Fuzzy Logic Based Assignable Cause Diagnosis Using Control Chart Patterns

SUJIKUMAR VIJAYAKUMAR

A Thesis

in

The Department

of

Mechanical and Industrial Engineering

Presented in partial fulfillment of the requirements for the
Degree of Master of Applied Science (Mechanical Engineering) at
Concordia University
Montreal, Quebec, Canada

August 2006

Your file Votre référence
ISBN: 978-0-494-20765-9
Our file Notre référence
ISBN: 978-0-494-20765-9

ABSTRACT

Fuzzy Logic Based Assignable Cause Diagnosis Using Control Chart Patterns

Sujikumar Vijayakumar

Control charts are widely used in manufacturing and non-manufacturing processes to monitor process average and to reduce variations in quality characteristic. The variations could be either due to common causes, which are inherent nature of the process and unavoidable, or assignable causes which can be diagnosed for rectification. The state of the process, i.e., whether or not the process is statistically in control, is traditionally judged using control limits and unnatural patterns exhibited on control charts. These unnatural patterns, in addition to help in determining the process state, also provide hints on possible assignable cause(s) whenever the process goes out of control. However, there are certain ambiguities associated with this traditional method, such as judging the process state when a point falls exactly on or very near to control limits and vagueness in interpreting unnatural patterns when multiple patterns co-exist on the control chart and relating them to assignable cause.

Fuzzy logic has been proved to be an excellent tool for handling such ambiguities and vagueness by quantifying the uncertainty mathematically. A fuzzy inference engine is developed for $\overline{X}$ chart, based on a chart pattern – cause relationship network. The domain of assignable causes is categorized based on the nature of the shift they can produce, and accordingly related to chart patterns. Each link in the network is represented by a fuzzy inference system which determines the intensity of each cause in the interval [0-1] based on degree of presence of each pattern. All the evidence supporting each cause from the unnatural patterns are aggregated using fuzzy connective operators (max, algebraic sum) and causes are prioritized accordingly so that when process goes out of control, the investigation can be done for the cause having highest priority. The developed fuzzy inference engine is tested with different combinations of unnatural patterns and the results are compared with manual interpretation of control charts and with results from a control chart software tool (MINITAB).

This thesis is
dedicated to my parents

Acknowledgements

I thank my supervisor, Dr. Kudret Demirli for his wholehearted support. His able guidance has been instrumental in the completion of this work, and without it, I would not have been able to come this far. I am thankful to Dr. Yong Zeng, Dr. Akif Bulgak and Dr. Ali Akgunduz for their suggestions in the final stage.

I thank my parents Mrs. Suganthy and Mr. Vijayakumar, for their love and support and Dr. Vaidehi Marthandam for her prayers. I cannot adequately express my gratitude to them.

I thank my sister Mrs. Shabna and my brother in law Mr. Sreeman for their moral support. Their encouragement and pep-talks went a long way into making this work successful.

I thank my friend Ms. Pramodha for her constructive criticism of this text at every stage. Her suggestions on the linguistics of this thesis were truly helpful.

I finally thank all my other friends for their encouragement.

TABLE OF CONTENTS

List of Figures

List of Tables

Abbreviations

Alg_Sum	Algebraic Sum
ARL	Average Run Length
ARL_1	Out of control Average Run Length
ARL_o	In-control Average Run Length
CL	Center Line
CUSUM	Cumulative Sum
EWMA	Exponentially Weighted Moving Average
FIR	Fast Initial Response
FIS	Fuzzy Inference System
FR1	Freak 1
FR2	Freak 2
GMA	Geometric Moving Average
GMP	Generalized Modus Ponens
i/p	Input
LCL	Lower Control Limit
MF	Membership Function
MR	Moving Range
MSM	Maximal Similarity Method
o/p	Output
OCAP	Out of Control Action Plan

OCL	Out of Control Limit
R	Run
SL	Sigma Line
SNo	Sample number
SPC	Statistical Process Control
T	Trend
UCL	Upper Control Limit

List of Symbols

$\oplus$	Aggregation operator
μ	Process mean or Target mean
$\mu_A(x)$	Degree of membership of element x in the fuzzy set A
$\mu_B(x)$	Degree of membership of element x in the fuzzy set B
$C1$	Isolated Causes
$C1*$	Prioritized Isolated Causes
$C1_A$, $C1_B$, $C1_C$	Output fuzzy sets for Isolated causes (C1)
$C2$	Shift Causes
$C2*$	Prioritized Shift Causes
$C2_A$, $C2_B$, $C2_C$	Output fuzzy sets for Shift causes (C2)
$C3$	Gradual Causes
$C3*$	Prioritized Gradual Causes
$C3_A$, $C3_B$, $C3_C$	Output fuzzy sets for Gradual causes (C3)
C_i^-	Cumulative sum of deviations below mean
C_i^+	Cumulative sum of deviations above mean
$FR1_A$, $FR1_B$, $FR1_C$	Input fuzzy sets for FR1 pattern
$FR2_A$, $FR2_B$, $FR2_C$	Input fuzzy sets for FR2 pattern

H	Decision interval
K	Reference value or Slack value
L	Width of the control limits in EWMA charts
n	Sample size
OCL_1, OCL_2, OCL_3	Input fuzzy sets for OCL pattern
R	Range
R_1, R_2, R_3	Input fuzzy sets for Run pattern
T_1, T_2, T_3	Input fuzzy sets for Trend pattern
β	Probability of type II error
λ	Constant for EWMA control chart
σ	Standard deviation
α	Probability of type I error
$\bar{X}$	Sample Mean

Chapter 1

Introduction

1.1 Background

Statistical process control (SPC) techniques are widely used in manufacturing and other processes to monitor and control process variability. SPC uses statistical signals to detect variations in the process, identify the sources of variation, improve performance and to maintain control of processes at required quality levels. A collection of several tools exist in SPC such as Histograms, Check Sheets, Control charts, Pareto charts, Cause-effect diagrams, Scatter diagrams and Defect concentration diagrams. Out of these, Control charts, developed by Dr. Walter A. Shewhart in the1920s, are considered as one of the most technically sophisticated SPC tools. Control charts are used to monitor variations in product quality. Causes for variations in the process are of two types. The first type is '*Common cause variation*', which are inherent nature of the process. Little can be done to eliminate these variations. The other is '*Assignable cause variation*', which indicates that there is a specific cause that can be identified and eliminated to restrict the variations within the required limits.

The presence of such assignable causes can be detected by unnatural patterns from the control charts. Tests for these unnatural patterns are governed by various rules such as the traditional Western Electric rules [1], Nelson rules [2] and other additional sensitizing rules. Once an unnatural pattern is detected, the process is said to be Out-of-Control due to some assignable cause(s). Actions should be initiated to identify the cause and remove it. Careful investigation is required to identify the root-cause of the problem and to prevent recurrence, instead of taking shallow measures for a cosmetic solution. Process experts use tools such as cause-effect diagrams (Ishikawa diagram), check sheets etc., to investigate the root-cause. The idea of using control chart patterns as signals to identify the cause has been proven as a promising method for faster and easier diagnosis. A matrix is prepared relating the control chart patterns to assignable causes that were found in the past. This acts as a guiding tool while searching for an assignable cause. However, there are uncertainties, ambiguities and vagueness associated in the process of determining whether or not the process is in control, and in searching for an assignable cause. This will be discussed in the following section.

1.2 Problem Definition

Generally, operators lack the knowledge to interpret signals from control charts and to use them for identifying assignable causes. It is essential to clearly identify the unnatural patterns exhibited on control charts, as these unnatural patterns indicate not only the presence of an assignable cause but also the nature of such a cause. Ambiguities and vagueness may arise in determining the process state and in searching for the assignable cause from the control chart patterns.

1.2.1 Ambiguities and vagueness in determining the process state using control charts

A process is declared as 'out of control' when one or more points fall outside the control limits or when a non-random pattern is observed on the control chart. Such a decision is supported by rules (Western Electric [1], Nelson's [2] or other sensitizing rules etc) built on statistical techniques which are based on the low probability of occurrence of non-random patterns. However, it may be difficult to make a decision about the process state when a point falls exactly on one of the control limits, or very near the control limits. There is some uncertainty and vagueness associated in determining whether or not there is a shift in the process mean when a point falls exactly on the control limits or near the control limits.

1.2.2 Problems in using sensitizing rules

Control charts are implemented to a process in two phases. Phase 1 involves establishing trial control limits and removing initial process variations and once the established limits are tested in phase 2, these control limits are used to monitor future production. In phase 1, sensitizing rules [1] [2] are used to increase chart sensitivity in order to detect the process shift earlier. Sensitizing rules are not usually used in phase 2 as they lead to problems of increased false alarms and decreased average run length. However, sensitizing rules help to detect any unnatural pattern present, which present useful information in assignable cause diagnosis, in the control chart. Such information is lost by abandoning the use of sensitizing rules, costing extra effort and time in investigating the assignable cause.

1.2.3 Ambiguities and vagueness in diagnosing the assignable cause using control chart patterns

As mentioned earlier, patterns in the control can give indications on the assignable cause present. Each non-random pattern has a set of causes associated with it. Sometimes there is scope for ambiguity when multiple non-random patterns co-exist for an out of control situation. For instance, the pattern 'Trend-up' is said to have occurred when an increase is observed for seven consecutive points. If the seventh point lies outside the upper control limit (UCL), then the process is said to be out of control by two tests, hence causing an ambiguity in distinguishing between the assignable causes of these two patterns which are the Trend-up and the point outside of the control limit.

Another problem is that there are chances to get misleading results due to misinterpretation of non-random patterns. For example, if six consecutive points are increasing and the sixth point falls beyond the upper control limit, the test indicates that the process is out of control because of one point falling beyond the control limit, while the actual pattern could be a run-up. So while diagnosing for an assignable cause, we might be misguided into looking for the cause(s) for one point falling beyond control limit instead of looking for the causes for a run-up. If we could quantify the uncertainty to indicate the extent to which each non-random pattern exists, and the degree to which each associated cause is present, it will facilitate easy decision making. Fuzzy logic has been proven as an excellent tool for handling such uncertainties. The following section surveys the application of fuzzy logic in control charting and in the subsequent section the methodology is proposed to handle such problems using fuzzy logic.

1.3 Literature Survey

The theory of fuzzy sets, introduced by Dr. Lotfi A. Zadeh in 1965 [3], has found extensive applications in various fields of engineering and science especially over the past two decades. Until then, modeling complex physical systems using the traditional mathematical models had always been a challenge, especially, when the system had to deal with imprecision, vagueness, ambiguities and uncertainties. Fuzzy sets and fuzzy logic yielded impressive results in handling such systems. In this section we review the applications of fuzzy sets and fuzzy logic in the field of SPC, focusing more on control charting and causal diagnosis.

Researchers started exploring the possibility of extending fuzzy logic to control charting in the mid 1990s. Kahraman *et al* [4] used triangular membership functions to define various unnatural patterns. A fuzzy test is defined for each unnatural pattern using the membership value of the i^{th} sample, number of points for the unnatural pattern considered and a threshold limit to confirm the presence of the unnatural pattern. But the membership function used is a simple triangular fuzzy membership function, common to define all the patterns. Zalila *et al* [5] proposed a fuzzy supervision method for SPC which could alert operators on the process state using visual signals. These visual signals are generated using a fuzzy rule base which monitors the process center and state of dispersion. The output from this system is in the form of colored light signals indicating the process state.

Wang and Rowlands [6] developed a fuzzy rule based inference system based on zone rules in control charts. The input variables are the degree of membership of a point in each zone represented as fuzzy sets, and the output is the process state, mapped by eleven fuzzy If-Then rules. This approach provides improved results in terms of interpretation of data and consistency, as the numeric output from the fuzzy system indicates whether or not action should be taken, if the process is out-of-control. Another excellent application of fuzzy logic to control charting for individuals was developed by Tannock [7]. In this approach, two fuzzy sets namely, centered fuzzy set and random fuzzy set are used. Three typical unnatural patterns: shift, trend and cyclical patterns are examined using these two fuzzy sets. The membership function of the centered fuzzy set assumes the OC curve of the equivalent Shewhart control chart, which considers the mean and standard deviation of the incoming distribution. The membership of the random set is determined by calculating the correlation coefficient of the series window at sample number n with the previous window $n-1$. The absolute value of correlation coefficient is subtracted from unity to obtain the degree of membership, such that highly correlated data are not considered to be very random. However, all these approaches [4], [5], [6] and [7] use fuzzy logic only in analyzing the control chart patterns to determine the process state. Diagnosing the assignable cause from the signals from the patterns has not been explored in these approaches.

Hsu and Chen [8] suggested a new approach using fuzzy logic and genetic algorithms to diagnose the assignable cause using the signals from unnatural patterns. The system comprises of a knowledge bank and a reasoning mechanism. The knowledge bank contains the membership functions of unnatural symptoms described by Nelson's rules and knowledge of cause-symptom relations. The fuzzy cause-symptom relation matrix is constructed with the help of a new approach called maximal similarity method (MSM). In the MSM approach, an optimization problem is formulated for constructing the fuzzy relational matrix, and genetic algorithms are used for faster search techniques. However, this system involves increased computational complexities due to the global search for the optimal solution for constructing the fuzzy relational matrix.

Several other methods have been developed for causal diagnosis based on observed fuzzy symptoms by integrating fuzzy logic with other approaches such as neural networks, belief networks based on Bayesian theory, Dempster-Shafer theory (Evidence theory) and possibility theory. Literature available in these areas is summarized in Table1.1. However, there are certain problems associated with these methods, such as the large volume of data required to train neural networks; knowledge of prior probability and conditional probability distributions required for belief networks based on Bayesian theory; subjective judgment on degrees of belief and computational complexities in evidence propagation in networks for belief networks based on Dempster-Shafer theory, and data required to train possibilistic networks.

	Fuzzy-Neural Networks	Fuzzy-Bayesian belief networks	Fuzzy-Dempster Shafer belief networks	Possibilistic belief networks
Pattern Recognition	[9] [10]	-	-	-
Causal Diagnosis	[11]	[12] [13] [14] [15] [16]	[17] [18]	[19] [20]

Table 1.1 Literatures in Fuzzy Logic with Other Approaches in Pattern Recognition & Causal Diagnosis

1.4 Motivation and Proposed Methodology

Considering all the shortcomings mentioned in the previous sections, our objective is to develop a unified system that can monitor the process state by analyzing the control chart patterns and to utilize the signals given by these unnatural patterns in diagnosing the assignable cause, with the help of fuzzy logic, to overcome the problems mentioned in Section 1.2. The main purpose of the system is to provide a framework to quantify the uncertainty to indicate to what extent each non-random pattern exists and to what degree the associated cause could be present, so as to facilitate easy decision making.

A fuzzy inference engine is developed for $\overline{X}$ control chart, composing of two modules, Fuzzy Inference System (FIS) modules and Aggregation module. The FIS modules are developed based on a chart pattern-cause relationship network shown in

Figure 4.7. The assignable cause domain is categorized according to nature of shift each cause can produce and is related to the chart patterns accordingly. Each link in the network is modeled by a fuzzy inference system (FIS), taking the chart pattern as an input variable and the respective cause as an output variable, each represented by fuzzy membership functions. The influence of each pattern in its respective cause is quantified through the fuzzy inference system with the help of fuzzy *If-Then* rules in the interval [0-1].

Once the evidence for each cause from its respective pattern is obtained, the values are aggregated in the aggregation module to determine the total evidence on each cause. The causes are then prioritized according to their value of aggregated evidence and listed in order for each point plotted on the control chart. On observing a point falling outside three sigma limits, the process is said to be out of control and the investigation for assignable cause can be initiated beginning with cause having highest priority. In this way, the aforementioned ambiguities can be resolved by quantifying the uncertainty using fuzzy logic, enabling a more guided approach in monitoring the process and diagnosing the assignable cause through SPC.

1.5 Thesis Outline

Chapter 2 contains a brief review on various types of control charts in use such as Shewhart control charts, CUSUM (Cumulative Sum) control charts and EWMA (Exponentially Weighted Moving Average) control charts, selection of type of control charts and the stages in implementation of control charts. In Chapter 3, various types of unnatural patterns detected on a control chart, rules for detecting these patterns and statistical basis for defining these patterns are discussed. Usage these sensitizing rules and their effect on average run length and false alarm are briefly outlined. Also, traditional methods for taking action during an out-of-control situation and the advantage of using chart pattern as hints to diagnose the assignable cause are mentioned.

In Chapter 4, the design of the fuzzy inference engine is explained. Its modules in the FIS component, along with membership functions, rule base and response curves are detailed. Finally, the Aggregation module is explained. In Chapter 5, the developed fuzzy inference engine is tested with various test cases containing different combinations of unnatural patterns. The results of the fuzzy inference engine are compared with manual interpretation and results from statistical software (MINITAB). Finally, the summary, conclusions and future work are presented in Chapter 6.

Chapter 2

Review of Control Charts

2.1 Shewhart Control charts

Control charts are mainly of two types, variable control chart and attribute control chart. A quality characteristic that can be measured on a numerical scale is called a variable. A quality characteristic which cannot be measure on a numerical scale and can only be characterized as *'conforming'* or *'non-conforming'* is called an attribute.

2.1.1 Variable control charts

While controlling a variable quality characteristic, it is necessary to monitor both the mean value of the quality characteristic and its variation simultaneously. Most commonly used variable control charts are $\overline{X}$ chart, which monitors the mean value, and R chart, which monitors the variation. Samples of smaller sizes (usually 3 to 7) are collected and the mean and range of each sample is calculated and plotted on the chart. Sometimes, S chart is used instead of R chart, when it is desired to estimate the process standard deviation directly and when the sample size is moderately large (10 or 12). Since $\overline{X}$ and R charts are generally used with smaller sample size, the magnitude of shift detected in the process is from moderate to large say, on the order of 2σ or larger. Selecting the samples or subgroups plays an important role in the use of $\overline{X}$ and R charts. As $\overline{X}$ chart monitors *between-sample* variability, samples should be selected in such a way that, if assignable causes are present, chances for shifts in process average between samples is maximized. On the other hand, *R* chart monitors the *within-sample* variability. Hence samples should be selected in such a way that it minimizes the chances for variations within the sample. The Sample data collected keeping in mind this idea, is termed as *rational subgroup*. There are occasions when sampling is not necessary, in cases where 100% inspection is done where the production rate is very low, or automated inspection methods are used. In such circumstances, it is economically feasible to plot every single unit produced on the control chart. Control charts for individuals are used in such occasions. The usage of CUSUM or EWMA charts is recommended when it is desired to detect process shifts of smaller magnitude.

2.1.2 Attribute control charts

Control charts for attributes are used where the outcome of inspection is either 'defective' or 'non-defective'. Four types of attribute charts are commonly used, *p* chart, *np* chart, *c* chart and *u* chart. The *p* chart is called the control chart for fraction non-conforming. The percentage of nonconforming units is calculated per sample, where the sample size is large, say 100. Since the *p* chart uses samples of data collected over a period of time, the sample size could vary. Therefore, variable-width control limits or control limits calculated based on average sample size or control limits for standardized control chart are used. The *np* control chart is plotted based on the number of non-conforming units in each sample. This is more advantageous than the *p* chart in that the percentage of non-conforming units need not be calculated. However, this requires equal sample size. The *c* chart, also known as defects-per-unit chart, is plotted by calculating the number of non-conformities found in each unit. Sometimes a product, by virtue of its nature, cannot be classified as a 'non-conforming unit' even if it has one or two non-conformities. In such cases, c charts are used, following a Poisson distribution. The u chart, also known as average defects per unit chart, is plotted based on average number of non-conformities per unit.

In Shewhart charts, whether or not the process is in control, is determined by a point falling outside the control limits and from the tests for various unnatural patterns. These unnatural patterns, in addition to determining the process state, provide information on nature of assignable cause that could be present. The details of various unnatural patterns and its relationship with the type of assignable cause are discussed in Chapter 3.

2.2 Other Control Charts

The variable and attribute control charts presented in the previous section, called Shewhart Control charts, are incapable of detecting shifts magnitude lower than say 1.5σ or less. Despite the use of additional sensitizing rules, these charts fail to monitor the process state by looking at the entire sequence of points. Moreover, usage of additional sensitizing rules increases the sensitivity of these charts, thereby reducing the average run length (number of in-control points plotted before getting an out of control point), and increasing the frequency of false alarms. CUSUM (cumulative sum) and EWMA (Exponentially Weighted Moving Average) charts are used as alternatives where detection of smaller shifts is of interest.

2.2.1 CUSUM control chart

CUSUM stands for cumulative sum. Like Shewhart control charts, CUSUM charts are also used to monitor both process mean and variability. The cumulative sum of the deviation from the target value both above and below the target is monitored continuously though out the sequence of points. Control limits are calculated using tabular or algorithmic CUSUM or by V-mask method, although former is generally preferred due to certain advantages over the latter, as V-mask method is not suitable for one-sided process control, difficulty in determining extent of backward arms of V-mask making the interpretation of chart difficult, ambiguities associated with $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ in V-mask procedure, non-availability of the fast initial response (FIR) feature used in CUSUM to increase chart sensitivity at process start-up.

CUSUM charts are desirable because of their ability to detect even small shifts of desired magnitude, as control limits are calculated based on the magnitude of the shift intended to be detected. Control limits depend on selection of two parameters: K (*reference value* or *slack value* or *allowance*) and H (*decision interval*). Usually, the value of K is chosen as half the magnitude of the shift to be detected, and H five times the process standard deviation. The cumulative sum of deviations that are greater than K are accumulated as C_i^+ for deviations above mean, and C_i^- for deviations below mean, for the i^{th} sample. If either C_i^+ or C_i^- exceeds the value of H, the process is said to be out of control. Even though CUSUM charts are capable of detecting smaller magnitude of shift, they are not effective in detecting shifts of larger magnitude. Hence CUSUM charts are combined with Shewhart control charts to improve its sensitivity for larger shifts. Combination of these charts is achieved by placing Shewhart control limit in CUSUM chart at 3.5 σ from the center line.

The only criterion to determine whether or not the process is in-control in CUSUM charts is to check if the accumulated deviations exceed the decision interval as there are no other unnatural patterns defined to be monitored to check the process state as in Shewhart control charts. Since there are no different unnatural patterns to be monitored as in Shewhart control charts, the possibility of narrowing down the domain of assignable cause(s) to be looked for, based on the type of unnatural patterns exhibited on the control chart is ruled out.

2.2.2 EWMA control charts

Exponentially weighted moving average (EWMA) charts are easier to set up and operate when compared with CUSUM charts. This method uses the weighted average of all previous samples with their weights decreasing in geometric progression with their ages. Hence EWMA charts are also known as Geometric Moving Average (GMA) charts. These charts are considered as ideal control charts for individual observations, as it uses weighted average of all samples, thereby making it insensitive to normality assumption. Control limits are calculated based on selection of values for two parameters: L, the width of the control limit in units of multiple of standard deviation and λ, a constant between 0 and 1. Usually, the value of L is chosen as 3, and λ between 0.05 and 0.25. The general idea is to use smaller values of λ to detect smaller magnitude of shifts. Nevertheless, EWMA charts are more capable than CUSUM charts in detecting larger shifts when choosing $\lambda > 0.1$.

The control limits in EWMA charts are calculated for every point plotted on the chart and it reaches steady state as more points are plotted. Like CUSUM charts, EWMA charts can also be used with Shewhart control charts to increase the sensitivity for larger shifts. Also, like CUSUM charts, an out of control situation in EWMA charts is identified by a point falling outside the control limit and there are no other unnatural patterns to be looked for. Hence, when an out of control situation is identified, like CUSUM charts, EWMA charts also do not provide any insight on assignable causes, as there are no unnatural patterns exhibited on the control chart that could describe the nature of assignable cause present. Due to these shortcomings, Shewhart charts are widely used except in cases where the intended magnitude of shift to be detected is very small.

The decision on the type of control chart to be used, hence, mainly depends on the type of quality characteristic to be controlled. It also depends on other factors such as magnitude of shift intended to be detected and the cost of collecting samples. A graphical guide for selection on type of control chart is given by Montgomery [21] as in Figure 2.1.

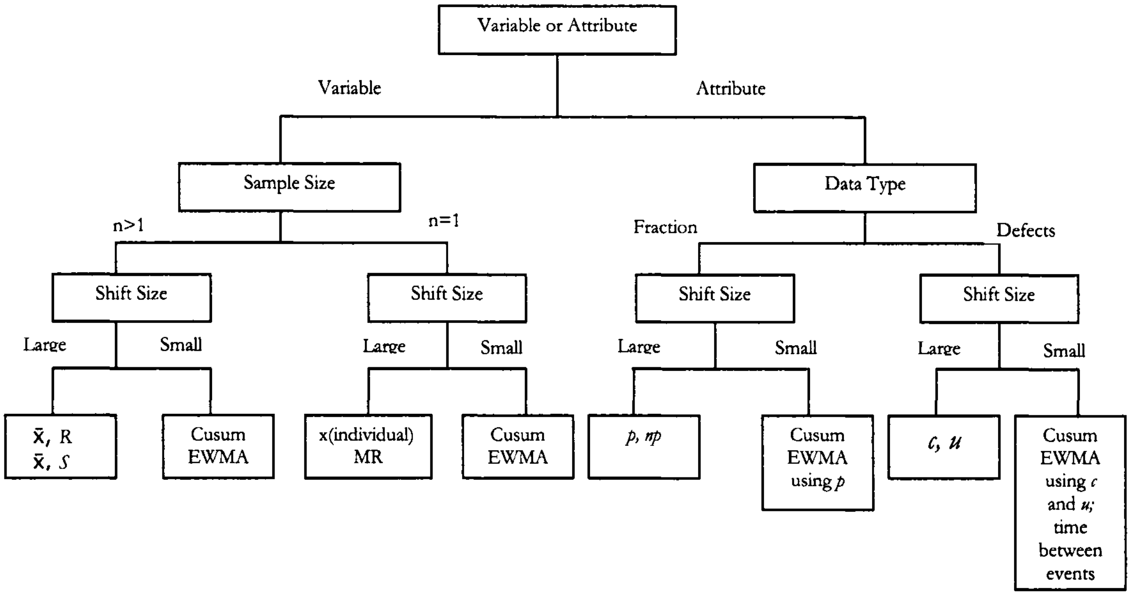

Figure 2.1 Selection of Control chart

Once the type of control chart to be used is decided, it is necessary to select the chart parameters, such as sample size, sampling frequency and control limits, called design of control chart. While selecting these parameters, in addition to considering statistical criteria, economical factors such as cost of sampling, cost associated with investigating out of control situations, cost of correcting assignable causes and cost of allowing non-conforming units to reach the customer are to be considered. Such a process is called the economical design of control chart, and extensive research is being done in this area to improve the performance of the control charts. However, it is beyond the scope of this work and our problem is focused on the ambiguities in analyzing unnatural patterns exhibited by the control charts and using these unnatural patterns to determine the nature of assignable causes that could be present. Various unnatural patterns that can be observed on Shewhart control charts, their relationships with the nature of the assignable cause, and how to make use of these patterns in detecting the assignable cause are discussed in Chapter 3. The following section explains the stages in implementation of control and how decision rules are used in each stage for monitoring the process.

2.3 Stages in Control Chart Implementation

There are two stages in implementing control charts. They are generally termed as Phase-I and Phase-II. Phase-I is concerned with the initial setting up of the control chart and fixing of parameters such as control limits, and bringing process mean and standard deviation in-control. Once the process mean and standard deviation are brought under control, Phase-II, which is monitoring the process mean and standard deviation, begins.

2.3.1 Phase I

Phase I of the control chart implementation involves selecting the appropriate type of control chart and selecting control chart parameters such as control limits, sample size and sampling frequency. Phase I continues until the process is initially brought under control. For example, if we are using $\overline{X}$ and R chart, a trial run is made, and at least 25 to 30 samples are collected. From these samples, the mean values and control limits for $\overline{X}$ and R chart are calculated and charts are plotted. While setting up these control charts, the R chart should be brought under control first before analyzing the $\overline{X}$ chart, as the control limits of the $\overline{X}$ chart depend on the value of $\overline{R}$. It is meaningful to adjust the process mean to any desired value, by adjusting the input process parameters once the process variability is brought under control. If the R chart is not under control, then the out of control points on the R chart are eliminated and new values for $\overline{R}$ and hence, new values for control limits on both $\overline{X}$ and R chart are calculated. This is an iterative process in phase I and repeated until reasonable control limits are established and the process is brought under control.

2.3.2 Phase II

Once a reliable set of control limits are established from phase-I and the process is brought under control, then the control charts are used to monitor future production and to look for any assignable cause that might arise in future. While monitoring the production for an in-control process, even though the use of sensitizing rules to determine whether or not the process is under control could help to detect smaller shift in process mean earlier, its is not generally advised [21]. The reason for this is that due to the increased the sensitivity of the control chart, there is a possibility of reduced Average Run Length (ARL) and increased probability of false alarms. Instead, it is generally advised to use CUSUM or EWMA charts, if the magnitude of shift to be detected is very small. Use of various sensitizing rules and its effect on Average run length and false alarms are discussed in chapter 3. Generally, once the process reaches an in-control state from phase-I, only 3σ limits are used to decide whether the process is in control. These limits are also known as action limits. The search for assignable cause is initiated only when a point falls beyond action limit. It is customary to use 2σ limits as warning limits.

Some of the zone rules/sensitizing rules discussed in chapter 3, such as two or more consecutive points falling beyond warning limits (2σ), indicate that process mean has shifted and hence search for assignable cause should be initiated. But, as mentioned earlier, the use of sensitizing rules might increase the frequency of out-of-control signals, and initiating the search for assignable cause every time will not be cost-effective and feasible. Nevertheless, signals from the sensitizing rules are treated as warning signals and instead of searching for assignable cause every time, actions such as increasing the sampling frequency, sample size or sometimes 100% inspection are initiated to make sure that process mean has really shifted and that it is not a false alarm. If the control chart continues to give unnatural patterns, it increases the confidence that there is really some assignable cause that had changed the process mean. Though the use of sensitizing rules has been criticized for an in-control process in phase-II, it could help us detect the unnatural patterns exhibited on control chart, which provides hints on the nature of the assignable cause that could be present.

Chapter 3

Control Chart Patterns and Assignable Causes

When a process is in-control, points plotted on control chart follow a random pattern in the normal distribution. About 68% of the points plotted are distributed randomly within 1σ from centerline and 28% of them between 1σ and 2σ, 4% of them between 2σ and 3σ and 0.27% of the points beyond 3σ. When the process goes out of control, the random behavior of the chart changes and some unnatural patterns are exhibited on the control chart. These unnatural patterns can be detected by a set of rules called zone rules or sensitizing rules. The following sections discuss various sensitizing rules and the statistical reasons for framing these rules in terms of its low probability of occurrence.

