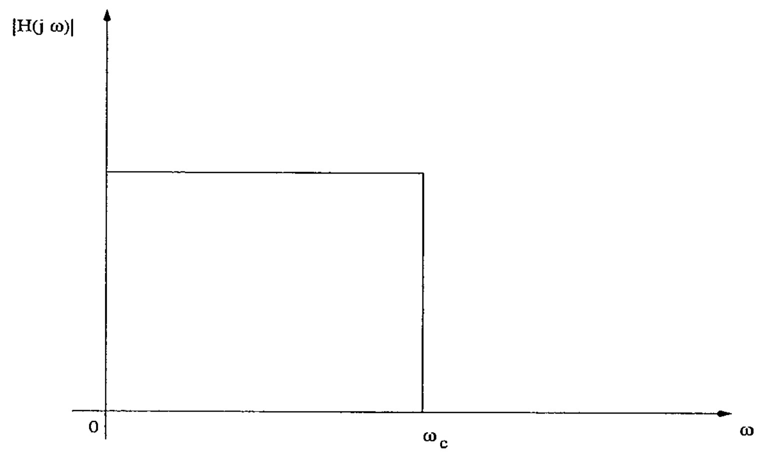

Figure 2.1: An ideal low-pass magnitude response

The ideal low-pass amplitude response is shown in fig 2.1, in which ω_c is the normalized cutoff frequency. The amplitude function $|H(j\omega)|$ is an even function since it is the square root of the sum of the squares of the real and imaginary parts of $|H(j\omega)|$, which are respectively, even and odd functions. Therefore $|H(j\omega)|$ is a function of ω^2. A realizable $|H(j\omega)|^2$ which approximates the ideal case of fig 2.1 is given by

$$|H(j\omega)|^2 = \frac{A}{1 + f(\omega^2)} \tag{2.1}$$

whe

$$f(\omega^2) \gg 1, \quad \omega > \omega_c$$

$$0 \leq f(\omega^2) \ll 1, \quad 0 \leq \omega < \omega_c \tag{2.2}$$

This is evidently true since in the passband, $0 \leq \omega < 1$, we have $|H(j\omega)|^2 \approx 1$, and in the stopband, $\omega > 1$, we have $|H(j\omega)|^2 \approx 0$, when $f(\omega)^2$ is given by eqn.(2.2).

One function suitable for use in eqns. (2.1) and (2.2) is given by

$$f(\omega^2) = \omega^{2n}; \quad n = 1, 2, 3, ... \tag{2.3}$$

It was first suggested by Butterworth. In this case

$$|H(j\omega)| = \frac{1}{\sqrt{1+\omega^{2n}}}; \quad n = 1, 2, 3, ... \tag{2.4}$$

which is defined as the amplitude response of the $nth-order\ Butterworth$ filter. This is a montonically decreasing function and thus it attains its maximum value, $|H(j\omega)|_{max} = 1$, at $\omega = 0$. The approximation in eqn. (2.4) is better for higher values of n, since for $n1 \gg n2$, we have $\omega^{2n_1} \gg \omega^{2n_2}$ for $\omega > 1$. It is particularly good for near $\omega = 0$, by expanding eqn. (2.4) using the binomial theorem; this results in

$$|H(j\omega)| = 1 - \frac{1}{2}\omega^{2n} + \frac{3}{8}\omega^{4n} - \frac{5}{16}\omega^{6n} + \frac{35}{128}\omega^{8n} - ... \tag{2.5}$$

which is valid for ω near 0. The first $2n - 1$ derivatives of $|H(j\omega)|$ in eqn (2.5) will contain a factor ω, and thus will be zero at $\omega = 0$. Therefore for n large, the function $|H(j\omega)|$ near $\omega = 0$ is exceedingly flat, or as it is $maximally\ flat$.

For $\omega \gg 1$, the Butterworth amplitude function may be approximated by

$$|H(j\omega)| \approx \frac{1}{\omega^n}$$

with the loss in dB given by

$$\alpha_{dB}(\omega) \approx 20log_{10}\omega^n = 20nlog_{10}\omega \tag{2.6}$$

Thus if the loss is plotted versus ω in decades, then for large ω, the loss $\alpha_{dB}(\omega)$ has a slope of $20n$ dB/decade. The loss thus increases rapidly for large n, which indicates a good approximation to the ideal case [23].

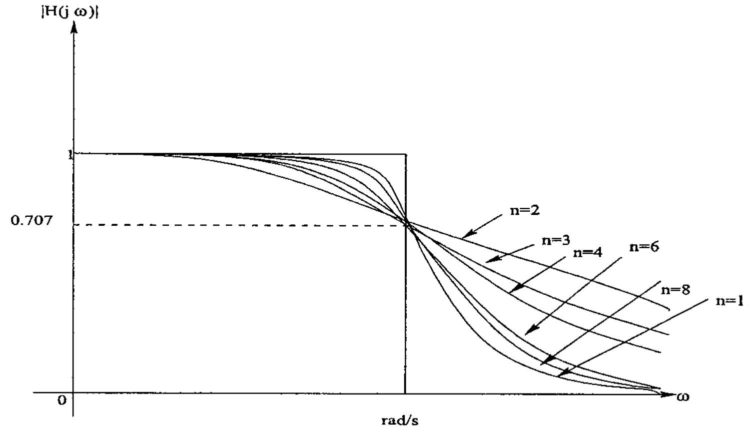

Figure 2.2: Butterworth amplitude responses.

Plots of $|H(j\omega)|$ and $\alpha_{dB}(\omega)$ are shown in fig. 2.2 and 2.3 for various value of n. Evidently the approximation to the ideal amplitude improves as n increases.

The Transfer function of Butterworth filter whose amplitude is given in eqn (2.4) is obtained by replacing ω^2 by $-s^2$ in $|H(j\omega)|^2$

$$H(s)H(-s) = \frac{1}{1 + (-s^2)^n}$$

so that

$$H(s) = \frac{1}{Q(s)} \tag{2.7}$$

where $Q(s)$ is the *Hurwitz* polynomial satisfying

$$Q(s)Q(-s) = 1 + (-s^2)^n$$

As an example, if $n = 2$, the case of the second-order Butterworth filter becomes

$$Q(s)Q(-s) = 1 + s^4$$

which may writen as

$$Q(s)Q(-s) = (s^2 + \sqrt{2}s + 1)(s^2 - \sqrt{2}s + 1)$$

The first factor in the right member is the *Hurwitz* factor and is therefore $Q(s)$. Thus

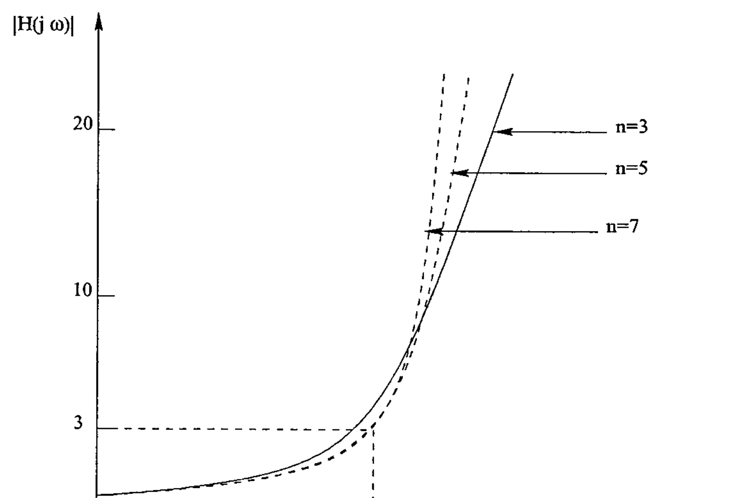

Figure 2.3: Butterworth loss curves.

the Transfer function of the second-order Butterworth low-pass filte

$$H(s) = \frac{K}{(s^2 + \sqrt{2}s + 1)} \tag{2.8}$$

where the constant K should be of any real numbers.

2.2 Two-Dimensional Band-pass fi lter

2.2.1 Low-Pass to Band-Pass Transformation

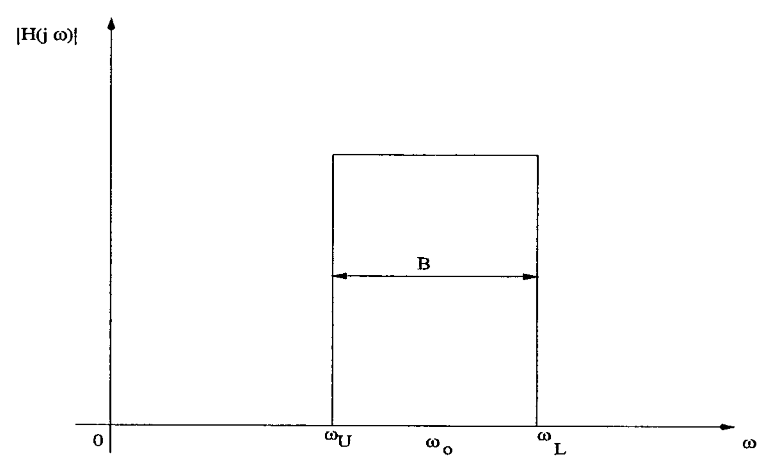

Figure 2.4: Ideal band-pass filters

To transform a low-pass prototype to a bandpass filter with the center frequency ω_o and bandwidth $B = \omega_U - \omega_L$, the transformation must map the ideal low-pass response of fig 2.1 into ideal bandpass response of fig 2.4. That is, (i) $s = j0$ must map into $s = j\omega_o$, (ii) $s = j1$ must map into $s = j\omega_U$, and (iii) $s = -j1$ must map into $s = j\omega_L$. Since $S = j\infty$

maps into $s = j\infty$, the function $F(s)$ must be a rational fraction,

$$S = F(s) = \frac{as^2 + bs + c}{ds + e} \tag{2.9}$$

Applying mapping (i) above to eqn. (2.9) results in

$$\frac{-a\omega_o^2 + jb\omega_o + c}{jd\omega_o + e} = 0$$

Equating the real and imaginary parts to 0 results in

$$c = a\omega_o^2, \quad b = 0 \tag{2.10}$$

Applying mapping (ii) and using eqn. (2.10) results in,

$$e = 0 \quad d = \frac{a(\omega_U^2 - \omega_o^2)}{\omega_U} \tag{2.11}$$

Mapping (c) results finally in

$$d = \frac{a(\omega_o^2 - \omega_L^2)}{\omega_L} \tag{2.12}$$

Equating the two values of d in eqn (2.11) and (2.12) yields

$$\omega_o^2 = \frac{\omega_L\omega_U^2 + \omega_U\omega_L^2}{\omega_U + \omega_L} = \omega_L\omega_U \tag{2.13}$$

Thus under this transformation ω_0 is the geometric mean of ω_L and ω_U. Using (2.10), (2.11) and (2.12), the transformation eqn. (2.9) becomes

$$s = \frac{a(s^2 + \omega_o^2)}{\left[\frac{a(\omega_U^2 - \omega_o^2)s}{\omega_U}\right]}$$

$$= \frac{s^2 + \omega_o^2}{(\omega_U - \omega_L)s}$$

or

$$S = \frac{s^2 + \omega_o^2}{Bs} \tag{2.14}$$

Thus a transfer function of a 4^{th} *order* bandpass filter with center frequency ω_o and bandwidth B is obtained by replacing S by eqn. (2.14) in (2.8). The numerator will be constant times s^n and the denominator will be a 4^{th} degree polynomial. This transformation can be applied to any order[2, 21, 30].

2.2.2 Band-Pass Filter Structure

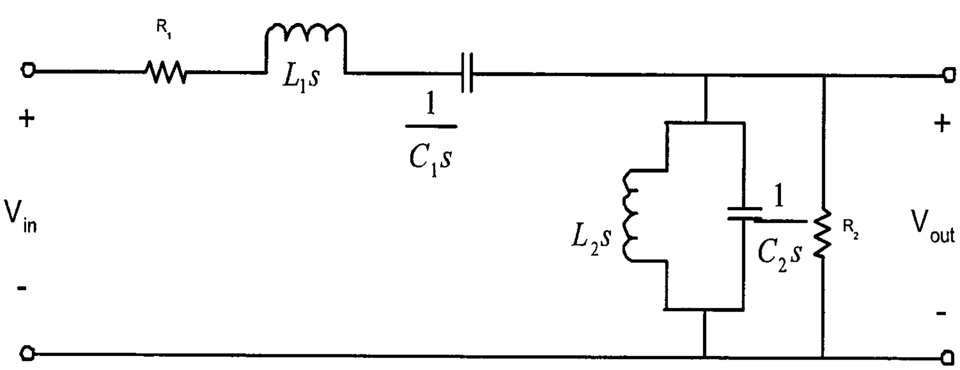

Figure 2.5: Band-Pass Filter structure

The fig. 2.5 shows the 4^{th} *order* Band-Pass filter structure.

The transfer function of this structure is given by

$$H_B(s) = \frac{R_2L_2C_1s^2}{(s^2L_1C_1 + sR_1C_1 + 1)(s^2R_2L_2C_2 + L_2s + R_2) + R_2L_2C_1s^2} \tag{2.15}$$

If the above band-pass structure is considered as $2-D$ Band-Pass filter, there are several ways of converting this into a 2-D band-pass transfer function. However, one of the ways is to convert the series arm into an impedeance in s_1 and the shunt arm into an impedance s_2. Then, the transfer function will be given as

$$H_B(s_1, s_2) = \frac{R_2L_2C_1s_1s_2}{(s_1^2L_1C_1 + s_1R_1C_1 + 1)(s_2^2R_2L_2C_2 + L_2s_2 + R_2) + R_2L_2C_1s_1s_2} \tag{2.16}$$

2.3 Determining the paramater values of the Band-Pass structure

The 4^{th} $order$ Band-Pass filter transfer function is given by

$$H_B(s) = \frac{R_2L_2C_1s^2}{(s^2L_1C_1 + sR_1C_1 + 1)(s^2R_2L_2C_2 + L_2s + R_2) + R_2L_2C_1s^2} \tag{2.17}$$

The *second order* Butterworth Low-Pass filter is shown in fig. 2.6 and its transfer function is given by [2, 30]

$$H_L(S) = \frac{K}{(S^2 + \sqrt{2}S + 1)} \tag{2.18}$$

By applying Band-Pass frequency transformation to the Low-Pass Butterworth filter in

(2.18) ,

$$H_B(s) = \frac{KB^2s^2}{s^4 + \sqrt{2}Bs^3 + (2\omega_o^2 + B^2)s^2 + \sqrt{2}\omega_o^2 Bs + \omega_o^4} \tag{2.19}$$

Because of the transformation the elements of the Low-Pass filter will change to Band-Pass filter elements. The inductance of the low-pass filter will transfrom into series combination of inductance and capacitance. Comparing the eqns. (2.16) and (2.19), we get

$$\frac{Ls}{B} + \frac{L\omega_o^2}{Bs} = L_1 s + \frac{1}{C_1 s} \tag{2.20}$$

From eqn. (2.20), the L_1 and C_1 values are

$$L_1 = \frac{L}{B}$$

$$C_1 = \frac{B}{\sqrt{2}\omega_o^2}$$

And also the capacitance of the Low-Pass filter will transform into parallel combination

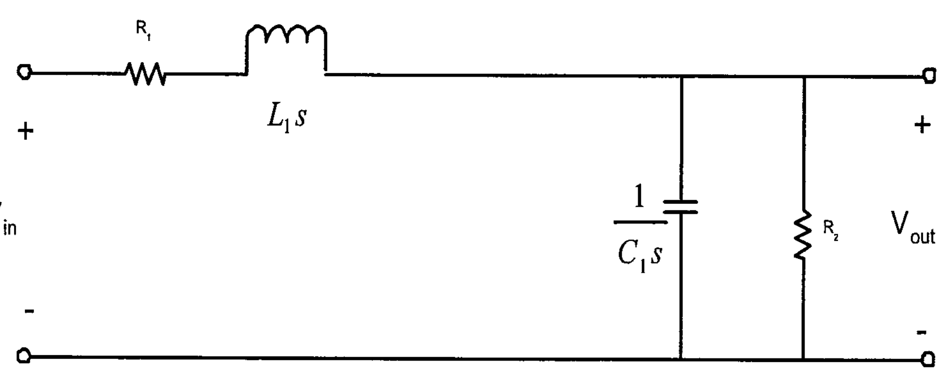

Figure 2.6: Low-Pass Filter Structure

of Inductance and capacitance.

$$\frac{1}{\frac{Cs}{B}+\frac{C\omega_o^2}{Bs}}=\frac{1}{C_2 s+\frac{1}{L_2 s}}$$

$$L_2=\frac{B}{\sqrt{2}\omega_o^2}$$

$$C_2=\frac{C}{B}$$

Therefore, the parameter values of the Band-Pass structure are

$$L_1=\frac{L}{B}$$

$$L_2=\frac{B}{\sqrt{2}\omega_o^2}$$

$$C_1=\frac{B}{\sqrt{2}\omega_o^2}$$

$$C_2=\frac{C}{B}$$

where L and C are the values of Inductance and capacitance of the Low -Pass filter structure shown in fig. 2.6. The Values of L and C are $\sqrt{2}$ each.

2.4 Digital Transfer Function

2.4.1 Band-Pass Limits for the Coefficeints of Generalized Bilinear Transformation

In the previous section, we have obtained the 2-D analog transfer function, eqn (2.16), for the circuit shown in fig. 2.5. Substituting the values of all the parameters in eqn (2.16), it becomes

$$H_B(s_1, s_2) = \frac{B^2 s_1 s_2}{4\omega_o^4 s_1^2 s_2^2 + (s_1 s_2^2 + s_2 s_1^2)2\sqrt{2}B\omega_o^4 + s_1 s_2(B^2 + 2B^2\omega_o^4) + (s_1^2 + s_2^2)4\omega_o^4 + (s_1 + s_2)2\sqrt{2}B\omega_o^6 + 4\omega_o^8} \tag{2.21}$$

When the generalized bilinear transformation is applied, it has to be ensured that the resulting 2-D digital filter is always stable. To satisfy the conditions of obtaining stable 2-D band-pass digital filters, the coefficients of the double generalized bilinear transformations should meet the following requirements [8, 26]

$$(i)\ k_i > 0 \qquad i = 1, 2 \tag{2.22}$$

$$(ii)\ 0 < |a_i| \leq 1 \qquad i = 1, 2 \tag{2.23}$$

$$(iii)\ 0 < |b_i| \leq 1 \qquad i = 1, 2 \tag{2.24}$$

With these constraints from eqns (2.22-2.24), we can obtain 2-D stable band-pass digital filter from its analog counterpart by the application of the generalized bilinear transformations.

2.4.2 2-D Band-Pass Digital Transfer Function from Double Generalized Bilinear Transformation

A familiar technique of designing a discrete filter is to start from an analog filter and then apply the bilinear transformation in order to obtain the discrete transfer function[8, 28]. This can also be applied to 2-D systems by

$$s_i = k_i \frac{z_i - a_i}{z_i + b_i} \tag{2.25}$$

using bilinear transformations to both the variables, where $i = 1, 2...$

The $1 - D$ digital band-pass filter transfer function is obtained by applying the eqn (2.25) to eqn (2.16), the transfer function is

$$H(z) = \frac{R_2 L_2 C_1 k^2 (z-a)^2}{\{[k^2(z-a)^2 L_1 C_1 + k(z-a) R_1 C_1 + (z+b)^2]} \tag{2.26}$$

$$[k^2(z-a)^2 R_2 L_2 C_2 + k(z-a) L_2 + R_2(z+b)^2] + R_2 L_2 C_1 k^2 (z-a)^2\}$$

The 2-D digital band-pass filter transfer function is obtained by applying the eqn (2.25) to eqn (2.17), the transfer function is

$$H(z_1, z_2) = \frac{R_2 L_2 C_1 k_1 k_2 (z_1 - a_1)(z_2 - a_2)(z_1 + b_1)(z_2 + b_2)}{\{[k_1^2(z_1 - a_1)^2 L_1 C_1 + k_1(z_1 - a_1) R_1 C_1 + (z_1 + b_1)^2]} \tag{2.27}$$

$$[k_2^2(z_2 - a_2)^2 R_2 L_2 C_2 + k_2(z_2 - a_2) L_2 + R_2(z_2 + b_2)^2]$$
$$+ R_2 L_2 C_1 k_1 k_2 (z_1 - a_1)(z_2 - a_2)(z_1 + b_1)(z_2 +$$
$$b_2)\}$$

The MATLAB function *d2band.m* (refer to APPENDIX) can be employed to get the contour and 3-D magnitude plots of the resulting 2-D band-pass digital filters with the transfer function (2.27). In this function, it is made certain that stability conditions are satisfied. As the changeable coefficients of the double generalized bilinear transformations can change the stability of the resulting 2-D digital filters, it is necessary to introduce stability constraints in discrete domain.

2.4.3 Stability Conditions of the 2-D Digital Band-Pass Filter

Starting from a 2-D analog transfer function with a VSHP as its denominator, applying the well-known used bilinear transformation will always results in 2-D stable digital filter. However, when the generalized bilinear transformation is applied, it has to be ensured that the resulting 2-D digital filter is always stable[6, 17, 18].

The inverse generalized bilinear transformation is

$$z_i \to \frac{b_i s_i + k_i a_i}{k_i - s_i},\ i = 1, 2 \tag{2.28}$$

Substituting eqn 2.28 to 2.25, we get

$$s_i \to k_i \frac{[s_i(1 + a_i) + (1 - a_i)]}{[s_i(1 - b_i) + (1 + b_i)]} \tag{2.29}$$

where the eqn (2.29) can be rewritten as

$$s_1 \to \frac{s_1\alpha_1 + \alpha_2}{s_1\alpha_4 + \alpha_3};\quad s_2 \to \frac{s_2\beta_1 + \beta_2}{s_2\beta_4 + \beta_3}$$

where

$$\alpha_1 = k_1 + k_1 a_1; \alpha_2 = k_1 - k_1 a_1; \alpha_3 = 1 + b_1; \alpha_4 = 1 - b_1 \tag{2.30}$$

$$\beta_1 = k_2 + k_2 a_2; \beta_2 = k_2 - k_2 a_2; \beta_3 = 1 + b_2; \beta_4 = 1 - b_2 \tag{2.31}$$

Applying (2.29) to $\frac{1}{D_B(s_1,s_2)}$, where $D_B(s_1, s_2)$ is the denomoinator in (2.21), we can obtain

$$D_B(s_1, s_2) = \zeta_1 s_1^2 s_2^2 + \zeta_2 s_1 s_2^2 + \zeta_3 s_1^2 s_2 + \zeta_4 s_1 s_2 + \zeta_5 s_1^2 + \zeta_6 s_2^2 + \zeta_7 s_1 + \zeta_8 s_2 + \zeta_9 \tag{2.32}$$

where

$$\zeta_1 = \alpha_1^2\beta_1^2\gamma_1+\alpha_1\alpha_4\beta_1^2\gamma_2+\alpha_1^2\beta_1\beta_4\gamma_3+\alpha_1\alpha_4\beta_1\beta_4\gamma_4+\alpha_1^2\beta_4^2\gamma_5+\alpha_4^2\beta_1^2\gamma_6+\alpha_1\alpha_4\beta_4^2\gamma_7+\alpha_4^2\beta_1\beta_4\gamma_8+\alpha_4^2\beta_4^2\gamma_9 \tag{2.33}$$

$$\begin{aligned}\zeta_2 = {} & 2\alpha_1\alpha_2\beta_1^2\gamma_1+(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_1^2\gamma_2+2\alpha_1\alpha_2\beta_1\beta_4\gamma_3+(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_1\beta_4\gamma_4+2\alpha_1\alpha_2\beta_4^2\gamma_5+2\alpha_3\alpha_4\beta_1^2\gamma_6+ \\ & (\alpha_2\alpha_4 + \alpha_1\alpha_3)\beta_4^2\gamma_7 + 2\alpha_3\alpha_4\beta_1\beta_4\gamma_8 + 2\alpha_3\alpha_4\beta_4^2\gamma_9\end{aligned} \tag{2.34}$$

$$\begin{aligned}\zeta_3 = {} & 2\alpha_1^2\beta_1\beta_2\gamma_1+2\alpha_1\alpha_4\beta_1\beta_2\gamma_2+\alpha_1^2(\beta_2\beta_4+\beta_1\beta_3)\gamma_3+\alpha_1\alpha_4(\beta_2\beta_4+\beta_1\beta_3)\gamma_4+2\alpha_1^2\beta_3\beta_4\gamma_5+2\alpha_4^2\beta_1\beta_2\gamma_6+ \\ & 2\alpha_1\alpha_4\beta_3\beta_4\gamma_7 + \alpha_4^2(\beta_2\beta_4 + \beta_1\beta_3)\gamma_8 + 2\alpha_4^2\beta_3\beta_4\gamma_9\end{aligned} \tag{2.35}$$

$$\begin{aligned}\zeta_4 = {} & 4\alpha_1\alpha_2\beta_1\beta_2\gamma_1+2\beta_1\beta_2(\alpha_2\alpha_4+\alpha_1\alpha_3)\gamma_2+2\alpha_1\alpha_2(\beta_2\beta_4+\beta_1\beta_3)\gamma_3+(\alpha_2\alpha_4+\alpha_1\alpha_3)(\beta_2\beta_4+\beta_1\beta_3)\gamma_4+ \\ & 4\alpha_1\alpha_2\beta_3\beta_4\gamma_5+4\alpha_3\alpha_4\beta_1\beta_2\gamma_6+2(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_3\beta_4\gamma_7+2\alpha_3\alpha_4(\beta_1\beta_3+\beta_2\beta_4)\gamma_8+4\alpha_3\alpha_4\beta_3\beta_4\gamma_9\end{aligned} \tag{2.36}$$

$$\zeta_5 = \alpha_1^2\beta_2^2\gamma_1+\alpha_1\alpha_4\beta_2^2\gamma_2+\alpha_1^2\beta_2\beta_3\gamma_3+\alpha_1\alpha_4\beta_2\beta_3\gamma_4+\alpha_1^2\beta_3^2\gamma_5+\alpha_4^2\beta_2^2\gamma_6+\alpha_1\alpha_4\beta_3^2\gamma_7+\alpha_4^2\beta_2\beta_3\gamma_8+\alpha_4^2\beta_3^2\gamma_9 \tag{2.37}$$

$$\zeta_6 = \alpha_2^2\beta_1^2\gamma_1+\alpha_2\alpha_3\beta_1^2\gamma_2+\alpha_2^2\beta_1\beta_4\gamma_3+\alpha_2\alpha_3\beta_1\beta_4\gamma_4+\alpha_2^2\beta_4^2\gamma_5+\alpha_3^2\beta_1^2\gamma_6+\alpha_3\alpha_3\beta_4^2\gamma_7+\alpha_3^2\beta_1\beta_4\gamma_8+\alpha_3^2\beta_4^2\gamma_9 \tag{2.38}$$

$$\begin{aligned}\zeta_7 = {} & 2\alpha_1\alpha_2\beta_2^2\gamma_1+(\alpha_1\alpha_3+\alpha_2\alpha_4)\beta_2^2\gamma_2+2\alpha_1\alpha_2\beta_2\beta_3\gamma_3+(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_2\beta_3\gamma_4+2\alpha_1\alpha_2\beta_3^2\gamma_5+ \\ & 2\alpha_3\alpha_4\beta_2^2\gamma_6 + (\alpha_1\alpha_3 + \alpha_2\alpha_4)\beta_3^2\gamma_7 + 2\alpha_3\alpha_4\beta_2\beta_3\gamma_8 + 2\alpha_3\alpha_4\beta_3^2\gamma_9\end{aligned} \tag{2.39}$$

$$\zeta_8 = 2\beta_1\beta_2\alpha_2^2\gamma_1+2\alpha_2\alpha_3\beta_1\beta_2\gamma_2+(\beta_1\beta_3+\beta_2\beta_4)\alpha_2^2\gamma_3+\alpha_2\alpha_3(\beta_2\beta_4+\beta_1\beta_3)\gamma_4+2\alpha_2^2\beta_3\beta_4\gamma_5+2\alpha_3^2\beta_1\beta_2\gamma_6+$$

$$2\alpha_2\alpha_3\beta_3\beta_4\gamma_7 + (\beta_1\beta_3 + \beta_2\beta_4)\alpha_3^2\gamma_8 + 2\alpha_3^2\beta_3\beta_4\gamma_9 \quad (2.40)$$

$$\zeta_9 = \alpha_2^2\beta_2^2\gamma_1+\alpha_2\alpha_3\beta_2^2\gamma_2+\alpha_2^2\beta_2\beta_3\gamma_3+\alpha_2\alpha_3\beta_2\beta_3\gamma_4+\alpha_3^2\beta_3^2\gamma_5+\alpha_3^2\beta_2^2\gamma_6+\alpha_2\alpha_3\beta_3^2\gamma_7+\alpha_3^2\beta_2\beta_3\gamma_8+\alpha_3^2\beta_3^2\gamma_9 \quad (2.41)$$

where $\gamma_1 = 4\omega_o^4; \gamma_2 = 2\sqrt{2}B\omega_o^4; \gamma_3 = 2\sqrt{2}B\omega_o^4; \gamma_4 = B^2 + 2B^2\omega_o^4$

$\gamma_5 = 4\omega_o^4; \gamma_6 = 4\omega_o^4; \gamma_7 = 2\sqrt{2}B\omega_o^6; \gamma_8 = 2\sqrt{2}B\omega_o^6; \gamma_9 = 4\omega_o^8$

In order that $D_B(s_1, s_2)$ is a Very Strict Hurwitz Polynomial (VSHP), the necessary and sufficient conditions are every polynomial coefficient needs to be positive and satisfy VSHP conditons [8, 9]. Hence, the stability conditions in (2.33)-(2.41) becomes

$$\alpha_1^2\beta_1^2\gamma_1+\alpha_1\alpha_4\beta_1^2\gamma_2+\alpha_1^2\beta_1\beta_4\gamma_3+\alpha_1\alpha_4\beta_1\beta_4\gamma_4+\alpha_1^2\beta_4^2\gamma_5+\alpha_4^2\beta_1^2\gamma_6+\alpha_1\alpha_4\beta_4^2\gamma_7+\alpha_4^2\beta_1\beta_4\gamma_8+\alpha_4^2\beta_4^2\gamma_9 > 0 \quad (2.42)$$

$$2\alpha_1\alpha_2\beta_1^2\gamma_1+(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_1^2\gamma_2+2\alpha_1\alpha_2\beta_1\beta_4\gamma_3+(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_1\beta_4\gamma_4+2\alpha_1\alpha_2\beta_4^2\gamma_5+2\alpha_3\alpha_4\beta_1^2\gamma_6+$$

$$(\alpha_2\alpha_4 + \alpha_1\alpha_3)\beta_4^2\gamma_7 + 2\alpha_3\alpha_4\beta_1\beta_4\gamma_8 + 2\alpha_3\alpha_4\beta_4^2\gamma_9 > 0 \quad (2.43)$$

$$2\alpha_1\alpha_2\beta_1^2\gamma_1+(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_1^2\gamma_2+2\alpha_1\alpha_2\beta_1\beta_4\gamma_3+(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_1\beta_4\gamma_4+2\alpha_1\alpha_2\beta_4^2\gamma_5+2\alpha_3\alpha_4\beta_1^2\gamma_6+$$

$$(\alpha_2\alpha_4 + \alpha_1\alpha_3)\beta_4^2\gamma_7 + 2\alpha_3\alpha_4\beta_1\beta_4\gamma_8 + 2\alpha_3\alpha_4\beta_4^2\gamma_9 > 0 \quad (2.44)$$

$$4\alpha_1\alpha_2\beta_1\beta_2\gamma_1+2\beta_1\beta_2(\alpha_2\alpha_4+\alpha_1\alpha_3)\gamma_2+2\alpha_1\alpha_2(\beta_2\beta_4+\beta_1\beta_3)\gamma_3+(\alpha_2\alpha_4+\alpha_1\alpha_3)(\beta_2\beta_4+\beta_1\beta_3)\gamma_4+4\alpha_1\alpha_2\beta_3\beta_4$$

$$4\alpha_3\alpha_4\beta_1\beta_2\gamma_6+2(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_3\beta_4\gamma_7+2\alpha_3\alpha_4(\beta_1\beta_3+\beta_2\beta_4)\gamma_8+4\alpha_3\alpha_4\beta_3\beta_4\gamma_9 > 0 \quad (2.45)$$

$$\alpha_1^2\beta_2^2\gamma_1+\alpha_1\alpha_4\beta_2^2\gamma_2+\alpha_1^2\beta_2\beta_3\gamma_3+\alpha_1\alpha_4\beta_2\beta_3\gamma_4+\alpha_1^2\beta_3^2\gamma_5+\alpha_4^2\beta_2^2\gamma_6+\alpha_1\alpha_4\beta_3^2\gamma_7+\alpha_4^2\beta_2\beta_3\gamma_8+\alpha_4^2\beta_3^2\gamma_9 > 0 \quad (2.46)$$

$$\alpha_2^2\beta_1^2\gamma_1+\alpha_2\alpha_3\beta_1^2\gamma_2+\alpha_2^2\beta_1\beta_4\gamma_3+\alpha_2\alpha_3\beta_1\beta_4\gamma_4+\alpha_2^2\beta_4^2\gamma_5+\alpha_3^2\beta_1^2\gamma_6+\alpha_3\alpha_3\beta_4^2\gamma_7+\alpha_3^2\beta_1\beta_4\gamma_8+\alpha_3^2\beta_4^2\gamma_9 > 0 \tag{2.47}$$

$$2\alpha_1\alpha_2\beta_2^2\gamma_1 + (\alpha_1\alpha_3 + \alpha_2\alpha_4)\beta_2^2\gamma_2 + 2\alpha_1\alpha_2\beta_2\beta_3\gamma_3 + (\alpha_2\alpha_4 + \alpha_1\alpha_3)\beta_2\beta_3\gamma_4 + 2\alpha_1\alpha_2\beta_3^2\gamma_5 +$$

$$2\alpha_3\alpha_4\beta_2^2\gamma_6 + (\alpha_1\alpha_3 + \alpha_2\alpha_4)\beta_3^2\gamma_7 + 2\alpha_3\alpha_4\beta_2\beta_3\gamma_8 + 2\alpha_3\alpha_4\beta_3^2\gamma_9 > 0 \tag{2.48}$$

$$2\beta_1\beta_2\alpha_2^2\gamma_1 + 2\alpha_2\alpha_3\beta_1\beta_2\gamma_2 + (\beta_1\beta_3 + \beta_2\beta_4)\alpha_2^2\gamma_3 + \alpha_2\alpha_3(\beta_2\beta_4 + \beta_1\beta_3)\gamma_4 + 2\alpha_2^2\beta_3\beta_4\gamma_5 +$$

$$2\alpha_3^2\beta_1\beta_2\gamma_6 + 2\alpha_2\alpha_3\beta_3\beta_4\gamma_7 + (\beta_1\beta_3 + \beta_2\beta_4)\alpha_3^2\gamma_8 + 2\alpha_3^2\beta_3\beta_4\gamma_9 > 0 \tag{2.49}$$

$$\alpha_2^2\beta_2^2\gamma_1+\alpha_2\alpha_3\beta_2^2\gamma_2+\alpha_2^2\beta_2\beta_3\gamma_3+\alpha_2\alpha_3\beta_2\beta_3\gamma_4+\alpha_3^2\beta_3^2\gamma_5+\alpha_3^2\beta_2^2\gamma_6+\alpha_2\alpha_3\beta_3^2\gamma_7+\alpha_3^2\beta_2\beta_3\gamma_8+\alpha_3^2\beta_3^2\gamma_9 > 0 \tag{2.50}$$

It can be observed from the eqns (2.42-2.50) is the function of $\alpha's$ and $\beta's$, which is given in eqns (2.30) and (2.31) respectively. As stable generalized bilinear transformation requires to satisfy the eqns (2.22) - (2.24), on applying these conditions to the eqns (2.30) and (2.31), the $\alpha's\ and\ \beta's$ in these equations will always be a positive. Since $\alpha's\ and\ \beta's$ are positive the eqns (2.42-2.50) will also be positve, which will make all the coefficients in the eqn (2.32) positive. Thus the denominator of the band-pass filter transfer function given in eqn (2.21) will have the positive coefficients for each and every polynomials and hence it is VSHP.

2.5 Frequency Response of the 2-D Band -Pass fi lters

The $second - order$ 1-D Lowpass Butterworth filter is transformed to a 1-D Bandpass filter by applying Lowpass to Bandpass transformation. The resultant transfer function is digitized by applying generalized bilinear transformation. Eqn (2.27) gives the transfer function of the 2-D digital band-pass filter transfer function. With the input coefficient of the generalized bilinear transformation, we can obtain the contour and 3-D magnitude plots of the resulting 2-D digital filter[22]. In this, the stability is taken care such that the $2-D$ band-pass filter is stable with these input arguments.

To investigate the manner in which each coefficient of generalized bilinear transformation effects the magnitude response of the resulting $2-D$ band-pass filter, we change the value of some of the coefficients or fixing some of the coefficient to the specific values. It is possible to obtain a $2-D$ band-pass filter when the coefficients are in the limits of $k_i > 0$, $0 < |a_i| \leq 1 and 0 < |b_i| \leq 1$ where $i = 1, 2$. Fig.2.7 shows the standard band-pass filter, which can be obtained by making all the coefficient values to be equal to one. In the following, we will see the effect of these coefficients to the frequency responses of the $2-D$ band-pass filter[25, 28].

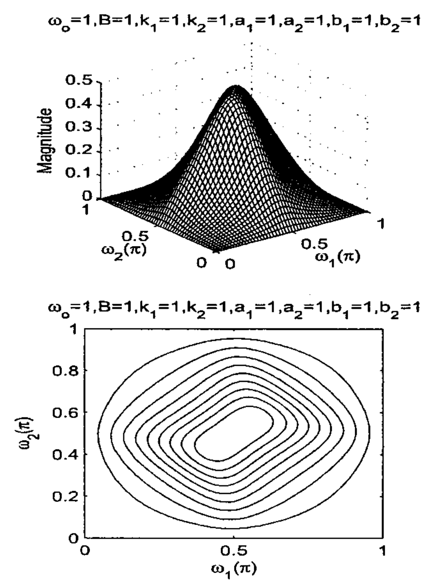

Figure 2.7: Frequency response of the standard 2-D band-pass filter with all the coefficient values as unity

2.5.1 Frequency Response of 2-D Band-Pass filters with various k_1 values

To study the manner how k_1 effects the frequency response behaviour of the resulting $2-D$ band-pass filter and to separate the effect of the other coefficents, we vary the values of k_1, and fixing all the other coefficients of the generalized bilinear transformation to be $k_2 = 1, a_1 = 1, a_2 = 1, b_1 = 1$ and $b_2 = 1$. The value of k_1 are varied from 0.1 to 100 and the 3-D magnitude and contour plots for the band-pass filter with the values of $k_1 = 0.1, k_1 = 0.75, k_1 = 1.5, k_1 = 3, k_1 = 5, k_1 = 10, k_1 = 50$ and $k_1 = 100$ are shown in figs. 2.8 and 2.9.

It can be observed that the coefficient k_1 mainly effects the center frequency(ω_c) and the bandwidth of the band-pass filter response at $\omega_1 - axis$. As k_1 increases, the center frequency moves to the minimum value from the maximum value in the $\omega_1 - axis$. In the diagrams, we can observe that the center frequency for $k_1 = 0.1$ is at 0.9π rad/sec. As the values of k_1 increases, the center frequency will start moving towards the minimum value, as it can be seen for the value of $k_1 = 5$ the center frequency reaches 0.2π rad/sec. If k_1 increases beyond 5, the center frequency will start moving towards the origin, and increase in the value of k_1 beyond 50 will decrease the gain of the response at $\omega_1 - axis$. Also, the bandwidth of the band-pass filter varies for the increase in the value of k_1. It can be observed that, at the lowest possible value of k_1 the bandwidth of this filter is less than 0.5. As k_1 increases, the bandwidth of the filter start increasing and it attains the maximum value at $k_1 = 1$ and decreases thereafter for the increase in value of k_1. We can simply mention it like k_1 is inversely proportional to ω_c at $\omega_1 - axis$, bandwidth increases upto $k_1 = 1$ and decreases for increase in value of $k_1 > 1$, and the coefficient k_1 has no effects on the gains in the pass-band and stop-band for limited value of k_1. Also, the changes in k_1

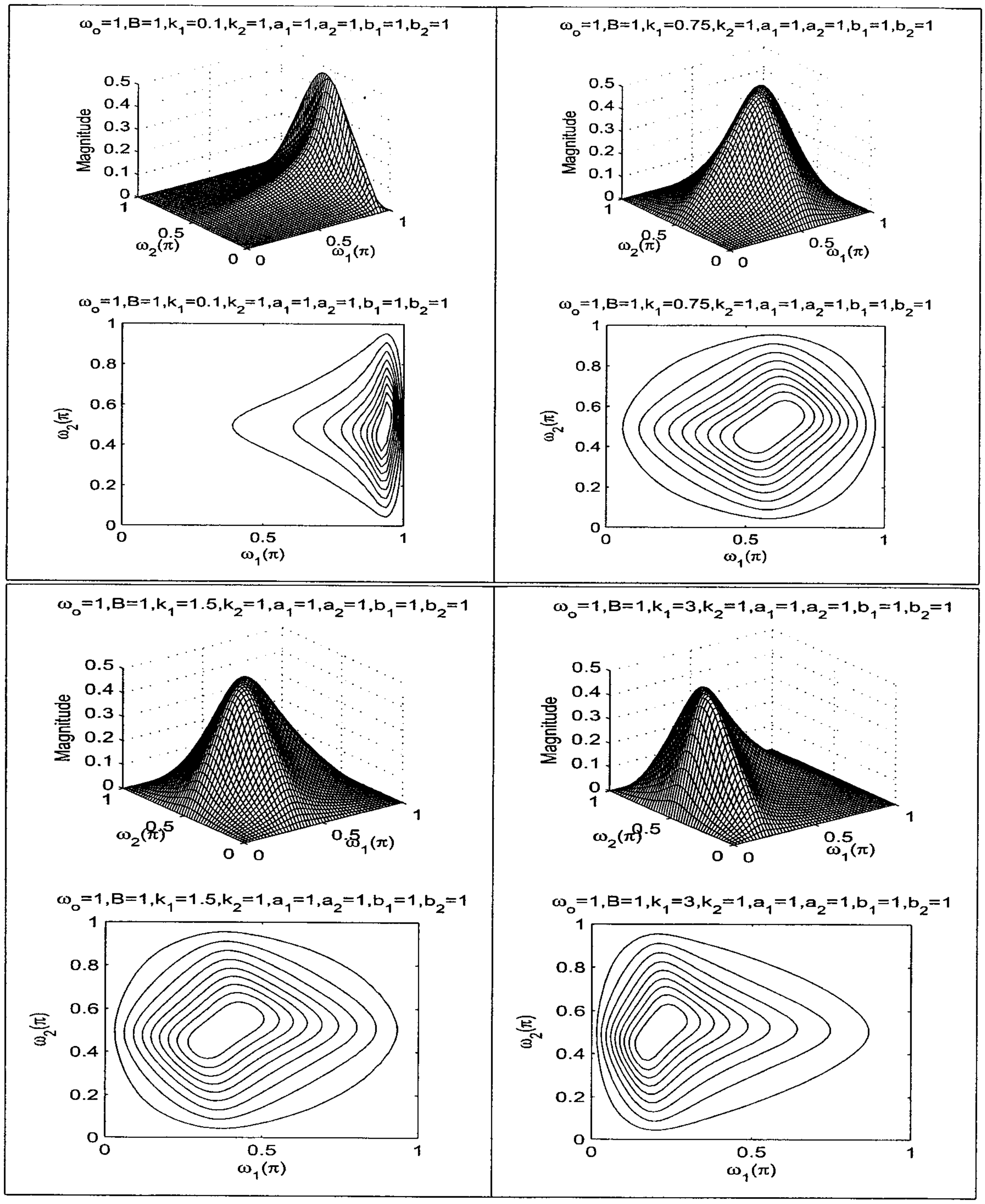

Figure 2.8: Frequency response of the band-pass filter with k_1 =0.1, 0.75, 1.5 and 3 and all the other coefficients as unity.

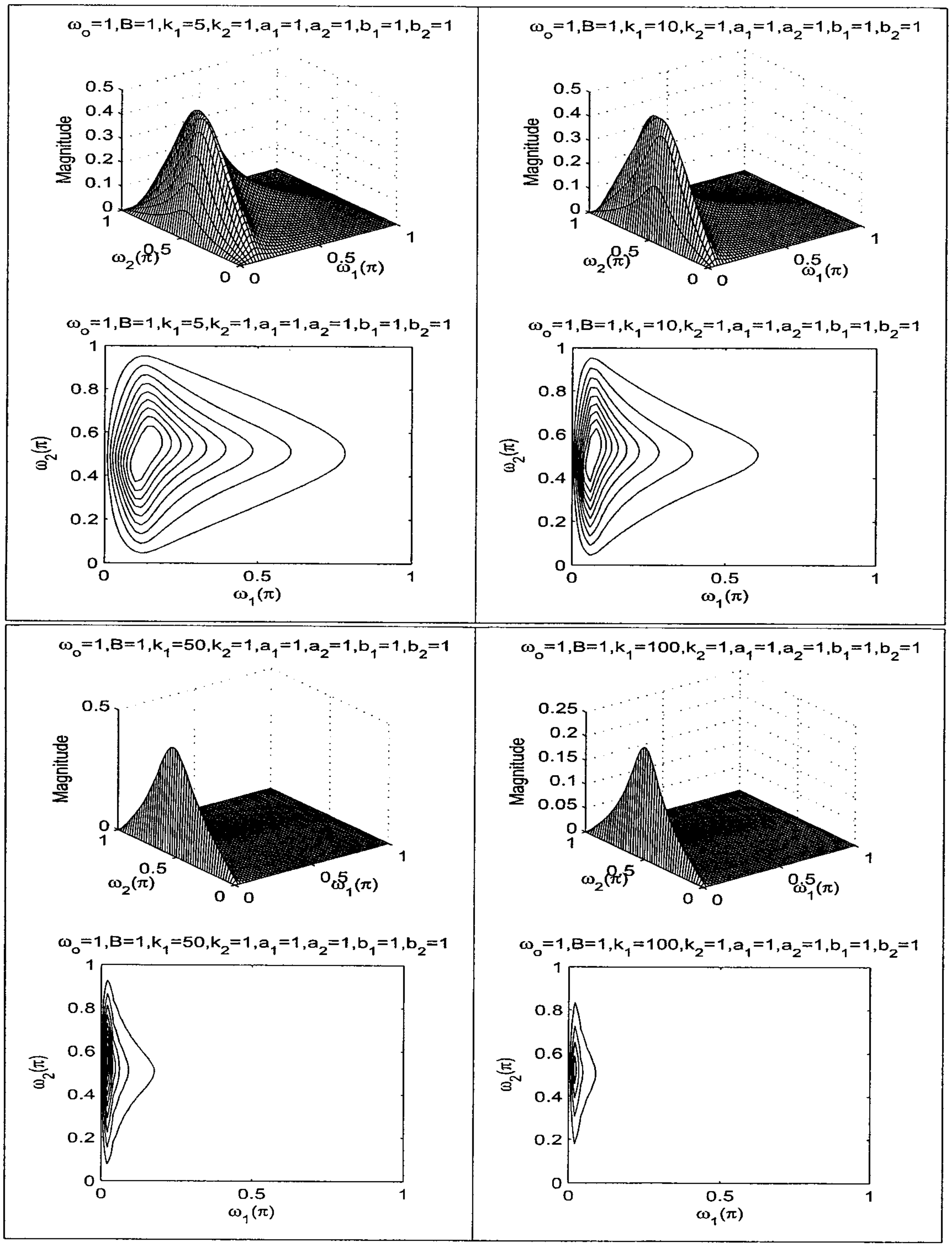

Figure 2.9: Frequency response of the band-pass filter with k_1 =5,10,50 and 100 and all the other coefficient as unity.

values did not make any changes in $\omega_2 - axis$.

2.5.2 Frequency Response of 2-D Band-Pass filters with various k_2 values

In section 2.5.1, the effect of the coefficient of k_1 was analyzed. In this section, effect of the k_2 will be analyzed. To study the manner how k_2 effects the frequency response behaviour of the resulting $2 - D$ band-pass filter and to separate the effect of the other coefficents, we vary the values of k_2, and fixing the other coefficients of the generalized bilinear transformation to be $k_1 = 1, a_1 = 1, a_2 = 1, b_1 = 1$ and $b_2 = 1$.The value of k_2 is varied from 0.1 to 100 and the 3-D magnitude response and contour plots for the band-pass filter with the values of $k_2 = 0.1, k_2 = 0.75, k_2 = 1.5, k_2 = 3, k_2 = 5, k_2 = 10, k_2 = 50$ and $k_2 = 100$ are shown in fig. 2.10 and 2.11.

It can be observed that the coefficient k_2 mainly effects the center frequency(ω_c) and the bandwidth of the band-pass filter response at $\omega_2 - axis$. As k_2 increases, the center frequency moves to the minimum value from the maximum value in the $\omega_2 - axis$. In the diagrams, it can be observed that for the lowest possible value of k_2 the center frequency will be at maximum in the $\omega_2 - axis$, as it can be seen from the diagrams, for $k_2 = 0.1$ the ω_c is at 0.9π rad/sec. As the values of k_2 increases, the center frequency will start moving towards the minimum value, as it can be seen for the value of $k_2 = 5$ the center frequency reaches 0.2π rad/sec. If k_2 increases beyond 5, the center frequency will start moving towards the origin, and increase in the value of k_2 beyond 50 will decrease the gain of the response at $\omega_2 - axis$. Also, the bandwidth of the band-pass filter varies for the increase in the value of k_2. It can be observed that, at the lowest possible value of k_2 the bandwidth of this filter is less than 0.5. As k_2 increases, the bandwidth of the filter start increasing and it

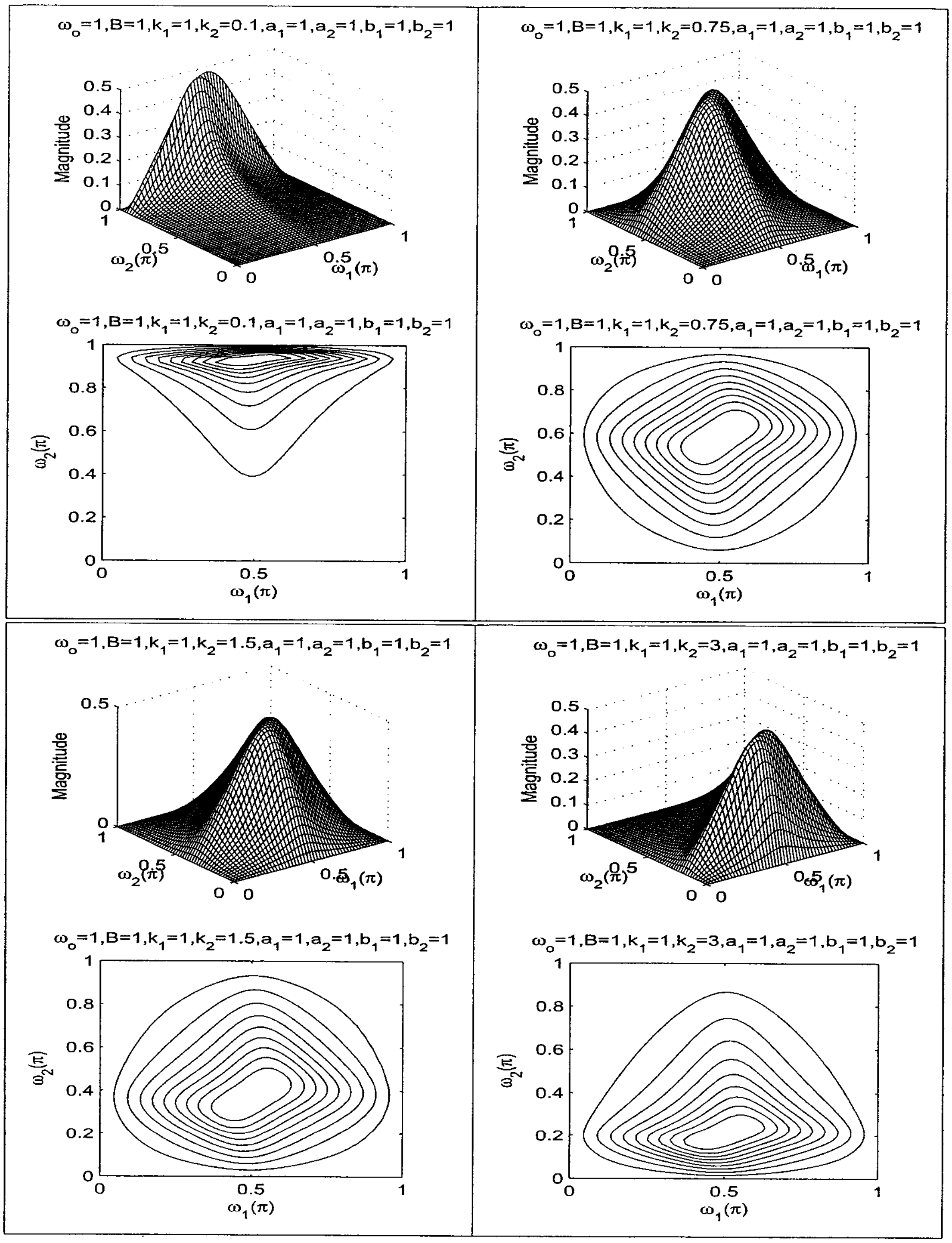

Figure 2.10: Frequency response for the band-pass filter with the value of $k_2 = 0.1, 0.75, 1.5$ and 3, and all the other coefficients as unity.

