NOTE TO USERS

This reproduction is the best copy available.
Three Essays on Expectation Driven Business Cycles

Shen Guo

A Thesis
In the Department
of
Economics

Presented in Partial Fulfillment of the Requirements
For the Degree of Doctor of Philosophy at
Concordia University
Montreal, Quebec, Canada

July 2009

© Shen Guo, 2009
NOTICE:

The author has granted a non-exclusive license allowing Library and Archives Canada to reproduce, publish, archive, preserve, conserve, communicate to the public by telecommunication or on the Internet, loan, distribute and sell theses worldwide, for commercial or non-commercial purposes, in microform, paper, electronic and/or any other formats.

The author retains copyright ownership and moral rights in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

In compliance with the Canadian Privacy Act some supporting forms may have been removed from this thesis.

While these forms may be included in the document page count, their removal does not represent any loss of content from the thesis.
ABSTRACT

Three Essays on the Expectation Driven Business Cycles

Shen Guo, Ph.D.

Concordia University, 2009

This thesis studies business cycles driven by agents' expectation of future technology changes.

The first chapter explores the effects which nominal rigidities and monetary policies have on the generation of Pigou cycles. The optimal response of the central bank is analyzed under circumstances when agents receive a signal indicating the technology change in the future. To achieve these objectives, I introduce nominal rigidities and monetary policy into a standard two-sector model with non-durable and durable goods. The optimal reaction of the central bank is found by solving the Ramsey optimization problem. I find that nominal rigidities tend to amplify the responses to the expectation and monetary policies affect the expectation driven business cycles by affecting the real interest rate and user cost of durable goods. Another interesting result is that a simple policy rule reacting to the inflation rates in both non-durable and durable sector with appropriate weights can closely mimic the performance of the Ramsey policy.

The second chapter estimates a sticky price two-sector model with home production and capital adjustment costs to assess the significance of the news shocks in generating aggregate fluctuations. The analysis suggests that news shocks account for about 34% of the fluctuations in the aggregate output, 25% of the fluctuations in consumption-sector output and 38% of the fluctuations in investment-sector output.

The third chapter explores the booms and busts induced by news shocks in a model economy with financial market frictions. With the presence of financial market
frictions, firms have to pay an external finance premium which depends inversely on their net values. This provides firms with an incentive to build up capital stocks now to lower the external finance premium in the future. When firms receive news indicating a future technology improvement, they anticipate the need for more capital and so more external finance in the future; they could lower their future external finance costs by building up their capital and net values now. By adding financial market frictions into an otherwise standard RBC model, the model in chapter 3 succeeds in generating a boom when a news shock hits the economy.
Acknowledgements

I want to express my special appreciation to my thesis supervisor Paul Gomme for his excellent guidance.

I am indebted to members of my committee, Stéphane Auray and Tatyana Koreshkova for their superb advice.

I want to thank Jaleel Ahmad, Syed Ahsan, Gordon Fisher, Nikolay Gospodinov, Carol Lau, Damba Lkhagvasuren, Christian Sigouin, William Sims for their encouragement and support.

I also want to thank participants of 42nd Annual Meeting of the Canadian Economics Association and 4th Dynare conference for their valuable comments.

Finally, I want to thank Lucy Gilson and Elise M. Melancon for their superb administrative assistance during my study at Concordia.
Contents

List of Figures viii

List of Tables x

Introduction 1

Chapter 1. Nominal Rigidity, Monetary Policy and Expectation Driven Business Cycles 4

1.1. Introduction 4
1.2. Model 8
1.3. Calibration and Solution 20
1.4. Simulation Results 21
1.5. Approximate Ramsey Policy Using Simple Policy Rules 28
1.6. Conclusion 38

Chapter 2. Exploring the Significance of News Shocks in Estimated Dynamic Stochastic General Equilibrium Model 41

2.1. Introduction 41
2.2. Model 45
2.3. Data and Calibration 55
2.4. A Preview of Impulse Responses 59
2.5. Estimation Results 63
2.6. Model Evaluation 67
2.7. Conclusion 79
List of Figures

1.1 Responses to an Unrealized News Shock. 23
1.2 Responses to an Unrealized News Shock: Higher Elasticity of Substitution between Durables and Non-durables. 29
1.3 Responses to an Unrealized News Shock to the Durable Sector. 30
1.4 Approximate the Ramsey Policy with Estimated Simple Rules. 32
1.5 Approximate the Ramsey Policy with Estimated Simple Rules. 35
1.6 Approximate the Ramsey Policy with Estimated Simple Rules. 36
1.7 Responses to an Unrealized News Shock under a Weakly Inflation Target Rule. 38
2.1 Responses to News Shocks: Flexible Prices Vs. Rigid Prices. 61
2.2 Responses under Various Elasticity of Substitution. 62
2.3 Responses to Consumption-sector Shocks. 72
2.4 Responses to Investment-sector Shocks. 73
2.5 Responses to Investment-sector Shocks in the Model without Capital Adjustment Costs. 74
3.1 Responses to News Shocks: Firms Can Produce Capital. 102
3.2 Responses to News Shocks: Hikes in the Real Interest Rate. 103
3.3 Responses to News Shocks: With Vs. Without Habit Formation. 106
3.4 Responses to News Shocks: Firms Can Only Purchase Capital in the Market. 107
3.5 Responses to News Shocks: Without Time-to-build Technology Constraint. 108
List of Tables

1.1 The Implications of Various Realizations of News Shock 15
1.2 Baseline Parameter Values 21
1.3 Estimation of Simple Policy Rules 33
1.4 Estimation of Simple Policy Rules 34
2.1 Calibrated Values of Parameters 58
2.2 Maximum Likelihood Estimation and Standard Error 64
2.3 Standard Deviations of Selected Variables for Three Models and U.S. Data 68
2.4 Correlations of Selected Variables for U.S. Data 69
2.5 Correlations of Selected Variables for Model with All Features 69
2.6 Correlations of Selected Variables for Model without Homework 70
2.7 Correlations of Selected Variables for Model with Flexible Prices 70
2.8 Correlations of Selected Variables for Model without Capital Adjustment Costs 70
2.9 Forecast Error Variance Decomposition: Output in Consumption Sector 78
2.10 Forecast Error Variance Decomposition: Hours Worked in Consumption Sector 78
2.11 Forecast Error Variance Decomposition: Output in Investment Sector 78
2.12 Forecast Error Variance Decomposition: Hours Worked in Investment Sector 78
2.13 Forecast Error Variance Decomposition: Aggregate Output 79
2.1 Forecast Error Variance Decomposition: Total Hours Worked  

3.1 Calibrated Values of Parameters  

3.2 Business Cycle Moments: U.S. Economy  

3.3 Business Cycle Moments: Baseline Model  

3.4 Business Cycle Moments: Alternative Model  

xi
Introduction

This thesis studies business cycles driven by individuals’ expectation of future technology changes.

Classical real business cycle models assume that fluctuations in the macroeconomic variables are caused by unexpected technology changes. However, it is well known that real business cycle models have difficulties explaining recessions. In real business cycle models, busts of the economy are explained by technological regress, which contradicts the fact that the technological regress rarely occurs in the real world.

Beaudry and Portier (2004) explore a theory of business cycles (Pigou cycles) in which booms and busts of the economy are caused by agents’ expectation of future technology changes. They introduce a new source of business cycles into the literatures, that is, fluctuations of the economy caused by news shocks: good news about future technology leads to a boom and an unrealized expectation could generate a bust. They find that their framework with news shocks can explain recessions without relying on technological regress. Since their seminal paper, researchers’ interests in news shocks have grown.¹

This thesis consists of three essays, contained in Chapters 1, 2 and 3. Chapter 1 explores the central bank’s optimal reaction to news shocks. Chapter 2 investigates the significance of news shocks in explaining business cycle fluctuations. Chapter 3 studies the role financial market frictions play in generating expectation driven business cycles.

Chapter 1 explores the effects nominal rigidities and monetary policies have on the generation of Pigou cycles, that is, business cycles driven by agents' expectations of future technology. The optimal response of the central bank is analyzed under circumstances when agents receive a signal indicating the technology change in the future. To achieve these objectives, I introduce nominal rigidities and monetary policy into a standard two-sector model with non-durable and durable goods. The optimal reaction of the central bank is found by solving the Ramsey optimization problem. Furthermore, the Ramsey optimal policy can be approximated by simple operational policy rules. I find that nominal rigidities tend to amplify the responses to the expectation and monetary policies affect the expectation driven business cycles by affecting the real interest rate and user cost of durable goods. Another interesting result is that a simple policy rule reacting to the inflation rates in both non-durable and durable sector with appropriate weights can closely mimic the performance of the Ramsey policy.

Chapter 2 estimates a sticky price two-sector model with home production and capital adjustment costs to assess the significance of the news shocks in generating aggregate fluctuations. All model features are proven to be essential to generate the boom when agents expect a future technology improvement. The main result of this chapter is that news shocks play a substantial role in accounting for aggregate fluctuations. The analysis suggests that news shocks account for about 34% of the fluctuations in the aggregate output, 25% of the fluctuations in consumption-sector output and 38% of the fluctuations in investment-sector output.

Chapter 3 explores the booms and busts induced by news shocks in a model economy with financial market frictions. Firms can accumulate capital through either purchase of the existing capital or producing new capital by themselves. Firms need to borrow from financial intermediaries to finance their purchases of capital. With the
presence of financial market frictions, firms have to pay an external finance premium which depends inversely on their net values. This provides firms with an incentive to build up capital stocks now to lower the external finance premium in the future. When firms receive news indicating a technology improvement in the future, they anticipate the need for more capital and so more external finance in the future; they could lower their future external finance costs by building up their capital and net values now. By adding financial market frictions into an otherwise standard real business cycle model, this model in chapter 3 succeeds in generating a boom when a news shock hits the economy.
CHAPTER 1

Nominal Rigidity, Monetary Policy and Expectation Driven Business Cycles

1.1. Introduction

The objectives of this chapter are: (1) to study the effect of nominal rigidities and monetary policies on expectation driven business cycles (Pigou cycles); (2) to find the Ramsey optimal policy when agents in the economy receive a signal indicating future technology change. I am particularly interested in studying whether the central bank can mimic the Ramsey optimal policy with a relatively simple rule (one targeting only a few macroeconomic variables).

Beaudry and Portier (2004) formalize Pigou (1926)'s idea and define Pigou cycles as: (i) agents receive signals or news indicating that technology will change in the near future and start to react, and (ii) later, the expectation does not realize, which leads to a reversion of previous reactions. The economy is said to be hit by a news shock. For example, an optimistic forecast of future technological improvement leads to a boom defined as an increase in aggregate output, employment, investment and consumption. When agents notice that their expectation does not realize, the economy incurs a recession defined as a fall in all the same aggregate variables. They also illustrate that standard one-sector and two-sector equilibrium models used in the macroeconomic literature cannot produce Pigou cycles. Of course, their largest contribution is to find a particular multi-sector model in which Pigou cycles can arise. Their finding is that expectation driven business cycles can arise in neoclassical models when one allows for a sufficiently rich description of inter-sector production technology. In particular,
the key assumption giving rise to the Pigou cycles is that non-durable goods and durable goods exhibit enough complementarities in the production of the final goods.

In this chapter, I formulate a dynamic general equilibrium model with two sectors that produce durable and non-durable goods respectively. One main difference with Beaudry and Portier's model consists in incorporating nominal price rigidity and monetary policy. If prices are flexible, firms will adjust their prices only when the expected change in technology realized. By contrast, if firms cannot adjust their prices every period, they will start to adjust their prices once they receive the signal indicating a change in the technology. The price adjustment will happen before the realization of the technology change. The introduction of nominal rigidities causes the rise of the relative price of durable goods to non-durable goods before the realization of the news about the technology improvement in non-durable goods sector, which boosts the demand of non-durable goods. Barsky, House and Kimball (2007) point out that one of the features of two sector models with long-lived durable goods is that relative price is the key determinant of the demand of non-durable goods. Furthermore, if non-durable goods and durable goods are assumed to be complements, the boom in the non-durable goods sector could easily spread to the durable goods sector. So, the nominal rigidities amplify the extent of boom and bust when a news shock hits the economy.

The introduction of monetary policy also plays an important role in generating Pigou cycles. Central banks can have an impact on the real interest rate by controlling the nominal interest rate. Furthermore, agents' decision of durable goods accumulation is very sensitive to the change in the real interest rate. Notice that if the depreciation rate is small, the durable goods stock is much greater than durable goods production at each period. A small fluctuation in the durable goods stock
translates into a much greater fluctuations in durable goods output and labor supply in this sector. The demand of durable goods is determined by the following two forces: it is affected by the demand of non-durable goods depending on whether they are complements or substitutes; in the meanwhile, it is also determined by the real interest rate controlled by the monetary policy. The interaction of these two forces determines the stock of durable goods and its production.

In this chapter, I also explore the optimal reaction of the central bank to news shocks by solving the Ramsey optimization problem. The optimal reaction of the central bank is to raise the real interest rate to prevent agents from accumulating too much durable goods during the boom period when agents expect an improvement in the technology and lower the real interest rate to shorten the bust period after the agents' expectations do not realize. Another interesting observation is that Ramsey policy fails to restore the outputs in both sectors to their potential levels, the output levels in the case of flexible prices. The main target of Ramsey policy is to restore the labor supply to its potential level by adjusting the production and labor inputs in the durable goods sector. If central banks are assumed to be committed to simple implementable rules, the coefficients in the simple rules can be estimated using the simulated interest rates under the Ramsey policy. Comparing the impulse responses indicates that Ramsey optimal policy can be approximated by a simple policy rule targeting inflation rates in both non-durable and durable sectors with appropriate weights.

This structure also allows me to study the following problem: is complementarity between non-durable goods and durable goods still a necessary condition to generate expectation driven business cycles after the introduction of nominal rigidities and monetary policy? My finding indicates that monetary policy plays an important role in generating Pigou cycles. In particular, a weak inflation targeting policy rule
helps generate Pigou cycles without assuming complementarities between non-durable goods and durable goods.

The framework in this chapter is closely related to the recent development of two sector models with nominal rigidities. Aoki (2001) studies optimal monetary policy responses to relative-price changes in a two-sector framework with a flexible-price sector and a sticky-price sector. Benigno (2004) evaluates monetary policy in a currency area where price rigidities may differ between countries. Barsky, House and Kimball (2007) explore the comovements of non-durable and durable goods sectors responding to a monetary shock in a two sector model with nominal rigidities and long-lived durable goods. Erceg and Levin (2006) study the optimal monetary policy in a two sector model with durable goods. They highlight the distinction between the non-durable and durable sectors in that the durable goods sector is much more interest-sensitive than the non-durable sector. Monacelli (2008,2009) introduces collateral constraints into a two-sector model with non-durable and durable goods to study the comovements in these two sectors in response to monetary policy shocks and optimal monetary issues. Other related literatures include Christiano, Ilut, Motto and Rostagno (2008) and Jaimovich and Rebelo's (forthcoming) research on the possibility of generating expectation driven business cycles in one sector models. They succeed in generating booms and busts of consumption, investment and output by adding investment adjustment costs, variable utilization of capital and habit persistence in preference into a standard one sector model. However, it is not that straightforward to get corresponding booms and busts of asset prices in their frameworks. Asset prices unexpectedly slump during the booms when all the other variables rise as expected. To solve this problem, Christiano, Ilut, Motto and Rostagno (2008) extend their model by adding sticky prices, sticky wages and standard Taylor-rule
monetary policies. Compared with their frameworks, my framework involves fewer real and nominal rigidities.

The remainder of this chapter is organized as follows: section 2 outlines the dynamic general equilibrium model and defines the Ramsey optimal policy. section 3 describes the parameter calibration and solution methods. In section 4, experiments and simulations are conducted in models with three different features: the model with flexible prices, the model with nominal rigidities and exogenous money supply, and the model with nominal rigidities and Ramsey optimal policy. Through the comparison of the responses in these three cases, we can study the effects of nominal rigidities and monetary policies have on the generation of the Pigou cycles and explore the optimal policy reactions to the news shock. Simulated interest rates under the Ramsey policy are used to estimate simple interest rate rules in section 5. Section 6 concludes.

1.2. Model

The economy is composed of two sectors: a non-durable goods sector and a durable goods sector. There are two types of firms in each sector: final goods firms produce final goods using intermediate goods; intermediate goods firms are monopolistic competitors that each produces a differentiated product using labor. These intermediate goods firms determine their prices following a Calvo-type staggered price adjustment. Households supply labor to both sectors and derive utility from consumption of non-durable final goods and services of durable final goods. The central bank conducts monetary policy.
1.2.1. Households

The household purchases a non-durable good \( C_t \) and a durable good \( D_t \) in the market and derives its utility from the consumption of a final good \( X_t \). \( X_t \) is defined as a CES composite of non-durable goods \( C_t \) and durable goods \( D_t \).

\[
X_t = \left[ (1 - \alpha)^{\frac{1}{\eta}} C_t^{\frac{\eta - 1}{\eta}} + \alpha^{\frac{1}{\eta}} D_t^{\frac{\eta - 1}{\eta}} \right]^{\frac{\eta}{\eta - 1}}
\]

where \( \alpha \) is the share of durable goods in the composite consumption index. \( \eta > 0 \) is the elasticity of substitution between non-durable goods and durable goods. In the case \( \eta \to 0 \), non-durable goods and durable goods are perfect complements; whereas if \( \eta \to \infty \), the two goods are perfect substitutes.

The household maximizes the following expected utility

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( \log(X_t) - \frac{N_t^{1+\sigma}}{1+\sigma} \right)
\]

Subject to the budget constraint (in nominal terms):

\[
P_{c,t} C_t + P_{d,t} (D_t - (1 - \delta) D_{t-1}) + B_t + M_t = W_t N_t + R_{t-1} B_{t-1} + M_{t-1} + T_t + \Pi_{c,t} + \Pi_{d,t}
\]

where \( N_t \) is total labor supply; \( B_t \) is end-of-period \( t \) nominal bond and \( R_{t-1} \) is the nominal interest rate on the bond stipulated at period \( t - 1 \). \( M_t \) corresponds to the nominal money balances held and \( W_t \) to the nominal wage. Labor is assumed to be perfectly mobile across sectors, implying that the nominal wage rate is common across
sectors. $\Pi_{c,t}$ and $\Pi_{d,t}$ are dividends received from the ownership of intermediate firms in both sectors. $T_t$ is lump-sum transfers net of taxes.

Following Barsky, House and Kimball (2007), money demand is assumed to be proportional to nominal purchases:

\begin{equation}
M_t = P_{c,t}C_t + P_{d,t}(D_t - (1 - \delta)D_{t-1})
\end{equation}

The household chooses $\{N_t, B_t, D_t, C_t\}$ to maximize (1.2) subject to (1.3). Defining $U_{k,t}(k \in C, N, D)$ as the marginal utility of the respective variables, first order conditions for household’s decision problem read:

\begin{equation}
\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_{c,t}}
\end{equation}

\begin{equation}
q_t U_{c,t} = U_{d,t} + \beta(1 - \delta)E_t(U_{c,t+1}q_{t+1})
\end{equation}

\begin{equation}
U_{c,t} = \beta E_t(R_t \frac{P_{c,t}}{P_{c,t+1}}U_{c,t+1})
\end{equation}

where $q_t = P_{d,t}/P_{c,t}$ is the relative price of the durable good. Equation (1.5) links the real wage to the household’s marginal rate of substitution between consumption and leisure. Equation (1.6) requires that the household equates the marginal utility of current non-durable consumption to the marginal gain of durable services. The marginal gain of durable services includes two parts: (i) the direct utility gain of an additional unit of durable; (ii) the expected utility stemming from the consumption of the resale value of the durable purchased in previous period. Equation (1.7) equates the cost of sacrificing one unit of consumption to the benefit of investing this money to the bond market. For future reference, the term $\beta U_{c,t+1}/U_{c,t}$ is defined as a stochastic discount factor $\Delta_t$. 

10
Barsky, House and Kimball (2007) point out that one feature of the two sector model with long-lived durable goods is that the shadow price of the durable goods is approximately constant following a temporary shock, which can be shown in the following equation:

\[(1.8) \quad \gamma_t = q_t U_{c,t} = U_{d,t} + \beta(1 - \delta)E_t(U_{d,t+1}) + \beta^2(1 - \delta)^2 E_t(U_{d,t+2}) + \ldots \]

where \( \gamma_t \) is the shadow value of the durable goods. If durables are long-lasting, meaning that \( \delta \) is close to zero, then, the ratio of the stock of durables to its investment flow is very high. Even large fluctuations in purchases of durable goods have only minor effects on the stock. It is reasonable to treat the shadow value of durable goods as roughly constant in response to a news shock, since news shock only causes a temporary and modest deviation of the durable goods stock from the steady state. From (1.8), it is straightforward to show that the marginal utility of non-durable goods, and thus the consumption of non-durable goods are determined by the price of the durable good relative to the price of the non-durable good.

The equation (1.6) can be rewritten as

\[(1.9) \quad U_{c,t} \left\{ q_t - \beta(1 - \delta)E_t\left(\frac{U_{c,t+1}}{U_{c,t}}q_{t+1}\right) \right\} = U_{c,t}uc_t = U_{d,t} \]

The term in the curly brace is defined as the user cost of durable goods. For one unit of durable goods, the user purchases it at the price \( q_t \). Next period, after the depreciation, the remaining durable goods stock \( 1 - \delta \) can be sold at price \( q_{t+1} \). Then, the user cost \( uc_t \) is the purchasing price of durable goods minus the present value of resale revenue.
1.2.2. Final Good Producers

In each sector $j$ ($j \in \{c, d\}$, where $c$ denotes the non-durable sector and $d$ denotes the durable sector), a perfectly competitive final good producer purchases $Y_{j,t}(i)$ units of intermediate good $i$. The production function that transforms intermediate goods into final good is given by

$$Y_{j,t} = \left[ \int_0^1 Y_{j,t}(i) \frac{e_j^{-1}}{e_j} di \right]^{-\frac{e_j}{e_j - 1}} (e_j > 1, j \in c, d)$$

where $e_j$ is the elasticity of substitution between differentiated intermediate goods in sector $j$.

Demand functions for intermediate good $i$ in sector $j$ can be derived from the following cost minimization problem: choose $Y_{j,t}(i)$ to minimize

$$\min \int_0^1 P_{j,t}(i) Y_{j,t}(i) di$$

subject to

$$\left[ \int_0^1 Y_{j,t}(i) \frac{e_j^{-1}}{e_j} di \right]^{-\frac{e_j}{e_j - 1}} \geq Y_{j,t}$$

It is straightforward to show that demand function for intermediate good $i$ in sector $j$ is

$$Y_{j,t}(i) = \left( \frac{P_{j,t}(i)}{P_{j,t}} \right)^{-e_j} Y_{j,t}$$
where \( P_{jt} = \left( \int_0^1 P_{jt}(i)^{1-\varepsilon_j} di \right)^{\frac{1}{1-\varepsilon_j}} \) is the price of final good \( j \).

1.2.3. Intermediate Goods Producers

In both of the non-durable and durable sectors, there is a continuum of monopolistically competitive intermediate goods producers indexed by \( i \in [0, 1] \). Each intermediate goods producer faces the demand curve (1.13) for its product. It uses only labor to produce output according to the following technology

\[
Y_{jt}(i) = A_{jt} N_{jt}(i)
\]

where \( A_{jt} \) is the technology in sector \( j \) and \( N_{jt}(i) \) is the labor hired by firm \( i \) in sector \( j \).

Intermediate goods producers set nominal prices on a staggered basis. Following Calvo (1983), firms are assumed to adjust their prices infrequently and that opportunities to adjust arrive following an exogenous Poisson process. Each period, there is a constant probability \( 1 - \omega_j \) that the firm can re-optimize its price, the remaining \( \omega_j \) fraction update prices by a steady-state inflation rate \( \pi \).

