A Dynamic Programming Approach for Economic Optimization of Inspection Strategies in a Multi-Stage Manufacturing System

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Abstract

In the past several decades, significant amount of resources have been spent by many manufactures for product quality improvement. Quality improvement efforts involve mitigating the impact of manufacturing variation through robust design, statistical process control (SPC), and inspection. This research focuses on manufacturing system design issues related to inspection. The problem of determining the optimal inspection strategy for a given multi-stage manufacturing system i.e. inspection strategy that will result in the lowest total processing cost is modeled as a joint optimization of inspection location and type. Three inspection options considered in this work are no inspection, full inspection and sampling inspection. This thesis presents a generic mathematical model and employs dynamic programming to identify the optimal inspection plan with minimum total processing cost. Numerical examples are presented to describe the solution procedure. The conclusions are supported by a factorial experiment. A sensitivity analysis is also conducted to gauge the impact of inspection errors on the inspection strategy. The developed model is extensible and applicable to solving manufacturing and inspection allocation problems.

Keywords: Quality improvement, multistage manufacturing system, inspection strategy, full inspection, sampling inspection, dynamic programming.

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Chapter One

Introduction

1.1 Motivation

In recent years, the strategic importance of total quality management has been widely accepted by all industries. Improving product, process and service quality is nowadays a key issue in many organizations to improve or at least maintain-profitability and competitiveness. Companies cannot survive without providing high quality products. To produce high quality products, the design and manufacturing community are using a variety of tools to improve quality throughout the product development cycle: e.g., Six Sigma, process improvement, inspection, statistical process control (SPC), process change, and robust design. Because no single approach is superior in every case, management often utilizes a combination of approaches to achieve the highest quality at the lowest cost. To evaluate the different approaches, companies need to understand the cause-and-effect relationship between feature-level variation and system-level product quality. In addition, they must be able to model the quality costs and the impact of different variation reduction strategies. One of these strategies, inspection system design and analysis, is the focus of this thesis. Inspection of products to prevent non-conforming items from reaching the customer is performed in virtually every production system. In particular, the determination of inspection strategy in a multi-stage production system where raw material is transformed into a product in a series of distinct processing stages has attracted much attention due to popularity of the production system. This is because the multi-stage production system presents various possibilities for inspection. The problem of determining the inspection strategy is an "inspection allocation problem". A typical inspection allocation problem is to determine the location and the correct inspection strategy: no inspection, full inspection, or sampling inspection, in a serial multi-stage production system.

In manufacturing processes, inspection is normally associated with rework for defective or nonconforming items which are repairable. This adds to the cost of production since some or all of the processing steps in making a product must be duplicated. Moreover, error propagation as nonconforming units progressing undetected through subsequent manufacturing steps increases production costs by increasing the amount of reprocessing needed to restore a unit to defect-free status. Inspection could be performed after each processing step to significantly reduce error propagation and thus lower the chances of having to send a unit back through multiple processing steps for rework. This would also reduce the likelihood of shipping defective units. However, the cost of inspecting after each step in the production process might outweigh the savings resulted from the early detection of non-conforming units. On the other hand, inspection could be used after the final processing step and at no other location. This would of course reduce the cost of the inspection activity itself. However, if the item is found to have acquired a defect at stage one, it might be necessary to repeat all processing steps, increasing the cost required to produce the unit. In order to minimize the cost of processing, in terms of manufacturing, inspection, rework and penalties for shipping defective units, it is necessary to balance the cost of accurately detecting defects through inspection. Thus, development of efficient economic inspection strategies ensures the required output quality while minimizing the total production cost. Therefore, a cost model must be developed to reflect the dependence of processing costs on process defect probabilities, rework and penalties in addition to manufacturing and inspection costs. Based on this model, a solution technique for determining the optimal location and combination of inspection can be formulated yielding to a systematic approach in minimizing total production cost for the considered production system.

1.2 Research Background

This research concentrates on an inspection allocation problem. It generalizes and extends the earlier work of Oppermann et al. (2003). The objective of their work was to develop models to decrease the quality cost in technological processes. They developed a quality cost model for a production system processing single type of parts. Oppermann et al. (2003) presented quality cost models to compare the quality behaviors of different technological processes and different inspection strategies (no inspection, full inspection and sampling inspection). All inspection operations were considered to be error free. They suggested dynamic programming as one of the possible solution approaches for solving multiple combinations of technological processes.

1.3 Scope and Objectives of this Thesis

The purpose of this study is to develop a method that addresses the inspection allocation problem to decide where to allocate an inspection operation and what inspection strategy to adopt in a serial manufacturing system so that the total expected production cost is minimized. Using the expression for expected total cost, an optimization procedure is developed to achieve the objective. Assumptions are made for not including a few costs or variables because inclusion of such costs and variables would not change the course or outcome of the model significantly. This is done to avoid unnecessary complexity and redundant constraints in developing the model.

The environment under consideration is a serial manufacturing system processing multiple parts. The considered production cost includes the costs of manufacturing, inspection, rework and penalties if a defective unit goes undetected and is shipped to customer. In the system considered all defective units are assumed to be repairable and reworked items are not necessarily restored to a defect-free status. In other words, rework procedures are not error free. An inspection operation can be performed after each manufacturing station. Three inspection options considered in this work are – no inspection, full inspection and sampling inspection. The full inspection operation is considered to be affected by inspection errors and screens all the incoming units. In sampling inspection, a sample from the lot (calculated on the basis of the lot size and AQL) is screened for defective units. Several important issues must be addressed in dealing with the inspection allocation problem.

1.4 Research Methodology

This research presents a framework of methodologies in the development of a cost model for optimal location of inspection stations in a serial production system processing multiple parts. This research uses analytical approach to study the optimal inspection problem. First, mathematical equations are developed to model the cost and impact of inspection. Second, a dynamic programming based optimization approach is used to solve the developed model.

The mathematical model formulated is used to minimize the total production cost. The solution of the model is used to compare the behavior of different manufacturing scenarios and different inspection strategies adopted. The model acts as a measurement tool to compare different inspection strategies at each processing stage. The model also contains the influence of defect rates at each processing stage and the inspection error associated with the full inspection on inspection processes. Model in this thesis research is formulated, coded and solved in optimization software LINGO, version 8, resident on a Pentium-4 machine for all the variants of the problem.

In real production lines, a decision between the inspection strategies for any possible production system is to be made. In this research, dynamic programming is used to formulate an expression for the total production cost which is integral to comparing different inspection strategies. One of the key features of the inspection allocation problem is that it can be broken into stages and at each stage a decision will be made whether to allocate an inspection station, which is also one of the basic characteristics of a dynamic programming model. So, for the model formulated in this thesis work, dynamic programming as solution approach is more pertinent. The decision, what inspection strategy to adopt is based on a simple comparison between the costs of the three possibilities- without inspection, full inspection and sampling inspection after each manufacturing station.

The developed model is extensively tested by several hypothetical example problems with realistic features and the results are verified to ascertain the robustness of the model.

1.5 Research Contributions

This research is aimed at extending the work of Oppermann et al. (2003), as they were among the first to consider both the full inspection and sampling inspection plan simultaneously. They considered a single manufacturing station and evaluated inspection allocation possibilities for different inspection policies. In this thesis, the work presented in Oppermann et al. (2003) is extended to a serial manufacturing system processing multiple parts and each part can have a different manufacturing sequence. The inspection operation is subjected to inspection errors. Dynamic programming approach is applied to perform optimization. Experimental designs and sensitivity analysis are used to study and analyze the effects on total cost and thus the inspection strategy to determine which variables are most influential. The main contribution of this thesis is developing the model and methodology to solve inspection allocation problem under different inspection strategies for serial manufacturing system processing multiple parts. This type of formulation is not seen in the existing literature.

1.6 Organization of the Thesis

Following the introductory chapter one, we shall review the literature on the earlier work done in the area of inspection allocation in chapter two. Chapter three presents the problem description and model formulation for the system under study; the solution approach adopted is also presented. Chapter four presents the numerical examples solved using the model and the analysis of results using experimental design. Concluding remarks are presented in chapter five; directions for future research work are also discussed within the limit of this thesis work.

Chapter Two

Literature Review

Over the last few decades, significant progress has been made in the area of quality system planning. More specifically, the problem of determining optimal locations of inspection stations and inspection policies so as to minimize related production cost has received much attention from many researchers. Lindsay and Bishop (1964) were the first authors to study the inspection allocation and cost minimization problem. They developed a general inspection screening program in which inspection levels and locations were treated as variables. They proposed a cost minimization model for inspection allocation in a single line and multistage production process with perfect inspection and used dynamic programming to solve it. A few years later, Eppen and Hurst (1974) proposed a method for the location of inspection stations taking into account imperfect inspection.

There is a broad range of literature studying inspection allocation problems. This chapter reviews some of the past and recent work in this area. The literature is categorized based on the solution approach as shown in Table 2.1.

Table 2.1: Categorization of Literature

Index	Solution Approach	Authors (Year)
1	Dynamic Programming	Bai and Yun(1986), Eppen and Hurst(1974), Lindsay and Bishop(1964), Oppermann et al. (2003), Penn and Raviv(2003), Penn and Raviv (2004)
2	Meta-Heuristics (Genetic Algorithm, Simulated Annealing, etc)	Chen and Thornton(1999),Feng and Kapur(2006), Hassan and Pham(2000), Kakade et al. (2004),Taneja and Viswanadham (1994),Taneja et al. (1996)
3	Heuristics	Lee and Unnikrishnan (1998), Peters and Williams (1984), Rabinowitz and Yahalom (2001), Rau and Chu (2005), Rau. et al. (2005), Raz and Avinadav (2003), Saxena et al. (1990), Shiau (2002), Shiau (2003)
4	Other Interesting Methods	Narahari and Khan (1995), Rabinowitz and Emmons (1997), Van Volsem and Van Landeghem. (2003), Veatch (1999), Zhou and Zhao (2002)

2.1 Dynamic Programming

2.1.1. Bai and Yun (1986): In this paper, the authors focused on an inspection allocation problem for a serial multistage production system producing identical parts. The rate of production was constrained by the rate of inspection and only a limited number of automated inspection stations were available. A cost model was developed to determine the inspection level and the locations of inspection stations in the line. A solution procedure based on dynamic programming was proposed for solving small size problems. For large size problems a heuristic allocation algorithm was presented. Numerical examples were presented to show the computational efficiency of the solution procedure. The authors reported that the developed method offered optimal or near optimal solutions in less time even when the number of stages and inspection machines were large.

2.1.2. Eppen and Hurst (1974): In this paper, the authors suggested a method for allocating inspection stations in a multistage production process with imperfect inspection. They developed a model for allocating inspection stations with minimum inspection cost. In the model the authors considered the inspection to be 100% or full inspection (each part manufactured is inspected) with known probabilities of accepting bad items and rejecting good items. They assumed that the probability of discovering a defective item is independent of the type of error or the stage where the defect was produced. Dynamic programming was applied to solve the model and inspection policies for the process were suggested.

2.1.3. Lindsay and Bishop (1964): In this paper, the authors developed a general inspection screening program in which inspection levels and locations were treated as variables. They proposed a cost minimization model for inspection allocation in a multistage single line system assuming perfect inspection. The authors assumed the inspection to be 100% perfect. A series of consecutive inspection level decisions are to be made and the outcome of each depends on the prior decisions. They applied dynamic programming approach to solve the model. Computational experiments were conducted to judge the performance of the model. The authors concluded that application of dynamic programming to inspection allocation problem produced expected results.

2.1.4. Oppermann et al. (2003): In this paper, the authors focused on an SMT (Surface Mount Technology) production line. The objective of their work was to develop models to decrease the quality cost in the processes. They discussed a single combination of technological and quality process. They described quality cost models based on batch level statistical quality control to compare the quality behaviors of different technological processes and of different inspection strategies (no inspection, full inspection and sampling inspection). All the inspection operations were considered to be error free. The decision criteria for the selection of a particular inspection strategy were based on the comparison of the quality cost incurred for each policy. The authors suggested dynamic programming as one possible approach for solving such problems.

2.1.5. Penn and Raviv (2003): The focus in this paper was unreliable serial production lines with known probability of failures for each operation. The authors developed a cost minimization model under certain throughput requirement and included holding costs in the objective function. The aim was to decide where and if to install inspection stations on the line at a given production rate. The authors also developed a model for profit maximization which selected simultaneously the inspection station configuration and production rate. The authors used a polynomial time dynamic programming algorithm for solving the model assuming exponentially distributed processing time and Poisson distribution for jobs arriving into a system. The profit maximization model was solved using a branch and bound technique under the same assumptions. The authors performed numerous computational experiments to test the efficiency of the algorithm. The test problems were varied in three areas; success probabilities, tendency of processing rates along the line and tendency of holding costs along the line. The authors reported that the algorithms developed to solve the models are efficient and can be used over a wide range of manufacturing environment.

2.1.6. Penn and Raviv (2004): This paper is an extension of the previous work done by Penn and Raviv (2003). In this paper, the authors discussed problems related to inspection stations in unreliable serial production lines. They assumed the system to be under any arrival process with zero holding costs. They proposed two algorithms, one for operational cost minimization and the other for profit maximization. The cost minimization model was solved using an $O(N^2)$ time dynamic programming algorithm and the profit maximization model was solved by an $O(N^4)$ time algorithm, where N stands for the number of stations. The authors recommended the use of branch and bound technique if the holding costs are relatively high. The authors concluded that polynomial time dynamic programming algorithms are efficient to solve the inspection station configuration problems.

2.2 Meta-Heuristics (Genetic Algorithm, Simulated Annealing, etc)

2.2.1. Chen and Thornton (1999): In this paper, the authors focused on the allocation of inspection stations in a complex assembly process with multi-characteristic specifications using a combination of modeling, simulation and simulated annealing. The authors developed a model to predict the cost of the product due to variations introduced by the manufacturing process, inspection strategy and the final product requirement. The authors used Monte Carlo simulation technique to calculate the expected cost of the inspection plan and simulated annealing to find the optimized inspection plan at the lowest possible cost. The inspection strategy included the location of inspection stations, inspection limits and if the product should be reworked or scrapped. The authors also presented a case study on aircraft wing contour to evaluate the performance of their method. They concluded that a quantitative inspection plan can be successfully developed by following their approach.

2.2.2. Feng and Kapur (2006): In this paper, the authors investigated the economical and statistical effects of inspection error on the design of specifications due to imperfect measurement systems. The authors presented three models for single quality characteristic. They were, Model 1: no inspection error; Model 2: with inspection error and constant inspection cost; and Model 3: with inspection error and variable inspection cost. Each model minimized the expected total cost which was a function of inspection cost, scrap cost and quality loss cost. They used genetic algorithm to find the optimal solution for minimum expected total cost. They conducted numerical tests on their model and concluded that based on the practical situations one of the three models can be used to make effective decisions for inspection. The authors also successfully extended their model to bivariate quality characteristic.

2.2.3. Hassan and Pham (2000): In this paper, the authors used simulated annealing to find optimal locations of inspection stations in a serial multistage production system. The authors used the transfer function model developed by Raz and Kapsi (1991). Experiments were conducted to judge the performance and results found by simulated annealing and were compared to those by genetic algorithm. They concluded that the cooling rate used in the simulated annealing is the most significant factor affecting the quality of the solution.

2.2.4. Kakade et al. (2004): In this paper, the authors discussed an optimization model for allocating inspection efforts in a serial multistage production system. The authors focused on assembly lines producing printed circuit board (PCBs) using surface mount technology. The total cost considered is the summation of inspection cost, rework cost and penalty cost. The inspection was assumed to be perfect. The authors used a combination of simulated annealing and branch and bound as the solution method for the model. To test the performance of the algorithm, experiments were conducted in three different groups with varying test conditions. The experimental results demonstrated that the proposed solution method offered significant improvements over simple simulated annealing method. For small and medium size problems the results generated by the algorithm were close to optimal solutions.

