Portfolio Credit Risk Modelling for a Canadian SME Loans Portfolio

Jade Michel Haddad

A Thesis
in
The Department
of
Economics

Presented in Partial Fulfillment of the Requirements
For the Degree of
Doctor of Philosophy (Economics) at
Concordia University
Montreal, Quebec, Canada

December 2012

CONCORDIA UNIVERSITY
SCHOOL OF GRADUATE STUDIES

This is to certify that the thesis prepared

By: Jade Michel Haddad

Entitled: Portfolio Credit Risk Modelling for a Canadian SME Loans Portfolio

and submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy (Economics)

complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

Signed by the final examining committee:

Dr. Anne-Marie Croteau	**Chair**
Dr. Richard Guay	**External Examiner**
Dr. Michel Magnan	**External to Program**
Dr. Nikolay Gospodinov	**Examiner**
Dr. Gordon Fisher	**Examiner**
Dr. Bryan Campbell	**Thesis Supervisor**

Approved by

____

Chair of Department or Graduate Program Director

____

Dean of Faculty

ABSTRACT

Portfolio Credit Risk Modelling for a Canadian SME Loans Portfolio

Jade Michel Haddad

The Basel II Capital Accords make strong and controversial assumptions on the behaviour of Small and Medium Enterprises (SMEs) in a credit portfolio. Benefiting from a rich, and as such rare, dataset of default and credit risk events, we measure the portfolio credit risk characteristics of one of the riskiest segments of the Canadian SME market. The depth of our data allows for robust segmentations of the data along dual dimensions, including risk grade and size of borrowers, not commonly found in the literature. This, in turn, allows for an SME-specific calibration of models for portfolio credit risk. In particular, we use the Merton-type asset value model (*AVM*) and the *CreditRisk*$^+$ frameworks to present empirical estimates of the correlations that underline the relationship among borrower segments in the portfolio. In addition, we present loss distribution estimates for our SME portfolio under various extensions to the *AVM* and *CreditRisk*$^+$. These extensions include a *Multiple Correlated Sectors* implementation of *CreditRisk*$^+$ and simulation-based, as well as analytical implementations of both frameworks. Our results allow for a thorough testing of Basel II assumptions for portfolio credit risk and its application to SME borrowers. In particular, we present evidence in contrast to Basel II specifications on SME asset correlations, and quantify the impact of the single sector and infinite granularity assumptions in the Basel II Internal Ratings Based (IRB) approach to portfolio credit risk. Our work is undertaken within a consistent calibration of the *AVM* and *CreditRisk*$^+$ frameworks and presents an SME-specific calibration refinement for *CreditRisk*$^+$. Finally, we focus on capital allocations under the Basel II framework and present a partial implementation analysis quantifying the impact of the application of various Basel II conventions to our SME portfolio. Capital allocations from our internally-calibrated portfolio credit risk frameworks reveal a misallocation of capital among SME segments under Basel II. Given our thorough assessment of both Basel II and the credit risk characteristics underlying SME portfolios, we provide suggestions for an improved SME portfolio credit risk management framework.

ACKNOWLEDGEMENTS

This thesis would not have been possible without the help and guidance of my supervisor, Professor Bryan Campbell to whom I owe a tremendous debt of gratitude. Throughout countless hours of discussion and debate, Professor Campbell continuously provided an elucidating perspective on the knot of challenges and opportunities that are a hallmark of applied work. Professor Campbell brought out the best in me, and having finally arrived at the end of this pursuit, I find myself privileged to be standing at the threshold of new ones with the knowledge and expertise obtained under his guidance.

To Dominique Davies, who has been a mentor, a friend, a supporter and an advisor throughout the most challenging of times, thank you. Your spirit, professionalism and candour were a defining factor in the successful completion of this thesis.

I would also like to thank my committee members Professor Gordon Fisher, Professor Nikolay Gospodinov, Professor Michel Magnan and Professor Richard Guay for their enthusiastic and incisive engagement. A special thanks to Professor Fisher and Professor Effrosiny Diamantoudi whose sound counsel and advice were pivotal in my early years at the Department of Economics.

I would like to thank the *Financing Company* for their contributions to this thesis, which are in no way or form limited to the unique database on which this research was based. I would also like to thank the Center for Intrauniversity Research and Analysis of Organizations (CIRANO) for always providing an open door and a fertile environment for research, critical thinking and engagement.

To my parents, Michel and Antoinette, thank you for believing in me and never letting me give up. You have instilled in me the values and character that have made me the man I am today. To my siblings, Nicholas and Rouba, you have enriched my life and lightened this journey innumerably.

Last but certainly not least, some words of thank you for my wife, Zeina. Zeina, you have been an anchor in my life. Your persistence and support were the pillars on which I could withstand the long nights spent in pursuit of this thesis, and its successful completion is only meaningful to me through the reflections of pride and love in your eyes.

DEDICATION

To my beautiful wife and loving parents, who have always shined a light upon the path I walk in life. This would not have been possible were it not for their unwavering dedication to education and self-fulfillment

TABLE OF CONTENTS

Chapter 1. Introduction

The Basel II treatment of portfolio credit risks places strong and controversial assumptions on the behaviour of Small and Medium Enterprise (SME) borrowers within a credit portfolio. In particular, specific assumptions on the sensitivity of these borrowers to systematic developments, as represented by their asset correlations, have resulted in lower capital charges for SMEs as compared to larger borrowers and disjointed capital allocations among SME segments.

These assumptions include decreasing asset correlations with decreasing borrower size, for SME borrowers treated under the Corporate asset class, while SME borrowers treated under the Retail asset class are, by default, assigned generally lower asset correlations. Under both treatments, a negative relationship between asset correlation and Probability of Default (PD) results in diminished capital charges for SME borrowers, generally considered to be of a riskier nature than larger borrowers.

Empirical evidence on these strong assumptions has been mixed. Jacobson, Linde, and Roszbach (2005) reject the claim that SME borrowers require less capital than larger borrowers. Focusing on asset correlations, Duellmann and Scheule (2003) and Lopez (2004) find evidence of increasing asset correlation with borrower size, while Dietsch and Petey (2004) find evidence to the contrary, rejecting that assumption. On the relationship between asset correlations and PD the results appear weaker but nonetheless contradictory, with Lopez (2004) showing signs of a negative relationship and Gordy

(2000) and Dietsch and Petey (2004) finding a positive relationship. This work, and especially that focused on SME borrowers, has been marked by aggregated data sets and generally limited historical span.

We present a unique and rich source of Canadian SME credit data, and use it to estimate the portfolio credit risk characteristics of this crucial segment of the financial sector. The availability of such a rich dataset, made up of over 25,000 SME borrowers spanning a time period from 1997 to 2010, concentrated within a single portfolio presents a significant contrast with the existing literature which has typically relied on aggregated data sets of SME borrowers with, in many cases, limited historical span. In and of itself, the use of aggregated data presents a potential for a dilution of risk characteristics and, for single institutions looking to benefit from SME portfolio credit risk analysis, a potential for a misrepresentation of the risks as they may relate to a single lending entity; see, e.g., Basurto and Padilla (2006) and Dietsch and Petey (2004). For example, Dietsch and Petey (2002) estimate portfolio credit risk over a database of 220,000 French SME borrowers, accounting for more than two thirds of all French SMEs, Dietsch and Petey (2002, p. 305), spanning the period 1995 to 1999; Dietsch and Petey (2004) estimate SME portfolio credit risk characteristics over an aggregated database of 440,000 French and 280,000 German borrowers spanning the periods 1995 to 2001, and 1997 to 2001, respectively; Duellmann and Scheule (2003) use another aggregated database of over 53,000 predominantly small and private-owned German borrowers spanning the period 1991 to 2000, while Jacobson, Linde, and Roszbach (2005) study the riskiness of SME borrowers as compared to larger borrowers over an aggregated database of approximately 60,000 Swedish borrowers spanning the period 1997 to 2000.

We begin with a comprehensive description of the makeup of our SME portfolio, and the credit risk characteristics that define it. An understanding of such characteristics allows for a clearer understanding of where risks typically lay and the risk patterns that emerge within such portfolios. In particular, we present a dual dimension segmentation of the data along risk grade, size, and industry segments. The availability of such segmentations within a coherent SME portfolio is rare in the literature, and allows for involved empirical work in outlining robust relationships between size and risk grade segments of a portfolio. Ultimately, it is the study of these patterns that is a major driver of existing guidelines for portfolio credit risk measurement and management. This work is undertaken in Chapter 2.

Having studied the unique characteristics that underpin a high-risk SME loans portfolio we proceed, in Chapter 3, with a partial implementation analysis aimed at studying the impact of the various assumptions on the behaviours of SME borrowers as found in the Basel II Pillar 1 minimum capital regulatory framework. Specifically, we look at capital results for SME segments under the Standardized and Internal Risk Based (IRB) frameworks of Basel II, and examine the impact of the application of capital calculation adjustments based on borrower size, probability of default, Corporate versus Retail asset classification, and loan maturity. This analysis allows for a deeper understanding of the drivers of Basel II capital allocations across SME segments with respect to the assumptions made within the framework.

In Chapter 4 assumptions on the level and relationships of asset correlations in Basel II across SME segments are tested. We present a careful and robust partition of our data into homogenous segments of borrowers, by Risk and Size Groups in particular, and use a single factor implementation of the asset value model (*AVM*) to non-parametrically estimate correlations within these segments and evaluate patterns across them. The portfolio credit risk *AVM* framework is based on the work of Merton (1974) and Vasicek (2002) and is commercialized in such products as J.P. Morgan's *CreditMetrics*; see, J.P. Morgan (1997). Its application to the estimation of correlations derived from credit data is presented in Gordy (2000). The unique depth of our data allows for various SME segmentations to be presented and allows for an empirical testing of hypothesised patterns across segments free of theoretically imposed constraints. These points are emphasised in order to dispel purely theoretical assumptions or poor data quality as justifications for the imposition of patterns and relationships on results, as is found elsewhere in the literature.

Our work in Chapters 3 and 4 also serves to highlight the reduced data requirements in the Basel II framework as compared to the non-neglible requirements typically needed for the calibration of modern portfolio credit risk models. The Basel II Accords presented portfolio credit risk managers with an unprecedented degree of integration of internal bank rating and monitoring systems into regulatory capital adequacy frameworks. This was accomplished by the Accord's adoption of modern methods for portfolio credit risk measurement, familiar to modellers and managers in the field, within strict restrictions on the applied framework of these methods. In particular, these restrictions include an assumption of infinitely granular loans portfolios over which the regulatory framework is applied and a single factor underlying risks in the portfolio.

Our use of a single factor *AVM* in Chapter 4 allows for direct comparison to the asymptotic single risk factor model (ASRF) implemented in the Basle II IRB framework, and allows us to test the impact of the Basel II infinite granularity assumption. Previous tests of this assumption, namely those found in Gordy and Lutkebohmert (2007), have focused on the size of the portfolio and the PDs measured within it as potential factors in the estimation error that may arise in the application of the infinite granularity assumption to real-world portfolios. Our work will build on correlation analysis undertaken in Chapter 4 to extend the impact analysis of this assumption to this crucial credit risk variable.

In Chapter 5 we take a broader view of the underlying credit risks in an SME portfolio by using the *CreditRisk*$^+$ framework to extend our analysis from a single sector framework to one in which multiple sectors are modelled as risk drivers within an SME portfolio; here *CreditRisk*$^+$ refers to the actuarial-type model for default risk commercialized and released to the public by Credit Suisse, see Credit Suisse (1997). The use of multiple risk factors allows for the introduction of correlations between risk factors, calibrated from historical time series within our SME dataset, into the analysis of portfolio credit risk;

here we use methods presented in Burgisser, Kurth, Wagner, and Wolf (1999) and Akkaya, Kurth, and Wagner (2004). With this framework in place, we are thus able to challenge another assumption of the Basel II framework - that of a single risk factor - and provide evidence from a portfolio with distinct characteristics where such evidence has previously been found inconclusive; see, for example, BCBS (2006b). Our analysis quantifies the impact resultant from the use of three assumptions on the nature of risk factors driving the portfolio credit risk: that of a single risk factor, multiple correlated risk factors, and multiple independent risk factors. We are thus able to present results and impacts as derived explicitly from an SME loans portfolio with the unique credit risk characteristics derived in Chapter 2. Our ability to present a uniform calibration method for the three implementation assumptions in Chapter 5 underlines the robustness of our results on the impact of a single sector assumption with respect to other studies in the literature in which results have been undermined by incompatible calibration techniques; see, for example, Lesko, Schlottmann, and Vorgrimler (2004). The relevance and importance of these calibration methods is further investigated in Chapter 6, with specific recommendations for the application of the *CreditRisk*$^+$ framework to SME portfolios.

Our work in this Thesis, and in Chapter 6 in particular, presents a consistent calibration of the *AVM* and the *CreditRisk*$^+$ frameworks for the estimation of portfolio credit risk in a real-world SME environment. This work builds on findings in Koyluoglu and Hickman (1998) in which a single general framework with harmonized input parameters is shown to underline both models. In addition, we add to results in Gordy (2000) wherein a mapping between the asset value model and the *CreditRisk*$^+$ model is presented, by providing consistent calibrations specific to SME portfolios.

In particular, we present a simulation-based implementation of the *CreditRisk*$^+$ within a mathematically consistent framework to the *AVM*, and present an empirical SME-specific calibration of the *CreditRisk*$^+$ single sector normalized volatility consistent with the calibration obtained in the *AVM*. This SME-specific calibration can be traced back to the robust segmentations presented in Chapter 4 and the statistical default rate characteristics associated with these segments. The end-result calibration differs from those typically presented using Corporate data generally provided by external rating agencies such as Standard & Poor's; see, for example, Gordy (2000). In addition, our work on the *CreditRisk*$^+$ calibration methods reveals significant restrictions on calculated default correlations and, by implication, resultant capital charges in a portfolio.

From a prudential point of view, a not insignificant testament to the Basel II treatment of portfolio credit risk lies in the minimal changes to the framework considered in the post-2008 financial crisis transition to Basel III; for the Basel III implementation according to the Office of the Superintendent of Financial Institutions Canada (OSFI), see OSFI (2012). For both the *CreditRisk*$^+$ and the *AVM* we implement a prudential adjustment to our model parameters. In particular, we are able to boost asset correlations in the *AVM* to values observed in Basel II, a mapping between the two models allows this boost to be extended to the *CreditRisk*$^+$ framework. This exercise allows for Economic Capital level, and allocation, comparisons in line with prudential levels pursued by banks and regulators, and highlights a significant pragmatic component of modern portfolio credit risk management.

Despite the lack of change in the prescriptions for the treatment of portfolio credit risk for Small and Medium Enterprise loans, the transition from Basel II to new capital adequacy and risk management guidelines under Basel III certainly bears further comment. In particular, as the primary global regulatory initiative in response to the post-2008 financial crisis, Basel III introduced a number of amendments to the Basel II framework aimed at increasing the resilience of both, individual banks and the financial sector as a whole; see BCBS (2010). These amendments include:

- *An increase in the quality and quantity of capital* – with greater emphasis on common equity and retained earnings as the basis of "going concern" capital;
- *Greater risk awareness, recognition and coverage* – especially as these risks relate to complex securitizations and credit derivative products;
- *Supplementary measures to risk-based capital requirements* – aimed at countering model risk and measurement error in risk-based measures, reducing procyclicality effects and constraining excess leverage in the financial system;
- *Addressing systemic risk and interconnectedness* – through the identification of globally and domestically systematically important financial, and;
- *Introduction of strong global liquidity standards* – aimed at ensuring that financial institutions are capable of withstanding extended periods of illiquidity in the market place, as well as ensuring a robust funding structure for institutions over the long run.

Due to the dynamic and interactive nature of the issues covered in this Thesis, we present in Figure 1.1 a schematic for the structure of the Thesis which should help the reader trace the issues discussed and the contributions presented across various Chapters and Sections. For example, the impacts of various calibration methods presented in Chapter 6 can be traced to robust segment characteristics described in Chapter 4, while patterns across estimated asset correlations in Chapter 4 are elucidated by portfolio characteristics described in Chapter 2. In Chapter 5 we tackle the Basel II single factor assumption described in Chapter 3 and calculate industry-based capital charges reflective of characteristics highlighted in Chapter 2. Average asset correlations in Chapter 3 form the basis for the prudential adjustment presented and applied in Chapters 4 and 6.

To recap, Chapter 2 presents the *Financing Company* portfolio of SME loans and explores the portfolio characteristics as well as the general structure of data inputs into models of portfolio credit risk. Chapter 3 introduces the Basel II regulatory framework for the treatment of portfolio credit risk; we analyze the approaches and assumptions of the models used in the framework, as well as the data requirements in the application of the framework. We present an evaluation of the impact of various aspects of the Basel II framework through a partial implementation analysis, and describe capital allocations across SME segments under Basel II. Chapter 4 presents the asset value model (*AVM*) as a tool for both, portfolio credit risk estimation and empirical estimation of SME correlations directly from data in our SME portfolio. Our work in Chapter 4 is bolstered by our ability to segment our data by both Risk and Size Groups, thereby providing empirical evidence on any relationship or pattern that may exist among correlations across these segments. Results are generated for asset correlation levels and patterns as well as resultant capital allocations. Chapter 5 introduces the *CreditRisk*$^{+}$ framework and presents results on the overestimation that may arise from the use of a single factor model. Finally Chapter 6 presents a comparison between the results and models of Chapters 3 to 5, as well as consistent calibration of the *AVM* and *CreditRisk*$^+$ frameworks and a comparative analysis of capital allocations as compared to Basel II.

The major contributions of this Thesis can be found in full detail in the concluding section of Chapter 6, and are presented here in summary form as follows:

1. *A Comprehensive Analysis of an SME loans portfolio within a financial institution*
   SME portfolios in the literature have typically been of an aggregated nature with limited historical time span. Our in depth analysis of a rich and heavily populated SME loans portfolio provides a unique database over which significant analysis can be performed.

2. *A Detailed Schematic for SME Portfolio Credit Risk Input Data and Structure*
   The significant depth of our unique database allows for a dual-dimension segmentation of our SME portfolio. As such, we are able to estimate credit risk measures, such as probabilities of default (PDs) and correlations, for homogenous segments of borrowers dually defined by risk grade and size. These credit risk measures form the underlying basis on which our work in this Thesis is conducted, both in testing the assumptions and relationships inherent in the Basel II treatment of SME portfolio credit risk, and in establishing internally-calibrated models of our own. The elevated data requirements accompanying the estimation of these models, and our ability to meet them in a robust manner, highlights the unique and important data source on which our results are based.

3. *A View of Conceptual & Pragmatic Implications of Basel II treatments for SMEs*
   We engage in a Partial Implementation exercise to test the impact of the assumptions within each implementation on capital charges. Our focus is on SME borrower Size segments and we find a U-shaped capital allocation with increasing borrower size with the source being a size adjustment applied to asset correlations.

4. *Empirical findings concerning SME Asset Correlations*
   These results run counter to Basel II specifications of a negative relationship between asset correlations and PD, and the Corporate asset class assumption of a positive relationship between asset correlation and Size. Our results appear to provide some support to specifications under the Retail-Other treatment, however this support remains limited by that treatment's programming of a negative

relationship of asset correlations with PD. In addition, our results on the lack of clear relationships between asset correlations and size and PD, contrast with the literature wherein such relationships have been deduced from generally weak empirical evidence. Our work in defining robust risk-size segments of SME borrowers allows us to present stronger evidence the presence of such relationships, or lack thereof.

5. *Increased GranularityEffectsin SME credit portfolios with Low Asset Correlations*
   Our findings appear within the context of a very large portfolio and show an approximation error, or granularity effect, of approximately 6% on Economic Capital. This result is the first, to our knowledge, to show empirical evidence of a link between asset correlation values and the granularity effect in a credit portfolio.

6. *Empirical Evidence on Economic Capital Impact of Single Sector Assumption*
   Our results show that for our portfolio of SME borrowers the use of a single risk factor can increase EC figures by approximately 40%. The assumption of independence across multiple sectors in our portfolio was shown to underestimate Economic Capital charges by approximately 60%.

7. *A Consistent Calibration of Single Sector* CreditRisk$^+$ *and Asset Value Models for SME Portfolio Credit Risk*
   Our results show that a calibration of the risk factor weights according to segment-specific ratios of the PD standard deviation to its unconditional mean, in the presence of a fixed sector normalized volatility figure of 0.5, generates segment-specific default correlations consistent with those observed in the *AVM*. In such a setting, the accompanying *CreditRisk*$^+$ loss distribution displays fatter tails that of the *AVM* implementation, and therefore produces higher EC values. Alternatively, we show that a fixed unitary weight setting for the *CreditRisk*$^+$ model can provide a comparable loss distribution to the *AVM*, with thinner tails. We also reveal implications of various calibration methods on default correlations and highlight the potential for strict restrictions as a result.

8. *An SME portfolio-specific calibration refinement for* CreditRisk$^+$ *models*
   Fixed sector normalized volatility values of 0.5 and 0.25 are not commonly found in the literature, which has tended to focus on calibrations from Corporate borrowers. These calibrations, along with the unitary weight calibration, therefore present SME-specific calibrations of the *Single Sector CreditRisk*$^+$ model.

9. *A thorough Assessment of Basel II approaches to SME credit risk modelling*
   Our results reveal that Basel II leads to misallocation of capital charges, such that in some cases, smaller and riskier SME borrowers are charged less than larger and safer SME borrowers. These Basel II capital charges can represent cases of under- or over-charging of capital to borrowers as compared to the capital charges they would incur under internally-calibrated models of portfolio credit risk.

10. *Suggestions for an SME portfolio credit risk management framework*

The misallocation of capital across SME borrower segments in Basel II may be alleviated through the removal of size-based adjustments within SME segments. Such a case exists in the Retail-Other treatment of SME borrowers, but is limited in its applicability to all SME segments due to exposure limits and other restrictions on its use. Our adoption of a simulation-based implementation methodology in the *CreditRisk*$^+$ framework, and our successful calibration of the model to our SME portfolio in a manner consistent with that of the *AVM* provides another avenue for SME portfolio credit risk measurement and management, and presents practitioners with a variety of settings to which the model structure can be set without some of the drawbacks usually associated with original model and its suitability to SME portfolios. The *AVM* provides a useful a direct avenue for allowing the data to talk, and revealing patterns and relationships across segments while allowing for the estimation of portfolio credit risks.

Chapter 2. A Canadian Small & Medium Enterprise Loans Portfolio

Our study of Small and Medium Enterprise (SME) portfolio credit risk is centred on the unique characteristics of our Canadian portfolio and the *Financing Company* in which it resides. In particular, we note that the *Financing Company* that is the source of our data is a specialized SME financer that specifically targets high risk niches within the SME loans market, both in terms of borrowers of diminished size (e.g., assets, sales, etc.) and industries which have historically faced some level of under-servicing from Canadian banks.

The *Financing Company* portfolio, therefore, is one in which small borrowers make up the vast majority and where exposures to medium-sized enterprises, can present a significantly large exposure for the portfolio. In addition, we note a marked concentration of our SME loans portfolio in the Manufacturing sector, widely considered a source of elevated risk. This manufacturing focus has resulted in an Ontario- and Quebec-centered geographical dispersion among borrowers in the portfolio.

This concentration in the riskiest segments of the Canadian credit market has given rise to a rich and unique database through which extensive segmentation and analyses of credit behaviour can be observed. In particular, we note that Size Buckets – even within industry-defined "small borrowers" segments, for example – can provide distinct information on default behaviour, with the smallest Sizes showing exceptionally elevated default rates. In addition, we observe that default rates suggest a reaction to macroeconomic events, with the degree and shape of this reaction potentially differing between industry-based segments, as well between size-based and risk-based segments.

These heuristic observations form the basis of further study on the portfolio and the behaviour of its credit risk components. Chapter 2 is thus organized as follows: Section 2.1 provides an overview and introduction to the *Financing Company*'s mission and operational scope; Section 2.2 provides a brief review of the treatment of SMEs in the credit risk literature; Section 2.3 explores in detail the *Financing Company* risk management systems and takes a snapshot of the portfolio at the heart of the thesis; Section 2.4 concentrates on analysing the Finance Company's default rates; finally, Section 2.5 provides a summary in the form of ten stylized facts related to the *Financing Company* portfolio, both in terms of the portfolio we evaluate and its history, and in terms of the underlying credit risk drivers – namely the default rates – which define the shape and characteristics of the loss distribution generated by this unique portfolio of the riskiest segments of the Canadian SME credit market.

Section 2.1. The *Financing Company*

The *Financing Company* whose loan portfolio is studied in this Thesis provides specialized lending solutions to Canadian SMEs facing difficulties in obtaining financing from traditional sources. As such, the *Financing Company* focuses on the capital needs of those segments of the market facing constraints in financial servicing.

This focus has resulted in a significant presence of the *Financing Company* in areas where the market has generally failed to provide adequate access to financing. Industry Canada classifies gaps in financing to SMEs as the following: the *risk gap*, characterized by conventional lenders' unwillingness to supply financially riskier loans even as demand for those loans generates higher interest rates; the *size gap*, reflecting chartered banks' preference for larger-sized business loans over relatively higher cost small-sized business loans; the *flexibility gap*, describing the lack of flexibility in repayment terms and conditions for companies with distinctive growth and revenue streams; and, the *knowledge gap*, reflecting an observed reticence on the part of lenders to provide loans to businesses operating in a knowledge-based industry (KBI) – such as the arts, computer services, electronics and biochemical industries – with entrepreneurs' lack of tangible assets and lenders' lack of know-how in these industries seen as possible factors in the widening of this gap; see Industry Canada (2001). Thus, the *Financing Company* is strategically positioned to provide financial services and support to a segment of the SME loans market that, is at least qualitatively, riskier than the rest of the market.

In order to account properly for the elevated risk levels inherent in the segment of the loans market in which it operates, and by extension its loans portfolio, the *Financing Company* must ensure that it maintains internal risk systems capable of accurately measuring and managing those risks. These systems must thereby provide sensible quantitative assessments of the credit risk as measured by Expected Loss and/or Provisions for credit losses, and by Unexpected Losses and/or Economic Capital.

Section 2.2. SME Loans Credit Risk Measurement

The *Financing Company*'s data is that of a portfolio of SME credit exposures, where SMEs are defined as enterprises with fewer than 500 employees and less than CAD $50 million in annual revenues. In this Section we underline the significance of SMEs in the broader economy, and review the credit risk measurement tools and conventions, as presented in the literature, used in their evaluation.

Industry Canada relates enterprise size to the number of employees. As such, small enterprises are defined as those employing up to 99 employees, while businesses with 100 to 500 employees are regarded as medium. Approximately 97% of the businesses serviced by the Finance Company can be considered small, 2% medium-sized and 0.3% large. According to Altman and Sabato (2005), SMEs account for over 97 percent of the total number of firms in OECD countries, while they employ approximately half of the entire workforce and account for over 99 percent of all employers. In Canada the story is similar with SMEs accounting for over 99 percent of all businesses in the country, employing approximately 50 percent of the labour force, or about 5 million people, and, in the case of small businesses with fewer than 50 employees, accounting for 26 percent of national GDP, Industry Canada (2009).

In addition to their significant economic presence, SMEs pose an interesting challenge to credit risk management through their characteristics. In the same paper, Altman and Sabato (2005) point out that SME credit risk profiles can differ significantly from those of corporate borrowers. This feature is especially relevant in relation to those exposures' default correlation structures and overall credit quality. These differences can be exacerbated in the presence of banks' information deficiencies when it comes to SME borrowers. These deficiencies arise, in part, due to the high cost-to-dollar value ratio of extensive monitoring systems for these small-sized loans (along with other small-sized "Retail" loans such as personal, credit card and residential mortgage loans) as compared to "Wholesale" loans (in which large corporate as well as sovereign loans are classified).

The retail credit market is typically used by small unrated borrowers to access funds. These borrowers require loans that are relatively miniscule when compared to loan sizes in the wholesale market. As such, the loss on any single retail loan has minimal effects on a bank's solvency. Loans in the wholesale market, on the other hand, are usually made to agency-rated borrowers on a syndicated basis, and for which there generally exists a secondary market. And while the most significant drivers of risk for retail loans remain the Probability of Default (PD), Loss Given Default (LGD), Exposure and Default Correlation, the characteristics of these drivers differ significantly from those of wholesale portfolios.

In an extensive survey of credit risk management practices for retail credit products, RMA (2000) finds that lower default correlations between retail credit products, as compared to corporate credit products, lead to lower economic capital requirements for those products; while, by contrast, those same retail products generally require a higher level of provisioning due to higher PD and LGD estimates and, consequently, higher expected losses than corporate credit products. This result is reinforced in Dietsch and Petey (2004) as underscored by Altman and Sabato (2005).

In Dietsch and Petey (2004) the level, volatility and correlation of default rates for German and French SMEs are studied, where SMEs are defined as "incorporated firms with turnover under €40 million", Dietsch and Petey (2004, p. 776). By segregating obligors into size buckets according to turnover (Bucket 1: <€1 million, Bucket 2: €1m - €7m, Bucket 3: €7m - €40m), as well as risk rating categories, the authors are able to observe and measure default characteristics by size and credit quality. As such, the authors generally observe decreasing default rates by size for a given credit quality in the French sample. The German sample, however, does not show evidence of a similar monotonic decrease in default rates with size.

In terms of default correlations, Dietsch and Petey (2004) find that default correlations are higher for smaller firms than for larger ones, although the robustness of this finding is questioned by the authors. A more solid finding is that of higher correlations for obligors with lower credit quality, independent of size. To explain this result, the authors point to the possibility of a diversification effect operating across firms of similar size but different industry. The authors find higher default rate volatility for larger size buckets and argue that this can be construed as a form of higher sensitivity to economic conditions. In addition, RMA (2000) also finds that constrained data collection systems may apply to banks' retail loans portfolios and limit the availability of individual loan tracking data for more than two years.

Retail loans portfolios can therefore pose specific challenges which require an alternative approach than a simple scaling down of approaches developed for wholesale portfolio modeling; Allen, DeLong, and Saunders (2003). Again RMA (2000) finds that it is not uncommon for a bank to apply different approaches for tracking and management of its retail credit products than those used for its wholesale credit products. For instance, where Mark-to-Market (MTM) models – in which credit losses due to credit exposures' upgrade or downgrade are measured and accounted for – might be used to measure credit risk for a bank's corporate loans portfolio, the same bank might opt to use a Default Mode (DM) – in which credit losses are strictly defined in terms of defaults – for its retail loans portfolios.

Section 2.3. The *Financing Company* Portfolio and Risk Management Systems

The *Financing Company*'s loans portfolio as of March 2009 is composed of over 35,000 loans to over 25,000 borrowers, totalling over \$10 billion in dollars Outstanding (\$OS). Loans are segregated into a Performing Loans Portfolio, containing those loans not classified as impaired according to the *Financing Company* criteria, and an Impaired Loans Portfolio. The *Financing Company*'s Impaired Loans Portfolio, which accounts for approximately 5% of the *Financing Company*'s loans portfolio by number of loans and dollars outstanding, will not be discussed in this thesis. Loans and borrowers classified as Performing but subject to “watch list” monitoring are also excluded from our analysis.

Our study of the *Financing Company* Performing loans portfolio will therefore examine both the number of borrowers in the Bank’s portfolio as well as the $OS to these borrowers. Our analysis will concentrate on the characteristics of the portfolio as defined by the credit quality, size, industry, and geographical region. Our characterization of each of these dimensions follows the internal *Financing Company* risk classification and management systems and terminology, while some adjustments may be applied to ensure anonymity for the Finance Company.