When a firm gets a chance to re-optimize its price, it sets the price \( P_{jt}^* \) to maximize the following expected discounted profit

\[
E_t \sum_{i=0}^\infty \omega_j^i \Delta_{t,t+i} \left( \frac{(1 + \tau_j)\pi^i P_{jt}^*}{P_{jt+t+i}} Y_{jt+t+i} - MC_{jt+t+i} Y_{jt+t+i} \right)
\]

where \( \Delta_{t,t+i} = \Pi_{s=t}^{t+i} \Delta_s \), \( \Delta_s \) is the stochastic discount factor defined in household’s decision problem and \( MC_{jt} = W_t/(A_{jt} P_{jt}) \) is the real marginal cost. The firm’s output is subsidized at a fixed rate \( \tau_j \). When a firm gets a chance to re-optimize its
price in period $t$, it has to take into account that it may not get the next opportunity to re-optimize its price until period $t + i$ with a probability $\omega_i^j$. The term in the bracket denotes the firm’s profit (in real terms) in period $t + i$ if it does not get a chance to re-optimize its price. Note that each firm adjusting its price in period $t$ faces the same profit maximization problem in (1.15), so all firms will set the same price $P_{j,t}^*$.

Using the definition of $\Delta_s$ and demand curve in (1.13), it is straightforward to derive $P_{j,t}^*$ from the maximization of (1.15)

\begin{equation}
(1.16) \quad P_{j,t}^* = \left( \frac{1}{1 + \tau_j} \right) \left( \frac{\varepsilon_j}{\varepsilon_j - 1} \right) \frac{E_t \sum_{i=0}^{\infty} \omega_i^j \beta^i (\pi^t)^{\varepsilon_i} \lambda_{t+i} M C_{j,t+i} Y_{j,t+i} P_{j,t+i}^*}{E_t \sum_{i=0}^{\infty} \omega_i^j \beta^i (\pi^t)^{1-\varepsilon_i} \lambda_{t+i} Y_{j,t+i}^2 P_{j,t+i}^{\varepsilon - 1}}
\end{equation}

Following Rotemberg and Woodford (1997), we set $\tau_j = 1/(\varepsilon_j - 1)$ to offset the inefficiency of steady-state output due to the markup pricing of monopolistically competitive producers.

The sector $j$’s price index satisfies

\begin{equation}
(1.17) \quad P_{j,t} = [(1 - \omega_j)(P_{j,t}^*)^{1-\varepsilon_j} + \omega_j (P_{j,t-1}^* \pi)^{1-\varepsilon_j}]^{1-\varepsilon_j}
\end{equation}

For future reference, the sectoral inflation rate $\pi_{j,t} = P_{j,t}/P_{j,t-1}$, and the sectoral real interest rate is defined as $R_t/E_t(\pi_{j,t+1})$. Final consumption good $X_t$’s price index satisfies

\begin{equation}
(1.18) \quad P_t = [(1 - \alpha)P_{c,t}^{1-\eta} + \alpha P_{d,t}^{1-\eta}]^{\frac{1}{1-\eta}}
\end{equation}

The aggregate inflation $\pi_t = P_t/P_{t-1}$. 

14
1.2.4. News Shocks

News Shocks are modeled as in Christiano, Ilut, Motto and Rostagno (2008). Up until period $t$, the economy is at a steady state. In period $t$, a signal arrives that suggests technology in sector $j$ will improve in period $t + p$. Then, in period $t + p$, the expected rise in technology in fact does not occur. A time series representation for productivity which captures this unrealized optimistic expectation is:

\[
\ln(A_{jt}) = \rho_j \ln(A_{jt-1}) + \varepsilon_{jt-p} + \zeta_{jt}
\]

(1.19)

with $0 < \rho_j < 1$, $\varepsilon_{jt}$ and $\zeta_{jt}$ are uncorrelated over time and with each other. To see that this setup can capture the unrealized expectation, suppose that a signal arrives at period $t - p$ indicating a future technology improvement at period $t$, that is, $\varepsilon_{jt-p}$ has a high value. This shifts up the expected value of $\ln(A_{jt})$. However, at period $t$, if the realization of $\zeta_{jt} = -\varepsilon_{jt-p}$, then the high expected value of $\ln(A_{jt})$ does not materialize. In this case, the signal turns out to be not informative. If the realization of $\zeta_{jt}$ is zero, then the high expected value of $\ln(A_{jt})$ does materialize. In this case, the signal is perfectly informative. In general, $\varepsilon_{jt}$ and $\zeta_{jt}$ are assumed to be drawn from normal distribution $N(0, \sigma_e)$ and $N(0, \sigma_\zeta)$ respectively. The implications of various realizations of $\varepsilon_{jt}$ and $\zeta_{jt}$ are shown in Table 1.1.

<table>
<thead>
<tr>
<th>Different combination of $\varepsilon_{jt-p}$ and $\zeta_{jt}$</th>
<th>Implications</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{jt-p} &gt; 0, \zeta_{jt} = 0$</td>
<td>High expectation fully realized</td>
</tr>
<tr>
<td>$\varepsilon_{jt-p} &gt; 0, -\varepsilon_{jt-p} &lt; \zeta_{jt} &lt; 0$</td>
<td>High expectation partially realized</td>
</tr>
<tr>
<td>$\varepsilon_{jt-p} &gt; 0, \zeta_{jt} = -\varepsilon_{jt-p}$</td>
<td>High expectation not realized at all</td>
</tr>
<tr>
<td>$\varepsilon_{jt-p} &gt; 0, \zeta_{jt} &lt; -\varepsilon_{jt-p}$</td>
<td>Expect a rise in TFP, actually TFP drops</td>
</tr>
<tr>
<td>$\varepsilon_{jt-p} &gt; 0, \zeta_{jt} &gt; 0$</td>
<td>TFP rises more than expected</td>
</tr>
</tbody>
</table>

Table 1.1. The Implications of Various Realizations of News Shock
For the purposes of solving and simulating the model, it is useful to specify the evolution of the exogenous shocks as:

\[
\begin{bmatrix}
\ln(A_{j,t}) \\
\varepsilon_{j,t} \\
\varepsilon_{j,t-1} \\
\vdots \\
\varepsilon_{j,t-p+1}
\end{bmatrix}
= 
\begin{bmatrix}
\rho_j & 0 & \cdots & 0 & 1 \\
0 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\ln(A_{j,t-1}) \\
\varepsilon_{j,t-1} \\
\varepsilon_{j,t-2} \\
\vdots \\
\varepsilon_{j,t-p}
\end{bmatrix}
+ 
\begin{bmatrix}
\zeta_{j,t} \\
\varepsilon_{j,t} \\
\varepsilon_{j,t} \\
\vdots \\
0
\end{bmatrix}
\]

(1.20)

1.2.5. Monetary Policy

The final actor in the economy is the government. Two different behavioral assumptions are made. First, an exogenous money supply rule; and second, a Ramsey optimal monetary policy.

1.2.5.1. Exogenous Monetary Supply Rule. To highlight the effects of different monetary policies on the responses to news about the future technology change, an exogenous monetary supply rule is added to the model as a baseline case. Money is injected into the economy through lump sum transfer \(T_t\).

Money supply is assumed to follow an exogenous money growth rule:

\[
g_t = \rho_g g + (1 - \rho_g) g_{t-1} + \xi_t,
\]

(1.21)

where \(M_t = g_t M_{t-1}\), \(\xi_t \sim N(0, \sigma^2_\xi)\), \(g\) is the steady-state money growth rate. The money supply shock, \(\xi_t\), is assumed to be independent of the news and productivity shocks.

1.2.5.2. Ramsey Optimal Monetary Policy. Alternatively, central bank is assumed to follow a Ramsey optimal monetary policy using interest rate as a policy instrument. The Ramsey approach to the study of optimal policy is mainly used...
in the field of public finance. Recently, researchers begin to adopt this approach to characterize optimal monetary policies in models with nominal rigidities. For example, Levin, Onatski, Williams and Williams (2006) investigate the design of monetary policy when the central bank faces uncertainty about the true structure of the economy. They find the optimal policy regime that maximizes household welfare using the Ramsey approach and then evaluate the performance of alternative simple policy rules relative to this benchmark.

The Ramsey optimal policy under commitment can be computed by formulating an infinite horizon Lagrangian problem, in which the central bank maximizes the conditional expected social welfare subject to the full set of non-linear constraints implied by the private sector's behavioral equations and the market-clearing conditions of the model economy.

In our case, the central bank's optimization problem can be described as

\[
(1.22) \quad \max_{\{\Omega_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U(X_t, N_t)
\]

subject to (1.5)-(1.7), (1.16) and (1.30)-(1.32), where \( \Omega_t \) is a set of the choice variables of the Ramsey problem \( \{N_{c,t}, D_{d,t}, C_t, D_t, B_t, R_t, P_{c,t}^*, P_{d,t}^*\} \).

Solving this problem generates the first order conditions that characterize the Ramsey optimal policy. These first order conditions are bundled together with the private sector's behavioral equations and the market-clearing conditions to analyze the behavior of the economy under the Ramsey optimal policy.

---

\(^1\)Dynare-Matlab procedures developed by Levin and Lopez-Salido (2004) are used to derive these first order conditions.
1.2.6. Aggregation and Equilibrium

Having described the behaviors of individual agents, let us turn to the aggregation of individual outputs. Recall that each intermediate goods producer’s production function is as follows:

\[(1.23) \quad Y_{j,t}(i) = A_{j,t}N_{j,t}(i),\]

Take the integration on both sides of (1.23), we get:

\[(1.24) \quad \int_0^1 Y_{j,t}(i) di = \int_0^1 A_{j,t}N_{j,t}(i) di = A_{j,t}N_{j,t},\]

where \(\int_0^1 N_{j,t}(i) di = N_{j,t}.\)

Recall that the demand function for intermediate good \(i\) in sector \(j\) is \(Y_{j,t}(i) = (\frac{P_i}{P_{j,t}})^{-\epsilon_j} Y_{j,t}.\) Then the equation (1.24) can be rewritten as:

\[(1.25) \quad \left(\int_0^1 \left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\epsilon_j} di\right) Y_{j,t} = s_{j,t}Y_{j,t} = A_{j,t}N_{j,t}\]

where the term \(s_{j,t} = \int_0^1 \left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\epsilon_j} di\) captures the resource costs due to the inefficient price dispersion. The price dispersion is caused by the Calvo-type staggered price adjustment: at time \(t\), of all the intermediate goods producers, \((1 - \omega_j)\) percentage of them get an opportunity to re-optimize their prices to \(P_{j,t}^*;\) \((1 - \omega_j)\omega_j\) percentage of them get an opportunity to re-optimize their prices at time \(t - 1\) and no chance to re-optimize their prices at time \(t\), their price at time \(t\) is \(P_{j,t-1}^*\); likewise, \((1 - \omega_j)\omega_j^i\) percentage of intermediate goods producers will set their prices to \(P_{j,t-1}^*\). We can write the price dispersion \(s_{j,t}\) in the recursive form:
When the steady-state inflation rate is not equal to zero, the nominal variables such as $P_j^t$, $P_j^{t-1}$, $P_t$, $W_t$ and $M_t$ all grow at the same rate. To induce the stationarity, we divide nominal variables $P_j^t$, $P_j^{t-1}$ by $P_t$, and define the transformed variables as $P_{j,t} = P_j^t/P_t$. Further, we rewrite $W_t/P_{c,t}$ as $\hat{W}_t$ and rewrite $M_t/P_{c,t} = \hat{M}_t$.

In equilibrium, households choose $N_t$, $C_t$, $D_t$, $B_t$ to satisfy their first order conditions (1.5)-(1.7), taking goods prices, the wage rate and the interest rate given; intermediate goods firms in sector $j$ choose prices to maximize the expected discounted profits (1.15), then decide the labor inputs $N_{j,t}$ to meet the demand for their products; final goods producers decide the purchase of intermediate goods based on
the demand function (1.13). In the meanwhile, all markets should clear:

\[(1.30) \quad \text{Non-durables: } Y_{c,t} = C_t \]
\[(1.31) \quad \text{Durables: } Y_{d,t} = D_t - (1 - \delta)D_{t-1} \]
\[(1.32) \quad \text{Labor: } N_t = N_{c,t} + N_{d,t} \]

### 1.3. Calibration and Solution

The model is calibrated at a quarterly frequency. Some parameter values are typical in the business cycle literature. The discount factor $\beta$ is set to be 0.99, consistent with a steady state annualized real interest rate of about 4%. Following Gomme and Rupert (2007), the quarterly depreciation rate of the durable stock $\delta$ is set to 0.058, implying an annual depreciation rate of 21.3%. $\alpha$, the parameter in the composite consumption index $X_t$, is chosen so that the steady state share of durable goods output in total output is 0.25. The parameter $\sigma$ is set to be 1 so that the elasticity of labor supply is unity. The parameter $\nu$ in the utility function is set so that steady-state labor supply is $1/3$. The autoregressive coefficient in the productivity process $\rho_j$ is set to 0.95. The total innovation to sector $j$ productivity is the sum of its news and productivity shocks. Since these shocks are assumed to be independent, the standard deviation of the total innovation is $\sqrt{\sigma_e^2 + \sigma_\zeta^2}$. Prescott (1986) finds that the standard deviation of the total innovation is 0.00763. We attribute half of this standard deviation to each of the news and productivity shocks. Thus, $\sigma_e = \sigma_\zeta = 0.00763/\sqrt{2} = 0.0054$.

Following Beaudry and Portier (2004), the elasticity of substitution between non-durable goods and durable goods, $\eta$, is set to be 0.2 in the baseline case, implying a strong complementarity between the non-durable goods and durable goods.
The parameter $\omega_j$ determines how long a price contract will last.\footnote{The expected time between price adjustments is $1/(1 - \omega_j)$.} The empirical evidence surveyed by Taylor (1999) suggests that nominal price contracts on average last for a year, implying $\omega_j = 0.75$. Bils and Klenow (2004) argue that the observed frequency of price adjustment in the U.S. is much higher, or in the order of two quarters, implying $\omega_j = 0.5$. In recent literatures of two-sector models with nominal rigidities, Erceg and Levin (2006) assume symmetric price rigidities in non-durable and durable sectors. Barsky, House and Kimball (2007) and Monacelli (2009) study the cases with asymmetric price rigidities. They argue that the prices of durable goods adjust more frequently than non-durable goods in that the menu cost relative to the unit price of durable goods is much lower. I set $\omega_j = 0.75$ for both sectors in the baseline calibration, but allow for asymmetric price rigidities in the approximation of the Ramsey policy.

The parameters $\varepsilon_j$ ($j \in \{c, d\}$) measure the elasticity of substitution between differentiated intermediate goods. Following Monacelli (2009), I set both parameters to 8, which yields a steady state mark-up of 15\% for intermediate goods producers.

The values of all the baseline parameters are summarized in Table 1.2.

The model is solved by taking a log-linear approximation of the equilibrium conditions in the neighborhood of the steady state.\footnote{Dynare-Matlab has been used to solve the model. For details of Dynare, refer to http://www.cepremap.cnrs.fr/dynare/.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$\eta$</th>
<th>$\sigma$</th>
<th>$\varepsilon_c$</th>
<th>$\varepsilon_d$</th>
<th>$\omega_c$</th>
<th>$\omega_d$</th>
<th>$\rho_c$</th>
<th>$\rho_d$</th>
<th>$\sigma_e$</th>
<th>$\sigma_\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>0.87</td>
<td>0.058</td>
<td>0.2</td>
<td>1</td>
<td>0.8</td>
<td>8</td>
<td>0.75</td>
<td>0.75</td>
<td>0.95</td>
<td>0.95</td>
<td>0.0054</td>
<td>0.0054</td>
</tr>
</tbody>
</table>

Table 1.2. Baseline Parameter Values

1.4. Simulation Results

In this section, responses to news about the future technology improvement are explored in three cases: model with an exogenous money supply rule; a Ramsey
optimal policy; and flexible prices. Through the comparison of the responses in these three cases, we can highlight the effects of adding nominal rigidities and different monetary policies on the responses to a news shock. In each of three cases, we focus on the comovements of non-durable goods sector and durable goods sector. The key issue is whether the model can generate a boom of production in both sectors after agents receive news about the future technology improvement.

Agents are assumed to receive a signal at period $t$ suggesting technology in sector $j$, $A_j$, will be increased by 1% at period $t + 4$, that is, a high $\varepsilon_{j,1}$ leads to an upward revision in the expectation of $\log(A_{j,5})$. In periods 1 to 4, agents will act on that expectation. In period 5, the realization of $\log(A_{j,5})$ is determined by the realization of $\zeta_{j,5}$. If it happens that $\zeta_{j,5} = -\varepsilon_{j,1}$, then the expected positive move in $\log(A_{j,5})$ does not occur. Following Beaudry and Portier (2004), I first assume that agents receive a signal suggesting the technology will increase in the non-durable sector.

1.4.1. Responses under flexible prices

The solid line in figure 1.1 illustrates responses to a news shock described above in which the news shock is exactly offset by the productivity shock; this is referred to as an unfulfilled expectation. When households receive the signal about the future technology improvement in the non-durable sector, they also expect an increase in the future demand of durable goods since the non-durable goods and durable goods are complements. Household starts to accumulate durable goods immediately after they receive the signal about the future technology improvement in non-durable sector, which boost the production of durable goods and the labor demand in that sector. In the meanwhile, the total labor supply does not rise as much as the labor demand in durable sector, which causes a drop in non-durable goods production. The relative price remains unchanged in the model with flexible prices since prices in both sectors
are a constant markup over the marginal cost, which is the ratio of the wage to the marginal product. In my model, marginal product of labor is just the technology level and wage is assumed to be equal in two sectors. As long as the technology does not change, the relative price will remain unchanged.
In a similar two sector model without any nominal rigidities, Beaudry and Portier (2004) succeed in generating the positive comovements between non-durable goods and durable goods, and an increase of the relative price after agents receive news about future technology improvement. Two differences between their model and the one presented in this chapter can explain the discrepancy. First, Beaudry and Portier assume a decreasing marginal product of labor while I assumed a constant one. With a decreasing marginal product of labor, the marginal cost in durable sector increases when labor employed in durable goods sector rises, which causes the increase in the relative price. The increase in the price of the durable goods relative to the non-durable goods dampens the incentive to accumulate the durable goods and boosts the demand of non-durable goods, both of which help generating the positive comovements. Second, as pointed out in Beaudry and Portier, an infinite elasticity of labor supply assumed in their model also helps generating the positive comovements.

1.4.2. Responses under exogenous money supply rule

The lines with circles in figure 1.1 show responses to unfulfilled news about a future technology improvement in the model with nominal rigidities and an exogenous money supply. Outputs, labor supplies and relative price experience a boom after agents receive a signal indicating the future technology improvement in the non-durable sector and a bust when the expectation is not realized. Comparing responses in the model with nominal rigidities with those with flexible prices, it is clear that nominal rigidities play an important role in generating the rise of relative price and the positive comovements of non-durable goods and durable goods. The underlying mechanism is as follows: with sticky prices, producers decide their prices based on the weighted average of the current and future marginal costs. Non-durable goods producers expect that marginal costs will drop in the future due to technology improvements and starts
to lower their prices once they obtain the chance of price adjustment, which causes a rise in the relative price. The low price of non-durable goods boosts the demand of non-durable goods and also spurs the demand of durable goods due to the assumed complementarity between non-durable and durable goods. As pointed out in Barsky, House and Kimball (2007), one of the features of the two sector models with long-lived durable goods is that the demand of non-durable goods is determined by the relative price. The co-movement between non-durable goods production and the relative price in figure 1.1 is consistent with their conclusion. In sum, the introduction of nominal rigidities causes the rise of the relative price, and thus the boom of the non-durable goods consumption which amplifies the responses to the news shock.

1.4.3. Responses under Ramsey optimal policy

The lines with stars in Figure 1.1 demonstrate impulse responses to a news shock when the central bank employs the Ramsey optimal policy. To highlight the effect of the Ramsey optimal monetary policy on the responses, we focus on comparing the responses under an exogenous money supply and those under the Ramsey policy. First, under the Ramsey policy, the news about a rise in the non-durable sector technology only generates a boom in the non-durable sector. Meanwhile, durable goods sector experiences a moderate bust. There is almost no responses in the total labor supply. It seems that Ramsey monetary policy does not have any effect on the movement of the relative price; consequently, the response of non-durable goods is similar to that under an exogenous money supply. The bust in the durable goods sector could be attributed to the rise of the nominal interest rate under the Ramsey policy. If the central bank follows the Ramsey optimal policy, the annualized nominal

\[ ^4 \text{This is true only when the degrees of nominal rigidities are assumed to be the same in two sectors. When asymmetric nominal rigidities are assumed, monetary policy has some effect on the relative price.} \]
interest rate should rise to about 4.3% (compared to its steady state value of 4%) in the boom periods and immediately drops to about 3.9% once agents realize that their expectations have not been realized. The rise in the nominal interest rate boosts the real interest rate in durable goods sector and the user cost of durable goods, which dampens the demand of durable goods. The demand of durable goods is determined by two forces: the complementarity between non-durable and durable goods pushes their demand up, while the rise of the user cost pushes their demand down. The net effect of these two forces is that durable goods decline by a moderate amount.

Comparing responses under the Ramsey policy with those under flexible prices, we can observe that the Ramsey policy does not lead to economy to behave as under flexible prices. The Ramsey policy involves a trade off that cannot fully remove the distortion caused by the nominal rigidities. First, consumption of non-durable goods is determined chiefly by the relative price. Since monetary policy has no impact on the relative price when the price nominal rigidities are the same in the two sectors, it has no impact on the consumption of non-durable goods. Regardless of the monetary policy rule applied, the relative price of durable goods and the consumption of non-durable goods will increase when agents expect an improvement in technology in the non-durable sector. Suppose that the Ramsey policy tried to restore the combination of non-durable goods and durable goods, \( X_t \), to its flexible price level; then durable goods production must fall sharply in order to have a sufficient impact on the stock of durables. But such a large decline in durable production will push total labor supply far below its flexible price level. Second, if Ramsey policy tries to restore the total labor supply to its flexible price level, labor has to move from the durable goods sector to the non-durable goods sector, which lowers output of durable goods. However, the stock of durable goods is almost unchanged since the investment flow into durables is small relative to its stock. In this case, the Ramsey policy fails to
restore the combination of non-durable goods and durable goods, $X_t$, to the flexible price level. In sum, the Ramsey policy can not restore both the combination of non-durable goods and durable goods, $X_t$, and the total labor supply to their flexible price levels. It turns out that in evaluating these trade offs, the Ramsey policy ends up with a total quantity of labor close to its flexible price level. The Ramsey policy implies an increase in the nominal interest rate so that the real interest rate and user cost of durable goods increase. This increase in the real interest rate reduces the incentive to accumulate durable goods. The muted response of durable goods production allows more labor to be allocated to the non-durable goods sector. It is also interesting to note that the Ramsey optimal policy fails to stabilize the inflations in both sectors due to the variations in the relative price. For if the relative price fluctuates while the inflation in one sector remains constant, there must be a variable inflation rate in another sector.