2.2.5. Taneja and Viswanadham (1994): In this paper, the authors discussed an inspection allocation problem for both serial and non-serial manufacturing systems with inspection errors where repeated inspection is allowed. The authors suggested two possible solution approaches, genetic algorithm and neural networks for solving the problem. They used a genetic algorithm based approach to determine the locations of inspection stations. The authors considered the total cost includes manufacturing cost, inspection cost and scrapping cost. They also considered a penalty cost for a non-conforming item reaching the customer. The authors solved the model using exterior penalty method and the genetic algorithm to minimize the total cost. They also presented experimental results for different cases of the problem.

2.2.6. Taneja et al. (1996): This paper is an extension of the previous work done by the authors in Taneja and Viswanadham (1994). In this paper the authors presented two stochastic search algorithms for solving the inspection allocation problem, one based on genetic algorithm and the other on simulated annealing. The production systems under consideration were both serial and non-serial with inspection errors. A mathematical model was formulated and solved using both stochastic search algorithms for different cases. Experimental results were reported and the performances of the two stochastic search algorithms were compared. The results showed that the genetic algorithm performed better for small to medium size problems but for large size problems performance of simulated annealing was better. The authors concluded that applications of genetic algorithm and simulated annealing lead to considerable reduction in computation time as compared to extensive search techniques while yielding near optimal solution.

2.2.7. Van Volsem et al. (2005): In this paper, the authors considered the inspection allocation problem for a multistage production system with constant production and inspection rate, perfect inspection and perfect rework. The authors presented a discrete event simulation model to calculate inspection costs and a genetic algorithm to optimize the inspection strategy. The costs considered were inspection cost, rework cost and penalty cost. The authors suggested full inspection, sampling inspection and no inspection as three possible inspection strategies for the production system. The authors also presented an example to show the computational efficiency of the algorithm. The results illustrated the effectiveness and efficiency of the evolutionary algorithm in solving the inspection allocation problems

2.3 Heuristics

2.3.1. Lee and Unnikrishnan (1998): In this paper, the authors presented a mathematical model for solving the inspection station allocation problem considering a multistage serial manufacturing system with inspection errors. The system under consideration processes different part types with distinctive processing sequences. The authors considered manufacturing cost, inspection cost, internal failure cost and external failure cost as the constituents of the total cost of the system. They presented three different heuristic solution methods for solving the inspection station allocation problem; sequential plan selection method (SPS), time constraint solution method (TCS) and manufacturing cost and nonconforming probability selection method (CNS). An example was presented with a production process of six manufacturing stations and three inspection stations processing four part types. The results obtained from TCS are reasonably close to the optimal solution as compared to those obtained from SPS and CNS. Also the computation time and memory requirement for TCS are much lower compared to other methods. The authors also conducted a two factorial experiment to evaluate the performance of the three heuristics to determine the factors affecting their performances.

2.3.2. Peters and Williams (1984): In this paper, the authors made an experimental assessment of five normative heuristics to evaluate their efficiencies. This paper contributes in the identification of cost and process characteristics that affect the operative conditions for the heuristics and in determining the strength and direction of the effects. The five heuristics are based on five rules of thumb : 1) locate inspection station prior to all processing operations; 2) locate inspection station before those processing operations of relatively high cost; 3) locate inspection station before processing operations that may make the later detection of defective items difficult and costly; 4) locate inspection station after those processing operations likely to generate a relatively high proportion of defective items and 5) locate inspection station after completion of all processing operations. Each heuristic was evaluated on a 13 stage serial production system. The results showed that a range of economic and operating factors affect the applicability of these heuristics. The authors concluded that the cost of processing at each operation did not have a significant impact on the performance of the second heuristic. Also the process constraints imposed on the operating conditions had a significant effect on the performance of four of the five heuristics.

2.3.3. Rabinowitz and Yahalom (2001): The focus of this paper is to determine the inspection capacity and rate. The authors considered three levels of decisions affecting the inspection policy: inspection capacity, assignment of attributes to inspections and inspection schedule. The authors considered three problems regarding inspection, restoration and processing. They used a heuristic method to solve the problem and tested the sensitivity of the solution for different scenarios. Numerical experiments were conducted to judge the performance of the method. The authors concluded that process imperfection had the most significant effect on the inspection policy. Also, they concluded that inspection duration had a negative impact on the inspection policy when inspection duration was a significant portion of manufacturing duration.

2.3.4. Rau and Chu (2005): In this paper, the authors studied an inspection allocation problem for a serial manufacturing system with two types of workstations; workstation of attribute data (WAD) and workstation of variable data (WVD).The authors also considered three possible ways for the treatment of the defective items: repair, rework and scrap. The inspection considered for the production process was imperfect. The authors considered the total cost including processing cost, inspection cost, rework cost, repair cost, scrap cost and penalty cost. They developed a model to maximize the total profit and to determine the optimal inspection policy for the production process. The authors used experimental heuristics and rules of thumb suggested in Peters and Williams (1984) for solving the model as the computation time with optimization methods based on complete enumeration grows exponentially with the number of workstations. They concluded that the performance of the heuristic method was very close to the optimization methods based on complete enumeration, but the former takes much less computation time as compared to the latter.

2.3.5. Rau et al. (2005): This paper is an extension of the earlier work done by the authors in Rau and Chu (2005). In this paper the authors developed a mathematical model considering layered fabrication to find an optimal solution for allocating inspection stations in re-entrant production systems. They considered workstations with variable data only. The authors used rule of thumbs suggested by Peters and Williams (1984) as well as the characteristics of the model developed to solve the inspection allocation problem. From the results obtained the authors concluded that the mathematical model developed is extensible and applicable. They also solved the model with a complete enumeration method and compared their performances. The heuristic algorithm proposed offered acceptable results in much less computational time as compared to complete enumeration method.

2.3.6. Raz and Avinadav (2003): In this paper, the authors considered the problem of selecting inspection operations out of a set of available inspections to maximize the profit per item produced. They considered that the inspection has errors. They used revenue, penalty cost and inspection cost to calculate the profit per item. They used a branch and bound algorithm to obtain the optimal solution of the problem. Various experiments were conducted to test the efficiency of the algorithm. They suggested the use of a greedy heuristic to overcome the disadvantage of the branch and bound algorithm. They also conducted experiments to compare the performance of the two methods. From the results obtained they concluded that the use of heuristics is advantageous when the problem size is large.

2.3.7. Saxena et al. (1990): In this paper, the authors discussed the performance of four inspection allocation heuristics on the basis of job completion time in a serial manufacturing system. The aim of the paper was to find which system parameter had the most significant effect on the performance of the heuristic, favorable range and the effect on the cost. The four heuristics discussed were: place an inspection station before the processing station with longest processing time and place another at the end of the processing station, place an inspection station after the processing station likely to generate a high proportion of defective items and place another at the end of the process, place an inspection station after each processing station and place an inspection station at the end of the whole process. The authors used simulation to simulate a serial production system with 100% inspection policy. Experiments were conducted to test the performance of the heuristics. They concluded that inspection time was the most significant factor affecting a particular heuristic. When inspection time is a high percentage of the processing time, it was suggested to place an inspection station after each processing station so that shorter manufacturing lead time can be achieved. When inspection time was a small percentage of the processing time, they concluded that it is better to place an inspection station after an operation likely to produce maximum percentage of defective units and to place another at the end.

2.3.8. Shiau (2002): In this paper, the author proposed an inspection planning strategy to allocate inspection stations in a multistage manufacturing system with limited inspection resources. He classified the inspection stations in different classes with each class having the same inspection capability and usage. The author developed a cost model considering inspection error which deals with inspection capability, manufacturing capability and tolerances. As the problem size becomes large, it becomes difficult to solve the model by complete enumeration method. Two heuristic methods, earliest stage assignment method and hybrid weighting assignment method, were proposed to solve the model. A case study was presented to measure the performance of the two heuristics and to compare them to the enumeration method. The results showed that the two heuristics gave acceptable performances comparing to enumeration method. The performance of the hybrid weighting assignment method was better than that of the earliest stage assignment method in terms of computation.

2.3.9. Shiau (2003): This paper is an extension of the earlier work done by Shiau (2002). In this paper the author discussed the inspection station allocation strategy in a multistage production system with inspection error considering limited inspection resources. The inspection allocation problem is solved using a cost model in which manufacturing capability, inspection capability and tolerances are considered. Heuristic methods are introduced for large size problems based on two decision criteria: sequence order of workstations and tolerance interval. A case study was presented to measure the performances of the heuristics and the results were compared with optimal solutions generated by enumeration method that generated an optimal solution. The author concluded that both heuristics produced acceptable results compared with the enumeration method, although for time efficiency sequence order method should be preferred over tolerance interval method.

2.4 Other Interesting Methods

2.4.1. Narahari and Khan (1995): In this paper, the authors discussed a re-entrant manufacturing system. They proposed a probabilistic model for locating inspection stations based on cycle time and throughput. The authors developed an analytic method based on mean value analysis (MVA) to compute cycle times and throughput rates. Numerical examples were provided to judge the efficiency of the method. The authors conducted simulations to validate the proposed method. They also compared different ways of allocating inspection stations and concluded that a small number of strategically located inspection stations perform better than a large number of poorly located stations.

2.4.2. Rabinowitz and Emmons (1997): In this paper, the authors considered a multistage production system with a single inspection facility to perform multiple inspection tasks. The main focus area in this paper was the scheduling of inspection and maximization of the fraction of good items produced by the production system. The author proposed an optimal inspection schedule for a two stage production system and heuristics for a system with more than two stages. The authors divided the heuristics into two categories as static and dynamic. Numerical experiments were conducted to evaluate the performance of the two heuristics. The authors reported that both heuristics performed well.

2.4.3. Van Volsem and Van Landeghem (2003): This paper studied the impact of various cost parameters on selection of an optimal inspection policy. The system under consideration is a multistage production system with constant production rate and inspection rate, perfect inspection and perfect rework. The total inspection cost consists of test cost, rework cost and penalty cost. The authors considered full inspection, sampling inspection and no inspection as three possible inspection strategies for the production system. They used simulation to solve the problem. Two types of problems were considered. In the first problem, the test cost was fixed and rework and penalty costs were varied. In the second problem, test and penalty costs were fixed and rework cost was varied. They concluded that the rework cost did not have a significant impact on the total inspection cost and its influence on the inspection policy was not significant.

2.4.4. Veatch (1999): The main objective of this paper was to find inspection strategies for a multistage production system with time varying quality. The authors formulated a cost of quality (CoQ) model that allows various repair options and sampling plans. The model emphasized the dependency between defects and costs. The model was used to analyze the CoQ of a thermal printer for digital photographs. They found that inspection is cost effective only for parts that have a poor record of quality or a very high unit cost. They also reported that sampling inspection is cost effective when there is a significant variation in the defect rate between lots. The authors concluded that the developed the CoQ model is efficient and widely applicable to a wide range of assembly processes.

2.4.5. Zhou and Zhao (2002): This paper focused on a mathematical model formulation that determines the number and locations of inspection stations. The costs considered were training cost, tool cost, transportation cost and cost of opening and operating inspection stations. The model has two types of constraints: all demands must be satisfied and capacity limits at machining and inspection stations cannot be exceeded. Five heuristic algorithms based on tripartite graph representation of the problem were developed to find feasible solutions. They were random search algorithm, cubic greedy algorithm, edge greedy algorithm, single matching algorithm and double matching algorithm. The authors also performed experiments to verify the algorithms and compared their performances. From the results the authors concluded that the double matching algorithm produced the best results. They produced near optimal solution and had best computational efficiency.

2.5 Summary

Much research work has been done for modeling and solving inspection allocation problems due to their importance and difficulties in finding optimal solutions. Depending upon different criteria such as serial or non-serial production system, perfect inspection or imperfect inspection, different inspection allocation models have been suggested using dynamic programming and various heuristics.

In the next chapter, a detailed mathematical model for inspection allocation problem considering a serial manufacturing system processing multiple part types is presented. After each manufacturing station one of the following three options can be chosen: no inspection, full inspection or sampling inspection. The dynamic programming approach employed to solve the model will also be discussed in detail.

Chapter Three

Model Formulation and Solution Approach

In this chapter, details of the problems studied in this research and the mathematical model developed to solve the problem are discussed. It further includes,

- Detailed description of the general characteristics of the inspection allocation problem.
- Assumptions made related to different stages of production.
- Notations used in the mathematical model.
- Explanations of the various parameters used in the model.
- Formulation of the mathematical model.
- Application of dynamic programming approach to solve the model.

3.1 Problem Introduction

As discussed in the previous chapters, the problem considered in this research is to determine an optimal inspection policy so as to minimize the total processing cost for a given serial manufacturing system to provide the desired quality of finished product. The characteristics of the production environment include the number of processing stages in the given manufacturing system, the probability of producing defective units at each processing stage, the inspection error at each stage, the sampling inspection plans and the production costs associated with each stage.

Consider a multi-stage manufacturing process as shown in Figure: 3.1. There are K manufacturing stages through which the parts to be processed follow certain sequences. Each stage of the manufacturing process receives batches of items to be processed. They may contain non-conforming or defective items. After each of the processing stations, one of the three actions for quality control may be chosen: no inspection (N), full inspection (F) and sampling inspection (S). It is possible to place an inspection station after each manufacturing station. The first option, no inspection obviously does not necessitate any further inspection decision. The undetected defective items will continue to be processed in the manufacturing line and may be shipped to the customer. If full inspection is chosen, then the inspection station may detect the defective items. The defective items are reworked to become conforming units. The full inspection operation subjects to two types of inspection errors. Type I error is the probability of classifying a conforming part as non-conforming, and Type II error is the probability of classifying a non-conforming part as conforming. Finally, the sampling inspection option requires a decision to be made on the parameters of the sampling plan based on the required quality level. The sampling procedure described by Oppermann et al. (2003) is followed in this research work. The acceptance criterion is normally a maximum number of defectives items in a sample. If the number of defective items in a sample is higher than the acceptance criterion, the sample is rejected and a 100% inspection of the whole batch is carried out. The sampling plan is assumed to be free of inspection errors.

Thus, in a multi-stage production system the inspection strategy addresses:

1. The number and locations of inspection stations;
2. The inspection policies to be used at the inspection stations.

The purpose is to determine the optimal location of inspection stations and the type of inspection policies in the serial multi-stage production system such that the total cost of the manufacturing process is minimized. The total cost comprises the manufacturing cost at each stage, the inspection cost for all inspected items, the rework cost associated with defective items and the penalty cost incurred by the defective items. In the next section, the assumptions used in developing the mathematical programming model are presented.

