ANALYSIS OF ISLANDING DETECTION CAPABILITIES OF
MULTI-INVERTER SYSTEMS

Yongzheng Zhang

A Thesis
in
The Department
of
Electrical and Computer Engineering

Presented in Partial Fulfillment of the Requirements
for the Degree of Master of Applied Science at
Concordia University
Montréal, Québec, Canada

December 2006

Copyright © Yongzheng Zhang, 2006
NOTICE:
The author has granted a non-exclusive license allowing Library and Archives Canada to reproduce, publish, archive, preserve, conserve, communicate to the public by telecommunication or on the Internet, loan, distribute and sell theses worldwide, for commercial or non-commercial purposes, in microform, paper, electronic and/or any other formats.

The author retains copyright ownership and moral rights in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author’s permission.

In compliance with the Canadian Privacy Act some supporting forms may have been removed from this thesis.

While these forms may be included in the document page count, their removal does not represent any loss of content from the thesis.

AVIS:
L’auteur a accordé une licence non exclusive permettant à la Bibliothèque et Archives Canada de reproduire, publier, archiver, sauvegarder, conserver, transmettre au public par télécommunication ou par l’Internet, prêter, distribuer et vendre des thèses partout dans le monde, à des fins commerciales ou autres, sur support microforme, papier, électronique et/ou autres formats.

L’auteur conserve la propriété du droit d’auteur et des droits moraux qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

Conformément à la loi canadienne sur la protection de la vie privée, quelques formulaires secondaires ont été enlevés de cette thèse.

Bien que ces formulaires aient inclus dans la pagination, il n’y aura aucun contenu manquant.
ABSTRACT

Analysis of Islanding Detection Capabilities of Multi-Inverter Systems

Yongzheng Zhang

Islanding is one important concern for grid connected distributed resources due to personnel and equipment safety. Inverter-resident islanding detection methods (IDMs) employ locally measured parameters to detect islanding. Passive type IDMs only monitor local variables such as voltage and frequency while active IDMs inject disturbances into the supply system and detect islanding based on system responses. Although very effective in systems with a single inverter, it is believed that active frequency drifting IDMs might have their effectiveness reduced in multi-inverter systems. This thesis investigates the islanding detection capabilities of multi-inverter systems. Three possible scenarios with passive-active, different and same type of active IDMs were discussed. Their performance was assessed using the concept of non-detection zones (NDZs) in a quality factor ($Q_l$) vs. load resonant frequency ($f_0$) load parameter space extended to multi-inverter systems. This thesis also proposes the use of a small dedicated reactive power source (STATCOM) as the sole active component in a multi-inverter system. In this way, interactions between inverters with active IDMs are prevented while effective islanding detection capability is provided by the islanding detection enhancer. The simulation and experiment results were presented to validate the theoretical analysis and the effectiveness of the proposed techniques.
ACKNOWLEDGMENTS

The author would like to express his sincere gratitude to his supervisor, Dr. Luiz A. C. Lopes for his invaluable guidance, advice, friendship and financial support throughout the course of this study.

Special thanks to Mr. Joseph Woods for his technical support, friendship, kindness, and unconditional help.

The author would also like to thank his colleagues in the P. D. Ziogas Power Electronics Laboratory. Smart suggestions from and helpful discussions with Huili Sun, Zhixiang Luo and Neil Savio D’Souza are unforgettable.

Last but not least, the author is very grateful towards his parents whose constant support made it possible to finish the project.
To my parents
LIST OF CONTENT

LIST OF FIGURE .............................................................................. viii
LIST OF TABLE ........................................................................... xii
LIST OF ACRONYMS .................................................................... xiii
LIST OF PRINCIPAL SYMBOLS .................................................... xiv

CHAPTER 1 ..................................................................................... 1

1.1 INTRODUCTION ........................................................................ 1

1.2 ISLANDING DETECTION METHODS .......................................... 2
  1.2.1 PASSIVE INVERTER-RESIDENT METHODS ............................. 4
  1.2.2 ACTIVE INVERTER-RESIDENT METHODS ............................... 7

1.3 ASSESSMENT OF ISLANDING DETECTION METHODS WITH NDZ ................................................................. 11
  1.3.1 PARALLEL RLC LOAD FREQUENCY CHARACTERISTICS ........... 12
  1.3.2 CONCEPT BEHIND THE $Q_f \times f_0$ NDZ .................................. 14
  1.3.3 NDZs in The $Q_f$ vs. $f_0$ Load Parameter Space ....................... 15

1.4 THESIS SCOPE AND CONTRIBUTIONS ..................................... 18

1.5 THESIS OUTLINE ..................................................................... 19

CHAPTER 2 ..................................................................................... 20

ACTIVE FREQUENCY DRIFTING IDMS IN MULTI-INVERTER SYSTEMS ................................................................. 20

2.1 INTRODUCTION ........................................................................ 20

2.2 NDZS OF MULTI-INVERTER SYSTEMS ...................................... 21
  2.2.1 ACTIVE AND PASSIVE IDM INVERTERS SYSTEM ...................... 21
  2.2.2 SYSTEM WITH AFD AND SMS IDM INVERTERS ....................... 23
  2.2.3 SYSTEM WITH TWO SIMILAR ACTIVE (SMS) IDM INVERTERS BUT WITH FREQUENCY MEASUREMENT ERRORS .............................................. 26

2.3 SIMULATION RESULT AND ANALYSIS ................................... 28
  2.3.1 SIMULATION SCHEMATICS AND PARAMETERS SETUP .............. 28
  2.3.2 SIMULATION RESULTS OF PASSIVE IDM AND SMS IDM INVERTERS SYSTEM .............................................. 40
  2.3.3 SIMULATION RESULTS OF AFD IDM AND SMS IDM INVERTERS SYSTEM .................................................. 41
  2.3.4 SIMULATION RESULTS OF TWO SMS IDMS WITH MEASUREMENT ERRORS INVERTERS SYSTEM ................................. 44

2.4 EXPERIMENT RESULT AND ANALYSIS .................................. 45
  2.4.1 EXPERIMENT SETUP .............................................................. 45
  2.4.2 LOCAL LOAD PARAMETERS SELECTION AND VALIDATION ........... 47
  2.4.3 PASSIVE AND SMS IDM INVERTERS ...................................... 48
  2.4.4 ONE AFD AND ONE SMS IDM INVERTERS ................................ 51
  2.4.5 TWO SMS IDM INVERTERS WITH FREQUENCY MEASUREMENT ERRORS ...................................................... 57

2.5 CONCLUSION ............................................................................ 59
CHAPTER 3

THE ISLANDING DETECTION ENHANCER

3.1 INTRODUCTION

3.2 PROPOSED ISLANDING DETECTION ENHANCER
   3.2.1 LINEAR REACTIVE CURRENT CONTROL
   3.2.2 SINUSOIDAL REACTIVE CURRENT CONTROL

3.3 DESIGN OF THE IDE

3.4 PERFORMANCE ASSESSMENT OF THE IDE
   3.4.1 UNITY POWER FACTOR INVERTER AND THE IDE
   3.4.2 INVERTER WITH SMS IDM AND THE IDE

3.5 SIMULATION AND EXPERIMENTAL RESULTS
   3.5.1 THE IDE IN A SYSTEM WITH ONE INVERTER
   3.5.2 THE IDE IN A SYSTEM WITH MULTIPLE UPF INVERTERS

3.6 LIMITATIONS OF THE PROPOSED TECHNIQUE

3.7 CONCLUSION

CHAPTER 4

CONCLUSION

4.1 SUMMARY

4.2 SUGGESTIONS FOR FUTURE WORK

REFERENCES

APPENDIX

THREE-LEVEL HYSTERESIS CURRENT CONTROLLER

A1. INTRODUCTION

A2. CONTROL STRATEGY

A3. IMPLEMENTATION OF THE CONTROL LOGIC

A4. SIMULATION RESULT
LIST OF FIGURE

Fig. 1-1 Classification of Islanding Detection Methods for PV Utility-interactive System 3

Fig. 1-2 The circuit for assessing anti-islanding features of grid-connected inverters [6]. 5

Fig. 1-3 Current and voltage waveforms of upward AFD IDM ........................................... 10

Fig. 1-4 Variation of the load angle of a RLC load with frequency for different values of resonant frequency ($f_0$) and quality factor ($Q_f$) .................................................................. 14

Fig. 1-5 NDZs of AFD IDM for different values of $\delta$ .......................................................... 17

Fig. 1-6 NDZs of SMS IDM for different $\theta_m$ and with $f_m - f_g = 3$ Hz .................................. 18

Fig. 2-1 NDZ of a multi-inverter system with both AFD and SMS IDMs ................................. 25

Fig. 2-2 NDZ of a multi-inverter system with 2 equal power SMS IDM inverters ($\theta_m = 10^\circ$ and $f_m - f_g = 3$Hz) and frequency measurement error of $\Delta f_e$ .............. 28

Fig. 2-3 The circuit for assessing anti-islanding features of grid-connected multi-inverter system .......................................................................................................................... 29

Fig. 2-4 MATLAB Simulink model of the grid-connected PV system ..................................... 31

Fig. 2-5 Sine wave considered for measurement [20] .............................................................. 33

Fig. 2-6 Frequency measurement block [20] ........................................................................ 35

Fig. 2-7 UVP/OVP & UFP/OFP Block .................................................................................... 37

Fig. 2-8 Reference Current with IDMs Block .......................................................................... 39

Fig. 2-9 Responses of systems with different power shares of active and passive inverters supplying local loads with different values of $Q_f$ .................................................. 41

viii
Fig. 2.10 Responses of systems with different power shares of SMS and AFD IDM

inverters ($K_{AFD_{pu}}$) supplying a local load with $Q_f=3$ and $f_0=59.8$ Hz ........................................... 42

Fig. 2.11 Responses of systems with different power shares of SMS and AFD IDM

inverters ($K_{AFD_{pu}}$) supplying a local load with $Q_f=3$ and $f_0=59.6$ Hz ........................................... 43

Fig. 2.12 Responses of systems with different power shares of SMS and AFD IDM

inverters ($K_{AFD_{pu}}$) supplying a local load with $Q_f=3$ and $f_0=59.2$ Hz ........................................... 43

Fig. 2.13 Simulation results of two SMS IDM inverters supplying one local load

$P_{load}=1$ kW , $Q_f=2.58, f_0=60.02$ Hz with different $Af_f$ ................................................................. 45

Fig. 2.14 Schematic of the grid-connected multi-inverter system laboratory set-up ...... 47

Fig. 2.15 Experiment results for a system with single SMS IDM inverter with load $f_0$

=60.02 Hz, $Q_f=2.58$ ................................................................................................................................ 49

Fig. 2.16 Experiment results for a system with passive and SMS IDM inverters with load

$f_0=60.02$ Hz, $Q_f=2.58$ ................................................................................................................................ 50

Fig. 2.17 Experiment results for a system with passive and SMS IDM inverters with load

$f_0=60.02$ Hz, $Q_f=1.19$ ................................................................................................................................ 50

Fig. 2.18 Experiment results for a system with AFD and SMS IDM inverters with

different power share $K_{AFD_{pu}}$ with load $f_0=59.8$ Hz, $Q_f=3$ ....................................................... 53

Fig. 2.19 Experimental results for a system with AFD and SMS IDM inverters with

different power share $K_{AFD_{pu}}$ with load $f_0=59.6$ Hz, $Q_f=3$ ....................................................... 55

Fig. 2.20 Experimental results for a system with AFD and SMS IDM inverters with

different power share $K_{AFD_{pu}}$ with load $f_0=59.2$ Hz, $Q_f=3$ ....................................................... 57
Fig. 2-21 Experimental results for the system with two SMS IDM inverters with $\Delta f_e=0.5$ Hz ................................................................. 58

Fig. 2-22 Experimental results for the system with two SMS IDM inverters with $\Delta f_e=0.8$ Hz ................................................................. 58

Fig. 3-1 Phase angle curves for the modified local load with the IDE operating with various values of $k_{IDE}$ and loads with $Q_f = 2.5$ and 7.5 ........................................... 64

Fig. 3-2 Phase angle curves for the modified local load with the IDE operating with various values of $I_m$ and loads with $Q_f = 2.5$ and 7.5 ........................................... 66

Fig. 3-3 Normalized reactive power of the IDE for loads with $Q_f = 2.5$ ....................... 68

Fig. 3-4 The NDZs of systems with one inverter and different types of IDMs.............. 71

Fig. 3-5 MATLAB-Simulink model of the grid-connected PV system with IDE......... 73

Fig. 3-6 Comparison of the theoretical and simulated NDZs for a system with the IDE and a unity power factor inverter .................................................. 73

Fig. 3-7 Experimental results for a system with one inverter and the IDE ............ 75

Fig. 3-8 Equivalent circuit of the multi-inverter system.......................................... 77

Fig. 3-9 Simulation results for a multi-UPF-inverter system with one IDE........... 77

Fig. A-1 Single-phase Voltage Source Converter ............................................... 90

Fig. A-2 Current trajectory of the proposed three-level HCC.............................. 92

Fig. A-3 Proposed three-level HCC state transition diagram .................................. 93

Fig. A-4 Proposed implementation for a three-level HCC ..................................... 94
Fig. A-5 Switching patterns for the APF with the three-level HCC

95
LIST OF TABLE

Table 1-Response to abnormal voltages and frequencies [5].............................. 4

Table 2-Parameters of the local RLC loads considered for the experimental tests......... 51

Table 3-Parameters of the inverters and loads of Fig. 3-8 ..................................... 76
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>Alternating Current</td>
</tr>
<tr>
<td>AFD</td>
<td>Active Frequency Drift</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>DG</td>
<td>Distributed Generation</td>
</tr>
<tr>
<td>DSP</td>
<td>Digital Signal Processor</td>
</tr>
<tr>
<td>HCC</td>
<td>Hysteresis Current Control</td>
</tr>
<tr>
<td>IDM</td>
<td>Islanding Detection Method</td>
</tr>
<tr>
<td>IDE</td>
<td>Islanding Detection Enhancer</td>
</tr>
<tr>
<td>MPPT</td>
<td>Maximum Power Point Tracking</td>
</tr>
<tr>
<td>NDZ</td>
<td>Non Detection Zone</td>
</tr>
<tr>
<td>PCC</td>
<td>Point of Common Coupling</td>
</tr>
<tr>
<td>PV</td>
<td>Photovoltaic</td>
</tr>
<tr>
<td>RMS</td>
<td>Root-Mean-Square value</td>
</tr>
<tr>
<td>SFS</td>
<td>Sandia Frequency Shift</td>
</tr>
<tr>
<td>SMS</td>
<td>Slip-Mode phase Shift</td>
</tr>
<tr>
<td>STATCOM</td>
<td>STATic synchronous COMpensator</td>
</tr>
<tr>
<td>THD</td>
<td>Total Harmonic Distortion</td>
</tr>
<tr>
<td>UFP/OFP</td>
<td>Under Frequency Protection and Over Frequency Protection</td>
</tr>
<tr>
<td>UPF</td>
<td>Unity Power Factor</td>
</tr>
<tr>
<td>UVP/OVP</td>
<td>Under Voltage Protection and Over Voltage Protection</td>
</tr>
</tbody>
</table>
# List of Principal Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega )</td>
<td>Angular frequency</td>
</tr>
<tr>
<td>( \omega_0 )</td>
<td>Load resonant angular frequency</td>
</tr>
<tr>
<td>( \omega_k )</td>
<td>Angular grid frequency</td>
</tr>
<tr>
<td>( \omega_i )</td>
<td>Current angular frequency</td>
</tr>
<tr>
<td>( \phi_{load} )</td>
<td>Load phase angle</td>
</tr>
<tr>
<td>( \Delta P )</td>
<td>Real power mismatch</td>
</tr>
<tr>
<td>( \Delta f_e )</td>
<td>Frequency measurement error</td>
</tr>
<tr>
<td>( \Delta Q )</td>
<td>Reactive power mismatch</td>
</tr>
<tr>
<td>( C )</td>
<td>Load Capacitance</td>
</tr>
<tr>
<td>( f )</td>
<td>Frequency</td>
</tr>
<tr>
<td>( f_0 )</td>
<td>Load resonant frequency</td>
</tr>
<tr>
<td>( f_{\text{max min}} )</td>
<td>Frequency between 59.3Hz to 60.5 Hz</td>
</tr>
<tr>
<td>( f_g )</td>
<td>Grid frequency</td>
</tr>
<tr>
<td>( f_i )</td>
<td>Current frequency</td>
</tr>
<tr>
<td>( f_{is} )</td>
<td>Islanding frequency</td>
</tr>
<tr>
<td>( f_{(k)} )</td>
<td>Present current frequency</td>
</tr>
<tr>
<td>( f_m )</td>
<td>Frequency at which ( \theta_m ) occurs for SMS</td>
</tr>
<tr>
<td>( f_{max} )</td>
<td>Maximum frequency</td>
</tr>
<tr>
<td>( f_{min} )</td>
<td>Minimum frequency</td>
</tr>
<tr>
<td>( f_{nom} )</td>
<td>Normal frequency that is not considered as islanding</td>
</tr>
<tr>
<td>( f_{0\text{max}} )</td>
<td>Maximum value of load resonant frequency that will result in islanding operation</td>
</tr>
</tbody>
</table>
$f_{0\text{min}}$ Minimum value of load resonant frequency that will result in islanding operation

