THREE ESSAYS ON OLIGOPOLISTIC COMPETITION, PRODUCT DIFFERENTIATION AND INTERNATIONAL TRADE

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Abstract

Three Essays on Oligopolistic Competition, Product Differentiation and International Trade

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This Thesis presents three essays in the area of strategic trade theory and policy.

The first essay presents an analysis of trade and welfare between countries with asymmetric conditions. A two-period two-country address model of product differentiation is examined in which firms face an initial period of autarky. Trade takes place in the subsequent period and firms fully anticipate switches in trade regimes. Results suggest that historical (domestic) conditions matter a lot on the international market place. Firms that come from countries with a larger market tend to develop longer product lines, which puts that country in a dominant position in international competition. The model is also used to analyse gains / losses from trade in relation to country size.

The second essay investigates the differential effects of specific and ad-valorem tariffs on quality, price and welfare in an oligopolistic industry consisting of foreign and domestic firms. These effects are shown to depend on the location of the home and foreign

firms in the quality spectrum. Both tariffs are ranked and conditions for either tariff to be welfare superior are derived.

Finally, the third essay presents an analysis of trade policy with endogenous market structure. A "third market model" is specified. Using a simple framework in which industry structure is derived endogenously as the outcome of product line decisions by firms, we show that governments have an incentive to affect the equilibrium product composition by setting non-zero subsidy rates in order to maximize domestic welfare. Subsidies may be uniform or non-uniform across goods and the optimal policy exhibits strong discontinuities as domestic welfare maximization implies a switch of regimes.

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ESSAY 1

STRATEGIC PRODUCT CHOICE, TRADE AND WELFARE

1-1 INTRODUCTION

Recent models of monopolistic competition emphasize the result that trade benefits all partner countries. Krugman (1979,80) and Dixit and Norman (1980) develop Chamberlinian trade models of monopolistic competition where all products enter the utility function symmetrically. The integration of two identical economies entails an increase in the total number of products although this number is smaller than the sum of those in the two national markets before trade, an increase in their scale of production and a decrease in their equilibrium prices (rationalization of production effect). Gains from trade arise through greater exploitation of economies of scale and the increased product diversity that access to each other's market makes possible. Lancaster (1980) and Helpman (1981) derive similar results using Lancaster's characteristic approach to product differentiation. Schmitt (1990) shows that the market structure and welfare effects of a complete tariff reduction do not generalize to bilateral trade liberalization from all intermediate tariff rates. In particular there exists a tariff rate which maximizes welfare in both countries.[1]

Oligopoly models that deal with entry conditions and strategic interactions between firms tend to reach different conclusions with respect to trade liberalization issues.[2] In Shaked and Sutton (1984)'s model of vertical product differentiation, the equilibrium number of firms is a function of consumer heterogeneity but is independant of the size of the economy. The integration of two separate countries entails the exit of low-quality producers

[1] See Lancaster (1984) for a similar analysis.

[2] In Lancaster (1980), Helpman (1981) and Schmitt (1990), strategic interactions between firms are disregarded since firms can relocate costlessly in the product space.

but benefits consumers both in the short run (price effect with qualities given at the autarkic level) and in the long run (quality effect).[3] Overall effects are unclear however, as losses borne by producers may outweigh consumer gains. Motta (1992) consider trade between countries that differ in size. He shows that Shaked and Sutton (1984)'s analysis applies to "new firms" only and that the exit of firms may be detrimental to the welfare of the small country in the short run and to that of the big country in the long run. If a firm's costs are sunk when trade occurs, however, the countries gain from trade, both in the short run and in the long run.[4] Mercenier and Schmitt (1996) introduce entry barriers in an applied general equilibrium evaluation of "Europe 1992" (completion of the internal market in the European Community). Because incumbent firms can earn pure profits in equilibrium, economic integration is less likely to lead to rationalization of production and to the exit of firms. So models that incorporate the Chamberlinian assumption of long run zero profits tend to overestimate the welfare gains from trade liberalization.

Product choices can be used strategically as in Eaton and Kierskowski (1984). They consider a model of industrial structure with sequential entry where the pure profit equilibrium is endogenously determined. In particular, they show that when entry and product choice decisions are made prior to other economic decisions, the choice of product becomes an instrument for protecting a firm's profit through entry deterrence; so trade can

[3] Gabszewicz and al (1981) consider the case where the economies may differ with respect to the distribution of income so that the combined market may support a larger number of firms.

[4] Firms already operating before trade face less severe entry barriers on the combined market when the quality gap in autarky is not too large

lead to a Pareto-inferior outcome.[5] Their result, however, is based on an temporal asymmetry in the choices open to the firms in the post-trade situation. Motta, Thisse and Cabrales (1997) show how trade can give an advantage to a firm coming from a larger country in a quality race, when the firms' choices at the beginning of the trade game are given by autarkic conditions. The leader of the industry in the trade situation is the firm that provided the higher quality at autarky, i-e the firm which belongs to the larger (or richer) country.

This paper also analyses the role of historical (domestic) conditions in determining a country's international performance. In what follows, firms face an initial period of autarky as in Motta (1992) and Motta, et. al. (1997), but are fully able to foresee trade. This is not an unreasonable assumption to make. Many period negotiations are required before an agreement on trade is reached. For example the negotiations for NAFTA began in 1989, whereas the treaty was ratified in 1992. With (product specific) sunk costs that lock in the firm's choices, conditions in the home market feed through to the trading period and the welfare effects need to be qualified.

A two period, two country address model of product differentiation is examined, in which firms, one in each country, make entry and product choice decisions in closed economies (Period 1) anticipating that trade will occur at some future period (Period 2).[6]

[5]Schmitt (1996) looks at product choices in international trade and shows that depending on tariff rates and consumer densities firms may choose product imitation and forclose trade. Welfare effects are not investigated however.

[6]This is proposed as an alternative model specification in Eaton and Kierskowski (1984) but not formalized.

Product choices are irreversible, so firms face a trade-off between exploiting optimally their domestic markets and minimizing price competition on the combined market. Firms perceive this trade-off differently as a function of their relative market size, with the result that asymmetric product choices can emerge in equilibrium. With fixed costs that must be incurred in period 2, the firms' choices are constrained to product configurations which are sustainable in the trading period. Domestic market conditions again help explain any asymmetries with respect to the firms' equilibrium strategies.

It is shown that depending on the level of product specific sunk costs and consumer densities, the bigger country dominates the product spectrum, a case referred to as 'market dominance'. In the absence of fixed production costs that further constrain the firm's choices, the smaller country always gains from trade because of strong economies of scale. The larger country can lose from trade for market size differentials large enough. With fixed costs that must be incurred in period 2 however, the smaller country can also lose from trade. This is the case when it is dominated at the trading equilibrium and sunk costs are small. So depending on set-up costs, market dominance can have strong implications for the small country's welfare. Both countries can actually lose from trade. This is the case when the countries differ substantially in size and the larger country dominates the product spectrum. Symmetric equilibria where both countries gain from trade exist for market size differentials low enough. So, in general, domestic market conditions which constrain the firms' strategies determine the countries' relative benefits from a complete trade liberalization.

The issue of possible losses from trade is of particular interest when studying programmes of integration between countries that differ in size (e.g. between Canada and the United States). For example, a Quebec entrepreneur involved in the Coffin industry recently expressed his concern over integration with the U.S. He made the argument that Quebec's coffin industry is traditionally oriented towards producing a relatively small product line and that direct competition with U.S. firms offering a much wider line might threaten the industry's very existence. The argument readily extends to the broadcasting industry in Canada, where the availability of satellite technology may force Canadian broadcasters to redesign their programming.[7]

The paper proceeds as follows. The model is outlined in section 2. Section 3 and 4 discuss equilibrium product choices in international trade. The welfare analysis is presented in section 5 while section 6 offers some concluding remarks.

1-2 THE MODEL

The model is an extension of the spatial approach in the Hotelling's tradition to the case of two markets experiencing market integration. It is interpreted as a model of competition between differentiated products in a one-dimensional characteristic space.

There are two markets $i = 1,2$ identical in all respects, except for possible size differences, as represented by their consumer's densities D_i, $i = 1,2$. The continuum of

[7] As an example in a closed economy environment. Sealink company recently proposed Mini-Cruises in an effort to minimize new competition from the Euro-Tunnel.

possible characteristics is represented in each market i by a circle of unit circumference. There are two periods: an initial period when markets are isolated, and a subsequent period characterized by free trade conditions. There is no discounting. Each firm (in each market) supplies a number $n_i \leq 2$ of products, $i = 1, 2$.[8] There is thus a total of $n = \sum_{i=1}^{2} n_i$ of products, distributed on the circles with characteristics x_j, (j=1,...,n) with $0 \leq x_1 \leq ... \leq x_j \leq ... \leq x_n \leq 1$. Product choices are irreversible.

Consumer's preferences are uniformly distributed with density D_i over the circle in each market such that without loss of generality $D_2 = \alpha D_1$, where α is a measure of relative size of the markets, $\alpha \in [0.1]$. Country 1's consumer density is normalized to one without loss of generality so that the combined market density is $D_I = (1+\alpha)$.

An individual consumer's demand for good j in any market i is:

$$q_j = \begin{cases} 0 & if \quad v(r, x_j, p_j, x^c) = r - g(x^c; p, x) \leq 0 \\ 1 & if \quad v(r, x_j, p_j, x^c) = r - g(x^c; p, x) > 0 \end{cases} \tag{1}$$

where

$$g(x^c; p, x) = \min_{j=1,...,n} \{f(x^c, j; p, x)\}$$

[8] The number of firms in each market and the length of their product lines are limited by product specific set-up costs.

and

$$f(x^c, p, x) = p_j + c(x^c - x_j)^2$$

denotes the delivered price of product j offered to consumer x^c under the price characteristic vector (x, p) where $x=(x_1 \ldots x_n)$ and $p=(p_1 \ldots p_n)$ are the vectors of characteristics and prices respectively. That is the consumer buys the good which is at the closest psychic distance from his most preferred characteristic, if he buys at all. $v(\cdot)$ is the representative consumer's indirect utility function which depends on the reservation price r, the individual's most preferred characteristic $x^c \in [0,1]$, the mill price of product j, p_j ,and it's product characteristic x_j. The term $c(x^c - x_j)^2$, where $c > 0$ represents the "psychic cost" associated with consuming a product with characteristics other than the most preferred. This term is quadratic to ensure the existence of an equilibrium.[9] c is assumed to be identical in both countries and can thus be normalized to unity without loss of generality. The reservation price is assumed to be non-binding and sufficiently large so that consumers always buy one unit of the differentiated commodity and markets are covered.

Production takes place at constant marginal cost, set equal to zero without loss of generality. Production requires a fixed sunk cost K per product, which must be financed up-front. For multiproduct equilibrium to make sense, there are exit costs, assumed to be high enough. These costs which are product specific ensure that product choices carry a

[9] See d'Aspremont and al (1979) for a discussion about the existence of equilibrium in spatial models.

commitment value.[10]

In the initial period, firms face no competition in their respective markets. Since the consumer's reservation price is assumed to be non-binding each firm's initial-period revenue is ultimately a function of the vector of product characteristics for products j, with $j \in B_i$, where B_i is the set of products supplied by firm i, i=1,2.

Second period revenues depend on the total number of products, their respective characteristics and the consumer's densities. Suppose there are $n = \sum_{n=1}^{2} n_i$ products distributed on the circle with product characteristics denoted by x_j with $0 \le x_1 \le \ldots \le x_j \le \ldots \le x_n \le 1$. By convention, we denote $x_{n+1} = x_1 + 1$ and $p_{n+1} = p_1$ where p_j is the mill price of product j with $j = 1, \ldots, n$.

Given the characteristics and the prices of the n products, assuming $x_j \le x_{j+1}$, the consumer who is indifferent between product j and j+1, whoever owns them, has ideal variety z_j with:

$$z_j = \frac{p_{j+1} - p_j}{2(x_{j+1} - x_j)} + \frac{x_{j+1} + x_j}{2} \qquad j = 1,\ldots,n \quad (2)$$

[10]See Judd (1985) for a discussion on this subject. Exit costs like severance pay to employees can play the role of a barrier to exit and ensure the existence of multi-product equilibria in spatial models. Analternative explanation for exit costs lies in cross product learning effects associated with the supply of a product line (eg. Boeing). A firm can withdraw from a market all-together but is committed to a given product line.

By convention, we denote $z_0 = z_n - 1$. Given (x, p) the vector of chosen characteristics and prices, the market share of product j is given by:

$$S_j(x, p) = z_j - z_{j-1} \qquad j = 1,\dots,n \quad (3)$$

Assuming that firm i produces every product $j \in B_i$ with $\bigcup_{i=1}^{2} B_i = \{1,\dots,n\}$, firm i's second-period revenue is:

$$\Pi_i(x, p) = (1+\alpha) \sum_{j \in B_i} p_j S_j(x, p) \qquad j = 1,\dots,n \quad i=1,2 \quad (4)$$

An equilibrium is the solution to the two period game. In the first period, both firms simultaneously choose the number and characteristics of their products in closed economies, pay all necessary costs and receive initial-period revenue. Anticipated market integration occurs in period 2 and firms then compete simultaneously over price in the second period given their first-period choices. In choosing their product configuration, firms must take into account subsequent price competition and its impact on profits. We look for the subgame perfect equilibrium of the game.

1-3 PRODUCT CHOICE UNDER MARKET INTEGRATION

We begin by characterizing the equilibrium product choices for each possible product configuration. Attention is restricted to symmetric equilibria when studying a multiproduct firm's locations, i-e product locations such that firms charge equal equilibrium prices for their products. We consider first the case with no fixed production costs that constrain the firm's choices in period two. Such costs will be introduced in section 4 to further study strategic product choices in international trade. Since each firm can supply a maximum of two products there are four product configurations to consider.

Proposition 1 :

(i) When both firms offer one product each, the equilibrium product choices exhibit maximal inter-firm differentiation.

(ii) When one firm chooses two products while the rival firm chooses only one, the equilibrium product choices entail maximum intra-firm differentiation when firm 1 is the two product supplier and depends on α when it is firm 2. In either case the one product supplier always chooses to locate in the middle of the larger side of the market.

(iii) When both firms offer two products each, the equilibrium product choices exhibit interlaced locations in the product space for $\alpha \in [0.0693, 1]$; i-e both firms position their products at the end of two orthogonal diameters. When $\alpha \in [0, 0.0693]$, firm 2 collapses it's two products at a single point (in the middle of the large size of the market). Firm 1 maximizes intra-firm differentiation for all $\alpha \in [0,1]$.

Proof : see appendix I.

Proposition 1 describes the firm's equilibrium choices of product characteristics in all four possible product configurations. The intuition for the results in proposition 1 is as follows: in deciding on their products' optimal locations firms must take into account both their autarkic markets and the combined market since these locations are irreversible once chosen. In particular they must take into account the effect of their choices on second period equilibrium prices.

When both firms choose one product, the equilibrium product choice exhibits maximal product differentiation. Given the number of products, the equilibrium locations are optimal for both periods. When firms operate under autarky any location is optimal. This is so because the product space is represented by a circle and the farthest consumer is always at distance $x_j + 1/2$, provided it is profitable to cover the whole market. So each firm has an incentive to differentiate it's product as much as possible from it's rival's to soften price competition on the integrated market.[11]

In the three product case, the equilibrium symmetric locations depend on which firm is the two product supplier. Let β_1 represent the distance between the two product firm's varieties. Firm 1 chooses to maximize the distance between it's products. $\beta_1=1/2$ for all $\alpha \in [0,1]$. Firm 2's behaviour, when it is the two product supplier, is different however. It chooses maximal intra-firm differentiation, $\beta_1=1/2$, for $\alpha \in [0.9459, 1]$ and $\beta_1 \in (0,1/2)$ for $\alpha \in [0, 0.9459]$ with $\beta_1=0$ a corner solution for $\alpha \in [0, 0.0910]$. This behavior can be

[11] The result established by d'Aspremont et Al (1979) holds in this two period framework. Note however that the equilibrium would be different if segments rather than circles were used in the analysis.

readily explained. $\beta_1=1/2$ is the optimal choice of a two product monopolist, enabling the firm to extract the highest possible price on the autarkic market. $\beta_1=0$ is on the other hand the best response of a firm when it shares the combined market (see Martinez-Giralt and Neven (1988)). As already mentioned the firms' optimal product choices trade off first and second period revenue. As firm 2's domestic market size, α, decreases, the relative importance of its autarkic revenue in its total revenue decreases. This implies that the equilibrium distance between its products is positively related to α. In the limit, when $\alpha <$ 0.0910, firm 2 is solely concerned with its revenues on the integrated market and collapses it's two products at the same point, in the middle of the larger side of the market. Firm 1 which operates from the bigger country faces a different trade-off when it is the two product supplier. In particular the relative importance of its autarkic revenues is consistent with $\beta_1=1/2$ for all $\alpha \in [0,1]$. The best response of the one product firm whichever it is, is to locate in the middle of the large size of the market, implying that it is located equidistant from the rival's products. This is because, again, any product choice is equally good from a first period point of view so the firm is interested in maximizing market share and minimizing price competition.

Similar arguments apply when both firms offer two products, except that we have to consider two possible product configurations. Let β_1, β_4 represent the distance between firm 2 and firm 1's products respectively in a configuration with neighboring products. Given this product pattern, firm 1 again chooses to maximize intra-firm differentiation for all $\beta_1 \in (0,1/2)$ and $\alpha \in [0,1]$. Firm 2's behavior depends on α. It chooses maximal product spacing for $\alpha \in [0.9837, 1]$, $\beta_1 \in (0,1/2)$, for $\alpha \in [0.1163, 0.9837]$, with

β_1=0 (a corner solution) for $\alpha \in [0, 0.1163]$. Firm 2's optimal product spacing, β_1, is decreasing with the size of its domestic market.

Now, the only symmetric equilibrium with interlaced product locations implies that both firms position their products at the end of two orthogonal diameters. For $\alpha \in [0.0693, 1]$, firm 2 prefers the equilibrium with interlaced product locations to the neighboring equilibrium. The former product configuration provides firm 2 with a better trade-off between its per period revenues for those values of α. To establish that the interlaced product pattern is an equilibrium for $\alpha \in [0.0693, 1]$, it is sufficient to note that firm 1 has no incentive to switch to the neighboring product pattern given firm 2's behavior since it would forego all of its revenue on the combined market. When $\alpha \in [0, 0.0693]$, however, firm 2 is essentially concerned with its revenues on the integrated market and chooses to collapse it's two products at a single point in the middle of the large side of the market. Part (ii) of proposition 1 shows that firm 1 maximizes intra-firm product differentiation in this case.

Having discussed equilibrium characteristics in the product space for each of the four possible product configurations we now turn to the question of the equilibrium number of products. Each firm must incur a product specific sunk cost per product K and the firm's product lines, once chosen, are fixed.

Given the optimal product characteristics in each of the possible product configurations and the resulting second period equilibrium prices, the equilibrium number of products chosen by each firm is determined as a Nash equilibrium in the number of products and is a function of the set-up cost K and relative consumer density α. [12]

Table 1-1

The payoff matrix of the first period game, given Bertrand competition in the second period, $\alpha \in (0,1)$

		Firm 2: 1	Firm 2: 2
Firm 1	1	$r + \frac{\alpha - 1}{8} - K,\ \alpha r + \frac{1-\alpha}{8} - K$	$\Pi_1(1/2)$, $\Pi_2(2/1)$
	2	$r + \frac{13 - 49\alpha}{576} - 2K,\ \alpha r + \frac{25 - 119\alpha}{576} - K$	$\Pi_1(2/2)$, $\Pi_2(2/2)$

where:

$$\Pi_1(1\ 2) = \begin{cases} r + \frac{\alpha - 1}{8} - K & \alpha \in [0.\ 0.910] \\ r - \frac{1}{4} + \frac{1}{243(\alpha+1)^2}\left[10393\alpha^3 + 10362\alpha^2 + 3153\alpha + 268 - \left(829\alpha^2 + 614\alpha + 109\right)\sqrt{157\alpha^2 + 80\alpha + 4}\right] - K & \\ & \alpha \in [0.910.\ 0.9459] \\ r + \frac{\alpha - 1}{32} - K & \alpha \in [0.9459.\ 1] \end{cases} \quad (5)$$

[12] A more natural way to proceed given the set-up of the game is to define best response functions in (n_i, β), but the approach followed above is equivalent.

$$\Pi_2(2,1) = \begin{cases} \alpha r + \dfrac{1-\alpha}{8} - 2K & \alpha \in [0,\ 0.910] \\ \alpha r + \dfrac{1}{243(\alpha+1)^2}\left[-3926\alpha^3 - 2976\alpha^2 - 492\alpha + 16 + \left(314\alpha^2 + 160\alpha + 8\right)\sqrt{157\alpha^2 + 80\alpha + 4}\right] - 2K & \alpha \in [0.910,\ 0.9459] \\ \alpha r + \dfrac{49 + 13\alpha}{576} - 2K & \alpha \in [0.9459,\ 1] \end{cases} \tag{6}$$

$$\Pi_1(2,2) = \begin{cases} r + \dfrac{13 + 49\alpha}{576} - 2K & \alpha \in [0,\ 0.0693] \\ r + \dfrac{\alpha - 1}{32} - 2K & \alpha \in [0.0693,\ 1] \end{cases} \tag{7}$$

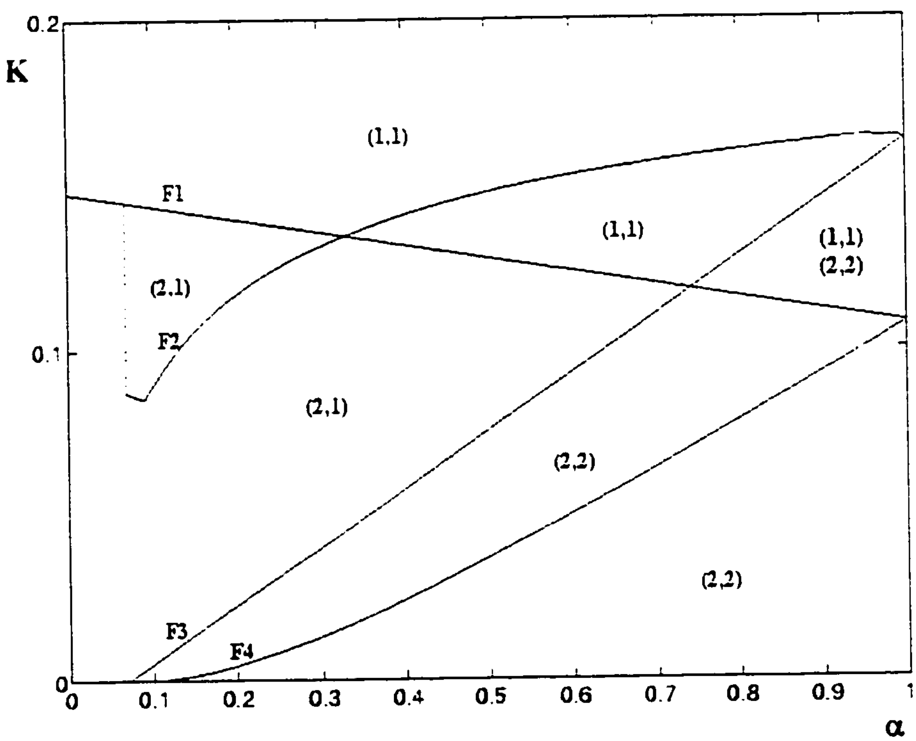
$$\Pi_2(2,2) = \begin{cases} \alpha r + \dfrac{25 - 119\alpha}{576} - 2K & \alpha \in [0,\ 0.0693] \\ \alpha r + \dfrac{1-\alpha}{32} - 2K & \alpha \in [0.0693,\ 1] \end{cases} \tag{8}$$

and where $\Pi_i(r/s)$ is the profit of firm i when it supplies r products and it's rival s products, defined on $\alpha \in [0,1]$. The payoffs are the reduced form profit functions obtained by substituting the equilibrium characteristics and prices into (A1), (A2), (A3), (A4), (A9) and (A11) of appendix 1.

It is worthwhile to note that the firm from the small country has a dominant strategy for $\alpha \in [0, 0.0693]$. That is, firm 2 chooses one product whatever firm 1 does. In this case the Nash equilibrium can be easily obtained by deletion of dominated strategies.

Note also that firm 2 will never choose two products when firm 1 supplies one product for $\alpha \in [0, 0.0910]$. The fact that firm 2 requires a lower market size to collapse its two products at the same point when firm 1 offers two products rather than one may seem counter-intuitive at first. This is because firm 2 has the additional strategy to choose an interlaced pattern in the four product configuration.

Figure 1-1:
The equilibrium number of products
as a function of the set-up cost per product *K*,
and the relative market size α.
F=0



The curves represent the following conditions and their equations are given in Appendix II:

F1: $\Pi_1(1/1) = \Pi_1(2/1)$ F3: $\Pi_2(1/2) = \Pi_2(2/2)$

F2: $\Pi_1(1/2) = \Pi_1(2/2)$ F4: $\Pi_2(1/1) = \Pi_2(2/1)$

Figure 1-1 shows the (K, α) space and is divided into the regions corresponding to the various equilibrium configurations. The loci in figure 1-1 partition the parameter space into zones that correspond to the firm's different patterns of optimal replies. For example the boundary labelled F4 gives the (K, α) for which firm 2 is indifferent between supplying one or two products given that firm 1 supplies one product. Above (below) this boundary, firm 2's best response to firm 1 supplying one product is to offer one (two) product(s). This figure allows one to determine which equilibria are possible given the values of the two parameters α and K.

There are three different Nash equilibria of the game. Depending on values of the set-up cost per product K, and relative country size α, each country supports one firm supplying one product, (1,1), each firm in each country supplies two products, (2,2), the firm from country 1 offers two products while that from the second country offers one product, (2,1), all emerge as equilibrium product configurations. The emergence of various equilibria as a function of K and α is quite intuitive. Because the number of products is assumed to be irreversible (high exit costs), the firms have to trade off the gains from having an optimal configuration of products from the first period point of view against severe price competition in the second period. Relative market size directly influences this trade-off as it ultimately determines the relative importance of autarky versus combined market profits; though in a different manner for each firm. The lower α, the relatively more firm 2 is concerned with its trade profits as depicted by the positive slopes of F3 and F4. On the other hand, the larger market justifies that firm 1 is essentially concerned with it's pre-trade profit for all $\alpha \in [0,1]$. In general, the gain from a larger product line is likely to outweigh the loss for K sufficiently low, while the opposite holds for K sufficiently high. For intermediate values of K, we get multiple equilibria for countries that tend to be similar.

Proposition 2: *There exist values of the parameters α and K for which the firm from the bigger country dominates the product spectrum in equilibrium. The minimum market size differential consistent with the 'no market dominance' result is $\alpha = 25.81\%$.*

Corollary: *The smaller country can never be dominant in international trade.*

For countries that differ substantially in size and for intermediate values of K, the model generates one asymmetric equilibrium which exhibits "market dominance" by the firm from the bigger market. So, only the firm which has the largest market can become the dominant firm in international trade. The corollary follows from the observation that there are no regions in figure 1-1 in which both firm 1's best reply to firm 2 supplying two products is to supply one product and firm 2's best reply to firm 1 supplying one product is to offer two products. This is because, for those values of K, increased second period price competition from a wider product line is much less costly to firm 1 that it is to firm 2. From figure 1-1 the minimum market size differential consistent with the no market dominance result is 25.81%. As countries become more similar, firms tend to face the same trade-offs resulting in symmetric equilibria.