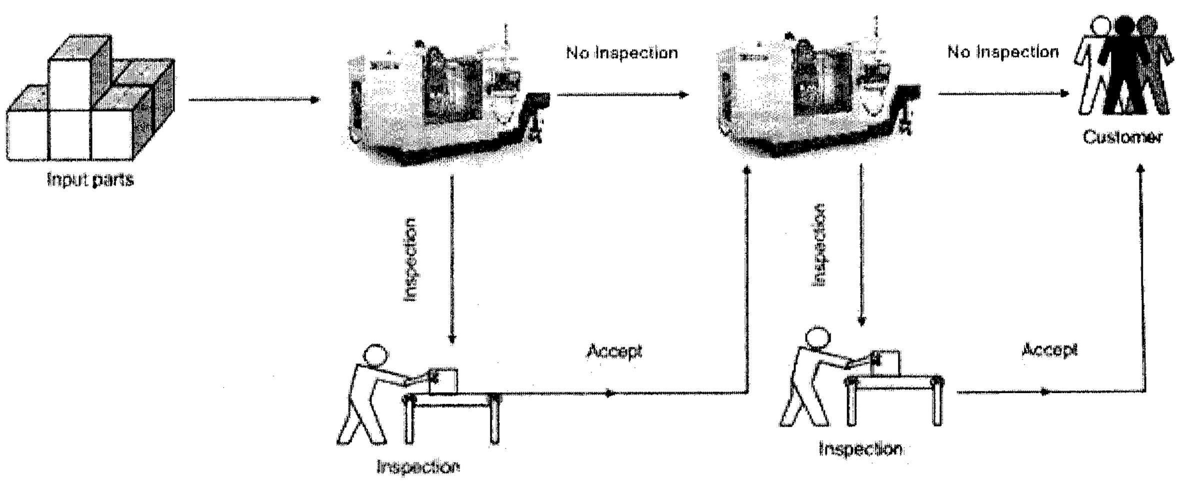

Figure 3.1: A Serial Manufacturing System

3.2 Model Assumptions

1. The system has K manufacturing stations arranged serially and processes I different part types. Each part type follows a given manufacturing sequence.
2. Defective parts are generated only at the manufacturing stations and each manufacturing station has a specific probability of producing defective parts for each part type.
3. One of the three quality control actions may be taken: no inspection, full inspection and sampling inspection at each manufacturing stage. The selection of these options should minimize the total manufacturing cost.
4. For no inspection, a penalty cost is incurred at each stage for the all undetected non-conforming units.
5. Two types of inspection errors are considered for the full inspection operation. Type I error is the probability of classifying a conforming part as non-conforming, and Type II error is the probability of classifying a non-conforming part as conforming. The probabilities of these two types of errors are known at each stage.
6. The sampling inspection is assumed to be free of inspection errors. A 100% inspection policy is applied to the whole lot rejected by sampling inspection.
7. The units identified as non-conforming by the inspection operation are assumed to be reworkable.
8. The production system has reached a steady state and system breakdown is not considered.

Before the model is presented, we first give the notations used in the model.

3.3 Model Notations

Indices:

i = index of part type, $i = 1,\ldots. I$.

k = index for manufacturing station, $k = 1,\ldots. K$.

Parameters:

n_i = number of units of part type i entering the system.

$\alpha_{i,k}$ = probability that the inspector at manufacturing station k erroneously classifies a conforming unit of part type i as a non-conforming unit.

$\beta_{i,k}$ = probability that the inspector at manufacturing station k erroneously classifies a non-conforming unit of part type i as a conforming unit.

$Z_{i,k}$ = probability of a non-conforming unit of part type i processing at manufacturing station k.

$s_{i,k}$ = sample size for batch processing part type i at manufacturing station k.

$MCP_{i,k}$ = unit manufacturing cost for processing part type i at manufacturing station k.

$ICP_{i,k}$ = unit inspection cost for part type i at manufacturing station k.

$RCP_{i,k}$ = unit rework cost for part type i at manufacturing station k.

$PCP_{i,k}$ = unit penalty cost for part type i at manufacturing station k.

Variables:

Continuous Variables:

$NC_{i,k}$ = number of conforming units of part type i leaving manufacturing station k.

$NNC_{i,k}$ = number of non-conforming units of part type i leaving manufacturing station k.

$NR_{i,k}$ = number of units of part type i rejected at inspection station after manufacturing station k.

Binary Variable:

Inspection Option:

No inspection: $NI_{i,k} = \begin{cases} 1, \text{ If no inspection is performed}, \\ 0, \text{Otherwise} \end{cases}$

Full inspection: $FI_{i,k} = \begin{cases} 1, \text{ If full inspection is performed}, \\ 0, \text{Otherwise} \end{cases}$

Sampling Inspection: $SI_{i,k} = \begin{cases} 1, \text{ If sampling inspection is performed}, \\ 0, \text{Otherwise} \end{cases}$

In this thesis, erroneously classifying a conforming item as non-conforming is called as Type-I error and labeling a non-conforming unit as conforming is called a Type-II error. Although these definitions are similar to definitions used in statistical analysis, it should be noted that they are defined in terms of inspector fallibility rather than sampling error.

3.4 Cost Structure

The objective function of the developed model is to minimize the total cost of production involved in the manufacturing and quality control processes. Specifically, they include manufacturing cost, inspection cost, reworking cost and penalty cost. They are discussed below.

3.4.1 Manufacturing Cost

The manufacturing cost of part type i at station k is the multiplication of the unit processing cost at station k and the number of part type i processed at station k. The unit processing cost is composed of machine setup cost, material cost and overhead cost. The number of units of part type i processed at station k is the total number of units flowing into station k from the preceding station. These units may be conforming and are designated as non-conforming after full inspection, or they may be non-conforming but are designated as conforming due to Type-II inspection error.

Therefore, the manufacturing cost ($MC_{i,k}$) is defined as follows:

For the first manufacturing station it is,

$$MC_{i,1} = [NC_{i,1} + NNC_{i,1}] \times MCP_{i,1} \tag{3.1}$$

For all other stations it is calculated as follows,

$$\sum_{i=1}^{I}\sum_{k=2}^{K} MC_{i,k} = \sum_{i-1}^{I}\sum_{k=2}^{K}\left[NC_{i,k-1} \times (1-\alpha_{i,k-1}) + NNC_{i,k-1} \times \beta_{i,k-1}\right] \times MCP_{i,k} \tag{3.2}$$

Where, the number of conforming units and number of non-conforming units produced at a manufacturing station are calculated as follows,

- **Calculation of the Number of Conforming Units ($NC_{i,k}$)**

The number of conforming units of part type i produced at station k is equal to the number of conforming units flowing from the immediately preceding manufacturing station which can be either a manufacturing station or an inspection station.

For the first manufacturing station it is,

$$NC_{i,1} = n_i \times (1 - Z_{i,1}) \tag{3.3}$$

For all other stations it is calculated as follows,

$$\sum_{i=1}^{I}\sum_{k=2}^{K} NC_{i,k} = \sum_{i-1}^{I}\sum_{k=2}^{K}\left[NC_{i,k-1} \times (1-\alpha_{i,k-1}) + NNC_{i,k-1} \times \beta_{i,k-1} + NR_{i,k-1}\right] \times (1 - Z_{i,k}) \tag{3.4}$$

- **Calculation of the Number of Non-Conforming Units ($NNC_{i,k}$)**

The number of non-conforming units of part type i produced at station k is equal to the number of non-conforming units flowing from the immediately preceding manufacturing station which can be either a manufacturing station or an inspection station.

For the first manufacturing station it is,

$$NNC_{i,1} = n_i \times Z_{i,1} \tag{3.5}$$

For all other stations it is calculated as follows,

$$\sum_{i=1}^{I}\sum_{k=2}^{K} NNC_{i,k} = \sum_{i-1}^{I}\sum_{k=2}^{K}\left[NC_{i,k-1} \times (1-\alpha_{i,k-1}) + NNC_{i,k-1} \times \beta_{i,k-1} + NR_{i,k-1}\right] \times Z_{i,k} \tag{3.6}$$

$\alpha_{i,k-1} = 0$ and $\beta_{i,k-1} = 1$, if at station $(k-1)$, no inspection or sampling inspection is performed. The number of parts reworked is calculated as follows,

- **Calculation of the Number of Reworked Units ($NR_{i,k}$)**

The number of parts identified as reworkable at an inspection station, in case of full inspection is affected by the types of inspection errors. It is calculated as follows,

$$\sum_{i=1}^{I}\sum_{k=1}^{K} NR_{i,k} = \sum_{i-1}^{I}\sum_{k=1}^{K}\left[NC_{i,k} \times \alpha_{i,k} + NNC_{i,k} \times (1-\beta_{i,k})\right] \tag{3.7}$$

3.4.2 Inspection Cost

The inspection operation in the system under consideration can be either full inspection or sampling inspection. In case of full inspection, the inspection cost ($IC_{i,k}$) is the product of the unit inspection cost multiplied by the entire lot size processed at workstation k. For sampling inspection, it is the product of the sample size and unit inspection cost for part type i manufactured at station k.

Therefore,

For no inspection, we have

$$IC_{i,k} = 0 \tag{3.8}$$

For full inspection, we have

$$\sum_{i=1}^{I}\sum_{k=1}^{K} IC_{i,k} = \sum_{i-1}^{I}\sum_{k=1}^{K}\left[NC_{i,k} + NNC_{i,k}\right] \times ICP_{i,k} \tag{3.9}$$

And for sampling inspection, we have

$$\sum_{i=1}^{I}\sum_{k=1}^{K} IC_{i,k} = \sum_{i-1}^{I}\sum_{k=1}^{K} s_{i,k} \times ICP_{i,k} \tag{3.10}$$

3.4.3 Rework Cost

At an inspection station, units identified by the inspection operation as non-conforming can be reworked. In case of full inspection, the rework cost ($RC_{i,k}$) is the cost of reworking a unit of part type i manufactured at station k identified as a non-conforming unit by an inspection station. For sampling inspection, it is the product of the number of non-conforming items in the sample and the unit rework cost.

For no inspection, it is

$$RC_{i,k} = 0 \tag{3.11}$$

For full inspection, we have

$$\sum_{i=1}^{I}\sum_{k=1}^{K} RC_{i,k} = \sum_{i-1}^{I}\sum_{k=1}^{K}\left[NC_{i,k} \times \alpha_{i,k} + NNC_{i,k} \times \left(1 - \beta_{i,k}\right)\right] \times RCP_{i,k} \tag{3.12}$$

And for sampling inspection, we have

$$\sum_{i=1}^{I}\sum_{k=1}^{K} RC_{i,k} = \sum_{i=1}^{I}\sum_{k=1}^{K} s_{i,k} \times Z_{i,k} \times RCP_{i,k} \tag{3.13}$$

3.4.4 Penalty Cost

This is the cost incurred when a non-conforming unit goes undetected and is shipped to the customer. In case of no inspection policy, the penalty cost ($PC_{i,k}$) constitutes the cost for further processing the unit in the manufacturing system, the warranty cost and the cost of repairing the defective unit after sales. The penalty cost is the product of the number of non-conforming units of part type i manufactured at station k and the unit penalty cost. For sampling inspection policy, it is the product of the number of defective units in the accepted batch and unit penalty cost.

For no inspection,

$$\sum_{i=1}^{I}\sum_{k=1}^{K} PC_{i,k} = \sum_{i=1}^{I}\sum_{k=1}^{K} NNC_{i,k} \times PCP_{i,k} \tag{3.14}$$

For full inspection,

$$PC_{i,k} = 0 \tag{3.15}$$

For sampling inspection,

$$\sum_{i=1}^{I}\sum_{k=1}^{K} PC_{i,k} = \sum_{i=1}^{I}\sum_{k=1}^{K} [(NC_{i,k} + NNC_{i,k}) - s_{i,k}] \times PCP_{i,k} \tag{3.16}$$

3.4.5 Cost of No Inspection

The cost incurred if no inspection is performed at a particular stage is the sum of manufacturing cost and penalty cost at that stage.

$$\sum_{i=1}^{I}\sum_{k=1}^{K} CN_{i,k} = \sum_{i=1}^{I}\sum_{k=1}^{K} (MC_{i,k} + PC_{i,k})$$

For the first manufacturing station it is,

$$CN_{i,1} = ([NC_{i,1} + NNC_{i,1}] \times MCP_{i,1} + NNC_{i,1} \times PCP_{i,1}) \tag{3.17}$$

For all other stations it is calculated as follows,

$$\sum_{i=1}^{I}\sum_{k=2}^{K} CN_{i,k} = \sum_{i=1}^{I}\sum_{k=2}^{K}([NC_{i,k-1} \times (1- \alpha_{i,k-1}) + NNC_{i,k-1} \times \beta_{i,k-1}] \times MCP_{i,k} + NNC_{i,k} \times PCP_{i,k}) \quad \dots (3.18)$$

3.4.6 Cost of Full Inspection

This cost is the sum of the manufacturing cost, inspection cost and reworking cost at a particular stage.

$$\sum_{i=1}^{I}\sum_{k=1}^{K} CF_{i,k} = \sum_{i=1}^{I}\sum_{k=1}^{K}(MC_{i,k} + IC_{i,k} + RC_{i,k})$$

For the first manufacturing station it is,

$$CF_{i,1} = ([NC_{i,1} + NNC_{i,1}] \times MCP_{i,1} + [NC_{i,1} + NNC_{i,1}] \times ICP_{i,1} + [NC_{i,1} \times \alpha_{i,1} + NNC_{i,1} \times (1- \beta_{i,1})] \times RCP_{i,1}]) \quad (3.19)$$

For all other stations it is calculated as follows,

$$\sum_{i=1}^{I}\sum_{k=2}^{K} CF_{i,k} = \sum_{i=1}^{I}\sum_{k=2}^{K}([NC_{i,k-1} \times (1- \alpha_{i,k-1}) + NNC_{i,k-1} \times \beta_{i,k-1}] \times MCP_{i,k} + [NC_{i,k} + NNC_{i,k}] \times ICP_{i,k} + [NC_{i,k} \times \alpha_{i,k} + NNC_{i,k} \times (1- \beta_{i,k})] \times RCP_{i,k}) \quad (3.20)$$

3.4.7 Cost of Sampling Inspection

This cost is incurred if at a particular stage, sampling inspection is performed. This cost is the sum of the manufacturing cost, inspection cost, rework cost and penalty cost at that stage. In addition, if the sample is rejected then a 100% inspection will be carried out. This additional cost will also contribute to the cost of sampling inspection.

$$\sum_{i=1}^{I}\sum_{k=1}^{K} CS_{i,k} = \sum_{i=1}^{I}\sum_{k=1}^{K}(MC_{i,k} + IC_{i,k} + RC_{i,k} + PC_{i,k})$$

For the first manufacturing station it is,

$$CS_{i,1}=([NC_{i,1} + NNC_{i,1}] \times MCP_{i,1} + P_a \times (s_{i,1} \times ICP_{i,1} + s_{i,1} \times Z_{i,1} \times RCP_{i,1} + ([NC_{i,1} + NNC_{i,1}] - s_{i,1}) \times Z_{i,1} \times PCP_{i,1}) + (1-P_a) \times [[NC_{i,1} + NNC_{i,1}] \times (ICP_{i,1} + Z_{i,k} \times RCP_{i,1})]) \quad (3.21)$$

For all other stations it is calculated as follows,

$$\sum_{i=1}^{I}\sum_{k=2}^{K} CS_{i,k} = \sum_{i=1}^{I}\sum_{k=2}^{K}([NC_{i,k-1} \times (1-\alpha_{i,k-1}) + NNC_{i,k-1} \times \beta_{i,k-1}] \times MCP_{i,k} + P_a \times (s_{i,k} \times ICP_{i,k} + s_{i,k} \times Z_{i,k} \times RCP_{i,k} + (NC_{i,k} + NNC_{i,k} - s_{i,k}) \times Z_{i,k} \times PCP_{i,k}) + (1-P_a) \times [(NC_{i,k} + NNC_{i,k}) \times (ICP_{i,k} + Z_{i,k} \times RCP_{i,k})]) \quad (3.22)$$

Where, the probability of acceptance (P_a) of the sample is calculated as follows,

- **Calculation of Probability of Acceptance of the Sample (P_a)**

The probability of acceptance of the sample is the probability that *d*, the number of defectives, is less than or equal to *Ac*, the acceptance number. The sample size and the acceptance number for the sample are calculated using Military Standard 105E tables. For a given batch size and AQL these tables give the values of sample size and the acceptance number for the sample. With these values it is possible to calculate the probability of acceptance. Actually the use of hypergeometric distribution is necessary for the exact calculation of P_a. When $n_{i,k} \geq 10 \times s_{i,k}$, it is possible to use binomial distribution as a good estimate to calculate the probability of acceptance [Oppermann et. al (2003)].