$f_{PCC}$ PCC voltage frequency

$f_{SMS1}$ Frequency of output current of the first SMS IDM inverter with $\Delta f_e$

$f_{SMS2}$ Frequency of output current of the second SMS IDM inverter with $\Delta f_e$

$f_v$ Voltage frequency

$f_{v(k-1)}$ Voltage frequency in the previous cycle

$h$ Bandwidth of hysteresis control

$I_{ac}$ Grid ac current

$i_{ac}$ Instantaneous grid ac current

$i_{AFD}$ Instantaneous output current of PV inverter equipped with AFD IDM

$I_{IDE}$ RMS value of the IDE current

$i_{load}$ Instantaneous load current

$I_{load}$ Load current

$I_M$ Maximum magnitude of $I_{IDE}$ with sinusoidal control method

$i_{PV}$ Instantaneous PV inverter output current

$I_{PV}$ PV inverter output current

$I_{PV\text{max}}$ Maximum value of PV inverter output current

$i_{PV\text{ref}}$ Instantaneous PV inverter reference current

$i_{SMS}$ Instantaneous output current of PV inverter equipped with SMS IDM

$k_{AFD\text{pu}}$ Ratio of the active power provided by AFD IDM inverter(s) to the total active power supplied for the load

$k_{IDE}$ Gain of IDE current with linear control method
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{UPF,pu}$</td>
<td>Ratio of $P_{UPF}$ to $P_{load}$</td>
</tr>
<tr>
<td>$L$</td>
<td>Load inductance</td>
</tr>
<tr>
<td>$L_{line}$</td>
<td>Line equivalent inductance</td>
</tr>
<tr>
<td>$P_{active}$</td>
<td>Output power of PV inverter equipped with active IDM</td>
</tr>
<tr>
<td>$P_{load}$</td>
<td>Load real power</td>
</tr>
<tr>
<td>$P_{PV}$</td>
<td>PV inverter output real power</td>
</tr>
<tr>
<td>$P_{SMS1}$</td>
<td>Output power of the first SMS IDM inverter with $\Delta f_e$</td>
</tr>
<tr>
<td>$P_{SMS2}$</td>
<td>Output power of the second SMS IDM inverter with $\Delta f_e$</td>
</tr>
<tr>
<td>$P_{UPF}$</td>
<td>Output power of PV inverter equipped with UPF passive IDM</td>
</tr>
<tr>
<td>$Q_f$</td>
<td>Quality factor</td>
</tr>
<tr>
<td>$Q_{f,eq}$</td>
<td>Equivalent quality factor</td>
</tr>
<tr>
<td>$Q_{load}$</td>
<td>Load reactive power</td>
</tr>
<tr>
<td>$Q_{PV}$</td>
<td>PV inverter output reactive power</td>
</tr>
<tr>
<td>$R$</td>
<td>Load resistance</td>
</tr>
<tr>
<td>$R_{eq}$</td>
<td>Equivalent resistance</td>
</tr>
<tr>
<td>$S$</td>
<td>Switching state</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Simulation sampling time</td>
</tr>
<tr>
<td>$T_i$</td>
<td>Period of sinusoidal part of the current</td>
</tr>
<tr>
<td>$T_{PCC}$</td>
<td>Period of PCC voltage</td>
</tr>
<tr>
<td>$t_z$</td>
<td>Dead time of PV inverter current</td>
</tr>
<tr>
<td>$V_{dc}$</td>
<td>DC side voltage of simulation</td>
</tr>
<tr>
<td>$V_g$</td>
<td>Grid voltage</td>
</tr>
</tbody>
</table>
\( v_{nom} \)  Normal voltage that is not considered as islanding

\( v_{PCC} \)  Instantaneous PCC voltage

\( V_{PCC} \)  PCC voltage

\( Z_{load, eq} \)  Equivalent load impedance

\( \delta f \)  Frequency drift

\( \theta_{AFD} \)  Equivalent phase angle of current and voltage of inverter with AFD

\( \theta_{inv} \)  Equivalent phase angle of inverter current and voltage

\( \theta_{load} \)  Phase angle of load current and voltage \( (\theta_{load} = -\phi_{load}) \)

\( \theta_{load, eq} \)  Equivalent phase angle of load current and voltage

\( \theta_{m} \)  Maximum phase shift of SMS (°)

\( \theta_{m,eq} \)  Equivalent maximum phase shift

\( \theta_{SMS} \)  Phase angle of current and voltage of inverter with SMS

\( \theta_{SMS1} \)  Maximum phase shift of the first SMS IDM inverter with \( \Delta f_e \)

\( \theta_{SMS2} \)  Maximum phase shift of the second SMS IDM inverter with \( \Delta f_e \)
CHAPTER 1
INTRODUCTION

1.1 INTRODUCTION

With the current worldwide renewable energy and green power initiatives combined with the effects of the energy deregulation, Distributed Generation (DG) technologies are becoming an attractive niche supplementary energy option with reasonable profitable investment. The premise of DG is to provide electricity to a customer at a reduced cost, higher reliability and efficiency while reducing transmission and distribution losses. However the integration of DG into the main electricity networks is currently changing the paradigm which we used to live with and raises a number of issues, among which one critical concern lies in the inadvertent islanding [1, 2].

Islanding occurs when a portion of the distribution system becomes electrically isolated from the remainder of the power system, yet continues to be energized by distributed generators. The island is an unregulated power system, which may creates a safety hazard for the maintenance worker and may cause damage to power generation and power supply facilities as a result of unsynchronized re-closure [3]. In [4], it was shown that the probability of a power match is negligible with low penetration of DG in the distribution network, however the odds of islanding would increase significantly for passive inverters and very high DG penetration as what is expected to take place for the "net-zero" houses. Consequently, the ability to quickly detect and eliminate inadvertent DG supported islands is the critical safety requirement for both the distributed generators and utilities.
As one type of DG, the inverter-based generator is commonly connected to the secondary feeder due to its relatively small size (typically in the range of less than one kilowatt to a few megawatts) and actually works as an interface between the system and the generator, which can be photovoltaic panels, fuel cells, micro-turbines etc. Since it is the inverter that interacts with the supply system, all inverter-based DGs have operating characteristics with respect to grid interaction primarily determined by the inverter topology and controls [1]. Thus the inverter-based DGs are required to include a means to shut down when the utility source is not present in order to prevent islanding.

For successful integration of DG with utilities, clear interconnection requirements must be formulated, such as the IEEE standard 929-2000 [5] and IEEE standard 1547-2003 [6], which are meant to mandate control and protection measures to minimize the probability of an inadvertent island, and to minimize the duration of an island’s existence. As a result, many inverter specific anti-islanding techniques have been proposed and a number have been implemented in actual DG projects [3] by incorporating into the controls of inverters used in utility-interactive DG applications.

However it is anticipated that the non-islanding inverters certified for independent operation can interfere with each other and yield islanding when operated in parallel. This thesis focuses on the assessment of islanding detection features of systems based on multiple photovoltaic (PV) inverters and proposes an effective way for preventing islanding in the system with multiple inverters.

1.2 Islanding Detection Methods

Islanding detection methods (IDMs) for inverter applications may be divided into two major categories [1,3,8] shown in Fig.1-1: non inverter-resident methods and
inverter-resident methods. The methods not resident in the inverter are generally controlled by the utility or have communications between the inverter and the utility to cause the inverters to shut down when necessary [7]. For the relatively low power PV inverter system, it is usually not cost-effective, and there is little incentive, to communicate with utility control systems. Therefore, islanding detection and protection schemes for PV inverter systems have focused on inverter-resident methods rather than communication based techniques [1]. The inverter-resident methods can be further divided into two sub-types. One is passive methods resident in the inverter, the other is active methods resident in the inverter. In this section, we review some existing islanding detection methods resident in the inverter.

![Diagram of Islanding Detection Methods for PV Utility-Interactive System](image)

**Fig. 1-1** Classification of Islanding Detection Methods for PV Utility-Interactive System
1.2.1 Passive Inverter-Resident Methods

Passive methods for detecting an islanding condition basically monitor selected parameters such as voltage and frequency and/or their characteristics and cause the inverter to cease converting power when there is a transition from normal specified conditions [8].

1.2.1.1 Under/Over Voltage and Under/Over Frequency

The standard protections, over/under frequency protection methods (OFP/UFP) and over/under voltage protection methods (OVP/UVP), are commonly used as one passive method. All grid-connected PV inverters are required to have OFP/UFP and OVP/UVP that cause the PV inverter to stop supplying power to the utility grid if the frequency or amplitude of the voltage at the point of common coupling (PCC) between the customer and the utility strays outside of prescribed limits. Also, several other islanding prevention methods act to produce an abnormal voltage frequency or amplitude, and rely on the OFP/UFP and OVP/UVP to actually deactivate the inverter. The UVP/OVP and UFP/OFP trip thresholds and the allowed time delay are usually set according to the IEEE Std. 929-2000 shown in Table 1 [5].

Table 1-Response to abnormal voltages and frequencies [5]

<table>
<thead>
<tr>
<th>Voltage (at PCC) (V)</th>
<th>Voltage (at PCC) of the nominal value</th>
<th>Frequency (Hz)</th>
<th>Maximum Trip Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V &lt; 60$</td>
<td>$(V &lt; 50%)$</td>
<td>$f_{\text{nom}}$</td>
<td>6 cycles</td>
</tr>
<tr>
<td>$60 \leq V &lt; 106$</td>
<td>$(50% \leq V &lt; 88%)$</td>
<td>$f_{\text{nom}}$</td>
<td>120 cycles</td>
</tr>
<tr>
<td>$106 \leq V \leq 132$</td>
<td>$(88% \leq V \leq 110%)$</td>
<td>$f_{\text{nom}}$</td>
<td>Normal Operation</td>
</tr>
<tr>
<td>$132 &lt; V &lt; 165$</td>
<td>$(110% &lt; V &lt; 137%)$</td>
<td>$f_{\text{nom}}$</td>
<td>120 cycles</td>
</tr>
<tr>
<td>$165 \leq V$</td>
<td>$(137% \leq V)$</td>
<td>$f_{\text{nom}}$</td>
<td>2 cycles</td>
</tr>
<tr>
<td>$V_{\text{nom}}$</td>
<td>$f &lt; 59.3$</td>
<td></td>
<td>6 cycles</td>
</tr>
<tr>
<td>$V_{\text{nom}}$</td>
<td>$59.3 \leq f \leq 60.5$</td>
<td></td>
<td>Normal Operation</td>
</tr>
<tr>
<td>$V_{\text{nom}}$</td>
<td>$f &gt; 60.5$</td>
<td></td>
<td>6 cycles</td>
</tr>
</tbody>
</table>
The circuit recommended in the IEEE Std. 929-2000 for assessing the islanding detection features of a given inverter is shown in Fig. 1-2 [5]. As a parallel source to a utility service, the grid-connected nonislanding PV inverter is connected to a local load, represented by a parallel RLC circuit at the PCC. The nonislanding inverter includes a means to shut down when the utility source is not present (the nonislanding feature) in order to prevent the islanding. The utility grid provides a reference ac-bus where voltage and frequency are considered to be constant. When the grid is connected, the inverter supplies active and reactive power, $P_{pv} + jQ_{pv}$, and the local load consumes active and reactive power, $P_{load} + jQ_{load}$. The utility grid supplies or absorbs the deficit or surplus of active power $\Delta P = P_{load} - P_{pv}$ and the reactive power $\Delta Q = Q_{load} - Q_{pv}$ of the local inverter-load set.

![Diagram]

**Fig. 1-2** The circuit for assessing anti-islanding features of grid-connected inverters [6]

The behavior of the system at the time of utility disconnection (islanding) will depend on $\Delta P$ and $\Delta Q$ at the instant before the switch opens to form the island. If $\Delta P \neq 0$, the RMS value of the voltage at PCC ($V_{PCC}$) will change, and the OVP/UVP can shut down the
inverter to prevent islanding, if \( V_{PCC} \) go outside of the pre-defined window. If \( \Delta Q \neq 0 \), the load voltage will show a sudden shift in phase, and then the inverter's control system will cause the frequency of the inverter output current, and thus the frequency of \( V_{PCC} \) to change until the reactive power balances between the PV inverter and local load occurs. This change in frequency is detected by the OFP/UFP. The OFP/UFP protection allows the inverter to disconnect if system frequency drops below or rises above pre-defined frequency values. While the method is simple, the ability of islanding detection is not guaranteed for all load conditions, especially for source-load balanced conditions when PV power penetration is high.

1.2.1.2 Voltage Phase Jump Detection

The inverter control system monitors the phase relationship of the inverter terminal voltage and output current for sudden changes. The inverter control system normally controls the output current to keep a very small phase difference between the voltage and current (unity power factor operation). A sudden change indicates that the distribution network is no longer maintaining the voltage at the inverter terminals and it has shifted in phase to match the phase angle of the local load \([1,3,7]\). However this method fails when the load power factor is unity.

1.2.1.3 Voltage Harmonics Detection

In this method, the inverter controller monitors the total harmonic distortion (THD) of the inverter terminal voltage and shuts down the inverter if the THD exceeds a threshold. The rationale is that, in normal operation, the distribution network acts as a stiff (low impedance) voltage source, maintaining a low distortion voltage (THD \( \approx 0 \)) on the inverter
terminals. Two mechanisms are expected to cause an increase in voltage THD when an island occurs. First, the impedance at the inverter terminals increases because the low impedance distribution network is disconnected and only the local load remains. As a result, current harmonics in the inverter output current will cause increased levels of voltage harmonics in the terminal voltage. Second, non-linear loads within the island, particularly distribution step-down transformers, will be excited by the output current of the inverter. The voltage response of the non-linear loads to the current excitation can be highly distorted [1,3,8]. However, setting an appropriate harmonic threshold to disconnect the inverter is nearly impossible due to presence of nonlinear loads. This renders the harmonic monitoring method as less practical.

1.2.2 Active Inverter-Resident Methods

Active methods try to overcome the shortcomings of passive method by introducing perturbations in the inverter output. The perturbations are selected such that the response of the inverter is distinct when islanding occurs, so the active methods are generally considered more effective than the passive ones [9-19].

1.2.2.1 Load insertion techniques

The inverter can periodically connect a load impedance across its output terminals and monitor changes that occur. For example, a capacitor can be inserted across the output to increase the reactive current and the resulting phase shift in terminal voltage can be measured to calculate the effective line impedance. This has similarities to the passive phase jump detection method but since detection of the phase jump can be correlated with the insertion of the impedance, it is more resistant to false trips due to random phase jumps
in the grid voltage. As with other impedance measurement techniques, there are concerns
about interference among multiple units and the ability to reliably detect impedance
changes with practical values of the inserted load.

1.2.2.2 Voltage shift techniques

Voltage shift techniques apply positive feedback to the current or active power
regulation control loop of the inverter to cause the inverter terminal voltage to rapidly shift
to the under/over voltage detection threshold if the distribution network is not present to
maintain the voltage. Without this positive feedback, the terminal voltage changes to a new
stable operating point, largely determined by the resistance of the local load, when an
island occurs and the distribution network is disconnected. This operating point may be
within the under/over frequency limits of the inverter if the load is closely matched to the
inverter’s active and reactive power output. The positive feedback introduces instability
that drives the inverter terminal voltage towards one of the voltage limits.