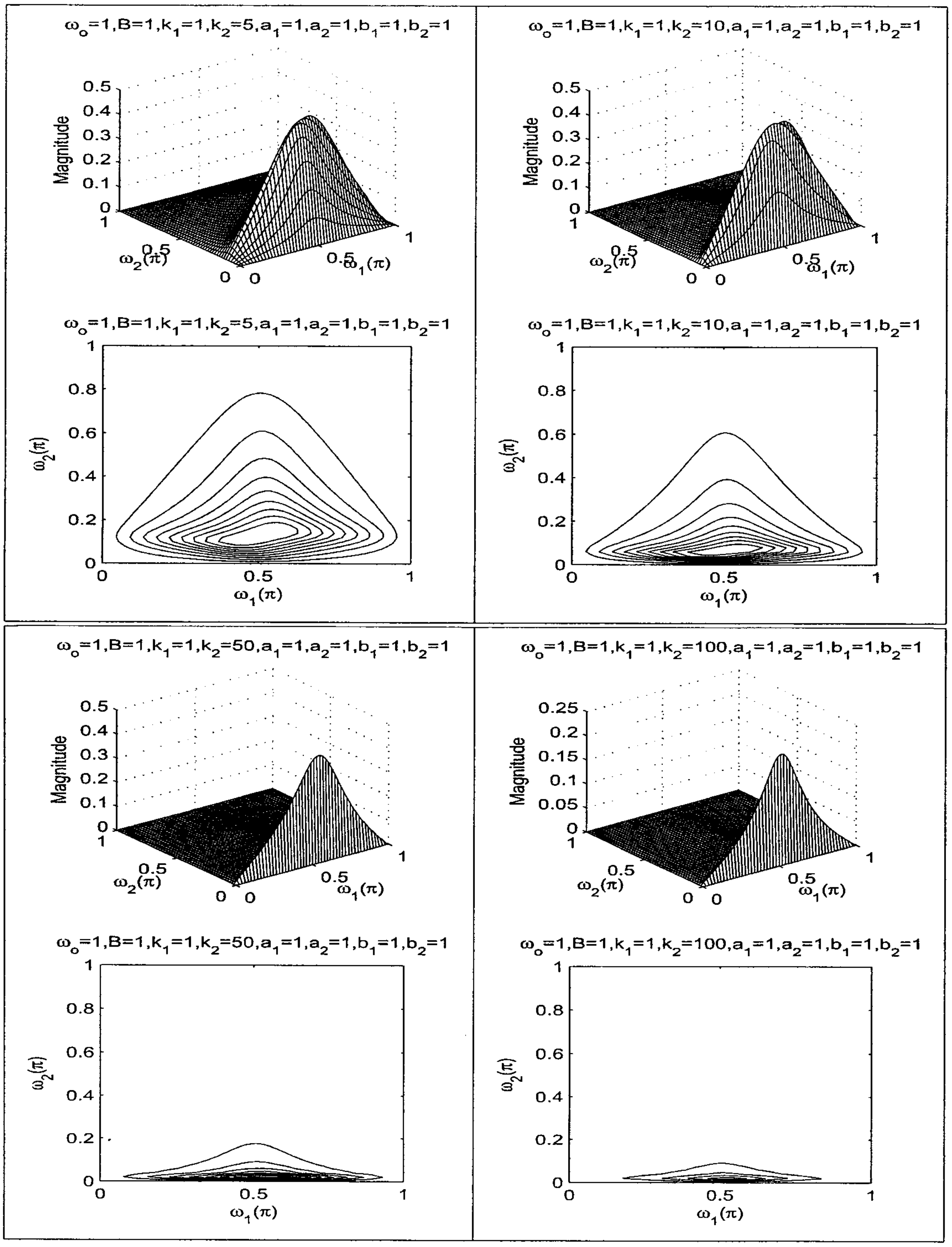

Figure 2.11: Frequency response for the band-pass filter with the value of k_2 = 5,10,50 and 100 and all the other coefficients as unity.

attain the maximum value at $k_2 = 1$ and decreases thereafter for the increase in value of k_2. We can simply mention it like k_2 is inversely proportional to ω_c at $\omega_2 - axis$, bandwidth increases upto $k_2 = 1$ and decreases for the value of $k_2 > 1$, and the coefficient k_2 has no effects on the gains in the pass-band and stop-band for limited value of k_2. Also, the changes in k_2 values does not make any changes in $\omega_1 - axis$. It is evident from this section that, the frequency response in $\omega_1 - axis$ is similar to the frequency responses in $\omega_2 - axis$ with respect to the similar changes in the values of k_1 and k_2.

2.5.3 Frequency Response of 2-D Band-Pass filters with various a_1 values

We can get the stable range of a_1 with the other specified coefficient of the double generalized bilinear transformation. There are many combinations possible for the coefficients. To analyze the response properly we fix the other coefficient values to be equal to unity, when studying the effect of a_1 on the frequency response of the band-pass filter. The range of a_1 varies from 0 to 1 and the other coefficient values are specified as $k_1 = 1, k_2 = 1, a_2 = 1, b_1 = 1$ and $b_2 = 1$.

The 3-D magnitude and contour plots with representative values of a_1 are given in fig 2.12. By making the $a_1 = 1$, it resembles the standard band-pass filter. It can be observed from the diagram that the coefficient a_1 effects the gain of the pass-band and the center frequency (ω_c). At the lowest possible value of a_1, the gain of the passband will be less than 0.2. As the a_1 increases, the gain increases and reach the maximum value at $a_1 = 1$. The center frequency of the band-pass filter decreases as the a_1 increases at $\omega_1 - axis$. The bandwidth of the passband will be approximately equivalent to the design bandwidth value, when the value of $a_1$is varied between 0 and 1. Also, it is evident from the diagram,

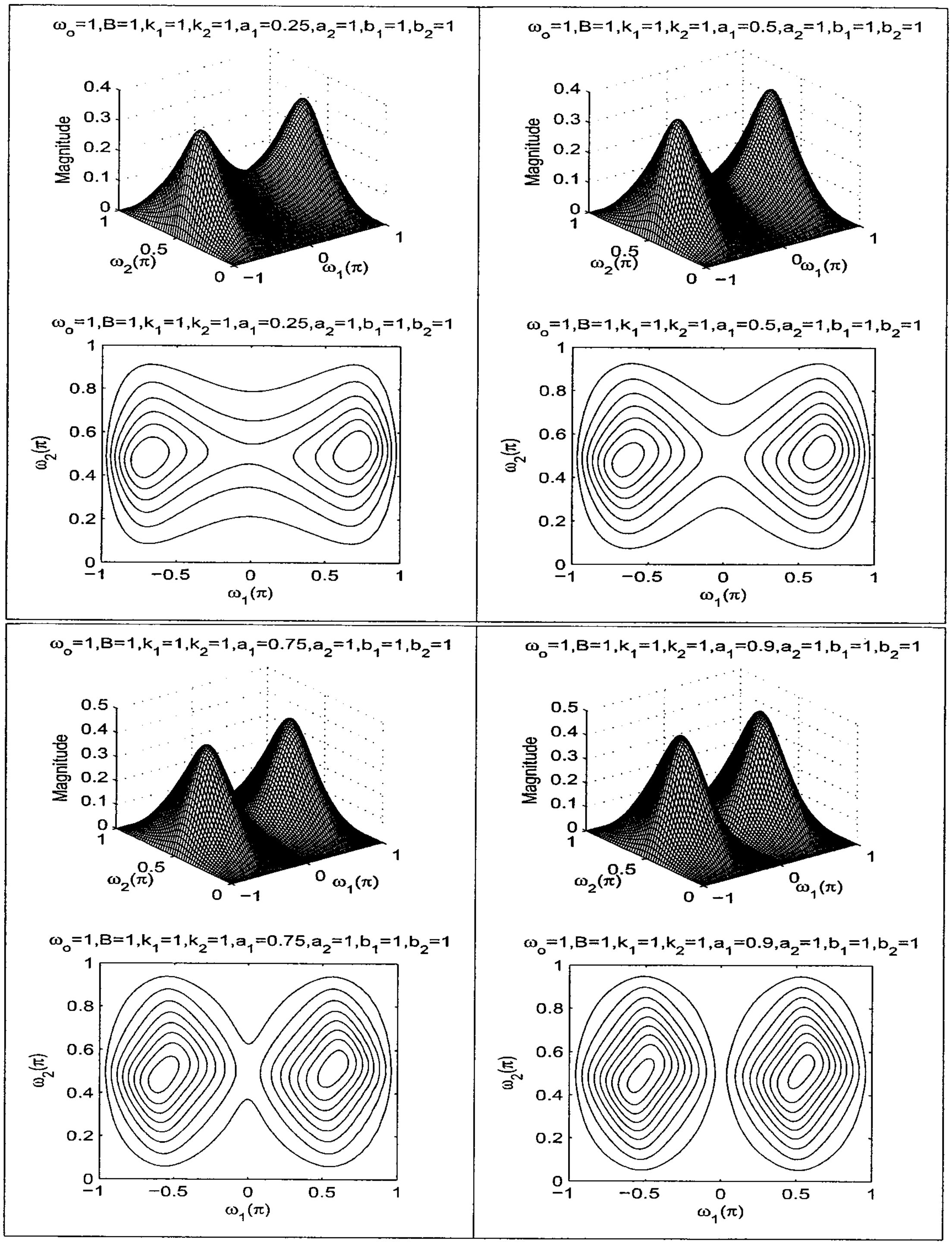

Figure 2.12: Frequency response for the band-pass filter with the value of a_1 = 0.25,0.5,075 and 0.9 and all the other coefficients as unity

the lower stopband has the non-zero gain at $\omega_1 - axis$. So we can relate on stating that the increase in the coefficient a_1 gives rise to the gain in the pass-band and it decreases the center frequency. The change in value of a_1 will not effect the response in $\omega_2 - axis$.

2.5.4 Frequecny Response of 2-D Band-Pass filters with various a_2 values

In section 2.5.3, the effects of the coefficient a_1 are analyzed. In this section, the effects of the coefficient a_2 are analyzed. We can get the stable range of a_2 with the other specified coefficient of the double generalized bilinear transformation. There are many combinations possible for the coefficients. To study the response properly we fix the other coefficient values to be equal to unity, when studying the effect of a_2 on the frequency response of the band-pass filter. The range of a_2 varies from 0 to 1 and the other coefficient values are specified as $k_1 = 1, k_2 = 1, a_2 = 1, b_1 = 1$ and $b_2 = 1$.

The 3-D magnitude and the contour plots with the various values of a_2 are shown in fig 2.13. By making the $a_2 = 1$, it is that of the standard band-pass filter. It can be seen that the coefficient a_2 effects the gain of the pass-band and also the center frequency (ω_c). It is observed that the gain of the passband increases as the value of a_2 increases, and it reaches the maximum value at $a_2 = 1$ in the $\omega_2 - axis$. It can also be observed the center frequency decreases as the a_2 increases at $\omega_2 - axis$. The bandwidth of the passband will be approximately equivalent to the design bandwidth value, when the value of $a_1$is varied between 0 and 1. The lower stopband in the band-pass filter will have the non-zero values in the $\omega_2 - axis$. So, we can conclude this on stating that the increase in the coefficient a_2 will give rise to the gain in the passband and decreases the center frequency. It is evident from the fig 2.12 and 2.13, the response in the $\omega_1 - axis$ is same as that of the response in

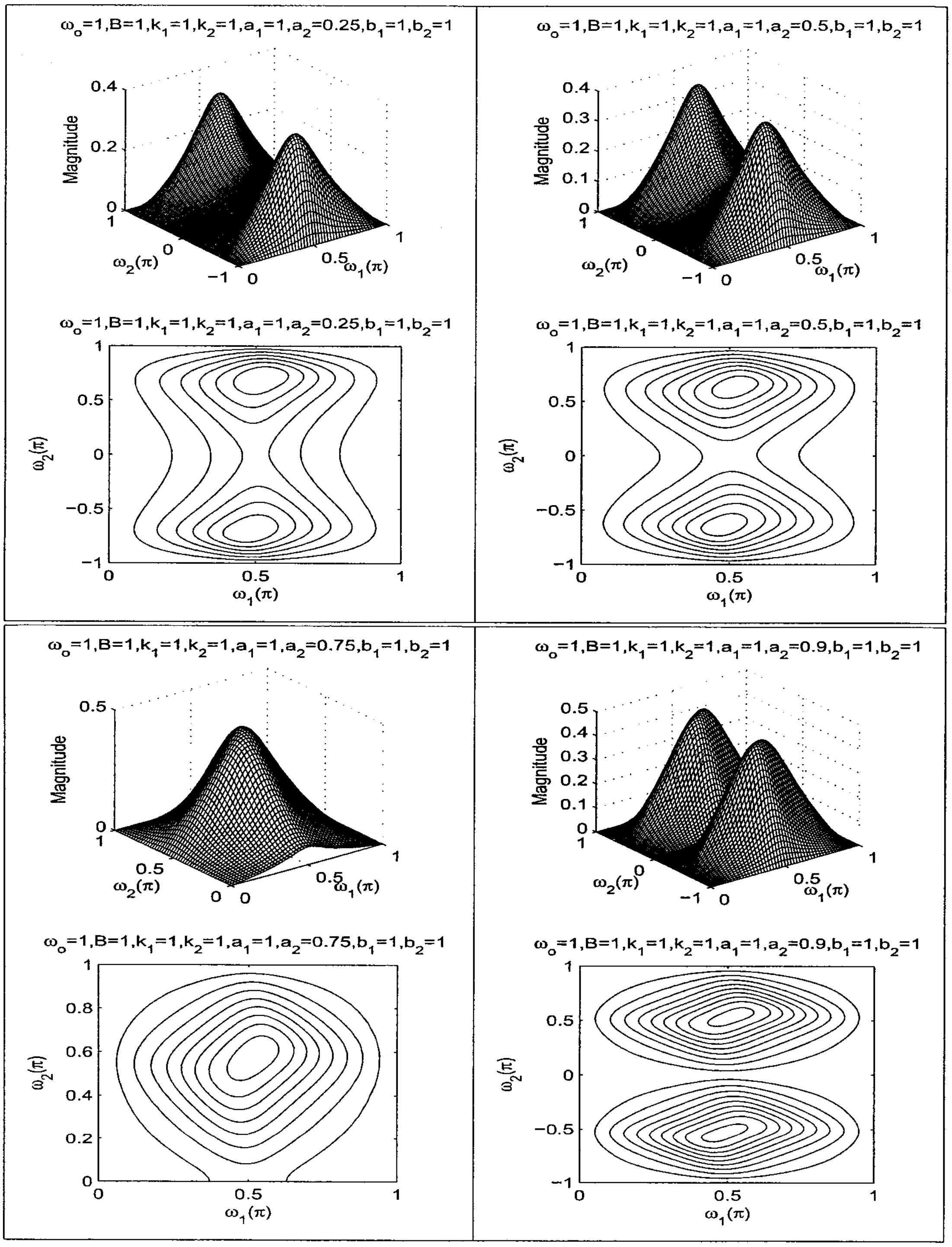

Figure 2.13: Frequency response for the band-pass filter with the value of $a_2 = 0.25, 0.5, 075$ and 0.9 and all the other coefficients as unity.

the $\omega_2 - axis$ with respect to similar changes in a_1 and a_2.

2.5.5 Frequecny Response of 2-D Band-Pass filters with various b_1 values

From the section 2.5.1 to 2.5.4, we analyzed the effect in the value of k_1, k_2, a_1 and a_2 respectively. In this section, the effect of b_1 is analyzed. We can get the stable range of b_1 with the other specified coefficient of the double generalized bilinear transformation. There are many combinations possible for the coefficients. To study the response properly we fix the other coefficient values to be equal to unity, when studying the effect of b_1 on the frequency response of the band-pass filter. The range of b_1 varies from 0 to 1 and the other coefficient values are specified as $k_1 = 1, k_2 = 1, a_1 = 1, a_2 = 1$ and $b_2 = 1$.

The 3-D magnitude and contour plots with representative values of b_1 are given in fig. 2.14. It is observed from the diagram, change in the value of b_1 will effect the gain in the pass-band and the center frequency(ω_c). At the lowest possible value of b_1, the response will have the lowest gain at the passband. As the b_1 increases, the gain of the passband increases and it reaches the maximum value at $b_1 = 1$. It can also be observed the center frequency increase as the b_1 increases in $\omega_1 - axis$. The bandwidth of the passband will be approximately equivalent to the design bandwidth value, when the value of $b_1$is varied between 0 and 1. Also, it is evident from the diagram that the upper stopband has the non-zero gain at $\omega_1 - axis$ for $b_1 < 1$. So, we can relate this on stating that the gain in the passband and in the center frequency results by the increase in the coefficient of b_1.

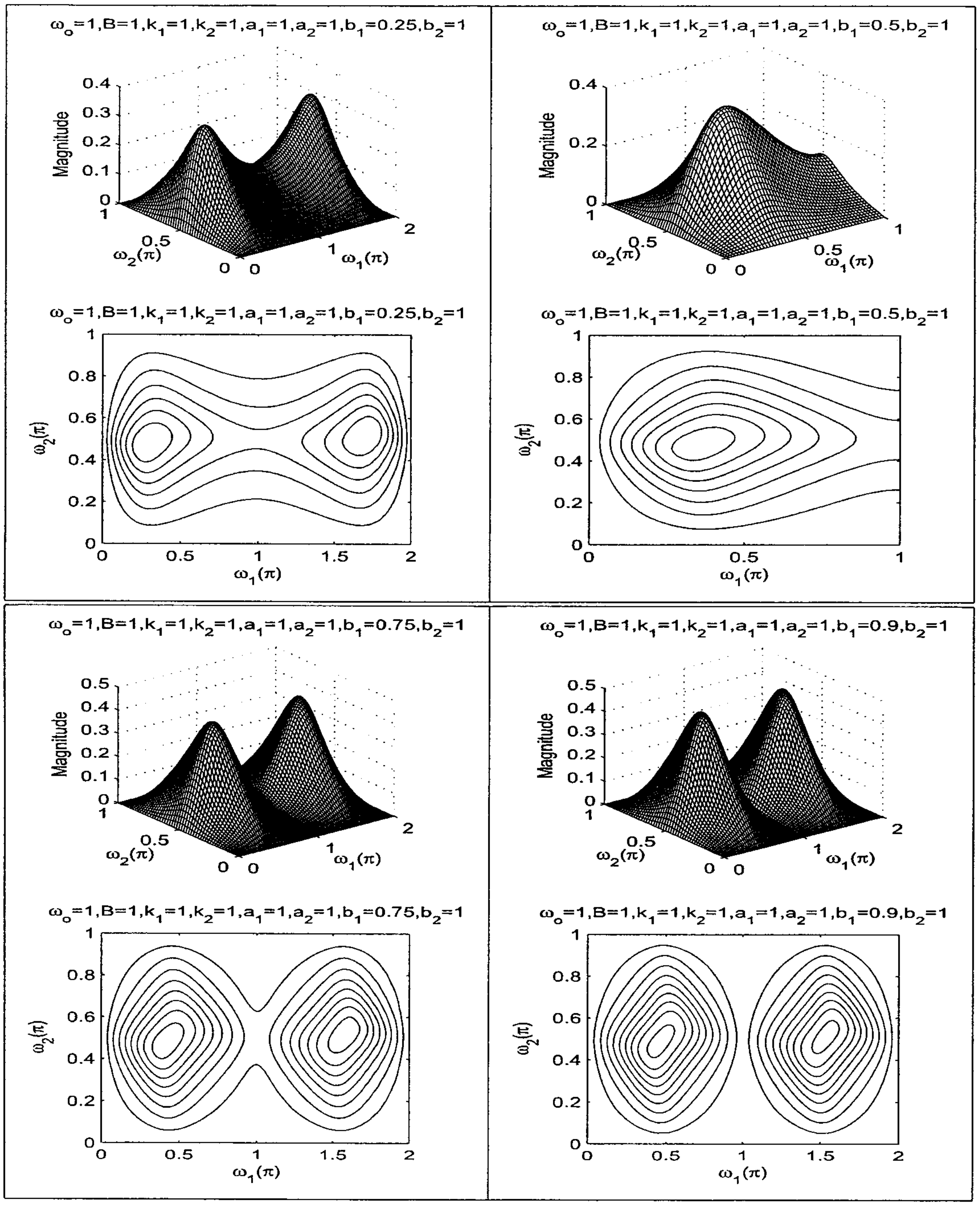

Figure 2.14: Frequency response for the band-pass filter with the value of $b_1 = 0.25, 0.5, 075$ and 0.9 and all the other coefficients as unity

2.5.6 Frequecny Response of 2-D Band-Pass filters with various b_2 value

In the previous section, we analyzed the effect of b_1 on the 2-D band-pass filters. In this, we continue our analyzes by varying the value of b_2. We can get the stable range of b_2 with the other specified coefficient of the double generalized bilinear transformation. There are many combinations possible for the coefficients. To study the response properly we fix the other coefficient values to be equal to unity, when studying the effect of b_2 on the frequency response of the band-pass filter. The range of b_2 varies from 0 to 1 and the other coefficient values are specified as $k_1 = 1, k_2 = 1, a_1 = 1, a_2 = 1$ and $b_1 = 1$.

The 3-D magnitude and contour plots with representative values of b_2 are given in fig. 2.15. It is observed from the diagram, change in the value of b_2 will effect the gain in the pass-band and the center frequency(ω_c). At the lowest possible value of b_2, the response will have the lowest gain at the passband. As the b_2 increases, the gain of the passband increases and it reaches the maximum value at $b_2 = 1$. It can also be observed the center frequency increase as the b_2 increases in $\omega_2 - axis$. The bandwidth of the passband will be approximately equivalent to the design bandwidth value, when the value of $b_2$is varied between 0 and 1. Also, it is evident from the diagram that the upper stopband has the non-zero gain at $\omega_2 - axis$ for $b_2 < 1$. So, we can relate this on stating that the gain in the passband and in the center frequency results by the increase in the coefficient of b_2. From the fig. 2.14 and 2.15, the response in the $\omega_1 - axis$ is similar to the response in the $\omega_2 - axis$ with the same in values of b_1 and b_2

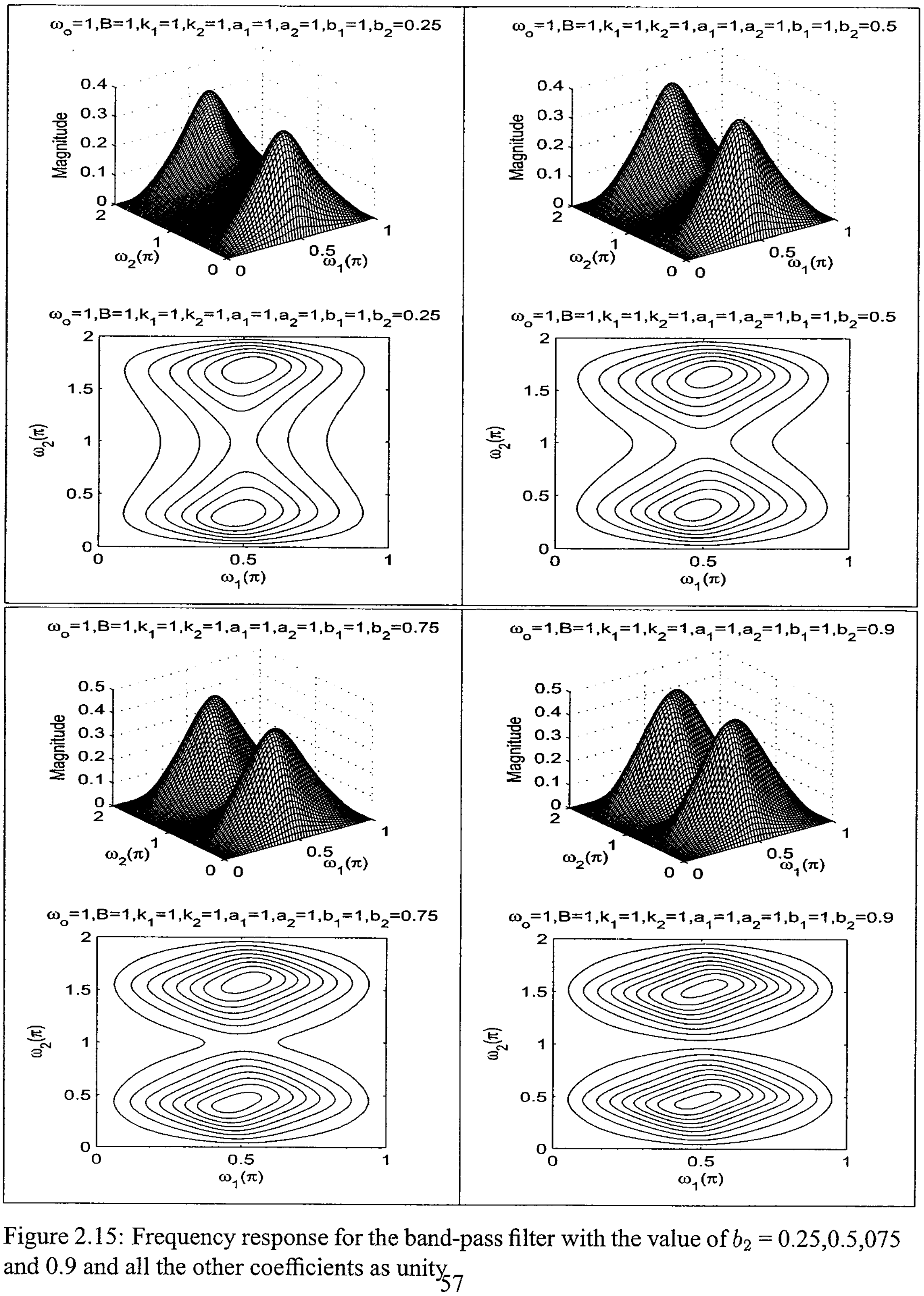

Figure 2.15: Frequency response for the band-pass filter with the value of $b_2 = 0.25,0.5,075$ and 0.9 and all the other coefficients as unity.

2.5.7 Frequecny Response of 2-D Band-Pass fi lters with various k_1 values and fi xed ω_o with same values of a_1 and b_1

In section 2.5.1, we saw the frequency response of the band-pass filter for the various k_1 values with a unity for the other coefficients. In this, we are going to assign the same values for a_1 and b_1 ranging from 0 to 1, and for remaining coefficients are considered to be unity such as $k_2 = 1,\ a_2 = 1,\ b_2 = 1$ and the theortical center frequency (ω_o) is considered as unity. The 3-D magnitude and the contour plots are shown from figs. 2.16, 2.17 and 2.18 for various values of k_1 with the coefficient values a_1 and b_1 are 0.5.

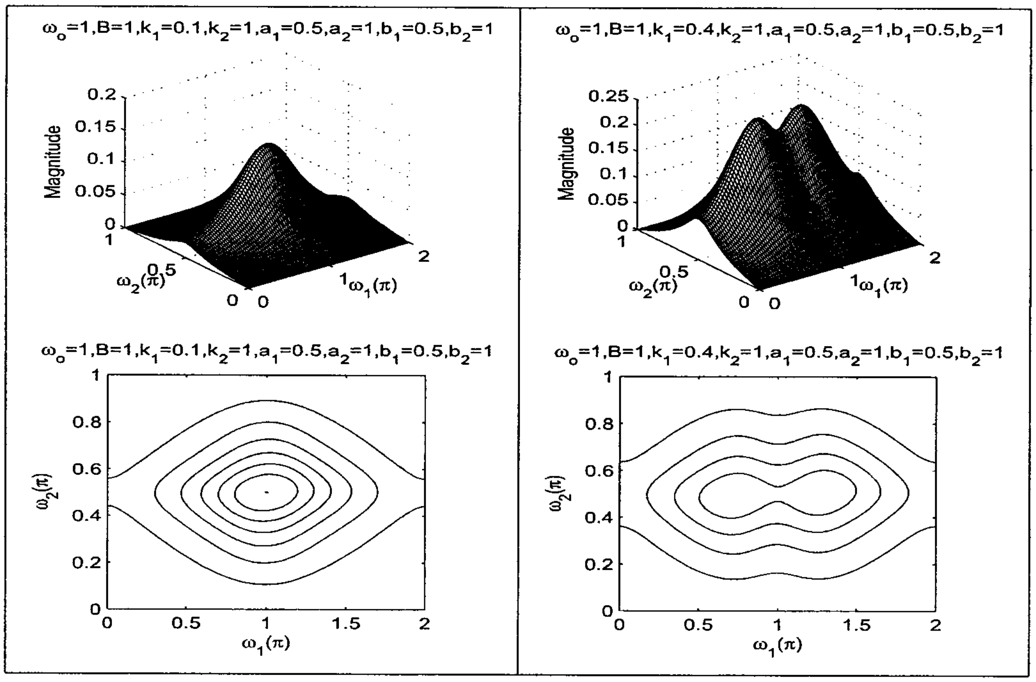

Figure 2.16: Frequency response for the band-pass filter with the value of $k_1 = 0.1$ and 0.4 for $a_1 = b_1 = 0.5$ and all the other coefficients as unity

The effect of response will be in the $\omega_1 - axis$ and there will not be any change in response in the $\omega_2 - axis$. If the value of k_1 happens to be equal to the therotical center

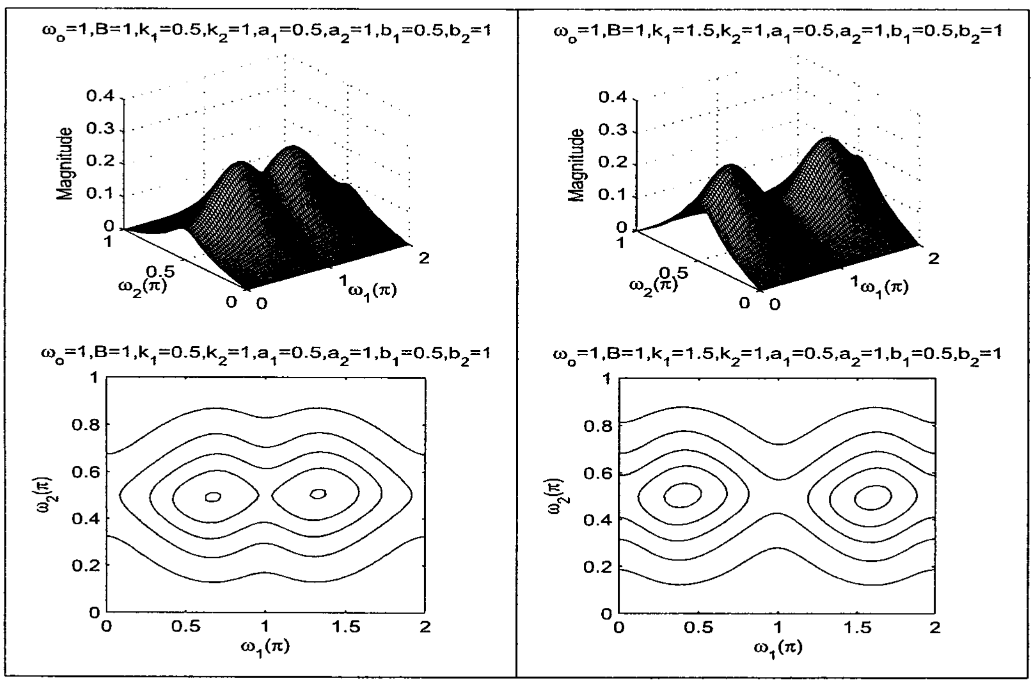

Figure 2.17: Frequency response for the band-pass filter with the value of $k_1 = 0.5$ and 1.5 for $a_1 = b_1 = 0.5$ and all the other coefficients as unity

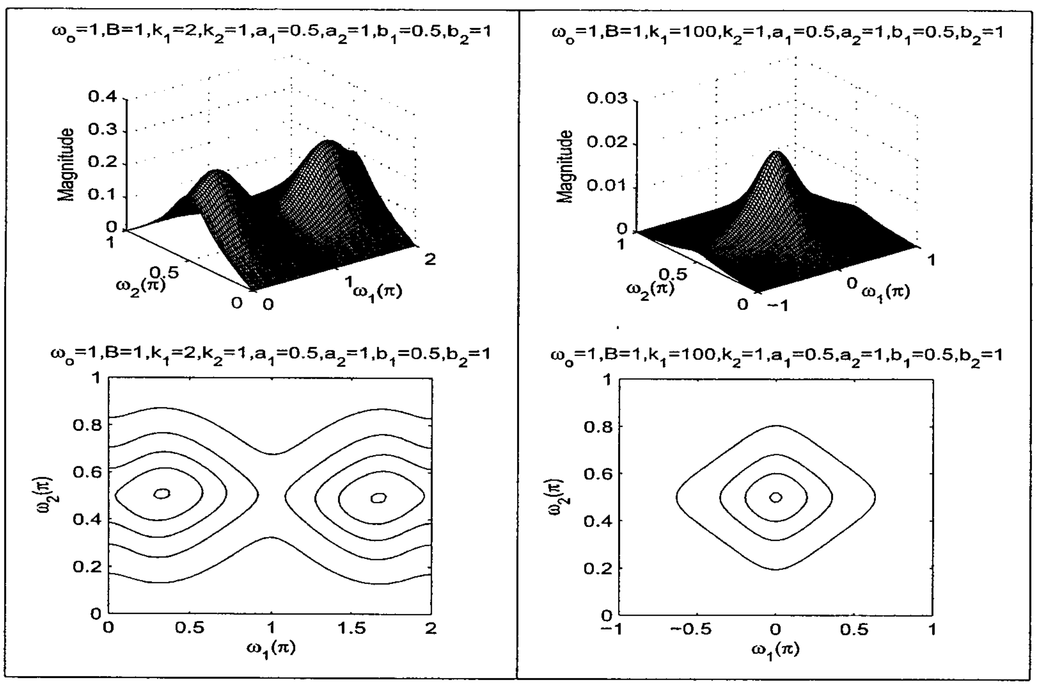

Figure 2.18: Frequency response for the band-pass filter with the value of $k_1 = 2$ and 100 for $a_1 = b_1 = 0.5$ and all the other coefficients as unity

frequency (ω_o), then the response will be similar to the standard band-pass filter shown in the figure 2.7 but gain will be decreased. This decrease in gain is due to the change in the a_1 and b_1 values. In this section, we can see three types of filter response namely band-pass, high-pass and low-pass response. The band-pass response are seen between $1 - a_1 or(b_1) \leq k_1 \leq 1 + a_1 or(b_1)$. The high-pass response are seen in the range of k_1 less than $1 - a_1 or(b_1)$. The low-pass response are seen in the range of k_1 greater than $1 + a_1 or(b_1)$. Bandwidth for all the cases will remain close to the design bandwidth. And also in all the response shown from fig 2.16, 2.17 and 2.18 the gain increase and reach the maximum value for k_1 equal to unity and then it start decreasing for the higher values of k_1.

2.5.8 Frequecny Response of 2-D Band-Pass filters with various k_2 values and fixed ω_o with same value of a_2 and b_2

In section 2.5.2, we saw the frequency response of the band-pass filter for the various k_2 values with a unity for the other coefficients. In this, we are going to assign the same values for a_2 and b_2 ranging from 0 to 1, and for remaining coefficients are considered to be unity such as $k_1 = 1, a_1 = 1, b_1 = 1$ and the theortical center frequency (ω_o) is considered as unity. The 3-D magnitude and contour plots are shown from figure 2.29 to 2.31 for various values of k_2 with the coefficient values a_2 and b_2 are 0.5.

The effect of response will be in the $\omega_2 - axis$ and there will not be any change in response in the $\omega_1 - axis$. If the value of k_2 happens to be equal to the therotical center frequency (ω_o), then the response will be similar to the standard band-pass filter shown in the fig 2.7 but with of decreased gain. This decrease in gain is due to the change in the a_2 and b_2 values. In this, we can see three types of filter response namely band-pass, high-pass

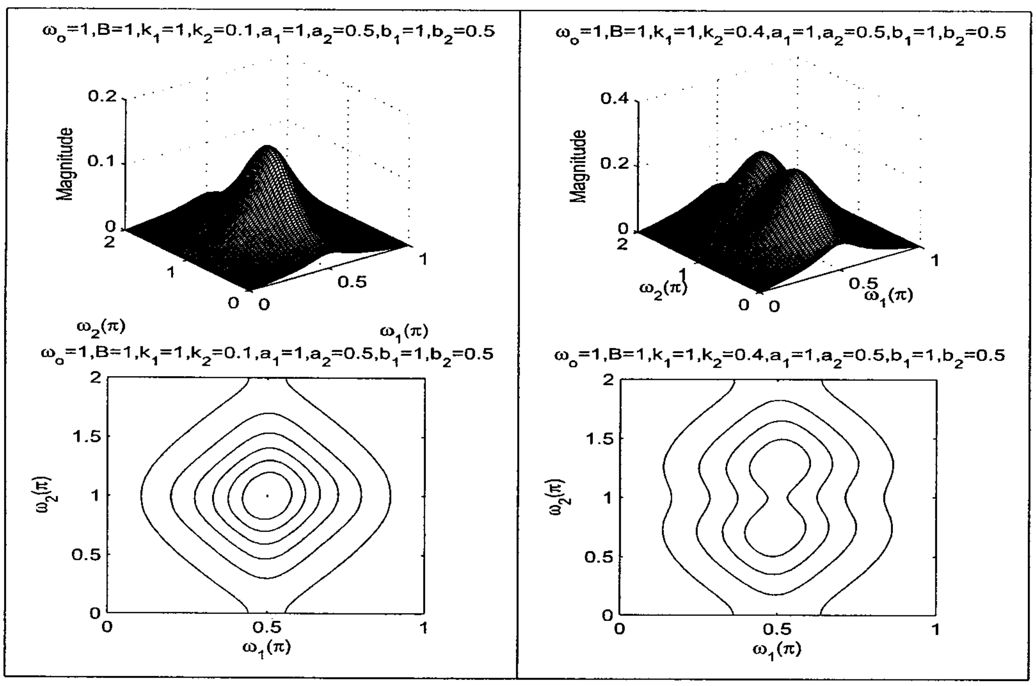

Figure 2.19: Frequency response for the band-pass filter with the value of $k_2 = 0.1$ and 0.4 for $a_2 = b_2 = 0.5$ and all the other coefficients as unity

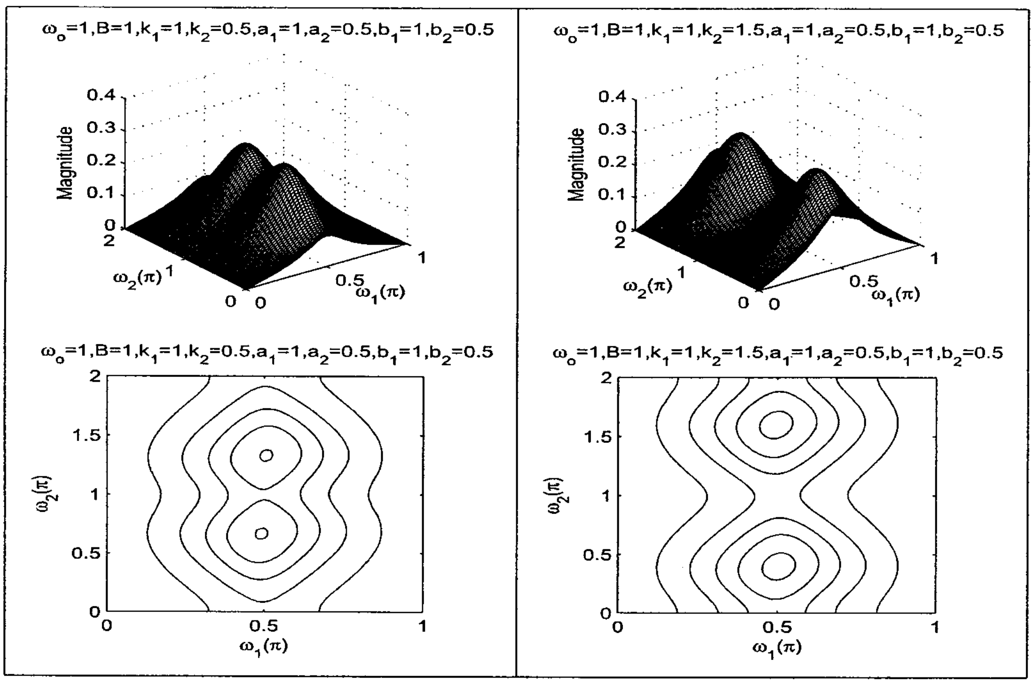

Figure 2.20: Frequency response for the band-pass filter with the value of $k_2 = 0.5$ and 1.5 for $a_2 = b_2 = 0.5$ and all the other coefficients as unity

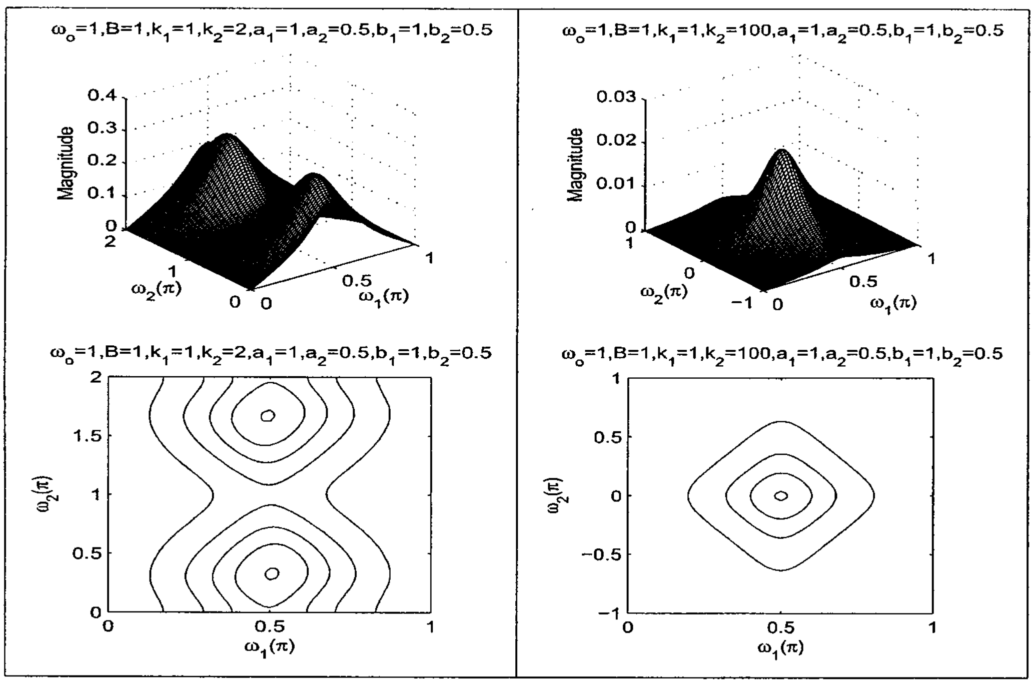

Figure 2.21: Frequency response for the band-pass filter with the value of $k_2 = 2$ and 100 for $a_2 = b_2 = 0.5$ and all the other coefficients as unity

and low-pass response. The band-pass response are seen between $1 - a_2 or(b_2) \leq k_2 \leq 1 + a_2 or(b_2)$. The high-pass response are seen in the range of k_2 less than $1 - a_2(b_2)$. The low-pass response are seen in the range of k_2 greater than $1 + a_2(b_2)$. Bandwidth for all the cases remain closer to the designed values. Also in all the responses shown in the figs. 2.19, 2.20 and 2.21, the gain increases and reaches the maximum value for k_2 equal to unity and then it starts decreasing for the higher values of k_2.

2.5.9 Frequency response for different values of B and ω_o with unity for other coeffi cients

2.5.9.1 Frequency response for different values of B

In this section, we will see the frequency response for the various values of B with the other coefficients values are fixed to be unity. The 3-D magnitude and the contour plot for various values of B are plotted in fig 2.22, it is same for both $\omega_1 - axis$ and $\omega_2 - axis$. From these plots, it is clear that the bandwidth increases as the value of B is varied in the range of 0.5 to 3.

2.5.9.2 Frequency response for different values of ω_o

From section 2.5.1 to 2.5.8, the frequency response of the fixed value ω_o are shown. In this section, we will see the frequency response for the various values of ω_o with the other coefficients values are fixed to be unity. The 3-D magnitude and the contour plot for various values of ω_o are plotted in fig 2.23, it is same for both $\omega_1 - axis$ and $\omega_2 - axis$. From these plots, it is clear that the center frequency shifts from the minimum to maximum value when ω_o is varied in the range of 0 to 3. Bandwidth of the response starts with larger

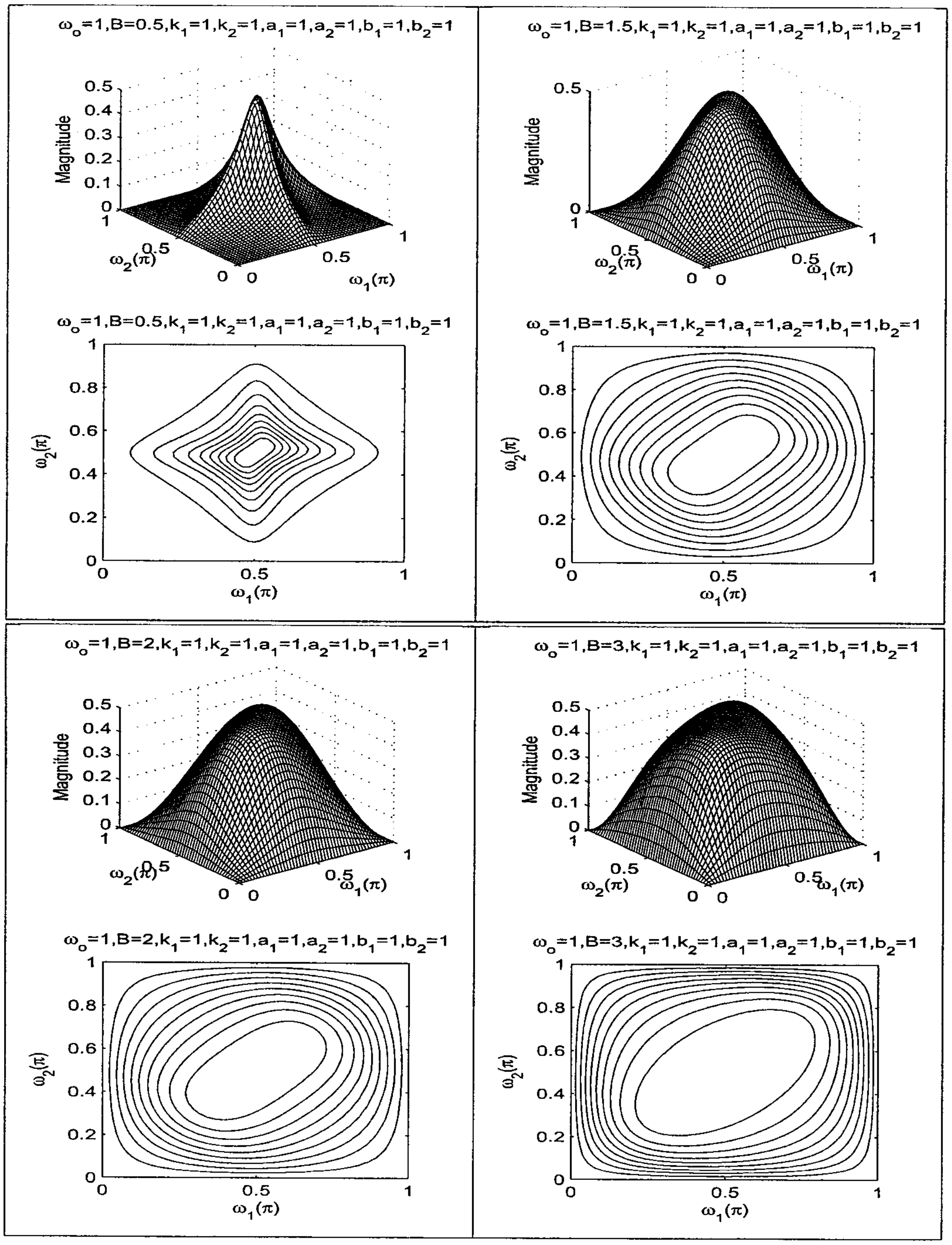

Figure 2.22: Frequency response of band-pass filter with various values of B

bandwidth for the minimum value of ω_o, it decreases for the increase in ω_o and reaches the minimum bandwidth for the highest possible value of ω_o.

2.5.10 Frequency Response of 2-D Band-Pass filters with fixed k values and various ω_o with same values of a_1 and b_1

In section 2.5.9, the frequency response of the various value ω_o are discussed. In this, we will see the frequency response for the various values of ω_o with fixed value of k_1 and with same values of a_1 and b_1. The 3-D magnitude and the contour plots for specified values of k_1 such as 0.25, 0.5, 1,2 to 5 are shown from figs. 2.24 to 2.28 with the ω_o values as $0.5, 1.5, 2$ and 3. The a_1 and b_1 values are taken as 0.75.

It can be seen that the effect of ω_o and k_1 are in the $\omega_1 - axis$. Whenever the values of k_1 and ω_o are same the center frequency of the response is similar to standard band-pass filter but with the decrease in gain. For the k_1 less than unity and for the ω_o in the range of $0.1\pi \leq \omega_o \leq \pi$, the frequency response will start from a band-pass filter for the lower value ω_o and slowly it transform to a high-pass filter. Figs. 2.24 and 2.25 shows the transformation of band-pass to high-pass filter in the $\omega_1 - axis$. For the k_1 value around unity, the frequency response will be of band-pass filter only, but the center frequency will shift from lower to higher value for the change in value of ω_o, fig 2.26 shows the frequency response for $k_1 = 1$. And finally, for the values of k_1 greater than unity, the frequency response will start from low-pass filter for the lower value of ω_o, and slowly get transformed into a band-pass filter for higher value of ω_o, fig 2.27 and 2.28 shows the response for $k_1 = 2$ and 5 respectively. The bandwidth of the freqeuncy response varies accordingly to the value of ω_o. As ω_o increases the bandwidth decreases, bandwidth increases as the ω_o decreases. Thus on varing the values of k_1 and ω_o, different types of

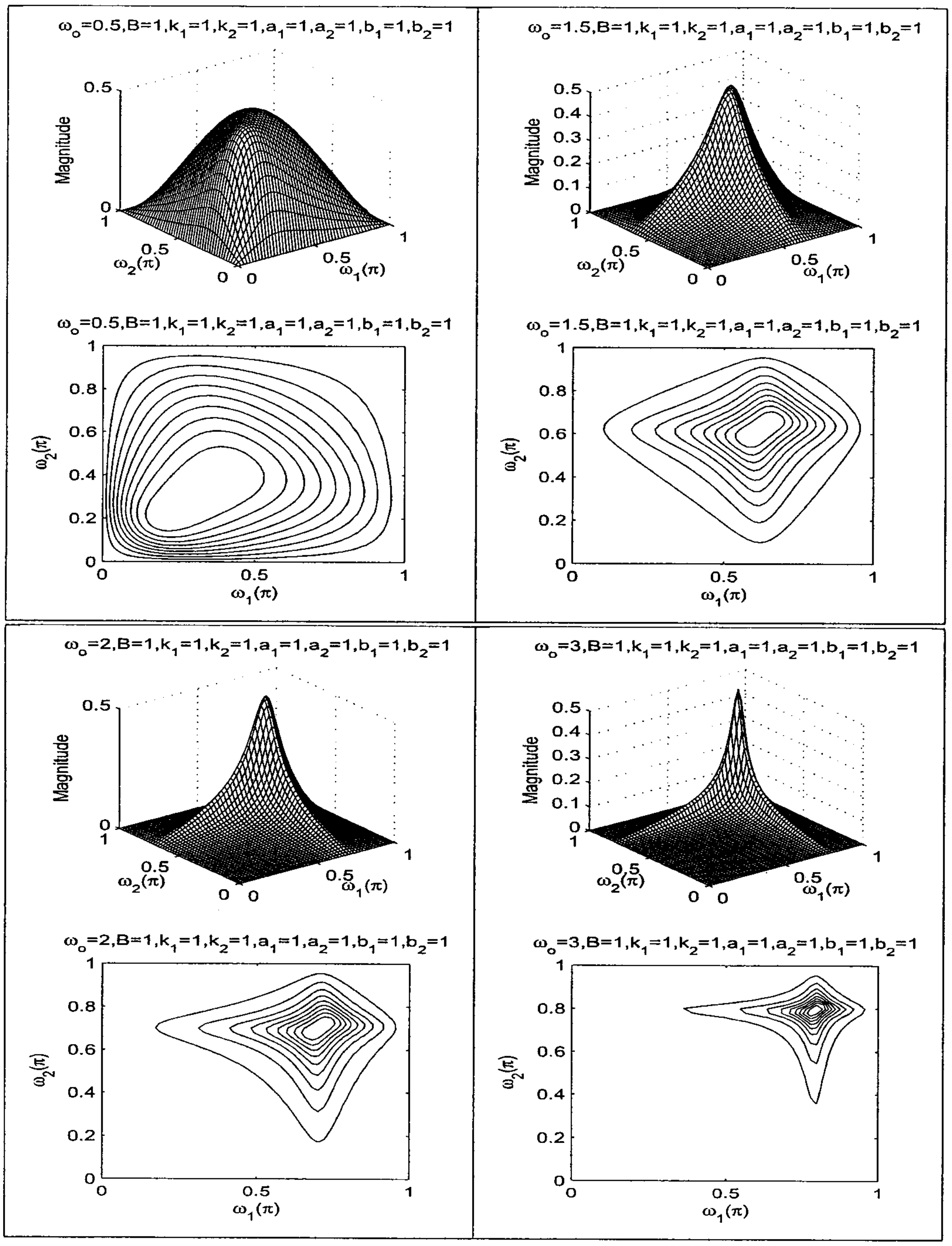

Figure 2.23: Frequency response for the band-pass filter with the value of ω_o = 0.5,1.5,2 and 3 and all the other coefficients as unity.

