The Determination of an Optimal Hedge Ratio and a Generalized Measure of Risk

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A Thesis

In

The John Molson School of Business

Presented in Partial Fulfillment of the Requirements
for the Degree of Master of Science in Administration at
Concordia University
Montreal, Quebec, Canada

December 2005

Your file Votre référence
ISBN: 0-494-14371-1
Our file Notre référence
ISBN: 0-494-14371-1

Abstract

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The use of futures contracts as hedging instruments to reduce risk has been the focus of much research. Various risk measures have been developed and have subsequently been employed in an effort to create hedging strategies and to calculate optimal hedge ratios. This thesis proposes a more generalized risk model to measure the risk of hedged assets. The five-parameter model presented herein assumes that each investor has a different target return, level of risk aversion, and degree of sensitivity to lower and higher partial moments. The optimal hedging activity for each investor should then seek to minimize the unique generalized risk measure. This paper utilizes an out-of-sample test on a hedged position in the S&P500 index in the period from December 1982 to December 2004. Tests are conducted to determine whether the change of target returns and sensitivity parameters will affect optimal hedge ratios. In addition, whether hedging effectiveness changes significantly in-sample versus out-of-sample, and between each model and a naïve hedging strategy is investigated. Also, mean returns of hedged portfolios are compared for various models.

This thesis makes three important contributions. First, this study is the first to implement both higher and lower partial moments in the determination of optimal hedge ratios. Second, an out-of-sample test is considered while most studies use only in-sample tests. Third, this thesis is the first to use discontinuous sample periods to separate market conditions and to analyze hedging performance in bull and bear markets.

Acknowledgements

I would like to thank Dr. Ian Rakita for his help, guidance and encouragement. He has shown both confidence in my abilities and patience in my limitations; his comments have not only enhanced the contribution of this thesis but also improved my research skills. His contribution to this paper could not possibly be overstated.

Dedications

I would like to dedicate this thesis to Mom and Dad, who always support me emotionally and financially. I can feel your concern and love all the time even though you are in China thousands of miles away. I miss you so much. This thesis is also dedicated to my wife Xinhua, who gives me ongoing support and feels proud of even my trivial achievements. Without your help, I would get nowhere. Love you always.

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1. INTRODUCTION

A futures contract is a promise to deliver a specific amount of a commodity (or asset) at a preset time in the future. Futures contracts are popular in financial markets where they are used as hedging instruments although they are also employed for speculative purposes. As a hedging tool, futures contracts are important for risk reduction. The most widely used risk measure appearing in the finance literature is the standard deviation or variance. These statistics are two-sided in their application as they take into account both positive and negative deviations from the sample mean. A survey by Adams and Montesi (1995), however, indicates that corporate managers are more concerned with variability in losses than they are regarding variability in gains. This finding is consistent with that of Mao (1970). A negative deviation from the mean may be called "downside risk" while a positive deviation may be termed "upside potential" (Lee and Rao, 1988).

One appropriate measure of downside risk is the Fishburn risk measure (Fishburn, 1977), which is also known as the lower partial moment (Bawa, 1975, 1978). Herein risk is measured as a probability-weighted power function of the shortfall from a specific target return. The power applied in the calculation is called the "order" of the lower partial moment (LPM). According to this approach, if a derivative instrument is to reduce risk, it should result in a smaller LPM rather than a smaller standard deviation or variance.

Previous articles argue that although downside risk may be the main concern of investors, it is inappropriate to ignore higher partial moments totally. As a result, a more generalized risk model is proposed to measure the risk of hedged assets. A five-parameter model developed in this thesis assumes that each investor has a different target return, a different level of risk aversion and a different degree of sensitivity to lower and higher partial moments. The hedging activity for each investor should minimize this unique generalized risk and the process of determining an optimal hedge ratio should follow this guideline.

The purpose of this thesis is fourfold. The first purpose is to determine whether the optimal hedge ratio obtained by minimizing the generalized risk model is significantly different from the one obtained through the minimum-variance model. Here the two risk sensitivity parameters vary from 0 to 1, and then the change of proportion of hedge ratios is compared to that of the minimum-variance ratio. The power term, or risk-aversion parameter, is fixed and set to 2. This means the investor is assumed to exhibit equal risk-aversion for returns below and above target returns while sensitivities to risks are permitted to vary.

The second purpose of the thesis is to investigate whether the change in the target return will affect the optimal hedge ratio. Daily target returns in the model vary from

-0.001 to 0.001 and optimal hedge ratios are compared with those of optimal hedge ratios when the target return is 0.

The third purpose is to compare the hedging effectiveness using in-sample data with that of out-of-sample data. Most previous research applies an in-sample approach exclusively. This procedure makes the questionable assumption that the futures position that produces the optimal hedge ratio and the measure of hedging effectiveness can be determined in the same period. Because in-sample hedging is impossible in practice, investors are more interested in out-of-sample hedging performance. This thesis assumes that a hedger first uses a specific period to calculate the model-based hedge ratio. Then the ratio is applied to hedge a position in the following period. The hedging performance between in-sample and out-of-sample positions is subsequently compared.

The final purpose of this study is to examine the out-of-sample returns for each model. Although all of the models applied in the thesis focus on risk reduction, it will be of interest to observe the return performance of each model and to compare the return of each model with that of a naïve hedge.

The remainder of this thesis is organized as follows. Section 2 briefly reviews previous theoretical and empirical research. Section 3 introduces the models used in this thesis. Section 4 explains the methodology and describes the data. Section 5 discusses the most important results. Finally, Section 6 gives a summary and outlines future potential research directions.

2. LITERATURE REVIEW

Hedging relies on the combination of a position in the spot market with one in the futures market in order to form a portfolio that will reduce the fluctuation in value. For example, if a portfolio consisting of C_s units of a long spot position and C_f units of a short futures position is created, and S_t and F_t are defined as the spot and futures prices at time t respectively, then a hedged portfolio can be created. The portfolio return R_h, is

$R_h = (C_s S_t R_S - C_f F_t R_f) / C_S S_t = R_S - hR_f$, where $h = C_f F_t / C_S S_t$, is the hedge ratio.

One of the key theoretical issues in hedging is the determination of the optimal hedge ratio. The specification of this ratio depends on how the concept of "optimization" is defined. For example, the most widely used hedging strategy is based on the assumption that investors only care about the risk associated with hedging and that the variance of the underlying asset is the appropriate method of measuring risk. Thus the minimum variance (MV) hedge ratio is estimated by minimizing the variance of the hedged portfolio.

It is clearly unrealistic to assume that all investors have a unique preference in the determination of the hedged portfolio. The expected return has also been incorporated along with the risk (variance) to form new strategies. These strategies are consistent with the expected utility maximization principle assuming a quadratic utility function or that returns are jointly normal. Other scholars (Cheung et al. (1990), Kolb and Okunev (1992), Lien and Luo (1993a), Shalit (1995), and Lien and Shaffer (1999)) have attempted to eliminate the assumptions regarding the utility function and return distribution, and accordingly, the method of minimization of the mean extended-Gini (MEG) coefficient was introduced.

Recently the generalized semi-variance (GSV) or lower partial moment was proposed as a possible approach to be used in hedging. The hedge ratios based on these concepts are consistent with the concept of stochastic dominance and the risk perception of investing managers. But when returns of spot and futures are jointly normally distributed and follow a pure martingale process, the minimum GSV ratio reduces to t the MV ratio.

Apart from considering various measures of risk and employing returns in the derivation of the optimal hedging ratio, researchers also differ on whether hedge ratios should be considered as being static or dynamic. Static hedge ratios are typically estimated using unconditional probability distributions, while dynamic ratios are estimated using conditional models such as ARCH (autoregressive conditional heteroscedastic, Engle (1982)) and GARCH (generalized autoregressive conditional heteroscedastic, Bollerslev (1986)).

2.1 Classical Theory

Traditional hedging theory emphasizes the risk avoidance potential that futures markets can provide. Hedgers believe that futures markets and spot markets are highly correlated and move in the same direction with similar magnitudes. Thus investment risk is eliminated if an equal contract value of the opposite sign is invested in the futures market for each unit of value held in the spot market. In this case, when the hedge ratio equals 1 the strategy is called "naïve hedging."

Working (1953 and 1962) challenged the view of hedgers as simply being risk minimizers. He argued that hedgers may also function as speculators and are concerned with relative price changes of spot and futures markets. According to Working, holders of long positions in the spot market will hedge if the basis (the difference between spot and futures prices) is expected to fall and will not hedge if the basis is expected to rise.

