An Empirical Comparison of Alternative Stochastic Volatility Option Pricing Models: Canadian Evidence

A Thesis
In
The John Molson School
Of
Business

Tiezhu Gao

Degree of Master of Science in Administration at
Concordia University
Montreal, Quebec, Canada

Your file Votre référence
ISBN: 0-494-14367-3
Our file Notre référence
ISBN: 0-494-14367-3

ABSTRACT

An Empirical Comparison of Alternative Stochastic Volatility Option Pricing Models: Canadian Evidence

Tiezhu Gao

In this thesis, I empirically compare the pricing performance of three classes of stochastic volatility option pricing models and the traditional Black-Scholes (1973) model in the pricing of S&P Canada 60 Index Options. The stochastic volatility models that I study are as follows: 1) the ad hoc Black and Scholes (1973) procedure that fits the implied volatility surface, 2) Madan et al.'s (1998) variance gamma model, and 3) Heston's (1993) continuous-time stochastic volatility model. I find that Heston's continuous-time stochastic volatility model outperforms the other models in terms of in-sample pricing and out-of-sample pricing. Second, the addition of the stochastic volatility term to the stochastic volatility model and variance gamma model does not resolve the "volatility smiles" effects, but it reduces the effects. Third, the Black-Scholes model performs adequately in pricing options, with the advantage of simplicity, although it suffers from the shortcoming of the "volatility smiles" effect. Finally, although it includes more parameters, the ad hoc Black and Scholes model does not perform as well as expected.

ACKNOWLEDGEMENT

I would like to thank Dr. Latha Shanker for her supervision and expertise on derivatives throughout the research. Special thanks are dedicated to Dr. Ian Rakita and Dr. Sergei Isaenko for their valuable comments.

To my family I dedicate my thesis. Without their support, I would not be able to finish this work.

And last, I would like to express my gratitude to the John Molson School of Business for providing me with the research grant to help facilitate this study.

TABLE OF CONTENTS

I. Introduction

Since Black and Scholes (1973) published their seminal article on option pricing, numerous empirical studies have found that this famous model results in systematic biases across moneyness and maturity. It is well known that after the October 1987 crash, the implied volatility computed from options on the stock index in the US market inferred from the Black-Scholes model (BS) appears to be different across exercise prices. This is the so-called "volatility smile". Given the Black-Scholes model's assumptions, all option prices on the same underlying security with the same expiration date but with different exercise prices should have the same implied volatility. However, the "volatility smile" pattern suggests that the Black-Scholes model tends to misprice deep in-the-money (ITM) and deep out-of-money (OTM) options.

In the last two decades, option pricing has experienced an explosion of new models that each relaxes some of the restrictive Black-Scholes assumptions. Hull and White (1987), Johnson and Shanno (1987), Scott (1987), Wiggins (1987), Melino and Turnbull (1990, 1995), Stein and Stein (1991) and Heston (1993) suggest a continuous-time stochastic volatility model. Merton (1976), Bates (1991) and Naik and Lee (1990) propose a jump-diffusion model. Duan (1995) and Heston and Nandi (2000) develop an option pricing model based on the GARCH process. Recently, Madan et al. (1998) use a three-parameter stochastic process, termed the variance gamma process, as an alternative model for the dynamics of log stock prices.

Most of the previous studies compared a range of stochastic volatility models using US data. For example, Bakshi et al. (1997) evaluated the performance of alternative models for the S&P 500 index option contracts. They examined how much additional parameters improve pricing performance. They showed that the stochastic volatility term provides a great improvement over Black-Scholes model (henceforth BS).

In this thesis, I use Canadian data to compare alternative stochastic volatility models to gauge their relative pricing performance. My thesis offers further evidence on whether the addition of the stochastic volatility term can improve the performance of option pricing models.

The first class of stochastic volatility models is the ad hoc Black and Scholes procedure (henceforth AHBS) proposed by Dumas et al. (1998). Assuming that option prices are given for all strikes and for all maturities, AHBS fits a volatility function for the underlying asset price process to the prices of option contracts. Once the volatility function is determined, it can be used to price other derivative assets.

The second class of stochastic volatility models is the variance gamma option pricing model (henceforth VG). The variance gamma process derived by Madan and Milne (1991) is aimed at providing a model for a log-return distribution that offers a physical interpretation and incorporates both long-tailness and skewness characteristics in a log-return distribution.

The third class of stochastic volatility models is the continuous-time stochastic volatility model (henceforth SV) of Heston (1993) which models the square of the volatility process with mean-reverting dynamics, allowing for changes in the underlying asset price to be contemporaneously correlated with changes in the volatility process.

In my opinion, the significance of this thesis lies in two aspects. First, I examine whether it is possible to improve upon the Black-Scholes model by allowing stochastic volatility terms in pricing S&P Canada 60 Index Options. Second, previous studies have examined the performance of stochastic volatility option pricing models in major markets such as S&P500 and FTSE 100. In this thesis I investigate if the pricing performance of stochastic volatility models in the Canadian market is consistent with other major markets.

The rest of the thesis is arranged as follows. In section 2, alternative stochastic volatility option pricing models are reviewed. In section 3, the data used for this analysis are described. The methodology, including estimation methods and models comparison methods, is described in section 4. In section 5, I outline some empirical findings to evaluate the pricing performance of alternative models. Section 6 summarizes the results and reviews the conclusions.

II. Models

According to option pricing theory, European options are priced by evaluating the expectation of the discounted terminal payoff of the option at maturity under an equivalent risk neutral measure Q. Hence the price of a European call with a strike price of K and maturity t is given by

$$C(t,\tau;K) = e^{-r\tau} E_t^Q[\max(S_{t+\tau} - K, 0)] \qquad (1)$$

where $E_t^Q[.]$ stands for the conditional expectation under the risk-neutral density.

Bakshi and Madan(2000) show that the above equation can be decomposed into two components as

$$C = SP_1 - Ke^{-r\tau} P_2 \qquad (2)$$

where $P_1 = E_t^Q[\frac{S_{t+\tau}}{S_t} 1_{[S_{t+\tau}>K]}]$ and $P_2 = E_t^Q[1_{[S_{t+\tau}>K]}]$

and the indication function $1_{[S_{t+\tau}>K]}$ is unity when $S_{t+\tau} > K$.

The price of a European put can be determined from the put-call parity relationship.

In the later part of this section, I only present the probability P_1 and P_2 of each model.

2.1 Ad hoc Black and Scholes (AHBS) Model

Because VG and SV have more parameters than BS, they may have the capability to price options with greater precision. Therefore, I follow Dumas et al. (1998) and construct the AHBS in which each option has its own implied volatility depending on a strike price and time to maturity. However, I consider only the function of the strike price because the liquidity of the S&P/TSX 60 index options market is concentrated in the nearest expiration contract. Even if there are options with multiple maturities in a specific day, only the function of the strike price is applied.

Specifically, the following specification is adopted for the BS implied volatilities:

$$\alpha_n = \beta_1 + \beta_2 (S / K_n) + \beta_3 (S / K_n)^2 \qquad (3)$$

where α_n is the implied volatility for an nth option of strike K_n and spot price S.

A four-step procedure is conducted to obtain the model's option prices. First, the BS implied volatility is extracted from each option price. Second, the β_i (i =1, 2, 3) are estimated by ordinary least squares. Third, using estimated parameters from the second step, each option's moneyness is plugged into the equation to obtain the model-implied volatility for each option. Finally, the volatility estimates computed in the third step are used to price options with the BS formula.

AHBS, although theoretically inconsistent, can be a more challenging benchmark than the simple BS for any competing option valuation model.