1-4 PRODUCT CHOICE IN INTERNATIONAL TRADE : THE CASE WITH FIXED PODUCTION COSTS

An interesting case arises when firms must face additional fixed production costs post trade. In general fixed costs affect the firm's best response functions since some product configurations may not be sustainable in the trading equilibrium. In what follows we consider values of F that do not exceed $25(1+\alpha)/576$, where $25(1+\alpha)/576$ represents the revenue that a one product firm realizes on the combined market against a two product rival with maximum intra-firm product differentiation.[13]

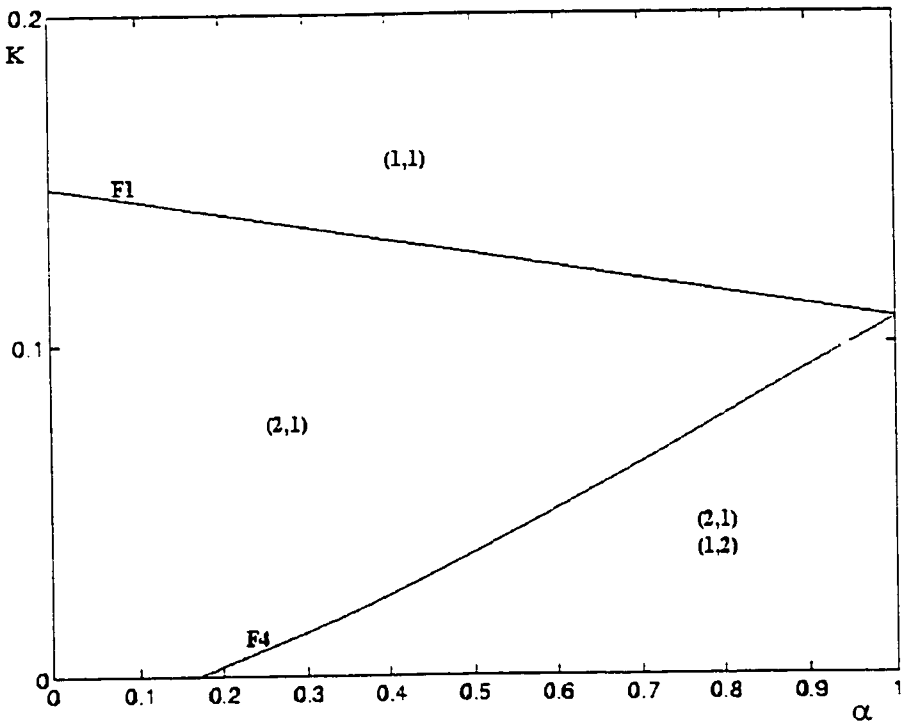
We start by looking at how the firm's best responses are affected by F. We have established earlier that when both firms offer two products each, the equilibrium symmetric product choices exhibit interlaced locations in the product space, with firms positionning their products at the end of two orthogonal diameters. Given this product configuration, a firm's revenue post trade is equal to $(1+\alpha)/32$. Therefore if $(1+\alpha)/32 < F < 25(1+\alpha)/576$, any one firm can achieve market dominance by an appropriate choice of product line. Here, the existence of fixed costs in the range $[25(1+\alpha)/576, (1+\alpha)/32]$ essentially constrains the firm's optimal replies to a two product rival. This does not imply, however, that the equilibrium product configuration necessarily entails market dominance by any one firm. As discussed previously, the firm's equilibrium decisions depend also on the set-up cost per product and the relative market density.

[13]No equilibrium exists for $F > 25(1+\alpha)/576$. For those values of F, a two-product firm induces a rival firm to exit before the competition period. Since both firms always have an incentive to monopolize the world market (for all α and K), the exit stage is characterized by multiple equilibria and there is no strategy profile that forms an equilibrium for the game considered here (at least in pure strategies).

We now turn to the firm's choices of the number of products to supply given F

Figure 1-2:
Equilibrium product choices as
a function of the set-up cost per product K
and the relative market size α,

$$F > \frac{1+\alpha}{32}$$



The curves represent the following conditions :

F1: $\Pi_1(1/1)=\Pi_1(2/1)$

F4: $\Pi_2(1/1)=\Pi_2(2/1)$

Figure 2-2 partitions the (K, α) space in regions where alternative product configurations arise as Nash equilibria of the game for the case with fixed production costs. The loci in figure 2 represent the firm's equilibrium decisions given F. The difference with figure 1 is that the boundaries defined by F2 and F3 which partition the parameter space according to the firm's pattern of optimal replies to a two product rival are no longer relevant because of F.

Depending on the parameters of the model, K and α, (1,1), the symmetric equilibrium with single product firms, (2,1) and (1,2) exhibiting market dominance by firm 1 and 2 respectively all arise as Nash equilibria of the game given F. As before firms equilibrium decisions reflect a trade off between exploiting their domestic markets optimally and minimizing price competition post trade, except that here firms are constrained to product configurations which are sustainable in the trading equilibrium. It is interesting to note that the larger country is able to exercice a dominant position in trade over a large region of the parameter space. This is because in the region above locus F4, firm 2 prefers to supply one product only given that firm 1 offers one product. So here again, relative domestic market conditions which constrain the firm's strategies puts the larger country in a dominant position in international competition. For lower values of K given α, both countries respectively achieve market dominance in the trading equilibrium.

It is worth mentioning at this point that depending on the values for the parameters K and α the existence of fixed costs $F \in [1+\alpha/32, 25(1+\alpha)/576]$ may or may not affect the equilibrium product configurations. Comparing figures 1-1 and 1-2 reveals that the product equilibrium is unaltered by F above locus F3 in figure 1-1. This is because firm 2's dominant strategy is to supply one product only in this region even in the absence of fixed costs. Above F1, the single product structure is a dominant strategy for both firms and F plays no role in that region also except that of eliminating multiple equilibria. This is not the case below min (F3, F1) and the larger country is able to extend substantially the region over which it exercises a dominant position in trade. Below F4 in figure 1-2, the existence of fixed costs produces multiple equilibria.

1-5 WELFARE IMPLICATIONS

In this section we explore the welfare implications of the opening of trade for both countries. We start by characterizing the autarkic equilibrium.

The two-period revenue that firm i can realize in autarky depends on the number of products, their characteristics and its market density. With one product at x_j, the farthest consumer is always at distance $x_j + 1/2$. The maximum price that firm i can set, provided it is profitable to cover the market, is given by $p_j = r - 1/4$. Firm i's revenue for two periods with one product is thus:

$$\Pi_i^A = 2\alpha_i\left(r - \frac{1}{4}\right), \qquad i = 1,2 \tag{9}$$

where $\alpha_i = \begin{cases} 1 & i = 1 \\ \alpha & i = 2 \end{cases}$

Let firm i own two products symmetrically located and $\beta \in (0,1/2)$ be the distance between them. In this case, the farthest consumer is at distance $(1-\beta)/2$ and the maximum price firm i can set is $p_j = r - (1-\beta)^2/4$. Firms i's revenue for two periods with two products is given by:

$$\Pi_i^{d} = 2\alpha_i\left(r - \frac{(1-\beta)^2}{4}\right), \qquad i = 1,2 \qquad (10)$$

Clearly β=1/2 maximizes the firm's revenue with two products. This product configuration minimizes the distance between the firm's products and the farthest consumer enabling it in turn to charge the highest possible price given that all consumers buy in equilibrium. Thus expression (10) becomes $\Pi_i^{d} = 2\alpha_i\,(r - 1/16)$ and firm i chooses to supply two products if and only if

$$K < \frac{3\alpha_i}{8} \qquad i = 1,2 \qquad (11)$$

Our objective is to compare the country's welfare under the alternative regimes of autarky and international trade. Following the literature on spatial differentiation we define social welfare loss functions.[14]

Because consumers in each country can possibly be served by two products at most in autarky, the two-period welfare loss function can take two different forms:

[14] The social loss functions that follows are defined given the firm's optimal product characteristics.

$$WL_i^A = 2\left(2\alpha_i \int_0^{1/2} x^2 dx\right) + K + F = \frac{192\alpha_i}{1152} + K + F \qquad i = 1,2 \qquad (12)$$

whenever one product is sold in market i. The social loss with two products given the firm's optimal behaviour is

$$WL_i^A = 2\left(4\alpha_i \int_0^{1/4} x^2 dx\right) + 2K + F = \frac{48\alpha_i}{24} + 2K + F \qquad i = 1,2 \qquad (13)$$

With perfectly inelastic consumer demands, prices do not affect the sum of consumer surplus and the firm's profits. Thus social welfare loss in autarky is represented by the consumer's aggregate disutility from consuming a product distant from their most preferred characteristic plus the resource product specific cost plus fixed production costs if any.

In the trade regime, there are additional elements influencing a country's welfare. First, the country's imports represents domestic payments to the foreign firm and must be considered as a loss. Second, the country's exports represent foreign payments to the home firm and constitute a social gain for the home country. Thus the social loss function in the trade regime is equal to the consumer's disutility plus net domestic payments plus the per product set-up cost plus fixed production costs if any. Depending on the equilibrium configuration, the social loss function in the trade regime can take four different forms:

In the trade equilibrium with single product firms,

$$
\begin{aligned}
WL_i &= 2\alpha_i \int_0^{1/2} x^2 dx + 4\alpha_i \int_0^{1/4} x^2 dx + \frac{\alpha_i}{8} - \frac{\alpha_k}{8} + K + F \\
&= \frac{120\alpha_i + 144(\alpha_i - \alpha_k)}{1152} + K + F \qquad i = 1,2; \quad k = 1,2; \quad i \neq k
\end{aligned}
\tag{14}
$$

The first and second terms represent first and second period consumer's disutility and the third and fourth terms are the country's net domestic payments. As before the resource cost per product and the fixed production costs enter the welfare loss function.

When the firm from the bigger country dominates the product spectrum,

$$
\begin{aligned}
WL_1 &= 4\int_0^{1/4} x^2 dx + 2\left(\int_0^{5/24} x^2 dx + \int_0^{1/24} x^2 dx + \int_0^{1/4} x^2 dx\right) + \frac{25 - 49\alpha}{576} + 2K + F \\
&= \frac{93 - 98\alpha}{1152} + 2K + F \\
WL_2 &= 2\alpha \int_0^{1/2} x^2 dx + 2\alpha\left(\int_0^{5/24} x^2 dx + \int_0^{1/24} x^2 dx + \int_0^{1/4} x^2 dx\right) + \frac{49 - 25\alpha}{576} + K + F \\
&= \frac{213\alpha - 50}{1152} + K + F
\end{aligned}
\tag{15}
$$

Where WL_1, WL_2 represent social loss for country one and two respectively.

When firm 2 dominates the product spectrum, the consumer's disutility and net domestic payments for both countries depend on the equilibrium product characteristics which are themselves a function of the relative size of country 2, α. In this case,

$$WL_1 = 2\int_0^{1/2} x^2 dx + 2\left(\int_0^{\frac{3-\beta_1}{12}} x^2 dx + \int_0^{\frac{3-5\beta_1}{12}} x^2 dx + \int_0^{\frac{\beta_1}{2}} x^2 dx \right) + \frac{(1-\beta_1)(3+\beta_1)^2 - \alpha(1-\beta_1)(3-\beta_1)^2}{72} + K + F$$

$$WL_2 = 2\alpha \int_0^{\frac{\beta_1}{2}} x^2 dx + 2\alpha \int_0^{\frac{1-\beta_1}{2}} x^2 dx + 2\left(\int_0^{\frac{3-\beta_1}{12}} x^2 dx + \int_0^{\frac{3-5\beta_1}{12}} x^2 dx + \int_0^{\frac{\beta_1}{2}} x^2 dx \right) + \frac{\alpha(1-\beta_1)(3-\beta_1)^2 - (1-\beta_1)(3+\beta_1)^2}{72} + 2K + F \qquad (16)$$

Substituting for β_1 (α) given by (A8) in Appendix I, welfare loss as a function of α gives:

$$W_1(\alpha) = \begin{cases} \dfrac{264 - 144\alpha}{1152} - K - F & \alpha \in [0, 0.0910] \\[2ex] \dfrac{-20786\alpha^4 - 39735\alpha^3 - 17979\alpha^2 - 1841\alpha - 105 - \left(414.5\alpha^3 - 688\alpha^2 - 195.5\alpha - 3\right)\sqrt{(46\alpha - 10)^2 - 36(11\alpha - 1)(\alpha - 1)}}{486(\alpha - 1)^2} - K - F & \alpha \in [0.3068, 0.9459] \\[2ex] \dfrac{216 - 50\alpha}{1152} - K - F & \alpha \in [0.9459, 1] \end{cases}$$

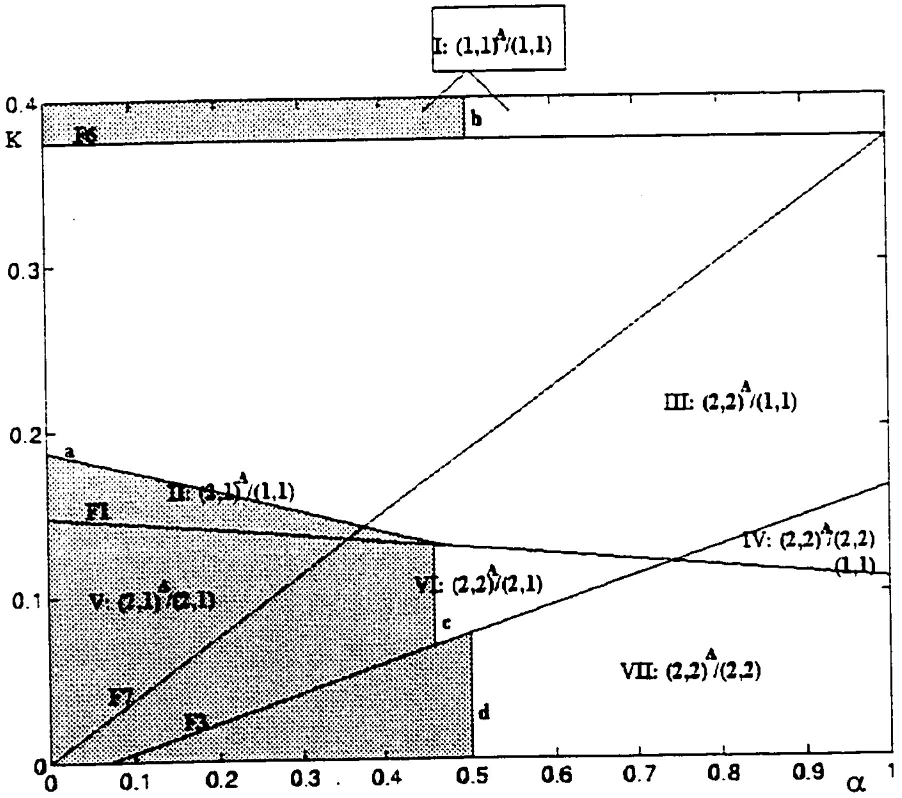
(17)

$$W_2(\alpha) = \begin{cases} \dfrac{264\alpha - 144}{1152} - 2K - F & \alpha \in [0, 0.0910] \\[2ex] \dfrac{8574\alpha^4 - 20425\alpha^3 - 10236\alpha^2 - 1269\alpha - 8 - \left(170.25\alpha^3 - 376.5\alpha^2 - 115.25\alpha - 1\right)\sqrt{(46\alpha - 10)^2 - 36(11\alpha - 1)(\alpha - 1)}}{486(\alpha - 1)^2} - 2K - F & \alpha \in [0.3068, 0.9459] \\[2ex] \dfrac{93\alpha - 98}{1152} - 2K - F & \alpha \in [0.9459, 1] \end{cases}$$

Finally with two firms carrying two products each:

$$WL_i = 4\alpha_i \int_0^{1/4} x^2 dx + 8\alpha_i \int_0^{1/8} x^2 dx + \frac{\alpha_i}{32} - \frac{\alpha_k}{32} + 2K + F$$
$$= \frac{30\alpha_i + 36(\alpha_i - \alpha_k)}{1152} + 2K + F \quad i = 1,2;\ k = 1,2;\quad i \neq k \tag{18}$$

Figure 1-3:
The effects of trade on the country's welfare as a function of the set-up cost per product *K* and the relative market size *α*;
***F*=0**



The curves represent the following conditions:

F1: $\Pi_1(1/1) = \Pi_1(2/1)$ a: $W_1(1,1) = W_1^A(2)$, $K=(216 - 144\alpha)/1152$

F3: $\Pi_2(1/2) = \Pi_2(2/2)$ b: $W_1(1,1) = W_1^A(1)$, $\alpha=1/2$

F6: $\Pi_1^{A}(1) = \Pi_1^{A}(2)$, $K=3/8$ c : $W_1(2,1) = W_1^{A}(2)$, $\alpha=45/98$

F7: $\Pi_2^{A}(1) = \Pi_2^{A}(2)$, $K=3\alpha/8$ d: $W_1(2,2) = W_1^{A}(2)$, $\alpha=1/2$

Figure 1-3 characterizes regions of the parameters (K, α) according to the equilibrium product configurations which arise in the alternative regimes for the case of no fixed costs. The first expression in brackets represents the firm's autarkic choices and the second expression, their choices given free trade in period 2. For example, in region III, both firms supply two products in autarky and one only in free trade. The figure shows also gains / losses from trade for both countries, using the relevant welfare loss functions (12) through (18) in each of the regions. The shaded areas represent regions where the bigger country loses from trade. The remaining areas are characterized by mutual gains from trade.

The social welfare loss function incorporates any adjustments in the country's industrial structure pre-trade. Combining net domestic payments and resource costs, domestic welfare can be interpreted as a measure of product diversity and economies of scale. In this model, trade can affect the degree of economies of scale in two ways. First, given the number of products, trade affects the scale of production and so the firm's average cost of production. Second, since trade can affect the firm's equilibrium product decisions, it can have further effects on the degree of economies of scale. This is the case in region III for example, where trade reduces both firm's product lines (from two products to one), increasing the scale of production of the remaining products. Fewer products also imply less product diversity however. There is thus a trade off between product diversity and scale of production and trade may affect this trade-off positively or negatively depending on the parameters K and α.

Characterizing now these effects for both countries, the smaller country always gains from trade whereas the larger country may gain or lose from the opening of trade.

In region I through IV and VII, firms share the global market equally in the equilibrium with international trade. Trade increases the scale of production for the smaller country and decreases it for the bigger country for all $\alpha \in [0,1]$. In regions I and VII, trade does not affect the firm's equilibrium product decisions. Both countries gain from increased product diversity in period 2, the "city lights" effect referred to in the trade literature. Country 2 always gains from trade for all $\alpha \in [0,1]$ whereas country 1 loses when $\alpha < 1/2$. This is not a surprising result. When switching from autarky to trade, both firms lose some business at home (trade diversion) and gain some business abroad (prices do not affect the country's aggregate surplus). Net domestic payments are negative for the smaller country and positive for the bigger country in a symmetric equilibrium with international trade for all $\alpha \in [0,1]$. So country 2's welfare unambiguously rises. In country 1, gains from product diversity outweigh losses from economies of scale when they are not too severe, which is the case for $\alpha > 1/2$.

In region III, both firms have reduced their product lines as a result of the opening of trade while this is the case for firm 1 only in region II. The saving of one lot of set-up costs with the consequent gain in economies of scale (accounting for second period scale effects) more than offset higher consumers' 'psychic costs' in period one.[15] As a result country 1 is able to secure gains from trade for all $\alpha \in [0,1]$ above locus a. This is related to excess product diversity associated here with monopoly power. The opposite holds below this locus in regions II and III where total consumer and producer costs are minimized in autarky. Given α, the smaller country enjoys even stonger gains from trade when the number of domestically produced goods decreases from two to one. So trade hurts the larger country when the market size differential is large enough, unless there is an adjustment in the country's industrial structure with the consequent gain in economies of scale.

In region V and VI, the larger country dominates in trade. Trade provides consumers in both countries with more product variety in period 2. In region VI, international competition has constrained firm 2's equilibrium product decision. From (13) and (15) it is worth noting that the second period sum of consumer and producer surplus is higher had country 2 remained in autarky, for $\alpha > 50/93$. This does not imply however that trade is welfare decreasing for the smaller country because of the impact it has on the country's industrial structure. Looking at overall welfare, the consumers' aggregate disutility increases compared to autarky and net domestic payments are positive for $\alpha > 25/49$. Country 2 secures gains from trade despite market dominance however, by saving on one lot of set-up costs. So as before, gains in economies of scale from a reduction in the product line outweighs the net increase in the consumers' 'psychic costs' (over the two periods). In region V the number of products supplied by firm 2 is the same in autarky and

[15] Firm one's average cost of production in the trade regime is $K / [(3+\alpha)/2]$ and is $2K/2$ in autarky. $K / [(3+\alpha)/2] < 2K/2$ implies $\alpha > -1$.

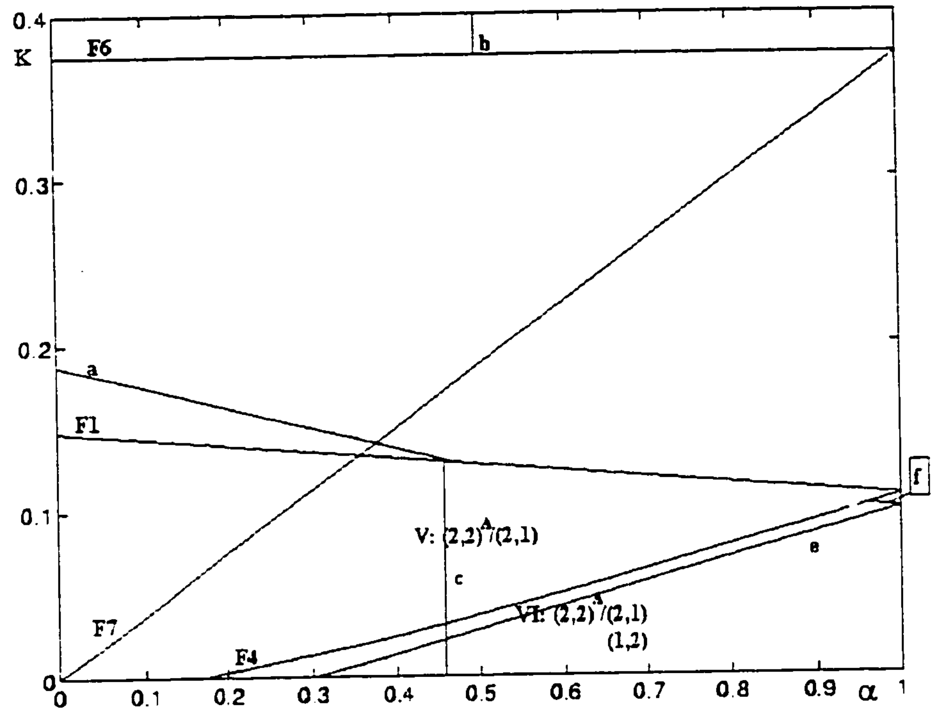
trade and it is the access to a larger market that drives welfare gains. An interesting result is that the bigger country's relative size disadvantage is minimized as a result of market dominance. This is the case for $\alpha \in [45/98, 1/2]$ in region VI, where it would have suffered losses from trade in a symmetric equilibrium with international trade and provided the number of domestically produced goods is constant across trade regimes.

Finally in region IV, both countries gain from trade. (1,1) Pareto dominates (2,2) because it generates stronger economies of scale for both countries and this effect outweighs the increase in consumers' aggregate disutility.

Figure 1-4 below, presents the analysis of welfare for the case with fixed production costs. The difference with the previous case is that (2,2) is no longer an equilibrium in the trade regime. Looking at country 2, net domestic payments are now positive for $\alpha \in [25/49, 1]$ in region V. Overall welfare may decline as a result of trade. This happens in region VI below locus e and provided (2,1) is the observed equilibrium. In this region K is low and autarky provides the smaller country with a better mix between product diversity and economies of scale.

Figure 1-4:
The effects of trade on the country's welfare as a function of the set-up cost per product K and the relative market size α;

$$F = \frac{1+\alpha}{32}$$



The curves represent the following conditions:

F1: $\Pi_1(1/1) = \Pi_1(2/1)$

a: $W_1(1.1) = W_1^{A}(2)$, $K = (216 - 144\alpha)/1152$

F4: $\Pi_2(1/1) = \Pi_2(2/1)$

b: $W_1(1.1) = W_1^{A}(1)$, $\alpha = 1/2$

F6: $\Pi_1^{A}(1) = \Pi_1^{A}(2)$, $K = 3/8$

c: $W_1(2.1) = W_1^{A}(2)$, $\alpha = 45/98$

F7: $\Pi_2^{A}(1) = \Pi_2^{A}(2)$, $K = 3\alpha/8$

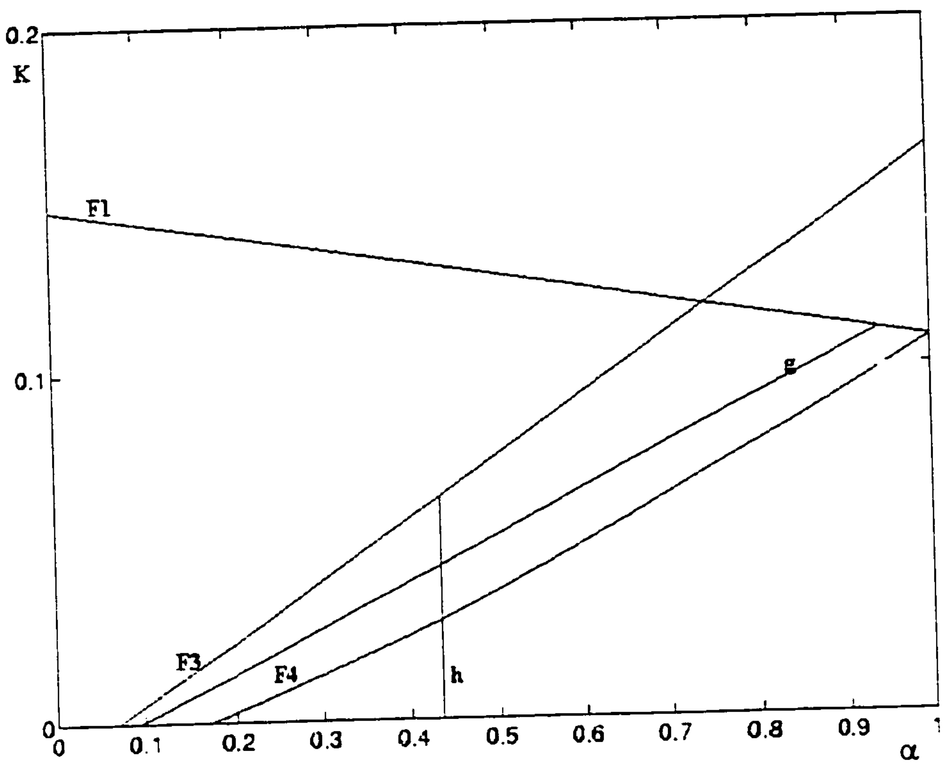
e: $W_2(2.1) = W_2^{A}(2)$, $K = 165\alpha - 50/1152$

f: $W_1(1.2) = W_1^{A}(2)$, $K = 165 - 50\alpha/1152$

When (1,2) is observed in region VI, country 1 experiences losses from trade for all $\alpha \in [0,1]$ except in the small area above locus *f*. Losses from trade also exist for this country when $\alpha < 45/98$ given the (2,1) product equilibrium. The remaining regions do not present any differences with respect to the case with F=0 except for region IV in figure 3 where multiple equilibria are eliminated. So our previous results hold there. Interestingly in the region below locus *e* and to the left of *c*, both countries lose from trade. In this region K is low and country 2 loses because of market dominance. Also country 2 is 'too small' and the bigger country loses from trade despite market dominance.

