Generation of 2-D Digital Filter from an active analog network with Application in Image Processing

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A Thesis

in

The Department

of

Electrical and Computer Engineering

Presented in Fulfillment of the Requirements for the

Degree of Master of Applied Science at

Concordia University

Montreal, Quebec, Canada, H3G 1M8

July 2008

Your file Votre référence
ISBN: 978-0-494-45348-3
Our file Notre référence
ISBN: 978-0-494-45348-3

Abstract

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A new approach to generate 2-D filters having variable magnitude characteristics from an active analog doubly terminated network has been proposed. An active analog circuit with reactance elements in T section in the feedback has been considered. Its stability has been ensured and a 2-D analog lowpass filter has been generated. The impedance values for the filter is obtained in comparison with type I chebyshev filter with 1db ripple in the pass band.

The 2-D analog lowpass filter has been transformed to digital domain by applying the generalized bilinear transformation. Similarly 2-D digital high pass filter has been obtained. The 2-D digital bandpass filter has been obtained by cascading the lowpass and highpass filters.

The 2-D digital filters are studied under five different cases. These five different cases are based on the coefficients of generalized bilinear transformation and the op-amp gain parameter. The effect of each generalized bilinear transformation coefficient and the op-amp gain on the filter output is studied by individually varying them.

Finally, performance comparison between the infinite gain and finite gain configuration has been done for the lowpass filter with a basic image processing application. A basic application for 2-D digital highpass filter with finite gain has been illustrated in image processing.

ACKNOWLEDGEMENTS

I thank, Dr. Venkat Ramachandran, my supervisor, for his continuous support throughout the length of this endeavour. It is Dr. Venkat Ramachandran's vision and ideas that provided the foundation for this thesis. I am deeply indebted to him for providing me an opportunity to work with him. He has been extremely helpful, understanding and extra-ordinarily patient in the critical review of my thesis.

Thanks to my parents, brother and friends for their support and encouragement that has brought me this far.

Dedicated to my Mother,

Late Mrs. Usha Sundharesan

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List of Symbols and Abbreviations

GBT : Generalized bilinear transformation.

VSHP : Very Strict Hurwitz Polynomial.

IIR : Infinite Impulse Response.

FIR : Finite Impulse Response.

VSHP : Very Strict Hurwitz Polynomial.

BIBO : Bounded Input Bounded Output.

H_a : Transfer function of a filter in analog domain.

H_d : Transfer function of a filter in digital domain.

N : Numerator of the transfer function.

D : Denominator of the transfer function.

s_1, s_2 : Laplace domain parameter in two dimensions.

z_1, z_2 : Z-domain parameter in two dimensions.

a_1, a_2 : Gain affecting coefficients of GBT.

b_1, b_2 : Polarity affecting coefficients of GBT.

k_1, k_2 : Bandwidth affecting coefficients of GBT.

ω_1, ω_2 : Spatial Frequencies.

$*$: Convolution.

k : Operational amplifier gain.

$\uparrow$	: Increases.
$\downarrow$	: Decreases.

Chapter 1

Introduction

Digital filter is just like a filter which operates on digital signals. It's a computation which takes one sequence of numbers i.e. the input signal and produces a new sequence of numbers i.e. the filtered output signal. Thus digital filter can be considered as a mathematical equation which translates one digital signal to another. [1]

Digital filter can provide any required degree of precision. Digital filter characteristics can be easily changed, they are much more reliable and repeatable, they are free from component drift and no tuning is required.

Design of Multi-dimensional filter has increasingly attracted considerable attention during the recent years and is still receiving significant interest by both theorists and practitioners. Multi-dimensional signal processing has many applications in modern day devices and many practical systems because of which, this subject is still being investigated in important areas as moving-objects recognition, robotics, medical imaging and so on. [2]

Two dimensional (2-D) digital systems have gained lot of attention due to its high efficiency, high speed computations, permitting high quality image processing and analysis, also providing greater application flexibility and adaptability. The 2-D digital filters have numerous applications in various fields such as image processing, video signal processing and seismic signal processing [3].

The 2-D digital filter can be classified into 2-D Finite Impulse Response (FIR) filter or non recursive filter and Infinite Impulse Response (IIR) filter or recursive filter. In this work, concentration would be given to the design of 2-D IIR filter or recursive filters.

1.1 *Research Objectives*

The primary objective of this research is to design a stable 2-D active analog LC filter and obtain its equivalent in digital domain by applying transformation which hasn't been done so far in the literature and to study its characteristics. Its lowpass, highpass and bandpass configurations would be developed and its characteristics would be studied.

The preliminary goal of this thesis is to develop a suitable structure for 2-D analog active IIR stable filter. The stability is ensured by making sure that the transfer function polynomial is a Very Strict Hurwitz Polynomial (VSHP).

Once a stable 2-D analog filter has been developed, suitable transformation would be applied to obtain the equivalent 2-D digital lowpass filter. After obtaining the 2-D digital lowpass filter, suitable frequency transformation are applied to get the equivalent highpass filter and bandpass filters.

At last the 2-D digital active LC lowpass, highpass and bandpass filters applications are demonstrated with the image processing applications.

1.2 *Infinite Impulse Response Filters (IIR Filters)*

The transfer function of 2-D IIR filters can be described by using 2-D z-transform [3] and can be expressed as a ratio of two-variable polynomials as follows:

$$H_d(z_1, z_2) = \frac{N_d(z_1, z_2)}{D_d(z_1, z_2)} = \frac{\sum_{i=0}^{I} \sum_{j=0}^{J} a_{ij} z_1^{-i} z_2^{-j}}{\sum_{k=0}^{K} \sum_{l=0}^{L} b_{kl} z_1^{-k} z_2^{-l}} \tag{1.1}$$

where $b_{00}=1$, a_{ij} and b_{kl} are real coefficients. For any input signal $X(z_1,z_2)$, the output $Y(z_1,z_2)$ of the filter is given by

$$Y(z_1, z_2) = H(z_1, z_2) * X(z_1, z_2) \tag{1.2}$$

In the 2-D IIR filter, one important problem to be dealt with is stability. According to the stability theorem [4][5], the 2-D IIR filter is guaranteed to be stable in the bounded-input bounded-output (BIBO) sense, if there exists no value of z_1 and z_2 for which $D(z_1,z_2) = 0$ for both $|z_1| \geq 1$and $|z_2| \geq 1$ [3]. This means it is highly preferable that the given analog transfer function must have VSHP denominator [4]. Therefore, the design of a 2-D IIR filter requires obtaining the coefficients a_{ij} and b_{kl} in eqn. (1.1) so that $H(e^{j\omega_1 t_1}, e^{j\omega_2 t_2})$ approximates a given response $G(j\omega_1, j\omega_2)$ where ω_1 and ω_2 are horizontal and vertical spatial frequencies respectively, which also ensures the stability of the filter.

1.3 *Different methods of designing a 2-D IIR filters [3, 7, 8]*

One of the methods of designing a IIR filter can be classified into three steps: the design using analog prototype filter, the design using digital frequency transformation and last one is computer-aided design. In the first step, an analog filter is designed to the (analog) specification and the analog filter transfer function is transformed into digital system function using transformation. In the second step, it's assumed that a digital low-pass filter can be designed. The desired digital filter is obtained from the digital low-pass filter by digital frequency transformation. The last step uses some algorithm to choose the coefficients so that the response is as close as possible to the desired filter.

Steps one and two would be used to get the 2-D active LC IIR filter. First an analog filter is designed and it is made sure that its transfer function is a VSHP in order to ensure the stability of the filter. Then its equivalent lowpass digital filter is obtained by applying generalized bilinear transformation (GBT).

Highpass and bandpass filters are obtained from the lowpass by using the second step mentioned above.

1.4 *2-D Stability Criteria - Very Strict Hurwitz Polynomial [4]*

In one-dimensional (1-D) systems (both analog and discrete), a filter having required specifications with the transfer function having no common factors between the numerator and the denominator is designed. Let

$$H_a(s) = \frac{N_a(s)}{D_a(s)} \tag{1.3}$$

be a transfer function in the analog domain with $N_a(s)$ and $D_a(s)$ being relatively prime. In order that the function is stable, $D_a(s)$ should be a strictly Hurwitz polynomial (SHP). A SHP contains its zeros strictly in the left half of the s-plane. Similarly, if

$$H_d(z) = \frac{N_d(z)}{D_d(z)} \tag{1.4}$$

is a transfer function in the discrete domain with $N_d(z)$ and $D_d(z)$ relatively prime, then $D_d(z)$ should be a Schur polynomial in order that $H_d(z)$ shall be stable. A Schur polynomial contains its zeros strictly within the unit circle.

In the case of 2-D analog systems, there is a possibility that both even and odd parts of a polynomial may become zero simultaneously at a specified set of points, but not in their neighbourhood. This phenomenon is known as singularities, which makes the filter unstable. There are two kinds of singularities which have to be avoided.

Consider a 2-D analog system $H_a(s_1,s_2)$ such as

$$H_a(s_1,s_2) = \frac{N_a(s_1,s_2)}{D_a(s_1,s_2)} \tag{1.5}$$

Two kinds of singularities might arise for the above analog system which has to be avoided,

(i) $D_a(s_{10},s_{20}) = 0$ and $N_a(s_{10},s_{20}) \neq 0$, this leads to non-essential singularity of the first kind at (s_{10},s_{20}).

(ii) $D_a(s_{10},s_{20}) = 0$ and $N_a(s_{10},s_{20}) = 0$, this leads to non-essential singularity of the second kind at (s_{10},s_{20}).

The similar situation exists in the case of 2-D discrete systems also. This leads to a class of polynomials called Very Strict Hurwitz Polynomials (VSHP) which doesn't contain singularities of the type mentioned above. A VSHP is defined as follows:

"$D_a(s_1,s_2)$ is a VSHP, if $\dfrac{1}{D_a(s_1,s_2)}$ does not possess any singularities in the region $\{(s_1,s_2) \mid \text{Re } s_1 \geq 0, \text{Re } s2 \geq 0, |s_1| \leq \infty, \text{and } |s_2| \leq \infty\}$"

The different methods of generating a VSHP and its properties have been discussed in detail in [4].

1.5 *Generation of VSHP [8]*

When a VSHP is used in the denominator of a 2-D analog transfer function, it is ensured that the resulting 2-D digital transfer function obtained by the application of the well-known bilinear transformation is stable. Therefore, VSHP is highly useful in the 2-D digital filter design. A two-variable VSHP is generated and it is assigned to the denominator of the 2-D analog transfer function, then double bilinear transformation is applied to obtain the transfer function in digital domain. Here, the method used to generate a VSHP is reviewed.

One of the simplest methods of generating a VSHP is to start from the VSHP

$$D_a(s_1,s_2) = a_{11}s_1s_2 + a_{10}s_1 + a_{01}s_2 + a_{00} \tag{1.6}$$

The reactance function is

$$G_{a_1}(s_1,s_2) = \frac{a_{11}s_1s_2 + a_{00}}{a_{10}s_1 + a_{01}s_2} \tag{1.7}$$

On applying the transformation

$$s_1 = \frac{b_{11}s_1s_2 + b_{00}}{b_{10}s` + b_{01}s_2} \tag{1.8}$$

Where $b_{11}>0$, $b_{10}>0$, $b_{01}>0$ and $b_{00}>0$

This result in

$$G_{a2}(s_1,s_2) = \frac{P_{a2}(s_1,s_2)}{Q_{a2}(s_1,s_2)} \tag{1.9}$$

Where

$$P_{a2}(s_1,s_2) = a_{11}b_{11}s_1s_2^2 + a_{00}b_{10}s_1 + (a_{11}b_{00} + a_{00}b_{01})s_2 \tag{1.10}$$

$$Q_{a2}(s_1,s_2) = a_{01}s_2^2 + (a_{10}b_{11} + a_{01}b_{10})s_1s_2 + a_{10}b_{00} \tag{1.11}$$

The polynomial $D_{a2}(s_1,s_2)=P_{a2}(s_1,s_2)+Q_{a2}(s_1,s_2)$ is a VSHP in which s_1 is of unity degree and s_2 is of second degree. When the transformations s_1 as in eqn. (1.8) and

$$s_2 = \frac{c_{11}s_1s_2 + c_{00}}{c_{10}s_1 + c_{01}s_2} \tag{1.12}$$

Where $c_{11}>0$, $c_{10}>0$, $c_{01}>0$ and $c_{00}>0$ are applied for $G_{a1}(s_1,s_2)$ in eqn. (1.9), the resulting VSHP contains s_1 and s_2 of second degree each. The resulting reactance function is given by

$$G_{a3}(s_1,s_2) = \frac{P_{a3}(s_1,s_2)}{Q_{a3}(s_1,s_2)} \tag{1.13}$$

Where

$$\begin{aligned} P_{a3}(s_1,s_2) = {} & a_{11}b_{11}c_{11}s_1^2s_2^2 + a_{00}b_{00}c_{10}s_1^2 + (a_{00}b_{01}c_{01} + a_{00}b_{01}c_{01} + a_{11}b_{00}c_{11} + a_{11}b_{11}c_{00})s_1s_2 \\ & + a_{00}b_{01}c_{01}s_2^2 + a_{11}b_{00}c_{00} \end{aligned} \tag{1.14}$$

and

$$\begin{aligned} Q_{a3}(s_1,s_2) = {} & (a_{10}b_{11}c_{10} + a_{01}b_{10}c_{11})s_1^2s_2 + (a_{10}b_{11}c_{01} + a_{01}b_{10}c_{11})s_1s_2^2 + (a_{10}b_{00}c_{10} + a_{01}b_{10}c_{00})s_1 \\ & + (a_{10}b_{00}c_{01} + a_{01}b_{01}c_{00})s_2 \end{aligned} \tag{1.15}$$

These transformations are applied again if a higher order VSHP is required.

1.6 *Bilinear Transformation [6],[12]*

Most often used transformation method in the literature is GBT method, in order to transform a analog filter into a digital filter.

$$s_i = k_i \frac{z_i - a_i}{z_i + b_i}, \quad where \quad i = 1,2 \tag{1.16}$$

The bilinear transformation maps the entire (s_1, s_2) biplane on to the entire (z_1, z_2) biplane, on one-to-one basis. The stability condition for the bilinear transformation is

$$k_i \neq 0, \quad |a_i| \leq 1, \quad |b_i| \leq 1 \tag{1.17}$$

1.7 *Verification of VSHP [4]*

In order to determine whether a given two-variable polynomial $D_a(s_1, s_2)$ is a VSHP or not, whether it is a SHP or not has to be determined. To ensure this the following procedures would be followed:

i. Determine that $D_a(s_1, 1)$ is SHP in s_1.

ii. From the given polynomial $D_a(s_1,s_2)$, formulate

$D_a(j\omega_1,j\omega_2)=[A_p(\omega_2)\omega_2]$

$$\begin{aligned} D_a(j\omega_1, j\omega_2) = & \begin{bmatrix} Ap(\omega_2)\omega_1^p + A_{p-1}(\omega_2)\omega_1^{p-1} + \dots + A_2(\omega_2)\omega_1 \\ + A_1(\omega_2)\omega_1 + A_0(\omega_2) \end{bmatrix} \\ & + j\begin{bmatrix} B_p(\omega_2)\omega_1^p + B_{p-1}(\omega_2)\omega_1^{p-1} + \dots\dots\dots + \\ B_1(\omega_2)\omega_1 + B_0(\omega_2) \end{bmatrix} \end{aligned} \tag{1.18}$$

where $A_i(\omega_2)$ and $B_i(\omega_2)$, $i=0,1,1,2\dots\dots p$ are polynomials in ω_2.

iii. Now (1.18) shall be rearranged in the form of Inners as follows:

$$\left[\begin{array}{ccc|cccc|ccc}
B_p & B_{p-1} & B_{p-2} & --- & --- & --- & --- & 0 & 0 & 0 \\
0 & B_p & B_{p-1} & --- & --- & --- & --- & 0 & 0 & 0 \\
0 & 0 & B_p & --- & --- & --- & --- & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & B_p & B_{p-1} & B_{p-2} & B_{p-3} & --- & --- & --- \\
0 & 0 & 0 & 0 & B_p & B_{p-1} & B_{p-2} & --- & --- & --- \\
0 & 0 & 0 & 0 & A_p & A_{p-1} & A_{p-2} & --- & --- & --- \\
0 & 0 & 0 & A_p & A_{p-1} & A_{p-2} & A_{p-3} & --- & --- & --- \\
\hline
0 & 0 & A_p & --- & --- & --- & --- & 0 & 0 & 0 \\
0 & A_p & A_{p-1} & --- & --- & --- & --- & 0 & 0 & 0 \\
A_p & A_{p-1} & A_{p-2} & --- & --- & --- & --- & 0 & 0 & 0
\end{array}\right]$$

iv. In order that $D_a(s_1,s_2)$ is a SHP, it is required that the inner determinants $\Delta_k>0$, $k=1,2,\ldots.p$, for all ω_2.

v. If the certain conditions of $D_a(s_1,s_2)$ are satisfied then it can be conclude that the given $D_a(s_1,s_2)$ is a VSHP. The conditions are:

$$D_a\left(s_1,\frac{1}{s_2}\right)\neq\frac{0}{0},\ as\ s_1\rightarrow 0\ and\ s_2\rightarrow 0 \tag{1.19}$$

$$D_a\left(\frac{1}{s_1},s_2\right)\neq\frac{0}{0},\ as\ s_1\rightarrow 0\ and\ s_2\rightarrow 0 \tag{1.20}$$

$$D_a\left(\frac{1}{s_1},\frac{1}{s_2}\right)\neq\frac{0}{0},\ as\ s_1\rightarrow 0\ and\ s_2\rightarrow 0 \tag{1.21}$$

In order to determine whether the generated polynomial is VSHP or not condition (v) would be used to verify. Once a VSHP is generated and is used in the denominator of a 2-D analog transfer function, it is guaranteed that the resulting 2-D digital transfer function obtained by applying the well-known bilinear transformation is stable [10], [11],[12].

1.7 *Organization of the thesis*

In chapter 1, IIR filter, a special type of polynomial for 2-D filter called VSHP, one of the methods of generating a VSHP that would be used in this thesis, methods to test whether obtained polynomial is VSHP and the transformation used to obtain a transfer function in digital domain from an analog domain transfer function are discussed.

Chapter 2 describes the method to obtain the transfer function of the 2-D active digital filter with infinite gain op-amp and finite gain op-amp. Stability of the transfer function is tested and the impedance values are found.

Chapter 3 outlines the method to obtain the transfer function of the lowpass filter in digital domain by applying the GBT to the analog transfer function obtained in chapter 2 for both infinite gain and finite gain cases. Then effect of GBT coefficient and the op-amp gain parameter k on the filter output characteristics would be studied.

Chapter 4 presents the method to obtain the transfer function of the highpass filter in digital domain by applying suitable GBT to the analog transfer function obtained in chapter 2. Then the effect of the individual GBT coefficients and op-amp parameter k on the 2-D digital highpass filter output would be studied.

Chapter 5 is a description of the method to obtain the bandpass filter in digital domain from the transfer function of the lowpass and highpass filter in the digital domain. Then effect of individual GBT coefficients and op-amp gain parameter k on the filter output would be examined.

In chapter 6 few applications of 2-D digital filters in image processing for image restoration and image enhancement has been discussed. The performance of the 2-D active digital filter in lowpass configuration would be compared for the filter with finite gain op-amp and infinite gain op-amp.

Chapter 7 summarizes the work done and outlines future initiatives that can be undertaken.

Chapter 2

Design of 2-D active IIR filter with T section in the feedback

2.1 Introduction

In the literature of the two dimensional filter design and analysis, so far study has been carried out in the passive domain only [14], [15], [16]. Filters have been designed by first ensuring the stability. In this work the possibilities of designing a 2-D active filter would be examined and its characteristics would be studied. In order to ensure stability, we would be using VSHP criteria.

The presence of operational amplifier in the 2-D active filter leads to two cases, operational amplifier with infinite gain and the other one with finite gain. Practical applications involving 1-D active filter, op-amp with finite gain are of interest. Detailed study of design procedures and characteristics of the 2-D active filter, with infinite gain op-amp and finite gain op-amp would be done.

Active networks containing impedances such as inductors and capacitances will have frequency and phase response which may be advantageous to the engineers in the filter design applications. An active network with a T network in the feedback path as shown in fig.2.1 would be considered.

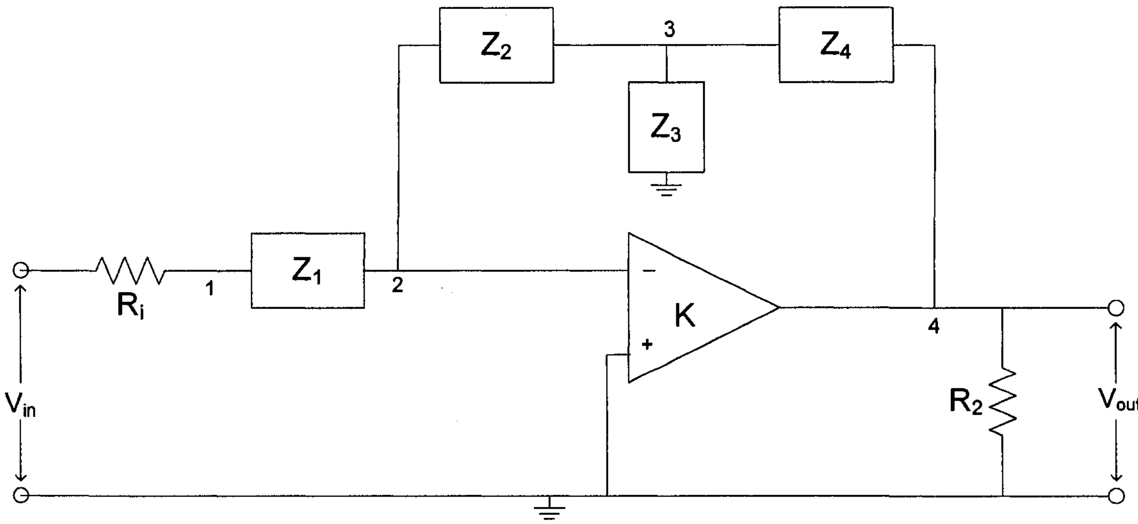


Figure 2.1: An active filter with T network in the feedback path.

The 2-D analog filter is obtained by substituting impedance values for Z_1, Z_2, Z_3 and Z_4. The stability of the transfer function is ensured by ensuring the denominator polynomial is VSHP. The various method of testing VSHP is described later in this chapter and one of the methods would be employed.

Various possibilities of generating a VSHP by fitting different impedance values for Z_1, Z_2, Z_3 and Z_4 are tried out. Once the transfer function with a VSHP polynomial is obtained and then the impedances values are found out, then generalized bilinear transformation is applied to obtain the equivalent digital filter.

By applying node voltage analysis to the circuit diagram in fig. 2.1 the transfer function of the filter in analog domain for the two cases are obtained. The transfer function of the filter with infinite gain op-amp is

$$\frac{V_O}{V_i} = -\frac{Z_2 Z_3 + Z_3 Z_4 + Z_4 Z_2}{Z_3 (Z_1 + R_i)} \tag{2.1}$$

Transfer function for op-amp with finite gain k is

$$\frac{V_O}{V_i} = -\frac{kZ_1\Delta Z}{Z_1Z_3(k+1)+\Delta Z(R_i+1)+Z_1Z_4} \tag{2.2}$$

where, $\Delta Z = Z_2Z_3 + Z_3Z_4 + Z_4Z_2$

Different values are substituted for impedances Z_1, Z_2, Z_3 and Z_4 in the above two equations and various possibilities of generating a VSHP are obtained.

2.2 *Stability test of the transfer function with infinite gain operational amplifier.*

The transfer function of the filter with infinite gain op-amp is given by eqn. (2.1). The transfer function has four impedance variables Z_1, Z_2, Z_3 and Z_4 and these

CASE	Z1	Z2	Z3	Z4	VSHP/NOT VSHP
1	$1/S_1C_1$	$1/S_2C_2$	S_2L_3	S_1L_4	VSHP
2	$1/S_1C_1$	$1/S_2C_2$	S_2L_3	$1/S_1C_4$	Not Vshp $H(1/S_1,S_2)=0/0$
3	$1/S_1C_1$	$1/S_2C_2$	$1/S_2C_3$	$1/S_1C_4$	Not Vshp $H(S_1,1/S_2)=0/0$
4	$1/S_1C_1$	$1/S_2C_2$	$1/S_2C_3$	S_1L_4	Not Vshp $H(1/S_1,S_2)=0/0$
5	$1/S_1C_1$	S_2L_2	S_2L_3	S_1L_4	Not Vshp $H(1/S_1,S_2)=0/0$
6	$1/S_1C_1$	S_2L_2	S_2L_3	$1/S_1C_4$	Not Vshp $H(1/S_1,S_2)=0/0$
7	$1/S_1C_1$	S_2L_2	$1/S_2C_3$	$1/S_1C_4$	Not Vshp $H(S_1,1/S_2)=0/0$
8	$1/S_1C_1$	S_2L_2	$1/S_2C_3$	S_1L_4	Not Vshp $H(S_1,1/S_2)=0/0$
9	S_1L_1	S_2L_2	S_2L_3	S_1L_4	Not Vshp $H(1/S_1,S_2)=0/0$
10	S_1L_1	S_2L_2	S_2L_3	$1/S_1C_4$	Not Vshp $H(1/S_1,S_2)=0/0$
11	S_1L_1	S_2L_2	$1/S_2C_3$	$1/S_1C_4$	Not Vshp $H(1/S_1,1/S_2)=0/0$
12	S_1L_1	S_2L_2	$1/S_2C_3$	S_1L_4	Not Vshp $H(S_1,1/S_2)=0/0$
13	S_1L_1	$1/S_2C_2$	S_2L_3	S_1L_4	VSHP
14	S_1L_1	$1/S_2C_2$	S_2L_3	$1/S_1C_4$	Not Vshp $H(1/S_1,S_2)=0/0$
15	S_1L_1	$1/S_2C_2$	$1/S_2C_3$	$1/S_1C_4$	Not Vshp $H(1/S_1,S_2)=0/0$
16	S_1L_1	$1/S_2C_2$	$1/S_2C_3$	S_1L_4	Not Vshp $H(1/S_1,S_2)=0/0$

Table 2.1: Impedance combinations for stable filter with infinite gain op-amp.

impedances are reactance's in two dimension (s_1, s_2). These impedances would be either inductor or capacitor or combination of inductor and resistor or capacitor and resistor. Various possibilities for these impedances to generate a fourth order VSHP is tested out in the Table 2.1. In Table 2.1, for the cases 1-16 impedances Z_1 and Z_4 are considered to be in s_1 domain and impedances Z_2 and Z_3 are considered to be in s_2 domain.

For each and every case the impedance value is substituted in the transfer function and tested for VSHP test cases. The test cases are as follows [4]

$$\frac{V_o}{V_{in}}\left(\frac{1}{s_1},s_2\right)\neq\frac{0}{0},\quad \frac{V_o}{V_{in}}\left(s_1,\frac{1}{s_2}\right)\neq\frac{0}{0},\quad \frac{V_o}{V_{in}}\left(\frac{1}{s_1},\frac{1}{s_2}\right)\neq\frac{0}{0}. \tag{2.3}$$

CASE	Z1	Z2	Z3	Z4	VSHP/NOT VSHP
1	$1/S_1C_1$	$1/S_2C_2$	S_1L_3	S_2L_4	Not Vshp Ha(S_1,1/S_2)=0/0
2	$1/S_1C_1$	$1/S_2C_2$	S_1L_3	$1/S_2C_4$	Not Vshp Ha(1/S_1,S_2)=0/0
3	$1/S_1C_1$	$1/S_2C_2$	$1/S_1C_3$	$1/S_2C_4$	Not Vshp Ha(S_1,1/S_2)=0/0
4	$1/S_1C_1$	$1/S_2C_2$	$1/S_2C_3$	S_2L_4	Not Vshp Ha(S_1,1/S_2)=0/0
5	$1/S_1C_1$	S_2L_2	S_1L_3	S_2L_4	Not Vshp Ha(S_1,1/S_2)=0/0
6	$1/S_1C_1$	S_2L_2	S_1L_3	$1/S_2C_4$	Not Vshp Ha(S_1,1/S_2)=0/0
7	$1/S_1C_1$	S_2L_2	$1/S_1C_3$	$1/S_2C_4$	Not Vshp Ha(1/S_1,S_2)=0/0
8	$1/S_1C_1$	S_2L_2	$1/S_2C_3$	S_2L_4	Not Vshp Ha(S_1,1/S_2)=0/0
9	S_1L_1	S_2L_2	S_1L_3	S_2L_4	Not Vshp Ha(1/S_1,1/S_2)=0/0
10	S_1L_1	S_2L_2	S_1L_3	$1/S_2C_4$	Not Vshp Ha(S_1,1/S_2)=0/0
11	S_1L_1	S_2L_2	$1/S_1C_3$	$1/S_2C_4$	Not Vshp Ha(1/S_1,S_2)=0/0
12	S_1L_1	S_2L_2	$1/S_2C_3$	S_2L_4	Not Vshp Ha(1/S_1,1/S_2)=0/0
13	S_1L_1	$1/S_2C_2$	S_1L_3	S_2L_4	Not Vshp Ha(1/S_1,S_2)=0/0
14	S_1L_1	$1/S_2C_2$	S_1L_3	$1/S_2C_4$	Not Vshp Ha(1/S_1,S_2)=0/0
15	S_1L_1	$1/S_2C_2$	$1/S_1C_3$	$1/S_2C_4$	VSHP
16	S_1L_1	$1/S_2C_2$	$1/S_2C_3$	S_2L_4	Not Vshp Ha(1/S_1,S_2)=0/0

Table 2.2: Impedance combinations for stable filter with infinite gain op-amp.

If all three conditions are satisfied then the corresponding impedance combination would make it possible for the transfer function to have a VSHP polynomial at the denominator, which ensures stability. The procedure to obtain a VSHP polynomial would be explained in the next section.

In Table 2.2, the impedances values are kept similar to Table 2.1 but impedances Z_1 and Z_3 are considered in s_1 domain and impedances Z_2 and Z_4 are considered in s_2 domain.

2.3 *Stability test of the transfer function with finite gain operational amplifier.*

Various combinations tried for the infinite gain op-amp case will also be tried out for the

CASE	Z1	Z2	Z3	Z4	VSHP/NOT VSHP
1	$1/S_1C_1$	$1/S_2C_2$	S_2L_3	S_1L_4	VSHP
2	$1/S_1C_1$	$1/S_2C_2$	S_2L_3	$1/S_1C_4$	Not Vshp $Ha(1/s_1,s_2)=0/0$
3	$1/S_1C_1$	$1/S_2C_2$	$1/S_2C_3$	$1/S_1C_4$	Not Vshp $Ha(1/s_1,1/s_2)=0/0$
4	$1/S_1C_1$	$1/S_2C_2$	$1/S_2C_3$	S_1L_4	Not Vshp $Ha(1/s_1,s_2)=0/0$
5	$1/S_1C_1$	S_2L_2	S_2L_3	S_1L_4	Not Vshp $Ha(1/s_1,s_2)=0/0$
6	$1/S_1C_1$	S_2L_2	S_2L_3	$1/S_1C_4$	Not Vshp $Ha(1/s_1,s_2)=0/0$
7	$1/S_1C_1$	S_2L_2	$1/S_2C_3$	$1/S_1C_4$	Not Vshp $Ha(1/s_1,s_2)=0/0$
8	$1/S_1C_1$	S_2L_2	$1/S_2C_3$	S_1L_4	Not Vshp $Ha(s_1,1/s_2)=0/0$
9	S_1L_1	S_2L_2	S_2L_3	S_1L_4	Not Vshp $Ha(1/s_1,1/s_2)=0/0$
10	S_1L_1	S_2L_2	S_2L_3	$1/S_1C_4$	Not Vshp $Ha(1/s_1,s_2)=0/0$
11	S_1L_1	S_2L_2	$1/S_2C_3$	$1/S_1C_4$	Not Vshp $Ha(1/s_1,1/s_2)=0/0$
12	S_1L_1	S_2L_2	$1/S_2C_3$	S_1L_4	Not Vshp $Ha(s_1,1/s_2)=0/0$
13	S_1L_1	$1/S_2C_2$	S_2L_3	S_1L_4	Not Vshp $Ha(s_1,1/s_2)=0/0$
14	S_1L_1	$1/S_2C_2$	S_2L_3	$1/S_1C_4$	Not Vshp $Ha(1/s_1,s_2)=0/0$
15	S_1L_1	$1/S_2C_2$	$1/S_2C_3$	$1/S_1C_4$	Not Vshp $Ha(s_1,1/s_2)=0/0$
16	S_1L_1	$1/S_2C_2$	$1/S_2C_3$	S_1L_4	Not Vshp $Ha(1/s_1,s_2)=0/0$

Table 2.3: Impedance combinations for stable filter with finite gain op-amp.

finite op-amp gain and the transfer function are obtained for the cases where VSHP are obtained after substituting the impedance values.