1.2.2.3 Frequency shift techniques

The main concept behind the operation of frequency drifting IDMs is that the
frequency of an islanded system drifts until reactive power balance is achieved. That is,
when the reactive power consumed by the load is equal to that produced by the local
source, PV inverter. The reactive power of inverters equipped with frequency drifting
IDMs is controlled as a function of the system frequency with positive feedback. It can be
shown that a parallel RLC load becomes more capacitive when the system frequency
increases, thus supplying more reactive power. If the inverter increases the amount of
reactive power it absorbs as the frequency increases, the load of an islanded system is
forced to supply more reactive power and the system frequency tends to increase. Therefore, operation within the normal frequency range (59.3 Hz to 60.5 Hz for a 60Hz system) can be made unstable when the grid is disconnected by controlling the inverter with a rate of change of reactive power different with that presented by the local load. In such a case, either the UFP or the OFP of the inverter will trip preventing islanding. Following two frequently used frequency drift methods will be introduced in detail. They are the active frequency drift (AFD) method and the slip mode frequency shift (SMS) method.

In active frequency drift (AFD), the waveform of the inverter current is slightly distorted presenting a zero current segment for a drift-up operation. AFD can be implemented by forcing the current frequency to be \( f_i \) (< 1.5 Hz) above the voltage frequency in the previous cycle \( f_i(k) = f_i(k-1) + \delta f \) and keeping the inverter current equal to zero from the end of its negative semi-cycle until the positive zero crossing of the voltage. When the grid is present, it maintains the voltage frequency. Otherwise, the frequency of the voltage at the PCC tends to drift upward, reaching values superior to the resonant frequency of the local RLC load \( f_0 \). In both cases, the fundamental component of the current leads the voltage by a small angle \( \theta_{AFD} \).

For AFD implemented with a constant frequency drift, the current in each cycle is expressed by

\[
i_{PV(k)} = \sqrt{2}I \sin[2\pi(f_{r(k-1)} + \delta f)]t
\]  \hspace{1cm} (1-1)

In the steady state and under islanding conditions, from Fig. 1-3, one can see that although the PV inverter output current has the same zero crossing point with the PCC voltage, the current waveform is neither odd nor even. In order to simplify the calculation
of the phase angle between the fundamental components of the voltage and the current, the start of the fundamental current has been moved by $-t_z/2$. Thus, the fundamental component of the inverter output current leads the PCC voltage by an angle proportional to $-t_z/2$. The phase angle of the inverter $\theta_{inv}$ can be approximated by

$$\frac{t_z/2}{T_v} = \frac{\theta_{AFD}}{2\pi}$$

(1-2)

where, $t_z = \frac{1}{f_v} - \frac{1}{f_i} = \frac{1}{f} - \frac{1}{f + \delta f} = \frac{\delta f}{f(f + \delta f)}$.

The inverter angle for the AFD IDM is given by

$$\theta_{AFD} = \pi f t_z = -\frac{\pi \delta f}{f + \delta f}$$

(1-3)

This phase angle plays an important role in mapping the no detection zone (NDZ), which will be discussed in the next section.

![Graph](image)

**Fig. 1-3** Current and voltage waveforms of upward AFD IDM

In the slip mode frequency shift (SMS) method, the phase angle of the current is controlled as a function of the deviation of the frequency of the voltage in the previous cycle $f_{d(k-1)}$ from the nominal operating frequency of the utility grid $f_g$. By applying
positive feedback to the current angle, one manages to vary the short-term frequency of the load voltage, when the grid is disconnected, until reactive power balance is achieved. The inverter current is then defined as

$$i_{PV(k)} = \sqrt{2}I \sin\left(2\pi f_{v(k-1)}t + \theta_{SMS}\right)$$  \hspace{1cm} (1-4)

The phase shift $\theta_{SMS}$ is set to be a sinusoidal function of the deviation of the frequency from the grid frequency $f_g$, then

$$\theta_{SMS} = \theta_m \sin\left(\frac{\pi}{2}\frac{f_{v(k-1)} - f_g}{f_m - f_g}\right),$$  \hspace{1cm} (1-5)

where $f_m$ is the frequency at which the maximum phase shift $\theta_m$ occurs.

Comparing with the voltage shift techniques, the frequency shift techniques can reduce the impact on the dc side of the inverters since reactive power is not reflected to the dc side of the inverter and the amount of reactive power required for effective frequency shifting is smaller than that of active power required for effective voltage drifting [8]. However like other active islanding detection methods, the frequency shift methods, such as AFD and Sandia Frequency Shift (SFS), slightly degrade the power quality of the PV system by injecting low frequency current harmonics.

1.3 ASSESSMENT OF ISLANDING DETECTION METHODS WITH NDZ

One of the main limitations with local detection schemes is that each scheme has an operating region where islanding conditions cannot be detected. This region is called the non-detection zone (NDZ). NDZ is one of the most important indexes to assess the performance of islanding detection methods (IDMs). This is defined as the range of local
loads (that is, loads inside the potential island) for which the islanding detection methods (IDMs) under consideration can be made to fail to detect islanding [9-13].

Traditionally the effectiveness of the passive (monitoring only) IDM can be represented by a non-detection zone (NDZ) defined in a power imbalance or mismatch space (ΔP vs. ΔQ) [13]. However the use of NDZs in the power mismatch ΔP vs. ΔQ space is not adequate for assessing the performance of most of active IDMs since for a given reactive power mismatch different combinations of L and C are possible [9]. Some of these combinations result in islanding while others do not. However a new load parameter space based on the quality factor and resonant frequency of the load ($Q_f$ vs. $f_0$) can be used for plotting the NDZs of these frequency drifting IDMs. The assessment of the performance of a given IDM applied to a given load can be done by inspection. One can easily identify if a given IDM design meets the design specifications and for what cases it fails. This Section is organized in three parts. At first the characteristics of the local load are presented; then the concept and the principle behind the plotting NDZ in $Q_f \times f_0$ load parameter space will be explained; at last the NDZs of the different IDMs are analyzed in the $Q_f \times f_0$ load parameter space.

1.3.1 Parallel RLC Load Frequency Characteristics

The values of the frequency and magnitude of the voltage at the PCC after grid disconnects (islanding condition) heavily depend upon the local load characteristics. It is important to derive the equations related to the load that are required to analyze the operating principles and assess the performance of common IDMs [9]. Usually the local load (the load inside the potential island) is modeled as a parallel RLC circuit. This is done because for most IDMs it is some type of RLC load that causes the most difficulty in
detection. In general, nonlinear loads such as harmonic-producing loads or constant-power loads do not present as much difficulty in islanding detection [10,14].

According to the IEEE Std. 929-2000 [4], the quality factor \( Q_f \) is defined as: two pi times the ratio of the maximum stored energy to the energy dissipated per cycle at a given frequency.

\[
Q_f = 2\pi \frac{\text{Maximum Energy Stored}}{\text{Total Energy Lost In One Period}}
\]  

(1-6)

For a parallel RLC load

\[
Q_f = \frac{2\pi \left( \frac{1}{2} CR^2 I^2 \right)}{\pi R I^2 / \omega_0} = \omega_0 RC = \frac{R}{\omega_0 L} = R \sqrt{\frac{C}{L}}
\]

(1-7)

where, \( \omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}} \).

The magnitude and phase of a parallel RLC load at an arbitrary frequency \( f \), as a function of the quality factor \( Q_f \) and resonant frequency \( f_0 \) of the load are given by:

\[
|Z_{load}| = \left( \frac{1}{R^2 + \left( \frac{1}{\omega L} - \omega C \right)^2} \right) \left( 1 + Q_f^2 \left( \frac{f_0 - f}{f_0} \right) \right)^{-1}
\]

\[
\phi_{load} = \tan^{-1} \left( \frac{1}{\omega L} \frac{\omega_0 LC}{\omega C} \right) = \tan^{-1} \left( Q_f \left( \frac{f_0 - f}{f_0} \right) \right)
\]

(1-8)

(1-9)

The load current voltage (C-V) phase angle \( \theta_{load} \), that represents the angle by which the current leads the voltage, is given by

\[
\theta_{load} = -\phi_{load} = \tan^{-1} \left( Q_f \left( \frac{f_0 - f}{f_0} \right) \right).
\]

(1-10)

The load phase angle \( \theta_{load} \) \( \theta_{load} = -\phi_{load} \) vs. frequency characteristic curves of loads with different quality factors and different resonant frequencies \( f_0 \) are shown in Fig. 1-4. There one sees that the larger \( Q_f \), the larger is the variation of \( \theta_{load} \) for a certain variation
of the system frequency around \( f_0 \). In addition, the intersection of the \( \theta_{load} \) vs. \( f \) curves and the zero phase angles always takes place at \( f_0 \). Further, one can see that a load with \( f_0 < f_g \) is net capacitive at grid frequency since the current leads the voltage (\( \theta_{load} > 0^\circ \)). Conversely, a load with \( f_0 > f_g \) is net inductive at grid frequency.

![Graph showing load angle variation with frequency for different values of resonant frequency (\( f_0 \)) and quality factor (\( Q_f \)).](image)

**Fig. 1-4** Variation of the load angle of a RLC load with frequency for different values of resonant frequency (\( f_0 \)) and quality factor (\( Q_f \)).

### 1.3.2 Concept Behind the \( Q_f \times f_0 \) NDZ

The principle behind the \( Q_f \times f_0 \) load parameter space is called the phase criteria [10]. It says that the frequency of an islanded system in the steady-state condition is that where the angle \( \theta_{load} \) by which the load current leads the PCC voltage is equal to that of the inverter \( \theta_{inv} \), that is

\[
\theta_{load}(f) = \theta_{inv}(f)
\]  

(1-11)

In order to calculate the boundaries for islanding detection one needs to obtain an equation that represents the steady-state frequency of the voltage across the PCC as a function of the load parameters (\( Q_f \) and \( f_0 \)) and also as a function of the main parameters of the active IDM methods. Therefore, from (1-10) and (1-11),

14
\[
\tan^{-1}\left[ \frac{Q_f}{f_0} \left( f_0 - \frac{f_0}{f} \right) \right] = \theta_{inv}(f).
\]

Thus, one can get

\[
f_0^2 + \frac{\tan \theta_{inv}(f)}{Q_f} f_0 - f^2 = 0.
\]

Then the resonant frequency \( f_0 \) which will cause the islanding operation can be solved as the function of the islanding frequency \( f_{ix} \) and the inverter equivalent angle, and the negative value can be neglected:

\[
f_0 = \frac{f_{ix}}{2Q_f} \left( -\tan \theta_{inv}(f_{ix}) + \sqrt{\tan^2 \theta_{inv}(f_{ix}) + 4Q_f^2} \right).
\]

In order to calculate the NDZ, one imposes the islanding frequency to be a threshold frequency (\( f_{max} \) and \( f_{min} \)), varies the value of \( Q_f \) and then calculates the resonant frequency of the load at the threshold of the NDZ. Thus, if we can find equivalent inverter angle for some IDM, the NDZ for the corresponding IDM can be plotted by applying Equation (1-14) [18].

### 1.3.3 NDZs in The \( Q_f \) vs. \( f_0 \) Load Parameter Space

#### 1.3.3.1 NDZ of Passive IDM

Since the passive IDM does not apply any disturbances to the current, for a stand-alone and unity power factor with current control inverter, its inverter angle is always equal to zero, that is

\[
\theta_{inv} = 0
\]

Applying equation (1-14), one can get

\[
f_0 = f_{ix}
\]
So the final frequency of the inverter system with passive IDM after grid disconnected will be the resonant frequency of the load \( f_0 \). If \( f_{\text{min}} \leq f_0 \leq f_{\text{max}} \), UFP/OFP will not trip and islanding will occur regardless of the \( Q_f \) of the load.

1.3.3.2 NDZ of Active Frequency Drift (AFD) IDM

The operation principle of this method was discussed in Section 1.2.2.3. The phase angle of the inverter with AFD IDM was calculated and shown in Equation 1-3. Applying the phase criteria \( (\theta_{\text{load}} = \theta_{\text{AFD}}) \),

\[
- \tan^{-1} \left[ \frac{Q_f \left( \frac{f_0 - f_u}{f_u} \right)}{\frac{f_u}{f_0} + \delta f} \right] = \frac{\pi \delta f}{f_u + \delta f}
\]  

(1-17)

Thus,

\[
f_0^2 - f_u \tan \left[ \theta_{\text{AFD}} \left( \frac{f_u}{f_0} \right) \right] f_0 - f_u^2 = 0
\]  

(1-18)

In order to calculate the NDZ of the AFD IDM, one imposes the islanding frequency to be a threshold frequency \( (f_{\text{min}} \) or \( f_{\text{max}} \), varies the value of \( Q_f \) and then calculates the resonant frequency \( f_0 \) of the load at the threshold of the NDZ.

Fig. 1-5 shows the NDZ for AFD IDM for different values of \( \delta f \) for a normal frequency range of 59.3Hz \( \leq f \leq 60.5Hz \). There one sees that the main effect of the AFD is to shift the NDZ to lower values of \( f_0 \) as \( \delta f \) increases. For \( Q_f = 2.5 \) and \( \delta f = 0.5 \) Hz the method will fail to detect islanding for loads with 58.99Hz \( \leq f_0 \leq 60.19Hz \). According to Fig.1-5, no islanding would occur for inductive loads with \( Q_f < 3 \) if speed-up AFD IDM with \( \delta f = 1 \) Hz is used. For loads with high \( Q_f \) this active IDM does not present any improvement with respect to the passive UFP/OFP because the frequency drifting capacity is significantly reduced.
Fig. 1-5 NDZs of AFD IDM for different values of $\delta f$

### 1.3.3.3 NDZ of Slip Mode Frequency Shift (SMS) IDM

The operation principle of this method was discussed in Section 1.2.2.3. The phase angle of the inverter with AFD IDM was calculated and shown in Equation 1-5. The NDZ of the SMS IDM in the $Q_f$ vs. $f_0$ space is derived using the phase criteria:

$$-\tan^{-1}\left[Q_f\left(\frac{f_0}{f_{is}} - \frac{f_{is}}{f_0}\right)\right] = \theta_m \sin\left(\frac{\pi}{2} \frac{f_{is} - f_s}{f_m - f_s}\right)$$  \hspace{1cm} (1-19)

Thus,

$$f_0^2 - f_{is} \tan[\theta_{SMS}(f_{is})] \frac{Q_f}{f_0} - f_s^2 = 0$$  \hspace{1cm} (1-20)

The resulting NDZ for SMS IDM in the $Q_f$ vs. $f_0$ space obtained from the solution of (1-20) is shown in Fig. 1-6 for different values of $\theta_m$. There one sees that the NDZ is null for $Q_f < 2.5$ when $\theta_m = 10^\circ$ and $f_m - f_s = 3$Hz, as per the design specifications. Besides, as $\theta_m$ decreases, the maximum $Q_f$ of the loads for which islanding can be prevented also decreases. It is worth mentioning that although large values of $\theta_m$ allow islanding
detection of loads with large $Q_f$, the maximum amount of active power the inverter can inject, without exceeding its rated apparent power, is reduced.

![Diagram showing resonant frequency vs quality factor](image)

**Fig. 1-6** NDZs of SMS IDM for different $\theta_m$ and with $f_m - f_g = 3$ Hz.

### 1.4 Thesis Scope and Contributions

A number of islanding detection methods have been implemented in actual project applications [3]. Although very effective in systems with a single inverter, it is believed that frequency drifting active IDMs might have their effectiveness reduced in multi-inverter systems. This thesis assesses the islanding detection capabilities of systems with multiple inverters. The main contributions of this thesis are the following:

1. Extend the concept of phase criteria used for single inverter systems to the multi-inverter systems. Based on that, the NDZs of the multi-inverter systems are deduced and used for the analysis of anti-islanding capacity of the multi-inverter systems.
2. Proposes a method based on the use of STATCOM to reduce or eliminate the NDZ of multi-inverter systems. In this case, all inverters can be of the passive IDM type, so that interference between active IDMs is prevented.

3. A multi-inverter system laboratory prototype was built and the experimental results verified the theoretical analysis and computer simulation.

1.5 Thesis Outline

The contents of this thesis are organized in four Chapters.

The first Chapter introduces the topic of anti-islanding and some common used IDMs, also explains how to create NDZs based on the phase criteria for the single inverter systems.