Finally. Figure 1-5 below compares the country's welfare when F=0 and F=(1+α)/32. With fixed production costs $F \in [(1+\alpha)/32, 25(1+\alpha)/576]$, the firm's choices are constrained to product configurations which are sustainable in the trading equilibrium. As a result, the region in which country 1 is dominant in trade is substantially extended to include the area between loci F3 and F4 in figure 1-5. Producers from the smaller country lose some profit (net of F) relative to the case with F=0. Producers in the larger country gain. This result substantiates somewhat some of the concerns about NAFTA that have been expressed by the business community in Canada. The smaller country's gains from trade are lower for $K < (147\alpha-14)/1152$ (locus *g*), because of market dominance. Surprisingly, there is a cut-off level of $\alpha < 27/62$ for which country 1 is worse off with respect to the case with F=0 despite market dominance. For these values of α, producer gains are smaller and do not compensate for the consumer' increased disutility because of firm 2's shorter product line.

Figure 1-5:
The effects of *F* on the country's gains from trade as a function of the set-up cost per product *K* and the relative market size *α*;



The curves represent the following conditions:

F1: $\Pi_1(1/1) = \Pi_1(2/1)$

F3: $\Pi_2(1/2) = \Pi_2(2/2)$

F4: $\Pi_2(1/1) = \Pi_2(2/1)$

g: $W_2(2,1) = W_2(2,2)$, $K=147\alpha-14/1157$

h: $W_1(2,1) = W_1(2,2)$, $\alpha=27/62$

Proposition 3: When two countries which differ by their (population) size are joined via trade:

(i) When $F=0$, the smaller country always gains from trade. Gains/losses from trade for the bigger country are a function of the relative market size α. Market dominance reduces the critical level of α for which the larger country loses from trade.

(ii) When $F=(1-\alpha)/32$, losses from trade exists for the smaller country if K $(165\alpha - 50)/1152$ and provided (2,1) is the observed equilibrium. The larger country loses from trade if α $45/98$. When (1,2) is the observed equilibrium, the larger country loses from trade except for K $(165\text{-}50\alpha)/1152$.

(iii) There exists a region of the parameter space where the larger country is dominant in trade. Gains from trade are minimized/maximized for the smaller/larger country respectively if α $27/62$ and $K < (147\alpha\text{-}14)/1152$.

So here, we provide examples of how trade can actually hurt the countries involved. Market asymmetries play a central role : they determine the firm's strategies and ultimately the distribution of the gains from trade between integrating countries.

1-6 CONCLUSION

In this paper we have developed a model of international trade with two production periods. Trade takes place in period two and firms face an initial period of autarky. The analysis carried out must be interpreted as a short to medium run one, i-e the period of time it takes for product specific costs to fully depreciate and during which firm's choices are locked in.

Because firms are forward looking, trade can affect the country's industrial structure before it is actually implemented. The country's relative market size matters. It determines the equilibrium product configurations, some of which exhibit market dominance. With no fixed production costs in the trade period, only the larger country can be dominant in international trade. The smaller country always gains because of strong economies of scale. The bigger country may lose from bilateral market access when the market size differential is large. With fixed production costs that further constrain the firms' choices, the smaller country can also lose from trade if it is dominated in the trading equilibrium. Also, there are regions of the parameters where the smaller / bigger country's gains are minimized / maximized because of market dominance. So market dominance which arises because of asymmetric domestic conditions can have strong implications for a small country's welfare. One implication of those results is that both countries have incentives to use trade policy to protect their respective markets.

An interesting extension would be to consider the potential for anti-competitive behaviour on the part of firms. In particular one could allow firms the possibility of longer product lines. With sunk costs and perfect foresight, firms can monopolize the integrated market by an appropriate choice of product line, a strategy of predatory product choice. Our analysis, which relates the firms' competitive strategies to the conditions on their home market, suggests that equilibria where only the firm from the bigger market finds monopolization profitable are possible. In such a case trade is likely to be welfare decreasing for the smaller country because of the strong producer loss. Further research along those lines is needed.

Appendix I: Proof of proposition 1

Proof of (i): let x_1 and x_2, with $x_1 \leq x_2$ be firm one and two's respective product characteristics. The revenues of the two firms can then be expressed as a function of distances β_1, β_2, with $\beta_2 = x_2 - x_1$ and $\beta_1 = 1 - \beta_2$. Having substituted for equilibrium prices,

$$\Pi_i(\beta_2) = \alpha_i\left(r - \frac{1}{4}\right) + \frac{(1+\alpha)(1-\beta_2)\beta_2}{2} \qquad \text{(A1)}$$

where $\beta_2 \in (0, 1/2)$, $\alpha_2 (=\alpha) \in [0,1]$, $\alpha_i = \begin{cases} 1 & i = 1 \\ \alpha & i = 2 \end{cases}$

The first part of the above expression represents autarkic (monopoly) revenues and is equal to the maximal price the firm can charge, provided it is profitable to cover the whole market (r sufficiently large). revenues are equal to a constant here since the farthest consumer is always at distance $x_j + 1/2$ for all x_j.

From (A1), $\frac{\partial \Pi_i}{\partial \beta_2}(\beta_2, \alpha) = 0$ implies $\beta_2 = 1/2$, $i=1,2$.

Hence, both firms maximize product differentiation for all $\alpha \in [0,1]$.

Proof of (ii): We start by showing that the one product supplier, whichever he is, always chooses to locate in the middle of the large side of the market.

Let the one product firm own x_1, and the two product firm, x_2 and x_3, with $0 \leq x_1 \leq x_2 \leq x_3 \leq 1$. Let $\beta_2 = x_2 - x_1$, $\beta_1 = x_3 - x_2$ and $\beta_3 = 1 - \beta_1 - \beta_2$, firm one's profit when it is the one product supplier is :

$$\Pi_1(\beta_1, \beta_2, \alpha) = r - \frac{1}{4} + \frac{(1+\alpha)\beta_2(1-\beta_1-\beta_2)(3-\beta_1)^2}{18(1-\beta_1)} \qquad \text{(A2)}$$

Where $\beta_1 \in (0,1/2)$, $\beta_2 \in (0,1/2)$,

and $\frac{\partial \Pi_1}{\partial \beta_2}(\beta_1, \beta_2, \alpha) = 0$ implies $\beta_2 = \frac{1-\beta_1}{2}$ for all $\beta_1 \in (0,1/2)$ and $\alpha \in [0,1]$.

Since $\beta_3 = 1 - \beta_1 - \beta_2$, $\beta_2 = (1-\beta_1)/2$ also implies $\beta_2 = \beta_3$, and firm one locates in the middle of the large size of the market.

The same result holds if firm two is the one product supplier since α is added to the first part of expression (A2) and changes nothing to the optimization problem.

We now turn to the two product firm problem. We restrict our attention to symmetric locations in the product space, i-e locations such that $\beta_2 = \beta_3$ (Martinez-Giralt and Neven (1988) show that symmetric locations arise in equilibrium).

Firm one's profit function when it is the two product supplier is:

$$\Pi_1(\beta_1, \alpha) = r - \frac{(1-\beta_1)^2}{4} + \frac{(1+\alpha)(1-\beta_1)(3+\beta_1)^2}{72} \qquad \text{(A3)}$$

for $\beta_1 \in (0,1/2)$,

where $r - \frac{(1-\beta_1)^2}{4}$ is in general the two product firm's autarkic revenues, noting that $\beta_1=(1-2\beta_2)/2$.

Note that $\frac{\partial \Pi_1}{\partial \beta_1}(\beta_1,\alpha)\Big|_{\beta_1=1/2} < 0$ implies $\alpha > 1.0571$, so that for $\alpha \in [0,1]$, $\frac{\partial \Pi_1}{\partial \beta_1}(\beta_1,\alpha)\Big|_{\beta_1=1/2} > 0$ which implies $\frac{\partial \Pi_1}{\partial \beta_1}(\beta_1,\alpha)\Big|_{\beta_1=1/2} > 0$ for all $\beta_1 \in (0,1/2)$ when $\alpha \in [0,1]$.

Thus $\beta_1=1/2 = \arg\max \Pi_1(\beta_1, \alpha)$. So firm 1 always maximizes intra-firm differentiation when it is the two product supplier.

If firm two is the two-product supplier, it's revenue function is:

$$\Pi_2(\beta_1,\alpha) = \alpha\left(r - \frac{(1-\beta_1)^2}{4}\right) + \frac{(1+\alpha)(1-\beta_1)(3+\beta_1)^2}{72} \qquad \text{(A4)}$$

for $\beta_1 \in (0, 1/2)$.

Note that $\frac{\partial \Pi_2}{\partial \beta_1}(\beta_1,\alpha)\Big|_{\beta_1=0} = 0$ for $\alpha = 0.0910$, and $\frac{\partial \Pi_2}{\partial \beta_1}(\beta_1,\alpha)\Big|_{\beta_1=1/2} = 0$ for $\alpha = 0.9459$, which implies that $\frac{\partial \Pi_2}{\partial \beta_1}(\beta_1,\alpha) < 0$ for all $\beta_1 \in (0,1/2)$ when $\alpha < 0.0910$ and $\frac{\partial \Pi_2}{\partial \beta_1}(\beta_1,\alpha) > 0$ for all $\beta_1 \in (0,1/2)$ when $\alpha > 0.9459$. So $\beta_1=0$ is a corner solution for $\alpha \in [0, 0.0910]$ and $\beta_1=1/2$ is a corner solution for $\alpha \in [0.9459, 1]$.

Now from (A4),

$$\beta_1(\alpha) = \frac{-(46\alpha+10)+\sqrt{(46\alpha+10)^2+36(11\alpha-1)(\alpha+1)}}{6(\alpha+1)} = \arg\max \Pi_2(\beta_1, \alpha) \tag{A5}$$

for all $\alpha \in [0.0910, 0.9459]$, which implies that $\beta_1 \in (0, 1/2)$ is an interior solution for that range of α.

Substituting (A5) into (A4), firm two's reduced form profit function is:

$$\Pi_2(\alpha) = \alpha r + \frac{1}{243(\alpha+1)^2}\left(-3926\alpha^3 - 2976\alpha^2 - 492\alpha + 16 + (314\alpha^2 + 160\alpha + 8)\sqrt{157\alpha^2 + 80\alpha + 4}\right) \tag{A6}$$

for $\alpha \in [0.0910, 0.9459]$

and by substituting (A5) into (A2), noting that $\beta_2 = (1-\beta_1)/2$, firm one's reduced form profit function is:

$$\Pi_1(\alpha) = r - \frac{1}{4} + \frac{1}{243(\alpha+1)^2}\left(10393\alpha^3 + 10362\alpha^2 + 3153\alpha + 268 - (829\alpha^2 + 614\alpha + 109)\sqrt{157\alpha^2 + 80\alpha + 4}\right) \tag{A7}$$

for $\alpha \in [0.0910, 0.9459]$

Firm two's behaviour when it is the two product supplier is summarized as:

$$\beta_1(\alpha)=\begin{cases} 0 & \alpha\in[0,0.0910] \\ (0,1/2) & \alpha\in[0.0910,0.9459] \\ 1/2 & \alpha\in[0.9459,1] \end{cases} \tag{A8}$$

So firm two chooses to collapse it's two outlets at the same point for $\alpha < 0.0910$, maximizes intra-firm differentiation for $\alpha > 0.9459$ and chooses interior solutions for $\alpha \in [0.0910, 0.9459]$

Proof of (iii): Let firm one own x_1 and x_4 and firm two x_2 and x_3, with $0 \le x_1 \le x_2 \le x_3 \le x_4 \le 1$ and $\beta_4 = x_4 - x_1$, $\beta_1 = x_3 - x_2$, $\beta_2 = x_2 - x_1$, and $\beta_3 = 1 - \beta_1 - \beta_2 - \beta_4$. We start by considering symmetric neighbouring locations i-e, locations such that $\beta_2 = \beta_3$.

The revenue of firm one may be expressed as:

$$\Pi_1(\beta_1,\beta_4,\alpha)=r-\frac{(1-\beta_4)^2}{4}+\frac{\begin{aligned}(1+\alpha)[&144-528\beta_1+736\beta_1^2-480\beta_1^3+144\beta_1^4-16\beta_1^5\\&+\beta_4\left(-336+1048\beta_1-1248\beta_1^2+720\beta_1^3-208\beta_1^4+24\beta_1^5\right)\\&+\beta_4^2\left(160-384\beta_1+369\beta_1^2-199\beta_1^3+63\beta_1^4-9\beta_1^5\right)\\&+\beta_4^3\left(96-240\beta_1+197\beta_1^2-66\beta_1^3+9\beta_1^4\right)\\&+\beta_4^4\left(-48+80\beta_1-45\beta_1^2+9\beta_1^3\right)\\&+\beta_4^5\left(-16+24\beta_1-9\beta_1^2\right)]\end{aligned}}{72\left[4(1-\beta_1)+\beta_4(-4+3\beta_1)\right]^2} \tag{A9}$$

for $\beta_1 \in (0,1/2)$, $\beta_4 \in (0,1/2)$, $\alpha \in [0,1]$

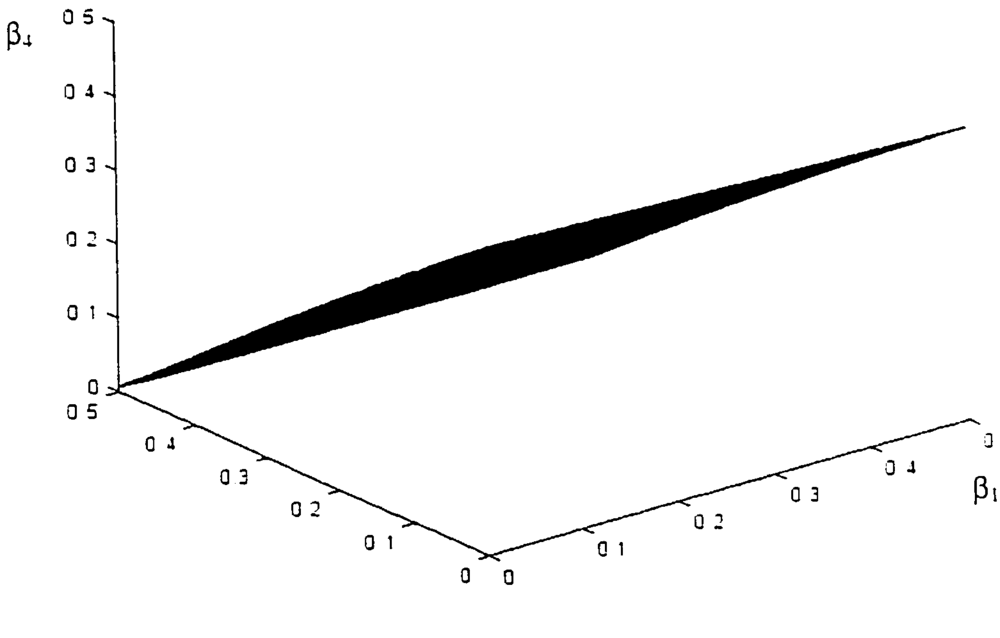
and,

$$\frac{\partial \Pi_1}{\partial \beta_4}\left(\beta_1, \beta_4, \alpha\right) = \frac{\left(1-\beta_4\right)}{2} + \frac{\begin{aligned}(1+\alpha)[&-192+448\beta_1-128\beta_1^2-384\beta_1^3+320\beta_1^4-64\beta_1^5 \\ &+\beta_4\left(-64+848\beta_1-2112\beta_1^2+2080\beta_1^3-896\beta_1^4+144\beta_1^5\right) \\ &+\beta_4^2\left(1152-4032\beta_1+5244\beta_1^2-3156\beta_1^3+900\beta_1^4-108\beta_1^5\right) \\ &+\beta_4^3\left(-1152+3296\beta_1-3508\beta_1^2+1719\beta_1^3-378\beta_1^4+27\beta_1^5\right) \\ &+\beta_4^4\left(64-128\beta_1+180\beta_1^2-162\beta_1^3+54\beta_1^4\right) \\ &+\beta_4^5\left(192-432\beta_1+324\beta_1^2-81\beta_1^3\right)]\end{aligned}}{72\left[4\left(1-\beta_1\right)+\beta_4\left(-4+3\beta_1\right)\right]^3} \tag{A10}$$

for all $\beta_1 \in (0,1/2)$, $\beta_4 \in (0,1/2)$, $\alpha \in [0,1]$

Figure A1

Graph of $\frac{\partial \Pi_1}{\partial \beta_4}(\beta_1, \beta_4, \alpha)$, $\beta_1 \in (0.1/2)$, $\beta_4 \in (0,1/2)$, $\alpha = 1$.



The first order derivative in (A10) summarizes firm one's behaviour. We used numerical methods to show that $\frac{\partial \Pi_1}{\partial \beta_4}(\beta_1, \beta_4, \alpha)\big|_{\alpha=0} > 0$ for all $\beta_1 \in (0,1/2)$, $\beta_4 \in (0,1/2)$, so that $\beta_4=1/2$ is globally a solution to (A10) for $\alpha=1$. See graph above which depicts (A10) for $\alpha=1$.

Now, if the derivative in (A10) is always positive for all $\beta_1 \in (0,1/2)$, $\beta_4 \in (0,1/2)$ and $\alpha=1$ it must be the case that it is also positive for all $\beta_1 \in (0,1/2)$, $\beta_4 \in (0,1/2)$ and $\alpha \in [0,1]$ since $\alpha > 0$. Intuitively, $\alpha=1$ corresponds to the biggest possible combined market. In making it's optimal product choice decision, firm one must consider both it's autarkic as well as the combined market. If it chooses to maximize intra-firm differentiation when the combined market is largest, it must do the same thing as $\alpha < 1$, since it's autarkic revenues become even more relatively important in it's total revenues and $\beta_4 = 1/2$ is an optimal autarkic choice.

We now turn to firm two's product choice problem. Firm two's revenue function is:

$$\Pi_2(\beta_1, \beta_4, \alpha) = \alpha\left(r - \frac{(1-\beta_1)^2}{4}\right) + \frac{\begin{array}{l}(1+\alpha)[144 - 528\beta_1 + 736\beta_1^2 - 480\beta_1^3 + 144\beta_1^4 - 16\beta_1^5 \\ + \beta_4\left(-336 + 1048\beta_1 - 1248\beta_1^2 + 720\beta_1^3 - 208\beta_1^4 + 24\beta_1^5\right) \\ + \beta_4^2\left(160 - 384\beta_1 + 369\beta_1^2 - 199\beta_1^3 + 63\beta_1^4 - 9\beta_1^5\right) \\ + \beta_4^3\left(96 - 240\beta_1 + 197\beta_1^2 - 66\beta_1^3 + 9\beta_1^4\right) \\ + \beta_4^4\left(-48 + 80\beta_1 - 45\beta_1^2 + 9\beta_1^3\right) \\ + \beta_4^5\left(-16 + 24\beta_1 - 9\beta_1^2\right)]\end{array}}{72[4(1-\beta_1) + \beta_4(-4 + 3\beta_1)]^2} \quad \text{(A11)}$$

for all $\beta_1 \in (0,1/2)$, $\beta_4 \in (0,1/2)$, $\alpha \in [0,1]$.

Since firm one has a dominant strategy (β_4=1/2), firm two's profit function can be simplified to:

$$\Pi 2(\beta_1,\alpha)=\alpha\left(r-\frac{(1-\beta_1)^2}{4}\right)+\frac{(1+\alpha)(12.5-46.25\beta_1+39.03125\beta_1^2+17.5625\beta_1^3-18.125\beta_1^4-6.25\beta_1^5)}{72(2-2.5\beta_1)^2}$$

(A12)

for all $\beta_1 \in (0,1/2)$, $\alpha \in [0,1]$, β_4=1/2.

and,

$$\frac{\partial \Pi 2}{\partial \beta_1}(\beta_1,\alpha)=\alpha\frac{(1-\beta_1)}{2}+\frac{(1+\alpha)(-30+40.5\beta_1+105.375\beta_1^2-188.90625\beta_1^3+28.625\beta_1^4+46.875\beta_1^5)}{72(2-2.5\beta_1)^3}$$

(A13)

for all $\beta_1 \in (0,1/2)$, $\alpha \in [0,1]$.

Note that $\frac{\partial \Pi_2}{\partial \beta_1}(\beta_1,\alpha)\Big|_{\beta_1=1/2}=0$ for $\alpha = 0.9837$, which implies $\frac{\partial \Pi_2}{\partial \beta_1}(\beta_1,\alpha)>0$ for all $\beta_1 \in (0,1/2)$ when $\alpha \in [0.9837, 1]$. So β_1=1/2 is a corner solution for $\alpha \in [0.9837, 1]$. Note that it takes a higher market for firm 2 to maximize intra-firm differentiation when firm 1 offers two products rather than one. Similarly, $\frac{\partial \Pi_2}{\partial \beta_1}(\beta_1,\alpha)\Big|_{\beta_1=0}=0$ for $\alpha = 0.1163$, which implies $\frac{\partial \Pi_2}{\partial \beta_1}(\beta_1,\alpha)<0$ for all $\beta_1 \in (0,1/2)$ when $\alpha \in [0, 0.1163]$. So $\beta_1 = 0$ is a corner solution in that range of α. Finally $\beta_1 \in (0,1/2)$ which solves (A13) implies $\alpha \in [0.1163, 0.9837]$.

We now turn to the interlaced products pattern. Let firm one own x_1 and x_3 and firm two x_2 and x_4 with $0 \leq x_1 \leq x_2 \leq x_3 \leq x_4 \leq 1$. The only symmetric equilibrium with interlaced locations in the product space implies that both firms position their products at the end of two orthogonal diameters. The proof is done in Marinez-Giralt and Neven (1988) in a one period model and will not be reproduced here.. Since this location pattern exhibits maximal intra-firm differentiation it also maximizes autarkic revenues, and hence is a location equilibrium for the two period model. In this case firm i's reduced form profit is:

$$\Pi_i(\alpha) = \alpha_i\left(r - \frac{1}{16}\right) + (1+\alpha)0.03125 \qquad i = 1,2 \tag{A14}$$

with $\alpha_i = \begin{cases} 1 & i = 1 \\ \alpha & i = 2 \end{cases}$

From (A12) and (A14),

$$\Pi_2(\beta_1 = 0, \alpha) \geq \alpha\left(r - \frac{1}{16}\right) + (1+\alpha)0.03125 \tag{A15}$$

implies $\alpha \leq 0.0693$. So for $\alpha \in [0.0693, 0.1163]$, firm two always prefers the interlaced product configuration to β_1=0. Also, the value of α such that $\beta_1 \in (0,1/2)$ is preferred to an interlaced product configuration is lower than 0.0693 and decreases with β_1. This in turn implies that firm two prefers the interlaced product configuration to any $\beta_1 \in (0,1/2)$ for $\alpha \in [0.0693, 1]$. In the limit, as $\beta_1=1/2$ for $\alpha \in [0.9837, 1]$, firm 2 is optimally located from the first period point of view but makes no profit on the integrated market. An interlaced product configuration clearly dominates in this case. So firm two has a dominant strategy which is to choose an interlaced pattern for $\alpha \in [0.0693, 1]$.

To establish that the interlaced product configuration is an equilibrium for $\alpha \in [0.0693, 1]$, from(A10), firm one has no incentive to switch to a neighbouring location given firm two's behaviour, as it would forego all of it's second period revenues

In the four product case, firm's behaviour is summarized as follows:

$$\begin{cases} \beta_4 = 1/2 \\ \\ \beta_1 = 0 \end{cases} \qquad \textit{for}\ \alpha \in (0, 0.0692899)$$

$$\textit{Interlaced products for}\ \alpha \in (0.0692899, 1)$$

So when both firms offer two products each, they choose an interlaced product configuration for $\alpha \in [0.0693, 1]$, while firm two chooses to collapse it's two products at one point in the middle of the large size of the market for $\alpha \in [0, 0.0693]$. Firm 1 maximizes intra-firm differentiation for all $\alpha \in [0,1]$.

Appendix II: Best response functions in (α, K) space for firms 1 and 2, F=0.

$$F1: K = \frac{85 - 23\alpha}{576} \qquad \alpha \in [0,1]$$

$$F2: \quad K = \begin{cases} \dfrac{85 - 23\alpha}{576} & \alpha \in (0,\ 0.0692890) \\[2ex] \dfrac{3 - 3\alpha}{32} & \alpha \in (\ 0.0692890,\ 0.090909) \\[2ex] \dfrac{\left[-10385.406\alpha^3 - 10293.656\alpha^2 - 3039.0938\alpha - 214.84375 + \left(829\alpha^2 + 614\alpha + 109\right)\sqrt{157\alpha^2 + 80\alpha + 4}\right]}{243(\alpha + 1)^2} & \alpha \in (0.090909,\ 0.9459459) \\[2ex] \dfrac{101 - 7\alpha}{576} & \alpha \in (0.9459459,\ 1) \end{cases}$$

$$F3. \quad K = \begin{cases} 0 & \alpha \in (0,\ 0.0692890) \\[2ex] \dfrac{101\alpha - 7}{576} & \alpha \in (0.0692890,\ 1) \end{cases}$$

$$F4: \quad K = \begin{cases} 0 & \alpha \in (0,\ 0.90909) \\[2ex] \dfrac{\left[-3895.625\alpha^3 - 2945.625\alpha^2 - 522.375\alpha - 14.375 + \left(314\alpha^2 + 160\alpha + 8\right)\sqrt{157\alpha^2 + 80\alpha + 4}\right]}{243(\alpha + 1)^2} & \alpha \in (0.090909,\ 0.9459459) \\[2ex] \dfrac{85\alpha - 23}{576} & \alpha \in (0.9459459,\ 1) \end{cases}$$

where F1 through F4 are derived from table 1 in the text and represent firms' best response functions mapped in the (α,K) space for the case F=0.

