Modular Hub Location Problems

Faezeh Sadat Mirzaghafour

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By: Faezeh Sadat Mirzaghafour

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Signed by the final examining committee:

Dr. Z. C. Chen Chair

Dr. M. Kazemi Zanjani Examiner

Dr. S. Chauhan Examiner

Dr. I. Contreras Supervisor

Approved by ________________
Chair of Department or Graduate Program Director

____
Dean of Faculty

Date February 14, 2013

Abstract

Hub location problems deal with the location of a set of hub facilities and the design of the network so as to provide the most cost-effective way to route a set of commodities through the network. In this thesis we present the *Modular Hub Location Problem* (MHLP). The MHLP differs from *classical hub location problems* in the way the economies of scale are modeled. The MHLP considers a step-wise cost function to model the flow dependency of transportation costs at the links of the network. We propose four variants of the MHLP: single allocation and multiple allocation versions with the assumption of having direct connections or not for each case. Computational experiments are performed on benchmark instances in order to evaluate the efficiency and limitations of the considered models.

Acknowledgement

I would like to take this opportunity to express my gratitude and regards to my supervisor Dr. Ivan Contreras for his exemplary guidance, monitoring and constant encouragement during this thesis. I really appreciate the time that he spent with me considering his busy schedule that made it possible to develop and complete this work.

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Chapter 1: Introduction

Hub-and-spoke networks play an important role in different industries such as air transportation, ground transportation, marine transshipment, postal delivery and telecommunication. They are also applicable in many-to-many distribution systems such as parcel delivery. In order to have both efficiency and effectiveness, these systems build their routes via hubs. Instead of using direct links among all pairs of origins and destinations, a hub network enables a better service by establishing hub facilities between origins and destinations and using smaller number of links. These networks make benefit of economies of scale attained through aggregation of flows at hubs. Figure 1, illustrates the difference between a network with direct links between origin/destination (O/D) nodes and a hub network with fewer links to connect O/D points. What exactly hub facilities do is to act as sorting and/or consolidation centers. As a sorting center, flows can be redirected at a hub. Though, it is the hub's mission to consolidate the flow. Hub Location Problems (HLPs) concern the location of hub facilities and the routing of commodities through the network. HLPs are known as a difficult group within discrete or continuous location problems.

Discrete location problems constitute a class of combinatorial optimization problems which most of them are known to be *NP*-hard. The main purpose in discrete location problems is to choose some facilities, from a discrete set of possible locations, in relation to some customers interacting with the facilities, in an optimal way with respect to some criteria such as transportation costs and service quality.

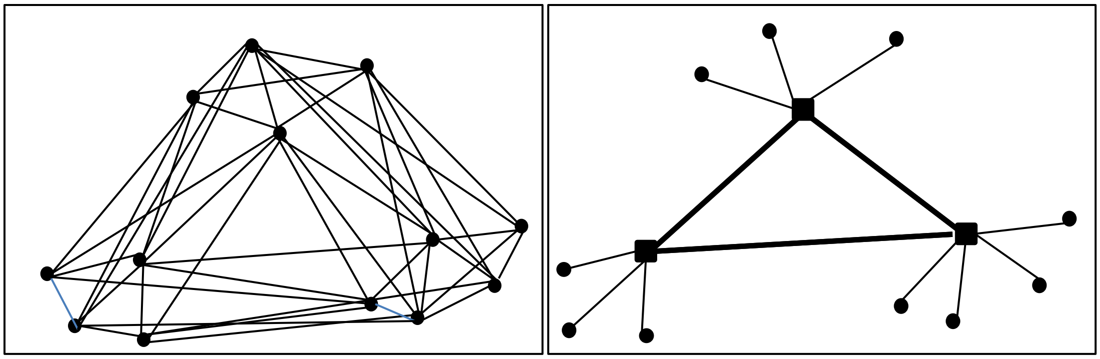

Figure 1 Network with direct links vs. Hub network

In the case of transportation systems, hub location problems have applications in postal operations, rapid transit systems, package delivery systems, air passenger travel and air freight travel. Commodities such as passengers, packages and goods correspond to demand which are traveling along the links by means of vehicles (i.e. trucks, trains, airplanes, etc.). Hubs, or transshipment points, help improve the performance of these systems by consolidating and re-routing the flow to their destinations. Generally speaking, HLPs consist of locating hubs facilities among a set of potential nodes in a network and allocating the non-hub points to those hub facilities so as to minimize the total flow cost.

For a comprehensive review of applications in transportation systems refer to Campbell et al. (2002) and Campbell (2005). As an example of an application of hub-and-spoke networks, consider the case of less than truck load (LTL) companies. In the absence of hub-and-spoke networks, when the structure is point-to-point deliveries, truck companies load trucks with commodities to answer a specific demand between a specific pair of origin and destination nodes. This means that for any O/D pair they need a single truck to satisfy the demand and thus, they require a considerable amount of trucks to provide such direct service. However, when LTL companies rely on hub-and-spoke networks, the shipments are routed via hub nodes and thus, economies of scale on transportation cost can be applied by consolidating the flow at break-bulk terminals (hubs). A similar situation arises in the case of airline services. Passengers with different destinations from one city are combined on a flight to a hub. Once there, they are regrouped with passengers arriving from other cities onto flights to other hubs or directly to common destinations. Airline firms take advantage of the economies of scale by allowing greater traffic volume at hubs. Yet another example of an application of hub-and-spoke networks appears in postal delivery, in which parcels and ordinary mails are brought together from different districts to be sorted at sorting facilities (hubs) and then distributed to their destinations.

HLPs are receiving increasing attention in the literature because of the variety of applications they have. It has been 25 years after the seminal work of O’Kelly (1986) in hub location. Before 1986, there existed some publications related to many-to-many distribution systems, but O’Kelly (1986) made this area a new field of research within facility location. Early reviews were put forward by Campbell (1994); O’Kelly and Miller (1994). Later, Klincewicz (1998) provided a survey on the location of hubs and the design of hub networks in telecommunication applications. Bryan and O’Kelly (1999) presented a review in air transportation companies. The recent paper by Campbell and O’Kelly (2012) provides an excellent survey on the literature of hub location research.

Several variants of HLPs have been proposed in the literature. They differ on a number of assumptions such as the way to select the number of hubs to be allocated, the way hubs are interconnected, the way non-hub nodes are allocated to hub nodes, and capacity constraints on the hubs, among others. However, most of these models share in common three particular assumptions. The first one is that it is possible to fully interconnect hubs (without any installation costs) with more effective, higher volume pathways that allow a discount factor α, $0 < \alpha < 1$ to be applied to the transportation cost of the flows between any pair of hubs. The second one is that this discount factor is assumed to be independent on the amount of flow that is actually send by an arc between a pair of hubs and it is the same for all inter-hub arcs. Finally, the third one is that all flows have to be consolidated by hubs. Thus, the paths between O/D pairs must include at least one hub node.

These classical HLPs have a series of attractive theoretical features, but the above assumptions in which they rely could lead to serious unrealistic results, in particular when dealing with transportation networks. For instance, the assumption of full interconnection between the set of hub nodes could very easily lead to solutions where inter-hub arcs send a much lower flow than non-inter-hub arcs, yet the transportation cost is only discounted on the inter-hub arcs. Also, it may happens that the amount of flow that is actually routed between inter-hub arcs is quite different, yet applying the same discount factor. These two assumptions of full interconnection and flow-independent costs not only miscalculates the overall transportation cost of the network, but more important, could also erroneously select the optimal set of hub nodes and the assignment of O/D nodes to hubs.

Several authors have pointed out these anomalies (see for instance, O'Kelly and Bryan, 1998; Kimms, 2006) and different hub location models that are able to capture more properly discounted costs. A flow-dependent hub location model that is able to capture flow economies of scale using a convex function was originally proposed in O'Kelly and

Bryan (1998) and later extended in Bryan (1998). A network design model using threshold-based discounting was proposed by Podnar et al. (2002). However, they focused on the design of the network rather than on the location of hub facilities.

There are few studies in the literature in which they relax the assumption of fully interconnection between hubs and focus on the location of discounted hub arcs in order to minimize the total transportation cost (see O'Kelly and Miller,1994; Campbell et al., 2004 a,b). Apart from hub arc location models, some papers have proposed different models that do not consider a complete network between hub nodes but rather, particular topological structures such as star-star networks (Labbe and Yaman, 2008), tree-star networks (Contreras et al., 2009, 2010), and cycle-star networks (Contreras et al, 2012).

Finally, the third assumption considering that all flows must be consolidated by hubs could also be unrealistic in some applications. As mentioned, hub facilities are generally used for consolidation and/or sorting proposes but, in some applications like freight transportation, hub nodes are used only for consolidation proposes. Therefore, both in terms of efficiency (low costs) and effectiveness (high levels of service) it could be that a direct connection between two particular non-hub nodes is the best option for routing their associated demand. There are a few models that have been proposed in the literature that explicitly consider direct interconnection between non-hub nodes (Aykin, 1994, 1995; Sung & Jin, 2001). However, it is worth mentioning that the incorporation of direct connections considerably increases the difficulty of HLPs.

The main goal of this thesis is to propose a new hub location model able to overcome the above mentioned disadvantages of classical hub location models. In particular, it does not assume either a fully interconnection between hub nodes or a particular topological structure. Instead, it considers the design of the entire hub network as part of the model. Also, it allows a direct connection between O/D points in order to construct a more effective and efficient network. Finally, and what is more important, it approaches the flow dependency problem of transportation costs by using modular costs on the links of the network. In this way, the overall transportation cost is not measured in terms of per unit cost and thus, we avoid the nonlinearities when dealing with flow dependent discounted costs.

The proposed problem, referred to as the *Modular Hub Location Problem* (MHLP), is especially suited for the design of freight transportation systems, in particular for large-scale trucking networks. Such networks provide freight transportation service between many origin and destinations. We study four different variants of the MHLP. One version restricts nodes to be allocated to a unique hub (single allocation pattern) while the other allows a node to interact with multiple hubs (multiple allocation pattern). Figure 2 depicts the differences of single and multiple allocation patterns. For these two models, we may allow to have direct connections or not. We propose integer programming formulations for the four variants of the MHLP. We perform a set of computational experiments to assess the performance of the proposed formulations when solved by using a general purpose solver (such as CPLEX). Moreover, we develop a comparative study on the topological structure of the networks obtained with the well-known uncapacitated hub location problem and the proposed one.

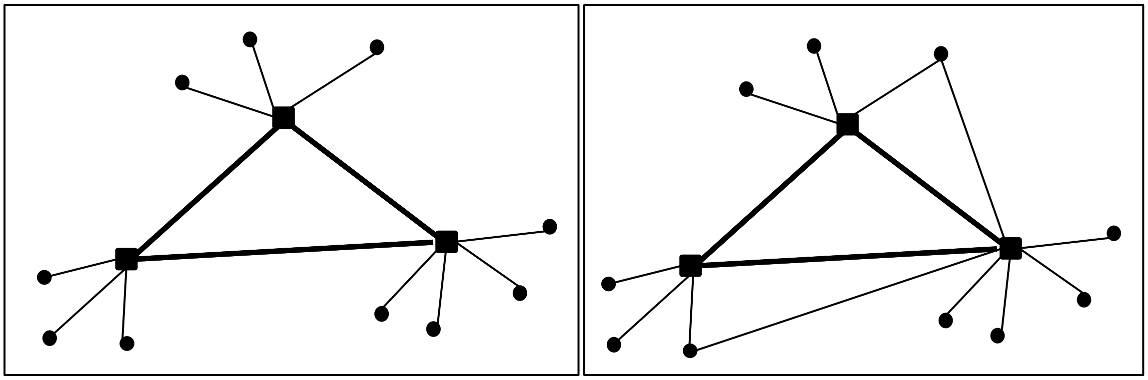

Figure 2 Single allocation vs. Multiple allocation

The remainder of the thesis is organized as follows. In chapter 2, we provide a review on hub location and freight transportation research. Chapter 3 gives the formal definition and mathematical programming formulation for the classical hub location problem and for the new proposed models. Chapter 4 presents the computational results. Finally, in chapter 5 we summarize our conclusions.

Chapter 2: Literature Review

Facility location is an important field of research within operations research. Hub location problems are an important class of facility location problems. Classical facility location problems and hub location problems share in common some characteristics but they also contain significant differences. On the one side, in hub location problems service demand is between pairs of users and the facilities are used as intermediate locations in the routes that connect pairs of users. Hub facilities act as sorting and consolidation centers and they have to be connected to each other in order to connect O/D pairs. On the other side, in classical facility location problems service is given to or from the facilities. It is thus not required to connect the facilities to each other; one just needs to allocate demand nodes to facilities. To better illustrate their differences, see Figure 3.

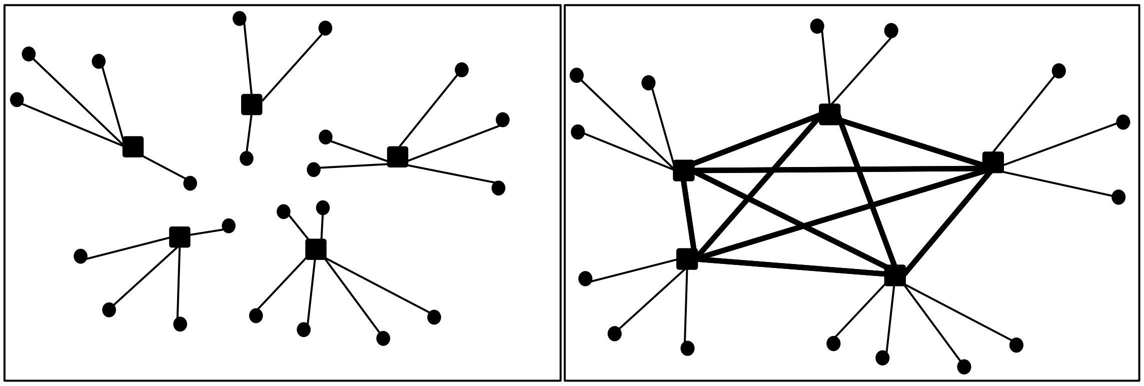

Figure 3 Facility Location Problem vs. Hub Location Problem

In this chapter we formally introduce hub location problems and present their taxonomy such as objective functions, network components, and constraints. Applications of HLPs in different area such as transportation and telecommunication are also discussed. In this thesis our main focus is on transportation applications and thus, the last section of this chapter is dedicated to introduce some relevant details related to trucking carriers.