2.2 Minimum-Variance Theory

Johnson (1960) and Stein (1961) are the first to view hedging as a simple application of basic portfolio theory. Ederington (1979) developed the theory further in estimating the optimal hedge ratio required to minimize the variance of a portfolio.

In Ederington's (1979) model, let R be the return on a portfolio including both spot holdings C_s, and futures holdings, C_f, and let R_s and R_f be returns in the spot and futures markets, respectively. Then:

$$\mathrm{E(R)} = C_s R_s + C_f R_f$$

Let hedge ratio $h = -C_f / C_s$, thus

$$Var(R) = C_s^2 \sigma_s^2 + C_f{}^2 \sigma_f^2 - 2C_s C_f Cov(R_{s,} R_f) = C_s^2 \left(\sigma_s^2 + h^2 \sigma_f^2 - 2h\sigma_{sf} \right),$$

where σ_s^2 and σ_f^2 represent the respective variances of the spot and futures prices and σ_{sf} represents the covariance between spot and futures prices.

Then by taking the derivative with respect to *h*, Ederington (1979) obtained the optimal hedge ratio as: $h^* = \rho \frac{\sigma_s}{\sigma_f}$,

where ρ is the correlation coefficient between R_s and R_f.

2.3 The Optimal Mean-Variance Hedge Ratio

Hsin et al. (1994) assume that the hedger has a negative exponential utility function with constant absolute risk aversion (ARA; Pratt (1964)) and determine the optimal hedge ratio by maximizing the following utility function:

$$U(R) = E\left(R_h\right) - 0.5 A\sigma_h^2,$$

where A is the risk aversion parameter.

The hedge ratio considers both risk and return and is consistent with the mean-variance framework. The optimal ratio is given by:

$$h = -\left[\frac{E\left(R_f\right)}{A\sigma_f^2} - \rho\frac{\sigma_s}{\sigma_f}\right].$$

Different investors have different risk aversion parameters thereby producing different hedge ratios in the mean-variance framework.

2.4 The Sharpe Hedge Ratio

Another way of incorporating the portfolio return into the hedging strategy is to use the Sharpe ratio. Howard and D'Antonio (1984) assume only mean and variance are relevant in choosing a portfolio, and that an optimal hedge ratio can be obtained by maximizing the ratio of the portfolio's excess return to its volatility:

$$\max\theta = \frac{E(R_g) - R_F}{\sigma_h}.$$

Here R_F is the risk-free interest rate. In this case, the optimal hedge ratio be:

$$h = -\frac{(\sigma_s/\sigma_f)\left[(\sigma_s/\sigma_f)\left(E(R_f)/E(R_s) - R_F\right) - \rho\right]}{\left[1 - (\sigma_s/\sigma_f)\left(E(R_f)\rho/E(R_s) - R_F\right)\right]}.$$

Hsin et al. (1994) point out that the Sharpe ratio is a valid approach only when the excess return is positive. This can lead to problems since negative returns are common in hedging activities. Chen et al. (2001) point out that the Sharpe ratio is a highly non-linear function of the hedge ratio and the ratio derived by equating the first derivative to zero may lead to a ratio that minimizes rather than maximizes the Sharpe ratio. Moreover, the Sharpe ratio may also be monotonically increasing with the hedge ratio.

2.5 The Minimum Mean Extended-Gini Coefficient Hedge Ratio and The Optimum Mean MEG Hedge Ratio

The Minimum mean extended-Gini coefficient approach of deriving the optimal hedge ratio is consistent with the concept of stochastic dominance and involves the use of the MEG coefficient. Cheung et al. (1990), Kolb and Okunev (1992), Lien and Luo (1993a), Shalit (1995), and Lien and Shaffer (1999) all apply this approach.

The approach minimizes the MEG coefficient $\Gamma_v(R_h)$ defined as follows:

$$\Gamma_\nu(R_h) = -\nu\ \mathrm{Cov}(R_h, (1 - G(R_h))_{\nu-1})$$

where G is the cumulative probability distribution and ν is the risk aversion parameter.

Note that $0 \leq v < 1$ is consistent with risk seekers, $v = 1$ is consistent with risk-neutral investors, and $v > 1$ is consistent with risk-averse investors. Shalit (1995) has shown that if the futures and spot returns are jointly normally distributed, then the minimum-MEG hedge ratio would be the same as the MV hedge ratio.

Instead of minimizing the MEG coefficient, Kolb and Okunev (1993) consider maximizing the utility function defined as follows:

$$\mathrm{U}(\mathrm{R_h}) = E(R_h) - \Gamma_v(\mathrm{R_h})$$

This is called as the M-MEG hedge ratio. The difference between the MEG and M-MEG hedge ratios is that the MEG hedge ratio ignores the expected return on the hedged portfolio.

2.6 The α-t Model and the Mean-Generalized Semi-Variance Hedge Ratio

Crum et al. (1981) argue that managers perceive risk as the failure to obtain a specific target. Fishburn (1977) formalizes this perception of risk in his α-t model. He considers a utility function under the target return, t, weighted by a measure for risk aversion, α. The model can be written as:

$$V_\alpha(R_h) = \int_{-\infty}^{t} (t - R_h)^\alpha \, dF(R_h)$$

where $F(R_h)$ is the probability distribution function of the returns of the hedged portfolio.

In this model risk is defined in such a way that investors regard only the returns below t to be risky. $\alpha < 1$ represents a risk-seeking investor and $\alpha > 1$ represents a risk-averse investor.

The hedge ratio determined by minimizing the α-t model is called the minimum Generalized Semi-Variance (GSV) hedge ratio. Both Fishburn (1977) and Bawa (1978) conclude that the GSV hedge ratio is consistent with the concept of stochastic dominance. Lien and Tse (1998) showed that the GSV ratio would be the same as the MV ratio when futures and spot returns are jointly normally distributed and futures prices follow a pure martingale process.

Chen et al. (2001) convert the GSV hedge ratio into a mean-GSV (M-GSV) hedge ratio by incorporating the mean return in the α-t model. The M-GSV hedge ratio is obtained by maximizing the following mean-risk utility function, which is similar to the conventional mean-variance-based utility function and is given by:

$$\mathrm{U}(\mathrm{R}_{\mathrm{h}}) = E(R_h) - V_\alpha(R_h)$$

2.7 The Dynamic Hedge Ratio

In all of the methodologies summarized above the optimal hedge ratio is determined during the hedging period, but dynamic theory argues that it could be beneficial to change the hedge ratio over time. This involves calculating the hedge ratio based on conditional information instead of unconditional information.

The adjustment to the hedge ratio based on new information can be implemented using conditional models such as ARCH and GARCH, (e.g. Kroner and Sultan (1993)). Alternatively, Lien and Luo (1993b) calculate the hedge ratio by considering the investors' wealth level at the end of the hedging activity via a multi-period model.

3. MODELS

Stone (1973) proves that risk measures such as VaR, variance, semi-variance, or mean absolute deviation are all related to the same generalized loss function - a monotonic concave utility function. Possibly for reasons of mathematical simplicity, Markowitz (1952) chose the standard deviation as the appropriate measure of risk and demonstrates that in the absence of perfect positive correlation, the standard deviation of a portfolio is less than a direct weighted linear combination of the standard deviations of the respective stocks taken individually. Since then the standard deviation and/or variance have become the most widely used measures of risk.

What the Markowitz approach may not capture is the fact that some investors seemingly exhibit a taste for risk. This is true in particular for gamblers. Other researchers have attempted to account for the observed attraction to risk by developing a theory of the gambling effect. The most famous of them is Fishburn (1980) who developed a series of axioms on this topic, which beyond the focus of this thesis.

Prospect Theory, formulated by Kahneman and Tversky (1979), represents a further development in the use of subjective probabilities. The basic tenets of Prospect Theory are that individuals view gains and losses not simply as mirror equivalents of each other but as entirely different experiences. The other inherent difference between Prospect Theory and Expected Utility Theory is that the reference point of Prospect Theory is dependent on the preference of individual investors. Kahneman and Tverksy conduct numerous surveys and find that individuals suffer greater "aggravation" from a loss than from a gain. Thaler (1985) conducts additional surveys within both the general student population and amongst MBA students who have been educated in economics. Findings in the two cases are similar. The survey of Adams and Montesi (1995) suggests that corporate managers are mostly concerned with one-sided risk, in which case only the shortfall from a target level is regarded as risk. Benartzi and Thaler (1995) also argue that, based on the psychology of decision making, individuals are more sensitive to reductions in their levels of wealth than they are to similar increases.

