An $\mathcal{H}_\infty$ Dynamic Routing Control of Networked Multi-Agent Systems

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A Thesis

in

The Department

of

Electrical and Computer Engineering

Presented in Partial Fulfillment of the Requirements

for the Degree of Doctor of Philosophy at

Concordia University

Montréal, Québec, Canada

September 2008

Your file Votre référence
ISBN: 978-0-494-45644-6
Our file Notre référence
ISBN: 978-0-494-45644-6

ABSTRACT

An $\mathcal{H}_\infty$ Dynamic Routing Control of Networked Multi-Agent Systems

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This research aims to introduce an analytical solution to the routing problem of Networked Multi-Agent Systems (NMAS) by taking advantage of control theory machinery. Routing problem can be defined as that of finding a route for messages among networked agents by adjusting the output flow of each link according to the traffic information of the network, such that some objective functions are minimized. In this research, a new objective function, namely *worst-case queueing length* is introduced based on which a novel routing methodology is presented. The propagating, transmitting and processing delays are inevitable characteristics of the queueing dynamics which is considered in the model of the network. The proposed dynamic optimization problem is formulated as a feedback control problem.

First, a centralized $\mathcal{H}_\infty$ optimal control scheme is proposed which can maintain a robust performance of the routing strategy in the presence of multiple and unknown time-varying delays for a fixed network topology. The routing problem is formulated as an $\mathcal{H}_\infty$ optimal control problem for a time-delayed system. The resulting optimization problem is then recast as a minimization problem involving Linear Matrix Inequality (LMI) constraints. The physical constraints are also formulated as LMI feasibility conditions. The proposed centralized routing scheme is then reformulated in a decentralized framework. This modification yields an algorithm that obtains the "fastest route", provides robustness against multiple unknown time-varying delays, and enhances the scalability of the algorithm to large scale traffic networks. By stochastically changing the network topology due to the nodes' mobility the overall network model is described by a Markovian jump process. The proposed Markovian jump dynamics can also support changing number of nodes due to adding new nodes to the network or deleting them because of their low energy or faults/failures. The resulting problem which involves Markovian jump dynamics due to the time-varying delays appearing in control is more challenging to solve. The problem is further complicated by the fact that the interconnected terms also change at each switching mode. To stabilize this system, an $\mathcal{H}_\infty$ controller is presented for the Markovian jump system for mode-dependent interconnected terms. Finally, the LMIs corresponding to the associated physical constrains are properly modified for the mobile networks.

Dedicated to

My parents, Zahra and Alireza

with love and gratitude.

ACKNOWLEDGEMENTS

I owe my achievements to all individuals who taught me even a word and stood by me through this journey. Specifically, I would like to express my sincere gratitude and special thank to my Ph.D. supervisor, Professor Kash Khorasani for giving me the chance to pursue my studies in Concordia and have a stimulating and rewarding experience, for his support and guidance in every stage of my research, and also for his advices to be prepared in my future career.

I wish to convey my deep and special appreciation to my M.Sc supervisor Dr. HeidarAli Talebi who encouraged me to keep moving forward and continue my studies in Ph.D program. He has always been very supportive through many tough moments during my study. I was honored to be his student, taking his advice, and participating to his classes, especially the Robust Control which constitute the pillars of my thesis.

My special thanks are due to my distinguished committee members, Dr. B Boulet, Dr. L. Narayanan, Dr. W. E. Lynch, Dr. Sh. HashtrudiZad, and Dr. A. G. Aghdam for devoting their valuable time in guiding and evaluating my work.

I wish to thank my friends and colleagues Nader Meskin, Ruiru Chen, and Elham Semsar with whom I've shared useful discussions on research. I warmly thank all of my friends specially Parisa, Fariba, Chakameh, Negin, Nazanin, Hojat, Hani, AhmadReza, Behzad who provided me a warm and friendly environment and memorable times in Montreal. At last not the least, my deepest and sincere acknowledgements go to my wonderful parents Zahra and Alireza for their unconditional love, and continuous encouragement to embrace new challenges, and also my lovely supportive sisters Azadeh and Maryam.

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List of Symbols and Abbreviations

LMI	Linear Matrix Inequality
NMAS	Networked Multi-Agent Systems
UGV	Unmanned Ground Vehicles
UAV	Unmanned Aerial Vehicles
UUV	Unmanned Underwater Vehicles
ESA	European Space Agency
PROBA	Project for On-Board Autonomy
MANETs	Mobile Ad Hoc Networks
NASA	National Aeronautics and Space Administration
TBRPF	Topology Broadcast based on Reverse-Path Forwarding
TFI	Terrestrial Planet Finder
TDMA	Time-Division Multiple Access
FDMA	Frequency-Division Multiple Access
ARPANET	Advanced Research Projects Agency Network
OLSR	Optimized Link State Routing Protocol
AODV	Ad hoc On Demand Distance Vector
DSR	Dynamic Source Routing Protocol
ZRP	Zone Routing Protocol
QoS	Quality of Service
DOF	Degrees of Freedom
DS1	Deep Space 1
IDEA	Intelligent Distributed Execution Architecture
MEMS	Micro Electro Mechanical Systems
DRI	Delay Rate Independent
DRD	Delay Rate Dependent

MJLS	Markovian Jump Linear System
vec	vector
diag	diagonal
$*$	The entries implied by the symmetry
$\lambda_{min}(.)$	The smallest eigenvalue of a positive-definite matrix (.)
$\lambda_{max}(.)$	The largest eigenvalue of the positive-definite matrix (.)

Chapter 1

Introduction

A machine capable of emulating human behavior has been a dream for ages. That is why the concept of "intelligent agent" came to existence. An intelligent agent is defined as an entity which can observe, make a decision and direct its activities towards achieving goals. Although, having an agent as intelligent as a human still seems a fiction, researchers have been trying to develop methodologies to capture certain human behaviors such as autonomy, learning, cooperation and collaboration. An example of the intelligent agents are robots. Nowadays, robots play a key role in space missions, factories, military and civilian missions. Over the past decades, considerable efforts and investments have been made worldwide to provide suitable and satisfactory solutions to the scientific and technological problems associated to robotic applications. The major challenge is to develop even more intelligent robotic systems capable of perceiving, reasoning, learning, evolving, interacting and co-operating with their environment and with other robotic systems. Autonomous mobile robots can operate on hostile, tedious, or hard-to-access environments that are usually partially or completely unknown, very complex or poorly structured. In the not too distant future, the use of robots in space, civilian security applications, military, and industry will become mandatory, either because it is the only available solution (searching for life elsewhere during long space missions) or for protecting human lives (security/defence).

Networked Multi-Agent Systems (NMAS) which represent human communities are composed of intelligent agents. Due to limited view of each agent, they require to communicate to each other. Indeed, NMAS are able to solve the problems which are difficult or impossible for individual agent to solve. Networked multi-agent robots are commonly known as Unmanned Ground Vehicles (UGV), Unmanned Aerial Vehicles (UAV), and Unmanned Underwater vehicles (UUV). The objective is that intelligent autonomous systems from land, air, and sea collaborate together to achieve surveillance and reconnaissance missions. Maximum area coverage and faster and more reliable operation will necessitate a large number of autonomous vehicles which in turn begs a more complex communication among the vehicles. To studying communication over NMAS, each agent is regarded as a single node. Each node which may consist of several sensors, actuators and decision makers, should communicate and exchange information, process the received information and make a decision autonomously while collaborating with other nodes to accomplish a mission. Indeed, the progress in networking and communication is a necessary aspect of autonomy for NMAS.

1.1 Motivation and Applications

Networked Multi-Agent Systems (NMAS) have found applications in various fields of research. They are capable of self-deployment, i.e., starting from some compact initial configuration the vehicles can be spread out such that the area covered by the network is maximized. In general, prior models of the environment is not available, hence each node shares its local information with others to obtain a global map of the environment. Several research laboratories have focused on developing self-deployment algorithms of NMAS. In [2], a self-deploying algorithm was proposed and tested on 100 nodes equipped with a scanning laser range with 360 degree field-of-view and a retro-reflective beacon. The laser returns some intensive information, and can therefore distinguish between vehicles and obstacles. The network is placed in a complex simulated environment that represents a single floor in a large hospital. In their starting configuration, the vehicles are crammed into the single room as shown in the top graph of Figure 1.1. The vehicles spread out to cover a sizable portion of the environment (Figure 1.1-b). The coverage area in the final configuration is in excess of $500m^2$. Finally, Figure 1.1-c shows the map found based on the mobile networked system.

Among the wealth of applications of NMAS, pursuit-evasion game (PEG) can be considered as one of the most complicated scenarios [3]. PEGs can address the problems of controlling a swarm of autonomous agents in the pursuit of one or more evaders. The typically defined scenarios for PEGs are search and rescue operations, surveillance, localization and tracking of moving parts in warehouse, and search and capture missions. The pursuer, the cooperative mobile agent, enters the field and attempts to intercept the other pursuers and its own autonomous control capabilities. Usually in capture missions, the evaders actively avoid detection, whereas in rescue operations their motion is approximately random.

In search and capture missions, the playing field is abstracted to contain a finite set of nodes, and the allowed motion for pursuers and evaders are represented by edges connecting nodes. An evader is captured if both the evader and one of the pursuers occupy the same node. However, pursuers have relatively small detection range. They usually employ computer vision or ultrasonic sensors, providing only local observability over the area of interest. To behave autonomously, each pursuer should have enough information to be able to make a proper decision. Therefore,

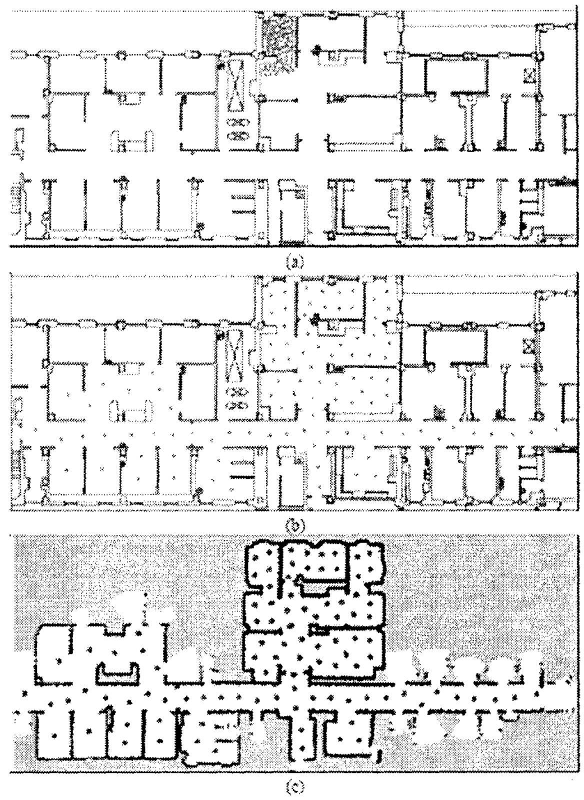

Figure 1.1: Deployment Experiment for 100 UGV. (a) Initial network configuration. (b) Final configuration after 300 seconds. (c) The discovered map by NMAS, visible space is marked in black (occupied) or white (free), unseen spaces is marked in gray [2].

it should communicate to other pursuers to share their local detected information. They collaborate and cooperate to accomplish the mission.

The above system can be considered as a network where each pursuer serves as a single node. When the number of nodes (pursuers) increases, the fully connected communication topology is not an appropriate and economic design. Therefore, proper routing and exchanging large amount of messages among the nodes play a key role in fulfilling the mission. The PEG can be more complicated when the environment is unknown. In this framework, a self-deployment algorithm should be added to the pursuit phase. To have a more precise view over the field and complete visibility, some static sensor networks may also be added to the mobile network

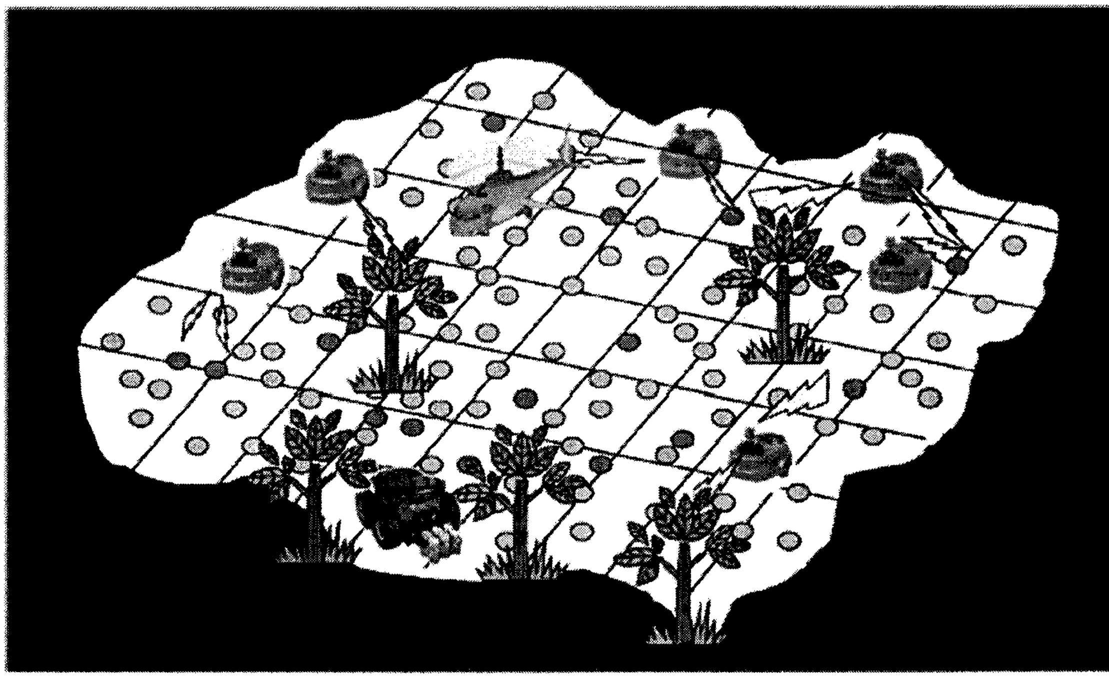

Figure 1.2: Pursuit-evasion-game with UAV and UGV [3]

vehicle. In [3], PEG has been experimented by some UAV and UGV pursuers. They cooperate together to catch the evaders (Figure 1.2).

NMAS are not only investigated in research laboratories, but also are planned to be employed in real world missions. They pose admirable performance in hazardous missions. For instance, consider a scenario involving a hazardous material leak in a damaged structure. The NMAS equipped with chemical sensors can rapidly be deployed throughout the environment and return real-time data indicating the location and concentration of hazards, then prevent the leakage and clean the area. In the longer term, other major civilian applications of NMAS will include maritime surveillance (traffic control and monitoring of ship movements, supervision of illegal fishery), crime monitoring, search and rescue operations, environmental monitoring (fire detection and fire fighting, oil spill discovery, etc) and surveillance of hazardous materials.

Mobile autonomous vehicles will also play a significant role in future space missions. NASA is currently exploiting the return on experiment of the Deep Space 1 (DS1) mission through the Intelligent Distributed Execution Architecture (IDEA). DS1 flew autonomously around a complex asteroid without human assistance. The probe performed on board planing, scheduling and diagnosis, combined with smart execution control. IDEA extends DS1 concepts to multiple robots or platforms. They exchange their sensed information and introduce more complex missions [9]. The other project at NASA is the Autonomous Spacecraft Free-Flying Robots. These robots are being developed to provide flexible, close-range mobile sensing inside and outside of in-space vehicles, stations and potential space facilities. The 6 DOF (degrees of freedom) robots are equipped with cameras, voice dialogue system and environmental sensors. They are expected to be applied in exploration missions as robotic networks over the next decades. Figure 1.3 shows such robot. NASA's

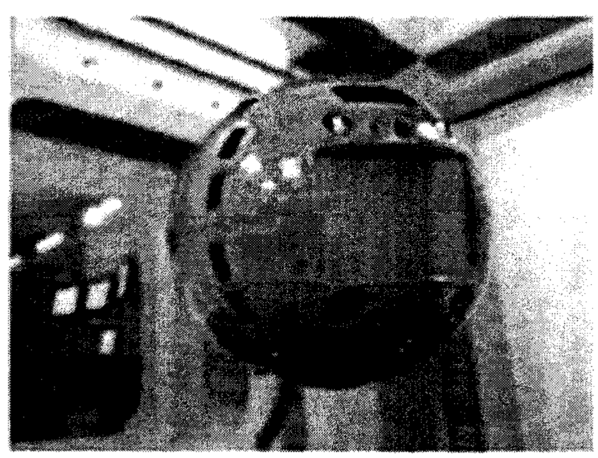

Figure 1.3: Free-flying spacecraft mobile robot [4].

vision for space exploration also includes future missions of planet exploration with development of space robots and reliable autonomous spacecraft. Scenarios in which robots operate with minimal Earth communication are essential for ensuring autonomy. Mission success will be achieved through robot communication, collaboration and cooperation with one another as a team.

Coordinated maneuver is an example of NMAS which is important in military as well as civilian applications. Space applications benefit from formation control of satellites to perform distributed observations and information fusion. Groups of UAV perform search, rescue and attack mission collectively in restricted areas where human intervention is dangerous. They may involve obstacle avoidance and possibly adversarial vehicles. To avoid obstacles, the group of vehicles require performing split/rejoin maneuvers. In addition, due to the environmental restrictions, the information flow and/or formation of the group need to change. For instance, in Figure 1.4 UAV should change their formation to line to pass through the mountains [5]. Furthermore, in recent years, lightweight, low-cost microsatellites and nano-satellites provide an opportunity for space agencies such as NASA's Ames Research Center to gather the required data and information more precisely with lower cost. However, to sense and gather the data, these satellites should follow some predefined formations. Indeed, many future space missions involve formation flying spacecraft performing imaging, inspection, assembly, and servicing missions. Advanced sensing, communication and computation are needed to design and develop the control technologies for such scenarios.

European Space Agency's (ESA) Project for On-Board Autonomy (PROBA)-3 mission which is planned to be a lunched in 2009 will perform spacecraft formation flight. To achieve high angular resolution in astronomical images, increasing apertures of telescopes or increasing baselines of interferometers is required which results in increasing the mass of the support structure of the telescopes, accordingly. The propellant for launch and navigation of long baseline space telescopes will exceed technical and financial boundaries. By combining satellites in autonomous formation flight to behave like a rigid body, one can deal with the mass constraint. Precision formation flying team at NASA is also working on formation flight which is planned

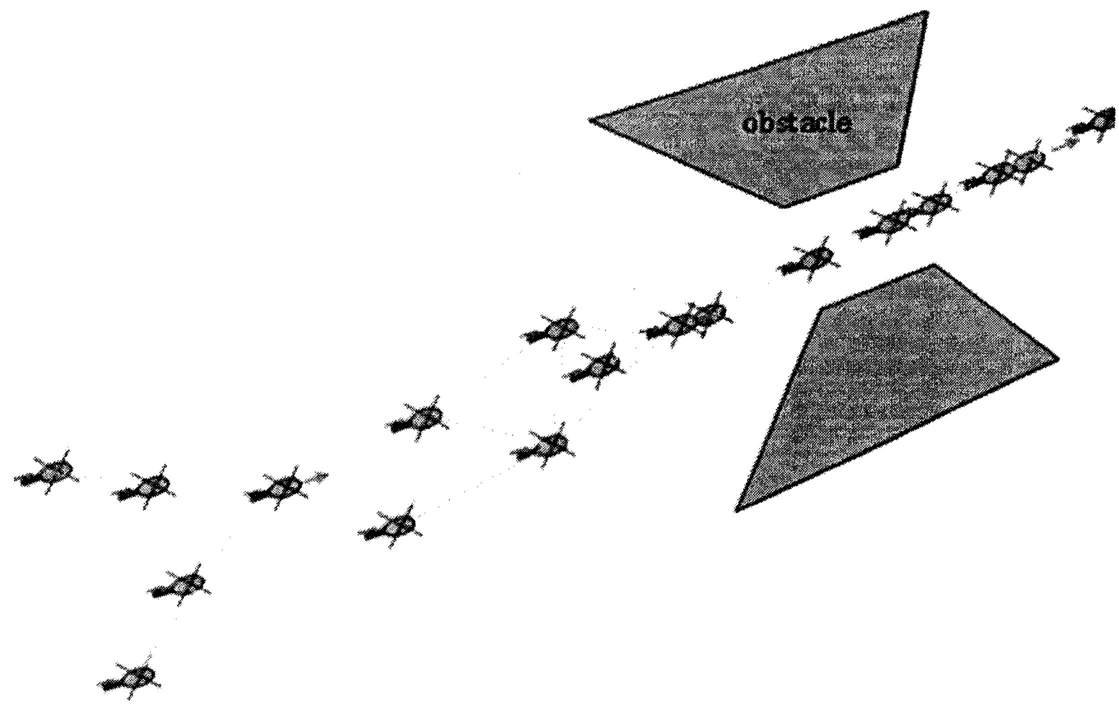

Figure 1.4: Reconfiguration of UAV [5].

to gather more precise information in future missions such as Terrestrial Planet Finder (TFI) [10]. Fig. 1.5 depicts an impression of spacecraft formation flight.

1.2 Challenges

NMAS pose considerable technical issues in different fields. Among them are:

1. Sensor Management and Data Processing: Sensors are key components in any autonomous system. Systems in general employ a wide range of different sensors relying on different physical phenomena. UGV mainly rely on electro-optical sensors, but can also employ radars and acoustics. UAV most often employ electro-optical sensors and/or radars. UUV on the other hand, mainly rely on hydro-acoustic sensors, i.e., sonar and sometimes electro-optical sensors. When a node receives information from other nodes, this information

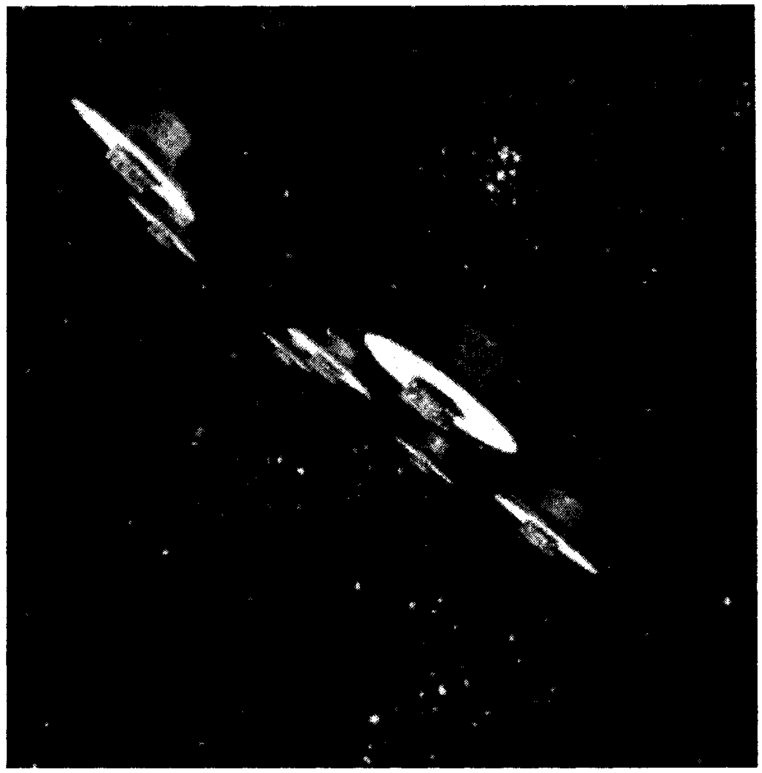

Figure 1.5: A Formation flying space interferometer [6].

has to be combined and fused with local detected information. Sensor management deals with collaborative signal and information processing over a network which is related to distributed information fusion (another new area of research). Fusion approaches range from simple rules of picking the best result to model-based techniques that consider how the information is generated. There is a tradeoff between performance and robustness. Simple fusion rules are robust but suboptimal, while more sophisticated fusion rules exhibit better performance but may be sensitive to the underlying models.

2. Location determination is the primary issue in control of NMAS. NMAS need to find their situation using internal and external sensors. Identifying the state of a vehicle is a major problem to build reliable situation awareness. A methodology is required for cooperation and coordination operation of NMAS

involved in complex and dangerous security operations or managing collective protection and handling missions such as infiltration.

3. Power consumption management is an important issue in developing autonomous systems. This affects the overall system life-cycle: mission management, design, dimensioning and requirement tradeoff. Micro Electro Mechanical Systems (MEMS) technology and smart materials create opportunities to design cheap, small, highly reconfigurable and highly redundant micro robots which consume less energy. The effort is to develop a range of new, very small and highly distributed micro and nano-robots that develop new forms of climbing, walking or flying locomotion and new actuation, sensing, control and perception skills. However, to allow miniature robots to engage in missions for extended periods of time in varying environments, novel methodologies are required for data analysis, and communication on such a small scale but wide range. Furthermore, new concepts for reducing power requirement, regulating energy consumption and providing rapid recharging are also needed.

4. Security is a main factor in designing NMAS. Since the NMAS may operate in a hostile environment, a communication security layer must provide known guarantees for access control, message integrity, and confidentiality.

5. Task allocation between the nodes is another challenging issue. Several algorithms have been proposed to allocate the tasks among the nodes. To allocate the tasks, different criteria can be used such as minimizing the energy consumption in the entire network and maximizing the nodes life-time.

6. Navigation: The system navigation should be designed such that the NMAS be able to adapt its behavior to the changes in the environment as well as perform fault detection, isolation and recovery. Reasoning and planning for complex tasks are other issues that should be considered in the navigation.

7. Communication: Since the sensors have limited range for detection, each node should share its information with others. Therefore, reliable communication plays a key role in NMAS. Furthermore, an efficient communication strategy is essential for collaboration and coordination. Since the NMAS are mobile, the communication is much more complicated and challenging since the mobility might result in continuous changes in the network topology. As the number of nodes and the complexity of the missions are increased, a more reliable communication is required. Various key issues pertaining to a fast and reliable communication can be listed as follows:

   - **Routing:** The routing problem can be defined as that of determining a route for packets from a given source to a destination through other nodes so that

     (a) certain physical constraints are satisfied.

     (b) desired performance specifications and requirements are guaranteed.

     (c) certain objective functions are optimized.

     In Section 1.3, several issues concerning the routing problem are introduced.

   - **Resource allocation:** Based on the physical characteristics of the network each node has specific resources that can be shared among its outgoing links, by means of some multiplexing scheme such as TDMA (Time-Division Multiple Access) and FDMA (Frequency-Division Multiple Access). The main resources in a wireless network can be listed as:

     (a) Transmitting power

     (b) Bandwidth

     (c) Time-slot fraction

On the other hand, one of the key constraints in making the routing decision is the capacity of transmission links. According to Shannon's capacity formula, the capacity of an individual link can be related to the resources available to that link [11]- [14]. In [15] a lower bound on the acceptable capacity of ad-hoc wireless network for a large number of nodes considering fading, interference and node mobility has been defined.

- **Energy consumption and lifetime**: Another important factor which is vital to NMAS, is the consumption of the energy within each node of the network. The energy consumed at each node, can be considered as the summation of the energy required for sensing, transmitting, and receiving. Lifetime of nodes is another issue specifically in wireless sensor networks which has been addressed by several researchers. In addition to minimizing the energy consumption, there is a great concern to keep the nodes alive as long as possible. Saving a node lifetime can be a decision factor in both task allocating [16], [17], and routing [18], [19] problems.

- **Congestion control:** When the performance of the network degrades due to the saturation of resources, congestion is said to have occurred in the network. Indeed, congestion control refers to the set of actions taken by network managers to minimize the intensity, spread and duration of congestion. In this case, the routing paths are fixed and predefined, and congestion is avoided by adjusting the control flow and resource allocation. Usually, congestion control is designed to support Quality of Service (QoS) [20], [21], [22]. In [20] a generic integrated dynamic congestion control scheme is proposed for differentiated service and its stability is studied through Lyapunov's direct method. Using queueing length information for feedback, the flow rate is adaptively regulated.

The research proposed in this thesis aims at addressing the routing problem. In the following section the challenges for solving the routing problem for mobile wireless networks are discussed in further details.

1.2.1 Routing Problem for Mobile Wireless Networks

Contrary to cellular networks, where the nodes are restricted to communicate with a few strategically placed base stations, in mobile ad hoc networks (MANETs) the nodes can directly communicate with each other. However, due to limited wireless channel each node can effectively communicate with only certain finite nodes, typically those that lie in its vicinity or in its neighboring set. Consequently, it is necessary that nodes cooperate with one another to forward data packets to their final destinations. However, the restrictive requirements imposed on the network make solving the routing problem in MANETs nontrivial. Some of the challenges are listed below:

1. **Variable Capacity:** The wireless channel is prone to bit errors due to interference from other transmissions, thermal noise, and multi-path fading. Therefore, the channel capacity is changing in MANETs. Moreover, because of node mobility, a mobile wireless network experiences random variations in the channel strength.

2. **Limited buffer size:** In crowded networks, the messages cannot be routed immediately and should wait in buffers to be routed in sequence. However, the buffer size is limited and if the messages are not routed properly, more messages should be kept in buffers and the buffers may overflow. Therefore, there is a higher chance that of packets and messages are dropped out.

3. **Variable Topology:** The nodes' neighboring sets are variable in MANETs. This is due to the fact that the nodes are mobile, the nodes might run out

of power, and/or the new nodes might be added to the network. The routing algorithm should adapt itself to such changes in the neighboring sets.

In the following section, the routing methodologies introduced in the literature will be investigated in more details.

1.3 Routing Methodologies

As new applications emerge, traffic networks become more complex and crowded. This fact does indeed justify the necessity for considering the routing problem as a subject of still significant interest in the research community. Routing algorithms can be classified based on several attributes. Some of them are defined as follows:

A. Responsiveness

1. **Static Routing [23]:** In static routing, paths are pre-computed based on the network topology, link capacities, other information and various assumptions regarding them. Once the computation is completed, the paths are loaded to the routing table and remain fixed for a relatively long period of time. Static routing may suffice if the network size is small, the traffic load does not change appreciably, or the network topology is relatively fixed. Static routing may become cumbersome as the network size increases. If the traffic load changes, the pre-computed paths may easily become suboptimal. The biggest disadvantage of static routing is its inability to react rapidly to network failures. In [23] an elegant static routing algorithms was introduced.

2. **Quasi-static Routing [24]:** Quasi-static algorithms take advantage of static algorithms but they change the routes at given intervals of time and/or whenever some drastic changes occur, for example establishing some new sessions or changing the fraction of traffic due to the node failure. In these algorithms, usually the external traffic arrival rates is considered to be stationary. They

may fail to produce satisfactory results when the rates change appreciably within a relatively short period of time. Gallager [24] has proposed a decentralized quasi-static algorithm which also avoids route oscillations.

3. **Dynamic Routing [11]:** In dynamic routing algorithms, the messages or packets are routed according to the instantaneous state of the queues at the output links of a node. The routing of a particular message or packet is not determined when it enters the network; instead, each node that receives the message selects the next node to which the message is routed on its path to the destination. Since the amount of messages entering a network at various nodes may vary from time to time, a dynamic routing strategy which can adapt to such variations can be superior to static and quasi-static routing algorithms. One disadvantage of dynamic routing is the additional computational complexity in the node. Another issue that should be considered in dynamic routing is the stability of the entire system. Several researchers have formulated the routing problem as a feedback control problem where generally a dynamic routing problem is solved based on dynamic queueing models [11], [22], [25], [26], and [27]. It is worth noting that since the topology of the MANETs is varying, static or quasi-static routing algorithms fail to yield satisfactory performance, while dynamic routing can properly adapt the routing decision with these changes.

B. Distribution of Routing Decision

1. **Centralized Routing [28]:** In centralized algorithms, a central decision maker defines the routing tables [28]. To solve the routing problem, the input flows of all nodes must be available to the central node. Although a centralized routing strategy benefits from possibly finding a global optimum solution, the main problem in these methods is that failure of the central node may cause

the failure of the entire network. Complexity is another disadvantage of this type of algorithms.

2. **Decentralized Routing [22]:** Decentralized algorithms, which can be implemented locally at individual nodes and require a minimum amount of information from the other nodes, are desirable in practice. In decentralized routing algorithms, each node routes its messages based on its local information from its neighboring set [22], [26], [29]. Hence, the overhead of exchanging routing information with local nodes is minimized. Moreover, they can react more rapidly to a local disturbance with slower fine tuning in the rest of the network. Therefore, decentralized routing algorithms generally scale better and are more suitable for practical implementations than the centralized routing algorithms.

C. Source-Destination v.s. Destination

In routing algorithms, the flows can be identified based on their source-destination pair or by their destination only. Indeed, most of the routing that are based on user optimality find the routes for each source-destination messages [26]. To decrease the complexity and dimensions of the problem, one can use destination based routing, i.e., all flows with the same destination are considered as one single commodity regardless of their sources (e.g. [11], [25], [28]). In other words, in a network with N nodes, a source-destination method results in total of $N \times N \times (N-1)$ queues. However, in a destination base method, only $N \times (N-1)$ queues are required.

D. Traffic Flow Model

To solve a routing problem, one must model the traffic flow at each node and then decide what portion of the capacity of each link should be used for each type of traffic. Most routing algorithms consider one of the following two methods for traffic flow model.

1. **Static Model [11]:** Static model is based on conservation law, which is the sum of external incoming flow and traffic coming from other nodes is equal to the sum of the outgoing flow from that node [11], [24].

2. **Dynamic Model [28]:** Physical constraints such as limited bandwidth cause traffic congestion. Hence, not all input flows can be immediately routed and some messages have to be kept in queues, waiting to be routed to their destinations. Therefore, a static model cannot accurately describe the traffic flow of such networks. In [28], a queueing model was proposed to describe the dynamic nature of network traffic flow. Therefore, in a network with N nodes, the traffic at node i can be modeled as

$$\dot{q}_i(t) = \sum f_{in_i}(t) + r_i(t) - \sum f_{out_i}(t) \tag{1.1}$$

where q_i is the message queueing length at node i, $r_i(t)$ is external input flow entering node i, $f_{in_i}(t)$ is the input traffic flow routed from other nodes to node i, and $f_{out_i}(t)$ is the output traffic flow routed from node i to other nodes. In [20], [22], [29] and [30] also some dynamical models for the network traffic are introduced.

E. Performance Measures

Routing problem is usually considered as an optimization problem, where an objective function based on some performance measure is minimized/maximized. Several objectives have been identified in routing problems. All existing algorithms support at least one of the following goals in transmitting the messages or packets.

1. **Shortest Path [31]:** Early routing algorithms, such as those implemented in the Advanced Research Projects Agency Network (ARPANET), were based on finding the shortest path from the initial node to the destination node [31]. This objective is still an attractive requirement to achieve [32]. In these

algorithms, the length of a path is usually proportional to the message flow rate on that path. However, these algorithms do not yield acceptable performance in crowded networks that usually have link congestion.

2. **Link Congestion [33]:** For complex and crowded networks, link congestion becomes an important factor in making appropriate routing decisions, given that the message flow rate on a link should be related to the capacity of that link. In recent years a number of congestion control algorithms have been developed in the literature [29], [33], and [34]. In [33], a congestion adaptive routing algorithm was introduced. However, performance of such routing algorithms might deteriorate due to the link congestion despite the use of the controller. Furthermore, the algorithms could be inherently less stable, since routing changes can directly affect link utilization and congestion. In [34], a stable routing algorithm using a congestion control approach was introduced.

3. **Delay [35]:** Optimum routes can be obtained so that the *total delay* in the network is minimized. Indeed, associated with a given network problem (e.g. traffic flows), total delay minimization implies determining a route that messages have to travel to reach their destination in the **shortest time** (also known as the "fastest route") as opposed to the shortest distance.

   It is well-known that the queueing delay is among the primary sources of delay in routing problems, which is defined as the total time that messages have to spend in the queue before reaching their destinations. The problem of determining a centralized routing controller that is based on minimization of a measure of the total delay (namely, the network total queueing length) was studied in [35]. Several other researchers have also attempted to address the problem of delay-constrained routing in recent years. For instance, in [36] a routing-based admission control mechanism that considers an end-to-end delay for the IP traffic flows was introduced. In [37], a routing algorithm was proposed to minimize an average of the queueing delay by using capacity allocation. In [38], a set of paths between the source and the destination nodes were indexed based on their energy consumption in an increasing order of priority. An estimate of the end-to-end delay along each of the ordered paths was obtained and then the route with the lowest index that satisfies a specific delay constraint was selected. In [29] end-to-end delay was considered as an issue for developing routing algorithm and it was investigated how rapidly routing can respond to changes, without compromising stability. In [26], the following types of delays were also considered in the dynamic model of the network flow:

(a) Transmitting delay: The time between starting and ending the transmission of a message. It depends on the length of the message.

(b) Propagating delay: The time for propagating a message on each link.

(c) Processing delay: The time that each message from upstream nodes or outside of the network should spend at each node to be received, identified by its destination, inserted to the appropriate queue, and performed the routing calculations (in dynamic routing).

In [25], the dynamic routing problem was defined as a team optimization problem based on the discrete-time version of the model introduced by Segall in [28], and an approximate solution to the problem using neural networks was obtained. As an alternative, minimizing the queueing length could also be considered as an objective of routing problem. A decentralized controller was proposed in [26] that guarantees the boundedness of the queueing length.

Most algorithms developed in the literature so far, generally assume that the various delays in the network (i.e., transmission, propagation, and processing) are fixed and known *a priori*, an assumption that is **not** valid in practice. In fact, delays are unknown and could vary due to unpredictable circumstances such as unpredicted traffic flows and congestion.

4. **Throughput [39]:** Throughput is defined as the average number of messages successfully delivered per unit of time. Maximizing the throughput can also be an objective to be achieved by a routing algorithm [39].

5. **Packet Loss [40]:** Packet loss occurs when buffer sizes are limited so that an incoming message may not be accommodated at a node [40] [41]. Specifically for messages with high priority, packet loss could be a main concern that should be considered in the routing problem.

6. **Energy [42]:** Energy is an issue in every engineering design. In routing problem, limited energy supply is an important factor. This is especially crucial for wireless sensor networks where a battery provides the energy required for each node. An efficient routing algorithm should consume minimum energy and keep the nodes alive as long as possible. Indeed, the total energy consumed at each node is the sum of the energy consumed for sensing, transmitting and receiving. The total energy should not exceed the available energy for each sensor node [39], [42]. In [39], a nonlinear optimization model for static wireless sensor networks was presented for two problems: 1- Maximizing the total information gathered subject to energy constraints (on sensing, transmission and reception), and 2- Minimization of energy usage subject to information constraints.

   Furthermore, in a multi-hop wireless network, energy/power conservation is of great importance not only for increasing the devices lifetime, but also for avoiding excessive interference and for gaining better coexistence with other systems. In [19] a dynamic programming algorithm was introduced to minimize energy consumption, subject to packet delay constraints.

A popular strategy to conserve energy is by sending some of the nodes to sleep in some period in order to operate them at lower duty cycles. However, this should not lead to deterioration of performance of the wireless sensor network through for example a loss of connectivity [43], [44].

F. Nodes Communication Capabilities

1. **Homogeneous Ad hoc Networks [45]:** In homogeneous ad hoc networks, all nodes are assumed to have the same communication capabilities and characteristics. Although a homogeneous network model is simple and easy to analyze, it misses important characteristics of many realistic MANETs such as military battlefield networks. In addition, a homogeneous ad hoc network suffers from poor performance limits and scalability.

2. **Heterogeneous Ad hoc Networks [27]:** In heterogeneous ad hoc networks, multiple types of nodes exist which have different communication characteristics in terms of transmission power, data rate, processing capability, reliability, etc. Hence, a heterogeneous network model is more realistic and provides many advantages such as leading to more efficient routing protocol design [27], [46].

G. Response to Changing Topology

1. **Proactive:** Proactive protocols, such as TBRPF (Topology Broadcast based on Reverse-Path Forwarding) [47] and OLSR (Optimized Link State Routing protocol) [48] exchange routing information periodically between hosts and constantly maintain a set of available routes for all nodes in the network. They are also called table-driven methods.

2. **Reactive:** Reactive protocols, such as AODV (Ad hoc On Demand Distance

Vector [49] [50]), DSR (Dynamic Source Routing Protocol [51], [52]) postpone route discovery until a particular route is required, propagate routing information only on demand and release when the transmission no longer takes place. Reactive methods are also called on-demand methods.

3. **Hybrid:** There are also a few hybrid protocols, such as ZRP (Zone Routing Protocol) [53] and [54], that combine proactive and reactive routing strategies.

The tradeoffs between proactive and reactive routing strategies are quite complex and depend on many factors, such as the size of the network, the mobility, the data traffic, the application of the network, pattern for sending the messages and so on. Reactive protocols make low routing overhead, however may suffer from increased latency due to on-demand route discovery and route maintenance. In other words, such routing approaches can dramatically reduce routing overhead when a network is relatively static and the active traffic is light since they do not need to update route information periodically and maintain routes on which there is no traffic. However, the source node has to wait until a route to the destination can be discovered which increases the response time. Furthermore, when a link is disconnected due to failure or node mobility, which often occurs in MANETs, the delay and overhead due to new route establishment may be significant. On the other hand, proactive protocols can provide good reliability and low latency through frequent dissemination of routing information and crowded networks. They can provide route information and establish a session, quickly. Hybrid methods, on the other hand combine proactive and reactive methods to find efficient routes, without much control overhead. However, hybrid methods are usually too complex to be implemented in practice, especially, when the network's characteristics, such as the mobility pattern and the traffic pattern are expected to be dynamic.

H. Routing Based on Topology or Position

1. **Topology-Based [33]:**

   Topology-based routing protocols use the information about the links that exist in the network to perform packet forwarding [33].

