OFCDM SYSTEMS OVER FADING CHANNELS

BY

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Along with the fast growing demand of information exchange, telecommunication systems are required to provide fast and reliable service to high-data-rate applications such as video conference, real-time broadcasting, and on-line gaming. In downlink transmission, orthogonal frequency and code division multiplexing (OFCDM) has been an attractive technique for high-data-rate applications. With two-dimensional spreading, in both time domain and frequency domain, OFCDM achieves diversity gains in multiuser scenarios. Moreover, the adjustable spreading factors (SF) give OFCDM systems the flexibility in transmission rate and diversity gain.

In this thesis, we focus on the downlink of OFCDM communication systems. The performance of OFCDM systems is investigated over Ricean fading channels with Rayleigh fading as special case. Code division multiple access (CDMA) technique is used to support multiuser communications, where users can transmit at the same time using the same frequency with the help of code sequences. We compare different
combining methods that are employed to achieve diversity gain. Moreover, channel
correlation is examined to see its effect on the system performance.

We also propose to combine multiple-input and multiple-output (MIMO) tech­
niques, specifically space-time block coding (STBC), with OFCDM systems. By
adding spatial diversity, a MIMO system can provide more reliable transmission com­
pared to a single-input and single-output (SISO) system. The space-time scheme
used in our study is Alamouti scheme [1], which employs $N = 2$ and $M$ antennas
at the transmitter side and receiver side respectively. In the thesis, we explain the
system structure, transmission and detection methods, and system performance of
such MIMO-OFCDM systems.

In our study, the expressions of system bit error rate (BER) are considered under
the condition that no multi-code interference (MCI) is present. The accuracy of the
BER expressions is verified when compared with the simulated ones for both SISO
and MIMO-OFCDM systems with different combining methods. These comparisons
are carried over different channels and with different system parameters to explore
the benefits of OFCDM based systems. Both analytical and simulation results show
the large diversity gains achieved when incorporating STBC with OFCDM.
ACKNOWLEDGMENTS

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<td>AWGN</td>
<td>additive white Gaussian noise</td>
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<td>BER</td>
<td>bit error rate</td>
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<td>BPSK</td>
<td>binary phase-shift keying</td>
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<td>CDMA</td>
<td>code division multiple access</td>
</tr>
<tr>
<td>CSI</td>
<td>channel state information</td>
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<td>DAB</td>
<td>digital audio broadcasting</td>
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<td>DVB</td>
<td>digital video broadcasting</td>
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<td>DS-CDMA</td>
<td>direct-sequence code division multiple access</td>
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<td>EGC</td>
<td>equal gain combining</td>
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<td>FDMA</td>
<td>frequency division multiple access</td>
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<td>FFT</td>
<td>fast Fourier transform</td>
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<td>IFFT</td>
<td>inverse fast Fourier transform</td>
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<td>ICI</td>
<td>intercarrier interference</td>
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<td>ISI</td>
<td>intersymbol interference</td>
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<tr>
<td>LOS</td>
<td>line-of-sight</td>
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<tr>
<td>Acronym</td>
<td>Description</td>
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<tr>
<td>MAI</td>
<td>multiple access interference</td>
</tr>
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<td>MC-CDMA</td>
<td>multicarrier code division multiple access</td>
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<tr>
<td>MC-DS-CDMA</td>
<td>multicarrier direct-sequence code division multiple access</td>
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<td>MCI</td>
<td>multi-code interference</td>
</tr>
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<td>MIMO</td>
<td>multiple-input and multiple-output</td>
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<td>MMSE</td>
<td>minimum mean-square error</td>
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<td>MRC</td>
<td>maximal ratio combining</td>
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<td>OFDM</td>
<td>orthogonal frequency-division multiplexing</td>
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<td>OFCDM</td>
<td>orthogonal frequency and code division multiplexing</td>
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<tr>
<td>RV</td>
<td>random variables</td>
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<tr>
<td>PAPR</td>
<td>peak-to-average power ratio</td>
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<td>PDF</td>
<td>probability density function</td>
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<td>OVSF</td>
<td>orthogonal variable spreading factor</td>
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<td>SC</td>
<td>selection combining</td>
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<td>SDMA</td>
<td>space division multiple access</td>
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<td>SF</td>
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<td>SISO</td>
<td>single-input and single-output</td>
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<td>SNR</td>
<td>signal-to-noise ratio</td>
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<td>SSMA</td>
<td>spread spectrum multiple access</td>
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<td>STBC</td>
<td>space-time block coding</td>
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<td>Acronym</td>
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<td>STTC</td>
<td>space-time turbo coding</td>
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<td>time division multiple access</td>
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<td>WLAN</td>
<td>Wireless Local Area Networks</td>
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<td>WLL</td>
<td>wireless local loop</td>
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LIST OF SYMBOLS

ζ ............... system load

κ ............... ratio of the dominant component to the total power of scattered waves in Ricean fading channel

μ_{h_1,h_2} ............... correlation between channels h_1 and h_2

η(l_i,j) ............... AWGN on the l_i-th sub-carrier during the j-th time slot

σ_f^2 ............... variance of AWGN that approximates MCI when K_c > 50

σ_n^2 ............... variance of the noise added to each OFCDM symbol

\tilde{\rho} ............... ratio of total transmitted signal power for each symbol to the background noise power

C ............... overall spreading code

C^f ............... frequency domain spreading code
$C^T$ .............. time domain spreading code

$d_k$ .............. transmitted data symbol of the $k$-th user

$\hat{d}_k$ .............. hard decision of the $k$-th user's data symbol

$E_s$ .............. transmitted signal energy

$\vec{e}$ .............. error vector between transmitted and received data symbol

$K$ .............. total number of spreading codes used

$K_e$ .............. number of interfering codes

$h_{l_i}$ .............. channel fading coefficient of the $l_i$-th sub-carrier

$L$ .............. total number of sub-carriers of one OFCDM frame

$l_i$ .............. index of the $i$-th sub-carrier transmitting the $k$-th user's symbols

$M$ .............. number of receive antennas

$N$ .............. number of transmit antennas

$N_B$ .............. number of data symbols spread with the same spreading code

$N_F$ .............. frequency domain spreading factor

$N_S$ .............. length of overall spreading code

$N_T$ .............. time domain spreading factor

$n_0$ .............. AWGN noise
$P_{e|h}$ ................ probability of bit error conditioned on the channel

$r_k(l_i)$ .............. output of the $k$-th user time domain desproader on the $l_i$-th sub-carrier

$w(l_i)$ .............. combining weight of the $l_i$-th sub-carrier

$\bar{Y}$ .............. transformed received signals

$y_k$ .............. output of the $k$-th user frequency domain desproader

$z_k$ .............. decision variable of the $k$-th user's data symbol
CHAPTER 1

INTRODUCTION

1.1 Introduction

In wireless communication systems, electromagnetic energy that is carrying data propagates from transmit antennas to receive antennas. Major problems raised in electromagnetic wave propagation are the multipath fading and multiple access interference (MAI). When the transmitted signal arrives at the receiver through different propagation paths with different delays, intersymbol interference may occur. Moreover, signals via different paths may be destructive due to phase difference, and this phenomenon is called signal fading. Reflection, diffraction, and scattering are three major mechanisms that cause multipath propagation [2]. Multiple-input and multiple-output (MIMO) techniques such as space-time trellis coding (STTC) and Space-time block coding (STBC) are introduced and widely studied to overcome the problem of fading through diversity [3][4]. MIMO techniques implement multiple transmit and receive antennas to transmit the same signal through independent channels to provide reliable wireless transmission. STTC and STBC are two different MIMO coding techniques used to achieve spatial diversity for reliable transmission [2].

Compared to a single-input and single-output (SISO) system, a MIMO system can support high-data-rate applications and improve received signal quality [5]. At the transmitter side, spatial diversity can be achieved by simple yet efficient STBC. The
first STBC introduced by Alamouti reaches full transmit diversity with two transmit antennas [1].

In a transmission system where the transmitted signal experiences multipath fading and noise degradation, a combining technique is needed to identify and combine the components from the same transmitted signal. These combining techniques can generally be grouped into selection combining (SC), gain combining, and hybrid selection/gain combining [5]. In selection combining, the received signal from the transmission path with the highest signal-to-noise ratio (SNR) is selected for detection. However in gain combining, signals from all transmission paths are combined to detect the transmitted data. Some typical gain combining techniques are maximal ratio combining (MRC), equal gain combining (EGC), and minimum mean-square error (MMSE) combining.

System bandwidth, transmission rate, and error probability are three main characteristics in evaluating wireless communication systems. To optimize the use of available bandwidth is one of the major topics in the study of wireless communication systems. Multiple-access systems provide a solution for optimizing frequency use and maximizing flexibility. For instance, frequency-division multiple access (FDMA), time-division multiple access (TDMA), spread spectrum multiple access (SSMA), and space-division multiple access (SDMA) are well investigated and implemented in different communication systems [6]. Also, orthogonal frequency-division multiplexing (OFDM) is one useful multiplexing scheme that has attracted many researchers [7]. It divides data into a set of parallel streams to be transmitted using mutually orthogonal frequency bands. Because of this orthogonality, the data streams can be
transmitted at the same time to increase the system transmission rate. On the other hand, CDMA is one form of "spread-spectrum" signaling [8] that is broadly used in telecommunication systems. In a CDMA system, users are assigned orthogonal codes so that different users can share the same frequency band at the same time with little or no interference. One integration of OFDM and CDMA techniques, called OFCDM, takes advantage of both OFDM and CDMA systems to gain frequency diversity and achieve high but flexible transmission rates [9].

With two-dimensional spreading, OFCDM is an attractive transmission technique for high-data-rate applications. In OFCDM, spreading in frequency domain provides frequency diversity gain. On the other hand, a large time domain spreading factor allows more users to access the system at the same time, whereas a small time domain spreading factor is more suitable for high-data-rate applications [9]. Combined with MIMO technology, a MIMO-OFCDM system can provide high reliability with flexible transmission rates. We can therefore expect a great potential from MIMO-OFCDM systems. Recently, many studies have been carried out for OFCDM systems on how to gain frequency domain diversity and on the effect of multi-code interference (MCI) on achieving this diversity [9]-[12]. In [9], the authors presented the system structure of SISO-OFCDM. They also explained the detection method using MMSE combining and iterative MCI cancellation scheme. In [13], the authors briefly mention the MIMO-OFCDM system structure. However, the system performance in terms of BER under different fading channels has not been addressed. Therefore, this thesis focuses on the system performance of OFCDM systems over slow fading channels. Semi-analytical results of BER are obtained for both SISO and MIMO-OFCDM sys-
tems using different combining techniques. We also explain the system structures and
detection methods for both SISO-OFCDM and MIMO-OFCDM systems in detail.

1.2 Thesis Contribution

The major contributions of this thesis are listed as follows:

1. The study of OFCDM systems, including the system structure, transmission
techniques involved, and detection methods. OFCDM systems over Rayleigh fading and Ricean fading channels with MRC, EGC, and MMSE combining are investigated.

2. We illustrate how to extend SISO-OFCDM systems to MIMO-OFCDM systems by adding spatial diversity. In MIMO-OFCDM systems, we employ Alamouti scheme to achieve transmit diversity of $N = 2$ and use MRC, EGC, and MMSE combining to obtain receive diversity $M$, where $M$ is the number of receive antennas.

3. We present semi-analytical results for the BER of both SISO-OFCDM and MIMO-OFCDM systems over fading channels. The results obtained are applicable for OFCDM systems over any channel model.

1.3 Thesis Outline

The contents of this thesis are outlined as follows:

In Chapter 2, we give a review of some background of major concepts involved in this thesis. We present OFDM, CDMA, and OFCDM communication techniques.
Finally, we review some commonly used detection methods.

Chapter 3 focuses on SISO-OFCDM systems with two dimensional spreading over slow fading channels. MRC, EGC, and MMSE combining techniques used at the receiver are studied. The performance of different multiuser scenarios is investigated where the BER is analyzed. We also derive closed-form semi-analytical expressions for the BER in systems with different combining techniques.

In Chapter 4, we extend the results in Chapter 3 to MIMO-OFCDM systems when using STBC. For $M$ receive antennas, gain combining techniques are used to extract receive diversity. The same as SISO-OFCDM systems, the performance of different combining schemes are studied for multiuser scenarios. BER semi-analytical results are then presented.

In Chapter 5 we present our conclusions and possible future directions in this area.
CHAPTER 2

BACKGROUNDS

In this chapter, we explain some key concepts related to the work in this thesis. These concepts include multipath fading, transmission techniques including OFDM, CDMA, and STBC, detection and combining methods such as MRC, EGC, and MMSE.

2.1 Multipath Fading

In wireless communication systems, the physical path from transmit antennas to receive antennas is not always a straight line of sight but subject to multipath propagation [2]. Each copy of the original electromagnetic wave goes through a unique propagation path, and along these paths signals may arrive at the receiver side at different times with different attenuation and different phase. Thus, the overall received signal may be constructive or destructive causing signal fading.

Multipath fading channels can be grouped into frequency flat-fading channels and frequency-selective fading channels. In flat-fading channels, the coherence bandwidth of the channel is larger than the signal bandwidth, and the signal on all frequency bands will experience the same fading amplitude. However, in frequency-selective fading channels, the bandwidth of the channel is smaller than the signal bandwidth, and different frequency components of the same signal may experience different fading amplitudes [14].
Because of the uniqueness of each multipath fading channel, some representative channel models are used for study and research. Two commonly used fading channel models are Rayleigh fading and Ricean fading channels.

2.1.1 Rayleigh Fading

Rayleigh fading model is used to characterize physical channels with large amount of randomly moving scattered signal components received by the receive antennas. When the scattering mechanism dominates the electromagnetic wave propagation, the channel fading coefficient follows a Rayleigh distribution. It is known that Rayleigh distribution is closely related to central chi-square distribution. Suppose we have two independent Gaussian distributed random variables (RV) \( X_1 \) and \( X_2 \), each with zero-mean and variance \( \sigma^2 \). Then the RV \( Y = X_1^2 + X_2^2 \) follows chi-square distribution with two degrees of freedom. Let us define a RV \( R = \sqrt{Y} = \sqrt{X_1^2 + X_2^2} \). Thus, the distribution of \( R \) is said to have a Rayleigh distribution [14].