2.2 Variance Gamma Model

The variance gamma process is obtained by evaluating Brownian motion with drift at a random time given by a gamma process. Let

$$b(t;\theta,\sigma)=\theta t+\sigma W(t) \qquad (4)$$

where W(t) is a standard Brownian motion. The process b(t; ?, s) is a Brownian motion with drift ? and volatility s. The gamma process ?(t; μ, ?) with mean rate μ and variance rate ? is the process of independent gamma increments over non-overlapping intervals. The VG process, X(t; s, ?, ?), is defined in terms of Brownian motion with drift b(t; ?, s) and the gamma process with unit mean rate, c(t; 1, ?) as X(t; s, ?, ?)=b(?(t; 1, ?), ? ,s).

Thus, the assumed process of the underlying asset, S_t, is given by replacing the role of Brownian motion in the original Black–Scholes geometric Brownian motion model by the variance gamma process as follows:

$$S_t=S_0\exp[mt+X(t;\sigma,\upsilon,\theta)+wt] \qquad (5)$$

where S_0 is the initial stock price, m is the mean rate of stock return, and

$w=(1/v)\ln(1-\theta v-\sigma^2 v/2)$.

Based on the above process, Madan et al. (1998) derive risk neutral probabilities for the price of a European option as follows:

$$P_1 = \varphi[d\sqrt{\tfrac{1-c_1}{v}}, (\alpha+\sigma)\sqrt{\tfrac{v}{1-c_1}}, \tfrac{\tau}{v}] \qquad (6)$$

$$P_2 = \varphi[d\sqrt{\tfrac{1-c_2}{v}}, \alpha\sqrt{\tfrac{v}{1-c_2}}, \tfrac{\tau}{v}] \qquad (7)$$

2.3 Stochastic Volatility Model

Heston (1993) provided a closed-form solution for pricing a European style option when volatility follows a mean-reverting square-root process. The actual diffusion processes for the underlying asset and its volatility are specified as

$$dS = \mu S dt + \sqrt{v_t} S dW_S \qquad (8)$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}\, dW_v \qquad (9)$$

where dW_S and dW_V have an arbitrary correlation ?, v_t is the instantaneous variance. κ is the speed of adjustment to the long-run mean θ, and σ is the variation coefficient of variance.

Given the dynamics in equation 8 and 9 above, Heston (1993) shows that risk neutral probabilities for pricing a European call option with t periods to maturity is given by

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-i\phi \ln(K)} f_j\left(t, \tau, S(t), R(t), V(t); \phi\right)}{i\phi}\right] d\phi \text{ (j=1,2)} \qquad (10)$$

where Re[.] denotes the real part of complex variables, i is the imaginary number, $\sqrt{-1}$, $f_j(x,v_t,\tau;\phi)=\exp[C(\tau;\phi)+D(\tau;\phi)v_t+i\phi x]$ and $C(\tau;\phi)$ and $D(\tau;\phi)$ are functions of $\theta,\kappa,\rho,\sigma,v_t$.

The risk neutral probability distribution function is addressed in Appendix A.

III. Data

3.1 Data Source

The data I adopt to compare the alternative classes of options models are the S&P/TSX 60 index options traded on the Montreal Exchange, where three consecutive near-term delivery months and one additional month from the quarterly cycle (March, June, September and December) make up four contract months. The expiration day is the second Thursday of each contract month. Each option contract month has at least five strike prices. The number of strike prices may, however, increase according to the price movement. Trading in the S&P/TSX 60 index options is fully automated. The exercise style of the S&P/TSX 60 index options is European and thus contracts can be exercised only on the expiration dates. Hence my test results are not affected by the complication that arises due to the early exercise feature of American options. Moreover, it is important to note that liquidity is concentrated in the nearest expiration contract.

The sample period extends from January 3, 2004 through December 31, 2005. Price data on SXO came from the transaction records provided by the Montreal Exchange (www.m-x.ca). Intradaily data were used to avoid imperfect synchronization of closing prices with the underlying index price. Midpoints of bid and ask quotes were used. The analysis used the last quote for a particular option in terms of exercise price, maturity, and type between 2:30 p.m. and 3:00 p.m. every trading day in the sample period. Contemporaneous index levels are obtained from the website of Finance.yahoo.com. The 3-month Treasure bill rate, the source of which is www.bankofcanada.ca, proxies as the risk-free rate.

3.2 Data Filtering

I apply the following rules to filter data needed for the test.

1). For each day in the sample, only the last reported transaction price, which has to occur between 2:30 p.m. and 3:00 p.m., of each option contract is employed in the empirical test.

2) An option of a particular moneyness and maturity is represented only once in the sample. This means that although the same option may be quoted again during the time window, only the last record of that option is included in the sample.

3) Prices lower than 0.6 are deleted to decrease the impact of price discreteness on option valuation.

4) Similarly, only options with number of days to expiration between 6 and 90 are included. Very short term options have substantial time decay that could interfere with the ability to isolate the volatility parameters. Very long term options are not included because they are not actively traded.

5) Prices not satisfying the following arbitrage restriction are excluded:

$$C_{t,r} \geq S_t - \sum_{s-1}^{r} e^{-r_{t,r}S} D_{t+S} - KB_{t,r} \qquad (11)$$

$$P_{t,r} \geq KB_{t,r} - S_t + \sum_{s-1}^{r} e^{-r_{t,r}S} D_{t+S} \qquad (12)$$

{Insert Table B1 Here}

{Insert Table B2 Here}

I divide the option data into several categories according to moneyness, S/K. Table B1 describes certain sample properties of the S&P/TSX 60 index option prices used in the study. Summary statistics are reported for the option price and the total number of observations, according to each moneyness-option type category. Because the liquidity of the S&P/TSX 60 index option contracts is concentrated in the nearest expiration contract, I do not observe the maturity category separately. The data pool includes 3244 call and 3511 put option observations, with out of the money (OTM) options comprising 30% of call option observations and 32% of put option observations.

IV. Methodology

I follow the estimation method used in previous research, (e.g., Bakshi et al. (1997), Bates (1991, 1996a, c), Dumas et al. (1995), Longstaff (1995), Madan and Chang (1996), and Nandi(1996), Bakshi et al. (2000), Kirgiz, (2001)) and estimate parameters of each model for each sample day. In applying option pricing models, one difficulty is that the spot volatility and the structural parameters are unobservable. Although in theory, econometric tools (such as maximum likelihood or the generalized methods of moments) can be applied to obtain the required estimates, such an estimation is difficult, because of the requirement of a long time series of historical data. To avoid this difficulty, previous scholars have tried to use option-implied volatility based on the model. Not only has this practice reduced data requirements greatly, but it has also led to significant performance improvement (e.g. Bates (1996 a,b,c), Bodurtha and Courtadon (1987), and Melino and Turnbull (1990, 1995)). Following this tradition, I adapt the following steps to each of the alternative stochastic models.

The first step is to collect N option prices on the same stock, taken from the same day. For each n=1,…,N, let τ_n and K_n be respectively the time-to-expiration and the strike price of the n-th option; let $O_i\,(t,\tau_n;K_n)$ be its observed price, and $O_i^*(t,\tau_n;K_n)$ its model price as determined by, for example, formula (2) using observed spot prices and risk free rates. The difference between $O_i\,(t,\tau_n;K_n)$ and

$O_i^*(t,\tau_n;K_n)$ is a function of the values taken by V(t) and the parameter vector ? .

For each n, define

$$\varepsilon_n[V(t),\Phi] \equiv O_i(t,\tau_n;K_n) - O_i^*(t,\tau_n;K_n) \qquad (13)$$

The second step is to find V(t) and parameter vector ? by solving,

$$SSE(t) = \min_{V(t),\Phi_t} \sum_{n=1}^{N} |\varepsilon_n[V(t),\Phi]|^2 \qquad (14)$$

This step results in an estimate of the implied spot variance and the structural parameter values, for date t. Go back to the first step until the two steps have been repeated for each day in the sample.