2. **Position-Based [41]:** In position-based routing algorithms, on the other hand, the physical position of the participating nodes are also required to be known. Hence, these methods are more costly, since the nodes should be equipped with Global Positioning System (GPS) or some other type of positioning service [41], [55].

1.3.1 Networked Multi-Agent Systems (NMAS) Characteristics

Networked Multi-Agent Systems (NMAS) can be considered as a subset of MANETs. However, due to the missions defined for such networks, NMAS have some characteristics which make them different from other type of MANETs, some of which are listed below:

1. Multi-agent systems usually move in groups and they should communicate with their leaders and supervisors. Therefore, destination nodes might be known *a priori*.

2. Messages are required to be sent periodically, except for reporting unpredictable events.

3. Type of the messages are limited to: video, audio, and data information gathered by senors or commands issued by the leader or supervisor.

4. Although the nodes should stay connected for the entire mission, usually minimum connection between the nodes is desired.

5. Although each vehicle makes decision and behaves autonomously, they should collaborate and cooperate with each other to accomplish a global mission. Therefore, the network topology does not change as randomly and as fast as some other type of wireless networks such as cellular networks.

6. Since the vehicles should react based on their received information, NMAS are sensitive to delays which affect the performance of the entire mission and may even lead to instability.

1.4 The Proposed Approach

This research aims to introduce an analytical solution to the routing problem for NMAS by taking advantage of control theory machinery. The proposed framework is based on the fluid flow model of the traffic network. Therefore, from control perspective, the dynamic routing problem for NMAS is defined as designing a stabilizing decentralized controller for a dynamical system with multiple, unknown, and time-varying delay functions. Time-delay has been known to be a major source of instability. Moreover, complications arise in practical cases where there is limited *a priori* knowledge about transmitting, propagating, and processing delays and the fact that they vary according to the traffic flow and other disturbances in the network. Random changes in the network topology makes the system dynamics time-varying and more challenging to describe. Furthermore, the presence of some physical constraints such as limited link capacity, and limited buffer size make it more complicated to investigate. To tackle this problem, an $\mathcal{H}_\infty$ robust control scheme is proposed for which a measure of the queueing length, namely the "*worst-case queueing length*" is introduced as the objective function. Our proposed routing schemes will guarantee the desired routing performance in the presence of unknown time-varying delays and other network uncertainties by minimizing this objective function. The resulting optimization problem is then reformulated as a linear objective minimization problem involving Linear Matrix Inequality (LMI) constraints. Indeed, the LMI specification which can deal with convex and quasi-convex optimization problems with a variety of design specifications and constraints has been the motivation for using this method in solving the routing problem. The use of LMI also facilitates the inclusion of several physical constraints imposed on the network.

Toward this end, this research has been accomplished in three phases. In the first step, a centralized dynamic routing strategy is proposed for networks with fixed topology in the presence of multiple unknown, time-varying delays and physical constraints of the network.

In the next step, our proposed centralized control scheme is extended as a decentralized routing strategy. Unlike most of the existing work in the literature, delay functions are considered to be unknown *a priori* and fast time-varying. Therefore, our proposed decentralized routing strategy not only determines the fastest route in the presence of the unknown time-varying delays, but also is scalable to potentially large traffic networks.

Finally, the proposed routing control is modified such that it can also incorporate the changes in the network topology. To deal with changing network topology, we propose to formulate the mobile network routing problem as a Markovian jump switching system. In fact, changes in the neighboring sets are independent from the previous neighboring sets. Therefore, a Markovian jump process is a viable framework for describing and modeling the mobility behavior. A decentralized $\mathcal{H}_\infty$ control scheme is then introduced for the Markovian jump system with time-varying delays.

On the other hand, during the network operation, the number of destination nodes may also vary. Changes in the number of destination nodes can be represented as changes in the number of states in the system dynamics. However, in the Markovian jump or in other switching systems, usually the number of states are fixed. In this thesis, a singular Markovian jump system is introduced for incorporating changing number of destination nodes.

Consequently, a unified multi-objective optimization framework is introduced for dynamic routing for NMAS. The proposed $\mathcal{H}_\infty$ control strategy behaves stable in the presence of multiple, unknown time-varying delays, provides the desired routing performance by minimizing a measure of the queueing length, handles stochastic changes in the network topology, supports changes in the number of the destination nodes, and also incorporates various physical constraints. It is worth noting that our routing algorithm is topology-based, decentralized and is applicable to heterogenous NMAS.

1.5 Thesis Contributions

The contributions of this thesis are in two main directions, namely of routing and $\mathcal{H}_\infty$ control of time-delayed systems which have been explained briefly as follows:

- Contributions on $\mathcal{H}_\infty$ control of time-delayed systems

  1. For both slow and fast time-varying delayed systems a centralized $\mathcal{H}_\infty$ control strategy is introduced. The time-delay functions appearing in system inputs are not known *a priori*. The stability conditions are then transformed to Linear Matrix Inequality (LMI) feasibility conditions by imposing *no constraints* on the nature of the variation of the delay functions.

  2. The results obtained for designing centralized $\mathcal{H}_\infty$ control is extended for interconnected systems and a decentralized $\mathcal{H}_\infty$ control strategy for both slow and fast time-varying delayed systems is developed.

3. A decentralized $\mathcal{H}_\infty$ controller is proposed for Markovian Jump Linear Systems (MJLSs) with time-varying delays for mode-dependent as well as mode-independent interconnections terms. Unlike many existing methods in the literature for MJLSs with delays in *control*, the proposed state feedback controllers are changing at each mode which provides less conservative and better performance in the response of parameter variations. In the considered dynamics it is assumed that the interconnected terms are changing at each switching mode. LMI constraints provided to design a delay-dependent $\mathcal{H}_\infty$ controller guarantee stochastic stability of the overall system.

4. A decentralized $\mathcal{H}_\infty$ controller for *singular* MJLSs with time-varying delays for mode-dependent and mode-independent interconnections is presented. The provided solution can guarantee that the closed-loop system is stochastically stable and also the solution is piecewise regular and piecewise impulse-free.

- Contributions on routing of NMAS

1. The concept of the *worst-case queueing length* is proposed as an objective function for the routing problem and stabilizing $\mathcal{H}_\infty$ control framework is introduced for routing of traffic networks. Therefore, the proposed routing algorithm is able to cope with multiple, unknown, and time-varying transmitting, processing, and propagating delays and also minimize the certain objective function.

2. The LMI technique is applied to satisfy the incorporating physical constraints in centralized routing problem including capacity constraint, buffer size constraint, and nonnegativity of flows and queues. An optimization problem is introduced for centralized routing of networks with fixed

topology that incorporates the LMIs for stabilizing $\mathcal{H}_\infty$ control and the associated physical constraints.

3. The LMI conditions are provided to satisfy the physical constraints imposed on decentralized routing strategy. By employing the introduced decentralized $\mathcal{H}_\infty$ and LMIs representing the physical constraints, an optimal solution is developed for decentralized routing of NMAS.

4. Stochastically changing the network topology is modeled by Markovian Jump Linear System (MJLS) with time-varying delays. In other words, variable network topology is presented by switching system where the switching rule is governed based on Markovian probability transitions.

5. The corresponding physical constraints for networks with variable topology is represented by LMI conditions and then an optimization solution is provided for decentralized routing for networks with variable topology.

6. Changing number of destination nodes are described by singular MJLS with time-varying delays. Indeed, when a destination node becomes inactive (active) the queuing dynamics switches from regular to singular (singular to regular). This definition enables us to analyze the stability of the overall system in a unified control strategy, avoid losing the remained messages in the corresponding queues to the destination nodes which become inactive, and route these messages and clear the certain queues quickly.

7. The obtained LMI conditions corresponding to physical constraints for variable topology are modified for singular MJLS dynamics and by applying the proposed decentralized $\mathcal{H}_\infty$ for singular MJLS with time-varying delays, a routing strategy is provided for the networks with variable topology and variable number of destination nodes.

1.6 Structure of the Thesis

The remainder of the thesis is organized as follows: In Chapter 2, the basic definitions and concepts on the $\mathcal{H}_\infty$ optimal control and the LMI technique are given. Also, a description of the traffic model in terms of the queueing dynamics and the associated physical constraints are provided. The routing problem is then formulated as an $\mathcal{H}_\infty$ robust control framework. Chapter 3 provides centralized as well as decentralized $\mathcal{H}_\infty$ control design for multiple time-varying delayed systems. In Chapter 4, the introduced centralized and decentralized controllers in Chapter 3 are applied to solve the routing problem of NMAS with fixed topology considering physical constraints. Chapter 5 provides a decentralized $\mathcal{H}_\infty$ control scheme for the Markovian jump systems with time-varying delays in control. The results are then extended to decentralized Markovian jump systems when the interconnected subsystems are mode-dependent. An $\mathcal{H}_\infty$ control scheme is also proposed for the singular Markovian jump systems with time-delays. In Chapter 6, the dynamics of mobile NMAS are modeled by a Markovian jump switching system. The stabilizing controller established for the Markovian jump system with mode-dependent interconnected terms in Chapter 5 is employed to solve the routing problem in mobile NMAS. To address the nodes mobility and changing nodes destination in a unified routing control, the $\mathcal{H}_\infty$ controller designed for the singular Markovian jump system in Chapter 5 is proposed. The simulation results obtained from our introduced routing algorithm for mobile NMAS is then compared and bench marked with some popular routing algorithms in the literature such as AODV [49] and OLSR [48]. Finally, in Chapter 7, conclusions and future work are presented.

The results of Chapters 2 and 3 were published in [56] [57], [58], and [59]. The results of Chapters 5 and 6 are provided to submit as two more journal and two conference papers.

Chapter 2

Problem Formulation

In this chapter, the dynamics of the communication network is expressed based on the fluid flow model for fixed as well as variable network topology. The dynamics are stated as standard state space representation and a novel routing objective is introduced based on the minimization of the *worst-case queueing length.* In fact, the proposed notion of *worst-case queueing length* can be interpreted as another measure of the delay in the network.

The chapter is organized as follows. In Section 2.1 a brief introduction to Poisson process, $\mathcal{H}_\infty$ control, and Linear Matrix Inequality (LMI) technique are given. The dynamical model of traffic flow in communication networks in presented in Section 2.2. In Section 2.2.1, the centralized dynamic model of the traffic flow and the physical constraints imposed on the routing problem are presented. The introduced dynamics is then modified for a decentralized strategy in Section 2.2.2 and also for mobile networks in Section 2.2.3.

2.1 Basic Definitions and Concepts

The norm of a vector $x \in R^n$ and the spectral norm of a matrix $A \in R^{m \times n}$ are denoted as

$$\|x\| = \sqrt{x^T x}\ ,\ \ \|A\|_s = \sqrt{\lambda_{max}(A^T A)}$$

where $\lambda_{max}(.)$ denotes the largest eigenvalue of the positive-definite or positive-semidefinite matrix (.). We denote the smallest eigenvalue of a positive-definite matrix (.) by $\lambda_{\min}(.)$.

2.1.1 Poisson Process [1]

In this thesis it is assumed that the packets arrive to the network according to a Poisson process. A Poisson process, named after the French mathematician Simon-Denis Poisson (1781-1840), is a stochastic process which is used for modeling random events in time that occur to a large extent independently of one another. The Poisson process is a collection $\{N(t) : t \geq 0\}$ of random variables, where $N(t)$ is the number of events that have occurred up to time t (starting from time 0). The number of events between time a and time b is given as $N(b) - N(a)$ and has the following distribution:

$$f(x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

where λ is a positive real number, equal to the rate of expected number of occurrences that occur during the given interval and x is the number of occurrences of an event.

2.1.2 Optimal $\mathcal{H}_\infty$ Control Problem

The objective of $\mathcal{H}_\infty$ optimal control design is to simultaneously guarantee the internal stability of the closed-loop system while achieving certain performance specifications despite uncertainties and modeling inaccuracies. Consider the following state space representation of a dynamical system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + B_w w(t) \\ z(t) &= Cx(t) \end{aligned} \tag{2.1}$$

where z is the regulated output, x is the state, w is the exogenous input, and u is the control signal.

The $\mathcal{L}_2$ norm of a signal $z(t)$ (denoted by $\|z(t)\|_2$) which reflects the energy content of the signal is defined as [60]:

$$\|z(t)\|_2 = \left(\int_0^\infty z^T(t)z(t)dt\right)^{\frac{1}{2}} \tag{2.2}$$

Accordingly, the $\mathcal{H}_\infty$ norm of system (2.1) which reflects the worst-case $\mathcal{L}_2$ gain of the system due to the exogenous signal w is defined as [60]

$$\sup_{w \in L_2} \frac{\|z(t)\|_2}{\|w(t)\|_2}$$

The standard $\mathcal{H}_\infty$ control problem is concerned with finding an admissible (stabilizing) controller such that the $\mathcal{H}_\infty$ norm of system (2.1) is minimized. Considering the controller as a state feedback $u(t) = Kx(t)$, the $\mathcal{H}_\infty$ control problem is to find the state feedback gain K such that

$$\min_{K \text{ stabilizing}} \quad \sup_{w \in L_2} \frac{\|z(t)\|_2}{\|w(t)\|_2} = \gamma, \quad \gamma > 0 \tag{2.3}$$

Now by considering the definition of the $\mathcal{L}_2$ norm in (2.2), the $\mathcal{H}_\infty$ control objective function (2.3) can be expressed as

$$\begin{aligned} \min \gamma \qquad s.t. \qquad & J(w) < 0 \qquad\qquad (2.4) \\ J(w) \quad = \quad & \int_0^\infty (z^T(s)z(s) - \gamma w^T(s)w(s))ds, \quad \gamma > 0 \end{aligned}$$

Moreover, in this thesis the stabilizing control gain K is obtained to guarantee *internal stability* or *ultimate boundedness* of the closed-loop system. Internal stability and ultimate boundedness are defined as follows:

Definition 2.1. Internal Stabilizability [61]:

Consider a nonlinear system modeled by equations of the form

$$\begin{aligned} \dot{x}(t) &= f(x) + g_1(x)w + g_2(x)u \\ z(t) &= h_1(x) + k_{12}(x)u \\ y(t) &= h_2(x) + k_{21}(x)w \end{aligned} \tag{2.5}$$

where z is the regulated output, x is the state, w is the exogenous input, y is measured output, and u is the control signal. It is also assumed that $h_1(0) = 0,\ h_2(0) = 0,\ f(0) = 0$. The control action to (2.5) is to be provided by a controller, which processes the measured variable y, generates the appropriate control input u such that it

- achieves closed-loop asymptotic stability (when $w = 0$).
- attenuates the influence of the exogenous input w on the penalty variable z, i.e., satisfies objective (2.4).

Definition 2.2. Ultimate Boundedness [62]: Consider the following nonlinear dynamics

$$\dot{x}(t) = f(t, x) \tag{2.6}$$

where $f : [0, \infty) \times D \to \mathcal{R}^n$ is piecewise continuous in t and locally Lipschitz in x on $[0, \infty) \times D$, and $D \subset \mathcal{R}^n$ is a domain that contains the origin. The solution of (2.6) is uniformally ultimately bounded with ultimate bound b, if there exist positive constants b and c, independent of $t_0 \geq 0$, and for every $a \in (0, c)$, there is

$T = T(a, b) \geq 0$, independent of t_0, such that

$$\|x(t_0)\| \leq a \Rightarrow \|x(t)\| \leq b, \forall t \geq t_0 + T$$

One of the popular methods for solving the $\mathcal{H}_\infty$ optimal control and obtaining internal stability is the Linear Matrix Inequality (LMI) methodology. The LMI methodology is introduced briefly in the next section.

2.1.3 Linear Matrix Inequality (LMI)

The LMI methodology for solving convex and quasi-convex optimization problems in the presence of design constraints has attracted a great deal of interest in the past decade. LMI techniques have emerged as powerful design tools in control engineering. The applications range from control engineering such as multi-model/multi-objective state feedback design, robust pole placement, control of stochastic systems, multi-criterion LQG/LQR, $\mathcal{H}_\infty$ control, positive orthant stability to system identification and structural design [63]. The main factors that make LMI techniques appealing can be stated as follows [64]:

- A variety of design specifications and constraints can be expressed in terms of LMI feasibility conditions which make it suitable for multi-objective optimization problems.

- LMI algorithms formulate the problem in terms of a convex optimization problem. Hence, usually exact solutions can be found [64].

- While most of the problems with multiple constraints or objectives lack analytical solutions in terms of matrix equations, they often remain tractable in the LMI framework. This makes LMI-based design a valuable alternative to classical analytical methods.

A linear matrix inequality (LMI) is a matrix inequality of the form

$$A(x) = A_0 + x_1 A_1 + ... + x_N A_N < 0$$

where $x = (x_1, ..., x_N)$ is a vector of unknown scalars (the so-called decision or optimization variables), $A_0, ..., A_N$ are given symmetric matrices, and < 0 stands for negative definite, i.e., the largest eigenvalue of $A(x)$ is negative. The LMI solution set, also called the feasible set, is a convex subset of R^N [64]. In other words, finding a solution x to the LMI condition, if any, is a convex optimization problem. In most control applications, the LMIs are defined as finding the solution to X for any constraint of the form

$$L(X_1, ..., X_n) < R(X_1, ..., X_n)$$

where $L(.)$ and $R(.)$ are affine functions of some structured matrix variables $X_1, ..., X_n$. Many engineering optimization problems involve matrix inequalities which are not linear in their variables. To convert such problems to linear matrix inequalities and use LMI optimization techniques the so-called Schur complement is usually employed. In the following lemma, Schur complement states that a certain quadratic matrix inequality is equivalent to an LMI.

Lemma 2.1. (Schur Complement) Given the matrix M as shown below:

$$M = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix}$$

where R and Q are symmetric.

- M is positive definite if and only if

$$R \geq 0, \quad Q - SR^{-1}S^T \geq 0 \tag{2.7}$$

or

$$Q \geq 0, \quad R - S^T Q^{-1} S \geq 0$$

- M is negative definite if and only if

$$R \leq 0, \quad Q - SR^{-1}S^T \leq 0 \tag{2.8}$$

or

$$Q \leq 0, \quad R - S^T Q^{-1} S \leq 0$$

For the proof of this lemma refer to [65].

The following lemma is used in this thesis

Lemma 2.2. [65] For any matrices U, $V \in \mathbb{R}^{n \times n}$ with $V > 0$, we have

$$UV^{-1}U^T \geq U + U^T - V \tag{2.9}$$

Proof: Note that $V > 0$ yields

$$(U - V)V^{-1}(U - V)^T > 0$$

Therefore,

$$UV^{-1}U^T - UV^{-1}V^T - VV^{-1}U^T + V \geq 0$$

which leads to (2.9). ■

2.1.4 Centralized and Decentralized Controller

Different definitions have been given for centralized and decentralized controllers in the literature. To avoid ambiguity, the following definitions are adopted for these concepts in this research.

Definition 2.3. A controller is classified as **centralized** if a central (global) controller is responsible for making decisions for the entire system.

Definition 2.4. [66] A controller is classified as **decentralized** if the controller for each subsystem (through the partitioning of the overall system) is capable of making decisions using only local information.

2.2 Communication Network Model

Conservation law states that the sum of incoming traffic flow is equal to the outgoing traffic flow of a node. This is the basis of a static model of the traffic network used for developing some early routing algorithms [11], [24], [37]. Static model of the traffic network nevertheless, cannot express an accurate behavior of the traffic. In other words, since all received messages cannot be routed instantaneously, (e.g., due to the limited capacity of the links), they are put into a queue and eventually are sent out to a downstream node sequentially. Using a dynamical models can provide a more precise description of the network operations and gives us an opportunity to develop control techniques whose properties can be studied analytically. Moreover, the fluid-flow models allow one to use control machinery to investigate stability [20], [22], [25], [26], [29] and [30]. Segal [28] proposed a queueing model based on the conservation law to describe the dynamic nature of the traffic flow. They also introduced the notion of "total queueing delay" as the total time that messages have to spend in the queue before reaching their destinations and developed a centralized routing controller to minimize the total queueing delay. In [26], other types of delay, namely transmission delay, propagation delay, and processing delay were also considered in the dynamic model of the network flow, and a decentralized controller was proposed that guarantees the boundedness of the queueing lengths and the delays. In [25], a discrete-time version of the model presented in [28] is considered as the network model and a neural network control strategy was employed to solve the dynamic routing problem. In [29], another fluid flow dynamics was introduced without considering the effects of the queueing delay in the routing algorithm. In [22], the dynamical model used by [28] was considered. A dual decomposition was used to vertically decompose the system problem into three protocol layers where congestion control, routing and scheduling problems are jointly solved in order to solve the network utility maximization problem. At each sequence, after gathering the local information each node adjusts its sending rate and determines the number of bits that should be sent to each destination. The routing strategy can adapt itself based on congestion and network traffic. However, it is assumed that the network topology is fixed. Furthermore, none of the transmission, propagation, processing, and queueing delays were considered in routing algorithm. Since the sending rate and scheduling is determined online, the processing delay to obtain a suitable routing is not negligible and will deteriorate the performance of the system in practice and may even lead to instability.

In this research, similar to the mainstream of the existing work, the dynamic model of [28] is considered. However, most algorithms developed in the literature assume that the various delays in the network (i.e., transmission, propagation, and processing) are fixed and known *a priori*, an assumption that is not valid in practice. In the model considered here the delays are unknown and time-varying which makes it more realistic for real-world traffic network applications. In fact, delays are unknown and could vary significantly due to unpredictable circumstances such as unpredicted traffic flows and congestion. The following sections describe the dynamical model for centralized, decentralized and mobile traffic networks considered in this research.

2.2.1 Centralized Dynamic Model of the Traffic Network

Consider a data communication network as a directed graph (N, L) consisting of a set N of n nodes and a set L of l oriented links. Each node receives messages from both the upstream nodes within the network and from outside the network. Each message has a destination node $d \in N$ such that it is absorbed as soon as it arrives at that node. It is assumed that the network is "connected", i.e., each node of the network must be reachable from other nodes. Let us consider $\bar{d}$ as number of destinations. In the worst-case where all the nodes are source as well as destination, at each node $i \in N$, there will be $n-1$ queues in which messages are stored for all destinations, $1, 2, ..., i-1, i+1, ..., n$. The communication network dynamics can be expressed by the following queueing model that can be derived based on the fluid flow conservation principle, namely

$$\dot{q}_i^d(t) = \sum_{k \in \wp(i), k \neq d} f_{ki}^d(t - \tau_{ki}^d(t)) + w_i^d(t) - \sum_{j \in \aleph(i)} f_{ij}^d(t) \tag{2.10}$$

where

q_i^d: message queueing length at node i destined to node d

$\wp(i)$: set of upstream neighbors of node i

$\aleph(i)$: set of downstream neighbors of node i

$f_{ki}^d(t)$: input traffic flow routed from node $k \in \wp(i)$ to node i destined to node d

$f_{ij}^d(t)$: output traffic flow routed from node i to node $j \in \aleph(i)$ destined to node d

$w_i^d(t)$: external input flow entering node i destined to node d

$\tau_{ki}^d(t)$: total unknown time-varying and bounded delay in transmitting, propagating, and processing of messages with destination d routed from node k to node i.

It is worth noting that in a wireless network with multiple channels available for transmission, usually the links that share a common node cannot transmit or receive, simultaneously, whereas the links that do not share nodes can do so [22], [67]. This issue can also be modeled as a delay in message transmission. Therefore, it is considered as part of $\tau^d_{ki}(t)$.

Physical Constraints

Physical characteristics in a traffic network impose certain constraints that should be considered in the routing problem. A typical set of constraints can be given as follows

- The non-negativity constraints

$$f^d_{ij}(t) \geq 0 \tag{2.11}$$

$$q^d_i(t) \geq 0 \tag{2.12}$$

- Capacity constraint: the total flow in each link cannot exceed the capacity of that link denoted by c_{ij},

$$\sum_{d \in N^i} f^d_{ij}(t) \leq c_{ij} \quad i \in N, \quad j \in \aleph(i) \tag{2.13}$$

  where $N^i = N \backslash \{i\}$.

- Buffer size constraint: to avoid packet loss, the length of the queue should always remain smaller than the maximum value specified for the buffer $q^d_{max_i}$ that is

$$q^d_i(t) \leq q^d_{max_i} \tag{2.14}$$

Standard State Space Representation

In this section, the routing problem is described and formulated in the $\mathcal{H}_\infty$ optimal control framework. The routing problem is in fact concerned with adjusting the output flow of each queue, $f_{ij}^d(t)$, according to the network traffic information, such that certain objective functions are minimized. Let us define

$$\begin{aligned} x(t) &= vec\{q_i^d(t)\} \in \Re^{n(n-1)} \\ w(t) &= vec\{w_i^d(t)\} \in \Re^{n(n-1)} \\ u(t) &= vec\{f_{ij}^d(t)\} \in \Re^{l(n-1)}, \quad i, j, d = 1, ..., n \end{aligned} \tag{2.15}$$

where the operation *vec* stands for a vector. Since the input flows for each queue are due to its upstream neighbors, they are indeed the output flows of those nodes with some delays. It should be noted that the time-varying delay functions associated with these nodes are not known *a priori* and are different from one another due to differences in the traffic load in each link and other network uncertainties. Let us define

$$\begin{aligned} u(t-\tau(t)) &= vec\{f_{ji}^d(t-\tau_{ji}^d(t))\} \in \Re^{l(n-1)} \\ \tau(t) &= vec\{\tau_{ji}^d(t)\}, \quad j, i, d = 1, ..., n. \end{aligned} \tag{2.16}$$

In view of the above notations, equation (2.10) can be rewritten in the standard state space representation as

$$\dot{x}(t) = Bu(t) + B_d u(t-\tau(t)) + B_w w(t) \tag{2.17}$$

where $B \in \Re^{n(n-1)\times l(n-1)}$ and $B_d \in \Re^{n(n-1)\times l(n-1)}$ represent the network connectivity (downstream and upstream nodes, respectively). In fact, $\{B_{ij}(B_{d_{ij}})\}$ is equal to $-1(1)$, if the flow u_j as defined in (2.15) is a downstream (upstream) flow of node i and is zero otherwise. Moreover, $B_w = I_{n(n-1)\times n(n-1)}$. The following example should

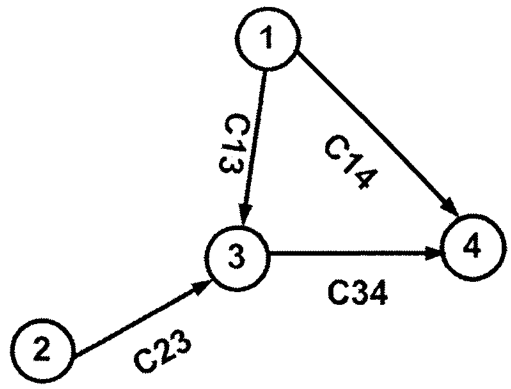

Figure 2.1: A Sample Network Topology.

clarify the definitions of B and B_d.

Example 2.1. Consider the network shown in Fig. 2.1. The network consists of 4 nodes where all messages are routed to one destination, namely node 4. Therefore, there is only one queue at each node. Let us define

$$u(t) = \begin{bmatrix} f_{13}^4(t) \\ f_{14}^4(t) \\ f_{23}^4(t) \\ f_{34}^4(t) \end{bmatrix}, u(t-\tau(t)) = \begin{bmatrix} f_{13}^4(t-\tau_{13}^4(t)) \\ f_{14}^4(t-\tau_{14}^4(t)) \\ f_{23}^4(t-\tau_{23}^4(t)) \\ f_{34}^4(t-\tau_{34}^4(t)) \end{bmatrix}$$

Corresponding to above situation, $B_w = I_{3\times 3}$ and the matrices B and B_d are given by

$$B = \begin{bmatrix} -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, B_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

It should be emphasized that the time-delays are indeed a major source of instability for the entire network. Classical control theory is not sufficiently capable of addressing stability and performance issues of time-delayed systems. Complications do arise in practical situations when there is no *a priori* knowledge about transmitting, propagating, and processing delays. Furthermore, the time-delay functions vary according to the traffic flow and other stimuli and disturbances in the network. Based on a given traffic flow characteristics the amount of delay may be substantially high.

The $\mathcal{H}_\infty$ robust optimal control design approach is a suitable framework for dealing with system uncertainties and unknown time-delays. According to the $\mathcal{H}_\infty$ robust optimal control definition (2.3), our proposed $\mathcal{H}_\infty$ routing algorithm is actually a state feedback controller of the form $u(t) = Kx(t)$ that simultaneously guarantees the stability of the network in the presence of time-varying delays and minimizes the *worst-case queueing length* due to the external inputs. Specifically, by selecting the regulated output as $z(t) = Cx(t)$, where C is a weight matrix that is full rank, and considering the $\mathcal{H}_\infty$ objective function (2.4) the routing problem can be recast into the following optimization problem

$$\begin{aligned} \min \gamma \qquad s.t. \qquad & J(w) < 0 \\ J(w) \quad = \quad & \int_0^\infty (z^T(s)z(s) - \gamma w^T(s)w(s))ds, \quad \gamma > 0 \end{aligned} \tag{2.18}$$

In other words, by using the objective function (2.18) the messages are routed such that the network is simultaneously stabilized subject to unknown transmitting, propagating, and processing delays $\tau(t)$, and the queueing length, x, is minimized subject to the presence of the external input w. It is worth noting that by minimizing the worst-case queueing length, one can indeed accomplish a measure of the minimum queueing delay.

The following specifications are also defined to be met

- Routing avoid permanent loops.

- Global stability is preserved, i.e., queues remain bounded in presence of bounded external flows and become clear when the external flows converge to zero.

Now, by using the definitions of u and x, the constraints (2.11)-(2.14) can be expressed as:

$$u \geq 0 \tag{2.19}$$

$$x \geq 0 \tag{2.20}$$

$$G_k u \leq c_k, \quad k = 1, ..., l \tag{2.21}$$

$$Q_{di} x \leq x_{max_{di}}, \quad d = 1, ..., \bar{d}, \; i = 1, ..., n \tag{2.22}$$

where $x_{max_{di}} = q^d_{max_i}$, c_k is the capacity of the link k, and G_k should be defined such that multiplying G_k with u yields the total flows that should go through the link k, and Q_{di} should be defined such that $Q_{di}x$ leads to the queueing length of the buffer di, for $d = 1, ..., \bar{d}, \; i = 1, ..., n$.

2.2.2 Decentralized Dynamic Model of the Traffic Network

According to the definition given in Section 2.1.4, a decentralized controller requires that the system dynamics is decomposed into local subsystems. In the routing problem, each node of the traffic network is considered as a subsystem that includes all its queues corresponding to different destinations. Consequently, the decentralized dynamic model of the traffic flow at each node, or subsystem is given by

$$\dot{x}_i(t) = B_i u_i(t) + \sum_{j \in \wp(i)} B_{dij} u_j(t - \tau_{ij}(t)) + B_{wi} w_i(t) \tag{2.23}$$

where

- $x_i = vec\{q^d_i(t)\} \in \Re^n \;\; for \;\; d = 1, ..., n$ denotes the queueing length in node i for different destinations,
- $u_i(t) = vec\{f^d_{ij}(t)\} \in \Re^l$, denotes the flows sent from node i,

- $\tau_{ij}(t)$ denotes the *unknown but bounded time-varying* total delay in transmission, propagation, and processing, and

- $w_i(t) \in \Re^n$ denotes the external input flow for node i.

- $\wp(i)$: set of upstream neighbors of node i. i.e., interconnected subsystems to node i.

The matrices B_i, B_{dij} and B_{wi} are defined for node i similar to that in Section 2.2.1. At each node (subsystem) the routing problem is to determine an $\mathcal{H}_\infty$ state feedback controller $u_i = K_i x_i$, such that the following global objective function is minimized:

$$J(w) = \int_0^\infty (z^T z - \gamma w^T w) ds < 0, \quad \gamma > 0 \tag{2.24}$$

where $z = vec\{z_i\}$ and $w = vec\{w_i\}$.

Physical Constraints

The physical constraints (2.19)-(2.22) can also be modified for each subsystem accordingly:

$$u_i \geq 0 \tag{2.25}$$

$$x_i \geq 0 \tag{2.26}$$

$$G_{k_i} u_i \leq c_{k_i}, \quad k_i = 1, ..., li, \quad i = 1, ..., n \tag{2.27}$$

$$Q_{di} x_i \leq x_{max_{di}}, \quad d = 1, ..., \bar{d}, \quad i = 1, ..., n \tag{2.28}$$

where $x_{max_{di}} = q^d_{max_i}$ and li is number of links in subsystem i.

2.2.3 Decentralized Dynamic Model of Mobile Traffic Network

When the topology of a network changes due to either node mobility, loss of node power, or addition of new nodes, the neighboring sets and consequently the connectivity matrices B_i and B_{dij} in (2.23) will change. Indeed, the connectivity between two nodes is determined by their radio range which in turn is a function of the antenna pattern, power level, geographic terrain, etc. To achieve an efficient routing strategy certain physical parameters such as the distance between the nodes or maximizing the nodes life-time, and fading nodes should be considered in defining the neighborhood sets. In [68], the distance d_{ij} between every two nodes i and j is calculated and if the distance is less than or equal to their radio range, R, then the nodes are connected. Keeping the nodes alive as long as possible is another issue in defining the neighborhood sets [69], [13]. In view of the above, the dynamics of the network characterizing the traffic flow will become *time varying.* Consequently, the dynamics of the system (2.23) is now modified to the following representation for mobile networks

$$\dot{x}_i(t) = B_i(r(t))u_i(t) + B_{w_i}(r(t))w_i(t) + \sum_{j \in \wp_{r(t)}(i)} B_{dij}(r(t))u_j(t - \tau_{ji}(t)) \quad (2.29)$$

where $r(t)$ is a function representing the rule for changing the neighboring sets. In mobile networks, the objective function defined in (2.24) is considered to be minimized for the routing problem.

Physical Constraints

For mobile networks the physical constraints given in (2.25), (2.26), and (2.28) will remain the same. However, the node mobility causes the link capacity to also be changing in MANETs. Therefore, the capacity constraints can be expressed as follows:

$$G_{k_i} u_i \leq c_{k_i}(r(t)), \quad k_i = 1, ..., li, \quad i = 1, ..., n \tag{2.30}$$

It is rather important to note that for achieving a good performance for any routing protocol for MANETs, the rate of topology change must not be higher than the rate of state information propagation. Otherwise, the routing information may not always remain stale and the routing scheme could even completely fail.

2.3 Conclusions

In this chapter, after revisiting the basic definitions and concepts, the centralized as well as decentralized dynamics of traffic networks was expressed based on fluid flow model. The dynamics were presented in a standard state space representation and transiting, propagating and processing delays are considered as unknown time varyings delays appeared in control.

Finally, by introducing the *worst-case queueing length* as routing objective, the routing problem was defined as an $\mathcal{H}_\infty$ optimization problem incorporating the physical constraints such as buffer size and capacity constraints.

Chapter 3

An H_∞ Control Strategy for Time-Varying Delayed Systems

Real world systems never respond instantaneously to exogenous signals and some time must elapse before any effect is apparent. Indeed, many practical systems such as chemical processes, communication networks, tele-operation systems, biological systems and economical systems exhibit considerable amount of delays in responding to input signals. The dynamic equations governing such systems include delay functions in inputs, states and/or time-derivative of the system states. Delayed systems represent a class of infinite-dimensional systems, where the presence of delay may cause complex behaviors including oscillatory responses, instability and poor performance associated with the closed-loop system. Stability analysis and control of time-delay systems have attracted much interest in the literature in the past decade. Research is focused on both frequency domain [70]- [72] and time domain approaches [65], [73]. In [74], the advantages and disadvantages of time and frequency domain approaches for treating time-delayed systems have been investigated.

During the past several decades, different aspects of time-delayed systems have been investigated, the majority of which can be classified into two broad categories, namely stability analysis and design of a stabilizing feedback controller.

Stability of time-delayed systems can be studied based on two different criteria: delay-dependent and delay-independent stability.

- **Delay-independent stability** introduces some conditions for stability of the time-delayed system independent of the delay, τ. Hence, to design a delay-independent controller, no *a priori* knowledge about the delay is required [63], [75]. However, delay-independent controllers tend to generally yield conservative results.

- **Delay-dependent stability** seeks stability conditions that guarantees the stability of the system for all delays satisfying $0 < \tau < \tau^*$, where τ^* is the upper bound of the delay. Hence, to design a delay-dependent controller, the upper bound of the delay should be known [76], [77].

In most practical cases however, the delays are not only unknown, but also varying with time. Treating time-varying delays requires a more involved and deeper analysis since their presence may induce further complex behaviors. Time-varying delayed systems may be categorized into slow time-varying and fast time-varying delayed systems. Most of the current results in the literature, however deal with uniformly bounded differentiable delays with a bounded delay-derivatives, i.e., $|\dot{\tau}| \leq 1$ [78], [79]. In other words, the resulting LMI condition is feasible only if the delay function grows slowly. However, there are several time-varying delayed systems which violate this condition and belong to the second class, namely fast time-varying delayed systems.

Stability of time-delayed systems with time-varying delays can be tackled in two different ways, namely

- **Delay Rate Independent (DRI)**: In this approach, stability conditions are obtained independent of the rate of change of the delay function ($\dot{\tau}$) [80].

- **Delay Rate Dependent (DRD)**: Alternatively, an *a priori* knowledge of the upper bound of the time-derivative of the delay function can be used to obtain stability results leading to the DRD approaches [81], [82].

From another perspective, the stability results developed in the time-domain are mainly based on Lyapunov's direct method using either Lyapunov-Krasovskii functionals or Lyapunov-Razumikhin functions:

- **Lyapunov-Krasovskii (L-K) functionals** [75]: To study the stability of delay free systems, a Lyapunov function candidate $V(t, x(t))$ is considered which presents a potential measure of quantifying the deviation of the state $x(t)$ from the trivial solution 0. In this procedure, $x(t)$ is needed to specify the system's future evolution beyond t. In time-delayed systems, the state required for the same purpose at time t is the value of $x(t)$ in the interval $[t-\tau,\ t]$. Let, $x_\tau(t),\ t \geq t_0$ denotes the restriction of $x(.)$ to the interval $[t-\tau,\ t]$, translated to $[-\tau, 0]$. Therefore, the corresponding Lyapunov function candidate will be a functional $V(t, x_\tau)$ depending on x_τ which also measures the deviation of x_τ from the trivial solution 0.

  Theorem 3.1. ***Lyapunov- Krasovskii Stability Theorem*** *[75] : Suppose $\dot{x}(t) = f(t, x_\tau)$ and $f : \mathcal{R} \times \mathcal{C} \rightarrow \mathcal{R}^n$ maps $\mathcal{R}\times$ (bounded set in $\mathcal{C}$) into a bounded set in $\mathcal{R}^n$, and that $u, v, w : \bar{\mathcal{R}}_+ \rightarrow \bar{\mathcal{R}}_+$ are continuous nondecreasing functions, where additionally $u(s)$ and $v(s)$ are positive for $s > 0$, and $u(0) = v(0) = 0$. If there exists a continuous differentiable functional $V : \mathcal{R} \times \mathcal{C} \rightarrow \mathcal{R}$ such that*

$$\begin{aligned} u(\|\phi(0)\|) &\leq V(t, \phi) \leq v(\|\phi\|_c) \\ \dot{V}(t, \phi) &\leq -w(\|\phi(0)\|) \end{aligned}$$

  then the trivial solution of $\dot{x}(t) = f(t, x_\tau)$ is uniformly stable. If $w(s) > 0$ for all $s > 0$, then it is uniformly asymptotically stable. If in addition, $lim_{s\rightarrow\infty} u(s) = \infty$ then it is globally uniformly asymptotically stable.

The details of the proof is given in [74].

For example, a Lyapunov-Krasovskii candidate can be a quadratic function of the form

$$V(t,x) = x^T(t)Px(t) + \int_{t-\tau}^{t} x^T(\theta)Qx(\theta)d\theta$$

where $0 < P = P^T \in \mathcal{R}^{n\times n}$ and $0 < Q = Q^T \in \mathcal{R}^{n\times n}$ are weighting matrices. The first term of the functional is concerned with the present state, whereas the second term accumulates the effects of the delayed state [75].

In [63], [76], and [81] the Lyapunov-Krasovski functional has been considered to develop LMI conditions for stability of time-delayed systems.

- **Lyapunov-Razumikhin (L-R)** [74]: The key idea behind the Razumikhin theorem also focuses on a function $V(x)$ that is representative of the size of $x(t)$. Let us first define

$$\bar{V}(x_\tau) = \max_{\theta\in[-\tau,0]} V(x(t+\theta))$$

which serves as a measure of the size of x_τ. If $V(x(t)) < \bar{V}(x_\tau)$, then $\dot{V}(x) > 0$ does force $\bar{V}(x_\tau)$ not to grow. For not making $\bar{V}(x_\tau)$ grow, it is merely necessary that $\dot{V}(x(t))$ be not positive whenever $V(x(t)) = \bar{V}(x_\tau)$.

