Quantification of Two Gestalt Laws Using Curve Reconstruction

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ABSTRACT

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Visual perception is the ability to interpret, process, and comprehend all the information received through the sense of sight by association with earlier experiences. Researchers have long struggled to explain what visual processing does to create what we actually see, and brought many theoretical approaches explaining how human beings see the world. The theoretical approaches of visual perception differ widely and their coverage ranges from early theories such as Gestalt theory to recent computational theory in the field of Artificial Intelligence. According to the characteristics of visual perception, human beings tend to classify the ambient environment objects into different categories described by various symbols or objects. Similar symbols or even quite dissimilar symbols may be perceived as belonging together or belonging to different groups according to people's judgment. It must follow certain rules when human beings set up relationships between those objects and symbols, and finally obtain the unambiguous perceptual results through the process of visual perception. To find out the mechanisms underlying these properties of visual perception, this present thesis conducts experiments on perception using curve reconstructions as test cases. The perception model developed through the experiment is implemented in a curve reconstruction algorithm. It is assumed that a good perception model will reconstruct curves in the same manner as human beings perceive them. In the present thesis, a series of methods from Design of Experiments (DOE), ANOVA and the multivariate nonlinear regression model are applied to investigate the relationships between the points and curves. The results show that our perception model conforms to the pattern human perceives the points.

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Chapter 1 Introduction

1.1 Visual perception

Visual perception is the ability to interpret, process, and comprehend all the information received through the sense of sight by association with earlier experiences. It is the most strikingly dominant perception among all other perceptions such as touch, hearing, taste, etc. The act of seeing starts when the lenses of the eyes focus an image of the outside world onto retina. The retina converts and transmits this information as an electrochemical signal along the optic nerve until it reaches the visual cortex of the brain, as that shown in Figure 1 [1].

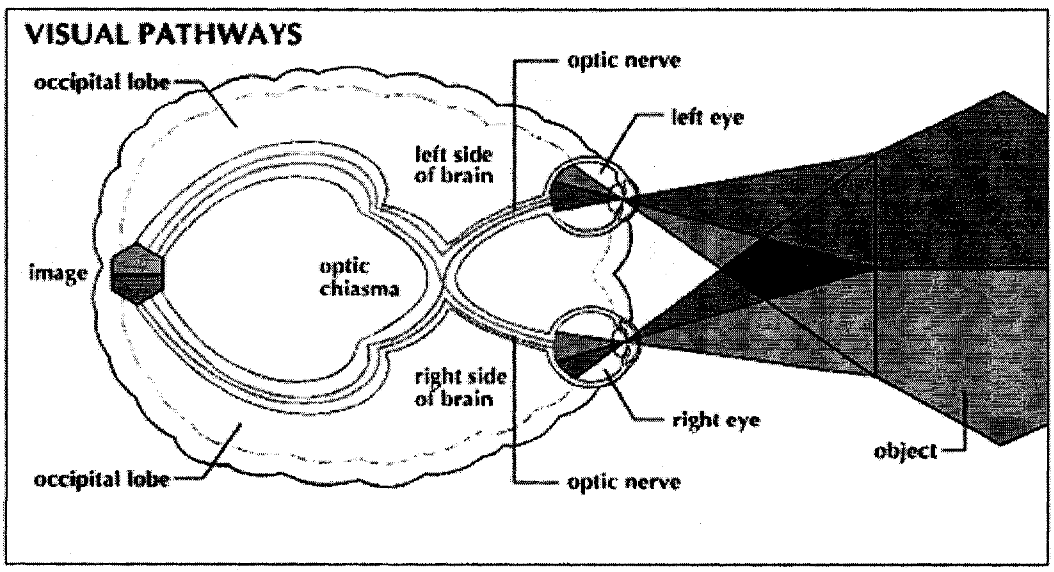


Figure 1 Physiological mechanism of visual pathways [1]

Visual perception is an active process, during which human beings can extract relevant information from the ambient environment and guide their actions. It involves investigation at many different levels of the visual system. The fundamental perceptual phenomena include spatial perception, shape and object perception, depth perception, visual motion perception, color perception, etc [2]. A comprehensive understanding of visual perception must involve an interdisciplinary approach such as biological, neurophysiological, and psychophysiological approaches. Physiology especially the eye and brain anatomy with countless experiments by scientists and researchers show that vision is a complicated process that requires numerous components of the human eye and brain to work together. Although researchers have used a wide variety of procedures and techniques to research on visual perception and have made great progresses in recent years, still there are a lot problems remain unresolved. Understanding the mechanisms of visual process is a key subject of investigation in visual perception.

The major problem in visual perception is that what people see is not simply a translation of retinal stimuli (i.e., the image on the retina). Thus, researchers interested in perception have long struggled to explain what visual processing does to create what we actually see. They have done various experiments ranging from simple single object to complex colorful moving objects, and came out with seven major theoretical approaches: threshold, Gestalt, probabilistic functionalism, neurophysiological, empiricism, ecological, and computational approach [3]. These approaches differ by the phenomena that they seek to explain, and are valid only under certain specific circumstances.

The difference between "seeing" and "seeing as" is the difference between vision (sensation) and visual perception. They are not the same thing because sensation is not identical to perception. Perception requires attention and awareness. However, the current thesis focuses on the perceptual process of vision, not on the anatomy of vision system. Hence, we treat vision as visual perception throughout the thesis.

1.2 Perception on individual collection

One of the most difficult problems that the visual perception has to solve is to group different elements of the environment into individual objects [4]. Applying the proper theories to explain the basic simple visual phenomena is a feasible way to research on the mechanism of visual perception. According to the characteristics of visual perception, human beings tend to classify the ambient environment objects into different categories described by various symbols, as shown in the Figure 2 a). Figure 2 c) shows the result of the subcategory process by focusing on specific objects and excluding all other irrelative objects. Therefore, to the objects belonging to the same category, human beings can abstract them as the collection of elements located at different spatial position. The collection of the elements is also defined as the point set [5], dot figures [6], dot-patterns [7], unorganized points [8], or sample points [9], etc.

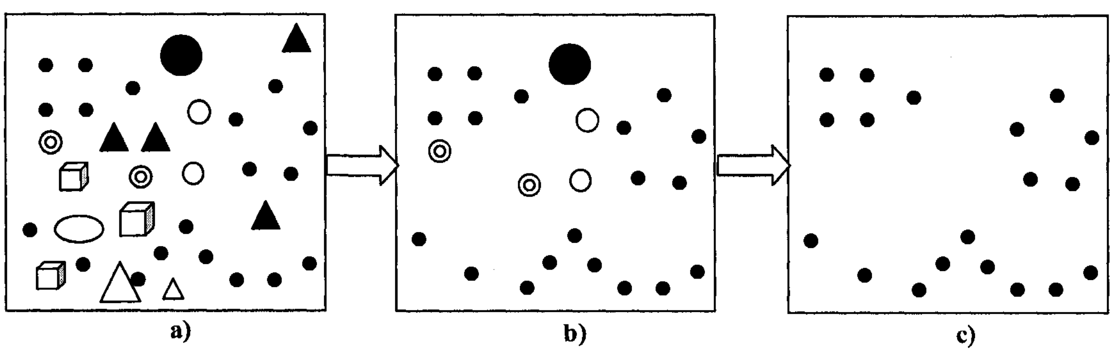


Figure 2 Process of subcategory

Different people can perceive the same collection of points with different patterns, even the same person may have different perceptual results because the varied spatio-temporal locations. However, in most cases, people can get the same perceptual patterns or shapes over the same points collection. In general, perception on collection of points includes two aspects, proximity (segregation) and connection [10]. Proximity means to distinguish how many parts in the points collection and get the topological shapes of different parts. Connection means careful comparison and judgment are applied on each part to connect the individual elements. People are wondering how to balance the weight of segregation and connection on various circumstances, and how to quantify and formulate the mechanism involved in these visual processes. Before the further analyses and experiments, the theories of visual perception and research on sample points reconstruction are introduced.

1.3 Theories of visual perception

When people segregate and connect the points to create shapes or patterns that are intuitive to visual perception, they follow some basic rules unconsciously or involve much conscious awareness based on their knowledge and experience. Visual perception should set up relationships between the points collection and connected patterns, and finally get the unambiguous perceptual results. To explain the nature of this mechanism, two relevant theoretical methods of computational theory and Gestalt theory are introduced in the following sections.

1.3.1 Computational theory

Computational theory is the most important development in visual perception theory in recent years. The developments of information theory, cybernetics, digital computers created a new discipline, artificial intelligence. This is an engineering approach and it treats organisms as machines that are controlled by processes, and some of these processes are perceptual. Visual perception thus offered an obvious challenge to researchers in this new discipline. Among all researchers contributed to computational theory, Marr integrated results from psychology, artificial intelligence, and neurophysiology into new models of visual processing. He is recognized as founder of the computational approach to visual perception. [3]

1.3.1.1 Three level of vision

Ullman categorizes the processes involved in vision into three levels: low-level, intermediate-level and high-level vision [4]. The hierarchical structure of three categories can be illustrated in Figure 3.

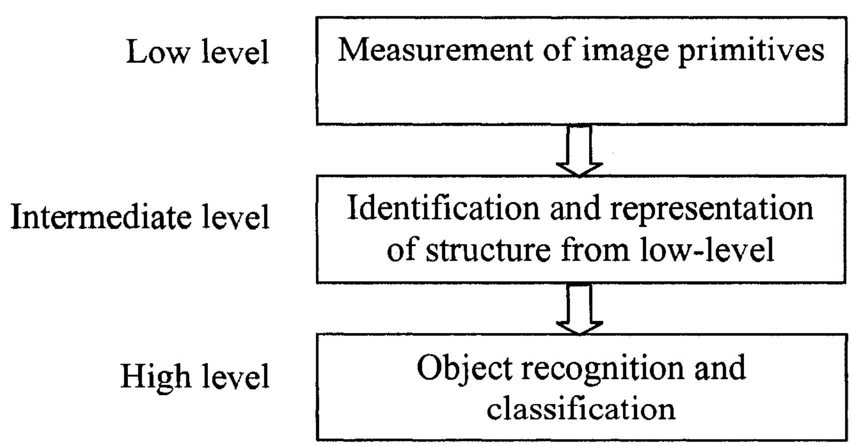


Figure 3 Hierarchical structure of three level of vision

Low-level vision is usually associated with the extraction of certain physical properties of the environment, such as depth, three-dimensional shape, object boundaries, or surface material properties. Low-level visual processes are considered bottom-up in nature. This means that they are determined by the data and are relatively independent of the experience or knowledge associated with specific objects. Low-level visual perception is data-driven, parallel, and provides input to higher-level perception processes.

Intermediate-level visual operations are sometimes data-driven and other times goal-driven, sometimes carried out in parallel and other times carried out serially. Processes that do not depend on knowledge about specific objects are sometimes called intermediate-level visual perception within the domain of higher-level perception. For example, the extraction of shape properties and spatial relations is often confined to selected elements in the scene and does not proceed uniformly and in parallel across the image. At the same time, the processing is largely independent of knowledge associated with specific objects.

High-level visual perception, in contrast, is concerned with problems such as the extraction of shape properties and spatial relations, and with object recognition and classification. It is concerned primarily with the interpretation and use of the information in the image, rather than the direct recovery of physical properties. High-level vision is knowledge and experience dependent, often serial, and often initiated voluntarily in accordance with our perceptual goals.

From Ullman' three categories of vision, we can find that low-level and intermediate-level vision deals with the tasks of filtering, segmentation, grouping, edge detection, etc. The tasks at these two levels extract a large amount of local data structure, which characterized by relatively simple operations at the early stages of visual perception. For the low-level and intermediate-level visual perception, the most significant achievement is Marr's computational theory [11].

1.3.1.2 Marr's computational approach to visual perception

The aim of the Marr' computational approach is to understand vision completely, that is, to understand how descriptions of the world may efficiently and reliably be obtained from images of it. According to Marr's theory, visual processing is the step-by-step recovery of information about the distal environment. Marr's analysis of the visual system has been quite successful in explaining the mechanisms of the visual system. It is the state-of-the-art theory in visual perception research by making great progress in the areas of low level and intermediate level vision.

Marr developed a multi-level theory of vision, which analyzed the process of vision at different levels of abstraction: the computational level, algorithmic level and the level of implementation. Among these three levels, he claimed that the computational level is the most crucial one for understanding vision.

The computational level addresses the goal that the computation must achieve and the problems that the visual system must solve at a high level of abstraction. The algorithmic level attempts to explain how the computational theory can be implemented. The implementation level attempts to explain how the representation and algorithm be realized physically.

Marr's theory is that visual perception is organized as an information-processing system and that this system comprises successive stages. He uses his knowledge of computer science to formulate a guiding principle, Modular Design. His stages of vision include:

1) The image: The image is a spatial distribution of intensity values across the retina and is the starting point of the process of seeing.

2) The primal sketch: This stage is to take the raw intensity values of the visual image and make explicit certain forms of information contained therein.

3) The 2-1/2 D sketch: The orientation and rough depth of visible surfaces are made explicit.

4) The 3-D model representation: The shapes and their orientation become clear as tokens of 3-D objects are organized in an object-centered framework.

1.3.1.3 Recent research on three levels of perception

Many researchers have developed various theories explaining exactly how human beings extract different level of features from the image produced by the human vision. Mather [2] summarizes the different continuous stages of visual processing in Figure 4, with assertion that the image of any kind of objects can have lot of low-level, intermediate-level, and high-level properties.

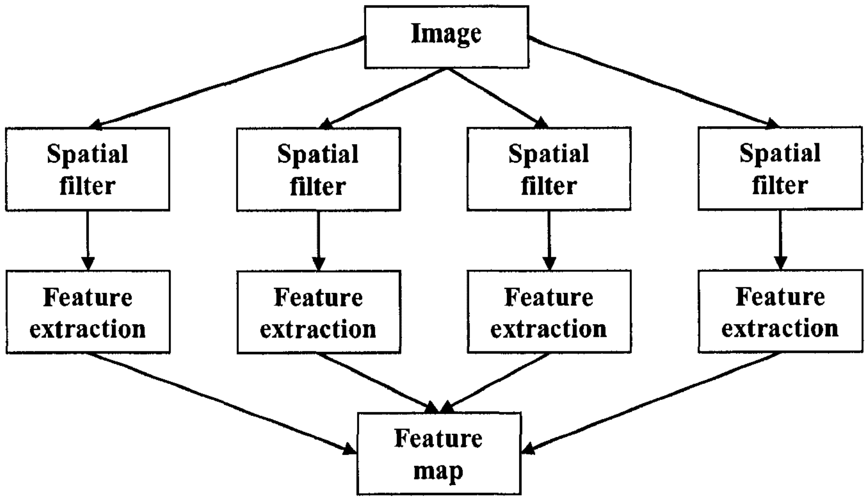


Figure 4 Processing stages involved in edge localization in visual system [2]

Rouw et al [12] find that people have better performance when evaluating the high-level properties than the low-level properties. However, people can also extract relatively low-level visual features from the image, even when these low-level features are embedded in high-level ones. The low-level visual properties are the basic structure to the high-level properties that arise from specific ways of organizing low-level properties. How to organize and formulate the low-level prosperities of the image through human vision is one of the hot topics in vision research.

1.3.2 Gestalt theory

In the 30s and 40s, Gestalt psychology [13] was applied to visual perception and then became part of our permanent knowledge of perception. Among all the researchers of Gestalt psychologist, Max Wertheimer, Wolfgang Köhler, and Kurt Koffka are credited with developing the theories that gave rise to the school of Gestalt psychology. The Gestalt psychologists tried to explain human perception of grouping objects, perceiving parts of objects, and forming the whole objects. Their aim was to investigate the global and holistic processes involved in perceiving structure in the ambient environment. Gestalt psychologists have developed six laws that govern human perception: proximity, similarity, good continuation, closure, good form, and figure/ground [10, 14]. As shown in Figure 5.

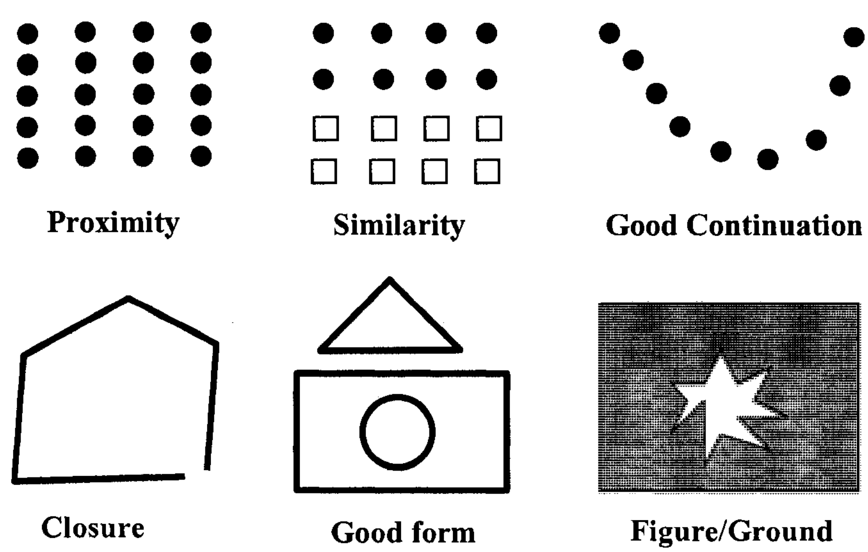


Figure 5 Gestalt laws of human perception

Proximity is a principle of organization holding that individual objects that are near to one another are perceived as belonging together as a unit. In visual perception, it is mainly referred to the elements with almost the same distance among each other or along certain directions. The spatial proximity of elements can induce the human beings to perceive them as a coherent object than other elements that are far apart. Proximity means objects that are relatively close to one another tend to be grouped together. It also means segregating various objects into different groups when the object or objects do not have proximity with other objects.