1.4.4. Model features help generating boom and bust

The previous experiments show that we succeeds in generating a boom in both sectors in the model with nominal rigidities and an exogenous money supply, but fails to do so in the model with flexible prices or in the model with nominal rigidities and the Ramsey policy. These results indicate that both nominal rigidities and monetary policies play significant roles in generating Pigou cycles. Beaudry and Portier (2004) attributes their success in generating Pigou cycles to the following features of their model: (1) the complementarity between non-durable goods and durable goods; and (2) an expectation of technology improvement in non-durable sector instead of durable sector. To see whether these features are still indispensable in generating Pigou cycles in our framework, we change these features of the model to alternative ones and compare the differences of impulse responses under alternative assumptions.
First, we change the elasticity of substitution between non-durable goods and durable goods, $\eta$, from 0.2 to 2 leaving the other parameters in the model unchanged. In this case, the non-durable goods and durable goods are good substitutes. The impulse responses in 1.2 show that the model fails to generate a boom in the durable goods sector when agents expect an productivity improvement in the non-durable sector in all three model variants. An expected productivity improvement in the non-durable sector leads to a lower price of non-durable goods relative to that of durable goods. Therefore, households will reduce the demand for durable goods since they can be easily replaced by the non-durable goods that are less expensive. This experiment verifies that complementarity between non-durable goods and durable goods still plays a significant role in generating Pigou cycles in a two-sector model.

Next, we assume that agents receive a signal suggesting a technology rise in the durable sector instead of non-durable sector. The impulse responses in 1.3 are also consistent with Beaudry and Portier’s result: macro aggregates move downwards together when agents expect a rise of technology in durable sector. Since agents expect a technology improvement in the durable sector and so a drop in the relative price of durable goods, they postpone their investment in durable sector until the realization of the technology improvement. This observation verifies that the assumption that agents receive signals about improvements in the non-durable sector is essential to generate Pigou cycles.

1.5. Approximate Ramsey Policy Using Simple Policy Rules

1.5.1. Baseline parameters

Following Taylor (1993), numerous researches have estimated or evaluated a multitude of simple monetary policy rules. The policies are simple in the sense that
they involve only a few observable macroeconomic variables. In terms of maximizing agents' welfare, the performances of simple rules are inferior to that of Ramsey optimal policy since simple rules only react to a few selected observable variables.
Figure 1.3. Responses to news at period 1 of future positive shock on the technology in durable sector and no realization of that shock at period 5 (line with circles: exogenous money supply, line with stars: Ramsey optimal policy, solid line: flexible prices)

However, simple rules are easier to explain to the public, so it appears more transparent. To see whether the nominal interest rate associated with Ramsey policy can be approximated by a simple policy rule, I estimate the following regressions using artificial time series generated from the model:
\[ (1.33) \quad R_t = \alpha_0 + \alpha_1 \pi_t + \alpha_2 x_t + \epsilon_t \]

\[ (1.34) \quad R_t = \beta_0 + \beta_1 \pi_{c,t} + \beta_2 \pi_{d,t} + \beta_3 y_{c,t} + \beta_4 y_{d,t} + \eta_t \]

where \( R_t \) is the annualized nominal interest rate, \( \pi_t \) is the CPI inflation rate and \( x_t \) is the output gap measured as the percentage deviation of final consumption good \( X_t \) from its steady state, \( \pi_{c,t} \) and \( \pi_{d,t} \) are annualized inflation rates in non-durable and durable sector respectively, \( y_{c,t} \) and \( y_{d,t} \) are output gaps in non-durable and durable sector measured as percent deviation from the steady state. Equation (1.33) is the original Taylor rule specification. The objective of estimating (1.34) is to see the additional benefit of targeting sectoral inflation and output gap. To generate artificial time series, shocks \( \xi_t \) and \( \zeta_t \) are drawn from the normal distribution, then time series of size 1000 for \( R_t, \pi_t, x_t, \pi_{c,t}, \pi_{d,t}, y_{c,t}, y_{d,t} \) are generated using the decision rules.

The OLS estimate of the simple rule (1.33) is:

\[ (1.35) \quad R_t = 4.002 + 1.407 \pi_t - 0.084 x_t, \quad R^2 = 0.932 \]

The OLS estimate of the simple rule (1.34) is:

\[ (1.36) \quad R_t = 4.004 - 1.129 \pi_{c,t} + 0.834 \pi_{d,t} - 0.005 y_{c,t} - 0.072 y_{d,t}, \quad R^2 = 0.962 \]

Both of these two estimated simple rules fit quite well the Ramsey policy. It seems that targeting sectoral inflation and output gaps does not bring significant additional
Figure 1.4. Responses to news at period 1 of future positive shock on the technology in non-durable sector and no realization of that shock at period 5 (line with circles: estimated simple interest rule, line with stars: Ramsey optimal policy).

benefit. Estimated simple rule (1.35) does not resemble the Taylor rule in that the coefficient on output gap is negative. The coefficient on inflation is greater than 1, which indicates that central bank raises nominal interest rate more than the inflation

---

5 Targeting sectoral inflation and output gap will bring additional benefit when asymmetric nominal rigidity is assumed in the next subsection.
rate to ensure an increase in the real interest rate. Figure 1.4 shows impulse responses of selected variables under the Ramsey optimal policy and the estimated rule (1.35).

1.5.2. Asymmetric nominal rigidities

Through the above experiments, we observe that an estimated simple rule can approximate the Ramsey policy quite well in the case of baseline parameters. In this section, sensitive analysis is conducted to explore how the interest rate reaction function will change under alternative parameters. In the baseline calibration, I assume symmetric nominal rigidity in two sectors. Since I am not certain about the degree of nominal rigidities in both sectors, I will do an experiment to see how optimal monetary policy will change when the frequencies to reset the prices \((\omega_j, j \in \{c, d\})\) are different between non-durable sector and durable sector. This experiment also shows how the central bank decides the weights given to each sector when the optimal policy is to target the inflation rates of both sectors. In this experiment, all the parameters will remain the baseline value except for the degree of nominal rigidity.

Table 1.3 lists the estimation results of simple policy rules defined in (1.33) and (1.34). First, in general, the estimated policy rules fit the Ramsey policy closely. However, the fit becomes worse when the asymmetry of nominal rigidities gets larger. Second, the improvement in fit of reacting to sectoral inflation and output gaps become more and more significant with the increase in the asymmetry of nominal rigidities. Third, when the nominal rigidities are symmetric in two sectors, Ramsey optimal
policy almost targets the inflation in durable sector only; when the nominal rigidity in non-durable sector becomes greater relative to that in durable sector, more weight will be given to inflation in non-durable sector. To see this trend more clearly, I estimate another form of simple rule that only reacts to inflation in both sectors. The results in Table 1.4 shows a clear shift of weights from durable sector inflation to non-durable sector inflation as the nominal rigidity in non-durable sector becomes relatively greater. Another interesting result is that it only loses a little fit by targeting only inflations compared with targeting both the inflation and output gap. Figure 1.5 and 1.6 show the impulse responses of selected variables under Ramsey optimal policy and estimated simple rules.

The results in Table 1.4 suggests that the Ramsey optimal policy puts much more weight on the durable sector inflation when the degree of nominal rigidities are the same in the two sectors or even when the degree of nominal rigidity in non-durable good sector is moderately larger. Notice that the relative share of durable good output to non-durable good output is 1 to 3. So, even when the coefficient on the non-durable good inflation is three times that of the durable good inflation in the interest rate reaction function, we can still say that the policy puts the same weight on the two sectors. Only when the degree of nominal rigidity is much larger in the non-durable good sector than in durable good sector (e.g. $\omega_c = 0.75, \omega_d = 0.3$), does the optimal policy begin to put more weight on non-durable sector inflation.
1.5.3. What if central bank follows a weakly inflation target rule

In this subsection, I did an experiment to study the responses of economy to a news shock if the central bank employs a simple rule with the coefficients of inflation different with the estimated policy that closely approximates the Ramsey optimal policy. In particular, I set the policy reaction function as:
Figure 1.6. Responses to news at period 1 of future positive shock on the technology in non-durable sector and no realization of that shock at period 5 (line with circles: estimated simple interest rule, line with stars: Ramsey optimal policy).

\[(1.37) \quad R_t = R_{ss} + 1.01\pi_t\]

compared with the estimated rule in (1.35), the interest rate in simple policy rule (1.37) only weakly reacts to inflation. To highlight the effect of monetary policy on
the generation of Pigou cycles, non-durable goods and durable goods are assumed to be substitutes ($\eta = 2$). Figure 1.7 shows the impulse responses of selected variables under both of the Ramsey policy and weakly inflation-targeting (WIT) rule defined in (1.37). The most interesting observation is that Pigou cycles can arise when the central bank follows WIT rule but fails to arise when the central bank follows the Ramsey policy. In particular, a considerable boom in the durable good sector occurs under the WIT rule while a slight bust in the durable sector occurs under the Ramsey policy. The underlying mechanism is as follows: recall non-durable goods and durable goods are assumed to be substitutes in this case; agents would rather postpone their accumulation of durable goods since they can be easily replaced by non-durable goods once the expected technology improvement is realized. However, a WIT rule causes a relatively lower real interest rate in durable sector and a lower user cost, which spurs agents' incentive to accumulate durable goods. Figure 1.7 shows that a little difference in the real interest rate can cause a great difference in user cost. When agents expect the technology in the non-durable sector will improve in the future, they also expect that the relative price of durable goods will rise, that is, a chance to capture the capital gain. A relative higher real interest rate under the Ramsey policy dampens the discounted present value of the capital gain and depresses the agents' incentives to capture the capital gain. By contrast, a relative lower real interest rate under WIT rule gives the agents more incentives to capture the expected capital gain by accumulating durable goods. The conclusion is that monetary policy has a significant effect on the generation of Pigou cycles by affecting the real interest rate and user cost of durable goods. The assumption of a strong complementarity between non-durable goods and durable goods is not necessary any more to generate Pigou cycles when the monetary policy rule employed by central bank fails to raise the real interest rate enough to dampen the desire to capture the capital gain.
Figure 1.7. Responses to news at period 1 of future positive shock on the technology in non-durable sector and no realization of that shock at period 5 (lines with stars: Ramsey policy, lines with circles: simple policy rule weakly targeting inflation).

1.6. Conclusion

In this chapter, I explored the expectation driven business cycles (Pigou cycles) and optimal monetary policy in a two-sector economy with nominal rigidities in both non-durable goods and durable goods sectors. My main findings are as follows: (1) nominal rigidities tend to magnify the extent of the boom and bust when a news
shock hits the economy and no active monetary policy is employed; (2) The Ramsey optimal policy requires the central bank to raise the nominal interest rate before the realization of the news, so that the real interest rate and user cost of durable goods are raised to dampen the agents' incentive to accumulate durable goods; (3) The Ramsey optimal policy succeeds in restoring the total labor supply to its flexible price level but fails to do so for the output of non-durable goods and durable goods. (4) The Ramsey optimal policy is closely mimicked by a simple policy rule targeting inflations in both durable and non-durable sectors. More weight should be given to the durable good sector unless the degree of nominal rigidity in the durable good sector is much smaller than that in the non-durable good sector.

Monetary policy also plays an important role in generating Pigou cycles by altering the real interest rate. A rise in the real interest rate reduces the present value of the capital gain and increases the user cost, which causes a decline in durable goods accumulation. When agents receive a signal indicating a technology improvement in the non-durable sector, they expect that the relative price of durable goods will increase and a capital gain can be captured. The reaction of a central bank following the Ramsey optimal policy is to raise the real interest rate which has the effect of reducing the present value of the capital gain and prevent the agents from accumulating too much durable goods. If the central bank does not raise the real interest rate enough, a high expected capital gain stimulates the accumulation of durable goods. Under such a circumstance, Pigou cycles can arise without the assumption of a strong complementarity between non-durable goods and durable goods.

In this chapter, the coefficients of simple policy rules are estimated using the simulated interest rates under the Ramsey optimal policy. The fit is measured simply by R-square or visual closeness of the IRFs. A possible extension is to define an
appropriate measurement of welfare and calculate the welfare loss for alternative implementable simple rules relative to the welfare under the Ramsey optimal policy.
CHAPTER 2

Exploring the Significance of News Shocks in Estimated Dynamic Stochastic General Equilibrium Model

2.1. Introduction

Beaudry and Portier (2004) explore a theory of business cycles (Pigou cycles) in which booms and busts of the economy are caused by agents' expectation of future technology changes. In the classical real business cycles models, booms and busts are attributed to the sudden changes of the technology levels (technology shocks). Beaudry and Portier introduced a new source of business cycles into the literatures, that is, fluctuations of the economy caused by news shocks: good news about future technology leads to a boom and an unrealized expectation could generate a bust. Since their seminal paper, researchers' interests in news shocks have grown.\footnote{For example, Den Haan and Kaltenbrunner (2004), Jaimovich and Rebelo (2007, forthcoming), Christiano, Hut , Motto and Rostagno (2008) and Devereux and Engel (2006).} However, one question remains: how important is news shocks in generating aggregate fluctuations?

This chapter estimates a dynamic general equilibrium model with both news shocks and technology shocks using maximum likelihood and explores the relative importance of news shocks in generating aggregate fluctuations in the United States data.

In this chapter, I introduce price nominal rigidities, home production and capital adjustment into a two sector dynamic general equilibrium model with durable goods and non-durable goods. As in Benhabib, Rogerson and Wright (1991), home production is incorporated into the model to induce the positive comovements of hours worked in two market sectors. The underlying mechanism is as follows: in a model with home production, when the technology or the expectation of the technology in
market sectors improves, households will substitute market produced consumption goods for home produced goods; in the meanwhile, labor will flow from home production into market sectors, rather than moving between the two market sectors. Thus, it is easier to achieve the positive comovements of hours worked in two market sectors. However, it is well known that the addition of home production alone cannot fix the problem of negative comovements of investments in household and market capitals. Adding capital adjustment costs can solve this problem by restricting the inter-sectoral flows of capitals and smooth the fluctuations in the sectoral investments. Incorporating nominal rigidities is essential for the consumption-sector technology changes (both expected and unexpected) to have distinct impact on business cycles. In the model without nominal rigidities, the economy does not react to the consumption sector news shock at all before the news is realized. In this case, it is impossible to distinguish the consumption sector news shock and technology shock since they will generate the same responses in the economy.

To highlight the importance of the addition of nominal rigidities, home production and capital adjustment cost, the baseline model with all features is estimated along with three restricted versions of the model: one without home production, one without nominal rigidities and one without capital adjustment cost. The estimation results show that all three features significantly improve the fit of the model based on likelihood ratio tests. A study of the moments of the simulated data indicates that the model with all features best captures the standard business cycle statistics, especially the positive correlation in hours worked and outputs in two market sectors. By contrast, the restricted versions of the model can only generate either weak positive comovements or negative comovements. In particular, the model without home production only generates weak positive comovements between outputs and hours worked within the same sector, and weak positive comovement across two market
sectors. The model without nominal rigidities generates strong positive comovements between outputs and hours worked within the same sector, but negative comovements across two market sectors. The model without capital adjustment costs can generate positive comovements between outputs and hours worked within and across market sectors, but fails to generate positive comovements among sectoral investments. These findings justify the use of the baseline model to explore the responses to sectoral news shocks and technology shocks, and examine the relative importance of the various shocks in explaining aggregate fluctuations.

The main result of this chapter is that news shocks play a substantial role in accounting for aggregate fluctuations. A variance decomposition conducted using the estimated baseline model reveals that news shocks are able to explain approximately 34% of the variations in aggregate output, 25% of the fluctuations in the consumption-sector output and about 38% of the fluctuations in the investment-sector output in the long run. This result indicates that a substantial proportion of economic fluctuations can be attributed to news shocks.

Another interesting result is that consumption-sector shocks explain almost none of the fluctuations in the investment sector. This result is in line with previous research on sector-specific technology shocks. Kimball (1994) first point out that consumption-sector technology shocks only affect consumption and have no impact on the allocation of employment and capital when the utility is logarithmic and additively time-separable. Theoretically, this "consumption-sector neutrality" no longer holds when nominal rigidities are added to the model in that the investment-sector outputs and hours worked do react to the consumption-sector shocks. However, quantitatively, "consumption-sector neutrality" still holds in that investment sector responses induced by consumption-sector shocks are quite small compared with the responses induced by investment-sector shocks.
In the related literature, Beaudry and Portier (2006) identify news shocks using the information of the stock prices and total factor productivity in a vector error correction model and claimed that news shocks can explain about 50% of business cycle fluctuations. By contrast, the current chapter identifies the news shock in an estimated dynamic general equilibrium model. An independent and contemporaneous work by Schmitt-Grohé and Uribe (2008) also explores the significance of news shocks in generating business cycle fluctuations in a dynamic general equilibrium model. The main differences between their work and the current one consist of the followings: (i) they construct a one-sector real business cycle model while the current work is based on a two-sector model with nominal rigidities; (ii) they assume that the technology innovation can be anticipated one, two, and three quarters in advance while the current work assume that the expectation of technology changes is formed four quarters in advance only. (iii) they consider the news about stationary and non-stationary technology changes while the current work focuses on the news about the stationary technology changes (iv) their results indicate that news shocks can account for two thirds of the variance of output which is higher than the one estimated in the current work.

Another novelty about this research is the use of sectoral data on outputs and hours worked instead of aggregate data to estimate the model. The two-sector framework and the sectoral data allow me to explore the different roles played by sectoral news shocks in generating fluctuations in different sectors. Meanwhile, the usage of sectoral data in the estimation highlights the importance of modelling sectoral co-movements. In the literature, researchers have long noticed the comovements of sectoral outputs and inputs in the data. For example, Benhabib, Rogerson and Wright (1991) emphasized that one of the benefits of introducing home production

---

is to induce a positive comovement in sectoral hours worked. Huffman and Wynne (1999) showed that the introduction of intratemporal adjustment costs is capable of generating positive comovements of sectoral capital, employment and output. Hornstein and Praschnik (1997) obtained the sectoral comovements in a model in which consumption goods can be used as intermediate goods in the production of investment goods. Greenwood et al.(1991), Fisher (1997), Chang (2000) and Gomme et al.(2001) provided various solutions to the negative comovements of sectoral investments. In the current work, home production and capital adjustment costs are incorporated into the model to induce sectoral co-movements.

The rest of the chapter is organized as follows. Section 2 presents the baseline model with nominal rigidities, home production and capital adjustment costs, where the model without these features is nested within this framework. Section 3 describes the data and the calibration procedures. Section 4 examines the importance of the addition of home production and nominal rigidities through a preview of impulse responses. Section 5 lists the estimation results. Section 6 evaluates the baseline model and three restricted versions of the model by studying the business cycle moments and impulse responses, a variance decomposition is conducted to detect the relative significance of various shocks in explaining the aggregate fluctuations. Section 7 concludes.

2.2. Model

The baseline model includes production in both the market and at home. Market production is composed of two sectors: a consumption goods sector and an investment goods sector. The consumption goods sector produces consumption goods and services, while the investment goods sector provides inputs to capital producers who
transform these inputs to installed capitals. Households purchase capital from capital producers and rent them to market sectors or use them in household production. Households' time is allocated among the production of consumption goods, investment goods and home production. There are two types of firms in each market sector: final goods firms produce final goods using intermediate goods; intermediate goods firms are monopolistic competitors that each produces a differentiated product using labor and capital. These intermediate goods firms determine their prices following a Calvo-type staggered price adjustment. Aggregate fluctuations are assumed to be caused by four shocks hitting the economy: news shocks and technology shocks in both of the market sectors.

2.2.1. Households and Home Production

The representative household derives utility from the consumption of a composition of the goods purchased in the market $C_m$ and the goods produced at home $C_h$, and derives disutility from hours worked in the market sectors $N_m$ and hours worked in home production $N_h$. Preferences are given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln (X(C_{mt}, C_{ht})) - \gamma \frac{(N_{mt} + N_{ht})^{1+\sigma}}{1+\sigma} \right]
\]

where $\beta$ denotes the discount factor, $\gamma$ determines the steady state labor supply, and $\sigma$ determines the elasticity of labor supply. The consumption aggregator $X(C_{mt}, C_{ht})$ is defined as a CES composite of market consumption $C_m$ and home consumption $C_h$:

\[
X(C_{mt}, C_{ht}) = \left[ (1 - \omega)C_{mt}^{\frac{\eta-1}{\eta}} + \omega C_{ht}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}
\]
where $\omega$ determines the share of home produced goods in the composite consumption index. $\eta$ is the elasticity of substitution between the market consumption goods and home produced goods. As $\eta \to 0$, the two consumption goods are perfect complements; as $\eta \to \infty$, the two consumption goods are perfect substitutes.

Home production is a Cobb-Douglas function of the time spent in home work $N_{ht}$ and household capital $K_{ht}$

\begin{equation}
C_{ht} = N_{ht}^{\alpha}K_{ht}^{1-\alpha}
\end{equation}

where $\alpha$ is the labor share parameter. To focus on the effects of technology shocks and news shocks in the market sectors, the technology of home production is assumed to be nonstochastic. McGrattan, Rogerson and Wright (1997) demonstrated that home production matters as long as there exist fluctuations in relative productivities across home and market productions. Even if the home sector is nonstochastic, a stochastic market sector is enough to guarantee the existence of the fluctuations in the relative productivity.

The household faces the following budget constraint

\begin{equation}
P_{ct}C_{mt} + Q_{ct}^{n}[K_{ct}-(1-\delta_c)K_{ct-1}] + Q_{dt}^{n}[K_{dt}-(1-\delta_d)K_{dt-1}] + Q_{ht}^{n}[K_{ht}-(1-\delta_h)K_{ht-1}] + \text{B}_t \\
= W_tN_{mt} + R_t^{c}K_{ct} + R_t^{d}K_{dt} + B_{t-1}R_{t-1} + D_t
\end{equation}

where $P_{ct}$ is price of consumption goods and $Q_{xt}^{n}$ ($x \in \{c, d, h\}$), where $c$ denotes consumption goods sector, $d$ denotes investment goods sector and $h$ denotes home production) are prices of installed capital in the different sectors; $K_{xt}$ ($x \in \{c, d, h\}$) denote the capital stocks in different sectors and $\delta_x$ are the corresponding depreciation
rates; $B_t$ is the bonds held by households; $W_t$ is the wage rate paid in market sectors; $R_t^c$ and $R_t^d$ are the rental rates of market capital $K_{ct}$ and $K_{dt}$; $R_t$ denotes the nominal interest rate; $D_t$ is the dividend from goods producing firms. Labor is assumed to flow freely among home and the two market sectors, so the wage rates are the same across sectors. The prices of installed capital may differ across sectors owing to capital adjustment costs.

Households maximize the utility function (2.1) subject to its budget constraint (2.4). The first order conditions (expressed in general function form) with respect to consumption of market produced goods, hours worked at home, hours worked in market production, home capital stocks, market capital stocks and bonds holding are:

\[(2.5) \quad U_1(X_t, N_{mt}, N_{ht})X_1(C_{mt}, C_{ht}) = \Lambda_t\]
\[(2.6) \quad U_1(X_t, N_{mt}, N_{ht})X_2(C_{mt}, C_{ht})C_h1(N_{ht}, K_{ht}) = U_3(X_t, N_{mt}, N_{ht})\]
\[(2.7) \quad \Lambda_t W_t / P_{ct} = U_2(X_t, N_{mt}, N_{ht})\]
\[(2.8) \quad \Lambda_t Q_{ht} = \beta E_t [U_1(X_t, N_{mt}, N_{ht})X_2(C_{mt}, C_{ht})C_h2(N_{ht}, K_{ht}) + (1 - \delta_h)\Lambda_{t+1}Q_{ht+1}]\]
\[(2.9) \quad \Lambda_t Q_{xt} = \beta E_t [\Lambda_{t+1}R_{t+1}^c/P_{ct+1} + (1 - \delta_x)\Lambda_{t+1}Q_{xt+1}] \quad (x \in c, d)\]
\[(2.10) \quad \Lambda_t = \beta E_t \left( R_t \frac{P_{ct}}{P_{ct+1}} \Lambda_{t+1} \right)\]

where $F_n(X, Y, Z)$ denotes the derivative of function $F$ with respect to the $n$th argument; $Q_{xt} = Q_{xt}^n/P_{ct}$ is the relative price of capital goods; $\Lambda_t$ is the Lagrangian multiplier associated with the budget constraint (2.4), which is equal to the marginal utility of consumption of market goods. First order conditions (2.6) and (2.7) imply that households will allocate hours worked between market and home production so
that the marginal benefits of working an extra hour at home and in the market are
equal to each other. Equations (2.8) and (2.9) indicate that the marginal cost of
investing one unit of capital, that is, the utility loss from less current consumption,
should be equal to the marginal gain of one extra unit of capital. The marginal gain
of one extra unit of capital consists of two parts: expected rental income or home
production increase at next period and the discounted present value of resale price of
the used capital. First order condition (2.10) equates the cost of sacrificing one unit
of consumption to the benefit of investing this money to the bond market. For future
reference, the term $\beta \Lambda_{t+1}/\Lambda_t$ is defined as a stochastic discount factor $\Delta_t$.