Many researchers indicate that investors treat gains and losses differently. Thus it may be more appropriate to measure risk asymmetrically. An investor may be risk-averse with respect to losses but risk-loving with respect to gains. For example, a manager of an investment fund may be stimulated by a potential bonus and knowingly take on more risk to achieve this goal. At the same time the manager might be very conservative with respect to losses because an extreme negative result could lead to a loss of employment.

Even though a manager may be risk-averse with respect to both losses and gains, sensitivity to these two alternatives may be different. For instance, even though a variance in profit is not welcome to a financial controller, greater sensitivity to possible loss is likely because the consequences of unexpected losses can be disastrous. It seems that there are undesirable aspects to both variance and semi-variance in that variance measurement assumes that investors are indifferent between gains and losses, and the semi-variance model ignores gains completely. A more generalized risk model is provided below:

$$U(\lambda,\theta)=a\int_{-\infty}^{r}(r-Y(\lambda))^{\alpha}\,dFY(\lambda)+b\int_{r}^{+\infty}(r-Y(\theta))^{\beta}\,dGY(\theta)$$

where r = target return, $Y(\lambda)$ = returns below r, $Y(\theta)$ = returns above r,

and α and β are respective measurements of risk aversion below and above the target return. If α (β) > 1, then the investor is risk-averse with respect to losses (gains). If α (β) < 1, then the investor is risk-loving with respect to losses (gains). Finally, risk neutrality is implied when α (β) = 1.

In the model, a and b are rates of value either acquired or lost per marginal unit of gain and loss. When $a > b$, the investor is more sensitive to losses than to gains, and vice versa.

$FY(\lambda)$ and $GY(\theta)$ are the probability distributions of $Y(\lambda)$ and $Y(\theta)$.

When $a = b = 1$, r is the expected return of Y, $\alpha=\beta= 2$, and $FY(\lambda)=GY(\theta)$, the familiar variance becomes the risk measure. When $a = 1$ and $b = 0$, the semi-variance measure obtains.

In this thesis, optimal hedge ratios are obtained by minimizing the generalized risk model. Numerically, the model tested is:

$$\sigma^2 = \frac{1}{N}\left\{a\sum_{i=1}^{N}[r-R_t]^2 U(r-R_t) + b\sum_{i=1}^{N}[r-R_t]^2 U'(r-R_t)\right\}, \qquad (1)$$

where $U(r-R_t)=\begin{cases}1, r \le R_t \\ 0, r > R_t\end{cases}$, $U'(E(R)-R)=\begin{cases}1, r \ge R_t \\ 0, r < R_t\end{cases}$, and N refers to the number of returns.

The values of *a* and *b* are set to 0, 0.25, 0.5, 0.75 and 1, respectively, and *r* (the daily return) is set to -0.001, -0.0005, 0, 0.0005 and 0.001 respectively (which correspond to annual returns of -0.223, -0.118, 0, 0.134 and 0.286 respectively). The risk-aversion parameters α and β are set to 2. The impact of changes to the optimal hedge ratios and the degree of hedging effectiveness can then be observed.

Analogous to the model of hedging effectiveness in Ederington (1979), the measure for the generalized risk model is also defined as the proportional reduction in risk:

$$HE = 1 - \frac{\sigma_{rp}^2}{\sigma_{rs}^2} \qquad (2)$$

where σ_{rp}^2 is the risk of the portfolio, hedged by futures contracts and σ_{rs}^2 is the risk of the spot position without hedging.

4. SAMPLE AND METHODOLOGY

Most of the empirical studies summarized earlier assume that the model-based hedge ratio and hedging effectiveness are based on data from the same period. This is not a realistic assumption. Therefore this study deals with this deficiency by utilizing an out-of-sample test. Malliaris and Urrutia (1991b), Benet (1992), and Geppert (1995) also test hedging effectiveness with out-of-sample tests, but they all use a moving-window approach, which allows parts of previous sample periods to overlap subsequent sample periods. This thesis does not use a moving-window approach, thus the overlapping-samples problem is avoided.

Ideally, a hedger would first use an estimation period to calculate the model-based hedge ratio by minimizing equation (1), and would then apply the ratio in the following period. In each sample period, 32 models are applied and a hedge ratio for each model is obtained. The first two models are variance and semi-variance models, in which $a = b = 1$, and $a = 1$, $b = 0$ respectively, and r is set to 0. In the other thirty models a and b in (1) vary over the range of values 0.25, 0.5, 0.75 and 1 respectively, and r varies over the range of values -0.001, -0.0005, 0, 0.0005 and 0.001. The hedging effectiveness and returns are calculated in- and out-of-sample after an optimal hedge ratio is obtained, as well as for a naïve hedge, where hedge ratio h is set to 1. Finally the hedge ratios in different models are compared, and the hedging effectiveness and returns are compared in- and out-of-sample.

Daily data on spot and futures prices of the S&P 500 index in the International Monetary Market (IMM) of the Chicago Mercantile Exchange (CME) are used. The data are obtained from Datastream International Limited. Six-month T-bill rates are obtained from the web site of the Federal Reserve Bank of St. Louis. For the hedges, data are from December 1982 to December 2004. For each contract, prices three months before the delivery month are used. Model-based hedge ratios are calculated by using daily data for the first two months of a period. Subsequently, a hedge ratio is determined for the last (third) month. This approach is illustrated in Figure 1. Model-based hedge ratios are calculated by using (1).

The data set consists of 89 non-overlapping observations. Tests are conducted to determine whether model-based hedge ratios follow a random walk, and whether the change of a, b and r affect optimal hedge ratios. Tests are also conducted to determine whether hedging effectiveness changes significantly in-sample versus out-of-sample, and between each model and a naïve hedging strategy. Also, the average returns of the hedged portfolios are compared across various models and out-of sample.

To check the robustness of the results, the whole period is divided into three parts and the tests are repeated in the sub-periods. The first sub-period is called the steadily rising period (Figure 2), which is from December 1982 to December 1994. In this period the S&P 500 rose steadily from 138.72 to 459.27, that is, the index increased by a factor of 2.31 in 12 years (an annualized compound growth rate of 7.23 percent). The standard deviation of daily returns for this period was 0.0096 (0.1524 annually). The second sub-period is denoted as the rapidly rising period, which extends from March 1995 to September 2000 (Figure 3). In this period the S&P 500 index increased rapidly from 485.64 to 1436.51, a 1.96 fold increase in five years (an annualized compound growth rate of 14.41 percent). The standard deviation of daily returns is also slightly higher than that of the first period, at 0.0105 (0.1559 annually). The third sub-period is called the fluctuating period, which begins in December 2000 and ends in December 2004. In this period the market experienced both upward and downward fluctuations with the index beginning at 1315.23 and ending at 1211.92 (Figure 4). The standard deviation of daily returns is largest in this period, at 0.0123 (0.1953 annually).

Because markets fluctuate, it is very possible that hedging activity in the rapidly rising period may still be occurring during a bear market. Thus in the second robustness test the condition of the market is inspected every six months. Subsequently a bull (bear) market is defined as one whose return is higher (lower) than that of the six-month T-bill rate in the same period.[1] Then the bull (bear) samples from discontinuous periods are combined together and the hedging performances are compared. In summary, there are 53 samples extracted from bull markets and 36 from bear markets.

5. EMPIRICAL RESULTS

Results are reported in this section for the thirty-two models and five target returns. Model-based hedging is compared with naïve hedging in the whole period, the three sub-periods and both bull and bear market conditions. Three issues are addressed. First, it is to be determined whether different models and different target returns produce hedge ratios that are different from the naïve hedge. Second, whether or not out-of-sample hedging effectiveness will decline compared with in-sample hedging is investigated. Finally, the issue of whether or not returns of model-based hedging are found to be reasonable is examined.

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[1] Jog (1997) used a similar approach in their study.

5.1 Hedge Ratios Determined by Various Risk Sensitivity Parameters and Target Returns

This section reports the hedge ratios of different risk sensitivity parameters and target returns. The absolute proportional difference of each model compared with a naïve hedge is calculated in the three sub-periods and two market conditions, and is reported in Tables 1 to 7.

5.1.1 Hedge Ratios Determined by Various Risk Sensitivity Parameters

In this part the optimal hedge ratios are obtained by minimizing Equation (1). The target return is set to 0, and sensitivity parameters a and b vary from 0 to 1. When $a = 1$ and $b = 0$, the minimum semi-variance model results. Similarly when $a = b = 1$, the minimum-variance (MV) model is obtained. The statistical properties of the optimal hedge ratios of each model are reported in Table 1.