For each borrower in the *Financing Company* credit portfolio our data consists of an internally assigned Risk Rating (RR), Size segmentation, Industry, and the dollars Outstanding at a given time. At the loan level, our data consists of the Security Coverage Interval (SCI) and months to maturity (Maturity). The separate assessment of default risk, through the RR system, and of collateral and recovery risk, through the SCI system, highlights the *Financing Company*’s use of a two-dimensional system for the evaluation of credit risks in its portfolio. To assign Risk Ratings, the *Financing Company* uses two separate scoring methodologies for borrowers on either side of a $250,000 size threshold. The scoring methodologies are then reconciled to a common Risk Rating. This method aims to exploit the discrepancies in available financial records for borrowers on either side of the threshold so that, for example, small borrowers with inadequate financial data to be approved under a credit scoring model for larger, corporate borrowers may still qualify for financing when such factors as the owner's credit history are evaluated. The Size measurement is based on the borrower's maximal total commitment to the *Financing Company* at last authorization, including $OS to other borrowers with common ownership on the *Financing Company* books.

In the subsequent subsections, portfolio characteristics will be evaluated along overall dimensions, such as Risk Rating or Size, as well as for segments at cross-sections of these overall dimensions, e.g., the *>$5,000,000* – 5 RR Size-Risk Rating segment. This analysis is extended to borrowers as well as $OS in the portfolio.

Subsection 2.3.1. The Portfolio at a Glance: Borrower Concentration

Table 2.1 provides key figures and percentiles along various benchmark $OS values. In Table 2.1 we observe that just over 55% of the *Financing Company* borrowers have $OS of $150,000 or less, while approximately two-thirds of borrowers have $OS of $250,000 or less. These borrowers' cumulative $OS account for approximately 14% of the overall portfolio $OS. On the other end of the spectrum we observer that less than 3% of the *Financing Company* borrowers have $OS greater than $3,000,000, accounting for approximately 25% of overall portfolio $OS, while approximately 10% of borrowers have $OS greater than $1,000,000, accounting for approximately 60% of overall portfolio $OS. In an extensive examination of bank balance sheets, Carey (2000) finds that the largest 10% of credit exposures generally account for approximately 40% of total exposure in a bank's commercial loan portfolios. Our results in Table 2.1 show, by contrast, that the largest 10% of the *Financing Company* clients account for approximately 60% of overall dollars outstanding, indicating a significant over-concentration of $OS among a relatively small proportion of borrowers for the *Financing Company* portfolio as compared to those of commercial institutions.

Subsection 2.3.2. The Portfolio at a Glance: Risk Systems

In a survey of the fifty largest US banking organisations' internal risk rating and credit scoring systems Treacy and Carey (2000) find that approximately 60% of those institutions use one-dimensional ratings systems in which a single rating incorporates both the PD and the LGD. The remaining 40% of those banking institutions use two-dimensional systems appraising obligors' credit worthiness (e.g., Risk Rating) on one scale and the risk of exposure loss on another.

For each loan, the *Financing Company* documents the percentage of dollars authorized that is secured by borrower collateral. This percentage, when extended into intervals referred to as Security Coverage Intervals (SCI), provides the segmentation along which the *Financing Company* applies its Loss Given Default (LGD) measurement and estimation. The intervals over which security coverage categories are organised, and along which LGD estimates are made, can vary depending on the granularity of security coverage categories required. While Risk Rating, and therefore credit quality and/or default, evaluations are made at the borrower level, Security Coverage evaluations are made at the loan level. A common convention is to classify loans with less than 30% SC as unsecured and those with 70% SC as secured. An SC greater than 100% is not uncommon among loans.

For its PD evaluation, the *Financing Company* uses internal credit scoring and risk rating systems to assign to each borrower a RR ranging from 1 (least risky rating) to 9 (riskiest rating) for borrowers in the Performing portfolio. The 10 RR acts as a watch-list grouping for borrowers still performing but with extremely elevated risks warranting enhanced monitoring – this can include previously defaulted or impaired borrowers who have been cured. Technically part of the Performing portfolio, the 10 RR is excluded from our analysis. Performing portfolio RRs, consisting of 9 rating grades, are assigned at authorization and are reviewed at one-year intervals for most obligors. In their survey of the largest 50 U.S. banking institutions, Treacy and Carey (2000) find that the number of ratings for performing borrowers varies between two and a figure in the low twenties, with a median of five performing loans grades. Banks with eight or more grades accounted for only 8% of banks surveyed. These results indicate that the *Financing Company*'s risk rating systems are comparable to the top quantile of banks' internal rating systems in terms of granularity of risk rating grades. However, direct comparisons of banks' internal risk rating systems are not only constrained by the varying number of ratings and the diversity of classification systems and criteria employed at these institutions, but also by the activities of the banks and the composition of their portfolios. For instance, according to Allen, DeLong, and Saunders (2003), banks with significant activity in the large corporate loans market tend to have more risk rating grades for investment-grade instruments than those for sub-investment grade ones. For banks with a predominantly middle-market loans portfolio, the number of investment-grade and sub-investment-grade rating grades tends to be more balanced, with middle-market portfolios here approximated by a test portfolio with 2,500 obligors with average exposures of £894,000 and an investment grade to non-investment grade loan ratio of 2:1, Allen and Saunders (2002, p. 144).

Tables 2.2 and Figures 2.1 segregate borrowers and $OS into Risk Ratings and Size Buckets. The main findings evidenced by the table are presented such that findings under points (a) will deal with concentrations of borrowers while those under points (b) will deal with concentrations of $OS. The findings are as follows:

1. a) The Size Bucket with the largest number of borrowers, for the portfolio as a whole, is the *$250,000 - $1,000,000* Size Bucket, containing 30% of the overall number of borrowers in our portfolio. b) The Size Bucket with the largest amount of $OS, for the portfolio as a whole is the *$1,000,000 - $3,000,000* Size Bucket, containing 31% of the overall $OS in the portfolio.

2. a) The RR with the largest number of borrowers, for the portfolio as a whole, is the 8 RR, containing 16% of the overall number of borrowers in our portfolio. b) The RR with the largest amount of $OS is the 3 RR, containing 13.5% of the overall $OS in the portfolio.

3. a) The RR and Size Bucket segment containing the highest number of borrowers is the *≤$100,000* Size Bucket - 9 RR segment, containing 7% of the overall number of borrowers in the portfolio. b) The RR and Size Bucket segment containing the largest amount of $OS is the *$1,000,000 - $3,000,000* Size Bucket – 6 RR segment, containing 4% of the overall portfolio $OS.

4. a) Within RRs, we observe that for the 1 to 7 RR the *$250,000-$1,000,000* Size Buckets are the most heavily populated; while for the 8 and 9 RRs, it is the *≤$100,000* Size Bucket that contains the largest number of borrowers. b) Similarly, we find that the 1 to 7 RRs show the highest concentration of $OS in Size Buckets of *$1,000,000 or greater*, while the 8 and 9 RRs show the highest concentration of $OS in the smaller *$250,000 - $1,000,000* Size Bucket.

5. a) Examining our data by Size Bucket, we observe that for those borrowers in Size Buckets of *$250,000 or less*, the RR containing the largest number of borrowers are the 8 and 9 RRs. For those Size Buckets in the *$250,000 to $3,000,000* range, the Risk Rating with the highest number of borrowers is the 7 RR, while in Size Buckets of *$3,000,000 or more* it is the 3 RR that accounts for the highest number of borrowers. Finally, we note a weakly inverse relationship between the concentration of borrowers in Size Buckets of *$250,000 or less,* and credit worthiness. This inverse relationship results in the proportion of borrowers in Size Buckets of *$250,000 or less* increasing from 33% for the 1 RR to 83% for the 9 RR. This concentration of approximately 80% of borrowers in the smallest

> Size Buckets for the 9 RR, along with one of 60% for the 8 RR, represents a significant departure from the small-Size Bucket concentration range of 32% to 45% observed in lower RRs. For the portfolio as a whole, the proportion of borrowers in Size Buckets of *$250,000 or less* is 49%. b) Examining the distribution of $OS between Size Buckets for given RRs, we observe a monotonic increase in credit qualities containing the highest proportion of $OS by Size Bucket. This is exemplified by the *≤$100,000* Size Bucket having the 9 RR as the RR with the highest proportion of $OS while the *>$5,000,000* Size Bucket has the 2 RR as the RR with the highest proportion of $OS.

To summarize, we observe that approximately 1 in 5 borrowers in our portfolio can be classified in the smallest and riskiest segments (Size Bucket of *$250,000 or less* and Risk Rating of 8 or worse). In addition, we find that the riskiness of the borrowers decreases monotonically with size so that Size Buckets of *$250,000 or less* are the most likely to have the highest proportion of borrowers in the 8 or 9 RRs, while the Size Buckets of *$3,000,000 or more* are the least likely. This result is reinforced by the finding that for the 1 to 7 RRs, it is the *$250,000 - $1,000,000* Size Bucket that contains the highest proportion of borrowers, while for the 8 and 9 RRs, it is the *≤$100,000* Size Bucket.

Figures 2.1A and 2.1B provide a visual representation of the distribution of borrowers across RRs and Size Buckets. From Figure 2.1A we observe that for the 8 and 9 RRs the number of borrowers is generally decreasing with Size Bucket, so that the Size Buckets with the most borrowers are the smallest. This pattern is generally transformed as RR decreases, so that borrowers in RRs from 1 – 7 display a generally bell shaped distribution within RRs and across Size Buckets, with significant skewness towards higher value Size Buckets. By contrast, Treacy and Carey (2000) find that among the largest 50 U.S. banks, 36% of them assign more than half of their rated loans to a single risk grade, the *Financing Company*'s risk rating system therefore seems to provide more adequate segregation and classification of obligor default risk.

Subsection 2.3.3. The Portfolio at a Glance: Industry Concentration

The *Financing Company* borrowers are also grouped along industry, providing a measure of the *Financing Company*'s activity throughout the various sectors of the Canadian economy. For our immediate purposes, we have chosen a NAICS-based 11 industry classification system of the *Financing Company* credit portfolio. Borrowers are classified as accordingly belonging in one of the following industries: Business Services (BUS); Construction (CON); Manufacturing (MAN); Non-Business Services (NBUS); Resources (RES); Retail (RET); Supplier of Premises (SOP); Tourism (TOU); Transportation and Storage (TRS); Wholesale (WHS); Other (OTH). A full description of the industries and their composition is given in the Appendix to Chapter 2. Table 2.3 provides a breakdown of the distribution of borrowers across Size Buckets and Industries, while Figure 2.2 provides a visual representation. The main findings revealed by the table can be listed as follows:

1. a) The Industry with the largest number of borrowers is the Manufacturing industry, containing 24% of all borrowers in the portfolio. b) The Manufacturing industry is also found to account for the highest proportion of $OS in the *Financing Company* portfolio, totalling 31% of the overall $OS.

2. a) We observe that the predominance of the Manufacturing industry carries through to all Size Buckets, where it accounts for a majority of borrowers in each. b) For all Size Buckets, the Manufacturing industry is also found to account for the highest proportion of $OS, albeit to varying degrees. For instance, while the MAN industry accounts for the highest proportion of $OS in the *≤$100,000* Size Bucket, it is only 3% greater (in terms of overall $OS for that industry) than the industry with the second highest proportion of $OS (RET). For the *≥$5,000,000* Size Bucket, however, the difference between the largest and second largest industries, MAN and RET, respectively, is 24%. As such, Table 2.3 allows us to document a positive relationship between concentration of clients in the MAN industry and Size Bucket. An explanation can perhaps be found in the MAN industry as one in which fixed and start-up costs are the most elevated, compared to Retail, Wholesale and the Services industries, for instance.

3. a) Within Industries, we observe that the Size Bucket with the highest proportion of borrowers in almost all industries is the *$250,000 - $1,000,000* Size Bucket. Exceptions can be found in the Business Services, Non-Business Services, Wholesale, and Other industries, for which the *≤$100,000* Size Bucket accounts

for the highest proportion of borrowers. b) In addition, we observe that for almost all industries the Size Bucket with the highest proportion of $OS is the *$1,000,000 - $3,000,000* Size Bucket, with the exceptions being the NBUS industry (for which the *$250,000 - $1,000,000* Size Bucket is the largest), and the SOP and TRS industries (for which the *≥$5,000,000* Size Bucket is the largest).

4. a) Unsurprisingly, we observe that the Size Bucket-Industry segment with the highest number of borrowers is the *$250,000-$1,000,000* Size Bucket in the Manufacturing industry, containing 7.5% of all borrowers in the portfolio. b) In addition, this result holds true for the $OS wherein the *250,000-$1,000,000* Size Bucket in the Manufacturing industry accounts for 10% of all $OS.

5. a) Table 2.3 shows that the industry with the highest proportion of borrowers in Size Buckets of *$250,000 or less* is the BUS industry, with 65% of borrowers in those Size Buckets, while the industry with the lowest proportion is the SOP, with 18% of borrowers in those Size Buckets. Excluding both SOP and BUS industries, we observe that this proportion varies between approximately 40% and 60%, with a mean and median of 50% across all industries. As such, the SOP industry presents a significant outlier in its proportion of borrowers in Size Buckets of *$250,000 or less*. Conversely, out of all the industries, the SOP industry has the highest proportion of borrowers in the *250,000-$1,000,000* Size Bucket. This result may suggest a significant size threshold for that industry. b) Similarly, in terms of $OS, we observe that the BUS and SOP industries account

for the industries with the highest and lowest proportion of $OS in Size Buckets of *$250,000 or less*, respectively. In addition, we find both the TRS and MAN industries exhibit significantly lower concentrations of $OS in Size Buckets of *$250,000 or less* (5%) as compared to the mean and median (9%) across all industries.

Segregating our industries along Risk Ratings, as in Table 2.4 and Figure 2.3, we observe the following:

6. a) For all industries barring the CON and WHS industries, the 8 RR accounts for the highest proportion of borrowers, with proportions ranging from 24% for the SOP industry and 14% for the MAN industry. For the CON and WHS industries, we observe the highest concentration of borrowers in the 9 and 3 RRs, respectively. Taking the two highest RRs of 8 and 9 together, we observe a concentration of 31% of the overall portfolio, with the highest concentrations across industries being in the SOP, TOU, NBUS and OTH industries (approximately 35%), and the lowest being MAN and WHS industries (23% and 25%, respectively). b) Conversely, a broad pattern is hard to detect when examining the distribution of $OS across RRs for each industry, with industries showing a large variety of predominant concentrations among RRs, and these concentrations being limited to a range of approximately 15% to 20%. One surprising observation is that of the 2 RR being the RR with the highest concentration of $OS for the CON industry – which is the only industry to have the 9 RR contain the highest concentration of borrowers. The industry with the

highest risk top concentration of $OS is the RES industry, for which 17% of $OS have a RR of 8. The CON industry is joined by the MAN industry, for which 14% of $OS have a RR of 2. The CON industry is the industry exhibiting the highest concentration level in any one RR within an industry, at 21.4%, while the MAN industry exhibits the lowest at 14.1%. For the portfolio as a whole, the 3 RR contains the largest concentration of $OS, accounting for 16.0% of the overall portfolio $OS.

7. a) Unsurprisingly, we observe that the MAN industry accounts for the highest proportion of borrowers for all RRs. We observe that the proportion of borrowers in the MAN industry decreases with increasing RR so that for the 1 RR 30% of borrowers are in the MAN industry, while for the 9 RR, that figure is 17%. For the portfolio as a whole, the concentration of borrowers in the MAN industry is 24%. b) In addition, the MAN industry accounts for the highest concentration of $OS for all RRs. We observe that the proportion of $OS in the MAN industry decreases with increasing RR, so that 40% of $OS in the 1 RR are attributed to the MAN industry while the same can be said of only 22% in the 9 RR. For the whole portfolio, 31% of $OS can be attributed to the MAN industry.

8. a) The RR-Industry segment with the highest proportion of borrowers is the 8 RR in the MAN industry, containing 831 borrowers and accounting for 3.3% of all borrowers in the portfolio. b) The RR-Industry segment with the highest

concentration of $OS is the 2 RR in the MAN industry, containing $464m in $OS and accounting for 4.4% of the overall portfolio $OS.

Summarizing, we observe that the MAN industry is the largest in terms of both, number of borrowers and $OS to those borrowers, accounting for nearly a quarter of the portfolio in the former and nearly a third in terms of the latter. This relationship is replicated in each Size Bucket for the portfolio. A bell shaped distribution of borrowers (with skewness towards higher value Size Buckets) is exhibited for most industries, including MAN, TOU, SOP and TRS. However, for industries such as BUS, RET and WHS, we observe a decreasing proportion of borrowers with Size Bucket. This phenomenon, along with that observed in (3.a.), could be explained by the low start-up and operational costs in such industries as RET and BUS. This argument is reinforced with the large predominance of borrowers in Size Buckets of *$250,000 or less* in the low-cost BUS industry and disproportionately low proportion of borrowers in Size Buckets of *$250,000 or less* for the high-cost SOP industry. For almost all industries, the 8 Risk Rating accounts for the highest concentration of borrowers. The lack of a bell shaped curve in defining borrower distributions is evident in Figure 2.3a, where at best, we observe an almost bimodal distribution, with the first mode centered on the 8 RR and the second mode around the 4 RR.

Subsection 2.3.4. The Portfolio at a Glance: Geographic Distribution

Table 2.5 provides a snapshot of the *Financing Company*'s presence (in terms of its Performing portfolio), by Industry, across twelve Canadian regions: Newfoundland & Labrador (N. & L.); Prince Edward Island (P.E.I.); Nova Scotia (N.S.); New Brunswick (N.B.) – collectively, these four provinces are referred to as the Maritimes; Quebec (QC); Ontario (ON); Manitoba (MN), Saskatchewan (SK), Alberta (AL) – collectively, these three provinces are referred to as the Prairies; British Columbia (B.C.); the Yukon (YK), Northwest Territories and Nunavut (N.W.T.). Table 2.5 and Figure 2.4 describe the distribution of borrowers and $OS across geographical regions and industries, while Table 2.6 and Figure 2.5 describe the distribution of borrowers and $OS across geographical regions and Size Buckets. Results from Table 2.5 and Figure 2.4 can be described as follows:

1. a) Nearly two thirds of the *Financing Company* borrowers are concentrated in the provinces of Quebec and Ontario, which contain approximately 34% and 27% of borrowers, respectively. b) Quebec and Ontario jointly account for approximately 70% of overall $OS in the *Financing Company* portfolio, with the former accounting for 38% of $OS and the latter accounting for 30%.

2. a) MAN is the predominant industry, in terms of borrowers, for the Prairies, New Brunswick, B.C., Ontario and Quebec, in the *Financing Company* portfolio. For provinces and territories where MAN is not the predominant industry among the

Financing Company borrowers, we observe RET to be. Two exceptions to this general observation are found in P.E.I. and the Yukon, for which the Tourism industry accounts for the highest concentration of the *Financing Company* borrowers. b) The MAN industry is similarly observed to have the highest concentration of $OS in the Prairies, New Brunswick, B.C., Ontario and Quebec regions. For most other provinces or territories, RET accounts for the highest proportion of $OS, with the exception being the P.E.I. where the TOU industry accounts for 33% of $OS.

3. a) For almost all industries, Quebec and Ontario are the provinces in which the highest concentration of borrowers is found and which make up a majority of all borrowers for those industries. The exception is Resources industry wherein the highest concentration of borrowers is found in Newfoundland and Labrador. b) Similarly, the Resources industry presents the only exception to the finding that for all industries, Quebec and Ontario account for the highest concentration of $OS. For the Resources industry, the highest concentration of $OS can be found in province of Alberta. It should be noted that the MAN industry shows an extremely elevated $OS concentration in the provinces of Ontario and Quebec. These two provinces combined account for 65% of $OS in the *Financing Company*'s MAN industry portfolio.

4. a) Accordingly, borrowers in the MAN industry in the province of Quebec account for the highest concentration of borrowers in any Industry-Geography

segment. This segment is observed to contain approximately 10% of all borrowers on the *Financing Company*'s portfolio. b) The MAN industry in Quebec is the Industry-Geography segment with the highest concentration of $OS in the portfolio, containing 13% of the overall portfolio $OS.

Results from Table 2.6 and Figure 2.5 are described as follows:

5. a) For most geographical regions we observe approximately 80% of borrowers to be concentrated in Size Buckets of *$1,000,000 or less*. Specifically, we note that Manitoba and New Brunswick show the highest concentrations of borrowers in the *≤$100,000* Size Bucket (33% and 31%, respectively); Alberta and B.C. in the *$100,000 - $250,000* Size Bucket (29% and 28%, respectively); and the remaining regions in the *$250,000 - $1,000,000* Size Bucket (ranging between 30% and 44%). Interestingly, in the region of the Northwest Territories and Nunavut, we observe the lowest concentration of borrowers in the *≤$100,000* Size Bucket (8.0%). In addition, this region is shown to contain the highest concentration of borrowers in the *$1,000,000 - $3,000,000* Size Bucket (35%). The Yukon exhibits similar traits but to a lesser degree. We observe that the province of Manitoba has the highest concentration of its borrowers in Size Buckets of *$250,000 or less*, while the Northwest Territories and Nunavut have the lowest. In addition we observe that P.E.I. has the lowest concentration of its borrowers in the *≥$5,000,000* Size Bucket, while the Yukon, followed by Alberta has the highest. b) For all regions barring AL, YK, and N.& L. we observe the

$1,000,000 - $3,000,000 Size Bucket accounts for the highest concentration of $OS. In the case of the former 2 regions (AL & YK) we observe the >*$5,000,000* Size Bucket to be the largest, while in the case of the latter (N. & L.) it is the *$250,000 - $1,000,000* Size Bucket.

6. For all Size Buckets, the provinces of Ontario and Quebec account for a majority of borrowers, with Quebec, in particular, accounting for the highest number of borrowers and the highest concentration of $OS.

7. a) Borrowers in the *$250,000 - $1,000,000* Size Bucket in the province of Quebec account for the highest proportion of borrowers in the Bank, with approximately 11% of the overall number of borrowers in our portfolio. b) Also in the province of Quebec the *$1,000,000 - $3,000,000* Size Bucket accounts for the highest concentration of $OS, with approximately 10% of the overall portfolio $OS.

Summarizing, we observe that approximately two thirds of the *Financing Company* borrowers and $OS are located in QC and ON. The MAN and RET industries are predominant in almost all provinces and territories excluding P.E.I. and YK, for which the TOU industry accounts for the highest percentage of borrowers – and in the case of P.E.I. the highest percentage of $OS. For all industries except RES and for all Size Buckets, we observe that QC and ON account for the highest concentration of borrowers and $OS. For the RES industry, N. & L. accounts for the highest concentration of borrowers while AL accounts for the highest concentration of $OS. In addition, we observe that AL is the province/territory with the highest concentration of borrowers in the *>$5,000,000* Size Bucket and YK is the province/territory with the highest concentration of $OS in the *>$5,000,000* Size Bucket. In turn, P.E.I. and N.W.T. are the provinces/territories with the lowest concentrations of borrowers and $OS, respectively. A brief note should be made on the sparseness of observations in the Northwest Territories and Nunavut, Yukon, and P.E.I. regions.

Subsection 2.3.5. Portfolio at a Glance: Historical Tracking

This subsection traces the development of the *Financing Company* portfolio over the period starting in 1997 and ending in 2010. Table 2.7 provides annual distributions of borrowers and $OS across industries. Figure 2.6 describes the evolution of borrowers and $OS over the evaluation period. In both, the concentrations of borrowers and $OS we observe that the MAN industry has had a continuing dominant role in the *Financing Company* portfolio. In addition, we note that the concentration of borrowers and $OS in the MAN industry grew from 1997, reaching a peak of 27%, in terms of the number of borrowers, in 2005 and a peak of 36%, in terms of $OS, in 2004. Over the same period, the *Financing Company* portfolio witnessed significant decreases in its concentration in the TOU and SOP industry; while the BUS, NBUS and CON industries witnessed growth, most significantly in terms of the concentration of borrowers in those industries.

These movements in industry concentration and overall portfolio riskiness can be partially attributed to over-riding policy drives predominant throughout the 1995 to 2005 period. On the one hand, the *Financing Company* engaged in a persistent policy of intervention and support to the continuously deteriorating economic conditions in southern Ontario's auto parts manufacturing sector. That policy translated into a growth of over 150% of the *Financing Company*'s Ontario – Manufacturing portfolio over that period. On the other hand, shifting internal policy directives regarding the *Financing Company*'s financing of realty, along with the elevated volatility experienced by the *Financing Company* portfolio – and the overall Canadian economy – in the Supplier of Premises and reality sectors led to a decrease of exposure in that sector. As can be seen in Figure 2.4, these policy shifts – had significant effects on the *Financing Company*'s regional diversification across Canada.

In Table 2.8 and Figure 2.7 we observe the evolution of the *Financing Company*'s borrower and $OS distribution across Risk Ratings over the 1997-2010 sample period. In both cases we observe a gradual increase in the proportion of the *Financing Company* portfolio assigned to borrowers in the RRs of 1 to 4 while simultaneously observing a decrease in the proportion of borrowers and $OS held in the RRs of 5 to 8. The most stark decreases were those for the 7 RR in terms of both $OS and number of borrowers with 1997 concentrations of 28% and 22%, reaching a peak of 30% in 2000 and 26% in 1998, and ending at 13% in 2010 for the number of borrowers and $OS, respectively. Overall, we observe a general migration in the credit quality of the portfolio with growth in both the number of borrowers and the $OS in the 1-4 RR and decreases in the concentrations of borrowers and $OS in the 6-8 RRs. Of note is the significant increase in the proportion of borrowers in the 9 RR over the sample period. As can be seen in

Table 2.8, the proportion in the number of borrowers in the 9 RR rose from 1% in 1997 to 12% in 2010, reaching a peak of 14% in 2008.

Observing the evolution of the *Financing Company* portfolio by Size Bucket, as in Table 2.9 and Figure 2.9, we note the historical predominance of the *$250,000 - $1,000,000* Size Bucket in terms of number of borrowers, which accounted for 35% of the portfolio, on average over the evaluation period. In terms of $OS, this predominance only holds at the beginning of our observation period. We observe a steep decline in the percentage of $OS in the *$250,000 - $1,000,000* Size Bucket while simultaneously observing a steep rise in the $OS of the *>$5,000,000* Size Bucket. This rise in the $OS concentration in the *>$5,000,000* Size Bucket, as well as the moderate rise in the *$3,000,000 - $5,000,000* Size Bucket, is accompanied by a sustained, albeit low-level, increase in the concentration of borrowers in those Size Buckets.

Section 2.4. Historical SME Default Rates

Following the dimensions highlighted in the preceding section, we are able to segregate our obligor default data along lines suitable for the modeling of the portfolio loss distribution. In our development and application of various portfolio credit risk models, it is these factors (or risk measures) – default rate level, volatility, correlations, along with the dollar exposure and number of obligors in each industry – that we will use in order to derive and describe the portfolio loss distribution.

Subsection 2.4.1. Default Rates by Risk Rating

Table 2.10 and Figure 2.10 provide a description of default rates segmented by Risk Rating over the evaluation period starting in January 1997 and ending in December 2010. Table 2.10 shows an average default rate of 4.6% for the portfolio as a whole, as well as monotonically increasing average PDs for RRs 1 to 9. Specifically, we observe an average default rate of 0.7% for the 1 RR and one of 12.7% for the 9 RR. However, while this monotonicity holds on average over the evaluation period, we observe that for RR 1 to 6, significant "breaches" can be noted at various years throughout the time series. Between the 6 to 9 RRs, we observe consistent monotonicity in the realized default rates. For all RRs we observe a spike in the realized default rates observed in the 2008-2009 period. Examining these default rate spikes in relation to the average default rate for each Risk Rating, we observe that in addition to a general sensitivity to the 2008-2009 macroeconomic environment, we can qualify that sensitivity by Risk Rating. Specifically, we observe that maximum default rates for the 2 – 6 RRs occurred in the 2008-2009 period, while for other RRs maximum annual default rates occurred in the years 1997 and 1998, a period not necessarily associated with an adverse macroeconomic environment. This suggests an elevated sensitivity among 2 to 6 RRs while lower sensitivity may exist for the safest and riskiest borrowers.

Figure 2.10 allows us to observe similar patterns in the behaviour of default rates for the 3 and 4 RRs, and perhaps to a lesser degree for the 2 and 5 RRs. In addition, default rates for the 6 to 8 RRs appear to follow similar patterns. For the 1 RR we observe both high volatility and a lack of similarity in pattern when compared to other Risk Ratings. We suspect this may be due to these safer borrowers' greater ability to absorb shocks in the macroeconomic environment – thereby generating patterns differing from those of riskier borrowers. Defaults among these safer borrowers can thus be generally related to idiosyncratic factors as opposed to common macroeconomic ones. For the 9 RR we observe extremely elevated default rates for years prior to 2001. We suspect this elevated default rate may be due to a retro-active internal re-rating processes initiated by the *Financing Company* for years prior to 2001 in which borrowers known to have defaulted may have re-rated to the highest risk rating for those years.

Table 2.13 provides correlation values for the Risk Rating default rates over the evaluation period. Strong correlations of 81% and 90% are observed between the 4 RR and the 3 and 5 RRs, respectively, as well as between the 7 RR and the 6 and 8 RRs – equal to 81% and 82%, respectively. The 9 RR exhibits negative correlations with the 1 RR and 5 RR, which is also negatively correlated with the 8 RR. These correlation values reflect the similarity in patterns observed in Figure 2.10. In addition, Table 2.13 provides a measure of the correlation between the default rates of each RR and that of the overall default rate. These values indicate that the 6 and 7 RRs show the strongest correlation to the overall default rate, while the 1 RR exhibits negative correlation with the overall default rate.

Subsection 2.4.2. Default Rates by Industry

Segmenting the default rate data by Industry, Figure 2.11 and Table 2.11 indicate a significant increase in default rates across almost all industries coinciding with the 2008-2009 economic slowdown. Historically the BUS and MAN industries manifest the highest average default rates over the evaluation period, while the SOP industry manifests the lowest average default rate – equal to half the portfolio-wide average. In terms of volatilities, we observe that the SOP industry has the highest observed volatility, while NBUS has the least volatile default rate, along with that for the MAN industry. This observation is made more interesting by the fact that the MAN and NBUS industries are on opposite ends of the average default rate spectrum, with the NBUS industry having the second lowest average default rate. Figure 2.11 allows us to observe some patterns for the annual default rates of industries, and through it we observe that NBUS and OTH industries share very similar patterns of annual default rates. This, we speculate, could be due to the high degree of heterogeneity in those industries. Similar patterns can be observed for the BUS and SOP industries, even while these industries represent the riskiest and least risky industries, respectively, in our portfolio. We suspect that this similarity in pattern may be due to internal *Financing Company* policies aimed at adjusting for the riskiness in both of these industries over the evaluation period.

Table 2.14 provides the default rate correlation matrix for our industries over the evaluation period. Overall, we observe positive correlations between almost all industries with some exceptions noted for the TRS and SOP industries (as well as a negative correlation between the OTH and RES industries). The highest correlation recorded is that between the SOP industry and the BUS industry (91%), while the lowest correlation recorded is that between the TOU and SOP industries (1%). In addition Table 2.14 provides the correlation between each Industry's default rate and that of the overall portfolio. As such, the MAN industry shows highest correlation with the overall default rate, equal to 92% - this is due to the predominance of the MAN industry in the *Financing Company* portfolio.