In the table 2.1, 2.2, 2.3 and 2.4, the different cases with VSHP indicates that a VSHP polynomial can be obtained. Case 1 of Table 2.1 and Table 2.3 would be considered. In both these cases, a fourth order filter would be obtained by substituting the impedance values. Throughout this work, only these two cases would be considered for the comparative study as well as the importance of controlling the GBT coefficient

CASE	Z1	Z2	Z3	Z4	VSHP/NOT VSHP
1	$1/S_1C_1$	$1/S_2C_2$	S_1L_3	S_2L_4	VSHP
2	$1/S_1C_1$	$1/S_2C_2$	S_1L_3	$1/S_2C_4$	Not Vshp $Ha(1/s_1,s_2)=0/0$
3	$1/S_1C_1$	$1/S_2C_2$	$1/S_1C_3$	$1/S_2C_4$	Not Vshp $Ha(1/s_1,1/s_2)=0/0$
4	$1/S_1C_1$	$1/S_2C_2$	$1/S_2C_3$	S_2L_4	Not Vshp $Ha(1/s_1,1/s_2)=0/0$
5	$1/S_1C_1$	S_2L_2	S_1L_3	S_2L_4	Not Vshp $Ha(1/s_1,s_2)=0/0$
6	$1/S_1C_1$	S_2L_2	S_1L_3	$1/S_2C_4$	Not Vshp $Ha(s_1,1/s_2)=0/0$
7	$1/S_1C_1$	S_2L_2	$1/S_1C_3$	$1/S_2C_4$	Not Vshp $Ha(1/s_1,s_2)=0/0$
8	$1/S_1C_1$	S_2L_2	$1/S_2C_3$	S_2L_4	Not Vshp $Ha(1/s_1,s_2)=0/0$
9	S_1L_1	S_2L_2	S_1L_3	S_2L_4	Not Vshp $Ha(1/s_1,1/s_2)=0/0$
10	S_1L_1	S_2L_2	S_1L_3	$1/S_2C_4$	Not Vshp $Ha(s_1,1/s_2)=0/0$
11	S_1L_1	S_2L_2	$1/S_1C_3$	$1/S_2C_4$	Not Vshp $Ha(1/s_1,1/s_2)=0/0$
12	S_1L_1	S_2L_2	$1/S_2C_3$	S_2L_4	VSHP
13	S_1L_1	$1/S_2C_2$	S_1L_3	S_2L_4	Not Vshp $Ha(s_1,1/s_2)=0/0$
14	S_1L_1	$1/S_2C_2$	S_1L_3	$1/S_2C_4$	Not Vshp $Ha(1/s_1,s_2)=0/0$
15	S_1L_1	$1/S_2C_2$	$1/S_1C_3$	$1/S_2C_4$	Not Vshp $Ha(s_1,1/s_2)=0/0$
16	S_1L_1	$1/S_2C_2$	$1/S_2C_3$	S_2L_4	Not Vshp $Ha(s_1,1/s_2)=0/0$

Table 2.4: Impedance combinations for stable filter with infinite gain op-amp.

and the gain parameter 'k' in the finite gain case will studied. Fourth order type I Chebyshev filter with 1db ripple would be considered to obtain the impedance v*alues.*

2.4 *Generation of impedance values for filter [6]*

Consider the transfer function of a 1-D fourth order type I chebyshev filter with 1db ripple. In order to get the impedance values, the transfer function of the filter is expanded by continued fraction.

The transfer function of the fourth order 1-D lowpass type I chebyshev filter (analog) with 1db ripple in the pass band is

$$\frac{V_0}{V_{in}} = \frac{1}{s^4 + 0.7014s^3 + 1.2745s^2 + 0.6667s + 0.2720} \tag{2.4}$$

$$\frac{V_0}{V_{in}} = \frac{0.2720}{3.676s^4 + 2.578s^3 + 4.685s^2 + 2.452s + 1} \tag{2.5}$$

After expanding the above transfer function eqn. (2.5) by continued fraction expansion we get

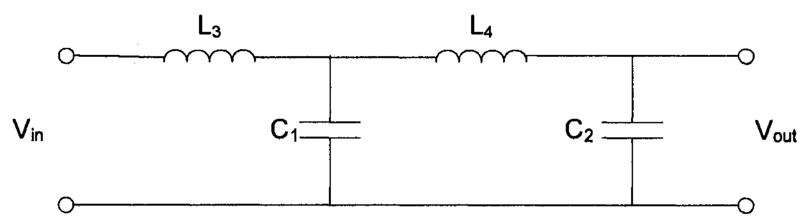


Fig 2.2 LC filter section

$$L_3=1.426,\ L_4=1.051,\ C_1=2.168,\ C_2=1.131 \tag{2.6}$$

The above filter is a two stage LC filter section. The value of the inductor and the capacitor in the first stage will be considered for the s_1 domain and the values in the second stage would be considered for the s_2 section.

2.5 *Transfer function of the active filter with infinite gain op-amp (Table 2.1, case 1)*

Transfer function of the 2-D analog active filter with infinite gain op-amp is obtained by substituting the impedance values for Z_1, Z_2, Z_3 and Z_4 from (Table 2.1, case 1) in the eqn. (2.1). The impedance value for case 1 in Table 2.1is

$$Z_1 = \frac{1}{s_1 C_1}, \quad Z_2 = \frac{1}{s_2 C_2}, \quad Z_3 = s_2 L_3, \quad Z_4 = s_1 L_4. \tag{2.7}$$

By substituting the above impedance value in eqn. (2.1), we get

$$\frac{V_o}{V_{in}}(s_1, s_2) = -\frac{s_1 s_2 C_1 L_3 + s_1^2 s_2^2 L_3 L_4 C_2 C_1 + s_1^2 L_4 C_1}{s_2^2 C_2 L_3 + s_1 s_2^2 C_2 C_1 R_i L_3} \tag{2.8}$$

Then the test conditions specified in eqn. (2.3) are tested below

$$\frac{V_o}{V_{in}}\left(\frac{1}{s_1}, s_2\right)_{@ s_1=0, s_2=0} = \frac{L_4 C_1}{0} \neq \frac{0}{0} \tag{2.9}$$

$$\frac{V_o}{V_{in}}\left(s_1, \frac{1}{s_2}\right)_{@ s_1=0, s_2=0} = \frac{0}{C_2 L_3} \neq \frac{0}{0} \tag{2.10}$$

$$\frac{V_o}{V_{in}}\left(\frac{1}{s_1}, \frac{1}{s_2}\right)_{@ s_1=0, s_2=0} = \frac{L_3 L_4 C_2 C_1}{0} \neq \frac{0}{0} \tag{2.11}$$

From eqn. (2.9), (2.10) and (2.11), it's clear that the case 1 in Table 2.1 satisfies the test conditions mentioned in eqn. (2.3). It's possible to obtain a 2-D stable active IIR filter for these impedance values. In order to obtain that, the following method would be adhered

The transfer function obtained would be of the nature

$$\frac{V_o}{V_{in}}(s_1, s_2) = \frac{P(s_1, s_2)}{Q(s_1, s_2)} \tag{2.12}$$

In order to obtain a VSHP polynomial in the denominator the following transformation would be done [9]

$$H_a(s_1,s_2) = \frac{N_a(s_1,s2)}{D_a(s_1,s_2)} = \frac{D_a(0,0)}{P(s_1,s_2)+Q(s_1,s_2)} \tag{2.13}$$

After obtaining a transfer function of the form specified in eqn. (2.13), test conditions specified in eqn. (2.3) should be satisfied.

After applying the transformation mentioned in eqn. (2.13) to eqn. (2.8)

$$H_a(s_1,s_2) = -\frac{0}{\begin{array}{l} s_1^2\left[s_2^2 L_3 L_4 C_2 C_1 + L_4 C_1\right] + \\ s_1\left[s_2^2 C_2 C_1 R_i L_3 + s_2 C_1 L_3\right] + \\ \left[s_2^2 C_2 L_3\right] \end{array}} = 0 \tag{2.14}$$

Eqn. (2.14) is obtained, which tends to become '0'. This trivial condition occurs because the transfer function of the 2-D active IIR filter with infinite gain op- amp in eqn. (2.8) is purely reactance function, which isn't practically realizable. In order to obtain a practically realizable transfer function, resistance is added either in series or in parallel to the impedances, such that the denominator obtained is a VSHP polynomial.

After trying different combination, the impedance combination which would lead to a VSHP polynomial in the denominator and a non-zero value for the numerator is obtained. The combination for the impedance would be

$$Z_1 = \frac{1}{s_1 C_1} \| R_1, \quad Z_2 = \frac{1}{s_2 C_2}, \quad Z_3 = s_2 L_3 + R_3, \quad Z_4 = s_1 L_3. \tag{2.15}$$

By substituting these impedance values in eqn. 2.1, we get

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = \frac{P(s_1,s_2)}{Q(s_1,s_2)} = -\frac{\begin{array}{c} s_1^2 s_2^2 L_3 L_4 C_2 C_1 R_1 + s_1^2 s_2 L_4 C_2 C_1 R_1 + s_1 s_2^2 L_3 L_4 C_2 + s_1^2 C_1 L_4 R_1 \\ + s_1 s_2 (L_4 C_2 R_3 + C_1 L_3 R_1) + s_1 (L_4 + C_1 R_1 R_3) + s_2 L_3 + R_3 \end{array}}{\begin{array}{c} s_1 s_2^2 C_1 C_2 L_3 R_1 + s_2^2 (L_3 C_2 R_i + L_3 C_2 R_1) \\ + s_1 s_2 C_1 C_2 R_1 R_i R_3 + s_2 C_2 R_3 (R_1 + R_i) \end{array}} \tag{2.16}$$

Next apply the transformation in eqn. (2.13) to eqn. (2.16), we get

$$H_a(s_1,s_2) = -\frac{R_3}{s_1^2 \begin{bmatrix} L_3 L_4 C_2 C_1 R_1 s_s^s \\ + L_4 C_2 C_1 R_1 R_3 s_2 \\ + C_1 L_4 R_1 \end{bmatrix} + s_1 \begin{bmatrix} \begin{pmatrix} L_3 L_4 C_2 \\ + C_1 C_2 L_3 R_1 R_i \end{pmatrix} s_2^2 \\ + \begin{pmatrix} L_4 C_2 R_3 + \\ C_1 L_3 R_1 \\ + C_1 C_2 R_1 R_i R_3 \end{pmatrix} s_2 \\ + (L_4 + C_1 R_1 R_3) \end{bmatrix} + \begin{bmatrix} \begin{pmatrix} L_3 C_2 R_1 + \\ L_3 C_2 R_i \end{pmatrix} s_2^2 \\ + \begin{pmatrix} L_3 + \\ C_2 R_3 R_1 \\ + C_2 R_i R_3 \end{pmatrix} s_2 \\ + R_3 \end{bmatrix}} \tag{2.17}$$

The resistances values are considered to be unity. Therefore

$$R_1 = 1,\ R_i = 1,\ R_3 = 1 \tag{2.18}$$

Substituting the impedance values in eqn. (2.6) and the resistance values in eqn. (2.18) in eqn. (2.17), we get

$$H_a(s_1,s_2) = -\frac{1}{s_1^2 \begin{bmatrix} 3.6749 s_2^2 \\ + 2.5771 s_2 \\ + 2.2786 \end{bmatrix} + s_1 \begin{bmatrix} 5.1916 s_2^2 \\ + 6.7323 s_2 \\ + 3.2190 \end{bmatrix} + \begin{bmatrix} 3.2256 s_2^2 \\ + 3.6880 s_2 \\ + 1 \end{bmatrix}} \tag{2.19}$$

Since the stability of the filter is defined by the denominator of the transfer function, the numerator is considered to be a constant. For a stable filter, the filter's transfer function should be free of singularities of first and second kind (sec. 1.4) and moreover denominator polynomial is VSHP. Further based on the test methodologies described in sec.(1.6), we get

$$D_a(s_1,s_2)=D_a\left(\frac{1}{s_1},s_2\right)=D_a\left(s_1,\frac{1}{s_2}\right)=D_a\left(\frac{1}{s_1},\frac{1}{s_2}\right)\neq\frac{0}{0} \tag{2.20}$$

As the non essential singularities of the first and second kind are eliminated, the denominator polynomial $D_a(s_1,s_2)$ is a VSHP. The generalized bilinear transformation is applied to obtain the discrete domain equivalent, which must also be stable.

2.6 *Transfer function of the active filter with finite gain op-amp (Table 2.3, case 1)*

By substituting the impedance values for Z_1, Z_2, Z_3 and Z_4 from (Table 2.3, case 1) in the eqn. (2.2) and the transfer function of the filter is obtained. But by substituting these values you end up getting a transfer function in terms of reactance's which is not acceptable for practical application. So, resistances are added either in series or parallel to this impedance to get the appropriate transfer function. After trying out various combinations, one of the combination would be $Z_1=(1/s_1C_1)$, $Z_2=1/s_2C_2$, $Z_3=R_3+s_2L_3$, $Z_4=s_1L_4$. By substituting these values in eqn. (2.2) we get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}=-\frac{k(s_1s_2^2L_3L_4C_2+s_1s_2C_2L_4R_3+s_1L_4+s_2L_3+R_3)}{\begin{array}{l}(s_1^2s_2^2L_3L_4C_2C_1+s_1^2s_2C_1C_2L_4R_3+s_1^2L_4C_1+s_1s_2L_3C_1+s_1C_1R_3)(R_i+1)\\+(s_2^2L_3C_2+s_2C_2R_3)(k+1)+s_1s_2L_4C_2\end{array}} \tag{2.21}$$

The same procedure as in sec. (2.4) would be employed and we end up getting

$$H_{af}(s_1,s_2)=-\frac{kR_3}{s_1^2\left[\begin{pmatrix}L_3L_4C_2C_1s_2^2\\+C_1C_2L_4R_3s_2\\+L_4C_1\end{pmatrix}(R_i+1)\right]+s_1\left[\begin{array}{l}kL_3L_4C_2s_2^2\\+\begin{pmatrix}L_4C_2\\+kC_2L_4R_3\\+L_3C_1(R_i+1)\end{pmatrix}s_2\\+C_1R_3(R_i+1)+kL_4\end{array}\right]+\left[\begin{array}{l}L_3C_2(k+1)s_2^2\\+\begin{pmatrix}C_2R_3(k+1)\\+L_3k\end{pmatrix}s_2\\+kR_3\end{array}\right]} \tag{2.22}$$

In this case also the resistance values are considered to be unity, so

$$R_i=1,\ R_3=1 \tag{2.23}$$

Substituting the impedance values in eqn. (2.6) and the resistance values in eqn. (2.23) in eqn. (2.22), we get

$$H_{af}(s_1,s_2) = -\frac{k}{s_1^2\begin{bmatrix}7.3498s_2^2\\+5.1541s_2\\+4.5571\end{bmatrix}+s_1\begin{bmatrix}1.6951ks_2^2\\+(7.3718+1.1887k)s_2\\+4.3360+1.051k\end{bmatrix}+\begin{bmatrix}(1.6128k+1.6128)s_2^2\\+(0.2950k+2.168)s_2\\+k\end{bmatrix}} \tag{2.24}$$

GBT is applied to obtain the digital equivalent lowpass, highpass and bandpass filter, and these things would be discussed in the next chapters.

2.7 *Summary*

In this chapter, the active filter with T network in the feedback path was introduced. The main challenge of designing a filter is to ensure its stability. In order to ensure stability, 2-D active filter transfer function stability is ensured by ensuring the polynomial is VSHP.

The active filter consists of four impedances. In order to obtain a 2-D active filter, two impedances was considered in s_1 domain and the other two impedances was considered in s_2 domain. These impedances could be either an inductor or a capacitor in that domain. As it's an active filter, the possibility of filter with finite gain and infinite gain was also tried out. All possibilities were tabulated in Table 2.1, 2.2, 2.3 and 2.4. For each case, the possibility of generating a VSHP polynomial was found out.

For the infinite gain case out of the 16 cases each in Table 2.1 and Table 2.2, only three cases were obtained in which a VSHP can be obtained. Similarly, for the finite gain case out of the 16 cases each in Table 2.3 and Table 2.4, there was three possibilities of generating a VSHP. The resulting transfer function would be a reactance function, so in order to avoid that resistance were added in series or parallel with these impedances to generate appropriate transfer function.

The inductor and capacitor values for these impedances was obtained from the fourth order type I chebyshev filter with 1db ripple. These values are then substituted in the transfer function of the 2-D active filter for the infinite gain case and the finite gain case for further analysis.

This chapter mainly describes the method for choosing the transfer function which is VSHP in order to ensure stability from the different combinations of the impedances values for the filter. Among the total 32 combination for the 2-D active filter with infinite gain, three combinations which would be suitable for designing stable 2-D active filter was obtained. Similarly for the 2-D active filter with finite gain case, three combinations which are suited for designing 2-D digital filter were obtained. But only case 1 in Table 2.1 and Table 2.3 would be considered further in this thesis.

Chapter 3

Two Dimensional Lowpass Filter

3.1 *Introduction*

A popular method to design a 2-D digital filter is to start from the analog part i.e., first obtain the analog transfer function for the filter and then apply the double bilinear transformations to the analog transfer function to obtain the equivalent digital filter. The denominator of the analog transfer function should be a VSHP, so that by applying double bilinear transformation a stable digital transfer function can be obtained. The stability conditions to be satisfied for the coefficients of GBT are discussed in [6].

To obtain a stable lowpass digital filter by applying the double bilinear transformation, the following conditions are to be satisfied $|a_1| \leq 1, |b_1| \leq 1, a_1b_1 < 0, |a_2| \leq 1, |b_2| \leq 1, a_2b_2 < 0$. In this chapter, these conditions would be fulfilled for the coefficients of bilinear transformation to obtain a stable digital lowpass filter.

In chapter 2, the transfer functions of the filter with infinite gain op-amp and finite gain op-amp was obtained. Eqn. (2.19) represents the stable transfer function for the filter with infinite gain op-amp and eqn. (2.24) represents the stable transfer function of the filter with finite gain op-amp. Both the equations represent the transfer function in analog domain.

In this chapter the transfer function of the lowpass filter in digital domain for the both cases would be obtained by applying bilinear transformation and importance of each GBT coefficients and op-amp gain parameter on the filter output would be discussed in the following sections.

3.2 *Transfer function of 2-D active digital lowpass filter*

The generalized bilinear transformation is applied to the transfer function in analog domain to obtain the equivalent digital lowpass filter [12]. Transformation to be applied for both the infinite gain and finite gain cases is

$$s_i = k_i \frac{(z_i - a_i)}{(z_i + b_i)}, \quad where\, k_i > 0, 0 \le a_i \le 1\; and\; 0 \le b_i \le 1 \tag{3.1}$$

$$s_1 = k_1 \frac{(z_1 - a_1)}{(z_1 + b_1)} = k_1 \frac{a}{c}, \quad a = z_1 - a_1, \quad c = z_1 + b_1 \tag{3.2}$$

$$s_2 = k_2 \frac{(z_2 - a_2)}{(z_2 + b_2)} = k_2 \frac{b}{d}, \quad b = z_2 - a_2, \quad d = z_2 + b_2 \tag{3.3}$$

where $k_1>0$, $k_2>0$, $0 \le a_1 \le 1$, $0 \le a_2 \le 1$ and $b_1=b_2=1$ in particular to obtain a lowpass filter transfer function. By substituting eqn. (3.2), (3.3) in eqn. (2.19), we get

$$H_{dL}(z_1, z_2) = -\frac{c^2 d^2}{k_1^2 a^2 \begin{bmatrix} 3.6749 k_2^2 b^2 + \\ 2.5771 k_2 bd + \\ 2.2786 d^2 \end{bmatrix} + k_1 ac \begin{bmatrix} 5.1916 k_2^2 b^2 + \\ 6.732 k_2 bd + \\ 3.2190 d^2 \end{bmatrix} + c^2 \begin{bmatrix} 3.2256 k_2^2 b^2 + \\ 3.688 k_2 bd + \\ d^2 \end{bmatrix}} \tag{3.4}$$

Eqn. (3.4) represents the transfer function of the 2-D IIR filter with infinite gain op-amp in digital domain, for case 1 in Table 2.1.

Now by substituting eqn. (3.2), (3.3) in eqn. (2.24), transfer function of the 2-D IIR filter with finite gain op-amp, for case 1 in Table 2.3.

After applying the transformation the transfer function is as given below:

$$H_{dfL}(z_1, z_2) = -\frac{kc^2d^2}{k_1^2a^2\begin{bmatrix}7.3498k_2^2b^2 + \\ 5.1541k_2bd + \\ 4.5571d^2\end{bmatrix} + k_1ac\begin{bmatrix}1.6951kk_2^2b^2 + \\ (7.3718 + 1.1887k)k_2bd + \\ (4.3360 + 1.051k)d^2\end{bmatrix} + c^2\begin{bmatrix}(1.6128 + 1.6128k)k_2^2b^2 + \\ (2.168 + 0.295k)k_2bd + \\ + kd^2\end{bmatrix}} \tag{3.5}$$

3.3 *Frequency response of 2-D active digital lowpass filter with infinite gain*

MATLAB has been used to plot the contour and 3-D amplitude responses of the filter transfer function given by eqn. (3.4). In simulation runs, the GBT coefficient b_1 & b_2 values are kept equal to unity and the other GBT coefficients k_1, k_2, a_1 and a_2 would be varied. Frequency response of 2-D digital lowpass filter with infinite gain will be studied under four different scenarios i.e. by varying each GBT coefficient individually in each scenario.

Let us consider all the GBT coefficients to be equal to unity i.e. k_1=1, k_2=1, a_1=1 and a_2=1. For this condition, the 3-D amplitude-frequency response and contour plot of the 2-D digital lowpass filter with infinite gain is show in Fig. 3.1. The contour plots are elliptical in nature and the magnitude of amplitude-frequency response is close to unity.

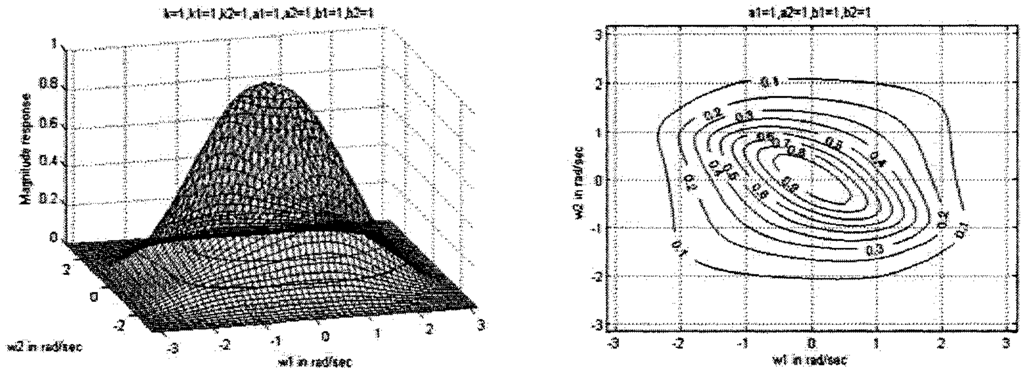


Figure 3.1: 3-D amplitude-frequency response and contour response of the 2-D digital lowpass filter with infinite gain op-amp when all the coefficients are unity.

Impedances values for the filter were obtained from a fourth order chebyshev filter with 1-db ripple in the pass band in chapter 2. In the next few sections the GBT coefficients would be individually varied to study its effect on the filter output characteristics.

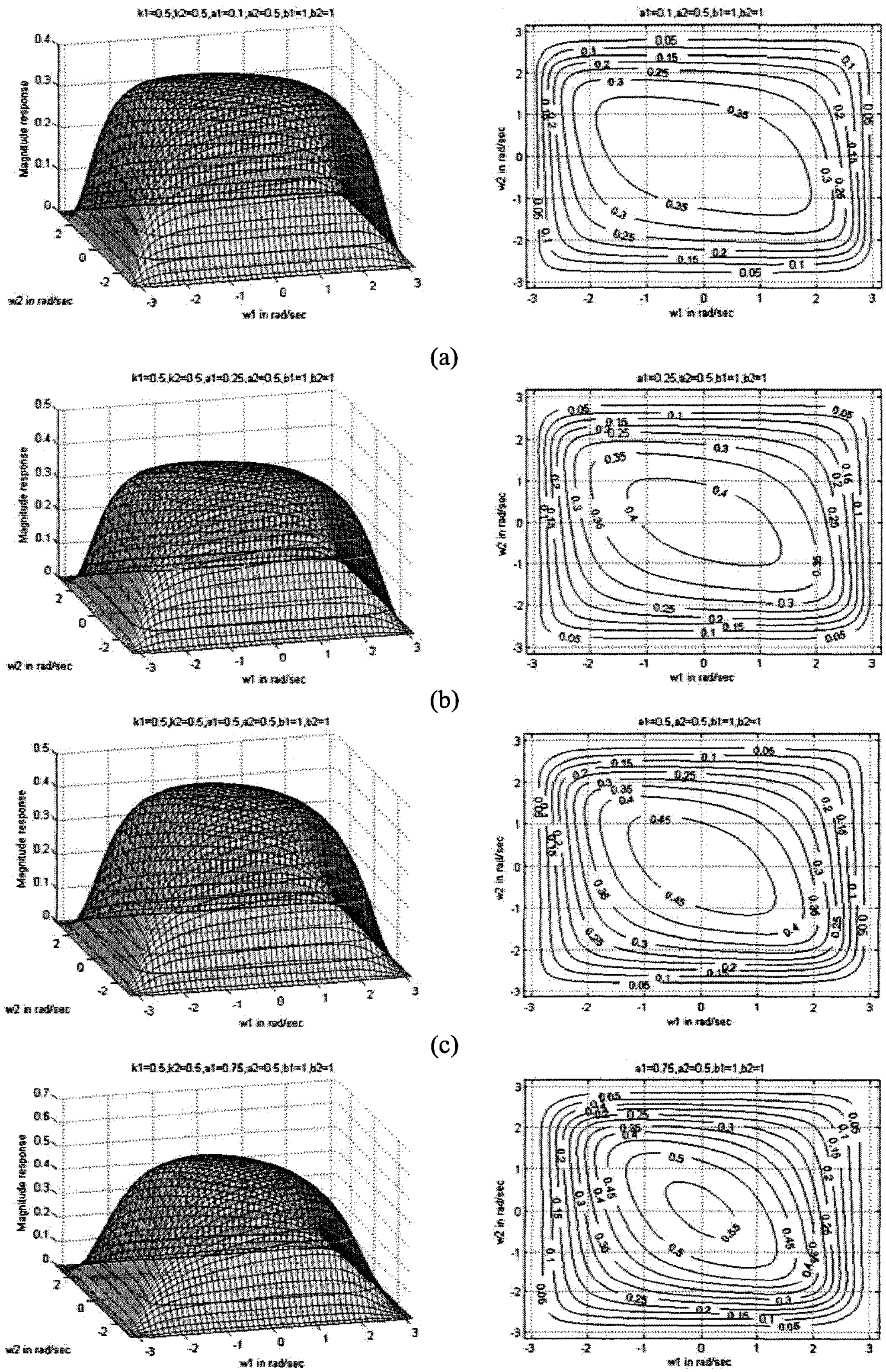
3.3.1 *Case 1*

In case 1, the effect of GBT coefficient a_1 is studied. So a_1 is varied and other parameters are set as given below:

$$k_1=k_2=\text{constant},\ b_1=b_2=1,\ a_2=\text{constant}.$$

$$k_1=k_2=0.5,\ b_1=b_2=1,\ a_2=0.5,\ \text{vary } a_1.$$

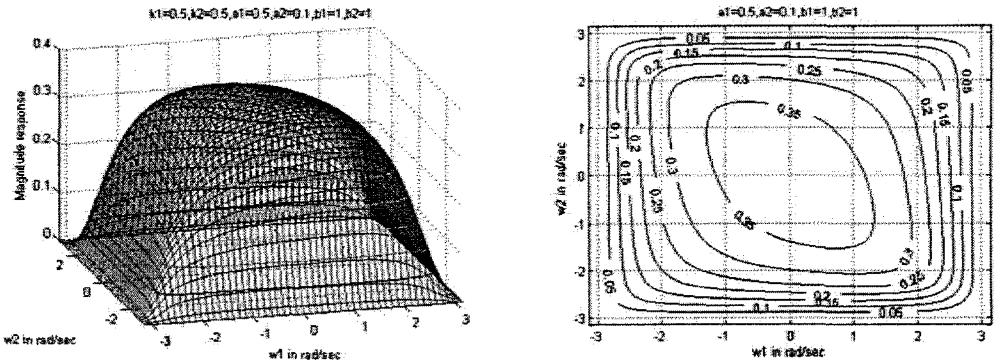
Figs. 3.2 (a), (b), (c) and (d) are obtained by varying the GBT coefficient a_1 from 0.1 to 0.75. As the value of a_1 is increased, there is a gradual increase in the magnitude of the amplitude-frequency response from 0.35 to 0.55 and there is very minute decrease in the pass bandwidth along the ω_1 axis and the pass bandwidth along the ω_2 axis remains constant. The GBT coefficient a_1 has a perceptible effect on the magnitude of the amplitude-frequency response.



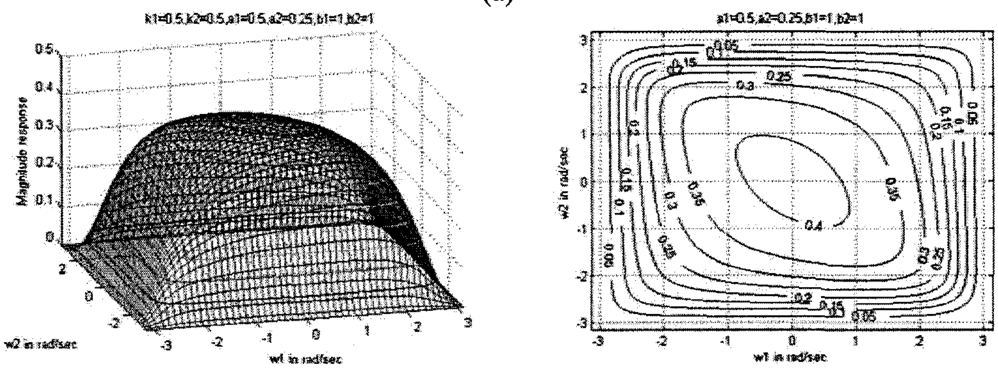
(d)

Figure 3.2: 3-D amplitude-frequency response and contour response of the 2-D digital lowpass filter with infinite gain op-amp for case 1 (varying a_1).

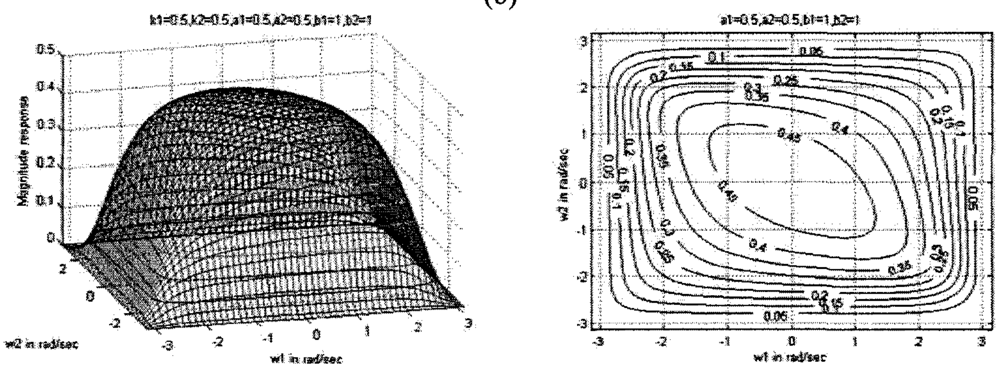
3.3.2 *Case 2*



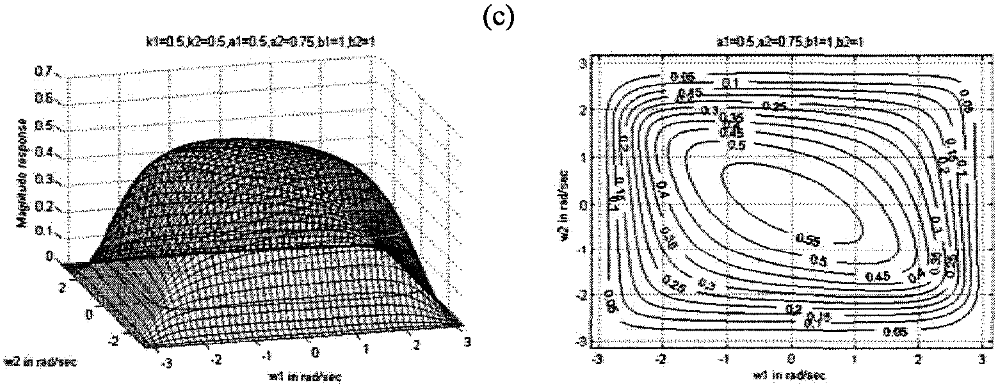
(a)



(b)



(c)



(d)

Figure 3.3: 3-D amplitude-frequency response and contour response of the 2-D digital lowpass filter with infinite gain op-amp for case 2 (varying a_2).