Good continuation means object shape tends to vary smoothly, with relatively few very sharp corners or edges. The Gestalt psychologists argued that perceptual organization tends to preserve smooth continuity rather than yielding abrupt changes. When human beings attempt to delineate the contours of objects, it makes sense to bias grouping in favor of contours that vary smoothly rather than sharply [15].

For the combination of proximity and good continuation, even quite dissimilar shapes or objects can be perceived together to form a pattern. As shown in Figure 6 a), despite there are a lot of different shapes in the pattern, it can be perceived intuitively as two unconnected curves intersected at the middle of them. If we just consider the property of proximity and good continuation of the pattern, we can treat Figure 6 a) as the pattern shown in Figure 6 b), which has only identical shapes. The method of grouping dissimilar shapes is a complementary way to the method of sub-categorizing same shapes used in Figure 2. Both methods can result in patterns of points collection to represent the environment.

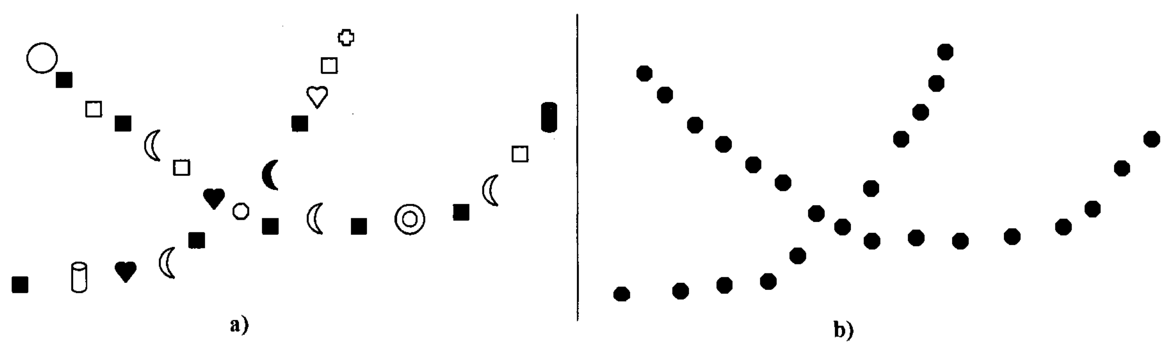


Figure 6 Different shapes group together by proximity good continuation

Gestalt theory captures some consistent aspects of perceptual organization. It is reasonable to say that Gestalt theory explains some characteristics of the visual system because they reflect the properties of real-world objects. The major problem of the Gestalt theory is that it is more descriptive than explanatory. It cannot explain how humans see continuous contours by simply stating that the brain prefers good continuity. Bruce, Green and Georgeson [16] point out that the physiological theory of the Gestalts has fallen by the wayside, leaving us with a set of descriptive principles, but without a model of perceptual processing. Some of their laws of perceptual organization sound vague and inadequate. It does not explain what proximity and good continuation exactly means. Recently, scientists have done lot of research on Gestalt perceptual principles by apply them in various fields such as contour detection, image property analysis, and object recognition.

In 1985, Smit, Vos and Van Oeffelen offered a model that named CODE-2 attempting to replacing the intuitive qualitative approach of the gestalt theory by formal mathematical models especially in contour detection [17]. In 1996, Kovks summarized the Gestalt laws of pattern connectivity in early processing of visual contours and surfaces [18]. Buhmann et al discussed using Gestalt rules of grouping to capture the simple properties of the world for object recognition [19]. In 2001, Sigman et al showed that Gestalt laws of good continuation can predict the arrangement of segments in natural scenes [20]. Besides the contour detection and object recognition, Gestalt laws can also be used in other fields such as in user interface manipulation. Dehmeshki & Stuerzlinger proposed the Gestalt laws of proximal groups and regular groups with empirical formulas to group object selection that is based on human perception [21].

Nevertheless, there is no further research on the properties and relationships of proximity and good continuation in the context of visual perception. In present thesis, we are attempting to quantify and formulate these two Gestalt laws through various experiments, trying to demonstrate how visual perception balance the effect of proximity and good continuation in various circumstances.

1.4 Other research about objects segregation and connection

Norman summarized constructivist approach and ecological approach to visual perception and linked two approaches together by proposing a dual-process approach [22]. The constructivist approach sees the visual perception is indirect and it is a multistage with mediational processes. It relies on inferential types and analyzes the recognition and identification of the visual input. Memory, stored schemata, and past experience play an important role. The ecological approach argues that the perception is a single, direct, and immediate stage process, and there are no role for memory and related phenomena in perception. Dual-process approach suggests ecological approach and the constructivist approach are valid descriptions of perception, but of different aspects. There is a great deal of cross talk between them, and they normally function in synergy.

Chen addressed the fundamental question of "What are primitives of visual perception" by propose a theory of topological structure and functional hierarchy in visual perception [23]. Chen's theory holds that the global nature of perceptual organization can be described in terms of topological invariants, global topological perception is prior to the perception of other featural properties, and the primitives of visual form perception are geometric invariants at different levels of structural stability. The dominant theories of global topological features and the ecological approach both focus on assimilating the information of the pattern at the instantaneous time scale. However, in many cases, the information extracted from this unconsciously short-time process is not enough for the visual perception. Careful observation with consciousness also places a very important role through the whole processes of the perception.

To the processes of segregating and connecting the individual points, we can segregate the points collection into different groups in a very short time, while spend a considerable long time on considering how to connect the points together in each group. As shown in Figure 7 a), the points collection can be immediately segregated into three groups, which are circled by dashed lines. While connecting points in each group to create a certain shape will take a longer time than segregating them, as show in Figure 7 b).

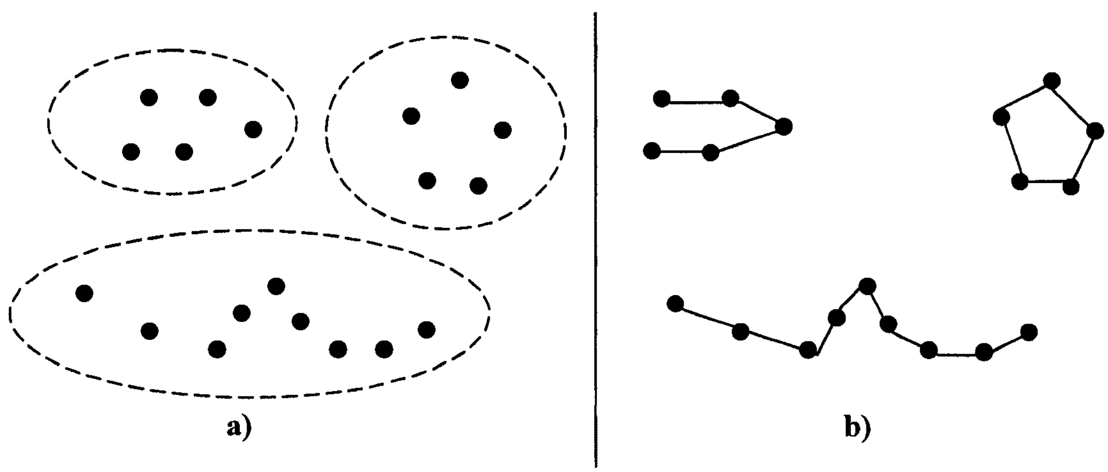


Figure 7 Processes of segregating and connecting on points collection

According to Norman's dual-process approach, the first phase of segregation is corresponding to ecological approach for its single, direct, and immediate stage process. The second phase of connection is corresponding to constructivist approach for its indirect, multistage, and mediatorial processes. For Chen's topological theory, the segregating is an unconsciously short-time process. While in the connection process, it needs careful consideration with consciousness to connect the points in each group. Two different theories about the same perceptual results explain a somewhat different aspect of visual perception. However, they are similar to each other and overlap in some perception process.

1.5 Relation of visual perception to curve reconstruction

Every object consists of atoms. If we treat an atom as a point, we can find that everything in the universe consists of small points. Several points can be connected to form a curve. A set of similar curves can be combined together to form a plane or a facet. Different planes can be piled up to construct a small part such as cube, ball, or other simple objects. Any huge complicated object can be integrated from various types of small parts. The process can be illustrated in Figure 8.

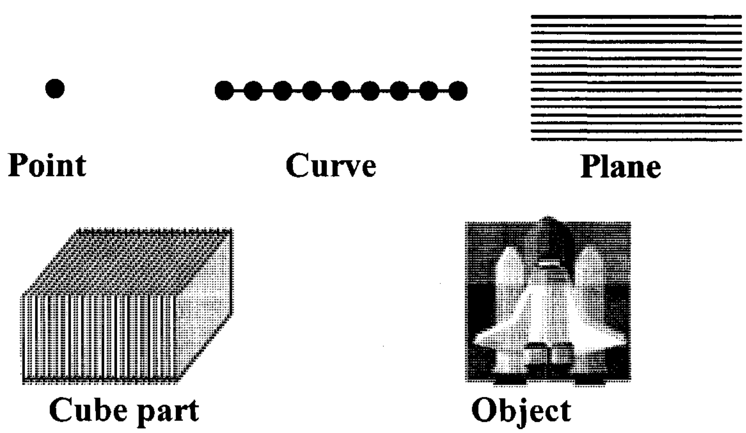


Figure 8 Create an object from a point

Based on the analysis of computational theory and Gestalt theory, retrieving the low level properties of information from the image provided by the environment is the primary step of visual perception. Research on the principle of this process can help people to understand the mechanism of the human visual perception. It can also help us to understand why we perceive the environment by various kinds of objects on some occasions, and why some different components can be integrated into one object on other circumstances. To quantify and formulate the mechanism involved in these processes, it is feasible to start from the simple experiment by using dot to represent the object, and thus various objects can be represented by a collection of points. Therefore, the research of visual perception on low-level image grouping process is accordance with the visual perception on points collection, and we can use the case of curve reconstruction to research on visual perception.

1.6 Objective

In this thesis, we focus on quantifying and formulating the visual perception's characteristics of proximity and good connection involved in the process of grouping the environment objects. We represent the environment objects with a set of points and use curve reconstruction as the test case. Our work uses the methods of DOE (design of experiments) and statistic analysis to find the algorithm of curve reconstruction that is based on human visual perception. The algorithm can group the unorganized points to form shapes in the way human beings perceive them. We list all the factors corresponding to the characteristics of the curves and points in the points collection, and then analyze the influence of factors by keeping the major factors and discarding the minor ones in the next round of experiments. Finally, a multivariable nonlinear regression model is applied to quantify the relationships between the connectivity value of the reference point and constructed curve.

1.7 Contributions

The major contributions of this thesis are summarized as follows:

1. The thesis reasonably quantified and formulated the visual perception's characteristics of proximity and good continuation.

2. The thesis found the major factors of points collection that affect the proximity and good continuation.

3. The thesis applied an appropriate multivariable nonlinear regression model to get the visual perception based curve reconstruction algorithm.

1.8 Organization

The rest of this thesis is organized as follows. Chapter 2 begins with a brief review of the current algorithms about curve reconstruction. We also present and analyze two algorithms published recently, DISCUR and VICUR, which are human-vision-related algorithms. In chapter 3, we define the factors of the points collection and analyze the property of the connectivity area and connectivity value. Chapter 4 implements the experiments designed in chapter 3 by doing the experiments to analyze the effect of the factors. A non-linear regression model is applied on the experiment data to get the connectivity formula. Chapter 5 demonstrates the effectiveness and efficiency of our algorithm by comparing with other algorithms. Finally, the conclusion and the proposition of future research are given in chapter 6.

Chapter 2 Problem Formulation

2.1 Curve reconstruction

From the analysis in the previous chapter, we can conclude that it is reasonable to research the proximity and good continuation of Gestalt laws by representing the objects with a set of points. To a set of points, human beings tend to perceive a certain pattern out of them by segregating them in different groups and connecting the points together in each group. We find that the process and mechanism of separating the points into different groups and connecting points together in Gestalt psychology are similar to those of curve reconstruction. Before further analysis, we first introduce the mechanism of curve reconstruction.

Curve reconstruction is the problem of constructing a polygonal line from a set of sample points. The conventional curve reconstruction processes are illustrated in Figure 9. Let T be a set of curves in the plane. Let P be a finite set of points from T, which is called the sample set of T. The objective is to reconstruct T from the sample set P. In recent years, the problem of the curve reconstruction mainly deals with generating a set of curves from a collection of points, and the original curves are unknown.

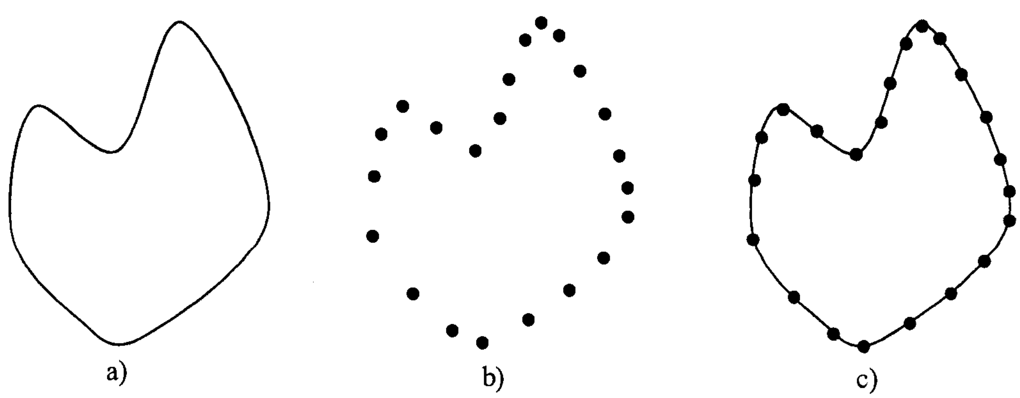


Figure 9 Conventional curve reconstruction problem

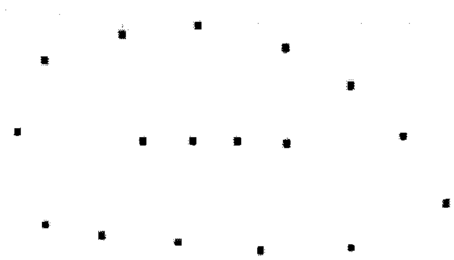
This problem is motivated by the requirements of computing and analyzing the scanning results of modern scanning devices such as fMRI (Functional magnetic resonance imaging) and laser scanner that generate a collection of points from the inner structure or the surface of the object. The curve reconstruction algorithms that can compute a digital model of a geometric shape from a collection of points boost the developments of scanning technology in CAD, medical imaging, geographic data processing, and other industrial fields.

Many researchers have been focusing on the curve reconstruction problems for a long time. As early as 1983, Edelsbrunner developed an algorithm that can generalize the convex hull of a finite set of points in plane and capture the intuitive notions of "fine shape" and "crude shape" of points sets [24]. Nowadays, Dey consider the curve reconstruction as the basic step to the complex surface reconstruction, and summarize the techniques related to the curve sampling and reconstruction in 2006 [9].

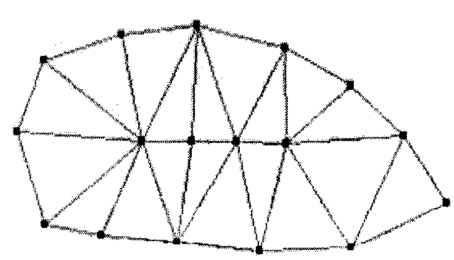
The algorithm used to compute the shapes from unorganized points is a classical problem in low-level computer vision, pattern recognition, image processing, and cluster analysis. In 1971, Zahn investigated the behavior of clustering algorithms on point sets in the plane where the set can be seen and the reader can get an intuitive feel for what the algorithm is doing. The algorithms can construct the curve like graph from the points by including edges between pairs of points satisfying a criterion that is based primarily on distance but also emphasizes the degree to which the edge in question is parallel to the expected direction of tracks [25].

In 1992, Brandt and Algazi studied solutions to boundary reconstruction from images based on Delaunay complexes. They exhibited a corresponding continuous, regular shape such that the sequence of points describing its boundary constitutes a sufficiently dense sampling for an accurate skeleton approximation. Additionally, they bounded the regeneration error from the sampling density and the regularity parameter. Their approach opened significant new possibilities for shape analysis by the exaction of Euclidean skeleton [26]. Figueiredo and Gomes proved that Euclidean minimal spanning trees correctly reconstruct differentiable arcs from sufficiently dense samples in 1994. The proof was based on a combinatorial characterization of minimal spanning paths and on a description of the local geometry of arcs inside tubular neighborhoods [27].