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ESSAY 2

TARIFF POLICY WITH ENDOGENOUS QUALITY AND OLIGOPOLISTIC COMPETITION

2-1 INTRODUCTION

The effects of trade policy on quality differentiated goods industries have been extensively analysed in the trade literature. More specifically, the non equivalence of quantity based restrictions such as a specific tariff (or quota) and ad valorem tariffs has been established even for a perfectly competitive environment. The general conclusion is that specific tariffs (quotas) Pareto dominate ad valorem tariffs in quality differentiated good's industries.[1]

Using the Swan (1970) specification, where consumers care only about the total amount of services (i-e physical units times the unit quality content), Rodriguez (1979) presents a partial equilibrium analysis of competitive foreign suppliers and shows that, when the objective is to set a particular level of imports, a specific tariff (quota) is welfare superior to an ad valorem tariff. Ad valorem tariffs apply to the final price of the good whereas specific tariffs (quotas) often discriminate on the basis of quality. Hence, their incidence can be diminished by shifting to a higher quality content.[2] This is somewhat related to Falvey (1978) who shows that ad valorem tariffs leave relative prices unaffected and consequently do not affect the product composition of imports while specific tariffs induce a shift towards the relatively more expensive good. Wall (1994) shows that an ad valorem tariff may actually increase the sales and market shares of some imported qualities.

[1] By contrast, homogeneous product models of imperfect competition conclude that ad valorem tariffs dominate specific tariffs and quotas because they generate better terms of trade, while it is well known that they are equivalent under perfectly competitive conditions. See Helpman and Krugman (1989, Ch 4, pp 65-67).

[2] See also Santoni and Van Cott (1980).

Das and Donnenfeld (1987) and Krishna (1987) consider variants of trade policy in the case of a foreign monopolist. In Das and Donnenfeld (1987), consumers have unit demands and varying intensities of preferences over quality. They find that quotas and specific tariffs are equivalent and Pareto dominate ad valorem tariffs because they lead to a higher level of quality, which directly benefits infra-marginal consumers. Ad valorem tariffs lead to quality downgrading. Using a more general model associated with the work of Spence (1975), Krishna (1987) highlights the role of demand conditions in determining quality. She shows that welfare comparisons of various trade restrictions on imports from a foreign monopoly firm depend critically on the valuation of quality increments by the marginal consumer, relative to that of all consumers on average. Krishna's general conclusion is that there is a tendency for specific tariffs (or quotas) to dominate ad valorem tariffs in the 'normal case', (when the marginal value of quality falls as absolute willingness to pay falls, and a tariff is needed to reduce imports).[3]

In this paper we consider duopolistic competition between a home and a foreign firm competing on the domestic market. The focus is on tariff policy. We examine a Mussa-Rosen type model of vertical product differentiation where the domestic and the foreign firms choose quality and price.[4] In choosing quality firms trade off the extraction of consumer surplus against the intensity of price competition and the costs of quality. Given

[3]This is not surprising since Mussa-Rosen type models are a particular case of the Spence formulation if we reinterpret Spence's inverse demand function as being composed of unit demand consumers ranked by decreasing order of willingness to pay for quality.

[4]Reitzes (1991) considers also duopolistic competition and investigates the scope for quality altering policies in a model where products contain multiple attributes and consumers have similar preference for quality.

qualities, prices determine the allocation of consumers between firms. Tariffs are investigated given their potential to affect the firm's product selection. Das and Donnenfeld (1989) also analyse an international duopoly but restrict their attention to quotas and minimum quality requirements and assume that firms compete over quality and quantity.

Results show that the effects of both types of tariffs hinge essentially on the location of the home and foreign firms in the product spectrum. A specific tariff induces quality upgrading and higher prices (in particular, a worsening of the importing country's terms of trade) when the foreign firm produces the higher quality variant, and quality downgrading together with an improvement in the terms of trade in the converse situation. An ad valorem tariff may deteriorate the quality of imports and improve the domestic country's terms of trade even when the foreign firm is located in the higher segment of the product spectrum. Furthermore, there exist values of the parameters for which ad valorem tariffs generates both higher qualities and better terms of trade, and so unambiguously improve domestic welfare.

The comparison between specific and ad valorem tariffs is shown to depend on the parameters of the model, such as the preference parameter of the lowest type consumer and the unit production cost Both types of tariffs yield an increase in domestic profits (at the expense of foreign profits) and welfare evaluated at initial equilibrium values. Contrary to previous work however an ad valorem tariff may or may not dominate a specific tariff based on its price, quality and ultimately wefare effects. So one can not extrapolate from the knowledge about alternative market structures (perfect competition, monopoly) to draw conclusions about the effects of trade restrictions in an ologopolistic setting.

The plan of the paper is as follows: Section 2 presents the model. In Section 3, we explore the impact of a specific tariff. Section 4 is devoted to the analysis of an ad valorem tariff. Finally, Section 5 concludes.

2-2 THE MODEL

We consider a Mussa-Rosen type model of vertical product differentiation. There is one home and one foreign firm competing on the domestic market.

There is a continuum of consumers whose types are identified by $\theta \in [a,b] \subset R_+$; the parameter θ is the consumer's marginal willingness to pay for quality and is uniformly distributed over $[a,b]$. We let $b = a+1$ with no loss of generality. The density is 1. The surplus of a consumer of type θ is given by:

$$U_\theta(p,q) = \begin{cases} \theta q - p & \textit{if he buys one unit of quality } q \textit{ at price } p \\ 0 & \textit{otherwise} \end{cases} \quad (1)$$

Each firm produces one variant of the good. let (q_i, p_i), $i = 1,2$, be the quality-price pairs where firms are labelled such that $q_1 < q_2$. The marginal cost of production of good i is independant of quantity but quadratic in quality. $MC(q_i) = cq_i^2$, $c > 0$.[5] The market boundary between firms is given by

$$\overline{\theta} = \frac{p_2 - p_1}{q_2 - q_1} \tag{2}$$

where $\overline{\theta}$ is the index of the consumer who is indifferent between buying variant 1 at price p_1 and variant 2 at price p_2. So the market demands are $D_1 = (\overline{\theta} - a)$ for variant 1 and $D_2 = (a+1-\overline{\theta})$ for variant 2. The market equilibrium results from a two stage game where firms choose qualities non cooperatively and simultaneously in the first stage. In the second stage, firms simultaneously choose prices given their quality choices. The equilibrium concept used is that of a subgame perfect equilibrium. There are two pure Nash equilibria in location, with the domestic firm producing the higher / lower quality product alternatively. The paper is concerned with the question of how imposing a small tariff perturbs a duopoly equilibrium in which all potential consumers make a purchase.[6] In particular, differences that arise given the firm's quality ranking in the product spectrum are highlighted.

2-3 ANALYSIS OF A SPECIFIC TARIFF

We begin our analysis of tariff policy with a specific tariff given by t. We examine the effects of such a tariff on the quality, prices of imports and of domestic

[5]This assumption rules out corner solutions.

[6] For the purpose of the analysis we require that a be large enough.

production, and then derive welfare implications We consider first the case where the foreign firm produces the high quality variant.

In this case the profits of the home and domestic firms are given by:

$$\Pi_1 = \left(\frac{p_2 - p_1}{q_2 - q_1} - a\right)\left(p_1 - cq_1^2\right)$$

$$\Pi_2 = \left(a + 1 + \frac{p_1 - p_2}{q_2 - q_1}\right)\left(p_2 - cq_2^2 - t\right) \tag{3}$$

Maximizing Π_i with respect to p_i and solving for the Nash equilibrium in prices gives:

$$p_1 = \frac{c}{3}\left(q_2^2 + 2q_1^2\right) + \frac{t}{3} + \frac{1-a}{3}\left(q_2 - q_1\right)$$

$$p_2 = \frac{c}{3}\left(2q_2^2 + q_1^2\right) + \frac{2t}{3} + \frac{2+a}{3}\left(q_2 - q_1\right) \tag{4}$$

Substituting (4) back into (3) gives the firm's profits as a function of qualities:

$$\Pi_1 = (q_2 - q_1)\left(\frac{c}{3}(q_2 + q_1) + \frac{t}{3(q_2 - q_1)} + \frac{1-a}{3}\right)^2$$

$$\Pi_2 = (q_2 - q_1)\left(\frac{-c}{3}(q_2 + q_1) + \frac{t}{3(q_2 - q_1)} + \frac{2+a}{3}\right)^2 \tag{5}$$

Solving for the Nash equilibrium in qualities and corresponding prices yields:

$$q_1 = \frac{4a-1}{8c} - \frac{2t}{3}, \qquad q_2 = \frac{4a+5}{8c} + \frac{2t}{3},$$

$$p_1(t) = \frac{16a^2 - 8a + 25}{64c} + \frac{4ct^2}{9} - \frac{t}{3}\left(\frac{4a-3}{2}\right), \qquad p_2(t) = \frac{16a^2 + 40a + 49}{64c} + \frac{4ct^2}{9} - \frac{t}{3}\left(\frac{4a+7}{2}\right). \tag{6}$$

$$\bar{\theta}(t) = \frac{2a+1}{2} - \frac{8ct}{9}$$

The free trade equilibrium is easily obtained by setting $t = 0$ in (6). Imposition of a specific tariff leads to quality upgrading by both firms. This is because the market boundary between firms shifts to the right with the tariff. As a result, the low quality domestic firm is now facing new consumers with a higher marginal willingness to pay and thus marginal revenue from quality increases. The foreign firm is left with a smaller market but the consumer's average willingness to pay has risen inducing it in turn to raise quality.

From (6), noting that the foreign price is $h_2 = p_2 - t$, the effects of a specific tariff on the price for the domestic product and on the foreign price evaluated at t = 0 are given by:

$$\frac{dp_1(t)}{dt} = \frac{4a+3}{6}$$

$$\frac{dh_2(t)}{dt} = \frac{4a+1}{6} \tag{7}$$

Prices are a function of the costs of production and the quality gap. Since the latter is invariant with respect to the specific tariff, prices rise because of the higher production costs brought about by the higher quality levels. The second expresion in (7) implies a terms of trade loss due to higher priced imports.

From (5), having substituted for equilibrium qualities, the effects of a small specific tariff on the firm' profitability are given by:

$$\frac{d\Pi_1}{dt} = \frac{2}{3}$$

$$\frac{d\Pi_2}{dt} = \frac{-2}{3} \tag{8}$$

The domestic firm's profits increase because it sells higher quality at a higher price, and therefore extracts more surplus from consumers. Also, the domestic firm's market share is increased at the expense of it's rival, so the domestic firm extracts additional surplus from higher valuation consumers. These effects outweight the increases in unit production costs.

We now proceed to examine the effect of a specific tariff on domestic welfare. Domestic welfare is defined as domestic consumer and producer surplus attached to variant 1, consumer surplus from imports and tariff revenues. Using foreign prices, i.e., $h_2 = p_2 - t$, domestic welfare is given by:

$$W(t) = \int_{a}^{\bar{\theta}} \left[\theta q_1(t) - cq_1^2(t)\right] d\vartheta + \int_{\bar{\theta}}^{a+1} \left[\theta q_2(t) - h_2(t)\right] d\theta \tag{9}$$

The effects of a marginal increase in the specific tariff are obtained by differentiating expression (9) and evaluating it at t=0:

$$\left(\frac{dW(t)}{dt}\right) = \frac{2}{3} \tag{10}$$

A small increase in the tariff rate improves domestic welfare. The increase in the domestic firm's profits combined with the positive effects of higher qualities outweigh losses from higher prices, in particular a worsening in the importing country's terms of trade. Brander and Spencer (1984) found that a worsenning in the terms of trade is not sufficient to break the case for a positive tariff in an oligopolistic market because of profit shifting effects. Here there is an additional positive effect through the improvement of both qualities available to consumers. Alternatively, noting that a change in the domestic product's price does not affect domestic welfare, the positive effects of a small specific tariff on the social value of variant 1 (given the allocation between firms) implies that the domestic firm is trading off more efficiently how it serves its customers with the costs of quality from a welfare point of view.

Proposition 1: *Assume that the market is covered in equilibrium. When the foreign firm produces the higher quality variant, a small specific tariff:*

(i) raises the market boundary between firms thus the quality of both the domestic and the foreign firm

(ii) worsens the importing country's terms of trade and increases the price of the domestic product

(iii) increases the profits of the domestic firm and reduces the profits of the foreign firm

(iv) increases domestic welfare.

We now turn to the case where the foreign firm is located in the lower segment of the quality spectrum. In this case the firm's profits are given by:

$$\Pi_1 = \left(\frac{p_2 - p_1}{q_2 - q_1} - a\right)\left(p_1 - cq_1^2 - t\right)$$

$$\Pi_2 = \left(a + 1 + \frac{p_1 - p_2}{q_2 - q_1}\right)\left(p_2 - cq_2^2\right) \tag{11}$$

solving for equilibrium prices and qualities yields:

$$q_1 = \frac{4a-1}{8c} - \frac{2t}{3}, \qquad q_2 = \frac{4a+5}{8c} - \frac{2t}{3},$$

$$p_1(t) = \frac{16a^2 - 8a + 25}{64c} + \frac{4ct^2}{9} - \frac{t}{3}\left(\frac{4a-3}{2}\right). \qquad p_2(t) = \frac{16a^2 + 40a + 49}{64c} + \frac{4ct^2}{9} - \frac{t}{3}\left(\frac{4a+1}{2}\right). \tag{12}$$

$$\bar{\theta}(t) = \frac{2a+1}{2} - \frac{8ct}{9}$$

So here, a specific tariff has opposing effects on qualities because the market boundary between firms is shifted to the left. This decreases marginal revenue of quality for both firms, since they now face market segments with a lower average willingness to pay. From (10), having substituted for equilibrium prices and qualities, with $h_1(t) = p_1(t) - t$,

$$\frac{dh_1(t)}{dt} = \frac{-4a - 3}{6}$$

$$\frac{dp_2(t)}{dt} = \frac{-4a - 1}{6} \tag{13}$$

Thus a small specific tariff reduces both the foreign price and the price for the domestic product with a consequent improvement in the importing country's terms of trade. Again since the quality gap is invariant with respect to the tariff, the reason for the price decreases must be found in the unit production cost decrease induced by lower equilibrium qualities.

It can be checked that the effects on the firm's profits are symmetrical to the case where the foreign firm specializes in the higher quality product :

$$\frac{d\Pi_1}{dt} = \frac{-2}{3}$$

$$\frac{d\Pi_2}{dt} = \frac{2}{3} \tag{14}$$

Here the domestic firm's profits increase despite a decrease in price since unit production costs decrease and the firm is able to cover a larger share of the market.

Turning now to the overall welfare effects, using $p_1 = h_1 - t$, we define domestic welfare as:

$$W(t) = \int_{a}^{\bar{\theta}} \left[\theta q_1(t) - c q_1^2(t)\right] d\theta + \int_{\bar{\theta}}^{a+1} \left[\theta q_2(t) - h_2(t)\right] d\theta \tag{15}$$

Differentiating (15) with respect to t and evaluating this derivative at t = 0 gives the effects of a slight increase in the specific tariff rate on welfare :

$$\frac{dW(t)}{dt} = \frac{2}{3} \tag{16}$$

When the foreign firm produces the lower quality variant a marginal increase in the tariff rate from the free trade equilibrium improves welfare but the underlying effects are different however. The gains to the domestic firm combined with an improvment in the country's terms of trade and a decrease in the price for the domestic quality more than offset the negative effects of quality downgrading.[7]

Proposition 2: *Assume that the market is covered in equilibrium. When the foreign firm produces the lower quality variant, a small specific tariff :*

(i) lowers the market boundary between firms thus the quality of both the domestic and the foreign firm

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[7] As before a similar welfare interpetation can be given. Since p_1 represents a transfer of surplus, the decrease in the domestic higher quality trades off more efficiently how consumers are served and the costs of quality (given the allocation of consumers between firms).

(ii) improves the importing country's terms of trade and lowers the price of the domestic product

(iii) increases the profits of the domestic firm and reduces the profits of the foreign firm

(iv) increases domestic welfare.

From proposition 1 and 2 the relative effects of a specific tariff are seen to depend critically on the firm's quality ranking in the product space. Depending on whether the foreign firm produces the low or high quality variant, a specific tariff has opposing effects on the firm's product quality and on the domestic country's terms of trade as well as on the price for the domestic product. As will be argued later this distinction has important implications for the design of trade policies.

2-4 ANALYSIS OF AN AD VALOREM TARIFF.

Das and Donnenfeld (1987) investigated the effects of specific and ad valorem tariffs with endogenous quality in a monopolistic framework. It is interesting to consider these instruments in an oligopolistic setting.

Let $\tau = 1/(1+T)$ where T is the ad valorem tariff. As before we start by considering first the case where the foreign firm specializes in the higher quality product.

In this case the firm's profit functions are given by:

$$\Pi_1 = \left(\frac{p_2 - p_1}{q_2 - q_1} - a\right)\left(p_1 - cq_1^2\right)$$

(17)

$$\Pi_2 = \left(a + 1 + \frac{p_1 - p_2}{q_2 - q_1}\right)\left(\frac{1}{\tau}\left[p_2 - \tau c q_2^2\right]\right)$$

Maximizing Π_i with respect to p_i gives the second stage reaction functions in prices. Solving them yields the Nash equilibrium in prices:

$$p_1 = \frac{c}{3}\left(\tau q_2^2 + 2q_1^2\right) + \frac{1-a}{3}\left(q_2 - q_1\right)$$

(18)

$$p_2 = \frac{c}{3}\left(2\tau q_2^2 + q_1^2\right) + \frac{2+a}{3}\left(q_2 - q_1\right)$$

Substituting (18) back into (17) gives the firm's profits as a function of qualities:

$$\Pi_1 = \left(q_2 - q_1\right)\left(\frac{c\left(\tau q_2^2 - q_1^2\right)}{3\left(q_2 - q_1\right)} + \frac{1-a}{3}\right)^2$$

(19)

$$\Pi_2 = \left(\frac{q_2 - q_1}{\tau}\right)\left(\frac{c\left(q_1^2 - \tau q_2^2\right)}{3\left(q_2 - q_1\right)} + \frac{2+a}{3}\right)^2$$

Solving for the Nash equilibrium in qualities yields:[8]

$$q_1(\tau)=\frac{a-1}{6c}+\frac{3-\left[4a^2(1-\tau)^2+8a\left(-\tau^2-\tau+2\right)+16-11\tau+4\tau^2\right]^{1/2}}{12c(\tau-1)}$$

$$q_2(\tau)=\frac{a-1}{6c\tau}+\frac{9\tau-6-\left[4a^2(1-\tau)^2+8a\left(-\tau^2-\tau+2\right)+16-11\tau+4\tau^2\right]^{1/2}}{12c\tau(\tau-1)} \qquad (20)$$

The market boundary between firms can be expressed as a function of qualities only and is given by $\bar{\theta}(\tau)=\frac{c\left(\tau q_2^2-q_1^2\right)}{3\left(q_2-q_1\right)}+\frac{2a+1}{3}$

Differentiating (20) with respect to τ and taking the limit as τ tends to one leads to[9] :

[8]The two functions in (20) are defined in R+ only if $\left[4a^2(1-\tau)^2+8a\left(-\tau^2-\tau+2\right)+16-11\tau+4\tau^2\right]>0$. This happens for τ not too large. The intuition behind this result is that for τ sufficiently high, the high quality foreign firm may want to select a lower quality than the domestic firm. The expression in brackets is positive for $\tau=1$, however and the effects of an ad valorem tariff around the free trade equilibrium point can be investigated.

[9]The functions in (20) are continuous and differentiable.

$$\frac{dq_1(\tau)}{d\tau} = \frac{16a^2 + 16a - 5}{96c}$$

$$\frac{dq_2(\tau)}{d\tau} = \frac{16a^2 - 32a - 65}{96c} \qquad (21)$$

$$\frac{d\bar{\theta}(\tau)}{d\tau} = \frac{16a^2 + 16a - 5}{72c}$$

Thus, an increase in the ad valorem tariff, starting from the free trade equilibrium results in a domestic quality increase.[10] The effect on foreign quality however depends on the taste parameter of the lowest type consumer. It is positive if $a > 13/4$, negative otherwise. Following the tariff some consumers switch from imports to domestic production and both firms raise qualities for relatively large values of a When a is low (the degree of consumer heterogeneity is high), the incentive to differentiate products is weakened as competition for consumers is less intense and the foreign firm prefers to downgrade quality in order to evade part of the tariff.

Making use of the fact that prices and profits can be expressed as a function of qualities only and using (22), noting that the foreign price $h_2 = p_2/\tau$:

[10]The assumption that all consumers buy in equilibrium implies that a be greater than 5/4.

$$\frac{dp_1(\tau)}{d\tau} = \frac{64a^3 + 112a^2 - 68a - 185}{384c}$$

(22)

$$\frac{dh_2(\tau)}{d\tau} = \frac{64a^3 - 112a^2 - 580a - 569}{384c}$$

The price for the domestic product increases for all feasible values of a. The interesting thing is that the domestic country can experience a terms of trade gain for some values of a ($a < 4.3225$). When $a < 13/4$ the norrowing of the quality gap, which intensifies price competition, reinforces the fall in the foreign price brought about by lower costs. For $13/4 < a < 4.3225$, the foreign price still decreases since as can be seen from (21) the domestic quality increase is stronger than the foreign quality increase, reducing the quality gap. For $a > 4.3225$, the foreign price increases as the cost effect becomes relatively stronger.

The ad valorem tariff also induces profit-shifting effects in favor of the domestic firm. From (19) together with (21):

$$\frac{d\Pi_1(\tau)}{d\tau} = \frac{4a^2 + a - 5}{24c}$$

(23)

$$\frac{d\Pi_2(\tau)}{d\tau} = \frac{-4a^2 - 7a - 5}{24c}$$

The first expression is positive for $a > 1/4$ and the second expression negative for all values of a. Again the domestic firm's market share increases as a result of the tariff.

The above results can be combined to investigate the overall welfare effect for a marginal increase in the ad valorem tariff from the free trade situation. As before, domestic welfare is the total surplus attached to variant 1 plus consumer surplus from imports and tariff revenues. Using the fact that the foreign price $h_2 = p_2/\tau$, domestic welfare can be written as:

$$W(t) = \int_a^{\bar{\theta}} \left[\theta q_1(t) - c q_1^2(t)\right] d\vartheta + \int_{\bar{\theta}}^{a+1} \left[\theta q_2(t) - h_2(t)\right] d\theta \tag{24}$$

where $\bar{\theta}$ is as defined in (20). Using the fact that h_2 can be expressed as a function of qualities only together with (21), the effects on domestic welfare of a marginal increase in the ad valorem tariff rate is given by:

$$\frac{dW(\tau)}{d\tau} = \frac{16a^2 + 40a + 43}{96c} \tag{25}$$

So, an increase in the ad valorem tariff from zero unambiguously improves domestic welfare. Interestingly, there is a range of a for which both qualities, the country's terms of trade and domestic profit all increase as a result of the tariff. As previously noted, since the increase in the price of the domestic product is merely a transfer of surplus from consumers to producers, the welfare gains attached to the lower domestic quality adjustment can be interpreted as bringing about a better trade off between product efficiency and the costs of quality.

Proposition 3: *Assume that the market is covered in equilibrium. When the foreign firm produces the higher quality variant, a small ad valorem tariff :*

(i) increases the market boundary between firms, raises the quality of the domestic firm while it may raise or lower the quality of imports as a function of a.

(ii) may improve or worsen the importing country's terms of trade and increases the price of the domestic product

(iii) increases the profits of the domestic firm and reduces the profits of the foreign firm

(iv) increases domestic welfare.

The final case to consider involves the foreign firm producing the lower quality variant. In this case the firm's profit functions are given by:

$$\Pi_1 = \left(\frac{p_2 - p_1}{q_2 - q_1} - a\right)\left(\frac{1}{\tau}\left[p_1 - \tau c q_1^2\right]\right)$$

$$\Pi_2 = \left(a + 1 + \frac{p_1 - p_2}{q_2 - q_1}\right)\left(p_2 - c q_2^2\right) \qquad (26)$$

Stage two reaction functions in prices are given by :

$$p_1 = \frac{c}{3}\left(2\tau q_1^2 + q_2^2\right) + \frac{1-a}{3}\left(q_2 - q_1\right)$$

$$p_2 = \frac{c}{3}\left(\tau q_1^2 + 2q_2^2\right) + \frac{2+a}{3}\left(q_2 - q_1\right) \tag{27}$$

Substituting those equilibrium prices back into (26) gives the firm's profits as a function of qualities:

$$\Pi_1 = \left(\frac{q_2 - q_1}{\tau}\right)\left(\frac{c\left(q_2^2 - \tau q_1^2\right)}{3\left(q_2 - q_1\right)} + \frac{1-a}{3}\right)^2$$

$$\Pi_2 = \left(q_2 - q_1\right)\left(\frac{c\left(\tau q_1^2 - q_2^2\right)}{3\left(q_2 - q_1\right)} + \frac{a+2}{3}\right)^2 \tag{28}$$

Solving for the Nash equilibrium in qualities yields:

$$q_1(\tau) = \frac{a-1}{6c\tau} + \frac{3\tau - \left[4a^2(1-\tau)^2 + 8a\left(2\tau^2 - \tau + 1\right) + 16\tau^2 - 11\tau + 4\right]^{1/2}}{12c\tau(1-\tau)}$$

$$q_2(\tau) = \frac{a-1}{6c} + \frac{9 - 6\tau - \left[4a^2(1-\tau)^2 + 8a\left(2\tau^2 - \tau + 1\right) + 16\tau^2 - 11\tau + 4\right]^{1/2}}{12c(1-\tau)} \tag{29}$$

Where as before the market boundary between firms is given by

$$\bar{\theta}(\tau) = \frac{c\left(q_2^2 - q_1^2\right)}{3\left(q_2 - q_1\right)} + \frac{2a+1}{3}$$

Differentiating (29) with respect to τ and taking the limit as τ tends to one. again gives the effects of a slight increase in the ad valorem tariff:

$$\frac{dq_1(\tau)}{d\tau} = \frac{-16a^2 - 64a + 17}{96c}$$

$$\frac{dq_2(\tau)}{d\tau} = \frac{-16a^2 - 16a + 5}{96c} \qquad (30)$$

$$\frac{dq_1(\tau)}{d\tau} = \frac{-16a^2 - 16a + 5}{96c}$$

A small ad valorem tariff induces both firms to downgrade quality. Both qualities decrease because some consumers substitute imports for the high quality domestic good. The foreign firm is now serving a smaller market in which consumers with a higher willingness to pay dropped out. inducing the foreign firm to lower quality. The domestic firm in turn lowers it's quality because it inherits these low valuation consumers.

From (27) using (30) and given that $h_1 = p_1 / \tau$:

$$\frac{dh_1(\tau)}{d\tau} = \frac{-64a^3 - 304a^2 + 164a - 165}{384c}$$

(31)

$$\frac{dp_2(\tau)}{d\tau} = \frac{-64a^3 - 80a^2 + 100a - 19}{384c}$$

Both expressions in (31) are negative for feasible values of a. Both the foreign price h_1 and the price for the domestic product decrease because qualities decrease. reducing unit production costs.

From (28), the effects on the profitability of the lower quality foreign firm and higher quality domestic firm are found to be:

$$\frac{d\Pi_1(\tau)}{d\tau} = \frac{-4a^2 - 7a - 4}{24c}$$

(32)

$$\frac{d\Pi_2(\tau)}{d\tau} = \frac{4a^2 + 7a - 2}{24c}$$

Examination of the above expressions reveals that foreign profit decrease and domestic profits increase for feasible values of a, implying once again that an increase in the ad valorem tariff from zero induces profit shifting to the benefit of the domestic firm.