In case 2, the effect of GBT coefficient a_2 on the 2-D digital filter output is studied. In order to so, a_2 is varied and other parameters are set as given below:

$k_1=k_2=\text{constant}, b_1=b_2=1, a_1=\text{constant}.$

$k_1=k_2=0.5, b_1=b_2=1, a_1=0.5, \text{vary } a_2.$

In Figs. 3.3 (a), (b), (c) and (d), there is a steady increase in the magnitude of the amplitude-frequency response from 0.35 to 0.55 of the filter as the value of a_2 is increased from 0.1 to 0.75. There is very small decrease in the pass bandwidth along the ω_2 axis and the pass bandwidth along the ω_1 axis remains constant. The contour plots are elliptical in nature. The GBT coefficient a_2 has a noticeable effect on the magnitude of the amplitude-frequency response of the filter.

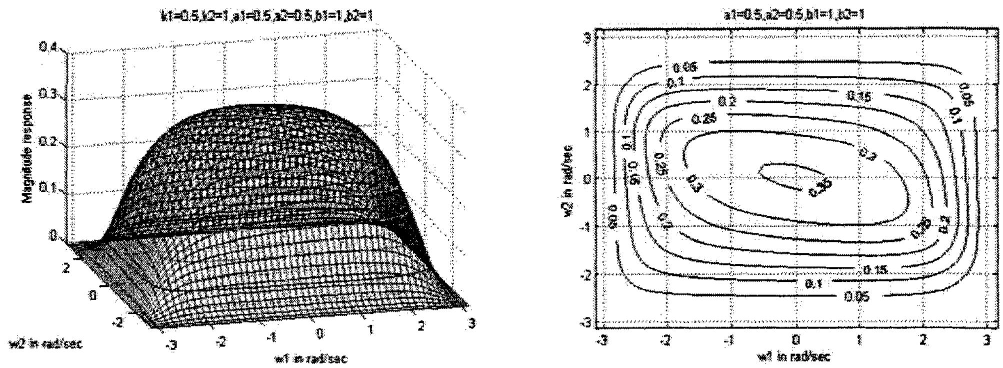
3.3.3 *Case 3*

In case 3, the effect of GBT coefficient k_1 on the 2-D digital lowpass filter is studied. In order to study the effect of k_1 parameter, the value of k_1 is gradually increased and other parameter values are set as given below:

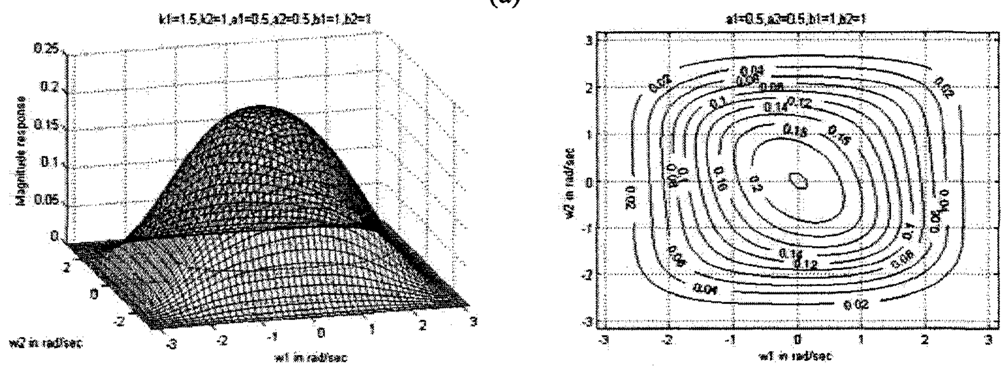
$k_2=\text{constant}, b_1=b_2=1, a_1=a_2=\text{constant}.$

$k_2=1, b_1=b_2=1, a_1=a_2=0.5, \text{vary } k_1.$

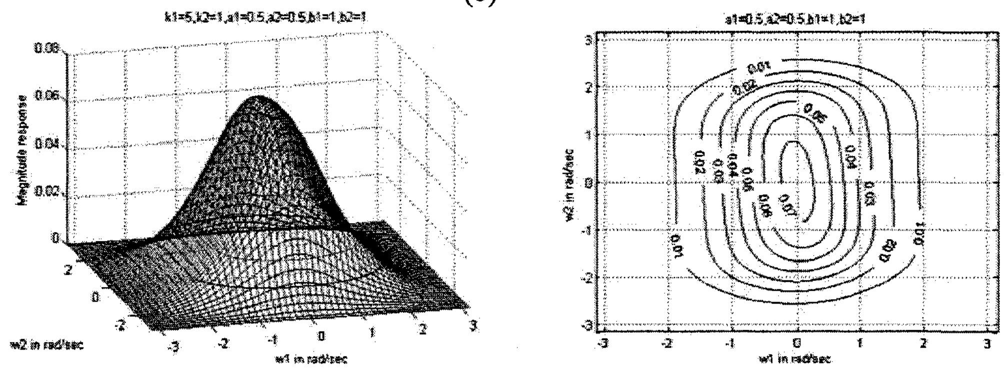
Figures 3.4 (a), (b), (c) and (d) are obtained by varying the GBT coefficient k_1. It's observed that as the value of k_1 is increased from 0.5 to 10 the pass bandwidth along the ω_1 axis decreases and moreover the magnitude of the amplitude-frequency response decreases from 0.35 to 0.025. The pass bandwidth along the ω_2 axis remains constant. The contour plots are elliptical in nature, but there is a rotation in orientation along the clockwise direction.



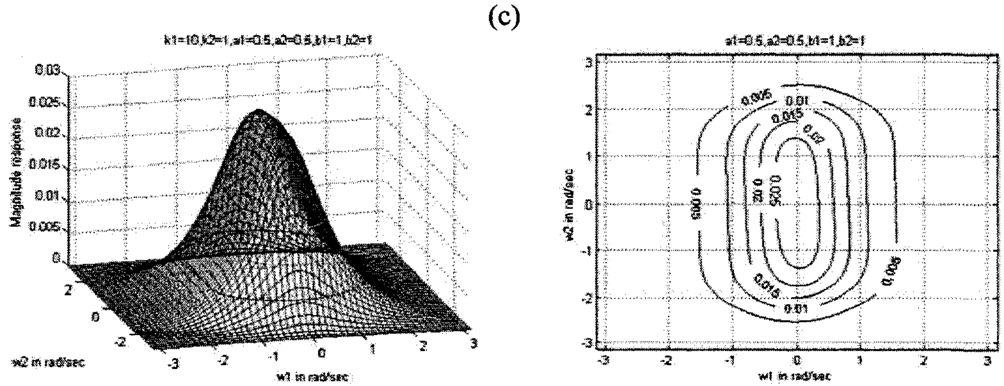
(a)



(b)



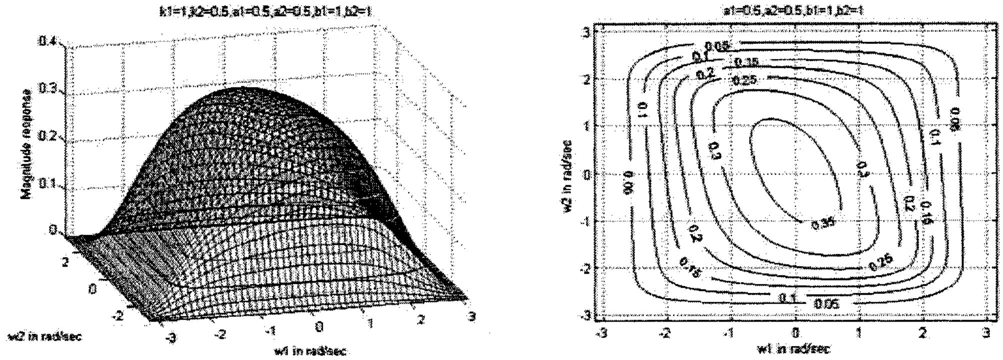
(c)



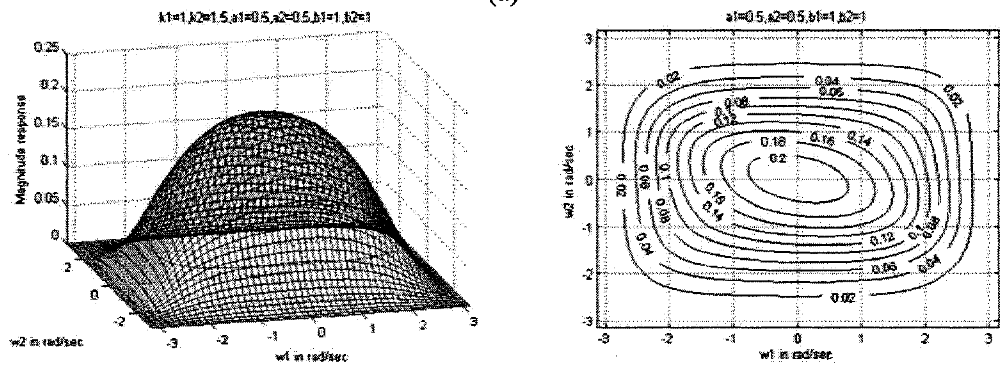
(d)

Figure 3.4: 3-D amplitude-frequency response and contour response of the 2-D digital lowpass filter with infinite gain op-amp for case 3 (varying k_1).

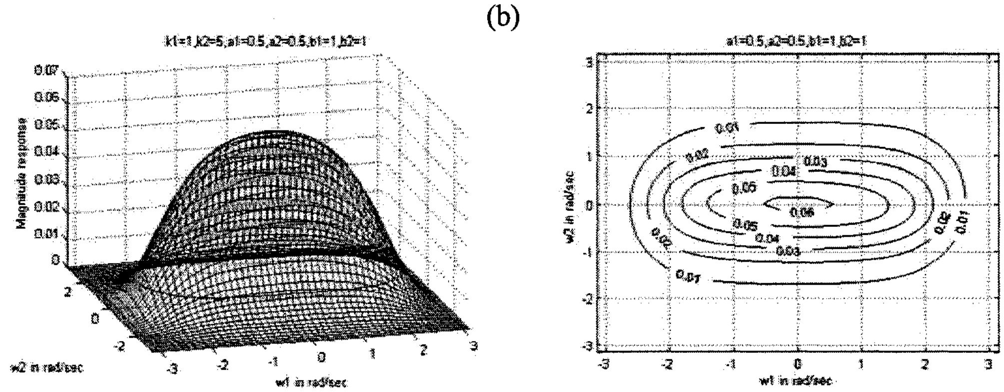
3.3.4 *Case 4*



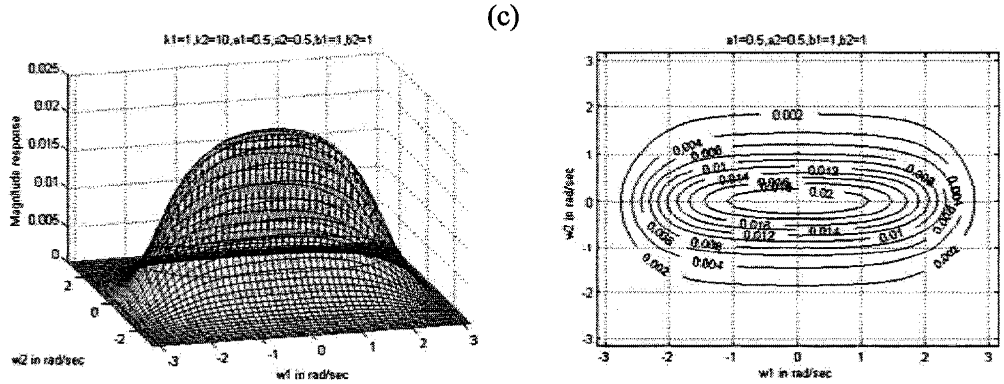
(a)



(b)



(c)



(d)

Figure 3.5: 3-D amplitude-frequency response and contour response of the 2-D digital lowpass filter with infinite gain op-amp for case 4 (varying k_2).

In case 4, the importance of GBT coefficient k_2 on the 2-D digital lowpass filter with infinite gain op-amp is studied by varying k_2 parameter and keeping the other parameters values set as mentioned below:

k_1=constant, $b_1=b_2=1$, $a_1=a_2$=constant.

$k_1=1$, $b_1=b_2=1$, $a_1=a_2=0.5$, vary k_2.

Figs. 3.5 (a), (b), (c) and (d) are obtained by varying the coefficient k_2 of GBT. It is observed that as the value of k_2 is increased from 0.5 to 10, the pass bandwidth along the ω_2 axis decreases and moreover magnitude of the amplitude-frequency response decreases from 0.35 to 0.02. The pass bandwidth along the ω_1 axis remains constant. The contour plots are elliptical in nature, as the value of k_2 is increased there is an anticlockwise rotation in the orientation of the contour plots.

3.4 *Frequency response of 2-D active digital lowpass filter with finite gain*

MATLAB has been used to plot the 3-D amplitude-frequency response and contour plots of the filter for the transfer function in eqn. (3.5). For the simulation runs, let the GBT coefficient b_1 & b_2 be equal to unity. So we are left with parameters k_1, k_2, a_1 and a_2 as varying parameters. In this case, we would be having one more parameter 'k' gain of the op-amp. The frequency response of the 2-D digital lowpass filter with finite gain is studied under five different cases based on the GBT coefficients which can be varied and the gain of the op-amp. By varying these parameters individually, importance of individual parameter on the filter output characteristics is studied.

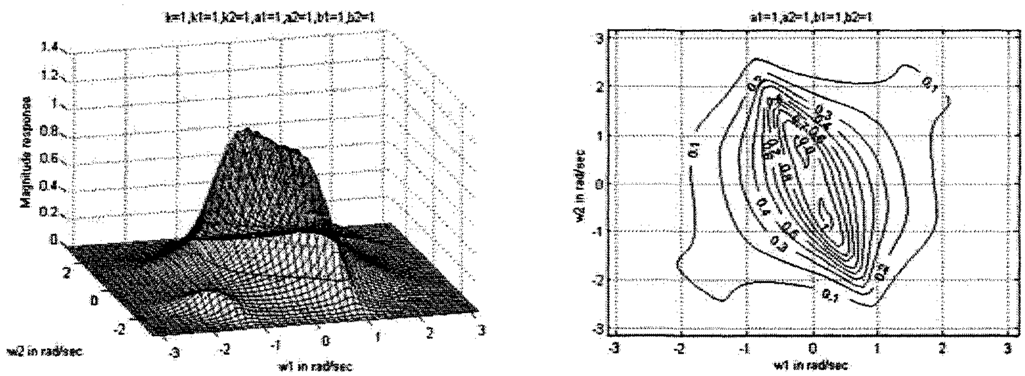


Figure 3.6: 3-D amplitude –frequency response and contour response of the 2-D digital lowpass filter with unity gain op-amp and all the GBT coefficients are unity.

Initially, let us consider all the GBT coefficients to be unity i.e. $k_1=1$, $k_2=1$, $a_1=1$ and $a_2=1$and also gain of the op-amp k=1. For this condition, the 3-D amplitude-frequency response and contour plots of the 2-D digital lowpass filter with finite gain is show in Fig. 3.6. It is observed that the contour plots are elliptical in nature. Ripples are also present in the output.

3.4.1 *Case 1*

In case 1, the effect of GBT coefficient a_1 on the filter output characteristics is studied. In order to study that, all other parameters are set as given below:

$$k=k_1=k_2=\text{constant},\ b_1=b_2=1,\ a_2=\text{constant, vary } a_1.$$

$$k=1,\ k_1=0.5,\ k_2=0.5,\ b_1=1,\ b_2=1,\ a_2=0.5,\ \text{vary } a_1.$$

From Figs. 3.7 (a), (b), (c) and (d), it is observed that, as the value of a_1 is gradually increased from 0.1 to 0.75 the pass bandwidth decreases a bit along the ω_1 axis and remains constant along the ω_2 axis. The magnitude of amplitude-frequency response increases from 0.3 to 0.55. The contour plots are elliptical in nature. Therefore the GBT coefficient a_1 mainly affects the magnitude of the amplitude-frequency response.

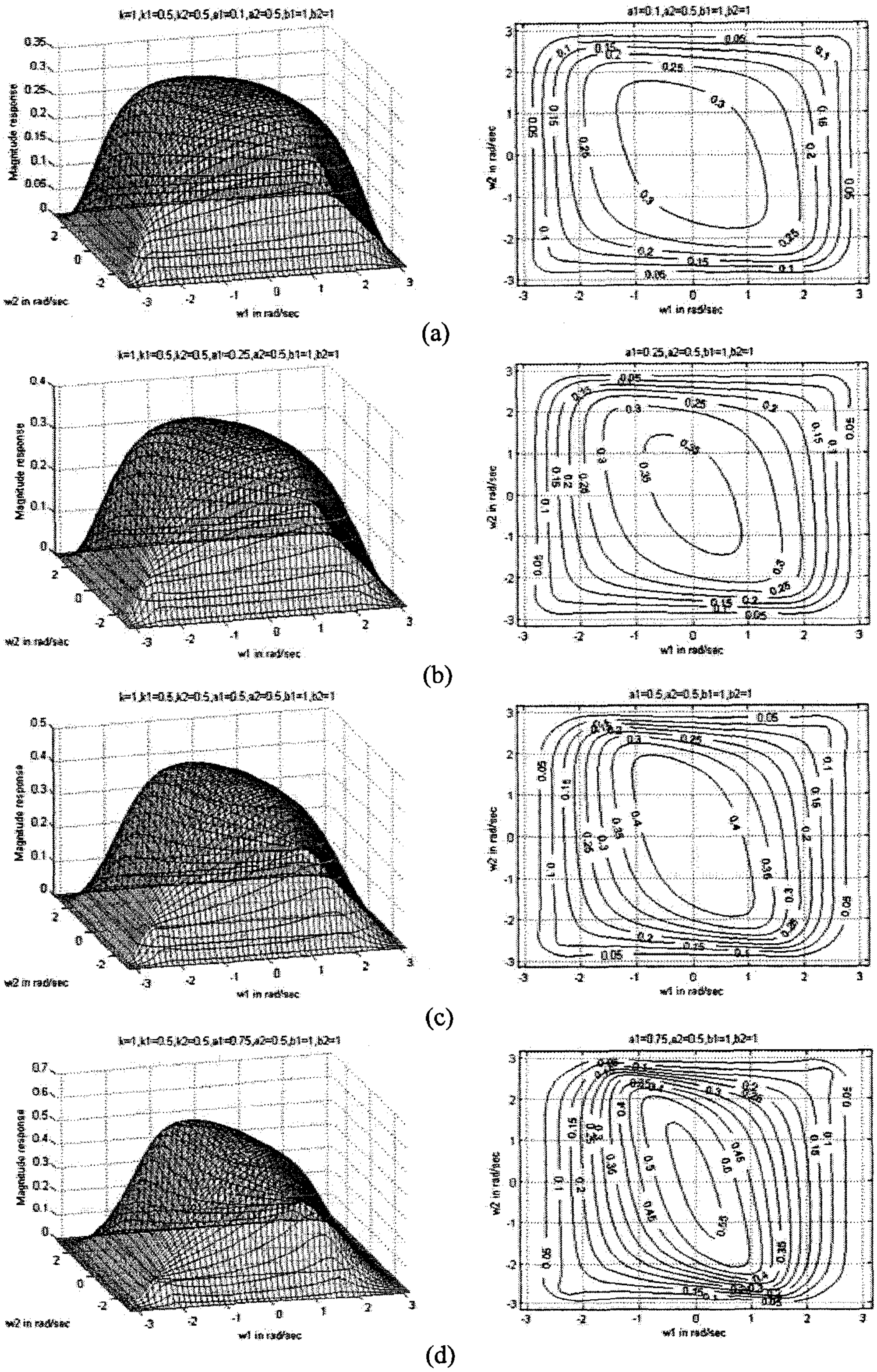
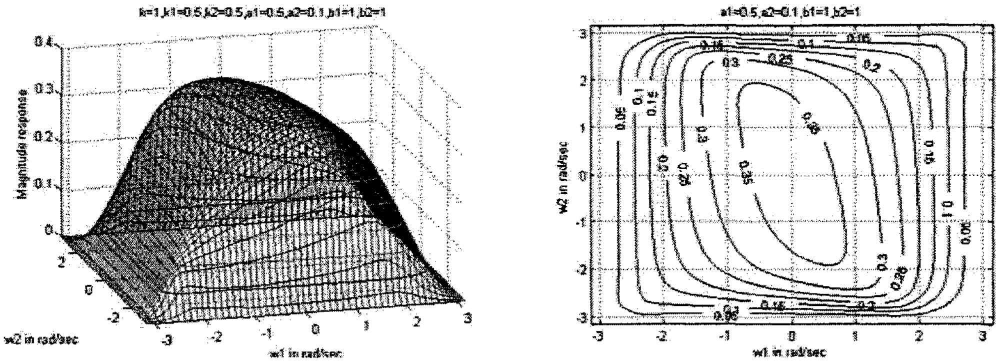
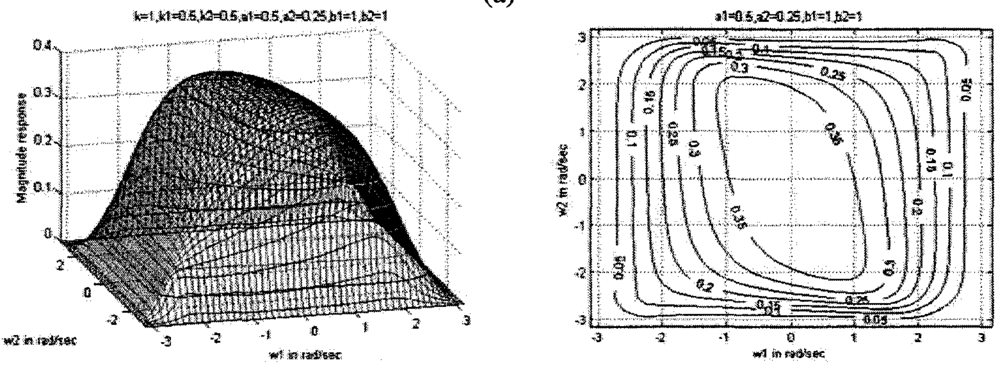


Figure 3.7: 3-D amplitude-frequency response and contour response of the 2-D digital lowpass filter with finite gain op-amp for case 1 (varying a_1).

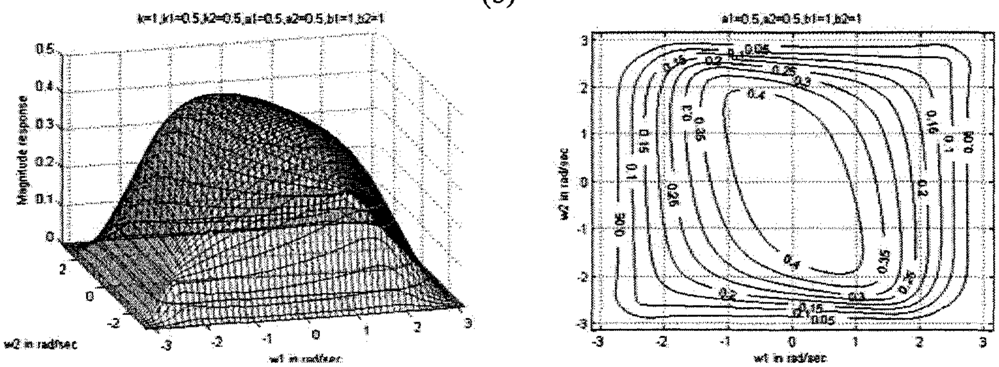
3.4.2 *Case 2*



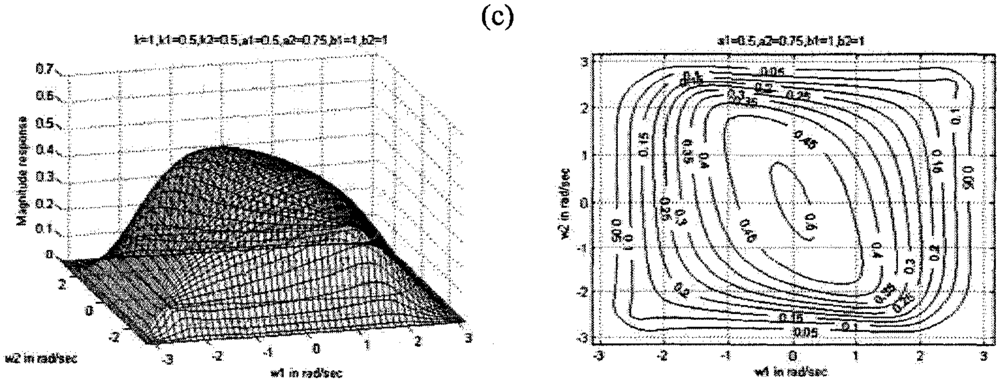
(a)



(b)



(c)



(d)

Figure 3.8: 3-D amplitude-frequency response and contour response of the 2-D digital lowpass filter with finite gain op-amp for case 2 (varying a_2).

In case 2, GBT coefficient a_2 is varied and other parameter are set as given below:

$k=k_1=k_2$=constant, $b_1=b_2=1$, a_1=constant, vary a_2.

$k=1$, $k_1=k_2=0.5$, $a_1=0.5$, $b_1=1$, $b_2=1$ and vary a_2.

Figs. 3.8 (a), (b), (c) and (d) are obtained by varying the GBT coefficient a_2 in the filter transfer function. As the value of a_2 is gradually increased from 0.1 to 0.75 the pass bandwidth decreases a bit along the ω_2 axis and there is also a gradual increase in the magnitude of the amplitude response from 0.35 to 0.5. The contour plots are elliptical in nature. The parameter a_2 mainly affects the magnitude of the amplitude-frequency response.

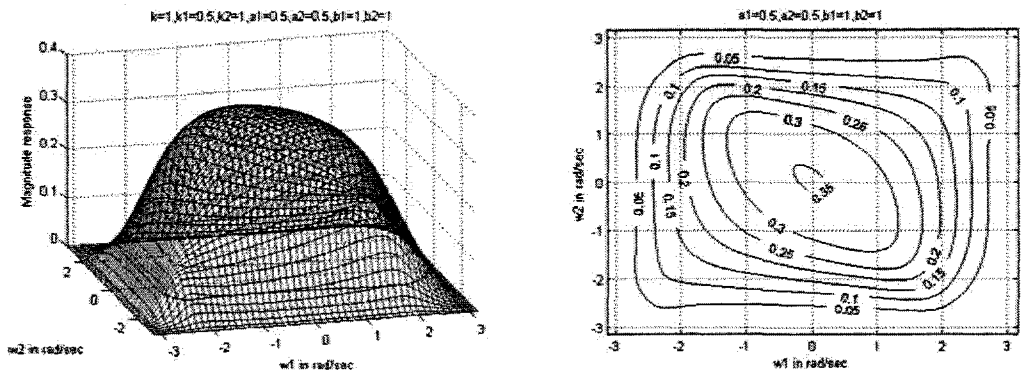
3.4.3 *Case 3*

In case 3, the effect of GBT coefficient k_1 on the digital filter output characteristics is studied. In order to study the effect of k_1 parameter, value of k_1 is varied and all other parameters are set as given below:

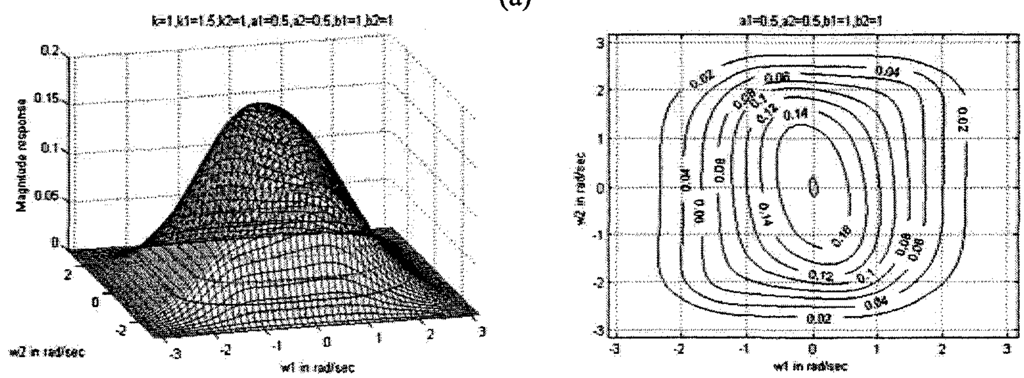
k=constant, k_2=constant, $b_1=b_2=1$, $a_1=a_2$=constant, vary k_1.

$k=1$, $k_2=1$, $b_1=b_2=1$, $a_1=a_2=0.5$, vary k_1.

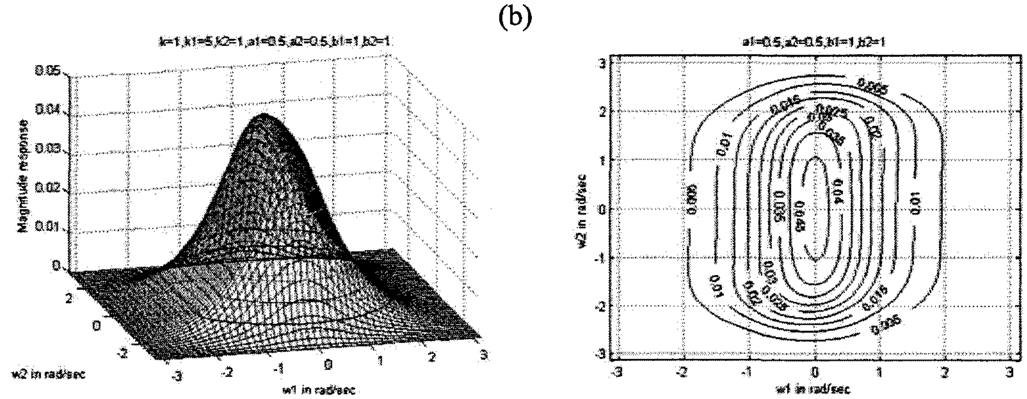
From Figs. 3.9 (a), (b), (c) and (d), it can be seen that as the value of k_1 is increased from 0.5 to 10 the pass bandwidth along the ω_1 axis decreases and more over the amplitude of the magnitude response also decreases from 0.35 to 0.016. The contour plots are elliptical in nature.



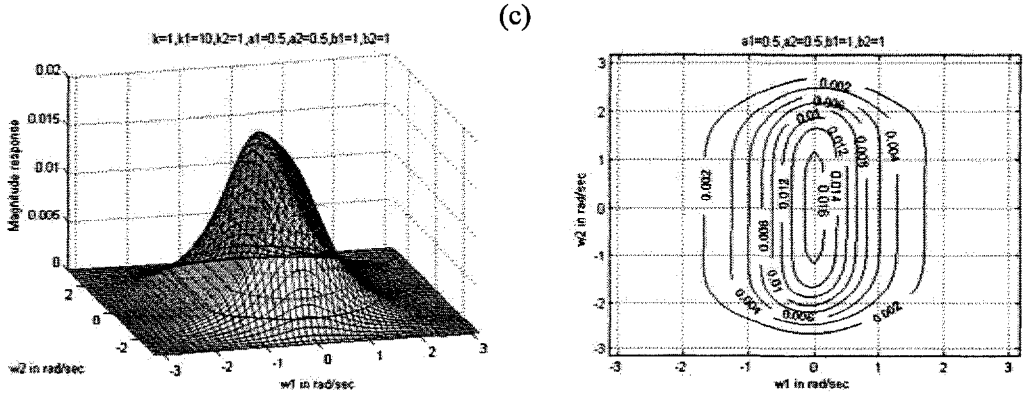
(a)



(b)



(c)



(d)

Figure 3.9: 3-D amplitude-frequency response and contour response of the 2-D digital lowpass filter with finite gain op-amp for case 3 (varying k_1).

Therefore the GBT coefficient k_1 affects the pass bandwidth along the ω_1 axis and the magnitude of the amplitude-frequency response. As the value of k_1 is increased, the pass bandwidth along the ω_1 axis and the magnitude of the amplitude-frequency response decreases.

