EFFICIENT PROVISIONING OF
PASSIVE OPTICAL NETWORKS

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Abstract

Efficient Provisioning of
Passive Optical Networks

Ali Asghar Shaikh

Passive Optical Networks (PONs) have evolved to provide much higher bandwidth in access networks. They involve low cost equipment, fiber infrastructure, maintenance, and appear to be one of the best candidates for a major component of the next-generation access networks. A PON is capable of delivering bundled voice, data, video and other multimedia services. It is a point-to-multipoint optical network, where an optical line terminal, located at the central office, is connected to multiple ONUs through some intermediate nodes equipped with either optical splitters or array waveguide gratings. In this thesis, we have focused on PONs with splitters only and have developed different optimization models for multicasting and wavelength assignment, i.e., for efficient provisioning of PONs. We consider two different objectives: Maximizing the weighted number of granted requests and maximizing the weighted number of served ONU's under the condition of a limited number of wavelengths. Variants include combinations of the two objectives in order to capture a cost/revenue concern through appropriate definitions of the weights. We developed different models dealing with either a tree or a light mesh topology as well as with traffic-grooming, i.e., Time Division Multiple Access (TDMA) or without traffic-grooming. Models correspond
to integer linear programs and for some of them, we have favored a column generation modeling in order to solve the linear programming relaxation and obtain a scalable solution process. Simulations have been conducted to validate all the models (with/with traffic-grooming). We have also evaluated the performance of the traffic-grooming models to observe the grade of service in terms of the number of served ONUs, bandwidth, bandwidth usage/waste, as well as efficient provisioning of PONs, under three different traffic scenarios for the objective of maximizing the weighted number of ONUs.
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I dedicate my thesis to parents, who always pray and always wishing me well in my career.
Contents

List of Figures .................................................. x

List of Tables .................................................. xii

1 Introduction .................................................. 1
   1.1 Motivation .................................................. 1
   1.2 Contributions ............................................. 2
   1.3 Plan of Thesis ............................................. 3

2 Definitions and Literature Review ....................... 4
   2.1 Passive Optical Networks (PONs) ....................... 5
      2.1.1 Wavelength Division Multiplexing (WDM) PONs .. 7
      2.1.2 PON Standards ....................................... 9
   2.2 PON Equipment .......................................... 11
      2.2.1 Optical Fiber ....................................... 11
      2.2.2 Optical Line Terminal (OLT) ....................... 13
      2.2.3 Optical Splitter .................................... 14
      2.2.4 Array Waveguide Grating (AWG) ................... 15

vii
2.2.5 Optical Network Unit (ONU) ........................................... 16

2.3 PON Topologies and Layout .......................................... 17

2.3.1 PON Physical Topologies ........................................... 17

2.3.2 PON Logical Topology ............................................. 20

2.3.3 PON Layout/Architecture .......................................... 21

2.4 PON Survivability ...................................................... 22

2.4.1 Protection ............................................................. 24

2.4.2 Restoration ........................................................... 24

2.4.3 Survivable WDM-PON Architecture .............................. 25

2.5 Multicast Routing and Wavelength Assignment .................. 30

2.5.1 Network Planning and Traffic Assumptions .................... 32

2.5.2 Previous Mathematical Models on WDM Core Networks ...... 35

3 MC-WA Optimization Models for PON Networks .................. 38

3.1 Motivation ............................................................. 39

3.2 Column Generation Overview ....................................... 41

3.3 Assumptions and Physical Constraints ............................ 42

3.4 Definitions and Notations ........................................... 43

3.4.1 Network Topology .................................................. 43

3.4.2 Traffic ............................................................... 44

3.4.3 Cost/Benefit ......................................................... 47

3.5 Maximizing the Weighted Number of Granted Requests ........ 48

3.5.1 MC-WA on WDM-PON Tree Network Topology ................. 50

3.5.2 MC-WA on WDM-PON Light Mesh Network Topology .......... 52
4 Simulation

4.1 Data Instances ............................................................. 86
  4.1.1 Network Topology Instances .................................. 87
  4.1.2 Traffic Instances ............................................... 89

4.2 Implementation of Models ........................................... 92
  4.2.1 Data Structures ................................................. 92
  4.2.2 A Heuristic Algorithm ........................................ 96
  4.2.3 Cost ............................................................ 97

4.3 Simulation ............................................................ 98
  4.3.1 Model Validations ............................................... 98
  4.3.2 PON Performance Evaluation .................................. 99

5 Conclusion and Future Work ........................................... 108
  5.1 Conclusion ........................................................ 108
  5.2 Future Work ....................................................... 110

Bibliography ............................................................... 111
List of Figures

1   (a) Multiple unicast routing, (b) Multicast routing (taken from [46]) .... 5
2   A PON diagram ............................................................... 6
3   Commulative global growth of FTTx for the years 2005-2012 (taken from [33]) 7
4   An Optical fiber (taken from [21]) ..................................... 12
5   A 1:4 fixed power optical splitter ...................................... 14
6   An array waveguide grating ............................................. 16
7   A PON tree topology ..................................................... 18
8   A PON light mesh topology ............................................. 19
9   Unicast vs. multicast (taken from [46]) ............................. 20
10  A PON architecture ..................................................... 21
11  A home and remote node model (taken from [5]) ................... 23
12  A star-ring architecture (taken from [43]) .......................... 26
13  A 1:1 protection schemes: Taken from [39] for (a) and [40] for (b) ..... 27
14  A 1:1 protection schemes: (Taken from [47]) ........................ 28
15  A 1:N protection scheme (taken from [38]) .......................... 29
16  An interconnection fibers among ONUs (taken from [38]) .......... 29
17  A 1+1 protection scheme (taken from [45]) .......................... 31
<table>
<thead>
<tr>
<th>Page</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>A PON with 4 ONUs</td>
</tr>
<tr>
<td>19</td>
<td>Traffic examples originated at the OLT</td>
</tr>
<tr>
<td>20</td>
<td>Traffic examples originated at the ONU</td>
</tr>
<tr>
<td>21</td>
<td>A tree topology example</td>
</tr>
<tr>
<td>22</td>
<td>How to solve a column generation formulation</td>
</tr>
<tr>
<td>23</td>
<td>A light mesh topology example</td>
</tr>
<tr>
<td>24</td>
<td>A 32-ONU tree topology</td>
</tr>
<tr>
<td>25</td>
<td>A 32-ONU light mesh topology</td>
</tr>
<tr>
<td>26</td>
<td>Traffic load</td>
</tr>
<tr>
<td>27</td>
<td>A Class diagram</td>
</tr>
<tr>
<td>28</td>
<td>Bandwidth on the feeder fibers</td>
</tr>
<tr>
<td>29</td>
<td>An effective bandwidth usage on the distribution fibers</td>
</tr>
<tr>
<td>30</td>
<td>bandwidth waste on the distribution fibers</td>
</tr>
<tr>
<td>31</td>
<td>Grade of service (GoS)</td>
</tr>
</tbody>
</table>

40 45 45 64 68 73 87 88 92 94 105 105 106 107
# List of Tables

1. Standard diameters of optical fibers ........................................... 12
2. A wavelength assignment for interconnection among ONU\(s\) (taken from [38]) 30
3. Traffic for tree network .......................................................... 65
4. A few generated configurations for Example 2 .............................. 65
5. Traffic for light mesh network topology ........................................ 74
6. A few generated configurations for Example 3 ................................ 74
7. An experimental results of the tree network topology ....................... 103
8. An experimental results of the light mesh network topology .............. 104
### Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>10GEPON</td>
<td>10 Gigabit Ethernet Passive Optical Network</td>
</tr>
<tr>
<td>APON</td>
<td>Asynchronous Transfer Mode Passive Optical Network</td>
</tr>
<tr>
<td>APS</td>
<td>Automatic Protection Switching</td>
</tr>
<tr>
<td>AWG</td>
<td>Array Waveguide Grating</td>
</tr>
<tr>
<td>BPON</td>
<td>Broadband Passive Optical Network</td>
</tr>
<tr>
<td>BRAND</td>
<td>The Broadband for Rural and Northern Development Pilot Program</td>
</tr>
<tr>
<td>CATV</td>
<td>Communication Antenna Television</td>
</tr>
<tr>
<td>CO</td>
<td>Central Office</td>
</tr>
<tr>
<td>CWDM</td>
<td>Coarse Wavelength Division Multiplexing</td>
</tr>
<tr>
<td>DBA</td>
<td>Dynamic Bandwidth Allocation</td>
</tr>
<tr>
<td>DSL</td>
<td>Digital Subscriber Line</td>
</tr>
<tr>
<td>EPON</td>
<td>Ethernet Passive Optical Network</td>
</tr>
<tr>
<td>FSAN</td>
<td>Full Service Access Network</td>
</tr>
<tr>
<td>FTTH</td>
<td>Fiber-To-The-Home</td>
</tr>
<tr>
<td>FTTB</td>
<td>Fiber-To-The-Building</td>
</tr>
<tr>
<td>GEAPON</td>
<td>Gigabit Ethernet Passive Optical Network</td>
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<td>GPON</td>
<td>Gigabit Passive Optical Network</td>
</tr>
<tr>
<td>HDTV</td>
<td>High Definition TeleVision</td>
</tr>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
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<tr>
<td>ILP</td>
<td>Integer Linear Program/Integer Linear Programming</td>
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<tr>
<td>IP</td>
<td>Internet Protocol</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>ITU</td>
<td>International Telecommunication Union</td>
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<tr>
<td>LAN</td>
<td>Local Area Network</td>
</tr>
<tr>
<td>LP</td>
<td>Linear Program/Linear Programming</td>
</tr>
<tr>
<td>MC</td>
<td>Multicast Capable</td>
</tr>
<tr>
<td>MC-GRWA</td>
<td>Multicast Routing and Wavelength Assignment with Grooming</td>
</tr>
<tr>
<td>MC-GWA</td>
<td>Multicasting and Wavelength Assignment with Grooming</td>
</tr>
<tr>
<td>MC-OXC</td>
<td>Multicast Optical Cross-Connect</td>
</tr>
<tr>
<td>MC-RWA</td>
<td>Multicast Routing and Wavelength Assignment</td>
</tr>
<tr>
<td>MC-WA</td>
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</tr>
<tr>
<td>MI</td>
<td>Multicast Incapable</td>
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<tr>
<td>MMF</td>
<td>Multi-Mode Fiber</td>
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<tr>
<td>NE</td>
<td>Network Engineering</td>
</tr>
<tr>
<td>NP</td>
<td>Network Planning</td>
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<tr>
<td>NTT</td>
<td>Nippon Telegraph &amp; Telephone Corporation</td>
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<tr>
<td>ODN</td>
<td>Optical Distribution Network</td>
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<tr>
<td>OEO</td>
<td>Optical-Electrical-Optical</td>
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<tr>
<td>OLT</td>
<td>Optical Line Terminal</td>
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<tr>
<td>ONU</td>
<td>Optical Network Unit</td>
</tr>
<tr>
<td>OXC</td>
<td>Optical Cross-Connect</td>
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<tr>
<td>PON</td>
<td>Passive Optical Network</td>
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<tr>
<td>S&amp;D</td>
<td>Splitter-and-Delivery</td>
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<tr>
<td>SDTV</td>
<td>StanDard TeleVision</td>
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<tr>
<td>SMF</td>
<td>Single-Mode Fiber</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
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<td>---------</td>
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<tr>
<td>TDMA</td>
<td>Time Division Multiple Access</td>
</tr>
<tr>
<td>TDM</td>
<td>Time Division Multiplexing</td>
</tr>
<tr>
<td>TE</td>
<td>Traffic Engineering</td>
</tr>
<tr>
<td>TPS</td>
<td>Triple Play Service</td>
</tr>
<tr>
<td>VDSL</td>
<td>Very-high-bit-rate Digital Subscriber Line</td>
</tr>
<tr>
<td>VoD</td>
<td>Video on Demand</td>
</tr>
<tr>
<td>WAN</td>
<td>Wide Area Network</td>
</tr>
<tr>
<td>WDM</td>
<td>Wavelength Division Multiplexing</td>
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</table>
Chapter 1

Introduction

In this section we introduce the problem our thesis and my contribution in the this thesis.

In the paragraph, we explain the organization of the thesis.

1.1 Motivation

The next generation multimedia applications such as online games, Video on Demand (VoD), High Definition TeleVision (HDTV), and video conferences require very high bandwidth which, in addition, is expected to be multiplied for each generation technology. Providing a cost effective network to the subscribers, is a major challenge for service providers, with the additional feature of efficiently provisioning multicasting services in access networks. By using multicast techniques, we can save costs on bandwidth and on network in terms of optical fiber and other equipment deployment as well as in the network maintenance. Multicast means sending throughout a network, as much as possible, a single copy of data in terms of bandwidth requirement, to multiple users. Passive Optical Networks (PONs) are one of the most extreme networks in subscriber access networks. A PON is
comprised of optical components which provide cost effective huge bandwidth. A PON is a point-to-multipoint optical network, where an Optical Line Terminal (OLT) at the central office is connected to many Optical Network Units (ONUs) through one or multiple optical splitter/array waveguide grating nodes as intermediate nodes. The network between the OLT and the set of ONUs is passive, i.e., it does not require any power supply.

We have developed new mathematical models for the objective of either maximizing the weighted number of granted requests or maximizing the weighted number of served ONUs on the available set of wavelengths (with given transport capacities) of each link for tree and light mesh topologies with/without traffic-grooming. When maximizing the weighted number of granted requests, we either accept or reject the whole request (service) for all its destinations and when maximizing the weighted number of served ONUs, we may grant partially multicast requests, i.e., all or only a subset of the ONU destinations are served for each request. All models correspond to integer linear programs.

We have worked on two objectives; the first one (maximizing the weighted number of requests) is for the aggregated traffic usually used by network providers to split the available bandwidth among different service providers. The second one (maximizing the weighted number of ONUs) is used by the service providers to assign the requested bandwidth to individual subscribers/users in order to maximize their revenues.

1.2 Contributions

The objective of this thesis was to design tools for efficient provisioning of WDM-PON networks with multimedia traffic. For this reason, we develop optimization models for the
multicasting and wavelength assignment (MC-WA) in PON networks with/without traffic-grooming for tree and light mesh network topologies. All models have been solved using CPLEX and its Concert C++ library using object-oriented programming techniques. Simulations have been conducted to validate all the models (with/without traffic-grooming). We have also evaluated the performance of the traffic-grooming models to observe the grade of service in terms of served ONUs, bandwidth, bandwidth usage/waste, as well as efficient provisioning of PONs, under three different traffic scenarios for the objective of maximizing the weighted number of ONUs.

1.3 Plan of Thesis

The thesis is organized into five chapters. Chapter 1 introduces the reader to the problem of efficient provisioning in PONs. Chapter 2 describes PON networks and their equipment. It also contains a description of PON topologies, and their layout as well as a literature review on multicast routing and wavelength assignment in WDM-PON networks. Chapter 3 describes the new proposed optimization models for Multicasting and Wavelength Assignment without traffic-grooming (MC-WA) and Multicasting and Wavelength Assignment with traffic-grooming (MC-GWA). Chapter 4 discusses the implementation of the models and their validation. All the models have been validated. A performance, of the traffic-grooming models, have been evaluated, in terms of served ONUs, bandwidth, bandwidth usage/waste, to find the best way for efficient provisioning of PONs on different traffic scenarios. We have also discuss the results of the experiments. Finally, Chapter 5 concludes and discusses avenues for future work.
Chapter 2

Definitions and Literature Review

As the development of multiuser applications such as video conferences, HDTV, video on demand and online games continues to increase, it becomes crucial to provision efficiently multicast requests in order that it is profitable in terms of bandwidth efficiency and network costs. This led to the study of the Multicast Routing and Wavelength Assignment (MC-RWA) problem with/without traffic-grooming.

Multicast means one-to-many or many-to-many communications. In this thesis, we will only consider one-to-many communications. Multicast is implemented to avoid sending a separate copy to each receiver, as shown in Figure 1(a). One-to-many multicast is implemented using a multicasting tree, in which the root represents the multicast source. Following the links in the tree, the same multicast message is transmitted only once whenever it is possible, as shown in Figure 1(b). Multicast trees are computed by multicast routing algorithms. There are two or three basic issues with respect to multicast provisioning in wavelength-routed networks (WDM): the routing problem, the wavelength assignment problem and possibly the traffic-grooming problem.
2.1 Passive Optical Networks (PONs)

The access network, also known as the first-mile or the last mile network, connects the service provider Central Offices (COs) to residential and business subscribers, which are dominant in an access network. It is also referred as subscriber access network or local loop [2]. The bandwidth is available through, e.g., Digital Subscriber Line (DSL), Community Antenna TeleVision (CATV) and Very-high-bit-rate DSL (VDSL), is limited as compared to the demand of nowadays subscribers for video on demand or HDTV.

A PON is a point-to-multipoint optical network in downstream direction and multipoint to-point in upstream direction, where an Optical Line Terminal (OLT) at the Central Office (CO) is connected to many Optical Network Units (ONUs) at remote nodes through one or multiple optical splitters or Array Waveguide Gratings (AWGs). The network between the OLT and the ONU is passive, i.e., it does not require any power supply. The PON technology uses a multiple-star architecture or, in other words, a tree architecture; the first star topology is centered at the OLT, and the other ones at the optical splitter levels. Most often, there is only one splitter level, as shown in Figure 2.
The term passive optical network mostly refers to a set of links connecting an OLT to a set of ONUs, including fibers and optical splitters/array waveguide gratings, as shown in Figure 2.

Many telecom operators are considering to deploy PONs using a fiber-to-the-x (FTTX) model (where $x =$ building (B), curb (C), home (H), premises (P), etc) in order to provide Triple Play Services (TPSs), i.e., voice, data and video, at a cheaper subscription cost than if each service was deployed separately by different providers.

PONs are at their initial stages of deployment in many parts of the world [34]. PONs are deployed in few countries, and are widely used in Asia where service providers offer TPSs. In the future, each customer may demand other services such as, e.g., video on demand, online games or HDTV.
At the end of 2007, there were nearly 29 million subscribers connected with FTTx infrastructure worldwide. Most of the subscribers are receiving service via Fiber-To-The-Home (FTTH) or Fiber-to-The-Building (FTTB). The global growth of FTTx for the years 2005-2012 is shown in Figure 3. The growth is expected to continue at a very fast pace with the number of FTTx subscribers expected to grow to over 100 million by the end of 2012 [33].

