

DYNAMIC PORTFOLIO OPTIMIZATION ACROSS ASSET CLASSES

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ABSTRACT

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Vincent Godin

This study provides evidence that a dynamic portfolio strategy, grounded on an asymmetric GARCH model and applied to investments in equities, real estate, and commodities, outperforms static strategies in terms of wealth, Sharpe ratios, and expected utility even when short selling restrictions are imposed on real estate and equities. For small investors, the benefits are subsumed by transactions costs; for large investors, the dynamic strategy remains marginally feasible.

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INTRODUCTION

The heart of financial diversification is that the addition to a portfolio of a security whose return is less than perfectly correlated with at least one other security already in the portfolio will reduce that portfolio's riskiness. The benefits of portfolio diversification have traditionally been illustrated with equities, with the work of Markowitz (1952) the cornerstone. The growth and availability of mutual funds and exchange-traded funds have made it easier for investors of relatively modest means to diversify holdings of stocks and bonds — albeit in a limited way, and not exactly in the sense of Markowitz or an equilibrium model such as the Capital Asset Pricing Model. Even with the universe of financial investments constantly expanding to include, among others, commodities, real estate, index funds, and exotic derivatives, economic theory tells us that rational investors will take advantage of new opportunities to diversify. And while exploring the benefits of naively adding individual securities to a portfolio may no longer be of much interest, it is still relevant to ask whether greater diversification can be had through a branching out into a limited number of other asset *classes* and whether it is practical to do so. The reason for this is that the number of choices can overwhelm [Fox (2005)], and opportunities to diversify may be overlooked because we anchor on a class of assets with which we are familiar or are passed over because of our affinity for the status quo [Kahneman, Knetsch, and Thaler (1991)].

There are hints that investors do not fully integrate the various asset classes into their portfolio strategies but instead deal with them in isolation or in a compartmentalized way. *The Economist*, for example, has highlighted on more than one occasion that investors shift money from class to class because of perceived advantage or fashion, in

effect treating the individual securities within the classes indiscriminately, and further that these shifting investment tides may actually change the correlations between them [Buttonwood (2006)]. The enthusiasm that positive roll yields created for commodities derivatives may have led ultimately to those roll yields turning negative and the correlations between commodities to increase. This raises another issue concerning portfolio strategies generally, and that is whether there is any benefit to treating the covariability between asset classes as time-varying.

There is evidence that investment in different asset classes, tangible assets, such as real estate, positions in commodities, either directly or via futures, and through the use of international equities or bonds, enhances diversification and protects against downside risk. Fama and Schwert (1977), for example, find that residential real estate is a good hedge against inflation. The rationale for inclusion of commodities or real estate in portfolios is that their systematic risks differ from that of equities and bonds and that their correlation with equities is usually low. This may have contributed to the doubling in the number of financial futures contracts traded between 1999 and 2005 and in a fourteen-fold increase in contracts' value between 1998 and 2006 [Raghavan (2007); Domanski and Heath (2006)]. For real estate, the Bank for International Settlement cited in its 74th annual report that the creation of REITs helped to finance project from 1997 to 2003 and that the boost in REITs' prices was primarily due to strong demand from institutional investors seeking higher yielding investment opportunities. Since both asset classes have shown increased popularity it is more than rationale to evaluate their impact on a portfolio.

This study explores the mean-variance efficiency and practicality of managing a portfolio comprised of three asset classes: equities, real estate, and commodities. The joint time-varying means and covariances of weekly returns of the S&P TSX Composite Index, the S&P TSX REITs Index, and the Goldman Sachs Commodities Futures Index is estimated for the period January 5, 2000 to December 31, 2003 using an asymmetric GARCH model. The performance of three portfolio strategies is compared: a simple buy-and-hold in which mean-variance efficient portfolios are formed using information about the unconditional distribution of joint returns, but where no rebalancing is done from week to week to maintain the initial allocations; portfolios that are, again, based on the unconditional distributions but, in this case, are rebalanced every week to maintain the optimal mean-variance investment allocations; and portfolios that are formed and rebalanced as often as weekly to maintain their efficiency using updated information about the conditional distribution of joint returns in an asymmetric GARCH model.

In general, it is found that using the information from the evolving covariance structure of returns results in superior portfolio performance as measured by final wealth, Sharpe ratios, *ex-ante* and *ex-poste* utility. Disallowing short selling of the TSX and REITs diminishes the benefits of dynamic investing but does not eliminate it. However, in either case, the benefits are largely subsumed by the higher transactions costs of having to rebalance regularly.

The next section reviews the relevant literature. This is followed by a description of the real estate and commodities asset classes. The fourth section describes the data and methods. The fifth presents the results and discusses them. The final section draws conclusions about the present research and offers suggestions for future work.

BACKGROUND

The process of allocating investments among asset classes can be motivated through four aspects of investment theory. These are diversification, market integration, strategic asset allocation, and dynamic portfolio rebalancing.

A. Portfolio diversification

In a capital market equilibrium [Sharpe (1964), Lintner (1965), Mossin (1966)], an investor should hold all or almost all assets in order to eliminate non-systematic risk. However, market frictions make it prohibitively costly to hold and maintain large portfolios, and the identification of the “right” market portfolio to hold is problematic [Roll (1977)]. Statman (1987) demonstrates that virtually all systematic risk can be eliminated with naïve investment in as few as 30 to 50 randomly-selected securities because the average covariance of a portfolio approaches the covariance of the market in the limit; but even these smaller portfolios may be too costly to put together and maintain. All this stems from the work of Markowitz (1952, 1959) who developed the mathematics linking the covariances between assets and the standard deviations of portfolios formed from them. Portfolios along a Markowitz efficient frontier¹ — those yielding the highest expected return for a given level of risk — can be comprised of as few as two securities and as many as one wants, and so the fundamental questions for portfolio management are which and how many securities to include and how sophisticated one chooses to be in measuring their co-movements.

¹ The mathematics for which is reviewed in subsection D of “Data and Methods”.

B. Market Integration

A natural application of portfolio theory is to international investment or any markets that are separated geographically. To the extent that the systematic risks of such markets are not perfectly correlated, they are said to be less than integrated, and diversification can be enhanced by spreading investments across borders.

Lowenfeld (1909), in perhaps the earliest study of diversification, finds that patterns of returns across investment instruments change over time and can affect diversification. In a similar vein, Goetzmann, Li and Rouwenhorst (2005), in a study of the integration of markets over a 150-year period, find that markets have generally become more integrated, reducing the potential for diversification as a consequence, and that correlations seem to increase during bad times (e.g., the Great Depression). Longin and Solnik (2001) likewise find that correlations across markets tend to increase during bear markets and decline or reverse during bulls. Antoniou, Pescetto and Stevens (2007), in a multivariate GARCH analysis of the volatility movements of many markets, again find that correlations across markets and across asset classes change over time. They emphasize that the benefits of diversification are not maximized if the co-movements between markets and asset classes are not updated regularly; weekly observations were made in their study.

While this thesis is not concerned with international diversification per se, what can be learned from the research on market integration is that the covariability of returns across geographic markets and asset classes appears to change over time, and that the benefits of diversification are more fully captured if covariances are reassessed with some regularity.

C. Strategic asset allocation

The question of which assets to include in a portfolio has no clear answer. The average of all investors portfolios' returns represents what should be considered the market return [Sharpe (1991)] and the variation in performance of individual portfolios is explained almost entirely by variations in asset allocation [Ibbotson and Kaplan (2000)] as opposed to, say, varying levels of success in attempts to pick winners.

To what extent should non-financial assets such as commodities and real estate be included in the menu of investment possibilities? According to Edwards and Park (1996) an investment in either a commodity pool or a commodity trading fund can enhance portfolio performance. Additionally, they find that active management of commodities exposure outperforms buy-and-hold. Furthermore, correlations across asset classes have changed significantly over time [Bodie and Rosanky (1980), Edwards and Park (1996), Elton et. al. (1987), Chance (1994)] suggesting that dynamic portfolio strategies might outperform static ones.

D. Dynamic portfolio strategies

While economic theory may offer a number of reasons as to why the covariance structure of returns may change over time, it is silent on the best way to model these changes empirically. De Goeij and Marquering (2004) contend that multivariate GARCH models perform well when the measuring stick is investor wealth, and that asymmetric GARCH models result in greater improvements in utility than their symmetric counterparts. Fleming et al. (1998), have identified volatility spilling over from one asset class to another and note that this has increased since the crash of October 1987. They

recommend the use of generalized method of moments (GMM) [Hanson (1982)], instead of GARCH models, because they consider information flows to be stochastic events which, in turn cause volatilities to evolve stochastically. For the purposes of this study, volatilities, apart from the usually random error, are assumed to evolve non-stochastically, and so a GARCH model is adopted.

DESCRIPTION OF ASSET CLASSES

This study considers the inclusion of real estate and commodities in an otherwise all-equity portfolio. While the choice is somewhat arbitrary, there is evidence to support characteristics favourable to portfolio investment.

A. Real estate

The obvious way to invest in real estate is to buy land or buildings. However, the markets for both can be illiquid, the minimum investment substantial, and chunkiness or indivisibility may limit diversification. Benveniste et al. (2001) find that the cost of liquidity can represent a discount of between 12 and 22% in the value of real property. They also find, on the other hand, that if the property is very liquid, it can be traded at a premium relative to its net asset value. Katmarian and Lowrie (1997) note that the main problems of a direct investment in real estate are that investors face the illiquidity of the real estate market and the difficulty of diversifying across the different types of real estate available. The common types of income-generating real estate investments (i.e., buildings and not land, although land can generate income if it is rented for some use) are office buildings, shopping plazas, industrial structures, and lodgings. Real estate mutual funds appeared in the 1980s, ostensibly to make real estate investing more liquid and accessible to less wealthy investors. However, those funds were required to have at least 150 holders throughout their existence, which could create problems for the investors as well as the managers of the funds. The main problem was that each time that an investor wanted to withdraw money from the fund, the fund manager had to either find a new

investor to invest in the fund or sell some assets to pay back the initial investor. Because the funds were restricted according to the type of real estate held and the income generated, liquidity was often an issue when the sale of assets was needed. Their open-ended structure made it difficult for an individual investor to enter or exist without affecting the structure or investments of the fund. For these reasons, real estate mutual funds have not been particularly successful in Canada.

In the mid 1990s, closed-ended real estate funds appeared in Canada, making entry and exit easier. Being exchange-traded, their structure paved the way for real estate investment trusts (REITs). The first REIT to be listed on the Toronto Stock Exchange appeared in 1993. Since then, 18 others have been listed through initial public offerings.

REITs are securitized portfolios of real estate; they give small investors access to real estate investments that they might not have otherwise. Even though they are traded as regular equities on exchanges, REITs have features that differentiate them from regular equities. They are required to distribute between 75 to 95% of their income to fund holders. The value of the distribution will usually represent 7 to 13% of their unit price, which is bigger than any high yield equity stock. Like other equities, REITs can enjoy capital appreciation. They do so when the underlying real estate held appreciates or is expected to appreciate or when an increase in income growth is expected. Unlike regular companies, REITs only pay taxes on income not distributed to unit holders, so there is no double-taxation. Avoidance of double-taxation has made trusts popular in Canada, so much so that many companies had switched from the corporate form to trusts, and prompting the Canadian government to pass a law limiting the trust form to real estate companies that meet certain criteria.