2.1 Hub location

Hub-and-spoke networks are widely applied in a variety of applications such as transportation, postal delivery, urban traffic, express delivery service, distribution, and telecommunications. In order to reduce the number of connections in these networks, commodities are routed through consolidation centers called hubs before reaching their final destinations. This feature not only makes the network to have less links but more importantly, to concentrate flow along inter-hub connections. This helps reduce transportation costs by applying economies of scale between hubs. HLPs focus on the determination of the location of hub facilities and on the routing of flows through the network so as to minimize the total set-up and transportation cost.

2.1.1 Taxonomy of hub location problems

To solve more realistic problems, several works have extended different variants of the classical HLPs by analyzing single allocation models and multiple allocation ones, capacitated versus non-capacitated models, models in which direct connections among non-hub nodes in permitted, and formulations which better present discount factor.

Objective

Most HLPs have cost related objective functions. Models appearing in the design of transportation networks usually minimize the cost related to the transshipment of products while other models arising in the design of telecommunication networks mostly focus on the fixed cost of establishing hubs and links. Beside these models with cost related objectives, there are some other models that consider other objectives such as service level (or travel time) and congestion. Similar to facility location problems, HLPs can be classified into four categories according to their objective function,

1. p-hub median problems,
2. Hub location problems,
3. p-Hub center problems,
4. p-Hub covering problems.

1. *p-hub median problems* consider the location of exactly p hubs so as to minimize the total transportation cost for sending the commodities through the network. For these types of problems, single allocation and multiple allocation variants have been addressed in the literature. Campbell et al. (2002), Skorin-Kapov et al. (1997) and Ernst and Krishnamoorthy (1996) are some references dealing with single assignment patterns, whereas Campbell (1992,1996,2002), Ernst and Krishnamoorthy (1998), and Boland et al. (2004), deal with the p-hub median problem with multiple assignments. Cetiner & Sural (2010) discuss the design hub-and-spoke networks in postal delivery systems. They consider a combination of hubbing and routing problem. First, they determine the location of hubs. Then, they clarify the routes among the nodes. Yaman (2011) propose the *r*-allocation p-hub median problem in which each node can be connected to at most *r* hubs.

2. In *hub location problems*, the number of hubs is not known in advance and thus the locational decisions include the number and location of hubs. In order to determine the optimal number of hubs two types of costs have been incorporated; a fixed cost for opening hubs and a variable transportation cost. The interaction between these costs is interesting; while we see an increase in the fixed establishment cost, we will face a reduction in the flow costs. As the number of hubs increase, the routes become shorter

and thus, the total transportation cost is reduced. Some relevant references of these problems are Campbell (1994), Bonald et al. (2004), Alumur and Kara (2008b) and Contreras et al. (2011).

3. *p-Hub Center Problems* (p-HCP) can be classified as minimax problems (minimize the maximum distance or cost between any pair of nodes). The first reference in the hub literature dealing with hub center problem is by Campbell (1994). Campbell et al. (2002), Kara and Tansel (2000), and Ernst et al. (2002) introduced single and multiple assignment versions for p-HCPs. For more recent references, see Campbell, et al. (2007), and Hamacher and Meyer (2006).

4. In *hub covering problems*, demand is not covered unless the origin and destination points are within a specified distance of a hub node. Campbell (2002) proposed these models based on three coverage criteria. Hubs *k* and *m* cover an origin/destination pair (i,j) if:

- For any path from i to j using hubs k and m, the routing cost does not exceed a specified value.
- For any link in the path from i to j via hubs k and m, the cost is within a specified value.
- Each of collection and distribution links has different specified values.

For a comprehensive review of hub covering problems, the interested reader is referred to Campbell (1994). A more recent paper dealing with a hub covering model was proposed by Hwang and Lee (2012).

Network Components

A hub network consists of two types of nodes connected by one or more type of arcs. The two types of nodes are non-hub nodes and hub nodes.

Hub nodes: they are going to be chosen among a set of nodes, which are in the set of O/D nodes or in a different set of potential hubs. Their mission is to act as a switching or sorting center and centralization and supporting center. When hub nodes act as sorting centers, demands with the same origin but different destinations are consolidated and combined with other flows having the same destination, to be sent to their final points or to other hub nodes.

Non-hub nodes: As the name clearly states, any other node which is not a hub is a non-hub node. Usually, origins and destinations are non-hub nodes.

Demand: that is the flow routed through the hub network between any origin and destination pair of nodes. In general, the demand is prescribed as an input data for the model. In order to build more realistic models, some works (see, for instance, Marianov et al., 1999) have considered demand values to be competitive. Regarding the considered application, demand corresponds to different objects. In telecommunication applications, data transmissions are the demand. In other applications, such as air transportation, postal operations and trucking systems, the demand corresponds to commodities (or goods) that need to be routed through the network.

Arcs: these are the links that connect the nodes in the system. Each link has a dedicated transportation rate. Arcs are weighted by some discount factors denoted X, α and γ, to represent the collection, transfer and distribution costs per unit of flow, respectively. The discount factor α is used for the inter-hub links. Therefore, we can divide arcs into four groups (see Figure 4):

a. Arcs between two hub nodes (transfer links): these arcs usually have an associated discount factor α for the flows traveling via them (Campbell, 1996). In hub networks, the parameter α is defined as a number that varies between 0 and 1.

b. Arcs connecting a non-hub node, usually an origin, to a hub node (collection links).

c. Arcs connecting hub nodes to non-hub nodes, normally destination (distribution links).

d. Links between two non-hub nodes (dashed line): not all of the models have direct connections between non-hub nodes. Aykin (1994, 1995) allows such connections in their models.

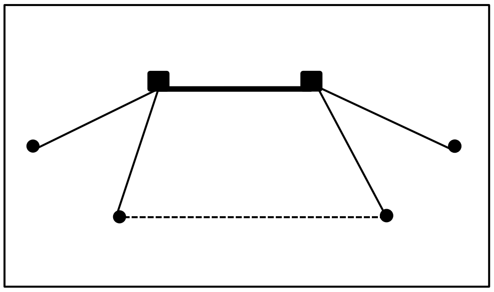

Figure 4 Different Division of Arcs

Network topology: Finally, the last component that we discuss is the topology of the network. According to the way the nodes are connected to each other, we can have different topologies. In most HLPs the hubs are connected via a complete graph (see Figure 5).

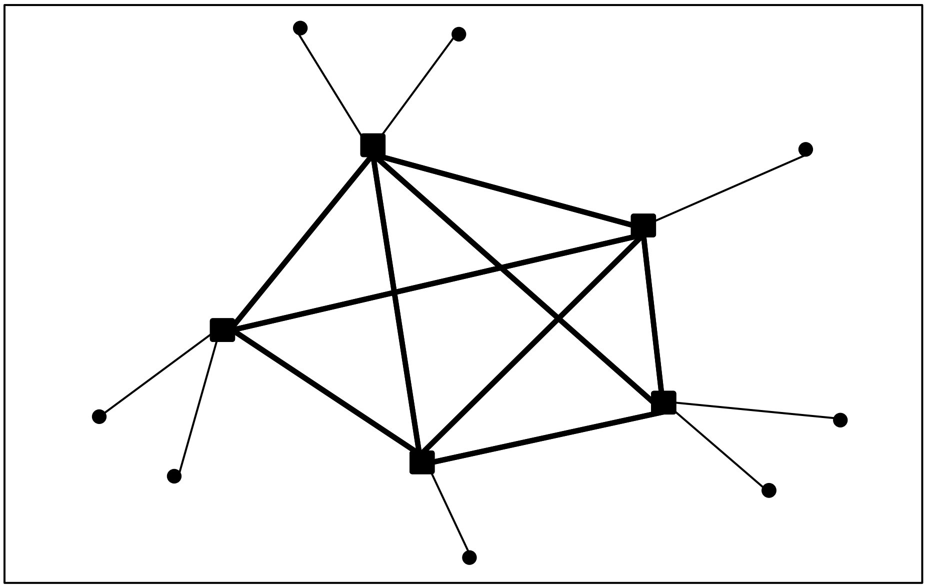

Figure 5 Complete Hub Network

Another topology that has been studied is the tree-star network (see Figure 6). In this case all hubs are connected by means of a tree. For each pair of node there must be a link to send the flow. It is a single-allocation hub location problem which means each node must be allocated to one single hub. An interested reader refers to Chou (1990) and Kim & Tcha (1992). For a more recent reference see Lee *et al.* (1996) and Contreras et al. (2009, 2010).

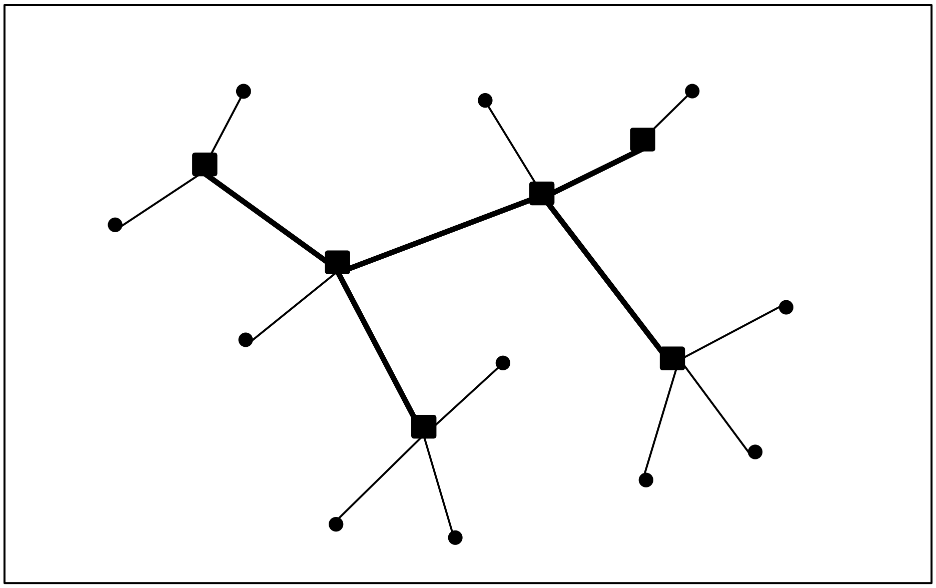

Figure 6 Tree-Star Network

The ring-star topology (see Figure 7) has also been studied, especially in telecommunication networks. Models that consider such topologies can be reviewd in Lee *et al.* (1993), Klincewicz (1998) and Contreras *et al.* (2012).

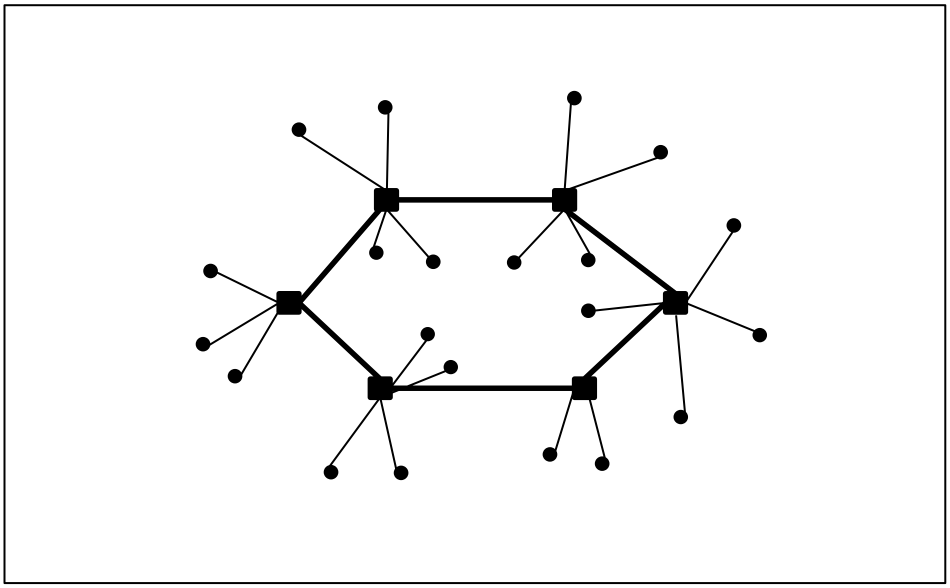

Figure 7 Ring-Star Network

Another topology commonly used in telecommunication networks is the star-star netwroks (see Figure 8). Labbe and Yaman (2008) and Yaman (2008) studied some HLPs considering this topology.

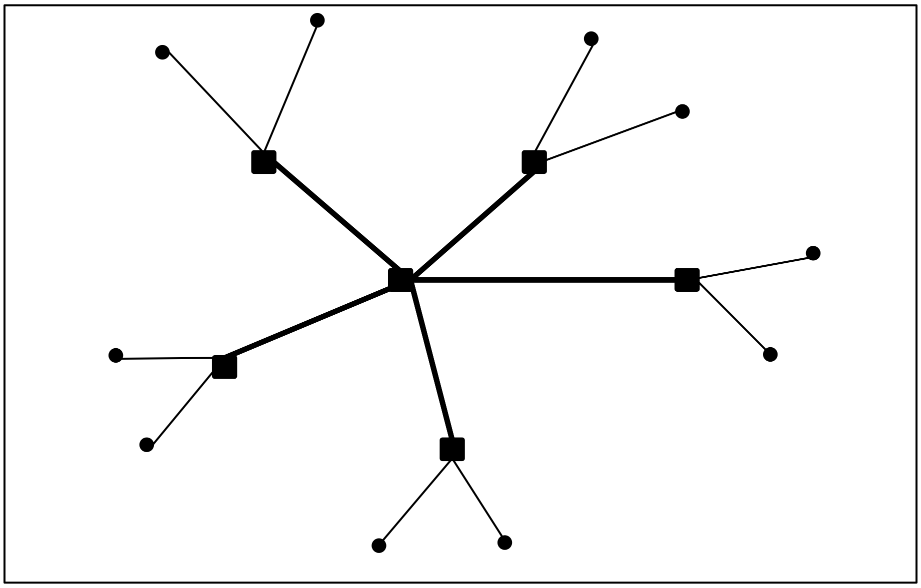

Figure 8 Star-Star Network

Incomplete hub networks (See Figure 9) without any particular topological structure have also been considered (see, for instance, Alumur et al., 2009 and Gelareh, 2008).