![Cumulative FTTx](image)

Figure 3: Cumulative global growth of FTTx for the years 2005-2012 (taken from [33])

### 2.1.1 Wavelength Division Multiplexing (WDM) PONs

Recent developments in optical technologies have made the realization of Wavelength Division Multiplexing Passive Optical Networks (WDM-PONs) feasible and cost effective. Wavelength Division Multiplexing supports the propagation of multiple laser beams through a single optical fiber link provided that each laser beam uses a distinct optical wavelength. Multiple wavelengths are multiplexed on a single optical fiber through Coarse Wavelength Division Multiplexer (CWDM).

Traditional single-wavelength PONs, also referred as Time-Division-Multiplexed PONs
(TDM-PONs), combine the high capacity provided by optical fibers with the low installation and maintenance cost of a passive infrastructure. As a consequence, the number of ONU\(s\) is limited because of the optical splitter attenuation and working bit rate of the transceivers in the Central Office (CO). The number of ONU\(s\) in TDM-PON depends on the distance, and on the splitting ratio of the splitters, e.g., for 32 ONU\(s\) at maximum distance of 20 km or 64 ONU\(s\) at maximum of 10 km from the OLT.

A WDM-PON solution provides scalability because it can support multiple wavelengths over the same optical fiber. In other words, different wavelengths may be used to support different independent PON subnetworks, all operating under the control of a single OLT over the same optical fiber infrastructure [2].

Two key network functions of an OLT are: Transmitting signals to ONU\(s\) and receiving signals from ONU\(s\). To control the signals coming from different ONU\(s\) on the same wavelength, the simplest method is TDMA, wherein each user transmits information within a specific assigned time slot at a prearranged data rate. However, this does not make efficient use of the bandwidth available since, many times, slots will be empty when one or several subnetwork users do not have information to be sent back to the central office. A more efficient process is Dynamic Bandwidth Allocation (DBA), wherein time slots of an idle connection or bandwidth of low-utilization users are dynamically reassigned to more active customers [16], see also, e.g., [14], [42] for specific algorithms.

In WDM-PONs, each optical fiber has multiple bidirectional wavelengths. Transmissions on different wavelengths in the same link do not interfere with each other. Each wavelength can be used either upstream (from ONU\(s\) to CO) or downstream (from CO to ONU\(s\)).
Deploying WDM-PONs adds a new dimension to TDM-PONs. The benefits of the WDM wavelength dimension are:

- To increase the network capacity;
- To improve the network scalability by accommodating more end users;
- To separate the services;
- To separate the service providers.

2.1.2 PON Standards

PON standardization has began in the 1990s when carriers anticipated fast growth in bandwidth demands. In 1995, the Full Service Access Network (FSAN) consortium was formed by seven global telecommunication operators including British Telecom and Bell South to standardize common requirements and services for passive optical network systems. FSAN recommendations were later adopted by the International Telecommunication Union (ITU) [9]. PON has different flavors, e.g., APON, BPON, GPON, EPON/GEPON, 10GEPON.

APON (ATM Passive Optical Network): The first passive optical network standard that uses ATM (Asynchronous Transfer Mode) size cells for transport. APON is the initial PON specification defined by the FSAN.

BPON (Broadband Passive Optical Network): A standard based on APON, BPON supports Wavelength Division Multiplexing (WDM). Typical APON/BPON supports 622 Mbps for downstream and 155 Mbps for upstream. APON/BPON standard is G.983.1, it has been set by ITU in 1998 [24].
GPON (Gigabit Passive Optical Network): A standard that is an evolution of BPON. It supports higher rates and enhanced security. GPON supports 2,488 Mbps for downstream and 1,244 Mbps for upstream. It supports Triple Play Services (TPSs): Voice, data and video. GPON was set as a standard by ITU-G.984.1 [23].

EPON/GEAPON (Ethernet/Gigabit Ethernet Passive Optical Network): Standards based on Ethernet. They support 1 Gbps for both downstream and upstream. EPON/GEAPON have been standardized by the Institute of Electrical and Electronics Engineers (IEEE) as 802.3ah [1].

10GEPON (10 Gigabit Ethernet Passive Optical Network): Advanced standard of GEAPON. It supports 10 Gbps for both downstream and upstream.

GPON deployments are mainly in the trial phase — no commercial deployments have been reported up to April 2007. On the other hand, EPON had already approximately 8 million subscribers, occurring mainly in the Asian market (Japan, Korea, China), by the end of March 2007. The growth rate in the Asian Market is an additional 3 to 4 million subscriber ports per 6 months. Only small deployments are present in the USA (mainly cable operators) and South America. In Europe, Sweden, Italy, Denmark and Netherlands are leaders in FTTH subscribers [29].

Canada, despite its low population density, remains one of the global leaders in broadband initiatives and government initiated programs such as "The Broadband for Rural and Northern Development Pilot Program(BRAND)", which started in 2002 and has since funded 58 rural broadband projects for 884 communities with approximately 105 million dollars. The largest telecom operator, Bell Canada, moving steadily forward with Greenfield
FTTH and selected rural rehabilitation sites [27].

Nippon Telegraph & Telephone Corporation NTT\textsuperscript{1} is currently evaluating 10GEPON but has not announced when they will start deploying it in any kind of volume [33].

2.2 PON Equipment

The main equipment used in PONs are optical fibers, OLT, optical splitters, AWGS, and ONUs, discussed each component separately in the following sections.

2.2.1 Optical Fiber

Optical fiber is used for long distance telecommunication and networking. It has very low attenuation and high capacity as compared to electrical cables. The transport capacity of each optical fiber is defined by its number of available wavelengths and the transport capacity of each wavelength. Two common types of optical fibers are single-mode fiber and multi-mode fiber.

- Single-Mode Fiber (SMF) is used for long distance, e.g., long-distance telephony and multichannel television broadcast systems because it has less signal loss as compared to multi-mode fiber. Standard SMF has small core diameter of about 10 $\mu$m, so light can propagate in one mode. It is usually used for long-distance unidirectional traffic on each fiber. SMF is more expensive than the multi-mode fiber.

- Multi-Mode Fiber (MMF) is used for shorter distance, e.g., LAN, video surveillance because it is less expensive and easier to install. MMF has higher attenuation than SMF. MMF has large core diameter as compared to SMF, and two standard diameter

\textsuperscript{1}NTT is a Japan-based telecommunication provider offering fixed and mobile voice related services.
sizes which are $50 \, \mu m$ and $62.5 \, \mu m$. In MMF, light can propagate in multi-mode; light can travel in multiple waves [9].

In optical networks, data is converted to bits of light called *photons* and then transmitted over fiber. In traditional (cable) networks, data is converted to *electrons* that travel through copper cable. Optical networks are faster than traditional networks because a photon’s weight is less than an electron’s one, and further, unlike electrons, photons do not affect one another when they move in a fiber (because they have no electric charge) [13].

Most optical fibers have three layers: Core, cladding, and jacket (buffer coating), as shown in Figure 4.

![Optical Fiber Diagram](image)

Figure 4: An Optical fiber (taken from [21])

The common diameters of optical fiber cables (SMF and MMF) are shown in Table 1.

<table>
<thead>
<tr>
<th>Core ($\mu m$)</th>
<th>Cladding ($\mu m$)</th>
<th>Jacket ($\mu m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>125</td>
<td>250</td>
</tr>
<tr>
<td>50</td>
<td>125</td>
<td>250</td>
</tr>
<tr>
<td>62.5</td>
<td>125</td>
<td>250</td>
</tr>
<tr>
<td>100</td>
<td>140</td>
<td>250</td>
</tr>
</tbody>
</table>

Table 1: Standard diameters of optical fibers
2.2.2 Optical Line Terminal (OLT)

The OLT is an active device, located in a central office, which controls the bidirectional flow of information across Optical Distribution Network (ODN). The term ODN refers to the collection of fibers and passive optical splitters or couplers that lie between the OLT and the various ONUs. An OLT must be able to support transmission distance across the ODN up to 20 km. In the downstream direction, the OLT function is to take the voice, data, video, and other multimedia traffic from a long-haul or metro network and to broadcast/multicast/unicast it to all the ONUs modules on the ODN. In the reverse (upstream) direction, the OLT accepts and distributes multiple types of voice and data traffic from the network users [16].

A PON has symmetric and asymmetric traffic; some applications need more bandwidth in the downstream than upstream direction, such as Internet browsing. The telephone, Online game and video conference applications have symmetric traffic, i.e., the same amount of bandwidth is required in the downstream and upstream directions. Other applications, such as Standard Television (SDTV), HDTV, audio stream, only have traffic in the downstream direction.

The OLT component contains transponders and wavelength multiplexers/demultiplexers. A transponder can read electrical signals and convert signals to optical signals or vice versa. Wavelength multiplexers receive multiple signals (wavelengths) from multiple optical fibers and combined them in order to send on a single optical fiber. Wavelength demultiplexers receive multiple signals from a single optical fiber and distribute them on multiple optical fibers [35].

A typical OLT can support more than one PON subnetworks, e.g., 4 PON subnetworks
with 32 ONUs on each outgoing link of the OLT [20], 8 PON subnetworks with 32 ONUs on each outgoing link of the OLT [22], 22 PON subnetworks with 32 ONUs on each outgoing link of the OLT [34], 32 PON subnetworks with 32 ONUs on each outgoing link of the OLT [19].

2.2.3 Optical Splitter

An optical splitter is a passive optical device with an input signal and \( k \) output signals. The size of an optical splitter corresponds to the number \( k \) (2, 4, 8, 16, 32, 64) of output ports. Through loss is the amount of attenuation the signal receives as it passes from input to output. A 1:2 has a typical 3/3.5 dB loss, 1:16 has 12 dB, 1:32 has 15 dB and so on, where dB is the measurement unit of the power. There are two types of optical splitters. The first type corresponds to a fixed power splitter as shown in Figure 5. The main drawback of a fixed power splitter is that it introduces unnecessary power losses when an input signal needs to be split into a number of outputs less than \( k \). Another type corresponds to the reconfigurable power splitters that can be split into any number of output signals, where any means less than or equal to \( k \) signals. Note, however, that configurable power splitters are, at least for today, active optical devices and therefore will not be considered in this thesis.

An optical splitter includes a combiner which reads multiple inputs (channels/wavelengths) and combined them to send them on a single output in the upstream direction.

![Figure 5: A 1:4 fixed power optical splitter](image)

14
An optical splitter is an equipment used at a remote node in a PON placed near subscribers premises/streets/buildings. It is connected with a feeder fiber on one side and a series of distribution fibers on the other side. An optical splitter can be considered as a defining feature of a PON, since it is one of the key technologies that allows access network to be electrically passive. A major cost advantage, of point-to-multipoint PON architectures through optical splitters, is to reduce the fiber requirements as compared to a point-to-point architectures. In point-to-point architectures, each ONU has a separate fiber to connect it to the central office. This cost reduction is achieved using a splitter to take one fiber from the CO in order to serve 32 ONUs [10].

In the past few years, significant improvements in reliability, cost per port, and insertion loss have been demonstrated with Planner Lightwave Circuit (PLC)-based splitters [9]. Typically PLC-based optical splitters are used for high split counts and Fused Biconic Tapered (FBT)-based optical splitters are used for low split counts [3]. PLC-based splitters are available in 1x4, 1x8, 1x16, 1x32 and FBT-based in 1x2, 1x4, 1x8 splitting ratio. Insertion loss in PLC-based splitter is low compared with FBT-based splitter.

2.2.4 Array Waveguide Grating (AWG)

An Array Waveguide Grating (AWG) is a passive optical component, also called passive waveguide router. The function of an AWG is multiplexing in the upstream direction: Multiplex multiple channels (wavelengths) from different fibers into a single optical fiber, and demultiplexing in the downstream direction: Demultiplex multiple channels (wavelengths) from a single optical fiber to multiple optical fibers, see Figure 6 for an illustration.

The difference between an optical splitter and an AWG is that an optical splitter demultiplex downstream signals and send them to all its outgoing ports, unlike AWG, which
demultiplex downstream signals and send them, each to a different outgoing port. Both AWGs and optical splitters can be used as multiplexers by upstream signals in the reverse direction in a WDM-PON network.

2.2.5 Optical Network Unit (ONU)

In an optical fiber telecommunication network, an Optical Network Unit (ONU), also referred as Optical Network Termination (ONT) or Network Interface Device (NID), is provided in the subscriber neighborhood for terminating the optical fiber transmission line and for providing electrical signals over metallic lines to the subscribers. An ONU is located at subscriber’s premises and shared by one or a small number of customers, so that the cost of each ONU is a key element in WDM-PONS.

To lower the cost of WDM-PONS, it is highly desirable to develop low-cost and colorless ONUs. The ONUs should be colorless (in other words, no specific wavelength on its input/output ports) in order to decrease the costs of operation, administration, and maintenance (OA&M) functions. It also reduces stock inventory issues [8] [30].

In this thesis, we consider each ONU is equipped with a single transmitter and a single receiver, for transmitting (upstream) and receiving (downstream), to keep a low cost for an ONU. Note that in practice, if the ONU is associated with, e.g., a business office, 2 or 3
transmitters or receivers could be necessary in order to handle all the traffic. Transmitting
and receiving is performed on separate channels (wavelengths). A typical ONU has interfaces
for telephone lines, analog TV, and high-speed Ethernet data. An ONU can serve one or
more subscribers. The downstream function of an ONU is to receive optical signals, convert
them into electrical signals and send them to subscriber(s). The upstream function consists
in receiving electrical signals from subscriber(s), converting them into optical signals and
sending them to the OLT on the ONU assigned time slots.

2.3 PON Topologies and Layout

In this section we discuss WDM-PON physical and logical network topologies. We also discuss
about WDM-PON layouts/architectures.

2.3.1 PON Physical Topologies

There are various suitable physical network topologies for PON in access networks: tree, ring,
bus, or even other ones when adding reliability concerns, discussed in Section 2.4. In this
thesis, we consider two types of network topologies: one is tree and another is light mesh.
Light mesh is an extended tree topology, where some nodes at the leaf-level are connected
with multiple links to intermediate (remote) nodes. The following sections describe these
two topologies.

Tree Network Topology

A tree network is rooted at the OLT, ONUs are the leaves and the intermediate nodes equipped
with passive equipment (i.e., optical splitters in this thesis) connected through the optical
fibers, as shown in Figure 7.

![Figure 7: A PON tree topology](image)

**Light Mesh Network Topology**

A *light mesh network* can be obtained from a tree topology rooted at an OLT, where some ONUs have more than one path from the OLT; some ONUs are connected with more than one link to the remote nodes. The difference between tree and light mesh network is that in a light mesh network, each ONU has one or more than one potential route through different links as shown in Figure 8, but each ONU can receive its request(s) on at most one wavelength and can send its request(s) on at most one wavelength, again for economical reasons each ONU is equipped with a single transmitter and a single receiver. In other words, at a given time, only one route is active and established, and the alternate route(s) are used when a
failure occurs. Figure 8 is showing that all the ONUs are connected with last (2nd) level optical splitters, but it is not fixed/restricted as any ONU can be connected with any level of splitters. Additionally it shows that the few ONUs are connected to two splitters, even more or all of the ONUs can be connected to multiple optical splitters.

The reason to provide multiple links to some ONUs is a network reliability. Reliability, i.e., alternate paths, is offered to some or all ONUs and has a series impact on the network cost, due to the increased equipment cost and the additional loss concern to be taken into an account. The reliability/survivability of the WDM-PON is discussed in Section 2.4.

Figure 8: A PON light mesh topology
2.3.2 PON Logical Topology

A lightpath represents a direct optical connection without any intermediate electronics. A lightpath is an all optical communication channel set up between end-to-end nodes of an uplink/downlink request which may span more than one optical fiber link and pass through some intermediate nodes. In the passive optical networks, at each intermediate node, an optical splitter or an AWG routes incoming signals all optically to its corresponding output port by multiplex/demultiplex signals. A lightpath is a point-to-point (unicast) connection. A light-tree is a tree that is rooted from source and contains some of the destinations. In other words, a light-tree is a point-to-multipoint (multicast) connection. The same wavelength is used throughout the light-tree. A light-tree is an extended concept of the lightpath developed for efficient implementation of multicasting in WDM optical networks [35].

For example, consider a set of three logical connections: From transmitter A to receiver B, from A to C, and from A to D, as shown in Figure 9(a), where each connection represent a lightpath. Now change these three unicast (point-to-point) connections in one multicast (point-to-multipoint) connection, as shown in Figure 9(b). We refer to such a point-to-multipoint extension of lightpaths as a light-tree [6]

![Diagram](image-url)

Figure 9: Unicast vs. multicast (taken from [46])
2.3.3 PON Layout/Architecture

A basic PON structure with an optical splitter is shown in Figure 10. In the diagram, each outgoing link at the OLT has an optical splitter at the intermediate node and set of ONUs at the leaf nodes. The Figure also shows only one link of the OLT, we assume that each outgoing link of the OLT has same structure.

The main equipment of a PON are its optical splitter devices. Each optical fiber is bidirectional; any wavelength can be used for either upstream or downstream traffic. Multi-mode fiber is used for bidirectional traffic on a single fiber. In other words, to keep a low cost access networks, single multi-mode fiber can be used on each outgoing PON link for both downstream and upstream traffic.

![Figure 10: A PON architecture](image)

In the downstream direction, the OLT will broadcast or multicast or unicast a message on links, where at each intermediate node, i.e., an optical splitter will split the message on all outgoing links then finally to each ONU that is configured to read the specified wavelength and reject the messages sent on the other wavelengths. For the upstream traffic, each ONU will send its request(s) on their specified wavelength on assigned time slot by using the
TDMA technique. ONUs cannot communicate each other directly, every traffic has to go through the OLT, then OLT will broadcast or unicast or multicast messages on required link(s).

An OLT uses tunable lasers and receivers to decrease total transceiver count and to generate optical carriers onto which ONUs can modulate their upstream traffic [15].

In addition, there is another issue to reduce a cost of the network, i.e., minimize the fiber length and the number of remote nodes. Tran et al. [5] proposed a home and remote node distribution model shown in Figure 11. All homes are distributed on a square grid with a uniform density. Each remote node aggregates a set of $n^2$ homes forming a square on this grid (shown in Figure 11(a)) and each OLT aggregates a square array of $N^2$ remote nodes (shown in Figure 11(b)). This model is considered for urban (1976 houses/km$^2$) and suburban (177 houses/km$^2$) areas.