3.1 Basic Western Electric Rules

The Western Electric Handbook (1956) [1] suggests the following set of rules to detect the unnatural patterns on a control chart. If one of the following situations are observed on the control chart, it indicates that process mean has been shifted.

- One point outside three sigma control limits

- Two out of three consecutive points beyond two sigma limit on same side of center line
- Four out of five consecutive points beyond one sigma limit on same side of center line
- Eight consecutive points plot on one side of center line

3.2 Using Sensitizing Rules

Sensitizing rules will be of more help in phase-I, initially to bring the process in-control. Once the process gets stabilized, use of these zone-rules/sensitizing rules, other than point falling beyond three sigma limits could lead to increased chart sensitivity resulting in decreased average run length (from 370 to 91.25, with these four Western Electric Rules) and increased false alarms. If there are 'n' decision rules used to judge the process state, each having probability of type I error α_i , then the overall probability of false alarm α is given by the expression,

$$\alpha = 1 - \prod_{i=1}^{n} (1 - \alpha_i) \qquad (3.1)$$

Equation (3.1) requires that the sensitizing rules be independent. But this requirement is not satisfied in the case of sensitizing rules as there are many possibilities for different unnatural patterns to co-exist, having common points in them.

Hence to avoid the problem of increased false alarms, instead of concluding the process is out of control resulting from tests by sensitizing rules, they shall be taken as warning signals. Action such as increasing sampling frequency or sample size, called *adaptive sampling* measures shall be taken to make sure whether or not the process mean has indeed shifted. If these unnatural patterns persist, the shift in the process mean becomes apparent. By this way we can detect smaller shifts earlier, without compromising for increased false alarms. Hence the search for assignable cause can be initiated with confidence using the hints provided by these unnatural patterns.

Alternatively, it is generally suggested that, if smaller shifts are of interest, CUSUM or EWMA charts be adopted. But the problem with CUSUM or EWMA charts is, first, these charts are not capable of detecting shifts of larger magnitude. Second, the criterion to decide whether or not the process is under control in CUSUM or EWMA charts, is governed by a single rule, a point falling beyond the control limits and there are no other rules, that require us to detect any other unnatural patterns, that could be of great help in determining the assignable cause. For cases, where both small and large shifts are of interest, combination of CUSUM/EWMA and Shewhart charts is used. From the viewpoint of assignable cause diagnosis, usage of these sensitizing rules could help us in narrowing down the assignable cause list. The definitions for various unnatural patterns and the statistics behind the definition of these patterns are discussed in the subsequent sections.

3.3 Control Chart Patterns

Apart from the basic Western Electric rules, there are many other rules define various unnatural patterns that can be detected on Shewhart control charts. These patterns are defined based on their lower probability of occurrence. Most commonly found unnatural patterns are Out of Control Limits (OCL), Freaks, Run, Trend, Cycle, Stratification, Instability, Grouping and Stable mixtures.

3.3.1 Out of Control Limits (OCL)

Pattern definition:

One or more points fall beyond three sigma control limit.

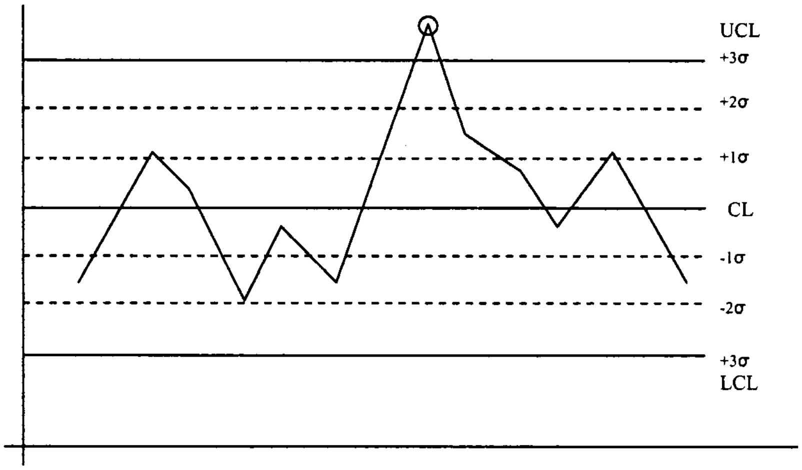

Figure 3.1 OCL Pattern

This is the conventional rule for concluding that process has gone out-of-control. For a normal distribution curve, 99.73% of all points fall within three sigma limits. Hence, the probability of a point falling beyond three sigma control limits is 0.0027. Due to this low probability, occurrence of such event is considered as rare. Even though the probability is very small, there is a possibility to observe a point beyond three sigma control limits, even when the process remains under control. The frequency of such an occurrence is given by ARL_0 (Average Run Length), which is defined as average number of points to be plotted within control limits before getting an out of control signal. It is given by the expression

$$1/\alpha = 1/0.0027 \sim 370$$

A false alarm is expected once every 370 points on the average. If the frequency of observing a point beyond three sigma limits is higher, then there is an assignable cause associated with this pattern.

3.3.2 Freaks

The second and third rules of Western Electric rules define freak patterns. For our convenience we shall name them as Freak 1 and Freak 2.

Freak 1 Pattern Definition:

Four out of five consecutive points fall beyond one sigma limit on the same side of the center line.

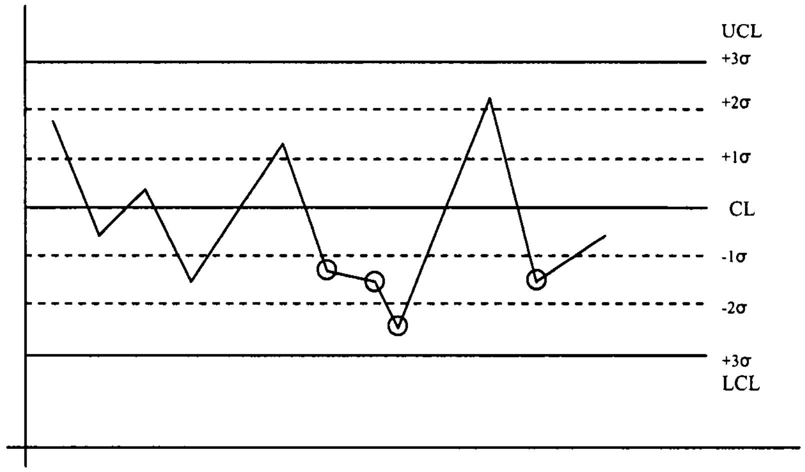

Figure 3.2 Freak 1 Pattern

From the properties of normal distribution curve, the probability of observing a point beyond one sigma is 0.16. Hence the probability of observing four out of five consecutive points beyond one sigma is given by,

$$P\ (4\ out\ of\ 5\ beyond\ 1\sigma) = 5\ (0.16 \times 0.16 \times 0.16 \times 0.16 \times 0.84) = 0.0028$$

This is same as the probability of observing a point beyond three sigma control limit. Hence, it is also considered as an unnatural pattern

Freak 2 Pattern Definition

Two out of three consecutive points fall beyond two sigma limit on the same side of center line.

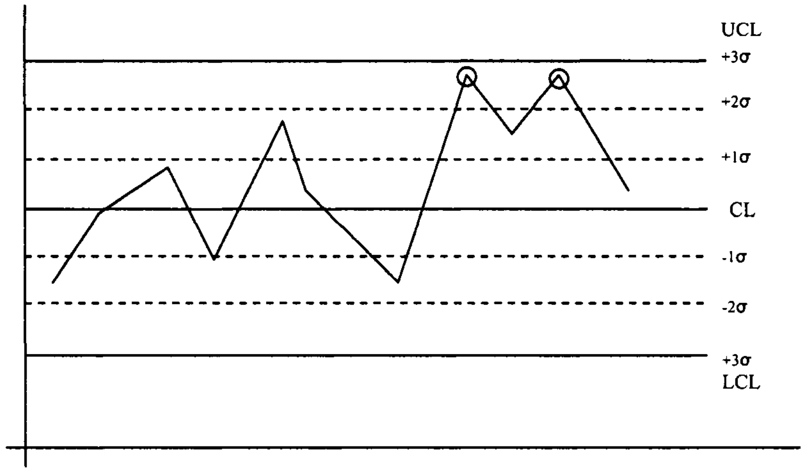

Figure 3.3 Freak 2 Pattern

From the properties of normal distribution, the probability of observing one point beyond two sigma limits is 0.023. Hence the probability of observing two out of three consecutive points beyond two sigma limits is given by

$$P\ (2\ out\ of\ 3\ points\ beyond\ 2\sigma) = 3\ (0.023 \times 0.023 \times 0.997) = 0.0016$$

The probability of occurrence of this event is very small compared to the probability of observing a point beyond three sigma limit. Hence it is considered as an unnatural pattern.

3.3.3 Run

This pattern is defined by the fourth rule of Western Electric rules.

Pattern Definition:

Eight or more consecutive points on one side of centerline

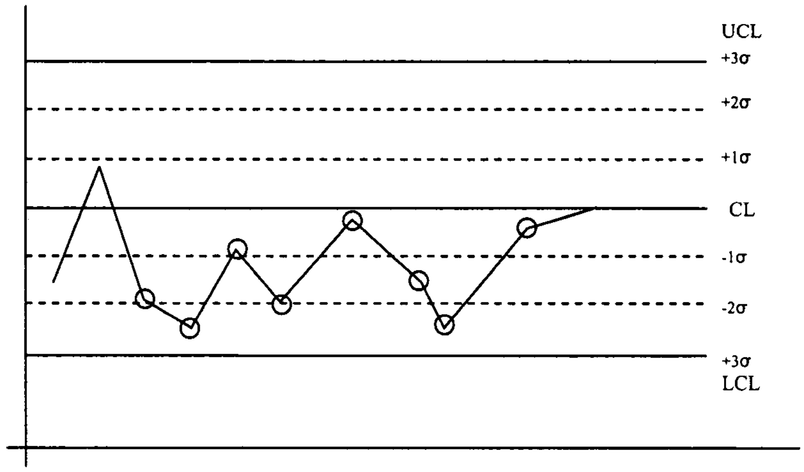

Figure 3.4 Run Pattern

From the properties of normal distribution curve, the probability of observing a point on one side of centerline is 0.5. Hence the probability of observing eight consecutive points on one side of center line is given by,

$$P\ (8\ points\ on\ one\ side) = (0.5)^8 = 0.0039$$

Though the Western Electric rules suggests eight consecutive points on onside of centerline, many other literature [22] [23] suggest seven consecutive points on one side (with probability 0.008) shall be considered in this work as a test for earlier detection of shift.

3.3.4 Trend

Pattern Definition:

Seven or more points continuously increasing or decreasing

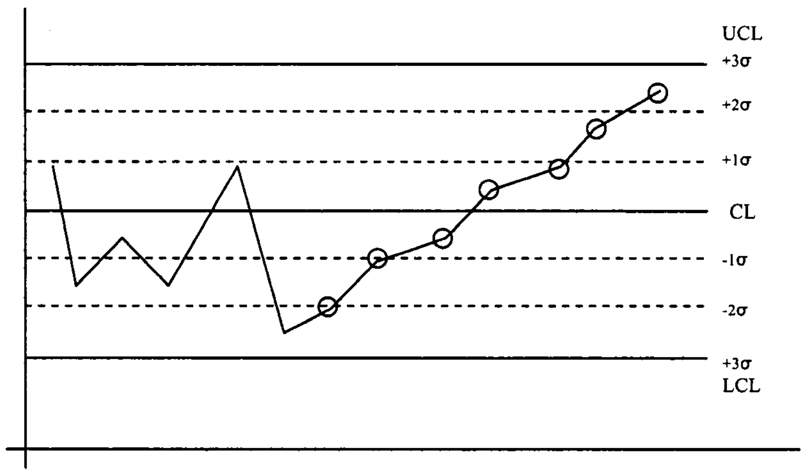

Figure 3.5 Trend Pattern

The probability of seven consecutive points increasing (or decreasing) can be calculated as follows.

$P(7\ increasing) = P\ (1^{st} > base\ point) \times P(2^{nd} > 1^{st}) \times \ldots\ldots \times P\ (7^{th} > 6^{th})$

$P\ (1^{st} > base\ point) < 0.5$

$P\ (2^{nd} > 1^{st}) < 0.5$

....

....

$P\ (7^{th} > 6^{th}) < 0.5$

Therefore,

$$P\ (7\ increasing) < (0.5)^7 = 0.008$$

From this it is confirmed that the probability of getting seven points increasing or decreasing in a row is always less than 0.008, making this pattern a rare event and thus forming yet another unnatural pattern.

3.3.5 Cycle

This kind of pattern exhibits systematic changes in the process. It can be identified by repetitive patterns observed on the control chart over a period of time. Though some literature defines fifteen points in a row alternating up and down as a cyclic pattern, it is not necessary that every other point fluctuates up and down to represent a cyclic pattern. It could be any number of points representing repetitive forms, say, sinusoidal, wavy, etc.

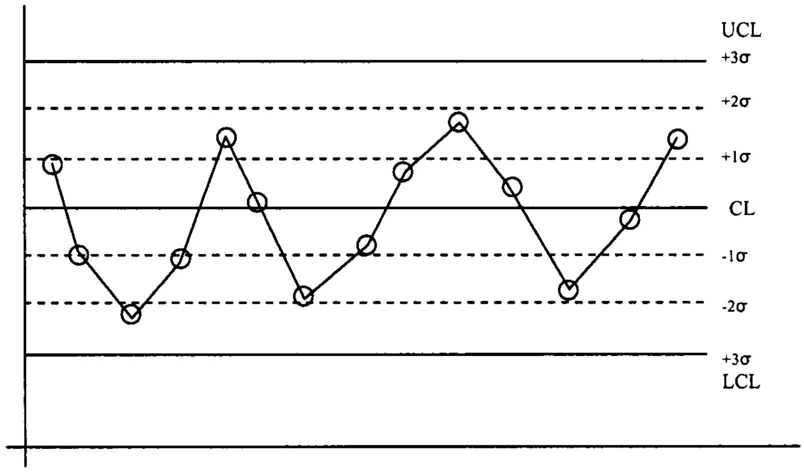

Figure 3.6 Cyclic Pattern

3.3.6 Stratification

This pattern arises when too many points are crowded near center line with absence of points near the control limits.

Pattern Definition

If fifteen or more consecutive points falls between $\pm 1\sigma$ *zone*

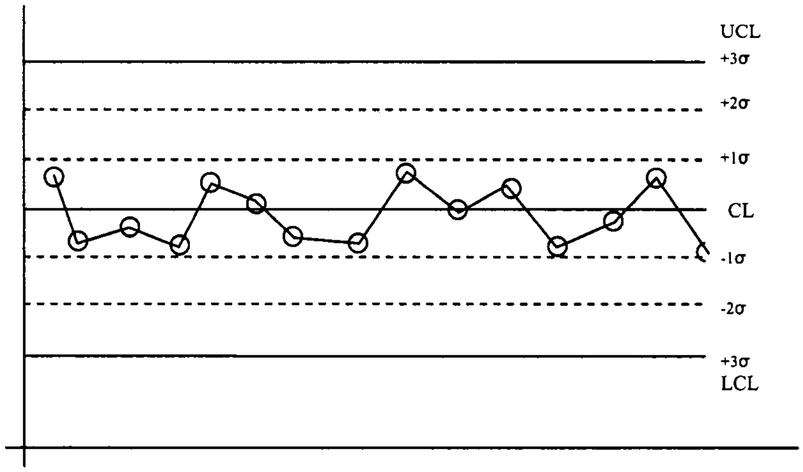

Fig 3.7 Stratification

From the normal distribution curve properties, the probability of observing a point within $\pm 1\sigma$ zone is 0.6826. Hence the probability of observing 15 points consecutively is given by,

$$P\ (15\ points\ in\ \pm 1\sigma) = (0.6826)^{15} = 0.00325$$

This is almost equal to the probability of getting a point out of control limits. Hence there could be some assignable cause present resulting this rare event.

3.3.7 Instability

This type of pattern on the control chart can be recognized by erratic fluctuations or zigzag distribution of points with frequent ups and downs. This pattern is also known as unstable mixture. There is no exact definition to detect this pattern clearly. Although the characteristics of the pattern can be explained as, more than one-third of the plotted points lying outside $\pm 1\sigma$ limits, with an erratic zigzag pattern.

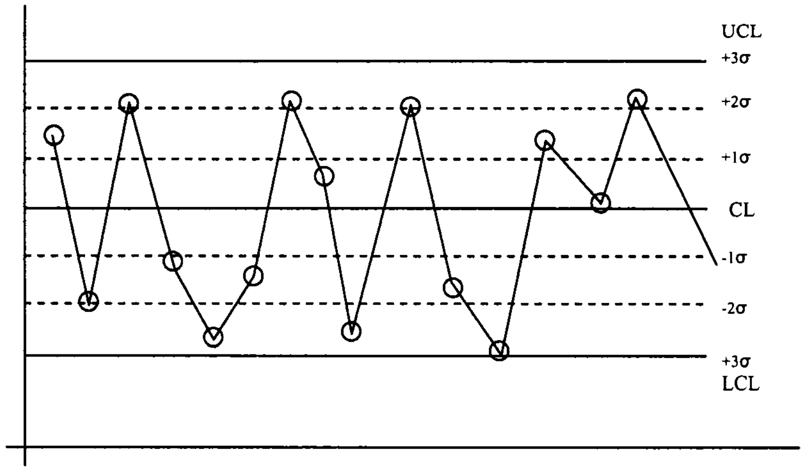

Figure 3.8 Instability

3.3.8 Stable Mixtures

Stable mixture pattern also contains erratic zigzag pattern, but it differs from unstable pattern in a manner that there will be no or very few points in the center of the control chart. Many points will be crowded near the upper and lower control limits, fluctuating up and down. Again, there is no exact rule to define this pattern. This pattern generally indicates mixture of two distributions.

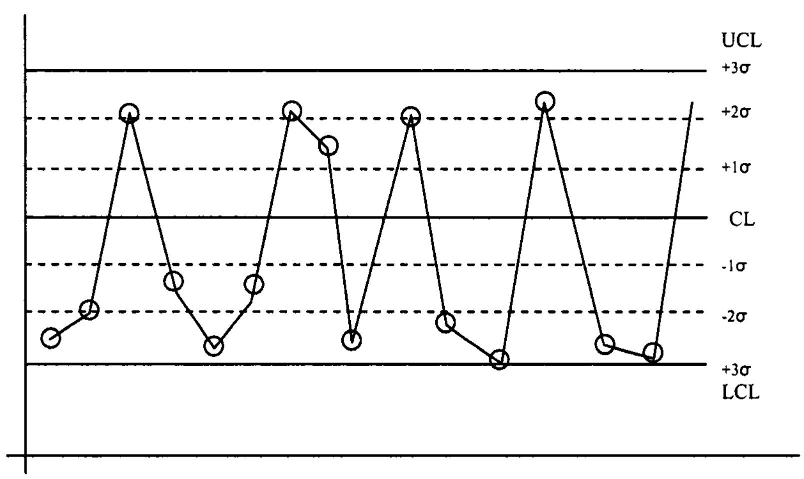

Figure 3.9 Stable Mixture

3.3.9 Grouping

This pattern is classified by clusters of points on the control chart, indicating the presence of several distributions. This pattern also has no exact definition and can be identified as groups or bunches of points on the control chart.

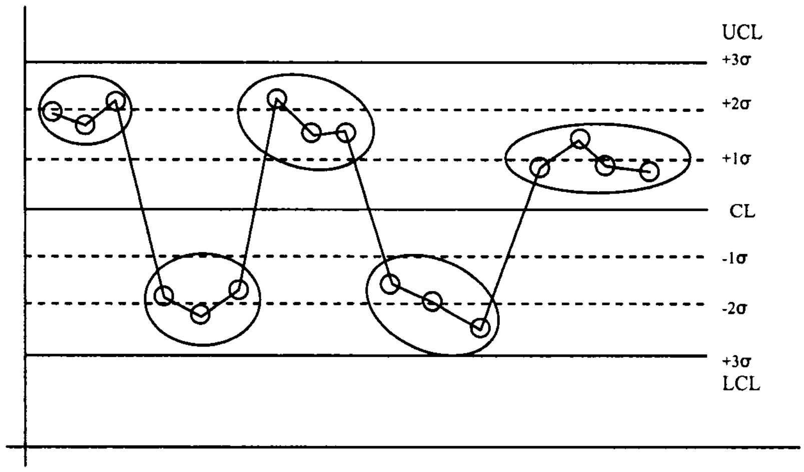

Figure 3.10 Grouping

3.4 Average Run Length and False Alarm

Average run length is defined as the average number of points plotted on the chart before getting one point outside three sigma limits. Since there is a possibility of getting a point outside three sigma control limits when the process is in control and out of control, ARL is accordingly further defined as in-control ARL (ARL_0) and out of control ARL (ARL_1).

- **ARL_0**

In-control ARL (ARL_0) is defined as the average number of points plotted on the control chart before getting a point outside three sigma limits when the process is in-control. It is calculated by the expression,

$$ARL = 1/\alpha \qquad (3.2)$$

where 'α' is the probability of getting a point outside of three sigma control limits when process is in control, in other words, the probability of false alarm.

- **ARL_1**

Out of control ARL (ARL1) is defined as the average number of points plotted on the control chart before getting a point outside of three sigma limits when the process is out of control. It is given by the expression

$$ARL = 1/(1-\beta) \qquad (3.3)$$

where 'β' is the probability of getting a point inside three sigma control limits when process is out of control

In our future discussions ARL refers to in-control average run length. As mentioned earlier, for a Shewhart control chart with three sigma control limits, under normal distribution, the value of average run length is 370, given by the equation (3.2). As per the discussion in the Section 3.2, the value of average run length reduces with increase in number of sensitizing rules. The equation for computing the overall probability of false alarm based on the number of sensitizing rules used is given by equation (3.1). However, calculation of overall probability of false alarm and the average run length using equation (3.1) will not be appropriate, due to lack of independence in sensitizing rules.

Only the patterns Out of Control limit (OCL), Freaks, Run and Trend are taken in to consideration in this work due to certain reasons such as lack of exact definitions for other unnatural patterns like cycle, stable and unstable mixtures, grouping etc, and the complexity associated in identifying those patterns.

The problem of false alarms can be eliminated to a considerable extent by two methods. First, by using adaptive sampling measures, the alarm signals given by the control chart can be verified to see whether or not it is a true indication of process shift. Second, on getting an out-of-control signal, it can be verified for false alarm by checking it with average run length. If the number of in-control points plotted on the control chart after the previous out of control point is close to average run length then the chances of getting false alarm is high. Even after the adaptive sampling measures continue to show unnatural patterns, it is a clear indication of presence of assignable cause and actions should be initiated to search and remove the assignable cause.

3.5 Assignable Cause Diagnosis

3.5.1 OCAP and Root cause analysis

OCAP stands for Out-of-Control Action Plan, which is basically a document containing the sequence of steps to be carried out when an out-of-control situation arises, including various checkpoints and corrective action. It contains list of potential causes and also the actions taken in the past to eliminate the assignable cause found, which can act as a step-by-step guide to how to act on an out-of-control situation.

There is another well known traditional and effective tool to identify the underlying root-cause, commonly known as 'Fishbone diagram' or 'Ishikawa diagram' or 'Cause-Effect diagram'. Usually a team, involving people who have good knowledge about the process such as managers, process engineers, operators will conduct a brainstorming to list out all the possible causes and put them categorically in the fishbone diagram. A typical model of a fishbone diagram is given in Figure 3.11.

Generally the list of possible causes that could have resulted in an out-of-control situation is categorized under Machines, Methods, Materials, Measurement and Man (widely known as 5Ms). Each of the potential causes is analyzed and unlikely causes are eliminated, finally narrowing the list down to most likely causes. Then further investigations are conducted to identify the right cause and necessary corrective action is implemented. One of the biggest drawbacks of this method is that it is time consuming and it requires involvement of too many people having expertise about process under investigation.

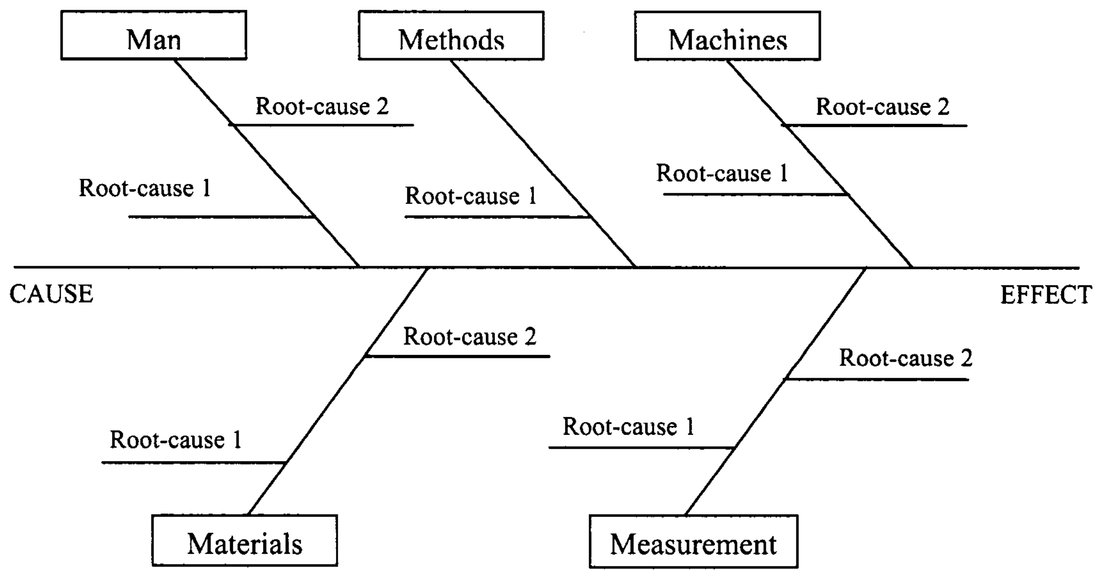

Figure 3.11 Cause-Effect Diagram

3.5.2 Use of chart patterns in assignable cause search

In order to simplify the process of assignable cause search, hints provided by the control charts through unnatural patterns can be used. This section explains the relationship between different unnatural patterns and the respective assignable cause producing that pattern. This chart pattern-assignable cause relationship can be established either from past experience or from the guidelines on general chart pattern – cause relationships. These general guidelines on chart pattern-cause relationships are available in the literature. [22] has en extensive discussion on various unnatural patterns that can be detected on Shewhart control charts with suggestions on list of causes for each pattern.

[21] and [23] also contain comprehensive suggestions on use of these unnatural patterns in finding assignable causes.

The Chart pattern-Cause relationship is established based on the nature of shift produced by the cause. Accordingly, assignable causes are categorized in to three divisions based on three modes of shift exhibited by the unnatural patterns. They are,

- **Isolated Causes**

  Isolated causes are those that will cause a single measurement to vary drastically, resulting in one particular point falling outside the control limits producing an OCL (out of control limits) pattern on the control chart. These causes have one-time effect. The possible causes that comes under this category are

  - A mistake in measurement, recording or plotting
  - Damage in handling
  - Defect in raw material used for that unit alone
  - False alarm

- **Shift causes**

  These causes produce a considerable shift in the process mean. These can be identified by the Freak and Run patterns on the control chart. They indicate that some event has taken place that has affected a few samples causing a drift in the mean. Usual causes that could produce this effect are,

- Tool break
- Change in raw material or supplier
- Change in inspection methods or standards
- Adjustments made in machine settings
- Introduction of new workers or inspectors

- **Gradual causes**

  These causes tend to change the process mean gradually over time, and produce increasing or decreasing trends on the control chart and are identified by the trend pattern. Trend patterns are produced by causes such as,

  - Gradual introduction of new raw-material
  - Loosening fixtures
  - Operator fatigue
  - Machine tool wear
  - Gauge wear
  - Environmental changes

By categorizing the whole domain of the assignable causes into sub-categories, the search for assignable causes using chart patterns becomes more focused as these chart patterns indicate what type of cause could have changed the process. It helps to reach at the root cause sooner, minimizing the effort required to investigate the whole domain of potential causes, provided the unnatural patterns shown on the control chart are interpreted correctly.

Interpretation of unnatural patterns becomes complex if more than one patterns co-exist. In such cases, it would again increase the list of causes to be investigated. Also, in situations such as having four points continuously increasing and fifth point falling beyond the control limit, the user would tend to interpret that as a OCL pattern, which actually could be a trend pattern identified pre-maturely. This could mislead the assignable cause search wasting effort, time and money. To handle such ambiguities, a fuzzy inference engine is developed which could prioritize the causes based on the unnatural patterns present.

Chapter 4

Fuzzy Logic Based Assignable Cause Diagnosis in $\overline{\mathrm{X}}$ Chart

Fuzzy set theory and fuzzy logic has been an excellent tool for handling ambiguities and vagueness associated with real-world applications. Before presenting the design of fuzzy inference engine, a brief introduction to fuzzy set theory and terminologies are presented.

4.1 Introduction to Fuzzy Set Theory

Fuzzy set theory was introduced by Dr. Lotfi Zadeh in 1965 [3], considering the problems associated with classical two valued logic (0 and 1). It provides a natural framework for representing and manipulating the ambiguities associated with real-world applications. The classical set theory allows the entities to be represented by crisp values, either zero or one, forcing us to consider either true or false. But in real world situations there is always some uncertainty prevails.

For example, consider the statement, *'The distance is far'*. In classical logic, this has to be represented as either, 1, considering the statement as true or, 0, by considering it false, which requires us to define the term *'far'* in exact numbers. In reality, the understanding of the term *'far'* is subjective and varies with the context. Hence, if the value of distance is given say, $\mathcal{X}$, then the truth or compatibility of the statement with *'far'* is a matter of degree between 0 and 1, determined by the *membership function*. Fuzzy set theory allows us to define the term *'far'* mathematically and evaluate to what degree the statement is compatible with the term *'far'* when a value of distance is given. The fuzzy sets and other important concepts in fuzzy set theory are defined as follows,

4.1.1 Fuzzy set

Let X be the universal set. A fuzzy subset A, is characterized by its membership function

$$\mu_A: X \longrightarrow [0\text{-}1] \tag{4.1}$$

which associates with each element x of X a real number $\mu_A(x)$ in the interval [0-1], with $\mu_A(x)$ representing the grade of membership of element x in the fuzzy set A.

4.1.2 Membership functions

Membership function is the characteristic function represented mathematically that assigns the grade of membership to its elements. It could be defined as a discrete set or as a continuous function. Some of the most common membership functions for continuous universe of discourse are triangular and trapezoidal membership functions.