The equation of the binomial distribution is:

$$p_d = \binom{s_{i,k}}{d}(Z_{i,k})^d \times (1 - Z_{i,k})^{s_{i,k}-d} \quad (3.23)$$

Thus, the probability of acceptance of the sample is calculated by:

$$P_a = p_0 + p_1 + \ldots + p_{Ac} = \sum_{d=0}^{Ac} p_d \tag{3.24}$$

3.4.8 Total Cost

This study is based on the minimization of total production cost to determine the optimal inspection policy and the location of inspection stations in a manufacturing line. The objective function for the inspection allocation problem for a manufacturing system producing i part types is expressed as follows:

$$\sum_{i=1}^{I}\sum_{k=1}^{K} TC_{i,k} = \sum_{i=1}^{I}\sum_{k=1}^{K}(CN_{i,k} \times NI_{i,k} + CF_{i,k} \times FI_{i,k} + CS_{i,k} \times SI_{i,k}) \tag{3.25}$$

Thus, the objective function of the inspection allocation problem is,

$$\text{Minimize } TC = \sum_{i=1}^{I}\sum_{k=1}^{K} TC_{i,k} \tag{3.26}$$

S.t, $\quad NI_{i,k} + FI_{i,k} + SI_{i,k} = 1 \quad (3.27)$

$NI_{i,k} \in \{0, 1\}$

$FI_{i,k} \in \{0, 1\}$

$SI_{i,k} \in \{0, 1\}$

Where, the constraint in equation 3.27 ensures that only one inspection policy is selected at a manufacturing stage.

3.5 Solution Approach

Many planning and control problems in manufacturing, telecommunications and capital budgeting call for a sequence of decisions to be made at different stages. The initial decision is followed by a second, the second by a third, and so on perhaps infinitely. Generally, a number of technological processes are arranged serially in a production line. In real production lines, we have to make a decision between the inspection strategies. The decision whether to inspect or not is based on a simple comparison between the costs of the two possibilities, with or without an inspection station after a manufacturing station. In this research, dynamic programming is used to solve the inspection allocation problem. The details involved in implementing this approach are discussed in the following section.

- **Characteristics of Dynamic Programming Approach to Inspection Allocation Problem**

In inspection allocation problem, each manufacturing station is considered as a stage and a decision had to be made whether to allocate an inspection station after each manufacturing station and the type of inspection policy to be adopted. The state at any stage is simply the stage where the parts are at the beginning of the stage i.e. whether the parts are at a manufacturing station (no inspection) or at an inspection station (full inspection or sampling inspection). To make a correct decision at any stage, we do not need to know how we got to this current location. The decision is to select from the three options: no inspection, full sampling inspection and sampling inspection after a manufacturing station. The current state of the system will convey all the information about its previous behavior necessary for determining the optimal inspection policy henceforth.

We now describe how dynamic programming is used to solve the model developed previously. Let $TC_{i,k}(S_{i,k};X_{i,k})$ is the total cost of the best overall policy for the remaining stages, given that the parts are in state $S_{i,k}$ ready to start stage k and selects $X_{i,k}$ as the immediate decision. Given $S_{i,k}$ and k, let $X^*_{i,k}$ denote the value of $X_{i,k}$ that minimizes $TC_{i,k}(S_{i,k};X_{i,k})$ and let $TC^*_{i,k}(S_{i,k};X_{i,k})$ be the corresponding minimum value. Thus,

$$TC^*_{i,k}(S_{i,k};X_{i,k}) = \min\left[TC_{i,k}(S_{i,k};X_{i,k})\right] \tag{3.28}$$

Where, $X_{i,k} \in \{NI_{i,k}\ ;FI_{i,k};SI_{i,k}\}$

The objective is to find $TC_{i,k}(S_{i,k};X_{i,k})$ and corresponding inspection policy.

$TC_{i,k}(S_{i,k};X_{i,k})$ = Immediate cost + Minimum future cost

Immediate cost $= TC_{i,k}$

Minimum future cost $= TC_{i,k-1}(S_{i,k-1};X_{i,k-1})$

For: $i = 1,\ldots\ldots,I\ ;\ k = 2,\ldots\ldots,K$

$$TC_{i,k}(S_{i,k};X_{i,k}) = TC_{i,k} + TC_{i,k-1}(S_{i,k-1};X_{i,k-1}) \tag{3.29}$$

$$TC^*_{i,k}(S_{i,k};X_{i,k}) = \min\{[CN_{i,k}\times NI_{i,k} + CF_{i,k}\times FI_{i,k} + CS_{i,k}\times SI_{i,k}] + TC_{i,k-1}(S_{i,k-1};X_{i,k-1})\}$$

The initial condition for $TC_{i,1}$ is: For: $i = 1,\ldots\ldots,I\ ;\ k = 1$

$$TC_{i,1}(S_{i,1};X_{i,1}) = \min\{CN_{i,1}\times NI_{i,1} + CF_{i,1}\times FI_{i,1} + CS_{i,1}\times SI_{i,1}\}$$

As discussed previously, one of the features of the inspection allocation problem is that it can be broken into stages and at each stage a decision had to be made whether to allocate an inspection station, which is also one of the basic characteristic of a dynamic programming model. Additionally, dynamic programming yields exact optimal results in less computational time. On the negative side, the computational effort increases with the increase in problem size and formulating a dynamic programming can be difficult. The model presented above can be solved to find the optimal solution of the inspection allocation problem using various available software packages. The widely used packages are LINGO, CPLEX, among others.

In this research, the model is coded in LINGO resident on a Pentium-4 machine operating at 2.6 GHZ and 512 MB RAM. The model requires as inputs the number of parts types (i) and manufacturing stations (k), a set of parameter values ($n_i, Z_{i,k}, MCP_{i,k}$,etc.) and assignment of inspection errors ($\alpha_{i,k}, \beta_{i,k}$) . This information is used to compute the total cost for each possible inspection configuration. The configuration which yields the minimum value is identified as the optimal solution.

In any model there is an issue of randomness and thus statistical analysis should be performed. By just running the model once, one cannot predict how valid the results might be. The estimates could differ greatly from the corresponding true characteristics and there could be a significant probability of making wrong inferences about the system under study. Thus, appropriate statistical methods must be used to analyze the output from the model. In this research two-level factorial experiments are used to study and analyze the system. Design of Experiments is used to evaluate and analyze the various parameters and their interactions on the total processing cost and thus their effect on the selection of a particular inspection strategy. The test problem instances studied for the above explained model are presented in Chapter Four.

Chapter Four

Numerical Examples and Analysis

This chapter presents several numerical examples to validate the developed model and its solution method in the previous chapter. The main purpose of this thesis research is to develop a mathematical model for inspection allocation problem. The model developed has been tested for various instances of the problem. The computational results of the example problems presented in this chapter validate the model and identify the sensitive parameters of the model. A further investigation of the model is conducted by varying the sampling parameters and probabilities of inspection errors. The data used in this example are realistic but hypothetical. The range for various parameters considered is presented in Table 4.1. The values of all the input parameters are considered from published work in literature. The model is programmed and solved by LINGO optimization software, version 8, for the optimal solution. The AQL and the probabilities of Type I and Type II errors are assumed to be constant and known for each inspection operation. The cost parameters and the probability of processing a non-conforming unit are different for each manufacturing stage. The reworking cost and penalty cost are dependent on the manufacturing cost at a particular stage and are also shown in Table 4.1.

Table 4.1 Data Values

Parameter	Range	
	Min	Max
Manufacturing Cost	10	500
Inspection Cost	1	15
Probability of Type I error	0.02	0.15
Probability of Type II error	0.02	0.15
Probability of processing a non-conforming unit	0	0.10
AQL	1%	2.5%
Reworking Cost = (45 % to 55 %) × Manufacturing Cost		
Penalty Cost = (2 to 5) × Manufacturing Cost		

As a rule of thumb, the relation between the four different types of costs was set so that always, Inspection Cost < Reworking Cost < Manufacturing Cost < Penalty cost. This simple rule is based on common sense; if inspection cost was larger then reworking cost, inspection would be uneconomical. Similarly, if reworking cost were larger then manufacturing cost and penalty cost, rework would be uneconomical. Design of Experiments (DOE) is used to conduct and analyze tests to evaluate the parameters that impact the total processing cost and thus the inspection policy.

4.1 Example Problems

The model is tested for two sets of problems corresponding to single and multiple part types processed by a serial production system. For these two sets of problems, dynamic programming methodology is used to obtain optimal solutions. The model is tested by different sets of problem instances using LINGO. The single part type problem is used to validate and test the effectiveness of the dynamic programming method. The multiple part type problem is used to show the ability of dynamic programming in handling complex problems. All the results are obtained using a Pentium 4 platform with 2.6 GHZ and 512 GB RAM. The comparison and analysis of the results are done in the following sections.

4.1.1 Numerical Examples for Serial Production System Processing Single Part

Consider a serial six stage manufacturing system processing a single part type with an option of allocating an inspection station after each manufacturing station. The data values for the problem are given in Table 4.2 and Table 4.3. All the costs are unit costs incurred for producing one unit of each part type i at manufacturing station k.

Table 4.2: General Data for a Six Stage Inspection Problem

Number of Manufacturing Stations	6
Number of Units Manufactured	500
Probability of Type-I Error	0.02
Probability of Type-II Error	0.05
AQL	1.5%
Sample Size	50
Acceptance Number	2

Table 4.3: Data for Each Stage of a Six Stage Inspection Problem

Manufacturing Station	**1**	**2**	**3**	**4**	**5**	**6**
Manufacturing Cost	21	45	29	24	15	70
Inspection Cost	1	11	13	3	8	10
Reworking Cost	10	22	15	11	7	35
Penalty Cost	42	90	60	50	30	140
Probability of Defect	0.09	0.04	0.01	0.08	0.03	0.06
Pa	0.16	0.68	0.99	0.23	0.81	0.42

4.1.1.1 Creating a Sampling Plan to Determine the Probability of Acceptance

The first step is to create a sampling plan for the considered manufacturing process. An acceptance sampling plan is a statement of the sample size (n) to be used and the associated acceptance criteria (*Ac*) for the given lot. The major types of sampling procedures and their applications are shown in Table A1 in the appendix. The selection of a sampling plan depends on both the objective and the history of the organization whose product is sampled. There are two widely used ways of picking (*n, Ac*):

1. Use tables (such as MIL STD 105E) indexed on either the AQL or the LTPD desired.
2. Specify two desired points on the OC curve and solve for the (*n, Ac*) that uniquely determines an OC curve going through these points.

In this research work, Military Standard 105E tables with normal inspection procedures are used to calculate the values of the sample size and the acceptance number for a given lot size and AQL. The standard includes three types of inspection (normal, tightened, and reduced inspection). The type of inspection that should be applied depends on the quality of the last batches inspected. At the beginning of inspection, normal inspection is used. The AQL represents the poorest level of quality for the vendor's process that the consumer would consider to be acceptable as a process average. The AQL is a property of the vendor's manufacturing process; it is not a property of the sampling plan. Furthermore, the AQL is usually not intended to be a specification on the product. It is simply a standard against which to judge the lots. It is common to use an AQL of 1% for major defects and 2.5% for minor defects. After the values of sample size and acceptance number for a lot are obtained the probability of acceptance of the lot (P_a) is calculated with equation (3.23).

The P_a values at each manufacturing stage are calculated depending on the values of probability of defect at each stage and are shown in Table 4.3. After the data in Table 4.2 and Table 4.3 were taken by the model in Chapter Three, the model was then solved in LINGO to obtain the optimal solution. As discussed in the previous chapter dynamic programming is used to solve the inspection allocation problem. Each manufacturing station represents a stage and after each stage a decision is to be made whether to allocate an inspection station after the manufacturing operation and the type of inspection policy to adopt. The objective function minimizes the total cost at which we have an optimal inspection policy. The total cost of processing at each stage was calculated by LINGO. They are shown in Tables 4.4 to 4.9. First we construct the function $TC_{1,1}{}^*(S_{1,1}; X_{1,1})$ as shown in Table 4.4.

Table 4.4: Processing Cost at Stage 1 for the Different Inspection Policies

	$S_{1,1}$	$TC^*_{1,1}(S_{1,1}; X_{1,1})$	$X^*_{1,1}$
Stage 1	No Inspection	14280	$NI_{i,k}$
	Full Inspection	11518	$FI_{i,k}$
	Sampling Inspection	11585	$SI_{i,k}$

At the first stage, the probability of defect is high and the inspection cost is low, so a rational decision would be to allocate an inspection station after the first manufacturing station and perform a full inspection operation. Equipped with $TC^*_{1,1}(S_{1,1}; X_{1,1})$ and $X^*_{1,1}$ we are ready to calculate $TC^*_{1,2}(S_{1,2}; X_{1,2})$ and $X^*_{1,2}$ as shown in Table 4.5.

Table 4.5: Processing Cost at Stage 2 for the Different Inspection Policies

$X_{1,2}$ \ $S_{1,2}$		$TC_{1,2}(S_{1,2};X_{1,2})= TC_{1,2}+TC^*_{1,1}(S_{1,1};X_{1,1})$			$TC^*_{1,2}$	$X^*_{1,2}$
		Stage 1				
		No Inspection	Full Inspection	Sampling Inspection		
Stage 2	No Inspection	23766+14280 =38046	23766+11518 =35284	23766+11585 =35351	35284	$NI_{i,k}$
	Full Inspection	26295+14280 =40575	26295+11518 =37813	26295+11585 =37880	37813	$FI_{i,k}$
	Sampling Inspection	23573+14280 =37853	23573+11518 =35091	23573+11585 =35158	35091	$SI_{i,k}$

At the second stage, the inspection cost and penalty cost are high which result in higher processing cost for no inspection and full inspection. Hence performing sampling inspection is the most economical choice at this stage. In the next stage, as presented in Table 4.6, $TC^*_{1,3}(S_{1,3};X_{1,3})$ and $X^*_{1,3}$ are calculated.

Table 4.6: Processing Cost at Stage 3 for the Different Inspection Policies

$X_{1,3}$ \ $S_{1,3}$		$TC_{1,3}(S_{1,3};X_{1,3})= TC_{1,3}+TC^*_{1,2}(S_{1,2};X_{1,2})$			$TC^*_{1,3}$	$X^*_{1,3}$
		Stage 2				
		No Inspection	Full Inspection	Sampling Inspection		
Stage 3	No Inspection	15110+35284 =50394	15110+37813 =52923	15110+35091 =50201	50201	$NI_{i,k}$
	Full Inspection	21219+35284 =56503	21219+37813 =59032	21219+35091 =56310	56310	$FI_{i,k}$
	Sampling Inspection	15488+35284 =50772	15488+37813 =53301	15488+35091 =50579	50579	$SI_{i,k}$

At the third stage, high inspection cost and low probability of defect results in selection of no inspection policy .We now turn to calculate $TC^*_{1,4}(S_{1,4};X_{1,4})$ and $X^*_{1,4}$ as shown in Table 4.7.