Theorem 3.2. ***Lyapunov- Razumikhin Stability Theorem*** *[74]: Suppose $\dot{x}(t) = f(t, x_\tau)$ and $f : \mathcal{R}\times\mathcal{C} \to \mathcal{R}^n$ maps $\mathcal{R}\times$ (bounded set in $\mathcal{C}$) into a bounded sets in $\mathcal{R}^n$, and that $u, v, w : \bar{\mathcal{R}}_+ \to \bar{\mathcal{R}}_+$ are continues nondecreasing functions, where additionally $u(s)$ and $v(s)$ are positive for $s > 0$, and $u(0) = v(0) = 0$, and v strictly increasing. If there exists a continuously differentiable function $V : \mathcal{R}\times\mathcal{R}^n \to \mathcal{R}$ such that*

$$\begin{aligned} u(\|x\|) &\leq V(t,x) \leq v(\|x\|_c) \text{ for } t \in \mathcal{R} \text{ and } x \in \mathcal{R}^n \qquad (3.1) \\ \dot{V}(t,x(t)) &\leq -w(\|x(t)\|) \text{ whenever } V(t+\theta, x(t+\theta)) \leq V(t,x(t)) \text{ for } \theta \in [-\tau, 0] \end{aligned}$$

then the trivial solution of $\dot{x}(t) = f(t, x_\tau)$ *is uniformly stable. If* $w(s) > 0$ *for* $s > 0$*, and there exist a continuous nondecreasing function* $p(s) > 0$ *for* $s > 0$ *such that (3.1) is given by*

$$\dot{V}(t, x(t)) \leq -w(\|x(t)\|) \text{ if } V(t+\theta, x(t+\theta)) \leq p(V(t, x(t))) \text{ for } \theta \in [-\tau, 0]$$

then it is uniformly asymptotically stable. If in addition, $lim_{s\to\infty} u(s) = \infty$ *then it is globally uniformly asymptotically stable.*

For details of the proof, refer to [74].

Lyapunov-Razumikhin function has been used to design robust $\mathcal{H}_\infty$ controllers in [75], [83], [84]. Lyapunov-Razumikhin stability criteria are in general more conservative than those of Lyapunov-Krasovskii.

In [82], a delay rate dependent (DRD) LMI condition is introduced for stability analysis of fast time-varying delayed systems. The free weighting method was used by considering *a priori* knowledge of both the lower bound and the upper bound of the delay, leading to delay rate dependent conditions. However, similar to most of the results introduced for stability analysis in the literature, the LMI conditions are not extendable for design of a *stabilizing state feedback control.* In other words, the resulting conditions are neither linear in parameters nor bilinear in parameters, for which a changing variable technique can be used.

In many practical cases designing a stabilizing controller based on DRI and DRD approaches for time-delayed system is also of great interest. Stabilization or robust stabilization problem of time-delayed systems with fixed delays or slow time-varying delays has been addressed, e.g. in [81]. It should be noted that in studying the stability or designing delay-dependent controllers, generally certain model transformations and/or bounding techniques on the cross product terms are assumed that ultimately lead to conservative results. Since the transformed and the original systems are not always equivalent, some constraints should be satisfied in order to make them equal [85], [86]. Therefore, the Achille's heels and challenge in this area of research is to develop novel methods for deriving less conservative stability conditions in designing delay-dependent $\mathcal{H}_\infty$ controllers. In [81], the so-called descriptor model transformation was introduced that is equivalent to the original system. It does not depend on additional assumptions on stability of the transformed system and requires bounding of fewer cross-terms. The authors extended their work to time-varying delay cases in [87]. However, to deal with the fast time-varying delays and to relax the constraint $|\dot{\tau}| \leq 1$, a DRI stability condition was obtained where the terms containing the upper bound of the time derivatives of the delay functions were simply ignored which could generally lead to conservative results. In [88] a DRD output feedback stabilization was presented for time-varying state-delayed systems with saturating actuators.

The free weighting matrix method was employed in [89] to investigate robust stability and $\mathcal{H}_\infty$ control problem for systems with interval time-varying delays. The developed robust control, however, was also independent of the time derivative of the delay function. Moreover, as stated in [82], the results were obtained by neglecting some useful terms in the derivative of the Lyapunov functional leading to more conservatism. In addition, due to the significance of the neglected terms for slow time-varying delays, the criteria are only applicable to systems with fast time-varying delays.

In this chapter, a delay-dependent $\mathcal{H}_\infty$ controller is introduced for time-delayed systems applicable to both slow and fast time-varying delays. The results are also extended for decentralized controllers. In addition, free weighting matrices are utilized to facilitate the solution of the resulting LMI condition.

The remainder of this chapter is organized as follows: In Section 3.1, a new centralized $\mathcal{H}_\infty$ control strategy in presented for time-delayed systems. An LMI condition is obtained to stabilize a time-delayed system and then, the LMI solution and the H_∞ controller are introduced in Theorem 3.4 to simultaneously guarantee stability of the closed-loop system and to satisfy the $\mathcal{H}_\infty$ objective function. The results are then extended to uncertain time-delayed systems in Section 3.1.4. In Section 3.2, a decentralized H_∞ controller is presented for time-varying delayed systems. It is shown that the proposed controller guarantees the internal stability of the system. Simulation results are provided in Section 3.3 to evaluate performance of our proposed controller as compared to other stability conditions and $\mathcal{H}_\infty$ controllers that are developed in the literature.

3.1 Centralized $\mathcal{H}_\infty$ Control of Time-Varying Delayed Systems

3.1.1 Problem Formulation

Consider the following time-varying delayed system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + B_d u(t-\tau(t)) + B_w w(t) \\ x(t) &= \phi(t), \quad t \in [-h, 0] \\ z(t) &= Cx(t) \end{aligned} \tag{3.2}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $u(t) \in \mathcal{R}^m$ is the input vector, $w(t) \in \mathcal{L}_2^q[0,\infty)$ is the exogenous disturbance signal, $z(t) \in \mathcal{R}^p$ is the regulated output to be attenuated, and $\tau(t)$ is an unknown time-varying delay. The matrices A, B, B_d, B_w and C are constant matrices of appropriate dimensions and ϕ is a continuously differentiable initial function. We assume that the system is controllable. For definition of stability, stabilizability and controllability of time-delayed systems refer to Appendix A.

To design a controller for the above system, the following assumption should hold:

Assumption 3.1. The delay $\tau(t)$ is an unknown differentiable function that for all $t \geq 0$ satisfies

$$0 \leq \tau(t) \leq h, \quad |\dot{\tau}(t)| \leq d, \quad d > 0$$

In this section, an $\mathcal{H}_\infty$ state feedback controller, $u(t) = Kx(t)$, is designed for system (3.2). The objective is to find a stabilizing controller K such that the $\mathcal{H}_\infty$ cost function given below is minimized:

$$J(w) = \int_0^\infty (z^T z - \gamma w^T w) ds, \quad \gamma > 0 \tag{3.3}$$

Indeed, making $J(w) < 0$ implies that the L_2-gain of system (3.2) is less than γ. In the remainder of this section at first, the time-delayed system (3.2) is stabilized and then an $\mathcal{H}_\infty$ controller is designed to make the objective function (3.3) negative.

3.1.2 Stabilization of Time-Varying Delayed Systems

In this section, a stabilizing controller is presented for the unforced system (3.2) (i.e., with $w = 0$) such that the upper bound of delay and its time derivative will appear in the LMI conditions. By applying the free weighting method [82], the constraint $d < 1$ is relaxed. Moreover, a design parameter ρ is introduced in the proposed Lyapunov-Krasovskii functional to improve the performance of the designed controller for both slow and fast time-varying delays.

Theorem 3.3. *Under the conditions of Assumption 3.1, the state feedback $u = Kx$ guarantees that the unforced system (3.2) is asymptotically stable, if for some design parameter $\rho > 0$ there exist matrices $\bar{N}_1, \bar{N}_2, M$ and positive definite matrices*

$Y_1, Y_2, \bar{S}, \bar{N}_3$, such that the LMI conditions (3.4) hold with $K = MY_1^{-1}$:

$$U_1(h,d) = \begin{bmatrix} \theta_1 & \theta_2 & h\bar{N}_1^T & 0 & \theta_3 \\ * & \theta_4 & 0 & h\bar{N}_2^T & \theta_5 \\ * & * & -h\bar{N}_3 & 0 & 0 \\ * & * & * & -h\bar{N}_3 & 0 \\ * & * & * & * & \theta_6 \end{bmatrix} < 0 \tag{3.4}$$

$$2Y_1 - Y_2 - \bar{N}_3 \geq 0$$

where

$$\begin{aligned} \theta_1 &= A^T Y_1 B^T M + Y_1 A + MB + \frac{1}{\rho}\bar{S} + \bar{N}_1^T + \bar{N}_1 \\ \theta_2 &= B_d M - \bar{N}_1 + \bar{N}_2 \\ \theta_3 &= 2hM^T B^T \\ \theta_4 &= -2\bar{N}_2 - \frac{(1-d)}{\rho}\bar{S} \\ \theta_5 &= 2hM^T B_d^T \\ \theta_6 &= -2hY_2 \end{aligned} \tag{3.5}$$

and $*$ denotes the entries implied by the symmetry.

Proof: To study the stability properties of the closed-loop system, the following Lyapunov-Krasovskii functional candidate is considered

$$\begin{aligned}
V &= V_1 + V_2 + V_3 \\
V_1 &= x^T(t)Px(t) \\
V_2 &= 2\int_0^h (h-\sigma)\dot{x}^T(t-\sigma)R\dot{x}(t-\sigma)d\sigma \\
V_3 &= \frac{1}{\rho}\int_{t-\tau}^{t} x^T(s)Sx(s)ds
\end{aligned} \tag{3.6}$$

where $\rho > 0$, and $P, R\ S$ are positive definite matrices. The time derivative of V along the system trajectories (3.2) and considering Assumption 3.1 is given by

$$\begin{aligned}
\dot{V} &= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \qquad (3.7) \\
\dot{V}_1 &= x^T(t)((A+BK)^T P + P(A+BK)x(t) + x^T(t-\tau)(B_dK)^T Px(t) \\
&+ x^T(t)P(B_dK)x(t-\tau) \\
\dot{V}_2 &= 2(h+t)\dot{x}^T(t)R\dot{x}(t) \\
&- 2t\dot{x}^T(t-h)R\dot{x}(t-h) - 2t[\dot{x}^T(t)R\dot{x}(t) - \dot{x}^T(t-h)R\dot{x}(t-h)] \\
&- 2\int_{t-h}^{t} \dot{x}^T(s)R\dot{x}(s)ds \leq 2h\dot{x}^T(t)R\dot{x}(t) - 2\int_{t-\tau(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds \\
\dot{V}_3 &= \frac{1}{\rho}[x^T(t)Sx(t) - (1-\dot{\tau})x^T(t-\tau(t))Sx(t-\tau(t)) \\
&\leq \frac{1}{\rho}[x^T(t)Sx(t) - (1-d)x^T(t-\tau(t))Sx(t-\tau(t))]
\end{aligned}$$

From the Leibniz-Newton formula, for any matrices N_1 and N_2 with appropriate dimensions the following equation is true

$$2[N_1x(t) + N_2x(t-\tau)]^T[x(t) - x(t-\tau(t)) - \int_{t-\tau}^{t} \dot{x}(s)ds] = 0 \tag{3.8}$$

Now, by adding the left hand side of (3.8) to (3.7), and adding it to and subtracting it from $hx^T(t)(N_1^T N_3^{-1} N_1)x(t)hx^T(t-\tau(t))(N_2^T N_3^{-1} N_2)x(t-\tau(t))$ and also considering that $0 \leq N_3 \leq R$, we have

$$\begin{aligned}
\dot{V} &\leq x^T(t)[(A+BK)^T P + P(A+BK) + N_1 + N_1^T + \frac{S}{\rho}]x(t) \\
&+ x^T(t)[PB_dK - N_1 + N_2]x(t-\tau) + x^T(t-\tau)[PB_dK - N_1 + N_2]^T x(t) \\
&- x^T(t-\tau)(N_2^T + N_2 + \frac{(1-d)}{\rho}S)x(t-\tau) + hx^T(t)N_1^T N_3^{-1} N_1 x(t) \\
&+ 2h\dot{x}^T(t)R\dot{x}(t) + hx^T(t-\tau)N_2^T N_3^{-1} N_2 x(t-\tau) \\
&- \int_{t-\tau}^{t} [N_1 x(t) + N_3\dot{x}(s)]^T N_3^{-1}[N_1 x(t) + N_3\dot{x}(s)]ds \\
&- \int_{t-\tau}^{t} [N_2 x(t-\tau) + N_3\dot{x}(s)]^T N_3^{-1}[N_2 x(t-\tau) + N_3\dot{x}(s)]ds \leq \bar{x}^T U_1 \bar{x} \qquad (3.9)
\end{aligned}$$

where $\bar{x} = [x(t) \quad x(t-\tau)]$,

$$U_1(h,d) = \begin{bmatrix} \Omega_1 & \Omega_2 & hN_1^T & 0 & \Omega_3 \\ * & \Omega_4 & 0 & hN_2^T & \Omega_5 \\ * & * & -hN_3 & 0 & 0 \\ * & * & * & -hN_3 & 0 \\ * & * & * & * & \Omega_6 \end{bmatrix} \qquad (3.10)$$

$$\begin{aligned}
\Omega_1 &= (A+BK)^T P + P(A+BK) + 2N_1 + \frac{1}{\rho}S \\
\Omega_2 &= PB_dK - N_1 + N_2 \\
\Omega_3 &= 2hK^T B^T R^T \\
\Omega_4 &= -2N_2 - \frac{(1-d)}{\rho}S \\
\Omega_5 &= 2hK^T B_d^T R^T \\
\Omega_6 &= -2hR
\end{aligned}$$

To make the bilinear matrix equation (3.10) linear, the following matrices should be defined $Y_1 = P^{-1}$, $Y_2 = R^{-1}$, $\bar{S} = Y_1^T S Y_1$, $\bar{N}_i = Y_1^T N_i Y_1$, $for\ i = 1,2,3$, $K = MY_1^{-1}$, $\Delta = \text{diag}\{Y_1,\ Y_1,\ Y_1,\ Y_1,\ Y_2\}$. By multiplying (3.10) with Δ^T and Δ from the left and right, respectively, the LMI condition (3.4) is obtained which guarantees the stability of the closed-loop system. Furthermore, by applying Lemma 2.2 implies that $2Y_1 - Y_2 - \bar{N}_3 \geq 0$ suffices in ensuring $N_3 \leq R$. This completes the proof of the theorem. ■

Remark 3.1. In cases when only stability properties of a time-delayed system is the subject of concern, no state feedback gain , K will be required. Therefore, the dynamics of the system is reduced to

$$\dot{x}(t) = Ax(t) + A_d x(t - \tau(t)) \tag{3.11}$$

By substituting $B_d M$ by $Y_1 A_d$ and then setting $B = 0$ in the LMI condition (3.4), one can obtain a delay-dependent criterion for analyzing the stability conditions of system (3.11) using the results in Theorem 3.3.

3.1.3 $\mathcal{H}_\infty$ Delay-Dependent Controller

The following theorem now introduces a robust $\mathcal{H}_\infty$ controller for system (3.2).

Theorem 3.4. *Consider system (3.2). Under the conditions of Assumption 3.1 and for $w \in L_2[0, \infty)$, there exists a memoryless state-feedback controller, $u = Kx$, such that the closed-loop system is internally stable and its L_2-gain is less than γ, if for some design parameter $\rho > 0$ there exist matrices $\bar{N}_1, \bar{N}_2, M$ and positive definite matrices $\bar{Y}_1, Y_2, \bar{S}, \bar{N}_3$, satisfying the LMI conditions (3.12) with $K = MY_1^{-1}$:*

$$W_1(h,d) = \begin{bmatrix} \theta_1 & \theta_2 & B_w & Y_1^T C^T & h\bar{N}_1^T & 0 & \theta_3 \\ * & \theta_4 & 0 & 0 & 0 & h\bar{N}_2^T & \theta_5 \\ * & * & -\gamma I & 0 & 0 & 0 & \theta_6 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -h\bar{N}_3 & 0 & 0 \\ * & * & * & * & * & -h\bar{N}_3 & 0 \\ * & * & * & * & * & * & \theta_7 \end{bmatrix} < 0$$

$$2Y_1 - Y_2 - \bar{N}_3 \geq 0 \tag{3.12}$$

where

$$\begin{aligned}
\theta_1 &= A^T Y_1 + B^T M + Y_1 A + MB + \frac{1}{\rho}\bar{S} + \bar{N}_1^T + \bar{N}_1 \\
\theta_2 &= B_d M - \bar{N}_1 + \bar{N}_2 \\
\theta_3 &= 2hM^T B^T \\
\theta_4 &= -2\bar{N}_2 - \frac{(1-d)}{\rho}\bar{S} \\
\theta_5 &= 2hM^T B_d^T \\
\theta_6 &= 2hB_w^T \\
\theta_7 &= -2hY_2
\end{aligned}$$

In fact, the LMI condition (3.12) simultaneously guarantees $J(w) < 0$ and asymptotic stability of the unforced closed-loop system.

Proof: It has been shown in [90] that the $\mathcal{H}_\infty$ objective function is satisfied, i.e., $J < 0$, if the associated Hamiltonian defined below is negative definite

$$J_1 = \frac{dV}{dt} + z^T z - \gamma w^T w \tag{3.13}$$

where $V(x) \geq 0$ is a storage function with $V(x(0)) = 0$. For our problem, the

Lyapunov-Krasovskii functional V defined in (3.6) may serve this purpose. The problem is now reduced to showing that the Hamiltonian J_1 is negative definite. By substituting (3.9) into (3.13), using Assumption 3.1, and then by taking a similar approach as that given in Theorem 3.3, (i.e., replacing $\dot{\tau}$ and τ by d and h, respectively), one obtains the following equation:

$$J_1 = \bar{x}^T W_1(\tau, \dot{\tau})\bar{x} \leq \bar{x}^T W_1(h, d)\bar{x} \tag{3.14}$$

where

$$\bar{x} = [x(t)^T \quad x^T(t-\tau) \quad w^T(t)]^T$$

and

$$W_1(h,d) = \begin{bmatrix} \Omega_1 & \Omega_2 & PB_w & C^T & hN_1^T & 0 & \Omega_3 \\ * & \Omega_4 & 0 & 0 & 0 & hN_2^T & \Omega_5 \\ * & * & -\gamma I & 0 & 0 & 0 & \Omega_6 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -hN_3 & 0 & 0 \\ * & * & * & * & * & -hN_3 & 0 \\ * & * & * & * & * & * & \Omega_7 \end{bmatrix} \tag{3.15}$$

$$\begin{aligned} \Omega_1 &= (A+BK)^T P + P(A+BK) + 2N_1 + \frac{1}{\rho}S \\ \Omega_2 &= PB_d K - N_1 + N_2 \\ \Omega_3 &= 2h(K^T B^T + A^T)R^T \\ \Omega_4 &= -2N_2 - \frac{(1-d)}{\rho}S \\ \Omega_5 &= 2hK^T B_d^T R^T \\ \Omega_6 &= 2hB_w^T R^T \\ \Omega_7 &= -2hR \end{aligned} \tag{3.16}$$

Now, by taking a similar approach as that given in Theorem 3.3, defining similar parameters and letting $\Delta_2 = \text{diag}\{Y_1,\ Y_1,\ I,\ I,\ Y_1,\ Y_1,\ Y_2\}$ and multiplying (3.15) with Δ_2^T and Δ_2 from the left and right, respectively, one obtains the LMI condition (3.12). This condition guarantees that the Hamiltonian J_1 in (3.13), and hence J in (3.3) are negative definite. Consequently, we can conclude that the closed-loop system is L_2-stable with its L_2-gain less than γ. Furthermore, by eliminating the third and the fourth rows and columns of (3.12) (corresponding to the term involving w), one may get (3.4) which implies that $\dot{V} < 0$, and consequently one may guarantee the asymptotic stability of the closed-loop system when $w = 0$. ■

Remark 3.2. Theorem 3.4 provides an LMI condition that internally stabilizes the closed-loop system and satisfies $J(w) < 0$. Now to obtain the $\mathcal{H}_\infty$ controller that achieves the minimum disturbance attenuation factor γ, the following optimization problem should be solved:

$$\min_{\bar{N}_1,\bar{N}_2,M,\bar{N}_3>0,Y_1>0,Y_2>0,\text{ and } \bar{S}>0} \gamma \quad \text{subject to (3.12) for} \tag{3.17}$$

Remark 3.3. By adopting a similar approach, and after performing some algebraic manipulations, the results of Theorem 3.4 can be extended to multiple delayed systems.

Remark 3.4. The stability criteria developed in [89] are only applicable to systems with fast time-varying delay. In fact, in some cases, the upper bound of the time derivative of the delay, d is known and may be small. In our Theorem 3.4, d is considered in the LMI by applying the Lyapunov functional V_3 and the first term of $\dot{V}_3$ appears in $\theta_1 = A^T Y_1 + B^T M + Y_1 A + MB + \frac{1}{\rho}\bar{S} + \bar{N}_1^T + \bar{N}_1$ and $\theta_4 = -2\bar{N}_2 - \frac{(1-d)}{\rho}\bar{S}$. For $d < 1$ the second term of θ_4 is negative which increases the chance of finding less conservative conditions as compared to the results which are independent of d [89]. However, for $d > 1$ this term will be positive and even though the free weight N_2

may help to keep θ_4 negative, the LMI solution is more difficult to hold. The adverse effect of increasing d can be moderated by adjusting ρ. Since, the free-weighting matrix method is also applied in developing the resulting LMI condition, hence, our proposed control strategy can provide less conservative LMI conditions for both slow and fast time-varying delays. Moreover, in cases where d is not known *a priori*, we can ignore V_3. Therefore, the corresponding terms in θ_1 and θ_4 will be eliminated from the LMI condition (3.12).

3.1.4 $\mathcal{H}_\infty$ Delay-Dependent Controller for Uncertain Delayed Systems

The results in the previous section can also be extended to the time-delayed systems when the dynamics is affected by the uncertain terms as follows:

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t) \\ &+ (B_d + \Delta B_d)u(t - \tau(t)) + B_w w(t) \end{aligned} \qquad (3.18)$$

We now require the following assumption to be imposed on the above system.

Assumption 3.2. The uncertainties $\Delta A(t)$, $\Delta B_d(t)$ and $\Delta B(t)$ are time-varying norm-bounded matrices that are to satisfy the following conditions:

$$\begin{aligned} \Delta A(t) &= H_a E(t) N_a \\ \Delta B_d(t) &= H_d E(t) N_d \\ \Delta B(t) &= H_b E(t) N_b \end{aligned} \qquad (3.19)$$

where H_a, H_d, H_b and N_a, N_d, N_b are constant matrices, and $E(t)$ is a real uncertain matrix with Lebesgue measurable entry which additionally meets the following requirement:

$$E(t)E^T(t) \leq I \qquad (3.20)$$

To cope with the uncertain terms the following lemma is used.

Lemma 3.1. [91] : Let H, N and E be real matrices of appropriate dimensions, with E satisfying (3.20), then for any $\varepsilon > 0$:

$$H^T E(t) N + N^T E^T(t) H \leq \varepsilon H^T H + \varepsilon^{-1} N^T N \tag{3.21}$$

The following corollary provides the modified LMI condition that guarantees closed-loop system stability as well as $\mathcal{H}_\infty$ performance requirements for system (3.18).

Corollary 3.4.1. *Consider the uncertain system (3.18), subject to conditions of Assumptions 3.1 and 3.2 and with $w \in L_2[0, \infty)$. This system is internally stabilizable by state-feedback controller $u = Kx$ and system L_2-gain remains less than γ, if for some design parameters $\rho > 0$ and $\varepsilon > 0$, there exist scalar $\gamma > 0$, matrices M, N_1, N_2 and symmetric positive definite matrices Y_1, Y_2, N_3, $\bar{Q}$, $\bar{S}$ satisfying the LMI condition (3.22). Consequently, the state feedback gain is selected as $K = MY_1^{-1}$.*

$$\begin{bmatrix} \tilde{\theta}_1 & \tilde{\theta}_2 & B_w & Y_1^T C^T & h\bar{N}_1^T & 0 & \tilde{\theta}_3 & Y_1 N_a^T & M^T N_b & 0 \\ * & \tilde{\theta}_4 & 0 & 0 & 0 & h\bar{N}_2^T & \tilde{\theta}_5 & 0 & 0 & N_d \\ * & * & -\gamma I & 0 & 0 & 0 & \tilde{\theta}_6 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -h\bar{N}_3 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -h\bar{N}_3 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \tilde{\theta}_7 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{3.22}$$

where

$$
\begin{aligned}
2Y_1 - Y_2 - \bar{N}_3 &\geq 0 \\
\tilde{\theta}_1 &= \theta_1 + \varepsilon(H_a H_a^T + H_b H_b^T + H_d H_d^T) \\
\tilde{\theta}_3 &= \theta_3 + 2h\varepsilon(H_a H_a^T + H_b H_b^T + H_d H_d^T) \\
\tilde{\theta}_5 &= 2hMB_d^T \\
\tilde{\theta}_6 &= 2hB_w^T \\
\tilde{\theta}_7 &= -2hY_2 + 2h\varepsilon(H_a H_a^T + H_b H_b^T + H_d H_d^T)
\end{aligned}
$$

Proof: To satisfy the $\mathcal{H}_\infty$ objective function, let us consider the Lyapunov-Krasovskii functional V defined in (3.6). Now, by taking the time-derivative of V along the system trajectories (3.18) and substituting it into (3.13), using Assumption 3.1, and then by taking a similar approach as that given in Theorem 3.4, one obtains the following equation:

$$
J_1 = \bar{x}^T W_2(\tau, \dot{\tau})\bar{x} \leq \bar{x}^T W_2(h, d)\bar{x} \tag{3.23}
$$

where

$$
\bar{x} = [x(t)^T \quad x^T(t-\tau) \quad w^T(t)]^T
$$

and

$$W_2(h,d) = \begin{bmatrix} \Omega_1 & \Omega_2 & PB_w & C^T & hN_1^T & 0 & \Omega_3 \\ * & \Omega_4 & 0 & 0 & 0 & hN_2^T & \Omega_5 \\ * & * & -\gamma I & 0 & 0 & 0 & \Omega_6 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -hN_3 & 0 & 0 \\ * & * & * & * & * & -hN_3 & 0 \\ * & * & * & * & * & * & \Omega_7 \end{bmatrix} \tag{3.24}$$

$$\begin{aligned} \Omega_1 &= (A+\Delta A+(B+\Delta B)K)^T P + P(A+\Delta A+(B+\Delta B)K) + 2N_1 + \frac{1}{\rho}S \\ \Omega_2 &= P(B_d+\Delta B_d)K - N_1 + N_2 \\ \Omega_3 &= 2h(K^T(B+\Delta B)^T + (A+\Delta A)^T)R^T \\ \Omega_4 &= -2N_2 - \frac{(1-d)}{\rho}S \\ \Omega_5 &= 2hK^T(B_d+\Delta B_d)^T R^T \\ \Omega_6 &= 2hB_w^T R^T \\ \Omega_7 &= -2hR \end{aligned} \tag{3.25}$$

In view of Lemma 3.21, one obtains

$$W_2 \ \leq \ W_1 + \varepsilon H^T H + \varepsilon^{-1} N^T N \tag{3.26}$$

where

$$H = \begin{bmatrix} H_a^T P & 0 & 0 & 0 & 0 & 0 & 2hH_a^T R \\ H_b^T P & 0 & 0 & 0 & 0 & 0 & 2hH_b^T R \\ H_d^T P & 0 & 0 & 0 & 0 & 0 & 2hH_d^T R \end{bmatrix}$$

$$N = \begin{bmatrix} N_a & 0 & 0 & 0 & 0 & 0 & 0 \\ N_b K & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_d K & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now, by applying the Schur complement, following similar lines as mentioned in Theorem 3.4, defining similar parameters and letting $\Delta_3 = \text{diag}\{Y_1,\ Y_1,\ I,\ I,\ Y_1,\ Y_1,\ Y_2,\ I,\ I,\ I\}$ and multiplying the resulting matrix with Δ_3^T and Δ_3 from the left and right, respectively, one obtains the LMI condition (3.22). This condition guarantees that the Hamiltonian J_1 in (3.13) is negative definite. Therefore, our proposed control in (3.12) is enhanced according to the condition provided in (3.22). This completes the proof of this corollary. ■

3.2 Decentralized $\mathcal{H}_\infty$ Control of Time-Varying Delayed Systems

Many practical systems in real world are composed of a set of interconnected subsystems such as power systems, digital communication networks, economic systems and urban traffic networks. Transferring information among subsystems could cause delays which adversely affect stability and deteriorate performance of the system. Control of such large-scale systems can become quite complicated owing to high dimensionality of system equations of motion, uncertainties and time-delays. Decentralized control strategy is among the best choices for controlling such systems. In other words, each subsystem is required to control its behavior by using its local and limited information received from other subsystems to accomplish a global objective.

A necessary and sufficient condition for stability of a class of interconnected system with delay was provided in [92]. Robust control of large scale time-delay systems has been one of the active areas of research in the past few years and a number of results are available [93]- [95]. The problem of robust control for a class of interconnected systems was investigated by using decentralized sliding mode control in [96], and by adaptive control strategies in [95] and [97]. A decentralized robust control scheme for a large-scale system with control delays was introduced in [98]. However, no delays were considered in the interconnection terms of their model and furthermore the delays were not time-varying. In [99], a decentralized stabilizing controller is developed for an interconnected constant time-delay systems. It should be noted that, in most practical large scale systems, interconnection delays vary in time and also are not known *a priori*. Stabilization of a class of time-varying large scale systems subject to time-varying delays was investigated in [100] and [101]. To the best of author's knowledge, all of the existing decentralized control strategies in the literature for time-varying delay systems relied on the condition $|\dot{\tau}| \leq 1$.

In this section, a decentralized $\mathcal{H}_\infty$ controller is presented for both fast and slow time-varying delayed systems, i.e it is not restricted to the condition $|\dot{\tau}| \leq 1$.

3.2.1 Problem Formulation

Consider a time-varying delayed system that is composed of n interconnected subsystems. The ith subsystem is described by

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i + B_i u_i(t) + \sum_{j \in \wp(i)} B_{dij} u_j(t - \tau_{ij}(t)) + B_{wi} w_i(t) \qquad (3.27) \\ x_i(t) &= \phi_i(t), \quad t \in [-h_i, 0], \quad i = 1, ..., n \\ z_i(t) &= C_i x_i(t) \end{aligned}$$

where $x_i(t) \in \mathcal{R}^{\bar{d}}$ is the state vector, $u_i(t) \in \mathcal{R}^m$ is the input vector, $w_i(t) \in \mathcal{L}_2^q[0, \infty)$ is the exogenous disturbance signal, $z_i(t) \in \mathcal{R}^p$ is the regulated output, and $\tau_i(t)$ is the unknown time-varying delay function of the ith subsystem. The matrices A_i, B_i, B_{dij}, B_{wi} and C_i are constant, known matrices of appropriate dimensions and ϕ_i is a continuously differentiable initial function. Matrix B_{dij}, $i \neq j$ represents coupling terms and interconnections that exist among all subsystems.

Our objective is to find a stabilizing $\mathcal{H}_\infty$ state feedback controller, $u_i = K_i x_i$, for each subsystem such that the following conditions are satisfied:

- System (3.27) is asymptotically stable with $w(t) = 0$.
- Under zero initial conditions the following global objective function is minimized

$$J(w) = \int_0^\infty (z^T z - \gamma w^T w) ds, \quad \gamma > 0 \tag{3.28}$$

where $z(t) = vec\{z_i(t)\}$ and $w(t) = vec\{w_i(t)\}$. In other word, achieving $J(w) < 0$ guarantees that L_2-gain of the entire system is less than γ.

To deal with time-varying delayed systems, the following standard assumption should be imposed on system (3.27).

Assumption 3.3. The delays $\tau_{ij}(t)$ are unknown differentiable functions that for all $t \geq 0$ satisfy

$$0 \leq \max\{\tau_{ij}(t)\} \leq h_{ij}, \quad \max\{|\dot{\tau}_{ij}(t)|\} \leq d_{ij},\ d_{ij} > 0, h_i = \max\{h_{ij}\}$$

For simplicity it is also assumed that the delay between two nodes in both directions are the same, i.e., $\tau_{ji} = \tau_{ij}$.

3.2.2 A Decentralized $\mathcal{H}_\infty$ Delay-Dependent Controller

The result stated in Theorem 3.5 provides a delay-dependent criterion for robust stability of the uncertain interconnected system (3.27).

Theorem 3.5. *Consider the uncertain interconnected system (3.27), subject to conditions of Assumptions 3.3 and with $w \in L_2[0,\infty)$. This system is internally stabilizable by local state-feedback controllers $u_i = K_i x_i$ and system's L_2-gain is less than γ, if for some design parameter $\rho > 0$ there exist matrices $M_i, \bar{N}_i, \bar{Z}_i$ and symmetric positive definite matrices Y_{i1}, Y_{i2}, $\bar{S}_i$, $\bar{U}_i$ for $i = 1, ..., n$ that satisfy the LMI condition (3.29). Consequently, the state feedback gain is selected as $K_i = M_i Y_{i1}^{-1}$.*

$$W_{i1} = \begin{bmatrix} \theta_{i1} & \theta_{i2} & B_{wi} & Y_{i1}^T C_i^T & \theta_{i3} & m_i h_{ji} \bar{N}_i^T & 0 \\ * & \theta_{i4} & 0 & 0 & \theta_{i5} & 0 & \theta_{i6} \\ * & * & -\gamma I & 0 & \theta_{i7} & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & \theta_{i8} & 0 & 0 \\ * & * & * & * & * & \theta_{i9} & 0 \\ * & * & * & * & * & * & \theta_{i10} \end{bmatrix} < 0$$

$$2Y_{i1} - Y_{i2} - \bar{U}_i \;\; \geq \;\; 0, \;\; \bar{N}_i \leq \bar{Z}_i \tag{3.29}$$

where

$$\begin{aligned} \theta_{i1} &= Y_{i1}^T A_i^T + M_i^T B_i^T + B_i M_i + A_i Y_{i1} + m_i[3\bar{N}_i - \bar{Z}_i + \frac{1}{\rho}\bar{S}_i] \\ \theta_{i2} &= \tilde{B}_{dij} \bar{M}_j \\ \theta_{i3} &= 2m_i h_{ji}(M_i^T B_i^T + Y_{i1}^T A_i^T) \end{aligned}$$

$$
\begin{aligned}
\theta_{i4} &= -\operatorname{diag}\{3\bar{Z}_j - \bar{N}_j + \frac{(1-d_{ji})}{\rho}\bar{S}_j\} \\
\theta_{i5} &= 2m_i h_{ji} \bar{M}_j^T \tilde{B}_{dij}^T \\
\theta_{i6} &= \operatorname{diag}\{h_{ji}\bar{Z}_j\} \\
\theta_{i7} &= 2m_i h_{ji} B_{wi}^T \\
\theta_{i8} &= -2h_{ji}Y_{i2}, \\
\theta_{i9} &= -m_i h_{ji}\bar{U}_i \\
\theta_{i10} &= -\operatorname{diag}\{h_{ji}\bar{U}_j\}
\end{aligned}
$$

and m_i = *number of subsystems that subsystem* i *belongs to their* $\wp(.)$ *(in the case that* $\wp(.) = N$ *then,* $m_i = N, i = 1, ..., N$*),* $\bar{M}_j = \operatorname{diag}\{M_j\}$*, and*
$\tilde{B}_{dij} = vec\{B_{dij}\}$ *for* $j \in \wp(i)$*.*

Proof: In the case of decentralized control, the Lyapunov-Krasovskii functional V that is introduced in (3.6) is modified as follows

$$
\begin{aligned}
V &= \bar{V}_1 + \bar{V}_2 + \bar{V}_3 \\
\bar{V}_1 &= \sum_{i=1}^{n} x_i^T P_i x_i \\
\bar{V}_2 &= 2\sum_{i=1}^{n} \sum_{j\in\wp(i)} \int_0^{h_{ji}} (h_{ji}-\sigma)\dot{x}_j^T(t-\sigma)R_j\dot{x}_j(t-\sigma)d\sigma \\
\bar{V}_3 &= \sum_{i=1}^{n} \sum_{j\in\wp(i)} \frac{1}{\rho}\int_{t-\tau_{ji}}^{t} x_j^T(s)S_jx_j(s)ds
\end{aligned}
$$

Now, taking the time-derivative of V along the system trajectories in (3.27) and substituting it in (3.13) yields

$$
J_1 = \dot{\bar{V}}_1 + \dot{\bar{V}}_2 + \dot{\bar{V}}_3 + z^T z - \gamma w^T w \tag{3.30}
$$

On the other hand, considering the Leibniz–Newton formula, the following equation holds for any N_j and Z_j with appropriate dimensions

$$2\sum_{i}^{n}\sum_{j\in\wp(i)}[N_jx_j(t)+Z_jx_j(t-\tau_{ji}(t))]^T$$
$$[x_j(t)-x_j(t-\tau_{ji}(t))-\int_{t-\tau_{ji}(t)}^{t}\dot{x}_j(s)ds]=0 \tag{3.31}$$

Now, by applying Assumption 3.3, adding the left hand side of (3.31) to (3.30) and then adding it to and subtracting it from $\sum_{i=1}^{n}\sum_{j\in\wp(i)}x_j^T(t)(h_{ji}N_jU_j^{-1}N_j-N_j+Z_j)x_j(t)+x_j^T(t-\tau_{ji}(t))(h_{ji}Z_jU_j^{-1}Z_j-N_j+Z_j)x_j(t-\tau_{ji}(t))$, using the fact that for any $L_i\in\{N_i,U_i,S_i,Z_i,R_i\}$ the following equation holds

$$\sum_{i=1}^{n}\sum_{j\in\wp(i)}x_j^T(t)L_jx_j(t)=\sum_{i=1}^{n}m_ix_i^T(t)L_ix_i(t)$$

and assuming that $0\le N_i\le Z_i,\ 0<U_i<R_i$ one can follow along the similar lines as in the proof of Theorem 3.4 to get

$$J_1 \ \le \ \sum_{i=1}^{n}\bar{X}_i^T(t)W_{i1}(h_{ji},d_{ji})\bar{X}_i(t) \tag{3.32}$$

where

$$\begin{aligned}\bar{X}_i(t,\beta) &= [x_i(t)^T \quad X_j^T(t-\tau_i(t)) \quad w_i^T(t)]^T,\\ X_j^T(t-\tau_i(t)) &= vec\{x_j^T(t-\tau_{ji}(t))\} \ \ for \ \ j\in\wp(i)\end{aligned}$$

and

$$W_{i1}(h,d)=\begin{bmatrix}\Omega_{i1} & \Omega_{i2} & P_iB_{wi} & C_i^T & \Omega_{i3} & m_ih_{ji}N_i^T & 0\\ * & \Omega_{i4} & 0 & 0 & \Omega_{i5} & 0 & \Omega_{i6}\\ * & * & -\gamma I & 0 & \Omega_{i7} & 0 & 0\\ * & * & * & -I & 0 & 0 & 0\\ * & * & * & * & \Omega_{i8} & 0 & 0\\ * & * & * & * & * & \Omega_{i9} & 0\\ * & * & * & * & * & * & \Omega_{i10}\end{bmatrix} \tag{3.33}$$

$$
\begin{aligned}
\Omega_{i1} &= (A_i + B_iK_i)^T P_i + P_i(A_i + B_iK_i) + m_i[3N_i - Z_i + \frac{1}{\rho}S_i] \\
\Omega_{i2} &= P_iB_{dij}K_j \\
\Omega_{i3} &= 2m_ih_{ji}(K_i^TB_i^T + A_i^T)R_i^T \\
\Omega_{i4} &= -\operatorname{diag}\{3Z_j - N_j + \frac{(1-d_{ji})}{\rho}S_j\} \\
\Omega_{i5} &= 2m_ih_{ji}K_j^T\tilde{B}_{dij}^TR_i^T \\
\Omega_{i6} &= \operatorname{diag}\{h_{ji}Z_j\} \\
\Omega_{i7} &= 2m_ih_{ji}B_{wi}^TR_i^T \\
\Omega_{i8} &= -2h_{ji}R_i \\
\Omega_{i9} &= -m_ih_{ji}U_i \\
\Omega_{i10} &= -\operatorname{diag}\{h_{ji}U_j\}
\end{aligned}
$$

Now, by defining the parameters $K_i = M_iY_{i1}^{-1}$, $P_i = Y_{i1}^{-1}$, $R_i = Y_{i2}^{-1}$, $Y_{i1}^TS_iY_{i1} = \bar{S}_i$, $Y_{i1}^TZ_iY_{i1} = \bar{Z}_i$, $Y_{i1}^TN_iY_{i1} = \bar{N}_i$, $Y_{i1}^TU_iY_{i1} = \bar{U}_i$, $\bar{Y}_{i1} = vec\{Y_{j1}\}$ $for\ i = 1, ..., n$, $j \in \wp(i)$, and $\Delta_{i2} = \operatorname{diag}\{Y_{i1},\ \bar{Y}_{i1},\ I,\ I,\ Y_{i2},\ Y_{i1},\ \bar{Y}_{i1}\}$, and pre and post multiplying (3.33) by Δ_{i2} and its transpose respectively, one can obtain the LMI conditions in (3.29). These conditions guarantee that the Hamiltonian J_1 in (3.13), and consequently J in (3.28) are negative definite. Therefore, the L_2-gain of the closed-loop system is less than γ. Moreover, by eliminating the $(n+2)$nd row and column (corresponding to the terms involving w_i) and the $(n+3)$rd row and column of (3.29), the LMI condition is obtained which implies that $\dot{V} < 0$, and therefore asymptotic stability of the unforced closed-loop system is guaranteed. Furthermore, applying Lemma 2.2 implies that $2Y_{i1} - Y_{i2} - \bar{U}_i \geq 0$ guarantees $U_i < R_i$. This completes the proof of Theorem 3.5. ■

Table 3.1: Allowed upper bound of the delay for different d for stability where h_1 is the lower bound of the delay

Method	Delay upper bound for d=0.5	Delay upper bound for d=2.1
Fridman et.al. [102], Wu et.al. [103], Han [104]	2	Not defined (feasible)
He et.al. [82]	2 for $h_1 = 0$, 4.47 for $h_1 = 4.4697$	Not defined (feasible)
Proposed method	5.3	0.67

3.3 Simulation Results

In this section, two numerical examples are provided to demonstrate the improved control performance as well as the reduced conservative behavior of the stability results that are obtained by using our proposed $\mathcal{H}_\infty$ control strategy.