Similarly, Khan [37] proposed an algorithm which may be employed to achieve cost savings on fiber and other equipment in the last mile network. This algorithm has two steps: First step concentrate on minimizing the distribution fiber length to find the location of ONUs and second step takes as input the locations of ONUs which are used to optimize the splitter locations and reduce both the fiber and equipment cost.

2.4 PON Survivability

This section explores the design issues with respect to survivability in WDM-PON networks.

WDM-PON networks are prone to failures of links, e.g., a feeder fiber or a distribution fiber. Any fiber cut causes a link failure. Sometimes, it is also possible that a head end node (e.g., an OLT), an intermediate node (e.g., an optical splitter/array waveguide grating), a
terminal node (e.g., an ONU), are to be failed. Therefore, it is imperative that these networks have a protection and a fault tolerance/restoration capability. Protection refers to spare components that may be used in case of working component failures. Fault tolerance refers to the ability of the network to reconfigure and re-establish communication upon failures. Restoration refers to the process of rerouting affected traffic around a component failure.

![Diagram of home and remote node model](image)

Figure 11: A home and remote node model (taken from [5])
2.4.1 Protection

Common network protection techniques include implementing duplicate network resources such as fiber links, remote nodes, and ONUs, to provide network resource redundancy. Network resource redundancy causes networks to be expensive. Resource sharing techniques are useful to reduce spare capacity requirements.

The ITU-T G.983.1 recommendations on passive optical networks (PONs) has suggested some possible fiber duplication and protection switching scenarios for reliable data delivery [24].

2.4.2 Restoration

Two common levels of restoration are the centrally controlled one (at OLT) and the self-protected one (at ONU) through Automatic Protection Switching (APS). At the centrally controlled level, all protection switchings can be performed at the OLT, after the fault alarms are collected. A self-protected switching can be performed at individual ONUs, to realize distributed control. In this case, the protection switches are incorporated in the individual ONUs, which continuously monitor the status of their attached fiber links or components. APS will be triggered only at the affected ONU when any fault is detected. Self-protected techniques are controlled at terminal nodes, i.e., ONUs, through APS. Sometimes APS is also implemented at the remote nodes [43].

Once a network is detecting a failure, the required time period to perform protection switching and restore the affected traffic is known as the traffic restoration time. This time period should be kept small, say in microseconds.
2.4.3 Survivable WDM-PON Architecture

While many studies have been conducted for optimal provisioning of working and backup traffic in WDM core networks, none is yet available for WDM-PON networks. However, several architectures have been proposed, which attempt to minimize the network resource redundancy.

Recent studies, [43], [40], [39], [47], [38], and [45], proposed various protection architectures for WDM-PONs which can provide 1+1, 1:1, or 1:N protection capability. All of these schemes assume AWG equipment at the intermediate nodes except [43] and protect in both directions i.e., upstream and downstream. However, AWGs can be replaced by optical splitters in all of these architectures. A self-protected mechanism is implemented in [40], [39], [38], and [45]. A centrally controlled and the remote node restoration designs are proposed in [43] and [47], respectively.

A star-ring architecture is proposed in [43]. All the remote nodes are connected to the OLT in a star architecture. A star topology is implemented at the OLT level and a ring topology is implemented at the level of the remote nodes. ONUs are divided on different subsets and, in each ONU subset, ONUs are connected to a ring. Each ring is connected to two remote nodes, as shown in Figure 12. Each remote node contains two CWDMs (for upstream and downstream directions) and several optical switches.
Figure 12: A star-ring architecture (taken from [43])

Bidirectional 1:1 protection against any fiber cut between the remote node and the ONUs is proposed in [39]. In there, the architecture protects feeder fibers and distribution fibers but there is no protection against failure for remote nodes. Each feeder fiber is protected with a spare feeder fiber; each remote node is connected to the OLT with two fibers. Each distribution fiber is protected by grouping two adjacent ONUs, connected them to the same port of an AWG but through separate links. If any link between a remote node and an ONU fails, the ONU will communicate to the remote node through another (adjacent in the same group) ONU. ONUs belonging to the same group are connected with each other through a simple fiber, as shown in Figure 13(a). The same distribution fibers protection scheme is also proposed in [40] but there is no protection against failure for feeder fibers. In addition, like [39], there is no protection against failure for remote nodes, as shown in Figure 13(b). This architecture is again discussed in [47], with the difference that each ONU is connected
to different ports of a remote node; each port at the remote node is dedicated to a single ONU, as shown in Figure 14.

Figure 13: A 1:1 protection schemes: Taken from [39] for (a) and [40] for (b)
Furthermore, a 1:N protection scheme has been proposed in [38]. This scheme protects feeder fibers and distribution fibers, but there is no protection against failure for remote nodes, as shown in Figure 15(e).

The proposed architecture has three feeder fibers, e.g., FF1, FF2, and FF3, between an OLT and a remote node. In normal operation, FF1 and FF2 are used to carry the downstream and upstream traffic respectively. In case of a feeder fiber, either FF1 or FF2, failures the FF3 is used to carry the affected, either downstream or upstream traffic to/from the remote node. The distribution fibers are protected through interconnection among the ONUs, connected to the same remote node, as shown in Figure 16.
Figure 15: A 1:N protection scheme (taken from [38])

Figure 16: An interconnection fibers among ONUs (taken from [38])
In case of distribution fiber failure occurs, in the left most or right most four distribution fibers, the scheme enters in the protection operation mode, described in next. Due to the two possible wavelength (FF1 and FF2) assignments for FF3, as shown in Table 2, the interconnection fibers can be classified into two different types, as shown in Figure 16. The first type provides protection for the leftmost four distribution fibers (shown in bold lines), while the second type protects the rightmost four distribution fibers (shown in dotted lines). The interconnection pattern can be changed as the wavelength assignment scheme changes.

The author claims that his scheme requires the half number of wavelengths than schemes proposed in [39] and [43].

<table>
<thead>
<tr>
<th>ONU #</th>
<th>Working stream</th>
<th>Protection stream</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda_1 )</td>
<td>( \lambda_5 )</td>
</tr>
<tr>
<td>1</td>
<td>( \lambda_2 )</td>
<td>( \lambda_6 )</td>
</tr>
<tr>
<td>2</td>
<td>( \lambda_3 )</td>
<td>( \lambda_7 )</td>
</tr>
<tr>
<td>3</td>
<td>( \lambda_4 )</td>
<td>( \lambda_8 )</td>
</tr>
<tr>
<td>4</td>
<td>( \lambda_5 )</td>
<td>( \lambda_1 )</td>
</tr>
<tr>
<td>5</td>
<td>( \lambda_6 )</td>
<td>( \lambda_2 )</td>
</tr>
<tr>
<td>6</td>
<td>( \lambda_7 )</td>
<td>( \lambda_3 )</td>
</tr>
<tr>
<td>7</td>
<td>( \lambda_8 )</td>
<td>( \lambda_4 )</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: A wavelength assignment for interconnection among ONUs (taken from [38])

2.5 Multicast Routing and Wavelength Assignment

Routing and Wavelength Assignment (RWA) refers to selecting a suitable route (i.e., light-path) and allocating an available wavelength respectively for each unicast connection. Multicast routing and wavelength assignment (MC-RWA) consists in selecting suitable routes or trees (often called light-trees), and allocating available wavelengths to each multicast request. Note that it might be convenient to serve a multicast request using more than
one light-tree, in order to overcome the severity of the wavelength continuity constraint in light-trees.

![Optical Line Terminal (OLT) Diagram](image)

Figure 17: A 1+1 protection scheme (taken from [45])

Multicast is becoming increasingly important in recent years, due to more efficient bandwidth usage and due to the increasing popularity of point-to-multipoint multimedia applications such as HDTV, video conferencing, on-line gaming, telemedicine, and video-on-demand. Multicast in PONs is possible by use of optical splitters. Compared with implementing multicast at electronic layer, optical multicast technology has some advantages [17][44], e.g.,
Optical-Electrical-Optical (OEO) and store-and-forward packet switching technology are not required. Multicast in electronic layer is implemented at higher layer (e.g., network layer). At the network layer, incoming packets are converted from optical to electronic signals to determine the destination(s) of the packet. After determining the packet destination(s), packets are converted back in optical signals to transmit on optical fibers. This mechanism is called store-and-forward packet switching technology.

2.5.1 Network Planning and Traffic Assumptions

There are 3 common terms related to the bandwidth and the traffic in the communication networks: Traffic Engineering (TE), Network Engineering (NE), and Network Planning (NP), defined as follows, see [6]:

- **TE** refers to "Put the traffic where the bandwidth is", and known as a routing problem, where the traffic to be routed in the networks. Thus TE is an online dynamic problem, whose decision-making time is very quick, usually in milliseconds.

- **NE** refers to "Put the bandwidth where the traffic is", and known as adding the more capacity. As a network continue its operation, and as increase the traffic, certain parts of the network may become more congested. Thus, to remove the congestion, the network operator needs to incur capital expenditure to handle this capacity-exhaust problem. The NE decision-making is perhaps weeks or months.

- **NP** refers to "Put the bandwidth where the traffic is forecasted to be", NP problem is quite similar to the NE, with the difference that NP starts from the scratch to design a cost effective network. A network planner may be given annual traffic forecasts over N years. The NP problem would be to design a low cost network for first year with
the planning of future that how/where to add more capacity in subsequent years.

The MC-RWA is defined as optimization problem which tries to accommodate a whole set of requests in optical networks. Typically requests may be of two types: static and dynamic [18]. In a static scenario, the set of requests is known in advance while the dynamic traffic, requests arrive in some random manner; a lightpath is setup for each connection request as it arrives, and the lightpath is released after some finite amount of time.

One of the major issue, in optical networks, is the bandwidth underutilization problem. In the backbone network the bandwidth is very high, but many customers are content with a lower bandwidth. Only a fraction of customers are expected to have a need for such a high bandwidth [6]. Since multiple requests can be combined on a single wavelength, this process known as traffic-grooming. Similarly, in the passive optical networks some services, e.g., audio stream, telephone, etc., are required low bandwidth. Therefore, these services, required low bandwidth, can be combined on a single wavelength, and even multiple subscribers services can be combined on a single wavelength. The traffic-grooming problem can be formulated as follows. Given a network configuration (including physical topology, number of wavelengths, and the transport capacity of each wavelength) and a set of connection requests with different bandwidth granularities, we need to determine how to set up light-trees/lightpaths to satisfy the connection requests.

This thesis concerns an efficient provisioning for PONs. It deals with network planning where static traffic are used.
Static Traffic

If the set of multicast requests is known before-hand and if no traffic variations take place over a long timescale, the problem is known as static MC-RWA problem. The static MC-RWA is appropriate for connections that are assumed to remain in the network for a relatively long period, and their provisioning can be performed off-line. In that context, the most important optimization objective is that either the number of wavelengths needed to accommodate all the requests is minimized or the number of granted requests is maximized for a limited number of wavelengths.

The solution to the static MC-RWA problem consists of set of long-lived light trees that creates a logical (or virtual) topology, which is embedded on to the PON. This problem is known as virtual topology design problem. In the virtual topology design problem, the routing and wavelength assignment problem can be formulated as integer linear programming (ILP) problem as we will see in Chapter 3.

Static Traffic-Grooming

In the static traffic-grooming problem all the traffic demands are known in advance. Assigning network resources to successfully carry the connection requests in WDM networks is well known as the routing and wavelength assignment (RWA) problem. For the static traffic-grooming problem, it is assumed that a set of connection demands are given, and they need to be established on the network [6]. Traffic-grooming examples, in the passive optical networks, are discussed in Section 3.5.3 and 3.5.4 on a tree and a light mesh network topologies, respectively.
2.5.2 Previous Mathematical Models on WDM Core Networks

No study has yet appeared on finding the optimal provisioning of multicast/unicast requests in a PON network, using optimization models, based on ILP programs. However, several such studies have appeared in the case of mesh WDM core networks.

Wavelength-routed WDM networks have emerged as a key technology for next-generation wide area networks (WANS). A wavelength-routed network adopts a mesh topology and each node in the network is equipped with an active device, called an Optical Cross-Connect (OXC), which has the capability to switch connections based on wavelengths carrying data. Multicast Optical Cross-Connect (MC-OXC) has the capability of multicasting. A MC-OXC contains light-splitter(s); where a light splitter performs light splitting which is relatively efficient than copying packet at the Internet protocol (IP) layer. In a WDM network, if all the nodes are multicast capable, the network is called full light splitting; however if only a portion of the nodes are multicast capable while other nodes have no light splitting (multicast incapable), the network is called partial light splitting [46].

Many optimization models have been reported for the multicasting in WDM mesh networks. All the nodes in WDM networks contain active devices. Some of the reported optimization models are listed below:

- Find the logical topology for multicast traffic, that minimizes the cost with a limited number of transmitters and receivers at each node. There is no constraint on the maximum number of wavelengths that each fiber can carry, since new transmission equipment allow the use of a very high number of wavelengths (up to 128), and in addition, the use of wavelength converters can be instrumental to the relaxation of the constraint limiting the number of wavelengths on each fiber. The cost of each link
depends on the congestion of the link [31].

- Find the optimal routing and wavelength assignment with minimum total costs on a given topology. The given network topology is fully mesh, there is no wavelength conversion and some nodes have splitting capability. The cost of each link is given with network topology [11].

- Maximizing the number of served destinations under the condition of minimizing total wavelength cost. The cost of a wavelength depends on the traffic load on the links and there is no wavelength conversion [26].

- Minimizing the number of wavelengths used for given multicast traffic with Multicast Capable (MC) and Multicast Incapable (MI) nodes. It is required that each MI node can reach an MC node in one hop, in the WDM optical mesh network [4].

- Minimizing the total number of fibers used for establishing multicast sessions with a subset of Splitter-and-Delivery (S&D) cross-connects assuming remaining nodes do not have splitting capability [36].

- Minimizing the total cost of multicast sessions. The author proposed two different optimization models. First model considers every node is equipped with a wavelength converter and a splitter. Second model considers every node is equipped with only a splitter [32].

- Maximizing the number of users that can be served by first accommodating only complete multicast groups (requests), then trying to serve as many users as possible by allowing partial accommodation (partial request) in WDM mesh networks [25].
To the best of our knowledge, no optimization model for efficient provisioning of multicast traffic in the passive optical networks has yet been reported. The following characters are different in WDM-PON than in WDM mesh core/metropolitan networks:

- All the traffic goes through root node, i.e., OLT;

- Intermediate nodes are equipped with passive components, i.e., optical splitters or AWGS;

- Source and destination can be an ONU or an OLT, it means an intermediate node cannot be a source or a destination.

For those reasons, models developed in the context of WDM core networks cannot be reused and new models have to be developed for PON networks.
Chapter 3

MC-WA Optimization Models for PON Networks

Optimization means "the action of finding the best solution". Optimization often involves some mathematical modeling in order to find the best solution to a problem. A mathematical program corresponds to the optimization of an objective function, (either minimizing or maximizing) subject to a set of constraints with an analytical expression. Two particular often encountered programs are linear and integer linear programs.

**Linear Program (LP):** The study of maximizing or minimizing a linear objective function subject to constraints which can be expressed by linear inequalities or equalities.

**Integer Linear Program (ILP):** If the unknown variables of a linear program are all required to have integer values, then the model is called an integer linear program.

Optimization models can help to solve problems when there are

- many possible solutions,
- limited available resources.
An optimization model is made of two parts. The first part is the goal or the objective we want to optimize, and the other part is made of a set of constraints which corresponds to a set of limitations or restrictions. We may have limited resources to satisfy the demand or we may look at using less resources to satisfy the demand.

3.1 Motivation

It is clear that a protection scheme will help to improve reliability/survivability in the passive optical networks. A protection must be provided to all or some ONU$s$, consequently, in this thesis, we consider light mesh network topology that concerns with a reliability. In other words, the light mesh network topology concerns with the network reliability, where all or few ONU$s$ has multiple paths. While PON efficient provisioning in the light mesh network topology, there are two techniques to consider the multiple-path ONUs:

1. During optimization, consider only one potential path, know as working path and reserve the alternate path(s) as spare path(s), that/those is/are to be used in case of working path failures, due to a fiber or the remote node.

2. During optimization, consider all paths as working paths, but use only one potential path, and keep the other ones as a spare path(s), that/those is/are to be used in the case of working path failures.

In the proposed mathematical models, we consider the second one, that increases the grade of service as shown in the following example.

Example 1: Consider a network topology shown in Figure 18; two links, each link has 2 wavelengths ($\lambda_1$ & $\lambda_2$), four ONU$s$, ONU$_2$ and ONU$_3$ have two paths.
In traffic, there are two requests: $r_1$ and $r_2$. The bandwidth required for each request is equal to the transport capacity of a wavelength. The source of $r_1$ is an ONU$_2$ and destinations are ONU$_1$ and ONU$_4$. The source of $r_2$ is an OLT and destinations are ONU$_2$ and ONU$_3$.

A solution on the first technique for this particular example, where we consider to use only one potential path (shown as complete lines arrow).

- To provision request $r_1$, two wavelengths on link-1 are needed; one for ONU$_2$ in the upstream direction and the other for ONU$_1$ in the downstream direction. In addition, one wavelength is needed on link-2 for ONU$_4$ in the downstream direction.

- To provision request $r_2$, one wavelength is needed on each link in the downstream direction.

Consequently, to provision both requests, they need three wavelengths on link-1, but in our topology we have only two wavelengths on each link, so far it can provision any one request, but not both.
A Solution on the second technique for this particular example, where we consider to use any one path (shown as complete and dashed lines arrow), which is an optimal, as working path and keep other ones as spare path(s).

- To provision the request \( r_1 \), use \( \lambda_1 \) on the link-1 (for ONU\(_1\)) and link-2 (for ONU\(_4\)) in the downstream direction, and \( \lambda_2 \) on the link-1 (for ONU\(_1\)) in the upstream direction.

- To provision the request \( r_2 \), use \( \lambda_2 \) on the link-2 (for ONU\(_2\) and ONU\(_3\)) in the downstream direction.

Consequently, they need two wavelengths on each link. Therefore, both requests can be provisioned, and it shows that grade of service has been increased as compare to the previous technique. The solution is also shown in the Figure 18. Note that, we assume that there are some spare wavelengths on each link that would be used in the case of link failures.