The probability density function (PDF) can be derived from the PDF of chi-square distributed RV \( Y \) to give [14]

\[
p_{Rayleigh}(r) = \frac{r}{\sigma^2} \cdot e^{(-\frac{r^2}{2\sigma^2})}, \quad r \geq 0. \tag{2.1}
\]

From (2.1) we can see that the Rayleigh PDF is characterized by \( \sigma^2 \), the variance at each dimension. Let us define \( \Omega = \sigma^2 \), the PDF of the Rayleigh RV is shown in Figure 2.1 for different values of \( \Omega \).
2.1.2 Ricean Fading

In some cases, besides the scattering paths, there exists a straight path from the transmitter to the receiver, known as line-of-sight (LOS). When the presence of LOS cannot be ignored, Ricean fading is more suitable for modeling these channels. In this case, the Ricean fading channel model is a summation of LOS and Rayleigh distributed scattered components. Different from Rayleigh distribution, the Ricean distribution is closely related to noncentral chi-square distribution [14].

Suppose we have two independent Gaussian distributed RVs $X_1$ and $X_2$ with non-zero means $m_1$ and $m_2$ respectively, and variance $\sigma^2$. Then, the RV $Y = X_1^2 + X_2^2$ follows noncentral chi-square distribution with two degrees of freedom. Similar to the

![Figure 2.1: PDF of Rayleigh distribution with different $\Omega$.](image)
Rayleigh distribution, let us define a RV \( R = \sqrt{Y} = \sqrt{X_1^2 + X_2^2} \), then the distribution of \( R \) is defined as a Ricean distribution. The relationship between the noncentrality parameter \( s \) of the Ricean distribution and the mean of \( X_1 \) and \( X_2 \) is [14]

\[
s^2 = m_1^2 + m_2^2. \tag{2.2}
\]

The PDF of the Ricean RV can be derived from the PDF of noncentral chi-square distributed RV \( Y \), to give [14]

\[
p_{\text{Ricean}}(r) = \frac{r}{\sigma^2} \cdot e^{-\frac{r^2 + s^2}{2\sigma^2}} \cdot I_0\left(\frac{rs}{\sigma^2}\right), \quad r \geq 0. \tag{2.3}
\]

In (2.3), \( I_0 \) is the 0-th order modified Bessel function of the first kind.

The PDF of the Ricean distribution is characterized by the ratio of the dominant component and the total power of scattered waves. We can denote this ratio by \( \kappa \), and for the normalized envelope, the Ricean PDF is given by [15]

\[
p_{\text{Ricean}}(r) = \frac{2(1 + \kappa)}{e^{\kappa + (1 + \kappa)r^2}} \cdot r \cdot I_0(2r \cdot \sqrt{\kappa(1 + \kappa)}). \tag{2.4}
\]

The PDF of Ricean RV with different values of \( \kappa \) is shown in Figure 2.2. From (2.3) we can see that when \( \kappa = 0 \), the Ricean distribution reduces to the Rayleigh distribution.

### 2.1.3 Channel Correlation

In wireless communication systems, different channels may have certain degrees of similarity, known as channel correlation. Suppose we have two channels \( h_1, h_2 \), and we take \( L \) samples from each channel marked as \( s^1_l \) and \( s^2_l \), where \( l = 1, 2, \ldots, L \). When
the variance of the channel distributions are \( \sigma_{h_1}^2 \) and \( \sigma_{h_2}^2 \), and the mean values are \( \overline{m_{h_1}} \) and \( \overline{m_{h_2}} \) respectively, the correlation between these two channels is given by [7]

\[
\mu_{h_1,h_2} = \frac{\sum_{i=1}^{L} (s_i^1 - \overline{m_{h_1}})(s_i^2 - \overline{m_{h_2}})}{(L-1)\sigma_{h_1}\sigma_{h_2}}. 
\] (2.5)

Spatial channels can have correlation, and sub-carriers in the same OFDM system may also have correlation. In an OFCDM system, each transmitted data spreads in frequency domain to obtain frequency diversity gain. In order to achieve maximum frequency diversity, all the sub-carriers dedicated to the same transmitted data have to be mutually independent. If sub-carriers are correlated, the frequency diversity will
be reduced and the BER performance will be degraded. In most cases presented in this thesis, we assume to use ideal interleavers to maximize the frequency diversity, where all the sub-carriers are independent. However, we will also show the BER performance degradation due to correlation in Rayleigh fading channels. The unit power correlated Rayleigh fading channels, denoted as $h_1$ and $h_2$, with given correlation coefficient can be generated according to the scheme introduced in [16]. The steps are briefly given below [16]:

1. For a given correlation, $\rho_{h_1,h_2}$, find the root $\lambda$, $0 \leq \lambda \leq 1$, of the empirical equation

$$0.2155\lambda^3 + 0.7352\lambda^2 + 0.0465\lambda - 0.0026 - \rho_{h_1,h_2} = 0. \quad (2.6)$$

2. Generate two unit power uncorrelated Rayleigh fading signals, $\omega_1$ and $\omega_2$.

3. Calculate the coloring matrix $L$ by

$$L = \begin{bmatrix} 1 & 0 \\ \frac{\lambda(1+i)}{\sqrt{2}} & \sqrt{1 - \lambda^2} \end{bmatrix}, \quad (2.7)$$

where $i$ is the square root of $-1$.

4. Calculate $X$ by

$$X = L \cdot \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \quad (2.8)$$

5. $x_1$ and $x_2$ are the desired Rayleigh fading channel coefficients.

Figure 2.3 shows Rayleigh fading envelopes with channel correlation coefficient equals to 0.95 and 0.05 respectively. In our work when correlation is considered, the
correlated Rayleigh fading channels are generated according to the scheme described above.

2.2 OFDM

As the name suggests, OFDM system employs a number of orthogonal frequencies to transmit signals. In OFDM systems, user data is split into streams and loaded onto mutually orthogonal sub-carriers that are then multiplexed to generate an OFDM symbol for transmission [6]. The key feature is the orthogonality in frequency domain among those sub-carriers. Suppose we have a base sub-carrier with frequency $f$ and period $T$, where $f = 1/T$, then the frequency of each sub-carrier in the OFDM
system is equal to $f$ multiplied by a unique positive integer $k$. Let $m$ and $n$ be two different positive integers, then $mf$ and $nf$ can represent two different sub-carriers in the OFDM system. Because of the orthogonality, the sub-carriers carrying data in an OFDM system can be multiplexed together to transmit simultaneously. At the receiver side, matched filters corresponding to each sub-carrier are used to demultiplex and extract the data.

One advantage of OFDM systems is that it can minimize intersymbol interference (ISI). As suggested in Figure 2.4, compared to single carrier systems, an OFDM system extends the data symbol duration by allowing a data stream to transmit in parallel; therefore, ISI is reduced. Another advantage is that OFDM can increase the transmission rate, which is more suitable for high-data-rate applications. From Figure 2.5 we can see that by allowing data streams to transmit in parallel, an OFDM system takes a small fraction of time to transmit the same amount of data compared to a single carrier system. The third advantage is that OFDM helps to handle severe
channel fading and reduce outage probability [17]. This can be explained as follows. In a communication system, some frequency bands may encounter severe interference or fading. If all data is transmitted through those deteriorated frequency bands, outage will occur. However, an OFDM system employs different frequency bands to transmit the data from the same user. Even if one or few sub-carriers encounter severe fading, the whole OFDM system can still maintain a reasonable performance.

In OFDM systems, the serial data is split into parallel streams and mapped onto different, mutually orthogonal sub-carriers. Then by inverse fast Fourier transform (IFFT) operation, the frequency domain signal is converted into time domain for transmission. Orthogonality is the key characteristic of OFDM technique. However, the physical channel in time domain may have variations during one OFDM frame, which destroys the orthogonality and power leakage among sub-carriers. This effect is known as intercarrier interference (ICI) [18]. In [19], some techniques are presented to
reduce ICI. Another disadvantage of OFDM is that it has high peak-to-average power ratio (PAPR). Many profound studies have been carried out, and different solutions of peak-to-average power control are proposed for both SISO and MIMO-OFDM systems [20]-[24].

During time domain transmission of OFDM system, ISI may occur due to multi-path delays. ISI can be mitigated by inserting guard intervals (a redundant symbol extension) between transmitted sequences [6]. However, inserting guard intervals causes deduction in transmission efficiency. In [25] and [26], the authors proposed coding techniques for OFDM systems to restore the transmission efficiency without sacrificing system performance.

After experiencing channel fading and noise, the received time domain signal is converted back into frequency domain by fast Fourier transform (FFT) [6][17]. Matched filters are then used to extract the data streams from each sub-carrier. Those received parallel data streams are converted back to serial data and then used for detection.

Because of the advantages of OFDM, many high-speed wireless transmission schemes such as MC-CDMA and OFCDM are based on OFDM technique [9][27][28]. New studies mainly focus on MIMO-OFDM systems that are more suitable for high-data-rate applications than traditional OFDM systems [29]-[32].

2.3 CDMA

Similar to OFDM, which uses orthogonal frequencies to transmit data at the same time, CDMA uses orthogonal codes to transmit data using the same frequency
at the same time. Now, CDMA becomes a widely used channel multiple access method that allows multiple users to access the channel at the same time. In a basic CDMA system, the data from each user is assigned a unique code, which is also called signature waveform. For a length-$L$ code, $C_k$, where $C_k = \left[ c_0^k \ c_1^k \ \ldots \ c_{L-1}^k \right]$, the component $c_l^k$ ($l = 0, 1, \ldots, L - 1$) is called a chip [14]. The most important property of the codes is due to its orthogonality. Suppose we have two length-$L$ codes $C_i$ and $C_j$ ($i \neq j$) assigned to two different users. The orthogonality implies that

$$C_i C_j^T = \sum_{l=0}^{L-1} c_l^i \cdot c_l^j = 0,$$  \hspace{1cm} (2.9)

where $(\cdot)^T$ represents transpose operation.

Because of the orthogonality, the implementation of CDMA is simple. At the transmitter side, each user's data is spread by multiplying a unique code from a code set [14]. Then, the spread signals from different users are multiplexed and transmitted at the same time. The received signal at the receiver side is given by

$$r = \sum_{k=0}^{K-1} \sqrt{E_s} \cdot h \cdot d_k \cdot C_k + n_0,$$  \hspace{1cm} (2.10)

where $K$ is the number of users, $E_s$ is the transmitted signal energy for each symbol, $h$ is the channel coefficient, $d_k$ is the transmitted data for the $k$-th user, $C_k$ is the unique code assigned to the $k$-th user, and $n_0$ is the additive white Gaussian noise (AWGN). Here we assume all users' signals are synchronized during transmission, and we keep this assumption throughout the thesis. At the receiver side, the received signal is applied to a filter matched to the user's code followed by a decision to recover the transmitted data.
One common spreading technique is direct-sequence code division multiple access (DS-CDMA). A DS-CDMA signal is generated by direct multiplication of user data by its code sequence [33]. Spread by orthogonal codes, data from different users can be multiplexed and transmitted at the same time. At the receiver side, the received data is applied to a bank of matched filters to recover each user data.

By using orthogonal codes, CDMA system can support multi-user scenario and can provide flexible transmission rate by varying the length of the spreading codes [34][35]. Nowadays, this multi-user transmission scheme is combined with MIMO technology to provide reliable service for both uplink and downlink transmission [36]-[39].
2.4 Orthogonal Variable Spreading Factor

Orthogonal variable spreading factor (OVSF) is a simple technique used to generate orthogonal codes for CDMA systems. Figure 2.6 shows how OVSF codes are generated by using a complete binary tree. Depending on the level in the tree where an OVSF code is located, the spreading factor of the OVSF code is determined. An OVSF code can be assigned to a user if and only if the code and all of its ancestor OVSF codes and descendant OVSF codes in the tree have not been assigned to other users. Once the level (length) of the OVSF code is chosen, the total data rate that can be assigned to users is fixed [40]. Hence, an OVSF code with small SF is ideal for high-data-rate applications.

2.5 MC-CDMA

One combination of OFDM with CDMA techniques is MC-CDMA. Basic concepts of MC-CDMA can be found in [41] and [42]. In brief, MC-CDMA system spread each symbol by a unique spreading code, then transmit spread symbols with an OFDM system.

According to [41], MC-CDMA can be categorized into two groups. One group spreads data symbols in frequency domain, then the spread replicas of the same symbol are combined to recover the transmitted symbol. Normally, MC-CDMA refers to this group. Another group spreads data symbols in time domain to form data streams. Then different streams are transmitted in parallel in an OFDM system. This MC-CDMA scheme is also called multicarrier direct-sequence code division multiple
access (MC-DS-CDMA) [43]. Frame structures of MC-CDMA and MC-DS-CDMA are compared in Figure 2.7.

MC-CDMA has the same drawbacks as OFDM since it employs OFDM system to transmit data symbols. However, by implementing CDMA scheme to multiplex different symbols, MC-CDMA system can lower the symbol rate in each sub-carrier. Thus, symbol duration is enlarged, which makes it easier to quasi-synchronize the transmissions [41].

With the advantages offered by MIMO technology, the combination of MIMO and MC-CDMA is widely studied to exploit the benefits of MC-CDMA in multi-user scenarios [44]-[46]. For instance, [44] presents an example of MIMO-MC-CDMA system using STBC, and [46] explains how to implement space-time coding and spatial multiplexing to benefit from both spatial diversity and at the same time improve spectral efficiency.

2.6 OFCDM

Another integration of OFDM and CDMA is called OFCDM. Typical implementation of CDMA into OFDM-based system includes MC-DS-CDMA, MC-CDMA, and OFCDM. Figure 2.7 presents the difference among these three schemes. MC-DS-CDMA systems spread symbols in time domain only, so they cannot achieve frequency domain diversity. On the other hand, MC-CDMA systems spread symbols in frequency domain only, so they cannot adapt to variable transmission rates. In an OFCDM system, symbols are spread in both frequency and time domain. Therefore, OFCDM systems can deliver frequency diversity gain and can be adapted to appli-
Figure 2.7: Different spreading schemes in multicarrier systems

Communications with different transmission rates, which is preferred in the 4G transmission systems [28][47].