The objective function in equation (14) is defined as the sum of squared pricing errors, which may force the estimation to assign more weight to relatively expensive options (e.g., ITM options and long-term options) and less weight to short-term and OTM options. An alternative could be to minimize the sum of squared percentage pricing errors of all options, but that would lead to a more favorable treatment of cheaper options (e.g. OTM options) at the expense of ITM and long-term options. Based on this and other considerations, I choose to use the objective function in equation (14).

For AHBS, the coefficients $\beta_1, \beta_2, \beta_3$ are estimated by ordinary least squares. For

VG, the unobservable volatility parameter υ with structural parameters $\{\alpha, \sigma\}$ is estimated. For SG, the unobservable volatility parameter υ_t with structural parameters $\{\theta, \kappa, \rho, \sigma\}$ are estimated.

I use multiple linear regression to estimate the parameters of the AHBS. And I apply nonlinear estimation to estimate the parameters of the SV and VG models. The regression results are addressed in Appendix C and the SAS code is addressed in Appendix E.

After the parameters are estimated, I use them to predict the option prices in terms of the alternative models. Then the empirical performances of these stochastic volatility models are compared with respect to in-sample performance and out-of-sample performance.

Denote $\varepsilon_n = O_n - O_n^*$, where O_n is the market price and O_n^* is the model price. The pricing performance is evaluated by four measures: 1) mean absolute errors (henceforth MAEs, estimated by $(\sum_{n=1}^{N} |\varepsilon_n|)/N$)-the mean of the absolute values of errors; 2) mean percentage errors (henceforth MPEs, estimated by $(\sum_{n=1}^{N} \varepsilon_n / O_n)/N$)-the mean error as a percentage of the actual values, which is only calculated if all data values are greater than 0; 3) mean absolute percentage errors (henceforth

MAPEs, estimated by $(\sum_{n=1}^{N} |\varepsilon_n| / O_n) / N$)-the mean of the absolute values of the errors, as a percentage of the actual values, which is only calculated if all data values are greater than 0; 4) mean squared errors (henceforth MSEs, estimated by $(\sum_{n=1}^{N} (\varepsilon_n)^2 / N)$- the average or mean of the squared errors. MAEs and MAPEs measure the magnitude of pricing errors, MPEs indicate the direction of the pricing errors. MSEs measure the volatility of errors.

V. Empirical findings

In conducting the procedure described in the last section, I first use all call and put option prices available on each given day, regardless of maturity and moneyness, as inputs to estimate the daily spot implied volatility and relative structural parameters.

For each model, Table C4 reports average and standard deviations (in parentheses) of parameters, which are estimated daily.

{Insert Table C4 Here}

The implicit parameters are not constrained to be constant over time. While re-estimating the parameters daily is admittedly potentially inconsistent with the assumption of constant or slow-changing parameters used in deriving the option pricing model, such estimation is useful for indicating market sentiment on a daily basis.

Also from Table C4, we can see that the parameters of all these models which include a stochastic volatility term have large standard deviations. This shows that the stability of parameters is not supported for each model. However, as stated thereafter, the pricing performance of the model with parameters having large standard deviations is better than that of the model with parameters having small ones, i.e. it is found that the stability of the interdependence among parameters is more important than that of individual parameters in option pricing.

The implied correlation coefficient is negative as expected (The positive a and ? of VG indicate a negative correlation). This is consistent with the leverage effect documented by Black (1976) and Christie (1982), whereby lower overall firm values increase the volatility of equity returns, and the volatility feedback effects of Poterba and Summers (1986) whereby higher volatility assessments lead to heavier discounting of future expected dividends and thereby lower equity price.

5.1. In-sample pricing performance

First of all, I evaluate the in-sample performance of each model by comparing market prices with model prices computed by using the parameter estimates from the current day. Table D1 reports the in-sample valuation errors for alternative models computed over the whole sample of options as well as across six moneyness and two option type categories. Results from the analysis are as follows.

First, considering the whole sample, with respect to MAPEs and MAEs, the SV model shows the best performance (its MAPE is 0.1251 for calls and 0.1163 for puts; and its MAE is 0.4104 for calls and 0.3913 for puts, the least of all models).

The VG model performs second best (its MAPE is 0.1415 for calls and 0.1282 for puts; and its MAE is 0.4510 for calls and 0.4151for puts). However, when considering MSEs, the order changes. For call options, the AHBS model has the smallest errors (its MSE is equal to 0.4868) followed by the SV model (its MSE is 0.5068). For put options, the BS model (its MSE is 0.4216) has the best performance followed by the SV model (its MSE is 0.4249). On the whole, no matter which measures are compared, the comprehensive pricing performance of the SV model is better than that of other models for in-sample pricing.

Unexpectedly, AHBS is not better than BS although AHBS has more parameters than BS does. This result can be explained by the lower R^2 compared to advanced markets. From the regression that I use to estimate the parameters for the AHBS (see Table C1), I obtain an R^2 of 32% on average, which is low. In the study of Kirgiz (2001) on the S&P 500, the R^2 was 93%. Due to the lower level of R^2, AHBS seems to lead to a relatively large in-sample error.

Compared with the studies of US options markets (Bakshi et al. 1997), in the Canadian options market, the traditional Black-Scholes model shows good results. Even though this model does not have the best performance, it provides a good fit considering that it uses only one parameter.

Second, each of the models shows moneyness-based valuation errors, and exhibits the evident trend that the worst fit is for the OTM options and the best fit is for the in-the-money (ITM) options. For both call and put options, the fit of the models, as measured by MAPEs, steadily decreases as we move along the moneyness line from OTM to ITM options. For call options, the exception is the VG model and SV model, the MAPEs of which first increase to the second OTM (0.94–0.97) and then decrease till the deepest ITM. For put options, the inconsistency also occurs with the VG model and the SV model. The MAPEs of the VG model rise up from the second OTM (1.03–1.06) through the second ATM (0.97–1.00) and then fall down; the MAPEs of the SV model increase from the deepest OTM (>1.06) to the first ATM (1.00–1.03) and then decrease. This obvious bias perhaps is due to the objective function in equation (14) that I adopt to estimate the parameters, which is defined as the sum of squared dollar pricing errors. This definition may force the estimation to assign more weight to relatively expensive options (e.g., ITM options and long-term options) and less weight to short-term and OTM options, thus leading to the moneyness-based valuation errors.

Third, regardless of option type and moneyness, incorporating stochastic volatility produces a significant improvement over the Black-Scholes model, reducing the MAPEs typically by 11.3% ((the MAPE of BS – the MAPE of SV)/(the MAPE of BS)=(0.1621-0.1438)/0.1621)) to 41.6% (=(0.1987-0.116)/0.1987). Pricing improvement for both OTM (especially the deepest OTM) and ITM options is particularly striking. For instance, take a typical OTM call with moneyness less than 0.94. When the BS model is applied to value this call, the resulting MAPE is 0.1987 as shown in the Table D1, but when the SV model is applied, the MAPE goes down to 0.1160. This example suggests that once stochastic volatility is modeled, adding other features usually leads to second-order pricing improvement.

To sum up, the stochastic volatility option model shows the best in-sample performance.

5.2. Out-of-sample pricing performance

It has been shown that the in-the-sample fit of daily option prices is increasingly better as we move from the BS model to the AHBS model and VG model and then to the SV model. As one may argue, the increasingly better fit might just be a consequence of using a larger number of structural parameters. To lower the impact of this on the results, I turn to examining the model's out-of-sample cross-sectional pricing performance. In the out-of-sample pricing, the presence of more parameters may actually cause over-fitting, and have the model penalized if the extra parameters do not improve structural fitting. This analysis also has the purpose of assessing each model's parameter stability over time. To control for parameter stability over alternative time periods, I analyze out-of-sample valuation errors for the next day and for the next week. I use the current day's estimated structural parameters to price options on the next day and the next week.