The second Chapter further utilizes the phase criteria to produce the new NDZs for the assessment of multi-inverter systems. Three different case studies are presented to analyze how the frequency drifting methods can interfere with each other in the multi-inverter system. The simulation and experiment results are presented to validate the theoretical analysis.

The third Chapter proposes a method to eliminate the potential islanding in multi-inverter systems. The effectiveness of the proposed scheme is demonstrated with simulation and experimental results.

The forth Chapter summarizes the work carried out in this thesis and the final conclusions. Suggestions for future work on this topic are presented.
CHAPTER 2
ACTIVE FREQUENCY DRIFTING IDMS IN MULTI-INVERTER SYSTEMS

2.1 INTRODUCTION

From the previous chapter one can clearly find that active IDMs can effectively reduce the NDZ of the passive methods in single-inverter systems. However, it is believed that in systems with multiple inverters equipped with active IDMs, non-islanding inverters certified for independent operation can interfere with each other averaging their effects out and failing to detect islanding [21,22]. With increased deployment or even expansion of PV systems, it is very likely that actual islands will contain multiple inverters. Therefore, it is of paramount importance to investigate not only the way IDMs affect each other but also whether and when islanding becomes more likely to occur in multi-inverter systems.

This chapter discusses three possible scenarios that can be found in a system with multiple PV inverters. The first is a system with one inverter equipped with an active frequency drifting IDM while the others present standard passive IDM. A procedure for identifying how the share of power supplied by the passive IDM affects the NDZ of a nonislanding active IDM inverter (for net inductive loads with quality factor smaller or equal to 2.5) is presented. The second case concerns a system with nonislanding active IDM inverters of two different types that are very common: active frequency drift (AFD) and slip mode frequency shift (SMS). An expression for the NDZ of this system in a load parameter space is derived and the effects of the ratio of the power capacity of the
inverters with AFD and SMS IDMs on the size of the NDZ are identified. The last case involves a system with two inverters equipped with the same type of active IDM (SMS) but with errors in the system’s frequency measurements. The impact of the magnitude of this error on the NDZ of the system is analyzed. Both simulation and experiment results will be shown to demonstrate the effectiveness of the theoretical analysis at the end of this chapter.

2.2 NDZs of Multi-Inverter Systems

From the previous chapter we learned that the phase criteria is the basic theory to analyze the islanded system in a $Q_f$ vs. $f_0$ load parameter space. With it we can obtain the expression to calculate the NDZs for different IDMs of single inverter system. This theory is still applicable for the study of islanding of the multi-inverter system. The following approach is employed for assessing the islanding detection capabilities of multi-inverter systems with NDZs in the $Q_f$ vs. $f_0$ load parameter space. All active IDM inverters are lumped together so that an angle for the current of an equivalent active IDM inverter can be calculated and then used for computing the NDZ applying the equation (1-14). On the other hand, the passive IDM inverters do not affect directly the NDZ curve of the multi-inverter system. They are combined with the local load, leading to an equivalent load with the same $f_0$ of the original local load but with a higher $Q_f$. The advantages of this approach can be demonstrated with the following case studies.

2.2.1 Active and Passive IDM Inverters System

This system is composed of one active and one or more passive IDM inverters. The analysis of this configuration can be carried out by recognizing that the UPF passive IDM
inverter(s) only share the local load’s active power with the inverter equipped with active frequency drifting IDM. The latter has to absorb (supply) all reactive power supplied (absorbed) by the local load, as the frequency of the island drifts from the rated value. Since the load power is shared by the passive and the active frequency drifting inverters, 

\[ P_{\text{active}} + P_{\text{UPF}} = P_{\text{load}} = \frac{V^2}{R} \]  

(2-1)

Therefore, one can replace the UPF passive IDM inverter(s) with a negative resistance, resulting in an increase in the equivalent resistance of the local load. For the moment only active IDM inverter supplies the active power for the equivalent load.

\[ P_{\text{active}} = P_{\text{load}} - P_{\text{UPF}} = \frac{V^2}{R_{eq}} \]  

(2-2)

Comparing equation (2-1) with equation (2-2), one can get,

\[ R_{eq} = \frac{R}{1 - \frac{P_{\text{UPF}}}{P_{\text{load}}}} = \frac{R}{1 - K_{\text{UPF} \, pu}} \]  

(2-3)

where \( K_{\text{UPF} \, pu} = \frac{P_{\text{UPF}}}{P_{\text{load}}} \), the ratio of the active power provided by UPF passive IDM inverter(s) to the total active power supplied for the load. Thus the equivalent resistance of the local load \( R_{eq} \) can be computed as a function of \( K_{\text{UPF} \, pu} \).

With this equivalent resistance, the equivalent quality factor of the local load plus the passive inverter(s) is given by

\[ Q_{f \, pu} = R_{eq} \sqrt{\frac{C}{L}} = \frac{1}{1 - K_{\text{UPF} \, pu}} \cdot (R \sqrt{\frac{C}{L}}) \]  

(2-4)

\[ = \frac{Q_f}{1 - K_{\text{UPF} \, pu}} \]
Thus, for the analysis of islanding detection capabilities, the active IDM inverter sees
an equivalent load with the same $f_0$ as the original load but with a higher $Q_f$. The NDZ
plots in the load parameter space developed for the single active IDM inverters can still
be used provided that the value of $Q_f$ for the equivalent load is recalculated as in (2-4). As
an example, consider the case of a nonislanding active IDM inverter designed for a load
with $Q_f \leq 2.6$ and that operates in parallel with UPF inverters that supply half of the
load’s active power ($K_{UPF, pu} = 0.5$). The equivalent load (UPF inverter + original load)
presents a $Q_f$ twice as large as that of the original load, what should increase the odds of
islanding. In such a case, the active IDM inverter will only be able to prevent islanding
for a local load with $Q_f \leq 1.3$.

2.2.2 SYSTEM WITH AFD AND SMS IDM INVERTERS

In this analysis it is assumed that if more than one AFD IDM inverter exist, they
have identical settings ($\delta_f$). The same applies to the settings of the SMS inverter(s), i.e.
$\theta_m$ and $f_m - f_g$. It is also assumed that the AFD IDM inverter(s) supply $K_{AFD, pu}$ of the
load’s active power while the SMS IDM inverter(s) supply $1 - K_{AFD, pu}$. The currents of
these IDMs are defined as

$$i_{AFD} = I_{AFD} \sin(2\pi f t) = I \cdot K_{AFD, pu} \cdot \sin(2\pi f t + \theta_{AFD})$$  
(2-5)

$$i_{SMS} = I_{SMS} \sin(2\pi f t + \theta_{SMS}) = I \cdot (1 - K_{AFD, pu}) \cdot \sin(2\pi f t + \theta_{SMS})$$  
(2-6)

where $\theta_{AFD}$ and $\theta_{SMS}$ are the phase angles of the AFD and SMS IDM inverters
respectively, which are shown in the equation (1-3) and (1-5). The total current supplied
by the inverters is,

$$i_{inv} = i_{AFD} + i_{SMS}$$

$$= I \cdot K_{AFD, pu} \cdot \sin(2\pi f t + \theta_{AFD}) + I \cdot (1 - K_{AFD, pu}) \cdot \sin(2\pi f t + \theta_{SMS})$$  
(2-7)
Assume $\sin^2 \beta = K_{AFD \ pu}$, $(0 < k < 1, \beta \in (0, \pi/2))$, $\omega = 2\pi f$

\[
i_{inv} = I \cdot [\sin^2 (\beta) \cdot \sin (\omega t + \theta_{AFD}) + \cos^2 (\beta) \cdot \sin (\omega t + \theta_{SMS})]
= I \cdot [\sin (\omega t) \cdot (\sin^2 \beta \cdot \cos \theta_{AFD} + \cos^2 \beta \cdot \cos \theta_{SMS})]
+ \cos (\omega t) \cdot (\sin^2 \beta \cdot \sin \theta_{AFD} + \cos^2 \beta \cdot \sin \theta_{SMS})]
\]

\[= I \cdot \sqrt{A^2 + B^2} \cdot \sin (\omega t + \arctan \frac{B}{A})
\]

Where $A = \sin^2 \beta \cdot \cos \theta_{AFD} + \cos^2 \beta \cdot \cos \theta_{SMS}$, $B = \sin^2 \beta \cdot \sin \theta_{AFD} + \cos^2 \beta \cdot \sin \theta_{SMS}$

\[\sqrt{A^2 + B^2}
= \sqrt{(\sin^2 \beta \cdot \cos \theta_{AFD} + \cos^2 \beta \cdot \cos \theta_{SMS})^2 + (\sin^2 \beta \cdot \sin \theta_{AFD} + \cos^2 \beta \cdot \sin \theta_{SMS})^2}
= \sqrt{\frac{1}{2} \sin^2 2\beta \cdot [1 - \cos (\theta_{AFD} - \theta_{SMS})]}
\approx 1
\]

The angle of the current of the equivalent active IDM inverter can be calculated as

\[\theta_{inv}(f) = \arctan \frac{B}{A}
= \arctan \frac{\sin^2 \beta \cdot \sin \theta_{AFD} + \cos^2 \beta \cdot \sin \theta_{SMS}}{\sin^2 \beta \cdot \cos \theta_{AFD} + \cos^2 \beta \cdot \cos \theta_{SMS}}
\]

\[\theta_{inv}(f) = \frac{\tan^2 \beta \cdot \sin \theta_{AFD} + \sin \theta_{SMS}}{\tan^2 \beta \cdot \cos \theta_{AFD} + \cos \theta_{SMS}}
\]

The NDZ for this system can be calculated using the phase criteria ($\theta_{inv} = \theta_{load}$) as described in the previous chapter. By substituting equation (2-10) into (1-14) and replacing the islanding frequency $f_{io}$ by the frequency threshold $f_{max}$ and $f_{min}$, the boundary of the NDZ for this system will be obtained as

\[f_{0\ max} = \frac{f_{max}}{2Q_f} \left( - \tan \theta_{inv}(f_{max}, K_{AFD \ pu}) + \sqrt{\tan^2 \theta_{inv}(f_{max}, K_{AFD \ pu}) + 4Q_f^2} \right), (f_{0\ max} > f_{0\ min})
\]

\[f_{0\ min} = \frac{f_{min}}{2Q_f} \left( - \tan \theta_{inv}(f_{min}, K_{AFD \ pu}) + \sqrt{\tan^2 \theta_{inv}(f_{min}, K_{AFD \ pu}) + 4Q_f^2} \right), (f_{0\ min} < f_{0\ max})
\]
Fig. 2-1 shows the resulting NDZs for a system with different shares of active power supplied by SMS and AFD IDM inverters. The parameters of the IDMUs are $\theta_m = 10^\circ, f_m - f_g = 3$ Hz for the SMS and $\Delta f = 1$ Hz for the AFD. It is worth mentioning that these two inverters are nonislanding when they operate alone in a local system with net inductive loads with $Q_f \leq 2.5$. Nonetheless, islanding can occur when the SMS and the AFD IDM inverters operate together supplying inductive local loads with $Q_f \leq 2.5$. This also can be explained by the fact that when local loads presents $f_0 < f_g$, the AFD IDM attempts to drift the frequency up while the SMS IDM attempts to drift it down. As a result, they might offset each other and the islanding frequency can remain within the normal frequency range ($59.3$ Hz $\leq f \leq 60.5$ Hz) resulting in islanding.

![Resonant Frequency vs Quality Factor](image)

**Fig. 2-1** NDZ of a multi-inverter system with both AFD and SMS IDMUs.

Figure 2-1 shows three load cases (A, B and C) for which islanding detection depends on the power ratings of the AFD and SMS IDM inverters. According to those
NDZs, load A would be islanded for the three inverter arrangements, load B would be islanded for two while load C only for one inverter combination.

2.2.3 System with Two Similar Active (SMS) IDM Inverters But With Frequency Measurement Errors

SMS IDM inverters have been chosen for this analysis because they have the potential for interfering with each other under certain conditions. It is assumed that there are frequency measuring errors $\Delta f_c$ in both SMS IDMs caused by miscalibrated sensors. As a worst case scenario for inverters of same capacity, it is assumed that the errors present the same magnitude and opposite signs. To simplify the notation, $f_{r(k,1)}$ is replaced in this analysis by $f$. Thus, the frequencies of the output current from the two SMS IDM inverters become,

\begin{align}
    f_{SMS1} &= f + \Delta f_c \\
    f_{SMS2} &= f - \Delta f_c
\end{align}

(2-12) (2-13)

The problem in this case can arise due to the fact that whenever the measured voltage frequency ($f$) is above the central frequency ($f_g$) the SMS IDM attempts to increase the system’s frequency by absorbing reactive power. Otherwise it attempts to decrease it. Therefore, for the conditions mentioned above and at nominal grid frequency, the two SMS IDMs tend to offset each other, possibly leading to islanding.

In this analysis, it is assumed that except the error in the frequency measurements, the inverters present the same capacity ($P_{SMS1} = P_{SMS2} = 0.5P_{load}$) and the same parameters for the SMS IDM ($\theta_m = 10^\circ$, $f_m - f_g = 3\text{Hz}$). The NDZ of such a system is obtained for different values of frequency measurement error ($\Delta f_c$) to verify its impact on the variation of the size of the NDZ.
The current injected by the two inverters is given by

\[ i_{\text{inv}} = i_{\text{SMS1}} + i_{\text{SMS2}} \]

\[ = \frac{I}{2} \sin(2\pi f_{\text{SMS1}} t + \theta_{\text{SMS1}}) + \frac{I}{2} \sin(2\pi f_{\text{SMS2}} t + \theta_{\text{SMS2}}) \]  \hspace{1cm} (2-14)

\[ \theta_{\text{SMS1}} = \theta_m \sin \left( \frac{\pi}{2} \frac{(f + \Delta f_e) - f_g}{f_m - f_g} \right) \]  \hspace{1cm} (2-15)

\[ \theta_{\text{SMS2}} = \theta_m \sin \left( \frac{\pi}{2} \frac{(f - \Delta f_e) - f_g}{f_m - f_g} \right) \]  \hspace{1cm} (2-16)

For relatively small values of \( \Delta f_e \) the approximation below is valid.

\[ i_{\text{inv}} \approx I \cos \left( \frac{\theta_{\text{SMS1}} - \theta_{\text{SMS2}}}{2} \right) \sin(2\pi f t + \frac{\theta_{\text{SMS1}} + \theta_{\text{SMS2}}}{2}) \]  \hspace{1cm} (2-17)

The angle of the current of the equivalent SMS IDM inverter can be calculated as

\[ \theta_{\text{inv}} = \theta_{\text{SMS1}} + \theta_{\text{SMS2}} \]

\[ = \frac{1}{2} \theta_m \left[ \sin \left( \frac{\pi}{2} \frac{(f + \Delta f_e) - f_g}{f_m - f_g} \right) + \sin \left( \frac{\pi}{2} \frac{(f - \Delta f_e) - f_g}{f_m - f_g} \right) \right] \]

\[ = \frac{1}{2} \theta_m \cos \left( \frac{\pi}{2} \frac{\Delta f_e}{f_m - f_g} \right) \sin \left( \frac{\pi}{2} \frac{f - f_g}{f_m - f_g} \right) \]

\[ = \theta_{m,eq} \sin \left( \frac{\pi}{2} \frac{f - f_g}{f_m - f_g} \right) \]  \hspace{1cm} (2-18)

where,

\[ \theta_{m,eq} = \theta_m \cos \left( \frac{\pi}{2} \frac{\Delta f_e}{f_m - f_g} \right) \]  \hspace{1cm} (2-19)

By comparing equation (2-18) and (2-19) to (1-11), one concludes that the frequency measurement error results in a reduction of the magnitude of the maximum phase shifting angle \( \theta_{m,eq} \). As mention in Section 1.3.3.3, the smaller the value of \( \theta_m \), the smaller is the largest value of \( Q_f \) for which islanding can be prevented with SMS IDM. However, even for relatively large values of error (- 0.5 Hz), the magnitude of \( \theta_{m,eq} \) decreases by
only 3.4 % for SMS IDMs with \( f_m - f_g = 3 \text{Hz} \). Thus, the frequency measurement error should not affect much the NDZ of multi-inverter systems with SMS IDM. This can be seen in Fig. 2-2 that depicts the NDZs of the multi-inverter system with two SMS IDM inverters and different values for \( \Delta f_e \).