In frequency-selective channels, signal transmitted on some sub-carriers may experience deep fade or outage. Therefore, if the spreading factors are too small, the OFCDM system cannot fully benefit from the multipath diversity. On the other hand, based on the OVSF code assignment criteria, when assigning the OVSF code with a large spreading factor to a user may preclude a larger number of OVSF codes with small spreading factors [40]. Therefore, the length of spreading codes in frequency domain and time domain should be properly chosen to satisfy the system requirements.

In the frequency domain, the sub-carriers of an OFDM system are expected to experience different fading and hence the orthogonality is no longer preserved. Therefore, at the receiver side, a combining technique is employed for de-spreading. In these OFCDM systems, gain combining methods such as MRC, EGC, and MMSE combi-
2.7 MIMO Systems

Nowadays, there is a rapid growth in the demand of high-speed and reliable wireless communication services. Increasing the diversity in the frequency domain might be a solution. However in reality, the frequency spectrum is limited and shared by different users, whose number is also increasing rapidly. On the other hand, high-speed applications put strong constrain on transmission time, which restricts the time domain diversity within certain degree. Moreover, spatial diversity has attracted research attention since it can help to adapt high-speed applications in multipath environments. Spatial diversity is achieved by implementing suitable transmitting and receiving schemes with multiple transmit and multiple receive antennas, known MIMO systems. Compared to SISO systems, MIMO systems support higher data rates with better reliability. Therefore, MIMO technology has already been included in the 4G standards [28][48].

Figure 2.8 presents a brief structure of a MIMO system. At the transmitter
side, data is transmitted from \( N \) transmit antennas according to some transmission technique. After transmission through different channels, \( M \) receive antennas pick up the signals which are then sent for detection. The channel matrix of a MIMO system is defined as \( N \times M \) matrix \( \mathbf{H} \):

\[
\mathbf{H} = \begin{bmatrix}
    h_{1,1} & h_{2,1} & \ldots & h_{M,1} \\
    h_{1,2} & h_{2,2} & \ldots & h_{M,2} \\
    \vdots & \vdots & \ddots & \vdots \\
    h_{1,N} & h_{2,N} & \ldots & h_{M,N}
\end{bmatrix}.
\] (2.11)

The element \( h_{i,j} \) \((i = 1, 2, \ldots, M, j = 1, 2, \ldots, N)\) represents the fading coefficient corresponding to the link from the \( j \)-th transmit antenna to the \( i \)-th receive antenna. When all channel coefficients are independent, maximum spatial diversity of \( N \times M \) is obtained. To achieve this maximum spatial diversity, the antenna spacing should be adjusted to be much larger than the transmitted signal wavelength [14]. If the antennas at either transmitter side or receiver side are not well physically separated, correlation between channels will degrade the system performance [5].

In a MIMO system, user data, \( d_{0} \), is first spread over the \( N \) transmit antennas according to a transmit scheme to achieve transmit diversity gain [2][5]. Normally the transmitter side has no knowledge of the channel state information (CSI), hence the transmitted power is equally allocated to all transmit antennas. At the receiver side, the received signal picked up by the \( i \)-th receive antenna is given by

\[
r_{i} = \sum_{i=1}^{N} \sqrt{\frac{E_{s}}{N}} \cdot d_{0,j} \cdot h_{i,j} + n_{i},
\] (2.12)

where \( E_{s} \) is the total transmitted energy for \( d_{0} \), \( d_{0,j} \) is the user data spread on the
\( j \)-th transmit antenna according to the selected space-time coding scheme, and \( n_i \) is the AWGN at the \( i \)-th receiver antenna. We can also represent the MIMO system in matrix form as:

\[
\mathbf{r} = \sqrt{\frac{E_s}{N}} \cdot \mathbf{d}_0 \cdot \mathbf{H} + \mathbf{n},
\]

where

\[
\mathbf{r} = \begin{bmatrix} r_1 & r_2 & \ldots & r_M \end{bmatrix},
\]

\[
\mathbf{d}_0 = \begin{bmatrix} d_{0,1} & d_{0,2} & \ldots & d_{0,N} \end{bmatrix},
\]

\[
\mathbf{n} = \begin{bmatrix} n_1 & n_2 & \ldots & n_M \end{bmatrix}.
\]

In MIMO systems, maximum likelihood detection can be used at the receiver side to give data estimates [2]:

\[
\hat{d}_0 = \arg \min_{d_0} \sum_{i=1}^{M} \left| r_i - \sum_{j=1}^{N} \sqrt{\frac{E_s}{N}} \cdot d_{0,j} \cdot h_{i,j} \right|^2.
\]

### 2.7.1 STTC

First introduced in [3], STTC transmits multiple copies of each data symbol during different time slots over different transmit antennas so that to achieve diversity. As suggested by its name, the transmit scheme of STTC is based on trellis coding. Figure 2.9 presents an example STTC encoding method for QPSK symbols using two transmit antennas. Normally the encoder is initialized to state \( S_0 \). At each step of the trellis, when current state is \( S_a \) (\( a = 0, 1, 2, 3 \)) and the input is \( b \) (\( b = 0, 1, 2, 3 \)), the encoder output is \( \begin{bmatrix} a & b \end{bmatrix} \). The two symbols of the encoder output are mapped to the two transmit antennas respectively for transmission. Then the encoder changes to state \( S_6 \) according to the four-state code trellis and starts next step of encoding.
In a general MIMO system with $N$ transmit antennas and $M$ receive antennas, the received signal at the $m$-th ($m = 1, 2, ..., M$) receive antenna is given by

$$y_m(t) = \sqrt{E_s} \sum_{n=1}^{N} h_{m,n} x_n(t) + \eta_m(t),$$

where $t$ represents the time slot, $h_{m,n}$ is the channel fading coefficient from the $n$-th transmit antenna to the $m$-th receive antenna, $x_n(t)$ is the encoded symbol transmitted from the $n$-th transmit antenna during $t$, and $\eta_m(t)$ represents the noise added to the $m$-th receive antenna during $t$. Then, the decoding of STTC is based on Viterbi algorithm [2][3].

Since STTC is based on trellis code, it can provide coding gain on top of the diversity gain. By using more complex decoder, STTC may deliver better BER performance than STBC introduced in the next section.
2.7.2 STBC

In some cases, receive diversity is hard to achieve. For example, the receive antennas in a mobile phone may not be separated far enough to pick up signals through independent paths [5]. Therefore, transmit diversity becomes crucial in order to achieve spatial diversity. STBC can help to achieve transmit diversity in a simple way. With STBC, a MIMO system transmits multiple copies of the same data across different transmit antennas during different time slots. At the receiver side, the received signals from different receive antennas are combined through a linear process to extract the signal for detection [2]. The simplest STBC scheme was introduced by Alamouti [1], which exploits full transmit diversity with two transmit antennas [2]. A system using Alamouti scheme with one receive antenna is show in Figure 2.10.

In Alamouti scheme, two symbols are transmitted at the same time from two different transmit antennas. It takes two time slots to transmit two symbols, so Alamouti scheme is full rate. Suppose user data $x_1$ and $x_2$ will be transmitted according to Alamouti scheme. During the first time slot, $T_1$, $x_1$ is transmitted from
the first transmit antenna and $x_2$ is transmitted from the second transmit antenna. During the second time slot, $T_2$, $-x_2^*$ is transmitted from the first transmit antenna and $x_1^*$ is transmitted from the second transmit antenna, where $(\cdot)^*$ represents complex conjugate operation.

At the receiver side, the received signal for the two consecutive time slots is given by

$$y_{T_1} = \sqrt{\frac{E_s}{2}} (h_{1,1} \cdot x_1 + h_{2,1} \cdot x_2) + n_1$$

(2.19)

$$y_{T_2} = \sqrt{\frac{E_s}{2}} (-h_{1,1} \cdot x_2^* + h_{2,1} \cdot x_1^*) + n_2,$$

(2.20)

where $n_1$ and $n_2$ are AWGN samples. One important assumption is that channel coefficients should be fixed during the two consecutive time slots.

Assuming the receiver side has perfect knowledge of the channel state information, the maximum likelihood estimates are [2]

$$\hat{x}_1 = \arg \max_{x_1} P(x_1 | y_{T_1}, y_{T_2}, h_{1,1}, h_{2,1})$$

(2.21)

$$\hat{x}_2 = \arg \max_{x_2} P(x_2 | y_{T_1}, y_{T_2}, h_{1,1}, h_{2,1}).$$

(2.22)

By simple conversion, and according to Bayes’ rule, the optimal decoding can be written as

$$\hat{x}_1 = \arg \max_{x_1} P(h_{1,1}^* y_{T_1} + h_{1,2}^* y_{T_2}^* | x_1, h_{1,1}, h_{2,1})$$

(2.23)

$$\hat{x}_2 = \arg \max_{x_2} P(h_{1,2}^* y_{T_1} - h_{1,1}^* y_{T_2}^* | x_2, h_{1,1}, h_{2,1}).$$

(2.24)

Therefore, with different combination of $y_{T_1}$ and $y_{T_1}$, the detector can decouple $x_1$ and $x_2$ to give [2]:

$$\hat{x}_1 = \arg \min_{x_1} \left| h_{1,1}^* y_{T_1} + h_{2,1}^* y_{T_2} - \sqrt{\frac{E_s}{2}} (|h_{1,1}|^2 + |h_{2,1}|^2) \cdot x_1 \right|$$

(2.25)
\[ \hat{x}_2 = \arg \min_{x_2} \left| h_{1,1}^* y_{T_1} - h_{1,1} y_{T_2} - \sqrt{\frac{E_s}{2}} (|h_{1,1}|^2 + |h_{2,1}|^2) \cdot x_2 \right|. \quad (2.26) \]

For a system with \( M \) receive antennas, the optimal decision rule is given by [2]:

\[ \hat{x}_1 = \arg \min_{x_1} \left| \sum_{j=1}^{M} \left[ h_{1,j}^* y_{T_1} - h_{1,j} y_{T_2} - \sqrt{\frac{E_s}{2}} (|h_{1,j}|^2 + |h_{2,j}|^2) \cdot x_1 \right] \right|^2 \quad (2.27) \]

\[ \hat{x}_2 = \arg \min_{x_2} \left| \sum_{j=1}^{M} \left[ h_{2,j}^* y_{T_1} - h_{1,j} y_{T_2} - \sqrt{\frac{E_s}{2}} (|h_{1,j}|^2 + |h_{2,j}|^2) \cdot x_2 \right] \right|^2, \quad (2.28) \]

where \( y_{T_1}(i) \) and \( y_{T_2}(i) \) represent the received signal at the \( i \)-th receive antenna during the first and second time slots, respectively.

Let us denote \( \rho \) as the ratio of the transmitted signal power to the noise power, then \( \rho \) is given by

\[ \rho = \frac{E_s}{2N_0}, \quad (2.29) \]

where \( E_s \) is the transmitted signal energy for each symbol during \( T_1 \) and \( T_2 \), and \( N_0 \) is the two-sided noise spectral density of \( n_1 \) and \( n_2 \). Using Alamouti scheme with \( M \) receive antennas, the BER is given by [2]

\[ P_b = \left[ \frac{1}{2} \left( 1 - \sqrt{\frac{\rho}{2 + \rho}} \right) \right] ^{2M} \sum_{k=0}^{2M-1} \binom{2M-1}{k} \left[ \frac{1}{2} \left( 1 + \sqrt{\frac{\rho}{2 + \rho}} \right) \right] ^k. \quad (2.30) \]

At high SNR, (2.30) reduces to [2]

\[ P_b \approx \left( \frac{4M - 1}{2M} \right) \left( \frac{1}{2\rho} \right) ^{2M}, \quad (2.31) \]

and hence a diversity order of \( 2M \) is achieved.

Alamouti scheme is the basic space-time block code that uses two transmit antennas. With two transmit antennas and \( M \) receive antennas, it can achieve full-rate with full-diversity of \( 2M \) for both real and complex signals. First introduced in 1999
[4], the general space-time block codes based on the theory of orthogonal designs prove to achieve full spatial diversity of $N \times M$. In [4], it was shown that the number of full-rate full-diversity STBC is limited. For complex symbols, the Alamouti scheme is the only full-rate full-diversity STBC. For real symbols, we can find full-rate full-diversity STBC only for 2, 4, and 8 transmit antennas [2][4].

The MIMO-OFCDM systems studied in this thesis are based on two transmit antennas, where Alamouti scheme is employed to achieve transmit diversity.

2.8 Combining Methods

When the transmitted data reaches the receiver side through multiple paths or multiple receive antennas, a suitable combining method is needed to extract data. One group of combining methods is called gain combining, where the received signals from all diversity branches are combined to form the sufficient statistics for data detection. Since the channel gain from all paths are used, this method is called gain combining. A second group of combining methods is selection combining, where the combiner picks the path with the highest SNR and then uses the received signal from this path for detection. Another group of combining methods is hybrid selection/gain combining, where the receiver selects a given number of paths with the highest SNR. Then gain combining methods are used to combine the received signals from these selected branches for detection [5].
2.8.1 Gain Combining

Gain combining methods take the received signals from all branches into account. Simple but efficient gain combining methods are introduced and widely used, which include MRC, EGC, and MMSE combining. In these gain combining methods, the signal used for detection is a linear combination of the received signal from all branches. Hence, the complexity grows linearly with the number of paths. To implement these combining methods, a perfect knowledge of CSI is needed. In a communication system with $L$ paths, let $y_l$ be the received signal from the $l$-th ($l = 1, 2, ..., L$) path and $w_l$ be the combining weight of the $l$-th path, then the signal used for detection is given by [5]

$$z = W \cdot Y = \sum_{l=1}^{L} w_l \cdot y_l, \quad (2.32)$$

where $W = \begin{bmatrix} w_1 & w_2 & ... & w_L \end{bmatrix}$, $Y = \begin{bmatrix} y_1 & y_2 & ... & y_L \end{bmatrix}^T$, and $y_l$ is given by

$$y_l = \sqrt{\frac{E_s}{L}} h_l d_0 + n_l, \quad (2.33)$$

with $h_l$ being the channel coefficient of the $l$-th path and $n_l$ being the AWGN of the $l$-th branch with a two-sided power spectral density $N_0$.