The highest default rates attained by the portfolio are in the Business Services sector (10.1%) in 1998 and (9.9%) in 1997. The lowest default rates are mostly found in the Supplier of Premises industry, with the portfolio-wide lowest being (0.5%) for that industry in 2008 (the presence of this portfolio-wide low point in a year with otherwise exceptional spikes in default rates is peculiar). Contrary to the case of default rates by Risk Rating (see Subsection 2.4.1), we find that not all industries exhibited elevated default rates in the 2008-2009 macroeconomic recessionary period. Specifically, we observe that for all industries except the BUS, RES and SOP, spikes in the default rate are observed in the 2008-2009 period, with the largest increase in default rates relative to the annual average default rate occurring in the TRS (approximately 70% increase), WHS (approximately 40% increase), and RET (approximately 40% increase). In general, we observe elevated default rates for almost all industries in the 1998-1999 period.

Subsection 2.4.3. Default Rates by Size Bucket

Table 2.12 provides an interesting result in showing monotonically decreasing average annual default rates with Size. For borrowers in the ≤ *$100,000* Size Bucket we observe an annual default rate of 8.3%. This value decreases to 1.2% for the >*$5,000,000* Size Bucket, a value equivalent to approximately 25% of the portfolio-wide average of 4.6%. We observe the lowest default rate volatility for the smallest and riskiest Size Bucket (≤ *$100,000*) while the least risky Size Bucket (>*$5,000,000*) exhibits the highest volatility. Figure 2.12 graphically illustrates the default rates by Size Bucket over the evaluation period. As can be seen in Figure 2.12 and Table 2.12, the default rates for our Size Buckets show strong ordering so that the rank of riskiness by Size Bucket remains constant throughout the evaluation period with very few exceptions. Spikes in default rates are observed for all Size Buckets except the *$1,000,000 - $3,000,000* and >*$5,000,000* Size Buckets over the 2008-2009 period. Table 2.15 gives the default rate correlations over the evaluation period. In addition to the correlation between each Size Bucket default rate, we also observe in Table 2.15 the correlation between each Size Bucket's default rate and that of the overall portfolio. The strongest correlation observed between Size Buckets is 69% between the *$100,000 - $250,000* Size Bucket and the *$3,000,000 - $5,000,000* Size Bucket. Negative correlations are observed between the ≤ *$100,000* Size Bucket and the *$3,000,000 - $5,000,000* and >*$5,000,000* Size Buckets, as well as between the *$250,000 - $1,000,000* and >*$5,000,000* Size Buckets. In addition, the >*$5,000,000* Size Bucket is the only one to manifest a negative correlation with the portfolio-wide default rate. Meanwhile the *$250,000 - $1,000,000* Size Bucket shows the strongest correlation with the portfolio-wide default rate, equal to 86%; as with the case of the industry default correlations (see Table 2.14), we attribute this to the *$250,000 - $1,000,000* Size Bucket being the historically predominant one in the *Financing Company* portfolio.

Across all Size Buckets we observe elevated annual default rates over the 2008-2009 macroeconomic recessionary period, with the degree of elevation the most pronounced (at approximately 50%) for the *$100,000 - $250,000* and the *$3,000,000 - $5,000,000* Size Buckets. In addition, we observe elevated annual default rates for all Size Buckets, except the >*$5,000,000* Size Bucket, during the 1998 calendar year.

Section 2.5. Loans Portfolio Stylized Facts

In this chapter we have carefully analyzed the unique underlying risk characteristics of the *Financing Company* performing loans portfolio. Our analysis has revealed an SME portfolio with sufficient depth to allow for a granular analysis of the credit risk characteristics of Small and Medium Enterprises borrowers within a portfolio context. This granularity thus opens the door to the study of SME credit risk not as a homogenous body, but as a layered heterogonous one in which credit risk behaviour may vary among different classes of borrowers. For the *Financing Company* portfolio, small borrowers – small even by the standards of Small and Medium Enterprise – have been shown to play a dominant role, and exhibited a high risk profile when compared to the rest of the portfolio.

To conclude, we highlight the *Financing Company* loans portfolio's distinguishing characteristics through the following stylized facts:

1. *Large Number of Small Borrowers* – the *Financing Company*'s loans portfolio is composed of a large number of small $OS borrowers, with approximately 55% of borrowers owing the Bank $150,000 or less (see Table 2.1), and approximately 50% of borrowers classified in Size Buckets of *$250,000 or less*, (see, e.g., Table 2.2), as of March 2009. The cumulative $OS of those borrowers highlighted in both cases above accounts for less than 10% of the overall portfolio $OS.

2. *High $OS Concentration Among Top Borrowers* – In addition to the large number of small borrowers, the *Financing Company* portfolio also contains a small number of large borrowers whose cumulative $OS accounts for a large percentage of the overall portfolio $OS. In particular, we note that for the March 2009 portfolio, the top 10% of borrowers (by $OS) account for approximately 60% of overall portfolio $OS (see Table 2.1), while the top 3% of the *Financing Company* borrowers account for approximately 25% of the overall portfolio $OS – this figure also holds for borrowers in the *>$5,000,000* Size Bucket (see, e.g., Table 2.2).

3. *Manufacturing Dominant Industry in the Portfolio* – As of March 2009, the Manufacturing industry has the highest concentration of borrowers and $OS, accounting for approximately a quarter of all borrowers and a third of all $OS in the portfolio (see, e.g., Table 2.3).

4. *Québec and Ontario Largest Regional Concentrations* – Combined, these two provinces account for 60% of borrowers and 64% of overall portfolio $OS (see, e.g., Table 4.0) – with Manufacturing being the predominant industry in those two geographical regions.

5. *Small Borrowers Riskier Than Large Ones (Part 1)* – As of March 2009, the *≤ $100,000* Size Bucket is the only Size Bucket in which borrowers in the 8-9 Risk Ratings accounted for the majority, in both numbers and $OS. The proportion of borrowers in the 8-9 Risk Ratings is monotonically decreasing with increasing Size Buckets (see Table 2.2).

6. *Small Borrowers Riskier Than Large Ones (Part 2)* – We observe monotonically decreasing average default rates with increasing Size Bucket, such that the smallest Size Bucket (≤ *$100,000*) exhibits average default rates approximately twice those of the overall average, while the largest Size Bucket (*>$5,000,000*) exhibits average default rates approximately one quarter those of the overall average (see Table 2.12). In addition, we observe that default rate volatilities are generally increasing with Size Bucket so that the smallest Size Bucket has default

rate volatilities approximately equal to that of the overall default rate, while the largest Size Bucket has default rate volatilities approximately four times those of the overall default rate (also see Table 2.12).

7. Supplier of Premises *the Least Risky Industry* – The Supplier or Premises industry exhibits the lowest average annual default rate, approximately equal to half that of the overall portfolio-wide average default rate. The Non-Business Services industry exhibits the second lowest average default rate, as well as one of the lowest default rate volatilities – in contrast to the Supplier of Premises industry, which has the highest default rate volatility (approximately four times that of the overall default rate volatility). The Non-Business Services industry shares its low default rate volatility with the Manufacturing industry (see Table 2.11 and Figure 2.11).

8. Business Services *and* Manufacturing *Most Risky Industries* – The Business Services and Manufacturing industries account for the highest average default rates among industries. However, we observe a divergence in the volatility of each industry's default rate volatility, with Manufacturing exhibiting one of the lowest default rate volatilities and the Business Services industry exhibiting a default rate volatility approximately three times that amount (see Table 2.11 and Figure 2.11).

9. *Default Rates react to Macroeconomic Environment to Varying Degrees* – Observing the *Financing Company* annual default rates by Size Bucket and Risk Rating, we note significant elevations in the default rates for all segregations in periods of macroeconomic stress. Observing the *Financing Company* annual default rates by Industry, we observe a significant degree of divergence in the default rates over such periods of macroeconomic stress, with default rates in the Transport and Storage industry and the Wholesale industry appearing to show the highest sensitivity to stressed macroeconomic conditions. The Supplier of Premises, Business Services and Resources industries show the lowest sensitivity. For almost all segments we observe elevated default rates in 1998.

10. *Segments Exhibit Negative Correlation* – We observe negative correlations between the safest and riskiest borrowers, as well as between the smallest and the largest borrowers. In particular, we observe negative correlation between borrowers in the 1 and 9 Risk Ratings. In addition, we observe negative correlations between borrowers in the (≤ *$100,000*) and (>*$5,000,000*) Size Buckets.

Chapter 3. Small and Medium Enterprises under Basel II

The Basel II Capital Accords marked an important step in the regulatory recognition of SMEs as borrowers who have divergent credit risk characteristics from those of their Corporate counterparts; see, for example, Hennek and Truck (2006), Altman and Sabato (2005), Dietsch and Petey (2004) and Jacobson, Linde, and Roszbach (2005). This recognition was reflected in both the Standardized Approach, as well as the Internal Ratings Based method, which allows banks to input internally measured components of credit risk into the risk capital measurement mechanism for banks' banking book exposures. This regulatory capital mechanism, also referred to as the risk-weighting function is based on asymptotic approximations of existing models for portfolio credit risk in use at financial institutions; see Gordy and Howells (2006). For a review of the Basel II risk-weighting function, see, for example, BCBS (2005); for a review of broadly applied commercial portfolio credit risk models see Crouhy, Galia, and Mark (2000).

In addition to internally measured components, the risk-weighting function incorporates pre-calibrated risk components and adjustments which play a crucial role in determining final capital requirements and allocations. For SME exposures, these pre-calibrated components have been set to values that effectively reduce the capital required for SMEs with respect to their Corporate counterparts. Specifically, the Basel II Accords allow for the recognition of SMEs through the following:

Standardized Approach

ii. Almost exclusively unrated by external agencies, SME borrower loans classified as Corporate exposures generally warrant a 100% risk weight. These borrowers will also, in general, exhibit elevated risk profiles, so that for an externally rated Corporate borrower loan with a comparably elevated risk profile, a higher risk weight of 150% could be applied; see BCBS (2006a, p. 23).

iii. Basel II allows for the classification of SME borrower loans under the Retail-Other asset class, with a 75% risk weight, given some restrictions; see BCBS (2006a, p. 23).

Internal Ratings Based Approach

i. For SMEs in the Corporate asset class, a size-based adjustment can be applied within the risk-weighting function to lower capital charges through lower asset correlations; see BCBS (2006a, p. 64).

ii. Within the Corporate asset class, SME properties such as shorter terms (maturities) and higher PDs generally lend themselves to lower capital charges within the Internal Ratings Based Approach framework; see BCBS (2006a, p. 64).

iii. For SMEs in the Retail asset class, Size and Term-to-Maturity have no impact on capital charges, however, lower overall capital charges are obtained through lower overall asset correlations; see BCBS (2006a, p. 77).

In this Chapter we use the unique characteristics of the *Financing Company* portfolio of SME borrowers to measure and explore the effects of the various adjustments and assumed relations within the Basel II framework. Given the broadly pre-calibrated nature of the Basel II capital charge mechanism, we are able to disassemble the Basel II IRB mechanism and apply each assumption or pre-calibrated relationship individually, thereby assessing its impact on the *Financing Company* required capital calculation. This exercise will be referred to as the Partial Implementation exercise. The results of this exercise provide a useful benchmark against which portfolio credit risk relationships and models may be compared. An in-depth review of the Basel II IRB risk weighting function is found in BCBS (2005); for the Canadian implementation of Basel II as dictated by the Office of the Superintendent of Financial Institutions, see OSFI (2011). For an interesting review and analysis of the development of the Basel II IRB framework, from an SME capital charge perspective, throughout the consultative period see Hennek and Truck (2006); for the comprehensive final version of the Basel II Accord see BCBS (2006a), and; for a review of the history of the Basel Accords and the international banking regulatory structure see Hull (2010) and Jorion (2002).

Chapter 2 presented a general abundance of default data in the *Financing Company*'s historical portfolio. Here we construct additional segmentations to those observed in Chapter 2 and use them to explore the SME relationships modeled in Basel II. In particular, we use dual segmentations to estimate probabilities of default for various Size and Risk Rating segmentations. In Chapter 4 this work is extended to the estimation of asset and default correlations, and ultimately capital charges based exclusively on internally-calibrated measures of SME credit risk.

More specifically, Chapter 3 is organized as follows: In Section 3.1 we focus on the Basel II IRB approach to the treatment of SME exposures and examine the divergence in the analysis of SME borrowers as compared to their larger, less risky Corporate counterparts. In addition, we describe the Basel II Accord's Pillar 1 treatment of credit risk, including two broad methods, the Standardized Approach (SA) method and the

Internal Ratings Based (IRB) method. A detailed analysis of the risk-weighting function proposed under the IRB approach is conducted, including the theory on which it is based, the functional form in which it is presented and used, and the underlying practical assumptions it makes about the behaviour of borrowers and credit risk components. In addition, we highlight some practical and conceptual differences in the treatment of SMEs as either Retail or Corporate borrowers.

Having adequately described the treatment of credit risk and SME credit risk in particular, we turn our attention to an implementation of the Basel II credit risk treatments on the *Financing Company* portfolio. In Section 3.2 we build on the work done in Chapter 2 by adding dual-dimension segmentations to our portfolio estimation of probabilities of default. In Section 3.3 the Basel II IRB model is applied to the *Financing Company* portfolio in sequential manner, allowing for analysis of the assumptions made in the Basel II framework and their impact on an SME portfolio. Given the particular nature of the *Financing Company* portfolio, we are thus able to not only test many of the assumptions made about the behaviour of SME credit risk components and the relationships between them, but also establish a standardized benchmark for the capital needed to meet the credit risk inherent in the *Financing Company* portfolio. Finally, Section 3.4 presents the Chapter's conclusions.

Section 3.1. The Basel II Capital Accords Credit Risk Framework

In the following subsections we detail the Basel II IRB framework for credit risk capital requirements as presented in BCBS (2006a) and tailored to the Canadian financial sector in OSFI (2011). Our discussion of Basel II deals exclusively with its application to banking book loans – with special emphasis on loans to SME borrowers, such as those found in the *Financing Company* portfolio.

In particular, we note that the Basel II modeling of banking book credit risk relies on two broad approaches for the modeling of banking book credit risk: the Standardized Approach (SA) – discussed in Subsection 3.1.1 and; the Internal Ratings Based (IRB) Approach – for which two implementations, the Foundation (FIRB) and the Advanced (AIRB) are permitted, depending on a bank's ability to meet supervisory standards for each – discussed in Subsection 3.1.2. Our discussion of these two IRB implementations will focus on the internal risk-weighting formula, which we will be analyzed in detail in Subsection 3.1.3.

Under the IRB approach, exposures classified under the Corporate and Retail asset class are subjected to the Corporate asset class IRB risk-weighting function. Within this function, loans to SMEs benefit directly from favourable treatment through size-adjustments reducing the overall capital requirement, all other things equal; see BCBS (2006a, p. 64). Under both the SA and IRB methods, SMEs eligible to be classified as Retail exposures require less regulatory capital as compared to the Corporate asset class, all other things equal. This is evident from the risk weights assigned under the SA method and the calibration of the risk-weighting function under the IRB method.

In the following, we survey the key features of the Standardized Approach and the Internal Ratings Based Approach, as presented in Part 2, Sections II and III, of BCBS (2006a), and explore the IRB risk weight function in greater detail. Our discussion of the IRB risk-weighting function will serve as a bridge to the single-factor model presented in Chapter 4. Together, Sections 3.1 and 3.2 will form the foundation of our implementation of the Basel II framework on the *Financing Company* portfolio in Sections 3.3, and, subsequently, our ability to test the assumptions on the behaviour of SME borrowers within this framework in Chapter 4.

Subsection 3.1.1. Basel II Credit Risk Models – Standardized Approach

The Standardized Approach assigns risk weights to exposures according to exposure classification. For Corporate borrowers these risk-weights rely almost exclusively on external credit ratings provided by credit rating agencies recognized by national supervisory bodies, see BCBS (2006a, p. 23).

Table 3.1 provides the SA risk weights for exposures under the Corporate and Retail asset class. Given the overwhelming predominance of unrated borrowers among SMEs, we expect that most SME borrowers classified under the Corporate asset class obtain SA risk weights of 100%. Alternatively, SME borrower exposures classified as Retail – Other, are accorded a risk weight of 75%, see BCBS (2006a, p. 23).

In order to calculate capital charges for a given portfolio under the SA approach, exposures are multiplied first by the corresponding risk weight and second by the minimum regulatory capital requirement. Under Basel II that minimum total regulatory capital requirement is set at 8%, while the Canadian regulator has set it at 10%; see, in particular, footnote 8, OSFI (2011, p. 7). In this Thesis, the calculation of portfolio capital requirements under the SA method will use the Basel II minimum capital ratio of 8% so that for each loan we multiply the corresponding $OS and SA risk weights by 8% to generate capital charges for that loan. In Section 3.3 capital charges under the SA method are presented, alongside capital charges from other Basel II implementations; see in particular Table 3.8.

Subsection 3.1.2. Basel II Credit Risk Models – IRB Approach

Our review of the Basel II IRB risk-weighting model closely follows the presentation given in BCBS (2005) in describing the underlying mathematical framework and economic basis of the model and its individual components.

The Basel II IRB framework provides a comparable, standardized measure of portfolio credit risk across a large number banks varying in size and makeup. This assignment is accomplished through a risk-weighting function for the estimation of portfolio credit risk capital requirements; see BCBS (2006a, pp. 52-119). The functional form of the risk-weighting function allows for the input of certain internally estimated credit risk components in the calculation of an exposure's portfolio credit risk capital requirement, while other components and credit risk relations are generated from BCBS pre-calibrations; see, for example, BCBS (2005, pp. 9-11).

Specifically, the IRB risk-weighting function allows banks, meeting the necessary data requirements, to input internally measured banking book exposures' PDs (in both the AIRB and FIRB implementations) as well as their LGDs, EADs, and Maturities (in the AIRB implementation).

A pre-calibrated risk component of particular significance to SMEs within the risk-weighting function is the asset correlation. The asset correlation used in the risk-weighting function describes the sensitivity of a given credit exposure to overall economic conditions and therefore, by extension, to the behaviour of other credit exposures in the portfolio; see BCBS (2005, pp. 12-15). The more sensitive an exposure is to possible downturns in the economy, the higher the likelihood that it will default as a result of them. The presence of highly sensitive exposures in a bank's portfolio increases the likelihood of a high number of defaults and losses in the event of a downturn. Our work in Chapter 4 will ultimately center on the estimation of asset correlations with a framework comparable to that of the IRB approach presented in this Chapter. The SME asset correlations estimated in this framework can be found in Section 4.2.2.

In addition to the inclusion of the above credit risk component, the Basel II IRB risk-weighting function captures the effects of several credit risk relationships as adjustments to SMEs' sensitivity to the overall state of the economy. These pre-calibrated adjustments are such that an SME's risk of default, under the framework, arises largely due to idiosyncratic factors – which elevate its riskiness – but which are less related to systematic factors in the economy, especially when compared to larger corporate borrowers, BCBS (2005, p. 12). This emphasis on idiosyncratic risk factors in SMEs as opposed to systematic factors thereby reduces SME contributions to portfolio unexpected losses. In this Subsection we will explore the mechanics of these relationships as modeled under Basel II. In particular, in this Thesis we will study the relationship between SME borrower size and capital requirement as defined under the Corporate asset class IRB formula, and compare our results to the Retail specification which does not include this relationship.

Another phenomena captured by the IRB risk-weighting function is that of increased riskiness with increased maturity. Specifically, a maturity-dependent adjustment of the measured risk for the maturity of a given exposure such that the longer the maturity and the lower the PD, the higher the risk of a potential deterioration in the quality of the borrower BCBS (2005, pp. 9-11). In Jacobson, Linde, and Roszbach (2005), the authors note that SMEs may benefit from reduced capital charges under the Basel II IRB approach due to lower maturities as compared to corporate loans. Our study of SME borrower characteristics and capital charges within a Basel II framework will provide further empirical evidence of this phenomenon. Finally, the IRB risk-weighting function provides for an inverse relationship between asset correlation values and PD. Our work in this Thesis will directly test this assumption.

Subsection 3.1.3. Basel II Credit Risk Models – IRB Risk Weight Function

In Vasicek (2002) a natural extension of the Merton (1974) asset value model to a specific asymptotic single risk factor (ASRF) portfolio credit risk model is given; see BCBS (2005, pp. 3-6) and Appendix B. A necessary condition of this extension is that the portfolio on which the resultant ASRF model is applied must contain a large enough number of obligors such that the idiosyncratic risks associated with each obligor is diversified away and the only significant risks affecting portfolio losses are systematic; BCBS (2005). Building on Vasicek (2002), Gordy (2003) showed that the ASRF model can be additive in the capital charges to the exposures in the portfolio to which it is applied, and that it is uniquely portfolio invariant.

As such, the ASRF model presents regulators with a model that is applicable across a variety of portfolios with relative ease, and therefore forms the underlying basis of the IRB risk weighting function. This resultant IRB risk weight function, see BCBS (2006a, p. 64), is given by:

$$K = \left[LGD \times N\left[\frac{1}{\sqrt{1-R}} \times N^{-1}(PD) + \sqrt{\frac{R}{1-R}} \times N^{-1}(0.999) \right] - PD \times LGD \right] \times \left[\frac{1 + (M - 2.5) \times b(PD)}{1 - 1.5 \times b(PD)} \right]. \quad (3.1)$$

where, (*K*) represents the percent capital charge in excess of EL; (*PD*) is the average Probability of Default defined over a given segment; (*LGD*) is the downturn Loss Given Default; (*R*) is the asset correlation, or the single factor weighting; ($N[\cdot]$) is the cumulative distribution function (CDF) for a standard normal variable, and ($N^{-1}[\cdot]$) is the inverse CDF of a standard normal variable; (*M*) is the loan Term-to-Maturity; and *b(PD)* is a smoothed regression maturity function, such that the slope of the adjustment function with respect to (*M*) decreases as the (*PD*) increases – specifically:

$$b(PD) = \left(0.11852 - 0.05478 \times ln(PD)\right)^2. \tag{3.2}$$

Within this functional form, the IRB capital framework is thus able to provide the required capital amount using a small set of parameters. In particular, the PD, internally calculated and specified under Basel II according to risk grade, is a central focus of IRB methodology and will be discussed extensively in Section 3.2. As can be seen in Equation (3.1), the degree to which an obligor is sensitive to an extreme realization of the systematic factor is determined through the asset correlation *(R)*. The pre-calibrated asset correlations presented in the IRB framework are specified according to a series of pre-defined asset classes, defined according to banking book exposures exhibiting divergent characteristics. They include Corporate and Retail exposure asset classes, each of which defines specific assumptions and relationships modelled into the pre-calibrated asset classes. Discussion of the asset correlation conventions in Basel II will form an integral part of this Chapter, as well as this Thesis as a whole, with further discussion and testing of Basel II assumptions performed in Chapter 4.

For the Corporate asset class, Equation (3.3) provides the asset correlation, as found in BCBS (2006a, p. 64):

$$R = 0.12 \times \frac{(1-exp(-50 \times PD))}{(1-exp(-50))} + 0.24 \times \left[1 - \frac{(1-exp(-50 \times PD))}{(1-exp(-50))}\right] - 0.04 \times \left[1 - \frac{S-6.25}{56.25}\right], \quad (3.3)$$

where (*S*) is the borrower size as measured by sales (applicable to Corporate SME exposures).

The calibration of Equation (3.3) takes into account two systematic dependencies. The first dependency modeled in Equation (3.3) is the positive relationship between asset correlations and firm size, and is applied in the shape of a size-adjustment applicable to exposures deemed to be SMEs under the Corporate asset class. Specifically, borrowers with annual sales ranging between \$6.25m and \$56.25m, as specified in OSFI (2011, p. 149), receive a negative adjustment ranging between 4% and 0%. Borrowers with sales of \$6.25m or less obtain a size adjustment of 4% while those with \$56.25m or more receive no adjustment. While the BCBS puts forward both intuition and empirical evidence as justification for this relationship, there is a concession that the empirical evidence supporting it is not conclusive, BCBS (2005, p. 12).

The second dependency is the inverse relationship between PD, or the riskiness of the borrower, and asset correlations. The underlying intuition supporting this relationship is that the more risky the borrower, the higher the likelihood that his default is due to idiosyncratic factors than to a realization of the systematic factor - as such, the borrower exhibits a decreased overall correlation to the systematic factor, BCBS (2005, p. 12).

Therefore taking into account these two systematic dependencies within the asset correlation framework, two limits on the asset correlations are established. For non-SME Corporate exposures asset correlations are set between 12%, corresponding to a PD of 100%, and 24%, corresponding to a PD of 0%. For SME Corporate exposures, these two limits then become 8% (for 100% PD) and 20% (for 0% PD).

$$R = 0.03 \times \frac{\left(1 - exp(-35 \times PD)\right)}{\left(1 - exp(-35)\right)} + 0.16 \times \left[1 - \frac{\left(1 - exp(-35 \times PD)\right)}{\left(1 - exp(-35)\right)}\right]. \tag{3.4}$$

Equation (3.4) presents the asset correlation equation under the Retail-Other exposure treatment to which borrowers classified as SMEs can be subjected; see BCBS (2006a, p. 77). For a borrower to be eligible for Retail-Other specification, a financial institution's exposure to that borrower may not be greater than \$1.25m, OSFI (2011, p. 40). A quick comparison of Equations (3.3) and (3.4) reveals the absence of a size adjustment for SME borrowers. For the so-called Retail-SME asset class, asset correlation limits are set to 3%, for 100% PD, and 16%, for 0% PD. These lower limits are a reflection of empirical and intuitive evidence suggesting that idiosyncratic factors play a larger role in the default behaviour of retail borrowers.

In addition to the absence of a size-asset correlation relationship in Equation (3.4), we also observe a distinction in the calibration of the relationship between asset correlation and PD. Specifically, we note that the Corporate asset class correlation and the Retail-Other asset class correlation functions are based on an exponential weighting function with a “k-factor” set to 50 for Corporate exposures and 35 for Retail-Other exposures. This k-factor determines the pace of decrease of the asset correlation with respect to the PD such that given the above calibrations, the asset correlation decreases quicker for Corporate borrowers as opposed to Retail borrowers. The resultant relationship between PD and asset correlation can be seen in Figure 3.1 which plots the two asset correlation functions across PDs, excluding – in the case of the Corporate asset correlation function – any size adjustments therein. A steeper slope is evident for the Corporate asset correlation function, as well as higher correlation values when holding PDs constant.

Finally, the asset correlations derived for the Retail asset class are applied to the IRB risk-weighting function given in Equation (3.3) with the exclusion of the Maturity adjustment (last bracket on the right-hand side of the equation). The maturity adjustment found in Equation (3.1), and detailed in Equation (3.2), reflects empirical and intuitive evidence as to the riskiness of long-term credit exposures as opposed to short-term credit exposures. In addition, the maturity adjustment includes a recognition of the higher potential for low risk (low PD) exposures to deteriorate as opposed to high risk (high PD) exposures which can be considered to have already deteriorated. A standard maturity of 2.5 years is assumed and adjustments are smoothed via the statistical regression represented as *b(PD)* in Equation (3.1) and given in Equation (3.2).

The Basel II IRB framework therefore presents two specifications for SME asset correlations and capital charge allocation, and thus two conceptual frameworks for SME behaviour in default. Under the Corporate asset class, SME asset correlations are highly sensitive to PD values, and can range between 8% and 20%, depending on Size (as well as PD). Capital charges for SMEs under this specification can be increased in longer maturities. Under the Retail asset class, SME asset correlations are measured independently of Size; asset correlations are still inversely related to PDs but compared to Corporate exposures this relationship is significantly dampened. Asset correlations are lower than those for SMEs classified as Corporate exposures and range between 3% and 16% (depending on PD), and capital charges are not determined by maturity.

Within a portfolio of SME borrowers, these two conceptual frameworks are distinguished by the $1.25m Retail-SME eligibility threshold. By segmenting our portfolio into homogenous subportfolios by Size and Risk Rating, and explicitly measuring the asset correlations in our SME portfolio, our work in Chapter 4 will directly test these assumptions and conceptual frameworks on the behaviour of asset correlations. To do so, we will use a comparable model to that used in the Basel II IRB approach to measure patterns in asset correlations among SME borrowers. In Section 3.3 we will implement various forms of the IRB risk-weighting function given in Equation (3.1) and present results for the aggregate capital charges by portfolio and segment.

Section 3.2. Key Components of the Basel II IRB Approach SME

In Section 3.2 dual segmentations are applied to the data in the calculation of probabilities of default. The application of dual segments to our data provides us with a useful convention in that it allows for the formation of homogenous segments of SME borrowers on which research on credit relationships and characteristics can be undertaken. The significant depth of the *Financing Company* portfolio differentiates this data from other studies in which this dual segmentation has been applied to aggregated data sets; see, for example Dietsch and Petey (2004) and Duellman and Scheule (2003). In Chapter 4 we extend the work undertaken in this Section by using the dual segmentation convention to measure explicitly asset and default correlations from our SME data..

Our estimates of PDs within Risk Rating – Size Bucket segments reconfirm overall patterns in PDs delineated in Chapter 2, namely those of increasing PD with Risk Rating and decreasing PD with Size. In Subsection 3.2.1 we introduce a correspondence between our Size Buckets and sales-based measures of Size used in Basel II and related literature. Our data work in this Chapter will be extended in Chapter 4 wherein our estimation of internally-calibrated asset correlations will result in more extensive data requirements than those found in Basel II. Specifically, as seen in Section 3.1, Basel II requires that financial institutions using the IRB approach enter internally measured values for the probability of default. Data requirements for this estimation are generally limited to an estimation period of at least five years; see, in particular, BCBS (2006, p.

102). Given that best practice measures call for the use of weighted averages in generating the PD from default rates, there is no need for significant populations of defaults in each segment for which PD estimates are generated. This is not the case when estimating asset and default correlations. The more strenuous data requirements encountered in Chapters 4 and 5 will force us to collapse the Risk Ratings and Size Buckets with the lowest numbers of defaults into more heavily populated amalgamated Risk and Size Groups. When needed, results presented in this Chapter will be replicated along Risk and Size Group dimensions defined in Section 4.2.

As in Chapter 2, the *Financing Company* historical defaults are compiled from January 1997 to December 2010, covering a period of 14 years; see Section 2.4, Table 2.10 and Figure 2.10 for further discussion. In addition to the single segmentation by Risk Rating or Size Bucket, for the purposes of this exercise, we prepare the data presented in Chapter 2 along dual segmentations of Size Bucket and Risk Rating. As before, internal *Financing Company* Risk Ratings of 1 (least risky) to 9 (most risky) were used, along with Size Buckets ranging from *≤$100,000* to *>$5,000,000*. In order to properly identify the discussion as pertaining to either the single segmentation or the dual segmentation of the data, we will generally use the "overall" adjective when referring to the single segmentation, and the "segment" adjective when referring to the dual segmentation.

PD Estimation for Dual Segmentations

Table 3.3 gives the estimated PD by Size Bucket and RR. These figures, on the overall or single segmentation level, correspond to the weighted average annual default rate values calculated in Chapter 2 and presented in Tables 2.10 and 2.12. Figure 3.2 gives a visual representation of the data found in Table 3.3. Specifically, we observe that the overall *Financing Company* PD is equal to 4.6%. As expected, PDs generally increase with RR, such that the RR with the lowest PD is the 1 RR (0.7%) and the RR with the highest PD is the 9 RR (12.7%). In terms of Size, we observe that overall, the PD decreases monotonically with Size Bucket, so that the highest PD is found to be 8.3% for the smallest Size Bucket (≤*$100,000*) and the lowest PD is found to be 1.2% for the largest Size Bucket (>*$5,000,000*). These overall values correspond to the average PD values given in the second last rows of Tables 2.10 and 2.12.