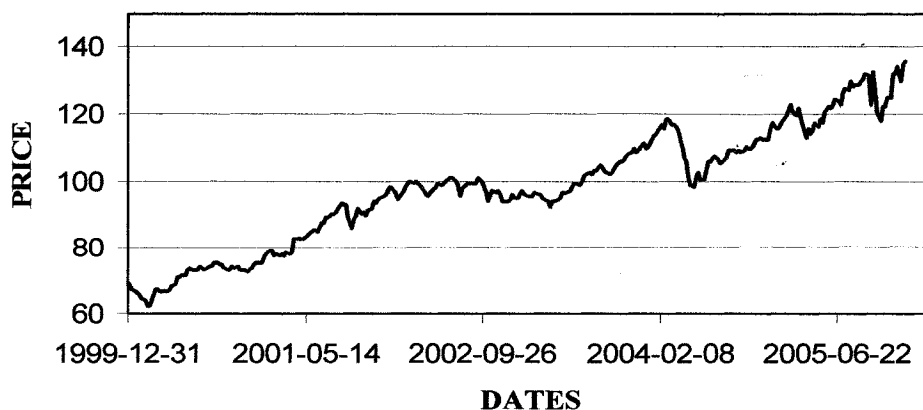
While REITs have existed in Canada since 1993, an index was not created until October 16, 2002. The TSX Group has synthetically reconstituted it back to January 1, 1999. As of December 31, 2005 the index contains the 12 REITs that are presented in Table 1 below. The time series of weekly index prices is shown in Figure 1. REITs have earned a mean weekly return of 0.2276% for the period of this study in comparison to 0.1292% for the TSX composite index.

Table 1. Constituents of the S&P REITS Index

Name	% Weight
RioCan Real Estate Investment Trust	25.48
H&R Real Estate Investment Trust	14.03
Summit Real Estate Investment Trust	10.08
Calloway Real Estate Investment Trust	7.99
Canadian Real Estate Investment Trust	7.83
Boardwalk Real Estate Investment Trust	6.31
Canadian Apartment Properties REIT	5.47
Primaris Retail Real Estate Investment Trust	4.99
Retirement Residences Real Estate Investment Trust	4.93
Chartwell Seniors Housing Real Estate Investment Trust	4.86
Legacy Hotels Real Estate Investment Trust	4.37
InnVest Real Estate Investment Trust	3.67

As of December 31, 2005. Source: Bloomberg

Figure 1. S&P TSX REIT



Weekly prices for the S&P TSX REIT Index, December 31, 1999 to December 31, 2005. Source: Datastream.

B. Commodities

Stock markets tend to react negatively to unexpected changes in the rate of inflation of an economy. Individual investors as well as institutions may try to protect themselves from the detrimental effects of inflation. Ankrum and Hansel (1993) note that pension plans have frequently sought partial insulation from the effects of inflation by devoting a fraction of their portfolios to real assets. There is also evidence that commodities futures can be a good hedge against inflation [Edwards and Park (1996)]. However, the inflation-hedging effectiveness of commodities can vary over time. Chance (1994) finds that the correlation between commodities and other securities is marked by considerable instability, where estimated coefficients of correlation vary in sign and magnitude, depending on the period analyzed and the asset in question. For these reasons, the inclusion of commodities as an alternative asset class in a portfolio strategy merits further exploration.

Direct investment in commodities is impractical for retail investors for the same reasons that real estate is, and the inconvenience extends to institutional investors as well. The viable alternative is commodity futures, although these too are pricey for the average retail investor. Commodity futures are, however, liquid, and the structure of futures markets allows investors to move from one real asset to another quite easily. Their popularity has increased dramatically. *The Wall Street Journal* in July 2007 reports that “Interest in mutual funds that buy commodities, a proxy for individual-investor activity, has soared: Assets in specialty-natural-resource and precious-metals funds surged to \$103.4 billion in May from \$8.1 billion at year-end 2001” [Raghavan (2007)]. Additionally, Domanski and Heath (2007) highlights that according to the Bank for

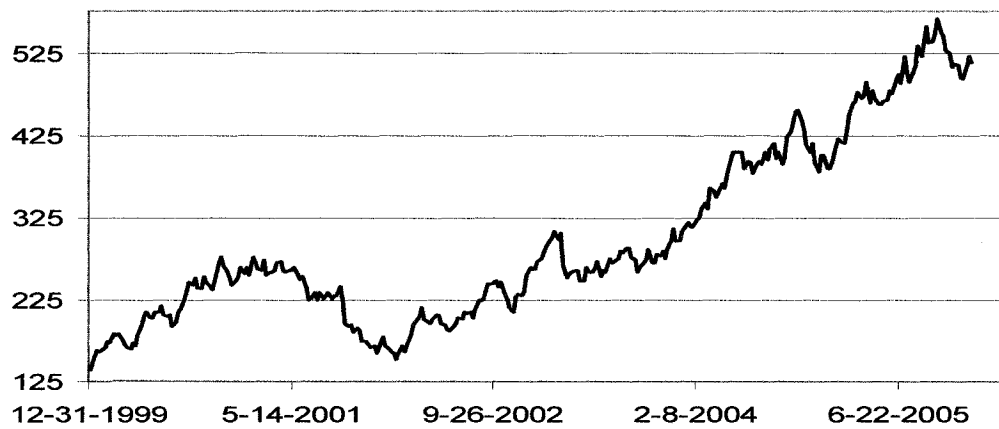
International Settlements' statistics, the value of derivatives contracts traded over-the-counter showed a fourteen-fold increase between 1998 and 2006. This craze for commodities makes them an asset class more important in the investment universe and then making it worthwhile to be analyzed in relation to equities.

Investors who are indifferent as to the specific underlying real assets can invest in commodity indices, the most well known perhaps being the Goldman Sachs Commodities Index (GSCI). This index was created in 1990, but it is possible to obtain simulated historical values back to 1970. Since 1992 there are futures traded on it, making it easily accessible to institutional and wealthier retail investors. For retail investors, Barclays Bank in July 2006 created the iShares S&P GSCI Commodity-Indexed Trust, which tracks the Goldman Sachs Commodities Index total return. This is an investment pool that tracks a long position in the GSCI futures contract, but with a minimum investment that is lower than direct investment in the futures contracts. The idea behind the introduction of investment vehicles such as the iShares was to permit individual or institutional investors to access investment in equity indices or sub-indices, real estate, fixed income, specific sectors or industries of the equity market, international equity indices and commodities. iShares are traded as regular equities on many exchanges. The main advantages of using iShares are that their management cost is lower than regular index mutual funds (0.20 versus 0.83%), their pricing is transparent because they are exchange traded, and they are liquid because of participation by specialists who supply or redeem shares of the market. For the iShares that track the GSCI, the objective is to replicate the return of the GSCI Total Return Index. The holdings of these iShares are

mainly futures contracts on the GSCI Total Return Index, futures contracts of the GSCI Excess Return Index and US Treasury T-Bills.

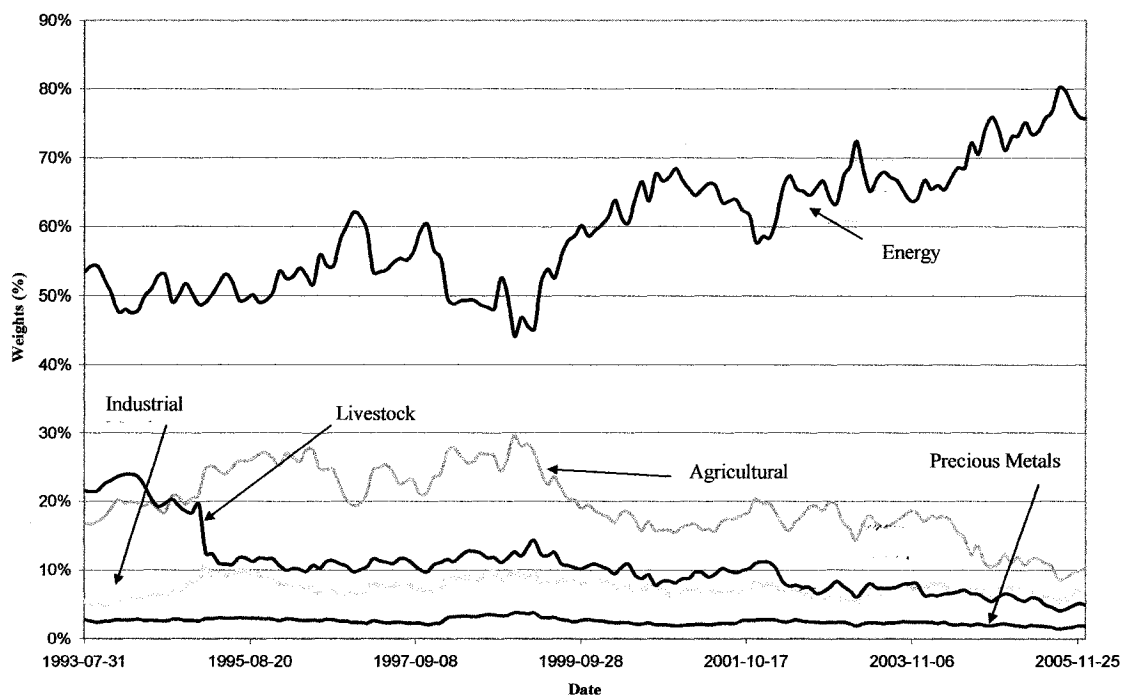
The GSCI index is a world production weighted index and includes 24 commodities. The commodity weights in the index are based on the average production of each commodity during the last five years. A rebalancing of the weights is made monthly according to criteria such as worldwide production, availability of future contracts for a specific commodity and the liquidity of that same contract. The 24 commodities held in the index are divided into five sub-categories: energy products, industrial metals, agricultural products, livestock products, and precious metals. Figure 2 shows the time series for the GSCI Index prices from December 29, 1999 to December 19, 2005. During that period, the average weekly return of the future contracts on that index has been 0.4833%. In Figure 3, it can be seen that the weights of the sub-categories has shown some variation, reflecting principally the impact of higher energy consumption and the higher price of the commodities in that category.

Figure 2. Goldman-Sachs Commodities Index



Prices of the GSCI Index in Canadian dollars, December 31, 1999 to December 19, 2005.
Source: Bloomberg

Figure 3. GSCI Index Sub-Category Weights



Weights of the five sub-categories of assets constituting the Goldman-Sachs Commodities Index, November, 1993 to December 19, 2005. Source: Goldman-Sachs.

DATA AND METHODS

A. Data

This study explores portfolios formed from three classes of assets: equities, commodities, and real estate. Equities are represented by the S&P TSX Composite Index excluding dividends (denoted TSX or asset 1), obtained from Datastream. Real estate is represented by the Canadian real estate investment trust index (denoted REIT or asset 2) created by the TSX group which was obtained, again, from Datastream. Commodities are represented by futures contracts traded on the Goldman-Sachs Commodity Index futures (denoted GSCI or asset 3), and were obtained from Bloomberg. The futures series is constructed to mimic an ongoing position in the front month contract, created by rolling over the position into the next one month contract on the first day of every month. Prices for the GSCI were converted from US to Canadian dollars using mid-market rates obtained from Morgan Stanley Capital International (MSCI) through Datastream.