![Diagram of NDZ](image)

**Fig. 2-2** NDZ of a multi-inverter system with 2 equal power SMS IDM inverters (\( \theta_m = 10^\circ \) and \( f_m - f_g = 3 \text{Hz} \)) and frequency measurement error of \( \Delta f_e \).

### 2.3 Simulation Result and Analysis

#### 2.3.1 Simulation Schematics and Parameters Setup

The theoretical analysis and NDZs derived in the previous sections are validated by means of simulation results by use of Matlab Simulink. The simulated multi-inverter
system is similar to that presented in Fig. 1-2 but comprises two or more PV inverter stations connected in parallel at the PCC, as shown from the Fig2-3.

![Diagram](image)

**Fig. 2-3** The circuit for assessing anti-islanding features of grid-connected multi-inverter system

The system described in Fig. 2-3 was modeled with MATLAB Simulink as shown in Fig. 2-4. The PV array and current controlled voltage source inverters are simplified as two controllable current sources, which are connected to the PCC. The local load is represented by a parallel $RLC$ block with variable resonant frequency and quality factor. The grid breaker, that connects the grid (AC voltage source) to the PCC, is preset to open at a prescribed time. The reference current with IDMs block is used to generate the inverter reference current with the islanding detection methods that match the local load’s active power. There are two sets of reference currents provided for two PV inverters system respectively. By selecting the output terminal from the reference current block, one can switch different islanding detection methods for related simulation test conveniently. The frequency measurement block measures the frequency of the voltage at PCC, and UVP/OVP & UFP/OFP block would generate a fault signal to shut down the inverter if its frequency or voltage exceeds the IEEE Std. 929-2000 limits, shown in Table 1. The
manual switch is used to deactivate the frequency and the voltage protection thus allowing the measurement of the steady-state values of frequency and voltage across the load under islanding condition.

The Matlab Simulink model of the grid-connected PV system is discrete system sampled at simulation sampling rate of 3240 samples per cycle of 60 Hz, i.e. 
\[ T_s = 5.144 \times 10^{-6} \text{ s} \]. In the model, the value of the grid voltage is preset as \( V_g = 120 V \), 
\( f_g = 60 Hz \).
Fig. 2-4 MATLAB Simulink model of the grid-connected PV system.
2.3.1.1 **Frequency Measurement Block** [20]

Based upon the concept of zero crossings, the frequency of voltage at PCC can be measured and calculated. Considered the inconsistency between the lower estimation errors and shorter time delay by and large, one complete cycle usually is utilized for the calculation. The measurement method is based upon counting the number of samples in one complete cycle.

Consider the following sinusoidal waveform for the purpose of the measurement:

\[ v(t) = V_m \sin \omega t, \quad (2-20) \]

where \( \omega = 2\pi f \) is the angular frequency.

The frequency can be obtained by:

\[ f = \frac{1}{NT_s}. \quad (2-21) \]

where \( N \) is the number of samples per cycle, \( T_s \) is the sampling interval.

From the Fig. 2-5, one may find that the duration of one cycle of the waveform does not necessarily divide evenly over the sampling interval. In order to obtain a more accurate estimate of the frequency, a simple method is introduced as follow. Assume that \( y_{-1}, y_0, \ldots, y_k, y_{k+1} \), shown in the Fig. 2-5, are the values of the sinusoidal waveform at the sampling time \( t_{-1}, t_0, \ldots, t_k, t_{k+1} \) respectively. It is hereby assumed that the samples \( y_0, \ldots, y_k \) are within interval of one complete cycle of the waveform, and that the two samples \( y_{-1}, y_{k+1} \) are outside this cycle.
Fig. 2-5 Sine wave considered for measurement [20]

According to the notation in Fig. 2-5, the value of $N$ is:

$$N = k + \frac{t_{12}}{T_s} + \frac{t_{13}}{T_s},$$

(2-22)

where $k$ is the integer number of sampling periods in one cycle.

If the sample interval $T_s$ is short enough, one can assume that the sine function in the range of a zero crossing is replaced approximately by a straight line, and by using the trigonometric identity, the number of the stripes in one cycle can be approximately by:

$$N \approx k + k' = k + \frac{|y_0|}{|y_0 + |y_{-1}|} + \frac{|y_k|}{|y_k + |y_{k+1}|}.$$  

(2-23)

With equation (2-21) and (2-23), one can see that the value of the frequency depends on the values of the two samples surrounding the first zero-crossing ($y_0, y_1$), the values of the two samples surrounding the last zero-crossing ($y_k, y_{k+1}$), the number of samples between these zero crossings ($k$) and the sampling interval ($T_s$).

The Frequency Measurement block is shown in Fig. 2-6. Fig. 2-6(a) shows the main circuit to measure the frequency of PCC voltage, and Fig. 2-6(b), (c) and (d) shows the detail of the 3 subsystems of the main circuit. This block has one input PCC voltage $v_{PCC}$ and one output frequency of the PCC voltage $f_{PCC}$. The Hit Crossing block
compares the input signal to the hit crossing offset value, which is set as 0 here. If the input signal rises above the offset value, the block outputs 1, and Simulink takes a time step before and after the hit crossing time, which indicates the sampling interval, referred to the interval A or B in Fig. 2-5, when the default sample is larger than or equal to zero and the last sample is less than zero. The output of Hit Crossing activates the subsystem \( k^+ \), where the value \( \frac{Y_k}{Y_k + Y_{k+1}} \) and \( \frac{Y_{k+1}}{Y_k + Y_{k+1}} \) are calculated, as shown in Fig. 2-6(b). In this subsystem, the default value of the input signal is named \( Y_{k+1} \), and the previous sample’s value is named \( Y_k \) obtained by applying the Unit Delay block. After this interval, the subsystem Counter, shown in Fig. 2-6(d), starts to count the integer numbers of one cycle. The Enable and the Trigger blocks are placed to make this subsystem a enabled and triggered subsystem, and the Unit Delay used in this subsystem is used here to prevent the error of the ambiguous execution order due to use of subsystem in a loop during the simulation. The total integer number \( K_1 \) of one cycle and the value of \( \frac{Y_{k+1}}{Y_k + Y_{k+1}} \) will be held in the subsystem Hold, shown in Fig. 2-6(c), and only update when Hold block is triggered by the Hit Crossing block output. Notice that \( \frac{Y_0}{Y_0 + Y_{-1}} \) in Equation (2-6), is obtained by applying a Unit Delay block to get the previous value of \( \frac{Y_{k+1}}{Y_k + Y_{k+1}} \), when Hold block is triggered. Thus, the value \( N = K_1 + K_2 + K_3 \), the frequency of the last cycle can be obtained by Equation (2-21). The Switch is used here to set the initial value of the frequency as \( 60 \text{Hz} \).
(a) Frequency measurement block main circuit

(b) $k'$

(c) Hold

(d) Counter

Fig. 2-6 Frequency measurement block [20]
2.3.1.2 UVP/OVP & UFP/OFP Block [20]

The UVP/OVP & UFP/OFP block is shown in Fig.2-7. The RMS value of the voltage is calculated using the definition of the RMS. The block Discrete Variable Frequency Mean Value is an existing block in Matlab Simulink, which computes the mean value of the input signal over a cycle of a specified frequency. The RMS value and the frequency of the PCC voltage are sent to the Voltage Thresholds and the Frequency Thresholds. If the RMS value or the frequency exceeds the limits, the Voltage Thresholds or Frequency Thresholds blocks will output 1 to enable the Abnormal Voltage Cycles Counter or Abnormal Frequency Cycles Counter, where the values of the cycles are different according to the Table 1. Then, the counter will be triggered to count the cycles of the continuous abnormal voltage at the zero crossing point which represents the start of one cycle. Once the number of continuous abnormal PCC voltage cycle exceeds the limitation of IEEE Std.929-2000, shown in Table 1, the resettable J-K Flip Flop, will be triggered, a low fault signal will be generated to turn off the inverter switches thereby ceasing the inverter to energize the utility. If the continuous normal voltage and frequency have been maintained by the utility for a minimum of five minutes, at the time the Normal Voltage Cycles Counter block will reset the J-K Flip Flop in order to automatically reconnect the PV system to the utility.
2.3.1.3 Reference Current Block

Fig. 2-8 shows the Reference Current block. The inputs of this block are the PCC voltage $v_{PCC}$ and the frequency $f_{PCC}$. The outputs are the three reference currents: passive (UPF), AFD and SMS. The time $t$ is obtained by use of the block Discrete-Time Integrator to integrate the input signal 1 per cycle, and this time $t$ is reset when a new cycle starts.

For AFD, a fixed frequency drift $df$ is added to the measured frequency of previous cycle of PCC voltage to obtain the current frequency $f = f_{PCC} + df$. The Saturation block in this branch is to limit the input signal upper and lower saturation values. In this case, the lower limit is set to be 0, and the upper limit is set to be $2\pi$. If the value of $2\pi(f_{PCC} + df)t$ exceeds the upper limit, the Saturation block outputs the value $2\pi$ to create the zero current segment of the distorted AFD waveform, shown in Fig. 1-2. The distorted inverter output current of one cycle is

$$i_{AFD} = I_{PV_{\text{max}}} \sin \left[2\pi(f_{PCC} + df)t\right], \text{ when } 0 \leq t \leq T_i;$$

$$i_{AFD} = 0, \text{ when } T_i < t \leq T_{PCC}. \quad (2-24)$$

For SMS, an additional phase shift is added to the current cycle, and the value is

$$\theta_{SMS} = \frac{\pi}{2} \frac{f_{PCC} - f_e}{3}. \text{ The PV system inverter with SMS output current is}$$

$$i_{SMS} = I_{PV_{\text{max}}} \sin \left(2\pi f_{PCC} + \frac{2\pi}{360} \theta_{SMS}\right). \quad (2-25)$$

The reference current of a SMS IDM with a frequency measurement error is implemented utilizing the SMS model with an additional error constant $\Delta f_e$ added to or subtracted from the correct measurement frequency. The $\Delta f_e$ can be modified manually depending on different testing applications.
Fig. 2-8 Reference Current with IDMs Block.
2.3.2 Simulation Results of Passive IDM and SMS IDM Inverters System

The first case to be investigated is a system composed of passive (UPF) and active frequency drifting IDM inverters. As we learned from Section 2.2.1 for a passive and active IDM inverters system, one can obtain $Q_{f_{eq}}$ by combining the UPF inverter with the load, and analyze the system utilizing the NDZ from that single active IDM. Without loss of generality, a nonislanding SMS IDM inverter, whose NDZ is shown in Fig. 1-6, for a local load with $Q_f \leq 2.6$ ($\theta_m = 10^\circ, f_m - f_g = 3$Hz) is used in this study. Fig. 2-9 (a) shows the variation of the local system's frequency due to a single SMS IDM inverter supplying a 1 kW load with $Q_f = 2.58$ and $f_0 = 60.02$ Hz. As the grid breaker opens at $t = 0.1$ s, the frequency increases continuously until the UFP-OFP trips at 0.49 s, 6 line cycles after the system’s frequency goes over 60.5 Hz. It is worth mentioning that for a passive UPF inverter, the system’s voltage and frequency would remain unchanged due to the zero active and active power imbalance and islanding would occur. Fig. 2-9 (b) shows the variation of the local system’s frequency when the 1 kW, $Q_f = 2.58$ and $f_0 = 60.02$ Hz local load is supplied by two inverters rated at 500 VA: One with SMS IDM ($\theta_m = 10^\circ, f_m - f_g = 3$Hz) and the other with passive IDM. There one sees that the system frequency decreases very little remaining within the normal frequency range and leading to islanding. Fig.2-9 (c) shows the frequency variation of the previous system but with a local load with $Q_f = 1.19$. The frequency drifts outside the normal frequency range and islanding is detected. Thus, the analysis presented in Section 2.2.1 is validated.
Fig. 2-9 Responses of systems with different power shares of active and passive inverters supplying local loads with different values of $Q_f$.

2.3.3 Simulation Results of AFD IDM and SMS IDM Inverters System

The second scenario includes only active frequency drifting IDM inverters: One with SMS IDM ($\theta_m = 10^\circ, f_m - f_s = 3$Hz and the other with AFD IDM ($\delta_f = 1$ Hz). The load power and quality factor are set to 1 kW and 3, respectively. The power shares of the AFD and SMS as well as the load’s $f_0$ are varied so as to verify the validity of the NDZs defined in Section 2.2.2. The results shown in Fig. 2-10 refer to a load represented by point A in Fig.2-1. One sees that the system’s frequency remains within the normal operating range for all three combinations of SMS and AFD inverters. Thus islanding occurs as anticipated, since this point lies in the NDZs of the three systems. Fig. 2-11 and Fig. 2-12 show the responses of the three systems to loads represented by points B and C in Fig. 2-1. The results also validate the analysis carried out with the NDZs. The system’s
frequency remains in the normal frequency range for the cases where the loads are located in the NDZs of the inverter combinations.

It is worth mentioning that the use drift-down AFD IDM would not change the overall performance of the system significantly. In such a case there would be inductive loads \( f_0 > f_i \) for which islanding would occur. As explained before, the problem in this scenario is that one method always drifts the frequency in one direction (AFD) while the other (SMS) can drift it up or down depending on the load's \( f_0 \).

![Graph showing responses of systems with different power shares of SMS and AFD IDM inverters (\( K_{AFD_{pu}} \)) supplying a local load with \( Q=3 \) and \( f_i=59.8 \) Hz](image)

**Fig. 2-10** Responses of systems with different power shares of SMS and AFD IDM inverters (\( K_{AFD_{pu}} \)) supplying a local load with \( Q=3 \) and \( f_i=59.8 \) Hz
Fig. 2-11 Responses of systems with different power shares of SMS and AFD IDM
inverters ($K_{AFD_{pu}}$) supplying a local load with $Q_f=3$ and $f_0=59.6$ Hz

Fig. 2-12 Responses of systems with different power shares of SMS and AFD IDM
inverters ($K_{AFD_{pu}}$) supplying a local load with $Q_f=3$ and $f_0=59.2$ Hz
2.3.4 Simulation Results of Two SMS IDMs with Measurement Errors Inverters System

The final case to be investigated is of a system with two SMS IDM inverters. If they are identical they would always attempt to drift the frequency in the same direction and the NDZ of the system would be the same as of a single unit. However as discussed in Section 2.2.3, they can in principle offset each other in the presence of opposite frequency measurement errors ($\Delta f_e$). Fig. 2-13 shows the response of the system’s frequency for two 500 VA SMS IDM inverters with different values of $\Delta f_e$ supplying a 1 kW load with $f_0 = 60.02$ Hz and 2.58, the point located between the two curves $\Delta f_e = 0$ and 0.5 Hz shown in the Fig. 2-2. There one can see that the frequency drifts less for larger values of $\Delta f_e$ because of the partial offset of the two SMS IDMs. Besides for a load $Q_f = 2.58$, islanding would only occur for very large values of $\Delta f_e$ ($\sim 0.8$ Hz) which are not very likely to happen in practical systems. Thus, one can say that SMS IDM inverters are appropriate for multi-inverter systems. They can be designed to be nonislanding for net inductive as well as net capacitive loads with a $Q_f$ lower than a given value and they present low sensitivity for offsetting due to frequency measurement errors.
Fig. 2-13 Simulation results of two SMS IDM inverters supplying one local load

\[ P_{\text{load}} = 1\, \text{kW}, \quad Q_f = 2.58, \quad f_0 = 60.02 \, \text{Hz} \text{ with different } \Delta f_e \]

2.4 EXPERIMENT RESULT AND ANALYSIS

2.4.1 EXPERIMENT SETUP

A 1 kW topology system was implemented to validate the theoretical analysis. Fig. 2-14 depicts the configuration of the grid-connected PV multi-inverter system laboratory set-up [24,25].

Within the DC Part, a single phase diode bridge rectifier with a 600 \( \mu \text{F} \) dc bus capacitor is utilized to emulate the PV array DC output. Usually a dc bus voltage regulating loop is included in a system like this to control the active power the inverter injects into the ac grid under transient conditions. However, since the worst case for islanding detection is when there is no variation of the power generated by the PV source and consumed by the local load, this feedback loop is omitted in this study. The PV inverters inject constant active power into the grid. Thus the PV source is assumed to
supply constant power to the Inverter Part. The dc bus voltage is controlled at 190 V via the VARIAC before the diode rectifier.