2.8.1.1 MRC

In MRC, the output SNR is maximized by choosing the combining weight $w_l$ as $h_l^*$ [14]. MRC is optimal when the noise on different paths is uncorrelated Gaussian noise. In communication systems involving CDMA, MRC is optimal when no inter-code interference is present [5]. After combining, the signal used for detection is given
Let $\rho$ be the ratio of signal power (averaged over diversity branches) to the noise power,

$$\rho = \frac{E_s}{L \cdot N_0}. \quad (2.35)$$

The instantaneous SNR of the $l$-th diversity branch, $\rho_l$, is defined as the ratio the instantaneous signal power at the $l$-th branch to the noise power of that branch [49]. When MRC is used, $\rho_l$ is given by

$$\rho_l = \rho \cdot \frac{|h_l|^4}{|h_l|^2} = \rho \cdot |h_l|^2. \quad (2.36)$$

We can define the output SNR as the ratio of the output signal power to the output noise power at the combiner [50]. Then the average output SNR for MRC can be obtained by

$$\rho_{out} = E \left\{ \sum_{l=1}^{L} \rho_l \right\} = E \left\{ \rho \cdot \sum_{l=1}^{L} |h_l|^2 \right\}, \quad (2.37)$$

where $E(\cdot)$ represents expectation operation. When paths are mutually independent and are Rayleigh distributed,

$$\rho_{out} = \rho \cdot L, \quad (2.38)$$

and the symbol error rate is given by [14]

$$P_e = \left[ 1 - \frac{\sqrt{\rho/(1+\rho)}}{2} \right] \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left[ 1 + \frac{\sqrt{\rho/(1+\rho)}}{2} \right]^k. \quad (2.39)$$
where $\rho$ is defined in (2.35). At high SNR, the symbol error rate can be approximated as [14]

$$P_e \approx (4\rho)^{-L} \left( \frac{2L - 1}{L} \right).$$

(2.40)

### 2.8.1.2 EGC

In EGC, the combining weight is chosen to compensate for the phase of the channel coefficient [51]:

$$w_i = \frac{h_i^*}{|h_i|}.$$  

(2.41)

That is the received signals from different paths are co-phased and linearly combined to give

$$z = \sum_{i=1}^{L} w_i \cdot (\sqrt{\frac{E_s}{L}} h_i d_0 + n_i)$$

$$= \sum_{i=1}^{L} \sqrt{\frac{E_s}{L}} |h_i| d_0 + w_i \cdot n_i.$$  

(2.42)

In this case, the average output SNR is given by

$$\rho_{out} = E \left\{ \frac{\left| \sum_{i=1}^{L} \sqrt{\frac{E_s}{L}} |h_i| \right|^2}{\sum_{i=1}^{L} |w_i \cdot n_i|^2} \right\} = \frac{\rho}{L} \cdot E \left\{ \left( \sum_{i=1}^{L} |h_i| \right)^2 \right\}$$

$$= \frac{\rho}{L} \cdot E \left\{ \sum_{i=1}^{L} |h_i|^2 + \sum_{i=1}^{L} \sum_{j=1 \atop i \neq j}^{L} |h_i| \cdot |h_j| \right\}.$$  

(2.43)

From the property of complex Gaussian RV we know that for any complex Gaussian RV $s$ with zero mean and variance $\sigma^2$, the expected value of $|s|$ is given by [5]

$$E \{ |s| \} = \sigma \sqrt{\frac{\pi}{2}}.$$  

(2.44)
Therefore, when paths are mutually independent and are Rayleigh distributed, the average output SNR can be simplified to [5]

\[
\rho_{out} = \frac{\rho}{L} \cdot E \left\{ \sum_{i=1}^{L} |h_i|^2 + \sum_{i=1}^{L} \sum_{j=1 \atop i \neq j}^{L} |h_i| \cdot |h_j| \right\} \\
= \frac{\rho}{L} \cdot E \left\{ \sum_{i=1}^{L} |h_i|^2 \right\} + E \left\{ \sum_{i=1}^{L} \sum_{j=1 \atop i \neq j}^{L} |h_i| \cdot |h_j| \right\} \\
= \frac{\rho}{L} \cdot \left( L + \frac{L(L-1)}{2} \cdot \sqrt{\frac{\pi}{2}} \cdot \sqrt{\frac{\pi}{2}} \right) \\
= \rho \cdot \left( 1 + (L-1) \frac{\pi}{4} \right). \quad (2.45)
\]

Although MRC outperforms EGC in systems without MCI, EGC can achieve full diversity with lower implementation complexity.

2.8.1.3 MMSE Combining

In a multipath fading channel, when the noise on different propagation paths are correlated or when the system has non-Gaussian interference, MMSE combining method becomes the optimal gain combining method [5]. MMSE combining minimizes the mean square error between the transmitted symbol, \(d_0\), and the combiner output, \(z\), [14]:

\[
w^* = \arg \min_w \{ |z - d_0|^2 \}. \quad (2.46)
\]

Hence, the combining weight on the \(l\)-th branch is given by [9][10]

\[
w_l = \frac{h_l^*}{(1 + K_c) |h_l|^2 + \left( \frac{E_x}{L \cdot N_0} \right)^{-1}}, \quad (2.47)
\]

where \(K_c\) is the number of effective interference code channels. It is noted that MMSE combining takes care of the channel phase, the channel gain, the noise power, and the
degree of interference. Therefore, when MCI is high, MMSE combining outperforms both MRC and EGC.

2.8.2 Selection Combining

In selection combining (SC), the combiner selects the signal transmitted on the best reliable channel. The selection criteria can be the highest SNR, the highest absolute power, the lowest error rate, or other criteria, or the combination of these. In a communication system with \( L \) independent paths, if the selection is always done based on highest SNR criteria, it can be shown that the diversity gain is equal to \( L \) [5]. However, selection combining methods incur some loss in SNR compared to gain combining methods (i.e. not all received signal power is taken into account for decision making in selection combining). Let \( s_l \) represent the absolute value of the channel coefficient \( h_l \):

\[
s_l = |h_l|, \quad l = 1, 2, \ldots, L.
\]

(2.48)

Suppose \( h_{\text{max}} \) is the channel coefficient with the largest channel gain within these \( L \) independent paths, then the possibility that all channel coefficients are less than \( S \) is given by

\[
P(|h_{\text{max}}| \leq S) = P(s_{\text{max}} \leq S)
= P(s_1, s_2, \ldots, s_L \leq S).
\]

(2.49)

Taking the derivative of (2.49), we can obtain \( p_{s_{\text{max}}}(s) \), the PDF of \( s \). Since the combiner in SC selects only the received signal from the best path, the instantaneous received signal power is \( \frac{P_t}{L} |h_{\text{max}}|^2 \), where \( \frac{P_t}{L} \) is the average transmitted power allocated
to each diversity branch for each symbol. Therefore, the average signal power used for decision making at the SC combiner output can be calculated by

\[ P_{\text{selection}} = \int_0^{+\infty} \frac{P_s}{L} \cdot s^2 \cdot P_{\text{max}}(s) ds. \] (2.50)

We can see that for each symbol, \( P_{\text{selection}} \) is only a portion of \( P_s \times L \), the total transmitted signal power, and the ratio between them is

\[ \frac{P_{\text{selection}}}{P_s} = \int_0^{+\infty} s^2 \cdot P_{\text{max}}(s) ds. \] (2.51)

Nevertheless, the advantage of selection combining is that it achieves the same diversity gain as a full system but with less hardware (i.e. RF chains).

2.8.3 Hybrid Selection/Gain Combining

In hybrid selection/gain combining, the combiner selects the best \( L_s \) paths out of \( L \) paths and then uses a gain combining method to combine the signals for detection [5][49]. Same as SC, hybrid selection/gain combining can extract diversity gain of \( L \) and uses less hardware to implement. However, hybrid selection/gain combining allows to choose the number of paths. As the number of selected paths increases, the received SNR gain increases, but the complexity of detection grows as well. This flexibility makes hybrid selection/gain combining adaptable in different applications.

2.9 Conclusion

In this chapter, key concepts related to this thesis were reviewed. First, commonly used channel models were presented. Then different technologies such as OFDM, CDMA, which are used in wireless communications to support multi-user environment
were discussed. Finally we studied how to achieve transmit and receive diversity with STBC and different combining schemes. Having understood those concepts, we will be able to present the study in this thesis.
CHAPTER 3

SISO-OFCDM SYSTEMS

In this chapter, we will study the performance of SISO-OFCDM systems. First we will introduce the system model of SISO-OFCDM. Then we will look into the detection algorithm and present system performance evaluations for MRC, EGC, and MMSE combining methods. Finally we provide simulation results for different SISO-OFCDM systems.

3.1 System Model

The transmitter structure of an OFCDM system is shown in Figure 3.1. The user data is firstly processed by a modulator such as BPSK. When using QPSK or higher order modulation, Grey coding should be employed. The modulated symbols are then converted from a serial symbol stream into \( N_B \) parallel symbol streams, where \( N_B \) is the number of user data that will be two-dimensionally spread with the same spreading code. The spreading is provided by a spreading code generator, and each group of \( N_B \) symbols will be spread with a unique code. In the OFCDM system, let the spreading factor in frequency domain and time domain be \( N_F \) and \( N_T \) respectively, then the length of spreading code, \( N_S \), is given by

\[
N_S = N_T \times N_F. \quad \text{(3.1)}
\]
Since the overall spreading factor is $N_S$, a maximum $N_S$ different codes are available for spreading. Let $K$ be the number of actually transmitted groups, and each group of data symbol is assigned with a unique spreading code, then $K \leq N_S$. And if different data symbols are from different system users, a maximum $K \times N_B$ users can access the system at the same time. The system load, denoted by $\zeta$, is defined as the number of spreading codes assigned divided by the total number of spreading codes available, which is given by

$$\zeta = \frac{K}{N_S}.$$  \hspace{1cm} (3.2)

When the system is running at full load, $N_S \times N_B$ symbols can be transmitted at the same time.

The overall spreading code $C$ is a combination of frequency domain spreading code $C^F$ and time domain spreading code $C^T$. Here, both $C^F$ and $C^T$ are OVSF codes. Let $C(k)$ represent the $k$-th ($k = 0, 1, ..., K - 1$) spreading code, $C^F(i)$ represent the $i$-th ($i = 0, 1, ..., N_F - 1$) frequency domain OVSF spreading code of length $N_F$, and so on.
$CT(j)$ represent the $j$-th ($j = 0, 1, ..., N_T - 1$) time domain OVSF spreading code of length $N_T$, denoted respectively as

$$C(k) = \begin{bmatrix} c_0(k) & c_1(k) & \ldots & c_{NF-1}(k) \\ c_{NF}(k) & c_{NF+1}(k) & \ldots & c_{2NF-1}(k) \\ \vdots & \vdots & \ddots & \vdots \\ c_{NTNF}(k) & c_{NTNF+1}(k) & \ldots & c_{NS-1}(k) \end{bmatrix}, \quad k = 0, 1, ..., K - 1 \quad (3.3)$$

$$CF(i) = \begin{bmatrix} c_F^F(i) & c_F^i(i) & \ldots & c_F^F(i-1) \end{bmatrix}, \quad i = 0, 1, ..., N_F - 1 \quad (3.4)$$

$$CT(j) = \begin{bmatrix} c_F^T(j) & c_T^T(j) & \ldots & c_F^{T-1}(j) \end{bmatrix}, \quad j = 0, 1, ..., N_T - 1. \quad (3.5)$$

Our study focuses on downlink transmission in OFCDM systems, where we assume perfect synchronization. Also, we assume each sub-carrier experiences flat fading in our study, so the orthogonality of the OVSF code in time domain is preserved. However, the orthogonality in frequency domain is distorted because of different fading among different sub-carriers. With an example, it is easier to understand why the orthogonality is lost. Assuming a simple OFCDM system with $N_F = 2$ and $N_T = 1$, then $CF(0) = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $CF(1) = \begin{bmatrix} 1 & -1 \end{bmatrix}$, and $CT(0) = [1]$. When two data symbols, denoted as $\alpha_1$ and $\alpha_2$, are spread with $CF(0)$ and $CF(1)$ respectively and then transmitted, the received signal is given by

$$\beta_1 = \sqrt{E_s} \cdot h_1 \cdot [\alpha_1 c_F^F(0) + \alpha_2 c_F^F(0)] \cdot c_0^T(0) + n_1$$

$$= \sqrt{E_s} \cdot h_1 \cdot [\alpha_1 + \alpha_2] + n_1 \quad (3.6)$$

$$\beta_2 = \sqrt{E_s} \cdot h_2 \cdot [\alpha_1 c_F^F(1) + \alpha_2 c_F^F(1)] \cdot c_0^T(0) + n_2$$

$$= \sqrt{E_s} \cdot h_2 \cdot [\alpha_1 - \alpha_2] + n_2, \quad (3.7)$$
where $h_1$ and $h_2$ are the two corresponding fading coefficients, and $n_1$ and $n_2$ are additive noise terms at the receiver side. If we use decoding technique for CDMA to detect $\alpha_1$ and $\alpha_2$, the decision variables will be

$$\hat{\alpha}_1 = \frac{1}{2} [\beta_1 c_F^c(0) + \beta_2 c_F^c(1)]$$
$$= \sqrt{E_s} \cdot \left( \frac{h_1 + h_2}{2} \right) \cdot \alpha_1 + \sqrt{E_s} \cdot \left( \frac{h_1 - h_2}{2} \right) \cdot \alpha_2 + \frac{n_1 + n_2}{2} \quad (3.8)$$

$$\hat{\alpha}_2 = \frac{1}{2} [\beta_1 c_F^c(0) + \beta_2 c_F^c(1)]$$
$$= \sqrt{E_s} \cdot \left( \frac{h_1 + h_2}{2} \right) \cdot \alpha_2 + \sqrt{E_s} \cdot \left( \frac{h_1 - h_2}{2} \right) \cdot \alpha_1 + \frac{n_1 + n_2}{2} \quad (3.9)$$

The terms $\sqrt{E_s} \cdot \left( \frac{h_1 - h_2}{2} \right) \cdot \alpha_2$ in (3.8) and $\sqrt{E_s} \cdot \left( \frac{h_1 - h_2}{2} \right) \cdot \alpha_1$ in (3.9) represent MCI, which is introduced as long as the channels $h_1$ and $h_2$ are not identical.