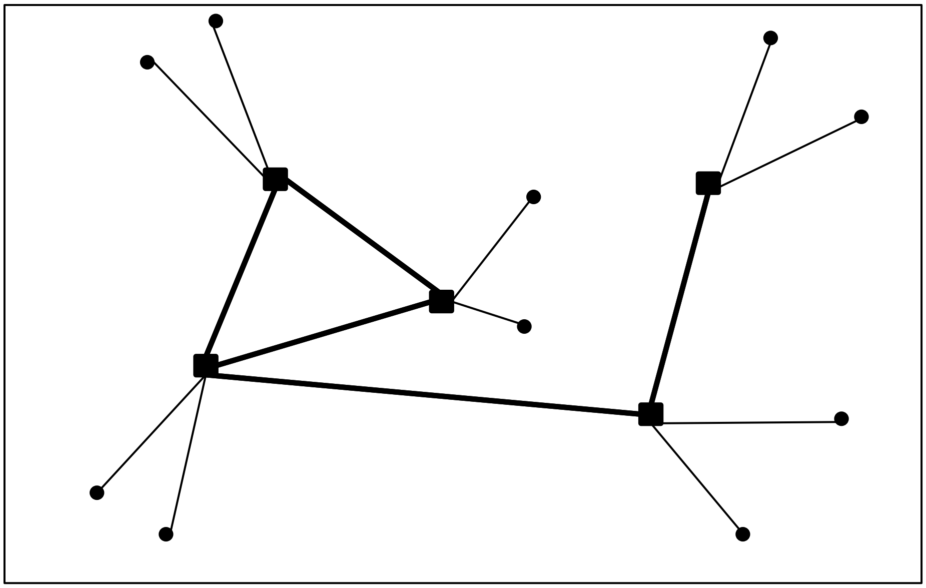

Figure 9 Incomplete Hub Network

Constraints

- *Capacity on nodes*: Imagine a maritime transportation company in which the docks are the hubs that ships come in to load and unload their commodity. The docks have a limited space for a number of ships to side and transfer their products. Capacity on the nodes has applications in airline and postal carriers as well. In airline systems, airports have restrictions for the number of passengers arriving in and departing and also the number of aircrafts using the facility. For postal carriers, capacity can be defined as the maximum number of ordinary mails that can be sorted in a sorting facility at each turn.

- *Capacity on arcs*: The capacity of an arc represents an upper bound on the total flow that can be passed on the arc. Bryan (1998) found out the correlation between inter-hub arc's capacities and the piecewise linear cost function proposed by O'Kelly and Bryan (1998a) for a prescribed number of hub nodes. Recently, Yaman (2008) has considered modular arc capacities in which there exist several capacity levels for the arcs.

- *Performance constraints*: In order to control the flow in the system, we could include performance constraints. Mostly, they put limitations on particular components of the network to be able to handle the traffic and congestion (Elhedhli and Wu, 2010). Camargo et al. (2011) uses a Benders decomposition algorithm to solve a single allocation hub location problem, while considering performance constraints to check the results under congestion. Marianov and Serra (2000) model transportation networks with these constraints. For more details about the applications of these constraints in telecommunication networks, see Klincewicz (1998). Telecommunications network design is the area in which these constraints, such as limitation on queue length or delays, are commonly used.

2.1.2 Applications of hub networks

As mentioned, hub networks are used in different telecommunication and transportation industries to reduce their transportation cost and enhance their frequency of service. These areas are somehow similar with small differences in their components. In transportation networks the product that flows on the links are tangible while in telecommunication networks the main products that are being moved are data packages. Another important difference between these two is the way they dedicate their capital. In telecommunication networks the main concern is on the expenses of building the network while in transportation networks the focus is on the cost of distribution.

In what follows, a brief literature review on different areas of application is presented:

- *Air transportation*: passengers or commodities from a unique city are combined on a flight to a hub. At the hub, they are regrouped, regarding their destinations, with other commodities or passengers. The greater the traffic volume at hubs, the better taking advantage of the economy of scale. We can divide air transportation into two categories: passenger transportation and freight transportation. Several authors have worked on passenger airlines. Bryan and O'Kelly (1999) provide an overview on this topic. O'Kelly (1998b) shows diversity between these two categories of freight and passenger networks. Regarding the number of stops (or transshipments) that are going to be used in the system, three types of assumptions are available. O/D paths with one, two, and any number of stops have been considered (Drenzer and Drenzer, 2001; Sasaki et al., 1997; Sasaki, et al., 1999; Jaillet et al., 1996). In the case of airline networks it would be interesting for airline companies to compete with each other by their ticket prices. Skorin-Kapov.D (1998) and Marianov et al. (1999) give some hints regarding the topic. Another issue for airline networks is using large aircrafts in order to have discount for large number of passengers. These aircrafts can place on any links or arcs on the network (see Campbell et al., 2004a; Campbell et al., 2004b; Jaillet et al., 1996).

- *Rapid transit*: Given that it is unrealistic to assume a fully interconnected network in public transportation systems, more flexible networks need to be devised. HLPs that include additional decision variables for the location of hub-arcs have been proposed (Campbell et al., 2000a, 2000b; Marin, 2007; Labbe et al., 2004; Laporte and Rodrigez-Martin, 2007).

- *Postal networks*: in these networks post code districts are the nodes, flow represents mail volume and hub nodes act as sorting and consolidation centers. The mail must be routed via one or at most two hubs in order to be sent to its destination. For fundamental discussions about postal networks, the interested reader is referred to Ernst and Krishnamoorthy (1996, 1999).

- *Trucking*: trucking can be divided to truckload (TL) and less than truck load (LTL) carriers. LTL carriers consolidate many small shipments from many different shippers to make efficient vehicle loads and route shipments via a network of consolidation or break-bulk (or hub) terminals. Each terminal collects shipments from its local service region by using local delivery trucks. Then, shipments are sorted at the terminal and loaded into line-haul trucks (inter-hub links), which carry the shipments to terminals near their destinations. Finally, the freight is then transshipped from the line-haul truck to a local delivery truck for transport to the final destination. For a review of motor carrier network design see Campbell (2005). Taha and Taylor (1994); Taha et al. (1996); Taylor et al. (1995); Taylor et al. (1999); Powel (1986); Powell and Sheffi (1983); Nagy and Salhi (1998) discuss other hub-and-spoke network models appearing in the trucking industry.

- *Telecommunication*: In telecommunication networks such as distributed computer processing, video conferences, and computer communication the concentration is mostly on the establishment cost rather than the flow cost of the networks. That is, the optimal solution is determined by minimizing the fixed costs of establishing the network. In these applications, the demand corresponds to data transmissions that are routed over a variety of physical media or through the air. Hub facilities correspond to electronic devices such as switches, gates, multiplexors, concentrators, etc. In this area a survey on the design of hub networks and the location of hubs is given by Klincewicz (1998). Chung et al. (1992) model telecommunication networks for large-scale data. Yoon et al. (1998) proposed more general network stucture for telecommunication networks. For more information an interested reader is referred to Hu (1974) and Gendron et al. (1999).

2.2 Transportation

As mentioned, transportation is one of the important fields in which hub location models are applied. Transportation includes air passenger travel, air freight travel, express shipments, large trucking systems, postal and rapid transit systems. Depending on the application, demand is defined as flows of travelers or commodities among origins and destinations. Considering available facts and figures, transportation accounts for approximately 10% of the US gross national product (GNP). For Canada, United Kingdom and France, transportation represents 16%, 15% and 9% of national expenditure, respectively (Crainic and Laporte, 1997).

Transportation can be divided into four categories:

- air transportation
- marine transportation
- rail transportation
- road transportation

In each of these classes, the components of hub and spoke are different. As an example, aviation uses its well-equipped airports as its hubs to consolidate its flow which are passengers who may change from one aircraft to another to arrive to their destination.

For many transportation hub networks, building the links is not the main concern as they are usually public infrastructures like highways, air space and the ocean. The focus is rather on the optimization of transportation costs. This leads to take into account the economies of scale, which are the savings that a company obtains from the consolidation of flows. That is, company's average cost per unit may fall down as the scale of output has a growth. As a long-term notion, *Economies of scale* means to reduce the cost in unit of products due to the increase in the size of a company and other levels of service (Campbell and O'Kelly, 2012).

In case of road transportation, *trucking* is the most relevant way of transportation. Generally, motor carriers in the United States account for 81% of the cargo bill ($372 billion per year in revenues), 60% of the freight volume (6.7 billion tons per year) and around 430 billion miles traveled per year. In a greater domain, within North America the statistics read 64% of the commodity trade value and 32% of merchandise trade weight. More significant, in the European Union 75% of inland freight in scale of ton-km use truck transport while for the total freight ton-km, it counts 44.5% (Campbell, 2005). Among the industries in UK economy, supply chain is the 5th largest sector and transportation is an important part of supply chain in which freight transportation dedicated 84% of it.

We can classify truck transportation into two main categories of:

- TL carriers
- LTL carriers

TL transport usually sends fully loaded trucks from origin to destination and occasionally they use terminals as load substitution to let drivers more frequently go back home. In *LTL* carriers the case is somehow different in a way that they use break-bulk (hub) terminals to consolidate and combine the commodity into efficient vehicle loads. This method is much more similar to postal carrier in which the letters and parcels must first send to some special offices (hubs), later according to their destination address they will classify and deliver. According to the concept of LTL carriers, companies try to fulfill as much commodity as they can on their trucks to run efficiency to their systems. LTL carriage is characterized as multiple shipments combined into a single truck for multiple deliveries within a multi-user network. This is the place where we can use the definition of 'synergism' that helps companies to work together to have better turnover than the sum of their individual outputs. Since efficiency and effectiveness are the two main goals among all the companies, they make effort bringing these goals to their companies by means of synergism and collaboration. Thus, they figure out it is more beneficial to cooperate with other companies in order to be able to compete in the market. To be able to visualize this in transportation, imagine that each company has its own trucks and facilities which having them means they should pay for buying them, afterwards they need spaces and lands to keep their trucks. Thus what most companies do is to send and receive their orders by getting help from LTL companies instead they just pay for each time of hauling and they do not need to buy the trucks themselves.

Chapter 3: Problem Definition and Formulations

Hub location problems have received increasing attention in the literature due to their diverse applications. HLPs are known to belong to the challenging class of NP-hard problems. The solution of HLPs requires a two-level decision process. The first-level decision deals with the selection of a set of nodes to locate the hub facilities whereas the second-level decision considers the design of the network, which is usually determined by the allocation pattern of the non-hub nodes to the hub nodes. One of the fundamental HLPs that have received more attention in the literature is the *Uncapacitated Hub Location Problem* (UHLP). For this reason, we will use the UHLP as a benchmark model to compare the topological structure of the hub networks obtained with this model and our proposed one.

In this chapter we discuss in detail the UHLP and our proposed model, referred to as the *Modular Hub Location Problem* (MHLP). In Section 3.1 the formal description and mathematical formulation of the UHLP is presented, both for the single assignment and multiple assignment versions. In Section 3.2 we introduce the MHLP and study four different variants of it. They differ on the way nodes are allocated to hubs (single and multiple allocation) and on the assumption of having direct connections among non-nodes or not.

3.1 The Uncapacitated Hub Location Problem

Consider the complete graph $G = (N, A)$, where $N = \{1, 2, \ldots, n\}$ is the set of nodes, which correspond to origins, destinations and potential hubs and A is the set of arcs. Let W_{ij} denote the amount of flow between nodes i and j. The fixed set-up cost for locating a hub at node i is denoted as f_i. Let d_{ij} denote the distances between nodes i and j which are assumed to be symmetric and to satisfy the triangular inequality. Distances represent the transportation cost per unit of flow on the arcs. It is assumed that hubs are fully interconnected at no cost with more effective, higher volume pathways, which allow a constant discount factor α $(0<\alpha<1)$ to be applied to inter-hub transportation costs. It is assumed that each route from each origin to destination must include at least one hub node. The capacity on the incoming and outgoing flow at the hubs and the amount of flow being transferred through the inter-hub arcs are assumed to be unlimited. The UHLP consists of selecting a set of nodes to locate the hub facilities and assigning the non-hub nodes to the hubs while minimizing the total set-up costs and transportation costs.

Given that hub nodes are assumed to be fully interconnected and transportation costs satisfy the triangle inequality, every path between an origin and a destination node will have at least one hub and at most two. Therefore, the transportation cost of routing the flow between O/D pairs for a particular path is given as follows. A path between an O/D pair is of the form i-j-k-m, where i and j, respectively, represent the origin and destination nodes; and k and m are the hubs to which i and j are allocated, respectively. Thus, the transportation cost for routing the flow W_{ij} along the path i-j-k-m, is given by

$$F_{ijkm} = W_{ij} (d_{ik} + \alpha d_{km} + d_{mj}). \tag{1}$$

We next present Mixed Integer Programming (MIP) formulations for two well-known variants of the HLPs. In section 3.1.1, we consider the HLP with multiple assignments, where it is assumed that each non-hub node can be allocated to more than one hub. In section 3.1.2, we consider the HLP with single assignments, in which it is assumed that each non-hub node has to be allocated to exactly one hub node.