Both of these patterns, that of the positive relationship between PD and RR and that of the negative relationship between PD and Size are generally observed within Risk Ratings and Size Bucket segmentations. By way of confirmation, we note that the lowest PD by Risk Rating and Size (0.2%) is found in the largest and safest (>*$5,000,000* - 1 RR) segment. We note that this "lowest PD" is still significantly higher than the minimum PD of 0.03% stipulated in the Basel II IRB approach; see BCBS (2006, p. 67). In general, exceptions to the patterns noted above are limited, within RRs, to Size Buckets of *$3,000,000 or more* – for example, we note that the highest PD by Risk Rating and Size (14.3%) is found in the *$1,000,000 - $3,000,000* Size Bucket (9 RR), thereby breaking the pattern of increasing PD by decreasing Size. The next highest PD is (13.2%) in the smallest (*≤$100,000*) and riskiest (9 RR) segment with all other PDs following the pattern. These exceptions may be generally attributed to low default and healthy borrower counts in the larger Size Buckets.

Direct comparisons with other research is challenging in that in many cases the size granularity differs and in our cases the standard measure of size differs. Conducting the same exercise, Dietsch and Petey (2004) find three broad categories of SMEs as defined by size, each with particular risk characteristics. Defining their Size Buckets by turnover, they find that small (or very small) firms with turnover less than €1m can be characterized, by their estimated PDs, as being less risky than medium-sized SMEs with turnover of €1m to €7m. In addition, these medium-sized SMEs are found to be riskier than larger SMEs with turnover of €7m to €40m, thereby forming an inverse-U shape of PDs in relation to Size.

Table 3.2 includes a broad one-time mapping from the *Financing Company* Size Buckets to average annual sales per borrower over data collected from 2009 to 2011. The Table shows that on average, the *Financing Company* borrower sales amount to approximately $5m, while those of small borrowers (in Size Buckets of *$250,000 or below*) amount to $3.6m annually. Two broad borrower Size Buckets, that of the *$1,000,000 - $5,000,000* Size Bucket and that of the *>$5,000,000* Size Bucket are identified with annual sales figures greater than $6.25m – the formal threshold below which borrowers receive a maximum size-based reduction of capital charge. This mapping therefore corresponds to the broad brush strokes with which we approach the calibration of the AIRB model with respect to the size adjustment, keeping in mind that our implementation would, in most cases for borrowers in Size Buckets of *$1,000,000 or more*, result in a higher size adjustment than that available using annual sales figures by borrower. In later terminology, the impact of loan Term-to-Maturity on Basel II capital requirements will be evaluated under Cases 6a and 6b.

As previously noted, Dietsch and Petey (2004) find that PDs vary with Size. As is clear from the discussion above, this result corresponds to our findings which show significant variations in PDs across Size, even within RRs. In fact, our findings indicate that Size plays a bigger role the lower the RR, so that less risky companies are much likelier to default if they're small, while for riskier companies size plays a smaller role in determining their PD.

From Table 3.3 we can show that the ratio of PDs for the *≤$100,000* Size Bucket to those for the *>$5,000,000* Size Bucket gradually decreases from approximately 13 for the 1 RR and 11 for the 2 RR, to the range 5 – 6 for the 3 – 7 RRs, and finally to 2 for the 8 – 9 RRs. To ensure that our results are not being biased by the low default counts in the *>$5,000,000* Size Bucket, we conduct the same exercise using the *≤$100,000* and *$250,000 - $1,000,000* Size Buckets. The results confirm our initial findings and show a ratio of approximately 6 for the 1 RR, decreasing to a range of 2 – 3 for the 2 – 7 RRs, and finally decreasing to approximately 1.5 for the 8 – 9 RRs.

We extend our analysis and segmentation to the industry level. Table 3.4 provides the PD for Industry-RR segments; see Figure 3.3 for a visual representation. Overall, we observe that the BUS and MAN industries exhibit the highest PD. In addition, we observe that while the pattern of increasing PD with RR is upheld for all industries, exceptions are most notably observed for the 2 RR – wherein a drop in PD is observed when compared to the 1 RR. Observing the data by segment, we note that the highest PDs are observed in the MAN-9 RR segment (15.4%) and the WHS-9 RR segment (14.0%). For most Risk Ratings, the BUS industry exhibits the highest PDs while the SOP industry exhibits the lowest PDs.

For completeness we explore the PD of the Industry-Size segments. Table 3.5 shows that the *≤$100,000* Size Bucket in the MAN industry exhibits the highest PD (10.3%), followed by *≤$100,000* Size Bucket in the WHS industry. Generally, we observe that for most industries, PDs decrease with increasing Size. We note that for the *>$5,000,000* Size Bucket the OTH, RES and TRS industries exhibit data deficiencies and so are assigned overall *>$5,000,000* Size Bucket average PDs in Table 3.5. Figure 3.4 provides a visual representation of PDs by Industry and Size Bucket.

Summarizing results for Size and RR segments, we write:

1. PDs are observed to be generally decreasing with Size when controlling for credit quality, as well as overall.

2. The lower the RR, the bigger a role Size plays in defining the riskiness of the borrower: The discrepancy between small and large SME borrowers' probability of default grows wider as the credit quality of the borrowers improves.

Finally, we note that the data time series used in our exercise is considerably longer that than used in Dietsch and Petey (2004), which was measured over the years 1995 to 2001 for French borrowers and 1997 to 2001 for German borrowers.

Section 3.3. Full and Partial Implementation of Basel II

In the following we implement the Basel II Standardized Approach and IRB approach for the modeling of portfolio credit risk, as described in Section 3.1, under various assumptions. Our objective is to show explicitly the effects of the various components and relationships in the IRB framework on the *Financing Company* SME portfolio correlations and capital charges. To do so, we define our various implementations cases along with their specific assumptions. For each case the overall portfolio capital charge is calculated and presented, along with a breakdown of capital charges by Size Bucket. This allows for direct comparisons between the various cases both at the overall level and by segment. In addition, we calculate average asset correlations obtained under each implementation case for our Risk Rating and Size Bucket segments. This comparison reveals a significant discount in Basel II capital charges as they pertain to the smallest borrowers.

The data used to calculate the capital figures in the subsequent Subsections is comprised of the *Financing Company* portfolio, including internally calibrated PDs and LGDs, as described in Chapter 2. In Subsection 3.3.1 we define the various implementations performed in this Section. In Subsection 3.3.2 average asset correlations are calculated while in Subsection 3.3.3 overall capital charges are presented and contrasted. Capital charges by Size Bucket segment are discussed in Subsection 3.3.3 and Subsection 3.3.4 presents conclusions for the Chapter.

Subsection 3.3.1. Defining the Basel II Cases

We start with an implementation of the Standardized Approach (SA), in which Corporate and Retail credit exposure risk-weight lookup tables – see, for example, Table 3.1 – are used to classify loans in the *Financing Company* portfolio. As discussed in Subsection 3.1.1, capital charges are calculated by multiplying the risk-weighted asset by 8%. For the IRB approach, we implement the FIRB and the AIRB approaches under various cases.

We proceed by delineating our implementations as the follows: Cases 1a and 1b present the FIRB implementation with the internally calibrated maturities for Case 1a and a maturity of 2.5 years for Case 1b; Case 2 presents the full AIRB implementation on the *Financing Company* portfolio; Case 3 presents what we refer to as the "naïve" implementation of the AIRB approach, classifying all of our exposures as Corporate exposures and withholding any maturity or size adjustments; Case 4 applies a size adjustment to the "naïve" implementation; Case 5 applies Retail – Other classifications, where applicable, to the "naïve" implementation, and Cases 6a and 6b in which the maturity adjustment is applied to the "naïve" implementation, using a maturity ceiling of

5 years (as per Basel II) in the former and no ceiling in the latter. For a review of Basel II calibrations see BCBS (2005). Cases 7a and 7d apply Case 3 and 5 specifications, respectively, to PDs calibrated for dual segmetnations of Risk Rating and Size Bucket.

Our choice of cases focuses primarily on the treatment of Basel II IRB assumptions on credit characteristics and, in particular, how they relate to SMEs; see *(i) – (iii)* in the introduction to this Chapter. Results for this exercise are generated using partial implementations of Equation (3.1), as well as Equations (3.2) and (3.3), for loans classified as Corporate, and Equation (3.4) for loans classified as Retail-Other; see Subsection 3.1.3. Capital charges are calculated on the loan level and then aggregated by segment (e.g., RR-Size segments). Results are presented as dollar-weighted percentages of aggregated exposures ($OS) for each segment, as well as for the overall portfolio.

Tables 3.6 and 3.7 present asset correlations under the various cases described above, allowing for an incremental description of Basel II asset correlation construction for the *Financing Company* SME portfolio. In addition, the use of PD calibrations taking into account borrower Size allows for comparisons with Basel II calibrations in which asset correlation relationship with Size is explicitly modelled. In Chapter 4 we take this analysis further by presenting asset correlations calibrated from internal data with no pre-specified relationships. Table 3.8 presents capital charges under the various cases and allows for the independent observance of the effects of the various adjustments and treatments under the IRB framework. Through Table 3.8 our work establishes the portfolio credit risk characteristics of the *Financing Company* portfolio as compared to a benchmark value provided by the Basel II pre-calibration of risk components.

Recall that under the AIRB approach the components are all internally measured, while under the FIRB approach only the PD is internally measured. This signifies that under the FIRB the LGD is externally calibrated – Canadian banks are required to use the same measure of maturity as under AIRB – see OSFI (2011, p. 161). Cases 1a and 1b (the Foundation IRB cases), use pre-set fixed LGDs of 45% for secured loans, and 75% for unsecured loans along with internally estimated PDs by Risk Rating. For all other cases, the *Financing Company* downturn LGDs of 73% and 41% were used to obtain risk capital charges. Note our choice of pre-set fixed LGDs uses a more conservative interpretation of Basel II guidelines than the portfolio-wide use of 45% would; see OSFI (2011, pp. 152-153) for details. No assumptions are made on the type of collateral used.

Subsection 3.3.2. Average Asset Correlations under various Basel II cases

Tables 3.6 and 3.7 present average asset correlations by Risk Ratings and Size Bucket segment under various implementations. Average asset correlations for each segment are calculated as straight averages across all loans in a given segment. In particular, we observe average asset correlations for Cases 2 to 5, encompassing the full AIRB model implementation (Case 2), the naive model implementation (Case 3), the partial size-based adjustment implementation (Case 4) and the Retail-Other asset class implementation (Case 5), as well as for Case 7a which repeats the exercise in Case 3 using dual segment

PDs (by Risk Rating and Size Bucket) calibrated in this Chapter; see Section 3.2 and Table 3.5.

In particular, Table 3.6 provides an incremental description of Basel II IRB asset correlations through our partial implementation exercise. Starting with Case 3, we observe our "base case" or "naive" asset correlations, based on the Corporate asset correlation function given in Equation (3.3) - excluding size adjustments – and PDs calibrated by Risk Rating. Overall, we observe decreasing asset correlations by Risk Rating (and, by extension, PD), and increasing asset correlation with Size. This second pattern is attributed to the presence of an inverse relationship between asset correlation and PD in Equation (3.3) and the predominance of high risk borrowers in the smaller Size Buckets – thereby allowing those Size Buckets to benefit from the inverse PD relationship; see Table 2.2A and Section 2.2 for further descriptions of borrower distributions by Risk Rating and Size Bucket.

The inclusion of the size adjustment in Case 4 yields asset correlations identical to those in Case 2 for borrowers in Size Buckets *greater than $1,000,000*. The inclusion of size adjustments in Case 4 yields increasing asset correlations with increasing Size at the segment level such that borrowers in Size Buckets of *$1,000,000 or less* benefit from a maximum correlation discount of 4 percentage points while borrowers in the *>$5,000,000* Size Bucket benefit from a size-based asset correlation discount of only 1.2 percentage points. The application of the Retail-Other asset classification for qualifying borrowers in Case 5 results in asset correlations identical to those of Case 2 for borrowers in Size

Buckets of *$1,000,000 or less*. Combining these asset correlations with Case 4 asset correlations for borrowers in Size Buckets *greater than $1,000,000*, we obtain the Case 2 results.

As expected, Case 2 correlations for borrowers in Size Buckets of *$1,000,000 or less*, generally eligible for Retail-Other treatment, range between 3% and 12% and are flat across Size Buckets. Borrowers in Size Buckets *greater than $1,000,000*, and treated under the Corporate asset class, show increasing asset correlations with Size. In both cases the relationship between asset correlation and PD is maintained, however, we observe greater relative discrepancy among smaller borrowers for which the maximum asset correlations (found under RR 1) are four times those of the minimum correlations (found under RR 9). For larger borrowers, that relative discrepancy is on the scale of two times. This result, consistent with Basel II IRB specifications discussed in Section 3.1, can be contrasted with our findings in Section 3.2.1 which showed that the relative riskiness of borrowers is dampened by increasing Size. The correlation patterns presented in Case 2 represent the mixture of Basel II pre-calibrated asset correlation relationship as they are applied to various segments of our SME portfolio.

Table 3.7 once again presents average asset correlation results for Cases 2 and 3, alongside average asset correlations obtained from the application of Case 3 and Case 5 specifications to PDs calibrated by Risk Rating and Size Bucket under Cases 7a and 7d, respectively. The integration of the Size dimension on the calibration of PDs results in several patterns on asset correlations. Comparing asset correlations under Case 7a to those under Case 3, we observe the emergence of patterns by Size within Risk Ratings. Overall, average asset correlation values are higher for all Risk Ratings while the two smallest Size Buckets exhibit lower asset correlations. This pattern is generally repeated with the *≤$100,000* and *$100,000 - $250,000* Size Buckets exhibiting lower asset correlations under Cases 7a and 7b and all other Size Buckets exhibiting generally higher asset correlations, when compared to Cases 3 and 5, respectively.

As in Case 3, the underlying dynamics driving these patterns are the PD levels within Risk Rating – Size Bucket segments relative to overall Risk Rating PD levels; see Table 3.3. Comparing to asset correlations under Case 2 we observe that internal calibrations incorporating Size generally amplifies the decreasing relationship between PD and asset correlations, resulting in greater discrepancies in asset correlation by Size. In Chapter 4 the Basel II assumption of decreasing asset correlations with PD will be removed and we will calibrate asset correlations from internal data without pre-set assumptions on behaviour and relationship with other credit risk factors.

Subsection 3.3.3. Overall portfolio capital charges under various Basel II cases

Instead of using the standard 2.5 year maturity (M) as specified in Basel II we follow the OSFI convention and use minimum of the actual loan maturity and 5 years in Case 1a. In Case 1b, internal estimates for the average PD were used while Basel II estimates are used for the other risk components – LGD and M (set at 2.5 years).

As expected the use of higher values for maturity generates higher capital requirements for the portfolio. Table 3.8 allows us to observe a decrease of 1.4% in the overall capital requirement for the portfolio as a percent of the total portfolio $OS due to the fixing of maturities at 2.5 years, as observed in the difference between Cases 1a and 1b.

Case 2 presents the implementation of the AIRB approach and indicates an immediate drop in the required capital amount, as compared to Case 1a, from 8.8% to 8.1%. This result is not unexpected given that the change implemented is one of replacing one set of LGD figures (the pre-calibrated Basel II set) with another, lower value set (internally calibrated). In both cases the same maturity and size adjustments are maintained, alongside Retail-Other classifications.

In Case 3, we withhold any maturity or size adjustments in our calculation of capital charges. Internally measured PD and LGD are used in conjunction with the Corporate claims risk-weighting function. As a result – and as a base of comparison for other cases/implementations – a “base-case” required capital value of 8.5% is achieved, a value 0.4 percentage points greater than that obtained under a full AIRB implementation, i.e., Case 2.

In Case 4, the application of the size-adjustment to the Corporate claims asset correlation function results in a decrease in the required capital of 1.6 percentage points as compared to the “naïve” implementation. The application of the Basel II size-adjustment to the *Financing Company* portfolio is based on a calibration of the *Financing Company* Size

Buckets to average annual sales for those borrowers with sales figures. Table 3.2 shows that for all borrowers in the *Financing Company* portfolio, across all Size Buckets, average annual sales figures are less than \$62.5m, thereby qualifying them for a size adjustment. In addition, for borrowers in Size Buckets of *\$1,000,000 or less* annual sales figures are such that those borrowers qualify for a maximum reduction in asset correlation of 4%. The resultant decrease is generally anticipated given that approximately 75% of the *Financing Company* loans qualify for the maximal size adjustment, and 100% of the *Financing Company* loans are subject to some size adjustment resulting in lower asset correlations and thus, portfolio capital requirements two percentage points lower than those obtained in the “naïve” implementation. In Subsection 3.3.3 we will examine in capital allocations by Size Bucket as well as average PDs across Risk Ratings and Size Bucket segments.

In Case 5, the *Financing Company* Size Bucket classifications, based on a borrower’s maximum commitment to the *Financing Company* at the time of his last authorization, along with the borrower’s \$OS as of March 2009, are used to determine eligibility for classification under Retail–Other exposure. Facilities to borrowers classified within Size Buckets of *\$1,000,000 or less*, with \$OS of \$1.25m or less, qualify for treatment as Retail–Other exposures. For the *Financing Company* portfolio, this classification applies to 75% of loans and 79% of borrowers. The Retail-Other classification entails a reduction in the asset correlation limits of 8% and 24% under the Corporate exposure asset type to limits of 3% and 16%. The resultant decreases in the capital requirements can therefore be quantified as 1.6 percentage points as compared to the "naïve" implementation of the IRB model (Case 3).

Case 6 details the effects of the maturity adjustment. In Case 6a we implement the maturity adjustment while maintaining a maximum maturity of 5 years (or 60 months). This matches the implementation in Case 2 and the broad maturity assumption implicit in the Basel II methodology. Case 6b allows for the maturity to vary to its fullest without a ceiling of 5 years. Implemented in isolation (i.e., no size or retail treatments) on the whole portfolio we observe a 3.7 percentage point increase in the required capital (as compared to Case 3) for Case 6a and an increase of 11.7 percentage points for Case 6b. Table 3.2 provides a breakdown of maturities (in months) for the *Financing Company* portfolio and shows that loans to smaller SME borrowers generally have significantly shorter terms as compared to loans to larger SME borrowers.

Our study of Size effects is enhanced by the use of internal calibrations by both Size and RR, and these results support our initial findings. Case 7a maintains the Case 3 specifications but uses PDs calibrated by RR and Size Bucket, while Case 7d uses the same PD calibration along with Case 5 specifications for borrowers eligible for Retail-Other treatment. Comparing overall capital charges, we observe a 1.6 percentage point drop under Case 7d as compared to Case 7a. This drop matches that of Case 5 as compared to Case 3

Thus far, results as given in this Subsection have described capital charges as they pertain to the overall portfolio. These results have centered on generally expected mechanics within Basel II, as applied to the *Financing Company* portfolio. In particular, we observe that the removal of maturity and size adjustments, as well as the withdrawal of the Retail-Other classification, results in higher portfolio capital charges. In addition, we observe that all *Financing Company* borrowers qualify for size adjustments while 75% qualify for the maximal adjustment. Table 3.2 demonstrates that large SME borrowers do in fact exhibit longer loan terms, and by extension higher Basel II capital charges, as compared to smaller borrowers, thereby supporting the assertion of Basel II maturity-based benefits to smaller borrowers – see (*b.ii*) in the introductory note to this Chapter and Jacobson, Linde, and Roszbach (2005). In turn, we observe that approximately 75% of loans and 79% of borrowers qualify for complete exclusion from the maturity adjustment through qualification for Retail-Other classification.

Additionally, we observe that the application of Size-based adjustments, whether through the Corporate asset class asset correlation function, as in Case 4, or through the pre-calibrated overall asset correlation range of the Retail-Other asset class, presents a significant decrease in capital charges. To gain a better understanding of these changes we move to Section 3.3.4.

Subsection 3.3.4. Capital charges under various Basel II cases by Size Bucket

In Jacobson, Linde, and Roszbach (2005) the notion that SMEs are less risky than Corporate loans is challenged. The authors argue that SMEs should not be accorded special treatment (reduced sensitivity to systemic risk; i.e., reduced asset correlations) under the Basel II IRB approach, and note that in most cases, SMEs not only have higher Expected Loss figures, but also higher Unexpected Loss or Value at Risk figures as well. In this Section our central interest has been to test the tenets under which SME capital charges are calculated given two alternative conceptual frameworks for the behaviour of defaulting SME borrowers. These tests will underline the assumptions in the two conceptual frameworks under study, and calculate their impacts on capital charges and asset correlations. In Chapter 4 these results will be compared against internally calibrated asset correlations and capital charges.

In addition to overall results, Table 3.8 also allows for a breakdown of the results by Size Bucket so that for each Bucket, the average capital charge is displayed. In particular, Table 3.8 allows us to observe several patterns and patterns in the calculation of capital charges across borrower Size Buckets.

Taking the same approach as in Subsection 3.3.2 we attempt to explain patterns in Basel II AIRB (Case 2) capital allocation by Size Bucket through a sequential review of our partial implementations. Specifically, we start by observing capital charges under Case 3, wherein a pattern of decreasing capital charges with increasing Size Bucket is immediately clear. This pattern matches that of significantly decreasing PDs with increasing Size in Table 3.3, and moderately increasing asset correlations in Table 3.6.

As expected, the application of the size-adjustment in Case 4 yields significant capital charge deductions for the smallest borrowers. Specifically, we observe a drop in capital charges of 3.9 percentage points for borrowers in the *≤$100,000* Size Bucket, decreasing to 0.5 percentage points for borrowers in the *>$5,000,000* Size Bucket. This discrepancy in size-based deductions yields an alternate capital charge allocation in Case 4 when compared to Case 3 in that we now observe an uptick in capital charges for the largest borrowers as compared to those in the *$3,000,000 - $5,000,000* Size Bucket. Segmenting our portfolio into borrowers in Size Buckets of *$1,000,000 or less* and borrowers in Size Buckets *greater than $1,000,000*, we can then classify two capital allocation patterns: Specifically, we observe decreasing capital charges with Size for the smallest borrowers and a U-shaped pattern in capital charges for larger SME borrowers due to a dissipation of Basel II size-based asset correlation reductions. As can be seen in Table 3.8, this pattern will be repeated under all cases in which the size adjustment is applied – including Cases 1a, 1b and 2. We will return to this interesting characteristic of the Basel II capital allocation framework in Chapter 4.

The application of the Retail-Other capital treatment to eligible borrowers results in large drops in capital charges to borrowers in Size Buckets of *$1,000,000 or less* when compared to Case 3. As in Case 4, these capital charge deductions are decreasing with Size, with a maximum deduction of 9.5 percentage points for borrowers in the *≤$100,000*

Size Bucket, decreasing to a capital deduction of 5 percentage points for borrowers in the *$250,000 - $1,000,000* Size Bucket, and 0 percentage points for larger borrowers. Cases 6a and 6b present capital charges including an adjustment for loan term maturity. As discussed in Section 3.1, this adjustment increases with loan term and decreases with PD; see Equation (3.2). In Case 6b, wherein maturities are not capped, we observe decreasing capital charge increases, over Case 3, with increasing Size. This result corresponds to our finding of increasing loan term maturities with increasing Size. Case 6a presents generally flat capital charge increases, of approximately 4 percentage points, across Size Buckets. This result can be traced to the imposition of a maximum term maturity of five years in Case 6a. Another interpretation can be found in an examination of this increase in capital charges as a percentage of Case 3 capital charges. This comparison yields an increase percentage change, from 25% for the smallest borrowers to 50% for the largest.

Turning to capital charge results under the SA implementation, two flat levels of capital charges are observed, with capital charges of 6.0% applied to borrowers in Size Buckets of *$1,000,000 or less* and charges of 8.0% applied to larger borrowers. In Cases 1 and 2, borrowers are once again segregated into two groups, with smaller borrowers in Size Buckets of *$1,000,000 or less* receiving lower capital charges than their larger counterparts. However, instead of flat capital charges across the two groups, we observe distinct patterns in each group. For the smaller borrowers, we observe decreasing capital charges with Size, while for the larger borrowers we observe U-shaped capital charges.

Finally, Table 3.9 presents Case 2 capital charges by Risk Rating and Size Bucket segment. Results show U-shaped capital charges such that capital charges for borrowers in Size Buckets of *$1,000,000 or less* have decreasing capital charges with increasing Size and borrowers in Size Buckets *greater than $1,000,000* have increasing capital charges with Size. These two patterns are generally repeated within each Risk Rating, as well as overall, and provide a decent summary of the discussion in this Subsection. Capital charges will be further explored in Chapter 4 using internally calibrated asset correlations and probabilities of default.

Section 3.4. Summary of Basel II Partial Implementations

Our review of Basel II approaches to portfolio credit risk focused on the prudential guidelines' treatment of SME borrowers. Specifically, we observed that SME borrowers were accorded significant discounts in capital charges as programmed into the Basel II portfolio credit risk frameworks. These discounts were quantified by our use of a partial implementation exercise in which specific assumptions and calibrations in the Basel II framework were toggled on and off. Proceeding in such a manner we were able to establish capital charge sensitivities to certain assumptions as compared to natural capital benchmarks. Two benchmarks against which we compare resultant capital charges are the full Basel II AIRB implementation (Case 2) and the naïve implementation of the Basel II framework (Case 3).

In particular, we find that the removal of almost all Basel II pre-calibrations, as under Case 3, yields very large increases in capital charges to the smallest borrowers and accords the largest borrowers significant discounts, as compared to Case 2. In addition, we find that the application of Case 3 results in significant change of size-based capital charge patterns as compared to Case 2. Staring from Case 3 and working our way back to Case 2, we attribute these changes largely to the use of two different treatments, the Retail-Other and the Corporate asset class treatments, on our portfolio of SME borrowers. In particular, we find that the introduction of the Retail-Other classification (as in Case 5) yields, for the smallest borrowers, a reduction of capital charges from 16.6% to 7.1% - levels consistent with Case 2. For larger borrowers, we find that the introduction of size-based adjustments to asset correlations for the Corporate asset class yields a reversal of capital charge patterns so that the largest borrowers are allocated capital charges higher (6.9%) than most smaller borrowers (ranging from 6.3% to 6.8%). Case 2 capital charges are finally obtained by accounting for increased capital requirements for longer loan term-to-maturities.

A central objective of this Thesis is to determine which of the two benchmarks outlined above, and their associated contradictory capital charge patterns, more closely approximates reality as observed in the *Financing Company* portfolio of SME borrowers. This objective is pursued in Chapter 4 through the estimation of a single factor asset correlation model, as done in Section 4.3. In Chapter 5, this analysis is enhanced by the use of an alternate model in which correlations are explored through single- and multi-factor models for portfolio credit risk.

Finally, and in conjunction with our determination of SME capital charge patterns, our work in this Thesis will also seek to estimate SME credit portfolio correlations and thereby test which of the Basel II SME asset correlation settings most closely approximates those found in a real-world SME loans portfolio. Namely, our work in this Chapter revealed two settings for SME asset correlations. The first setting, obtained under the Retail-Other treatment, SME borrower asset correlations show no relation to borrower Size and show significantly lower sensitivities to borrower PDs, as well as generally low overall values. This setting nevertheless yields a pattern of decreasing capital charges with increasing Size. In the second setting, obtained under the Corporate asset class treatment, Size and PD relationships with asset correlation play a more prominent role, such that capital charges form a U-shaped pattern with increasing Size.

Presented in this way our results allow us to underscore two broad frameworks for the study of portfolio credit risk: patterns in capital charges across various size segments in our SME portfolio, and; asset (and default) correlations patterns and levels that operate across this portfolio.

Chapter 4. An Asset Value Model for Portfolio Credit Risk

In Chapter 3 we saw that Basel II presented a generally lower capital charge for SMEs. However, this lower capital setting for SME borrowers, and in particular, the mechanisms through which it has been implemented, has come under scrutiny in the portfolio credit risk modelling and management literature. For example, Jacobson, Linde, and Roszbach (2005) and Dietsch and Petey (2004) test some of the SME assumptions made in Basel II on specially constructed data sets of aggregated credit portfolios. Despite the breadth of these data sets, however, the time series over which probabilities of default and correlations are measured have a generally restricted span.

Chapter 2 presented a general abundance of default data in the *Financing Company*'s historical portfolio. Here we construct additional segmentations to those observed in Chapter 2 and use them to explore the SME relationships modeled in Basel II. In particular, we use dual segmentations to estimate probabilities of default and correlations for various Size and Risk Rating segmentations.

This depth of SME default data is then used in a calibration exercise wherein explicit segmentations of borrowers according to size and credit quality allow for direct tests of Basel II pre-calibrations and assumed relationships. This work is similar both in spirit and technique to that of Dietsch and Petey (2004). We use the internal calibration methods of the Gordy (2000) single-factor portfolio credit risk model to measure asset correlations by risk and size segment, and explore the relationship between correlations – and by extension portfolio credit risk charges – along these borrower dimensions.

The use of this technique is doubly informative given the genesis of the Basel II IRB model as an implementation of work by Merton (1974), Vasicek (2002), and Gordy (2003). In addition, the estimation of asset correlations under various calibrations within a single-factor credit risk framework provides an easily implementable and comparable avenue along which to explore SME portfolio credit risk portfolio characteristics. To that end, our results in this Chapter demonstrate that for the *Financing Company* SME portfolio, the relationship between asset correlation and size, as well as those between asset correlation and probability of default, can differ to a large extent from those pre-calibrated in the Basel II portfolio credit risk mechanism.

In estimating asset correlations, greater emphasis is placed on the data quality of the time series over which defaults are counted. In particular, it becomes imperative that the default series be adequately populated, with both healthy and defaulted borrowers, such that an accurate understanding of default behaviour in a given segment is achievable. To that end we introduce several amalgamations of our previously defined Risk Ratings and Size Buckets, refered to as Risk Groups and Size Groups. The aim of these amalgamated groups is to bolster our estimation of correlations and their relationship with other credit characteristics among SME borrowers. In order to ensure an unbiasedness in our construction of Risk Groups we present several definitions and use cross comparisons between them as a check on the robustness of our conclusions.

Having estimated asset correlations reflective of the SME credit characteristics found in the Financing Compnary portfolio, we apply them in the estimation of the portfolio credit risk as defined by the value-at-risk (VaR) and Economic Capital (EC) – or Credit-VaR (CVaR) – at a given confidence level.

In particular, EC values are generated in both asymptotic and non-asymptotic implementations of a single-factor framework; here asymptotic refers to a portfolio with a large number of borrowers such that idiosyncratic risks are assumed to be sufficiently diversified so as not to contribute to portfolio risk. This approximating asymptotic portfolio loss distribution is shown to hold even if borrower exposures are not uniform but with a large number of borrowers not one or a few of which are significantly larger than the rest Vasicek (2002).. The asymptotic framework is directly comparable to a stripped down – or naïve – version of the Basel IRB model which is itself based on an asymptotic single factor model; Gordy (2003) showed that the asymptotic single factor model employed in the regulatory capital mechanism is uniquely able to provide a portfolio invariant framework for capital calculation, a necessary condition for regulators looking to apply a consistent standardized model across varying financial institutions. The non-asymptotic implementation uses Monte Carlo simulations to generate and allocate capital charges while taking into account explicit idiosyncratic risks present in the portfolio. The juxtopostion of these asymptotic and non-asymptotic implementations within the same framework yields an interesting comparison and discussion of the "granularity effect" due to the application of an asymptotic model to a finite real-world

SME portfolio. This granularity effect has been broadly discussed and dealt with in Gordy and Lutkebohmert (2007), BCBS (2006b), Tarashev and Zhu (2007), and other notable papers. Tarashev and Zhu (2007) perform an analysis similar to that performed in this paper.

Our analysis in this Chapter therefore focuses on two central aspects of portfolio credit risk modelling while using Basel II minimum capital regulations as a backdrop. The first is the estimation of correlations as they relate to credit events in an SME portfolio. The second focus of this Chapter is on capital charges generated by an SME portfolio using internally estimated asset correlations.