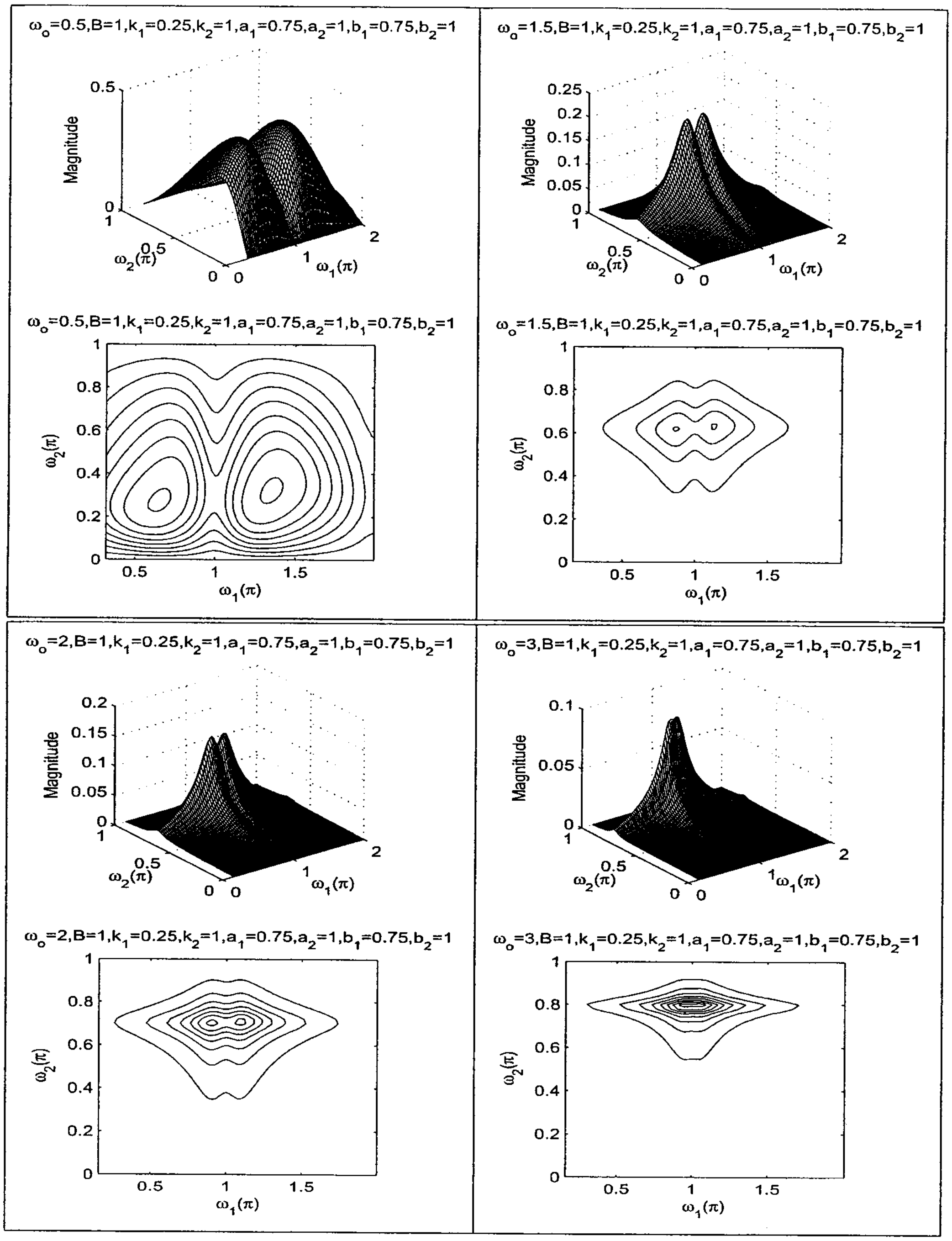

Figure 2.24: Frequency response for the band-pass filter with the value of $k_1 = 0.25$ for $a_1 = b_1 = 0.75$ and all the other coefficients as unity

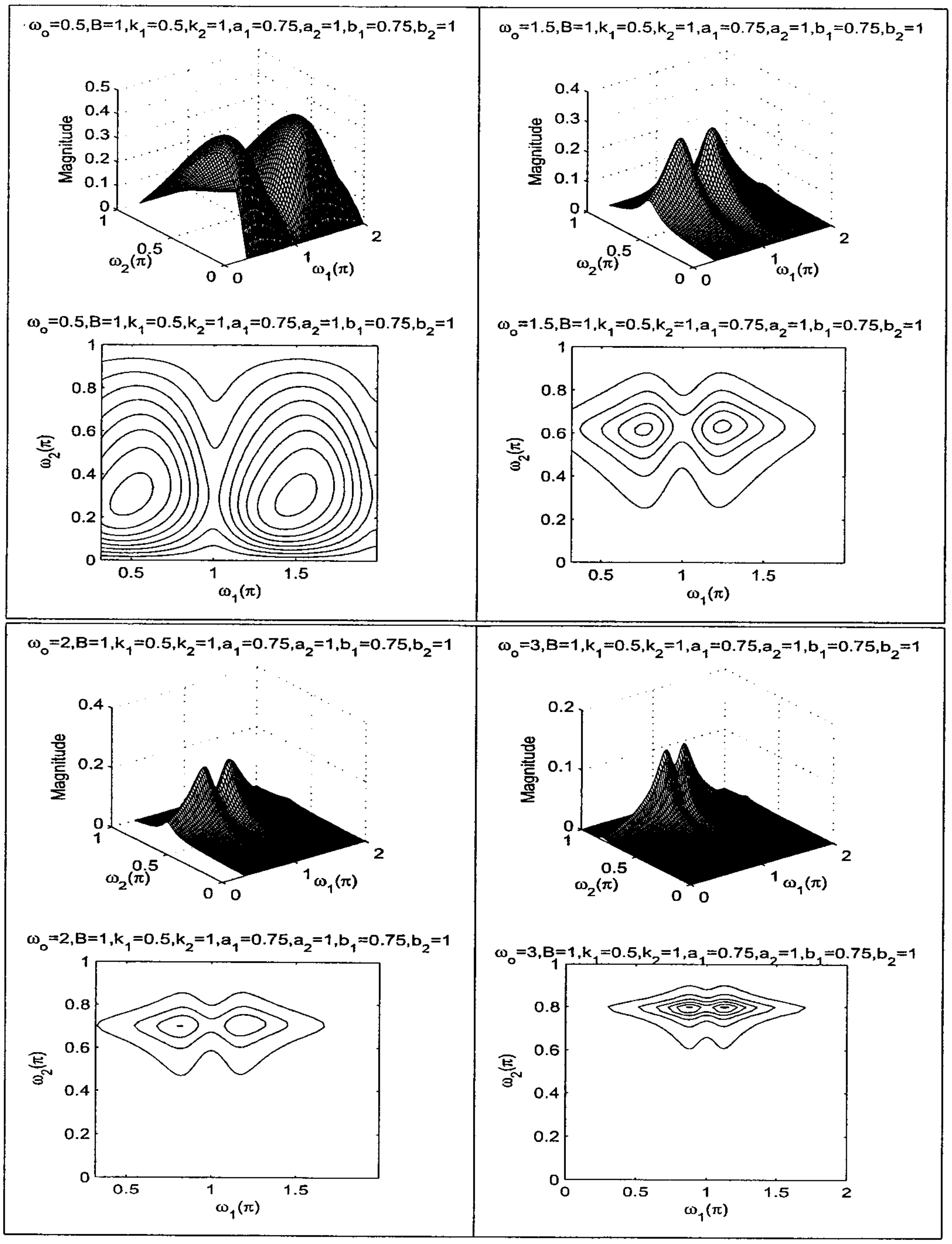

Figure 2.25: Frequency response for the band-pass filter with the value of $k_1 = 0.5$ for $a_1 = b_1 = 0.75$ and all the other coefficients as unity

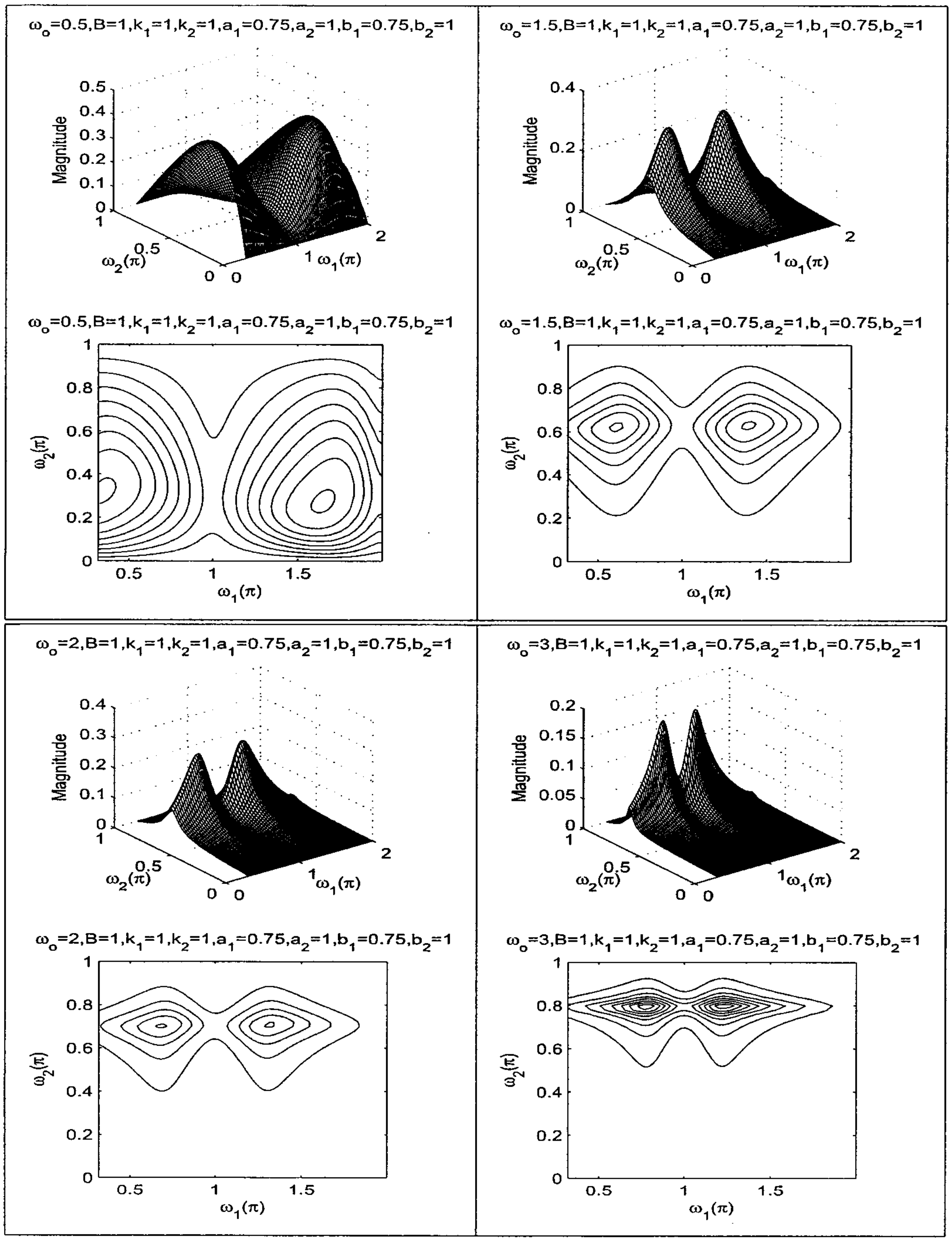

Figure 2.26: Frequency response for the band-pass filter with the value of $k_1 = 1$ for $a_1 = b_1 = 0.75$ and all the other coefficients as unity

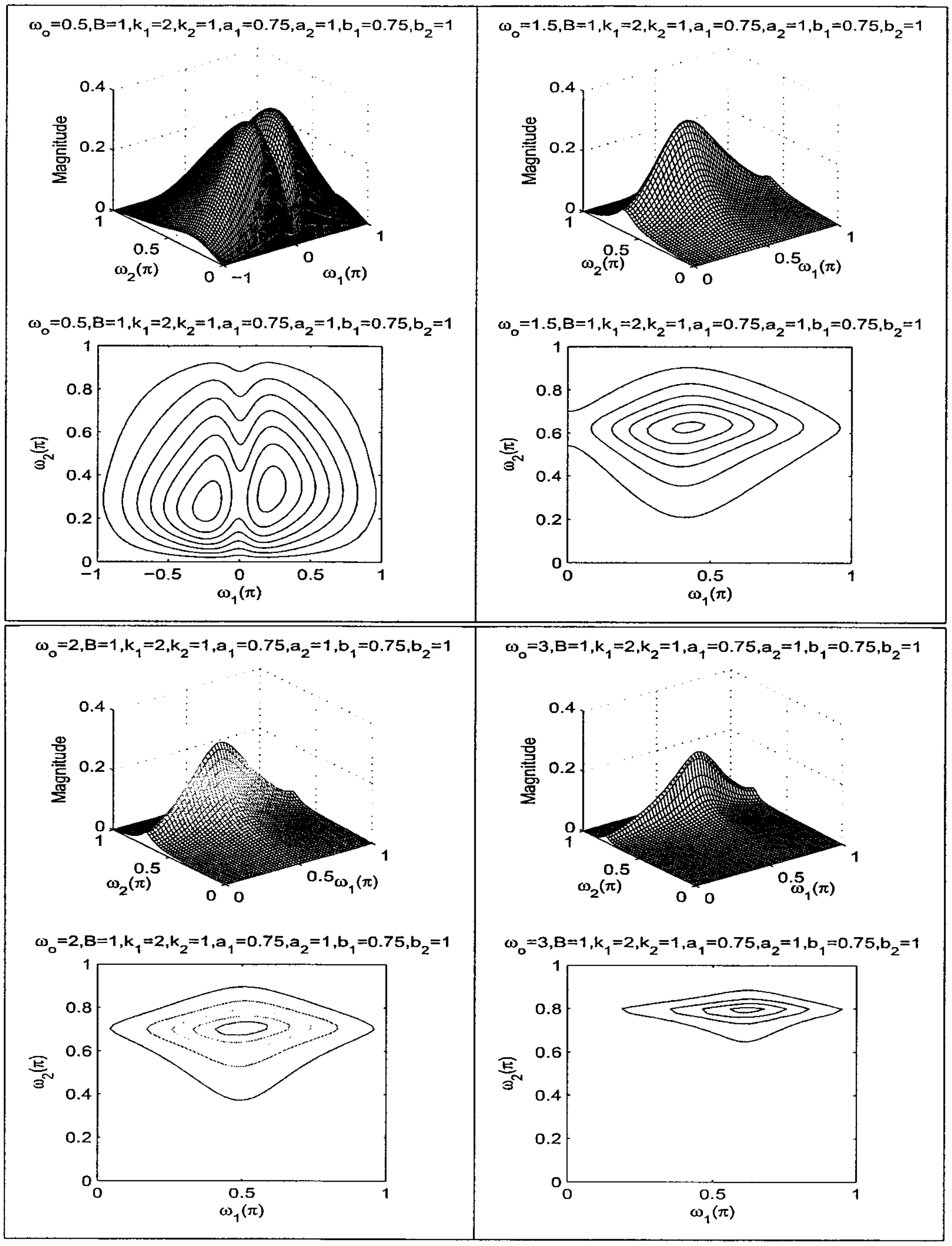

Figure 2.27: Frequency response for the band-pass filter with the value of $k_1 = 2$ for $a_1 = b_1 = 0.75$ and all the other coefficients as unity

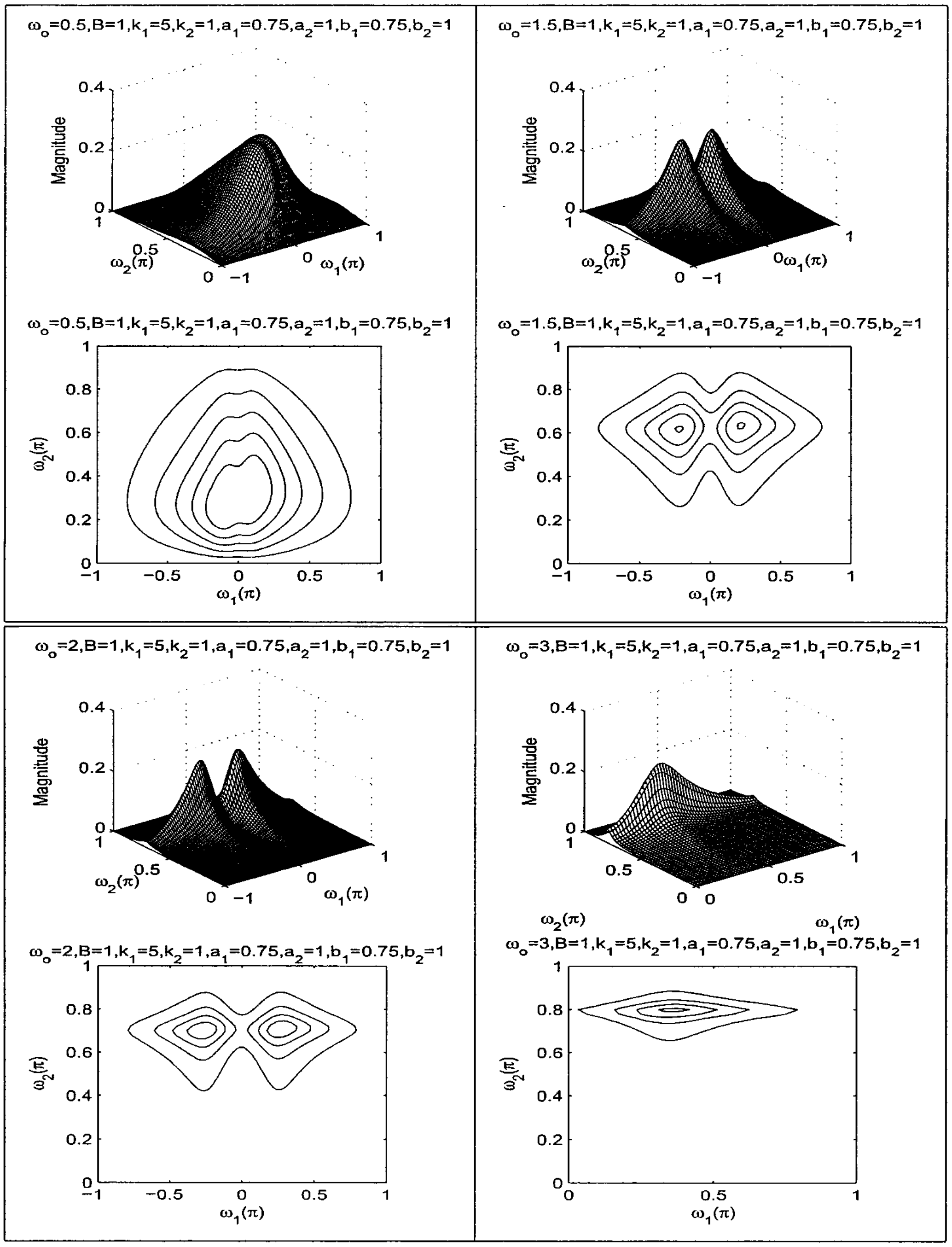

Figure 2.28: Frequency response for the band-pass filter with the value of $k_1 = 5$ for $a_1 = b_1 = 0.75$ and all the other coefficients as unity

filter response are obtained with the bandwidth inversely related to ω_o.

2.5.11 Frequecny Response of 2-D Band-Pass filters with fixed k_2 values and various ω_o with same values of a_2 and b_2

In section 2.5.9, the frequency response of the various value ω_o are discussed. In this, we will see the frequency response for the various values of ω_o with fixed value of k_2 and with same values of a_2 and b_2. The 3-D magnitude and the contour plots for specified values of k_2 such as 0.25, 0.5, 1,2 to 5 are shown from fig. 2.29 to 2.33 with the ω_o values as 0.5, 1.5, 2 and 3. The a_2 and b_2 values are taken as 0.75.

It can be seen that the effect of ω_o and k_2 are in the $\omega_2 - axis$. Whenever the values of k_2 and ω_o are same the center frequency of the response is similar to standard band-pass filter but with the decrease in gain. For the k_2 less than unity and for the ω_o in the range of $0.1\pi \leq \omega_o \leq 3\pi$, the frequency response will start from a band-pass filter for the lower value ω_o and slowly it transform to a high-pass filter. The fig. 2.29 and 2.30shows the transformation of band-pass to high-pass filter in the $\omega_2 - axis$. For the k_2 value around unity, the frequency response will be of band-pass filter only, but the center frequency will shift from lower to higher value for the change in value of ω_o, fig 2.31 shows the frequency response for $k_2 = 1$. And finally, for the values of k_2 greater than unity, the frequency response will start from band-pass filter for the lower value of $\omega_o > 0.3\pi$, and slowly get transformed into a highpass filter for higher value of ω_o, fig. 2.32 and 2.33shows the response for $k_2 = 2$ and 5 respectively. The bandwidth of the freqeuncy response varies accordingly to the value of ω_o. As ω_o increases the bandwidth decreases, bandwidth increases as the ω_o decreases. Thus on varing the values of k_2 and ω_o, different types of filter response are obtained with the bandwidth inversely related to ω_o.

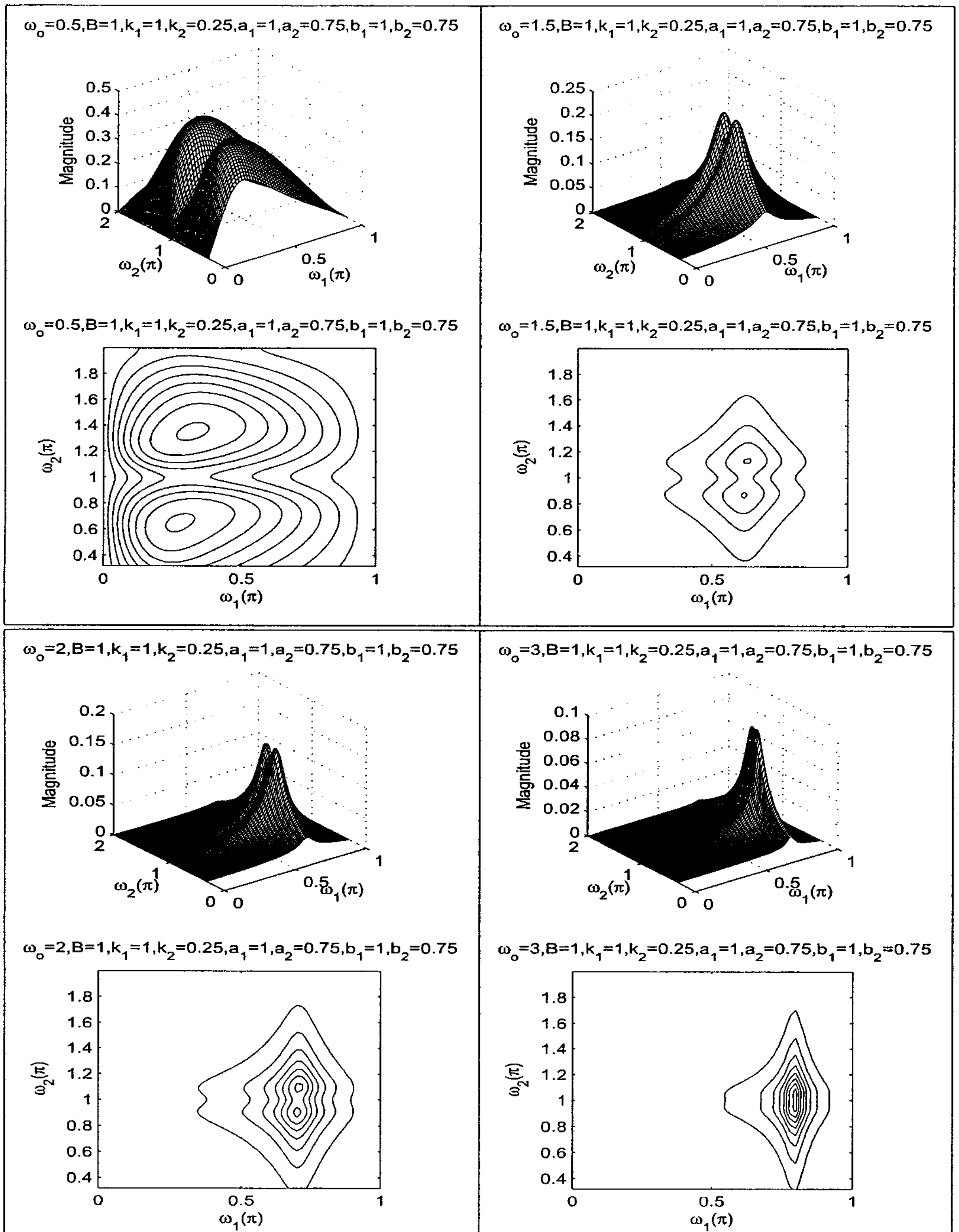

Figure 2.29: Frequency response for the band-pass filter with the value of $k_2 = 0.25$ for $a_2 = b_2 = 0.75$ and all the other coefficients as unity

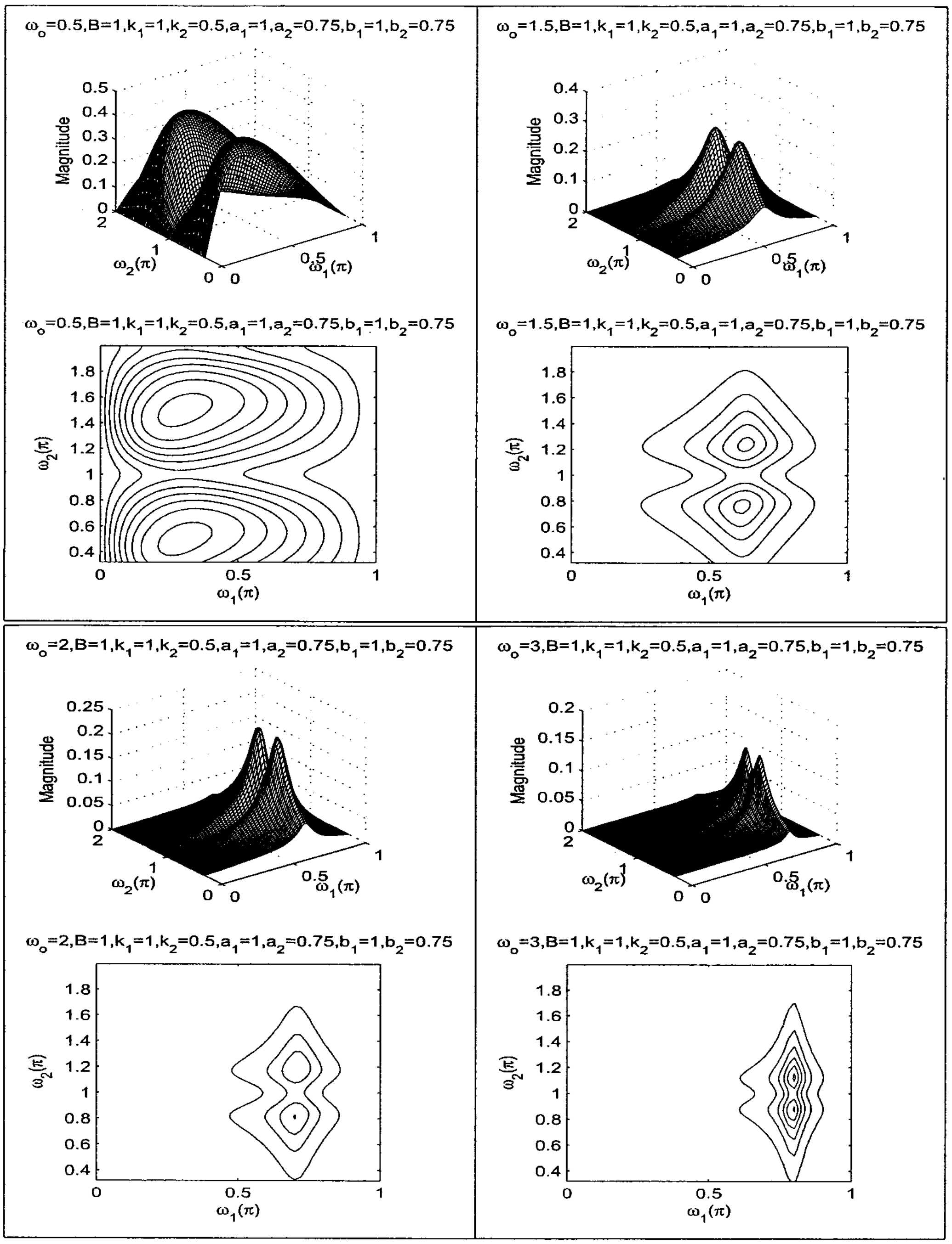

Figure 2.30: Frequency response for the band-pass filter with the value of $k_2 = 0.5$ for $a_2 = b_2 = 0.75$ and all the other coefficients as unity

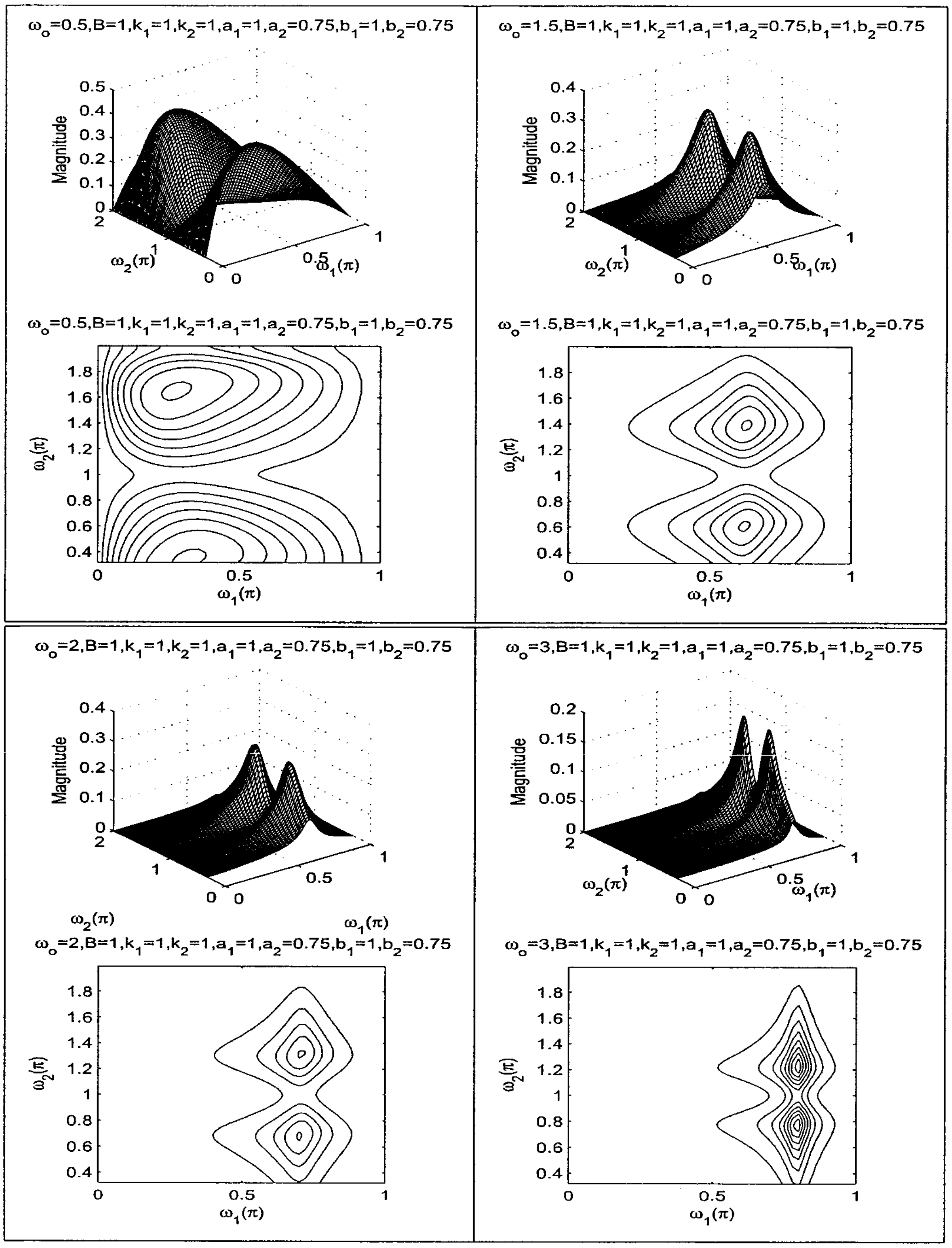

Figure 2.31: Frequency response for the band-pass filter with the value of $k_2 = 1$ for $a_2 = b_2 = 0.75$ and all the other coefficients as unity

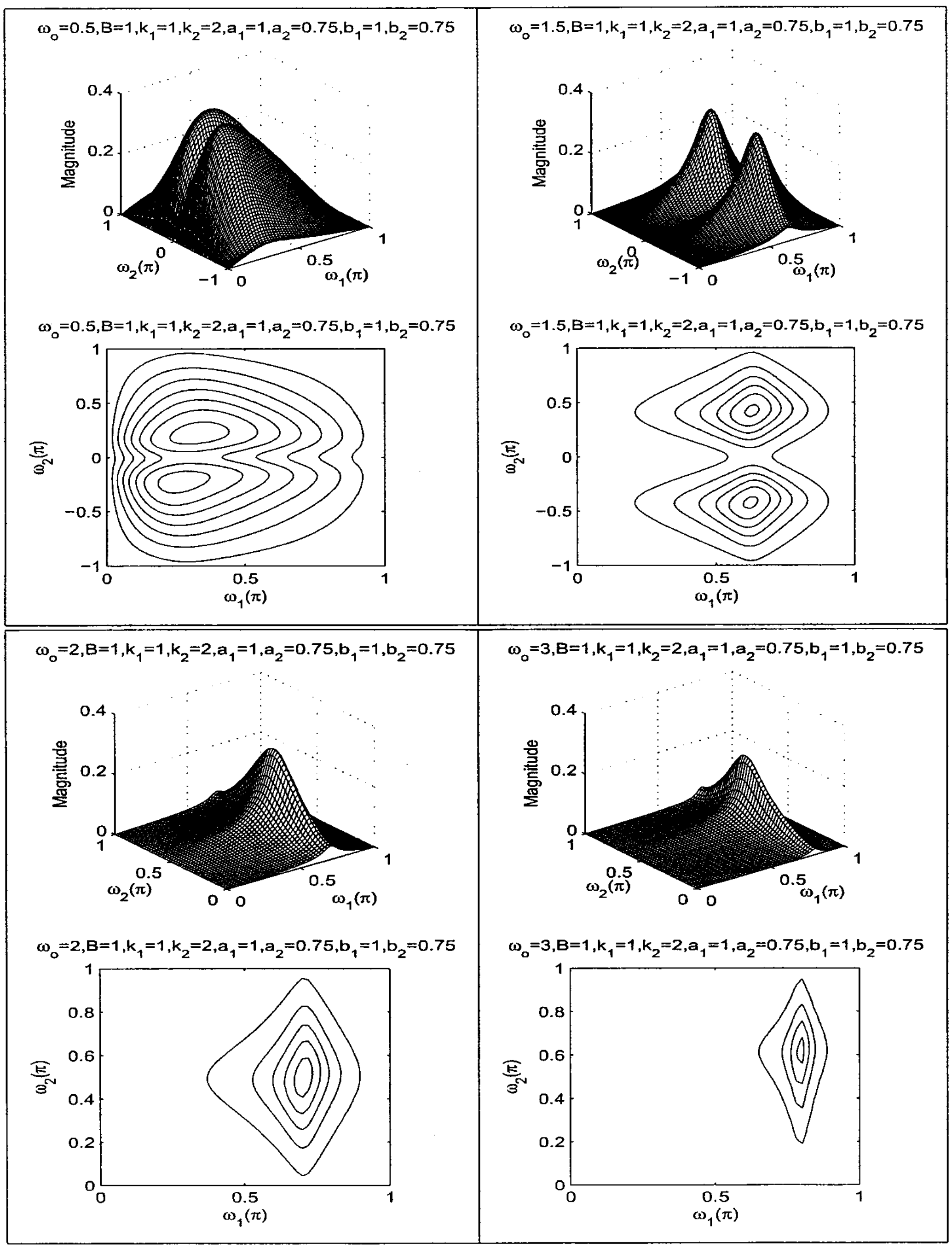

Figure 2.32: Frequency response for the band-pass filter with the value of $k_2 = 2$ for $a_2 = b_2 = 0.75$ and all the other coefficients as unity

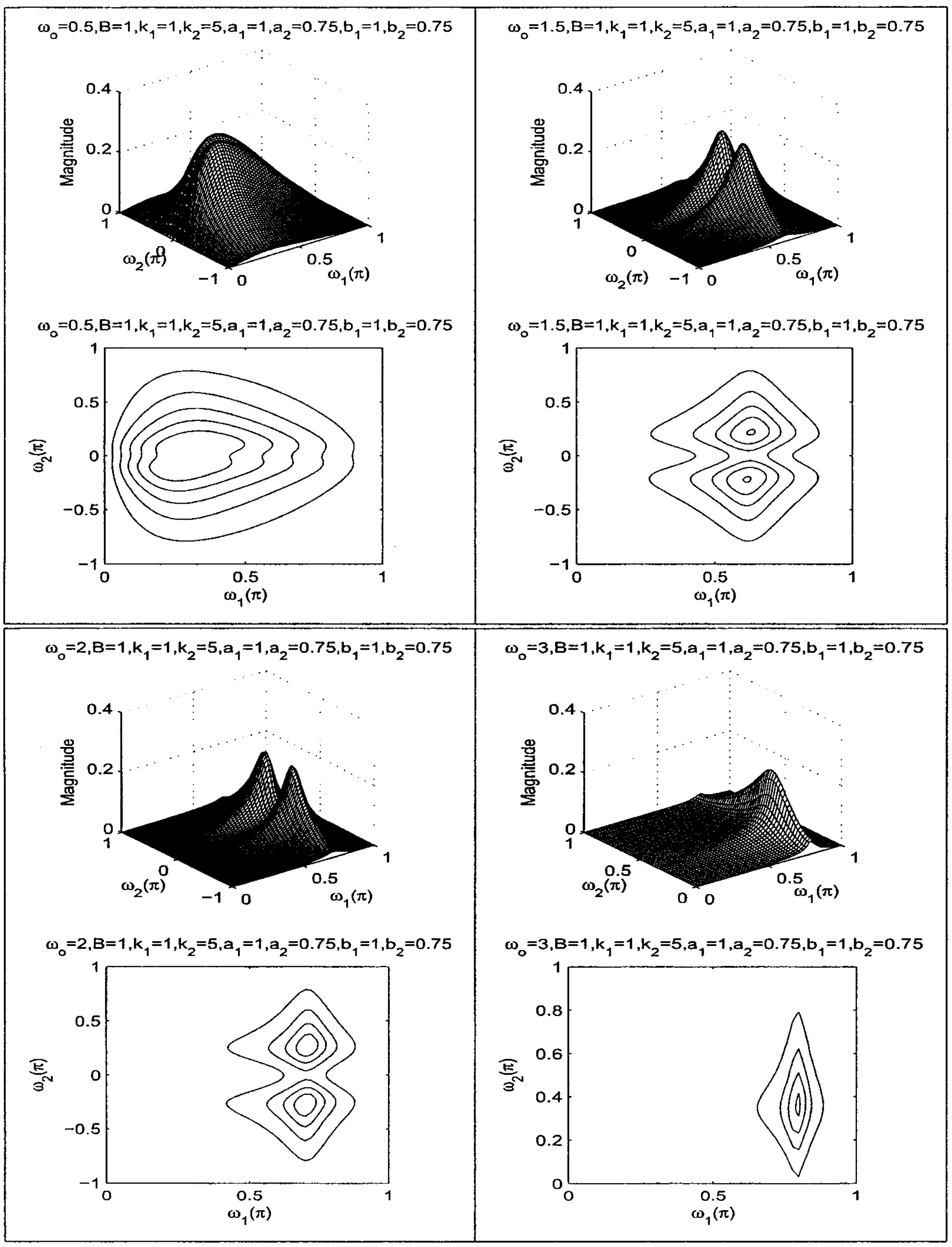

Figure 2.33: Frequency response for the band-pass filter with the value of $k_2 = 5$ for $a_2 = b_2 = 0.75$ and all the other coefficients as unity

2.5.12 Frequency Response of 2-D Band-Pass filters with various k_1 and ω_o values with different values of a_1 and b_1

In section 2.5.7 and section 2.5.10, we saw the frequency response for various k_1 and ω_o values with the same values of a_1 and b_1. In this section we will see the effect different values of a_1 and b_1, for various values of k_1 and ω_o. Fig 2.34 to 2.37 shows the 3-D magnitude and contour plots for different k_1, a_1 and b_1 values for ω_o ranging from $0.1\pi < \omega_o < \pi$.

It can be seen from the figure that the for the k_1 values less than unity, the response will start from a band-pass filter for lower value of ω_o and it slowly transform into a high-pass filter for higher value of ω_o, for the different values of a_1 and b_1. Fig 2.34 and 2.36 shows the response for the coefficient values $k_1 = 0.25$ for the different values of a_1 and b_1 for $\omega_o = 0.5,\ 1.5,\ 2$ and 3. Also, the frequency response for the k_1 greater than unity, the response will start from a low-pass filter for lower value of ω_o and slowly it moves on into a band-pass filter for higher value of ω_o. Fig. 2.35 and 2.37 shows the response for $k_1 = 3$ for the different values of a_1 and b_1 for $\omega_o = 0.5, 1.5, 2$ and 3. When $a_1 < b_1$, the bandwidth decreases but when $a_1 > b_1$, bandwidth is approximately the same. Bandwidth of the response for $a_1 < b_1$ varies inversely to the change in ω_o. The gain of the passband decreases for all the cases if the value of a_1 and b_1 is less than unity.

2.5.13 Frequency Response of 2-D Band-Pass filters with various k_2 and ω_o values with different values of a_2 and b_2

In section 2.5.8 and section 2.5.11, we saw the frequency response for various k_2 and ω_o values with the same values of a_2 and b_2. In this section we will see the effect different values of a_2 and b_2, for various values of k_2 and ω_o. Fig2.38 to 2.41 shows the 3-D magnitude

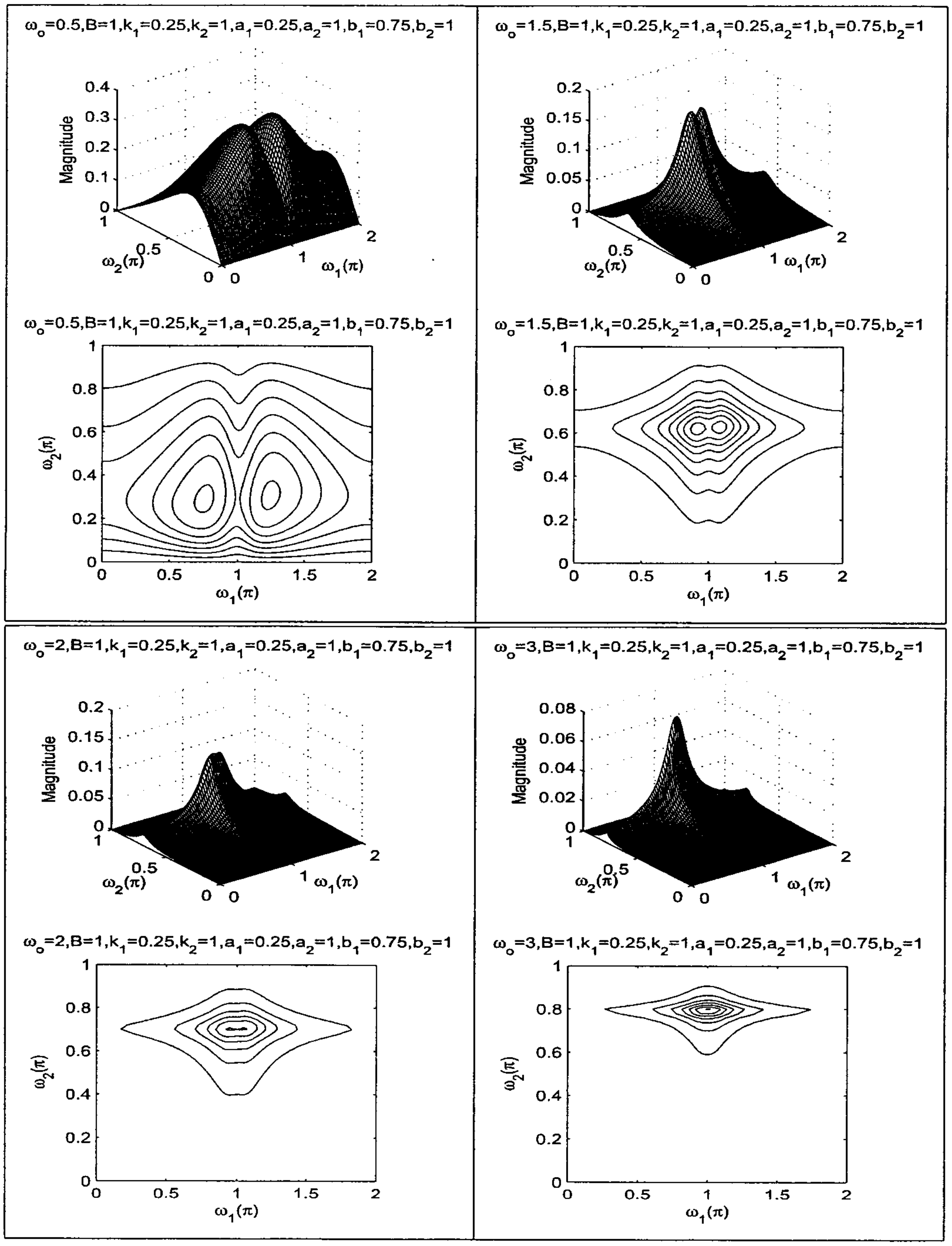

Figure 2.34: Frequency response for the band-pass filter with the value of $k_1 = 0.25$ for $a_1 = 0.25, b_2 = 0.75$ and all the other coefficients as unity

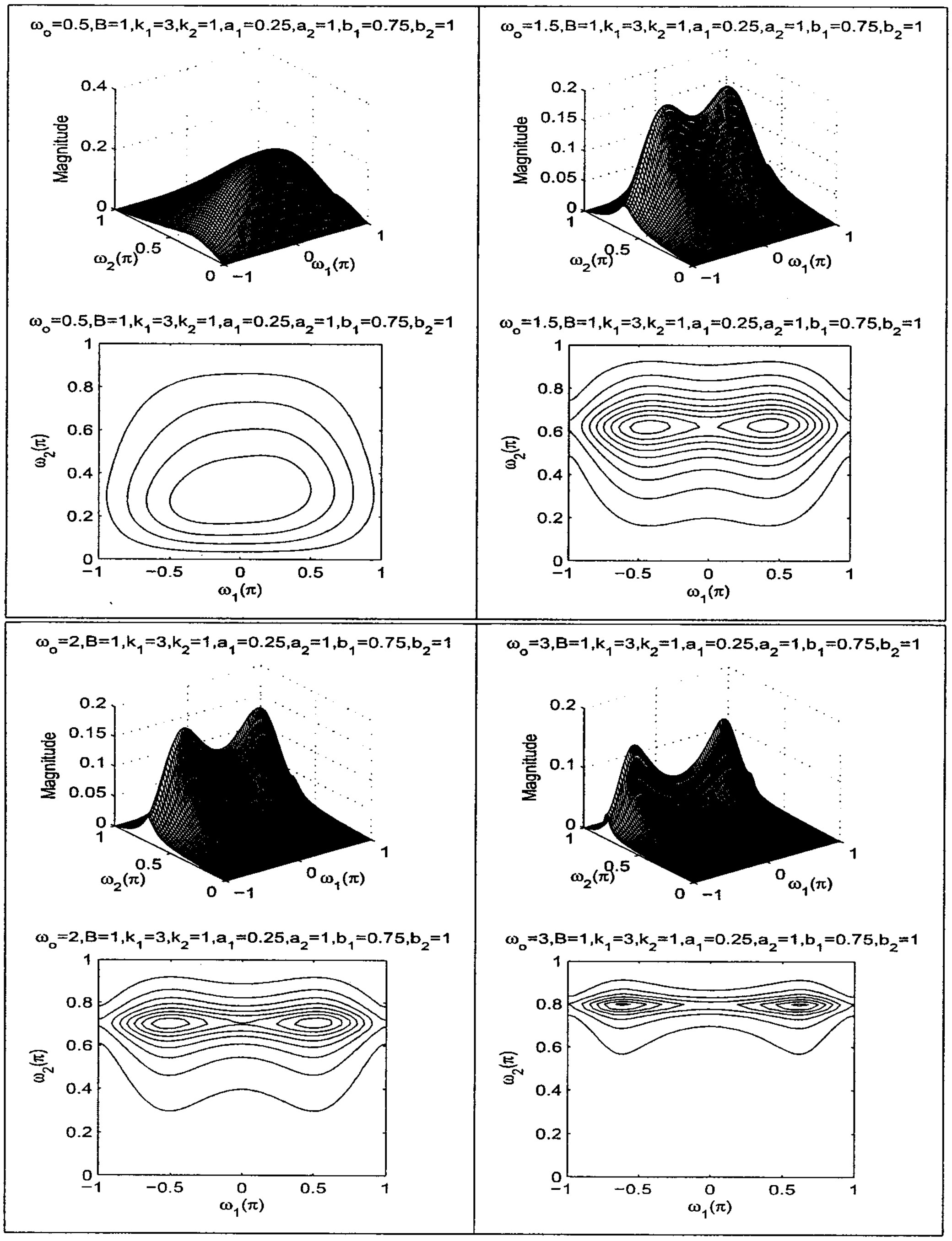

Figure 2.35: Frequency response for the band-pass filter with the value of $k_1 = 3$ for $a_1 = 0.25, b_1 = 0.75$ and all the other coefficients as unity

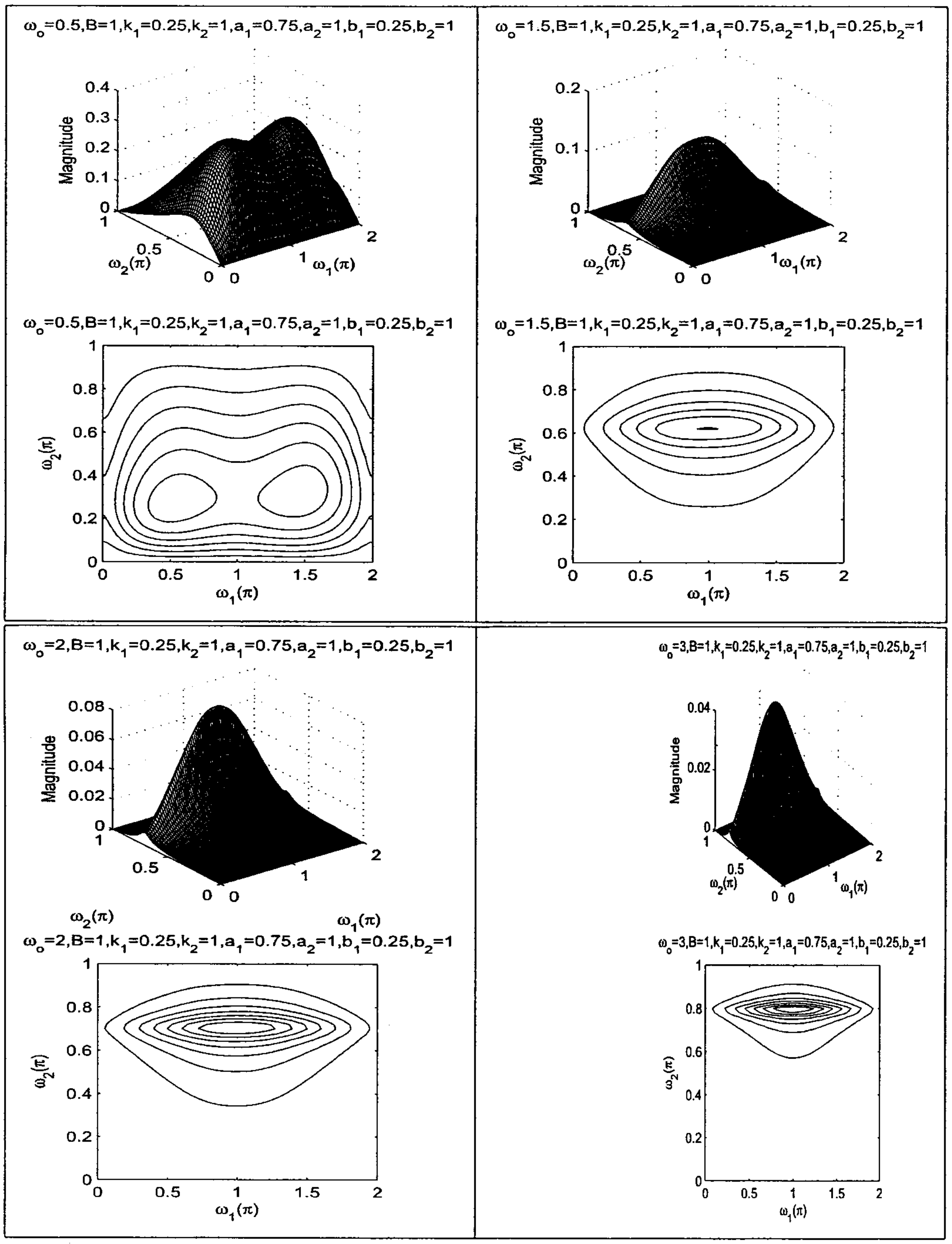

Figure 2.36: Frequency response for the band-pass filter with the value of $k_1 = 0.25$ for $a_1 = 0.75, b_1 = 0.25$ and all the other coefficients as unity

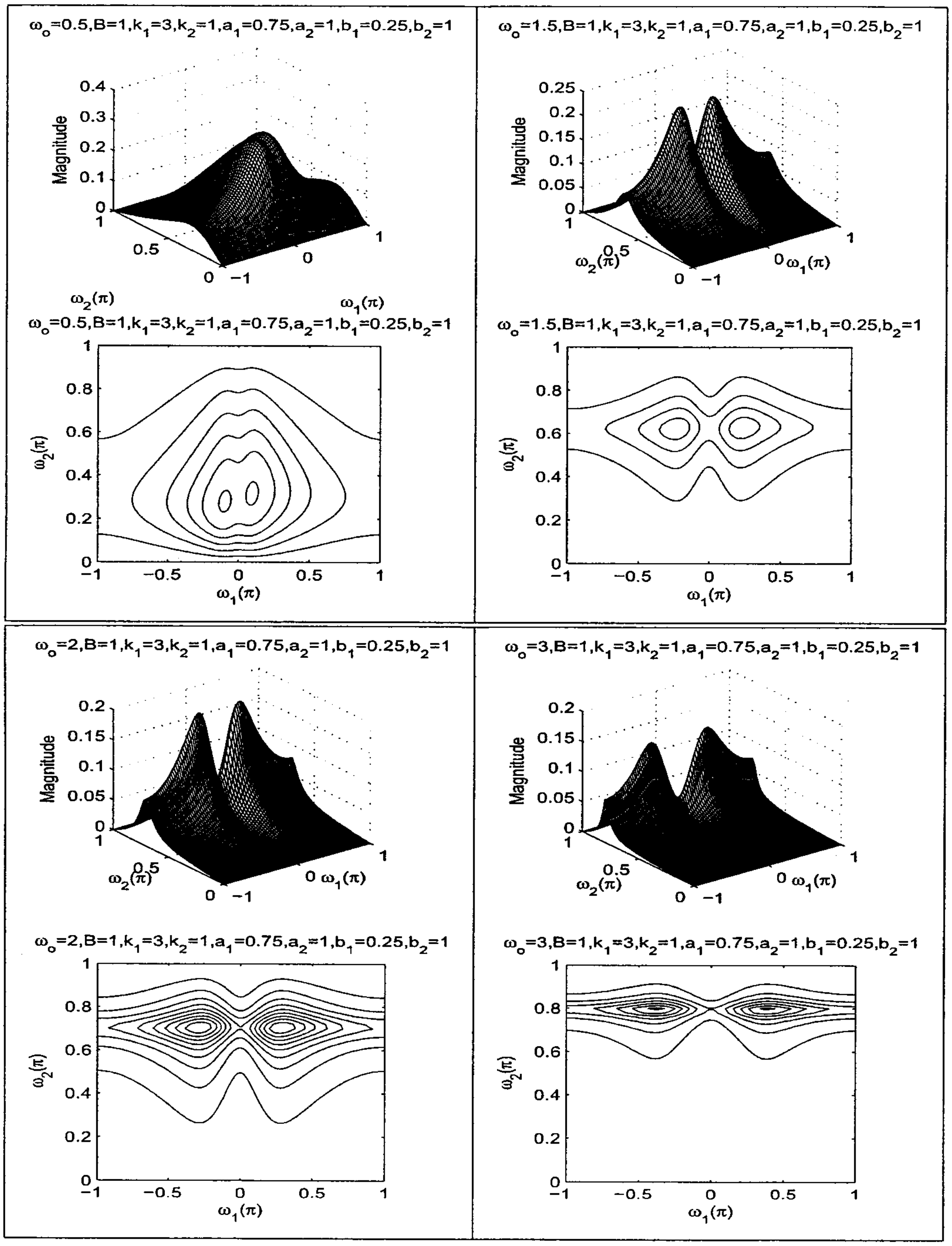

Figure 2.37: Frequency response for the band-pass filter with the value of $k_1 = 3$ for $a_1 = 0.75, b_1 = 0.25$ and all the other coefficients as unity

and contour plots for different k_2, a_2 and b_2 values for ω_o ranging from $0.1\pi < \omega_o < 3$.