Table 4.7: Processing Cost at Stage 4 for the Different Inspection Policies

	$S_{1,4}$ \ $X_{1,4}$	$TC_{1,4}(S_{1,4};X_{1,4})= TC_{1,4}+TC^*_{1,3}(S_{1,3};X_{1,3})$			$TC^*_{1,4}$	$X^*_{1,4}$
		Stage 3				
		No Inspection	Full Inspection	Sampling Inspection		
Stage 4	No Inspection	16000+50201 =66201	16000+56310 =72310	16000+50579 =66579	66201	$NI_{i,k}$
	Full Inspection	13952+50201 =64153	13952+56310 =70262	13952+50579 =64531	64153	$FI_{i,k}$
	Sampling Inspection	14019+50201 =64220	14019+56310 =70329	14019+50579 =64598	64220	$SI_{i,k}$

At stage four, the high probability of defect and low inspection cost result in selection of full inspection policy. At stage 5, as shown in Table 4.8 we find $TC^*_{1,5}(S_{1,5};X_{1,5})$ and $X_{1,5}^*$.

Table 4.8: Processing Cost at Stage 5 for the Different Inspection Policies

	$S_{1,5}$ \ $X_{1,5}$	$TC_{1,5}\ (S_{1,5};X_{1,5})= TC_{1,5}+TC^*_{1,4}(S_{1,4};X_{1,4})$			$TC^*_{1,5}$	$X_{1,5}^*$
		Stage 4				
		No Inspection	Full Inspection	Sampling Inspection		
Stage 5	No Inspection	7812+66201= 74013	7812+64153= 71965	7812+64220= 72032	71965	$NI_{i,k}$
	Full Inspection	10959+66201 =77160	10959+64153 =75112	10959+64220 =75179	75112	$FI_{i,k}$
	Sampling Inspection	8276+66201= 74477	8276+64153= 72429	8276+64220= 72496	72429	$SI_{i,k}$

Now at stage five, the probability of defect and the penalty cost are low which would result in selection of the no inspection policy. Finally $TC^{*}{}_{1,6}(S_{1,6};X_{1,6})$ and $X^{*}{}_{1,6}$ are calculated in Table 4.9.

Table 4.9: Processing Cost at Stage 6 for the Different Inspection Policies

	$X_{1,6}$ \ $S_{1,6}$	$TC_{1,6}(S_{1,6};X_{1,6})=TC_{1,6}+TC_{1,5}{}^{*}(S_{1,5};X_{1,5})$			$TC^{*}{}_{1,6}$	$X^{*}{}_{1,6}$
		Stage 5				
		No Inspection	Full Inspection	Sampling Inspection		
Stage 6	No Inspection	43400+71965 =115365	43400+75112 =118512	43400+72429 =115829	115365	$NI_{i,k}$
	Full Inspection	41326+71965 =113291	41326+75112 =116438	41326+72429 =113755	113291	$FI_{i,k}$
	Sampling Inspection	40350+71965 =112315	40350+75112 =115462	40350+72429 =112779	112315	$SI_{i,k}$

At stage six, high inspection cost and penalty cost results in the selection of the sampling inspection policy as the most economic option. Now, the overall optimal solution obtained for the six stage inspection allocation problem is,

Total cost= 112385

Cost per unit=224

Inspection policy = {FI, SI, NI, FI, NI, SI}

Total inspection operations= 4

For the purpose of comparison, we review the optimal solution obtained above with three different scenarios. The objective function in all scenarios is to minimize the total processing cost of the system.

Case 1: No Inspection Policy

Now, consider if there was no inspection at all in the manufacturing system. Without inspection, the solution obtained from solving the model is,

Total cost = 123410

Cost per unit =247

Inspection policy = {NI, NI, NI, NI, NI, NI}

Total inspection operations= 0

The cost per unit without inspection is high as compared to that from the optimal solution (247>224). All the non-conforming units produced in the system are processed and are delivered to the customers. This scenario results in a higher cost per unit due to high penalty cost imposed on processing of a defective unit. So, selection of no inspection policy can be justified only if the probability of defect and penalty cost are low.

Case 2: Full Inspection Policy

If full inspection operation is performed after each manufacturing operation, then the solution obtained is:

Total cost= 122484

Cost per unit=245

Inspection policy = {FI, FI, FI, FI, FI, FI}

Total inspection operations= 6

Here, as we are performing full inspection operation at each stage, the cost per unit of processing a unit is higher as compared to that obtained for the optimal solution (245>224).

In this scenario, we are inspecting all the units processed in the system which result in higher cost per unit. Therefore, performing full inspection at each stage may not always desirable.

Case 3: Sampling Inspection Policy

If we choose to do sampling inspection after each manufacturing stage, the solution obtained from solving the model is

Total cost= 116267

Cost per unit=232

Inspection policy = {SI, SI, SI, SI, SI, SI}

Total inspection operations= 6

In this scenario, the cost per unit is higher as compared to that obtained from optimal solution (232>224). Moreover, lots with low probability of acceptance would also be accepted which is not desirable.

The results for other tested problems for the system processing single part type are shown in Table 4.10. The data for all these problems are given in the appendices.

Table 4.10: Results for Different Scenarios of Single Part Type Inspection Problem

		Optimal solution
Total Cost	k=4 ; n_i=300	257,350
	k=8; n_i=1000	159,0472
	k=10; n_i=5000	878,5074
Cost per Unit	k=4 ; n_i=300	857
	k=8; n_i=1000	1590
	k=10; n_i=5000	1757
Inspection Policy	k=4 ; n_i=300	{SI,FI,SI,FI}
	k=8; n_i=1000	{SI,FI,SI,FI,FI,NI,SI,NI}
	k=10; n_i=5000	{SI,FI,FI,SI FI,FI,FI,NI,SI,SI}
Total Inspection Operations	k=4 ; n_i=300	4
	k=8; n_i=1000	7
	k=10; n_i=5000	9

From the above results, one can observe that the model developed is effective in solving the inspection allocation problem. Depending on actual parameters and system requirements the method would produce satisfactory results.

4.1.2 Numerical Example for Processing Multiple Part Types

The model is now extended to a multistage manufacturing system processing multiple part types with each part having a different sequence of processing. Consider a five stage serial manufacturing system processing two part types with an option of allocating an inspection station after each manufacturing station. The manufacturing sequences for the two parts types are given in Table 4.11.

Table 4.11: Manufacturing Sequences

Manufacturing Station	1	2	3	4	5
Part Type-A	*	*	*	*	-
Part type-B	-	*	*	*	*

Where, * indicates manufacturing is performed on a particular part type at the corresponding processing station. The data values for the problem are given in Tables 4.12 and 4.13.

Table 4.12: General Data for a Two Part Type- Five Stage Inspection Problem

Part Type	A	B
Number of Units Manufactured	300	500
Probability of Type-I Error	0.01	0.05
Probability of Type-II Error	0.14	0.08
AQL	1%	1%
Sample Size	13	13
Acceptance Number	1	1

Table 4.13: Data for Each Stage of a Two Part Type Five Stage Inspection Problem

Manufacturing Station		1	2	3	4	5
Manufacturing Cost	A	194	310	123	412	-
	B	-	556	178	268	329
Inspection Cost	A	9	12	4	4	-
	B	-	10	13	7	5
Reworking Cost	A	100	150	50	200	-
	B	-	250	90	130	180
Penalty Cost	A	390	620	250	820	-
	B	-	900	350	540	650
Probability of Defect	A	0.04	0.10	0.03	0.07	-
	B	-	0.09	0.07	0.01	0.03
P_a	A	0.40	0.04	0.55	0.13	-
	B	-	0.05	0.13	0.91	0.55

The optimal solution obtained for the inspection problem for part A and part B at each stage of manufacturing processing are shown in Tables 4.14 and 4.15.

Table 4.14: Processing Costs at Each Manufacturing Stage for Part A

Manufacturing station	Processing Cost	Inspection Policy
1	62360	SI
2	98742	FI
3	42650	SI
4	127092	FI
5	-	-

For part A, at the first stage the high inspection cost and penalty cost result in higher processing cost for full inspection and no inspection policy. So performing sampling inspection is the most cost-effective option at this stage. At stage two, high probability of defect and penalty cost results in the selection of full inspection as the most viable option. At stage three, sampling inspection is selected as the most economic inspection choice. Finally, at stage four high probability of defect and low inspection cost results in the selection of the full inspection plan. So, the optimal solution obtained for the inspection allocation problem for part A is,

Total cost= 330844

Cost per unit= 1103

Total inspection operations= 4

Table 4.15: Processing Costs at Each Manufacturing Stage for Part B

Manufacturing station	Processing Cost	Inspection Policy
1	-	-
2	256275	FI
3	78248	SI
4	122320	NI
5	134200	NI

For part B, at the second stage, the probability of defect occurrence is high which results in the selection of the full inspection policy. At stage three, the high inspection cost and penalty cost results in the selection of the sampling inspection plan. At stages four and five, no inspection policy is selected as a result of low probability of defect and high reworking cost.

So the overall inspection plan for part B is,

Total cost= 591043

Cost per unit=1182

Total inspection operations= 2

The results for other tested problems for the system manufacturing multiple part types are shown in Table 4.16. The data for all these problems are given in the appendices.

Table 4.16: Results for Different Scenarios of Multiple Part Type Inspection Problem

		Optimal Solution
Total cost	$i=2$; k=5 ; n_i=300,500	A-330844 B-591043
	$i=3$; k=4; n_i=1000,700,1200	A-533275, B-540140, C-717972
	$i=2$; k=6; n_i=800, 500	A-1221873, B- 836778
Cost per Unit	$i=2$; k=5 ; n_i=300,500	A-1103, B-1182
	$i=3$; k=4; n_i=1000,700,1200	A-533, B-771, C- 598
	$i=2$; k=6; n_i=800, 500	A-1527, B-1673
Inspection Policy	$i=2$; k=5 ; n_i=300,500	A-{SI, FI, SI, FI} B-{FI, SI, NI, NI}
	$i=3$; k=4; n_i=1000,700,1200	A-{FI ,NI, FI,} B-{FI, FI, SI} C-{FI, FI}
	$i=2$; k=6; n_i=800, 500	A-{NI, FI, FI, FI, FI, SI} B-{NI, FI, SI, FI, FI, FI}
Total Inspection Operations	$i=2$; k=5 ; n_i=300,500	A-4, B-2
	$i=3$; k=4; n_i=1000,700,1200	A-2, B-3, C-2
	$i=2$; k=6; n_i=800, 500	A-5, B-5

The results demonstrate the ability of the model to handle complex problems. It can be concluded that the model is highly extensible and applicable, so it can serve as a production planning tool to solve inspection allocation problems.

4.2 Experimental Design and Analysis

A Design of Experiment (DOE) is a structured and organized method for determining the relationship between factors (X_s) affecting a process and the output of that process (Y). Design of Experiment involves designing a set of experiments, in which all relevant factors are varied systematically. When the results of these experiments are analyzed, they help to identify optimal conditions, the factors that most influence the results, and those that do not, as well as details such as the existence of interactions and synergies between the factors.

Experiments are conducted to evaluate and analyze the various parameters and their interactions on the total processing cost. In this study, factors such as cost parameters, inspection errors and probability of defect are used to conduct a thorough investigation of the interactions on the total processing cost along with other system parameters. Two-level fractional factorial experimental designs are used to study and analyze the effect on total processing cost and to determine which variables are most influential. The experimental design is implemented using statistical software Minitab-Release 15. Minitab uses analysis of variance to decide which factors have an effect on the response.

4.2.1 Effect on Total Processing Cost

Here we analyze the effect of the considered factors on the total processing cost under a fixed unit manufacturing cost of 100 processing 100 units. A 2^{7-2} fractional factorial experiment was conducted for the factors chosen. The experiment requires 32 runs and the sequences of the experiments are randomized to ensure that variation between runs and biases are eliminated at all conditions. Table 4.18 provides the design matrix and an overview of the DOE factors used to access the performance of total processing cost under various operating conditions. It also shows the response values. The factors chosen for analyzing the effect on total processing cost are inspection cost (ICP), rework cost (RCP), penalty cost (PCP), probability of defect (Z), probability of Type-I error (Type-I), probability of Type-II error (Type-II) and Acceptable Quality Level (AQL). The levels of factors under consideration are taken from the previous published work and are shown in Table 4.19.

Table 4.17: 2^{7-2} Fraction Factorial Design

Factors:7
Runs:32
Resolution: IV
Fraction: 1/4

Table 4.18: The Design Matrix and DOE Factors

Std Order	Run Order	ICP	RCP	PCP	Z	TYPE-I	TYPE-II	AQL	TC
1	31	Low	Low	Low	Low	Low	High	High	20132
2	28	High	Low	Low	Low	Low	Low	Low	20124
3	23	Low	High	Low	Low	Low	Low	Low	20156
4	21	High	High	Low	Low	Low	High	High	20400
5	26	Low	Low	High	Low	Low	Low	High	20170
6	9	High	Low	High	Low	Low	High	Low	21000
7	8	Low	High	High	Low	Low	High	Low	20228
8	20	High	High	High	Low	Low	Low	High	21000
9	14	Low	Low	Low	High	Low	Low	Low	20084
10	18	High	Low	Low	High	Low	High	High	22897
11	2	Low	High	Low	High	Low	High	High	20303
12	7	High	High	Low	High	Low	Low	Low	22978
13	29	Low	Low	High	High	Low	High	Low	20097
14	3	High	Low	High	High	Low	Low	High	22884
15	27	Low	High	High	High	Low	Low	High	20316
16	4	High	High	High	High	Low	High	Low	23103
17	30	Low	Low	Low	Low	High	High	Low	19423
18	12	High	Low	Low	Low	High	Low	High	20400
19	25	Low	High	Low	Low	High	Low	High	19577
20	16	High	High	Low	Low	High	High	Low	20400

Table 4.18: The Design Matrix and DOE Factors (Contd.)

Std Order	Run Order	ICP	RCP	PCP	Z	TYPE-I	TYPE-II	AQL	TC
21	22	Low	Low	High	Low	High	Low	Low	19646
22	13	High	Low	High	Low	High	High	High	21000
23	32	Low	High	High	Low	High	High	High	19816
24	6	High	High	High	Low	High	Low	Low	21000
25	5	Low	Low	Low	High	High	Low	High	19820
26	24	High	Low	Low	High	High	High	Low	22250
27	1	Low	High	Low	High	High	High	Low	20053
28	19	High	High	Low	High	High	Low	High	22451
29	15	Low	Low	High	High	High	High	High	19980
30	10	High	Low	High	High	High	Low	Low	22767
31	11	Low	High	High	High	High	Low	Low	20433
32	17	High	High	High	High	High	High	High	23220

Table 4.19: The Levels of Factors

Notation	ICP	RCP	PCP	Z	TYPE-I	TYPE-II	AQL
Low	1	45	200	0.01	0.02	0.02	1
High	15	55	500	0.10	0.15	0.15	2.5

A sampling plan created for the given batch size and the corresponding AQL values following Military Standard 105E is shown in Table 4.20.

Table 4.20: Sampling Plan

Batch Size=100	Sample Size	Acceptance Number
AQL		
1 %	13	0
2.5%	13	1

In this analysis, totally confounded patterns were not taken into consideration. Table 4.21 contains the estimated effects and the coefficients of the experiments. The probability values close to zero are considered to be significant. Figure 4.1 presents the normal probability plot of the effects estimates from the experiments. It shows that the main effects A, B, C, D, E and the interaction AC and AD are significant at 95% level. The points lying on the straight line can be interpreted as random noise. On the other hand, points corresponding to A, B, C, D, E, AC and AD appear to be falling off the straight line, hence the significant factors.

