Integrating Capacitated Lot-Sizing and Lot Streaming In Flowshop Schedules with Time Varying Demand

Alipasha Bayat

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ABSTRACT

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Any reasonable production planning contains three important decisions on lot size, lead time, and capacity. The common approach in the literature is to divide the planning problem into lot sizing, lot sequencing, and lot splitting sub-problems. Very few studies, to the best of our knowledge, have been conducted on the interdependencies and three-way interaction of lead-time, lot size, and actual capacity usage. A particular lot size calculated by the sub-problem method, however, will likely yield an infeasible solution or at least result in schedule instability (nervousness). This is just because in most capacitated lot sizing models, the capacity constraints in the model only take into consideration the available time on each work station, ignoring the sequencing of lots, sublot sizes, and their effect on makespan and lead times. In this thesis we bridge the gap between lot sizing and scheduling in flowshops, and examine the use of the lot splitting and sequencing techniques to reduce schedule instability. A mixed integer programming formulation is presented, which enables us to simultaneously find the optimal lot sizes as well as the corresponding sublot sizes and sequence of jobs. With this model, small size problems can be solved within a reasonable time. The computational results confirm that this model can be advantageous in dampening the scheduling nervousness. For larger size instances, a Genetic algorithm is proposed.

Acknowledgements

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List of Acronyms

CDS	Campbell, Dudek, and Smith Algorithm
CLSP	Capacitated Lot Sizing Problem
CLSLP	Capacitated Lot Sizing and Loading Problem
Cmax	Minimizing the Maximum Makespan
EOQ	Economic Order Quantity
Fm	Flow shop with m Machines
GA	Genetic Algorithms
LFL	Lot For Lot
LUC	Least Unit Cost
MILP	Mixed Integer Linear Programming
MIP	Mixed Integer Programming
MRP	Material Requirement Planning
MSP	Master Schedule Plan
NEH	Nawaz, Enscore, and Ham algorithm
NP-hard	nondeterministic polynomial-time hard
PPB	Part-Period Balance
SA	Simulated Annealing
$ST_{si,b}$	Sequence Independent Setup Times for sublots
TRC	Total Relevant Cost
TRCUT	Total Relevant Cost per Unit of Time
TS	Tabu Search

ULS	Uniform Lot Size
VLS	Variable Lot Size
WIP	Work In Process

CHAPTER 1

INTRODUCTION

1.1 Motivation

The aim of lot-sizing research is to minimize setup and holding costs by determining production lot size and inventory levels to meet a given demand. Lot streaming (Biskup and Feldmann, 2006), on the other hand, aims to divide lots into sublots in order to allow overlapping process in a multi-stage production system to accelerate the flow of material and reduce the lead-time.

The common approach in the literature (known as the "hierarchical approach") is to divide planning problem into lot sizing, lot splitting, and sequencing sub-problems. First, using the master production plan, supported by bill of materials, and production structure, the lot size for each product or part would be found. Then, after orders are released to shop floor, lot splitting and sequencing techniques will be used to find the optimal production sequence and sublot sizes. A particular lot size calculated by this common practice might, however, gives an infeasible, or at least a poor solution in respect to other aspects such as work in process (WIP), inventory holding cost, and set up costs. This

infeasibility forces the planner to frequently change the master production plan which results in schedule instability (nervousness). This is because most capacitated lot sizing models have four major unrealistic assumptions:

First, conventional capacitated lot sizing models, also known as CLSP, assume that the time spent by each lot on each work center is small and negligible. This only makes physical sense when a single item transfers from one work center to another (lots of size one). However, when the production is by lots, the processing time of a lot depends on the size of the lot and may not be negligible. For example, the time spent on machine 1 by a lot of "*n*" items with "*P*" being the processing time is "n*P". Additionally, machine 2 would be idle for "n*P" units of time since the first job scheduled on machine 2 cannot start until it is completed on machine 1. Thus, the capacity available for production depends on the size of the production sublots.

Second, in most capacitated lot sizing models, the capacity constraints are nothing more than the available time of each machine. Again, this assumption is only valid, if there is zero idling time. In a flow shop in which production is done in lots, assuming zero idling times means ignoring the effects of sequencing and sublot sizes, on makespan and lead time.

Third, in presence of setup times, the limited capacity available for production may be reduced by changeovers, causing sequence- dependent setup times. Lot sizing and capacity usage therefore depend on both the sequence and the size of the sublots.

Fourth, conventional capacitated lot-sizing models consider the lead time as a fixed and controllable input data, when it is really an output of other decisions, especially lot sizing and sequencing (Lasserre, 2003).

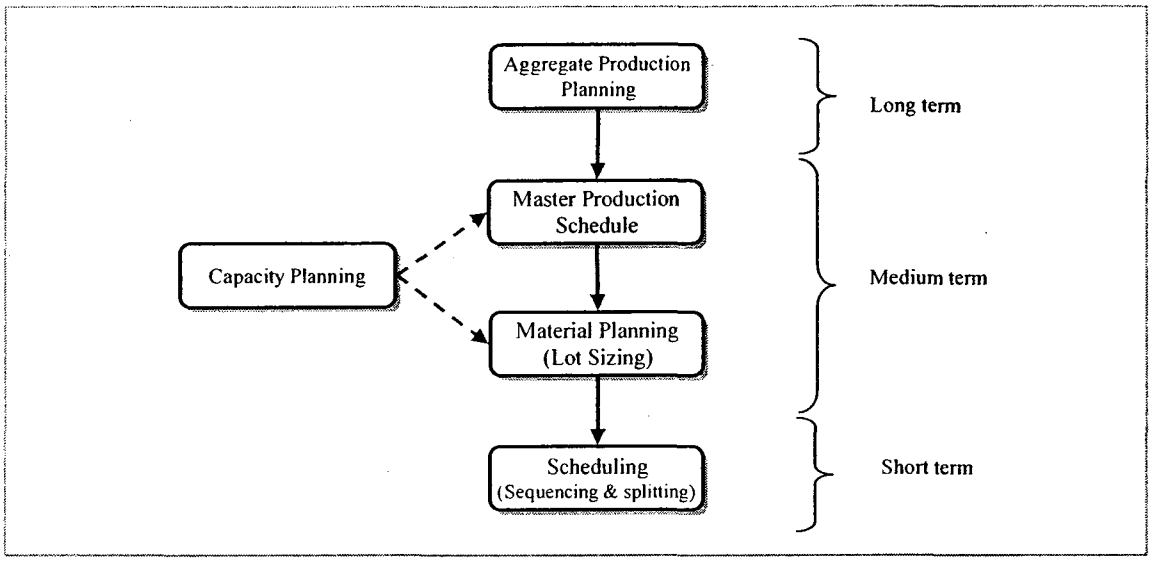

Figure 1-1: Hierarchical Planning Approach

Making any of the above assumptions would result in failure to plan capacity and in scheduling instability. Scheduling instability is a severe problem in any production planning system. It can be caused by one of the following reasons:

- Uncertain events occurring within or outside the production system such as forecast errors and machine breakdowns,
- Operating factors such as lotsizing rules and capacity utilization

Traditionally, there are two options for dealing with instability: first to maintain safety stock, safety lead time and safety capacity, and second to change the master production

schedule (MPS). There is a large cost associated with first option, and frequent rescheduling leads to system instability.

Ho (1998) concluded that although forecast errors have impact on instability, use of an appropriate lot sizing rule may neutralize the negative impact of forecast errors. He also examined dampening effect of different lot sizing rules (Ho, 2005). He concluded that part period balancing rule and least total cost rule turn out to be two effective lot sizing rules to reduce system nervousness (Ho, 1986; Ho and Law, 1995; Ho, 2005).

1.2 Objectives and Contributions

In this thesis we focus on the problem of finding the optimal lot size for multiple items with time varying demand in a multistage production system, when lot streaming and sequencing are applied simultaneously to ensure order's feasibility and reduce the need for rescheduling of the master production plan. The main contribution of this thesis is the integration of sequencing and scheduling decisions with lot sizing. We make our decision on the sequence of production and the sublot sizes simultaneously in advance, thus, we have better schedule because we know the idling times of machines and we can fill them with jobs. Moreover this can help us to have better estimate of the lead times. In order achieve our goal we propose the following:

- Formulated the problem as a mixed integer linear programming problem model
- Developed a genetic algorithm (GA)
- Generated random problems to evaluate and compare the linear model and genetic algorithm

1.3 Overview of this Thesis

Chapter 2 is an introduction to existing literature on the three subjects of lot sizing, scheduling, and lead time. It introduces some of the works done on the integration of these three subjects. This literature review focuses mainly on the flowshop problem.

Chapter 3 presents a mixed integer linear formulation of the problem. Later in Chapter 5 this exact solution will be used as a benchmark for evaluating the genetic algorithm.

Chapter 4 provides a study on design of genetic algorithms (GA), their design factors, and their parameters. Then, it presents our proposed genetic algorithm to solve the lot sizing problem (combined with scheduling) in more detail. It also presents numerical examples and comparison with the exact solution presented in chapter 3 in order to illustrate the efficiency of genetic algorithm.

Chapter 5 is the summary of this thesis and provides some direction for future research works.

CHAPTER 2

LITERATURE REVIEW

The three research areas of lot sizing, scheduling (sequencing and lot streaming), and lead time have direct impacts on this study. The literature on the above mentioned areas is vast, though, very few studies are available on the combination of the three. This chapter discusses the literature of these three areas, and presents the related studies on the integration of scheduling and lot sizing.

2.1 Lot-Sizing

In this section, we will discuss the literature of deterministic and dynamic capacitated lot sizing problems. By deterministic, we mean that demand is considered as known, as opposed to probabilistic demand, and by dynamic we mean that the demand is time varying over the planning horizon. The aim of lot sizing research is to minimize setup and holding costs by determining production lot sizes and inventory levels to meet a given demand. In other words, lot sizing is about answering the questions of when and how many to produce. The goal is to find a feasible production plan which meets the

demand as stated in master schedule plan (MSP), while having the minimum setup and inventory holding cost.

Based on the literature, solution methods of lot sizing problems can be classified into three main categories: simple and common sense techniques, exact optimization methods, and heuristic methods.

2.1.1 Simple and Common Sense Techniques

Lot sizing research began with simple techniques like EOQ, fixed order quantity, and lot for lot (LFL) models and developed further to handle various cases. The EOQ model is single level, single item production with no capacity constraints. The demand is deterministic and assumed to have constant rate. EOQ model is a continuous time model and it is very easy to solve. When the demand variation is small, we can ignore the time variability of demand and use the basic EOQ model.

In the lot for lot method, which is considered to be the simplest method, order quantities are the same as demand in each period. This it is not the best method when the setup cost is high.

2.1.2 Exact Optimization Methods

The most cited optimization method for solving lotsizing problems is the Wagner-Whitin algorithm, which uses integer or mixed programming. Development of the dynamic method of Wagner-Whitin constitutes a major improvement in solving lot sizing problems. Wagner-Whitin (Wagner, 1958) assumes a finite planning horizon which is divided into discrete periods. Demand is time-varying and considered to be deterministic

and known for each period. But still it was un-capacitated and single level and a single item problem.

The next generation of models is a combination of dynamic methods like Wagner-Whitin and the capacity constraints, called the capacitated lot sizing problem (CLSP). Single level lot-sizing problem is reviewed by many scholars. In the next sections, we present the mathematical formulation and the basic assumptions of this model.

2.1.2.1 Assumptions and Limitations

To find the best amount and timing of production, following assumptions were made for solving the single item capacitated lot sizing problem:

- The demand rate is known and deterministic,
- The cost factors do not change with time. In other words inflation is negligible
- No shortage is allowed
- The carrying cost is only applicable to the inventory that is carried over from one period to the next period
- The production has to be completed at the end of each period to meet the demand of the next period
- Jobs should be processed in sequence on m machines or stages

2.1.2.2 Mathematical Model

The single item capacitated lot sizing problem (CLSP) can be formulated as follows:

(2-1) $$\textit{Minimize} \qquad TRC = \sum_{j=1}^{t} AY_j + I_j h$$

$$S.T.$$

$$I_j = I_1 + Q_j - D_j$$

$$j = 1,\ldots, t$$
$$Q_j \geq 0$$
$$I_j \geq 0$$
$$Y_j = \{0,1\}$$

Demand for each period (D_j) is given. The objective function minimizes the total relevant cost of production (TRC) which is the summation of setup (A) and inventory holding costs (h). In each period with production a set up cost will be incurred and this is modeled by using a binary coefficient in the objective function. The first constraint is flow-balance which shows the requirement in each period will produce at least one period before needed period. As experiments showed, lot-sizing problems are NP-hard which is why this field is dominated by heuristic techniques. The heuristic and meta-heuristic models will be discussed in the next sections.

2.1.3 Heuristics

Heuristic methods are iterative procedures to solve a problem but do not grantee an optimal solution. Heuristic methods are used for NP-hard problems when it is computationally infeasible to find an optimal solution. The most common heuristic approaches used to solve the lot sizing problem for dynamic demand are the Silver-Meal (Meal, 1969), Least Unit Cost (LUC), and Part-Period Balance (PPB) methods.

Silver-Meal heuristic considers a lot size that equals to the demand of some *m* periods ahead. For example, if m = 3 the lot size equals to the demand of the 3 next periods. The average holding and set up cost per period for m period span. Inventory costs (holding and setup) per units of time of placing an order at period t, if the order covers m periods, can be expressed as follows:

$$(2\text{-}2) \qquad TRCUT(t) = \frac{(setup\ \ cost) + (Total\ \ Carring\ \ Costs\ \ to\ \ end\ \ of\ \ period\ \ T)}{T}$$

Silver-Meal heuristic starts from ordering for one period ahead and increase the number of periods (*t*) systematically to include the demand of next *t* periods ahead. It computes the cost for each *t =1, 2, 3*, etc, until the average cost per period starts to increase. The best *t* is the last one before the average cost per period increases. Then, an order to meet the demand of the next *t* periods must be placed. After that, it moves to period *t+1* and repeat the procedure until all demands are included in orders.

The Least Unit Cost heuristic is quite similar to the Silver-Meal heuristic, except that it accumulates demands until the cost per unit increase. In other words it divides the cost by number of units instead of dividing the cost by number of periods (*t*).

The idea behind the Part Period Balancing is to select the number of periods covered by a setup such that the setup cost is nearly equal to the holding cost.

In the recent years, meta-heuristics such as tabu search, genetic algorithm, and simulated annealing have been used extensively for solving the lot sizing problems as well as its variants. Design of meta-heuristic algorithms revolves around choosing ways to represent a solution, evaluate each solution and define a neighborhood. Reza and Akgunduz

(2007), Ozdamar *et al.* (2002), Meyr (2000), and Sikora (1996), are a few examples of the huge number of the existing literature on different variants of lot sizing. We will discuss the design of their algorithms in more detail in the next sections and especially those who use genetic algorithm in chapter 4.

2.1.4 General Characteristics of Lot Sizing Problems

There are many comprehensive reviews on the lot-sizing literature. Karimi *et al.* (2003) classified the lot sizing problems into four categories based on the number of products (single-item or multi-item) and the capacity (un-capacitated, capacitated). They also introduced eight characteristic for lot sizing models. The following characteristics are to be noted in any lot sizing models:

Planning horizon: Planning horizon can be discrete or continuous and it can also be finite or infinite. For finite planning horizon with deterministic and time varying demand, dynamic models like Wagner-Whitin are usually used.

Planning horizon can be categorized as either big bucket or small bucket. Big bucket horizons are those where the time period is long enough to produce multi items, while for small bucket problems the time period is so short that only one item can be produced in each time period.

Production level: Production level can be either single or multi-level. Multi-level production can further be categorized as serial, assembly, disassembly, and general. In a single level production system, the final product with independent demand is directly produced from raw or purchased materials, whereas in a multi-level production system, in

which items have the hierarchal relationship and output of one item is the input for another one.

Number of products: There are two types of production with respect to the number of products, namely single item and multi-item. Presence of sequence dependent setup times and costs increase the complexity of planning for production of multi-item.

Demand for the end item: Demand can be deterministic (known in advance) or probabilistic (not known exactly). It can be static (i.e. it will not change over time), or time varying and dynamic.