- **Triangular membership function**

Triangular membership functions are the simplest form of membership functions to represent and manipulate. A single point, $x=\alpha$, at peak, has the highest value of degree of membership. They are represented by the general equation,

$$\mu_A(x) = \begin{cases} (x-(a-b))/c & if (a-b) \le x \le a \\ 1-(x-a)/c & if a \le x \le (a+c) \\ 0 & otherwise \end{cases} \tag{4.2}$$

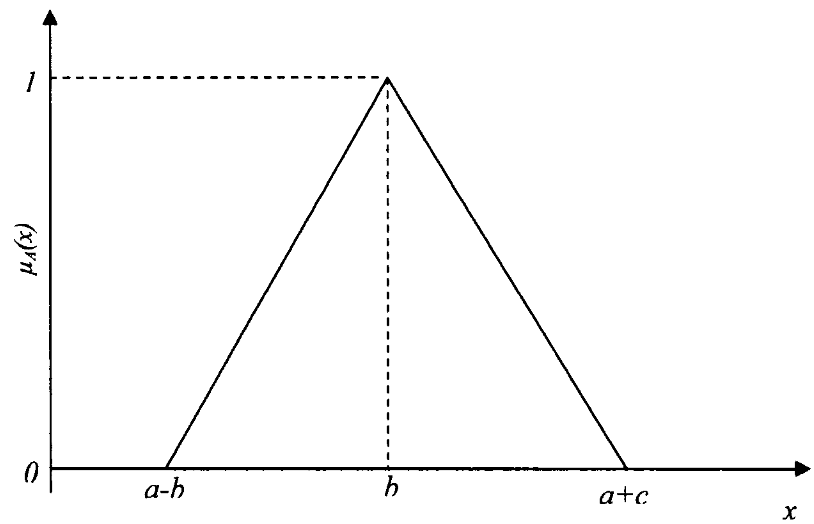

Fig 4.1 Triangular Membership function

- **Trapezoidal membership function**

This kind of membership function is suitable to define a fuzzy set where more than one element has highest degree of compatibility to the fuzzy set defined. The general equation for trapezoidal function is given by,

$$\mu_A(x) = \begin{cases} (x-(a-c))/c & if (a-c) \le x \le a \\ 1 & if a \le x \le b \\ 1-(x-b)/d & if b \le x \le (b+d) \\ 0 & otherwise \end{cases} \tag{4.3}$$

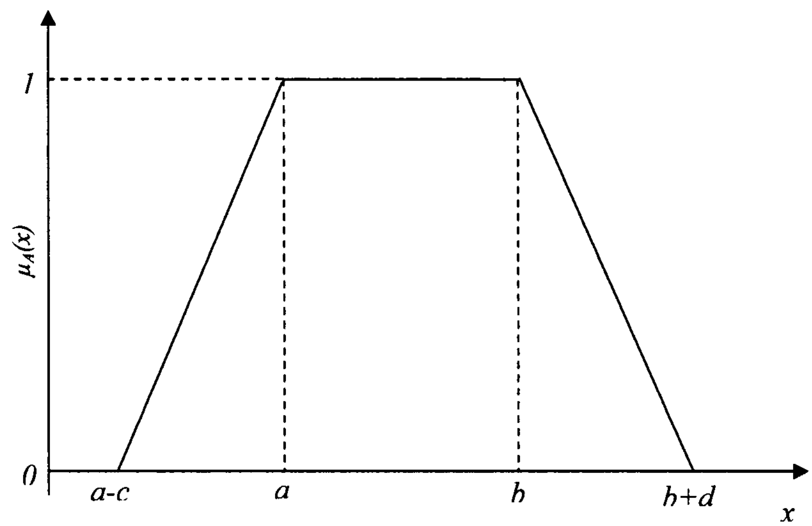

Fig 4.2 Trapezoidal Membership function

Triangular and trapezoidal membership functions are used in our fuzzy inference systems, due to its simplicity in representation and manipulation. There are other membership functions available such as Gaussian membership functions, sigmoid membership functions, Pi-shaped membership functions, S-shaped membership functions, Z-shaped membership functions etc.

4.1.3 Fuzzy Inference System (FIS)

Fuzzy inference system maps input variables to output variables through a fuzzy rule base. The components of a fuzzy inference system involve input variables and its membership functions, fuzzy rule base, output variables and its membership functions. The steps involved in fuzzy inference system are fuzzification, inference and defuzzification. The input value is fuzzified by the membership functions defined for the input variables and the degree of membership is calculated. The commonly used methods of inference are Mamdani and Sugeno type of inference. Out of these two, Mamdani-type [24] is more commonly used as compared to Sugeno-type inference [25] [26]. These methods differ in the way the output value is calculated.

In Mamdani-type inference, the output variables are defined as fuzzy sets and final output value is determined by a defuzzification method, whereas in Sugeno model, the output values are defined as a polynomial of input variables and the final output value is determined using weighted average of data points. Sugeno type of reasoning is best suitable to cases where a structured fuzzy model has to be developed from a set of available input and output data.

The fuzzy inference Engine developed in our model uses Mamdani-type of inference. A typical Mamdani-type inference system, from MATLAB [27] fuzzy logic toolbox, is shown in the figure 4.3.

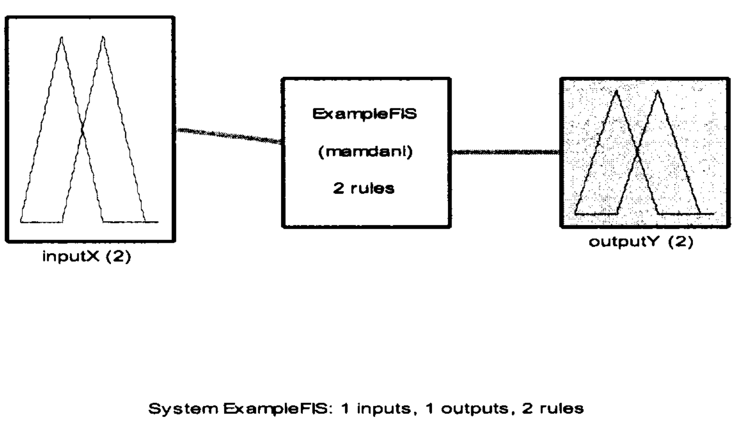

Figure 4.3 Fuzzy Inference System

4.1.4 Fuzzy rule base and inference

This forms the core of the inference engine, which actually does mapping of input variables to output variables. A collection of fuzzy rules is called a fuzzy rule-base. A fuzzy rule is represented by an ***If –Then*** statement containing input fuzzy sets in the antecedent and output fuzzy sets in the consequent. The general syntax of the fuzzy ***If-Then*** rule is,

If X is A ***Then*** *Y is B*

where A is a fuzzy set of input variable X and B is the fuzzy set of output variable Y. If there are more than one input/output variable, they can be combined using *AND/OR* operators, which are essentially a t-norm or t-conorm operators [28]. Most commonly used t-norm and t-conorm operators are '*min*' and '*max*' functions respectively.

The inference part consists of combination of two operations implication and composition. In Mamdani type of inference '*min*' operator is used for implication and '*max-min*' operator is used for composition. The inference process can be explained with a following example. Consider that input variable, *inputX*, shown in the FIS Figure 4.3, has two fuzzy sets, A and B defined as triangular membership functions shown in the Figure 4.4. The output variable, *outputY*, has two fuzzy sets C and D, again as triangular membership functions as shown in Figure 4.5. Let fuzzy-rule base is defined as

If inputX is A ***Then*** *outputY is C*

***If** inputX is B **Then** outputY is D*

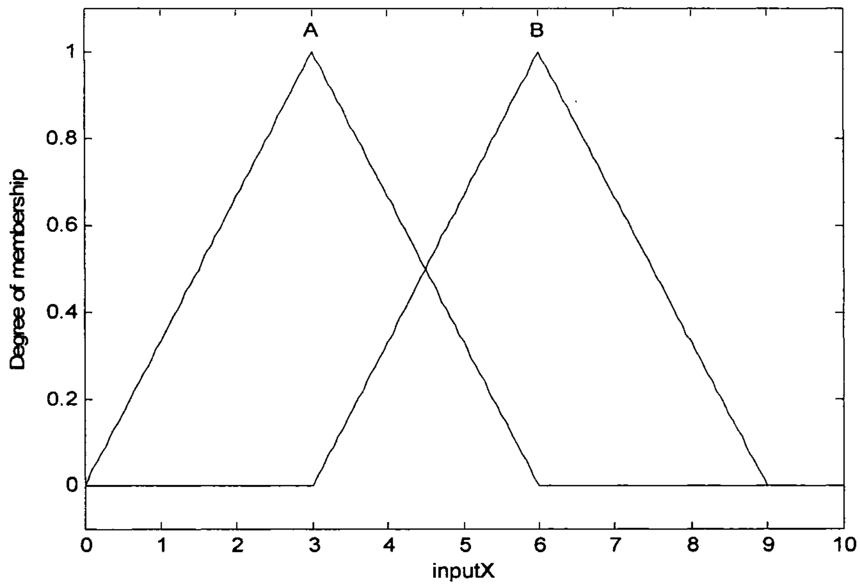

Figure 4.4 *inputX* membership functions

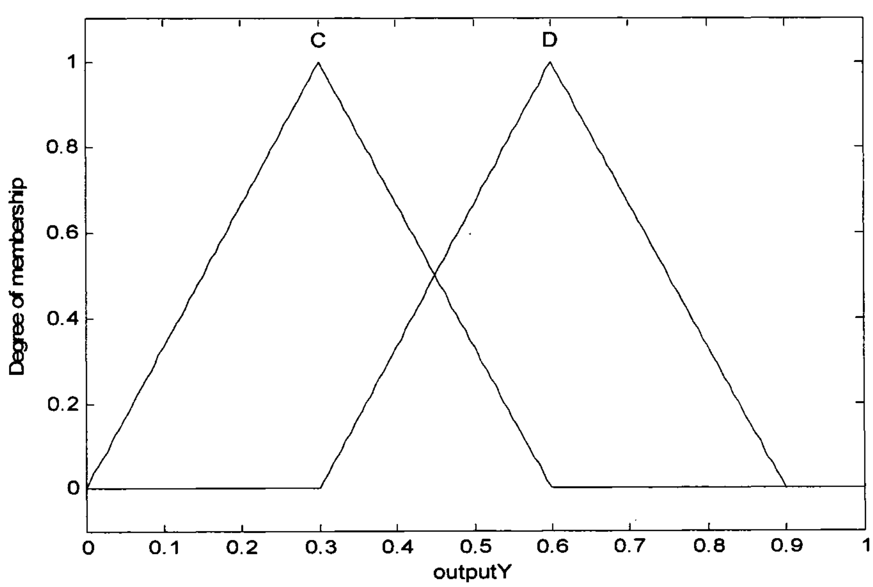

Figure 4.5 outputY membership functions

If the value of input variable is given, say 5, it is fuzzified and its the degrees of membership function in each input fuzzy set ($\mu_A(x)$ and $\mu_B(x)$) is calculated to determine the firing strength with which projected on the output fuzzy set, represented as shaded region in the output fuzzy sets in figure 4.6.

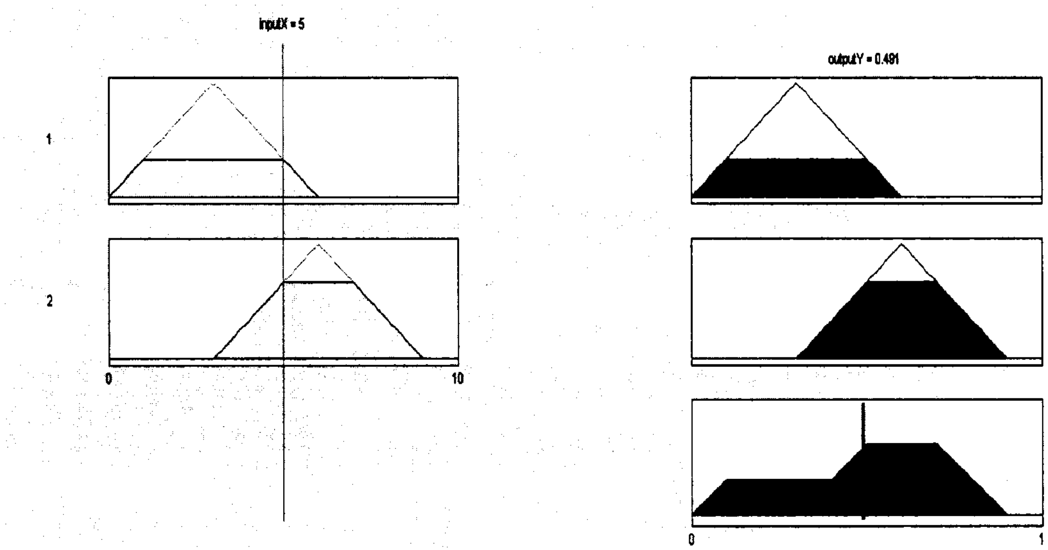

Figure 4.6 Fuzzy Inference & Defuzzification

This is achieved by implication operation and the shaded regions in the output fuzzy set are aggregated by 'max-min' operator, called composition.

4.1.5 Defuzzification

After implication and aggregation the output is still in the form of a fuzzy set and it has to be defuzzified to get a single crisp output value. There are various methods supported to defuzzify the output fuzzy sets such as, centroid, bisector, middle of maximum (average of maximum value of the output sets), largest of maximum and smallest of maximum. Among them, the most popular method is centroid. In this method, the final single crisp output value is determined by calculating the center of area of aggregated shaded region in output fuzzy sets. It is represented by a vertical line in the aggregated output region in the Figure 4.6.

4.2 Design of Fuzzy Inference Engine

4.2.1 Network model of chart pattern-cause relationship and components of fuzzy inference engine

The fuzzy inference engine is developed keeping in mind the problems due to ambiguities associated with interpretation of control chart patterns in

- determining whether or not the process is in-control
- if the process goes out of control, ambiguities in relating the control chart patterns to assignable causes.

To resolve this, the fuzzy inference engine is designed based on the chart pattern–cause relationship, represented as a network in Figure 4.7, determine the intensity of the causes based on the patterns exhibited on the control chart. The network is developed based on the discussions in Section 3.5.2., on using the control chart patterns to minimize the effort spent in assignable cause search by categorizing the causes according to the nature of shift they could produce.

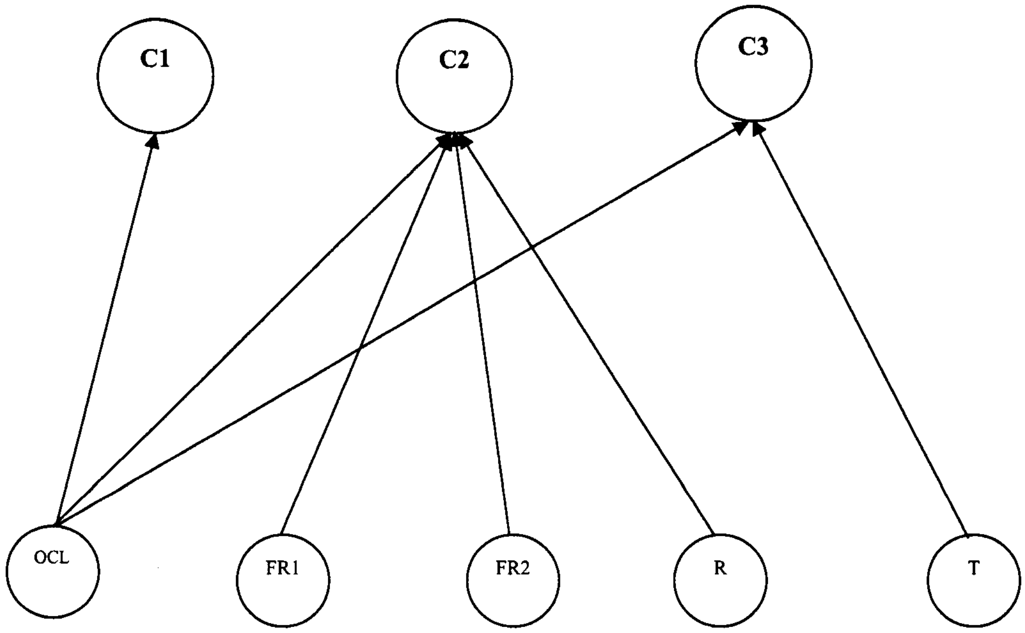

Figure 4.7 Network Model of Chart pattern – Cause Relationship

C1 – Isolated causes
- **A mistake in measurement, recording or plotting**
- **Defect in raw material used for that unit alone**
- **Damage in handling**
- **False Alarm**

OCL – Out of control limit
FR1 – Freak1
FR2- Freak2
R-Run
T-Trend

C2 - Shift Causes
- **Tool break**
- **Change in raw material**
- **Adjustments made in machine settings**
- **Change in inspection method or standards**
- **Introduction of new workers or inspectors**

C3 - Gradual Causes
- **Gradual induction of new raw material**
- **Loosening fixtures**
- **Operator fatigue**
- **Machine tool wear**
- **Gauge wear**
- **Environmental changes**

The top nodes in the network C1, C2 and C3 represent isolated causes, shift causes and gradual causes respectively. The lower nodes represent unnatural patterns Out of Control-limits (OCL), Freak 1 (FR1), Freak 2 (FR2), Run (R) and Trend (T). The links represent the causal relationship between the respective nodes, each link modeled by a separate fuzzy inference system (FIS). The whole inference engine is developed using the software, MATLAB. Each FIS has the pattern as input variable and the cause as an output variable. Each FIS determines the intensity of the cause, on the scale of [0-1], based on the degree of presence of its input pattern. Then finally the output from all the FIS is aggregated, by the method described in Section 4.3., and the causes are ranked in order, based on their intensity. Hence the assignable cause investigation shall be started with the causes having highest intensity of presence.

Any assignable cause, if present, may produce a point outside the control limit, at the beginning of its characteristic pattern, in the middle or as a final point completing the definition of its characteristic pattern. Therefore once observing a point beyond control limits and concluding prematurely that the assignable cause is an isolated cause, could be misleading. Due to the possibility that OCL pattern might arise within the patterns of other causes it has been linked to other causes with lower influence of causality modeled by fuzzy if-then rules and membership functions. Once obtaining further evidence from the presence of other underlying patterns, the intensities of the causes from the presence of other underlying patterns are also collected and aggregated using the methods described in Section 4.3. The degree of presence of OCL determines intensity of isolated causes, degree of presence of FR1, FR2 and Run determines the intensity of presence of shift causes and degree of presence of Trend patterns determines the intensity of gradual causes, thus, dividing the fuzzy inference engine in to seven individual fuzzy inference systems as sub-modules and one aggregation module. The block diagram of the fuzzy inference engine design is given in Figure 4.8.

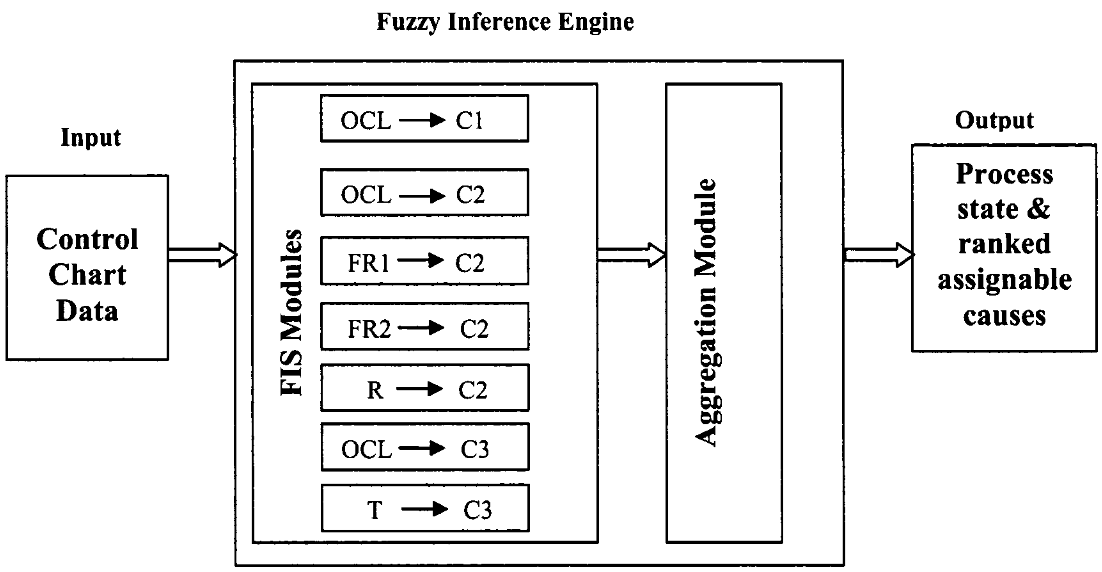

Figure 4.8. Design of Fuzzy Inference Engine

The following assumptions were made in designing the fuzzy inference engine.

- The process is in phase-II of control chart implementation, i.e., initial variations in the process are removed and the process is brought under control and the data is monitored to look for any assignable cause that might arise in future.
- The R-chart is brought under control before looking for unnatural patterns in $\bar{X}$-chart.
- Adaptive sampling measures are taken before investigating for assignable causes to ensure whether or not it's a false alarm.

As it is assumed that the process is operating in phase II, the estimates of process mean and standard deviation are known, and hence the control chart data, input to the fuzzy inference engine constitutes process mean (μ), standard deviation (σ), sample size (n) and sample measurements (x_i). The data is checked by each pattern recognizing FIS modules and its intensity on the assignable cause is determined. The design of each FIS module is explained in the following sections.

4.2.2 *OCL – C1* FIS module

The function of this FIS module is to determine the intensity of *OCL* pattern on isolated causes (C_1). The fuzzy inference system for this module is shown in the Figure 4.9. It has one input variable (*input_OCL*), one output variable (*C1*) and rule base contains three fuzzy *if-then* rules.

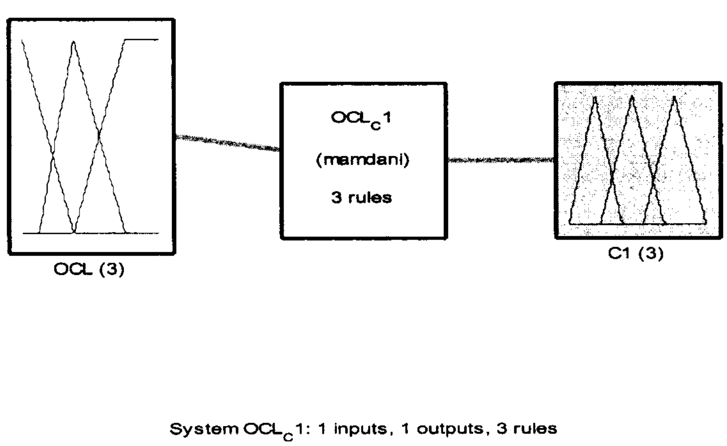

Figure 4.9 *OCL-C1* FIS

The sample measurement (x_i) from the input data is taken and the dispersion from the process mean in terms of multiple of standard deviation unit is calculated using the equation,

$$input\_OCL = \frac{|x_i - \mu|}{\sigma} \tag{4.4}$$

4.2.2a) Input membership functions

The input variable is fuzzified by three membership functions, OCL1, OCL2 and OCL3, two triangular and one trapezoidal function respectively, defined as shown in the Figure 4.10.

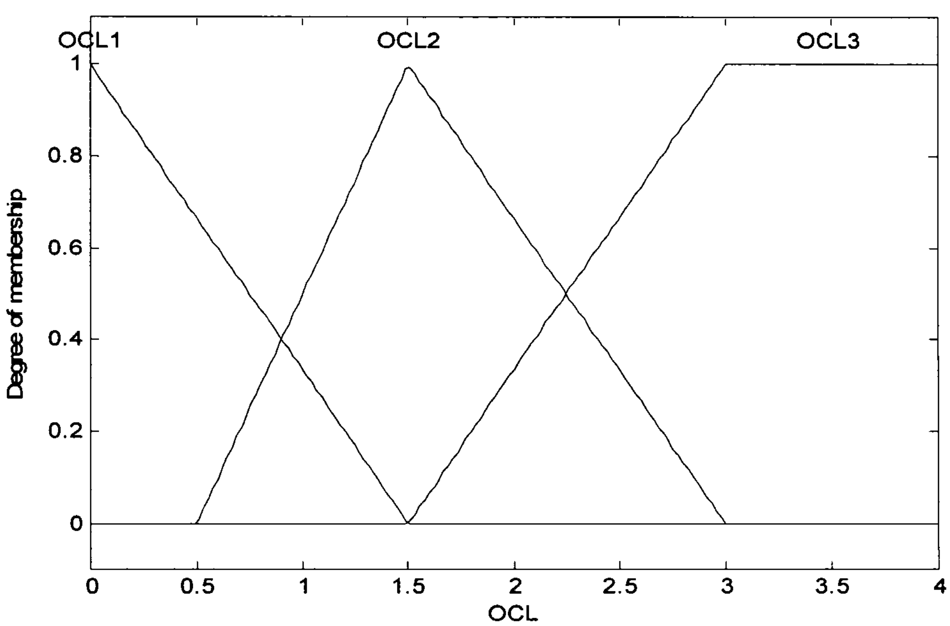

Figure 4.10 Input membership functions of *OCL-C1* FIS

The range for the input data is taken as [0-4] in multiple of standard deviation units, so that it allots the degree of membership for every point in the control chart, from zero for a point lying on center line up to 4σ deviation, in three different fuzzy sets.

4.2.2b) Output membership functions

The membership functions for the output variable '*C1*' are defined over the range [0-1.3], measuring the intensity of the cause present according to the degree of pattern present. '*C1*' has three triangular membership functions, namely, $C1_A$, $C1_B$ and $C1_C$, defined as shown in the Figure 4.11.

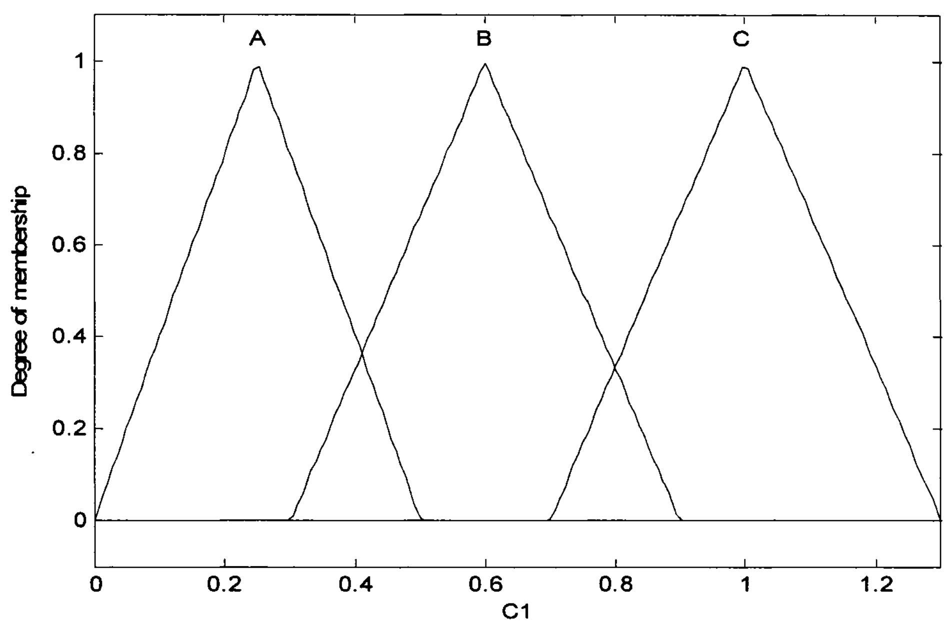

Figure 4.11 Output Membership Functions of OCL-C1 FIS

4.2.2c) Fuzzy rule-base

The intensity of the cause increases with deviation from the center line, reaching the maximum value of 1 when deviation is 3σ from the center line, and thus the three input fuzzy sets are mapped to the three output fuzzy sets by the following *if-then* rules.

- **If** *(input_OCL* is *OCL1)* **then** *(C1* is *A)*
- **If** *(input_OCL* is *OCL2)* **then** *(C1* is *B)*
- **If** *(input_OCL* is *OCL3)* **then** *(C1* is *C)*

The inference process with the above mentioned rules is explained, with an input value of 3 in standard deviation units, as shown in the Figure 4.12. It reaches the maximum value 1.0.

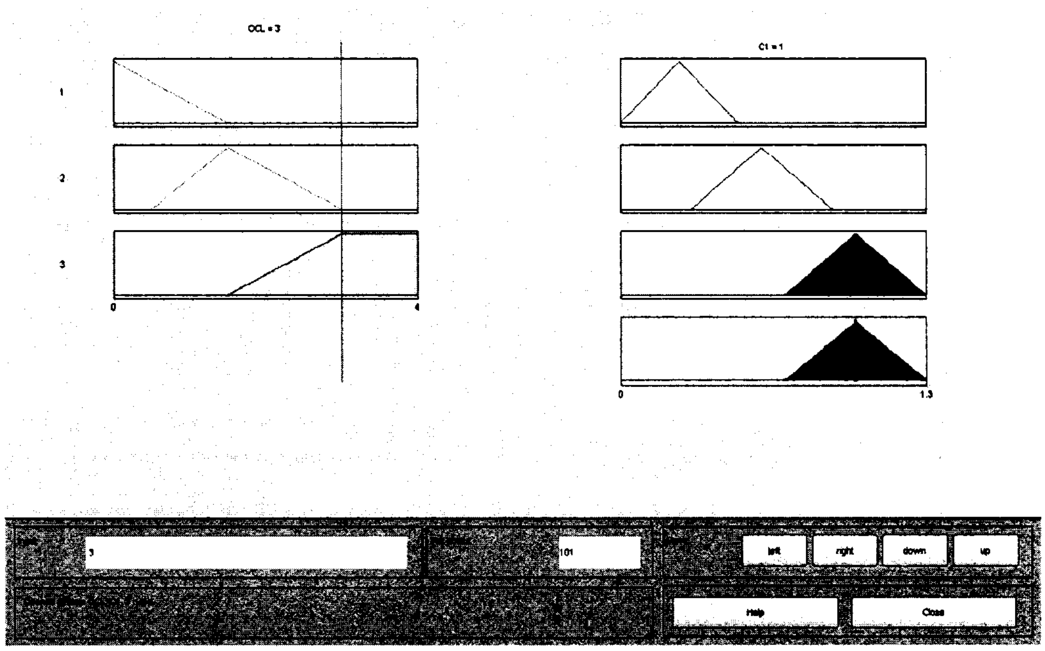

Figure 4.12 Inference and Defuzzication in *OCL-C1* FIS

4.2.2d) Response curve

The values of output generated by this FIS for each input in the specified range can be obtained from the response curve as plotted in Figure 4.13.