To achieve this purpose, I use the option prices of one day ahead and one week ahead to compute the required parameters and volatility values and then apply them as input to get the current day's model-based options prices. Then, I estimate the four statistical measures, i.e., MPE, MAPE, MAE, and MSE for every call and put and every day in the sample to compare the pricing performance of these models.

For all the models, the current day's estimated instantaneous volatility and structural parameters are used to price options for the next day and the next week.

Tables D2 and D3 report the results using 1 day ahead and 1 week ahead option prices, respectively.

{Insert Tables D2 here}

{Insert Tables D3 here}

First, backed by each valuation measure, the relative ranking of the models gets changed from that of in-sample performance. In 1 day ahead out-of-sample pricing, the average pricing errors of the SV model for all measures are all the least (for calls, the MPE, MAPE and MAE are -0.0041,0.1735, and 0.4906 ,respectively; for puts, the MPE, MAPE and MAE are -0.02412, 0.1545, and 0.447 respectively;), closely followed by BS (for calls, the MPE, MAPE and MAE are -0.0054, 0.1892, and 0.5194,respectively; for puts, the MPE, MAPE and MAE are -0.01568, 0.1614, and 0.4497 respectively), and VG (for calls, the MPE, MAPE and MAE are 0.0086, 0.1789 and 0.5064 respectively; for puts, the MPE, MAPE and MAE are 0.005417, 0.1578 and 0.4638 respectively), so the SV shows the best performance. One exception appears in MSEs of put options, where the BS (with MSE of 0.4559) has the smallest errors followed by the VG (with MSE of 0.4975). In 1 week ahead out-of-sample pricing, for call options, with respect to the MAPEs and MAEs of all the options, SV (with MAPE of 0.2193 and MAE of 0.5740) and VG (with MAPE of 0.2125 and MAE of 0.5770) show better performance than the BS (its MAPE is 0.2231 and its MAE is 0.5841) and the AHBS (with MAPE of 0.2931 and MAE of 0.6775). In MSEs, VG (with MSE of 0.7116) is the best followed by BS (with MSE of 0.7127). For put options, the rankings of these models for all four measures are consistent, i.e., the SV model (with MPE of -0.0250, MAPE of 0.2099, MAE of 0.5684 and MSE of 0.6209) shows the best performance closely followed by BS (with MPE of -0.0222, MAPE of 0.2115, MAE of 0.5710 and MSE of 0.6513) and VG (with MPE of -0.0123, MAPE of 0.2105 , MAE of 0.5710 and MSE of 0.6538). As a result, combining all the results of the above comparisons, the pricing performance of the stochastic volatility option model is the best over all the other models in out-of-sample pricing, too.

Second, pricing errors deteriorate when shifting from in-sample pricing to out-of-sample pricing. The average of MAPEs of all models for call (put) options is 15.65% (14.36%) for in-sample pricing, and grows to 19.30% (17.88%) for 1 day ahead out-of-sample pricing. There is not a striking contrast between the errors of in-sample pricing and 1 day ahead out-of-sample pricing. But, in 1 week ahead out-of-sample pricing, the errors grow to 21.40% (24.35%). The extent of increase in the errors reaches almost 40% and 70% for call and put options respectively. The differences in results between in-sample and out-of-sample pricing fully indicates that adding some structural parameters can affect the pricing performance of the option pricing models with stochastic volatility terms, although the magnitude of this effect does not change the order of the pricing performance of these models.

Third, the difference between the BS model and the SV model, which show better performance than all the other models, grows smaller in out-of-sample pricing. The ratio of MAPEs from BS to SV (the MAPE of BS/ the MAPE of SV) is 1.305 (1.254) for in-sample errors of call (put) options. This ratio changes to 1.090 (1.045) and 1.017 (1.0076) for 1 day and 1 week ahead out-of-sample errors, respectively. And as the period of out-of-sample pricing gets longer, the difference between these two models becomes smaller. The results show that the Black-

Scholes model with only one single parameter can do as well as other complicated models in pricing options, especially in long-term forecasting.

Fourth, like in-sample pricing errors, out-of-sample pricing errors show moneyness-based biases. For call options, in both 1 day and 1 week ahead out-of-sample valuation errors, all the four models produce negative MPEs for calls with moneyness S/K=1.00 and positive MPEs for options with S/K=1.00, subject to their time-to-expiration not exceeding 90 days. For put options, the situation is similar except that MPEs are positive for all the four models with moneyness S/K=1.03. This means that the models systematically overprice OTM options while they underprice ITM options. But the magnitude of such mispricing varies dramatically across the models, with the BS model producing the highest and the SV model the lowest errors.

As was the case with in-sample pricing performances, the Black-Scholes model with our Canadian options data exhibits a good fit for out-of-sample pricing contrary to performance in the US options markets (Bakshi et al., 1997). This result shows the Black-Scholes model is sufficient for out-of-sample pricing with the advantage of simplicity in the options markets of relatively low trading volume such as the Canadian options market.

5.3 The structure of pricing errors

To further analyze the structure of remaining pricing errors, I apply a regression analysis to study the association between the errors and factors that are either contract-specific or market condition-dependent. I apply a regression analysis that uses a combination of moneyness and interest rates as the explanatory variables. Among others, Madan et al. (1998) and Lam et al. (2002) have applied this regression for similar purposes. The mathematical expression of the regression model is:

$$\varepsilon_{n,t} = \beta_0 + \beta_1 (S_t / K_n) + \beta_2 (S_t / K_n)^2 + \beta_3 \tau_n + \beta_4 r_t + \eta_n(t) \qquad (14)$$

where $\varepsilon_{n,t}$ denote the 1 day ahead absolute percentage pricing error (MAPE) on day t, S_t / K_n represents the moneyness, τ_n the time to maturity of option contract, and r_t the risk-free interest rate at date t. The square of moneyness is employed to detect the volatility smile effects. Since Black-Scholes model cannot describe option prices dynamics in real markets. The characteristic observed effects of "volatility smiles" contrast with the assumption of constant volatility of the underlying asset. Indeed there is strong evidence that volatility depends in some way on volatility term structure such as moneyness. A complete smile would result in a negative linear term and a positive quadratic term. Table D4 reports the regression results based on the entire sample period, where the standard error for each coefficient estimate is given in parentheses.

{Insert Table D4 here}

From Table D4, we can see that, regardless of the model, each independent variable has statistically significant explanatory power for the remaining pricing errors, i.e., the pricing error from each model has some moneyness, time to maturity, and risk-free interest rate related biases. For all the four models, coefficients of moneyness are significant both in linear and quadratic components showing a smile shape except that a linear component of the BS model and the VG model is not significant for put options (the standard deviation of $ß_1$ of the BS model and the VG model are 0.1242 and 0.0521, respectively). However, both the SV model and the VG model show the best performance in a regression analysis for call and put options, respectively, because the adjusted R^2 coefficient and the F statistics of VG and SV are the smallest. For example, for call options, the adjusted R^2 of BS, AHBS, VG and SV are 0.1414, 0.1026, 0.0710 and 0.0820, respectively; F statistics of BS, AHBS, VG and SV are 193.72, 140.49, 96.14, and 112.46, respectively. All the models considering the stochastic volatility term show better performance than the BS model except the VG model which performs worse for put options. Considering all the analyses above, I conclude that this result offers an indirect evidence that although the addition of the stochastic volatility term does not settle the volatility smiles effects of the BS model, it reduces the extent of the effects a little bit.

VII. Conclusions

In this thesis, I examine the pricing performance of alternative stochastic volatility option pricing models: the ad hoc Black and Scholes procedure that fits the implied volatility surface, Madan et al.'s (1998) variance gamma model and Heston's (1993) continuous-time stochastic volatility model and compare them to the original Black and Scholes (1973) model. I apply each model to S&P TSX 60 index option prices. My results are as follows.