All three series are daily prices or values covering the period December 29, 1999 to December 19, 2005. For the analyses, Wednesday-to-Wednesday simple weekly returns (percentage changes in value for the GSCI) were created, resulting in a total of 310 observations. Wednesday was chosen to minimize the incidence of missing observations due to holidays. Only the first 208 observations were used in this study. The remaining 102 observations were set aside for future work on out-of-sample forecasts.

B. Generalized autoregressive conditional heteroskedasticity (GARCH)

The generation of a time-varying minimum-variance frontier requires estimation of the covariances and mean returns of the three assets. This was done using an

asymmetric GARCH (1,1) specification for the covariance matrix shown in equations (1), (2), and (3).

$$(1) \quad H_t = C'C + A'e_{t-1}e_{t-1}'A + G'H_{t-1}G + D'H_{t-1}D$$

$$(2) \quad \bar{R}_t = \begin{pmatrix} \bar{R}_{1t} \\ \bar{R}_{2t} \\ \bar{R}_{3t} \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} + \begin{pmatrix} b_{1t}\sigma_{1t-1}^2 \\ b_{2t}\sigma_{1t-1}^2 \\ b_{3t}\sigma_{1t-1}^2 \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{pmatrix}$$

$$(3) \quad e_{it} \sim t(\eta)$$

H_t is the conditional 3x3 covariance matrix at week t , with coefficient estimates in the elements of matrices C , A , G , and D . C is symmetric, and its elements are estimated constants; A holds the estimated coefficients for products of lagged residuals; G the estimated coefficients for lagged covariances; and D the estimated coefficients for dummy variables which take the value 1 when lagged return is positive and zero otherwise. Inclusion of the asymmetries captured by D is motivated by the observation that security returns tend to be more volatile after a price drop than after a rise. Equation (1), excluding the last term, is commonly known as the BEK or BEKK GARCH model and is attributed to Engle and Kroner (1995). Reversing the signs of all coefficients does not affect on the function's value. The use of quadratic forms in the specification ensures positive definiteness of the covariance matrix, solving the problem of negative variance estimates that can occur in some other specifications.

The mean equations are given in (2), where a dependence on own lagged variance is included. This specification was first introduced by Engle, Lilien and Robin (1987) to capture unspecified sources of risk that might be priced in a market. Accordingly, riskier assets should provide a higher return to compensate. It should be noted, that in the

analysis of portfolio performance, a conservative unconditional estimate of the means based on simple averages over the sample period was used; this stacks the deck against any dynamic strategy considered. The full model represented by (1) and (2) was estimated by maximum likelihood under the assumption that the errors in (3) are approximately t -distributed with estimated shape parameter η to account for fat tails. The number of parameters estimated is 40: 33 in (1), 6 in (2), and η .

To test the validity of the model, restricted versions were estimated with some of the coefficients set to zero, and likelihood ratio tests done to determine whether these restricted models display significantly less explanatory power than the full model. The likelihood ratio test consists of testing the significance of the difference between the log likelihood values of the models analyzed. The empirical statistic, LR , for the test is computed as

$$(4) \quad LR = -2[L(\beta_R) - L(\beta_{UR})] \sim \chi_m^2,$$

where $L(\beta_R)$ and $L(\beta_{UR})$ are the log-likelihoods of the restricted and unrestricted model.

LR is distributed as χ^2 with m degrees of freedom equal to the number of restrictions.

C. Background on GARCH models

The GARCH model was developed by Bollerslev (1986) and has the feature of calculating a variance evolution recursively. An important aspect of the GARCH model is that it assumes mean-reversion in the variances of the securities analyzed by the model.

Bollerslev finds that the GARCH model has advantages over Engle's (1982) original ARCH model. The rationale for Engle's model was that conventional time series and econometric models operate under an assumption of constant variance. The ARCH

process he introduced allowed the conditional variance to change over time as a function of past errors leaving the unconditional variance constant. However one of the issues with the original ARCH model is that a long lag structure was required by the model, and to avoid the problem of negative variance, the lag structure often has to be fixed to a definite period. The GARCH model, however, is more flexible with respect to lag structure and can be framed to avoid the problem of negative variance estimates.

A GARCH model can be used when there is reason to believe that the variance of the error terms is not a function of an independent variable but instead varies over time in relation to the movement of the lagged errors. Examples of GARCH models analyzing time series include studies on interest rates, inflation, and stock market returns [Baillie, R., Chung, C-F., and Tieslau, M. (1996); Demirtas, K.O. (2006); De Goeij P. and Marquering, W. (2004)].

The frequency of the data used by a GARCH model is sometimes a concern. When using daily or weekly data, there is reason to expect that the variance of the error terms will depend on past volatilities going back a large number of periods. The problem in this case is that a large number of parameters must be estimated, and this may be difficult to do it with precision. Since the present analysis is made using weekly data, it will be important to look at the fit of the model to gauge its validity.

D. Portfolio formation

Regardless of the strategy being assessed, the investor chooses a portfolio on the minimum-variance frontier for at least one of the 208 weeks. The portfolio problem is stated as

$$(5) \quad \underset{(x)}{\text{Min}} \sigma_p^2 = x' H_i x,$$

where x is a 3×1 vector of investment weights, or the percentage of wealth allocated to each asset class such that $\sum_{i=1}^3 x_i = 1$. Following standard efficient set mathematics [see Huang and Litzenburger (1988)] the solution to the problem in (5) results in the familiar parabolic minimum-variance frontier

$$(6) \quad \sigma_p^2 = (\bar{R}_p \quad 1) Q^{-1} \begin{pmatrix} \bar{R}_p \\ 1 \end{pmatrix},$$

where \bar{R}_p is the desired expected return on a portfolio,

$$Q = \begin{pmatrix} \bar{R}' H^{-1} \bar{R} & \bar{R}' H^{-1} l \\ l' H^{-1} \bar{R} & l' H^{-1} l \end{pmatrix} = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix}; \quad \bar{R} \text{ is the vector of expected returns on the three}$$

assets; and l is a vector of 1's. The vector of optimal investment weights that yields a portfolio with desired expected return \bar{R}_p on the minimum-variance frontier is given by

$$(7) \quad x = H^{-1} (\bar{R} \quad 1) Q^{-1} \begin{pmatrix} \bar{R}_p \\ 1 \end{pmatrix}.$$

Examination of (7) reveals that the weights are a linear function of expected portfolio return, and so can be written more compactly as

$$(8) \quad x = g + h \bar{R}_p,$$

where g and h are the vector of weights for frontier portfolios with expected returns of zero and 100% respectively. All risk-averse, expected utility maximizers choose their portfolios according to (8) to lie on the efficient portion of the frontier. In this study, the representative investor is assumed to have quadratic tastes of the form

$$(9) \quad u(W) = W - \frac{\gamma}{2}W^2,$$

where $W = W_0(1 + R_p)$ is uncertain end-of-period wealth resulting from uncertain portfolio return, R_p , on current wealth, W_0 , and $\gamma > 0$ is a risk-preference parameter.

Without loss of generality, the investor's current wealth can be set equal to one dollar, and the utility function written in terms of uncertain portfolio return as

$$(10) \quad u(R_p) = \left(1 - \frac{\gamma}{2}\right) + (1 - \gamma)R_p - \frac{\gamma}{2}R_p^2.$$

The Pratt-Arrow coefficient of absolute risk aversion, ϕ , is positively related to preference parameter γ and negatively related to uncertain return,

$$(11) \quad \phi(\gamma, R_p) \equiv -\frac{u''(R_p)}{u'(R_p)} = \frac{\gamma}{(1 - \gamma) - \gamma R_p}.$$

Expected utility, V , is a linear function of expected return and variance of return.

$$(12) \quad V(\bar{R}_p, \sigma^2) \equiv E[u(R_p)] = \left(1 - \frac{\gamma}{2}\right) + (1 - \gamma)\bar{R}_p - \frac{\gamma}{2}\bar{R}_p^2 - \frac{\gamma}{2}\sigma^2$$

Equating the marginal rate of substitution of any indifference curve from (12) with the slope of the minimum-variance frontier in (6) yields the utility-maximizing expected portfolio return that will be chosen by the investor.

$$(13) \quad \bar{R}_p = \frac{d + \gamma(q_{11} - d)}{\gamma(q_{21} + d)} = \frac{1}{q_{21}} \left(\frac{d}{\phi} + q_{11} \right), \text{ where } d = \det(Q)$$

It can be seen in (13) that a more risk averse investor chooses a portfolio with a lower expected return because it is commensurately less risky. Further, the choice of portfolio in (13) is carried out by allocating wealth among the three assets according to the weights in (8).

E. Investment strategies

Three investment strategies were compared to see whether differences in the type and amount of information used to construct and maintain portfolios could result in better portfolio performance.

Buy-and-hold. The investor chooses an efficient portfolio in the first week based on unconditional estimates of the mean returns of each asset and the covariance matrix using the entire sample of 208 observations. Nothing else is done for the remainder of the investment period. The investment weights drift, and consequently so does the expected return on the portfolio, according to the actual returns earned by each asset week by week. Notably, there is no rebalancing to maintain the initial efficient allocation.

Static. This strategy differs from buy-and-hold only in that the portfolio is rebalanced each week to maintain the initial efficient allocation. The strategy is referred to as static because, like buy-and-hold, no attempt is made to incorporate conditioning information to update the estimates of the asset means or the covariances over time.

Dynamic. Under this strategy the portfolio is selected using the conditional estimates of the means and covariance matrix generated by the GARCH model presented in equations (1), (2), and (3). The portfolio is rebalanced each week as that information is updated.

The three strategies can be thought of as differing in the amount of information used. The static strategy makes more use than the buy-and-hold of the same information, and the dynamic strategy uses more information than the static.

F. Range of portfolios considered and evaluation of the strategies

For each strategy a number of portfolios were formed and strategy performance compared along a number of dimensions. One set of portfolios used a range of target annual expected returns, and the other was a set that would be chosen by the utility maximizer in (10) with range of coefficients of absolute risk aversion, ϕ . For both sets of portfolios, we compare the case where there is no constraint on short positions for any of the assets with the case where short selling of the TSX and REIT is not allowed because the majority of investors cannot easily short-sell equities or REITs. For the GSCI, we consider that short-selling is allowed because by definition, it is no more difficult to go short than long in futures contracts. It should be noted that, for computational convenience, portfolios formed under utility maximization were done so unconstrained, and then expected and realized utility calculated with the short-sales constraints imposed in those cases. The choices are, therefore, second-best in this regard.

Target expected return. For each of the three strategies, portfolios were formed with target expected returns in the range of 10 to 20%. Each of these is utility maximizing *initially* for *some* level of risk aversion; for example, a quadratic utility maximizer with coefficient of absolute risk aversion, ϕ , equal to 20, implied by $\gamma =$

$\frac{20}{21}$ at her current level of wealth prefers an expected annual return of about 15.9% when

facing the unconditional distribution of returns. But they are not utility maximizing for every week of the investment period unless the investor rebalances according to her preferences. The comparison of ex-poste utility between the three strategies is still useful in this regard because it resembles real-life professionally managed investments, that is,

those in which individual investors do not, or cannot, make the allocations precisely or rebalance them, as is the case for mutual or pension funds.