For the sample points reconstruction methods, numerous methods focused on the Delaunay approach that reconstructs a shape from the Delaunay complex of the points. A Delaunay triangulation for a set P of points in the plane is a triangulation $DT(P)$ such that no point in P is inside the circumcircle of any triangle in $DT(P)$. Delaunay triangulations maximize the minimum angle of all the angles of the triangles in the triangulation [28]. As illustrated in Figure 10 b).



a)



b)

Figure 10 Delaunay triangulations of point set

Besides the Delaunay approach to get the possible connection between neighbor points, there are some other solutions. Veltkamp generalized the β-skeletons and used it for the polygonal boundary reconstruction and polyhedral surface reconstruction [29]. Fuchs et al presents a general solution to the problem of surface constructing use triangular tiles and slices [30]. Nevertheless, most algorithms to curve reconstruction use restricted Delaunay complexes and they differ in the methods of choosing the appropriate edge between the points base on computation.

The curve reconstruction methods used in two dimension circumstances are the basis to three dimensions reconstructions such as surfaces reconstruction. It can serve as a prelude to the research on geometry and topology of shapes and forms, furthermore, to the object construction and three-dimensional object recognition.

2.2 Existing algorithms

The curve reconstruction can be classified into two categories: closed curves and open curves. The uniformly sampled simple closed curves category can use algorithms such as minimum spanning tree [27], alpha shapes [24, 31], and r-regular shapes [8]. The non-uniformly sampled simple closed curves category can use the crust and skeleton method developed by Amenta, Bern and Eppstein [32], and refined by Gold [33].

For simple open curves, Dey, Mehlhorn and Ramos introduced a method using Voronoi and Delaunay disks of edges [34]. To reconstruct curves with sharp corners, Dey et al developed an algorithm based on two predefined parameters: a threshold distance for filtering Delaunay triangles and an angle to adjust the smoothness and sharp corners in the reconstruction of curves [35]. Many researchers have developed various curve reconstruction algorithms over a long period. The most typical algorithms for curve reconstruction are list below.

Alpha-shapes: In 1983, Edelsbrunner, Kirkpatrick, and Seidel introduced the α-shape, which seems to capture the intuitive notions of "fine shape" and "crude shape" of point sets, to a finite set of points in the plane. It is shown that α-shapes are subgraphs of the closest point or furthest point of Delaunay triangulation [24].

Crust: In 1998, Amenta, Bern, and Eppstein defined the crust of a point set. They found that if a smooth curve is sampled densely enough; the graph on the samples is a polygonalization of the curve, with no extraneous edges. The required sampling density varies with the Local Feature Size on the curve, so that areas of less detail can be sampled less densely [32]. The curst is the first provable curve reconstruction algorithm for simple close smooth curves. However, the Curst algorithm has two drawbacks. First, it is unable to handle the open curve. Secondly, it requires the curve to be quite smooth. Continue on the work of Amenta et al. Dey and Kumar presented a modified nearest (NN-Crust) algorithm, which is base on Crust algorithm but with better sampling density [36]. NN-Curst is simple and the sampling density is to 1/3 from 0.252 as required by Curst algorithm.

A-shape: In 1997, Melkemi define the shape hull of a finite point set *S* by a family of straight-line graphs and people refer to it as an *A*-shape of *S*, where *A* is a point set that controls the level of detail reflected by the shape. Conceptually, the *A*-shape corresponds to a transcription of the A-shape of a continuous bounded domain say D. By A-shape, we simply mean the boundary of some dilation of D with a level controlled by a set of curves A. Accordingly, the sets S and A may be interpreted as samples of the continuous ones D and A respectively. As a consequence, the A-shape is defined via closely related rules [5].

Gathan: Giesen presented the first algorithm that can successfully reconstruct corners in the traveling salesman problem [37]. The main shortcoming of Giesen's method is that it cannot handle the sample points with several components. To the shortcomings of Giesen's method, Dey and Wenger developed a heuristic algorithm named Gathan, the sanskrit for "construction", which can properly handle opened or closed curves with corners, multi-components [35]. Later they developed a new algorithm called GathanG for Gathan with guarantees since it is a guaranteed version [38]. Nevertheless, their algorithm needs the precondition of densely sampled points around corner.

All algorithms mentioned above have been proven correct for certain types of curves by using rigorous mathematical procedure. The reconstruction results can be homeomorphic to the original curve and correct under certain preconditions. However, in many cases, they cannot construct the curve natural to human eyes, thus make it look unreasonable, as shown in Figure 11 and Figure 12.

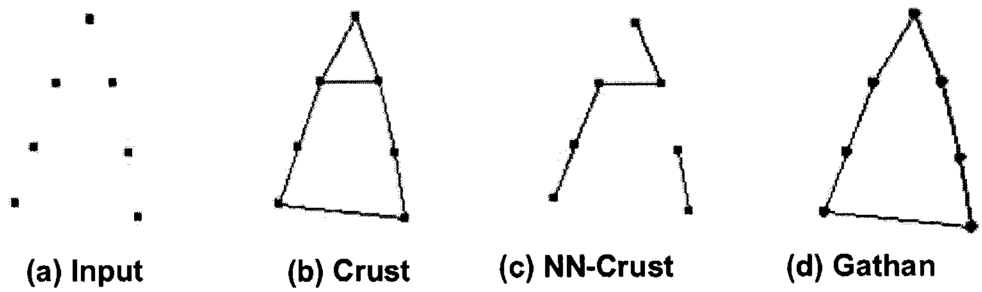


Figure 11 Existing reconstruction result for sharp corner

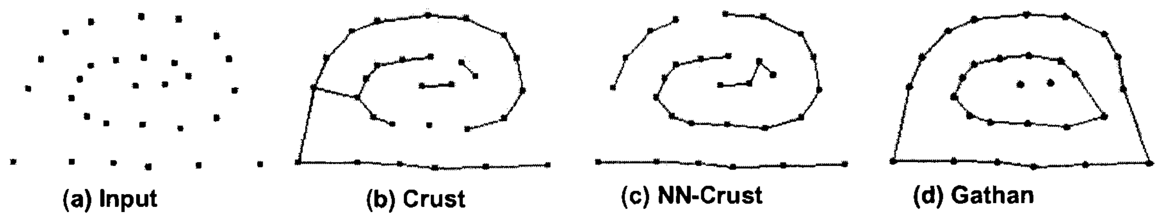


Figure 12 Existing reconstruction result for spiral line and a straight line

Moreover, the general problem of curve construction is to construct the curve from unorganized points collection of which people have no information about the original curves. The required algorithm should be able to distinguish and reconstruct multiple simple curves that may be open, closed, and/or with sharp corners. As a result, some concepts that applied to problems of the conventional reconstruction, such as uniformly distributed or non-uniformly distributed sampling, cannot be applied directly to the general problem of constructing curves from unorganized points. Among all the research to solve this problem, visual perception based algorithm comes up in recent years and it turns out to be an intuitive and effective method.

2.3 Curve reconstruction based on human vision function

Most curve reconstruction algorithms mentioned in the previous section are based on concepts from computational geometry. The application of the curve and surface reconstruction in computer vision, biomedical imaging, and reverse engineering motivates the requirements of connecting points in a way that is natural to the human eyes. The present thesis aims to design a new algorithm that reconstructs curves from points in the same way that human beings perceive them. The human visual perception of curve shapes includes not only topological aspects, such as the identification of connected components and the differentiation between open and closed curves, but also geometrical aspects, such as qualitative measures of degree of edge angle, length of the edge, etc.

Researchers have been investigated for a long time to the problem of detecting boundaries of points set in terms of human perception [7, 39] . Recently, Papari and Petkov [40] proposed an algorithm that groups points similarly to how human observers do. They identified groups as the connected components of a Reduced Delaunay Graph (RDG). Their method can be seen as an algorithmic equivalent of the gestalt law of perceptual grouping according to proximity. More recently, detailed observation and research related to the characteristics of human vision have been applied on the curve reconstruction. These kinds of visual perception based algorithms can handle many problems that the conventional methods cannot solve properly.

2.3.1 DISCUR

Zeng et al have developed a distance-based parameter-free algorithm for curve reconstruction named DISCUR [41]. Algorithm DISCUR can reconstruct multiple simple curves that may be open, closed, and/ or with sharp corners while it requires no parameters in the algorithm.

The algorithm originates from two observations made concerning the human visual system: 1) two closest neighbors tend to be connected, and 2) sampling points tend to be connected into a smooth curve. DISCUR constructs curves through the competition of close neighbors against smooth property of the curve. DISCUR is a guaranteed algorithm that can correctly reconstruct non-smooth open curves.

- **Notations of DISCUR:**

For a finite set of points $S = \{s_1, s_2, \ldots, s_m\}$ in R^n, the Euclidean distance between two points s_i and s_j is denoted by $d(s_i, s_j) = \|s_i - s_j\|$. $|S|$ is the total number of points in the finite set S.

A polyline is a continuous and piecewise linear curve. It can be denoted by $T = [y_1, y_2 \ldots y_m]$, where $y_1, y_2, \ldots, y_m$ are vertices on the polyline and $y_i \neq y_{i+1}$ for all $i = 1, \ldots, m-1$. If $y_1 \neq y_m$, then T is an open curve; otherwise, T is a closed curve. The kth element of T and the numbers of vertices in T can be denoted by $T[k]$ and $|T|$, respectively. For any open polyline T, a point x can be added to its head or tail by $[x|T]$ or $[T|x]$, respectively. Two open polylines T^1 and T^2 can be combined to an open polyline as $[T^1|T^2]$ or $[T^2|T^1]$ by connecting their nearest endpoints. For any closed curve T, $[y_1, y_2, \ldots y_m = y_1]$, $[y_2, y_3, \ldots, y_{m-1}, y_1, y_2]$ and so on are considered the same. Since only simple curves are considered, it is assumed that any two vertices y_i and y_j on a curve are different if $i \neq j$.

A point is called a free point if there is no polyline connected to it; endpoint or boundary if there is only one polyline connected to it; an interior point if there are two polylines connected to it [41]. For any two points p and q in a point set, a distance between p and q is an Euclidean distance, denoted by $d(p, q) = \|p - q\|$.

A polyline $[y_1, y_2 \ldots y_m]$ is called $< h_d, \sigma_d >$ - distributed, where

$$h_d = \frac{\sum_{i=1}^{m-1} \|y_i - y_{i+1}\|}{m-1} \tag{2.1}$$

$$\sigma_d = \sqrt{\frac{\sum_{i=1}^{m-1} (\|y_i - y_{i+1}\| - h_d)^2}{m-1}} \tag{2.2}$$

Symbols σ_d and h_d are standard deviation of distance and the distance mean of the curve. They are used to determine whether a sample point should be connected to a polyline T.

- **Requirement of DISCUR:**

Since DISCUR aims to deal with the problem where the curves are unknown in advance, it is assumed that the sampling points should be connected in a pattern that is natural to human perception. DISCUR has made two observations about human vision as given below:

• Nearness: the human visual system tends to connect two nearest neighbors

• Smoothness: the human visual system tends to connect sampling points into a smooth curve.

The nearness observation implies that two nearest neighbors may be connected if they are close enough. DISCUR aims to develop an algorithm that can imply smoothness by considering only nearness. It assumes that if a curve can be reconstructed in the way that human beings perceive it, then this curve has the best smoothness among all its possible reconstructions.

- **Simulation of nearness**

From both human vision and statistical points of view, after a new point p is connected to an end point q of an already reconstructed curve segment T_q , the new edge should not introduce an abrupt change to the statistical properties of the curve segment T_q. To achieve this objective, a dynamic function needs to be constructed to define the range of this change. The research group names this function as vision function. Based on the range defined by the vision function at the point being considered for connection to the existing curve segment T_q, it can be determined whether or not the point p should be connected to the sample q. A general form of this vision function can be written as the following:

$$E[p, T_q] = f(p, V) \qquad (2.3)$$

where E is connectivity value between p and endpoint q, V is a vector that comprises statistical properties of the curve segment T_q such as the distance mean, distance standard deviation, angle mean, and angle standard deviation. They assume that if $d(p, q) < E[p, T_q]$, then p and q can be connected. The function $f(p, V)$ can be obtained through experiments or through observations. A concrete form of Eq. (2.3) that considers only distance in the equation is given based on observations about human vision.

$$E[p, T_q] = h_d \frac{h}{s}(1+\frac{h_d}{\sigma_d})^{\frac{h_d}{\sigma_d}} \tag{2.4}$$

where $h = \frac{l+l_0}{2}$, $s = \frac{|l-l_0|}{\sqrt{2}}$, $l_0 = d(x_{i-1}, x_i)$ (resp. $l_0 = d(x_2, x_1)$), and $l = d(x_k, x_i)$ (resp. $l = d(x_k, x_1)$). The main steps of DISCUR is illustrated in Figure 13.

Algorithm DISCUR (Sample Set: S)

1: Step 1 – Delaunay triangulation and initialization

2: Step 2 – Determining the connectivity of Delaunay edges

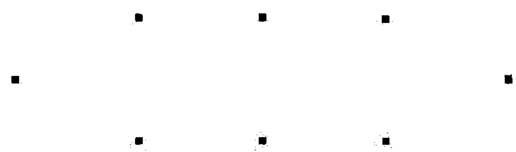
3: Step 3 – Updating the connectivity of Delaunay edges

4: Output the reconstructed curves

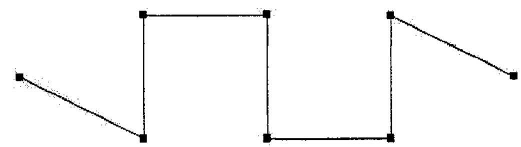
Figure 13 Main steps of DISCUR

As a parameter free algorithm, DISCUR brings two major advantages. First, without parameter, reconstruction of multiple curves with multiple features is made an automatic process. There is no need of multiple parameters for various parts with complex features. Second, the algorithm can be used when the original curve is not known. In this case, the sampling points will be connected as human eyes naturally perceive them. This makes it more useful for engineering applications than those dependent on known curve features.

However, the algorithm has some shortcomings. Firstly, DISCUR uses only distance to quantify the two observations about the human visual system, which means it use only proximity property of Gestalt laws and neglect the important characteristics of good continuation. It can construct sample points as human perceive them in many cases. However, in some cases such as that given in Figure 14 a), if any two adjacent sampling points have the same distance, the curve cannot be reconstructed into a visually acceptable result.



a) Sampling point



b) Reconstructed result by DISCUR

Figure 14 Example of wrong connections by DISCUR

Secondly, even though DISCUR can correctly construct non-smooth curve, a very dense sampling is required near the sharp corner area. Thirdly, algorithm DISCUR uses the Equation (2.4) to determine the connectivity between two points. This equation is based on observations about human vision, it does not have any experimental support. The equation can be replaced by better equation that is obtained experimentally base on human vision function.

2.3.2 VICUR

Nguyen and Zeng presented a new algorithm named VICUR for curve reconstruction problem [42]. Their algorithm can construct curves that look natural to human vision from a set of unorganized points. VICUR is based on two connectivity criteria: proximity and good continuation form the prominent Gestalt principles of visual perception.

Nguyen and Zeng observed that human eyes tend to connect a point to an existing curve when the point lies within a certain area determined by the characteristics of the curve. They name this area as connectivity area. They also define a criterion to evaluate the possibility of the connectivity for each reference point in the connectivity area and give a concrete form of the function obtained through observation and experiment as follows.

$$E[p, T_{P1}] = (c(\frac{\alpha_s}{\overline{\alpha}} - 1)^2 + (1-c)(\frac{d_s}{2(\overline{d} + \sigma)})^2 + 1)^{-1} \qquad (2.5)$$

where $E[p, T_{p1}]$ is the connectivity value between sample p and curve endpoint p_1, T_{p1} is a segment of the curve. α_s is candidate angle, c is user-defined parameter, $\overline{\alpha}$ is average angle, d_s is the length of candidate segment. $\overline{d}$ is the average length of the curve segment, and σ is the standard deviation of the edge lengths. They assume that if p has the highest value of E, then p can be connected to the curve.

Despite the fact that VICUR can handle well many examples, the algorithm has some shortcomings. First, VICUR is sensitive to vertex position. Little difference of the degree or length that cannot detect by human vision may result in significant difference. Secondly, there are some wrong connections in favor of sharp corners. Finally, the parameters used in VICUR are not accurate to handle all sample sets.

2.4 Three basic rules for connection

Based on the definitions and analysis above, three basic rules for connecting the point to curve, curve to curve, and point to point are proposed as following:

- **Rule 1:** Point-curve connectivity. For an even $< h_d, \sigma_d > -$*distributed* curve $T = [p_1, p_2, \ldots, p_n]$, $n > 1$, which is partially reconstructed from a sample set S. To the connectivity area $A(p_n)$ which is around p_n, if there exists a sampling point $p_i \in A(p_n)$, such that $E[p_i, T_n] > E[p_j, T_n]$ for all $p_j \in A(p_n)$, then p_i and p_n should be connected. The rule also applies on the connectivity area $A(p_1)$.

- **Rule 2**: Curve-curve connectivity. For two even-$< h_d, \sigma_d >$*-distributed* $T = [p_1, p_2, \ldots, p_n]$, $T' = [q_1, q_2, \ldots, q_m]$, $n, m > 1$, which are partially reconstructed from a sample set S, if p_1 (or p_n) can be connected to T' by Rule 1 or q_1 (or q_m) can be connected to T by Rule 1, then these two curves can be connected.