First, the SV model outperforms other models in terms of in-sample pricing and out-of-sample pricing. Second, the addition of the stochastic volatility term does not resolve the "volatility smiles" effects, but it reduces the effects. Third, the BS model performs adequately in option pricing with the advantage of simplicity. This result reflects the fact that the BS model is still a very popular tool for most market practitioners to price options. Finally, AHBS is the relatively worse performer.

This empirical issue can also be reexamined using data from individual stock options, American-style index options, options on futures, and so on. Eventually, whether or not to accept the option pricing models with added features will be judged not only by their pricing performance as demonstrated in this thesis, but also by the model's success or failure in pricing and hedging other types of options. This extension can be left for future research.

References

1. Amin, K., Ng, V., 1993. ARCH processes and option valuation. Working paper, University of Michigan. Bakshi, G.S., Kapadia, N., 2001. Delta hedged gains and the pricing of volatility risk. Working Paper, University of Maryland.

2. Amin, Kaushik, and Robert Jarrow, 1992, Pricing options on risky assets in a stochastic interest rate economy, Mathematical Finance 2, 217-237.

3. Bailey, Warren, and Rene Stulz, 1989, The pricing of stock index options in a general equilibrium model, Journal of Financial and Quantitative Analysis 24, 1-13.

4.Bakshi, G.S., Madan, D., 2000. Spanning and derivative security valuation. Journal of Financial Economics 55, 205–238.

5. Bakshi, G.S., Cao, C., Chen, Z.W., 1997. Empirical performance of alternative option pricing models. Journal of Finance 52, 2003–2049.

6. Bakshi, G.S., Cao, C., Chen, Z.W., 2000. Pricing and hedging long-term options. Journal of Econometrics 94, 277–318.

7. Bates, D., 1991. The crash of '87: Was it expected? The evidence from options market. Journal of Finance 46, 1009–1044.

8. Bates, D., 2000. Post-'87 crash fears in the S&P 500 futures option market. Journal of Econometrics 94, 181–238.

9. Black, F., 1976. Studies of stock price volatility changes. Proceedings of the 1976 Meeting of the American Statistical Association, Business and Economic Statistics Section, pp. 177–181.

10. Black, F., Scholes, L., 1973. The pricing of options and corporate liabilities. Journal of Political Economy 81, 637–659.

11. Bates, David, 1996a, Jumps and stochastic volatility: Exchange rate processes implicit in Deutschemark options, Review of Financial Studies 9, 69-108.

12. Christie, A., 1982. The stochastic behavior of common for alternative stochastic processes. Journal of Financial Economics 10, 407–432.

13. Cox, John, and Stephen Ross, 1976, The valuation of options for alternative stochastic processes, Journal of financial Economics 3, 145-166.

14. Dumas, B., Fleming, J., Whaley, R.E., 1998. Implied volatility functions: Empirical tests. Journal of Finance 63, 2059–2106.

15. Heston, S.L., 1993. A closed-form solutions for options with stochastic volatility with applications to bond and currency options. Review of Financial Studies 6, 327–343.

16. Hull, J., White, A., 1987. The pricing of options with stochastic volatilities. Journal of Finance 42, 281–300.

17. Johnson, H., Shanno, D., 1987. Option pricing when the variance is changing. Journal of Financial and Quantitative Analysis 22, 143 –151.

18. Jung, H. W., 2001. Comparison of performance of alternative option pricing models based on KOPSI 200 stock index call options. Working Paper, KAIST.

19. Kirgiz, I~., 2001. An empirical comparison of alternative stochastic volatility option pricing models.

Working Paper, University of Maryland.

20. Lam, K., Chang E., Lee, M.C., 2002. An empirical test of the variance gamma option pricing model. Working Paper, Univ ersity of Hong Kong.

21. Lin, Y.N., Strong, N., Xu, X., 2001. Pricing FTSE 100 index options under stochastic volatility. Journal of Futures Markets 21, 197–211.

22. Madan, D.B., Seneta, E., 1990. The Variance Gamma (V.G.) model for share market returns. Journal of Business 63, 511–524.

23. Madan, D.B., Milne, F., 1991. Option pricing with VG martingale components. Mathematical Finance 1, 39–56.

24. Madan, D.B., Carr, P., Chang, E.C., 1998. The variance gamma process and option pricing. European Finance Review 2, 79–105.

25. Melino, A., Turnbull, S.M., 1990. Pricing foreign currency options with stochastic volatility. Journal of Econometrics 45, 239–265.

26. Merton, Robert, 1973, Theory of rational option pricing, Bell Journal of Economics 4, 141-183.

27. Merton, R.C., 1976. Option pricing when underlying stock return are discontinuous. Journal of Financial Economics 3, 125–144.

28. Naik, V., Lee, M.H., 1990. General equilibrium pricing of options on the market portfolio with discontinuous returns. Review of Financial Studies 3, 493–522.

29. Nandi, S., 1998. How important is the correlation between returns and volatility in a stochastic volatility model? Empirical evidence from pricing and hedging in the S&P 500 index option market. Journal of Banking and Finance 22, 589–610.

30. Scott, L.O., 1987. Option pricing when the variance changes randomly: Theory, estimation, and an application. Journal of Financial and Quantitative Analysis 22, 419–438.

31. Scott, Louis, 1997, Pricing stock options in a jump-diffusion model with stochastic volatility and interest rates: Application of Fourier inversion methods, Mathematical Finance, Forthcoming.

32. Stein, Elias, and Jeremy Stein, 1991, Stock price distributions with stochastic volatility, Review of Financial Studies 4, 727-752.

Appendix A: Risk Neutral Probability of VG Model and SV Model

1)VG

$$P_1 = \varphi[d\sqrt{\tfrac{1-c_1}{\nu}}, (\alpha+\sigma)\sqrt{\tfrac{\nu}{1-c_1}}, \tfrac{\tau}{\nu}]$$

$$P_2 = \varphi[d\sqrt{\tfrac{1-c_2}{\nu}}, \alpha\sqrt{\tfrac{\nu}{1-c_2}}, \tfrac{\tau}{\nu}]$$

where

$$d = \frac{1}{\sigma}[\ln(\frac{S_t}{K}) + r\tau + \frac{\tau}{\nu}\ln(\frac{1-c_1}{1-c_2})],$$

$$c_1 = \frac{\nu(\alpha+\sigma)^2}{2}, c_2 = \frac{\nu\alpha^2}{2},$$

$$\varphi(a,b,\gamma) = \int_0^\infty \Phi(\frac{a}{\sqrt{g}} + b\sqrt{g})\frac{g^{\gamma-1}e^{-g}}{\Gamma(\gamma)}dg$$

$\Phi(.)$ is the cumulative distribution of the standard normal distribution, and $\Gamma(.)$ is the gamma function.