Expected utility. Portfolios formed are utility maximizing for investors with coefficients of absolute risk aversion, ϕ , in the range 10 to 50, and are rebalanced in any given week *only if* rebalancing yields a higher expected utility than doing nothing. The rebalancing is therefore conditional on expected utility and is applicable only to the dynamic strategy. Utility under conditional rebalancing for the dynamic strategy is compared to utility experienced under buy-and-hold.

The strategies were evaluated by comparing final wealth, Sharpe ratios without a riskless asset, and both by *ex-ante* and *ex-poste* utility.

Wealth. $W_{208} = \$100 \prod_{t=1}^{208} (1 + R_{pt})$ is simply the cumulative total return on an initial investment of \$100.

Sharpe ratio. The Sharpe ratio was introduced by Sharpe (1966) in order to quantify the ratio of reward-to-variability of mutual funds. The idea was to obtain a standardized measure to evaluate the performance of different securities or funds. The version of the Sharpe ratio used in this study excludes a risk-free asset and is given by

$\frac{\mu_p}{\sigma_p}$. A higher Sharpe ratio indicates ex-poste first-degree stochastic dominance of a

given strategy, and therefore would be preferred by any risk averter.

Ex-poste utility. Realized utility is computed as the mean of (10) using the portfolio returns realized under each strategy.

Ex-ante utility. Expected utility in (12) is computed for the dynamic strategy under conditional rebalancing using expected returns and standard deviations.

Implied costs of rebalancing. The cost of rebalancing portfolios under the dynamic strategies can be computed as the difference between Sharpe ratios under the buy-and-hold and the dynamic strategies; in other words, it is the break-even or maximum average weekly transactions cost that would make an investor indifferent between dynamic rebalancing and doing nothing.²

For comparison under target expected returns, the break-even transaction costs are equal to

$$(14) \quad TC = \mu_D - \left(\frac{\mu_B}{\sigma_B} \right) \sigma_D$$

The weekly transaction costs are considered to be the average weekly return of the dynamic strategy, μ_D , minus the product of the average Sharpe ratio of the buy-and-hold strategy $\left(\frac{\mu_B}{\sigma_B} \right)$ and the standard deviation of the dynamic strategy, σ_D . Equation (14)

assumes that the portfolio will be rebalanced every week. An expected utility maximizer subject to short-selling constraints will only rebalance when doing so yields higher utility, so the inferred transactions cost is scaled by the total number of weeks the portfolio is rebalanced, τ_D , out of the total number of weeks in the investment period, η_D .

$$(15) \quad TC = \left[\mu_D - \left(\frac{\mu_B}{\sigma_B} \right) \sigma_D \right] \cdot \frac{\eta_D}{\tau_D}$$

² Another way to infer the break-even transaction cost is by taking the difference in average ex-post utility under the two strategies.

RESULTS AND DISCUSSION

A. Descriptive statistics

Table 2 shows the average weekly returns for each asset class. For the TSX, there is a mild inverse relationship between the average returns and standard deviations, consistent with our modeling of asymmetries in D . While the average return tends to increase overtime, the standard deviation tends to decrease. For REITs, oscillating weekly returns and standard deviation are shown, and for the GSCI, a wider range of weekly returns than for the two other asset classes is observed. However, the variances of the returns for the latter are higher than for the two others. Table 2 also shows that average returns and standard deviations change dramatically over the period, providing informal support of a dynamic model. Table 3 reports zero-investment Sharpe ratios, which show similar variability as the means and standard deviations; notable is that not one asset dominates.

Table 2. Mean weekly returns and standard deviations

<i>Returns</i>			
Year	TSX	REITS	GSCI
2000	0.1317%	0.1439%	1.1124%
2001	-0.2651%	0.5216%	-0.5371%
2002	-0.2284%	0.0001%	0.8180%
2003	0.4222%	0.2697%	0.2786%
2000-2003	0.0372%	0.2349%	0.4389%
<i>Standard deviations</i>			
Year	TSX	REITS	GSCI
2000	3.0415%	1.3231%	3.9806%
2001	2.8382%	1.2661%	3.9294%
2002	2.3641%	1.4669%	3.8753%
2003	1.3894%	0.8864%	3.5942%
2000-2003	2.5031%	1.2671%	3.8972%

Mean weekly returns and standard deviations for the three asset classes, January 3, 2000 to December 31, 2003 (N= 208). TSX = S&P TSX Index; REITS = S&P TSX REIT Index; GSCI = futures on the Goldman Sachs Commodity Index.

Table 3. Sharpe ratios

Year	TSX	REITS	GSCI
2000	0.0433	0.1088	0.2794
2001	-0.0934	0.4119	-0.1367
2002	-0.0966	0.0001	0.2111
2003	0.3039	0.3043	0.0775
2000-2003	0.0148	0.1854	0.1126

Sharpe ratios excluding a risk-free asset, weekly data, January 3, 2000 to December 31, 2003 (N= 208). TSX = S&P TSX Index; REITS = S&P TSX REIT Index; GSCI = futures on the Goldman Sachs Commodity Index.

Finally, Table 4 shows correlation coefficients for sub-periods of the total sample, indicating, again, what appears to be time variation in the co-movements of returns.

Table 4. Correlation coefficients of weekly returns

2000			
	TSX	REIT	GSCI
TSX	1		
REIT	0.1440	1	
GSCI	0.1158	-0.0105	1
2001			
	TSX	REIT	GSCI
TSX	1		
REIT	0.4210	1	
GSCI	0.1555	0.2044	1
2002			
	TSX	REIT	GSCI
TSX	1		
REIT	0.4852	1	
GSCI	0.2376	0.0725	1
2003			
	TSX	REIT	GSCI
TSX	1		
REIT	0.2647	1	
GSCI	-0.0217	0.1937	1
2000-2003			
	TSX	REIT	GSCI
TSX	1		
REIT	0.3207	1	
GSCI	0.1376	0.0789	1

Correlations for each year and for the total sample, January 3, 2000 to December 31, 2003 (N= 208). TSX = S&P TSX Index; REITS = S&P TSX REIT Index; GSCI = futures on the Goldman Sachs Commodity Index.

B. GARCH Estimation – in sample

Maximum likelihood estimates of the model in equations (1), (2), and (3) are shown in Table 5. These estimates are used for the dynamic portfolio strategy. We refer to this as the Full model. Fifteen out of the 40 estimated coefficients are significant, and those that occur in each term of the specification. Hence we should consider that the corrections for the asymmetry and the GARCH-in-mean are useful in the modeling of the covariance matrix of the three assets.

In order to gauge the appropriateness of the Full model, likelihood ratio tests were performed on nested or restricted models. The alternative models considered were one which excludes asymmetries (restricting the coefficients in matrix D to zero)³; excluding dependence on own-variance in the mean equations (restricting the coefficients in vector b to zero) and excluding both asymmetries and GARCH-in-mean. The likelihood values, likelihood ratio statistics, and corresponding test results are presented in the Tables 6, 7, and 8. The hypotheses implied by the restrictions are:

H_0 = *The restricted model does not have significantly less explanatory power*

H_a = *The restricted model does have significantly less explanatory power*

A likelihood ratio test statistic bigger than its Chi-square cut-off requires us to reject the null. Considering the p -values from Table 8, we reject the null that the explanatory power of the restricted models is insignificantly less than that of the Full model. This therefore supports the use of the Full GARCH specification in constructing the dynamic strategy.

³ A non-italicized letter D is used in later tables to refer to the dynamic investment strategy.

Table 5. GARCH estimates – Full Model

		<i>Coefficient</i>	<i>Std. error</i>	<i>t-stat.</i>	<i>p-value</i>
1	α_1	0.0008	0.0030	0.2586	0.7960
2	b_1	0.5290	4.7241	0.1120	0.9108
3	α_2	0.0044	0.0011	4.0360	**0.0001
4	b_2	-11.1996	6.9065	-1.6216	0.1049
5	α_3	0.0183	0.0065	2.8151	**0.0049
6	b_3	-7.6701	3.6570	-2.0974	*0.0360
7	C(1,1)	0.0025	0.0034	0.7433	0.4573
8	C(2,1)	0.0052	0.0023	2.2342	*0.0255
9	C(2,2)	-0.0016	0.0064	-0.2447	0.8067
10	C(3,1)	0.0117	0.0384	0.3057	0.7598
11	C(3,2)	0.0302	0.0155	1.9465	0.0516
12	C(3,3)	0.0001	0.1666	0.0007	0.9994
13	A(1,1)	-0.4767	0.1321	-3.6085	**0.0003
14	A(1,2)	0.1128	0.0440	2.5629	*0.0104
15	A(1,3)	-0.8058	0.1680	-4.7949	**0.0000
16	A(2,1)	-0.2122	0.1939	-1.0947	0.2737
17	A(2,2)	-0.1486	0.0696	-2.1350	*0.0328
18	A(2,3)	0.0043	0.2345	0.0181	0.9855
19	A(3,1)	0.0194	0.0684	0.2837	0.7766
20	A(3,2)	0.1414	0.0248	5.7003	**0.0000
21	A(3,3)	-0.0260	0.0814	-0.3196	0.7493
22	B(1,1)	0.6074	0.1123	5.4081	**0.0000
23	B(1,2)	0.0910	0.0604	1.5076	0.1317
24	B(1,3)	0.1131	0.2202	0.5140	0.6073
25	B(2,1)	0.1364	0.2551	0.5347	0.5929
26	B(2,2)	0.3480	0.1282	2.7153	**0.0066
27	B(2,3)	0.6557	0.4196	1.5626	0.1181
28	B(3,1)	-0.3662	0.0552	-6.6360	**0.0000
29	B(3,2)	0.0127	0.0331	0.3847	0.7004
30	B(3,3)	0.2355	0.1439	1.6365	0.1017
31	D(1,1)	-0.4731	0.1334	-3.5467	**0.0004
32	D(1,2)	-0.1144	0.0693	-1.6517	0.0986
33	D(1,3)	0.4917	0.2515	1.9548	0.0506
34	D(2,1)	-0.4147	0.3890	-1.0661	0.2864
35	D(2,2)	-0.9864	0.2336	-4.2218	**0.0000
36	D(2,3)	-0.7023	0.4248	-1.6535	0.0982
37	D(3,1)	0.0891	0.1139	0.7824	0.4340
38	D(3,2)	0.1487	0.0420	3.5368	**0.0004
39	D(3,3)	-0.0306	0.0882	-0.3467	0.7288
40	η	8.4990	2.7399	3.1019	**0.0019

Maximum likelihood estimates of the coefficients of the tri-variate, asymmetric GARCH(1,1) model described in equations (1), (2), and (3) for 208 observations of the TSX index (asset 1), REITs (asset 2), and GSCI (asset 3) for the period January 5, 2000 to December 31, 2003. Likelihood value = 1,541. (** = significant at 99% and * = significant at 95%).