Our results here will show that, contrary to Basel prescriptions, asset correlations can not be shown to increase with probability of default, nor can they be shown to strictly increase with size within SME segments. These results compare starkly with findings of a generally increasing relationship between asset correlations and PDs in Dietsch and Petey (2004), Duellmann and Scheule (2003) and Gordy (2000) and a decreasing relationship in Lopez (2004) and, of course, BCBS (2006a). For the relationship with Size, these results, in their general rejection of decreasing asset correlations with decreasing Size, run counter to the relationship programmed into the Basel II Corporate exposure class IRB function for SME borrowers, while the lack of a programeed relationship for the Retail-Other asset class suggests the recognition of a potential absence of such a relationship for the smallest borrowers. Following Duellmann and

Scheule (2003), we frame our results on Size patterns with several hypotheses on the relationship found in the literature.

In addition to these patterns, asset correlations estimated in this Chapter will also be characterized by generally low values. However, these values correspond in scale to those found in the literature. Frye (2008) and Chernih, Henrard, and Vanduffel (2010) demonstrate that asset correlations derived from loss data – such as that employed here – consistently generate values to scale with those found in this Chapter, while asset correlations derived from market equity data generally produces asset correlation values on scale with those applied within Basel II.

Applying these correlations within a single factor portfolio credit risk framework, we explore capital charge patterns along risk and size segmentations. Our results will show that the overall value of measured asset correlations can have a significant impact not only on the overall portfolio capital charge but also on capital charge patterns by segment.

In order to test and adjust for the results obtained with internally calibrated asset correlations, we apply a log odds adjustment to estimated asset correlations rendering their overall portfolio value on par with that obtained in Basel II. Our results show that for asset correlations at the scale of those found in Basel II, SME capital charges by Size should display a decreasing pattern. As a final note, however, we echo Duellman and Scheule (2003, p. 21) and observe that macro-prudential factors, as well as micro-prudential ones, play an important role in the preset calibration of Basel II parameters and capital charges. In this Thesis, these factors, e.g., the avoidance of pro-cyclical effects and the encouragement of looser credit conditions for small borrowers, are not considered as we focus on the micro-prudential factors reflected in the credit characteristics of an SME portfolio.

More specifically, Chapter 4 is organized as follows: In Section 4.1 we expand on the asymptotic single factor origins of the IRB risk weighting function, examined in Chapter 3 and Appendices A and B, and introduce a single factor model used for the estimation of SME correlations in our portfolio. In addition, a Monte Carlo simulation procedure is built around the model allowing for the non-asymptotic estimation of portfolio credit risk for our portfolio. In Section 4.2 we build on the work done in Chapters 2 and 3 by adding alternative dual-dimension segmentations. These segmentations are used in the estimation of PD, PD volatility and both asset and default correlations. Correlations here are estimated using the model introduced in Section 4.1. In Section 4.3, PD and correlation estimates derived in Section 4.2 are applied in both the asymptotic and non-asymptotic single factor models. The results, presented in this way, provide an interesting avenue of study when compared to each other and the results obtained in Chapter 3. Finally, Section 4.4 presents the Chapter's conclusions.

Section 4.1. A Single Factor Model for Portfolio Credit Risk

Recalling Section 3.1, Equation (3.1) can be reformulated as:

$$K = [LGD \times V(PD, R, \alpha) - PD \times LGD] \times \left[\frac{1 + (M - 2.5) \times b(PD)}{1 - 1.5 \times b(PD)}\right], \qquad (4.1)$$

where,

$$V(PD, R, \alpha) = N\left[\frac{N^{-1}(PD) + \sqrt{R} \times N^{-1}(\alpha)}{\sqrt{1 - R}}\right], \qquad (4.2)$$

and, $\alpha = 99.9\%$. Equation (4.2) is commonly referred to as the Vasicek distribution or function, as given in Equation (B.7). In Appendix B, a full derivation of Equation (4.2) is given. As previously noted, the Basel II IRB model is based on the Vasicek (2002) asymptotic approximation of the single risk factor model based on Merton (1974). In this Chapter we use the underlying Merton (1974) framework in two capacities: as a model for the estimation of portfolio credit risk loss distributions; and as a tool for the measurement and estimation of default correlations within a given credit portfolio.

In the first capacity, and in contrast to Chapter 3, we present the IRB risk weighting function as an asymptotic version of a single factor asset value model (*AVM*) for the determination of portfolio credit risk. In this asymptotic framework idiosyncratic risks are assumed away. We also present the framework as a non-asymptotic single factor asset value model for portfolio credit risk in which a Monte Carlo simulation procedure is used to generate portfolio loss distribution, and in which systematic and idiosyncratic risks in the portfolio are explicitly modelled; see Subsection 4.1.1.

In the second capacity, we use the framework, as presented in Gordy (2000), along with internally estimated PDs and PD volatilities, to non-parametrically estimate internally-calibrated asset (and, by extension, default) correlations for our SME portfolio; see Subsections 4.1.2 and 4.2.2. These correlations and their relationship to other credit risk measures and factors (e.g., PD, Size, etc…) are then evaluated and compared to values and relationships programmed into the Basel II IRB framework; see Section 3.1. We compare results generated using the internally calibrated correlations and both the asymptotic and non-asymptotic versions of this framework; see Section 4.3.

More specifically, asset correlation values, estimated by borrower segment, denote the dependence of borrowers in a given segment to a single underlying latent factor. Borrowers in different segments are allowed to have divergent dependencies on the same single factor, while borrowers in the same segment share the same dependence. Segments are defined along Risk and Size dimensions. Proceeding in this manner, we are able to test Basel II assumptions on the relationship between asset correlation values and Size, and asset correlation values and riskiness (as represented by the segment PDs).

Having generated our internally calibrated asset correlations, we define a non-asymptotic single factor model for portfolio credit risk. The internally calibrated asset correlations are used within this framework and a Monte Carlo simulation procedure is used to generate a portfolio loss distribution for our portfolio of SME borrowers.

In Subsection 4.3.2, we input the internally estimated asset correlations from Subsections 4.2.2 and 4.2.3 into an asymptotic single factor model. The capital charges generated within this asymptotic framework are contrasted against those obtained in the simulation-based non-asymptotic framework. This comparison technique provides us with an estimate of the capital estimation error, in an SME setting, due to the application of the asymptotic framework to a real-world portfolio of finite granularity. This work is similar in technique to that used in Tarashev and Zhu (2007).

Finally, in addition to a comparison of overall capital charges for the portfolio, we examine two simple and commonly applied capital allocation schemes within our non-asymptotic framework and compare them to the allocations under the asymptotic framework. In order to ensure efficient comparability, allocation schemes are applied to the same simulation-based VaR value.

Our results will indicate that the portfolio characteristics of a real-world portfolio with SME characteristics – high PD, low correlation values – may display a higher granularity effect when asset correlations are estimated from default data. In addition, we will find that single exposure size is an important factor in determining capital charges, one that is not properly accounted for in some analytical and asymptotic allocation schemes; see, for example, Heitfield, Burton, and Chomsisengphet (2006). In particular, we will challenge the inclusion of size effects through correlation parameters, as is done in the Basel II IRB framework.

Subsection 4.1.1. Internal Estimation of Asset Correlations

In this Subsection we present the asset correlation estimation methods used in Gordy (2000). In that paper the author proposes a calibration technique that allows for the common calibration of a single factor asset value model and a single factor implementation of the *CreditRisk*$^+$ framework; see Credit Suisse (1997). As such, the single factor model presented below plays several important roles in this Thesis. On the one hand, it establishes the estimation technique used in Chapter 4 for the internal estimation of asset correlations for an SME portfolio. This is doubly informative, not just as a stand alone model, but also as a basis for comparison with other studies such as Dietsch and Petey (2004) who also use this technique towards the estimation of SME correlations.

On the other hand, this model and calibration method serves as a bridge to an alternative model for portfolio credit risk, as found in Chapter 5's presentation of the *CreditRisk*$^+$ framework. Given the above, we now turn to the single factor asset value model presented below; for a review of the underlying mathematical and concentual concepts related to the asset value framework, see Appendix A.

For a given segment (ζ_A) of borrowers sharing some common characteristic, e.g., same RR, we define for each borrower (i) the standard normal latent factor $y_{\zeta_A,i}$, such that:

$$y_{\zeta_A,i} = w_{\zeta_A}x + \sqrt{\left(1 - w_{\zeta_A}^2\right)} \cdot e_i, \tag{4.3}$$

where x is the systematic factor, e_i is the idiosyncratic factor and each is an independent standard normal variate. Characterising the systematic factor as being representative of the state of the economy, borrowers' dependence on the business cycle can be measured by the weighting w_{ζ_A} on x. Given two borrowers from two different segments, (ζ_A) and (ζ_B), the covariance between their latent factors is then defined as:

$$Cov\left[y_{\zeta_A,i}; y_{\zeta_B,J}\right] = w_{\zeta_A} \cdot w_{\zeta_B} \tag{4.4}$$

Borrower, (i)'s status at the end of a given time horizon is set to default if:

$$w_{\zeta_A}x + \sqrt{\left(1 - w_{\zeta_A}^2\right)}e_i < N^{-1}\left(\bar{p}_{\zeta_A}\right), \tag{4.5}$$

where $N^{-1}(\cdot)$ denotes the inverse cumulative standard normal distribution function and $\bar{p}_{\zeta_A}$ is the unconditional, or long-term, probability of default for segment (ζ_A).

In contrast to the Merton (1974) asset value model's dependence on externally measured asset correlations in the determination of default correlations, the single latent factor model relies on conditional default rate dynamics to determine asset correlations – see, for example, Dietsch and Petey (2002) and Dietsch and Petey (2004) – and, by extension, default correlations. Following Gordy (2000), and using Equation (4.5), borrower (i)'s probability of default conditional on a realization of the single systematic factor is given by:

$$p_{(\zeta_A)}(x) = \Pr\left[e_i < \frac{N^{-1}(\bar{p}_{\zeta_A}) - w_{\zeta_A} x}{\sqrt{1 - w_{\zeta_A}^2}} \middle| x \right] = N\left[\frac{N^{-1}(\bar{p}_{\zeta_A}) - w_{\zeta_A} x}{\sqrt{1 - w_{\zeta_A}^2}} \right], \tag{4.6}$$

with the joint probability of default for two borrowers (i) and (k) in the same segment (ζ_A) is then given by:

$$\begin{aligned} &\Pr\left[y_{\zeta_A,i} < N^{-1}(\bar{p}_{\zeta_A})\ and\ y_{\zeta_A,k} < N^{-1}(\bar{p}_{\zeta_A}) \middle| x \right] \\ &= \Pr\left[y_{\zeta_A,i} < N^{-1}(\bar{p}_{\zeta_A}) \middle| x \right] \cdot \Pr\left[y_{\zeta_A,k} < N^{-1}(\bar{p}_{\zeta_A}) \middle| x \right] = \bar{p}_{\zeta_A}^2. \end{aligned} \tag{4.7}$$

Having explicitly established a link between our single factor latent variable model and the Vasicek distribution through Equations (4.2) and (4.6), we now use Equation (4.7), to define the variance of the conditional probability of default as a function of the asset correlation and the unconditional probability of default. Then, using the method proposed in Gordy (2000), and our empirically calibrated unconditional probability of defualt and conditional probability variance we will estimate the representative asset correlation for segment (ζ_A), $w_{\zeta_A}^2$.

Specifically, we write the variance for the conditional probability of default $p_{(\zeta_A)}(x)$ as the following:

$$Var\left[p_{(\zeta_A)}(x)\right] = E\left[p_{(\zeta_A)}(x)^2\right] - \left(E\left[p_{(\zeta_A)}(x)\right]\right)^2$$
$$= E\left[\Pr\left[y_{\zeta_A,i} < N^{-1}\left(\bar{p}_{\zeta_A}\right) and \;\; y_{\zeta_A,k} < N^{-1}\left(\bar{p}_{\zeta_A}\right)\middle| x\right]\right] - \bar{p}_{\zeta_A}^2. \tag{4.8}$$

Given the assumptions of standard normality for the latent variables $y_{\zeta_A,i}$ and $y_{\zeta_A,k}$, and the correlation $w_{\zeta_A}^2$, based on Equation (4.4), the unconditional expectation in Equation (4.8) is given by:

$$Bivnorm\left(N^{-1}\left(\bar{p}_{\zeta_A}\right), N^{-1}\left(\bar{p}_{\zeta_A}\right), w_{\zeta_A}^2\right), \tag{4.9}$$

so that:

$$Var\left[p_{(\zeta_A)}(x)\right] = Bivnorm\left(N^{-1}\left(\bar{p}_{\zeta_A}\right), N^{-1}\left(\bar{p}_{\zeta_A}\right), w_{\zeta_A}^2\right) - \bar{p}_{\zeta_A}^2. \tag{4.10}$$

Here we assume serial independence for the systematic factor realizations and conditional independence between borrower defaults; Dietsch and Petey (2004). To calculate $\left(w_{\zeta_A}^2\right)$, we first calculate the conditional variance $Var\left[p_{(\zeta_A)}(x)\right]$ as a function of the data-derived unconditional variance $Var\left[p_{(\zeta_A)}\right]$, the average number of healthy borrowers in a given segment $H_{(\zeta_A)}$ across the beginning of one year periods, and the unconditional probability of default for that segment, $\left(\bar{p}_{\zeta_A}\right)$; see, for example, Dietsch and Petey (2002, p. 313). The resultant equation is given below:

$$Var\left[p_{(\zeta_A)}(x)\right] = \frac{Var\left[p_{(\zeta_A)}\right] - E\left[1/H_{(\zeta_A)}\right] \cdot \bar{p}_{\zeta_A} \cdot \left(1 - \bar{p}_{\zeta_A}\right)}{1 - E\left[1/H_{(\zeta_A)}\right]}. \tag{4.11}$$

Given the joint probability of default found in Equation (4.9), we can calculate the default correlation (DC) between two borrowers in the same segment (ζ_A) as:

$$DC_{\zeta_A} = \frac{Bivnorm\left(N^{-1}\left(\bar{p}_{\zeta_A}\right), N^{-1}\left(\bar{p}_{\zeta_A}\right), w_{\zeta_A}^2\right) - \bar{p}_{\zeta_A}^2}{\left(\bar{p}_{\zeta_A} \cdot \left(1 - \bar{p}_{\zeta_A}\right)\right)}. \tag{4.12}$$

Default correlation results will serve as a basis for comparison to default correlations calculated in Chapter 5 according to various segments.

To estimate the unconditional probability of default and the conditional probability of default variance we use the methods described in Chapter 2 on the portfolio using the segmentation to be presented in Section 4.2. Applying Equation (4.10) the representative asset correlation is then estimated for each segment.

Subsection 4.1.2. The Single Factor Portfolio Credit Risk Model

The question of credit portfolio allocations is an integral part of any economic capital methodology and is a continuing focus of research in the field, see, for example Mausser and Rosen (2008), Heitfield, Burton, and Chomsisengphet (2006) and Garcia, Alderweireld, and Leonard (2006). In this Subsection we elaborate on the portfolio loss distribution generating mechanism described in the introduction to this Section. This is done through a discussion of the value-at-risk (VaR) and EL figures and their allocation to obligors in the portfolio.

We use a Monte Carlo simulation method to draw realizations of the single systematic factor x, as well as realizations of the idiosyncratic factor e_i, for each borrower (i). For each previously defined segment (e.g., Risk and Size) the corresponding internally estimated asset correlation is used to define the movement of the latent factor of a borrower in that segment according to the systematic factor and that borrower's randomly generated idiosyncratic factor. Default is assigned to borrower (i) if Equation (4.5) is found to hold. Taking the exposure as given and multiplying by a given LGD a loss is calculated for a given loan. Aggregating across borrowers we obtain a portfolio loss for a given draw of the systematic factor x. Portfolio EC values are derived over 150,000 simulations, the 99.9% VaR and the portfolio EL. For each loan, capital is allocated according to average VaR contributions as measured across 300 realizations of a 99.9% VaR. For each segment, EC values are given as the dollar-weighted average across all loans for borrowers in that segment. EC values are presented along with the percentage change in EC in going from an asymptotic implementation to a simulation-based implementation. Taking the 99.9% VaR of the portfolio loss distribution and subtracting the portfolio EL yields an Economic Capital or risk capital charge for the portfolio. The portfolio EL is calculated as the sum of the individual obligor ELs.

Specifically, we define the VaR contribution of an obligor j as:

$$C_j^{VaR_\alpha} = E\left[L_j \middle| L = VaR_\alpha\right]; \tag{4.12}$$

see, for example, Mausser and Rosen (2008. Pg 691). Taking Equation (4.12), we then proceed by simply repeating our simulation procedure; for each execution, we save the VaR_α run, maintaining our realized obligor loss under an $(L = VaR_\alpha)$ simulation. For each obligor, we then average over our realizations and obtain a set of $\left(C_j^{VaR_\alpha}\right)$ such that:

$$L = \sum_{j=1}^{N} C_j^{VaR_\alpha} \cdot L_j. \tag{4.13}$$

Section 4.2. PD and Asset Correlation Estimation for SME Segments

In Chapter 2 we constructed Risk Ratings and Size Buckets that largely reflected the status quo within the *Financing Company* SME loans portfolio. Organizing our portfolio along these credit risk dimensions, we were able to obtain a breakdown of portfolio segments, in terms of borrower and $OS distributions, as well as default rates over time by segment; for borrower and $OS distributions by RR and SB see Tables 2.2, for default rate time series and average rates by RR and SB see Tables 2.10 and 2.12, respectively.

In Chapter 3, it was shown that these time series of default rates, along with loan $OS and estimates of LGD, were adequate for providing the necessary inputs needed for Basel

II IRB capital charge estimation. Specifically, as noted in Section 3.2, Basel II requires that at least five years of default rate data be available in specifying probabilities of default by risk grade; see BCBS (2006, p. 102). For the Risk Ratings specified in Chapter 2, recall, the average PD is calculated as the weighted mean of default rates over our sample period of 12 years. This method represents industry best practice and is widely applied in the literature; see, e.g., Standard & Poor's (2011, p. 2) and Dietsch and Petey (2002, p. 311).

In Chapter 4, we seek to calibrate the Merton-type asset value model (*AVM*) – an asymptotic implementation of which is used in the Basel II IRB approach; see, for example, Section 4.1 – to the internal data of the *Financing Company*. This calibration involves the estimation of asset correlations for various segments of our portfolio, and will, in turn, provide us with empirical evidence on the SME credit risk relationships outlined in the Basel II IRB approach and programmed into functions defining the asset correlation for the Corporate and Retail asset classes; see Section 3.1.2 and, in particular, Equations (3.1) to (3.5).

To that end, we move away from the Risk Ratings defined in Chapter 2 and organize our borrowers into four Risk Groups (RGs) according to Risk Rating: Low Risk (comprising RRs 1-3); Medium Risk (comprising RRs 4-5); High Risk (comprising RRs 6-7); and Very High Risk (comprising RRs 8-9). Collectively, these Risk Groups will be referred to as RGs. Our definition of the RGs is primarily driven by the need to have significant numbers of default in every year of our default rates in order to obtain robust estimates of default rates volatilities and, by extension, correlations. As such, the Low Risk RG aggregates across the most RRs in order to garner enough defaults every year to render the data series suitable for analysis. To ensure that our work doesn't suffer from biases in the construction of RGs, we propose various constructions of adequately populated RGs and present PD and correlation results obtained for each of these alternative Risk and Size groupings. These alternative groupings and their associated auxiliary Tables will be referred to extensively throughout the remainder of this Section.

In addition, we define four Size Groups according to the Size Buckets over which they are aggregated. Given the high populations of defaults among the smallest borrowers, the focus of the Size Group definitions will be on the largest borrowers in the portfolio. To that end, we define the following Size Groups: the *≤$100,000 Size Group*; the *$100,000 - $250,000 Size Group*; the *$250,000 – $1,000,000 Size Group*; and the *≥$1,000,000 Size Group*, comprised of the *$1,000,000 – $3,000,000*, *$3,000,000 – $5,000,000*, and *≥$5,000,000 Size Buckets*. Auxiliary Tables present alternative definitions of the largest SG alongside results by PD and correlation. Auxiliary Tables will also include results for the original RRs and SBs defined in Chapter 2.

Our work will show that for SME borrowers, relationships between asset correlation and Size and PD cannot be accorded simple linear attributes. As seen in Chapter 3, this property among SME borrowers finds some support in the formulation of asset correlations in Basel II under the Retail-Other asset class, as well as support from the persistence of contradictory empirical evidence and hypotheses on these relationships in the data in the literature; see, for example, BCBS (2005) and Duellman and Scheule (2003). Our results will show low overall values for asset correlations for SMEs. These results are in line with other literature in which asset correlations are estimated from default data; see Chernih, Henrard, and Vanduffel (2010) and Frye (2008).

Whether due to calculation methodology or to underlying data type, quality or quantity, the presence of low asset correlations is not uncommon in the literature; see papers cited above. When comparing to prudentially conservative regulatory asset correlation values, these low asset correlations can be a source of glaring incongruity. In this Section we will present an ad hoc asset correlation boosting methodology, the application of which follows similar exercises in the literature; see, for example, Dietsch and Petey (2002).

In Subsection 4.2.1 we construct our Risk and Size Groups and estimate probabilities of default using dual segmentations. In Subsection 4.2.2 we apply the methodologies presented in Subsection 4.1.1 to estimate asset and default correlations overall for the portfolio as well as by Risk and Size Group segments. In Subsection 4.2.3 we present an ad hoc asset correlation boosting methodology.

Subsection 4.2.1. PD Estimation by Risk and Size Groups

To construct our Risk and Size Groups we evaluate the number of defaults in the Risk Ratings and Size Buckets, both at the overall single dimension level and as segments of each other in a dual dimension setting. Overall, our analysis indicates that sufficient populations of default are attainable when the 1 to 3 Risk Ratings are collapsed into one another, forming the 1-3 Risk Group, and when the 8 and 9 RRs are collapsed into one another, forming the 8-9 RR. We also find that, overall, the *≤$100,000*, *$100,000 - $250,000*, and *$250,000 - $1,000,000* Size Buckets have sufficient populations of defaults. For the larger Size Buckets, we find that sufficient populations of default are attainable when the *$1,000,000 - $3,000,000*, *$3,000,000 - $5,000,000,* and *>$5,000,000* Size Buckets are collapsed into one another.

Turning to Risk Rating – Size Bucket segments, we observe that an amalgamation of the 4 and 5 Risk Ratings, into the 4-5 Risk Group, yields sufficient defaults by Risk Group – Size Group segment. Our final five Risk Groups are therefore given by the 1-3 Risk Group, the 4-5 Risk Group, the 6 and 7 Risk Groups (Ratings) individually, and the 8-9 Risk Group. Our final four Size Groups are given by the *≤$100,000*, *$100,000 - $250,000*, and *$250,000 - $1,000,000* Size Groups (Buckets) and the *>$1,000,000* Size Group.

Table 4.1 provides probability of default estimates, along with PD variance and normalized standard deviations under the newly defined Risk Groups and Size Groups. For comparative purposes, Table 4.1A provides similar measures and statistics for the full range of Risk Ratings and Size Buckets presented in Chapter 2. Going the other way, we reduce our RGs to three, as opposed to the 9 RRs and the 4 RGs defined above, by amalgamating the 6 and 7 RGs, forming the 6-7 RG.

Overall, PDs are shown to increase monotonically with Risk Group such that PDs for the 1-3 RG and the 8-9 RG are equal to 1.30% and 8.75%. PDs by overall Size Group are shown to decrease monotonically as Size increase. We therefore observe PDs for the smallest borrowers equal to 8.32% and PDs for the largest borrowers equal to 2.37%. This pattern in default rates by Size is not surprising given the distributions of borrowers by Risk Rating in each Size Group and Bucket; see, specifically, Table 2.2A.

Observing our data by RG-SG segment, we observe that overall RG patterns are observed in all SGs. We similarly observe a repeat of overall Size patterns in all RGs except the 8-9 RG where a U-shape pattern is observed, such that, unlike other RGs, we observe an increase in the relative riskiness of borrowers in the *>$1,000,000* Size Group as compared to those in the *$250,000 - $1,000,000* SG. This U-shape for the largest riskiest borrowers may be reflective of a willingness to tolerate elevated risk characteristics among smaller borrowers while acknowledging the severe circumstances under which a larger borrower would find himself in the elevated risk grouping. Put another way, high PDs among larger borrowers in the 8-9 RG may be reflective of deteriorating financial conditions among those borrowers while decreasing PD patterns with increasing Size among other borrowers in the 8-9 RG may be reflective of the risk appetite at or near authorization for smaller borrowers. These results are supported by findings in Table 4.1A for the 8 and 9 RRs.

In addition to results for the original Risk Ratings and Size Buckets, we present results for various alternative Risk and Size groupings. In particular, we add the *>$250,000* Size

Group, amalgamating the *\$250,000 - \$1,000,000* and the *>\$1,000,000* Size Groups; as well as adding the 6-7 Risk Group. Results for these additional Risk and Size Groups are presented in Table 4.1B and show supporting results. In the case of the Size-based PD patterns in the 8-9 RG, we observe that the amalgamation of the *\$250,000 - \$1,000,000* and the *>\$1,000,000* Size Groups results in strictly decreasing pattern, without the up tic found in the *>\$1,000,000* Size Group in Table 4.1. In Table 4.1A we present PD result, PD variance and PD normalized standard deviations under the original Risk Rating and Size Bucket segments defined in Chapter 2 and find that pattern results by segment lose their monotonicity but maintain overall patterns.

In the next section we use the PD, PD variance and normalized standard deviation estimates of the segment-specific default rate time series to estimate asset and default correlations. These estimates can be observed in the bottom panels of the Tables discussed above.

Subsection 4.2.2. SME Single Factor Model Correlations by Risk & Size Group

In Subsection 4.2.2 we use the single factor model described in Subsection 4.1.1 and our unique data set to challenge Basel II assumptions on the credit risk behaviour of SME borrowers. Namely, we test the validity of a negative relationship between PD and asset correlation for SMEs throughout the IRB framework, as well as the positive relationship between borrower Size and asset correlation for the Corporate asset class. Implicitly our estimates of SME asset correlations may be interpreted as a reflect on the lower Basel II settings of asset correlations for SME borrowers – e.g., asset correlations in the range of 3% to 16% for SMEs under the Retail-Other asset class as opposed to asset correlations in the 8% to 24% range for borrowers in the Corporate asset class; see Section 3.1. However, we cite work presented in Frye (2008) and Chernih, Henrard, and Vanduffel (2010) that may cause pause on that comparability.

Size and Risk Groups present homogenous sets of borrowers for which representative asset correlations can be estimated. We follow in the steps of Dietsch and Petey (2002, 2004) and apply non-parametric internal calibration techniques found in Gordy (2000) to the *Financing Company* default rates and portfolio. For the Size and Risk Group segments, Table 4.2 depicts the internally calibrated *Financing Company* SME asset and default correlations using the data presented in Table 4.1. As in Section 4.2.1, we present results both on the overall level and the segment level, unlike Section 4.2.1, however, we do not find the presence of strong relationships between asset correlation and either RG or SG.

Specifically, we observe an overall portfolio asset correlation of 0.34%, the lowest observed value in our portfolio. For the overall RGs, we do not observe evidence of either monotonically increasing or decreasing patterns of asset correlation with PD. Specifically, we observe the lowest asset correlation values at the 8-9 RG (0.93%) and the 1-3 RG (0.98%), while the highest asset correlation values are observed at the 4-5 RG (1.49%) and the 7 RG (1.30%). In Tables B.5 and B.6, asset and default correlation results are presented for the 6-7 RG and the *>$250,000* Size Group, among others, respectively. For the overall RGs, results using the 6-7 RG show an inverse-U pattern in asset correlations, so that the two lowest values at the 1-3 RG and the 8-9 RG are maintained. Under the full set of Risk Ratings and Size Buckets defined in Chapter 2, Table 4.2A asset correlations show no relationship with PD values at the overall RR level.

For the overall SGs, we do not observe a pattern in asset correlation by Size over the four fixed SGs. In particular, we observe the highest asset correlation for the *\$100,000 - \$250,000* SG (0.77%) and the lowest asset correlation in the *≤\$100,000* SG (0.34%). Once again turning to alternative SG segmentations in Tables B.4 and B.6, we observe an inverse-U pattern over three SGs in Table 4.2C, while no discernible evidence of a pattern is visible over the six SBs presented in Table 4.2A.

Controlling for Size, and observing results horizontally across Table 4.2, we do not observe any discernible patterns in asset correlation with respect to PD among RG-SG segments. This result is confirmed in Table 4.2B, in which the 6 and 7 RGs are amalgamated, for the three smallest borrower SGs, and emphasized by the reversal of the only PD pattern that does appear: For the largest borrowers in Table 4.2, those in the *>\$1,000,000* SG, we observe a U-shaped pattern, while in Table 4.2B, wherein the two largest SGs are replaced by the *>\$250,000* Size Group, an inverse-U pattern for the largest borrowers is observed, with no discernible pattern for the other SGs.

Controlling for RG we examine asset correlation results vertically by RG-SG segment in Tables 4.2 and B.5. In both Tables 4.2 and B.5, the 1-3 RG displays the opposite pattern to that found for the majority of other RGs, namely that of a U-shaped relationship between asset correlation and Size. In Table 4.2C this combination of patterns is upheld over the three SGs evaluated with the 6-7 RG providing an alternative, increasing, pattern over SGs.

Empirical work in Gordy (2000, p. 134), using the same framework used here in Chapter 4, estimates asset correlations for various S&P risk grades and shows an increasing relationship with increasing PD. In Dietsch and Petey (2002, p. 312) the authors evaluate asset correlations in an *AVM* framework across risk grades while controlling for Industry. Their results indicate a uniquely inverse-U pattern across risk grades, this pattern is characterized as generally increasing given low borrower counts in the highest risk grade.

In Dietsch and Petey (2004, p. 780), SME asset correlations are evaluated over aggregated data sets of borrowers in France and Germany using three SME Size groupings and eight risk grades. Results indicate a generally increasing pattern with increasing PD, overall, but no strict relationship - within Size groups the results show even less homogeneity in pattern. Examining results by overall Size, the authors observe decreasing asset correlations with increasing Size over the three SME Size groups. Examining results by Size and PD segments, the authors observe a mixture of patterns; see Table 4, Dietsch and Petey (2004, p. 780).

Lopez (2004, p. 273) finds evidence of decreasing asset correlations with increasing PD at the overall level for datasets of borrowers worldwide, in the US, Japan, and Europe. These results find some support when controlling for Size, however, a universally monotonic relationship is not clear; see Table 4, Lopez (2004, p. 275). Examining overall results by Size, the author finds evidence of strictly increasing relationship across all geographically-defined portfolios; see Table 3, Lopez (2004, p. 274). This result is upheld when controlling for PD; see Table 4, Lopez (2004, p. 275).

Our results fail to confirm the presence of strict relationships between asset correlations and either PD or Size. Taken in conjunction with low overall asset correlation values and comparing to Basel II IRB asset correlation settings under the Corporate and Retail asset classes, it is possible to say that our evidence resembles the latter more than former. This assertion is bolstered by the lack of Size-adjustment in the Retail-Other setting, the lower overall asset correlation values, and the decreased sensitivity of asset correlation to PD. Nevertheless, our results maintain a general break with Basel II precepts. In Chapter 5 we present the *CreditRisk*$^+$ framework for the estimation of portfolio credit risk and implied values of default correlation. We will examine default correlation results obtained in this Subsection within the context of a comparison of these results with those obtained under the *CreditRisk*$^+$ framework.

In the following paragraphs we present a review of theoretical and empirical evidence on the relationships between asset correlations and both PD and Size as found in Duellman and Scheule (2003, pp. 3-6). The aim of this review is not to present new material but to use the existing material to enrich the results presented in this Chapter.