Example 3.1. In this example, the stability of system $\dot{x} = Ax(t) + A_d x(t - \tau(t))$ which was studied in [82] is investigated where the numerical parameters are defined as $A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}$ and $A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$. In Table 3.1, the maximum acceptable upper bound of the delay and its time derivative obtained by applying our proposed method mentioned in Theorem 3.3 and Remark 3.1 is compared with some recent work that have appeared in the literature. From Table 3.1, it can be concluded that our proposed method could offer less conservative performance for both slow ($d < 1$) and fast ($d > 1$) time-varying delay functions. It should be mentioned that none of the selected methods above could find a feasible LMI solution for $d > 2$.

Example 3.2. Consider the example studied in [87] with dynamics

$$\begin{aligned} \dot{x} &= Ax(t) + A_d x(t - \tau(t)) + Bu(t) + B_w w(t) \\ y &= Cx(t) \end{aligned}$$

where $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, A_d = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix}, \quad B_w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [0 \;\; 1].$

For constant delays using descriptor method and iterative search of the controller gain [105] the above system is stabilizable for $\tau \in [0, 3.2]$ and alternatively using [91] the system is stabilizable for all $\tau < 1$. The introduced descriptor method in [87] fails to obtain feasible solution for a fast time-varying delay function. The reported state feedback gain $K = [-74.8 \;\; -105.5]$ in [87] was achieved by choosing $h = 2,\ d = 0.2$ which results in a minimum attainable disturbance attenuation factor of $\gamma = 6$. Our proposed method can provide a stabilizing controller with $K = [-9.4942 \;\; -74.4322]$ for a time-varying delay function with $d = 1.1,\ h = 0.5$ and result in a minimum attainable attenuation factor of $\gamma = 0.12$. The above comparisons clearly show the merit of our proposed control strategy.

3.4 Conclusions

In this chapter, centralized and decentralized robust delay-dependent $\mathcal{H}_\infty$ controllers are introduced for time-varying delayed systems. By taking advantage of the upper bound of the delay and its derivative with no constraints and limitation on the variational behavior of the unknown delays, we have obtained stability results for both slow and fast time-varying delay functions. By relaxing the constraint on the variational behavior of delay function, the proposed approach is applicable to a larger class of time-varying delayed systems with larger acceptable range of delay variations. The $\mathcal{H}_\infty$ controllers proposed in this chapter is applied for centralized as well as decentralized routing of networks with fixed topology in the next chapter.

Chapter 4

$\mathcal{H}_\infty$ Control Strategy for Routing of Networks with Fixed Topology

As mentioned in Chapter 2, the dynamic routing problem is subject to different sources of *unknown and time-varying* delays such as transmitting, propagating, and processing delays. On the other hand, it is well known that maintaining the stability of time-delayed feedback systems in general is not a trivial problem owing to infinite dimensional nature of time-delay systems. $\mathcal{H}_\infty$ control scheme has been shown to be an efficient tool for dealing with system uncertainties and disturbances. Moreover, the routing problem is subject to various physical constraints which increase the complexity of the problem and render the possibility of finding a solution using standard approaches rather limited. This chapter is organized into two main parts. In the first part, considering the dynamical model of traffic network presented in Chapter 2, and employing the introduced $\mathcal{H}_\infty$ optimal control schemes in Chapter 3, a new centralized routing strategy is introduced which is based on minimization of the *worst-case queueing length.* The proposed $\mathcal{H}_\infty$ optimal control scheme can maintain a robust performance and guarantee an optimal routing strategy (in the sense of minimization of the introduced performance index) in the presence of multiple and unknown time-varying delays. The physical constraints are also formulated as LMI feasibility conditions.

When the network nodes are distributed over a large area, exchanges of individual information and communications among the nodes and the centralized routing controller will impose costly overheads that would result in significant delays. Consequently, decentralized routing controllers that can be implemented locally at individual nodes with minimal overheads are highly desirable. In the second part of this chapter, our proposed centralized control scheme is modified for a decentralized routing strategy. Unlike [25] and [26], the delays are considered to be unknown *a priori* and fast time-varying. Therefore, the proposed decentralized routing strategy not only determines the fastest route in the presence of the unknown time-varying delays, but also is scalable to potentially large traffic networks.

The rest of this chapter is organized as follows. In Section 4.1, the traffic model given in Chapter 2 is briefly recalled. The introduced $\mathcal{H}_\infty$ robust control in Chapter 3 is modified for a *centralized* routing problem. Then, the physical constraints are formulated as LMI feasibility conditions. The proposed centralized routing controller is provided in Theorem 4.3. The above results are subsequently extended to a *decentralized* routing control strategy in Section 4.2. By invoking the stability results presented in Section 3.2, LMI conditions are proposed that guarantee the stability as well as an $\mathcal{H}_\infty$ performance of the traffic network. Finally, Theorem 4.4 presents a decentralized routing controller which simultaneously minimizes the *worst-case queueing length* and satisfies the physical constraints. Simulation results are presented in Section 4.3 to evaluate and illustrate the performance and capabilities of the proposed robust routing control strategies.

4.1 Centralized Routing Control Scheme

Let us redefine the dynamical model of the traffic network and routing objective proposed in Chapter 2. By applying the flow conservation law, a network traffic dynamics was expressed in the following standard state space representation (2.17):

$$\dot{x}(t) = Bu(t) + B_d u(t - \tau(t)) + B_w w(t) \tag{4.1}$$

where $x(t)$ denotes that queueing lengths in nodes, $u(t)$ denotes the flows sent, $\tau(t)$ is an unknown but bounded time-varying total delay in transmitting, propagating, and processing messages, $w(t)$ is the external input flow, B and B_d represent network connectivity matrices. A typical set of constraints imposed on traffic networks can be given as

$$u \geq 0 \tag{4.2}$$

$$x \geq 0 \tag{4.3}$$

$$G_k u \leq c_k, \quad k = 1, ..., l \tag{4.4}$$

$$Q_{dj} x \leq x_{max_{dj}} \tag{4.5}$$

According to the H_∞ robust optimal control definition (2.3), our proposed H_∞ routing algorithm is indeed, a state feedback controller of the form $u(t) = Kx(t)$ that simultaneously guarantees the stability of the network in the presence of time-varying delays and minimizes the *worst-case queueing length* due to the external input flows. By selecting the regulated output as $z(t) = Cx(t)$, the routing problem objective function can be cast as

$$\begin{aligned} \min \gamma \quad s.t. \quad & J(w) < 0 \quad (4.6) \\ J(w) = & \int_0^\infty (z^T(s)z(s) - \gamma w^T(s)w(s))ds, \quad \gamma > 0 \end{aligned}$$

The goal of this section is to design a centralized state feedback $\mathcal{H}_\infty$ controller for the network model (4.1) subject to the constraints (4.2)-(4.5). The $\mathcal{H}_\infty$ control problem specified in (4.6) is formulated as an optimization problem involving LMIs. First, by employing the results presented in Section 3.1, the LMI constraints that ensure robust stability and $\mathcal{H}_\infty$ performance in the presence of uncertain time-varying delays are obtained. Subsequently, the physical constraints (4.2)-(4.5) are expressed as LMI feasibility conditions.

Theorem 3.3 establishes a basis for design of an $\mathcal{H}_\infty$ state feedback controller. Comparing the traffic network dynamics (4.1) with the dynamics (3.2), one can conclude that by simply substituting $A = 0$, the LMI conditions (3.4) can be applied for the centralized routing problem. Hence, considering the traffic network dynamics (4.1) and for $w \in L_2[0, \infty)$, if for some design parameter $\rho > 0$ there exist matrices $\bar{N}_1, \bar{N}_2, M$ and positive definite matrices $Y_1, Y_2, \bar{S}, \bar{N}_3$, to satisfy the following LMI conditions, the existence of a *centralized* memoryless robust state-feedback controller of the form $u = Kx$ that makes the closed-loop system internally stable and $J(w) < 0$ will be guaranteed, where

$$W_1(h,d) = \begin{bmatrix} \theta_1 & \theta_2 & B_w & Y_1^T C^T & h\bar{N}_1^T & 0 & \theta_3 \\ * & \theta_4 & 0 & 0 & 0 & h\bar{N}_2^T & \theta_5 \\ * & * & -\gamma I & 0 & 0 & 0 & \theta_6 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -h\bar{N}_3 & 0 & 0 \\ * & * & * & * & * & -h\bar{N}_3 & 0 \\ * & * & * & * & * & * & \theta_7 \end{bmatrix} < 0$$

$$2Y_1 - Y_2 - \bar{N}_3 \geq 0 \tag{4.7}$$

$$
\begin{aligned}
\theta_1 &= B^T M + MB + \frac{1}{\rho}\bar{S} + \bar{N}_1^T + \bar{N}_1 \\
\theta_2 &= B_d M - \bar{N}_1 + \bar{N}_2 \\
\theta_3 &= 2hM^T B^T \\
\theta_4 &= -2\bar{N}_2 - \frac{(1-d)}{\rho}\bar{S} \\
\theta_5 &= 2hM^T B_d^T \\
\theta_6 &= 2hB_w^T \\
\theta_7 &= -2hY_2
\end{aligned}
$$

and the state feedback controller gain is given by

$$K = MY_1^{-1} \tag{4.8}$$

Furthermore, the LMI conditions (4.7) simultaneously guarantee not only $J(w) < 0$, but also asymptotic stability of the unforced ($w = 0$) closed-loop system.

Remark 4.1. Under the situations when the input flow w does not belong to the $\mathcal{L}_2$ space, i.e., $w \notin \mathcal{L}_2$, one can filter w through a shaping filter before applying it to the system. The use of a shaping filter, however, might remove some information from the input signal. Therefore, one needs to employ decoding or interpolation techniques so that the missing information could be recovered at destination nodes. It should be noted that if loss of input signal information cannot be tolerated, the filtering should be avoided, in which case it is no longer possible to guarantee that the queueing length remains in $\mathcal{L}_2$. However, our proposed routing methodology can still be modified such that the boundedness of the queueing length is guaranteed for a bounded input flow w. This issue has been investigated in the following Theorem.

Theorem 4.1. *Consider the traffic network (4.1). Under the conditions of Assumption 3.1 and for $\|w\| < \bar{w}$, there exists a memoryless state-feedback controller,*

$u = Kx$, such that the closed-loop system is ultimately bounded, if for some design parameter $\rho > 0$, there exist matrices $\bar{N}_1, \bar{N}_2, M$ and positive definite matrices $\bar{Y}_1, Y_2, \bar{S}, \bar{N}_3$, satisfying the LMI conditions (3.4) with $K = MY_1^{-1}$.

Proof: By selecting a similar Lyapunov-Krasovskii functional V defined in (3.6) and following along the lines given in the proof of Theorem 3.3 and substituting $A = 0$, one can obtain

$$\dot{V} \leq \bar{x}^T U_1 \bar{x} + \bar{x}^T \begin{bmatrix} 0 \\ PB_w \end{bmatrix} w(t) + w(t)^T \begin{bmatrix} 0 \\ PB_w \end{bmatrix} \bar{x} \tag{4.9}$$

where $\bar{x} = \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}$. Now, the LMI conditions (3.4) (i.e., $U_1 < 0$) yields

$$\dot{V} \leq -\lambda_{min}(-U_1)\|\bar{x}\|^2 + \|\bar{x}\|(\begin{bmatrix} 0 \\ PB_w\bar{w} \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{w}B_wP \end{bmatrix}) \tag{4.10}$$

Therefore, the negative semi-definiteness of $\dot{V}$ is guaranteed provided that the following condition is satisfied

$$\|\bar{x}\| > \frac{\| \begin{bmatrix} 0 \\ PB_w\bar{w} \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{w}B_wP \end{bmatrix} \|}{\lambda_{min}(-U_1)} = b \tag{4.11}$$

In fact, $\dot{V}$ is negative definite outside the ball with radius b described as $\chi = \{\bar{x} \mid \|\bar{x}\| > b\}$, and $\bar{x}$ is uniformly ultimately bounded. The region inside the ball is attractive, since the increase of $\dot{V}$ for smaller values of $\|\bar{x}\|$ will increase V and $\bar{x}$, which forces $\bar{x}$ outside the ball χ where $\dot{V}$ is negative semi-definite and that results in reducing V and $\bar{x}$. The above analysis shows the ultimate boundedness of $\bar{x}$ and therefore, $x(t)$ (queue length). ■

4.1.1 LMI Conditions for Incorporating the Physical Constraints

In this section, the physical constraints are represented as LMI feasibility conditions. These constraints will be taken into account for determining a complete solution to the robust dynamic routing problem.

Capacity Constraint

As mentioned in (4.4), the capacity constraint can be expressed in terms of the control input u, namely

$$G_k u \leq c_k, \quad k = 1, ..., l \tag{4.12}$$

Now, consider the following ellipsoid for a selected $\varrho > 0$

$$\Sigma = \{x(t) | x^T(t) Y_1^{-1} x(t) \leq \varrho, \; Y_1 = Y_1^T > 0\} \tag{4.13}$$

If the stability condition (4.7) is satisfied, then it follows from the definition of $V(t)$ in (3.6) that

$$x^T(t) Y_1^{-1} x(t) \leq V(t)$$

On the other hand, by integrating $J_1 < 0$ defined in (3.13), from 0 to t and considering $V(0) = 0$, one can get

$$V(t) \leq -\int_0^t z^T(t) z(t) dt + \int_0^t \gamma w^T(t) w(t) dt \leq \gamma L \tag{4.14}$$

where $L = \int_0^\infty w^T(t) w(t) dt$ is an upper bound on the energy of the external input $w(t)$. Therefore, $x(t)$ belongs to an invariant set Σ for all $t > 0$, if

$$\gamma L \leq \varrho \tag{4.15}$$

Using (4.8), the state feedback controller u can be defined as

$$u = MY_1^{-1}x$$

Therefore, (4.12) can be written as

$$G_k MY_1^{-1}x \leq c_k \tag{4.16}$$

By squaring (4.16), we get

$$x^T(G_k MY_1^{-1})^T G_k MY_1^{-1}x \leq c_k^2 \tag{4.17}$$

Note that from (4.13) we have $x(t)^T Y_1^{-1} x(t) < \varrho$, therefore (4.17) will be satisfied if

$$(G_k MY_1^{-1})^T(\varrho/c_k^2)G_k MY_1^{-1} \leq Y_1^{-1} \tag{4.18}$$

Now, by applying the Schur complement (2.7) to (4.18), the capacity constraints can be expressed according to the following LMI conditions

$$W_{c1} \triangleq \gamma \leq \varrho/L \tag{4.19}$$

$$W_{c2k} \triangleq \begin{bmatrix} Y_1 & M^T G_k^T \\ G_k M & c_k^2/\varrho \end{bmatrix} \geq 0 \quad k = 1, ..., l \tag{4.20}$$

Upper Bound on the Buffer Size

The constraint on the queue buffer size (4.5) is redefined as

$$Q_{di}x \leq x_{max_{di}} \; for \; d = 1, ..., \bar{d}, \; i = 1, ..., n \tag{4.21}$$

where $x_{max_{di}} = q_{max_i}^d$. Adopting the similar lines used for capacity constraint and considering the ellipsoid (4.13), equation (4.21) is satisfied if

$$Q_{di}^T(\varrho/x_{max_{di}}^2)Q_{di} \leq Y_1^{-1} \tag{4.22}$$

which can be expressed by the following LMI condition

$$W_{c3} \triangleq \begin{bmatrix} Y_1 & Y_1^T Q_{di}^T \\ Q_{di} Y_1 & x_{max_{di}}^2/\varrho \end{bmatrix} \geq 0, \; d = 1, ..., \bar{d}, \; i = 1, ..., n \qquad (4.23)$$

Non-negative Orthant Stability

The non-negativity constraint (4.3) can be expressed in terms of the non-negative orthant stability condition given by the following theorem:

Theorem 4.2. *[106] The linear time-delayed system $\dot{x} = Ax(t) + A_d x(t - \tau(t))$ is non-negative if and only if $A \in \mathcal{R}^{n\times n}$ is essentially nonnegative, i.e., its off-diagonal entries are non-negative and $A_d \in \mathcal{R}^{n\times n}$ is non-negative, i.e., all its elements are non-negative. For a discrete-time system $x(t+1) = \bar{A}x(t) + \bar{A}_d x(t - \tau(t))$, the non-negative orthant condition is achieved if and only if both $\bar{A}$ and $\bar{A}_d$ are non-negative.*

The proof of this theorem is given in [106].

By substituting the state feedback controller $u = Kx$ into the dynamical model (4.1), the closed-loop dynamics is obtained according to

$$\dot{x}(t) = BKx(t) + B_d K x(t - \tau(t)) + B_w w(t) \qquad (4.24)$$

The design problem for the gain K can now be formulated according to the Theorem 4.2 conditions, namely that the off-diagonal entries of BK and all entries of $B_d K$ should be non-negative. Now, by selecting the positive definite matrix Y_1 to be a diagonal matrix and by setting $K = MY_1^{-1}$, the (essential) non-negativity of $(B_d K)$ and BK, which ensures the non-negativity constraint (4.3), can be expressed as follows

$$W_{c4} \triangleq (BM)_{ij} \geq 0, \quad i \neq j \qquad (4.25)$$

$$W_{c5} \triangleq (B_d M)_{ij} \geq 0, \quad i, j = 1, ..., \bar{d} \qquad (4.26)$$

Once the non-negativity condition $x \geq 0$ is satisfied, the second non-negativity condition $u \geq 0$, as given by (4.2) can be easily satisfied if we specify $K_{ij} > 0$. Hence, by using (4.8) and noting that Y_1 is a diagonal positive definite matrix, (4.2) is satisfied if the following LMI condition holds

$$W_{c6} \triangleq M_{ij} \geq 0, \quad i = 1, ..., \bar{d}, j = 1, ..., l(n-1) \tag{4.27}$$

Remark 4.2. It should be noted that since the elements of B are -1 or 0, satisfying condition (4.27) results in a square matrix BM with negative or zero elements. On the other hand, satisfying W_{c4} leads to a diagonal negative definite matrix BM. This is also validated by the fact that the queues at each node are decoupled from each other. Therefore, BK should always be diagonal. Moreover since the elements of B_d are 1 and 0, satisfying condition (4.27) results in a square matrix B_dM with positive or zero elements. Therefore, W_{c5} is trivially satisfied.

The results above are summarized by the following theorem:

Theorem 4.3. *An $\mathcal{H}_\infty$* centralized *routing controller for the traffic network with dynamical queueing model (4.1) is obtained by solving the optimization problem:*

$$\min_{Y_1, Y_2, \bar{S}, M, \bar{N}_i, i=1,2,3} \gamma \tag{4.28}$$

subject to the positive definite matrices $Y_1, Y_2, \bar{S}, \bar{N}_3$ and the LMI conditions W_1, W_{c1}, W_{c2k}, W_{c3}, W_{c4}, and W_{c6} as governed by equations (4.7), (4.19), (4.20), (4.23), (4.25), and (4.27), respectively.

Proof: Follows from the constructive derivations given above in this section.

■

It should be mentioned that due to existing several constraints and objectives in routing problem, (4.28) leads to a suboptimal solution.

Remark 4.3. It is worth noting that since the routing control provided by Theorem 4.3 is internally stable, the states (queues) are L_2. It guarantees that the achieved routing is loop free.

4.2 Decentralized Routing Control Scheme

Decentralized decision making is an attractive feature, especially for large scale systems such as traffic networks. In such systems, each subsystem is capable of making its own autonomous decisions. In this section, the robust centralized routing control strategy introduced in Section 4.1 is modified and developed as a decentralized control scheme. Specifically, the problem is reformulated so that each node only requires its own local information to route the received messages while ensuring that a global objective function or performance index is optimized and desired specifications are satisfied.

Robust control of large scale time-delay systems has witnessed an intense research activity in the past few years. In [99], a decentralized stabilizing controller is developed for interconnected fixed time-delay systems. Stabilization of a class of time-varying large scale systems subject to time-varying delays was investigated in [100] and [101]. The existing decentralized control strategies in the literature for time-varying delay systems all assume that the delay functions vary slowly with time. In this section, a robust decentralized routing control strategy for unknown and time-varying delays is proposed. The properties of our decentralized robust control scheme and a brief discussion on its complexity and scalability are also provided.

First, let us revisit the decentralized dynamic model of the traffic flow at each node, or subsystem given in (2.23)

$$\dot{x}_i(t) = B_i u_i(t) + \sum_{j \in \wp(i)} B_{dij} u_j(t - \tau_{ij}(t)) + B_{wi} w_i(t) \tag{4.29}$$

where $x_i = vec\{q_i^d(t)\} \in \Re^n \;\; for \;\; d = 1, ..., \bar{d}$ denotes the queueing length in node i for different destinations, $u_i(t) = vec\{f_{ij}^d(t)\} \in \Re^l$ denotes the flows sent from node i, $\tau_{ij}(t)$ denotes the unknown but bounded time-varying total delay in transmission, propagation, and processing, $w_i(t) \in \Re^n$ denotes the external input flow for node i, and the matrices B_i, B_{dij} and B_{wi} are defined for the node i similar to that in Section 2.2.2.

The set of constraints are also given by

$$u_i \geq 0 \tag{4.30}$$

$$x_i \geq 0 \tag{4.31}$$

$$G_{k_i} u_i < c_{k_i}, \quad k_i = 1, ..., li, \quad i = 1, ..., n \tag{4.32}$$

$$Q_{di} x_i < x_{max_{di}} \tag{4.33}$$

At each node, the routing problem is defined as determining an $\mathcal{H}_\infty$ state feedback controller $u_i = K_i x_i$, such that the following objective function is minimized:

$$J(w) = \int_0^\infty (z^T z - \gamma w^T w) ds < 0, \quad \gamma > 0 \tag{4.34}$$

By substituting $A_i = 0$ in LMI conditions (4.35), the results of Theorem 3.5 can be used for decentralized routing problem. Hence, it can be concluded that the fluid flow model of a traffic network governed by the system (4.29) with $w \in L_2[0, \infty)$ is internally stabilizable by a *decentralized* state feedback controllers of the form $u_i = K_i x_i$ with an L_2-gain that is less than γ, if for some design parameter $\rho > 0$ there exist matrices $M_i, \bar{N}_i, \bar{Z}_i$ and symmetric positive definite matrices Y_{i1}, Y_{i2}, $\bar{S}_i$, $\bar{U}_i$ for $i = 1, ..., n$ such that the following LMI conditions are satisfied

$$W_{i1} = \begin{bmatrix} \theta_{i1} & \theta_{i2} & B_{wi} & Y_{i1}^T C_i^T & \theta_{i3} & m_i h_{ji} \bar{N}_i^T & 0 \\ * & \theta_{i4} & 0 & 0 & \theta_{i5} & 0 & \theta_{i6} \\ * & * & -\gamma I & 0 & \theta_{i7} & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & \theta_{i8} & 0 & 0 \\ * & * & * & * & * & \theta_{i9} & 0 \\ * & * & * & * & * & 0 & \theta_{i10} \end{bmatrix} < 0$$

$$2Y_{i1} - Y_{i2} - \bar{U}_i \;\geq\; 0,\; \bar{N}_i \leq \bar{Z}_i \tag{4.35}$$

where

$$\begin{aligned}
\theta_{i1} &= M_i^T B_i^T + M_i B_i + m_i[3\bar{N}_i - \bar{Z}_i + \frac{1}{\rho}\bar{S}_i] \\
\theta_{i2} &= \tilde{B}_{dij} \bar{M}_j \\
\theta_{i3} &= 2m_i h_{ji} M_i^T B_i^T \\
\theta_{i4} &= -\operatorname{diag}\{3\bar{Z}_j - \bar{N}_j + \frac{(1-d_{ji})}{\rho}\bar{S}_j\} \\
\theta_{i5} &= 2m_i h_{ji} \bar{M}_j^T \tilde{B}_{dij}^T \\
\theta_{i6} &= \operatorname{diag}\{h_{ji}\bar{Z}_j\} \\
\theta_{i7} &= 2m_i h_{ji} B_{wi}^T \\
\theta_{i8} &= -2h_{ji} Y_{i2} \\
\theta_{i9} &= -m_i h_{ji} \bar{U}_i \\
\theta_{i10} &= -\operatorname{diag}\{h_{ji}\bar{U}_j\}
\end{aligned}$$

and m_i = number of downstream nodes for i, $\bar{M}_j = \operatorname{diag}\{M_j\}$, and $\tilde{B}_{dij} = vec\{B_{dij}\}$ for $j \in \wp(i)$. Moreover, the robust decentralized state feedback controller gain is given by $K_i = M_i Y_i^{-1}$.

4.2.1 LMI Conditions for Incorporating the Physical Constraints

The associated centralized LMI conditions given previously in Section 4.1.1 for the physical constraints have to be now redefined for the decentralized routing problem. The corresponding LMI conditions for the decentralized problem are developed below.

Capacity Constraint

The capacity constraint for each subsystem is defined as

$$G_{k_i} u_i \leq c_{k_i}, \quad k_i = 1, ..., li, \quad i = 1, ..., n$$

Now, consider the following ellipsoid for a selected $\varrho_i > 0$

$$\Sigma_i = \{x_i(t) | x_i^T(t) Y_{i1}^{-1} x_i(t) \leq \varrho_i, \ Y_{i1} = Y_{i1}^T > 0\} \tag{4.36}$$

Provided that the stability conditions defined in (4.35) are satisfied, and by following along similar lines as those given in Section 4.1.1, the capacity constraints for the subsystem i can be modified according to the following LMI conditions

$$W_{c1i} \triangleq \gamma \leq \max_i \{\varrho_i / L_i\} \tag{4.37}$$

$$W_{c2ik_i} \triangleq \begin{bmatrix} Y_{i1} & M_i^T G_{k_i}^T \\ G_{k_i} M_i & c_{k_i}^2 / \varrho_i \end{bmatrix} \geq 0, \quad k_i = 1, ..., li \tag{4.38}$$

where $L_i = \int_0^\infty w_i^T(t) w_i(t) dt$ is an upper bound on the energy of the external input $w_i(t)$.

Upper Bound on the Buffer Size

The constraint on the queue buffer size for each subsystem can be defined as follows

$$Q_{di} x_i \leq x_{max_{di}}, d = 1, ..., \bar{d}, \ i = 1, ..., n \tag{4.39}$$

Following along the similar lines as that stated in Subsection 4.1.1, and considering the ellipsoid (4.36), equation (4.39) can be satisfied by the following LMI condition

$$W_{c3i} \triangleq \begin{bmatrix} Y_{i1} & Y_{i1}^T Q_{di} \\ Q_{di} Y_{i1} & x_{max_{di}}^2 / \varrho_i \end{bmatrix} \geq 0, \; d = 1, ..., \bar{d}, \; i = 1, ..., n \tag{4.40}$$

Non-negative Orthant Stability

The non-negativeness condition of the queueing length for the decentralized routing control scheme for subsystem i can be expressed according to

$$W_{c4i} \triangleq (B_i M_i)_{sr} \geq 0, \quad s \neq r \tag{4.41}$$

$$W_{c5i} \triangleq (B_{dij} M_j)_{sr} \geq 0, \; r, s = 1, ..., \bar{d}, i = 1, ..., n, j \in \wp(i) \tag{4.42}$$

Provided that the non-negativity condition $x_i \geq 0$ is satisfied, the second non-negativity condition $u_i \geq 0$ can be achieved if $K_{i(sr)} > 0$. Hence, by noting that Y_{i1} is a positive definite diagonal matrix, $u_i \geq 0$ is satisfied if the following LMI conditions hold

$$W_{c6i} \triangleq M_{i(sr)} \geq 0, r, s = 1, ..., \bar{d}, i = 1, ..., n \tag{4.43}$$

Similar to lines mentioned in Remark 4.2, satisfying condition W_{c6i} can trivially guarantee W_{c5i}. Consequently, the above results are summarized by the following theorem.

Theorem 4.4. *A decentralized $\mathcal{H}_\infty$ routing controller for the traffic network with the dynamical queueing model (4.29) is obtained by solving the following optimization problem:*

$$\min_{M_i, Y_{i1}, Y_{i2}, \bar{S}_i, \bar{U}_i, \bar{N}_i, \bar{Z}_i} \gamma \tag{4.44}$$

subject to the positive definite matrices $Y_{i1}, Y_{i2}, \bar{S}_i, \bar{U}_i$, and the LMI conditions for

W_{i1}, W_{c1i}, W_{c2ik_i}, W_{c3i}, W_{c4i}, and W_{c6i} for $i = 1, ..., n$, as governed by equations (4.35), (4.37),(4.38), (4.40), (4.41), and (4.43), respectively.

Proof: Follows from the constructive derivations that are given above in this section. ■

4.2.2 Computational Complexity and Scalability

As the number of nodes in a network increases, there would be a significant increase in the number of different possible paths from a given source node to a given destination node. Therefore, in general, it is difficult to implement a centralized routing algorithm for an arbitrarily large traffic network. Centralized routing controllers are also vulnerable to failures in the network and introduce a large communication overhead on the network. In view of these drawbacks and limitations, decentralized controllers that can be constructed locally at individual nodes are highly desirable for practical purposes and implementation.

Specifically, in order to apply our proposed centralized routing method, six $(n-1)\bar{d} \times (n-1)\bar{d}$ and one $(l-1)\bar{d} \times (n-1)\bar{d}$ matrices should be obtained such that the LMI conditions in (4.7) are satisfied, where n is the number of nodes, $\bar{d}$ is the number of destination nodes, and l is the number of links. On the other hand, by utilizing our proposed decentralized routing method for each node, there are 7 matrices that should be obtained for satisfying the LMI conditions in (4.35). Therefore, there would be a total of $6(n-\bar{d})$ matrices with dimension $\bar{d} \times \bar{d}$, $6\bar{d}$ matrices with dimension $(\bar{d}-1)\times(\bar{d}-1)$, $n-\bar{d}$ matrices with dimension $m_i\bar{d}\times k$, and $\bar{d}$ matrices with dimension $m_i(\bar{d}-1)\times(\bar{d}-1)$ where m_i is number of downstream nodes for i. Even though the number of the required matrices in the decentralized control scheme is higher than the centralized controller method, given that the dimensions of matrices are lower than the centralized case, a solution to the decentralized scheme can be obtained by LMI technique much faster and more efficiently. Moreover, the implementation of the decentralized controller is computationally less expensive as compared to the centralized method. To summarize, for networks that are not *crowded*, the centralized method is more desirable due to its optimality (vis-a-vis the use of the full information set) and accuracy. However, by increasing the number of nodes in the network it becomes difficult and sometimes even impossible (due to ill-conditioning and curse of dimensionality) to design and implement a centralized controller. Consequently, the decentralized method is more suitable and appropriate for these large scale traffic networks.

4.3 Simulation Results

To evaluate the performance of the proposed traffic network routing control strategies, simulations are conducted in this section on two examples. The results obtained by applying our proposed decentralized routing strategy are compared with those of the centralized scheme as well as another conventional optimal control scheme, namely Model Predictive Control (MPC) method. The following section briefly explains the MPC framework which is applied for the routing problem.

4.3.1 Formulating the Routing Problem in MPC Framework

To solve the dynamic routing problem, the MPC algorithm proposed in [7] is considered. This algorithm can deal with **known constant delays** and physical constraints. The block diagram of the MPC feedback controller is shown in Figure 4.1. In this figure, d is the disturbance, r is the reference value, u is the manipulated variable (input signal to the plant), $\hat{y}$ is the pure output (before being corrupted by measurement noise), y is the measured output, and z is the noise. Now, consider

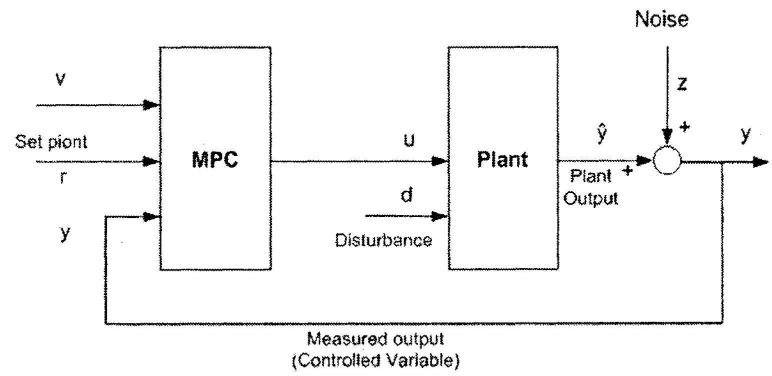

Figure 4.1: Block diagram of the MPC feedback controller

the state space representation of the queue model as follows:

$$\begin{aligned} x(k+1) &= B_u u(k) + B_d u(k+D) + B_w d(k) \\ \hat{y}(k) &= x(k) \end{aligned} \tag{4.45}$$

Therefore, the MPC control signal at time k is obtained by solving the following optimization problem [107]:

$$J = \min_{\Delta u(k|k),\ldots\Delta u(m-1+k|k)} \sum_{i=0}^{p-1} \sum_{j=1}^{n_y} |\bar{w}^y_{i+1}(y_j(k+i+1|k)|^2 + \sum_{j=1}^{n_u} |\bar{w}^u_{i,j}(u_j(k+i|k)|^2 \tag{4.46}$$

subject to the following constraints:

$$\begin{aligned} u_{jmin}(i) &\leq u_j(k+i|k) \leq u_{jmax}(i) \\ y_{jmin}(i) &\leq y_j(k+i+1|k) \leq u_{jmax}(i), i = 1, \ldots, p-1 \end{aligned} \tag{4.47}$$

where, the subscript $(.)_j$ denotes the jth component of a vector, $(k+i|k)$ denotes the predicted value for time $k+i$ based on the information available at time k, $u(k) = u(k-1) + \Delta u(k|k)$, and $\bar{w}$ are the weights.

To calculate the next move u_k, the controller operates in two phases:

1.) **Estimation:** To make an intelligent move, the controller needs to know the current state. All past and current measurements are used to accomplish the estimation step.

2.) **Optimization:** Values of setpoints and constraints are specified over a finite horizon of future sampling instants $k+1, ...k+P$, where $P \geq 1$ is the prediction horizon shown in Figure 4.2(a). The controller computes M moves $u_k, ..., u_{k+M+1}$, where $1 \leq M \leq P$ is the control horizon. This is illustrated in Figure 4.2(b).

The Dantzig-Wolfe's algorithm can be used to solve the above constrained optimization problem [108].

Example 4.1. Consider the network shown in Fig. 4.3 adopted from [109]. The capacity of each link is also indicated in the figure, the unit of which is kbit/s All nodes are assumed to be sources as well as destinations. From Fig. 4.3, it follows that there are 6 queues (the states of the system) and 8 output flows for these queues (the input signals). The initial values of all states are set to 0. The external input is considered to be the following pulse function:

$$w(t) = \begin{cases} 1 \ kbit/s. & 0 < t < 50 \\ 0 & otherwise \end{cases}$$

and the delay is assumed to be 50 ms for each input flow. The results obtained by using our proposed decentralized and centralized $\mathcal{H}_\infty$ controllers for the queueing lengths are shown in Fig. 4.4 and for the flow links in Fig. 4.5, respectively. To compare the performance of our proposed $\mathcal{H}_\infty$ controllers with another benchmark dynamic routing strategy, the Model Predictive Controller (MPC) scheme discussed in Section 4.3.1 were applied. The weights of the objective function are defined as:

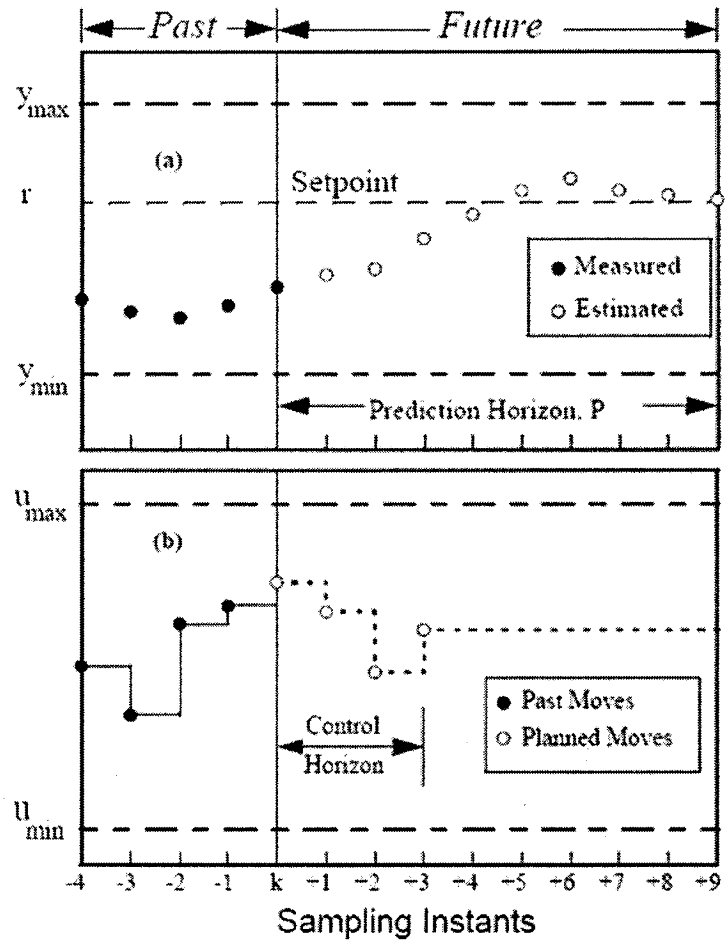

Figure 4.2: The MPC controller state at the kth sampling instant [7]

$\bar{w}^u_{i,j} = 5$ $\bar{w}^y_{i,j} = 7$. The control horizon is 2 and the prediction horizon is 10. The results obtained by using the MPC are depicted in Figs. 4.4 and 4.5. These figures clearly demonstrate the superiority of the $\mathcal{H}_\infty$ controller over the MPC methodology despite the fact that all delays are assumed to be **known** *a priori* for the MPC controller, whereas for the proposed $\mathcal{H}_\infty$ controllers, *no a priori* knowledge about the delays, except their upper bound, was assumed. It can be stated that overall the decentralized control performance is comparable with that of the centralized method. However, since there are only three nodes in the network, the performance difference is not quite significant.

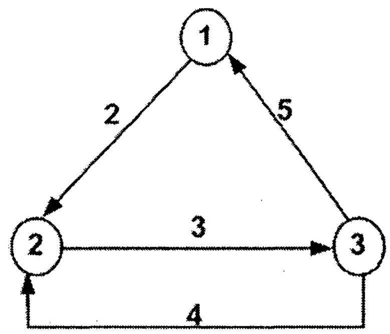

Figure 4.3: Network topology of Example 4.1

It is also important to investigate the effects of jitter on the performance of the proposed centralized and decentralized routing strategies. It should be noted that different definitions for jitter is given in the literature [110], [111]. We are considering the following definition for jitter [110]: Jitter is an unwanted variation of one or more characteristics of a periodic signal in electronic and telecommunication systems. Jitter may be seen in the amplitude, frequency, or phase of a signal.

Therefore, the following input signal is added to the above signal w for node 3 to model the jitter in the amplitude:

$$\begin{aligned} \acute{w}_3^1(t) &= \begin{cases} pp & 20k < t < 23k \\ 0 & otherwise \end{cases} \\ \acute{w}_3^2(t) &= \begin{cases} pp & 40k < t < 43k \\ 0 & otherwise \end{cases} \end{aligned} \quad (4.48)$$

where pp is a random signal with a Poisson distribution and rate of 3 kbit per second and $k = 1, 2, \ldots$. Fig. 4.6-a and Fig. 4.6-b illustrate the queueing length of node 3 and Fig. 4.6-c and Fig. 4.6-d show their corresponding flow link, for the centralized and decentralized methods, respectively. By considering the results obtained in Fig. 4.6, it can be concluded that the effects of the jitter do not deteriorate the stability

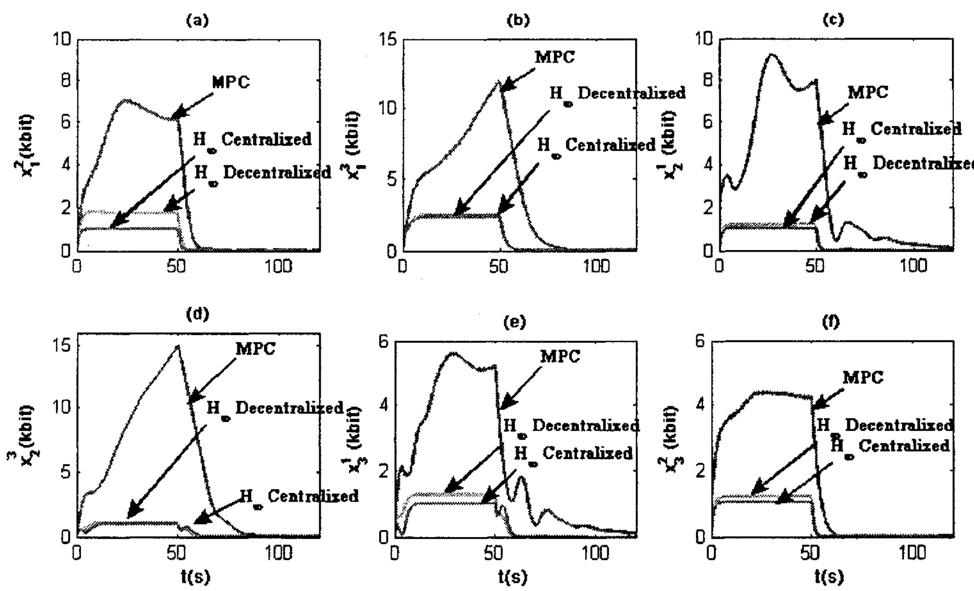

Figure 4.4: The response of the queueing lengths of the network in Example 4.1 by using the MPC scheme, the centralized $\mathcal{H}_\infty$ controller, and the decentralized $\mathcal{H}_\infty$ controller, where x_i^d denotes the queueing length of node i destined to node d.

of our routing strategies and the additional disturbances are attenuated after a short transient time.