From the example above we can see that MCI is generated when different users' data is spread with different frequency domain spreading codes and the same time domain spreading code. To overcome this, the spreading code should be designed in such a way to take advantage of the orthogonality in time domain. Specifically, the spreading code generator minimizes the number of frequency domain OVSF spreading codes used. Therefore, the level of MCI is minimized. Here we use an example to illustrate how to generate the spreading codes. When $N_F = 2$ and $N_T = 4$, $C(0)$ is generated by $C_F(0) = \begin{bmatrix} 1 & 1 \end{bmatrix}$ and $C_T(0) = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$:

$$C(0) = [C_F(0)]^T \cdot C_T(0)$$
$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$
$C(1)$ is generated by $C^F(0)$ and $C^T(1) = \begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix}$:

\[
C(1) = [C^F(0)]^T \cdot C^T(1) = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix},
\]

$C(2)$ is generated by $C^F(0)$ and $C^T(2) = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}$:

\[
C(2) = [C^F(0)]^T \cdot C^T(2) = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix},
\]

and so forth. Until the time domain OVSF spreading code is used up, the code generator will change the frequency domain OVSF spreading code and restarts the cycle. Therefore, $C(N_T)$ is generated by $C^F(1)$ and $C^T(0)$, $C(N_T + 1)$ is generated by $C^F(1)$ and $C^T(1)$, $C(N_T + 2)$ is generated by $C^F(1)$ and $C^T(2)$, and so forth. In the example when $N_F = 2$ and $N_T = 4$, $C(N_T) = C(4)$ is given by

\[
C(4) = [C^F(1)]^T \cdot C^T(0) = \begin{bmatrix} 1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \end{bmatrix},
\]

$C(N_T + 1) = C(5)$ is given by

\[
C(5) = [C^F(1)]^T \cdot C^T(1) = \begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \end{bmatrix}.
\]
By assigning spreading codes according to this method, one can see that MCI will
be present if \( K > N_T \). When \( K \leq N_T \), data symbols transmitted by the same \( N_F \)
carriers are spread with the same frequency domain spreading code but different time
domain spreading codes. In this case, no MCI is present in the system. However,
when \( K > N_T \), at least two data symbols are spread with different frequency domain
spreading codes but the same time domain spreading code, and MCI occurs. For any
given spreading code \( C(k) \) \( (k = 0, 1, ..., K - 1) \), let \( K_c \) be the number of interfering
codes. When the spreading codes are generated according to the method explained
above, \( K_c \) is given by the number of codes that have the same time domain spreading
code as \( C(k) \) but different frequency domain spreading codes.

Take the system with \( N_F = 2 \) and \( N_T = 4 \) for example. When \( K \leq 4 \), the
spreading codes \( C(0), C(1), ..., C(K - 1) \) are generated from \( C^T(0) \) with different
time domain spreading codes, so no MCI is present and \( K_c = 0 \). On the other
hand, consider the case when \( K = 6 \). \( C(0) \) and \( C(4) \) are generated from the same
time domain spreading code \( C^T(0) \) with different frequency domain spreading codes
\( C^F(0) \) and \( C^F(1) \) respectively. Thus, \( C(0) \) and \( C(4) \) interfere with each other. Also,
since \( C(1) \) and \( C(5) \) are generated from \( C^T(1) \) with \( C^F(0) \) and \( C^F(1) \) respectively,
they interfere with each other. Therefore, \( K_c = 1 \) for \( C(0), C(1), C(4) \) and \( C(5) \), but
\( K_c = 0 \) for \( C(2) \) and \( C(3) \). One can generalize the relationship between \( K_c \) and \( K \) as

\[
\left\lfloor \frac{K - 1}{N_T} \right\rfloor - 1 \leq K_c \leq \left\lfloor \frac{K - 1}{N_T} \right\rfloor, \text{ when } K > N_T.
\]

(3.10)

where the operator \( \lfloor x \rfloor \) represents the integer portion of \( x \) with \( \lfloor x \rfloor \leq x \).

From this spreading codes generating method, one can see that when the code
\( C(k) \ (k = 0, 1, ..., K - 1) \) is generated by \( C^F(i) \) and \( C^T(j) \), the relationship between \( k, i, \) and \( j \) is

\[ i = \left\lfloor \frac{k}{N_T} \right\rfloor , \quad (3.11) \]

\[ j = k \mod N_T \]

\[ = k - N_T \cdot \left\lfloor \frac{k}{N_T} \right\rfloor . \quad (3.12) \]

The spreading factor in frequency domain is \( N_F \), where \( N_F \) sub-carriers are dedicated to transmit the same data symbol. In order to minimize the correlation among those \( N_F \) sub-carriers, one should ensure that sub-carriers are separated enough using interleaving. The interleaving should be carried out as follows. In a group of \( N_B \) symbols that is spread with the same spreading code, the first symbol is spread into the zeroth, \( N_B \)-th, \((2N_B)\)-th, ..., and \([(N_F - 1) \cdot N_B]\)-th sub-carriers, the second symbol is spread into the first, \((N_B + 1)\)-th, \((2N_B + 1)\)-th, ..., and \([(N_F - 1) \cdot N_B + 1]\)-th sub-carriers, and so forth. Finally, the \( N_B \)-th symbol is spread into the \((N_B - 1)\)-th, \((2N_B - 1)\)-th, ..., and \((L - 1)\)-th sub-carriers, where \( L = N_F \cdot N_B \) is the total number of sub-carriers employed in the OFCDM system.

Let us take for example the \( j \)-th \((j = 1, 2, ..., N_B)\) symbol in the \( k \)-th \((k = 0, 1, ..., K - 1)\) group. The two-dimensional spreading and frequency interleaving is presented in Figure 3.2. The spreading code assigned to this symbol is \( C(k) \) and the time domain spreading factor is \( N_T \). Then the first \( N_T \) chips of \( C(k) \), \( \{c_0(k), c_1(k), c_2(k), ..., c_{N_T-1}(k)\} \), are transmitted on the \((j - 1)\)-th sub-carrier; the second \( N_T \) chips of \( C(k) \), \( \{c_{N_T}(k), c_{N_T+1}(k), c_{N_T+2}(k), ..., c_{2N_T-1}(k)\} \), on the \((j - 1 + N_B)\)-th sub-carrier; \( \{c_{2N_T}(k), c_{2N_T+1}(k), c_{2N_T+2}(k), ..., c_{3N_T-1}(k)\} \) on the \((j - 1 + 2N_B)\)-th sub-carrier; and so forth.
sub-carrier; and so forth. Finally, the last $N_T$ chips of $C(k)$ are transmitted on the
$(j - 1 + (N_F - 1)N_B)$-th sub-carrier.

As shown in Figure 3.2, a signal transmitted during one time slot on one sub-carrier in OFCDM system is called an OFCDM symbol. After two-dimensional spreading and frequency interleaving for a group of $N_B$ modulated symbols, an OFCDM frame consisting of $L \times N_T$ OFCDM symbols is constructed. Assigned with different spreading codes, in total $K$ OFCDM frames are multiplexed to build a super-frame, and this step is done by a frame multiplexer, shown in Figure 3.3.

Let the indices $l_0, l_1, \ldots , l_{N_F - 1}$ represent the $N_F$ sub-carriers carrying the $k$-th user data after frequency interleaving. In an OFCDM super-frame, the OFCDM symbol on the $l_i$-th ($i = 0, 1, \ldots , N_F - 1$) sub-carrier during the $j$-th ($j = 0, 1, \ldots , N_T - 1$) time
Figure 3.3: Multiplexing $K$ frames into a superframe

A slot can be expressed as

$$s(l_i, j) = \sqrt{\frac{E_s}{N_S}} \sum_{k=0}^{K-1} d_k c_{\left\lfloor \frac{j}{N_B} \right\rfloor N_T + j}(k),$$

(3.13)

where $d_k$ is the $k$-th user data on the $l_i$-th sub-carrier, $c_{\left\lfloor \frac{j}{N_B} \right\rfloor N_T + j}(k)$ is the $\left\lfloor \frac{j}{N_B} \right\rfloor N_T + j$-th chip of the $k$-th user’s spreading code, $\left\lfloor \frac{j}{N_B} \right\rfloor$ is the chip index of the frequency domain spreading code, $j$ is the chip index of the time domain spreading code, and $E_s$ is the total transmitted signal energy for each symbol. Since each data symbol is spread on $N_S$ OFCDM symbols, the total transmitted power has to be averaged over these $N_S$ OFCDM symbols. After frame multiplexing, the OFCDM super-frame is transmitted through a SISO-OFDM transmission system. The transmitted OFCDM symbols on different sub-carriers experience different fades. In this work, within one OFCDM super-frame, the OFCDM symbols transmitted on the same sub-carrier experience the same fading (i.e. fading is fixed for one OFCDM frame duration).
The receiver structure is shown in Figure 3.4. Let $r(i, j)$ denote the received OFCDM symbol on the $i$-th sub-carrier during the $j$-th time slot:

$$r(i, j) = \sum_{k=0}^{K-1} \left( \sqrt{\frac{E_s}{N_S}} \cdot h_{ik} \cdot d_k \cdot c_{[\frac{i}{N_R}]N_T+j}(k) \right) + \eta(i, j)$$

$$= \sqrt{\frac{E_s}{N_S}} \cdot h_{ik} \cdot \sum_{k=0}^{K-1} d_k \cdot c_{[\frac{i}{N_R}]N_T+j}(k) + \eta(i, j), \quad (3.14)$$

where $\eta(i, j)$ is the AWGN on the $i$-th sub-carrier during the $j$-th time slot.

3.2 Detection Algorithm

At the receiver side, the received signal is first frequency deinterleaved to form $N_B$ groups. Each group has $N_F$ received signals from the $N_F$ sub-carriers carrying the same user data. In each group, there are $K$ users’ data multiplexed by different spreading codes. Then, time domain despreading is carried out. After despreading the $N_F$ signals in time domain, to sort out each of the $K$ users’ data, the output of
the \( k \)-th user desreader is

\[
r_k(l_i) = \frac{1}{N_T} \sum_{j=0}^{N_T-1} r(l_i, j) c^T_j(k)
\]

\[
= \sqrt{\frac{E_s}{N_S}} \cdot h_{l_i} \cdot d_k c_{[\left\lfloor \frac{l_i}{B} \right\rfloor]}(k) + \eta(l_i),
\]

(3.15)

where \( c^T_j(k) \) is the \( j \)-th chip of the \( k \)-th time domain OVSF code, and \( \eta(l_i) \) is the equivalent noise added to the \( l_i \)-th sub-carrier.

Let \( \sigma^2_n \) represent the noise energy added to each OFCDM symbol:

\[
E \{|\eta(l_i, j)|^2\} = \sigma^2_n,
\]

(3.16)

then the variance of \( \eta(l_i) \) at each dimension is given by

\[
E \{|\eta(l_i)|^2\} = E \left\{ \frac{1}{N_T} \sum_{i=0}^{N_T-1} |\eta(l_i, j)|^2 \right\}
\]

\[
= \frac{1}{N_T^2} E \left\{ \sum_{i=0}^{N_T-1} |\eta(l_i, j)|^2 \right\}
\]

\[
= \frac{1}{N_T} E \left\{ |\eta(l_i, j)|^2 \right\}
\]

\[
= \frac{\sigma^2_n}{N_T}.
\]

(3.17)

From the design of the spreading code described above, one can see that the system will introduce MCI if the number of multiplexed frames \( K \) is greater than the number of available time domain spreading codes \( N_T \). In the case when MCI is present, an interference canceller is used to cancel the MCI in an iterative way after time domain despreading \cite{9}. In this thesis, we focus on OFCDM systems with no MCI. In what follows, we analyze the performance of this system in terms of its BER.

After time domain despreading, a suitable combining method is employed to combine the \( N_F \) signals, \( r_k(l_i) (i = 0,1,...,N_F - 1) \), for the \( k \)-th user. In what
follows, we study MRC, EGC, and MMSE combining methods. In these methods, the combining weight \( w(l_i) \) \((i = 0, 1, ..., N_F - 1)\) of each sub-carrier calculated and used in frequency domain despreading is given by

- for MRC [14],
  \[
  w(l_i) = h_{li}^*,
  \]
  \[ (3.18) \]

- for EGC [51],
  \[
  w(l_i) = \frac{h_{li}^*}{|h_{li}|},
  \]
  \[ (3.19) \]

- for MMSE combining [9][10],
  
  \[
  w(l_i) = \frac{h_{li}^*}{(1 + K_c)|h_{li}|^2 + \left(\frac{E_s}{N_F N_0}\right)^{-1}}
  = \frac{h_{li}^*}{(1 + K_c)|h_{li}|^2 + \left(\frac{\bar{\rho}}{N_F}\right)^{-1}},
  \]
  \[ (3.20) \]

where \( E_s \) is the total transmitted energy for each data symbol over all the \( N_F \) sub-carriers, \( N_0 \) is the power spectral density of the AWGN added to each OFCDM symbol, \( \bar{\rho} \) is defined as the ratio of total transmitted power of each data symbol to the background noise power, and \( K_c \) is the number of interfering codes. For any transmitted data symbol, \( K_c \) is given by the number of data symbols that are spread with the same time domain spreading code but different frequency domain spreading codes. In an OFCDM system with no MCI, the combining weight of the MMSE combining becomes

\[
w(l_i) = \frac{h_{li}^*}{|h_{li}|^2 + \left(\frac{\bar{\rho}}{N_F}\right)^{-1}}.
\]

\[ (3.21) \]
In general, the output of the frequency domain despreader is given by

\[ y_k = \sum_{i=0}^{N_F-1} r_k(l_i) \cdot c_{[N_H]}^{F_{l_i}}(k) \cdot w(l_i). \]  

(3.22)

This output is then normalized to form the decision variable, \( z_k \), which will be used in MCI cancellation for systems with MCI:

\[ z_k = \frac{1}{\sqrt{\frac{E_b}{N_0}}} \cdot \frac{y_k}{\sum_{i=0}^{N_F-1} h_{li} \cdot w(l_i)}. \]  

(3.23)

Finally a hard decision based on minimum distance criteria is made to recover the corresponding user data:

\[ \hat{d}_k = \arg \min_{\{s\}} |z_k - s|^2, \]  

(3.24)

where \( \{s\} \) is the set of all possible transmitted data symbols.