2.2.2. Final Goods Producers

In each sector $x$ ($x \in \{c, d\}$), a perfectly competitive final goods producer purchases
$Y_{xt}(i)$ units of intermediate goods $i$. The production function that transforms inter­
mediate goods into final goods is given by

$$\begin{align*}
(2.11) \quad Y_{xt} &= \left[ \int_{0}^{1} Y_{xt}(i)^{\frac{\varepsilon_x - 1}{\varepsilon_x}} di \right]^{\frac{\varepsilon_x}{\varepsilon_x - 1}}
\end{align*}$$

where $\varepsilon_x > 1$ is the elasticity of substitution between differentiated intermediate
goods.

The demand function for intermediate good $i$ can be derived from the following
cost minimization problem: taking the prices of intermediate goods $P_{xt}(i)$ as given,
choose $Y_{xt}(i)$ to minimize the cost of producing $Y_{xt}$ unit of final good:

$$\begin{align*}
(2.12) \quad \min & \int_{0}^{1} P_{xt}(i)Y_{xt}(i) di \\
\text{s.t.} \quad Y_{xt} &= \left[ \int_{0}^{1} Y_{xt}(i)^{\frac{\varepsilon_x - 1}{\varepsilon_x}} di \right]^{\frac{\varepsilon_x}{\varepsilon_x - 1}}
\end{align*}$$
It is straightforward to show that demand function for intermediate good \(i\) in sector \(x\) is

\[
(2.13) \quad Y_{xt}(i) = \left( \frac{P_{xt}(i)}{P_{xt}} \right)^{-\varepsilon_x} Y_{xt}
\]

where \(P_{xt} = \left( \int_0^1 P_{xt}(i)^{1-\varepsilon_x} di \right)^{\frac{1}{1-\varepsilon_x}}\) is the price of final good.

### 2.2.3. Intermediate Goods Producers and Nominal Price Setting

In each sector, there exists a continuum of monopolistically competitive intermediate goods producers indexed by \(i \in [0, 1]\). The demand of each intermediate good is determined by the demand function (2.13). Intermediate goods producers hire labor and rent capital from households as inputs. The production function of intermediate good \(i\) is given by

\[
(2.14) \quad Y_{xt}(i) = A_{xt} N_{xt}^{\phi_x}(i) K_{xt}^{1-\phi_x}(i) \quad (x \in c, d)
\]

where \(A_{xt}\) is the productivity in sector \(x\). \(N_{xt}(i)\) and \(K_{xt}(i)\) are the labor and capital employed in firm \(i\) sector \(x\). \(\phi_x\) is the labor’s share of income in sector \(x\).

Intermediate goods firms choose an appropriate combination of labor and capital to minimize their cost. At the optimum, firms in each sector adjust the two inputs to equalize marginal cost \(MC_t\) across the two factors, i.e.

\[
(2.15) \quad MC_{xt} = \frac{W_t}{\phi_x Y_{xt}(i)/N_{xt}(i)} = \frac{R_t^x}{(1 - \phi_x) Y_{xt}(i)/K_{xt}(i)}
\]
intermediate goods producers set nominal prices on a staggered basis. Following Calvo (1983), firms are assumed to adjust their prices infrequently and that opportunities to adjust arrive following a Poisson process. Each period, only a fraction $1 - \theta_x$ of the firms can adjust their prices, the remaining fraction $\theta_x$ keep their prices fixed.

When a firm gets a chance to adjust its price, it sets the price $P^*_x$ to maximize the following expected discounted profit

\begin{equation}
E_t \sum_{j=0}^{\infty} \theta_x^j \Delta_{t,t+j} \left[ P^*_x(i) Y_{xt+j}(i) - MC_{xt+j} Y_{xt+j}(i) \right]
\end{equation}

where $\Delta_{t,t+j} = \Pi_{n=t}^{t+j} \Delta_s$, $\Delta_s$ is the stochastic discount factor defined in household’s decision problem. Firms act in interests of their owners, the households. $P^*_x(i)$ denotes the optimal price chosen by firm $i$ at period $t$. When a firm gets a chance to adjust its price at period $t$, it has to take into account that the opportunity of resetting its price will not come until period $t + j$ with a probability $\theta_x^j$. The term in the bracket denotes the firm’s profit at period $t + j$ if it does not get a chance to adjust its price. The stochastic discount factor represents household’s relative valuation of cash across time. Note that each firm within a sector adjusting its price at period $t$ faces the same profit maximization problem in (2.16), so all firms within a sector will set the same price $P^*_x$.

Using the definition of $\Delta_{t,t+j}$ and demand function in (2.13), it is straightforward to derive $P^*_x$ from the maximization (2.16)

\begin{equation}
P^*_x = \left( \frac{\varepsilon_x}{\varepsilon_x - 1} \right) \frac{E_t \sum_{j=0}^{\infty} \theta_x^j \beta^j \Lambda_{t+j} MC_{xt+j} Y_{xt+j} P^e_{xt+j}^{e-1}}{E_t \sum_{j=0}^{\infty} \theta_x^j \beta^j \Lambda_{t+j} Y_{xt+j} P^e_{xt+j}^{e-1}}
\end{equation}
The price index in sector \( x \) satisfies

\[(2.18) \quad P_{xt} = [(1 - \theta_x)(P^*_{xt})^{1-\varepsilon} + \theta_x(P_{xt-1})^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \]

The aggregate price index satisfies

\[(2.19) \quad P_t = (P_{ct} Y_{ct} + P_{dt} Y_{dt}) / (Y_{ct} + Y_{dt}) \]

The aggregate inflation is defined as \( \Pi_t = \frac{P_t}{P_{t-1}} \).

2.2.4. Capital producers

Capital producers combine investment-sector final goods and existing capital stock to produce capital goods. This process entails a capital adjustment cost. Capital stocks in the household and the two market sectors evolve according to separate accumulation equations:

\[(2.20) \quad K_{xt+1} = (1 - \delta_x)K_{xt} + I_{xt} - \frac{\chi_x}{2} \left( \frac{I_{xt}}{K_{xt}} - \delta_x \right)^2 K_{xt} \quad (x \in c, d, h) \]

The problem of a capital producing firm is to maximize the profit

\[(2.21) \quad Q^n_{xt} \left[ I_{xt} - \frac{\chi_x}{2} \left( \frac{I_{xt}}{K_{xt}} - \delta_x \right)^2 K_{xt} \right] - P_{dt}I_{xt} \quad (x \in c, d, h) \]

which implies that the capital prices \( Q^n_{xt} \) are set to satisfy the following equation:
2.2.5. News Shocks

As in Christiano, Ilut, Motto and Rostagno (2008), productivity in the consumption goods sector follows the following process:

\[
\ln(A_t) = (1 - \rho) \ln(A_{t-1}) + \psi_{t-p} + \xi_t
\]

where \(\psi_{t-p}\) is news shock, which is a signal received at period \(t - p\) indicating a change in the future productivity in the consumption sector. A signal received at period \(t - p\) will change agents' expectation of future productivity at period \(t\). The term \(\xi_t\) captures the difference between the realized productivity and the expected productivity, which can be thought of as conventional technology shock. \(\psi_{t-p}\) and \(\xi_t\) are assumed to be uncorrelated over time and with each other, normally distributed with standard deviation \(\sigma_{\psi}\) and \(\sigma_{\xi}\). The parameter \(p\), which measures the time periods between the signal and the realization of the productivity change, is set to be 4 quarters as in Beaudry and Portier (2006).

One objective of this chapter is to quantify the variances of \(\psi_{t-p}\) and \(\xi_t\). The relative magnitude of the variance of these two shocks indicates the precision of the signal of future productivity. The greater is the variance of the news shock relative to that of the technology shock, the more precise is the signal. One extreme case is that the variance of news shock is zero, which indicates that agents do not receive any news about future productivity, the term \(\xi_t\) becomes conventional technology shock.
Another extreme case is that variance of the technology shock is zero, which indicates that the news about the productivity changes are perfectly accurate. Realized productivity is exactly equal to what the signal indicates.

Likewise, productivity in the investment goods sector is assumed to follow the following process:

\[
\ln(A_{dt}) = (1 - \rho_d) \ln(A_d) + \rho_d \ln(A_{dt-1}) + \kappa_{t-p} + \zeta_t
\]

where \( \kappa_{t-p} \) and \( \zeta_t \) are assumed to be uncorrelated over time and with each other, normally distributed with standard deviation \( \sigma_\kappa \) and \( \sigma_\zeta \). For simplicity, the two shocks in the investment sector are not related to the two shocks in the consumption sector.

2.2.6. Monetary Policy

The central bank conducts monetary policy by following a Taylor rule:

\[
\ln(R_t) = \rho_R \ln(R_{t-1}) + (1 - \rho_R) \ln(R) + \rho_R (\ln(Y_t) - \ln(Y)) + \rho_n \ln(\Pi_t)
\]

where aggregate output \( Y_t = Y_{ct} + Y_{dt} \), \( \Pi_t \) is the deviation of aggregate inflation from the steady state zero, \( R \) and \( Y \) denote the steady-state nominal interest rate and aggregate output respectively. The central bank is assumed to adjust the nominal interest rate in response to deviations of inflation and output from their respective steady-state levels. \( \rho_R \) captures the degree of interest rate smoothing.

2.2.7. Equilibrium and Model Solving

An equilibrium for this economy is a set of prices \( \{P_{ct}, P_{dt}, Q_{ct}, Q_{dt}, Q_{ht}, R_{t}, R^c_{t}, R^d_{t}, W_t\}_{t=0}^\infty \); an allocation \( \{C_{mt}, N_{ht}, N_{mt}, K_{ht}, K^s_{ct}, K^s_{dt}, B_t\}_{t=0}^\infty \) for the representative household, an
allocation \( \{N_{ct}, N_{dt}, K^d_{ct}, K^d_{dt}, Y^s_{ct}(i), Y^s_{dt}(i)\}_{t=0}^\infty \) for the intermediate goods producers, an allocation \( \{Y_{ct}, Y_{dt}, Y^d_{ct}(i), Y^d_{dt}(i)\}_{t=0}^\infty \) for the final goods producers, and an allocation \( \{I_{ct}, I_{dt}, I_{ht}\}_{t=0}^\infty \) for the capital producers, such that

a. \( \{C_{mt}, N_{ht}, N_{mt}, K_{ht}, K^*_c, K^*_d, B_t\} \) solves the household’s problem (2.1)-(2.4) given the stated prices;

b. \( \{N_{ct}, N_{dt}, K^d_{ct}, K^d_{dt}, Y^s_{ct}(i), Y^s_{dt}(i)\} \) solves the intermediate goods producer’s problem (2.16) given the stated prices;

c. \( \{Y_{ct}, Y_{dt}, Y^d_{ct}(i), Y^d_{dt}(i)\}_{t=0}^\infty \) solves the final goods producer’s problem (2.12) given the stated prices;

d. \( \{I_{ct}, I_{dt}, I_{ht}\}_{t=0}^\infty \) solves the capital producer’s problem (2.21) given the stated prices;

e. all markets clear: \( Y_{ct} = C_{mt}, Y_{dt} = I_{ct} + I_{dt} + I_{ht}, N_{mt} = N_{ct} + N_{dt}, K^*_c = K^d_{ct}, K^*_d = K^d_{dt}, Y^s_{ct}(i) = Y^d_{ct}(i), Y^s_{dt}(i) = Y^d_{dt}(i). \)

Equations (A1)-(A17) in the Appendix A.1 summarize the equilibrium of the model. The model can be solved by first log-linearizing (A1)-(A17) around steady state and applying a QZ decomposition method. The solution is expressed in a state space form, then maximum likelihood estimates of the parameters, together with the variance of shocks, can be obtained by using Kalman filter algorithm as described by Hamilton (1994, chapter 13).

### 2.3. Data and Calibration

An important task before the estimation is to choose the observable variables and find the corresponding data. Since one objective of this chapter is to study the comovements of consumption and investment sectors, sectoral data are used in the

---

\footnote{See Sims(2002) for more discussion of the QZ decomposition.}
estimation instead of aggregate data. The four observable variables used in the estimation are: output in the consumption goods sector, \( Y_{ct} \) (\( Y_{ct} = C_{mt} \)), output in the investment goods sector, \( Y_{dt} \) (\( Y_{dt} = I_{ct} + I_{dt} + I_{ht} \)), hours worked in the consumption goods sector, \( N_{ct} \), and hours worked in the investment goods sector, \( N_{dt} \). One challenge in connecting these variables defined in the model to the data is to match the sectoral data on output to the sectoral data on hours worked. According to the definitions of variables \( Y_{ct} \) and \( Y_{dt} \), output of the consumption sector, \( Y_{ct} \), corresponds to the sum of the personal consumption of non-durable goods and services; output of the investment sector \( Y_{dt} \) corresponds to the sum of personal consumption of durable goods and gross private fixed domestic investment. These data are available in National Income and Product Accounts (NIPA) at a quarterly frequency. However, sectoral data on hours worked are not available directly and have to be constructed from the data at the industry level reported by Bureau of labor Statistics (BLS). BLS reports the data on the number of employees and average weekly hours worked in industries classified based on the North American Industry Classification System (NAICS). Total weekly hours worked by different industries can be calculated by multiplying the number of employees and average weekly hours worked.\(^4\) The problem is how to combine the data on industry level into two sectors: consumption sector or investment sector. Following Huffman and Wynne (1999), I used the 2005 input-output tables reported by Bureau of Economic Analysis (BEA) to determine whether an industry should be classified as consumption sector or investment sector. If more than 50% of an industry's final output is allocated to personal consumption, the industry is classified as consumption sector; if more than 50% of an industry's final output is used as investment or intermediate goods, the industry is classified as investment sector. Based on this criterion, mining, utilities, construction, durable

\(^4\)In order to catch the inter-sectoral labor movements, hours worked are measured as a mix of intensive and extensive margins.
goods manufacturing,\textsuperscript{5} wholesale trade, transportation and warehousing are grouped into the investment sector; while non-durable goods manufacturing and services other than wholesale trade, transportation and warehousing\textsuperscript{6} are grouped into consumption sector. Then, hours worked in the consumption sector is calculated by summing up hours worked in industries that are grouped into consumption sector; hours worked in the investment sector is the sum of hours worked in those industries that are grouped into investment sector.

Following the usual practice in maximum likelihood estimation literature, the values of some parameters will be set before the estimation since the value of the likelihood function does not change significantly with these parameters. As in Rotemberg and Woodford (1992) and Ireland (2001), the elasticity of demand for intermediate goods $\varepsilon_x$, is set to be 6 so that the steady-state markup of price over marginal cost is 20%. Values for other parameters are chosen prior to estimation based on a rigorous calibration following the recipe by Gomme and Rupert (2007). First, labor’s shares of income $\phi_c$ and $\phi_d$ are measured by the ratio of compensation of employees in sector $x$ to (sectoral output $Y_x$ less proprietors’ income in sector $x$). NIPA tables section 6 reports the compensation of employees and proprietors’ income by industry. Sectoral data can be calculated by summing up the data at the industrial level based on the classification criterion defined previously. Second, depreciation rates of market capital $\delta_c, \delta_d$ (or household capital $\delta_h$) can be measured by dividing the depreciation of private fixed assets (or household capital) by stock of private fixed assets (or stock of household capital). The Stock of household capital is composed of stock of durable goods and residential capital owned by households. Data on depreciation and stock

\textsuperscript{5}Huffman and Wynne (1999) did not separate the manufacturing into non-durable goods and durable goods manufacturing.

\textsuperscript{6}Includes information, professional and business services, finance, insurance and real estate, retail trade, education and health, arts, entertainment, recreation, accommodation, food services, other services.
Table 2.1. Calibrated Values of Parameters

<table>
<thead>
<tr>
<th>( \phi_c )</th>
<th>( \phi_d )</th>
<th>( \delta_c )</th>
<th>( \delta_d )</th>
<th>( \delta_h )</th>
<th>( \gamma )</th>
<th>( \omega )</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( A_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.570</td>
<td>0.631</td>
<td>0.0162</td>
<td>0.0188</td>
<td>0.0160</td>
<td>2.1949</td>
<td>0.5524</td>
<td>0.5074</td>
<td>0.9882</td>
<td>0.7251</td>
</tr>
</tbody>
</table>

of fixed assets by industry, depreciation and stock of durable goods and residential capital are available in BEA fixed assets tables.\(^7\)

With the values of parameters \( \phi_c, \phi_d, \delta_c, \delta_d \) and \( \delta_h \) at hand, the values of the following five parameters \( \gamma \) (determines the steady state labor supply), \( \omega \) (determines the share of home produced goods in the composite consumption index), \( \alpha \) (the labor share parameter in the home production function), \( \beta \) (the discount factor), and \( A_d \)\(^8\) (the investment-sector productivity) are chosen to match five calibration targets such as time-use evidence and the investment shares of output.\(^9\) First, studies based on Time-Use Survey indicate that the ratio of the time spent by a typical married couple in working in market to the time spent in household work is about 4/3. Second, based on the calculation using data from BLS, hours worked in consumption sector account for 9.69% of disposable time. Third, according to the calculation using the data from BEA fixed assets tables, investment-aggregate output ratios in consumption sector, investment sector and household are 8.49%, 5.29% and 14.32% respectively. It is worth noting that the values of \( \gamma \) and \( \omega \) will vary with the choice of \( \sigma \) and \( \eta \) (to be estimated); while \( \alpha, \beta, \) and \( A_d \) will not. Table 2.1 summarizes the values of parameters when the parameter governing the elasticity of labor supply, \( \sigma \), is set to be 0 and the parameter measuring the elasticity of substitution between the market consumption goods and home produced goods, \( \eta \), is set to be 1.001 temporarily.

\(^7\)In BEA fixed assets tables section 3, capital stock and depreciation in real estate industry include the residential capital owned by households and corresponding depreciation. Data on the residential stock owned by households and related depreciation can be found in BEA fixed assets tables section 5. They were then removed from real estate industry and counted as part of household capital.

\(^8\)\( A_c \) is normalized to 1.

\(^9\)Refer to Gomme and Rupert (2007) for reasons to prefer to target investment-output ratios rather than capital output ratios.
The measured labor shares and depreciation rates of capital in market sectors are close to those reported in Huffman and Wynne (1999). Their estimation of the labor share in consumption sector is 0.59, and 0.66 in investment sector, while my estimation is 0.57 and 0.63 respectively. The difference in the estimated labor share could stem from the difference in time periods used, sector classification criterion and calculation formula.\textsuperscript{10} The measured labor share in home production, $\alpha$, is 0.55, which is lower than reported in the literature: for example, 0.622 reported in Gomme and Rupert (2007) and 0.68 reported in Greenwood, Rogerson and Wright (1995). The value of the discount factor is close to the value 0.99 used in literatures.

Quarterly data from 1972Q1-2006Q4 are used for both model calibration and estimation.\textsuperscript{11} It is worth noting that all the data used in the calibration are nominal. Since the calibration targets are ratios, using nominal or real data will not change the results. For the estimation, data on outputs in the two market sectors are transformed to real terms by dividing the deflator in the corresponding sector. Hours worked and outputs in consumption and investment sectors are log-transformed and H-P detrended to induce stationarity.

2.4. A Preview of Impulse Responses

Before estimating the model, it is interesting to study the impulse responses of selected variables to shocks. To focus the attention on the effects of addition of nominal rigidities and home production on responses, the capital adjustment costs are temporarily set to zero. Besides those parameters already calibrated, the parameters to be estimated are set as follows: $\rho_R = 0$, $\rho_y = 0.125$, $\rho_w = 1.5$, $\theta_c = 0.75$, $\theta_d = 0.75$, $\rho_c = 0.95$, $\rho_d = 0.95$.

\textsuperscript{10}For example, Huffman and Wynne(1999) calculate the labor share as the ratio of sum of the compensation of employees plus proprietors' income to sectoral output. By doing so, they assume that all the proprietors' income belongs to labor. As in Gomme and Rupert(2007), I assume that labor share of proprietors' income is the same as for the economy as a whole.

\textsuperscript{11}Appendix A.2 provides a detailed description of the data source.
Figure 2.1, which plots the responses of selected variables to a 1% innovation to the news shock in the consumption sector, demonstrates the effects of nominal rigidities on the responses. When the prices are flexible, the economy does not react to the consumption sector news shock at all before the news are realized. Even when the news about the technology improvement is realized, the only reaction is that consumption jumps up. This phenomenon that technology change that affects the consumption goods sector alone and has no impact on employment or capital accumulation has been referred to by Kimball (1994) as "consumption-technology neutrality". When prices are flexible, the price of consumption goods will drop immediately once the technology is improved. The increase in demand induced by the price slump exactly absorbs the increased production capacity. There is no change in either employment or capital allocation. It is interesting to note that this neutrality does not hold when nominal rigidities are introduced. As shown in Figure 2.1, a realization of expected technology improvement in consumption sector leads to a boom in consumption and investment outputs, a decline in hours worked in the consumption sector and a rise of hours worked in the investment sector. The underlying mechanism is as follows: after the technology improves, the demand for consumption goods will increase due to the drop in the price. However, nominal rigidities prevent the price of consumption goods from falling, thus depressing the demand. In this case, the increase in the demand for consumption goods is less than the increase of the technology, which means that consumption-sector producers need less inputs after the technology improvement. Then, resources flow from the production of consumption goods to the production of investment goods.

As demonstrated in chapter 1, adding nominal rigidities also facilitates the generation of expectation-driven business cycles. Figure 2.1 shows that the economy does not react to the consumption sector news shock at all when prices are flexible. By
contrast, a positive consumption sector news shock generates a boom in both market sectors before the realization of the technology improvement. When prices are rigid, consumption goods producers will lower their prices once they get a signal indicating an improvement in the consumption-sector technology. The lower price, relative to investment goods, boosts the demand for consumption goods, which further raises the demand for the labor and capital inputs. Thus, the addition of nominal rigidities is essential for the consumption-sector technology changes (both expected and unexpected) to have impact on business cycles.
Figure 2.2. Responses of selected variables to consumption-sector technology shocks in models with various elasticity of substitution between market consumption goods and home produced goods.

Solid lines: $\eta = 6$; lines with triangles: $\eta = 2$; lines with stars: $\eta = 1$; lines with circles: $\eta = 0.25$.

Figure 2.2, which plots the responses of selected variables to a 1% innovation to technology shock in the consumption sector, demonstrates the effects of various elasticities of substitution between market goods and home produced goods. When the parameter $\eta$ is close to 1, as the lines with stars in Figure 2.2 show, technology changes in the market have no impact on hours spent on home work. A rise of technology in consumption sector causes a decline of hours worked in consumption sector and a rise in hours worked in investment sector. So, the model fails to generate positive comovements of hours worked. When the parameter $\eta$ is greater than 1,
home goods and market goods become good substitutes. In this case, home produced goods are more easily replaced with cheaper market consumption goods when the consumption-sector technology improves. Hours worked will shift from home work to market consumption goods production. When the elasticity of substitution becomes high enough, hours worked in the consumption sector will rise with the technology improvement (as the solid lines in Figure 2.2 show). Thus, a high elasticity of substitution between market goods and home produced goods could generate a positive comovements of hours worked in two market sectors.