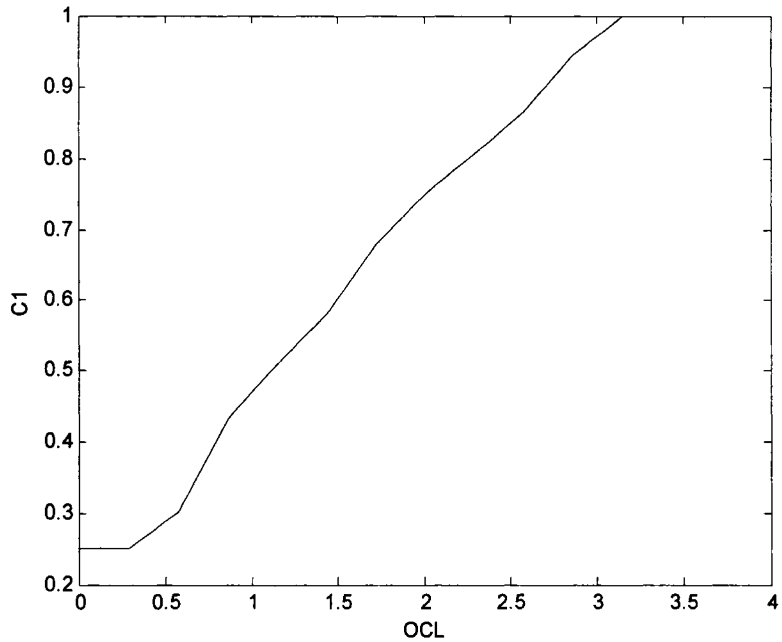

Figure 4.13 Response Curve for *OCL-C1* FIS

4.2.3 *OCL-C2* FIS module

This FIS module determines the intensity of *OCL* pattern it imparts on shift causes (C_2). Design of *OCL-C2* FIS module is shown in Figure 4.14. The input variable is the same as *OCL-C1* module, calculated from equation (4.4) and the input membership functions for *OCL* is same as defined in Figure 4.10. Also the output membership functions for *C2* is defined in the same way for *C1*, and it is shown in Figure 4.15.

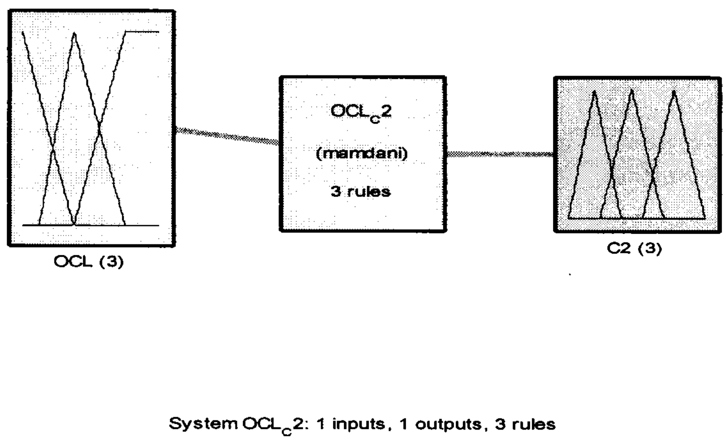

Figure 4.14 ***OCL-C2*** **FIS**

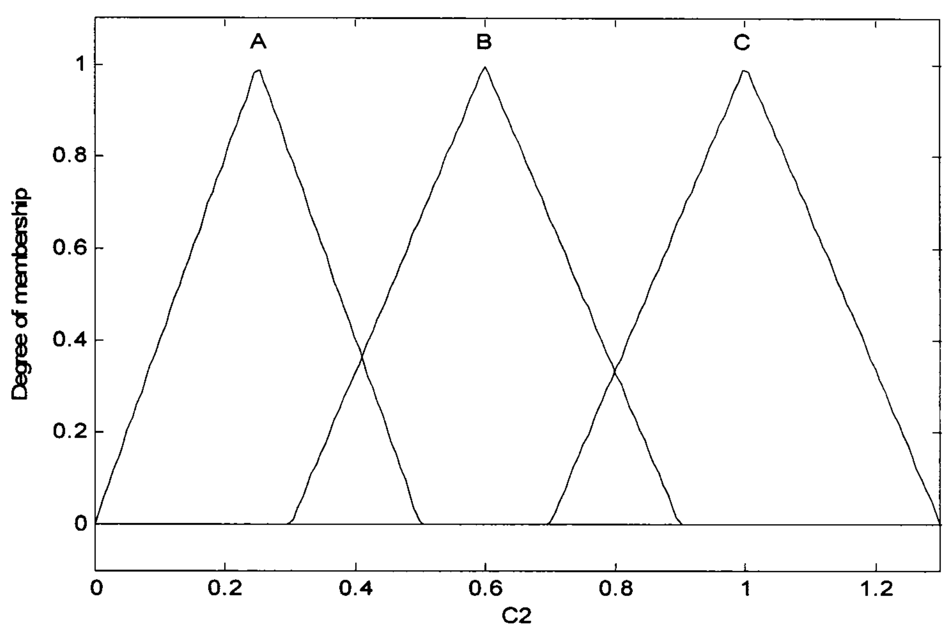

Figure 4.15. Output Membership Functions for C2

4.2.3a) Fuzzy rule-base

As explained earlier, *OCL* pattern could be one of the any points in other unnatural pattern. But the amount of intensity it imparts on shift or gradual causes is relatively lower than their respective characteristic patterns such as freaks, run or trend, and lower than the intensity it imparts on isolated causes. This is achieved through the rule-base established for *OCL-C2* FIS. The presence of shift causes or gradual causes will be determined on aggregating additional evidences from their respective characteristic patterns.

- **If** *(input_OCL* is *OCL1)* **then** *(C2* is *A)*
- **If** *(input_OCL* is *OCL2)* **then** *(C2* is *B)*
- **If** *(input_OCL* is *OCL3)* **then** *(C2* is *B)*

The highest fuzzy set in the input variable *OCL3* is mapped to lower fuzzy set of output variable *C2*, thus restricting the amount of intensity it imparts to *C2*. This can be viewed through rule viewer, in Figure 4.16., for an input value of 3 in standard deviation units, the intensity of *C2* obtained is 0.6, as compared to 1.0 it imparted for *C1* in *OCL-C1* module.

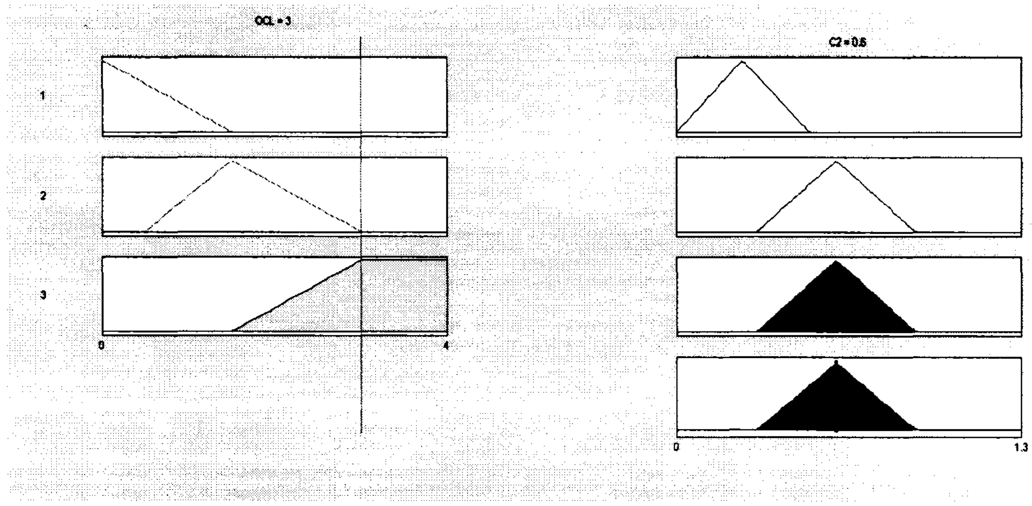

Figure 4.16Inference and Defuzzification in *OCL-C2* FIS

4.2.3b) Response curve

The response curve as shown in the Figure 4.17., shows that *C2* remains constant at 0.6, after 1.5 standard deviation units of input. Further presence of shift causes can only be confirmed on obtaining additional evidence from the respective characteristic patterns of shift causes.

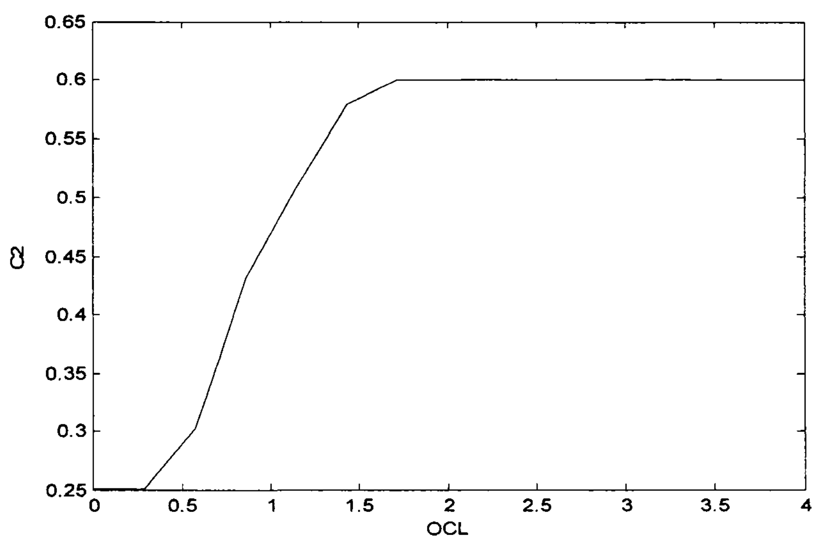

Figure 4.17 Response curve for *OCL-C2* FIS

4.2.4 *OCL-C3* FIS module

The function of this FIS module is to determine the intensity of gradual causes by the *OCL* pattern. The design of this module is represented in Figure 4.18. This module is designed in a similar way as that of *OCL-C2* module. The input variable is calculated by equation (4.4) and the input membership functions are same as that of Figure 4.10. The output membership function for *C3* is also designed similar to that of *C2* & *C1*, as shown in the Figure 4.19.

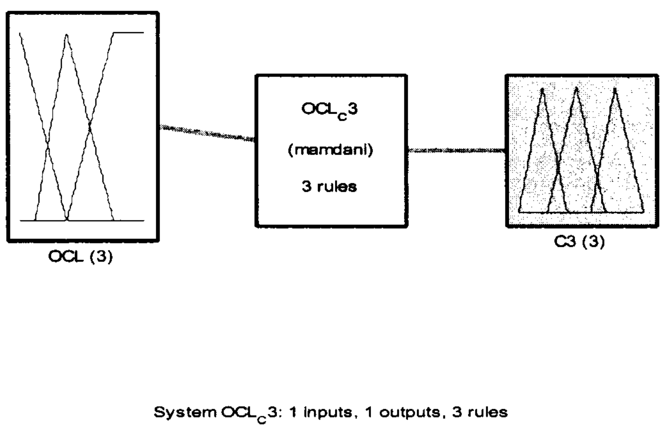

Figure 4.18. ***OCL-C3*** **FIS**

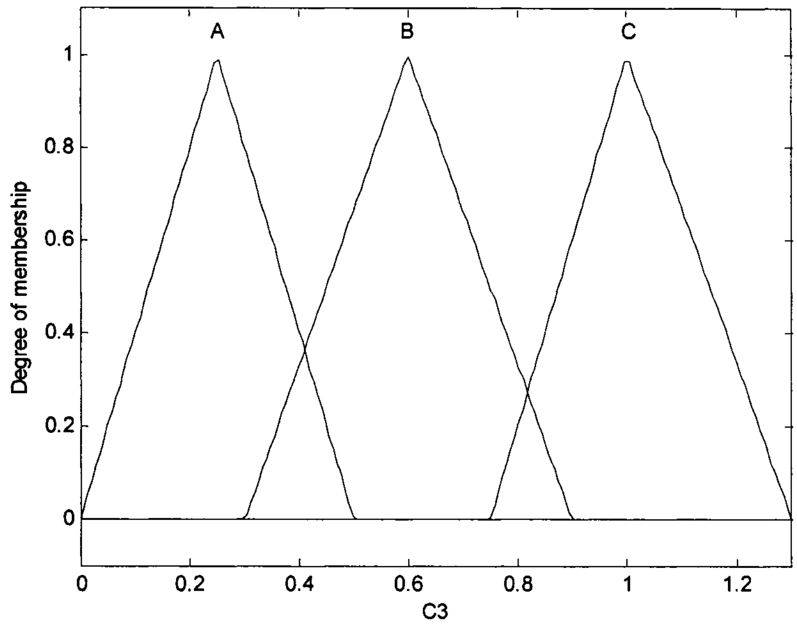

Figure 4.19. Output Membership functions for *OCL-C3*FIS

4.2.4a) Fuzzy rule-base

The fuzzy rule-base is designed similar to that of *OCL-C2* FIS module.

- **If** *(input_OCL* is *OCL1)* **then** *(C3* is *A)*
- **If** *(input_OCL* is *OCL2)* **then** *(C3* is *B)*
- **If** *(input_OCL* is *OCL3)* **then** *(C3* is *B)*

As explained earlier, *OCL* pattern does not impart the same strength of intensity to gradual causes as Trend pattern does. The presence of gradual causes will be confirmed only if further evidence from the trend pattern is reinforced.

4.2.4b) Response curve

The response of this *OCL-C3* module is same as that of the *OCL-C2* FIS module. It is shown in the Figure 4.20.

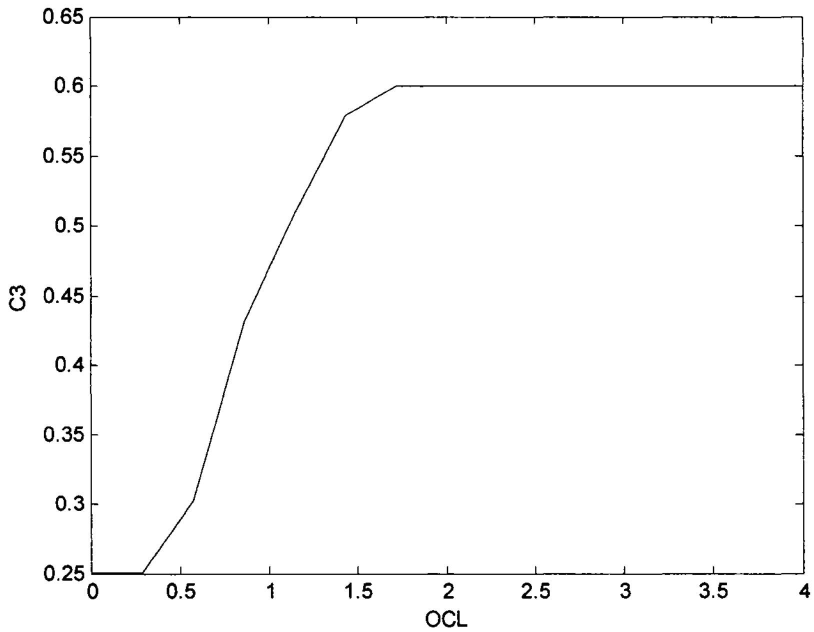

Figure 4.20 Response Curve for *OCL-C3* FIS module

4.2.5 *FR1-C2* module

Freak 1 pattern is produced when four out of five consecutive points fall beyond one sigma control limit on one side of center line. This pattern is an indication of presence of shift cause. The design of FR1-C2 FIS is shown in the Figure 4.21. The input variable '*FR1*' is determined by maintaining a sample-window of five consecutive samples and checking each point for a value greater than one sigma. The number of points above and below the one sigma lines, within the sample-window, is stored separately and the higher value is taken as the input.

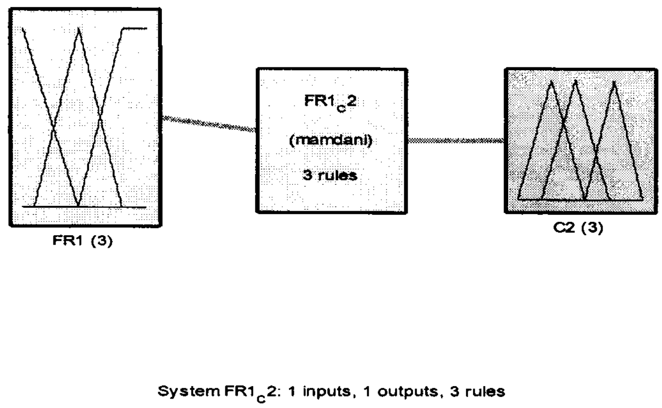

Figure 4.21 ***FR1-C2*** **FIS**

4.2.5a) Input membership functions

As the sample-window is fixed as five, the input data range is [0-5] and three membership functions $FR1_A$, $FR1_B$ and $FR1_C$ are developed as shown in Figure 4.19.

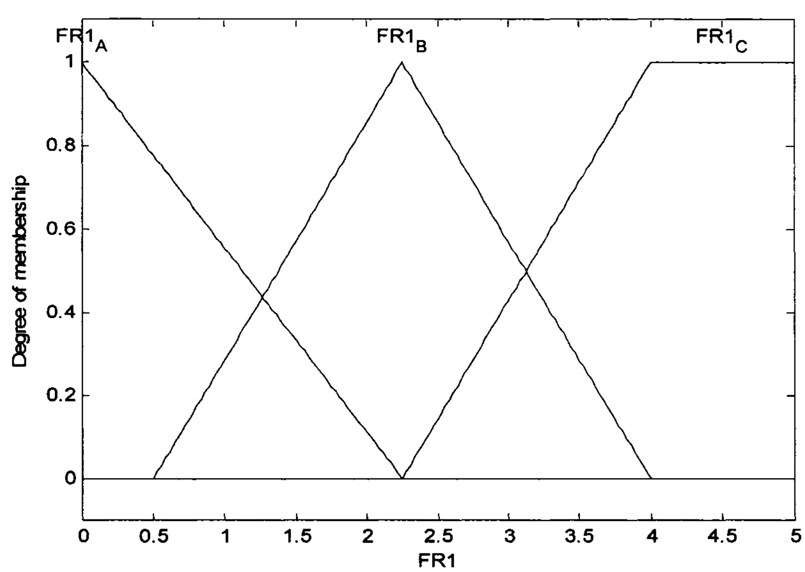

Figure 4.22. Input Membership Functions for ***FR1-C2*** **FIS**

4.2.5b) Output membership functions

The output variable, *C2*, is represented by the membership functions, *C2A*, *C2B* and *C2C*, same as that of *OCL-C2* FIS module, as in Figure 4.15.

4.2.5c) Fuzzy rule-base

More the number of points completing the definition of FR1 pattern, stronger the indication of presence of shift causes. Hence the three input membership functions are mapped to three output membership functions respectively as,

- **If** *(FR1* is $FR1_A$*)* **then** *(*C_2 is *A)*
- **If** *(FR1* is $FR1_B$*)* **then** *(*C_2 is *B)*
- **If** *(FR1* is $FR1_C$*)* **then** *(*C_2 is *C)*

For example, if the input value for '*FR1*' is 3, the inference process is shown in the Figure 4.23, gives 0.75 for the output variable *C2.*

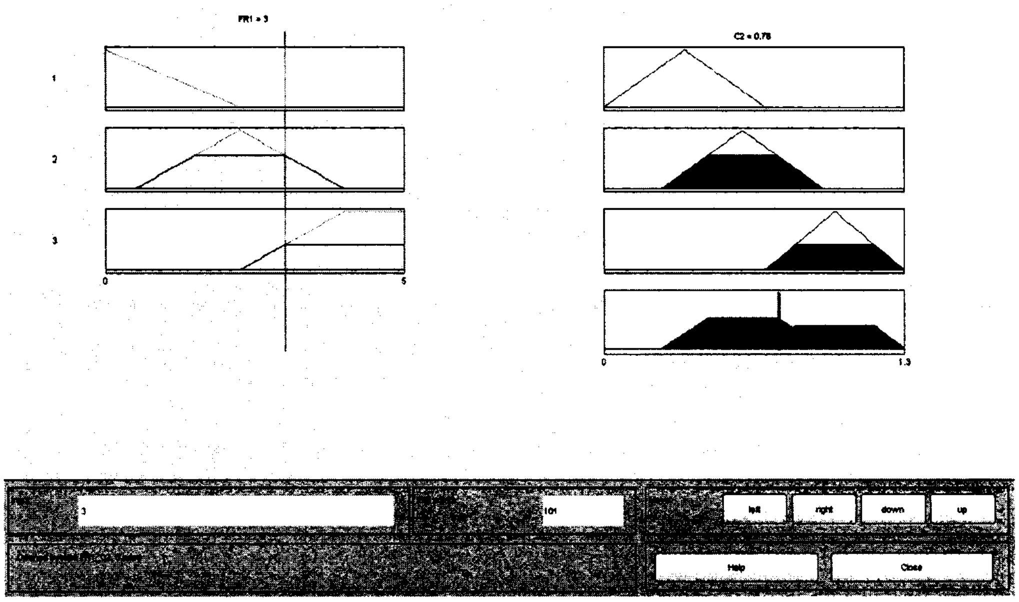

Figure 4.23. Inference and Defzzification in FR1-C2 FIS

4.2.5d) Response curve

The response curve increases with the increase in number of points defining the pattern and it reaches the maximum value 1.0 when the input for *FR1* is four. The response curve is shown in the Figure 4.24.

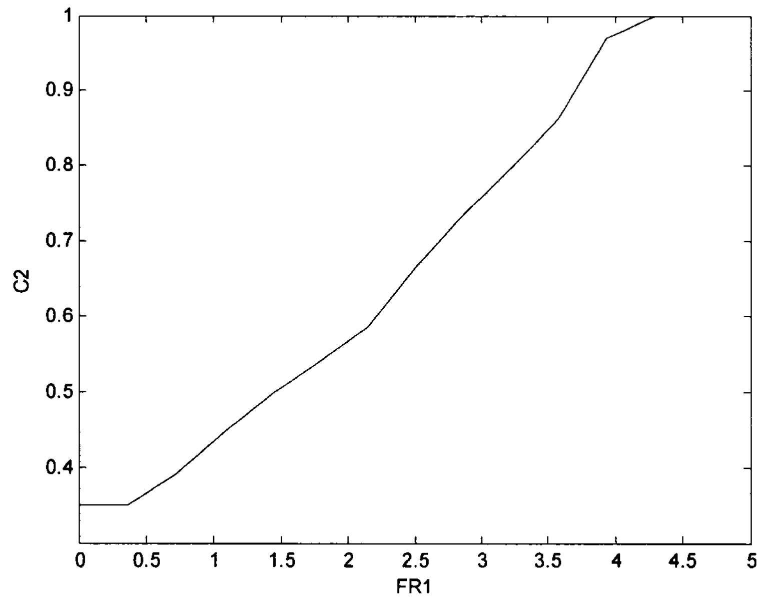

Figure 4.24. Response Curve for *FR1-C2* FIS

4.2.6 *FR2-C2* module

Freak 2 pattern is produced when two out of three consecutive points fall beyond two sigma control limit on one-side of center line. This pattern is also an indication of presence of shift cause. The design of *FR2-C2* FIS is shown in the Figure 4.25. The input variable '*FR1*' is determined by maintaining a time-frame window of three consecutive samples and checking each point for a value greater than two sigma. The number of points above and below the two sigma lines, within the time-frame window, are stored separately and the higher value is taken as the input. The input variable membership functions are shown in the Figure 4.26. The output membership functions for C2 are same as in Figure 4.15.

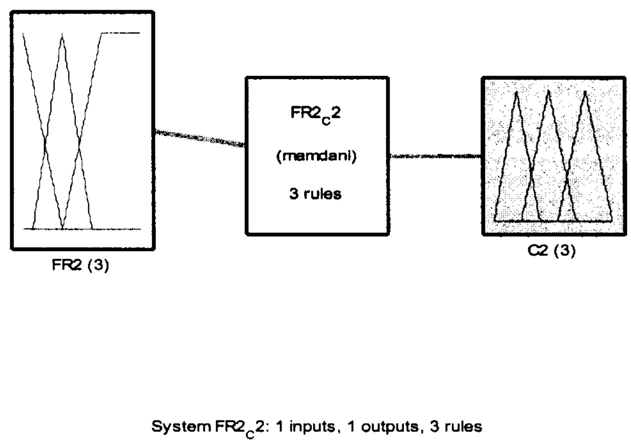

Figure 4.25 FR2-C2 FIS

4.2.6a) Input membership functions

Since the sample-window is taken as three, the input data range is taken as [0-3] with three membership functions as shown in Figure 4.26.

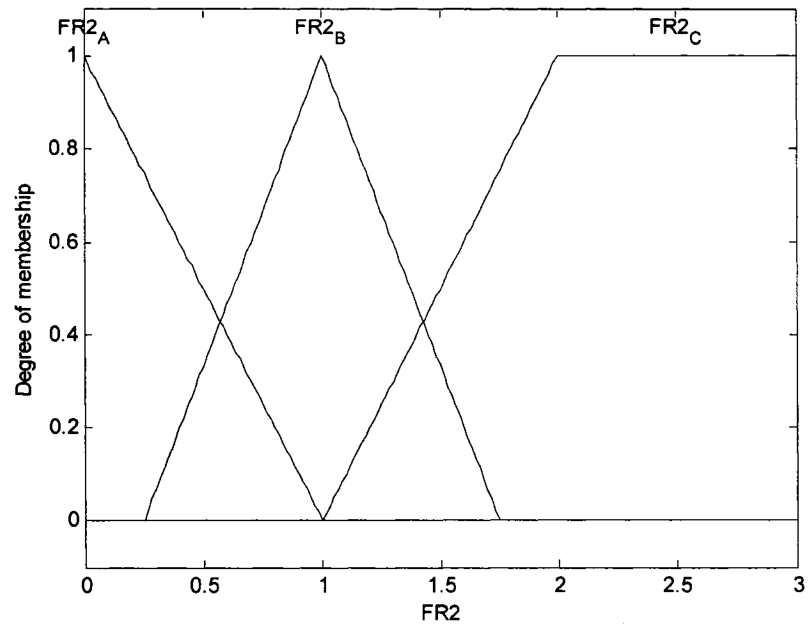

Figure 4.26 Input membership functions of FR2-C2 FIS

4.2.6b) Fuzzy rule-base

Similar to FR1-C2 module, FR2 pattern also has the same fuzzy relation with C2. More clear the presence of FR2, higher the indication of presence of FR2. The fuzzy rule base for this module is defined as,

- **If** *(FR2* is *$FR2_A$)* **then** *(C_2* is *A)*
- **If** *(FR2* is *$FR2_B$)* **then** *(C_2* is *B)*
- **If** *(FR2* is *$FR2_C$)* **then** *(C_2* is *C)*

Since two points out of three falls beyond two sigma limit, when the input value becomes two, giving rise to a complete freak2 pattern, the intensity of C2 becomes 1.0. This inference is shown in the rule viewer diagram in Figure 4.27.

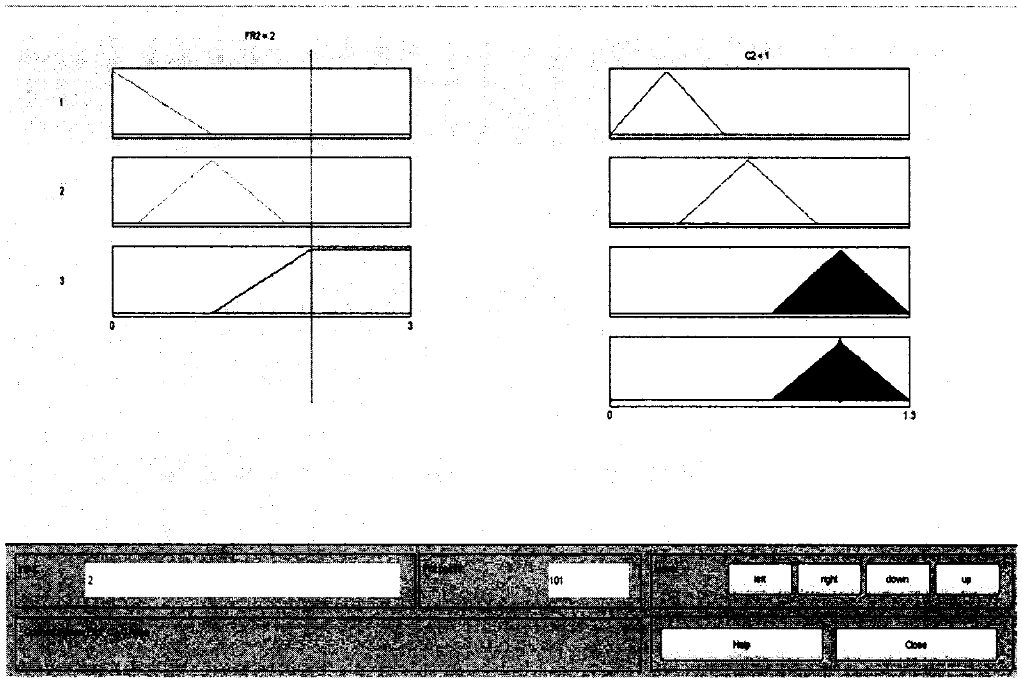

Figure 4.27. Inference and Defuzzification in FR2-C2 FIS

4.2.6c) Response curve

FR2 pattern also has the same effect on C2 as that of FR1 pattern. More complete the definition of FR2 pattern on the control chart, clearer the indication of presence of shift cause. Hence, as per the definition of FR2, the pattern is defined completely when the input value reaches two, and the corresponding output value for C2 reaches maximum of 1.0, which can be seen from the Figure 4.28.

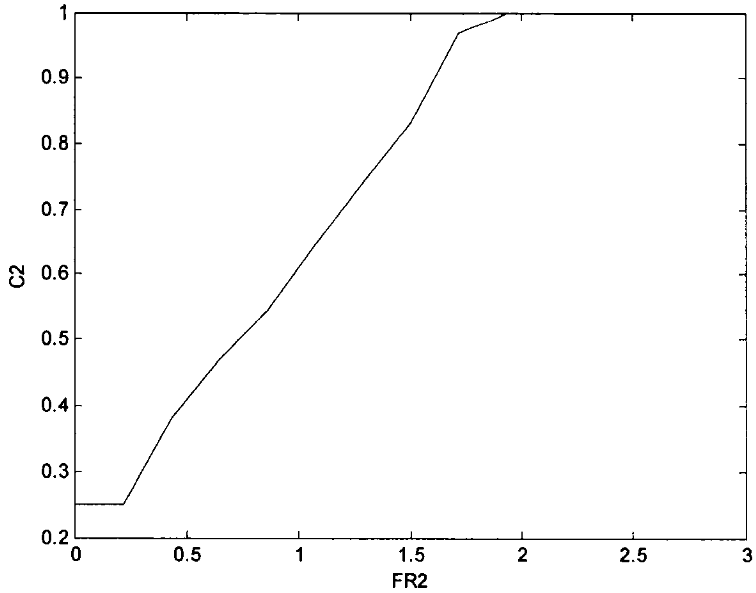

Figure 4.28. Response Curve for FR2-C2 FIS

4.2.7 *R-C2* FIS module

This FIS determines the intensity of shift cause presence due to Run pattern. The design of FIS is shown in the Figure 4.29. The Run pattern is defined as seven or more consecutive points falling on one side of center line. Here the sample-window is maintained for seven samples and '*consecutive*' points above and below the center line are calculated and highest value is taken as the input variable '*R*'.