It can be seen from the figure that the for the k_2 values less than unity, the response will start from a band-pass filter for lower value of ω_o and it slowly transform into a high-pass filter for higher value of ω_o, for the different values of a_2 and b_2. Fig 2.38 and 2.40 shows the response for the coefficient values $k_2 = 0.25$ for the different values of a_2 and b_2 for $\omega_o = 0.5,\ 1.5,\ 2$ and 3. Also, the frequency response for the k_2 greater than unity, the response will start from a low-pass filter for lower value of ω_o and slowly it moves on into a band-pass filter for higher value of ω_o. Fig. 2.39 and 2.41 shows the response for $k_2 = 3$ for the different values of a_2 and b_2 for $\omega_o = 0.5, 1.5, 2$ and 3. When $a_2 < b_2$, the bandwidth decreases but when $a_2 > b_2$, bandwidth is approximately the same. Bandwidth of the response for $a_2 < b_2$ varies inversely to the change in ω_o. The gain of the passband decreases for all the cases if the value of a_2 and b_2 is less than unity.

2.5.14 Frequency Response of 2-D Band-Pass filters with various k_1, k_2 and ω_o values with different values of a_1, a_2, b_1 and b_2

In this section, we can see the effect of all the coefficient such as k_1, k_2, a_1, a_2, b_1, b_2 and ω_o. Fig 2.42 shows the combination of all the effect in $\omega_1 - axis$ and $\omega_2 - axis$. The coefficients k_1, a_1 and b_1 contributes for the effect in $\omega_1 - axis$, similarly the coefficient k_2, a_2 and b_2 contributes for the effect in $\omega_2 - axis$. The effects of each coefficient are studied upto the section 2.5.6. The fig. 2.42 shows the combination of the effect discussed in section 2.5.12 and 2.5.13. Bandwidth changes inversely to the the value of ω_o.

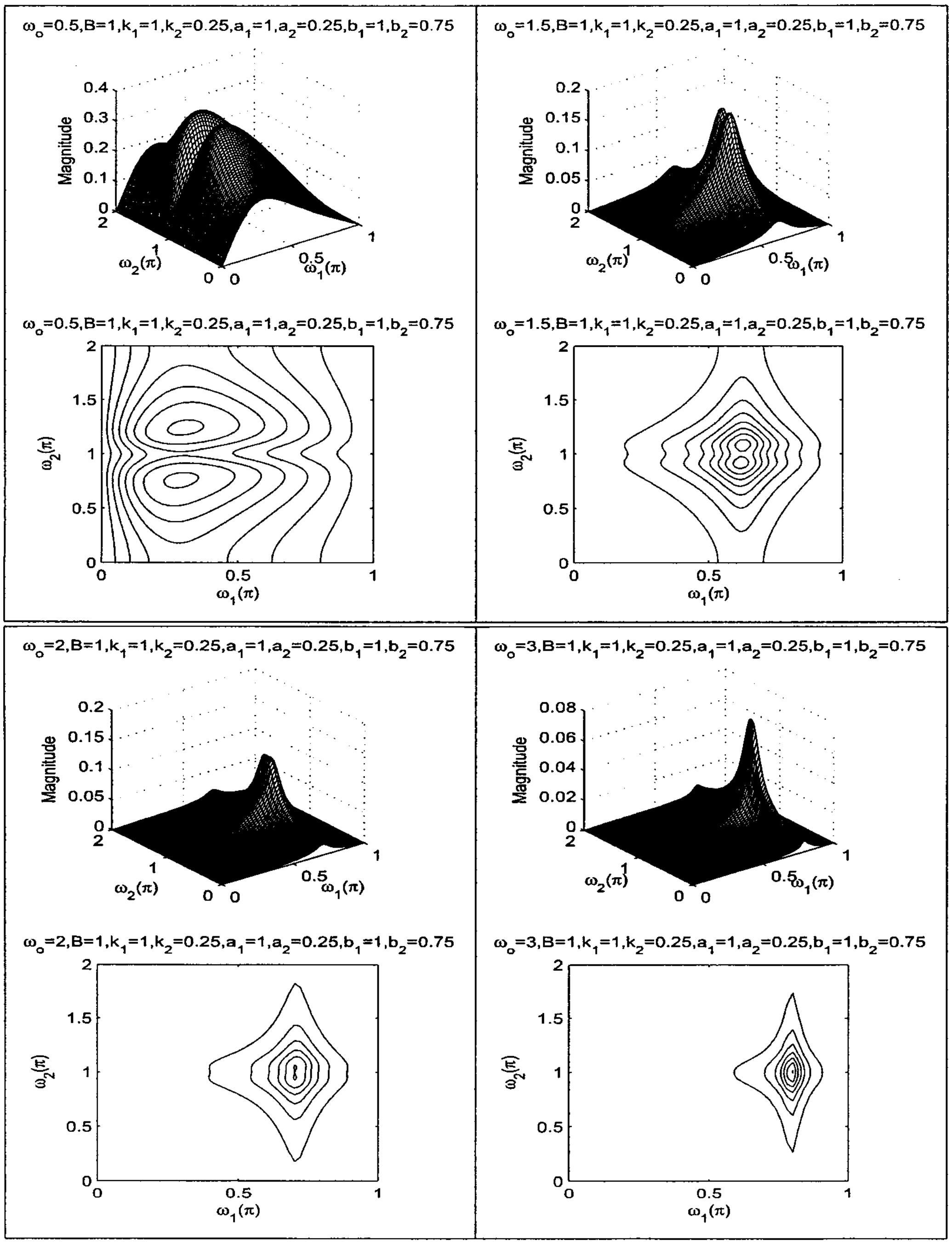

Figure 2.38: Frequency response for the band-pass filter with the value of $k_2 = 0.25$ for $a_2 = 0.25, b_2 = 0.75$ and all the other coefficients as unity

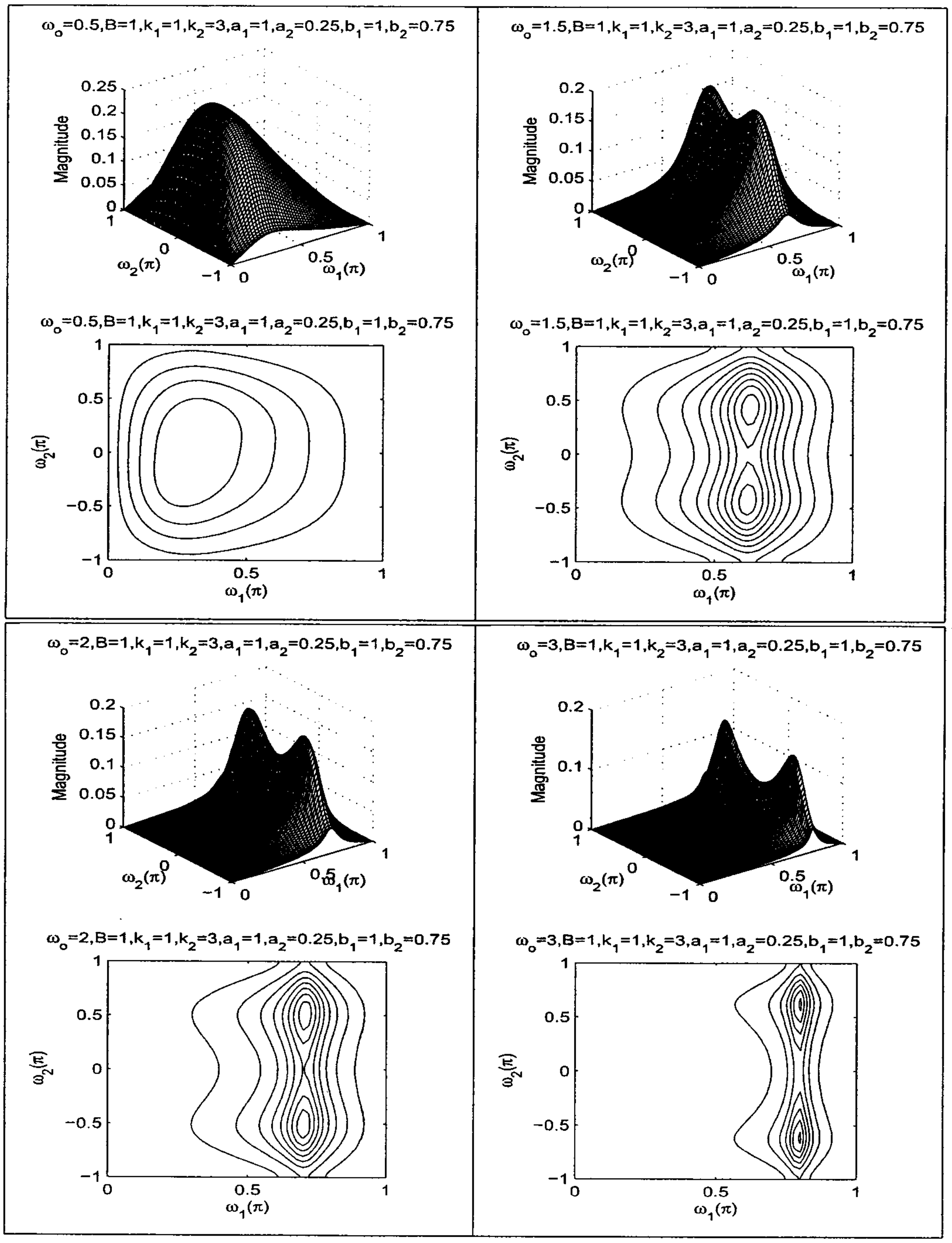

Figure 2.39: Frequency response for the band-pass filter with the value of $k_2 = 3$ for $a_2 = 0.25, b_2 = 0.75$ and all the other coefficients as unity

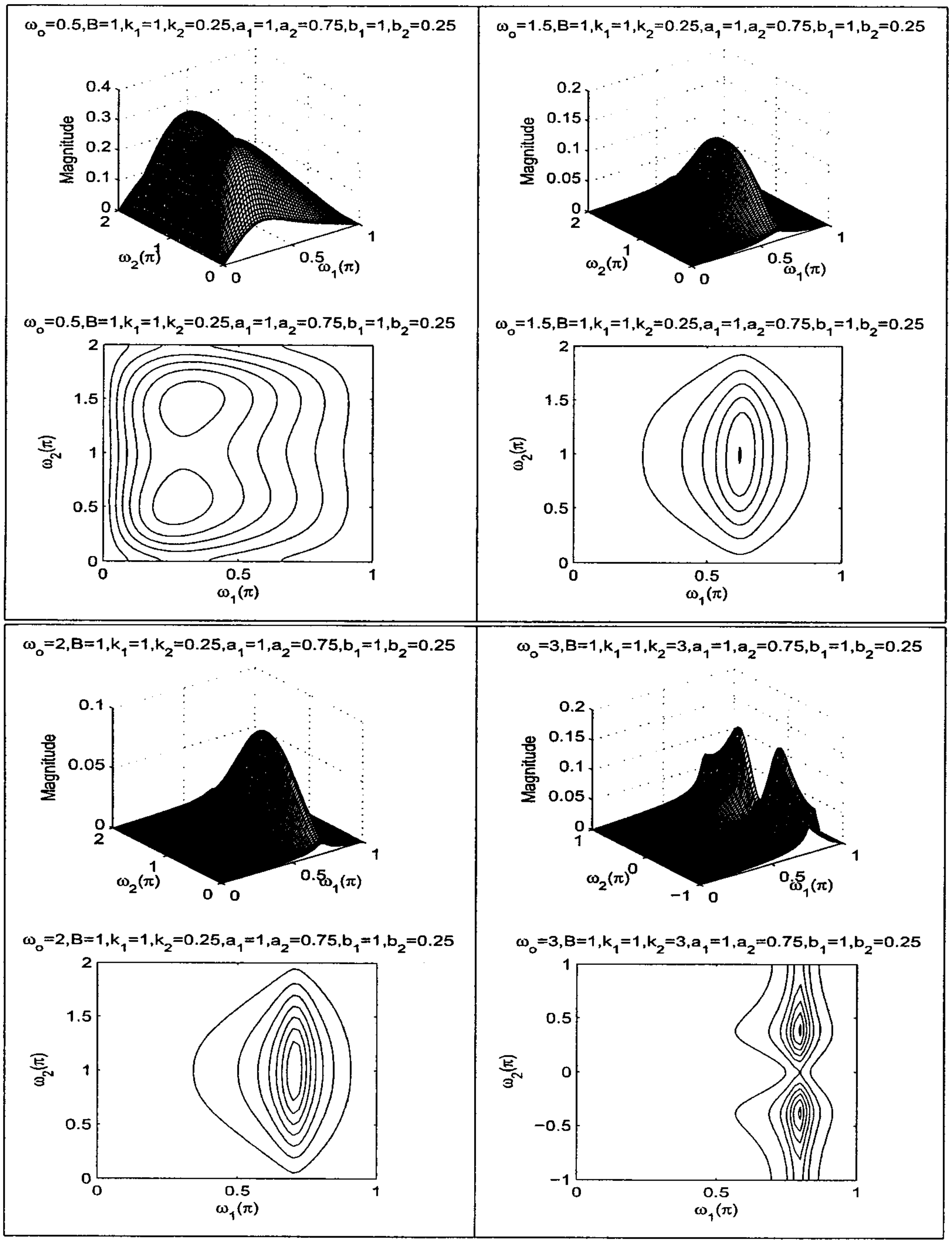

Figure 2.40: Frequency response for the band-pass filter with the value of $k_1 = 0.25$ for $a_1 = 0.75, b_1 = 0.25$ and all the other coefficients as unity

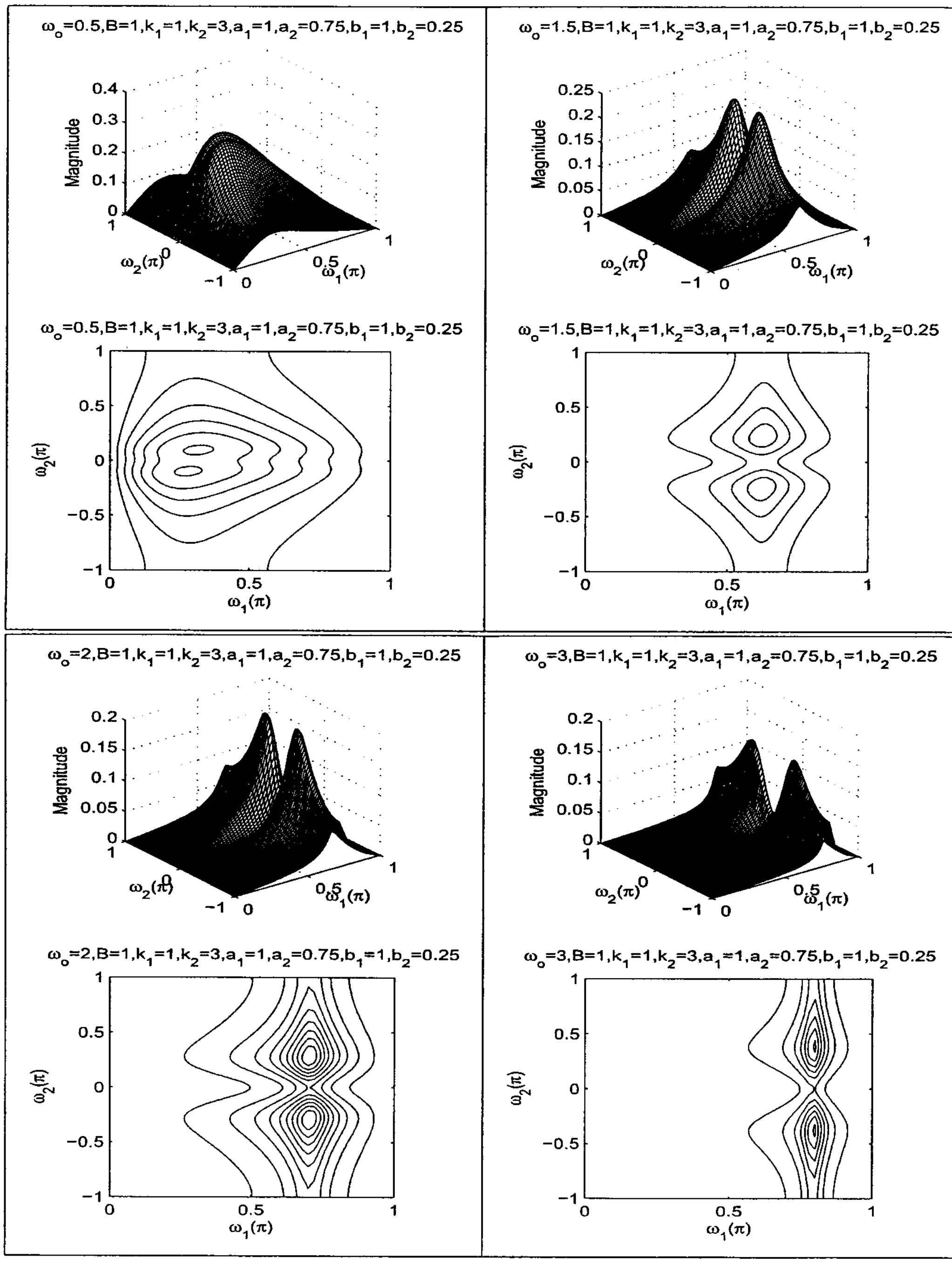

Figure 2.41: Frequency response for the band-pass filter with the value of $k_2 = 3$ for $a_2 = 0.75, b_2 = 0.25$ and all the other coefficients as unity

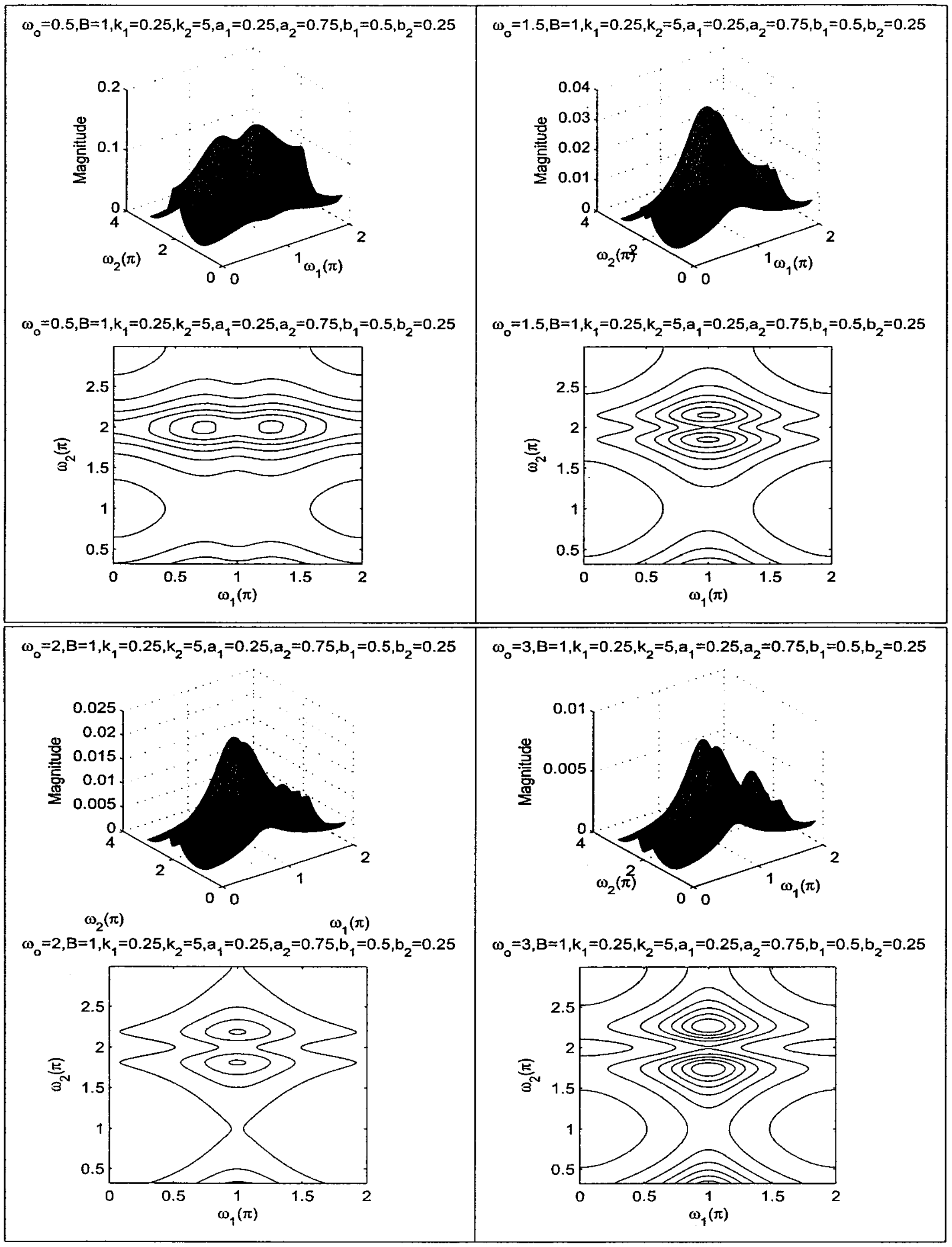

Figure 2.42: Frequency response of a band-pass filter with the coeffecient values of $k_1 = 0.25, k_2 = 5, a_1 = 0.25, b_1 = 0.5, a_2 = 0.75$ and $b_2 = 0.25$

2.6 Summary and Conclusion

This chapter discusses the effects of the coefficient of the digital band-pass filter obtained by applying the generalized bilinear transformation to the band-pass transfer function which is transformed from low-pass butterworth filter. The detailed analysis on each of the coefficient is discussed in section 2.5. The summary of the effect of the coefficient are as follows. On increasing k_1 and k_2 as discussed in sections 2.5.1 and 2.5.2 respectively, the center frequency moves from maximum to minimum and bandwidth increases upto $k_1 = k_2 = 1$, then decreases in $\omega_1 - axis$ and $\omega_2 - axis$ respectively. Sections 2.5.3 and 2.5.4 discusses about the frequecny response on varying a_1 and a_2 respectively. The increase in a_1 and a_2 will increase in gain and center frequency shift from maximum to minimum frequency with the negligible amount of change in bandwidth at $\omega_1 - axis$ and $\omega_2 - axis$ respectively. Section 2.5.5 and 2.5.6 discuss about the frequecny response by varying b_1 and b_2 respectively. The increase in b_1 and b_2 will increase in gain and center frequency shift from minimum to maximum frequency with the neglible amount of change in bandwidth at $\omega_1 - axis$ and $\omega_2 - axis$ respectively. Sections 2.5.7 and 2.5.8 discuss the effect of the same value of $a_1,\ b_1$ and $a_2,\ b_2$. The different types of filter response are obtained with decreased gain. The effect of each of the coefficients of the band-pass filter are studied with the fixed value of center frequency (ω_o). In section 2.5.9, the effect of the coefficients, the center frequency and Bandwidth are analyzed. Sections 2.5.10 and 2.5.11 give the effect of the band-pass filter response for the combination of $k_1,\ k_2$ with various values of ω_o with same value of $a_1,\ b_1$and $a_2,\ b_2$. Sections 2.5.12 and 2.5.13 discuss the efeect of the band-pass filter for the various combination of $k_1,\ k_2$ and ω_o with the different values of $a_1,\ b_1$ and $a_2,\ b_2$. Finally, the combination of all the coefficients with ω_o are plotted in ω_1 and $\omega_2 - axis$. Thus the effects of the band-pass filter were analyzed for the various combinations of the coefficient of the generalized bilinear transformation along with the combination of the center frequency ω_o, and the selective 3-D magnitude and contour plots of the band-pass filter were plotted.

Chapter 3

Two-Dimensional Band-Stop Filters from Low-Pass Butterworth fi lters

In this chapter, the 2-D Band-stop filters are analyzed. The low-pass butterworth filter is transformed to a band-stop filter, and the same is transformed to a digital filter by using the generalized Bilinear transformation. Then, sensitivity analyses are performed by varying the coefficient of the generalized bilinear transformation and the effects are observed in the design of 2-D band-stop filter. Section 3.1 describes the low-pass to band-stop transformation. The band- stop structure and its transfer function and the determining of parameter values are explained in sections 3.2 and 3.3 respectively. Section 3.4 gives the stability condtion and Band-Stop digital transfer function. Finally, section 3.5 gives the detail frequency response of band -stop filter on varying the Bilinear Transformation coeffiecient.

3.1 Low-Pass to Band-Stop Transformation

The low-pass filter response is explained in the section 2.1. In the case of a band-stop filter the transformation $F(s)$ must map low-pass response to a band-stop response as shown in the fig (3.1). The result is a band-stop filter with the center frequency ω_o and width B of the band rejected. We may note that the transformation could be effected by first making a low-pass to high-pass transformation with $\omega_c = 1$, and then making the mappings (a),(b) and (c) of the low-pass to band-pass transformation explained in the section 2.2. The resulting transformation then must be [21]

$$S = \frac{Bs}{s^2 + \omega_o^2} \tag{3.1}$$

3.2 Band-Stop Filter Structure

The fig 3.2 shows the $4th\ order$ Band-Stop filter structure.

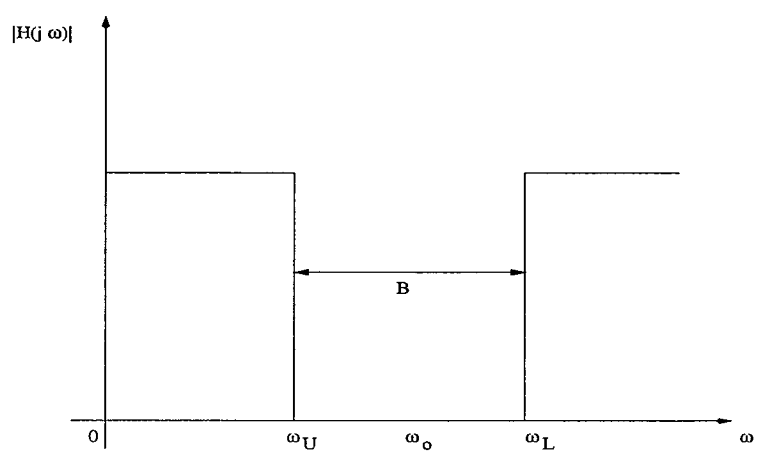

Figure 3.1: An ideal band-stop amplitude response.

The Transfer function of this structure is given by

$$H_{BS}(s) = \frac{R_2 + R_2(L_2C_2 + L_1C_1)s^2 + R_2L_1C_1L_2C_2s^4}{(R_1 + R_2)L_1C_1L_2C_2s^4 + (R_1R_2C_1 + L_2)L_1C_2s^3 + [(R_1 + R_2)(L_1C_1 + L_2C_2)} \tag{3.2}$$

$$+R_2L_1C_2]s^2 + (L_1 + R_1R_2C_2)s +$$

$$R_1 + R_2$$

If the above band-stop structure is considered as $2 - D$ Band-stop filter, the transfer function will be given as

$$H_{BS}(s_1, s_2) = \frac{R_2 + R_2L_2C_2s_2^2 + R_2L_1C_1s_1^2 + R_2L_1C_1L_2C_2s_1^2s_2^2}{(R_1 + R_2)L_1C_1L_2C_2s_1^2s_2^2 + R_1R_2L_1C_1C_2s_1^2s_2 + L_1L_2C_2s_1s_2^2} \tag{3.3}$$

$$+(R_1 + R_2)L_1C_1s_1^2 + (R_1 + R_2)L_2C_2s_2^2 + R_2L_1C_2s_1s_2 + L_1s_1$$

$$+R_1R_2C_2s_2 + R_1 + R_2$$

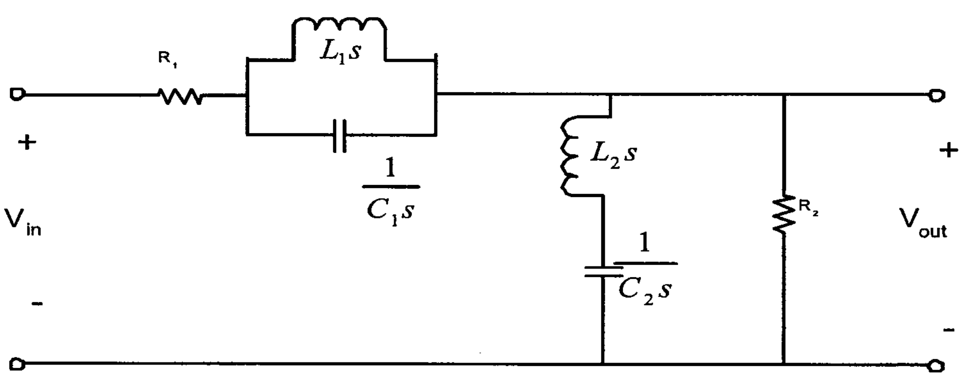

Figure 3.2: Band-Stop filter structure

3.3 Determining the paramater values of the band-stop structure

The $4th\ order$ band-stop filter transfer function is given by

$$H_{BS}(s) = \frac{R_2 + R_2(L_2C_2 + L_1C_1)s^2 + R_2L_1C_1L_2C_2s^4}{(R_1+R_2)L_1C_1L_2C_2s^4 + (R_1R_2C_1 + L_2)L_1C_2s^3 + [(R_1+R_2)(L_1C_1 + L_2C_2)} \tag{3.4}$$

$$+R_2L_1C_2]s^2 + (L_1 + R_1R_2C_2)s +$$

$R_1 + R_2$

The *second order* Butterworth Low-Pass filter is shown in figure 2.6 and its transfer function is given by

$$H_L(S) = \frac{K}{(S^2 + \sqrt{2}S + 1)} \tag{3.5}$$

By applying Band-Stop frequency transformation eqn (3.1) to the Low-Pass Butterworth filter in (2.18),

$$H_B(s) = \frac{K(s^4 + 2s^2\omega_o^2 + \omega_o^4)}{s^4 + \sqrt{2}Bs^3 + (2\omega_o^2 + B^2)s^2 + \sqrt{2}\omega_o^2Bs + \omega_o^4} \tag{3.6}$$

Because of the transformation the elements of the Low-Pass filter will change to Band-Stop filter elements. The Inductance of the Low-Pass filter will transfrom into parallel s combination of Inductance and capacitance.

$$\frac{LsB}{s^2 + \omega_o^2} = \frac{1}{C_1s + \frac{1}{L_1s}} \tag{3.7}$$

$$\frac{1}{\frac{s}{LB} + \frac{\omega_o^2}{LBs}} = \frac{1}{C_1s + \frac{1}{L_1s}} \tag{3.8}$$

From (2.20), the L_1 and C_1 values are

$$L_1 = \frac{LB}{\omega_o^2}$$

$$C_1 = \frac{1}{LB}$$

And also the capacitance of the Low-Pass filter will transform into series combination of Inductance and capacitance in band-stop filter design.

$$\frac{s^2 + \omega_o^2}{CBs} = L_2 s + \frac{1}{C_2 s} \tag{3.9}$$

$$\frac{s}{CB} + \frac{\omega_o^2}{CBs} = L_2 s + \frac{1}{C_2 s}$$

$$L_2 = \frac{1}{CB}$$

$$C_2 = \frac{CB}{\omega_o^2}$$

Therefore, the parameter values of the Band-Stop structure are

$$L_1 = \frac{LB}{\omega_o^2}$$

$$L_2 = \frac{1}{CB}$$

$$C_1 = \frac{1}{LB}$$

$$C_2 = \frac{CB}{\omega_o^2}$$

where L and C are the values of Inductance and capacitance of the Low -Pass filter structure shown in figure 2.6. The Values of L and C are of $\sqrt{2}$ values.

3.4 Digital Transfer Function

3.4.1 Band-Stop Limits for the Coefficeints of Generalized Bilinear Transformation

In the previous section, we have obtained the 2-D analog transfer function eqn (3.3)for the circuit in fig. 3.2. Substituting the values of all the parameters in eqn (3.3), it becomes

$$H_{BS}(s_1, s_2) = \frac{s_1 s_2^2 + \omega_o^2 s_1^2 + \omega_o^2 s_2^2 + \omega_o^4}{2s_1^2 s_2^2 + s_1 s_2^2 \sqrt{2}B + s_1^2 s_2 \sqrt{2}B + 2s_1 s_2 B^2 + 2\omega_o^2 s_1^2 + 2\omega_o^2 s_2^2 + \sqrt{2}B\omega_o^2 s_1 + \sqrt{2}B\omega_o^2 s_2 + \cdots} \tag{3.10}$$

When the generalized bilinear transformation is applied, it has to be ensured that the resulting 2-D digital filter is always stable. To satisfy the conditions of obtaining stable 2-D band-stop digital filters, the coefficients of the double generalized bilinear transformations should meet the requirements given in eqns (2.22-2.24). With these constraints, we can obtain 2-D stable band-stop digital filter from its analog counterpart by the application of the generalized bilinear transformations.

3.4.2 2-D Band-Stop Digital Transfer Function

Generalized Bilinear Transformation is explained in section 2.4. The $1-D$ digital Band-Stop filter transfer function is obtained by applying the equation 2.25 to equation 3.2, the transfer function is

$$H(z) = \frac{R_2L_1C_1L_2C_2k^4(z-a)^4 + R_2k_1(L_2C_2+L_1C_1)(z-a)^2(z+b)^2 + R_2(z+b)^4}{(R_1+R_2)L_1C_1L_2C_2k^4(z-a)^4 + (R_1R_2C_1+L_2)L_1C_2k^3(z-a)^3(z+b)} \tag{3.11}$$

$$[(R_1+R_2)(L_1C_1+L_2C_2)+R_2L_1C_2]k^2(z-a)^2(z+b)^2$$

$$+(L_1+R_1R_2C_2)k(z-a)(z+b)^3+(R_1+R_2)(z+b)^4$$

The 2-D digital Band-Stop filter transfer function is obtained by appyling the equation??to equation3.3, the transfer function is

$$R_2(z_1+b_1)^2(z_2+b_2)^2 + R_2L_2C_2k_2^2(z_2-a_2)^2(z_1+b_1)^2+$$

$$H_{BS}(z_1,z_2) = \frac{R_2L_1C_1k_1^2(z_1-a_1)^2(z_2+b_2)^2 + R_2L_1C_1L_2C_2k_1^2k_2^2(z_1-a_1)^2(z_2-a_2)^2}{(R_1+R_2)L_1C_1L_2C_2k_1^2k_2^2(z_1-a_1)^2(z_2-a_2)^2 + R_1R_2L_1C_1C_2k_1^2k_2} \tag{3.12}$$

$$(z_1-a_1)^2(z_2-a_2)(z_z+b_2)+L_1L_2C_2k_1k_2^2(z_1-a_1)(z_1+b_1)(z_2-a_2)^2+(R_1+R_2)L_1C_1k_1^2$$

$$(z_1-a_1)^2(z_2+b_2)^2+(R_1+R_2)L_2C_2k_2^2(z_2-a_2)^2(z_1+b_1)^2+R_2L_1C_2k_1k_2(z_1-$$

$$a_1)(z_2-a_2)$$

$$(z_1+b_1)(z_2+b_2)+R_1R_2C_2k_2(z_2-a_2)(z_2+b_2)(z_1+b_1)^2+L_1k_1(z_1-a_1)$$

$$(z_1+b_1)(z_2+b_2)^2+(R_1+R_2)(z_1+b_1)^2(z_2+b_2)^2$$

The MATLAB function *d2stop.m* (refer to APPENDIX) can be employed to get the contour and 3-D magnitude plots of the resulting 2-D band-stop digital filters with the transfer function (3.12). In this function, it is made certain that stability conditions are satisfied. As the changeable coefficients of the double generalized bilinear transformations can change the stability of the resulting 2-D digital filters, it is necessary to introduce stability constraints in discrete domain.

3.4.3 Stability Conditions of the 2-D Digital Band-Pass Filter

Starting from a 2-D analog transfer function with a VSHP as its denominator, applying the well-known bilinear transformation will always results in 2-D stable digital filter. However, when the generalized bilinear transformation is applied, it has to be ensured that the resulting 2-D digital filter is always stable[17, 18, 19].

The inverse generalized bilinear transformation is

$$z_i \rightarrow \frac{b_i s_i + k_i a_i}{k_i - s_i},\ i = 1, 2 \tag{3.13}$$

Substituting eqn (3.13) to (2.25), we get

$$s_i \rightarrow k_i \frac{[s_i(1 + a_i) + (1 - a_i)]}{[s_i(1 - b_i) + (1 + b_i)]} \tag{3.14}$$

where the eqn (3.14) can be rewritten as

$$s_1 \rightarrow \frac{s_1\alpha_1 + \alpha_2}{s_1\alpha_4 + \alpha_3};\quad s_2 \rightarrow \frac{s_2\beta_1 + \beta_2}{s_2\beta_4 + \beta_3}$$

where

$$\alpha_1 = k_1 + k_1 a_1; \alpha_2 = k_1 - k_1 a_1; \alpha_3 = 1 + b_1; \alpha_4 = 1 - b_1 \tag{3.15}$$

$$\beta_1 = k_2 + k_2 a_2; \beta_2 = k_2 - k_2 a_2; \beta_3 = 1 + b_2; \beta_4 = 1 - b_2 \tag{3.16}$$

Applying (3.14) to $\frac{1}{D_{BS}(s_1,s_2)}$, where $D_{BS}(s_1, s_2)$ is expressed as

$$D_{BS}(s_1, s_2) = \zeta_1 s_1^2 s_2^2 + \zeta_2 s_1 s_2^2 + \zeta_3 s_1^2 s_2 + \zeta_4 s_1 s_2 + \zeta_5 s_1^2 + \zeta_6 s_2^2 + \zeta_7 s_1 + \zeta_8 s_2 + \zeta_9 \tag{3.17}$$

where

$$\zeta_1 = \alpha_1^2\beta_1^2\gamma_1+\alpha_1\alpha_4\beta_1^2\gamma_2+\alpha_1^2\beta_1\beta_4\gamma_3+\alpha_1\alpha_4\beta_1\beta_4\gamma_4+\alpha_1^2\beta_4^2\gamma_5+\alpha_4^2\beta_1^2\gamma_6+\alpha_1\alpha_4\beta_4^2\gamma_7+\alpha_4^2\beta_1\beta_4\gamma_8+\alpha_4^2\beta_4^2\gamma_9 \tag{3.18}$$

$$\zeta_2 = 2\alpha_1\alpha_2\beta_1^2\gamma_1+(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_1^2\gamma_2+2\alpha_1\alpha_2\beta_1\beta_4\gamma_3+(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_1\beta_4\gamma_4+2\alpha_1\alpha_2\beta_4^2\gamma_5+2\alpha_3\alpha_4\beta_1^2\gamma_6+$$

$$(\alpha_2\alpha_4 + \alpha_1\alpha_3)\beta_4^2\gamma_7 + 2\alpha_3\alpha_4\beta_1\beta_4\gamma_8 + 2\alpha_3\alpha_4\beta_4^2\gamma_9 \tag{3.19}$$

$$\zeta_3 = 2\alpha_1^2\beta_1\beta_2\gamma_1+2\alpha_1\alpha_4\beta_1\beta_2\gamma_2+\alpha_1^2(\beta_2\beta_4+\beta_1\beta_3)\gamma_3+\alpha_1\alpha_4(\beta_2\beta_4+\beta_1\beta_3)\gamma_4+2\alpha_1^2\beta_3\beta_4\gamma_5+2\alpha_4^2\beta_1\beta_2\gamma_6+$$

$$2\alpha_1\alpha_4\beta_3\beta_4\gamma_7 + \alpha_4^2(\beta_2\beta_4 + \beta_1\beta_3)\gamma_8 + 2\alpha_4^2\beta_3\beta_4\gamma_9 \tag{3.20}$$

$$\zeta_4 = 4\alpha_1\alpha_2\beta_1\beta_2\gamma_1+2\beta_1\beta_2(\alpha_2\alpha_4+\alpha_1\alpha_3)\gamma_2+2\alpha_1\alpha_2(\beta_2\beta_4+\beta_1\beta_3)\gamma_3+(\alpha_2\alpha_4+\alpha_1\alpha_3)(\beta_2\beta_4+\beta_1\beta_3)\gamma_4+$$

$$4\alpha_1\alpha_2\beta_3\beta_4\gamma_5+4\alpha_3\alpha_4\beta_1\beta_2\gamma_6+2(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_3\beta_4\gamma_7+2\alpha_3\alpha_4(\beta_1\beta_3+\beta_2\beta_4)\gamma_8+4\alpha_3\alpha_4\beta_3\beta_4\gamma_9 \tag{3.21}$$

$$\zeta_5 = \alpha_1^2\beta_2^2\gamma_1+\alpha_1\alpha_4\beta_2^2\gamma_2+\alpha_1^2\beta_2\beta_3\gamma_3+\alpha_1\alpha_4\beta_2\beta_3\gamma_4+\alpha_1^2\beta_3^2\gamma_5+\alpha_4^2\beta_2^2\gamma_6+\alpha_1\alpha_4\beta_3^2\gamma_7+\alpha_4^2\beta_2\beta_3\gamma_8+\alpha_4^2\beta_3^2\gamma_9 \tag{3.22}$$

$$\zeta_6 = \alpha_2^2\beta_1^2\gamma_1+\alpha_2\alpha_3\beta_1^2\gamma_2+\alpha_2^2\beta_1\beta_4\gamma_3+\alpha_2\alpha_3\beta_1\beta_4\gamma_4+\alpha_2^2\beta_4^2\gamma_5+\alpha_3^2\beta_1^2\gamma_6+\alpha_3\alpha_3\beta_4^2\gamma_7+\alpha_3^2\beta_1\beta_4\gamma_8+\alpha_3^2\beta_4^2\gamma_9 \tag{3.23}$$

$$\zeta_7 = 2\alpha_1\alpha_2\beta_2^2\gamma_1+(\alpha_1\alpha_3+\alpha_2\alpha_4)\beta_2^2\gamma_2+2\alpha_1\alpha_2\beta_2\beta_3\gamma_3+(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_2\beta_3\gamma_4+2\alpha_1\alpha_2\beta_3^2\gamma_5+$$

$$2\alpha_3\alpha_4\beta_2^2\gamma_6 + (\alpha_1\alpha_3 + \alpha_2\alpha_4)\beta_3^2\gamma_7 + 2\alpha_3\alpha_4\beta_2\beta_3\gamma_8 + 2\alpha_3\alpha_4\beta_3^2\gamma_9 \tag{3.24}$$

$$\zeta_8 = 2\beta_1\beta_2\alpha_2^2\gamma_1+2\alpha_2\alpha_3\beta_1\beta_2\gamma_2+(\beta_1\beta_3+\beta_2\beta_4)\alpha_2^2\gamma_3+\alpha_2\alpha_3(\beta_2\beta_4+\beta_1\beta_3)\gamma_4+2\alpha_2^2\beta_3\beta_4\gamma_5+2\alpha_3^2\beta_1\beta_2\gamma_6+$$

$$2\alpha_2\alpha_3\beta_3\beta_4\gamma_7 + (\beta_1\beta_3 + \beta_2\beta_4)\alpha_3^2\gamma_8 + 2\alpha_3^2\beta_3\beta_4\gamma_9 \tag{3.25}$$

$$\zeta_9 = \alpha_2^2\beta_2^2\gamma_1+\alpha_2\alpha_3\beta_2^2\gamma_2+\alpha_2^2\beta_2\beta_3\gamma_3+\alpha_2\alpha_3\beta_2\beta_3\gamma_4+\alpha_3^2\beta_3^2\gamma_5+\alpha_3^2\beta_2^2\gamma_6+\alpha_2\alpha_3\beta_3^2\gamma_7+\alpha_3^2\beta_2\beta_3\gamma_8+\alpha_3^2\beta_3^2\gamma_9 \tag{3.26}$$

where $\gamma_1 = 2; \gamma_2 = \sqrt{2}B; \gamma_3 = \sqrt{2}B; \gamma_4 = 2B^2$

$\gamma_5 = 2\omega_o^2; \gamma_6 = 2\omega_o^2; \gamma_7 = \sqrt{2}B\omega_o^2; \gamma_8 = \sqrt{2}B\omega_o^2; \gamma_8 = 2\omega_o^4$

In order that $D_B(s_1, s_2)$ is a Very Strict Hurwitz Polynomial (VSHP), the necessary and sufficient conditions are every polynomial coefficient needs to be positive and satisfy VSHP conditons [8, 9]. Hence, the stability conditions in (3.18)-(3.26) becomes

$$\alpha_1^2\beta_1^2\gamma_1+\alpha_1\alpha_4\beta_1^2\gamma_2+\alpha_1^2\beta_1\beta_4\gamma_3+\alpha_1\alpha_4\beta_1\beta_4\gamma_4+\alpha_1^2\beta_4^2\gamma_5+\alpha_4^2\beta_1^2\gamma_6+\alpha_1\alpha_4\beta_4^2\gamma_7+\alpha_4^2\beta_1\beta_4\gamma_8+\alpha_4^2\beta_4^2\gamma_9 > 0 \tag{3.27}$$

$$2\alpha_1\alpha_2\beta_1^2\gamma_1+(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_1^2\gamma_2+2\alpha_1\alpha_2\beta_1\beta_4\gamma_3+(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_1\beta_4\gamma_4+2\alpha_1\alpha_2\beta_4^2\gamma_5+2\alpha_3\alpha_4\beta_1^2\gamma_6+$$

$$(\alpha_2\alpha_4 + \alpha_1\alpha_3)\beta_4^2\gamma_7 + 2\alpha_3\alpha_4\beta_1\beta_4\gamma_8 + 2\alpha_3\alpha_4\beta_4^2\gamma_9 > 0 \tag{3.28}$$

$$2\alpha_1\alpha_2\beta_1^2\gamma_1+(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_1^2\gamma_2+2\alpha_1\alpha_2\beta_1\beta_4\gamma_3+(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_1\beta_4\gamma_4+2\alpha_1\alpha_2\beta_4^2\gamma_5+2\alpha_3\alpha_4\beta_1^2\gamma_6+$$

$$(\alpha_2\alpha_4 + \alpha_1\alpha_3)\beta_4^2\gamma_7 + 2\alpha_3\alpha_4\beta_1\beta_4\gamma_8 + 2\alpha_3\alpha_4\beta_4^2\gamma_9 > 0 \tag{3.29}$$

$$4\alpha_1\alpha_2\beta_1\beta_2\gamma_1+2\beta_1\beta_2(\alpha_2\alpha_4+\alpha_1\alpha_3)\gamma_2+2\alpha_1\alpha_2(\beta_2\beta_4+\beta_1\beta_3)\gamma_3+(\alpha_2\alpha_4+\alpha_1\alpha_3)(\beta_2\beta_4+\beta_1\beta_3)\gamma_4+4\alpha_1\alpha_2\beta_3\beta_4$$

$$4\alpha_3\alpha_4\beta_1\beta_2\gamma_6+2(\alpha_2\alpha_4+\alpha_1\alpha_3)\beta_3\beta_4\gamma_7+2\alpha_3\alpha_4(\beta_1\beta_3+\beta_2\beta_4)\gamma_8+4\alpha_3\alpha_4\beta_3\beta_4\gamma_9 > 0 \tag{3.30}$$

$$\alpha_1^2\beta_2^2\gamma_1+\alpha_1\alpha_4\beta_2^2\gamma_2+\alpha_1^2\beta_2\beta_3\gamma_3+\alpha_1\alpha_4\beta_2\beta_3\gamma_4+\alpha_1^2\beta_3^2\gamma_5+\alpha_4^2\beta_2^2\gamma_6+\alpha_1\alpha_4\beta_3^2\gamma_7+\alpha_4^2\beta_2\beta_3\gamma_8+\alpha_4^2\beta_3^2\gamma_9 > 0 \tag{3.31}$$

$$\alpha_2^2\beta_1^2\gamma_1+\alpha_2\alpha_3\beta_1^2\gamma_2+\alpha_2^2\beta_1\beta_4\gamma_3+\alpha_2\alpha_3\beta_1\beta_4\gamma_4+\alpha_2^2\beta_4^2\gamma_5+\alpha_3^2\beta_1^2\gamma_6+\alpha_3\alpha_3\beta_4^2\gamma_7+\alpha_3^2\beta_1\beta_4\gamma_8+\alpha_3^2\beta_4^2\gamma_9 > 0 \tag{3.32}$$

$$2\alpha_1\alpha_2\beta_2^2\gamma_1 + (\alpha_1\alpha_3 + \alpha_2\alpha_4)\beta_2^2\gamma_2 + 2\alpha_1\alpha_2\beta_2\beta_3\gamma_3 + (\alpha_2\alpha_4 + \alpha_1\alpha_3)\beta_2\beta_3\gamma_4 + 2\alpha_1\alpha_2\beta_3^2\gamma_5+$$

$$2\alpha_3\alpha_4\beta_2^2\gamma_6 + (\alpha_1\alpha_3 + \alpha_2\alpha_4)\beta_3^2\gamma_7 + 2\alpha_3\alpha_4\beta_2\beta_3\gamma_8 + 2\alpha_3\alpha_4\beta_3^2\gamma_9 > 0 \tag{3.33}$$

$$2\beta_1\beta_2\alpha_2^2\gamma_1 + 2\alpha_2\alpha_3\beta_1\beta_2\gamma_2 + (\beta_1\beta_3 + \beta_2\beta_4)\alpha_2^2\gamma_3 + \alpha_2\alpha_3(\beta_2\beta_4 + \beta_1\beta_3)\gamma_4 + 2\alpha_2^2\beta_3\beta_4\gamma_5 +$$

$$2\alpha_3^2\beta_1\beta_2\gamma_6 + 2\alpha_2\alpha_3\beta_3\beta_4\gamma_7 + (\beta_1\beta_3 + \beta_2\beta_4)\alpha_3^2\gamma_8 + 2\alpha_3^2\beta_3\beta_4\gamma_9 > 0 \tag{3.34}$$

$$\alpha_2^2\beta_2^2\gamma_1+\alpha_2\alpha_3\beta_2^2\gamma_2+\alpha_2^2\beta_2\beta_3\gamma_3+\alpha_2\alpha_3\beta_2\beta_3\gamma_4+\alpha_3^2\beta_3^2\gamma_5+\alpha_3^2\beta_2^2\gamma_6+\alpha_2\alpha_3\beta_3^2\gamma_7+\alpha_3^2\beta_2\beta_3\gamma_8+\alpha_3^2\beta_3^2\gamma_9 > 0 \tag{3.35}$$

It can be observed from the eqns (3.27-3.35) is the function of $\alpha's$ and $\beta's$, which is given in eqns (3.15) and (3.16) respectively. As stable generalized bilinear transformation requires to satisfy the eqns (2.22) - (2.24), on applying these conditions to the eqns (3.15) and (3.16), the $\alpha's\ and\ \beta's$ in these equations will always be a positive. Since $\alpha's\ and\ \beta's$ are positive the eqns (3.27-3.35) will also be positve, which will make all the coefficients in the eqn (3.17) positive. Thus the denominator of the band-stop filter transfer function given in eqn (3.10) will have the positive coefficients for each and every polynomials and hence it is VSHP.