3.3 Performance Evaluation

In this section, we present semi-analytical results for the BER of OFCDM systems over flat fading channels with MRC, EGC, and MMSE combining. We assume binary phase-shift keying (BPSK) is used to modulate user data.

In what follows, we use the notation \( e^* \) to represent the error vector between the
decision variable \( z_k \) and the transmitted BPSK symbol \( d_k \), where \( \vec{e} \) is given by

\[
\vec{e} = z_k - d_k
\]

\[
= \frac{\sum_{i=0}^{N_F-1} r_k(l_i)c^F_{\frac{1}{\sqrt{N_F}}}(k)w(l_i)}{\sqrt{\frac{E_s}{N_s} \sum_{i=0}^{N_F-1} h(l_i)w(l_i)}} - d_k
\]

\[
= \frac{\sum_{i=0}^{N_F-1} \left[ \left( \sqrt{\frac{E_s}{N_s}}h_i d_k c^F_{\frac{1}{\sqrt{N_F}}}(k) + \bar{\eta}(l_i) \right)c^F_{\frac{1}{\sqrt{N_F}}}(k) \right]}{\sqrt{\frac{E_s}{N_s} \sum_{i=0}^{N_F-1} h_i w(l_i)}}
\]

\[
= \frac{\sum_{i=0}^{N_F-1} \bar{\eta}(l_i) \cdot w(l_i)}{\sqrt{\frac{E_s}{N_s} \sum_{i=0}^{N_F-1} h_i \cdot w(l_i)}},
\]

with \( \bar{\eta}(l_i) \) being the equivalent complex Gaussian noise on the \( l_i \)-th sub-carrier, with zero mean and variance \( \sigma^2_{\bar{\eta}} = \frac{\sigma^2_{\eta}}{N_T} \). Let \( \bar{\rho} \) be the ratio of total transmitted signal power for each symbol to the background noise power. Since the additive noise term \( \eta(l,i) \) is complex AWGN with variance 0.5 per dimension, then \( \bar{\rho} = E_s \), and the error vector, \( \vec{e}' \), becomes

\[
\vec{e}' = \frac{\sum_{i=0}^{N_F-1} \bar{\eta}(l_i) \cdot w(l_i)}{\sqrt{\frac{E_s}{N_s} \sum_{i=0}^{N_F-1} h_i \cdot w(l_i)}},
\]

with \( \sigma^2_{\bar{\eta}} = \frac{1}{N_T} \). Conditioned on the channel, \( \sum_{i=0}^{N_F-1} \bar{\eta}(l_i) \cdot w(l_i) \) is Gaussian with zero mean and variance

\[
\sigma^2_{\bar{\eta}} = \frac{1}{N_T} \sum_{i=0}^{N_F-1} |w(l_i)|^2.
\]

In the case when MCI is present, if \( K_c \) is greater than 50 [12], the MCI can be considered as an additive Gaussian noise added to each OFCDM symbol with zero
mean and variance $\sigma_i^2$ given by [11][12]

$$\sigma_i^2 = \frac{2K_e E_s}{N_F N_S} \left[ \sum_{i=0}^{N_F-1} |h_i \cdot w(l_i)|^2 - \left\{ \sum_{i=0}^{N_F-1} |h_i \cdot w(l_i)| \right\}^2 \right]. \tag{3.28}$$

Since $\sigma_\eta^2$ and $\sigma_i^2$ are independent, the total system noise can be approximated as $(\sigma_\eta^2 + \sigma_i^2)$ when evaluating the probability of bit error.

In what follows, we evaluate the performance of MRC, EGC, and MMSE combining respectively when no MCI is present. MRC maximizes the output SNR [5], so it performs better than EGC and MMSE combining in OFCDM systems without MCI. EGC is simple to implement since only the phases of the channels are compensated. MMSE combining corrects the phase shift and the attenuation of the channel fading at the same time, which makes it the only adaptive method among these three methods. Therefore, in the presence of MCI, MMSE combining becomes optimal and outperforms both MRC and EGC [5].

3.3.1 MRC

With MRC, the combining weight of each sub-carrier is given by [14]:

$$w(l) = h_i^*, \tag{3.29}$$

and the error vector, $\vec{e}^*$, becomes

$$\vec{e}^* = \frac{\sum_{i=0}^{N_F-1} \bar{\eta}(l_i) \cdot w(l_i)}{\sqrt{\frac{\bar{\rho}}{N_S} \cdot \sum_{i=0}^{N_F-1} h_i \cdot w(l_i)}} = \frac{\sum_{i=0}^{N_F-1} \bar{\eta}(l_i) h_i^*}{\sqrt{\frac{\bar{\rho}}{N_S} \cdot \sum_{i=0}^{N_F-1} |h_i|^2}}, \tag{3.30}$$

with

$$\sigma_\eta^2 = \frac{1}{N_T} \sum_{i=0}^{N_T-1} |w(l_i)|^2 = \frac{1}{N_T} \sum_{i=0}^{N_T-1} |h_i|^2. \tag{3.31}$$
When BPSK modulation is used, the probability of bit error conditioned on the channel is given by

\[ P_{e|h} = P(\text{Re}(\mathbf{e}) > 1) \]

\[ = P \left( \text{Re} \left( \frac{\sum_{i=0}^{N_F-1} \bar{\eta}(l_i) h_i^*}{\sqrt{\frac{\rho}{N_S} \sum_{i=0}^{N_F-1} |h_i|^2}} > 1 \right) \right) \]

\[ = Q \left( \sqrt{\frac{\rho}{N_S} \sum_{i=0}^{N_F-1} |h_i|^2} \right) / \sqrt{\frac{\rho}{N_S} \sum_{i=0}^{N_F-1} |h_i|^2} \]

\[ = Q \left( \sqrt{\frac{2\rho}{N_F} \sum_{i=0}^{N_F-1} |h_i|^2} \right), \quad (3.32) \]

where \( Q(\cdot) \) is defined as [14]

\[ Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{v^2}{2}} dv. \quad (3.33) \]

### 3.3.2 EGC

As mentioned before, with EGC, the combining weight of each sub-carrier is chosen to compensate for the phase shift of the fading coefficient [51]:

\[ w(l) = \frac{h_i^*}{|h_i|}. \quad (3.34) \]

The error vector becomes

\[ \mathbf{e} = \frac{\sum_{i=0}^{N_F-1} \bar{\eta}(l_i) \cdot w(l_i)}{\sqrt{\frac{\rho}{N_S} \sum_{i=0}^{N_F-1} h_i \cdot w(l_i)}} = \frac{\sum_{i=0}^{N_F-1} \bar{\eta}(l_i) h_i^*}{\sqrt{\frac{\rho}{N_S} \sum_{i=0}^{N_F-1} |h_i|^2}}, \quad (3.35) \]
\[ \sigma_\tilde{\eta}^2 = \frac{1}{N_T} \sum_{i=0}^{N_F-1} \left| w(l_i) \right|^2 = \frac{1}{N_T} \sum_{i=0}^{N_F-1} \left| h_i^* \right|^2 = \frac{N_F}{N_T}. \]  

(3.36)

Similar to the approach in MRC, the probability of bit error conditioned on the channel with EGC is given by

\[ P_e | h = P \left( \text{Re}(\tilde{\mathbf{e}}) > 1 \right) = P \left( \text{Re} \left( \frac{\sum_{i=0}^{N_F-1} \tilde{\eta}(m_i) h_i^*}{\sqrt{\frac{\rho}{N_T} \sum_{i=0}^{N_F-1} \left| h_i \right|^2}} \right) > 1 \right) = Q \left( \frac{\sqrt{\frac{\rho}{N_T} \sum_{i=0}^{N_F-1} \left| h_i \right|^2}}{\sqrt{\frac{N_F}{2N_T}}} \right) = Q \left( \frac{\sqrt{2\rho}}{N_F} \sum_{i=0}^{N_F-1} \left| h_i \right|^2 \right). \]  

(3.37)

### 3.3.3 MMSE Combining

For MMSE combining with no MCI, the combining weight is given by\[9\][10],

\[ w(l) = \frac{h_i^*}{\left| h_i \right|^2 + \left( \frac{\rho}{N_F} \right)^{-1}}. \]  

(3.38)

Then the error vector \( \tilde{\mathbf{e}} \) becomes

\[ \tilde{\mathbf{e}} = \frac{\sum_{i=0}^{N_F-1} \tilde{\eta}(l_i) \cdot w(l_i)}{\sqrt{\frac{\rho}{N_T} \sum_{i=0}^{N_F-1} h_i \cdot w(l_i)}} = \frac{\sum_{i=0}^{N_F-1} \tilde{\eta}(l_i) \frac{h_i^*}{\left| h_i \right|^2 + \left( \frac{\rho}{N_F} \right)^{-1}}}{\sqrt{\frac{\rho}{N_T} \sum_{i=0}^{N_F-1} \left| h_i \right|^2 + \left( \frac{\rho}{N_F} \right)^{-1}}}, \]  

(3.39)

and

\[ \sigma_\tilde{\eta}^2 = \frac{1}{N_T} \sum_{i=0}^{N_F-1} \left| w(l_i) \right|^2 = \frac{1}{N_T} \sum_{i=0}^{N_F-1} \frac{\left| h_i \right|^2}{\left[ \left| h_i \right|^2 + \left( \frac{\rho}{N_F} \right)^{-1} \right]^2}. \]  

(3.40)
The probability of bit error conditioned on the channel is given by

\[ P_{e|h} = P(\text{Re}(\bar{e^*}_k) > 1) \]

\[ = P\left( \text{Re}\left( \frac{\sum_{i=0}^{N_F-1} \frac{h_i^*}{|h_i|^2 + (\bar{\chi}_F)^{-1}}}{\sqrt{\frac{\rho}{N_S}} \sum_{i=0}^{N_F-1} |h_i|^2} \right) > 1 \right) \]

\[ = Q\left( \sqrt{\frac{\rho}{N_S}} \cdot \sum_{i=0}^{N_F-1} \frac{|h_i|^2}{|h_i|^2 + (\bar{\chi}_F)^{-1}} \right) \]

\[ = Q\left( \sqrt{2} \sum_{i=0}^{N_F-1} \frac{1}{1 + \left(\frac{\rho}{\bar{\chi}_F}ight)^{-1}|h_i|^2} \right) \]

\[ = Q\left( \sum_{i=0}^{N_F-1} \frac{1}{\bar{\chi}_F^2|h_i|^2 + 2 + (\bar{\chi}_F^{-1})^2|h_i|^2} \right) . \tag{3.41} \]

### 3.4 Simulation Results

Both simulation and semi-analytical results are presented in this section. Each data symbol is transmitted over 8 or 16 sub-carriers \((N_F = 8 \text{ or } 16)\), and the time domain spreading factor is \(N_T = 8\). Since we are interested in the case without MCI, the number of multiplexed frames \(K = 8\) in order to maximize the transmission rate under the condition of no MCI. We assume CSI is perfectly known at the receiver side, and the channel is fixed for the duration of \(N_T\) OFCDM symbols. We simulate the system under Rayleigh fading and Ricean fading channels. All the curves of semi-analytical results are generated from Monte Carlo approach, where the BER expression are averaged over 100000 channel realizations.

Figure 3.5 shows the performance of a SISO-OFCDM system over Rayleigh fading channel with MRC. The frequency domain spreading factor is \(N_F = 8\), and the sub-
Figure 3.5: BER for SISO-OFCDM system using MRC over Rayleigh fading channel: 
$N_F = 8$, $N_T = 8$, $K = 8$

carriers of interest are separated enough to minimize the effect of correlation. Also simulation results are provided when the sub-carriers carrying the same information have correlation coefficients of 0, 0.25, and 0.5. From this figure we can see that the analytical results agree with the simulated ones. It is to be noted that our results are valid for a general OFCDM system with any system parameters. From these results, one can also see the effect of correlation on the overall diversity order.

In Figure 3.6, we present the performance of a SISO-OFCDM system over Ricean fading channel. EGC is used to achieve frequency diversity. The frequency domain spreading factor is $N_F = 16$. Different values of $\kappa$ (i.e. the ratio of dominant component to the total power of scattered waves) are used in modeling the Ricean channel. As a reference, we include the performance over Gaussian channel. From these results
Figure 3.6: BER for SISO-OFCDM system using EGC over Ricean fading channel: $N_F = 16$, $N_T = 8$, $K = 8$

we can see the agreement between the analytical results and simulated ones.

Figure 3.7 presents the performance of a SISO-OFCDM system over Rayleigh fading channels using MMSE combining with $N_F = 16$. Different degree of correlation between adjacent sub-carriers transmitting the same information are simulated to show the effect of correlation along with the accuracy of our analytical results.