3.1.1 The Uncapacitated Hub Location Problem with Multiple Assignments

Several MIP have been proposed for the *Uncapacitated Hub Location Problem with Multiple Assignments* (UHLPMA). We next present a strong path-based formulation proposed in Skorin-Kapov et al. (1997). One of the decisions in hub location is to determine the location of the hub facilities. Thus, for each $k \in N$, we define the following binary decision variables:

$$Z_k = \begin{cases} 1, & \text{if a hub facility is located at node } k \\ 0, & \text{otherwise} \end{cases}$$

The other decision is related to the routing of flows through the network. Therefore, for each O/D pair *(i,j)* and each pair of potential hub nodes *(k,m)*, we define the following binary decision variables:

$$x_{ijkm} = \begin{cases} 1, & \text{if the flow between nodes i and j transits via the first hub k and a second hub m} \\ 0, & \text{otherwise} \end{cases}$$

Using these two sets of decision variables, (Z_k, x_{ijkm}), the UHLPMA can be formulated as:

$$min \quad \sum_{k \in N} f_k\, z_k \;+\; \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{m \in N} F_{ijkm}\, x_{ijkm} \tag{2}$$

$$s.t. \quad \sum_{k\in N} \sum_{m\in N} x_{ijkm} = 1 \qquad \forall i,j \in N \quad (3)$$

$$\sum_{m\in N} x_{ijkm} \leq z_k \qquad \forall i,j,k \in N \quad (4)$$

$$\sum_{k\in N} x_{ijkm} \leq z_m \qquad \forall i,j,m \in N \quad (5)$$

$$z_k \in \{0,1\} \qquad \forall k \in N \quad (6)$$

$$x_{ijkm} \geq 0 \qquad \forall i,j,k,m \in N \quad (7)$$

The objective function minimizes the total cost of establishing the hubs and transportation costs. For each pair of node, constraint (3) guarantees that there is a unique path connecting them. By means of constraints (4) and (5), we prohibit the flow to be routed via a node which is not a hub. Finally, constraints (6) and (7) are the classical integrality and non-negativity constraints. Given that there are no capacity constraints on the hub nodes, there is no need to explicitly state the integrality on the x_{ijkm} variables because there always exists an optimal solution of (2)-(7) in which all x_{ijkm} variables are integer.

3.1.2 The Uncapacitated Hub Location Problem with Single Assignments

Several MIP have also been proposed for the *Uncapacitated Hub Location Problem with Single Assignments* (UHLPSA). We next present a strong path-based formulation proposed in Skorin-Kapov et al. (1997). In this single assignment variant of the HLP, each non-hub node has to be allocated to exactly one hub. For that reason, a new set of binary decision variables is introduced. For each pair $i, k \in N$, we have

$$z_{ik} = \begin{cases} 1, & \text{if node } i \text{ is assigned to hub } k \\ 0, & \text{otherwise} \end{cases}$$

Moreover, variable Z_{kk} represents the establishment or not of a hub at node k. The paths between O/D pairs can also be tracked with the X_{ijkm} variables defined above for the UHLPMA. Using these two sets of decision variables, (z_{ik}, X_{ijkm}), the UHLPSA can be formulated as:

$$min \quad \sum_{k\in N} f_k\, z_{kk} \;+\; \sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{m\in N} F_{ijkm}\, x_{ijkm} \qquad (8)$$

$$s.t. \quad \sum_{k\in N} z_{ik} = 1 \qquad \forall i \in N \qquad (9)$$

$$z_{ik} \leq z_{kk} \qquad \forall i,k \in N \qquad (10)$$

$$\sum_{m\in N} x_{ijkm} = z_{ik} \qquad \forall i,j,k \in N \qquad (11)$$

$$\sum_{k\in N} x_{ijkm} = z_{jm} \qquad \forall i,j,m \in N \qquad (12)$$

$$z_{kk} \in \{0,1\} \qquad \forall k \in N \qquad (13)$$

$$x_{ijkm} \geq 0 \qquad \forall i,j,k,m \in N \qquad (14)$$

Constraint (9) ensures that every non-hub node is allocated to one single hub, while constraint (10) guarantees that no flow is assigned to a node which is not a hub. Constraints (11) state that if node i is assigned to hub k then all the flow from node i to any other node j must go through some other hub m. A similar interpretation for constraint (12) states that the flow arriving to a node j assigned to hub m from some node i. Constraints (13) and (14) are the integrality and non-negativity constraints. Given that there are no capacity constraints on the hub nodes, there is no need to explicitly state the integrality on the x_{ijkm} variables because there always exists an optimal solution of (8)-(14) in which all x_{ijkm} variables are integer.

3.2 The Modular Hub Location Problem

This subsection describes in detail the *Modular Hub Location Problem* (MHLP). As mentioned in Chapter 1, the main goal of the MHLP is to overcome the major disadvantages of classical hub location models (such as the UHLPMA and UHLPSA). In particular, one of the key features of the MHLP is the way economies of scale between hub nodes are modeled. Rather than assuming constant discount factors on the transportation cost between hubs (flow-independent discounted costs), that may lead to the underestimation (or overestimation) of the total transportation cost of the hub network, we consider flow-dependent discounted costs by using modular costs on the links of the network. In this way, we do not only avoid the nonlinearities that usually appear in other hub location models when dealing with flow-dependent discounted costs (see O'Kelly and Bryan, 1998; Bryan, 1998), but most importantly, we are able to obtain a more accurate estimation of them.

Another key feature of the MHLP is that it no longer assumes a fully interconnected hub network at no cost (see, for instance, Alumur and Kara, 2008; Campbell and O'Kelly, 2012) or a particular topological structure (Contreras et al., 2009,2010; Yaman, 2008). Instead, it considers a set-up cost for the installation of access arcs and inter-hub arcs and allows the model to select the most appropriate topological structure for the considered instance. Finally, in order to have a more effective and efficient network, the MHLP can be extended to allow direct connections between O/D pairs. This means that some flows between O/D pairs may not be routed via hub nodes, if it is convenient to do so. As we will see in Chapter 4, the relaxation of the above mentioned assumptions dramatically increases the difficulty of the problem. This is not surprising considering that the MHLP is a much more realistic problem than the UHLPMA and UHLPSA. We next provide the formal description of the MHLP.

Let $G= (N, A)$ be a complete graph, where $N= \{1, 2, \ldots, n\}$ is the set of nodes that serve as origins and destinations of flow as well as potential hub locations and A is the set of arcs. Let W_{ij} denote the amount of flow between nodes i and j. For each node i, f_i denotes the fixed set-up cost for locating a hub at node i and d_{ij} denotes the distance between nodes i and j. Distances are not assumed to be symmetric nor to satisfy the triangular inequality. In order to appropriately estimate the transportation costs on both access and inter-hub arcs, the amount of flow that is routed on each arc is used to explicitly determine the number of link facilities (i.e., trucks, airplanes, etc.) with a given capacity that will be needed to route the flow on that arc. That is, we assume that transportation costs on arcs can be modeled using a step-wise function (see Figure 10). In particular, for each pair of hub nodes (k,m) let $c_{km}=l_c+bd_{km}$ denote the transportation cost for using one facility link with capacity B on inter-hub arc (k,m), where l_c and b represent the fixed and variable costs, respectively. In a similar way, for each pair of non-hub node and hub node (k,m) let $q_{km}=l_q+pd_{km}$ denote the transportation cost for using one facility link with capacity H on access arc (k,m), where l_q and p represent the fixed and variable costs, respectively. In order to properly represent economies of scale when consolidating flows at hub facilities and using more efficient path ways between hubs, we assume that $\frac{c_{km}}{B}<\frac{q_{km}}{H}$, where $B > H$, $b> p$ and $l_c>l_q$.

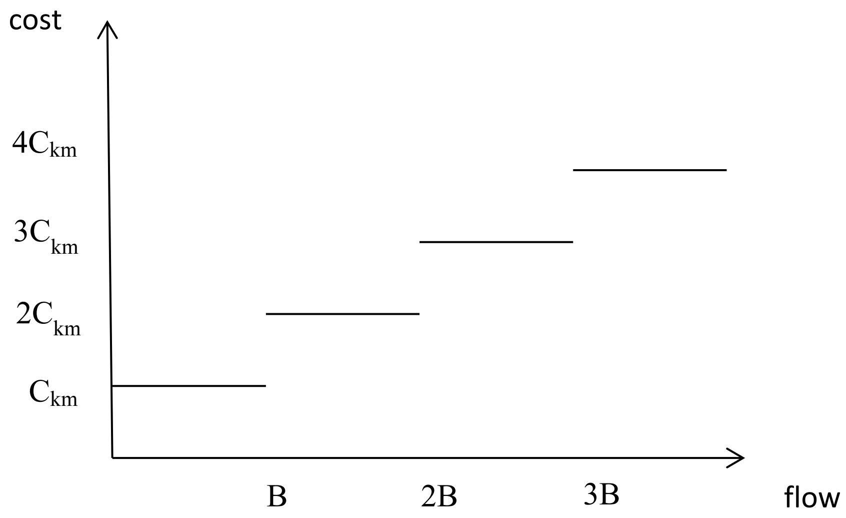

Figure 10 Step-wise Function

The assumption of modular (or step-wise) costs is consistent with practice. In the case of freight transportation, trucking companies send commodities between break-bulk terminals (inter-hub links) and between an end-of-line terminal and a break-bulk terminal (access links), by using one or more trucks. The number and capacity of trucks and distance can be used to provide an accurate estimation of the transportation cost between terminals. Fixed costs could represent the leasing or buying cost of one truck whereas the variable costs may represent the average fuel and labor costs for using it to transit a given distance. The consolidation of flows at hubs allows trucking companies to use large trucks (line-haul trucks), most commonly fully loaded between hub facilities. Local delivery trucks are used between end-of line and break bulk terminals, usually partially loaded, to route the commodities from/to their O/D nodes. Even though both fixed and variable costs for line-haul trucks are greater than local delivery trucks, the per unit flow transportation cost of inter-hub arcs is lower than the access arcs because of the increased capacity on the trucks.

Broadly speaking, the MHLP consists of: *i)* locating a set of hub facilities, *ii)* installing a set of facility links to construct the hub network, and *iii)* routing the flow through the network, so that the total set-up and transportation costs are minimized. We study four different variants of the MHLP that differ on the allocation pattern of nodes to hubs or the structure of O/D paths. In particular, Cases A and B consider a multiple assignment pattern of nodes to hubs. However, the former one does not allow direct connections between non-hub nodes (as in the case of most HLPs), whereas the latter one does allow them. Cases C and D consider a single assignment pattern of nodes to hubs. Similarly to previous cases, they differ on whether direct connections between non-hub nodes are allowed (Case D) or not (Case C). The rest of the section presents MIP formulations for these cases.

3.2.1 Case A: Multiple Assignments without Direct Connections

In this version of the MHLP, referred to as the *Modular Hub Location Problem with Multiple Assignments* (MHLP-MA), it is assumed that flows must be routed via at least one hub node (similar to the vast majority of HLPs). Moreover, by considering a multiple allocation patter the non-hub nodes can be allocated to more than one hub node, if convenient. A fundamental difference with respect to classical HLPs that assume full interconnection between hub nodes is that now, paths between O/D nodes could contain more than two hub facilities. Therefore, we need to explicitly determine the set of access and inter-hub arcs that will be part of a given O/D path, making the modeling of these paths much more challenging than other HLPs.

To model the MHLPMA we define several sets of decision variables to determine the number and location of hubs and links and the O/D paths. In particular, for each k in N we define the following locational decision variables:

$$Z_k=\begin{cases}1, & \text{if a hub facility is located at node } k\\ 0, & \text{otherwise}\end{cases}$$

For each pair of nodes i and j and each pair of hubs k and m, we define the following path variables:

$$X_{ijkm}=\begin{cases}1, & \text{if the interhub arc } (k,m) \text{is used in the path between } i \text{ and } j\\ 0, & \text{otherwise}\end{cases}$$

For each pair of nodes *i* and *j* and hub nodes *k* and *m*, we define the following allocation variables:

$$a_{ijk}=\begin{cases}1, & \text{if the flow between } i \text{ and } j \text{ uses access arc } (i,k)\\ 0, & \text{otherwise}\end{cases}$$

$$S_{ijm}=\begin{cases}1, & \text{if the flow between } i \text{ and } j \text{ uses access arc } (m,j)\\ 0, & \text{otherwise}\end{cases}$$

Finally, for each pair of nodes *k* and *m*, we define the following network design variables:

y_{km}= number of facility links between hub nodes *k* and *m*

v_{mk}^{1}= number of facility links between non-hub node *m* and hub node *k*

v_{mk}^{2}= number of facility links between hub node *m* and none-hub node *k*

Using these sets of variables, the MHLPMA can be stated as follows:

$$min \sum_{k\in N} f_k\, z_k + \sum_{k\in N} \sum_{m\in N} q_{km} v^1_{km} + \sum_{k\in N} \sum_{m\in N} q_{km} v^2_{km} + \sum_{k\in N} \sum_{m\in N} c_{km} y_{km} \tag{15}$$

$$s.t. \;\; \sum_{k\in N} a_{ijk} = 1 \qquad \forall i,j \in N \tag{16}$$

$$\sum_{k\in N} S_{ijk} = 1 \qquad \forall i,j \in N \tag{17}$$

$$y_{km} \leq Q z_k \qquad \forall k,m \in N \tag{18}$$

$$y_{km} \leq Q z_m \qquad \forall k,m \in N \tag{19}$$

$$\sum_{i\in N} \sum_{j\in N} W_{ij} x_{ijkm} \leq B y_{km} \qquad \forall k,m \in N \tag{20}$$

$$\sum_{j\in N} W_{ij}\, a_{ijk} \leq H v^1_{ik} \qquad \forall i,k \in N; i \neq k \tag{21}$$

$$\sum_{i\in N} W_{ij}\, S_{ijk} \leq H v^2_{mj} \qquad \forall m,j \in N; m \neq j \tag{22}$$

$$v^1_{mk} \leq Q z_k \qquad \forall k,m \in N \tag{23}$$

$$v^2_{mk} \leq Q z_m \qquad \forall k,m \in N \tag{24}$$

$$a_{ijk} + \sum_{m\in N} x_{ijmk} - \sum_{m\in N} x_{ijkm} - S_{ijk} = 0 \qquad \forall i,j,k \in N; i \neq j \tag{25}$$

$$z_k \in \{0,1\} \qquad \forall k \in N \tag{26}$$

$$v^1_{mk}, v^2_{mk}, y_{km} \in \mathbb{Z}^+ \qquad \forall k,m \in N \tag{27}$$

$$x_{ijkm} \geq 0 \qquad \forall i,k,m \in N \tag{28}$$

The objective function minimizes the total cost of establishing the hubs and the transportation costs due to three different types of facility links. Constraints (16) and (17) sate that for each pair of *i* and *j* we must use at least on hub node for routing the flow. Constraints (18) and (19) guarantee that there would not be an inter-hub arc between two nodes, unless they are both hub nodes. Constraints (20)-(22) limit the amount of capacity that can be sent between hub nodes, non-hub nodes and hub nodes, hub nodes and non-hub nodes, respectively. Moreover, constraints (23) and (24) impose that access links are installed only when exactly one end point of a link is a hub (either the origin or the destination). Constraints (25) are the well-known flow conservation constraints that ensure that the total number of arcs exiting each node is equal to the ones entering it. Finally, constraints (26), (27) and (28) are the classical integrity and non-negativity constraints. See Figure 11 for an example of a solution network for the MHLPMA.