More on the relationship between SME asset correlations and PD

In Duellman and Scheule (2003) a brief outline of two theoretical arguments for the relationship between asset correlations and PD is presented.

In the first argument, it is proposed that borrowers with elevated sensitivities to macroeconomic developments may choose more conservative capital structures, thereby reducing overall riskiness. This theory then indicates that borrowers with higher asset correlations may display lower probabilities of default.

In the second argument, it is proposed that if an increase in a borrower's credit risk is initiated by idiosyncratic events, then the relative importance of idiosyncratic risks to systematic risks increases.

More on the relationship between SME asset correlations and Size

Duellman and Scheule (2003) present three tentative explanations for the presence of discrepancies in asset correlations by borrower size. The first explanation, referred to as the "business sector argument" presents size discrepancies as proxies for varying dependencies across industries. Here, asset correlations are seen as a measure of dependence on a global business cycle. This argument is bolstered by variances in predominant borrower sizes across industries. More specifically, if highly cyclical industries are dominated by large borrowers and less cyclical industries are dominated by small borrowers, then we should expect that smaller borrowers to display lower asset correlations than larger borrowers.

In Duellman and Scheule (2003), three sectors considered to be highly cyclical (recall they use German data) are presented: manufacturing; construction; and automotive, along with three sectors considered less cyclical: transport & communication services; health & financial services; and other public & personal services. In the case of the first three, SMEs account for a small (approximately 15%) percentage of borrowers, while in the last three SMEs account for a significant percent of borrowers (between 30% and 40%).

The second explanation presents large borrowers as better diversified firms as compared to their smaller counterparts. This better diversification reduces idiosyncratic risks among large borrowers, thereby increasing their correlation to systematic risks relative to smaller borrowers. This hypothesis is contested by Roll (1988) which presented empirical work suggesting that small firms displayed higher diversification than larger borrowers.

Contrary to the first two hypotheses, the third hypothesis, referred to as the "financial accelerator" hypothesis, suggests that asset correlations are in fact inversely related to borrower size. The hypothesis, put forward in Bernanke and Gertler (1995) and Bernanke, Gertler, and Gilchrist (1996) holds that smaller borrowers' reliance on bank loans for financing, as compared to larger borrowers who can access capital markets, renders them more vulnerable to macroeconomic shocks and their effects on credit-market conditions. In particular, empirical work in Bernanke, Gertler, and Gilchrist (1996) suggests this negative relationship between borrower size and asset correlation holds even when controlling for industry. This effect may be mitigated, however, by the presence of strong bank-borrower relationships ensuring the availability of credit even periods of economic downturn, Duellman and Scheule (2003, pg. 21) and von Kalckreuth (2001).

Subsection 4.2.3. Boosted SME Single Factor Model Asset Correlations

The Basel II IRB framework reviewed in Section 3.1 provides for asset correlations ranging from 3% (applied to Retail-Other exposures; see Equation (3.4) for details) to 30% (applied to the High Volatility Commercial Real Estate (HVCRE) asset class; see BCBS (2006a, p. 66) for details). For SMEs, the range maximum is reduced to 20%; see Subsection 3.1.3. Compared to these prudential regulatory levels, internally calibrated asset correlations derived in this Chapter within a single factor framework appear to be of a significantly lower level. This discrepancy in the overall level of internally calibrated asset correlations and those found in the Basel II regulatory framework retains a sharp focus both in the academic literature and practical implementations of portfolio credit risk frameworks.

In particular, Chernih, Henrard, and Vanduffel (2010) review asset correlation results found in the literature and segregate them by source data type. Their survey – replicated in Table 4.3 – suggest that the type of source data (i.e., default data vs. market-based equity data) may play a significant role in the setting of overall asset correlation levels; see Table 1 and Table 2 in Chernih, Henrard, and Vanduffel (2010, p. 53). Citing this work, Frye (2008) observes that the maximum asset correlation obtained with observed defaults as the source data, is approximately 10%, with that figure dropping to 2.3% for some studies; see, for example, Hamerle, Liebig, and Roesch (2003b). This maximum figure of 10% asset correlations, when estimated over default data, compares to a minimum of 10% asset correlations when estimated over equity data, and is attributed to observed and conceptual differences in the underlying data; see Frye (2008). In addition, working with both default data, based on the observed number of defaults, and loss data, derived from provisions data, Duellman and Scheule (2003) show the default data provides the lowest overall levels of asset correlation.

Commenting on low asset correlation levels obtained in their respective studies, Dietsch and Petey (2004) and Duellman and Scheule (2003) suggest that the use of aggregated data may engender some over-diversification within their data sets and therefore be a possible source of low correlation values. Dietsch and Petey (2004) also suggest their shortened time series as a potential source of reduced asset correlations due to the lack of a full economic cycle over the time period considered.

In contrast, our research benefits from the use of non-aggreagted data, specific to one institution targeting high-risk SME borrowers. In addition, we benefit from a time series with 12 years of data. Despite our longer time series, however, we observe that the period covered is comprised of a prolonged period of economic growth along with low volatilities in our observed default rates. In particular, Table 4.4 compares normalized default rate volatilise obtained in our study with those observed in Standard & Poor's (2011) over the period 1981 to 2010. As can be seen in Table 4.4, *Financing Company* normalized default rate volatilities are considerably lower than those observed over the Corporate defaults studied in Standard & Poor's (2011). The presence of lower volatilities may be a significant contributor to low asset correlation values.

While the 2008 - 2010 period added volatility to our data, the tameness of the Canadian 2001 - 2002 economic slowdown may explain lower asset correlations compared to other studies. Hamerle, Liebig, and Roesch (2003b) segregate their data into country and industry and estimate asset correlations using three models. Results for Canada reveal a maximum observed asset correlation of approximately 0.6% as found in the Agriculture sector; see Exhibit 2 in that paper, pg 22. For the Canadian Manufacturing and Services sectors, two sectors that together make up approximately half of the *Financing Company*'s lending portfolio (see, for example, Tables 2.5A and 2.5B), the authors estimate asset maximum asset correlations of approximately 0.3%. These results compare to roughly equivalent results for France, maximum asset correlations of approximately 2.0% for Germany and Japan, and 2.3% for the United States.

Finally, the asset correlation estimation algorithm described in Subsection 4.1.1 corresponds to that found in Gordy (2000) and Dietsch and Petey (2004). Duellman and Scheule (2003) estimate asset correlations within a similar single factor framework using three algorithms, the third of which most closely resembles that used in this Chapter, and shows that this method provides for the highest values as compared to other algorithms. In comparison, Gordy and Heitfield (2002) find that this methodology presents the greatest degree of inefficiency when compared to more restricted methodologies.

Possible solutions in dealing with this phenomenon of low asset correlations may include the choice of time periods in which one or more full economic cycles are represented; the choice of high (or maximal) volatility periods, or the application of ad-hoc "conservatism" adjustments to the estimated correlations using external data which may provide required characteristics.

Asset correlation boost retaining observed relationships between SME borrowers

We perform an ad-hoc conservatism factor adjustment to the low level of asset corrlations obtained in our estimation. This exercise is similar to that in Dietsch and Petey (2002) wherein an SME portfolio credit risk model is designed and estimated from SME default data. Given findings of low overall asset correlation values, averaging approximately 2%, the authors input Basel II IRB asset correlations equal to 20% for Corporate borrowers and 8% for Retail borrowers; see Dietsch and Petey (2002, pp. 307-308).

In this Subsection, we take the average asset correlations across all loans in the portfolio under the Partial Implementation cases described in Subsection 3.3.2; specifically, see Table 3.6. For Cases 2, 3 and 4 average asset correlations are found to equal 7.5%, 15.0% and 11.3%, respectively. Next, a bounded log odds ratio adjustment is applied to all segments such that the overall estimated asset correlation of 0.34% is equal to pre-specified value. For example, suppose that we want to adjust our estimated segment asset correlations $\{a_1, a_2, \dots\}$ such that the overall asset correlation A is equal to some value B, subject to the condition that no segment asset correlation $\{b_1, b_2, \dots\}$ is less than some lower boundary value L or greater than some upper boundary value U. The applied adjustment to each segment asset correlation would then be given by the following:

$$b_i = \frac{L + U \times exp(ln(a_i/(1 - a_i)) + X)}{1 + exp(ln(a_i/(1 - a_i)) + X)}, \tag{4.15}$$

where,

$$X = ln\left(\frac{B - L}{U - B}\right) - ln\left(\frac{A}{1 - A}\right). \tag{4.16}$$

The idea of these boosts, ultimately, is to provide internally measured asset correlations that can be practically applied within a prudentially concordant portfolio credit risk framework. An important aspect of this practicality is the level at which asset correlations are set with respect to the international regulatory requirements presented in

Basel II and reviewed in Section 3.1. To that end, we use 3% and 30% as the lower and upper bounds, respectively, in our adjustment. Another important aspect is the embodiment of the credit characteristics in the patterns and relative differences of asset correlations among different segments of borrowers in an SME portfolio. Our work up to Subsection 4.2.2 focused on the latter, in this subsection we addressed the former and presented a simple and common method for the augmentation of asset correlation levels to those present in the nationally applied regulatory frameworks.

Table 4.7 presents the results of our boost by Risk and Size Group segmentations. In both cases, the overall portfolio asset correlation is adjusted to the average asset correlation obtained in the full AIRB implementation (Case 2), equal to 7.4%; see Subsection 3.4.1. As we will see later in this Chapter, the effects of this boost on the resultant loss distribution are non-negligible. Capital charge results using these boosted values are given in Section 4. 3. The boosted asset correlations presented in Table 4.7 range between 9.0% (for the 7 – *>$1,000,000* SG segment) and 18.6% (RR 4-5 – SG *$250,000 - $1,000,000* SG segment). As discussed above, patterns observed in Subsection 4.2.2 are maintained.

RG- and SG-based Partial Implementation Average Asset Correlations

Before moving on capital charge results we present an abridged restatement of Table 3.6 under RG and RG-SG calibrations of our PDs. Specifically, Table 4.12 presents restated average asset correlations under Cases 2, 3 and 7a. Results are presented by RG and SG segments and show the same patterns observed in Table 3.6. In Table 4.12 the Corporate asset class specification is now applied to one RG (>*$1,000,000*) so that under Case 2, controlling for RG, we observe identical asset correlations for the smallest three SGs – treated under the Retail-Other asset class – and a significantly higher average asset correlation for the largest SG. Restated average asset correlations are found to be generally lower than those calculated in Chapter 3.

Comparing results in Table 4.7 to those in Case 2 of Table 4.12 we find, as expected, a lack of Basel II-imposed patterns in our internally calibrated results. In addition, we find that for the vast majority of SG and RG segments, internally-calibrated boosted asset correlations are higher than the average asset correlation in Case 2. Exceptions to this observation occur in the RG 7 – >*$1,000,000* SG segment (in which we find the lowest boosted asset correlations, recall) and the overall >*$1,000,000* SG. Table 4.13 presents discrepancies between internally-calibrated boosted asset correlations and Case 2 average asset correlations as ratios of the former to the latter. Table 4.13 shows that the greatest discrepancies occur for smaller borrowers with high PDs, reflecting Basel II pre-calibrations, with the most closely matched asset correlations are those for the >*$1,000,000* SG.

Subsection 4.2.4. Summary of PD and Asset Correlation Results

In this Subsection we review the results obtained in Subsections 4.2.1 to 4.2.3, categorizing results as relating to Probability of Default and Asset Correlation, both boosted and not boosted. In addition, we present some quick results on the corresponding estimated Default Correlations. Results on internally calibrated *Financing Company* SME Probabilities of Default by Risk and Size Group are found in Tables 4.1, and B.1 to B.3; results on internally calibrated *Financing Company* SME asset and default correlations are found in Table 4.2, and B.4 to B.6; while, results on boosted *Financing Company* SME asset and default correlations are found in Table 4.7.

Results for Probabilities of Default

1. Probabilities of Default increase with overall RG and within all RG-SG segments when controlling for Size.

2. Probabilities of Default decrease with overall SG. This result holds within all Risk Groups except riskiest, RG 8-9, and is potentially indicative of alternative risk appetites in different Size Groups.

Results for Asset Correlations

1. Estimated asset correlations are much lower than those found in Basel II, such that all segments exhibit asset correlations lower than the Basel II programmed minimum of three percent. This is not uncommon in the literature; possible explanations may include reduced default rate means and volatilities over the time period of measurement, and the use of default data versus other sources such as loss data or market-based data.

2. Our results suggest that asset correlations are closer in value and behaviour to those found in Basel II IRB Retail-Other specification as opposed to those found in the Corporate asset class specifications for SMEs. Specifically, we find that there is no fixed relationship between asset correlations and Size, nor is there a fixed relationship between PD and asset correlation.

3. These results run counter to the Size-asset correlation relationship programmed into the Basel II IRB Corporate asset class risk-weighting function, and the PD-

asset correlation relationship found in both the Retail-Other risk-weighting function.

4. Boosted asset corrlations maintain the patterns observed in the internally-calibrated SME asset correlations, but are adjusted to average asset correlations obtained under Case 2, or the full Basel II AIRB implementation.

Results for Default Correlations

1. Internally calibrated default correlations show an increasing pattern with increasing Probability of Default, overall. When controlling for Size this pattern is harder to discern and is only found to hold under the *$100,000 - $250,000* SG.

2. Similarly to asset correlations, internally-calibrated default correlations show an inverse-U pattern with Size. This pattern holds at the overall level and when controlling for RG.

3. Boosted default correlations display similar patterns to internally calibrated default correlations reviewed above.

Section 4.3. Internally Calibrated Single Factor Model Capital Charges

In Section 4.3 we calculate capital charges for the *Financing Company* portfolio using the internally-calibrated asset correlations derived in Subsection 4.2.2 and the boosted asset correlations calculated in Subsection 4.2.3, in contrast to the pre-calibrated values presented in Basel II and applied in Section 3.3. The use of PDs and asset correlations estimated along the dual segmentations presented in Section 4.2 will directly incorporate Size as a measurement dimension. This exercise will provide further insights on the credit characteristics of SME borrowers as based on a real-world SME portfolio and as compared to the SME settings programmed into the Basel II regulatory capital mechanisms.

Our results will show that the full Basel II implementation (Case 2, as defined in Chapter 3) may suffer from misallocation of capital. This is especially evident in comparison to an internally calibrated simulation-based implementation of the single factor *AVM* on our SME portfolio. Specifically, we observe that smaller SMEs are severely undercharged under Basel II while larger SME borrowers have a significant surplus in capital charges. These results are amplified when Size is explicitly used as a dimension in PD and asset correlation calibration.

Internally-calibrated capital charges are calculated using two single factor models. The first is the asymptotic single factor model derived in Appendix B and evaluated in Section 4.1 as the basis for the IRB risk weighting function. This model is essentially the Case 3 model implemented with internally calibrated asset correlations instead of those programmed into the IRB Corporate risk weight function. The second model is the asymptotic single factor model's non-asymptotic counterpart, introduced in Subsection 4.1.2, in which a Monte Carlo simulation procedure is included for the estimation of a portfolio loss distribution. In this model, the internally-calibrated asset correlations are used to generate loss scenarios for the portfolio, the portfolio capital charge is then calculated as the value-at-risk figure at the 99.9% confidence interval less the Expected Loss of the portfolio.

The ensuing comparison between results from identically-calibrated asymptotic and non-asymptotic models allows for the study of the approximation error generated by the application of an asymptotic model to a finitely grained portfolio. This effect, better known as the granularity effect, has been a source of research since the introduction of the Basel II accord's IRB risk weight function. In particular, BCBS (2006b) reviews model-based methods for the measurement and mitigation of granularity effects, as presented in Gordy and Lutkebohmert (2007), Emmer and Tasche (2003), and Vasicek (2002). Our study of the granularity effect is similar in style to that conducted in Tarashev and Zhu (2007).

Our results will that show low overall asset correlation values can have serious impacts on a portfolio's measured granularity effect, so that even finely granular portfolios will display significant underestimation of overall portfolio capital charges when idiosyncratic risks are not explicitly modelled – to our knowledge this is the first explicit link between asset correlation values and granularity effects in the literature.

Results generated in this Chapter, will also shed light on the allocations of capital to borrowers of different Size under asymptotic and non-asymptotic implementations of the single factor framework. We will show that an asymptotic model and allocation scheme cannot conceptually account for significant idiosyncratic risks among the largest SME borrowers. This result is shown to dissipate with elevated asset correlation values.

Subsection 4.3.1. Defining Internally Calibrated Cases for Analysis

We start by defining the Cases that we will implement. These will be less involved than the Cases defined in Chapter 3 and will focus on two calibrations of SME asset correlations, those along the single dimension RG, and those along the dual dimension RG-SG.

Continuing from Section 3.3, we recall Case 7a as the naïve model used in Case 3 but using PDs calibrated along the RG-SG dual segmentations of Chapter 4. This implementation allows for a direct comparison with Case 3 in which Size was not a dimension accounted for in the PD calibration, we will only present an asymptotic implementation of this Case. Case 7b represents the first fully internally-calibrated implementation of the single factor asset value model (*AVM*) in both its asymptotic and simulation-based forms. Case 7b takes us from the naïve model implementation of Case 7a – in which Corporate asset class correlations are maintained, along with a negative relationship with PD – to a fully calibrated model in which internally-calibrated asset correlations are used. As in Case 7a, a dual RG-SG segmentation is used. Recall, PD values are presented in Table 4.1 and asset correlation values in Table 4.2, both are derived in Section 4.2. Case 10 uses the same single factor *AVM* framework used in Case 7b with calibrations of PD and asset correlations by Risk Group alone. Both Case 10 and Case 7b will be implemented in both their asymptotic and simulation-based forms.

Results generated in the asymptotic cases are derived as in Section 3.3, at the individual loan level, using Equation (4.2), the loan $OS and LGD, and subtracting the loan EL – as discussed in Section 4.1. Excluding pre-calibrated asset correlation and maturity adjustments, this is equivalent to the Basel II IRB risk weighting formula presented in Equation (3.1). Results are aggregated by segment and presented as dollar-weighted averages of capital charges as a percent of $OS. In the simulation-based implementation of the single factor model, portfolio capital charges are generated for the portfolio as a whole and then allocated back to obligors.

For comparative purposes, we regenerate Tables 3.6 to 3.8 under the Risk and Size Groups defined in Section 4.2. Average asset correlation results for the Basel II partial implementation exercise using RGs and SGs are given in Table 4.5 while capital results are given in Table 4.6. In Table 4.7 boosted asset correlations, and corresponding default correlations, are given according to the methodology presented in Section 4.2.3, the PD and internally-calibrated asset correlations of presented in Tables 4.1 and 4.2, and the restated average asset correlations of Table 4.5.

In the following Subsections we will present *Financing Company* portfolio capital charges derived using internally-calibrated PD and asset correlation values, both boosted and not boosted, under asymptotic and simulation-based implementations of the *AVM*. Jumping straight to a comparison of boosted capital charges with Basel II (Case 2) charges, Subsection 4.3.2 discusses capital charges generated under simulation-based implementations with boosted asset correlation values calibrated under RG (Case 10) and RG-SG (Case 7b) segmentations; see Tables 4.8 and 4.9.

Subsection 4.3.2. Internally Calibrated Capital Charges versus Basel II

In this Subsection we will compare internally calibrated simulation-based capital charges using boosted asset correlations calibrated to RG-SG segments, to Basel II (Case 2) capital charges. Our results will show a lower capital charge of 7% as compared to 8.2% under Basel II. In addition, observing capital charges by overall SG, we note that internally-calibrated capital charges display a strictly decreasing pattern with increasing Size, as opposed to the Basel II U-shaped pattern in capital charges by Size. This decreasing pattern has particularly strong implications for the smallest SME borrowers who, under the internal calibration, receive capital charges equal to up to three times those under Basel II. By contrast, capital charges for the largest SME borrowers are approximately half what they would be under Basel II.

In order to adequately assess the impact of using internally calibrated asset correlations in the calculation of capital charges we turn again to Case 2, the full AIRB implementation. In Table 4.6 we observe that the use of Retail-Other asset classification results in significantly lower capital charges to the smallest SME borrowers as compared to their larger SME counterparts. Specifically, we observe that the smallest borrowers, those in the *≤$100,000* Size Group, are charged approximately three quarters the capital assigned to the largest SME borrowers, those in the *>$1,000,000* Size Group. This discount in capital charges is enhanced by the negative relationship between asset correlations and PD, the positive relationship between asset correlations and Size, and the presence of lower maturities for smaller SME borrowers; see Chapter 3.

In Case 3, recall, the use of the Retail-Other classification is dropped, as are Size and Maturity effects, however, the Corporate asset class correlation function's inverse PD relation is maintained. This implementation reveals a sharp increase in capital charges to smaller SMEs so that the smallest borrowers are charged over twice the capital of the largest SME borrowers.

Table 4.8 presents capital charges by Risk Group and Size Group under Cases 2, 7b and 10, while Table 4.9 presents cross ratios of segment capital charges across Cases. Examining results by overall RG in Table 4.8 and using tabulated ratios in Table 4.9, we observe that for all RGs except the riskiest (RG 8-9), borrowers generally obtain a reduction in capital charges varying between 50% and 10%. For borrowers in the 8-9 RG, capital charges are increased by 70%. Examining results by RG-SG segment we observe that the highest surcharges are generally applied to those borrowers most benefitting from Basel II pre-calibrations, namely, the smallest and riskiest borrowers. Specifically, the highest surcharge observed is one of 280% for the 7 RG – *≤$100,000* SG segment while the highest capital reduction observed is one of 60% for the 1-3 RG – > *$1,000,000* SG.

These results, obtained in Case 7b and compared to Case 2, take into account Size as a dimension in the calibration of PDs and asset correlations, and use a simulation-based model and allocation scheme to obtain capital results. Case 10 presents a case in which Size is not a dimension in calibration, and show an underlying base for results observed above. Namely, we observe that overall portfolio capital charges under Case 10 are identical to those of Case 2, and equal to 8.2%. For the smallest borrowers, overall SG capital charges are 30% higher under the boosted asset correlations implementation of Case 10 when compared to the Basel II implementation. For borrowers in the largest Size Group, this result is reversed so that the largest borrowers, as an overall group, receive a discount of 30% to capital charges.

Examining results by overall RG, we observe discounts ranging between 20% and 30% for all RGs except RG 8-9. For the riskiest borrowers we observe a surcharge of 10% over Basel II capital charges. Additionally, we observe, under both Case 10 and Case 7b, and for all RG – SG segments as well as overall, a greater dispersion in capital charge results as compared to Base II (Case 2). Specifically, we observe a range of capital charges from 3.6% (RG 1-3 – SG > *$1,000,000*) to 24.8% (RG 8-9 – SG ≤ *$100,000*) under Case 7b and 4.6% (RG 1-3 – SG > *$1,000,000*) to 21.1% (RG 8-9 – SG ≤ *$100,000*) under Case 10. This compares to capital charges ranging from 3.5% (RG 1-3 – SG *$250,000 - $1,000,000*) to 13.2% (RG 8-9 – SG > *$1,000,000*) under Case 2.

Our exercise in this Subsection compared capital charges under prudentially high pre-calibrated asset correlations (i.e., Case 2) to internally calibrated capital charges adjusted to prudential levels. Our use of Size as an explicit calibration dimension provided an alternative to pre-calibrated relationships in Basel II. Our comparison showed that pre-calibrations in Basel II provide discounts to smaller borrowers that are not reflective of their true relative riskiness, while providing a surcharge of capital to larger SME borrowers. We use a RG calibration of PD and asset correlation parameters in order to gauge the relative impact of internal calibrations and Size dimensionality in those calibrations. Our results show that the use of internal calibrations of parameters, removing pre-specified Basel II calibrations, yields the underlying changes observed while the addition of the Size dimensionality amplifies these underlying reversals.

These results are dependent on the specifications of the bounded asset correlation boost, such that if we were to use an upper bound of 20% asset correlation we would obtain different capital levels, but we expect that the patterns and overall results obtained in this Subsection would be maintained; see Subsection 4.2.3.

Subsection 4.3.3. Asymptotic versus Simulation-based Single Factor Model

In order to study the various aspects of the difference in Case 2 and the simulation-based results presented above, we study results obtained under an asymptotic implementation of the single risk factor model and the simulation-based implementation shown above. This exercise is carried out on both boosted and non-boosted model calibrations, and is linked to granularity effects studied in the literature. In particular, granularity effect results derived in this Subsection are compared to results presented in BCBS (2006b) and Gordy and Lutkebohmert (2007).

Our results show that when low asset correlations are used, the asymptotic implementation of the single factor model can lead to significant undercapitalization. On a segment level, we also observe misallocation of capital when asset correlations are low, such that smaller borrowers are overcharged under the asymptotic model and larger borrowers are undercharged. These effects are generally dissipated with boosted asset correlations and, to our knowledge, are the first indication

A Brief Review of Granularity Effects

A common measure of single name exposure concentration, the Herfindahl-Hirschman Index (HHI) is calculated as the sum of the squared exposure shares in the portfolio. A value of zero for the HHI is indicative of full or infinite granularity, while a value of one indicates monopoly. According to BCBS (2006b, pp. 9-10) EU exposure rules dictate that the maximum HHI in a bank's banking book portfolio be 0.0156. For such a portfolio the granularity effect is measured at 13% to 21% when compared to a perfectly granular portfolio. However, it should be noted that the HHI is best suited to portfolios displaying heterogeneity in exposure size and little else, with the HHI becoming unreliable as a measure of granularity when applied to portfolios heterogeneous in credit characteristics such as PD, LGD and default correlations, see BCBS (2006b, p. 10). In particular, Gordy and Lutkebohmert (2007) note that the granularity adjustments can be found to generally increase with increasing PDs in the portfolio, so that the lower credit quality portfolios generally encompass a greater approximation error in the application of asymptotic frameworks; see Figure 2, Gordy and Lutkebohmert (2007, p.17).

In their study of the granularity effect on corporate loans portfolios, Gordy and Lutkebohmert (2007) observe an increase of 1.5% to 4% in VaR for large portfolios (4000 exposures or more) and an increase of 4% to 8% in small portfolios (1000 to 4000 exposures). In addition, the authors calculate the basis points addition to EC as 0.018% for a reference portfolio of 6000 homogenous exposures, with a PD of 1%, an LGD of 45%, and an HHI of 0.00017; see Table 3, Gordy and Lutkebohmert (2007, p. 16).

Our results will indicate that asset correlations, rather than PDs (which are unaffected by our boosts) are the primary actors in increasing the granularity effect from minimal levels of 2% to 4%, to levels of 6%. We measure the *Financing Company* HHI at 0.00017 when measured across loans, and 0.00024 when measured against borrower exposures. Given our portfolio of over 35,000 exposures and over 25,000 borrowers, we observe that our results for a comparable segmentation (RG) to that used in other studies place us just above the range observed in Gordy and Lutkebohmert (2007).

Low SME portfolio asset correlation values can generate significant Granularity Effects

Tables 4.10 to 4.13 provide the capital charge for Risk and Size segments under both the simulation-based and the asymptotic implementations of the single factor model, using RG-calibrated asset correlations and PDs – Table 4.10 and 4.12 – and RG-SG calibrated asset correlations and PDs – Tables 4.11 and 4.13. In Tables 4.10 and 4.11, internally estimated asset correlations are used; in Tables 4.12 and 4.13 boosted asset correlations are input into the two implementations. EC results are compared across implementations in each Table, along with the granularity effect obtained under implementations using the estimated and boosted asset correlations (bottom panels of Tables).

Granularity effect results reveal that given low asset correlations, there can be significant underestimation of capital charges when an asymptotic model is applied to a real-world granular portfolio of SME borrowers. Specifically, Tables 4.10 and 4.11 reveal capital surcharges of 6% when idiosyncratic risks are explicitly modelled, as in the simulation-based approach, as compared to asymptotic implementations. This granularity effect, however, is significantly mitigated when boosted asset correlations are used to generate EC results; see Tables 4.12 and 4.13. Specifically, we observe capital surcharges of 3% and 4% for the "boosted" implementations of Cases 10 (RG-calibration) and 7b (RG-SG calibration), respectively.

In addition to overall granularity effect results, Tables 4.10 to 4.13 allow us to observe granularity effects by Size and Risk Group. Specifically, in Tables 4.10 and 4.11 we observe that when correlations are low the impact of idiosyncratic effects is specific to the largest borrowers, such that these borrowers' contribution to portfolio riskiness (as measured by the EC) may be severely underestimated in asymptotic implementations. These effects are mitigated by the use of boosted asset correlations; see Tables 4.12 and 4.13.

Subsection 4.3.4. Summary of Capital Charges for Internally Calibrated Models

In Section 4.3 we used the internally calibrated PDs and asset correlations estimated in Section 4.2 and applied them within asymptotic and non-asymptotic frameworks of the single factor asset value model discussed in Section 4.1. These results were compared to each other and to the Basel II partial implementation capital charge results obtained in Section 3.4. Our main results are as follows:

1. *Basel II Capital Charges Can Distort Capital Allocations for SMEs*: A fully calibrated internal model for SME portfolio credit risk will reveal significant misallocation of capital under Basel II. This misallocation is manifested in significant capital charge discounts to the smallest SME borrowers and significant capital surcharges to the largest SME borrowers.

2. *Granularity Effects are Amplified by Low Estimated Asset Correlation Values:* The use of low asset correlation values (see Subsection 4.3.3) can play a significant role in the generation of approximation errors due to the application an asymptotic framework, such as that found in the Basel II IRB approach, to a real-world finite portfolio. Intuitively, this result is explained by the predominance of idiosyncratic risks in portfolios with low dependence on systematic factors and the lack of accounting for these idiosyncratic factors in asymptotic frameworks.

Section 4.4. Conclusion

In Chapter 4 we applied a dual segmentation system to the *Financing Company* database of Canadian SME loans. This dual segmentation differed in two ways from the segmentations presented in Chapter 2 and used in the Basel II implementations of Chapter 3. The first difference was the application of this dual segmentation to the estimation of portfolio correlations, and not just probabilities of default. This application of the data drives the second difference, which is the re-definition of some Size and Risk

Groups to ensure robust estimation. Presented in this way, our work with the data in this Chapter underscored the minimal data requirements under Basel II as compared to the exigent data requirements when estimating the credit risk relationships and properties for an SME portfolio. Our work in Chapter 4 therefore allowed us to explore the credit risk characteristics of this distinct group of borrowers, and test the assumptions made on them in the Basel II framework. For both the estimation of portfolio credit risk and the estimation of correlations within our SME portfolio, a single factor asset value model was used.

On the relationship between asset correlation and Size, our results showed that the simple increasing relationship pre-calibrated into the Basel II framework could not be supported empirically for SME borrowers. Similarly to the literature, our internally calibrated asset correlations were significantly lower than those found in Basel II. While arguments for over-diversification as a potential root of low asset corrlation values do not apply in our case, we suggest that the use of default data – as opposed to market data – may be linked to our finding of low asset correlation values. Reservations on the overall value of asset correlations do not take away from patterns estimated across credit quality and size. Taken as a whole, our resuts on asset correlations for SME borrowers reveal low overall values, and no significant relationship with Size and PD. This results points to settings for SME borrowers closer to those found in the Retail-Other treatment of Basel II rather than the Corporate asset class treatment. Maintaining these patterns, or lack thereof, we apply a widely used ad hoc bounded log odds adjustment to boost asset correlation values to levels in line with Basel II prudential guidelines.

Internally estimated asset correlations were applied in the estimation of capital charges on our portfolio of SME borrowers, within a single factor model, implemented both in its asymptotic and simulation-based forms. This comparison of simulation-based and asymptotic internally-calibrated models reveals significant underestimation of portfolio capital charges when asset correlation values are low. In addition, the underestimation of capital charges due to the application of an asymptotic framework to a large real-world finite SME portfolio was found to be concentrated on larger borrowers.