3.5 Frequency Response of the 2-D Band -Stop fi lters

The $second - order$ low-pass butterworth filter is transformed to a band-stop filter by applying low-pass to band-stop transformation. The resultant transfer function is digitized by appying bilinear transformation. With the input coeeficient of the generalized bilinear transformation, we can obtain 3-D magnitude and contour plot of the resulting 2-D digital band-stop filter. In this, the stability is taken care such that the 2-D band-stop filter is stable with these input arguments.

To analyze the manner in which each coefficient of generalized bilinear transformation effects the magnitude response of the resulting band-stop filter, we chnage the value of some of the coefficients or fixing some of the coefficient to a specified values. It is possible to obtain a 2-D band-stop filter when the coefficient are in the limits of $k_i > 0,\ 0 < |a_i| \leq 1\ and\ 0 < |b_i| \leq 1$, where $i = 1, 2$. Figure 3.3 shows the standard band-stop filter, which can be obtained by making all the coefficient values to be equal to one. In the following, we will see the effect of these coefficient to the frequency responses of the 2-D band-stop filter[25, 28].

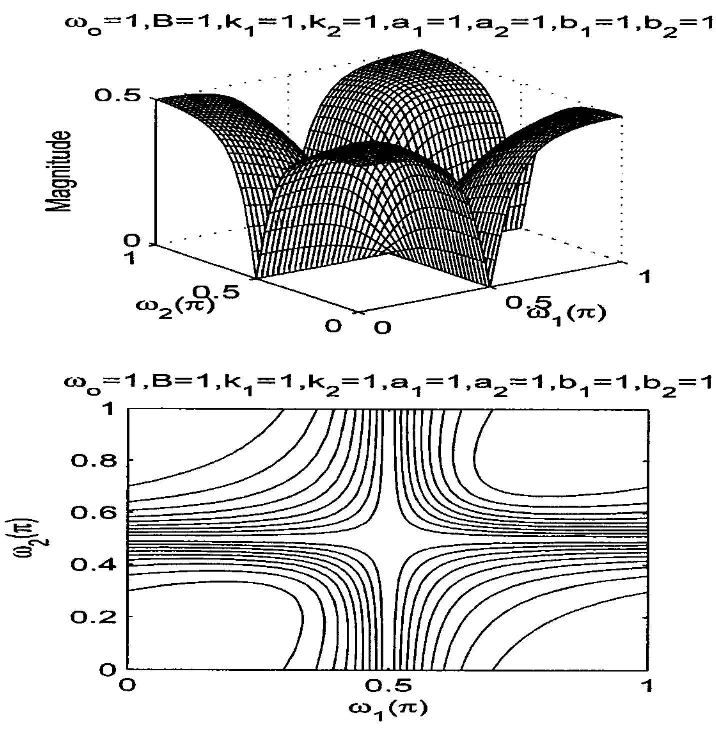

Figure 3.3: 2-D Standard band-stop filter

3.5.1 Frequency Response of 2-D Band-Stop filters with various k_1 values

To study the manner how k_1 effects the frequency response behaviour of the resulting $2-D$ band-stop filter and to separate the effect of the other coefficents, we vary the values of k_1, and fixing the other coefficients of the generalized bilinear transformation to be $k_2 = 1, a_1 = 1, a_2 = 1, b_1 = 1$ and $b_2 = 1$.The value of k_1 is varied from 0.1 to 100 and the resulting frequency response of the band-stop filter are plotted in figures 3.4,3.5and3.6 for the varies value of k_1. The contour and 3-D magnitude response plots for the filter with the values of $k_1 = 0.1,\ k_1 = 0.5,\ k_1 = 0.75,\ k_1 = 1.5,\ k_1 = 3,\ k_1 = 10,\ k_1 = 50$ and $k_1 = 100$ are shown in figures 3.4,3.5 and 3.6.

It can be observed that the coefficient k_1 mainly effects the center frequency(ω_c) of the filter response at $\omega_1 - axis$ and also it effects the bandwidth of lower, upper passband and the bandwidth of stopband of the band-stop filter. At the lowest possible positive value of k_1, the lower passband will have the maximum bandwidth in the available frequency range, the bandwidth of the upper passband is almost negligible and the bandwidth of the stopband is also less. As k_1 increases from 0.1, the center frequency moves to the minimum value from the maximum value and simultaneously the bandwidth of the lower passband decreases, the upper passband increases and the bandwidth of the stopband increases upto $k_1 \leq 2$, after that the the bandwidth of the stopband decreases. In the above shown diagram we can observe that the center frequency for $k_1 = 0.1$ is at 2.8 rad/sec. As we increase the coefficient values of k_1, for example when we reach $k_1 = 3$ the center frequency will start moving towards the minimum value reaches at the minimum value of 0.7 rad/sec and also we can observe that the bandwidth of the lower passband decreases and upper passband

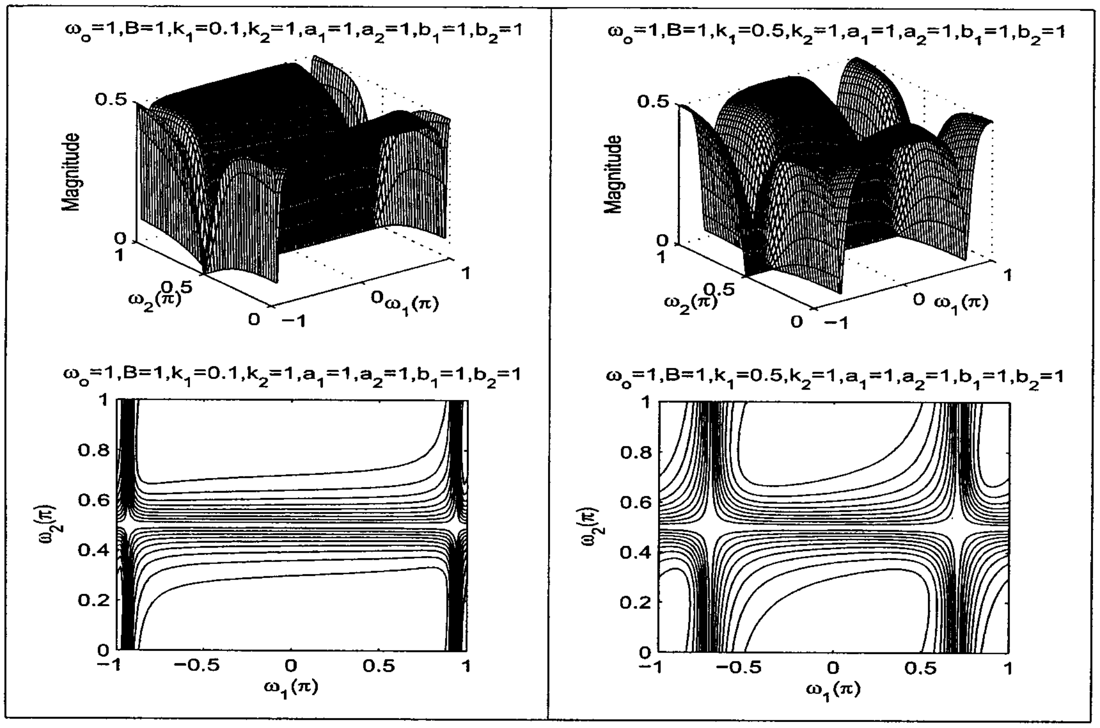

Figure 3.4: Frequency response of the band-stop filter with k_1 =0.1 and 0.5 and all the other coefficients as unity.

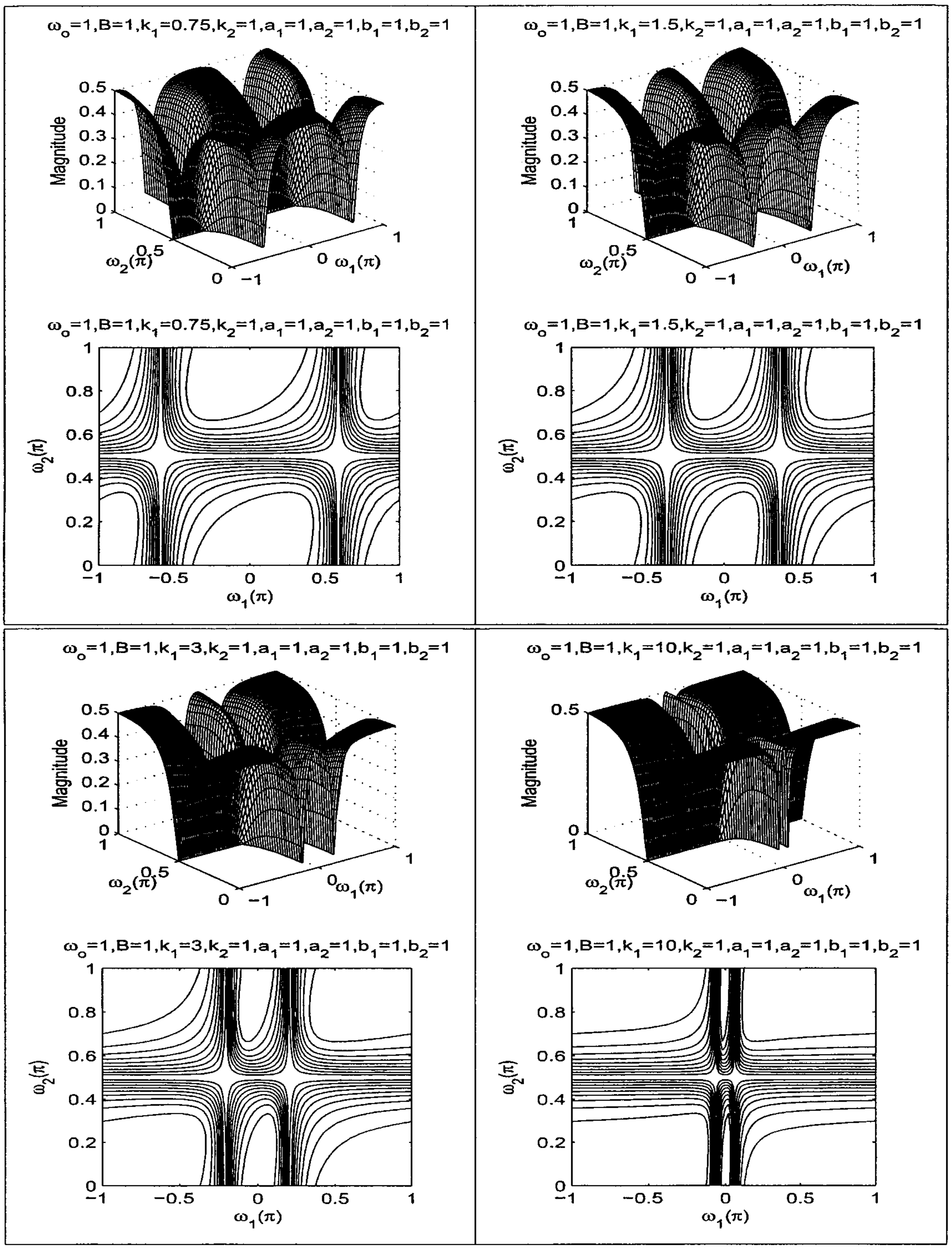

Figure 3.5: Frequency response of the band-stop filter with k_1 =0.75,1.5 and 3 and all the other coefficients as unity.

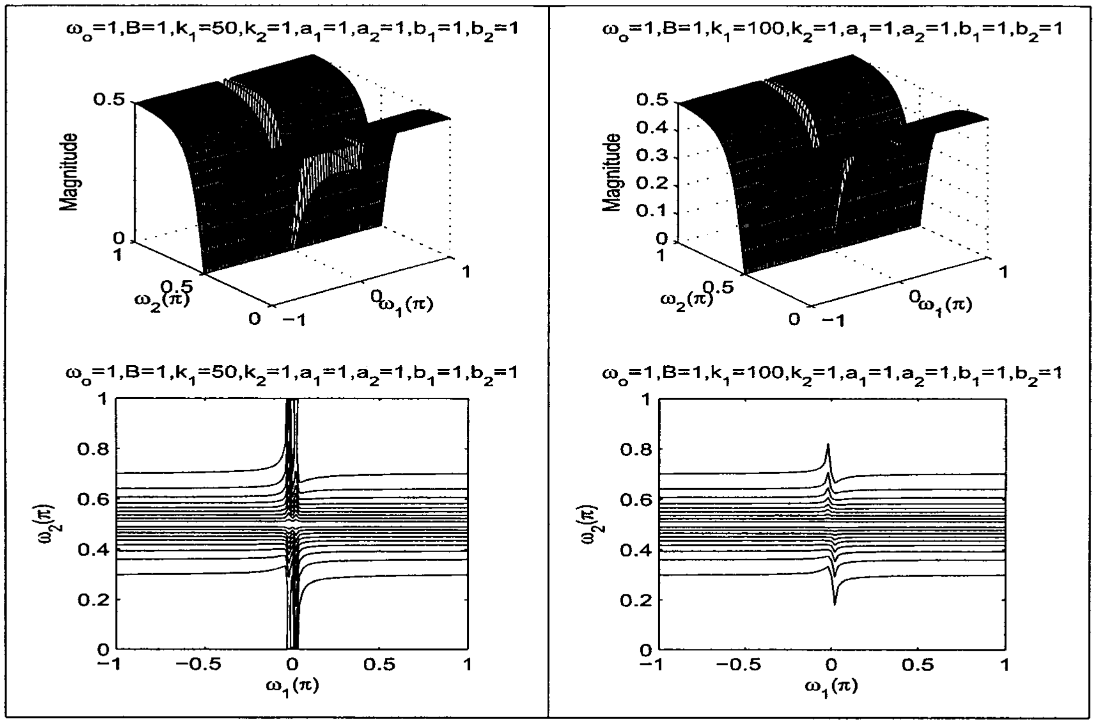

Figure 3.6: Frequency response of the band-stop filter with k_1 =50 and 100 and all the other coefficients as unity.

increases compared to the previous value of k_1. If we increase k_1 beyond 5, the center frequency will start moving towards the origin as shown in the figures 3.5and 3.6. If we increase the k_1 value after 50, the response will become as the all- pass filter at $\omega_1 - axis$. We can simply mention it like k_1 is inversely proportional to ω_c at $\omega_1 - axis$ and the coefficient k_1 has no effects on the gains in the pass-band and stop-band. And also the changes in k_1 values did not make any changes in $\omega_2 - axis$.

3.5.2 Frequency Response of 2-D Band-Stop filters with various k_2 values

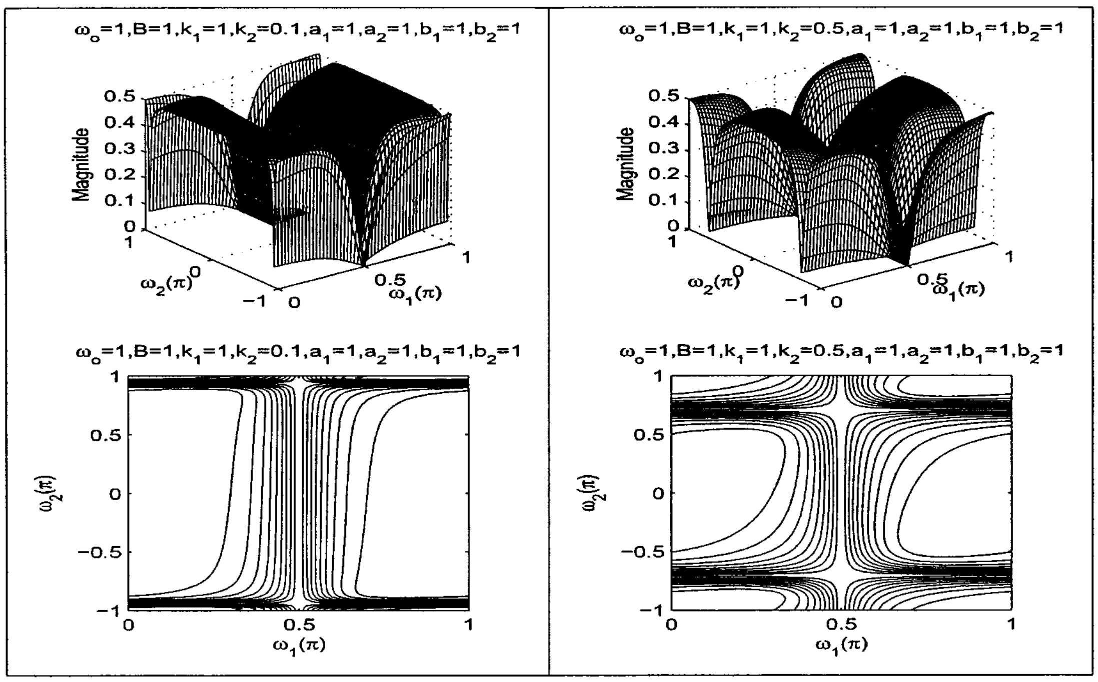

Figure 3.7: Frequency response of the band-stop filter with k_2 =0.1 and 0.5 and all the other coefficients as unity.

To study the manner how k_2 effects the frequency response behaviour of the resulting

$2-D$ band-stop filter and to separate the effect of the other coefficents, we vary the values of k_2, and fixing the other coefficients of the generalized bilinear transformation to be $k_1 = 1,\ a_1 = 1,\ a_2 = 1,\ b_1 = 1$ and $b_2 = 1$.The value of k_2 is varied from 0.1 to 100 and the resulting frequency response of the band-stop filter are plotted in figure 3.7, 3.8and3.9 for the varies value of k_1. The contour and 3-D magnitude response plots for the filter with the values of $k_1 = 0.1,\ k_1 = 0.5,\ k_1 = 0.75,\ k_1 = 1.5,\ k_1 = 3,\ k_1 = 10,\ k_1 = 50$ and $k_1 = 100$ are shown in figure 3.73.8and3.9.

It can be observed that the coefficient k_2 mainly effects the center frequency(ω_c) of the filter response at $\omega_2 - axis$ and also it effects the bandwidth of lower, upper passband and the bandwidth of stopband of the band-stop filter. At the lowest possible positive value of k_2, the lower passband will have the maximum bandwidth in the available frequency range, the bandwidth of the upper passband is almost negligible and the bandwidth of the stopband is also less. As k_2 increases from 0.1, the center frequency moves to the minimum value from the maximum value and simultaneously the bandwidth of the lower passband decreases, the upper passband increases and the bandwidth of the stopband increases upto $k_2 \leq 2$, after that the the bandwidth of the stopband decreases. In the above shown diagram we can observe that the center frequency for $k_2 = 0.1$ is at 2.8 rad/sec. As we increase the coefficient values of k_2, for example when we reach $k_2 = 3$ the center frequency will start moving towards the minimum value at reaches the minimum value of 0.7 rad/sec and also we can observe that the bandwidth of the lower passband decreases and upper passband increases compared to the previous value of k_2. If we increase k_2 beyond 5, the center frequency will start moving towards the origin as shown in the figure 3.8 and 3.9. If we increase the k_1 value after 50, the response will become as the all- pass filter at $\omega_2 - axis$. We can simply mention it like k_2 is inversely proportional to ω_c at $\omega_2 - axis$ and the

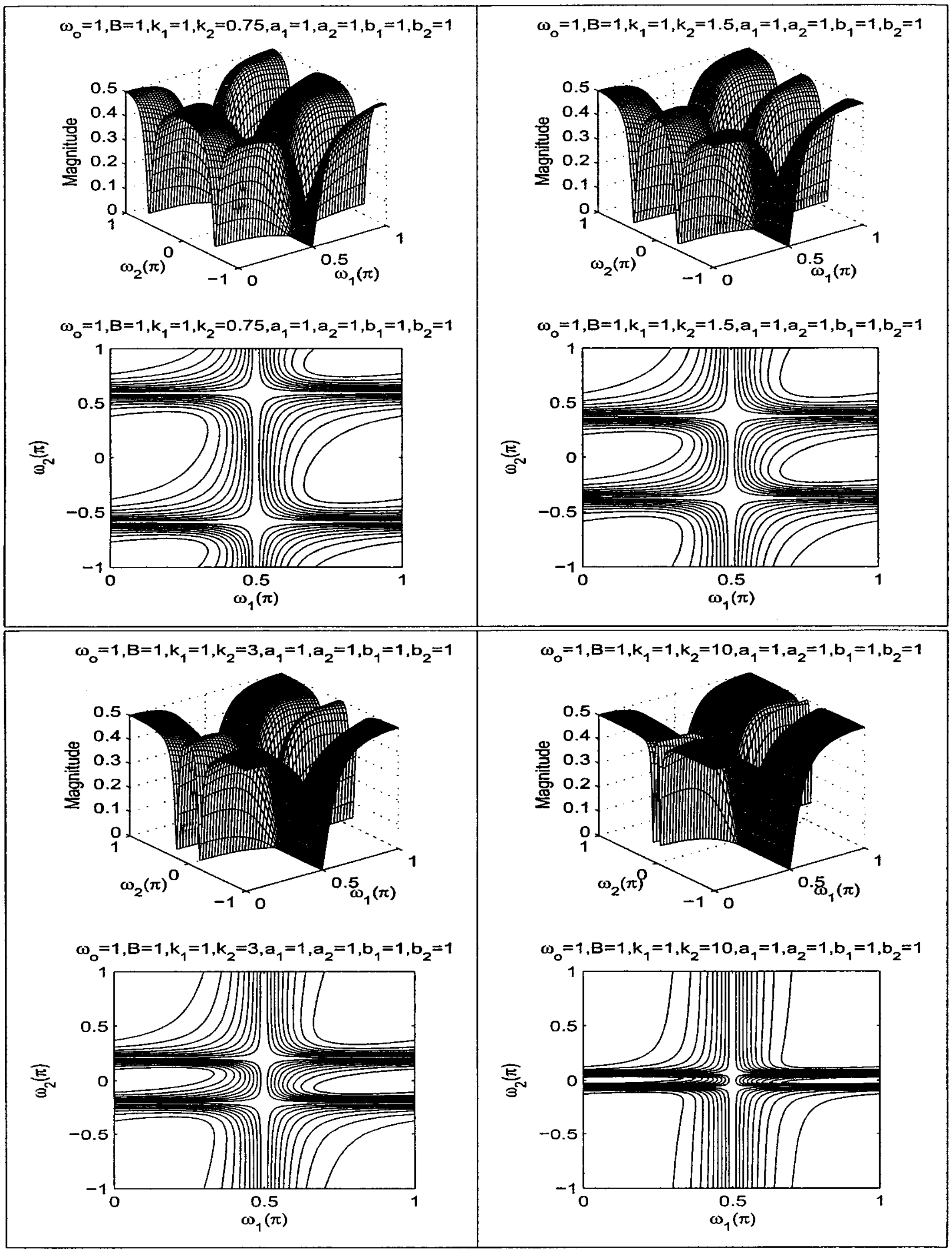

Figure 3.8: Frequency response of the band-stop filter with k_2 =0.75,1.5,3 and 5 and all the other coefficients as unity.

coefficient k_2 has no effects on the gains in the pass-band and stop-band. And also the changes in k_2 values does not make any changes in $\omega_1 - axis$. The responses is similar to the response obtained in the $\omega_1 - axis$ on varying the value of k_1, which is described in the section 3.5.1.

3.5.3 Frequency Response of 2-D Band-Stop filters with various a_1 values

We can get the stable range of a_1 with the other specified coefficient of the double generalized bilinear transformation. There are many combinations possible for the coefficients. To study the response properly we fix the other coefficient values to be equal to unity, when studying the effect of a_1 on the frequency response of the Band-Stop fil-

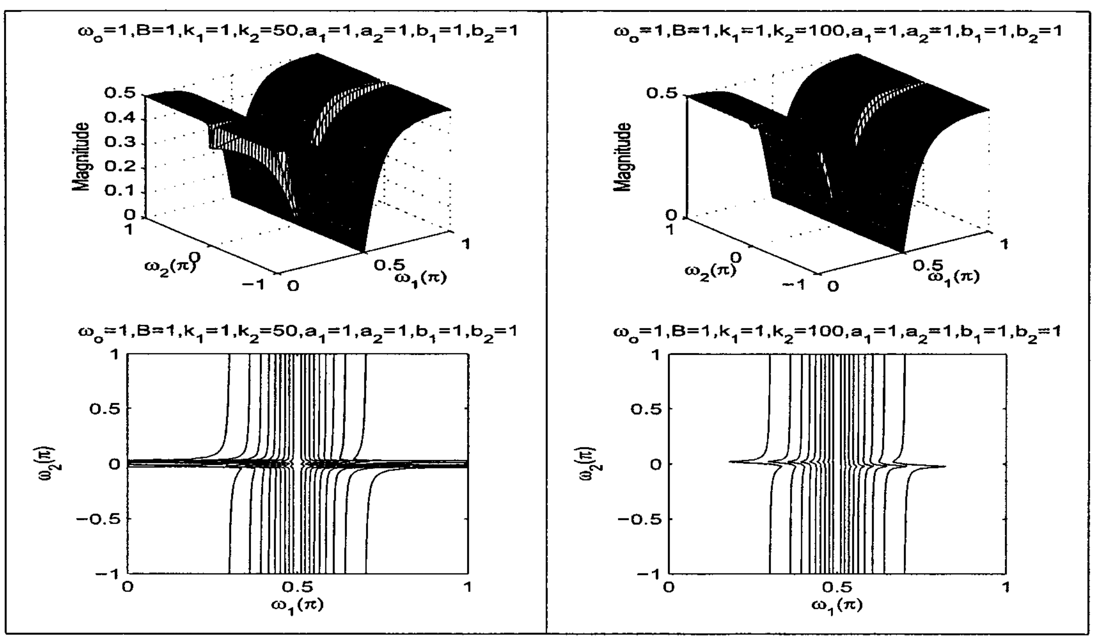

Figure 3.9: Frequency response of the band-stop filter with k_2 =50 and 100 and all the other coefficients as unity.

ter. The range of a_1 varies from 0 to 1 and the other coefficient values are specified as $k_1 = 1,\ k_2 = 1,\ a_2 = 1,\ b_1 = 1$ and $b_2 = 1$.

The contour and 3-D plots with representative values of a_1 are given in figure 3.10. By making the $a_1 = 1$ it resembles the standard band-stop filter. It can be observed that the center frequency (ω_c) of the stopband, bandwidth of the lower passband and the bandwidth of the upper passband changes on increasing the value of a_1, at $\omega_1 - axis$. At the lowest possible value of a_1, the gain of the upper passband will be similar to the gain obtain in the standard band-stop filter, but the gain of the lower passband is less than half of the original gain obtain in the standard band-stop filter. Also, the center frequency of the stopband will be at maximum value and the bandwidth of the lower passband will be greater than the bandwidth of the upper passband. As we keep on increasing the value of a_1, the center frequency of the stopband moves from the maximum value to the minimum value, the bandwidth of the lower passband decreases with the increase in gain of the lower passband and the bandwidth of the upper passband increases. And it can be observed that the last part of the lower passband and the beginning part of the upper passband merges in the stopband producing the small gain and also the maximum gain of the stopband decreases as the coffecient a_1 increases. For simplicity, it can be mentioned as increase in a_1 can make the lower passband to decrease in bandwidth with the increase in gain, the upper passband to increase in bandwidth with a constant gain and the center frequency of the stopband will move from maximum to minimum frequency.

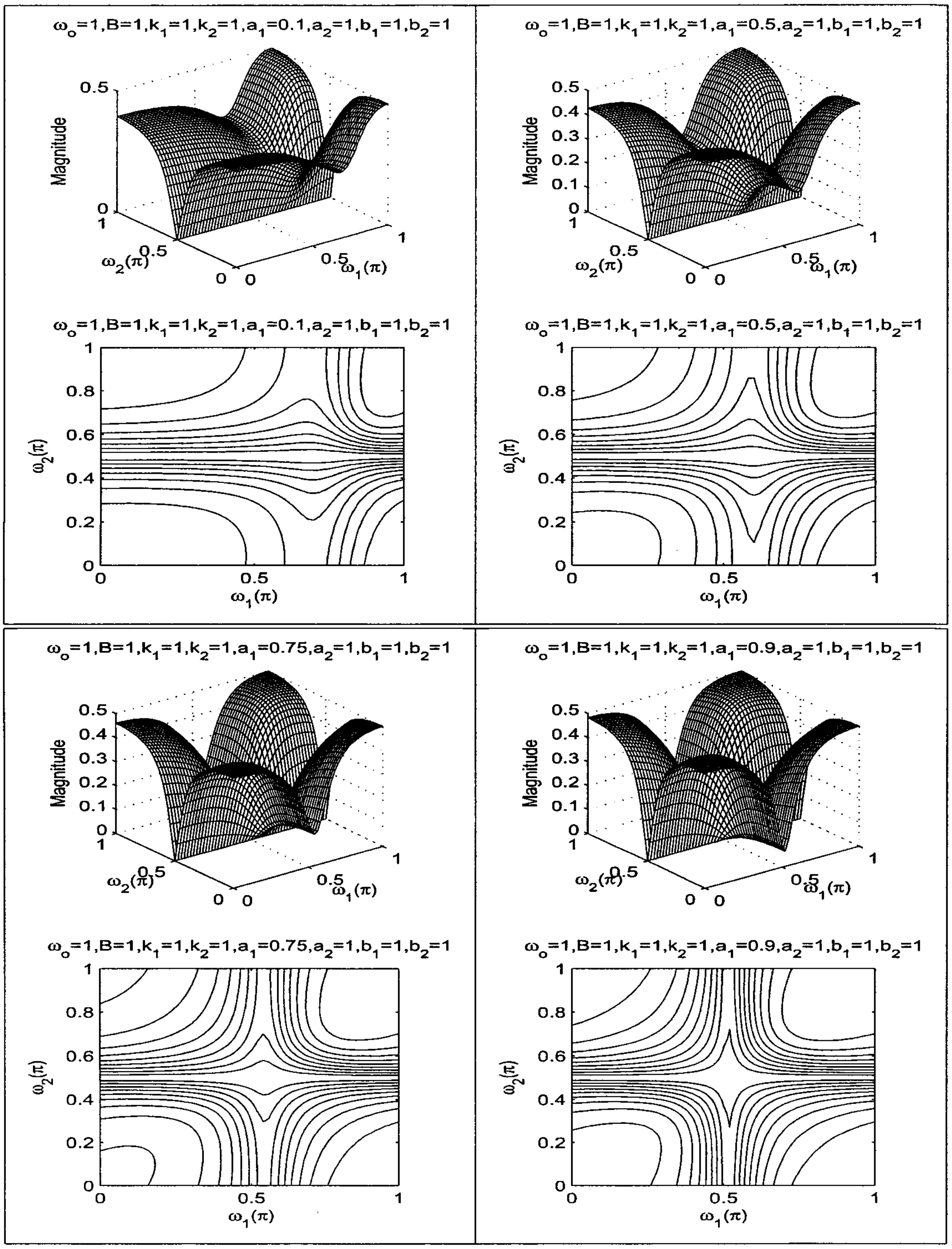

Figure 3.10: Frequency response of the band-stop filter with a_1 =0.1, 0.5,0.75 and 0.9 and all the other coefficients as unity.

3.5.4 Frequency Response of 2-D Band-Stop filters with various a_2 values

We can get the stable range of a_2 with the other specified coefficient of the double generalized bilinear transformation. There are many combinations possible for the coefficients. To study the response properly we fix the other coefficient values to be equal to unity, when studying the effect of a_2 on the frequency response of the Band-Stop filter. The range of a_2 varies from 0 to 1 and the other coefficient values are specified as $k_1 = 1,\ k_2 = 1,\ a_1 = 1,\ b_1 = 1$ and $b_2 = 1$.

The contour and 3-D plots with representative values of a_2 are given in figure 3.11. By making the $a_2 = 1$ it resembles the standard band-stop filter. It can be observed that the center frequency (ω_c) of the stopband, bandwidth of the lower passband and the bandwidth of the upper passband changes on increasing the value of a_2, at $\omega_2 - axis$. At the lowest possible value of a_2, the gain of the upper passband will be similar to the gain obtain in the standard band-stop filter, but the gain of the lower passband is less than half of the original gain obtain in the standard band-stop filter. And also, the center frequency of the stopband will be at maximum value and the bandwidth of the lower passband will be greater than the bandwidth of the upper passband. As we keep on increasing the value of a_2, the center frequency of the stopband moves from the maximum value to the minimum value, the bandwidth of the lower passband decreases with the increase in gain of the lower passband and the bandwidth of the upper passband increases. And it can be observed that the last part of the lower passband and the beginning part of the upper passband merges in the stopband producing the small gain and also the maximum gain of the stopband decreases as the coefficient a_2 increases. For simplicity, it can be mentioned as increase in a_2 can make the lower passband to decrease in bandwidth with the increase in gain, the upper passband to

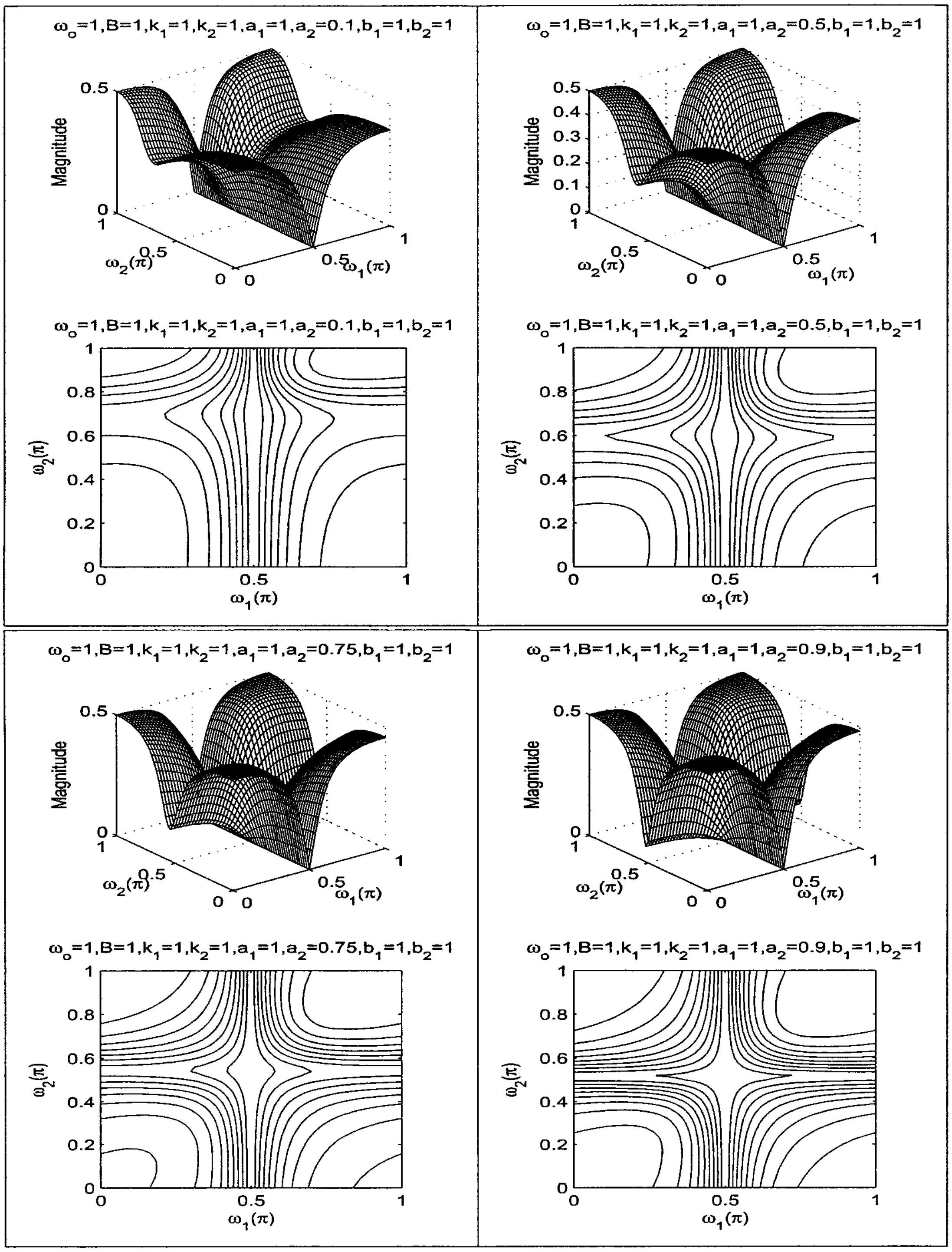

Figure 3.11: Frequency response of the band-stop filter with a_2 =0.1,0.5,0.75 and 0.9 and all the other coefficients as unity.

increase in bandwidth with a constant gain and the center frequency of the stopband will move from maximum to minimum frequency. The frequency response analyzed in this section looks similar to the response obtained in $\omega_1 - axis$ on varying the a_2.

3.5.5 Frequency Response of 2-D Band-Stop filters with various b_1 values

We can get the stable range of b_1 with the other specified coefficient of the double generalized bilinear transformation. There are many combinations possible for the coefficients. To study the response properly we fix the other coefficient values to be equal to unity, when studying the effect of b_1 on the frequency response of the Band-Stop filter. The range of b_1 varies from 0 to 1 and the other coefficient values are specified as $k_1 = 1,\ k_2 = 1,\ a_1 = 1,\ a_2 = 1$ and $b_2 = 1$.

The contour and 3-D plots with representative values of b_1 are given in figure 3.12. By making the $b_1 = 1$ it resembles the standard band-stop filter. It can be observed that the center frequency (ω_c) of the stopband, bandwidth of the lower passband and the bandwidth of the upper passband changes on increasing the value of b_1, at $\omega_1 - axis$. At the lowest possible value of b_1, the gain of the lower passband is lesser than the half of the maximum gain. For example $b_1 = 0.1$, the center frequency of the stopband will be at minimum value and also the bandwidth of the lower passband will be lesser than the bandwidth of the upper passband. As we keep on increasing the value of b_1, the center frequency of the stopband moves from the minimum value to the maximum value, the bandwidth of the lower passband increases and the bandwidth of the upper passband decreases with the increase in gain. And it can be observed that the last part of the lower passband and the beginning part of the upper passband merges in the stopband producing the small gain and

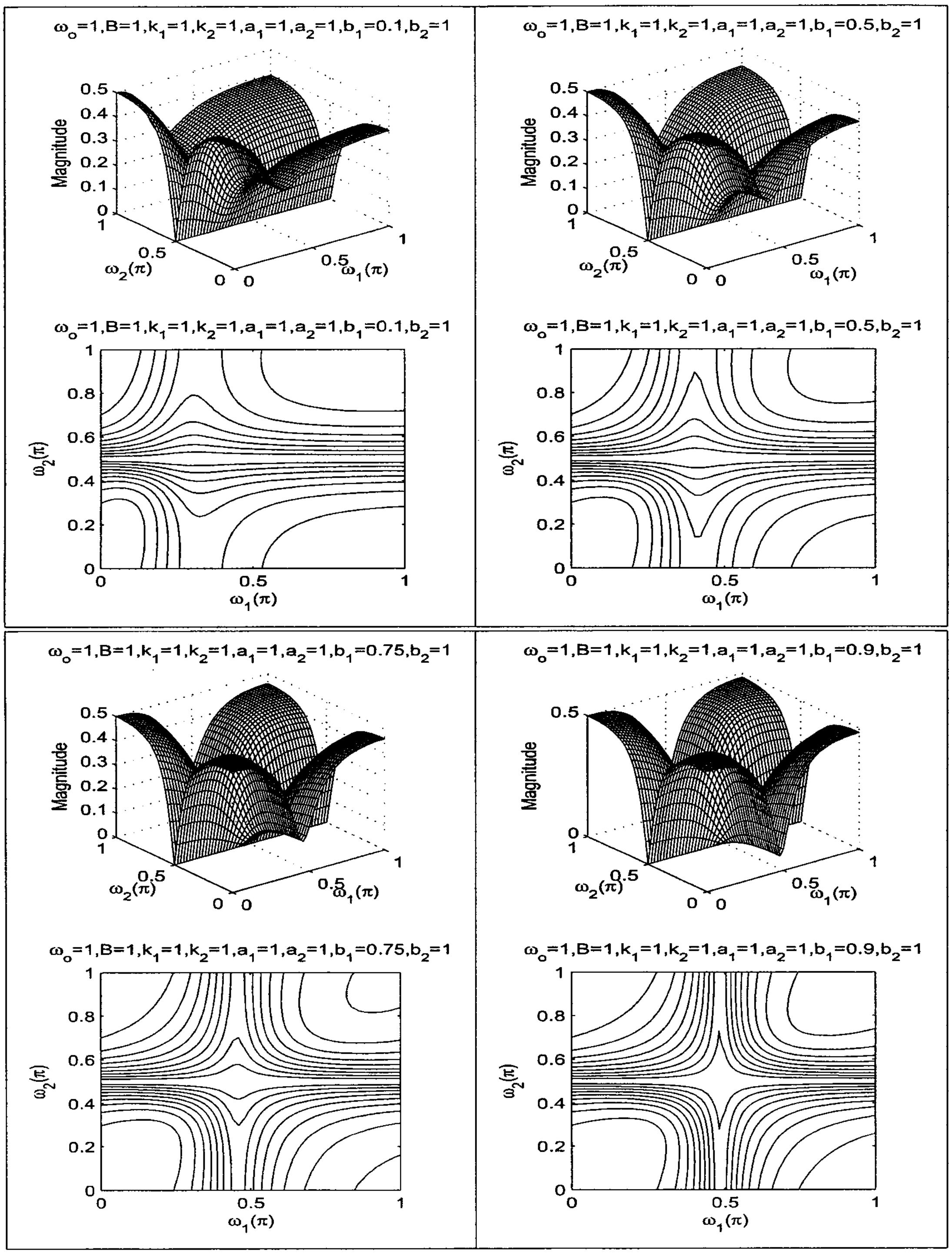

Figure 3.12: Frequency response of the band-stop filter with b_1 =0.1,0.5,0.75 and 0.9 and all the other coefficients as unity.

also the maximum gain of the stopband decreases as the coffecient b_1 increases. We can conclude this by saying that the increase in b_1 will increase the bandwidth of the lower passband, decrease in bandwidth of upper passband and shifting the center frequency from minimum to maximum frequency.

3.5.6 Frequency Response of 2-D Band-Stop filters with various b_2 values

We can get the stable range of b_2 with the other specified coefficient of the double generalized bilinear transformation. There are many combinations possible for the coefficients. To study the response properly we fix the other coefficient values to be equal to unity, when studying the effect of b_2 on the frequency response of the Band-Stop filter. The range of b_2 varies from 0 to 1 and the other coefficient values are specified as $k_1 = 1,\ k_2 = 1,\ a_1 = 1,\ a_2 = 1$ and $b_1 = 1$.

The contour and 3-D plots with representative values of b_1 are given in figure 3.13. By making the $b_2 = 1$ it resembles the standard Band-Stop filter. It can be observed that the center frequency (ω_c) of the stopband, bandwidth of the lower passband and the bandwidth of the upper passband changes on increasing the value of b_2, at $\omega_2 - axis$. At the lowest possible value of b_2, the gain of the lower passband is lesser than the half of the maximum gain. For example $b_2 = 0.1$, the center frequency of the stopband will be at minimum value and also the bandwidth of the lower passband will be lesser than the bandwidth of the upper passband. As we keep on increasing the value of b_2, the center frequency of the stopband moves from the minimum value to the maximum value, the bandwidth of the lower passband increases and the bandwidth of the upper passband decreases with the increase in gain. And it can be observed that the last part of the lower passband and the

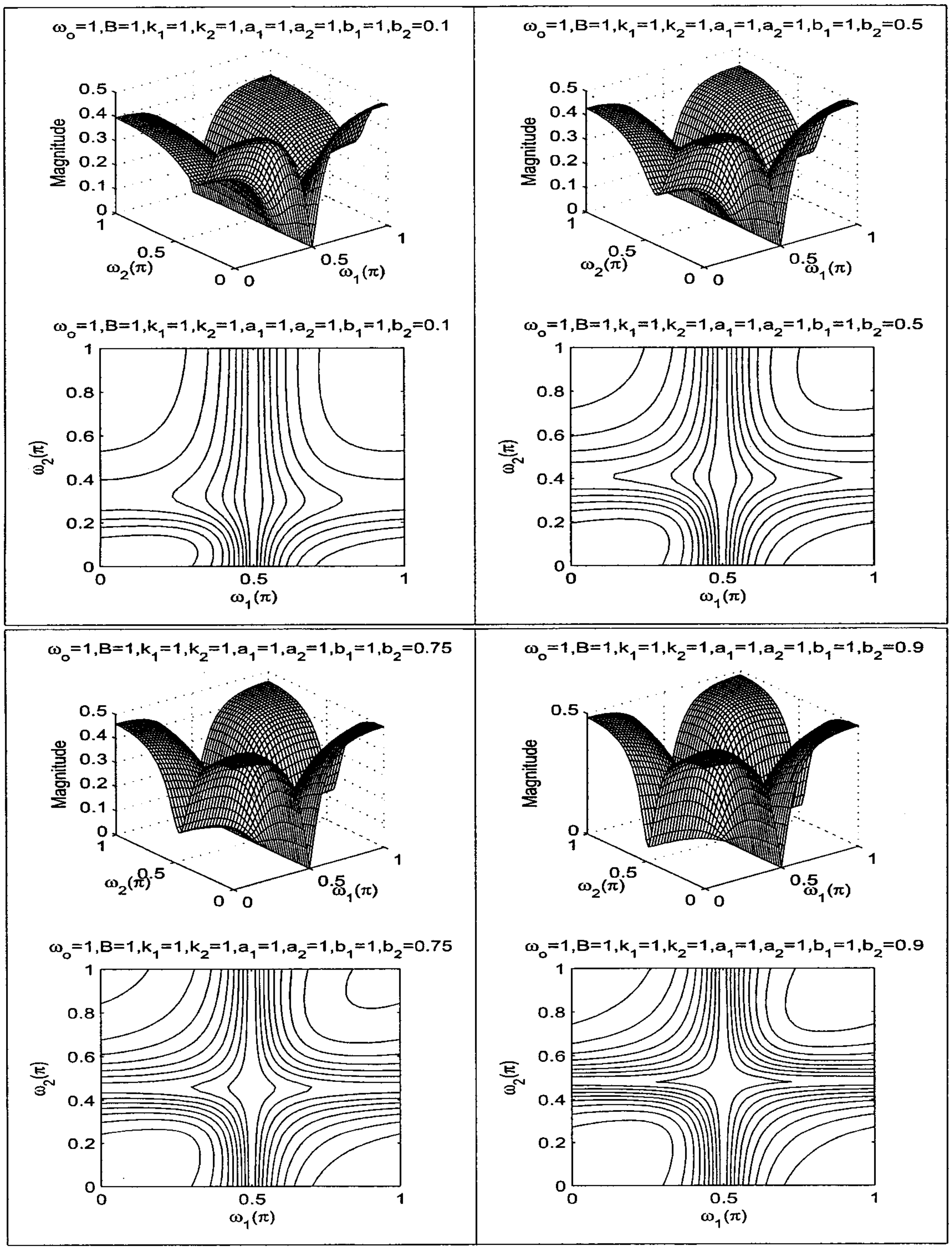

Figure 3.13: Frequency response of the band-stop filter with b_2 =0.1,0.5,0.75 and 0.9 and all the other coefficients as unity.

beginning part of the upper passband merges in the stopband producing the small gain and also the maximum gain of the stopband decreases as the coffecient b_2 increases. We can conclude this by saying that the increase in b_2 will increase the bandwidth of the lower passband, decrease in bandwidth of upper passband and shifting the center frequency from minimum to maximum frequency.

3.5.7 Frequency Response of 2-D Band-Stop filters with various k_1 values and fixed ω_o with same values of a_1 and b_1

In section 3.5.1 we saw the frequency response of the band-stop filter for the various k_1 values with a unity for the other coefficients. In this, we are going to assign the same values for a_1 and b_1 ranging from 0 to 1, and for remaining coefficient it is considered as unity such as $k_2 = 1$, $a_2 = 1$, $b_2 = 1$ and the theortical center frequency (ω_o) is considered as unity. The contour and the 3-D magnitude plot are shown from figure 3.14,3.15 and 3.16 for various values of k_1 with the coefficient values a_1 and b_1 are 0.75.

The effect of response will be in the $\omega_1 - axis$ and there will not be any change in response in the $\omega_2 - axis$. If the value of k_1 happens to be equal to the therotical center frequency (ω_o), then the response will be similar to the standard band-stop filter shown in the figure 3.3 but with a reduced stopband. This reduced stopband is due to change in the values of a_1 and b_1. It can be observed that the frequencey response is much similar to the frequency response obtained in the section 3.6.1, the only difference will be lower passband cutoff frequency extends and it merges with upper cutoff frequency which results in the reduced bandwidth at the stopband. The center frequency of the stopband moves from the maximum to minimum on increase in the coefficient value of k_1.

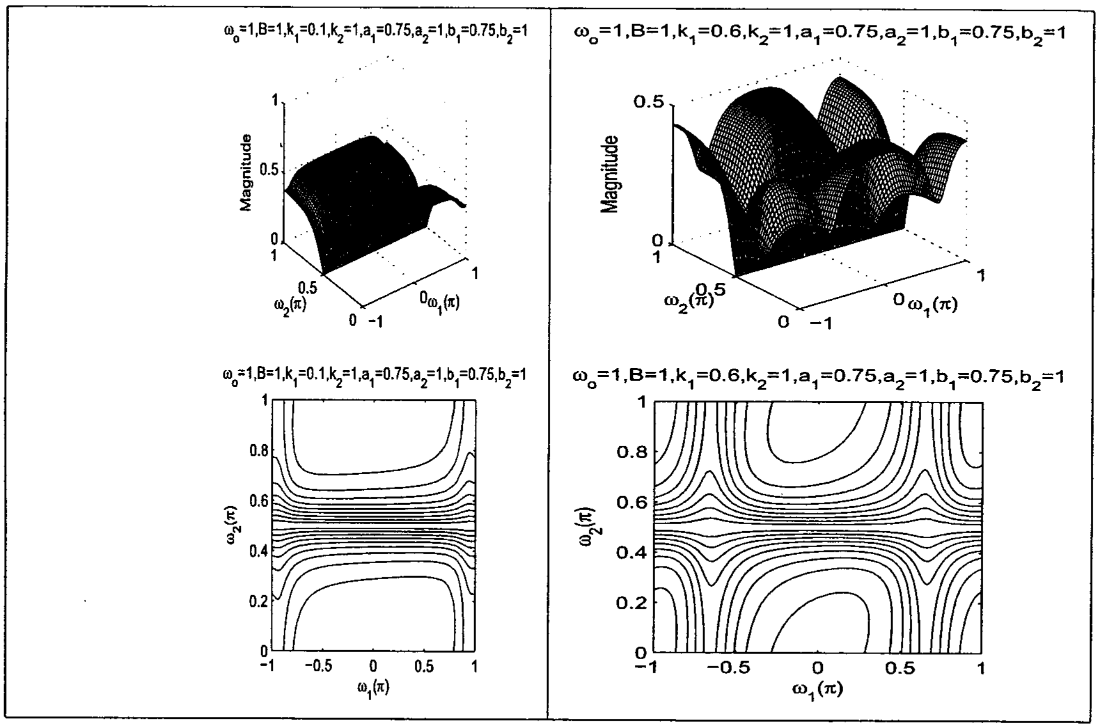

Figure 3.14: Frequency response for the band-stop filter with the value of $k_1 = 0.1$ and 0.6 for $a_1 = b_1 = 0.75$ and all the other coefficients as unity.

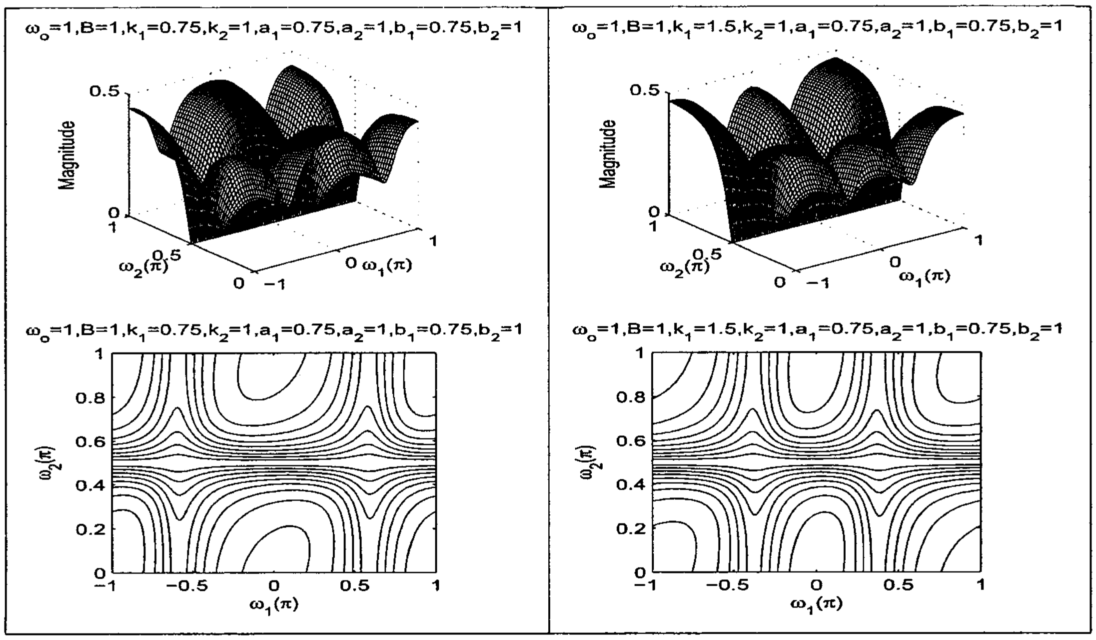

Figure 3.15: Frequency response for the band-stop filter with the value of $k_1 = 0.75$ and 1.5 for $a_1 = b_1 = 0.75$ and all the other coefficients as unity.