2.5. Estimation Results

In this section, the remaining parameters $\rho_R$, $\rho_y$, $\rho_\pi$, $\theta_c$, $\theta_d$, $\sigma$, $\eta$, $\rho_c$, $\rho_d$, $\chi_x$ together with the standard deviation of shocks $\sigma_y$, $\sigma_\xi$, $\sigma_\kappa$ and $\sigma_\xi$ are estimated using maximum likelihood method. Four versions of model will be estimated. The first is the baseline model in which all model features are included. The second is a restricted version of the model where the interaction between the market production and home production is reduced to minimal. This can be achieved by setting the elasticity of substitution between market produced goods and home produced goods $\eta$ to 1 (Cobb-Douglas home production function). The third restricted version of the model is the one where prices are flexible. The last one is the model without capital adjustment costs (i.e. $\chi_x = 0$).

Table 2.2 (the second and third columns) lists maximum likelihood estimates of the baseline model's parameters along with their standard errors, computed by taking the square roots of the diagonal elements of the inverted Hessian matrix evaluated at the estimates. First, the estimate of the parameter $\sigma$ translates to an elasticity of labor supply of 1.12, which is greater than 0.5, the upper bound of the range of estimates from micro-data studies. Second, the elasticity of substitution between
Table 2.2. Maximum Likelihood Estimation and Standard Error

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline Model</th>
<th>No home work</th>
<th>Flexible price</th>
<th>No capital adj. cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>Std. error</td>
<td>Estimate</td>
<td>Std. error</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.8926</td>
<td>0.2255</td>
<td>0.0000</td>
<td>0.3613</td>
</tr>
<tr>
<td>$\eta$</td>
<td>4.4144</td>
<td>0.5674</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>0.9597</td>
<td>0.0163</td>
<td>0.9977</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\rho_d$</td>
<td>0.4518</td>
<td>0.0565</td>
<td>0.4261</td>
<td>0.4259</td>
</tr>
<tr>
<td>$\chi_c$</td>
<td>13.267</td>
<td>1.7647</td>
<td>18.977</td>
<td>2.1399</td>
</tr>
<tr>
<td>$\chi_d$</td>
<td>8.6958</td>
<td>1.1775</td>
<td>10.957</td>
<td>2.8338</td>
</tr>
<tr>
<td>$\chi_h$</td>
<td>23.426</td>
<td>3.1138</td>
<td>22.473</td>
<td>7.1389</td>
</tr>
<tr>
<td>$\theta_c$</td>
<td>0.8383</td>
<td>0.0212</td>
<td>0.7523</td>
<td>0.0583</td>
</tr>
<tr>
<td>$\theta_d$</td>
<td>0.5179</td>
<td>0.0117</td>
<td>0.4848</td>
<td>0.0157</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>0.0000</td>
<td>0.0053</td>
<td>0.0000</td>
<td>0.1459</td>
</tr>
<tr>
<td>$\rho_\pi$</td>
<td>1.0397</td>
<td>0.0182</td>
<td>1.0338</td>
<td>0.2222</td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>0.0000</td>
<td>0.0662</td>
<td>0.0000</td>
<td>0.1460</td>
</tr>
<tr>
<td>$\sigma_\psi$</td>
<td>0.0017</td>
<td>0.0001</td>
<td>0.0036</td>
<td>0.0021</td>
</tr>
<tr>
<td>$\sigma_\xi$</td>
<td>0.0040</td>
<td>0.0003</td>
<td>0.0059</td>
<td>0.0022</td>
</tr>
<tr>
<td>$\sigma_\kappa$</td>
<td>0.0381</td>
<td>0.0026</td>
<td>0.0348</td>
<td>0.0037</td>
</tr>
<tr>
<td>$\sigma_\zeta$</td>
<td>0.0533</td>
<td>0.0034</td>
<td>0.0531</td>
<td>0.0112</td>
</tr>
</tbody>
</table>

likelihood value

<table>
<thead>
<tr>
<th>Baseline Model</th>
<th>No home work</th>
<th>Flexible price</th>
<th>No capital adj. cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1772</td>
<td>1688</td>
<td>1463</td>
<td>1649</td>
</tr>
</tbody>
</table>

$\sigma$: elasticity of labor supply; $\eta$: elasticity of substitution between $C_m$ and $C_h$;
$\rho_c$, $\rho_d$: persistence of technology shock in C-sector and I-sector;
$\chi_c$, $\chi_d$, $\chi_h$: capital adjustment cost in C-sector, I-sector and home production;
$\theta_c$, $\theta_d$: degree of nominal rigidity in C-sector and I-sector;
$\rho_y$, $\rho_\pi$, $\rho_R$: parameters in the monetary policy rule;
$\sigma_\psi$, $\sigma_\xi$, $\sigma_\kappa$, $\sigma_\zeta$: standard deviation of shocks.
market consumption goods and home produced goods, \( \eta \), has a point estimate of 4.4, implying that market goods and home goods are good substitutes. This result is higher than 4, the upper bound of range of values estimated by Rupert, Rogerson and Wright (1995) using microeconomic data, and much greater than the value 2.33 estimated by McGrattan, Rogerson and Wright (1997) in a dynamic general equilibrium model. Third, the estimated persistence of technology shock in consumption sector is 0.96, which is close to the value calibrated in literatures; while the persistency of technology shock in investment sector is estimated to be 0.45, which is much lower than that in consumption sector. Fourth, the parameters governing the magnitude of capital adjustments in the consumption sector, investment sector and home production are estimated to be 13.3, 8.7 and 23.4 respectively. By comparison, Ireland and Schuh (2007) assume that the magnitudes of capital adjustment costs are equal across the two market sectors, and estimated the parameter governing the magnitude of capital adjustment cost to be 17.33. Fifth, the estimates of parameters determining the degree of nominal rigidities, \( \theta_c \) and \( \theta_d \), are 0.84 and 0.52 respectively, indicating that the consumption-sector producers adjust their prices approximately once every six quarters and investment-sector producers keep their prices unchanged for about 2 quarters. The estimate of the degree of nominal rigidity in the consumption sector is much larger than those found in survey evidence but still in the range of those estimated in models with Calvo-type staggered prices. The empirical evidence surveyed by Taylor (1999) suggests that nominal price contracts last on average for a year. Bils and Klenow (2004) argue that the observed frequency of price adjustment in the U.S. is much higher, on the order of two quarters.\(^{12}\) Christiano, Eichenbaum and Evans (2005)'s estimates of the parameter governing the price rigidity range from 0.28 to 0.92, depending on different modelling features. The estimated parameters \( \theta_c \) and

\(^{12}\text{See Eichenbaum and Fisher (2007) for a reconciliation of the price adjustment frequency estimated in model with those found in survey evidence.}\)
\( \theta_d \) imply that investment-sector producers change their prices more frequently than those in the consumption sector. This is consistent with the belief that the prices of investment goods should change more frequently since the menu costs only account for a small percentage of the prices of investment goods. Sixth, the estimated policy reactions to inflation and output gap are 1.04 and 0 respectively. These two estimates are much lower than those estimated in the literature (e.g. 1.5 and 0.5 in the original Taylor rule; and 1.72 and 0.34 estimated by Clarida, Gali and Gertler (2000)). The estimate of parameter \( \rho_R \) hits the lower bound 0, indicating that the Fed has no intention to smooth the interest rates. In contrast, this parameter is estimated to be 0.71 in Clarida et al. (2000). Last, but the most important, the standard deviation of news shocks and technology shocks are 0.0017 and 0.0040 respectively in consumption sector, 0.0381 and 0.0533 in investment sector. By contrast, Huffman and Wynne (1999) calculate Solow residuals in the consumption sector and investment sector to be 0.011 and 0.0252 respectively. It is worth noting that the estimated standard deviation of news shock is lower than that of technology shocks in both sectors. If the technology shock is thought of as an unexpected technology change or a revision to the expectation of the technology, then a relatively large standard deviation of technology shock means that the news about future technology changes are not that informative in the sense that the revisions of the expectations are relatively large.

The fourth and fifth columns in Table 2.2 list the estimates of the parameters along with the standard errors, when \( \eta \) is set to be 1. As shown in figure 2.2, adding home production has no effect on market variables when the elasticity of the substitution between the market goods and home produced goods is set to be close to unity. The sixth and seventh columns in Table 2.2 list the estimates of the parameters along with the standard errors in the model with flexible prices. The last two columns in Table 2.2 list the estimation results when the parameters governing the capital adjustment
costs are set to zero. Since all these restricted versions of model are nested within
the baseline model, a likelihood ratio test can be applied to test whether these model
features can help improve the fit of the model significantly. It is straightforward to
show that all the model features can improve the fit of the model significantly.

2.6. Model Evaluation

This section evaluates the performance of the baseline model and three restricted
versions of the model using the calibrated and estimated values for the parameters.
The focus of this analysis is to show that the model with the features of home pro-
duction, nominal rigidities and capital adjustment costs can best account for a set of
statistics that are meant to capture the business cycle facts. The reason why these
features are important is examined by a study of the impulse responses of related
variables to four shocks. At the end, a variance decomposition is conducted to reveal
the relative significance of four shocks to explain the business cycle fluctuations.

2.6.1. Cyclical Implications

This subsection compares the business cycles properties of models with different fea-
tures to find the one that best matches the business cycle moments of the U.S. data.
Following the usual practice in the literature, both the U.S. and model-simulated data
are log-transformed and H-P filtered. The simulated moments are averages over 1000
replications of 140 observations (the same number of observations as in U.S. data).

The first row in Table 2.3 lists the standard deviation of the selected variables for
the U.S. data. The second to fifth rows list the standard deviations of simulated data
from the baseline model and three restricted versions of the model. The standard

13For example, to test whether the addition of household production improves the fit significantly,
calculate the likelihood ratio=2*(1772-1688)=168. Even at a 0.5% significance level, \( \chi^2(1) \) is 7.88,
which is far less than 168. Then, the hypothesis that home production does not improve the fit can
be easily rejected.
Table 2.3. Standard Deviations of Selected Variables for Three Models and U.S. Data

<table>
<thead>
<tr>
<th></th>
<th>( Y_c )</th>
<th>( Y_d )</th>
<th>( N_c )</th>
<th>( N_d )</th>
<th>( I_c )</th>
<th>( I_d )</th>
<th>( I_h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data</td>
<td>0.82</td>
<td>5.99</td>
<td>1.16</td>
<td>2.95</td>
<td>7.34</td>
<td>10.13</td>
<td>6.94</td>
</tr>
<tr>
<td>Model with all features</td>
<td>0.88</td>
<td>7.44</td>
<td>1.15</td>
<td>3.56</td>
<td>8.85</td>
<td>12.01</td>
<td>4.95</td>
</tr>
<tr>
<td>Model without home production</td>
<td>0.78</td>
<td>7.21</td>
<td>0.73</td>
<td>3.66</td>
<td>6.96</td>
<td>10.95</td>
<td>5.99</td>
</tr>
<tr>
<td>Model with flexible prices</td>
<td>11.92</td>
<td>9.43</td>
<td>8.98</td>
<td>7.38</td>
<td>233.20</td>
<td>71.87</td>
<td>136.52</td>
</tr>
<tr>
<td>Model without capital adj.</td>
<td>1.97</td>
<td>4.20</td>
<td>1.53</td>
<td>3.83</td>
<td>50.22</td>
<td>131.14</td>
<td>65.24</td>
</tr>
</tbody>
</table>

deviations of the simulated data from the model with all features and the model without home production match the U.S. data better than the other models. However, the match is not perfect in that the baseline model generates more variation than the data. In particular, the standard deviation of actual investment is 5.99%, whereas the baseline model generates a corresponding 7.44%, about 25% more than the actual one. The standard deviation of hours worked in investment sector is about 20% more than that of the actual data. The match is better for output and hours worked in the consumption sector. Although sectoral investment data have not been used in the estimation, it is interesting to see that only those models with capital adjustment costs can generate appropriate level of variations in sectoral investments,\(^{14}\) while the model without capital adjustment costs generate way too much variations in sectoral investments, about 10 times those of U.S. data.

Tables 2.4-2.8 show the contemporaneous correlations of selected variables for the U.S. data and four models. As shown in Table 2.4, there exist strong positive comovements between almost all selected variables in U.S. data. In particular, the data exhibit a positive correlation between outputs and hours worked within the same sector; the positive correlation also exists between outputs and hours worked across two market sectors. The baseline model with all features best captures these strong positive comovements (as shown in Table 2.5), while the restricted versions of the model

\(^{14}\)Note that model with flexible prices also generates too much variations in sectoral investments because the estimated parameters governing the magnitude of capital adjustment costs are close to zero.
Table 2.4. Correlations of Selected Variables for U.S. Data

<table>
<thead>
<tr>
<th>U.S. data</th>
<th>$Y_c$</th>
<th>$Y_d$</th>
<th>$N_c$</th>
<th>$N_d$</th>
<th>$I_c$</th>
<th>$I_d$</th>
<th>$I_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_c$</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_d$</td>
<td>0.75</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_c$</td>
<td>0.73</td>
<td>0.81</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_d$</td>
<td>0.65</td>
<td>0.81</td>
<td>0.94</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_c$</td>
<td>0.32</td>
<td>0.75</td>
<td>0.78</td>
<td>0.82</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_d$</td>
<td>0.40</td>
<td>0.55</td>
<td>0.36</td>
<td>0.63</td>
<td>0.60</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$I_h$</td>
<td>-0.26</td>
<td>0.77</td>
<td>0.58</td>
<td>0.53</td>
<td>0.38</td>
<td>0.01</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 2.5. Correlations of Selected Variables for Model with All Features

<table>
<thead>
<tr>
<th>Model with all features</th>
<th>$Y_c$</th>
<th>$Y_d$</th>
<th>$N_c$</th>
<th>$N_d$</th>
<th>$I_c$</th>
<th>$I_d$</th>
<th>$I_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_c$</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_d$</td>
<td>0.41</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_c$</td>
<td>0.76</td>
<td>0.59</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_d$</td>
<td>0.68</td>
<td>0.52</td>
<td>0.85</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_c$</td>
<td>0.44</td>
<td>0.99</td>
<td>0.60</td>
<td>0.52</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_d$</td>
<td>0.40</td>
<td>0.99</td>
<td>0.58</td>
<td>0.51</td>
<td>0.99</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$I_h$</td>
<td>0.40</td>
<td>0.99</td>
<td>0.59</td>
<td>0.53</td>
<td>0.99</td>
<td>0.98</td>
<td>1.00</td>
</tr>
</tbody>
</table>

can only generate either weak positive comovements or negative comovements. In particular, the model without home production only generates weak positive comovements between outputs and hours worked within the same sector, and weak positive comovement across two market sectors (as shown in Table 2.6). The model without nominal rigidities generates strong positive comovements between outputs and hours worked within the same sector, but negative comovements across two market sectors (as shown in Table 2.7). The model without capital adjustment costs can generate positive comovements between outputs and hours worked within and across market sectors, but fails to generate positive comovements among sectoral investments (as shown in Table 2.8). These results imply that home production, nominal rigidities and capital adjustment costs all play significant role in generating the positive comovements.
Table 2.6. Correlations of Selected Variables for Model without Homework

<table>
<thead>
<tr>
<th>Model without homework</th>
<th>$Y_c$</th>
<th>$Y_d$</th>
<th>$N_c$</th>
<th>$N_d$</th>
<th>$I_c$</th>
<th>$I_d$</th>
<th>$I_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_c$</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_d$</td>
<td>0.09</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_c$</td>
<td>0.00</td>
<td>0.19</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_d$</td>
<td>0.12</td>
<td>0.52</td>
<td>0.08</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_c$</td>
<td>0.09</td>
<td>0.99</td>
<td>0.19</td>
<td>0.52</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_d$</td>
<td>0.08</td>
<td>0.99</td>
<td>0.20</td>
<td>0.51</td>
<td>0.99</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$I_h$</td>
<td>0.09</td>
<td>0.99</td>
<td>0.18</td>
<td>0.52</td>
<td>0.99</td>
<td>0.99</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 2.7. Correlations of Selected Variables for Model with Flexible Prices

<table>
<thead>
<tr>
<th>Model with flexible prices</th>
<th>$Y_c$</th>
<th>$Y_d$</th>
<th>$N_c$</th>
<th>$N_d$</th>
<th>$I_c$</th>
<th>$I_d$</th>
<th>$I_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_c$</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_d$</td>
<td>-0.09</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_c$</td>
<td>0.99</td>
<td>-0.06</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_d$</td>
<td>-0.14</td>
<td>0.96</td>
<td>-0.11</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_c$</td>
<td>0.11</td>
<td>-0.05</td>
<td>0.06</td>
<td>-0.06</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_d$</td>
<td>0.05</td>
<td>0.62</td>
<td>0.07</td>
<td>0.45</td>
<td>-0.15</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$I_h$</td>
<td>-0.14</td>
<td>0.06</td>
<td>-0.08</td>
<td>0.11</td>
<td>-0.98</td>
<td>0.04</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 2.8. Correlations of Selected Variables for Model without Capital Adjustment Costs

<table>
<thead>
<tr>
<th>Model without capital adj.</th>
<th>$Y_c$</th>
<th>$Y_d$</th>
<th>$N_c$</th>
<th>$N_d$</th>
<th>$I_c$</th>
<th>$I_d$</th>
<th>$I_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_c$</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_d$</td>
<td>0.15</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_c$</td>
<td>0.82</td>
<td>0.05</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_d$</td>
<td>0.41</td>
<td>0.48</td>
<td>0.56</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_c$</td>
<td>-0.03</td>
<td>0.23</td>
<td>-0.35</td>
<td>-0.10</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_d$</td>
<td>-0.34</td>
<td>0.21</td>
<td>-0.57</td>
<td>-0.54</td>
<td>0.43</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$I_h$</td>
<td>0.29</td>
<td>-0.13</td>
<td>0.59</td>
<td>0.51</td>
<td>-0.74</td>
<td>-0.91</td>
<td>1.00</td>
</tr>
</tbody>
</table>

In sum, the model with all features best matches the data not only because it achieves the greatest maximum likelihood value, but also because it can best match those statistics characterizing business cycles.

2.6.2. Impulse Responses

Figures 2.3-2.4 show the impulse responses of selected variables to four shocks in the baseline model evaluated at the estimated parameters. All responses are with respect to a 1% innovation.
Figure 2.3 displays the impulse responses to consumption-sector news shocks and technology shocks. When the prices are rigid, producers determine their prices based on the weighted average of the current and future marginal costs. Under this circumstance, consumption-sector producers will lower their prices once they receive the signal indicating an technology improvement in their sector. The relatively low price boosts demand for consumption goods; correspondingly, hours worked and capital employed in consumption sector increase. In the meanwhile, since the estimated elasticity of substitution between market consumption and home produced goods is very high, households will replace home goods with market goods that are relative cheap now. Thus, both hours worked at home and demand for durable goods will fall. The demand for investment goods is determined by the sum of the demand for durable goods and the demand for capital in the market sectors. In Figure 2.3, output in the investment sector drops below its steady state level after the news shock, which can be explained by the fact that the decline of durable goods demand outweighs the increase in demand for capital in consumption sector. Lines with stars in Figure 2.3 plot the impulse responses to the consumption-sector technology shock. The underlying mechanism is similar to those used to explain the responses to news shock.

Figure 2.4 shows the impulse responses to investment-sector news shock and technology shocks. As shown by lines with stars in Figure 2.4, a technology shock in the investment sector leads to rises in all market variables: an increase in the supply of investment goods is accompanied by the corresponding increase of the capital stocks and durable goods stocks. Since the estimated degree of nominal rigidity in the investment sector is very small, the fast drop of the price of the investment goods boosts their demand so that the increase in investment-sector output is more than the increase in the productivity, which causes a rise in the hours worked in this sector. Meanwhile, the rise in the installed capital boosts labor productivity and wage rates.
Hours previously worked at home are then switched to the market consumption sector. As illustrated by solid lines in Figure 2.4, an expected improvement of technology in investment sector generates a boom before the realization of the expectation; that is, outputs, hours worked and investments in both sectors all rise. The boom before the realization of the good news is mainly caused by the existence of capital adjustment costs that provide an incentive to increase investments before the realization of the expected improvement of technology. As shown in Figure 2.5, the model without capital adjustment costs fails to generate a boom before the realization of the news about the future technology improvement in the investment sector.
Through the study of the impulse responses, it has been found that the baseline model generates positive within-sector comovements (between outputs and hours in the same sector) and inter-sector comovements (between outputs and hours in the two market sectors) when the economy is hit by consumption-sector technology shocks, investment-sector news shocks and technology shocks; as to the consumption-sector news shocks, the baseline model only generates the positive within-sector comovements of outputs and hours, the inter-sectoral comovements are negative. The positive
Figure 2.5 displays the responses to investment-sector news shocks and technology shocks in the model where the parameters governing the capital adjustment costs are set to zero and the remaining parameters are held at the estimated values in the baseline model. As in Beaudry and Portier (2004), an expected improvement of technology in investment sector generates a short period of bust before the realization of the expectation in a model without adjustment costs. Without adjustment
costs, firms will wait until the realization of the improvement of the investment-sector technology to increase investments, since the investment goods can be produced more productively after the realization of the technology improvement, and firms incur no adjustment costs by increasing their investment suddenly. Thus, the first effect of capital adjustment costs on responses is to generate a boom before the realization of the investment-sector news shock. The second effect is to achieve positive comovements of household investments and market investments. As shown in Figure 2.5, without the capital adjustment costs, an increase in productivity boosts demand for market investments, and part of household investments are reallocated to the market sectors. This inter-sectoral capital movement causes three problems: first, it generates a negative correlation in household investments and market investments, which is not consistent with the positive correlations in the data; second, the model without capital adjustment costs generates huge fluctuations in sectoral investments, as shown in Table 2.3; third, a decrease in household capital is accompanied by a decrease in household production and time allocated to home work, which boosts demand for market consumption dramatically since home produced goods and market consumption goods are good substitutes. As shown in Figure 2.5, the magnitude of the response in consumption sector output is close to that in the investment sector, which contradicts to the fact that the fluctuations in the consumption sector is much smaller than that in the investment sector. Introducing capital adjustment cost solves these problems by restricting the reallocation of household investments and smoothing the fluctuations in the investments. The business cycle statistics listed in Tables 2.3-2.8 clearly show that models with capital adjustment costs are able to generate appropriate level of fluctuations and the positive comovements of sectoral investments; while those models without capital adjustment costs generate implausibly large standard
deviations of sectoral investments and negative correlations between household and market investments.

2.6.3. Variance Decomposition

One of the objectives of this chapter is to examine the significance of news shock in explaining the fluctuations of macroeconomic variables. Tables 2.9-2.14 answer this question by decomposing the forecast error variance of outputs and hours worked in both sectors, as well as aggregate output and hours worked, into components attributable to each of the four shocks. The percentage of each variable's forecast error variance due to four shocks are reported for several forecast horizons.

Table 2.9 reveals that the consumption-sector technology shock accounts for most of the fluctuations of output in consumption sector at all horizons, about 55% in the short run and 60% in the long run. Investment-sector technology shocks explain about 35% in the short run and 15% in the long run. News shocks in both sectors account for about 25% of the fluctuations in the consumption-sector output.

Table 2.10 indicates that investment-sector technology shock accounts for a bulk of the fluctuations of consumption-sector hours worked, about 80% in the short run and 45% in the long run. Consumption-sector news shock explains about 15%-20% and investment-sector news shock explains about 30%.