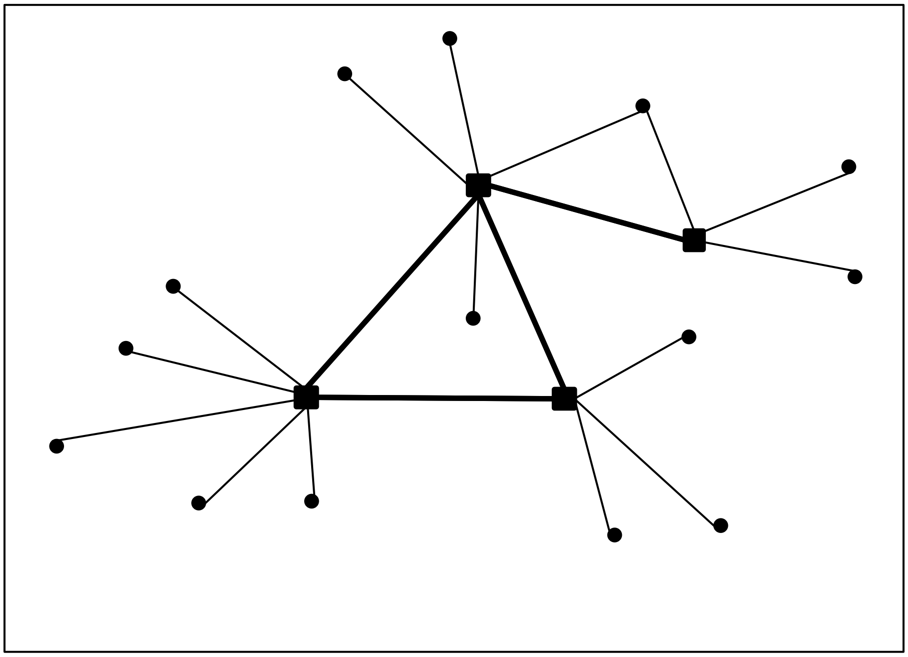

Figure 11 Network Structure for Case A

3.2.2 Case B: Multiple Assignments with Direct Connections

In this version of the MHLP, referred to as the *Modular Hub Location Problem with Multiple Assignments* and Direct Connections (MHLP-MAD), it is allowed to directly connect an O/D pairs even if there are not hub nodes. That is, we do not longer assume that flows should visit at least one hub node. It is now possible to directly send a given commodity from its origin to its destination node.

To model the MHLP-MAD we can adapt and extend the previous formulation of the MHLP-MA to incorporate the direct connection decisions by using the following set of decision variables. For each *k* and *m*, we define

P_{ij}= number of facility links between non-hub nodes *k* and *m,* and for each i and j, we define

$$m_{ij}=\begin{cases}1, & \text{if the flow is between } i \text{ and } j \text{ goes directly}\\ 0, & \text{otherwise}\end{cases}$$

Using the variables from the previous section and the ones mentioned above, the MHLP-MAD can be formulated as:

$$min \sum_{k\in N} f_k\, z_k + \sum_{k\in N}\sum_{m\in N} q_{km} v_{km}^1 + \sum_{k\in N}\sum_{m\in N} q_{km} v_{km}^2 + \sum_{k\in N}\sum_{m\in N} c_{km} y_{km} + \sum_{m\in N} q_{km}\, P_{km} \qquad (29)$$

$$s.t.\ \sum_{k\in N} a_{ijk} + m_{ij} = 1 \qquad \forall i,j \in N \qquad (30)$$

$$\sum_{k\in N} S_{ijk} + m_{ij} = 1 \qquad \forall i,j \in N \qquad (31)$$

$$y_{km} \leq Q z_k \qquad \forall k,m \in N \qquad (32)$$

$$y_{km} \leq Q z_m \qquad \forall k,m \in N \qquad (33)$$

$$\sum_{i\in N}\sum_{j\in N} W_{ij} x_{ijkm} \leq B y_{km} \qquad \forall k,m \in N \qquad (34)$$

$$\sum_{j\in N} W_{ij}\, a_{ijk} \leq H v_{ik}^1 \qquad \forall i,k \in N; i \neq k \qquad (35)$$

$$\sum_{i\in N} W_{ij}\, S_{ijk} \leq H v_{mj}^2 \qquad \forall m,j \in N; m \neq j \qquad (36)$$

$$W_{ij}.m_{ij} \leq H.P_{ij} \qquad \forall i,j \in N \qquad (37)$$

$$v_{mk}^1 \leq Q z_k \qquad \forall k,m \in N \qquad (38)$$

$$v_{mk}^2 \le Qz_m \qquad \forall k,m \in N \tag{39}$$

$$a_{ijk}+\sum_{m\in N} x_{ijmk}-\sum_{m\in N} x_{ijkm}-S_{ijk}=0 \qquad \forall i,j,k \in N; i \neq j \tag{40}$$

$$m_{ij} \le 1-z_i \qquad \forall i,j \in N \tag{41}$$

$$m_{ij} \le 1-z_j \qquad \forall i,j \in N \tag{42}$$

$$z_k \in \{0,1\} \qquad \forall k \in N \tag{43}$$

$$v_{mk}^1, v_{mk}^2, y_{km} \in \mathbb{Z}^+ \qquad \forall k,m \in N \tag{44}$$

$$x_{ijkm} \ge 0 \qquad \forall i,j,k,m \in N \tag{45}$$

Once more, the objective function minimizes the total cost of establishing the hubs and transportation cost. Constraints (30) and (31) sate that for each pair of *i* and *j* we must either use at least on hub node for routing the flow or connect the nodes directly. Constraints (32)-(36), (38)-(40), and (43)-(45) have the same meaning as in the MHLPMA. Constraints (37) are the capacity constraints on the direct connection links. Finally, constraints (41) and (42) ensure that a direct connection can be used just in the cases, where *i* and *j* are not hubs.

As can be seen in Figure 12, in the MHLP-MAD, some nodes may be allocated to more than one hub. Moreover, the flow may be routed directly from an origin to its destination by not using any hub as a consolidation center (dotted lines).

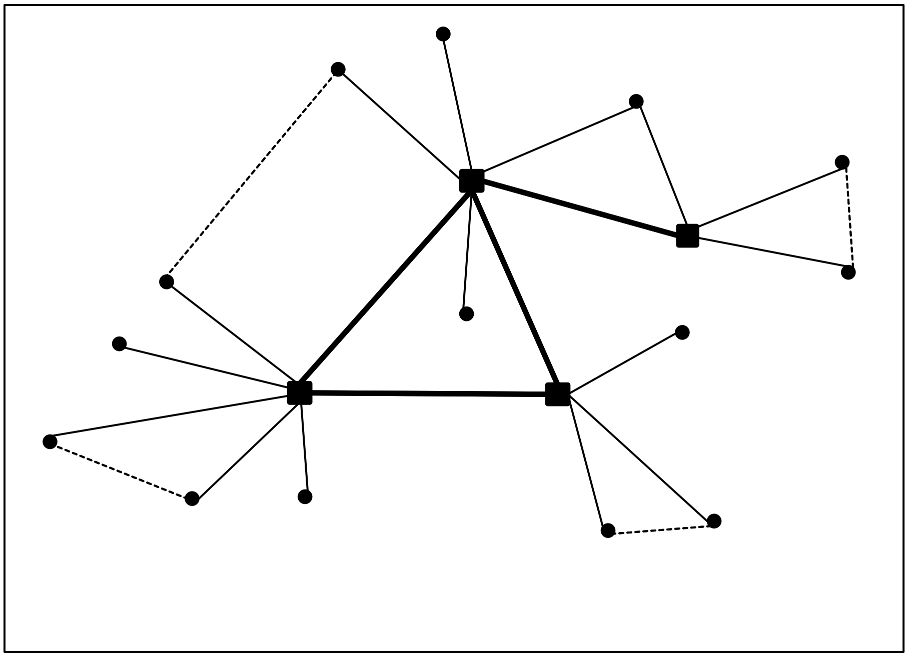

Figure 12 Network Structure for Case B

3.2.3 Case C: Single Assignments without Direct Connection

In this version of the MHLP, referred to as the *Modular Hub Location Problem with Single Assignments* (MHLP-SA), it is assumed that every non-hub node is allocated to exactly one hub facility. Moreover, it is assumed that flows must be routed via at least one hub node, that is, direct connections between non-hub nodes are not allowed. As we have observe in 3.1.2, single allocation HLPs use an extra set of allocation variables to formulate the problem. In particular, for each pair $i, k \in N$, we have

$$Z_{ik} = \begin{cases} 1, & \text{if node } i \text{ is assigned to hub } k \\ 0, & \text{otherwise} \end{cases}$$

In the following model O_i and D_j denote the total amount of flow that is leaving and entering each node, respectively.

$$O_i = \sum_{j \in N} W_{ij} \qquad D_j = \sum_{i \in N} W_{ij}$$

Using the variables from the previous MHLP models and the ones mentioned above, the MHLP-SA can be formulated as:

$$min \sum_{k\in N} f_k\, z_k + \sum_{k\in N} \sum_{m\in N} q_{km} v_{km}^1 + \sum_{k\in N} \sum_{m\in N} q_{km} v_{km}^2 + \sum_{k\in N} \sum_{m\in N} c_{km} y_{km} \tag{46}$$

$$s.t.\ \sum_{k\in N} Z_{ik} = 1 \qquad \forall i \in N \tag{47}$$

$$y_{km} \leq Q z_k \qquad \forall k, m \in N \tag{48}$$

$$y_{km} \leq Q z_m \qquad \forall k, m \in N \tag{49}$$

$$\sum_{i\in N} \sum_{j\in N} W_{ij} x_{ijkm} \leq B y_{km} \qquad \forall k, m \in N \tag{50}$$

$$O_i Z_{ik} \leq H v_{ik}^1 \qquad \forall i, k \in N; i \neq k \tag{51}$$

$$D_j Z_{jm} \leq H v_{mj}^2 \qquad \forall m, j \in N; m \neq j \tag{52}$$

$$v_{mk}^1 \leq Q z_k \qquad \forall k, m \in N \tag{53}$$

$$v_{mk}^2 \leq Q z_m \qquad \forall k, m \in N \tag{54}$$

$$Z_{ik} + \sum_{m\in N} x_{ijmk} - \sum_{m\in N} x_{ijkm} - Z_{jk} = 0 \qquad \forall i, j, k \in N; i \neq j \tag{55}$$

$$z_k \in \{0,1\} \qquad \forall k \in N \tag{56}$$

$$v_{mk}^1, v_{mk}^2, y_{km} \in \mathbb{Z}^+ \qquad \forall k, m \in N \tag{57}$$

$$x_{ijkm} \geq 0 \qquad \forall i, k, m \in N \tag{58}$$

Constraints (47) ensure that each non-hub is allocated to a single hub node. Constraints (48) and (49) have the same interpretation as before. Constraints (50)-(52) are capacity constraints on the amount of flow that is being routed on inter-hub arcs and access arcs, respectively. Constraints (53) and (54) ensure that access links are used only if the end node or the starting node is a hub, respectively. Constraints (55) are the well-known flow conservation constraints to keep track of the paths between hub nodes. Finally, constraints (56)-(58) are the classical integrality and non-negativity constraints.

Figure 13 illustrates an example of a solution network for the MHLP-SA. In this case, O/D paths are enforced to have at least one hub node on its route to the destination.

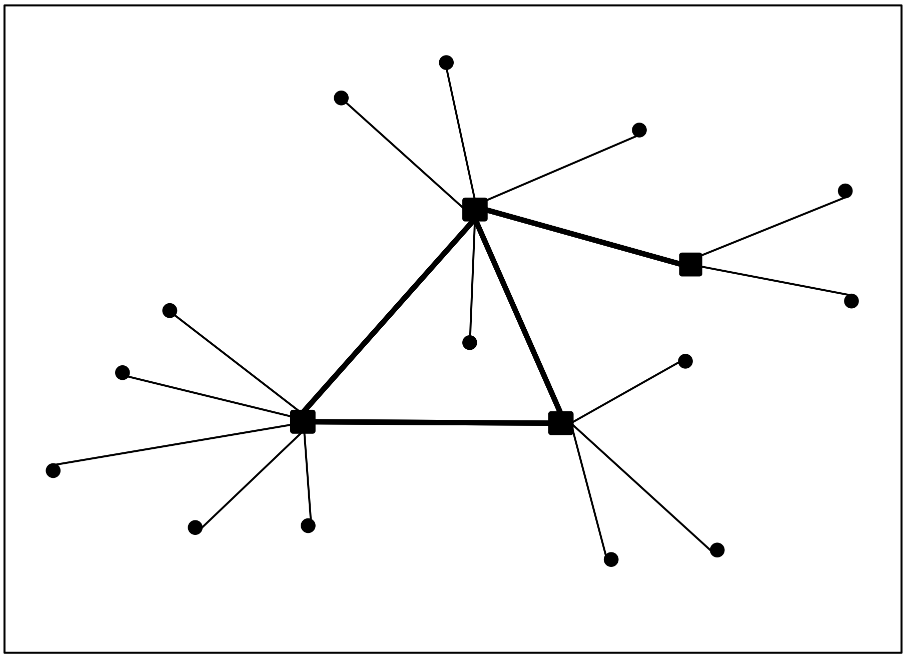

Figure 13 Network Structure for Case C

3.2.4 Case D: Single Assignments with Direct Connection

In this version of the MHLP, referred to as the *Modular Hub Location Problem with Single Assignments and Direct Connections* (MHLP-SAD), it is assumed that every non-hub node is allocated to exactly one hub facility. Moreover, it is assumed that direct connections between non-hub nodes are allowed. That is, we do not longer assume that flows should visit at least one hub node. It is now possible to directly send a given commodity from its origin to its destination node. Similar to the multiple allocation cases, we can extend the previous formulation of the MHLP-SA to incorporate the direct connection decisions by using the $P_{ij}$and m_{ij} decision variables previously defined.