To examine now the overall welfare effects we define the domestic country welfare function as:

$$W(t) = \int_{a}^{\bar{\theta}} \left[\theta q_1(t) - c q_1^2(t)\right] d\vartheta + \int_{\bar{\theta}}^{a+1} \left[\theta q_2(t) - h_2(t)\right] d\theta \tag{33}$$

As previously, using the fact that h_l can be expressed as a function of qualities alone and making use of (30), the effect on domestic welfare of a marginal increase in the ad valorem tariff rate is given by :

$$\frac{dW(\tau)}{d\tau} = \frac{16a^2 - 8a + 19}{96c} \tag{34}$$

A small ad valorem tariff improves welfare. The increase in domestic profit and the improvement in the importing country's terms of trade together with a lower price for the domestic product outweight the negative effects from quality downgrading.

Proposition 4: *Assume that the market is covered in equilibrium. When the foreign firm produces the lower quality variant, a small ad valorem tariff :*

(i) lowers the market boundary between firms and thus the quality of both the domestic and the foreign firm

(ii) improves the importing country's terms of trade and decreases the price of the domestic product

(iii) increases the profits of the domestic firm and reduces the profits of the foreign firm

(iv) increases domestic welfare.

The relation between the effects of a tariff and the firm's quality ranking in the product spectrum is not solely a feature of the specific tariff case. Depending on whether the foreign firm produces the low or the high quality product, an ad valorem tariff has opposite effects on domestic quality while it may or may not have opposite effects on foreign quality. An ad valorem tariff improves the importing country's terms of trade when the foreign firm produces the low quality variant while it may or may not improve it when the foreign firm supplies the higher quality variant.

Combining propositions 1 through 4, a small tariff whether specific or ad valorem always shifts some profits to the domestic firm and improves domestic welfare. Terms of trade effects of both tariffs may differ when the foreign firm specializes in the higher quality variant. So if the domestic country imports high quality goods and the domestic government considers improving the country's terms of trade as an important policy goal, an ad valorem may be the preferred instrument.[11] Furthermore there are values of the parameters for which an ad valorem tariff improves both the qualities produced by the firms and the importing country's terms of trade.

The differential effects of a small increase in both tariffs can now be examined by referring to expressions (10), (16) and (25), (34). The effect of a specific tariff

[11] If the country is constrained in its foreign exchange reserves for instance.

on welfare is a constant while that of an ad valorem tariff depends on the lowest type consumer's marginal willingness to pay, a, and the firm's unit production cost c. An ad-valorem tariff acts as a tax on the foreign firm's revenues. It has a direct effect on the foreign firm's trade off between the extraction of consumer surplus and the costs of quality. A specific tariff does not have such a direct effect. An ad valorem tariff dominates a specific tariff evaluated at initial equilibrium values of welfare for large values of the ratio $a c$, i.e., as the marginal willingness to pay for quality of the lowest type consumer increases or as the magnitude of the firm's marginal production cost (c) decreases. This is because for larger $a c$ ratios an ad valorem tariff generates stronger effects on quality, prices and domestic profits, all of which influence domestic welfare. The opposite holds for low $a c$ ratios where a specific tariff provides a relatively better instrument from a welfare point of view.

2-5 CONCLUSION

In this paper we have examined the effects of tariffs on the quality choices of home and foreign firms and ultimately on the welfare of the domestic country in an oligopolistic market. Our analysis reveals that these effects depend on the location of domestic and foreign firms in the quality spectrum. When the domestic firm specializes in the higher quality variant, specific and ad valorem tariffs have similar effects : they result in a lower quality provided by both firms, an increase in the domestic firm's profits and an improvement in the domestic country's terms of trade. Their effects are different when the domestic firm is the low quality producer. A specific tariff induces both firms to raise product quality and deteriorates the domestic country's terms of trade. An ad valorem may have a positive or negative impact on the quality of imports and on the terms of trade, depending on the consumers' average willingness to pay.

A comparison of both tariffs reveals that ad valorem tariffs may or may not dominate specific tariffs as a function of the average valuation of consumers for quality and the firm's unit production cost. This result differs from previous analysis using alternative market structures and indicates that the type of market structure assumed has important implications from a policy perspective.

An analysis of trade policies should also include quotas. Das and Donnenfeld (1989) compares quotas and minimum quality standards in a similar setting, but firms are assumed to compete in qualities and quantities. When firms use prices as their strategic variables and products are substitutes, Krishna (1989) shows that a quota set at the free trade level enables both the domestic and the foreign firm to achieve higher prices in equilibrium, thus acting as a facilitating device. Here, given qualities, a quota set at the free trade level will affect the Nash equilibrium in prices in a similar way (since it affects the post-quota reaction functions). So the firms's quality choices will ultimately reflect the change in the nature of price competition. In the present context however, a quota may not relax price competition as much because firms also compete in quality space. So one should expect the effects of a quota to differ substantially from those of a tariff. Further research along those lines is needed.

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ESSAY 3

STRATEGIC TRADE POLICY WITH MULTIPRODUCT FIRMS

3-1 INTRODUCTION

There now exists a large literature on trade and trade policy under conditions of imperfect competition and increasing returns to scale. Dixit (1984), Brander and Spencer (1985) and Eaton and Grossman (1986), examine the rent-shifting motive for intervention and establish that government intervention is desirable on welfare grounds, though the international non cooperative policy equilibrium is suboptimal. All these papers however consider trade policy in a given market sructure.

Brander and Spencer (1981) examine the potential for trade policy to extract foreign monopoly rents under a threat of domestic entry. Dixit and Kyle (1985) discuss the use of protection and subsidies for entry promotion and deterrence.[1] The set of strategies available to the firms is kept relatively simple however. In Dixit and Kyle (1985), trade policy is assumed to be all-or-nothing (i.e either free trade or entry deterrence policy). Horstmann and Markusen (1992) and Motta (1992) show how a tariff can induce a shift in market structure by causing an exporter to enter the market as a multinational, but trade policy is not endogenized.[2]

[1] Venables (1985) and Horstman and Markusen (1986) consider also international oligopolistic equilibria under free entry, but use a zero profit assumption. Profit shifting effects disapear as there are no profits to shift.

[2] Schmitt (1991) examines unilateral import policy in a reciprocal-markets location model where each firm supplies one product only. He finds that a domestic tariff which is sufficiently large lowers domestic welfare if it induces the foreign firm to select a product attribute which eliminates domestic exports.

This paper looks at optimal trade policy with endogenous market structure.[3] It focuses on an international export market. A model of product line competition is examined, in which firms (domestic and foreign active on a third market) choose the number of feasible product lines to supply. There is a degree of substitutability between products which can vary. There are product specific scale economies modeled by fixed costs (equal across products). The output of each firm in the equilibrium product configurations is given by a Cournot-Nash equilibrium.

Depending on the level of fixed costs and product substitutability, duopolistic competition in both products, international duopoly with product specialization, domestic monopoly and foreign monopoly all arise as initial equilibrium market structures. In such a context small policy changes can produce large welfare effects as the equilibrium market structure shifts, implying discontinuous jumps in prices, quantities and profits. Trade policy is investigated given the potential to affect the firm's product composition.

Depending on the parameters of the model, rent-shifting, product line reducing and entry deterrence all constitute motives for intervention. Over a range of the parameters of the model, entry deterrence is achieved by product proliferation, so trade policy can help a firm commit to a product line. The optimal policy usually involves subsidies (which can be uniform or non uniform) but tax-subsidy combinations can also

[3] It is concerned with unilateral intervention. Though not a very realistic assumption, it is useful to get the analysis of trade policy with endogenous market structures clarified before proceeding of to the more general case where both governments are active.

arise in equilibrium. Interestingly, the optimal policy is shown to exihibit strong discontinuities as national rent maximization implies a switch of regimes.

This framework seems particularly suited for the analysis of international non cooperative rivalries in the aero-space industy, where national governments have largely intervened for entry promotion and deterrence, the best example being perhaps the Boeing-Airbus competition. This paper shows that governments have an incentive to affect industry structure in order to maximize domestic welfare. This incentive comes in addition to the usual argument for active trade policy arising in a given non-competitive market structure.

The paper proceeds as follows : Section 2 presents the model. In Section 3, we investigate strategic trade policy with multiproducts firms. Discontinuities in the optimal policy are examined in Section 4. Section 5 concludes.

3-2 THE MODEL

We consider a model of competition between multiproduct firms, one domestic and one foreign. We build on a model developed by De Fraja (1992) who examines the interplay between the degree of substitutability and economies of scale on the product line decisions of firms. We incorporate a policy stage to look at unilateral optimal export policies. Free entry is assumed but the number of firms is limited by increasing returns to scale. In the spirit of Brander and Spencer (1985), we assume that both firms produce only for a third market: there is no consumption in the producing countries. This abstraction is made to underline the role of export policies in raising GNP and their relationship to conduct and market structure. At the same time, it avoids the problem of possible consumer surplus changes in the home country.[4]

There are two identical firms labelled H and F, where H stands for the domestic firm and F for the foreign firm. The firms can produce two goods labelled 1 and 2. We assume that the technology is characterized by constant and equal marginal costs which are assumed equal to zero. Economies of scale are modelled by a fixed entry cost $F > 0$, per product. There are no economies of scope. This simplification can be justified on the grounds that it does not affect the main results under consideration.

[4]The inclusion of domestic consumption does not affect the analysis if firms can price discriminate between markets and, if each firm enjoys a monopoly in its home market. If marginal costs are constant (as assumed here) an export policy does not affect the profit maximizing product line and level of sales in the domestic market.

Consumers in the third country are assumed to possess preferences over the two goods represented by the following quadratic utility function:

$$U(q_1, q_2, m) = a(q_1 + q_2) - \frac{b}{2}(q_1^2 + q_2^2) - \frac{\sigma}{2}(q_1 + q_2)^2 + m \tag{1}$$

Where p_1 and p_2 are the prices, and m is exogenously given income.

Maximizing utility with respect to prices p_1, p_2 and income m yields the following demand functions for good 1 and 2 respectively:

$$p_1 = a - (b - \sigma)q_1 - \sigma q_1 \tag{2}$$

$$p_2 = a - \sigma q_1 - (b - \sigma)q_2 \qquad a, b \geq 0 \tag{3}$$

Parameter σ is an index of substitutability between commodities 1 and 2. $\sigma \in [0 , b/2]$, so products may be perfect or imperfect substitutes in consumption. $\sigma = 0$ implies zero cross elasticities of demand. To ensure that the cross substitution effects are not stronger than the own price effects, $\sigma \leq b/2$.

The domestic government, G^H sets a particular trade policy instrument belonging to the set $\{s_1, s_2\}$ where s_1 and s_2 are production subsidies on good 1 and 2 respectively, and where s_1 and s_2 may be negative (taxes).

The market equilibrium results from a three stage game. In the first stage, the domestic government G^H, chooses a policy $\{s_1,s_2\}$ that maximizes domestic welfare, perfectly anticipating the competition between the firms, including any effect on market structure. Domestic welfare is simply domestic profits net of subsidies. The opportunity cost of a dollar of public funds is one. It is assumed for the present time that the government in country F, G^F, is inactive in order to focus on the potential gains from a unilateral intervention from free trade. In the second stage of the game both firms simultaneously decide their product range, choosing one element in the set $\Gamma = \{\phi, \{1\},\{2\},\{1,2\}\}$. Finally, in the third stage, having observed the choices of the previous stages, firms decide simultaneously the quantity to supply for each of the products that they have decided to produce in the second stage. The solution concept adopted is subgame perfect equilibrium. The game is solved backwards.

Table 1
The payoff matrix of the second stage given Cournot competition in the third stage

Firm2

Firm1

	(1, 2)	(1)	(2)	∅
(1, 2)	$\frac{2+4s_1+4s_2}{9}+\frac{4(1-\sigma)(s_1^2+s_2^2)-8\sigma s_1 s_2}{9(1-2\sigma)}-2F$ $\frac{2}{9}+\frac{s_1(s_1-2)+s_2(s_2-2)+\sigma(s_1+s_2)(4-s_1-s_2)}{9(1-2\sigma)}-2F$	$\frac{13-18\sigma}{3\sigma(1-\sigma)}+\frac{s_2}{2}+\frac{(8-9\sigma)s_1}{18(1-\sigma)}+\frac{(1-\sigma)s_1^2}{4(1-2\sigma)}+\frac{[4(\sigma-2)(3\sigma-2)-3\sigma^2]s_2^2}{3\sigma(1-\sigma)(1-2\sigma)}-\frac{\sigma s_1 s_2}{2(1-2\sigma)}-2F$ $\frac{(1-s_1)^2}{9(1-\sigma)}-F$	$\frac{13-18\sigma}{3\sigma(1-\sigma)}+\frac{s_1}{2}+\frac{(8-9\sigma)s_2}{18(1-\sigma)}+\frac{(1-\sigma)s_2^2}{4(1-2\sigma)}+\frac{[4(\sigma-2)(3\sigma-2)-3\sigma^2]s_1^2}{3\sigma(1-\sigma)(1-2\sigma)}-\frac{\sigma s_1 s_2}{2(1-2\sigma)}-2F$ $\frac{(1-s_2)^2}{9(1-\sigma)}-F$	$\frac{1+s_1+s_2}{2}+\frac{(1-\sigma)(s_1^2+s_2^2)-2\sigma s_1 s_2}{4(1-2\sigma)}-2F$ 0
(1)	$\frac{(1+2s_1)^2}{9(1-\sigma)}-F$ $\frac{13-18\sigma}{3\sigma(1-\sigma)}-\frac{s_1(2-s_1)}{9(1-\sigma)}-2F$	$\frac{(1+2s_1)^2}{9(1-\sigma)}-F$ $\frac{(1-s_1)^2}{9(1-\sigma)}-F$	$(1-\sigma)\left(\frac{1}{2-\sigma}+\frac{2s_1(1-\sigma)}{(2-3\sigma)(2-\sigma)}\right)^2-F$ $(1-\sigma)\left(\frac{1}{2-\sigma}-\frac{\sigma s_1}{(2-3\sigma)(2-\sigma)}\right)^2-F$	$\frac{(1+s_1)^2}{4(1-\sigma)}-F$ 0
(2)	$\frac{(1+2s_2)^2}{9(1-\sigma)}-F$ $\frac{13-18\sigma}{3\sigma(1-\sigma)}-\frac{s_2(2-s_2)}{9(1-\sigma)}-2F$	$(1-\sigma)\left(\frac{1}{2-\sigma}+\frac{2(1-\sigma)s_2}{(2-3\sigma)(2-\sigma)}\right)^2-F$ $(1-\sigma)\left(\frac{1}{2-\sigma}+\frac{\sigma s_2}{(2-3\sigma)(2-\sigma)}\right)^2-F$	$\frac{(1+2s_2)^2}{9(1-\sigma)}-F$ $\frac{(1+s_2)^2}{9(1-\sigma)}-F$	$\frac{(1+s_2)^2}{4(1-\sigma)}-F$ 0
∅	0 $\frac{1}{2}-2F$	0 $\frac{1}{4(1-\sigma)}-F$	0 $\frac{1}{4(1-\sigma)}-F$	0 0

3-3 STRATEGIC TRADE POLICY AND OPTIMAL PRODUCT COMPOSITION.

The firm's second stage payoffs associated with all possible product configurations given the vector of policies $\{s_1, s_2\}$ are presented in Table 3-1.[5]

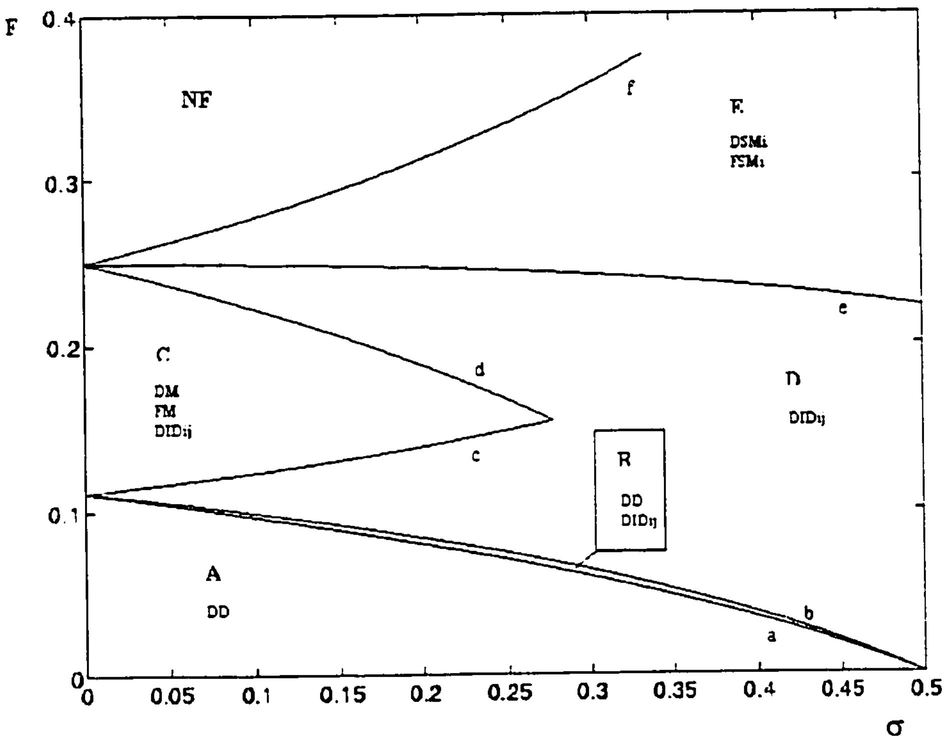
The second stage payoff matrix is obtained by substituting the equilibrium quantities into the profit functions. In the second stage, when the scope of production is fixed, there are sixteen possibilities to consider : Duopoly in both goods (DD); Domestic monopoly in good i / Duopoly in good j (DMD_{ij}); Domestic monopoly in both goods (DM); Foreign monopoly in good i / Duopoly in good j (FMD_{ij}); Differentiated duopoly (DID_{ij}) where the home firm produces good i and the foreign firm good j; Duopoly in good i only (SD_i), good j not produced; Domestic monopoly in good i only (DSM_i), good j not produced; Foreign monopoly in both goods (FM); Foreign monopoly in good i only (FSM_i), good j not produced; $i = 1,2$, $j = 1,2$, $i \neq j$; No firms (NF).

The first thing to note is that the subgame formed by the last two stages of the game generates a number of Nash equilibria. Depending on F and σ, DD, DID_{ij}, DM, FM, DSM_i FSM_i all emerge as Nash equilibria. Figure 3-1 partitions the (F, σ) space in the regions corresponding to the various equilibrium configurations. It is seen that an increase in product substitutability always induces the firms to reduce their product lines.

[5]Table 1 is set-up using $a = b = 1$, and the figures which follow are drawn using those values but the existence, shape, and relative position of the regions are valid foe all admissible parameter values (a,b).

The role of the fixed cost is less clear cut however. When moving from region D to C, a reduction in fixed costs implies a shorter product line for a firm.[6]

Figure 3-1 : The equilibrium market structure as a function of F and σ, $s_1=s_2=0$



The curves represent the firm's best reply functions. and their equations are given below :

$$a : F = \frac{(1-2\sigma)(16-20\sigma+9\sigma^2)}{36(1-\sigma)(2-\sigma)^2} \qquad d : F = \frac{1-2\sigma}{4(1-\sigma)}$$

$$b : F = \frac{1-2\sigma}{9(1-\sigma)} \qquad e : F = \frac{1-\sigma}{(2-\sigma)^2}$$

$$c : F = \frac{1}{9(1-\sigma)} \qquad f : F = \frac{1}{4(1-\sigma)}$$

[6]See De Fraja (1992)

Given the structure of the game outlined above, the domestic government can influence the firm's product line decisions to maximize domestic welfare. To find the policy that maximizes welfare given the ability to affect the firm's product composition, we first characterize the optimal policy given the market structure, as a function of the parameters of the model. We then proceed to compare the optimal values to decide which one gives the largest national rent.

In region A of figure 1, the parameters (F,σ) are such that firms are initially duopolists on both markets. It was established by the profit shifting literature that a small production subsidy given to the home firm will increase the profit net of subsidy (i-e national rent). The argument extends readily to the case of multiproduct competition. The optimal profit shifting policy involves a uniform subsidy s^s_{DD} that enables the home firm to achieve a Stackelberg position on both markets. The existence of fixed costs given σ, however makes it appropriate to define $\bar{s}(\sigma,F)$, where $\bar{s}(\sigma,F)$ is the uniform single good limit subsidy given that the home firm is active on both markets. The policy that supports DD as the unique equilibrium product configuration is given by $s_{DD} = \min(s^s_{DD}, \bar{s}(\sigma,F))$. This constraint has the effect of partitioning the (F,σ) space in subregions. When $F(\sigma) > F_0(\sigma)$, where $F_0(\sigma)$ defines the parameter values for which s^s_{DD} makes the foreign firm indifferent between supplying one or two products when the domestic firm offers the entire product line, the limit subsidy is smaller than the optimal profit shifting subsidy and the optimal subsidy s_{DD} is the limit subsidy. (See Appendix I-A)

The set of policies s_{DMD} that implements DMD as an equilibrium in region A involves also several subsidy combinations as a function of F and σ.[7] The home government is able to impose a non uniform subsidy combination $(\bar{s}_i(\sigma,F), \bar{s}_j = \frac{1}{4})$ on a sub domain of (F,σ), where $\bar{s}_j$ maximizes domestic welfare given that $\bar{s}_i$ deters the foreign firm from market i, $i = 1,2; j = 1,2; i \neq j$. It is interesting to note at this point that the home country is never able to impose the optimal (non uniform) subsidy given DMD_{ij}. This illustrates the role of endogenous product selection in constraining the set of feasible policies available to the home country.[8] Above $F_0(\sigma)$, the constraint $s_j \leq s_i$ binds and the home government must resort to a uniform policy consistent with DMD. Let s^{s}_{DMD} denote the optimal uniform policy and $\bar{\bar{s}}(\sigma,F)$ the uniform subsidy that deters the foreign firm from both markets. Then s_{DMD} = max $(\bar{s}(\sigma,F)$; min$(s^{s}_{DMD}$,$\bar{\bar{s}}(\sigma,F)$). Again s_{DMD} defines subregions within region A where alternative policies must be used to produce DMD. For $F(\sigma) \in (F_0(\sigma), F_{11}(\sigma))$, where $F_{11}(\sigma)$ represents the parameter values for which s^{s}_{DMD} makes the foreign firm indifferent between supplying one or two products when the domestic firm offers the entire product line, the optimal (uniform) subsidy s^{s}_{DMD} does not induce the foreign firm to choose one product and the limit subsidy $\bar{s}(\sigma,F)$ must be used. Above $F_{12}(\sigma)$ and below locus a, where $F_{12}(\sigma)$ represents the parameter

[7] In what follows. DMD is used to indicate that both DID_{12} and DID_{21} arise as equilibria. which is the case when uniform subsidies are used to deter the foreign firm from one market.

[8] The optimal subsidy given DMD_{ij} solves $\arg\max_{s_i, s_j} W^{H}_{DMDij}$ and is given by $(s_i, s_j) = \left(\frac{\sigma}{4(1-\sigma)}, \frac{1}{4}\right)$. It can be checked that $s_i = \frac{\sigma}{4(1-\sigma)}$ is never consistent with DMD_{ij} for all values of F and σ where a non uniform policy is feasible.

combinations for which the optimal uniform subsidy given DMD, s^s_{DMD}, is equal to the two-good entry deterring (uniform) subsidy $\bar{\bar{s}}(\sigma,F)$, s^s_{DMD} deters the foreign firm from both markets and so, $\bar{\bar{s}}(\sigma,F)$ must be used. Here product substitutability must be low for s^s_{DMD} to deter the foreign firm all-together. Thus DMD is a unique equilibrium product configuration if s_{DMD} = $(\bar{s}_i, 1/4)$ when $\bar{s}_i$ $\geq$ 1/4 and s_{DMD} = max $(\bar{s}(\sigma,F)$: $\min(s^s_{DMD}, \bar{\bar{s}}(\sigma,F))$ when $\bar{s}_i < 1/4$. (See Appendix I-B)

Since the foreign firm can supply either of the two goods, the policy that deters it from producing at all involves equal subsidies on both goods. So in region A, $s_{DM}=\bar{\bar{s}}(\sigma,F)$ generates DM as the unique equilibrium configuration. (See Appendix I-C)

The domestic government can also induce both firms to specialize. The policy that induces DID$_{ij}$ in region A, denoted $(s^i_{DID_{ij}}, s^j_{DID_{ij}})$ involves an export subsidy on good i and an export tax on good j. $s^i_{DID_{ij}}$ = max $(\bar{s}_i(\sigma,F), s^s_i)$ deters the foreign firm from the production of good i. Again $s^i_{DID_{ij}}$ defines subregions of the parameters (F,σ) where the single good entry deterring subsidy $\bar{s}_i(\sigma,F)$ must be used for $F(\sigma) \leq F_{IS}(\sigma)$, where $F_{IS}(\sigma)$ represents the parameter values for which s^s_i makes the foreign firm indifferent between supplying both products and product j when the domestic firm offers product i only. The optimal profit shifting subsidy s^s_i given DID$_{ij}$ can be achieved over a small portion of region A without inducing the foreign firm to enter market i. The export tax $s^j_{DID_{ij}} = \min\left(0; \min\left(s^{00}_j\left(\sigma, F, s^i_{DID_{ij}}\right), s_j\left(\sigma, s^i_{DID_{ij}}\right)\right)\right)$ is used to deter the home firm from good

j whatever the foreign firm does and so guarantees the uniqueness of DID_{ij}. (See Appendix I-D)

Finally, the policy that implements DSM_i is also given by a combination of subsidies and taxes, where $s^i_{DSM_i} = \tilde{\bar{s}}_i(\sigma,F)$ deters the foreign firm from market j while $s^j_{DSMi} = \min\left(0; \min\left(s_j^{00}\left(\sigma, F, \tilde{\bar{s}}_i(\sigma,F)\right), s_j\left(\sigma, \tilde{\bar{s}}_i(\sigma,F)\right)\right)\right)$ ensures that the home firm is only active in market i. (See Appendix I-E)

Region B of figure 1 displays initially multiple equilibria. DD and DID_{ij}, $i = 1,2$, $j = 1,2$, $i \neq j$ arise as Nash equilibria of the subgame played by the firms. So here in addition to implementing a particular product configuration, the home country's policies can be used to eliminate the multiplicity problem. In this region $s_{DD} = \bar{s}(\sigma,F)$, implements DD as the unique equilibrium in a subregion only, defined by $F(\sigma) \leq F_3(\sigma)$, where $F_3(\sigma)$ represents the parameter combinations for which the single good (uniform) entry deterring subsidy, $\bar{s}(\sigma,F)$, is equal to the two-good (uniform) entry inducing subsidy, $s^0(\sigma,F)$, and $F_3(\sigma)$ lies entirely between locuses a and b. Above $F_3(\sigma)$ the single good limit subsidy does not eliminate DID_{ij}. However, as will be shown below, DD is never optimal in this region and so the multiplicity problem can be ignored.

$s_{DMD} = \min\ (s^s_{DMD}\ , \bar{\bar{s}}(\sigma,F)$ deters the foreign firm from one market and eliminates DID_{ij} since the home firm finds it now profitable to offer both products given the subsidies.