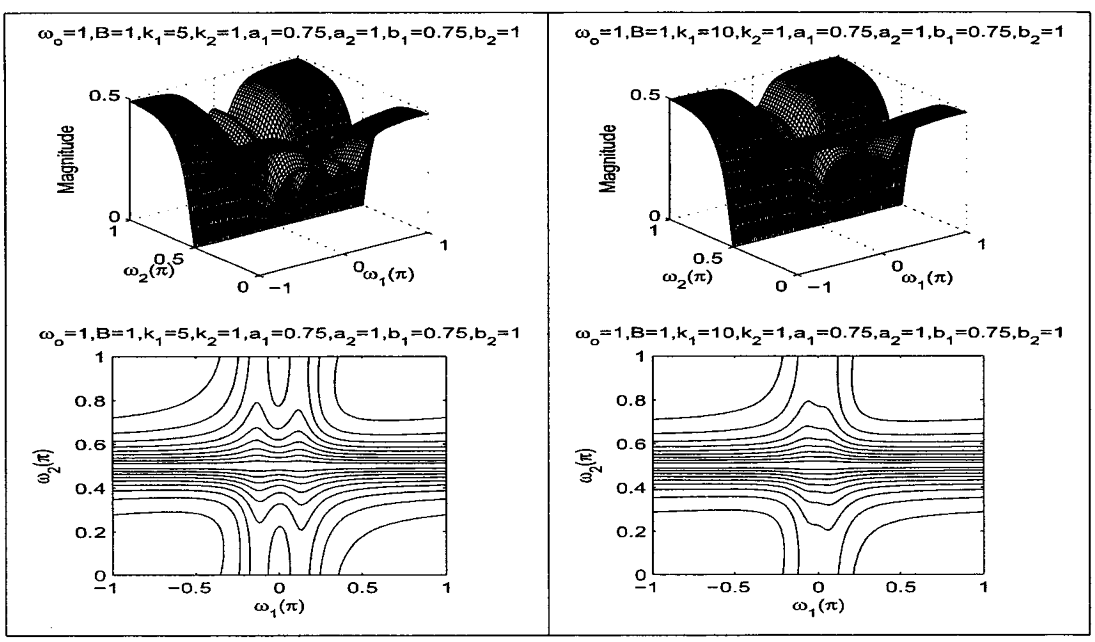

Figure 3.16: Frequency response for the band-stop filter with the value of $k_1 = 5$ and 10 for $a_1 = b_1 = 0.75$ and all the other coefficients as unity.

3.5.8 Frequency response of different values of B with all the other coefficient as unity.

In this section, we will see the frequency response for the various values of B with the other coefficients values are fixed to be unity. The 3-D magnitude and the contour plot for various values of B are plotted in fig 3.17, it is same for both $\omega_1 - axis$ and $\omega_2 - axis$. From these plots, it is clear that the bandwidth increases as the value of B is varied in the range of 0.5 to 3.

3.5.9 Frequency Response of 2-D Band-Stop filters with fixed k_1 values and various ω_o with same values of a_1 and b_1

In section 3.5.7, the frequency response of the fixed value ω_o are shown. In this, we will see the frequency response for the various values of ω_o with fixed value of k_1 and with same values of a_1 and b_1. Before seeing the effect of ω_o and k_1, we will see the effect of ω_o for the other constant coefficient such as $k_1 = 1,\ k_2 = 1,\ a_1 = 1,\ a_2 = 1,\ b_1 = 1$ and $b_2 = 1$. The contour and the 3-D plot for various values of ω_o are plotted in figure 3.18 and it is same for both $\omega_1 - axis$ and $\omega_2 - axis$. From these plots, it can be observed that the center frequency(ω_c) of the stopband is directly proportional to the theortical center frequency(ω_o), i.e as the ω_o increases ω_c also increases. It can also be observed that the bandwidth of the lower passband increases, the bandwith of the upper passband decreases and the bandwidth of the stopband decreases upon the increase in the value of ω_o.

The contour and 3-D plot for specified values of k_1 such as 0.5, 2 and 5 are shown from figure 3.19, 3.20 and 3.21 with the ω_o values as $0.5, 1.5, 2$ and 3. The a_1 and b_1 values are

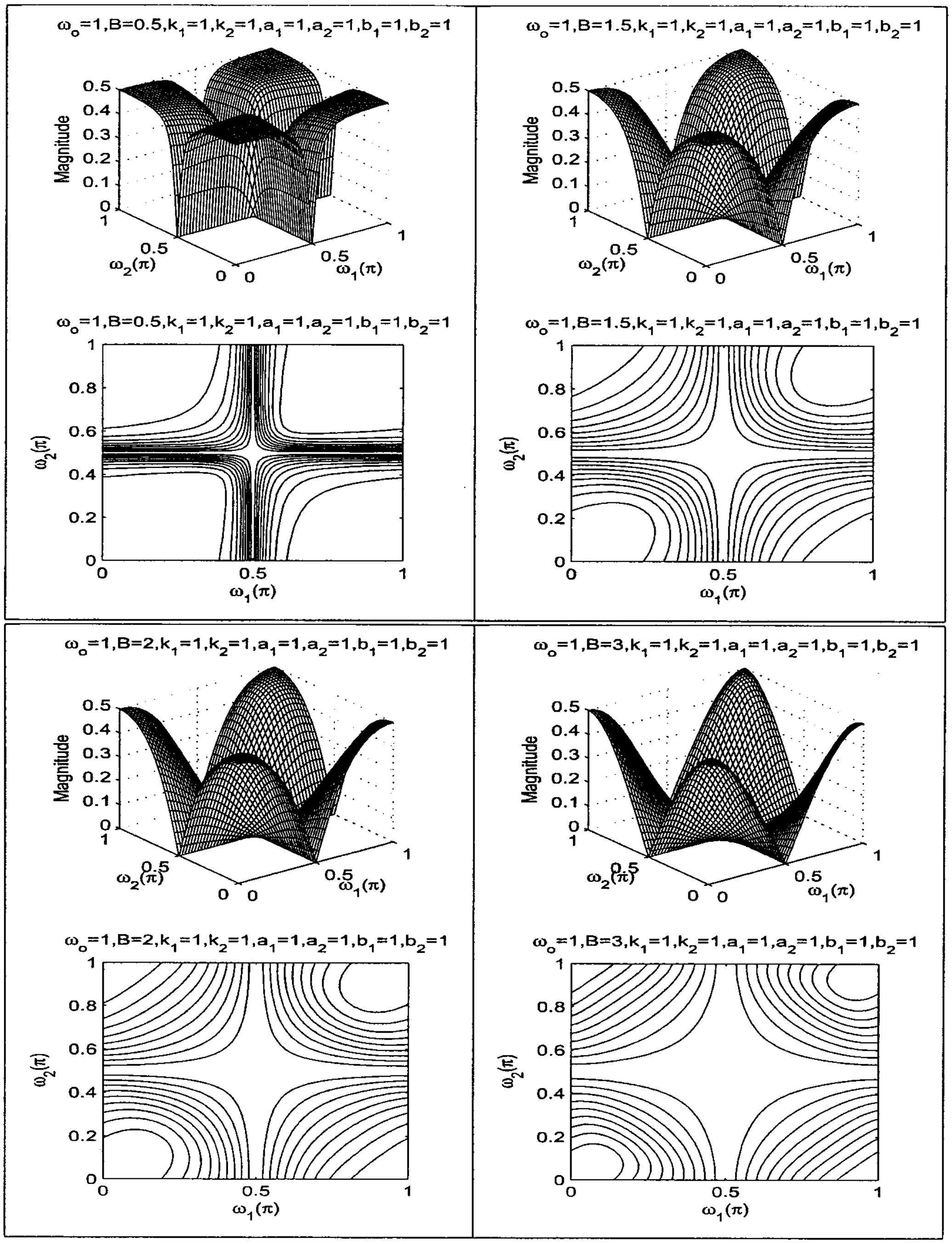

Figure 3.17: Frequency response of band-stop filter with values of $B = 0.5, 1.5, 2$ and 3 and all the other coefficient as unity.

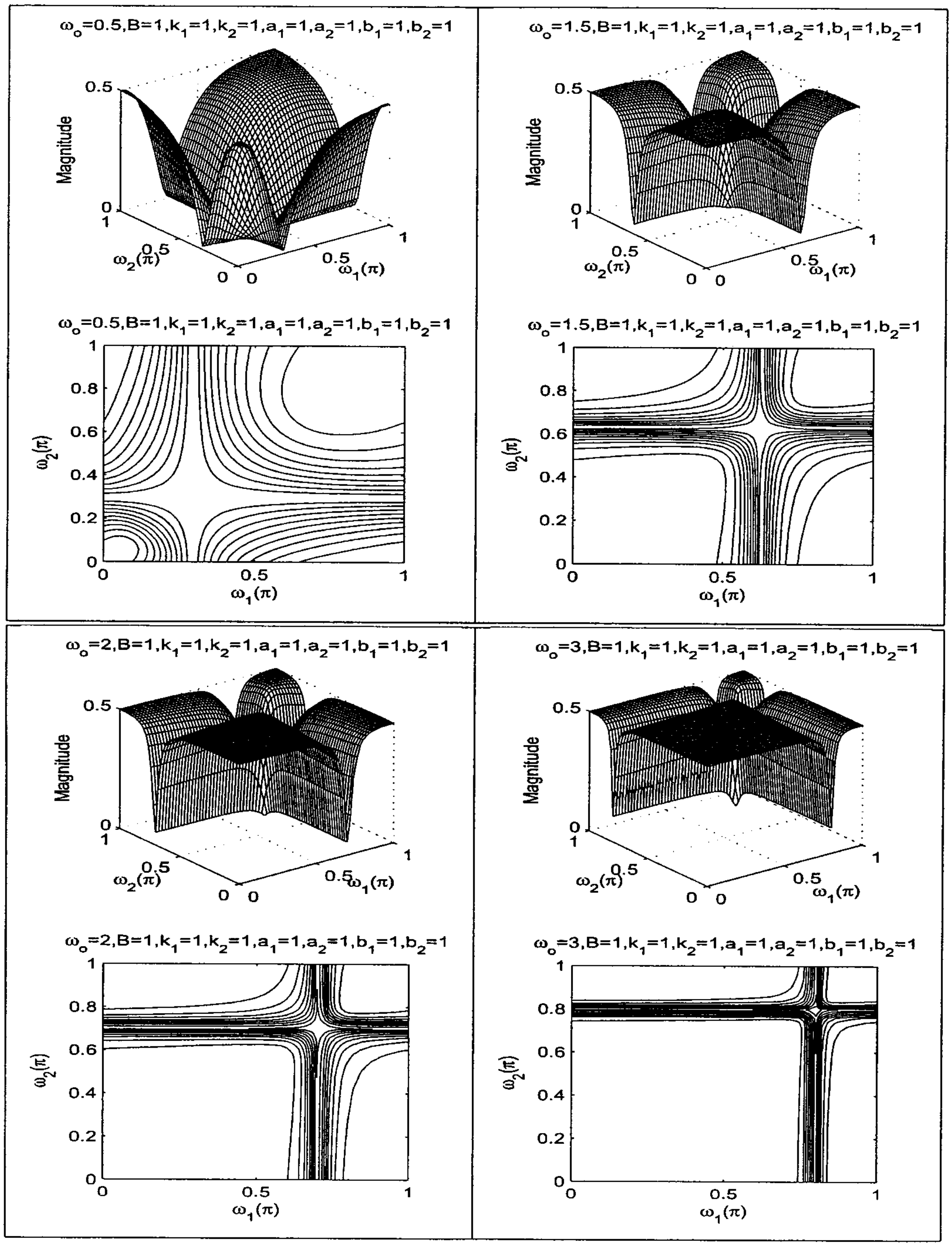

Figure 3.18: Frequency response for the band-stop filter with the value of ω_o = 0.5,1.5,2 and 3 and all the other coefficients as unity.

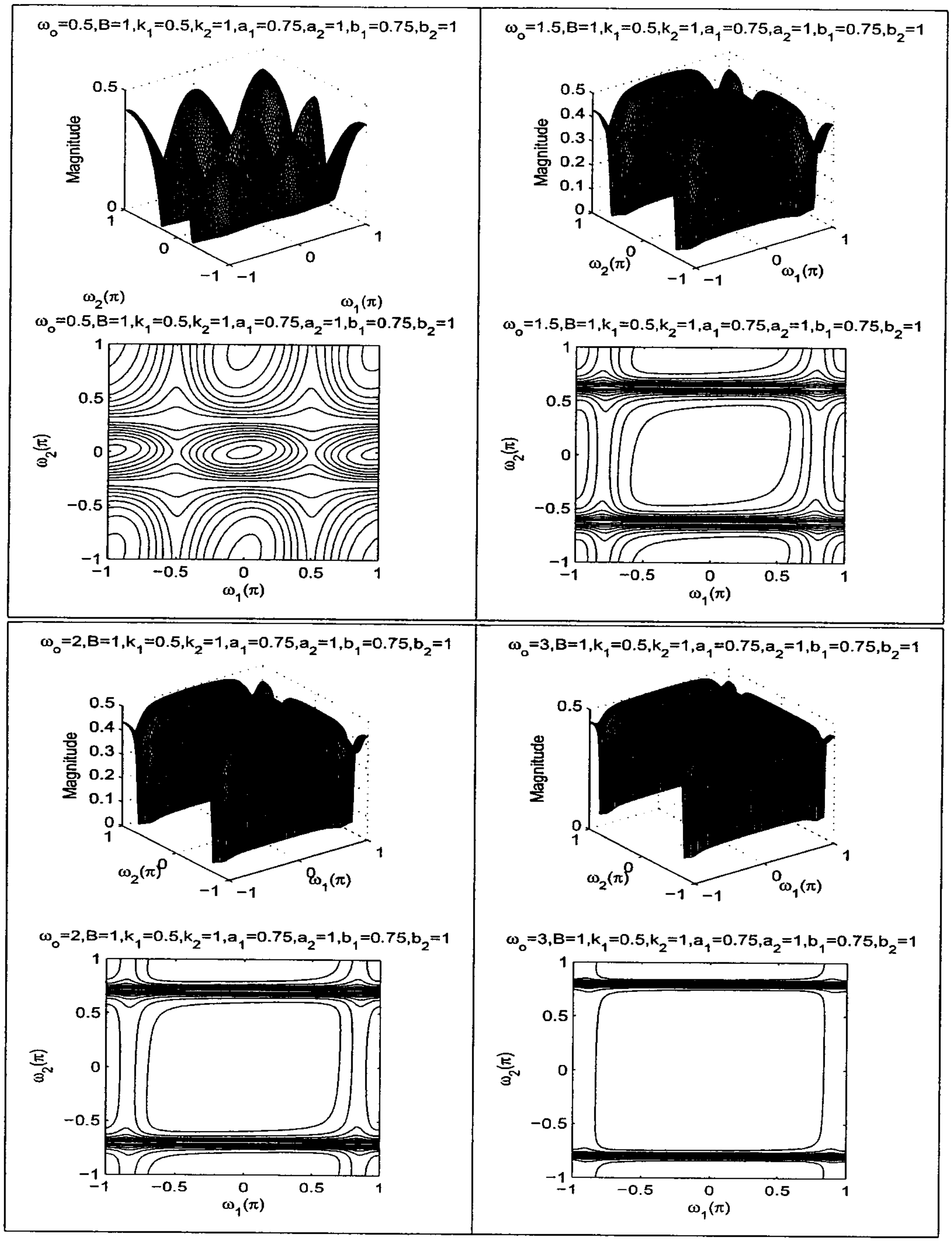

Figure 3.19: Frequency response for the band-stop filter with the value of $k_1 = 0.5$ for $a_1 = b_1 = 0.75$ and all the other coefficients as unity.

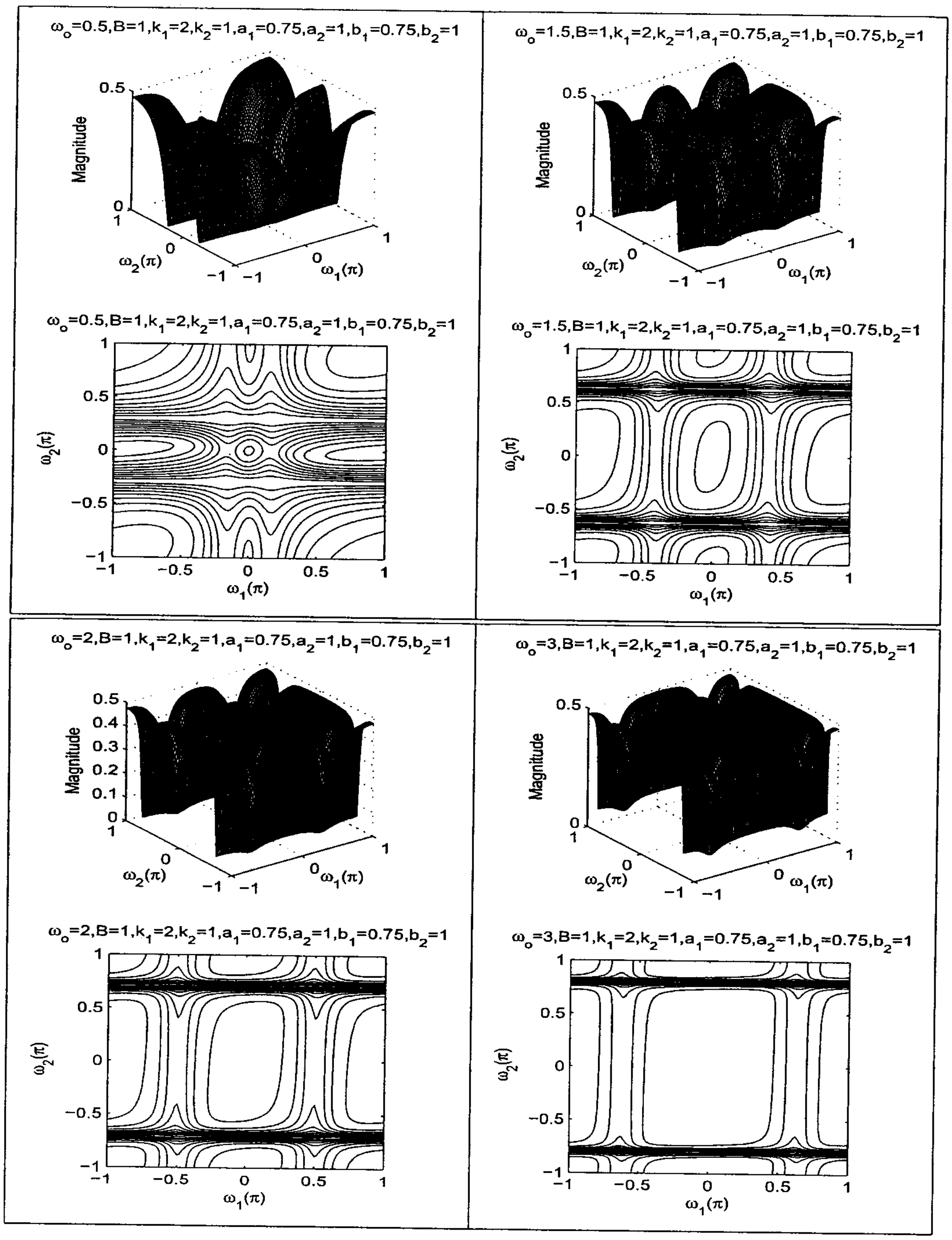

Figure 3.20: Frequency response for the band-stop filter with the value of $k_1 = 2$ for $a_1 = b_1 = 0.75$ and all the other coefficients as unity.

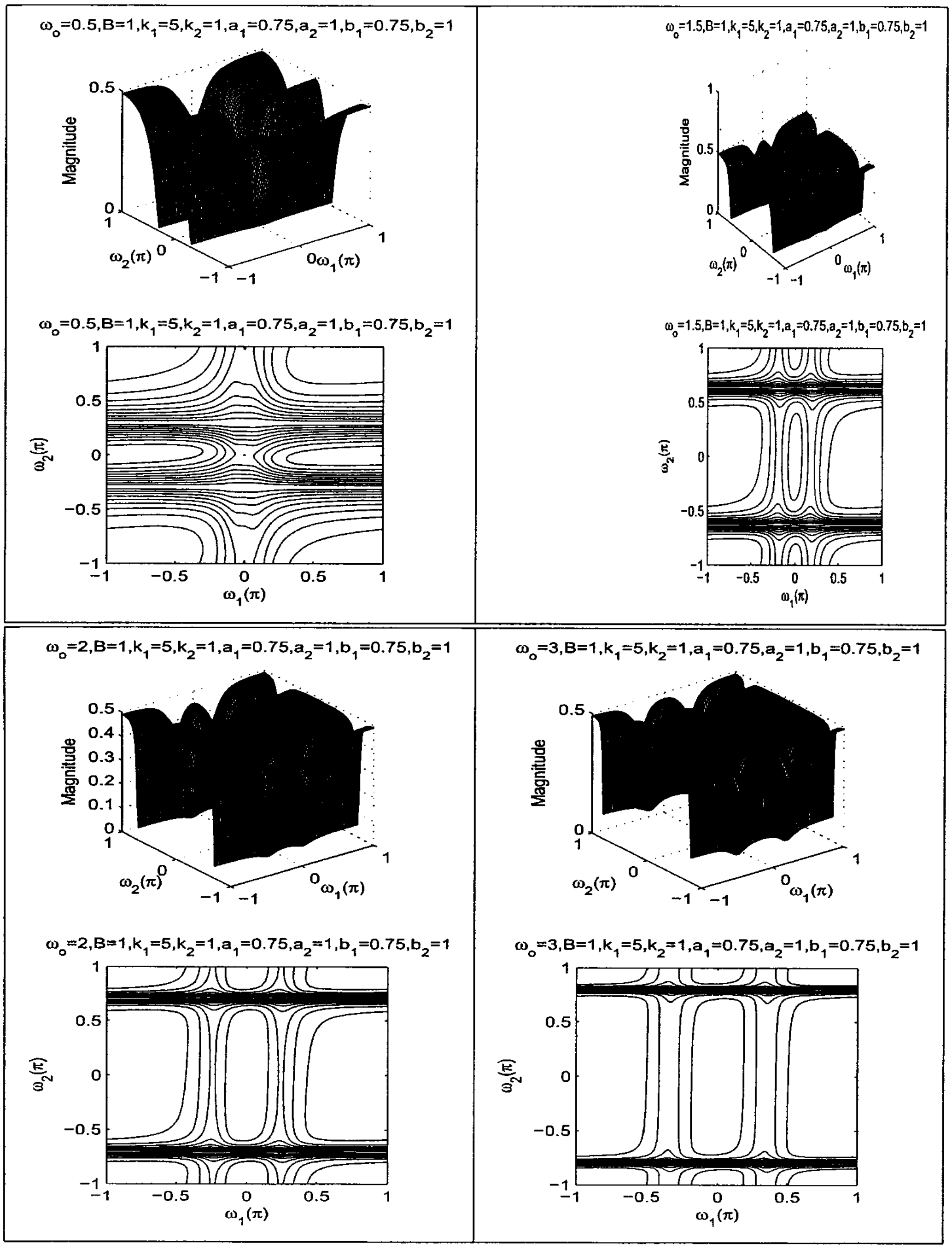

Figure 3.21: Frequency response for the band-stop filter with the value of $k_1 = 5$ for $a_1 = b_1 = 0.75$ and all the other coefficients as unity.

taken as 0.75. It can be seen that the effect of ω_o and k_1 are in the $\omega_1 - axis$. Whenever the values of k_1 and ω_o are same the frequency response will resemble similar to standard band-stop filter but with the reduced stopband. It can be seen that the frequency response of the band-stop filter will be the combined effect of section 3.5.1, section 3.5.7 and the frequency response of the various value of ω_o shown in the figure 3.18.The section 3.5.1 describes that the increase in value of k_1 makes the center frequency of the stopband to move from maximum to minimum value. The section 3.5.7 describes that the increase in value of k_1 with the same value of a_1 and b_1 makes the center frequency to move from maximum to minimum value but with the reduced stop band.When $k_1 < 1$, the frequency responses starts from band-stop filter ans slowly transforms into a low-pass filter response. When $1 \leq k_1 \leq 2$, the filter response will be a band-stop filter. When $k_1 > 2$, the filter response will get transformed to band-stop filter from a high-pass filter response on increase in value of ω_o. In this section, it is observed that the change in the value of k_1 with the various value of ω_o makes the center frequency to move from maximum to minimum value with proportional to the value of ω_o.

3.5.10 Frequency Response of 2-D Band-Stop filters with various k_2 values and fixed ω_o with same values of a_2 and b_2

In section 3.6.1 we saw the frequency response of the band-stop filter for the various k_1 values with a unity for the other coefficients. In this, we are going to assign the same values for a_1 and b_1 ranging from 0 to 1, and for remaining coefficient it is considered as unity such as $k_2 = 1$, $a_2 = 1$, $b_2 = 1$ and the theortical center frequency (ω_o) is considered as unity. The contour and the 3-D magnitude plot are shown from figure 3.22,3.23and3.24 for various values of k_1 with the coefficient values a_1 and b_1 are 0.75.

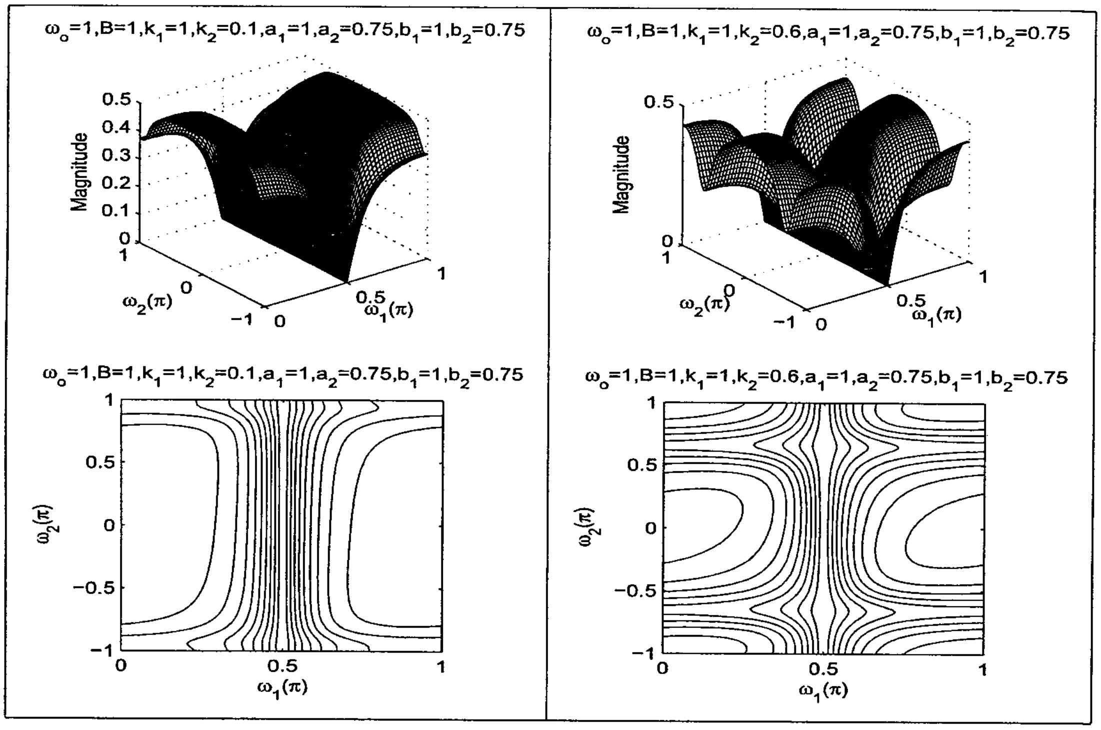

Figure 3.22: Frequency response for the band-stop filter with the value of $k_2 = 0.1$ and 0.6 for $a_2 = b_2 = 0.75$ and all the other coefficients as unity.

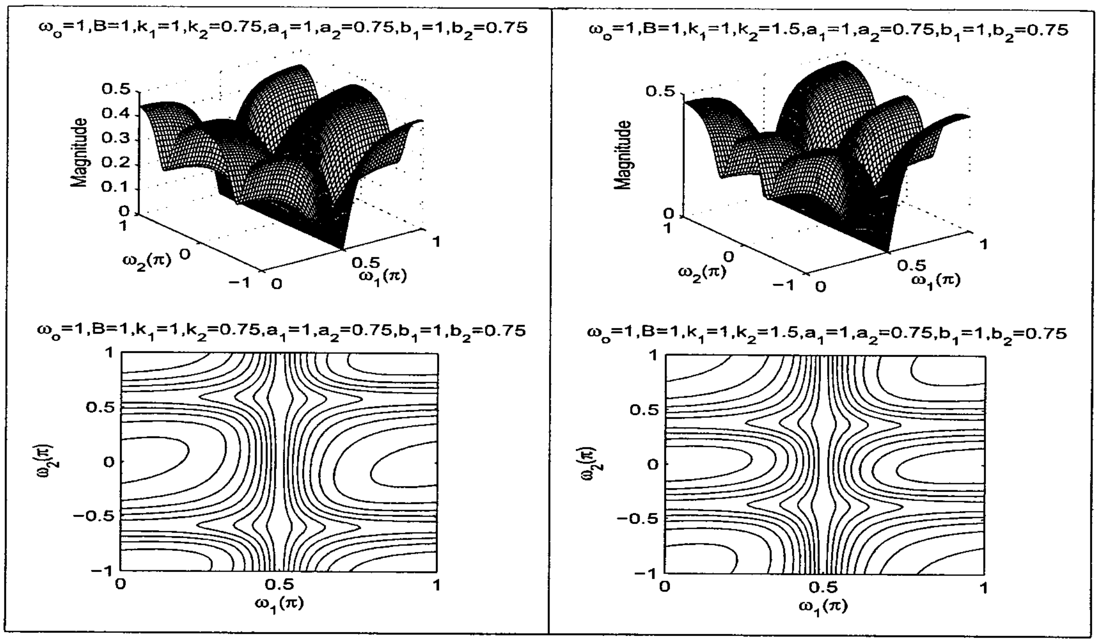

Figure 3.23: Frequency response for the band-stop filter with the value of $k_2 = 0.75$ and 1.5 for $a_2 = b_2 = 0.75$ and all the other coefficients as unity.

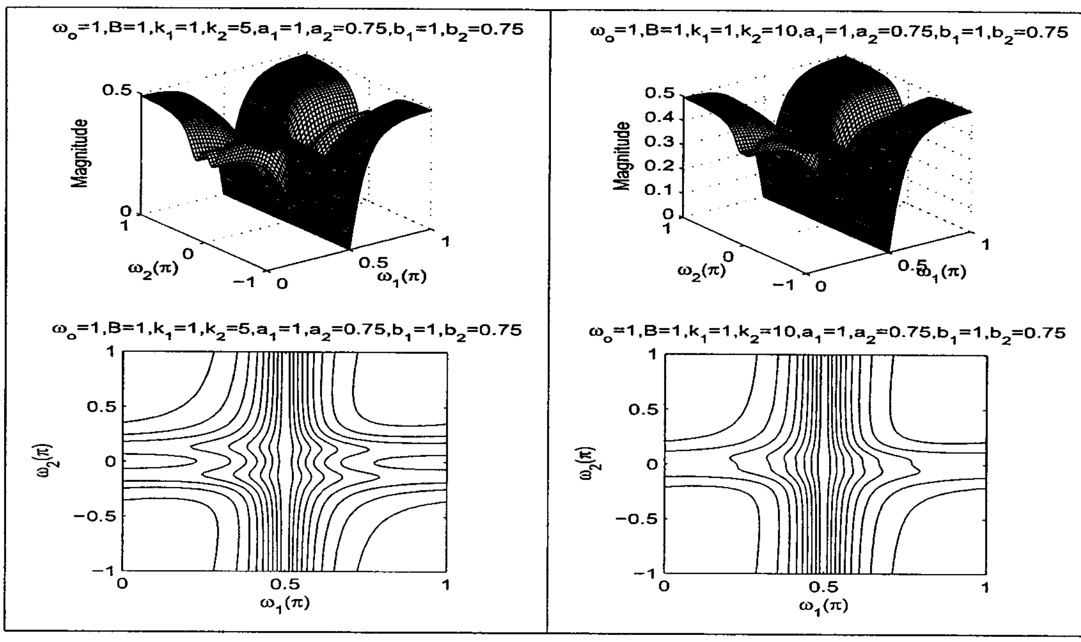

Figure 3.24: Frequency response for the band-stop filter with the value of $k_2 = 5$ and 10 for $a_2 = b_2 = 0.75$ and all the other coefficients as unity.

The effect of response will be in the $\omega_2 - axis$ and there will not be any change in response in the $\omega_1 - axis$. If the value of k_2 happens to be equal to the therotical center frequency (ω_o), then the response will be similar to the standard band-stop filter shown in the figure 3.4 but with a reduced stopband. This reduced stopband is due to change in the values of a_2 and b_2. It can be observed that the frequencey response is much similar to the frequency response obtained in the section 3.5.2, the only difference will be lower passband cutoff frequency extends and it merges with upper cutoff frequency which results in the reduced bandwidth at the stopband. The center frequency of the stopband moves from the maximum to minimum on increase in the coefficient value of k_2.

3.5.11 Frequency Response of 2-D Band-Stop filters with fixed k_2 values and various ω_o with same values of a_2 and b_2

In section 3.5.9, the frequency response of the fixed value ω_o are shown. In this, we will see the frequency response for the various values of ω_o with fixed value of k_2 and with same values of a_1 and b_1. The contour and the 3-D plot for various values of ω_o are plotted in figure 3.18 and it is same for both $\omega_1 - axis$ and $\omega_2 - axis$. From these plots, it can be observed that the center frequency(ω_c) of the stopband is directly proportional to the theortical center frequency(ω_o), i.e as the ω_o increases ω_c also increases. It can also be observed that the bandwidth of the lower passband increases, the bandwith of the upper passband decreases and the bandwidth of the stopband decreases upon the increase in the value of ω_o.

The contour and 3-D plot for specified values of k_2 such as $0.5, 2$ and 5 are shown from figures 3.25, 3.26 and3.27 with the ω_o values as $0.5, 1.5, 2$ and 3. The a_2 and b_2 values are taken as 0.75. It can be seen that the effect of ω_o and k_2 are in the $\omega_2 - axis$. Whenever

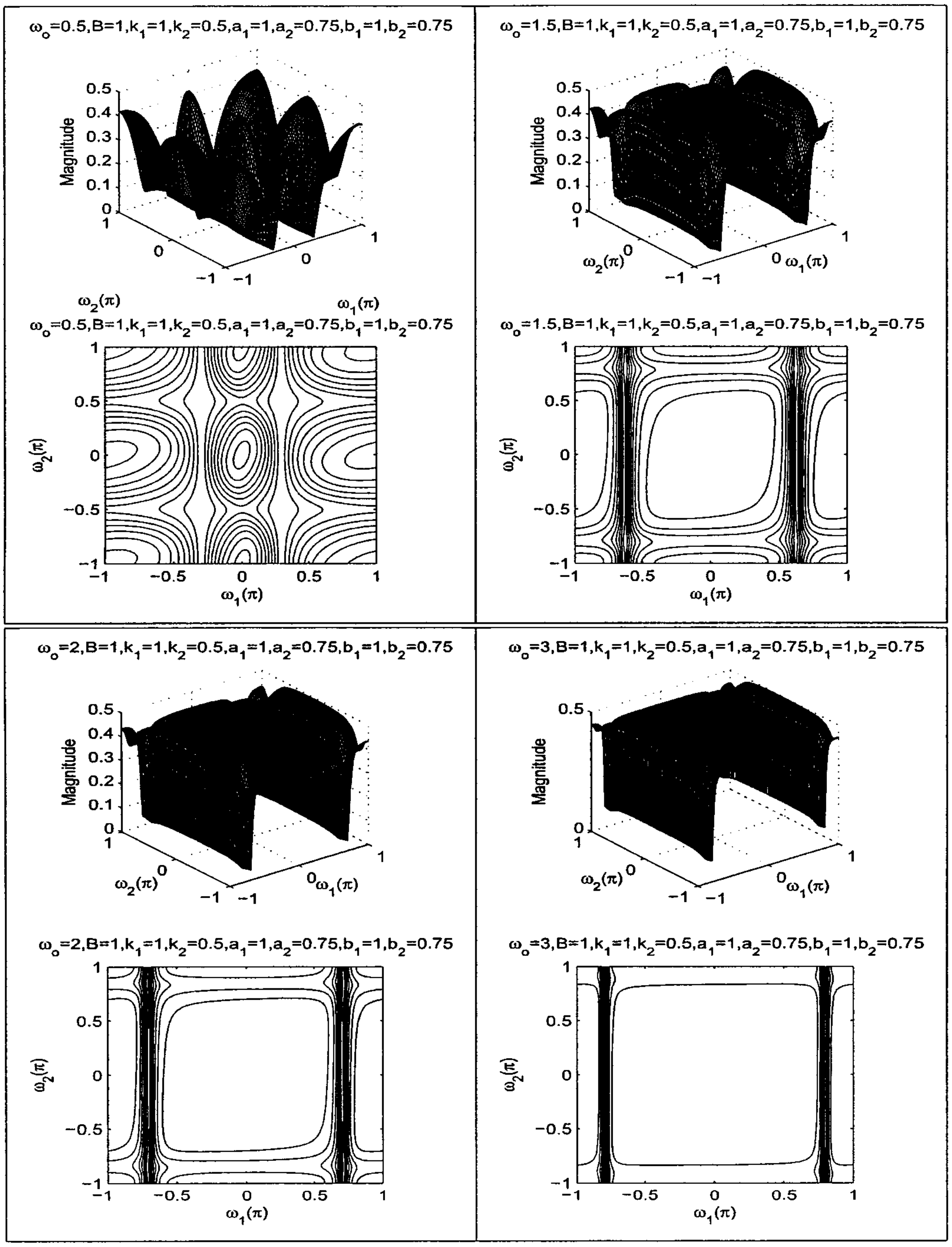

Figure 3.25: Frequency response for the band-stop filter with the value of $k_2 = 0.5$ for $a_2 = b_2 = 0.75$ and all the other coefficients as unity.

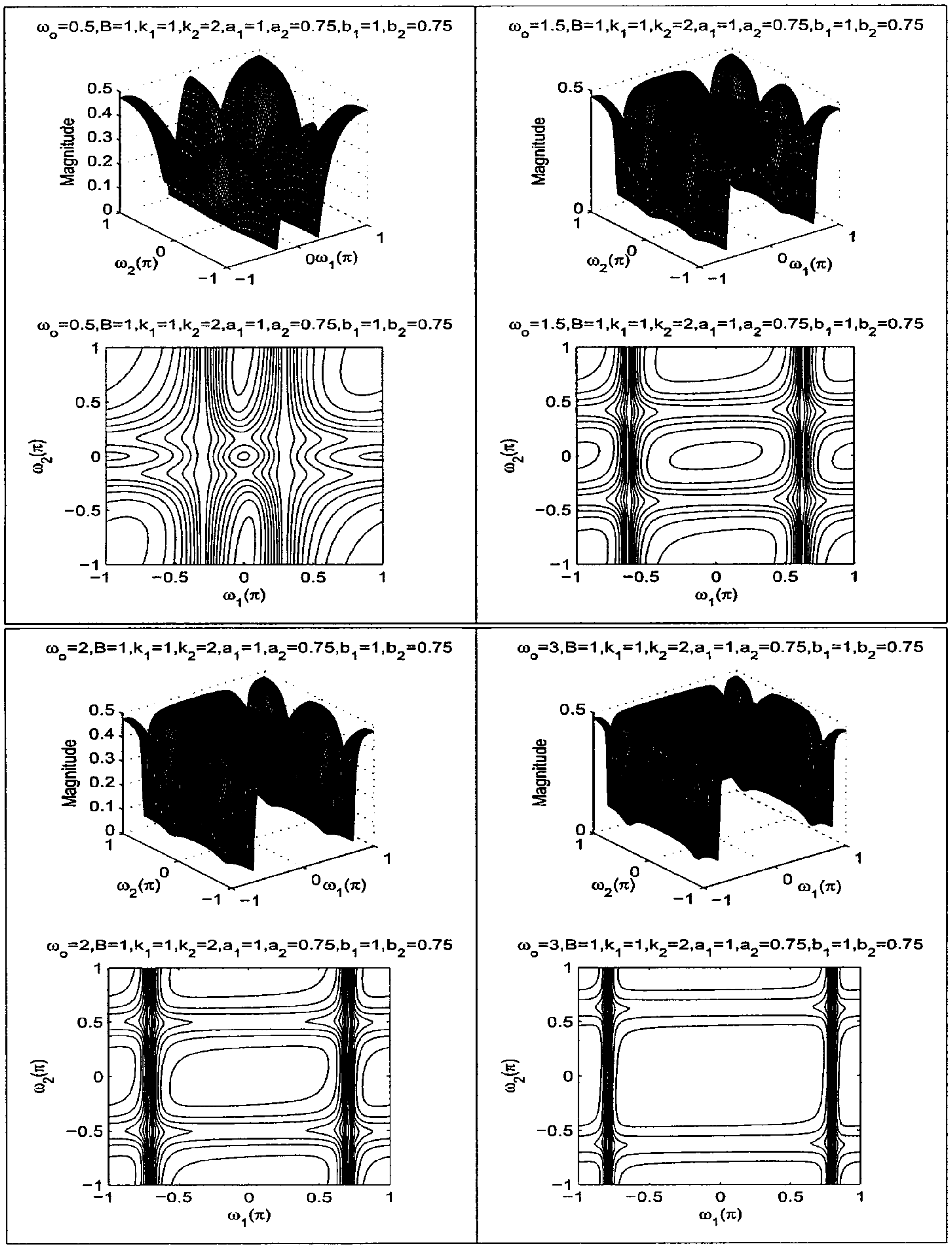

Figure 3.26: Frequency response for the band-stop filter with the value of $k_2 = 2$ for $a_2 = b_2 = 0.75$ and all the other coefficients as unity.

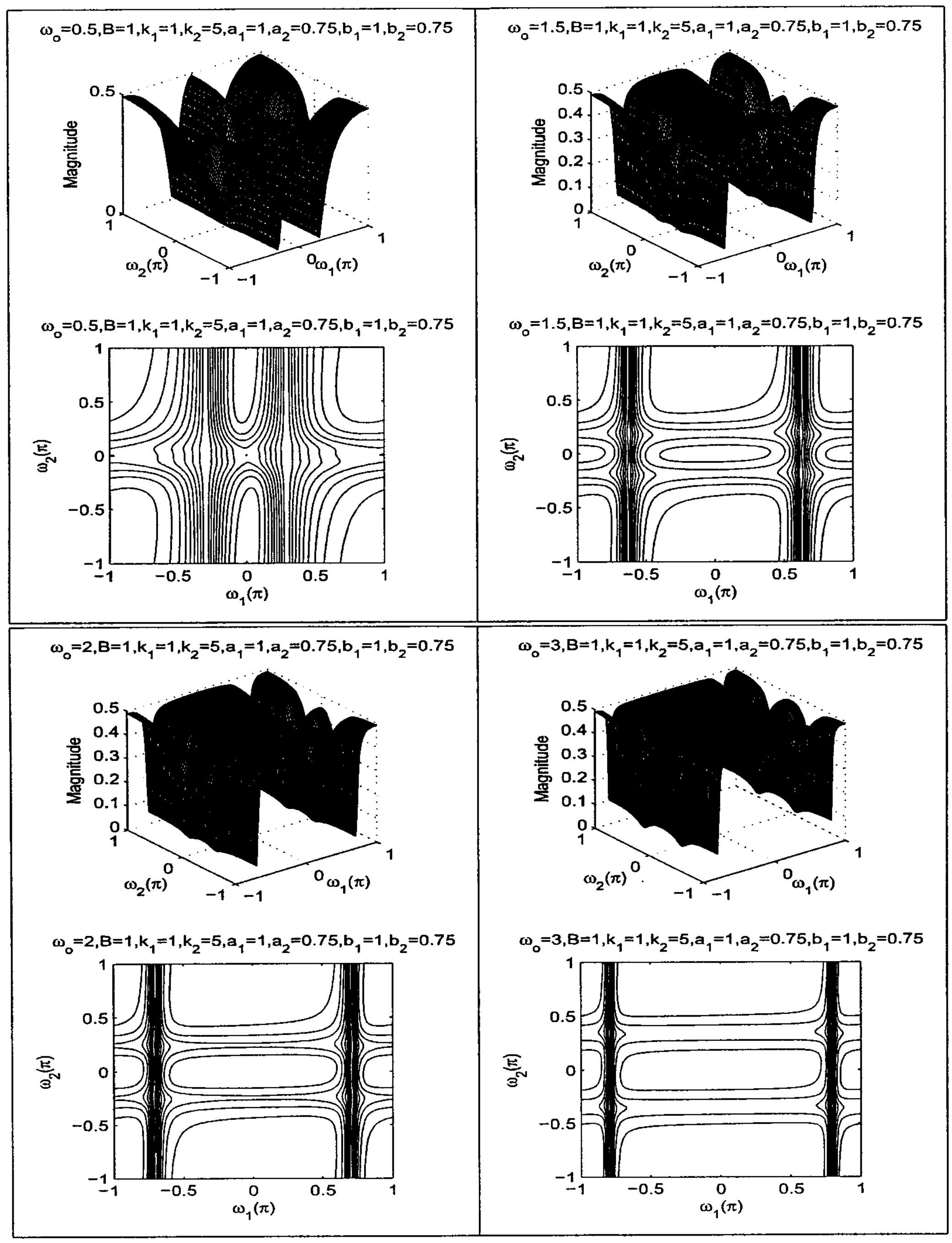

Figure 3.27: Frequency response for the band-stop filter with the value of $k_2 = 5$ for $a_2 = b_2 = 0.75$ and all the other coefficients as unity.

the values of k_2 and ω_o are same the frequency response will resemble similar to standard band-stop filter but with the reduced stopband. It can be seen that the frequency response of the band-stop filter will be the combined effect of section 3.5.2, section 3.5.9 and the frequency response of the various value of ω_o shown in the figure 3.18.The section 3.5.2 describes that the increase in value of k_2 makes the center frequency of the stopband to move from maximum to minimum value. The section 3.5.9 describes that the increase in value of k_2 with the same value of a_2 and b_2 makes the center frequency to move from maximum to minimum value but with the reduced stop band.When $k_2 < 1$, the frequency responses starts from band-stop filter and slowly transforms into a low-pass filter response. When $1 \leq k_2 \leq 2$, the filter response will be a band-stop filter. When $k_2 > 2$, the filter response will get transformed to band-stop filter from a high-pass filter response on increase in value of ω_o. In this section, it is observed that the change in the value of k_2 with the various value of ω_o makes the center frequency to move from maximum to minimum value with proportional to the value of ω_o.

3.5.12 Frequency Response of 2-D Band-Stop filters with various k_1 and ω_o values with different values of a_1 and b_1

In section 3.5.7 and section 3.5.8, we observed the frequency response for various k_1 and ω_o values with the same values of a_1 and b_1. In this section we will see the effect different values of a_1 and b_1, for various values of k_1 and ω_o. Figure 3.28,3.29,3.30and3.31 shows 3-D magnitude and the contour plot for different k_1, a_1 and b_1 values for ω_o ranging from $0.5 < \omega_o < 3$.

From the section 3.5.3, on increasing the value of a_1 from the lowest value, the lower

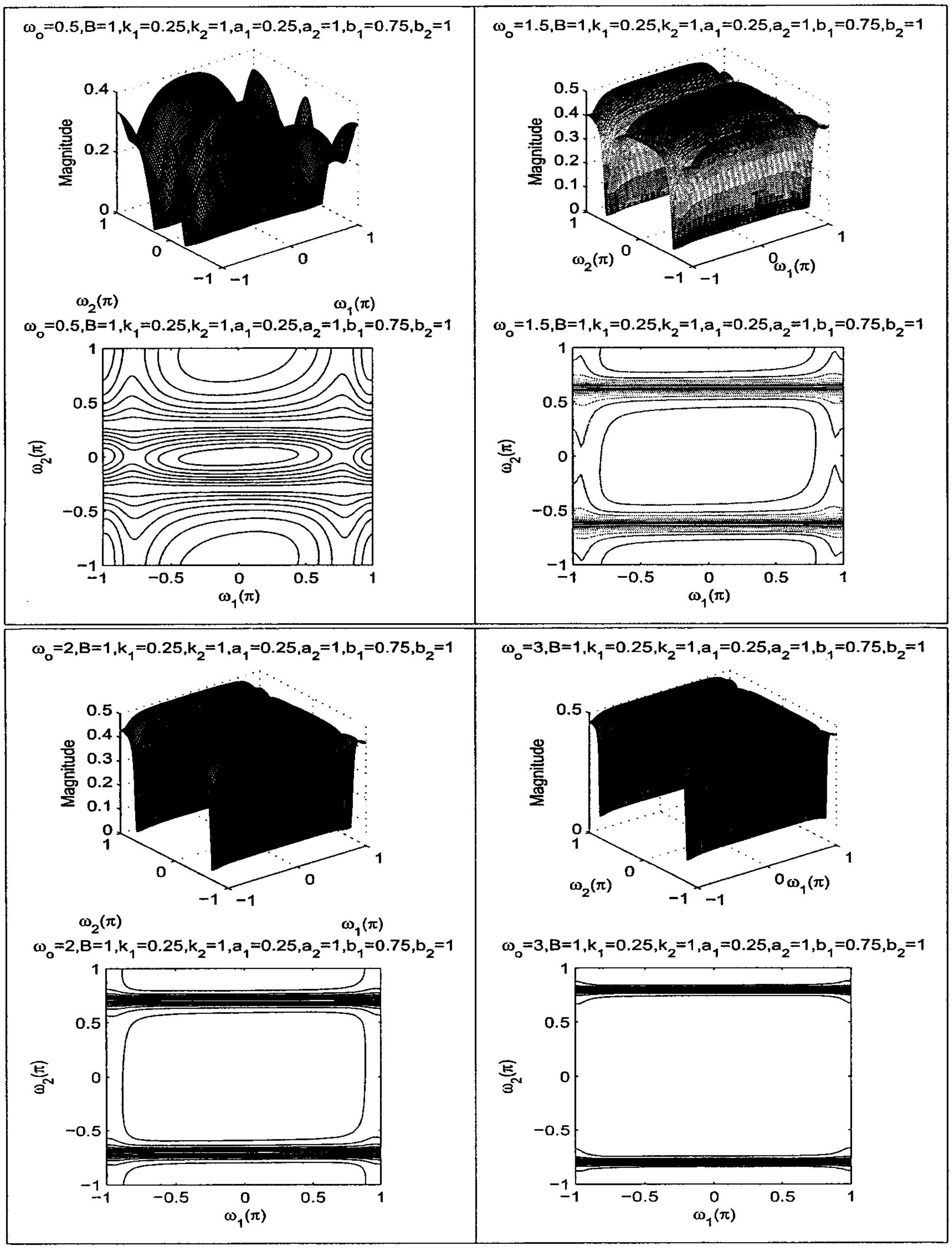

Figure 3.28: Frequency response for the band-stop filter with the value of $k_1 = 0.25$ for $a_1 = 0.25, b_2 = 0.75$ and $\omega_o = 0.5, 1.5, 2$ and 3 with all the other coefficients as unity.

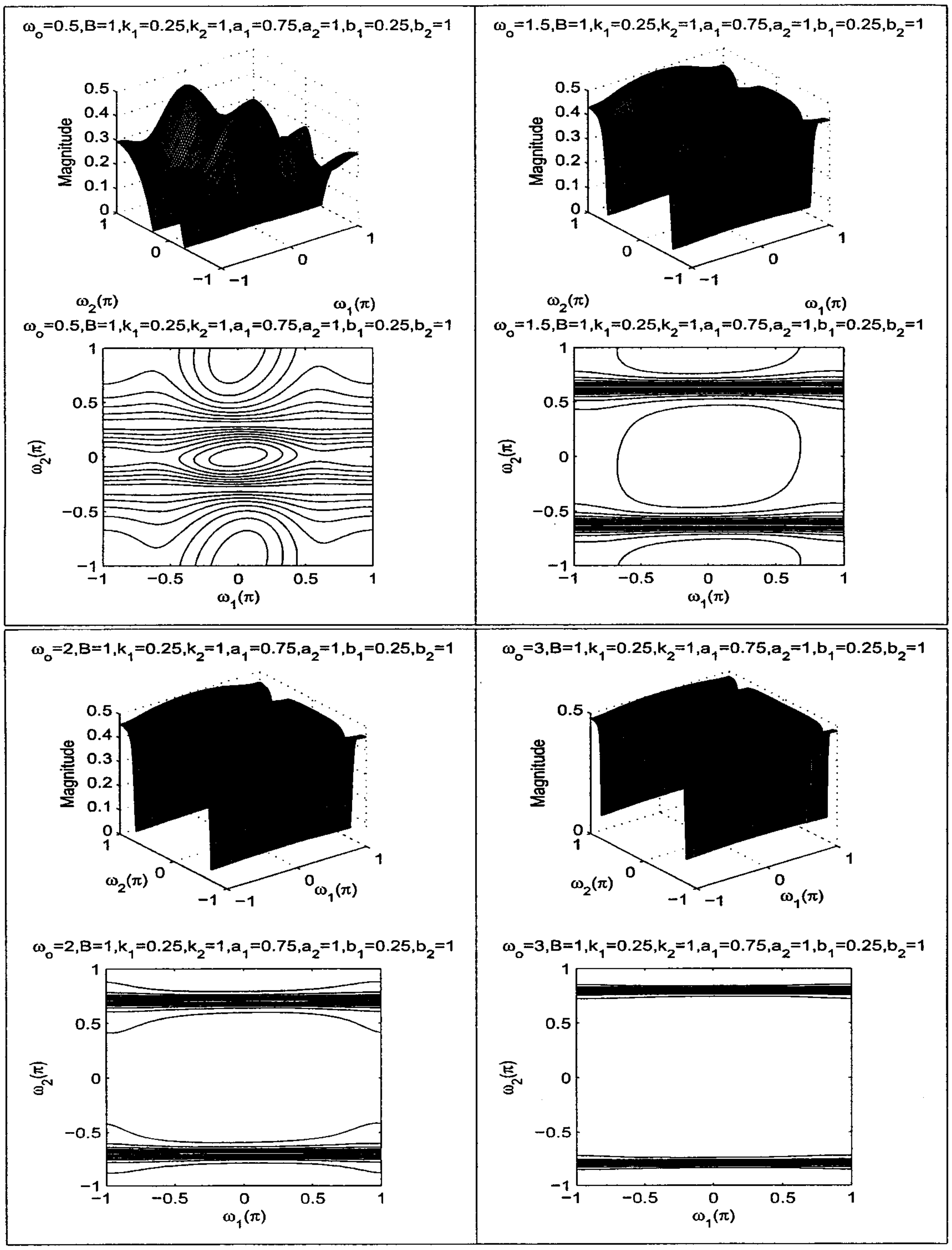

Figure 3.29: Frequency response for the band-stop filter with the value of $k_1 = 0.25$ for $a_1 = 0.75, b_2 = 0.25$ and $\omega_o = 0.5, 1.5, 2$ and 3 with all the other coefficients as unity.