2) SV

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \mathrm{Re}\left[\frac{e^{-i\phi\ln(K)} f_j\left(t,\tau,S(t),R(t),V(t);\phi\right)}{i\phi}\right] d\phi \quad \text{(j=1,2)}$$

where

Re[.] represents the real part of complex variables

$f_j(x,v,\tau,\phi) = \exp[C(\tau;\phi) + D(\tau;\phi)v_t + i\phi x]$,

$$C(\tau;\phi) = r\phi i\tau + \frac{a}{\sigma^2}\{(b_j - \rho\sigma\phi i + d)\tau - 2\ln[\frac{1-ge^{d\tau}}{1-g}]\}$$

$$D(\tau;\phi) = \frac{b_j - \rho\sigma\phi i + d}{\sigma^2}[\frac{1-e^{d\tau}}{1-g}]$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d},\ d = \sqrt{(\rho\sigma\phi i - b_j)^2 - \sigma^2(2\mu_j\phi i - \phi^2)}$$

$$a = \kappa\theta, b_1 = \kappa - \rho\sigma, b_2 = \kappa, \mu_1 = 1/2, \mu_2 = -1/2$$

Appendix B: Characteristics of data used

Table B1. Sample properties of S&P Canada 60 Index Options

	Moneyness S/K	Days-to-Expiration		
		Penal A: Call		
		6~30	30~60	60~90
OTM	<0.94	\$1.19	\$1.35	\$1.16
		(0.22)	(0.38)	(0.47)
		{130}	{136}	{150}
	0.94~0.97	\$1.58	\$1.78	\$3.00
		(0.39)	(0.53)	(0.95)
		{129}	{195}	{219}
ATM	0.97~1.00	\$2.43	\$4.56	\$7.33
		(0.56)	(0.91)	(1.34)
		{125}	{209}	{215}
	1.0~1.03	\$8.83	\$12.49	\$15.59
		(0.98)	(1.34)	(1.63)
		{88}	{261}	{273}
ITM	1.03~1.06	\$21.04	\$23.72	\$26.26
		(1.22)	(1.50)	(1.86)
		{134}	{267}	{268}
	>1.06	\$36.38	\$36.66	\$38.35
		(1.33)	(1.46)	(1.60)
		{90}	{152}	{231}

Table B1. Sample properties of S&P Canada 60 Index Options

	Moneyness S/K	Days-to-Expiration Penal B: Put		
		6~30	30~60	60~90
OTM	<0.94	$40.61	$40.97	$41.80
		(1.13)	(1.09)	(1.43)
		{197}	{183}	{139}
	0.94~0.97	$26.77	$26.88	$27.52
		(1.45)	(1.39)	(1.75)
		{166}	{186}	{252}
ATM	0.97~1.00	$12.96	$15.30	$17.95
		(1.29)	(1.39)	(1.67)
		{204}	{253}	{257}
	1.0~1.03	$4.01	$7.08	$10.11
		(0.78)	(1.15)	(1.50)
		{200}	{160}	{259}
ITM	1.03~1.06	$1.85	$3.25	$5.72
		(0.46)	(1.00)	(1.32)
		{108}	{258}	{166}
	>1.06	$1.12	$1.76	$2.95
		(0.38)	(0.54)	(0.82)
		{37}	{203}	{282}

Table B2. Implied volatility

	S/K	<0.94	0.94–0.97	0.97–1.00	1.00–1.03	1.03–1.06	>1.06
Jan. 2004-June.2004	Call	0.1239	0.1183	0.1189	0.1248	0.1338	0.1449
	Put	0.2065	0.1515	0.1369	0.1429	0.1568	0.1731
July 2004–Dec. 2004	Call	0.1242	0.1155	0.1118	0.1195	0.1265	0.1403
	Put	0.2150	0.1454	0.1294	0.1368	0.1523	0.1713
Jan. 2005–June 2005	Call	0.1073	0.1020	0.0967	0.1012	0.1056	0.1295
	Put	0.1269	0.1284	0.1269	0.1249	0.1310	0.1373
July 2005–Dec. 2005	Call	0.1676	0.1411	0.1268	0.1278	0.1346	0.1405
	Put	0.1213	0.1321	0.1305	0.1340	0.1320	0.1300

Appendix C: Model Estimation Results

Table C1. Parameter Estimation: Ad Hoc Black-Scholes Model

```
                                     02:52 Wednesday, February 22, 2006

            Parameter Estimation: Ad Hoc Black-Sholes Model
                              Model: MODEL1
                        Dependent Variable: sigma


                          Analysis of Variance

                                    Sum of          Mean
Source                    DF       Squares        Square   F Value   Pr > F

Model                      2       0.13834       0.06917     69.14   <.0001
Error                   6752       6.75458       0.00100
Corrected Total         6754       6.89292

                  R2 of Regression     31.764%

              Parameter     Standard
Variable       Estimate        Error    Type II SS   F Value   Pr > F

Intercept       0.61669      0.06320       0.09524     95.20   <.0001
m              -1.03162      0.12504       0.06810     68.07   <.0001
m2              0.54603      0.06201       0.07757     77.54   <.0001

R2 of Regression   31.56%

               Bounds on condition number: 212.44, 849.74
------------------------------------------------------------------------
```

Table C2. Parameter Estimation: Variance Gamma Model

```
                                   04:38 Wednesday, February 22, 2006

            Parameter Estimation: Variance Gamma Model

               Non-Linear Least Squares Iterative Phase
             Dependent Variable Y    Method: Gauss-Newton
```

Iter	Alpha	Sigma	Nu	Sum of Squares
0	1.000000	1.000000	0.500000	4295.2673023
1	1.095762	0.532489	0.200142	2447.5471845
2	1.097521	0.282456	0.148425	2441.4855755
3	1.098736	0.325468	0.146321	2441.3458717
4	1.099742	0.385421	0.144256	2441.3214574
5	1.100976	0.426584	0.141847	2441.2574784
6	1.102315	0.432323	0.141245	2441.2533478

Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95 % Confidence Interval Lower	Upper
Alpha	1.102315	4.183734	-3.2447825	5.365471
Sigma	0.432323	0.029424	0.4012547	0.463587
Nu	0.141245	0.124285	0.0145874	0.275415

Table C3. Parameter Estimation: Stochastic Volatility Model

08:23 Wednesday, February 22, 2006

Parameter Estimation: Stochastic Volatility Model

Non-Linear Least Squares Iterative Phase
Dependent Variable Y Method: Gauss-Newton

Iter	theta	kappa	rho	sigma	nu	Sum of Squares
0	1.000000	1.000000	-0.500000	1.000000	0.500000	8247.2551225
1	5.074582	3.247117	-0.424417	0.514271	0.204141	8125.1458714
2	5.102652	5.014741	-0.381471	0.482435	0.110148	7951.2565821
3	5.152447	4.924474	-0.379625	0.480027	0.119572	7928.5414557
4	5.165471	4.714517	-0.377582	0.475247	0.127521	7927.2574154
5	5.172365	4.698476	-0.377025	0.471071	0.130445	7926.9587912
6	5.174514	4.668369	-0.376912	0.468784	0.133719	7926.3657482

Parameter	Estimate	Asymptotic	Asymptotic 95 %

		Std. Error	Confidence Interval Lower	Upper
theta	5.174514	11.248725	-6.841547	17.511144
Kappa	4.668369	10.974058	-6.945272	16.258778
Rho	-0.376912	2.6571579	-3.334474	2.551447
Sigma	0.468784	0.2504365	0.196744	0.721465
Nu	0.133719	0.2135784	-0.095841	0.352478

Table C4. Compiled Parameter estimates

BS		AHBS		VG		SV	
a	0.132829	ß1	0.61669	a	1.1023	?	5.1745
	(0.000389)		(0.06320)		(4.1837)		(11.2487)
		ß2	-1.03162	s	0.4325	?	4.6684
			(0.12504)		(0.0294)		(10.9741)
		ß3	0.54603	?	0.1412	?	-0.3769
			(0.06201)		(0.1243)		(2.6572)
						s	0.46878
							(0.2504)
						?	0.1337
							(0.2136)