3.4.4 *Case 4*

In case 4, the effect of GBT coefficient k_2 on the digital filter output is studied. Value of k_2 is varied and other parameter values are set as given below:

$$k=\text{constant},\ k_1=\text{constant},\ b_1=b_2=1,\ a_1=a_2=\text{constant},\ \text{vary } k_2$$

$$k=1,\ k_1=1,\ b_1=b_2=1,\ a_1=a_2=0.5,\ \text{vary } k_2.$$

In this case the value of k_2 is varied from 0.5 to 10, as the value of k_2 is increased gradually the pass bandwidth along the ω_2 axis decreases to a greater extend and moreover the magnitude of the amplitude-frequency plot decreases from 0.3 to 0.025 which is observed in Figs. 3.10 (a), (b), (c) and (d). The pass bandwidth along the ω_1 axis remains constant. The contour plots are elliptical in nature.

The GBT coefficient k_2 affects the pass bandwidth along ω_2 axis and the magnitude of the amplitude-frequency plot.

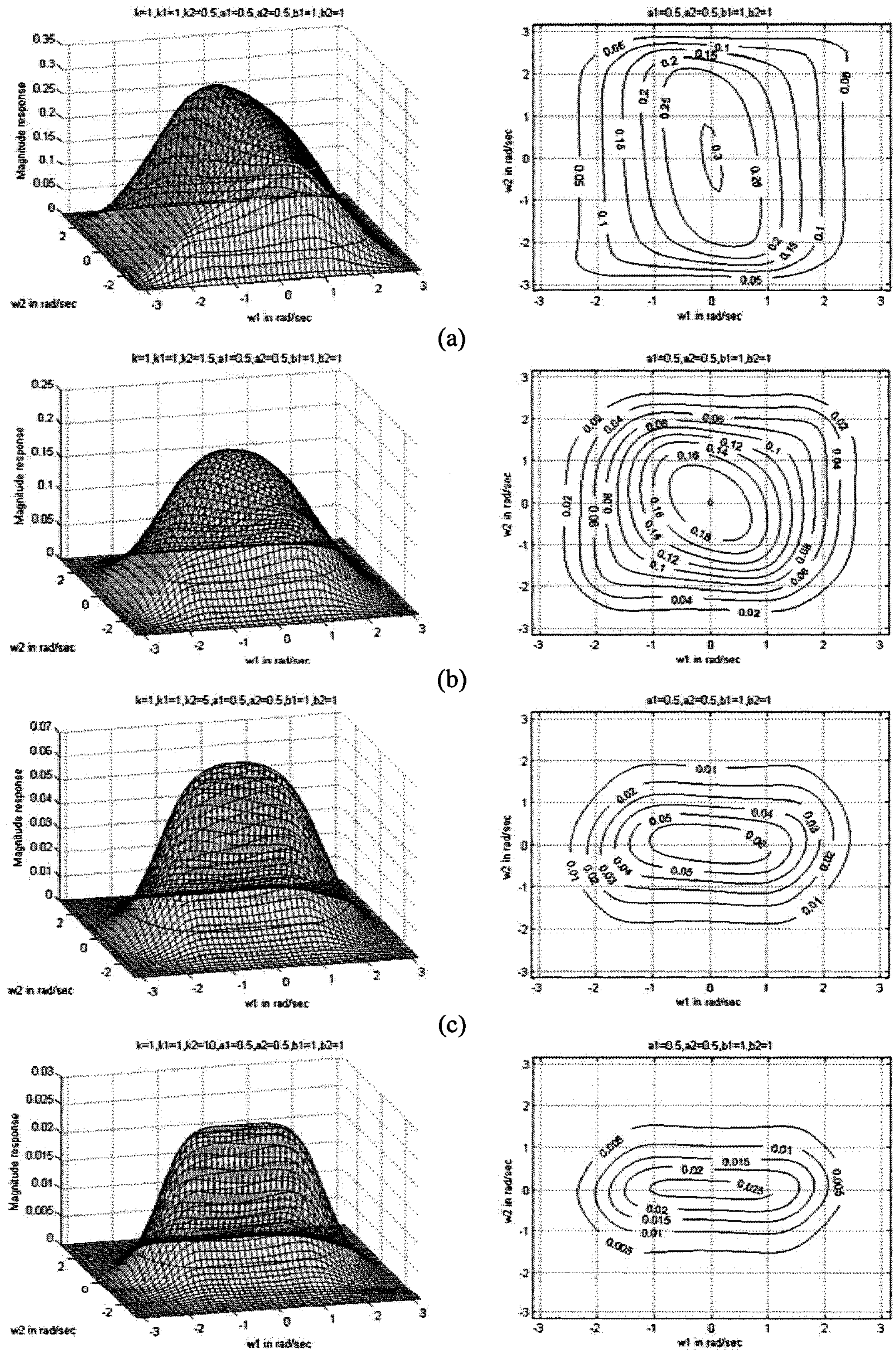
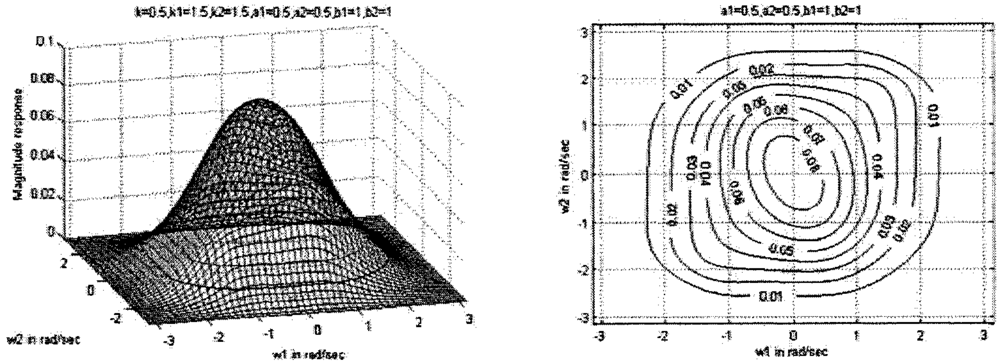
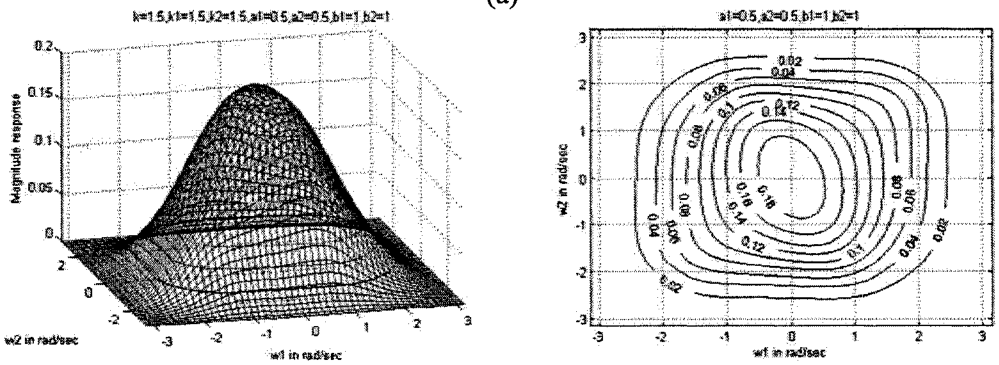


Figure 3.10: 3-D amplitude-frequency response and contour response of the 2-D digital lowpass filter with finite gain op-amp for case 4 (varying k_2).

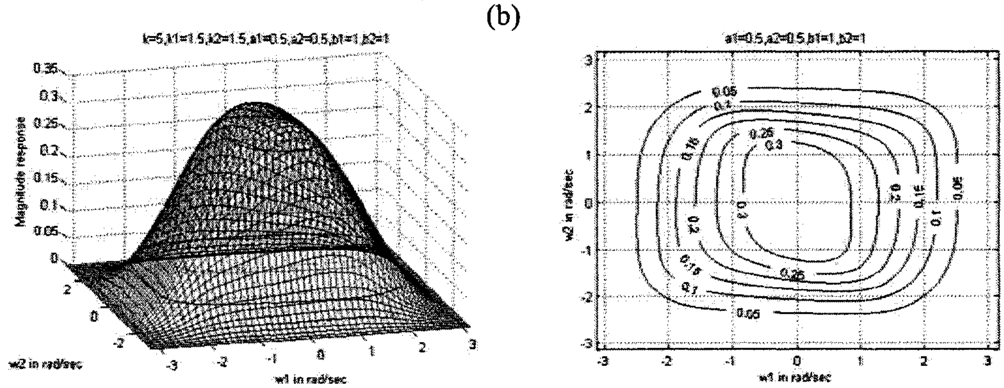
3.4.5 *Case 5*



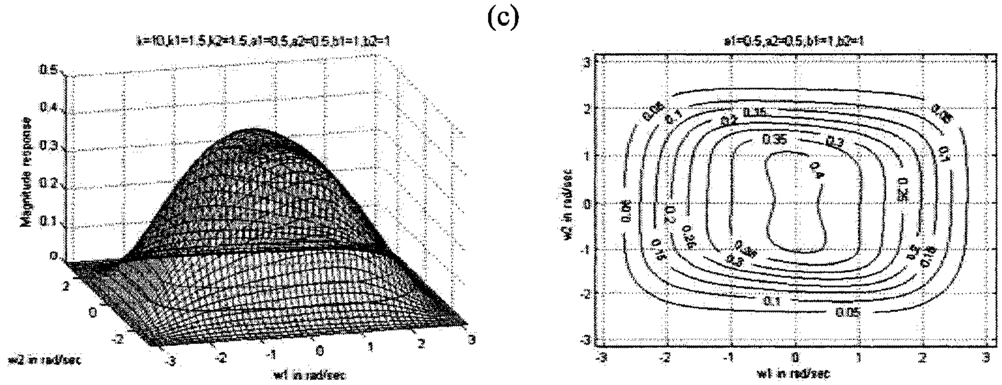
(a)



(b)



(c)



(d)

Figure 3.11: 3-D amplitude-frequency response and contour response of the 2-D digital lowpass filter with finite gain op-amp for case 5 (varying k).

In case 5, the effect of op-amp gain parameter k on the digital filter output characteristics is examined. Value of k is varied and other parameters are set as given below:

k_1=k_2=constant, b_1=b_2=1, a_1=a_2=constant, vary k.

k_1=k_2=1.5, a_1=a_2=0.5, b_1=b_2=1, vary k.

From Figs. 3.11 (a), (b), (c) and (d), it is observed that as the value of op-amp gain k is increased from 0.5 to 10, the magnitude of the amplitude-frequency response increases from 0.08 to 0.4. The pass bandwidth along the ω_1 axis increases and the pass bandwidth along the ω_2 axis remains constant.

3.5 *Summary*

In this chapter, the transfer function of the 2-D digital lowpass filter was obtained by applying the GBT to the analog transfer function obtained in chapter 2 represented by eqn. (2.19) and eqn. (2.24). Eqn. (3.4) and (3.5) represent the transfer function of the 2-D lowpass filter in digital domain with infinite gain op-amp and finite gain op-amp.

The transfer function for both the cases was obtained and the effect of each GBT coefficient and op-amp gain parameter on the filter output characteristics was examined. Table 3.1 summarizes the effect of each GBT coefficients on the filter output characteristic of 2-D digital lowpass filter with infinite gain op-amp. Initially the GBT coefficient a_1 was varied from 0.1 to 0.75. As the value of a_1 is increased, the magnitude of the amplitude-frequency response increased. The pass bandwidth along the ω_1 axis decreased a bit, but the pass bandwidth along the ω_2 axis remained constant. Next, we increased the value of a_2 from 0.1 to 0.75, the magnitude of amplitude-frequency

GBT coefficient	Value of GBT coefficient	Amplitude of 3D magnitude response	Pass bandwidth along ω_1 axis	Pass bandwidth along ω_2 axis
a_1	↑	↑	↓	Constant
a_2	↑	↑	Constant	↓
k_1	↑	↓	↓	Constant
k_2	↑	↓	Constant	↓

Table 3.1: Summary of the effects of GBT coefficients on 2D digital lowpass filter with infinite gain.

response increased. But as far as pass bandwidths are considered, the pass bandwidth along the ω_1 axis remained constant and the pass bandwidth along the ω_2 axis decreased a bit.

Then the GBT coefficient k_1 was varied from 0.5 to 10. It was observed that the magnitude of the amplitude-frequency response decreased and the pass bandwidth along the ω_1 axis also decreased. But the pass bandwidth along the ω_2 axis remained constant.

Finally, we varied the GBT coefficient k_2 of the 2-D digital lowpass filter with infinite gain. Value of k_2 was increased from 0.5 to 10 and it was observed that the magnitude of the amplitude-frequency response decreased and the pass bandwidth along the ω_2 axis also decreased. But the pass bandwidth along the ω_1 axis remained constant.

Table 3.2 sums up the effect of GBT coefficients and op-amp gain on the filter output characteristic of 2-D digital lowpass filter with finite gain op-amp. As the GBT coefficient a_1 and a_2 value was varied from 0.1 to 0.75, the magnitude of the amplitude-frequency response increased. Moreover, when the value of a_1 was increased the pass bandwidth along ω_1 axis decreased and when the value of a_2 was increased the pass bandwidth along the ω_2 axis decreased.

GBT coefficient	**Value of GBT coefficient**	**Amplitude of 3D magnitude response**	**Pass bandwidth along ω_1 axis**	**Pass bandwidth along ω_2 axis**
a_1	↑	↑	↓	Constant
a_2	↑	↑	Constant	↓
k_1	↑	↓	↓	Constant
k_2	↑	↓	Constant	↓
k	↑	↑	↑	Constant

Table 3.2: Summary of the effects of GBT coefficients on 2D digital lowpass filter with finite gain.

Next the GBT coefficients k_1 and k_2 was varied from 0.5 to 10, the magnitude of the amplitude-frequency response decreased in the both the cases. The pass bandwidth along the ω_1 axis decreased, as the value of k_1 was increased. Similarly, the pass bandwidth along the ω_2 axis also decreased as the value of k_2 was increased.

Finally, the effect of op-amp gain 'k' on the filter output characteristics was analyzed. As the value of k was increased from 0.5 to 10, the magnitude of amplitude-frequency response increased and the pass bandwidth along the ω_1 axis decreased. But the pass bandwidth along the ω_2 axis remains constant.

Thus the effect of each and every individual parameter on the 2-D digital lowpass filter output was studied successfully for the infinite gain and finite gain cases.

Chapter 4

Two Dimensional Active Highpass Filter

4.1 Introduction

In this chapter, the method of obtaining the transfer function of 2-D digital highpass filter from its analog transfer function by applying double bilinear transformation has been discussed. After generating the transfer function in digital domain, the effects of each GBT coefficient and the op-amp gain parameters on the filter output characteristics are studied.

To obtain the transfer function of the highpass digital filter by applying the double bilinear transformation, the following conditions are to be satisfied $k_i > 0, 0 \le |a_1| \le 1, -1 \le |b_1| \le 0$ [12]. In this whole chapter, these conditions would be satisfied for coefficients of GBT to obtain a stable 2-D digital highpass filter.

4.2 Transfer function of 2-D active digital highpass filter

Below mentioned generalized bilinear transformation is applied to obtain the transfer function of the highpass filter in digital domain. These conditions are discussed in [7], [12].

$$s_i = k_i \frac{(z_i + a_i)}{(z_i + b_i)}, \quad where k_i > 0, 0 \le a_i \le 1 \; and -1 \le b_i \le 0 \tag{4.1}$$

$$s_1 = k_1 \frac{(z_1 + a_1)}{(z_1 + b_1)} = k_1 \frac{e}{g}, \quad e = z_1 + a_1, \quad g = z_1 + b_1. \tag{4.2}$$

$$s_2 = k_2 \frac{(z_2 + a_2)}{(z_2 + b_2)} = k_2 \frac{f}{h}, \quad f = z_2 + a_2, \quad h = z_2 + b_2. \tag{4.3}$$

Where $k_1>0$, $k_2>0$, $0 \le a_1 \le 1$, $0 \le a_2 \le 1$ and $b_1=b_2=-1$ in particular to obtain a digital highpass filter transfer function. By substituting eqn. (4.2), (4.3) in eqn. (2.19), we get

$$H_{dH}(z_1, z_2) = \frac{g^2h^2}{k_1^2e^2\begin{bmatrix}3.6749k_2^2f^2 + \\ 2.5771k_2fh + \\ 2.2786h^2\end{bmatrix} + k_1ac\begin{bmatrix}5.1916k_2^2f^2 + \\ 6.7323k_2fh + \\ 3.2190h^2\end{bmatrix} + c^2\begin{bmatrix}3.2256k_2^2f^2 + \\ 3.688k_2fh + \\ h^2\end{bmatrix}} \tag{4.4}$$

Eqn. (4.4) represents the transfer function of the 2-D IIR highpass filter with infinite gain op-amp in digital domain, for case 1 in Table 2.1.

Next eqn. (4.2), (4.3) are substituted in eqn. (2.24) to obtain the digital equivalent transfer function for the 2-D IIR highpass filter with finite gain op-amp, for case 1 in Table 2.3. After applying the transformation, we get

$$H_{dfH}(z_1, z_2) = \frac{kg^2h^2}{\begin{gathered}k_1^2a^2\begin{bmatrix}7.3498k_2^2f^2 + \\ 5.1541k_2fh + \\ 45571h^2\end{bmatrix} + k_1ac\begin{bmatrix}1.6951kk_2^2f^2 + \\ (7.3718 + 1.1887k)k_2fh + \\ (4.3360 + 1.051k)h^2\end{bmatrix} \\ + c^2\begin{bmatrix}(1.6128 + 1.6128k)k_2^2f^2 + \\ (2.168 + 0.295k)k_2fh + \\ kh^2\end{bmatrix}\end{gathered}} \tag{4.5}$$

4.3 *Frequency response of 2-D active digital lowpass filter with infinite gain.*

MATLAB is used to plot the 3-D amplitude-frequency response and the contour plots of the filter transfer function in eqn. (4.4). The GBT coefficients b_1 and b_2 are set to negative unity and the rest of the GBT coefficients k_1, k_2, a_1 and a_2 are varied and the corresponding filter responses are obtained. At the end, the effect of each GBT coefficients on the filter output response can be judged.

Initially, let all the GBT coefficients value be set to unity i.e. k_1=1, k_2=1,a_1=1 and a_2=1. Amplitude-frequency response and contour plot of the 2-D digital highpass filter with infinite gain op-amp is shown in fig. 4.1. In the next sections, GBT coefficients would be varied individually and the effect of individual coefficient on the filter output characteristics will be studied.

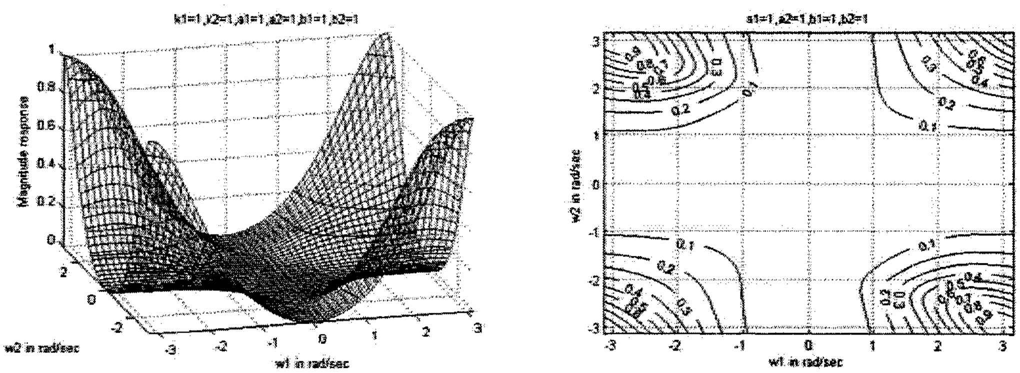


Figure 4.1: 3-D amplitude –frequency response and contour response of the 2-D digital highpass filter with infinite gain op-amp and all coefficient value equal to unity

4.3.1 *Case 1*

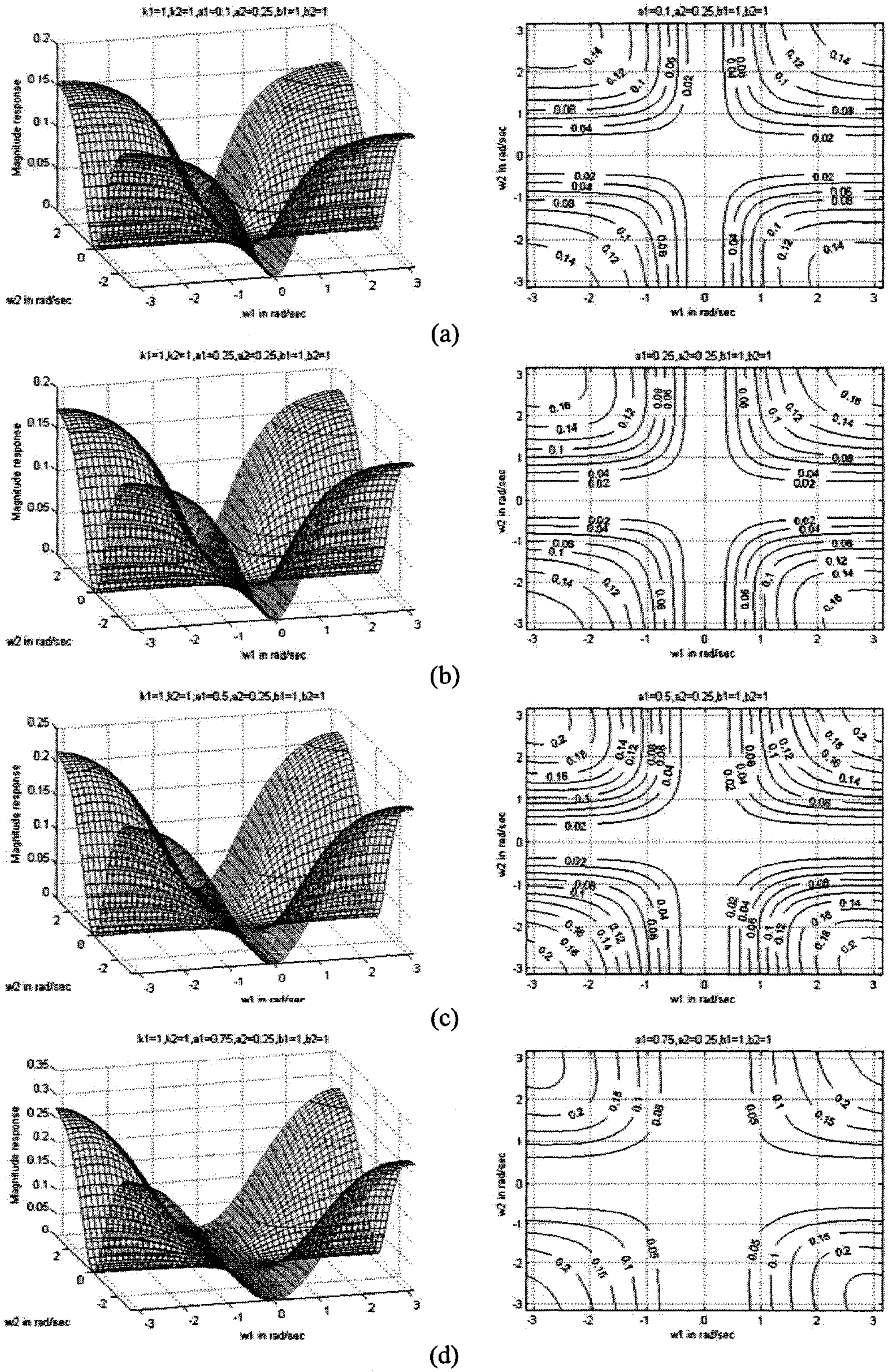


Figure 4.2: 3-D amplitude-frequency response and contour response of the 2-D digital highpass filter with infinite gain op-amp for case 1 (varying a_1).

In case 1, the effect of GBT coefficient a_1 on the filter output characteristics is studied. Value of a_1 is varied and other parameters are set as given below:

$k_1=k_2=\text{constant}$, $b_1=b_2=-1$, $a_2=\text{constant}$, vary a_1.

$k_1=k_2=1$, $b_1=b_2=-1$, $a_2=0.25$, vary a_1.

Value of the GBT coefficient a_1 is varied from 0.1 to 0.75 and the response of the filter is obtained. Figs. 4.2 (a), (b), (c) and (d) represents the response of the filter obtained for four different value of a_1. The magnitude of amplitude-frequency response increases from 0.14 to 0.22, as the value of a_1 is varied from 0.1 to 0.75. The contour plots in the first and third quadrant are symmetrical; the second and fourth quadrant contour plots are also symmetrical in nature. The stop bandwidth along the ω_1 axis increases as the value of a_1 is increased and along the ω_2 axis remains constant.

To sum up, GBT coefficient a_1 mainly affects the magnitude of the amplitude-frequency response and the stop bandwidth along the ω_1 axis.

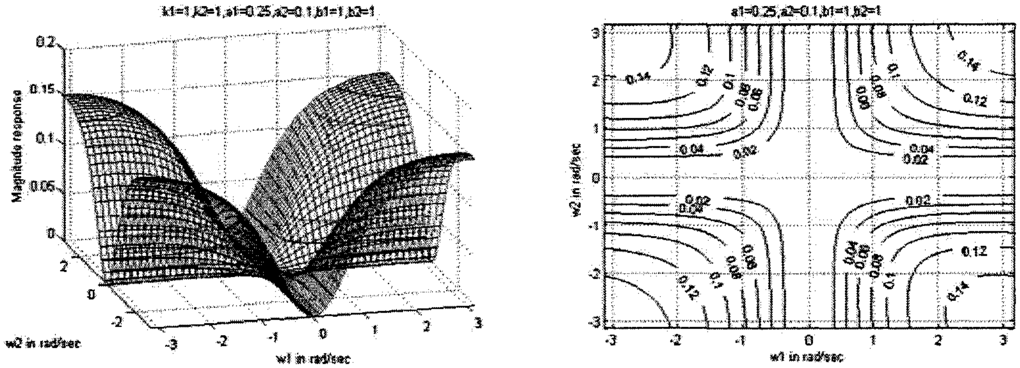
4.3.2 *Case 2*

In case 2, the effect of GBT coefficient a_2 on the 2-D digital highpass filter output is studied. In order to study that, value of a_2 is varied and other parameter values are set as given below:

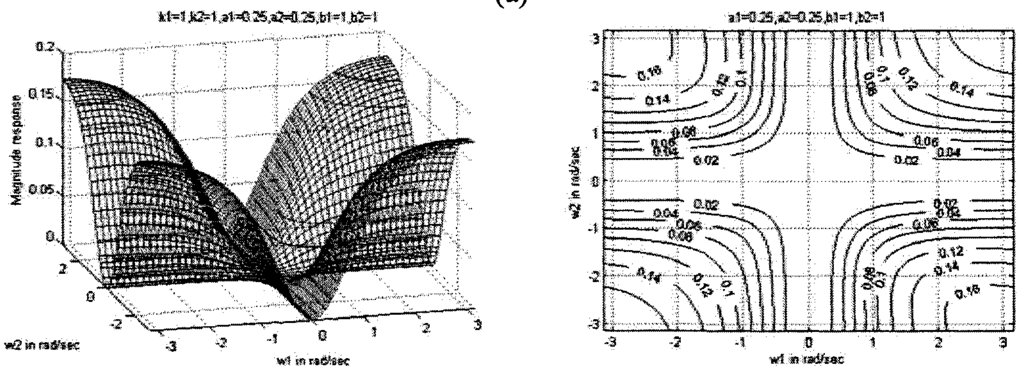
$k_1=k_2=\text{constant}$, $b_1=b_2=-1$, $a_1=\text{constant}$, vary a_2.

$k_1=k_2=1$, $b_1=b_2=-1$, $a_1=0.25$, vary a_2.

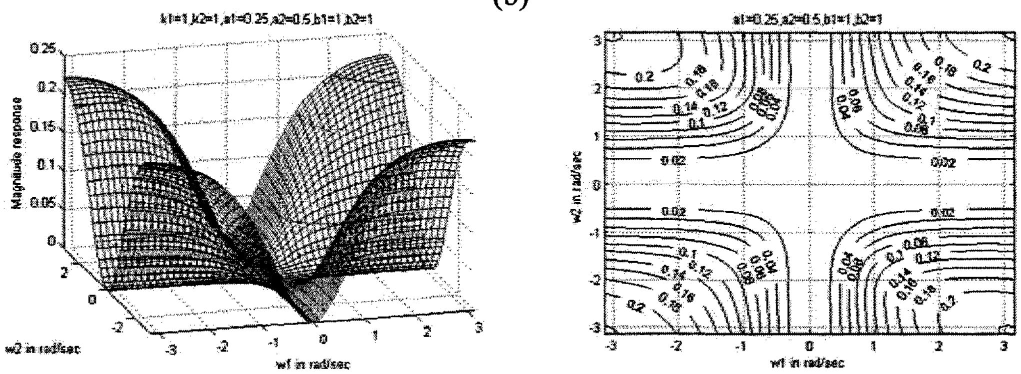
In case 2, the value of GBT coefficient a_2 is increased gradually from 0.1 to 0.75 and the filter output response is obtained. Figs. 4.3 (a), (b), (c) and (d) shows the filter



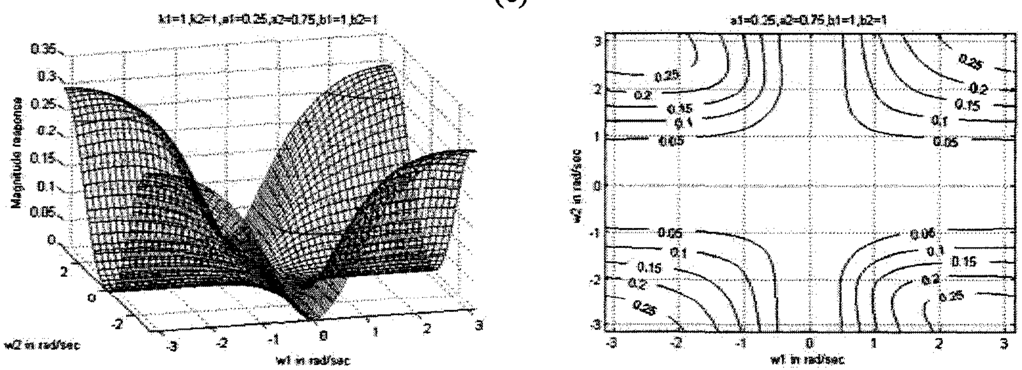
(a)



(b)



(c)



(d)

Figure 4.3: 3-D amplitude-frequency response and contour response of the 2-D digital highpass filter with infinite gain op-amp for case 2 (varying a_2).

output response for a_2=0.1, 0.25, 0.5 and 0.75. As the value of a_2 is increased the magnitude of amplitude response increases from 0.14 to 0.28. The stop bandwidth along the ω_2 axis increases and along ω_1 axis remains constant.

4.3.3 *Case 3*

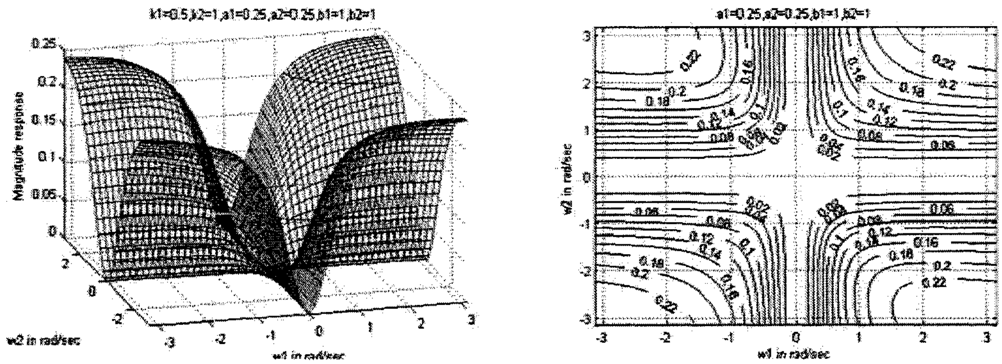
In case 3, the effect of GBT coefficient k_1 on the 2-D digital highpass filter output is studied. The value of k_1 is varied gradually and other parameter values are set as given below:

$$k_2=\text{constant},\ b_1=b_2=-1,\ a_1=a_2=\text{constant}.$$

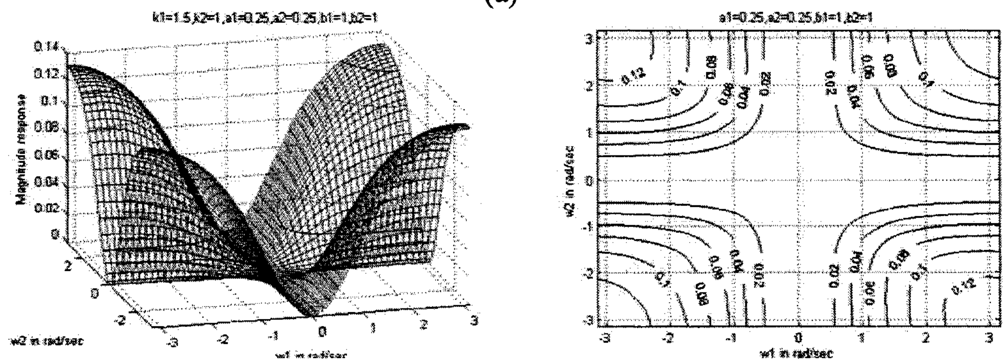
$$k_2=1,\ b_1=b_2=-1,\ a_1=a_2=0.25,\ \text{vary } k_1.$$

Figs. 4.4 (a), (b), (c) and (d) represents the output response of the 2-D digital highpass filter obtained by varying the value of the GBT coefficient k_1. It is seen that as the value of k_1 is increased from 0.5 to 10 the magnitude of amplitude-frequency response decreases from 0.22 to 0.012.