In this region, the home country can use the single good optimal profit shifting subsidy s_i^s given DID_{ij} while an appropriate tax on good j eliminates DD and DID_{21} if DID_{12} is targeted. Finally, both DM and DSM_i are induced by the same policies used in region A.

It is apparent that duopoly on both markets can not exist above locus b. Given that the foreign firm is active in good j, the home firm will supply the entire product line if s_{DMD} = max (max $(s^0(\sigma,F), s_{SD}^0(\sigma,F))$; min $(s_{DMD}^s, \bar{\bar{s}}(\sigma,F)))$ for $F(\sigma) \leq$ min $(F_{14}(\sigma), F'_{14}(\sigma))$ in region D. $s^0(\sigma,F)$ and $s_{SD}^0(\sigma,F)$ are the subsidies on both goods which induce the home firm to provide the entire product line over good i and j respectively when the foreign firm produces good j only. $F_{14}(\sigma)$ and $F'_{14}(\sigma)$ define the parameter values for which $\bar{\bar{s}}(\sigma,F)$ makes the domestic firm indifferent between supplying the entire product line and good i and j respectively when the foreign firm supplies good j only. The fact that the two-good entry inducing subsidy is given by max $(s^0(\sigma,F), s_{SD}^0(\sigma,F))$ can be readily explained: given the foreign firm's behaviour and for subsidy levels large enough $(s > (2-3\sigma)/2)$, the home firm prefers competition in the same good over differentiated product competition. This implies in turn that $s_{SD}^0(\sigma,F) > s^0(\sigma,F)$. So in general, the two good entry inducing subsidy is given by max $(s^0(\sigma,F), s_{SD}^0(\sigma,F))$ which defines sub regions of the parameter space. When $F(\sigma) \leq F_{15}(\sigma)$, $s_{SD}^0(\sigma,F) \leq s^0(\sigma,F) \leq (2-3\sigma)/2$ while $s_{SD}^0(\sigma,F) > s^0(\sigma,F) > (2-3\sigma)/2$ for $F(\sigma) > F_{15}(\sigma)$, where $F_{15}(\sigma)$ characterizes the parameter values for which $s^0(\sigma,F) = s_{SD}^0(\sigma,F)$. (See Appendix I-B). When both F and σ are not too high, the home government can use the optimal subsidy s_{DMD}^s given DMD. For higher values of F and σ the entry inducing subsidy max $(s^0(\sigma,F), s^0_{SD}(\sigma,F))$ must be used for the home firm to supply both products. When σ is low, $\bar{\bar{s}}(\sigma,F)$ is used, not to deter the foreign firm all-together. Above min $(F_{14}(\sigma), F'_{14}(\sigma))$ which lies entirely between loci *b* and *c* firms can not coexist in a DMD configuration since in this region max $(s^0(\sigma,F), s^0_{SD}(\sigma,F)) \geq \bar{\bar{s}}(\sigma,F)$.

Alternatively, the home government can take DID_{ij} as given and impose the optimal profit shifting subsidy for $F(\sigma) \leq F_{19}(\sigma)$, where $F_{19}(\sigma)$ defines the parameter values for which s_i^s makes the foreign firm indifferent between supplying good *j* and not producing at all when the domestic firm supplies good *i*. For $F(\sigma) > F_{19}(\sigma)$, $s_i^s > \tilde{\bar{s}}_i(\sigma,F)$ and thus $\tilde{\bar{s}}_i(\sigma,F)$ must be used given DID_{ij} . Thus in region D, $s^l_{DID_{ij}}$ = min (s_i^s , $\tilde{\bar{s}}_i(\sigma,F)$).[9] As before $s^l_{DID_{ij}}$ is set so as to eliminate DID_{21} if DID_{12} is the targeted product configuration.

In region C however, $s^l_{DID_{ij}}$ must eliminate the foreign monopoly. Here, s_i^s is sufficiently high to eliminate FM for $F(\sigma) \leq F_{20}(\sigma)$. $F_{20}(\sigma)$ defines the parameter values for which s_i^s makes the domestic firm indifferent between supplying one good and not producing at all when the foreign firm supplies the entire product line. For $F(\sigma)$ above $F_{20}(\sigma)$ and below locus *d*, the entry inducing subsidy $\hat{s}_i(\sigma,F)$ must be used where

[9] It is worth noting that $\tilde{\bar{s}}_i(\sigma,F)$ will never be optimal, since the profit of the corresponding limit output is always larger in a one firm equilibrium than the profit of approximately the same production produced in a two firm differentiated product equilibrium.

$\hat{s}_i(\sigma, F)$ is the single good entry inducing subsidy given that the foreign firm is active on both markets. So in region C, $s^i_{DID_{ij}} = \max(s^s_i,\ \hat{s}_i(\sigma, F))$ while an appropriate tax $s^j_{DID_{ij}}$ eliminates DM as well as DID_{21} if DID_{12} is targeted. There is one interesting point worth noticing here. Given σ for example, when moving from region C to D the home government is able to switch from $\hat{s}_i(\sigma, F)$ to s^s_i since above locus *d* the foreign firm responds to the home firm not producing by supplying one product only. Thus the welfare function associated with DID_{ij} is discontinuous across locus *d*.

Regions C and D can be combined to consider DM and DSM_i since both s_{DM} and $(s^i_{DSM_i}, s^j_{DSM_i})$ eliminate DID_{ij} and FM. In general the policy $s_{DM} = \bar{\bar{s}}(\sigma, F)$ which deters the foreign firm from both markets allows in turn the home firm to commit to the entire product line over a large region of the parameter space (F, σ). Above min $(F_{14}(\sigma), F'_{14}(\sigma))$ where min $(F_{14}(\sigma), F'_{14}(\sigma))$ lies entirely between loci *b* and *c*, the entry inducing subsidy max $(s^0(\sigma,F), s^0_{SD}(\sigma, F))$ must be used to commit the home firm to both products.[10] As F increases given σ so does the required entry inducing subsidy. When $s^0(\sigma,F)$ is the relevant policy (i.e to the left of $F_{15}(\sigma)$), there exists a critical boundary denoted $F_{16}(\sigma)$ above which $s^0(\sigma,F)$ becomes larger than $\bar{\bar{s}}_i(\sigma, F)$, the entry deterring subsidy given that the foreign firm supplies one good only. $F_{16}(\sigma)$ characterizes the parameter values for which $s^0(\sigma,F)$ makes the foreign firm indifferent between supplying good *j* and not producing at all when the domestic firm supplies good *i*. However as F

[10]In region C, max $(s^0(\sigma,F), s^0_{SD}(\sigma, F)) = s^0(\sigma,F)$ and $s^0(\sigma,F) > \hat{s}_i(\sigma, F)$ which implies that the foreign monopoly is eliminated.

continues to increase given σ, $\tilde{\tilde{s}}_i(\sigma, F)$ decreases and there exists another critical boundary denoted $F_{17}(\sigma)$ such that $\tilde{\tilde{s}}_i(\sigma, F)$ becomes smaller than $s^{00}(\sigma,F)$, where $s^{00}(\sigma,F)$ is the entry inducing subsidy given that the foreign firm is inactive. $F_{17}(\sigma)$ represents the parameter values for which $\tilde{\tilde{s}}_i(\sigma, F)$ makes the domestic firm indifferent between supplying the entire product line and one product when the foreign firm is inactive. Thus $s_{DM} = \max\ (\bar{\bar{s}}(\sigma, F)\ ;\ \min(s^0(\sigma,F)\ ;\ \max(\tilde{\tilde{s}}_i(\sigma, F), s^{00}(\sigma,F))))$ for $F(\sigma) \leq \min\ (F_{15}(\sigma)\ ;\ \frac{1-\sigma}{(2-\sigma)^2})$ where $\frac{1-\sigma}{(2-\sigma)^2}$ is locus e in figure 1. Above $F_{16}(\sigma)$, the entry inducing subsidy $s^0(\sigma,F)$ is no longer needed since at these subsidy levels, the foreign firm ceases production even if the home firm produces one good only. The entry inducing subsidy $s^{00}(\sigma,F)$ is required above $F_{17}(\sigma)$ however given that the foreign firm is not active. Finally $s_{DM} = \max\ (\bar{\bar{s}}(\sigma, F)\ ;\ (s^0_{SD}(\sigma, F))$ for $F(\sigma) \geq F_{15}(\sigma)$. [11]The DM equilibrium need not be considered above locus e since both firms choose to be single-product provided the rival is not active in this region. Note that in region C, the foreign firm's best response to the home firm producing both products is not to produce at all and $s^0(\sigma,F)$ is used to eliminate DID_{ij} and FM.

[11] Note that domestic welfare associated with DM increases in the region where $\tilde{\tilde{s}}_i(\sigma, F)$ is used since $\frac{\partial \tilde{\tilde{s}}(\sigma, F)}{\partial F} < 0$ given σ. Also, since $s^0_{SD}(\sigma, F) = s^{00}(\sigma,F)$ for all σ and F, $\tilde{\tilde{s}}_i(\sigma, F)$ can never be used for $F(\sigma) \geq F_{15}(\sigma)$ since this would require $s^0_{SD}(\sigma, F) < \tilde{\tilde{s}}_i(\sigma, F) < s^{00}(\sigma,F)$.

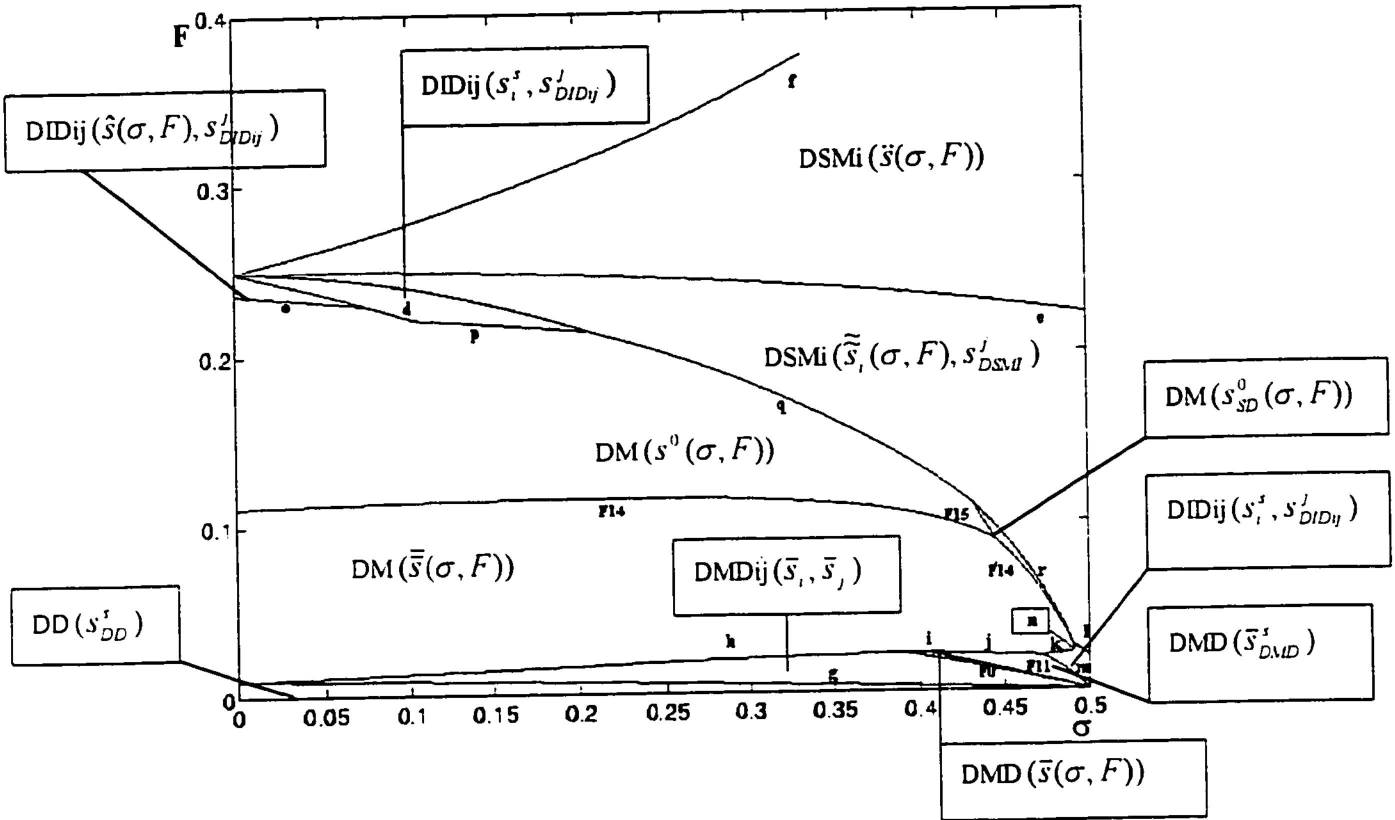
Finally in region E of figure 1, DSM_i and FSM_i, $i = 1,2$ are initial equilibria. Here a uniform subsidy on both goods $\tilde{s}_i(\sigma, F) = \frac{(2-\sigma)(2-3\sigma)}{2(1-\sigma)}\left(\frac{F}{1-\sigma}\right)^{\frac{1}{2}} - \frac{(2-3\sigma)}{2(1-\sigma)}$ is sufficient to eliminate FSM_i, $i = 1,2$; where $\tilde{s}_i(\sigma, F)$ is i's entry inducing subsidy when the foreign firm produces j only, $i \neq j$. Only the subsidy associated with the good produced in equilibrium enters the domestic welfare function however.

We now turn to the analysis of the optimal policy given the possibility to affect the firm's product composition. A natural way to proceed would be to compare the optimal values for the subsidies given the market structure to decide which subsidy gives largest national rent.[12] However since there are many of these optimal values this method proves rapidly inefficient. An equivalent way of characterizing the optimal policy is to compare the welfare associated with the different product configurations supported by the relevant policies.[13] Such comparisons define regions of the parameters space (F, σ) where each product configuration is welfare optimal. It is then easy to infer the optimal policy that underlies it.

[12] For example when interested by the comparison between s^s_{DD} and s_{DMD}, it is always possible to find critical levels of s such that the national rent of a domestic monopoly-duopoly structure is equal to the national rent of a double duopoly structure (with the domestic firm achieving a stackelberg position on both markets). The subsidy s is referred as the stackelberg equivalent subsidy. Now if s_{DMD} is lower than this critical s, and s_{DMD} is feasible, then s_{DMD} is welfare superior to s^s_{DD}.

[13] Welfare functions for the different product configurations are given in appendix I.

Figure 3-2 : Optimal product configurations and implied policies



$g: W^H_{DMDij}(\bar{s}_i, \bar{s}_j) = W^H_{DD}(s^s_{DD})$

$h: W^H_{DM}(\bar{\bar{s}}) = W^H_{DMDij}(\bar{s}_i, \bar{s}_j)$

$i: W^H_{DM}(\bar{\bar{s}}) = W^H_{DMD}(\bar{s})$

$j: W^H_{DM}(\bar{\bar{s}}) = W^H_{DMD}\left(s^S_{DMD}\right)$

$F_0(\sigma): \bar{s}(\sigma, F) = \bar{s}_i(\sigma, F)$

$F_{11}(\sigma): s^s_{DMD} = \bar{s}(\sigma, F)$

$k: W^H_{DM}(\bar{\bar{s}}) = W^H_{DIDij}\left(s^s\right)$

$l: W^H_{DM}\left(s^0_{SD}\right) = W^H_{DIDij}\left(s^s\right)$

$m: W^H_{DIDij}(s^s) = W^H_{DMD}(s^s_{DMD})$

$n: W^H_{DSMi}\left(\tilde{\bar{s}}_i, s_j^{DSMi}\right) = W^H_{DIDij}\left(s^s, s_j^{DIDij}\right)$

$F_{14}(\sigma): \bar{\bar{s}}(\sigma, F) = s^0(\sigma, F)$

$F'_{14}(\sigma): \bar{\bar{s}}(\sigma, F) = s^0_{SD}(\sigma, F)$

$F_{15}(\sigma): s^0(\sigma, F) = s^0_{SD}(\sigma, F)$

$o: W^H_{DIDij}(\hat{s}_i) = W^H_{DM}(s^0)$

$p: W^H_{DIDij}(s^s) = W^H_{DM}(s^0)$

$q: W^H_{DSMi}\left(\tilde{\bar{s}}_i, s_j^{DSMi}\right) = W^H_{DM}\left(s^0\right)$

$r: W^H_{DSMi}\left(\tilde{\bar{s}}_i, s_j^{DSMi}\right) = W^H_{DM}\left(s^0_{SD}\right)$

Figure 3-2 partitions the parameter space (F, σ) into regions where alternative product configurations supported by the relevant policies maximise domestic welfare.

The relative benefit of each policy is seen to be a function of the exogenous parameters of the model F and σ. In general, given σ, entry deterring policies become the optimal policy as the fixed cost per product F increases. DMD_{ij} where the foreign firm is deterred from market i only enters first. As F increases further the home goverment finds it profitable to deter the foreign firm all together achieving DM when σ is not too high and SM for high substitutability between products. This result is not surprising since the limit subsidy gets smaller when the fixed costs get larger. For moderate values of σ and above min $(F_{14}(\sigma), F'_{14}(\sigma))$ the limit subsidy is too low to support DM and the entry inducing subsidy max $(s^0(\sigma,F), s^0_{SD}(\sigma, F))$ becomes the optimal policy. Given F now, the home goverment induces the domestic firm to specialize as products become better substitutes, achieving DID_{ij} when F is low and entry deterrence costly or DSM_i when F is higher. The fact that DID_{ij} appears as the optimal product configuration in the upper left corner of figure 2 may seem counter intuitive at first. In this region fixed costs are too high to justify an entry inducing policy and low product substitutability makes entry deterrence too costly given duopolistic competition in differentiated products.[14]

It is possible to identify several motives for trade policy by comparing figures 3-1 and 3-2. When DD is the initial market structure trade policy can be used to shift

[14] In the limit as $\sigma = 0$, it is impossible to implement DSM_i and the optimal policy is $\hat{s}_i(\sigma, F)$ for F sufficiently high.

profits to the domestic firm (s^s_{DD}).[15] It involves a combination of rent shifting and entry deterrence for higher values of F and σ. In particular the home goverment is able to impose the optimal rent shifting subsidy $\bar{s}_j$ and the policy displays non uniform subsidies $(\bar{s}_i, \bar{s}_j)$ in a subregion of the parameters. Trade policy is also used to deter the foreign firm all-together using $\bar{\bar{s}}(\sigma, F)$. So, economies of scale coupled with some degree of product substitutability serve as an argument for entry deterring policies.

In regions B, C and E of figure 3-1, trade policy is used to eliminate some equilibria. In region C, the entry inducing subsidy $s^0(\sigma,F)$ eliminates both DID_{ij} and FM. In region E, $\ddot{s}_i(\sigma, F)$ensures that the single product domestic monopoly is the only equilibrium.

In part of region D, trade policy induces product proliferation for purposes of entry deterrence, using $\bar{\bar{s}}(\sigma, F)$ when $F(\sigma)$ is not too high, max $(s^0(\sigma,F), s^0_{SD}(\sigma, F))$ otherwise. In this region the level of fixed costs given σ is consistent with the domestic firm being multiproduct. As F given σ increases further, $(\tilde{\bar{s}}_i(\sigma, F)\ ;\ s^j_{DSMi})$ which induces DSM_i, becomes the optimal tax-subsidy policy package. Finally, a tax-subsidy combination is used with DID_{ij} being the associated product configuration. The home government can impose the optimal profit shifting subsidy s^s_i except in part of the upper left region of figure 3-2 where the subsidy must be higher ($\hat{s}$) to eliminate FM. The tax is merely used to signal that the domestic firm will not produce good j in equilibrium. So

[15] The only difference with Brander and Spencer (1985) is that firms can be multiproduct.

when F is low and products are good substitutes ($\sigma = 1/2$ in the limit) the optimal policy boils down to the standard profit shifting motive for intervention examined by Brander and Spencer (1985) except that a tax must also be used here because of endogenous product selection on the part of firms.[16] So in general, in addition to manipulating price/output decisions, trade policy can be used to help the domestic firm commit to a product line.

3-4 DISCONTINUITIES IN THE OPTIMAL POLICY

An interesting point not sufficiently addressed by the previous literature is that the optimal policy displays strong discontinuities with respect to the parameters of the model. Such discontinuities obviously derive from the discrete variations in the welfare level that correspond to market structure changes, as the figure below reveals.

[16] In Brander and Spencer (1985), products are homogenous.

Figure 3-3: Welfare discontinuities, F=1/25, σ= 1/3

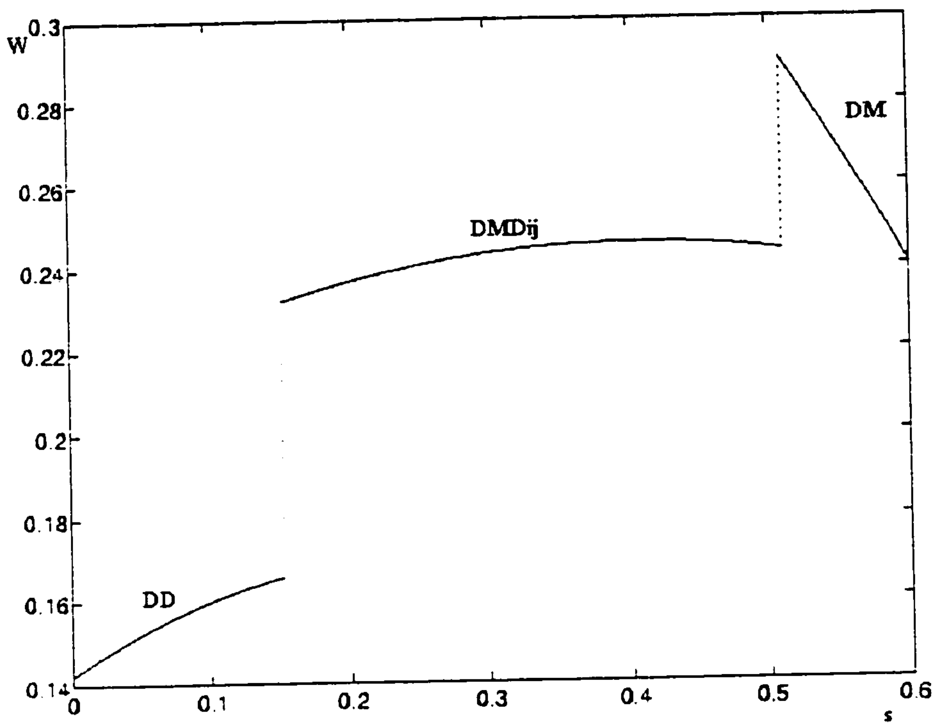
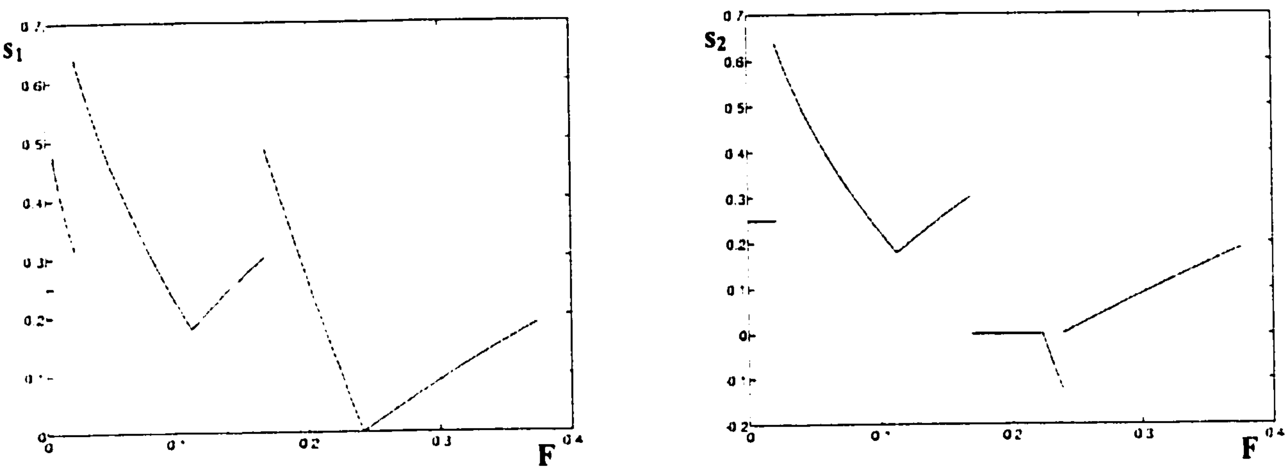


Figure 3-3 shows domestic welfare as a function of subsidies for given values of the parameters. It is drawn for σ= 1/3 and F = 1/25, so that the double duopoly market structure is the initial equilibrium. It clearly reveals the effects of a (uniform) subsidy combination as the equilibrium market structure shifts. Given these values for the parameters, the domestic government can not implement s^{s}_{DD}, the optimal rent shifting subsidy combination associated with DD. As s increases from zero, the equilibrium market structure shifts to DMD_{ij}, long before s^{s}_{DD} is reached. Increasing s further is optimal since domestic welfare attains its highest level when DM becomes the equilibrium market structure.[17]

The discontinuities in the optimal export policy are depicted in the next figure. We let $\sigma = 1/3$ and characterize the optimal export policy as a function of the fixed costs only.

Figure 3-4: Optimal policies as a function of F, $\sigma = 1/3$



policy on market 1

policy on market 2

Starting with F low, the optimal trade policy consists of a uniform subsidy $s^s_{DD} = 1/4$ on both goods. The economies of scale are so small that we can ignore them. As

[17] The optimal uniform policy is the lowest s which produces DM. We can safely consider uniform subsidies only, since we have established in the previous section that a non uniform subsidy combination is not optimal for these parameter values.

discussed previously this case is similar to the case with standard profit shifting where fixed costs are zero except that here firms are multiproduct.

As F increases the domestic government finds it profitable to deter the foreign firm from one market (say market 1). It can use a non uniform subsidy combination $(\bar{s}_1, \bar{s}_2)=\left(\frac{5-4\sqrt{18F}}{8}, \frac{1}{4}\right)$ on the range of F consistent with $\bar{s}_2 \leq \bar{s}_1$ where $\bar{s}_2$ is the optimal profit schifting subsidy on market 2 given that $\bar{s}_1$ deters the foreign firm from market 1.

For higher levels of F, the domestic government switches to $\bar{\bar{s}}=1-\sqrt{6F}$ as DM maximizes national rent in that range of the fixed cost. The entry inducing subsidy $s^0 = \frac{-83+10\sqrt{1+1314F}}{219}$ is required to sustain DM at higher levels of F, where DM is still optimal.

F eventually reaches a level where entry deterrence by product proliferation is no longer optimal and the home government induces DSM$_i$. The optimal policy consists of a tax-subsidy combination where $\tilde{\bar{s}}_1 = \frac{6-5\sqrt{6F}}{2}$ and $s_2 = \min\left(0; s_2 = \frac{29-25\sqrt{6F}}{8}\right)$ is set so as to ensure that DSM$_i$ is the unique equilibrium. Finally, above $F=\frac{6}{25}$, a uniform subsidy $\ddot{s} = \frac{5\sqrt{6F}-6}{8}$ eliminates FSM$_i$, and the domestic firm secures a monopoly position on good i.