Setup: Setup can be simple (independent of scheduling decisions) or complex. Complex setups can further be categorized as setup with carry over, sequence dependent, and family or major setup.

Deterioration of items: For items which can deteriorate, another constraint would restrict the inventory holding time and add to complexity of the problem.

Inventory shortage: Backlogging and lost sales are two inventory characteristics which can be allowed or not. Backlogging means that the demand of a period can be satisfied in future.

2.2 Scheduling (Sequencing and Lot-Streaming)

A production setting, in which n-job should proceed in sequence of m machines or stages and all the jobs are processed in the same sequence, is called a flowshop. Processing times of different jobs on different machines may vary. All jobs are assumed to be

available at time 0. There are a number of performance measures that can be used to evaluate the particular production schedule in a flowshop environment, such as average idling time of machinery, total waiting times of jobs, or machinery utilization. However, the most common objective is to minimize the makespan.

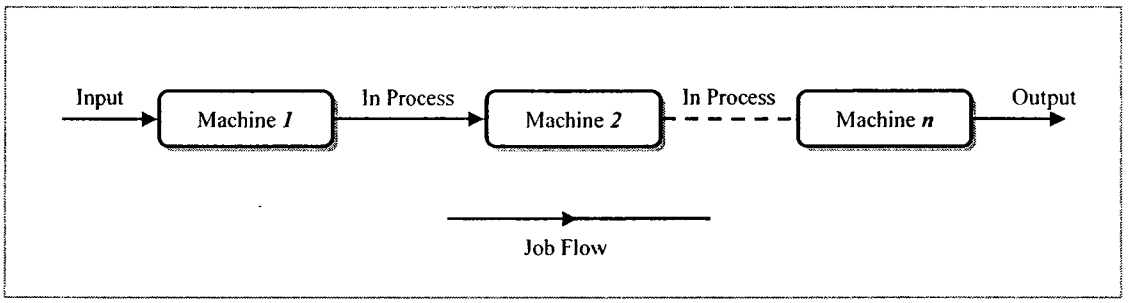

Figure 2-1: m-Machine Flowshop

Throughout the literature in flowshops, the term scheduling is often used purely to denote sequencing of jobs. However, we want to make a distinction between sequencing and scheduling. For the purpose of this thesis, "scheduling" means allocating the resources to perform different jobs over time with respect to constraints like machines' capacities and time bucket length, whereas, sequencing is concerned with the arrangement in which a set of jobs should be performed. In other words, sequencing only ranks the jobs to be performed. In this thesis, by scheduling, we mean the combination of sequencing and lot streaming techniques which enable us to execute jobs in a right order on different machines without any capacity violation within time buckets and to find a feasible production schedule. The literature on scheduling is vast; but we are only concerned with flowshop scheduling with the objective of minimizing the makespan. We are particularly interested in production settings in which production takes place in batches as opposed to producing individually (unit by unit) that is where literature of both the lot streaming and the sequencing problems merge together.

We follow the three-field classes of scheduling problems introduced by Graham *et al.* (1979). We use $Fm/ST_{si,b}/Cmax$ to denote the problem studied in this thesis. In this classification problems can be specified in terms of the three-field classification α /β / γ where α specifies the production environment, β specifies the job characteristics, and γ describes the objective function.

Fm represents flowshops with m-machines, in the second field, "$ST_{si,\ b}$" sequence independent setup times for sublots, and "*Cmax*" represents the performance measure of minimization of makespan. In the rest of this section, the important notions and definitions will first be discussed and then the literature of lot streaming and sequencing will be presented briefly.

Sequencing research starts with the very early works of Johnson's 1954 paper on two machine flow-shop scheduling (Johnson, 1954). Reiter (1966) introduced the concept of lot streaming.

Lot streaming is the process of splitting a job or a lot to allow overlapping between successive operations in a multi-stage production system. This usually results in a shorter makespan. In the context of multi-product lot streaming in addition to the sublot sizes and the number sublots the production sequence of items is another decision variable.

Chang and Chiu (2005) classified lot streaming problems into four categories based on number of products (single or multiple) and the performance measurements (time-related and cost-related). They described lot streaming problems with three main dimensions, namely system configuration, sub-lot-related feature, and performance measurement.

They also introduced seven sub-dimensions which has several levels as shown in appendix 2.

2.2.1 Notions and Classifications

In the rest of this section, we introduce the necessary definitions and classifications.

1. In an *m*-machine flowshop, a schedule is called permutation if the job sequence stays the same on all the machines or stages. Otherwise, it is a non-permutation schedule.
2. Sequencing problem in a flowshop can be categorized further to those with independent setup and those with sequence dependent setup times.
3. A setup is attached if it cannot be started before the production lot or sublot becomes available on the machine. If it can starts before availability of sublot a setup is detached.
4. Lot or sublot sizes can be consistent, equal, or variable. Consistent batches have the same size over the different stages or machines. If all the sublots' sizes are equal it would be called equal sublots otherwise, it would be called variable sublot size.
5. Idling can be allowed or not. Machines can be idle between sublots if intermittent idling is allowed. Non-idling means that sublots should be processed one after the other on a particular machine or stage.
6. Setup, production, and transportation are the three types of activities that involve a series of operation.

7. Sublot size can be a real number or an integer (discrete or continuous). For discrete sublots, the number of item in a sublot has to be integer number. While, for continuous sublots size there is no restriction on the sublot size. For example, in chemical industries sublots take continuous numbers.

In multi-product setting, if intermingling sublots (preemptions) is allowed, the sequence of sublots of a given product may be interrupted by sublots of another product. For non-intermingling sublots, no interruption in the sequence of sublots of a product is allowed (Defersha and Chen 2008). In other words, production of a particular product on one machine or stage can be started once the last sublot of the previous product is finished. Figure 2-2, in the next page, shows the benefits of lot streaming in reducing the makespan (Chang and Chiu, 2005).

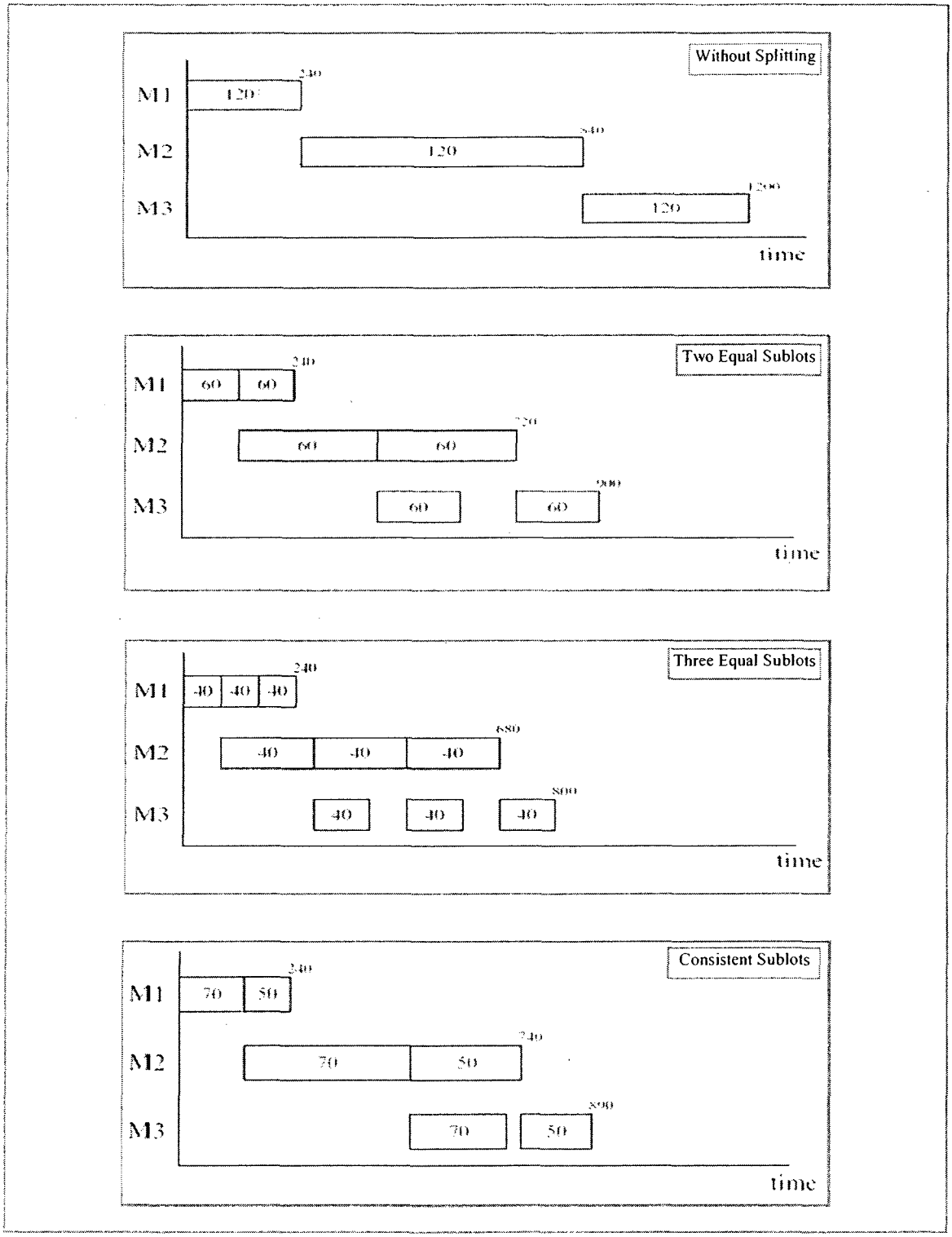

Figure 2-2: Gantt Charts of Different Sublot Sizes

2.2.2 Common Solution Methods

There are two main approaches to solve flowshop scheduling problems namely, the exact and heuristic approaches. There are a huge number of studies on both the exact and heuristic solution procedures developed to solve the scheduling problems in flowshops. The exact algorithms are capable of determining the optimal solutions. However, heuristics approaches are also very prominent in t2he literature since such problems are proven to be NP-complete.

2.2.2.1 Exact techniques

Sequencing research starts with the very early works of Johnson's 1954 paper on two machine flow-shop scheduling (Johnson, 1954). Johnson developed a procedure that minimized the makespan for jobs on two machines. For a two-machine setting, Johnson's solution is proven be an optimal solution and it can be extended to a three-machine flowshop problem as well. The Johnson's algorithm also known as Johnson rule is as follows:

1. Determine the shortest processing time among all jobs yet to be sequenced on machine 1 and 2.
2. Schedule the job associated with the shortest processing time first in the sequence, if it occurs on the first machine.
3. Schedule the job associated with the shortest processing time last in the sequence, if it occurs on the second machine.
4. Continue until all the remaining jobs are scheduled.

Essentially, Johnson's procedure minimizes the idle time on machine 2 by scheduling a job with the shortest processing time on machine 1.

2.2.2.2 Heuristic methods

In this section we discuss some heuristic and meta-heuristic methods to minimize the makespan in flowshops. Palmer's method, NEH heuristic, and CDS algorithm are three examples of such heuristic methods developed through application of Johnson's algorithm.

Palmer (1965) developed a heuristic based on the idea of placing jobs, which have increasing processing times from one stage to the next stage in earlier positions in the sequence. He assigns jobs to later positions in the sequence when they have decreasing processing times as they move from one stage to the next as they progress through the flowshop.

The Nawaz, Enscore, and Ham algorithm, known as NEH, is one of the most cited greedy constructive methods in literature of flowshop sequencing problem (Nawaz, Enscore, and Ham, 1983). The NEH algorithm takes three simple steps as follow:

1. Schedule the first two jobs so as to minimize the partial makespan as if there were only two jobs
2. Order the remaining jobs by decreasing sums of total job processing times.
3. Place the remaining jobs in one of the slots between already scheduled jobs such that the total makespan is minimized at each step.

Campbell *et al.* (1970) developed an algorithm; known as the CDS algorithm which uses Johnson's rules. The algorithm breaks the *m*-machine problem into several two-machine problems, then each of two-machine problems is solved using Johnson's two-machine algorithm.

Sriskandarajah and Wagneur (1999) considered the problem of minimizing makespan in two-machine and no-wait flow shop with multiple products. Their objective is to determine sublot sizes and sequence the jobs simultaneously. They develop a heuristic procedure to solve sequencing and lot streaming of multiple products. Computational results are shown to be close to the optimal. Later, Kumar *et al.* (2000) extends the two-machine model of Sriskandarajah and Wagneur to *m*-machines lot streaming problem.

Lot streaming for multiple products in a multi-stage flow shop with equal sublots and no intermingling is discussed by Kalir and Sarin (2001). Their objective was to minimize the makespan by finding the right sequence for jobs. They developed a heuristic procedure for the lot-streaming and sequencing problems. They used an example to show the importance of sequencing in lot streaming problems.

Hall *et al.* (2003) studied the problem of minimizing the makespan in flowshop with lot streaming. Their model required the sublots to be processed consecutively on each machine. Setups occur only between sublots of different families. Setups are either all detached or all attached. They showed that this problem can be formulated as a general traveling salesman problem. Furthermore, they proposed a heuristic for multiple products, no wait processing and lot streaming flowshop problem.

Biskup and Feldmann (2006) introduced the very first MIP formulation for single item in multi-stage production settings with variable sublots sizes. They showed that significant makespan reduction can be achieved by using variable sublot sizes. Defersha and Chen (2008) extended the single item lot streaming model of Biskup and Feldmann (2006) to multiple products and proposed a genetic algorithm. Their model is outstanding in tackling the sequencing and splitting problem simultaneously. Their model also outperforms that of earlier approaches because their model of variable sublots also allows intermingling of sublots.

2.3 Integrated Lot-Sizing and Scheduling

In the most of the literature of production planning, lot sizing and scheduling problems are solved independently and, for the sake of simplifying the problem, the interdependencies of lot size, sequence, and capacity usage are ignored. The problem discussed here is the lot sizing and scheduling of n-product in a flowshop consist of m machines with the objective of minimizing total inventory holding cost and setup costs. Demand has a deterministic and dynamic pattern over a finite planning horizon and must be met without back-logging. Production takes place in lots. In this condition, lot sizing and scheduling of jobs should be done simultaneously because we encounter sequence dependent setup times and production which takes place in lots.

First, in presence of sequence dependent setup times, the limited capacity available for production may be reduced by changeovers causing sequence-dependent setup times. Thus, lot sizing and capacity usage depends on both the sequence and the size of the sublots.

Second, when the production is by lots, the processing time of a lot may not be negligible, and it would be naive to ignore the machines' idling times. For example, machine 2 would be idle since the job scheduled on machine 2 cannot start until it is completed on machine 1. Therefore, the capacity available for production depends on the size of the production sublots and the sequence of jobs.

There are a few studies available on the combination of lot sizing and sequencing and fewer with lot streaming in mind. In the rest of this section, we will discuss literature on studies on the combination of lot sizing and scheduling.

Szendrovits (1975) presented a model of constant lot size in multistage manufacturing system with constant and continuous demand for products. He came up with a modified EPQ model that assumes a uniform lot size proceeding though *m* machines or stages, with one setup at each machine. His model allows lot splitting to reduce the manufacturing cycle time. The objective of his model is to minimize the sum of the fixed production costs and the holding costs of both the WIP (Work in Process) and finished product inventories. Szendrovits' work resolves the relationship between the production lot size, the manufacturing cycle time and the average work in process.

Goyal (1978) stated that transportation cost of sublots should be noted in Szendrovits model and showed that the total amount of WIP with consistent sublots is less than constant sublots.

Drezner *et al.* (1984) proposed a model with variable lot size (VLS). Szendrovits and Golden (1984) compared the VLS model with ULS, in that they showed that the ULS can

give a better result in minimizing the cost in some situations but it depends on the sublot transportation cost.

Smith and Ritzman (1988) presented a mixed integer programming formulation for lot sizing problem considering the sequencing problem. Their formulation considers sequence dependent set-up times and capacity constraints. However, the MIP formulation is not always able to solve real size (large) problems.

Sikora *et al.* (1995) studied the problem of flowshops with finite scheduling horizon with objective of minimizing the makespan and inventory holding cost. The flowshop they discused has the following characteristics:

- Sequence dependent setup times,
- Limited intermediate buffer space,
- capacity constraints,
- In addition jobs are assigned with due dates that have to be met.