### 3.2 Column Generation Overview

Column generation techniques offer solution methods for linear programs with a very large number of variables (e.g., exponential), where the constraints can be expressed implicitly. They rely on a decomposition of the initial linear program into the *master problem* and the *pricing problem*. The master problem corresponds to a linear program solved in practice with a restricted constraint matrix, with respect to the number of variables (or columns) of the initial constraint matrix leading to the so-called restricted master problem. The pricing problem is defined by the optimization of the so-called reduced cost (refer to [41] if not familiar with linear programming) subject to the implicit constraints expressed by the coefficients of the constraint matrix of the master problem.
The column generation solution scheme is similar to that of the simplex algorithm: it is an iterative process where, at each step, we attempt to add one or more columns to the constraint matrix of the master problem in order to improve the value of the objective function. The search of such a column is made through the solution of the pricing problem. If its outcome corresponds to one or more column(s) with a negative reduced cost (assuming we deal with minimization), then it entails an empowerment of the value of the restricted master problem objective function; otherwise, if no solution of the pricing problem can be identified with a negative reduced cost, we then conclude that the current solution of the master problem is indeed optimal.

Column generation can be combined with branch-and-bound techniques for solving integer linear programs with a large number of variables leading to the so-called branch-and-price methods, see [7] for a nice overview. Branching rules have to be devised properly in order to avoid generating a huge number of subproblems in the search tree associated with the branch-and-bound, either by branching on the variables of the master problem using cuts, or by branching on the variables of the pricing problem using classical branching schemes or cuts.

3.3 Assumptions and Physical Constraints

This section describes assumptions considered in mathematical models.

**Wavelength continuity constraint:** There is no wavelength conversion at intermediate nodes, so a lightpath must use the same wavelength on all the links along its path, from OLT to destination ONUs and from source ONU to OLT.
**Attenuation:** All the ONU's are reached within acceptable attenuation.

**Link:** Each link has a single optical fiber.

**Fiber:** All the optical fibers are bidirectional and each fiber has a limited number of wavelengths. In case of the light mesh network topology, we also assume that each fiber has some spare wavelengths.

**ONU:** Each ONU can receive its request(s) on at most one wavelength and send its request(s) on at most one wavelength. Indeed, we assume each ONU to be equipped with only one emitter and one receiver.

**Traffic:** Traffic is static; all the requests are in hand in advance.

### 3.4 Definitions and Notations

This section describes notations related to the network topologies and the traffic, used in the proposed mathematical models.

#### 3.4.1 Network Topology

Let us consider an optical access WDM network represented by an undirected graph $G = (V, E)$ where the node set $V$ contains the OLT node at the root, the set of intermediate nodes where the optical splitters are located, the set of leaf nodes where the ONU's are located, and where the link set $E = \{\ell_1, \ell_2, \ldots, \ell_n\}$ is such that each link is associated with a physical fiber link of the physical network.

$O$: Set of available ONU's in the physical network, i.e., $O = \{ONU_1, ONU_2, \ldots, ONU_n\}$. 

43
\( O^\ell \):  Set of ONUs that can be reached by a path starting at the OLT with link \( \ell \in \omega(\text{OLT}) \).

\( W \):  Number of wavelengths on each optical fiber.

\( \Lambda \):  Set of available wavelengths, i.e., \( \Lambda = \{\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_W\} \) on the optical fiber link, where \( W = |\Lambda| \).

\( B_{\ell} \):  Transport capacity of a wavelength on link \( \ell \) (we assume all the wavelengths to have the same transport capacity on a given link).

### 3.4.2 Traffic

The following paragraph describes the traffic and their notations used in our proposed models.

There are usually two types of requests in terms of origination (source of request \( r \)), one is originated at the OLT and another is originated at the ONU. A request originated at the OLT may have unidirectional traffic or bidirectional traffic. If it is bidirectional traffic then the request can be decomposed in two linked requests, e.g., \( x_{r_1} \) and \( x_{r_2} \), as shown in figure 19(a), and a new constraint, i.e., \( x_{r_1} = x_{r_2} \) can be added in the proposed models, which assures that the both requests must be granted or rejected. However, if a request has unidirectional traffic, as shown in Figure 19(b), then there is no need to decompose the request.

A request originated at the ONU has bidirectional traffic, i.e., upstream and downstream as shown in Figure 20 (a), (b), and (c). Additionally, a unicast request may have same ONU as a source and a destination, as shown in Figure 20 (a). Note that we do not need to decomposed the bidirectional request originated at the ONU because as we discussed
previously, every traffic must go through the OLT. Therefore, we have same ONU as a source and a destination, and the traffic will go to/come from the OLT. In contrast, the bidirectional request originated at the OLT must need destination, i.e., ONU, for that reason the request must be decomposed in two sub requests.

Figure 19: Traffic examples originated at the OLT

Figure 20: Traffic examples originated at the ONU

$r$: Generic index of a request defined by its source and destination nodes, i.e., $x_r = (s_r, D_r)$, where $s_r = OLT$ or $s_r = ONU_i$ and $D_r$ is the set of destination ONUs. Note that if $r$ is a unicast request, $D_r$ contains a unique ONU.
\( b_r^D \): Bandwidth required for request \( r \) in the downstream direction.

\( b_r^U \): Bandwidth required for request \( r \) in the upstream direction.

Note that in some unicast requests we need two separate parameters (\( b_r^D \) and \( b_r^U \)) for upstream and downstream traffic. This type of unicast request, with asymmetric traffic, has a same ONU as a source and a destination, as shown in Figure 20(a). In other words, if a request has bidirectional symmetric traffic (see Figure 20(b) and (c)) or a unidirectional traffic (see Figure 19(b)) then one parameter, e.g., \( b_r \), is needed. The common example of asymmetric traffic is Internet Browsing where it needs more bandwidth in the downstream direction than the upstream direction.

\( \text{COST}_r \): Request cost.

\( \text{COST}_{ri} \): ONU cost.

\( \mathcal{R} \): Set of requests, i.e., \( \mathcal{R} = \{r_1, r_2, r_3, \ldots, r_n\} \).

\( \mathcal{R}^{\text{CORE}} \): Set of requests, such that each request is originated at an OLT, e.g. \( r = (\text{OLT}, D_r) \).

\( \mathcal{R}^{\text{ACCESS}} \): Set of requests, such that each request is originated at an ONU, e.g. \( r = (\text{ONU}_i, D_r) \).

\( \mathbf{D} \): Downstream traffic (OLT to ONU).

\( \mathbf{U} \): Upstream traffic (ONU to OLT).
3.4.3 Cost/Benefit

A cost can be used for different purposes, such as revenues, priority, traffic classes, network resources, etc. In the proposed mathematical models, the cost is used as benefit/price denoted as $\text{COST}_r$ and $\text{COST}_{ri}$, that represent the cost of each request $r$ and the cost of each ONU$_i$ of request $r$, respectively. We used cost as revenues, that represents weight of the request/ONU for generating the revenues or giving the priority to the different classes of traffic.

In addition, apart from revenues and classes of traffic, there are other factors related to the cost, which might be in interest of service providers/network operators. For example, if there are three multicast requests, requested from 3 different service providers to the network operator, i.e., $r_1$ (3 destinations), $r_2$ (3 destinations), and $r_3$ (6 destinations). Assume aggregate cost of $r_1$ and $r_2$ is equal to the cost of request $r_3$. In other words, each destination ONU has same cost. Suppose we have two solutions for this particular example: first solution is provision of $r_1$ and $r_2$, and alternate solution is provision of $r_3$. For the network operators, if he wants give more priority to serve one large service provider instead of two small service providers, the objective function for maximizing the weighted number of granted requests under the condition of giving priority to multicast requests with higher number of destination ONUs, is as follows:

$$\max \sum_{r \in R} \text{COST}_r \cdot x_r$$

where

$$\text{COST}_r = \begin{cases} 
\text{COST}^0_r & \text{if request } r \text{ is unicast} \\
\text{COST}^0_r - \frac{\text{COST}_{\min}}{n+1} & \text{if request } r \text{ is multicast.}
\end{cases} \quad r \in R.$$
\[
\left( \min_{r \in R} \text{COST}_r^0 \right) = \text{COST}_{\min} \quad \text{and} \quad n=\text{number of requests in the given traffic set.}
\]

Another Example for service providers, if there are three multicast requests, requested from users/subscribers, i.e., \( r_1 \) (4 destinations), \( r_2 \) (4 destinations), and \( r_3 \) (4 destinations). Assume each destination ONU has same cost. Suppose we have two solutions for this particular example: first solution is provision of two destination ONUs from each request \( r_1 \) and \( r_2 \), and alternate solution is provision of 4 destination ONUs of \( r_3 \). Assume these multicast requests are for online games, and service provider wants to give the priority to serve one request in a better way instead of served two requests badly. The objective function in the described situation for the maximizing the weighted number of served ONUs can be written as follows:

\[
\max \sum_{r \in R} \text{COST}_{r} \; x_{ri} - \sum_{r \in R} \text{COST}_{r} \; x_{r}
\]

where

\[
\text{COST}_{r} = \begin{cases} 
0 & \text{if request } r \text{ is unicast} \\
\frac{\text{COST}_{\min}}{n+1} & \text{if request } r \text{ is multicast.}
\end{cases} 
\]

\[
\left( \min_{r \in R} \text{COST}_r^0 \right) = \text{COST}_{\min} \quad \text{and} \quad n=\text{number of destination ONUs in a given traffic set.}
\]

### 3.5 Maximizing the Weighted Number of Granted Requests

In the objective of maximizing the weighted number of granted requests, we accept only whole request: If all the destination ONUs in the request \( r \) can be served then the request is granted, otherwise, the request is rejected. We have developed models for two types of problems: one is Multicasting & Wavelength Assignment without traffic-grooming (MC-WA) and the other one is Multicasting & Wavelength Assignment with traffic-grooming.
(MC-GWA). In the former, each ONU can receive/transmit at most one request, while in the latter, each ONU can receive/transmit multiple requests using the technique of Time Division Multiple Access (TDMA) on a single wavelength, as we assume only one receiver and one transmitter per ONU.

Objective

The objective is to maximize the grade of service, i.e., to maximize the weighted number of granted requests:

$$\max \sum_{r \in \mathcal{R}} \text{COST}_r \ x_r$$

where \( \text{COST}_r \) (refer Section 3.4.3) is the cost of request \( r, r \in \mathcal{R} \), and a decision vector of binary variables, such that each component \( x_r \) is defined as follows:

$$x_r = \begin{cases} 
1 & \text{if request } r \text{ is granted} \\
0 & \text{otherwise.}
\end{cases} \quad r \in \mathcal{R}.$$  

binary means variable may have a value either 0 or 1. Note that all the variables, in the proposed models, are binary.

In terms of service providers revenues, the cost of the request is based on the type of service. For unicast request, e.g., telephone, Internet, etc., has a fixed cost for each request. However, for multicast request, video, audio streams, etc., has a cost depends on the number of ONUs in a request. A cost is a weight/priority of the request/ONU. Consequently, in order to maximize the revenues, request \( r \) or \( \text{ONU}_i \) with a higher cost will get a higher priority during the solution of the optimization problem.
3.5.1 MC-WA on WDM-PON Tree Network Topology

We are given a network with a tree topology rooted at an OLT and where the ONUs are located on the tree leaves. For two different links \( \ell \) and \( \ell' \) of \( \omega(OLT) \), we have \( O^\ell \cap O^{\ell'} = \emptyset \) which implies that the routes are unique for every \( r \in R \).

Variables

In order to set the optimization model we need only one decision vector of binary variables, such that each component \( x_r \), used for a request \( r \).

Constraints

Each optical fiber has a limited number of wavelengths, i.e., \( W \), for both downstream and upstream traffic. We assume required bandwidth for each request (which is aggregated traffic requested from service provider) is less or equal to the transport capacity of each wavelength and there is no TDMA. In other words, Each request \( r \) which goes from OLT to ONU needs one wavelength for downstream, while those requests which goes from ONU to ONU(s) need up to two wavelengths; one for upstream (ONU to OLT) and another one for downstream (OLT to ONU(s)), which is not necessarily the same as the one used for upstream. In order to ensure that the number of available wavelengths is not exceeded for both downstream and upstream traffic, we only need to consider the wavelengths of each outgoing (first level) optical fiber at the OLT, due to two reasons: First reason is wavelength continuity constraint and the other one is all level fibers are equipped with the same transport capacity, so if a request \( r \) is granted on first level fiber, the request will also be granted on the remaining level fibers. The following set of capacity constraints ensures
that the limited number of wavelengths, $W$, is not exceeded:

$$
\sum_{r \in \mathcal{R} : D_r \cap \mathcal{O}^\ell \neq \emptyset} x_r + \sum_{r \in \mathcal{R}^{\text{ACCESS}} : s_r \in \mathcal{O}^\ell} x_r \leq W \quad \ell \in \omega(\text{OLT})
$$

(1)

for all outgoing links of the OLT.

Each ONU can receive its request on at most one wavelength, as we assume only one receiver per ONU; at most one request can be granted to each ONU in the downstream direction:

$$
\sum_{r \in \mathcal{R} : \text{ONU}_i \in D_r} x_r \leq 1 \quad \text{ONU}_i \in \mathcal{O}.
$$

(2)

Each ONU can send its request on at most one wavelength, as we assume only one transmitter per ONU; at most one request can be granted to each ONU in the upstream direction:

$$
\sum_{r \in \mathcal{R}^{\text{ACCESS}} : s_r = \text{ONU}_i} x_r \leq 1 \quad \text{ONU}_i \in \mathcal{O}.
$$

(3)

We do not need to add explicitly the constraints stating which wavelengths to assign to a given ONU, one for the downstream signal, another one for the upstream signal due to the following result.

\textbf{Theorem 3.5.1.} Given a solution to the MC-WA problem, one can always associate wavelength to the granted requests using a greedy algorithm.

\textit{Proof.} The set of used wavelengths can be split between the wavelengths for downstream requests and for those upstream requests. On link $\ell$, there are $W_D^\ell = \sum_{r \in \mathcal{R} : D_r \cap \mathcal{O}^\ell \neq \emptyset} x_r$ downstream wavelengths, and $W_U^\ell = \sum_{r \in \mathcal{R}^{\text{ACCESS}} : s_r \in \mathcal{O}^\ell} x_r$ upstream wavelengths. Equation (1) guarantees that we do not exceed the number of available wavelengths, i.e., $W$. 

51
On link $\ell$, we may assign the first $W^D$ wavelengths to the downstream traffic, so that the $j^{th}$ granted core request $r_j$ on link $\ell \in \omega(\text{OLT})$ is assigned wavelength $\lambda_j$ and the remaining wavelengths to the upstream traffic. This means that if $\text{ONU}_i \in D_{r_j}$, then $\text{ONU}_i$ receives signal on $\lambda_j$. Similarly, again on link $\ell$, the $j^{th}$ granted access request $r_j$ on link $\ell \in \omega(\text{OLT})$ is assigned wavelength $\lambda_{j+W^P}$, and $\text{ONU}_{ij}$ sends signal on $\ell \in \omega(\text{OLT})$. Equations (2) and (3) ensure that each $\text{ONU}$ receives signal on at most one wavelength for downstream traffic and sends signal on at most one wavelength for upstream traffic.

### 3.5.2 MC-WA on WDM-PON Light Mesh Network Topology

Light mesh topology is an extension of the tree topology where some $\text{ONUs}$ have more than one potential paths from $\text{OLT}$, see Section 2.3.1. Such a topology may be of interest in order to provide some sort of resiliency in case of link failure, as discussed in Section 2.4. We consider that not necessarily all $\text{ONUs}$ have more than one link disjoint paths on which they can be reached from the $\text{OLT}$. Indeed, today, it would be probably too costly to design a WDM-PON network which would be protected against all link failures. Therefore, we remain with the assumptions that each $\text{ONU}$ is equipped with a single transmitter and a single receiver, meaning that it can receive its request(s) on at most one wavelength and sends its request(s) on at most one wavelength, but with the possibility to switch over another link (and wavelength) in case of a link failure.

The main difference between a tree and a light mesh topology is that, we do not have any more $O^\ell \cap O^{\ell'} = \emptyset$ for two different links $\ell$ and $\ell'$ of $\omega(\text{OLT})$. It means that we may have several potential routes from the $\text{OLT}$ to a given $\text{ONU}$, however we have few routes (we consider at most two routes).

In this thesis, we assign at most one wavelength to each provisioned $\text{ONU}$ for each
direction (if applicable). We assume each link has some spare wavelengths. In case of link failure, the ONUs with more than one potential path will be assigned the spare wavelength on one of the alternate link.

Variables

In order to set the MC-WA model for light mesh topology, we use a first decision vector with binary variables, such that each component \( x_r \), used for request \( r \). We also set two decision vectors with binary variables in order to check whether a given link belongs to the route of a given request, and if this is the case, whether the route traverses the link in the downstream or the upstream direction. Let \( x^D_\ell = (x^D_{r\ell})_{r \in R} \) and \( x^U_\ell = (x^U_{r\ell})_{r \in R} \), for each link \( \ell \in \omega(OLT) \) be such that, for a given \( \ell \), \( x^D_{r\ell} \) (resp. \( x^U_{r\ell} \)) is equal to 1 if request \( r \) is routed through \( \ell \) in the downstream (resp. upstream) direction, and 0 otherwise.

Two other vectors with binary variables, such that each component \( x^i_{\ell i} \) and \( x^d_{\ell i} \), are used in order to check whether each destination ONU \(_i\) and source ONU \(_i\) with respect to request \( r \) on link \( \ell \) is served or not. If \( x^i_{\ell i} = 1 \), it means destination ONU \(_i\) of request \( r \) is served on link \( \ell \) in the downstream direction, 0 otherwise (respectively \( x^d_{\ell i} \) in the upstream direction).

Capacity constraints

Each fiber has a limited transport capacity in terms of the number of wavelengths, \( W \). Again there is no TDMA, and each request (again, which is aggregated traffic requested from service provider) required one wavelength in each direction. We must consider the requests of type (OLT, \( D_r \)) with only a downstream direction, and both the downstream
components and the upstream components of the requests of type \((\text{ONU}_i, D_r)\):

\[
\sum_{r \in R : D_r \cap \mathcal{O}^f \neq \emptyset} x^D_{r\ell} + \sum_{r \in \mathcal{R}_{\text{access}}, \ell \in \mathcal{O}^f} x^U_{r\ell} \leq W \quad \ell \in \omega(\text{OLT})
\]  \hspace{1cm} (4)

for all outgoing links at the OLT.