Using the variables from the previous MHLP models the MHLP-SAD can be formulated as:

$$min \sum_{k\in N} f_k\, z_k + \sum_{k\in N}\sum_{m\in N} q_{km} v_{km}^1 + \sum_{k\in N}\sum_{m\in N} q_{km} v_{km}^2 + \sum_{k\in N}\sum_{m\in N} c_{km} y_{km} + \sum_{m\in N} q_{km}\, P_{km} \tag{59}$$

$$s.t.\ \ \sum_{k\in N} Z_{ik} = 1 \qquad \forall i \in N \tag{60}$$

$$\sum_{k\in N} a_{ijk} + m_{ij} = 1 \qquad \forall i,j \in N \tag{61}$$

$$\sum_{k\in N} S_{ijk} + m_{ij} = 1 \qquad \forall i,j \in N \tag{62}$$

$$a_{ijk} + \sum_{m\in N} x_{ijmk} - \sum_{m\in N} x_{ijkm} - S_{ijk} = 0 \qquad \forall i,j,k \in N; i \neq j \tag{63}$$

$$a_{ijk} \leq z_{ik} \qquad \forall i,j,k \in N \tag{64}$$

$$S_{ijm} \leq z_{jm} \qquad \forall i,j,m \in N \tag{65}$$

$$y_{km} \leq Q z_k \qquad \forall k,m \in N \tag{66}$$

$$y_{km} \leq Q z_m \qquad \forall k,m \in N \tag{67}$$

$$\sum_{i\in N}\sum_{j\in N} W_{ij} x_{ijkm} \leq B y_{km} \qquad \forall k,m \in N \tag{68}$$

$$\sum_{j\in N} W_{ij}\, a_{ijk} \leq H v_{ik}^1 \qquad \forall i,k \in N; i \neq k \tag{69}$$

$$\sum_{i\in N} W_{ij}\, S_{ijk} \leq H v_{mj}^2 \qquad \forall m,j \in N; m \neq j \tag{70}$$

$$W_{ij}.m_{ij} \leq H.P_{ij} \qquad \forall i,j \in N \tag{71}$$

$$v_{mk}^1 \leq Q z_k \qquad \forall k,m \in N \tag{72}$$

$$v_{mk}^2 \leq Q z_m \qquad \forall k,m \in N \tag{73}$$

$$m_{ij} \leq 1 - z_i \qquad \forall i,j \in N \tag{74}$$

$$m_{ij} \leq 1 - z_j \qquad \forall i,j \in N \tag{75}$$

$$z_k \in \{0,1\} \qquad \forall k \in N \tag{76}$$

$$v_{mk}^1, v_{mk}^2, y_{km} \in \mathbb{Z}^+ \qquad \forall k,m \in N \tag{77}$$

$$x_{ijkm} \geq 0 \qquad \forall i, k, m \in N \quad (78)$$

Again, the objective function minimizes the same quantity. Constraints (60) guarantee the allocation of each non-hub node to a single hub node. Constraints (61) and (62) ensure that for each pair of i and j we must either use direct connections or go through hubs for routing the flow. Constraints (63) are the flow conservation constraints that ensure the equality of arcs exiting and entering each node. Constraints (64) and (65) impose that access links are installed only when exactly one end point of a link is a hub. Constraints (66)-(67), (68)-(71), (72)-(73), (74)-(75), and (76)-(78) have the same meaning as in the previous models.

See Figure 14 for an example of a solution network for the MHLP-SAD. Observe that each node is allocated to only one hub node. Moreover, non-hub nodes can make direct connections among themselves.

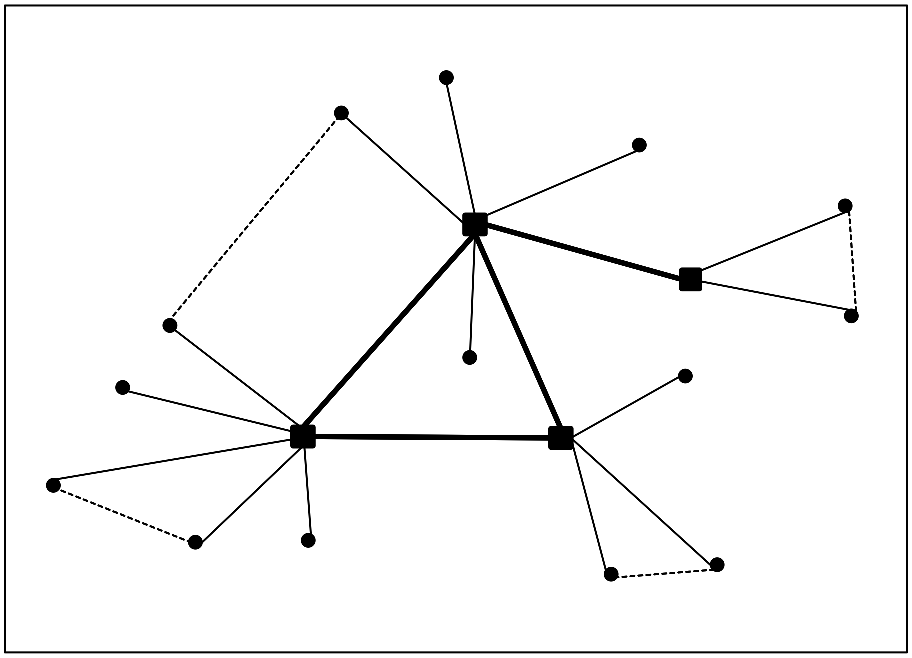

Figure 14 Network Structure for Case D

Chapter 4: Computational Results

In this chapter we present a set of computational experiments to analyze and compare the proposed formulations for the considered MHLPs. We use the classical UHLPMA and UHLPSA problems as a benchmark to analyze the differences, if any, in the topological structure of the obtained hub networks when using a different approach to estimate the economies of scale as in MHLPs. Several tables and figures are used to illustrate the location of hub nodes and the allocation patterns in the network.

For these computational experiments, we use the well-known Australian Post *(AP)* set of instances. They are the most commonly used in the hub location literature. It consists of Euclidean distances d_{ij} between cities in Australia and the values of W_{ij} representing passenger flows between pairs of cities. Each instance has a strictly positive flow between every pair of nodes. From this set of instances, due to the complexity of the model, we have selected instances containing 10 nodes. In order to obtain optimal solutions, the mathematical programming formulations were modeled with OPL and solved using CPLEX 12.2© Optimization Studio. All computational experiments were run on an HP PC with a Pentium® Dual-Core CPU E5500 processor running at a 2.80 GHz and 4 GB of RAM under a Windows 7 environment.

The remaining of this chapter is structured as follows. In the first part we focus on the computational experiments for the well-known UHLPMA and UHLPSA. The second part is devoted to the four different variants of the MHLP we have introduced. Finally, the last part presents a comparison of the obtained hub networks with both classes of problems.

4.1 Uncapacitated Hub Location Problems

The aim of the first set of experiments is to analyze and compare the structure of hub networks obtained with the classical UHLPMA and UHLPSA problems. To obtain the optimal hub networks, we use the MIP formulations presented in Sections 3.1.1 and 3.1.2 to solve them with CPLEX. We have generated three different instances from the AP data set by considering a 10-node instance with the following values for the inter-hub discount factor α = { 0.2, 0.6, 0.8 }.

The detailed results of the UHLPMA and UHLPSA are given in Tables 1 and 2, respectively. For a considered value of α, the optimal location of hubs, the optimal value, and CPU time (in seconds), the *LP* %gap, and the number of explored nodes in the Branch and Bound tree (B&B), are provided.

Table 1 Computational Result for UHLPMA

Discount Factor	Location of hubs	Optimal Solution Value	CPU time (sec)	LP Gap (%)	B&B nodes
0.2	7, 4, 1	201373.08	0.65	0.00	0
0.6	7, 4, 1	201642.80	0.42	0.00	0
0.8	7, 4, 1	201756.46	0.39	0.00	0

The results of Table 1 show that, regardless the value of the discount factor, the location and number of hubs are the same for the UHLPMA. However, the total cost increases as α increases its value. Moreover, these problems considering 10 nodes are relatively easy to solve, as CPLEX is able to optimally solve them in less than one second. LP gap and the number of explored nodes in the B&B tree are equal to 0. These results provide an indication that the considered formulation for the UHLPMA is strong.

For multiple allocation models, the allocation of non-hub nodes to hub nodes changes as α changes its value (Figure 15).

Table 2 Computational Result for UHLPSA

Discount Factor	Location of hubs	Optimal Solution value	CPU time (sec)	LP Gap (%)	B&B nodes
0.2	7, 4, 1	201383.12	5.84	0.00	0
0.6	7, 4, 1	201749.41	5.58	0.00	0
0.8	7, 4, 1	201932.55	5.45	0.00	0

Similar results can be observed in Table 2 for the UHLPSA. For both UHLPMA and UHLPSA the number and location of the hubs are the same. The optimal set of hubs does not change when varying the discount factor α. Moreover, the allocation of non-hub nodes to hub nodes does not change for single allocation models (Figure 16). However, because of the single allocation assumption, observe that the total cost slightly increased with respect to the UHLPMA. The CPU time to solve these instances with CPLEX has also increased; however, we can still optimally solve them in less than 6 seconds.

Figures 15 and 16 illustrate the structure of the obtained hub networks with the UHLPMA and the UHLPSA, respectively. In these figures, the rectangles represent hub nodes while the circles represent the non-hub nodes. Access arcs are represented with solid lines while inter-hub links are represented by bold lines.

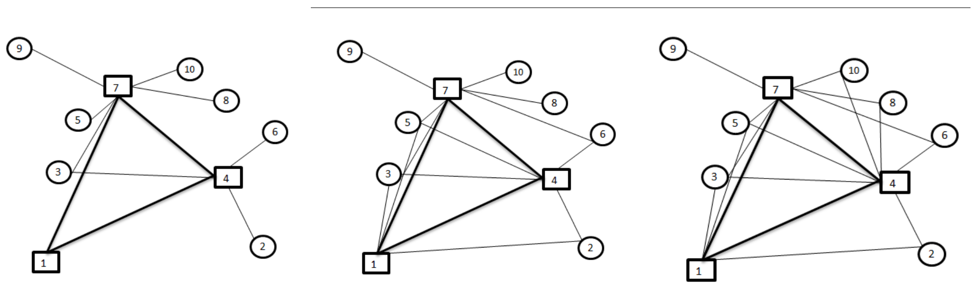

Figure 15 Network Structure for the UHLPMA

In the case of the UHLPMA, note that nodes 2, 3, 5, 6, 8 and 10 are allocated to more than one hub along the three cases, whereas in the UHLPSA, they are allocated to exactly one hub. We also note that, in the case of UHLPSA, node 3 is not allocated to its closest hub (node 1), but to node 4.

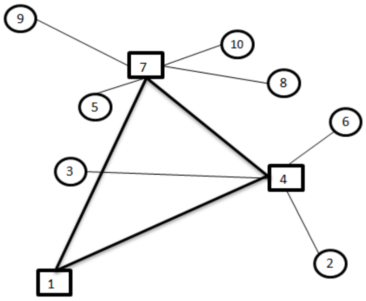

Figure 16 Network Structure for the UHLPSA

4.2 Modular Hub Location Problems

The goal of the second set of experiments is twofold. The first is to compare the structure of hub networks obtained when using the four variants of the MHLP. The second is to analyze the capabilities and limitations of the proposed formulations for optimally solving MHLPs when using a general purpose solver such as CPLEX. We have generated four different instances from the AP data set by considering a 10-node instance and different values for the parameters *B*, *H, b,* and *p*. As mentioned in Section 3.2, *B* and *H* represent the capacity of the facility links between the hub nodes and among non-hub nodes and hub nodes, respectively, and *b* and *p* represent the variable cost per unit traveled distance. In the case of ground transportation applications, capacities are related to the number of pallets that can be loaded into a truck whereas the variable cost represents the fuel cost for using a fully loaded truck.

The detailed results of the MHLP-MAD, MHLP-MA, MHLP-SAD, and MHLP-SA are given in Tables 3, 4, 5, and 6, respectively. For a particular configuration of *α*, *B*, *H, b,* and *p,* the optimal location of hubs, the optimal value, the CPU time (in seconds), the *LP* %gap, and the number of explored nodes in B&B, are provided. The configuration of the parameters has been chosen in such a way that we obtain an equivalent discount factor for the inter-hub arcs of α = {0.2, 0.6} (see first column of Tables 3, 4, 5, and 6).

In the first two rows of Table 3, the capacities are the same but there is an increase in the transportation cost in row 2, which results in an increase in the number of hubs and the total cost of the obtained hub network. Comparing CPU times, it is harder to solve the model with higher prices. When we increase the price, while the capacity remains the same, the model tends to choose more hubs from the nodes. In rows three and four, the prices are equal but the capacities are not. By increasing the capacity of facility links, the number of hubs decreases. Therefore, the total cost and the CPU time decrease.

Table 3 Computational Results for MHLP - MAD

α	B	H	b	p	Hub locations	Optimal Solution Value	CPU time	LP Gap	B&B nodes
0.2	750	100	300	200	7, 4	183540.58	42	34.60	324
0.2	750	100	600	400	7, 4, 3	296790.80	209	23.40	2688
0.6	200	100	500	400	7, 4, 3	331200.71	996	24.30	13925
0.6	300	150	500	400	7, 4	253860.24	65	30.63	426

Table 4 Computational Results for MHLP -MA

α	B	H	b	p	Hub locations	Optimal Solution Value	CPU time	LP Gap	B&B nodes
0.2	750	100	300	200	7, 4	188659.88	7	31.50	14
0.2	750	100	600	400	7, 4, 3	301382.97	239	22.10	2420
0.6	200	100	500	400	7, 4, 3	335792.88	258	22.10	4382
0.6	300	150	500	400	7, 4	260634.68	109	29.90	1286

From Tables 3 and 4, we observe that the cost in MHLP-MAD is lower than that of the MHLP-MA. This reduction in the total cost is caused by the incorporation of direct links between some non-hub nodes. Figures 17 and 18 depict the optimal hub network of the instances previously considered (Tables 3 and 4).