Table 4.21: Estimated Effects and Coefficients for TC

Term	Effect	Coef.	SE Coef.	T	P
Constant		20878.4	27.51	758.91	0.000
ICP	1727.5	863.7	27.51	31.40	0.000
RCP	172.5	86.3	27.51	3.14	0.020
PCP	325.8	162.9	27.51	5.92	0.001
Z	1197.7	598.9	27.51	21.77	0.000
TYPE-I	-227.2	-113.6	27.51	-4.13	0.006
TYPE-II	31.0	15.5	27.51	0.56	0.594
AQL	39.0	19.5	27.51	0.71	0.505
ICP*RCP	-18.8	-9.4	27.51	-0.34	0.745
ICP*PCP	183.5	91.8	27.51	3.34	0.016
ICP*Z	955.5	477.8	27.51	17.37	0.000
ICP*TYPE-I	115.0	57.5	27.51	2.09	0.082
ICP*TYPE-II	52.3	26.1	27.51	0.95	0.379
ICP*AQL	39.8	19.9	27.51	0.72	0.497
RCP*PCP	24.0	12.0	27.51	0.44	0.678
RCP*Z	87.3	43.6	27.51	1.59	0.164
RCP*TYPE-I	35.5	17.7	27.51	0.65	0.543
RCP*TYPE-II	-79.5	-39.8	27.51	-1.44	0.199
RCP*AQL	-97.5	-48.8	27.51	-1.77	0.319
PCP*Z	-80.2	-40.1	27.51	-1.46	0.195
PCP*TYPE-I	110.2	55.1	27.51	2.00	0.092
PCP*TYPE-II	-2.5	-1.3	27.51	-0.05	0.965
PCP*AQL	-25.0	-12.5	27.51	-0.45	0.666
Z*TYPE-I	16.3	8.1	27.51	0.30	0.778
Z*TYPE-II	-9.8	-4.9	27.51	-0.18	0.865
Z*AQL	-25.8	-12.9	27.51	-0.47	0.656

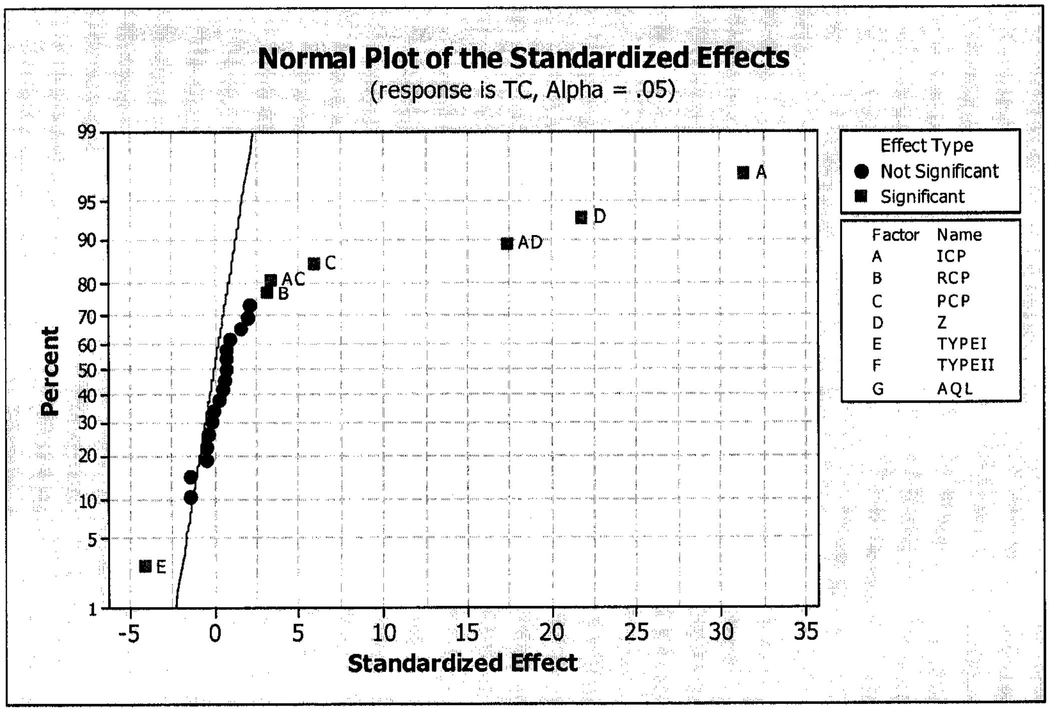

Figure 4.1: Normal Probability Plot of Effects

From the DOE analysis, we can identify the coefficients of the regression equation as shown in Equation 4.1 for calculating the total processing cost.

$$TC = 20874.4 + \left(\frac{1727.5}{2}\right) \times ICP + \left(\frac{172.5}{2}\right) \times RCP + \left(\frac{325.8}{2}\right) \times PCP + \left(\frac{1197.7}{2}\right) \times Z$$
$$- \left(\frac{227.2}{2}\right) \times Type - I + \left(\frac{183.5}{2}\right) \times ICP \times PCP + \left(\frac{955.5}{2}\right) \times ICP \times Z$$

.... (4.1)

If we shift from the lower inspection cost values to the higher values, the main effect will be to increase the total processing cost by an amount of 1727.50. The main effect of RCP causes an increase by an amount of 172.5 in the total cost when RCP increases. A shift from lower penalty cost to higher penalty cost increases the total processing cost by 325.8. An increase in Z will increase the total cost by 1197.70. The total processing cost decreases by an amount of 227.2 if we shift from lower value of Type-I error to the higher value. A simultaneous increase in inspection cost and probability of processing a defective unit increases the total cost. This interaction effect is 955.5. The interaction effect of inspection cost and penalty cost has an effect of 183.5 on the total cost.

Table 4.22: Analysis of Variance for TC

Source	DF	Seq. SS	Adj. SS	Adj. MS	F	P
Main Effects	7	36870841	36870841	5267263	217.48	0.000
2-Way Interactions	18	8316504	8316504	462028	19.08	0.001
Residual Error	6	145316	145316	24219		
Total	31	45332661				

The analysis of variance again confirms the results obtained previously using the normal probability plot showing that main effects A, B, C, D and E, and two way interaction effects AC and AD are significant. The probability values very close to zero are considered to be significant. Now, to check the assumption of normal distribution, residual analysis was performed using multiple regression analysis.

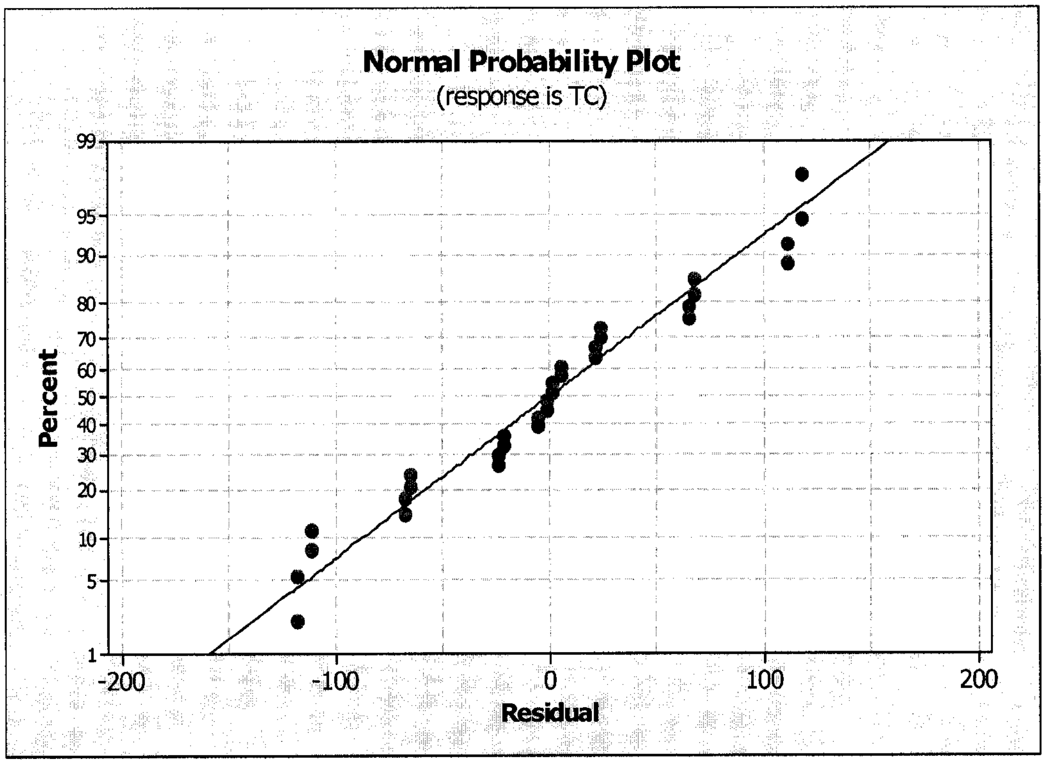

Figure 4.2: Normal Probability Plot of Residuals

Figure 4.2 is the normal probability plot of the residuals and the plot is satisfactory. As a diagnostic check, the residual plot confirms that the model is adequate. From the plot it can be concluded that all the points lie on the probability line and the deviation may be interpreted as noise. This plot confirms the assumptions that the effects of A, B, C, D, E, AC and AD can be explained as noise.

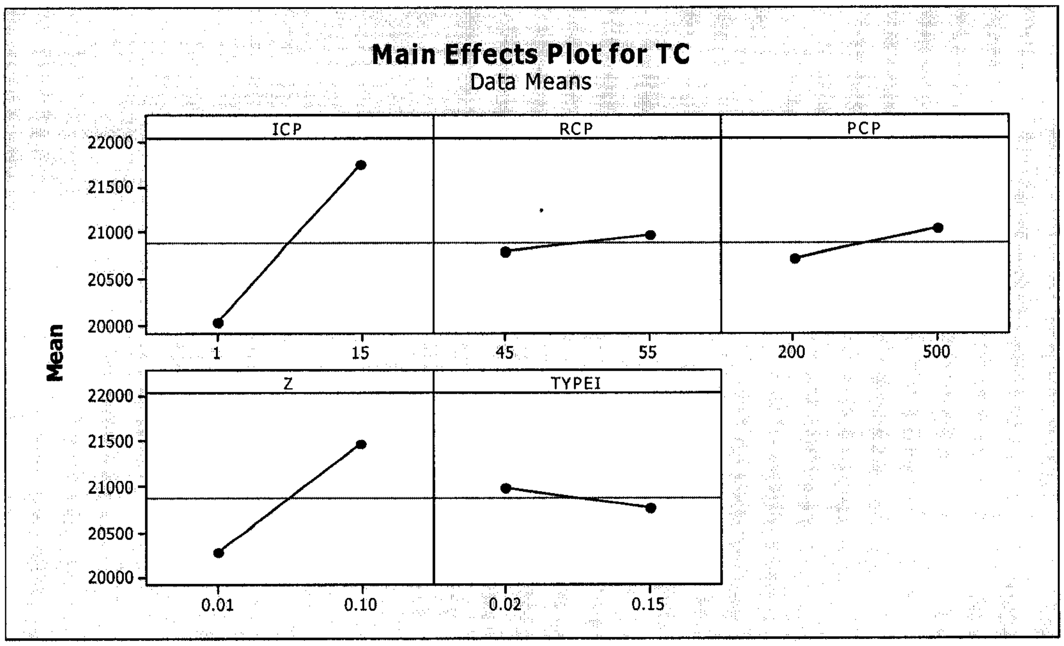

Figure 4.3: Main Effects Plot for TC

Figure 4.3 shows the main effects plots for the total cost. The factor of unit inspection cost, rework cost, penalty cost and probability of processing a defective unit has a positive effect on the cost per unit. The total cost increases with a shift from the lower values to the higher values. The probability of Type-I error has a negative impact on the total cost. The total cost decreases with the increase in Type-I error. High values of Type-I error increases the cost of performing full inspection which leads to the selection of other low-priced inspection option. Thus, high probability of Type-I error reduces the total processing cost.

Figure 4.4 shows the interaction plot for AC. According to the figure, if the penalty cost is more then the total cost is also higher at higher values of inspection cost. High penalty cost results in selection of either full inspection or sampling inspection. It results in the increase of total cost when inspection cost is also high. Figure 4.5 shows the interaction plot of AD. According to the figure, the total cost is low at lower values of inspection cost and probability of defect. As the probability of defect increases full inspection is performed and the high inspection cost increases the total cost.

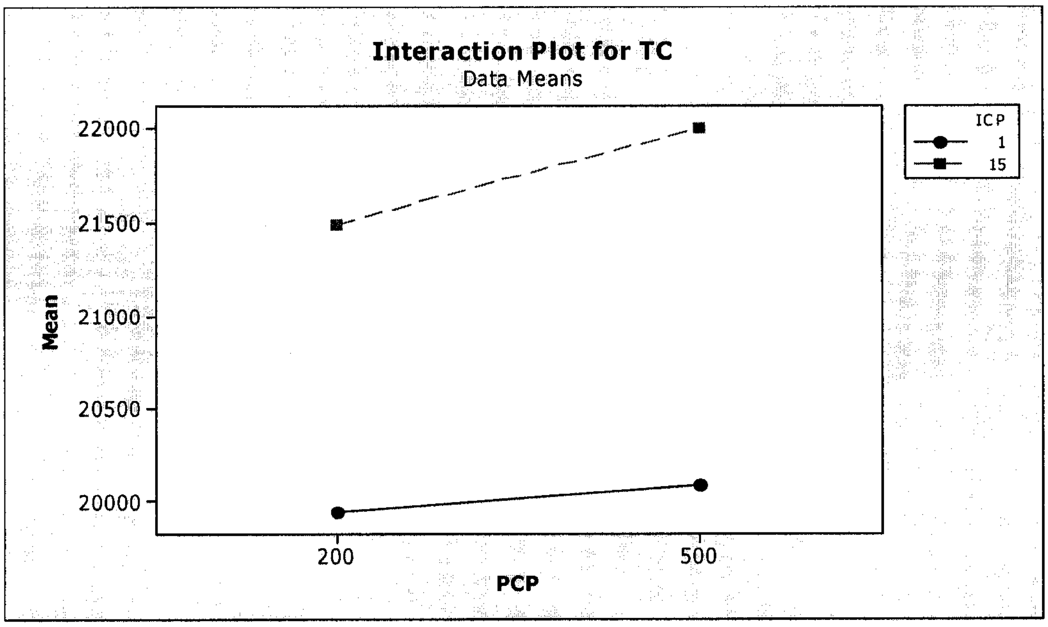

Figure 4.4: Interaction Plot (AC) for TC

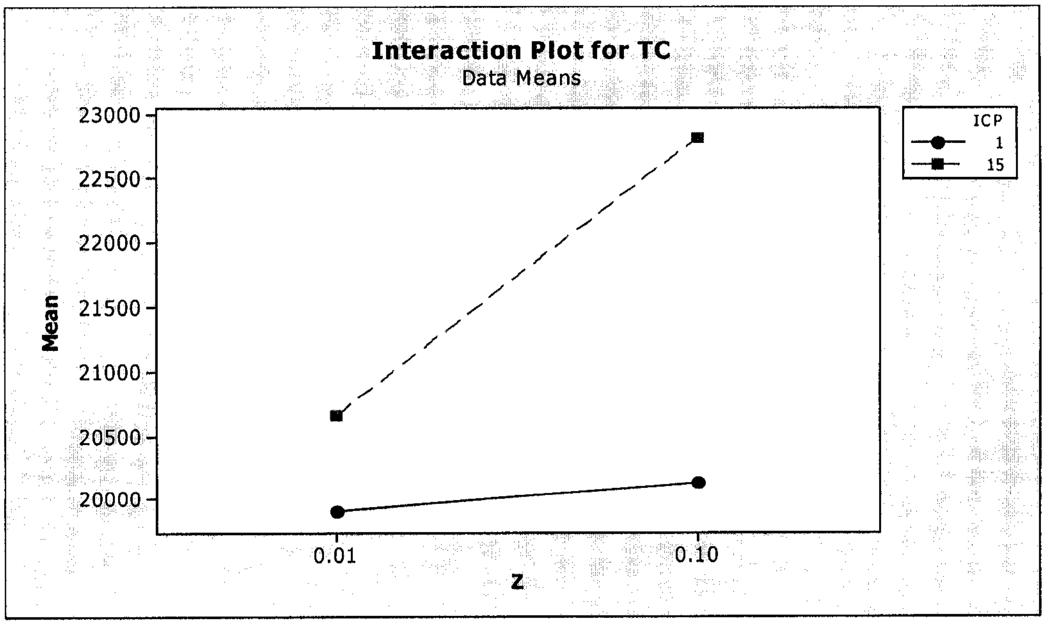

Figure 4.5: Interaction Plot (AD) for TC

The results suggest that the other parameters such as Type-II inspection error and AQL do not have a direct impact on the minimized total production cost. It is the inspection cost and the probability of processing a defective unit at a particular stage that will affect the total cost more significantly. This will in turn affect the inspection decision at that stage. If the probability of processing a defective unit is high, a more rational decision will be to allocate an inspection station performing full inspection after that stage irrespective of the costs so as to minimize the number of non-conforming units being forwarded in the system. If the probability of processing a defective unit is low, it would be more economical not to perform an inspection provided that the penalty cost is also low. In the intermediate range of probability of processing a defective unit, the decision on which type of inspection policy to adopt will depend on the cost parameters and the probability of acceptance of the sample at that stage. The fact that Type-I error has a negative impact on the total cost implies that at a higher value of Type-I error, the total cost is also higher which would affect the selection of an economic inspection plan.