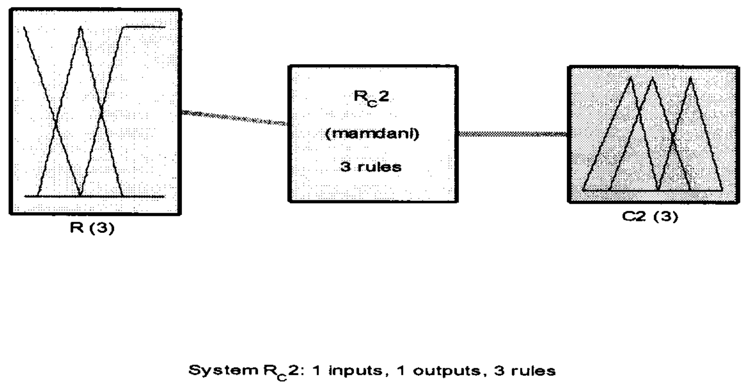

Figure 4.29. R-C2 FIS

4.2.7a) Input membership functions

As the sample window is taken as seven, the input variable in terms of number of consecutive samples on the same side of centerline, within the sample-window is represented using three membership functions as shown in the Figure 4.30 with the input data range [0-10]. The output membership function for shift cause variable is same as represented in Figure 4.15.

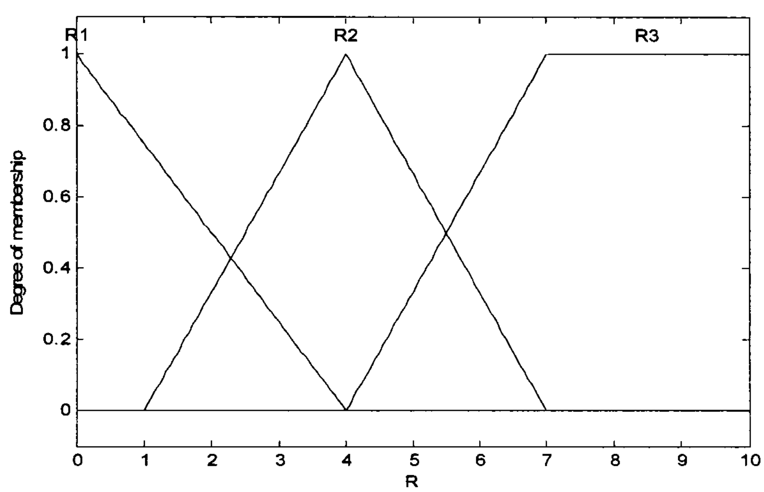

Figure 4.30 Input Membership Functions of *R-C2* FIS

4.2.7b) Fuzzy rule-base

The three input membership functions of Run variable is mapped to three output membership functions as

- **If** *(R* is *R_1)* **then** *(C_2* is *A)*
- **If** *(R* is *R_2)* **then** *(C_2* is *B)*
- **If** *(R* is *R_3)* **then** *(C_2* is *C)*

The inference process for the input value seven is shown in the Figure 4.31.

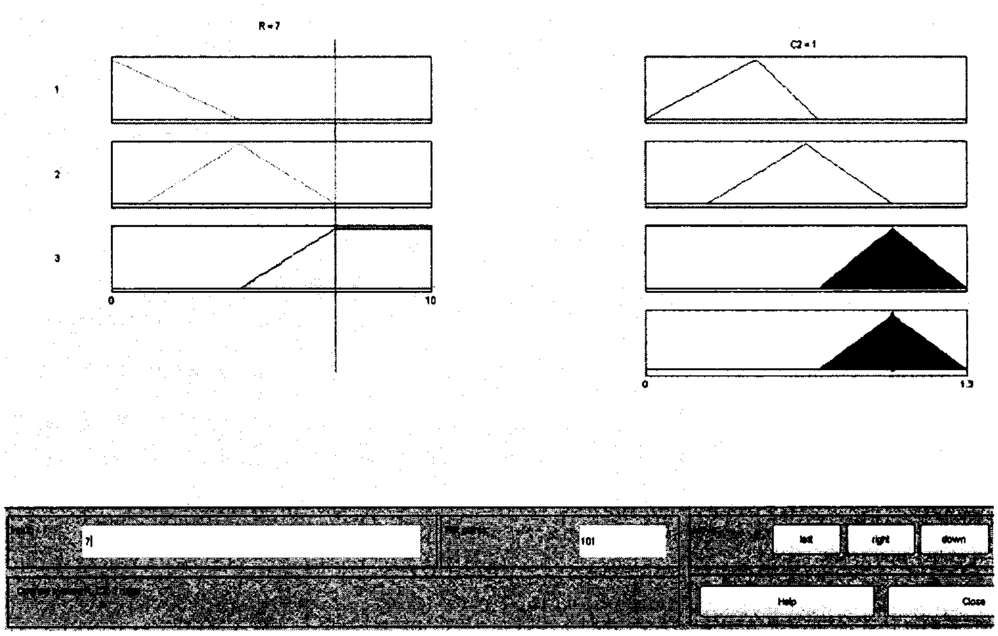

Figure 4.31. Inference and Defuzzification in *R-C2* FIS

4.2.7c) Response curve

From the Figure 4.32., it can be seen that the value of C2 increases with the increase in value of R and reaches the maximum of 1.0 when the input value is seven, number of points indicate the clear presence of Run pattern.

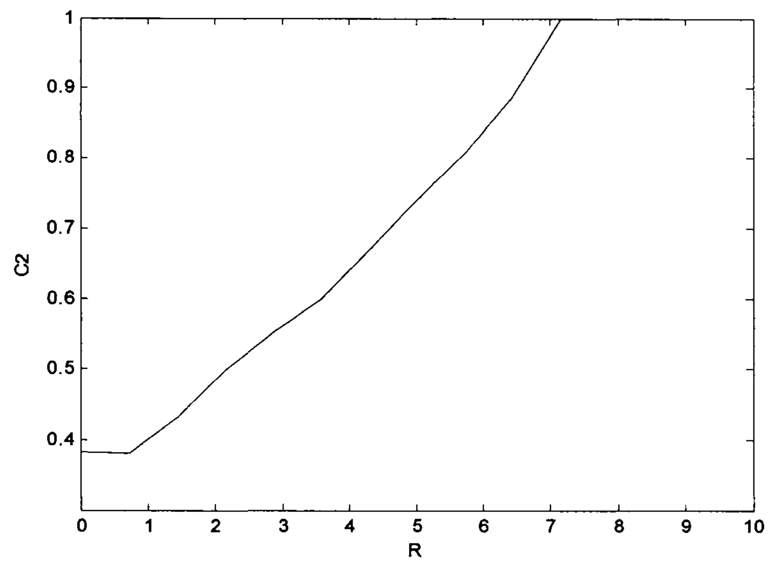

Figure 4.32. Response Curve for R-C2 FIS

4.2.8 *T-C3* FIS module

This module determines the intensity of gradual causes based on the presence of trend pattern. A trend pattern is observed when seven or more consecutive points are continuously increasing or decreasing. The design of T-C3 module is shown in the Figure 4.33. The input value is calculated by maintaining a sample window of length eight, and checking each sample with the previous for an increase or decrease in the value, and simultaneously, storing the number of consecutive increasing points and decreasing points within the sample window in separate variables.

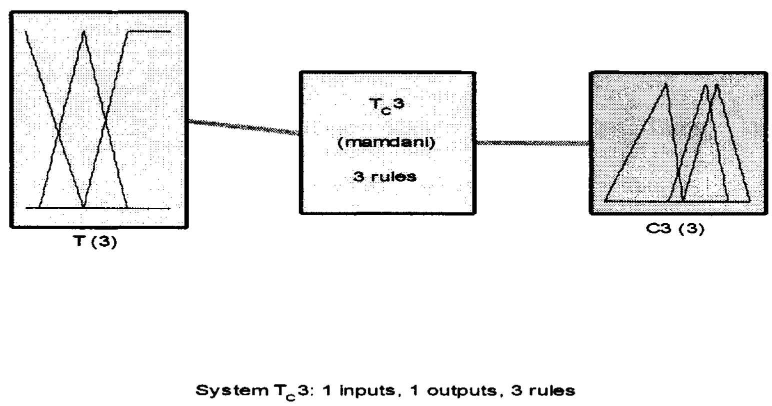

Figure 4.33 T-C3 FIS

4.2.8a) Input membership functions

The sample window length for the trend pattern is eight and hence the input data range is taken as [0-10]. Three membership functions are defined over that interval to represent the input variable as shown in the Figure 4.34.

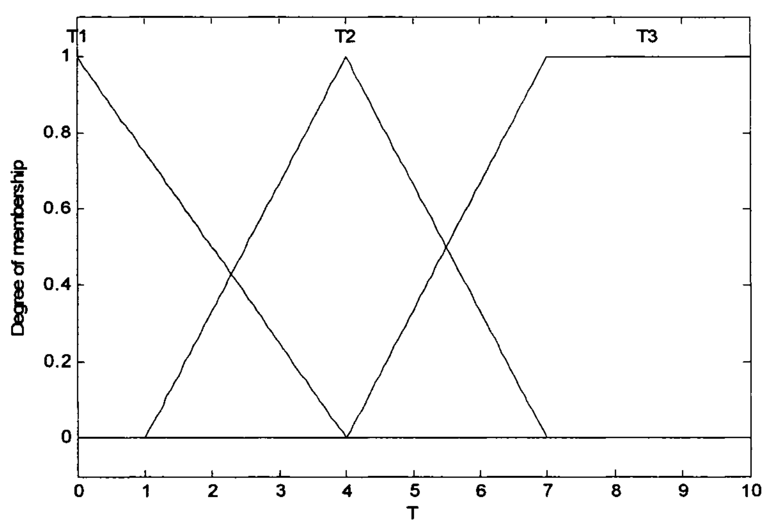

Figure 4.34 Input Membership functions for T-C3 FIS

4.2.8b) Output membership functions

The output variable C3, gradual cause, is represented by three triangular membership functions as shown in the Figure 4.35. The membership functions of C3 is slightly different than those of C2 and C1, as these membership functions are fine tuned for earlier detection of trend pattern, due to its longer sample window length compared to other patterns.

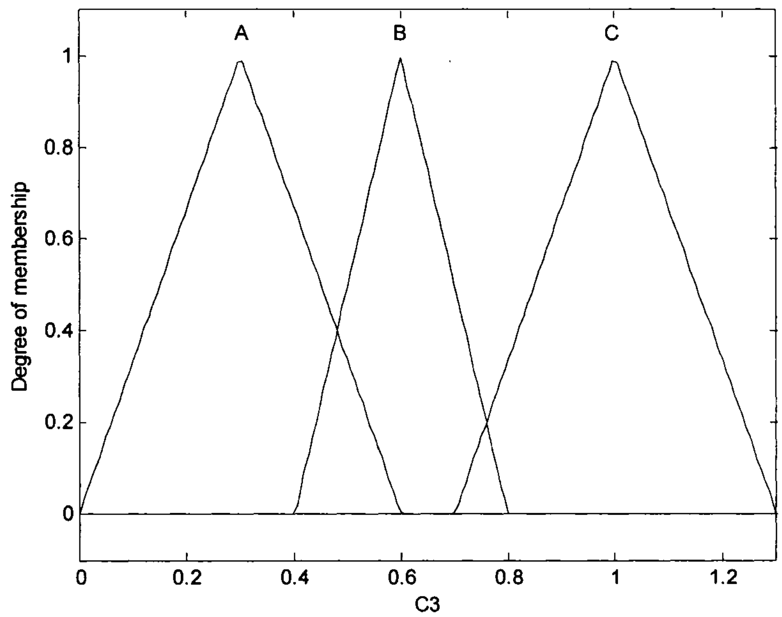

Figure 4.35 Output Membership Functions of T-C3

4.2.8c) Fuzzy rule-base

The fuzzy rule-base is formed in a similar way to that of previous modules, mapping the three input membership functions to three output membership functions, so that when the input increases showing the presence of trend pattern stronger, C3 increases correspondingly.

- **If** *(T* is *T1)* **then** *(C_3* is *A)*
- **If** *(T* is *T2)* **then** *(C_3* is *B)*
- **If** *(T* is *T3)* **then** *(C_3* is *C)*

The inference and defuzzification values for the input value of seven are shown in the Figure 4.36.

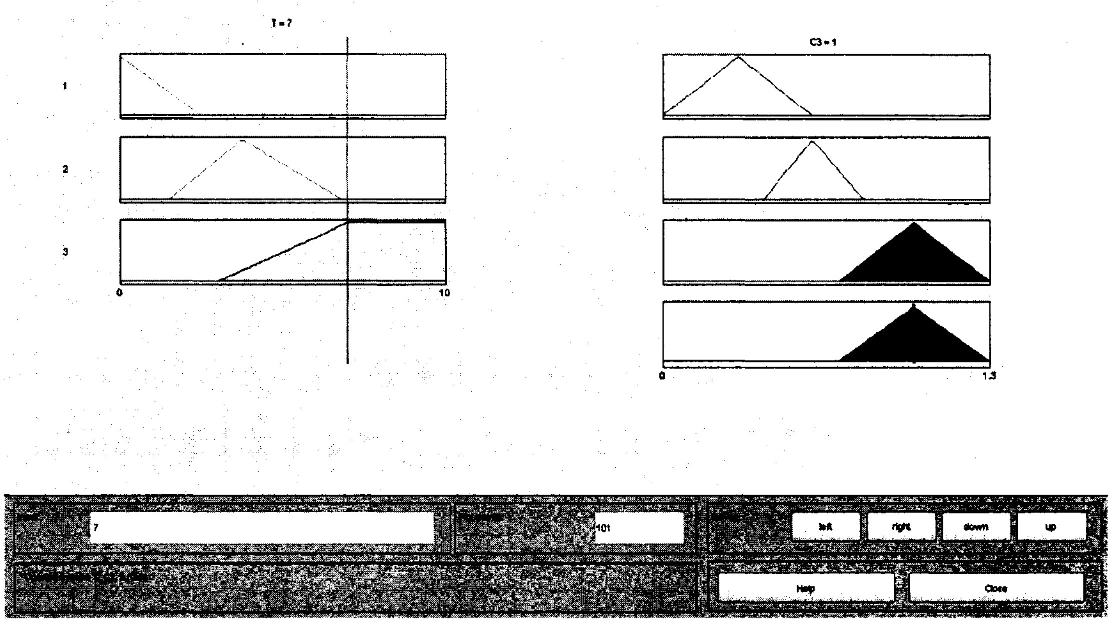

Figure 4.36 Inference and Defuzzification of T-C3 FIS

4.2.8d) Response curve

The response curve for this module is shown in the Figure 4.37. From the graph it can be seen that as the input value reaches seven, on detection of a complete defined pattern of trend, the value of gradual cause reaches maximum of 1.0. As described in the

Section 4.2.8b., the membership functions are fine tuned so that the inference system gives higher response at the initial stage itself on detecting the trend pattern.

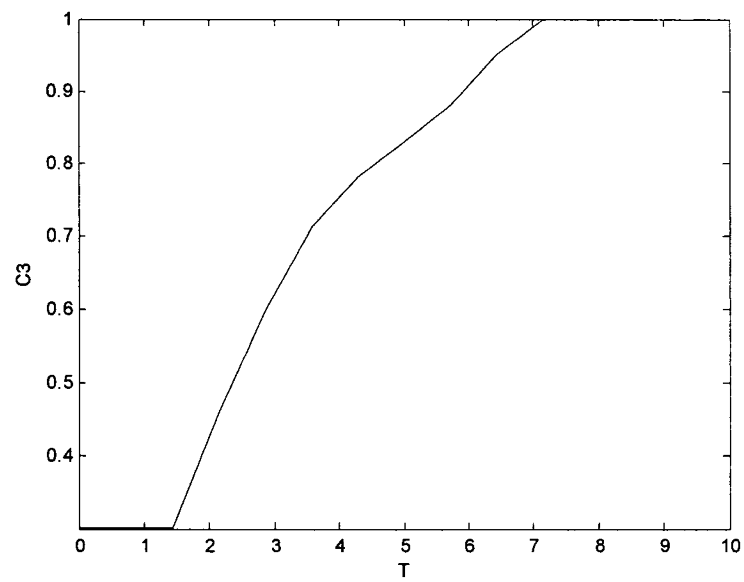

Figure 4.37 Response Curve of T-C3 FIS

4.3 Aggregation of Results from FIS Modules

The output from each FIS module gives the evidence on presence of each cause in the range [0-1], based on degree of presence of its characteristic pattern. All the evidence has to be aggregated to compute the overall evidence and to rank the causes, so that the search for assignable cause shall be made beginning with the cause having highest likelihood of presence. As each cause has varying number of input patterns, aggregation has to be carried out in such a way that all the aggregated evidence is also within the range [0-1].i.e.,

$$[0\text{-}1] \oplus [0\text{-}1] \longrightarrow [0\text{-}1] \qquad (4.5)$$

Mere summing up of the evidence will lead to incorrect results. This requires certain properties to be fulfilled while choosing the aggregation operator.

Let '$\oplus$' represents the aggregation operator and it should be

- Monotonic, which requires continuously increasing in the interval [0-1]

  $$A \oplus B < C \oplus D, \text{ where } A < C \text{ and } B < D \tag{4.6}$$

- Commutative, which requires

  $$A \oplus B = B \oplus A \tag{4.7}$$

- Associative, which requires the aggregated value should not depend on the order of aggregation

  $$A \oplus (B \oplus C) = (A \oplus B) \oplus C \tag{4.8}$$

- Existence of a unit 0, which requires the aggregated values be within the interval [0-1]

  $$(0 \oplus B)=B \text{ for all } B \text{ in } [0\text{-}1] \tag{4.9}$$

In fuzzy set theory, t-norms and t-conorms are used as connective operators for '*AND*' and '*OR*' respectively, satisfying the above mentioned properties. A detailed discussion on different t-norms and t-conorms and their properties as fuzzy connectives are available in the literature Gupta and Qi [28], Dubois and Prade [29], Fodor and Reubens [30] and Mizumoto [31]. However, t-norms are not suitable for our case as they are not optimistic aggregators in the sense, the weight of the evidence is reduced than the weight of its highest operand after aggregation. Hence T-conorms are used to aggregate the evidence from each FIS. There are two type of evidence each causal node is getting,

- Evidence from its characteristic patterns
- Evidence from other patterns which may appear as a part in its characteristic pattern (OCL in our case).

From the network model in Figure 4.7, isolated causes node, C1, has input only from its characteristic pattern, which is OCL. Shift causes and Gradual causes receives input from OCL pattern apart from its own characteristic patterns. Also, shift causes node has three of its own characteristic patterns. Hence the aggregation is carried out in two stages. In the initial stage, the evidence from the characteristic patterns of a particular cause is combined using '*Max*' operator. In the second stage, this evidence is combined with the evidence from other patterns using *'Algebraic sum'* operator,

Let the output from each FIS be identified under the name of the FIS itself.

- *OCL_C1*
- *OCL_C2*
- *OCL_C3*
- *FR1_C2*
- *FR2_C2*
- *R_C2*
- *T_C3*

4.3.1 *Max* Aggregation

In the network we have considered, only shift cause (*C2*) has more than one characteristic pattern (*FR1, FR2* and *Run*). As all these patterns indicate the presence of same cause, maximum of the evidence is taken, to avoid over imposing of the evidence. Let 'Max_C2' be the aggregated evidence for *C2*.

$$Max\_C2 = FR1\_C2 \oplus FR2\_C2 \oplus R\_C2$$

$$= Max\ (FR1\_C2,\ FR2\_C2,\ R\_C2) \tag{4.10}$$

4.3.2 *Algebraic sum* aggregation

This is the second stage of aggregation. The algebraic sum operator is defined by Equation (4.5).

$$a \oplus b = a + b - ab \tag{4.11}$$

Nodes C2 and C3 uses this step of aggregations to combine the evidence from its characteristic patterns (Max_C2 for *C2* and T_C3 for *C3*) with OCL pattern as described below.

Let $C2^*$ and $C3^*$ be the final values of *C2* and *C3*, respectively, after aggregation.

$$C2^* = Max\_C2 \oplus OCL\_C2 \tag{4.12}$$

$$C3^* = T\_C3 \oplus OCL\_C3 \tag{4.13}$$

Since isolated causes, C1, has only one input, from its characteristic pattern OCL, no aggregation process is involved with C1 node. Hence,

$$C1^* = OCL\_C1 \tag{4.14}$$

where C1* is the final value of isolated causes.

Thus the fuzzy inference engine gives the final output as *C1*, C2* and C3** in the range [0-1] after aggregation, representing its degree of presence. When any of these values reaches its maximum, it indicates that process mean is shifted and the assignable cause investigation could be initiated with the cause having maximum value. It also indicates what pattern or combination of patterns caused the process to go out-of-control with the weighted assignable cause domain that could have possibly caused the out-of control situation. The developed FIS engine is tested with control chart data containing various combinations of patterns and the results are discussed in the next chapter.

Chapter 5

Test Cases and Results

The fuzzy inference engine developed is tested with control chart data containing various combinations of unnatural patterns. Each test case is developed considering a particular unnatural pattern or a combination of unnatural patterns taken from the network in Figure 4.7. The test cases and test data were developed considering the assumptions made in Section 4.2.1. As per these assumptions, process is operating in phase-II. The process average and standard deviation are known. The control chart data (test data) for each test case is developed using the software MINITAB [32] using random data generated under normal distribution by specifying mean and standard deviation.

Control charts are plotted using MINITAB. Sensitizing rules (tests) pertaining to the unnatural patterns that are interested in detecting (OCL, FR1, FR2, Run & Trend) are selected to determine the intensity of assignable causes.

The Disadvantages in using common control charting software are,

- They can only detect whether or not the process in under control. No further information on possible assignable causes is given
- Use of sensitizing rules in determining the process state could lead to increased false alarms.

- If it is decided to not use these sensitizing rules, then useful information on the type of unnatural patterns present to diagnose the assignable cause cannot be obtained.

These pitfalls can be overcome by the fuzzy inference engine developed. The output from the fuzzy inference engine is compared with the manual interpretation of charts and using the chart in determining the process state and analyzing chart patterns in assignable cause search.

Each test case is discussed along with the results from the fuzzy inference engine. Initially, to understand the performance of the fuzzy inference engine, simple test cases are considered containing only one unnatural pattern at a time. The initial three test cases were taken considering one characteristic pattern for each cause at a time. The later test cases taken contain different combinations of all the patterns. Since it is assumed that the process is operating in phase II, the test data was generated with process mean 50.0 and standard deviation 1.0 with a sample size of 5 for all the test cases.

5.1 Test Case-1 (OCL)

In this first test case, a common and well known unnatural pattern, OCL, a point falling beyond outer control limits is considered. The test data was generated using MINTIAB with the mean value as 50.0, standard deviation as 1.0 and a sample size of five. The data generated is given in the Table 5.1 and its control chart, plotted using MINITAB is given in Figure 5.1

<u>Data:</u> $\mu = 50, \sigma = 1.0, n = 5$

Sample No	X_1	X_2	X_3	X_4	X_5	Mean
1.	50.7387	49.5947	49.5849	49.2320	50.9549	50.0211
2.	49.3567	50.8175	49.3406	49.4535	48.8772	49.5691
3.	49.4348	51.1365	51.0497	50.0943	51.1910	50.5813
4.	49.1990	50.5684	49.3381	51.1209	50.4775	50.1408
5.	49.0848	49.3550	49.2542	49.2287	49.7714	49.3388
6.	50.2280	50.4704	49.9426	49.9962	53.1599	50.7594
7.	50.6425	52.1969	50.0847	49.7479	49.7048	50.4753
8.	50.8166	51.8717	51.6397	53.4504	50.2349	51.6027

Table 5.1 Data for Test Case 1

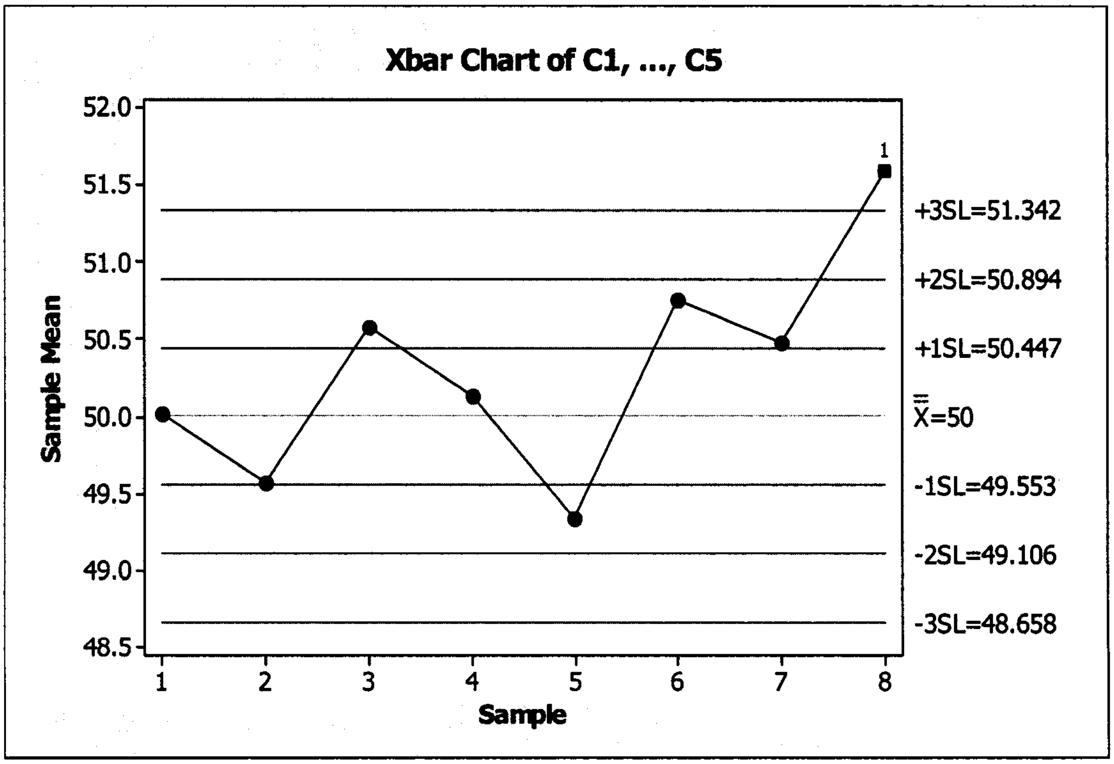

Figure 5.1. $\overline{X}$ Chart for Test Case 1

MINITAB Result:

Results for: Case1_OCL.MTW

Xbar Chart of C1, ..., C5

Test Results for Xbar Chart of C1, ..., C5

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  8
```

Results from Fuzzy Inference Engine

Mean:50.0000 Sigma:1.0000 Sample_Size:5

SNo	Cause	OCL	FR1	FR2	Run	Trend	Max	Alg_ Sum	Cause Priority
1	i/p values	0.04	0	0	1	0			C2*(Run), C3*, C1*
	C1	0.25					0.25	0.25	
	C2	0.25	0.35	0.25	0.38		0.38	0.53	
	C3	0.25				0.3	0.30	0.47	
2	i/p values	0.96	0	0	1	1			C2*(Run), C3*, C1*
	C1	0.46					0.46	0.46	
	C2	0.46	0.35	0.25	0.38		0.38	0.66	
	C3	0.46				0.3	0.3	0.62	
3	i/p values	1.3	1	0	1	1			C2*(FR1), C3*, C1*
	C1	0.54					0.54	0.54	
	C2	0.54	0.43	0.25	0.38		0.43	0.74	
	C3	0.54				0.3	0.30	0.68	
4	i/p values	0.31	1	0	2	1			C2*(Run), C3*, C1*
	C1	0.25					0.25	0.25	
	C2	0.25	0.43	0.25	0.48		0.48	0.61	
	C3	0.25				0.30	0.30	0.47	

5	i/p values	1.47	1	0	1	2			
	C1	0.59					0.59	0.59	**C2*(FR1), C3*, C1***
	C2	0.59	0.43	0.25	0.38		0.43	0.77	
	C3	0.59				0.41	0.41	0.76	
6	i/p values	1.69	2	0	1	1			
	C1	0.67					0.67	0.67	**C2*(FR1), C3*, C1***
	C2	0.60	0.56	0.25	0.38		0.56	0.82	
	C3	0.60				0.30	0.30	0.72	
7	i/p values	1.06	3	0	2	1			
	C1	0.48					0.48	0.48	**C2*(FR1), C3*, C1***
	C2	0.48	0.75	0.25	0.48		0.75	0.87	
	C3	0.48				0.30	0.30	0.64	
8	i/p values	3.58	3	1	3	1			
	C1	1					1	1	**C1*, C2*(FR1), C3***
	C2	0.6	0.75	0.60	0.56		0.75	0.90	
	C3	0.60					0.30	0.72	
Process out of control: C1-1.0000									

Table 5.2 Results from Fuzzy Inference Engine for Test Case 1

The results from the fuzzy inference engine for every point plotted on the control chart are given in the Table 5.2. The first column indicates sample number and the first row of every sample in the table gives the input values of each unnatural pattern. For example, sample 1 has OCL pattern input of 0.047 (measure of dispersion from center line in terms of standard deviation units, calculated as per equation 4.4), FR1 pattern input 0 (determined as per Section 4.2.5), FR2 pattern input 0 (determined as per Section 4.2.6), Run pattern input 1 (as per Section 4.2.7) and Trend pattern input of 0 (as per Section 4.2.8).

The subsequent rows C1, C2 and C3 show the values each cause received from its unnatural patterns as represented in the network diagram though its corresponding FIS module. "Max" column represents the *Max* aggregation of characteristic patterns of same cause, which in our case, only for C2, maximum of FR1, FR2 and Run pattern values is taken. For C1 and C3 the value remains the same, as it contains only one characteristic pattern. In 'Alg_Sum' column, algebraic sum aggregation is performed as explained in Section 4.3.2. Finally, causes are prioritized based on the aggregated evidence.

When the aggregated evidence for a cause reaches the maximum of 1.0, the process is said to be out of control and the cause having the maximum value is given the highest priority. The ambiguities in situations where all the causes reach maximum of 1.0 when all patterns co-exist together, as in some of the cases tested later, is resolved by the fuzzy inference engine. This is done by prioritizing the causes based on the probabilistic values of its characteristic patterns. The cause of the characteristic pattern having highest probability of occurrence (as per Section 3.3) is given the highest priority. The prioritized causes are listed in order (starting with the highest) in the last column of Table 5.2. Since shift causes (C2) has many characteristic patterns, the pattern influencing C2 to the most (the pattern corresponding to C2 in the 'Max' column) is given in the brackets next to C2*.

The results from test case 1 show that sample 8 has the isolated cause at maximum value of 1.0, hence making the process out of control. Accordingly cause priority is ordered as C1*, C2* and C3* as per their value of aggregated evidence. The investigation for assignable cause is conducted starting with the cause having highest priority. The performance of the fuzzy inference engine is tested with various other test cases, with different combination of patterns in the following test cases.

5.2 Test Case 2 (Run)

In this test case the shift cause characteristic pattern 'Run' is considered. The parameters of the control chart remaining the same, the sample data is given in the Table 5.2 and the control chart from MINITAB is given in Figure 5.2 with the output generated from MINITAB. The test rule for detecting a Run pattern in MINITAB is defined as nine or more consecutive points on one side of center line. Observing nine consecutive points on one side of center line has a lower probability ($0.5^9 = 0.0019$) than observing eight consecutive points ($0.5^8 = 0.0039$) on one side of the center line. It means chances of detecting the shift is more in latter case compared to former. For this test case MINITAB shows that the process is in-control and no unnatural pattern is detected. But the output from fuzzy inference engine show that run pattern is detected and hence presence of shift causes with maximum value of 1.0.

<u>Data:</u> $\mu = 50, \sigma = 1.0, n = 5$

Sample No	**X_1**	**X_2**	**X_3**	**X_4**	**X_5**	**Mean**
1.	45.8504	51.5015	50.0084	49.6865	49.9424	49.3979
2.	51.4859	52.5719	50.4062	48.4897	50.1307	50.6169
3.	50.5363	53.1283	49.5771	50.2932	50.0845	50.7239

Sample No	X_1	X_2	X_3	X_4	X_5	Mean
4.	52.8048	52.7215	51.7870	48.5275	49.3996	51.0481
5.	49.1027	50.8896	48.9389	49.7564	48.2330	49.3841
6.	48.8958	52.4114	51.0855	48.3104	49.0380	49.9482
7.	51.2322	50.4726	49.1889	49.6465	48.7237	49.8528
8.	47.0161	49.0403	50.7732	51.1620	47.0539	49.0091
9.	50.0082	52.0126	50.2318	50.6496	52.3304	51.0465
10.	50.1550	49.3009	51.4115	51.0635	50.2256	50.4313
11.	48.7763	48.5632	48.1658	50.1236	49.4177	49.0093
12.	50.5327	50.7294	50.8167	51.4960	51.3382	50.9826
13.	50.8436	51.0526	48.3579	49.9998	50.8643	50.2237
14.	50.7566	50.0173	49.2853	50.0981	50.6453	50.1605
15.	49.8980	51.4502	49.1314	50.2680	50.2477	50.1990
16.	50.5441	49.8908	50.3852	53.5696	48.4198	50.5619
17.	49.8169	52.9305	50.8563	50.5612	48.1090	50.4548
18.	48.9334	51.2682	50.6664	50.1519	50.4290	50.2898

Table 5.3 Data for Test Case 2

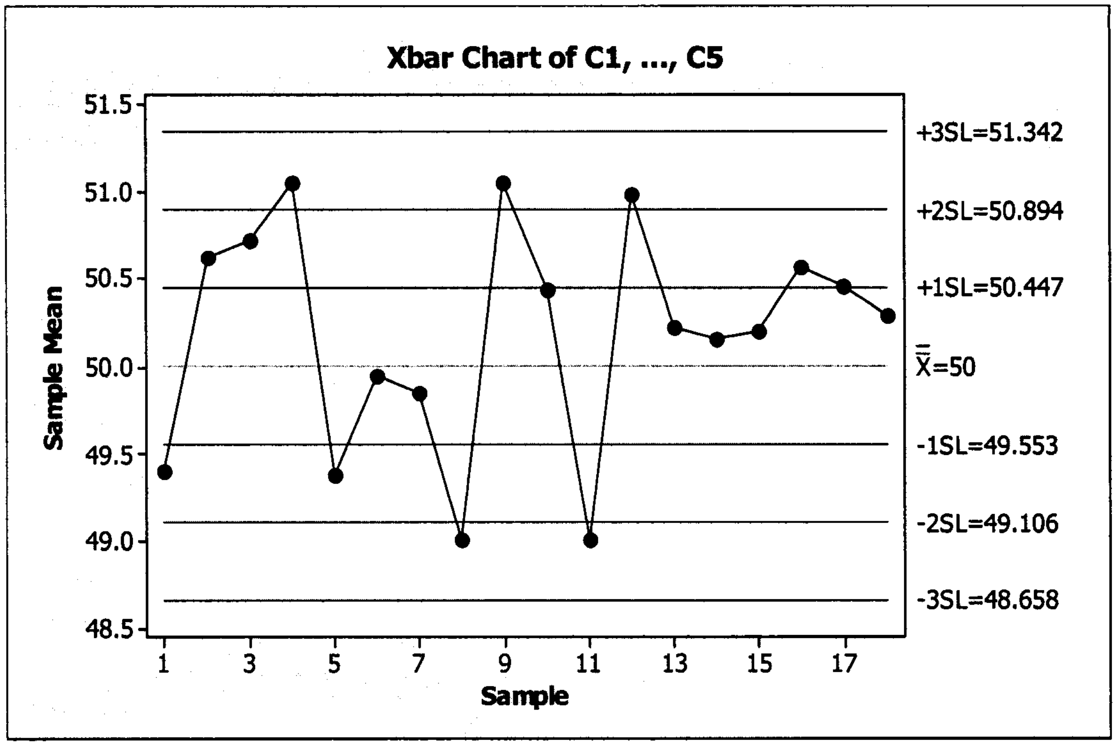

Figure 5.2. $\overline{X}$ Chart for Test Case 2

MINITAB Result:

[No patterns detected in MINITAB result window]

Results from Fuzzy Inference Engine

Mean:50.0000 Sigma:1.0000 Sample_Size:5									
SNo	Cause	OCL	FR1	FR2	Run	Trend	Max	Alg_ Sum	Cause Priority
1	i/p values	1.34	1	0	1	0			**C2*(FR1), C3*, C1***
	C1	0.55					0.55	0.55	
	C2	0.55	0.43	0.25	0.38		0.43	0.75	
	C3	0.55				0.30	0.30	0.69	
2	i/p values	1.37	1	0	1	1			**C2*(FR1), C3*, C1***
	C1	0.56					0.56	0.56	
	C2	0.56	0.43	0.25	0.38		0.43	0.75	
	C3	0.56				0.30	0.30	0.69	
3	i/p values	1.61	2	0	2	2			**C2*(FR1), C3*, C1***
	C1	0.64					0.64	0.64	
	C2	0.60	0.56	0.25	0.48		0.56	0.82	
	C3	0.60				0.41	0.41	0.76	
4	i/p values	2.34	3	1	3	3			**C2*(FR1), C3*, C1***
	C1	0.81					0.81	0.81	
	C2	0.60	0.75	0.60	0.56		0.75	0.90	
	C3	0.60				0.60	0.60	0.84	
5	i/p values	1.37	3	1	1	1			**C2*(FR1), C3*, C1***
	C1	0.56					0.56	0.56	
	C2	0.56	0.75	0.60	0.38		0.75	0.89	
	C3	0.56				0.30	0.30	0.69	
6	i/p values	0.11	3	1	2	1			**C2*(FR1), C3*, C1***
	C1	0.25					0.25	0.25	
	C2	0.25	0.75	0.60	0.48		0.75	0.81	
	C3	0.25				0.3	0.30	0.47	
7	i/p values	0.32	2	0	3	1			**C2*(FR1& Run), C3*, C1***
	C1	0.25					0.25	0.25	
	C2	0.25	0.56	0.25	0.56		0.56	0.67	
	C3	0.25				0.3	0.30	0.47	

8	i/p values	2.21	2	1	4	2			**C2*(Run),C1*, C3***
	C1	0.79					0.79	0.79	
	C2	0.60	0.56	0.60	0.63		0.63	0.85	
	C3	0.60				0.41	0.41	0.76	
9	i/p values	2.34	2	1	1	1			**C2*(FR2),C1*, C3***
	C1	0.81					0.81	0.81	
	C2	0.60	0.56	0.60	0.38		0.60	0.84	
	C3	0.60				0.30	0.30	0.72	
10	i/p values	0.96	1	1	2	1			**C2*(FR2), C3*, C1**
	C1	0.46					0.46	0.46	
	C2	0.46	0.43	0.60	0.48		0.60	0.78	
	C3	0.46				0.30	0.30	0.62	
11	i/p values	2.21	2	1	1	2			**C2*(FR2),C1*, C3***
	C1	0.79					0.79	0.79	
	C2	0.60	0.56	0.60	0.38		0.60	0.84	
	C3	0.60				0.41	0.41	0.76	
12	i/p values	2.19	2	1	1	1			**C2*(FR2),C1*, C3***
	C1	0.78					0.78	0.78	
	C2	0.60	0.56	0.60	0.38		0.60	0.84	
	C3	0.60				0.3	0.30	0.72	
13	i/p values	0.50	2	1	2	1			**C2*(FR2),C1*, C3***
	C1	0.25					0.25	0.25	
	C2	0.25	0.56	0.60	0.48		0.60	0.70	
	C3	0.25				0.30	0.30	0.47	
14	i/p values	0.35	1	1	3	2			**C2*(FR2), C3*, C1**
	C1	0.25					0.25	0.25	
	C2	0.25	0.43	0.60	0.56		0.60	0.70	
	C3	0.25				0.41	0.41	0.56	
15	i/p values	0.44	1	0	4	1			**C2*(Run), C3*, C1**
	C1	0.25					0.25	0.25	
	C2	0.2	0.43	0.25	0.633		0.63	0.72	
	C3	0.25				0.30	0.30	0.47	
16	i/p values	1.25	2	0	5	2			

	C1	0.53					0.53	0.53	**C2*(Run), C3*, C1**
	C2	0.53	0.56	0.25	0.74		0.74	0.88	
	C3	0.53				0.41	0.41	0.72	
17	i/p values	1.01	2	0	6	1			**C2*(Run), C3*, C1**
	C1	0.47					0.47	0.47	
	C2	0.47	0.56	0.25	0.83		0.83	0.91	
	C3	0.47				0.30	0.30	0.63	
18	i/p values	0.64	2	0	7	2			**C2*(Run), C3*, C1**
	C1	0.34					0.34	0.34	
	C2	0.34	0.56	0.25	1		1	1	
	C3	0.34				0.41	0.41	0.61	
Process out of control:C2-1.0000									

Table 5.4 Results from Fuzzy Inference Engine for Test Case 2

From the results in Table 5.4, C2 reaches the maximum value 1.0 for sample 18 due to the run pattern. The causes are ordered in the sequence C2*, C3* and C1* according to their aggregated values. It has to be noted that output from MINITAB has not detected any pattern.

5.3 Test Case 3 (Trend)

In this test case, only the characteristic pattern for gradual cause, 'Trend', is considered. The test data is given in Table 5.5 and the MINITAB results and control chart which shows a decreasing trend are given in Figure 5.3. The test for 'Trend' pattern in MINITAB is designed for detecting six consecutive points, increasing or decreasing, as opposed to seven points suggested in literature. Seven consecutive increasing or decreasing points is considered as Trend pattern in this work.

The probability of occurrence of six points in a row all increasing or decreasing is more than that of the occurrence of seven points, and hence concluding the process is out of control on observing six points may lead to increased false alarms and reduced average run length. The output from MINITAB shows that the process is out of control from sample 15. The output from fuzzy inference engine, as given in the Table 5.6, shows that process went out of control at sample 16, reaching the maximum value for gradual cause, 1.0, at sample 16.

<u>Data:</u> $\mu = 50, \sigma = 1.0, n = 5$

Sample No	X_1	X_2	X_3	X_4	X_5	**Mean**
1.	45.8504	51.5015	50.0084	49.6865	49.9424	49.3979
2.	51.4859	52.5719	50.4062	48.4897	50.1307	50.6169
3.	50.5363	53.1283	49.5771	50.2932	50.0845	50.7239
4.	52.8048	52.7215	51.7870	48.5275	49.3996	51.0481
5.	49.1027	50.8896	48.9389	49.7564	48.2330	49.3841
6.	48.8958	52.4114	51.0855	48.3104	49.0380	49.9482
7.	51.2322	50.4726	49.1889	49.6465	48.7237	49.8528
8.	47.0161	49.0403	50.7732	51.1620	47.0539	49.0091
9.	50.0082	52.0126	50.2318	50.6496	52.3304	51.0465
10.	50.1550	49.3009	51.4115	51.0635	50.2256	50.4313

Sample No	X_1	X_2	X_3	X_4	X_5	Mean
11.	48.7763	48.5632	48.1658	50.1236	49.4177	49.0093
12.	50.5327	50.7294	50.8167	51.4960	51.3382	50.9826
13.	50.8436	51.0526	48.3579	49.9998	50.8643	50.2237
14.	50.7566	50.0173	49.2853	50.0981	50.6453	50.1605
15.	49.8980	51.4502	49.1314	50.2680	50.2477	50.1990
16.	50.5441	49.8908	50.3852	53.5696	48.4198	50.5619

Table 5.5 Data for Test Case 3

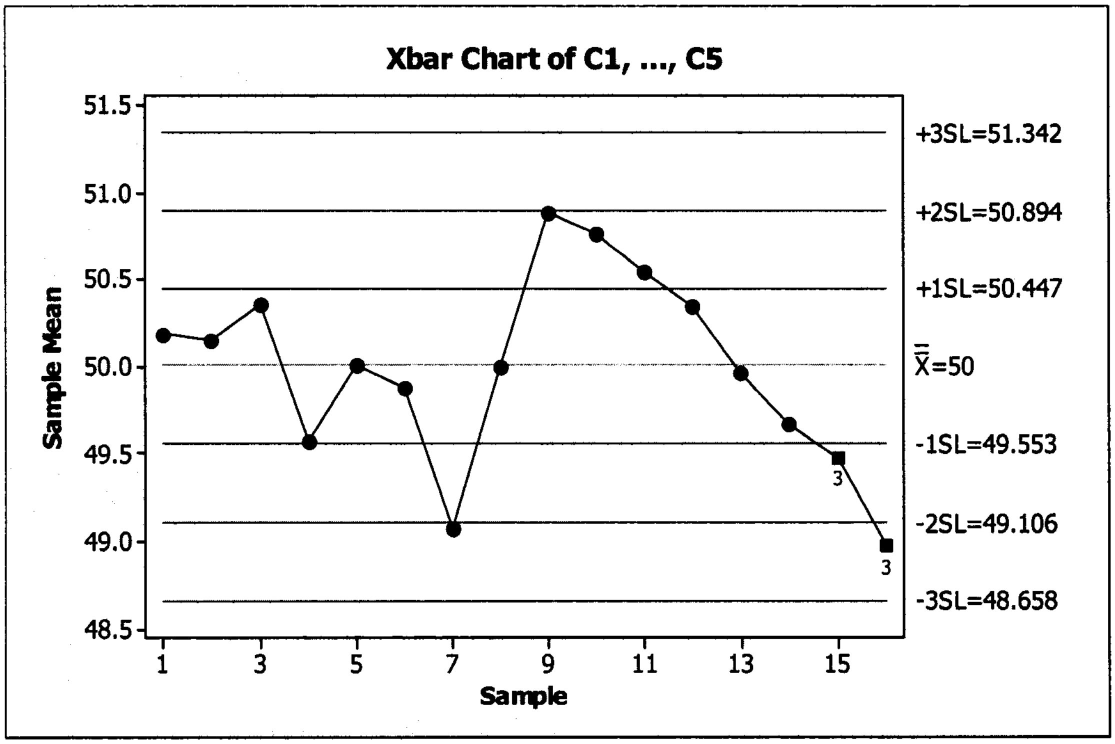

Figure 5.3. $\overline{X}$ Chart for Test Case 3

MINITAB Result:

Xbar Chart of Case3_Trend.MTW

Test Results for Xbar Chart of C1, ..., C5

```
TEST 3. 6 points in a row all increasing or all decreasing.
Test Failed at points:  15, 16
```

Results from Fuzzy Inference Engine

Mean:50.0000 Sigma:1.0000 Sample_Size:5									
SNo	**Cause**	**OCL**	**FR1**	**FR2**	**Run**	**Trend**	**Max**	**Alg_ Sum**	**Cause Priority**
1	i/p values	0.41	0	0	1	0			**C2*(Run),C3*, C1***
	C1	0.25					0.25	0.25	
	C2	0.25	0.35	0.25	0.38		0.38	0.53	
	C3	0.25				0.30	0.30	0.47	
2	i/p values	0.34	0	0	2	1			**C2*(Run), C3*, C1***
	C1	0.25					0.25	0.25	
	C2	0.25	0.35	0.25	0.48		0.48	0.61	
	C3	0.25				0.30	0.30	0.47	
3	i/p values	0.79	0	0	3	1			**C2*(Run), C3*, C1***
	C1	0.40					0.40	0.40	
	C2	0.40	0.35	0.25	0.56		0.56	0.74	
	C3	0.40				0.30	0.30	0.58	
4	i/p values	0.97	0	0	1	1			**C2*(Run), C3*, C1***
	C1	0.46					0.46	0.46	
	C2	0.46	0.35	0.25	0.38		0.38	0.67	
	C3	0.46				0.30	0.30	0.62	
5	i/p values	0.002	0	0	1	1			**C2*(Run), C3*, C1***
	C1	0.25					0.25	0.25	
	C2	0.25	0.35	0.25	0.38		0.38	0.53	
	C3	0.25				0.30	0.30	0.47	

6	i/p values	0.28	0	0	1	1			**C2*(Run), C3*, C1***
	C1	0.25					0.25	0.25	
	C2	0.25	0.35	0.25	0.38		0.38	0.53	
	C3	0.25				0.30	0.30	0.47	
7	i/p values	2.07	1	1	2	2			**C2*(FR2), C3*, C1***
	C1	0.76					0.76	0.76	
	C2	0.60	0.43	0.60	0.48		0.60	0.84	
	C3	0.60				0.41	0.41	0.76	
8	i/p values	0.01	1	1	3	1			**C2*(FR2), C3*, C1***
	C1	0.25					0.25	0.25	
	C2	0.25	0.43	0.60	0.56		0.60	0.70	
	C3	0.25				0.30	0.30	0.47	
9	i/p values	1.97	1	1	1	2			**C2*(FR2), C3*, C1***
	C1	0.74					0.74	0.74	
	C2	0.60	0.43	0.60	0.38		0.60	0.84	
	C3	0.60				0.41	0.41	0.76	
10	i/p values	1.70	2	0	2	1			**C2*(FR1), C3*, C1***
	C1	0.67					0.67	0.67	
	C2	0.60	0.56	0.25	0.48		0.56	0.82	
	C3	0.60				0.30	0.30	0.72	
11	i/p values	1.20	3	0	3	2			**C2*(FR1), C3*, C1***
	C1	0.52					0.52	0.52	
	C2	0.52	0.75	0.25	0.56		0.75	0.88	
	C3	0.52				0.41	0.41	0.72	
12	i/p values	0.76	3	0	4	3			**C2*(FR1), C3*, C1***
	C1	0.40					0.40	0.40	
	C2	0.40	0.75	0.25	0.63		0.75	0.85	
	C3	0.40				0.60	0.60	0.76	
13	i/p values	0.09	3	0	1	4			**C2*(FR1), C3*, C1***
	C1	0.25					0.25	0.25	
	C2	0.25	0.75	0.25	0.38		0.75	0.81	
	C3	0.25				0.75	0.75	0.81	

14	i/p values	0.74	2	0	2	5			
	C1	0.38					0.38	0.38	**C3*, C2*(FR1), C1***
	C2	0.38	0.56	0.25	0.48		0.56	0.73	
	C3	0.38				0.83	0.83	0.89	
15	i/p values	1.17	1	0	3	6			**C3*, C2*(Run), C1***
	C1	0.51					0.51	0.51	
	C2	0.51	0.43	0.25	0.56		0.56	0.78	
	C3	0.51				0.90	0.90	0.95	
16	i/p values	2.30	2	1	4	7			**C3*, C2*(Run), C1***
	C1	0.80					0.80	0.80	
	C2	0.60	0.56	0.60	0.63		0.63	0.85	
	C3	0.60				1	1	1	
Process out of control:C3-1.0000									

Table 5.6 Results from Fuzzy Inference Engine for Test Case 3

From the results of the fuzzy inference engine Table 5.6, the trend pattern is detected by the fuzzy inference engine at sample16, thereby prioritizing the gradual causes (C3*) over the other two causes.

5.4 Test Case 4 (OCL and FR2)

In this case the combination of two unnatural patterns, OCL and Freak 2 is considered. *OCL* is the characteristic pattern of isolated causes (*C2*) and Freak 2 is the characteristic pattern of shift causes. Accordingly, the output from MINITAB shows the presence of two unnatural patterns and so does the fuzzy inference engine, giving the values of *C1* (isolated causes) and *C2* (shift causes) at the maximum value 1.0. In such circumstances where more than one cause reaches the maximum value 1.0, prioritization is done based on the higher probability of occurrence of the patterns. The test data is given in the Table 5.7 and control chart is shown in the Figure 5.4. The results from the fuzzy inference engine are shown in Table 5.8.

<u>Data:</u> $\mu = 50, \sigma = 1.0, n = 5$

Sample No	X_1	X_2	X_3	X_4	X_5	Mean
1.	50.7387	49.5947	49.5849	49.2320	50.9549	50.0211
2.	49.3567	50.8175	49.3406	49.4535	48.8772	49.5691
3.	49.4348	51.1365	51.0497	50.0943	51.1910	50.5813
4.	49.1990	50.5684	49.3381	51.1209	50.4775	50.1408
5.	49.0848	49.3550	49.2542	49.2287	49.7714	49.3388
6.	50.2658	51.4124	52.0780	49.9458	51.0676	50.9539
7.	50.3587	51.4124	52.0780	51.2457	51.0676	51.2325
8.	50.8166	51.8717	51.6397	53.4504	50.2349	51.6027

Table 5.7 Data for Test Case 4

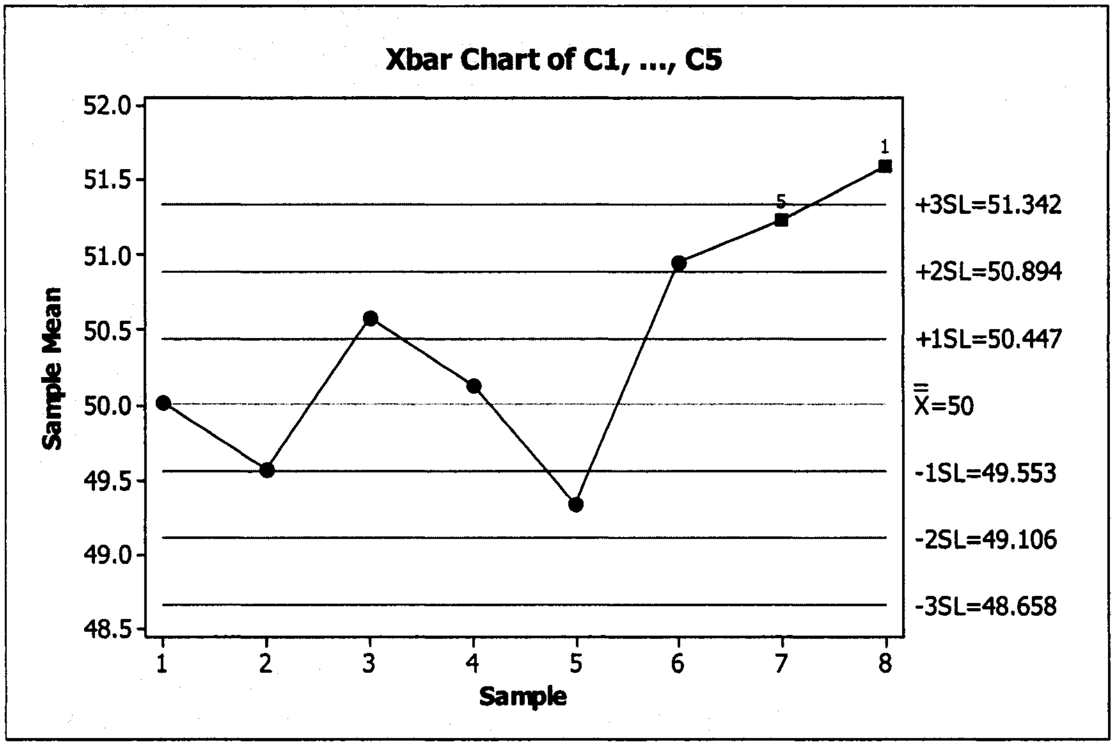

Figure 5.4. $\overline{X}$ Chart for Test Case 4

MINITAB Result:

Test Results for Xbar Chart of C1, ..., C5

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  8

TEST 5. 2 out of 3 points more than 2 standard deviations from center line
(on
    one side of CL).
Test Failed at points:  7, 8
```

Results from Fuzzy Inference Engine

Mean:50.0000 Sigma:1.0000 Sample_Size:5									
SNo	Cause	OCL	FR1	FR2	Run	Trend	Max	Alg_ Sum	Cause Priority
1	i/p values	0.04	0	0	1	0			C2*(Run), C3*, C1*
	C1	0.25					0.25	0.25	
	C2	0.25	0.35	0.25	0.38		0.38	0.53	
	C3	0.25				0.3	0.30	0.47	
2	i/p values	0.96	0	0	1	1			C2*(Run), C3*, C1*
	C1	0.46					0.46	0.46	
	C2	0.46	0.35	0.25	0.38		0.38	0.66	
	C3	0.46				0.3	0.30	0.62	
3	i/p values	1.3	1	0	1	1			C2*(FR1), C3*, C1*
	C1	0.54					0.54	0.54	
	C2	0.54	0.43	0.25	0.38		0.43	0.74	
	C3	0.54				0.3	0.30	0.68	
4	i/p values	0.31	1	0	2	1			C2*(Run), C3*, C1*
	C1	0.25					0.25	0.25	
	C2	0.25	0.43	0.25	0.48		0.48	0.61	
	C3	0.25				0.3	0.30	0.47	
5	i/p values	1.47	1	0	1	2			C2*(FR1), C3*, C1*
	C1	0.59					0.59	0.59	
	C2	0.59	0.43	0.25	0.38		0.43	0.77	
	C3	0.59				0.41	0.41	0.76	
6	i/p values	2.13	2	1	1	1			C2*(FR2), C1*, C3*
	C1	0.77					0.77	0.77	
	C2	0.60	0.56	0.60	0.38		0.60	0.84	
	C3	0.60				0.30	0.30	0.72	
7	i/p values	2.75	3	2	2	2			C2*(FR2), C1*, C3*
	C1	0.91					0.91	0.91	
	C2	0.60	0.75	1	0.48		1	1	
	C3	0.60				0.41	0.41	0.76	

Process out of control:C2-1.0000									
8	i/p values	3.58	3	3	3	3			**C1*, C2*(FR2), C3***
	C1	1					1	1	
	C2	0.60	0.75	1	0.56		1	1	
	C3	0.60				0.60	0.60	0.84	
Process out of control:C1-1.0000									
Process out of control:C2-1.0000									

Table 5.8 Results from Fuzzy Inference Engine for Test Case 4

The results from fuzzy inference engine are shown in Table 5.8. In this case the FR2 pattern is identified at sample 7 causing C2* to reach the maximum value1.0 and hence it is given the highest priority. At sample 8, both the isolated and shift causes reach the maximum value of 1.0 due to OCL and FR2 patterns. In such circumstances prioritization is done based on probabilistic values of the patterns (as per Section 3.3). OCL pattern has higher probability of occurrence (0.0027) compared to FR2 pattern (0.0016) and hence C1* is prioritized over C2*.

5.5 Test Case 5 (OCL, FR1 and FR2)

In this test case the combination of three unnatural patterns OCL, FR1 and FR2, is considered, in which OCL is the characteristic pattern of isolated causes and both FR1 and FR2 are the characteristic patterns for shift causes. The outputs from both MINITAB and the fuzzy inference engine show the presence of all three patterns and the fuzzy inference engine prioritizes C2 again, as it receives additional evidence supporting C2 from FR1, as similar to the previous test case. The test data is shown in Table 5.9 and the control chart is shown in Figure 5.5.