- **Rule 3:** Point-point connectivity. For the any shortest edge $e = [q_1, q_2]$ in Delaunay triangles of sample points, where q_1 and q_2 are both free points. Let $B(q_1, r)$ be a ball centered at q_1 with radius $r = \frac{1}{2}([q_1, q_{k1}] + [q_1, q_{k2}])*\omega$, where $[q_{k1}, q_{k2}]$ are the shortest and second shortest neighbor to q_1, $\omega = 1.849$ (the value of ω comes from experiments on sharp corner cases). If there exist a $q_t \neq q_2$ in B such that angle $q_t q_1 q_i$ larger than the maximum angle that formed by edge $[q_1 q_2]$ with q_1 as the vertex, where $q_i \in B$ and $q_i \neq q_2$, then q_1 and q_2 can not be connected, otherwise, q_1 and q_2 can be connected. The rule also applies q_2.

For Rule 3, two closest points are not necessarily two adjacent points on a curve, as illustrated in Figure 15. In Figure 15 a), point A and B are nearest neighbors to each other but they are not adjacent on the curve if there has a sharp corner.

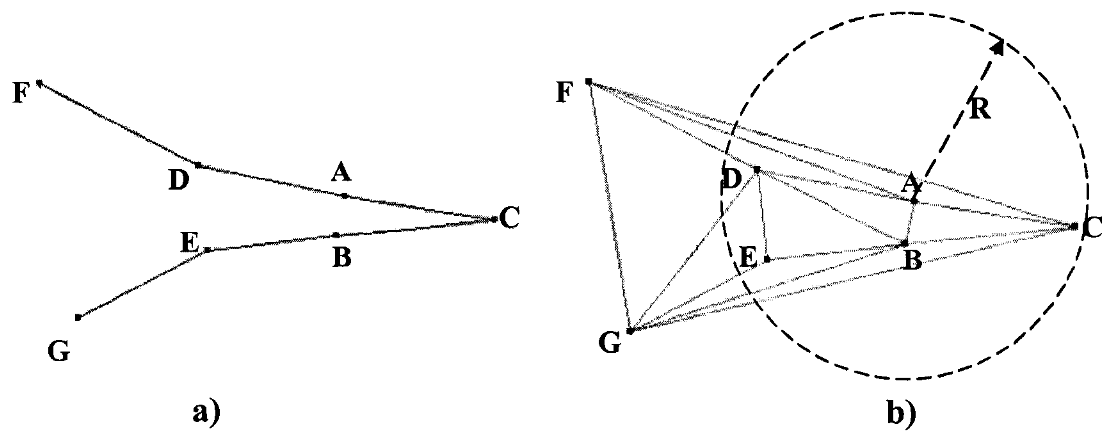


Figure 15 Two nearest neighbors are not adjacent

Hence, any attempt to connect shortest edge first may result in wrong connection for the entire curve. If the two points incident to the shortest edge does not belong to the same ray of the corner angle, the maximum angle formed by shortest edge with other edges must be smaller than the maximum angle formed by any adjacent edges that do not include the shortest edge. As shown in Figure 15 b), the triangles connect all the points are Delaunay triangles, and edge AB is the shortest edge. For point A, the maximum angle formed by edge AB with A as the vertex in the dashed circle is $\angle BAC$. The maximum angle formed by any adjacent edges, which do not include the edge AB, is $\angle DAC$. It is obvious that $\angle DAC > \angle BAC$, so point A and B cannot be connected. To limit the number of adjacent short edges around two closest points, which also means to quantify the radius of the dashed circle R, we set ω equals to 1.849. We get this value by lot of experiments on sharp corner cases.

2.5 Curve reconstruction procedures

The curve reconstruction algorithm takes a set of sample point as input and constructs curves in three steps:

Step 1: Compute Delaunay diagram on the set of sample point. Let D be a set of Delaunay edges and R be temporarily removed edge, R is initially empty.

Step 2: Find the shortest Delaunay edge $e=[p_1, p_2]$. There are three possibilities for p_1, p_2:

- Both are free points: if p_1 and p_2 satisfies Rule 3, then connect p_1 to p_2. Otherwise, put $[p_1\, p_2]$ into R.
- p_1 is free point and p_2 is endpoint of curve T: if p_1 and p_2 satisfies Rule 1, connect p_1 to p_2. If p_1 and p_2 do not satisfy Rule 1, put $[p_1\, p_2]$ into R.
- p_1 is endpoint of curve T_1 , p_2 is endpoint of curve T_2. If p_1 and p_2 satisfies Rule 2, connect p_1 to p_2 to form T. If p_1 and p_2 do not satisfy Rule 2, put $[p_1\, p_2]$ into R.

Step 3: Remove edge $[p_1\, p_2]$ from D. Repeat from step 2 until D becomes empty. As long as D is empty. Put all edges in R back into D and repeat from step 2. When there is a connection between p_1 and p_2 to form a new curve T that has the new properties because its length changes, explore T for any incident edges and connect the edge that has the largest connectivity value. Then repeat from the second part of step 2 until D and R are empty.

The main ideas of the algorithm are as following: firstly, it searches the shortest Delaunay edge. If the two vertices incident to the edge are free points, check Rule 3 to avoid wrong connection may occur in case the points near the sharp corner area. If one of the vertices is endpoint of a curve or both vertices are endpoints, check Rule 1 and Rule 2 respectively to see if the curve can be extended. When the vertices do not satisfy the corresponding rule, they are put into *R* so that they can be reconsidered later. The reason for such reconsideration is that when the curve extends during the process, the change of the curve's characteristics lead to the change of connectivity area and connectivity value. Therefore, the vertices do not satisfy the rule at present may satisfy the same rule later.

Figure 16 shows the connecting sequence of the visual perception based curve reconstruction algorithm. Figure 16 a) is the input points, and Figure 16 b) is the first step of connecting two points. Figure 16 f) is the reconstructed curve.

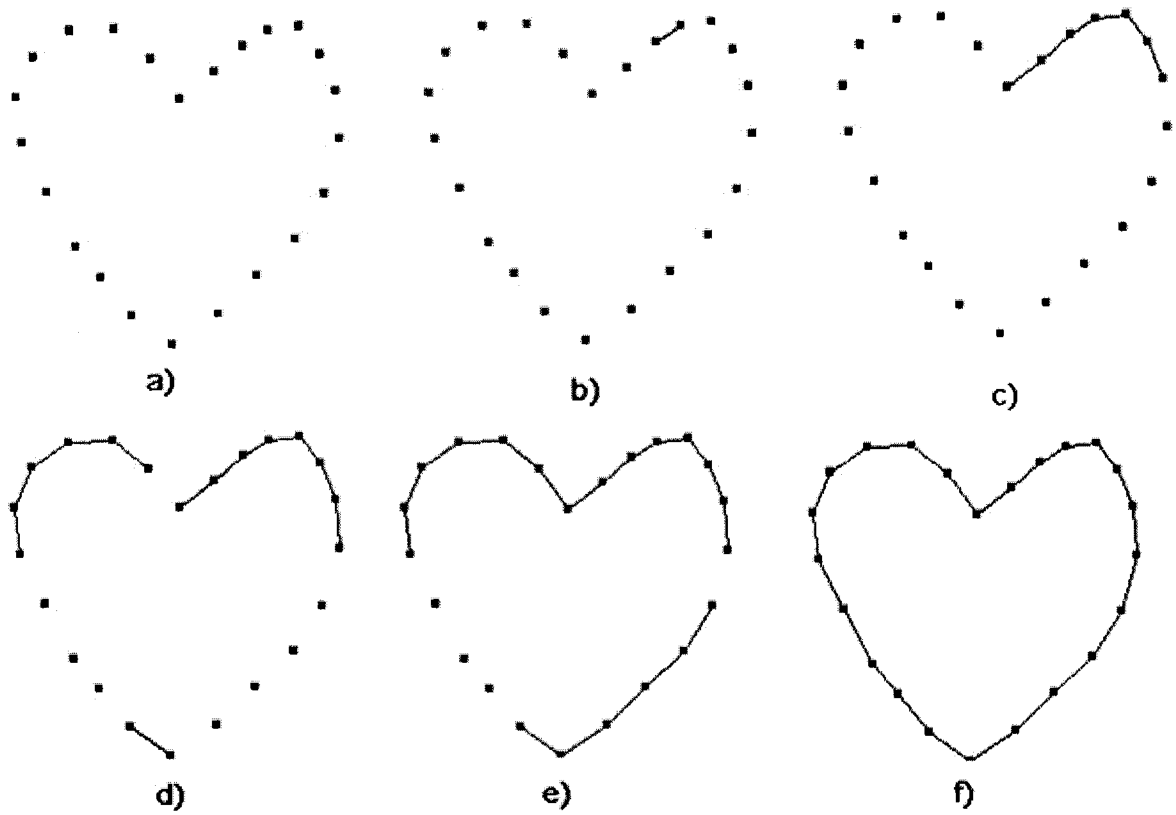


Figure 16 Connecting sequence of the visual perception based curve reconstruction

2.6 Review of previous work

Li from our research lab in his master's thesis, attempted to extend the DISCUR algorithm by including the smoothness requirement based on the experiment of human vision [43]. In order to find the appropriate formula to represent the general form of vision function $E[p, T_q] = f(p, V)$, his work used the characteristics of the curve as the input to represent the coordinates information of all sample points p. The output of his formula is the possibility of whether these sample points can be connected to form a curve; and he treated the possibility as the connectivity value.

Li proposed a framework for doing experiments: point size, straight line, curved line, factor analysis, etc. First, he listed all the factors of the sample points such as point size, points count, distance between points, etc. Then he used ANOVA method to analyze the importance of each factor and get regression formula from the experiment results. Instead of doing one experiment with all the factors, he included some part of the factors in each experiment, then excluded the minor factors and kept major factors in the next round of experiment in which new factors are added.

His assumption of connectivity value is show in Figure 17, which is a user interface of the software he developed to display the curve and collect data from participants. In the figure, the participants were asked to decide if these points are suitable to be connected to form a curve or not. While my assumption is that if a reference point is suitable to be connected to a already existed curve. The difference of the assumptions results in a very different experiment design even there are some similar experiment process and methods.

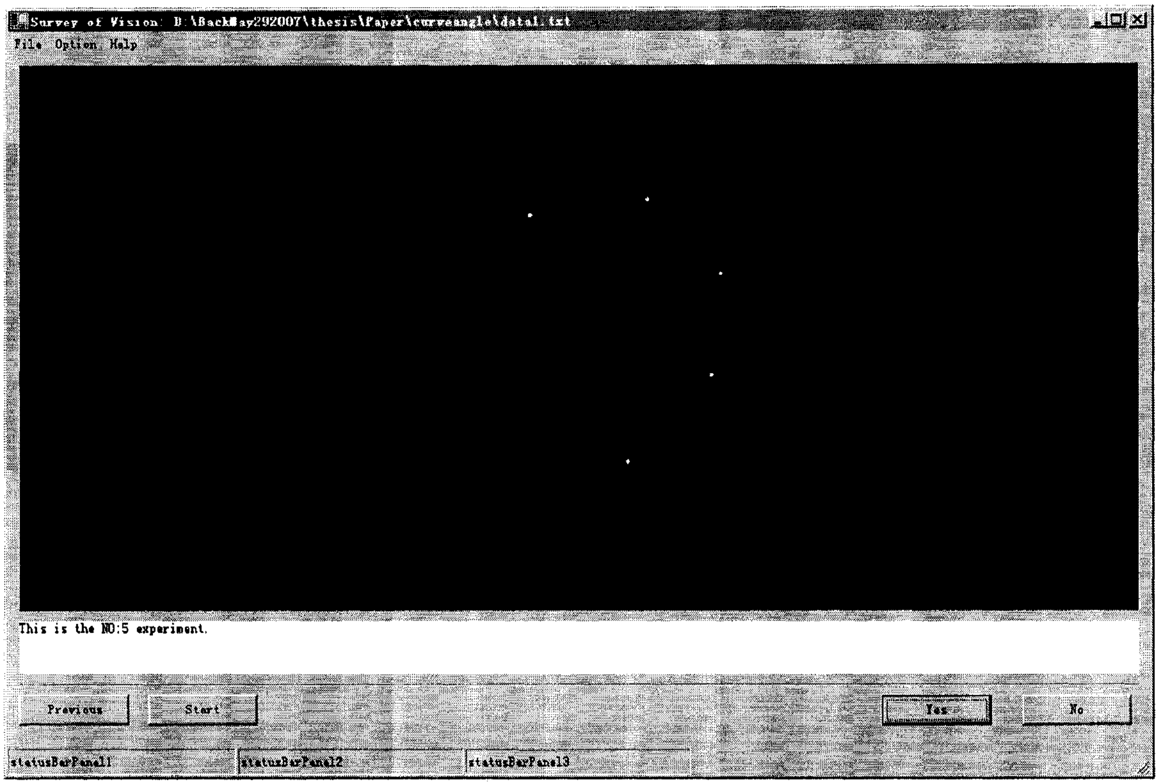


Figure 17 Interface of collecting data [43]

Li's work presents some characteristics of the vision based curve reconstruction, and it can work properly in some cases. However, his work has some problems and shortcomings.

First, the curve segment he chooses to calculate the connectivity value includes only 5 points or fewer than 5 points, which cannot properly represent the characteristics of the curve. If there is an outlier near the endpoint, which changes the properties of the curve significantly, his method of including this outlier in the calculation will result into wrong connection. Furthermore, it is possible for a curve that has more than 5 points to change other properties when the curve extends.

Secondly, his work neglected the important characteristics of reference point. Therefore, the output of his algorithm cannot represent the connectivity value of the reference point.

Thirdly, there is no sharp corner checking when the first two closest points are connected; as a result, the reconstruction result can be wrong in his algorithm, which makes the rest of connection wrong.

At last, his work choose the quadratic regression model, which allows only three values for each factor, thus cannot demonstrate the significant effect of the important factors such as reference angle, thus results in inaccurate results. Furthermore, the quadratic regression model is not accurate enough to be used in such a complicated situation.

In my experiment design, I follow his experiment steps and include experiments of straight line, curving line, point count by using a different assumption of connectivity value. The experiment processes and analysis methods are similar but the judgment results have been changed to the max distance of the reference point. I also modify his software by adding the participants' manipulation of reference point according to my assumption.

2.7 Motivations for new experiment design

As mentioned before, DISCUR and VICUR are based on researcher's observations. DISCUR is focused on the proximity of the sample points. VICUR considers the good continuation and solves the problems of sharp corner, curve segment, points count existed in Li's work. However, DISCUR and VICUR use observational results about visual perception; scientific experiment can be more convincing.

The success of the DISCUR, VICUR, and Li's vision function become motivation for understanding the mechanism of human visual perception. To formulate the visual perception on proximity and good continuation and quantify the effect of the characteristics of the curve and reference point to connectivity value, a serial of experiments are designed and implemented to identify how human beings perceive curves from the points collection. The objective of the experiment is to quantify the two Gestalt laws so curves can be reconstructed in the way human beings perceive them. In the following chapters, we will focus on the experiment design, the factor analysis, data collection, data regression, and formula validations.

Chapter 3 Experiment Design for Quantify Proximity and Good Continuation Laws

3.1 Notations and definitions of points set

As mentioned previously, problems of curve reconstruction can be viewed as combination of connection between points and connection between a point and a curve. In the case of point-curve connection, the point is called reference point; the edge between reference point and endpoint is called reference edge. The angle at endpoint is reference angle and is defined as the clockwise rotation from reference edge to curve segment incident to curve endpoint. Figure 18 shows a curve, a reference point, and some other characteristics.

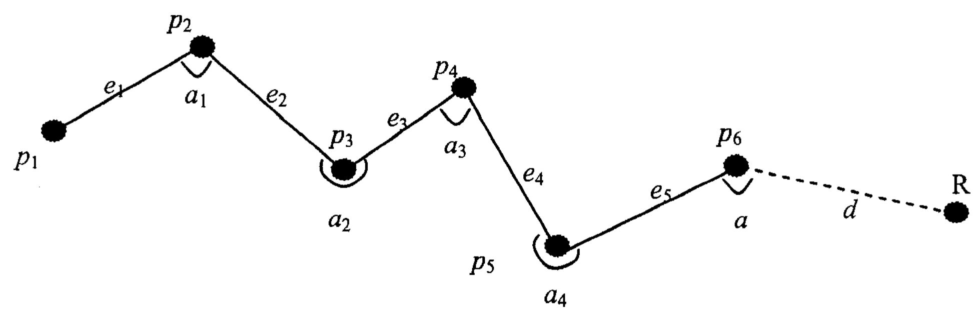


Figure 18 Characteristics of the curve and the reference point

In the Figure 18, R is the reference point, d is the reference edge, p_6 is the endpoint, and a is the reference angle. e_5 is the head edge, and the extension line of the curve means the extension of the head edge where reference angle equals to 180 degrees.

An outlier is the vertex where the curve changes its property dramatically. We define the vertex where the curve changes its winding direction as the outlier. There can be several outliers in a curve. For example, we check every angle of the curve from the endpoint to start point, if the first angle of the curve is less than 180 degrees, then the point where the angle is larger than 180 degrees is the outlier point, and vice versa. If the curve has outliers when we analyze the connectivity of the curve with the reference point, the curve needs to separate into individual segments that have the same winding direction because the outlier changes the characteristics of the curve significantly.

3.2 Connectivity area

When the participant is asked to decide if a reference point should be connected to a curve or not, the characteristics of the curve and the reference point must be evaluated first. If the reference point is far away from the curve, obviously the reference point is not suitable to be connected to the curve. If the point is very close to the curve's endpoint, then the reference point is suitable to be connected to the curve. Based on the above analysis, it can be assumed that there must exist a connectivity area A to the curve near the endpoint, all the reference point located in this area can be connected to the curve.

