

Collaborative Techniques for Achieving Spatial Diversity in Wireless Networks

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# ABSTRACT

Collaborative Techniques for Achieving Spatial Diversity in Wireless Networks

Hesam Khoshnevis

Recently, there has been lots of interest in techniques that achieve spatial diversity to improve the reliability and/or the rate of the transmission in wireless networks. To achieve spatial diversity the transmitters need to be equipped with more than one antenna. To obtain maximum diversity gain, the fading among these antennas should be uncorrelated and hence the antennas should be well separated. This is usually not possible due to the cost and the small size of the wireless devices. In this case, the only way to achieve spatial diversity is a new technique which is known as *cooperative diversity*. In this technique, diversity is achieved through collaboration between the transmitting nodes in the network.

In this thesis, we develop a collaboration protocol and a practical coding strategy for the collaborative communication in a three node network with no knowledge of channel state information (CSI) at the transmitter side. Unlike most other works, we have used a variable time-fraction scheme as the basis of our protocol and will show that this protocol achieves full diversity while providing a noticeable coding gain. Then, assuming the availability of the channel state information at the transmitters via a simple feedback from the destination to the relay or to both the source and the relay, we develop

collaboration protocols for both feedback scenarios. The performance of all of these protocols has been obtained using Monte Carlo simulations and has been compared with the outage probability of a single transmitter scenario. We have also obtained constant gain contours that demonstrate the loci of the relay that guarantees a minimum gain for all three scenarios.

*To my parents*

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## List of Acronym

AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BLAST	Bell Laboratories Layered Space-Time
BPSK	Binary Phase-Shift Keying
BS	Base Station
CDMA	Code-Division Multiple-Access
CSI	Channel State Information
FDMA	Frequency-division multiple access
FER	Frame Error Rate
MIMO	Multiple-Input, Multiple-Output
MRC	Maximal Ratio Combining
SC	Selection Combining
SNR	Signal to Noise Ratio
STBC	Space-Time Block Codes
STC	Space Time Codes
STTC	Space-Time Trellis Codes
TDMA	Time-Division Multiple Access
WLAN	Wireless Local Area Network

# List of Symbols

$a_i(t)$	Fading amplitude
$B$	Number of block in a frame
$\mathbf{B}(c, e)$	Codeword difference matrix
$B_c$	Coherence bandwidth
$B_d$	Doppler spread
$BM$	Branch metrics
$C_{AWGN}$	AWGN channel capacity
$C_h$	Instantaneous channel capacity in fading channel
$D$	Distance
$d(c, e)$	Hamming distance of $c$ and $e$
$d_{min}$	Minimum Hamming distance
$E_s$	Symbol energy
$E_b$	Bit energy
$E[x]$	Expected value of $x$
$h$	Fading coefficient
$L$	Number of channels
$N_t$	Number of transmit antennas
$N_r$	Number of receive antennas
$N_0/2$	Additive white Gaussian noise variance
$\bar{P}_r$	Average received power at the receiver



$P_t$	Transmitted power
$P_{b,Ray}$	Bit error probability over a Rayleigh fading channel
$P_e$	Probability of error
$P_{Outage}$	Outage probability
$P_{b,AWGN}$	Bit error probability over an AWGN channel
$PG_{i,j}$	Path gain between nodes i and j
$PL(d)$	Path loss at distance d
$Q(x)$	Complementary error function
$rank$	Rank of a matrix
$T_m$	Delay spread
$W$	Signal band width
$y(t)$	Received signal at time t
$\alpha$	Fading amplitude
$\tau_i(t)$	Propagation delay
$\sigma^2$	Variance
$\Omega$	Average fading power
$\gamma$	Instantaneous received SNR
$\bar{\gamma}$	Average received SNR
$\gamma_{min}$	Minimum required SNR
$\gamma_{SC,eff}$	Effective received SNR at the decoder with selection combining
$\gamma_{MRC,eff}$	Effective received SNR at the decoder with maximal ratio combining
$(\Delta t)_c$	Coherence time

# **CHAPTER ONE**

## **1 Introduction**

### **1.1 Motivation**

With the advancements in technology, the new portable electronic devices become more powerful and capable of running sophisticated applications. Many of these devices/applications need to communicate with each other and/or to the Internet to perform their services. Since these devices are mobile, this communication has to be done over a wireless channel. To support these services the demand for reliable, high data rate, low power communication over wireless channels has increased considerably. However the design of such a system is a very difficult task, due to various characteristics of wireless channels. One of the most challenging problems in the design of a wireless communication system for mobile devices is the random variations of the wireless channel gains, which is called fading. It has been shown that the most effective way to increase the transmission rate and/or reduce the power consumption of a communication system in a fading environment is to use a type of diversity technique. This technique

is called spatial diversity and can be employed by installing several antennas on the transmitting and/or receiving nodes and forming a multiple-input, multiple-output (MIMO) channel [1] [2] [3] [4]. To be able to provide a full diversity gain, these antennas have to be well separated from each other to minimize the amount of correlation between the resulting channels [5]. Due to the size and cost of the mobile wireless devices, this is not possible in most cases. But on the other hand, in an urban environment, we are surrounded by tens if not hundreds of such devices at all time. If these wireless terminals cooperate with each other to transmit and/or receive their information, they can form a virtual multiple antenna system. In this case, proper spatial separation between antennas is usually guaranteed and the resulting channels can be modeled as independent fading channels. This way of achieving diversity is known as *cooperative diversity* [6] and it has been shown that it can provide a diversity gain equal to that of a multiple antenna system [7] [8] [9] [6] [10].

## 1.2 Contributions

In this thesis we have initially assumed no knowledge of channel state information (CSI) at the transmitter side and developed a cooperation protocol and a practical coding strategy for the collaborative communication in a three node network. This network consists of a source, a relay, and an ultimate destination node. It is assumed that the relay is idle at the time of collaboration and can allocate all its resources to help the source node. The transmission of one frame of data is divided into two phases: the exchange phase and the collaborative (cooperation) phase. In the exchange phase the source transmits its information to both the relay and the destination. Then, if

the relay decodes the source message without error, they both transmit in the collaboration phase. And if the relay cannot decode the source message at the end of the exchange phase, it will remain silent and the source will be the only one who transmits during the second phase. One of the most important features of our protocol is its variable time-fraction. In a fixed time fraction protocol the relay may not be able to decode the source message at the end of the fixed-length exchange phase and therefore, remains silent during the collaboration phase. On the other hand when the source-relay channel is in good shape, the relay may not need to spend all that time to understand the message and could start the collaboration earlier. Our coding scheme allows the relay to increase the length of the exchange phase in gradual steps. This is being done despite the fact that the source does not know whether the relay collaborates or not. This, on the average, increases the chance of the relay to collaborate with the source by setting the length of the exchange phase closer to the optimum value. Hence the chance that the diversity gain be achieved by the systems increases.

At the destination, we used a simple decoder that utilizes all the received information to decode the message, including the information obtained during the exchange phase. This guarantees the achievement of the full diversity advantage regardless of the channel code used by the source and the relay in the collaboration phase. It means that unlike the scheme proposed in [9], the focus in the channel code design can be concentrated on the coding gain instead of diversity advantage. Using our design criteria, we have selected a channel code and obtained the performance of our protocol in different environments via Monte Carlo simulation and showed that our system achieves

the full diversity advantage. Then we moved the relay in a plane around the source and the destination and used the simulation results to obtain the loci of the relay that guarantees a certain gain in the system.

Furthermore, assuming the availability of the channel state information at the transmitters via a simple feedback from the destination to the relay or to both the source and the relay, we developed collaboration protocols for both feedback scenarios. The performance of these protocols has been obtained using Monte Carlo simulations and has been compared with the previous results.

### **1.3 Thesis Outline**

This thesis is organized as follows. In Chapter 2, we represent a brief background on the wireless communication in fading environments, diversity techniques, MIMO channels, and space-time codes. Then, we continue with a review of the existing literature on cooperative diversity. Chapter 3 contains our system model and our proposed protocol and the coding strategies. In Chapter 4, we provide details of the simulation parameters; the encoder and the decoder used for the simulations and present the simulation results for the case where there is no CSI available at the transmitters. In Chapter 5, we assume limited knowledge of channel state information at the transmitter side via a simple feedback from the destination first to the relay and later on to both the source and the relay. Using this information, we design a protocol for each feedback scenario that improves the performance of the system. Then, we present the simulation results for the two feedback scenarios and will compare them with the results obtained in

Chapter 4. Finally the concluding remarks and suggestions for future work are given in Chapter 6.

# CHAPTER TWO

## 2 Background and Related Work

In this chapter we present the background and literature review of wireless communication in fading environments. We start by a brief introduction and modeling of large-scale and small-scale fading and continue by explaining different diversity techniques as an effective tool to combat the severe effects of fading on the performance of communication systems. Then, MIMO systems and space-time codes are explained as ways to employ spatial diversity. Finally, we review the existing literature on cooperative diversity as a new useful type of spatial diversity for wireless networks. Later, in Chapter 3 we will define our collaborative protocol as a practical and efficient way of implementing cooperative diversity.

### 2.1 The Wireless Channels and Fading

One of the main challenges in designing a reliable communication system over a wireless channel is the variation of the characteristics of the channel over time and

frequency. These variations can be generally divided into two major categories, namely large-scale fading and small-scale fading. Large-scale fading is mainly the result of path loss according to the distance and the medium between the transmitter and the receiver and shadowing by large objects like buildings and mountains in the environment. Small-scale fading is the consequence of multipath nature of wireless channels. We start by a brief explanation of path loss and will continue by a more thorough introduction to the small-scale multipath fading.

### 2.1.1 Path Loss

Path loss can be defined as the drop in the average received power density of an electromagnetic wave as it moves through space. It includes the effects of propagation losses, transmitting antenna gain, and absorption losses when signal travels through different medium. Path loss is usually characterized by path loss exponent. To define path loss exponent, assume that  $\bar{P}_r$  is the average received power at the receiver over a long period of time and let  $P_t$  be the transmitted power. We can write

$$\bar{P}_r = \frac{c}{d^\beta} P_t, \quad (2.1)$$

where  $c$  is a constant and  $d$  is the distance between the transmitter and the receiver. Then  $\beta$  will be defined as the path loss exponent of the channel [11]. Path loss at a distance  $d$  from the transmitter can also be defined as a relative value corresponding to a known path loss at reference distance  $d_0$  that has been obtained via actual measurements. This way it can be presented as



$$PL(d) = \left( \frac{d}{d_0} \right)^\beta PL(d_0). \quad (2.2)$$

Value of  $\beta$  varies depending on the nature of the environment. Table 2.1 shows some typical path loss exponents measured in different environments.

Table 2.1 Path Loss Exponent for Different Environments [12]

Environment	Path Loss Exponent, $\beta$
Free Space	2
Urban area	2.7 to 3.5
In building Line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6

### 2.1.2 Small-Scale Fading

Small-scale fading or multipath fading can be defined as the random variation of the received signal amplitude and phase during a short period of time or travel distance and is the result of the reception of several versions of the transmitted signal through different paths on the receive antenna. Each of these replicas of the transmitted signal, in general, has a different time varying amplitude, phase shift and delay. The variations in different multipath signals are the results of propagation, scattering, diffraction, and reflection of signal via/through different materials. Also, they can be caused by the relative motion of the transmitter and the receiver or their surrounding objects. Different multipath copies of the transmitted signal can add together in a destructive or

constructive manner and cause random fluctuations in the received signal amplitude and phase, which is called fading. If  $x(t)$  is the transmitted signal and  $y(t)$  is the received signal, we can model the multipath wireless channel as

$$y(t) = \sum_i^L a_i(t)x(t - \tau_i(t)) + n(t). \quad (2.3)$$

In the above equation,  $a_i(t)$  is the amplitude and  $\tau_i(t)$  in the propagation delay of the  $i^{th}$  component of the multipath received signals and  $n(t)$  is the Additive White Gaussian Noise (AWGN). Note that we have assumed that there are a finite number of multipath components,  $L$ . If  $L$  grows the summation will lead to integration.

One of the important characteristics of a fading channel is the time between the reception of the first and the last multipath version of the signal and is called *delay spread* or *multipath spread* of the channel ( $T_m$ ). Based on Equation (2.3), it can be viewed as the difference between the minimum and maximum value of  $\tau_i(t)$ . If the delay spread of the channel is considerably smaller than the transmitted signal duration, we can assume that all multipath components of the received signal have arrived almost at the same time and hence the shape of the transmitted symbol is intact. In the frequency domain we can consider this as an equal treatment of different frequency components of the transmitted signal and therefore, we can say that the channel has a flat frequency response. Furthermore the *coherence bandwidth* of the channel ( $B_c$ ) can be defined as

$$B_c \approx \frac{1}{T_m}. \quad (2.4)$$

If the bandwidth of the transmitted signal is smaller than the coherence bandwidth of the channel, then the channel is said to be a *flat fading* channel and otherwise it is called *frequency selective fading* channel. If we consider a flat fading channel, Equation (2.3) can be written as

$$y(t) = x(t) \sum_i^L a_i(t) + n(t). \quad (2.5)$$

If we define the channel fading coefficient as  $h(t) = \sum_i^L a_i(t)$ , the above equation becomes

$$y(t) = h(t)x(t) + n(t). \quad (2.6)$$

The time variation of the received amplitudes is the effect of the relative motion of the transmitter and the receiver or the movement of the objects in the surrounding environment. *Doppler spread* and *coherence time* of the channel are parameters that are defined to quantify these variations. Doppler spread ( $B_d$ ) is defined as the range of frequencies for which the Doppler power spectrum is non-zero. Coherence time ( $(\Delta t)_c$ ) of the channel can be defined as the average time over which the attenuation factor or the channel fading coefficient remains essentially constant and is the inverse of Doppler spread.

$$(\Delta t)_c \approx \frac{1}{B_d} \quad (2.7)$$

If the duration of the transmitted symbol considerably smaller than the coherence time of the channel, then the channel is said to be a *slow fading* channel and otherwise it will be

called a *fast fading* channel. In a fast fading channel a single symbol may undergo several different fades and hence the received symbol can be severely distorted.

In a flat fading channel if we assume that the number of received multipath signals,  $L$ , is large, the central limit theorem can be applied and we can approximate the distribution of the channel impulse response by a Gaussian process. If there is no line-of-sight between the transmitter and the receiver, the Gaussian process will be zero-mean and therefore the absolute value of the channel gain at any time instant is a Rayleigh random variable and its phase is uniformly distributed over the interval  $(0, 2\pi)$ . Then the channel will be called a Rayleigh flat fading channel. If the Gaussian process has a non-zero mean then the absolute value of the channel can be modeled with a Rician distribution and the channel will be called a Rician fading channel and it is used when there is a line-of-sight between the transmitter and the receiver. There is also a general model for the fading channel based on the experimental observations and it is called Nakagami- $m$  fading. Both of the above mentioned theoretical models can be represented (or approximated) as special cases of the Nakagami- $m$  fading. One can refer to the [12] and [13] for a detailed explanation of these models. Since Rayleigh distribution provides a reliable model for the “worst case” fading scenario [11] (there is no line-of-sight), we will use a Rayleigh fading distribution to model our fading channels throughout this thesis.

To show the severe effect of fading on wireless communication, let us derive the performance of a simple communication system over a slow flat fading Rayleigh channel and compare it with the performance of the same system over an AWGN channel.

Assume we use binary phase-shift keying (BPSK) modulation with average received signal to noise ratio  $SNR = \frac{E_b}{N_0}$ . We can model the received signal as

$$y = hx + n \quad (2.8)$$

where  $h$  is the channel coefficient,  $y$  is the received signal,  $x$  is the transmitted signal with values  $\pm\sqrt{E_b}$  and  $n$  is the additive white Gaussian noise with mean zero and variance  $N_0/2$  per dimension. The conditional bit error rate of the channel for a specific value of  $h$  can be written as

$$P_b(h) = Q\left(\sqrt{2|h|^2 \frac{E_b}{N_0}}\right) \quad (2.9)$$

where  $Q(x)$  is complementary error function. Since we assumed Rayleigh fading distribution, the probability distribution function of  $|h|$  is given by

$$p_{|h|}(u) = \frac{2u}{\Omega} e^{-u^2/\Omega} \quad u \geq 0 \quad (2.10)$$

where  $\Omega = E(h^2)$  and can be represented as the average fading power. Then  $\gamma = h^2 E_b/N_0$  has a chi-square probability distribution with two degrees of freedom. We have

$$p(\gamma) = \frac{1}{\gamma} \exp\left(-\frac{\gamma}{\gamma}\right) \quad (2.11)$$

where  $\bar{\gamma}$  is the average  $\gamma$ . Now we can average the conditional bit error rate obtained in (2.9) over the probability distribution of  $\gamma$  and obtain the average bit error rate of the system over a Rayleigh fading channel. We have

$$P_b = \int_0^{\infty} P_b(\gamma) p(\gamma) d\gamma. \quad (2.12)$$

Assuming that the average fading power is equal one ( $\Omega = 1$ ) the result of the above integration is obtained as

$$P_{b, \text{Rayleigh}} = \frac{1}{2} \left( 1 - \sqrt{\frac{\bar{\gamma}}{1 + \bar{\gamma}}} \right). \quad (2.13)$$

And for large  $\bar{\gamma}$  we have

$$P_{b, \text{Rayleigh}} \approx \frac{1}{4\bar{\gamma}}. \quad (2.14)$$

From basic communications, we know that the bit error rate of a BPSK modulation in AWGN channel is upper bounded by

$$P_{b, \text{AWGN}} = Q\left(\sqrt{2\frac{E_b}{N_0}}\right) \leq \frac{1}{2} \exp\left(-\frac{E_b}{N_0}\right). \quad (2.15)$$

Both approximations are tight at high SNRs. It can be seen that the bit error rate decreases exponentially with SNR in AWGN channel, while it only decays with the inverse of SNR over Rayleigh fading channel. The obtained bit error rates are depicted in

Figure 2.1 The figure shows the severe effect of fading in the performance of a wireless communication system.

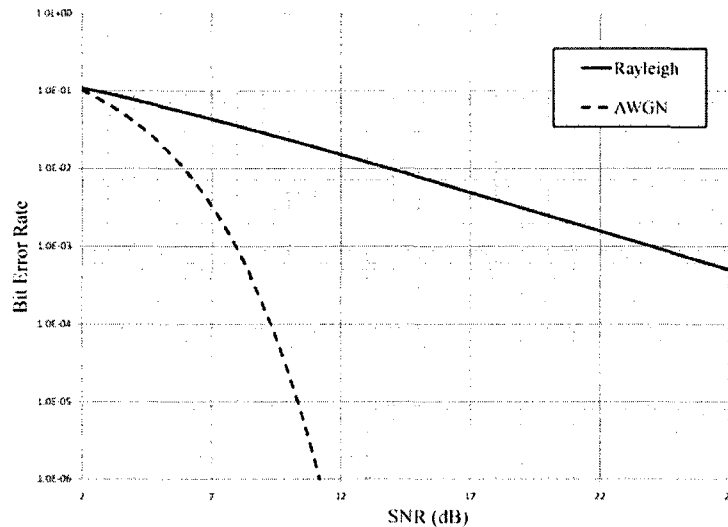


Figure 2.1 Bit error rates of BPSK modulation over AWGN and Rayleigh fading channels

To obtain the bit error rate of the fading channel we averaged the conditional error rate over all possible fading values. This means we have to transmit an infinite number of bits in order to achieve the error rate of Equation (2.13). Since this is not practical in many applications, let us define another measure for analyzing the performance of a communication system in a Fading environment. Assume that a specific transmission system requires a minimum frame error rate (FER) to function properly and that to achieve that frame error rate a minimum signal to noise ratio ( $\gamma_{min}$ ) is needed. In a fading environment the instantaneous SNR fluctuates with respect to variations of the channel coefficients. If we know the distribution of the channel coefficients, we can calculate the probability of the event when the instantaneous SNR falls below the minimum required SNR. This Probability is called the *outage probability* and indicates

the percentage of the time that the performance of the system is not satisfactory. Outage probability is a good measure of performance of the system when it is not possible to average over all fading coefficients.