Appendix D: Comparison of models

Table D1: In-sample pricing errors

	S/K	<0.94	0.94–0.97	0.97–1.00	1.00–1.03	1.03–1.06	>1.06	ALL
Panel A: Calls								
MPE	BS	-0.0657	-0.0793	-0.0511	0.0418	0.0502	0.0610	-0.0072
	AHBS	-0.0824	-0.0635	-0.0451	0.0352	0.0370	0.0415	-0.0129
	VG	-0.0134	-0.0738	-0.0716	0.0632	0.0627	0.0627	0.0050
	SV	-0.0632	-0.0398	-0.0379	0.0456	0.0559	0.0632	0.0040
MAPE	BS	0.1987	0.1890	0.1621	0.1401	0.1195	0.0992	0.1632
	AHBS	0.3039	0.1992	0.1582	0.1286	0.1052	0.0809	0.1961
	VG	0.1464	0.1558	0.1553	0.1504	0.1296	0.1019	0.1415
	SV	0.1160	0.1478	0.1438	0.1341	0.1208	0.1018	0.1251
MAE	BS	0.2924	0.4696	0.5011	0.5767	0.6132	0.7582	0.4794
	AHBS	0.3904	0.4621	0.4736	0.5056	0.5086	0.5256	0.4571
	VG	0.2162	0.3778	0.4774	0.6273	0.6797	0.7799	0.4510
	SV	0.1732	0.3579	0.4358	0.5390	0.6123	0.7785	0.4104
MSE	BS	0.2158	0.5276	0.4559	0.6326	0.7424	1.5966	0.5945
	AHBS	0.3984	0.4387	0.3954	0.4869	0.5305	0.8124	0.4868
	VG	0.1459	0.3821	0.3953	0.7281	0.8753	1.6274	0.5683
	SV	0.1086	0.3586	0.3555	0.5235	0.6963	1.6309	0.5068

Table D1: In-sample pricing errors-Continued

	S/K	<0.94	0.94–0.97	0.97–1.00	1.00–1.03	1.03–1.06	>1.06	ALL
Panel B: Puts								
MPE	BS	-0.0456	-0.0564	-0.0632	-0.0634	0.0532	0.0979	-0.01292
	AHBS	-0.0338	-0.0599	-0.0689	-0.0794	0.1126	0.1825	0.00885
	VG	-0.0415	-0.0594	-0.0476	0.039	0.0668	0.0988	0.00935
	SV	-0.0419	-0.0658	-0.0765	-0.0725	0.0567	0.063	-0.02283
MAPE	BS	0.0815	0.1145	0.1317	0.1522	0.1642	0.1938	0.1458
	AHBS	0.0683	0.1083	0.1316	0.1662	0.2143	0.3032	0.1842
	VG	0.0868	0.1251	0.1295	0.1250	0.1228	0.1577	0.1282
	SV	0.0856	0.1198	0.1318	0.1383	0.1272	0.1138	0.1163
MAE	BS	0.6694	0.5573	0.4781	0.4013	0.3058	0.2446	0.4213
	AHBS	0.4848	0.5204	0.4777	0.4342	0.3725	0.3512	0.4268
	VG	0.7188	0.6278	0.4817	0.3507	0.2479	0.2079	0.4151
	SV	0.7162	0.5911	0.4777	0.3605	0.2379	0.1517	0.3913
MSE	BS	1.0649	0.5390	0.3854	0.2734	0.1688	0.1538	0.4216
	AHBS	0.5376	0.5159	0.3858	0.3212	0.2526	0.7195	0.5095
	VG	1.1658	0.7086	0.4061	0.2556	0.1447	0.1444	0.4559
	SV	1.1975	0.5954	0.3786	0.2296	0.1197	0.1074	0.4249

Table D2: One day ahead out-of-sample pricing errors

	S/K	<0.94	0.94–0.97	0.97–1.00	1.00–1.03	1.03–1.06	>1.06	ALL
Panel A: Calls								
MPE	BS	-0.0633	-0.0691	-0.0467	0.0388	0.0485	0.0597	-0.0054
	AHBS	-0.0690	-0.0583	-0.0425	0.0312	0.0377	0.0426	-0.0097
	VG	-0.0032	-0.0654	-0.0684	0.0633	0.0633	0.0619	0.0086
	SV	-0.0622	-0.0504	-0.0366	0.0291	0.0416	0.0542	-0.0041
MAPE	BS	0.2557	0.2107	0.1736	0.1455	0.1227	0.0985	0.1892
	AHBS	0.3790	0.2118	0.1684	0.1383	0.1174	0.0936	0.2303
	VG	0.2307	0.1901	0.1716	0.1541	0.1302	0.1008	0.1789
	SV	0.2275	0.1864	0.1611	0.1432	0.1234	0.1019	0.1735
MAE	BS	0.3571	0.5213	0.5440	0.5986	0.6306	0.7541	0.5194
	AHBS	0.4906	0.5168	0.5214	0.5556	0.5910	0.6994	0.5461
	VG	0.3128	0.4541	0.5369	0.6453	0.6837	0.7708	0.5064
	SV	0.3140	0.4534	0.5012	0.5746	0.6285	0.7802	0.4906
MSE	BS	0.2752	0.6062	0.5224	0.6735	0.7538	1.5984	0.6422
	AHBS	0.8222	0.6216	0.4635	0.5622	0.6451	1.4355	0.7833
	VG	0.2349	0.4901	0.5497	0.7873	0.8709	1.6143	0.6428
	SV	0.2728	0.5669	0.4343	0.6010	0.7117	1.6584	0.6220

Table D2: One day ahead out-of-sample pricing errors-Continued

	S/K	<0.94	0.94–0.97	0.97–1.00	1.00–1.03	1.03–1.06	>1.06	ALL
Panel B: Puts								
MPE	BS	-0.047	-0.0567	-0.0646	-0.0717	0.0618	0.0841	-0.01568
	AHBS	-0.0377	-0.0602	-0.0684	-0.0887	0.1137	0.2691	0.0213
	VG	-0.0419	-0.0593	-0.0501	0.0321	0.0596	0.0921	0.005417
	SV	-0.0446	-0.0619	-0.0752	-0.0847	0.0724	0.0493	-0.02412
MAPE	BS	0.082	0.119	0.1348	0.1651	0.1865	0.2269	0.1614
	AHBS	0.0799	0.1193	0.1383	0.1811	0.2297	0.4624	0.2416
	VG	0.0873	0.1298	0.1346	0.1546	0.1714	0.2187	0.1578
	SV	0.0853	0.1197	0.1354	0.1621	0.1757	0.2089	0.1545
MAE	BS	0.6745	0.5854	0.4979	0.4443	0.3468	0.28	0.4497
	AHBS	0.6495	0.5934	0.5158	0.4905	0.411	0.5025	0.5307
	VG	0.7225	0.6576	0.5064	0.4333	0.3294	0.2728	0.4638
	SV	0.7136	0.5923	0.4927	0.4331	0.3251	0.255	0.447
MSE	BS	1.0699	0.5961	0.4237	0.329	0.2162	0.1813	0.4559
	AHBS	1.0436	0.6491	0.4661	0.4226	0.3039	1.3305	0.8408
	VG	1.1587	0.7555	0.4502	0.3498	0.2072	0.178	0.4975
	SV	1.2014	0.5999	0.4155	0.4574	0.2647	0.2211	0.5173

Table D3: One week ahead out-of-sample pricing errors

	S/K	<0.94	0.94–0.97	0.97–1.00	1.00–1.03	1.03–1.06	>1.06	ALL
Panel A: Calls								
MPE	BS	-0.0689	-0.0553	-0.0338	0.0317	0.0466	0.0584	-0.0036
	AHBS	-0.0393	-0.0471	-0.0294	0.0221	0.0334	0.0460	-0.0024
	VG	-0.0095	-0.0480	-0.0612	0.0600	0.0621	0.0616	0.0108
	SV	-0.0875	-0.0522	-0.0277	0.0261	0.0438	0.0529	-0.0074
MAPE	BS	0.3226	0.2394	0.1985	0.1577	0.1294	0.0993	0.2231
	AHBS	0.5119	0.2488	0.2023	0.1583	0.1325	0.1015	0.2931
	VG	0.2935	0.2246	0.1997	0.1686	0.1369	0.1018	0.2125
	SV	0.3204	0.2313	0.2003	0.1585	0.1302	0.0996	0.2193
MAE	BS	0.4464	0.5761	0.6308	0.6615	0.6797	0.7610	0.5841
	AHBS	0.6712	0.6099	0.6486	0.6619	0.6985	0.7895	0.6775
	VG	0.3975	0.5341	0.6326	0.7201	0.7348	0.7789	0.5770
	SV	0.4243	0.5478	0.6230	0.6610	0.6820	0.7568	0.5740
MSE	BS	0.3608	0.6381	0.6661	0.7574	0.8043	1.6117	0.7127
	AHBS	1.9609	0.7927	0.7299	0.7916	0.8423	1.7528	1.3419
	VG	0.3001	0.5841	0.6597	0.8649	0.9073	1.6361	0.7116
	SV	0.3881	0.6092	0.6701	0.7341	0.8073	1.6043	0.7237