In the next section, the impact of inspection errors and AQL on the selection of full inspection plan and sampling inspection plan will be discussed.

4.3 Impact of Inspection Errors on Full Inspection Cost

In an effort to gain insight into the impact of inspection errors on the full inspection operation, the model was run under variety of conditions of probability of defect and inspection errors. Three scenarios were generated representing probability of defects ranging from lower to higher values. In these scenarios, the inspection error moves from a better quality of 0.02 through three intermediate levels to a poor quality level of 0.15. For each scenario, the cost of performing full inspection operation was calculated. The data for the problem are given in the appendices. Tables 4.23 to 4.25 and the corresponding Figures 4.6 to 4.8 summarize the key variables for different combinations of Type-I and Type-II errors for different levels of probability of defect.

Table 4.23: Full Inspection Cost For Different Levels of Inspection Errors at Z=0.01

Type-II → / Type-I ↓	0.02	0.05	0.08	0.12	0.15
0.02	10997	10996	10994	10992	10991
0.05	11096	11095	11093	11091	11090
0.08	11244	11243	11242	11240	11238
0.12	11442	11441	11440	11438	11436
0.15	11591	11590	11588	11586	11585

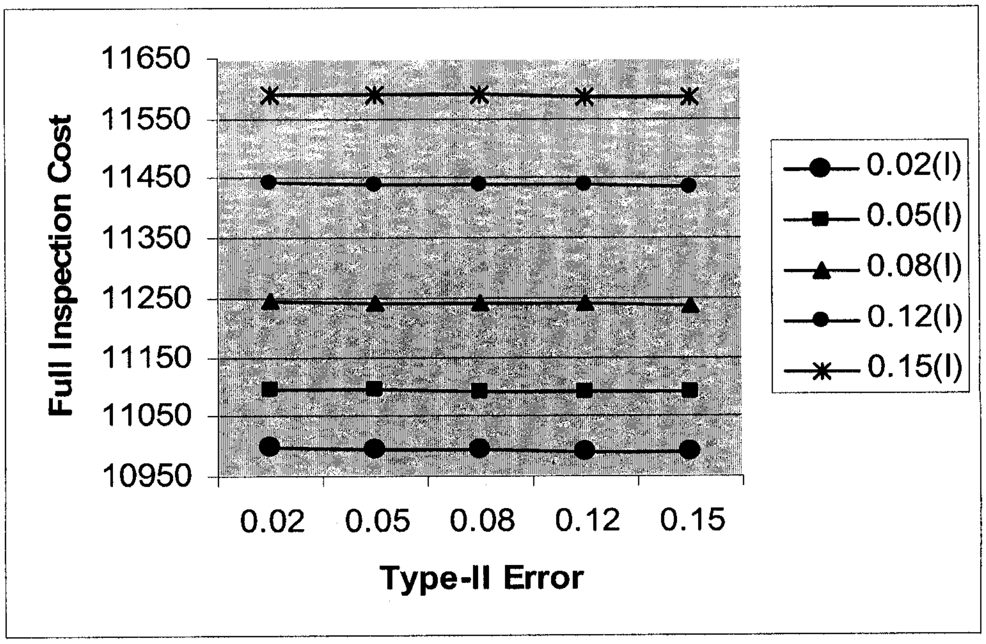

Figure 4.6: Full Inspection Cost w.r.t Varying Inspection Errors at Z=0.01

Table 4.24: Full Inspection Cost For Different Levels of Inspection Errors at Z=0.05

Type-II → / Type-I ↓	0.02	0.05	0.08	0.12	0.15
0.02	11140	11132	11125	11115	11107
0.05	11282	11275	11267	11257	11250
0.08	11422	11417	11410	11400	11392
0.12	11612	11607	11600	11590	11582
0.15	11757	11750	11742	11732	11725

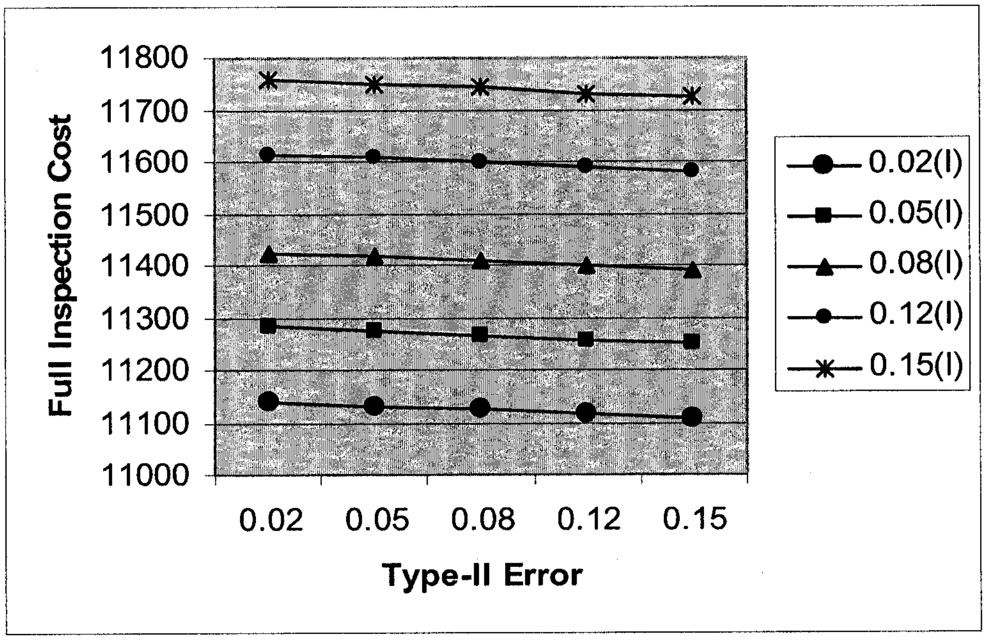

Figure 4.7: Full Inspection Cost w.r.t Varying Inspection Errors at Z=0.05

Table 4.25: Full Inspection Cost For Different Levels of Inspection Errors at Z=0.10

Type-II → / Type-I ↓	0.02	0.05	0.08	0.12	0.15
0.02	11380	11365	11350	11330	11315
0.05	11515	11500	11485	11465	11450
0.08	11650	11635	11620	11600	11585
0.12	11830	11815	11800	11780	11765
0.15	11971	11965	11950	11930	11915

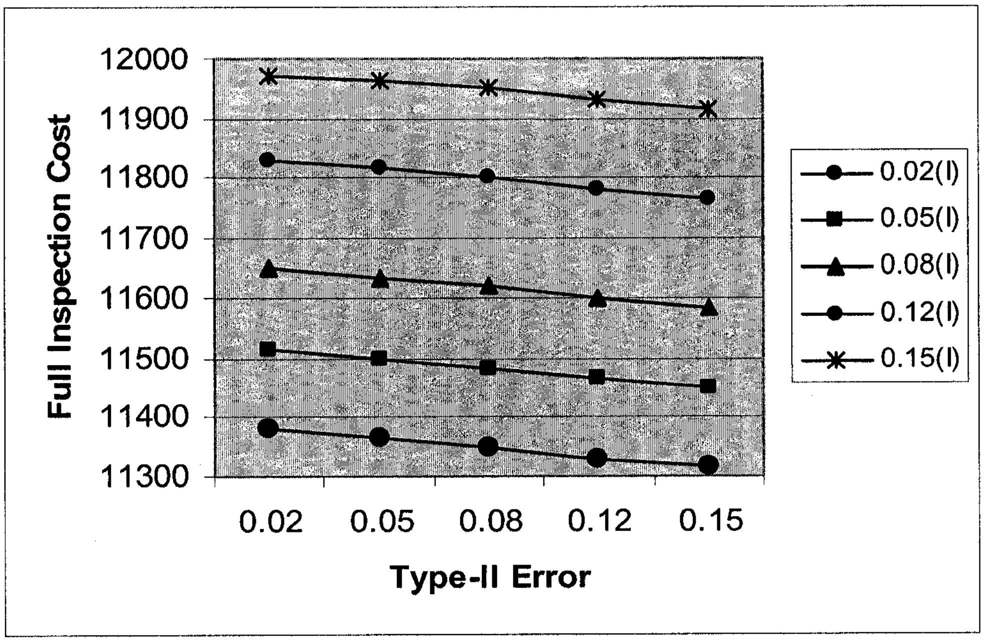

Figure 4.8: Full Inspection Cost w.r.t Varying Inspection Errors at Z=0.10

From the analytical results, it can be observed that the modeled system is more sensitive to Type-I error than Type-II error. In all the three scenarios, the full inspection cost increases with the increase in Type-I error. This will generally be true as long as the yield of the process exceeds 50% due to the greater exposure of Type-I error as well as the fact that Type-I inspection error erodes the base. That is, such errors reduce the number of conforming units which arrive to the customer. Hence, higher values of inspection errors and probability of defect occurrence results in an increase in the cost of full inspection. This will lead to the selection of the other available inspection options depending upon the system parameters.

4.4 Impact of AQL on Sampling Inspection Cost

In this section, the effects of the variation of the AQL and thus the acceptance criteria for a given sample on the cost of performing sampling inspection are observed. Two scenarios, one for AQL=1% and the other for AQL= 2.5 % with acceptance numbers 0 and 1, respectively, are considered. The data for the problem is given in the appendices. Sampling inspection cost for the two types of variations are given in Table 4.26 and Table 4.27. Figure 4.9 compares the sampling inspection cost with the variation in the probability of defects.

Table 4.26: Sampling Inspection Cost for Different Levels of Probability of Defect at AQL=1%

Z	*Pa*	Sampling Inspection Cost
0	1	10104
0.01	0.8775	10353
0.02	0.769	10565
0.03	0.6735	10745
0.04	0.5882	10897
0.05	0.5133	11027
0.06	0.4473	11171
0.07	0.3892	11317
0.08	0.3382	11450
0.09	0.2934	11570
0.1	0.2541	11678

Table 4.27: Sampling Inspection Cost for Different Levels of Probability of Defect at AQL=2.5%

Z	*Pa*	Sampling Inspection Cost
0	1	10104
0.01	0.9927	10288
0.02	0.9731	10476
0.03	0.9436	10662
0.04	0.9068	10842
0.05	0.8645	11012
0.06	0.8185	11138
0.07	0.7702	11234
0.08	0.7206	11317
0.09	0.6707	11390
0.1	0.6213	11454

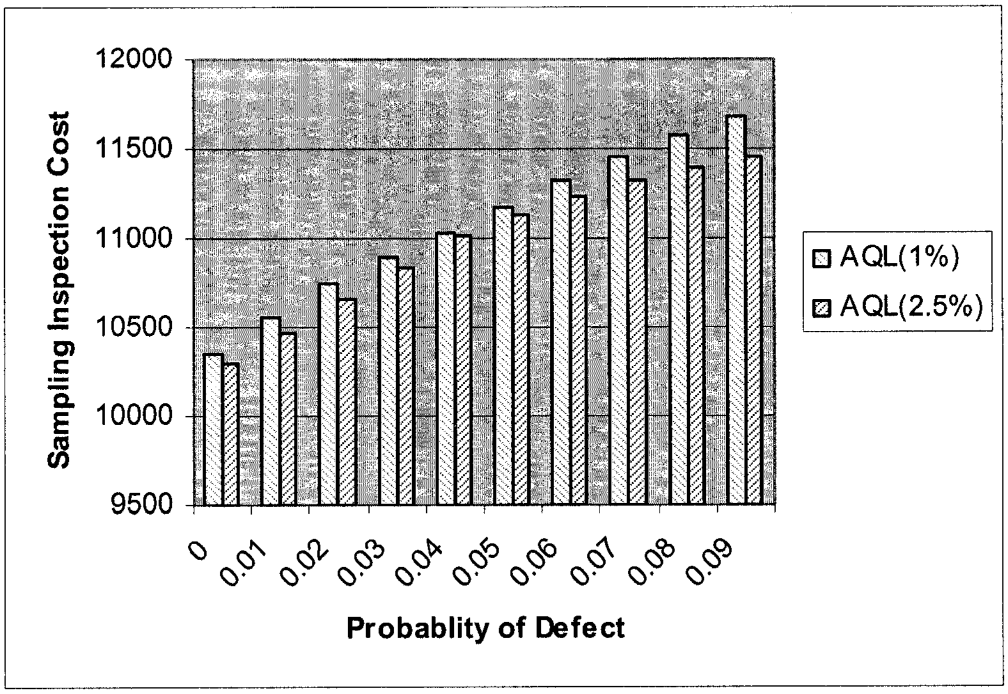

Figure 4.9: Sampling Inspection Cost w.r.t Varying Probability of Defect

In the two scenarios, it can be seen that sampling inspection cost increases with the increase in the probability of defect. The cost of performing sampling inspection is higher when AQL= 1% AQL than when AQL=2.5 %.

The costs of no inspection, at the three mentioned defect levels (0.01, 0.05, and 0.10) are 10350, 11750 and 13500, respectively. A comparison of the cost values for the three inspection options at same defect levels indicates the effect of inspection errors and AQL on the selection of a particular inspection plan. For the same defect levels, lower values of Type-I error result in lower full inspection cost as compared to sampling inspection cost and no inspection cost. This will lead to the selection of full inspection as the best option. At higher values of Type-I error, the full inspection cost is also higher which results in the selection of either the sampling inspection or no inspection as the best option.

4.5 Summary

The inspection allocation model developed in Chapter Three is solved using dynamic programming. The methodology is tested using problems of different sizes. Data used for the tested problems are realistic but hypothetical. The model is tested for manufacturing systems processing both single and multiple part types. The results obtained are reasonable for the variations in the most important parameters. The methodology developed is efficient and adequate in handling different problem scenarios.

In this chapter, insights are gained into the effects of changing the model parameters such as inspection cost, reworking cost, penalty cost, and probability of defect and inspection errors. An experimental design was conducted to mitigate the problem and was also used to quantify the results. The results suggested that inspection cost, probability of processing a non-conforming unit and the probability of Type-I error play a major role in determining the total processing cost, the optimal locations of inspection station and the type of inspection policy adopted. The other factors which have an influence on determining the total cost and inspection policy are reworking cost and penalty cost.