As shown in Figure 19, the dash line forms the right side connectivity area of the curve T, and this area is named as A_1. Point P_1 and P_2 located in A_1, both of them are suitable to be connected to the curve T. Point P_3 located outside of the A_1, and then it is not suitable to be connected to the curve. It is obvious that there is another connectivity area A_2 at the other side of the curve, and different curve has different shape of connectivity areas because the connectivity area is determined by the various characteristics of the curve.

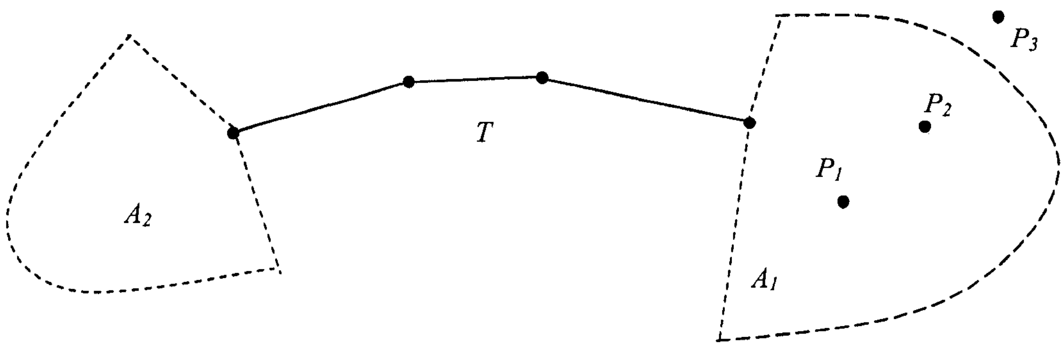


Figure 19 Connectivity area of the curve

3.3 Connectivity value

We define the connectivity value (E) as the trend of the reference point to be connected to the cure. The higher value the trend is, the more suitable the reference point should be connected to the curve. On the edge of the connectivity area, the reference point can or cannot be connected to the curve. Therefore, it is reasonable to define its connectivity value equals to zero. For the reference points with the same reference angle located in the connectivity area, the connectivity value is larger when the reference point is closer to the endpoint, while the connectivity value is smaller when the reference point is further away to the endpoint. On the position of the endpoint, the connectivity value increase to its maximum and we define this max value as one. We also define the connectivity value equals to zero when the reference point located outside of the connectivity area. The simple demonstration of the relationship for the connectivity value and the position of the reference point is shown in Figure 20. In Figure 20, the endpoint is located at coordinate (X=0, Y=0); The Z axis stand for the connectivity value. The connectivity area in Figure 19 corresponds to the area of the curving surface where connectivity value equals to zero.

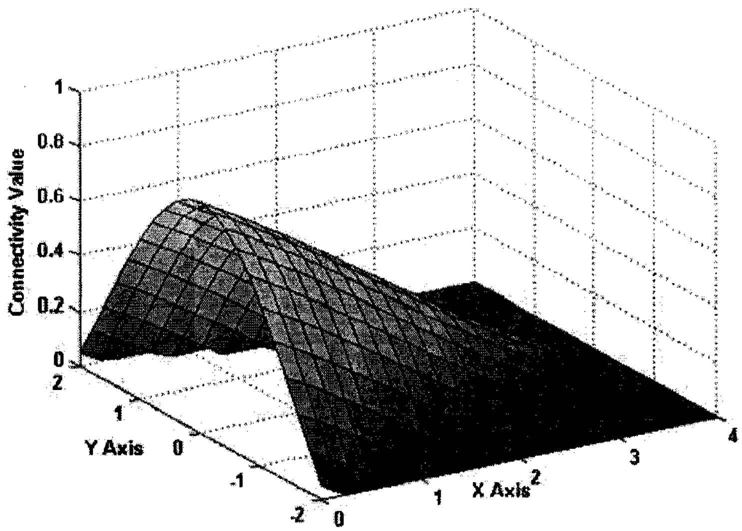


Figure 20 Relationship of the connectivity value with position of reference point

In general, it can be assumed that when there are two reference points with the same distance to the endpoint, the larger the reference angle departs from the extension line, the smaller the connectivity value is. As shown in Figure 21, points P_4 and P_5 have the same distance to P_1, and points P_6 and P_7 have the same distance to P_1 with shorter distance. From the definition and analysis previously, if we use $E(P)$ to stand the connectivity value of point P, we can get that $E(P_1) = 1$, $E(P_2) = E(P_3) = 0$. Because P_4 and P_5 have the same distance to the P_1, we can get $E(P_4) > E(P_5)$. We can also get $E(P_6) > E(P_7)$ for the same reason. For the same direction along the extension line, we can get $E(P_6) > E(P_4)$ and $E(P_7) > E(P_5)$. For any other points located in the connectivity area, it has the connectivity value less than 1 and larger than 0, and it has only one connectivity value corresponding to its position.

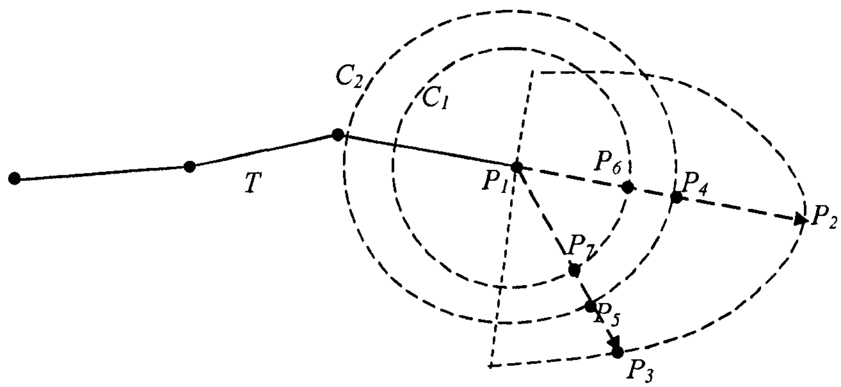


Figure 21 Connectivity value of the curve

It is very hard to tell which value is larger for E (P_7) and E (P_4) in Figure 21 because P_7 is located with a shorter distance to P_1 but with a larger angle away from extension line. As the fact that the connectivity value decrease continuously along the extension line, we can assume that there exist a set of points, which can have the same connectivity value. We can also assume that different connectivity value corresponding to different set of points. As show in Figure 22, there are three sets of points. Points P_4 and P_5 have the same connectivity value, so they are on the same isoline, which has the same connectivity value. We can also treat the edge of the connectivity area as an isoline with the connectivity value equal to zero.

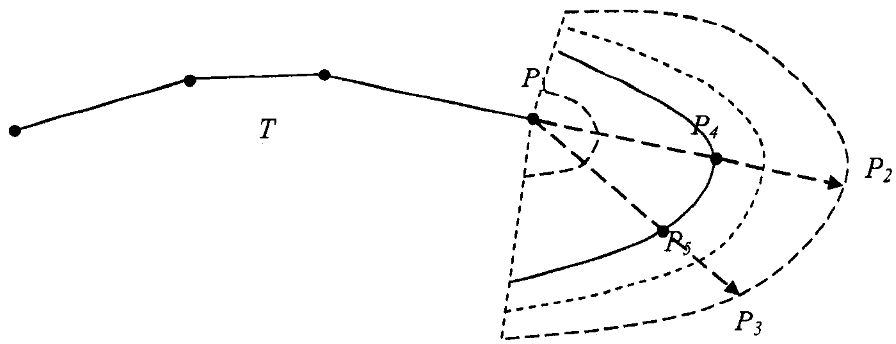


Figure 22 Three sets of points with different connectivity values.

For all the isolines in the connectivity area with different connectivity values, we can conclude that these isolines are parallel to each other, and there are no intersections among themselves. If there is an intersection, it means that there exists one point that has two connectivity values in the connectivity area, thus it contradicts to the characteristic of the reference point we mentioned before.

3.4 Comparison of connectivity value for two reference points

For any reference points in the curve's connectivity area, the reference point with larger connectivity value E is more suitable to connect to the curve than the reference point with less E. To get the relationship of the E with the characteristics of the curve and reference point, it is necessary to assign a certain value to a reference point in the experiment. In Figure 23, the reference point located on the extension line of curve's head edge is defined as contrastive reference point (CRP), which can be used to compare the connectivity with other reference point. On the contrary, the reference point dose not locate on the extension line is defined as static reference points (SRP) because its position does not change when we compare the connectivity in the experiments. CRP is moved to an appropriate position until the participant think that the CRP and SRP have the same connectivity value, which mean both CRP and SRP can be connected to the curve with same preference for the participant.

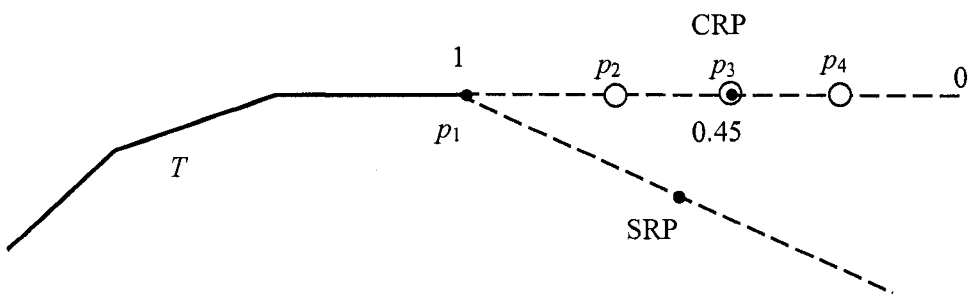


Figure 23 Comparison the connectivity value of SRP with CRP

In Figure 23, there is one SRP and one CRP. The length of the extension line is the maximum distance of the reference point on that reference angle (which equals to 180 degrees). The maximum distance can be calculated by using the function of connectivity area. The E equals to 1 on the start point of the extension line, and equals to 0 on the end point of the extension line. To simplify the model, linear function is adopted here to calculate the E at different position on extension line. For example, $E(p_2) = 0.7$, $E(p_3) = 0.45$, $E(p_4) = 0.2$. If the CRP should be located at p_3, it can be assumed that the E (SRP) = 0.45.

3.5 Factor analysis

The human vision function should be a simple formula and must be intuitive and effective. From the point of view of curve reconstruction algorithm, the inputs of the function are the curve and the reference point. The output is the connectivity value of the reference point to the curve. To find the function, first, all potential factors in the context of curve reconstruction should be enumerated. Then the experiment is designed and implemented to exclude the minor factors and keep the major ones. Finally, the experiment data of the major factors are used to construct the function and then to evaluate the correctness of the function. In the experiment, a lot of curve and reference point combinations are generate to decide if the reference point should or should not be connected to the curve in each combination.

The following relevant factors are apparent to the characteristics of the curve and reference point: edge lengths, curve angles, distance of the reference point, and the reference angle. Beside the above factors about the characteristics of the curve, there are also following factors such as point size, point color, background color, shape of curve, point count, etc

As the present thesis focuses on the geometric factors in the context of curve reconstruction, the color of the point, curve and the background are excluded from the experiment. Furthermore, when people reconstruct curves from two sets of unorganized points that have the same pattern figure but different in the pattern size, people tend to reconstruct the curve in the same way, as show in Figure 24. Therefore, the absolute distances of the edges are not essential and only the curve edges with relative length should be focused on.

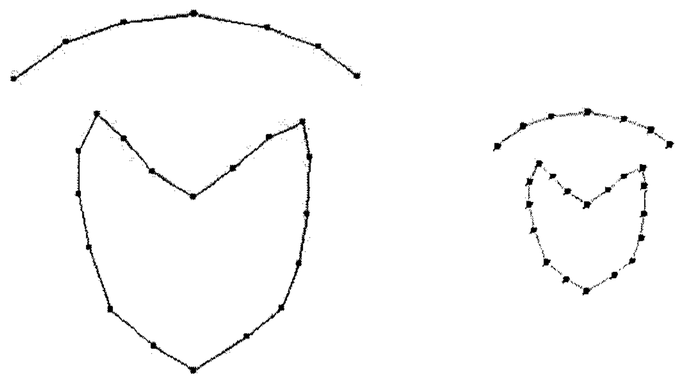


Figure 24 Two set of points with deferent pattern size but same reconstruction result

The ideal way to analyze the effect of factors is to include all the factors in the experiment. However, if there are too many factors and the experiment considers all factors, the sample number for the experiment would be huge. Suppose that the 10 factors are considered, for example, edge lengths, curve angles, etc. and each factor has four levels on average, the total samples will be 1048576 (4^{10}). If each sample takes 10 second to finish, it would take 4 months without a break. For this consideration, the experiment should start with the some part of the factors and then add more factors to it in the later. The implementation of the experiment is listed in the next section.

Chapter 4 Data Collection, Processing, and Analysis

The procedures of implementing experiment for constructing the visual perception based function are listed below:

1) The simplest situation of curve reconstruction is considered at the beginning, that is, a straight line constructed by several points. The relationships among point size, variance of edge lengths, and reference angle are analyzed and the major factors are kept by using analysis of variance (ANOVA).

2) Keeping the major factors from the previous step and other factors are added to the experiment to identify the major factors.

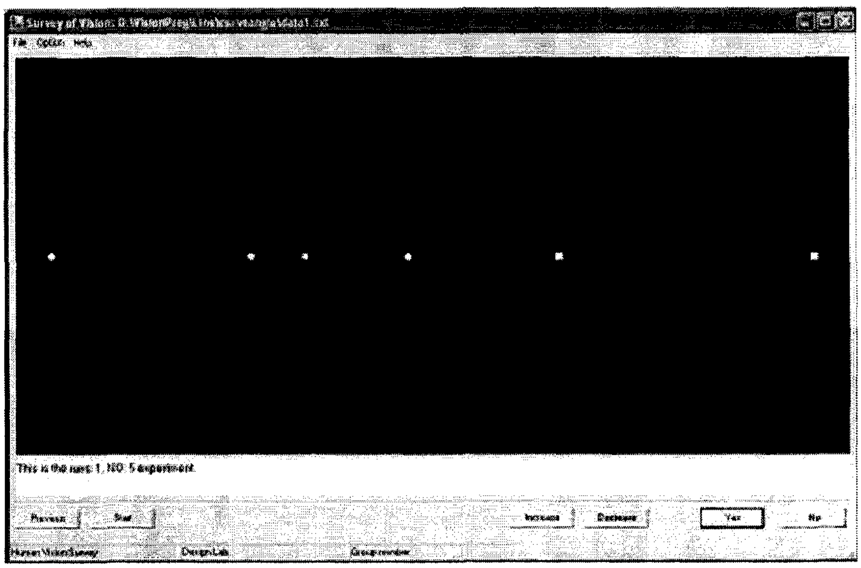
3) Repeat the procedure 2 until all factors are considered and only the most significant factors are kept, the function of connectivity area of the curve can be constructed by multivariable nonlinear regression model.

4) The reference points of different location are compared with standard points in the connectivity area to determine their connectivity values. Then, a multivariable nonlinear regression model is applied to construct the function.

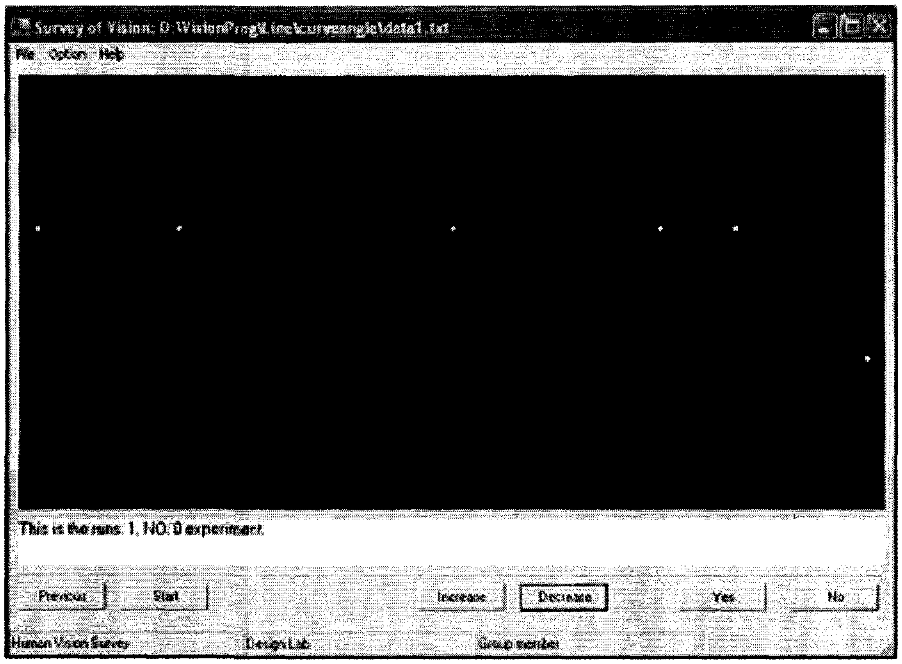
4.1 Experiment of straight line

First, we need to consider the influence of the point size. We need to consider this factor because the sample points are drawn on the computer screen. Either a small dot or a big disk can represent the curve point and the point size may affect the participant's judgment. Thus, we should do the experiment to evaluate effect of the point size.