Consider the same slow flat Rayleigh fading channel and assume that the fading coefficient  $h$  remains essentially constant during the transmission of a fixed length frame of data (*quasi-static fading*). If we need  $\gamma_{min}$  to achieve the required frame error rate, the outage probability can be written as

$$P_{outage} = P[\gamma \leq \gamma_{min}] = \int_0^{\gamma_{min}} p(\gamma) d\gamma = \int_0^{\gamma_{min}} \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}} d\gamma = 1 - e^{-\frac{\gamma_{min}}{\bar{\gamma}}} \quad (2.16)$$

where  $\bar{\gamma}$  is the average received signal to noise ratio. At high signal to noise ratios, the above can be approximated by

$$P_{outage} \approx \frac{\gamma_{min}}{\bar{\gamma}}. \quad (2.17)$$

It can be seen that the outage probability only decays inversely with the signal to noise ratio which is not acceptable in most communication systems. For a quasi-static Fading channel, outage probability can be defined in terms of the transmission rate and instantaneous channel capacity. Capacity of a channel is defined as the maximum amount of information that can be reliably transmitted through a communication channel. Shannon proved that if the information transmission rate exceeds this limit, it is impossible to decode the received message with an arbitrary small probability of error [14]. Capacity of an AWGN channel can be written as



$$C_{AWGN} = \log\left(1 + \frac{E_b}{N_o}\right) \quad (2.18)$$

where the capacity is measured in bits per channel use and  $E_b/N_o$  is the information signal to noise ratio. In a fading channel the instantaneous received signal to noise ratio ( $\gamma$ ) fluctuates with the variation of the channel gain, therefore, the instantaneous channel capacity is a function of the fading coefficient. We have

$$C_h = \log\left(1 + |h|^2 \frac{E_b}{N_o}\right). \quad (2.19)$$

Now we define an outage as the event when the instantaneous capacity ( $C_h$ ) of the channel falls below the information transmission rate ( $R$ ). Based on the definition of the channel capacity, this means that when an outage occurs, it is impossible to decode the received message without error. Now the *information outage probability* or simply the *outage probability* can be written as

$$P_{outage} = P[C_h < R]. \quad (2.20)$$

For every outage probability, we can define an *outage capacity* as the maximum transmission rate that can be achieved with probability  $(1 - P_{outage})$ . In block fading environments, the outage probability can be considered as the lower bound to the frame error rate of the system and hence it is a useful tool to evaluate the performance of a coded system.

## 2.2 Diversity Techniques

In the previous section, we explained the undesirable effects of fading on the performance of a communication system. Now we will present some techniques to combat the effects of channel fading. These techniques are referred to as *diversity techniques*. Diversity techniques in general refer to the ways of improving performance of the system by providing the receiver with more than one replica of the same information through independently fading channels. To make it more clear let us give an example. Assume that in a given communication system, the received signal to noise ratio has to be above a certain threshold  $\gamma$  for the receiver to be able to decode the message with a required error rate. Now assume that our system works in a fading environment and the received signal to noise ratio falls below the required threshold with probability  $p$ , then we can say that  $P_{outage} = p$ . If we supply the receiver with  $L$  replicas of the same information, each received through an independent fading channel, the probability that all the received replicas fall below the assumed threshold is  $p^L$  and therefore,  $P_{outage}|_L = p^L$ . Since  $p < 1$ ,  $p^L < p$ . Therefore, if  $L$  independent diversity channels (branches) are available, the outage probability obtained in the previous section will decay inversely with the  $L$ th power of the average signal to noise ratio. In this case the system is said to provide a diversity of order  $L$ . In general diversity order is defined as

$$\text{Diversity Gain} = - \lim_{SNR \rightarrow \infty} \frac{\text{Log}(P_e)}{\text{Log}(SNR)} \quad (2.21)$$

where  $P_e$  is the error rate of the system. Diversity can be achieved via several methods. The most famous types of diversity are *frequency diversity*, *time diversity*, and *space diversity*.

Frequency diversity consists in transmitting the same signal via  $L$  different carrier frequencies. If the separation between each two of these carrier frequencies is more than the coherence bandwidth of the channel, then each of the received signals will encounter an independent fade. Therefore, we achieve a diversity of order  $L$ . Another method to implement frequency diversity is to transmit a wideband signal. If the bandwidth of the transmitted signal ( $W$ ) is much larger than the coherence bandwidth of the channel, the received signal will encounter several independent fades in frequency domain. The achieved order of diversity can be approximated by  $L = W/B_c$ .

Another way to achieve diversity is to transmit the same information several times. This is called time diversity if the transmitted signals are received via independent fades. To ensure this, the time separation between the consecutive transmissions should be equal or greater than the coherence time of the channel. If we transmit the same information  $L$  times, this can be considered as a repetition code with rate  $1/L$ .

Diversity can also be achieved by transmitting the same signal through spatially separated channels. For example if we have one transmit antenna and  $L$  receive antennas, we have  $L$  channels. If the spatial separation between the receive antennas is large enough (more than half of the wavelength [5]), the channels can be modeled as independent fading channels. Then the received replicas of the signal have encountered

different fading coefficients and we can achieve a diversity of order  $L$ . This diversity technique is called space or antenna diversity.

Certainly nothing can be achieved for free and there are costs associated with different diversity techniques. In frequency diversity, the system needs extra bandwidth as well as one extra transmitter/receiver pair for every extra carrier frequency. The cost of extra hardware can be justified or reduced through selection diversity in many applications but in most wireless communication systems bandwidth is extremely precious and it is not possible to allocate extra bandwidth to the system. To achieve time diversity the transmitter has to send the same signal several times. This means that we have to reduce the transmission rate or increase the bandwidth. In the former case, delay will also be added. The added delay is proportional to the coherence time of the channel as well as the number of times that the signal is being repeated. Many communication systems cannot tolerate such a delay and hence cannot employ time diversity to combat fading. Spatial diversity does not have a burden on the system in terms of extra bandwidth or delay but it has its own limitation: It is not possible to have sufficient separation between antennas in many mobile wireless communication systems due to their relatively small sizes.

To achieve spatial diversity, there should be more than one independent fading channel between the transmitter and the receiver. This can be achieved by employing several antennas at the receiver (receive diversity), several antennas at the transmitter (transmit diversity), or both at the receiver and the transmitter (transmit and receive diversity). Different types of spatial diversity are depicted in Figure 2.2.

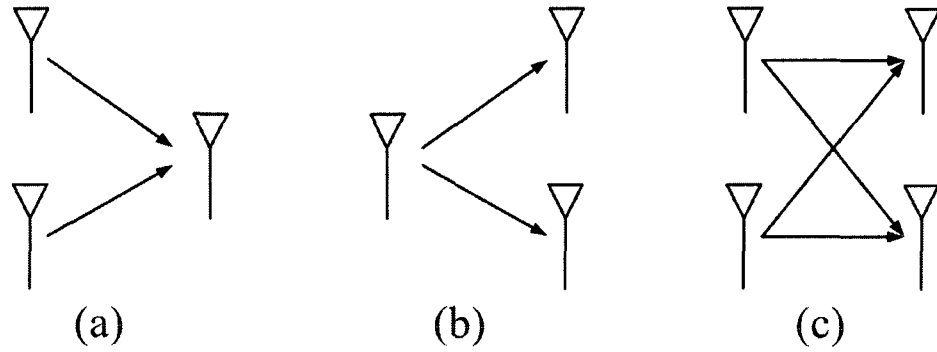


Figure 2.2 (a) Transmit diversity, (b) receive diversity, (c) transmit and receive diversity

### 2.2.1 Combining methods

Consider a wireless communication system with one transmit and  $L$  receive antennas. If the receive antennas are sufficiently separated from each other, this system provides us with a spatial diversity of order  $L$ . A simple channel model for such a system is depicted in Figure 2.3. In such a system the receiver has access to  $L$  different versions of the transmitted signal, namely  $y_1, y_2, \dots, y_L$ . There are several methods to use these received signals and achieve the available diversity gain. The simplest method to do this is called *selection combining*. In this method, the diversity branch that has the highest signal to noise ratio is selected. This method is easy to implement as the system only needs to measure the received signal power at each antenna and make its decision. Under this scheme the overall received signal can be modeled as

$$y = \left( \max_{i=1,2,\dots,L} |h_i| \right) x + n \quad (2.22)$$

where  $n$  is a complex Gaussian random variable with variance  $N_0/2$  per dimension. And the effective received signal to noise ratio can be written as

$$\gamma_{SC,eff} = \left( \max_{i=1,2,\dots,L} |h_i|^2 \right) \frac{E_b}{N_0}. \quad (2.23)$$

In the environments where, the channel does not change very fast, the system can use several antennas and only one receiver stage and switch between the antennas only when the received SNR falls below a certain threshold. This method is called *scanning* or *switch-and-stay diversity*.

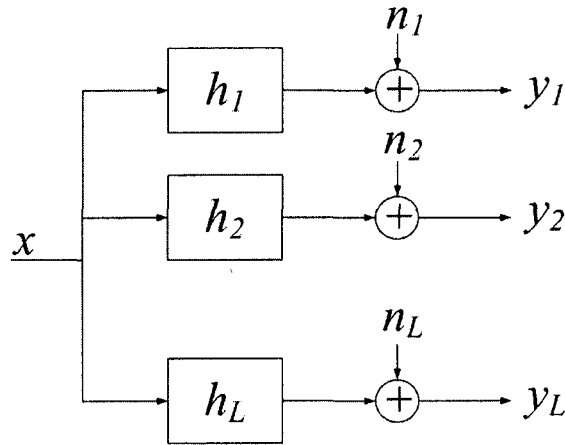


Figure 2.3 Channel model for a system with  $L$ th order receive diversity

None of the above mentioned methods is optimal as they do not use the received signal from all branches. The optimal reception method is called *maximal ratio combining* (MRC) [15]. In this method all of the received signals are co-phased and weighted according to their individual signal to noise ratios and then combined (summed) together. Note that this method requires a perfect knowledge of the channel coefficients at the receiver. The received signal from each branch can be written as

$$y_i = h_i x + n_i \quad (2.24)$$

where  $n_i$ 's are independent samples of a complex Gaussian random variable with variance  $N_0/2$  per dimension. Then, the output of the combiner can be written as

$$y = \left( \sum_{i=1}^L |h_i|^2 \right) x + \left( \sum_{i=1}^L h_i n_i \right) \quad (2.25)$$

The effective signal to noise ration of the combined output is

$$\gamma_{MRC,eff} = \frac{\left( \sum_{i=1}^L |h_i|^2 \right)^2 E_b}{N_0 \sum_{i=1}^L |h_i|^2} = \sum_{i=1}^L |h_i|^2 \frac{E_b}{N_0} = \sum_{i=1}^L \gamma_i. \quad (2.26)$$

It can be seen that the effective signal to noise ratio at the output of the MRC is the sum of signal to noise ratios of all channels. Therefore a system with maximal ratio combiner can achieve a certain error rate at a lower energy level than a system that uses selection diversity. When the receiver cannot estimate the channel coefficients fast enough to do the maximal ratio combining, it is possible to only co-phase the received signals and combine them together. This method is called equal gain combining and its performance is slightly lower than that of the maximal ratio combining.

## 2.3 Multiple-input, multiple-output (MIMO) channels

In the previous section, we only talked about using multiple antennas at the receiver. It is possible to have multiple antennas both on the transmitter and the receiver. This results in a multiple-input, multiple-output (MIMO) system (Figure 2.4). In their influential works, Telatar [3] and Foschini and Gans [2] independently derived the

equations of the capacity of MIMO channels. They showed that the capacity of a wireless channel in a fading environment can be increased considerably by employing multiple transmitting and/or receiving antennas. Consider a system with  $N_t$  transmit and  $N_r$  receive antennas and assume that,  $N_t = N_r = n$ . It is possible to show that the channel capacity grows linearly with  $n$  when all fading channels are mutually independent.

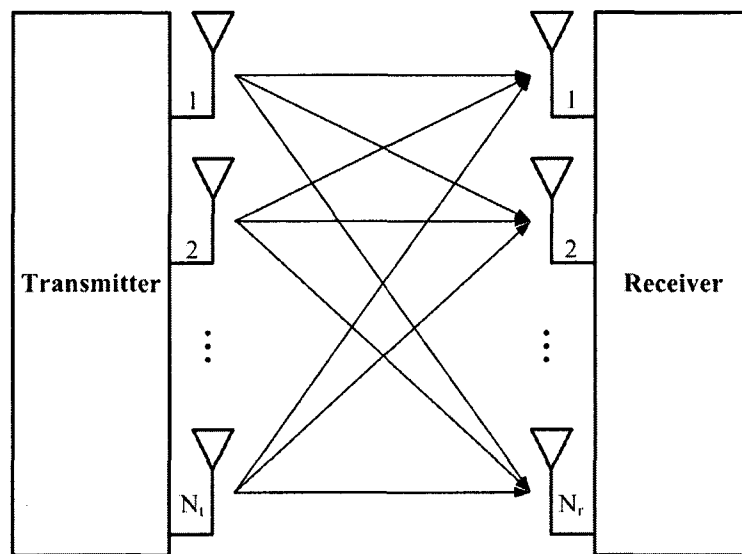


Figure 2.4 A typical MIMO channel model

A MIMO system also provides spatial diversity. It can combine transmit diversity with receive diversity. It has been shown that for a MIMO system with  $N_t$  transmitter and  $N_r$  receiver antennas and independent fading between all antenna pairs, one can design codes with a probability of error that decays with  $SNR^{-N_r N_t}$  [4; 16]. Which means that the system can achieve a diversity gain of  $N_t N_r$ . In the previous section we explained how to combine the received signals from different channels to achieve



diversity gain. In the following section the coding strategies that provide transmit diversity will be explained.

## 2.4 Space-time codes

To start, let us assume a system with several transmitting and one receiving antenna. This is a common scenario in wireless mobile networks. For example, it is easily possible to employ several antennas on the base station (BS) in a mobile cellular network or on the router in a wireless local area network (WLAN) but according to the size limitation of the mobile devices it is usually not possible to incorporate more than one antenna on those and maintain the proper spacing between antennas, not to mention the added cost of extra antenna on every single device. If there are  $N_t$  transmit antennas, it is easy to show that the system can achieve a diversity gain of  $N_t$ . The simplest way would be to retransmit the same symbol once from each antenna. This way the receiver will get  $N_t$  replicas of the transmitted signal each through an independent fading channel. Using one of the explained combining methods, it is strait forward to show that the system achieves a diversity gain of  $N_t$ . But this is equal to a repetition code and even though it achieves the diversity gain, the transmission rate of the system is reduced by a factor of  $1/N_t$ . There has been lots of research around design of proper coding schemes that provide transmit diversity, while maintaining the transmission rate. To achieve this, these codes utilize both available degrees of freedom in a MIMO channel, namely space and time. As a result, this family of codes is called space-time codes. Space-time codes are first presented by Tarokh et al. in [4]. Space time codes (STC) can be divided into

two main categories of space-time block codes (STBC) and space-time trellis codes (STTC) [4] [16] [17]. In [4], authors derived the design criteria for their proposed family of codes under the assumption of Rayleigh and Rician fading channels and used their criteria to design several space-time trellis codes. And in [16], they have extended their design to block codes. One of the simplest and most practical space-time block codes is proposed by Alamouti in [17]. Alamouti scheme provides a simple transmit diversity scheme for two transmit and one receive antennas, but it can be expanded to the case of multiple receive antennas. For the case of one receive antenna, it achieves a diversity gain of 2, equal to that of one transmit and two receive antennas, while maintaining the same transmission rate. And when extended to the case of  $N_r$  receive antennas; it achieves a diversity gain of  $2N_r$  at high SNR. As a result of its decoding simplicity and excellent performance, Alamouti scheme has been added to the many third generation (3G) cellular standards.

As stated in the previous section, the capacity of a MIMO channel increases linearly with the number of transmit antennas in a fading environment. Therefore it is possible to employ a MIMO channel to increase the transmission rate of the system. It can be shown that the transmission rate can be as high as  $\min\{N_t, N_r\}$  symbol per channel use [1]. This higher transmission rate can be achieved by use of spatial multiplexing. But then it is no more possible to achieve the diversity gain of  $N_t N_r$ . In [18] it has been proven that for a diversity multiplexing gain of  $l = 0, 1, \dots, \min\{N_t, N_r\}$ , the maximum achievable diversity gain can be written as

$$\text{Diversity Gain}(l) = (N_r - l)(N_t - l). \quad (2.27)$$

When the block length of the code is more than or equal to  $N_t + N_r - 1$ . Therefore there is a tradeoff between the diversity and spatial multiplexing. For some wireless applications it can be useful to increase rate while maintaining some level of diversity. There are coding schemes that tradeoff diversity to achieve a higher transmission rate, some examples are Bell Laboratories Layered Space-Time (BLAST) codes [1] and multilayered space-time codes presented in [4].

## 2.5 Cooperative Diversity

Nowadays, most wireless handheld devices need reliable high data rate communication protocols to support various high-bandwidth demanding multimedia applications. This has to be done without any increase or even if possible, with decrease of the power consumption of the system, as available power is always extremely limited in wireless devices. Considering what has been explained in the previous sections and knowing that these wireless devices usually work in rich scattering urban environments, using multiple transmit and/or receive antennas seems to be the answer to the problem. But wireless devices are usually small and it may not be possible to employ the extra antennas on the device and maintain the proper spacing between them to guarantee independent fading channels. Also the size and cost of the hardware itself is an important issue. To overcome these problems the idea of cooperation between nodes in a wireless network has been introduced recently. Cooperative communication can result in a virtual MIMO channel between the transmitting and the receiving nodes, by using the idle resources available in the network.

The main idea of the cooperation goes back to the introduction of the relay channel by Van der Meulen [19] where he models a three terminal wireless channel. In [20] Cover and El Gamal derived upper and lower bounds on the channel capacity for the specific case of AWGN channels (without fading) and showed that these two bounds converge together for specific case of degraded relay channel. Since then, there has been lots of work on the capacity of the relay channels in different environments and under different conditions. In [21] the authors proposed a novel coding scheme for the relay channel and derived a rate region for the case of AWGN channel with multiple relays. Authors in [22] have studied the capacity of multiple-access relay channels. Capacity gain for transmitter and receiver cooperation is presented in [23]. Some recent results on the coding strategies for multi-node relay channels that achieve the ergodic capacity under certain conditions are presented in [24].