Data: $\mu = 50, \sigma = 1.0, n = 5$

Sample No	X_1	X_2	X_3	X_4	X_5	Mean
1.	50.7387	49.5947	49.5849	49.2320	50.9549	50.0211
2.	49.3567	50.8175	49.3406	49.4535	48.8772	49.5691
3.	49.4348	51.1365	51.0497	50.0943	51.1910	50.5813
4.	49.1990	50.5684	49.3381	51.1209	50.4775	50.1408
5.	49.0848	49.3550	49.2542	49.2287	49.7714	49.3388
6.	50.2280	50.4704	49.9426	49.9962	53.1599	50.7594
7.	50.6425	52.1969	50.0847	49.7479	49.7048	50.4753
8.	50.1587	51.4124	52.0780	49.8457	51.0676	50.9125
9.	50.8166	51.8717	51.6397	53.4504	50.2349	51.6027

Table 5.9 Data for Test Case 5

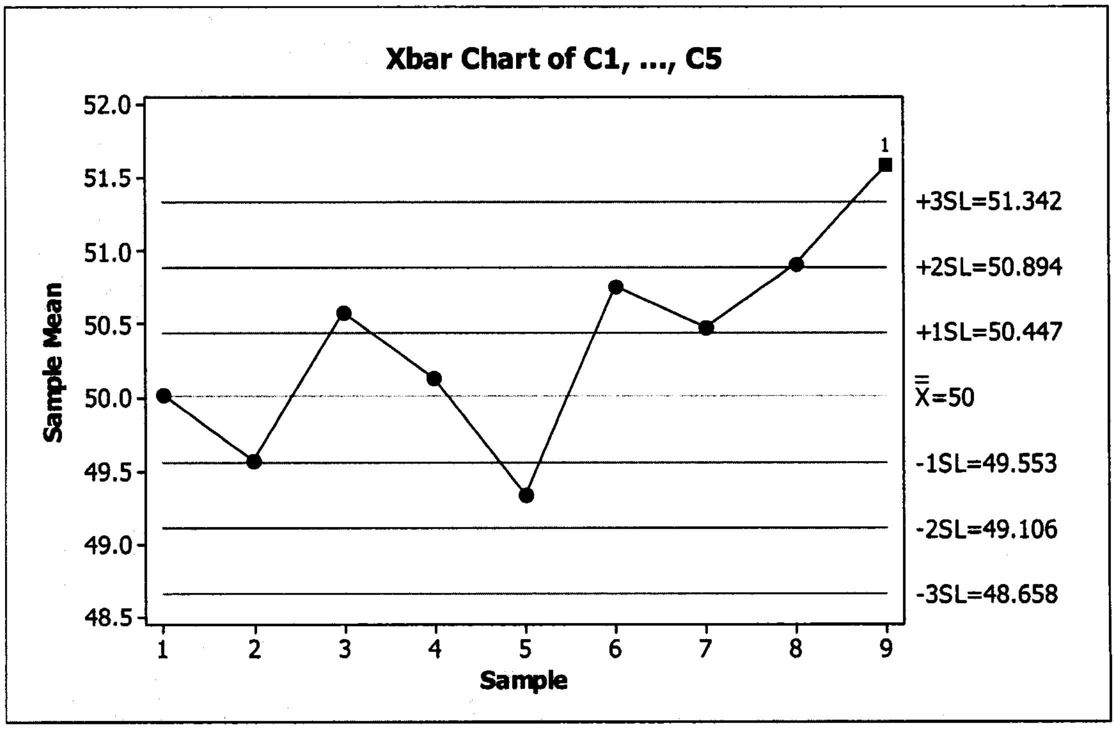

Figure 5.5 $\overline{X}$ Chart for Test Case 5

MINITAB Result

Results for: Case2_FR2_FR1_OCL.MTW
Xbar Chart of C1, ..., C5

Test Results for Xbar Chart of C1, ..., C5

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  9

TEST 5. 2 out of 3 points more than 2 standard deviations from center line
(on
     one side of CL).
Test Failed at points:  9

TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  9
```

Results from Fuzzy Inference Engine

Mean:50.0000 Sigma:1.0000 Sample_Size:5

SNo	Cause	OCL	FR1	FR2	Run	Trend	Max	Alg_ Sum	Cause Priority
1	i/p values	0.04	0	0	1	0			C2*(Run), C3*, C1*
	C1	0.25					0.25	0.25	
	C2	0.25	0.35	0.25	0.38		0.38	0.53	
	C3	0.25				0.30	0.30	0.47	
2	i/p values	0.96	0	0	1	1			C2*(Run), C3*, C1*
	C1	0.46					0.46	0.46	
	C2	0.46	0.35	0.25	0.38		0.38	0.66	
	C3	0.46				0.30	0.30	0.62	
3	i/p values	1.3	1	0	1	1			C2*(FR1), C3*, C1*
	C1	0.54					0.54	0.54	
	C2	0.54	0.43	0.2	0.38		0.43	0.74	
	C3	0.54				0.30	0.30	0.68	
4	i/p values	0.31	1	0	2	1			C2*(Run), C3*, C1*
	C1	0.25					0.25	0.25	
	C2	0.25	0.43	0.25	0.48		0.48	0.61	
	C3	0.25				0.30	0.30	0.47	

5	i/p values	1.47	1	0	1	2			
	C1	0.59					0.59	0.59	**C2*(FR1), C3*, C1***
	C2	0.59	0.43	0.25	0.38		0.43	0.77	
	C3	0.59				0.41	0.41	0.76	
6	i/p values	1.69	2	0	1	1			
	C1	0.67					0.67	0.67	**C2*(FR1), C3*, C1***
	C2	0.60	0.56	0.25	0.38		0.56	0.82	
	C3	0.60				0.30	0.30	0.72	
7	i/p values	1.06	3	0	2	1			
	C1	0.48					0.48	0.48	**C2*(FR1), C3*, C1***
	C2	0.48	0.75	0.25	0.48		0.75	0.87	
	C3	0.48				0.30	0.30	0.64	
8	i/p values	2.04	3	1	3	1			
	C1	0.75					0.75	0.75	**C2*(FR1), C1*, C3***
	C2	0.60	0.75	0.60	0.56		0.75	0.90	
	C3	0.60				0.30	0.30	0.72	
9	i/p values	3.58	4	2	4	2			
	C1	1					1	1	**C2*(FR1&FR2), C1*, C3***
	C2	0.60	1	1	0.63		1	1	
	C3	0.60				0.41	0.41	0.76	
Process out of control:C1-1.0000									
Process out of control:C2-1.0000									

Table 5.10 Results from Fuzzy Inference Engine for Test Case 5

The results from the fuzzy inference engine are shown in the Table 5.10. In this case both FR1 and FR2 are identified and along with OCL at sample 9, thus giving both C1 and C2 at maximum value 1.0. Since FR1 (the characteristic pattern of shift causes) has the higher probability of occurrence (0.0028) compared to OCL (0.0027 as per Section 3.3), shift causes (C2*) are prioritized over isolated causes (C1*).

5.6 Test Case 6 (OCL, FR2 and Run)

On a further attempt to test with more characteristic patterns of shift causes (C2), another combination of shift cause patterns, FR2 and Run along with OCL pattern is considered. Again, as described earlier, MINITAB test for run is detected at ninth consecutive point falling on one-side of centerline and hence the result from MINITAB shows the presence of only two out of three patterns considered. On the other hand, results from the fuzzy inference engine show the presence of all three patterns and since OCL is preceded by other two unnatural patterns FR2 and Run, presence of shift causes are confirmed with a maximum value of 1.0. The test data is given in Table 5.11 and the control chart is shown in Figure 5.6

Data: $\mu = 50, \sigma = 1.0, n = 5$

Sample No	X_1	X_2	X_3	X_4	X_5	Mean
1.	50.2681	48.5294	49.0512	50.0176	50.7473	49.7227
2.	51.1256	49.6924	50.4742	51.0484	49.0525	50.2786
3.	50.2721	49.1841	49.7142	49.9592	49.4854	49.7230
4.	48.7486	49.7143	48.8453	49.4014	48.8658	49.1151
5.	49.2317	50.2178	47.8083	50.7124	48.4915	49.2923
6.	51.4859	52.5719	50.4062	48.4897	50.1307	50.6169
7.	50.5363	53.1283	49.5771	50.2932	50.0845	50.7239
8.	50.1550	49.3009	51.4115	51.0635	50.2256	50.4313
9.	52.8048	52.7215	51.7870	48.5275	49.3996	51.0481
10.	50.7566	50.0173	49.2853	50.0981	50.6453	50.1605
11.	50.0082	52.0126	50.2318	50.6496	52.3304	51.0465
12.	50.8166	51.8717	51.6397	53.4504	50.2349	51.6027

Table 5.11 Data for Test Case 6

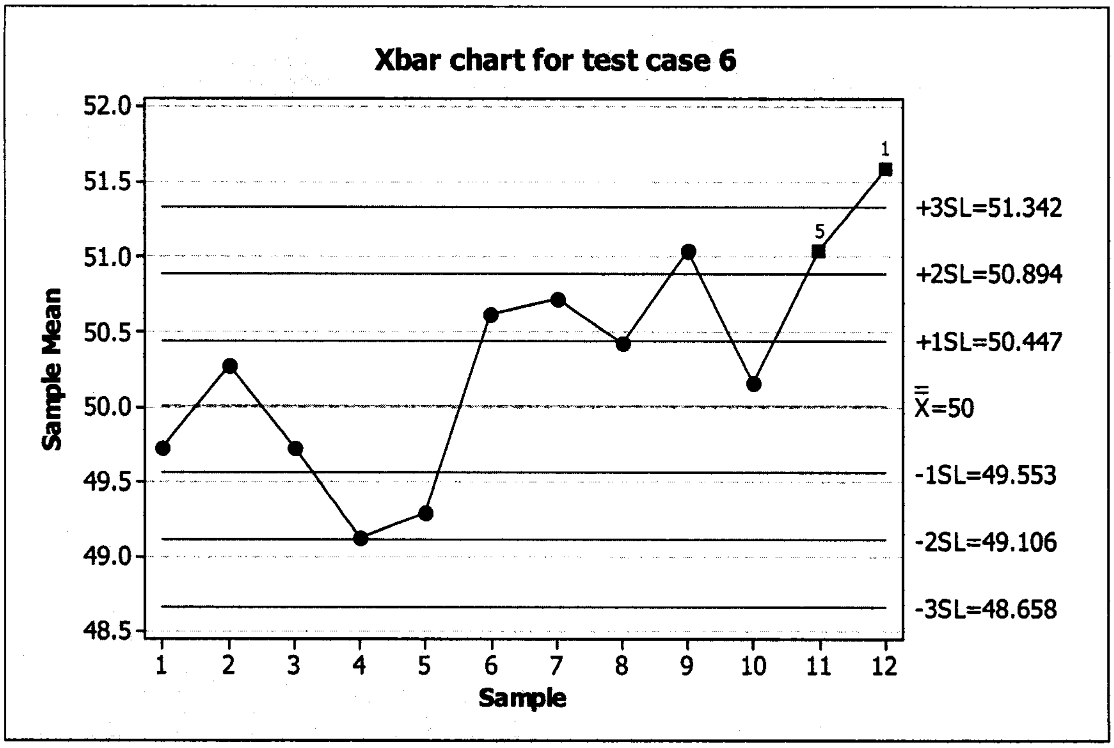

Figure 5.6. $\overline{X}$ Chart for Test Case 6

MINITAB Result:

Test Results for Xbar Chart of C1, ..., C5

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12

TEST 5. 2 out of 3 points more than 2 standard deviations from center line
(on
    one side of CL).
Test Failed at points:  11, 12
```

Results from Fuzzy Inference Engine

Mean:50.0000 Sigma:1.0000 Sample_Size:5									
SNo	Cause	OCL	FR1	FR2	Run	Trend	Max	Alg_ Sum	Cause Priority
1	i/p values	0.62	0	0	1	0			C2*(Run), C3*, C1*
	C1	0.33					0.33	0.33	
	C2	0.33	0.35	0.25	0.38		0.38	0.58	
	C3	0.33				0.30	0.30	0.53	
2	i/p values	0.62	0	0	1	1			C2*(Run), C3*, C1*
	C1	0.33					0.33	0.33	
	C2	0.33	0.35	0.25	0.38		0.38	0.58	
	C3	0.33				0.30	0.30	0.53	
3	i/p values	0.61	0	0	1	1			C2*(Run), C3*, C1*
	C1	0.33					0.33	0.33	
	C2	0.33	0.35	0.25	0.38		0.38	0.58	
	C3	0.33				0.30	0.30	0.53	
4	i/p values	1.97	1	0	2	2			C2*(Run), C3*, C1*
	C1	0.74					0.74	0.74	
	C2	0.60	0.43	0.25	0.48		0.48	0.79	
	C3	0.60				0.41	0.41	0.76	
5	i/p values	1.58	2	0	3	1			C2*(FR1&Run), C3*, C1*
	C1	0.63					0.63	0.63	
	C2	0.60	0.56	0.25	0.56		0.56	0.82	
	C3	0.60				0.30	0.30	0.72	
6	i/p values	1.37	2	0	1	2			C2*(FR1), C3*, C1*
	C1	0.56					0.56	0.56	
	C2	0.56	0.56	0.25	0.38		0.56	0.81	
	C3	0.56				0.41	0.41	0.74	
7	i/p values	1.61	2	0	2	3			C3*, C2*(FR1), C1*
	C1	0.64					0.64	0.64	
	C2	0.60	0.56	0.25	0.48		0.56	0.82	
	C3	0.60				0.60	0.60	0.84	

8	i/p values	0.96	2	0	3	1			**C2*(FR1&Run), C3*, C1***
	C1	0.46					0.46	0.46	
	C2	0.46	0.56	0.25	0.56		0.56	0.76	
	C3	0.46				0.30	0.30	0.62	
9	i/p values	2.34	3	1	4	1			**C2*(FR1), C1*, C3***
	C1	0.81					0.81	0.81	
	C2	0.60	0.75	0.60	0.63		0.75	0.90	
	C3	0.60				0.30	0.30	0.72	
10	i/p values	0.35	3	1	5	1			**C2*(FR1), C3*, C1***
	C1	0.25					0.25	0.25	
	C2	0.25	0.75	0.60	0.74		0.75	0.81	
	C3	0.25				0.30	0.30	0.47	
11	i/p values	2.34	3	2	6	1			**C2*(FR2), C1*, C3***
	C1	0.81					0.81	0.81	
	C2	0.60	0.75	1	0.83		1	1	
	C3	0.60				0.30	0.30	0.72	
Process out of control:C2-1.0000									
12	i/p values	3.58	3	2	7	2			**C2*(FR2&Run), C1*, C3***
	C1	1					1	1	
	C2	0.60	0.75	1	1		1	1	
	C3	0.60				0.41	0.41	0.76	
Process out of control:C1-1.0000									
Process out of control:C2-1.0000									

Table 5.12 Results from Fuzzy Inference Engine for Test Case 6

The results from the fuzzy inference engine are shown in the Table 5.12. This case is also similar to that of previous test case, however with different combination of patterns. FR2 pattern is identified at sample 11 and FR2, Run along with OCL patterns are identified at sample 12. In this case, Run pattern of shift causes has the higher probability of occurrence (0.008 as mentioned in Section 3.3) thus making shift causes highest in the priority.

5.7 Test Case 7 (FR1, FR2 and Trend)

In this case a combination of characteristic patterns from both shift and gradual causes is considered to compare the response of the fuzzy inference engine with other methods. Here FR1 and FR2 patterns of shift causes and Trend pattern for gradual causes are considered. All three patterns are detected by MINITAB except for that the trend pattern is detected in its sixth point itself (sample 12). Now, the result from the fuzzy inference engine gives both the causes (*C2* and *C3*) a maximum value of 1.0, prioritizing *C3** over *C2**. Here this prioritization is done based on the probability of occurrence of characteristic patterns. As explained in section 3.3, trend pattern has the higher probability of occurrence than FR1 and FR2. Presence of trend pattern reinforces the gradual causes over shift causes. The test data is given in Table 5.13 and control chart is shown in Figure 5.7.

<u>Data:</u> $\mu = 50, \sigma = 1.0, n = 5$

Sample No	X_1	X_2	X_3	X_4	X_5	Mean
1.	50.7387	49.5947	49.5849	49.2320	50.9549	50.0211
2.	49.3567	50.8175	49.3406	49.4535	48.8772	49.5691
3.	49.4348	51.1365	51.0497	50.0943	51.1910	50.5813
4.	49.1990	50.5684	49.3381	51.1209	50.4775	50.1408
5.	49.0848	49.3550	49.2542	49.2287	49.7714	49.3388
6.	49.0409	48.7372	48.8515	48.8099	49.9312	49.0742
7.	49.3567	50.8175	49.3406	49.4535	48.8772	49.5691
8.	50.4653	50.0439	49.3196	50.2429	49.2853	49.8714
9.	49.1990	50.5684	49.3381	51.1209	50.4775	50.1408
10.	50.6425	52.1969	50.0847	49.7479	49.7048	50.4753

11.	52.5712	49.4842	51.0704	51.3932	49.2886	50.7615
12.	50.2658	51.4124	52.0780	49.9458	51.0676	50.9539
13.	50.3587	51.4124	52.0780	51.2457	51.0676	51.2325

Table 5.13 Data for Test Case 7

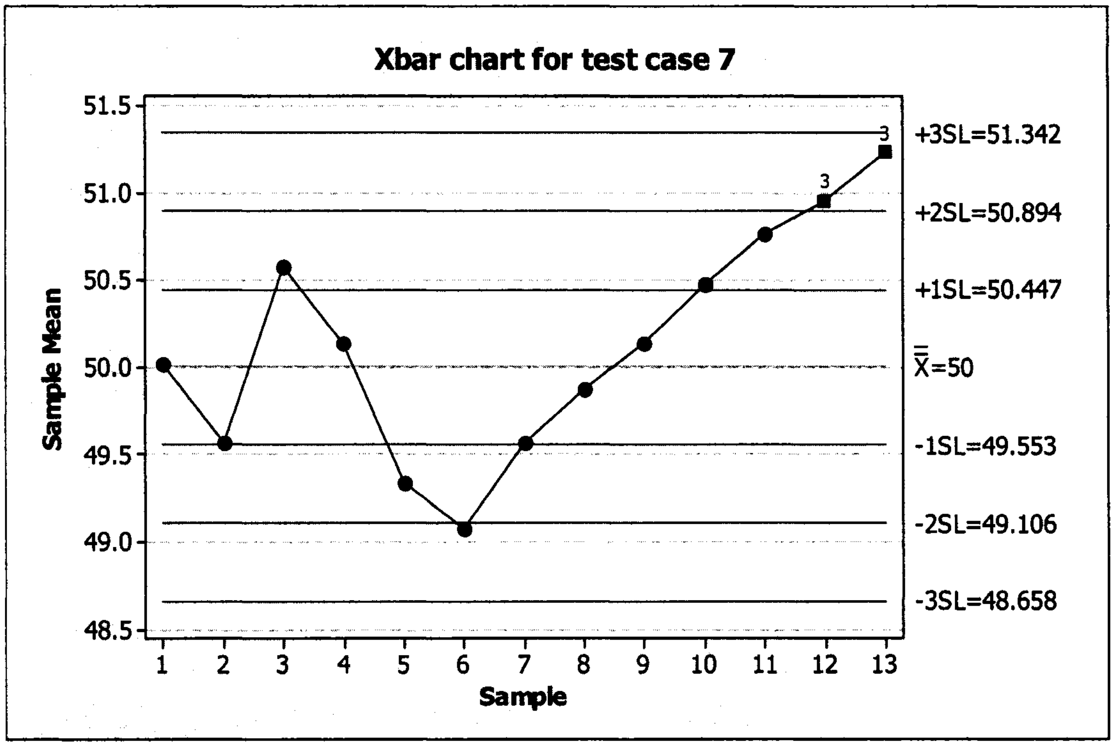

Figure 5.7. $\overline{X}$ Chart for Test Case 7

MINITAB Result:

Test Results for Xbar Chart of C1, ..., C5

```
TEST 3. 6 points in a row all increasing or all decreasing.
Test Failed at points:  12, 13

TEST 5. 2 out of 3 points more than 2 standard deviations from center line
(on
     one side of CL).
Test Failed at points:  13

TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  13
```

Results from Fuzzy Inference Engine

Mean:50.0000 Sigma:1.0000 Sample_Size:5									
SNo	Cause	OCL	FR1	FR2	Run	Trend	Max	Alg_ Sum	Cause Priority
1	i/p values	0.04	0	0	1	0			C2*(Run), C3*, C1*
	C1	0.25					0.25	0.25	
	C2	0.25	0.35	0.25	0.38		0.38	0.53	
	C3	0.25				0.30	0.30	0.47	
2	i/p values	0.96	0	0	1	1			C2*(Run), C3*, C1*
	C1	0.46					0.46	0.46	
	C2	0.46	0.35	0.25	0.38		0.38	0.66	
	C3	0.46				0.30	0.30	0.62	
3	i/p values	1.3	1	0	1	1			C2*(FR1), C3*, C1*
	C1	0.54					0.54	0.54	
	C2	0.54	0.43	0.25	0.38		0.43	0.74	
	C3	0.54				0.30	0.30	0.68	
4	i/p values	0.31	1	0	2	1			C2*(Run), C3*, C1*
	C1	0.25					0.25	0.25	
	C2	0.25	0.43	0.25	0.48		0.48	0.61	
	C3	0.25				0.30	0.30	0.47	
5	i/p values	1.47	1	0	1	2			C2*(FR1), C3*, C1*
	C1	0.59					0.59	0.59	
	C2	0.59	0.43	0.25	0.38		0.43	0.77	
	C3	0.59				0.41	0.41	0.76	
6	i/p values	2.07	2	1	2	3			C2*(FR2), C3*, C1*
	C1	0.76					0.76	0.76	
	C2	0.60	0.56	0.60	0.48		0.60	0.84	
	C3	0.60				0.60	0.60	0.84	
7	i/p values	0.96	2	1	3	1			C2*(FR2), C3*, C1*
	C1	0.46					0.46	0.46	
	C2	0.46	0.56	0.60	0.56		0.60	0.78	
	C3	0.46				0.30	0.30	0.62	

8	i/p values	0.28	2	1	4	2			**C2*(Run), C3*, C1***
	C1	0.25					0.25	0.25	
	C2	0.25	0.56	0.60	0.63		0.63	0.72	
	C3	0.25				0.41	0.41	0.56	
9	i/p values	0.31	2	0	1	3			**C3*, C2*(FR1), C1***
	C1	0.25					0.25	0.250	
	C2	0.25	0.56	0.25	0.38		0.56	0.67	
	C3	0.25				0.60	0.60	0.70	
10	i/p values	1.06	1	0	2	4			**C3*, C2*(FR1), C1***
	C1	0.48					0.48	0.48	
	C2	0.48	0.43	0.25	0.4		0.43	0.73	
	C3	0.48				0.75	0.75	0.87	
11	i/p values	1.70	2	0	3	5			**C3*, C2*(FR1&Run), C1***
	C1	0.67					0.67	0.67	
	C2	0.60	0.56	0.25	0.56		0.56	0.82	
	C3	0.60				0.83	0.83	0.93	
12	i/p values	2.13	3	1	4	6			**C3*, C2*(FR1), C1***
	C1	0.77					0.77	0.77	
	C2	0.60	0.75	0.60	0.63		0.75	0.90	
	C3	0.60				0.90	0.90	0.96	
13	i/p values	2.75	4	2	5	7			**C3*, C2*(FR1&FR2), C1***
	C1	0.91					0.91	0.91	
	C2	0.60	1	1	0.74		1	1	
	C3	0.60				1	1	1	
Process out of control:C2-1.0000									
Process out of control:C3-1.0000									

Table 5.14 Results from Fuzzy Inference Engine for Test Case 7

Now, the result from the fuzzy inference engine, as shown in Table 5.14 gives both the causes (*C2* and *C3*) a maximum value of 1.0 due to FR1, FR2 (for C2) and Trend (C3). As explained in the Section 3.3, Trend pattern has the higher probability of occurrence than FR1 and FR2. Hence C3* is prioritized over C2*.

5.8 Test Case 8 (OCL, FR1, FR2 and Trend)

In this case, the fuzzy inference engine is tested when an OCL pattern is preceded by the characteristic patterns of both shift and gradual causes. FR1 and FR2 patterns for shift causes and Trend pattern for gradual causes preceding the OCL pattern is considered in this case. The test data used in this case is similar to that of test case 7, except that the last point (sample13) falls outside the upper control limit. The test data is given in Table 5.15 and control chart is given in Figure 5.8.

<u>Data:</u> μ = 50, σ = 1.0, n = 5

Sample No	X_1	X_2	X_3	X_4	X_5	Mean
1.	50.7387	49.5947	49.5849	49.2320	50.9549	50.0211
2.	49.3567	50.8175	49.3406	49.4535	48.8772	49.5691
3.	49.4348	51.1365	51.0497	50.0943	51.1910	50.5813
4.	49.1990	50.5684	49.3381	51.1209	50.4775	50.1408
5.	49.0848	49.3550	49.2542	49.2287	49.7714	49.3388
6.	49.0409	48.7372	48.8515	48.8099	49.9312	49.0742
7.	49.3567	50.8175	49.3406	49.4535	48.8772	49.5691
8.	50.4653	50.0439	49.3196	50.2429	49.2853	49.8714
9.	49.1990	50.5684	49.3381	51.1209	50.4775	50.1408
10.	50.6425	52.1969	50.0847	49.7479	49.7048	50.4753
11.	52.5712	49.4842	51.0704	51.3932	49.2886	50.7615
12.	50.2658	51.4124	52.0780	49.9458	51.0676	50.9539
13.	50.3587	51.4124	52.5780	51.2457	51.1676	51.3525

Table 5.15 Data for Test Case 8

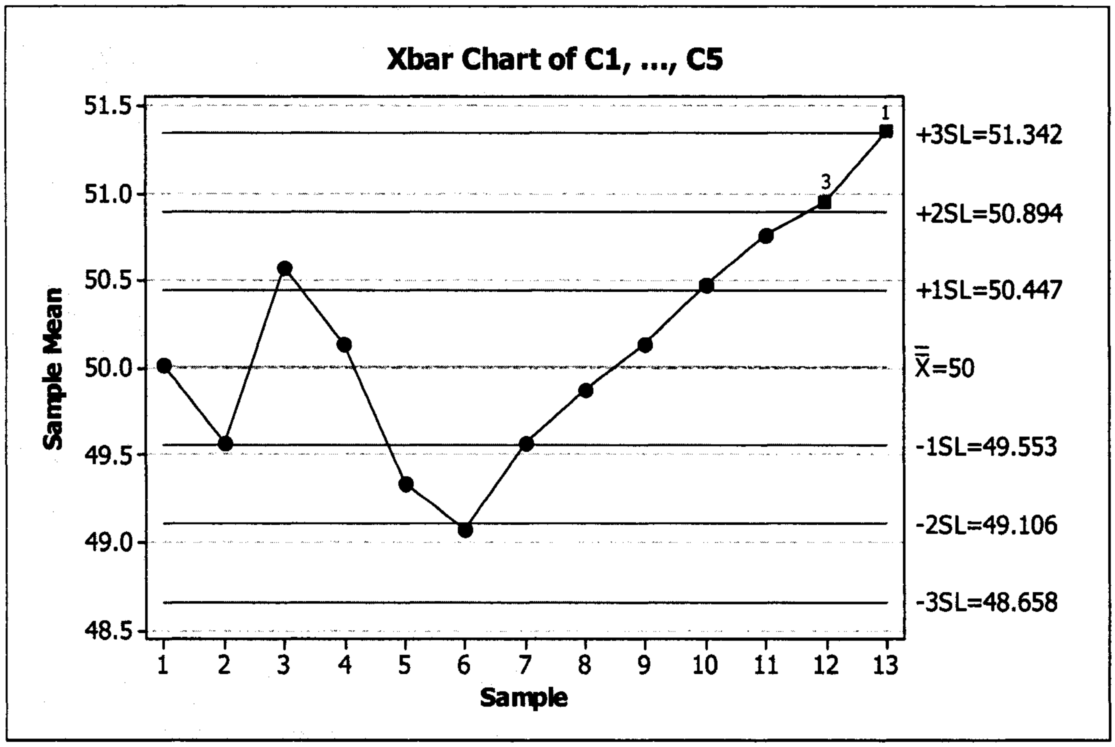

Figure 5.8. $\overline{X}$ Chart for Test Case 8

MINITAB Result:

Test Results for Xbar Chart of C1, ..., C5

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  13

TEST 3. 6 points in a row all increasing or all decreasing.
Test Failed at points:  12, 13

TEST 5. 2 out of 3 points more than 2 standard deviations from center line
(on
    one side of CL).
Test Failed at points:  13

TEST 6. 4 out of 5 points more than 1 standard deviation from center line
(on
    one side of CL).
Test Failed at points:  13
```

Results from Fuzzy Inference Engine

Mean:50.0000 Sigma:1.0000 Sample_Size:5									
SNo	Cause	OCL	FR1	FR2	Run	Trend	Max	Alg_ Sum	Cause Priority
1	i/p values	0.04	0	0	1	0			**C2*(Run), C3*, C1***
	C1	0.25					0.25	0.25	
	C2	0.25	0.35	0.25	0.38		0.38	0.53	
	C3	0.25				0.30	0.30	0.47	
2	i/p values	0.96	0	0	1	1			**C2*(Run), C3*, C1***
	C1	0.46					0.46	0.46	
	C2	0.46	0.35	0.25	0.38		0.38	0.66	
	C3	0.46				0.30	0.30	0.62	
3	i/p values	1.3	1	0	1	1			**C2*(FR1), C3*, C1***
	C1	0.54					0.54	0.54	
	C2	0.54	0.43	0.25	0.38		0.43	0.74	
	C3	0.54				0.30	0.30	0.68	
4	i/p values	0.31	1	0	2	1			**C2*(Run), C3*, C1***
	C1	0.25					0.25	0.25	
	C2	0.25	0.43	0.25	0.48		0.48	0.61	
	C3	0.25				0.30	0.30	0.47	
5	i/p values	1.47	1	0	1	2			**C2*(FR1), C3*, C1***
	C1	0.59					0.59	0.59	
	C2	0.59	0.43	0.25	0.38		0.43	0.77	
	C3	0.59				0.41	0.41	0.76	
6	i/p values	2.07	2	1	2	3			**C2*(FR2), C3*, C1***
	C1	0.76					0.76	0.76	
	C2	0.60	0.56	0.60	0.48		0.60	0.84	
	C3	0.60				0.60	0.60	0.84	
7	i/p values	0.96	2	1	3	1			**C2*(FR2), C3*, C1***
	C1	0.46					0.46	0.4	
	C2	0.46	0.56	0.60	0.56		0.60	0.78	
	C3	0.46				0.30	0.30	0.62	

8	i/p values	0.28	2	1	4	2			**C2*(Run), C3*, C1***
	C1	0.25					0.25	0.25	
	C2	0.25	0.56	0.60	0.63		0.63	0.72	
	C3	0.25				0.41	0.41	0.56	
9	i/p values	0.31	2	0	1	3			**C3*, C2*(FR1), C1***
	C1	0.25					0.25	0.25	
	C2	0.25	0.56	0.25	0.38		0.56	0.67	
	C3	0.25				0.60	0.60	0.70	
10	i/p values	1.06	1	0	2	4			**C3*, C2*(Run), C1***
	C1	0.48					0.48	0.48	
	C2	0.48	0.43	0.25	0.48		0.48	0.73	
	C3	0.48				0.75	0.75	0.87	
11	i/p values	1.70	2	0	3	5			**C3*, C2*(FR1 & Run), C1***
	C1	0.67					0.67	0.67	
	C2	0.60	0.56	0.25	0.56		0.56	0.82	
	C3	0.60				0.83	0.83	0.93	
12	i/p values	2.13	3	1	4	6			**C3*, C2*(FR1), C1***
	C1	0.77					0.77	0.77	
	C2	0.60	0.75	0.60	0.63		0.75	0.90	
	C3	0.60				0.90	0.90	0.96	
13	i/p values	3.02	4	2	5	7			**C3*, C2*(FR1 & FR2), C1***
	C1	1					1	1	
	C2	0.60	1	1	0.74		1	1	
	C3	0.60				1	1	1	
Process out of control:C1-1.0000									
Process out of control:C2-1.0000									
Process out of control:C3-1.0000									

Table 5.16 Results from Fuzzy Inference Engine for Test Case 8

The results from the fuzzy inference engine are given in the Table 5.16 for this case. Here all the patterns (OCL, FR1, FR2 and Trend) co-exist at sample 13 and thereby giving all three causes at maximum value. The causes are prioritized based on the probability of characteristic patterns. Gradual causes (C3*), with the trend pattern having highest probability of occurrence is the highest priority. C2* is prioritized next due to patterns with next higher probability of occurrence (FR1) and finally the C1* with OCL having lower probability of occurrence.

5.9 Test Case 9 (OCL with partial Run & partial FR1)

In this test case performance of the fuzzy inference engine is tested with an OCL pattern preceded by a partial Run and partial FR1 pattern. The OCL point (sample 11) is the sixth consecutive point falling on upper side of the center line, thus forming a partial run, and the third out of five consecutive points, falling beyond the one sigma control line, thus forming a partial FR1 pattern. The test data is given in the Table 5.17 and the control chart is shown in the Figure 5.9.

Data: $\mu = 50, \sigma = 1.0, n = 5$

Sample No	X_1	X_2	X_3	X_4	X_5	Mean
1.	50.2681	48.5294	49.0512	50.0176	50.7473	49.7227
2.	51.1256	49.6924	50.4742	51.0484	49.0525	50.2786
3.	50.2721	49.1841	49.7142	49.9592	49.4854	49.7230
4.	48.7486	49.7143	48.8453	49.4014	48.8658	49.1151
5.	49.2317	50.2178	47.8083	50.7124	48.4915	49.2923
6.	51.4859	52.5719	50.4062	48.4897	50.1307	50.6169
7.	50.5363	53.1283	49.5771	50.2932	50.0845	50.7239
8.	50.1550	49.3009	51.4115	51.0635	50.2256	50.4313

9.	52.8048	51.3215	51.7870	48.5275	49.3996	50.7681
10.	50.7566	50.0173	49.2853	50.0981	50.6453	50.1605
11.	50.8166	51.8717	51.6397	53.4504	50.2349	51.6027

Table 5.17 Data for Test Case 9

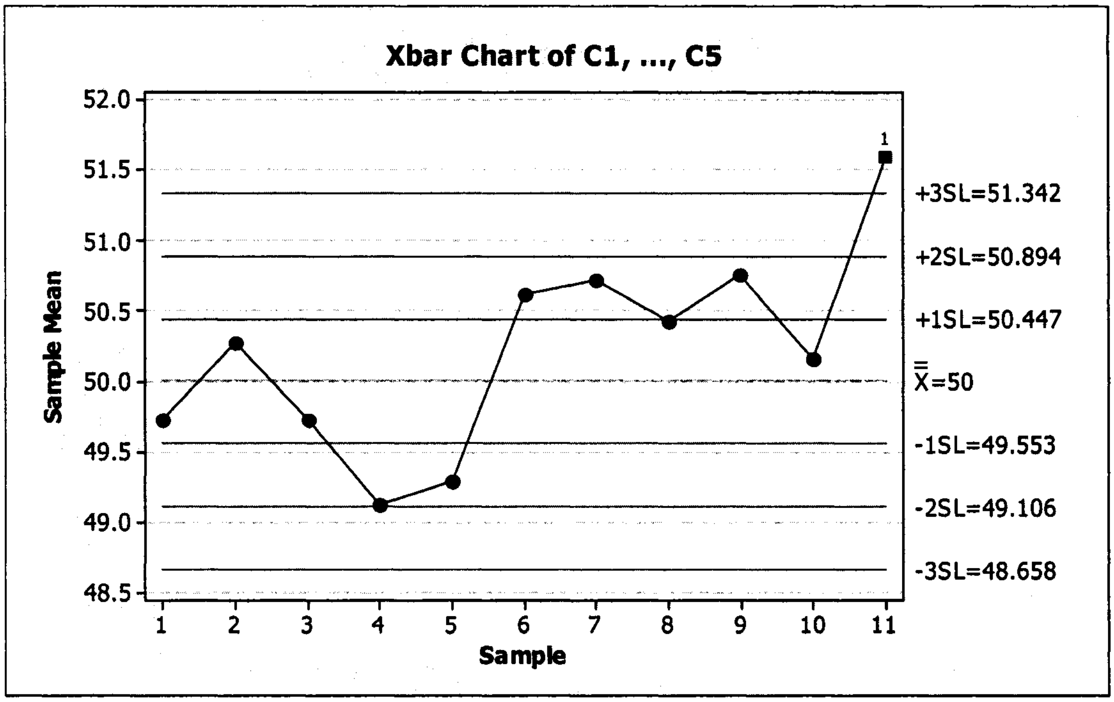

Figure 5.9 $\bar{X}$ Chart for Test Case 9

MINITAB Result:

Test Results for Xbar Chart of C1, ..., C5

Test Results for Xbar Chart of C1, ..., C5

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  11
```