Internally estimated asset correlations were also applied following a boost to their values to bring them to Basel II consistent levels. This boost could be presented as conservatism factor. Capital charge results using boosted asset correlations showed the dissipation of the granularity effect observed in implementations using low asset correlation values. To our knowledge, these results are the first to empirically link asset correlation value to the approximation error obtained from applying asymptotic models to finite real-world portfolios.

Finally, comparing boosted internally estimated capital charges to Basel II capital charges we find evidence of significant Basel II capital allocation distortions to SME borrowers, such that the smallest SME borrowers receive significant capital charge discounts and the largest SME borrowers receive a capital surcharge. This is especially true in cases in which SME borrowers were treated under two regimes, Retail-Other and Corporate. These capital misallocations were traced directly to asset correlation settings.

Chapter 5. SME Economic Capital under *CreditRisk*$^+$

In this Chapter we present *CreditRisk*$^+$, an analytical model which, in its basic form, brings together two sources of uncertainty to generate a loss distribution for a portfolio of credit exposures. These two uncertainties can be summarized as the uncertainty surrounding default rates and the uncertainty surrounding the distribution of exposures in the portfolio over which default would occur.

The basic form of the model allows for both a single sector and a multiple independent sectors implementation, with the latter allowing for the inclusion of idiosyncratic risks among obligors. A major factor of consideration in the previous Chapter, in this Chapter we put the issue of idiosyncratic risks in the portfolio aside and focus, instead, on the introduction of inter-sector correlations in our portfolio and their impact on EC. Here, inter-sector correlations are defined as correlations between borrowers in different sectors while intra-sector correlations are defined as the correlations between borrowers in the same sector. The study of multi-factor frameworks for portfolio credit risk, and their comparison to single factor frameworks, highlights and challenges a major assumption in the Basel II IRB framework, that of a single risk factor. Our results in this Chapter aim to shed light on EC impacts due to this assumption.

In particular, Chapter 5 estimates portfolio credit risk capital charges using various implementations of the *CreditRisk*$^+$ framework. Along with the *Single Sector* and *Multiple Sectors* implementations described in the *CreditRisk*$^+$ documentation; see Credit

Suisse (1997), we implement an extension to the *CreditRisk*$^+$ framework allowing for the inclusion of inter-sector correlations; as presented in Akkaya, Kurth, and Wagner (2004) and Burgisser, Kurth, Wagner, and Wolf (1999). These correlations are calibrated from the *Financing Company* data, maintaining the SME portfolio internal calibrations that characterize this Thesis. When referring to the *CreditRisk*$^+$ implementations as presented in Credit Suisse (1997), we will generally use the terms "original" or "basic" *CreditRisk*$^+$, otherwise we will use the terms *CreditRisk*$^+$ on its own, prefixing it with "simulation-based" or *Multiple Correlated Sectors* when necessary.

Chapter 3 of this Thesis explored the model underlying the regulatory capital framework advanced under Basel II, while Chapter 4 presented a simulation-based single factor asset value model, calibrated along various SME default rate segmentations. Economic Capital results were thus generated for the *Financing Company* under a single factor model fully calibrated from internal *Financing Company* data. In Chapter 5 we use the "actuarial-type" framework of the *CreditRisk*$^+$ model to once again calculate portfolio credit risk Economic Capital based purely on SME data derived from the *Financing Company* portfolio; for a complete description of the *CreditRisk*$^+$ model see Credit Suisse (1997), for a comprehensive review of academic literature on the *CreditRisk*$^+$ framework and extensions to the framework see Gundlach and Lehrbass (2004), and for a review of *CreditRisk*$^+$ implementations from a prudential benchmarking perspective see Avesani, Liu, Mirestean, and Salvati, (2006).

In its *Single Sector* implementation, *CreditRisk*$^+$ provides a framework comparable to the single factor model explored in Chapter 4. In Chapter 6 a comparison between the two models, as well as the Basel II AIRB model presented in Chapter 3, is conducted wherein we examine the resulting portfolio Economic Capital charges, as well as the EC charges by segments. In Chapter 5, we pursue an industry-based *Multiple Sectors* and *Multiple Correlated Sectors* implementations of *CreditRisk*$^+$, comparing the resultant Economic Capital figures to those obtained in a single sector setting. This analysis reveals over- and under-estimations of Economic Capital in relation to the assumptions of single or multiple, whether independent or correlated, risk factors in a portfolio. This work closely resembles that undertaken in Lesko, Schlottmann, and Vorgrimler (2004) and bears some similarities to exercises in Tarashev and Zhu (2007). In addition, we draw references from BCBS (2006b) wherein the question of multi- and single-sector assmuptions's impacts on EC is tackled.

A continuing focus of this Thesis is the behaviour of different borrower size-segments in our portfolio of SME borrowers. As such, we examine Economic Capital and default correlation results generated in this Chapter according to Size segments. As with Chapters 3 and 4, our analysis is enhanced by a data dual-segmentation system incorporating Size as a dimension of parameter measurement.

Chapter 5 is divided as follows: Section 5.1 provides a brief introduction and summary of the original analytical *CreditRisk*$^+$ model. Section 5.2 presents the *Multiple Correlated Sectors* implementation, and; Section 5.3 presents results for the *Single Sector, Multiple*

Sectors, and *Multiple Correlated Sectors* implementations as well as comparative results relative to each other. Finally, Section 5.4 presents a summary of findings and conclusion.

Section 5.1. The *CreditRisk*$^+$ Framework

The *CreditRisk*$^+$ model relies on statistical techniques, first developed in the actuarial sciences, to model the loss distribution for a portfolio of credit exposures. The model quantifies the risk of default without making any assumptions on the causes of default. This differs from the asset value model wherein the cause of default is specified as a drop in asset value below some default barrier; see, for example, Gupton, Finger, and Bhatia (1997).

In this Section we assume the reader has a general knowledge of the workings of the model and only highlight some pertinent aspects, a full derivation of the *CreditRisk*$^+$ model can be found in Credit Suisse (1997), Crouhy, Galia, and Mark (2000) and Gundlach (2004).

Subsection 5.1.1. The Original *CreditRisk*$^+$ Framework

Consider, once again, a portfolio of N obligors in which every obligor (i) has been assigned a *Probability of Default* (PD) over a given time horizon T (say one-year). This reflects the probability that the customer's status, at the end of the one year horizon will be either defaulted or performing. This two-state scenario can be depicted through the use of a Bernoulli distributed indicator variable, $D_i \sim B(1; PD_i)$:

$$D_i = \begin{cases} 1 & \text{if firm } n \text{ is in default at time } T \\ 0 & \text{otherwise} \end{cases} \quad and \;\; \mathbb{P}(D_i = 1) = PD_i. \tag{5.1}$$

In its most basic form, *CreditRisk*$^+$ assumes that an obligor's *Exposure at Default* (EAD) and *Loss Given Default* (LGD) are both given constants, and describes the loss of any obligor *n* through the *Loss Variable*:

$$L_i = EAD_i \cdot LGD_i \cdot D_i, \tag{5.2}$$

such that the *Expected Loss* (EL_i) of obligor (*i)* can be expressed as the expectation of its corresponding loss variable (L_i):

$$EL_i = \mathbb{E}[L_i] = EAD_i \cdot LGD_i \cdot \mathbb{E}(D_i) = EAD_i \cdot LGD_i \cdot PD_i. \tag{5.3}$$

For our portfolio of (*N*) obligors, this then allows for the definition of the *Portfolio Loss* as:

$$L = \sum_{i=1}^{N} L_i. \tag{5.4}$$

For a discrete random variable (*Y*), we define its probability generating function *(pgf)* as:

$$G_Y(z) = \mathbb{E}[z^Y] = \sum_{y=0}^{\infty} \mathbb{P}[Y = y]z^y \tag{5.5}$$

For our Bernoulli distributed default indicator for obligor *(i)*, we write:

$$G_{D_i}(z) = \mathbb{P}(D_i = 1)z^1 + \mathbb{P}(D_i = 0)z^0 = 1 + PD_i(z - 1). \tag{5.6}$$

Working with this representation of D_i's *pgf* we can move from a Bernoulli setting to a Poissonian one at the heart of the analytically tractable solution that the *CreditRisk*$^+$ framework brings to the problem of credit portfolio loss distribution generation. That transition results in the rewriting of Equation (5.6) as the *pgf* of a Poisson variable D_i with intensity PD_i:

$$G_{D_i}(z) = exp(-PD_i) \sum_{y=0}^{\infty} \frac{{PD_i}^y}{y!} z^y. \tag{5.7}$$

For our portfolio of (*N*) independent obligors, in which the risk of default for each obligor *(i)* is now given by a Poisson variable D_i, we define our loss variable L_i as:

$$L_i = E_i \times D_i, \tag{5.8}$$

where $E_i = EAD_i \times LGD_i$ is defined as the non-random *Exposure* for obligor (i). Therefore, maintaining our assumption of independent defaults in our portfolio, we can construct the portfolio loss distribution by defining the *pgf* of L_i:

$$G_{L_i}(z) = G_{E_i \times D_i}(z) = G_{D_i}(z^{E_i}) = \sum_{y=0}^{\infty} \mathbb{P}[D_i = y] z^{yE_i}. \tag{5.9}$$

Next, the portfolio loss distribution *pgf* can be written as:

$$G_L(z) = \prod_{i=1}^{N} G_{L_i}(z) = \prod_{i=1}^{N} \left[\sum_{y=0}^{\infty} (\mathbb{P}[D_i = y] z^{yE_i}) \right] = \prod_{i=1}^{N} \left[\sum_{y=0}^{\infty} \left(e^{-Pi} \cdot \frac{PD_i{}^y}{y!} z^{yE_i} \right) \right]$$

$$= \prod_{i=1}^{N} \left[e^{-PD_i + PD_i z^{E_i}} \right] = exp\left(\sum_{i=1}^{N} PD_i (z^{E_i} - 1) \right). \tag{5.10}$$

We note here that the portfolio loss distribution is not Poisson distributed, due to the inclusion of the variability of (E_i), in contrast to the portfolio distribution of default events.

Recall, the simulation-based *AVM* model of Chapter 4 presented borrower defaults as realizations of a normally distributed latent variable below some given default barrier. In the single sector implementation, the latent variable was assumed to depend on a single systematic factor and an idiosyncratic shock, both generated from standard normal distributions. Correlations between obligors were determined through common dependence on the single systematic factor, and were calibrated from PD averages and volatilities observed in the *Financing Company* SME portfolio.

The *CreditRisk*$^+$ model, in contrast, does not place assumptions on the cause of default, instead borrower default probabilities are modelled to vary over time, increasing or decreasing with gamma-distributed latent systematic factors. Borrowers' probability of default sensitivity to, and co-movements with, the systematic factor thereby generates correlations in defaults, Gordy (2000, p. 119). Whether calibrated to a single sector or a multi-sector analysis, the mean default rate stochasticity can be attributed to one or several background factors, each associated with a given sector.

In both the *Single Sector* and the *Multiple Sector* case, the *CreditRisk*$^+$ framework allows for a closed form solution for the loss distribution to be generated. In particular, let our portfolio be divided into *(K)* sectors, each with a Gamma distributed risk factor with a long-term mean of μ_k and a variance of σ_k^2, Credit Suisse (1997, p. 42):

$$X_k \sim \Gamma(\mathrm{\alpha_k}, \mathrm{\beta_k}), \quad for \quad k = 1, \ldots, K, \tag{5.11}$$

where,

$$\alpha_k = \mu_k^2 / \sigma_k^2 \qquad and \qquad \beta_k = \sigma_k^2 / \mu_k. \tag{5.12}$$

Assuming that the default rate of each obligor depends on only one factor, obligors are assigned to the sector with which they are associated. *CreditRisk*$^{+}$ includes a more general framework in which obligors can be associated with more than one sector. Under such a generalized framework, an obligor's dependence on a given sector is represented with a given weighting, such that the sum of an obligor's weights across the set of sectors should be less than or equal to one. Finally, this framework allows for the inclusion of an idiosyncratic sector capturing the volatility in obligors' default rates which may be due to idiosyncratic factors, as opposed to systematic ones; for more information on these aspects of the *CreditRisk*$^{+}$ framework, the reader can refer to Section A.12 of Credit Suisse (1997).

For each obligor (*i*) we introduce a series of sector weights $\{w_0, \dots, w_K\}$ satisfying:

$$\sum_{k=0}^{K} w_{i,k} = 1, \tag{5.13}$$

such that, $w_{i,k} \geq 0$. These weights, act as factor loadings, measuring obligor *(i)'*s sensitivity to each of the risk factors, while $\left(w_{i,0}\right)$ can be viewed as assigning a weight to an idiosyncratic sector with mean one and variance zero.

For a given borrower *(i)* in Risk Rating *(ζ)*, the probability of default, conditional on realizations of the systematic factor, is amplified or subdued according to a given sensitivity $w_{i,\zeta}$. More specifically, we write:

$$PD_{i,\zeta}(X) = \overline{PD}_{i,\zeta} \cdot \left(\sum_k w_{i,k} \cdot \frac{X_k}{\mu_k} \right), \tag{5.14}$$

where $\overline{PD}_{i,\zeta}$ is the unconditional long term average probability of default for a given Risk Rating *(ζ)*; see Gundlach (2004, pp. 16-17) and Gordy (2000, pp. 121-122). In addition, we write:

$$\mathbb{E}\big[PD_{i,\zeta}(X)\big] = \overline{PD}_{i,\zeta}, \ \ and \quad \mathbb{V}\big[PD_{i,\zeta}(X)\big] = \overline{PD}_{i,\zeta}^{2} \cdot \sum_{k=1}^{K} \left(\frac{w_{i,k}^2}{\mu_k^2} \cdot \mathbb{V}[X_k] \right). \tag{5.15}$$

For our portfolio of (*N*) obligors, we can now write our expected portfolio loss distribution conditional on the realization of our *K* sectors $x = \{x_1, \dots, x_K\}$ as:

$$\mathbb{E}[L|X = x] = \sum_{i=1}^{N} \mathbb{E}[L_i|x] = \sum_{i=1}^{N} \mathbb{E}\big[PD_{i,\zeta}(X)|x\big] \cdot E_i = \sum_{i=1}^{N} \left[E_i \cdot \overline{PD}_{i,\zeta} \cdot \left(\sum_k w_{i,k} \cdot \frac{x_k}{\mu_k} \right) \right]$$

$$= \sum_{k=0}^{K} \left[\frac{x_k}{\mu_k} \cdot \sum_{i=1}^{N} \left(w_{i,k} \cdot \overline{PD}_{i,\zeta} \cdot E_i \right) \right]. \tag{5.16}$$

In order to derive the portfolio loss distribution, we begin by deriving the *pgf* for obligor *n* conditional on x. Analogously to equation (5.10), we write:

$$G_{L_i}(z|X) = G_{D_i}(z^{E_i}|X) = \sum_{y=0}^{\infty} \mathbb{P}[D_i = y] z^{yE_i} = exp\left(PD_{i,\zeta}(X) \cdot (z^{E_i} - 1) \right). \tag{5.17}$$

Our assumption of conditional independence then allows us to write:

$$G_L(z|\mathrm{X}) = \prod_{i=1}^{N} G_{L_i}(z|\mathrm{X}) = exp\left(\sum_{i=1}^{N} PD_{i,\zeta}(X) \cdot (z^{E_i} - 1)\right)$$

$$= exp\left(\sum_{k=0}^{K} \frac{X_k}{\mu_k} \sum_{i=1}^{N} \overline{PD}_{i,\zeta} \cdot w_{i,k} \cdot (z^{E_i} - 1)\right). \tag{5.18}$$

Using the gamma distribution functional forms and integrating out *X,* we obtain:

$$G(z) = \prod_{k=0}^{K} G_k(z) = exp\left(\sum_{n=1}^{N} w_{i,0} \cdot \overline{PD}_{i,\zeta} \cdot (z^{E_i} - 1)\right) \cdot \prod_{k=1}^{K} \left(\frac{1 - p_k}{1 - \frac{p_k}{\mu_k} \sum_{i=1}^{N} w_{i,k} \cdot \overline{PD}_{i,\zeta} \cdot z^{E_i}}\right)^{\alpha_k}, \tag{5.19}$$

where,

$$p_k = \frac{\beta_k}{(1 + \beta_k)}. \tag{5.20}$$

Credit Suisse (1997, pp. 48-49) and Gundlach (2004, pp. 21-23) present the Panjer recursion used in the original *CreditRisk*$^+$ for generating portfolio losses from the *pgf*; for alternative solution schemes see, for example, Gordy (2002), Haaf, Reiss, and Schoenmakers (2004), and Merino and Nyfeler (2004). We will use the Panjer recursion in our estimation of portfolio losses according to the analytical *CreditRisk*$^+$ implementations.

What is left is the calibration of the sector factor parameters μ_k and σ_k. Kluge and Lehrbass (2004, p. 317) observe that gamma distributed sector factors can be normalized to any desired expected value. When obligor-specific default rate standard deviations are available, Credit Suisse (1997, pp. 51-52) shows that an appropriate and pragmatic calibration of the sector parameters can be undertaken as follows:

$$\mu_k = \sum_i w_{i,k} \cdot \overline{PD}_{i,\zeta} \quad and \quad \sigma_k = \sum_i w_{i,k} \cdot \overline{PDVol}_{i,\zeta}. \tag{5.21}$$

The settings in Equation (5.21) emphasise the importance of the distribution of borrowers across various PD-calibration segments within a given sector, as opposed to other outstanding sectoral characteristics; see Credit Suisse (1997, pp. 43). When default rate standard deviations are not available, Credit Suisse (1997, pp. 44) suggests the use of a single fixed ratio for σ_k/μ_k and suggests a value of order one in accordance with historical experience. This fixed ratio setting, and specifically the unitary volatility ratio setting, is widely applied in the literature. In Chapter 6 we explore the use and impact of these two calibration methods, on the loss distribution and resultant EC.

Subsection 5.1.2. Incorporating Inter-Sector Default Correlations

Akkaya, Kurth, and Wagner (2004) present a framework in which correlations between geographical or industry-specific sector factors can be integrated into the *CreditRisk*$^+$ framework while preserving the analytical solution method for the portfolio loss distribution. Their method builds on work in Burgisser, Kurth, Wagner, and Wolf (1999) and is accomplished through a moment matching method, as applied to the first and second moments of the *CreditRisk*$^+$ generated loss distribution, as well as a pre-defined correlation matrix between the pre-defined sector factors.

Specifically, Akkaya, Kurth, and Wagner (2004, p. 133) define the mean and variance of the *Multiple Sector* loss distribution as:

$$EL = \sum_{i=1}^{N} PD_i \cdot v_i \quad and \quad \sigma^{2\,(M)} = \sum_{k=1}^{K} {EL_k}^2 \left(\frac{\sigma_k}{\mu_k}\right)^2 + \sum_{i=1}^{N} EL_i v_i. \tag{5.22}$$

Under a *Single Sector* implementation the portfolio loss distribution variance can be defined as:

$$\sigma^{2\,(S)} = EL^2 \left(\frac{\sigma_{(S)}}{\mu_{(S)}}\right)^2 + \sum_{i=1}^{N} EL_i v_i. \tag{5.23}$$

Here we've used the convention of defining the *Single Sector* risk factor variance as $\left(\sigma_{(S)}^2\right)$ and the *Single Sector* portfolio loss distribution variance as $\left(\sigma^{2\,(S)}\right)$. For the *Multiple Sectors* implementation, we extend the use of the notation proposed in Subsection 5.1.1. In line with the *CreditRisk*$^+$ framework assumptions on the *Multiple Sector* implementation, Equation (5.22) denotes a situation in which inter-sector correlations are set to zero. Removing this restriction and allowing inter-sector correlations to be different from zero, we obtain the following functional form for the portfolio loss distribution variance:

$$\sigma^{2\,(MC)} = \sum_{k=1}^{K} EL_k{}^2 \left(\frac{\sigma_k}{\mu_k}\right)^2 + \sum_{k,l:k\neq l} Corr(X_k, X_l) EL_k EL_l \frac{\sigma_k}{\mu_k}\frac{\sigma_l}{\mu_l} + \sum_{i=1}^{N} EL_i v_i. \tag{5.24}$$

Proceeding with the moment matching method, Akkaya, Kurth, and Wagner (2004, p. 135) suggest setting Equations (5.24) and (5.23) equal to each other and solving for $\sigma^2_{(S^*)}$. Specifically, we have:

$$EL^2\left(\frac{\sigma_{(S^*)}}{\mu_{(S)}}\right)^2 + \sum_{i=1}^{I} EL_i v_i = \sum_{k=1}^{K} EL_k{}^2 \left(\frac{\sigma_k}{\mu_k}\right)^2 + \sum_{k\neq l} Corr(X_k, X_l) EL_k EL_l \frac{\sigma_k}{\mu_k}\frac{\sigma_l}{\mu_l} + \sum_{i=1}^{I} EL_i v_i$$

$$\sigma^2_{(S^*)} = \left(\frac{\mu_{(S)}}{EL}\right)^2 \left[\sum_{k=1}^{K} EL_k{}^2 \left(\frac{\sigma_k}{\mu_k}\right)^2 + \sum_{k\neq l} Corr(X_k, X_l) EL_k EL_l \frac{\sigma_k}{\mu_k}\frac{\sigma_l}{\mu_l}\right]. \tag{5.25}$$

Having derived the first and second moments, a single factor gamma distribution can be calibrated and a new loss distribution generated – as previously detailed in Subsection 5.1.1.

Subsection 5.1.3. Model Specification and Implementation

Throughout Chapters 4 and 5, we maintain equivalent parameter calibrations. To that end, we use the same parameter estimates as those presented in Table 4.1. Economic

Capital calculations are made under *Single Sector, Multiple Sector* and *Multiple Correlated Sectors* implementations of the *CreditRisk*$^+$ framework. Sectors here are defined along the eleven *Financing Company* industries of Business Services (BUS), Construction (CON), Manufacturing (MAN), Non-Business Services (NBUS), Resources (RES), Retail (RET), Supplier of Premises (SOP), Tourism (TOU), Transport & Storage (TRS), Wholesale (WHS), and Other (OTH).

Table 5.1 presents inter-sector correlations input into the *Multiple Correlated Sectors* implementation. Distinguishing between correlations floored at zero and minimal correlation values between industries, we observe the lowest correlations between the BUS and OTH industries (6%), the RET and RES industries (5%), and the SOP and TOU industries (1%). The highest correlations are observed between the BUS and SOP industries (91%), and the RET and TOU industries (80%).

As in Chapter 4, we use Risk Group, as well as Risk and Size Group, calibrations of PDs and PD variations to generate capital results. In the *Single Sector* implementation all borrowers are allotted to the same sector and correlations are calculated for the homogenous segments used for calibration; e.g., by Risk Group. In the *Multiple Sector* implementation default correlations between borrowers in different industries are set to zero, and default correlations for borrowers within the same sector are calculated as in the *Single Sector* implementation; see Appendix C. Given the zero inter-sector correlation restriction in the *Multiple Sector* implementation, we propose that correlations between borrowers in different industries be implied from a comparison of *Multiple Sector* and *Single Sector* implementations.

In the *CreditRisk*$^+$ framework intra-sector default correlations are intrinsically generated through analysis of risk factor volatilities. As noted in Subsection 5.1.1, the calibration these sector volatilities can be undertaken through the use of obligor-specific default rate volatilities, available, in our case, on a single- and dual-dimension basis across various segments of our portfolio. As such, and in reference to Equation (5.21), each obligor is assigned to a specific sector such that his sector weight is set to one for that particular sector. In Chapter 6, this unitary weight setting is contrasted against the unitary normalized volatility setting briefly introduced in Subsection 5.1.1.

We evaluate the credit risk over the *Financing Company* March 2009 performing portfolio, using Risk and Size Group segments as defined in Chapter 4. In order to properly identify the discussion as pertaining to either the single segmentation or the dual segmentation of the data, we will, as before, use the "overall" adjective when referring to the single segmentation. As in Chapter 2, *Financing Company* historical defaults are compiled from January 1997 to December 2010, covering 14 years (see Section 2.4 for further discussion).

As noted above, our data is prepared in such a way as to maximize comparability with the methodology presented in Chapter 4. As such, we proceed by segmenting our data by loan and documenting the loan \$OS – this will serve as the exposure; the LGD – this is set at either 41% or 73% depending on Security Coverage, see Subsection 3.3.1; the borrower industry code associated with that loan; the borrower Risk Group associated with that loan; the borrower Size Group for that loan, and; the calculated Exposure at Default – which we define as (*$OS x LGD*).

Having specified our portfolio input data, and specified our "*K"* sectors as corresponding to our eleven industries, we now proceed with the banding process used in *CreditRisk*$^{+}$. Specifically, we note that LGD-adjusted exposures are divided into exposure bands wherein the value of the individual exposures is approximated by an integer equal to the rounded value of the exposure given some Loss Unit; see, for example Crouhy, Galia, and Mark (2000, p. 110).

Specifically, for some Loss Unit *(F)*, obligor exposures are transformed such that:

$$v_i = round(E_i/F) \tag{5.26}$$

All exposures for which $v_i = v_j$ are then collected into band *j*. For each obligor *(i)*, we define $EL_i = (E_i \cdot PD_i)/F$, such that for each exposure band

$$EL_j = \sum_{i:v_i=v_j} EL_i \tag{5.27}$$

and

$$\mu_j = \frac{EL_j}{v_j} = \sum_{i:v_i=v_j} \frac{EL_i}{v_j} = \sum_{i:v_i=v_j} \frac{EL_i}{v_i}. \tag{5.28}$$

In a multiple sector setting, and for a given sector *(k)*, the maximum band value J^k is determined by evaluating the largest exposure as in Equation (5.26), and exposure bands are defined for $v_j^{(k)} = 1, \dots, J^k$. Defining:

$$EL_j^{(k)} = \sum_{n:v_i=v_j^{(k)}} EL_i^{(k)}, \quad \mu_j^{(k)} = \frac{EL_j^{(k)}}{v_j^{(k)}}, and\ \mu_k = \sum_{j=1}^{J^{(k)}} \mu_j^{(k)}\ and\ \sigma_k = \sum_{i \in k} \sigma_i^{(k)}, \tag{5.29}$$

and allowing us to reformulate the portfolio loss distribution given in Equation (5.19) as:

$$G(z) = \prod_{k=1}^{K} G_k(z) = \prod_{k=1}^{K} \left(\frac{1 - p_k}{1 - \frac{p_k}{\mu_k} \sum_{j=1}^{J^{(k)}} \left(\frac{EL_j^{(k)}}{v_j^{(k)}} \right) z^{v_j^{(k)}}} \right)^{\alpha_k}. \tag{5.30}$$

In most implementations a unit of exposure *(F)* is chosen to be \$10,000 unless stated otherwise. The rounding procedure used in the banding process; see Equation (5.26), effectively discards all loans with \$OS < \$5,000. In addition, for each loan, an adjustment is made to the (*PD*) so as to adjust for the approximation generated by the banding process: $APD_i = (PD_i \cdot E_i)/(v_i \cdot F)$; see Bluhm, Overbeck, and Wagner (2003, p. 152).

Having fully specified our data, the sector factor gamma distributions are calibrated and the portfolio loss distribution is generated using the Panjer Recursion. Portfolio EC results are then calculated as the 99.9% loss distribution VaR less the portfolio EL. For each loan, the VaR is allocated back using the procedures highlighted in Appendix C and the loan EL is subtracted to give us the loan EC. Segment results are presented as the $OS-weighted average capital charges as a percent of total segment $OS.

Section 5.2. Results

Section 5.2 explores the calculation of Economic Capital charges using the analytical implementations of *CreditRisk*$^+$. We present results under RG and RG-SG calibrations. In Figures 5.1 and 5.2, Economic Capital allocations obtained under RG-SG calibrations are presented by Industry and Risk Group, and Industry and Size Group, respectively are presented for the three implementations. In particular, the top panel of each Figure presents *Single Sector* implementation results; the middle panel presents *Multiple Sector* implementation results, and; the bottom panel presents EC allocations under the *Multiple Correlated Sectors* implementation. Figure 5.3 presents the analytically derived portfolio loss distributions under the three implementations and along the RG and RG-SG calibrations. Specifically, the middle panel provides loss distributions for the three implementations under the RG calibration, the bottom panel provides the loss distributions for the three implementations under the RG-SG calibration, and the top panel provides a combined plot of the other two panels.

Economic Capital results for our three implementations are presented for Risk and Size Group segments in Tables 5.2A and 5.2B, with the former depicting results under RG calibrations and the latter under RG-SG calibrations. Tables 5.3A and 5.3B present ratios of capital charges calculated in the various implementations for the portfolio segments presented in Table 5.2. The top panel of Tables 5.3A and 5.3B present the ratios of EC charges obtained under the *Single Sector* implementation to those obtained *Multiple Sector* implementation; the middle panel presents the ratios for the *Single Sector* implementation as compared to the *Multiple Correlated Sectors* implementation, and; the bottom panel presents the ratios for the *Multiple Correlated Sectors* implementation as compared to the *Multiple Sectors* implementation.

Similarly to Table 5.2, Tables 5.4 and 5.6 present EC capital charges under the various implementations. Tables 5.5 and 5.7 present ratios corresponding to the segments presented in Tables 5.4 and 5.6, respectively. By and large, our discussions will focus on the ratios of capital charges for various segments, with discussion of EC level results presented for completion. Below we describe outstanding ratio results, focusing on EC charges generated under RG-SG calibrations, contrasting them against those obtained under RG calibrations; next we provide a detailed description capital results. Our discussion is also peppered with results obtained under similar analyses in the literature.

Our results, based on RG-SG calibrations, show a ranking of EC results such that the highest EC charges both, for the overall portfolio and its segments, is observed in the *Single Sector* implementation, followed by the *Multiple Correlated Sectors* implementation, and finally the *Multiple [Independent] Sectors* implementation. Additionally, results show that the inclusion of inter-sector correlations has a greater effect the smaller the borrower segment. This result holds when controlling for either RG or Industry and therefore suggests greater sensitivity of smaller borrowers to systematic factors. When setting inter-sector correlations to 100% this relationship is generally maintained when controlling for Risk Group and Industry.

The inclusion of inter-sector correlations displays increasing EC impact with increasing RG, overall. However, this relationship is not replicated consistently when controlling for Size or Industry. These EC impact patterns are similarly observed when setting inter-sector correlations to 100% as under the *Single Sector* implementation.

Thus far we've described the impact on EC from the introduction inter-sector correlations and from the use of the single sector assumption, under an RG-SG calibration of the models. Observing results under an RG calibration we note similar EC impact patterns as those presented above. In addition, we note that while overall portfolio EC impacts under the RG-SG calibrations appear larger, segment-specific impacts are generally lower under the RG-SG calibration. We propose that this may suggest that the use of RG-SG calibrations may incorporate a small portion of the segment-specific effects observed in the RG implementation.

Overall, our results show that under each implementation the positive monotonic relationship between capital charges and Risk Group is respected. In addition, we observe that under almost all implementations, capital charges display a negative monotonic relationship with Size. The exception to this relationship is found in the *Multiple Sector* implementation wherein a U-shaped effect is observed. This Size-based U-shape relationship is reminiscent of results in Chapter 4 under internally estimated correlations.

When controlling for Industry and Size, we observe that the positive relationship between EC charge and Risk Group is maintained under all implementations.

Overall Results

Table 5.2 presents Economic Capital results by Risk and Size Group under the *Single Sector*, *Multiple Sector* and *Multiple Correlated Sectors* implementations. In Table 5.3 these results are compared to each other through ratios of capital charges obtained in each segment under each implementation.