They developed an integrated approach in order to address the problems of solving lotsizing and sequencing decisions independently. It is an iterative approach which does the sequencing for each period before an additional lot is scheduled for that period. Because of the interdependency between lotsizing and sequencing decision, they came up with the framework consist of 3 modules namely lotsizing module, sequencing module, and testing module. Figure 2-3 which shows their integrated framework is adopted from Sikora *et al.* (1995).

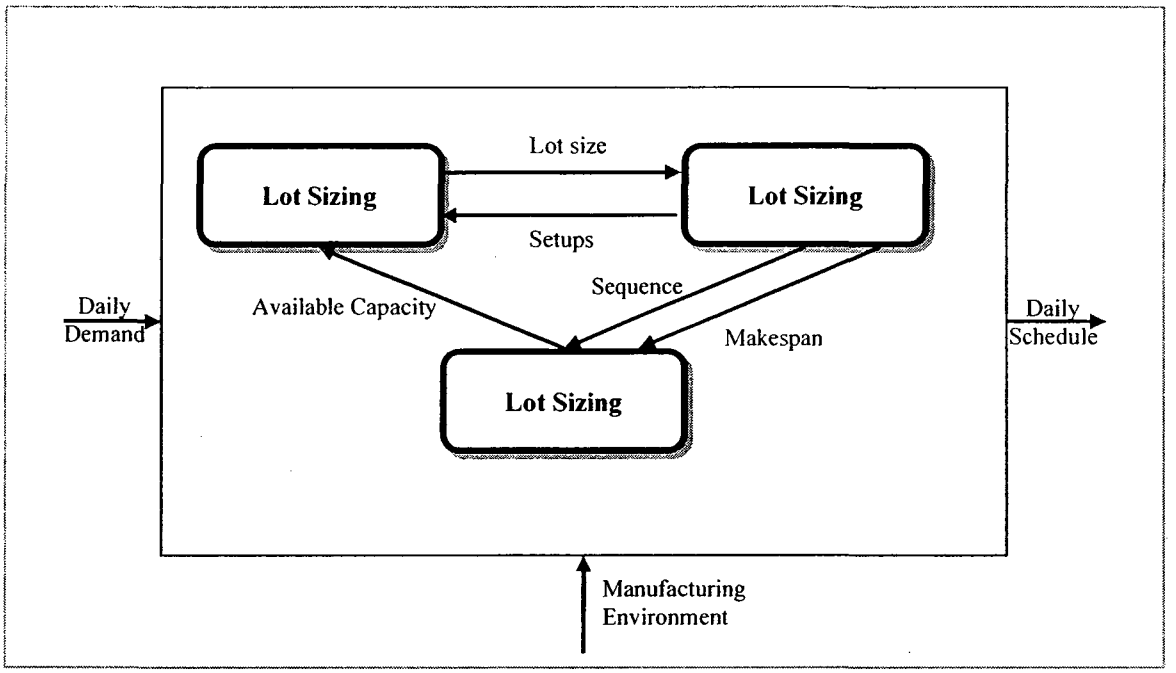

Figure 2-3: The Integrated Framework of Scheduling System

In each iteration, the lot sizing module calls the sequencing module to find the optimal sequences which minimize the makespan. Then, the testing module calculates the makespan and residual capacity on the bottleneck machine for the given lot size and sequence and information will be returned to lot sizing module before any additional lot be scheduled in the period.

The lot sizing module uses a modified version of Silver Meal heuristic that allows lot splitting in order to increase the capacity usage. It calculates b_i which is the fraction of the next period's $(T_i + 1)$ demand that can be produced in the current period without violating the capacity constraints. Then, it quantifies the TRCUT (total relevant cost per unit of time) based on this fraction (b_i), using the following formulation:

(2-3) $$b_i = \min\left\{1, \frac{remaining\ \ capacity}{k_i D_{i(T_i+1)}}\right\}$$

(2-4) $$TRCUT(T_i+1, b_i) = \frac{A_i + h_i \sum_{d=T^*}^{T_i} (d - T^*) D_{id} + h_i D_{i(T_i+1)} b_i}{T_i - T^*}$$

Where:

k_i Required capacity for producing one unit of product i

D_{id} Demand of item i in period d

A_i Setup cost for item i

h_i Inventory holding cost for item i

T^* The current period

T_i The number of periods ahead in planning horizon

The sequencing module uses an improved version of palmer heuristic and the pair-wise exchanged which incorporates the sequence dependent set up times and the limited buffer capacities.

Sikora (1996) developed a genetic algorithm for integrating lot sizing and sequencing problems in a flowshops with all the characteristics of the integrated approach described previously by Sikora *et al.* (1995). In their proposed genetic algorithm, each possible solution is represented by a string of paired values for each period in the time horizon. The first number of any pair indicates the type of job and the second number indicates the number of units of that job type that have to be produced (lot-size). The order of the pairs indicates the sequence of jobs.

In order to enable the genetic algorithm to manipulate both data of lot sizing and sequencing, they use separate crossover and mutation operators. Their GA uses tournament selection mechanism. The feasibility of the initial population is assured by scheduling the lot size s equal to demand in that period. Finally, experiments showed that their genetic algorithm outperforms both the integrated and sequential approaches.

Ozdamar and Birbil (1998) developed a hybrid heuristic consisting of meta-heuristics such as simulated annealing (SA), tabu search (TS) and genetic algorithms (GA) for solving the capacitated lot sizing and loading problem (CLSLP). They define the CLSLP problem as determining the lot size of product families and loading them on parallel machines.

Meyr (2000) introduced another model for solving lotsizing and scheduling problems simultaneously in a capacitated flowshop with sequence dependent setup times. Their model combines a simulated annealing heuristic with the mixed integer model to solve the problem by fixing the binary variable of setup sequence and solve the sub-problems by heuristic. Then, they discuss that solving each of sub-problems to optimality is too time consuming. They develop a mathematical solution which remodels the problem as a dual network flow problem. The computational tests prove that their procedure is efficient.

Ramasesh *et al.* (2000) proposed an EOQ model for a single item in multistage when lot streaming is applied. They develop a total cost function consisting of the inventory holding costs, setup cost, transportation cost, and also WIP carrying cost, and then they discuss a procedure to find the optimal production lot size that minimizes the total cost.

Their contribution is twofold; they capture more accurately the components of the manufacturing lead time that are encountered in a multistage production environment, and they use the lot streaming in a multistage system MRP.

Hoque and Goyal (2003) claimed that Ramasesh model (Ramasesh, 2000) is not only a special case of the model of Hoque and Kingsman (1995) but also is not able to find the minimum cost. They solved two examples, which one of them had been solved by Ramasesh (2000) and the results showed a significant reduction in cost.

2.4 Lead Time

Lead times in MRP systems represent the planned amount of time allowed for orders to flow through the manufacturing system (Kanet, 1986). Lead time is important because it can affect every component of an MRP-based manufacturing and it is related to capacity planning. Lead time structure or the way that lead times are estimated is under management's complete control, and can change the pattern of planned order releases as well as flow time and the amount of inventory.

Lead-time determination can change the pattern of planned order releases in an MRP system. Moreover, by changing the order release pattern the bottleneck facilities would change. This results in unpredictable lead times and capacity. Kanet (1986) compared different lead time structures but did not find any evidence to show which structure is better. In other studies, the results show that the commonly used structure (processing time + setup + waiting time) does not have a better result, especially when the criterion is tardiness (Kanet, 1981).

Many researchers believe that holding planned lead times down will decrease the inventory and eventually yield benefits. They argue that there is a vicious circle of lead time, which means that an increase in lead time encourages shop personnel to ignore deadlines. As a result more increases in lead time are required. Furthermore, it becomes increasingly difficult to plan the MPS because as lead times increase forecasts become more inaccurate. However, Kanet (1986) categorizes the inventory and creates an example that shows lowering the lead times does not always result in reduction in inventory levels. He also came up with a simple and very intuitive rule:

> "If storeroom inventory contains less early inventory than delayed inventory then lead times should be increased. If storeroom inventory contains earlier inventory than delayed inventory, then lead times should be reduced."

From the above mentioned studies we conclude that we cannot ignore the interaction of lead time, lot size, and capacity usage. Since different lot size, sublot size, and sequence has direct impact in on lead time we should find way to find the actual lead time while we are deciding on lot size.

CHAPTER 3

MATEMATICAL FORMULATION

As mentioned in previous chapters, our objective is to decide the sequence of the production and the sublot sizes while solving the lot sizing problem. This will result in better scheduling, since the sublot sizes and the sequence of production are known, we will know the idling times of machines and be able to fill them with jobs, and the orders released to shop floor will be definitely feasible.

In this chapter, a mixed integer linear programming (MILP) formulation is presented. It enables us to simultaneously find the optimal lot sizes, as well as the corresponding sublot sizes and sequences. This model takes the demand as an input and gives the optimal lot size, sublot size, and the sequence of production as an output. With this model, small size problems can be solved within a reasonable time. This model also calculates the makespan, in order to ensure that makespan is less than the available time in each period. Moreover, the lead time is explicitly computed in the model.

3.1 Assumptions and Limitations

To find the best production batch size will make a number of simplifying assumptions:

1) The demand is time varying, known and deterministic
2) The inventory holding cost is only applicable to the inventory that is carried over from one period to the next
3) Production must be completed at the end of each period to meet the demand,
4) Jobs should be processed in sequence M machines or stages,
5) Each production order (lot size) consists of Q identical items which would be divided into sublots
6) Items become available for processing at the next machine whenever the processing of the whole sublot has been finished on the previous machine
7) The entire production lot is produced with a single setup on each machine and transferred to the next machine in one or more sublots
8) The cost factors do not change with time. In other words, inflation is negligible
9) No shortage is allowed

The proposed linear programming formulation to solve the lot sizing problem with the above assumptions is a combination of the conventional model of lot sizing commonly abbreviated as CLSP (Karimi, Fatemi, and Wilson, 2003) and lot splitting with variable lot size quite similar to (Biskup and Feldmann, 2006).

3.2 Notations

Let the parameters be:

$D_{j,t}$ Demand for the Product j in period t

$St_{j,m}$ Setup time of machine m for the Product j

Sc_j Setup cost for the Product j

$r_{j,m}$ Processing time for one unit of the Product j on machine m

l^* Fixed minimum lead time (not include an estimate of delay due to machines idling times or capacity constraints)

h Inventory holding cost

$Cap_{m,t}$ Available time on machine m in period t

TB_t Available units of time (time bucket) in period t

S Number of sublots

s Indices for sublots, $s = 1,...,S$

N Number of products

j Indices for products, $j = 1,...,N$

M Number of machines

m Index for the machines, $m = 1,...,M$

A Sufficiently large number

Let the Decision variables be:

$L_{j,t}$ Lot Size (quantity to be produced from product j in each period t)

$U_{j,s,t}$ Size of the sublot s of the Product j in period t

$I_{j,t}$ Ending inventory of the Product j in period t

$X_{j,k,t}$ Binary variable, which takes the value 1 if the Product j on machine m has started before the Product k, takes 0 otherwise.

$Y_{j,t}$ Binary variable, which takes value 1 if production of the Product j take place in period t

$b_{j,s,m,t}$ Starting time of the sublot s of product j on machine m in period t

$P_{j,s,m,t}$ Processing time of sublot s of product j on machine m in period t

3.3 Mathematical Model

(3-1) $$MIN = \sum_{j=1}^{N}\sum_{t=1}^{T} I_{j,t} * h + \sum_{j=1}^{N}\sum_{t=1}^{T} Sc_{j} * Y_{j,t}$$

(3-2) $$I_{j,t-1} + L_{j,t-l^*} - I_{j,t} = D_{j,t} \qquad j = 1,...,N \;\; t = l^*,...,T$$

(3-3) $$\sum_{j=1}^{N}(L_{j,t} * r_{j,m} + St_{j,m} * Y_{j,t}) \le Cap_{m,t} \qquad t = 1,...,T \;\; m = 1,...,M$$

(3-4) $$b_{j,S,M,t} + P_{j,S,M,t} \le TB_t \qquad j = 1,...,N \;\; t = 1,...,T$$

(3-5) $$L_{j,t} - A * Y_{j,t} \le 0 \qquad j = 1,...,N \;\; t = 1,...,T$$

(3-6) $$\sum_{s=1}^{S} U_{j,s,t} = L_{j,t} \qquad j = 1,...,N \;\; t = 1,...,T$$

(3-7) $$P_{j,s,m,t} = U_{j,s,t} * r_{j,m} \qquad j = 1,...,N \;\; t = 1,...,T \;\; m = 1,...,M \;\; s = 1,...,S$$

(3-8) $$b_{j,S,m,t} + P_{j,S,m,t} + St_{k,m} * Y_{k,t} \le b_{k,s,m,t} + (1 - X_{j,k,t})A$$

$$j,k = 1,...,N \;\; j \le k \;\; s = 1,...,S \;\; m = 1,...,M \;\; t = 1,...,T$$

(3-9) $$b_{k,S,m,t} + P_{k,S,m,t} + St_{j,m} * Y_{j,t} \le b_{j,s,m,t} + x_{j,k,t} * A$$

$$j,k = 1,...,N \;\; j \le k \;\; s = 1,...,S \;\; t = 1,...,T \;\; m = 1,...,M$$

(3-10) $$b_{j,1,m,t} \ge b_{j,1,m-1,t} + P_{j,1,m-1,t} + St_{j,m} \qquad j = 1,...,N \;\; t = 1,...,T \;\; m = 2,...,M$$

(3-11) $$b_{j,1,1,t} \ge St_{j,1} \qquad j = 1,...,N \;\; t = 1,...,T$$

(3-12) $$b_{j,s,m,t} \ge b_{j,s-1,m,t} + P_{j,s-1,m,t}$$

$j = 1,..., N$, $t = 1,..., T$, $m = 1,..., M$ $s = 2,..., S$

(3-13) $Y_{j,t} \geq X_{j,k,t}$ $\quad j,k = 1,..., N \quad t = 1,..., T$

(3-14) $I_{j,t}, L_{j,t}, U_{j,s,t}, b_{j,s,m,t}, P_{j,s,m,t} \geq 0$ $\quad j = 1,..., N \quad t = 1,..., T \quad m = 1,..., M \quad s = 1,..., S$

(3-15) $Y_{j,t} = 0, or 1$ $\quad j = 1,..., N \quad t = 1,..., T$

(3-16) $X_{j,k,t} = 0, or 1$ $\quad j,k = 1,..., N \quad j \leq k \quad t = 1,..., T$

In the objective function, equation (3-1), we are trying to minimize inventory holding cost and setup cost. Note that setup time should not be included in setup cost. Constraint (3-2) is flow-balance which shows the requirement in each period will produce at least l^* periods before needed. l^* is a fixed minimum lead time which doesn't include an estimate of delay due to machine's idling times or capacity constraints. (3-3) is the capacity constraint in which CAP_m is the available capacity of machine *m*. Constraint (3-4) insures that the makespan is less than each period length (time bucket). Constraints (3-2), (3-3), and (3-4) together cause production to start even before l^* period, this additional lead time is because of machine idling times. This ensures that the lead time is explicitly calculated by model. Constraint (3-6) ensures that the sums of sublots are equal to lot size. With (3-7) the processing time of each sublot is calculated. Constraints (3-8), (3-9), and (3-13) determine the sequence of sublots. It means that if $X_{j,k,t}$ takes the value 1 then product *j* is started before the product *k* in period *t*. Constraint (3-10) is bound the sublot *s* on machine *m* to start after sublot *s* on the preceding machine (*m-1*) has been finished. As the model assumes that completions of jobs are consecutive, constraint (3-12) prevents sublot *s* on machine *m* from starting before sublot *s-1* on machine *m* finishes, meaning that overlapping operations on a single machine is not allowed. Constraint (3-14) is the

non negativity constraint. Constraints (3-15) and (3-16) restricts X and Y variables to binary form.