In the experiment, it is assumed that all the points on the screen should have the same size, because in the case when there are difference size of points in the sample points, people tend to subcategorize the points into different groups, and in each group the point size is similar or almost the same. In this section, the input includes a set of horizontally distributed points (which can be treated as an unconnected curve), together with a reference point located on various location. The output is the max distance of the reference point (within this distance, the reference point can be connected to the curve). The corresponding software is developed for this experiment, and the user interface is shown as below:



(a)



(b)

Figure 25 Survey of simple curve's extension

The size of point size in Figure 25 (a) equals to 9 pixels while in Figure 25 (b) equals to 2 pixels. The curve and the reference point are created with the following factors: point size (P_Size), standard deviation of the edge lengths (Dev_D), and the reference angle (Ref_A). The values of each factor are set as following in Table 1.

Factor	Values
P_Size	2, 9
Dev_D	2.7, 0
Ref_A	135, 180

Table 1 Factors and values for straight line

The straight line includes 5 points (the factor of point size will be analyzed later). The Data are generated and sorted randomly. One of the sample data are listed in the table below:

Seq	D_S	L1	L2	L3	L4	R_A	D_D	P_Size
1	3	178.72	63.83	236.17	121.28	-45	2.7	2
2	7	63.83	236.17	178.72	121.28	0	2.7	2
3	1	150	150	150	150	-45	0	2
4	8	150	150	150	150	0	0	2
5	4	178.72	63.83	121.28	236.17	-45	2.7	9
6	6	236.17	121.28	178.72	63.83	0	2.7	9
7	2	150	150	150	150	-45	0	9
8	5	150	150	150	150	0	0	9

Table 2 Sample data from experiment of straight line

In Table 2, Seq means the trial number, which is same to the Seq in Table 3. D_S means the sample sequence showed on the screen. L1 to L4 means the length of the edge. R_A stands for Ref_A and D_D stands for Dev_D.

In the experiment, participants are required to practice several times before the experiment to make sure that they are familiar with the experiment procedures. Then they are asked to make decisions on connectivity based on the points drawn on the screen. Four participants take the experiment on the computer under the common working environment and each participant take the experiment twice so there will be 8 group of data. The time spent on each trial is also recorded to identify outliers. If an experiment takes too much time, it should be redone. One of the experimental results is listed below.

Seq	Distance	Time
1	1.246667	1.34
2	1.593333	1.20
3	0.88	1.35
4	1.34	.90
5	0.993333	.99
6	1.766667	.80
7	1.08	.88
8	1.473333	1.04

Table 3 Sample result of experiment of straight line

In Table 3, Distance means the maximum distance the reference point can extend. Time means the seconds the participants spend on each trial. The data was analyzed by using ANOVAN in MATLAB (R2006A). One of The MATLAB analysis result is listed below.

Analysis of Variance

Source	Sum Sq.	d.f.	Mean Sq.	F	Prob>F
Ref_A	0.93767	1	0.93767	123.6	0
Dev_D	0.1778	1	0.1778	23.44	0.0013
P_Size	0.03547	1	0.03547	4.68	0.0626
Ref_A*Dev_D	0.01034	1	0.01034	1.36	0.2767
Ref_A*P_Size	0.00123	1	0.00123	0.16	0.6983
Dev_D*P_Size	0.01734	1	0.01734	2.29	0.1691
Ref_A*Dev_D*P_Size	0.01034	1	0.01034	1.36	0.2767
Error	0.06069	8	0.00759		
Total	1.25086	15			

Constrained (Type III) sums of squares.

Table 4 One analysis result of experiment for straight line

The F-ratios corresponding to the three factors of four participants are list below, which include the data in Table 4 (participant 1):

Participant	Ref_A	Dev_D	P_Size
1	123.6	23.44	4.68
2	191.81	50.53	1.02
3	268.21	61.08	4.17
4	198.85	44.31	0.95

Table 5 F-ratios result of three factors for straight line

In Table 5, F is the ratio of the Model Mean Square to the Error Mean Square. The F statistic follows an F distribution with p numerator degrees of freedom and n-p-1 denominator degrees of freedom. The null hypothesis is rejected if the F ratio is large [44]. In this experiment and all other experiments, only the factors with F value larger than certain threshold value are significant factors.

In Table 5, F-ratios of factor P_Size are less than $F_{0.025,1,8}$, which equals to 7.57 (The upper critical value is set to 2.5%, and it can be get from Matlab by using function finv(1-0.025,1,8)), and F-ratios of other factors are larger than 7.57. Therefore, it can be conclude that factors Ref_A and Dev_D are significant factors, the factor P_Size (point size) is not a significant factor and it can be excluded in the following experiments.

4.2 Experiments of curving line

The objective of the experiments in this section is to find out the significant factor to the curving line. After the factor of point size is eliminated, two other factors related to the curving line are included in the experiment, factor Mea_A means the mean of the curve angles, and factor Dev_A means the deviation of the curve angles.

If two factors Mea_A and Dev_A are added to the curve at the same time, there will be two kinds of curving lines as shown below.

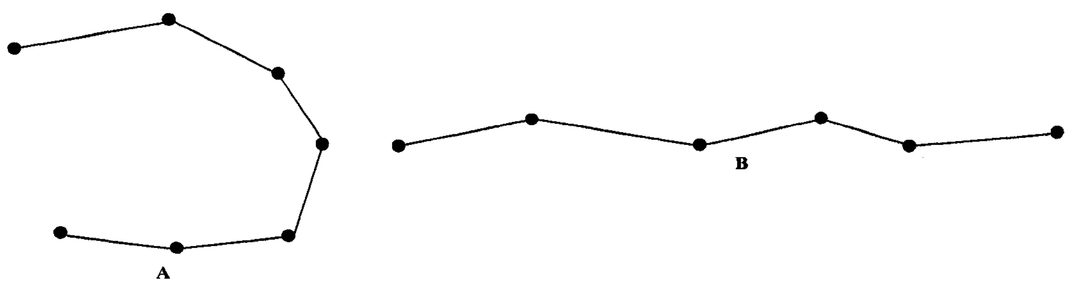


Figure 26 Two kinds of curving lines with angle deviation

In Figure 26, curving line B has the mean of angles equal to 180 degrees, but with the angle deviation not equal to zero. It is obvious that the curving line B has some outliers and it need to be separated into several segments, which have the same winding direction. To avoid the outlier in the curve, it is necessary to add Mea_A and Dev_A separately in the following two sections.

4.2.1 Curved line without angle deviation

In the experiment of curving line without angle deviation, only factor Mea_A (the mean of the curve angles) is added to evaluate its influence.

Together with two significant factors in the straight-line experiment, the curve and the reference point are created with the following factors and values, as shown in Table 6:

Factor	Values
Mea_A	140, 180
Dev_D	2.7, 0
Ref_A	110, 180

Table 6 Factors and values for curving line without angle deviation

The curve has 6 points. The input includes a set of curves, for each curve there is a reference point located on various directions. The user interface and the sample data are listed below:

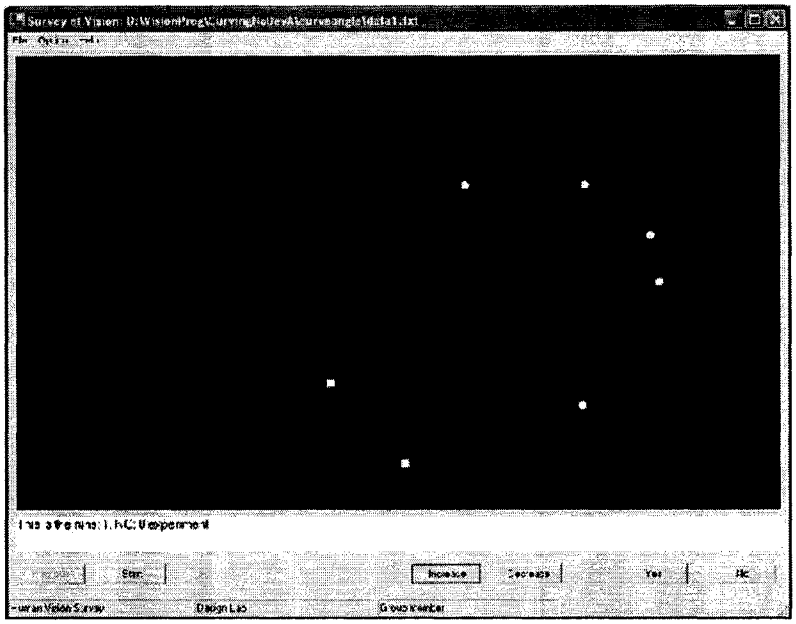


Figure 27 User interface of the curving line without angle deviation

Seq	D_S	L1	L2	L3	L4	L5	R_A	D_D	M_A	A1	A2	A3	A4
1	4	130	92.66	55.319	167.34	204.68	-70	2.7	40	-40	-80	-120	-160
2	6	204.68	92.66	130	55.319	167.34	0	2.7	40	-40	-80	-120	-160
3	2	130	130	130	130	130	-70	0	40	-40	-80	-120	-160
4	5	130	130	130	130	130	0	0	40	-40	-80	-120	-160
5	1	204.68	92.66	130	55.319	167.34	-70	2.7	0	0	0	0	0
6	8	204.68	92.66	55.319	167.34	130	0	2.7	0	0	0	0	0
7	7	130	130	130	130	130	-70	0	0	0	0	0	0
8	3	130	130	130	130	130	0	0	0	0	0	0	0

Table 7 Sample data from experiment of curving line without angle deviation

Seq	Distance	Time
1	1.361538	216
2	2.3	96
3	1.323077	51
4	1.784615	104
5	1.169231	57
6	2.007692	28
7	1.053846	58
8	1.638462	28

Table 8 Sample result of experiment of curving line without angle deviation

In Table 7, M_A means the mean of angles of the curve. The experiment process is similar to the previous ones. One of the ANOVA analysis results is listed below.

Analysis of Variance

Source	Sum Sq.	d.f.	Mean Sq.	F	Prob>F
Ref_A	1.94388	1	1.94388	429.78	0
Dev_D	0.17737	1	0.17737	39.22	0.0002
Mea_A	0.20773	1	0.20773	45.93	0.0001
Ref_A*Dev_D	0.07776	1	0.07776	17.19	0.0032
Ref_A*Mea_A	0.00563	1	0.00563	1.24	0.2971
Dev_D*Mea_A	0.00003	1	0.00003	0.01	0.9337
Ref_A*Dev_D*Mea_A	0.00018	1	0.00018	0.04	0.8463
Error	0.03618	8	0.00452		
Total	2.44875	15			

Constrained (Type III) sums of squares.

Table 9 One analysis result of curving line without angle deviation

The F-ratios corresponding to the three factors of four participants are list below In Table 10, which also include the data in Table 9:

Participant	Ref_A	Dev_D	Mea_A
1	429.78	39.22	45.93
2	232	8.87	33.21
3	471.11	60.64	57.8
4	197.29	42.24	45.95

Table 10 F-ratios result of curving line without angle deviation

In Table 10, all F-ratios are larger than $F_{0.025,1,8}$ which equal to 7.57, so it can be concluded that factors Ref_A, Dev_D, and Mea_A are all significant factors.

4.2.2 Curved line with angle deviation

As shown in Figure 25 (B), factor Dev_A cannot be applied to the curve of which the mean of angles equals to zero. To evaluate the effect of the Dev_A, it is unnecessary to change the value of the mean of angles (Mea_A) in the experiment.

In this section, the curve and the reference point are created with the following factors and values, as shown in Table 11:

Factor	Values
Dev_A	25, 0
Dev_D	2.7, 0
Ref_A	110, 180

Table 11 Factors and Values for curving line with angle deviation

The curve includes 6 points and the mean of angles equals to 155 degrees. The sample data are listed in Table 12.

Seq	D_S	L1	L2	L3	L4	L5	R_A	D_D	D_A	A2	A3	A4	A5
1	5	204.68	92.66	55.319	130	167.34	-70	2.7	25	-60	-70	-130	-140
2	8	92.66	130	55.319	204.68	167.34	-70	2.7	0	-35	-70	-105	-140
3	2	204.68	167.34	55.319	130	92.66	0	2.7	25	-60	-70	-130	-140
4	4	92.66	130	55.319	204.68	167.34	0	2.7	0	-35	-70	-105	-140
5	3	130	130	130	130	130	-70	0	25	-60	-70	-130	-140
6	7	130	130	130	130	130	-70	0	0	-35	-70	-105	-140
7	1	130	130	130	130	130	0	0	25	-10	-70	-80	-140
8	6	130	130	130	130	130	0	0	0	-35	-70	-105	-140

Table 12 Sample data from experiment of curving line with angle deviation

Four participants take the experiment and the experiment process is similar to the previous ones. The total F-ratios corresponding to the three factors of four participants are list below in Table 13.

Participant	Dev_A	Ref_A	Dev_D
1	1.82	20.55	19.6
2	0.19	22.04	9.09
3	1.95	168.94	62.51
4	1.06	26.05	40.64

Table 13 F-ratios result of three factors with angle deviation

In Table 13, all values of the Dev_A are less than $F_{0.025,1,8}$ which equal to 7.57, and F-rations of other two factors are larger than 7.57. It can be concluded that Ref_A and Dev_D are significant factors and Dev_A is not a significant factor. Therefore, factor Dev_A will be eliminated in the following experiments.

4.2.3 Factor of point count

At the beginning of curve reconstruction, two points are connected to form a curve, which has just one edge, then more and more points are connected to the curve and the characteristics of the curve change while the curve becomes longer and longer. Therefore, the point count in a curve must affect the characteristics of the curve.

To evaluate the effect of the points count (Pnt_C), two groups of experiments are designed and factor Pnt_C are set to different values in each experiments. In the first group of experiments, Pnt_C is set with (4, 8). In the second group Pnt_C is set with (3, 7). The curve and the reference point are created with the following factors and values in the first group of experiment, as shown in Table 14:

Factor	Values
Pnt_C	4, 8
Dev_D	2.7, 0
Ref_A	110, 180
Mea_A	150, 180

Table 14 Factors and values with points count equals (4, 8)

The experiment process is similar to the previous experiment processes and the ANOVA results are list below in Table 15.

Participant	Pnt_C	Ref_A	Dev_D	Mea_A
1	3.53	377.78	73.51	25.21

2	1.6	223.21	41.09	12.86
3	2.23	190.67	35.1	10.98
4	0.41	76.88	18.34	14.43

Table 15 F-ratios result of four factors with points count equals (4, 8)

In Table 15, all values of the Pnt_C are less than $F_{0.025,1,16}$ which equals to 6.12, and F-rations of other three factors are larger than 6.12. It can be concluded that Ref_A, Dev_D, and Mea_A are significant factors and Pnt_C is not a significant factor when the curve has more than 3 points.

Table 16 shows the result of the second group with Pnt_C equals to (3,7), Dev_D (0, 2), Ref_A (110, 180), Mea_A (130,180).

Participant	Pnt_C	Ref_A	Dev_D	Mea_A
1	31.28	120.97	8.29	32.34
2	43.43	114.59	13.4	16.08
3	20.73	93.42	7.73	26.99
4	24.19	95.93	10.04	1.93

Table 16 F-ratios result of four factors with points count equals (3, 7)

In Table 16, all values of the Pnt_C are larger than $F_{0.025,1,16}$ which equals to 6.12, and all F-rations of other three factors are larger than 6.12 except the Mea_A of the fourth participant, which can be treated as an outlier. It can be assumed that Pnt_C Ref_A, Dev_D, and Mea_A are significant factors when the curve has only 3 points.

From the analysis of points count, it can be concluded that when a curve has three points, the point count affect the connectivity value significantly, but when a curve has four points or more than four points, the change of the point count does not affect the connectivity value too much. However, it is inappropriate to trim the curve to four points curve because other characteristics of the curve also change when the curve extends.

4.3 Experiment of connectivity area

The experiment in this section is to find the max distances of reference points located at difference directions. From the analysis in the previous experiments, the factor Ref_A is the most significant factor, so it is necessary to set more values to this factor to make the experiment results more accurate. The factors and the values set to the factors in this part are listed below in Table 17.

Factor	Values
Ref_A	90, 135, 180, 225, 270
Dev_D	0.404, 0.5556, 0
Mea_A	135, 180
Pnt_C	2, 3, 4

Table 17 Factors and values for connectivity area

The factor Dev_D has three values because its value changes with the Pnt_C to make the curve deviation natural to participant's eyes. The experiment processes are similar to the previous ones, and the experiments result are shown in Table 18.