3-5 CONCLUSION

In this paper, we have examined optimal export policy (to a third market) when firms choose their product lines endogenously. Absent any government intervention and depending on the parameters of the model, equilibrium product configurations lead to six different equilibrium market structures: International duopoly on both markets, duopoly with product specialization, domestic (foreign) monopoly on both markets, and single good domestic (foreign) monopoly.

This paper shows that the domestic country has an incentive to affect the firm's equilibrium product composition by setting non zero subsidy rates in order to maximize national rent. This incentive comes in addition to the now standard rent shifting motive for intervention examined by Brander and Spencer (1985) in a given market structure. Depending on the parameters of the model, rent-shifting, product line reducing and entry deterrence all constitute motives for intervention. Over a range of the parameters of the model, entry deterrence is achieved by product proliferation, so trade policy can help a firm commit to a product line. In general entry deterring (inducing) policies become optimal as fixed costs get larger (smaller) and product substitutability higher (lower). So economies of scale coupled with some degree of substitutability within the product line can serve as an argument for intervention. The Brander and Spencer (1985) result emerges when products are close substitutes and fixed costs are small. The results would extend naturally to the case where we broaden the firm's product lines.

This paper illustrates an important point : when there exist several different market structures, the domestic country's motives for intervention change dramatically over narrow ranges of parameters. As a result, the optimal trade policy exhibits strong discontinuities with respect to the parameters of the model as national rent maximization implies a switch of regimes.

An important issue not considered here is that of retaliation by the foreign government. Trade policy and retaliation have been analysed in several papers, Brander and Spencer (1985), Dixit and Kyle (1985), de Meza (1986) and Neary (1990). They have found that the international non cooperative equilibrium involves such subsidies by producing countries but that they are jointly suboptimal. Incorporated in our setting, retaliation could lead to different and interesting results. For moderate values of the fixed costs and product substitutability such that duopoly on both markets is the initial equilibrium, governments in both countries may want to deter the foreign firm from one market only, generating an international duopoly with product specialization instead. With product specialization both firm's profits are higher and welfare (defined as profits less subsidies) may rise in both countries with respect to free trade. This may in turn weaken the case for negotiation on trade liberalization and competition policies on the part of producing nations. In other words, government intervention may be the preferred policy when firms engage in excessive head to head competition. Further analysis along those lines is needed.

Appendix I : Optimal policies as a Function of F and σ

LEMMA A1 : Let s_i, $i=1,2$; be a given policy, $\Pi^F_{DIDij} \geq \Pi^F_{SDi}$ implies $s_i \geq \dfrac{-(2-3\sigma)}{4-3\sigma}$, $i=1,2;\ j=1,\ 2;\ i\neq j$. Thus, given that the home firm supplies good i only the foreign firm always chooses to differentiate it's product for all $s_i \geq 0$.

A corrolary to lemma A1 is that SD_i is never observed in equilibrium for all $s_i \geq 0$.

LEMMA A2 : Let $s_i = s_j = s$, $i = 1,\ 2;\ j = 1,\ 2;\ i \neq j$, be a uniform subsidy on both goods, $\Pi^H_{DIDij} > \Pi^H_{SDi}$ implies $s < \dfrac{2-3\sigma}{2}$. Thus, given that the foreign firm supplies good i only, the home firm prefers differentiated product competition (provide good j) over duopolistic competition in the same good (provide good i) if $s < \dfrac{2-3\sigma}{2}$.

A) *DD*

The subsidy combination which maximises W^H_{DD} is given by

$$s_1^S = s_2^S = s_{DD}^S = \underset{s_1 s_2}{\arg\max}\ W_{DD}^H \qquad \text{(A1)}$$

which yields the optimal uniform profit shifting subsidy structure $s_1^S = s_2^S = s_{DD}^S = \dfrac{1}{4}$.

Let $\bar{s}(\sigma,F)=1-3\left(\frac{1-\sigma}{1-2\sigma}F\right)^{\frac{1}{2}}$, $\sigma\neq\frac{1}{2}$ be the uniform single good entry deterring subsidy when the home firm offers both products

$$s^{s}_{DD}\geq\bar{s}(\sigma,F)\text{ implies }F(\sigma)\geq F_0(\sigma)=\frac{1-2\sigma}{16(1-\sigma)}$$

where $F_0(\sigma)$ lies entirely below locus a in figure 1.

Let $\dot{s}(\sigma,F)=\frac{3\left(\frac{1-\sigma}{1-2\sigma}F\right)^{\frac{1}{2}}-1}{2}$, $\sigma\neq\frac{1}{2}$ be the (uniform) entry inducing subsidy such that the home firm's best response is to supply both products when the foreign firm produces both products also. Applying lemma 2, let $\max\left(s^{0}(\sigma,F),s^{0}_{SD}(\sigma,F)\right)$ be the entry inducing subsidy on both goods when the foreign firm produces one good only, where $s^{0}(\sigma,F)$ and $s^{0}_{SD}(\sigma,F)$ are the subsidies such that the home firm supplies both products over good i and j respectively when the foreign firm supplies good j only, $i=1,2;\ j=1,2;\ i\neq j$.

$$s^{0}(\sigma,F)=\frac{-(2-3\sigma)\left[(2-3\sigma)(2-\sigma)^2(17-18\sigma)-72(1-\sigma)^3\right]+6(2-3\sigma)(2-\sigma)\sqrt{\Delta}}{(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4},\quad\sigma\neq\frac{1}{2}$$

where

$$\Delta=\sigma^2(1-2\sigma)^2+(1-\sigma)\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]F$$

and

$$s^{0}_{SD}(\sigma,F)=2\left(\frac{1-\sigma}{1-2\sigma}F\right)^{\frac{1}{2}}-1,\ \sigma\neq\frac{1}{2}$$

Now,

$s^{s}_{DD}\geq\max\left(s^{0}(\sigma,F),s^{0}_{SD}(\sigma,F)\right)$ implies $F(\sigma)\leq\min\left(F_1(\sigma),F_1'(\sigma)\right)=F_1(\sigma)$ for all $\sigma\in(0,1/2)$

since $s^{0}\left(\sigma^{0},F\right)<\dfrac{2-3\sigma}{2}$ in this region.

$F_1(\sigma)$ is defined by :

$$F_1(\sigma)=\frac{\left[(2-3\sigma)^2(2-\sigma)^2(31-30\sigma)-48(1-\sigma)^3(5-7\sigma)\right]^2-64\sigma^2(1-2\sigma)^2(2-3\sigma)^2(2-\sigma)^2}{64(2-3\sigma)^2(2-\sigma)^2(1-\sigma)\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]}$$

It is defined for all $\sigma\neq 1/2$, and lies everywhere above $F_0(\sigma)$.

While,

$$s^{s}_{DD}\geq\hat{s}(\sigma,F)\quad\text{implies}\quad F(\sigma)\leq F_2(\sigma)=\frac{1-2\sigma}{4(1-\sigma)}$$

where $F_2(\sigma)$ coincides with locus d in figure 1. Thus s^{s}_{DD} generates DD as the unique equilibrium product configuration in the region defined by $F_0(\sigma)$.

Let $\tilde{s}_i(\sigma,F)$ be the entry deterring subsidy that induces the foreign firm to produce good j when the home firm produces good i only, $i=1,2;\ j=1,2;\ i\neq j$. $\tilde{s}_i(\sigma,F)$ is defined as

$$\tilde{s}_i(\sigma,F)=\frac{(2-3\sigma)[(4\sigma-5)(3\sigma-4)+(5\sigma-4)]}{2(4-5\sigma)(4-3\sigma)}-\frac{3(2-3\sigma)(2-\sigma)\sqrt{\Delta}}{2(1-2\sigma)(4-5\sigma)(4-3\sigma)},\quad i=1,2$$

$$\sigma\neq\frac{1}{2}$$

where

$$\Delta=\sigma^2(1-2\sigma)^2+[4(1-2\sigma)(1-\sigma)(5\sigma-4)(3\sigma-4)]F.$$

Lemma A1 shows that the foreign firm always chooses differentiated product competition for all $s_i \geq 0$. Furthermore it can be checked that $\bar{s}(\sigma,F)$ implies $\tilde{s}_i(\sigma,F)$ for all F and σ below locus b.

Now, $\bar{s}(\sigma,F)\geq \max\left(s^0(\sigma,F), s^0_{SD}(\sigma,F)\right)$ implies $F(\sigma)\leq \min\left(F_3(\sigma), F_3'(\sigma)\right)=F_3(\sigma)$ since $s^0(\sigma,F)<\dfrac{2-3\sigma}{2}$ in this region and $F_3(\sigma)$ is defined as :

$$\left(\frac{1-\sigma}{1-2\sigma}F_3(\sigma)\right)^{\frac{1}{2}}=\frac{-2\left[(2-3\sigma)^2(2-\sigma)^2(7-6\sigma)-12(1-\sigma)^3(4-5\sigma)\right]}{\left[(2-3\sigma)^2(2-\sigma)^2(10\sigma-21)+144(1-\sigma)^4\right]}$$

$$\frac{+2(2-3\sigma)(2-\sigma)(1-2\sigma)^{\frac{1}{2}}\left\{\begin{array}{l}4\left[(2-3\sigma)^2(2-\sigma)^2(7-6\sigma)-12(1-\sigma)^3(4-5\sigma)\right]^2\\-\sigma^2(1-2\sigma)\left[(2-3\sigma)^2(2-\sigma)^2(10\sigma-21)+144(1-\sigma)^4\right]\end{array}\right\}^{\frac{1}{2}}}{\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]^{\frac{1}{2}}\left[(2-3\sigma)^2(2-\sigma)^2(10\sigma-21)+144(1-\sigma)^4\right]},\sigma\neq\frac{1}{2}$$

where $F_3(\sigma)$ lies strictly between loci a and b in figure 1.

and $\bar{s}(\sigma,F)\geq\hat{s}(\sigma,F)$ implies $F(\sigma)\leq F_4(\sigma)=\dfrac{1-2\sigma}{9(1-\sigma)}$ where $F_4(\sigma)$ coincides with locus b in figure 1. Note that $F_4(\sigma)$ defines an upper bound on the existence of DD, since above $F_4(\sigma)$, $\hat{s}(\sigma,F)>\bar{s}(\sigma,F)$

So DD emerges as the unique equilibrium product configuration if:

$$s_{DD}=\begin{cases} s_{DD}^{s}=\dfrac{1}{4} & 0<F(\sigma)\leq F_0(\sigma) \\ \bar{s}(\sigma,F)=1-3\left(\dfrac{1-\sigma}{1-2\sigma}F\right)^{\frac{1}{2}} & F_0(\sigma)<F(\sigma)\leq F_3(\sigma) \end{cases} \qquad \text{(A2)}$$

For $F_3(\sigma)<F(\sigma)\leq F_4(\sigma)$, $\bar{s}(\sigma,F)$ generates multiple equilibria DD and DID_{ij}, since $\hat{s}(\sigma,F)<\bar{s}(\sigma,F)<s^{ij}(\sigma,F)$ in this region.

The associated domestic welfare function is given by :

$$W_{DD}^{H}=\frac{2}{9}+\frac{s_1(1-2s_1)+s_2(1-2s_2)+2\sigma(s_1+s_2)(s_1+s_2-1)}{9(1-2\sigma)}-2F$$

where $s_1=s_2$ are defined in (A2).

B) *DMD*

Given that the home firm is active on both markets, the foreign firm is deterred from market i, $i = 1, 2$, if

$$\left(\bar{s}_i(\sigma, F), \bar{s}_j\right) = \left(\frac{4 - 7\sigma - 12[(1-\sigma)(1-2\sigma)F]^{\frac{1}{2}}}{4(1-\sigma)}, \frac{1}{4}\right) \qquad \text{for } 0 < F < F_0(\sigma) = \frac{1-2\sigma}{16(1-\sigma)}$$

(A3)

where

$$\bar{s}_j = \frac{1}{4} = \arg\max_{s_j} W^H_{DMD_{ij}} \quad \text{s/t} \begin{cases} s_i = \dfrac{1 + \sigma(sj - 2) - 3[(1-\sigma)(1-2\sigma)F]^{\frac{1}{2}}}{1-\sigma} \\ s_j \leq s_i \end{cases}$$

(A3-1)

So $\left(\bar{s}_i(\sigma.F), \bar{s}_j\right)$ is the non uniform subsidy combination that deters the foreign firm from market i while $F_0(\sigma)$ is implied by (A3.1).

To establish that $\left(\bar{s}_i(\sigma, F), \bar{s}_j\right)$ generates DMD_{ij} as the unique equilibrium of the game it is sufficient to show that the home firm has a dominent strategy to supply both products given $\left(\bar{s}_i(\sigma, F), \bar{s}_j\right)$ on the region of the parameters defined by $F_0(\sigma)$ and provided the foreign firm is active.

Let $\dot{\bar{s}}_i(\sigma, F) = \dfrac{3 - 5\sigma - 6[(1-\sigma)(1-2\sigma)F]^{\frac{1}{2}}}{4\sigma}$, $\sigma \neq 0$ be the entry inducing subsidy given $\bar{s}_i$ when the foreign firm supplies both products.

From (A3.1), $s_i \geq s_j$ in the region defined by $F_0(\sigma)$ and,

$$\bar{s}_i(\sigma,F) \leq \hat{\bar{s}}_i(\sigma,F) \quad \text{implies} \quad F(\sigma) \leq F_5(\sigma) = \frac{(1-2\sigma)^3}{4(1-\sigma)(1-3\sigma)^2}, \quad \sigma \neq \frac{1}{3}$$

and $F_0(\sigma) \leq F_5(\sigma)$ implies $\sigma \leq 1$.[1]

Let $s_i(\sigma) = \dfrac{3(1-\sigma)(5-7\sigma)}{4(2-\sigma)(2-3\sigma)} - \dfrac{1}{2}$ be the subsidy on good i given $\bar{s}_i$, such that the home firm is indifferent between duopolistic competition in the same good over differentiated product competition when the foreign firm supplies good i only, $i=1,2; j=1,2; i \neq j$.

$$\bar{s}_i(\sigma,F) \gtrless s_i(\sigma) \text{ implies } F(\sigma) \gtrless F_6(\sigma) = \frac{\left[(1-\sigma)^2(7\sigma-5)+(2-3\sigma)^2(2-\sigma)\right]^2}{16(1-\sigma)(1-2\sigma)+(2-\sigma)(2-3\sigma)^2}, \quad \sigma \neq \frac{1}{2}$$

where $F_6(\sigma)$ lies everywhere below $F_0(\sigma)$.

Let $s^0_{SD_i}(\sigma,F) = \dfrac{5-9\sigma-8[(1-\sigma)(1-2\sigma)F]^{\frac{1}{2}}}{4\sigma}$, $\sigma \neq 0$ and $s^0_{i,i}(\sigma,F)$ be the subsidies on good i given $\bar{s}_i$, such that the home firm is indifferent between supplying both goods over good i and j respectively when the foreign firm supplies good i only, $i=1,2; j=1,2; i \neq j$.

---

[1]The fact that the subsidy to the domestic firm must be below a critical level can be understood as follows. Because products are substitutes a subsidy on good i given $\bar{s}_i$ puts the domestic firm at a strategic disadvantage on market 2. When this subsidy is too high the domestic firm abandons the production of good j.

Necessary conditions for the home firm to supply both products given $\bar{s}_i$ are $\bar{s}_i(\sigma,F) \le s^0_{SD_i}(\sigma,F)$ for $F(\sigma) < F_6(\sigma)$ and $\bar{s}_i(\sigma,F) \ge s^0_{I/i}(\sigma,F)$ for $F_6(\sigma) < F(\sigma) < F_0(\sigma)$

Now,

$$\bar{s}_i(\sigma,F) \le s^0_{SD_i}(\sigma,F) \quad \text{implies} \quad F(\sigma) \le F_7(\sigma) = \frac{(1-2\sigma)(5-8\sigma)^2}{16(1-\sigma)(2-5\sigma)^2},\ \sigma \ne \frac{2}{5}$$

and

$$\bar{s}_i(\sigma,F) \ge s^0_{I,i}(\sigma,F) \quad \text{implies} \quad F(\sigma) \le F_8(\sigma) \text{ where } F_8(\sigma) \text{ is defined by :}^2$$

$$\left[(1-\sigma)(1-2\sigma)F_8(\sigma)\right]^{\frac{1}{2}} = \frac{12(1-\sigma)\left[4(1-2\sigma)(8-9\sigma)-9\sigma(1-\sigma)\right]-12(4-7\sigma)\left[4(2-\sigma)(2-3\sigma)-3\sigma^2\right]}{144\left[4(2-\sigma)(2-3\sigma)-3\sigma^2\right]-576(1-\sigma^2)}$$

$$-\frac{\left\{(2-\sigma)^2(2-3\sigma)^2\left[-12(1-\sigma)\left[4(1-2\sigma)(8-9\sigma)-9\sigma(1-\sigma)\right]-12(4-7\sigma)\left[4(2-\sigma)(2-3\sigma)-3\sigma^2\right]\right]^2-\left[144\left[4(2-\sigma)(2-3\sigma)-3\sigma^2\right]-576(1-\sigma)^2\right]\left[(2-\sigma)^2(2-3\sigma)^2\left[(1-\sigma)^2\left[40(1-2\sigma)(7-9\sigma)-9(1-\sigma)^2\right]+2(4-7\sigma)(1-\sigma)\left[4(1-2\sigma)(8-9\sigma)-9\sigma(1-\sigma)\right]+(4-7\sigma)^2\left[4(2-\sigma)(2-3\sigma)-3\sigma^2\right]\right]-144(1-\sigma)^4(1-2\sigma)(5-7\sigma)^2\right]\right\}^{\frac{1}{2}}}{(2-\sigma)(2-3\sigma)\left[144\left[4(2-\sigma)(2-3\sigma)-3\sigma^2\right]-576(1-\sigma^2)\right]}$$

---

[2] $F_8(\sigma)$ is obtained by substituting $\bar{s}_i, \bar{s}_j$ in Π^H_{DMDij} and Π^H_{DIDij} and solving for $F(\sigma)$.

$F_0(\sigma) < F_7(\sigma)$ implies $\sigma < 1$, while $F_8(\sigma)$ lies everywhere above $F_0(\sigma)$. This establishes that the home firm's best response to the foreign firm producing good *i* is to supply both products given $\bar{s}_i, \bar{s}_j$ in the region defined by $F_0(\sigma)$.

Finally, let $s_i'(\sigma) = \dfrac{\sigma(2-3\sigma)}{4(1-\sigma)^2}$ be the subsidy on good *i* given $\bar{s}_j$, which induces the home firm to supply the differentiated product when the foreign firm supplies good *j* only $i=1,2;$ $j=1,2;$ $i\neq j$

$$\bar{s}_i(\sigma, F) \geq s_i'(\sigma) \quad \text{implies} \quad F(\sigma) \leq F_9(\sigma) = \frac{(1-2\sigma)(4-5\sigma)^2}{144(1-\sigma)^3}$$

and $F_0(\sigma) \cdot F_9(\sigma)$ implies $\sigma < \dfrac{1}{2}$.

Now let $s_{i\,j}''(\sigma, F)$ be the subsidy on good *i* given $\bar{s}_j$ such that the home firm supplies both products over product *i* when the foreign firm is in good *j* only. The home firm supplies both products if and only if

$$\bar{s}_i(\sigma, F) \leq s_{i,j}^0(\sigma, F) \quad \text{which implies} \quad F(\sigma) < F_{10}(\sigma)$$

where

$$(F_{10}(\sigma))^{\frac{1}{2}} = 2(1-2\sigma)^{\frac{1}{2}}\left[3(2-\sigma)^2(2-3\sigma)-6(1-\sigma)^2(8-13\sigma)\right]$$
$$\frac{-(2-\sigma)(2-3\sigma)\left\{(2-\sigma)^2(2-3\sigma)^2(11-32\sigma)+(1-\sigma)^2\left[144(1-2\sigma)(5\sigma-3)+5(8-13\sigma)^2\right]\right\}^{\frac{1}{2}}}{(1-\sigma)^{\frac{1}{2}}\left[5(2-\sigma)^2(2-3\sigma)^2-144(1-\sigma)^2(1-2\sigma)\right]}$$

and $F_{10}(\sigma)$ lies everywhere above $F_0(\sigma)$. Combining the above results, the home firm's dominant strategy is to supply both products under the subsidy combination $(\bar{s}_i, \bar{s}_j)$ in the region defined by $F_0(\sigma)$ provided the foreign firm is active. This establishes that $(\bar{s}_i, \bar{s}_j)$ generates DMD_{ij} as the unique equilibrium of the game in this region.

Now, $\bar{s}_i < \bar{s}_j$ implies $F(\sigma) > F_0(\sigma)$ and the foreign firm will choose good i when faced with $(\bar{s}_i, \bar{s}_j)$ as defined in (A3). So, for $F(\sigma) > F_0(\sigma)$, the constraint $s_i = s_j$ binds and the domestic government must resort to a uniform subsidy.

The optimal uniform subsidy combination solves

$$s^s_{DMD} = \arg\max_s W^H_{DMD_{ij}} \quad \text{s/t } s_i = s_j;\ i = 1, 2;\ j = 1, 2;\ i \neq j$$

and is given by $s^s_{DMD} = \dfrac{2}{17-18\sigma}$.

The uniform subsidy combination which deters the foreign firm from one market (whichever one) provided the home firm is active on both markets is obtained by setting $s_i = s_j$ in (A3.1) and is given by

$$\bar{s}_1 = \bar{s}_2 = \bar{s}(\sigma, F) = 1 - 3\left(\frac{1-\sigma}{1-2\sigma}F\right)^{\frac{1}{2}}, \quad \sigma \neq \frac{1}{2}.$$

We have shown above that $\bar{s}(\sigma, F) > \tilde{s}_i(\sigma, F)$ for all F and σ below locus b.[3]

Now,

$$s^{s}_{DMD} \leq \bar{s}(\sigma, F) \text{ implies } F(\sigma) \leq F_{11}(\sigma) = \frac{(1-2\sigma)(5-6\sigma)^2}{(1-\sigma)(17-18\sigma)^2},$$

where $F_{11}(\sigma)$ lies strictly between $F_0(\sigma)$ and locus b in figure1, while

$$s^{S}_{DMD} \geq \bar{\bar{s}}(\sigma, F) \text{ implies } F(\sigma) \geq F_{12}(\sigma) = \frac{(5-6\sigma)^2}{(1-\sigma)(17-18\sigma)^2},$$

where $\bar{\bar{s}}(\sigma, F) = 1 - 3[(1-\sigma)F]^{\frac{1}{2}}$ is the uniform subsidy combination that deters the foreign firm all-together, when the home firm supplies two products.

3 So $\bar{s}(\sigma, F)$ introduces the single product as a dominant strategy for the foreign firm provided the home firm is active.

$s^{s}_{DMD} \leq \max\left(s^0(\sigma,F), s^0_{SD}(\sigma,F)\right)$ implies $\quad F(\sigma) \geq \min\left(F_{13}(\sigma), F'_{13}(\sigma)\right) = F_{13}(\sigma)$

since $s^0(\sigma,F) \leq \dfrac{2-3\sigma}{2}$ in that region

and,

$$F_{13}(\sigma) = \frac{\left(\dfrac{(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4}{3(2-3\sigma)(2-\sigma)(17-18\sigma)} + \dfrac{(2-3\sigma)(2-\sigma)^2(17-18\sigma)-72(1-\sigma)^3}{6(2-\sigma)}\right)^2 - \sigma^2(1-2\sigma)^2}{(1-\sigma)\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]}$$

$$\sigma \neq \frac{1}{2}$$

while

$$\bar{\bar{s}}(\sigma,F) \geq \max\left(s_0(\sigma,F), s^0_{SD}(\sigma,F)\right) \text{ implies } \quad F(\sigma) \leq \min\left(F_{14}(\sigma), F'_{14}(\sigma)\right)$$

where

$$\left[(1-\sigma)F_{14}(\sigma)\right]^{\frac{1}{2}} = \frac{-2\left[(2-3\sigma)^2(2-\sigma)^2(7-6\sigma)-12(1-\sigma)^3(4-5\sigma)\right]}{(2-3\sigma)^2(2-\sigma)^2(18\sigma-21)+144(1-\sigma)^4}$$
$$- \frac{2(2-3\sigma)(2-\sigma)\sqrt{\Delta}}{\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]^{\frac{1}{2}}\left[(2-3\sigma)^2(2-\sigma)^2(18\sigma-21)+144(1-\sigma)^4\right]}$$

$$\sigma \neq \frac{1}{2}$$

where

$$\Delta = 4\left[(2-3\sigma)^2(2-\sigma)^2(7-6\sigma)-12(1-\sigma)^3(4-5\sigma)\right]^2$$
$$-\sigma^2(1-2\sigma)^2\left[(2-3\sigma)^2(2-\sigma)^2(18\sigma-21)+144(1-\sigma)^4\right]$$

and

$$F'_{14}(\sigma)=\frac{4}{\left[2\left(\frac{1-\sigma}{1-2\sigma}\right)^{\frac{1}{2}}+3(1-\sigma)^{\frac{1}{2}}\right]^{2}},\qquad \sigma\neq\frac{1}{2}$$

Finally,

$$s^{0}_{SD}(\sigma,F)\geq s^{0}(\sigma,F)\ \text{ implies }\ F(\sigma)\geq F_{15}(\sigma)=\frac{(1-2\sigma)(4-3\sigma)^{2}}{16(1-\sigma)}$$

There is a region of the parameter space where $F_{15}(\sigma)$ lies strictly between $F_{13}(\sigma)$ and $F'_{14}(\sigma)$.[4] The loci $F_{12}(\sigma), F_{13}(\sigma)$ and $\min(F_{14}(\sigma), F'_{14}(\sigma))$ all intersect at the same point in the (F,σ) space. To the left of that point $F_{13}(\sigma) > min\, F_{14}(\sigma), F'_{14}(\sigma) > F_{12}(\sigma)$ while $F_{12}(\sigma) > min(F_{14}(\sigma), F'_{14}(\sigma)) > F_{13}(\sigma)$ to the right of that point.

The boundary $min(F_{14}(\sigma), F'_{14}(\sigma))$ defines an upper limit on the existence of DMD as an equilibrium. Above $\min(F_{14}(\sigma), F'_{14}(\sigma))$, $\max(s^{0}(\sigma,F), s^{0}_{SD}(\sigma,F))$ and $\bar{\bar{s}}(s,F)$ cannot be both satisfied for if $\max(s^{0}(\sigma,F), s^{0}_{SD}(\sigma,F))$ is used, the foreign firm will be deterred and if $\bar{\bar{s}}(\sigma,F)$ is used, the home firm will respond by supplying one good only. Thus, only those portions of $F_{12}(\sigma)$ and $F_{13}(\sigma)$ below $\min(F_{14}(\sigma), F'_{14}(\sigma))$ are relevant.

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[4] It can be checked that $F_{14}(\sigma), F'_{14}(\sigma)$ and $F_{15}(\sigma)$ all intersect at the same point where $\bar{\bar{s}}(\sigma,F)=s^{0}(\sigma,F)=s^{0}_{SD}(\sigma,F)$.