The minimum and maximum hedge ratios are 0.59 (semi-variance model) and 1.09 ($a = 1$ $b = 0.5$) respectively. The standard deviation of hedge ratios is about 0.09. This suggests that the optimal hedge ratios fluctuate over time. Since optimal hedge ratios are used in an out-of-sample setting, variations in the optimal hedge ratios will decrease hedging effectiveness.

Table 1 also indicates that the null hypotheses that the first three autocorrelation coefficients, ρ_1, ρ_2 and ρ_3 are zero can be rejected in all cases. This finding implies that hedge ratios are autocorrelated and models such as ARCH and GARCH may be considered in future research.

Malliaris and Urrutia (1991a) use an augmented Dickey-Fuller test to test the random walk hypothesis. They cannot reject the hypothesis that hedge ratios follow a random walk. The results of the unit root tests in Table 1, however, suggest that the random walk hypothesis can be rejected for all cases. This implies that the hedge ratios do not follow a random walk, and the shocks on hedge ratios will have a transitory effect rather than a permanent impact.

Table 2 reports the proportional changes of hedge ratios of each model compared with the MV model. Because this research is concerned primarily with whether the optimal hedge ratios of generalized risk models are different from the Minimum-variance model, the difference of means is not an appropriate measure in that the ratio obtained from the generalized risk model is sometimes higher than that of the MV model and is sometimes lower than that of the MV model, even though their means may be very close. Thus Table 2 I still set the target return to 0, and compares the proportional difference of other models against the MV model. With H denoting the hedge ratio, the absolute proportional difference is measured with the following formula:

$$R_{\text{Proportion}} = \left| \left(H_{\text{othermodel}} - H_{\text{MV model}} \right) / H_{\text{MV model}} \right|$$

Although the mean difference of the hedge ratios for each model is trivial (smaller than 0.01 according to table 1), the absolute proportional difference is much larger. The average difference between the MV and semi-variance models is 4 percent, and the maximum difference is over 20 percent. For both the a = 1, b = 0.25 model and the a = 0.25, b = 1 model, the average difference is above 2 percent. The closest model to that of the MV model is when a = 0.75, b = 1, with a mean proportional difference of only 0.4 percent. For all models the absolute proportional change is significantly different from zero for both t and sign tests.[2]

Results in Table 2 show that all of the optimal hedge ratios obtained from generalized risk models are statistically different from those obtained in the MV model, and that most of the differences are large enough to generate a material impact on the hedged portfolios (average differences are 2 to 4 percent, and the maximum difference is 20 percent). The findings have practical implications in that if an investor indeed has an asymmetrical risk-aversion sensitivity to lower and higher partial moments, then the MV model is far from optimal since differences between the MV and generalized models are material.

Table 3 reports proportional changes of hedge ratios in the three sub-periods. The results in the three panels are very consistent with those displayed in Table 2. For all three periods the semi-variance model deviates from the MV model by the widest margin with the mean difference being about 4 percent. This amount is large enough to have a significant economic impact. For example, if a portfolio manager of a mutual fund uses minimum-variance models to hedge a one billion dollar portfolio, he may need to sell futures contracts worth on average $40,000,000 more in terms of underlying value than

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[2] Sign test values are the same in this and succeeding tables due to high correlations between models noted in Table 1. This tends to produce the exact same number of data points above and below the mean thereby yielding identical sign test values.

the amount he would need if generalized models were employed. The $a = 1$, $b = 0.25$ and $a = 0.25$, $b = 1$ models follows next with a mean difference of about 2 percent. The $a = 0.75$, $b = 1$ model is the one that is closest to the MV model with a difference of only about 0.4 percent. The difference in all cases for all three sub-periods departs significantly from 0 for both t and sign tests.

Table 4 reports the results in bull and bear market conditions. Again the results are consistent with those of Table 2. Most of the models generate deviations from the MV model large enough to be materially important to hedged portfolios. In conclusion, the results in Table 2 are robust with respect to the different sub-periods examined and to varying market conditions.

5.1.2 Hedge Ratios Determined By Various Target Returns

Table 5 reports the proportional difference in hedge ratios under various target returns compared with the ratio when r equals 0. The formula used in the comparison is:

$$R_{\text{Proportion}} = \left| \left(H_{\text{r=R}} - H_{\text{r=0}} \right) / H_{\text{r=0}} \right|$$

From Table 5 it can be observed that for all models the difference in hedge ratio performance between the r = R model and $r = 0$ model is significantly different from 0 for both t and sign tests. The model with $r = 0.001$ always deviates the most from the $r = 0$ model, and the deviation of the $r = 0.005$ model is always smallest. Even though the differences are statistically significant, the economic significance of the differences is minor. For instance, the largest deviation occurs when $r = -0.001$ for the $a = 0.25$, $b = 1$

model, and the average difference is only 0.55 percent. Investment managers will not adjust their portfolios for such trivial deviations. Thus it appears that the selection of the target return is not a key factor in the determination of optimal hedge ratios.

Tables 6 and 7 report the proportional difference of returns compared with the $R = 0$ model in continuous sample periods (steadily rising, rapidly rising, and fluctuating) and discontinuous sample periods (bear and bull). The proportional change is still different from 0 statistically but again does not have any strong economic impact. For example, the largest deviation in continuous and discontinuous sample periods is only 0.0067 (during the rapidly rising period) and 0.0057 (in the bull market) respectively.

5.2 Comparisons of Hedging Effectiveness

In this section three kinds of hedging effectiveness for various models are calculated by using Equation (2). The first one is in-sample hedging effectiveness and is determined in the period in which optimal hedge ratios are estimated. The second one is out-of-sample hedging effectiveness, which is obtained by applying the optimal hedge ratios in the testing period. The last one is out-of-sample naïve effectiveness, which is calculated by setting the optimal hedge ratio to 1 in the testing period.

The statistical properties of three categories of hedging effectiveness are reported in Table 8. All of the hedging approaches listed are above or slightly below 90 percent. This implies that significant risk reduction has been achieved compared with unhedged portfolios. Of the three hedging scenarios, the effectiveness of out-of-sample naïve hedging is the poorest, and its standard deviation is also the largest. This outcome was expected a priori.

Table 9 uses a *t*-test to compare the mean difference between the out-of-sample hedging scenario and the out-of-sample naïve hedging alternative as well as the mean difference between the out-of-sample and the in-sample hedging scenarios. From Panel A of Table 9 it can be observed that the difference of means between model effectiveness and naïve effectiveness is positive for all models with mean differences ranging from 0.0098 (in the a = 1, b = 0.75 model) to 0.0375 (in the semi-variance model). All the differences in means are significantly different from zero. The results suggest that hedge ratios obtained from generalized risk models do improve the hedging effectiveness compared with that of a naïve hedge. Moreover, because the results are obtained from out-of-sample tests, they are more convincing than results obtained from in-sample tests.

The second part of Table 9 displays the mean difference between in-sample hedging effectiveness and out-of-sample hedging effectiveness and permits the examination of whether or not hedging effectiveness declined in the out-of-sample period. As forecast, the difference in means between the two is positive. However, except for the semi-variance model, in which difference reaches 0.0289, the differences for all other models is much smaller than the difference between model effectiveness and naïve effectiveness and ranges from 0.0003 (in the a = 0.75, b = 1 model) to 0.0062 (in the a = 0.25, b = 1 model). Again except for the semi-variance model, the mean difference of other models is not significantly different from zero. These results imply that if an investor uses a two-month sample period to estimate the optimal hedge ratio and proceeds to apply the estimated hedge ratio in the third month, hedging effectiveness will not change dramatically compared to that of in-sample hedging.

Table 10 repeats the above tests in the three sub-periods. For the steadily rising period (Panel A), the results are similar to those of the whole period. However, for the rapidly rising period and the fluctuating period, results are somewhat inconsistent. The improvements for model hedging effectiveness compared with the naïve out-of-sample hedge in the rapidly rising period (Panel B) are all small and not significantly different from 0. On the other hand, the difference between in-sample hedging effectiveness and out-of-sample hedging effectiveness is relatively large compared with other periods (four of them are significantly different from 0). These results imply that in this period hedging effectiveness not only declined in the out-of-sample period but also did not outperform naïve hedging. In the fluctuating period (Panel C) both the improvement of model-hedging effectiveness and the difference between in-sample and out-of-sample results are not significantly different from 0.

Table 11 reports the results from bull and bear market conditions. The bull market (Panel A) shows favourable results for model hedging, in that there are improvements for model hedging while out-of-sample means are similar to similar means compared with in-sample results. Bear markets (Panel B) also provide evidence to support the position that there is no decline in out-of-sample hedging effectiveness, but the comparison between the model hedge and naïve hedge show mixed results - only three models (variance, $a = 1$, $b = 0.25$ and $a = 1$, $b = 0.5$) outperform the naïve hedge.