3.4 Computational Experiments

3.4.1 A Numerical Example

Through the following example we will show how simultaneous lot streaming can affect lot sizing process and how it ensures the feasibility of the orders released to shop floor and reduces scheduling nervousness. Assuming a production setting of a single product in which each lot of Q units must proceed in sequence of 4 machines with associated unit processing time of 8, 4, 8, and 4 respectively on each machine. The availabilities on each machine are 160, 80, 160, and 70 units of time respectively. Each lot may be divided into 4 sublots, though the entire production lot is produced with a single setup of $100 on each machine. The inventory holding cost of $10 is only applicable to inventory that is carried over from one period to the next period. For each period (Month) we assumed 200 working units of time. Demand over the planned horizon, order released by conventional lot sizing models, and order released by our model are as shown in table 3-1.

Month	Jan	Feb	Mar	Apr	May	Jun
Demand	10	5	10	20	15	20
Order released by conventional model	5	14	17	17	17	0
Order released by our model	6	16	16	16	16	0

Table 3-1: Demand and Order released in Planning Horizon

In order to compare the quality of planning, orders released on March are studied separately. The order released by a conventional lot sizing model for this month is 17 units. We applied lot streaming model of Biskup and Feldmann (2006) for this order size

(Q=17), and as shown in figure 3-1 (a), the optimal sublot size would be 5, 5, 5, and 2 with the makespan of 208 hours. This would be an infeasible order because there are only 200 available working hours per month. In other words, the conventional lot sizing model assigned a load that exceeds the shop floor's available time by 8 hours during the months of March, April, and May. Figure 3-1(b) illustrates order released by our model (Q=16) with four sublots of size 4 and the makespan of 200 hours. Given the net required values are the same as ones been proposed by conventional lotsizing models, our model is able to resolve the infeasibility without changing the Master Schedule Plan (MSP).

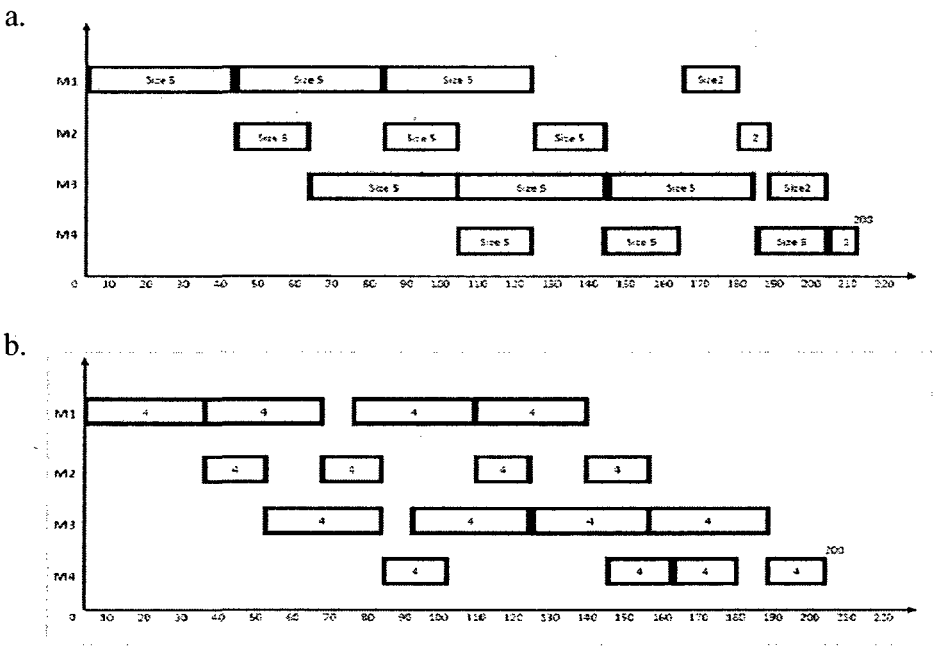

Figure 3-1: Gantt Chart of Production in Month of March

3.4.2 Analytical Examples

For testing, this mathematical model was coded for Lingo 10.0 (see the appendix 3). We also generated two sets of random problems with different number of machines, products,

and number of planning periods. Then, we solved each problem twice with different time bucket length. The required parameters of problems are as follows:

- Number of periods in the planning horizon is *6* or *12*.
- The number of products *N*:{ 3, 7, 9,10 ,12,14,15,17}
- The number of machines *M*:{4, 5, 6, 7, 8, 10, 12,15}
- Maximum number of sublots allowed is 4
- Inventory holding costs and setup costs are considered constant and equal to 10, 100 respectively.
- Required capacity of each machine would be estimated by following formula:

$$(3\text{-}17)\quad Cap_{m,t} = \frac{\sum_{t=1}^{T}\sum_{j=1}^{N} D_{j,t} * r_{j,m}}{T} * 1.8$$

It is important to note that average required capacity is multiplied by a utilization factor (u=1.8) to cope with effect of demand lumpiness. Moreover, this is an estimation of required capacity and the setup times are totally ignored.

- The production takes place in 1, 2, or 3 shifts in a day for 25 days a month. So, considering 8 hours in each shift, the available time in each month or period (time bucket) varied as multiple of 200.
- Demand for each product in each period is equal to *0* by probability of *0.2* or follows the Uniform distribution $D_{j,t} \sim U(1, 60)$ with probability of *0.8*. It should be noted that to avoid time-zero infeasibility the demand for first period t_1 considered as 0.

- Processing times of each units of product on each machine *m* follows the Uniform distribution $r_{j,m} \sim U(1, 5)$.
- Setup times follow Uniform distribution $St_{j,m} \sim U(5, 20)$.

We let Lingo to run for maximum of 3 hours on a 1.6 GHz Pentium PC with 1024 MB RAM before reporting a solution. Table 3-2 shows the computational result for the first set of problems.

Problem	*TB*	*N*	*M*	*T*	Iterations	Processing time	Objective value	Objective bound	State
1	400	3	4	6	2853	00:00:04	3510	3510	Optimal
2	200				72212	00:00:54	3610	3610	Optimal
3	400	3	4	12	101239	00:03:22	5910	5910	optimal
4	200				108702522	3:00:00	6230	6190	Feasible
5	600	7	5	6	111594	00:04:14	7370	7370	Optimal
6	400				4668066	3:00:00	9490	7400	Feasible
7	600	7	5	12	1234777	3:00:00	14200	14157	Feasible
8	400				28141429	3:00:00	-	14157	Unknown
9	600	9	6	6	158945	00:07:18	7360	7360	Optimal
10	400				8687203	3:00:00	-	7360	Unknown

Table 3-2: Computational Result for the First set of Problems

As Table 3-2 shows, we find the optimal solution for small problems at an early stage of problem running. There are two observations. First, as the number of machines, products, and periods increases, it becomes more difficult to find a feasible solution. Second, even for rather small problems when time bucket is tightly used for production and the ratio of makespans and available time bucket is smaller the complexity of problem increases significantly.

In the second set of problems, as the number of products increased, demand, processing and setup times decreased.

- Demand for each product in each period is equal to *0* by probability of *0.2* or follows the Uniform distribution $D_{j,t} \sim U(1, 20)$ with probability of *0.8*.
- Processing times of each units of product on each machine *m* follows the Uniform distribution $r_{j,m} \sim U(1, 3)$.
- Setup times follow Uniform distribution $St_{j,m} \sim U(1, 4)$.

Problem	TB	N	M	T	Iterations	Processing time	Objective value	Objective bound	State
11	300	10	7	6	1615550	3:00:00	8030	7817	Feasible
12	250				6866325	3:00:00	8290	7817	Feasible
13	500	12	8	6	1644858	3:00:00	11170	11150	Feasible
14	400				1385916	3:00:00	-	11150	Unknown
15	600	14	10	6	723880	3:00:00	14520	14420	Feasible
16	500				2277011	3:00:00	-	14420	Unknown
17	600	15	12	6	721757	3:00:00	-	11412	Unknown
18	450				1833381	3:00:00	-	11420	Unknown
19	600	17	15	6	268340	3:00:00	-	12979	Unknown
20	500				913001	3:00:00	-	12979	Unknown

Table 3-3: Computational Result for the Second Set of Problems

Computational result suggests, as we knew, that the problem is NP hard and it cannot be solved by branch and bound method in a reasonable amount of time. In the next chapter we attempt to develop a genetic algorithm to enable us to solve the real size problems in a reasonable time.

CHAPTER 4

GENETIC ALGORITHM

During the last decades we have seen a growing interest in heuristic algorithms which are based on evolution and survival of the fittest, especially for global optimization problems like lotsizing and other NP-hard problems. The best known algorithms of this kind include genetic algorithms, Tabu Search, and Simulated Annealing, as well as some hybrid algorithms which integrate various features of the above methods.

This chapter includes an introduction to genetic algorithms and a brief overview of the important genetic algorithms used in lot sizing and scheduling problems in the literature. Then we present the developed genetic algorithm.

4.1 Genetic Algorithms: An Overview

The Genetic Algorithm (GA), developed by Holland in 1975, is a one of the meta-heuristic methods used to solve complex optimization problems. GA searches the solution space randomly and do not guarantee an optimal solution. In practice, however, along with advantages; such as flexibility and straightforwardness, the results are usually extremely good. GA is an iterative heuristic search procedure following the principles of

genetic variation, natural selection, and survival of the fittest. In these algorithms, the initial population of individuals, which represent possible solutions, goes through a sequence of genetic operators such as crossover and mutation. The population members are evaluated based on a given fitness function. Highly fitted population members have a higher chance to reproduce through a crossover process with other highly fitted population members by exchanging pieces of their genetic information. This process produces new generation and continues until satisfying the termination condition. The termination of a genetic algorithm is defined by either reaching a number of specified iterations or finding the best individuals who represent a near optimum solution. The GA procedure is shown in Figure 4-1 (Nasaruddin *et al.* 2003).

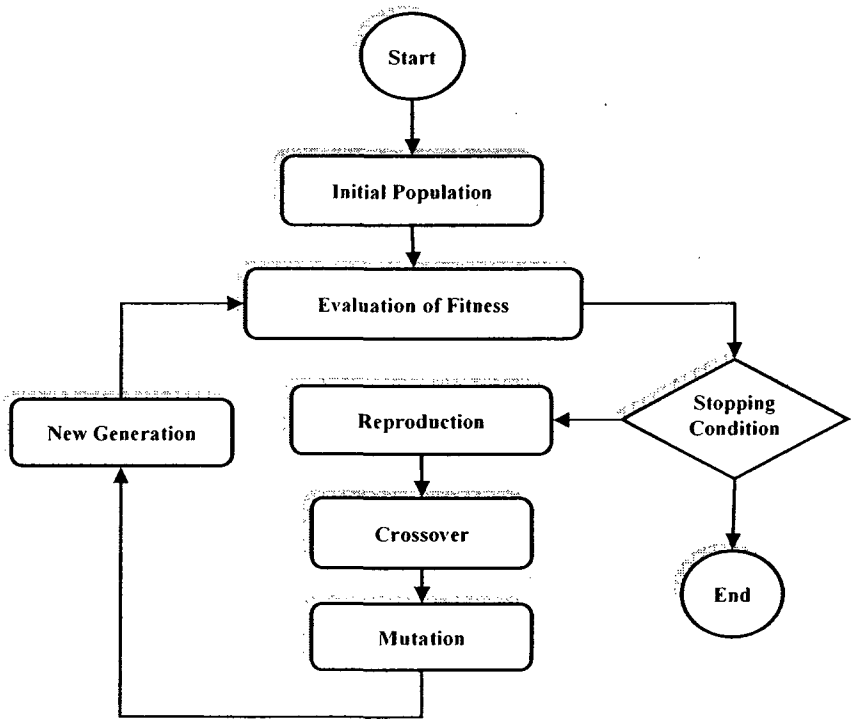

Figure 4-1: The GA Procedure

4.1.1 GA Design Factors

Design of a GA revolves around choosing four important factors which must be addressed systematically:

1. Solution representation
2. Evaluation of each solution
3. Genetic operators
4. Choice of a selection mechanism

These factors are interrelated and, used in the right combination, enable GA to solve optimization problems like capacitated lot sizing more efficiently. In the rest of this section we discuss them along with some of the fundamental terminologies used in the genetic algorithms.

4.1.1.1 Solution representation

In a GA, the possible solution of a problem is presented with a string of genes (chromosomes). Each gene in a chromosome can have a binary digit, integer, or real value. Our mixed integer programming model for CPLS problems contains two binary variables along with integer or continuous variables. The first set of binary variables is used to indicate the setups and the second is used for the sequencing decisions. The integer and continuous variables indicate the production order quantities. There are two methods for representing a solution (chromosome). In the first option, the solution representation includes both the binary variables and the production quantities (Ozdamar, Bilbil, and Portmann, 2002; Ponnambalam, and Reddy, 2003). In the second option, the

solution representation only shows the binary variables, and production quantities can be found by solving the relaxed MIP. A very good example of this is presented by William Hernandez and Sue (1999) and Hung and Chien (2000). It should be noted that in this article we use the three words "individual", "chromosome", and "possible solution" interchangeably.

4.1.1.2 Evaluation of each Solution

As mentioned before a GA is an iterative algorithm. In each iteration, all individuals (solutions) are evaluated and some individuals are selected for reproduction operators. The most common method for evaluating a solution is using the objective function. It is, however, possible to obtain an infeasible solution after applying the genetic operators. In capacitated lot sizing problems, capacity constraints cannot be violated otherwise the solution would be infeasible. In order to treat such infeasible solutions, one method is to impose penalty cost proportional to infeasibility. An example of this is imposing a backlog cost in Xie and Dong's paper (2002). Another good example is penalty for capacity violation in Ozdamar and Birbil (1998). Another method is using a decoder method. The decoder method uses the knowledge of lot sizing to avoid infeasible solution in the first place. A very good example of decoder method can be found in Xie and Dong (2002).

4.1.1.3 Genetic Operators

Two genetic operators, crossover and mutation, are used to explore the entire solution space. The choice or design of operator depends on the problem and the representation model used.

Mutation is the procedure by which individuals are randomly modified. The mutation operator forces the algorithm search a new solution area to escape from local optima by changing a single solution. The most common mutation operator is bit flip in which when mutation takes place the binary value switches from 1 to 0 or vice versa. Example of bit flip can be found in (Hernandez and Sure, 1999). When solution presentation includes production quantities the mutation operator can change the lot size by a random amount or switch the quantities of different periods (Ponnambalam and Reddy, 2003). An important issue here is that applying mutation should not lead to an infeasible solution, or that the infeasibility should be treated in some way. Applying a penalty for capacity violation or very high cost for demand which cannot be met is some ways can be seen throughout the literature.

Crossovers combine existing solutions (parents) and create new solutions (children). In crossover process, population members exchange pieces of their genetic information. This process produces new generation that are hopefully better than both of the parents. Crossover occurs according to a pre-definable probability. In the one point crossover the strings of the two parents are cut in two at some random point and are recombined into one new solution (Xie and Dong, 2002; Hernandez and Suer, 1999). There are also more complex crossovers in literature (Hung and Chien, 2000; Ozdamar, Bilbil, and, Portmann, 2002).

4.1.1.4 Choice of a Selection Mechanism

The selection mechanism is the procedure by which chromosomes are chosen from the current population in order to apply genetic operators (crossover and mutation) and

generate the new population. How to choose individuals for crossing and how many offspring each create are questions that should be answered by selection mechanism. The selection mechanism randomly selects chromosomes that have higher evaluation function or fitter individuals in a population in hopes that their off springs have higher fitness values.

Proportionate selection, ranking selection, elitist selection, and tournament selection are some of the commonly used mechanisms. In the popular proportionate reproduction the probability of selection of each chromosome is proportional to its fitness value (Hernandez and Suer, 1999; Hung and Chien 2000). In these mechanisms the probability of selection of i^{th} chromosome (P_i) from the population of size N is calculated as follows:

$$P_i = \frac{fi}{\sum_{i=1}^{N} f_i} \tag{4-1}$$

Two problems are associated with proportionate selection mechanism. The first one occurs when a few strings with high fitness values force the selection mechanism to allocate a large number of offspring to them and take over the population, which causes premature convergence. The second one occurs in the later stages of the GA, when the population has converged and the variance between strings fitness values becomes small, the proportionate selection scheme allocates approximately equal numbers of offspring to all strings; thereby reducing the chances of selecting better strings. To prevent those problems, ranking procedures can be included.