A sensitivity analysis was conducted to gauge the impact of inspection errors on full inspection and AQL on sampling inspection plan. The model proved to be more sensitive to Type-I error than to Type-II error. Nonetheless, many real world systems continue to pressure on the inspector to avoid Type-II errors which may be detected further down the line while failing to properly audit Type-I errors which may exist further among the discarded items at the inspection station. With the increase in the AQL, the cost of performing sampling inspection decreases. The above insights identify the forces affecting the objective function and the main interaction among these forces. Most of these phenomena in principle would remain valid beyond the specific example problems used in this study.

Chapter Five

Conclusions and Future Research

This chapter presents a summary of the research conducted in this thesis. It also presents several concluding remarks based on the problem modeling and results analysis. Future directions for research on this study are also discussed.

5.1 Concluding Summary

This research extends the work of Oppermann et al. (2003) by incorporating additional features in the model. They showed the mathematical background of the inspection allocation problem and the peculiarities of the inspection and repair process for a serial manufacturing system processing single part. In this thesis, a generic model is formulated to accommodate several scenarios simultaneously. The model is extended to a serial manufacturing system processing multiple part types with each part having a different sequence of manufacturing. The full inspection operation was considered to be subjected to two types of inspection errors. Problem size has been substantially increased due to the increase in planning horizon. While many different optimization methods are available, the dynamic programming optimization method is an appropriate method to perform optimization. The developed model is extensively tested by several hypothetical example problems with realistic features and the results are verified to ascertain the robustness of the model. Computational results showed that dynamic programming is an effective approach in solving such and similar problems. The performance of the proposed method on the sample problems indicates the feasibility of their implementation in a manufacturing environment.

In this work, design of experiments is used to analyze the effects of variation in manufacturing and inspection parameters on the selection and location of the inspection policy and thus the total processing cost. The results suggested that inspection cost, probability of processing a non-conforming unit and the probability of Type-I error play a major role in determining the optimal locations of inspection station and the type of inspection policy adopted, and thus the total processing cost. The other factors which have an influence were rework cost, penalty cost and interaction effects.

The method described above proved capable of providing insights into the influence of system parameters on the alterative inspection strategies. For the selected frames of problems and parameters a sensitivity analysis was conducted to determine the impact of inspection errors on full inspection and AQL on sampling inspection procedures. For full inspection option, it was found that the model was more sensitive to Type-I error as compared to the Type-II error. The full inspection cost increased with the increase in Type-I error. For sampling inspection option, inspection cost was lower at larger value of AQL.

It is important to note that the primary objective of this thesis work is to develop a general mathematical model for inspection allocation problem capable of handling multiple part types. The necessary steps in practice depend on the concrete quality of the investigated process and on the complexity of the investigated products.

5.2 Future Directions for Research

While the research and experimentation described in this thesis provide interesting and useful results the latent research possibilities in the inspection allocation problem are multifold and many problems remain to be solved. Some of the probable extensions to this work include the development of an integrated model to

- Study the effects of production scheduling and material handling operations on the quality.
- Extending the model to cases where there are constraints on the number of inspections, available inspection time and the inspection budget.
- Incorporating inspection error probabilities that vary according to the incoming fraction of non-conforming units at the inspection station.
- Accommodating features such as variable sampling parameters and effect of inspection errors on the sampling plan.
- Having a penalty cost function that increases with the fraction defective.

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Appendix A

Tables Utilized in Presenting the Discussion

Table A1: Acceptance Sampling Procedures [Montgomery, D. (2005)]

Table 14-1 Acceptance-Sampling Procedures

Objective	Attributes Procedure	Variables Procedure
Assure quality levels for consumer/producer	Select plan for specific OC curve	Select plan for specific OC curve
Maintain quality at a target	AQL system; MIL STD 105E, ANSI/ASQC Z1.4	AQL system; MIL STD 414, ANSI/ASQC Z1.9
Assure average outgoing quality level	AOQL system; Dodge–Romig plans	AOQL system
Reduce inspection, with small sample sizes, good-quality history	Chain sampling	Narrow-limit gauging
Reduce inspection after good-quality history	Skip-lot sampling; double sampling	Skip-lot sampling; double sampling
Assure quality no worse than target	LTPD plan; Dodge–Romig plans	LTPD plan; hypothesis testing

Table A2: General Data for Four Stage Manufacturing System

Number of Manufacturing Stations	4
Number of Units Manufactured	300
Probability of Type-I Error	0.08
Probability of Type-II Error	0.03
AQL	1%
Sample Size	50
Acceptance Number	1

Table A3: Data for Each Stage of a Four Stage Manufacturing System

Manufacturing Station	**1**	**2**	**3**	**4**
Manufacturing Cost	250	375	70	160
Inspection Cost	4	2	13	8
Reworking Cost	125	175	35	80
Penalty Cost	500	750	140	320
Probability of Defect	0.06	0.07	0.04	0.08
Pa	0.19	0.13	0.40	0.09

Table A4: General Data for Eight Stage Manufacturing System

Number of Manufacturing Stations	8
Number of Units Manufactured	1000
Probability of Type-I Error	0.05
Probability of Type-II Error	0.09
AQL	1.5%
Sample Size	80
Acceptance Number	3

Table A5: Data for Each Stage of an Eight Stage Manufacturing System

Manufacturing Station	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Manufacturing Cost	179	101	265	53	468	227	324	29
Inspection Cost	12	6	1	3	11	5	6	8
Reworking Cost	90	50	125	25	235	115	170	15
Penalty Cost	360	220	550	110	932	450	650	60
Probability of Defect	0.01	0.06	0.07	0.10	0.10	0.05	0.03	0.05
Pa	0.99	0.29	0.19	0.04	0.04	0.42	0.78	0.42

Table A6: General Data for Ten Stage Manufacturing System

Number of Manufacturing Stations	10
Number of Units Manufactured	5000
Probability of Type-I Error	0.12
Probability of Type-II Error	0.07
AQL	1%
Sample Size	200
Acceptance Number	5

Table A7: Data for Each Stage of a Ten Stage Manufacturing System

Manufacturing Station	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Manufacturing Cost	148	166	222	237	36	298	288	448	377	78
Inspection Cost	15	5	9	12	6	2	1	11	4	6
Reworking Cost	80	95	140	110	13	180	190	255	195	35
Penalty Cost	300	300	450	400	80	600	650	900	800	150
Probability of Defect	0.01	0.02	0.05	0.04	0.02	0.08	0.01	0.00	0.07	0.04
Pa	0.98	0.79	0.06	0.19	0.79	0.01	0.98	1	0.04	0.19

Table A8: Manufacturing Sequences

Manufacturing Station	**1**	**2**	**3**	**4**
Part Type-A	*	*	*	-
Part Type-B	-	*	*	*
Part Type-C	*	*	-	-

Table A9: General Data for Four Stage Three Part Type Manufacturing System

Part Type	**A**	**B**	**C**
Number of Units Manufactured	1000	700	1200
Probability of Type-I Error	0.05	0.06	0.02
Probability of Type-II Error	0.04	0.10	0.15
AQL	2.5%	2.5%	2.5%
Sample Size	80	80	80
Acceptance Number	5	5	5

Table A10: Data for Each Stage of a Four Stage Three Part Type Manufacturing System

Manufacturing Station		**1**	**2**	**3**	**4**
Manufacturing Cost	A	120	327	91	-
	B	-	415	300	80
	C	370	225	-	-
Inspection Cost	A	3	3	1	-
	B	-	7	4	1
	C	4	3	-	-
Reworking Cost	A	65	180	45	-
	B	-	220	145	35
	C	170	110	-	-
Penalty Cost	A	250	700	200	-
	B	-	900	650	125
	C	700	400	-	-
Probability of Defect	A	0.02	0.04	0.07	-
	B	-	0.08	0.06	0.01
	C	0.06	0.03	-	-
Pa	A	0.99	0.91	0.51	-
	B	-	0.38	0.65	0.99
	C	0.65	0.97	-	-

Table A11: Manufacturing Sequences

Manufacturing Station	1	2	3	4	5	6
Part Type-A	*	*	*	*	*	*
Part Type-B	*	*	*	*	*	*

Table A12: General Data for Six Stage Two Part Type Manufacturing System

Part Type	A	B
Number of Units Manufactured	800	500
Probability of Type-I Error	0.04	0.08
Probability of Type-II Error	0.08	0.04
AQL	1%	1%
Sample Size	80	80
Acceptance Number	1	1

Table A13: Data for Each Stage of a Six Stage Two Part Type Manufacturing System

Manufacturing Station		1	2	3	4	5	6
Manufacturing Cost	A	478	125	357	12	408	170
	B	445	242	182	47	388	355
Inspection Cost	A	7	5	3	9	1	10
	B	11	15	8	2	1	6
Reworking Cost	A	250	65	175	6	200	80
	B	220	121	90	25	190	170
Penalty Cost	A	950	250	700	24	800	340
	B	900	500	360	90	770	700
Probability of Defect	A	0.01	0.08	0.03	0.09	0.04	0.06
	B	0.01	0.08	0.03	0.04	0.03	0.06
Pa	A	0.99	0.23	0.81	0.16	0.68	0.42
	B	0.99	0.23	0.81	0.68	0.81	0.42

Table A14: Data for Sensitivity Analysis on Inspection Errors and AQL

Number of Units Manufactured	100
Manufacturing Cost	100
Inspection Cost	8
Reworking Cost	50
Penalty Cost	350
Sample Size	13

Appendix B

LINGO Code for Six Stage Manufacturing System Processing Single Part Type

MODEL:

SETS:

Part_types/1/: Num;

Manufacturing_stations/1, 2, 3, 4, 5, 6/;

Links(Part_types,Manufacturing_stations):Num_Conf,Num_Nonconf,Num_rework,F,CN,CF
,CS,Pa,Type_one,Type_two,NI,FI,SI,Manu_price,Insp_price,Rework_price,Prob_def,PC;

ENDSETS

DATA:

Num=500;

Manu_price= 21, 45, 29, 24, 15, 70;

Rework_price=10, 22, 15, 11, 7, 35;

Insp_price=1, 11, 13, 3, 8, 10;

Prob_def=0.09, 0.04, 0.01, 0.08, 0.03, 0.06;

PC=42, 90, 61, 50, 34, 140;

PA=0.16, 0.68, 0.99, 0.23, 0.81, 0.42;

Sample=50;

ENDDATA

! Processing Cost at stage 1;

```
@for (Part_types (i) :
@for (Manufacturing_stations (k) |k #EQ# 1:
F(i,k)= @min(links(i,k):C1(i,k)*NI(i,k)+C2(i,k)*FI(i,k)+C3(i,k)*SI(i,k))
););
! Processing Cost at other stages;
@for (Part_types (i) :
@for (Manufacturing_stations (k) |k #GE# 2:
F(i,k)=@min(links(i,k):(C1(i,k)*NI(i,k)+C2(i,k)*FI(i,k)+C3(i,k)*SI(i,k))+    F(i,k-1))
););
! Inspection Option;
@for (Part_types (i) :
@for (Manufacturing_stations (k) :
@BIN (NI(i,k));
@BIN (FI(i,k));
@BIN (SI(i,k))
););
! This constraint makes sure that only one inspection option is selected at a stage;
@for (Part_types (i) :
@for (Manufacturing_stations (k) :
NI (i, k) + FI (i, k) + SI (i, k) =1
););


@for (Part_types (i) :
```

```
@for (Manufacturing_stations (k):@GIN (Num_conf (i, k)), @GIN (Num_nonconf (i, k))
););
! Compute cost associated with each inspection option at stage 1;
@for (Part_types (i) :
@for (Manufacturing_stations (k) |k #EQ# 1:
CN(i,1)=((((Num_conf(i,k)+Num_nonconf(i,k))*Manu_price(i,k))+
        (Num_nonconf(i,k)*PC(i,k))));
CF (i, 1) = (((Num_conf (i, k) +Num_nonconf (i, k))*Manu_price (i, k)) +
        ((Num_conf (i, k) +Num_nonconf (i, k))*Insp_price (i, k))+
        (Num_rework (i, k)*Rework_price (i, k)));
CS (i, 1) = ((((Num_conf (i, k) +Num_nonconf (i, k))*Manu_price (i, k)) +
          Pa (i, k)*((Sample*Insp_price (i, k)) +
          (((Sample))*Prob_def (i, k)*Rework_price (i, k)) +
          (((Num_conf (i, k) +Num_nonconf (i, k)))-Sample)*Prob_def (i, k)*PC (i, k)) +
          ((1-Pa (i, k))*(Num_conf (i, k) +Num_nonconf (i, k))*Insp_price (i, k))
          +Prob_def (i, k)*Rework_price (i, k)))));
););
! Compute cost associated with each inspection option at remaining stages;
@for (Part_types (i) :
@for (Manufacturing_stations (k) |k #GE# 2:
CN (i, 1) = ((((Num_conf (i, k-1)*(1-Type_one (i, k-1)) + Num_nonconf (i, k-1)*
          Type_two (i, k-1)))*Manu_price (i, k)) + (Num_nonconf (i, k)*PC (i, k))));
CF (i, 1) = ((((Num_conf (i, k-1)*(1-Type_one (i, k-1)) + Num_nonconf (i, k-1)*
```

```
            Type_two (i, k-1)))*Manu_price (i, k))) +
             ((Num_conf (i, k) +Num_nonconf (i, k))*Insp_price (i, k))+
            (Num_rework (i, k)*Rework_price (i, k)));
CS (i, 1) = ((((Num_conf (i, k-1)*(1-Type_one (i, k-1)) + Num_nonconf (i, k-1)*
            Type_two (i, k-1)))*Manu_price (i, k))) +
            Pa (i, k)*((Sample*Insp_price (i, k)) +
            (((Sample))*Prob_def (i, k)*Rework_price (i, k)) +
            (((Num_conf (i, k) +Num_nonconf (i, k)))-Sample)*Prob_def (i, k)*PC (i, k)) +
            ((1-Pa (i, k))*(Num_conf (i, k) +Num_nonconf (i, k))*Insp_price (i, k))
             +Prob_def (i, k)*Rework_price (i, k)))));
););
! Number of Conforming and Non-conforming units produced at stage1;
@for (Part_types (i) :
Num_conf (i, 1) = Num (i)*(1 -Prob_def (i, 1));
Num_nonconf (i, 1) = Num (i) * Prob_def (i, 1);
);
! Number of Conforming units produced at remaining stages;
@for (Part_types (i) :
@for (Manufacturing_stations (k)| k #GE# 2:
Num_conf (i, k) = (((Num_conf (i, k-1)*(1-Type_one (i, k-1))) +
                    (Num_nonconf (i, k-1)* Type_two (i, k-1))) +
                     Num_rework (i, k-1))*(1- Prob_def (i, k)) ););
! Number of Non-conforming units produced at remaining stages;
```

```
@for (Part_types (i) :
@for (Manufacturing_stations (k)| k #GE# 2:
Num_nonconf (i, k) = (((Num_conf (i, k-1)*(Type_one (i, k-1))) +
                      (Num_nonconf (i, k-1)* (1-Type_two (i, k-1)))) +
                       Num_rework (i, k-1))*(Prob_def (i, k))
););
! Number of parts reworked at all stages;
@for (Part_types (i) :
@for (Manufacturing_stations (k) :
Num_rework(i,k)=((Num_conf(i,k)*Type_one(i,k)+Num_nonconf(i,k)*(1-Type_two(i,k))))
););
! Inspection errors are considered only if full inspection is performed;
@for (Part_types (i) :
@for (Manufacturing_stations (k) :
Type_one (i, k) =0.02*FI (i, k);
Type_two (i, k) =0.05*FI (i, k) + (1-FI (i, k))
););
END
```