Table 10 and Table 11 report conflicting results about bull markets. In table 10, hedging in rapidly rising markets provides unfavourable results for model hedging. However, Table 11 shows that model hedging performs rather well in bull markets. Two possible reasons may explain the difference. The first is that in the rapidly rising period there are some bear market periods and these results may have affected the whole period. The second possible explanation is that the small sample size (23) makes the t-test somewhat unreliable.

In summary, the out-of-sample model hedging performance seems to do well in bull markets but may not outperform the naïve hedge in bear markets for some models.

5.3 Returns of Each Model

Although the main purpose of hedging is to reduce risk, the return of hedged portfolios is also an important factor to be considered by investment managers. Table 12 reports the average daily returns of the S&P 500 index and the hedged portfolios in the various hedging periods.

All of the returns are positive, and the null hypothesis that the mean daily return is equal to 0 is rejected for all models. The return of the index is highest among all portfolios. It reached a level of 0.0498 percent, while the return for the naïve hedge, at around 25 percent of the index return, is the poorest performer. The returns for all other models are similar and are about 40 percent of the index return.

The risk of each model is measured according to the formula specified by each model. In Table 12 the row marked "standard deviation" is not the usual meaning of standard deviation but instead represents the square root of the generalized risk measure. The index return is the highest of all portfolios with the highest risk, thus it is difficult to judge whether it is superior to other models. It seems clear that the model hedge is much better than the naïve hedge because the return of the naïve hedge is only one half of that of the model hedge but its risk is still higher than that of the model hedge. In conclusion, although generalized risk models are designed to minimize risk, the returns of the hedged portfolios based on these models in out-of-sample tests are still reasonable.

6. Conclusions and Suggestions for Further Research

This thesis points out the inherent flaws of the two most widely used measures of risk, namely variance/standard deviation and semi-variance. These measures are used extensively in hedging activities. A more generalized measure of risk is proposed in an effort to obtain improved hedging results.

This study is the first attempt to consider both higher partial moments and lower partial moments in a hedging perspective. Two risk sensitivity parameters are provided to show the asymmetrical risk preference for the two moments, and the models are applied in out-of-sample tests to determine optimal hedge ratios for S&P 500 index portfolios.

This thesis also considers out-of-sample tests and non-overlapping windows while most other studies use in-sample tests exclusively. Finally, this thesis uses discontinuous sample periods to separate market conditions in an attempt to assess hedging performance under different market settings.

The main purpose of the thesis is to study whether the optimal hedge ratio obtained by minimizing the set of generalized risk models is significantly different from those obtained from the minimum-variance model. The results from 89 samples suggest that the absolute proportional differences between the MV model and the generalized risk models are significantly different from zero (the average difference reached 2 to 4 percent, and the maximum reached 20 percent), with the results being consistent in the three sub-periods as well. The results lend support to the position that if an investor's risk-aversion sensitivities to lower and higher partial moments are not equal, the MV model should not be considered as a preferred approach in obtaining optimal hedge ratios.

This thesis also investigates whether changing target returns can have an influence on optimal hedge ratios. Daily target returns in the model vary from -0.001 to 0.001 and their optimal hedge ratios are compared to a scenario where the target return is 0. The largest deviation is only 0.45 percent, which will not affect the hedged portfolio materially. Thus target returns levels do not appear to be a primary factor in the determination of optimal hedge ratios.

In this thesis, comparisons are made between differences in hedging effectiveness in-sample versus out-of-sample. The results show that over the whole period, tests of the difference between out-of-sample model hedging effectiveness and the out-of-sample naïve hedging effectiveness are positive and significantly different from zero for all models. This suggests that hedge ratios obtained from the generalized risk models improve the hedging effectiveness compared with that of naïve hedging. On the other hand, in-sample hedging effectiveness is not significantly different from that of out-of-sample hedging effectiveness (except for the semi-variance model), which implies that if an investor uses a two-month estimation period to determine the optimal hedge ratio and hedges in the third month, the hedging effectiveness will not change materially compared with that of in-sample hedging. However, the robustness test concludes that the previous finding is more likely to be valid in bull markets. Moreover model-based hedging performance is not particularly good and may not outperform a naïve hedge in bear markets.

Finally, this paper examines out-of-sample returns for each model, and finds that the generalized risk model outperforms the naïve hedge when target returns are considered.

Two suggestions are offered for further research. For reasons of simplicity, this thesis fixes the power variable (α and β, i.e. the risk-aversion parameters), to two, and only investigates the impact brought about by the change in sensitivity parameters (a and b) and target returns. In the future it would be interesting to study the impact of these risk-aversion parameters.

In a previous study, Bera, Garcia and Roh (1997) consider the hedge ratio to be time varying and adopt a random coefficient approach in their regression model. Additionally, bivariate GARCH models have been more widely adopted in an effort to examine dynamic hedging strategies (Baillie and Myers (1991); Myers (1991); Lien and Luo (1994)). Considering the finding that three orders of autocorrelation coefficients for hedge ratios (Table 1) are significantly different from zero, it would be useful in subsequent work to employ conditional models such as ARCH and GARCH to examine the generalized risk model.

Figure 1. Illustration of Procedure Used in Sample Collection and the Calculation of Optimal Hedge Ratios

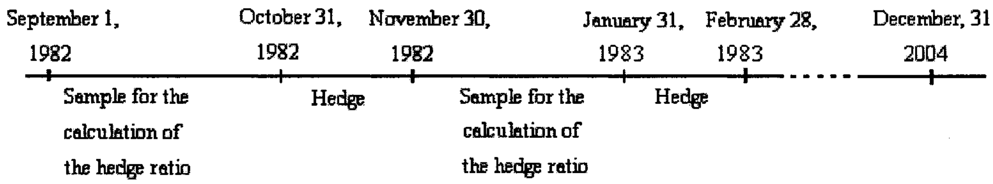

Figure 2. Steadily Rising Period 1982/12-1994/12 (49 Obs) (Index rose from 138.72 to 459.27)

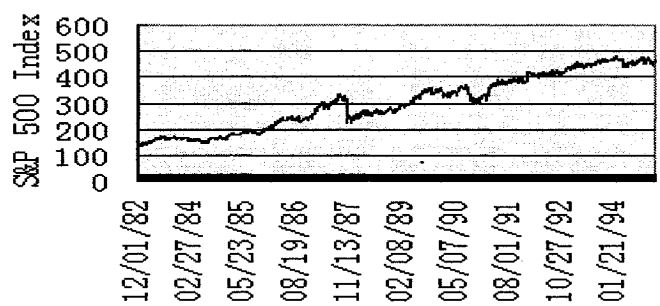

Figure 3: Rapidly Rising Period 1995/01-2000/09 (23 Obs) (Index rose from 459.27 to 1436.51)

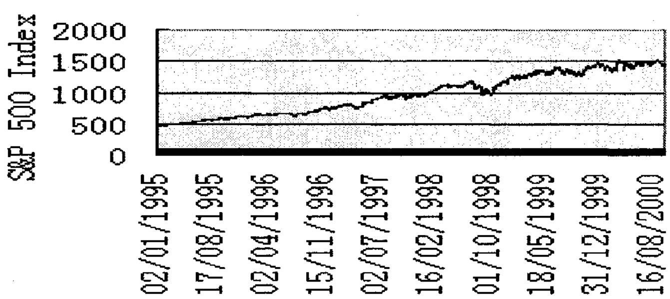

Figure 4. Fluctuating Period 2000/10-2004/12 (17 Obs) (Index decreased from 1436.23 to 1211.92)