Tables 2.11 and 2.12 show that investment-sector shocks account for almost all the fluctuations in investment-sector output and hours worked over all horizons. In the short run, the investment-sector technology shock dominates, accounting for almost all of the variations in investment-sector output and hours. As the time horizon lengthens, the variation explained by investment-sector technology shocks drops, while the portion explained by investment-sector news shock rises. In the long run, investment-sector news shock contributes to about 38% of the fluctuations in the
investment-sector output and about 47% of the variations in the investment-sector hours worked.

Tables 2.13 and 2.14 demonstrate that investment-sector technology shocks and news shocks account for the bulk of the fluctuations in the aggregate output and hours worked. The investment-sector technology shock causes most of the fluctuations in the short run. In the long run, the investment-sector news shocks can explain about 31% of the fluctuations in the aggregate output and about 43% of the fluctuations in the total hours worked; while consumption-sector news shocks only account for 3% of the fluctuations in the aggregate output and 2% of the fluctuations in the total hours worked.

In sum, news shocks (both sectors) account for about 25% of the fluctuations in output and 45% of the variations in hours worked in the consumption sector. As to the investment sector, the fluctuation in output explained by news shocks is about 38% and the variation in hours worked explained by news shocks is about 47%. News shocks in both sectors contribute to 34% of the variations in aggregate output and 45% of the variations in total hours worked. There results confirm Beaudry and Portier (2007)'s estimation that news shocks can account for a substantial portion of economic fluctuations. An independent and contemporaneous work by Schmitt-Grohé and Uribe (2008) estimates that news shocks can explain about two thirds of business-cycle fluctuations. At first glance, their result is much higher than that in the current work. However, their work considers the news about stationary and non-stationary technology changes while the current work only considers the news about the stationary technology changes. Their results indicate that the news about the stationary technology changes accounts for 37% of the fluctuation in the aggregate output, 27% of the fluctuation in the consumption and 45% of the fluctuation in the investment, which is very close to the results obtained in the current work.
Table 2.9. Forecast Error Variance Decomposition

<table>
<thead>
<tr>
<th>Quarters Ahead</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>20</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>News shocks in consumption sector</td>
<td>7.06</td>
<td>9.20</td>
<td>11.00</td>
<td>12.25</td>
<td>13.26</td>
<td>13.85</td>
</tr>
<tr>
<td>Technology shocks in consumption sector</td>
<td>55.50</td>
<td>55.90</td>
<td>55.51</td>
<td>58.03</td>
<td>60.01</td>
<td>60.46</td>
</tr>
<tr>
<td>News shocks in investment sector</td>
<td>0.39</td>
<td>2.49</td>
<td>12.09</td>
<td>11.34</td>
<td>10.53</td>
<td>10.72</td>
</tr>
<tr>
<td>Technology shocks in investment sector</td>
<td>36.93</td>
<td>32.39</td>
<td>21.38</td>
<td>18.35</td>
<td>16.18</td>
<td>14.95</td>
</tr>
</tbody>
</table>

Table 2.10. Forecast Error Variance Decomposition

<table>
<thead>
<tr>
<th>Quarters Ahead</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>20</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>News shocks in consumption sector</td>
<td>17.76</td>
<td>25.12</td>
<td>18.30</td>
<td>18.21</td>
<td>18.27</td>
<td>17.29</td>
</tr>
<tr>
<td>Technology shocks in consumption sector</td>
<td>2.20</td>
<td>3.04</td>
<td>4.22</td>
<td>5.70</td>
<td>7.81</td>
<td>11.23</td>
</tr>
<tr>
<td>News shocks in investment sector</td>
<td>1.01</td>
<td>6.97</td>
<td>30.08</td>
<td>29.96</td>
<td>29.13</td>
<td>28.23</td>
</tr>
<tr>
<td>Technology shocks in investment sector</td>
<td>79.00</td>
<td>64.96</td>
<td>47.38</td>
<td>46.11</td>
<td>44.78</td>
<td>43.22</td>
</tr>
</tbody>
</table>

Table 2.11. Forecast Error Variance Decomposition

<table>
<thead>
<tr>
<th>Quarters Ahead</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>20</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>News shocks in consumption sector</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Technology shocks in consumption sector</td>
<td>0.09</td>
<td>0.07</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>News shocks in investment sector</td>
<td>0.00</td>
<td>2.24</td>
<td>37.51</td>
<td>37.75</td>
<td>37.78</td>
<td>37.81</td>
</tr>
</tbody>
</table>

Table 2.12. Forecast Error Variance Decomposition

<table>
<thead>
<tr>
<th>Quarters Ahead</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>20</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>News shocks in consumption sector</td>
<td>0.06</td>
<td>0.05</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Technology shocks in consumption sector</td>
<td>1.68</td>
<td>0.82</td>
<td>0.55</td>
<td>0.55</td>
<td>0.56</td>
<td>0.56</td>
</tr>
<tr>
<td>News shocks in investment sector</td>
<td>0.08</td>
<td>23.09</td>
<td>46.94</td>
<td>47.26</td>
<td>47.24</td>
<td>47.19</td>
</tr>
<tr>
<td>Technology shocks in investment sector</td>
<td>98.19</td>
<td>76.04</td>
<td>52.47</td>
<td>52.15</td>
<td>52.17</td>
<td>52.21</td>
</tr>
</tbody>
</table>

It is also interesting to note that the impact of the consumption-sector shocks is limited in the consumption sector itself; while the investment-sector shocks have impacts on both sectors. Although introducing nominal rigidities makes it possible for consumption-sector shocks to have some effect on investment-sector variables, this effect is too small to compete with the large fluctuations caused by investment-sector shocks. Ireland and Schuh (2007) also found that shocks to consumption-sector technology levels play no role in explaining fluctuations in investments. As shown in
Table 2.13. Forecast Error Variance Decomposition

<table>
<thead>
<tr>
<th>Quarters Ahead</th>
<th>Aggregate Output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>News shocks in consumption sector</td>
<td>0.58</td>
</tr>
<tr>
<td>Technology shocks in consumption sector</td>
<td>5.33</td>
</tr>
<tr>
<td>News shocks in investment sector</td>
<td>0.02</td>
</tr>
<tr>
<td>Technology shocks in investment sector</td>
<td>94.07</td>
</tr>
</tbody>
</table>

Table 2.14. Forecast Error Variance Decomposition

<table>
<thead>
<tr>
<th>Quarters Ahead</th>
<th>Total hours worked</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>News shocks in consumption sector</td>
<td>3.74</td>
</tr>
<tr>
<td>Technology shocks in consumption sector</td>
<td>5.33</td>
</tr>
<tr>
<td>News shocks in investment sector</td>
<td>0.07</td>
</tr>
<tr>
<td>Technology shocks in investment sector</td>
<td>93.41</td>
</tr>
</tbody>
</table>

Tables 2.13 and 2.14, investment-sector news shocks play much more important role in explaining the business cycle fluctuations than consumption-sector news shocks.

2.7. Conclusion

Beaudry and Portier (2006) identify news shocks using the information of the stock prices and total factor productivity in a vector error correction model and claimed that news shocks can explain about 50% of business cycle fluctuations. This chapter estimates a two-sector dynamic general equilibrium model to assess the significance of the news shocks in generating aggregate fluctuations. The baseline model is incorporated with features of home production, nominal rigidities and capital adjustment costs. All these three model features play important role in generating positive co-movements between sectoral outputs, hours worked and investments. In particular, home production is introduced to induce the positive comovement of outputs and hours worked across two market sectors. Capital adjustment costs are incorporated to generate positive comovements between sectoral investments. To justify the incorporation of these three model features, the baseline model with all three features and three restricted versions of the model are estimated and evaluated. It is found that
the baseline model best fits the data and matches business cycle moments. The variance decomposition based on this estimated baseline model demonstrates that news shocks account for 34% of the variations in aggregate output and 45% of the variations in total hours worked. In particular, news shocks account for about 25% of the variations in outputs and 45% of the fluctuations in hours worked in the consumption sector. As to the investment sector, news shocks explain about 38% of the variations in outputs and 47% of the fluctuations in hours worked. The current research represents confirming evidence of Beaudry and Portier (2006) that news shocks may be an important source of economic fluctuations.
CHAPTER 3

News Shocks, Expectation Driven Business Cycles and
Financial Market Frictions

3.1. Introduction

Beaudry and Portier (2004) explore a theory of business cycles (Pigou cycles) in which booms and busts of the economy are caused by agents' expectation of future technology changes. In classical real business cycles models, booms and busts are attributed to the sudden changes of technology levels (technology shocks). Beaudry and Portier introduce a new possible source of business cycles into the literature, stating that fluctuations of the economy may be caused by news shocks: good news about future technology could lead to a boom while an unrealized expectation could generate a bust. Since their seminal paper, interest in news shocks has grown. The challenge faced by all these researchers is that, as shown in Beaudry and Portier (2004), it is difficult to generate a boom of all macroeconomic variables after agents receive news about future technology improvement (Pigou cycles) in classical RBC models. For example, in a classical one-sector RBC model, a news shock always causes consumption and investment to move to the opposite direction.

This chapter explores the booms and busts induced by news shocks in a model economy with financial market frictions. In the model economy, firms can accumulate capital either by purchasing existing capital, or by producing new capital themselves, subject to a time-to-build technology constraint as in Kydland and Prescott (1982).

Firms need to borrow from financial intermediaries to finance their purchases of capital. With the presence of financial market frictions, firms have to pay an external finance premium which depends inversely on their net values, i.e., firms pay a lower external finance premium when they have higher net values. This provides firms an incentive to build up capital stocks now to lower the external finance premium in the future. When firms receive news indicating a future technology improvement, they anticipate the need for more capital, and therefore more external finance in the future; they can lower their future external finance costs by building up their capital and net values now. By adding financial market frictions into an otherwise standard RBC model, this chapter succeeds in generating a boom in investment when a news shock hits the economy. However, the boom in the investment comes at the cost of a decrease in the consumption, which is not consistent with the positive comovements among macroeconomic variables observed in the data. This chapter further demonstrates that this problem can be solved by incorporating a habit formation in household's preference. The addition of time-to-build has two effects: first, it reduces the volatilities of investment so that the model's simulated moments are more in line with those observed in the data; second, with the time-to-build constraint, the capital price rises sharply when an optimistic expectation drives the demand for existing capital, which leads to a rise in firms' net values, therefore lowering the external finance premium and boosting investment.

The financial market friction is introduced into the model as in Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999). A firm combines funds borrowed from bankers and its own net assets to finance capital purchases. There is some friction present in the financial market: the firm's investment return is only observed by itself; and bankers incur a cost to observe it. This asymmetric information creates a moral hazard problem because the firm may misreport the return of the
investment. The optimal contract will be constructed in such a way that firms will always report the true return of investment. This financial market friction introduces a wedge between the cost of external funds and the opportunity cost of internal funds, termed “the external finance premium.” This premium is an endogenous variable and depends inversely on the net value of firms. Furthermore, firms’ net values can be boosted in two ways: firms can increase their net values by producing new capital using their own outputs; or firms’ net values could increase due to the capital appreciation. By contrast, Bernanke, Gertler and Gilchrist (1999) assume that firms cannot produce capital and rely on the capital appreciation to boost their net values. The procyclical behavior in firms’ net values in turn implies countercyclical movement in the premium for external funds. This countercyclical movement in the premium serves to amplify investment and hence overall aggregate activity, relative to the case of frictionless financial markets.

The main contribution of this chapter is to construct a model which can generate a boom in all macroeconomic variables such as consumption, investment, hours worked and output when a news shock hits the economy. Beaudry and Portier (2004) succeed in generating the positive comovements after a news shock in a two-sector model with durable and non-durable goods. The key assumption they make is that there exists a strong complementarity between the non-durable goods sector and durable goods sector. Den Haan and Kaltenbrunner (2004) argue that models in which expectations induce a boom in both consumption and investment must allow for “idle resources” in the aggregate economy, so that the economy is not at its full capacity when the change in growth expectations occurs. If the amount of “idle resources” can be reduced by an increase in growth expectations, then consumption and investment can increase at the same time. They incorporate labor market matching into their model so that there exists an “idle resource” in the economy: the pool of unemployed. Because
the number of vacancies posted by firms depends on firms' expectation about future profits, it follows that whenever profit expectations are suddenly revised upwards, more vacancies are posted, more jobs are generated, and employment rises. This will cause production to increase even though current productivity has remained unchanged. Christiano, Ilut, Motto and Rostagno (2008) also explore the possibility of generating Pigou cycles in one sector models. They succeed in generating booms of consumption, investment and output by adding investment adjustment costs, variable utilization of capital and habit persistence in preference into a standard one sector model. However, it is not a straightforward matter to get corresponding boom of asset prices in their frameworks. Asset prices slump during the booms when all the other variables rise. To solve this problem, Christiano, Ilut, Motto and Rostagno (2008) extended their model by adding sticky prices, sticky wages and Taylor-rule monetary policies. The financial market friction introduced in the current work provides firms with incentives to invest once they expect a technology improvement rather than wait until the realization of the improvement: they can lower their external finance costs in the future by building up their capital stocks now. By contrast, Christiano, Ilut, Motto and Rostagno (2008) allow firms to reduce their future investment adjustment costs by increasing current investment levels.

The model in the current work also succeeds in generating the lead-lag pattern between the external finance premium and output observed in U.S. data. The data show that the external finance premium leads output by three quarters. This lead-lag pattern can only be generated in the model where firms are not capable of producing new capital. In this case, capital appreciation is the only channel through which firms increase their net values and in turn lower their external finance premium. When a news shock hits the economy, agents' expectations of future technology improvement raise demand for new capital. The existing capital becomes more valuable due to the
fact that it takes time to build new capital. Since the boom in capital price is caused by the temporary scarcity of the existing capital due to time-to-build, the effect on capital price is the strongest at the moment when the news shock hits the economy. A boom in capital price in turn boosts firms’ net values and lowers the external finance premium. Therefore, the slump of the external finance premium leads the boom in output.

The rest of the chapter is organized as follows. Section 2 describes the optimal financial contract. Section 3 presents the general equilibrium model with financial market frictions, time-to-build and habit formation. Section 4 describes the calibration. Section 5 evaluates the model by studying the impulse responses and the business cycle moments. The importance of each model feature is analyzed by removing them one a time from the baseline model. Section 6 concludes the chapter.

3.2. The Optimal Financial Contract

This section describes the optimal financial contract problem with the presence of financial market frictions. At the end of period $t$, firms need to finance the purchase of new capital $K_{t+1}$, which can be used at period $t+1$. There are two sources of financing: firms’ own net worth $NV_{t+1}$ and funds borrowed from financial intermediaries. Given the price of capital $Q_t$, firms need to borrow $Q_tK_{t+1} - NV_{t+1}$. It is assumed that the return on capital is $\omega R_{t+1}^k Q_t K_{t+1}$, where $R_{t+1}^k$ is the aggregate return on capital and $\omega$ is an idiosyncratic shock to the return. The shock $\omega$ is assumed to be independent and identically distributed across time and across firms, with a probability density function $f(\omega)$ and a continuous cumulative density function $F(\omega)$. As in Carlstrom and Fuerst (1997) and Bernanke et al. (1999), the financial market friction is introduced to the model by assuming asymmetric information between firms and financial intermediaries. It is assumed that the idiosyncratic shock $\omega$ is private
information, which cannot be observed by financial intermediaries unless an auditing cost \( \mu R^k_{t+1} Q_t K_{t+1} \) is incurred. To avoid the reputation problem involved in multiperiod contracts, it is assumed that there is enough anonymity in financial markets that only one-period contracts between borrowers and lenders are feasible. Under this circumstance, borrowers have an incentive to misreport their return on capital. To prevent this misreporting, lenders have to audit all the borrowers who default and charge borrowers an interest rate higher than the risk-free rate to cover the audit costs.

The optimal contract is characterized by a threshold value \( \overline{\omega} \) such that if \( \omega > \overline{\omega} \), the borrower pays the lender the fixed amount \( \overline{\omega} R^k_{t+1} Q_t K_{t+1} \) and keeps the equity \( (\omega - \overline{\omega}) R^k_{t+1} Q_t K_{t+1} \). Alternatively, if \( \omega < \overline{\omega} \), the borrower cannot pay the contractual return and thus declare default. The lender then audits the borrower who defaults and receives \( (1 - \mu) \omega R^k_{t+1} Q_t K_{t+1} \), while the borrower is left with nothing.

The value of \( \overline{\omega} \) under the optimal contract is determined by the requirement that the lender receive an expected return equal to the opportunity cost of its funds, the risk-free rate \( R_{t+1} \). Accordingly, the loan contract must satisfy

\[
(3.1) \quad [1 - F(\overline{\omega})] \overline{\omega} R^k_{t+1} Q_t K_{t+1} + (1 - \mu) \int_0^{\overline{\omega}} \omega R^k_{t+1} Q_t K_{t+1} dF(\omega) = R_{t+1} (Q_t K_{t+1} - NV_{t+1})
\]

where \( F(\overline{\omega}) = \int_0^{\overline{\omega}} f(\omega) d\omega \) gives the probability of default. The first term on the left side of the equation denotes the return from those who do not default and the second term denotes the expected return from those who default. The right side of the equation denotes the lender's opportunity cost of lending.

When the aggregate return on capital \( R^k_{t+1} \) fluctuates, the risk-neutral borrower is willing to offer a state-contingent non-default payment that guarantees the lender an
expected return equal to the risk-free rate, that is, \( \bar{\omega} \) will depend on the realization of \( R_{t+1}^k \). The optimal contracting problem is for the borrower to choose \( K_{t+1} \) and \( \bar{\omega} \) to maximize the expected return:

\[
\int_{-\infty}^{\infty} \omega R_{t+1}^k Q_t K_{t+1} dF(\omega) - [1 - F(\bar{\omega})] \bar{\omega} R_{t+1}^k Q_t K_{t+1}
\]

subject to the constraint implied by equation (3.1).

This optimal contracting problem can be rewritten as:

\[
\max_{K_{t+1}, \bar{\omega}} (1 - \Gamma(\bar{\omega})) R_{t+1}^k Q_t K_{t+1}
\]

subject to:

\[
[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] R_{t+1}^k Q_t K_{t+1} = R_{t+1}(Q_t K_{t+1} - NV_{t+1})
\]

where \( \Gamma(\bar{\omega}) = \int_0^{\bar{\omega}} \omega f(\omega) d\omega + \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega \) is the lender's expected share of profit; and \( \mu G(\bar{\omega}) = \mu \int_0^{\bar{\omega}} \omega f(\omega) d\omega \) is the expected auditing cost. Furthermore, define the external finance premium \( s = \frac{R_{t+1}^k}{R_{t+1}} \) and use \( k = \frac{Q_t K_{t+1}}{NV_{t+1}} \), the capital/net value ratio as the choice variable. The first order conditions for the optimal contracting problem may be written as:
\( \bar{\omega} : \Gamma'(\bar{\omega}) - \lambda[\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})] = 0 \)  

(3.6) \[ k : [(1 - \Gamma(\bar{\omega})) + \lambda(\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]s - \lambda = 0 \]

(3.7) \[ \lambda : (\Gamma(\bar{\omega}) - \mu G(\bar{\omega}))sk - (k - 1) = 0 \]

where \( \lambda \) is the Lagrangian multiplier on the constraint (3.4).

From first order condition (3.5), \( \lambda \) can be expressed as a function of \( \bar{\omega} \),

\[
\lambda(\bar{\omega}) = \frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})}
\]

and from the first order condition (3.6), the external finance premium \( s \) can be expressed as a function of the threshold rate \( \bar{\omega} \),

\[
s(\bar{\omega}) = \frac{\lambda(\bar{\omega})}{(1 - \Gamma(\bar{\omega})) + \lambda(\bar{\omega})(\Gamma(\bar{\omega}) - \mu G(\bar{\omega}))}
\]

and from the first order condition (3.7), the capital/net value ratio \( k \) can also be expressed as a function of \( \bar{\omega} \),

\[
k(\bar{\omega}) = 1 + \frac{\lambda(\bar{\omega})(\Gamma(\bar{\omega}) - \mu G(\bar{\omega}))}{1 - \Gamma(\bar{\omega})}
\]

Bernanke et al. (1999) prove that \( s'(\bar{\omega}) > 0 \) and \( k'(\bar{\omega}) > 0 \) for a certain range of \( \bar{\omega} \). Thus, equation (3.9) and (3.10) together establish the monotonically increasing relationship between the external finance premium \( s \) and the borrower's capital/net value ratio \( k \). The underlying intuition is that borrowers with higher net values (lower
financial leverage) are less likely to default than those with lower net values (higher financial leverage), therefore, borrowers with higher net values only need to cover a smaller share of the auditing cost, hence pay lower external finance premiums. In the presence of asymmetric information, an optimal financial contract implies that borrower has to pay external finance premium that depends inversely on its net value.

3.3. The General Equilibrium Model

This section embeds the financial contract problem of section 2 into general equilibrium models. The baseline model economy is composed of households, firms and financial intermediaries. Firms produce output using labor and capital. Firms can accumulate capital in two ways: they can either use part of their own output to produce new capital subject to a time-to-build technology constraint, or they can increase their capital stock by trading in the market for existing capital. In the latter case, firms need to borrow from the financial intermediaries to finance their capital purchases. Due to the existence of asymmetric information between borrowers and lenders, lenders determine the lending rates based on the financial positions of borrowers. An alternative model is constructed, in which all the model features remain unchanged except one: firms cannot produce their own capital, and they can only accumulate capital by purchasing either the existing capital or purchasing new capital from capital producers. By comparing the impulse responses and business cycle moments in the baseline model and the alternative model, I can explore the pros and cons of these two models, both of which can generate a boom of all macroeconomic variables after agents receive news about future technology improvement (Pigou cycles).
3.3.1. The Baseline Model

3.3.1.1. Households. The representative household supplies labor $N_t$ to firms and allocates its income between consumption $C_t$ and savings in financial intermediary $D_t$. Households’ preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln(C_t - bC_{t-1}) + \gamma \frac{(1 - N_t)^{1-\sigma}}{1 - \sigma} \right)$$

where $\beta$ is the discount factor; $\sigma$ determines the elasticity of labor supply and $\gamma$ determines the steady state labor supply. It is assumed that households exhibit internal habit formation, that is, households’ preferences depend on their own consumption history. The parameter $b$ measures the strength of habit persistence in consumption preference.

The household budget constraint is given by

$$C_t + D_{t+1} = W_t N_t + R_t D_t$$

where $W_t$ is the real wage rate, and $R_t$ is the gross real interest rates paid by the financial intermediary.

The household maximizes the utility function (3.11) subject to its budget constraint (3.12). The first order conditions with respect to consumption, hours worked and savings are:

$$\frac{1}{(C_t - bC_{t-1})} - E_t \left( \frac{b\beta}{C_{t+1} - bC_t} \right) = \lambda_t$$

$$\lambda_t W_t = \gamma (1 - N_t)^{-\sigma}$$

$$\lambda_t = \beta E_t (R_{t+1} \lambda_{t+1})$$
where $\lambda_t$ is the Lagrangian multiplier associated with the budget constraint (3.12). The first term in the first order condition (3.13) captures the impact of one extra unit of consumption on current period's utility; the second term in (3.13) captures the impact of one extra unit of consumption on the next period's utility. Notice that an increase in current consumption has a negative effect on the next period's utility by raising the criterion by which consumers will judge their preferences. The first order condition (3.14) equates the marginal benefits of working an extra hour to the marginal disutility from working an extra hour. First order condition (3.15) equates the cost of sacrificing one unit of consumption to the benefit of saving this money in the financial intermediary.

3.3.1.2. Firms (also capital producers themselves). Firms produce output according to the production function given by

\begin{equation}
Y_t = A_t N_t^\alpha K_t^{1-\alpha}
\end{equation}

where $A_t$ is exogenous technology, $N_t$ is the amount of labor used, $K_t$ is the capital stock, and $\alpha$ is the labor's share of the income.