Example 4.2. Consider the network shown in Fig. 4.7 which is adopted from [25]. The capacity of each link, with unit of kbit/s, is also depicted in the figure. The destination nodes are 7 and 10 and overall there are 17 queues (the states of the system) and 35 output flows for these queues (the input signals). The initial values of all states are set to 0. The external input is considered as a Poisson distribution with the flow rate λ for 70 s For each input flow the delay is taken as a fast time-varying function $100 + 2|sin(t)|$ ms which is considered to be *unknown* to the controllers.

Each simulation is run for 100 s For $\lambda = 0.2$ kbit per second, the results obtained by using the proposed decentralized and centralized $\mathcal{H}_\infty$ controllers for the queueing lengths of nodes 1-3 are shown in Fig. 4.8. As can be seen from this figure, the results meet the specifications and the proposed control schemes satisfy

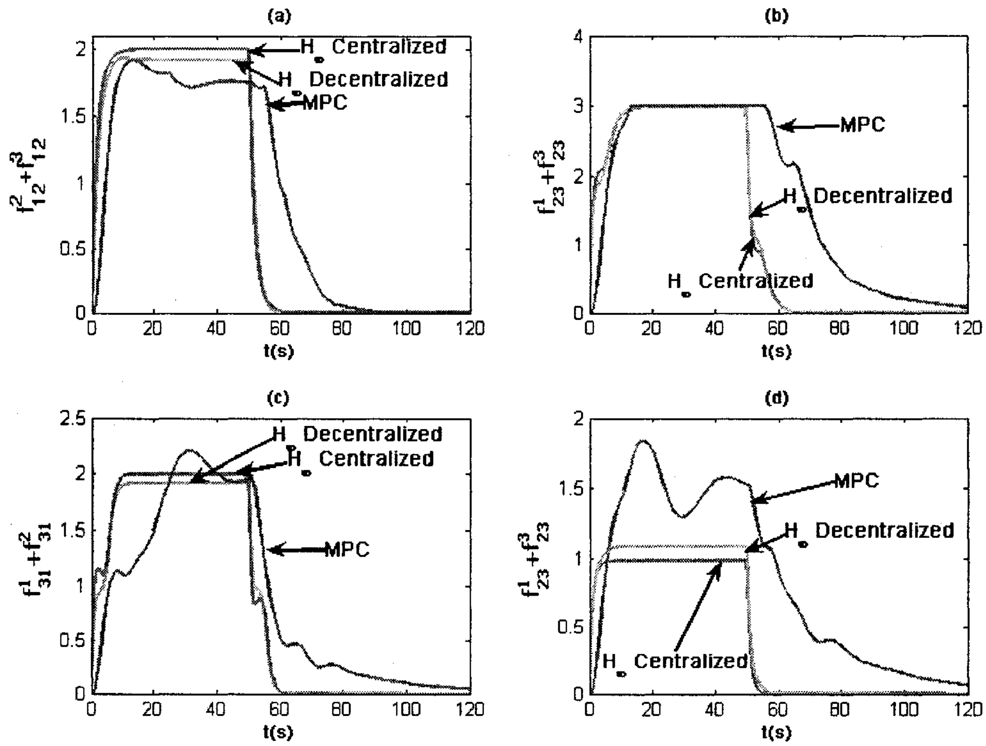

Figure 4.5: The response of output flows of the network in Example 4.1 by using the MPC scheme, the centralized $\mathcal{H}_\infty$ controller, and the decentralized $\mathcal{H}_\infty$ controller, where f_{ij}^d denotes the flow from node i to node j with destination d.

the physical constraints and the closed-loop system behaves robustly in the presence of fast time-varying delays. Similar results are also obtained for nodes 4-9. Note that the time derivative upper bound of the delay is taken as $d = 2$ and the upper bound of the delay is $h = 102$ ms Table 4.2 shows the resulting queueing delays for different input flow rates. It should be noted that our proposed routing algorithms can provide acceptable performance for higher values of the transmission, propagation and processing delays. By invoking the stability results obtained in Theorems 3.4 and 3.5, the maximum delay upper bound for which the routing controller can maintain its acceptable performance is found to be $h = 10$ s for the decentralized controller, and is found to be $h = 8$ s for the centralized controller. Queueing lengths

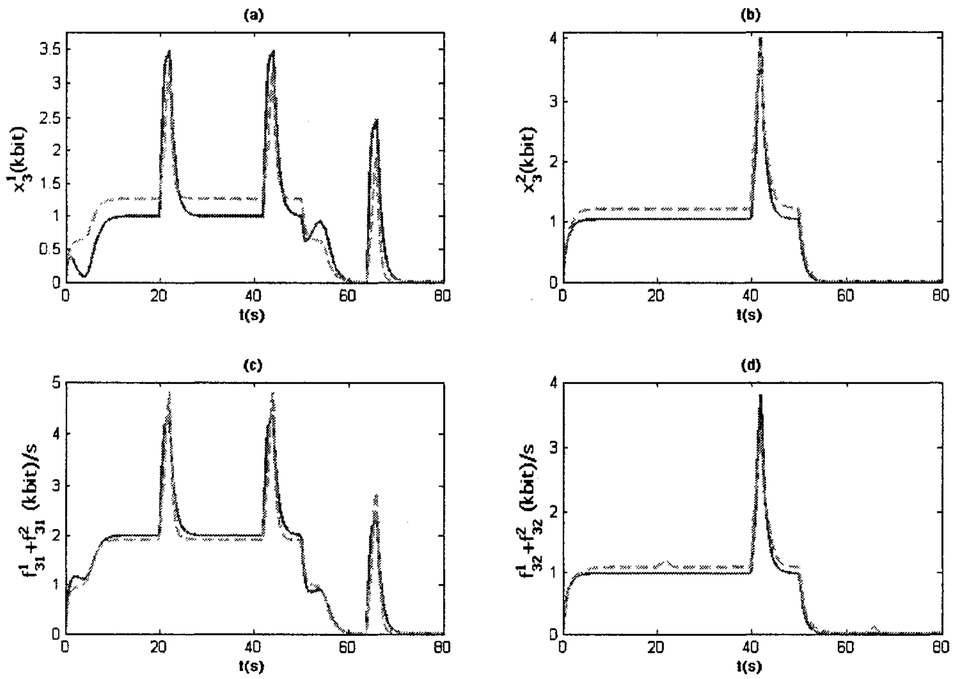

Figure 4.6: (a) and (b): queueing lengths of node 3 for Example 4.1 by using the centralized $\mathcal{H}_\infty$ controller (solid) and the decentralized $\mathcal{H}_\infty$ controller (dashed), (c) and (d): Link flows of the network in Example 4.1 for node 3 by using centralized $\mathcal{H}_\infty$ controller (solid) and the decentralized $\mathcal{H}_\infty$ controller (dashed).

by using decentralized method are 2.5 bits larger than using centralized method, on average. This deviation is small compare to order range of queueing length which is in *kbit*.

When the network traffic is heavy, congestion occurs and packets are dropped from the network, which can cause a decrease in the throughput performance. Fig. 4.9 depicts the performance of our proposed algorithms under different traffic loads. Indeed, by increasing λ to 0.3 kbit per second the decentralized method loses 3% of its packets while for $\lambda = 0.4$ kbit per second this loss is increased to 11%.

Generally, it can be concluded that the decentralized control strategy could fairly compete with the performance of the centralized method. However, as the number of nodes increases, one may *not* be able to easily solve the corresponding

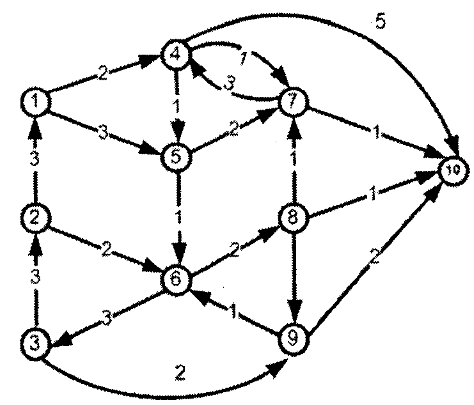

Figure 4.7: The network topology of Example 4.2

Table 4.1: Queueing length for different external flow rates

λ (kbits/s)	0.1	0.2	0.3	0.4
External input	187	289	425	561
queueing delay for the H_∞ centralized method	211.18	438.26	644.07	847.02
queueing delay for the H_∞ decentralized method	220.05	615.02	720.89	955.81

high dimensional LMI conditions associated with the centralized control strategy due to ill-conditioning and/or reductions in the size of the feasibility regions. On the other hand, the decentralized strategy is scalable and would provide an acceptable performance even in crowded networks.

4.4 Conclusions

In this chapter, a new methodology for analytical solutions to centralized and decentralized routing problems for fixed network topology was proposed. The method incorporates queueing dynamics and physical constraints that exist in the traffic network. The transmitting, propagating, and processing delays considered in the

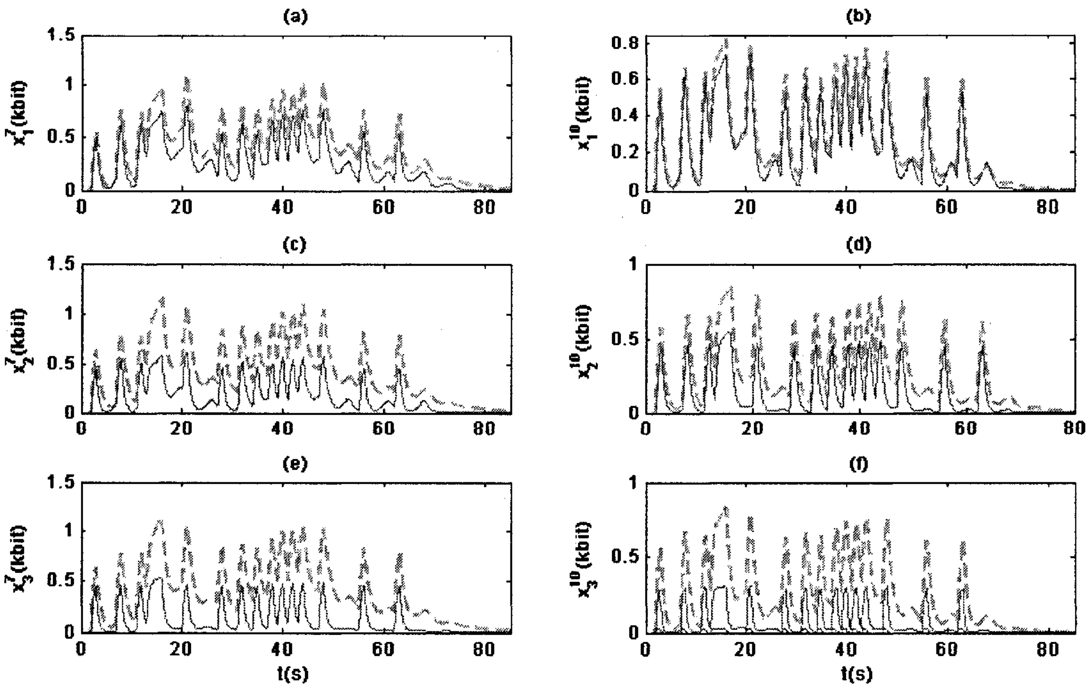

Figure 4.8: queueing lengths of nodes 1-3 for Example 4.2 by using the centralized $\mathcal{H}_\infty$ controller (solid) and the decentralized $\mathcal{H}_\infty$ controller (dashed), where x_i^d denotes the queueing length from node i destined to node d.

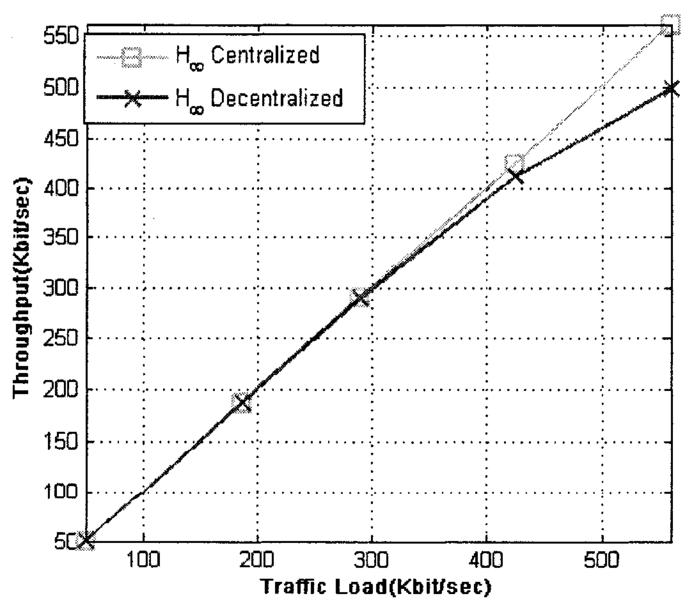

Figure 4.9: Throughput comparison between centralized and decentralized controllers.

dynamics of the network are assumed to be unknown and fast time-varying. By employing $\mathcal{H}_\infty$ robust control strategy, the proposed routing schemes will guarantee the desired routing performance in the presence of unknown *fast time-varying* delays and other network uncertainties by minimizing a measure of the queueing length (namely, the *worst-case queueing length*). Consequently, a measure of the queueing delay is also minimized. The $\mathcal{H}_\infty$ control problem is formulated in the LMI framework. By taking advantage of the LMI methodology, the network routing problem subject to various physical constraints has been solved through a unified multi-objective optimization framework. Simulation results presented do indeed confirm and demonstrate the effectiveness of the proposed routing strategies. It is worth noting that since the proposed decentralized routing controller can be implemented locally at each individual node, it is therefore *scalable* to large and crowded traffic networks.

Chapter 5

An $\mathcal{H}_\infty$ Controller for Markovian Jump Time-Delayed Systems

In this chapter, a decentralized $\mathcal{H}_\infty$ controller is proposed for regular as well as singular Markovian Jump Linear Systems (MJLSs) with time varying delays in control input. Markovian jump formulation can be used to represent various practical systems such as target tracking, manufacturing processes, fault-tolerant control systems, and other systems which experience abrupt changes in their structure and parameters, caused by phenomena such as sudden environmental disturbances and changing subsystem interconnections. Indeed, MJLSs are modeled by a set of linear systems where transitions among the models are determined by a Markov chain taking values in a finite set, introduced first by Krasovskii and Lidskii in 1961 [112]. Such systems, in fact, can be categorized as stochastic hybrid systems. This is due to the fact that the switching is governed by a stochastic process, whereas the states take continuous values and system parameters take discrete values. Time-delay systems with Markovian jump parameters using the stochastic Lyapunov functional approach was first developed by Kushner [113].

Recently, considerable attention has been devoted to Markovian systems with time-delays [114] [115] [117] [118] (and references therein). In [115], a sufficient condition for exponential estimates of a class of Markovian systems with fixed state delay was introduced. By employing LMI techniques a state feedback stabilizing controller was presented. The authors then extended the controller for time-varying delays [118]. However, their method results in a non-LMI condition, and therefore a non-convex optimization problem. In [65], a stabilizing control for MJLSs with input delays was presented. However, they failed to design a switching gain for state feedback and therefore, a fixed gain K should be found for all switching modes of the systems resulting in conservative conditions and reducing the possibility of finding a feasible solution. Moreover, the final conditions are nonlinear in parameters and cannot be solved by LMI techniques. In our proposed control scheme for MJLSs with input delay, the state feedback gains are different for each switching mode. Therefore, it provides a more reliable performance based on system specification at each mode and also increases the chance of finding a feasible solution. Moreover, the state feedback controller can be designed by solving LMI conditions which is easy to solve. The proposed control strategy for MJLSs is then extended to singular MJLSs.

Singular systems are also referred to as descriptor systems, implicit systems, generalized state space systems, differential-algebraic systems, or semi-state systems [81], [119], [120]. They represent an interesting and challenging class of dynamical systems, as they combine differential equations and algebraic equations. Indeed, various systems in economics, chemical process, mechanics and electrical systems are described as descriptor systems. Some practical examples in electrical systems are given in [119]. A great number of results based on the theory of regular systems (or state-space systems) have been extended to the area of singular systems [121], [122] [119] (and references therein). Recently, some LMI approaches have been introduced for singular systems with time-delays [120], [123].

Robust stability and stabilizability of singular systems with state delay was investigated in [120]. The authors, extended the results for $\mathcal{H}_\infty$ filtering of singular systems in [124]. An $\mathcal{H}_\infty$ control of singular systems with time-delay was addressed in [125]. In [119], the combination of MJLSs and singular systems were introduced as a new class of systems known as stochastic singular systems with random abrupt changes and control of such systems were investigated. However, the considered systems have not been affected by delay. For singularly perturbed jump systems, a delay-dependent stabilization was presented in [126], where it was assumed that the singular matrix is fixed during the system operation.

On the other hand, research on decentralized control of descriptor systems has received considerable interest in the last decades. In [127] decentralized stabilization and servomechanism problems were studied, and a constructive controller design procedure was given by transforming the descriptor system into a principal square subsystem and an equivalent proper state space model. A robust impulse control problem for uncertain singular systems by decentralized output feedback was presented in [128]. They introduced sufficient algebraic conditions for the existence of a robust decentralized controller to eliminate impulsive modes. By using LMI techniques, a decentralized $\mathcal{H}_\infty$ control for multichannel descriptor systems was also introduced in [129]. In [130], LMI conditions were developed for decentralized $\mathcal{H}_\infty$ control of multi-channel descriptor systems with time-delays in states. Whereas, to the best of author's knowledge, decentralized $\mathcal{H}_\infty$ control of singular MJLSs with delays is still an open problem.

In our decentralized $\mathcal{H}_\infty$ control for singular MJLSs, delays are considered in the inputs. It is also assumed that the interconnection terms in decentralized dynamics can change at each switching mode.

In switching systems, the number of system states is usually assumed to be fixed. In other words, for linear systems only the state matrix A and the input matrix B change at each case of switching. However, in some systems the number of states may also change in certain circumstances. For instance, in communication networks, the number of nodes may increase/decrease because of adding new nodes or deleting them due to faults or malfunctioning of the nodes. This behavior of the system can be expressed by a singular MJLSs. Specifically, in the switching modes when the number of states decreases, the dynamics are described as a singular system. Therefore, the overall systems behavior can be described in a unified dynamics and varying the number of states does not impact the integrity of the controller design problem.

It is worth noting that the result of this chapter is intended to be applied for routing problem of mobile networks in the next chapter. Since, decentralized routing control is preferred over the centralized routing control due to scalability, all the results in this chapter are based on a decentralized formulation.

The rest of this chapter is organized as follows: In Section 5.1 some preliminary definitions are given for the MJLSs. Section 5.1.1 establishes LMI delay-dependent conditions for decentralized $\mathcal{H}_\infty$ control of MJLSs. The results are then extended to decentralized MJLSs when the interconnected terms are mode-dependent. In Section 5.2, dynamics of singular systems with random abrupt changes and some preliminary definitions are given. In Section 5.2.1, an $\mathcal{H}_\infty$ control scheme is proposed for singular MJLSs with time-delay.

5.1 Markovian Jump Linear Systems with Time-Varying Delays

An MJLS can be considered as a hybrid system whose state vector has two components: $x(t)$ which is generally referred to as the state and r_t which represents the mode. MJLSs jump abruptly from one mode to another in a random manner and for this reason their switching is classified as stochastic switching. The switching between the modes is governed by a continuous-time Markov process with discrete and finite state space, whereas, at each mode the system evolves as a deterministic linear system. Definition of a Markov process is given in Appendix B.

A mathematical representation of a decentralized MJLS with input time-delay is described by the following dynamics:

$$\begin{aligned}\dot{x}_i(t) &= A_i(r(t))x_i(t) + B_i(r(t))u_i(t) + B_{w_i}(r(t))w_i(t) \\ &+ \sum_{j\in\wp(i)} B_{dij}(r(t))u_j(t-\tau_{ji}(t)) \\ x_i(t) &= \phi_i(t), \quad t\in[-h_i,0] \\ z_i(t) &= C_i(r(t))x_i(t)\end{aligned} \tag{5.1}$$

where $A_i(r(t))$, $B_i(r(t))$, $B_{w_i}(r(t))$, $B_{dij}(r(t))$ and $C_i(r(t))$ are constant matrices of appropriate dimensions which may change at each switching mode; ϕ_i is a continuously differentiable initial function; $x_i(t)$ and $u_i(t)$ represent the state and control input of subsystem i, respectively; and $r(t)$ is a continuous-time Markov process taking values in a finite space $\mathcal{S} = \{1, ..., M\}$ describing the switching between different modes whose evolution is governed by the following probability transitions:

$$\mathbb{P}[r(t+h) = l | r(t) = k] = \left\{ \begin{array}{cc} \pi_{kl}h + o(h) & k \neq l \\ 1 + \pi_{kk}h + o(h) & k = l \end{array} \right\}$$

where $\pi_{kl} > 0$ is the transition rate from mode k to mode l, $\pi_{kk} = -\sum_{l=1,l\neq k}^{N} \pi_{kl}$, $o(h)$ is a function satisfying $\lim_{h\to 0} \frac{o(h)}{h} = 0$.

Without loss of generality, it is assumed that the transition from mode l to k for all subsystems are the same.

Assumption 5.1. The delays $\tau_{ij}(t)$ are unknown differentiable functions that for all $t \geq 0$ satisfy

$$0 \leq \max\{\tau_{ij}(t)\} \leq h_{ij}, \qquad \max\{|\dot{\tau}_{ij}(t)|\} \leq d_{ij} < 1,\ d_{ij} > 0, h_i = \max\{h_{ij}\}$$

Denote $\mathcal{C}[-h_{ji}, 0]$ as the space of continuous functions on the interval $[-h_{ji}, 0]$.

Since the evolution of $x_i(t)$ in (5.1) depends on $x_i(s)$, $t - \tau_{ij} \leq s \leq t$, it is not a Markov process. In order to cast this model into the framework of the Markov process, let us define a process in $\mathcal{C}[-h_{ji}, 0]$ by

$$x_{is}(t) = x_i(s+t), \qquad t - \tau_{ij} \leq s \leq t \tag{5.2}$$

Let us now define the concept of stochastic stability, stochastic stabilizability and $\mathcal{H}_\infty$ control of stochastic systems [65].

Definition 5.1. The free nominal jump linear system (5.1) is said to be *stochastically stable* if when $w(t) \equiv 0$ and $u(t) \equiv 0$, for all $\phi \in L_2[-\tau, 0)$, and for an initial mode $r_0 \in S$ there exists a constant $M(\phi(.), r_0) > 0$ such that

$$\mathbb{E}[\int_0^\infty x(t)^T x(t) | \phi(.), r_0] \leq M(\phi, r_0)$$

where ϕ is the initial condition and $x(t) = vec\{x_i(t)\}$, for $i = 1, ..., n$.

Definition 5.2. The free nominal jump linear system (5.1) is said to be *stochastically stabilizable* if there exist linear state feedback $u_i = K_{ir} x_i$ such that the closed-loop system is stochastically stable, where K_{ir} are constant matrices.

Definition 5.3. Let $\gamma > 0$ be a positive constant. System (5.1) is said to be stochastically stable with γ-disturbance attenuation if there exits a constant $M(\phi, r_0)$ with $M(0, r_0) = 0$ such that

$$\|z\|_{\mathbb{E}_2} = \mathbb{E}\left\{\int_0^\infty z^T(s) z(s) ds\right\}^{1/2} \leq [\gamma \|w(t)\|_2 + M(\phi, r_0)]^{1/2} \tag{5.3}$$

To simplify the notation, we denote B_{ir} to represent $B_i(r(t))$ when $r(t) = r_t$. This notation is also applied to other matrices.

5.1.1 $\mathcal{H}_\infty$ Control Scheme for MJLSs with Time Varying Delayed Control

In this section, sufficient conditions are developed to guarantee $\mathcal{H}_\infty$ disturbance attenuation performance of the MJLSs with input delay. It is worth noting that unlike the current $\mathcal{H}_\infty$ control scheme for MJLSs with input delay in the literature, the control scheme proposed in this thesis is mode variable.

Theorem 5.1. *System (5.1) subject to the conditions of Assumption 5.1 is stochastically stabilizable with γ-disturbance attenuation by using the decentralized state feedback controllers $u_i = K_{ik}x_i$, if there exist matrices M_{ik}, and symmetric positive definite matrices Y_{ik}, $\bar{R}_{ik}$, $\bar{Q}_i$ for $i = 1, ..., n$, $k = 1, ..., M$ such that the following LMI conditions are satisfied*

$$L_{ik} = \begin{bmatrix} \theta_{i1} & \theta_{i2} & B_{w_{ik}} & Y_{ik}^T C_{ik}^T & \theta_{i3} & \theta_{i4} & h_{ji}\theta_{i4} \\ * & \theta_{i5} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & \theta_{i6} & 0 & 0 \\ * & * & * & * & * & \theta_{i7} & 0 \\ * & * & * & * & * & * & \theta_{i8} \end{bmatrix} < 0 \quad (5.4)$$

$$\begin{bmatrix} 2(1-\pi_{kk})I - \bar{Q}_i + \pi_{kk}\bar{R}_{ik} & \bar{\pi}_k \\ * & \tilde{R}_{ik} \end{bmatrix} \geq 0 \quad (5.5)$$

where

$$\begin{aligned}
\theta_{i1} &= Y_{ik}^T A_{ik}^T + A_{ik}Y_{ik} + M_{ik}^T B_{ik}^T + B_{ik}M_{ik} + \pi_{kk}Y_{ik}, \\
\theta_{i2} &= \tilde{B}_{dik}\tilde{R}_{ik}, \\
\theta_{i3} &= (\bar{\pi}_k Y_{ik})^T,
\end{aligned}$$

$$\begin{aligned}
\theta_{i4} &= m_i M_{ik}, \\
\theta_{i5} &= -(1-d_{ji})\tilde{R}_{ik}, \\
\theta_{i6} &= -\operatorname{diag}\{Y_{i1}, ..., Y_{i(k-1)}, Y_{i(k+1)}, ..., Y_{iM}\}, \\
\theta_{i7} &= -m_i \bar{R}_{ik}, \\
\theta_{i8} &= -m_i h_{ji}\bar{Q}_i, \\
\bar{\pi}_k &= [\sqrt{\pi_{k1}} \ \cdots \ \sqrt{\pi_{k(k-1)}} \ \sqrt{\pi_{k(k+1)}} \ \cdots \ \sqrt{\pi_{kM}}]^T
\end{aligned}$$

and m_i = *number of subsystems that subsystem i belongs to their* $\wp(.)$, $\tilde{R}_{ik} = \operatorname{diag}_{j\in\wp(i)}\{\bar{R}_{jk}\}$, *and* $\tilde{B}_{dik} = vec\{B_{dijk}\}$, *for* $j \in \wp(i)$. *Moreover, the robust decentralized state feedback controller gain is given by* $K_{ik} = M_{ik}Y_{ik}^{-1}$.

Proof: Consider the following Lyapunov-Krasovskii functional

$$\begin{aligned}
V(x_t, r_t) &= V_1 + V_2 + V_3 \\
V_1 &= \sum_{i=1}^{n} x_i^T(t)P_{ir_t}x_i(t) \\
V_2 &= \sum_{i=1}^{n}\sum_{j\in\wp(i)} \int_{t-\tau_{ji}}^{t} u_j^T(s)R_{jr_t}u_j(s)ds \\
V_3 &= \sum_{i=1}^{n}\sum_{j\in\wp(i)} \int_{0}^{h_{ji}} (h_{ji}-\sigma)u_j^T(t-\sigma)Q_j u_j(t-\sigma)d\sigma \qquad (5.6)
\end{aligned}$$

To achieve the $\mathcal{H}_\infty$ objective (5.3), one should show that

$$J_1 = \mathcal{A}V(x_t, r_t) + z^T(t)z(t) - \gamma w^T(t)w(t) < 0 \tag{5.7}$$

where $\mathcal{A}$ is the infinitesimal generator of $\{(x_{it}, r_t), t \geq 0\}$. Further details are given in Appendix B.

Therefore, one can get

$$\mathcal{A}V(x_t, r_t) = \mathcal{A}V_1(x_t, r_t) + \mathcal{A}V_2(x_t, r_t) + \mathcal{A}V_3(x_t, r_t) \tag{5.8}$$

Suppose $r_t = k \in S$ then

$$\begin{aligned}
\mathcal{A}V_1(x_t, r_t) &= \lim_{\Delta \to 0_+} \frac{1}{\Delta}\{\mathbb{E}[V_1(x_{t+\Delta}, r_{t+\Delta}, t+\Delta)|x_t, r_t = k] - V_1(x_t, k, t)\} \qquad (5.9)\\
&= \lim_{\Delta \to 0_+} \frac{1}{\Delta}\sum_{i=1}^{n}\{\sum_{l \neq k} \mathbb{P}[r_{t+\Delta} = l | r_t = k] x_i^T(t+\Delta)P_{il}x_i(t+\Delta)\\
&+ \mathbb{P}[r_{t+\Delta} = k | r_t = k] x_i^T(t+\Delta)P_{ik}x_i(t+\Delta) - x_i^T(t)P_{ir_t}x_i(t)\}\\
&= \lim_{\Delta \to 0_+} \frac{1}{\Delta}\sum_{i=1}^{n}\{\sum_{l \neq k}(\pi_{kl}\Delta + o(\Delta))x_i^T(t+\Delta)P_{ir_{t+\Delta}}x_i(t+\Delta)\\
&+ (1 + \pi_{kk}\Delta + o(\Delta))x_i^T(t+\Delta)P_{ir_{t+\Delta}}x_i(t+\Delta) - x_i^T(t)P_{ir_t}x_i(t)\}\\
&= \sum_{i=1}^{n}[\dot{x}_i^T(t)P_{ir_t}x_i(t) + x_i^T(t)P_{ir_t}\dot{x}_i(t) + x_i^T(t)\sum_{k=1}^{M}\pi_{r_tk}P_{ik}x_i(t)]\\
&= \sum_{i=1}^{n}[x_i^T(t)((A_{ir_t} + B_{ir_t}K_{ir_t})^T P_{ir_t} + P_{ir_t}(A_{ir_t} + B_{ir_t}K_{ir_t}))x_i(t)\\
&+ \sum_{j \in \wp(i)}[u_j^T(t-\tau_{ji})B_{dijr_t}^T P_{ir_t}x_i(t) + x_i^T(t)P_{ir_t}B_{dijr_t}u_j(t-\tau_{ji})]\\
&+ x_i^T(t)P_{ir_t}B_{wir_t}w_i(t) + w_i^T(t)B_{wir_t}^T P_{ir_t}x_i(t) + x_i^T(t)\sum_{k=1}^{M}\pi_{r_tk}P_{ik}x_i(t)]
\end{aligned}$$

$$
\begin{aligned}
\mathcal{A}V_2(x_t, r_t) &= \lim_{\Delta\to 0_+} \frac{1}{\Delta}\sum_{i=1}^{n}\sum_{j\in\wp(i)}\{\sum_{l\neq k}\mathbb{P}[r_{t+\Delta}=l|r_t=k]\int_{t+\Delta-\tau_{ji}}^{t+\Delta} u_j^T(s)R_{jl}u_j(s)ds \\
&+ \mathbb{P}[r_{t+\Delta}=k|r_t=k]\int_{t+\Delta-\tau_{ji}}^{t+\Delta} u_j^T(s)R_{jk}u_j(s)ds \\
&- \int_{t-\tau_{ji}}^{t} u_j^T(s)R_{jr_t}u_j(s)ds \qquad (5.10)\\
&= \lim_{\Delta\to 0_+} \frac{1}{\Delta}\sum_{i=1}^{n}\sum_{j\in\wp(i)}\{\sum_{l\neq k}(\pi_{kl}\Delta+o(\Delta))\int_{t+\Delta-\tau_{ji}}^{t+\Delta} u_j^T(s)R_{jl}u_j(s)ds \\
&+ (1+\pi_{kk}\Delta+o(\Delta))\int_{t+\Delta-\tau_{ji}}^{t+\Delta} u_j^T(s)R_{jl}u_j(s)ds \\
&- \int_{t+\Delta-\tau_{ji}}^{t+\Delta} u_j^T(s)R_{jr_t}u_j(s)ds\} \\
&= \sum_{i=1}^{n}\sum_{j\in\wp(i)}[u_j^T(t)R_{jr_t}u_j(t) - (1-\tau_{ji})u_j^T(t-\tau_{ji})R_{jr_t}u_j(t-\tau_{ji}) \\
&+ \int_{t-\tau_{ji}(t)}^{t} u_j^T(s)\sum_{k=1}^{M}\pi_{r_tk}R_{jk}u_j(s)ds]
\end{aligned}
$$

$$
\begin{aligned}
&\leq \sum_{i=1}^{n}\sum_{j\in\wp(i)}[u_j^T(t)R_{jr_t}u_j(t) - (1-d_{ji})u_j^T(t-\tau_{ji})R_{jr_t}u_j(t-\tau_{ji}) \\
&+ \int_{t-\tau_{ji}(t)}^{t} u_j^T(s)\sum_{k=1}^{M}\pi_{r_tk}R_{jk}u_j(s)ds]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}V_3(x_t, r_t) &= \sum_{i=1}^{n}\sum_{j\in\wp(i)}[h_{ji}u_j^T(t)Q_ju_j(t) - \int_{t-h_{ji}(t)}^{t} u_j^T(s)Q_ju_j(s)ds] \qquad (5.11)\\
&\leq \sum_{i=1}^{n}\sum_{j\in\wp(i)}[h_{ji}u_j^T(t)Q_ju_j(t) - \int_{t-\tau_{ji}(t)}^{t} u_j^T(s)Q_ju_j(s)ds]
\end{aligned}
$$

In view of (5.8), considering

$$
Q_i \geq \sum_{k=1}^{M}\pi_{r_tk}R_{ik} \qquad (5.12)
$$

and using the fact that

$$\sum_{i=1}^{n} \sum_{j \in \wp(i)} u_j^T(t) R_{jr_t} u_j(t) = \sum_{i=1}^{n} m_i u_i^T(t) R_{ir_t} u_i(t)$$

$$\sum_{i=1}^{n} \sum_{j \in \wp(i)} u_j^T(t) Q_j u_j(t) = \sum_{i=1}^{n} m_i u_i^T(t) Q_i u_i(t),$$

and by applying the Schur complement, and then substituting the result into (5.7), one gets

$$J_1 \leq \sum_{i=1}^{n} X_i^T(t) \bar{L}_{ir_t} X_i(t)$$

where

$$X_i = [x_i^T(t) \quad U_j^T(t - \tau_i(t)) \quad w_i^T(t)]^T,$$

$$U_j^T(t - \tau_i(t)) = vec\{u_j^T(t - \tau_{ji}(t))\} \quad for \quad j \in \wp(i)$$

$$\bar{L}_{ik} = \begin{bmatrix} \Omega_{i1} & \Omega_{i2} & P_{ik} B_{w_{ik}} & C_{ik}^T & \Omega_{i3} \\ * & \Omega_{i4} & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & \Omega_{i5} \end{bmatrix} < 0 \tag{5.13}$$

$$\Omega_{i1} = (A_{ik} + B_{ik} K_{ik})^T P_{ik} + P_{ik}(A_{ik} + B_{ik} K_{ik}) + m_i K_{ik}^T [h_{ji} Q_i + R_{ik}] K_{ik} + \pi_{kk} P_{ik},$$

$$\Omega_{i2} = P_{ik} \tilde{B}_{dik},$$

$$\Omega_{i3} = [\sqrt{\pi_{k1}} P_{i1} \ \dots \ \sqrt{\pi_{k(k-1)}} P_{i(k-1)} \ \sqrt{\pi_{k(k+1)}} P_{i(k+1)} \ \dots \ \sqrt{\pi_{kM}} P_{iM}]^T,$$

$$\Omega_{i4} = -\mathrm{diag}_{j \in \wp(i)}\{(1 - d_{ji}) R_{jk}\},$$

$$\Omega_{i5} = -\,\mathrm{diag}\{P_{i1}, \dots, P_{i(k-1)}, P_{i(k+1)}, \dots, P_{iM}\},$$

Now, let

$K_{ik} = M_{ik}Y_{ik}^{-1},\ P_{ik} = Y_{ik}^{-1},\ R_{ik} = \bar{R}_{ik}^{-1},\ Q_i = \bar{Q}_i^{-1},\ \bar{\bar{R}}_{ik} = vec\{\bar{R}_{jk}\}\ for\ j \in \wp(i),\ \bar{Y}_i = vec\{Y_{il}\}\ for\ l = 1, ..., (p-1), (p+1), ..., M$. Then, pre and post multiplying (5.13) by $\Delta_{ik} = diag\{Y_{ik},\ \bar{\bar{R}}_{ik},\ I,\ I,\ \bar{Y}_i\}$ and Δ_{ik}^T respectively, and applying the Schur complements lead to L_{ik} in (5.4). Therefore, (5.4) guarantees negative definiteness of J_1 in (5.7). Now by using Dynkin's formula (see Appendix B), one can get

$$\begin{aligned} J_T &= \mathbb{E}[\int_0^T [J_1 - \mathcal{A}V(x_t, r_t)dt] \\ &\leq \mathbb{E}\int_0^T \sum_{i=1}^{n} X_i^T(t) L_{ir_t} X_i(t)dt - \mathbb{E}[V(x_T, r_T) + V(x_0, r_0)] \end{aligned} \tag{5.14}$$

Using the fact that $L_{ir_t} < 0$ and $\mathbb{E}[V(x_T, r_T)] > 0$ yields

$$J_T \leq V(x_0, r_0)$$

and therefore

$$J_\infty \leq V(x_0, r_0)$$

In other words the $\mathcal{H}_\infty$ objective (5.3) is stratified according to the following inequality:

$$\|z\|_{\mathbb{E}_2} - \gamma\|w(t)\|_2 \leq V(x_0, r_0)$$

To investigate the stability and convergence of the states in the absence of the external input, w_i, let us eliminate the $(n+2)$nd row and column (corresponding to the terms involving w_i) and the $(n+3)$rd row and column of (5.4). Therefore, the LMI condition is obtained which implies that

$\mathcal{A}V(x, r_t) < \sum_{i=1}^{n} \bar{X}_i^T(t)\hat{L}_{ir1}(h_{ji}, d_{ji})\bar{X}_i(t) < 0$

where

$$\hat{L}_{ik} = \begin{bmatrix} \theta_{i1} & \theta_{i2} & \theta_{i3} & \theta_{i4} & h_{ji}\theta_{i4} \\ * & \theta_{i5} & 0 & 0 & 0 \\ * & * & \theta_{i6} & 0 & 0 \\ * & * & * & \theta_{i7} & 0 \\ * & * & * & * & \theta_{i8} \end{bmatrix} < 0$$

Hence, we have

$$\mathcal{A}V(x, r_t) \le -\alpha \sum_{i=1}^{n} \|\bar{X}_i\|^2 \le -\alpha \|x_t\|^2$$

where $\alpha = \min_{i,r}\{\lambda_{min}(-\hat{L}_{ir1})\} > 0$. By applying the Dynkin's formula, one can get

$$\mathbb{E}[V(x(t), r(t))] - \mathbb{E}[V(x_0, r_0)] = \mathbb{E}\int_0^t [\mathcal{A}V(x, r_t)]ds \le -\alpha \mathbb{E}\int_0^t x^T(s)x(s)ds$$

Since $\mathbb{E}[V(x(t), r(t))] \ge 0$, the above equation implies that

$$\mathbb{E}\int_0^t x^T(s)x(s)ds \le \alpha^{-1}\mathbb{E}[V(x_0, r_0)]$$

This proves the stochastic stability of the unforced system (5.1). On the other hand, the condition (5.12) should also be expressed based on the new LMI parameters $\bar{Q}_i$ and $\bar{R}_{il}$. Substituting Q_i and R_{il} by $\bar{Q}_i$ and $\bar{R}_{il}$ in (5.12) yields

$$\bar{Q}_i^{-1} \ge \sum_{l=1}^{M} \pi_{kl}\bar{R}_{il}^{-1} \tag{5.15}$$

Using the fact that $\pi_{kk} = -\sum_{l=1, l\neq k}^{N} \pi_{kl}$, where $\pi_{kl} > 0$ and applying the Schur complement lead to the following matrix inequality

$$\begin{bmatrix} \bar{Q}_i^{-1} - \pi_{kk}\bar{R}_{ik}^{-1} & \bar{\pi}_k \\ * & \bar{R}_i \end{bmatrix} > 0 \tag{5.16}$$

Furthermore, using Lemma 2.2 and considering the fact that $\pi_{kk}\bar{R}_{ik} < 0$, lead to $\bar{Q}_i^{-1} - \pi_{kk}\bar{R}_{ik}^{-1} > 2(1-\pi_{kk})I - \bar{Q}_i + \pi_{kk}\bar{R}_{ik}$. Therefore, to guarantee (5.16), it suffices to satisfy the LMI condition (5.5). This completes the proof of the theorem. ■

In system (5.1), it was assumed that the interconnections are fixed and do not change at each switching mode. In the next section, the proposed control scheme is modified for mode-dependent interconnections.