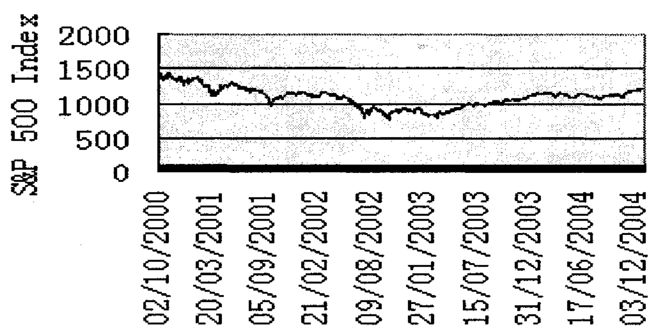

Table 1. Statistical Properties of the Optimal Hedge Ratios

	Minimum Variance	Semi-variance	a =0.25 b =1	a =0.5 b =1	a =0.75 b =1	a =1 b =0.25	a =1 b =0.5	a =1 b =0.75
				Summary Statistics				
Mean	0.8900	0.8836	0.8872	0.8895	0.8910	0.8974	0.8951	0.8938
Minimum	0.6256	0.5898	0.6381	0.6355	0.6338	0.6381	0.6355	0.6338
Maximum	1.0719	1.0569	1.0372	1.0535	1.0648	1.1007	1.0888	1.0797
Std. Dev.	0.0880	0.1009	0.0912	0.0880	0.0869	0.0887	0.0871	0.0867
				Correlations				
Variance	1.0000							
Semi-variance	0.9387	1.0000						
a =0.25, b =1	0.9827	0.9827	1.0000					
a =0.5, b =1	0.9958	0.9628	0.9952	1.0000				
a =0.75, b =1	0.9993	0.9458	0.9873	0.9981	1.0000			
a =1, b =0.25	0.9822	0.8640	0.9331	0.9628	0.9774	1.0000		
a =1, b =0.5	0.9883	0.8873	0.9490	0.9736	0.9848	0.9946	1.0000	
a =1, b =0.75	0.9935	0.9077	0.9629	0.9830	0.9912	0.9902	0.9986	1.0000
				Autocorrelation				
ρ_1	0.588***(a)	0.4912***	0.5669***	0.5830***	0.5836***	0.5324***	0.5589***	0.5762***
ρ_2	0.4731***	0.4196***	0.4589***	0.4697***	0.4714***	0.4466***	0.4648***	0.4765***
ρ_3	0.3848***	0.3650***	0.4006***	0.3980***	0.3903***	0.3235***	0.3590***	0.3796***
				Unit-Root Tests				
ADF Test	-24.80***(b)	-27.81***	-25.42***	-24.90***	-24.89***	-26.78***	-25.46***	-24.62***

The table entries present summary statistics for various optimal hedge ratios. All calculations are based on the sample period from December 1982 to December 2004, giving a total of 89 non-overlapping observations.

Here a and b are the risk sensitivity parameters in the generalized risks model $\sigma^2 = \frac{1}{N}\left\{a\sum_{t=1}^{N}[r-R_t]^2 U(r-R_t) + b\sum_{t=1}^{N}[r-R_t]^2 U'(r-R_t)\right\}$

***Indicates significance at the 1% level.

(a) The null hypothesis is that the first-order autocorrelation coefficient equals zero. (b) The augmented Dickey–Fuller (ADF) test assumes a null hypothesis that the optimal hedge ratios in each case follow a random walk.

Table 2. Proportional Changes Compared with the Minimum Variance Model for the Whole Period

	Minimum Semi-variance	a =0.25 b =1	a =0.5 b =1	a =0.75 b =1	a =1 b =0.25	a =1 b =0.5	a =1 b =0.75
Mean	0.0393	0.0209	0.0106	0.0044	0.0222	0.0151	0.0090
Maximum	0.2024	0.0931	0.0385	0.0222	0.1068	0.1254	0.1180
Minimum	0.0009	0.0006	0.0002	0.0001	0.0006	0.0003	0.0001
t-test	9.57***	11.23***	12.91***	10.79***	11.09***	8.17***	5.8***
Sign test	44.5***	44.5***	44.5***	44.5***	44.5***	44.5***	44.5***

This table sets target returns to 0, and compares the proportional change of other models against the minimum variance model. The formula used for comparison is

$$R_{\text{Proportion}} = \left| \left(H_{\text{othermodel}} - H_{\text{MV model}} \right) / H_{\text{MV model}} \right|.$$

Here a and b are the risk sensitivity parameters in the generalized risks model: $\sigma^2 = \frac{1}{N}\left\{ a\sum_{i=1}^{N}[r-R_t]^2 U(r-R_t) + b\sum_{i=1}^{N}[r-R_t]^2 U'(r-R_t) \right\}$

The null hypothesis is that the mean of the proportional difference equals zero.

***Indicates significance at the 1% level.

Table 3. Proportional Changes Compared with the Minimum Variance Model in Three Sub-periods

	Minimum Semi-variance	a =0.25 b =1	a =0.5 b =1	a =0.75 b =1	a =1 b =0.25	a =1 b =0.5	a =1 b =0.75
Panel A (Steadily rising period, 49 Obs)							
Mean	0.0417	0.0218	0.0110	0.0045	0.0235	0.0145	0.008
Maximum	0.2024	0.0931	0.0352	0.0222	0.1068	0.0722	0.0448
Minimum	0.0010	0.0007	0.0002	0.0001	0.0006	0.0003	0.0001
t-test	6.61***	8.23***	10.04***	7.93***	8.17***	7.79***	6.72***
Sign test	24.50***	24.50***	24.50***	24.50***	24.50***	24.50***	24.50***
Panel B (Rapidly rising period, 23 Obs)							
Mean	0.0380	0.0200	0.0101	0.0043	0.0241	0.0149	0.0083
Maximum	0.1362	0.0614	0.0274	0.0149	0.0795	0.0544	0.0346
Minimum	0.0009	0.0007	0.0004	0.0001	0.0012	0.0007	0.0003
t-test	5.73***	6.24***	7.19***	5.62***	6.37***	5.73***	6.72***
Sign test	11.50***	11.50***	11.50***	11.50***	11.50***	11.50***	11.50***
Panel C (Fluctuating period, 17 Obs)							
Mean	0.0343	0.0193	0.0103	0.0043	0.0162	0.0169	0.0124
Maximum	0.1224	0.0713	0.0385	0.0156	0.0668	0.1254	0.1180
Minimum	0.0059	0.0042	0.0027	0.0006	0.0020	0.0006	0.0005
t-test	4.52***	4.42***	4.42***	4.43***	4.18***	2.28**	1.76*
Sign test	8.50***	8.50***	8.50***	8.50***	8.50***	8.50***	8.50***

This table repeats tests of table 2 in three sub-periods. Panel A reports the results in the steadily rising period, from 1982/12 to 1994/12 (49 observations), when the index rose from 138.72 to 459.27. Panel B reports results in the rapidly rising period, from 1982/12 to 1994/12 (23 observations), when the index rose from 485.64 to 1436.51. Panel C reports results in the fluctuating period, from 2000/12 to 2004/12 (17 observations), when the index decreased from 1315.23 to 1211.92.

Here a and b are the risk sensitivity parameters in the generalized risks model: $\sigma^2 = \frac{1}{N}\left\{a\sum_{i=1}^{N}[r-R_t]^2 U(r-R_t) + b\sum_{i=1}^{N}[r-R_t]^2 U'(r-R_t)\right\}$

The null hypothesis is that the mean of the proportional difference equals zero.

*** Indicates significance at the 1% level. ** Indicates significance at the 5% level. * Indicates significance at the 10% level.

Table 4. Proportional Changes Compared with the Minimum Variance Model in Bull and Bear Market Contiditions (Discontinuous Periods)

	Minimum Semi-variance	a =0.25 b =1	a =0.5 b =1	a =0.75 b =1	a =1 b =0.25	a =1 b =0.5	a =1 b =0.75
				Panel A (Bull market, 53 Obs)			
Mean	0.0390	0.0207	0.0106	0.0047	0.0233	0.0145	0.0082
Median	0.0288	0.0178	0.0091	0.0034	0.0173	0.0107	0.0054
Maximum	0.1757	0.0618	0.0274	0.0222	0.0795	0.0544	0.0346
Minimum	0.0015	0.0011	0.0012	0.0002	0.0035	0.0009	0.0008
t-test	7.76***	9.98***	11.94***	8.35***	9.59***	9.02***	7.44***
Sign test	26.50***	26.50***	26.50***	26.50***	26.50***	26.50***	26.50***
				Panel B (Bear market, 36 Obs)			
Mean	0.0398	0.0211	0.0107	0.0040	0.0208	0.0159	0.0102
Median	0.0257	0.0155	0.0076	0.0032	0.0180	0.0110	0.0049
Maximum	0.2024	0.0931	0.0385	0.0156	0.1068	0.1254	0.1180
Minimum	0.0009	0.0006	0.0002	0.0001	0.0006	0.0003	0.0001
t-test	5.64***	6.09***	6.77***	6.84***	5.98***	4.05***	2.91***
Sign test	18.00***	18.00***	18.00***	18.00***	18.00***	18.00***	18.00***

This table repeats tests of table 2 in bull and bear market conditions. The market conditions are inspected every six months and a bull (bear) market is defined as one whose return is higher (lower) than that of the six-month T-bill rate. Panel A reports the results in bull market (53 observations) and panel B reports the results obtained from bear market (36 observations).