In the Inverter Part, the main element are two single phase voltage source inverters (VSIs), which are independently operating in parallel and connected to the PCC through 5 mH inductors. The inverters are built using a single-phase full bridge configuration with bipolar power transistors (FUJI EVK71-050, 75A 500V) and anti-parallel diodes as main switches. A digital signal processor (DSP) development kit (DS-1103, dSPACE) was employed to implement the reference current with the IDMs, including Passive, AFD and SMS. The real AC output current of each inverter is measured by Hall effect current sensors (LA55-P, LEM) and sent to hysteresis current controller. The gating signals generated by the three-level hysteresis current controller board, where the real measured inverter output current is compared with the reference current, are amplified and isolated by a gate drive system to control the inverter. The inner boundary and outer boundary of the hysteresis current controller are set as 0.2 A and 0.4 A respectively. The details of the three-level hysteresis current controller are shown in Appendix.

At the ac side the RLC local load is connected with the ac grid breaker and a single-phase 5 kVA VARIAC (variable autotransformer) at PCC. An ac voltage sensor (LV100, LEM) is employed to monitor the PCC voltage. It provides synchronism for the reference current of the inverter and allows the measurement of the frequency and magnitude of the voltage at the PCC. The RMS value of the AC grid is set to 120 V by a single-phase 5 kVA variable autotransformer that represents the utility grid. It is worth mentioning that the dc bus voltage of the inverter has to be higher than the peak value of the ac grid voltage for full control of the power flow.
Fig. 2-14 Schematic of the grid-connected multi-inverter system laboratory set-up

2.4.2 LOCAL LOAD PARAMETERS SELECTION AND VALIDATION

In order to obtain the correct experiment results, the local load need be selected and validated correctly. The approach used in this section for calculating the load used in the experimental tests is based on the specification of the desired resonant frequency $f_0$ and quality factor $Q_r$. In the load parameter $Q_r$ vs. $f_0$ space, there may exist several different combinations of $L$ and $C$ for a same resonant frequency $f_0$. However the inductor available in the laboratory has a maximum value of 32 mH and intermediary taps with inductances equal to 8 mH and 16 mH. We initially choose a value for the inductor and then calculate the value of the capacitor $C$ that yields the desired resonant frequency since the values of capacitors available in the laboratory is more diverse than of inductors. At last the value of the resistor $R$ is chosen so as to achieve a desired quality factor.

After deciding the theoretical value of the local load, one also need confirm the actual value of the resonant frequency and quality factor of a given $RLC$ load by preliminary tests, because the actual value of most power components is not exactly the
nominal value. The manufactures usually provide a tolerance range, such as ± 10%, for each component. There are three steps in the preliminary test to adjust the load.

1) Without applying any active IDMs or UVP/OVP and UFP/OFP, open the utility disconnect switch, and let the system operate in islanding condition to achieve the load resonant frequency.

2) Once the system runs at steady state, the value of the resistance $R$ can be verified through the preset value of the PV inverter current $I_{PV}$ and the measurable islanding PCC voltage $V_{PCC}$.

3) The value of resonant frequency of the load can be read from ControlDesk, dSPACE's well-established experiment software, which provides all the functions to control, monitor and automate experiments.

Based on these three steps, the parameters of the local load, $f_0$ and $Q_f$, can be verified by the experimentally.

### 2.4.3 Passive and SMS IDM Inverters

There are two loads will be used in this test. Following the procedure described in Section 2.4.2, one can obtain the real value of the load parameters. One is $f_0 = 60.02$ Hz and $Q_f = 2.58$ (L=15.14 mH, C= 462 μF, R=14.86 Ω), the other is $f_0 = 60$ Hz and $Q_f = 1.19$ (L=32.9 mH, C= 214 μF, R=14.83 Ω). First test is for a single inverter equipped with SMS IDM ($\theta_m = 10^\circ, f_m - f_g = 3$Hz) supplies power to the 1 kW load ($f_0 = 60.02$ Hz, $Q_f = 2.58$). The results can be seen in Fig. 2-15, which shows the four most relevant waveforms of the system, the grid current $i_{grid}$, the PCC voltage $V_{PCC}$, PV inverter current $i_{SMS}$ and system frequency $f$. When the grid breaker opens at t=0.245 s, the frequency of the
voltage at PCC keeps decreasing and at t=0.522 s the island is detected and the inverter is shut down.

Fig. 2-16 shows that for the same load, two inverters rated at 500 VA supply the power simultaneously, one with SMS IDM ($\theta_m = 10^\circ$, $f_m - f_g = 3\text{Hz}$) and the other with passive IDM. There one sees that after grid opens at t=0.18 s, the system frequency decreases very little remaining within the normal frequency range and leading to islanding. Fig.2-17 shows the frequency variation of the previous system but with a local load with $Q_r = 1.19$. The frequency drifts outside the normal frequency range and islanding is detected at t=0.47 s after the utility is disconnected at t=0.18 s. Thus, the analysis presented in Section 2.2.1 is validated by the experiment.

Fig. 2-15 Experiment results for a system with single SMS IDM inverter with load $f_0 = 60.02 \text{ Hz}$, $Q_r = 2.58$
Fig. 2.16 Experiment results for a system with passive and SMS IDM inverters with
load $f_0 = 60.02$ Hz, $Q_r = 2.58$

Fig. 2.17 Experiment results for a system with passive and SMS IDM inverters with
load $f_0 = 60.02$ Hz, $Q_r = 1.19$
2.4.4 **ONE AFD AND ONE SMS IDM INVERTERS**

There are three different local loads involved in this test. The real values of the loads are listed in the Table 2 after verified by experiment.

**Table 2-Parameters of the local RLC loads considered for the experimental tests.**

<table>
<thead>
<tr>
<th>LOAD</th>
<th>L (mH)</th>
<th>C (µF)</th>
<th>R (Ω)</th>
<th>Q_{coul}</th>
<th>f_{local} (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15.1</td>
<td>467</td>
<td>17.38</td>
<td>3</td>
<td>59.8</td>
</tr>
<tr>
<td>B</td>
<td>15.1</td>
<td>472</td>
<td>17.38</td>
<td>3</td>
<td>59.6</td>
</tr>
<tr>
<td>C</td>
<td>15.1</td>
<td>477</td>
<td>17.38</td>
<td>3</td>
<td>59.2</td>
</tr>
</tbody>
</table>

The 1 kW load is supplied by two active frequency drifting IDM inverters, SMS IDM ($\theta_m = 10^\circ, f_m - f_s = 3$Hz) and AFD IDM ($\delta = 1$ Hz). The Fig. 2-18 (a), (b) and (c) are the experiment results referred to a load represented by Point A in Fig. 2-1 for different power shares between SMS and AFD IDM inverters respectively. The difference in power injected by each inverter can be seen from the magnitudes of inverter output currents, $i_{AFD}$ and $i_{SMS}$. One can find that the system frequency varies within the limitation $f_{\min}$ and $f_{\max}$, thus the islanding detection fails for all cases as discussed at Section 2.2.2. Fig. 2-19 and Fig. 2-20 show the experiment results of the three systems with the loads represented by points B and C in Fig. 2-1. Islanding occurs when the loads are located in the NDZs mentioned in Section 2.2.2. The theoretical analysis is validated by the experiment.
(a) $K_{AFD}_{pu}=0.2$ for the load at testing point A

(b) $K_{AFD}_{pu}=0.5$ for the load at testing point A
Fig. 2-18 Experiment results for a system with AFD and SMS IDM inverters with different power share $K_{AFD\_pu}$ with load $f_0 = 59.8$ Hz, $Q_f = 3$
(a) $K_{AFD\ pu} = 0.2$ for the load at testing point B

(b) $K_{AFD\ pu} = 0.5$ for the load at testing point B
Fig. 2-19 Experimental results for a system with AFD and SMS IDM inverters with different power share $K_{AFD \ pu}$ with load $f_0 = 59.6$ Hz, $Q_f = 3$

(c) $K_{AFD \ pu} = 0.8$ for the load at testing point B
(a) $K_{AFD_{pu}}=0.2$ for the load at testing point C

(b) $K_{AFD_{pu}}=0.5$ for the load at testing point C
(c) $K_{AFD,pu}=0.8$ for the load at testing point C

**Fig. 2-20** Experimental results for a system with AFD and SMS IDM inverters with different power share $K_{AFD,pu}$ with load $f_0=59.2$ Hz, $Q_f=3$

### 2.4.5 Two SMS IDM Inverters with Frequency Measurement Errors

At the last test, the load is selected as $f_0=60$ Hz, $Q_f=2.58$ as same as the first load mentioned at Section 2.4.3. In the Fig. 2-21 one can see that when $\Delta f_c=0.5$ Hz, the system still can detect the islanding and shut down the system at $t=1.01$s. The results shown in the Fig. 2-22 for a $\Delta f_c=0.8$ Hz are accord with the NDZ boundaries theoretic calculation and simulation result.
Fig. 2-21 Experimental results for the system with two SMS IDM inverters with $\Delta f_e=0.5$ Hz

Fig. 2-22 Experimental results for the system with two SMS IDM inverters with $\Delta f_e=0.8$ Hz
2.5 CONCLUSION

The performance assessment of active frequency drifting IDM s of systems with a single inverter is usually done by means of NDZs in a load parameter space such as the $Q_f$ vs. $f_0$ space. This chapter has shown that NDZs can also be defined for multi-inverter systems. For systems with multiple active IDM inverters, a new NDZ has to be defined as a function of the type, parameters and rated power of the inverters. On the other hand, the influence of the passive IDM inverters can be represented by an increase on the value of $Q_f$ of the equivalent local load as a function of the share of the load’s active power supplied by these inverters (participation factor).

Three cases were presented to demonstrate the use of the proposed approach. It was shown that for systems with one type of identical active IDM inverters plus passive IDM inverters, islanding becomes more likely to happen as the share of the load’s active power supplied by passive inverters increases. The local load’s maximum $Q_f$ for which the active IDM inverter can provide islanding protection decreases proportionally to the participation factor of the passive IDM inverters. In systems with two different types of active IDMs, the NDZ of the combined inverters has to be redrawn. It was shown that when AFD and SMS IDM inverters are employed together, they can offset each other, increasing the NDZ and resulting in islanding for local load conditions which would be nonislanding if only one type of active IDM inverters were used. The effect of frequency measuring errors on the islanding detection capabilities of a multi-inverter system was investigated for a system with SMS IDM inverters. In this case, the inverters could theoretically attempt to drift the frequency in opposite directions thus offsetting each
other and increasing the combined NDZ. It was shown that this is not an issue for practical frequency measuring errors in the order of 0.1 to 0.3 Hz.
CHAPTER 3
THE ISLANDING DETECTION ENHANCER

3.1 INTRODUCTION

The previous chapter has indicated that the assessment of the islanding detection capability of systems with multiple inverters equipped with active IDMs can become very complicated, mostly if they employ different techniques [6-9]. Non-islanding inverters certified for independent operation can interfere with each other averaging their effects out and failing to detect islanding.

A simple and effective way for preventing islanding in multi-inverter systems was proposed in [25]. A switched capacitor placed in parallel with a number of inverters with passive IDM is switched on by a signal supplied by a nearby substation when the grid is disconnected. As a result, the system frequency drifts down, tripping the inverters’ UFP. Means for identifying grid disconnection are not clearly discussed. In principle a transfer circuit breaker could be used for a segment of the distribution system supplied by a single feeder. However, if the segment can be supplied by multiple feeders capacitors could be erroneously switched on, even when the power system was not islanded.

This chapter discusses the use of a small dedicated static synchronous compensator (STATCOM) to achieve enough reactive power imbalance, thus islanding detection, in a system with multiple inverters with passive IDM. It is called islanding detection enhancer (IDE) and its reactive power is varied as a function of the system frequency available locally. The NDZ for a system with one IDE and one unity power factor (UPF) inverter with passive IDM is derived analytically in a load parameter space. An IDE design
approach that eliminates the NDZ for loads with a quality factor smaller than a specified value is presented. The effectiveness of the IDE is demonstrated with experimental results for a system with one inverter and by simulation for a multi-inverter system.

3.2 PROPOSED ISLANDING DETECTION ENHANCER

The device proposed in this chapter, the IDE, was conceived for systems with multiple UPF inverters equipped with basic passive IDM. This approach works by changing the shape of the load angle curve so that no stable operating points occur within the normal frequency range. This can be done with a controllable reactive power source / sink such as a STATCOM placed in parallel with the local load.

Its reactive power is controlled with positive feedback as in a PV inverter with a frequency drifting IDM. However, since the STACOM is to be controlled independently of the inverter(s), it is not possible to define a phase angle that represents the active and reactive power injected into the islanded system. Recall that this was the inverter phase angle ($\theta_{inv}$) in systems with an inverter with active IDM. Instead, one should define an modified equivalent load phase angle ($\theta_{load,eq}$), for the STATCOM + local load, and compare it to a fixed inverter angle, 0° for UPF passive IDM inverters, for assessing the islanding detection capabilities of this system.

Operated as a STATCOM, the IDE does not consume any real power. Thus the equivalent impedance of IDE is

$$Z_{IDE} = jX_{IDE} = j \frac{V_{PCC}}{I_{IDE}(f)} \tag{3-1}$$

where $I_{IDE}(f)$ is the RMS value of the IDE output current.

The equivalent load impedance is

62
\[ Z_{load\_eq} = \frac{Z_{load}Z_{IDE}}{Z_{load} + Z_{IDE}} = \frac{1}{\frac{1}{R} - j\left(\frac{1}{\omega L} - \omega C + \frac{1}{X_{IDE}}\right)} = \frac{1}{\frac{1}{R} - j\left(\frac{1}{\omega L} - \omega C + \frac{I_{IDE}(f)}{V_{PCC}}\right)}. \]  

(3-2)

Thus the equivalent load phase angle is

\[ \phi_{load\_eq} = \arctan\left[R\left(\frac{1}{\omega L} - \omega C - \frac{I_{IDE}(f)}{V_{PCC}}\right)\right] = \arctan\left[Q_f\left(\frac{f_0}{f} - \frac{f}{f_0}\right) + R\frac{I_{IDE}(f)}{V_{PCC}}\right]. \]  

(3-3)

The load current voltage (C-V) phase angle (\(\theta_{load\_eq}\)), that represents the angle by which the current leads the voltage, is given by

\[ \theta_{load\_eq} = -\phi_{load\_eq} = -\arctan\left[Q_f\left(\frac{f_0}{f} - \frac{f}{f_0}\right) + R\frac{I_{IDE}(f)}{V_{PCC}}\right]. \]  

(3-4)

From the equation (3-4), one can see that the equivalent load phase angle (\(\theta_{load\_eq}\)) varies as a function of system frequency and different control methods of \(I_{IDE}(f)\). Two schemes have been considered for controlling the IDE. These are the linear and the sinusoidal reactive current control methods. In both cases, the reactive current supplied or absorbed by the STATCOM varies with the frequency of the voltage at the bus where the IDE is located.

### 3.2.1 Linear Reactive Current Control

The RMS value of the inductive component of the IDE current is controlled according to

\[ I_{IDE}(f) = k_{IDE}(f - f_g) \]  

(3-5)

where \(k_{IDE}\) is a gain and \(f_g\) is the rated grid frequency.

Figure 3-1 shows the phase angle curves of a UPF inverter (0\(^\circ\)) and of the modified local load (\(\theta_{load\_eq}\)) (RLC load in parallel with the linear IDE for different factors \(k_{IDE}\) and load \(Q_f\), shown from the equation (3-4)). For a load with \(f_0 = 60\) Hz and \(Q_f = 2.5\) that draws 1 kW at 120 V, the modified load angle curve changes from that of the original local load.
when \( k_{IDE} = 0 \), to one with a lower slope, as \( k_{IDE} \) increases. For \( k_{IDE} \) larger than a certain value, the slope of the equivalent load angle curve becomes negative and smaller than that of the UPF inverter (0°) in the normal frequency range (59.3 Hz to 60.5 Hz). Thus this becomes an unstable operation region when the grid is disconnected and islanding is prevented. The maximum reactive current that the IDE has to deal with continuously occurs at the lower end of the normal frequency range. For a gain \( k_{IDE} = 0.75 \), this current is 0.525 A. Figure 3-1 also shows the extreme cases \( (k_{IDE} = 0 \text{ and } 0.75) \) for a load with \( Q_f = 7.5 \). Since the slope of this RLC load angle curve is larger than before, the reactive power absorbed by the inverter with the previous values of \( k_{IDE} \) is not enough to make the slope of the modified load angle curve become negative. Thus, for a local load with a higher \( Q_f \), a higher value of \( k_{IDE} \) is required to make the normal frequency range unstable for islanded operation.