Both $\min(F_{14}(\sigma), F'_{14}(\sigma))$ and the portion of $F_{13}(\sigma)$ below $\min(F_{14}(\sigma), F'_{14}(\sigma))$ lie entirely in region C of Figure 1 (between loci *b* and *c*). This means that the home firm finds it profitable to supply both products in response to the foreign firm supplying only one in part of region C, when subsidised according to s^{s}_{DMD} and $\max(s^{0}(\sigma,F), s^{0}_{SD}(\sigma,F))$.

So the uniform subsidy combination, which sustains *DMD* as the unique equilibrium of the game is given by:

$$s_{DMD} = \begin{cases} \bar{s} = 1 - 3\left(\frac{1-\sigma}{1-2\sigma}F\right)^{\frac{1}{2}} & F_{0}(\sigma) < F(\sigma) \le F_{11}(\sigma) \\ s^{s}_{DMD} = \frac{2}{17-18\sigma} & F_{11}(\sigma) < F(\sigma) \le \min(F_{12}(\sigma), F_{13}(\sigma)) \\ \bar{\bar{s}} = 1 - 3[(1-\sigma)F]^{\frac{1}{2}} & F_{12}(\sigma) < F(\sigma) \le \min(F_{14}(\sigma), F'_{14}(\sigma)) = F_{14}(\sigma). \\ s^{0}(\sigma,F) & F_{13}(\sigma) < F(\sigma) \le \min(F_{14}(\sigma), F_{15}(\sigma)) \\ s^{0}_{SD} = 2\left(\frac{1-\sigma}{1-2\sigma}\right) & F_{15}(\sigma) < F(\sigma) \le \min(F_{14}(\sigma), F'_{14}(\sigma)) = F'_{14}(\sigma) \end{cases} \quad \text{(A4)}$$

The associated domestic welfare function is given by

$$W^{H}_{DMDij} = \frac{13-18\sigma}{3\sigma(1-\sigma)} + \frac{s_j}{9(1-\sigma)} - \frac{(1-\sigma)s_i^2}{4(1-2\sigma)} - \frac{(3\sigma^2 + 2(\sigma-2)(3\sigma-2))s_j^2}{3\sigma(1-\sigma)(1-2\sigma)} + \frac{\sigma s_i s_j}{2(1-2\sigma)} - 2F$$

where s_i, s_j are defined by (A3) and (A4).

C) *DM*

Let $\bar{\bar{s}}(\sigma,F) = 1-3[(1-\sigma)F]^{\frac{1}{2}}$ be the uniform subsidy on both goods that deters the foreign firm from both markets when the home firm supplies the full product line. It can be checked that $\bar{\bar{s}}(\sigma,F) > \tilde{\bar{s}}_i(\sigma,F)$ for all σ, F below locus c in figure 1. It was shown above that $\bar{\bar{s}}(\sigma,F) \geq \max(s^0(\sigma,F), s^0_{SD}(\sigma,F))$ implies $F(\sigma) \leq min(F_{14}(\sigma), F'_{14}(\sigma))$ and that $s^0(\sigma,F) \geq s^0_{SD}(\sigma,F)$ implies $F(\sigma) \leq F_{15}(\sigma)$.

Let $\tilde{\bar{s}}_i(\sigma,F) = \frac{(2-3\sigma)}{\sigma} - \frac{(2-3\sigma)(2-\sigma)}{\sigma}\left(\frac{F}{1-\sigma}\right)^{\frac{1}{2}}$, $\sigma \neq 0$ $i = 1, 2$, be the subsidy that deters the foreign firm all-together when the home firm supplies good i only[5] and $s^{00}(\sigma,F) = 2\left(\frac{1-\sigma}{1-2\sigma}F\right)^{\frac{1}{2}} - 1$ be the subsidy such that the home firm supplies the entire product line when the foreign firm is not active.

It is worth noticing first that $s^{00}(\sigma,F) = s^0_{SD}(\sigma,F)$.

Now,

$s^0(\sigma,F) \geq \bar{\bar{s}}(\sigma,F)$ implies $F(\sigma) > F_{16}(\sigma)$ where $F_{16}(\sigma)$ is defined by:

[5]Note that $\tilde{\bar{s}}_i(\sigma,F)$ is not defined for $\sigma = 0$, since when products are independent, a subsidy on good i produced by the home firm does not affect the production of good j by the foreign firm. Remember that by lemma 1 that the foreign firm always chooses to differentiate it's product for all $s_i \geq 0$, i=1,2: when both firms are single product suppliers.

$$\left(\frac{F_{16}(\sigma)}{1-\sigma}\right)^{\frac{1}{2}}=\frac{-(2-3\sigma)(2-\sigma)[(2-3\sigma)(25-18\sigma)+\sigma(17-18\sigma)]+72(1-\sigma)^3}{36\sigma^2(1-\sigma)^2-\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]}$$

$$+\frac{6\sigma\sqrt{\Delta}}{\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]^{\frac{1}{2}}\left(36\sigma^2(1-\sigma)^2-\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]\right)}$$

Where $\sigma \neq 1/2$, and

$$\begin{aligned}\Delta=&(1-\sigma)^2\left[(2-3\sigma)(2-\sigma)[(2-3\sigma)(25-18\sigma)+\sigma(17-18\sigma)]-72(1-\sigma)^3\right]^2\\&+\sigma^2(1-2\sigma)^2\left[(2-3\sigma)(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]\\&-36\sigma^4(1-\sigma)^2(1-2\sigma)^2\end{aligned}$$

and,

$$s^{00}(\sigma,F)\geq\bar{\bar{s}}(\sigma,F)\text{ implies }F(\sigma)>F_{17}(\sigma)=\left[\frac{2(1-\sigma)}{2\sigma\left(\frac{1-\sigma}{1-2\sigma}\right)^{\frac{1}{2}}+\frac{(2-3\sigma)(2-\sigma)}{(1-\sigma)^{\frac{1}{2}}}}\right]^2,\ \sigma\neq\frac{1}{2}.$$

Both $F_{16}(\sigma)$ and $F_{17}(\sigma)$ lie strictly between loci d and e in figure 1.[6] In the region to the right of $F_{15}(\sigma)$ where s^0_{SD} is the entry inducing subsidy, $\bar{\bar{s}}(\sigma,F)$ can never need to generate *DM*. This would require $s^{00}(\sigma,F)<\bar{\bar{s}}(\sigma,F)<s^0_{SD}(\sigma,F)$ which never holds since $s^{00}(\sigma,F)=s^0_{SD}(\sigma,F)$. Region C of figure 1 lies entirely between loci $F_{14}(\sigma)$ and $F_{16}(\sigma)$ and it can be checked that $s^0(\sigma,F)>\hat{s}(\sigma,F)$ in that region, where

[6] Again, all $F_{16}(\sigma)$, $F_{17}(\sigma)$ and $F_{15}(\sigma)$ intersect at the same point where $\bar{\bar{s}}_i(\sigma,F)=s^0(\sigma,F)=s^0_{SD}(\sigma,F)$.

$\hat{s}(\sigma, F) = \frac{[(1-\sigma)F]^{\frac{1}{2}}}{2} - 1$ is the single good entry inducing subsidy when the foreign firm provides the entire product line. So $s^0(\sigma, F)$ eliminates *FM* in that region.

Thus *DM* emerges as the unique equilibrium product configuration if:

$$s_{DM} = \begin{cases} \bar{s}(\sigma, F) = 1 - 3[(1-\sigma)F]^{\frac{1}{2}} & 0 < F(\sigma) \leq \min(F_{14}(\sigma), F'_{14}(\sigma)) \\ s^0(\sigma, F) & F_{14}(\sigma) < F(\sigma) \leq \min(F_{15}(\sigma), F_{16}(\sigma)) \\ \bar{s}(\sigma, F) = \frac{(2-3\sigma)}{\sigma} - \frac{(2-3\sigma)(2-\sigma)}{\sigma}\left(\frac{F}{1-\sigma}\right)^{\frac{1}{2}} & F_{16}(\sigma) < F(\sigma) \leq \min(F_{15}(\sigma), F_{17}(\sigma)) \\ s^{00}(\sigma, F) = s^0_{SD}(\sigma, F) = 2\left(\frac{1-\sigma}{1-2\alpha}\right)^{\frac{1}{2}} - 1 & \min(F_{15}(\sigma), F_{17}(\sigma)) < F(\sigma) \leq \frac{1}{4(1-\sigma)} \end{cases} \quad \text{(A5)}$$

The associated domestic welfare function is given by

$$W^H_{DM} = \frac{1}{2} + \frac{2\sigma s_1 s_2 - (1-\sigma)(s_1^2 + s_2^2)}{4(1-2\sigma)} - 2F$$

where $s_1 = s_2$ are defined by (A5).

D) DID_{ij}

By lemma A1, the foreign firm always chooses to differentiate it's product from the home firm's for all $s_i \geq 0$, $i = 1, 2$.

The subsidy that maximises national welfare under differentiated duopolistic competition solves.

$$s_i = \arg\max_{s_i} W^H_{DID_{ij}}, \quad i = 1, 2;\ j = 1, 2;\ i \neq j$$

and is given by:[7]

$$s_i^s = \frac{\sigma^2(3\sigma - 2)}{4(1-\sigma)\left(\sigma^2 - 2(1-\sigma)^2\right)}$$

$s_i^s \geq \tilde{s}_i(\sigma, F)$ implies $F \geq F_{18}(\sigma)$ where :

$$F_{18}(\sigma) = \frac{\left(\frac{\sigma^2(1-2\sigma)(4-5\sigma)(4-3\sigma)}{6(1-\sigma)(2-\sigma)\left(\sigma^2 - 2(1-\sigma)^2\right)} + \frac{(1-2\sigma)\left[(4\sigma-5)(3\sigma-4)+(5\sigma-4)\right]}{3(2-\sigma)}\right)^2 - \sigma^2(1-2\sigma)^2}{4(1-2\sigma)(1-\sigma)(5\sigma-4)(3\sigma-4)}, \sigma \neq \frac{1}{2}$$

$F_{18}(\sigma)$ lies strictly below locus a in figure 1.

And

$$s_i^s \geq \tilde{\tilde{s}}_i(\sigma, F) \quad \text{implies} \quad F \geq F_{19}(\sigma) = (1-\sigma)\left(\frac{1}{2-\sigma} + \frac{\sigma^3}{4(2-\sigma)(1-\sigma)\left(\sigma^2 - 2(1-\sigma)^2\right)}\right)^2.$$

where $F_{19}(\sigma)$ lies strictly between loci d and e in figure 1.

[7] It should be noted that $s^s_i = 0$ when $\sigma = 0$. When products are independant, a subsidy on good i does not affect the foreign production of good j.

Let $\hat{s}_i(\sigma,F)=\dfrac{3[(1-\sigma)F]^{\frac{1}{2}}-1}{2}$ be the single good entry inducing subsidy such that the home firm's best response is to supply good i when the foreign firm is in both goods, $i = 1, 2$.

Then,

$$s_i^s \geq \hat{s}_i(\sigma,F) \text{ implies } F \leq F_{:0}(\sigma)=\frac{\left(\sigma^3-4(1-\sigma)^3\right)^2}{36(1-\sigma)^3\left(\sigma^2-2(1-\sigma)^2\right)^2}.$$

while

$$\hat{s}_i(\sigma,F)\geq \tilde{\bar{s}}_i(\sigma,F) \text{ implies } F(\sigma)\geq F_{:1}(\sigma)=\frac{(4-5\sigma)^2}{\left[3\sigma(1-\sigma)^{\frac{1}{2}}+\dfrac{2(2-3\sigma)(2-\sigma)}{(1-\sigma)^{\frac{1}{2}}}\right]^2}$$

where $F_{:1}(\sigma)$ lies everywhere above locus d in figure 1.

Above $F_{:0}(\sigma)$ and below locus d in figure 1, the optimal subsidy s_i^s does not eliminate FM and the entry inducing subsidy $\hat{s}_i(\sigma,F)$ must be used. Furthermore, DID_{ij} need not be defined for parameter combinations above locus e since the foreign firm's best response to the domestic firm supplying product i is not to produce at all in that region.

Thus, DID_{ij} is supported as the unique equilibrium of the game by a vector of policies $\left(s^i_{DID_{ij}}, s^j_{DID_{ij}}\right)$, $i = 1, 2; j = 1, 2; i \neq j$, where

$$s^{i}_{DID_{ij}} = \begin{cases} \bar{s}_i(\sigma, F) & 0 < F(\sigma) \le F_{18}(\sigma) \\ s^s_i(\sigma, F) = \dfrac{\sigma^2(3\sigma - 2)}{4(1-\sigma)\left(\sigma^2 - 2(1-\sigma)^2\right)} & F_{18}(\sigma) < F(\sigma) \le F_{19}(\sigma) \\ & \text{to exclusion of } F_{20}(\sigma) < F(\sigma) \le d \\ \hat{s}_i(\sigma, F) = \dfrac{3[(1-\sigma)F]^{\frac{1}{2}} - 1}{2} & F_{20}(\sigma) < F(\sigma) \le d \\ \bar{\bar{s}}_i(\sigma, F) = \dfrac{2 - 3\sigma}{\sigma} - \dfrac{(2-3\sigma)(2-\sigma)}{\sigma}\left(\dfrac{F}{1-\sigma}\right)^{\frac{1}{2}} & F_{19}(\sigma) < F(\sigma) < e \end{cases} \tag{A6}$$

and

$$s^{j}_{DID_{ij}} = \min\left(0, \min\left(s^{(0)}_j\left(\sigma, F, s^{i}_{DID_{ij}}\right), s_j\left(\sigma, s^{i}_{DID_{ij}}\right)\right)\right) \tag{A7}$$

where $s^{(0)}_j(\sigma, F, s^{i}_{DID_{ij}}) = \dfrac{-(1-2\sigma) + \sigma s^{i}_{DID_{ij}} + 2[(1-\sigma)(1-2\sigma)F]^{\frac{1}{2}}}{(1-\sigma)}$, $i = 1,2$ $j = 1,2$ $i \neq j$ is the export tax on good j given $s^{i}_{DID_{ij}}$ such that the home firm prefers to supply good i only when the foreign firm is inactive and $s_j(\sigma, s^{i}_{DID_{ij}}) = \dfrac{(2-3\sigma)\left[2(2-\sigma)s^{i}_{DID_{ij}} - (1-2\sigma)\right]}{6(1-\sigma)^2}$, $i=1,2;\ j=1,2;\ i \neq j$ is the export tax on good i given $s^{i}_{DID_{ij}}$ such that the home firm prefers duopolistic competition in the same good over differentiated product competition. Expression (A7) must be understood as follows: the domestic government sets $s^{i}_{DID_{ij}}$ optimally as a function of the parameters σ, F and $s^{j}_{DID_{ij}}$ is set residually so as to generate the supply of good i only, as the home firm's dominant strategy.

For example, when $s^{i}_{DID_{ij}} = s^{s}_{i}$, $s_{j}(\sigma) \leq 0$ and constant and thus $s^{j}_{DID_{ij}} = s^{00}_{j}(\sigma, F, s^{s}_{i})$ for F low given σ while $s^{j}_{DID_{ij}} = s_{j}(\sigma, s^{s}_{i})$ for higher values of F given σ. For different parameter combinations $s^{i}_{DID_{ij}}$ can be quite large. Given that products are substitutes $\min\left(s^{00}_{j}(\sigma, F, s^{s}_{i}), s_{j}(\sigma)\right) > 0$ and $s^{j}_{DID_{ij}} = 0$ in this case.[8]

The associated domestic welfare function is given by

$$W^{H}_{DID_{ij}} = \frac{1-\sigma}{(2-\sigma)^{2}} + \frac{\sigma^{2} s_{l}}{(2-3\sigma)(2-\sigma)^{2}} + \frac{2(1-\sigma)\left(\sigma^{2} - 2(1-\sigma)^{2}\right)s_{l}^{2}}{(2-3\sigma)^{2}(2-\sigma)^{2}} - F$$

where s_{l} is given by (A6).

E) DSM_{i}

Below locus e in figure 1, both firms choose to be active against a single product rival.

Let $\tilde{\bar{s}}_{i}(\sigma, F) = \frac{(2-3\sigma)}{\sigma} - \frac{(2-3\sigma)(2-\sigma)}{\sigma}\left(\frac{F}{1-\sigma}\right)^{\frac{1}{2}}$ be the limit subsidy when the home firm produces good j only, $i = 1, 2; j = 1, 2; i \neq j$.

[8] Also, since $s^{00}_{j}(\sigma, F, s^{i}_{DIDij})$ is the limit export tax when the foreign firm is inactive, it is possible to find less costly instruments consistent with the firm's incentives. This is inconsequential with respect to the problem at hand since such taxes are merely used as threats and never borne by the home firm in equilibrium while their characterization would involve tedious algebra.

It was shown above that $\tilde{\tilde{s}}(\sigma,F) \geq \hat{s}_i(\sigma,F)$ implies $F(\sigma) \leq F_{21}(\sigma)$ where $F_{21}(\sigma)$ lies everywhere above locus *d*. Thus, $\tilde{\tilde{s}}(\sigma,F) \geq \hat{s}_i(\sigma,F)$ eliminates *FM* in the relevant region. Thus, below locus *e*, DSM_i is supported as the unique equilibrium of the game by a vector of policies $\left(s^i_{DSM_i} = \tilde{\tilde{s}}_i(\sigma,F);\ s^j_{DSM_i}\right)$, $i=1,2; j=1,2; i \neq j$ where

$$s^j_{DID_{ij}} = \min\left(0, \min\left(s_j^{00}\left(\sigma, F, \tilde{\tilde{s}}_i\ (\sigma,F)\right), s_j\left(\sigma, \tilde{\tilde{s}}_i\ (\sigma,F)\right)\right)\right).$$

As before, given $\tilde{\tilde{s}}_i\ (\sigma,F), s^j_{DSM_i}$ commits the home firm to good *i* whatever the foreign firm's choice.

Above locus *e* in figure 1, there are multiple equilibria DSM_i, FSM_i, $i=1,2$.

Let $\ddot{s}(\sigma,F) = \dfrac{(2-\sigma)(2-3\sigma)}{2(1-\sigma)}\left(\dfrac{F}{1-\sigma}\right)^{\frac{1}{2}} - \dfrac{(2-3\sigma)}{2(1-\sigma)}$ be the (uniform) subsidy combination such that the home firm supplies good *i* when the foreign supplies good *j* and $\ddot{s}_{SD}(\sigma,F) = \dfrac{3[(1-\sigma)]^{\frac{1}{2}}}{2} - \dfrac{1}{2}$, be the (uniform) subsidy such that the home firm chooses competition in the same product over nothing at all.[9] Applying Lemma 2 gives us the single good entry inducing subsidy as $\min\left(\ddot{s}(\sigma,F); \ddot{s}_{SD}(\sigma,F)\right)$

$$\ddot{s}(\sigma,F) \leq \ddot{s}_{SD}(\sigma,F) \quad \text{implies} \quad F(\sigma) \leq F_{22}(\sigma) = 1-\sigma$$

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[9] Note that $\ddot{s}_{SD}(\sigma,F) = \hat{s}(\sigma,F) = \dfrac{3[(1-\sigma)]^{\frac{1}{2}}}{2} - \dfrac{1}{2}$.

and $\frac{1}{4(1-\sigma)} < 1-\sigma$ implies $\sigma < \frac{1}{2}$ where $\frac{1}{4(1-\sigma)}$ is locus f in figure 1. Furthermore, it can be checked that $\ddot{s}(\sigma,F) < s^{00}(\sigma,F)$ above locus e. Thus, $\ddot{s}(\sigma,F)$ is sufficient to eliminate FSM_i in region E.

The associated domestic welfare function is given by:

$$W^H_{DSM_i} = \frac{1-s_1^2}{4(1-\sigma)} - F$$

where s_1 is defined above.

Appendix II : The Government induced optimal product composition as a Function of F and σ (Figure 2).

$$g: F = \frac{1.372583(1-2\sigma)}{144(1-\sigma)}$$

$$h: F = \left[3(1-\sigma)^{\frac{1}{2}}\left(2 - \frac{(1-2\sigma)^{\frac{1}{2}}}{1-2\sigma}\right) + 3\left(4(1-\sigma) + \frac{(1-2\sigma)}{2(1-\sigma)} - 4(1-2\sigma)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right]^2$$

$$i: \quad F = \left[\left(\frac{2(1-2\sigma)^{\frac{1}{2}}\left[(5-6\sigma) - 6(1-\sigma)(1-2\sigma)^{\frac{1}{2}}\right]}{(1-\sigma)^{\frac{1}{2}}[17 - 18\sigma - 18(1-\sigma)(1-2\sigma)]}\right)\right]^2$$

$$j: \quad F = \left[\frac{1 - \left(1 - \frac{(6\sigma-5)}{2(1-\sigma)(17-18\sigma)}\right)^{\frac{1}{2}}}{3(1-\sigma)^{\frac{1}{2}}}\right]^2$$

$$k: F^{\frac{1}{2}} = \frac{3(1-\sigma)^{\frac{1}{2}} - (1-\sigma)^{\frac{1}{2}}\left[9 - \frac{2(11-9\sigma)}{(2-\sigma)^2}\left(1 - \frac{\sigma^4}{8(1-\sigma)^2\left(\sigma^2 - 2(1-\sigma)^2\right)}\right)\right]^{\frac{1}{2}}}{(11-9\sigma)}$$

$$l:\left(\frac{F}{1-2\sigma}\right)^{\frac{1}{2}}=\frac{(1-\sigma)^{\frac{1}{2}}+(1-\sigma)^{\frac{1}{2}}\left[\frac{1+\sigma^2}{(2-\sigma)^2}+\frac{\sigma^4(3-4\sigma)}{8(1-\sigma)^2(2-\sigma)^2\left(\sigma^2-2(1-\sigma)^2\right)}\right]^{\frac{1}{2}}}{3-4\sigma}$$

$$m:F=\frac{13-18\sigma}{36(1-\sigma)}+\frac{17-18\sigma}{9(1-\sigma)(17-18\sigma)^2}+\frac{\sigma-1}{(2-\sigma)^2}+\frac{\sigma^4}{8(1-\sigma)(2-\sigma)^2\left(\sigma^2-2(1-\sigma)^2\right)}$$

$$n:\left(\frac{F}{1-\sigma}\right)^{\frac{1}{2}}=\frac{1}{2-\sigma}-\frac{\sigma\left\{4\sigma(4-3\sigma)+\frac{2\sigma^4}{\sigma^2-2(1-\sigma)^2}\right\}^{\frac{1}{2}}}{2(2-3\sigma)(2-\sigma)^2}$$

o :

$$\left[\frac{4(1-\sigma)^2\left(\sigma^2-2(1-\sigma)^2\right)+2(2-3\sigma)^2(2-\sigma)^2}{2(2-3\sigma)^2(2-\sigma)^2}+\frac{36(2-3\sigma)^2(2-\sigma)^2(1-\sigma)}{2\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]}\right]F$$

$$+\left(\frac{3\left[4(1-\sigma)^3-\sigma^3\right](\sigma-1)^{\frac{1}{2}}}{2(2-3\sigma)^2(2-\sigma)^2}\right)F^{\frac{1}{2}}$$

$$-\frac{12(2-3\sigma)^2(2-\sigma)^2\left[(2-3\sigma)^2(2-\sigma)^2(17-18\sigma)-72(1-\sigma)^3\right]\left(\sigma^2(1-2\sigma)^2+(1-\sigma)\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]F\right)^{\frac{1}{2}}}{2\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]^2}$$

$$-\frac{1}{2}-\frac{(2-3\sigma)^2\left[(2-3\sigma)(2-\sigma)^2(17-18\sigma)-72(1-\sigma)^3\right]^2+36\sigma^2(1-2\sigma)^2(2-3\sigma)^2(2-\sigma)^2}{2\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]^2}+\frac{(1-2\sigma)\left[2(1-\sigma)(3-4\sigma)-\sigma^2\right]}{2(2-3\sigma)^2(2-\sigma)^2}=0$$

$$\text{p:}\quad \frac{G^2-\sigma^2(1-2\sigma)^2}{(1-\sigma)\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]}$$

where G is defined by:

$$\left(\frac{36(2-3\sigma)^2(2-\sigma)^2}{2\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]^2}+\frac{1}{(1-\sigma)\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]}\right)G^2$$

$$-\frac{12(2-3\sigma)^2(2-\sigma)^2\left[(2-3\sigma)(2-\sigma)^2(17-18\sigma)-72(1-\sigma)^3\right]}{2\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]^2}\cdot G$$

$$+\frac{(2-3\sigma)^2\cdot\left[(2-3\sigma)(2-\sigma)^2(17-18\sigma)-72(1-\sigma)^3\right]^2}{2\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]^2}-\frac{\sigma^2(1-2\sigma)^2}{(1-\sigma)\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]}$$

$$-\frac{1}{2}+\frac{(1-\sigma)}{(2-\sigma)^2}-\frac{\sigma^4}{8(1-\sigma)(2-\sigma)^2\left(\sigma^2-2(1-\sigma)^2\right)}=0$$

q :

$$\left[\frac{-(2-3\sigma)^2(2-\sigma)^2+4\sigma^2(1-\sigma)}{4\sigma^2(1-\sigma)}-\frac{36(2-3\sigma)^2(2-\sigma)^2(1-\sigma)}{2\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]}\right]F$$

$$-\left[\frac{(2-3\sigma)^2(2-\sigma)}{2\sigma^2(1-\sigma)^{\frac{3}{2}}}\right]F^{\frac{1}{2}}$$

$$\frac{-12(2-3\sigma)^2(2-\sigma)\left[(2-3\sigma)(2-\sigma)^2(17-18\sigma)-72(1-\sigma)^3\right]\left(\sigma^2(1-2\sigma)^2+(1-\sigma)\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]F\right)^{\frac{1}{2}}}{2\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]^2}$$

$$-\frac{1}{2}+\frac{(2-3\sigma)^2\left[(2-3\sigma)(2-\sigma)^2(17-18\sigma)-72(1-\sigma)^3\right]^2+36\sigma^2(1-2\sigma)^2(2-3\sigma)^2(2-\sigma)^2}{2\left[(2-3\sigma)^2(2-\sigma)^2(25-18\sigma)-144(1-\sigma)^4\right]^2}-\frac{(1-2\sigma)}{\sigma^2}=0$$

Γ :

$$\left(\frac{F}{(1-\sigma)(1-2\sigma)}\right)^{\frac{1}{2}} = \frac{-(2-3\sigma)^2(2-\sigma)(1-2\sigma)^{\frac{1}{2}} - 4\sigma^2(1-\sigma)^2 - \left[\left[(2-3\sigma)^2(2-\sigma)(1-2\sigma)^{\frac{1}{2}} - 4\sigma^2(1-\sigma)^2\right] - 4(1-\sigma)(1-2\sigma)\left[(2-3\sigma)^2(2-\sigma)(1-2\sigma) - 4\sigma^2(1-\sigma)^2(3-4\sigma)\right]\right]^{\frac{1}{2}}}{-(2-3\sigma)^2(2-\sigma)^2(1-2\sigma) - 4\sigma^2(1-\sigma)^2(3-4\sigma)}$$

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