Elitist mechanism keeps the best chromosomes from the current population in the next population (Ozdamar and Bozyel, 2004). It copies the best solutions had found into the next generation without any change in their genetic information. An advantage of elitist is that good solutions are never lost. Moreover, in most cases it will result in faster convergence.

Tournament selection is another well known selection procedure. Tournament selection takes number of different chromosomes from the population and replaces the worst. An advantage of tournament is fewer sorting required by algorithm.

4.1.2 GA Parameters

The main parameters of the design of a GA are crossover and mutation probability, population size, number of generations, crossover and mutation type, and selection. Different values of parameters have effects on convergence, computation time, and may result in different solution. Low crossover frequency decreases the speed of convergence. High mutation rates introduce high diversity in the population.

4.2 Proposed Genetic Algorithm

As previously discussed, designing a GA is revolves around four important factors solution representation (chromosome), evaluation of each solution, genetic operators, and choice of a selection mechanism. The most important issue is how to fit the lot sizing and scheduling problem into the design of genetic algorithm. It is very important for chromosome representation and genetic operators to be able to produce and manipulate data both for order quantity and item sequencing. The genetic algorithm proposed

attempts to include the problem specific knowledge of lotsizing and lot streaming. In order to do that a decoding method for lot sizes procedure with penalty considerations is designed.

4.2.1 Chromosome Representation

Our GA has an initial population which consists of m possible solutions. Each possible solution is represented by a chromosome consists of n binary (0, 1) variables that each represents the production in a period. The gene t of a chromosome indicates if a production order has been placed in period t or not, which has the same interpretation of binary variables in MILP models. A value of 1 indicates that a production order and a setup has been placed and 0 otherwise. The production quantity in periods which production takes place must satisfy the demand of the subsequent periods with no production. The length of each chromosome (number of genes) is equal to the number of periods in planning horizon. For example, one possible chromosome (solution) for single item model can be as follows:

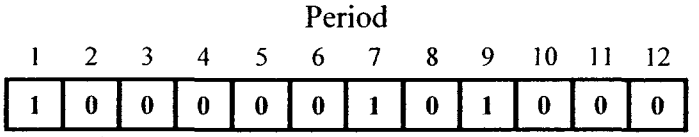

Figure 4-2: Chromosome for Single Item Lot Sizing

The chromosome shown in Figure 4-2 indicates that setups are scheduled only in period 1, 7, and 9. And production quantities in period 1, 7 and 9 should satisfy the demand of the subsequent periods till next setup.

For multi-item lot sizing problem, two different issues should be addressed. First is whether a production order should be placed in period *t* or not. The second is sequencing, which means determination of sequence in which products should be produced or the position of each item in the sequence of each period schedule. To do this, we used strings of *N* (number of items) values for each period in time horizon. As mentioned before we only encode the setup pattern and sequencing positions (binary variables of mathematical model) in chromosomes. The other variables such as order quantities, inventory levels, and makespans must be computed. In the following section we will show how these variables can be decoded. An example of a chromosome (a possible production plan) for a problem with three items and 12 periods can be as follows:

		Period											
		1	2	3	4	5	6	7	8	9	10	11	12
items	1	2	0	2	2	1	3	1	0	1	0	0	0
	2	3	0	3	0	1	2	3	0	2	3	0	2
	3	1	2	3	3	3	1	0	2	0	2	2	3

Figure 4-3: Chromosome for 3-Item Lot Sizing and Sequencing

A value of between *0* to *N* will be assigned randomly to each cell. The value of *0* indicates that production order will not be placed in the period *t* for an item *n*. Values larger than *0* indicate production order has been placed in period *t* and also the position of item in sequence of each period schedule is declared. For instance, the first column indicates the production for all three items are planned in period one. The sequencing indicates that item 2 is produced first, second item is 1, and the last item is 3. In the third Column, both items 2 and 3 have the same priority number (3). To break the tie, the item with smaller index number (item 2) would be first item to be produced, followed by item 3.

4.2.2 Decoding Algorithm

The details of orders released, inventory levels of items, and their corresponding makespan must be determined using a decoding algorithm. The decoding algorithm takes each chromosome or encoded solution and decodes it into a real solution. Decoding is performed to determine the order quantities, inventory levels, and makespans. Ultimately, the total relevant cost is determined by adding inventory holding cost and setup cost.

The decoding algorithm computes the solution as follows: The schedule is generated backward. It first computes the lot size in last period (T), and then it goes back to period $T-1$ and continues, until the production decisions are made for all the periods.

Now, consider a period t, when we had made our production decisions for all the subsequent periods in time horizon. The cumulative demand for each item j in period t (Cd_{jt}) which should be satisfied in period t or its previous periods is equal to the demands of item j in subsequent periods minus ordered quantities of item j in subsequent periods.

$$(4\text{-}2) \qquad Cd_{jt} = \sum_{s=t+1}^{T} (D_{js} - L_{js})$$

The minimum of Cd_{jt} or the quantity of item j that can be produced by the available capacity of machines will be assigned for production. This means that splitting of demand is allowed when cumulative demand cannot be completely scheduled for production due to capacity restrictions. The objective of demand splitting is to increase capacity usage and reduce the total relevant cost. Before making decision for next item, makespan for equal number sublots will be calculated and if the makespan exceeds the time bucket the

quantity of last item scheduled in the period is reduced by one and the makespan is recalculated. This procedure continues till a feasible production order is found. Then, cumulative demand for item j and the residual capacity on machines in the period t will be updated.

$$RCa_{mt} = Ca_{mt} - \sum_{j=1}^{N} r_{jm} * L_{jt} \qquad (4\text{-}3)$$

At this point, the possibility of production and amount of the item with the second position in the sequence of each period will be checked until decisions are made for the all items in the period t. The flowchart for the decoding algorithm is shown in figure 4-4 and the pseudo code for algorithm is presented in the appendix 4.

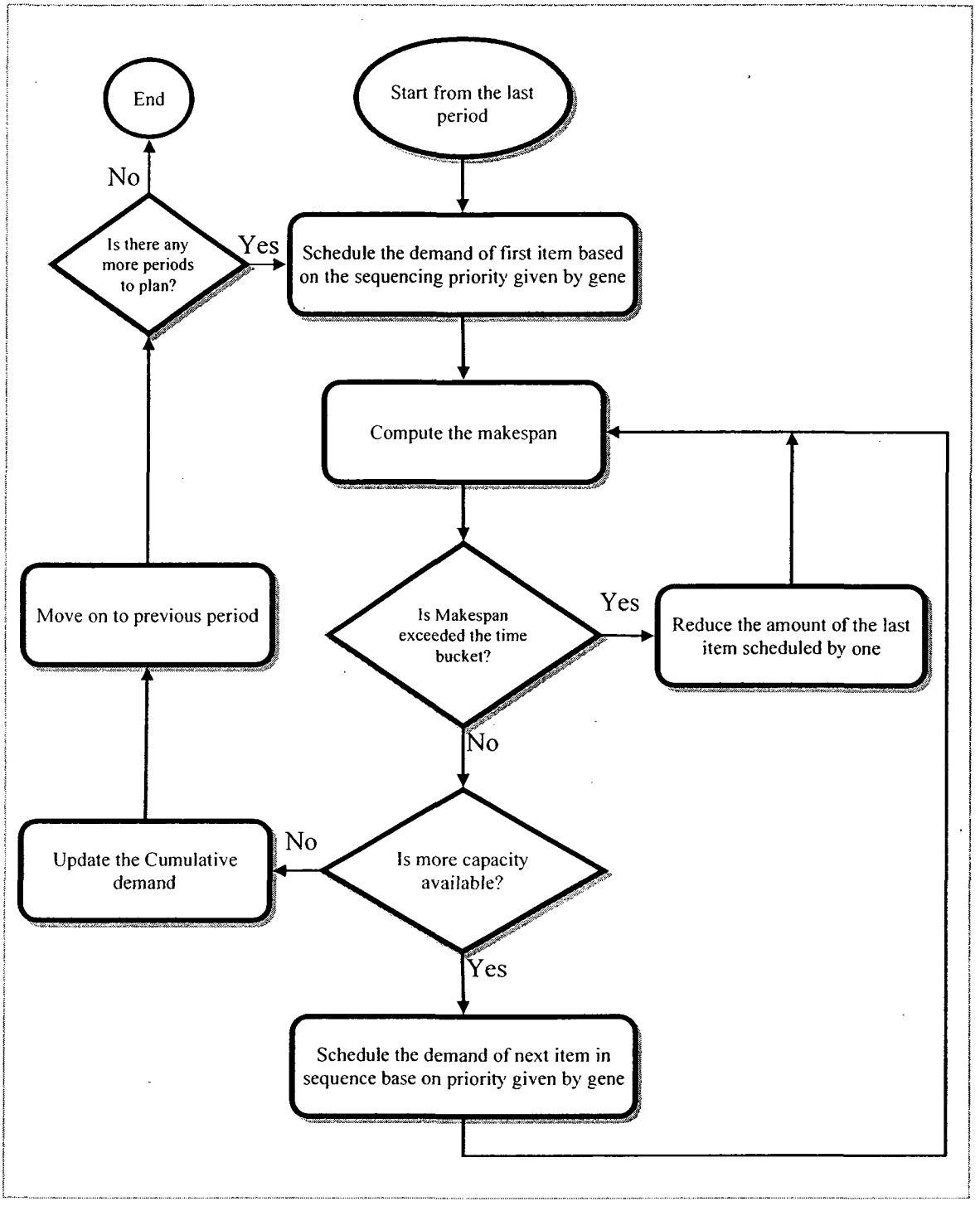

Figure 4-4: Flowchart of Decoding Algorithm

4.2.2.1 Numerical Example

Through the following example we will show how our decoding algorithm works. Assume a production setting of *3* products in which each lot of *q* units must proceed in sequence of *4* machines. The capacities on each machine are *299, 151, 229*, and 223 unit of time in each of *6* periods respectively. The time available in each period (time bucket) is 400 units of time. Each lot may be divided up to *4* sublots; although, the entire production lot is produced with a single setup of *$100* on each machine. The inventory holding cost of *$10* is only applicable to the inventory that is carried over from one period to the next period. For each period (Month) we assumed *350* working hours (time bucket). Demand over the planned horizon, unit processing times and setup times on each machine are as follows:

$$d_{jt} = \begin{bmatrix} 0 & 3 & 38 & 0 & 7 & 40 & 42 \\ 0 & 2 & 1 & 2 & 39 & 0 & 0 \\ 0 & 13 & 40 & 36 & 13 & 58 & 10 \end{bmatrix} \quad P_{jm} = \begin{bmatrix} 5 & 1 & 2 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 1 & 1 & 3 \end{bmatrix} \quad St_{jm} = \begin{bmatrix} 18 & 11 & 16 & 19 \\ 11 & 16 & 12 & 13 \\ 20 & 15 & 9 & 16 \end{bmatrix}$$

Now consider a possible solution (chromosome) as follows:

		Period						
		1	2	3	4	5	6	7
items	1	2	1	0	3	2	1	0
	2	3	0	2	1	2	2	0
	3	1	3	3	0	3	3	0

Figure 4-5: Chromosome 3-Item and 6 Periods

This chromosome specifies the sequence to schedule the items; for example, in the first period item 2 is produced first, followed by item 1 and finally item 3. In the third period,

only item 3 and item 2 will be produced and item 1 will not be scheduled. Table 4-1 shows the iterative process of decoding this solution into a real production schedule.

Period t	product	Current gene	Cumulative Demand		q_{it}		Residual Capacity on Machine M				makespan
			$d_{j,t>t+1}$	d_{jt+1}			M1	M2	M3	M4	
6	P1	1	0	10	10	P3	299	151	299	223	96
	P2	2	0	42	42	P2	269	126	280	177	317
	P3	3	0	0			51	75	184	78	
							51	**75**	**184**	**78**	
5	P1	2	0	40	58	P3	299	151	299	223	
	P2	2	0	0	14	P1	221	78	232	33	322
	P3	3	0	58			133	53	188	0	
							133	**53**	**188**	**0**	
4	P1	3	26	7	33	P1	299	151	229	223	
	P2	1	0	39	19	P2	116	107	217	171	344
	P3	0	0	13			67	34	110	139	
							67	**34**	**110**	**139**	
3	P1	0	0	0	49	P3	299	151	229	223	
	P2	2	20	2	22	P2	230	87	241	60	278
	P3	3	13	36			175	5	119	25	
							175	**5**	**119**	**25**	
2	P1	1	0	38	40	P3	299	151	229	223	
	P2	0	0	14	38	P1	239	96	250	87	312
	P3	3	0	40			31	47	158	30	
							31	**47**	**158**	**30**	
1	P1	2	0	3	3	P2	299	151	229	223	
	P2	3	14	2	16	P1	274	114	252	203	
	P3	1	0	13	13	P3	176	87	204	168	250
							143	**59**	**182**	**113**	

Table 4-1: Decoding Example

In period 6, Product 3 scheduled first with the demand of 10 units which will be produced in 4 sublots of size 2, 2, 2, and 4 and the makespan will be 96 units of time. Residual capacities on the 4 machines after producing 10 units of Product 3 are as 269, 126, 280, and 177 respectively. Followed by Product 3, Product 2 will be scheduled with demand of 42 units which will be produced in 4 sublots of size 10, 10, 10, and 12 and the total makespan would be 317 units of time. Still there is some capacity left but the demand for Product 1 is zero. So, we move back to period 5, Product 3 scheduled first with

cumulative demand of 58 units. Makespan is checked and then the capacity is up dated. The next product in the sequence is Product 1 with total cumulative demand of 40 units. However, the residual capacity of 33 units of time on machine 4 allows maximum production of 14 units of Product 1. So, the remaining demand (26 = 40 - 14) should be produced in the previous periods (1 to 4).

Moving back to period 4, first position belongs to Product 1 with cumulative demand of 33, second position belongs to Product 2 with cumulative demand of 39. However, even though we have enough capacity we cannot produce 39 unites of Product 2 because the makespan of producing 33 unites of Product 1 and 39 units of Product 2 (both with 4 sub-lots) would be larger than the available time bucket (350 units); therefore, in order to find a feasible schedule, the production quantity of Product 2 should be reduced. Thus, we can only produce 19 units of Product 2 in period 4 and 20 units remaining demand should be produced in the previous periods (1 to 3). In the same way, we move backward to period 2 and 1 to decode each chromosome and find the feasible production schedule.

4.2.3 Genetic Algorithm Procedure

The global genetic algorithm continues iteratively until the termination condition is reached. The termination condition we use for our genetic algorithm is a maximum generations. This means the GA will stop after of specific number of generations. The pseudo code for global genetic algorithm is shown in figure 4-6 and the programming code is presented the appendix 5.

```
Global Genetic Algorithm

generation_number ← 0
create initial_population randomly
While (generation_number < max generation) DO
        Decode solutions using the decoding algorithm
        Evaluate initial population and select parents from the population
        Apply crossover operator each pair and put them into new population
        Mutate child
        Add best chromosomes form old population into new population
        generation_number ← generation_number + 1
```

Figure 4-6: Global Genetic Algorithm

4.2.4 Fitness Function Evaluation

For evaluating solutions we used the objective function, which, as in the MIP model, minimize the total relevant cost as follows:

(4-4) *Total Relevant Cost = Inventory Holding Cost + Setup costs*

Infeasibility would not occur in any period other than the first period of each solution because decoding algorithm checks the feasibility in every backward step. In order to deal with possible infeasibility in the first period we impose two penalty costs, one for capacity violation and one for makespans (C) exceeds the available time in the period (TB) to avoid or reduce infeasible solutions.

$$(4\text{-}5) \qquad TC = TRC + P_1 * (C_1 - TB_1) + P_2 * \sum_{m=1}^{M} (\sum_{j=1}^{J} ((L_{j1} * r_{j1}) + St_{jm}) - Cap_{m,t})$$

Given that the objective function values might differ significantly, especially in the first population, we only use the objective function to sort the chromosomes, and fitness values would be assigned to them based on their rankings. This means the solution with lowest total relevant cost should have highest fitness value of N, and the highest total relent cost has the fitness of 1.