![Graph showing phase angle curves for modified local load with IDE operating with various values of \( k_{IDE} \) and loads with \( Q_f = 2.5 \) and 7.5](image)

**Fig. 3-1** Phase angle curves for the modified local load with the IDE operating with various values of \( k_{IDE} \) and loads with \( Q_f = 2.5 \) and 7.5.

64
3.2.2 **Sinusoidal Reactive Current Control**

The RMS value of the inductive component of the IDE current is controlled according to:

\[
I_{IDE}(f) = I_M \sin \left( \frac{\pi}{2} \frac{f - f_g}{f_m - f_g} \right)
\]  

(3-6)

where \( f_m \) is the frequency at which the maximum IDE current, \( I_M \), occurs.

Figure 3-2 shows the phase angle curves of a UPF inverter (0°) and of the modified local load (\( \theta_{load,eq} \)) for different factors \( I_M \) and load \( Q_f \). Factor \( f_m - f_g \) is taken as 3 Hz as common in SMS IDM. For a load with \( f_0 = 60 \) Hz and \( Q_f = 2.5 \) that draws 1 kW at 120 V, the modified load angle curve changes from that of the original local load, when \( I_M = 0 \), to one with a lower slope, as \( I_M \) increases. It becomes negative at \( f = 60 \) Hz for \( I_M \) larger than 1.5 A, thus making the rated grid frequency an unstable operation point for a UPF inverter. For frequencies further above the rated grid frequency, the slope of the modified load angle curve becomes positive again and this curve intersects that of the UPF inverter. These operating points are stable ones but islanding is prevented when they occur outside the normal frequency range (59.3 Hz to 60.5 Hz) since the UFP or the OFP will shut the inverter down. Figure 3-2 also shows that the IDE needs to operate with larger values of \( I_M \) if islanding detection is required for a load with \( Q_f = 7.5 \).

This strategy yields a large \( \frac{d\theta_{load,eq}}{df} \) at rated grid frequency. At the same time, the current that the IDE has to provide at the lower end of the normal frequency is lower than that for a linear relation between \( I_{IDE} \) and \( f \) with the same \( \frac{d\theta_{load,eq}}{df} \). Thus, one manages to reduce the power ratings of the IDE.

It should be noted that by reducing the slope of the modified load angle curve, the IDE allows inverters equipped with frequency drifting type active IDM to detect islanding for
loads with $Q_f$ larger than they were designed for. The reduction of the NDZ in such a case is discussed in Section 3.5.

![Diagram showing phase angle curves for modified local load with IDE operating with various values of $I_M$ and loads with $Q_f = 2.5$ and $7.5$.]

**Fig. 3-2** Phase angle curves for the modified local load with the IDE operating with various values of $I_M$ and loads with $Q_f = 2.5$ and $7.5$.

### 3.3 Design of the IDE

The design of the IDE consists primarily in determining the required value of the parameter $I_M$, used in the control strategy, and the rated power of the IDE. The main requirement for the selection of $I_M$ is that all operating points within the normal frequency range of the inverters be made unstable. For inverter resident frequency drifting IDMs, this can be accomplished by making $\frac{d\theta_{ax}}{df} > \frac{d\theta_{load}}{df}$ at the frequencies where the inverter and load angle curves intersect in that range. Since the IDE is to be controlled independently of the inverter(s), one needs a criterion equivalent to $\frac{d\theta_{ax}}{df} > \frac{d\theta_{load,eq}}{df}$ that is a function of the reactive current of the IDE. Given that the reactive current of UPF
inverter(s) is always zero and assuming that the magnitude of the voltages across all
elements in the islanded system are the same, reference to the equation (3-4) the criterion
can be stated as

$$\frac{d}{df} \left( -\arctan \left( Q_f \left( \frac{f_0}{f} - \frac{f}{f_0} \right) + R I_{\text{IDE}}(f) \frac{f}{V_{\text{PCC}}} \right) \right) < 0$$  \hspace{1cm} (3-7)

at the frequencies where the phase angle of the equivalent load is equal to zero.

Calculating the derivatives of (3-5) and substituting in (3-7) one gets

$$I_M > \frac{V Q_f}{R} \left( \frac{1}{f_0} + \frac{f_0}{f^2} \right) \frac{2(f_m - f_s)}{\cos \left( \frac{\pi}{2} \frac{f - f_g}{f_m - f_g} \right)}$$  \hspace{1cm} (3-8)

The critical operating point is $f = f_g$, where the reactive currents of the IDE and of a
load with $f_0 = f_g$ are zero. For $f_m - f_g = 3$ Hz and $f_g = 60$ Hz, the parameter $I_M$ is given by

$$I_M > \frac{V Q_f}{5 \pi R}$$  \hspace{1cm} (3-9)

The rated reactive power of the IDE has to be chosen so that it can operate
continuously while the grid is connected and the system frequency and voltage are within
the normal range. As to (3-5), the highest magnitude of reactive current that the IDE has to
deal with the system occurs at the low end frequency (59.3 Hz). This is the furthest point
from the rated frequency where the control scheme is centered ($f_g = 60$ Hz). Considering
the trip point for the OVP of the inverter as 110% of rated voltage, the rated power of the
IDE is given by

$$S_{\text{IDE}} \geq 0.028 P Q_f$$  \hspace{1cm} (3-10)

Figure 3-3 shows the reactive power of the IDE normalized with respect to the rated
load active power, for loads with different resonant frequencies and with $Q_f = 2.5$ when $V_g$
= 1.1 pu. As per the IEEE Std. 929-2000, the inverter’s OVP should trip after 120 line
cycles at this voltage level. The IDE operates with maximum reactive power for a load with $f_0 = 60^\circ$ Hz (59.99 Hz). The normalized reactive power is approximately 0.064 pu and varies linearly with $Q_f$. The discontinuity point at 60 Hz is due to the fact that in principle, the IDE will attempt to drift the system frequency downwards when $f_0$ of the local load is smaller than $f_g$ and upwards otherwise. Since the amount of frequency drifting from the vicinity of the rated frequency required to trip the UFP is larger than to trip the OFP, the magnitude of $I_{IDE}$ is larger in the first case than in the second.

![Fig. 3-3](image)

**Fig. 3-3** Normalized reactive power of the IDE for loads with $Q_f = 2.5$

As a design example, consider a system where a UPF inverter and a local load with $P_{rated} = 1$ kW, $Q_f = 2.5$ and $f_0 = 60$ Hz are connected to a $V_g = 120$ V and $f_g = 60$ Hz grid. One calculates $I_M$ from (3-9) as 1.46 A, (using a grid voltage of 1.1 pu). The power rating of the IDE according to (3-10) is approximately 70 VAr or 0.07 pu.
3.4 Performance Assessment of the IDE

The performance enhancement provided by the IDE can be demonstrated by comparing the NDZ of two cases in a $Q_f \times f_0$ load parameter space [18]. The first case deals with an UPF inverter while the second deals with an inverter equipped with the SMS frequency drifting IDM.

3.4.1 Unity Power Factor Inverter and the IDE

For the IDE operating with sinusoidal current variation, there will always be a stable operating point. This occurs where the phase angle of the equivalent load is equal to zero. Thus by equating (3-4)

$$I_{IDE}(f) = \frac{V Q_f}{R} \left( \frac{f}{f_0} - \frac{f_0}{f} \right)$$

(3-11)

The boundaries of the NDZ of a system with a UPF inverter and the IDE in the $Q_f \times f_0$ space can be calculated the following way. For a given value of $Q_f$ one computes the values of $f_0$ for which $f$ equals the limits of the normal line frequency range ($f_{\text{min}}$ and then $f_{\text{max}}$) used in (3-11). The operation is repeated for other values of $Q_f$ until the entire range of interest is covered.

Since the NDZ of the proposed scheme is located in the vicinity of the normal line frequency range, one can linearize (3-11) around the limit values of the normal frequency range ($f_{\text{min}}$ and $f_{\text{max}}$). By applying the Taylor series to the right hand side term of (3-11)

$$y = \frac{f_{\text{min, max}}}{f_0} - \frac{f_0}{f_{\text{min, max}}}$$

(3-12)

and ignoring quadratic and higher order terms, one finds

$$\Delta y = \frac{dy}{df_{\text{min, max}}} \Delta f_{\text{min, max}}$$

(3-13)
where
\[
\frac{dy}{df_{\text{min, max}}}_{f_{\text{min, max}} = f_0} = \frac{1}{f_0} + \frac{f_0}{f_{\text{min, max}}}_{f_{\text{min, max}} = f_0} = \frac{2}{f_0}.
\] (3-14)

Thus, one can simplify the Equation (3-11) by
\[
\frac{2}{f_0} (f_{\text{min, max}} - f_0) = \frac{I_{\text{IDE}}(f_{\text{min, max}}) R}{V Q_f}.
\] (3-15)

Solving for \(f_0\), one can obtain the following expressions for the maximum and minimum values of the load resonant frequency at the threshold of the NDZ as
\[
f_{0,\text{max}} = \frac{2Q_f f_{\text{max}}}{2Q_f + I_{\text{IDE}}(f_{\text{max}}) R} \quad \text{when} \quad f_0 > f_g
\] (3-16)
\[
f_{0,\text{min}} = \frac{2Q_f f_{\text{min}}}{2Q_f + I_{\text{IDE}}(f_{\text{min}}) R} \quad \text{when} \quad f_0 < f_g.
\] (3-17)

### 3.4.2 INVERTER WITH SMS IDM AND THE IDE

The performance improvement of the proposed IDE to a system with an inverter equipped with an active IDM can also be analyzed and demonstrated in the \(Q_f \times f_0\) space. The inverter-resident active IDM considered in this case was the SMS.

Utilizing the phase criteria and assuming that there is no active power imbalance prior to the opening of the grid and that the phase angle of the inverter is small enough, one can obtain,
\[
Q_f \left( \frac{f_0}{f} - \frac{f}{f_0} \right) + R \frac{I_{\text{IDE}}(f)}{V_{pcc}} = \theta_{\text{SMS}}
\] (3-18)

From the equation (3-12) and (3-13), one can simplify the equation (3-18) by
\[
f_0 = \frac{2Q_f f}{2Q_f + \left[ R I_{\text{IDE}}(f) \frac{1}{V_{pcc}} + \theta_{\text{SMS}} \right]}.
\] (3-19)
Thus the following expressions can be used to calculate the NDZ of such a system in the $Q_f \times f_0$ space

\[
\begin{align*}
    f_{0,\text{max}} &= \frac{2Q_f f_{\text{max}}}{2Q_f + \left[ I_{\text{IDE}} \left( \frac{f_{\text{max}}}{V} \right) R + \theta_{\text{inv}} \left( f_{\text{max}} \right) \right]} \\
    f_{0,\text{min}} &= \frac{2Q_f f_{\text{min}}}{2Q_f + \left[ I_{\text{IDE}} \left( \frac{f_{\text{min}}}{V} \right) R + \theta_{\text{inv}} \left( f_{\text{min}} \right) \right]}
\end{align*}
\] (3-20) (3-21)

**Fig. 3-4** The NDZs of systems with one inverter and different types of IDM.

Figure 3-4 shows the resulting NDZs of four cases. In the first one, there is no IDE, only an inverter with SMS IDM designed for preventing islanding for a load with $Q_f \leq 2.5 \ (\theta_m = 10^\circ)$ [18]. The second case refers to a system with a UPF inverter and the IDE designed as in Section 3.4. One sees in Fig. 3-4 that the NDZs of these two cases are very similar and null for loads with $Q_f < 2.5$. The NDZ of a system with passive IDM only can
be obtained from (3-16) and (3-17) by making $I_{IDE} = 0$. As Fig. 3-4 shows, any loads with resonant frequency in the normal range of line frequency (59.3Hz to 60.5Hz) will result in islanding, irrespectively of the $Q_f$. Finally, one can see the NDZ of a system where the IDE and the inverter with the SMS IDM are used together. It shows that the IDE can increase the islanding detection capability of the SMS inverter to loads with $Q_f$ of up to 5.1.

3.5 SIMULATION AND EXPERIMENTAL RESULTS

The effectiveness of the IDE is verified by simulation and by a prototype for a system with one inverter and by simulation for a multi-inverter system.

3.5.1 THE IDE IN A SYSTEM WITH ONE INVERTER

The single inverter system described in Fig. 1-2 with an IDE connected in parallel with the RLC load was modeled with MATLAB Simulink as shown in Fig. 3-5. The IDE is represented by a current source that is controlled with a sinusoidal function of the voltage frequency. The reference current block is used to output the inverter’s reference current. The UFP/OFP & UVP/OVP block measures the frequency and the RMS value of the PCC voltage, and shuts down the inverter if either the frequency or the voltage at the PCC exceeds the IEEE Std. 929-2000 limits for 6 line cycles.
Fig. 3-5 MATLAB-Simulink model of the grid-connected PV system with IDE

Fig. 3-6 Comparison of the theoretical and simulated NDZs for a system with the IDE and a unity power factor inverter

The validation of the theoretical NDZ of the IDE plus UPF inverter system was carried out by simulating the system designed in Section 3.3 and that was also used to generate
Fig. 3-4. Figure 3-6 shows a comparison of the theoretical and simulated NDZs for the case when there is a perfect match between the active power consumed by the load and that supplied by the inverter. The values of $L$ and $C$ used in the simulation were calculated to yield values of $f_0$ and $Q_f$ close to those predicted by the theoretical analysis for failure of the IDM. As one sees from Fig.3-6, the agreement between the theoretical and the simulation results is very good.

A 1 kW system was implemented to validate the theoretical analysis. The UPF inverter and the IDE are implemented with power transistors and are connected to the PCC through 5 mH inductors. The RMS value of the ac grid is set to 120 V by a single-phase 5 kVA variable autotransformer that represents the utility grid. Both converters use three-level hysteresis current control with 0.4 and 0.2 A tolerance bands, respectively. The control functions of the inverter and IDE are implemented in a digital signal processor (DSP) development kit (DS-1103, dSPACE) that operates with a sampling frequency of 18 kHz. The local load used in the experiment presents $f_0 = 59.8$ Hz and $Q_f = 2.53$ and consumes around 1 kW at rated voltage and frequency. The inverter operates with unit power factor injecting 8.3 A into the PCC and the factor $I_M$ of the IDE is equal to 1.5 A. Figure 3-7 shows some relevant waveforms. The grid current is mostly composed of low order harmonics that flow through the load (capacitor) while the grid is connected. When the grid breaker opens at $t = 0.3$ s, the magnitude of the IDE current starts to increase, drifting the frequency downwards until the inverter and IDE shut down at $t = 0.47$ s. It has been seen in other tests that the system frequency drops to 59.8 Hz when the UVP/OVP & UFP/OFP block and IDE are disabled, which is the load's resonance frequency.
3.5.2 **The IDE in a System with Multiple UPF Inverters**

The system considered for verifying the performance of the IDE when multiple independently-controlled UPF inverters are used is shown in Fig. 3-8. It represents the low voltage side of a radial residential distribution system supplying five homes from a 50 kVA, 14.4/25kV – 120/240 V, 60 Hz pole-mounted transformer. The parameters of the transformer and conductors were obtained from [26]. The lengths of the secondary circuits and of the service drops were assumed to be 100 m and 50 m respectively. The inverter and load characteristics assumed for the houses are shown in Table 3. There one can see cases with shortage and others with surplus of active power. The IDE ($I_m = 20$ A, $f_{m-n} = 3$ Hz and $Q_{IDE} = 1050$ VAr) was designed for a 15 kW power system with $Q_f = 2.5$.

Figure 3-9 shows some relevant waveforms including the currents of the inverter ($i_{PV5}$), load ($i_{load5}$) and service drop ($i_s$) of house #5. The voltage at all buses is
approximately at rated value before the grid breaker opens. After the grid is disconnected, the voltage magnitude decreases to 92% of rated value at all buses because of the overall shortage of active power, and so do the load currents. Nonetheless, $i_{load5}$ is still larger than $i_{PV5}$ with additional active power coming from other inverters through $i_5$. The IDE starts to supply an increasing amount of reactive power drifting the system frequency downwards. The magnitude of $i_5$ increases slightly due to the reactive power supplied by the IDE, showing that the IDE affects all inverters in the island. The system frequency keeps decreasing until the OFPs of the inverters trip 6 line cycles after it falls below 59.3 Hz. The energy stored in the load capacitors and inductors is consumed by the load resistors and the load voltage decreases to zero. The measured value of system frequency is irrelevant after the inverters shut down.