Table 6. Log Likelihood values

Name	# of parameters	Log likelihood value
Full Model	40	1541.3889
Restriction - D and b	27	1523.4911
Restriction - b	36	1534.1051
Restriction - D	30	1529.8300

Log likelihood values for each variation of the GARCH models in equations (1), (2), and (3). In the restricted versions, the coefficients of matrix D are set to zero, those of vector b are set to zero, or both are set to zero.

Table 7. Likelihood ratio statistics

Model 1	Model 2	Degrees of Freedom	LRT
Restriction- D and b	Full Model	13	35.7957
Restriction - b	Full Model	4	14.5677
Restriction - D	Full Model	10	23.1179

Likelihood ratio statistics (LRT), computed as in equation (4), for each variation of the GARCH models in equations (1), (2), and (3). In the restricted versions, the coefficients of matrix D are set to zero, those of vector b are set to zero, or both are set to zero.

Table 8: Likelihood ratio tests

Model 1	Model 2	df	LRT-stats	H_o	p-values
Restriction - D and b	Full Model	13	35.7957	Reject H_o	0.00034942
Restriction - b	Full Model	4	14.5677	Reject H_o	0.002226
Restriction - D	Full Model	10	23.1179	Reject H_o	0.00593683

Likelihood ratio statistics (LRT), computed as in equation (4), for each variation of the GARCH models in equations (1), (2), and (3). In the restricted versions, the coefficients of matrix D are set to zero, those of vector b are set to zero, or both are set to zero.

C. Comparison of Portfolio Performance

Unconstrained Target Return. In this section, the performance of portfolios formed and managed according to the three strategies is compared under the assumption that short selling of the TSX and REITs is not restricted. Table 9 shows that the dynamic strategy outperforms the static and buy-and-hold for all target returns as measured by final wealth, Sharpe ratios, and mean ex-poste utility.

Portfolios managed dynamically leave an investor 7.097% wealthier than the buy-and-hold and 5.306% than the static strategy. These two values can be considered as opportunity costs for not incorporating all new information with regard to each asset class.

Table 9. Performance of unconstrained portfolios by target return and strategy

E(R)	<i>Wealth (\$)</i>			<i>Sharpe Ratio</i>			$\phi = 20$		
	B	S	D	B	S	D	B	S	D
10%	144.90	143.85	152.28	0.1445	0.1409	0.1582	0.523817	0.523815	0.523823
11%	149.87	149.36	157.92	0.1628	0.1605	0.1774	0.523830	0.523829	0.523837
12%	154.80	154.93	163.63	0.1783	0.1784	0.1948	0.523840	0.523840	0.523848
13%	159.69	160.56	169.41	0.1902	0.1935	0.2095	0.523846	0.523849	0.523857
14%	164.54	166.26	175.23	0.1983	0.2049	0.2207	0.523850	0.523854	0.523863
15%	169.34	172.02	181.12	0.2028	0.2125	0.2280	0.523852	0.523858	0.523866
16%	174.10	177.82	187.05	0.2046	0.2165	0.2318	0.523851	0.523859	0.523867
17%	178.83	183.67	193.02	0.2043	0.2175	0.2326	0.523848	0.523857	0.523866
18%	183.51	189.57	199.03	0.2026	0.2164	0.2313	0.523843	0.523853	0.523863
19%	188.15	195.50	205.08	0.2001	0.2139	0.2284	0.523836	0.523847	0.523857
20%	192.76	201.47	211.15	0.1971	0.2105	0.2246	0.523828	0.523838	0.523849

Final wealth, Sharpe ratios, and mean ex-poste utility values for each target return, E(R). The utility results are from the perspective of an investor with a coefficient of absolute risk aversion of 20. The investment period is January 12, 2000 to December 31, 2003 (N = 208). B = Buy-and-Hold; S = Static; D = Dynamic.

That the Sharpe ratios are biggest throughout for the dynamic strategy implies first-degree stochastic dominance for all risk averters, at least ex-poste; that the ratios peak and then decline somewhat with target return is to be expected with the concavity of efficient frontiers.

Constrained Target Return. The dominance of the dynamic strategy is unaffected by a restriction on short selling the TSX and REITs (Table 10). The Sharpe ratios for the dynamic strategy peak at a 17% target return rather than 15% because the constraint is binding.

Table 10. Performance of constrained portfolios by target return and strategy

E(R)	<i>Final Wealth (\$)</i>			<i>Sharpe Ratio</i>			$\phi = 20$		
	B	S	D	B	S	D	B	S	D
10%	144.90	143.85	152.16	0.1445	0.1409	0.1580	0.523817	0.523815	0.523823
11%	149.87	149.36	157.65	0.1628	0.1605	0.1768	0.523830	0.523829	0.523837
12%	154.80	154.93	162.69	0.1783	0.1784	0.1927	0.523840	0.523840	0.523847
13%	159.69	160.56	168.60	0.1902	0.1935	0.2073	0.523846	0.523849	0.523855
14%	164.54	166.26	175.28	0.1983	0.2049	0.2189	0.523850	0.523854	0.523861
15%	168.96	171.93	181.37	0.1947	0.2094	0.2228	0.523845	0.523856	0.523862
16%	173.24	177.35	185.06	0.1809	0.2022	0.2123	0.523826	0.523847	0.523852
17%	177.47	182.49	187.62	0.1662	0.1892	0.1953	0.523798	0.523828	0.523831
18%	181.67	187.33	190.65	0.1535	0.1752	0.1787	0.523760	0.523799	0.523800
19%	185.86	191.87	194.58	0.1432	0.1621	0.1648	0.523715	0.523760	0.523762
20%	189.97	196.02	198.04	0.1352	0.1509	0.1526	0.523664	0.523713	0.523714

Final wealth, Sharpe ratios, and mean ex-poste utility values for each target return, E(R). The utility results are from the perspective of an investor with a coefficient of absolute risk aversion of 20. In this scenario, short selling of the TSX and REITs is assumed to be disallowed. The investment period is January 12, 2000 to December 31, 2003 (N = 208). B = Buy-and-Hold; S = Static; D = Dynamic.

The short selling constraints reduce terminal wealth by an average of 0.468% for the buy-and-hold strategy, 0.609% for the static strategy and 1.867% for the dynamic strategy. These reductions in terminal wealth can be interpreted as the costs of constraining the optimal weight in each asset class. Table 11 presents these costs for each target return. The values vary considerably from strategy to strategy, but since rebalancing from the conditional covariance matrix is used only for the dynamic strategy the impact is bigger than for the other two strategies.

Table 11. Wealth reductions when short-selling is constrained

E(R)	B	S	D
10%	0.000%	0.000%	-0.079%
11%	0.000%	0.000%	-0.172%
12%	0.000%	0.000%	-0.579%
13%	0.000%	0.000%	-0.478%
14%	0.000%	0.000%	0.025%
15%	-0.222%	-0.050%	0.141%
16%	-0.498%	-0.264%	-1.061%
17%	-0.758%	-0.643%	-2.799%
18%	-1.001%	-1.180%	-4.211%
19%	-1.218%	-1.856%	-5.115%
20%	-1.448%	-2.704%	-6.208%
M	-0.468%	-0.609%	-1.867%

Buy-and-Hold (B), the Static (S), and the Dynamic (D) strategies. The investment period is January 12, 2000 to December 31, 2003 (N=208).

Table 10 also shows that ex-poste utility is lower when short selling is constrained, which must be true as portfolios along an unconstrained efficient frontier are optimal for risk averters. Table 12 summarizes the differences between the unconstrained and constrained ex-poste utilities. Of note in comparing Tables 11 and 12 is that seemingly small reductions in mean utility translate into substantial reductions in mean wealth.

Table 12. Ex-poste utility reductions

E(R)	B	S	D
10%	0.000%	0.000%	0.000%
11%	0.000%	0.000%	0.000%
12%	0.000%	0.000%	0.000%
13%	0.000%	0.000%	0.000%
14%	0.000%	0.000%	0.000%
15%	-0.001%	0.000%	0.000%
16%	-0.002%	-0.001%	-0.002%
17%	-0.005%	-0.003%	-0.004%
18%	-0.008%	-0.005%	-0.006%
19%	-0.012%	-0.009%	-0.010%
20%	-0.016%	-0.013%	-0.014%
M	-0.004%	-0.003%	-0.003%

Percentage ex-poste utility reductions for a constrained investor. Buy-and-Hold (B), the Static (S), and the Dynamic (D) strategies. The investment period is January 12, 2000 to December 31, 2003 (N=208).

Expected Utility Maximization. If short selling is constrained, an investor may not necessarily rebalance a portfolio in a given week under a dynamic strategy because the expected utility may lower than doing nothing but maintaining the current position. In this section, we consider portfolios formed under expected utility maximization [equation (12)] where rebalancing in any given week is conditional on expected utility being higher than doing nothing.

Table 13. Expected utility maximization investment strategies

ϕ	<i>Final Wealth (\$)</i>		<i>Mean Expected Utility</i>		<i>Mean ex-poste Utility</i>		<i>Actual Mean Return</i>	
	D	B	D	B	D	B	D	B
10	183.30	185.252	0.545604	0.545539	0.545625	0.545517	0.303%	0.322%
20	186.04	171.617	0.523845	0.523835	0.523871	0.523835	0.307%	0.271%
30	178.11	167.072	0.516126	0.516117	0.516144	0.516128	0.285%	0.255%
40	175.59	164.580	0.512173	0.512163	0.512187	0.512175	0.278%	0.248%
50	177.90	163.061	0.509771	0.509759	0.509786	0.509772	0.284%	0.243%

Final wealth for the dynamic (D) and buy-and-hold (B) strategies, as well as average expected utility. The average ex-poste utility for the dynamic and buy-and-hold are also presented. Finally, the actual average weekly returns for the dynamic and the buy-and-hold are expressed. The investment period is January 12, 2000 to December 31, 2003 (N =208). All portfolios were built in accordance to the risk aversion levels (ϕ).

Table 13 shows that expected utility is higher under the dynamic strategy than the buy-and-hold for four of the five ϕ levels. On average, the dynamic strategy produces a 7.70% increase in wealth. For $\phi =10$, terminal wealth is almost the same as the buy-and-hold but slightly lower. A possible explanation is the low number of rebalancings (Table 14) that are made for that ϕ level. With only 10 rebalancings, the investment pattern is close to the buy-and-hold.

Table 14 shows a positive relationship between the average risk aversion level of the investor and the number of times a portfolio is rebalanced. The reason for this is that investors who are more risk averse have a higher sensitivity to changes in the covariance matrix of the assets. The higher sensitivity is a greater incentive to rebalance in order to stay closer of the efficient portfolio suggested by the updated covariance matrix. On the other hand, less risk averse investors will be willing to let their portfolios drift away from their initial investment weights without seeking immediate rebalancing, unless large change in the covariance matrix is observed.

Table 14. Number of rebalancings by level of risk aversion

ϕ	Rebalancings
10	26
20	66
30	100
40	134
50	145

The number of weeks the portfolio is rebalanced for each risk aversion level (ϕ) under a dynamic portfolio strategy. The maximum number of rebalancings is equal to the testing period (N=208).