User cooperation was first explained and studied in [7] and [8]. The authors assumed dedicated orthogonal sub-channels between the mobile users, developed a specific cooperation model for code-division multiple-access (CDMA) systems and derived the rate regions for communication to the base station under the assumption of the availability of some channel state information (CSI) at the transmitters. In [10] and [25], the authors integrated error control coding with cooperation and named it coded cooperation. It is assumed that both users want to transmit their own information to the destination, but each does it in its own fixed time slot. While user 1 transmits its information to the destination, user 2 remains silent and listens to user 1's message. Then, if user 2 can decode user 1's message, it will transmit that information after its own

data to the destination and if not, it will transmit its own information during the whole time slot. It has been shown that at high SNR the system achieve full diversity (diversity gain of 2 as there are 2 transmit and one receive antennas). This work has been continued in [26] by introducing a variable time fraction that changes based on the geometrical location of the nodes.

The term cooperative diversity was first introduced in [6] by Laneman, et al. Unlike most of earlier works, they considered a more realistic half-duplex transmission mode (i.e., network nodes cannot transmit and receive at the same), developed several cooperation protocols, and obtained and compared their outage probabilities. These protocols define the details of the cooperation scheme from different perspectives. From the overall network point of view, these protocols can be named as fixed, selection, and incremental relaying. Since they assume a half-duplex transmission, in all of the proposed schemes, in general, the transmission time can be divided into two phases. During the first phase the source transmits its information and the relay terminal listens, then during the second phase, the source remains silent and the relay processes and retransmits the information it received. Finally the destination combined both of the received signals and decodes the message using all received information. After reception of the source message the relay can either do amplify-and-forward or decode-and-forward. In amplify-and-forward, the relay simply amplifies its received signal and retransmits it to the destination. This can be viewed as a repetition code that is being transmitted from two antennas with the difference that the relay amplifies its own receiver noise as well as the signal. Despite of the noise amplification, fixed relaying

with amplify-and-forward achieves full diversity, i.e., diversity of order 2 since there are two transmitting antennas. On the other hand, the relay can decode the source message before forwarding the information to the destination. This is called decode-and-forward strategy. In the proposed decode-and-forward scheme the destination declares an error if the relay cannot decode the source message correctly. Therefore, in fixed relaying protocol the decode-and-forward scheme only achieves a diversity of order one.

Fixed relaying is simple to implement, but in general is a very wasteful protocol, especially when the source-destination channel is in good state. In selection relaying, it is assumed that all nodes have access to the channel state information (CSI) and use this information to improve their cooperation strategy. The source constantly monitors the source-relay channel gain and when it drops below a certain threshold, it will continue transmitting an uncoded or a coded version of its message in the second phase. The authors in [6], shown that selection relaying achieves full diversity with both amplify-and-forward and decode-and-forward strategies.

In incremental relaying, the destination sends a one bit feedback to the relay at the end of the first phase and asks for its cooperation when it is needed. This way the relay does not transmit unnecessary information to the destination, therefore the system efficiency increases. In [6], it is shown that the incremental relaying achieves a diversity of order 2 and it has the best spectral efficiency.

There is a general problem with amplify-and-forward and decode-and-forward strategies, none is bandwidth efficient especially when the number of the relaying terminals increases. The relays have to work in different time slots, time-division

multiple access (TDMA) or different frequency channels Frequency-division multiple access (FDMA) to avoid interference with each other. In [9], distributed space-time codes was proposed to combat this problem. In this scenario the source transmits its message in the first phase. Then, those relays who can decode the message without error will encode it using a specific code which is a column of the overall space-time code. This way all relays can cooperate at the same time and in the same frequency. Finally the authors have shown that distributed space-time coded protocols achieve a diversity gain equal to the number of the transmitting nodes (including the source) without increasing the bandwidth. Distributed space-time coded protocols have been further studied in many works including [27], [28] , [29], [30], [31], and [32].

In [33], the authors extended the decode-and-forward protocol of [6] by utilizing a variable-rate transmission scenario between nodes. This means that when the relay is closer to the source the source-relay transmission rate can be higher and when it is closer to the destination the relay-destination rate can be higher. Therefore the transmission rates are set according to the geometrical location of the relay. They have proposed and analyzed the performance of several protocols in different situations. For example when the relay is closer to the source, *transmit diversity protocol* has a better performance but when the relay is closer to the destination *receive diversity protocol* outperforms. In the transmit diversity protocol both source and relay transmit during the second phase using an STC, but in the receive diversity protocol the source remains silent during the second phase and the relay uses a regular channel code to transmit the information to the destination. Other proposed protocols are simplified transmit protocol, where the

destination ignores the received information in the first phase, and multi-hopping, where only one node transmits at any time and the receiver only uses the received information from the relay to decode the message. They have shown that all protocols except multi-hopping achieve full diversity.

In [34], Mitran et al. proposed a variable time-fraction cooperation protocol. In their scheme, unlike the previous works, the transmission time is not divided into two fixed time-fraction. Instead, they use an adaptive fraction of the transmission time to transmit the information to the relay based on the instantaneous source-relay channel state. It has been shown that, under certain conditions (when relays are located around the source within 1/3 of the distance between the source and the destination), this protocol can essentially achieve the performance of a real MIMO system with two transmit and one receive antennas. Note that, in the real MIMO scenario, both of the transmitting antennas are physically connected to the source and have *a priori* knowledge of its information, while in a cooperative scenario the relay has to obtain this information before cooperation. The authors have derived the outage probability of the system as

$$P_{outage} = P \left[ |H_{s,d}|^2 + \Delta |H_{r,d}|^2 < \frac{2^R - 1}{\gamma} \right] \quad (2.28)$$

where  $H_{s,d}$  is the fading matrix between the source and the destination nodes,  $H_{r,d}$  is the fading matrix between the relay and the destination nodes,  $R$  is the information transmission rate of the system,  $\gamma$  is the SNR and  $\Delta$  is the time fraction defined as



$$\Delta = \min \left( 1, \frac{R}{C(H_{s,r}, \gamma_r)} \right) \quad (2.29)$$

where  $\gamma_r$  is the average received SNR at the relay and  $C(H_{s,r}, \gamma_r)$  is the instantaneous capacity of the source-relay channel. The outage capacity of the channel can be obtained by averaging (2.28) over all possible channel fades. In [34], it has been shown that this capacity is very close to the capacity of the ideal MIMO system when the relays are located in the previously mentioned distance from the source.

## 2.6 Conclusion

In the previous section we have presented several cooperation protocols. Many of these protocols are the same in the aspect of having two phases for the communication. Some divide the transmission time between the two phases in a fixed manner and therefore waste the network resources when the source-relay channel is in good condition. It is shown that [34] in the performance of the collaborative system can be improved by using a variable time fraction in the exchange phase, when the source transmits its information to the relays. Although, the proposed protocol in [34] is useful for evaluating the performance of a variable time fraction scheme, it is not physically realizable since when the relay is very close to the source, it requires the transmission of the information from the source to the relay in an arbitrarily small fraction of time. In this work, we present a practical protocol and coding strategy to employ variable time fraction in order to increase the performance of the system.

## CHAPTER THREE

### 3 System Model and Channel Code Selection

In this chapter, we start by defining our communication network structure and explain the details of the corresponding channel parameters. Then we present our variable time-fraction collaborative protocol and explain and analyze the collaboration parameters. Finally the channel code design criteria for the collaborative protocol are explained.

#### 3.1 System Model

The network is set up as in Figure 3.1. Without loss of generality we set the distance between the source and the destination to one and normalize associated path gain,  $PG_{SD} = 0dB$ . All other path gains are measured using their relative distances and a path loss exponent of 2 [12]. We assume that all the nodes are located on the same plane and the relay can move freely on this plane. We further assume that the distance between the relay and the source is always greater than zero and for the rare case that they are co-

located it is fair to assume that the relay has full knowledge of the source message, hence the duration of the exchange phase is zero.

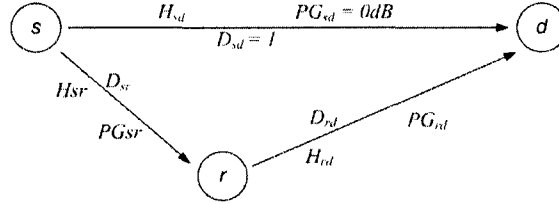


Figure 3.1 Network Model for two-phase communication

In this scheme, transmitters do not have access to channel state information (CSI). It means that the source is neither aware of its transmission channel's fading coefficients, nor of the relay's collaboration status. The relay is aware of  $s - r$  channel's fading coefficients (so it is able to determine its collaboration strategy) but not of its transmission channel's coefficients. Finally the destination has all the fading coefficients hence it can determine the relay's collaboration status as well. Lack of access to the channel state information at source means it cannot encode according to specific instantaneous channel realizations and it transmits to the destination regardless of whether relay collaborates with it or not.

However, the source must encode its data in a way for the relay to have the potential of retrieving all the information within a fraction of the whole frame. In our two-phase collaborative scheme this time fraction is variable. If we assume that each frame consists of  $n$  channel uses we want the relay to be able to decode the source message after  $\Delta n$  channel uses where  $\Delta < 1$ . If we let  $\Delta$  take any real value (as in [ [34]]) the frame length has to be infinite which is not practical. Instead we use a set of

quantized time fractions  $\Delta \in \{\Delta_1, \Delta_2, \dots, 1\}$  where  $\Delta = 1$  means that the relay could not decode the source message hence was unable to collaborate with the source before the end of the frame.

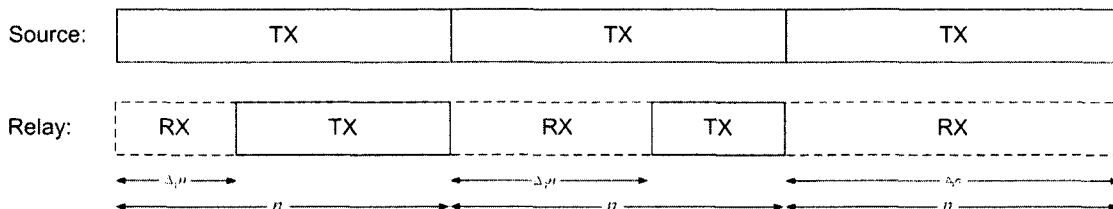


Figure 3.2 Collaborative communication with variable time-fraction

To make sure that our system does not require extra bandwidth compared to a non-collaborative system, the above set of time fractions have to be further constrained and the modulation order has to be chosen in order to comply with it. For example if we are willing to transmit  $k$  information bits in  $n$  channel uses from the source to the destination, source should be able to transmit all those information bits in  $\Delta_1 n$  time period to the relay and this requires a modulation scheme with a minimum of  $2^{\frac{k}{\Delta_1 n}}$  constellation points to avoid bandwidth expansion. If we use equal length time fractions the set can be written as  $\Delta \in \{1/P, 2/P, \dots, 1\}$  hence the minimum number of the constellation points will be  $2^{\frac{kP}{n}}$ .

In this work we always use BPSK modulation and assume that the source transmits its  $k$  information bits in  $B$  blocks. We further assume that each block consists of  $l$  bits and all of the blocks are of equal length. It is important to mention that having

equal length blocks is not a general requirement for our scheme. The above assumptions actually defines our set of  $\Delta$ 's as

$$\Delta \in \left\{ \frac{1}{B}, \frac{2}{B}, \dots, \frac{B-1}{B}, 1 \right\}. \quad (3.1)$$

$\Delta = 1$  represents the case where the relay could not decode the source message in time and remains silent during the frame. The frame length can be written as  $n = Bl$  and regardless of the collaboration status, the over-all transmission rate can be given as

$$R = \frac{k}{n} = \frac{k}{Bl}. \quad (3.2)$$

For our numerical results we have used  $B=3$ . This results in three different collaboration modes, which is shown in Fig. 3.3. In mode one, *(i)*, the relay can decode the source message without error after the first block, i.e., the received signal to noise ratio at the relay is high enough so that it can decode the message with an acceptable bit error rate. The relay then transmits an encoded version of the source message during the second and third blocks. In mode two, *(ii)*, the relay cannot decode the message after the first block but can decode it reliably after the reception of the second block. In this mode it will only collaborate with the source during the transmission of the last block. In mode three, *(iii)*, it is not possible for the relay to decode the message reliably after the reception of the second block, hence it will stay silent during the rest of the frame and there will be no collaboration for this frame.

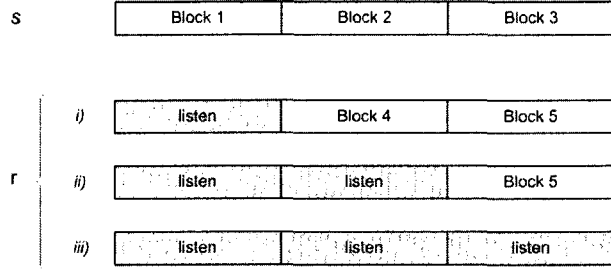


Figure 3.3 Three modes of operation based on  $s$ - $r$  channel. *i)*  $\Delta = 1/3$ , *ii)*  $\Delta = 2/3$ , *iii)*  $\Delta = 1$  (no collaboration).

Regardless of the relay collaboration status, the source uses an  $(n, k)$  code for transmission. During the first phase, both the relay and destination receive signals

$$y_t^d = h_{sd}x_t^s + \eta_t^d \quad (3.3)$$

$$y_t^r = \sqrt{PG_{sr}}h_{sr}x_t^s + \eta_t^r, 1 \leq t \leq \Delta n \quad (3.4)$$

where  $y_t^j$  is the received signal at node  $j \in \{s, r\}$  and time  $t$ ,  $x_t^j$  is the transmitted symbol at node  $j$  and time  $t$ , the noise components  $\eta_t^d$  and  $\eta_t^r$  are independent samples of a zero-mean complex Gaussian random variable with variance  $N_o / 2$  per dimension and the quasi-static fading coefficients  $h_{sd}$  and  $h_{rd}$  are assumed to be constant during a frame and are modeled as independent samples of complex Gaussian random variables with zero mean and variance 0.5 per dimension. In the second phase, the source and relay collaborate to transmit to the destination,

$$y_t^d = h_{sd}x_t^s + \sqrt{PG_{rd}}h_{rd}x_t^r + \eta_t^d, \Delta n \leq t \leq n \quad (3.5)$$

The information received at the destination during both phases will be used to decrease the decoding error. This will give us more options in defining the coding strategies for the collaborative phase.

As we wish to compare the frame error rate (FER) performance of our collaborative system with a system without collaboration, the signal-to-noise ratio must be normalized. In general to compare the performance of different protocols the received SNR should be equal in all systems but in a collaborative system the received SNR will change as the relay moves in the space and when the relay gets very close to the destination the received SNR increases radically. If we want to compensate for this increase we have to reduce the relay's transmit power drastically and this will cause a poor performance of the system even when channel between the relay and the destination is in a good condition. Therefore, to compare the performance of our collaborative system to a non-collaborative one, we use transmitted signal-to-noise ratio,  $SNR_{sr}^{Tx}$ . The subscript  $sr$  is added to show that the power is transmitted from both source and relay. Assuming all transmitters have equal power we can easily derive the transmitted  $SNR$  as

$$SNR_{sr}^{Tx} = \frac{(2 - E[\Delta])nE[x^2]}{kN}, \quad (3.6)$$

where  $E[x^2]$  is the transmitted symbol energy which can be normalized and  $E[\Delta]$  is the expected value of the time fraction that relay listens to the source for an specific codebook and a fixed relay position and will be derived later.

The time fraction  $\Delta$  depends on the channel code, the location of the relay (its distance to the source) and the source-relay channel fading coefficients. We assume that at the beginning of each frame the relay is aware of the  $s - r$  channel coefficients and of its received SNR. Utilizing this information the relay can calculate the required time fraction that ensures error free (i.e.  $BER \leq 10^{-5}$ ) decoding for each specific channel state and can determine the mode of collaboration. For a quasi-static fading channel where the fading coefficients change every frame, the relay has to decide on the collaboration status at the beginning of each frame. Since the source always uses the same error control code to transmit the information, the relay can use a table to determine the collaboration scenario based on the received signal-to-noise ratios,  $SNR_s^{Rx}$ 's (Table 3.1). The source-relay link has the following instantaneous received SNR

$$SNR_s^{Rx} = \frac{PG_{sr} \alpha_{sr}^2 E[x^2]}{N_o}, \quad (3.7)$$

where  $PG_{sr}$  is the path gain of the  $s - r$  channel and we have used  $h_{sr} = \alpha_{sr} e^{j\theta_{sr}}$ .

3.1 Relay's time fraction Table

Source-Relay Code Rate $R_{sr}$	Time fraction ( $\Delta$ )	Required $SNR_r^{Rx}$ for $BER \leq 10^{-5}$
$k/l$	$1/B$	$SNR_r^{Rx}  _{\Delta=1/B}$
$k/2l$	$2/B$	$SNR_r^{Rx}  _{\Delta=2/B}$
...	....	...
$k/(B-1)l$	$(B-1)/B$	$SNR_r^{Rx}  _{\Delta=(B-1)/B}$

As mentioned before, our assumption in this part of the work is that the relay does not have access to the source-destination or relay-destination channel state information



$(h_{sd}, h_{rd})$ , therefore it assumes that the source-destination channel is in worst state while trying to determine the collaboration mode and will try to maximize collaboration time. To achieve this, the relay has to select the minimum value for  $\Delta$  that guarantees an error free ( $BER \leq 10^{-5}$ ) reception at the relay. Rewriting (3.7) we have,

$$\alpha_{sr}^2 = \frac{N_o SNR_r^{Rx}}{PG_{sr} E[x^2]} = \frac{SNR_r^{Rx}}{\gamma}, \quad (3.8)$$

where  $\gamma = \frac{PG_{sr} E[x^2]}{N_o}$  and the transmitted bit energy can be normalized (i.e.,  $E[x^2] = 1$ ).

To maximize the collaboration time the time fraction  $\Delta$  is being selected as follows:

The relay knows  $\gamma$  and  $\alpha_{sr}$  and will select the smallest  $\Delta$  (largest  $SNR_r^{Rx}$  from Table 3.1) for which

$$SNR_r^{Rx} |_{\Delta_i} \leq \alpha_{sr}^2 \cdot \gamma. \quad (3.9)$$

Using the above method to obtain the time fraction,  $\Delta$ , for each channel realization, we can analytically compute  $E[\Delta]$ . We define  $\rho = \alpha_{sr}^2$  and note that the fading amplitude  $\alpha_{sr}$  has a Rayleigh distribution and  $\rho$  is a random variable with an exponential distribution and its pdf can be written as  $f(\rho) = e^{-\rho}$ . We have

$$E[\Delta] = \Delta_1 p(\Delta = \Delta_1) + \Delta_2 p(\Delta = \Delta_2) + \dots + p(\Delta = 1), \quad (3.10)$$

where  $p(\Delta = \Delta_1)$  is the probability of  $\Delta = \Delta_1$ . Substituting (3.1) in (3.10) we get

$$E[\Delta] = \frac{1}{B} p(\Delta = \frac{1}{B}) + \frac{2}{B} p(\Delta = \frac{2}{B}) + \dots + p(\Delta = 1). \quad (3.11)$$

Using (3.9) we have

$$\begin{aligned}
E[\Delta] = & \frac{1}{B} p\left(\rho \geq \frac{SNR_r^{Rx} |_{\Delta=1/B}}{\gamma}\right) + \frac{2}{B} p\left(\frac{SNR_r^{Rx} |_{\Delta=2/B}}{\gamma} \leq \rho \leq \frac{SNR_r^{Rx} |_{\Delta=1/B}}{\gamma}\right) \\
& + \dots + p\left(\rho \leq \frac{SNR_r^{Rx} |_{\Delta=(B-1)/B}}{\gamma}\right)
\end{aligned} \tag{3.12}$$

By straightforward algebra we get

$$E[\Delta] = 1 - \frac{1}{B} e^{-\frac{SNR_r^{Rx} |_{\Delta=1/B}}{\gamma}} - \frac{1}{B} e^{-\frac{SNR_r^{Rx} |_{\Delta=2/B}}{\gamma}} - \dots - \frac{1}{B} e^{-\frac{SNR_r^{Rx} |_{\Delta=(B-1)/B}}{\gamma}}. \tag{3.13}$$

## 3.2 Channel code selection

As the source has no knowledge of relay's collaboration, it always uses the same channel code and the source transmission rate stays the same regardless of the relay's ability to collaborate. The source's codebook rate can be written as  $k/n = k/Bl$ .