Each ONU can receive its request on at most one wavelength on each incoming fiber. A multicast request \(r\) is not necessarily provisioned with the same wavelength for all its ONU destinations:

\[
\sum_{r \in R : \text{ONU}_i \in \mathcal{O}^f} \sum_{\ell : \text{ONU}_i \in \mathcal{O}^f} x^f_{r\ell} \leq 1 \quad \text{ONU}_i \in \mathcal{O}.
\]  \hspace{1cm} (5)

Each ONU can send its request on at most one wavelength on each incoming fiber. Transmitting and receiving must be on the different wavelengths:

\[
\sum_{r \in \mathcal{R}_{\text{access}}} \sum_{\ell : \text{ONU}_i \in \mathcal{O}^f} x^f_{s_r} \leq 1 \quad s_r : \text{ONU}_i \in \mathcal{O}.
\]  \hspace{1cm} (6)

**Request constraints**

Each destination ONU\(_i\) of request \(r\) must be served in the downstream direction, if the request \(r\) is granted. Conversely, if for a given request \(r\), a single of its destination ONU\(_i\) \(\in D_r\) is not served in the downstream direction, then the request \(r\) is denied:

\[
x_r \leq \sum_{\ell : \text{ONU}_i \in \mathcal{O}^f \cap D_r} x^f_{r\ell} \quad \text{ONU}_i \in D_r, r \in \mathcal{R}.
\]  \hspace{1cm} (7)

For a request \(r\) originated at an ONU, the source \(s_r = \text{ONU}_i\) must be served to send its request in the upstream direction on at least one of its link \(\ell\) allowing it to reach the OLT,
if the request $r$ is granted. Conversely, if for a given request $r$, a source $s_r = \text{ONU}_i$ is not served in the upstream direction, then the request $r$ is denied:

$$x_r \leq \sum_{\ell: s_r \in \mathcal{O}^f} x_{s_r}^f \quad s_r \in \mathcal{R}^{\text{ACCESS}} : r \in \mathcal{R}^{\text{ACCESS}}.$$  \hspace{1cm} (8)

### Relationship among the variables

The destination $\text{ONU}_i$ of request $r$ can be served (downstream direction) using link $\ell$ only if request $r$ is granted on link $\ell$ for its downstream direction:

$$x_{r|\ell}^f \leq x_{r|\ell}^D \quad \ell: \text{ONU}_i \in \mathcal{O}^f, \text{ONU}_i \in \mathcal{D}_r, r \in \mathcal{R}.$$ \hspace{1cm} (9)

The source $s_r = \text{ONU}_i$ is to be served (upstream direction) using link $\ell$ only if request $r$ is granted on link $\ell$ for its upstream direction:

$$x_{s_r}^f \leq x_{r|\ell}^U \quad \ell: s_r \in \mathcal{O}^f, s_r \in \mathcal{R}^{\text{ACCESS}} : r \in \mathcal{R}^{\text{ACCESS}}.$$ \hspace{1cm} (10)

### 3.5.3 MC-GWA on WDM-PON Tree Network Topology

In the traffic-grooming models, we can serve several requests on a single wavelength. As we have stated earlier, we assume each ONU has a single receiver/transmitter but it can receive/transmit multiple services (requests) on a single wavelength through TDMA. Distinct wavelengths must be used for downstream (receive) and upstream (transmit) directions. As we have a large number of variables in the traffic-grooming models, we will use a column generation modeling. It then means that the original problem is divided into two sub-problems, the master problem and the pricing problem. The master problem deals with
maximizing the number of weighted requests under a given network condition, e.g., number of wavelengths (on each fiber) and transmitters/receivers (in each ONU). The pricing problem consider the transport capacity of each wavelength to provision the downstream and/or upstream components of the requests.

Let $C$ be the overall set of configurations. A configuration $C \in C$ corresponds to provisioned request components (either the uplink components of request(s), and/or one of its downlink components, i.e., a subset of served ONUs) assigned on the same wavelength. The master problem will take care of selecting these configurations. Each configuration is associated with one wavelength of all the links; in other words, each configuration associated to a single wavelength of $\ell_1, \ell_2, \ldots, \ell_n$. Furthermore, in a given configuration each link cannot have both of its upstream and downstream directions served. Additionally, each link may have more than one requests components, but all the request(s) components must be either uplink or downlink of that link.

**Master Problem**

In the master problem, we consider constraints related to the transport capacity of the fibers in terms of the number of wavelengths, capacity of ONUs in terms of the number of receivers/transmitters. All the configurations generated by the pricing problem are decided in the master problem for their selecting or rejecting, under the condition of a given fiber and ONU capacity. In addition, master problem will also make sure about the components of requests in order to grant the request $r$, e.g., all the destination ONUs must be served and if request is originated at an ONU, the source ONU must be served. A transport capacity of each wavelength is considered in the pricing problem.
Parameters

In order to define the configurations, we need the following parameters:

\[ C_{ri} = \begin{cases} 
1 & \text{if destination ONU}_i \text{ is served with respect to } r \text{ in } C \quad \text{ONU}_i \in \mathcal{D}_r, r \in \mathcal{R}. \\
0 & \text{otherwise.} 
\end{cases} \]

\[ C_r^U = \begin{cases} 
1 & \text{if source } s_r = \text{ONU}_i \text{ can be provisioned in } C \quad \text{ONU}_i = s_r : r \in \mathcal{R}^{ACCESS}. \\
0 & \text{otherwise.} 
\end{cases} \]

\[ C_i^D = \begin{cases} 
1 & \text{if destination } \text{ONU}_i \text{ is served by at least one request } r \text{ in } C \quad \text{ONU}_i \in \mathcal{O}. \\
0 & \text{otherwise.} 
\end{cases} \]

\[ C_i^U = \begin{cases} 
1 & \text{if source } \text{ONU}_i \text{ is served by at least one request } r \text{ in } C \quad \text{ONU}_i \in \mathcal{O}. \\
0 & \text{otherwise.} 
\end{cases} \]

It follows that each configuration is formally defined by the pricing problem.

Variables

We use a variable \( x^C, (C \in \mathcal{C}) \), to indicate if a configuration is selected and later assigned a given wavelength \( \lambda \):

\[ x^C = \begin{cases} 
1 & \text{if } C \text{ is selected} \quad C \in \mathcal{C}. \\
0 & \text{otherwise.} 
\end{cases} \]

We next need two decision vectors in relation with the requests and their destination ONUs.
The first decision vector is such that each component $x_r$ is used to indicate whether request $r$ is granted or not. The second vector is such that each component $x_{ri}$ is used to indicate whether ONU$_i$, when the destination of request $r$ (for $r \in \mathcal{R}, \text{ONU}_i \in \mathcal{D}_r$), is served or not in the downstream direction, describes as follows.

$$
x_{ri} = \begin{cases} 
1 & \text{if destination ONU}_i \text{ is served with respect to request } r \\
0 & \text{otherwise.}
\end{cases} \quad \text{ONU}_i \in \mathcal{D}_r, r \in \mathcal{R}.
$$

**Capacity constraints**

In each network link/fiber, there is a limited number of wavelengths (channels), $W$. Therefore, no more configurations than the available number of wavelengths can be used, as each configuration is associated with a wavelength of each link outgoing at the OLT:

$$
\sum_{C \in \mathcal{C}} x^C \leq W. \quad \text{(11)}
$$

Each ONU can receive its request(s) on at most one wavelength, as we assume only one receiver per ONU:

$$
\sum_{C \in \mathcal{C}} \left( \max_{r \in \mathcal{R}} C_{ri} \right) x^C \leq 1 \quad \text{ONU}_i \in \mathcal{O} \quad \text{(12)}
$$

where $\max_{r \in \mathcal{R}} C_{ri} = C_{i}^D$, and $\max_{r \in \mathcal{R}} C_{ri} = 1$ if at least one request $r$ is granted (downstream direction) in order to serve ONU$_i$, and 0 otherwise.

Each ONU can send its request(s) on at most one wavelength, as we assume only one
transmitter per ONU:

\[ \sum_{C \in C} \left( \max_{r \in R^{\text{ACCESS}}, s_r = \text{ONU}_i} C_r^U \right) x^C \leq 1 \quad \text{ONU}_i \in O \]  

(13)

where \( \max_{r \in R^{\text{ACCESS}}, s_r = \text{ONU}_i} C_r^U = C_i^U \), and \( \max_{r \in R^{\text{ACCESS}}, s_r = \text{ONU}_i} C_r^U = 1 \) if at least one request \( r \) is granted (upstream direction) in order to serve \( s_r = \text{ONU}_i \), and 0 otherwise.

**Request constraints**

In order to serve the destination \( \text{ONU}_i \) of request \( r \), it must be served in one of its associated granted configuration \( x^C \) in the downstream direction:

\[ \sum_{C \in C} C_{ri} x^C \geq x_{ri} \quad \text{ONU}_i \in D_r, r \in R. \]  

(14)

In order for a request to be granted, all of its destination ONUs must be served in the downstream direction, possibly in a different configuration (i.e., wavelength):

\[ x_r \leq x_{ri} \quad \text{ONU}_i \in D_r, r \in R. \]  

(15)

In order for a request to be granted, its source \( s_r = \text{ONU}_i \) of request \( r \) must be served, in one of its associated granted configuration \( x^C \) in the upstream direction:

\[ \sum_{C \in C} C_r^U x^C \geq x_r \quad r \in R^{\text{ACCESS}}. \]  

(16)
Pricing Problem

Each pricing problem corresponds to the generation of a configuration. Each configuration is associated with all the provisioning made on a particular wavelength of each link outgoing at the OLT.

Parameters

Following are five input parameter vectors for the pricing problem provided by the solution of the master problem. The first parameter, \( u_0 \), is the dual variable associated with constraint (11). The second parameter vector \( u^D = (u^D_i) \) is such that \( u^D_i \geq 0 \), is the dual variable associated with constraint (12-i). The third parameter vector \( u^U = (u^U_i) \) is such that \( u^U_i \geq 0 \), is the dual variable associated with constraint (13-i). The fourth parameter vector \( v^D = (v^D_{ri}) \) is such that \( v^D_{ri} \leq 0 \), is the dual variable associated with constraint (14-ri). The last parameter vector \( v^U = (v^U_r) \) is such that \( v^U_r \leq 0 \), is the dual variable associated with constraint (16-r).

Variables

In order to set the configuration, all the parameters, such that each component \( C^D_i \), \( C^U_i \), \( C_{ri} \), and \( C^U_r \), defined in the master problem are the variables in the pricing problem. The detail of these variables can be seen in the parameters of the master problem. In addition, another vector such that each component \( C^D_{r\ell} \) is used to determine request \( r \) on link \( \ell \) is granted or
not, described as follows:

\[ C_{rt}^{D} = \begin{cases} 
1 & \text{if request } r(r \in R) \text{ is granted in configuration } C, \text{ which entails the servicing of} \\
& \text{all or few } ONU_i \in D_r \text{ on link } \ell \text{ in the downstream direction} \\
0 & \text{otherwise.} 
\end{cases} \]

\( \text{ONU}_i \in D_r \cap O^\ell, r \in R. \)

We also use two decision vectors \( y^D \) and \( y^U \) such that each component \( y^D_\ell \) of \( y^D \) (respectively \( y^U_\ell \) of \( y^U \)) indicates whether the wavelength of link \( \ell \) that is associated to the configuration is used or not in the downstream direction (respectively in the upstream direction). Here is a description of \( y^D_\ell \) and \( y^U_\ell \):

\[ y^D_\ell = \begin{cases} 
1 & \text{if wavelength of link } \ell \text{ is used in the downstream direction} \\
0 & \text{otherwise.} 
\end{cases} \quad \ell \in \omega(OLT). \]

\[ y^U_\ell = \begin{cases} 
1 & \text{if wavelength of link } \ell \text{ is used in the upstream direction} \\
0 & \text{otherwise.} 
\end{cases} \quad \ell \in \omega(OLT). \]

**Objective**

The objective of the pricing problem is the reduced cost:

\[
\overline{C}_{\text{COST}} = -u_0 - \sum_{i:ONU_i \in D} u^D_i \left( \max_{r \in R} C_{ri} \right) - \sum_{i:ONU_i \in D} u^U_i \left( \max_{r \in R, s_r = ONU_i} C^U_r \right) \\
- \sum_{r \in R} \sum_{i:ONU_i \in D_r} y^D_{ri} C_{ri} - \sum_{r \in R_{\text{ACCESS}}} y^U_r C^U_r
\]

61
where \( u_{0}, u^{D}_{i}, u^{U}_{i}, v^{D}_{ri}, \) and \( v^{U}_{r} \) are dual values as defined in the set of parameters of the pricing problem.

It can be rewritten:

\[
\overline{C_{COST}} = -u_{0} - \sum_{i: ONU_{i} \in O} u^{D}_{i} C^{D}_{i} - \sum_{i: ONU_{i} \in O} u^{U}_{i} C^{U}_{i} - \sum_{r \in R} \sum_{i: ONU_{i} \in D_{r}} v^{D}_{ri} C_{ri} - \sum_{r \in R_{ACCESS}} v^{U}_{r} C^{U}_{r}
\]

using the definitions of the variables \( C^{D}_{i} \) and \( C^{U}_{i} \), for all \( ONU_{i} \) belonging to \( O \).

**Request constraints**

In order to serve \( ONU_{i} \) in the downstream direction, at least one request with \( ONU_{i} \) as a destination has to be served in configuration \( C \) in the downstream direction:

\[
C^{D}_{i} \geq C_{ri} \quad ONU_{i} \in D_{r}, r \in R. \tag{17}
\]

Note that constraints (17) entails that: \( C^{D}_{i} \geq \max_{r \in R} C_{ri} \) for \( ONU_{i} \in O \).

In order to serve \( ONU_{i} \) in the upstream direction, at least one request with \( ONU_{i} \) as a source has to be served in this configuration \( C \) in the upstream direction:

\[
C^{U}_{i} \geq C^{U}_{r} \quad i: ONU_{i} = s_{r}, r \in R_{ACCESS}. \tag{18}
\]

Similarly, constraints (18) entails that: \( C^{U}_{i} \geq \max_{r \in R_{ACCESS}, s_{r} = ONU_{i}} C^{U}_{r} \) for \( ONU_{i} \in O \)

In order to serve a given \( ONU_{i} \) in the downstream direction, there must be at least one
request \( r \) such that \( \text{ONU}_i \in \mathcal{D}_r \) is served in this configuration \( C \) in the downstream direction:

\[
C^D_i \leq \sum_{r \in \mathcal{R}} C_{ri} \quad \text{ONU}_i \in \mathcal{O}. \tag{19}
\]

In order to serve a given \( \text{ONU}_i \) in the upstream direction, there must be at least one request \( r \) such that \( \text{ONU}_i = s_r \) \((s_r \in \mathcal{R}^{\text{ACCESS}})\) is served in configuration \( C \) in the upstream direction:

\[
C^U_i \leq \sum_{r \in \mathcal{R}^{\text{ACCESS}}: \text{ONU}_i = s_r} C^U_r \quad \text{ONU}_i \in \mathcal{O}. \tag{20}
\]

**Capacity constraints**

A wavelength of link \( \ell \) can be used in either the downstream or the upstream direction or non of them; but not for both directions:

\[
y^D_\ell + y^U_\ell \leq 1 \quad \ell \in \omega(\text{OLT}). \tag{21}\]

The transport capacity of each wavelength is given. We must consider the transport capacity of wavelength of each link \( \ell \) outgoing at the OLT and it should not be exceeded. In this set of constraints, we consider this transport capacity, when it is used in the downstream direction. The parameter such that each component \( B_\ell \) is the transport capacity of a wavelength, as we assume all the wavelengths of all the fibers have same transport capacity.

Another parameter such that each component \( b^D_\ell \) is a required bandwidth of request \( r \) in the downstream direction:

\[
\sum_{r \in \mathcal{R}: \mathcal{D}_r \cap \mathcal{O}^\ell \neq \emptyset} b^D_r C^D_{ri} \leq B_\ell y^D_\ell \quad \ell \in \omega(\text{OLT}). \tag{22}\]
A given destination $\text{ONU}_i$ of request $r$ can be served in the downstream direction, only if request $r$ is granted on link $\ell$ in the downstream direction, such that $\text{ONU}_i$ receives its request $r$ on link $\ell$:

$$C_{ri} \leq C^D_{r\ell} \quad \text{ONU}_i \in \mathcal{D}_r, r \in \mathcal{R}. \quad (23)$$

Similarly, in this set of constraint we consider the transport capacity of each wavelength, when it is used in the upstream direction. The parameter such that each component $b^U_r$ is a required bandwidth of request $r$ in the upstream direction:

$$\sum_{r \in \mathcal{R}_{\text{ACCESS}}, b_r \in \mathcal{O}^U} b^U_r C^U_r \leq B_{\ell} b^U_{\ell} \quad \ell \in \omega(\text{OLT}). \quad (24)$$

**Example 2:** Consider a tree topology shown in Figure 21 with two outgoing links where each link has two wavelengths.

![Tree topology example](image)

Figure 21: A tree topology example

Suppose we are given a traffic with 3 requests $r_1, r_2$ and $r_3$. Table 3 shows the details of cost, required bandwidth, source and destinations for each request.
<table>
<thead>
<tr>
<th>Request</th>
<th>Cost</th>
<th>Required Bandwidth</th>
<th>Source</th>
<th>Destinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>r₁</td>
<td>3</td>
<td>0.5</td>
<td>ONU₂</td>
<td>ONU₁, ONU₂, ONU₄</td>
</tr>
<tr>
<td>r₂</td>
<td>3</td>
<td>0.5</td>
<td>OLT</td>
<td>ONU₂, ONU₃, ONU₄</td>
</tr>
<tr>
<td>r₃</td>
<td>2</td>
<td>0.5</td>
<td>OLT</td>
<td>ONU₁, ONU₃</td>
</tr>
</tbody>
</table>

Table 3: Traffic for tree network

Some of the generated configurations are depicted in Table 4.

<table>
<thead>
<tr>
<th></th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>r₁, ONU₁</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>r₁, ONU₂</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>r₁, ONU₄</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>r₂, ONU₂</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>r₂, ONU₃</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>r₂, ONU₄</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>r₃, ONU₁</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>r₃, ONU₃</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>r₁, ONU₂ (Up)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4: A few generated configurations for Example 2

Note that each configuration is associated to a single wavelength outgoing at the OLT in the network topology. The master problem will select the two optimal configurations, i.e., C₃ and C₄. The solution of selected configurations for this particular example is:

- In C₃, we serve r₁ and r₂ in the downstream direction on link-1 on λ₁, and r₁ and r₂ in the downstream direction on link-2 on λ₁.