4.2.5 Genetic Operators

4.2.5.1 Crossover

The crossover operator applies on two chromosomes (parents) to generate two new chromosomes (Children). A single cutting point is selected randomly from 2 to T-2. Then the two chromosomes exchange their genes from that point to the last position which is period T.

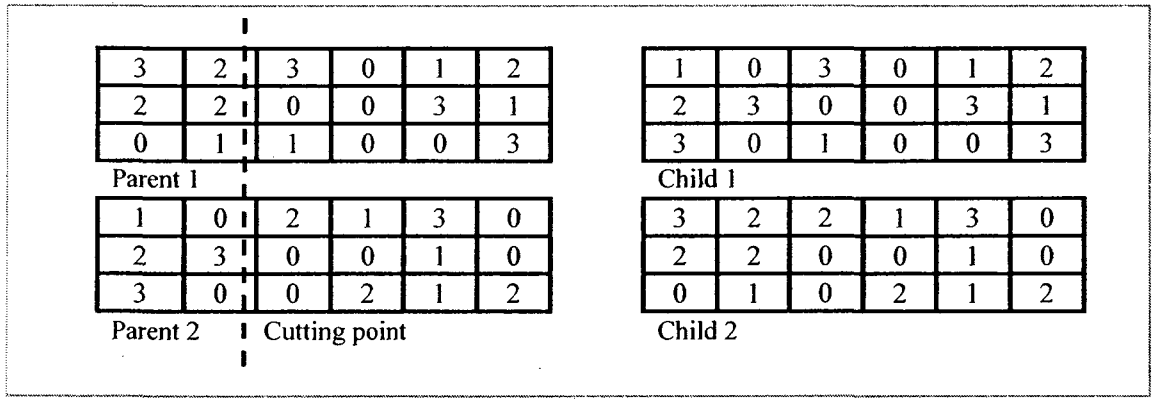

Figure 4-7: Crossover Operator

4.2.5.2 Mutation

The mutation operator has a low (e.g. 0.1) probability of being applied to each gene. When mutation takes place, the content of the gene switches from 0 to 1 and vice-versa

(in the single item case). For multi-item problems, when mutation takes place, another set of random numbers will be assigned to the gene regardless of its original content. Thus, the mutation operator not only might change the decision of whether or not to produce an item in particular period; but also, the sequence of items (products) can be changed randomly in each scheduled period.

4.2.6 Selection Mechanism

In order to be able to regulate selection pressure and population diversity, we used a combination of ranking and elitist selection mechanisms.

In the proportionate selection chromosomes with high fitness values have higher chances of being selected and might take over the population, causing premature convergence, whereas ranking selection results in low convergence. The ranking selection also maintains the selection pressure in the last generation when the fitness variance is low, and proved to successfully search the entire solution. We also combined the elitism mechanism into our search mechanism to keep some of the best individuals in the population and increase the convergence pressure.

First, we sort current population base on the objective function values and each chromosome is given a fitness value of 1 to N (Population size). Then the ¼ of best chromosomes and their off-springs are copied to the next generation. After copying the best chromosomes to the next population, the rest of chromosomes are selected by the ranking mechanism. Genetic operators (crossover and mutation) are applied to generate the new population. This procedure is repeated for each generation for a specific number of generations. The pseudo code for the selection mechanism is as follow:

Selection Mechanism	
Sort the Current Population	
Next Population ←	The ¼ of the best chromosomes of the current population + their offspring
Next population ←	Offspring's of the chromosomes between ¼ and ¾ in ranking

Figure 4-8: Selection Mechanism

4.3 Tuning GA Parameters

When developing a genetic algorithm or any other meta-heuristics, it is very important to choose the right parameters values. The main parameters of the design of a GA are crossover and mutation probability, population size, number of generations, crossover and mutation type, and selection operators. All of these parameters have an effect on the performance of GA algorithms not only in terms of quality of the solutions obtained but also in terms of the amount of time needed to find a solution and computational efforts. Thus, one of the major tasks in the design of any GA is to find the best choice of parameters in order optimize the performance of GA, which is called tuning. To do so, we have two choices:

1. Use one of the standard parameter settings;
2. Use experimental design and statistical tools available in it to compare the effect of each parameter value (main factor) and their interactions,

There are several well known Standard settings in literature. First works were published by De Jong (1975) who investigated different parameters' value to increase the performance and his results were the first rule for researchers. Table 4-2 shows His results.

Population size	50-100
Crossover rate	0.6
Mutation rate	0.001
Mutation types	bit flip
Number of Generation	1000

Table 4-2: De Jong Parameter Settings

Later, Grefensette (1986) published parameters which were different than those of De Jong. Grefensette parameters are used when, for any reason, such as the computational expense, a smaller population is required. His results are shown in the Table 4-3.

Population size	30
Crossover rate	0.95
Mutation rate	0.01
Mutation types	bit flip

Table 4-3: Grefensette Parameter Settings

In this thesis, we used a combination of settings recommended by De Jong (1975) and Grefensette (1986) with one difference which is larger population sizes. We set number of initial population equal to 800 with crossover rate of 60 % and mutation probability set at 1%. The type of mutation we used for single item is bit flip, however for multiple items production settings, because of the special structure representation we use different mutation type as we discussed before. The termination condition or maximum generation we use for our genetic algorithm 1000 generations.

4.4 Computational Results

We coded our genetic algorithm in visual C++ 6.0 and run same problems of Table 3-2 for comparison between linear model and GA results. Table 4-4 shows the result.

Problem	TB	N	M	T	GA Objective value	GA Processing time	Lingo Processing time	Lingo Objective value	Lingo Objective bound	State
1	400	3	4	6	3510	00:00:02	00:00:04	3510	3510	Optimal
2	200				3640	00:00:22	00:00:54	3610	3610	Optimal
3	400	3	4	12	5910	00:00:25	00:03:22	5910	5910	optimal
4	200				6540	00:00:25	3:00:00	6230	6190	Feasible
5	600	7	5	6	7370	00:00:41	00:04:14	7370	7370	Optimal
6	400				8350	00:01:22	3:00:00	9490	7400	Feasible
7	600	7	5	12	14180	00:01:09	3:00:00	14200	14157	Feasible
8	400				14990	00:03:45	3:00:00	-	14157	Unknown
9	600	9	6	6	7360	00:00:53	00:07:18	7360	7360	Optimal
10	400				7480	00:01:15	3:00:00	-	7360	Unknown
11	300	10	7	6	7840	00:00:12	3:00:00	8030	7817	Feasible
12	250				8000	00:00:17	3:00:00	8290	7817	Feasible
13	500	12	8	6	11160	00:01:40	3:00:00	11170	11150	Feasible
14	400				11330	00:02:12	3:00:00	-	11150	Unknown
15	600	14	10	6	14420	00:04:10	3:00:00	14520	14420	Feasible
16	500				14710	00:06:21	3:00:00	-	14420	Unknown
17	600	15	12	6	11420	00:04:21	3:00:00	-	11412	Unknown
18	450				11570	00:05:34	3:00:00	-	11420	Unknown
19	600	17	15	6	12980	00:06:31	3:00:00	-	12979	Unknown
20	500				13550	00:09:53	3:00:00	-	12979	Unknown

Table 4-4 comparison between linear and GA solution

As it shown in Table 4-4, GA gives solutions that are optimal or they are very close to the optimal. For problems which Lingo cannot find the optimal solution, GA provides better answer than the Best Objective find by lingo with an insignificant deviation from the objective bound. Moreover, GA found these solutions notably faster than Lingo and gives us this opportunity to find a reasonable solution in a shorter time while branch and bound technique used by Lingo fails to find even a feasible solution.

In order to visualize and compare the quality of GA's solution with that of Lingo, the percentage deviation of GA's solution from the Lingo objective bound is presented in table 4-5. We also plot the Lingo objective bound and GA objective value of all the twenty problems in figure 4-9.

Problem	GA Objective value	Lingo Objective bound	Percentage Deviation of GA from Lingo bound
1	3510	3510	0.00%
2	3640	3610	0.83%
3	5910	5910	0.00%
4	6540	6190	5.65%
5	7370	7370	0.00%
6	8350	7400	12.84%
7	14180	14157	0.16%
8	14990	14157	5.88%
9	7360	7360	0.00%
10	7480	7360	1.63%
11	7840	7817	0.29%
12	8000	7817	2.34%
13	11160	11150	0.09%
14	11330	11150	1.61%
15	14420	14420	0.00%
16	14710	14420	2.01%
17	11420	11412	0.07%
18	11570	11420	1.31%
19	12980	12979	0.01%
20	13550	12979	4.40%

Table 4-5: Comparison between Lingo objective bound and GA objective value

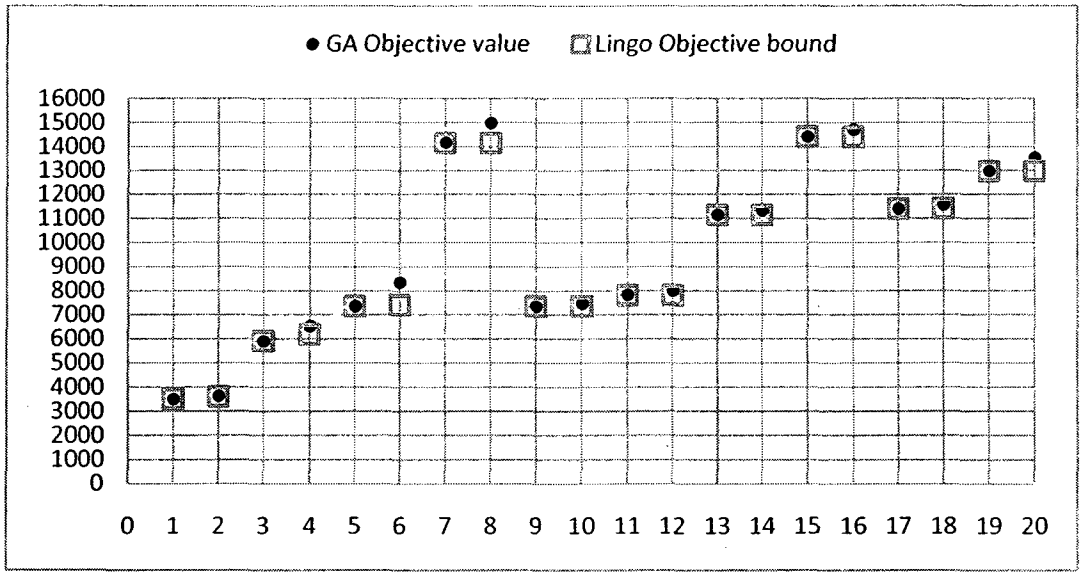

Figure 4-9: Comparison between Lingo objective bound and GA objective value

As it shown in table 4-5 and figure 4-9, the gap between GA objective value and objective bound of Lingo is negligible (i.e. average difference is less than 2%). The largest difference between the GA objective value and the Lingo objective bound belongs to the problem six with 12.8% deference. However, we should note that the best objective value found by Lingo is $9490 which is 12% higher compare to $8350 by GA version. Moreover, the objective bound of Lingo may become even tighter if we run the Lingo for longer period of time.

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

This thesis presents two solution procedures to find the optimal lot size for multiple items with time varying demand in a multistage production system, when lot streaming and sequencing are applied simultaneously to ensure orders feasibility and reduce the need for rescheduling of master production plan. Moreover, it considers machine capacity constraints, and limited time bucket length, and independent setup times in flowshop environment.

First, the problem is formulated as a mixed integer linear programming problem (MILP) and coded with Lingo 10 software. This model takes the demand as an input and gives the optimal lot size, corresponding sublot size, and the sequence of production as an output. This model also calculates the makespan in order to ensure that makespan of the production orders released to shop floor is less than available time in each period. A small set of problems is generated randomly to test the proposed model. The optimal solution for small and medium size problems could be found within a reasonable time. However for larger size problems, because of the NP-hard nature of lot sizing and lot streaming problems, finding even a feasible solution is hard.

Second, a genetic algorithm is developed in order to solve the larger size problems, when the linear model is unable to function. Then, GA solutions are compared with optimal solutions and for the larger size problem with the objective bound of linear model. It is concluded that GA results are acceptable. The proposed genetic algorithm attempts to include the problem specific knowledge of lotsizing and lot streaming. Because of that, this genetic algorithm only works on the two binary variables of mathematical model namely setup pattern and sequencing positions. Then, another heuristic algorithm is used to decode a solution represented by GA and find the other variables in model like order quantities, inventory levels, and makespans.

One thing that should be done in future is to use the Experimental Design and the statistical tools available in it to tune the GA main parameters. The main parameters of GAs are crossover and mutation probability, population size, number of generations, crossover and mutation type, and selection operators, which all affect the performance of GA algorithms not only in terms of quality of the solutions obtained but also in terms of the amount of time needed to find a solution and computational efforts. Experimental Design allows us to compare the effect of each parameter value and their interactions.

Another possible future work is to solve more problems to investigate the effect of different problem configurations like different ratios of holding cost to setup cost, or different capacity utilization rates on the complexity of the problem.

This thesis only considered independent setup times. However, in presence of dependent setup times, the limited capacity available for production may be reduced by changeovers that cause larger setup times; and the need for simultaneous lot sizing and scheduling

become more vital. Thus, including the dependent setup times and cost in the model would make it much more realistic and could be very useful.

Furthermore, like the studies Ho had done between years 1986 and 2005(Ho, 2005; Ho and law 1995; Ho, 1986). A thorough study of dampening effect of this model and the impact of it on reduction of the planning instability could be a future work.