A number of cases such as surplus of active power and equivalent loads with resonant frequencies above and below the rated line frequency were simulated. The results were always the same. The IDE was able to drift the frequency of the island outside the normal frequency range when the system power and the quality factor of the equivalent load were below those used in the design of the IDE.

**Table 3-Parameters of the inverters and loads of Fig. 3-8**

<table>
<thead>
<tr>
<th>House #</th>
<th>$P_{DYN} (kW)$</th>
<th>$P_{load} (kW)$</th>
<th>$f_0 (Hz)$</th>
<th>$Q_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>1.0</td>
<td>59.90</td>
<td>2.50</td>
</tr>
<tr>
<td>2</td>
<td>5.0</td>
<td>4.0</td>
<td>60.20</td>
<td>2.40</td>
</tr>
<tr>
<td>3</td>
<td>3.5</td>
<td>5.0</td>
<td>59.90</td>
<td>2.40</td>
</tr>
<tr>
<td>4</td>
<td>2.2</td>
<td>2.0</td>
<td>59.80</td>
<td>2.50</td>
</tr>
<tr>
<td>5</td>
<td>2.1</td>
<td>3.0</td>
<td>60.00</td>
<td>1.50</td>
</tr>
<tr>
<td>Total</td>
<td>13.8</td>
<td>15</td>
<td>59.95</td>
<td>2.21</td>
</tr>
</tbody>
</table>
Fig. 3-8 Equivalent circuit of the multi-inverter system

Fig. 3-9 Simulation results for a multi-UPF-inverter system with one IDE
3.6 LIMITATIONS OF THE PROPOSED TECHNIQUE

The approach of using a single active frequency drifting device to prevent islanding in a system with multiple passive IDM inverters presents some limitations. The IDE can only cause the shut down, by tripping of the UFP-OFP, of the passive IDM inverters that are connected to it during an islanding condition. For instance, if a breaker/fuse opens in the service line of house #5, that is downstream to the IDE, an island can result if the power imbalances there are small. By small it is meant less than those required for tripping the UVP-OVP and the UFP-OFP of the passive UPF inverter(s). Therefore, the IDE should be located in a position that includes as many possible islands with small power imbalances as possible. In fact, the ideal environment for the IDE would be a building with multiple ac photovoltaic modules [24], some on the roof top others in the façade, and a single load bus.

Another issue is the cost and ownership of the IDE in a system as that of Fig. 3-8. As mentioned in Section3.3, the required size of the IDE depends on the power rating of the active power injecting inverters. It is envisioned that the IDE be deployed in a modular fashion such as in UPSs for telecommunication applications. If more inverters are added to the system, the capacity of the IDE can be easily increased. The cost of the upgrading the IDE could be charged to the owners of the new inverters along with the utility inspection fee that takes place prior to the commissioning of the inverters.

Two of the main concerns of the utilities with inverter based distributed resources are islanding and power quality. There are a few measures that can reduce the harmonic distortion caused by inverters: Operation at high switching frequencies, output filters with large reactors and small capacitors and current references generated internally and not an image of the grid voltage [27]. Although the reference current for some active IDM, such
as AFD, is not a pure sinusoid, the chopping factor \( (d_f) \) is selected so that the total harmonic distortion of the injected current \( (\text{THD}_i) \) is very low [10]. The IDE should not present any extra burden on the power quality of the system for a number of reasons. It does not inject any active power into the grid so it will not cause voltage surges. It is implemented with a high frequency self-commutated converter using an internally generated sinusoidal reference current yielding a \( \text{THD}_i \) lower than that of AFD IDM inverters. Finally, due to its reduced power rating, as compared to the inverters (~ 7%), the magnitude of the current harmonics it injects into the grid is very small and so should be the voltage distortion it causes.

### 3.7 Conclusion

This chapter has discussed the use of a STATCOM as an islanding detection enhancer (IDE) for a system with multiple inverter based distributed resources. It shows that the IDE can be designed to drift outside the normal frequency range the frequency of an islanded system with UPF inverters and load with a quality factor lower than a specified value. Thus a non-islanding multi-inverter system can be implemented with inverters equipped with standard under and over frequency protections. By employing inverters with passive, technology neutral IDM, possible interactions between active IDMs that might interfere with each other, averaging their effects out and thus failing to detect islanding are prevented. The power rating of the IDE is very small, around 7% of the total active power generated in the island and for a load with quality factor smaller than 2.5, as considered in the test circuit of the IEEE Std. 929-2000. For compliance with IEEE Std. 1547-2003, where the quality factor of the test load is 1, the power rating of the IDE would be 2.8%.
Limitations of this technique and issues concerning ownership, upgrade of the IDE and its impact on the power quality were briefly discussed.
CHAPTER 4
CONCLUSION

4.1 SUMMARY

Grid interconnection of photovoltaic (PV) power generation systems has the advantage of effective utilization of generated power. However, the technical requirements from the utility power system side need to be satisfied to ensure the safety of the PV system installer and the reliability of the utility grid. The “Islanding Detection” topic has undergone extensive study and discussion.

This thesis firstly reviews some commonly used IDMs. In order to assess the islanding detection performances of different IDMs, the concept of NDZs was introduced subsequently. The performance of islanding detection of two frequency drifting methods AFD and SMS in a single-inverter system were assessed in detail, utilizing NDZ in the $Q_f \times f_0$ load parameter space based on the phase criteria.

It is believed that in systems with multiple inverters equipped with active IDMs, non-islanding inverters certified for independent operation can interfere with each other averaging their effects out and failing to detect islanding. With increased deployment or even expansion of PV systems, it is very likely that actual islands will contain multiple inverters. Therefore, the second Chapter extends the concept of phase criteria from single-inverter system to multi-inverter system. It shows that NDZ in $Q_f \times f_0$ load parameter space can still be defined for systems with multiple inverters equipped with different types of IDMs. Three possible scenarios that can be found in a system with multiple PV inverters were studied to show how the frequency drift methods average
their effects out and fail to detect islanding. It was shown that passive IDM inverters reduce the effectiveness of active IDMs’ as their share of the total load power increases. Besides, certain IDMs, such as AFD and SMS, tend to interfere with each other reducing their effectiveness. The shape of the NDZ of this type of system, as well as its size, varies with the power share of the inverters. Finally, it was shown that inverters with the same type of active IDMs present very little interference on each other, thus enabling the shape and size of the individual’s NDZ to be extended to the multi-inverter system. The simulation and experiment results were presented to validate the theoretical analysis.

The last issue discussed in this thesis was how to design an islanding detection enhancer for a system with multiple inverters to eliminate the potential islanding in the multi-inverter systems. The performance assessment of the IDE was given. A prototype of the proposed grid connected inverters system has been implemented with a digital signal processing (DSP) and two three-level hysteresis logic controllers. A number of tests that demonstrate the effectiveness of the proposed scheme have been presented.

4.2 SUGGESTIONS FOR FUTURE WORK

1) The assessment of the performance of active frequency drifting islanding detection methods (IDMs) is done by means of non-detection zones (NDZs) in a load parameter space. The phase criterion tells only if the frequency of an islanded system will drift out of the normal frequency range. However, it does not provide any information regarding how long the transient period lasts. The development of a means for calculating this time interval would be very important for the design of fast acting IDMs.

2) The procedure for investigating the interaction of active frequency drifting IDMs could be applied for other combinations of IDMs such as AFD – SFS and SFS – SMS.
3) The IDE can only drift the frequency of those inverters that are connected to it during an islanding condition. Therefore, it is important to develop a method for selecting the most appropriate location of the IDE in a system with multiple sources, loads and breakers would. Such a method should be based on known profiles of power generation and consumption in each node of the system.
REFERENCES


84


[27] Enslin, J.H.R.; Heskes, P.J.M.; “Harmonic interaction between a large number of
distributed power inverters and the distribution network”, Power Electronics, IEEE
Transactions on, Volume 19, Issue 6, Nov. 2004 Page(s):1586 - 1593

for a single phase voltage source inverter”, Power Electronics Specialists Conference,
vol.1

full-bridge inverters”, Industry Applications Society Annual Meeting, 1988.,
Conference Record of the 1988 IEEE, 2-7 Oct. 1988 Page(s):689 - 693 vol.1
APPENDIX

THREE-LEVEL HYSTERESIS CURRENT CONTROLLER

A1. INTRODUCTION

Hysteresis current control (HCC) is well-known for its excellent transient response, inherent peak current limiting, stability and robustness under varying load conditions. However, the average switching frequency of the classical two-level HCC is high since at any switching instant, four switches of the bridge switch simultaneously. It has been shown in [28, 29] that three-level HCC can be implemented with the use of zero states when the converter free-wheels the ac current. It is worth mentioning that the reduction in the switching frequency for the HCC due to the use of the zero state(s) is typically larger than that achieved by the unipolar PWM control, which also uses zero states, with respect to the bipolar one.

A2. CONTROL STRATEGY

Figure A-1 shows a full-bridge single-phase voltage source converter (VSC). The switching functions for the converter legs $S_A$ and $S_B$ are defined according to the state of the upper switch of each leg. If $U_A$ ($U_B$) is ON, $S_A$ ($S_B$) is 1, otherwise it is 0. The lower switches of each leg operate complementarily to the upper switches to prevent a dc bus short-circuit.

Unlike conventional two-level HCC where the converter switches between the positive state ($S_A S_B = 01$), when the ac current ($i$) is to increase, and the negative state ($S_A S_B = 10$), when the ac current is to decrease, three-level HCCs also use the zero state
($S_aS_B = 00$ or $11$). In such a state, the converter flywheels the AC side current whose gradient is determined by the ac source voltage ($v_o$) and inductor ($L$).

![Diagram of Single-phase Voltage Source Converter](image)

**Fig. A-1** Single-phase Voltage Source Converter

The change rate of the ac side current at each converter state is given by

$$\frac{di}{dt} = \begin{cases} \frac{v_o + v_c}{L} & \text{at the positive state} \\ \frac{v_o - v_c}{L} & \text{at the negative state} \\ \frac{v_o}{L} & \text{at a zero state.} \end{cases} \quad (A-1)$$

Recall that for this boost type converter the magnitude of the dc bus voltage must be larger than the peak value of the ac voltage source. As shown by trajectory $IJ$ in Fig.A-2, during a zero state, the trajectory of the ac side current will move towards the upper boundary ($i_{ref} + h$) when $v_o - L\frac{di_{ref}}{dt} > 0$, since the gradient of the ac side current is bigger than that of the reference current $i_{ref}$. It is worth noting that this statement applies even...
when \( v_o < 0 \) and \( L \frac{d i_{\text{ref}}}{dt} < 0 \) as illustrated by trajectory GH. Similarly, \( i \) will move towards the lower boundary \((i_{\text{ref}} - h)\) during a zero state when \( v_o - L \frac{d i_{\text{ref}}}{dt} < 0 \), as illustrated by trajectory BC. The three-level hysteresis control logic contemplates four possible crossings or transitions as shown in Fig.A-2.

- The ac side current increases and crosses the inner upper hysteresis boundary \((i_{\text{ref}} + h)\). If the converter is in the positive state \((S_A S_B = 01)\), the converter will be set to a zero state (as shown in points B and G). Otherwise, the converter will be set to a negative state \((S_A, S_B = 10)\), as shown in point J.

- The ac side current decreases and crosses the inner lower hysteresis boundary \((i_{\text{ref}} - h)\). If the converter is in the negative state \((S_A S_B = 10)\), the converter will be set to the zero state (as shown in point I). Otherwise, the converter will be set to the positive state \((S_A S_B = 01)\), as shown in points A and C.

- The ac side current increases and crosses the outer upper hysteresis boundary \((i_{\text{ref}} + ho)\). The converter will be set to the negative state \((S_A S_B = 10)\), as shown in point H.

- The ac side current decreases and crosses the outer lower hysteresis boundary \((i_{\text{ref}} - ho)\). The converter will be set to the positive state \((S_A S_B = 01)\).

It is worth noting that the converter has two zero states: \( S_A S_B = 00 \) and \( S_A S_B = 11 \), and they should occur alternately to prevent that one leg remains idle while the other leg switches at all transitions. In other words, the HCC control logic should output ‘00’ if the previous zero state was ‘11’, and vice-versa. In this way, the switching of the four switches is balanced.
\[ v_o - \frac{L d i_{ref}}{dt} < 0 \]

\[ v_o - \frac{L d i_{ref}}{dt} > 0 \]

Fig. A-2 Current trajectory of the proposed three-level HCC

A3. IMPLEMENTATION OF THE CONTROL LOGIC

Based on the proposed three-level hysteresis control logic, a sequential state machine has been used to represent this three-level controller, as shown in Fig.A-3. There are two inputs for the state machine: \( C_{in} \) and \( C_{out} \) which are the logical outputs of the inner and outer hysteresis comparators, respectively. The configuration of the comparators is as follows: \( C_{in} \) changes to “1” when the current error reaches the inner upper hysteresis boundary, and changes to “0” when it reaches the inner lower hysteresis boundary. \( C_{out} \) changes to “0” when the current error reaches the outer upper hysteresis boundary, and changes to “1” when it reaches the outer lower hysteresis boundary. The output \( U_A \) and \( U_B \) define the switching states of the converter legs. Moreover they also indicate the next switching state \( (S_A S_B) \) of the converter. In addition, another state variable \( S_Z \) is
introduced to denote the latest zero state of the system: \( S_Z = 1 \): when the latest zero state \( S_A S_B = 11 \) and \( S_Z = 0 \): when the latest zero state \( S_A S_B = 00 \)

![Diagram of HCC state transition](image)

**Fig. A-3** Proposed three-level HCC state transition diagram

There are six states in the proposed system. The logic expression of state variable \( S_Z \) and output \( U_A, U_B \) can be calculated from.

\[
\begin{align*}
S_Z^{n+1} &= C_{in} \oplus C_{out} \cdot S_Z^n + (C_{in} \oplus C_{out}) \cdot S_Z^n \\
U_A^{n+1} &= C_{in} \oplus C_{out} \cdot S_Z^n + (C_{in} \oplus C_{out}) \cdot C_{in} \\
U_B^{n+1} &= C_{in} \oplus C_{out} \cdot S_Z^n + (C_{in} \oplus C_{out}) \cdot C_{out}
\end{align*}
\] (A-2)

The state variable \( S_Z \) remains unchanged when the input signals \( C_{in} \) and \( C_{out} \) are the same; once the two inputs are different, the state \( S_Z \) inverts from the previous state to ensure that the two zero states appear alternately. A D-type flip-flop implements this function and balance the switching frequency of the two legs.
According to the state and output equations, a simple logic circuit composed of a D-type flip flop, two comparators along with a few logic gates can be used to implement the proposed logic as shown in Fig. A-4.

![Logic Circuit Diagram]

**Fig. A-4** Proposed implementation for a three-level HCC

### A4. SIMULATION RESULT

A unity power factor rectifier controlled with the three-level hysteresis current controller is tested by simulation using MATLAB/Simulink. The switches are modeled as ideal devices, so the dead time circuits are not used. Fig. A-5 shows the simulation results, including the current waveforms, gating signals generated for the switches $U_A$ and $U_B$ and the resulting operating states of the converter. Here one sees that the gating signals are balanced. This means that the switching balance circuit works as expected and opposite zero states appear alternately. There one can also see that the converter operates alternately with the zero state and negative state between points N and O and the ac side current follows the reference current. At point O, when the ac current reaches the lower inner boundary, the state of the inverter changes from 10 to 11. Between points O and P, the slope of the reference current becomes less negative and the actual current does not reach the upper inner boundary as before. The next crossing between ac current and one of the
inner boundaries is again with the lower boundary at point P. Since the outputs of the hysteresis comparators remain unchanged from the previous crossing, no commutations occur at point P and the ac current keeps decreasing. Then, it reaches the lower outer boundary, when the positive state is used to quickly increase the ac current. Similarly, inner boundary crossings without commutations also occur when the reference current is increasing (point M).

Fig. A-5 Switching patterns for the APF with the three-level HCC