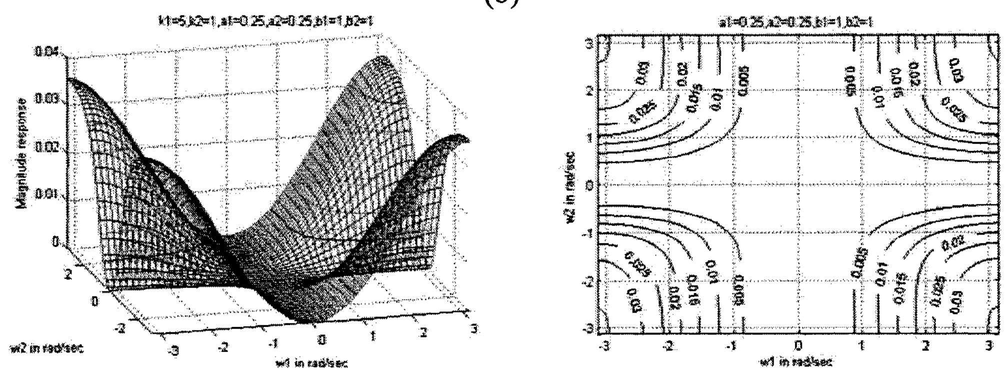
From the contour plots, it's inferred that when the value of $k_1<1$ the pass bandwidth along the ω_1 axis is larger. As the value of k_1 is increased, the pass bandwidth along the ω_1 axis decreases and the pass bandwidth along the ω_2 axis remain constant.



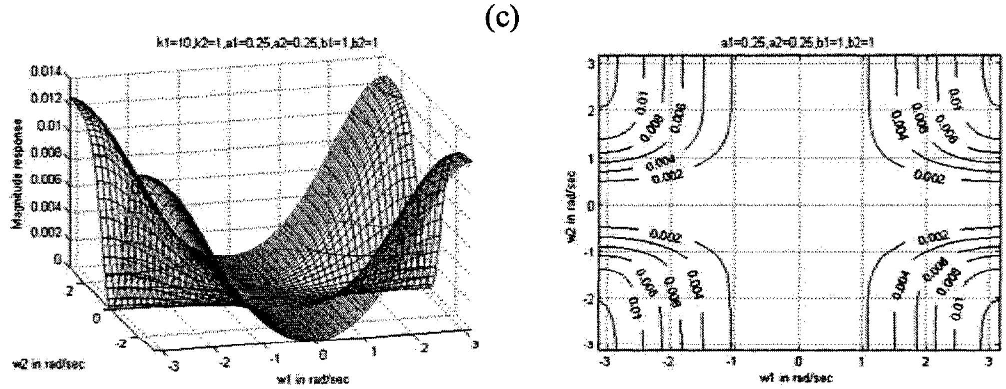
(a)



(b)



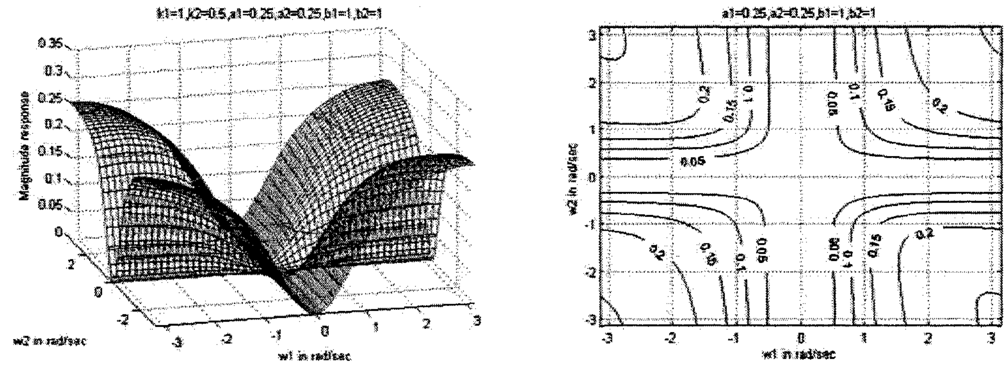
(c)



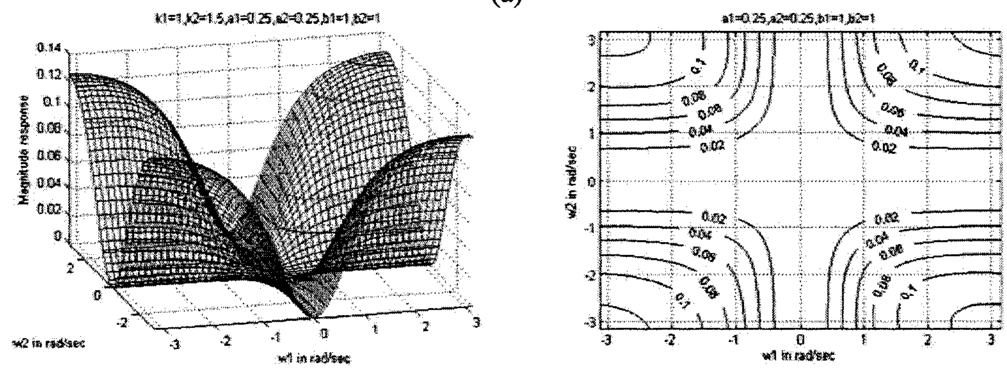
(d)

Figure 4.4: 3-D amplitude-frequency response and contour response of the 2-D digital highpass filter with infinite gain op-amp for case 3 (varying k_1).

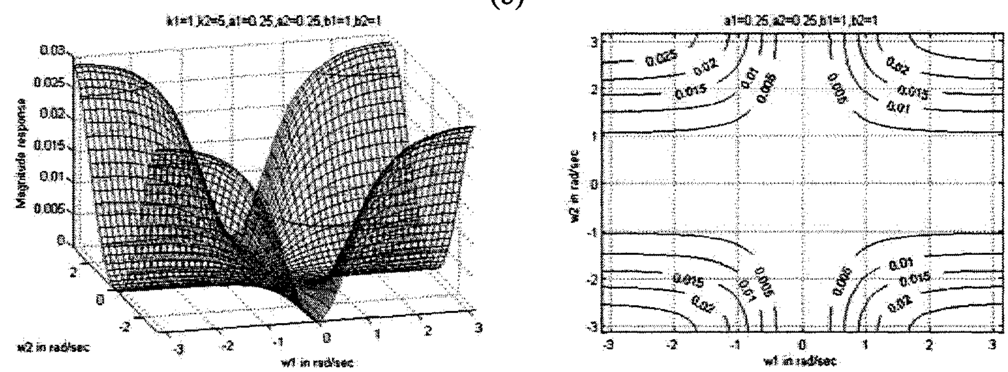
4.3.4 *Case 4*



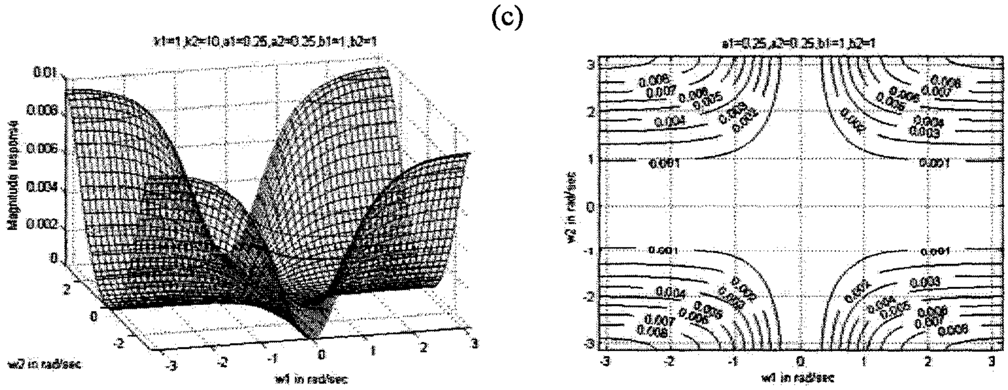
(a)



(b)



(c)



(d)

Figure 4.5: 3-D amplitude-frequency response and contour response of the 2-D digital highpass filter with infinite gain op-amp for case 4 (varying k_2).

In case 4, the effect of GBT coefficient k_2 on the 2-D digital highpass filter with infinite gain op-amp is studied. In order to study the effect of k_2 parameter, it's value is varied and other parameters are set as mentioned below:

k_1=constant, $b_1=b_2=-1$, $a_1=a_2$=constant.

$k_1=1$, $b_1=b_2=-1$, $a_1=a_2=0.25$, vary k_2.

The value of the GBT coefficient k_2 is varied in four steps and the output response of the 2-D digital highpass filter is obtained as shown in figs. 4.5 (a), (b), (c) and (d). It's seen that as the value of k_2 is increased from 0.5 to 10, the magnitude of the amplitude-frequency response decreases from 0.35 to 0.009 and as far as the contour plot is concerned, it is observed that the pass bandwidth along the ω_1 axis remains constant. But the pass bandwidth along the ω_2 axis decreases to a greater extend. Therefore, the GBT coefficient k_2 affect both the magnitude of amplitude-frequency response and the pass bandwidth along ω_2 axis.

4.4 *Frequency response of 2-D active digital highpass filter with finite gain*

In this section, the effect of GBT coefficients and the op-amp gain parameters on the output of 2-D digital highpass filter with finite gain op-amp is studied. In order to do so, the values of the GBT coefficients are varied and its effect on the output of the 2-D active digital highpass filter with finite gain is studied.

Before varying each GBT coefficient individually, the output of the filter is obtained by keeping all the GBT coefficients and the op-amp gain equal to unity. Fig. 4.6 represents the corresponding output.

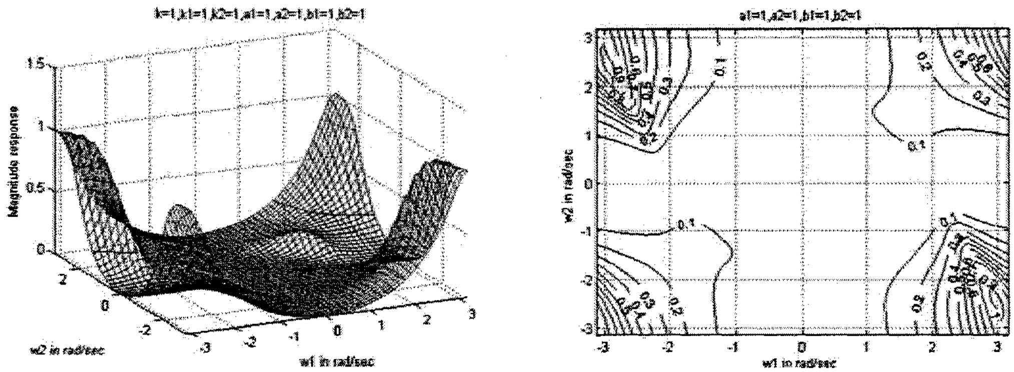


Figure 4.6: 3-D amplitude –frequency response and contour response of the 2-D digital highpass filter with all the GBT coefficients equal to unity

From fig. 4.6 it can be observed that, ripples are present in the frequency response in the first and third quadrant. The ripples present in the output can be minimized by reducing the values of GBT coefficient a_1 and a_2 less than unity. In the upcoming sections GBT coefficient b_1 and b_2 are set equal to negative unity and other parameters are varied and its effects on the amplitude-frequency response and the contour plot of the filter are studied.

4.4.1 *Case 1*

In case 1, the effect of GBT coefficient a_1 on the filter output characteristics is examined. In order to do so, value of a_1 is varied and other parameters are set as given below:

$$k=k_1=k_2=\text{constant},\ b_1=b_2=-1,\ a_2=\text{constant},\ \text{vary } a_1.$$

$$k=1,\ k_1=2,\ k_2=2,\ b_1=-1,\ b_2=-1,\ a_2=0.25,\ \text{vary } a_1.$$

In case 1, the GBT coefficient a_1 is increased gradually in four steps from 0.1 to 0.75 and other parameters are set at a constant value. The response of the filter is obtained and is shown in figs. 4.7 (a), (b), (c) and (d). It can be observed from the fig 4.7 that as the value of a_1 is increased from 0.1 to 0.75, the magnitude of 3-D amplitude plot

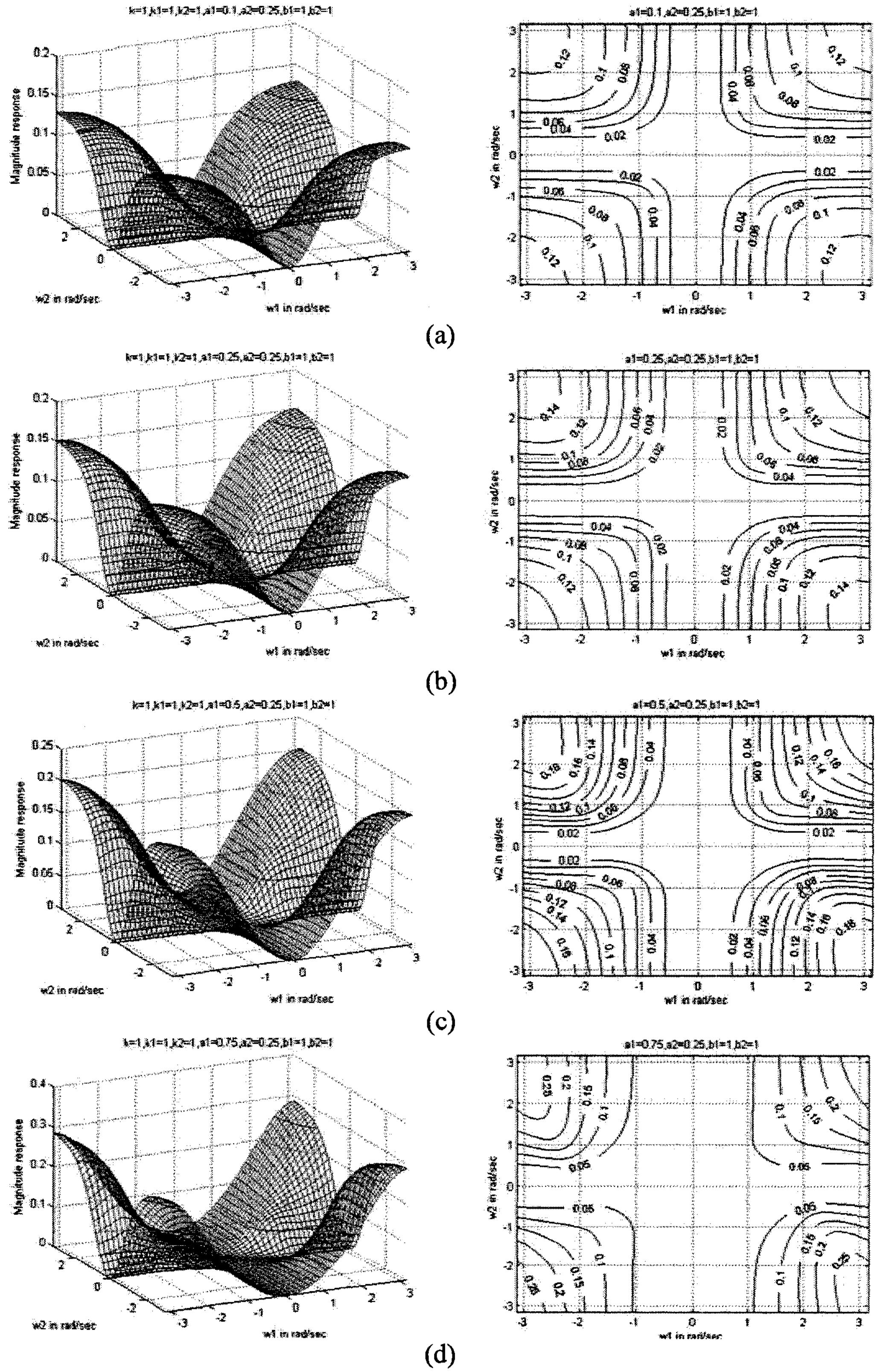


Figure 4.7: 3-D amplitude-frequency response and contour response of the 2-D digital highpass filter with finite gain op-amp for case 1 (varying a_1).

increases from 0.12 to 0.28 and the pass bandwidth along the ω_1 axis decreases. The pass bandwidth along the ω_2 axis remains constant.

To sum up the GBT coefficient a_1 affects the magnitude of the amplitude response and also the pass band width along the ω_1 axis.

4.4.2 *Case 2*

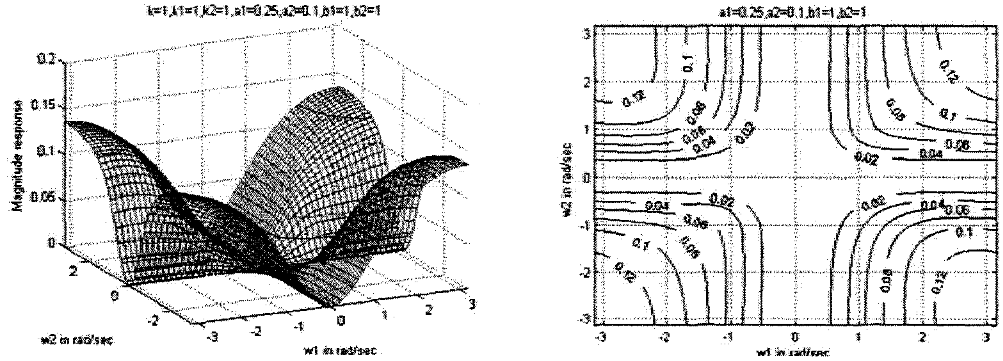
In case 2, the effect of GBT coefficient a_2 on the filter output characteristics is analyzed. In order to do so, value of a_2 is varied and other parameter values are set as given below:

$$k=k_1=k_2=\text{constant},\ b_1=b_2=-1,\ a_1=\text{constant, vary } a_2.$$

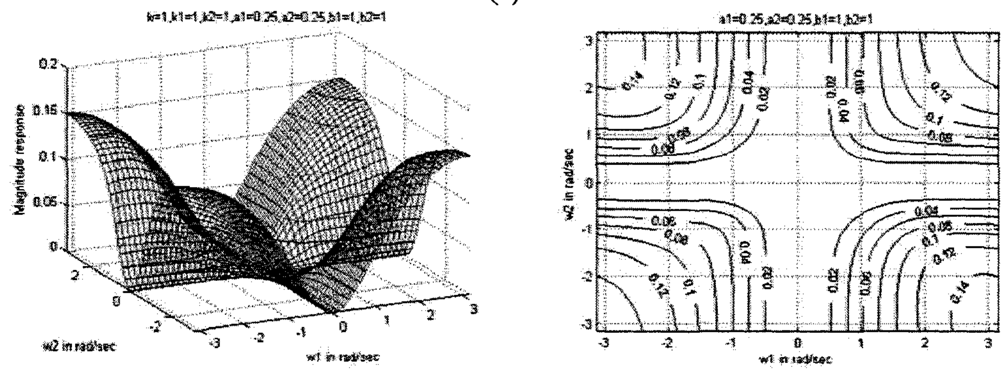
$$k=1,\ k_1=k_2=1,\ a_1=0.25,\ b_1=-1,\ b_2=-1 \text{ and vary } a_2.$$

Figs. 4.8 (a), (b), (c) and (d) shows the amplitude-frequency response of the 2-D digital highpass filter for different values of GBT coefficient a_2 in the range of 0.1 to 0.75. As the value of a_2 is increased from 0.1 to 0.75, it is observed that the magnitude of the amplitude response increases from 0.12 to 0.22. The pass bandwidth along the ω_2 axis decreases, but the pass band width along ω_1 axis remains constant.

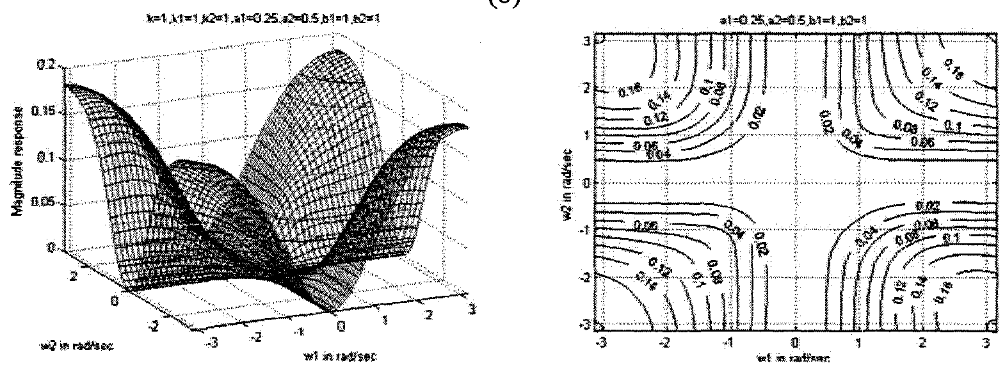
To sum up the GBT coefficient a_2 affects the magnitude of the amplitude-frequency response and the pass bandwidth along the ω_2 axis.



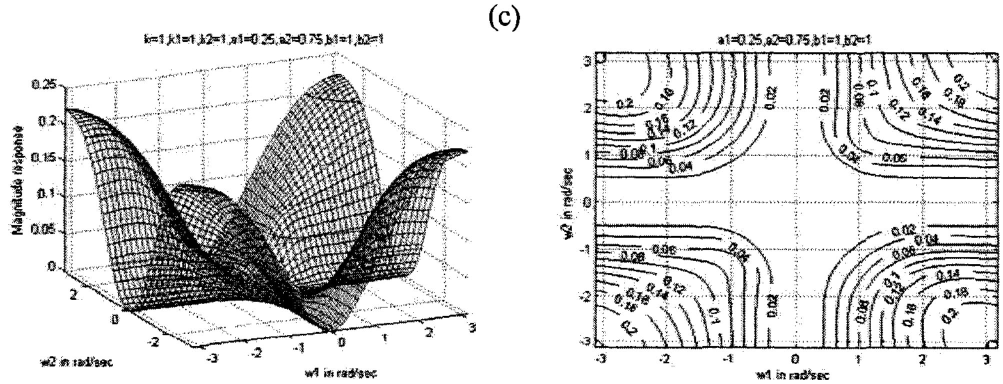
(a)



(b)



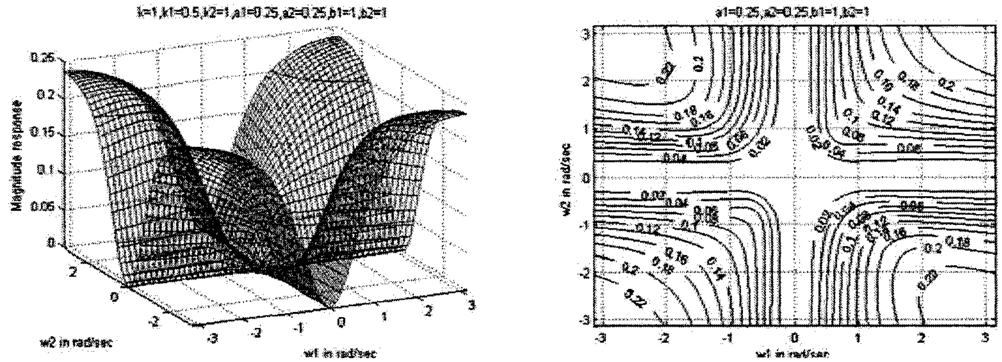
(c)



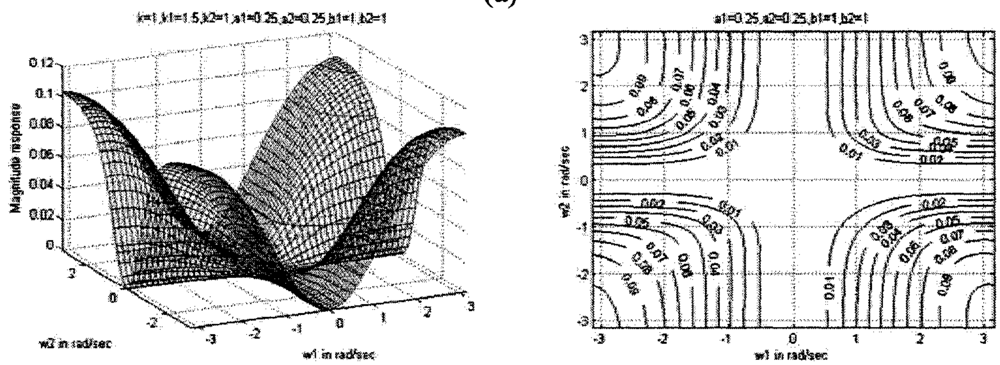
(d)

Figure 4.8: 3-D amplitude-frequency response and contour response of the 2-D digital highpass filter with finite gain op-amp for case 2 (varying a_2).

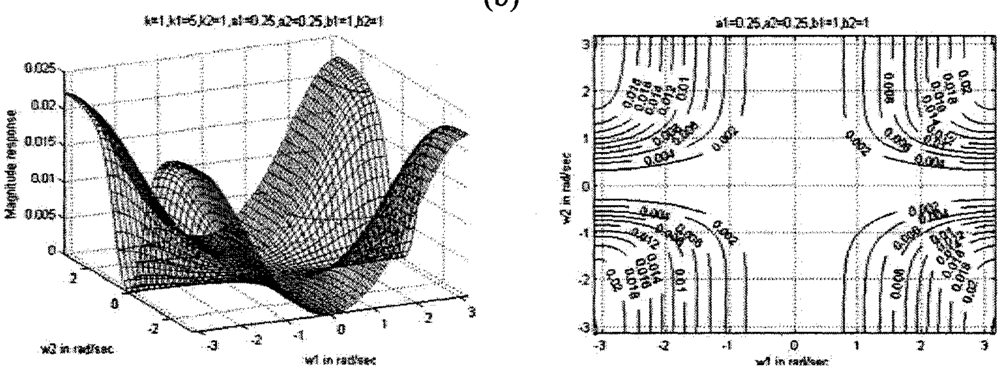
4.4.3 *Case 3*



(a)



(b)



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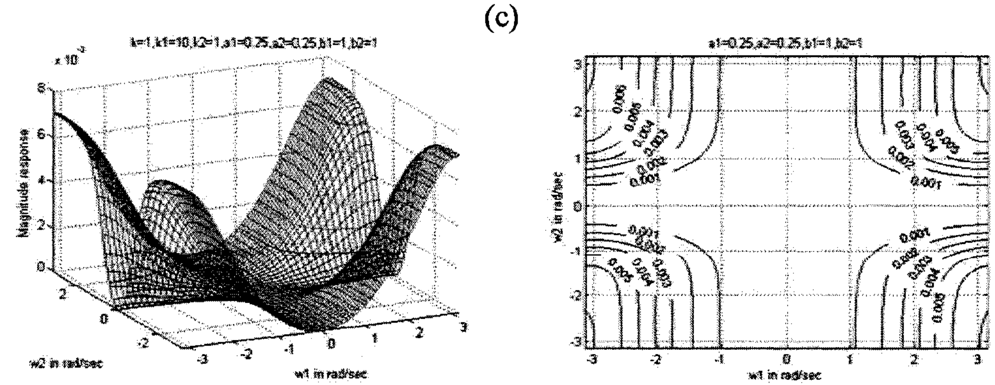


(d)

Figure 4.9: 3-D amplitude-frequency response and contour response of the 2-D digital highpass filter with finite gain op-amp for case 3 (varying k_1).

In case 3, the effect of GBT coefficient k_1 on the digital filter output characteristics is examined. In order to study the effect of k_1 parameter, the value of k_1 is varied and other coefficients of GBT are set as given below:

$$k=k_2=\text{constant},\ b_1=b_2=-1,\ a_1=a_2=\text{constant},\ \text{vary } k_1.$$

$$k=k_2=1,\ b_1=b_2=-1,\ a_1=a_2=0.25,\ \text{vary } k_1.$$

From figs. 4.9 (a), (b), (c) and (d) it can be seen that as the value of k_1 is increased from 0.5 to 10 the pass bandwidth along the ω_1 axis decreases and more over the magnitude of the amplitude-frequency response decreases from 0.75 to 0.008 and the pass bandwidth along the ω_2 axis remains constant.

Therefore the GBT coefficient k_1 affects the pass bandwidth along the ω_1 axis and the amplitude of the magnitude response.

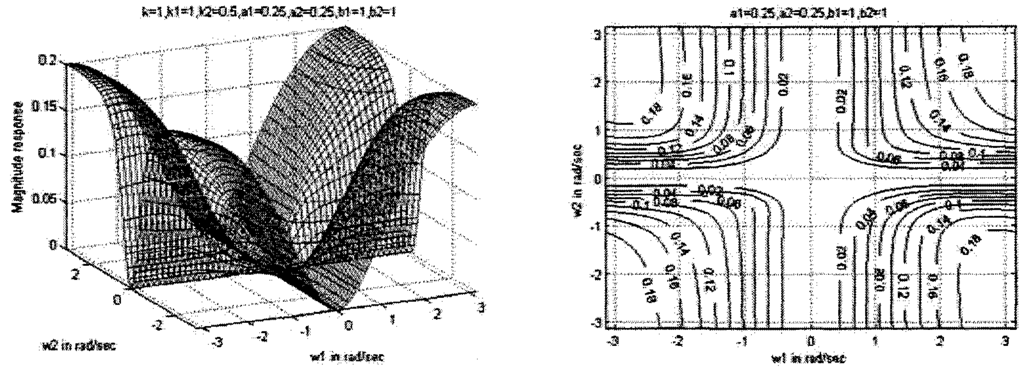
4.4.4 *Case 4*

In case 4, the effect of GBT coefficient k_2 on the digital filter output is studied. In order to study the effect of k_2, value of k_2 is varied and other parameters are set as given below:

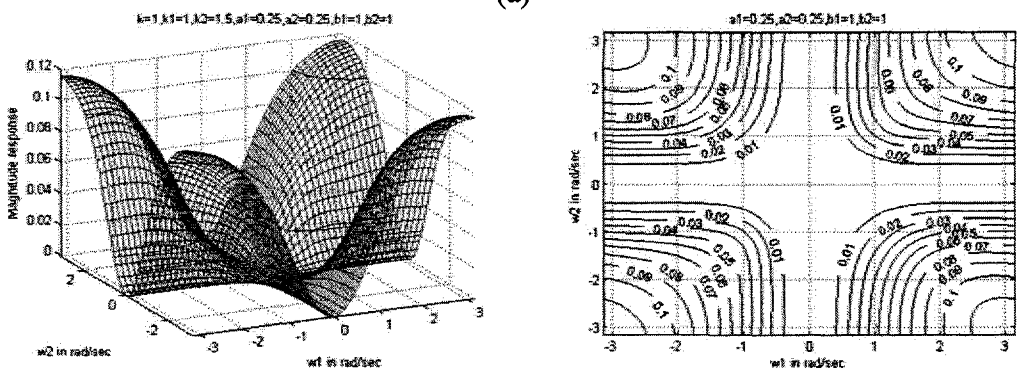
$$k=\text{constant},\ k_1=\text{constant},\ b_1=b_2=-1,\ a_1=a_2,\ \text{vary } k_2$$

$$k=1,\ k_1=5,\ b_1=-1,\ b_2=-1,\ a_1=0.5,\ a_2=0.5,\ \text{vary } k_2.$$

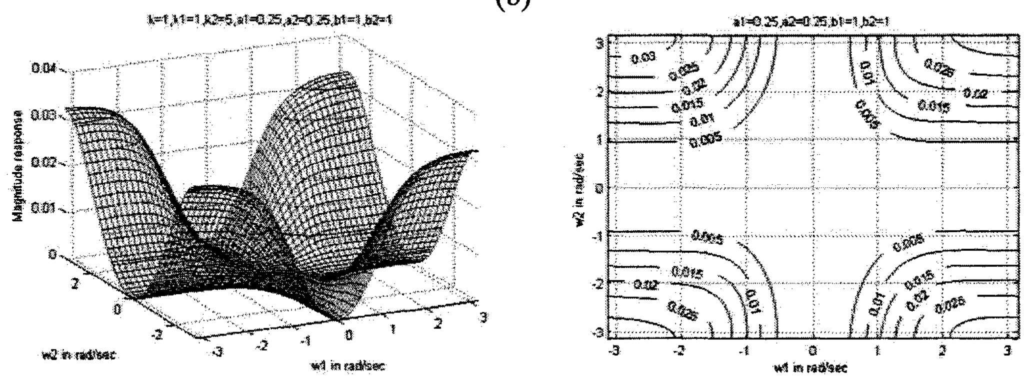
Fig. 4.10 shows the response of the 2-D digital highpass filter obtained by varying the value of GBT coefficient k_2. As the value of k_2 is increased from 0.5 to 10 the magnitude of amplitude-frequency response decreases from 0.5 to 0.015, the pass bandwidth along the ω_2 axis decreases and the pass bandwidth along the ω_1 axis remains constant.