- In C₄, we serve r₁ in the upstream direction on link-1 on λ₂.

Note that there is still one wavelength (λ₂) on link-2, where ONU₃, requested as one of r₃ destination, can be served, while another destination, i.e., ONU₁, requested in r₃ has no available wavelength on link-1 which prevents r₃ from being served. Observe that it is indeed
possible to grant any two requests. Consequently, requests with the higher cost/weight, i.e., $r_1$ and $r_2$ are granted, and rejected the lowest cost request, i.e., $r_3$.

**How to Solve the MC-GWA Model**

The generic algorithm to solve column generation formulation (proposed in the master problem and the pricing problem) are as follows:

1. Initialize a single configuration that satisfy all the constraints.

2. Solve the LP relaxation of the restricted master problem.

3. Use the dual values of the constraints of the restricted master problem in order to define the reduced cost, i.e., the objective function of the pricing problem.

4. Solve the pricing problem.

5. If the reduced cost of pricing problem is non negative\(^1\) ($\geq 0$), take objective variable values and put them in master problem as parameters of a new generated column and goto step 2.

6. Otherwise, if the solution of pricing problem is negative (less than zero), solve master problem at ILP.

The flow diagram of described algorithm is shown in Figure 22.

**How it works**

The restricted master problem is solved on the generated configurations by the pricing problem. Each configuration is associated to a single wavelength of all outgoing links

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\(^1\)In case of cycling problems, i.e., degeneracy problem, one needs to use perturbations techniques on the so-called Bland's rule [12] to overcome them.
at the OLT. The master problem provides the downstream and upstream components of the requests — based on the dual values of configuration constraints — to the pricing problem. Each configuration is defined in pricing problem, where it provisions ONUs on a single wavelength of each link. If pricing problem generates the configuration (in this case maximizes the objective, i.e., positive value), the new column will be added in the master problem based on generated configuration. This is an iterative process, and in each solution of the master problem, the objective value is going to be increased.

Initially, in the master problem, we have initialized one configuration which satisfied all the constraints but not necessarily optimal, so it gives some dual values which are not necessary to generate configurations in the pricing problem that led to optimal solution in the master problem. After some initial iterations based on dual values of master problem, the pricing problem generate some configuration that led to find an optimal solution in the master problem.

3.5.4 MC-GWA on WDM-PON Light Mesh Network Topology

As we have discussed in the Section 3.2, the solution problems with a large number of variables are divided into two steps. Like the tree traffic-grooming problem, light mesh traffic-grooming problem is also divided into two subproblems: Master problem and pricing problem, discussed each in the following sections. Again, the definition of configuration $C$ ($C \in \mathcal{C}$) is also same as defined in the Section 3.5.3.

Master Problem

In contrast with the tree network topology, the light mesh network topology has some ONUs with more than one paths. In the light mesh network topology, ONUs with multiple paths
are considered in the pricing problem. Consequently, there is no change in a master problem and the same master problem, described in Section 3.5.3 can be used in this problem. For objective, constraints and their parameters, variables description, refer to Section 3.5.3.

Figure 22: How to solve a column generation formulation

Pricing Problem

Again, each configuration $C$ is associated to one wavelength, the same for all the links in the PON. The parameters, such that each component $u^P_i$, $u^U_i$, $v^D_{ri}$, $v^U_{ri}$, and variables, such that each component $C^P_i$, $C^U_i$, $C_{ri}$, $C^V_i$, $y^P_i$, $y^U_i$, $C^D_{ri}$, are same as defined in Section 3.5.3. In addition, in a light mesh network topology, some ONUs has multiple paths, therefore, we
use two other variable vectors, in order to set a light mesh model, described as follows:

\[
C_{r}^{\ell} = \begin{cases} 
1 & \text{if destination ONU}_i \text{ of request } r \text{ is served on link } \ell \text{ in configuration } C \text{ in the downstream direction} \\
0 & \text{otherwise.}
\end{cases}
\]

\[\ell : \text{ONU}_i \in \mathcal{O}^{\ell}, \text{ONU}_i \in \mathcal{D}_r, r \in \mathcal{R}.\]

\[
C_{r}^U = \begin{cases} 
1 & \text{if source ONU}_i \text{ of request } r \text{ is served on link } \ell \text{ in configuration } C \text{ in the upstream direction} \\
0 & \text{otherwise.}
\end{cases}
\]

\[s_r \in \mathcal{O}^{\ell}, r \in \mathcal{R}^{\text{ACCESS}}.\]

\[
C_{r}^{U\ell} = \begin{cases} 
1 & \text{if request } r \text{ is granted on link } \ell \text{ in configuration } C \text{ in the upstream direction} \\
0 & \text{otherwise.}
\end{cases}
\]

\[\ell : s_r \in \mathcal{O}^{\ell}, r \in \mathcal{R}^{\text{ACCESS}}.\]

**Objective**

The expression of the reduced cost, i.e., the objective function of the pricing problem is expressed as follows:

\[
\overline{C}_{\text{COST}} = -u_0 - \sum_{i: \text{ONU}_i \in \mathcal{O}} u_i^D C_i^D - \sum_{i: \text{ONU}_i \in \mathcal{O}} u_i^U C_i^U - \sum_{r \in \mathcal{R}} \sum_{i: \text{ONU}_i \in \mathcal{D}_r} u_{ri}^D C_{ri}^D - \sum_{r \in \mathcal{R}^{\text{ACCESS}}} u_r^U C_r^U.
\]

69
Request constraints

The subset constraints related to the configuration, i.e., (25), (26), (27), and (28), to determine the upstream and downstream components provisioned in configuration $C$, are same as subset configuration constraints, i.e., (17), (18), (19), and (20), discussed in Section 3.5.3.

\[ C_i^D \geq C_{ri} \quad \text{ONU}_i \in \mathcal{D}, r \in \mathcal{R}. \]  
\[ \text{(25)} \]

\[ C_i^U \geq C_r^U \quad i : \text{ONU}_i = s_r, r \in \mathcal{R}^{\text{ACCESS}}. \]  
\[ \text{(26)} \]

\[ C_i^D \leq \sum_{r \in \mathcal{R}} C_{ri} \quad \text{ONU}_i \in \mathcal{O}. \]  
\[ \text{(27)} \]

\[ C_i^U \leq \sum_{r \in \mathcal{R}^{\text{ACCESS}}: \text{ONU}_i = s_r} C_r^U \quad \text{ONU}_i \in \mathcal{O}. \]  
\[ \text{(28)} \]

In order to serve destination $\text{ONU}_i$ of request $r$ on link $\ell$, it must be served in granted configuration $C$ in the downstream direction:

\[ C_{ri}^D \leq C_{ri} \quad \ell : \text{ONU}_i \in \mathcal{O}^\ell, \text{ONU}_i \in \mathcal{D}, r \in \mathcal{R}. \]  
\[ \text{(29)} \]

In order to serve source $s_r = \text{ONU}_i$ of request $r$ on link $\ell$, it must be served in granted configuration $C$ in the upstream direction:

\[ C_r^U \leq C_r^U \quad \ell : s_r \in \mathcal{O}^\ell, r \in \mathcal{R}^{\text{ACCESS}}. \]  
\[ \text{(30)} \]
In order to grant ONU\textsubscript{i} of request \textit{r}, it must be granted on at least one of its link in the downstream direction. In a light mesh network topology, some ONUs are connected to more than one intermediate node. In other words, some ONUs have multiple paths from the OLT through intermediate nodes:

\[
C_{ri} \leq \sum_{\ell: \text{ONU}_{i} \in O^\ell} C_{ri}^\ell \quad \text{ONU}_i \in \mathcal{D}_r, r \in \mathcal{R}. \tag{31}
\]

In order to grant source \textit{s}_\text{r} = \text{ONU}_i of request \textit{r}, it must be granted on at least one of its link in the upstream direction. Again, in a light mesh network topology, some ONU are connected to more than one intermediate node. In other words, some ONUs can reach to the OLT on multiple paths through intermediate nodes:

\[
C_{r}^{U} \leq \sum_{\ell: \text{s}_r \in O^\ell} C_{r}^{\ell} \quad r \in \mathcal{R}^{\text{ACCESS}}. \tag{32}
\]

Capacity constraints

A wavelength can be used in either the downstream or the upstream direction or none of them, but not for both directions:

\[
y_{\ell}^{D} + y_{\ell}^{U} \leq 1 \quad \ell \in \omega(\text{OLT}). \tag{33}
\]

A following set of constraints assure that the transport capacity of each wavelength, i.e.,
\[ \sum_{r \in \mathcal{R} : \text{ONU}_i \in \mathcal{D}_r \cap \mathcal{O}^\ell} b^D_r C^D_{rl} \leq B_\ell y^D_\ell \quad \ell \in \omega(\text{OLT}). \quad (34) \]

Similarly, a following set of constraints assures that the transport capacity of each wavelength can not be exceeded, when it is used in the upstream direction:

\[ \sum_{r \in \mathcal{R}^{\text{ACCESS}} : \text{ONU}_i \in \mathcal{O}^\ell} b^U_r C^U_r \leq B_\ell y^U_\ell \quad \ell \in \omega(\text{OLT}). \quad (35) \]

A given destination \text{ONU}_i of request \( r \) can be served in the downstream direction, only if request \( r \) is granted on link \( \ell \) in the downstream direction, which allows reaching \text{ONU}_i:

\[ C^D_{rl} \leq C^D_{r\ell} \quad \ell : \text{ONU}_i \in \mathcal{O}^\ell, \text{ONU}_i \in \mathcal{D}_r, r \in \mathcal{R}. \quad (36) \]

A given source \( s_r = \text{ONU}_i \) of request \( r \) can be served in the upstream direction, only if request \( r \) is granted on link \( \ell \) in the upstream direction:

\[ C^U_r \leq C^U_{r\ell} \quad \ell : s_r \in \mathcal{O}^\ell, r \in \mathcal{R}^{\text{ACCESS}}. \quad (37) \]

Each destination \text{ONU}_i of \( \mathcal{D}_r \) can be served on at most one link; it means that if \text{ONU}_i of \( \mathcal{D}_r \) has multiple paths down from the OLT, it can be served on at most one path, and the other ones can be used for backup purposes assuming a path switch mechanism (as discussed in Section 2.4.2) is available:

\[ \sum_{\ell : \text{ONU}_i \in \mathcal{O}^\ell} C^\ell_{rl} \leq 1 \quad \text{ONU}_i \in \mathcal{D}_r, r \in \mathcal{R}. \quad (38) \]
Each source $s_r = \text{ONU}_i$ of $\mathcal{R}^{\text{ACCESS}}$ can send its request(s) on at most one link; it means that if a given source $\text{ONU}_i = s_r$ of $\mathcal{R}^{\text{ACCESS}}$ has the choice among multiple paths to reach the OLT, it can send its request(s) on at most one path.

$$\sum_{\ell: s_r \in \mathcal{O}^\ell} C_{r}^{\ell} \leq 1 \quad r \in \mathcal{R}^{\text{ACCESS}}.$$ (39)

**Example 3:** Consider a light mesh topology shown in Figure 23 with two outgoing links where each link has two wavelengths.

![Light mesh topology](image)

Figure 23: A light mesh topology example

Suppose we are given a traffic with 3 requests $r_1$, $r_2$ and $r_3$. Table 5 shows the details of required bandwidth, source and destination for each request.
<table>
<thead>
<tr>
<th>Request</th>
<th>Cost</th>
<th>Required Bandwidth</th>
<th>Source</th>
<th>Destinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>3</td>
<td>0.5</td>
<td>ONU$_2$</td>
<td>ONU$_1$, ONU$_2$, ONU$_4$</td>
</tr>
<tr>
<td>$r_2$</td>
<td>3</td>
<td>0.5</td>
<td>OLT</td>
<td>ONU$_2$, ONU$_3$, ONU$_4$</td>
</tr>
<tr>
<td>$r_3$</td>
<td>2</td>
<td>0.5</td>
<td>OLT</td>
<td>ONU$_1$, ONU$_3$,</td>
</tr>
</tbody>
</table>

Table 5: Traffic for light mesh network topology

Some of the generated configurations are depicted in Table 6.

<table>
<thead>
<tr>
<th>Request</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$, ONU$_1$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$r_1$, ONU$_2$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$r_1$, ONU$_4$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$r_2$, ONU$_2$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$r_2$, ONU$_3$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$r_2$, ONU$_4$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$r_3$, ONU$_1$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$r_3$, ONU$_3$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$r_1$, ONU$_2$ (Up)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6: A few generated configurations for Example 3

Note that each configuration is associated with one wavelength outgoing at the OLT in the network topology. The master problem will select two optimal configurations, i.e., $C_3$ and $C_4$. The optimal solution of this particular example is:

- In $C_3$, we serve $r_1$ and $r_3$ in the downstream direction on link-1 on $\lambda_1$, and $r_1$ and $r_2$ in the downstream direction on link-2 on $\lambda_1$.  

74
In $C_4$, we serve $r_1$ for upstream on link-1 on $\lambda_2$ and $r_2$ and $r_3$ in the downstream direction on link-2 on $\lambda_2$.

Note that Example 2 uses the same traffic and similar networks as in this example. The basic difference between these two examples is that, in Example 3 ONU$_2$ has two paths whereas ONU$_2$ has a single path in Example 2. As a result, Example 2 grants two requests while Example 3 grants three requests, which shows that the light mesh network topology increases the grade of service as compared to the tree network.

### 3.6 Maximizing the Weighted Number of Served ONUs

In this section, we define the mathematical models for the tree and the light mesh topologies with/without traffic-grooming, where the objective is to maximize the weighted number of served ONUs subject to the available resources in the networks. A partially granted request is defined as a request that is granted to serve a few (at least one) destination ONUs. In this section, while maximizing the number of served ONUs we may grant partially requests and/or whole requests. In a weighted version, we can associate a cost to each served ONU that would represent weight/priority of ONU.

#### 3.6.1 MC-WA on WDM-PON Tree Network Topology

A new optimization model for the tree network without traffic-grooming, with the objective of maximizing the weighted number of served ONUs, is proposed as follows.

**Variables**

In order to set the optimization model, we need three decision vectors. The first vector such that each component $x_r$ corresponds to a request, second vector such that each component
$x_{ri}$ for destination ONU$_i$, and the last vector such that each component $x_{sr}$ for source ONU$_i = s_r$. The detailed definition of these variables is as follows:

$$
x_r = \begin{cases} 
1 & \text{if request } r \text{ is granted} \\
0 & \text{otherwise.} 
\end{cases}, \quad r \in \mathcal{R}.
$$

$$
x_{ri} = \begin{cases} 
1 & \text{if destination ONU$_i$ of request } r \text{ is served in the downstream direction} \\
0 & \text{otherwise.} 
\end{cases}, \quad \text{ONU}_i \in \mathcal{D}_r, r \in \mathcal{R}.
$$

$$
x_{sr} = \begin{cases} 
1 & \text{if source } s_r = \text{ONU}_i \text{ of request } r \text{ is served in the upstream direction} \\
0 & \text{otherwise.} 
\end{cases}, \quad \text{ONU}_i = s_r : r \in \mathcal{R}^{\text{ACCESS}}.
$$

**Objective**

The objective is to maximize the served destination ONUs and can be written as follows:

$$\max \sum_{r \in \mathcal{R}} \sum_{i: \text{ONU}_i \in \mathcal{D}_r} \text{COST}_r x_{ri}$$

where COST$_{ri}$ is the cost associated with ONU$_i$ with respect to request $r$.

**Capacity constraints**

Each fiber has a limited number of wavelengths and each wavelength can be used in either the downstream or the upstream direction. The set of the capacity constraints, in order to make sure the number of wavelengths cannot exceed than the available number of wavelengths
on each link, is defined as follows:

\[
\sum_{r \in \mathcal{R}: \Omega_r \cap \mathcal{O} \neq \emptyset} x_r + \sum_{r \in \mathcal{R}^{\text{ACCESS}}, s_r \in \mathcal{O}} x_r \leq W \quad \ell \in \omega(OLT).
\]

Note again that it is enough to reinforce the optical fiber capacity constraints on the adjacent links of the OLT.

Each ONU can receive its request on at most one wavelength; at most one request, in the downstream direction, can be granted to each ONU:

\[
\sum_{r \in \mathcal{R}, \text{ONU}_i \in \mathcal{D}_r} x_{ri} \leq 1 \quad \text{ONU}_i \in \mathcal{O}.
\]

Similarly, each ONU can send its request on at most one wavelength; at most one request, in the upstream direction, can be granted to each ONU:

\[
\sum_{r \in \mathcal{R}^{\text{ACCESS}}, s_r = \text{ONU}_i} x_{sr} \leq 1 \quad \text{ONU}_i \in \mathcal{O}.
\]

**Request constraints**

In order to grant the request \( r \), at least one of its destination \( \text{ONU}_i \) must be served; if none of the destination \( \text{ONU}_i \) is served, then \( x_r \) denied.

\[
x_r \leq \sum_{i: \text{ONU}_i \in \mathcal{D}_r} x_{ri} \quad r \in \mathcal{R}.
\]

In order to grant the request \( r \), its source \( s_r = \text{ONU}_i \) (if source is not an OLT) must be
served:

\[ x_r \leq x_{sr} \quad r \in \mathcal{R}^{\text{ACCESS}}. \]  

(44)

In order to serve the destination ONU \( i \) with respect to request \( r \), its request \( r \) must be granted:

\[ x_r \geq x_{ri} \quad i : \text{ONU}_i \in \mathcal{D}_r. \]  

(45)

3.6.2 MC-WA on WDM-PON Light Mesh Network Topology

This section describes the MC-WA (without traffic-grooming) problem on WDM-PON light mesh network topology with the objective of maximizing the weighted number of served ONUs.