5.1.2 $\mathcal{H}_\infty$ Control Scheme for MJLSs with Mode-Dependent Interconnections

In certain large scale systems such as mobile networks, interconnected terms may vary at each switching mode. In other words, the set of interconnections are changing at each mode, i.e., $\wp_{r_t}(.)$. This class of systems can be described by the following differential equations:

$$\begin{aligned} \dot{x}_i(t) &= A_{ir}x_i(t) + B_{ir}u_i(t) + B_{w_{ir}}w_i(t) \\ &+ \sum_{j\in\wp_{r(t)}(i)} B_{dijr}u_j(t-\tau_{ji}(t)) \\ x_i(t) &= \phi_i(t), \quad t\in[-h_i,0] \\ z_i(t) &= C_{ir}x_i(t) \end{aligned} \tag{5.17}$$

where $x_i(t)$, $A_i(r(t))$, $B_i(r(t))$, $B_{w_i}(r(t))$, $B_{dij}(r(t))$, $u_i(t)$ and $\tau_{ji}(t)$ are defined in (5.1), $\wp_{r(t)}(i)$ is the interconnection set of node i in switching mode $r(t)$. The following example describes the dynamics of a mobile network in which the interconnected terms change at each switching mode.

Example 5.1. Consider a mobile network with four nodes and two switching modes. During the network operation, in one mode nodes 2 and 3 are in the neighboring set of node 1, and in the other mode nodes 3 and 4 belong to the neighboring set of node 1. Therefore, $\wp_1(1) = \{2,3\}$ and $\wp_2(1) = \{3,4\}$ and the dynamics of node

1 in the first and the second modes are defined as follows, respectively.

$$\begin{aligned}\dot{x}_1(t) &= A_{11}x_1(t) + B_{11}u_1(t) + B_{w_{11}}w_1(t) \\ &+ B_{d121}u_2(t-\tau_{21}(t)) + B_{d131}u_3(t-\tau_{31}(t)) \\ z_1(t) &= C_{11}x_1(t)\end{aligned}$$

and

$$\begin{aligned}\dot{x}_1(t) &= A_{12}x_1(t) + B_{12}u_1(t) + B_{w_{12}}w_1(t) \\ &+ B_{d132}u_3(t-\tau_{31}(t)) + B_{d142}u_4(t-\tau_{41}(t)) \\ z_1(t) &= C_{12}x_1(t)\end{aligned}$$

The following corollary provides a stabilizing $\mathcal{H}_\infty$ control for system (5.17).

Corollary 5.1.1. *The Markovian jump system (5.17) subject to Assumption 5.1 is stochastically stabilizable with γ-disturbance attenuation by using the decentralized state feedback controllers $u_i = K_{ik}x_i$, if there exist matrices M_{ik} and symmetric positive definite matrices Y_{ik}, $\bar{R}_{ik}$, $\bar{Q}_i$ for $i = 1, ..., n,\ k = 1, ..., M$ such that the following LMI conditions hold*

$$\begin{bmatrix} \theta_{i1} & \theta_{i2} & B_{w_{ik}} & Y_{ik}^T C_{ik}^T & \theta_{i3} & m_{ik}M_{ik}^T & h_{ji}M_{ik}^T \\ * & \theta_{i4} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & \theta_{i5} & 0 & 0 \\ * & * & * & * & * & \theta_{i6} & 0 \\ * & * & * & * & * & * & \theta_{i7} \end{bmatrix} < 0 \qquad (5.18)$$

$$\begin{bmatrix} 2(1 - m_{ik}\pi_{kk})I - \bar{Q}_i + m_{ik}\pi_{kk}\bar{R}_{ik} & \tilde{\pi}_k \\ * & \tilde{R}_{ik} \end{bmatrix} \geq 0 \qquad (5.19)$$

where

$$\begin{aligned}
\theta_{i1} &= Y_{ik}^T A_{ik}^T + A_{ik} Y_{ik} + M_{ik}^T B_{ik}^T + B_{ik} M_{ik} + \pi_{kk} Y_{ik}, \\
\theta_{i2} &= \tilde{B}_{dik} \tilde{R}_{ik}, \\
\theta_{i3} &= (\bar{\pi}_k Y_{ik})^T, \\
\theta_{i4} &= -(1 - d_{ji}) \tilde{R}_{ik}, \\
\theta_{i5} &= -\operatorname{diag}\{Y_{i1}, ..., Y_{i(k-1)}, Y_{i(k+1)}, ..., Y_{iM}\}, \\
\theta_{i6} &= -m_{ik} \bar{R}_{ik}, \\
\theta_{i7} &= -h_{ji} \bar{Q}_i, \\
\bar{\pi}_k &= [\sqrt{\pi_{k1}} \ ... \ \sqrt{\pi_{k(k-1)}} \ \sqrt{\pi_{k(k+1)}} \ ... \ \sqrt{\pi_{kM}}]^T \\
\tilde{\pi}_k &= [\sqrt{m_{i1}\pi_{k1}} \ ... \ \sqrt{m_{i(k-1)}\pi_{k(k-1)}} \ \sqrt{m_{i(k+1)}\pi_{i(k+1)}} \ ... \ \sqrt{m_{iM}\pi_{kM}}]^T
\end{aligned}$$

and m_{ik} = *number of subsystems that subsystem* i *belongs to their* $\wp_k(.)$ *in mode* k, $\tilde{R}_{ik} = \operatorname{diag}_{k \in S}\{\bar{R}_{jk}\}$, *and* $\tilde{B}_{dik} = vec\{B_{dijk}\}$, *for* $j \in \wp_k(i)$. *Moreover, the robust decentralized state feedback controller gain is given by* $K_{ik} = M_{ik} Y_{ik}^{-1}$.

Proof: To study the stability of this system, let us consider the following Lyapunov-Krasovskii functional candidate

$$
\begin{aligned}
V(x_t, r_t) &= V_1 + V_2 + V_3 \\
V_1 &= \sum_{i=1}^{n} x_i^T(t) P_{ir_t} x_i(t) \\
V_2 &= \sum_{i=1}^{n} \sum_{j \in \wp_{r_t}(i)} \int_{t-\tau_{ji}}^{t} u_j^T(s) R_{jr_t} u_j(s) ds \\
V_3 &= \sum_{i=1}^{n} \int_{0}^{h_{ij}} (h_{ij} - \sigma) u_i^T(t-\sigma) Q_i u_i(t-\sigma) d\sigma \qquad (5.20)
\end{aligned}
$$

Therefore,

$$
\mathcal{A}V(x_t, r_t) = \mathcal{A}V_1(x_t, r_t) + \mathcal{A}V_2(x_t, r_t) + \mathcal{A}V_3(x_t, r_t) \qquad (5.21)
$$

where $\mathcal{A}V_1(x_t, r_t)$ is obtained in (5.9). Following the definition of the weak infinitesimal generator $\mathcal{A}$ in Appendix B, $\mathcal{A}V_2(x_t, r_t)$ and $\mathcal{A}V_3(x_t, r_t)$ are obtained below:

$$
\begin{aligned}
\mathcal{A}V_2(x_t, r_t) &= \lim_{\Delta \to 0_+} \frac{1}{\Delta} \sum_{i=1}^{n} \{ \sum_{l \neq k} \mathbb{P}[r_{t+\Delta} = l | r_t = k] \sum_{j \in \wp_l(i)} \int_{t+\Delta-\tau_{ji}}^{t+\Delta} u_j^T(s) R_{jl} u_j(s) ds \\
&+ \mathbb{P}[r_{t+\Delta} = k | r_t = k] \sum_{j \in \wp_k(i)} \int_{t+\Delta-\tau_{ji}}^{t+\Delta} u_j^T(s) R_{jk} u_j(s) ds \\
&- \sum_{j \in \wp_{r_t}(i)} \int_{t-\tau_{ji}}^{t} u_j^T(s) R_{jr_t} u_j(s) ds \} \\
&= \lim_{\Delta \to 0_+} \frac{1}{\Delta} \sum_{i=1}^{n} \{ \sum_{l \neq k} (\pi_{kl}\Delta + o(\Delta)) \sum_{j \in \wp_l(i)} \int_{t+\Delta-\tau_{ji}}^{t+\Delta} u_j^T(s) R_{jl} u_j(s) ds \\
&+ (1 + \pi_{kk}\Delta + o(\Delta)) \sum_{j \in \wp_k(i)} \int_{t+\Delta-\tau_{ji}}^{t+\Delta} u_j^T(s) R_{jl} u_j(s) ds
\end{aligned}
$$

$$
\begin{aligned}
& - \sum_{j\in\wp_{r_t}(i)} \int_{t+\Delta-\tau_{ji}}^{t+\Delta} u_j^T(s)R_{jr_t}u_j(s)ds\} \\
&= \sum_{i=1}^{n} \sum_{j\in\wp_{r_t}(i)} [u_j^T(t)R_{jr_t}u_j(t) - (1-\tau_{ji})u_j^T(t-\tau_{ji})R_{jr_t}u_j(t-\tau_{ji})] \\
&+ \sum_{i=1}^{n}\sum_{k=1}^{M} \sum_{j\in\wp_k(i)} \pi_{r_tk} \int_{t-\tau_{ji}(t)}^{t} u_j^T(s)R_{jk}u_j(s)ds \\
&\leq \sum_{i=1}^{n} \sum_{j\in\wp_{r_t}(i)} [u_j^T(t)R_{jr_t}u_j(t) - (1-d_{ji})u_j^T(t-\tau_{ji})R_{jr_t}u_j(t-\tau_{ji})] \\
&+ \sum_{i=1}^{n}\sum_{k=1}^{M} \sum_{j\in\wp_k(i)} \pi_{r_tk} \int_{t-\tau_{ji}(t)}^{t} u_j^T(s)R_{jk}u_j(s)ds \qquad (5.22)
\end{aligned}
$$

$$
\mathcal{A}V_3(x_t, r_t) \;\leq\; \sum_{i=1}^{n}[h_{ij}u_i^T(t)Q_iu_i(t) - \int_{t-\tau_{ij}(t)}^{t} u_i^T(s)Q_iu_i(s)ds] \qquad (5.23)
$$

Now assuming

$$
\sum_{i=1}^{n} \int_{t-\tau_{ij}(t)}^{t} u_i^T(s)Q_iu_i(s)ds \geq \sum_{i=1}^{n}\sum_{k=1}^{M} \pi_{r_tk} \sum_{j\in\wp_k(i)} \int_{t-\tau_{ji}(t)}^{T} u_j^T(s)R_{ik}u_j(s)ds \qquad (5.24)
$$

and following the similar lines as that given in Theorem 5.1, one can obtain condition (5.18) which guarantees the negative definiteness of J_1 in (5.7) and consequently J in (5.3). It can also be shown that by eliminating the $(n+2)$nd and $(n+3)$rd rows and columns of (5.18), the reduced LMI conditions can guarantee the stochastic stability of the unforced system. Consequently, the stability of the system dynamics (5.1) is guaranteed.

Towards this end, noting that

$$
\sum_{i=1}^{n} \int_{t-\tau_{ij}(t)}^{t} u_i^T(s) \sum_{k=1}^{M} \pi_{r_tk}m_{ik}R_{ik}u_i(s)ds = \sum_{i=1}^{n}\sum_{k=1}^{M} \pi_{r_tk} \sum_{j\in\wp_k(i)} \int_{t-\tau_{ji}(t)}^{t} u_j^T(s)R_{jk}u_j(s)ds
$$

then, by substituting Q_i and R_{ik} by $\bar{Q}_i$ and $\bar{R}_{ik}$ in (5.24); applying the Schur complement and Lemma 2.2; and following the similar lines as that given in Theorem 5.1 yields (5.19). Indeed, the LMI condition (5.19) is sufficient to guarantee (5.24).

This completes the proof of Corollary. ■

In the next section, we tackle the more challenging dynamics for MJLSs, namely singular MJLSs.

5.2 Singular Markovian Jump Linear Systems with Time-Delay

Singular systems arise in many practical systems such as electrical circuits, power systems, network, etc. that can not be described by regular dynamics. Singular systems, indeed, include a combination of differential equations and algebraic equations. In this chapter a class of singular linear systems with delay is considered which the dynamics abruptly changes based on Markovian process. For a decentralized singular MJLSs with time-varying delay system including n subsystems, the dynamics of subsystem i can be expressed as:

$$\begin{aligned} E(r(t))\dot{x}_i(t) &= A_i(r(t))x_i(t) + B_i(r(t))u_i(t) + B_{w_i}(r(t))w_i(t) \\ &+ \sum_{j \in \wp(i)} B_{dij}(r(t))u_j(t - \tau_{ji}) \\ x_i(t) &= \phi_i(t), \quad t \in [-h_i, 0] \\ z_i(t) &= C_i(r(t))x_i(t) \end{aligned} \tag{5.25}$$

where $x_i(t)$, $u_i(t)$, $A_i(r(t))$, $B_i(r(t))$, $B_{w_i}(r(t))$ and $B_{dij}(r(t))$ are defined in (5.1), $\tau_{ji} \leq h_{ji}$, and $E(r(t))$ is a diagonal singular matrix, i.e., $rank[E(r(t))] = n_1 \leq n$. Hence, for $n_1 < n$, $E(r(t))$ can be defined as follows

$$E(r(t)) = \begin{bmatrix} I_{n1} & 0 \\ 0 & 0 \end{bmatrix}$$

Remark 5.1. In case that $E(r(t))$ does not follow the specified structure, one can apply Jordan canonical form decomposition and define a transformation to achieve the desired structure [120], [121].

It should be noted that time response of singular systems even for the deterministic cases, and an arbitrary finite initial condition may exhibit impulses and therefore, non-causal behavior along with the derivatives of these impulses. In fact, singular systems contain three modes: finite dynamic, infinite dynamic and non-dynamic modes. The undesired impulsive behavior can be initiated by infinite dynamic modes. Moreover, due to the presence of infinite dynamic modes and non-dynamic modes, the existence and uniqueness of a solution to a given singular system is not always guaranteed. The regularity and impulse-free conditions ensure the existence and uniqueness of an impulse-free solution for the singular system. On the other hand, for singular MJLSs the regularity and impulse-free conditions should be investigated at each switching mode. Therefore, piecewise regularity and piecewise impulse-free conditions are considered for the singular MJLSs. The definition of piecewise regularity and piecewise impulse-free conditions are stated bellow.

Definition 5.4. [119], [120] The system $E(r(t))\dot{x}_i(t) = A_i(r(t))x_i(t)+B_i(r(t))u_i(t)+B_{w_i}(r(t))w_i(t)$ is said to be *piecewise regular* if the characteristic polynomial $det(sE_r - A_{ir})$ is not identically zero for $r = 1, ..., M, i = 1, ..., N$.

Definition 5.5. [119], [127] The system $E(r(t))\dot{x}_i(t) = A_i(r(t))x_i(t)+B_i(r(t))u_i(t)+B_{w_i}(r(t))w_i(t)$ is said to be *piecewise impulse-free*, if $deg(det(sE_r - A_{ir})) = rank(E_r)$ for $r = 1, ..., M, i = 1, ..., N$.

The following lemma provides a necessary and sufficient condition to satisfy regularity and piecewise impulsive-free conditions.

Lemma 5.1. [119] For stabilizing the system (5.25) by state feedback $u_i = K_{ir}x_i$ the piecewise regularity and piecewise impulse-free condition is guaranteed if $A_{cli} = A_{ir} + B_{ir}K_{ir}$ and $A_{cli} + A_{dcli}$ are nonsingular, where $A_{dcli} = \sum_{j\in\wp(i)} B_{dij}(r(t))K_{jr}$.

In next section, an $\mathcal{H}_\infty$ control is proposed to guarantee the stability as well as piecewise regularity and impulse-free behavior of the singular MJLSs (5.25).

5.2.1 An $\mathcal{H}_\infty$ Control Scheme for Singular MJLSs with Delay

This section deals with designing stabilizing controllers for singular MJLSs. The result of this section will be applied for routing of mobile networks with variable destination nodes in the next chapter. The following theorem provides sufficient condition to guarantee that the system (5.25) with state feedback $u_i = K_{ik}x_i$ is piecewise regular, impulse-free and stochastically stable when $w_i = 0$.

Theorem 5.2. *Under Assumption 5.1, the unforced singular system (5.25) is stochastically stabilizable, piecewise regular, and piecewise impulse-free by decentralized state feedback controllers $u_i = K_{ik}x_i$, if there exist matrices M_{ik}, nonsingular matrices Y_{ik} and symmetric positive definite matrices $\bar{R}_{ik}$, $\bar{Q}_i$ for $i = 1, ..., n$, $k = 1, ..., M$, such that the following LMI conditions are satisfied*

$$
\begin{aligned}
E_k Y_{ik}^T &= Y_{ik} E_k^T > 0 \\
L_{ik} &= \begin{bmatrix} \theta_{i1} & \theta_{i2} & \theta_{i3} & \theta_{i4} & h_{ji}\theta_{i4} \\ * & \theta_{i5} & 0 & 0 & 0 \\ * & * & \theta_{i6} & 0 & 0 \\ * & * & * & \theta_{i7} & 0 \\ * & * & * & * & \theta_{i8} \end{bmatrix} < 0 \qquad (5.26)
\end{aligned}
$$

$$
\begin{bmatrix} Y_{ik}^T A_{ik}^T + A_{ik}Y_{ik} + M_{ik}^T B_{ik}^T + B_{ik}M_{ik} & \tilde{B}_{dik}\tilde{M}_{jk} & Y_{ik} \\ * & -2Y_{jk} + I & 0 \\ * & * & -I \end{bmatrix} < 0 \qquad (5.27)
$$

$$
\begin{bmatrix} 2(1-\pi_{kk})I - \bar{Q}_i + \pi_{kk}\bar{R}_{ik} & \bar{\pi}_k \\ * & \tilde{R}_{ik} \end{bmatrix} \geq 0 \qquad (5.28)
$$

where

$$
\begin{aligned}
\theta_{i1} &= Y_{ik}^T A_{ik}^T + A_{ik} Y_{ik} + M_{ik}^T B_{ik}^T + B_{ik} M_{ik} + \pi_{kk} E_k Y_{ik}, \\
\theta_{i2} &= \tilde{B}_{dik} \tilde{R}_{ik}, \\
\theta_{i3} &= (\bar{\pi}_k Y_{ik})^T, \\
\theta_{i4} &= m_i M_{ik}, \\
\theta_{i5} &= -(1 - d_{ji}) \tilde{R}_{ik}, \\
\theta_{i6} &= -\operatorname{diag}\{Y_{i1}, ..., Y_{i(k-1)}, Y_{i(k+1)}, ..., Y_{iM}\}, \\
\theta_{i7} &= -m_i \bar{R}_{ik}, \\
\theta_{i8} &= -m_i h_{ji} \tilde{Q}_i, \\
\bar{\pi}_k &= [\sqrt{\pi_{k1}} E_1 \ ... \ \sqrt{\pi_{k(k-1)}} E_{(k-1)} \ \sqrt{\pi_{k(k+1)}} E_{(k+1)} \ ... \ \sqrt{\pi_{kM}} E_M]^T
\end{aligned}
$$

and $m_i =$ *number of subsystems that subsystem* i *belongs to their* $\wp(.)$, $\tilde{R}_{ik} = \operatorname{diag}_{j\in\wp(i)}\{\bar{R}_{jk}\}$, $\tilde{M}_{jk} = \operatorname{diag} j \in \wp(i)\{M_{jk}\}$, *and* $\tilde{B}_{dik} = vec\{B_{dijk}\}$, *for* $j \in \wp(i)$. *Moreover, the robust decentralized state feedback controller gain is given by* $K_{ik} = M_{ik} Y_{ik}^{-1}$.

Proof: To design a stabilizing controller for the system dynamics (5.25), let us consider the following Lyapunov-Krasovskii functional:

$$
\begin{aligned}
V(x_t, r_t) &= V_1 + V_2 + V_3 \\
V_1 &= \sum_{i=1}^{n} x_i^T(t) E_{r_t} P_{ir_t} x_i(t) \\
V_2 &= \sum_{i=1}^{n} \sum_{j\in\wp(i)} \int_{t-\tau_{ji}}^{t} u_j^T(s) R_{jr_t} u_j(s) ds \\
V_3 &= \sum_{i=1}^{n} \sum_{j\in\wp(i)} \int_{0}^{h_{ji}} (h_{ji} - \sigma) u_j^T(t-\sigma) Q_j u_j(t-\sigma) d\sigma
\end{aligned}
\tag{5.29}
$$

where R_{jr_t} and Q_j are positive definite matrices, and $E_k P_{ik}^T = P_{ik} E_k^T > 0$. A direct computation of the weak infinitesimal generator of $\{r_t, x_t\}$ over $V(x_t, r_t)$ leads to

$$\mathcal{A}V(x_t, r_t) = \mathcal{A}V_1(x_t, r_t) + \mathcal{A}V_2(x_t, r_t) + \mathcal{A}V_3(x_t, r_t) \tag{5.30}$$

where $\mathcal{A}V_2(x_t, r_t)$ and $\mathcal{A}V_3(x_t, r_t)$ are obtained in (5.10) and (5.11) respectively, and $\mathcal{A}V_1(x_t, r_t)$ is given bellow:

$$\begin{aligned}
\mathcal{A}V_1(x_t, r_t) &= \lim_{\Delta \to 0_+} \frac{1}{\Delta}\{\mathbb{E}[V_1(x_{t+\Delta}, r_{t+\Delta}, t+\Delta)|x_t, r_t = k] - V_1(x_t, k, t)\} \qquad (5.31)\\
&= \lim_{\Delta \to 0_+} \frac{1}{\Delta}\sum_{i=1}^{n}\{\sum_{l \neq k}\mathbb{P}[r_{t+\Delta} = l|r_t = k]x_i^T(t+\Delta)E_l P_{il} x_i(t+\Delta)\\
&+ \mathbb{P}[r_{t+\Delta} = k|r_t = k]x_i^T(t+\Delta)E_k P_{ik} x_i(t+\Delta)\\
&- x_i^T(t)E_{r_t}P_{ir_t}x_i(t)\}\\
&= \lim_{\Delta \to 0_+} \frac{1}{\Delta}\sum_{i=1}^{n}\{\sum_{l \neq k}(\pi_{kl}\Delta + o(\Delta))x_i^T(t+\Delta)E_{r_{(t+\Delta)}}P_{ir_{t+\Delta}}x_i(t+\Delta)\\
&+ (1+\pi_{kk}\Delta + o(\Delta))x_i^T(t+\Delta)E_{r_{(t+\Delta)}}P_{ir_{t+\Delta}}x_i(t+\Delta)\\
&- x_i^T(t)E_{r_t}P_{ir_t}x_i(t)\}\\
&= \sum_{i=1}^{n}[\dot{x}_i^T(t)E_{r_t}P_{ir_t}x_i(t) + x_i^T(t)E_{r_t}P_{ir_t}\dot{x}_i(t) + x_i^T(t)\sum_{k=1}^{M}\pi_{r_t k}E_k P_{ik}x_i(t)]\\
&= \sum_{i=1}^{n}[x_i^T(t)((A_{ir_t} + B_{ir_t}K_{ir_t})^T P_{ir_t} + P_{ir_t}(A_{ir_t} + B_{ir_t}K_{ir_t}))x_i(t)\\
&+ \sum_{j \in \wp(i)}[u_j^T(t-\tau)B_{dijr_t}^T P_{ir_t}x_i(t) + x_i^T(t)P_{ir_t}B_{dijr_t}u_j(t-\tau)]\\
&+ x_i^T(t)\sum_{k=1}^{M}\pi_{r_t k}E_k P_{ik}x_i(t)]
\end{aligned}$$

By substituting (5.31), (5.10) and (5.11) into (5.30), and assuming

$$Q_i \geq \sum_{k=1}^{M}\pi_{r_t k}R_{ik} \tag{5.32}$$

and applying the Schur complements, one can get

$$\mathcal{A}V(x_t, r_t) \leq \sum_{i=1}^{n}\bar{X}_i^T(t)\bar{L}_{ir_t}\bar{X}_i(t)$$

where

$$\begin{aligned}
\bar{X}_i &= [x_i^T(t) \quad U_j^T(t-\tau_i(t))]^T, \\
U_j^T(t-\tau_i(t)) &:= vec\{u_j^T(t-\tau_{ji}(t))\} \quad for \quad j \in \wp(i) \\
\bar{L}_{ik} &= \begin{bmatrix} \Omega_{i1} & \Omega_{i2} & \Omega_{i3} \\ * & \Omega_{i4} & 0 \\ * & * & \Omega_{i5} \end{bmatrix}
\end{aligned} \tag{5.33}$$

$$\begin{aligned}
\Omega_{i1} &= (A_{ik}+B_{ik}K_{ik})^T P_{ik} + P_{ik}(A_{ik}+B_{ik}K_{ik}) + m_i K_{ik}^T[h_{ji}Q_i + R_{ik}]K_{ik} \\
&+ \pi_{kk}E_k P_{ik}, \\
\Omega_{i2} &= P_{ik}\tilde{B}_{dik}, \\
\Omega_{i3} &= [\sqrt{\pi_{k1}}E_1 P_{i1} \ \dots \ \sqrt{\pi_{k(k-1)}}E_{(k-1)}P_{i(k-1)} \\
&\quad \sqrt{\pi_{k(k+1)}}E_{(k+1)}P_{i(k+1)} \ \dots \ \sqrt{\pi_{kM}}E_M P_{iM}], \\
\Omega_{i4} &= -\text{diag}_{j\in\wp(i)}\{(1-d_{ji})R_{jk}\}, \\
\Omega_{i5} &= -\text{diag}\{P_{i1}, \dots, P_{i(k-1)}, P_{i(k+1)}, \dots, P_{iM}\},
\end{aligned}$$

Hence, to get $\mathcal{A}V(x_t, r_t) < 0$ it suffices to show that $\bar{L}_{ik} < 0$. Therefore, it can be concluded that

$$\mathcal{A}V(x, r_t) \le -\alpha \sum_{i=1}^{n} \|\bar{X}_i\|^2 \le -\alpha\|x_t\|^2$$

where $\alpha = \min_{i,r}\{\lambda_{min}(-\bar{L}_{ir1})\} > 0$. Now, by applying the Dynkin's formula, see Appendix B, one can get

$$\mathbb{E}[V(x(t), r(t))] - \mathbb{E}[V(x_0, r_0)] = \mathbb{E}\int_0^t [\mathcal{A}V(x, r_t)]ds \le -\alpha\mathbb{E}\int_0^t x^T(s)x(s)ds$$

Since $\mathbb{E}[V(x(t), r(t))] \ge 0$, the above equation leads to

$$\mathbb{E}\int_0^t x^T(s)x(s)ds \le \alpha^{-1}\mathbb{E}[V(x_0, r_0)]$$

Therefore, $\bar{L}_{ir1} < 0$ guarantees stochastic stability of (5.25). However, $\bar{L}_{ir1}$ is a nonlinear matrix inequality due to the presence of terms such as "$P_{ik}B_{ik}K_{ik}$". Now, following the similar line as that given in the proof of Theorem 5.1, one can define $K_{ik} = M_{ik}Y_{ik}^{-1}$, $P_{ik} = Y_{ik}^{-1}$, $R_{ik} = \bar{R}_{ik}^{-1}$, $Q_i = \bar{Q}_i^{-1}$, $\bar{\bar{R}}_{ik} = vec\{\bar{R}_{jk}\}$ *for* $j \in \wp(i)$, $\bar{Y}_i = vec\{Y_{il}\}$ *for* $l = 1, ..., (p-1), (p+1), ..., M$, and for $i = 1, ..., n$. Then, pre and post multiplying $\bar{L}_{ir}$ by $\Delta_{ik} = \text{diag}\{Y_{ik},\ \bar{\bar{R}}_{ik},\ \bar{Y}_i\}$ and Δ_{ik}^T, respectively leads to L_{ir}. Therefore, negative definiteness of L_{ir}, (5.26), yields $\bar{L}_{ir1} < 0$. The LMI condition (5.28) is also obtained by following the similar lines as indicated in the proof of Theorem 5.1. Moreover, consider the following condition is satisfied.

$$\begin{bmatrix} (A_{ik} + B_{ik}K_{ik})^T P_{ik} + P_{ik}(A_{ik} + B_{ik}K_{ik}) + I & P_{ik}\tilde{B}_{dik}\tilde{K}_{jk} \\ * & -I \end{bmatrix} < 0 \qquad (5.34)$$

where $\tilde{K}_{jk} = \text{diag}\{j \in \wp(i)\}\{K_{jk}\}$. The condition (5.34) implies that $(A_{ik} + B_{ik}K_{ik})^T P_{ik} + P_{ik}(A_{ik} + B_{ik}K_{ik}) < 0$. Therefore, A_{cli} is nonsingular. Furthermore, applying Schur complement yields $A_{cli} + A_{dcli}$ is nonsingular. Therefore, the closed-loop system is piecewise regular and piecewise impulsive mode free. Now by substituting $P_{ik} = Y_{ik}^{-1}$ and pre and post multiplying (5.34) by $\text{diag}\{Y_{ik}, \bar{\bar{Y}}_{ik}\}$ and its transpose, respectively, where $\bar{\bar{Y}}_{ik} = vec\{\bar{Y}_{jk}\}$ *for* $j \in \wp(i)$, one can obtain (5.27). This completes the proof of the theorem. ■

In fact, the gain K_{ik} obtained from Theorem 5.2 can stochastically stabilize the system dynamics (5.25). In the next theorem the results are extended to designing an $\mathcal{H}_\infty$ controller for system dynamics (5.25).

Theorem 5.3. *Given that Assumption 5.1 holds, the system (5.25) is stochastically stabilizable with γ-disturbance attenuation, piecewise regular, and piecewise impulse-free by decentralized state feedback controllers $u_i = K_{ir}x_i$ with an L_2-gain less than γ, if there exist matrices M_{ik}, nonsingular matrices Y_{ik} and symmetric positive definite matrices $\bar{R}_{ik}$, $\bar{Q}_i$ for $i = 1, ..., n,\ k = 1, ..., M$, such that the following LMI conditions* *are satisfied*

$$E_kY_{ik}^T = Y_{ik}E_k^T > 0$$

$$L_{ik} = \begin{bmatrix} \theta_{i1} & \theta_{i2} & B_{w_{ik}} & Y_{ik}^T C_{ik}^T & \theta_{i3} & \theta_{i4} & h_{ji}\theta_{i4} \\ * & \theta_{i5} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & \theta_{i6} & 0 & 0 \\ * & * & * & * & * & \theta_{i7} & 0 \\ * & * & * & * & * & * & \theta_{i8} \end{bmatrix} < 0 \tag{5.35}$$

$$\begin{bmatrix} Y_{ik}^T A_{ik}^T + A_{ik}Y_{ik} + M_{ik}^T B_{ik}^T + B_{ik}M_{ik} & \tilde{B}_{dik}\tilde{M}_{jk} & Y_{ik} \\ * & -2Y_{jk} + I & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{5.36}$$

$$\begin{bmatrix} 2(1-\pi_{kk})I - \bar{Q}_i + \pi_{kk}\bar{R}_{ik} & \bar{\pi}_k \\ * & \tilde{R}_{ik} \end{bmatrix} \geq 0 \tag{5.37}$$

where

$$\begin{aligned}
\theta_{i1} &= Y_{ik}^T A_{ik}^T + A_{ik}Y_{ik} + M_{ik}^T B_{ik}^T + B_{ik}M_{ik} + \pi_{kk}E_kY_{ik},\\
\theta_{i2} &= \tilde{B}_{dik}\tilde{R}_{ik},\\
\theta_{i3} &= (\bar{\pi}_k Y_{ik})^T,\\
\theta_{i4} &= m_i M_{ik},\\
\theta_{i5} &= -(1-d_{ji})\tilde{R}_{ik},\\
\theta_{i6} &= -\operatorname{diag}\{Y_{i1}, ..., Y_{i(k-1)}, Y_{i(k+1)}, ..., Y_{iM}\},\\
\theta_{i7} &= -m_i\bar{R}_{ik},\\
\theta_{i8} &= -m_i h_{ji}\bar{Q}_i,\\
\bar{\pi}_k &= [\sqrt{\pi_{k1}}E_1 \; ... \; \sqrt{\pi_{k(k-1)}}E_{(k-1)} \; \sqrt{\pi_{k(k+1)}}E_{(k+1)} \; ... \; \sqrt{\pi_{kM}}E_M]^T
\end{aligned}$$

and m_i = *number of subsystems that subsystem* i *belongs to their* $\wp(.)$, *and* $\tilde{R}_{ik} = \operatorname{diag}_{j\in\wp(i)}\{\bar{R}_{jk}\}$,$\tilde{M}_{jk} = \operatorname{diag} j \in \wp(i)\{M_{jk}\}$,*and* $\tilde{B}_{dik} = vec\{B_{dijk}\}$, *for* $j \in \wp(i)$. *Moreover, the robust decentralized state feedback controller gain is given by* $K_{ik} = M_{ik}Y_{ik}^{-1}$.

Proof: To achieve the $\mathcal{H}_\infty$ objective (5.3), it suffices to establish

$$J_1 = \mathcal{A}V(x_t, r_t) + z^T(t)z(t) - \gamma w^T(t)w(t) < 0 \tag{5.38}$$

where the $V(x_t, r_t)$ introduced in Theorem 5.2 is selected for this problem. By applying the $\mathcal{A}$ generator and following the manipulations that were given in Theorem 5.2 yields

$$J_1 \;\le\; \sum_{i=1}^{n} X_i^T(t)\bar{L}_{ir_t}X_i(t)$$

where

$$
\begin{aligned}
X_i &= [x_i^T(t) \;\; U_j^T(t-\tau_i(t)) \;\; w_i^T(t)]^T, \\
U_j^T(t-\tau_i(t)) &:= vec\{u_j^T(t-\tau_{ji}(t))\} \;\; for \;\; j \in \wp(i)
\end{aligned}
$$

$$
\begin{aligned}
\bar{L}_{ik} &= \begin{bmatrix} \Omega_{i1} & \Omega_{i2} & P_{ik}B_{w_{ik}} & C_{ik}^T & \Omega_{i3} \\ * & \Omega_{i4} & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & \Omega_{i5} \end{bmatrix} < 0 \qquad (5.39) \\
\Omega_{i1} &= (A_{ik}+B_{ik}K_{ik})^T P_{ik} + P_{ik}(A_{ik}+B_{ik}K_{ik}) + m_i K_{ik}[h_{ji}Q_i + R_{ik}]K_{ik} \\
&+ \pi_{kk}E_k P_{ik} \\
\Omega_{i2} &= P_{ik}\tilde{B}_{dik}, \\
\Omega_{i3} &= [\sqrt{\pi_{k1}}E_1 P_{i1} \; ... \; \sqrt{\pi_{k(k-1)}}E_{(k-1)}P_{i(k-1)} \\
&\quad \sqrt{\pi_{k(k+1)}}E_{(k+1)}P_{i(k+1)} \; ... \; \sqrt{\pi_{kM}}E_M P_{iM}]^T, \\
\Omega_{i4} &= -\mathrm{diag}_{j\in\wp(i)}\{(1-d_{ji})R_{jk}\}, \\
\Omega_{i5} &= -\mathrm{diag}\{P_{i1},...,P_{i(k-1)},P_{i(k+1)},...,P_{iM}\},
\end{aligned}
$$

Now let us define $K_{ik} = M_{ik}Y_{ik}^{-1}$, $P_{ik} = Y_{ik}^{-1}$, $R_{ik} = \bar{R}_{ik}^{-1}$, $Q_i = \bar{Q}_i^{-1}$, $\bar{\bar{R}}_{ik} = vec\{\bar{R}_{jk}\}$ *for* $j \in \wp(i)$, $\bar{Y}_i = vec\{Y_{il}\}$ *for* $l = 1,...,(l-1),\ (l+1),...,M$ and $i = 1,...,n$. By pre and post multiplying (5.39) by $\Delta_{ik} = \mathrm{diag}\{Y_{ik},\ \bar{\bar{R}}_{ik},\ I,\ I,\ \bar{Y}_i\}$ and Δ_{ik}^T, respectively, L_{ik} in (5.35) is obtained. By using the Dynkin's formula, one can get

$$
\begin{aligned}
J_T &= \mathbb{E}[\int_0^T [J_1 - \mathcal{A}V(x_t,r_t)dt] \\
&\leq \mathbb{E}[\int_0^T \sum_{i=1}^n \eta_i^T(t)L_{ir_t}\eta_i(t)dt] - \mathbb{E}[V(x_T,r_T)] + V(x_0,r_0) \qquad (5.40)
\end{aligned}
$$

Since $L_{ir_t} < 0$ and $\mathbb{E}[V(x_T, r_T)] > 0$, one obtains

$$J_T \le V(x_0, r_0)$$

and therefore

$$J_\infty = \|z\|_{\mathbb{E}_2} - \gamma\|w(t)\|_2 \le V(x_0, r_0)$$

This completes the proof of the theorem. ■

If the set of interconnections also varies at each switching mode, i.e., the system dynamics is expressed by the following equations:

$$\begin{aligned} E_r\dot{x}_i(t) &= A_{ir}x_i(t) + B_{ir}u_i(t) + B_{w_{ir}}w_i(t) \\ &+ \sum_{j\in\wp_{r_t}(i)} B_{dijr}u_j(t - \tau_{ji}(t)) \\ x_i(t) &= \phi_i(t), \quad t \in [-h_i, 0] \\ z_i(t) &= C_{ir}x_i(t) \end{aligned} \tag{5.41}$$

the stabilizing $\mathcal{H}_\infty$ control can be obtained by following the Corollary 5.3.1 results.

Corollary 5.3.1. *The Singular Markovian jump system (5.41) subject to Assumption 5.1 is stochastically stabilizable with γ-disturbance attenuation, piecewise regular, and piecewise impulse-free by applying the decentralized state feedback controllers $u_i = K_{ik}x_i$, if there exist matrices M_{ik}, nonsingular matrices Y_{ik} and symmetric positive definite matrices $\bar{R}_{ik}$, $\bar{Q}_i$ for $i = 1, ..., n,\ k = 1, ..., M$ such that the following LMI conditions hold*

$$E_k Y_{ik}^T = Y_{ik} E_k^T > 0$$

$$\begin{bmatrix} \theta_{i1} & \theta_{i2} & B_{w_{ik}} & Y_{ik}^T C_{ik}^T & \theta_{i3} & m_{ik} M_{ik}^T & h_{ji} M_{ik}^T \\ * & \theta_{i4} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & \theta_{i5} & 0 & 0 \\ * & * & * & * & * & \theta_{i6} & 0 \\ * & * & * & * & * & * & \theta_{i7} \end{bmatrix} < 0 \qquad (5.42)$$

$$\begin{bmatrix} Y_{ik}^T A_{ik}^T + A_{ik} Y_{ik} + M_{ik}^T B_{ik}^T + B_{ik} M_{ik} & \tilde{B}_{dik} \tilde{M}_{jk} & Y_{ik} \\ * & -2Y_{jk} + I & 0 \\ * & * & -I \end{bmatrix} < 0 \qquad (5.43)$$

$$\begin{bmatrix} 2(1 - m_{ik}\pi_{kk})I - \bar{Q}_i + m_{ik}\pi_{kk}\bar{R}_{ik} & \tilde{\pi}_k \\ * & \tilde{R}_{ik} \end{bmatrix} \geq 0 \qquad (5.44)$$

where

$$
\begin{aligned}
\theta_{i1} &= Y_{ik}^T A_{ik}^T + A_{ik}Y_{ik} + M_{ik}^T B_{ik}^T + B_{ik}M_{ik} + \pi_{kk}E_kY_{ik}, \\
\theta_{i2} &= \tilde{B}_{dik}\tilde{R}_{ik}, \\
\theta_{i3} &= (\bar{\pi}_k Y_{ik})^T, \\
\theta_{i4} &= -(1-d_{ji})\tilde{R}_{ik}, \\
\theta_{i5} &= -\operatorname{diag}\{Y_{i1}, ..., Y_{i(k-1)}, Y_{i(k+1)}, ..., Y_{iM}\}, \\
\theta_{i6} &= -m_{ik}\bar{R}_{ik}, \\
\theta_{i7} &= -h_{ji}\bar{Q}_i, \\
\bar{\pi}_k &= [\sqrt{\pi_{k1}}E_1 \; ... \; \sqrt{\pi_{k(k-1)}}E_{(k-1)} \; \sqrt{\pi_{k(k+1)}}E_{(k+1)} \; ... \; \sqrt{\pi_{kM}}E_M]^T \\
\tilde{\pi}_k &= [\sqrt{m_{i1}\pi_{k1}} \; ... \; \sqrt{m_{i(k-1)}\pi_{k(k-1)}} \; \sqrt{m_{i(k+1)}\pi_{i(k+1)}} \; ... \; \sqrt{m_{iM}\pi_{kM}}]^T
\end{aligned}
$$

and m_{ik} = *number of subsystems that subsystem i belongs to their* $\wp_k(.)$ *in mode* k, $\tilde{R}_{ik} = \operatorname{diag}_{k\in\mathcal{S}}\{\bar{R}_{jk}\}$, $\tilde{M}_{jk} = \operatorname{diag} j \in \wp(i)\{M_{jk}\}$, *and* $\tilde{B}_{dik} = vec\{B_{dijk}\}$, *for* $j \in \wp(i)$. *Moreover, the robust decentralized state feedback controller gain is given by* $K_{ik} = M_{ik}Y_{ik}^{-1}$.