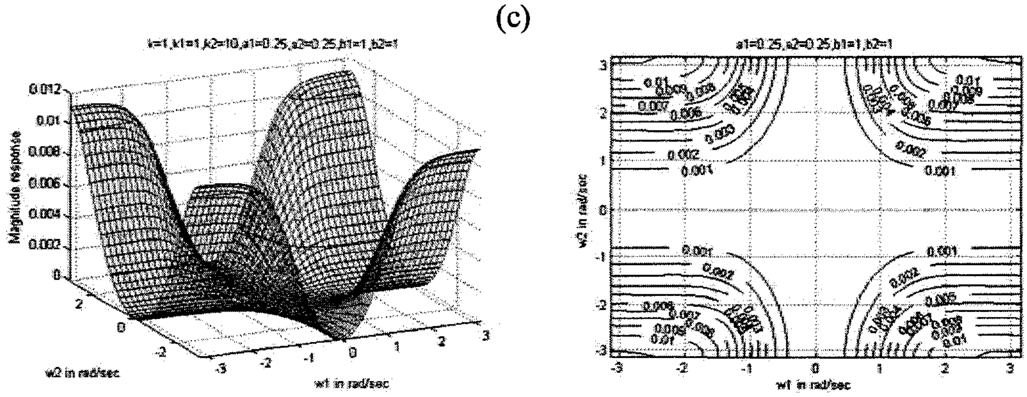
(a)



(b)



(c)



(d)

Figure 4.10: 3-D amplitude-frequency response and contour response of the 2-D digital highpass filter with finite gain op-amp for case 4 (varying k_2).

Therefore, the GBT coefficient k_2 affects the magnitude of the amplitude-frequency response and the pass bandwidth along the ω_2 axis.

4.4.5 *Case 5*

In case 5, the effect of op-amp gain parameter k on the digital filter output characteristics is analyzed. In order to do so, the value of k is varied and other GBT coefficients are set as given below:

$k_1=k_2=$constant, $b_1=b_2=-1$, $a_1=a$, vary k.

$k_1=k_2=1$, $a_1=a_2=0.25$, $b_1=b_2=-1$, vary k.

Figs. 4.11 (a), (b), (c) and (d) are obtained by varying the op-amp gain k and by keeping the GBT coefficient values at a constant value. It's observed that as the value of k is increased, the magnitude of the amplitude-frequency plot increases from 0.08 to 0.4, the pass bandwidth along the ω_1 axis and ω_2 axis remains constant.

The op-amp gain k mainly affects the magnitude of the 3-D amplitude-frequency plot.

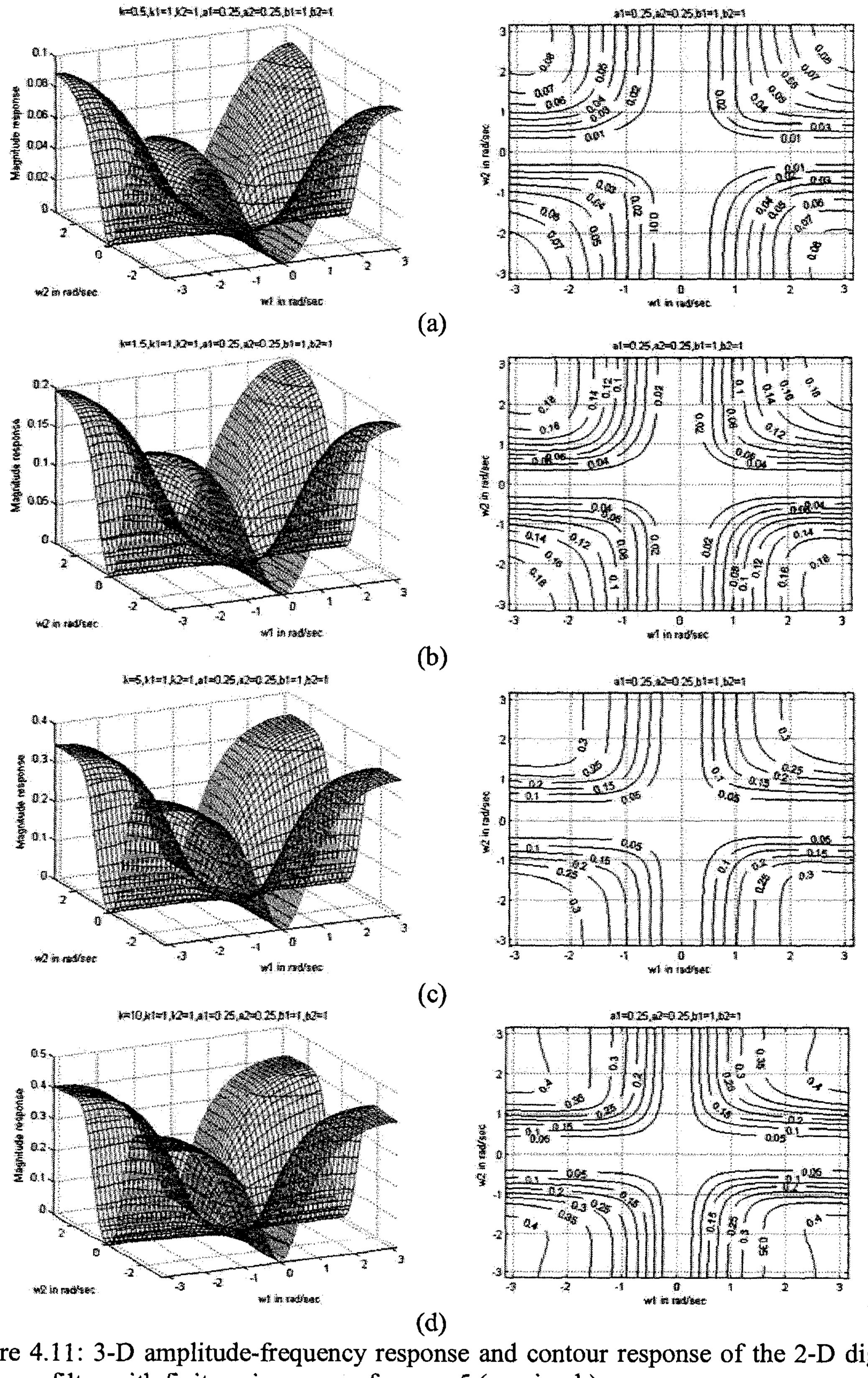


Figure 4.11: 3-D amplitude-frequency response and contour response of the 2-D digital highpass filter with finite gain op-amp for case 5 (varying k).

4.5 *Summary*

The transfer function of the 2-D digital highpass filter for infinite gain op-amp case was given by eqn. (4.4) and for finite gain op-amp case is given by eqn. (4.5). It was obtained by applying the highpass GBT to analog transfer function given by eqn. (2.19) and eqn. (2.24) in chapter 2.

The transfer function in the digital domain has GBT coefficients, in order to examine the effect of each GBT coefficient on the filter output, every GBT coefficient was varied individually and the output was examined. The result for the 2-D digital filter with infinite gain op-amp has been summarized in Table 4.1.

Initially the GBT coefficient a_1 was varied from 0.1 to 0.75 and it was observed that the magnitude of the amplitude-frequency plot also increased. The stop bandwidth along the ω_1 axis increased and along ω_2 axis remained constant. Next, the value of GBT coefficient a_2 was increased from 0.1 to 0.75, the magnitude of the amplitude-frequency response increased along with it, the stop bandwidth along the ω_1 axis remained constant and the stop bandwidth along the ω_2 axis increased.

The value of GBT coefficient k_1 is varied from 0.5 to 10, the magnitude of the amplitude-frequency response decreased, stop bandwidth along the ω_1 axis increased and

GBT coefficient	**Value of GBT Coefficient**	**Amplitude of 3D magnitude response**	**Stop bandwidth along ω_1 axis**	**Stop bandwidth along ω_2 axis**
a_1	↑	↑	↑	Constant
a_2	↑	↑	Constant	↑
k_1	↑	↓	↑	Constant
k_2	↑	↓	Constant	↑

Table 4.1: Summary of effects of GBT coefficients on infinite gain highpass filter.

stop bandwidth along the ω_2 axis remained constant. Finally, the value of GBT coefficient k_2 was varied from 0.5 to 10 and it was observed that the magnitude of the amplitude-frequency response decreased with the increase in k_2 value, stop bandwidth along the ω_2 axis increased and stop bandwidth along the ω_1 axis remained constant.

Table 4.2, summaries the effects of GBT coefficients and the op-amp gain parameter on the output of 2-D highpass filter with finite gain op-amp. GBT coefficients were varied individually; to start with the GBT coefficient a_1 was varied from 0.1 to 0.75. As the value of a_1 was increased, the magnitude of the amplitude-frequency response increased with it, the stop bandwidth along the ω_1 axis increased and the stop bandwidth along the ω_2 axis remained constant. Next the value of GBT coefficient a_2 was increased from 0.1 to 0.75. It was observed that with the increase in the value of a_2 the magnitude of the amplitude-frequency response increased, the stop bandwidth along the ω_1 axis remained constant and the stop bandwidth along the ω_2 axis decreased.

Followed by a_2, the value of GBT coefficient k_1 was increased from 0.5 to 10. It was observed that the magnitude of the amplitude-frequency response decreased, the stop

GBT coefficient	**Value of GBT coefficient**	**Amplitude of 3D magnitude response**	**Stop bandwidth along ω_1 axis**	**Stop bandwidth along ω_2 axis**
a_1	↑	↑	↑	Constant
a_2	↑	↑	Constant	↓
k_1	↑	↓	↑	Constant
k_2	↑	↓	Constant	↑
k	↑	↑	↓	Constant

Table 4.2: Summary of effects of GBT coefficients on finite gain highpass filter.

bandwidth along the ω_1 axis decreased and the stop bandwidth along the ω_2 axis remained constant. After that the value of GBT coefficient k_2 was increased from 0.5 to 10. It was observed that the magnitude of the amplitude-frequency response decreased, the stop bandwidth along the ω_1 axis remained constant and the stop bandwidth along the ω_2 axis increased.

Finally, the op-amp gain parameter k was varied from 0.5 to 10 and it was observed that the magnitude of the amplitude-frequency response increased and the stop bandwidth along the ω_1 and ω_2 axis remained constant.

Thus the effect of GBT coefficients and the op-amp gain parameters on the output of 2-D digital highpass filter for infinite gain and finite gain cases were studied.

Chapter 5

Two Dimensional Bandpass Filter

5.1 *Introduction*

The bandpass filter is obtained by cascading the lowpass filter and highpass filter, pass bandwidth of both the filters must overlap. In this chapter, the transfer function of the bandpass filter is obtained from the transfer function of the lowpass filter and highpass filter obtained in chapter 3 and 4 for both the infinite gain and finite gain cases.

5.2 *Transfer function of 2-D active digital bandpass filter*

The transfer function of the 2-D digital BPF with infinite gain op-amp is obtained by cascading the transfer function of the 2-D digital LPF with infinite gain op-amp given by eqn. (3.4) and the transfer function of the 2-D digital BPF with infinite gain op-amp given by eqn. (4.4). Similarly the transfer function is obtained for the finite gain case. It's required that $k_i>0$ and $0\leq a_i\leq 1$ for the bandpass filter to be stable. Mathematically,

For infinite gain op-amp case

$$H_{dBP}(z_1,z_2)=H_{dL}(z_1,z_2)\ \mathrm{x}\ H_{dH}(z_1,z_2) \tag{5.1}$$

For finite gain op-amp case

$$H_{dfBP}(z_1,z_2)=H_{dfL}(z_1,z_2) \times H_{dfH}(z_1,z_2) \tag{5.2}$$

5.3 Frequency response of 2-D digital bandpass filter with infinite gain

MATLAB is used to plot the 3-D amplitude-frequency response and the contour plots of the transfer function of the filter with infinite gain obtained in eqn. (5.1). For the bandpass filter, the GBT coefficient k_1, k_2, a_1 and a_2 are the variable parameters. These four GBT coefficients are varied individually and its effect on the filter output is analyzed.

Initially let all the GBT coefficient values be set to unity in the lowpass filter and highpass filter. Fig. 5.1 represents the 3-D amplitude-frequency response of 2-D digital bandpass filter with infinite gain.

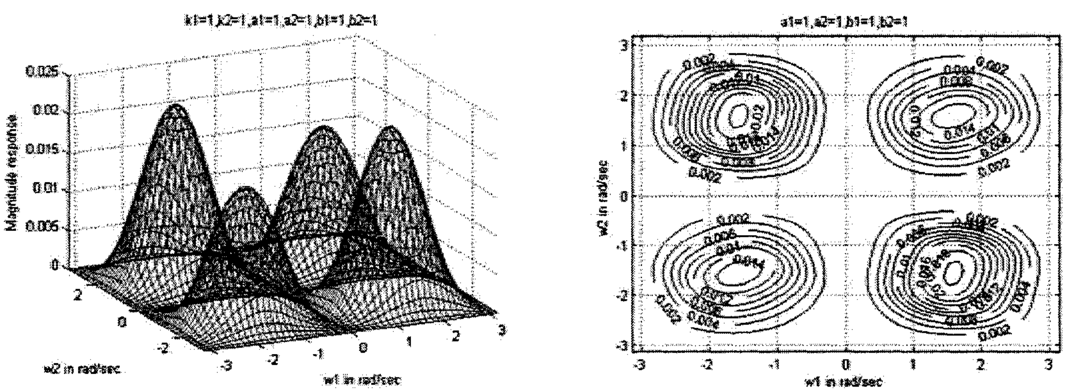


Figure 5.1: 3-D amplitude –frequency response and contour response of the 2-D digital bandpass filter with infinite gain op-amp and all coefficient value equal to unity.

Amplitude-frequency response in the first and third quadrant is symmetrical and also the output in the second and fourth quadrant is symmetrical. The magnitude of the amplitude-frequency response present in the second and fourth quadrant is larger than the magnitude of the amplitude-frequency in the first and third quadrant.

In the next sections the effect of individual GBT coefficients on the filter output is analyzed.

5.3.1 *Case 1*

In case 1, the effect of GBT coefficient a_1 on the filter output characteristics is studied. In order to study that, the value of a_1 is varied and other parameter values are set as given below:

k_1=constant, k_2=constant, $b_1=b_2=1$, a_2=constant.

$k_1=0.5$, $k_2=0.5$, $b_1=b_2=1$, $a_2=0.25$, vary a_1.

Fig. 5.2 shows the amplitude-frequency response of the 2-D digital bandpass filter obtained by varying the GBT coefficient a_1 from 0.1 to 0.75. From the amplitude-frequency response it's observed that symmetry is retained between first and third quadrant and between second and fourth quadrant. The magnitude of amplitude-frequency response in the first and third quadrant increases from 0.06 to 0.08 and in the second and fourth quadrant magnitude increases from 0.06 to 0.14.

The pass bandwidth along the ω_1 axis decreases with the increase in the value of a1 and the pass bandwidth along the ω_2 axis remain constant.

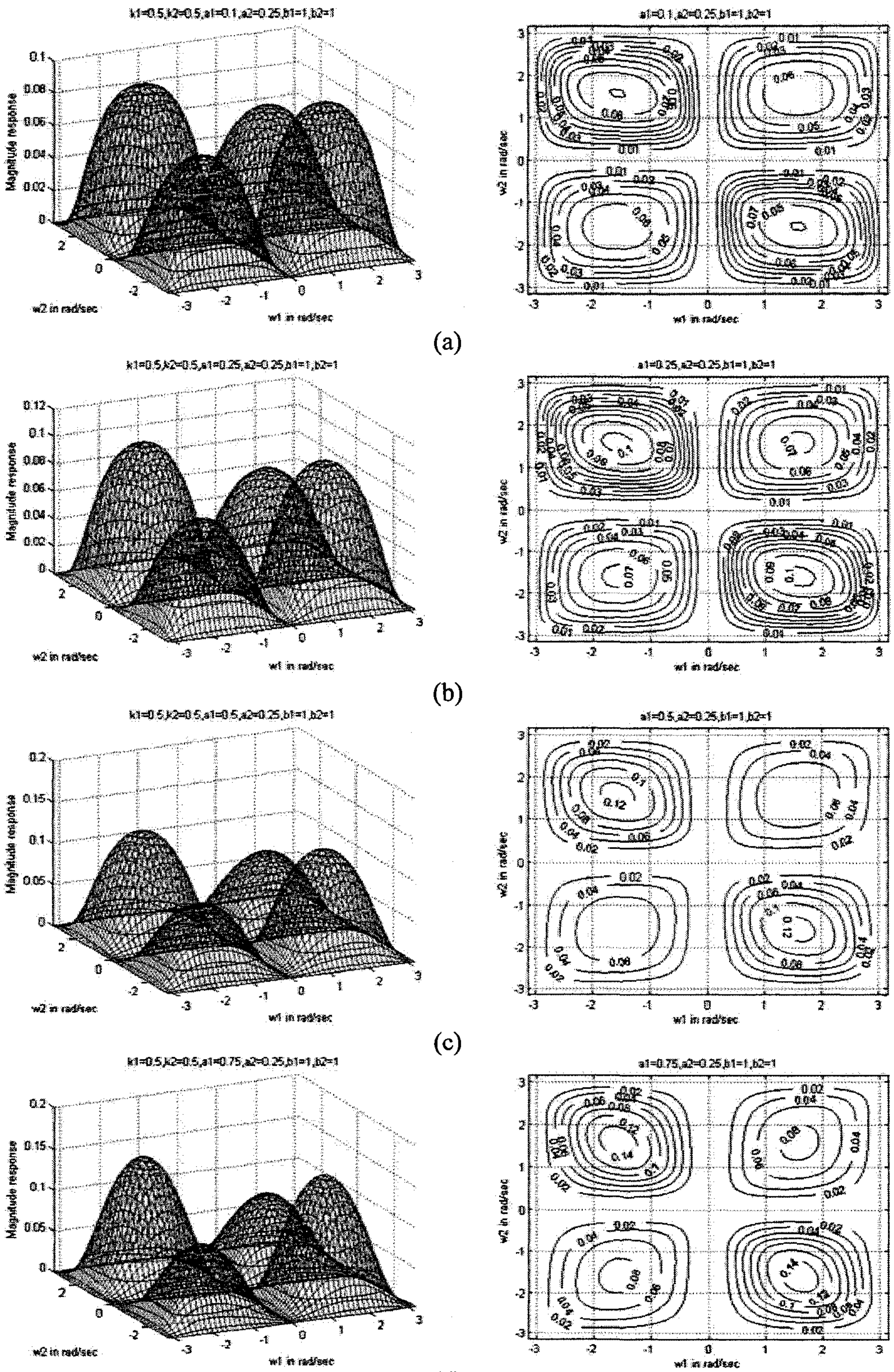


Figure 5.2: 3-D amplitude-frequency response and contour response of the 2-D digital bandpass filter with infinite gain op-amp for case 1 (varying a_1).

5.3.2 *Case 2*

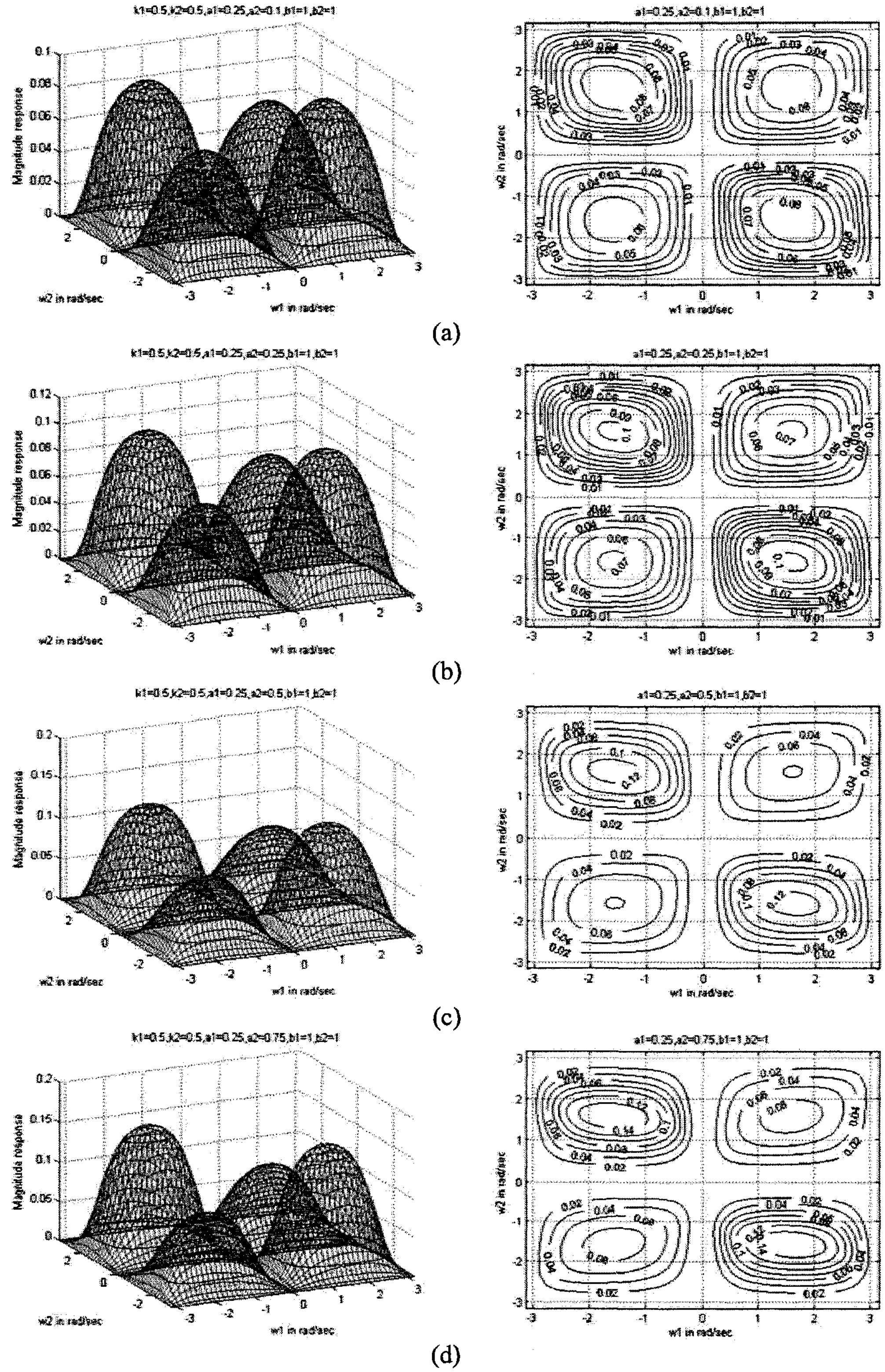


Figure 5.3: 3-D amplitude-frequency response and contour response of the 2-D digital bandpass filter with infinite gain op-amp for case 2 (varying a_2).

In case 2, the effect of GBT coefficient a_2 on the 2-D digital filter output characteristics are studied. In order to study that, the value of a_2 is varied and other parameters are set as given below:

$k_1=k_2=$constant, $b_1=b_2=1$, $a_1=$constant.

$k_1=k_2=0.5$, $b_1=b_2=1$, $a_1=0.25$, vary a_2.

Fig. 5.3 is obtained by varying the GBT coefficient a_2 from 0.1 to 0.75. Other GBT coefficients are set to above mentioned values. From figs. 5.3 (a), (b), (c) and (d) it is observed that as the value of a_2 increased the magnitude of the 3-D amplitude-frequency response in the first and third quadrant increases from 0.06 to 0.08 and in the second and fourth quadrant also the magnitude increases from 0.06 to 0.14.

The pass bandwidth along the ω_1 axis remains constant and the pass band width along the ω_2 axis decreases with the increase in the value of a_2.

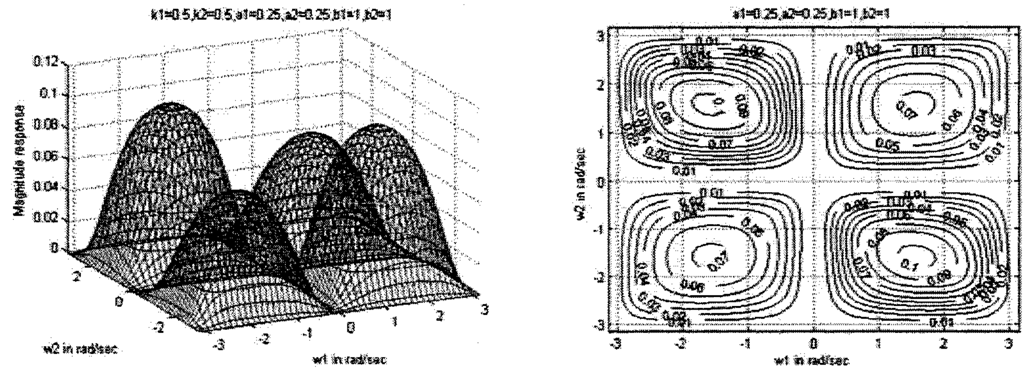
5.3.3 *Case 3*

In case 3, the effect of GBT coefficient k_1 on the 2-D digital bandpass filter output is analyzed. In order to study the effect of k_1 parameter, value of k_1 parameter is varied and other parameter values are set as given below:

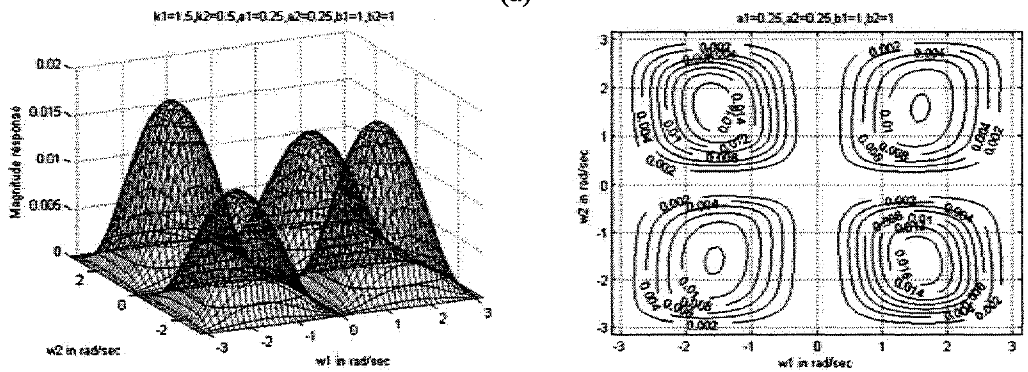
$k_2=$constant, $b_1=b_2=1$, $a_1=a_2=$constant, vary k_1.

$k_2=0.5$, $b_1=b_2=1$, $a_1=a_2=0.25$, vary k_1.

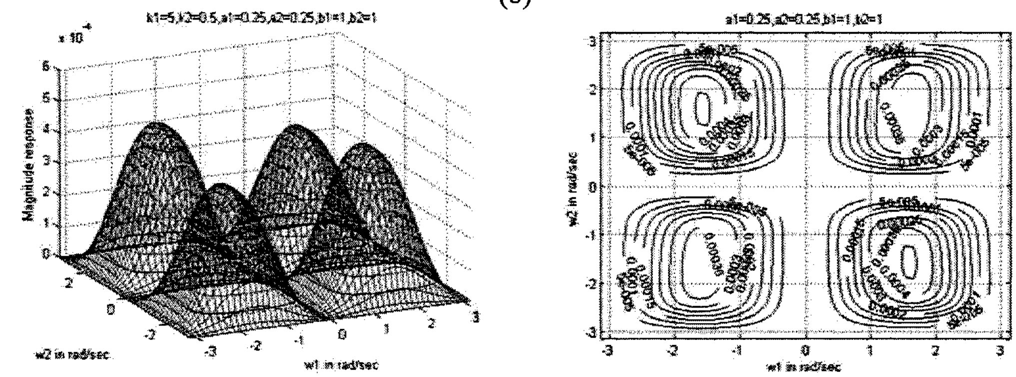
Figs. 5.4 (a), (b), (c) and (d) represent the amplitude-frequency response of the 2-D bandpass filter obtained by varying the GBT coefficient k_1 in the range of 0.5 to 10. The magnitude of amplitude-frequency response in all the four quadrants of the output



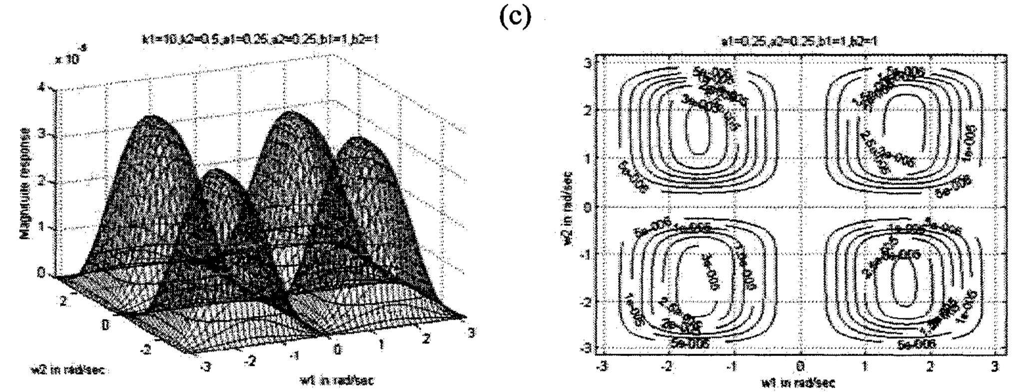
(a)



(b)



(c)



(d)

Figure 5.4: 3-D amplitude-frequency response and contour response of the 2-D digital bandpass filter with infinite gain op-amp for case 3 (varying k_1).

decreases as the value of k_1 is increased. Symmetry is preserved in first and third quadrant and in second and fourth quadrant. The magnitude of the amplitude-frequency response in the first and third quadrant decreases from 0.07 to 0.00003 and in the second and fourth quadrant magnitude decreases from 0.1 to 0.00003.

As far as contour plots are considered, as the value of k_1 is increased greater than unity there is a 90 degree rotation in the orientation of the contour plot along the clockwise direction. The contour plots are elliptical in nature at the center.

5.3.4 *Case 4*

In case 4, the effect of GBT coefficient k_2 on the 2-D digital bandpass filter with infinite gain op-amp is studied. In order to study the effect of k_2 parameter, value of k_2 is varied and other parameters values are set as mentioned below:

$$k_1=\text{constant},\ b_1=b_2=1,\ a_1=a_2=\text{constant}.$$

$$k_1=1,\ b_1=b_2=1,\ a_1=a_2=0.25,\ \text{vary } k_2.$$

Figs. 5.5 (a), (b), (c) and (d) represents the amplitude-frequency response of the 2-D digital bandpass filter obtained by varying the GBT coefficient k_2 and other parameters are set to the values as specified above. As the value of k_2 is increased from 0.5 to 10 the magnitude of the amplitude-frequency response in the first and third quadrant decreases from 0.025 to $6e^{-6}$ and the magnitude of the amplitude-frequency in the second and fourth quadrant decreases from 0.04 to $6e^{-6}$.