If we consider the overall (source and relay) transmission, the situation is different and the rate is variable. For example, if we use the collaboration scheme depicted in Figure 3.3, it is easy to see that each mode will require a different code rate. In general a collaboration scenario operating at a specific  $\Delta$  will transmit  $B$  blocks from the source and  $(1-\Delta)B$  blocks from the relay, hence uses a code of rate  $C_\Delta = \frac{k}{(2-\Delta)Bl}$ .

Then the over-all collaboration scheme uses a set of codebooks that can be written as  $C \in \{C_{\Delta=1/B}, C_{\Delta=2/B}, \dots, C_{\Delta=1}\}$ . Different types of channel codes can be utilized to obtain the

required variable rate codebook. For example, we can use a block code and set the first block to contain all the information bits and the other blocks can be different parity sets. Or a convolutional encoder with  $(2 - \Delta)B$  outputs can be used and we can add or drop output sequences in order to achieve the proper rate. Note that the source always uses the same number of output sequences and the source code book remains the same (Fig. 3.4.).

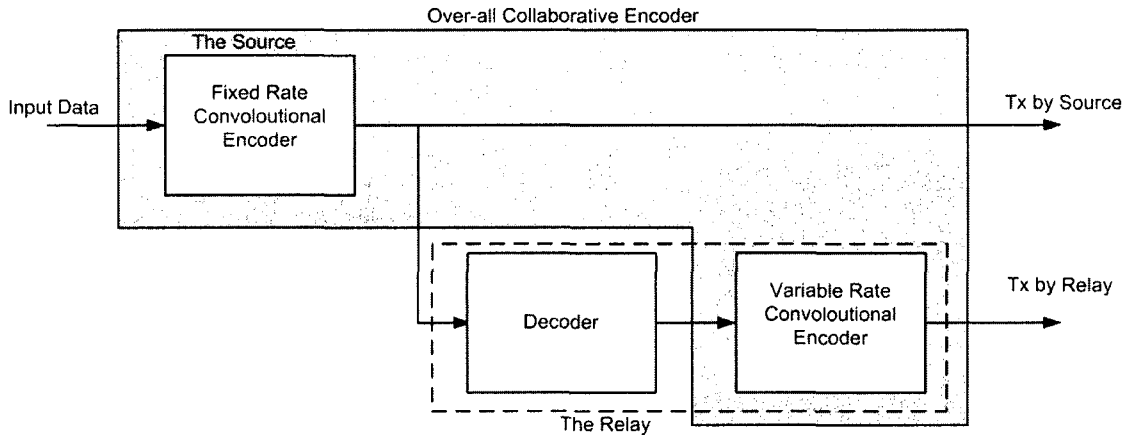


Figure 3.4 Convolutional encoder for collaborative communications

As mentioned before the destination is aware of  $\Delta$  and will adjust its decoder to use all the received data to improve the performance of the system. If we use a convolutional code the branch paths on the trellis will be labeled as  $a_i / c_{1,i}^s \ c_{2,i}^s \ \dots \ c_{B,i}^s \ c_{(1+\Delta)B,i}^r \ \dots \ c_{2B-2,i}^r \ c_{2B-1,i}^r$ , where  $a_i$  is the  $i^{th}$  input to the system and  $c_{j,i}^s$  and  $c_{j,i}^r$  are respectively the source's and relay's output bits in blocks  $j$  and  $k$  corresponding to the  $i^{th}$  input and  $j \in \{1, 2, \dots, B\}$  and  $k \in \{(1 + \Delta)B, \dots, 2B - 1\}$ . The maximum likelihood decoder utilizes the Viterbi algorithm with the following branch metric equation to minimize the path metrics

$$BM_i = \sum_{t=0}^{B-1} \left| y_{i+t}^d - h_{sd} c_{t+1,i}^s - \sqrt{PG_{rd}} h_{rd} c_{t+B,i}^r \right|^2. \quad (3.14)$$

It should be noted that the relay does not transmit during the first phase, i.e.

$$c_{k,i}^r = 0 \text{ for } k < \Delta B.$$

### 3.3 Modified Pairwise Error Probability

We need to obtain the probability of transmitting

$$\mathbf{c} = \begin{pmatrix} c_1^s & c_2^s & \dots & c_n^s \\ c_1^r & c_2^r & \dots & c_n^r \end{pmatrix} \quad (3.15)$$

and deciding in favor of

$$\mathbf{e} = \begin{pmatrix} e_1^s & e_2^s & \dots & e_n^s \\ e_1^r & e_2^r & \dots & e_n^r \end{pmatrix}. \quad (3.16)$$

Given an ideal channel state information and considering that in our system, the number of transmit antennas at the source,  $n_s$ , and at the relay,  $n_r$ , are both equal to one and there is only a single receive antenna, from [4] this probability can be well approximated by,

$$P(\mathbf{c} \rightarrow \mathbf{e} | h_{i,d}, i = r, s) \leq e^{\left( \frac{-d^2(\mathbf{c}, \mathbf{e}) E_s}{4N_s} \right)} \quad (3.17)$$

where

$$d^2(\mathbf{c}, \mathbf{e}) = \sum_{i=1}^n \left| \sum_{i \in \{s,r\}} h_{i,d} (c_i^i - e_i^i) \right|^2. \quad (3.18)$$

Since in general the path gain from the source to the destination is not the same as the path gain from the relay to the destination, we have to include the effect of this path gain difference in our equations, as in [35]. Hence Equations (3.17) and (3.18) become

$$P(\mathbf{c} \rightarrow \mathbf{e} | \sqrt{PG_{i,d}} h_{i,d}, i = r, s) \leq e^{\frac{-d^2(\mathbf{c}, \mathbf{e}) E_s}{4N_0}} \quad (3.19)$$

and

$$d^2(\mathbf{c}, \mathbf{e}) = \sum_{i=1}^n \left| \sum_{i \in \{s,r\}} \sqrt{PG_{i,d}} h_{i,d} (c_i^i - e_i^i) \right|^2. \quad (3.20)$$

By defining  $\mathbf{H} = (\sqrt{PG_{s,d}} h_{s,d}, \sqrt{PG_{r,d}} h_{r,d})$  and following the method used in [4], (3.20) can be written as

$$d^2(\mathbf{c}, \mathbf{e}) = \sum_{i \in \{s,r\}} \sum_{i' \in \{s,r\}} (\sqrt{PG_{i,d}} h_{i,d}) (\sqrt{PG_{i',d}} h_{i',d}) \sum_{i=1}^n (c_i^i - e_i^i) \overline{(c_{i'}^{i'} - e_{i'}^{i'})}. \quad (3.21)$$

By setting

$$\mathbf{A}(\mathbf{c}, \mathbf{e}) = \begin{bmatrix} \sum_{i=1}^n (c_i^s - e_i^s)^2 & \sum_{i=1}^n (c_i^s - e_i^s) \overline{(c_i^r - e_i^r)} \\ \sum_{i=1}^n (c_i^r - e_i^r) \overline{(c_i^s - e_i^s)} & \sum_{i=1}^n (c_i^r - e_i^r)^2 \end{bmatrix}, \quad (3.22)$$

Equation (3.21) can be compressed in the matrix form as

$$d^2(\mathbf{c}, \mathbf{e}) = \mathbf{H}\mathbf{A}\mathbf{H}^*, \quad (3.23)$$

where  $\mathbf{H}^*$  is the Hermitian (transposed conjugate) of  $\mathbf{H}$ . Note that  $\mathbf{A}(\mathbf{c}, \mathbf{e}) = \mathbf{B}(\mathbf{c}, \mathbf{e})\mathbf{B}^*(\mathbf{c}, \mathbf{e})$  where  $\mathbf{B}(\mathbf{c}, \mathbf{e})$  is the codeword difference matrix

$$\mathbf{B}(\mathbf{c}, \mathbf{e}) = \begin{bmatrix} c_1^s - e_1^s & c_2^s - e_2^s & \dots & c_n^s - e_n^s \\ c_1^r - e_1^r & c_2^r - e_2^r & \dots & c_n^r - e_n^r \end{bmatrix}. \quad (3.24)$$

Now (3.19) can be written as

$$P(\mathbf{c} \rightarrow \mathbf{e} | \sqrt{PG_{i,d}} h_{i,d}, i = r, s) \leq e^{\frac{(-\mathbf{H}\mathbf{A}(\mathbf{c}, \mathbf{e})\mathbf{H}^* E_s)}{4N_s}}. \quad (3.25)$$

Given that  $\mathbf{A}(\mathbf{c}, \mathbf{e})$  is Hermitian, we can have  $\mathbf{V}\mathbf{A}(\mathbf{c}, \mathbf{e})\mathbf{V}^* = \mathbf{D}$ , where  $\mathbf{V}$  is a unitary matrix composed of eigenvectors of  $\mathbf{A}$  and  $\mathbf{D}$  is a diagonal matrix with eigenvalues,  $\lambda_i$ , of  $\mathbf{A}$  as its diagonal elements. As  $\mathbf{A}$  is nonnegative definite, its eigenvalues,  $\lambda_i$ , are nonnegative real numbers.

By defining  $(\beta_1, \beta_2) = \mathbf{H}\mathbf{V}^*$ , equation (3.23) can be written as

$$d(\mathbf{c}, \mathbf{e}) = \sum_{i=1}^2 \lambda_i |\beta_i|^2. \quad (3.26)$$

Considering that  $h_{sd}, h_{rd}$  are independent samples of complex Gaussian random variables with zero mean and variance 0.5 per dimension and the fact that the set of the rows of  $\mathbf{V}$ ,  $\{\mathbf{v}_1, \mathbf{v}_2\}$ , is an orthonormal basis,  $\beta_i$ 's are independent complex Gaussian random variables with variance per dimension

$$\sigma_i^2 = \frac{PG_{sd}}{2} |v_{i,1}|^2 + \frac{PG_{rd}}{2} |v_{i,2}|^2, \quad (3.27)$$

where  $v_{i,j}$  is the  $j^{th}$  element of the  $i^{th}$  eigenvector [35].

Then the modified pairwise error probability can be written as,

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \left( \prod_{i=1}^{rank} 2\sigma_i^2 \lambda_i \right)^{-1} (E_s / 4N_o)^{-rank}, \quad (3.28)$$

where  $rank$  is the rank of the  $\mathbf{A}(\mathbf{c}, \mathbf{e})$  matrix [35].

Furthermore, in our system, we make sure that the collaboration phase, if it exists, is long enough for the relay to transmit all the information to the destination. Therefore, we can guarantee a rank, diversity advantage, of 2 regardless of the code construction. The pairwise error probability can be written as

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \left( \prod_{i=1}^2 2\sigma_i^2 \lambda_i \right)^{-1} (E_s / 4N_o)^{-2}. \quad (3.29)$$

As shown in [4], to maximize the coding gain, we have to maximize the minimum value of  $\prod_{i=1}^2 2\sigma_i^2 \lambda_i$  for all codeword difference matrices  $\mathbf{B}(\mathbf{c}, \mathbf{e})$ . As we have no feedback from the destination to the source and the relay, the transmitters are not aware of each other's channel path gain to the destination. For this reason they are not able to change their codebook and use the specific node configuration to their advantage. Given that on the average the worst possible assumption is that the two path gains are equal, we will use the coding criterion presented in [4], and we will try to maximize the minimum

of the determinant of  $A(\mathbf{c}, \mathbf{e})$  taken over all pairs of distinct codewords  $\mathbf{c}$  and  $\mathbf{e}$ .

### 3.4 Code Construction

First we label the codebooks that are being used by the source and the relay at different times.  $C_1$  corresponds to the codebook used by the source during the first phase,  $C_2$  is the source's codebook for the second, collaboration, phase and  $C_3$  is the codebook used by the relay in case of collaboration. These assignments are illustrated in Fig 3.5. Since in this thesis we use a BPSK modulation with constellation points at -1 and 1, the first phase (exchange phase) constitutes transmission of  $n_1 = \Delta n$  bits and the second phase (collaborative phase) is composed of  $n_2 = (1-\Delta)n$  bits. It should be mentioned that the length of the codewords in these codebooks depends on the value of  $\Delta$ , obtained from Table (3.1).

Phase \ Node	First Phase	Second Phase
Source	$C_1$	$C_2$
Relay	-----	$C_3$

$\longleftarrow n_1 = \Delta n \quad \longrightarrow \quad \longleftarrow n_2 = (1-\Delta)n \quad \longrightarrow$

#### 3.5 Two-phase Collaborative Communication

For code construction we start from the worst case scenario. In the worst case scenario the relay cannot decode the source message and will not collaborate, therefore the only transmitted signal is from the source to the destination ( $C_1, C_2$ ). The minimum hamming distance,  $d_{1,2min}$ , of the codewords in codebook  $C_{1,2} = \{C_1, C_2\}$ , an  $(n, k)$



code, has to be maximized to ensure the maximum bit error rate performance of the system in this situation. This problem has been studied in details in [36].

Now we consider the case where the relay can collaborate. The first constraint on the code construction is that the codewords in the codebook  $C_1$  have to contain all the information bits that are to be transmitted to the destination. This condition allows the relay to detect the source message. In a binary code, this can be achieved by including all the information bits in each codeword and ensuring that the minimum Hamming distance of the codewords in the codebook  $C_1$  is greater than 1 ( $d_{1\ min} \geq 1$ ).

As the relay does not transmit during the first phase, for all codewords  $c_i^r = 0$  for  $i \leq (1 - \Delta)n$ . Then the codeword difference matrix can be given as

$$\mathbf{B}(\mathbf{c}, \mathbf{e}) = \begin{bmatrix} c_1^s - e_1^s & \cdots & c_{n_1}^s - e_{n_1}^s & c_{n_1+1}^s - e_{n_1+1}^s & \cdots & c_n^s - e_n^s \\ 0 & 0 & 0 & c_{n_1+1}^r - e_{n_1+1}^r & \cdots & c_n^r - e_n^r \end{bmatrix}. \quad (3.30)$$

Since the minimum Hamming distance of the codewords in  $C_1$  is greater than 1, there is at least one nonzero  $c_i^s - e_i^s$  for,  $i \leq (1 - \Delta)n$ . Therefore, to guarantee that each codeword difference matrix is of full rank and thus the code achieves maximum diversity, at least one of  $c_i^r - e_i^r$  for  $(1 - \Delta)n < i \leq n$  have to be nonzero. This can be achieved by ensuring that the minimum Hamming distance of the codewords in  $C_3$  is greater than 1 ( $d_{3\ min} \geq 1$ ).

To minimize the pairwise error probability of the codebook, the determinant criterion as defined in [4] has been used. The matrix  $A(\mathbf{c}, \mathbf{e})$  can be written as

$$\mathbf{A}(\mathbf{c}, \mathbf{e}) = \mathbf{B}(\mathbf{c}, \mathbf{e})\mathbf{B}^H(\mathbf{c}, \mathbf{e}) = \begin{bmatrix} \sum_{i=1}^n (c_i^s - e_i^r)^2 & \sum_{i=1}^{n_2} (c_{n_1+i}^s - e_{n_1+i}^s)(c_{n_1+i}^r - e_{n_1+i}^r) \\ \sum_{i=1}^{n_2} (c_{n_1+i}^s - e_{n_1+i}^s)(c_{n_1+i}^r - e_{n_1+i}^r) & \sum_{i=1}^{n_2} (c_{n_1+i}^r - e_{n_1+i}^r)^2 \end{bmatrix}. \quad (3.31)$$

In the above matrix, the weight distribution of the codewords can be used. In Blocks 1,2 and 3, codewords  $\mathbf{c}$  and  $\mathbf{e}$  are separated by  $d_1$ ,  $d_2$  and  $d_3$  respectively.  $\mathbf{A}(\mathbf{c}, \mathbf{e})$  can be written as

$$\mathbf{A}(\mathbf{c}, \mathbf{e}) = \begin{bmatrix} 4(d_1 + d_2) & f \\ f & 4d_3 \end{bmatrix} \quad (3.32)$$

where  $|f| \leq 4d|_{d=\min(d_2, d_3)}$  [35]. And the determinant of  $\mathbf{A}(\mathbf{c}, \mathbf{e})$  is bounded by

$$\det A \geq 16d_3(d_1 + d_2) - 16d_{\min(d_2, d_3)}^2 \quad (3.33)$$

$$\det \mathbf{A}(\mathbf{c}, \mathbf{e}) \leq 16d_3(d_1 + d_2). \quad (3.34)$$

### 3.4.1 Block Codes

Here we analyze the use of an orthogonal code, like Alamouti codes that have been presented in [17] and will show that the use of no other block codes in the collaboration phase can provide a higher coding gain for the system.

The orthogonality of the code guarantees that  $f = 0$ . Hence, we have

$$\det \mathbf{A}(\mathbf{e}, \mathbf{c}) = 16d_3(d_1 + d_2). \quad (3.35)$$

It can be seen that the upper bound of the determinant have been achieved. Alamouti codes have the same minimum free distance for the codewords that are being transmitted from different antennas, this means  $d_2 = d_3$ . Let us define  $d_{1,2\min}$  as the minimum free distance of the overall code used by the source,  $C_{1,2} = \{C_1, C_2\}$ . By the definition of Hamming distance we have

$$(d_1 + d_2) \geq d_{1,2\min}. \quad (3.36)$$

Substituting (3.36) in (3.35) we get

$$\det \mathbf{A}(\mathbf{e}, \mathbf{c}) \geq 16d_2d_{1,2\min}. \quad (3.37)$$

Now consider a codebook that maximizes  $d_{1,2\min}$  and let us choose the codeword that has the minimum  $d_2$  ( $d_{2\min}$ ). If we select any other codeword from this codebook since it has a higher minimum free distance it will lead to a higher determinant of  $\mathbf{A}(\mathbf{c}, \mathbf{e})$ .

Using  $d_{2\min}$  we have

$$\det \mathbf{A}(\mathbf{e}, \mathbf{c}) \geq 16d_{2\min}d_{1,2\min}. \quad (3.38)$$

Equation (3.38) shows that to maximize  $\det \mathbf{A}(\mathbf{c}, \mathbf{e})$ , minimize pairwise error probability,  $d_{1,2\min}$  and  $d_{2\min}$  have to be maximized. This can be done by selecting a codebook that maximizes  $d_{1,2\min}$  and rearrange it to maximize  $d_{2\min}$  as well.