Transaction costs. The superiority of a dynamic strategy will depend on the transactions costs borne. The maximum weekly transaction cost that an investor will be willing to pay can be computed as the amount which deducted from the return of a dynamic strategy reduces its Sharpe ratio to that of a buy-and-hold [see equation (14)]. Table 15 shows that for unconstrained dynamic portfolios that are rebalanced every week to maintain their target returns, this break-even cost varies from 1.83 to 4.57 basis points. There is a positive relationship between target expected return and break-even cost for dynamic portfolios but a negative relationship between target return and the amount by which the return target is beat. A dynamic portfolio with a target return of 10% can bear only 1.83 basis points in transaction costs but beats its target by 1.09% on an annual basis (excluding transactions costs). On the other hand, a portfolio with a target return of 20% can bear transaction costs of 4.57 basis points, but beats its target return objective by only 0.55% annually.

Table 15. Transaction costs by target return when short selling is unconstrained

		<i>Sharpe ratio</i>		<i>Mean return</i>	
E(R)	TC	B	D	B	D
10%	1.83	0.1445	0.1582	9.72%	11.09%
11%	1.87	0.1628	0.1774	10.64%	12.10%
12%	2.08	0.1783	0.1948	11.54%	13.10%
13%	2.41	0.1902	0.2095	12.41%	14.09%
14%	2.82	0.1983	0.2207	13.26%	15.05%
15%	3.25	0.2028	0.2280	14.07%	16.01%
16%	3.64	0.2046	0.2318	14.87%	16.95%
17%	3.98	0.2043	0.2326	15.64%	17.87%
18%	4.24	0.2026	0.2313	16.39%	18.78%
19%	4.44	0.2001	0.2284	17.12%	19.67%
20%	4.57	0.1971	0.2246	17.83%	20.54%

Inferred maximum transaction cost (TC) per week in basis points which when applied to the dynamic strategy (D) reduces its Sharpe ratio to that of a buy-and-hold (B). The investment period is January 12, 2000 to December 31, 2003 (N =208).

Transactions costs aside, all unconstrained dynamic portfolios outperform their targets while all buy-and-hold portfolios under-perform. Hence, although the inclusion of new information in portfolio management involves costly rebalancing, the new information helps to more reliably meet the target return objective.

If investors cannot short sell the TSX or REIT, the break-even transaction cost that can be borne increases under the dynamic strategy as shown in Table 16. The maximum of 4.57 basis points for an unconstrained portfolio with a target return of 20% (Table 15) rises to a maximum of 4.71 basis points for a constrained portfolio with a target return of 17%. The reason for this is that the short selling constraint is a bigger restriction on the buy-and-hold than the dynamic strategy because when the constraint is binding on a buy-and-hold it remains binding for the entire investment period.⁴

⁴ This also suggests a certain implausibility of the buy-and-hold or any strategy that does not involve at least some updating of information in that short position held for so long come with an unacceptable amount of risk.

Table 16. Transaction costs by target return when short selling is constrained

		<i>Sharpe ratio</i>		<i>Mean return</i>	
E(R)	TC	B	D	B	D
10%	1.80	0.1445	0.1580	9.72%	11.06%
11%	1.80	0.1628	0.1768	10.64%	12.05%
12%	1.81	0.1783	0.1927	11.54%	12.94%
13%	2.13	0.1902	0.2073	12.41%	13.95%
14%	2.63	0.1983	0.2189	13.26%	15.06%
15%	3.73	0.1947	0.2228	14.01%	16.05%
16%	4.53	0.1809	0.2123	14.73%	16.64%
17%	4.71	0.1662	0.1953	15.42%	17.04%
18%	4.62	0.1535	0.1787	16.10%	17.51%
19%	4.48	0.1432	0.1648	16.76%	18.11%
20%	4.06	0.1352	0.1526	17.40%	18.63%

Inferred transaction cost (TC) per week in basis points which when applied to the dynamic strategy (D) reduces its Sharpe ratio to that of a buy-and-hold (B). The investment period is January 12, 2000 to December 31, 2003 (N=208).

As shown in Table 17, the short constraint is binding under the buy-and-hold strategy for all 208 weeks for target returns of 15% or more presents the numbers of short position for the unconstrained dynamic and buy-and-hold strategies.

Table 17. Weeks short under the unconstrained strategies

E(R)	TSX		REIT	
	B	D	B	D
10%	0	0	0	1
11%	0	1	0	0
12%	0	2	0	0
13%	0	35	0	0
14%	0	122	0	0
15%	208	172	0	0
16%	208	193	0	0
17%	208	203	0	0
18%	208	206	0	0
19%	208	207	0	0
20%	208	208	0	0

Number of weeks that a short position would have to be assumed in the TSX and REITs under the buy-and-hold (B) and dynamic (D) portfolio strategies. The investment period is January 12, 2000 to December 31, 2003 (N = 208).

Table 15 and 16 suggest that, not only is there great variability in relation to target return and to the constraints imposed on investment, the break-even transactions costs are also too small for the dynamic strategy to be feasible. However, these inferred costs impose too big a penalty on the dynamic strategy because an expected utility maximizer would only rebalance if expected utility were higher than simply staying the course for another week. In the absence of transactions costs and short selling constraints, expected utility must always be higher for a rebalanced position given the updated information for the week. If we admit short selling constraints, the investor would not rebalance every week even if transactions costs were zero. Then for those weeks when rebalancing does yield higher expected utility, the break-even transaction cost can be inferred. Table 18 shows that under conditional rebalancing the maximum transaction costs that an expected utility maximizer would be willing to bear varies between 6.15 and 70.41 basis points, a range that makes the dynamic strategy feasible.

Table 18. Inferred transactions costs under conditional rebalancing

ϕ	Sharpe (B)	Sharpe (D)	Rebalancings	TC
10	0.1446	0.2038	26	70.41
20	0.1866	0.2370	66	20.61
30	0.1985	0.2316	100	8.48
40	0.1967	0.2293	134	6.15
50	0.1948	0.2384	145	7.47

Inferred transaction cost (TC) per week in basis points which when applied to the dynamic strategy (D) reduces its Sharpe ratio to that of a buy-and-hold (B). The portfolio is rebalanced under the dynamic strategy if expected utility is higher than if nothing were done. The investment period is January 12, 2000 to December 31, 2003 (N=208).

To illustrate for $\phi = 20$, a 15% portfolio is optimal under the dynamic strategy. This portfolio yielded an actual annualized mean return of 16.05% (Table 10). The maximum transactions costs were 3.25 basis points per rebalancing, and the portfolio would have been rebalanced every week (208 times). On the other hand, if the portfolio was rebalanced only if expected utility is higher, then the actual annualized mean return would be 16.79%, and the latter portfolio could bear transactions cost that amount to a maximum of 20.61 basis points per rebalancing, a six-fold increase, because it is only rebalanced 66 weeks.

Are transactions costs of, say, 20.61 basis points realistic? In a real investment environment, it is known that the lowest transaction costs that an investor would pay are at the institutional level. At that level, it is considered that transaction costs of ten basis points to sell and ten to buy in a rebalancing process are the norm. This only leaves 0.6 basis points of slack for an investor with $\phi = 20$. Without any other friction, the advantage of incorporating new updated information is almost gone and the investment return becomes quite close to a buy-and-hold strategy. The advantages of a dynamic strategy are more tenuous. For the average risk aversion levels of 30, 40 or 50, the maximum transaction costs go below ten basis points, making the strategy even less feasible.

From an individual investors' standpoint, the conclusion would be worse. The reason is that usually an individual investor uses the services of an investment advisor. The use of an advisor will usually increase the minimum transaction costs to around 1 or 2% of the amount invested. Unfortunately, the advantages of dynamic portfolio management vanish with weekly data, although it remains to be seen whether the strategy fairs better with monthly or some other lower frequency data.

CONCLUSIONS

This study used an asymmetric GARCH model to analyze the movements in covariance between three asset classes, equities, real estate, and commodities, in order to compare the performance of a dynamic portfolio strategy over two others that do not rely on updating information.

Two main conclusions come out of the analysis. The first is that the dynamic strategy, in the absence of transactions costs, out-performs a buy-and-hold strategy with regard to wealth, utility, and Sharpe ratios at every target return level. Hence, there is support for the idea that information about assets' past volatilities can enhance portfolio performance. The second conclusion is that under realistic restrictions on short selling, portfolios that are rebalanced conditional on expected utility can bear transactions costs that put dynamic strategies on the border of feasibility but just barely. In order to extend the findings of the present study, we suggest the modeling of the expected returns of each asset class as an autoregressive process and with lower frequency data, and to provide evidence on robustness, out of sample forecasts of the GARCH model.

REFERENCES

- Ankrim and Hansel (1993), Commodities in asset allocation: a real asset alternative to real estate?, *Financial Analysts Journal*, 49, 20-29.
- Antoniou, A., Pescetto, G., and Stevens I. (2007), Market-wide and sectoral integration, *Managerial Finance*, 3, 173-194.
- Baillie, R., Chung, C-F., and Tieslau, M. (1996), Analysing inflation by the fractionally integrated ARFIMA-GARCH model, *Journal of Applied Econometrics*, 11, 23.
- Bank for International Settlements (2003), 74th Annual Report, Part VII: The Financial Sector, 122-141.
- Benveniste, L., Capozza D. R., and Seguin P. J. (2001), The value of liquidity, *Real Estate Economics*, 29, 633-660.
- Bodie, Z. (1983), Commodity futures as a hedge against inflation, *The Journal of Portfolio Management*, 12-17.
- Bodie, Z., and Rosansky, V. (1980), Risk and return in commodity futures, *Financial Analysts Journal*, 27-39.
- Bollerslev, T. (1986), Generalized autoregressive conditional heteroscedasticity, *Journal of Econometrics*, 31, 307-327.
- Buttonwood (2006), Roll Over, *The Economist*, October 12, 2006.
- Chance, D. (1994), Managed futures and their role in investment portfolios, *The Research Foundation of the Institute of Chartered Financial Analysts*, Charlottesville, Virginia.

- De Goeij, P, and Marquering, W. (2004), Modeling the conditional covariance between stock and bond returns: a multivariate GARCH approach, *Journal of Financial Econometrics*, 2 (4), 531-564.
- Demirtas, K.O. (2006), Nonlinear asymmetric models of the short-term interest rate, *The Journal of Futures Markets*, 26, 869.
- Domanski, D., and Heath, A. (2007), Financial investors and commodity markets, *BIS Quarterly Review*, March, 53-67.
- Edwards, F., and Park, J. (1996), Do managed futures make good investments?, *The Journal of Futures Markets*, 16, 475–517.
- Elton, E., Gruber, M., and Rentzler, J. (1987), Professionally managed, publicly traded commodity funds, *Journal of Business*, 177–199.
- Engle, R. F. (1982), Autoregressive conditional heteroscedasticity with estimates of the variance of UK inflation, *Econometrica*, 50, 987-1008.
- Engle, R. F., and Kroner, K. F. (1995), Multivariate simultaneous generalized arch, *Econometric Theory*, 11(1), 122-150.
- Engle R., Lilien, D.M., and Robins R.P. (1987), Estimating time varying risk premia in the term structure: the ARCH-M model, *Econometrica*, 55, 391-407.
- Fama, E. F., and Schwert G. W. (1977), Asset returns and inflation, *Journal of Financial Economics*, 5, 15-46.
- Fleming, J., Kirby, C., and Ostdiek, B. (1998), Information and volatility linkages in the stock, bond, and money markets, *Journal of Financial Economics*, 49, 111-137.
- Fox, J. (2005), Why Johnny can't save for retirement, *Fortune*, 151(6), 202-207.