The pass bandwidth along the ω_1 axis remains constant and the pass bandwidth along the ω_2 axis decreases with the increase in the value of k_2. Contour plots are elliptical in nature and are aligned along the ω_2 axis for low values of k_2 i.e. $k_2<1$. As the

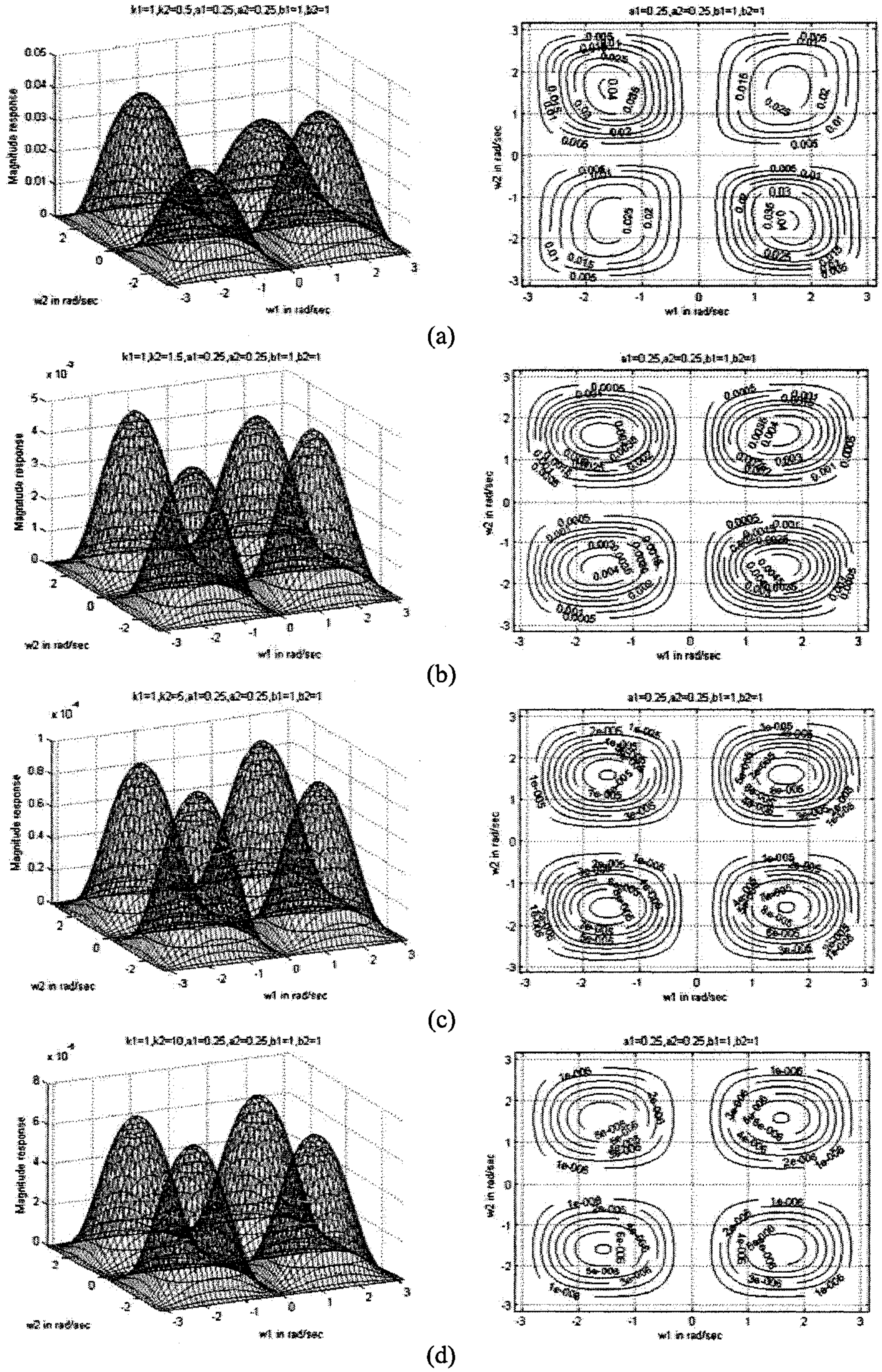


Figure 5.5: 3-D amplitude-frequency response and contour response of the 2-D digital bandpass filter with infinite gain op-amp for case 4 (varying k_2).

value of k_2 is increased i.e. $k_2>1$ the orientation of the contour plots is rotated by 90 degree in anti clockwise direction and gets aligned along the ω_1 axis.

5.4 *Frequency response of 2-D active digital bandpass filter with finite gain*

In this section, the effect of each GBT coefficient and the op-amp gain parameter 'k' on the amplitude-frequency response and contour plots of the 2-D digital bandpass filter with finite gain op-amp is examined. In order to do so, each of the parameter is varied individually by keeping other parameters constant.

Initially all the GBT coefficients and the op-amp gain parameter values are set equal to unity. Fig. 5.6 represents the corresponding output. Ripples are present in the output. Symmetry is retained between the first and third quadrants and between second and fourth quadrants. The magnitude of the amplitude-frequency response in the first and third quadrants is greater than the magnitude of the amplitude-frequency response in second and fourth quadrant.

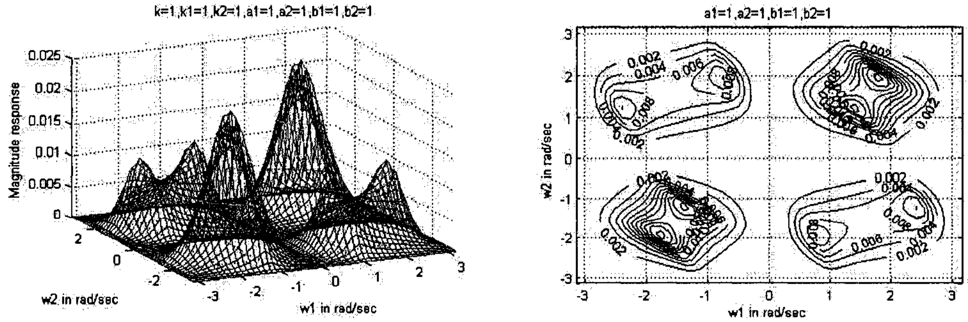


Figure 5.6: 3-D amplitude –frequency response and contour response of the 2-D digital highpass filter with finite gain op-amp and all coefficient value set equal to unity.

In the upcoming sections the effect of each GBT coefficient and op-amp gain parameter on the filter output would be studied.

5.4.1 *Case 1*

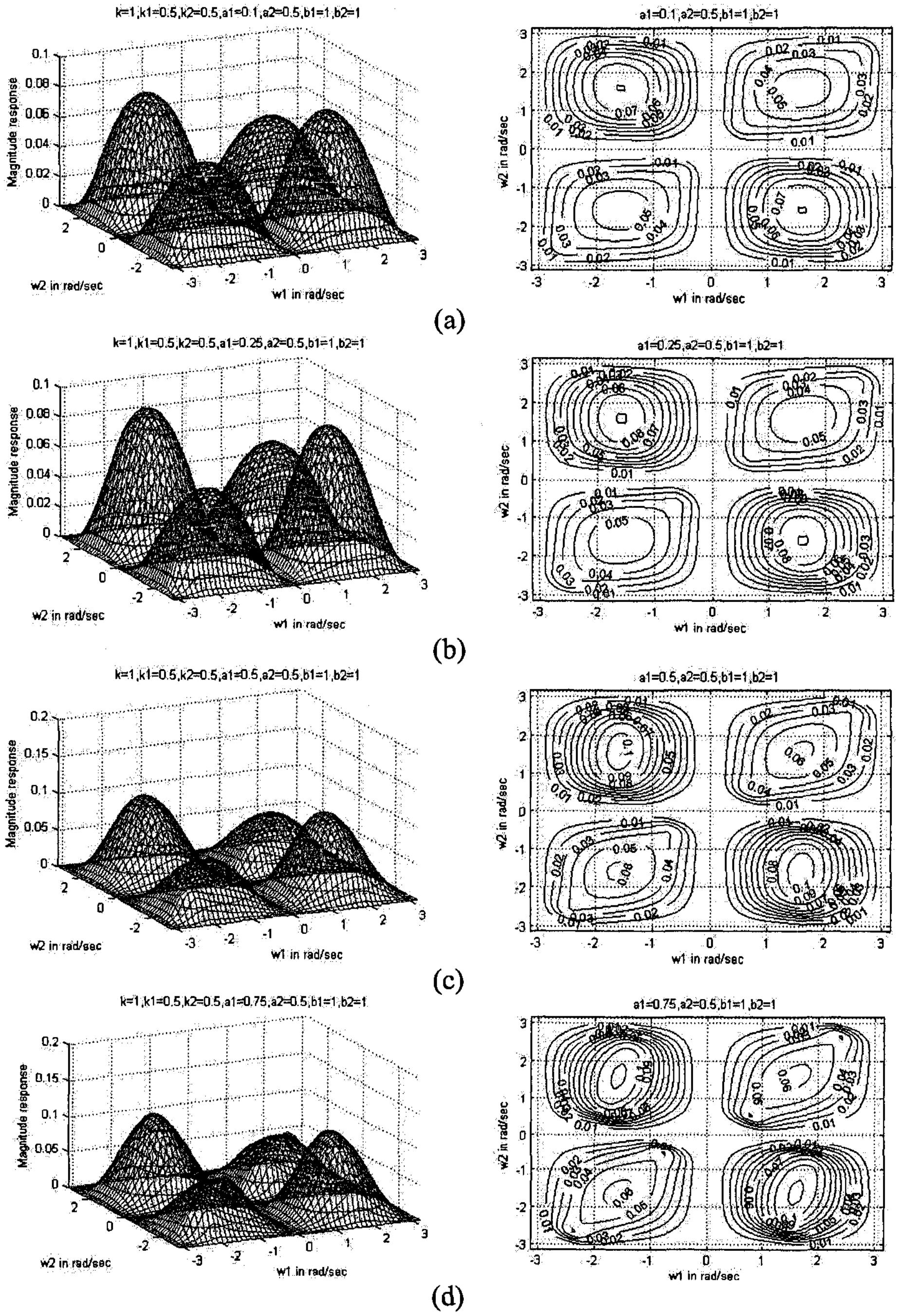


Figure 5.7: 3-D amplitude-frequency response and contour response of the 2-D digital bandpass filter with finite gain op-amp for case 1 (varying a_1).

In case 1, the effect of GBT coefficient a_1 on the filter output characteristics is studied. In order to study that, the value of a_1 is varied and other parameter values are set as given below:

k=constant, $k_1=k_2$=constant, $b_1=b_2=1$, a_2=constant, vary a_1.

$k=1$, $k_1=0.5$, $k_2=0.5$, $b_1=1$, $b_2=1$, $a_2=0.5$, vary a_1.

Fig. 5.7 (a), (b), (c) and (d) represent the amplitude-frequency response of the 2-D digital bandpass filter obtained by varying the GBT coefficient a_1 from 0.1 to 0.75. It's observed that the outputs in the first and third quadrants and in the second and fourth quadrant are symmetric. As the value of GBT coefficient a_1 is increased from 0.1 to 0.75, magnitude of the 3-D amplitude-frequency response in the first and third quadrants increases from 0.05 to 0.06 and the magnitude in the second and fourth quadrants increases from 0.07 to 0.1. The pass bandwidth along the ω_1 and ω_2 axis remains almost constant.

5.4.2 *Case 2*

In case 2, the effect of GBT coefficient a_2 on the filter output characteristics is examined. In order to study that, value of a_2 is varied and other parameter values are set as given below:

k=constant, $k_1=k_2$=constant, $b_1=b_2=1$, a_1=constant, vary a_2.

$k=1$, $k_1=k_2=0.5$, $a_1=0.5$, $b_1=1$, $b_2=1$ and vary a_2.

Figs. 5.8 (a), (b), (c) and (d) represent the amplitude-frequency response of the 2-D digital bandpass filter obtained by varying the GBT coefficient a_2 from 0.1 to 0.75. The pass band width along both ω_1 and ω_2 axis remains constant. The magnitude of the amplitude-frequency in the first and third quadrants increases from 0.05 to 0.06 and in the second and fourth quadrant the magnitude of amplitude-frequency response increases from 0.07 to 0.12.

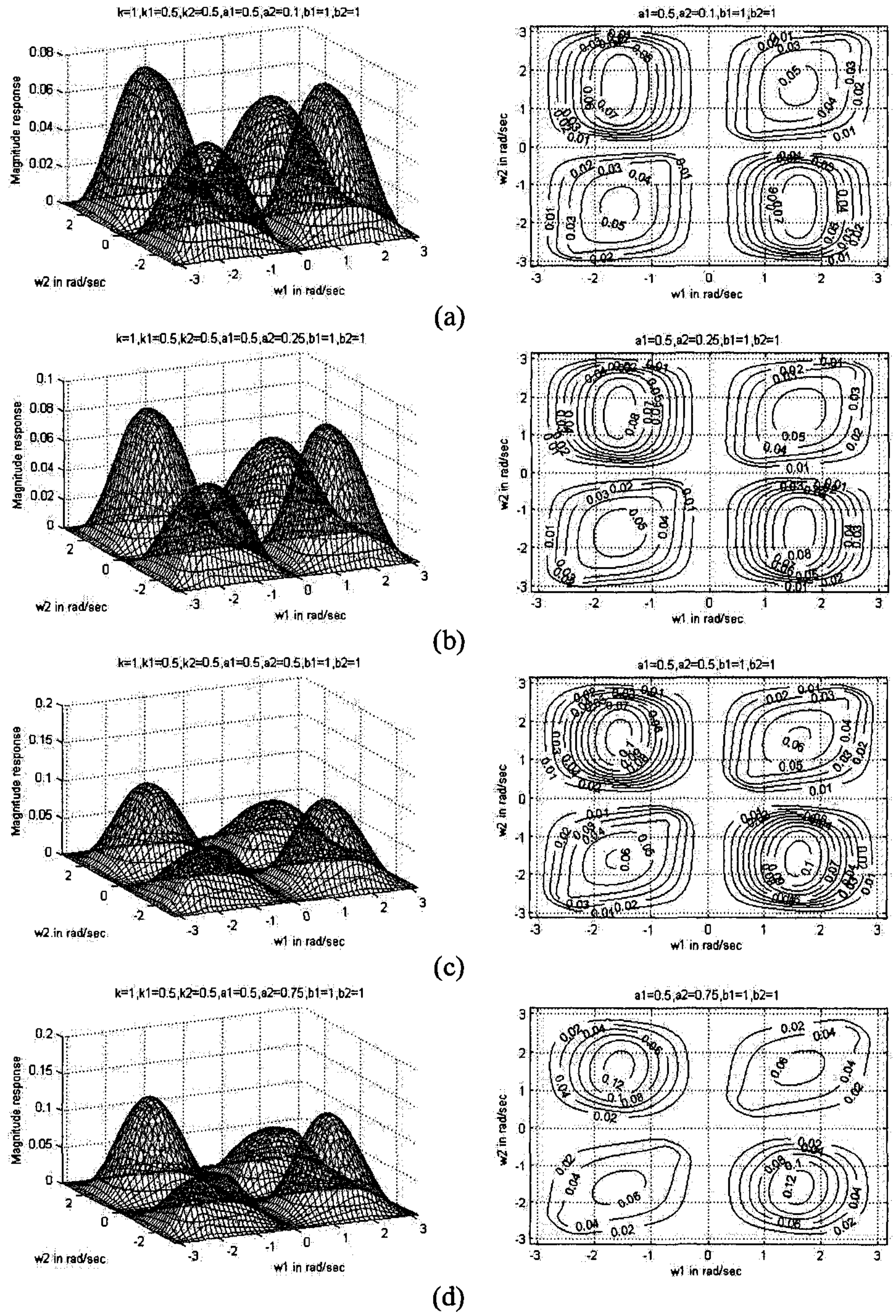
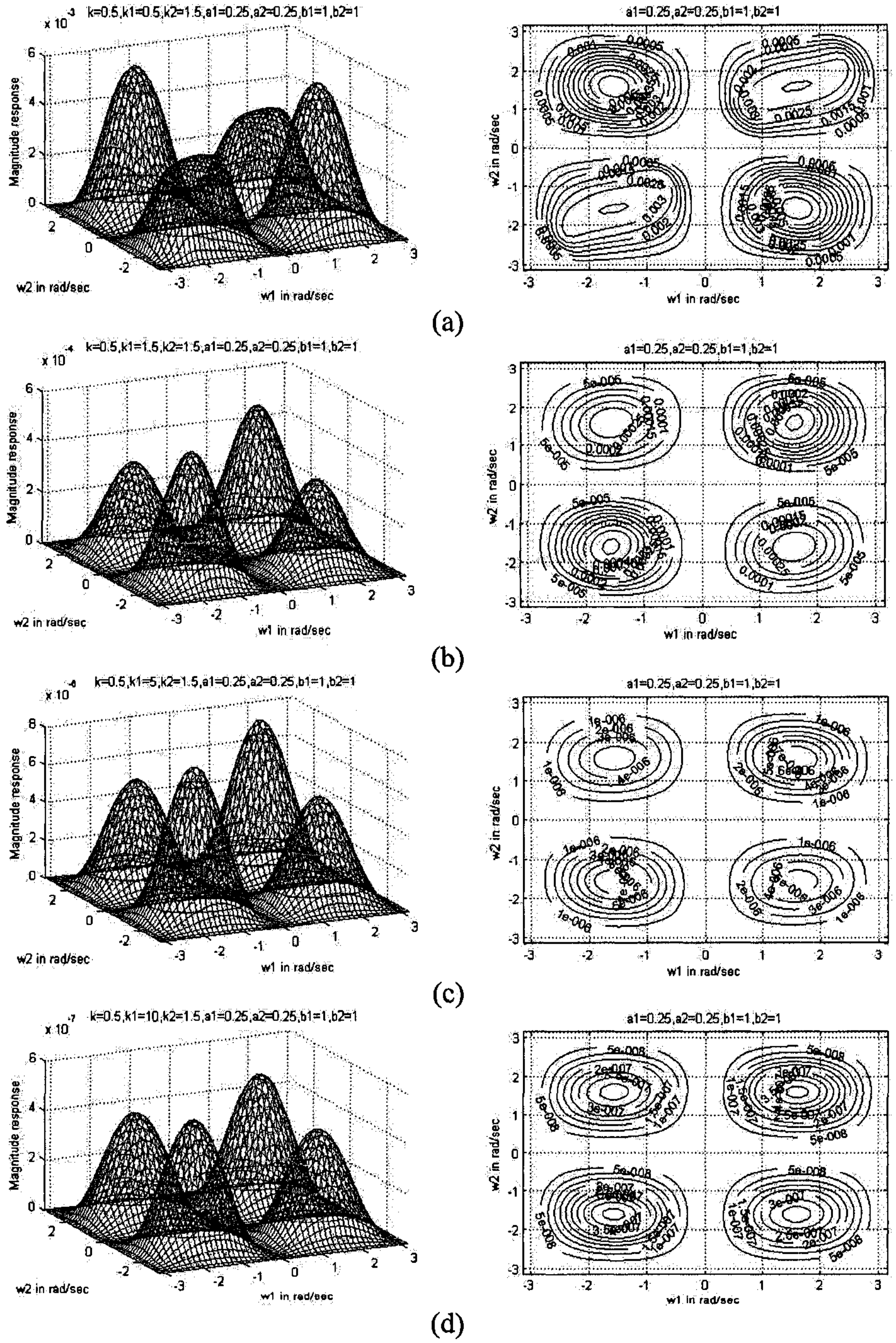


Figure 5.8: 3-D amplitude-frequency response and contour response of the 2-D digital bandpass filter with finite gain op-amp for case 2 (varying a_2).

5.4.3 *Case 3*



(d)

Figure 5.9: 3-D amplitude-frequency response and contour response of the 2-D digital highpass filter with finite gain op-amp for case 3 (varying k_1).

In case 3, the effect of GBT coefficient k_1 on the digital filter output characteristics is analyzed. In order to study the effect of k_1 parameter, the value of k_1 is varied and other GBT coefficients are set as given below:

$$k=\text{constant},\ k_2=\text{constant},\ b_1=b_2=1,\ a_1=a_2=\text{constant},\ \text{vary } k_1.$$

$$k=0.5,\ k_2=1.5,\ b_1=b_2=1,\ a_1=a_2=0.25,\ \text{vary } k_1.$$

Fig. 5.9 (a), (b), (c) and (d) represent the amplitude-frequency response of the bandpass filter obtained by varying the GBT coefficient k_1. As the value of k_1 is increased from 0.5 to 10 the pass bandwidth along the ω_1 axis remains constant and along ω_2 axis the pass bandwidth decreases. The magnitude of the amplitude-frequency response in the first and third quadrants decreases from 0.003 to $3.5e^{-7}$ and in the second and fourth quadrant it decreases from 0.005 to $3e^{-7}$.

5.4.4 *Case 4*

In case 4, the effect of GBT coefficient k_2 on the digital filter output is studied. In order to study the effect of k_2 parameter, the value of k_2 is varied and other GBT coefficient values are set as given below:

$$k=\text{constant},\ k_1=\text{constant},\ b_1=b_2=1,\ a_1=a_2,\ \text{vary } k_2$$

$$k=0.5,\ k_1=0.5,\ b_1=b_2=1,\ a_1=a_2=0.25,\ \text{vary } k_2.$$

Figs. 5.10 (a), (b), (c) and (d) are obtained by varying the GBT coefficient k_2 and keeping the other parameters at a constant value as said above. As the value of k_2 is increased from 0.5 to 10, the magnitude of the amplitude-frequency response in the first and third quadrants decreases from 0.015 to $1.4e^{-5}$ and in the second and fourth quadrant magnitude decreases from 0.025 to $1.4e^{-5}$. As far as the pass bandwidth is considered, it remains constant along the ω_2 axis and the pass bandwidth along the ω_1 axis decreases.

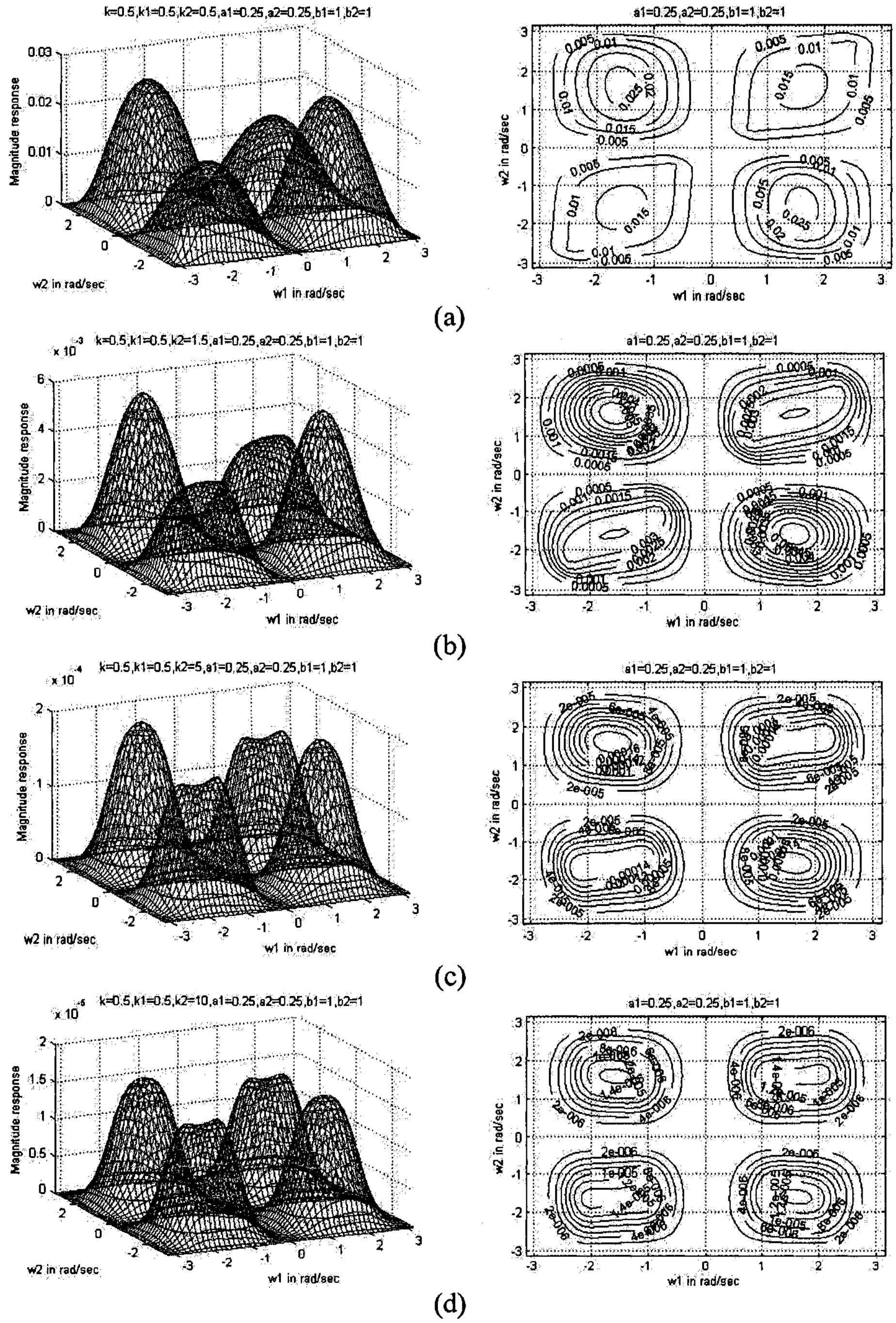
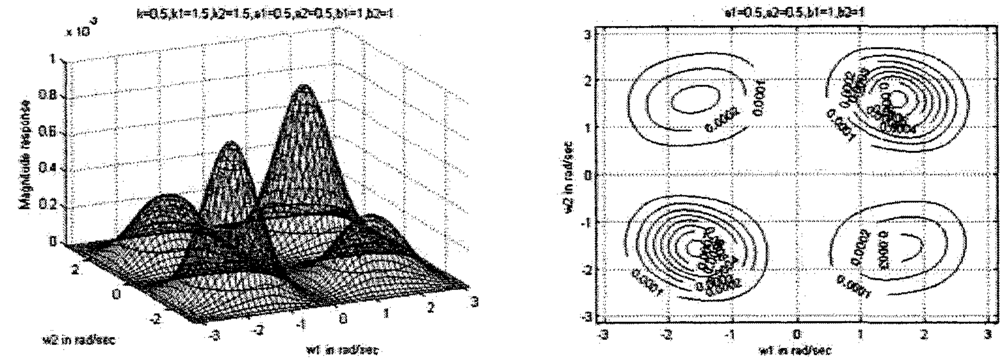
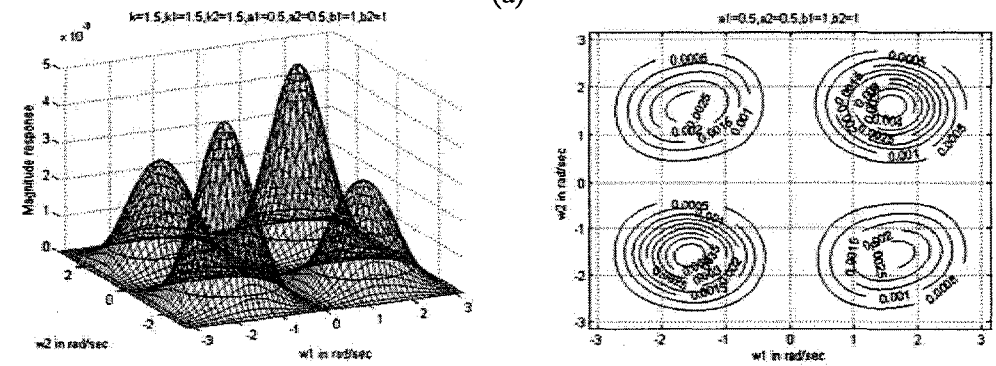


Figure 5.10: 3-D amplitude-frequency response and contour response of the 2-D digital bandpass filter with finite gain op-amp for case 4 (varying k_2).

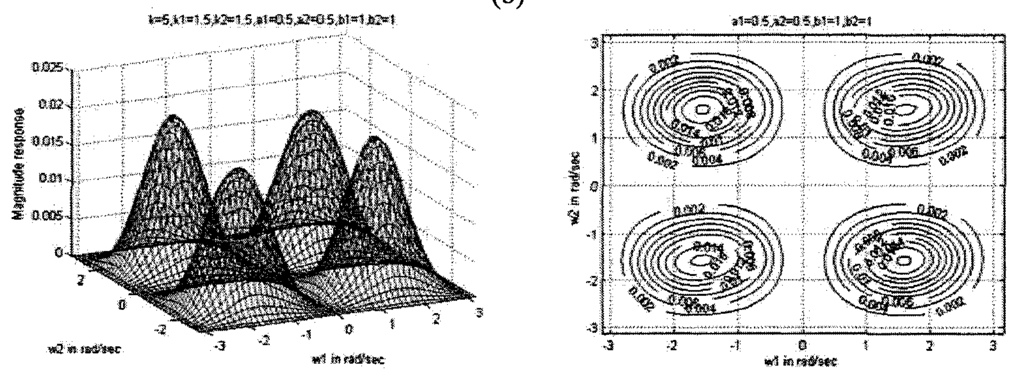
5.4.5 *Case 5*



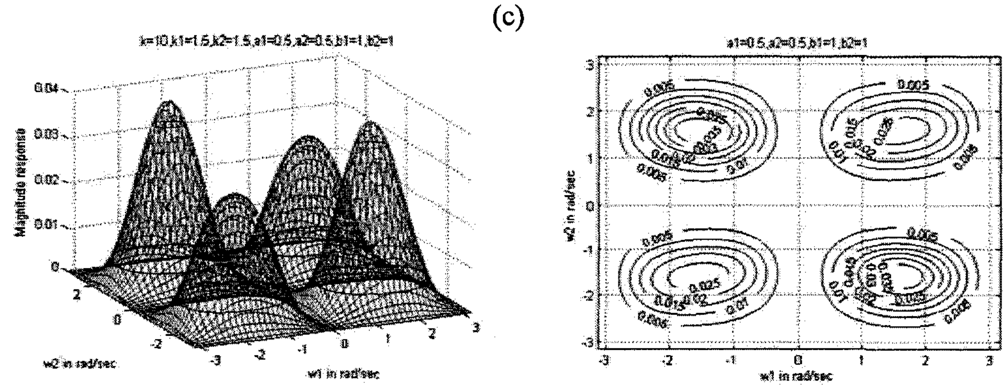
(a)



(b)



(c)



(d)

Figure 5.11: 3-D amplitude-frequency response and contour response of the 2-D digital bandpass filter with finite gain op-amp for case 5 (varying k).

In case 5, the effect of op-amp gain parameter k on the digital filter output characteristics is studied. In order to study the effect of k parameter, the value of k is varied and other GBT coefficient values are set as given below:

$k_1=k_2=\text{constant},\ b_1=b_2=1,\ a_1=a,$ vary k.

$k_1=k_2=1.5,\ a_1=a_2=0.5,\ b_1=b_2=1,$ vary k.

Fig. 5.11 (a), (b), (c) and (d) are obtained by varying the op-amp gain parameter k from 0.5 to 10, the magnitude of the amplitude-frequency response in the first and third quadrants increases from 0.0007 to 0.025 and in the second and fourth quadrants it increases from 0.0002 to 0.035, the pass bandwidth along the ω_1 axis increases with the increase in the value of k and the pass bandwidth along the ω_2 axis remains constant.

The op-amp gain k mainly affects the amplitude of the 3-D magnitude response in the 2-D digital bandpass filter output.

5.5 *Summary*

The transfer function of the 2-D digital bandpass filter was obtained in sec. 5.2 for infinite gain case and finite gain case. The effect of GBT coefficient on the filter output was individually examined for both the cases. The effect of gain parameter k was examined for the 2-D digital bandpass filter with finite gain op-amp. The effect of these parameters on the filter output for infinite gain op-amp case has been summarized in Table 4.1 and for the finite gain case has been summarized in Table 5.2.

GBT coefficient	Value of GBT coefficient	Amplitude of 3D magnitude response	Pass band width along ω_1 axis	Pass band width along ω_2 axis
a_1	↑	↑	↓	Constant
a_2	↑	↑	Constant	↓
k_1	↑	↓	↓	Constant
k_2	↑	↓	Constant	↓

Table 5.1: Summary of effects of GBT coefficients on infinite gain bandpass filter.

GBT coefficient	Value of GBT coefficient	Amplitude of 3D magnitude response	Pass band width along ω_1 axis	Pass band width along ω_2 axis
a_1	↑	↑	Constant	Constant
a_2	↑	↑	Constant	Constant
k_1	↑	↓	↓	Constant
k_2	↑	↓	Constant	↓
k	↑	↑	↑	Constant

Table 5.2: Summary of effects of GBT coefficients on finite gain bandpass filter.

Chapter 6

Application of 2-D digital filter in image processing

6.1 *Introduction*

An image is defined as a two-dimensional function, f(x,y), where x and y are spatial coordinates and the amplitude of f at any pair of coordinates (x,y) is called the intensity or gray level of the image at that point. When x, y and the amplitude values of f are all finite, discrete quantities, we call the image a digital image. The digital image is composed of a finite number of elements, each of which has a particular location and value. These elements are referred to as picture elements, image elements and pixels [13].

6.2 *Basics of filtering in Image Processing [13]*

Image processing is done either in spatial domain or frequency domain depending on the need of the application. Fourier transform of the spatial domain component provides its equivalent frequency domain component and inverse fourier transform of the frequency domain component gives its equivalent spatial domain component.

In this work, an image is considered as a 2-D intensity matrix and the designed 2-D digital filters are applied in the frequency domain for a standard application like image enhancement.

The 2-D fourier transformers are effective tools for image processing. The 2-D discrete Fourier transform pair is

$$F(\omega_1,\omega_2)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)e^{-j2\pi\left(x\omega_1/M+y\omega_2/N\right)} \tag{6.1}$$

$$f(x,y)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}F(\omega_1,\omega_2)e^{j2\pi\left(x\omega_1/M+y\omega_2/N\right)} \tag{6.2}$$

f (x,y) → Image of size MxN.