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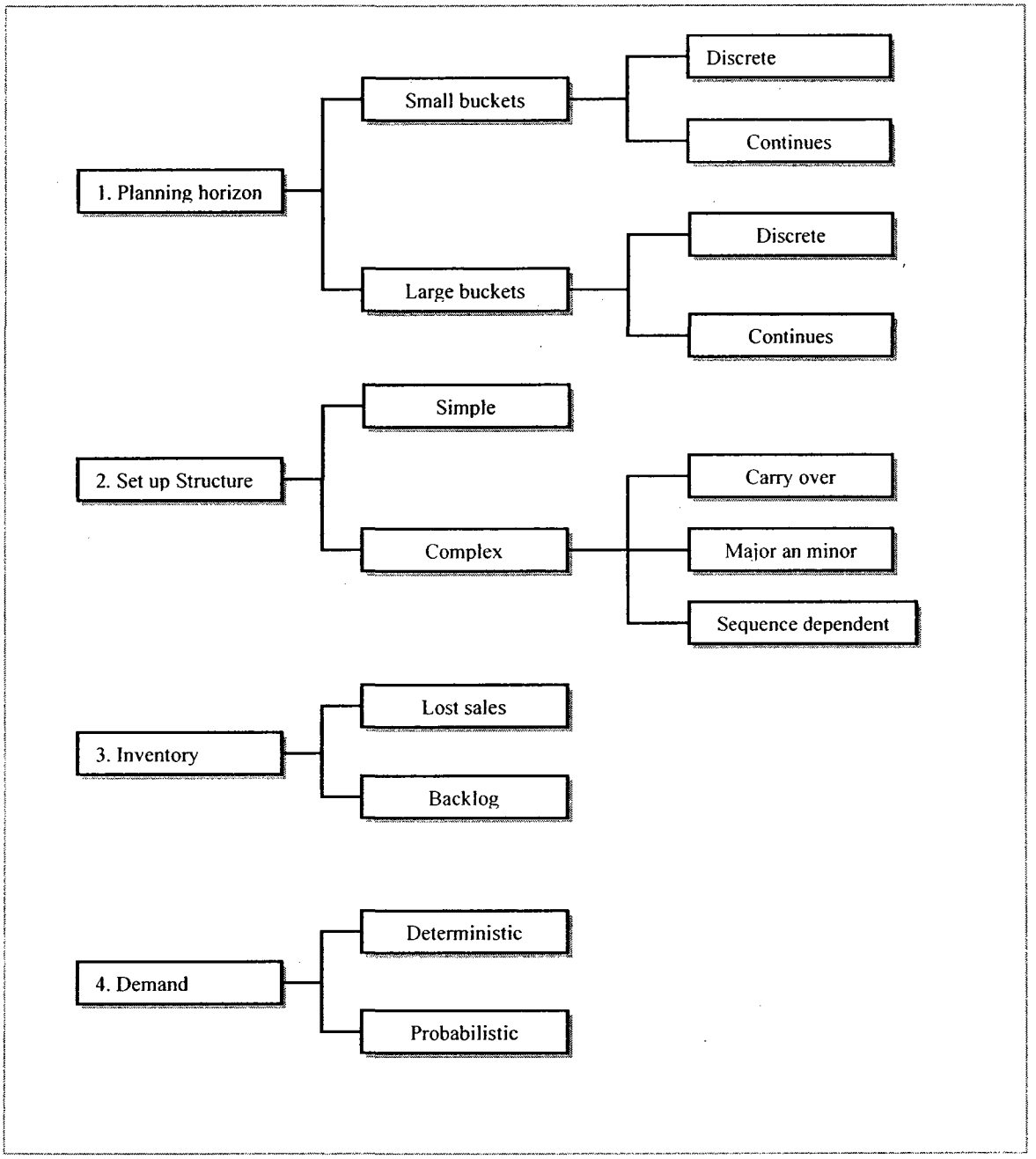
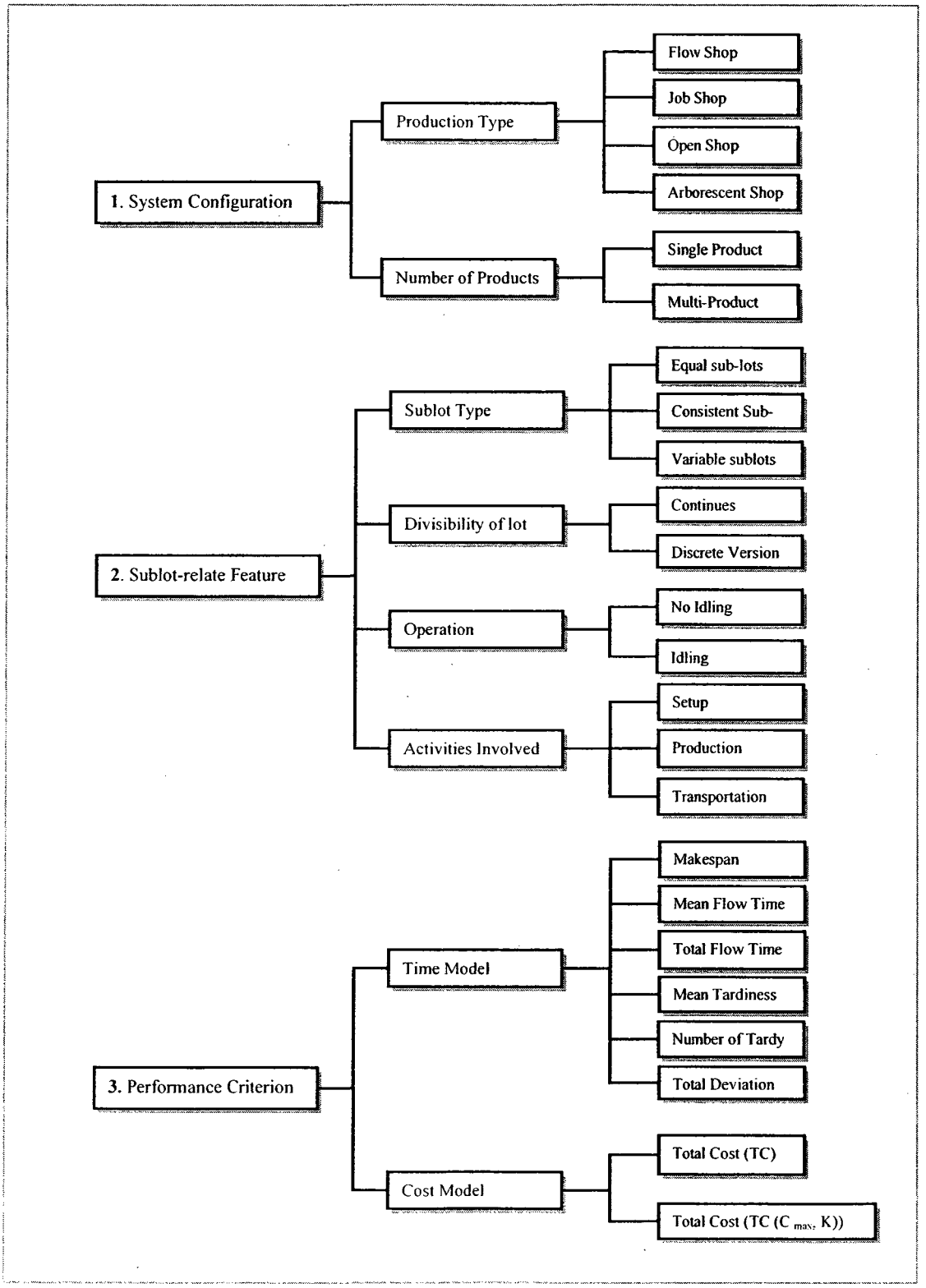
APPENDIX 1: Classification of Lot Sizing Problems



APPENDIX 2: Classification of Lot Streaming Problems



APPENDIX 3: Lingo Code

```
MODEL:
! MULTI PRODUCTS INTEGRATED LOT STREAMING AND LOT SIZING INTERMINGLING;

SETS:
MACHINE/1..12/:CAP;
PRODUCT/1..15/;
SUBLOT/1..4/;
PERIOD/1..7/:C,A_T;
UNIT_PROCESSING_TIME(PRODUCT,MACHINE):r,st;
PRODUCTION_LOT(PRODUCT, PERIOD):D,Q,Y,I,E,F;
FORMULA1(PRODUCT,SUBLOT,PERIOD):L;
FORMULA2(PRODUCT,SUBLOT,MACHINE,PERIOD):P,B;
FORMULA3(PRODUCT,PRODUCT,PERIOD):X;
END SETS

DATA:
A=1000000;
SETUP_COST=100;
H=10;
A_T=600;
r=@OLE('C:\Documents and Settings\Data.xls','Unite_Processing_Time');
D=@OLE('C:\Documents and Settings\Data.xls','Demand');
st=@OLE('C:\Documents and Settings\Data.xls','Setup_time');
CAP=@OLE('C:\Documents and Settings\Data.xls','Capacity');
@OLE('C:\Documents and Settings\Data.xls','L')=Q;
@OLE('C:\Documents and Settings\Data.xls','Inventory')=I;
@OLE('C:\Documents and Settings\Data.xls','Sublot_Size')=L;
@OLE('C:\Documents and Settings\Data.xls','v')=B;
@OLE('C:\Documents and Settings\Data.xls','P')=P;
END DATA

! OBJECTIVE FUNCTION;
! Minimizing the Total Relevant Cost;

MIN=@sum(PRODUCTION_LOT(j,t):H*I(j,t))+
@sum(PRODUCTION_LOT(j,t):SETUP_COST*Y(j,t));

! SUBJECT TO;

!1;
@FOR(PRODUCTION_LOT(j,t)|t#GE#2:I(j,t)=I(j,t-1)+Q(j,t)-D(j,t));

      !1-1;
      @for(PRODUCTION_LOT(j,t):I(j,1)=Q(j,1));
      @for(PRODUCTION_LOT(j,t)|t#GE#2:I(j,t-1)>=D(j,t));

!2;
@FOR(MACHINE(m):
@FOR(PERIOD(t):
                  @SUM(PRODUCT(j):Q(j,t)*r(j,m))+
@SUM(PRODUCT(j):st(j,m)*Y(j,t))<=CAP(m) ));

      !2-1;
      @FOR(PERIOD(t):
                  C(t) <= A_T(t));

!3;
@FOR(PRODUCTION_LOT(j,t):Q(j,t)-A*Y(j,t)<=0);
```

```
!4;
@for(PRODUCT(j):
       @for(PERIOD(t):
            @sum(SUBLOT(s):L(j,s,t))=Q(j,t)));

!5;
@FOR(FORMULA2(j,s,m,t):P(j,s,m,t)=L(j,s,t)*r(j,m));

!6;
@FOR(MACHINE(m):
@FOR(FORMULA3(j,k,t):
      @for(FORMULA2(j,s,m,t)|j#LT#k
:B(j,4,m,t)+P(j,4,m,t)+st(k,m)*Y(k,t)<=B(k,s,m,t)+(1-X(j,k,t))*A)));

!7;
@FOR(MACHINE(m):
@FOR(FORMULA3(j,k,t):
      @FOR(FORMULA2(j,s,m,t)|j#LT#k:B(k,4,m,t)+P(k,4,m,t)+st(j,m)*Y(j,t)
<=B(j,s,m,t)+X(j,k,t)*A)));

!8;
@FOR(FORMULA2(j,s,m,t)|m#GE#2:B(j,1,m,t)>=B(j,1,m-1,t)+P(j,1,m-
1,t)+st(j,m));

!9;
@FOR(FORMULA2(j,s,m,t):B(j,1,1,t)>=st(j,1));

!10;
@FOR(FORMULA2(j,s,m,t)|m#GE#2:B(j,s,m,t)>=B(j,s,m-1,t)+P(j,s,m-1,t));

!11;
@FOR(FORMULA2(j,s,m,t)|s#GE#2:B(j,s,m,t)>=B(j,s-1,m,t)+P(j,s-1,m,t));

!12;
@FOR(FORMULA3(j,k,t):Y(j,t)>=X(j,k,t));

!13;
@FOR(PRODUCTION_LOT(j,t):c(t)>=B(j,4,12,t)+P(j,4,12,t)) ;

!15;
@for(PRODUCTION_LOT(j,t):@BIN(Y(j,t)));

!16;
@FOR(FORMULA3(j,k,t)|j#LT#k:@BIN(X(j,k,t)));


! Integer;
@FOR(FORMULA1(j,s,t):@GIN(L(j,s,t)));


!costant sublot size;

!@FOR(FORMULA1(j,s,t)|s#GE#2 #AND# S#LE#3:L(j,s,t)=L(j,s-1,t));

!@FOR(FORMULA1(j,s,t)|s#EQ#4:L(j,s,t)>=L(j,s-1,t));
```

APPENDIX 4: Decoding Algorithm Pseudo Code

PRODUCTION QUANTITY

```
Input:      Ca_m; d_j,t; current_gen_i,j,1; P_m,j; S_m,j; TB
Output:     quantity_i,j,1

FOR product j ←1 UP TO J
    Cu_De_j ←0
FOR period t ←T DOWN TO 1
    Counter ←0
    FOR machine m ←0 UP TO M
        current_ca_m ←Ca_m
        FOR current_gen i ←1 DOWN TO 1
            IF current_gen_i,j,1 > 0
                Counter ←Counter +1
                Cu_De_j ←Cu_De_j + d_t+1, j
                quantity_j,i,t ←min(Cu_De_j, Capacity-check(current_ca_m, i))
                WHILE TRUE
                    small_sub ← quantity_j,i,t / 1
                    large_sub ←quantity_j,i,t - small_sub* (1-1)
                    FOR sublot i←1 UP TO 1
                        SL_i ← small_sub
                        SL_i-1← large_sub
                    END FOR
                    IF makespan (SL, counter, j) >TB
                        quantity_j,i,t ←quantity_j,i,t -1
                    END IF
                END WHILE
                Cu_De_j ←Cu_De_j - quantity_j,i,t
                FOR machine m ←0 UP TO M
                    current_ca_m ← current_ca_m – P_m,j * quantity_j,i,t +S_m,j
            ELSE IF current_gen_i,j,1 ← 0
                Cu_De_j ←Cu_De_j+ d_t+1, j
                quantity_j,i,t ←0
            END ELSE IF
        END FOR
    END FOR
END FOR
```

Capacity check

```
Input:      current_ca_m; P_m,j; S_m,j
Output:     max-pro

max-pro ← (current_ca0 – S0,j)/ P_m,j
FOR machine m ←1 UP TO M
    IF max-pro > (current_ca_m - S_m,j)/ P_m,j
        max-pro ← (current_ca_m - S_m,j)/ P_m,j
    END IF
END FOR
```