Now we show that a non-orthogonal block code in the collaboration phase cannot achieve a higher coding gain. Assume that there is a non-orthogonal block code that achieves a higher coding gain. For all codeword pairs in the codebook, we should have

$$16d_3(d_1 + d_2) - 16d_{\min(d_2, d_3)}^2 > 16d_{2\min}^* d_{1,2\min}^*. \quad (3.39)$$

where the  $d_{2\min}^*$  and  $d_{1,2\min}^*$  are the maximized minimum free distance of the codebooks  $C_2$  and  $C_{1,2} = \{C_1, C_2\}$  respectively. The above inequality can be written as

$$d_3(d_1 + d_2) - d_{2\min}^* d_{1,2\min}^* > d_{\min(d_2, d_3)}^2 \geq 0. \quad (3.40)$$

Since this has to hold true for the codeword that has  $d_1 + d_2 = d_{1,2\min}$  we have

$$d_3 d_{1,2\min} - d_{2\min}^* d_{1,2\min}^* > 0. \quad (3.41)$$

$d_{1,2\min}$  has to be less than or equal to  $d_{1,2\min}^*$ , since we have defined  $d_{1,2\min}^*$  as the maximum possible minimum free distance of the codebook  $C_{1,2}$ . Then the above inequality holds only if  $d_3 > d_{2\min}^*$ . But the codewords in codebooks  $C_2$  and  $C_3$  are of equal length (note that these are code books used by source and relay in the collaboration phase) and  $d_{2\min}^*$  is the maximum possible minimum free distance of a codebook with that length, therefore this is not possible. We see that it is not possible to have a non-orthogonal block code in the collaboration phase that achieves a higher coding gain.

### 3.4.2 Convolutional Codes

Here we explain the design parameters for the convolutional codes in our collaborative system. In contrast with block codes, there are no completely orthogonal convolutional codes. When the codewords used by the source and the relay in the collaborative phase are not orthogonal, the worst that can happen is that the

corresponding parts of the codeword difference matrix have nonzero value in all possible places. This means  $|f| = 4d|_{d=\min(d_2, d_3)}$ . Then we have

$$\det A = 16d_3(d_1 + d_2) - 16d_{\min(d_2, d_3)}^2. \quad (3.42)$$

To design a convolutional code for our collaborative system we first determine the maximum number of blocks that are to be transmitted from the source and the relay. If source transmits  $B$  blocks the number will be  $(2B - 1)$ , assuming that all the information bits are included in the first block. Then we select a convolutional code with rate  $R = \frac{1}{2B-1}$  that has the maximum free distance in the overall form. This means that the codebook  $C_{1,2,3} = \{C_1, C_2, C_3\}$  had the maximum  $d_{1,2,3 \text{ free}}$ . Then we use the  $B$  outputs that provide the highest  $d_{1,2 \text{ free}}$  as the source codebook and the rest of the outputs will be used by the relay when it collaborates. Note that each of the three codebooks ( $C_1, C_2, C_3$ ) can contain more than one output. These parameters have been considered in selecting the codes used for simulations in Chapter 4 and Chapter 5.

### 3.5 Chapter Summery

In this chapter we explained our communication network model and its parameters in details. We then introduced a novel collaborative protocol as a practical method of cooperative diversity in wireless networks. Then, the channel code selection and design criteria for our collaborative protocol are presented. In the next chapter we will use these code design strategies to construct a proper channel code for our system

and evaluate the performance of our variable time-fraction collaborative protocol via Monte Carlo simulations.

## **CHAPTER FOUR**

### **4 Simulation Results for the System with No Feedback**

In this chapter we will talk about the simulation parameters, the code used for the simulation and will provide and compare Monte Carlo simulation results obtained using different channel codes in our collaboration protocol. To obtain the results presented in this chapter, we assumed a system with no feedback from the receivers to the transmitters. Therefore the transmitters have no knowledge of their transmission channel state information.

#### **4.1 Simulation Parameters**

##### **4.1.1 General Parameters**

The first assumption in this chapter is that there is no feedback from the receivers to the transmitter. The source and the relay have no knowledge of their transmission channels. Hence the source and the relay assume the worst case scenario while trying to

transmit data to the destination. The source assumes that the relay cannot collaborate and will transmit all the parity bits to the destination. The relay assumes that the channel between the source and the destination is not good and the destination cannot decode the source message, therefore, the relay starts transmitting the parity bits to the destination as soon as it can decode the source message.

We set the frame length to 130 bits ( $l = 130$ ) and choose the number of information bits that are going to be transmitted ( $k$ ) accordingly to make sure that the trellis returns to state zero ( $k = 127$ ). We assume that the source transmits its  $k$  information bits in three blocks ( $B = 3$ ), hence we can write the set of  $\Delta$  as

$$\Delta \in \{\frac{1}{3}, \frac{2}{3}, 1\}. \quad (4.1)$$

This results in three possible collaboration modes that have been depicted in Figure 3.3. In mode one, (*i*), the relay can decode the source message without error after the first block, i.e., the received signal to noise ratio at the relay is high enough (Table 4.1) so that it can decode the message with an acceptable bit error rate ( $BER \leq 10^{-5}$ ). The relay then transmits an encoded version of the source message during the second and third blocks. In mode two, (*ii*), the relay cannot decode the message after the first block but can decode it reliably after the reception of the second block. In this mode it will only collaborate with the source during the transmission of the last block. In mode three, (*iii*), it is not possible for the relay to decode the message reliably after the reception of the second block, hence it will stay silent during the rest of the frame and there will be no collaboration for this frame.



It should be noted that we assume equal transmit power from the source and the relay. We further assume that the duration of symbols transmitted by the source and the relay is of the same length. Therefore the destination can use the same matched filter regardless of the value of  $\Delta$ .

#### 4.1.2 Path Gain

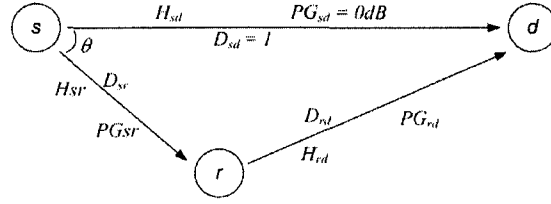


Figure 4.1 Network Model

We normalize the path gain between the source and the destination ( $PG_{sd} = 1 = 0dB$ ) and set the path loss exponent  $\beta = 2$ . We have also repeated the simulations with path loss exponent of 4 to show the consistency of our results in different environments. Now the other path gains can be defined as

$$PG_{sr} = PG_{sd} \left( \frac{D_{sd}}{D_{sr}} \right)^\beta = \left( \frac{1}{D_{sr}} \right)^2 \quad (4.2)$$

And

$$PG_{rd} = PG_{sd} \left( \frac{D_{sd}}{D_{rd}} \right)^\beta = \left( \frac{1}{D_{rd}} \right)^2 \quad (4.3)$$

where

$$\begin{aligned}
 D_{rd} &= \sqrt{D_{sd}^2 + D_{sr}^2 - 2D_{sr}D_{sd} \cos \theta} \\
 &= \sqrt{1 + D_{sr}^2 - 2D_{sr} \cos \theta}.
 \end{aligned}
 \tag{4.4}$$

## 4.2 Encoder

We have used an 8-state code (Figure 4.2) represented in octal form as (15,17) and repeated the output in different blocks in a way that the overall code can be presented as  $(g^{B_1}, g^{B_2}, g^{B_3}, g^{B_4}, g^{B_5}) = (15,17,15,15,17)$  where  $g^{B_i}$  is the generator polynomial of block  $i$ ,  $B_i$ . The minimum free distance of the constructed rate  $1/5$  code is  $d_{free} = 16$ .

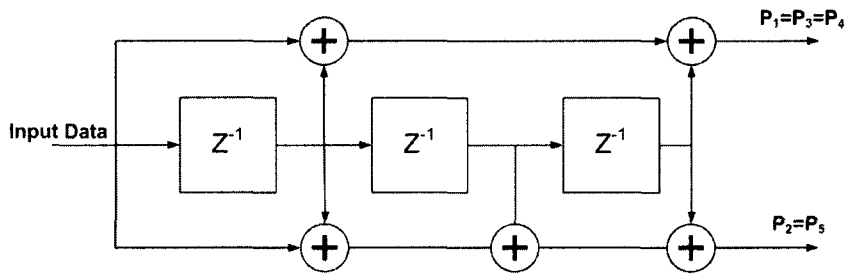


Figure 4.2 Encoder

The received signal to noise ratio required at the relay for being able to decode the source message reliably for each communication rate has been obtained from the simulation. These values can be found in Table 2.1.

Table 4.1 Relay's Required Received SNR

Source-Relay Nominal Code Rate $R_{sr}$	Source-Relay Actual Code Rate $R_{sr}$	Time fraction ( $\Delta$ )	Required $SNR_r^{Rx}$ for $BER \leq 10^{-5}$
1	$127/130$	$1/3$	8.7 dB
$1/2$	$127/260$	$2/3$	5.4 dB

### 4.3 Decoder

The trellis for this code is obtained from the following graph (Figure 4.3). The branch paths on the trellis are labeled as  $a_i / c_{1,i}^S c_{2,i}^S c_{3,i}^S c_{4,i}^R c_{5,i}^R$ , where  $a_i$  is the  $i^{th}$  input to the system and  $c_{j,i}^S$  and  $c_{k,i}^R$  are, respectively, the source's and relay's output bits in blocks  $j$  and  $k$  corresponding to the  $i^{th}$  input and  $j \in \{1, 2, \dots, B\}$  and  $k \in \{(1 + \Delta)B, \dots, 2B - 1\}$ .

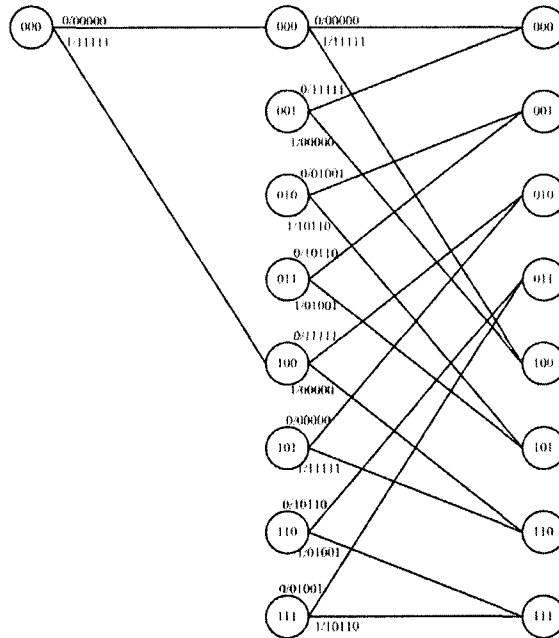


Figure 4.3 Trellis for the 8-state Code

#### 4.3.1 Branch Metric Equations

During the first phase (non collaborative part) the received signal at the destination can be written as

$$y_t^d = h_{sd}x_t^s + \eta_t^d, 1 \leq t \leq \Delta n \quad (4.5)$$

And for the second phase we have

$$y_t^d = h_{sd}x_t^s + \sqrt{PG_{rd}}h_{rd}x_t^r + \eta_t^d, \Delta n \leq t \leq n. \quad (4.6)$$

Now we define the received signals at the relay and the destination and the branch metrics for each mode of the collaboration.

#### 4.3.1.1 Mode (i) $\Delta = 1/3$

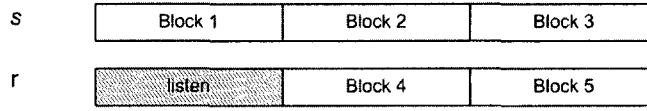


Figure 4.4 Mode (i), ( $\Delta = 1/3$ )

In this mode the relay can decode the source message without error after the first block and will transmit during the second and third blocks. The received signals at the destination for the three blocks can be written as

$$y_{1,i} = h_{sd}P_1 + \eta_1, \quad (4.7)$$

$$y_{2,i} = h_{sd}P_2 + \sqrt{PG_{rd}}h_{rd}P_4 + \eta_2 \quad (4.8)$$

and

$$y_{3,i} = h_{sd}P_3 + \sqrt{PG_{rd}}h_{rd}P_5 + \eta_3. \quad (4.9)$$

And the branch metric that must be minimized will be

$$BM = \left| y_{1,i} - h_{sd} c_{1,i}^s \right|^2 + \left| y_{2,i} - h_{sd} c_{2,i}^s - \sqrt{PG_{rd}} h_{rd} c_{4,i}^r \right|^2 + \left| y_{3,i} - h_{sd} c_{3,i}^s - \sqrt{PG_{rd}} h_{rd} c_{5,i}^r \right|^2. \quad (4.10)$$

#### 4.3.1.2 Mode (ii) $\Delta = 2/3$

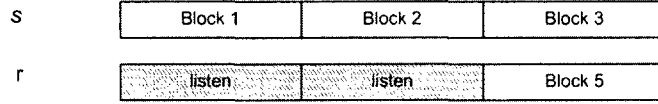


Figure 4.5 Mode (ii), ( $\Delta = 2/3$ )

In mode (ii) the relay needs to listen to the first and the second blocks before being able to decode the source message, therefore, it will only transmit during the third block. The received signals at the destination can be written as

$$y_{1,i} = h_{sd} P_1 + \eta_1, \quad (4.11)$$

$$y_{2,i} = h_{sd} P_2 + \eta_2 \quad (4.12)$$

and

$$y_{3,i} = h_{sd} P_3 + \sqrt{PG_{rd}} h_{rd} P_5 + \eta_3. \quad (4.13)$$

And the branch metric is

$$BM = \left| y_{1,i} - h_{sd} c_{1,i}^s \right|^2 + \left| y_{2,i} - h_{sd} c_{2,i}^s \right|^2 + \left| y_{3,i} - h_{sd} c_{3,i}^s - \sqrt{PG_{rd}} h_{rd} c_{5,i}^r \right|^2. \quad (4.14)$$

### 4.3.1.3 Mode (iii) $\Delta=1$

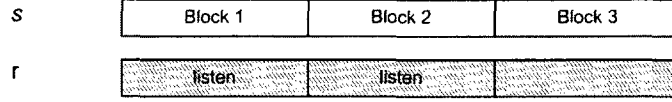


Figure 4.6 Mode (i), ( $\Delta=1/3$ )

Finally in mode (iii) the relay cannot decode the source message and will stay silent during the frame. The received signals and the branch metric are as follows:

$$y_{1,i} = h_{sd}P_1 + \eta_1, \quad (4.15)$$

$$y_{2,i} = h_{sd}P_2 + \eta_2 \quad (4.16)$$

$$y_{3,i} = h_{sd}P_3 + \eta_3 \quad (4.17)$$

and

$$BM = |y_{1,i} - h_{sd}c_{1,i}^s|^2 + |y_{2,i} - h_{sd}c_{2,i}^s|^2 + |y_{3,i} - h_{sd}c_{3,i}^s|^2. \quad (4.18)$$

## 4.4 Results

In Figure 4.7, we compare the FER performance of various codes in our Collaborative scheme. All codes use the same frame length of  $l = 130$  and the number of information bits is  $k = 127$ . The three extra bits are used to bring the code trellis to a known final state. As illustrated in Figure 4.1 the distance between the source and the

destination is normalized ( $D_{sd} = 1$ ). To obtain the results shown in Figure 4.7, we further assume that the relay is located on the line connecting the source and the destination and its distance to source is 0.3 ( $D_{sr} = 0.3$ ). We define an outage event as an error in the decoding of a frame and compare the frame error rate performance of different codes with the outage probability obtained in [34] that has been modified to include the effect of different path gains in the source-destination and the relay-destination channels.

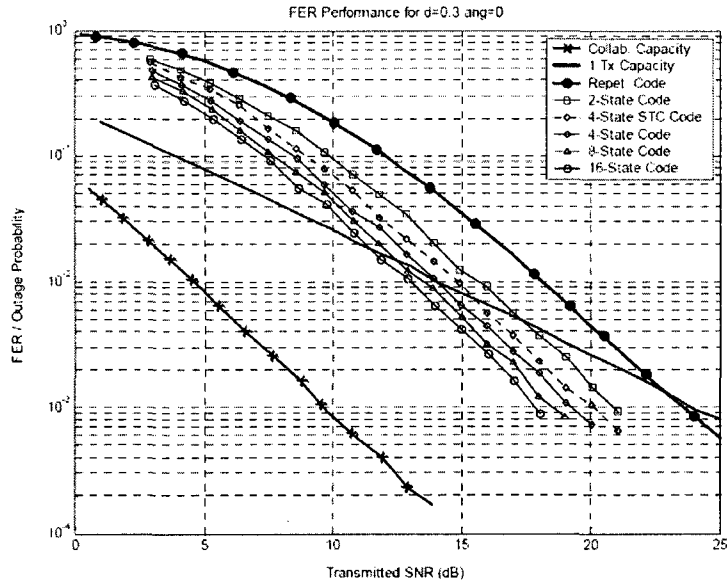


Figure 4.7 FER performance of different codes [35]

All of the codes maximize the  $d_{free}$  for their complexity level. The 2-state code is  $(2,3,3,1,3)_{octal}$  with  $d_{free} = 8$ , the 4-state code is  $(7,5,7,7,5)_{octal}$  with  $d_{free} = 13$ , the 4-state space-time trellis code (STTC) is  $(7,3,3,6,4)_{octal}$  with  $d_{free} = 10$ , the 8-state code is  $(15,17,15,15,17)_{octal}$  with  $d_{free} = 16$ , and the 16-state code is  $(37,27,33,25,27)_{octal}$  with  $d_{free} = 20$ . It can be observed that all of the codes achieve the diversity advantage and as the  $d_{free}$  increases the performance of the system improves (at  $FER=10^{-3}$  there is 3.5dB

difference between the 2-state and 16-state codes). We have also included a simple repetition code, where all the blocks transmitted from the source and the relay are identical. Even with this code the diversity advantage is obtained. We also notice that a 4-state code with maximized  $d_{free}$  outperforms the 4-state STTC code. The reason is that the design restrictions imposed by STTC [4] ensure achieving the full diversity but do not guarantee a maximized  $d_{free}$ .

For the rest of the results in this chapter and in the next chapter we use the previously mentioned 8-state code.

We set the source location to (0,0) and the destination location to (1,0) and moved the relay on the plane by changing  $D_{sr}$  with increments of 0.1 and  $\theta$  with steps  $\pi/8$ . For each location of the relay we have obtained the performance of our collaborative scheme through Monte Carlo simulation.

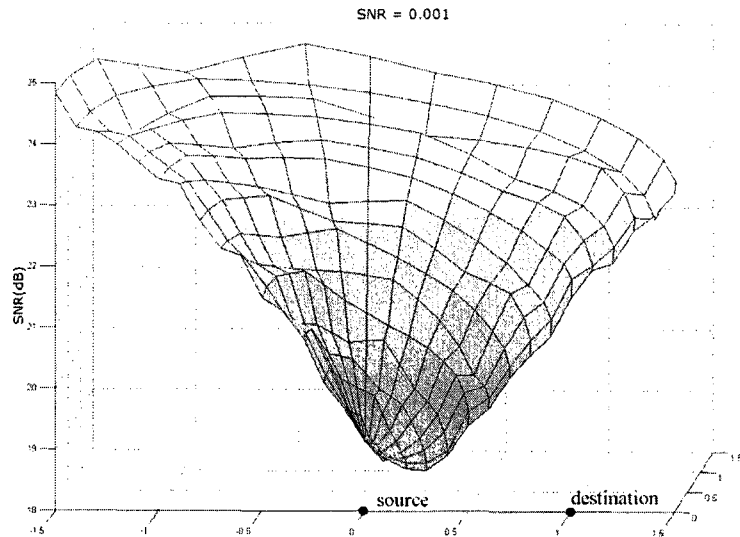


Figure 4.8 Transmitted SNR Required to achieve  $FER=10^{-3}$ , source is at (0,0), destination is at (1,0) and relay moves on the plane



The 3-D plot in Figure 4.8 shows the transmitted SNR required to achieve  $FER=10^{-3}$ . To be able to analyze the results we have presented a cross section of the 3-D plot in Figure 4.9. In this graph the relay moves on the line that connects the source and the destination and instead of transmitted SNR we have used the gain obtained versus the non-collaborative single transmitter outage probability. The single transmitter outage probability is obtained assuming the same distance, path gain and fading coefficients between the source and the destination. It indicates that 24 dB is required for achieving an outage of  $10^{-3}$ .