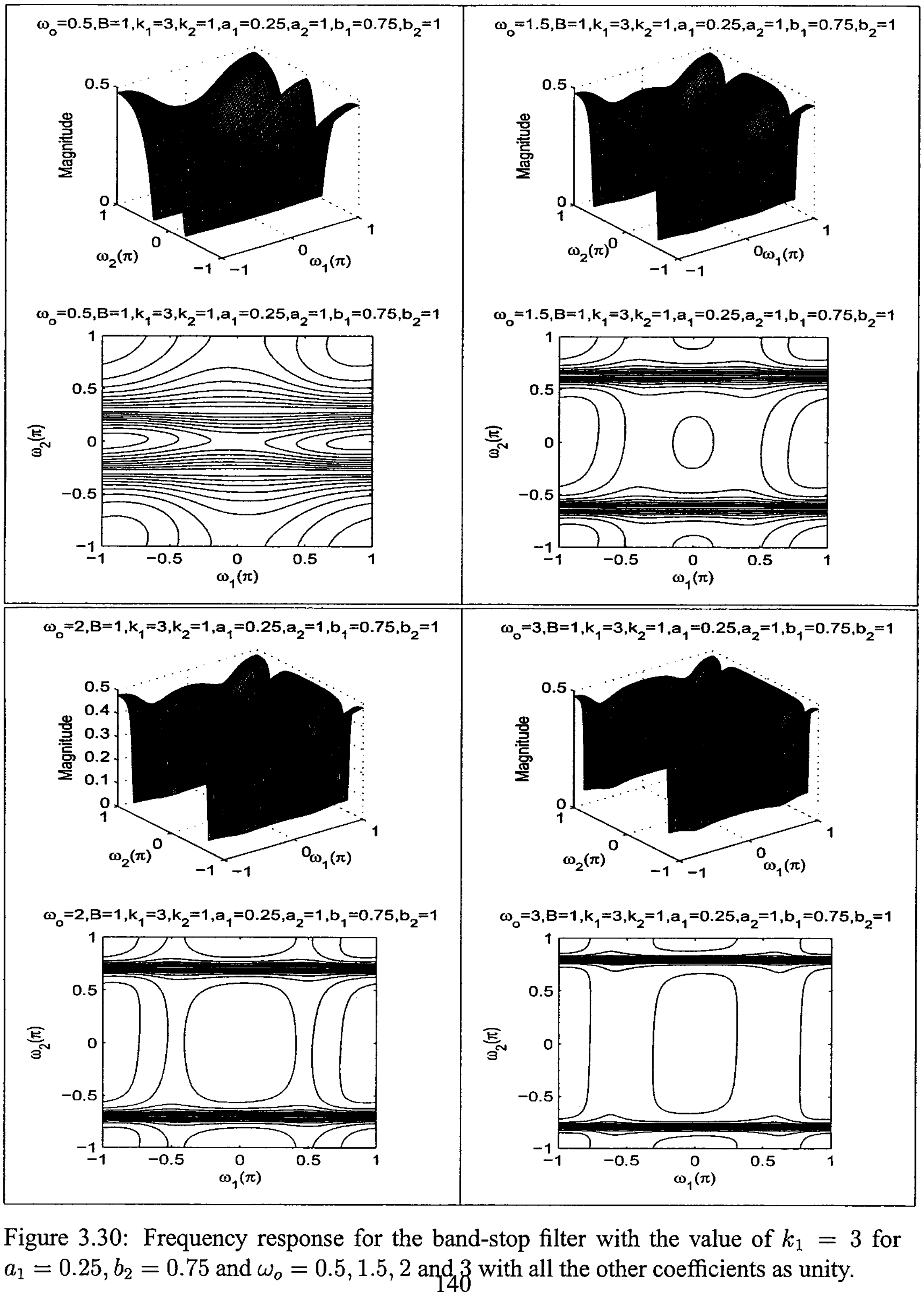

Figure 3.30: Frequency response for the band-stop filter with the value of $k_1 = 3$ for $a_1 = 0.25, b_2 = 0.75$ and $\omega_o = 0.5, 1.5, 2$ and 3 with all the other coefficients as unity.

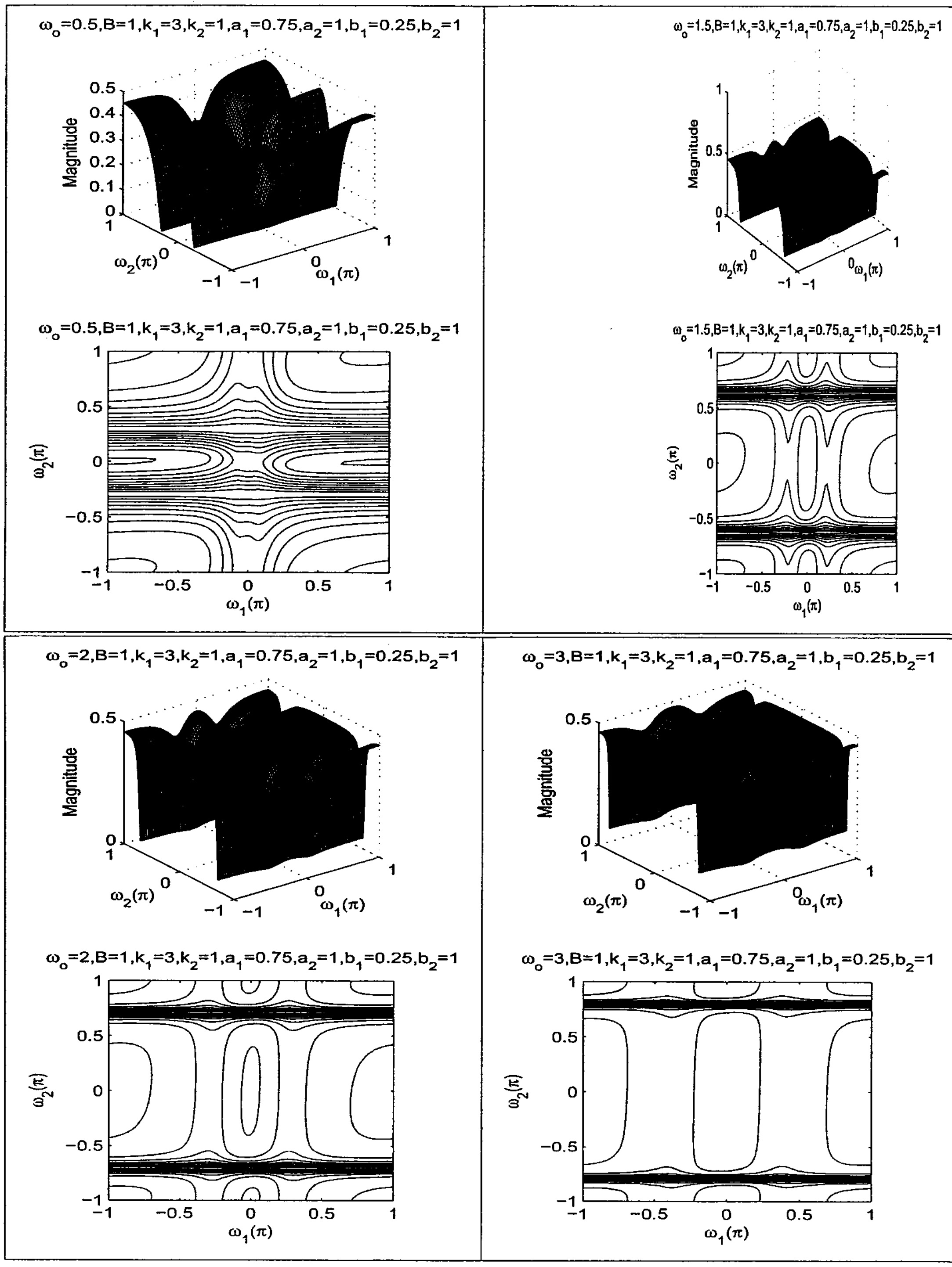

Figure 3.31: Frequency response for the band-stop filter with the value of $k_1 = 3$ for $a_1 = 0.75, b_2 = 0.25$ and $\omega_o = 0.5, 1.5, 2$ and 3 with all the other coefficients as unity.

passband increases in gain and decreases in bandwidth. From the section 3.5.5, on increasing the value of b_1 from the lowest value, the upper passband increases in gain but decreases in bandwidth. If we assign different value to both a_1 and b_1, response follows the way which has the larger value. If $a_1 > b_1$, then the frequency response will have the effect similar to the one which is explained in section 3.5.3, else the frequency response will have the effect similar to the one which is explained in section 3.5.5. Figure 3.28and 3.30 shows the effect of $a_1 < b_1$, it can be observed that the lower passband will have lesser gain compared to gain of upper passband, the center frequency of the stopband will move from maximum to minimum frequency upon increase in k_1. Figure 3.29 and 3.31 shows the effect of $a_1 > b_1$, it can be observed that the upper passband will have lesser gain compared to gain of lower passband, and the center frequency of the stopband will move from minimum to maximum frequency upon increase in k_1. When $k_1 < 1$ the filter response will start from low-pass and transform into a all-pass filter with the variation of ω_o between 0.5 and 3 rad/sec. When $k_1 > 1$ the filter response will start from high-pass filter and get transformed to the band-stop filter. In the above two cases, the variation of ω_o will shift the center frequency to move proportionally to the value of ω_o. It can also be observed that for the different values a_1, b_1, k_1 and ω_o all the frequency response will look like a all-pass filter this is due to the merging of passband from lower and upper frequencies resulting in neglible bandwidth for the stopband.

3.5.13 Frequency Response of 2-D Band-Stop filters with various k_2 and ω_o values with different values of a_2 and b_2

In section 3.5.8 and section 3.5.10, we observed the frequency response for various k_2 and ω_o values with the same values of a_2 and b_2. In this section we will see the effect different values of a_2 and b_2, for various values of k_2 and ω_o. Figure 3.32,3.33,3.34 and 3.35 shows 3-D magnitude and the contour plot for different k_2, a_2 and b_2 values for ω_o ranging from $0.5 < \omega_o < 3$.

From the section 3.5.4, on increasing the value of a_2 from the lowest value, the lower passband increases in gain and decreases in bandwidth. From the section 3.5.6, on increasing the value of b_2 from the lowest value, the upper passband increases in gain but decreases in bandwidth. If we assign different value to both a_2 and b_2, response follows the way which has the larger value. If $a_2 > b_2$, then the frequency response will have the effect similar to the one which is explained in section 3.5.4, else the frequency response will have the effect similar to the one which is explained in section 3.5.6. Figure 3.32 and3.34 shows the effect of $a_2 < b_2$, it can be observed that the lower passband will have lesser gain compared to gain of upper passband, the center frequency of the stopband will move from maximum to minimum frequency upon increase in k_2. Figure 3.33 and 3.35 shows the effect of $a_2 > b_2$, it can be observed that the upper passband will have lesser gain compared to gain of lower passband, and the center frequency of the stopband will move from minimum to maximum frequency upon increase in k_2.When $k_2 < 1$ the filter response will start from low-pass and transform into a all-pass filter with the variation of ω_o between 0.5 and 3 rad/sec. When $k_2 > 1$ the filter response will start from high-pass filter and get transformed to the band-stop filter. In the above two cases, the variation of ω_o will shift the center frequency to move proportionally to the value of ω_o. It can also be observed that for the different values a_2, b_2, k_2 and ω_o all the frequency response will look like a all-pass filter this is due to the merging of passband from lower and upper frequencies resulting in neglible bandwidth for the stopband.

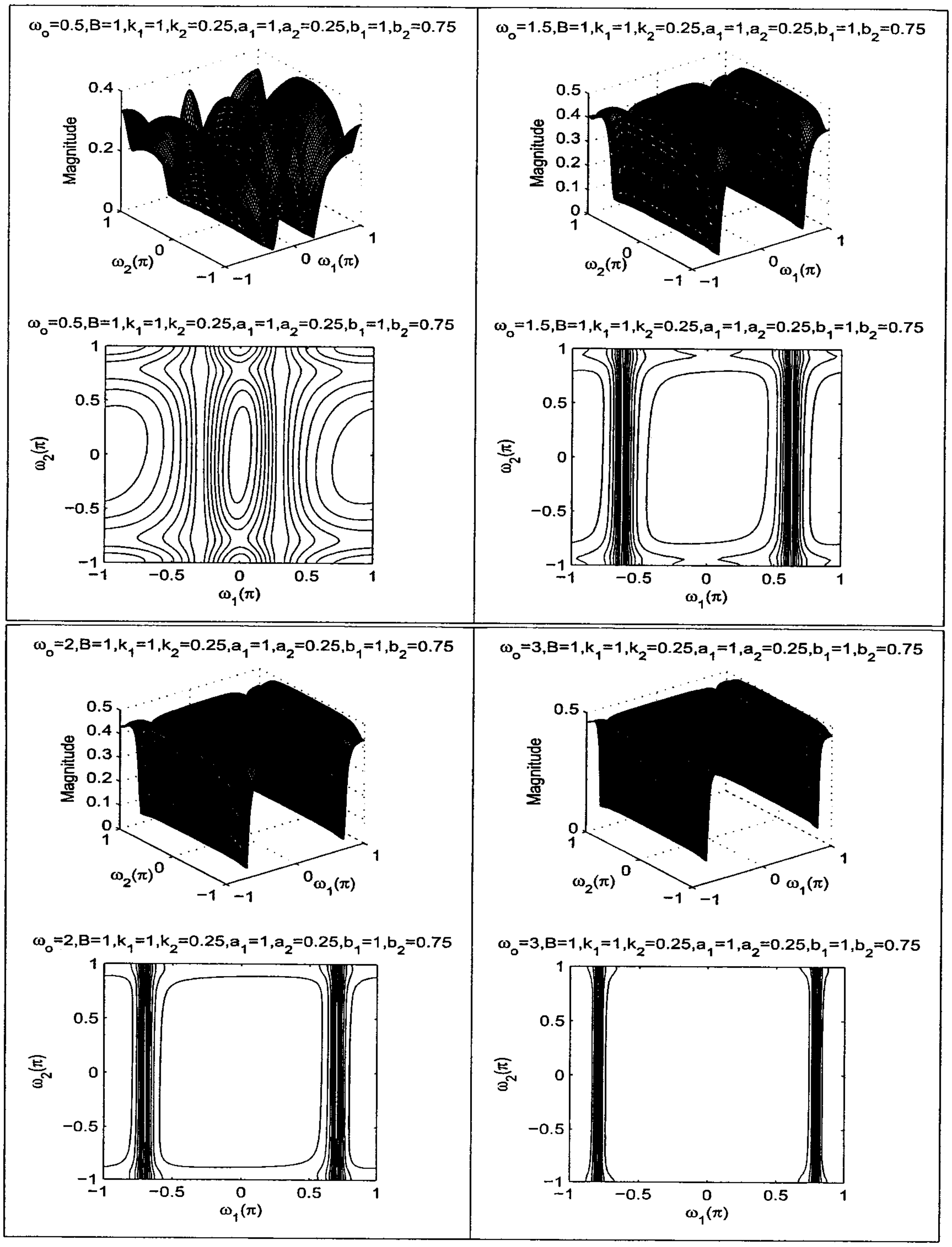

Figure 3.32: $k_2 = 0.25$, $a_2 = 0.25$, $b_2 = 0.75$ and $\omega_o = 0.5, 1.5, 2$ and 3

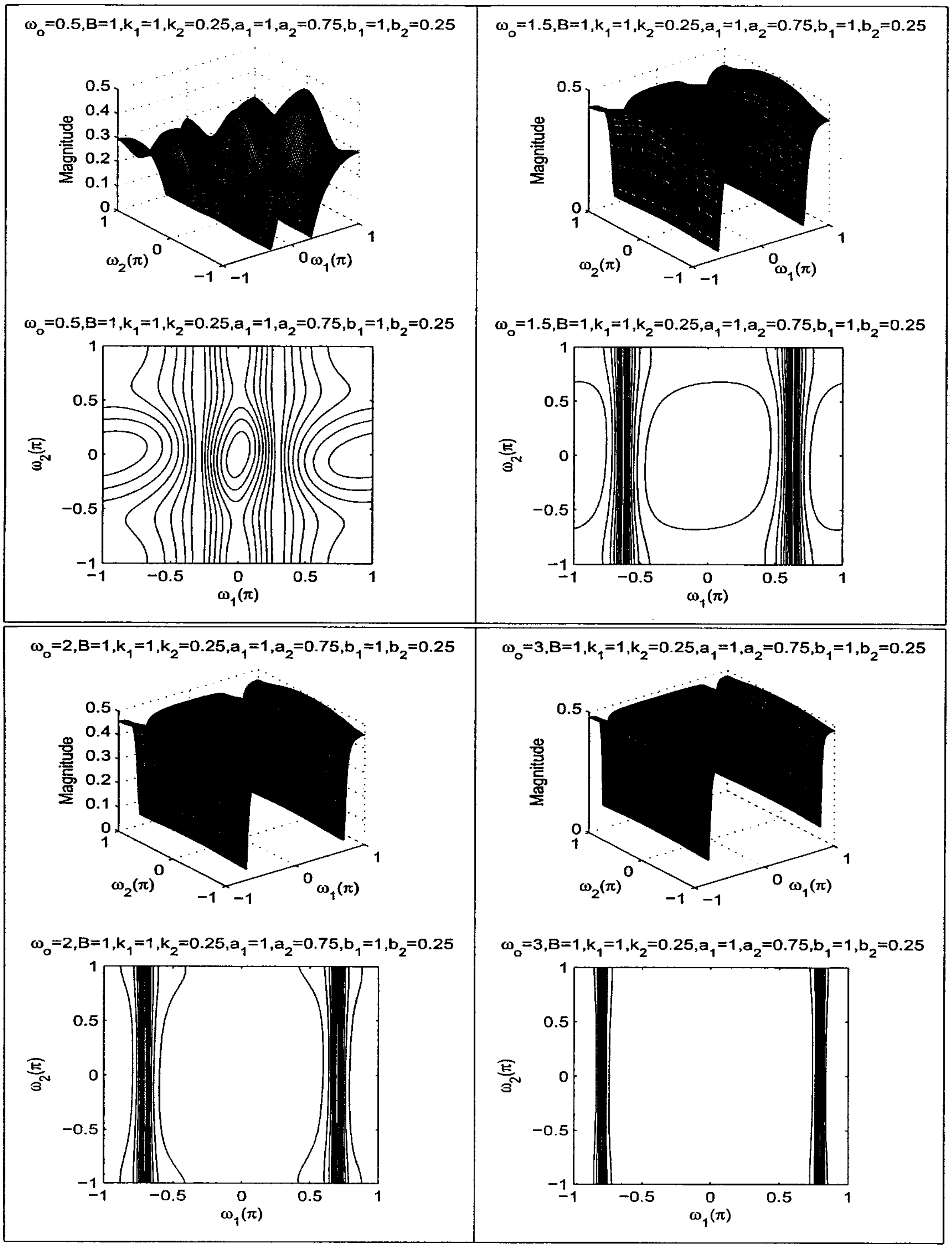

Figure 3.33: $k_2 = 0.25$, $a_2 = 0.75$ $b_2 = 0.25$ and $\omega_o = 0.5, 1.5, 2$ and 3

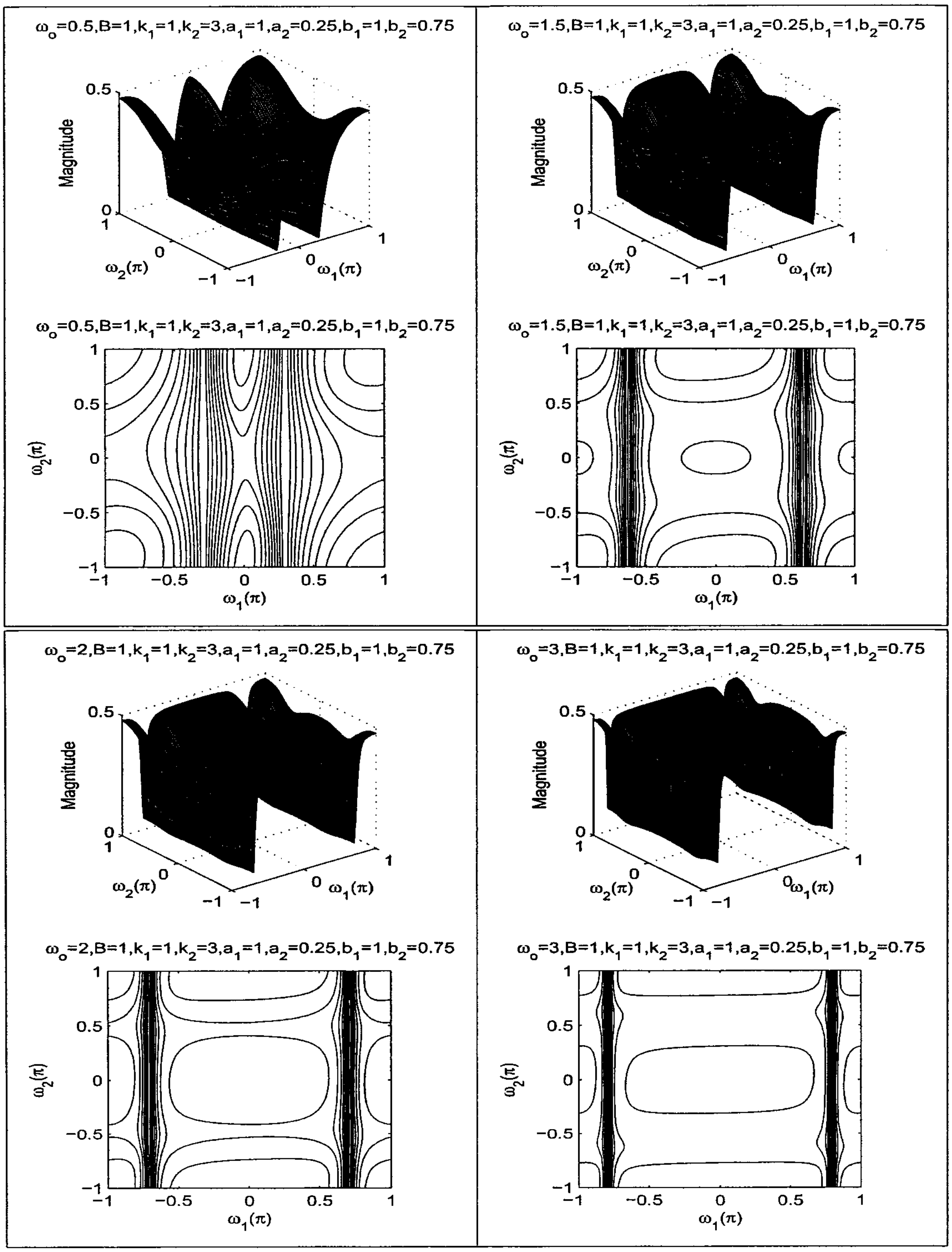

Figure 3.34: $k_2 = 3$, $a_2 = 0.25$ $b_2 = 0.75$ and $\omega_o = 0.5, 1.5, 2$ and 3

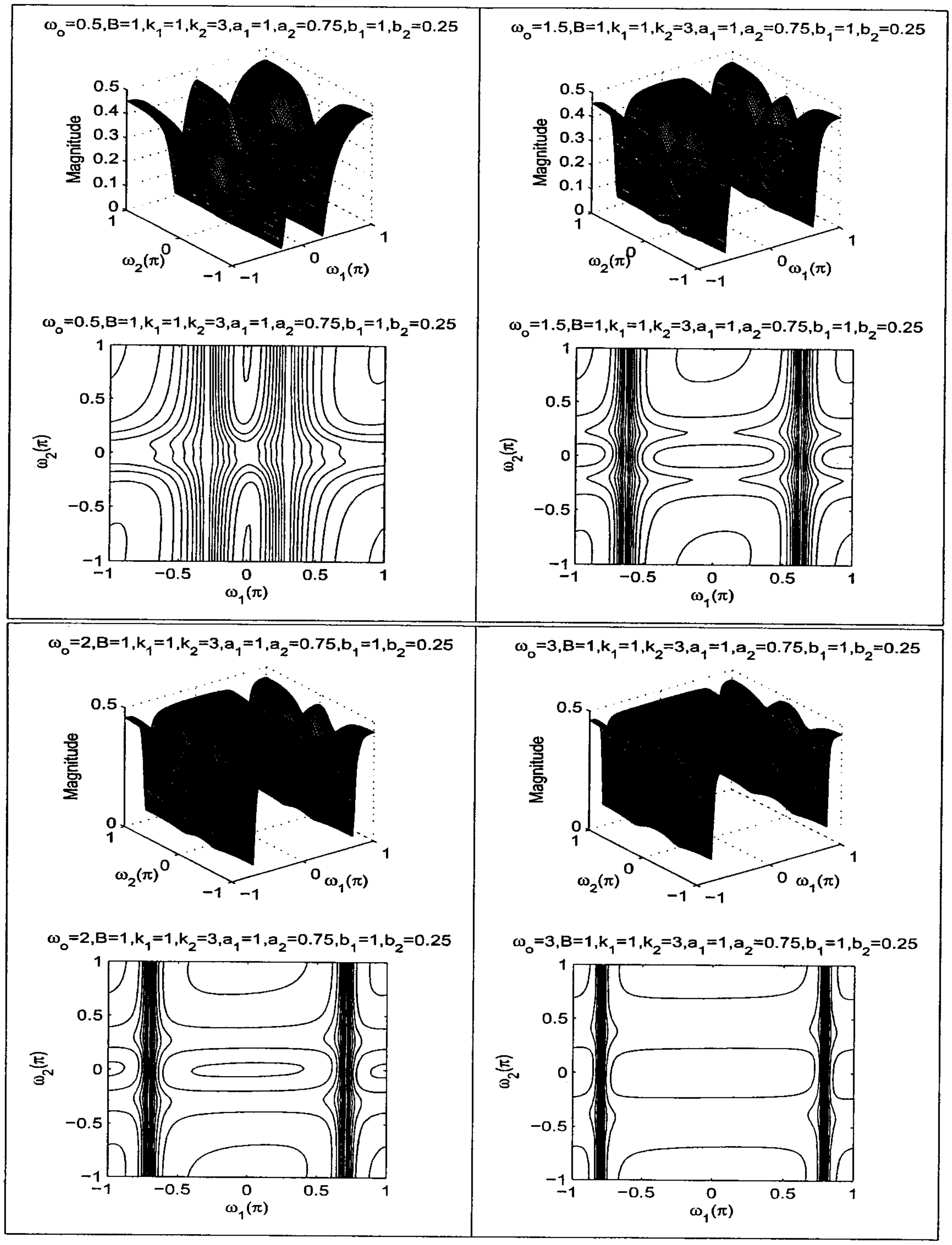

Figure 3.35: $k_2 = 3$, $a_2 = 0.75$ $b_2 = 0.25$ and $\omega_o = 0.5, 1.5, 2$ and 3

3.6 Frequency Response of 2-D and-stop filters with various k_1, k_2 and ω_o values with different values of a_1, a_2, b_1 and b_2

In this section, we can see the effect of all the coefficient such as $k_1,\ k_2,\ a_1,\ a_2,\ b_1,\ b_2$ and ω_o. Figure 3.36 shows the combination of all the effect in $\omega_1 - axis$ and $\omega_2 - axis$. The coefficients $k_1,\ a_1$ and b_1 contributes for the effect in $\omega_1 - axis$, similarly the coefficient $k_2,\ a_2$ and b_2 contributes for the effect in $\omega_2 - axis$. The effects of each coefficient are studied in the section 3.5. It also shows the combination of the effect discussed in section 3.5.11 and 3.5.12.

3.7 Summary and Conclusion

This chapter describes the transformation of low-pass to band-stop filter,and applying the generalized bilinear transformation to a band-stop transfer function. The effects of each coefficient of the generalized bilinear transformation are discussed and plots are shown for the same in each subsection in the section 3.5. Section 3.5.1 and 3.5.2 explians the effects of k_1 and k_2, on increase in value of k_1 and k_2, the center frequency of the band-stop filter moves from maximum to minimum value and also the bandwidth of the response changes. Section 3.5.3 and 3.5.4 describes the effects of a_1 and a_2, the pass-band gain and the center frequency of the stopband filter and the gain of lower and upper pass-band are controlled by a_1 and a_2 in the ω_1 and ω_2 axis respectively. The center frequency of the stopband and the gain of the stopband are controlled by b_1 and b_2 in the ω_1 and ω_2 axis respectively are analyzed in the section 3.5.3 and 3.5.4. The effect of varying B and ω_o will make the center

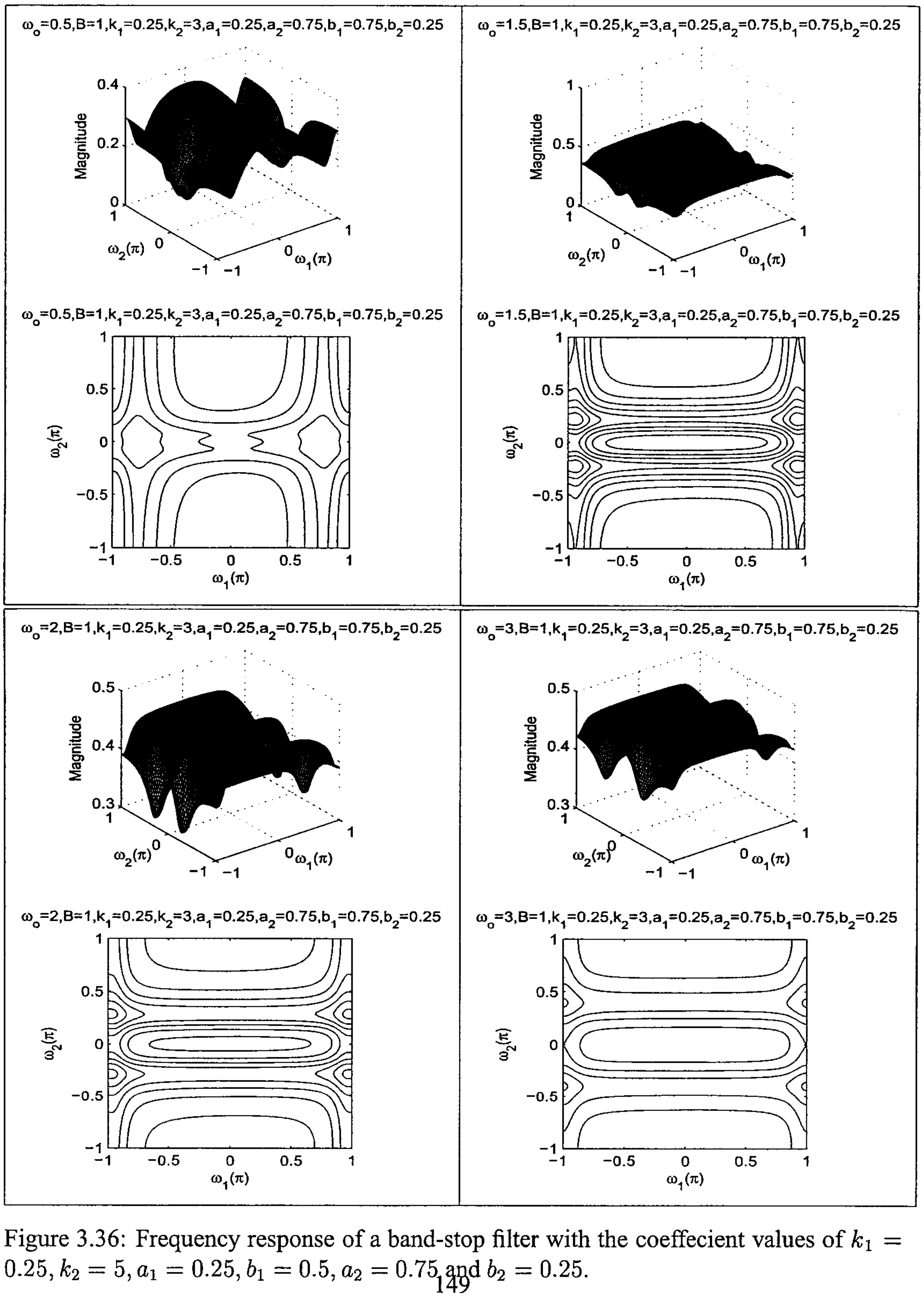

Figure 3.36: Frequency response of a band-stop filter with the coeffecient values of $k_1 = 0.25, k_2 = 5, a_1 = 0.25, b_1 = 0.5, a_2 = 0.75$ and $b_2 = 0.25$.

frequency of the stopband and the stop-band width varying proportionally are discussed in section 3.5.8 and 3.5.9. Section 3.5.7, 3.5.9, 3.5.10, 3.5.11, 3.5.12 and 3.5.13 explians the combination of all this effect, when the values of k_1, a_1, b_1 and ω_o are changed in the $\omega_1 - axis$, the same effect can be observed in the $\omega_2 - axis$ on varying the values of k_2, a_2, b_2 and ω_o. Finally the combination of all the coefficients are explained in section 3.6 and plotted in figure 3.36.

Chapter 4

An Application of 2-D Band-Pass and Band-Stop Filters

The application of digital signal processsing techniques in general and of digital filtering in particular has expanded in many important areas such as speech signal processing, digital telephony and communications, facsimile and TV image processing, radar-sonar systems, biomedicine,space research and operative systems,geoscience,etc. Digital filters are widely applied to 1-D and multidimensional signals. Techniques used for 2-D systems can generally be extended to multidimensional systems. Examples of 2-D systems include image processing, seismic signal processing, meteorology, etc. This chapter discusses about the application of digital filters in digital speech and image processing.

4.1 Digital Speech Processing

Speech signals are obviously one of the most important signals and therefore it is quite natural that their processing by means of digital techniques has received so much attention. Indeed the representation and transmission of speech signals by digital means can result in more efficient and often more economic techniques than in the corresponding analog case. The application of digital filtering to speech processing and representation was made possible many years ago by the relatively low frequency range of speech and consequent availabilty of low cost analog to digital (A-D) and digital-to-analog (D-A) converters. Sampling frequencies in the range 8KHz-100KHz are used with word-length of 7-12 bits, depending on the nature of the processing[7].

4.2 Digital Image Processing

The most common applications of digital filters in image processing are in the areas of digital image enhancement and restoration. To enhance images digitally, filters can be designed that reduce noise by smoothing, emphasize some region of spectrum, sharpen edges, and implement other functions that produce improved images.Such improvement may be intended to provide a subjectively better image for humans to look at, or that may be intended as a preprocessor for higher level tasks, such as segmentation and pattern recognition or for machine vision. Digital restoration filters may be used to invert a degradation process. For example, the effect of known camera defocusing, or some other limitations, can be reduced or removed by developing a restoration model on it. Digital filters may also be applied in postprocessing after image coding, and in preprocessing before image segmentation[33]. In this section, we will see small application in the field of image processing using the designed band-pass filter in chapter 2.

4.2.1 Image smoothing and noise reduction

Digital filters are applied in image enhancement to try and improve the subjective apperance of images without attempting directly to invert the effect of any degradation. They may also be employed to enhance certain properties of the images and to supress others [7]. The application of digital filters in image enhancement for three kinds of applications such as a preprocessor to provide an improved image, as a noise-reduction process in its own right, and as a postprocessor to reduce the effects of noise introduced by some other form of processing. In this section, the designed digital band-pass filters are used to obtain a smooth and noise reduced images.

The original images are added with the gaussian random noise with different noise level, which are then applied to a band-pass filter with the transfer function given in eqn (2.27), where the noise level is the number of times of the generated random noise. The smoothened and noise reduced images are shown in fig (4.1-4.9). Each figure consists of four images. The first one shows the original image, and under that is the smoothened image. Noisy images are shown to the right of original image. Noise reduced images are shown beneath the noisy images. Smoothened images are obtained on applying the band-pass filter to original image and the noise reduced images are obtained on applying the band-pass filter to the noisy images. The quality of the noise reduced images are measured in terms of Peak Signal to Noise Ratio. On varying the coefficients of the bilinear transformation an improved PSNR value of the noise reduced images can be obtained. PSNR is explained in the next sub-section. MATLAB program is in the appendix for this application.