Proof: The following Lyapunov-Krasovskii functional candidate is considered to study the stability of the system

$$
\begin{aligned}
V(x_t, r_t) &= V_1 + V_2 + V_3 \qquad (5.45) \\
V_1 &= \sum_{i=1}^{n} x_i^T(t)E_{r_t}P_{ir_t}x_i(t) \\
V_2 &= \sum_{i=1}^{n}\sum_{j\in\wp_{r_t}(i)} \int_{t-\tau_{ji}}^{t} u_j^T(s)R_{jr_t}u_j(s)ds \\
V_3 &= \sum_{i=1}^{n} \int_0^{h_{ij}} (h_{ij}-\sigma)u_i^T(t-\sigma)Q_iu_i(t-\sigma)d\sigma
\end{aligned}
$$

Therefore,

$$\mathcal{A}V(x_t, r_t) = \mathcal{A}V_1(x_t, r_t) + \mathcal{A}V_2(x_t, r_t) + \mathcal{A}V_3(x_t, r_t) \quad (5.46)$$

where $\mathcal{A}V_1(x_t, r_t)$, $\mathcal{A}V_2(x_t, r_t)$, and $\mathcal{A}V_3(x_t, r_t)$ are obtained in (5.31), (5.22) and (5.23), respectively. Following the constructive lines as invoked in Theorem 5.3, one can obtain the condition (5.42) which guarantees the negative definiteness of J_1 in (5.38) and consequently J in (5.3). The conditions (5.43) and (5.44) can also be obtained similar to the LMI conditions (5.36) and(5.19), respectively. This completes the proof of the corollary. ■

5.3 Conclusions

A decentralized $\mathcal{H}_\infty$ control scheme was proposed for MJLSs with time varying delay functions in control signals. It is worth noting that unlike many existing methods in the literature, the state feedback controllers proposed in this chapter are changing at each mode providing less conservative and better performance in the response of parameter variations. The results are extended to MJLSs with varying interconnected terms at each mode. A decentralized $\mathcal{H}_\infty$ control scheme is also introduced for the singular MJLSs with time-delay. It is assumed that the singular matrix E can change at each switching mode. It should be noted that all conditions for controller design achieved in this chapter are linear in parameter which can be easily solved by the available algorithms and softwares such as YALMIP [131], and Robust Control Toolbox of MATLAB [64].

Chapter 6

$\mathcal{H}_\infty$ Control Strategy for Routing Problems in Mobile Networks

In this chapter a Markovian jump switching strategy in proposed for routing problem in mobile NMAS. Changing the neighboring sets of nodes in mobile networks due to node's mobility, variations in the network topology, and left over energy resources can be described by a switching system. However, node mobility and therefore changing the network topology is not generally deterministic and involves random transitions. Since the network topology changes and the selection of new neighboring sets in the next transition step only depends on the existing neighboring sets, a Markovian jump process is a good candidate and a viable framework for describing and modeling the mobility behavior of networks. In this chapter, we propose to model the stochastically changing neighboring sets by a Markovian jump system and the $\mathcal{H}_\infty$ control introduced in Chapter 5 is applied for routing problem of mobile networks. In the proposed routing strategy, for each possible network topology a routing controller is designed and it is guaranteed that the $\mathcal{H}_\infty$ objective is minimized when the topology changes stochastically based on Markovian process.

On the other hand, in the queueing dynamics that are derived based on the fluid flow conservation principle in (2.29) each state of the subsystem (node) represents a queue corresponding to each destination. Another issue that should be considered in routing problem of either fixed or variable network topology is that the number of destination nodes may also vary. Hence, during the time that a node destination is not active, no external traffic is required to be routed to this destination, and the existing messages that are waiting in the corresponding queues should be routed as quickly as possible. Changes in the number of destination nodes can be represented as changes in the number of states. To address changing number of states, in this chapter it is proposed to express the system dynamics as a singular MJLS. Therefore, the $\mathcal{H}_\infty$ control scheme for singular MJLSs presented in Chapter 5 is applied for routing problem of networks with a variable number of destinations. Finally, our proposed Markovian jump model with time-varying delays will provide the appropriate queueing dynamics for the network routing traffic problem to address random mobility of the nodes, as well as varying the number of the destination nodes.

The rest of this chapter is organized as follows: In Section 6.1, the mobile network routing problem is described as a Markovian jump switching system and then the $\mathcal{H}_\infty$ controller introduced in Chapter 5 is modified properly to provide routing for mobile networks. Moreover, the LMI conditions provided in Chapter 4 to address the corresponding physical constraints for networks with fixed topology is modified for mobile networks. Routing problem for variable number of destinations is provided in Section 6.2. Finally, in Section 6.3 the performance and capabilities of our proposed robust routing control strategy is evaluated and compared with some popular routing algorithms such as AODV [49] and OLSR [48].

6.1 A Markovian Jump $\mathcal{H}_\infty$ Control Strategy for Routing Problems in Mobile Networks

Recall the dynamics of mobile networks as represented in (2.29):

$$\dot{x}_i(t) = B_i(r(t))u_i(t) + B_{w_i}(r(t))w_i(t) + \sum_{j\in\wp_{r(t)}(i)} B_{dij}(r(t))u_j(t-\tau_{ji}(t)) \quad (6.1)$$

where $r(t)$ is a function representing the rule for changing the neighboring sets. Let us consider $r(t)$ as a continuous-time Markov process taking values in a finite space $\mathcal{S} = \{1, ..., M\}$ which describes the switching between different modes, and whose evolution is governed by the following probability transitions:

$$\mathbb{P}[r(t+h) = k|r(t) = l] = \begin{Bmatrix} \pi_{kl}h + o(h) & k \neq l \\ 1 + \pi_{kk}h + o(h) & k = l \end{Bmatrix}$$

where $\pi_{kl} > 0$ is the transition rate from mode k to mode l, $\pi_{kk} = -\sum_{l=1,l\neq k}^{M} \pi_{kl}$, $o(h)$ is a function satisfying $\lim_{h\to 0} \frac{o(h)}{h} = 0$. As mentioned in Chapter 2, for mobile networks the physical constraints given in (2.25), (2.26), and(2.28) will remain the same, however, for capacity constraint the link capacity is also variable at each mode. Consequently, the typical set of constraints can be given as

$$u_i(t) \geq 0 \quad (6.2)$$

$$x_i(t) \geq 0 \quad (6.3)$$

$$G_{k_i}u_i(t) \leq c_{ki}(r(t)) \quad k_i = 1, ..., li, \;\; i = 1, ..., n \quad (6.4)$$

$$Q_{dji}x_i(t) \leq x_{max_{dji}} \quad (6.5)$$

Therefore, the $\mathcal{H}_\infty$ objective function defined in (2.24) that was considered to be minimized for the routing problem should be modified as $\mathcal{H}_\infty$ objective for the stochastic system as follows:

$$\min \gamma \qquad s.t. \qquad J(w) < 0 \tag{6.6}$$
$$J(w) \;=\; \mathbb{E}\{\int_0^{\infty} (z^T(s)z(s) - \gamma w^T(s)w(s))ds\}, \quad \gamma > 0$$

where $z_i(t) = C_{ir}x_i(t),\ z(t) = vec\{z_i(t)\},\ \ w(t) = vec\{w_i(t)\}$. In fact, at each node (subsystem) the routing problem is stated as finding an $\mathcal{H}_\infty$ state feedback control, $u_i = K_{ir}x_i$, such that it simultaneously guarantees the stochastic stability of the network traffic in the presence of time-varying delays and minimizes an objective function which is considered as the *worst-case queueing length* due to the external input flows.

Corollary 5.1.1 can provide such $\mathcal{H}_\infty$ control. Comparing the dynamical model (6.1) with (5.17), implies that by simply substituting $A_i = 0$, the LMI conditions (5.18) and (5.19) can be directly applied for mobile routing problem. Therefore, considering the mobile traffic network dynamics (6.5) and for $w_i \in L_2[0,\infty)$ the state feedback routing controllers $u_i = K_{ir}x_i$ with gain of $K_{ir} = M_{ir}Y_{ir}^{-1}$ can make the closed-loop system stochastically stable and $J(w) < 0$ will be guaranteed, if there exist matrices M_{ir}, and symmetric positive definite matrices $Y_{ir},\ \bar{R}_{ir},\ \bar{Q}_i$ for $i = 1,...,n,\ \ r = 1,...,M$ such that the following LMI conditions are satisfied

$$W_{ir1} = \begin{bmatrix} \theta_{i1} & \theta_{i2} & B_{wir} & Y_{ir}^T C_{ir}^T & \theta_{i3} & m_{ir}M_{ir}^T & h_{ji}M_{ir}^T \\ * & \theta_{i4} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & \theta_{i5} & 0 & 0 \\ * & * & * & * & * & \theta_{i6} & 0 \\ * & * & * & * & * & * & \theta_{i7} \end{bmatrix} < 0 \tag{6.7}$$

$$W_{ir2} = \begin{bmatrix} 2(1-m_{ir}\pi_{rr})I - \bar{Q}_i + m_{ir}\pi_{rr}\bar{R}_{ir} & \tilde{\pi}_r \\ * & \tilde{R}_{ir} \end{bmatrix} \geq 0 \tag{6.8}$$

where

$$\begin{aligned}
\theta_{i1} &= M_{ir}^T B_{ir}^T + B_{ir} M_{ir} + \pi_{rr} Y_{ir}, \\
\theta_{i2} &= \tilde{B}_{dir} \tilde{R}_{ir}, \\
\theta_{i3} &= (\bar{\pi}_r Y_{ir})^T, \\
\theta_{i4} &= -(1-d_{ji})\tilde{R}_{ir}, \\
\theta_{i5} &= -\operatorname{diag}\{Y_{i1}, ..., Y_{i(r-1)}, Y_{i(r+1)}, ..., Y_{iM}\}, \\
\theta_{i6} &= -m_{ir}\bar{R}_{ir}, \\
\theta_{i7} &= -h_{ji}\bar{Q}_i, \\
\bar{\pi}_k &= [\sqrt{\pi_{r1}} \; ... \; \sqrt{\pi_{r(r-1)}} \; \sqrt{\pi_{r(r+1)}} \; ... \; \sqrt{\pi_{rM}}]^T \\
\tilde{\pi}_r &= [\sqrt{m_{i1}\pi_{r1}} \; ... \; \sqrt{m_{i(r-1)}\pi_{r(r-1)}} \; \sqrt{m_{i(r+1)}\pi_{i(r+1)}} \; ... \; \sqrt{m_{iM}\pi_{rM}}]^T
\end{aligned}$$

and m_{ir} = number of subsystems that subsystem i belongs to their $\wp_r(.)$ in mode r, $\tilde{R}_{ir} = \operatorname{diag}_{j\in\wp_r(i)}\{\bar{R}_{jr}\}$, $\tilde{M}_{jr} = \operatorname{diag} j \in \wp_r(i)\{M_{jr}\}$, and $\tilde{B}_{dir} = vec\{B_{dijr}\}$, for $j \in \wp_r(i)$.

6.1.1 LMI Conditions for Incorporating the Physical Constraints

In this section, the associated LMI conditions for the physical constraints given for the fixed network topology in Section 4.2.1 is modified for the variable network topology.

Capacity Constraint

The capacity constraint for each subsystem is defined as

$$G_{k_i} u_i \;\leq\; c_{k_i}(r(t)) \quad k_i = 1, ..., li, \;\; i = 1, ..., n$$

Now, consider the following ellipsoid for a selected $\varrho_i > 0$

$$\Sigma_i = \{x_i(t) | x_i^T(t) Y_{ir}^{-1} x_i(t) \leq \varrho_{ir},\ Y_{ir} = Y_{ir}^T > 0\} \tag{6.9}$$

If the stability condition (6.7) is satisfied, then from the definition of $V(x_t, r_t)$ in (5.20), it follows that

$$x_i^T(t) Y_{ir}^{-1} x_i(t) \leq V(x_t, r_t)$$

On the other hand, by integrating $J_1 < 0$ defined in (5.7) from 0 to t one gets

$$\mathbb{E}[V(x_t, r_t)] \leq \mathbb{E}[-\int_0^t z^T(t) z(t) dt + \int_0^t \gamma w^T(t) w(t) dt] + V(x_0, r_0) \leq \gamma L_1 + L_2$$

where $L_1 = \int_0^\infty w_i^T(t) w_i(t) dt$ is an upper bound on the energy of the external input $w_i(t)$, and $L_2 = V(x_0, r_0)$. Therefore, $x_i(t)$ belongs to an invariant set Σ_i, if $\gamma L_1 + L_2 \leq \varrho_{ir}$.

Furthermore, by applying Corollary 5.1.1, the state feedback controller u_i is obtained from $u_i = M_{ir} Y_{ir}^{-1} x_i$. Therefore, (6.4) can be described as

$$G_{kir} M_{ir} Y_{ir}^{-1} x_i \leq c_{kir} \tag{6.10}$$

Now, squaring (6.10) yields

$$x_i^T(t) (G_{kir} M_{ir} Y_{ir}^{-1})^T G_{kir} M_{ir} Y_{ir}^{-1} x_i(t) \leq c_{kir}^2 \tag{6.11}$$

Furthermore, $x_i^T(t) Y_{ir}^{-1} x_i(t) \leq \varrho_{ir}$ implies that to satisfy (6.11) it suffices to show

$$(G_{kir} M_{ir} Y_{ir}^{-1})^T (\varrho_{ir} / c_{kir}^2) G_{kir} M_{ir} Y_{ir}^{-1} \leq Y_{ir}^{-1} \tag{6.12}$$

Hence, by applying the Schur complement to (6.12), the capacity constraints for mobile networks can be expressed as LMI conditions

$$W_{c1ir} \triangleq \gamma \leq \max_{i,r}\{(\varrho_{ir} - L_2)/L_1\}, \quad r = 1, ..., M \tag{6.13}$$

$$W_{c2irk_i} \triangleq \begin{bmatrix} Y_{ir} & M_{ir}^T G_{kir}^T \\ G_{kir} M_{ir} & c_{kir}^2/\varrho_{ir} \end{bmatrix} \geq 0, k_i = 1, ..., li, i = 1, ..., n \tag{6.14}$$

Upper Bound on the Buffer Size

The constraint on the queue buffer size for each subsystem can be described as follows

$$Q_{di} x_i \leq x_{max_{di}}, \; d = 1, ..., \bar{d}, \; i = 1, ..., n \tag{6.15}$$

Following along similar lines as stated in Subsection 4.2.1, and considering the ellipsoid (6.9), inequality (6.15) can be satisfied by the following LMI conditions

$$W_{c3ir} \triangleq \begin{bmatrix} Y_{ir} & Y_{ir}^T Q_{di}^T \\ Q_{di} Y_{ir} & x_{max_{di}}^2/\varrho_{ir} \end{bmatrix} \geq 0, d = 1, ..., \bar{d}, i = 1, ..., n, r = 1, ..., M \tag{6.16}$$

Non-negative Orthant Stability

By considering Theorem 4.2 and following the similar lines as that given for the non-negativeness condition of the queueing length of network with fixed topology in Subsection 4.2.1, the non-negativity constraint (6.3) for subsystem i can be expressed by the following LMI conditions

$$W_{c4ir} \triangleq (B_{ir} M_{ir})_{sm} \geq 0, \quad s \neq m, i = 1, ..., n \tag{6.17}$$

$$W_{c5ir} \triangleq (B_{dijr} M_{jr})_{sm} \geq 0, \; m, s = 1, ..., \bar{d}, r = 1, ..., M, j \in \wp_r(i) \tag{6.18}$$

and also $u_i \geq 0$ is satisfied if the following LMI conditions hold

$$W_{c6ir} \triangleq M_{ir(sm)} \geq 0, \; s, m = 1, ..., \bar{d}, i = 1, ..., n, r = 1, ..., M \tag{6.19}$$

Following along the lines as invoked in Remark 4.2, W_{c6ir} can trivially guarantee conditions W_{c5ir}. Consequently, the above results are summarized by the following theorem.

Theorem 6.1. *An $\mathcal{H}_\infty$ routing control for a traffic network governed by the dynamical queueing model (6.1) is obtained by solving the following optimization problem:*

$$\min_{M_{ir}, Y_{ir},\ \bar{R}_{ir},\ \bar{Q}_i} \gamma \tag{6.20}$$

subject to the selection of positive definite matrices Y_{ir}, $\bar{R}_{ir}$, $\bar{Q}_i$, and the LMI conditions for W_{ir1}, W_{ir2}, W_{c1ir}, W_{c2irk_i}, W_{c3ir}, W_{c4ir}, and W_{c6ir} for $i = 1, ..., n$, $r = 1, ..., M$, as described by equations (6.7), (6.8), (6.13), (6.14), (6.16), (6.17), and (6.19), respectively.

Proof: The proof follows along the lines that are given in this section. ■

Remark 6.1. It is worth noting that representing the network mobility by a MJLS will enable one to model the changes in the number of nodes due to new nodes additions or deletions them that may occur due to node low energy or presence of faults. For instance, if in a given switching mode the number of nodes is increased by one, the LMI conditions corresponding to the new subsystem (node) is added and the corresponding connection matrices B_{ir} and B_{dijr} of all nodes for which the new node is in their neighboring set will change, accordingly. On the other hand, if the number of nodes (subsystem) is decreased by one, the corresponding LMI conditions of that node is eliminated corresponding to that mode and the connection matrices B_{ir} and B_{dijr} of all nodes for which the missing node was in their neighboring set will change accordingly.

6.2 $\mathcal{H}_\infty$ Control Strategy for Routing Problem in Networks with Variable Destination Nodes

In networks, occasionally the number of destination nodes may vary. In other words, for some destination nodes no external traffic is required to be routed in certain periods. However, due to the system dynamics and time-delays there may still some messages present in queues that should be routed to these destinations as quickly as possible. Moreover, in the dynamical model (6.1) the states are defined as queueing length at each node corresponding to a destination node. Therefore, the number of states depends on the active destination nodes. On the other hand, simply neglecting and deleting the corresponding states associated with the inactive destinations can lead to loss of integrity and stability of the overall system. It also ignores the leftover messages that are kept in the eliminated queues. To cope with these issues, we propose to model the behavior of the network as a singular MJLS as expressed in (5.25):

$$\begin{aligned} E(r(t))\dot{x}_i(t) &= B_i(r(t))u_i(t) + B_{w_i}(r(t))w_i(t) \\ &+ \sum_{j \in \wp_{r(t)}(i)} B_{dij}(r(t))u_j(t-\tau_{ji}(t)) \\ x_i(t) &= \phi_i(t), \quad t \in [-h_i, 0] \\ z_i(t) &= C_i(r(t))x_i(t) \end{aligned} \tag{6.21}$$

where $E(r(t))$ is a diagonal matrix that is specified according to the following two scenarios:

(a) *Regular mobile networks:* In this case we have $E(r(t)) := E^I(r(t)) = I$;

(b) *Varying number of destination nodes:* In this case some destinations become inactive. Therefore, $E(r(t)) := E^D(r(t)) = \text{diag}\{e_j(r(t))\}$, where

$$e_j(r(t)) = \begin{cases} 1 & \text{if the queue is associated with an active destination} \\ 0 & \text{if the queue is associated with an inactive destination} \end{cases}$$

In other words, the activeness or inactiveness of a destination node can be defined as a mode of switching. Therefore, when a destination node become inactive (active), the dynamics switch from regular to singular (singular to regular). Furthermore, by applying the results obtained in Corollary 5.3.1 one can provide an $\mathcal{H}_\infty$ routing control scheme that guarantees the stability of the overall system. Hence, by simply substituting $A_{ir} = 0$ in the given LMI conditions in Corollary 5.3.1, the following results are obtained:

Theorem 6.2. *The fluid flow model of a traffic network governed by system (6.21) with $w \in L_2[0, \infty)$ is stochastically stabilizable, piecewise regular, and piecewise impulse-free by using the decentralized state feedback routing controllers of the form $u_i = K_{ir}x_i$ with an L_2-gain that is less than γ, if there exist matrices M_{ir}, nonsingular matrices Y_{ir} and symmetric positive definite matrices $\bar{R}_{ir}$, $\bar{Q}_i$ for $i = 1, ..., n$, $r = 1, ..., M$ such that the following LMI conditions hold*

$$\begin{aligned} E_r Y_{ir}^T &= Y_{ir} E_r^T > 0 \\ W_{ir1} &= \begin{bmatrix} \theta_{i1} & \theta_{i2} & B_{w_{ir}} & Y_{ir}^T C_{ir}^T & \theta_{i3} & m_{ir} M_{ir}^T & h_{ji} M_{ir}^T \\ * & \theta_{i4} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & \theta_{i5} & 0 & 0 \\ * & * & * & * & * & \theta_{i6} & 0 \\ * & * & * & * & * & * & \theta_{i7} \end{bmatrix} < 0 \qquad (6.22) \end{aligned}$$

$$W_{ir2} = \begin{bmatrix} M_{ir}^T B_{ir}^T + B_{ir} M_{ir} & \tilde{B}_{dir} \tilde{M}_{jr} & Y_{ir} \\ * & -2Y_{jr} + I & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (6.23)$$

$$W_{ir3} = \begin{bmatrix} 2(1 - m_{ir}\pi_{rr})I - \bar{Q}_i + m_{ir}\pi_{rr}\bar{R}_{ir} & \tilde{\pi}_r \\ * & \tilde{R}_{ir} \end{bmatrix} \geq 0 \quad (6.24)$$

where

$$\begin{aligned}
\theta_{i1} &= M_{ir}^T B_{ir}^T + B_{ir} M_{ir} + \pi_{rr} E_r Y_{ir}, \\
\theta_{i2} &= \tilde{B}_{dir} \tilde{R}_{ir}, \\
\theta_{i3} &= (\bar{\pi}_r Y_{ir})^T, \\
\theta_{i4} &= -(1 - d_{ji}) \tilde{R}_{ir}, \\
\theta_{i5} &= -\operatorname{diag}\{Y_{i1}, ..., Y_{i(r-1)}, Y_{i(r+1)}, ..., Y_{iM}\}, \\
\theta_{i6} &= -m_{ik} \bar{R}_{ik}, \\
\theta_{i7} &= -h_{ji} \bar{Q}_i, \\
\bar{\pi}_r &= [\sqrt{\pi_{r1}} E_1 \ \cdots \ \sqrt{\pi_{r(r-1)}} E_{(r-1)} \ \sqrt{\pi_{r(r+1)}} E_{(r+1)} \ \cdots \ \sqrt{\pi_{rM}} E_M]^T \\
\tilde{\pi}_r &= [\sqrt{m_{i1}\pi_{r1}} \ \cdots \ \sqrt{m_{i(r-1)}\pi_{r(r-1)}} \ \sqrt{m_{i(r+1)}\pi_{i(r+1)}} \ \cdots \ \sqrt{m_{iM}\pi_{rM}}]^T
\end{aligned}$$

and m_{ir} = *number of subsystems that subsystem* i *belongs to their* $\wp_r(.)$ *in mode* r, $\tilde{R}_{ir} = \operatorname{diag}_{j \in \wp_r(i)}\{\bar{R}_{jr}\}$, $\tilde{M}_{jr} = \operatorname{diag} j \in \wp(i)\{M_{jr}\}$, *and* $\tilde{B}_{dir} = vec\{B_{dijr}\}$, *for* $j \in \wp_r(i)$. *Moreover, the robust decentralized state feedback controller gain is given by* $K_{ir} = M_{ir} Y_{ir}^{-1}$.

Remark 6.2. It is worth noting that the input signal at each node, u_i can always be defined such that the matrix $B_i(r(t))$ is block diagonal, i.e., $B_i(r(t)) =$

diag$\{B_{1i}(r(t)),\ B_{2i}(r(t))\}$, where $B_{1i}(r(t))$ and $B_{2i}(r(t))$ correspond to active and inactive destinations, respectively.

Remark 6.3. The transition rate π_{ij} can be specified as a function of parameters that change the network topology, specifically due to node mobility speed, available node energy, and/or rate of change the activeness or inactiveness of destination nodes. When the precise values of the transition rates are not available, one can incorporate uncertainty terms to the nominal values and correspondingly modify the stabilizing controllers according to the results developed in [132].

6.2.1 LMI Conditions for Incorporating the Physical Constraints

The LMI conditions to guarantee the network physical constraints as discussed in Section 6.1.1 are now modified for the dynamical model (6.21).

Capacity Constraint

To guarantee the capacity constraint for each subsystem

$$G_{k_i}u_i \ \leq\ c_{k_i}(r(t)) \quad k_i = 1, ..., li, \ \ i = 1, ..., n$$

Let us consider the following ellipsoid for a selected $\varrho_i > 0$

$$\Sigma_i = \{x_i(t)|\int_{t-\tau_{ij}}^{t} x_i^T(s)K_{ir}^T\bar{R}_{ir}^{-1}K_{ir}x_i(s)ds \leq \varrho_{ir},\ \bar{R}_{ir} = \bar{R}_{ir}^T > 0\}\} \qquad (6.25)$$

Provided that the stability conditions defined in (6.22)-(6.24) are satisfied, then from the definition of $V(x_t, r_t)$ in (5.29), it follows that

$$\int_{t-\tau_{ij}}^{t} x_i^T(s)K_{ir}^T\bar{R}_{ir}^{-1}K_{ir}x_i(s)ds \ \leq\ V(x_t, r_t)$$

Therefore,

$$\mathbb{E}[\int_{t-\tau_{ij}}^{t} x_i^T(s)K_{ir}^T\bar{R}_{ir}^{-1}K_{ir}x_i(s)ds] \leq \mathbb{E}[V(x_t, r_t)]$$

On the other hand, by integrating $J_1 < 0$ defined in (5.7) from 0 to t one gets

$$\mathbb{E}[V(x_t, r_t)] \leq \mathbb{E}[-\int_0^t z^T(t)z(t)dt + \int_0^t \gamma w^T(t)w(t)dt] + V(x_0, r_0) \leq \gamma L_1 + L_2$$

where $L_1 = \int_0^\infty w_i^T(t)w_i(t)dt$ is an upper bound on the energy of the external input $w_i(t)$, and $L_2 = V(x_0, r_0)$. Therefore, $x_i(t)$ belongs to an invariant set Σ_i, if $\gamma L_1 + L_2 \leq \varrho_{ir}$. Furthermore, the state feedback controller u_i is obtained from $u_i = K_{ir}x_i$. Therefore, (6.25) can be modified as

$$G_{kir}K_{ir}x_i < c_{kir} \tag{6.26}$$

Now, by squaring (6.25), considering $h_i = \max\{h_{ij}\}$ and integrating both sides of the expression from $t - \tau_{ij}$ to t, we get

$$\int_{t-\tau_{ij}}^{t} x_i^T(s)(G_{kir}K_{ir})^T G_{kir}K_{ir}x_i(s)ds < h_i c_{kir}^2 \tag{6.27}$$

Note that from (6.25), we have $\int_{t-\tau_{ij}}^{t} x_i^T(s)K_{ir}^T\bar{R}_{ir}^{-1}K_{ir}x_i(s)ds \leq \varrho_{ir}$, therefore (6.27) will be satisfied if

$$G_{kir}^T(\varrho_{ir}/(h_i c_{kir}^2))G_{kir} < \bar{R}_{ir}^{-1} \tag{6.28}$$

Now by applying the Schur complement to (6.28), the capacity constraints for the subsystem i can be expressed as LMI conditions

$$W_{c1ir} \triangleq \gamma \leq \max_{i,r}\{(\varrho_{ir} - L_2)/L_1\}, r = 1, ..., M \tag{6.29}$$

$$W_{c2irk_i} \triangleq \begin{bmatrix} 2I - \bar{R}_{ir} & G_{kir}^T \\ G_{kir} & h_i c_{kir}^2/\varrho_{ir} \end{bmatrix} \geq 0, k_i = 1, ..., li, i = 1, ..., n \tag{6.30}$$

Upper Bound on the Buffer Size

For each subsystem the constraint on the queue buffer size is described as follows

$$Q_{di}x_i \leq x_{max_{di}},\ d = 1, ..., \bar{d},\ i = 1, ..., n \tag{6.31}$$

Now, considering the state feedback $u_i = M_{ir}Y_{ir}^{-1}x_i$, for selected matrices $\bar{M}_{ir}$ one can obtain the following equation

$$(\bar{M}_{ir}M_{ir}Y_{ir}^{-1})^{-1}\bar{M}_{ir}u_i = x_i$$

Therefore, (6.31) can be expressed as follows

$$Q_{di}(\bar{M}_{ir}M_{ir}Y_{ir}^{-1})^{-1}\bar{M}_{ir}u_i \leq x_{max_{di}},\ d = 1, ..., \bar{d},\ i = 1, ..., n, r = 1, ..., M \tag{6.32}$$

By squaring (6.32) and integrating both sides of the expression from $t - \tau_{ij}$ to t, we get

$$\int_{t-\tau_{ij}}^{t} u_i^T(s)(Q_{di}(\bar{M}_{ir}M_{ir}Y_{ir}^{-1})^{-1}\bar{M}_{ir})^T Q_{di}(\bar{M}_{ir}M_{ir}Y_{ir}^{-1})^{-1}\bar{M}_{ir}u_i(s)ds < h_i x_{max_{di}}^2 \tag{6.33}$$

Note that from (6.25) we have $\int_{t-\tau_{ij}}^{t} u_i^T(s)\bar{R}_{ir}^{-1}u_i(s)ds \leq \varrho_{ir}$, therefore (6.33) will be satisfied if

$$(Q_{di}(\bar{M}_{ir}M_{ir}Y_{ir}^{-1})^{-1}\bar{M}_{ir})^T(\varrho_{ir}/(h_i x_{max_{di}}^2))(Q_{di}(\bar{M}_{ir}M_{ir}Y_{ir}^{-1})^{-1}\bar{M}_{ir}) < \bar{R}_{ir}^{-1} \tag{6.34}$$

Now by applying the Schur complement to (6.34), and using Lemma 2.2, the constraint on the queue buffer size for each subsystem, (6.31) can be guaranteed by the following LMI conditions

$$\begin{aligned} W_{c3ir} &\triangleq \begin{bmatrix} 4I - 2Y_{ir} - (Q_{di}^T Q_{di})\varrho_{ir}/(h_i x_{max_{di}}^2) & (\bar{M}_{ir}M_{ir})^{-1}\bar{M}_{ir} \\ \bar{M}_{ir}^T(\bar{M}_{ir}M_{ir})^{-T} & 2I - \bar{R}_{ir} \end{bmatrix} \geq 0, \quad (6.35) \\ d &= 1, ..., \bar{d}, i = 1, ..., n, r = 1, ..., M \end{aligned}$$

Non-Negative Orthant Stability

In switching modes when the matrix E_{ir} is full rank, i.e., regular dynamics, the associated LMI conditions to guarantee non-negativeness of the states are defined similar to the conditions (6.17) and (6.18). However, if E_{ir} is a singular matrix, the state x_i is partitioned into two components as follows: $x_i = [x_{i1}^T \; x_{i2}^T]^T$, where x_{i1} is the queue associated with the active destinations and x_{i2} is the queue associated with the inactive destinations. We furthermore, partition the state vector gain into $K_{ir} = [K_{ir1} \;\; K_{ir2}]$ with appropriate dimensions for K_{ir1} and K_{ir2} according to x_{i1} and x_{i2}, respectively. It should be noted that the queueing dynamics corresponding to the inactive destinations do not receive any external stimuli, i.e., $w_{i2} = 0$. Consequently, the closed-loop dynamics of (6.21) can be expressed according to

$$\begin{aligned} \dot{x}_{i1} &= B_{ir1}K_{ir1}x_{i1}(t) + \sum_{i=1}^{n} B_{dijr1}K_{jr1}x_{j1}(t-\tau(t)) + B_{iw1}w_{i1}(t) \\ 0 &= B_{ir2}K_{ir2}x_{i2}(t) + \sum_{i=1}^{n} B_{dijr2}K_{jr2}x_{j2}(t-\tau(t)) \end{aligned} \tag{6.36}$$

By invoking Theorem 4.2 to the first equation of (6.36), the off-diagonal entries of $B_{ir1}K_{ir1}$ and all entries of $B_{dijr1}K_{jr1}$ should be non-negative. By selecting the positive definite matrix Y_{ir1} defined in Corollary 5.3.1 to be a diagonal matrix and by setting $K_{ir} = M_{ir}Y_{ir1}^{-1}$, the (essential) non-negativity of $B_{dijr1}K_{jr1}$ and $B_{ir1}K_{ir1}$, can be expressed as follows

$$W_{c4ir} \triangleq (B_{ir1}M_{ir1})_{sm} \geq 0, \quad s \neq m, i = 1, ..., n \tag{6.37}$$

$$W_{c5ir} \triangleq (B_{dijr1}M_{jr1})_{sm} \geq 0, \; m, s = 1, ..., \bar{d}, r = 1, ..., M, j \in \wp_r(i) \tag{6.38}$$

The above ensures the non-negativity constraint (6.3) for the first equation of (6.36). For the second set of equations in (6.36), the non-negative orthant condition is achieved by finding K_{ir2} such that $B_{dijr2}K_{jr2}$ is non-negative and $B_{ir2}K_{ir2}$ is negative. Defining a diagonal positive definite matrix Y_{ir1}, these conditions can then be expressed as follows

$$\begin{aligned} W_{c6ir} &\triangleq (B_{ir2}M_{ir2})_{sm} \leq 0, i=1,...,n, s,m=1,...,\bar{d}, r \in \bar{\mathcal{S}}, \\ W_{c7ir} &\triangleq (B_{dijr2}M_{jr2})_{sm} \geq 0, \quad i,s,m=1,...,\bar{d}, r=1,...,M, j \in \wp_r(i) \end{aligned} \quad (6.39)$$

where $\bar{\mathcal{S}}$ is the set of modes in which E_{ir} is singular. Therefore, W_{c4ir}, W_{c5ir} and W_{c7ir} are the same for the regular modes as well as the singular modes. However, W_{c6ir} changes from regular modes to singular modes. Specifically the W_{c6ir} conditions for both singular and regular modes are defined as follows:

$$W_{c6ir} = \begin{cases} (B_{ir2}M_{ir2})_{sm} \leq 0 & i=1,...,n, s,m=1,...,\bar{d}, r \in \bar{\mathcal{S}} \\ (B_{ir2}M_{ir2})_{sm} \geq 0 & i=1,...,n, \quad s,m=1,...,\bar{d}, r \in \mathcal{S}-\bar{\mathcal{S}}, \end{cases} \quad (6.40)$$

Provided that the non-negativity condition $x_i \geq 0$ is satisfied, $u_i \geq 0$ is guaranteed if the following LMI conditions hold

$$W_{c8ir} \triangleq M_{ir(sm)} \geq 0, \; s,m=1,...,\bar{d}, \; i=1,...,n, r=1,\ldots,M \quad (6.41)$$

Remark 6.4. It should be noted that since the elements of B_{ir} are either -1 or 0 satisfying condition (6.41) results in a square matrix $B_{ir}M_{ir}$ with negative or zero elements. Therefore, W_{c6ir} is trivially satisfied for the singular mode. On the other hand, satisfying W_{c6ir} for the regular mode and W_{c4ir} lead to a diagonal negative definite matrix $B_{ir}M_{ir}$. This is also validated by the fact that the dynamics of the queueing system is expressed in a decentralized framework and the queues at each node are decoupled from each other. Moreover, since the elements of B_{dijr} are either 1 or 0, W_{c8ir} trivially guarantees that the conditions W_{c5ir} and W_{c7ir} are satisfied.

To summarize, the following theorem provides our robust routing control strategy corresponding to the mobile network dynamics (6.21) that satisfies the associated physical constraints (6.2)-(6.5).

Theorem 6.3. *An $\mathcal{H}_\infty$ routing control strategy for a traffic network that is governed by the dynamical queueing model (6.21) is obtained by solving the following optimization problem:*

$$\min_{M_{ir}, Y_{ir}, \bar{R}_{ir}, \bar{Q}_i} \gamma \tag{6.42}$$

subject to the selection of positive definite matrices $\bar{R}_{ir}$, $\bar{Q}_i$, and the LMI conditions for $W_{ir1} - W_{ir3}$, W_{c1ir}, W_{c2irk_i}, W_{c3ir}, W_{c4ir}, W_{c6ir} and W_{c8ir} for $i = 1, ..., n$, $r = 1, ..., M$, as described by equations (6.22)-(6.24), (6.29), (6.30), (6.35), (6.37), (6.40), and (6.41), respectively.

Proof: The proof follows along the constructive lines that are given in this section. ■

6.3 Simulation Results

In this section, simulation results are provided to evaluate the performance of our proposed $\mathcal{H}_\infty$ routing control strategy in mobile NMAS. Agents in a network usually move in groups. Furthermore, nodes within a team should exchange information among one another and also communicate with other specific nodes designated as commanders or supervisors of the entire network. In the first two examples a mobile network containing 50 nodes are considered. By using the QualNet software which is a scalable simulator for network performance emulation and simulation [133], the routing algorithm developed in Section 6.1 is compared with two commonly used routing algorithms known as AODV [49] and OLSR [48]. A brief description of these algorithms are given in Appendix C. In the third example by employing the routing algorithm proposed in Section 6.2 to present simulation results for a mobile network with 20 nodes where the number of destination nodes are allowed to changes.

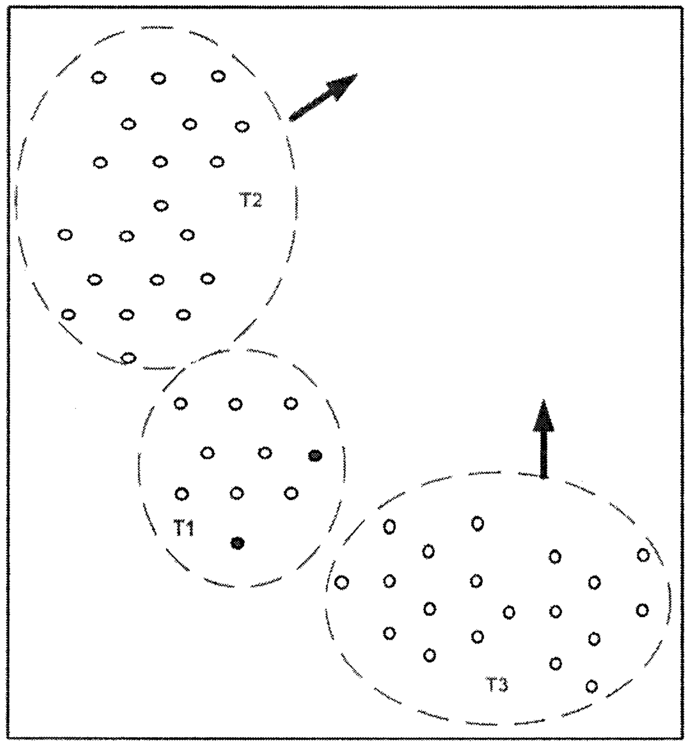

Figure 6.1: The initial configuration of the mobile network in Example 6.1. Destination nodes 7 and 10 are shown with dark circles. The arrows indicate the direction of each team motion in the plane.

Example 6.1. Fixed Destination Nodes

In this example, let us consider a scenario of NMAS having 50 nodes that are partitioned into three teams covering in an area of $8000m \times 12000m$. The first team T_1 that includes the nodes $1 - 10$ is fixed, the second team T_2 including the nodes $11 - 30$ moves towards north-east, and the third team T_3 containing the nodes $31 - 50$ moves towards north. Fig. 6.1 depicts the initial configuration of the nodes and also the direction of the team's motion. It is assumed that the network remains connected at all times. The nominal communication range for each node is considered to be 484 m, the capacity is 1 Mbps, and maximum buffer size is 450 kbit. The transition mode is selected as $\pi_{rj} = 0.002$ for $r \neq j$. The total simulation time duration is selected as 700 s fir each run. In this example, it is assumed that the destination nodes are 7 and 10 which are in the first team and fixed. They are shown as dark circles in Fig. 6.1. Therefore, each node has two states: the first

Table 6.1: Total messages that are lost for different node speeds and 139680 kbit traffic load

Second team max speed (m/s)	20	40	60
Our proposed method (kbit)	18848	31776	35360
OLSR (kbit)	22240	32361	40480
AODV (kbit)	25312	33056	37280

state is the queue associated with the destination node 10 and the second state is associated with the destination node 7. Node 10 does not route any message and is considered as a sink. Therefore, there are 49 subsystems and a total of 97 states (i.e., 49 queues corresponding to the destination node 10 and 48 queues corresponding to the destination node 7). For each input flow, the delay is taken as a time-varying function $\tau(t) = 3 + 0.8|sin(t)|$ s. Although this is considered to be *unknown* to the controllers. A total of 9 switching cases are defined based on the changes in the neighboring sets. The following three representative cases are considered for evaluating the performance of our routing algorithm:

Case A: Messages Received and Lost Under Different Node Mobility

In this case, the traffic load for each node is based on the well-known Poisson distribution with the rate of $\lambda = 300$ bytes per second for 600 s. The total messages that are arrived to the network is 139680 kbit. We assume that maximum speeds are 0, 10 m/s and 20 m/s for nodes in teams one, two and three, respectively. The simulations are repeated when the maximum speeds are increased by factors of two and three times of the above values. The maximum speed of the second team is used as an index of comparison. Fig. 6.2 depicts the total messages that are received in the destination nodes 10 and 7 as a function of the second team maximum speed using our proposed routing algorithm, AODV and OLSR algorithms. The total messages that are lost using our proposed routing algorithm, AODV, and OLSR corresponding to different nodes speeds are illustrated in Table 6.1. The results confirm that by increasing the speed of nodes the proportion of dropped messages

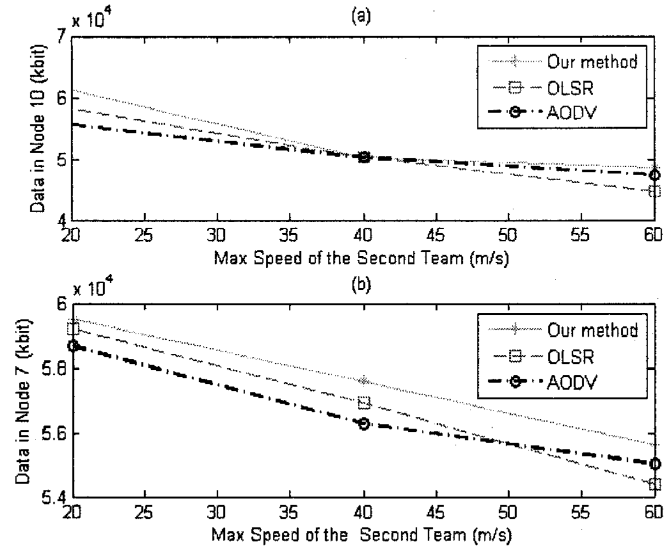

Figure 6.2: Received messages to the destination node 10 (a) and node 7 (b) for different node speeds and different routing algorithms in Example 6.1 by using our proposed routing (solid lines with star), AODV (dashed-dot lines with circles) and OLSR (dashed lines with squares) methods.

is also increased. It should be noted that, our proposed scheme can route messages with fewer loss when compared to AODV and OLSR methods.