The a and b are the risk sensitivity parameters in the generalized risks model: $\sigma^2 = \frac{1}{N}\left\{a\sum_{i=1}^{N}[r-R_t]^2 U(r-R_t) + b\sum_{i=1}^{N}[r-R_t]^2 U'(r-R_t)\right\}$

The null hypothesis is that the mean of the proportional difference equals zero.

*** Indicates significance at the 1% level.

Table 5. Proportional Changes in Hedge Ratios under Various Target Returns Compared with R = 0 (Whole Period)

Target Return	a =0.25 b =1	a =0.5 b =1	a =0.75 b =1	a =1 b =0.25	a =1 b =0.5	a =1 b =0.75
R = 0.0005	0.0015***	0.0009***	0.0004***	0.0018***	0.0012***	0.0005***
R = 0.001	0.004***	0.0027***	0.0010***	0.0045***	0.0030***	0.0015***
R = -0.0005	0.0018***	0.0009***	0.0005***	0.0017***	0.00086***	0.0005***
R = -0.001	0.0055***	0.0027***	0.0010***	0.0038***	0.0021***	0.0012***

This table reports the percentage difference of hedge ratios under various target returns compared with the ratio when R equals 0. The formula is $R_{\text{Proportion}} = \left|(H_{r=R} - H_{R=0}) / H_{R=0}\right|$.

The a and b are the risk sensitivity parameters in the generalized risks model: $\sigma^2 = \frac{1}{N}\left\{a\sum_{i=1}^{N}[r-R_t]^2 U(r-R_t) + b\sum_{i=1}^{N}[r-R_t]^2 U'(r-R_t)\right\}$

The null hypothesis is that the mean of the proportional difference equals zero.

*** Indicates significance at the 1% level.

Table 6. Change of Proportions in Hedge Ratios under Various Target Returns Compared with R = 0 in Three Sub-periods

Target Return	a =0.25 b =1	a =0.5 b =1	a =0.75 b =1	a =1 b =0.25	a =1 b =0.5	a =1 b =0.75
Panel A (Steadily rising period, 49 Obs)						
R = 0.0005	0.0017***	0.0010***	0.0004***	0.0018***	0.0012***	0.0005***
R = 0.001	0.0052***	0.0026***	0.0011***	0.0045***	0.0030***	0.0015***
R = -0.0005	0.0018***	0.0009***	0.0004***	0.0017***	0.0009***	0.0005***
R = -0.001	0.0055***	0.0033***	0.0011***	0.0038***	0.0021***	0.0012***
Panel B (Rapidly rising period, 23 Obs)						
R = 0.0005	0.0011***	0.0007***	0.0004***	0.0017***	0.0012***	0.0003***
R = 0.001	0.0037***	0.0033***	0.0010***	0.0045***	0.0031***	0.0017***
R = -0.0005	0.0016***	0.0010***	0.0007***	0.0018***	0.0011***	0.0007***
R = -0.001	0.0067***	0.0025***	0.0012***	0.0045***	0.0025***	0.0014***
Panel C (Fluctuating period, 17 Obs)						
R = 0.0005	0.0014***	0.0010***	0.0004***	0.0018***	0.0010***	0.0008***
R = 0.001	0.0031***	0.0020***	0.0007***	0.0028***	0.0027***	0.0010***
R = -0.0005	0.0023***	0.0009***	0.0003***	0.0011***	0.0006***	0.0003***
R = -0.001	0.0036***	0.0013***	0.0007***	0.0026***	0.0015***	0.0007***

This table repeats the tests in table 5 and reports the proportional difference of hedge ratios under various target returns compared with the ratio when R equals 0 in each sub-period.

Panel A reports results for the steadily rising period; Panel B reports the rapidly rising period; and Panel C provides results for the fluctuating period.

The null hypothesis is that the mean of proportional difference equals zero.

The a and b are the risk sensitivity parameters in the generalized risks model: $\sigma^2 = \frac{1}{N}\left\{a\sum_{i=1}^{N}[r-R_t]^2 U(r-R_t) + b\sum_{i=1}^{N}[r-R_t]^2 U'(r-R_t)\right\}$

*** Indicates significance at the 1% level.

Table 7. Change of Proportions in Hedge Ratios under Various Target Returns Compared with $R = 0$ in Bull and Bear Markets

Target Return	a =0.25 b =1	a =0.5 b =1	a =0.75 b =1	a =1 b =0.25	a =1 b =0.5	a =1 b =0.75
Panel A (Bull market, 53 Obs)						
R = 0.0005	0.0017***	0.0008***	0.0005***	0.0020***	0.0013***	0.0005***
R = 0.001	0.0038***	0.0028***	0.0009***	0.0046***	0.0027***	0.0014***
R = -0.0005	0.0019***	0.0010***	0.0005***	0.0019***	0.0010***	0.0005***
R = -0.001	0.0057***	0.0025***	0.0011***	0.0041***	0.0022***	0.0012***
Panel B (Bear market, 36 Obs)						
R = 0.0005	0.0012***	0.0011***	0.0003***	0.0016***	0.0010***	0.0004***
R = 0.001	0.0053***	0.0025***	0.0011***	0.0043***	0.0035***	0.0016***
R = -0.0005	0.0018***	0.0007***	0.0004***	0.0013***	0.0006***	0.0004***
R = -0.001	0.0051***	0.0030***	0.0010***	0.0034***	0.0019***	0.0012***

Table repeats the tests in table 5 and reports the proportional difference of hedge ratios under various target returns compared with the ratio when R equals 0 in varying market conditions.

Panel A reports the results for bull markets and Panel B reports the results for bear markets.

The a and b are the risk sensitivity parameters in the generalized risks model: $\sigma^2 = \frac{1}{N}\left\{a\sum_{i=1}^{N}[r-R_t]^2 U(r-R_t) + b\sum_{i=1}^{N}[r-R_t]^2 U'(r-R_t)\right\}$

The null hypothesis is that the mean of proportional difference equals zero.

*** Indicates significance at the 1% level.

Table 8. Statistical Properties of Hedging Effectiveness

	Variance	Semi-Variance	a =0.25 b =1	a =0.5 b =1	a =0.75 b =1	a =1 b =0.25	a =1 b =0.5	a =1 b =0.75
Panel A. In-sample Model Hedge								
Mean	0.9109	0.9051	0.9093	0.9108	0.9115	0.9116	0.9125	0.9127
Minimum	0.7074	0.6477	0.6781	0.6994	0.7138	0.6356	0.6689	0.6929
Maximum	0.9876	0.9921	0.9902	0.9889	0.9880	0.9889	0.9879	0.9872
Std. Dev.	0.0569	0.0708	0.0607	0.0575	0.0564	0.0596	0.0570	0.0560
Panel B. Out-of-sample Model Hedge								
Mean	0.9092	0.8762	0.9031	0.9089	0.9112	0.9088	0.9116	0.9121
Minimum	0.7743	-0.1218	0.6000	0.6277	0.6374	0.6832	0.6649	0.6532
Maximum	0.9871	0.9881	0.9879	0.9878	0.9882	0.9905	0.9896	0.9889
Std. Dev.	0.0545	0.1469	0.0715	0.0610	0.0577	0.0615	0.0574	0.0564
Panel C. Out-of-sample Naïve Hedge								
Mean	0.8948	0.8387	0.8900	0.8984	0.9012	0.8961	0.9011	0.9024
Minimum	0.6195	-0.5711	0.4989	0.6057	0.6158	0.5706	0.6346	0.6298
Maximum	0.9889	0.9889	0.9889	0.9889	0.9889	0.9888	0.9888	0.9888
Std. Dev.	0.0748	0.2403	0.0874	0.0725	0.0691	0.0866	0.0756	0.0709

This table reports the statistical properties of the hedging effectiveness of the three hedging strategies.

The formula used to calculate the hedging effectiveness is: $HE = 1 - \frac{\sigma_{rp}^2}{\sigma_{rs}^2}$

The first hedging strategy considers the minimization of generalized risks models using in-sample data; the second hedging strategy applies estimated optimal hedge ratios in out-of-sample periods; and the last strategy applies a naïve hedge by setting the hedge ratio to 1 in out-of-sample data.