Seq	X1	X2	X3	X4	Y		Seq	X1	X2	X3	X4	Y
1	0.5	0	1	2	0.903846		24	1.25	0	1	3	1.276923
2	0.75	0	1	2	1.326924		25	1.5	0	1	3	1.026923
3	1	0	1	2	1.696154		26	0.5	0.5556	0.75	4	1.592308
4	1.25	0	1	2	1.296154		27	0.75	0.5556	0.75	4	1.826923
5	1.5	0	1	2	0.915385		28	1	0.5556	0.75	4	2.053847
6	0.5	0.404	0.75	3	1.657693		29	1.25	0.5556	0.75	4	1.284615
7	0.75	0.404	0.75	3	1.8		30	1.5	0.5556	0.75	4	1.065385
8	1	0.404	0.75	3	1.823077		31	0.5	0	0.75	4	1.523077
9	1.25	0.404	0.75	3	1.238461		32	0.75	0	0.75	4	1.703847
10	1.5	0.404	0.75	3	0.903846		33	1	0	0.75	4	1.696154
11	0.5	0	0.75	3	1.434616		34	1.25	0	0.75	4	1.134615
12	0.75	0	0.75	3	1.496154		35	1.5	0	0.75	4	1.007692

13	1	0	0.75	3	1.688462		36	0.5	0.5556	1	4	1.261539
14	1.25	0	0.75	3	1.2		37	0.75	0.5556	1	4	1.707693
15	1.5	0	0.75	3	0.8		38	1	0.5556	1	4	2.065385
16	0.5	0.404	1	3	1.169231		39	1.25	0.5556	1	4	1.465385
17	0.75	0.404	1	3	1.484616		40	1.5	0.5556	1	4	1.192308
18	1	0.404	1	3	2.042308		41	0.5	0	1	4	1.134616
19	1.25	0.404	1	3	1.311539		42	0.75	0	1	4	1.523077
20	1.5	0.404	1	3	1.188462		43	1	0	1	4	1.861539
21	0.5	0	1	3	1.080769		44	1.25	0	1	4	1.334616
22	0.75	0	1	3	1.384615		45	1.5	0	1	4	1.092308
23	1	0	1	3	1.742308							

Table 18 Experiment data of connectivity area

In Table 18, X1 means the Ref_A divided by 180 degrees; X2 means the deviation of the curve's edge lengths divided by average length of the curve edges; X3 means the Mea_A divided by 180 degrees; X4 means the Pnt_C; Y means average value of four participants' data divided by the average length of the curve edges. Here the average value of four participants is used instead of using data of each participant because the experiment want to explore the majority judgment of the people not just only one person.

The Levenberg-Marquardt algorithm (LMA) provides a numerical solution to the problem of nonlinear regression [45-47]. In this section, we analyze the geometrical properties of the curve and reference point and compare the regression results of many different formulas. Then the following formula is chosen and the parameters are calculated by using Auto2fit (version 3.0) [48], which implements LMA effectively.

$$\begin{aligned} Y = {} & m1+m2*\sin(X1*PI/2)+m3*(X2)^2+m4*X2+m5*\cos((X3-X1)*PI/2) \\ & +m6*\sin(X3*PI/2)+m7*(X4)^2+m8*X4 \end{aligned} \tag{5.1}$$

In Formula (5.1), X1 means the Ref_A divided by 180 degrees; X2 means the deviation of the curve's edge lengths divided by mean of edge lengths; X3 means the Mea_A divided by

180 degrees; X4 means the Pnt_C; Y means the max distance of the reference point. The iterative calculation result is list in Table 19, and the comparison of regression result and original data is shown in Figure 28.

Name	Value
Correlation Coefficient (R)	0.91729
R-Square (R^2)	0.84142
m1	0.94953
m2	0.67170
m3	-0.64486
m4	0.62895
m5	1.25812
m6	-1.45642
m7	0.008654
m8	0.027499

Table 19 Regression result of connectivity area

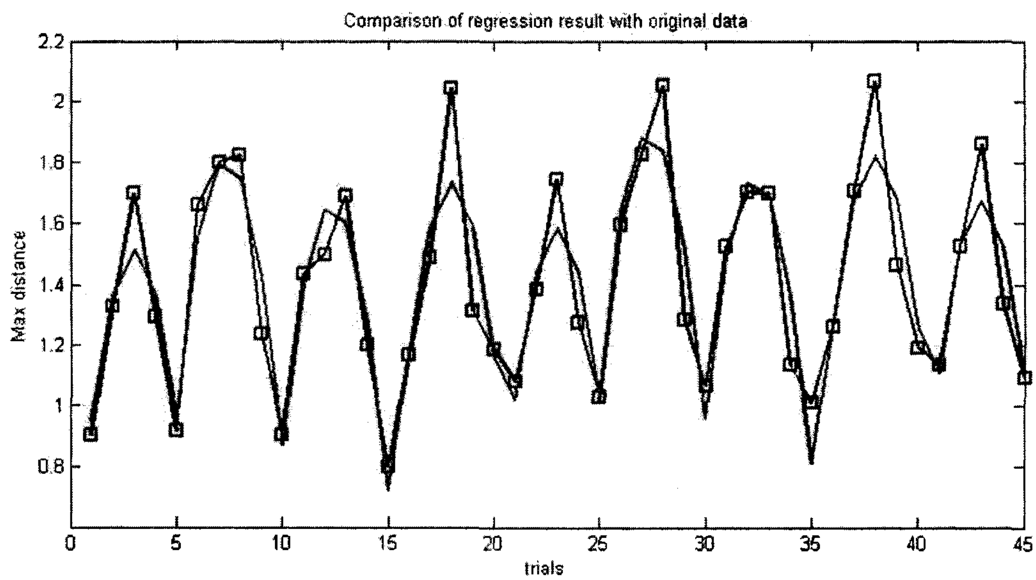


Figure 28 Comparison of regression result and original data for connectivity area

In Figure 28, the line with the small rectangle markers is the original data and the line without markers is regression result, from the figure, it can be assumed that the regression result is acceptable.

4.4 Experiment of connectivity value

In this part, the experiment is designed to find the connectivity value (*E*) of the different reference points in the curve's connectivity area. The input includes a curve, with a reference point located on various directions and different distance within the connectivity area. The output is the *E* of the reference point, and we get *E* by means of comparison of SRP with CRP mentioned in previous chapter. The factors and the values set to the factor are list below in Table 20 to get the *E* of the reference point.

Factor	Values
Ref_A	90, 135, 225, 270
Dev_D	0.404, 0.5556, 0
Mea_A	135, 180
Pnt_C	2, 3, 4
Ref_P	0.25, 0.5, 0.75, 1

Table 20 Factors and values for connectivity value

Ref_P means the position of the reference point to the max distance of that reference angle. We exclude the cases where Ref_A equals to 180 degrees because CRP and SRP are overlapped at the extension line if Ref_A equals to 180. The following formula is chosen and the experiments results are list below.

$$Y = m1+m2*x1^2+m3*x1+m4*\sin(x2*PI/2)+m5*x3^2+m6*x3 \qquad (5.2)$$

+m7*cos((x4-x2)*PI/2)+m8*x5^2+m9*x5

In Formula (5.2), X1 means the distance of reference point divided by mean of edge lengths. X2 means the Ref_A divided by 180 degrees; X3 means the deviation of the curve's edge lengths divided by mean of edge lengths; X4 means the Mea_A divided by 180 degrees; X5 means the Pnt_C; Y means the connectivity value of the reference point.

Name	Value
Correlation Coefficient (R)	0.95921
R-Square (R^2)	0.92009
m1	0.60381
m2	0.25396
m3	-1.09871
m4	0.35376
m5	-0.22202
m6	0.27243
m7	0.27791
m8	0.01845
m9	-0.09315

Table 21 Regression result of connectivity value

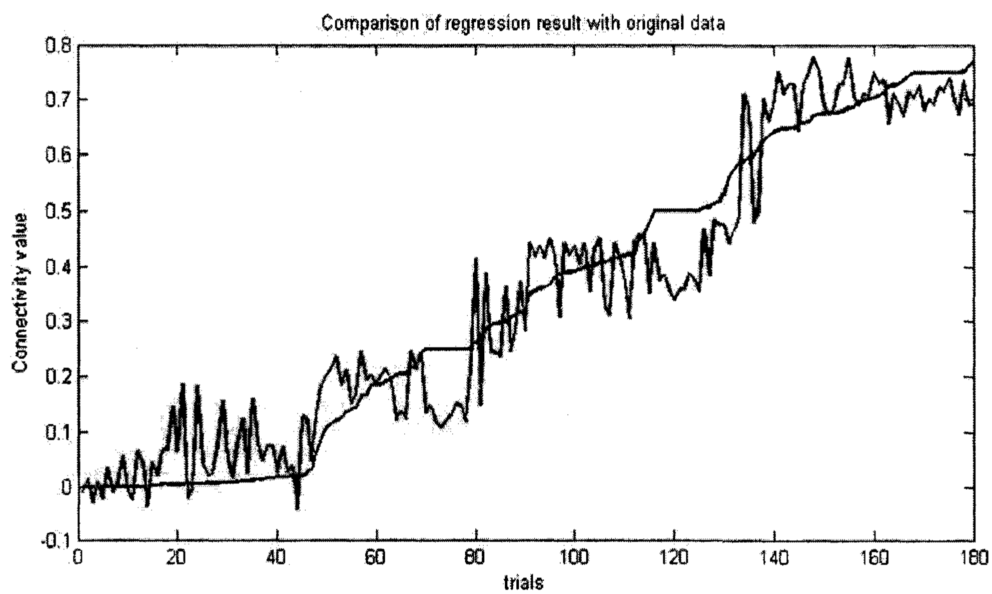


Figure 29 Comparison of regression result and original data for connectivity value

In Figure 29, the line ascending steadily is the original data and the line having a lot vibration is the regression result. From the figure, it can be assumed that the regression result is acceptable. Figure 30 shows that the error follows the normal distribution.

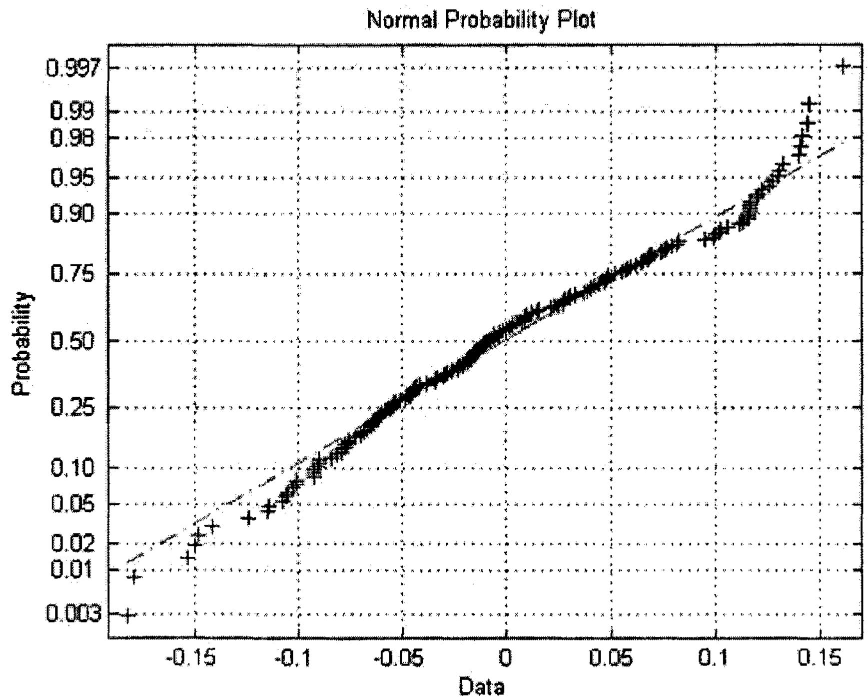


Figure 30 Residual from connectivity value

Chapter 5 Results and Comparisons

5.1 Results of visual perception based algorithm

In this part, the result of the visual perception based algorithm and comparisons with other algorithms are demonstrated. Three different points collections and reconstruction results are shown in Figure 31.

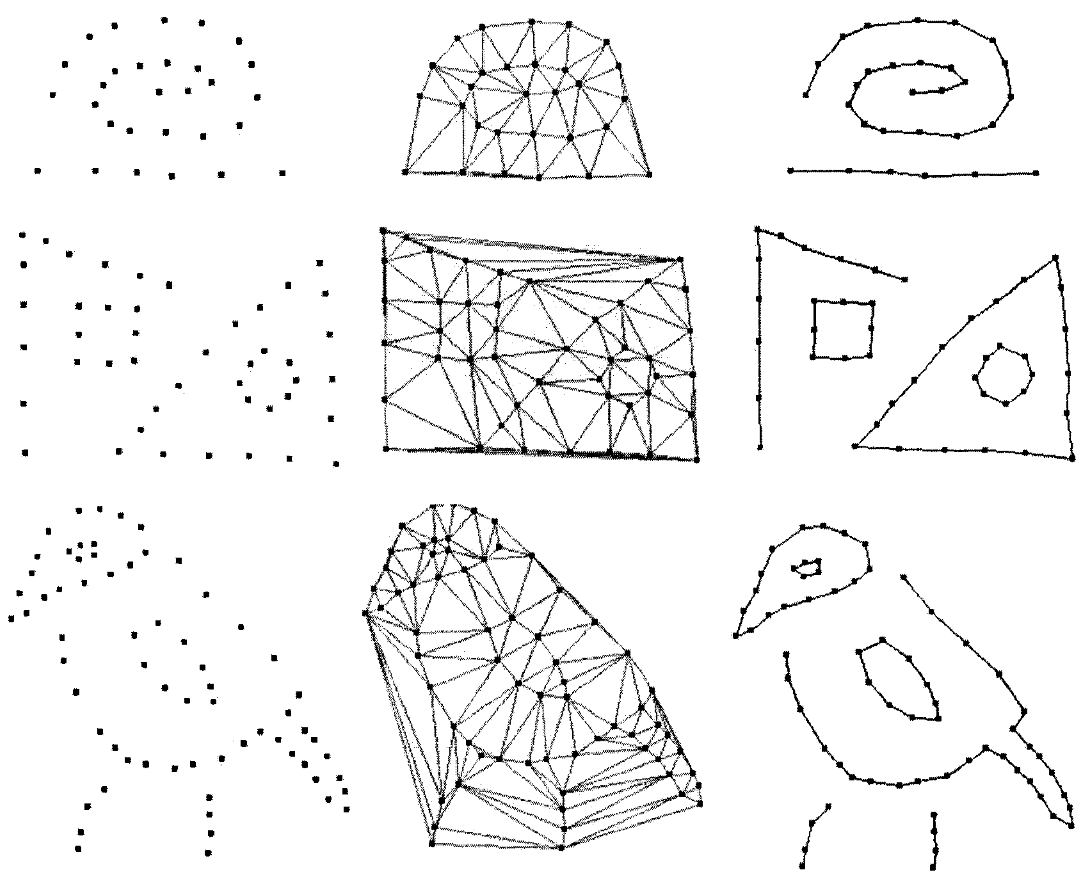
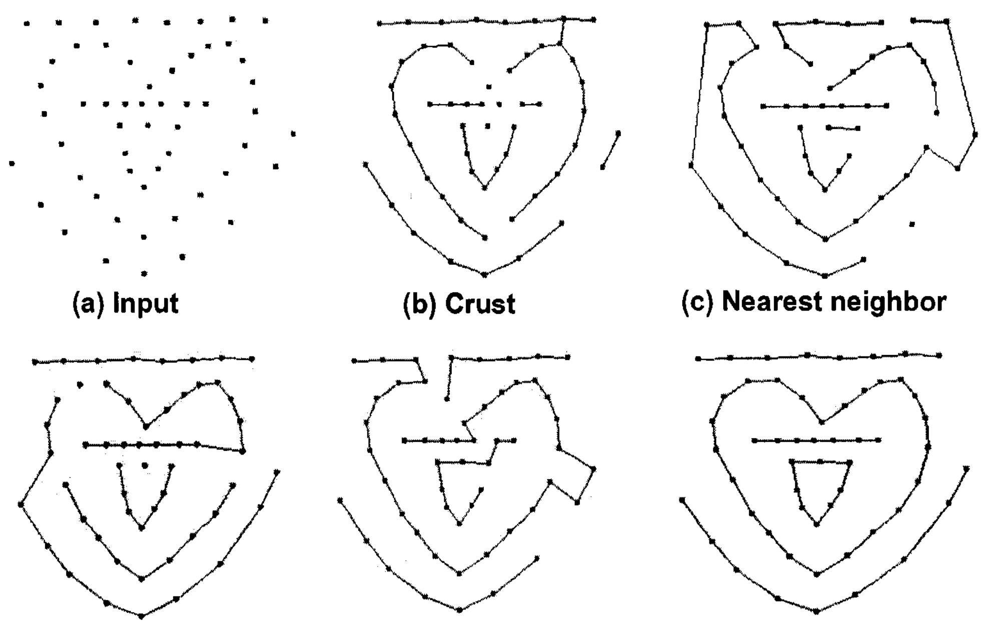


Figure 31 Reconstruction result of visual perception based algorithm

The Delaunay triangulation of each point set is shown in middle column in Figure 31. It is easy to find that visual perception based algorithm is natural to human eyes.

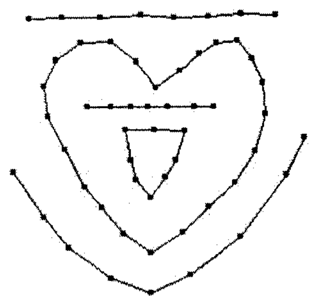
5.2 Comparison with existing algorithms

We create several set of points collections and use Curst, Nearest neighbor, Gathan, DISCUR, VICUR, and visual perception based algorithm to group the individual points. The first dot-patterns (points collection) are a complicated pattern, which include sharp corners, open curves and close curves. The construction results of different algorithms are illustrated in Figure 32. VICUR shows a reasonable connection, but it is required to set its parameter in the range of [0.2, 0.9]. The comparison shows that visual perception based algorithm constructs the individual points as the human perceive them, and the construction result is natural to human eyes.