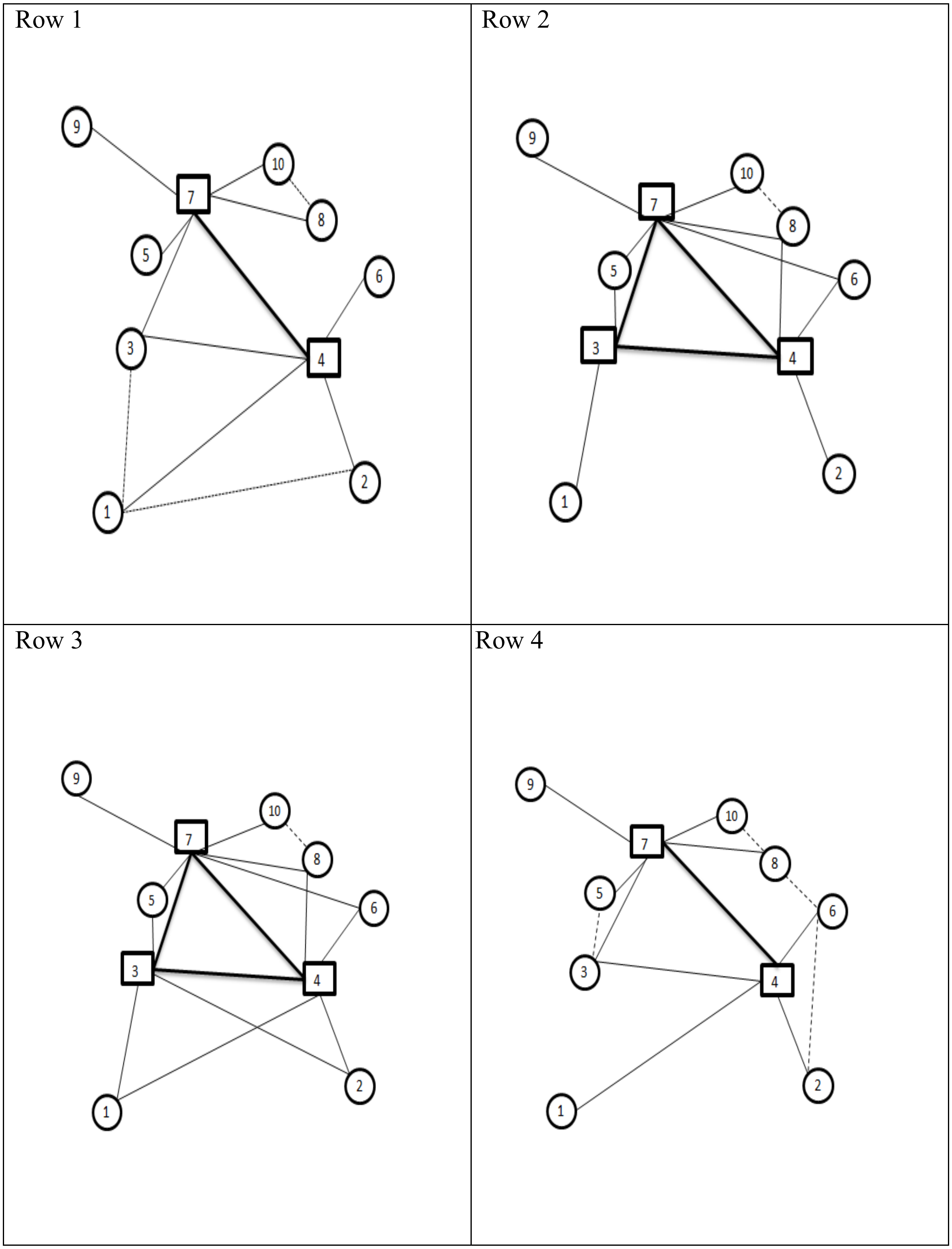

Figure 17 Network Structures for MHLP-MAD

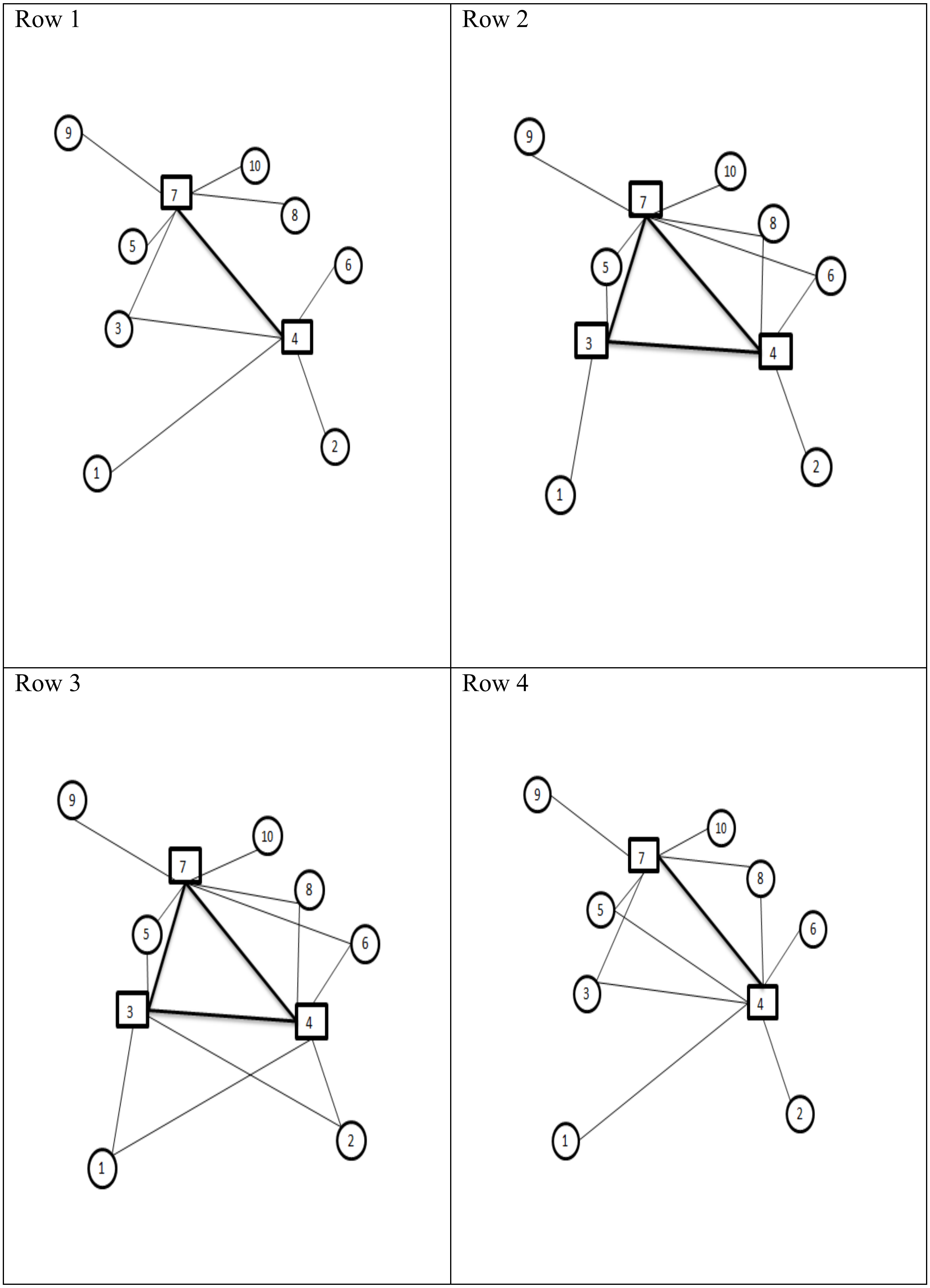

Figure 18 Network Structures for the MHLP-MA

Comparing Rows 1 and 2 of Figure 17, when the prices are higher the model tends to minimize the number of direct connections. Instead, the network will have more hubs. In Row 1, we have 2 hubs and all of the other non-hub nodes directly connect to either of them. Three direct links exists between pairs of nodes *(3,1)*, *(2,1)*, and *(10,8)*. In Row 2, as a result of the increase in price, we have three hubs which are fully interconnected. In this case just we have one direct link which is between nodes *10* and *8*. Three of the non-hub nodes *(5, 6, 8)* are connected to more than one hub; while in Row 1 just node 3 was connected to hubs *4* and *7*. Regarding rows 3 and 4, for situations with higher capacity the policy is different. When there is an increase in the capacity of the facility links, the number of hubs decrease which cause an increase in the number of direct connections among non-hub nodes. Comparing the obtained results between MHLP-MAD and MHLP-MA, we note that the allocation of non-hub nodes to hub nodes do not change in the four considered instances. Moreover, only a small subset of commodities is directly routed between non-hub nodes. These results provide a clear indication that the consolidation of flows at hubs still provides an important source of reduction for transportation costs.

Figure 18, illustrates the same results as Figure 17 for situations without direct connections. The location of hubs and the allocation pattern of the non-hub nodes to hub nodes are relatively the same as Figure 17 with the elimination of direct links. In Row 4 of Figure 18, nodes 5 and 8 are allocated to more than one hub which previously they were only allocated to hub 7.

Tables 5 and 6 provide the results for the MHLP-SAD and the MHLP-SA, respectively.

Table 5 Computational Results for MHLP-SAD

α	B	H	b	p	Hub locations	Optimal Solution Value	CPU time	LP Gap	B&B nodes
0.2	750	100	300	200	6, 5	186159.27	7200	15.80	135591
0.2	750	100	600	400	7, 4, 3	301754.50	2520	14.90	12174
0.6	200	100	500	400	7, 5, 4	347616.03	9550	12.80	182943
0.6	300	150	500	400	5	259785.49	756	13.70	8760

By comparing Tables 5 and 6, in situations with direct connections (MHLP-SAD), usually the CPU time is higher than the CPU time for the MHLP-SA, which is due to the incorporation of direct links between non-hub nodes. As for the MHLPs with multiple assignments, the cost decreased for the MHLP-SADs.

Figures 19 and 20 depict the optimal hub network of the single allocation instances previously considered (Tables 5 and 6).

Table 6 Computational Results for MHLP-SA

α	B	H	b	p	Hub locations	Optimal Solution Value	CPU time	LP Gap	B&B nodes
0.2	750	100	300	200	7, 4	193133.75	3	2.27	0
0.2	750	100	600	400	7, 4, 3	306346.68	148	7.75	2047
0.6	200	100	500	400	7, 4, 3	357850.85	51	4.46	1346
0.6	300	150	500	400	7, 4	275262.63	20	3.60	213