F (ω_1, ω_2) → Equivalent image of size MxN in frequency domain.

The basics of linear filtering in both the spatial and frequency domains is the convolution theorem, which may be written as

$$f(x,y)*h(x,y)\Leftrightarrow H(\omega_1,\omega_2)F(\omega_1,\omega_2) \tag{6.3}$$

and conversely

$$f(x,y)h(x,y)\Leftrightarrow H(\omega_1,\omega_2)*F(\omega_1,\omega_2) \tag{6.4}$$

In eqn. (6.3) and (6.4), the symbol '*' indicates convolution of the two functions and the expressions on the sides of the double arrow constitute a Fourier transform pair. In terms of filtering, we are interested in eqn. (6.3).

Filtering in the frequency domain consists of multiplying the transfer function of the filter H (ω_1, ω_2) with the frequency domain equivalent of the image F(ω_1, ω_2). The size of the image matrix and filter matrix has to be equal to get the best result or else both of them have to be a square matrix.

Filtering would be done in the frequency domain. To demonstrate the application of the filter in image restoration, an image is corrupted with Gaussian noise of zero mean and specified standard deviation. Then the corrupted image is conditioned with the 2-D digital filter to reduce the effect of noise. It's mathematically represented as

$$f_c(x,y) = f(x,y) + n(x,y) \tag{6.5}$$

f(x,y) → Input image.

n(x,y) → Gaussian noise with zero mean and standard deviation σ.

f_c(x,y) → Image + noise i.e. corrupted image.

$$fr(x,y) = IDFT\left[Fc(\omega_1,\omega_2)\cdot H(\omega_1,\omega_2)\right] \tag{6.6}$$

f_r(x,y) → Recovered image which is close to the original image.

$F_c(\omega_1,\omega_2)$ → Corrupted image equivalent in frequency domain.

$H(\omega_1,\omega_2)$ → Transfer function of the 2-D active filter.

Various quality measures are available in the literature, those that correlate well with visual perception are quite complicated to compute. Most image processing systems of today are designed to minimize the Mean Square Error (MSE), the quantitative measure between two images $f_1(x,y)$, $f_2(x,y)$ which is defined as

$$MSE = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[f_1(x,y) - f_2(x,y)\right]^2 \tag{6.7}$$

where MxN is the image dimension and its product gives total number of pixel in the image. The Peak Signal-to-Noise Ratio (PSNR) in decibels (dB) is more often used as a quality measure. The PSNR is defined as

$$PSNR = 10\log_{10}\left(\frac{\psi_{\max}^2}{MSE}\right) \tag{6.8}$$

where $\psi_{\max}$ is the peak (maximum) intensity value of the image. For eight bit gray image, $\psi_{\max}$=255.

6.3 *Image Restoration [13]*

The objective of image restoration is to improve an image in some predefined sense. Restoration attempts to reconstruct an image that has been degraded by using a priori knowledge of the degradation phenomenon. The restoration technique is oriented toward modeling the degradation and applying the inverse process in order to recover the original image. Image restoration for most part is an objective process.

An image degraded due to noise alone is considered. The noise in digital images arises during image acquisition and/or transmission. Noise which is independent of spatial coordinates and uncorrelated with respect to the image itself have been added to produce a degraded image. The lowpass filter in the spatial domain is equivalent to that of smoothing filter, as it blocks high frequencies corresponding to sharp intensity changes.

A 8-bit gray level image has been considered and is corrupted by Gaussian noise with zero mean and standard deviation of σ x 255 gray levels. The corrupted image is then conditioned with the 2-D digital lowpass filter of both configurations. The results are obtained individually for infinite gain case and finite gain case.

The original image, corrupted image and the recovered/filtered output image for infinite gain case and finite gain case are show in figs. 6.1 (a), (b), (c) and (d) for a Gaussian noise with σ=0.1. The quality of output image is measured in terms of mean square error (MSE) and Peak Signal to Noise Ratio (PSNR). The results are tabulated in Table 6.1 for both the cases 2-D active lowpass filter. The results are calculated in comparison with the original image. From the results it is inferred that the 2-D digital active filter with finite gain op-amp provides better performance when compared to infinite gain case.

Figure 6.1: Image restoration using lowpass filter with (a) Original Image (b) Image with Gaussian noise (c) Image filtered with infinite gain lowpass filter and (d) Image filtered with finite gain lowpass filter.

Parameter \ Filter Type	Finite Gain Filter	Infinite Gain Filter
Mean Square Error (MSE) With Noise	4564.029	4564.029
Peak Signal to Noise Ratio (PSNR) With Noise	11.4525	11.4525
Mean Square Error (MSE)	1004.9742	1399.6754
Peak Signal to Noise Ratio (PSNR)	18.1093	16.6705

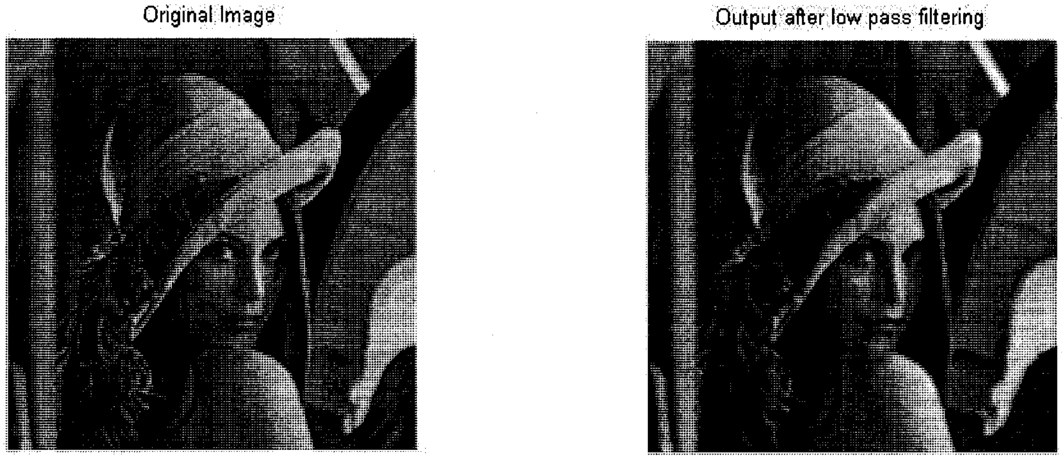
Table 6.1: Comparison of infinite gain and finite gain lowpass filter.

6.4 *Image Enhancement [13]*

The purpose of image enhancement is to improve interpretability or perception of information in images for human viewers or to provide better input for other automated image processing techniques. Image enhancement techniques can be done either in spatial domain or frequency domain.

Image enhancement has been done in frequency domain. In image processing the lowpass filter is expected to blur the image passed through it, as the high frequency components are lost as the image is passed through the lowpass filter which contributes to the sharpness of the image.

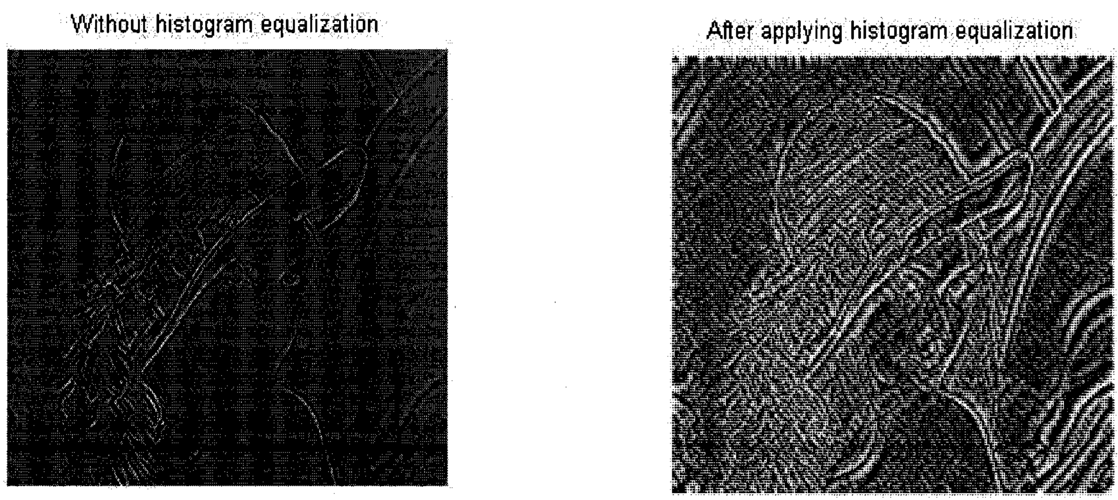
From Table 6.1 it is inferred that filter with finite gain op-amp provides better performance, so 2-D digital filter with finite op-amp has been used to show its application in image enhancement. Fig. 6.2 (a) is the original image which is passed through the 2-D digital lowpass filter. Fig. 6.2 (b) shows the blurred image obtained as the output of the 2-D digital lowpass filter.



(a) (b)

Figure 6.2: Image enhancement using lowpass filter (a) Original Image (b) Blurred image.

Fig. 6.3 (a) is the output obtained from a 2-D highpass filter. Highpass filter zero out the dc term, thus reduces the average value of an image to 0. The principal edges of the image is retained which can be seen from the fig. 6.3 (a) and histogram equalization is applied to the fig. 6.3 (a) in order to expand the gray scale region, by which the principal edges of the image are clearly visible.



(a) (b)

Figure 6.3: Image enhancement using highpass filter (a) Output of a highpass filter (b) Output after adjusting gray level.

6.5 *Summary and Discussion*

In image processing the most of the energy of a typical image is located at the low frequencies. The energy of the noise is often spread across the frequency axes in the case of a white noise or else in the higher frequency range depending on the distribution function. The 2-D digital lowpass filter provides a good noise removal property, but the high frequency component of an image such as edges are affected i.e. sharpness of the recovered image is lost. The 2-D digital highpass filter provides good edge detection in image processing.

For testing the functionality of filters, a standard image is corrupted by additive Gaussian noise with known variance and mean. The 2-D digital lowpass filter derived from infinite gain op-amp and finite gain op-amp is used to decrease the noise from the corrupted noise. The performance comparison of these two kinds of filter is done by comparing MSE and PSNR of the recovered images. MSE and PSNR provide the quantitative measures of the image restoration. Image enhancement properties of the 2-D digital lowpass filter and highpass filter are also shown.

Chapter 7

Conclusions

A new technique is presented for generating a 2-D digital filter. In order to generate a second order 2-D digital filter, a doubly terminated network constituting a op-amp, an input impedance and a reactance T network in the feedback path is considered. Various combinations have been tried out for the input impedance and the reactance T network which would lead to a stable transfer function. These combinations have been tried out for two different cases of the op-amp i.e. infinite gain and finite gain. The stability of the transfer function is checked by ensuing denominator polynomial of the transfer function is a VSHP.

The impedance value of the transfer function is obtained by continued fraction expansion of the fourth order 1-D chebhyshev filter with 1-dB ripple in the pass band. A stable 2-D active filter in the analog domain is obtained. GBT is applied to the analog transfer function to obtain the equivalent 2-D digital lowpass filter. Subsequently 2-D digital highpass filter and bandpass filters are obtained by applying suitable transformation.

The 2-D lowpass filter in digital domain is obtained for both the infinite gain op-amp and finite gain op-amp cases. The effect of the GBT co-efficient on the amplitude-frequency response of 2-D digital lowpass filter for infinite gain case and finite case are studied, for the finite gain case effect of op-amp gain 'k' is also studied and the results are tabulated. It is observed that the GBT co-efficient k_1 and k_2 mainly affects the pass bandwidth along the ω_1 and ω_2 axis, co-efficient a_1 and a_2 affects the magnitude of the amplitude-frequency response. For finite gain op-amp case the effect of op-amp gain parameter k on the filter output is studied. As the gain parameter k is increased the magnitude of the amplitude-frequency response also increases.

The effect of GBT coefficient on the amplitude-frequency response of a 2-D highpass digital filter for the infinite gain op-amp and finite gain op-amp case has been studied and the results are tabulated. It is observed the GBT co-efficient k_1 and k_2 affects the pass bandwidth along the ω_1 and ω_2 axis and the GBT co-efficient a_1 and a_2 affects the magnitude of amplitude-frequency response.

The 2-D bandpass filter is obtained by cascading the 2-D digital lowpass filter and 2-D digital highpass filter for the infinite gain case and finite gain case. The pass bandwidth of the bandpass filter is equal to the pass band area overlapped between the lowpass filter and highpass filter. The effect of GBT co-efficient on the amplitude-frequency response is studied by varying each parameter individually. It is inferred from the results that the GBT co-efficient a_1 and a_2 affects the magnitude of amplitude-frequency, the k_1 and k_2 affects the pass bandwidth.

At last, the application of 2-D digital lowpass filter in image processing for image restoration and image enhancement has been shown. Performance comparison has been done for the lowpass infinite gain and finite gain configuration with the image restoration application. From the results it is inferred that the 2-D digital filter with finite gain op-amp has better performance than the 2-D digital filter with infinite gain op-amp. Application of 2-D digital highpass filter has been illustrated.

Scope of Future Work

The impedance values for the filter are obtained by comparing with the fourth order type I chebyshev filter. The impedance values can also be obtained by comparing with other standard polynomial such as type II chebyshev or butterworth filter or any other standard filter.

In the design of filter the resistance value has been considered to be equal to unity, different values can be tried out.

For the T section in the analog circuit, only reactance combination has been tried out for the impedances. Other combinations can be tried out.

Study has been done based on the amplitude-frequency response of the filter, phase response of the filter can also be studied.

All pass filter and their combination can also be considered to generate a stable 2-D digital filters.

Depending upon the application requirement, suitable values for the co-efficient of the GBT can be determined based upon the properties of the filter like symmetry, amplitude characteristics and the response in the stop band.

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Certain reference has multiple references which has not been listed here.

Appendix

A. MATLAB code to plot the 3-D amplitude-frequency response and the contour response of the 2-D digital lowpass filter with infinite gain op-amp.

```
% Operational amplifier with infinite gain.
% Lowpass configuration
% Obtained by applying Generalized Bilinear Transformation
 clear all; clc
w1=-pi:pi/25:pi; w2=-pi:pi/25:pi;
z11=exp(-j.*w1); z21=exp(-j.*w2);
 [z1,z2]=meshgrid(z11,z21);
 % Input values for GBT coefficients
a1=input('Enter the value of a1=');
a2=input('Enter the value of a2=');
k1=input('Enter the value of k1=');
k2=input('Enter the value of k2=');
 %HLIG tranfer function of the filter with infinite gain.
a=z1-a1;  c=z1+1; b=z2-a2;  d=z2+1;
jj=1;
 d11=(k1^2).*(a.^2).*(3.6749.*k2^2.*(b.^2)+2.5771.*k2.*b.*d+2.2786.*d.^2);
```

```
d12=(k1.*a.*c).*(5.1916.*k2^2*(b.^2)+6.7323.*k2.*(b.*d)+3.2190.*(d.^2));
d13=(c.^2).*(3.2256.*k2.^2.*b.^2+3.688.*k2.*b.*d+d.^2);
NR=-(c.^2).*(d.^2); DR=d11+d12+d13;
 % Transfer function
HLIG=abs(NR./DR);
 % Magnitude plot
subplot(2,2,jj); contour3(w1,w2,HLIG);
surface(w1,w2,HLIG,'EdgeColor',[.2 .2 .2],'FaceColor','none');
grid on; view(-15,25);
title(['k1=',num2str(k1),',k2=',num2str(k2),',a1=',num2str(a1),',a2=',num2str(a2),',b1=',nu
m2str(b1),',b2=',num2str(b2)]);
xlabel('w1 in rad/sec'); ylabel('w2 in rad/sec'); zlabel('Magnitude response');
 % Contour plot
jj=jj+1; subplot(2,2,jj); [C,h]=contour(w1,w2,HLIG);
clabel(C,h); set(h,'linecolor','black'); grid on;
title(['a1=',num2str(a1),',a2=',num2str(a2),',b1=',num2str(b1),',b2=',num2str(b2)]);
xlabel('w1 in rad/sec'); ylabel('w2 in rad/sec');
```

B. MATLAB code to plot the 3-D amplitude-frequency response and the contour response of the 2-D digital lowpass filter with finite gain op-amp.

```
% Operational amplifier with finite gain.
% Lowpass configuration
% Obtained by applying Generalized Bilinear Transformation
```

```
clear all; clc
w1=0:pi/25:pi; w2=0:pi/25:pi;
z11=exp(-j.*w1); z21=exp(-j.*w2);
[z1,z2]=meshgrid(z11,z21);
jj=1;
% Input values for GBT coefficients
display('Enter the GBT coefficients values for finite gain filter');
a1=input('Enter the value of a1=');
a2=input('Enter the value of a2=');
k1=input('Enter the value of k1=');
k2=input('Enter the value of k2=');
% Input value for the gain of operational amplifier
k=input('Enter the value of k=');
%HLFG transfer function of the lowpass filter with finite gain
a=z1-a1; c=z1+1; b=z2-a2; d=z2+1;
dl1=(k1^2).*(a.^2).*(7.3498.*k2^2.*(b.^2)+5.1541.*k2.*b.*d+4.5571.*(d.^2));
dl2=(k1.*a.*c).*(1.6951.*k.*k2^2.*(b.^2)+(7.3718+1.1887*k).*k2.*(b.*d)+(4.3360+1.0
51*k).*(d.^2));
dl3=(c.^2).*((1.6128*k+1.16128).*k2.^2.*(b.^2)+(2.168+0.2950*k).*k2.*b.*d+k.*d.^2);
NR=-k.*(c.^2).*(d.^2); DR=dl1+dl2+dl3;
% Transfer function
HLFG=abs(NR./DR);
% Magnitude plot
```

```
subplot(2,2,jj); contour3(w1,w2,HLFG);
surface(w1,w2,HLFG,'EdgeColor',[.2 .2 .2],'FaceColor','none');
grid on; view(-15,25);
title(['k=',num2str(k),',k1=',num2str(k1),',k2=',num2str(k2),',a1=',num2str(a1),',a2=',num2
str(a2),',b1=',num2str(b1),',b2=',num2str(b2)]);
xlabel('w1 in rad/sec'); ylabel('w2 in rad/sec'); zlabel('Magnitude response');
 % Contour plot
jj=jj+1; subplot(2,2,jj);
[C,h]=contour(w1,w2,HLFG);
clabel(C,h); set(h,'linecolor','black'); grid on;
title(['a1=',num2str(a1),',a2=',num2str(a2),',b1=',num2str(b1),',b2=',num2str(b2)]);
xlabel('w1 in rad/sec'); ylabel('w2 in rad/sec');
```

C. MATLAB code to plot the 3-D amplitude-frequency response and the contour response of the 2-D digital highpass filter with infinite gain op-amp.

```
% Operational amplifier with infinite gain .
% GBT of Highpass filter
clear all; clc
w1=-pi:pi/25:pi; w2=-pi:pi/25:pi;
z11=exp(-j.*w1); z21=exp(-j.*w2);
[z1,z2]=meshgrid(z11,z21);
 % Input values for GBT coefficients
```

```
a1=input('Enter the value of a1=');

a2=input('Enter the value of a2=');

k1=input('Enter the value of k1=');

k2=input('Enter the value of k2=');

 %HHIG transfer function of the filter with infinite gain.

e=z1+a1; g=z1-1; f=z2+a2; h=z2-1;

d11=(k1^2).*(a.^2).*(3.6749.*k2^2.*(b.^2)+2.5771.*k2.*b.*d+2.2786.*d.^2);

d12=(k1.*a.*c).*(5.1916.*k2^2*(b.^2)+6.7323.*k2.*(b.*d)+3.2190.*(d.^2));

d13=(c.^2).*(3.2256.*k2.^2.*b.^2+3.688.*k2.*b.*d+d.^2);

NR=-(g.^2).*(h.^2); DR=d11+d12+d13;

 % Transfer Function

HHIG=abs(NR./DR);

 % Magnitude plot

jj=1; subplot(2,2,jj); contour3(w1,w2,HHIG);

surface(w1,w2,HHIG,'EdgeColor',[.2 .2 .2],'FaceColor','none');

grid on; view(-15,25);

title(['k1=',num2str(k1),',k2=',num2str(k2),',a1=',num2str(a1),',a2=',num2str(a2),',b1=',nu

m2str(b1),',b2=',num2str(b2)]);

xlabel('w1 in rad/sec'); ylabel('w2 in rad/sec'); zlabel('Magnitude response');

 % Contour plot

jj=jj+1; subplot(2,2,jj); [C,h]=contour(w1,w2,HHIG); clabel(C,h);

set(h,'linecolor','black'); grid on;

title(['a1=',num2str(a1),',a2=',num2str(a2),',b1=',num2str(b1),',b2=',num2str(b2)]);
```

```
xlabel('w1 in rad/sec'); ylabel('w2 in rad/sec');
```

D. MATLAB code to plot the 3-D amplitude-frequency response and the contour response of the 2-D digital highpass filter with finite gain op-amp.

```
% Operational amplifier with finite gain .
% GBT of Highpass filter
clear all; clc
w1=-pi:pi/25:pi; w2=-pi:pi/25:pi;
z11=exp(-j.*w1); z21=exp(-j.*w2);
[z1,z2]=meshgrid(z11,z21);
% Input values for GBT coefficients
a1=input('Enter the value of a1=');
a2=input('Enter the value of a2=');
k1=input('Enter the value of k1=');
k2=input('Enter the value of k2=');
k=input('Enter the value of k=');
%HHFG tranfer function of the filter with finite gain.
e=z1+a1; g=z1-1; f=z2+a2; h=z2-1;
dl1=(k1^2).*(e.^2).*(7.3498.*k2^2.*(f.^2)+5.1541.*k2.*f.*h+4.5571.*(h.^2));
dl2=(k1.*e.*g).*(1.6951.*k.*k2^2.*(f.^2)+(7.3718+1.1887*k).*k2.*(f.*h)+(4.3360+1.05
1*k).*(h.^2));
dl3=(g.^2).*((1.6128*k+1.16128).*k2.^2.*(f.^2)+(2.168+0.2950*k).*k2.*f.*h+k.*h.^2);
NR=-k.*(g.^2).*(h.^2); DR=d11+d12+d13;
```

```
% Transfer Function
HHFG=abs(NR./DR);
% Magnitude plot
jj=1;
subplot(2,2,jj); contour3(w1,w2,HHFG);
surface(w1,w2,HHFG,'EdgeColor',[.2 .2 .2],'FaceColor','none');
grid on; view(-15,25);
title(['k1=',num2str(k1),',k2=',num2str(k2),',a1=',num2str(a1),',a2=',num2str(a2),',b1=',nu
m2str(b1),',b2=',num2str(b2)]);
xlabel('w1 in rad/sec'); ylabel('w2 in rad/sec'); zlabel('Magnitude response');
 % Contour plot
jj=jj+1; subplot(2,2,jj); [C,h]=contour(w1,w2,HHFG);
clabel(C,h); set(h,'linecolor','black'); grid on;
title(['a1=',num2str(a1),',a2=',num2str(a2),',b1=',num2str(b1),',b2=',num2str(b2)]);
xlabel('w1 in rad/sec'); ylabel('w2 in rad/sec');
```

E. MATLAB code to plot the 3-D amplitude-frequency response and the contour response of the 2-D digital bandpass filter with infinite gain op-amp.

```
% Operational amplifier with infinite gain.
% Bandpass filter
 clear all; clc;
w1=-pi:pi/25:pi; w2=-pi:pi/25:pi;
```

```
z11=exp(-j.*w1); z12=exp(-j.*w2);

[z1,z2]=meshgrid(z11,z12);

 % Input values

a1=input('Enter the value of a1=');

a2=input('Enter the value of a2=');

b1=input('Enter the value of b1=');

b2=input('Enter the value of b2=');

k1=input('Enter the value of k1=');

k2=input('Enter the value of k2=');

 jj=1;

 % HBPIG transfer function of bandpass filter with infinte gain op-amp.

a=z1-a1; c=z1+1; b=z2-a2; d=z2+1;

d11=(k1^2).*(a.^2).*(3.6749.*k2^2.*(b.^2)+2.5771.*k2.*b.*d+2.2786.*d.^2);

d12=(k1.*a.*c).*(5.1916.*k2^2*(b.^2)+6.7323.*k2.*(b.*d)+3.2190.*(d.^2));

d13=(c.^2).*(3.2256.*k2.^2.*b.^2+3.688.*k2.*b.*d+d.^2);

NR=-(c.^2).*(d.^2); DR=d11+d12+d13;

HL=abs(NR./DR);

 e=z1+a1; g=z1-1; f=z2+a2; h=z2-1;

dh1=(k1^2).*(e.^2).*(3.6749.*k2^2.*(f.^2)+2.5771.*k2.*f.*h+2.2786.*h.^2);

dh2=(k1.*e.*g).*(5.1916.*k2^2*(f.^2)+6.7323.*k2.*(f.*h)+3.2190.*(h.^2));

dh3=(g.^2).*(3.2256.*k2.^2.*f.^2+3.688.*k2.*f.*h+h.^2);

NRH=-(g.^2).*(h.^2); DRH=dh1+dh2+dh3;

HH=abs(NRH./DRH);
```

```
% Transfer function of bandpass filter.
HBPIG=(HL.*HH);
 % Magnitude plot
subplot(2,2,jj); contour3(w1,w2,abs(HBPIG));
surface(w1,w2,abs(HBPIG),'EdgeColor',[.2 .2 .2],'FaceColor','none');
grid on; view(-25,25);
title([',k1=',num2str(k1),',k2=',num2str(k2),',a1=',num2str(a1),',a2=',num2str(a2),',b1=',nu
m2str(b1),',b2=',num2str(b2)]);
xlabel('w1 in rad/sec'); ylabel('w2 in rad/sec'); zlabel('Magnitude response');
 % Contour plot
jj=jj+1; subplot(2,2,jj); [C,h]=contour(w1,w2,abs(HBPIG));
clabel(C,h); set(h,'linecolor','black'); grid on;
title(['a1=',num2str(a1),',a2=',num2str(a2),',b1=',num2str(b1),',b2=',num2str(b2)]);
xlabel('w1 in rad/sec'); ylabel('w2 in rad/sec');
```

F. MATLAB code to plot the 3-D amplitude-frequency response and the contour response of the 2-D digital bandpass filter with finite gain op-amp.

```
% Operational amplifier with finite gain.
% Bandpass filter
 clear all; clc;
w1=-pi:pi/25:pi; w2=-pi:pi/25:pi;
z11=exp(-j.*w1); z12=exp(-j.*w2);
[z1,z2]=meshgrid(z11,z12);
```

```
% Input values
a1=input('Enter the value of a1=');
a2=input('Enter the value of a2=');
b1=input('Enter the value of b1=');
b2=input('Enter the value of b2=');
k1=input('Enter the value of k1=');
k2=input('Enter the value of k2=');
k=input('Enter the value of k=');
jj=1;
% HBPFG transfer function of bandpass filter with finte gain op-amp.
a=z1-a1; c=z1+1; b=z2-a2; d=z2+1;
dl1=(k1^2).*(a.^2).*(7.3498.*k2^2.*(b.^2)+5.1541.*k2.*b.*d+4.5571.*(d.^2));
dl2=(k1.*a.*c).*(-1.6951.*k.*k2^2.*(b.^2)+(7.3718-1.1887*k).*k2.*(b.*d)+(4.3360-
1.051*k).*(d.^2));
dl3=(c.^2).*((1.6128*k+1.16128).*k2.^2.*(b.^2)+(2.168-0.2950*k).*k2.*b.*d-k.*d.^2);
NR=-k.*(c.^2).*(d.^2); DR=dl1+dl2+dl3;
HL=(NR./DR);
e=z1+a1; g=z1-1; f=z2+a2; h=z2-1;
dh1=(k1^2).*(e.^2).*(7.3498.*k2^2.*(f.^2)+5.1541.*k2.*f.*h+4.5571.*(h.^2));
dh2=(k1.*e.*g).*(-1.6951.*k.*k2^2.*(f.^2)+(7.3718-1.1887*k).*k2.*(f.*h)+(4.3360-
1.051*k).*(h.^2));
dh3=(g.^2).*((1.6128*k+1.16128).*k2.^2.*(f.^2)+(2.168-0.2950*k).*k2.*f.*h-k.*h.^2);
NRh=-k.*(g.^2).*(h.^2);
```

```
DRh=dh1+dh2+dh3;

HH=(NRh./DRh);

% Transfer function of bandpass filter.

HBPFG=abs(HL.*HH);

 % Magnitude plot

subplot(2,2,jj); contour3(w1,w2,abs(HBPFG));

surface(w1,w2,abs(HBPFG),'EdgeColor',[.2 .2 .2],'FaceColor','none');

grid on; view(-25,25);

title(['k=',num2str(k),',k1=',num2str(k1),',k2=',num2str(k2),',a1=',num2str(a1),',a2=',num2
str(a2),',b1=',num2str(b1),',b2=',num2str(b2)]);

xlabel('w1 in rad/sec'); ylabel('w2 in rad/sec'); zlabel('Magnitude response');

 % Contour plot

jj=jj+1; subplot(2,2,jj); [C,h]=contour(w1,w2,abs(HBPFG));

clabel(C,h); set(h,'linecolor','black'); grid on;

title(['a1=',num2str(a1),',a2=',num2str(a2),',b1=',num2str(b1),',b2=',num2str(b2)]);

xlabel('w1 in rad/sec'); ylabel('w2 in rad/sec');
```

G. MATLAB code for performance comparison of lowpass filter with infinite gain and finite gain op-amp configuration.

```
% Filter Application to image processing.

clear all; clc;

% Generation of fiilter transfer function.

HLIG1=lfg();
```

```
original_image=imread('G:\My Documents\lena2.jpg');

% Data format of the image is changed.

image_d=mat2gray(original_image);

figure, imshow(image_d); title('Original Image');

% Gaussian noise added to the image.

deg_image= imnoise(image_d,'gaussian',0,.1); image_d_freq=fft2(deg_image);

HLIG1=fftshift(HLIG1);

fil_image_freq=HLIG1.*image_d_freq;  % Filtered image in frequency domain.

% Filtered image in spatial domain.

fil_image_spa=real(ifft2(fil_image_freq));

% Image data reformat

display(' The min value before reformatting'); min(fil_image_spa(:));

fil_image_spa_rf=fil_image_spa+abs(min(fil_image_spa(:)));

display('The min value after reformatting');

min(fil_image_spa_rf(:));

fil_image_spa_rf=fil_image_spa_rf./max(fil_image_spa_rf(:));

display('The max value after reformatting');

max(fil_image_spa_rf(:)); MN=size(original_image);

M=MN(1,1); N=MN(1,2);

display('MSE and PSNR of the image with noise')

sum=0;

for i=1:M

   for j=1:N
```

```
    sum=sum+(deg_image(i,j)*255-image_d(i,j)*255)^2;
  end
end
figure;
imshow(deg_image, []);
title('Image + Noise(with SD=0.1, Mean=0)');
MSE=sum*(1/(M*N)); PSNR=10*log10(255^2/MSE);
xlabel(['MSE=',num2str(MSE),', PSNR=',num2str(PSNR)]);
display('MSE and PSNR of the image after LPF');
sum=0;
for i=1:M
  for j=1:N
    sum=sum+(fil_image_spa_rf(i,j)*255-image_d(i,j)*255)^2;
  end
end
MSE=sum*(1/(M*N)); PSNR=10*log10(255^2/MSE);
figure, imshow(fil_image_spa_rf);
title('Output of lowpass filter with infinite gain');
xlabel(['MSE=',num2str(MSE),', PSNR=',num2str(PSNR)]);
```

H. MATLAB code to exhibit the performance of lowpass filter with finite gain op-amp configuration.

```
% Filter application for subjective processing.
```

```
% Filter Application to image processing.
% Lowpass filter
clear all; clc;
 HLIG1=lfg();
original_image=imread('G:\My Documents\lena2.jpg');
figure; imshow(original_image);
original_image=mat2gray(original_image);
original_freq=fft2(original_image);
HLIG1=fftshift(HLIG1);
fil_image_freq=HLIG1.*original_freq;
fil_image=real(ifft2(fil_image_freq));
figure; imshow(fil_image, []);
```

I. MATLAB code to exhibit the performance of highpass filter with finite gain op-amp configuration.

```
% Filter application for subjective processing.
% Filter Application to image processing.
% Highpass filter with finite gain.
clear all; clc;
 HLIG1=hfg();
original_image=imread('G:\My Documents\lena2.jpg');
figure; imshow(original_image); title('Original Image');
original_image=mat2gray(original_image);
```

```
original_freq=fft2(original_image);
HLIG1=fftshift(HLIG1);
fil_image_freq=HLIG1.*original_freq;
fil_image=real(ifft2(fil_image_freq));
figure; imshow(fil_image, []);
title('Without any hisequalization');
figure; imshow(histeq(gscale(fil_image),256),[]);
title('After applying histogram equalization');
display('Good Bye!!!');
```