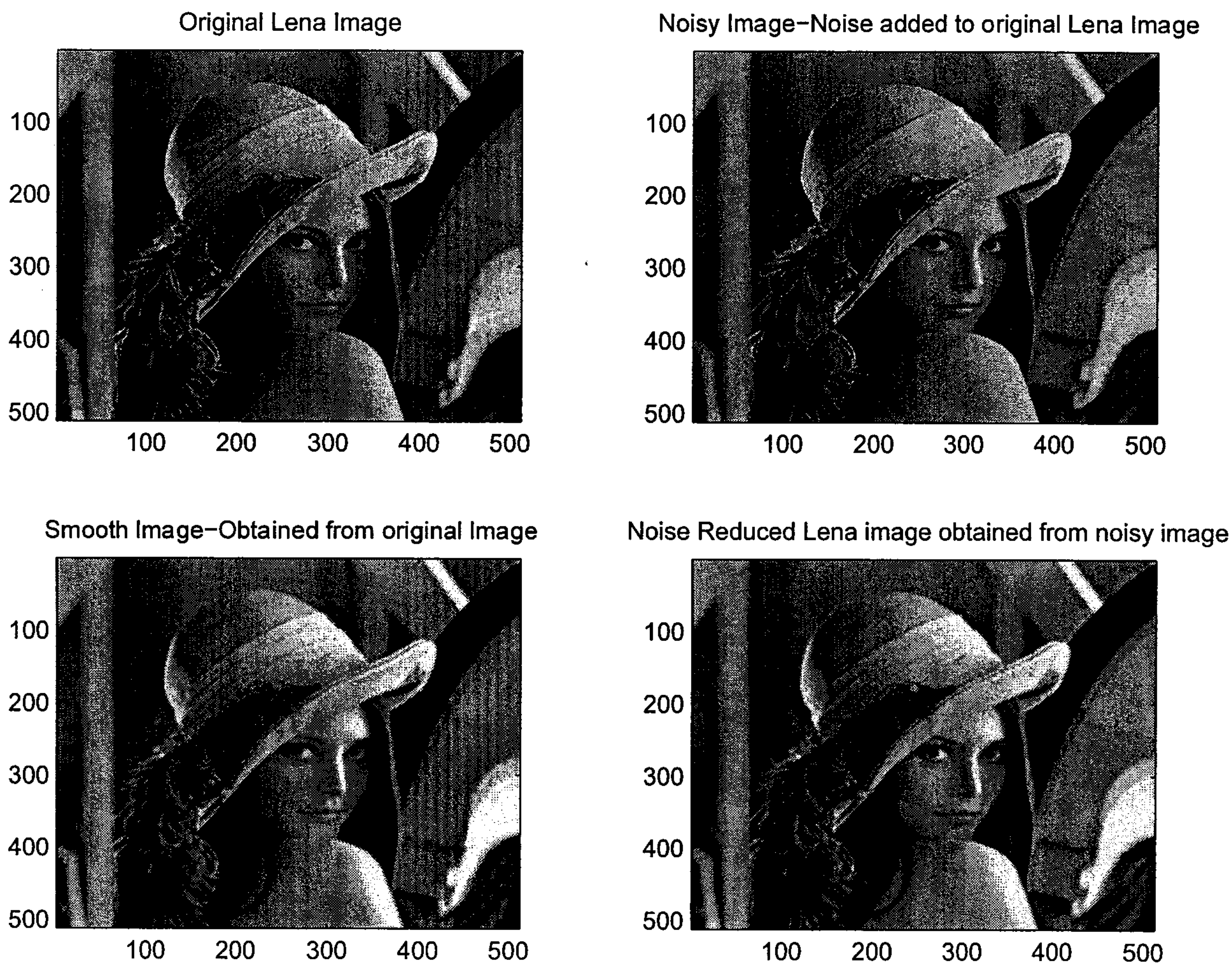

Figure 4.1: Smooth and noise reduced lena images at noise level is 5

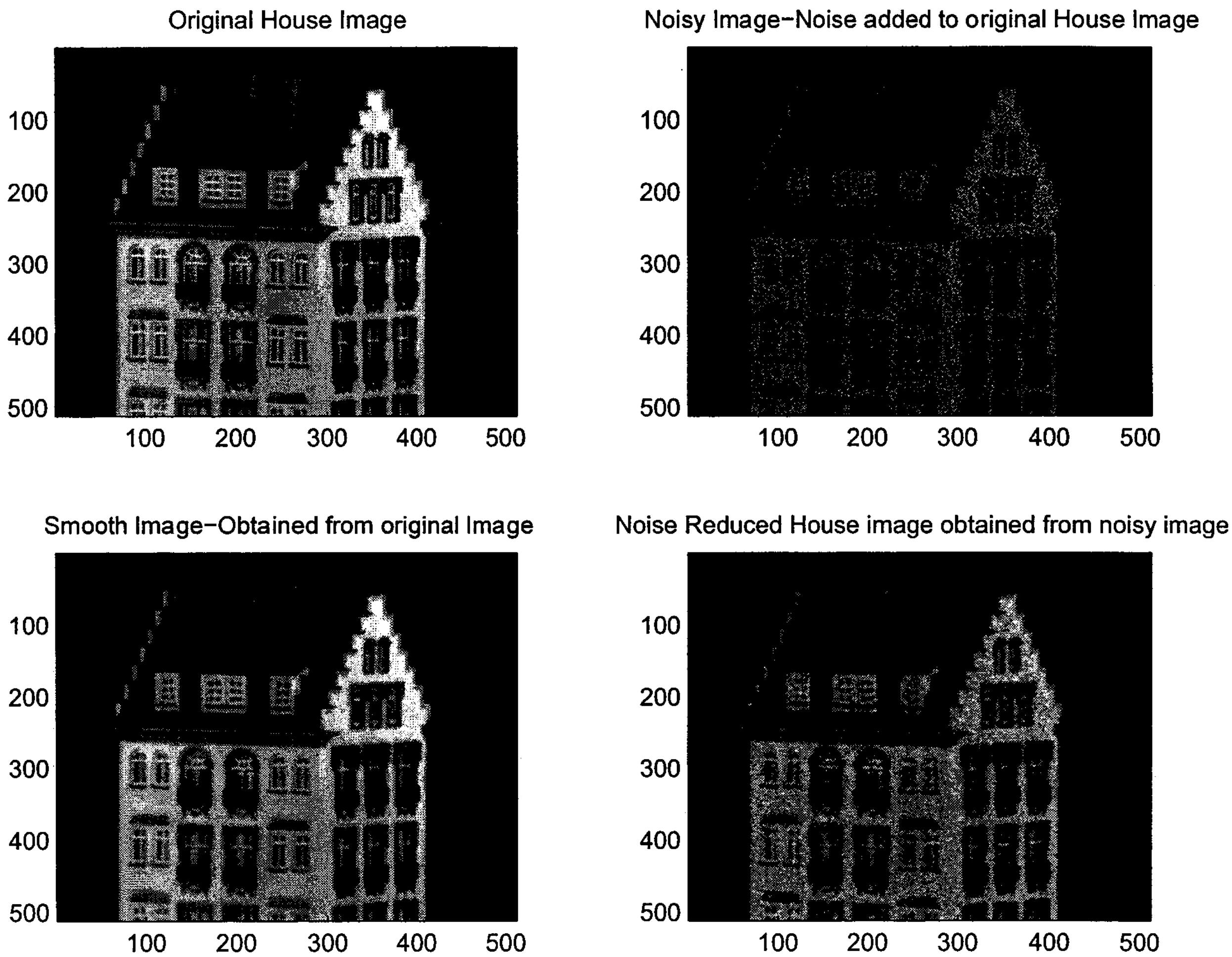

Figure 4.2: Smooth and noise reduced images of house at noise level is 5

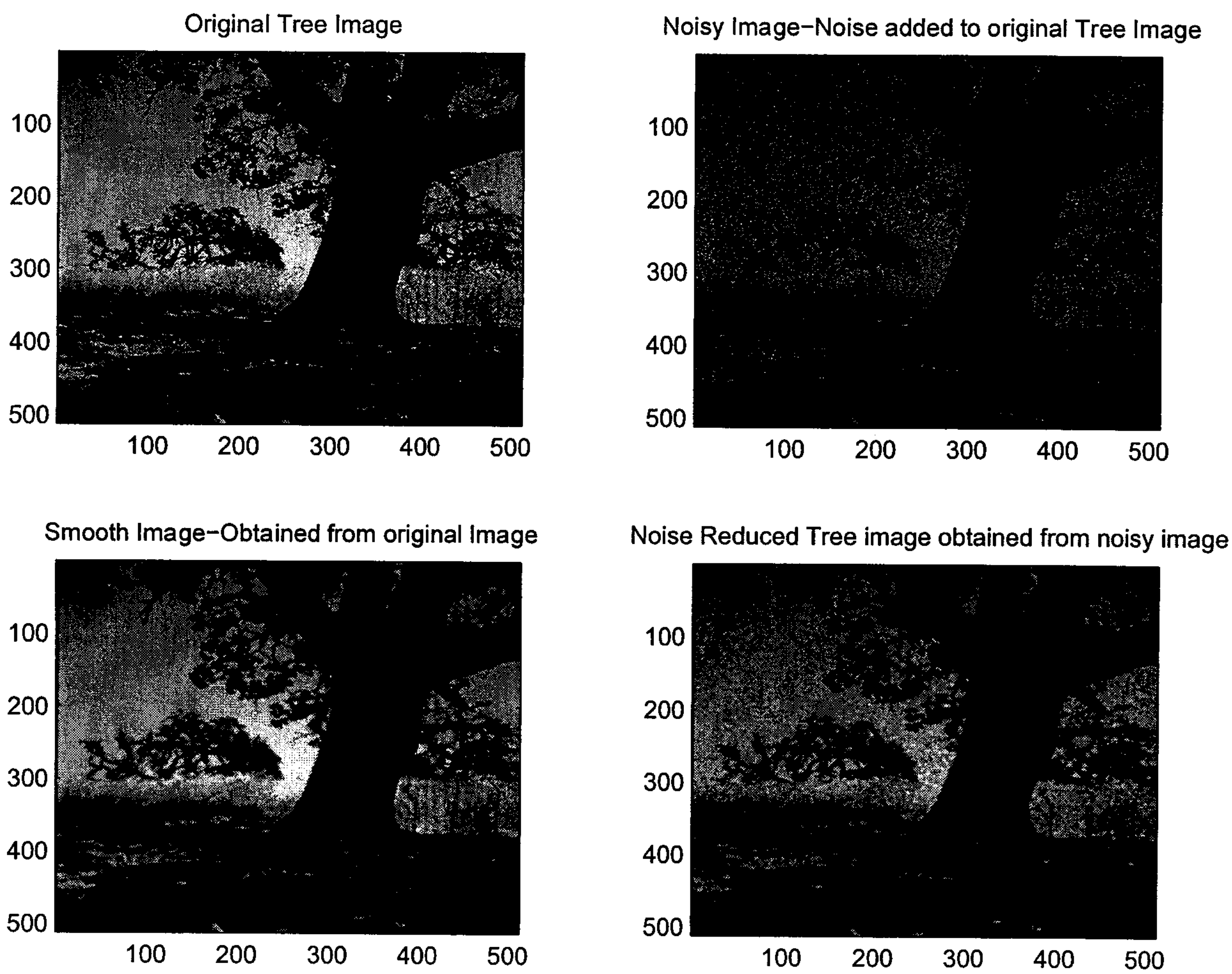

Figure 4.3: Smooth and noise reduced images of tree at noise level is 5

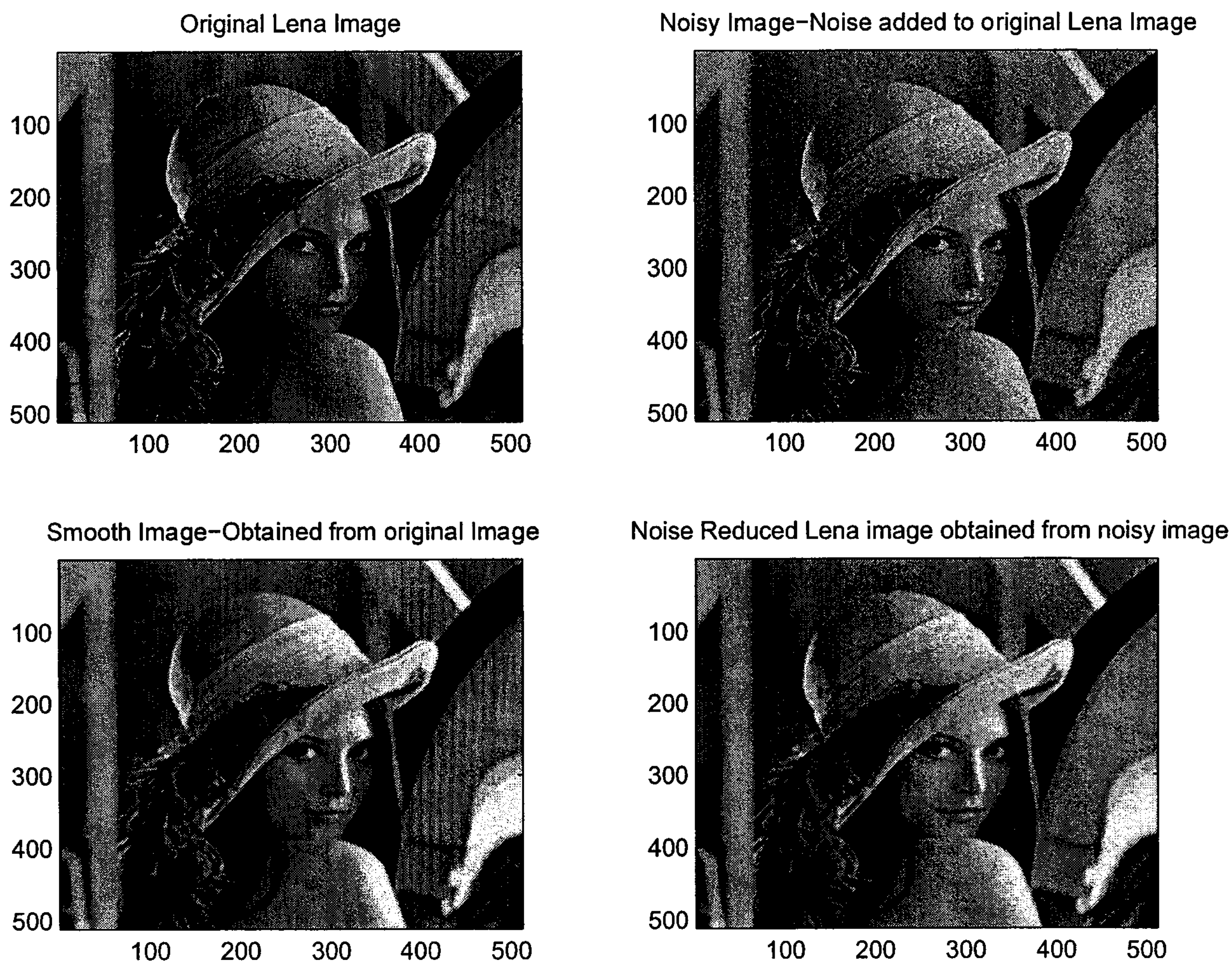

Figure 4.4: Smooth and noise reduced lena images at noise level is 15

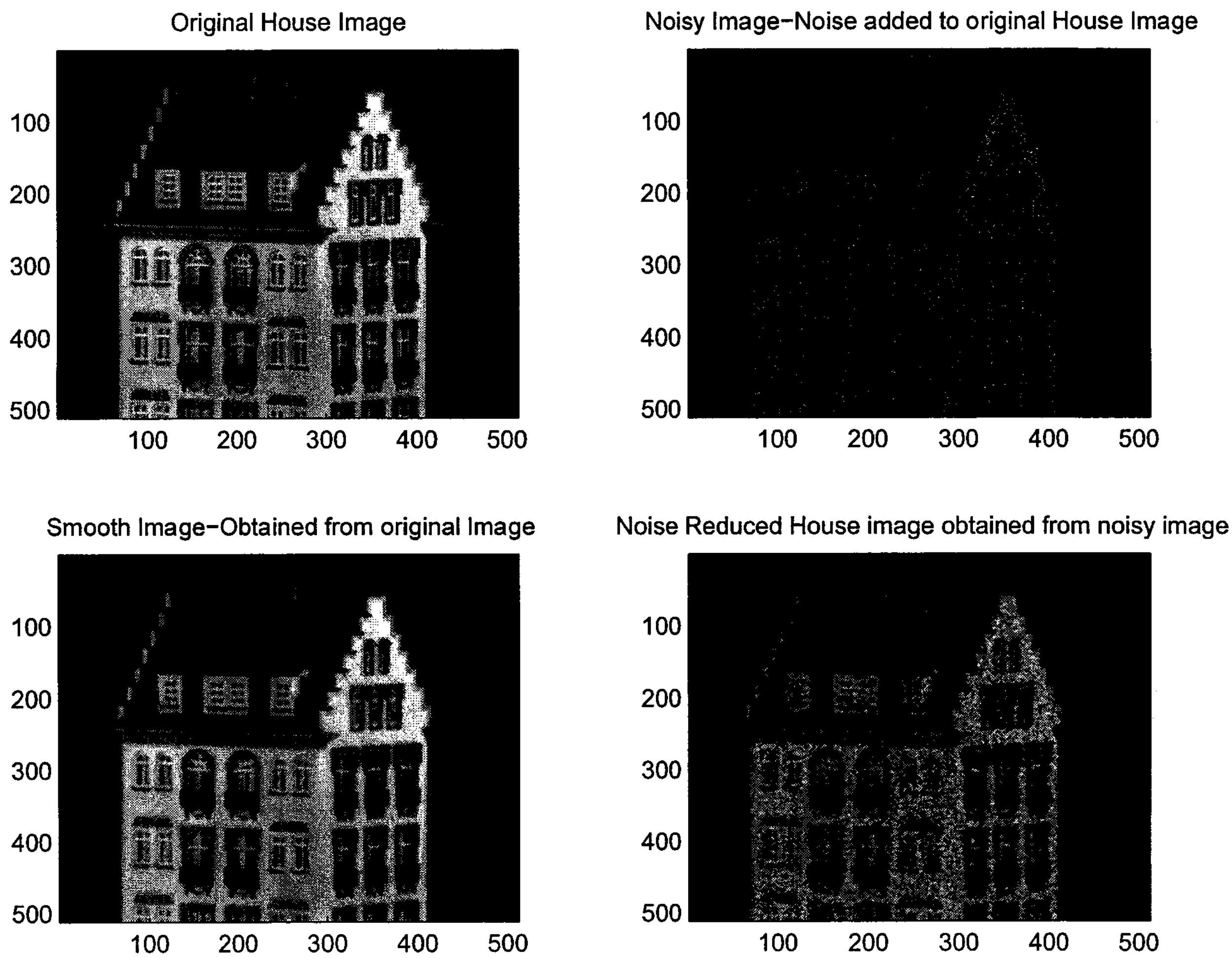

Figure 4.5: Smooth and noise reduced images of house at noise level is 10

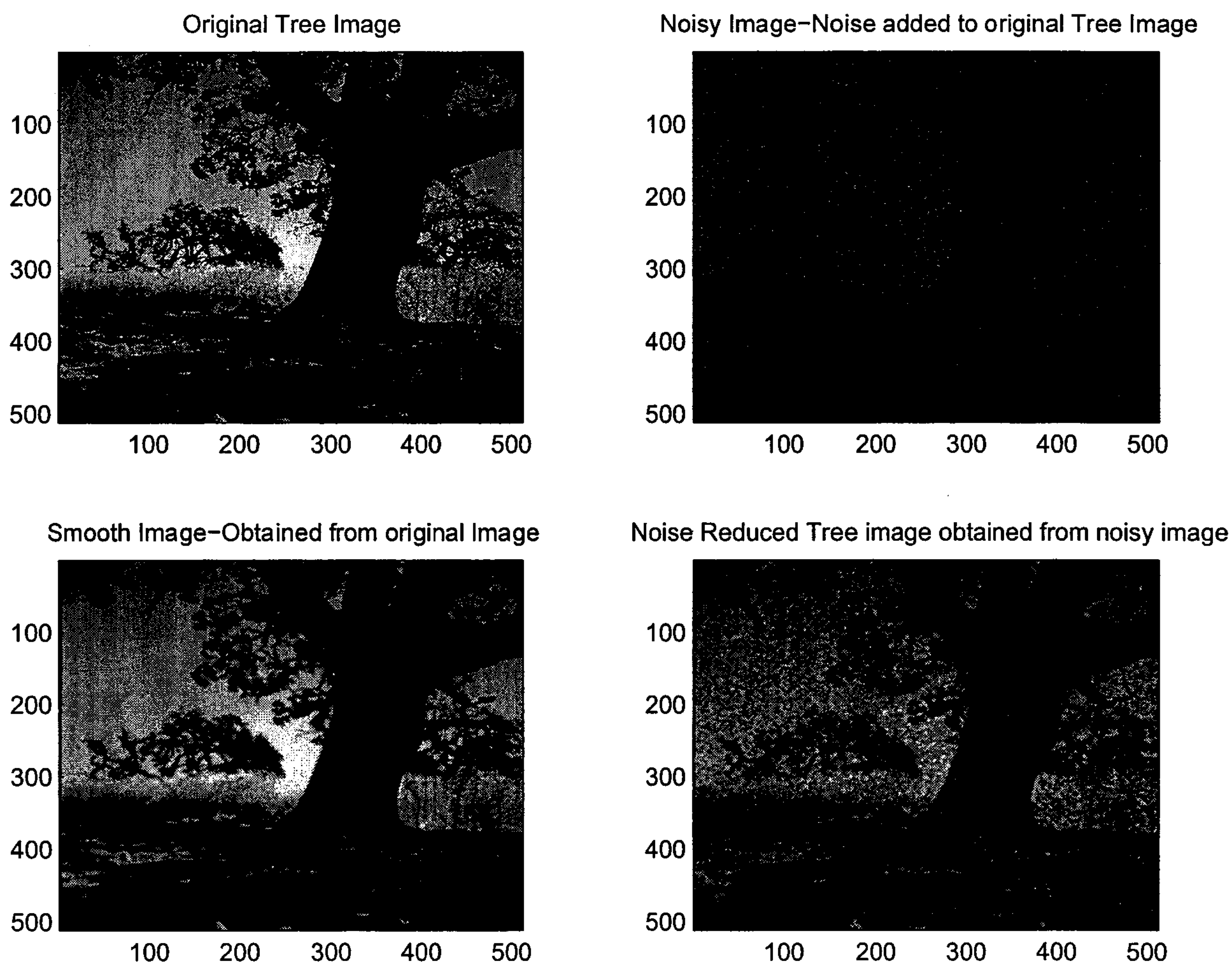

Figure 4.6: Smooth and noise reduced images of tree at noise level is 10

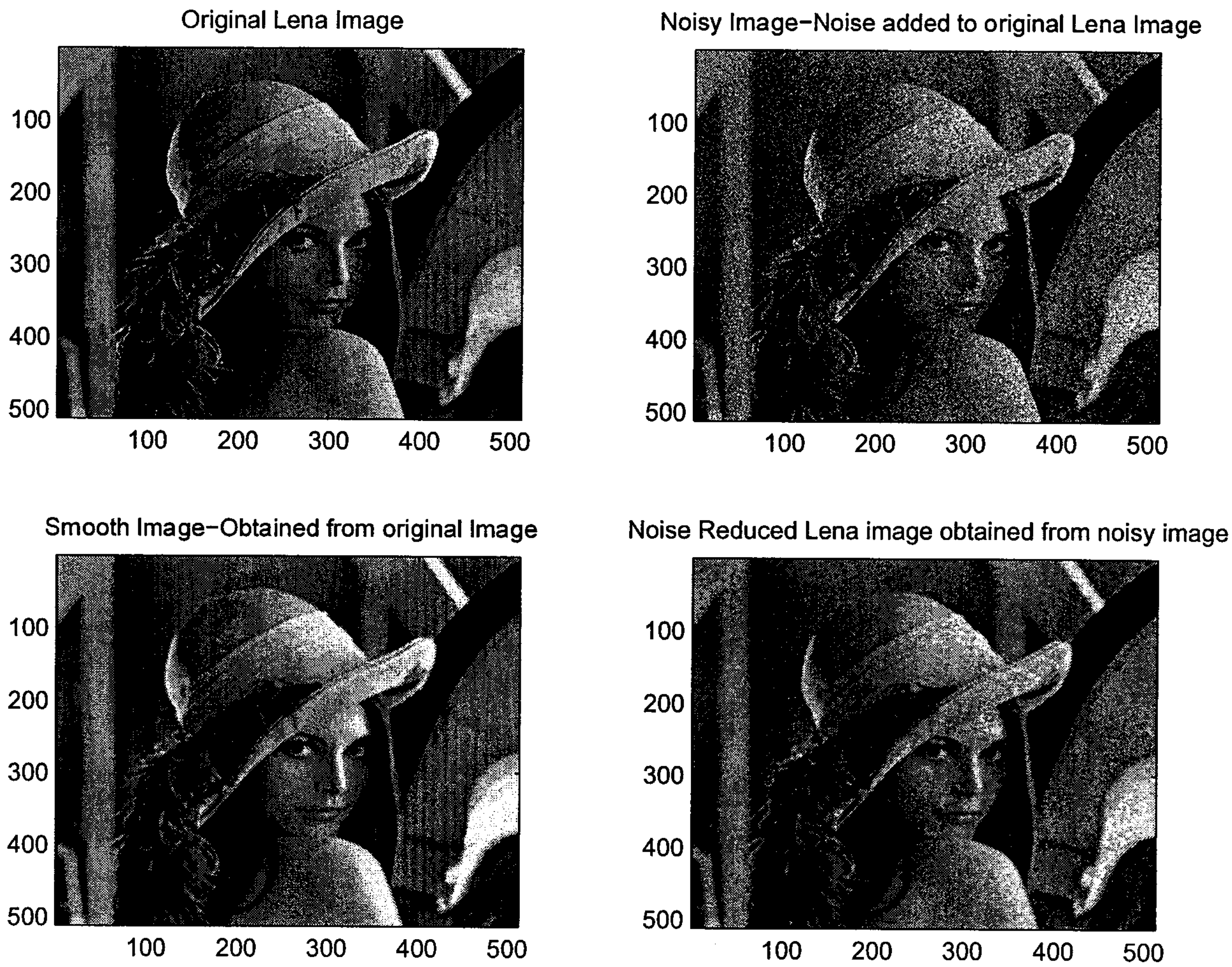

Figure 4.7: Smooth and noise reduced lena images at noise level is 30

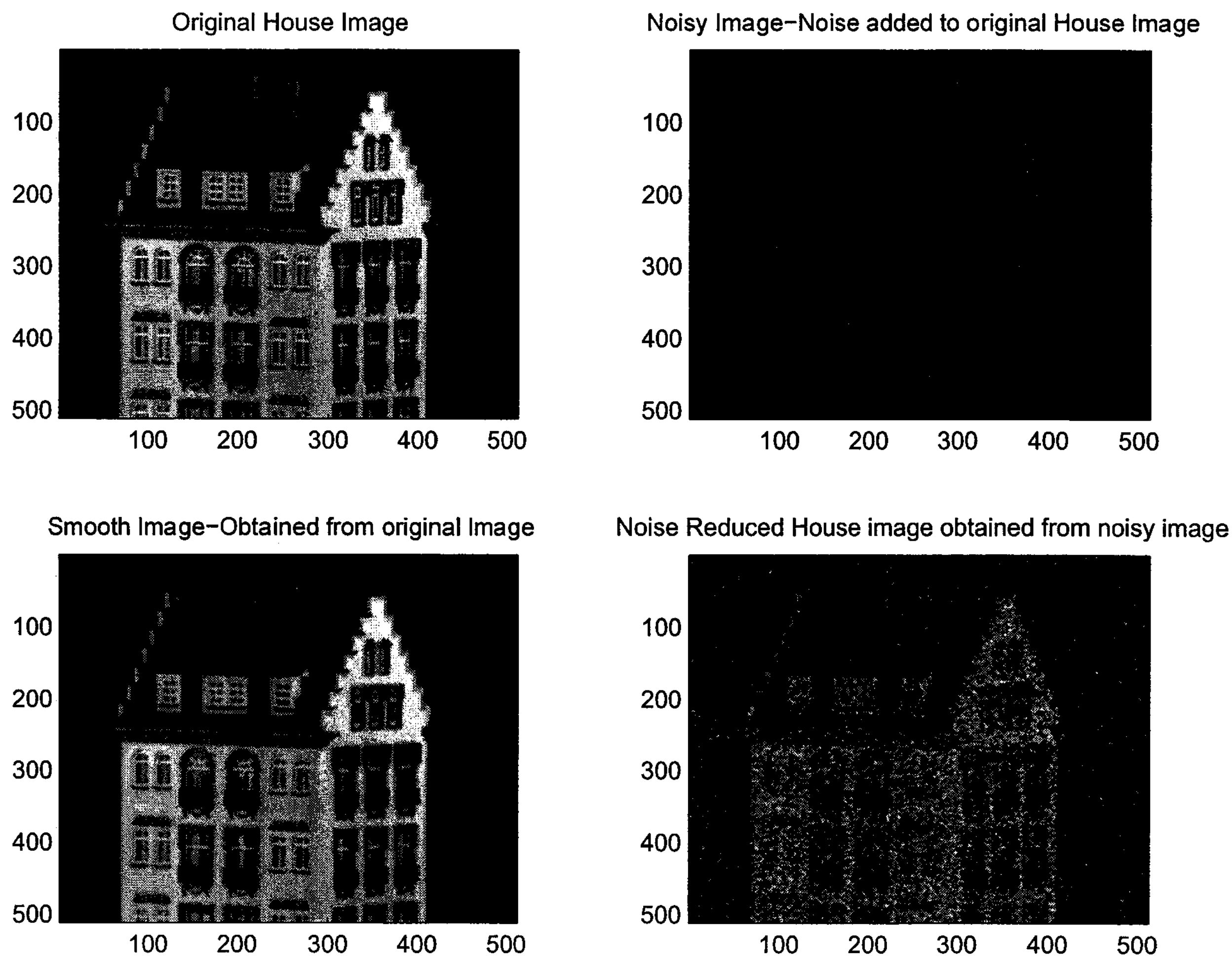

Figure 4.8: Smooth and noise reduced images of house at noise level is 20

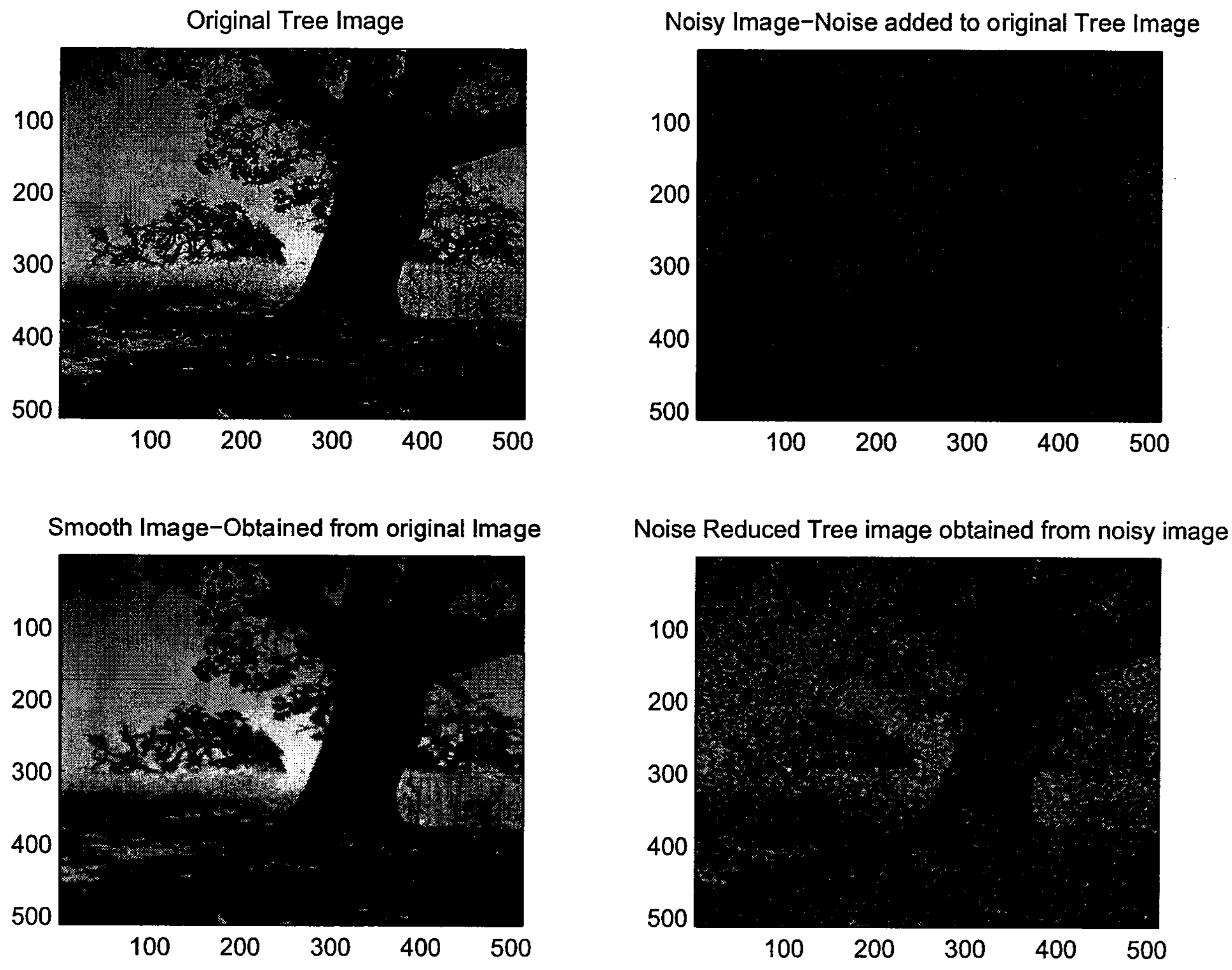

Figure 4.9: Smooth and noise reduced images of tree at noise level is 20

4.2.1.1 Peak Signal to Noise Ratio (PSNR)

Signal-to-noise (SNR) measures are estimates of the quality of a reconstructed image compared with an original image. The basic idea is to compute a single number that reflects the quality of the reconstructed image. Reconstructed images with higher metrics are judged better. In fact, traditional SNR measures do not equate with human subjective perception. Several research groups are working on perceptual measures, but for now we will use the signal-to-noise measures because they are easier to compute. Just remember that higher measures do not always mean better quality.

The actual metric we will compute is the peak signal-to-noise reduced image measure which is called PSNR. Assume we are given a source image f(i,j) that contains N by N pixels and a noise reduced image F(i,j) where F is noise reduced by applying the band-pass filter to f(i,j). Error metrics are computed on the luminance signal only so the pixel values f(i,j) range between black (0) and white (255).

First you compute the mean squared error (MSE) of the reconstructed image as follows

$$MSE = \frac{\Sigma[(f(i,j) - F(i,j)]^2}{N^2}$$

The summation is over all pixels. The root mean squared error (RMSE) is the square root of MSE.

PSNR in decibels (dB) is computed by using

$$PSNR = 20log_{10}(\frac{255}{RMSE})$$

Typical PSNR values range between 20 and 40. They are usually reported to two decimal points (e.g., 25.47). The actual value is not meaningful, but the comparison between two values for different reconstructed images gives one measure of quality. The MPEG committee used an informal threshold of 0.5 dB PSNR to decide whether to incorporate a coding optimization because they believed that an improvement of that magnitude would be visible.

4.2.2 Simulation results

The subjective test experiments are conducted to the three test model using the designed band-pass filter and the results are compared. The three test model of size 512 X 512 are taken and passed on to the band-pass filter to get a smooth image and the random noise are added to it at different levels starting from 5 to 30 then it is passed to the band-pass filter, the resultant will have noise reduced image. The tabular column below will give the Peak Signal to Noise Ratio at different noise levels.

TestModel	Lena	House	Tree
Smooth Image	33.65	39.99	38.60
Noise Level 5	33.55	35.09	34.69
Noise Level 10	33.12	30.58	30.49
Noise Level 20	31.57	25.54	25.57
Noise Level 30	29.55	22.45	22.51

Table 4.1: PSNR for Smooth and Noise Reduced images at different noise level

The PSNR values decreases as the noise level increases. This can be solved by passing the noise reduced image to the filter again after comparing with the original image.

4.2.3 Image analysis

Pictures such as radiographs,angiographs,tomographs,etc., are widely used in medicine for diagnosis, treatment monitoring and research. Consequently, the digital filters and image processing also find considerable application in this field. Image processing has two related functions here. First, it may be used to enhance and improve images which have been formed by X radiography or other means. Such pictures can be quite poor for a variety of reasons, and it is necessary to process and improve them. For example, there may be degradation blur and noise, occuring during the image formation process or as a result of uneven illumination.

The nature of the X-radiographic process itself may lead to distortion. In this case, image processing may actually be required to produce an image that physicians can study. But the most publicized application of image processing in medicine is the formation of X-ray images via computerized tomography. The basic radiographic problem involves reconstruction from projection measurements of the linear attenuation coefficient integrated along the path of a collimated X-ray beam. Digital filters are widely applied during the reconstruction process that produces the image.The second function of image processing is as a preprocessor for machine diagnois, reognition or measurement. In such an application, the output of the image processor is aimed at a subsequent set of algorithms, rather than for subjective human viewing[7, 33].

4.3 Summary

This chapter deals with the application of the digital filters especially digital speech and image processing. Apart from this two areas, these types of filters can also be implemented in the field of digital communications such as digital telephony, digital telemetry and digital transmission, radar-sonar systems and biomedicine. One simple image processing application is provided using the band-pass transfer function obtained in the chapter 2 and its results are discussed in terms of PSNR. There are many kinds of applications are possible using these types of filters, since our thesis goal is to study the variable characteristics of these filters, we did not give proper attention in implementing other types of applications.

Chapter 5

Conclusions and Future Works

5.1 Conslusions

In this thesis, a technique for designing 2-D digital filters having variable magnitude characteristics has been proposed. A second order Low-pass Butterworth filter is transformed to a Band-Pass and Band-Stop filters, through double generalized bilinear transformations, 2-D band-pass and band-stop digital filters are obtained. If one or more coefficients of the generalized bilinear transformations are changing, the resulting 2-D band-pass and band-stop filters have variable frequency responses. The effect of each coefficients are studied in detail for the 2-D band-pass andband-stop filters.

In chapter 2, 2-D variable recursive band-pass filter has been investigated. Starting from a 2^{nd} order 1-D Butterworth low-pass analog ladder network, the value of each element has been determined. The low-pass analog transfer function is transformed to a band-pass filter using the low-pass to band-pass transformation. Because of this transformation the 2^{nd} order low-pass filter will become a 4^{th} band-pass filter. In other words, the series inductor and parallel capictor in the low-pass network will get transformed to a series inductor and capacitor and parallel inductor and capacitor in the band-pass filter network. By assigning the series inductor and capacitor as the $s_1 - variable$ and the parellel inductor and capacitor as the $s_2 - variable$, the doubly terminated 2-D 4^{th} order band-pass analog network could be formed, and then the analog transfer function of the 2-D analog network has been obtained. Through the application of double generalized bilinear transformations to the 2-D analog transfer function, the 2-D digital transfer function has been dervied. With the coefficients of the double generalized bilinear transformation are in the stable ranges : $0 < k_i < \infty, 0 < a_i \leq 1, 0 < b_i \leq 1$ $(i = 1, 2)$, the band-pass digital filter with variable magnitude characteristics has been obtained.

The effect of $k_1, k_2, a_1, a_2, b_1, b_2, B$ and ω_o are studied. The coefficients k_1 controls the center frequencycenter frequency and the pass-band width of the resulting 2-D band-pass filter in $\omega_1 - axis$. when the value of k_1 is chosen near to its lower boundary, the filter center frequency will be closer to π radian and the pass-band width becomes narrower. On increasing the value of k_1 the center frequnecy of the filter will shift towards the origin and pass-band width also increase upto k_1 reaches unity, then on increase in k_1 after unity will make pass-band width to decrease gradually in $\omega_1 - axis$. The same phenomenon is observed for the coefficients k_2, which controla the center frequency and pass-band width in $\omega_2 - axis$. The pass-band gain and the center frequency is controlled by the coefficients a_1 and a_2 in the $\omega_1$and ω_2 axis respectively. The center frequency of the pass-band, the pass-band gain and the stop-band gain are controlled by the coefficients b_1 and b_2 in the $\omega_1$and ω_2 axis respectively. The effect of varying ω_o and B will make the center frequency of pass-band and the pass-band width varying proportionally. The combination of all this effect will be observed when the values of k_1, a_1, b_1 and ω_o are changed in the $\omega_1 - axis$. The same effect can be observed in the $\omega_2 - axis$ on varying the values of k_2, a_2, b_2 and ω_o.

2-D band-stop filter with variable magnitude characteristics has been studied in chapter 3. From the same analog 2^{nd} order 1-D Butterworth low-pass analog ladder network, the value of each electronic element has been determined. The low-pass analog transfer function is transformed to a band-stop filter using the low-pass to band-stop transformation. Because of this transformation the 2^{nd} order low-pass filter will become a 4^{th} order band-stop filter. In other words, the series inductor and parallel capictor in the low-pass network will get transformed to a parallel inductor and capacitor and series inductor and capacitor in the band-stop filter network. By assigning the parellel inductor and capacitor as the $s_1 - variable$ and the series inductor and capacitor as the $s_2 - variable$, the doubly terminated 2-D 4^{th} order band-stop analog network could be formed, and then the analog transfer function of the 2-D analog network has been obtained. Through the application of double generalized bilinear transformations to the 2-D analog transfer function, the 2-D digital transfer function has been dervied. With the coefficients of the double generalized bilinear transformation are in the stable ranges : $0 < k_i < \infty$, $0 < a_i \leq 1$, $0 < b_i \leq 1$ $(i = 1, 2)$, the band-stop digital filter with variable magnitude characteristics has been obtained.

The effect of $k_1, k_2, a_1, a_2, b_1, b_2, B$ and ω_o are studied. The coefficients k_1 controls the center frequency, lower pass-band and the upper pass-band width in the $\omega_1 - axis$. When the k_1 is near to the lower boundary the center frequency will be at π radian, on increasing the k_1 the center frequency moves towards origin and also varying the lower pass-band and upper pass-band width. The same pheonomenon is observed for the coefficients k_2 in the

$\omega_2 - axis$. The pass-band gain and the center frequency of the stopband filter and the gain of lower and upper pass-band are controlled by a_1 and a_2 in the ω_1 and ω_2 axis respectively. The center frequency of the stopband and the gain of the stopband are controlled by b_1 and b_2 in the ω_1 and ω_2 axis respectively. The effect of varying ω_o and B will make the center frequency of the stopband and the stop-band width varying proportionally. The combination of all this effect will be observed when the values of k_1, a_1, b_1 and ω_o are changed in the $\omega_1 - axis$.The same effect can be observed in the $\omega_2 - axis$ on varying the values of k_2, a_2, b_2 and ω_o.

Chapter 4 deals with the discussion of the application of band-pass and band-stop filter. Since, our thesis is mainly concerned with studying the variable magnitude characteristics of the band-pass and band-stop filters, proper attentions are not given to the implementations of the applications of these filters, although a application has been presented by using a band-pass filters and the results are discussed.

Hence, from chapter 2 and chapter 3 we can conclude that through the double generalized bilinear transformations, we can actually obtain 2-D band-pass and band-stop digital filters with the variable characteristics, which will introduce more changeable coefficients into our design. We thus have more freedom to design 2-D variable band-pass and band-stop digital filters, to meet the design specifications.

5.2 Directions for Future work

In this thesis, we have used the Butterworth 1-D 2^{nd} low-pass analog filters as our design starting point. Butterworth is frequently used analog prototype filter, which has the maximally flat magnitude. In future research, instead of using 1-D 2^{nd} order low-pass Butterworth filter one can use the 1-D $2^{nd} order$ chebyshev filter as a starting point and apply the transformation to band-pass or band-stop filters, digitize using bilinear transformation and can study the effect of these coefficients. In this thesis magnitude characteristics are discussed, for future research the same concept can be used to study phase response of 2-D recursive digital filters. 2-D digital filters has some kind of symmetry in its contour plot, for future research the same concept can be used to study symmetry in the contour plots.

Bibliography

[1] Andreas Antoniou, Digital filters : analysis and design 1990, McGraw-Hill series.

[2] David E. Johnson, Introduction to Filter Theory, 1976, Prentice -Hall, Inc.

[3] D.Childers and A.Durling, Digital Filtering and signal processing, 1975, West Publishing Company.

[4] Dan E. Dudgeon and Russell M.Mersereau, Multidimesional Digital Signal Processing, Prentice-hall signal processing series.

[5] B.A.Shenoi, Magnitude and Delay Approximation of 1-D and 2-D Digital Filters, Springer 1999

[6] Huang.T.S., "Stability of two-dimensional Recursive Filters ". IEEE Trans. Audio Electroacoustics, Vol AU-20, pp.158-163, 1972

[7] V.Cappellini, A.G. Constantinides and P. Emilliani Digital Filters and their applications 1978, Academic Press

[8] V.Ramachandran, C.S.Gargour, Ravi P. Ramachandran and F.Awad, "Variable magnitude characteristics of 1-D IIR discrete filters by a generalized bilinear transformation", IEEE Symp. on Circuits and Systems, Lansing MI, Aug 8-11,2000

[9] V.Ramachandran and C.S.gargour, “Generation of very strict hurwitz polynomials and applications to 2-D filter design”, Control and Dynamic systems, Vol. 69. pg 211-254,1995

[10] V.Ramachandran and C.S.gargour, “Generation of stable 2-D transfer functions having variable magnitude characteristics”, Control and Dynamic systems, Vol. 69,pg 255-297

[11] V.Ramachandran and M.Ahmadi, “Design of 2-D Stable Recursive Filters by Generation of VSHP Using Terminated n-port Gyrator Networks”, *Journal of the Franklin Institute,* Vol. 316, No. 5, 373-380, November 1983.

[12] M.A. Abiri, V.Ramachandran and M.Ahmadi, “An Alternative Approach in Generating a 2-variable Very Strict Hurwitz Polynomial (VSHP) and its Application”, Journal of the Franklin Institute, Vol. 324, No.2, pp. 187-203, 1987.

[13] V.Ramachandran and M.Ahmadi, “Design of 2-D Stable Analog and Recursive Digital Filters Using Properties of the Derivative of Even or Odd parts of Hurwitz Polynomials”, *Journal of the Franklin Institute,* Vol. 315, No. 4, 259-267, April 1983.

[14] M.Ahmadi, S.Golikeri, and V.Ramachandran, “A New Method for the Design of 2-Dimensional Stable Recursive Digital Filters Satisfying Prescribed Magnitude and Group Delay Response”, *Ascoustics,Speech,and signal processing*, IEEE Internation Conference on, ICASSP’83, Vol.8, pp.399-402, April 1983.

[15] J.S.Lim, “Two-dimensional signal and Image Processing”, Prentice Hall Inc.1990

[16] Alan V.oppenheim, Ronald W.schafer and John R.Buck, “Discrete-Time signal Processing” (Second Edition), Prentice-Hall Inc., 1998

[17] Brain T.O'Connor and Thomas S.Huang, "Stability of General Two-dimensional Recursive Digital filters", *IEEE Transactions on Acoustics,Speech, and Signal Processing,* ASSP-26,No.6,pp.550-560,Dec 1978

[18] John L.shanks, Sven Treital and James H.Justice, " Stability and Synthesis of Two-dimensional Recursive Filters," *IEEE Transactions on Audio and Electroacoustics,*AU-20,No.2,pp.115-128,June 1972

[19] Dennis Goodman, "An Alternative Proof of Huang's Theorem on Stability," *IEEE Transactions on Acoustics,Speech and Signal Processing,* ASSP-24, No.5, pp.426-427, Oct.1976

[20] T.B.Deng, "Design of Linear Phase Variable 2-D Digital Filters Using Real-Complex Decomposition", *IEEE Transaction on Circuits and Systems, Vol.45,* pp.330-339, March 1998

[21] James H.McClellan, "The Design of Two-Dimensional DIgital Filters by Transformations", *Proceeding of 7th Annual Princeton Conference on Information Sciences and Systems,* pp.247-251, 1973

[22] A.V.Oppenheim,W.F.G.Mechklenbrauker and R.M.Mersereau, "Variable cutoff linear phase digital filters", *IEEE Transactions on Circuits System,* vol.23, pp. 199-203, April 1976

[23] S.S.Abuja and S.C. Dutta Roy, "Variable-cutoff two-dimensional lowpass FIR digital filters," Electron.Letter, vol. 14, no. 14, pp.422-423, July 1978

[24] S.K.Mitra, Y.Neuvo, and H.Roivaninen, " Design of Recursive Digital Filters with Variable Characteristics", *Journal of Circuit Theory Applications,* Vol. 18, pp. 107-119, 1990

[25] C.S.Gargour, V.Ramachandran and Ravi P.Ramachandran, "Generation of a class of 2-D Transfer Function Yielding Variable Magnitude and Contour Characteristics", *ISCAS'2001,pp.797-800* Sydney, Australia, May 2001

[26] D.Goodman, "Some Difficulties with Double Bilinear Transformation in 2-D Digital Filter Design", *Proceeding of IEEE,* Vol. 65,pp.905-914,June 1978

[27] P.K.Rajan,H.C.Reddy,M.N.S.Swamy and V.Ramachandran, "Generation of Two-Dimensional Digital Functions Without Nonessential Singularties of the Second kind", *IEEE Transactions on Acoustics,Speech, and Signal Processing,* Vol. ASSP-28,No.2,pp.216-223, April 1980

[28] C.S.Gargour,V.Ramachandran,Ravi P.Ramachandran, "Modificationof filter responses by the generalized bilinear transformations and the inverse bilinear transformations", pp. 2043-2046, *Electrical and Computer Engineering,2003. IEEE CCECE 2003,Canadian Conference on,* Volume:3, May4-7,2003

[29] Thomas S.Huang, "Two-Dimensional Windows",*IEEE Transactions on Audio and Electroacoustics,*AU-20,No.1,pp.260-269,May 1978

[30] M.E.Valkenburg, "Analog Filter Design", Holt, Rinehart and Winston,1982

[31] M.N.S.Swamy and Harnath C.Reddy, "Two-variable Analog Ladders With Applications", in Multidimensional Systems: Techniques and Applications (Edited by S.G.Tzafestas) New york:Marcel Dekkar,1986,Chapter 6

[32] J.Little and C.Moler, "MATLAB", Sherborn,MA: The MathWorks

[33] R.King, M.Ahmadi,R. Gorgui-naguib,Alan Kwabwe and Mahmood Azimi-sadjadi "Digital Filtering in One and Two dimensions"Plenum Press, New york, 1989

Appendix

Programs

```
%Matlab program for determining whether the band-pass filter is stable for
    % given coefficients
    function val= BPstablerange(k1,k2,a1,a2,b1,b2,wo,B)
    % Setting values of Center frequency (wo) and Bandwifth
    B=1;
    wo=1;
    % Assigning the values to alpha's and beta's
    alpha1=k1+a1*k1;alpha2=k1-k1*a1;alpha3=1+b1;alpha4=1-b1;
    beta1=k2+a2*k2;beta2=k2-k2*a2;beta3=1+b2;beta4=1-b2;
    % Assigning the coefficients of the denominator's to c's
    c1=4*wo^4;c2=2*1.414*B*wo^4;c3=2*1.414*B*wo^4;c4=B*B+2*B*B*wo^4;
    c5=4*wo^4;c6=4*wo^4;c7=2*1.414*B*wo^6;c8=2*1.414*B*wo^6;c9=4*wo^8;
    % Checking the stabitiy conditions for the given values of the coefficients
    q1=[c1*alpha1^2*beta1^2+c2*alpha1*alpha4*beta1^2+c3*alpha1^2*beta1*beta4+...
    c4*alpha1*alpha4*beta1*beta4+c5*alpha1^2*beta4^2+c6*alpha4^2*beta1^2+...
```

```
c7*alpha1*alpha4*beta4^2+c8*alpha4^2*beta1*beta4+c9*alpha4^2*beta4^2];
q2=[c1*2*alpha1*alpha2*beta1^2+c2*(alpha2*alpha4+alpha1*alpha3)*beta1^2+...
c3*2*alpha1*alpha2*beta1*beta4+c4*(alpha2*alpha4+alpha1*alpha3)*beta1*beta4+...
c5*2*alpha1*alpha2*beta4^2+c6*2*alpha3*alpha4*beta1^2+...
c7*beta4^2*(alpha2*alpha4+alpha1*alpha3)+c8*2*alpha3*alpha4*beta1*beta4+...
c9*2*alpha3*alpha4*beta4^2];
q3=[c1*2*beta1*beta2*alpha1^2+c2*2*alpha1*alpha4*beta1*beta2+...
c3*alpha1^2*(beta2*beta4+beta1*beta3)+c4*alpha1*alpha4*(beta2*beta4+beta1+beta3)+...
c5*2*alpha1^2*beta3*beta4+c6*2*alpha4^2*beta1*beta2+c7*2*alpha1*alpha4*beta3*beta4+...
c8*alpha4^2*(beta2*beta4+beta1*beta3)+c9*2*alpha4^2*beta3*beta4];
q4=[c1*4*alpha1*alpha2*beta1*beta2+c2*beta1*beta2*(alpha2*alpha4+alpha1*alpha3)+...
c3*2*alpha1*alpha2*(beta2*beta4+beta1*beta3)+c4*(alpha2*alpha4+alpha1*alpha3)*...
(beta2*beta4+beta1*beta3)+c5*4*alpha1*alpha2*beta3*beta4+c6*4*beta1*beta2*alpha3*alpha4
c7*2*beta3*beta4*(alpha2*alpha4+alpha1*alpha3)+c8*2*alpha3*alpha4*(beta2*beta4+beta1*be
c9*4*alpha3*alpha4*beta3*beta4];
q5=[c1*alpha1^2*beta2^2+c2*alpha1*alpha4*beta2^2+c3*alpha1^2*beta2*beta3+...
c4*alpha1*alpha4*beta2*beta3+c5*alpha1^2*beta3^2+c6*alpha4^2*beta2^2+...
c7*alpha1*alpha4*beta3^2+c8*alpha4^2*beta2*beta3+c9*alpha4^2*beta3^2];
q6=[c1*alpha2^2*beta1^2+alpha2*alpha3*beta1^2+alpha2^2*beta1*beta4+...
alpha2*alpha3*beta1*beta4+alpha2^2*beta4^2+alpha3^2*beta1^2+beta4^2*alpha2*alpha3+...
beta1*beta4*alpha3^2+alpha3^2*beta4^2];
q7=[c1*2*alpha1*alpha2*beta2^2+c2*beta2^2*(alpha2*alpha4+alpha1*alpha3)+...
c3*2*alpha1*alpha2*beta2*beta3+c4*(alpha2*alpha4+alpha1*alpha3)*beta2*beta3+...
2*c5*alpha1*alpha2*beta3^2+c6*2*alpha3*alpha4*beta2^2+...
```

```
c7*beta3^2*(alpha2*alpha4+alpha1*alpha3)+2*c8*alpha3*alpha4*beta2*beta3+...
2*c9*alpha3*alpha4*beta3^2];
q8=[c1*2*alpha2^2*beta1*beta2+2*c2*beta1*beta2*alpha2*alpha3+...
c3*alpha2^2*(beta2*beta4+beta1*beta3)+c4*alpha2*alpha3*(beta2*beta4+beta1*beta3)+...
c5*2*alpha2^2*beta3*beta4+c6*2*alpha3^2*beta1*beta2+c7*2*beta3*beta4*alpha2*alpha3+...
c8*alpha3^2*(beta2*beta4+beta1*beta3)+2*c9*alpha3^2*beta3*beta4];
q9=[c1*alpha2^2*beta2^2+c2*alpha2*alpha3*beta2^2+c3*alpha2^2*beta2*beta3+...
c4*alpha2*alpha3*beta2*beta3+c5*alpha2^2*beta3^2+c6*alpha3^2*beta2^2+...
c7*alpha2*alpha3*beta3^2+c8*alpha3^2*beta2*beta3+c9*alpha3^2*beta3^2];
% Checking the coefficients of the denominator's are real or not
% If it is real 1 will be returned otherwise 0 will be returned
if (q1>0 & q2>0 & q3>0 & q4>0 & q5>0 & q6>0 & q7>0 & q8>0 & q9>0)
   val = 1;
else
   val=0;
end
```

```
% MATLAB code for displaying frequency response for the given values of the
% coefficients d2band.m
clear;clc;
% Assigning the values of center frequency and Bandwidth
wo=1;
B=1;
% Enter and checking for stability of the coefficients
k1=input('Enter the k1 coefficient value=');
if (k1<=0 | k1 >100)
   error('k1 range is between 1 to 100');
end

k2=input('Enter the k2 coefficient value=');
if (k2<=0 | k2 >100)
   error('k1 range is between 1 to 100');
end

a1=input('Enter the a1 coefficient value=');
if (a1<=0 | a1 >1)
   error('selection of a1 will make the filter unstable');
end

b1=input('Enter the b1 coefficient value=');
if (b1 <=0 | b1 >1)
```

```
   error('selection of b1 will make the filter unstable');
end

a2=input('Enter the a2 coefficient value=');
if (a2 <=0 | a2 >1)
   error('selection of a2 will make the filter unstable');
end

b2=input('Enter the b2 coefficient value=');
if (b2 <=0 | b2 >1)
   error('selection of b2 will make the filter unstable');
end
%calling BPstable to check the stability
val=BPstable(k1,k2,a1,a2,b1,b2,wo,B);
% If val is one then the filter is stable
if val==1
   display('Stable Band-Pass filter can be obtained');
% Setting w1 and w2 axis
w1=0:pi/50:pi;
w2=-pi:pi/50:pi;
z1=exp(-j*w1);
z2=exp(-j*w2);
[Z1,Z2]=meshgrid(z1,z2);
% Assigning the values to the filter parameters
```

```
R2=1;
R1=1;
L1=1.414214/B;
L2=B/(1.414214*wo*wo);
C1=B/(1.414214*wo*wo);
C2=1.414214/B;
% Applying Bilinear Transformation
S1=k1.*(Z1-a1)./(Z1+b1);
S2=k2.*(Z2-a2)./(Z2+b2);
% Transfer function of Band-Pass filter
H=(S1.*S2*R2*L2*C1)./(((S1.*S1*L1*C1+R1*S1.*C1+1).*(S2.*S2*R2*L2*C2+S2.*L2+R2))+...
R2*L2*C1.*S1.*S2);
figure;
% Plotting the magnitude and contour plots of the Band-Pass filters
subplot(211);
mesh(w1/pi,w2/pi,abs(H));
axis square;
xlabel('\omega_1(\pi)');
ylabel('\omega_2(\pi)');
zlabel('Magnitude');
title(['\omega_o=',num2str(wo), ',','B=',num2str(B),',','k_1=',num2str(k1),',','k_2=',num2str(k2)...
,',','a_1=',num2str(a1),',','a_2=',num2str(a2),',','b_1=',num2str(b1),',','b_2=',num2str(b2)]);
subplot(212);
[c1,c2]=contour(w1/pi,w2/pi,abs(H));
```

```
axis square;
title(['\omega_o=',num2str(wo), ',','B=',num2str(B),',','k_1=',num2str(k1),',','k_2=',num2str(k2)...
,',','a_1=',num2str(a1),',','a_2=',num2str(a2),',','b_1=',num2str(b1),',','b_2=',num2str(b2)]);
xlabel('\omega_1(\pi)');
ylabel('\omega_2(\pi)');
else
   error('Coefficients used will not provide an stable band-pass filter');
end
```

```
%Matlab Code for band-stop stable
function val= BSstable(k1,k2,a1,a2,b1,b2,wo,B)
% Setting values of Center frequency (wo) and Bandwidth
B=1;
wo=1;
% Assigning the values to alpha's and beta's
alpha1=k1+a1*k1;alpha2=k1-k1*a1;alpha3=1+b1;alpha4=1-b1;
beta1=k2+a2*k2;beta2=k2-k2*a2;beta3=1+b2;beta4=1-b2;
% Assigning the coefficients of the denominator's to c's
c1=2;c2=1.414*B;c3=1.414*B;c4=2*B*B;c5=2*wo^2;c6=2*wo^2;c7=1.414*B*wo^2;
c8=1.414*B*wo^2;c9=2*wo^4;
% Checking the stabitiy conditions for the given values of the coefficients
q1=[c1*alpha1^2*beta1^2+c2*alpha1*alpha4*beta1^2+c3*alpha1^2*beta1*beta4+...
c4*alpha1*alpha4*beta1*beta4+c5*alpha1^2*beta4^2+c6*alpha4^2*beta1^2+...
c7*alpha1*alpha4*beta4^2+c8*alpha4^2*beta1*beta4+c9*alpha4^2*beta4^2];
q2=[c1*2*alpha1*alpha2*beta1^2+c2*(alpha2*alpha4+alpha1*alpha3)*beta1^2+...
c3*2*alpha1*alpha2*beta1*beta4+c4*(alpha2*alpha4+alpha1*alpha3)*beta1*beta4+...
c5*2*alpha1*alpha2*beta4^2+c6*2*alpha3*alpha4*beta1^2+...
c7*beta4^2*(alpha2*alpha4+alpha1*alpha3)+c8*2*alpha3*alpha4*beta1*beta4...
+c9*2*alpha3*alpha4*beta4^2];
q3=[c1*2*beta1*beta2*alpha1^2+c2*2*alpha1*alpha4*beta1*beta2+...
c3*alpha1^2*(beta2*beta4+beta1*beta3)+c4*alpha1*alpha4*(beta2*beta4+beta1+beta3)+...
c5*2*alpha1^2*beta3*beta4+c6*2*alpha4^2*beta1*beta2+c7*2*alpha1*alpha4*beta3*beta4+...
c8*alpha4^2*(beta2*beta4+beta1*beta3)+c9*2*alpha4^2*beta3*beta4];
```

```
q4=[c1*4*alpha1*alpha2*beta1*beta2+c2*beta1*beta2*(alpha2*alpha4+alpha1*alpha3)+...
c3*2*alpha1*alpha2*(beta2*beta4+beta1*beta3)+c4*(alpha2*alpha4+alpha1*alpha3)*...
(beta2*beta4+beta1*beta3)+c5*4*alpha1*alpha2*beta3*beta4+c6*4*beta1*beta2*alpha3*alpha4
c7*2*beta3*beta4*(alpha2*alpha4+alpha1*alpha3)+c8*2*alpha3*alpha4*(beta2*beta4+beta1*be
c9*4*alpha3*alpha4*beta3*beta4];
q5=[c1*alpha1^2*beta2^2+c2*alpha1*alpha4*beta2^2+c3*alpha1^2*beta2*beta3+...
c4*alpha1*alpha4*beta2*beta3+c5*alpha1^2*beta3^2+c6*alpha4^2*beta2^2+...
c7*alpha1*alpha4*beta3^2+c8*alpha4^2*beta2*beta3+c9*alpha4^2*beta3^2];
q6=[c1*alpha2^2*beta1^2+alpha2*alpha3*beta1^2+alpha2^2*beta1*beta4+...
alpha2*alpha3*beta1*beta4+alpha2^2*beta4^2+alpha3^2*beta1^2+...
beta4^2*alpha2*alpha3+beta1*beta4*alpha3^2+alpha3^2*beta4^2];
q7=[c1*2*alpha1*alpha2*beta2^2+c2*beta2^2*(alpha2*alpha4+alpha1*alpha3)+...
c3*2*alpha1*alpha2*beta2*beta3+c4*(alpha2*alpha4+alpha1*alpha3)*beta2*beta3+...
2*c5*alpha1*alpha2*beta3^2+c6*2*alpha3*alpha4*beta2^2+c7*beta3^2*(alpha2*alpha4+...
alpha1*alpha3)+2*c8*alpha3*alpha4*beta2*beta3+2*c9*alpha3*alpha4*beta3^2];
q8=[c1*2*alpha2^2*beta1*beta2+2*c2*beta1*beta2*alpha2*alpha3+...
c3*alpha2^2*(beta2*beta4+beta1*beta3)+c4*alpha2*alpha3*(beta2*beta4+beta1*beta3)...
+c5*2*alpha2^2*beta3*beta4+c6*2*alpha3^2*beta1*beta2+c7*2*beta3*beta4*alpha2*alpha3...
+c8*alpha3^2*(beta2*beta4+beta1*beta3)+2*c9*alpha3^2*beta3*beta4];
q9=[c1*alpha2^2*beta2^2+c2*alpha2*alpha3*beta2^2+c3*alpha2^2*beta2*beta3+...
c4*alpha2*alpha3*beta2*beta3+c5*alpha2^2*beta3^2+c6*alpha3^2*beta2^2+...
c7*alpha2*alpha3*beta3^2+c8*alpha3^2*beta2*beta3+c9*alpha3^2*beta3^2];
% Checking the coefficients of the denominator's are real or not
% If it is real 1 will be returned otherwise 0 will be returned
```

```
if (q1>0 & q2>0 & q3>0 & q4>0 & q5>0 & q6>0 & q7>0 & q8>0 & q9>0)
   val = 1;
else
   val=0;
end
```

```
% MATLAB code for displaying frequency response for the given values of the
% coefficients
clear;clc;
% Assigning the values of center frequency and Bandwidth
wo=1;
B= 1;
% Enter and checking for stability of the coefficients
k1=input('Enter the k1 coefficient value=');
if (k1 <0 | k1 >100)
   error('k1 range is between 1 to 100');
end
k2=input('Enter the k2 coefficient value=');
if (k2 <0 | k2 >100)
   error('k1 range is between 1 to 100');
end
a1=input('Enter the a1 coefficient value=');
if (a1 <0 | a1 >1)
   error('selection of a1 will make the filter unstable');
end
b1=input('Enter the b1 coefficient value=');
if (b1 <0 | b1 >1)
   error('selection of b1 will make the filter unstable');
end
a2=input('Enter the a2 coefficient value=');
```

```
if (a2 <0 | a2 >1)
   error('selection of a2 will make the filter unstable');
end
b2=input('Enter the b2 coefficient value=');
if (b2 <0 | b2 >1)
   error('selection of b2 will make the filter unstable');
end
% Enter and checking for stability of the coefficients
val=BSstable(k1,k2,a1,a2,b1,b2,wo,B);
% If val is one then the filter is stable
if val==1
   display('Stable Band-Pass filter can be obtained');
% Setting w1 and w2 axis
w1=0:pi/50:pi;
w2=0:pi/50:pi;
z1=exp(-j*w1);
z2=exp(-j*w2);
[Z1,Z2]=meshgrid(z1,z2);
% Assigning the values to the filter parameters
R2=1;
R1=1;
L1=(1.414214*B)./(wo*wo);
L2=1./(1.414214*B);
C1=1./(1.414214*B);
```

```
C2=(1.414214*B)./(wo*wo);
% Applying Bilinear Transformation
S1=(k1.*(Z1-a1))./(Z1+b1);
S2=(k2.*(Z2-a2))./(Z2+b2);
% Transfer function of Band-Pass filter
H=((R2)+(R2*L2*C2.*S2.*S2)+(R2*L1*C1.*S1.*S1)+(R2*L1*C1*L2*C2.*S1.*S1.*S2.*S2))...
./((R1+L1.*S1+R1*L1*C1.*S1.*S1+R1*R2*C2.*S2+R2*L1*C2.*S1.*S2+R1*R2*L1*C1*...
C2.*S1.*S1.*S2+R1*L2*C2.*S2.*S2+L1*L2*C2.*S1.*S2.*S2+R1*L1*L2*C1*C2.*S1.*S1.*S2.*S2)
+(R2)+(R2*L2*C2.*S2.*S2)+(R2*L1*C1.*S1.*S1)+(R2*L1*C1*L2*C2.*S1.*S1.*S2.*S2));
figure;
% Plotting the magnitude and contour plots of the Band-Pass filters
subplot(211);
mesh(w1/pi,w2/pi,abs(H));
axis square;
title(['\omega_o=',num2str(wo), ',','B=',num2str(B),',','k_1=',num2str(k1),',','k_2=',num2str(k2)...
,',','a_1=',num2str(a1),',','a_2=',num2str(a2),',','b_1=',num2str(b1),',','b_2=',num2str(b2)]);
xlabel('\omega_1(\pi)');
ylabel('\omega_2(\pi)');
zlabel('Magnitude');
subplot(212);
[c1,c2]=contour(w1/pi,w2/pi,abs(H));
axis square;
title(['\omega_o=',num2str(wo), ',','B=',num2str(B),',','k_1=',num2str(k1),',','k_2=',num2str(k2)...
,',','a_1=',num2str(a1),',','a_2=',num2str(a2),',','b_1=',num2str(b1),',','b_2=',num2str(b2)]);
```

```
xlabel('\omega_1(\pi)');
ylabel('\omega_2(\pi)');
else
    error('Coefficients used will not provide an stable band-pass filter');
end
```

```
% Matlab code for image processing in smoothing and noise reduction
% Three images are used lena,tree an house for this program
clear;
clc;
% Intializing all the parameters
B=1;
L=1.414;C=1.414;
con=1.414;
wo=1;
%Checking the coefficients values to maintain stability conditions
k1=input('Enter the k1 coefficient value=');
if (k1 >0)
   error('k1 should be in positive');
end
k2=input('Enter the k2 coefficient value=');
if (k2 >0)
   error('k2 should be in positive');
end
a1=input('Enter the a1 coefficient value=');
if (a1<=0 | a1 >1)
   error('selection of a1 will make the filter unstable');
end

b1=input('Enter the b1 coefficient value=');
```

```
if (b1 <=0 | b1 >1)
   error('selection of b1 will make the filter unstable');
end

a2=input('Enter the a2 coefficient value=');
if (a2 <=0 | a2 >1)
   error('selection of a2 will make the filter unstable');
end

b2=input('Enter the b2 coefficient value=');
if (b2 <=0 | b2 >1)
   error('selection of b2 will make the filter unstable');
end
% noise level
noise=30;
% reading of the image
x=double(imread('lena512.bmp'));
% Adding noise to the image
x_rand=noise*randn(size(x));
y=double(uint8(x + x_rand));
% Transfer function of the band-pass filter
for i=1:5
   for j=1:5
      z1=(k1.*(i*(1+a1)+(1-a1)))./(i*(1-b1)+(1+b1));
```

```
        z2=(k2.*(j*(1+a2)+(1-a2)))./(j*(1-b2)+(1+b2));
        a11=B^2*z1*z2;
        a2=L*z1.*z1+z1*B+con*wo^2;
        a3=C*z2.*z2+z2*B+con*wo^2;
        h(i,j)=a11./((a2.*a3)+a11);
    end
end
% convoluting the transfer function with the image signal
f1=conv2(x,h,'same');
% Convoluting the transfer function with the noise image signal
rf1=conv2(y,h,'same');
% Outputing the original, noise , smooth and noise reduced images
figure;
subplot(2,2,1);imagesc(x);title('Lena');
subplot(2,2,2);imagesc(y);title('Noisy Lena');
subplot(2,2,3);imagesc(f1);title('Smooth');
subplot(2,2,4);imagesc(rf1);title('Noise Reduced Lena Image ');
colormap(gray);
%Calculating the PSNR values
psnr_bp=20*log10(255^2.*prod(size(x))./sumsqr(f1-x));
est_psnr_bp=20*log10(255^2*prod(size(y))/sumsqr(y-rf1));
```