- Goetzmann, W., Li, L., and Rouwenhorst, G. (2005), Long-term global market correlations, *Journal of Business*, 78, 1-38.
- Goldman Sachs, Goldman Sachs, The GSCI Manual: A Guide to the Goldman Sachs Commodity Index, 2007 Edition, www.gs.com.
- Hansen, L.P. (1982), Large sample properties of generalized method of moments estimators, *Econometrica*, 50, 1029-1054.
- Huang, C., and Litzenberger R. (1988), Foundations for financial economics, Prentice-Hall Publishing, Chapter 3.
- Ibbotson R.G., and Kaplan P.D. (2000), Does asset allocation policy explain 40, 90, or 100 percent of performance?, *Financial Analysts Journal*, 56, 26-33.
- Irwin, S., and Landa, D. (1987), Real estate, futures, and gold as portfolio assets, *The Journal of Portfolio Management*, 29-34.
- Kahneman, D., Knetsch, J., and Thaler, R. (1991), Anomalies: the endowment effect, loss aversion, and status quo bias, *The Journal of Economic Perspectives*, 5, 193-206
- Katmarian, S., and Lowrie, S. (1997), The REIT stuff, *Benefits Canada*, March.
- Lintner, J., (1965), The valuation of risky assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics*, 47: 13-57.
- Longin, F. and Solnik, B. (2001), Extreme correlation of international equity markets, *The Journal of Finance*, 56, 649-76.
- Lowenfeld, H. (1909), Investment an exact science, *Financial Review of Reviews*.
- Markowitz, H.M. (1952), Portfolio selection, *The Journal of Finance*, 16, 77-91.
- Markowitz, H.M., (1959), Portfolio selection: efficient diversification of investments, New York: Wiley.

- Mossin, J. (1966), Equilibrium in a capital asset market, *Econometrica*, 34, 768-783.
- Raghavan, A. (2007), How hot investments burn the little guy, *Wall Street Journal*, July 14, B1.
- Roll, R. (1977), A critique of the asset pricing theory's tests, *Journal of Financial Economics*, 4, 129-176.
- Sharpe, W. F. (1964), Capital asset prices: a theory of market equilibrium under conditions of risk, *The Journal of Finance*, 19, 425-442.
- Sharpe, W. F. (1966), Mutual fund performance, *The Journal of Business*, 39(1), 119-139.
- Sharpe, W. 1991, The arithmetic of active management, *Financial Analyst Journal*, 47(1), 7-9.
- Statman, M. (1987), How many stocks make a diversified portfolio?, *The Journal of Financial and Quantitative Analysis*, 22, 353-363.
- The REIT Guide: 8th Edition, Real Estate Investment Trusts, Deloitte, Canada – Deloitte & Touche LLP.

APPENDIX

Appendix A. Results for the three investment strategies

Table 19. Unconstrained buy-and-hold strategy

E(R) annual	<i>Target</i>		<i>Actual</i>			
	E(R)	σ	Mean	σ	Sharpe	Wealth
10%	0.0018	0.0130	0.0019	0.0129	0.1445	144.90
11%	0.0020	0.0125	0.0020	0.0124	0.1628	149.87
12%	0.0022	0.0122	0.0022	0.0122	0.1783	154.80
13%	0.0024	0.0122	0.0023	0.0122	0.1902	159.69
14%	0.0025	0.0123	0.0025	0.0125	0.1983	164.54
15%	0.0027	0.0127	0.0026	0.0129	0.2028	169.34
16%	0.0029	0.0132	0.0028	0.0135	0.2046	174.10
17%	0.0030	0.0139	0.0029	0.0142	0.2043	178.83
18%	0.0032	0.0147	0.0030	0.0150	0.2026	183.51
19%	0.0034	0.0157	0.0032	0.0158	0.2001	188.15
20%	0.0035	0.0167	0.0033	0.0167	0.1971	192.76

Descriptive statistics for the unconstrained buy-and-hold strategy. The left panel shows target annual expected portfolio return followed by the weekly equivalent and standard deviation. The right panel shows realized mean weekly returns, standard deviation, Sharpe ratios (excluding a riskless asset) and accumulated wealth for an initial investment of \$100. Portfolios were formed from the TSX, REIT and GSCI indexes for the period January 12, 2000 to December 31, 2003 (N = 208).

Table 20. Constrained buy-and-hold strategy

E(R) annual	<i>Target</i>		<i>Actual</i>			
	E(R)	σ	Mean	σ	Sharpe	Wealth
10%	0.0018	0.0130	0.0019	0.0129	0.1445	144.90
11%	0.0020	0.0125	0.0020	0.0124	0.1628	149.87
12%	0.0022	0.0122	0.0022	0.0122	0.1783	154.80
13%	0.0024	0.0122	0.0023	0.0122	0.1902	159.69
14%	0.0025	0.0123	0.0025	0.0125	0.1983	164.54
15%	0.0027	0.0127	0.0026	0.0134	0.1947	168.96
16%	0.0029	0.0132	0.0028	0.0153	0.1809	173.24
17%	0.0030	0.0139	0.0029	0.0175	0.1662	177.47
18%	0.0032	0.0147	0.0031	0.0200	0.1535	181.67
19%	0.0034	0.0157	0.0032	0.0226	0.1432	185.86
20%	0.0035	0.0167	0.0034	0.0252	0.1352	189.97

Descriptive statistics for the constrained buy-and-hold strategy. The left panel shows target annual expected portfolio return followed by the weekly equivalent and standard deviation. The right panel shows realized mean weekly returns, standard deviation, Sharpe ratios (excluding a riskless asset) and accumulated wealth for an initial investment of \$100. Portfolios were formed from the TSX, REIT and GSCI indexes for the period January 12, 2000 to December 31, 2003 (N = 208). No short-selling of the TSX or REIT was permitted.

Table 21. Unconstrained static strategy

E(R) annual	<i>Target</i>		<i>Actual</i>			
	E(R)	σ	Mean	σ	Sharpe	Wealth
10%	0.0018	0.0130	0.0018	0.0130	0.1409	143.85
11%	0.0020	0.0125	0.0020	0.0125	0.1605	149.36
12%	0.0022	0.0122	0.0022	0.0122	0.1784	154.93
13%	0.0024	0.0122	0.0024	0.0122	0.1935	160.56
14%	0.0025	0.0123	0.0025	0.0123	0.2049	166.26
15%	0.0027	0.0127	0.0027	0.0127	0.2125	172.02
16%	0.0029	0.0132	0.0029	0.0132	0.2165	177.82
17%	0.0030	0.0139	0.0030	0.0139	0.2175	183.67
18%	0.0032	0.0147	0.0032	0.0147	0.2164	189.57
19%	0.0034	0.0157	0.0034	0.0157	0.2139	195.50
20%	0.0035	0.0167	0.0035	0.0167	0.2105	201.47

Descriptive statistics for the unconstrained static strategy. The left panel shows target annual expected portfolio return followed by the weekly equivalent and standard deviation. The right panel shows realized mean weekly returns, standard deviation, Sharpe ratios (excluding a riskless asset) and accumulated wealth for an initial investment of \$100. Portfolios were formed from the TSX, REIT and GSCI indexes for the period January 12, 2000 to December 31, 2003 (N = 208).

Table 22. Constrained static strategy

E(R) annual	<i>Target</i>		<i>Actual</i>			
	E(R)	σ	Mean	σ	Sharpe	Wealth
10%	0.0018	0.0130	0.0018	0.0130	0.1409	143.85
11%	0.0020	0.0125	0.0020	0.0125	0.1605	149.36
12%	0.0022	0.0122	0.0022	0.0122	0.1784	154.93
13%	0.0024	0.0122	0.0024	0.0122	0.1935	160.56
14%	0.0025	0.0123	0.0025	0.0123	0.2049	166.26
15%	0.0027	0.0127	0.0027	0.0129	0.2094	171.93
16%	0.0029	0.0132	0.0029	0.0141	0.2022	177.35
17%	0.0030	0.0139	0.0030	0.0160	0.1892	182.49
18%	0.0032	0.0147	0.0032	0.0182	0.1752	187.33
19%	0.0034	0.0157	0.0034	0.0207	0.1621	191.87
20%	0.0035	0.0167	0.0035	0.0233	0.1509	196.02

Descriptive statistics for the constrained static strategy. The left panel shows target annual expected portfolio return followed by the weekly equivalent and standard deviation. The right panel shows realized mean weekly returns, standard deviation, Sharpe ratios (excluding a riskless asset) and accumulated wealth for an initial investment of \$100. Portfolios were formed from the TSX, REIT and GSCI indexes for the period January 12, 2000 to December 31, 2003 (N = 208). No short-selling of the TSX or REIT was permitted.

Table 23. Unconstrained dynamic strategy

E(R) annual	<i>Target</i>		<i>Actual</i>			
	E(R)	σ	Mean	σ	Sharpe	Wealth
10%	0.0018	0.0130	0.0021	0.0134	0.1582	152.28
11%	0.0020	0.0125	0.0023	0.0129	0.1774	157.92
12%	0.0022	0.0122	0.0024	0.0126	0.1948	163.63
13%	0.0024	0.0122	0.0026	0.0125	0.2095	169.41
14%	0.0025	0.0123	0.0028	0.0126	0.2207	175.23
15%	0.0027	0.0127	0.0029	0.0129	0.2280	181.12
16%	0.0029	0.0132	0.0031	0.0134	0.2318	187.05
17%	0.0030	0.0139	0.0033	0.0140	0.2326	193.02
18%	0.0032	0.0147	0.0034	0.0148	0.2313	199.03
19%	0.0034	0.0157	0.0036	0.0157	0.2284	205.08
20%	0.0035	0.0167	0.0037	0.0166	0.2246	211.15

Descriptive statistics for the unconstrained dynamic strategy. The left panel shows target annual expected portfolio return followed by the weekly equivalent and standard deviation. The right panel shows realized mean weekly returns, standard deviation, Sharpe ratios (excluding a riskless asset) and accumulated wealth for an initial investment of \$100. Portfolios were formed from the TSX, REIT and GSCI indexes for the period January 12, 2000 to December 31, 2003 (N = 208).

Table 24. Constrained dynamic strategy

E(R) annual	<i>Target</i>		<i>Actual</i>			
	E(R)	σ	Mean	σ	Sharpe	Wealth
10%	0.0018	0.0308	0.0021	0.0133	0.1580	152.16
11%	0.0020	0.0079	0.0023	0.0129	0.1768	157.65
12%	0.0022	0.0079	0.0024	0.0126	0.1927	162.69
13%	0.0024	0.0082	0.0026	0.0125	0.2073	168.60
14%	0.0025	0.0087	0.0028	0.0127	0.2189	175.28
15%	0.0027	0.0093	0.0030	0.0133	0.2228	181.37
16%	0.0029	0.0102	0.0031	0.0145	0.2123	185.06
17%	0.0030	0.0111	0.0032	0.0162	0.1953	187.62
18%	0.0032	0.0121	0.0033	0.0183	0.1787	190.65
19%	0.0034	0.0132	0.0034	0.0208	0.1648	194.58
20%	0.0035	0.0144	0.0036	0.0234	0.1526	198.04

Descriptive statistics for the constrained dynamic strategy. The left panel shows target annual expected portfolio return followed by the weekly equivalent and standard deviation. The right panel shows realized mean weekly returns, standard deviation, Sharpe ratios (excluding a riskless asset) and accumulated wealth for an initial investment of \$100. Portfolios were formed from the TSX, REIT and GSCI indexes for the period January 12, 2000 to December 31, 2003 (N = 208). No short-selling of the TSX or REIT was permitted.