The a and b are the risk sensitivity parameters in the generalized risks model: $\sigma^2 = \frac{1}{N}\left\{a\sum_{i=1}^{N}[r-R_t]^2 U(r-R_t) + b\sum_{i=1}^{N}[r-R_t]^2 U'(r-R_t)\right\}$

Table 9. Comparison of Hedging Effectiveness (Whole Period)

	Variance	Semi-Variance	a =0.25 b =1	a =0.5 b =1	a =0.75 b =1	a =1 b =0.25	a =1 b =0.5	a =1 b =0.75
Panel A. Difference between Model Hedge and Naïve Hedge								
Mean	0.0143	0.0375	0.0131	0.0104	0.01	0.0126	0.0106	0.0098
Lowest	0.0067	0.0022	0.0032	0.0023	0.0026	0.0044	0.0034	0.0028
Highest	0.022	0.0729	0.0231	0.0186	0.0174	0.0208	0.0177	0.0167
t-test	3.71***	2.11**	2.63***	2.55**	2.69***	3.07***	2.95***	2.80***
Panel B. Difference between In-sample Hedge and Out-of-sample Hedge								
Mean	0.0017	0.0289	0.0062	0.002	0.0003	0.0029	0.0009	0.0005
Lowest	-0.011	-0.004	-0.011	-0.013	-0.013	-0.011	-0.012	-0.013
Highest	0.014	0.0615	0.0231	0.0165	0.0141	0.0169	0.0141	0.0137
t-test	0.27	1.77*	0.73	0.27	0.04	0.40	0.13	0.08

Panel A of this table compares the mean difference between out-of-sample model hedging effectiveness and out-of-sample naive hedging effectiveness. Panel B compares the mean difference between hedging effectiveness of the model in-sample versus out-of-sample.

Here a and b are the risk sensitivity parameters in the generalized risks model: $\sigma^2 = \frac{1}{N}\left\{a\sum_{i=1}^{N}[r-R_i]^2 U(r-R_i)+b\sum_{i=1}^{N}[r-R_i]^2 U'(r-R_i)\right\}$

The null hypothesis is that the mean difference equals zero.

*** Indicates significance at the 1% level. ** Indicates significance at the 5% level.

Table 10. Comparison of Hedging Effectiveness (Three Sub-periods)

	Variance	Semi-Variance	a =0.25 b =1	a =0.5 b =1	a =0.75 b =1	a =1 b =0.25	a =1 b =0.5	a =1 b =0.75
Panel A (Steadily rising period, 49 Obs)								
Difference between Model Hedge and Naïve Hedge								
Mean	0.0216	0.0344	0.0199	0.0166	0.016	0.0197	0.0171	0.016
t -test	3.36***	2.62**	2.38**	2.38**	2.53**	2.90***	2.87***	2.73***
Difference between In-sample Hedge and Out-of-sample Hedge								
Mean	-0.003	0.0347	0.0029	-0.002	-0.004	0.0001	-0.003	-0.004
t-test	-0.28	1.21	0.20	-0.19	-0.36	0.01	-0.30	-0.38
Panel B (Rapidly rising period, 23 Obs)								
Difference between Model Hedge and Naïve Hedge								
Mean	0.0085	0.0749	0.0103	0.0061	0.0051	0.0056	0.0046	0.0043
t-test	1.71	1.19	1.55	1.40	1.28	1.03	0.99	1.03
Difference between In-sample Hedge and Out-of-sample Hedge								
Mean	0.0168	0.0332	0.0184	0.015	0.0135	0.0141	0.0132	0.0131
t-test	1.94*	2.26**	1.88*	1.73*	1.63	1.55	1.55	1.58
Panel C (Fluctuating period, 17 Obs)								
Difference between Model Hedge and Naïve Hedge								
Mean	0.00150	-0.00400	-0.00200	-0.00200	-0.00080	0.00150	-0.00012	-0.0007
t-test	0.56	-1.35	-0.93	-0.64	-0.30	0.48	-0.04	-0.29
Difference between In-sample Hedge and Out-of-sample Hedge								
Mean	-0.0050	0.0065	-0.0008	-0.0030	-0.0050	-0.0040	-0.0040	-0.0030
t-test	-0.59	0.55	-0.08	-0.34	-0.50	-0.57	-0.49	-0.38

This table repeats the tests of table 9. Panel A reports the results for the steadily rising period, from 1982/12 to 1994/12 (49 observations), when the index rose from 138.72 to 459.27. Panel B reports results in the rapidly rising period, from 1998/01 to 2000/09 (23 observations), when the index rose from 485.64 to 1436.51. Panel C reports results in the fluctuating period, from 2000/12 to 2004/12 (17 observations), when the index decreased from 1315.23 to 1211.92.

Here a and b are the risk sensitivity parameters in the generalized risks model: $\sigma^2 = \frac{1}{N}\left\{a\sum_{i=1}^{N}[r-R_t]^2 U(r-R_t) + b\sum_{i=1}^{N}[r-R_t]^2 U'(r-R_t)\right\}$

The null hypothesis is that the mean difference equals zero.

*** Indicates significance at the 1% level. ** Indicates significance at the 5% level. * Indicates significance at the 10% level.

Table 11. Comparison of Hedging Effectiveness (Bull and Bear markets)

	Variance	Semi-Variance	a =0.25 b =1	a =0.5 b =1	a =0.75 b =1	a =1 b =0.25	a =1 b =0.5	a =1 b =0.75
Panel A (Bull market, 53 Obs)								
Difference between Model Hedge and Naive Hedge								
Mean	0.0158	0.0571	0.0208	0.016	0.0144	0.0129	0.0235	0.0127
t -test	2.86***	2.02**	3.18***	2.81***	2.65**	2.22**	2.29**	2.36**
Difference between In-sample Hedge and Out-of-sample Hedge								
Mean	0.0043	0.0282	0.0094	0.0046	0.0022	-0.00001	-0.00014	0.0005
t -test	0.57	1.90*	1.09	0.60	0.29	-0.00	-0.02	0.07
Panel B (Bear market, 36 Obs)								
Difference between Model Hedge and Naive Hedge								
Mean	0.0121	0.0087	0.0019	0.0023	0.0035	0.0122	0.0078	0.0055
t -test	2.40**	0.65	0.26	0.42	0.81	2.18**	2.03**	1.58
Difference between In-sample Hedge and Out-of-sample Hedge								
Mean	-0.002	0.03	0.0014	-0.002	-0.002	0.0071	0.0024	0.0005
t -test	-0.18	0.87	0.08	-0.13	-0.18	0.53	0.19	0.04

This table repeats the tests of Table 9. Panel A reports the results in the bull market condition and Panel B reports those of the bear market condition.

Here a and b are the risk sensitivity parameters in the generalized risks model: $\sigma^2 = \frac{1}{N}\left\{a\sum_{i=1}^{N}[r-R_t]^2 U(r-R_t)+b\sum_{i=1}^{N}[r-R_t]^2 U'(r-R_t)\right\}$

The null hypothesis is that the mean difference equals zero.

***Indicates a significance level of 1%. **Indicates a significance level of 5%.

Table 12. Mean Daily Returns of the S&P 500 and Out-of-sample Hedged Portfolios (1982/12 – 2004/12)

	Index	Naïve	Variance	Semi-Variance	a =0.25 b =1	a =0.5 b =1	a =0.75 b =1	a =1 b =0.25	a =1 b =0.5	a =1 b =0.75
Mean	0.0005	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
Median	0.0007	0.00008	0.00012	0.0001	0.0001	0.00011	0.00012	0.00012	0.00012	0.00012
Daily Std. Dev.	0.0104	0.0029	0.0026	0.0019	0.0021	0.0023	0.0025	0.0025	0.0023	0.0021
Annualized Std. Dev.	0.1651	0.0460	0.0413	0.0302	0.0333	0.0365	0.0397	0.0397	0.0365	0.0333
Skewness	-0.4745	4.5142	0.6218	0.3039	0.5184	0.5683	0.6023	0.6822	0.6820	0.6876
Kurtosis	0.0629	31.0261	0.4213	1.7893	1.1604	0.7321	0.5278	0.2450	0.3222	0.4018
t test	2.3434	4.3230	5.4000	4.9047	5.1392	5.2840	5.3672	5.5614	5.6118	5.6346
p-value	0.0200	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001
Sign test	14.50	16.50	15.50	16.50	16.50	17.50	16.50	17.50	16.50	16.50
p-value	0.0028	0.0006	0.0013	0.0006	0.0006	0.0003	0.0006	0.0003	0.0006	0.0006

This table reports mean daily returns of S&P 500 index and hedged portfolios over the sample period.

Here a and b are the risk sensitivity parameters in the generalized risks model: $\sigma^2 = \frac{1}{N}\left\{a\sum_{i=1}^{N}[r-R_i]^2 U(r-R_i) + b\sum_{i=1}^{N}[r-R_i]^2 U'(r-R_i)\right\}$

The null hypothesis is that the mean of the daily returns equals zero.

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