In Figures 3.8, 3.9 we compare the performance of MRC, EGC, and MMSE combining methods. Figure 3.8 shows the performance of SISO-OFCDM systems without MCI (i.e. the number of multiplexed frames does not exceed the time domain spreading factor). We can see that when there is no MCI, MRC achieves the best performance since it maximizes the output SNR. MMSE and EGC perform slightly worse than MRC, but they all achieve the same diversity gain. Figure 3.9 presents
Figure 3.7: BER for SISO-OFCDM system using MMSE combining over Rayleigh fading channel: $N_F = 16$, $N_T = 8$, $K = 8$

the performance of a SISO-OFCDM system with MCI. In these results, the system is running at twenty-five percent of full load. Since the overall spreading factor $N_S = N_F \times N_T = 128$, from (3.2) one can see that twenty-five percent of full load implies that $K = 32$. In this system, every 4 symbols share the same time domain spreading code but with different frequency domain spreading codes. Hence, each transmitted symbol is interfered by other 3 symbols ($K_c = 3$). We can see that MMSE combining outperforms both MRC and EGC. This is due to the fact that among these three combining methods, MMSE combining is the only method that is adaptive, where MCI is accounted for during combining. It is clear that, only MMSE combining can achieve large diversity gains compared to MRC and EGC when MCI is present in the system.
Figure 3.8: BER for SISO-OFCDM system without MCI over Rayleigh fading channel: $N_F = 16, N_T = 8, K = 8$

Figure 3.9: BER for SISO-OFCDM system with MCI over Rayleigh fading channel: $N_F = 16, N_T = 8, K = 32$
By comparing Figures 3.8 and 3.9, we can see the effect of MCI on system performance. The degradation of BER for MRC and EGC is clearly shown in Figure 3.9. When MMSE combining is employed, BER is less than $10^{-5}$ when SNR equals 12 dB in system with no MCI. In contrast, BER is greater than $10^{-4}$ at the same SNR when MCI is present. Therefore, a MCI cancellation scheme can be implemented to reduce this SNR loss.

3.5 Conclusion

In this chapter we have provided closed-form semi-analytical results for the BER of SISO-OFCDM systems without MCI. Our analytical results are proved to be accurate when compared with simulations. The performance of MRC, EGC, and MMSE combining techniques were compared in Ricean fading channels and Rayleigh fading as a special case. Also, we examined the effect of sub-carrier correlation and MCI on the system performance. Both simulation and analytical results proved that MMSE combining is optimal compared to both MRC and EGC when MCI is present.
In the previous chapter, we studied SISO-OFCDM systems in terms of system structure, detection method, and their BER performance. In this chapter, we extend the results in chapter 3 to MIMO-OFCDM systems that employ STBC. Specifically, we study MIMO-OFCDM systems with two transmit and \( M \) receive antennas. Alamouti scheme is used to achieve transmit diversity, and gain combining techniques are used to extract receive diversity. The same as SISO-OFCDM systems, the performance of different combining schemes are studied for multiuser scenario, and BER semi-analytical results are then presented.

4.1 System Model

The transmitter structure of MIMO-OFCDM system is shown in Figure 4.1. In the MIMO-OFCDM system studied, the transmit diversity is achieved by hiring Alamouti scheme introduced in [1]. In order to implement Alamouti scheme, CSI is fixed over two frame duration, which is \( 2N_T \) times one OFCDM symbol duration. There are \( N = 2 \) transmit antennas and \( M \) receive antennas used in the MIMO-OFCDM system. At the beginning of each two frame duration, \( 2KN_B \) user data are modulated into BPSK symbols for transmission. Among these BPSK symbols, \( KN_B \) symbols are transmitted from the first transmit antenna, and the other \( KN_B \) sym-
symbols from the second transmit antenna. At each transmit antenna, the corresponding
$KN_B$ symbols are two-dimensionally spread and interleaved to form $K$ frames, and
these $K$ frames are multiplexed into a super-frame.

Let the indices $l_0, l_1, \ldots, l_{N_F-1}$ represent the $N_F$ sub-carriers carrying the $k$-th
user data after frequency interleaving. Within the OFCDM super-frame transmitted
from the $n$-th ($n = 1, 2$) transmit antenna, the transmitted signal on the $l_i$-th
($i = 0, 1, \ldots, N_F - 1$) sub-carrier during the $j$-th ($j = 0, 1, \ldots, N_T - 1$) OFCDM symbol
duration is a summation of $K$ OFCDM symbols and can be expressed as

$$S_n(l_i, j) = \sqrt{\frac{E_s}{2N_S}} \sum_{k=0}^{K-1} d_{n,k} c_{\left\lfloor \frac{l_i}{N_B} \right\rfloor N_T + j}(k),$$

(4.1)

where $d_{n,k}$ is the $k$-th user data that uses the $l_i$-th sub-carrier from the $n$-th transmit
antenna in the first half of the two-frame duration, and $c_{\left\lfloor \frac{l_i}{N_B} \right\rfloor N_T + j}(k)$ is the
($\left\lfloor \frac{l_i}{N_B} \right\rfloor N_T + j$)-th chip of the $k$-th user’s spreading code. We use $S_1$ and $S_2$
to represent the two super-frames at the two transmit antennas respectively. Then as shown
in Figure 4.2, the OFCDM super-frames $S_1$, $S_2$, $-S_1^*$ and $S_2^*$ are transmitted accord-

Figure 4.1: Transmitter structure of MIMO-OFCDM system
ing to Alamouti scheme through OFDM transmission system. During $T_1$, the first half of the two frame duration, $S_1$ is sent to the first transmit antenna and $S_2$ is sent to the second transmit antenna for transmission. During $T_2$, the second half of the two frame duration, $S_2^*$ is sent to the first transmit antenna and $-S_1^*$ is sent to the second transmit antenna for transmission.

After experiencing channel fading and noise distortion, the signal is picked up by $M$ receive antennas. Let us use $R_{m,1}$, $R_{m,2}$ to represent the received OFCDM superframes for the two consecutive frame duration $T_1$ and $T_2$ at the receiver side shown in Figure 4.2. $R_{m,1}(l_i,j)$ and $R_{m,2}(l_i,j)$ represent the received OFCDM signal at $m$-th ($m = 1, 2, ..., M$) receive antenna on the $l_i$-th sub-carrier during the $j$-th OFCDM symbol duration within the first and second frame duration respectively. We assume perfect synchronization. Therefore, $R_{m,1}(l_i,j)$ and $R_{m,2}(l_i,j)$ are given by

$$R_{m,1}(l_i,j) = \sqrt{\frac{E_s}{2NS}} \sum_{k=0}^{K-1} \left( h_{l_i}^{m,1} d_{1,k} + h_{l_i}^{m,2} d_{2,k} \right) c_{jT}^{l_i} N_{T+j}(k) + \eta_{m,1}(l_i,j) \quad (4.2)$$

$$R_{m,2}(l_i,j) = \sqrt{\frac{E_s}{2NS}} \sum_{k=0}^{K-1} \left( h_{l_i}^{m,1} d_{2,k} - h_{l_i}^{m,2} d_{1,k} \right) c_{jT}^{l_i} N_{T+j}(k) + \eta_{m,2}(l_i,j), \quad (4.3)$$

where $h_{l_i}^{m,n}$ ($m = 1, 2, ..., M$, $n = 1, 2$) is the fading coefficient on the $l_i$-th sub-carrier.
from the \( n \)-th (\( tr = 1, 2 \)) transmit antenna to the \( m \)-th receive antenna, \( \eta_{m,1}(l_i,j) \)
and \( \eta_{m,2}(l_i,j) \) are the noise terms added to the signal on the \( l_i \)-th sub-carrier during
the \( j \)-th OFCDM symbol duration at the \( m \)-th receive antenna.

4.2 Detection Algorithm

In this section, we propose the detection algorithm to recover the transmitted
data symbols. This algorithm extracts receive diversity based on the optimal decision
for Alamouti scheme with multiple receive antennas, which is given in (2.27) and
(2.28). Figure 4.3 presents the receiver structure of MIMO-OFCDM system.

At the receiver side, each transmitted OFCDM symbol pair are decoupled over
all receive antennas. These decoupled signals are then combined over all sub-carriers
of interest to form the decision variables.

The transformed received signals, denoted as \( \tilde{Y}_1(l_i,j) \) and \( \tilde{Y}_2(l_i,j) \), at the \( l_i \)-th
sub-carrier within the $j$-th OFCDM symbol duration are given by

$$
\tilde{Y}_1(l_i, j) = \sum_{m=1}^{M} \left[ (h_{i_t}^{m,1})^* R_{m,1}(l_i, j) + h_{i_t}^{m,2} [R_{m,2}(l_i, j)]^* \right]
$$

$$
= \sqrt{\frac{E_s}{2N_S}} \sum_{k=0}^{K} \sum_{m=1}^{M} \left( |h_{i_t}^{m,1}|^2 + |h_{i_t}^{m,2}|^2 \right) d_{1,k} |\frac{\mathcal{V}}{N_B}| N_{T} + j(k) + \eta_1^{eq}(l_i, j) \tag{4.4}
$$

$$
\tilde{Y}_2(l_i, j) = \sum_{m=1}^{M} \left[ (h_{i_t}^{m,2})^* R_{m,1}(l_i, j) - h_{m}^{m,1} [R_{m,2}(l_i, j)]^* \right]
$$

$$
= \sqrt{\frac{E_s}{2N_S}} \sum_{k=0}^{K} \sum_{m=1}^{M} \left( |h_{i_t}^{m,1}|^2 + |h_{i_t}^{m,2}|^2 \right) d_{2,k} |\frac{\mathcal{V}}{N_B}| N_{T} + j(k) + \eta_2^{eq}(l_i, j) \tag{4.5}
$$

where the equivalent noise after combining becomes

$$
\eta_1^{eq}(l_i, j) = \sum_{m=1}^{M} \left[ (h_{i_t}^{m,1})^* \cdot \eta_{m,1}(l_i, j) + h_{i_t}^{m,2} \cdot [\eta_{m,2}(l_i, j)]^* \right] \tag{4.6}
$$

$$
\eta_2^{eq}(l_i, j) = \sum_{m=1}^{M} \left[ (h_{i_t}^{m,2})^* \cdot \eta_{m,1}(l_i, j) - h_{i_t}^{m,1} \cdot [\eta_{m,2}(l_i, j)]^* \right]. \tag{4.7}
$$

The equivalent channel parameters to on the $l_i$-th sub-carrier is given by

$$
h_{i_t}^{eq} = \sum_{m=1}^{M} \left( |h_{i_t}^{m,1}|^2 + |h_{i_t}^{m,2}|^2 \right). \tag{4.8}
$$

$\eta_1^{eq}(l_i, j)$ and $\eta_2^{eq}(l_i, j)$ are both Gaussian with zero mean and the variance

$$
E \left\{ |\eta_1^{eq}(l_i, j)|^2 \right\} = E \left\{ |\eta_2^{eq}(l_i, j)|^2 \right\}
$$

$$
= \sum_{m=1}^{M} \left( |h_{i_t}^{m,1}|^2 + |h_{i_t}^{m,2}|^2 \right) E \left\{ |\eta_{m,1}(l_i, j)|^2 \right\}
$$

$$
= \sum_{m=1}^{M} \left( |h_{i_t}^{m,1}|^2 + |h_{i_t}^{m,2}|^2 \right) E \left\{ |\eta_{m,2}(l_i, j)|^2 \right\}
$$

$$
= \sum_{m=1}^{M} \left( |h_{i_t}^{m,1}|^2 + |h_{i_t}^{m,2}|^2 \right). \tag{4.9}
$$

The received signal transmitted from the $n$-th transmitting antenna during $T_1$ on

the $l_i$-th sub-carrier using the $K_T$-th time domain spreading code, after time domain
despreading is given by

\[
    r_n(l_i) = \frac{1}{N_T} \sum_{j=0}^{N_T-1} \tilde{Y}_n(l_i, j) \times e^T_j(k)
\]

\[
    = \sqrt{\frac{E_s}{2N_S}} h_{t_i}^{eq} d_{n,k} [c_{l_i}^{F} [\frac{1}{N_B}]](k) + \eta_n^{eq}(l_i). \quad (4.10)
\]

Then a suitable combining method is employed to extract frequency domain diversity over \( N_F \) sub-carriers of interest. The combining weight for the \( l_i \)-th sub-carrier is given below

- for MRC,

\[
    w^{eq} (l_i) = (h_{t_i}^{eq})^* = h_{t_i}^{eq}, \quad (4.11)
\]

- for EGC,

\[
    w^{eq} (l_i) = \frac{(h_{t_i}^{eq})^*}{|h_{t_i}^{eq}|} = 1, \quad (4.12)
\]

- for MMSE combining without MCI,

\[
    w^{eq} (l_i) = \frac{(h_{t_i}^{eq})^*}{(h_{t_i}^{eq})^2 + \left(\frac{E_s}{2N_FN_0}\right)^{-1}} = \frac{h_{t_i}^{eq}}{(h_{t_i}^{eq})^2 + \left(\frac{E_s}{2N_FN_0}\right)^{-1}}. \quad (4.13)
\]

The output of the frequency domain despreader is given by

\[
    y_{n,k} = \sum_{i=0}^{N_F-1} r_n(l_i) \cdot c_{l_i}^{F} [\frac{1}{N_B}](k) \cdot w^{eq}(l_i). \quad (4.14)
\]

Then this output is normalized to form the decision variable, which will be used in MCI cancellation if needed. The decision variable is given by

\[
    z_{n,k} = \frac{1}{\sqrt{\frac{E_s}{2N_S}}} \cdot \frac{y_{n,k}}{\sum_{i=0}^{N_F-1} h_{t_i}^{eq} \cdot w^{eq}(l_i)}. \quad (4.15)
\]
Finally a hard decision based on minimum distance criteria is made to recover the corresponding user data:

\[ \hat{d}_{n,k} = \arg\min_{\{s\}} |z_{n,k} - s|^2, \]  

(4.16)

where \( \{s\} \) is the set of all possible transmitted data symbols.

### 4.3 Performance Evaluation

In this section, we derive the BER expression for MIMO-OFCDM systems using Alamouti scheme with multiple receive antennas for MRC, EGC, and MMSE combining.