Results from Fuzzy Inference Engine

Mean:50.0000 Sigma:1.0000 Sample_Size:5									
SNo	Cause	OCL	FR1	FR2	Run	Trend	Max	Alg_ Sum	Cause Priority
1	i/p values	0.62	0	0	1	0			C2*(Run), C3*, C1*
	C1	0.33					0.33	0.33	
	C2	0.33	0.35	0.25	0.38		0.38	0.58	
	C3	0.33				0.30	0.30	0.53	
2	i/p values	0.62	0	0	1	1			C2*(Run), C3*, C1*
	C1	0.33					0.33	0.33	
	C2	0.33	0.35	0.25	0.38		0.38	0.58	
	C3	0.33				0.30	0.30	0.53	
3	i/p values	0.61	0	0	1	1			C2*(Run), C3*, C1*
	C1	0.33					0.33	0.33	
	C2	0.33	0.35	0.25	0.38		0.38	0.58	
	C3	0.33				0.30	0.30	0.53	
4	i/p values	1.97	1	0	2	2			C2*(Run), C3*, C1*
	C1	0.74					0.74	0.74	
	C2	0.60	0.43	0.25	0.48		0.48	0.79	
	C3	0.60				0.41	0.41	0.76	
5	i/p values	1.58	2	0	3	1			C2*(FR1&Run), C3*, C1*
	C1	0.63					0.63	0.63	
	C2	0.60	0.56	0.25	0.56		0.56	0.82	
	C3	0.60				0.30	0.30	0.72	
6	i/p values	1.37	2	0	1	2			C2*(FR1), C3*, C1*
	C1	0.56					0.56	0.56	
	C2	0.56	0.56	0.25	0.38		0.56	0.81	
	C3	0.56				0.41	0.41	0.74	
7	i/p values	1.61	2	0	2	3			C3*, C2*(FR1), C1*
	C1	0.64					0.64	0.64	
	C2	0.60	0.56	0.25	0.48		0.56	0.82	
	C3	0.60				0.60	0.60	0.84	

8	i/p values	0.96	2	0	3	1			
	C1	0.46					0.46	0.46	C2*(FR1&Run), C3*, C1*
	C2	0.46	0.56	0.25	0.56		0.56	0.76	
	C3	0.46				0.30	0.30	0.62	
9	i/p values	1.71	3	0	4	1			C2*(Run), C3*, C1*
	C1	0.67					0.67	0.67	
	C2	0.60	0.	0.25	0.63		0.75	0.90	
	C3	0.60				0.30	0.30	0.72	
10	i/p values	0.35	3	0	5	1			C2*(FR1), C3*, C1*
	C1	0.25					0.25	0.25	
	C2	0.25	0.75	0.25	0.74		0.75	0.81	
	C3	0.25				0.30	0.30	0.47	
11	i/p values	3.58	3	1	6	1			C1*, C2*(Run), C3*
	C1	1					1	1	
	C2	0.60	0.75	0.60	0.83		0.83	0.93	
	C3	0.60				0.30	0.30	0.72	
Process out of control:C1-1.0000									

Table 5.18 Results from Fuzzy Inference Engine for Test Case 9

The results from the fuzzy inference engine are shown in the Table 5.18. In this test case OCL is preceded by a partial Run pattern. The OCL pattern occurs at sample 11 along with FR1 and Run patterns lacking one point from their definition. Even though the FR1 and Run patterns are not observed completely, the degree of presence is calculated by the fuzzy inference engine and combined with OCL. Since OCL pattern is observed clearly without any ambiguity, it is given the highest priority. In spite of the fact that the FR1 and Run pattern are observed only partially, the very possibility of the presence of shift causes are not overlooked and hence is given the second highest in the causal priority order.

5.10 Test Case 10 (OCL with partial Trend)

Similar to the previous test, the fuzzy inference engine is tested for an OCL pattern preceded this time, by an incomplete trend pattern. The OCL point (sample 12) is the sixth consecutive increasing point. As per MINITAB's test rules (six point continuously increasing), the trend pattern as well as OCL pattern is detected. However, the trend pattern is defined as seven consecutive increasing or decreasing points for the fuzzy inference engine. Hence it is a partial pattern and comparatively, OCL is clearly identified and is given the higher value of 1.0. Nevertheless, the evidence from partial presence of trend pattern is also aggregated with the OCL pattern and gradual cause intensity is found to be second in order after OCL pattern's characteristic cause. The test data is given in Table 5.19 and control chart is shown in Figure 5.10.

<u>Data:</u> $\mu = 50, \sigma = 1.0, n = 5$

Sample No	X_1	X_2	X_3	X_4	X_5	Mean
1.	50.7387	49.5947	49.5849	49.2320	50.9549	50.0210
2.	49.3567	50.8175	49.3406	49.4535	48.8772	49.5691
3.	49.4348	51.1365	51.0497	50.0943	51.1910	50.5813
4.	49.1990	50.5684	49.3381	51.1209	50.4775	50.1408
5.	49.0848	49.3550	49.2542	49.2287	49.7714	49.3388
6.	49.0409	48.7372	48.8515	48.8099	49.9312	49.0741
7.	49.3567	50.8175	49.3406	49.4535	48.8772	49.5691
8.	50.4653	50.0439	49.3196	50.2429	49.2853	49.8714
9.	49.1990	50.5684	49.3381	51.1209	50.4775	50.1408
10.	50.6425	52.1969	50.0847	49.7479	49.7048	50.4754

11.	52.5712	49.4842	51.0704	51.3932	49.2886	50.7615
12.	50.3587	51.4124	52.5780	51.2457	51.1676	51.3525

Table 5.19 Data for Test Case 10

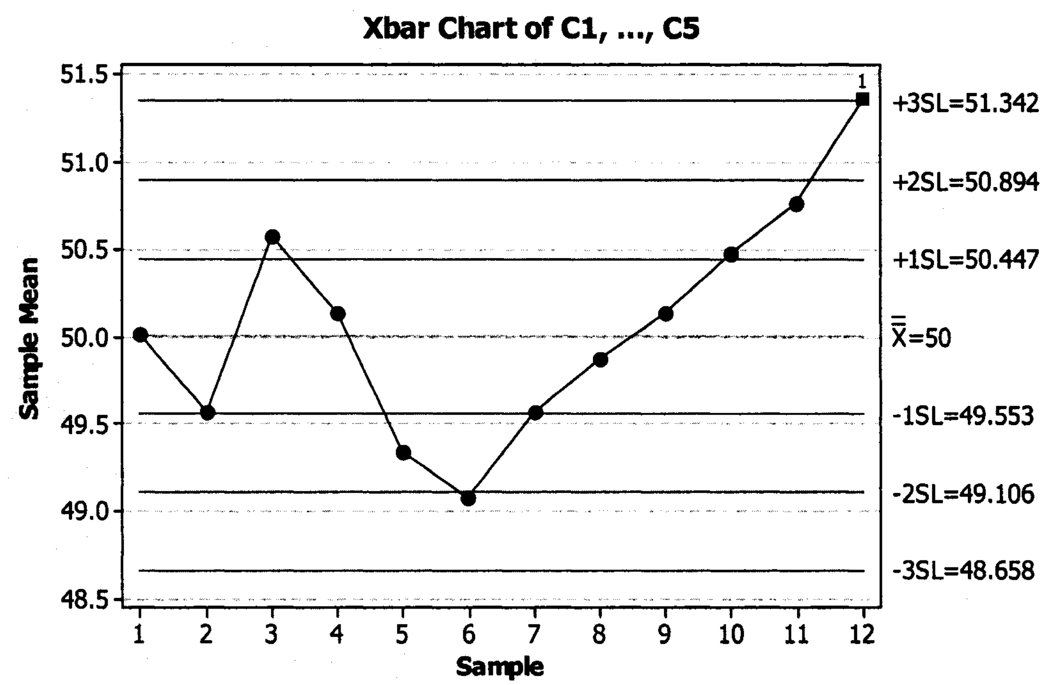

Figure 5.10 $\overline{X}$ Chart for Test Case 10

MINITAB Result:

Test Results for Xbar Chart of C1, ..., C5

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12

TEST 3. 6 points in a row all increasing or all decreasing.
Test Failed at points:  12
```

Results from Fuzzy Inference Engine

Mean:50.0000 Sigma:1.0000 Sample_Size:5									
SNo	Cause	OCL	FR1	FR2	Run	Trend	Max	Alg_ Sum	Cause Priority
1	i/p values	0.04	0	0	1	0			C2*(Run), C3*, C1*
	C1	0.25					0.25	0.25	
	C2	0.25	0.35	0.25	0.38		0.38	0.53	
	C3	0.25				0.30	0.30	0.47	
2	i/p values	0.96	0	0	1	1			C2*(Run), C3*, C1*
	C1	0.46					0.46	0.46	
	C2	0.46	0.35	0.25	0.38		0.38	0.66	
	C3	0.46				0.30	0.30	0.62	
3	i/p values	1.3	1	0	1	1			C2*(FR1), C3*, C1*
	C1	0.54					0.54	0.54	
	C2	0.54	0.43	0.25	0.38		0.43	0.74	
	C3	0.54				0.30	0.30	0.68	
4	i/p values	0.31	1	0	2	1			C2*(Run), C3*, C1*
	C1	0.25					0.25	0.25	
	C2	0.25	0.43	0.25	0.48		0.48	0.61	
	C3	0.25				0.30	0.30	0.47	
5	i/p values	1.47	1	0	1	2			C2*(FR1), C3*, C1*
	C1	0.59					0.59	0.59	
	C2	0.59	0.43	0.25	0.38		0.43	0.77	
	C3	0.59				0.41	0.41	0.76	
6	i/p values	2.07	2	1	2	3			C2*(FR2), C3*, C1*
	C1	0.76					0.76	0.76	
	C2	0.60	0.56	0.60	0.48		0.60	0.84	
	C3	0.60				0.60	0.60	0.84	
7	i/p values	0.96	2	1	3	1			C2*(FR2), C3*, C1*
	C1	0.46					0.46	0.46	
	C2	0.46	0.56	0.60	0.56		0.60	0.78	
	C3	0.46				0.30	0.30	0.62	

8	i/p values	0.28	2	1	4	2			C2*(Run), C3*, C1*
	C1	0.25					0.25	0.25	
	C2	0.25	0.56	0.60	0.63		0.63	0.72	
	C3	0.25				0.41	0.41	0.56	
9	i/p values	0.31	2	0	1	3			C2*(FR1), C3*, C1*
	C1	0.25					0.25	0.25	
	C2	0.25	0.56	0.25	0.38		0.56	0.67	
	C3	0.25				0.6	0.60	0.70	
10	i/p values	1.06	1	0	2	4			C3*, C2*(FR2), C1*
	C1	0.48					0.48	0.48	
	C2	0.48	0.43	0.25	0.48		0.48	0.73	
	C3	0.48				0.75	0.75	0.87	
11	i/p values	1.70	2	0	3	5			C3*, C2*(FR1&Run), C1*
	C1	0.67					0.67	0.67	
	C2	0.60	0.56	0.25	0.56		0.56	0.82	
	C3	0.60				0.83	0.83	0.93	
12	i/p values	3.02	3	1	4	6			C1*, C3*, C2*(FR1)
	C1	1					1	1	
	C2	0.60	0.75	0.60	0.63		0.75	0.90	
	C3	0.60				0.90	0.90	0.96	
Process out of control:C1-1.0000									

Table 5.20 Results from Fuzzy Inference Engine for Test Case 10

This test case considers another example of presence of OCL pattern preceded by another incomplete pattern, Trend. The results from the fuzzy inference engine are given in the Table 5.20. At sample 12, C1 takes the highest of the cause priority as OCL is identified clearly without any ambiguity. This time the second highest priority in cause order is given to gradual causes (C3) as it lacks just one point to form the pattern completely. It has to be noted that FR2 pattern of shift cause also lacks one point to form the pattern completely. Even though, both patterns lack one point for its complete definition at sample 12, the degree of influence of Trend pattern on gradual causes (0.90) is relatively higher compared with FR1 pattern on shift causes (0.75).

5.11 Comparison of Results

The results from the fuzzy inference engine for various test cases are summarized and compared with manual interpretation of control charts and MINITAB output in Table 5.21 in the aspects of pattern identification and cause indication.

Test case	Method	Out of control sample	Out of control pattern	Pattern Identification	Cause Identification (In Order)
1	Manual	#8	OCL	✓	×
	MINITAB	#8	OCL	✓	×
	Fuzzy	#8	OCL	✓	C1*, C2*, C3*
2	Manual	#18	Run	×	×
	MINITAB	#18	Run	×	×
	Fuzzy	#18	Run	✓	C2*, C3*, C1*
3	Manual	#16	Trend	×	×
	MINITAB	#15*	Trend	✓	×
		#16	Trend	✓	×
	Fuzzy	#16	Trend	✓	C3*, C2*, C1*

4	Manual	#8	OCL	✓	×
	MINITAB	#7	FR2	✓	×
		#8	FR2, OCL	✓	×
	Fuzzy	#7	FR2	✓	C2*, C1*, C3*
		#8	FR2, OCL	✓	C1*, C2*, C3*
5	Manual	#9	OCL	✓	×
	MINITAB	#9	OCL, FR1, FR2	✓	×
	Fuzzy	#9	OCL, FR1, FR2	✓	C2*, C1*, C3*
6	Manual	#11	FR2	×	×
		#12	OCL	✓	×
			Run	×	×
	MINITAB	#11	FR2	✓	×
		#12	OCL	✓	×
			Run	×	×
	Fuzzy	#11	FR2	✓	C2*, C1*, C3*
		#12	OCL, Run	✓	C2*, C1*, C3*
7	Manual	#13	FR1, FR2, Trend	×	×
	MINITAB	#12*	Trend	✓	×
		#13	FR1, FR2, Trend	✓	×
	Fuzzy	#13	FR1, FR2, Trend	✓	C3*, C2*, C1*

8	Manual	#13	OCL	✓	×
			FR1, FR2, Trend	×	×
	MINITAB	#12*	Trend	✓	×
		#13	OCL, FR1, FR2	✓	×
	Fuzzy	#13	OCL, FR1, FR2, Trend	✓	C3*, C2*, C1*
9	Manual	#11	OCL	✓	×
	MINITAB	#11	OCL	✓	×
	Fuzzy	#11	OCL	✓	C1*, C2*, C3*
10	Manual	#12	OCL	✓	×
	MINITAB	#12	OCL, Trend	✓	×
	Fuzzy	#12	OCL	✓	C1*, C3*, C2*

Table 5.21 Comparison of Results

Note: * In MINITAB, the rule for detecting Trend pattern is defined as six consecutive points increasing or decreasing whereas in our fuzzy inference engine it is defined as seven consecutive points increasing or decreasing.

Chapter 6

Summary, Conclusions and Future Work

6.1 Summary and Conclusions

Control charts have been proven an excellent tool for process monitoring and have been widely used in many industries. Even with sophisticated software, control charts are used only to detect the presence of assignable causes. Once the presence of assignable cause is found, considerable amount of time and energy is spent to identify the assignable cause through brainstorming, root-cause analysis etc, involving many people concerned with the process. Besides detecting the presence of assignable cause, control charts also provide useful information in determining the type of assignable cause through the patterns they exhibit. It requires lot of expertise to analyze these patterns produced on the control chart and to determine the nature of assignable cause present.

Even with technical expertise and past experience, it sometimes becomes difficult to achieve this due to the ambiguities in classifying patterns and mapping them to causes. Examples of such ambiguities are, situations such as multiple patterns existing simultaneously, inclusion of one pattern in another and same pattern indicating presence of multiple causes at different degrees. Fuzzy logic very efficiently handles such ambiguities by quantifying them mathematically. In this work the ambiguities in determining the assignable cause using control chart patterns is carried out with the help of fuzzy logic. A fuzzy inference engine is developed based on a chart pattern – cause relationship network, representing each link by a fuzzy inference system, with the objective as to resolve the aforementioned ambiguities and diagnosing the assignable cause(s) using chart patterns.

The chart-pattern relationship network is developed based on literature sources on control chart patterns and root cause analysis [21], [22] and [23] to provide a frame work for mapping the causes to chart patterns so that the ambiguities and vagueness in using chart patterns for assignable cause diagnosis can be handled by fuzzy logic. Unique network can also be developed for a specific process using past experience and inputs from process experts for more accurate results. Based on the nature of shift each cause can produce, the domain of possible causes is sub-categorized, so that effort spent in searching for assignable cause is greatly reduced.

The fuzzy inference system prioritizes each category of causes based on the degree of evidence imparted by its characteristic patterns. The value for each cause category is calculated for every single point plotted on the control chart. Usually for a control chart operating in phase 2, a single point falling outside three sigma control limit is alone considered as an out of control signal. In this phase sensitizing rules could increase the chart sensitivity, leading to increased false alarms and decreased average run-length and is therefore not advisable. This however leads to prohibiting us from obtaining useful hints for assignable cause diagnosis. But with this fuzzy inference system the intensity of the cause can be monitored directly, still keeping one point falling outside three sigma control limit as an out of control signal, thereby avoiding the problem of increased false alarms at the same time monitoring the chart patterns.

The developed fuzzy inference engine is then tested with different cases containing various combinations of unnatural patterns in which situations of aforementioned ambiguities may arise. The first three test cases considered are simple test cases containing only one pattern (from one cause in each test), to gain a better understanding of the performance of the fuzzy inference engine. In all of these test cases, the fuzzy inference engine identified the pattern and prioritized the respective cause with maximum value of 1.0.

In test case 1, OCL pattern is identified and C1* is prioritized with maximum value 1.0. In test case 2, Run pattern is identified and C2* is prioritized with maximum value 1.0 and in test case 3, Trend pattern is identified and C3* is prioritized with maximum value of 1.0. The later test cases include combinations of various unnatural patterns. In test case 4, both OCL and FR2 patterns are considered. Both the patterns are identified by the fuzzy inference engine and since OCL is preceded by FR2, C2* (shift causes) is prioritized over C1* (isolated causes). In test case 5 and 6, OCL is preceded by two shift cause patterns (FR1 and FR2 in test case 5 and FR2 and Run in test case 6). The fuzzy inference engine identified all three patterns (in each case) and C2* (shift causes) is prioritized due to the presence of shift cause patterns along with OCL. In test case 7, combination of two patterns from shift causes (FR1 and FR2) along with Trend pattern is considered. Here, since OCL pattern was not observed, actions to search for the assignable cause will not be initiated for a process operating in phase 2. However, the influence of presence of this combination of patterns on their respective causes was studied. All three patterns were identified by the fuzzy inference engine and due to higher probability of occurrence of Trend pattern C3* (gradual causes) is prioritized over C2* (shift causes). Test case 8 considers the same combination of patterns as in test case 7, but now with OCL as well. Here, actions for searching the assignable cause will be initiated with C3* at highest priority.

Test cases 9 and 10 represent the out of control situations where an OCL pattern is observed but other patterns underlie OCL without being defined completely. In test case 9, a Run of 6 points (7 points on same side of centerline for complete definition of Run) and 3 points in FR1 (4 out of 5 points beyond 1σ for complete definition of FR1) are observed when a point falls beyond 3σ control limit. In this case since the underlying shift patterns (Run and FR1) were not completely defined, C1* is prioritized due to its maximum value of 1.0 (from OCL) and shift causes (C2*) were assigned next priority over gradual causes (C3*). In test case 10, OCL is preceded by an increasing trend of 6 points. The fuzzy inference engine prioritizes C1* on observing OCL and C3* was given next higher priority, even though the pattern was not defined completely (as 7 consecutive increasing or decreasing points have to be observed to detect Trend pattern).

Thus the fuzzy inference engine keeps track of underlying patterns in addition to completely defined patterns and prioritizes the causes accordingly even when the pattern is not defined completely. The degree of presence of each pattern is monitored for every point plotted on the control chart and causes are prioritized accordingly. Once a point is observed beyond the three sigma control limit, the search for the assignable cause can be conducted beginning with the cause having highest priority thus ensuring that the search is carried out in a properly guided manner, even in ambiguous situations. This facilitates quicker identification of the exact cause.

6.2 Future Work

The fuzzy inference engine developed in this work analyzes the unnatural patterns exhibited on a $\overline{X}$ chart. Generally $\overline{X}$ chart is used in conjunction with R chart to monitor the process variation. It can therefore, be extended to monitor the R chart patterns also simultaneously so that any correlation present between these charts can be identified and be used in diagnosing the assignable cause. The chart pattern –cause relationship may vary depending upon the process nature. Hence provisions for customizing the chart pattern–cause network can be made so that user can define the network to be more specific to the process, using his past experience. Also, with the extended process knowledge and experience, the assignable cause domain can be classified more deeply to specific causes, facilitating to reach the root cause directly.

References

[1] Western Electric (1956). "Statistical Quality Control Handbook", Western Electric Corporation, Indianapolis, IN

[2] Nelson, L.S. (1984). "The Shewhart Control Chart – Test for Special Causes", Journal of Quality Technology, Vol. 16.

[3] Zadeh, L.A. (1965). "Fuzzy Sets", Information and Control, Vol.8. pp 338-353

[4] Kahraman, C., Tolga, E. and Ulukan, Z. (1995). "Using Triangular Fuzzy Numbers in the Tests of Control Charts for Unnatural Patterns", INRIA / IEEE symposium on emerging technologies and factories automation, Paris, FRANCE. pp 291-298.

[5] Zalila, Z., Fatene, M. and Kadhi, R. (1998). "Fuzzy Supervision in Statistical Process Control", Proceedings of IEEE International Conference on Systems, Man and Cybernetics, Vol.1. pp 638-643.

[6] Wang, L.R. and Rowlands, H. (1999). "A Fuzzy Logic Application in SPC Evaluation and Control", Proceedings of IEEE International Conference on Emerging Technologies and Factory Automation, Vol.1. pp 679-684.

[7] Tannock, J. D. T (2003). "A Fuzzy Control Charting Method for Individuals", International Journal of Production Research, Vol. 41, No. 5.

[8] Hsu, H. M. and Chen, Y. K. (2001). "A fuzzy reasoning based diagnosis system for $\overline{X}$ control charts", Journal of Intelligent Manufacturing, Vol. 12.

[9] Tontini, G. (1996). "Pattern Identification in Statistical Process Control Using Fuzzy Neural Networks", Proceedings of 5th IEEE International Conference on Fuzzy Systems, Vol.3. pp 2065-2070.

[10] Tontini, G (1998). "Robust Learning and Identification of Patterns in Statistical Process Control Charts Using a Hybrid RBF Fuzzy Artmap Neural Network", Proceedings of IEEE International Joint Conference on Neural Networks, Vol.3. pp 1694-1699.

[11] Wang, L.R. and Rowlands, H. (2001). "A Novel NN-Fuzzy-SPC Feedback Control System", Proceedings of IEEE International Conference on Emerging Technologies and Factory Automation, Vol.1. pp 417-423.

[12] Qiu, S., Agogino, A. M., Song, S., Wu, J. and Sitarama, S. (2001). "Fusion of Bayesian and Fuzzy Analysis for Print Defect Diagnosis", Proceedings of the ISCA 16th International Conference on Computers and Their Applications, (Seattle, WA, USA).

[13] Pan, H. and Liu, L. (1999). "Fuzzy Bayesian Network – A General Formalism for Representation, Inference and Learning with Hybrid Bayesian Networks", Proceedings of 6th International Conference on Neural Information Processing, Vol.1. pp 401-406.

[14] Baldwin, J. F. and Tomaso, E. D. (2003). "Inference and Learning in Fuzzy Bayesian Networks", Proceedings of 12th IEEE International Conference on Fuzzy Systems, Vol.1. pp 630-635.

[15] Looney, C.G. and Liang, L. R. (2002). "Inference via Fuzzy Belief Networks", Proceedings of ISCA CAINE International Conference, San Diego, CA.

[16] Pan, H. and McMichael, D. (1998). "Fuzzy Causal Probabilistic Networks – A New Ideal and Practical Inference Engine", Proceedings of 1st International conference on Multisource-Multisensor Information Fusion, Las Vegas, USA.

[17] Binaghi, E., Gallo, I. and Madella, P. (2000). "A Neural Model for fuzzy Dempster-Shafer classifiers", International Journal of Approximate Reasoning, Vol. 25.

[18] Denoeux, T. (2000). "Modelling Vague Beliefs Using Fuzzy-Valued Belief Structure", Fuzzy Sets and Systems, Vol. 116.

[19] Borgelt, C., Gebhardt, J., and Kruse, Rudolf. (1998). "Possibilistic Graphical Models", Proceedings of 3rd International Workshop on Computational Intelligence in Data Mining, Udine, Italy.

[20] Clark, D. F. and Kandel, A.(1990). "Fuzzy Belief Networks", Proceedings of ACM SIGSMALL/PC Symposium on Small Systems, Virginia, USA.

[21] Montgomery, D. C. (2005). "Introduction to Statistical Quality Control", 5th Edition, John Wiley, Hoboken, N J, USA

[22] Smith, G. M. (2004). "Statistical Process Control and Quality Improvement", 5th Edition, Pearson/Prentice Hall, N. J. USA.

[23] Doty, L. A. (1996). "Statistical Process Control", 2nd Edition, Industrial Press, NY. USA.

[24] Mamdani, E. H. and Assilian, S. (1975). "An Experiment in Linguistic Synthesis with a Fuzzy Logic Controller", International Journal of Man-Machine Studies, Vol. 7.

[25] Tagaki, T. and Sugeno, M. (1985). "Fuzzy Identification of Systems and its Applications to Modeling and Control", IEEE Transactions on Systems, Man, and Cybernetics, SMC-Vol. 15

[26] Sugeno, M. and Kang, G. T. (1988). "Structure identification of Fuzzy Model". Fuzzy Sets and Systems, Vol. 28.

[27] The MathWorks, Inc (2006). "Fuzzy LogicToolbox" User's Guide, Matlab®, Release 7.0.4, MA, USA

[28] Gupta, M. M. and Qi, J. (1991). "Theory of t-norms and Fuzzy Inference Methods", Fuzzy Sets and Systems. Vol. 31.

[29] Dubois, D. and Prade, H. (1985). "Fuzzy Set Aggregation Connectives", Information Sciences, Vol. 36

[30] Fodor, J.C. and Roubens, M. (1992). "Aggregation and Scoring Procedures in Multi-criteria Decision making Methods", IEEE international conference on Fuzzy systems, CA, USA.

[31] Mizumoto, M. (1989). "Pictorial representations of fuzzy connectives, Part I; cases of t-norms, t-conorms and averaging operators", Fuzzy sets and systems, Vol.31.

[32] Minitab Inc. (2003). MINITAB Statistical Software, Release 14 for Windows, State College, Pennsylvania. MINITAB®