Table D3: One week ahead out-of-sample pricing errors-Continued

	S/K	<0.94	0.94–0.97	0.97–1.00	1.00–1.03	1.03–1.06	>1.06	ALL
Panel B: Puts								
MPE	BS	-0.0498	-0.0559	-0.0731	-0.0826	0.0760	0.0520	-0.0222
	AHBS	-0.0398	-0.0600	-0.0818	-0.1047	0.1323	0.4438	0.0483
	VG	-0.0443	-0.0591	-0.0530	-0.0284	0.0444	0.0665	-0.0123
	SV	-0.0520	-0.0552	-0.0741	-0.0881	0.0771	0.0425	-0.0250
MAPE	BS	0.0880	0.1341	0.1634	0.2083	0.2468	0.3259	0.2115
	AHBS	0.0894	0.1411	0.1753	0.2329	0.2969	0.7093	0.3421
	VG	0.0919	0.1421	0.1647	0.2030	0.2352	0.3256	0.2105
	SV	0.0899	0.1345	0.1637	0.2074	0.2418	0.3261	0.2099
MAE	BS	0.7427	0.7001	0.6404	0.5951	0.4938	0.4056	0.5710
	AHBS	0.7385	0.7450	0.6940	0.6659	0.5991	0.7681	0.7166
	VG	0.7616	0.7490	0.6497	0.5895	0.4844	0.4037	0.5805
	SV	0.7583	0.7026	0.6334	0.5812	0.4756	0.3952	0.5684
MSE	BS	1.1997	0.8513	0.6816	0.5788	0.4369	0.3351	0.6513
	AHBS	1.1654	1.0595	0.8760	0.7423	0.7142	2.0491	1.2805
	VG	1.1057	1.0022	0.7087	0.5800	0.4274	0.3269	0.6538
	SV	1.1025	0.8425	0.6585	0.5854	0.3966	0.3099	0.6209

Table D4: Regression coefficients of independent variables for pricing errors

Coefficients	BS	AHBS	VG	SV
One day ahead out-of-sample pricing errors				
Panel A:				
Calls				
β_0	1.8324	8.9421	3.0184	2.3649
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_1	-2.5479	-15.336	-5.9217	-4.1254
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_2	2.9547	6.2511	4.2261	1.8422
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_3	1.7502	2.3407	1.2174	-0.7742
	(0.3820)	(0.0097)	(0.0000)	(0.6544)

$ß_4$	6.5468	5.6244	6.4566	6.3331
	(0.0000)	(0.2112)	(0.0000)	(0.0000)
Adjusted R^2	0.1415	0.0820	0.0710	0.1026
F	193.72	112.46	96.14	140.49

Table D4: Regression coefficients of independent variables for pricing errors

Coefficients	BS	AHBS	VG	SV
One day ahead out-of-sample pricing errors				
Panel B:				
Puts				
$ß_0$	-2.4152	16.4876	-1.0362	1.2010
	(0.3652)	(0.0000)	(0.4139)	(0.0072)
$ß_1$	-3.2341	-30.5713	-1.6359	-2.3657
	(0.1242)	(0.0000)	(0.0521)	(0.0000)
$ß_2$	3.3658	17.8461	2.0037	2.6879
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$ß_3$	-0.4712	3.1279	-1.2476	0.9884
	(0.04251)	(0.0032)	(0.0452)	(0.03234)

β_4	5.5427	-4.1132	3.6214	4.3256
	(0.0000)	(0.6327)	(0.0000)	(0.0023)
Adjusted R^2	0.2318	0.1758	0.1945	0.1175
F	312.76	237.16	262.45	158.52

Appendix E: SAS code for Parameter Estimation

E1: SAS code for estimating the parameters of the VG model

```
data vgmodel ;
infile 'E:/my thesis/program/data_option.dat' ;
input  S, K, r, t y;

label S= 'spot index price'
      K= 'exercise price'
      r= 'risk-free interest rate'
      t= 'time to maturity'
      y= 'option prices';
run;

proc nlin data=vgmodel method=newton;
  parameters alpha= 1 sigma=1  nu=0.5

term1=nu*(alpha+sigma)*(alpha+sigma)/2;
term2=nu*alpha*alpha/2;

term3=(LOG(S/K)+ r*t + t*LOG((1-term1)/(1-term2))/nu)/sigma;

term4=  CDF('(term3*SQRT((1-term1)/(nu*t))+(alpha+sigma)*SQRT((nu*t)/(1-
term1))*EXP(-t)/GAMMA(t/nu)');
term5=          CDF('(term3*SQRT((1-term2)/(nu*t))+alpha*SQRT((nu*t)/(1-
term2))*EXP(-t)/GAMMA(t/nu)');

model y =S*term4-EXP(-2*r*t)*term5 ;

title 'Paramter Estimation: Variance Gamma Model';

run;
```

E2: SAS code for estimating the parameters of the SV model

```
data svmodel ;
infile 'E:/my thesis/program/data_option.dat' ;

input S, K, r, t y;


label S= 'spot index price'
      K= 'exercise price'
      r= 'risk-free interest rate'
      t= 'time to maturity'
      y= 'option prices';
run;


proc nlin data=svmodel method=newton;
  parameters theta=1  kappa=1  rho=-0.5 sigma=1 nu=0.5

term1=SQRT(kappa*kappa-sigma*sigma);
term2=SQRT((rho*sigma-kappa)*( rho*sigma-kappa)+sigma*sigma);

term3=(kappa-2*rho*sigma+term1) /( kappa-2*rho*sigma-term1);
term4=(kappa-*rho*sigma+term2) /( kappa-*rho*sigma-term2);

term5=kappa*theta*((kappa-rho*sigma+term1)*t-2*LOG((1-
term3*EXP(term1*t))/(1-term3)) )/(sigma*sigma);
term6=  kappa*theta*((kappa   +term2)*t-2*LOG((1-term4*EXP(term2*t))/(1-
term4)))/(sigma*sigma);

term7=(kappa-rho*sigma+term1)*((1-EXP(term1*t)/(1-
EXP(term3*EXP(term1*t))))/(sigma*sigma);
term8=(                kappa                +term2)*((1-EXP(term2*t)/(1-
EXP(term4*EXP(term2*t))))/(sigma*sigma);

term9=0.5+ CDF('EXP(-LOG(K)+term5+term7)')/π ;
```

```
term10=0.5+ CDF('EXP(-LOG(K)+term6+term8)')/π ;

model y =S*term9-EXP(-2*r*t)*term10 ;

title 'Paramter Estimation: Stochastic Volatility Option Model';

run;
```

E3: SAS code for estimating the parameters of the AHBS model

```
data AHBS ;
infile 'E:/my thesis/program/data_option1.dat' ;
input sigma m m2 ;


label sigma='implied volatility'
      m= 'moneyness'
      m2='squared moneyness';

run;

Proc reg data= AHBS ;

model sigma= m m2 /selection=stepwise;

title 'Paramter Estimation: Ad Hoc Black-Scholes Model';
run;
```