Variables

In order to set the optimization model we need five decision vectors with binary variables. The description of these variables are as follows:

\[ x_r = \begin{cases} 
1 & \text{if request } r \text{ is granted} \\
0 & \text{otherwise.}
\end{cases} \quad r \in \mathcal{R}. \]

\[ x_{r\ell}^D = \begin{cases} 
1 & \text{if request } r \text{ is granted on link } \ell \text{ in the downstream direction} \\
0 & \text{otherwise.}
\end{cases} \quad \text{ONU}_i \in \mathcal{D}_r \cap \mathcal{O}^\ell, r \in \mathcal{R}. \]
\[ x_{rl}^U = \begin{cases} 1 & \text{if request } r \text{ is granted on link } \ell \text{ in the upstream direction} \\ 0 & \text{otherwise.} \end{cases} \]

\[ s_r \in \mathcal{O}^\ell, r \in \mathcal{R}^{ACCESS}. \]

\[ x_{ri}^f = \begin{cases} 1 & \text{if destination } \text{ONU}_r \text{ of request } r \text{ is served on link } \ell \text{ in the downstream direction} \\ 0 & \text{otherwise.} \end{cases} \]

\[ \text{ONU}_r \in \mathcal{D}_r, r \in \mathcal{R}. \]

\[ x_{sr}^f = \begin{cases} 1 & \text{if source } s_r = \text{ONU}_r \text{ of request } r \text{ is served on link } \ell \text{ in the upstream direction} \\ 0 & \text{otherwise.} \end{cases} \]

\[ \text{ONU}_r = s_r : r \in \mathcal{R}^{ACCESS}. \]

**Objective**

The objective is to maximize the served destination ONUs and can be written as follows:

\[
\max \sum_{r \in \mathcal{R}} \sum_{i : \text{ONU}_i \in \mathcal{D}_r} \sum_{\ell : \text{ONU}_i \in \mathcal{O}^\ell} \text{COST}_{ri} x_{ri}^f
\]

where \( \text{COST}_{ri} \) (see Section 3.4.3) is the cost associated with \( \text{ONU}_i \) with respect to request \( r \).

**Capacity constraints**

In view of the capacity of optical fiber, it has a limited number of wavelengths. We cannot exceed the number of available wavelengths on each link. Note that wavelengths are bidirectional, so they can be used either for downstream or upstream direction. Let us first assume that we are free to use the wavelengths downstream and upstream. The following
set of constraints assures that wavelengths used in the downstream and upstream direction cannot be exceeded on each link outgoing at the OLT:

\[
\sum_{r \in \mathcal{R} : D_r \cap \mathcal{O}^f \neq \emptyset} x^D_{r\ell} + \sum_{r \in \mathcal{R}_{\text{ACCESS}}} x^U_{r\ell} \leq W \quad \ell \in \omega(\text{OLT}). \tag{46}
\]

Each ONU can receive its request on at most one wavelength; and therefore it can receive at most one request in the downstream direction:

\[
\sum_{r \in \mathcal{R}} \sum_{\ell : \text{ONU}_i \in \mathcal{O}^f} x^D_{ri} \leq 1 \quad i : \text{ONU}_i \in \mathcal{O}. \tag{47}
\]

Similarly, each \( s_r = \text{ONU}_i \) can send its request on at most one wavelength; and consequently, it can send at most one request in the upstream direction:

\[
\sum_{r \in \mathcal{R}_{\text{ACCESS}}} \sum_{\ell : s_r \in \mathcal{O}^f} x^U_{s_r} \leq 1 \quad \text{ONU}_i \in \mathcal{O}. \tag{48}
\]

**Request constraints**

In order to serve destination \( \text{ONU}_i \) of request \( r \) in the downstream direction, its request \( r \) must be granted in the downstream direction:

\[
x_r \geq \sum_{\ell : \text{ONU}_i \in \mathcal{O}^f} x^D_{ri} \quad \text{ONU}_i \in \mathcal{D}_r, r \in \mathcal{R}. \tag{49}
\]

If none of the destinations \( \text{ONU}_i \) is served, then \( x_r \) denied; in order to grant request \( r \),
at least one of its destination \( \text{ONU}_i \) must be served in the downstream direction:

\[
x_r \leq \sum_{i: \text{ONU}_i \in \mathcal{D}_r} \sum_{\ell: \text{ONU}_i \in \mathcal{O}^\ell} x_{\ell i}^\ell \quad r \in \mathcal{R}.
\]

In order to serve source \( s_r = \text{ONU}_i \) (if source is not an OLT) of request \( r \), its request \( r \) must be granted in the upstream direction:

\[
x_r \leq \sum_{\ell: s_r \in \mathcal{O}^\ell} x_{s_r}^\ell \quad r \in \mathcal{R}^{\text{ACCESS}}.
\]

**Relationship among the variables**

In order to grant the request \( r \) on link \( \ell \) in the downstream direction, at least one of its destination \( \text{ONU}_i \) must be served on line \( \ell \) in the downstream direction:

\[
x_{r \ell}^D \leq \sum_{i: \text{ONU}_i \in \mathcal{O}^\ell} x_{\ell i}^\ell \quad \text{ONU}_i \in \mathcal{D}_r \cap \mathcal{O}^\ell, r \in \mathcal{R}.
\]

In order to grant the request \( r \) on link \( \ell \) in the upstream direction, its source \( s_r = \text{ONU}_i \) must be served on link \( \ell \) in the upstream direction and vice versa:

\[
x_{r \ell}^U = x_{s_r}^\ell \quad \ell: s_r \in \mathcal{O}^\ell, r \in \mathcal{R}^{\text{ACCESS}}.
\]

In order to serve the destination \( \text{ONU}_i \) of request \( r \) on link \( \ell \) in the downstream direction, its request \( r \) must be granted on link \( \ell \) in the downstream direction:

\[
x_{r \ell}^D \geq x_{\ell i}^\ell \quad i, \ell: \text{ONU}_i \in \mathcal{D}_r \cap \mathcal{O}^\ell, r \in \mathcal{R}.
\]
3.6.3 MC-GWA on WDM-PON Tree Network Topology

Again, this traffic-grooming problem is divided in two steps: Master problem and pricing problem. Let us discuss each problem separately. Note that the configuration $C$ has the same definition as we described in Section 3.5.3.

Master Problem

The master problem is quite similar as we discussed in Section 3.5.3 except that now our objective is different, i.e., to maximize the weighted number of destination ONUs. In this problem, the parameter vectors such that each component $C_{ri}$ and $C^U_{ri}$ for $ONU_i \in D_r, r \in R$ and the variable vectors such that each component $x^C_{ri}$, $x_{ri}$ are also same as defined in Section 3.5.3. Note that, in this problem we do not use the variable vector such that each component $x_r$, but we use new variable vector such that each component $x_{sr}$, defined as follows:

$$x_{sr} = \begin{cases} 1 & \text{if source } s_r = ONU_i \text{ of request } r \text{ is served in the upstream direction} \\ 0 & \text{otherwise.} \end{cases}$$

$$ONU_i = s_r : r \in R^{ACCESS}.$$  

Objective

The objective is to maximize the served (destination) ONUs and can be written as follows:

$$\max \sum_{r \in R} \text{COST}_{ri} x_{ri}$$

where $\text{COST}_{ri}$ (again, see Section 3.4.3 for detail) is the cost associated with $ONU_i$ with respect to request $r$.  

82
Capacity constraints

The set of capacity constraints, i.e., (55), (56), and (57), are same capacity constraints, i.e., (11), (12), and (13), defined in Section 3.5.3:

\[ \sum_{C \in \mathcal{C}} x^C \leq W. \quad (55) \]

\[ \sum_{C \in \mathcal{C}} \left( \max_{r \in \mathcal{R}} C_{ri} \right) x^C \leq 1 \quad \text{ONU}_i \in \mathcal{O}. \quad (56) \]

\[ \sum_{C \in \mathcal{C}} \left( \max_{r \in \mathcal{R}^{\text{ACCESS}}, s_r = \text{ONU}_i} C^{U}_r \right) x^C \leq 1 \quad \text{ONU}_i \in \mathcal{O}. \quad (57) \]

Request constraints

In order to serve destination \( \text{ONU}_i \) with respect to request \( r \) in the downstream direction, one of its associated configuration \( C \) must be granted:

\[ \sum_{C \in \mathcal{C}} C_{ri} x^C \geq x_{ri} \quad \text{ONU}_i \in \mathcal{D}_r, r \in \mathcal{R}. \quad (58) \]

In order to serve destination \( \text{ONU}_i \) with respect to request \( r \) in the downstream direction, the source \( s_r = \text{ONU}_i, (s_r \in \mathcal{R}^{\text{ACCESS}}) \) (if source in not an OLT) must be served in the upstream direction:

\[ x_{ri} \leq x_{sr}, \quad \text{ONU}_i \in \mathcal{D}_r, r \in \mathcal{R}^{\text{ACCESS}}. \quad (59) \]

In order to serve source \( s_r = \text{ONU}_i, (s_r \in \mathcal{R}^{\text{ACCESS}}) \) with respect to request \( r \) (if source in not an OLT) in the upstream direction, one of its associated configuration \( C \) must be
granted:

\[
\sum_{C \in \mathcal{C}} c_r^U x^C \geq x_{s_r}, \quad s_r \in \mathcal{R}^{ACCESS}. \tag{60}
\]

In order to serve source \( s_r = \text{ONU}_i \) with respect to request \( r \) (if source is not an OLT) in the upstream direction, at least one destination \( \text{ONU}_i \) of request \( r \) must be served in the downstream direction:

\[
x_{s_r} \leq \sum_{i: \text{ONU}_i \in \mathcal{D}_r} x_{ri}, \quad s_r \in \mathcal{R}^{ACCESS}. \tag{61}
\]

**Pricing Problem**

The pricing problem is the same as the pricing problem for a tree network topology when the objective is to maximize the weighted number of granted requests in Section 3.5.3. As mentioned previously, the number of granted requests/ONUs are considered in the master problem and the configuration is generated by the pricing problem, so that there is no change in the pricing problem while generating a configuration \( C \). Consequently, the pricing problem has same objective expression as well as constraints and descriptions of their parameters, variables, for details refer to Section 3.5.3.

**3.6.4 MC-GWA on WDM-PON Light Mesh Network Topology**

A light mesh traffic-grooming model, like other traffic-grooming models, is also divided in two subproblems called the master problem and the pricing problem, discussed in the following Sections. A configuration \( C \) has a same characteristics as described in Section 3.5.4.
Master Problem

The master problem is same as master problem for the tree network topology, described in Section 3.6.3, with the same objective. Furthermore, the difference is only in this light mesh network topology, a few or all ONUS has multiple paths to the OLT, so far this multiple paths constraints are considered in the pricing problem. For detail of variables, parameters, objective and constraints refer the master problem of Section 3.6.3.

Pricing Problem

Similarly, the pricing problem is same as the pricing problem for light mesh network topology for the objective of maximizing the weighted number of granted requests in Section 3.5.4. As mentioned previously, the pricing problem provide a configuration to the master problem, which make sure about the each upstream and downstream components of the requests, so far there is no change in the pricing problem. For details of objective expression, constraints and their parameters, variables, refer to Section 3.5.4.
Chapter 4

Simulation

For multicast efficient provisioning of PONs, we have evaluate the performance of the proposed traffic-grooming models with the objective of maximizing the number of ONUs. Performance evaluation has been conducted with experiments. We have analysed the results, in terms of grade of service and bandwidth usage/waste done under the different traffic scenarios on multiple instances, for a given network topology.

In this chapter, we first describe the network topologies and traffic scenarios, that are later used in the experiments. We also describe the implementation of all models described in Chapter 3. Finally, we discuss the result of simulations on different traffic scenarios and network topologies.

4.1 Data Instances

We discuss below the different network topologies and traffic instances we will use in Section 4.3 in the simulations for efficient provisioning of WDM-PONs.
4.1.1 Network Topology Instances

We use a tree network topology and a light mesh network topology. The topology of the tree network uses four outgoing links at the OLT node, each link has a bidirectional fiber with six wavelengths. Also, the end point of the feeder fiber, at the intermediate node on each link, is equipped with one 1:8 optical splitter. Consequently, each optical splitter has eight distribution fibers connecting to the ONU. The transport capacity of each wavelength (on both distribution and feeder fibers) is 1Gbps and the total number of ONU in the network is 32, as shown in Figure 24.

![Figure 24: A 32-ONU tree topology](image)

A light mesh network topology is an extension of a tree topology, where, e.g., twelve ONU (ONU6, to ONU11, and ONU22 to ONU27) are connected with two potential links to optical splitters, as shown in Figure 25. The remaining parameters of the light mesh network topology are the same as discussed in the tree network topology except that the optical
splitter has more split counts, i.e., 1:11. A light mesh network topology is concerned with reliability of the network, where some ONUs have multiple potential paths, and only one path is used, other alternate path is only considered in case of link failures.

![Diagram of a 32-ONU light mesh topology]

Figure 25: A 32-ONU light mesh topology

In the passive optical networks, there are some broadcast services. We assume all the ONUs, subscribe to these broadcast services. In practice these services need only downstream traffic such as video channels and audio channels, therefore if wavelength is used for downstream then we reduce the transport capacity of a wavelength to 800 Mbps, assuming a need of 200 Mbps for these broadcast services. For wavelengths used in the upstream direction, we set the transport capacity to 1 Gbps.
4.1.2 Traffic Instances

We have done large scale experiments for the MC-GWA problem with the objective of maximizing the weighted number of served ONUs on the tree and the light mesh network topologies under three different traffic scenarios. In the first one, we only considered Triple Play Services (TPSs), i.e., voice, Internet, video. In the second scenario, in addition to TPSs, we considered video conferences and audio streams for 10% of ONUs. In the third scenario, we considered the same services as in the second scenario, but video conferences and audio stream services are now assumed to be subscribed by 50% of the ONUs. In all of these three scenarios, we also considered broadcast services discussed previously. Next is the detailed description of these three scenarios:

First Scenario: We consider triple play services with four types of requests.

- First type of requests: Voice, i.e., telephone. We generated one request per ONU with a consumption of 1 Mbps for downstream and 1 Mbps for upstream. The required bandwidth for each user is 1 Mbps in each direction, and we assume 10 telephone users/subscribers are connected to each ONU, therefore, the required bandwidth is 10 Mbps in each direction, i.e., downstream and upstream direction.

- Second type of requests: Internet with browsing. We consider that 90% of the ONUs subscribe to this service. Bandwidth requirement is 20 Mbps for downstream and 1 Mbps for upstream for each user. We assume 5 users/subscribers are connected to each ONU, therefore, the required bandwidth per ONU is 100 Mbps in the downstream direction and 5 Mbps in the upstream direction.
• Third type of requests: They correspond to Internet with browsing and online games. We consider that 10% of the ONUs subscribe to this service. Bandwidth requirement is 20 Mbps for downstream and 20 Mbps for upstream per user. We assume 5 Internet browsing and 2 Internet browsing with online games users/subscribers are connected to each ONU, therefore, the required bandwidth is 140 Mbps in the downstream direction and 45 Mbps in the upstream direction.

• Fourth type of requests: HDTV video channels. We consider that 80% of the ONUs subscribe to this service. Each ONU can subscribe to 2 to 3 packages, each package contains 10 channels. Bandwidth requirement, for each HDTV channel, is 20 Mbps therefore a package consumes 200 Mbps for downstream. This type of requests does not need upstream bandwidth.

Second Scenario: In addition to triple play services, we consider two other services, i.e., Internet with video conferences and audio streams.

• First type of requests: Voice, i.e., telephone we generated one request per ONU.

• Second type of requests: Internet with browsing. We consider that 90% of the ONUs subscribe to this service.

• Third type of requests: Internet with browsing and online games. We consider that 10% of the ONUs subscribe to this service.

• Fourth type of requests: Internet with browsing and video conference. We consider that 10% of the ONUs subscribe to this service. Bandwidth requirement is 100 Mbps for downstream and 100 Mbps for upstream. Note that we consider that each ONU can make a video conference with 5 other ONUs simultaneously,
and the bandwidth required for each ONU is 20 Mbps for each direction, i.e., upstream and downstream.

- Fifth type of requests: HDTV video channels. We consider that 80% of the ONUs subscribe to this service. Each ONU can subscribe to 2 to 3 packages, each package contains 10 channels.

- Sixth type of requests: Audio stream channels. We consider that 10% of the ONUs subscribe this service. Each ONU can subscribe to 2 to 3 packages, each package contains 10 channels. We consider super audio channels. Bandwidth requirement is 10 Mbps per package for downstream. This type of request does not need upstream bandwidth.

**Third Scenario**: In this scenario we consider the same services as in the second scenario, with the difference that Internet browsing with video conferences/online games and audio streams channels are subscribed by 50% of ONUs.

In all of these requests, some of them are unicast requests, e.g., Internet browsing, Internet browsing with video conferences/online games and others are multicast requests, e.g., HDTV and audio stream channels. Observe that each unicast request originated from an ONU has also this ONU as its destination. Note that it corresponds to one upstream subrequest from the ONU to the OLT, and one downstream subrequest in the reverse direction. All the multicast requests considered in the three scenarios are unidirectional, i.e., downstream direction, and originated from the OLT. Each subtree induced by a feeder fiber (see Figure 26) is assigned a different load for its traffic, i.e., we consider a non-uniform distribution traffic over the subtrees. Traffic load distribution is such that one set of "opposite" subtrees receive each a 20% load, while the other subtrees receive each a 30% load.
uniform distribution traffic, then there may not be any significant difference in the result
of the light mesh topology compared to the tree network topology.

![Diagram of network topology](image)

Figure 26: Traffic load

4.2 Implementation of Models

This section describes the classes for implementation of all mathematical models. We also
propose the heuristic algorithm to solve the traffic-grooming models for a large experiments.

4.2.1 Data Structures

These models were implemented in C++ using Object-Oriented Programming (OOP) ap-
proach. The approach simplifies the design of the models and facilitates reuse of code
through inheritance. Many classes that implement models were created by extending oth-
ers. In describing the implementations, we give a class diagram that summaries the design
in Figure 27.
Classes are divided into two groups:

1. Classes that use the standard C++ library.

2. Classes that use CPLEX library in addition to the standard C++ library.

The following classes belong to the first group:

- **Main** - defines only one function — main — which defines the entry to the program.
  
The function takes parameter of model type and, depending on model type, it calls
  the appropriate model for reading the network and traffic instances and their solution.

- **Network** - abstract class implements generic network topology and traffic.

- **TreeNetwork** - implements tree network topology. It extends Network class by
  providing functions for manipulating tree network and traffic.

- **MeshNetwork** - implements light mesh network topology. It extends Network class
  by providing functions for manipulating light mesh network and traffic.

The following classes belong to the second group:

- **NetworkILO** - abstract class implements generic network topology regarding reading
  topology and traffic scenarios.