Let \( \overrightarrow{e_n} \) represent the error vector corresponding to the data symbols sent to the \( n \)-th transmit antenna during \( T_1 \), then \( \overrightarrow{e_n} \) is given by

\[
\overrightarrow{e_n}^2 = z_{n,k} - d_{n,k} = \sum_{i=0}^{N_F-1} r_n(l_i) \cdot c^F_{l_i} \cdot w^{eq}(l_i) - d_{n,k}
\]

\[
= \sqrt{\frac{E_s}{2N_S}} \sum_{i=0}^{N_F-1} h_{l_i}^{eq} \cdot w^{eq}(l_i)
\]

\[
\frac{N_F-1}{\sqrt{2N_S}} \sum_{i=0}^{N_F-1} \overline{h^{eq}}(l_i) \cdot w^{eq}(l_i)
\]

\[
\overrightarrow{e_n}^2 = \sum_{i=0}^{N_F-1} \overline{h^{eq}}(l_i) \cdot w^{eq}(l_i)
\]

(4.17)

where \( h_{l_i}^{eq} \) is given by (4.8), \( \bar{\rho} \) is the ratio of total transmitted power of each data symbol during \( T_1 \) and \( T_2 \) to the background noise power, and \( \overline{h^{eq}}(l_i) \) is the equivalent noise of the \( l_i \)-th sub-carrier at the receiver side during \( T_n \) (\( n = 1, 2 \)). Since the additive noise terms \( \eta_{m,1}(l_i, j) \) and \( \eta_{m,2}(l_i, j) \) are complex AWGN with 0.5 per dimension, it is easy to show that the equivalent noise, \( \overline{h^{eq}}(l_i) \), is complex Gaussian noise with zero mean and variance \( \frac{1}{2N_T} \cdot \sum_{m=1}^{M} (|h^{m,1}_{l_i}|^2 + |h^{m,2}_{l_i}|^2) = \frac{h^{eq}_{l_i}}{2N_T} \) per dimension.
Conditioned on the channel, \( \sum_{i=0}^{N_F-1} \frac{\eta_n^q(l_i)}{\eta_n(l_i)} \cdot w^q(l_i) \) is also Gaussian with zero mean and variance

\[
\sigma_n^2 = \frac{1}{N_T} \sum_{i=0}^{N_F-1} h_{l_i}^q |w^q(l_i)|^2. \quad (4.18)
\]

### 4.3.1 MRC

With MRC, since the equivalent channel coefficient \( h_{l_i}^q \) is a real number,

\[
w(l_i) = (h_{l_i}^q)^* = h_{l_i}^q. \quad (4.19)
\]

The error vector \( \bar{e}_n \) becomes

\[
\bar{e}_n = \frac{\sum_{i=0}^{N_F-1} \eta_n^q(l_i) \cdot w^q(l_i)}{\sqrt{\frac{p}{2NS}} \sum_{i=0}^{N_F-1} h_{l_i}^q \cdot w^q(l_i)} = \frac{\sum_{i=0}^{N_F-1} \eta_n^q(l_i) \cdot h_{l_i}^q}{\sqrt{\frac{p}{2NS}} \sum_{i=0}^{N_F-1} (h_{l_i}^q)^2}, \quad (4.20)
\]

and

\[
\sigma_n^2 = \frac{1}{N_T} \sum_{i=0}^{N_F-1} h_{l_i}^q |w(l_i)|^2 = \frac{1}{N_T} \sum_{i=0}^{N_F-1} (h_{l_i}^q)^3. \quad (4.21)
\]

When BPSK modulation is used, the probability of bit error conditioned on the channel is given by

\[
\text{BER}_{|h_i} = P\left(\text{Re}(\bar{e}_n^* > 1)\right)
\]

\[
= P\left(\text{Re}\left(\frac{\sum_{i=0}^{N_F-1} \eta_n^q(l_i) \cdot w^q(l_i)}{\sqrt{\frac{p}{2NS}} \sum_{i=0}^{N_F-1} h_{l_i}^q \cdot w^q(l_i)} > 1\right)\right)
\]

\[
= Q\left(\frac{\sqrt{\frac{p}{N_F}} \sum_{i=0}^{N_F-1} (h_{l_i}^q)^2}{\sqrt{\sum_{i=0}^{N_F-1} (h_{l_i}^q)^3}}\right). \quad (4.22)
\]
4.3.2 EGC

With EGC, the combining weight of each sub-carrier is:

$$w(l_i) = \frac{(h_{li}^{eq})^*}{h_{li}^{eq}} = 1.$$  \hspace{1cm} (4.23)

The error vector $\overrightarrow{e_n}$ becomes

$$\overrightarrow{e_n} = \frac{\sum_{i=0}^{N_F-1} \bar{h}_n^{eq}(l_i) \cdot w^{eq}(l_i)}{\sqrt{\frac{\rho}{2NS}}} = \frac{\sum_{i=0}^{N_F-1} \bar{h}_n^{eq}(l_i)}{\sqrt{\frac{\rho}{2NS}}} = \frac{\sum_{i=0}^{N_F-1} h_i^{eq}}{\sqrt{\frac{\rho}{2NS}}}.$$  \hspace{1cm} (4.24)

and

$$\sigma_n^2 = \frac{1}{NT} \sum_{i=0}^{N_F-1} h_i^{eq} |w(l_i)|^2 = \frac{1}{NT} \sum_{i=0}^{N_F-1} h_i^{eq}.$$  \hspace{1cm} (4.25)

Therefore, the probability of bit error conditioned on the channel is given by

$$BER|_h = P(\text{Re}(\bar{e}_n^+)| > 1)$$

$$= P \left( \text{Re} \left( \frac{\sum_{i=0}^{N_F-1} \bar{h}_n^{eq}(l_i)}{\sqrt{\frac{\rho}{2NS}}} \right) > 1 \right)$$

$$= Q \left( \frac{\frac{\rho}{2NS} \sum_{i=0}^{N_F-1} h_i^{eq}}{\sqrt{\frac{\rho}{2NS}}} \right).$$  \hspace{1cm} (4.26)

4.3.3 MMSE Combining

The combining weight of MMSE combining for the systems without MCI is

$$w(l_i) = \frac{(h_{li}^{eq})^*}{(h_{li}^{eq})^2 + \left( \frac{E_s}{2NFN_0} \right)^{-1} - 1} = \frac{h_{li}^{eq}}{(h_{li}^{eq})^2 + \left( \frac{\rho}{2NF} \right)^{-1}}.$$  \hspace{1cm} (4.27)

The error vector $\overrightarrow{e_n}$ is given by

$$\overrightarrow{e_n} = \frac{\sum_{i=0}^{N_F-1} \bar{h}_n^{eq}(l_i) \cdot w^{eq}(l_i)}{\sqrt{\frac{\rho}{2NS}}} = \frac{\sum_{i=0}^{N_F-1} \bar{h}_n^{eq}(l_i) \cdot \frac{h_i^{eq}}{(h_i^{eq})^2 + \left( \frac{\rho}{2NF} \right)^{-1}}}{\sqrt{\frac{\rho}{2NS}}}.$$  \hspace{1cm} (4.28)
and

\[ \sigma^2 = \frac{1}{N_T} \sum_{i=0}^{N_F-1} h'^2_{i} |w_{eq}(l_i)|^2 \]

\[ = \frac{1}{N_T} \sum_{i=0}^{N_F-1} \left[ (h'^2_{i})^2 + \left( \frac{\bar{\rho}}{2N_F} \right)^{-1} \right]. \quad (4.29) \]

Then, we can find the probability of bit error conditioned on the channel as

\[ BER|_h = P \left( \text{Re} \left( \sum_{i=0}^{N_F-1} \hat{\eta}_{eq}(l_i) \cdot \frac{h'^2_{i}}{(h'^2_{i})^2 + \left( \frac{\bar{\rho}}{2N_F} \right)^{-1}} \right) > 1 \right) \]

\[ = Q \left( \sqrt{2} \sum_{i=0}^{N_F-1} \frac{1}{1 + \left( \frac{\bar{\rho}}{2N_F} \right)^{-1}} \right). \quad (4.30) \]

4.4 Simulation Results

The simulation and semi-analytical results for MIMO-OFCDM systems are presented in this section. CSI is assumed to be perfectly known at the receiver side, and it is fixed for the duration of two consecutive OFCDM frames. Different system parameters are used when investigating the performance with MRC, EGC and MMSE combining.

Figure 4.4 presents the performance of MIMO-OFCDM systems with different number of receive antennas over Rayleigh fading channels using MRC. The analytical result for SISO-OFCDM system with \( N_F = 8 \) is shown as a reference. From this figure we can see the slope of the curves increases as the spatial diversity of the system increases. The MIMO-OFCDM systems with 1, 2, and 3 receive antennas have overall diversity orders of 16, 32, and 48 respectively, whereas the SISO-OFCDM system has
Figure 4.4: BER for MIMO-OFCDM system using MRC over Rayleigh fading channel: $N_F = 8$, $N_T = 8$, $K = 8$

a diversity order of 8. At the same time, we can see the accuracy of our semi-analytical results when compared with simulations.

In Figure 4.5 we investigate the performance degradation caused by correlation among sub-carriers of interest. The MIMO-OFCDM system employs two transmit antennas and two receive antennas with EGC. From this figure we can see that for SISO and MIMO-OFCDM systems, the impact on the overall diversity order is the same when sub-carriers of interest have correlation. This is because the correlation is only present among sub-carriers from the same transmit antenna to the same receive antenna. If spatial correlation exists in the MIMO system in addition to the correlation in frequency domain, the system performance will be degraded further.

Figure 4.6 shows the performance of MIMO-OFCDM systems over Rayleigh fading channels when MCI is introduced. The BER performance using different com-
Figure 4.5: BER for OFCDM system using EGC over Rayleigh fading channel: $N_F = 8$, $N_T = 8$, $K = 8$

Figure 4.6: BER for MIMO-OFCDM system with MCI over Rayleigh fading channel: 
$N_F = 16$, $N_T = 8$, $N = 2$, $M = 3$, $K = 32$
bining methods are compared. The results are presented for systems running at twenty-five percent of full load; that is every four spreading codes are interfering with each other. From this figure we can see that MMSE combining offers the best performance where it can deliver large diversity gain when MCI is present.

We present the performance of two MIMO-OFCDM systems over Ricean fading channels in Figs. 4.7 and 4.8, where three receive antennas with EGC and one receive antenna with MMSE combining are used in the two systems respectively. As a reference, we also present system performance over Gaussian channels with the same number of receive antennas. Systems with different values of $\kappa$ are compared in both figures, where $\kappa$ represents the ratio of the dominant component to the total power of scattered waves in Ricean fading channels. From these two figures we verified
Figure 4.8: BER for MIMO-OFCDM system using MMSE combining over Ricean fading channel: $M = 1$, $N_F = 8$, $N_T = 8$, $K = 8$

that as $\kappa$ increases from zero to infinity, the system performance converges to the performance on the Gaussian channel.

4.5 Conclusion

In this chapter we explained the transmission and detection methods for MIMO-OFCDM systems using Alamouti scheme with $M$ receive antennas. It was shown that by employing STBC, MIMO-OFCDM systems provide significantly better performance than SISO-OFCDM systems without affecting the system transmission rate. We also derived BER semi-analytical results for MIMO-OFCDM with MRC, EGC, and MMSE combining methods. The analytical results obtained in this chapter are shown to be accurate when compared with the simulated ones. In addition, More-
over, we compared different combining methods when MCI is present, from which we showed that MMSE combining outperforms both MRC and EGC. Same as in the SISO-OFCDM cases, the analytical results we have obtained in this chapter are also applicable for general MIMO-OFCDM systems with different parameters over different fading channels.
CHAPTER 5

CONCLUSIONS AND FUTURE WORKS

5.1 Conclusions

In this section, we present a summary of the study accomplished in this thesis.

In Chapter 3, we studied the system structure, transmission and detection method, and the performance of SISO-OFCDM systems in downlink transmission. The SISO-OFCDM systems studied in this thesis involve two dimensional spreading in both time and frequency domains. At the receiver side, MRC, EGC, and MMSE combining techniques are employed to extract diversity in frequency domain. Semi-analytical expressions conditioned on the channel are derived for MRC, EGC, and MMSE combining in systems with no MCI. The accuracy of the semi-analytical results are proved by comparison with the simulated ones. Also, we examined the effects of sub-carrier correlation and MCI on the system BER.

In Chapter 4, we extended the results in Chapter 3 to MIMO-OFCDM systems when using STBC. By hiring Alamouti scheme with $M$ receive antennas, we explained the signal structure, transmission and detection methods in MIMO-OFCDM systems. By implementing STBC, MIMO-OFCDM systems achieve spatial diversity on top of the frequency diversity, which shows significant improvement on the BER performance compared to SISO-OFCDM systems. Moreover, we derived the semi-analytical expressions of BER for different combining schemes in systems without
MCI. In addition, we checked the effect of sub-carrier correlation and MCI. The sub-carrier correlation has similar effect for both SISO and MIMO-OFCDM systems, and we confirmed MMSE combining is optimal when MCI is present in both SISO and MIMO-OFCDM systems.

5.2 Future Works

There are some future works that can be extended from this thesis.

1. We have derived the semi-analytical expressions for both SISO and MIMO-OFCDM systems. However, these expressions are conditioned on the channels. When the distribution of the channel coefficient is known, closed-form analytical results may be obtained.

2. In our study, we focused on systems without MCI. As mentioned in Chapter 3, we can approximate MCI as a Gaussian noise only when $K_c > 50$. However, how to include MCI in the semi-analytical expressions when $N_F < 50$ can be another extension of our study.

3. All the systems studied in this thesis assume downlink transmission, where the user data are perfectly synchronized. Therefore, the performance of up-link OFCDM transmission is also another interesting topic.
Bibliography


[33] Zhengdong Luo, Junshi Liu, Ming Zhao, Yuanan Liu, and Jinchun Gao, “Double-Orthogonal Coded Space-Time-Frequency Spreading CDMA Scheme,”


[44] Vincent Le Nir, Maryline HBlard and Rodolphe Le Gouable, “Space-Time Block Coding Applied to Turbo Coded Multicarrier CDMA,” The 57th IEEE Semian-