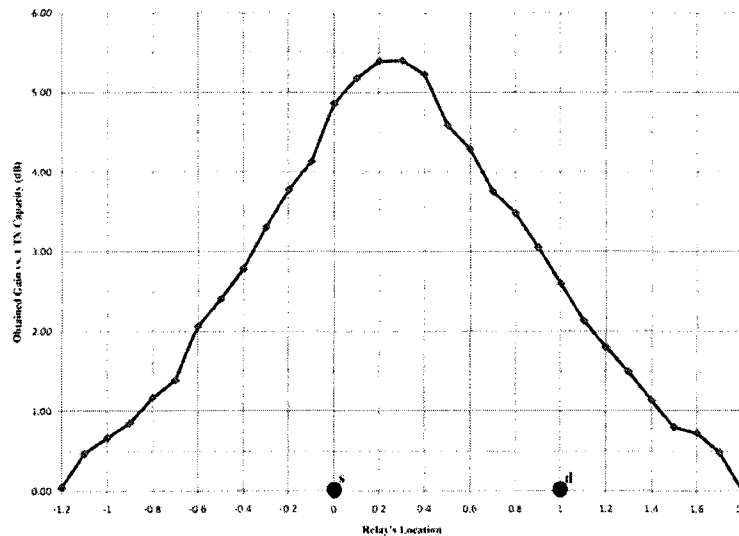


Figure 4.9 Gain vs. Non-Collaborative Single Transmitter Capacity

We note that the system is at the peak performance when the relay is between the source and the destination around  $d = 0.25$  and it starts degrading after  $d = 0.3$ . We also note that our collaborative scheme outperforms a non collaborative system even when the relay is further than the destination.

In Figure 4.10 we show the constant gain contours, i.e., the geometrical loci of the relay to achieve a specific amount of gain when compared to a non-collaborative system. When the relay is located inside a specific contour, the amount of gain written on that contour is guaranteed. For example if the relay is located in the 3dB contour, the overall transmit power required to achieve an FER of  $10^{-3}$  is half ( $-3dB$ ) of that needed when there is no relay. These contours can also be used when there is more than one relay in the vicinity and one has to choose a relay to collaborate.

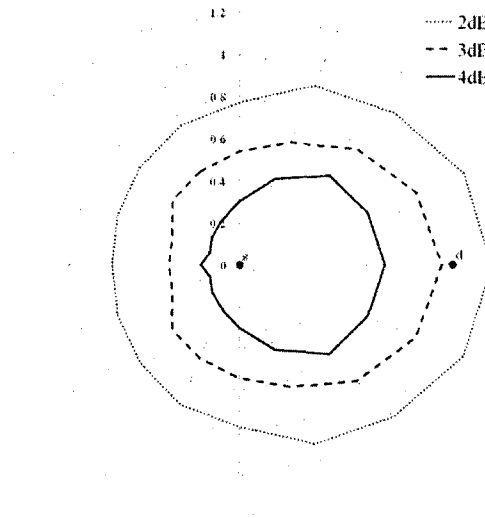


Figure 4.10 Constant gain contours

#### 4.4.1 Simulation Results for Path Loss Exponent of 4

To examine the performance of our system in different environments, we changed the path loss exponent from 2 to 4 and repeated the simulations. We have

$$PG_{sr} = PG_{sd} \left( \frac{D_{sd}}{D_{sr}} \right)^\beta = \left( \frac{1}{D_{sr}} \right)^4 \quad (4.19)$$

and

$$PG_{rd} = PG_{sd} \left( \frac{D_{sd}}{D_{rd}} \right)^\beta = \left( \frac{1}{D_{rd}} \right)^4. \quad (4.20)$$

It should be noted that we still assume that path gain between the source and the destination is  $PG_{sd} = 1 = 0dB$ . Therefore the values obtained from the simulations for transmitted SNR or achieved gain cannot be compared with the results derived with path loss exponent of  $\beta = 2$ .

Figure 4.11 shows the obtained gain versus the non-collaborative capacity when the relay moves on the line connecting the source and the destination. The best performance of the system is achieved at a further distance from the source and closer to the center point between the source and the destination. It should be noted that as it was expected, the area in which the relay can move to provide gain is reduced from (-1.2, 1.8) to (-0.7, 1.4).

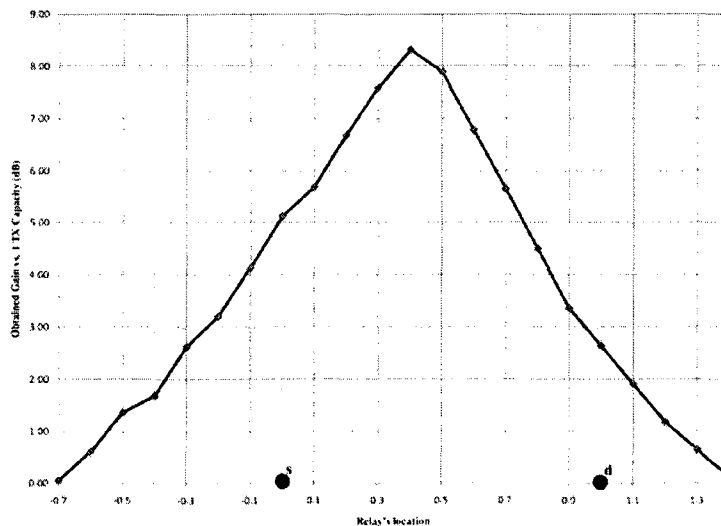


Figure 4.11 Gain vs. Non-Collaborative Single Transmitter Capacity (Path Loss Exponent 4)

In Figure 4.12 we show the geometrical loci of the relay to achieve a specific amount of gain when compared to a non-collaborative system.

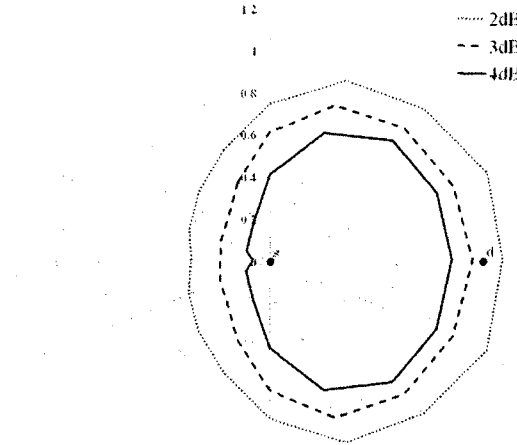


Figure 4.12 Constant Gain Contours (Path Loss Exponent 4)

## 4.5 Conclusion

In this chapter we presented the simulation parameters of our system in details. We compared the frame error rate performance of several codes with different  $d_{free}$  in our collaborative protocol with the non-collaborative single transmitter outage capacity and the collaborative outage capacity obtained in [34]. We demonstrated that our system achieves full diversity without using an STC in the second phase. It was seen that the performance of the system improves when the minimum free distance of the code increases. We have also provided the results when the relay takes different locations in the plane and presented the fixed gain contours and areas that guarantee a minimum gain

when compare with the non-collaborative single transmitter system. In the next chapter we will analyze the effect of knowledge of channel at the transmitter side by assuming that the destination sends a feedback containing the channel state information to the transmitters (the source and the relay). We will try to minimize overall transmitted power while maintaining the same performance (FER) by using that feedback.

## **CHAPTER FIVE**

### **5 Simulation Results for the System with Feedback**

In this chapter, we will introduce two feedback scenarios. In the first case, the source-destination and relay-destination channel state information is available only at the relay. In the second scenario, both the source and the relay know the channel state information. This assumes a feedback channel between the destination and the relay or both the source and the relay. Then we will provide Monte Carlo simulation results showing the improvement in the performance of our collaborative protocol in a system where the channel state information is at least partially available at the transmitters' side.

#### **5.1 CSI Available at Relay Only**

We start with a simple feedback scenario. In this mode, we assume that only the relay receives the channel state information from the destination, therefore, the source

does not have any knowledge about its transmission channel. In this scenario the relay decides about its collaboration mode at the beginning of each frame. It will only decode and forward the source message if its collaboration during this frame can improve the frame error rate performance of the system.

### 5.1.1 Modes of Collaboration

In this scenario there are three different modes of collaboration. The modes of collaboration are the same as those in a system with no knowledge of channel at the relay. But the frequency of their occurrence is different. The modes of collaboration for this scenario are plotted in Figure 5.1.

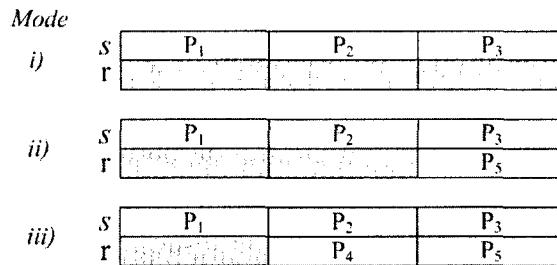


Figure 5.1 Collaboration Modes in Partial Feedback Scenario

When the relay has no knowledge about source-destination and relay-destination channels, the best it can do is to start transmitting the extra parities as soon as it can decode the source message. But there are times that the relay is able to decode the source message in time and also the quality of the source-destination channel is good enough for the destination to decode the message with a frame error rate less than the required value. Even when the source-destination channel is not good, it is possible that the relay is able

to decode the source message after the first block and hence it will transmit two parity blocks but the relay-destination channel is so good that the destination can decode the message after receiving only one block from the relay. In these situations, the relay is wasting some energy transmitting extra parity bits.

If the relay knows the source-destination and the relay-destination channel state information, it can calculate the received signal power from the source and the relay at the destination. In addition if the relay knows the frame error rate performance of different rates of the channel code being used in the system, it can evaluate the probability of error in decoding a frame at the destination, when the system is working in each of the three collaboration mode. Then the relay can decide whether its collaboration is needed or not, and if it is needed how many blocks should be transmitted in order to achieve the required frame error rate. Therefore, the system can achieve the same performance at a lower energy level.

### **5.1.2 Decision Criteria**

As mentioned in the previous section, the relay needs to know the frame error rate performance of each rate of the code (each mode of collaboration) at different received signal to noise ratios to decide on the collaboration mode. In this chapter the 8-state code which was used in the previous chapter is used. In mode  $i$  the relay does not transmit and the source transmits with rate  $1/3$ , hence we can present the code in octal form as  $(15, 17, 15)_{\text{octal}}$ . Figure 5.2 shows the achieved frame error rate versus the received bit SNR. Since the rate of the code is  $1/3$  the overall received SNR is 3 times (4.77dB) more than



the value depicted on the horizontal axis. It will be shown that, to be able to easily compare different modes and decide about the collaboration, it is more practical to use the received bit SNR.

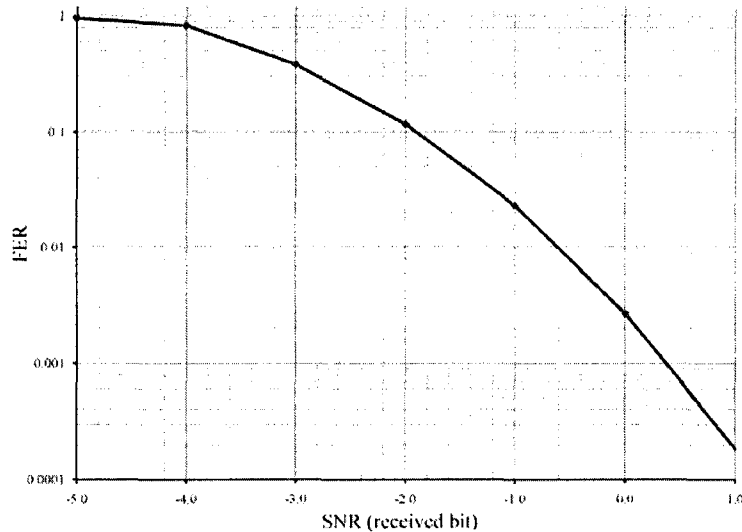


Figure 5.2 FER of the 8-State Code at Rate 1/3 (mode *i*)

In mode *ii* the source transmits with rate 1/3 and the relay transmits with rate 1, then the overall collaborative code can be written in octal form as  $(15, 17, 15, 17)_{\text{octal}}$  and the overall code rate is 1/4. As the relay and the source both transmit in this mode, the achievable frame error rate depends on two variables, received signal power from the source and the received signal power from the relay. Figure 5.3 shows the achieved frame error rate versus received bit SNR from the relay at the destination at different received bit SNR's from the source.

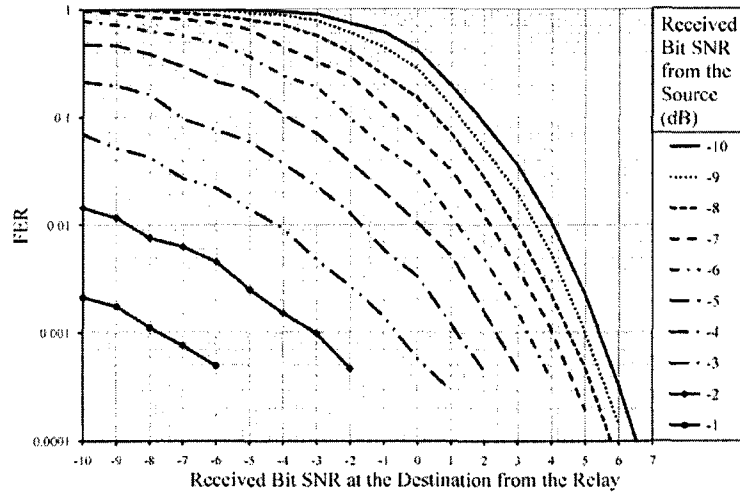


Figure 5.3 FER vs. received bit SNR from the relay at different received bit SNR from the source in mode *ii*

In mode *iii* the source transmits during all available three blocks (rate 1/3) and the relay transmits two blocks (rate 1/2), then the overall collaborative code has a rate of 1/5 and is written in octal form as  $(15, 17, 15, 15, 17)_{\text{octal}}$ . Figure 5.4 shows the achieved frame error rate versus received bit SNR from the relay at the destination at different received bit SNR's from the source.

Similar to the previous chapter we assume that a frame error rate of  $10^{-3}$  is required at the destination and we set the relay's decision criteria to match the same number. Using the above results we can plot  $\text{FER}=10^{-3}$  curves as in Figure 5.5. Note that the values on the two axes are linear (not dB).

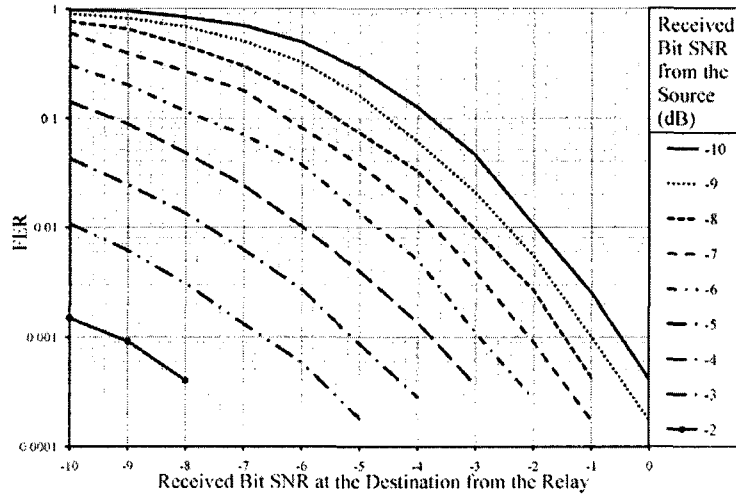


Figure 5.4 FER vs. received bit SNR from the relay at different received bit SNR from the source in mode *iii*

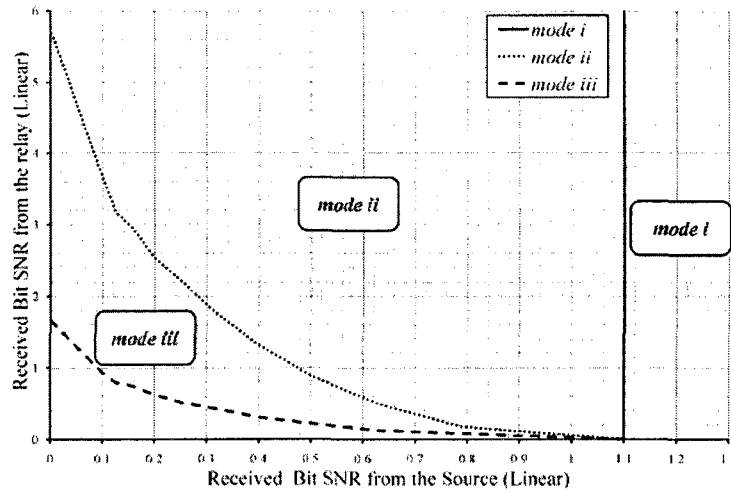


Figure 5.5 Collaboration mode areas when CSI is only available at the relay.

At the beginning of each frame, relay knows the relay-destination and the source-destination channel coefficients through the feedback it receives from the destination and can calculate the received signal power from each channel at the destination. These two values correspond to a point on Figure 5.5. During the transmission of the frame, the relay listens to the source till it can decode the message. Then the relay chooses an

available collaboration mode that guarantees a frame error rate less than  $10^{-3}$  and uses the least number of blocks for transmission. If the point falls below the  $\text{FER}=10^{-3}$  curve of mode *iii*, it will decide in favor of the highest possible mode.

The relay decides in favor of mode *i* either when its collaboration is not needed or when it cannot decode the source message reliably.

The relay decides in favor of mode *ii* when its collaboration is needed for only one block and it can reliably decode the source message after the first or the second block. Also when its collaboration is needed but the relay can only decode the source message after reception of the second block it has no option but to choose mode *ii*.

Finally the system will be in mode *iii* when the relay's collaboration is needed at the maximum possible rate and it can decode the source message without error after reception of the first block.

Here is an example. Let us assume that the relay can decode the source message after it receives the first block and the point corresponding to the transmission channel states is (1,3). If there was no feedback, the relay would start collaborating immediately and would transmit two parity blocks to the destination. However now the relay knows that even if it only transmits one block the probability of an error in the decoding of the frame at the destination is still less than  $10^{-3}$  but if it does not transmit that single block then the probability of a frame error will be higher than  $10^{-3}$ . Therefore it will decide on mode *ii*.

In the next section, the second feedback scenario will be explained and then the simulation results for both scenarios will be presented and compared with the results obtained in the previous chapter.

## **5.2 CSI available at Both Source and Relay**

Now we extend our feedback scenario to a case where both source and relay have access to the channel state information through a feedback channel. In this scenario we assume that both source and relay are aware of the source-destination, the relay-destination and the source-relay channel states. Here both the source and the relay decide about the collaboration mode. They both try to achieve the required frame error rate while minimizing the energy consumption by optimizing the number of transmitted blocks in each frame.