*Case B:*Messages Received and Lost Under Different Traffic Loads

The performance of our proposed routing algorithm is also evaluated based on different traffic loads with rates of $\lambda = [10 \quad 30 \quad 100 \quad 300 \quad 600]$ bytes per second, when the nodes maximum speeds are set to 0, 10, and 20 for teams one, two and three, respectively. Fig. 6.3 depicts the total messages that are received in the destination nodes 10 and 7, versus different traffic loads by using our proposed routing algorithm, AODV and OLSR, respectively. The total messages that are lost by using our proposed routing algorithm, AODV and OLSR with different traffic loads are indicated in Table 6.2. Fig. 6.3 and Table 6.2 do indeed confirm the superiority of the performance of our proposed routing algorithm in dealing with different traffic loads.

Case C: Maximum Queueing Length

Table 6.2: Total messages that have been lost for different traffic loads at the maximum speed of 20 m/s for the second team.

Traffic load (kbit)	4656	13968	46560	139680	279360
Our proposed method (kbit)	960	2847	5615	18848	49738
AODV (kbit)	977	5952	16514	22240	106856
OLSR (kbit)	2161	6352	18114	25312	66856

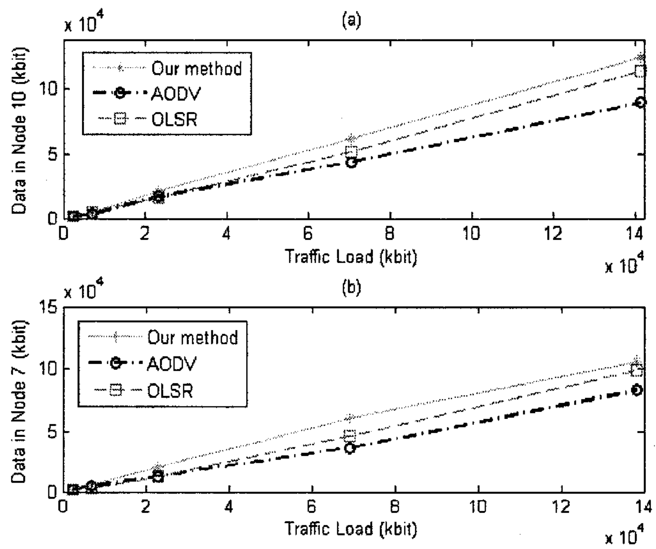

Figure 6.3: Received messages to the destination node 10 (a) and node 7 (b) for different traffic loads and different routing algorithms in Example 6.1 by using our proposed routing (solid lines with star), AODV (dashed-dot lines with circles) and OLSR (dashed lines with squares) methods.

The maximum queueing length of nodes in a mobile network can also be considered as an issue for evaluating the performance of a routing algorithm. Fig. 6.4 depicts the maximum queueing length obtained by using our proposed routing, OLSR and AODV methods for the input rate of $\lambda = 300$ bytes per second when the maximum speed of the second team is $20m/s$. As expected the AODV method which is a reactive algorithm keeps the messages longer in the queues for determining the optimal routes. However, the maximum queueing length obtained by using our proposed routing is comparable with the OLSR algorithm, in the sense of distribution as well as individual values. Moreover, the sum of the maximum queues obtained by using our routing algorithm is 2585 kbit, by using OLSR algorithm is

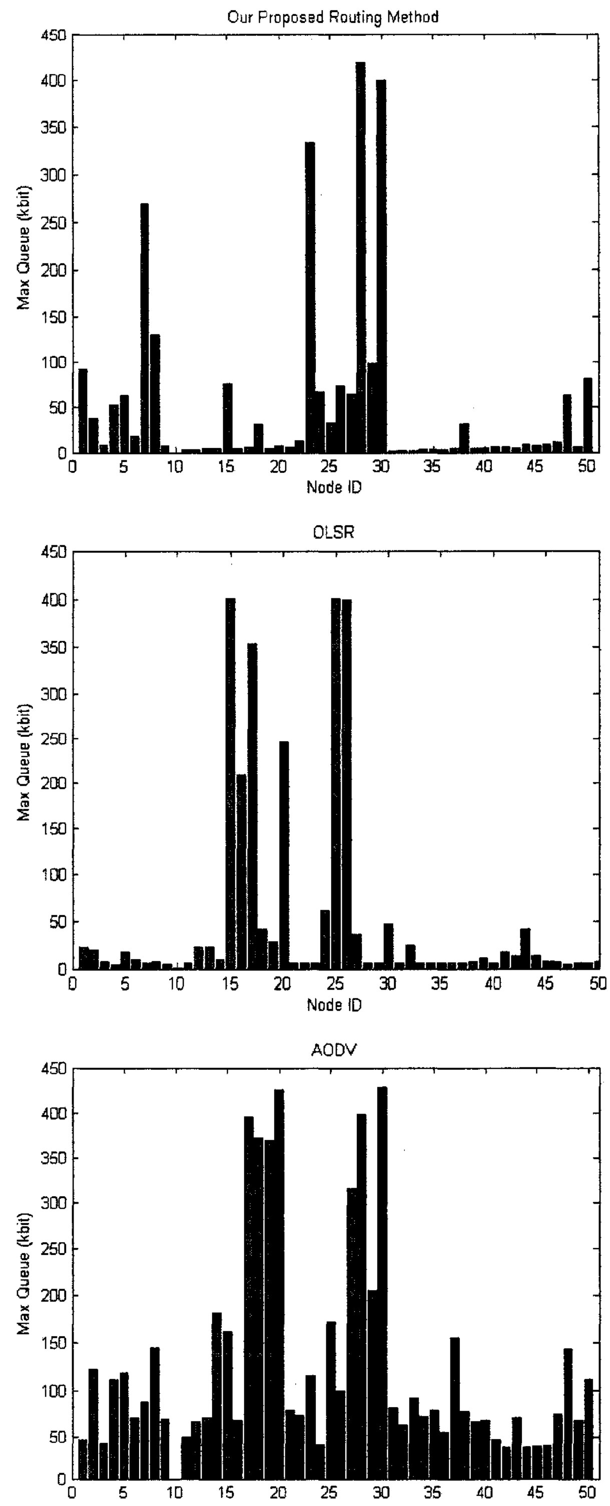

Figure 6.4: The maximum queueing length for the input rate $\lambda = 300$ and the max speed of the second team $20m/s$ for Example 6.1 by using our proposed routing, OLSR, and AODV methods from top to bottom, respectively.

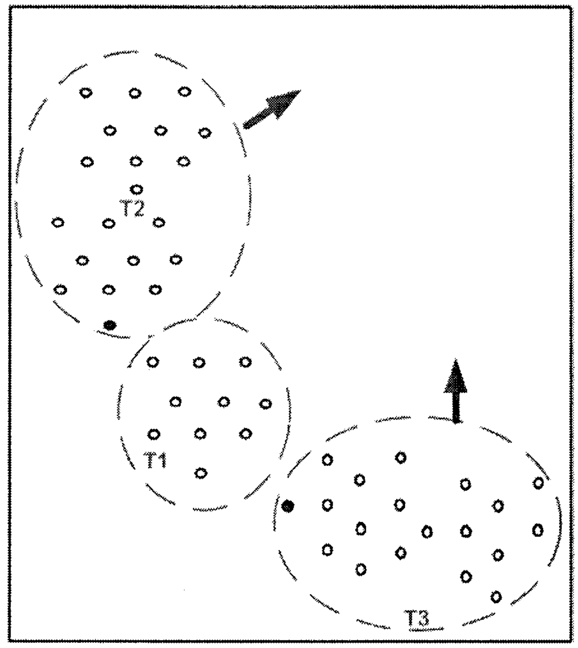

Figure 6.5: The initial configuration for the mobile network in Example 6.2. Destination nodes 30 and 50 are shown with dark circles. The arrows indicate the direction of each team motion in a plane.

2621 kbit, and by using AODV algorithm is 6372 kbit. This confirms that overall our proposed routing algorithm provides the shortest queues on average.

Example 6.2. Mobile Destination Nodes

In this example, the network topology and scenario considered in the previous example is repeated. However, the destination nodes are changed to nodes 30 and 50 which are in the second and third teams, respectively. Therefore, the destination nodes are mobile themselves. The external traffic load of node 10 will be routed to node 50 only. Therefore, similar to Example 6.1, there are 97 states, 49 states (queues) for the destination 50 and 48 states (queues) for the destination 30. Fig. 6.5 demonstrates the initial configuration of the nodes in this example and also the direction of the teams motion. Performance of our proposed routing algorithm is investigated by the following three cases:

Case A: Messages Received and Lost Under Different Node Mobility

Consider the rate of the traffic load for each node is equal to $\lambda = 300$ bytes per

Table 6.3: Total messages that are lost for different node speeds and 139680 kbit traffic load.

Second team max speed (m/s)	20	40	60
Our proposed method (kbit)	23331	33119	43623
OLSR (kbit)	25120	38080	47200
AODV (kbit)	31040	35200	46080

second for 600 s Therefore, the total arrival messages is 139680 kbit. The messages have been routed when the second team maximum speed is $20m/s$ and the third team maximum speed is $10m/s$. The simulations are also repeated when the maximum speeds are increased to two and three times faster. The maximum speed of the second team is used as an index of comparison. Fig. 6.6 depicts the total messages that are received in the destination nodes 30 and 50, by using our proposed routing algorithm, AODV and OLSR algorithms when the set of nodes maximum speed are [0, 10, 20] for team one, two, and three receptively which is then increased by two and three times. The total lost messages by using our proposed routing algorithm, AODV, and OLSR algorithms with different node speeds are illustrated in Table 6.3. Fig. 6.6 and Table 6.3 indicate the acceptable performance of our proposed routing algorithm in response to increasing the node speeds.

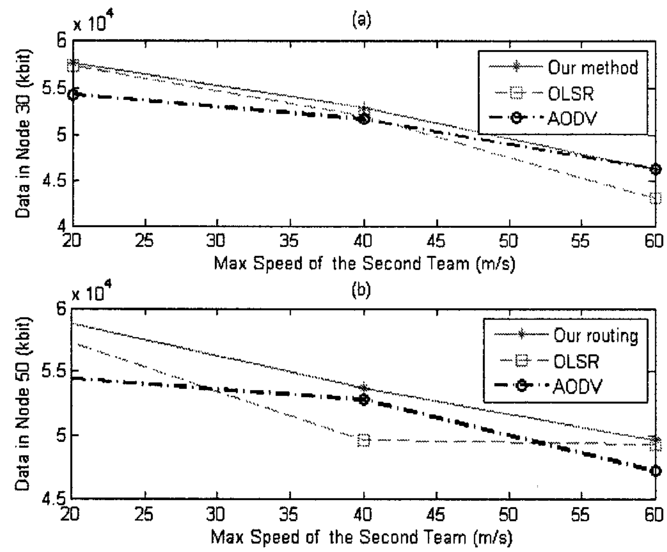

Figure 6.6: Received messages to the destination node 30 (a) and node 50 (b) for different node speeds and different routing algorithms in Example 6.2 by using Our proposed routing (solid lines with star), AODV (dashed-dot lines with circles) and OLSR (dashed lines with squares) methods.

Table 6.4: Total messages that are lost for different traffic loads and speed of 20 m/s for second team.

Traffic load (kbit)	4656	13968	46560	139680	279360
Our proposed method (kbit)	1193	2656	8181	23331	41716
AODV (kbit)	1216	4208	23120	31040	85600
OLSR (kbit)	3155	7488	8320	25120	79360

*Case B:*Messages Received and Lost Under Different Traffic Loads

The routing algorithm is now investigated according to different traffic loads with rates of $\lambda = [10\ \ 30\ \ 100\ \ 300\ \ 600]$ bytes per second. the maximum speed of team one, two, and three are assumed to be 0, 10 and 20, respectively. Fig. 6.7 depicts the total messages that are received in the destination nodes 30 and 50 by using our proposed routing algorithm, AODV and OLSR algorithms, versus different traffic loads. The total lost messages by using our proposed routing algorithm, AODV and OLSR algorithms with different traffic loads are illustrated in Table 6.4. Fig. 6.3 and Table 6.2 show that the routing performance of our proposed method routing is comparable with the OLSR and AODV algorithms in low traffic loads and as the

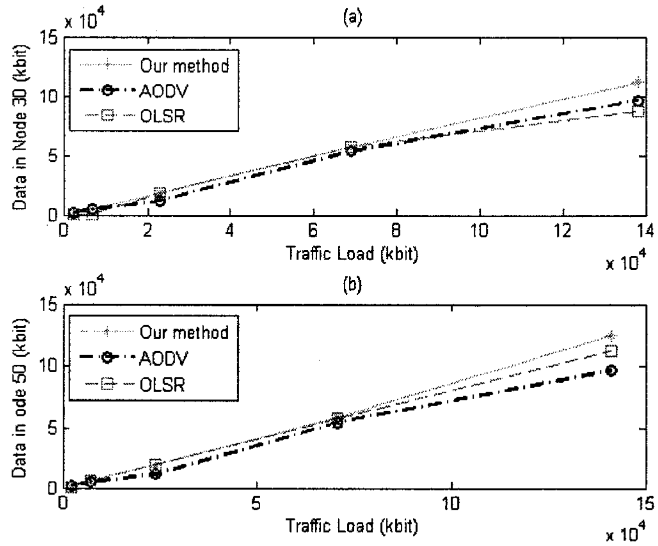

Figure 6.7: Received messages to the destination node 30 (a) and node 50 (b) for different traffic loads and different routing algorithms in Example 6.2 by using Our proposed routing (solid lines with star), AODV (dashed-dot lines with circles) and OLSR (dashed lines with squares) methods.

rate of input traffic is increased our proposed method can route the traffic better with less loss as compared with the other two algorithms.

Case C: Maximum Queueing Length

By considering the node maximum speeds set of [0 10 20] m/s for team one, two, and three respectively, the maximum queueing length by using our proposed routing, OLSR and AODV algorithms for input rate of $\lambda = 300$ bytes per second is demonstrated in Figs. 6.8. In mobile destination nodes, the AODV algorithm also provides longer queues, as compared to our proposed routing and the OLSR. In this case, sum of maximum queues by using our routing algorithm is 2757 kbit, by using the OLSR algorithm is 3297 kbit and by using the AODV algorithm is 6852 kbit. This confirms that overall our proposed routing algorithm keeps the messages less waiting in the queues.

Example 6.3. In this example, the performance of our routing algorithm for a variable number of destination nodes is investigated. Toward this end, let us consider

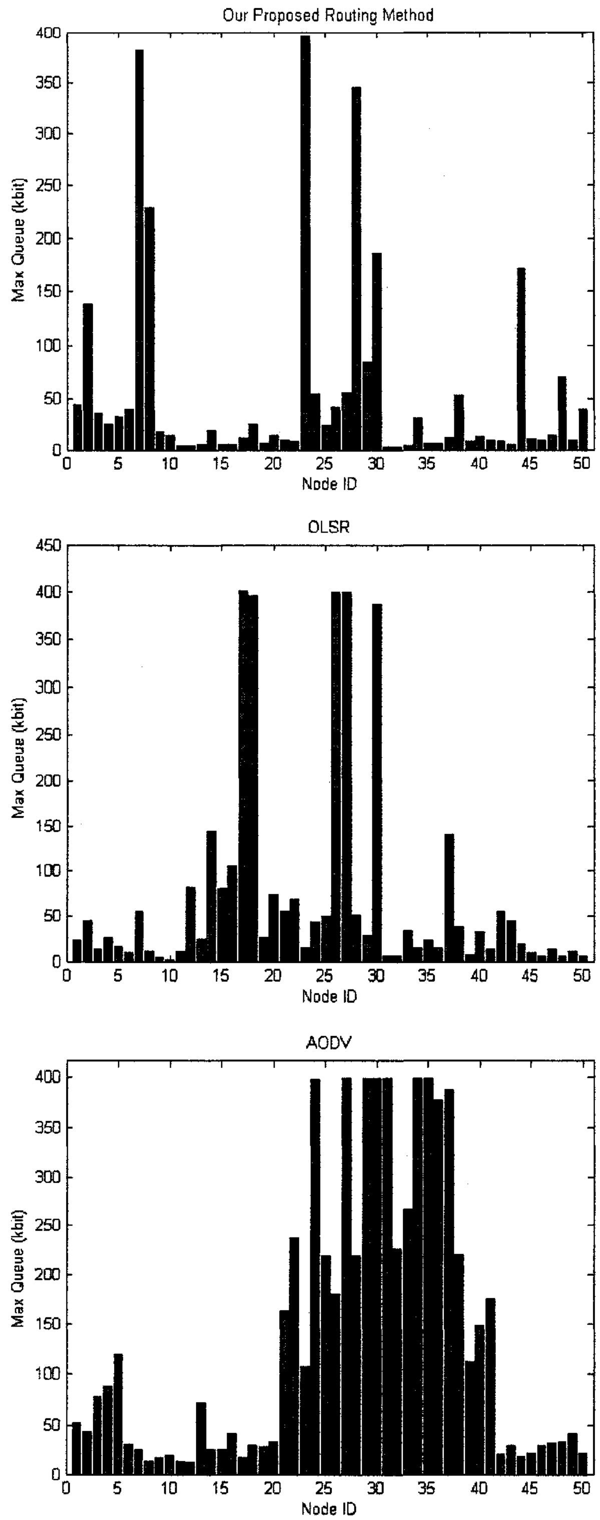

Figure 6.8: The maximum queueing length for the input rate $\lambda = 300$ and max speed of the second team is $20m/s$ in Example 6.2 by using our proposed routing, OLSR, and AODV methods from top to bottom, respectively.

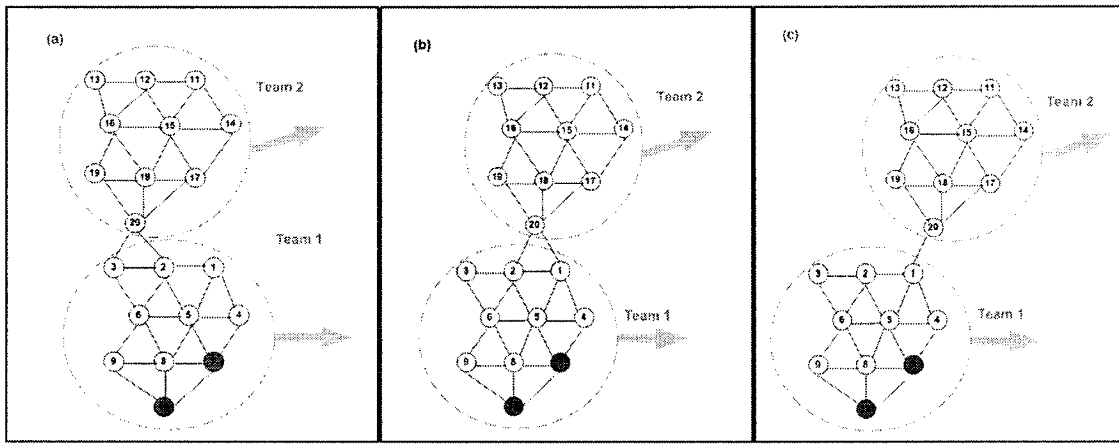

Figure 6.9: The schematic of the network configuration in three modes in Example 6.3. The destination nodes 7 and 10 are shown as dark circles. Arrows indicate the direction of motion of each team.

a scenario of NMAS having 20 nodes that are partitioned into two teams which are mobile in an area of $8000m \times 12000\ m$. The first team includes the nodes $1-10$ that moves towards east with the speed of 5 m/s and the second team includes the nodes $11-20$ that moves towards north-east with the speed of 25 m/s. Fig. 6.9 depicts the configuration of the nodes at three distinct modes and also the direction of the teams motion. It is assumed that the network remains connected at all times. The nominal communication range for each node is considered to be 450 m and the channel capacity is 1 Mbps. The maximum buffer size is set to 450 kbit. The pause time is set to 200 s and the total time for each simulation is 500 s.

The packet generation rate for each node is based on the well-known Poisson distribution with the rate of $\lambda = 200$ bytes per second for 400 s The transition mode is selected as $\pi_{rj} = 0.002$ for $r \neq j$. Nodes 7 and 10 are considered as destination nodes which are shown by dark circles in Fig. 6.9. Therefore, each mode should have two states: the first state is the queue associated with the destination node 10 and the second state is associated with the destination node 7. For each input flow, the delay is taken as a time-varying function $\tau(t) = 1 + .1|sin(t)|$ s which is considered to be *unknown* to the controllers. It is assumed that in certain time periods the destination node 7 is not active. In the first three modes, switching occurs due to the mobility and changing the neighboring sets. In other words, according to the network model given by (6.21) we have $E_{i1} = E_{i2} = E_{i3} = I$. In the 4th mode, it is assumed that the destination node 7 is inactive. Consequently, in model (6.21) we have $E_{i4} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

Our objective is to show that through modeling the inactive destination nodes by singular Markovian jump dynamics, we will be able to empty their associated queues faster. The simulation results for our $\mathcal{H}_\infty$ routing controller that is designed according to Theorem 6.3 for a singular Markovian jump dynamics is now compared with the performance that is achieved by using the $\mathcal{H}_\infty$ routing controller that is designed based on Theorem 6.1 for regular Markovian jump dynamics. In other words, when we ignore the fact that node 7 will be inactive for some periods of time. Fig. 6.10 depicts the queueing length of node 3 corresponding to the destination nodes 10 and 7 by using our proposed algorithms for the singular dynamics (sub-figures (a)-(c)) and the regular dynamics (sub-figures (d)-(f)).

Figs. 6.10-(a) and 6.10-(d) show the queueing length for the destination node 10 and Figs. 6.10-(b) and 6.10-(e) illustrate the queueing length for the entire simulation period of 500 s for the destination node 7. This confirms the stable behavior of our proposed routing algorithms for both regular as well as singular MJLSs. Figs. 6.10-(c) and 6.10-(f) depict the behavior of the routing algorithms when the destination node 7 becomes inactive at $t = 220$ s It shows that the singular Markovian jump dynamics could empty the queue in 4 seconds. However, by applying the regular Markovian jump dynamics, it took 24 seconds to route the messages and empty the queues. On the other hand, due to capacity constraint decreasing the queueing length of one destination node results in an increase of the queueing length of the other destination nods. Therefore, applying the singular Markovian jump dynamics

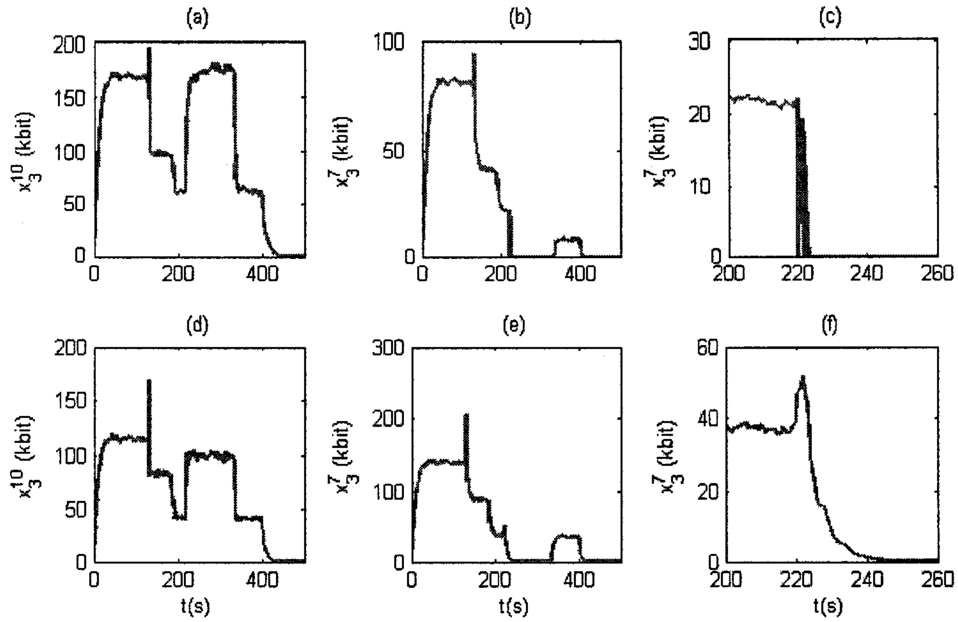

Figure 6.10: The queueing length of node 3 to destination nodes 10 and 7 in Example 6.3. Subplots (a)-(b) are obtained by using the singular Markovian jump dynamics, and subplots (d)-(e) are obtained by using the regular Markovian jump dynamics. Subplots (c) and (f) are zoomed version of (b) and (e), respectively.

is suitable when other destination nods have lower priority in receiving the messages.

6.4 Conclusions

In this chapter, Markovian jump dynamics with unknown and time-varying delays is introduced to model the data traffic in mobile networks. By applying the stabilizing controller for the MJLSs presented in the pervious chapter, an $\mathcal{H}_\infty$ routing control strategy is proposed which can simultaneously stabilize the system and provide a desired routing performance by minimizing the *worst-case queueing length.* By taking advantage of the Markovian jump and singular behavior in the network dynamics, changing neighboring sets due to node mobility and changes in the number of the destination nodes can be handled. LMI conditions are developed to address the physical conditions in the mobile network. Therefore, the mobile network routing problem subject to various physical constraints is solved through a unified multi-objective optimization framework.

Chapter 7

Conclusions and Future Work

This chapter contains a conclusion of the thesis followed by some suggestions for future research work.

7.1 Conclusions

This thesis endeavored to address routing problem of NMAS by taking advantage of control theory machinery. To achieve this goal, the following steps were pursued:

- The queuing model based on the conservation law is considered as dynamics of the traffic network which describes the required time for transmitting, propagating and processing the messages as an unknown time-varying delay function. The concept of the *worst-case queueing length* was proposed as an objective function for the routing problem and it was shown that the routing problem can be defined as designing a stabilizing $\mathcal{H}_\infty$ controller for the traffic network dynamics with time-varying delay functions such that physical constraints such as capacity constraint, buffer size constraint, and non-negativeness of queues and flows are satisfied.

- A stable robust $\mathcal{H}_\infty$ controller with an LMI solution was proposed as a powerful tool to deal with multiple time-varying delays when there is a limited *a priori* knowledge about the delays. The LMI conditions are then provided to address the physical constraints. Consequently, an optimization problem that incorporates the LMI conditions for designing an $\mathcal{H}_\infty$ controller and satisfying the associated physical constraints was introduced. This resulted in a centralized routing solution for networks with fixed topology.

- By defining each node as a subsystem, a decentralized traffic network dynamics was considered. The proposed $\mathcal{H}_\infty$ stabilizing control scheme and the corresponding LMI conditions associated with the physical conditions are modified as a decentralized routing problem.

- To address node mobility, and consequently a stochastic change of the network topology which could be due to changes in the neighboring sets and/or changes in the number of nodes (subsystems), a Markovian jump process was considered and proposed. An $\mathcal{H}_\infty$ control strategy was developed to stochastically stabilize Markovian jump linear systems with time-varying delays in the input. Moreover, changes in the neighboring sets result in changes in the interconnected terms at each switching mode. Therefore, the interconnection terms associated with the decentralized Markovian jump linear system is mode-dependent. The introduced LMIs for obtaining the $\mathcal{H}_\infty$ controller for the Markovian jump linear system is modified when the interconnection terms are mode-dependent. The LMIs are also provided to describe the physical conditions of the mobile network.

- Changing the number of destination nodes in certain periods is another problem that was investigated in this thesis. Associated with the considered decentralized network dynamics, by changing the number of destination nodes results in changes in the number of states. A decrease in the number of active destination nodes is represented as a switching mode in the Markovian jump linear systems, in which the corresponding inactive state destinations are defined as a singular dynamics. An $\mathcal{H}_\infty$ control scheme was presented for this dynamics to guarantee stability for the entire system. We showed that the proposed Markovian jump model with time-varying delays in Chapter 6 provides an appropriate queueing dynamics for the network routing traffic problem for addressing random mobility of the nodes, as well as varying the number of nodes and the destination nodes.

7.2 Future Work

To extend the current research, several open problems need to be addressed in the control domain as well as the network domain. Some of the most relevant problems are listed below:

- To stabilize and analyze the stability of singular Markovian jump linear systems with time-varying delays the constraints on the rate of delays could not be relaxed with the well-known methods in the literature such as descriptor methods and free weighting methods. Finding LMIs which can relax the constraint on rate of the delays can provide stability for singular systems with larger range of variations in the delay functions.

- Providing a stabilizing control for a singularly perturbed system with time-delays can be an interesting extension of our proposed control scheme in Chapter 5. In singularly perturbed systems, the system dynamics is decomposed into two time scales: slow and fast. Stabilizing such systems by incorporating time-delays is challenging and requires more investigation even for deterministic and centralized dynamics. Moreover, it can be shown that different priority of destination nodes can be expressed as a singularly perturbed Markovian jump linear systems.

- In Chapter 6, it is assumed that the transition rate π_{ij} for the mobile networks is given. Indeed, this transition rate can be defined as a function of parameters that change the network topology such as node velocity, node energy, and/or rate of activeness/inactiveness of destination nodes. Defining a model that can properly describe the relationship among the above parameters to π_{ij} helps in achieving a more precise routing control. On the other hand, when the precise values of the transition rates are not available, an uncertainty term can be incorporated to the nominal values. Some results for stabilizing Markovian jump linear systems with uncertain transition rates have been introduced in the literature recently [132]. However, considering such uncertainty requires more investigations for Singular Markovian jump linear systems.

- Another important factor which is vital in networked multi-agent systems is the consumption of energy in each node of the network. The energy consumed at each node can be considered as the sum of energy required for sensing, transmitting, and receiving. Finding an optimal solution to the routing problem which in addition to minimizing the queueing length also incorporates the goal of minimizing energy consumption and maximizing the nodes life-time is a more challenging problem that deserves further investigation.

Appendix A

Stability and Controllability of Time-Delayed Systems

Let us consider a linear and a nonstationary dynamical system described by following ordinary differential equation:

$$\dot{x}(t) \;=\; A(t)x(t) + \sum_{i=0}^{M} B_i(t)u(v_i(t)) + B_w w(t),\ t \geq t_0 \tag{A.1}$$

where $A(t)$ is an $(n \times n)$-dimensional matrix, $B_i(t), i = 1, ..., M$ are $(n \times m)$-dimensional matrices, and the functions $v_i : [t_0, \infty) \longrightarrow R, i = 0, 1, ..., M$ are absolutely continues and strictly increasing ($\dot{v} > 0$ almost everywhere in $[t_0, \infty)$) and satisfy the following inequalities:

$$v_M(t) < v_{M-1}(t) < \ldots < v_0(t) = t, t \in [t_0, \infty)$$

To be more specific, let us define $v_i(t)$ as

$$v_t(t) = t - h_i(t), i = 0, ..., M,\ t \in [t_0, \infty)$$

where $h_i(t) \geq h \geq 0, i = 0, ...M$ are time-variable delay functions. The initial value of x can also be defined as $x(t) = \phi(t),\ t \in [-h, 0]$.

The concepts of stability, stabilizability and controllability of time-delayed system (A.1) are defined as follows:

Definition A.1. [65] System (3.2) with $u \equiv 0$ and $w \equiv 0$ is said to be stable if for every positive ε there exits a positive δ, which may depend on the initial time and ε, such that if $\|\phi(.)\| < \delta$, then $\|x(t, t_0, \phi(.)\| < \varepsilon \quad \forall t \geq 0$, where $\|\phi(.)\| = \max_{s\in[-h,0]} \|\phi(s)\|$.

Definition A.2. [65] A dynamical time-delay system (3.2) with $w \equiv 0$ is said to be stabilizable if there exist a control law $u(t)$ such that the closed-loop system us stable in sense of Definition A.1.

A stronger notion than stabilizablity is controllability. A system is determined to be stabilizable when all uncontrollable states have stable dynamics. Thus, even though some of the state cannot be controlled, all the states will still remain bounded during the system's evolution. The concept of controllability for time-delayed systems is more complex than non-delayed systems and various definitions and corresponding criteria for controllability of time-delayed systems is introduced in the literature [134], [135], [136]. Here, the notion of relative controllability of time-delayed system (A.1) is presented [137].

Definition A.3. [65] The dynamical system (A.1) is said to be *relatively controllable* in $[t_0, t_1]$, if for any initial state $\phi(.)$ and any vector $x_1 \in R^n$, there exists a control $u \in L^2([t_0, t_1], R^m)$ such that the corresponding trajectory $x(t, \phi, u)$ of the dynamical system (A.1) satisfies the following condition

$$x(t_1, \phi, u) = x_1$$

Now let us define the inverse function of v_i as a leading function $r_i : [v_i(t_0), v_i(t_1)] \to [t_0, t_1]$. The following theorem formulates an algebraic condition for the relative controllability in $[t_0, t_1]$ of the dynamical system (A.1).

Theorem A.1. *[137] Assuming that matrix $A(t)$ and functions $v_i(t)$ are analytic on $[T_0, T_1]$ and the matrices $B_i(t)$ are analytic on $[r_i(t_0), t_1]$, and $w \equiv 0$, the dynamical* *system (A.1) is relatively controllable in* $[t_0, t_1]$ *if and only if*

$$rank Q_\infty(t_1) = n \tag{A.2}$$

where

$$Q_p(t) = [L^0 B_0(t) \ \dots \ L_0 B_M(t) \ L^1 B_0(t) \ \dots \ L^1 B_M(t) \ \dots \ L^p B_M(t)]$$

denote an $(n \times mMp)$*-dimensional matrix defined for* $t \in [r_M(t_0), t_1]$*, and the operator* L *is defined as shown below:*

$$\begin{aligned} L^0 B_i(t) &= B_i(t)\dot{r}_i(v_i(t)), \quad i = 1, \dots, M, \ t \in [r_i(t_0), t_1] \\ L^j B_i(t) &= -A_i(t)\dot{r}_i(v_i(t))L^{j-1}B_i(t) + \frac{d}{dt}L^{j-1}B_i(t), \ j = 0, \dots, p \end{aligned}$$

The proof of the theorem is given in [137].

Remark A.1. For a time-invariant dynamical system with a fixed delay function $h(t) = h_i$, the relatively controllable condition (A.2) is simplified to

$$rank[B_0, \dots, B_M, AB_0, \dots, AB_M, \dots, A^{n-1}B_0, \dots, A^{n-1}B_M] = n$$

Appendix B

The Markov Process

A Markov process, named after the Russian mathematician Andrey Markov, is a mathematical model for the random evolution of a memoryless system. Often the property of being "memoryless" is expressed such that conditional on the present state of the system, its future and past are independent. Mathematically, the Markov process is expressed as follows

Definition B.1. Let $\{r_t, t \geq 0\}$ be a stochastic process taking values in $\mathcal{S} = \{1, 2, , \ldots, M\}$. Then, $\{r_t, t \geq 0\}$ is said to be a *Markov process* with the state space $\mathcal{S}$ if

$$\mathbb{P}[r_t = i | r_w : w \leq s] = \mathbb{P}[r_t = i | r(s)]$$

holds for all $0 \leq s \leq t$ and $i \in \mathcal{S}$. For any $i, j \in \mathcal{S}$, let $\mathbb{P}_{ij}(s, t)$ denote the transition probability, i.e., $\mathbb{P}_{ij}(s, t) = \mathbb{P}(r(t) = j | r(s) = i)$. Matrix $\mathbb{P}(s, t) = (\mathbb{P}_{ij}(s, t)),\ i, j \in \mathcal{S}$ is said to be the transition matrix.

Definition B.2. Let $\mathcal{A}$ be the weak infinitesimal generator, acting on function $V : \{[-d, 0], \mathbb{R}^n) \times S \times \mathbb{R}_+ \to \mathbb{R}$. It is defined as follows [113]:

$$\mathcal{A}V(x_t, i, t) = \lim_{\Delta \to 0_+} \frac{1}{\Delta} \{\mathbb{E}[V(x_{t+\Delta}, r_{t+\Delta}, t + \Delta) | x_t, r_t = i] - V(x_t, i, t)\}$$

Definition B.3. Dynkin's formula: Let β be a random time with $\mathbb{E}[\beta(x_0, r_0)] < \infty$ and let $V(x_t, r_t)$ be in the domain of $\mathcal{A}$. Then

$$\mathbb{E}[V(x_\beta, r_\beta)|(x_0, r_0)] - V(x_0, r_0) = \mathbb{E}[\int_0^\beta \mathcal{A}V(x_s, r(s))ds|(x_0, r_0)] \tag{B.1}$$

Appendix C

Two Routing Algorithms for Mobile Networks

A brief description of two commonly used routing algorithms for mobile networks in the literature, namely Ad hoc On Demand Distance Vector (AODV) and Optimized Link State Routing Protocol (OLSR) is given in this appendix. It should be noted that AODV and OLSR algorithms are usually used as benchmarks for comparative studies in the literature [33], [54] [138].

C.1 Ad hoc On Demand Distance Vector Algorithm (AODV) [49] [50]

AODV is a reactive protocol. It is an on demand algorithm, implying that it builds routes between nodes only as desired by the source nodes. In other words, AODV requires the communication to be postponed until a route to the destination is found. The advantage of AODV is that it creates no extra traffic for communication along existing links. However, AODV requires more time to establish a connection, and the initial communication to establish a route is heavier than proactive protocols.

C.1.1 Technical Description

When a node requires a route towards a destination for which it does not have a route yet, three control messages are used to find the route: RREQ (Route REQuest), RREP (Route REPLY), RERR (Route ERRor). At first the source node broadcasts a RREQ. If a node receives a RREQ which it has not seen before it sets up a reverse route to the sender. If it does not know a route to the destination it rebroadcasts the updated RREQ especially incrementing the hop count. If it knows a route to the destination it creates a RREP which is returned to the origin node. A RREP includes a unique identifier, the destination IP address and sequence number, the source IP address, sequence number, and a hop count. When a node receives a RREP it checks if the hop count in the RREP for the emitter of the message is lower than the one in its own routing table or the destination sequence number in the message is higher than the one in its own routing table. If none of them is true it just throws the package away. Otherwise, it updates its routing table and if it is not the destination it reunicasts the RREP. Moreover, if a node realizes that other nodes are not any longer reachable it broadcasts a RERR containing a list of the unreachable nodes with there IP addresses and sequence number. Fig. C.1 demonstrates a routing establishment by using AODV algorithm.

C.2 Optimized Link State Routing Protocol (OLSR) [48], [139]

The Optimized Link State Routing Protocol (OLSR) is a proactive routing protocol. In this method, the routes to all destinations within the network are known and maintained before use. Having the routes available within the standard routing table, there is no route discovery delay associated with finding a new route. However, OLSR uses power and network resources in order to propagate data about possibly unused routes.

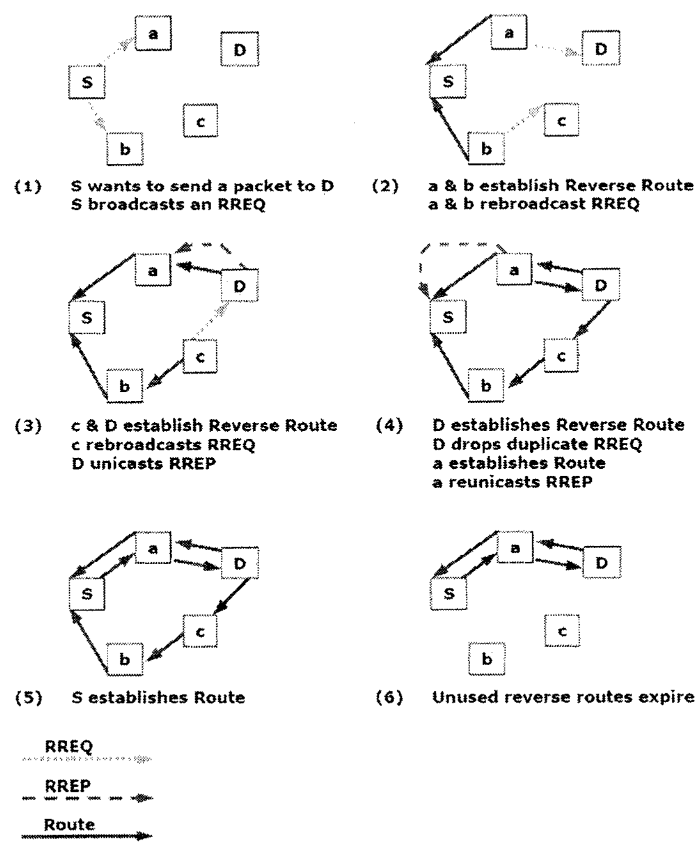

Figure C.1: An example for routing establishment by using AODV algorithm [8]

C.2.1 Technical Description

Every node sends periodically broadcast "Hello"-messages with information to specific nodes in the network to exchange neighborhood information. The information includes the nodes IP, sequence number and a list of the distance information of the nodes neighbors. After receiving this information a node builds itself a routing table. The node can then calculate with the shortest path algorithm the route to every node it wants to communicate. In the routing tables the information on the route to each node in the network is stored and the information is only updated when:

- a change in the neighborhood is detected.
- a route to any destination is expired.
- a better (shorter) route is detected for a destination.

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