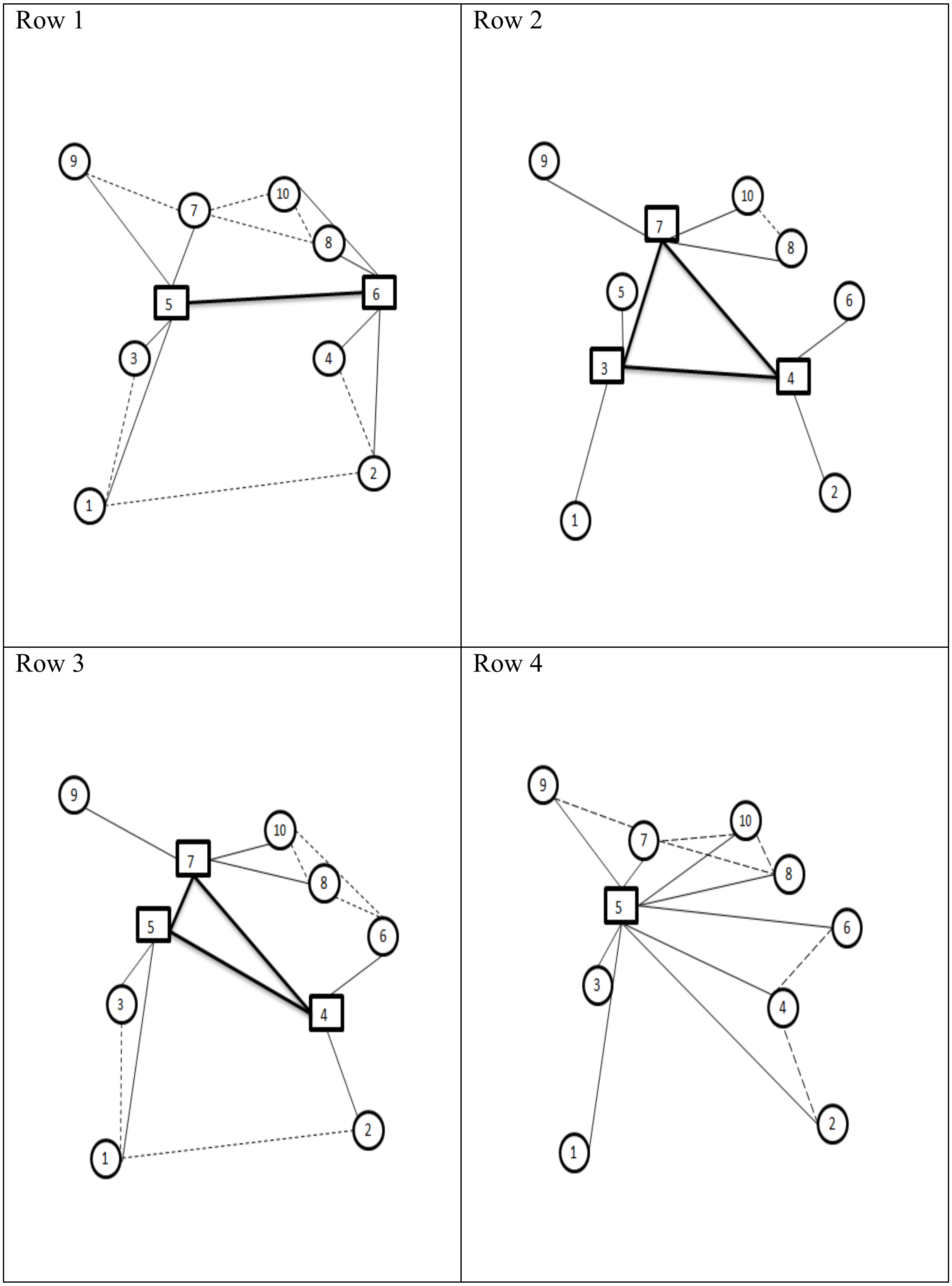

Figure 19 Network Structures for MHLP-SAD

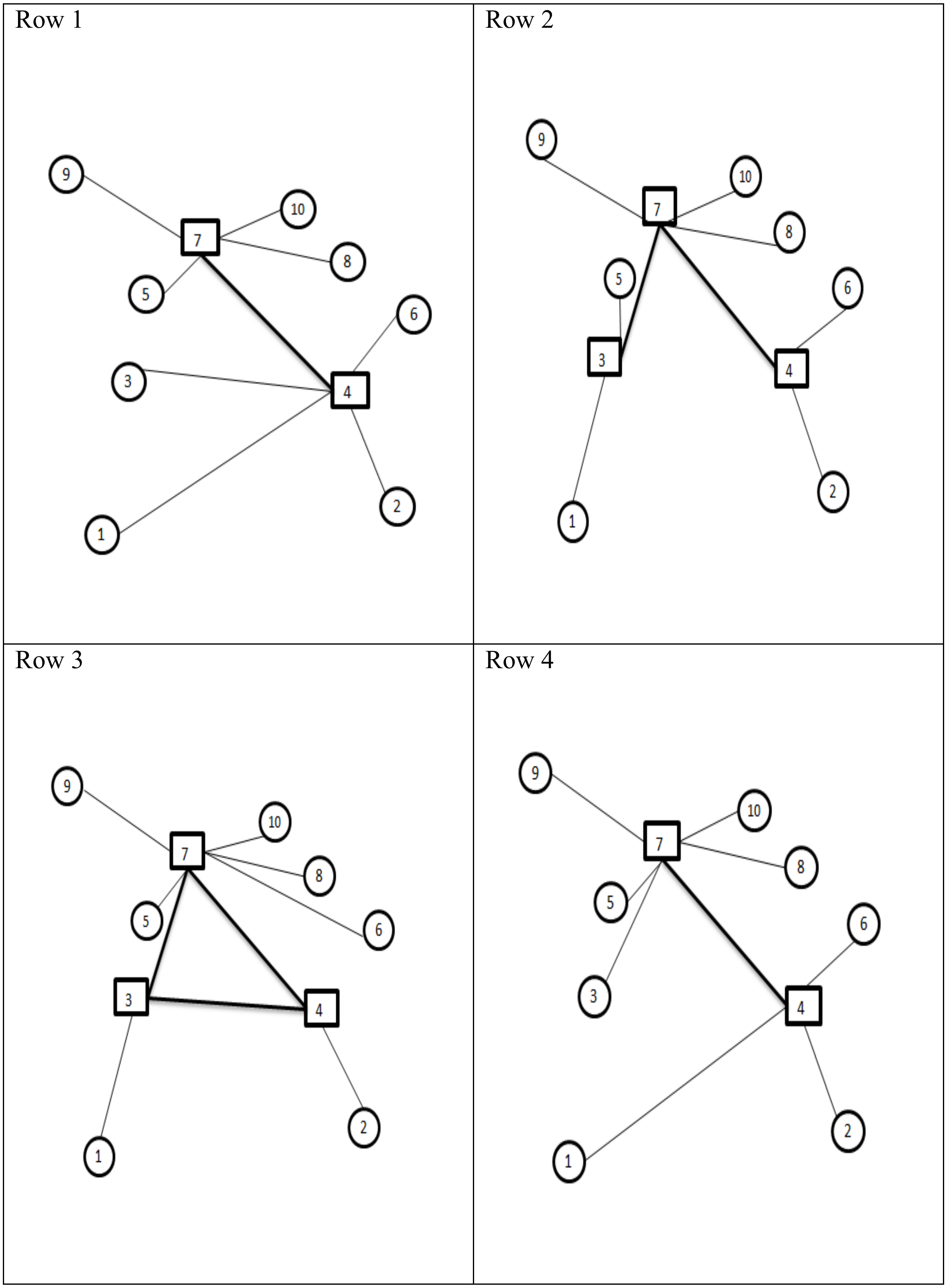

Figure 20 Network Structures for MHLP-SA

Figure 19 shows the optimal network structures for MHLP-SADs. The location and allocation pattern of this models have changed. Nodes 5 and 6 have now become hubs; they were never chosen as hubs in previously considered models. In Row 1, nodes 5 and 6 are connected to each other and the non-hub nodes are connected to only one hub node which maybe either of these hub nodes. There are some direct links between the non-hub nodes as well. In Row 2, the structure is similar to the previous models expect that the non-hub nodes are allocated to only one hub node. In Row 3, nodes 4 and 7 are still hubs; node 3 replaces its location as a hub with node 5. The hubs are fully interconnected and the non-hub nodes connect to hub nodes via access links. Also there exists direct links amongst the non-hub nodes. In Row 4, just we have one hub and all of the non-hub nodes directly connect to it. There are also some direct links between some of the non-hub nodes.

Figure 20 represents the MHLP-SA models. The location of the hubs is similar to the MHLP-MA models (Figure 18). The allocation pattern varies considerably in Row 2, which there is not a fully interconnected network among the hub nodes. In the rest of the Rows (1, 3, 4) just there is a difference in the structure which is due to the single allocation pattern that limits the non-hub nodes to be allocated to only one hub node.

Finally, we make a direct comparison between the four different models we have considered (MHLP-SA, MHLP-SAD, MHLP-MAD, MHLP-MA). We fixed the capacity of facility links *(B and H)* and the costs for transportation *(b and p), (B=750, H=100, b=600, p=400)*. The number and location of hubs, best cost and CPU time are given. Comparing the CPU time, the MHLP-SADs are taking longer times than the MHLP-MADs to be solved. Besides, MHLP-SAs are solved faster than the MHLP-MAs. In both multiple allocation and single allocation models, it is harder to solve the model while direct connections were permitted due to the CPU time. Locating the hubs and building the network in both versions of multiple assignment (MHLP-MAD and MHLP-MA), costs less than that of the single assignment versions (MHLP-SAD and MHLP-SA), respectively.

Table 7 Computational Results among the Four Possible Situations

Allocation method	Hub locations	Optimal Value	CPU time
MHLP-SAD	7, 4, 3	301754.50	2520
MHLP-SA	7, 4, 3	306346.68	148
MHLP-MAD	7, 4, 3	296790.80	174
MHLP-MA	7, 4, 3	301382.97	195

Figure 21 illustrates the differences of the optimal hub network obtained with the models MHLP-SAD, MHLP-SA, MHLP-MAD and MHLP-MA, respectively, for the above case with α value equal to *0.2*.

In all of the four situations, the number and locations of the hubs remain the same; just the network structures have slight changes. For three of the cases, instead of MHLP-SA, the hubs are fully interconnected. For the cases which allow direct connections, nodes *10* and *8* directly connect to each other. In multiple allocation versions, nodes *5*, *6* and *8* are connected to more than one hub node.

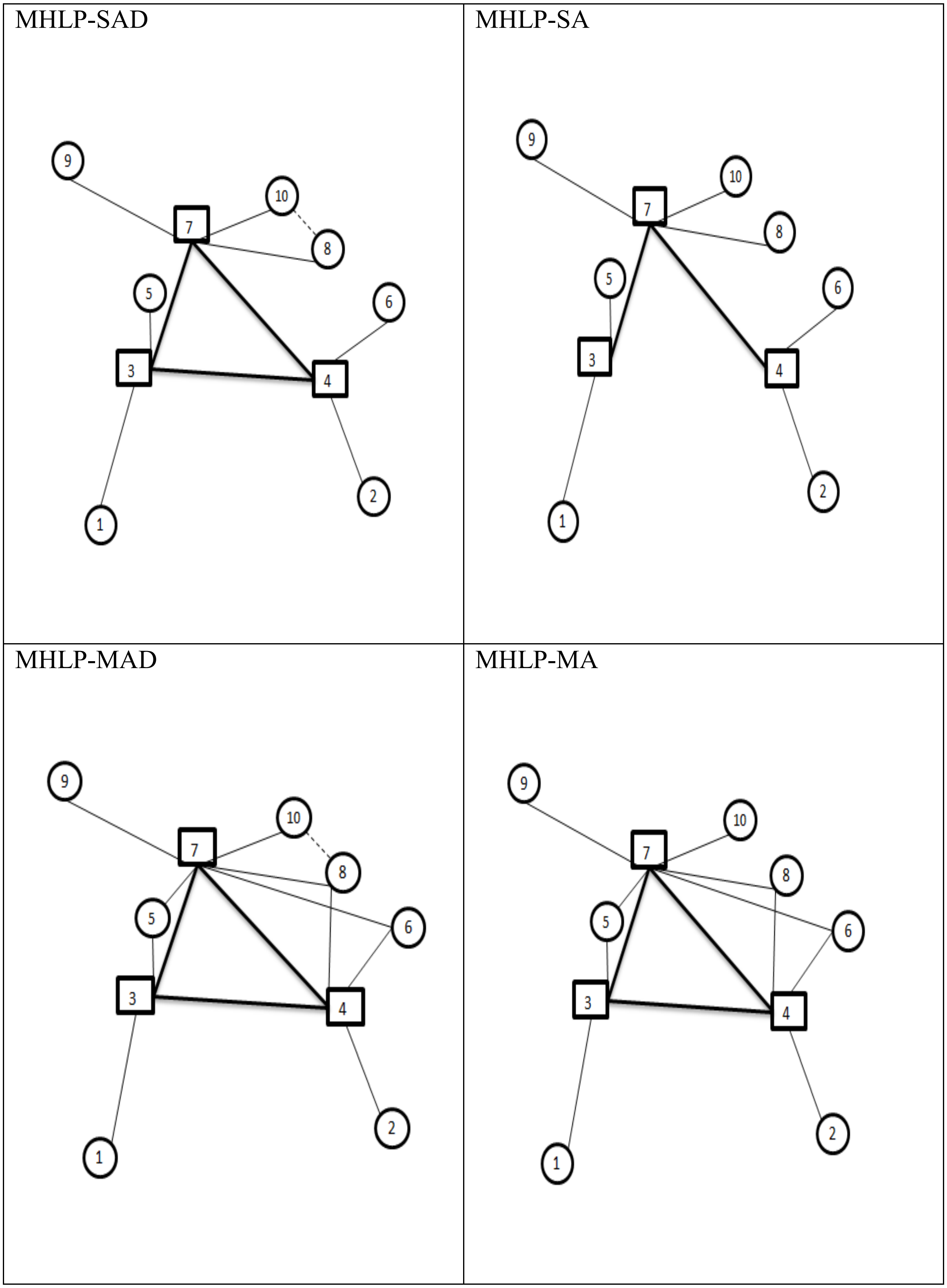

Figure 21 Network Structures for Four Situations with $\alpha=0.2$

In order to study the limitations of the proposed formulations, for optimally solving MHLPs when using a general purpose solver such as CPLEX, we test the four considered MHLP models with 20 node instances. Unfortunately, CPLEX was not able to solve any of these models in three days of CPU time. Given that the LP bounds for the proposed formulations are rather weak, all formulations take a huge amount of time and memory capacity to be solved to optimality. Only for the particular case of the MHLP-MAD, we were able to obtain the optimal solution in three days and a half (88 hours and 45 minutes). In the case of the other three models, the computer ran out of memory after three days.

4.3 Comparison between UHLP and MHLP

In order to compare the UHLP and MHLP, see Figures 15, 16 and the right column of Figure 21 (with single assignment on top and multiple assignment at the bottom). Comparing the MHLP-SA (the top cell in Figure 21) with Figure 16, the number of hubs is still the same; the location of hubs has changed from node *1* to *3*, but nodes *4* and *7* are still hubs. In the UHLPSA the hubs are fully interconnected while, in MHLP-SA there is not a fully interconnected network amongst the hubs. In UHLP most of the non-hub nodes are connected to more than one hub node, while in MHLP just nodes 5, 6 and 8 are connected to more than one hub nodes. The number and location of the hubs in these models have the same changes as in single allocation version.

Chapter 5: Conclusion

In this thesis we have introduced a new hub location model, referred to as the *Modular Hub Location Problem* (MHLP), able to overcome several disadvantages of well-known hub location models. The MHLP is suited for the design of freight transportation systems, in particular for large-scale trucking networks. It considers the design of the entire hub network as part of the model and thus, it does not assume either a fully interconnection between hub nodes or a particular topological structure. It also allows a direct connection between O/D points in order to construct a more cost-effective network. Finally, it approaches the flow dependency problem of transportation costs by using modular costs on the links of the network.

We have studied four different variants of the MHLP. Two versions restrict nodes to be allocated to a unique hub (single allocation pattern) while the others allow a node to interact with multiple hubs (multiple allocation pattern). For these versions, we may allow to have direct connections or not. We have proposed integer programming formulations for the four variants of the MHLP. We have performed a set of computational experiments to assess the performance of the proposed formulations when solved by using the state-of-the-art solver CPLEX. Moreover, we have developed a comparative study on the topological structure of the networks obtained with the well-known uncapacitated hub location problem and the proposed one. Computational results have shown that it may be possible to have different configurations for the hub networks when explicitly considering flow dependency for the computation of the total transportation cost. However, one of the major drawbacks of the proposed MHLP is the increased difficulty in modeling and solving these problems to optimality with a general purpose solver. A promising research direction is the development of specialized solution methodologies to approach larger, more realistic, size instances for these MHLPs. Decomposition techniques, such as Lagrangean relaxation and Benders decomposition, may be able to exploit the structure of these problem to obtain tight lower and upper bounds on the optimal solution value. In addition, metaheuristic solution methods should also be considered to efficiently obtain good feasible solutions. Other research directions could be the incorporation of capacity constraints at the hub facilities or service level constraints to limit the structure of O/D paths.

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