### **5.2.1 Modes of Collaboration**

Since not only the relay but also the source has access to all channel coefficients, there are several modes of transmission/collaboration in this scenario than the previous one. In Figure 5.6, all eight collaboration modes are presented. One may find other possible combinations but since we assume quasi-static fading (the channel state information remains constant for the entire frame), all of those combinations can be modeled as one of the depicted modes. It should be noted that the mode names in this section are not the same as the previous section, i.e., mode  $i$  here is different than mode  $i$  in the previous section.

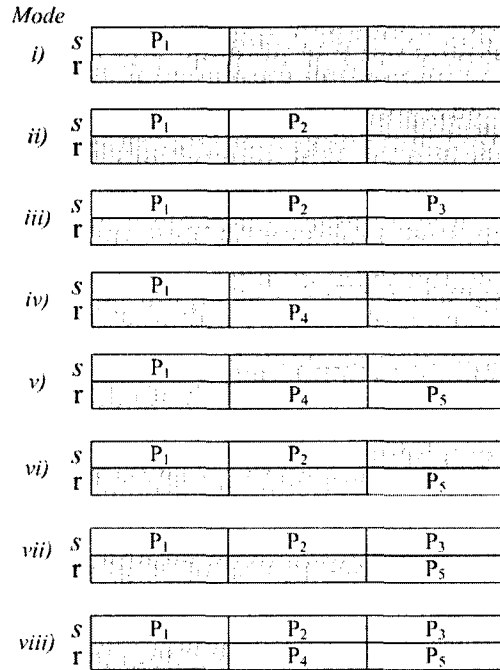


Figure 5.6 Collaboration modes when both source and relay have access to CSI

In no CSI scenario discussed in the previous chapter the source had no knowledge about source-destination, source-relay and relay-destination channels. So, the best it could do was to transmit all three blocks. However, in this scenario the source-destination channel could be in a considerably good state that the destination is capable of decoding the source message within the required frame error rate after reception of one or two blocks. Even when this is not the case, the source-relay and the relay-destination channels may be able to provide a reliable path for transmission at a lower energy level while satisfying the frame error rate requirement.

If both source and relay know the channel state information they both can calculate the received signal powers at the destination. In addition if they both have knowledge of the frame error rate performance of the system's variable rate channel code

in every possible collaboration mode, they can select the mode that guarantees the quality criterion and has the minimum number  $f$  blocks. Hence the amount of energy used to transmit that frame is reduced, without compromising the link reliability.

### 5.2.2 Decision Criteria

To be able to compare the results, we use the same 8-state trellis encoder that was explained in Chapter 4 and change the code rate by adding or dropping the outputs. In modes *i*, *ii* and *iii* the relay does not transmit. They represent the cases when either the source-destination channel is in a good state or the relay cannot decode the source message without error even after receiving two blocks from relay. In mode *i*, source transmits with rate 1 (actual rate 127/130). Figure 5.7 shows the achieved frame error rate versus the received SNR. In mode *ii*, source transmits with rate (1/2). The trellis code can be presented in octal form as (15, 17)<sub>octal</sub>. The FER performance of the code is depicted in Figure 5.8. It should be noted that the numbers on the horizontal axis are received bit SNR, and since the code rate is (1/3) the overall received SNR is 2 times (3dB) more than the presented value.

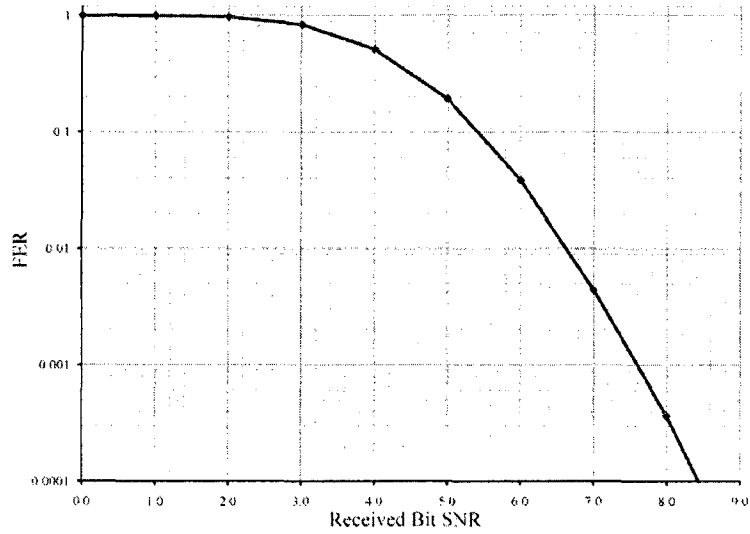


Figure 5.7 FER of Rate 1(mode *i*)

Mode *iii* is similar to mode *i* of the previous section. In this mode the source transmits with rate  $(1/3)$  and the code can be written in octal form as  $(15, 17, 15)_{\text{octal}}$ . The FER graph of this mode can be found in Figure 5.2.

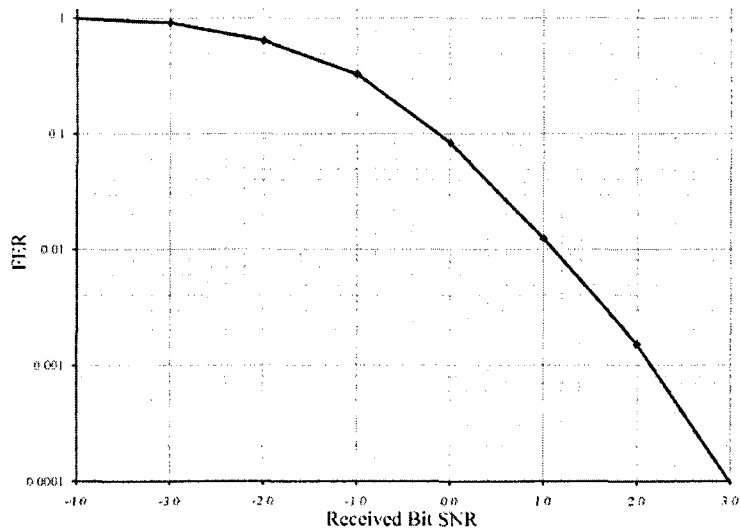


Figure 5.8 FER of the 8-State Code at Rate  $1/2$  (mode *ii*)



In mode  $iv$  source transmits one block and then the relay transmits another block. The overall code rate is  $(1/2)$  and the code can be shown in octal form as  $(15, 17)_{\text{octal}}$ . As the destination receives each block through a different channel, the frame error rate of the code depends on two received signal to noise ratios. Figure 5.9 shows the achieved FER versus received bit SNR from the relay at the destination at different received bit SNR's from the source.

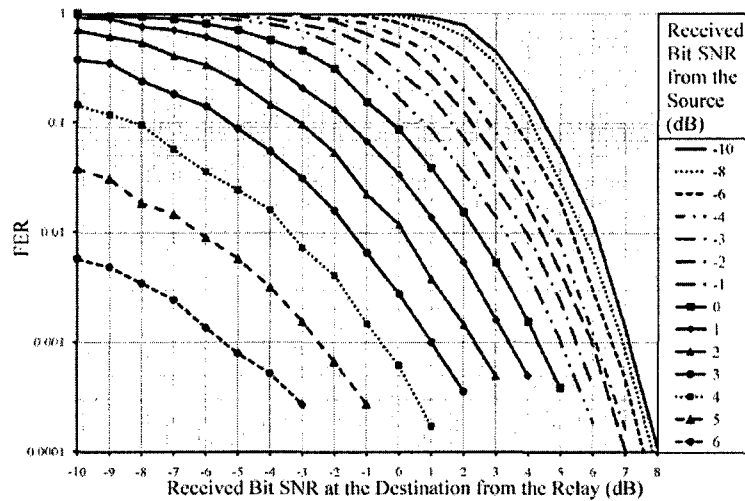


Figure 5.9 FER vs. received bit SNR from the relay at different received bit SNR from the source in mode  $iv$

In mode  $v$  the source transmits one block and the relay transmits two blocks. The overall code rate is  $1/3$ . The code can be written in octal form as  $(15, 17, 15)_{\text{octal}}$ . Figure 5.10 shows the achieved FER versus received bit SNR from the relay at the destination at different received bit SNR's from the source.

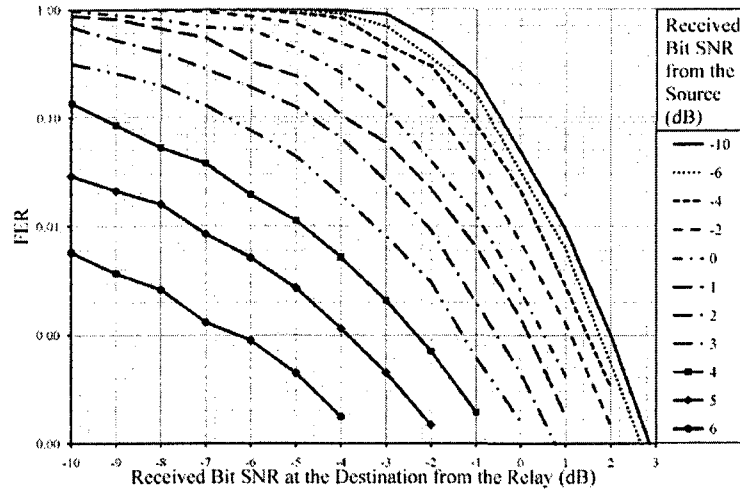


Figure 5.10 FER vs. received bit SNR from the relay at different received bit SNR from the source, mode  $v$

In mode  $vi$  the source transmits two blocks and the relay transmits only one block. The overall code rate is still  $1/3$ . The code can be written in octal form as  $(15, 17, 15)_{\text{octal}}$  but the second parity in this mode is being transmitted by the source instead of the relay as in mode  $v$ . Figure 5.11 shows the achieved FER versus received bit SNR from the relay at the destination at different received bit SNR's from the source.

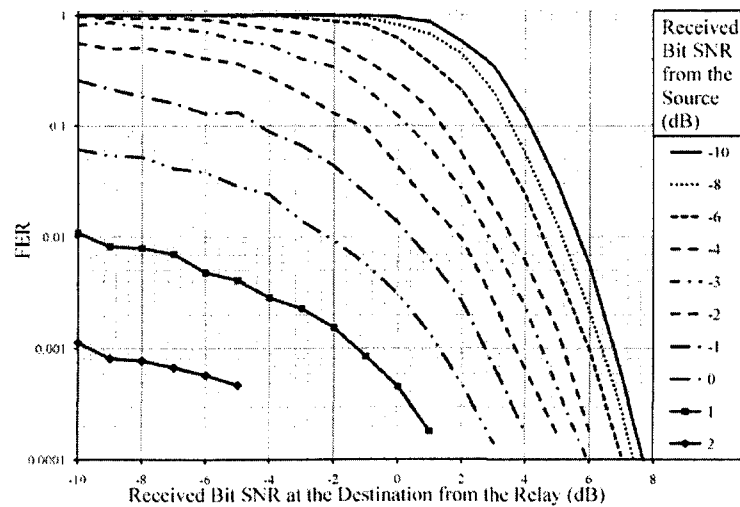


Figure 5.11 FER vs. received bit SNR from the relay at different received bit SNR from the source, mode  $vi$

Finally, modes *vii* and *viii* are exactly identical to modes *ii* and *iii* in the previous section when CSI was only available to the relay. The overall code rates are 1/4 and 1/5 and the codes can be presented in octal form as  $(15, 17, 15, 17)_{\text{octal}}$  and  $(15, 17, 15, 15, 17)_{\text{octal}}$ . The frame error rate performance of mode *vii* can be found in Figure 5.3 and the one for mode *viii* in shown in Figure 5.4.

Similar to the previous section, we use these results to plot  $\text{FER}=10^{-3}$  curves. Note that the values on the two axes are linear (not dB). Since there are mode collaboration modes, there are more curves in Figure 5.12 than in Figure 5.5.

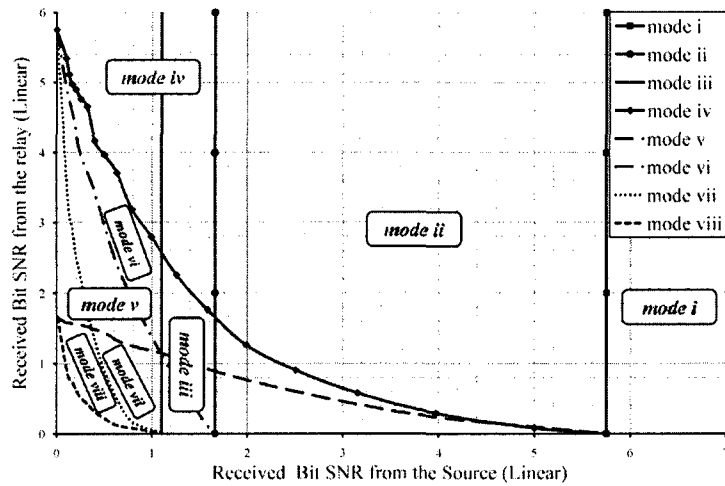


Figure 5.12 Collaboration mode areas when CSI is available at both source and relay.

At the beginning of each frame both source and relay receive channel state information through the feedback channel. They calculate the received signal power from each channel at the destination as well as the received signal power at the relay from the source. Then they use the data provided in Figure 5.12 to choose the collaboration

mode. They select an available collaboration mode that provides a frame error rate equal or less than  $10^{-3}$  and uses minimum number of blocks.

They decide in favor of mode  $i$  when the probability of error in decoding a frame at the destination is less than the required value after receiving only one block. This is when the points is located on the right side of the boundary of mode  $i$ .

Mode  $ii$  will be selected when the destination needs to receive two blocks from the source to decode the message reliably. This happens when the point in between the boundaries of mode  $i$  and  $ii$ .

Mode  $iii$  is used when the relay cannot decode the source message after the first block but the destination can do it when it receives all three blocks from the source. This is when the point is between the boundaries of mode  $ii$  and mode  $iii$  and below the boundary of mode  $iv$ .

They decide in favor of mode  $iv$ , when the destination cannot decode the source message reliably by receiving two blocks but it can do so if it receives one block from the source and one from the destination. This happens when the point is located to the left of the boundary of mode  $ii$  and above the boundary of mode  $iv$ .

Mode  $v$  is selected when the source-destination channel is bad but the relay can decode the source message after the first block and the destination can decode the message if it receives two blocks from the relay. This is when the source-relay channel rate is one and we are located below the boundary of mode  $iv$ , above that of mode  $v$  and to the left of the boundary of mode  $ii$ .

The system will choose mode  $vi$  when the source-destination channel is not good but the relay-destination channel is in a good state and the relay needs to receive two blocks from the source to decode its message reliably. This will happen when the point falls below the boundary of mode  $iv$ , above that of mode  $vi$  and to the left of the boundary of mode  $iii$ . Note that this mode will not occur if the relay can decode the source message after receiving only one block.

Mode  $vii$  is used when the relay collaboration is needed but only for one block or if it cannot collaborate for more than one block. This is below the boundary of mode  $v$  above that of mode  $vii$  and to the left of mode  $iii$ . And if the relay needs to receive two blocks from the source to decode its message it will be just below the boundary of mode  $v$  and to the left of mode  $iii$ .

Finally mode  $viii$  is used if none of the above cases is true.

### 5.3 Results

In Figure 5.13 the frame error rate performance of the two feedback scenarios are compared with the results obtained in the previous chapter when CSI was not available at the transmitters. Consistent to the previous chapter, we define an outage event as an error in the decoding of a frame and compare the simulated frame error rate curves to the outage probability of a single transmitter as well as the modified outage probability of the collaborative scheme presented in [34]. To obtain the results shown in the figure, we assumed that the relay is located on the line connecting the source and the destination and

its distance to source is 0.3 ( $D_{sr} = 0.3$ ). Note that the latter two curves are drawn based on no-feedback assumption. It can be seen that both feedback scenarios achieve the diversity advantage. At  $FER=10^{-3}$  the first feedback scenario outperforms the no-feedback scheme by almost 2dB. Further, when both source and relay have access to the channel state information (second feedback scenario), the performance of the system improves for about 5.5dB at high SNR.

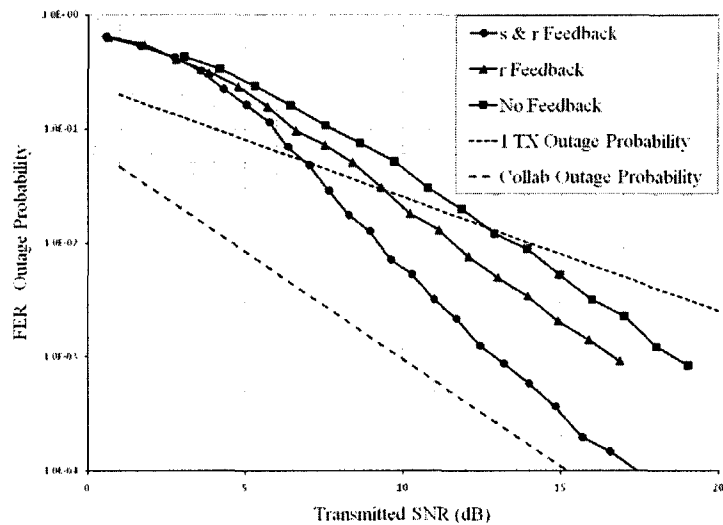


Figure 5.13 FER performance of different feedback scenarios (the relay is located at  $d=0.3$  between source and destination)

Now, we set the source location to  $(0,0)$ , the destination location to  $(1,0)$  and moved the relay on the plane by changing  $D_{sr}$  with increments of 0.1 and  $\theta$  with steps  $\pi/8$ . For each location of the relay we have obtained the performance of each of the feedback schemes through Monte Carlo simulation. Figure 5.14 shows the achieved gain of the two feedback and the no-feedback scenarios versus a single transmitter scenario at outage probability of  $10^{-3}$ , when the relay moves on the line connecting the source and

the destination. The single transmitter outage probability is obtained assuming the same distance, path gain and fading coefficients between the source and the destination.

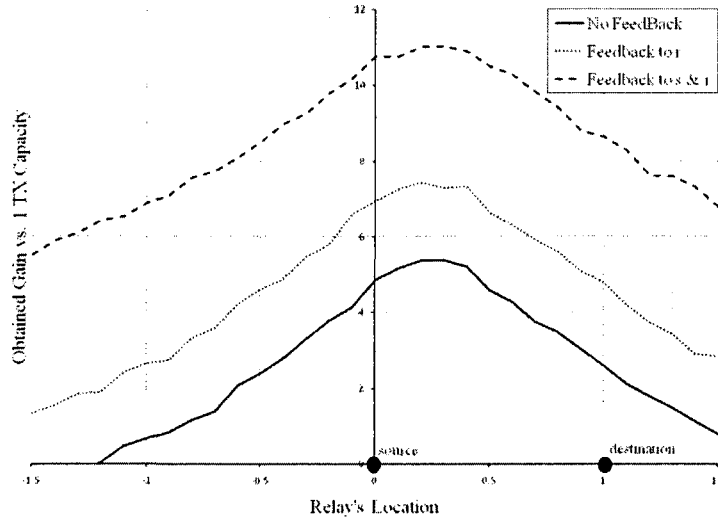


Figure 5.14 Gain vs. Non-Collaborative Single Transmitter Capacity with different feedback scenarios

Figure 5.15 show constant gain contours, the geometrical loci of the relay, to achieve a specific amount of gain when compared to a non-collaborative system for the feedback scenario where CSI is available only at the relay. Figure 5.16 shows the constant gain contours for the feedback scenario where CSI is available both at source and relay. The numbers written on the contours show the minimum achieved gain versus a single transmitter, when the relay is located inside that each contour. For example in Figure 5.16, if there is a relay located in the 8dB contour and the system is operating under the collaborative scheme where the CSI is available at source and relay, the overall required power to achieve a frame error rate of  $10^{-3}$  is 8dB less than a single transmitter with no knowledge of the channel.