Firms can accumulate capital in two ways: they can either use part of their own output to produce new capital subject to a time-to-build technology constraint or they can purchase existing capital in the market. As in Kydland and Prescott (1982), starting a project at date $t$ requires investment of resources at dates $t$, $t+1$, $\ldots$, $t+J-1$, with the capital finally being ready for use at date $t+J$. Let $s_{j,t}$ denote the number of projects $j$ periods from completion at date $t$. The laws of motion that describe the evolution of the incomplete projects are given by
It is assumed that a fixed fraction $\phi_j$ of resources are expended for a project $j$ periods from completion. The total investment expenditure at date $t$ is given by

$$I_t = \sum_{j=1}^{J} \phi_j s_{j,t}$$

with $0 \leq \phi_j \leq 1$, and $\sum_{j=1}^{J} \phi_j = 1$.

Alternatively, firms can increase their capital stock by purchasing existing capital in the market. Let $K_t^m$ denote the purchase of existing capital. Assume all the existing capital after the depreciation will be sold in the market; then the law of motion of the capital stock is given by

$$K_{t+1} = K_t^m + s_{1,t}$$

Firms need to borrow from financial intermediaries to finance capital purchases. At the end of period $t$, firms purchase existing capital $K_t^m$ in the market at price $Q_t$. The purchase of the capital is partly financed with firms' net value $NV_{t+1}$ and partly financed by borrowing from the financial intermediary $B_{t+1}$, that is,

$$Q_t K_t^m = NV_{t+1} + B_{t+1}$$

The solution to the optimal contracting problem shown in section 2 demonstrates that the external finance premium is an increasing function of the capital/net value.
ratio, that is:

\[
E_t(R^{k}_{t+1}) = f \left( \frac{Q_tK^m_t}{NV_{t+1}} \right) E_t(R_{t+1}) = \left( \frac{Q_tK^m_t}{NV_{t+1}} \right)^\eta E_t(R_{t+1})
\]

where \(E_t(R^{k}_{t+1})\) denotes the expected cost of external finance between \(t\) and \(t+1\); \(E_t(R_{t+1})\) denotes the expected risk free rate; \(f(.)\) is called the external finance premium, which measures the wedge between the cost of external finance and the risk free rate; \(Q_t\) is the price of the existing capital; and \(NV_{t+1}\) denotes firms’ net value at the end of period \(t\). The borrower’s financial position is summarized by its capital/net value ratio. A relative high capital/net value ratio indicates high financial leverage, that is, a low net value relative to the total funds needed to finance the capital purchase. According to the solution to the optimal financial contract, the parameter \(\eta\), the elasticity of the external finance premium with respect to financial leverage, should be positive.

The firm’s objective is to maximize the expected discounted profits

\[
E_0 \sum_{t=0}^{\infty} \Delta_{0,t} \left[ A_t N_t^\alpha K_t^{1-\alpha} + Q_t(1-\delta)K_t - E_{t-1}(R^k_t)(Q_{t-1}K^m_{t-1} - NV_t) - \sum_{j=1}^{J} \phi_{j,t} s_{j,t} - W_t N_t \right]
\]

where \(\Delta_{0,t}\) is the stochastic discount factor that represents household’s relative valuation of cash across time. \(\Delta_{0,t}\) is defined as:

\[
\Delta_{0,t} = \beta^t U_1(C_t, N_t) = \Pi_{s=0}^{t} \left( \frac{1}{R_s} \right)
\]

where \(U_1(C_t, N_t)\) is the marginal utility of consumption. The second equation in (3.23) is derived from the household’s first order condition (3.15).

At each period \(t\), a firm’s cash inflow includes the income from the sales of its output and its stock of existing capital after the depreciation. A firm’s cash outflow
includes the repayment of loan, the expenditure on the capital projects including the project started at the current period and all projects still incomplete, and the wage payment. After substituting $K_{t-1}$ with $K_t - s_{1,t-1}$, the firm is assumed to maximize the objective function in (3.22) with respect to $N_t$, $K_t$, $s_{j,t}$ subject to laws of motions described in (3.17). The corresponding first order conditions are:

\begin{align}
(3.24) \quad W_t &= \alpha A_t N_t^{\alpha-1} K_t^{1-\alpha} \\
(3.25) \quad E_t(R_{t+1}^k) &= E_t \left[ (1 - \alpha) \frac{Y_{t+1}}{K_{t+1}} + Q_{t+1}(1 - \delta) \right] \\
(3.26) \quad E_t \left( \frac{R_t^k Q_{t+J-1}}{\Pi_{s=t+1}^{t+J-1} R_s} \right) &= \phi_J + E_t \left( \frac{\phi_{J-1}}{\Pi_{s=t+1}^{t+J-1} R_s} \right) + \ldots + E_t \left( \frac{\phi_1}{\Pi_{s=t+1}^{t+J-1} R_s} \right)
\end{align}

First order condition (3.24) equates the real wage rate to the marginal product of labor; Equation (3.25) demonstrates that the expected rate of return on capital can be decomposed into two parts: the expected marginal increase in the production of outputs and capital appreciation. Firms will adjust their capital purchases so that the expected rate of return on capital equals the expected marginal cost of finance. In the model without financial market imperfections, the expected real return on capital will be equated to the expected real risk-free interest rate $E_t(R_{t+1})$. Equation (3.26) shows that the price of existing capital must adjust so that the firm is indifferent between starting a new capital project by itself and purchasing existing capital in the market. To increase its stock of capital at period $t+J$, the firm can either purchase a unit of existing capital at period $t+J-1$, or start a new capital project at period $t$. In the latter case, the firm will incur a cost of $\phi_J$ at current period $t$, and an expected cost of $E_t(\frac{\phi_{J-1}}{R_{t+1}^k})$ at period $t+1$, and so on. Notice that the left-hand side of equation (3.26) is the expected cost of purchasing a unit of existing capital at period $t+J-1$, discounted to the present value; the right-hand side of equation (3.26) denotes all the costs incurred by the firm to build up a unit of one-period-to-complete capital.
project, discounted to the present value. Since both of these options provide the firm with one unit of ready-to-use capital at period \( t + J \), the expected costs incurred should be equal to each other.

The profits earned by firms will be accumulated in net values over time. Following Bernanke et al. (1999), it is assumed that each firm has a constant probability of surviving to the next period. This assumption is intended to preclude the possibility that the entrepreneurs will ultimately accumulate enough wealth to be fully self-financing. Firms that do not survive will consume the residual equity. The evolution of the aggregate net value of firms can be described as:

\[
NV_{t+1} = \mu \left[ R_{t}^k Q_{t-1} K_t - E_{t-1}(R_{t}^k)(Q_{t-1} K_{t-1}^m - NV_t) \right]
\]

where \( \mu \) is the probability of surviving; \( R_{t}^k Q_{t-1} K_t \) is the realized return on capital; \( E_{t-1}(R_{t}^k) \) is the rate of return on capital anticipated in the previous period, which is also the actual external financing cost; and \( (Q_{t-1} K_{t-1}^m - NV_t) \) is the funds borrowed from the financial intermediary. Equation (3.27) suggests that there exist two sources of changes in firms' net values: the first source is the wedge between the realized and the expected rate of return on capital. When the economy is hit by a positive shock, the realized rate of return on capital is greater than the expected one, which boosts firms' net values. The second source is the wedge between the capital purchased \( K_{t-1}^m \) and the total capital used in the production \( K_t \). When firms produce more capital by themselves, this wedge will widen; therefore, firms' net values increase. According to equation (3.21), an increase in the net value lowers the producer's external finance premium and in turn spurs borrowing and capital investment. The rise in capital demand fuels the boom of the capital price, which further raises the producer's net value and reduces the external finance premium. This mechanism is called the "financial accelerator".
3.3.1.3. News Shocks. As in Beaudry and Portier (2004) and Christiano, Ilut, Motto and Rostagno (2008), productivity follows the following process:

\[(3.28) \quad \log(A_t) = (1 - \rho) \log(A) + \rho \log(A_{t-1}) + \psi_{t-p} + \xi_t\]

where \(\psi_{t-p}\) is called news shock, which is a signal received at period \(t - p\) indicating a change in the future productivity. A signal received at period \(t - p\) will change agents' expectation of future productivity at period \(t\). The term \(\xi_t\) is a conventional technology shock. \(\psi_{t-p}\) and \(\xi_t\) are assumed to be uncorrelated over time and with each other, and normally distributed with standard deviation \(\sigma_\psi\) and \(\sigma_\xi\).

3.3.1.4. Equilibrium. An equilibrium for this economy is a set of prices \(\{Q_t, R_t, R^k, W_t\}_{t=0}^{\infty}\), an allocation \(\{C_t, N^s_t, D_t\}_{t=0}^{\infty}\) for the representative household, an allocation \(\{N^d_t, B_t, K^m_t, s_t, s_J, Y_t\}_{t=0}^{\infty}\) for the firm, such that

- a. \(\{C_t, N^s_t, D_t\}\) solves the household's problem given the stated prices;
- b. \(\{N^d_t, K^m_t, s_t, s_J, Y_t\}\) solves the firm's problem given the stated prices;
- c. all markets clear: \(N^s_t = N^d_t, Y_t = C_t + I_t, D_t = B_t\).

Equations (A1)-(A14) in Appendix A.3 summarize the equilibrium of the model. The model can be solved by first log-linearizing (A1)-(A14) around steady state and applying a QZ decomposition method.²

3.3.2. The Alternative Model

This subsection presents a variant of the baseline model: keeping the other model features the same, I assume that firms now can only accumulate capital by purchasing it from capital producers.

3.3.2.1. Firms (purchasing capital from capital producers). Firms still need to finance their capital purchases by borrowing from financial intermediaries. The

²See Sims(2002) for more discussion of the QZ decomposition.
financial market frictions still exist so that firms have to pay an external finance premium based on their capital/net value ratios.

The firm’s objective is to maximize the expected discounted profits

\[
\max E_0 \sum_{t=0}^{\infty} \Delta_{0,t} [A_t N^\alpha K_t^{1-\alpha} + Q_t (1 - \delta) K_t - E_{t-1}(R^k_t)(Q_{t-1} K_t - NV_t) - W_t N_t]
\]

where all the notations are the same as described in previous subsection. The corresponding first order conditions are:

\[
W_t = \alpha A_t N_t^{\alpha-1} K_t^{1-\alpha}
\]

\[
E_t(R^k_{t+1}) = E_t \left[ \frac{(1 - \alpha)(Y_{t+1}/K_{t+1}) + Q_{t+1}(1 - \delta)}{Q_t} \right]
\]

Notice that these two first order conditions are the same as in (3.24) and (3.25).

Two different assumptions concerning firms’ capital accumulation behaviors mainly affect the evolution of firms’ net value. When firms are assumed to accumulate capital through trading with each other or purchasing from capital producers, firms’ net value evolves as follows:

\[
NV_{t+1} = \mu [R^k_t Q_{t-1} K_t - E_{t-1}(R^k_t)(Q_{t-1} K_t - NV_t)]
\]

Notice that firms choose to purchase \( K_t \) units of capital at price \( Q_{t-1} \) at period \( t - 1 \). They borrow \( Q_{t-1} K_t - NV_t \) from financial intermediaries at the cost of paying an interest rate \( E_{t-1}(R^k_t) \) depending on their capital/net value ratios. \( R^k_t Q_{t-1} K_t \) measures the realized return on capital, where all the capital used in production, \( K_t \), is purchased in the market. By contrast, when firms are assumed to be able to produce capital using their own outputs, firms’ net value evolves as follows:

\[
NV_{t+1} = \mu [R^k_t Q_{t-1} K_t - E_{t-1}(R^k_t)(Q_{t-1} K_t^{m_{t-1}} - NV_t)]
\]
Now, of all the capital used in the production at period $t$, $K_t$, only the part $K_{t-1}^{n}$ needs to be purchased in the market, the rest $(K_t - K_{t-1}^{n})$ is produced by the firm itself. The capital produced by the firm itself increases the firm’s net value directly. By contrast, when firms are assumed to purchase all the capital in the market, the main source of changes in firms’ net value is the wedge between the realized and the expected rate of return on capital.

3.3.2.2. Capital Producers. Capital producers generate new capital using outputs produced by firms subject to a time-to-build technology constraint as described previously. The capital producer maximizes the following expected discounted profit

\[
(3.34) \quad \max E_0 \sum_{t=0}^{\infty} \Delta_{0,t} \left[ Q_t s_{1,t} - \sum_{j=1}^{J} \phi_j s_{j,t} \right]
\]

subject to laws of motions described in (3.17). The corresponding first order condition is:

\[
(3.35) \quad E_t \left( \frac{Q_{t+J-1}}{\Pi_{s=t+1}^{t+J-1} R_s} \right) = \phi_{J} + E_t \left( \frac{\phi_{J-1}}{\Pi_{s=t+1}^{t+J-1} R_s} \right) + \ldots + E_t \left( \frac{\phi_1}{\Pi_{s=t+1}^{t+J-1} R_s} \right)
\]

3.4. Calibration

The values of parameters are either borrowed from the literature or calibrated to match certain long-run averages observed in the U.S. economy. The discount factor $\beta$ is set to 0.99 so that the steady-state annualized risk-free rate is about 4%. Following Gomme and Rupert (2007), the parameter determining the labor’s share of income $\alpha$ is set to 0.717. The depreciation rate of capital $\delta$ is set to 0.025. The parameter $\sigma$ is set to be 2 so that the elasticity of labor supply is unity. The parameter $\gamma$ is chosen so that steady-state hours worked account for 1/3 of the disposable time.

As in Kydland and Prescott (1982), assume it takes four quarters to complete an investment project, and each period 1/4 of the total resources are used. Thus $J = 4$
and $\phi_j = 1/4$ for $j = 1, 2, 3, 4$. The parameter $b$, measuring the strength of habit persistence in preferences, is set to 0.73 as in Boldrin, Christiano, and Fisher (2001).

Following Bernanke, Gertler and Gilchrist (1999), the steady-state ratio of capital to net value $K/NV$ is set to be 2, which indicates that half of firms' assets are borrowed; the steady-state annual external finance premium is set to be 4%, which approximates the historical average of the high yield spread. The firms' probability of surviving $\mu = 93.12\%$ can be derived from the evolution of firms' net value (3.27). The parameter $\eta$, the elasticity of external finance premium with respect to the capital/net value ratio is set to 0.05, which is also used in Bernanke, Gertler and Gilchrist (1999).

The parameter governing the persistence of technology shocks is set to be 0.964 as in Gomme and Rupert (2007). The time lag between the arrival of news on the productivity improvement and the realization of this news is assumed to be 4 quarters, that is, $p = 4$. In the absence of evidence to guide the choice of the standard deviation of news shocks, it is assumed that news shocks are perfectly informative so that all agents' expectations on future technology improvements are fully realized. In this case, the standard deviation of technology shocks is zero, and the standard deviation of news shocks is exactly equal to the standard deviation of realized technology innovations, which is estimated to be 0.0082 by Gomme and Rupert (2007). Table 3.1 summarizes the calibrated parameters.

### 3.5. Findings

#### 3.5.1. Impulse Responses

In this section, I am interested in studying two problems: first, whether the model can generate a boom of all macroeconomic variables after agents receive news about future
Table 3.1. Calibrated Values of Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibrated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>labor’s share of income</td>
<td>0.717</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>determinant of the elasticity of labor supply</td>
<td>2</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>leisure weight in the utility</td>
<td>1.128</td>
</tr>
<tr>
<td>$\phi_j$</td>
<td>percentage of resources used on projects each period</td>
<td>0.25</td>
</tr>
<tr>
<td>$b$</td>
<td>strength of habit persistence</td>
<td>0.73</td>
</tr>
<tr>
<td>$\mu$</td>
<td>firms’ probability of surviving</td>
<td>0.9312</td>
</tr>
<tr>
<td>$\eta$</td>
<td>elasticity of external finance premium w.r.t capital/net value ratio</td>
<td>0.05</td>
</tr>
<tr>
<td>$\rho$</td>
<td>persistence of technology shock</td>
<td>0.964</td>
</tr>
<tr>
<td>$\sigma_{\psi}$</td>
<td>standard deviation of news shock</td>
<td>0.0082</td>
</tr>
<tr>
<td>$\sigma_{\xi}$</td>
<td>standard deviation of technology shock</td>
<td>0.00</td>
</tr>
</tbody>
</table>
technology improvement; and second, whether the model can explain the countercyclical movement and the lead-lag pattern of the external finance premium. I explore the impulse responses of selected variables to news shocks in models with different combinations of model features. Assume that all agents in the economy receive news at period 1 indicating a future technology improvement at period 5. At period 5, the expected technology improvement is realized.

First, I study the scenario in which firms can produce capital by themselves. Figure 3.1 plots the responses of selected variables to a one standard deviation innovation to the news shock in the model with and without the financial accelerator. In the presence of financial market frictions (as shown in lines with stars), macroeconomic variables such as consumption, investment, output, capital prices and net values all experience a boom after agents receive a signal indicating future improvement of technology; in the meantime, the external finance premium drops.

Beaudry and Portier (2004) demonstrated that it is difficult to generate a boom in both consumption and investment in classical RBC models when agents receive news about future technology improvement. The baseline model in the current work succeeds in generating the boom by adding financial market frictions, time-to-build and habit formation into an otherwise standard RBC model. It is important to understand the contribution of each of these three features. First, the introduction of time-to-build generates a boom in firms' net values by boosting the capital price when a news shock hits the economy. Intuitively, agents' expectations of future technology improvement raise demand for the new capital. Since it takes time to build new capital, the existing capital becomes more valuable. The underlying mechanism is as follows: according to equation (3.27), the expression determining firms' net value, the net value will increase with the widening of the wedge between $r^c_t$, the realized capital return and $E_{t-1}(r^h_t)$, the external financing cost. Further, based on equation
(3.25), the expression determining the expected return on capital, the wedge between $R_t^k$ and $E_{t-1}^k(R_t^k)$ will widen when the capital price rises unexpectedly. According to equation (3.26), the expression governing the capital price, the capital price is determined by the expected real interest rates with different terms. Figure 3.1 shows that the real interest rate experiences a hike every four periods. An increase of real interest rates raises the cost of producing new capital, therefore boosting the price of existing capital. As shown in figure 3.1, the capital price shoots up when the news shock hits the economy, then it falls gradually. This pattern will repeat every four
Figure 3.2. Responses of selected variables to news shocks in the model in which firms can produce capital. The parameter governing the strength of habit persistence, b, is set to zero.

periods. When the capital price increases, the firm's net value will be boosted; when the capital price falls, the net value also decreases. According to equation (3.21), the expression determining the external finance premium, an increase in firms' net value helps lower the external finance premium, which boosts the investment.

The hikes in real interest rate deserve more discussion. As shown in figure 3.2, when habit formation is removed from the model, consumption experiences a jump every four periods after a news shock hits the economy. According to first order conditions determining households' optimal behaviors (3.13) and (3.15), every jump in consumption leads to a hike in the real interest rate. Further study of figure 3.2 shows
that the periodical jump in consumption is caused by the periodical start of the new capital project. Since starting a project requires the investment of resources at the next three periods, firms that already have projects under construction do not have extra resources to start another new project. This explains why hikes emerge in new capital projects every four periods. Furthermore, the periodical start of new capital projects leads to periodical completion of capital projects and periodical increase in capital stock. Every time firms finish their ongoing projects, they will adjust their new capital startups. It is at this time point that consumption and investment jump to new levels.

Figure 3.1 also demonstrates an upward trend in the firms' net value, besides the fluctuations caused by time-to-build. This can be explained by firms' accumulation of self-owned capital. As shown in equation (3.27), the production of self-owned capital can increase firms' productive capital, $K_t$, without increasing borrowing from financial intermediaries. Thus, firms' production of capital boosts their net value directly. The gradual increase in firms' net value leads to a gradual decrease in the external finance premium. The low external finance premium further boosts investment.

The presence of financial market frictions is the key factor in generating a boom in investment after agents expect a future technology improvement. As shown by the solid lines in Figure 3.1, the model without financial market frictions (set $\eta = 0$) fails to generate a boom in investment. In the model without financial market frictions, news about the future technology improvement instantaneously increases consumption and leisure through a wealth effect. Thus, hours worked decreases, as does output. The only way consumption can be increased while hours worked are decreased is by decreasing investment. By contrast, in the model with financial market frictions, firms increase investment since they have incentives to do so: they
can build up capital stocks now to lower the external finance premium in the future. When firms receive news indicating a future technology improvement, they expect a future rise in the external finance for capital purchases; they could lower their future external finance costs by building up their own capitals and net values now since the external finance premium depends inversely on the net value. By building up their own capital, they rely less on the purchase of the existing capital and avoid paying relatively high external finance costs. Thus, they can accumulate their net value more quickly. Furthermore, a rise in firms' net value lowers the external finance premium it has to pay and further boosts the investment.

The purpose of adding habit formation into the model is to induce an increase in consumption after a news shock. As shown in Figure 3.3 which displays the responses of selected variables to a one standard deviation innovation to the news shock in the model with and without habit formations. Without habit formation in preferences, households are willing to lower their current consumption and therefore more resources can be used in investment without a large increase in the hours worked and outputs. By contrast, introduction of habit formation increases households' desire to smooth consumption over time. Since households expect an increase of consumption in the future due to the anticipated technology improvement, they would rather start to increase their consumption gradually once they receive the news indicating the future technology improvement. Since there are incentives to increase both the consumption and the investment, output has to increase, as does the hours worked.

In the baseline model, firms have two channels to boost their net values: capital appreciation and new capital production. Next, I study the alternative model in which firms are not capable of producing new capital, and only purchase capital in the market. Figure 3.4 plots the responses of selected variables to a one standard deviation innovation to the news shock in the model where firms cannot build capital using their
The solid lines: without habit formation, $b = 0$; the lines with stars: with habit formation, $b = 0.73$.

Figure 3.3. Responses of selected variables to news shocks in the model in which firms can produce capital.

outputs. The solid lines correspond to the baseline parameters, where $\eta$, the elasticity of external finance premium with respect to the capital/net value ratio, is set to 0.05. Notice that the model fails to generate a boom in investment when agents receive news about the future technology improvement. Without the capability of producing capital using their own outputs, firms' net value can only be raised through the capital appreciation. Although a time-to-build technology constraint does generate a short period of boom in capital price, it only has a moderate effect on the net value, so the effect on the external finance premium is small. Only when a much greater elasticity
Figure 3.4. Responses of selected variables to news shocks in the model in which firms can only purchase capital in the market.

The solid lines: \( \eta = 0.05 \); the lines with stars: \( \eta = 0.25 \).

of external finance premium with respect to the capital/net value ratio is assumed, can the model generate a low enough external finance premium and provide enough incentives for firms to increase capital purchases. The lines with stars depict the responses in the model where \( \eta \), the elasticity of external finance premium with respect to the capital/net value ratio, is set to 0.25. Firms have incentives to increase capital purchases before the realization of the expected technology improvement, because they expect that an increase in the capital purchase will cause a boom in capital price due to the time-to-build constraint; a boom in the capital price will further
boost firms' net values, and therefore lower the external finance premium. The drop in the external finance premium will in turn stimulate the investment.

It is natural to raise the question: can a model without the time-to-build technology constraint still generate a boom in all macroeconomic variables, especially the investment, when agents receive news indicating future technology improvement? Figure 3.5 plots the responses of selected variables to a one standard deviation innovation to the news shock in the model where the time-to-build technology constraint is removed. Without the time-to-build constraint, the capital price remains constant.
Firms can only boost their net values through building their own capital. As shown by the solid lines in figure 3.5, the model without time-to-build fails to generate the boom in investment after agents expect a future technology improvement. Only when the elasticity of the external finance premium with respect to the capital/net value ratio is greater than 0.08, can the model generate a boom in investment. The results in figure 3.4 and 3.5 demonstrate that both firms' capability of building their own capital and time-to-build contribute to the baseline model's success in generating a boom in investment.