Appendix B. Ex-post utility by target expected return

Table 25. Mean ex-post utility under the unconstrained buy-and-hold strategy

E(R)	$\phi = 10$	$\phi = 20$	$\phi = 30$	$\phi = 40$	$\phi = 50$
10%	0.545547	0.523817	0.516107	0.512157	0.509757
11%	0.545566	0.523830	0.516118	0.512167	0.509766
12%	0.545583	0.523840	0.516125	0.512173	0.509771
13%	0.545596	0.523846	0.516129	0.512176	0.509774
14%	0.545606	0.523850	0.516131	0.512177	0.509773
15%	0.545614	0.523852	0.516130	0.512174	0.509770
16%	0.545619	0.523851	0.516126	0.512170	0.509765
17%	0.545623	0.523848	0.516121	0.512163	0.509758
18%	0.545624	0.523843	0.516114	0.512155	0.509749
19%	0.545624	0.523836	0.516105	0.512145	0.509738
20%	0.545622	0.523828	0.516095	0.512134	0.509726

Mean ex-post utility for an investor with quadratic tastes and coefficient of absolute risk aversion ϕ following the buy-and-hold portfolio strategy. E(R) is target expected return for portfolios comprised of the TSX, REIT, and GSCI indexes.

Table 26. Mean ex-post utility under the constrained buy-and-hold strategy

E(R)	$\phi = 10$	$\phi = 20$	$\phi = 30$	$\phi = 40$	$\phi = 50$
10%	0.545547	0.523817	0.516107	0.512157	0.509757
11%	0.545566	0.523830	0.516118	0.512167	0.509766
12%	0.545583	0.523840	0.516125	0.512173	0.509771
13%	0.545596	0.523846	0.516129	0.512176	0.509774
14%	0.545606	0.523850	0.516131	0.512177	0.509773
15%	0.545607	0.523845	0.516123	0.512168	0.509763
16%	0.545596	0.523826	0.516102	0.512145	0.509740
17%	0.545576	0.523798	0.516070	0.512112	0.509706
18%	0.545547	0.523760	0.516029	0.512070	0.509663
19%	0.545512	0.523715	0.515981	0.512019	0.509611
20%	0.545470	0.523664	0.515926	0.511963	0.509554

Mean ex-post utility for an investor with quadratic tastes and coefficient of absolute risk aversion ϕ following the buy-and-hold portfolio strategy. E(R) is target expected return for portfolios comprised of the TSX, REIT, and GSCI indexes. Portfolios are constrained so that there is no short-selling of the TSX or REIT.

Table 27: Mean ex-post utility under the unconstrained static strategy

E(R)	$\phi = 10$	$\phi = 20$	$\phi = 30$	$\phi = 40$	$\phi = 50$
10%	0.545543	0.523815	0.516105	0.512156	0.509755
11%	0.545564	0.523829	0.516116	0.512166	0.509765
12%	0.545583	0.523840	0.516125	0.512173	0.509771
13%	0.545599	0.523849	0.516131	0.512178	0.509775
14%	0.545612	0.523854	0.516134	0.512180	0.509776
15%	0.545623	0.523858	0.516135	0.512179	0.509774
16%	0.545631	0.523859	0.516133	0.512176	0.509770
17%	0.545637	0.523857	0.516129	0.512170	0.509764
18%	0.545641	0.523853	0.516122	0.512162	0.509755
19%	0.545643	0.523847	0.516113	0.512152	0.509744
20%	0.545642	0.523838	0.516102	0.512139	0.509730

Mean ex-post utility for an investor with quadratic tastes and coefficient of absolute risk aversion ϕ following the static portfolio strategy. E(R) is target expected return for portfolios comprised of the TSX, REIT, and GSCI indexes.

Table 28. Mean ex-post utility under the constrained static strategy

E(R)	$\phi = 10$	$\phi = 20$	$\phi = 30$	$\phi = 40$	$\phi = 50$
10%	0.545543	0.523815	0.516105	0.512156	0.509755
11%	0.545564	0.523829	0.516116	0.512166	0.509765
12%	0.545583	0.523840	0.516125	0.512173	0.509771
13%	0.545599	0.523849	0.516131	0.512178	0.509775
14%	0.545612	0.523854	0.516134	0.512180	0.509776
15%	0.545621	0.523856	0.516132	0.512177	0.509772
16%	0.545620	0.523847	0.516121	0.512163	0.509758
17%	0.545609	0.523828	0.516099	0.512140	0.509734
18%	0.545589	0.523799	0.516067	0.512106	0.509699
19%	0.545560	0.523760	0.516025	0.512063	0.509655
20%	0.545522	0.523713	0.515974	0.512010	0.509601

Mean ex-post utility for an investor with quadratic tastes and coefficient of absolute risk aversion ϕ following the static portfolio strategy. E(R) is target expected return for portfolios comprised of the TSX, REIT, and GSCI indexes. Portfolios are constrained so that there is no short-selling of the TSX or REIT.

Table 29. Mean ex-post utility under the unconstrained dynamic strategy

E(R)	$\phi = 10$	$\phi = 20$	$\phi = 30$	$\phi = 40$	$\phi = 50$
10%	0.545564	0.523823	0.516109	0.512157	0.509756
11%	0.545584	0.523837	0.516120	0.512168	0.509765
12%	0.545603	0.523848	0.516129	0.512175	0.509772
13%	0.545618	0.523857	0.516135	0.512180	0.509775
14%	0.545632	0.523863	0.516138	0.512182	0.509777
15%	0.545642	0.523866	0.516139	0.512181	0.509776
16%	0.545651	0.523867	0.516138	0.512179	0.509772
17%	0.545657	0.523866	0.516134	0.512173	0.509766
18%	0.545661	0.523863	0.516128	0.512166	0.509758
19%	0.545663	0.523857	0.516119	0.512156	0.509747
20%	0.545662	0.523849	0.516109	0.512144	0.509735

Mean ex-poste utility for an investor with quadratic tastes and coefficient of absolute risk aversion ϕ following the dynamic portfolio strategy. E(R) is target expected return for portfolios comprised of the TSX, REIT, and GSCI indexes.

Table 30. Mean ex-post utility under the constrained dynamic strategy

E(R)	$\phi = 10$	$\phi = 20$	$\phi = 30$	$\phi = 40$	$\phi = 50$
10%	0.545563	0.523823	0.516109	0.512157	0.509756
11%	0.545584	0.523837	0.516120	0.512167	0.509765
12%	0.545600	0.523847	0.516128	0.512174	0.509771
13%	0.545616	0.523855	0.516134	0.512179	0.509775
14%	0.545631	0.523861	0.516137	0.512180	0.509776
15%	0.545639	0.523862	0.516135	0.512177	0.509771
16%	0.545634	0.523852	0.516122	0.512163	0.509757
17%	0.545618	0.523831	0.516099	0.512140	0.509733
18%	0.545595	0.523800	0.516067	0.512106	0.509698
19%	0.545564	0.523762	0.516025	0.512063	0.509654
20%	0.545525	0.523714	0.515974	0.512010	0.509600

Mean ex-poste utility for an investor with quadratic tastes and coefficient of absolute risk aversion ϕ following the dynamic portfolio strategy. E(R) is target expected return for portfolios comprised of the TSX, REIT, and GSCI indexes. Portfolios are constrained so that there is no short-selling of the TSX or REIT.

Appendix C. Investment strategies ranked by ex-post utility

Table 31. Unconstrained portfolio strategies ranked by mean ex-post utility

E(R)	$\phi = 10$			$\phi = 20$			$\phi = 30$			$\phi = 40$			$\phi = 50$		
10%	D	B	S	D	B	S	D	B	S	B	D	S	B	D	S
11%	D	B	S	D	B	S	D	B	S	D	B	S	B	D	S
12%	D	S	B	D	B	S	D	B	S	D	B	S	D	B	S
13%	D	S	B	D	S	B	D	S	B	D	S	B	D	S	B
14%	D	S	B	D	S	B	D	S	B	D	S	B	D	S	B
15%	D	S	B	D	S	B	D	S	B	D	S	B	D	S	B
16%	D	S	B	D	S	B	D	S	B	D	S	B	D	S	B
17%	D	S	B	D	S	B	D	S	B	D	S	B	D	S	B
18%	D	S	B	D	S	B	D	S	B	D	S	B	D	S	B
19%	D	S	B	D	S	B	D	S	B	D	S	B	D	S	B
20%	D	S	B	D	S	B	D	S	B	D	S	B	D	S	B

Mean ex-post utility for an investor with quadratic tastes and coefficient of absolute risk aversion ϕ following either a buy-and-hold (B), static (S), or dynamic (D) portfolio strategy. E(R) is target expected return for portfolios comprised of the TSX, REIT, and GSCI indexes. The cell entries are in descending order of utility; so, for example, (D, S, B) means that the dynamic strategy yielded the highest utility for the given target return, static the second highest, and buy-and-hold the lowest.

Table 32. Constrained portfolio strategies ranked by mean ex-post utility

E(R)	$\phi = 10$			$\phi = 20$			$\phi = 30$			$\phi = 40$			$\phi = 50$		
10%	D	B	S	D	B	S	D	B	S	B	D	S	B	D	S
11%	D	B	S	D	B	S	D	B	S	D	B	S	B	D	S
12%	D	S	B	D	B	S	D	B	S	D	B	S	B	D	S
13%	D	S	B	D	S	B	D	S	B	D	S	B	S	D	B
14%	D	S	B	D	S	B	D	S	B	D	S	B	S	D	B
15%	D	S	B	D	S	B	D	S	B	D	S	B	S	D	B
16%	D	S	B	D	S	B	D	S	B	D	S	B	S	D	B
17%	D	S	B	D	S	B	D	S	B	S	D	B	S	D	B
18%	D	S	B	D	S	B	D	S	B	S	D	B	S	D	B
19%	D	S	B	D	S	B	D	S	B	S	D	B	S	D	B
20%	D	S	B	D	S	B	S	D	B	S	D	B	S	D	B

Mean ex-post utility for an investor with quadratic tastes and coefficient of absolute risk aversion ϕ following either a buy-and-hold (B), static (S), or dynamic (D) portfolio strategy. E(R) is target expected return for portfolios comprised of the TSX, REIT, and GSCI indexes. Portfolios are constrained so that there is no short-selling of the TSX or REIT. The cell entries are in descending order of utility; so, for example, (D, S, B) means that the dynamic strategy yielded the highest utility for the given target return, static the second highest, and buy-and-hold the lowest.