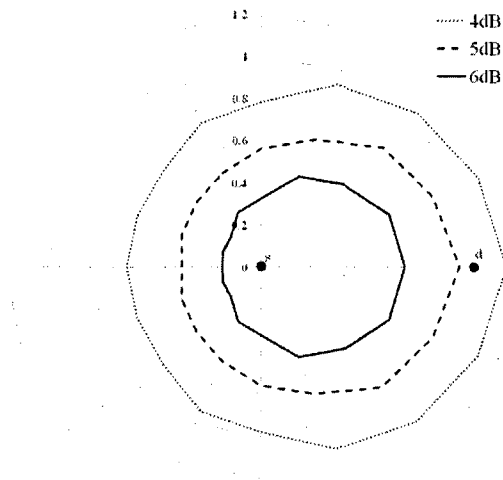


Figure 5.15 Constant gain contours, CSI available at relay

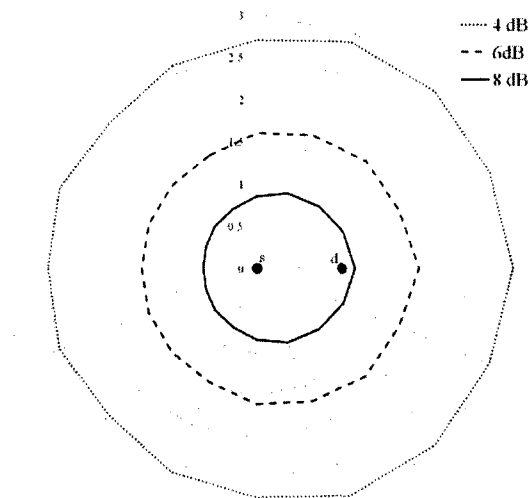


Figure 5.16 Constant gain contours, CSI available at both source and relay

Finally in Figure 5.17, the 4dB constant gain contours of the two feedback scenarios are compared with the no-feedback setting of Chapter 4. It can be seen that,



when CSI is available at both source and relay, the contour area is approximately 25 times of the contour obtained in previous chapter for the no-feedback scenario.

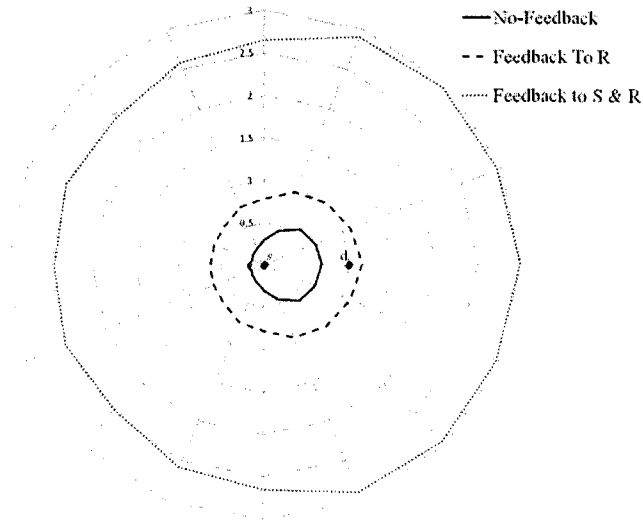


Figure 5.17 Comparison of 4dB gain contours in different feedback scenarios

## 5.4 Conclusion

In this chapter we presented two different feedback scenarios. In the first scenario only the relay is partially or fully aware of its transmission channel state while in the second scenario full or partial knowledge of channel state information is available both at the source and the relay. After that we proposed variable time-fraction collaborative protocols for each feedback scenario. Finally, we used the same channel code used in the previous chapter and presented Monte Carlo simulation results for the two scenarios and compared the results with the results obtained in the previous chapter.

## **CHAPTER SIX**

### **6 Concluding Remarks**

#### **6.1 Conclusion**

In this thesis we have developed and studied a new practical protocol for collaborative communication in fading environments. We designed our protocol assuming no knowledge of channel state information at the transmitter side (source and relay). Our variable time-fraction protocol, unlike the protocols presented in previous works, does not impose a fixed duration on the exchange and collaboration phases. Therefore, it benefits from all available diversity gain (number of transmitting nodes) in the network, while improving the coding gain of the system. The improvement in the coding gain is particularly considerable when the relay is relatively close to the source. In these situations in our system, the relay can understand the source message in a small fraction of the transmission time and can collaborate with the source during the rest of the frame, while in a fixed time-fraction protocol, it has to listen to the source for usually half

of the time and only collaborate, if possible, during the next half. On the other hand, the relay may need more time (receive more information) to decode the source message, but the fixed time-fraction will not allow this and the system will not benefit from the available diversity advantage in the network. However, in our collaborative protocol, the relay can adjust the length of the exchange phase.

In Chapter 3, we presented our communication network model and its parameters in details. We then explained our novel collaborative protocol as a practical method of cooperative diversity in wireless networks. The channel code selection and design criteria for our collaborative protocol are also presented in this chapter.

In Chapter 4, the simulation parameters of our system are presented in details. In this chapter the frame error rate of several codes with different minimum free Hamming distances in our collaborative protocol are compared with the non-collaborative single transmitter outage capacity and the collaborative outage capacity obtained in [34]. It is demonstrated that our collaborative protocol achieves full diversity without using an STC in the second phase. We have also showed that the performance of the system improves when the minimum free distance of the code increases. Fixed gain contours and areas that guarantee a minimum gain, compared to the non-collaborative single transmitter system are obtained via simulation. All of the mentioned results are obtained, assuming a path loss exponent  $\beta = 2$ . To evaluate our results in a different environment, we repeated the simulations for path loss exponent  $n = 4$  and showed that the results are pretty consistent in different environments.

In Chapter 5, we presented two simple feedback scenarios. In the first one, only the relay receives a feedback from the destination containing some information about the source-destination and relay destination channels. Using this information the relay tries to optimize its transmission, by re transmitting the source message only if it is necessary. Through simulation, we showed that this scheme can reach the performance of the no-feedback scenario at  $10^{-3}$  frame error rate, with 2 dB less SNR, while achieving the same diversity gain. In the second feedback scenario, both the source and the relay are aware of their transmission channel states through a feedback from the destination. This scheme also provides a full diversity advantage but the energy consumption of the system is dramatically improved, since the source and the relay will only transmit enough number of blocks to the destination to decode the message and avoid unnecessary transmission. It has been shown that this protocol can achieve the frame error rate of  $10^{-3}$  at almost 7 dB less signal to noise ratio. Finally, like Chapter 4, we obtained the constant gain contours for both of the feedback scenarios and compared them with the ones obtain for the no-feedback case.

## 6.2 Future Work

There are several ways to continue the research and obtain further important results in the framework of this thesis. We believe the following to be of the highest importance:

- One can analytically evaluate the performance of the proposed protocols, regardless of the specific selection of the channel codes. This can be done for both the non-feedback and feedback scenarios.

- In the second feedback scenario, where both source and relay have knowledge of their transmission channels, one can look into the code design algorithm and improve the coding gain of the system using the available information about the channel. This can be extended also to efficient power allocation between the source and the relay.
- In this thesis we only studied the performance of the system with three nodes (one relay). This work can be further extended to study and design of collaborative protocols in a network with several relay nodes. We believe that, the first step in this direction is the design of a proper relay selection protocol. The constant gain contours that are presented in this thesis can be of great help to design of such protocol.

## **Bibliography**

- [1] Foschini, G. J., "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas." Bell Labs Tech. J., 1996, Issue 2, Vol. 1, pp. 41-59.
  
- [2] G. J. Foschini, M. Gans., "On the limits on wireless communication in a fading environment when using multiple antennas." Wireless Personal Communications, March 1998, Vol. 6, pp. 311-335.
  
- [3] Telatar, E., "Capacity of multi-antenna gaussian Channels." European Transaction on Telecommunications, Nov./Dec. 1999, Issue 6, Vol. 10, pp. 585-595.
  
- [4] V. Tarokh, N. Seshadri and A.R. Calderbank., "Space-time codes for high data rate wireless communication: Performance criterion and code construction." IEEE Trans. Inform. Theory, March 1998, Issue 2, Vol. 44, pp. 744-765.
  
- [5] Jakes, W.C., "New Techniques for Mobile Radio." Bell Laboratory Record, December 1970, pp. 326-330.

- [6] J. N. Laneman, D. N. C. Tse, G. W. Wornell., "Cooperative diversity in wireless networks: efficient protocols and outage behavior." *IEEE Trans. Inform. Theory*, December 2004, Issue 12, Vol. 50, pp. 3062–3080.
- [7] A. Sendonaris, E. Erkip, and B. Aazhang., "User cooperation diversity, part I: System description." *IEEE Trans. Communications*, November 2003, Issue 11, Vol. 51, pp. 1927–1938.
- [8] A. Sendonaris, E. Erkip, B. Aazhang., "User cooperation diversity, part II: Implementation aspects and erformance analysis." *IEEE Trans. on Communications*, November 2003, Issue 11, Vol. 51, pp. 1939–1948.
- [9] J. N. Laneman, G. W. Wornell., "Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks." *IEEE Trans. Inform. Theory*, October 2003, Issue 10, Vol. 49, pp. 2415–2525.
- [10] T. E. Hunter, A. Nosratinia., "Performance analysis of coded cooperation diversity." 2003. *Int. Conf. on Comm. ICC03*. Vol. 4, pp. 2688–2692.
- [11] Tolga M. Duman, Ali Ghrayeb., *Coding for MIMO Communication Systems*. s.l. : John Wiley & Sons, 2007.
- [12] Rappaport, T.S., *Wireless Communications - Principles and Practice*. NJ : Prentice Hall, 1996.
- [13] David Tse, Pramod Viswanath., *Fundamentals of Wireless Communication*. s.l. : Cambridge University Press, 2005.

- [14] Shannon, C. E., "A mathematical theory of communication." Bell Systems Technical Journal, 1948, Vol. 27, pp. 379-423 and 623-656.
- [15] Kahn, L., "Ratio Squarer." Proceedings of IRE, November 1954, Vol. 42, p. 1074.
- [16] V. Tarokh, H. Jafarkhani, A. R. Calderbank., "Space-time block coding for wireless communications: performance results." Selected Areas in Communications, IEEE Journal on, March 1999, Issue 3, Vol. 17, pp. 451-460.
- [17] Alamouti, S., "A Simple Transmit Diversity Technique for Wireless Communications." IEEE Journal on Sel. Areas in Comm., October 1998, Issue 8, Vol. 16, pp. 1451-1458.
- [18] Lihong Zheng, D.N.C. Tse., "Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels." Information Theory, IEEE Transactions on, May 2003, Issue 5, Vol. 49, pp. 1073-1096.
- [19] Meulen, E. C. Van der., "Three-terminal communication channels." Advances in Applied Probability, s.l. : Applied Probability Trust, 1971, Issue 1, Vol. 3, pp. 120-154 .
- [20] T. Cover, A. E. Gamal., "Capacity theorems for the relay channel." IEEE Trans. on Information Theory, September 1979, Issue 5, Vol. 25, pp. 572-584.
- [21] L. L. Xie., P.R. Kumar., "A network information theory for wireless communication: Scaling laws and optimal operation." Information Theory, IEEE Transactions on, 2004, Issue 5, Vol. 50, pp. 748-767.



- [22] L. Sankaranarayanan, G. Kramer, and N. B. Mandayam., "Capacity theorems for the multiple-access relay channel." Allerton, IL : s.n., 2004. 42nd Annual Allerton Conference on Communication, Control and Computing.
- [23] Chris T. K. Ng, Andrea J. Goldsmith., "Capacity gain from transmitter and receiver cooperation." Adelaide, Australia : s.n., 2005. IEEE International Symposium on Information Theory (ISIT). pp. 397- 401.
- [24] G. Kramer, M. Gatspar, and P. Gupta., "Cooperative strategies and capacity theorems for relay networks." IEEE Trans. Information Theory, September 2005, Issue 9, Vol. 51, pp. 3037–3063.
- [25] T. E. Hunter, A. Nosratinia., "Diversity Through Coded Cooperation." IEEE Trans. on Wireless Comm., Feb 2006, Issue 2, Vol. 5, pp. 283-289.
- [26] T. Hunter, S. Sanayei and A. Nosratania,., "Outage Analysis of Coded Cooperation." IEEE Trans. on Info. Theory, Feb 2006, Issue 2, Vol. 52, pp. 375-391.
- [27] P. A. Anghel, G. Leus, M. Kaveh., "Distributed Space-Time Cooperative Systems with Regenerative Relays." Wireless Communications, IEEE Transactions on, November 2006, Issue 11, Vol. 5, pp. 3130 - 3141.
- [28] Y. Jing, B. Hassibi., "Distributed Space-Time Coding in Wireless Relay Networks." Wireless Communications, IEEE Transactions on, December 2006, Issue 12, Vol. 5, pp. 3524 - 3536.

- [29] Seddik, Karim G., Sadek, Ahmed K. and Liu, K. J. Ray., "Protocol-Aware Design Criteria and Performance Analysis for Distributed Space-Time Coding." 2006. Global Telecommunications Conference, IEEE .
- [30] Jing, Y. and Jafarkhani, H., "Distributed differential space-time coding for wireless relay networks." *Communications, IEEE Transactions on*, July 2008, Issue 7, Vol. 56, pp. 1092-1100.
- [31] Tourki, K. and Alouini, M.-S. and Deneire, L., "Blind Cooperative Diversity Using Distributed Space-Time Coding in Block Fading Channels." 2008. *Communications, 2008. ICC '08. IEEE International Conference on*. pp. 4596-4600.
- [32] Seddik, K.G., et al., "Design Criteria and Performance Analysis for Distributed Space-Time Coding." *Vehicular Technology, IEEE Transactions on*, July 2008, Issue 4, Vol. 54, pp. 2280-2292.
- [33] Ochiai, H. and Mitran, P. and Tarokh, V., "Variable-Rate Two-Phase Collaborative Communication Protocols for Wireless Networks." *Information Theory, IEEE Transactions on*, September 2006, Issue 9, Vol. 52, pp. 4299-4313.
- [34] P. Mitran, H. Ochiai and V. Tarokh., "Space-Time Diversity Enhancements Using Collaborative Communications." *IEEE Trans. On Info. Theory*, june 2005, Vol. 51, pp. 2041-2057.
- [35] P. Toohar, H. Khoshnevis, M. R. Soleymani., "Design of Collaborative Codes Achieving Space-Time Diversity." s.l. : IEEE, 2007. *ICC 2007*. pp. 5819-5824.

- [36] S. Lin, D. Costello Jr., *Error Control Coding*. NJ : Prentice-Hall, 1983.
- [37] Leon-Garcia, Alberto., *Probability and Random Processes for Electrical Engineering*. s.l. : Addison-Wesley, 1994.
- [38] Proakis, John G., *Digital Communications*. Fourth. s.l. : McGraw-Hill, 2001.
- [39] T. Cover, J. Thomas., *Elements of Information Theory*. s.l. : John Wiley Inc., 1991.

## APENDIX

### A Note on the Stop Criteria in the Simulations

Here we want to find the minimum number of error events we need to observe in our Monte Carlo simulations to make sure that the obtained performance is in a required distance to the real performance. We use the sample mean to obtain an estimate of the expected value of the number of error events,

$$\bar{e} = \frac{1}{n_b} \sum_{i=1}^{n_b} e_i = \frac{n_e}{n_b}, \quad (1)$$

where  $e \in \{0,1\}$  and  $e_i = 1$  represent an error event,  $n_b$  in the number of transmitted information bits and  $n_e$  is the number of error event. We know that if  $n_b \rightarrow \infty$  the above statistical value will converge to the real mean ( $\mu$ ). Here we need to find the proper value of  $n_b$  which guarantees that the statistical mean is close enough to the real mean. As the statistical mean is the sum of a large number of independent identically distributed random variables, we can consider it as a Gaussian random variable (central limit theorem). Then, we have

$$P(-z_{\alpha/2} < Z \leq z_{\alpha/2}) = \int_{-z_{\alpha/2}}^{z_{\alpha/2}} e^{-z^2/2} / \sqrt{2\pi} dz = 1 - \alpha, \quad (2)$$

where  $Z$  is normally distributed random variable with zero mean, unit variance and can be written as

$$Z = \frac{\bar{e} - \mu}{\sigma / \sqrt{n_b}}. \quad (3)$$

Therefore we have

$$P(\bar{e} - z_{\alpha/2} \sigma / \sqrt{n_b} < \mu \leq \bar{e} + z_{\alpha/2} \sigma / \sqrt{n_b}) = 1 - \alpha \quad (4)$$

We can see that with probability  $1 - \alpha$  the error in the sample mean ( $|\mu - \bar{e}|$ ) is smaller than  $z_{\alpha/2} \sigma / \sqrt{n_b}$ . The values for  $z_{\alpha/2}$  can be found in [37] page 290 for different given probabilities  $1 - \alpha$ . In all of the simulations in this thesis we used a 95% confidence interval ( $1 - \alpha = 0.95$ ) and a 10% tolerance ( $\varepsilon = 0.1$ ) on the  $\bar{e}$ . Therefore we have  $z_{\alpha/2} = 1.96$ . We want

$$\varepsilon \bar{e} \geq \frac{z_{\alpha/2} \sigma}{\sqrt{n_b}}. \quad (5)$$

We assume that the number of error events,  $n_e$ , is large enough that sample variance ( $\sigma_{stat}^2$ ) can be used as the real variance ( $\sigma^2$ ). We can write the statistical variance as

$$\sigma_{stat}^2 = \sum_{i=1}^{n_b} \frac{(e_i - \bar{e})^2}{n_b - 1}. \quad (6)$$

Using the fact that  $e \in \{0,1\}$ , we have

$$\begin{aligned}
 \sigma_{stat}^2 &= \sum_{i=1}^{n_e} \frac{(1-\bar{e})^2}{n_b-1} + \sum_{i=1}^{n_b-n_e} \frac{(\bar{e})^2}{n_b-1} \\
 &= \frac{(1-\bar{e})^2}{n_b-1} n_e + \frac{(\bar{e})^2}{n_b-1} (n_b - n_e) \\
 &= \frac{n_e - 2n_e\bar{e} + n_b\bar{e}^2}{n_b-1}
 \end{aligned} \tag{7}$$

Now we can use (7) in (6)

$$\sqrt{\frac{n_b(n_e-1)}{n_b-n_e}} \geq \frac{z_{\alpha/2}}{\varepsilon} = 19.6. \tag{8}$$

By approximating  $n_e$  by  $P_e n_b$  we get

$$n_b \geq 19.6^2 \frac{(1-P_e)}{P_e} + 1. \tag{9}$$

Which for small  $P_e$  can be approximated by

$$n_b \geq \frac{19.6^2}{P_e} \tag{10}$$

Or

$$n_e \geq 384.16 \tag{11}$$

Therefore, for a bit error rate of  $10^{-5}$ , we need to observe at least 385 error events.