3.5.2. Business Cycle Moments

Tables 3.2-3.4 display the business cycle moments for U.S. economy, the baseline model and the alternative model respectively. The detailed description of data can be found in Appendix A.4. All data have been detrended by taking logarithms and Hodrick-Prescott filtering. The simulated moments are averages over 1000 replications of 120 observations (the same number of observations as in U.S. data).

The baseline model's prediction for the volatility of output and the external finance premium is virtually the same as that seen in data. The volatility of consumption is less than that of output; in fact, the model predicts that consumption is too smooth relative to the data. The volatility of investment is greater than that of output, although the model predicts too much volatility in investment. The standard deviation of hours worked is about two thirds of that seen in the data. The baseline model matches the strong positive contemporaneous correlations between output and investment, and between output and hours worked. Furthermore, the model predicts that investment and hours worked are coincident with the cycle. The model predicts a slightly negative contemporaneous correlation between output and external finance.
premium, which is consistent with that seen in the data. However, the model's predictions are at odds with the data in the following two areas: first, the model predicts that consumption lags output, while they move coincidently in that data. Second, the model predicts that the external finance premium lags the output, while the data indicate that the premium leads the output. This result is consistent with the impulse responses to news shock in the baseline model as shown in figure 3.1: when agents receive news indicating future technology improvement, output rises smoothly; four quarters after the news shock, the realization of the technology improvement boosts output to a peak. Meanwhile, the external finance premium spikes down periodically, which can be explained by the periodical completion of new capital projects. In the baseline model where firms are assumed to be capable of producing new capital, firms' net values are boosted each time their new capital projects are completed, which in turn leads to drops in the external finance premium. Notice that the realized technology improvement at period 5 generates a boom in new capital projects, and these new capital projects will be completed at period 9, which leads to a slump of the external finance premium at that moment. This explains why the external finance premium lags output by four quarters in the baseline model.

Table 4 exhibits the business cycle moments for the alternative model in which firms are not capable of producing new capital. The elasticity of external finance premium with respect to the capital/net value ratio, \( \eta \), is set to 0.25. Compared with the baseline model, the alternative model performs better in matching the volatilities of investment and consumption, but worse in matching the volatilities in output, hours and external finance premium. The alternative model improves a little in matching the coincident movement of consumption: the alternative model predicts that consumption lags the cycle by two quarters, while the baseline model predicts that consumption lags the cycle by five quarters. The most prominent improvement of the
Table 3.2. Business Cycle Moments: U.S. Economy

<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
<th>Cross Correlation of GDP with</th>
<th>(x_{t-6})</th>
<th>(x_{t-5})</th>
<th>(x_{t-4})</th>
<th>(x_{t-3})</th>
<th>(x_{t-2})</th>
<th>(x_{t-1})</th>
<th>(x_t)</th>
<th>(x_{t+1})</th>
<th>(x_{t+2})</th>
<th>(x_{t+3})</th>
<th>(x_{t+4})</th>
<th>(x_{t+5})</th>
<th>(x_{t+6})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross domestic product</td>
<td>1.43</td>
<td>-0.14</td>
<td>0.00</td>
<td>0.21</td>
<td>0.40</td>
<td>0.64</td>
<td>0.81</td>
<td>1.00</td>
<td>0.81</td>
<td>0.64</td>
<td>0.40</td>
<td>0.21</td>
<td>0.00</td>
<td>-0.14</td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.71</td>
<td>-0.24</td>
<td>-0.18</td>
<td>-0.12</td>
<td>0.02</td>
<td>0.25</td>
<td>0.43</td>
<td>0.68</td>
<td>0.64</td>
<td>0.62</td>
<td>0.49</td>
<td>0.35</td>
<td>0.15</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>4.61</td>
<td>-0.08</td>
<td>0.07</td>
<td>0.31</td>
<td>0.49</td>
<td>0.68</td>
<td>0.81</td>
<td>0.93</td>
<td>0.70</td>
<td>0.50</td>
<td>0.27</td>
<td>0.10</td>
<td>-0.06</td>
<td>-0.20</td>
<td></td>
</tr>
<tr>
<td>Hours worked</td>
<td>1.66</td>
<td>-0.29</td>
<td>-0.13</td>
<td>0.07</td>
<td>0.27</td>
<td>0.47</td>
<td>0.63</td>
<td>0.80</td>
<td>0.85</td>
<td>0.81</td>
<td>0.70</td>
<td>0.52</td>
<td>0.34</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>External finance premium</td>
<td>0.25</td>
<td>-0.44</td>
<td>-0.54</td>
<td>-0.58</td>
<td>-0.60</td>
<td>-0.56</td>
<td>-0.45</td>
<td>-0.21</td>
<td>0.04</td>
<td>0.29</td>
<td>0.43</td>
<td>0.59</td>
<td>0.65</td>
<td>0.67</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3. Business Cycle Moments: Baseline Model

<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
<th>Cross Correlation of GDP with</th>
<th>(x_{t-6})</th>
<th>(x_{t-5})</th>
<th>(x_{t-4})</th>
<th>(x_{t-3})</th>
<th>(x_{t-2})</th>
<th>(x_{t-1})</th>
<th>(x_t)</th>
<th>(x_{t+1})</th>
<th>(x_{t+2})</th>
<th>(x_{t+3})</th>
<th>(x_{t+4})</th>
<th>(x_{t+5})</th>
<th>(x_{t+6})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross domestic product</td>
<td>1.40</td>
<td>-0.09</td>
<td>0.03</td>
<td>0.17</td>
<td>0.40</td>
<td>0.58</td>
<td>0.80</td>
<td>1.00</td>
<td>0.80</td>
<td>0.58</td>
<td>0.40</td>
<td>0.17</td>
<td>0.03</td>
<td>-0.09</td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.35</td>
<td>-0.51</td>
<td>-0.46</td>
<td>-0.36</td>
<td>-0.25</td>
<td>-0.12</td>
<td>0.04</td>
<td>0.20</td>
<td>0.29</td>
<td>0.36</td>
<td>0.42</td>
<td>0.48</td>
<td>0.49</td>
<td>0.48</td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>7.56</td>
<td>0.02</td>
<td>0.12</td>
<td>0.26</td>
<td>0.46</td>
<td>0.65</td>
<td>0.83</td>
<td>0.98</td>
<td>0.77</td>
<td>0.55</td>
<td>0.33</td>
<td>0.10</td>
<td>-0.05</td>
<td>-0.17</td>
<td></td>
</tr>
<tr>
<td>Hours worked</td>
<td>0.97</td>
<td>0.06</td>
<td>0.18</td>
<td>0.36</td>
<td>0.57</td>
<td>0.71</td>
<td>0.63</td>
<td>0.80</td>
<td>0.71</td>
<td>0.51</td>
<td>0.27</td>
<td>0.00</td>
<td>-0.10</td>
<td>-0.20</td>
<td></td>
</tr>
<tr>
<td>External finance premium</td>
<td>0.23</td>
<td>0.00</td>
<td>-0.04</td>
<td>-0.08</td>
<td>-0.10</td>
<td>-0.14</td>
<td>-0.17</td>
<td>-0.20</td>
<td>-0.27</td>
<td>-0.32</td>
<td>-0.37</td>
<td>-0.40</td>
<td>-0.38</td>
<td>-0.36</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4. Business Cycle Moments: Alternative Model

<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
<th>Cross Correlation of GDP with</th>
<th>(x_{t-6})</th>
<th>(x_{t-5})</th>
<th>(x_{t-4})</th>
<th>(x_{t-3})</th>
<th>(x_{t-2})</th>
<th>(x_{t-1})</th>
<th>(x_t)</th>
<th>(x_{t+1})</th>
<th>(x_{t+2})</th>
<th>(x_{t+3})</th>
<th>(x_{t+4})</th>
<th>(x_{t+5})</th>
<th>(x_{t+6})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross domestic product</td>
<td>1.20</td>
<td>-0.14</td>
<td>-0.01</td>
<td>0.16</td>
<td>0.40</td>
<td>0.61</td>
<td>0.82</td>
<td>1.00</td>
<td>0.82</td>
<td>0.61</td>
<td>0.40</td>
<td>0.16</td>
<td>-0.01</td>
<td>-0.14</td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.47</td>
<td>-0.52</td>
<td>-0.49</td>
<td>-0.42</td>
<td>-0.25</td>
<td>0.02</td>
<td>0.26</td>
<td>0.53</td>
<td>0.65</td>
<td>0.78</td>
<td>0.75</td>
<td>0.72</td>
<td>0.67</td>
<td>0.59</td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>5.67</td>
<td>0.02</td>
<td>0.16</td>
<td>0.33</td>
<td>0.53</td>
<td>0.70</td>
<td>0.84</td>
<td>0.95</td>
<td>0.70</td>
<td>0.43</td>
<td>0.16</td>
<td>-0.10</td>
<td>-0.26</td>
<td>-0.38</td>
<td></td>
</tr>
<tr>
<td>Hours worked</td>
<td>0.79</td>
<td>0.18</td>
<td>0.38</td>
<td>0.66</td>
<td>0.72</td>
<td>0.73</td>
<td>0.69</td>
<td>0.56</td>
<td>0.35</td>
<td>0.13</td>
<td>-0.12</td>
<td>-0.40</td>
<td>-0.43</td>
<td>-0.45</td>
<td></td>
</tr>
<tr>
<td>External finance premium</td>
<td>0.32</td>
<td>-0.49</td>
<td>-0.65</td>
<td>-0.78</td>
<td>-0.68</td>
<td>-0.57</td>
<td>-0.45</td>
<td>-0.26</td>
<td>-0.07</td>
<td>0.11</td>
<td>0.28</td>
<td>0.35</td>
<td>0.40</td>
<td>0.42</td>
<td></td>
</tr>
</tbody>
</table>
The alternative model predicts that the external finance premium leads the output by four quarters, which is very close to the three quarters leading periods observed in the data. This result is consistent with the impulse responses to news shock in the alternative model as shown in figure 3.4: the external finance premium slumps immediately after the news shock hits the economy, and moves back to the steady-state gradually. In the meantime, output reaches a peak four quarters later when the expected technology improvement materializes. In the alternative model where firms cannot produce new capital, capital appreciation is the only channel through which firms increase their net values and in turn lower their external finance premium. Since the boom in capital price is caused by the temporary scarcity of the existing capital due to time-to-build, the effect on capital price is the strongest when the news shock hits the economy. This explains why the external finance premium slumps immediately after the news shock hits the economy.

3.6. Conclusion

This chapter highlights the role that financial market frictions play in the propagation of news shocks. It is difficult to generate a boom driven by agents' expectation of future technology improvement in a standard real business cycle model. Beaudry and Portier (2004) succeed in generating the positive comovements after a news shock in a two-sector model with durable and non-durable goods. The key assumption they make is that there exists a strong complementarity between the non-durable goods sector and the durable goods sector. By adding financial market frictions, habit formation and time-to-build into an otherwise standard RBC model, this chapter succeeds in generating a boom when a news shock hits the economy. Due to the presence of financial market frictions, firms' external finance cost depends inversely
on their net values. Firms' net values can be boosted through either building up new capital or through appreciation of existing capital. With time-to-build, capital price increases when an optimistic expectation drives the demand for existing capital; firms also have an incentive to build up their capital stocks now to lower the external finance premium in the future. An increase in firms' net values lowers their external finance premium and in turn boosts the investment. The purpose of adding habit formation into the model is to induce an increase in consumption after a news shock.

The data shows that the external finance premium leads output by three quarters. This lead-lag pattern can only be generated in the model where firms are not capable of producing new capital. When a news shock hits the economy, agents' expectations of future technology improvement raise demand for new capital. The existing capital becomes more valuable due to the fact that it takes time to build new capital. Since the boom in capital price is caused by the temporary scarcity of the existing capital due to time-to-build, the effect on capital price is the strongest at the moment when the news shock hits the economy. A boom in capital price in turn boosts firms' net values and lowers the external finance premium. Therefore, the slump of the external finance premium leads the boom in output. By contrast, when firms are capable of producing new capital, their net values will be boosted periodically by the completion of new capital projects. Since the bulk of the new capital projects will be completed after the realization of technology improvement, the peak of firms' net values lags output, therefore the external finance premium also lags output.

The analysis in this chapter shows that the effect of financial market frictions depends on whether or not firms have the capability to produce new capital using their own outputs, and on the elasticity of the external finance premium with respect to the capital/net value ratio. When firms are capable of producing new capital, they can accumulate net values much faster so that a small elasticity of the external finance
premium with respect to the capital/net value ratio is enough to generate a boom. By contrast, when firms cannot produce new capital by themselves, they rely on capital appreciation to raise net values. In this case, their net values increase only moderately, so that a much greater elasticity of the external finance premium with respect to the capital/net value ratio is needed to generate a boom. This observation arouses interest in conducting further empirical research on the elasticity of the external finance premium with respect to the capital/net value ratio.
Concluding Remarks

This thesis studies three questions related to business cycles driven by individuals’ expectation of future technology changes.

The first question is: what is the central bank’s optimal reaction to news shocks? What should the central bank do when individuals’ expectations of future technology changes are causing booms and busts of the economy? According to my research in Chapter 1, the central bank should employ a policy rule reacting to the inflation rates in both non-durable goods and durable goods sector with appropriate weights. This policy recommendation implies that the central bank does not need to know when individuals are forming expectations or whether individuals’ expectations will be materialized. The central bank only needs to be concerned about one thing: inflation rates.

The second question is: how important are news shocks in generating business cycle fluctuations? Based on the estimation in chapter 2, news shocks play a substantial role in accounting for aggregate fluctuations. The analysis suggests that news shocks account for about 34% of the fluctuations in the aggregate output, 25% of the fluctuations in consumption-sector output and 38% of the fluctuations in investment-sector output. This result justifies researchers’ increasing interests in studying news shocks.

The third question is: what is the role that financial market frictions play in generating expectation driven business cycles? Chapter 3 answers this question by exploring the booms and busts induced by news shocks in a model economy with financial market frictions. With the presence of financial market frictions, firms have
to pay an external finance premium which depends inversely on their net values. This provides firms with an incentive to build up capital stocks now to lower the external finance premium in the future. When firms receive news indicating a technology improvement in the future, they anticipate the need for more capital and so more external finance in the future; they could lower their future external finance costs by building up their capital and net values now. Thus, financial market frictions induce the boom in the investment before the realization of news about future technology improvement.

In conclusion, this thesis sheds light on several important questions related to business cycles driven by news shocks. As a possible important source of business cycle fluctuations, news shocks definitely deserve more future research.
References


Appendix

A.1 A Summary of Equations Describing the Model Equilibrium in Chapter 2

(1) Households' decisions

(A1) \[ x_{jt}^{1-n}(1 - \omega)C_{mt}^{-\frac{1}{n}} = \Lambda_t \]

(A2) \[ x_{jt}^{\frac{1}{n}}\omega C_{ht}^{\frac{1}{n}}\alpha N_{ht}^{\alpha - 1}K_{ht}^{1-\alpha} = \gamma(N_{mt} + N_{ht})^{\sigma} \]

(A3) \[ \Lambda_t W_t / P_{ct} = \gamma(N_{mt} + N_{ht})^{\sigma} \]

(A4) \[ \Lambda_t Q_{ht} = \beta E_t (x_{t+1}^{1-n}\omega C_{ht+1}^{\frac{1}{n}}(1 - \alpha)N_{ht+1}^{\alpha}K_{ht+1}^{1-\alpha} + (1 - \delta_h)\Lambda_{t+1}Q_{ht+1}) \]

(A5) \[ \Lambda_t Q_{xt} = \beta E_t (\Lambda_{t+1}R_{t+1}^x / P_{ct+1} + (1 - \delta_x)\Lambda_{t+1}Q_{xt+1}) \quad (x \in c, d) \]

(A6) \[ \Lambda_t = \beta E_t (R_t \frac{P_{ct}}{P_{ct+1}}\Lambda_{t+1}) \]

(2) Intermediate goods producers' decisions

(A7) \[ MC_{xt} = \frac{W_t}{\phi_x Y_{xt}/N_{xt}} = \frac{R_t^x}{(1 - \phi_x)Y_{xt}/K_{xt}} \quad (x \in c, d) \]

(A8) \[ P_{xt} = \left( \frac{\epsilon_x}{\epsilon_x - 1} \right) \frac{E_t \sum_{j=0}^{\infty} \theta_x^j \beta_x \Lambda_{t+j}MC_{xt+j}Y_{xt+j}P_{xt+j}^{x-1}}{E_t \sum_{j=0}^{\infty} \theta_x^j \beta_x \Lambda_{t+j}Y_{xt+j}P_{xt+j}^{x-1}} \quad (x \in c, d) \]

(A9) \[ P_{xt} = \left[ (1 - \theta_x)(P_{xt})^{1-\epsilon_x} + \theta_x(P_{x,t-1})^{1-\epsilon_x} \right]^{\frac{1}{1-\epsilon_x}} \quad (x \in c, d) \]

(3) Productions, Resource constraints and capital goods accumulations:
(A10) \[ Y_{xt} = A_{xt} N_{xt}^{\phi_x} K_{xt}^{1-\phi_x} \quad (x \in c, d) \]

(A11) \[ Y_{ct} = C_{mt} \]

(A12) \[ Y_{dt} = I_{ct} + I_{dt} + I_{ht} \]

(A13) \[ N_{mt} = N_{ct} + N_{dt} \]

(A14) \[ K_{xt+1} = (1 - \delta_x) K_{xt} + I_{xt} - \frac{X_x}{2} \left( \frac{I_{xt}}{K_{xt}} - \delta_x \right)^2 K_{xt} \quad (x \in c, d, h) \]

(4) Monetary policy rule:

(A15) \[ \ln(R_t) = \rho_R \ln(R_{t-1}) + (1 - \rho_R)(\ln(R) + \rho_y(\ln(Y_t) - \ln(Y)) + \rho_\pi \ln(\Pi_t)) \]

(5) Technology process:

(A16) \[ \ln(A_{ct}) = (1 - \rho_c) \ln(A_c) + \rho_c \ln(A_{ct-1}) + \psi_{t-p} + \xi_t \]

(A17) \[ \ln(A_{dt}) = (1 - \rho_d) \ln(A_d) + \rho_d \ln(A_{dt-1}) + \kappa_{t-p} + \zeta_t \]
A.2 Description of Data Used in Chapter 2

The following data can be obtained from Bureau of Economic Analysis (BEA):

Personal consumption of non-durable goods, durable goods and services, gross private domestic investment: NIPA Table 1.1.5.

Compensation of employees and proprietors' income by industry: NIPA Table 6.2D and 6.12D.

Input-output table: BEA-industry-the use of commodities by industries before redefinitions 2005.

Depreciation of market capital by industry: BEA-fixed assets-Table 3.4ES.

Depreciation of durable goods: BEA-fixed assets-Table 8.4.

Depreciation of residential capital owned by households: BEA-fixed assets-Table 5.4.

Investment in market capital by industry: BEA-fixed assets-Table 3.7ES.

Investment in durable goods: BEA-fixed assets-Table 8.7.

Investment in residential capital owned by households: BEA-fixed assets-Table 5.7.

Stock of market capital by industry: BEA-fixed assets-Table 3.1ES.

Stock of durable goods: BEA-fixed assets-Table 8.1.

Stock of residential capital owned by households: BEA-fixed assets-Table 5.1.

Implicit price deflators for non-durable goods, durable goods, services and gross private domestic investment: NIPA Table 1.1.9.

The following data can be obtained from Bureau of Labor Statistics (BLS):

Employees on nonfarm payrolls by industry: Employment, Hours, and Earnings from the Current Employment Statistics survey (National) Table B-1.

Weekly hours worked by industry: Employment, Hours, and Earnings from the Current Employment Statistics survey (National) Table B-2.
All data are seasonal adjusted. Monthly data are transformed to quarterly data by calculating the average of three months in a quarter.
A.3 A Summary of Equations Describing the Model Equilibrium in Chapter 3

(1) Households’ decisions

(A1) \[ \frac{1}{(C_t - bC_{t-1})} - E_t \left( \frac{b\beta}{C_{t+1} - bC_t} \right) = \lambda_t \]

(A2) \[ \lambda_t W_t = \gamma (1 - N_t)^{-\sigma} \]

(A3) \[ \lambda_t = \beta E_t (R_{t+1}\lambda_{t+1}) \]

(2) Firms’ decisions

(A4) \[ W_t = \alpha A_t N_t^{\alpha-1}K_t^{1-\alpha} \]

(A5) \[ E_t(R_{t+1}^k) = E_t \left[ \frac{(1 - \alpha)(Y_{t+1}/K_{t+1}) + Q_{t+1}(1 - \delta)}{Q_t} \right] \]

(A6) \[ E_t \left( \frac{R_{t+1}^kQ_{t+1}R_{t+1}^k}{\Pi_{s=t+1}^{t+1}R_s^k} \right) = \phi_j + E_t \left( \frac{\phi_{j-1}}{\Pi_{s=t+1}^{t+1}R_s^k} \right) + \ldots + E_t \left( \frac{\phi_1}{\Pi_{s=t+1}^{t+1}R_s^k} \right) \]

(A7) \[ E_t(R_{t+1}^k) = \left( \frac{Q_t K_t^m}{NV_{t+1}} \right)^{\eta} E_t(R_{t+1}) \]

(A8) \[ NV_{t+1} = \mu \left[ R_t^k Q_{t-1}K_t - E_{t-1}(R_t^k)(Q_{t-1}K_{t-1}^m - NV_t) \right] \]
(3) Productions, resource constraints and capital goods accumulations:

(A9) \[ Y_t = A_t N_t^\alpha K_t^{1-\alpha} \]

(A10) \[ Y_t = C_t + I_t \]

(A11) \[ K_{t+1} = K_t^{\theta} + s_{1,t} \]

(A12) \[ I_t = \sum_{j=1}^{J} \phi_j s_{j,t} \]

(A13) \[ s_{j-1,t+1} = s_{j,t} \quad j = 2, \ldots, J \]

(4) Technology process:

(A14) \[ \log(A_t) = (1 - \rho) \log(A) + \rho \log(A_{t-1}) + \psi_{t-p} + \xi_t \]
A.4 Description of Data Used in Chapter 3

The following U.S. data from 1976:1q to 2006:4q are extracted from DRI Basic Economics (Citibase). The names of the series used here correspond to the names in the database.

GCN: personal consumption of non-durable goods
GCS: personal consumption of service
GCD: personal consumption of durable goods
GPI: gross domestic private investment
GDPQ: gross domestic product
GDP: nominal gross domestic product
P16: population above 16 years old
LBMN: hours of all persons in private business sector

The macroeconomic variables defined in the chapter are connected to the above data as follows:

\[
\text{GDP deflator} = \frac{\text{GDP}}{\text{GDPQ}}
\]

Real per capita consumption \( C_t = \frac{(\text{GCN}+\text{GCS})}{\text{P16}} \times \frac{1}{\text{GDP deflator}} \)

Real per capita investment \( I_t = \frac{(\text{GCD}+\text{GPI})}{\text{P16}} \times \frac{1}{\text{GDP deflator}} \)

Real per capita output \( Y_t = \frac{\text{GDPQ}}{\text{P16}} \)

Per capita hours worked \( N_t = \frac{\text{LBMN}}{\text{P16}} \)

External finance premium is measured by high yield spread. The high yield spread is measured as the difference between the high yield bond rate and the corresponding rate for the highest quality bonds, where the high yield bond rate is measured by Merrill Lynch U.S. High Yield Master II Index and the rate for highest quality bonds is measured by Moody’s Seasoned Aaa Corporate Bond Yield.