Makespan check

```
Input:      counter; P_m,j; S_m,j; SL_i
Output:     C_counter, 1, M

C_counter, 0, 0 ← C_counter-1, 0,0 + S_0,j + (SL_0* P_0,j)
FOR sublot i ← 1 UP TO 1
    C_counter, i, 0 ← C_counter-1, 0,0 + (SL_i* P_i,j)
END FOR
FOR machine m←1 UP TO M
    C_counter, 0, m ← max(C_counter-1, 0, m-1 ; C_counter-1, 1, m ) + S_m,j + (SL_0* P_m,i)
END FOR
FOR sublot i ← 1 UP TO 1
    C_counter, i, m ← max(C_counter, i-1, m ; C_counter, i, m-1 ) + S_m,j + (SL_i* P_m,i)
END FOR
```

```
# include "stdio.h"
# include "conio.h"
# include "math.h"
# include "string.h"
# include <stdlib.h>
# include <time.h>
# include <iostream>
# include <algorithm>
# include <time.h>


#define MX SUB 4
#define MX MACH 12
#define POP 800
#define PERIOD 7
#define T HOR 450
#define N PRDT 15
#define GEN 1000


int number of setup[POP],x[POP],cost[2][POP],penalty 2[POP],penalty 1[POP] ;
int current gen[POP][PERIOD][N PRDT],d[PERIOD][N_PRDT],inv[POP][PERIOD][N_PRD
T],quantity[POP][PERIOD][N PRDT];
int C[N PRDT+1][MX SUB][MX_MACH],P[MX_MACH][N_PRDT],S[MX_MACH][N_PRDT],Ca[MX_
MACH],ms[POP][PERIOD];
void intitial poppulation(),demand(),production_lot(),print_report(),inventor
y(),TR Cost();
void process time(), setup time(), Capacity(),print_report(int),print_report_
file(int), sublot(),sort b(int [POP][POP],int);
int  span(int [MX_SUB],int,int);



void initial_population(void)
{

    int j,i,k;

    srand(time( NULL ));
    //srand(1);

    for(i=0;i<POP;i++)
    {
        for (j=0;j<PERIOD;j++)
        {
            for (k=0;k<N_PRDT;k++)
            {
                current gen[i][j][k] =  rand()%2;
                if(current_gen[i][j][k]==1)
                {
                    current_gen[i][j][k]+=rand()%N_PRDT;
                }
            }
        }
    }
}


void Demand (void)
{
  int j,k;
  FILE * pFile;

  pFile = fopen ("Demand.txt","r");
  for (k=0;k<N_PRDT;k++)
```

```
  {
      for(j=0;j<PERIOD;j++)
      {
        fscanf (pFile, "%d",&d[j][k]);
      }
  }
  fclose (pFile);
}


void process_time(void)
{
  int j,k;
  FILE * pFile;

  pFile = fopen ("process.txt","r");
   for (k=0;k<N_PRDT;k++)
   {
      for(j=0;j<MX_MACH;j++)
      {
        fscanf (pFile, "%d",&P[j][k]);
      }
  }
  fclose (pFile);
}


void setup_time (void)
{
  int j,k;
  FILE * pFile;

  pFile = fopen ("setup.txt","r");
   for (k=0;k<N_PRDT;k++)
  {
      for(j=0;j<MX_MACH;j++)
      {
          fscanf (pFile, "%d",&S[j][k]);
      }
  }
  fclose (pFile);
}


void Capacity (void)
{
  int j;
  FILE * pFile;

  pFile = fopen ("Capacity.txt","r");
  for(j=0;j<MX_MACH;j++)
  {
      fscanf (pFile, "%d",&Ca[j]);
  }
  fclose (pFile);
}

int bottle_neck(int curr_ca[MX_MACH],int j)
{

    int i,bottle,b;

    b=curr ca[0]-S[0][j];
    if(b<0)b=0;
    bottle=b/P[0][j];
    for(i=1;i<MX_MACH;i++)
    {
        b=curr ca[i]-S[i][j];
        if(b<0)b=0;
```

```
        if(bottle>b/P[i][j])
            bottle=b/P[i][j];
    }

    return bottle;
}


void production_lot(void)
{
    int j,i,k,t,l,z,counter;
    int Cu De[N PRDT],current_ca[MX_MACH];
    int min(int,int);
    int small sub,large_sub;
    int SL[MX SUB];
    int bottle neck(int [MX MACH],int);
    //int S[MX_MACH][N_PRDT];

    for(i=0;i<POP;i++)
    {

        for(z=0;z<N PRDT;z++)
            Cu_De[z]=0;

        for(j=PERIOD-2;j>=1;j--)
        {

            counter=0;
            for(z=0;z<MX MACH;z++)
            current_ca[z]=Ca[z];


            for(t=N_PRDT;t>=0;t--)
            {

                for(l=0;l<N_PRDT;l++)
                {

                    if(current_gen[i][j][l]==t && t>0)
                    {
                        counter++;

                        Cu De[l]+=d[j+1][l];
                        quantity[i][j][l]=min(Cu_De[l],bottle_neck(current_ca
,l));

                        /*for(z=0;z<MX_MACH;z++)
                        {
                            current_ca[z]-=P[z][l]*quantity[i][j][l]+S[z][l];
                        }*/


                        while(1)
                        {
                            small sub=quantity[i][j][l]/MX SUB;
                            large_sub=quantity[i][j][l]-(small_sub)*(MX_SUB-1
);


                            for(k=0;k<MX_SUB-1;k++)
                            {
                                SL[k]=small sub;
                                SL[MX_SUB-1]=large_sub;
                            }

                            ms[i][j]=span(SL,counter,l);
                            if(ms[i][j]>T_HOR)
                            {
                                quantity[i][j][l]--;
```

```
                        }
                        else
                        {
                            Cu De[l]-=quantity[i][j][l];
                            break;
                        }
                    }

                    for(z=0;z<MX_MACH;z++)
                    {
                        current_ca[z]-=P[z][l]*quantity[i][j][l]+S[z][l];
                    }
                }
                else if(current_gen[i][j][l]==t && t==0)
                {
                    Cu De[l]+=d[j+1][l];
                    quantity[i][j][l]=0;
                }
            }// end of product seq search(l)
        }//end of seq(t)
    }//end of period(j)


    //first month
    j=0;
    counter=0;
    for(t=N_PRDT;t>=0;t--)
    {

        for(l=0;l<N_PRDT;l++)
        {

            if(current_gen[i][j][l]==t)
            {
                counter++;

                Cu De[l]+=d[j+1][l];
                quantity[i][j][l]=Cu_De[l];

                small sub=quantity[i][j][l]/MX SUB;
                large_sub=quantity[i][j][l]-(small_sub)*(MX_SUB-1);

                for(k=0;k<MX_SUB-1;k++)
                {
                    /*SL[k]=small sub;
                    SL[MX SUB-1]=large sub;*/
                    SL[k]=large sub;
                    SL[MX_SUB-1]=small_sub;

                }

                ms[i][j]=span(SL,counter,1);

            }
        }// end of product seq search(l)
    }//end of seq(t)


}//end of pop(i)
}


void inventory()
{
    int j,i,l,z;

    for(i=0;i<POP;i++)
    {
```

```
        for(z=0;z<N_PRDT;z++)
        {
            inv[i][0][z]=quantity[i][0][z];//-d[0][z];
        }

        for(j=1;j<PERIOD;j++)
        {
            for(l=0;l<N_PRDT;l++)
            {
                inv[i][j][l]= inv[i][j-1][l]+ quantity[i][j][l]-d[j][l];
            }
        }
    }
}


int max(int a,int b)
{
    int output;
    output=a;
    if(b>a)
        output=b;
    return output;
}

int min(int a,int b)
{
    int output;
    output=a;
    if(b<a)
        output=b;
    return output;
}



int span(int SL[MX_SUB],int counter,int l)
{
    int max(int,int);
    int i, j;


    C[counter][0][0]=C[counter-1][MX_SUB-1][0]+S[0][l]+(SL[0]*P[0][l]);
    for(i=1;i<MX_SUB;i++)
    {
            C[counter][i][0]= C[counter][i-1][0]+(SL[i]*P[0][l]);
    }

    for(j=1;j<MX_MACH;j++)
    {
            C[counter][0][j]=max(C[counter][0][j-1],C[counter-1][MX_SUB-1][j]
)+S[j][l]+(SL[0]*P[j][l]);
    }


    for(i=1;i<MX_SUB;i++)
    {
        for(j=1;j<MX_MACH;j++)
        {
            C[counter][i][j]=max(C[counter][i-1][j],C[counter][i][j-1])+SL[i]
*P[j][l];
        }
    }

    return C[counter][MX_SUB-1][MX_MACH-1];
}


void TR_Cost(void)
```

```
{
    int i,j,l,z;
    int TCU[MX_MACH];

    for(i=0;i<POP;i++)
    {
        number of setup[i]=0;
        for(j=0;j<PERIOD;j++)
        {
            for(l=0;l<N_PRDT;l++)
            {
                if(quantity[i][j][l]>0)
                    number_of_setup[i]+=1;
            }
        }
    }

    for(i=0;i<POP;i++)
    {
        x[i]=0;
        for(j=0;j<PERIOD;j++)
        {
            for(l=0;l<N_PRDT;l++)
            {
                x[i]+=inv[i][j][l];
            }
        }
    }

    for(i=0;i<POP;i++)
    {
        penalty 2[i]=0;
        for(j=0;j<PERIOD;j++)
        {
            if(ms[i][j]>T HOR)
                penalty_2[i]+=ms[i][j]-T_HOR;
        }

        penalty 1[i]=0;
        for(j=0;j<PERIOD;j++)
        {
            for(z=0;z<MX_MACH;z++)

            {
                TCU[z]=0;
                for(l=0;l<N_PRDT;l++)

                {
                    if(quantity[i][j][l]>0)

                    TCU[z]+=quantity[i][j][l]*P[z][l]+ S[z][l];
                }

                if(TCU[z]>Ca[z])
                penalty_1[i]+=TCU[z]- Ca[z];

            }
        }

        cost[0][i]=i;
        cost[1][i]=x[i]*10 + number_of_setup[i]*100 + penalty_2[i]*5000 + pen
alty_1[i]*5000;
    }
}


void print_report(int genno)
{
```

```
int i,j,k,t;


printf("Process time on machine j:\n");
for(j=0;j<MX_MACH;j++)
{
  for (k=0;k<N PRDT;k++)
    printf("%4d", P[j][k]);
    printf("\n");
}
printf("\n\n");


printf("Setup time on machine j: \n");
for(j=0;j<MX_MACH;j++)
{
  for (k=0;k<N PRDT;k++)
    printf("%4d", S[j][k]);
    printf("\n");
}
printf("\n\n");


printf("Capacity of machine j:    \n");
for(j=0;j<MX_MACH;j++)
{
    printf("%6d", Ca[j]);
}
printf("\n\n");



for(i=0;i<POP;i++)
{


printf("-----------------------------------------------------\n");
printf("-----------------------------------------------------\n");
printf("Generation: %d\n",genno);
printf("-----------------------------------------------------\n");
printf("-----------------------------------------------------\n");


    printf("Chromosome %d\n",i);
    for (k=0;k<N_PRDT;k++)
    {
        for(j=0;j<PERIOD;j++)
        printf("%5d", current_gen[i][j][k]);
        printf("\n");
    }
    printf("\n");

    printf("Demand for Products:""\n\n");
    printf("   Jan  Feb  Mar April May  Jun July  ""\n");
    for (k=0;k<N_PRDT;k++)
    {
       for(j=0;j<PERIOD;j++)
       printf("%5d",d[j][k]);
       printf("\n");
    }
    printf("\n");

    printf("Order Released for each Product:""\n\n");
    printf("   Jan  Feb  Mar April May  Jun July  ""\n");
    for (k=0;k<N_PRDT;k++)
    {
       for(j=0;j<PERIOD;j++)
       printf("%5d",quantity[i][j][k]);
```

```
            printf("\n");
        }
        printf("\n");

        printf("Makespans:""\n\n");
        for(j=0;j<PERIOD;j++)
        {
            printf("%5d", ms[i][j]);
        }
        printf("\n\n");

        printf("Inventory of each product at the end of each period:""\n\n");
        printf("   Jan  Feb  Mar April May  Jun July  ""\n");
        for (k=0;k<N_PRDT;k++)
        {
           for(j=0;j<PERIOD;j++)
           printf("%5d",inv[i][j][k]);
           printf("\n");
        }
        printf("\n");


        printf("\n-------------------------------------------------\n");
        printf("number of setups            %4d",number_of_setup[i]);
        printf("\n");
        printf("total Inventory             %4d",x[i]);
        printf("\n");
        printf("Capacity violation Penalty  %4d",penalty_1[i]);
        printf("\n");
        printf("time horizon Penalty        %4d",penalty_2[i]);
        printf("\n");
        for(t=1;t<POP;t++)
            if(cost[0][t]==i)
                printf("total relevant cost        $%4d",cost[1][t]);
        printf("\n");
        printf("\n\n\n");
    }
}


void print_report_file(int genno)
{

    int i,j,k,t;
    FILE *report;

    report=fopen("report.txt","w");


    fprintf(report,"Process time on machine j:\n");
    for(j=0;j<MX_MACH;j++)
    {
      for (k=0;k<N PRDT;k++)
        fprintf(report,"%4d", P[j][k]);
        fprintf(report,"\n");
    }
    fprintf(report,"\n\n");


    fprintf(report,"Setup time on machine j: \n");
    for(j=0;j<MX_MACH;j++)
    {
      for (k=0;k<N PRDT;k++)
        fprintf(report,"%4d", S[j][k]);
        fprintf(report,"\n");
    }
    fprintf(report,"\n\n");
```

```
    fprintf(report,"Capacity of machine j:     \n");
    for(j=0;j<MX_MACH;j++)
    {
        fprintf(report,"%6d", Ca[j]);
    }
    fprintf(report,"\n\n");


    for(i=0;i<POP;i++)
    {


    fprintf(report,"-------------------------------------------------\n");
    fprintf(report,"-------------------------------------------------\n");
    fprintf(report,"Generation: %d\n",genno);
    fprintf(report,"-------------------------------------------------\n");
    fprintf(report,"-------------------------------------------------\n");


        fprintf(report,"Chromosome %d\n",i);
        for (k=0;k<N_PRDT;k++)
        {
            for(j=0;j<PERIOD;j++)
            fprintf(report,"%5d", current_gen[i][j][k]);
            fprintf(report,"\n");
        }
        fprintf(report,"\n");

        fprintf(report,"Demand for Products:""\n\n");
        fprintf(report,"   Jan  Feb  Mar April May  Jun July  ""\n");
        for (k=0;k<N_PRDT;k++)
        {
           for(j=0;j<PERIOD;j++)
           fprintf(report,"%5d",d[j][k]);
           fprintf(report,"\n");
        }
        fprintf(report,"\n");

        fprintf(report,"Order Released for each Product:""\n\n");
        fprintf(report,"   Jan  Feb  Mar April May  Jun July  ""\n");
        for (k=0;k<N_PRDT;k++)
        {
           for(j=0;j<PERIOD;j++)
           fprintf(report,"%5d",quantity[i][j][k]);
           fprintf(report,"\n");
        }
        fprintf(report,"\n");

        fprintf(report,"Makespans:""\n\n");
        for(j=0;j<PERIOD;j++)
        {
            fprintf(report,"%5d", ms[i][j]);
        }
        fprintf(report,"\n\n");

        fprintf(report,"Inventory of each product at the end of each period:"
"\n\n");
        fprintf(report,"   Jan  Feb  Mar April May  Jun July  ""\n");
        for (k=0;k<N_PRDT;k++)
        {
           for(j=0;j<PERIOD;j++)
           fprintf(report,"%5d",inv[i][j][k]);
           fprintf(report,"\n");
        }
        fprintf(report,"\n");


        fprintf(report,"\n-------------------------------------------------\
```

```
n");
        fprintf(report,"number of setups            %4d",number_of_setup[i]);
        fprintf(report,"\n");
        fprintf(report,"total Inventory             %4d",x[i]);
        fprintf(report,"\n");
        fprintf(report,"Capacity violation Penalty  %4d",penalty_1[i]);
        fprintf(report,"\n");
        fprintf(report,"time horizon Penalty        %4d",penalty_2[i]);
        fprintf(report,"\n");
        for(t=1;t<POP;t++)
            if(cost[0][t]==i)
                fprintf(report,"total relevant cost        $%4d",cost[1][t]);
        fprintf(report,"\n");
        fprintf(report,"\n\n\n");
    }
}


void sort_b(int list[POP][POP],int count)
{

    int i,j,temp;
    for(i=0;i<count;i++)
        for(j=count;j>=i;--j)
            if(list[1][j-1]>list[1][j])
            {
                temp=list[0][j-1];
                list[0][j-1]=list[0][j];
                list[0][j]=temp;
                temp=list[1][j-1];
                list[1][j-1]=list[1][j];
                list[1][j]=temp;
            }
}


void gen()
{
    int i,j,current_gen_next[POP][PERIOD][N_PRDT],parent1,parent2,l,random_ba
d,bads[POP/4];

    //sort
    sort_b(cost,POP-1);


    //crossover first best 1/4
    for(i=0;i<(POP/4);i+=2)
    {
        parent1=cost[0][i];
        parent2=cost[0][i+1];

        for(j=0;j<PERIOD;j++)
        {
            for(l=0;l<N_PRDT;l++)
            {
                current gen next[i][j][l]  =current gen[parent1][j][l];
                current_gen_next[i+1][j][l]=current_gen[parent2][j][l];
            }
        }

        for(j=0;j<PERIOD;j++)
            if(j<(PERIOD/2))
            {
                for(l=0;l<N_PRDT;l++)
                {
                    current_gen_next[(POP/4)+i][j][l]=current_gen[parent1][j]
[l];
                    current_gen_next[(POP/4)+1+i][j][l]=current_gen[parent2][
j][l];
```

```
                }
            }
            else
            {
                for(l=0;l<N_PRDT;l++)
                {
                    current_gen_next[(POP/4)+i][j][l]=current_gen[parent2][j]
[l];
                    current_gen_next[(POP/4)+1+i][j][l]=current_gen[parent1][
j][l];
                }
            }
    }


    //version 1
    for(i=POP/2;i<POP;i++)
        for(j=0;j<PERIOD;j++)
            for(l=0;l<N PRDT;l++)
                current_gen_next[i][j][l]=current_gen[i-POP/4][j][l];


    //version 2
    /*for(i=POP/2;i<(3*POP/4);i++)
        for(j=0;j<PERIOD;j++)
            for(l=0;l<N PRDT;l++)
                current gen_next[i][j][l]=current_gen[i-POP/4][j][l];
    for(i=0;i<POP/4;i++)
    {
        random bad=rand()%(POP/2)+POP/2;
        for(j=0;j<i;j++)
            if(bads[j]==random_bad)
            {
                random_bad=rand()%(POP/2)+POP/2;
                j=-1;
            }
        bads[j]=random_bad;
    }
    for(i=(3*POP/4);i<POP;i++)
        for(j=0;j<PERIOD;j++)
            for(l=0;l<N PRDT;l++)
                current_gen_next[i][j][l]=current_gen[bads[i-(3*POP/4)]][j][l
];*/


    //change generations
    for(i=0;i<POP;i++)
        for(j=0;j<PERIOD;j++)
            for(l=0;l<N PRDT;l++)
                current_gen[i][j][l]=current_gen_next[i][j][l];

    //mutation
    for(i=0;i<POP;i++)
        for(j=0;j<PERIOD;j++)
            for(l=0;l<N PRDT;l++)
                if(rand()%25==0)
                {
                    current gen[i][j][l]=rand()%2;
                    if(current gen[i][j][l]==1)
                        current_gen[i][j][l]+=rand()%N_PRDT;
                }
}


void main(void)
{
    int i;
    time_t start,end;
```

```
    start = time (NULL);

    printf("Generating initial population.\n");
    initial population();
    Demand();
    setup time();
    process time ();
    Capacity();

    printf("running the GA\n");
    for(i=0;i<GEN;i++)
    {
        if(i>0)
            gen();
        production lot();
        inventory();
        TR Cost();
        //getch();
    }

    end = time (NULL);

    print report(i);
    print report file(i);
    printf("\nTime: %d\n",end-start);

}
```