(a) Input (b) Crust (c) Nearest neighbor

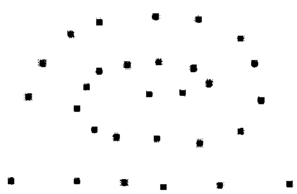
(d) Gathan (e) DISCUR (f) VICUR



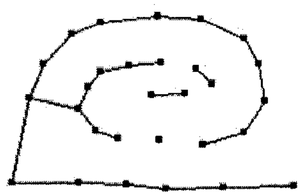
(g) Visual Perception

Figure 32 Comparison of connection results for complicated patterns

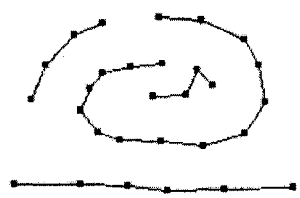
The second dot-patterns include a spiral line and a straight line. The construction results of different algorithms are illustrated in Figure 33. The comparison shows that construction result of visual perception based algorithm is natural to human eyes.



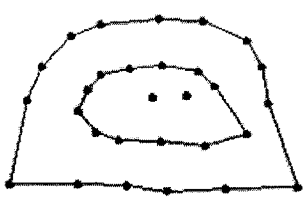
(a) Input



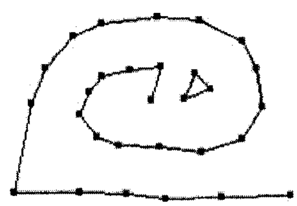
(b) Crust



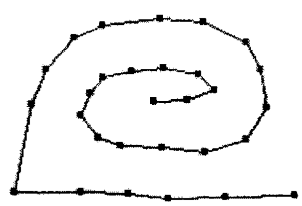
(c) Nearest neighbor



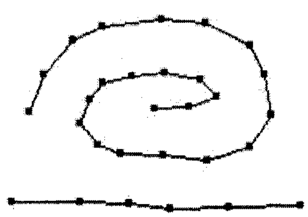
(d) Gathan



(e) DISCUR



(f) VICUR



(g) Visual Perception

Figure 33 The comparison of connection results for spiral line

The third dot-patterns include one simple sharp corner. The construction results are illustrated in Figure 34. The comparison shows that Gathan construct the points collection into a closed curve, VICUR and visual perception based algorithm results are natural to human eyes.

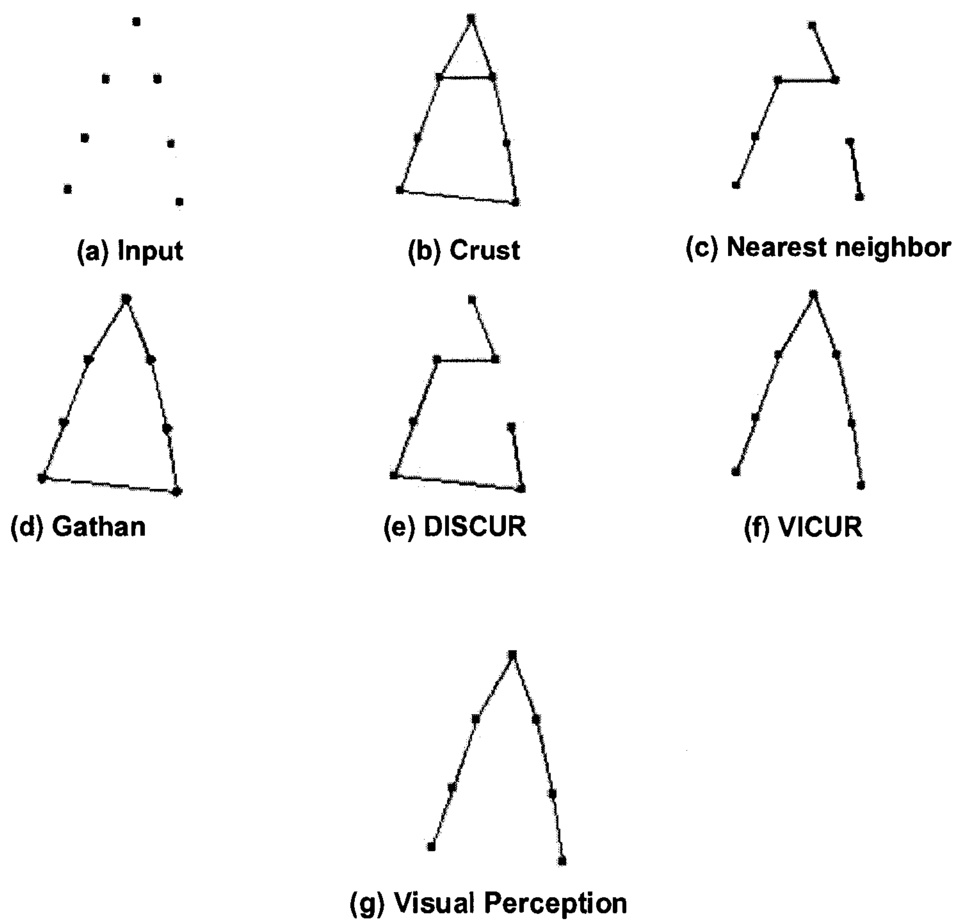


Figure 34 Comparison of connection results for sharp corner

The fourth dot-patterns include three vertical lines and one horizontal line. This kind of pattern is used to test the algorithm's ability to distinguish the boundaries and components of the points collection. The construction results are illustrated in Figure 35. The comparison shows that visual perception based algorithm result is natural to human eyes.

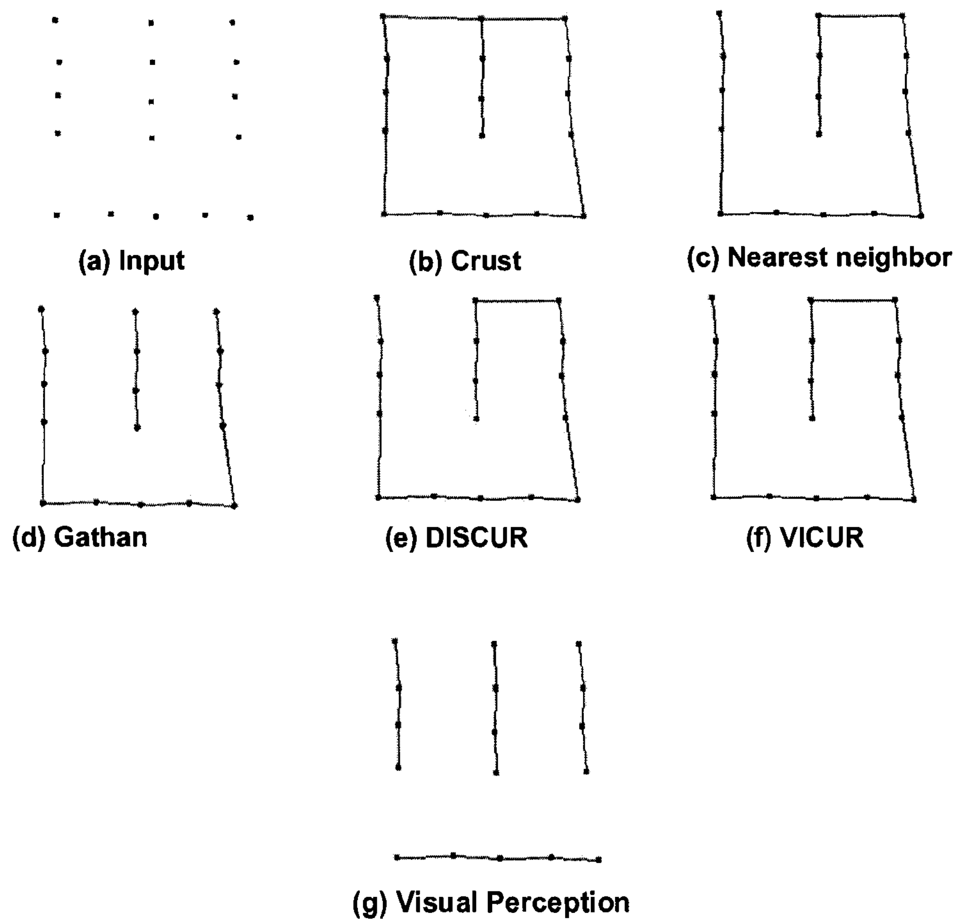
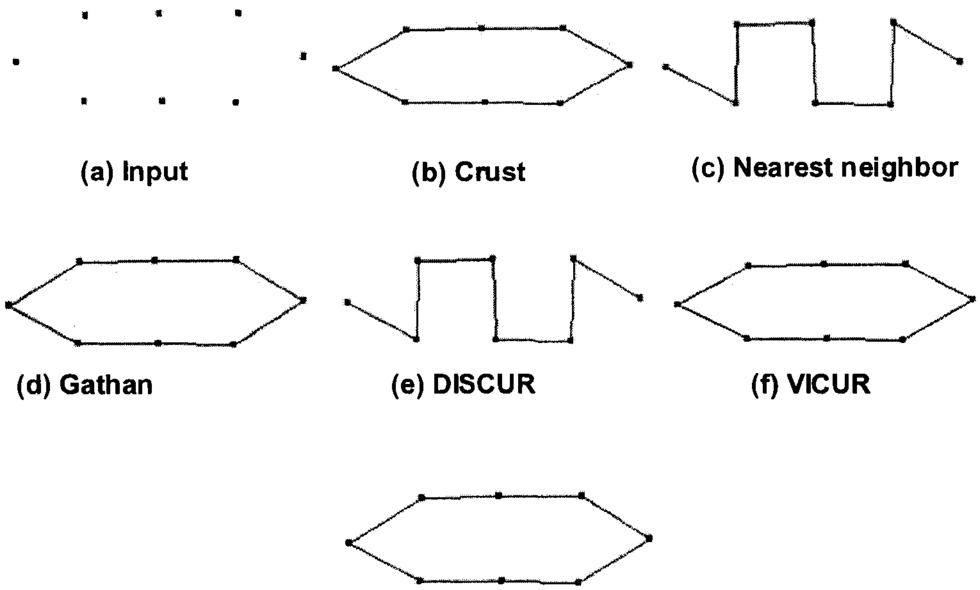


Figure 35 Comparison of connection results for boundaries and components

The fifth dot-patterns are a symmetric pattern. This pattern is used to test the algorithm's ability to handle the cases of the same distance among the points. The construction results are illustrated in Figure 36. The comparison shows that construction result of Gathan, VICUR and visual perception based algorithm is natural to human eyes.



(a) Input (b) Crust (c) Nearest neighbor

(d) Gathan (e) DISCUR (f) VICUR

(g) Visual Perception

Figure 36 Comparison of connection results for symmetric pattern

The sixth dot-patterns include a noisy point and a regular shape. This pattern is used to test the algorithm's ability to handle noisy points. The construction results are illustrated in Figure 37. VICUR can get the same result as Visual perception by set the smooth parameter less than 0.7. The comparison also shows that visual perception based algorithm result is natural to human eyes.

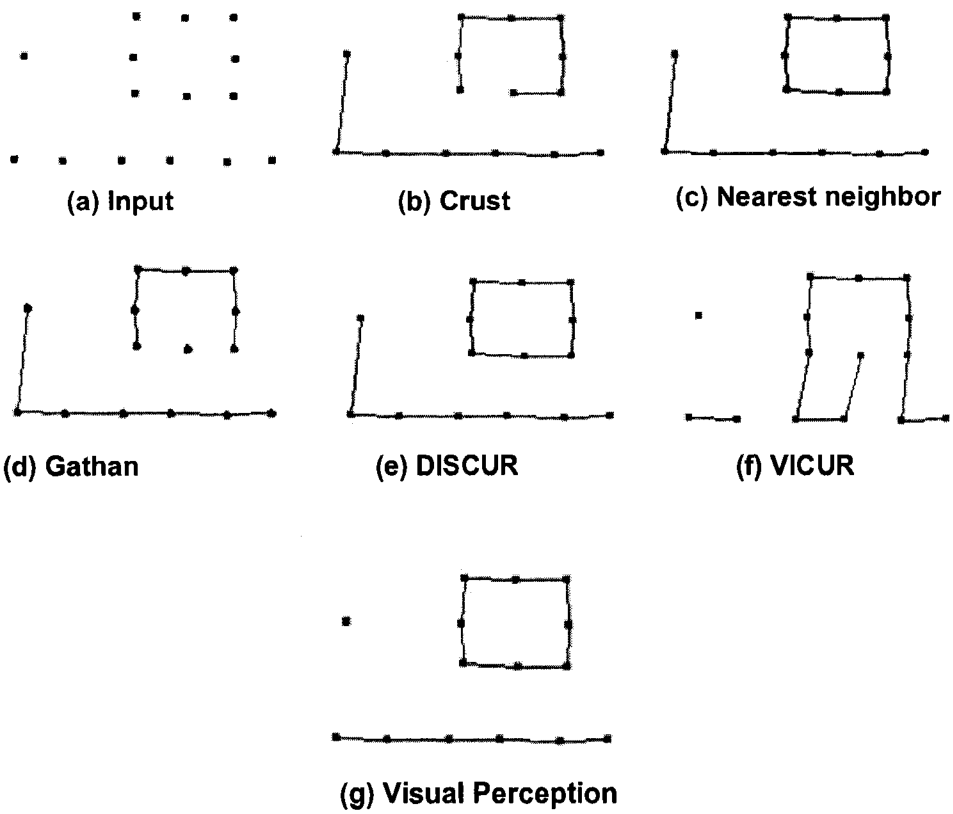


Figure 37 Comparison of connection results for spiral line

Based on all the comparisons above, we can conclude that the visual perception based algorithm for curve reconstruction is an effective and intuitive algorithm. We can use this algorithm in any situations where there are individual elements in the ambient environment. It can automatically construct sharp corner, distinguish boundaries and multiple components from the points collection. Compare to VICUR, which requires a parameter to determine the smoothness of the curve, visual perception based curve reconstruction algorithm, is a totally parameter-free algorithm. With this algorithm, construction of curve components from points collection with multiple features is worked follow the process as human perceives them.

However, this experiment only focuses on the basic segregation and connection problem of constructing patterns from points collection. It does not consider the complicated problems such as the branching, intersection, etc. Furthermore, the experiment processes require a lot of people and thus a long time to finish, and the inaccurate judgments incurred by the fatigue of the participants can affect the experiment result negatively. New methods to overcoming these shortcomings will be used in future experiments.

Chapter 6 Conclusions and Future Work

The purpose of this thesis to quantify and formulate the visual perception's characteristics of proximity and good continuation involved in the process of grouping the environment objects. Human beings perceive the environment objects by segregated groups on some occasions, or by one integrated object in other circumstances. This perceptual process is characterized by low-level and intermediate-level vision retrieved from the image at the beginning of the visual perception. Our research based on the phenomenon that human tends to group the individual elements together to form shapes or patterns natural to human eyes. To do research on the grouping property of Gestalt laws, we represent the environment objects with a set of points, and found two observations that are similar to the Gestalt laws of proximity and continuity in the curve reconstruction: 1) two closest neighbors tend to be connected, and 2) sampling points tend to be connected into a smooth curve. We use curve reconstruction as the test case to simulate the perceptual environment. In the experiment, the methods of DOE and statistics are used to exclude insignificant factors and keep major factors in several rounds of experiments. Finally, multivariable nonlinear regression model is applied to calculate the coefficients of the perceptual formula. The result shows that the algorithm can construct the points collection intuitively, and the constructed pattern are natural to human eyes, thus effectively formulate the relationships between proximity and good continuation.

In the future, researchers can design some dot-patterns, and asks the participants to connect the points to create patterns based on their perception, and then use software to extract the connection information from patterns and build the construction formula. Further work can also be focused on human perception combining with human experience and knowledge in some special domain. Furthermore, the EEG and Eyegaze systems can be used in the experiment to research the mechanism of visual perception.

Publications

Publications in Refereed Conferences

1. **Guang Qing He**, Jie Jin, A. Ben Hamza, Yong Zeng (2007), Using DOE Method to Determine the Precision of a 3D Scanner, *Flexible Automation and Intelligent Manufacturing'07*, Philadelphia, USA, June 18-20, 2007.

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Appendix: Experiment Steps of Connectivity Area

We list the experiment steps involved in connectivity area as an example to show how we do the experiment.

1. Set the following factors with corresponding values to the software for connectivity area: Ref_A (90, 135, 180, 225, 270), Dev_D (0.404, 0.5556, 0), Mea_A (135, 180), Pnt_C (2, 3, 4). Then run the software to generate a set of sample data and sort the data randomly.

2. Four participants are required to do the experiments. Each participant is assigned with the same sample data. The participant runs the software to demonstrate the curve and the reference point based on the data generated in step 1. There are 45 samples for each participant. In each sample, the participant is required to adjust the distance of the reference point along certain direction until he or she think the reference point reaches the largest distance while the reference point still can be connected to the curve. Then the participant clicks the "OK" button to continue the experiment on the next sample.

3. After all the participants finished their experiments (each participant do the experiment twice), the researcher collects the data and use ANOVA to analyze the data. Based on the ANOVA result, the researcher can find out which factors are significant factors and which factors are insignificant factors.

4. The researcher use Microsoft Excel to calculate the average values of four participants, and relate it with data generated in step 1. Now the research can get the data matrix of [45×5], number 45 means the number of samples, and number 5 means the four factors and one result.

The first four columns are corresponding to four factors of Ref_A, Dev_D, Mea_A, and Pnt_C. The last column is the result that means the max distance of reference point.

5. Put the matrix in the software Auto2fit, and define proper formula and coefficients for the matrix based on the properties of the factors and logical relationships between factors. Run the software to get the coefficients value. Try different formulas and select the one with highest correlation coefficient value.