- **TreeNetworkILO** - implements tree network topology. It extends NetworkILO by
  providing functions for reading a given network topology and traffic scenario and set
  them in the object by calling functions of Network/TreeNetwork classes.

- **MeshNetworkILO** - implements light mesh network topology. It extends Network-
  ILO by providing functions for reading a given network topology and traffic scenario
  and set them in the object by calling functions of Network/MeshNetwork classes.
Figure 27: A Class diagram
- **SolutionILO** - abstract class implements with/without traffic-grooming functions for solving the model and writing different simulation and result reports in files.

- **SolutionNGILO** - implements non-grooming traffic functions. It extends SolutionILO by providing functions for the solution and writing reports for non-grooming models.

- **SolutionGrILO** - implements traffic-grooming functions. It extends SolutionILO by providing functions for the solution and writing reports for traffic-grooming model.

- **ModelILO** - ModelILO is a parent of all model classes, those classes have prefix 'Model' in the name of class. This class contains constraints for all the models.

- **ModelTreeReqILO** - implements the tree network topology without traffic-grooming for maximizing the weighted number of granted requests.

- **ModelMeshReqILO** - implements the light mesh network topology without traffic-grooming for maximizing the weighted number of granted requests.

- **ModelTreeGrReqILO** - implements the tree network with traffic-grooming for maximizing the weighted number of granted requests.

- **ModelMeshGrReqILO** - implements the light mesh network topology with traffic-grooming for maximizing the weighted number of granted requests.

- **ModelTreeOnuILO** - implements the tree network topology without traffic-grooming for maximizing the weighted number of served ONUs.

- **ModelMeshOnuILO** - implements the light mesh network topology without traffic-grooming for maximizing the weighted number of served ONUs.
- **ModelTreeGrOnuILo** - implements the tree network topology with traffic-grooming for maximizing the weighted number of served ONUs.

- **ModelMeshGrOnuILo** - implements the light mesh network topology with traffic-grooming for maximizing the weighted number of served ONUs.

### 4.2.2 A Heuristic Algorithm

In order to get a scalable solution process of the traffic-grooming models described in Section 3.5.3, 3.5.4, 3.6.3 and 3.6.4, the following heuristic algorithm is proposed. This algorithm can be applied to solve the whole network or a subnetwork, in order to speed up the solution of the problem. In our experiments, we solve each subtree separately in the tree network topology and a group of two subtrees separately in the light mesh network topology. A subtree is defined by the set of links involved in the subgraph induced by a given feeder fiber.

1. Set $\text{threshold} = \text{totalOnus} \times MUL$, where totalOnus is the number of destination ONUs in all the requests and MUL is a constant equal to 3 and 4 in the tree and the light mesh network topologies, respectively. Note that constant values 3 and 4 have been analyzed from many experiments with a given topology and traffic. In other words, these values depend on the characteristics of the traffic instances.

2. Set $\text{iter} = 0$, where iter is an iteration counter.

3. Solve the LP relaxation of the master problem.

4. If $\text{iter} \geq \text{threshold}$ then goto step 5 else goto step 6.
5. If the solution value of master problem has increased by less than 1% since last
threshold iterations then round highest \( x^C \) to 1 and goto step 2, where \( x^C \) is a
configuration variable, generated by the pricing problem and added in the master
problem

6. Solve the ILP pricing problem.

7. If the reduced cost is positive then add new column to the master problem based on
generated configuration and \( iter \leftarrow iter + 1 \). Goto step 3.

8. If the solution of the master problem is not an ILP then round highest \( x^C \) to 1 and
goto step 2.

9. Otherwise, we have obtained a heuristic ILP solution, but unfortunately no accurate
gap, i.e., distance to the optimal value, is available.

4.2.3 Cost

As mentioned in Section 3.4.3, the cost can be defined in different ways. In our proposed
models the cost represents the weight of request/ONU, which can be used for generating the
revenues or giving the priority to some classes of traffic. For simplification of the problem,
in the following experiments, we take the number of destination ONUs as a cost of a request.
If there is one destination ONU (e.g., unicast) in a request, the cost is 1, if there are 6
destination ONUs (e.g., multicast) in a request, the cost is 6, and so on. In other words,
each destination ONU has a cost of 1.

A service provider may decide to give a higher priority to the requests with a higher
number of destinations. For example, if there are three multicast requests, i.e., \( r_1 \) (3 destinations), \( r_2 \) (3 destinations), and \( r_3 \) (6 destinations). Assume aggregating cost of \( r_1 \) and \( r_2 \) is equal to the cost of request \( r_3 \). Suppose we have two solutions for this particular example: first solution is to provision \( r_1 \) and \( r_2 \), and alternate one is to provision \( r_3 \). To give a priority to the alternate solution, the objective can be to maximize the weighted number of served ONUs as follows:

\[
\max_{r \in \mathcal{R}^\text{access}} \sum_{r_i \in \mathcal{R}} \text{COST}_{r_i} X_{r_i}
\]

where

\[
\text{COST}_{r_i} = \begin{cases} 
\text{COST}_{r}^0 & \text{if request } r \text{ is unicast} \\
\text{COST} + \frac{\text{COST}}{D_{\text{max}} + 1} \times m_r & \text{if request } r \text{ is multicast}
\end{cases} \quad r \in \mathcal{R}
\]

where \( \max_{r \in \mathcal{R}} |D_r| = D_{\text{max}} \) and \( m_r = \text{number of destination ONUs in the request } r \).

### 4.3 Simulation

All the models proposed in Chapter 3 have been tested and validated. The performance, i.e., bandwidth, bandwidth, and grade of service, have been evaluated for various traffic instances on the topologies described in Section 4.1.

#### 4.3.1 Model Validations

In order to verify the optimization models proposed in Chapter 3, we have simulated them on different sizes of network topology and traffic instances. Models are examined on small but complex examples of network topologies with different traffic scenarios.
4.3.2 PON Performance Evaluation

Simulation is also performed to evaluate the performance of the PON networks. In order to do that, we employ network topologies and traffic instances described in Section 4.1. Experiments have been conducted in order to analyse the results for the objective of maximizing the weighted number of served ONUs in the two topologies (tree and light mesh) with traffic-grooming. Each scenario runs on 5 different random traffic instances for each topology.

Grade of Service (GoS)

GoS has been analyzed for the number of served ONUs using the following formula:

\[
\text{GoS}_{\text{ONU}} = \frac{\sum_{r \in R} \sum_{\text{ONU} \in D_r} x_{ri}^{o}}{\sum_{r \in R} \sum_{\text{ONU} \in D_r} x_{ri}}
\]

where

\(x_{ri}^{o}\) is granted ONU with respect to \(r\) in a granted traffic set, and,

\(x_{ri}\) is requested ONU with respect to \(r\) in a given traffic set.

Note that, as we discussed in Section 4.2.3, we assign a cost 1 for each destination \(\text{ONU}_i\), therefore, it is not necessary to put the cost parameter in the calculation of the GoS.

Bandwidth

Bandwidth is a rate of data transfer, or a bit rate, usually measured by bits per second. Here, we refer bandwidth, a data transfer rate, at the feeder fibers in order to serve ONUs at the distribution fibers, and denoted as \(\text{Bandwidth}^{PP}\). In the case of PONS, observe that this does not necessarily correspond to the effective bandwidth that is delivered to the ONUs.
Bandwidth Usage

In our experiments, we calculate the bandwidth usage at three different levels, as discussed below.

First we calculate the bandwidth usage on the feeder fiber links alone. The feeder fiber links are the most important links in the access networks due to their long distances compared to the distribution fibers, and calculated as follows:

$$B_{usage}^{FF} = \frac{Bandwidth^{FF}}{\sum_{\ell \in \omega(OLT)} B_{\ell}}$$

where \( \sum_{\ell \in \omega(OLT)} B_{\ell} \) is the transport capacity of the used wavelengths at the feeder fibers.

We also calculate the bandwidth usage on both fibers, i.e., feeder fibers and distribution fibers. The formula is:

$$B_{usage}^{FF,DF} = \frac{Bandwidth^{FF} + \sum_{ONU \in O} B_{read(ONU)}^{DF} + \sum_{ONU \in O} B_{sent(ONU)}^{DF}}{\sum_{\ell \in \omega(OLT)} B_{\ell} + \sum_{ONU \in O} TC_{(ONU)}^{D} + \sum_{ONU \in O} TC_{(ONU)}^{U}}$$

where

- \( B_{read(ONU)}^{DF} \) is bandwidth read by the ONUs at the distribution fibers,
- \( B_{sent(ONU)}^{DF} \) is bandwidth sent by the ONUs at the distribution fibers,
- \( TC_{(ONU)}^{D} \) is transmitted bandwidth to the ONUs at the distribution fibers, and
- \( TC_{(ONU)}^{U} \) is bandwidth sent by the ONUs at the distribution fibers.

We also calculate the bandwidth usage on both fibers, but different weight is assigned to the feeder fibers and the distribution fibers. Suppose there is a 20 kms distance between the OLT and ONUs in the passive optical networks. We assume 15 kms is feeder fibers and 5 kms
is distribution fibers. In others words, the distance between OLT and optical splitters is 15 kms, and the distance between optical splitters and ONU is 5 kms. Consequently, we assign the weight 3 to the feeder fibers and weight 1 to the distribution fibers, and calculated as follows:

$$
\bar{B}_{usage}^{FF,DF} = \frac{3 \cdot Bandwidth^{FF} + \sum_{ONU_i \in \mathcal{O}} B_{read(ONU)}^{DF} + \sum_{ONU_i \in \mathcal{O}} B_{sent(ONU)}^{DF}}{3 \cdot \sum_{\ell \in \omega(OLT)} B_{\ell} + \sum_{ONU_i \in \mathcal{O}} TC_{D(ONU)}^{D} + \sum_{ONU_i \in \mathcal{O}} TC_{U(ONU)}^{U}}
$$

Effective Bandwidth Usage

Effective bandwidth usage has been calculated on the distribution fibers. On the distribution fibers all the splitted wavelengths are transmitted to ONU and each ONU will read its own configured wavelength and discards other wavelengths. The formula is:

$$
B_{Eff}^{DF} = \frac{\sum_{ONU_i \in \mathcal{O}} B_{read(ONU)}^{DF} + \sum_{ONU_i \in \mathcal{O}} B_{sent(ONU)}^{DF}}{\sum_{ONU_i \in \mathcal{O}} TC_{D(ONU)}^{D} + \sum_{ONU_i \in \mathcal{O}} TC_{U(ONU)}^{U}}
$$

Bandwidth Waste

In our experiments, we consider the bandwidth waste due to optical splitters. Passive Optical splitters split signals on all the outgoing links to the ONUs. Bandwidth waste is calculated at the distribution fibers, and calculated as follows:

$$
B_{waste}^{DF} = \frac{\sum_{ONU_i \in \mathcal{O}} B_{Rej(ONU)}^{DF}}{\sum_{ONU_i \in \mathcal{O}} TC_{D(ONU)}^{D} + \sum_{ONU_i \in \mathcal{O}} TC_{U(ONU)}^{U}}
$$

where $B_{Rej(ONU)}^{DF}$ is bandwidth discarded by the ONUs.
Experimental Results

The results based on the tree network topology are shown in Table 7, and the results for the light mesh network topology are presented in Table 8. These tables show the performance for efficient provisioning of the PONs in terms of bandwidth, bandwidth usage, bandwidth waste and GoS. Figure 28 to Figure 31 have been drawn from the experimental results, which are next discussed.

Observed from Figure 28, the bandwidth (\(Bandwidth^{FF}\)) in the light mesh network topology is little bit higher than that in the tree network topology. The reason is that the light mesh network topology serves more ONUs than the tree network topology does, thanks to the capability of the optical splitters. Additionally, if light mesh network topology serves more unicast requests then, we will consume more bandwidth, as it is seen in some scenarios. In other words, serving more ONUs in multicast requests does not necessary increase the amount of the consumed bandwidth.

Similarly, the bandwidth usages calculated on different levels (\(B_{usage}^{FF}, B_{usage}^{FF,DF}, \text{and } \bar{B}_{usage}^{DF}\)) do not vary much between the tree and the light mesh network topologies, as shown in Table 7 and 8. The reason is the same as we discussed for the bandwidth.

Figure 29 shows effective bandwidth usage (\(B_{Eff}^{DF}\)) comparison between both topologies. It shows that the tree network topology has higher effective bandwidth usage than the light mesh network topology. The reason is that the light mesh topology has higher splitter counts. As a result, the light mesh topology wastes more bandwidth on the feeder fibers.

The light mesh network topology has higher bandwidth waste (\(B_{waste}^{DF}\)) than in the tree network topology on the distribution fibers, as shown in Figure 30. The reason is again the same as discussed for effective bandwidth usage.
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Table 7: An experimental results of the tree network topology
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<td>76.67%</td>
<td>36.94%</td>
<td>49.35%</td>
<td>25.27%</td>
<td>74.73%</td>
<td>98.75%</td>
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<td>4</td>
<td>4</td>
<td>13665</td>
<td>62.11%</td>
<td>34.93%</td>
<td>44.10%</td>
<td>25.64%</td>
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<tr>
<td>5</td>
<td>5</td>
<td>14690</td>
<td>81.61%</td>
<td>35.92%</td>
<td>49.13%</td>
<td>24.26%</td>
<td>75.74%</td>
<td>100.00%</td>
<td>96.10%</td>
<td>98.05%</td>
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<tr>
<td>Average</td>
<td></td>
<td>14478</td>
<td>75.99%</td>
<td>35.76%</td>
<td>47.98%</td>
<td>24.50%</td>
<td>75.50%</td>
<td>98.50%</td>
<td>98.33%</td>
<td>98.42%</td>
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Table 8: An experimental results of the light mesh network topology
Figure 28: Bandwidth on the feeder fibers

Figure 29: An effective bandwidth usage on the distribution fibers
The Grade of Service (GoS) in the light mesh network topology is higher compared to the tree network topology, as shown in Figure 31. The reason is that some ONUs are connected to two optical splitters, which lead to increase the grade of service. Additionally, it is observed that twenty four wavelengths (six wavelengths on each link) are required in order to serve 32 ONUs in both directions by using optical splitters while 64 wavelengths are required to serve the 32 ONUs by using AWGs. Consequently, we can conclude that optical splitters save more than 60% bandwidth at the feeder fibers, but they waste more bandwidth on the distribution fibers. It is more important for the feeder fibers to save the bandwidth than for the distribution fibers because, usually, the feeder fibers have longer distances compared to the distribution fibers.

![Graph showing bandwidth waste on the distribution fibers](image)

Figure 30: bandwidth waste on the distribution fibers

It is also observed that there is no direct correlation between the values of the consumed bandwidth and the grade of service, because the grade of service does not distinguish among
the served ONUs, those belonging to a unicast request from those belonging to a multicast one. In conclusion, if the number of ONUs belonging to unicast requests is higher than the number of ONUs belonging to multicast requests, then the bandwidth consumption will be higher compared from one instance to another instance.

![Graph showing bandwidth consumption](image)

Figure 31: Grade of service (GoS)
Chapter 5

Conclusion and Future Work

This Chapter describes the conclusion of the thesis and discuss about the future work.

5.1 Conclusion

Recently, development on multimedia applications have received much attention from researchers attempting to design networks that provide higher bandwidth for access networks. Current access networks that are based on DSL/VDSL/CATV have limited bandwidth; they do not meet the requirements of new multimedia applications like video conferences, video on demand, online games, HDTV, etc., that require higher bandwidths. The passive optical networks provide adequate solution to higher bandwidth requirements in the access networks.

In this thesis, we have studied Passive Optical Networks (PON)s and their components. We also discussed different architectures of PON concerns to the reliability of the networks. The reliability of the network provide the alternate solution of the network in case of link failure due to optical fibers or other equipment failure. We also discussed tree and light mesh
network topologies considered in our optimization models. Furthermore, we reviewed related work on optimization models for multicast routing and wavelength assignment (MC-RWA) in WDM networks. So far, no optimization model has been found for optimal provisioning of PONS.

Later, we presented our eight optimization models for efficient provisioning of PONS. The proposed models use only optical splitters at intermediate nodes for multicasting and wavelength assignment (MC-WA). We discussed two objectives; the first objective deals with maximizing the weighted number of served ONUs while the other deals with maximizing the weighted number of granted requests. We considered only static traffic. Some models were developed for traffic-grooming and others were developed for without traffic-grooming. These optimization models have been modeled in integer linear programs, a without traffic-grooming model is comprised on a single model, while a with traffic-grooming model is comprised on two problems, called the master problem and the pricing problem, on the column generation technique. We also discussed the generic algorithm for solving column generation problems. In addition, we described some examples using column generation in tree and light mesh network topologies. It was shown that the light mesh topology increases the grade of service as compared to the tree network topology.

All models have been implemented in CPLEX ILOG concert libraries using C++ language. Model validation was done for all of them. However, experiments have been only done on the traffic-grooming models with the objective of maximizing the weighted number of served ONUs for tree and light mesh network topologies with unicast/multicast traffic. Experiments was done on different traffic scenarios in terms of multimedia services. For these large experiments, we have proposed heuristic algorithm. Based on the results of the
experiments, we were able to do efficient provisioning of PON. Finally, we have discussed on results of the experiments.

5.2 Future Work

An AWG and an optical splitter, used at the intermediate nodes, are two major components in passive optical networks. The optimization models with pure AWGs were also developed in the Master Thesis of P. Luo [28]. An AWG is more expensive than optical splitter. However, an optical splitter has more attenuation, so it can not serve more than 32 ONU, while an AWG can serve more than 32 ONU. Additionally, with optical splitters we use 6 wavelengths to serve 32 ONU and getting up to 98% grade of service as discussed in our experiments, while with AWGs we need 64 wavelengths to serve the same number of ONUs, two wavelengths for each ONU. Therefore, optical splitters are more usefull for multicast requests whereas AWGs are useful for unicast requests. These two MC-WA PON studies (optical splitters and AWGs) will lead to design a PON with mix of optical splitters and AWGs as intermediate nodes and develop optimization models for MC-WA PON, which should be profitable for service providers/cable operators and meet the requirement of new multimedia services, such as video Conferences, online games, telemedicines, VoD, etc, for subscribers in today and in the near future. In addition, the proposed models have been developed for static traffic based on one-to-many communications. This makes it difficult to accommodate efficiently many-to-many multicast communication, i.e., video conference and online games, where the traffic keeps changing frequently. To this end, we suggest MC-GRWA models with many-to-many multicast communication should be developed for dynamic traffic.
Bibliography


112