Under RG calibrations, the Economic Capital charge for the *Financing Company* portfolio is calculated at 1.2% under the *Single Sector* implementation, 0.5% under the

Multiple Sector implementation and 0.9% under the *Multiple Correlated Sectors* implementation. Under RG-SG calibrations, these overall portfolio capital charges are equal to 1.0%, 0.5% and 0.7%, respectively. Observe that overall capital charges are higher under the RG calibration, this result matches those obtained in previous Chapters.

Lesko, Schlottmann, and Vorgrimler (2004, p. 251) observe that under a unitary weight setting, and using the same *Multiple Correlated Sectors* methodology as that presented in Subsection 5.1.2, Burgisser, Kurth, Wagner, and Wolf (1999) obtained an increase of 25% in the portfolio loss distribution standard deviation, translating into 20% increase in the 99.0% VaR. Under equivalent settings, we find that the introduction of inter-sector correlations to the *CreditRisk*$^+$ framework, as in the *Multiple Correlated Sectors* implementation, results in an increase in EC of 65% at the 99.9% confidence level when compared to the *Multiple Sector* implementations, this is equivalent to an increase of 17% in the VaR. These results are roughly equivalent with those reviewed in Lesko, Schlottmann, and Vorgrimler (2004) and derived in Burgisser, Kurth, Wagner, and Wolf (1999).

BCBS (2006b) presents a review of the potential impact of the single risk factor assumption to EC calculations. Citing Duellmann and Masschelein (2006), BCBS (2006b) estimate an icnrease in EC, going from the most diversified case to the single sector case, equal to approximately 50%. This compares to values of approximately 130% under the RG and RG-SG calibrations for our portfolio. BCBS (2006b, p. 13) notes that the impact of the single risk factor assumption has been found to be positive or negative, and is heavily dependent on exposure concentrations within a portfolio and correlations between risk factors.

In Figures 5.1 and 5.2 we observe an outstanding characteristic of the implementations and their EC allocation in comparison to each other. In particular, we observe that for the *Multiple Sectors* and *Multiple Correlated Sectors* implementations, EC allocations for the MAN industry are significantly elevated. This marks a contrast with EC allocations observed under the *Single Sector* implementation wherein allocations by industry are generally uniform in shape. This characteristic can be attributed to higher normalized standard deviation values for the MAN industry as compared to the *Single Sector* value. This will be discussed in greater detail below.

Results by Risk and Size Group

This nominal ranking of EC results (highest in *Single Sector* implementation; lowest in *Multiple Sector* implementation) is generally maintained for all segments. Specifically, Tables 5.2 and 5.3 indicate that for every segment, a *Multiple Correlated Sectors* implementation of the *CreditRisk*$^+$ framework results in higher capital charges than the *Multiple Sector* implementation; see bottom panel of Tables 5.3. Given that the two implementations hold everything equal except the presence of inter-sector correlations – as defined in Table 5.1 – we can attribute the capital surcharge in the *Multiple Correlated Sectors* implementation with respect to the *Multiple Sector* implementation to inter-sector correlations. Specifically, we observe in the bottom panel of Tables 5.3, an inter-sector correlations capital surcharge of 65% over the independent sectors case under the RG calibration and 67% under the RG-SG calibration. The middle panel of Table 5.3 shows an EC surcharge of 39% resulting from the imposition of a single sector as compared to multiple correlated sectors.

Table 5.3 shows that for all RG-SG segmentations the nominal ranking of EC charges by implementation is maintained so that the highest EC charges are obtained in the *Single Sector* implementation, while the lowest are obtained in the *Multiple Sector* implementation. In particular, Table 5.3 shows an increasing inter-sector correlation EC surcharge with increasing Risk and decreasing Size. Specifically, under the RG calibration, we note that the highest inter-sector correlation EC surcharge (i.e., EC increase going from the *Multiple Sector* implementation to the *Multiple Correlated Sectors* implementation) is observed in the 8-9 RG for the *≤$100,000* Size Group (146%), while the lowest is observed in the 1-3 RG for the *>$1,000,000* Size Group (39%). Under the RG-SG calibration the equivalent figures, relevant for the same Risk and Size Group segments, are equal to 143% and 36%, respectively. Observing capital surcharges in going from the *Multiple Correlated Sectors* implementation to the *Single Sector* implementation, we note a generally positive, but not necessarily monotonic, relationship with RG when controlling for Size. For the *>$1,000,000* Size Group we observe lower surcharges than smaller borrowers.

Comparing results across Risk and Size Group segments within implementations, we observe that, both, overall and for all Size Groups, and under all implementations and calibrations, we observe increasing Economic Capital charges with increasing Risk. Under the RG-SG calibration, we note decreasing EC charges with Size for the *Single Sector* and *Multiple Correlated Sectors* implementations for Risk and Size Group segments. An exception is observed in the 8-9 RG wherein we observe a U-shaped pattern for those two implementations. For the *Multiple Sector* implementation the U-shaped pattern of capital charges by Size is observed for all RGs; see Table 5.2B. In addition, we observe a U-shaped pattern for all RG-calibrated implementations; see Table 5.2A.

Results by Industry and Risk Rating

Tables 5.4 and 5.5 present Economic Capital results by Industry and Risk Group for the three *CreditRisk*$^{+}$ analytical implementations. As can be seen, under all implementations and calibrations, the positive monotonic relationship between Risk Group and Economic Capital is respected, both at the overall and the segment-specific level.

In terms of overall capital charges by Industry under the RG calibration, we observe in Tables 5.4A, for the *Single Sector* implementation the highest capital charges are observed in the RES (1.5%) and SOP (1.4%) industries, while the industries with the lowest capital charges are the CON (1.1%) and WHS (1.1%) industries. Comparing the *Single Sector* implementation to the *Multiple Sectors* implementation we observe a shuffling of rankings. In particular, we observe the highest capital charge under the *Multiple Sectors* implementation in the MAN industry (0.8%), whereas that industry was allocated the third lowest capital amount under the *Single Sector* implementation. The industries with the lowest capital charges are the OTH (0.2%) and CON (0.3%) industries. For the *Multiple Correlated Sectors* implementation we observe the highest capital charges in the MAN (1.1%) industry and the lowest in the RES (0.6%). Under the RG-SG calibration, we observe only slight differences to the Industry ordering observed under the RG calibration.

Under both calibrations we observe that the introduction of inter-sector correlations has the smallest effect on the MAN industry, increasing capital charges by approximately 25%; see Tables 5.5A and 5.5B. Viewed another way, we can say that the introduction of inter-sector correlations has the greatest diversification effect on the MAN industry. Tables 5.5 show that the industry for which capital charges increase the most with the introduction of inter-sector correlations is the OTH industry, for which capital charges increase by approximately 220%. This result is not entirely surprising as we would expect borrowers classified into this heavily mixed industry to exhibit stronger correlations with other industries than with each other.

Results by Industry and Size Group

Tables 5.6 and 5.7 present results by Industry and Size Group. Results in Table 5.7 echo and extend some of the results observed in Tables 5.2 and 5.3 by showing generally decreasing capital charges by Size when controlling for Industry under the *Single Sector* and *Multiple Correlated Sectors* implementations while a U-shaped pattern is observed under the *Multiple Sectors* implementation. Examining results in the bottom panel of Tables 5.7, we observe that inter-sector correlations surcharges tend to decrease with increasing Size for all Industry-SG segments.

Inter-Sector and Intra-Sector Correlations

Our analysis of the various *CreditRisk*$^{+}$ implementations allows for some commentary on the measurement of inter-sector correlations in the *Financing Company* portfolio. Specifically, we observe that on average, the presence of inter-sector correlations accounts for a significant portion of risk capital charges. In particular, we observe that the inclusion of inter-sector correlation results in an increase in portfolio-wide capital surcharges of approximately 65% as compared to the case of independent sectors. Documenting our results by Size, we note that the impact of inter-sector correlations decreases monotonically with Size, so that the smallest borrowers exhibit the highest capital charge surplus due to inter-sector correlations, while the largest borrowers exhibit the lowest. In addition, we observe that the impact of inter-sector correlations increases with RG and, by extension, PD. As such, we find that the borrowers most affected by the inclusion of inter-sector correlations are those in the smallest Size Group and highest Risk Group. Conversely, those least affected are those in the largest Size Group and the lowest Risk Group.

Results in Chapter 3 and Chapter 4 dealt exclusively with intra-sector (or intra-segment) correlations, so that for a uniform group of borrowers, correlations were measured at a uniform level with respect to a single factor. *CreditRisk*$^{+}$ provides a framework through which intra-sector correlations are measured as shown in Equations (5.27).

Recall, PDs and PD volatilities are calibrated either along RG or RG-SG dimensions while sectors are defined by Industries. Under the RG calibration and the *Single Sector* implementation default correlations are calculated based on the RG PD and the square of the single sector volatility ratio (σ/μ). Under the *Multiple Sectors* implementation borrowers in different industries are completely independent so that their default correlation is zero; default correlations between borrowers in the same industry are dependent on the industry-specific ratio (σ_k/μ_k).

Table 5.8 provides the relevant ratio for the *Single Sector* implementation as well as for the eleven Industry sectors under the *Multiple Sector* implementation. The second column of Table 5.8 relates the relevant ratio to the implementation discussed in Subsection 5.4.1. Recall that for this exercise we relied exclusively on RG-calibrations, columns three to seven give the intra-sector default correlation for borrowers in the corresponding RG segment. As can be seen in Table 5.8, the SOP industry exhibits the lowest ratio (0.0433), while the MAN industry exhibits the highest ratio (0.0489). Returning to Table 5.4, we note that these ratio rankings correspond to the rankings of Industry by capital allocation under the *Multiple Sectors* implementation.

Section 5.3. Conclusion

In this Chapter we introduced the *CreditRisk*$^+$ framework for the calculation of portfolio credit risk. We tested the effect of various implementations of the *CreditRisk*$^+$ framework on the capital calculation for our portfolio, using various calibrations and assumptions on the behaviour of SME borrowers. Specifically, we used RG- and RG-SG calibrated PDs, along with sectors divided by *Financing Company* industry specifications, to calculate EC charges under *Single Sector*, *Multiple Sectors* and *Multiple Correlated Sectors* implementations of the analytical *CreditRisk*$^+$ model. For each obligor the sector weight was set to one for the industry to which the corresponding borrower is assigned, and zero otherwise, with all borrowers accorded a weight of one to the single sector in the *Single Sector* implementation. Under the *Multiple Correlated Sectors* implementation default correlations between sectors were proxied by *Financing Company* default rate correlations between industries.

Our results showed that the use of a *Single Sector* implementation can result in an overestimation of overall EC by approximately 40%, with the smallest and riskiest portions of the portfolio experiencing the highest degree overestimation. Our results also indicated that the assumption of independence between borrowers, as in the *Multiple Sectors* implementation, results in a 65% underestimation of overall EC, with the majority of this divergence, again, residing in capital allocations to smaller borrowers.

Chapter 6. A comparative analysis of portfolio credit risk models on a portfolio of Canadian SME loans

Thus far, the *CreditRisk*$^+$ framework has been presented as an alternative structure to that found in the asset value model. However, it can nonetheless be shown that *CreditRisk*$^+$ shares common conceptual and mathematical foundations with the *AVM* studied in Chapter 4; see, for example, Crouhy, Galia, and Mark (2000). In particular, Koyluoglu and Hickman (1998) show that both model types can be classified under a single general framework and are capable of providing similar results provided that input parameters are harmonized across the models. Gordy (2000) provides a mapping between the asset value model and the *CreditRisk*$^+$ model while Gordy and Lutkebohmert (2007) present an example of how applications or tests from one framework can be applied to another by using results generated in a *CreditRisk*$^+$ framework to propose amendments to the Basel II IRB framework.

Wieczerkowski (2003, pp. 45-46) summarizes results of numerous comparative studies between the *CreditRisk*$^+$ and the *AVM*, as represented by the commercialized *CreditMetrics* model, see J.P. Morgan (1997), frameworks:

1. In limiting *CreditMetrics* to a two-state model, both it and *CreditRisk*$^+$ can be considered factor models imposing conditional independence on defaults. The two models, however, differ in the modeling of the distributions of risk factors and conditional default probabilities.
2. The two models can be reformulated analogously to one another. In the case of *CreditRisk*$^+$, this reformulation can generate a two-state *CreditMetrics* framework

in which loss distributions are generated through Monte Carlo simulation. In the case of *CreditMetrics* a limited two-state model probability-generating function can be generated analogously to *CreditRisk*$^+$; however, no closed form solution has been found.

3. While it is possible to match the *CreditRisk*$^+$ and *CreditMetrics* models through factor transformations, this matching is not based on the standard parameterizations of the models.

For their part, Koyluoglu and Hickman (1998) identify three points of comparison between the frameworks – the default rate distribution, the conditional default dtribution, and the loss aggregation method – with only the first found to generate significant divergences in results; see Koyluoglu and Hickman (1998, p. 17).

In Chapter 6 we introduce a simulation-based implementation of the *CreditRisk*$^+$ framework. Our comparisons of the *AVM* and *CreditRisk*$^+$ will show that using the calibration and bridging techniques presented in Gordy (2000), it is possible to obtain comparable results for an SME portfolio under the two models. Our results emphasize the distinctiveness of the SME default rate volatilities and their importance in the calibration of portfolio credit risk models. In particular, we will show that a unitary setting to normalized SME default rate volatilities in the *CreditRisk*$^+$ framework may not be suitable in an SME portfolio, especially in the presence of low correlations.

Our work will show that a unitary risk factor weighting, allowing for the calibration of the single sector risk factor volatility from internal data, as discussed in Chapter 5, provides loss distribution characteristics comparable to those obtained under the *AVM*. In addition, we present a calibration refinement and show that a fixed sector normalized standard deviation setting of 0.5 for our SME portfolio provides for parameter settings harmonized with those found in the *AVM*. Given the scope of our work in the Thesis, especially as it applies to a real-world portfolio of SME borrowers, this result is an interesting contribution to the literature.

Similarly to Chapter 4, correlations calibrated from internal *Financing Company* SME data are generally low in value. We use simulation-based implementations of the *CreditRisk*$^{+}$ model and the calibration bridge presented in Gordy (2000) to extend the ad hoc correlation boost presented in Chapter 4 to the modeling framework used in this Chapter. This exercise, bearing similarities to that undertaken in Dietsch and Petey (2002), yields capital charges significantly higher than those obtained under the purely internal calibrations discussed above.

Finally, Chapter 6 will conclude the study of SME portfolio credit risk undertaken in this Thesis. Having studied the unique characteristics that underpin a high-risk SME loans portfolio in Chapter 2, we proceeded, in Chapter 3, with undertanding of the impact of the various assumptions on the behaviours of borrowers in an SME as found in the Basel II regulatory framework. In Chapter 4 these assumptions were tested through the estimation of SME correlations as found in our portfolio and within a comparable framework as that used in Basel II. Our results could not empirically support Basel II assertions on the relationship between SME borrower asset correlations and either probabilities of default or borrower size. In addition, our estimation of borrower asset correlations from internal *Financing Company* default data resulted in low correlation values as compared to those found in Basel II. Extending our analysis on this phenomenon, we demonstrate, empirically, the significant underestimation of portfolio credit risk that can arise from the presence of these low asset correlations in an ASRF framework. To our knowledge, this is the first demonstration of a relationship between "granularity effects" and asset correlation values in a portfolio credit risk context.

In Chapter 5 we take a broader view of the underlying credit risks in an SME portfolio by using the *CreditRisk*$^+$ framework to extend our analysis from a single sector framework to one in which multiple sectors, along with the accompanying portfolio diversification benefits are modelled. Our analysis quantified the overestimation resultant from the use of a single risk factor in a portfolio credti risk model, as opposed to an internally calibrated model with multiple correlated risk factors. In addition, we quantify the underestimation that could result from the use of zero correlation.

Chapter 6 brings us full circle by comparing the *AVM* and *CreditRisk*$^+$ framework and the resultant portfolio risk capital charges, obtained under prudentially boosted values to internally calibrated correlations, to Basel II. Specifically, Chapter 6 is divided as follows: Section 6.1 presents a single sector simulation-based *CreditRisk*$^+$ model, including calibration implementations and calibrations; Section 6.2 presents a comparative analysis of *AVM* and *CreditRisk*$^+$ results, and; Section 6.3 presents a discussion of results and conclusions. We conclude the Thesis in Section 6.4 with a review of the issues explored and a summary of the contributions presented.

Section 6.1. Specification of a Simulation-Based *CreditRisk*$^+$ Model

While presented in an alternative structure to that found in the asset value models reviewed in Chapter 4, *CreditRisk*$^+$ can be shown to share common conceptual and mathematical foundations with those studied in Chapter 4. In Gordy (2000) a single sector implementation is used to present a comparative mapping between a simulation-based *CreditRisk*$^+$ model and the *AVM* presented in Chapter 4. In this Section we elaborate on this comparison, while in Section 6.2 results based on various implementations of the simulation-based *CreditRisk*$^+$ are presented.

The simulation-based *AVM* model of Chapter 4 presented borrower defaults as realizations of a normally distributed latent variable below some given default barrier. The latent variable was assumed to depend on a single systematic factor and an idiosyncratic shock, both generated from standard normal distributions. Correlations between obligors were determined through common dependence on the single systematic factor, and were calibrated from PD averages and volatilities observed in the *Financing Company* SME portfolio.

In contrast, the *CreditRisk*$^+$ model does not place assumptions on the cause of default; in a single sector implementation, borrower default probabilities are modelled to vary over time, increasing or decreasing with a gamma-distributed latent systematic factor. Borrower probability of default co-movements with the systematic factor thereby generate correlations in defaults, Gordy (2000, p. 120). In this Section we will use methods presented in Gordy (2000) to present a simulation-baesd implementation of the *CreditRisk*$^+$ framework. Using this framework we are able to form direct comparisons in the simulated portfolio loss distribution using two alternative calibrations to the single risk factor distribution and the sensitivities to that factor.

In the first calibration we follow Gordy (2000) and fix the risk factor paramters and calibrate the sensitivies accordingly, this method will be referred to as the unitary normalized sector volatility setting. In the second calibration we will hold sensitivities constant and calibrate the shape and scale parameters of the gamma-distributed latent factor from *Financing Company* default rate data; this setting will be referred to as the unitary weight setting. These calibrations, as in Chapter 4, will depend solely on the PD averages and volatilities observed in the *Financing Company* SME portfolio.

We construct the *CreditRisk*$^+$ simulation-based implementation such that it uses the same simulation framework as that presented in Chapter 4. Analogously to Equation (5.11) and Equation (5.12), presented in Subsection 5.1.1, adhering to a single sector setting and dropping the *"k"* subscript of the sector parameters, we define a single gamma-distributed systematic factor (*X)* with shape and scale parameters:

$$\alpha = \mu^2/\sigma^2 \qquad and \qquad \beta = \sigma^2/\mu. \tag{6.1}$$

Allowing for the presence of an idiosyncratic factor with mean one and zero variance, for a given borrower *(i)* in risk grade *(ζ)* the probability of default, conditional on realizations of the systematic factor, is amplified or subdued according to a given sensitivity $w_{i,\zeta}$. More specifically, we write:

$$PD_{i,\zeta}(x) = \overline{PD}_{i,\zeta} \cdot \left(\frac{x \cdot w_{i,\zeta}}{\mu} + \left(1 - w_{i,\zeta}\right)\right), \tag{6.2}$$

where $\overline{PD}_{i,\zeta}$ is the unconditional long term average probability of default for a given risk grade *(ζ)*; see Gundlach (2004, pp. 16-17) and Gordy (2000, pp. 122-124); and:

$$\mu = \sum_{i} w_{i,\zeta} \cdot \overline{PD}_{i,\zeta} \ and \ \sigma = \sum_{i} w_{i,\zeta} \cdot \overline{PDVol}_{i,\zeta}. \tag{6.3}$$

To generate a distribution of defaults, Gordy (2000, p. 127) suggests the following specification for the latent variable y_i:

$$y_i = \left(\frac{x \cdot w_{i,\zeta}}{\mu} + \left(1 - w_{i,\zeta}\right)\right)^{-1} \cdot \epsilon_i, \tag{6.4}$$

such that the idioysyncratic risk factors ϵ_i are exponentially distributed with scale parameter equal to one. This specification is analogous to the *AVM* specification described in Subsection 4.1.1; see Equation (4.3). For a borrower *(i)*, default occurs if and only if $y_i \leq \overline{PD}_{i,\zeta}$, where:

$$
\begin{aligned}
Pr\left(y_i < \overline{PD}_{i,\zeta} \middle| X = x\right) &= Pr\left(\epsilon_i < \overline{PD}_{i,\zeta} \cdot \left(\frac{X \cdot w_{i,\zeta}}{\mu} + \left(1 - w_{i,\zeta}\right)\right) \middle| x\right) \\
&= 1 - exp\left(-\overline{PD}_{i,\zeta} \cdot \left(\frac{x \cdot w_{i,\zeta}}{\mu} + \left(1 - w_{i,\zeta}\right)\right)\right) \\
&\approx \overline{PD}_{i,\zeta} \cdot \left(\frac{x \cdot w_{i,\zeta}}{\mu} + \left(1 - w_{i,\zeta}\right)\right) = PD_{i,\zeta}(x). \qquad (6.5)
\end{aligned}
$$

Defaults can be simulated by drawing realizations of (ϵ_i) for each borrower and a realization of the systematic factor (x), and applying Equation (6.4). This method, while useful for descriptive purposes is known to generate a small approximation error and can be replaced by independent draws of a $Bernoulli\left(PD_{i,\zeta}(x)\right)$ variable, using the last line of Equation (6.5), above, and draws of systematic factor (x); see Gordy (2000, p. 142).

Losses are generated by multiplying the simulated borrower defaults by their exposures and their losses given default, individual losses are summed to obtain the portfolio loss for that draw of the systematic factor. The process is repeated times and the loss distribution is generated. Furthermore, we use the simulation-based allocation methodology presented Chapter 4 to present capital charges by segment. This process therefore describes a simulation-based implementation of the *CreditRisk*$^+$ framework in which an alternative systematic risk factor engine is used within the same portoflio loss distribution framework as that applied in Chapter 4. We thereby limit differences in implementation to the systematic risk factor distributional assumptions and calibration techniques of the asset value model (*AVM*) presented in Chapter 4 and the *CreditRisk*$^+$ model presented in Chapter 5 and extended in this Section.

As previously noted, under the *AVM* systematic and idiosyncratic risk factors were modeled as standard normal random variables, asset correlations representing obligor dependencies were then non-parametrically calibrated using historical PD and PD volatility values. In calibrating the simulation-based *CreditRisk*$^+$ we return to a subject touched on in Chapter 5, that of the unitary weight setting versus the unitary normalized volatility setting. In particular, *CreditRisk*$^+$ implementations allow for the choice of calibration of obligor sensitivities with given assumed distributional characteristics for the sector factor, or, alternatively, for the calibration of the factor scale parameters given fixed sensitivities. In this Subsection we will present a method incorporating both of these calibrations. Results based on these calibrations will be compared against each other as well as against those obtained under the *AVM* of Chapter 4. In addition, we will calibrate our model to reflect the boosted correlations obtained in Chapter 4. This exercise will be similar to Dietsch and Petey (2002).

In particular, we observe that for the *AVM*, the methods presented in Subsection 4.1.1 are used to calibrate asset correlations from default rate mean and volatility. For a single sector implementation of the *CreditRisk*$^+$ framework, with a borrower-specific idiosyncratic factor, see Credit Suisse (1997, p. 52), we can define a given segment (ζ)'s variance around its probability of default as:

$$Var[p_{\zeta}(x)] = Var\left[\overline{PD}_{\zeta} \cdot \left(\frac{X \cdot w_{\zeta}}{\mu} + (1 - w_{\zeta})\right)\right] = \overline{PD}_{\zeta}{}^{2} \cdot Var\left[\left(\frac{X \cdot w_{\zeta}}{\mu}\right)\right]$$

$$= \left(\frac{\overline{PD}_{\zeta} \cdot w_{\zeta}}{\mu}\right)^{2} \cdot Var(X) = \left(\frac{\overline{PD}_{\zeta} \cdot w_{\zeta} \cdot \sigma}{\mu}\right)^{2}, \qquad (6.6)$$

such that for the normalized volatility $\left(\sqrt{Var\left[p_{\zeta}(x)\right]}/\overline{PD}_{\zeta}\right)$ we can uniquely determine the weights *(w_{ζ})* or the normalized sector volatility (σ/μ); see Gordy (2000, p. 134):

$$\sqrt{Var\left[p_{\zeta}(x)\right]}/\overline{PD}_{\zeta} = w_{\zeta} \cdot \left(\frac{\sigma}{\mu}\right). \qquad (6.7)$$

In Section 6.2 we present cases of the simulation-based *CreditRisk*$^{+}$ implementation in which the normalized sector volatility is held fixed and the weights are calibrated, as well as cases in which the weights are fixed (at a value of one) and the sector volatility is calibrated. Recall, under the analytical implementations of *CreditRisk*$^{+}$ the weights are always fixed at a value of one.

Boosted Asset Correlations

In order to provide comparability between boosted asset correlation results in Chapter 3 and those obtainable under the *CreditRisk*$^{+}$ framework for the same boosted asset correlation values, we use the work in Gordy (2000) as a bridge, this is similar to the boosting and bridging in Dietsch and Petey (2002).

Specifically, in a single sector setting, for a given segment, we use Equation (4.10) to calibrate the default rate variance given the boosted asset correlation value and the segment PD. The next step then depends on our choice of calibration setting: Under the unitary normalized volatility setting we substitue this "boosted variance" into Equation (6.7) and thus obtain the segment weight; Under the unitary weight setting we take the square root of segment-specific boosted variances and substitute them into Equation (6.3) to obtain the systematic risk factor standard deviation. The boosted weight or sector volatility is then substituted into the simulation-based implementation of the *CreditRisk*$^+$ framework described above to generate capital charges for the portfolio.

Final Comparative Model Specifications

In this exercise we use identical data to calibrate the *CreditRisk*$^+$ and asset value models. For both models, data segmented by borrower and loan, LGD values are given as either 73% or 41%. In both models a conditional default framework is established such that a latent factor mimicking the business cycle is simulated, and borrowers default depending on a combination of their sensitivity to the systematic latent factor and idiosyncratic effects. Our database of historical SME defaults, spanning 12 years and containing information on borrower creditworthiness, size and industry, allows us to determine that sensitivity under the varying assumptions of the models implemented. Loss distributions are generated for various realizations of the latent and idiosyncratic factors for each obligor, a given condition for default, and fixed obligor-specific exposure and LGD values.

In order to combine the defaults with losses, we simply use the same simulation engine used Chapter 4, so that defaults are generated by borrower and then assigned to the corresponding loans with which LGD and exposure amounts are associated. Given the use of identical input data in the calibrations of the models presented in Chapter 3 and Chapter 4, we obtain results using the exact same inputs but with slight alternations to the conditional default and portfolio loss structure.

Section 6.2. Comparative Capital Charges: Basel II, *AVM* and *CreditRisk*$^+$

Our results will review the comparability of various calibrations of the *CreditRisk*$^+$ model to the *AVM* model in terms of correlations, loss distributions, and capital charges – both overall and at the segment-specific level. In turn, we present results along three *Comparisons*. *Comparison I* presents loss distributions under the *AVM* framework and various settings of the *CreditRisk*$^+$ framework. Using Figure 6.1 and Table 6.1A we are able to identify settings of the *CreditRisk*$^+$ framework under which calculated loss distributions are clearly divergent from that obtained under the *AVM*. *Comparison II* presents a comparison of segment-specific default correlations, along with other parameters, under the two frameworks and suggests a calibration refinement for

CreditRisk$^+$. Having observed comparative loss distribution results under the various settings and calibrations of the *CreditRisk*$^+$ and *AVM* frameworks, we apply the boosting method described in Section 6.1 to compare EC results under both frameworks to those obtained under the Basel II AIRB framework for portfolio credit risk; see Case 2 of the Basel II implementations of Chapter 3. This comparison, which we will refer to as *Comparison III*, uses Figures 6.2 and 6.3, and Tables 6.4 to 6.6, to clearly delineate loss distributions and EC allocations under the boosted and non-boosted implementations of the frameworks.

Fixed normalized standard deviation ratio (σ/μ) settings of the *Single Sector CreditRisk*$^+$ model include sector weights which have been calibrated according to Equation (6.7), such that for each segment, the sector weight and the idiosyncratic weight sum to one. The unitary sector weight setting of the *Single Sector CreditRisk*$^+$ model includes a sector weight that has been set to one and the single sector ratio that has been calibrated from the data according to Equation (6.3). This specification, matches that used in the analytical implementation of the *CreditRisk*$^+$ model and will be the primary base of comparison against the internally-calibrated *AVM* model. In addition, we will use this specification as a basis for comparisons using the boosted asset correlations of Chapter 4, using boosted asset correlations given in Table 4.7. This exercise shares similarities with Dietsch and Petey (2002).

Finally, we will examine default correlation results obtained in the *CreditRisk*$^+$ framework and compare them to those obtained in the *AVM* and pre-calibrated into the

Basel II. This work will be undertaken in Subsection 6.2.3. It should be noted that here we are interested in the default correlations among borrowers in the same portfolio segment and will not discuss the cross-correlations across segments. These values can be readily derived from results presented in this Subsection and the application of Equation (C.1) of Appendix C.

Figures presented in this Chapter will have two panels: the top panel will show loss distributions for “non-boosted” internally calibrated models; the lower panel will show loss distributions for “boosted” internally calibrated models. In Figure 6.2, a third (bottom) panel juxtaposes “boosted” and “non-boosted” model loss distributions, providing further perspective on results obtained under each calibration.

Comparison I *Loss Distributions under various settings of* CreditRisk^{+}

Figure 6.1; Table 6.1A; Table 6.2

We examine loss distributions under various settings of the *CreditRisk*$^{+}$ model, under an RG calibration, and compare them to a similarly calibrated *AVM* implementation. Settings include the unitary weight setting, and fixed normalized standard deviation settings with values for the ratio (σ/μ) of one, 0.5 and 0.25. Our choice is based on results our observation of normalized PD standard deviation values for our SME RG segments, as depicted in Table 4.4. As can been seen in Table 4.4, SME normalized PD standard deviation values are considerably lower to those observed for Corporate borrowers. These Corporate borrower normalized PD standard deviation values may suggest a calibration of one or greater for the *CreditRisk*$^+$ single sector risk factor normalized standard deviation, a fact that has resulted in such settings throughout the literature – even when that literature has claimed to present settings suitable for SME borrowers; see, for example, Gordy (2000) and Dietsch and Petey (2002).

Our results show that a unitary normalized standard deviation setting for *CreditRisk*$^+$ presents a loss distribution significantly different in shape and characteristics from that obtained under the *AVM*. The unitary weight setting provides a comparable loss distribution with thinner tails, and therefore lower capital charges. The 0.25 and 0.5 settings of the fixed normalized standard deviation setting provide loss distributions comparable in shape, both overall and at the tail, with the *AVM*, providing a thinner tail in the case of the former and a fatter tail in the case of the latter. Comparing capital charges, we observe 2.0% EC under an RG calibration of the unitary normalized standard deviation setting, as compared to 1.2% under the unitary weight implementation and 1.4% under the *AVM*. We observe 99.9% percentile EC values of 1.3% and 1.5% under the 0.25 and 0.5 settings, respectively.

We conclude that the unitary normalized standard deviation setting for *CreditRisk*$^+$ is not appropriate for our SME portfolio. The unitary weight setting provides an approximate fit with thinner tails, while the 0.25 and 0.5 fixed normalized standard deviation settings also provide acceptable fits.

For completeness Table 6.2 provides comparisons of capital allocations under non-boosted implementations of the models, for various calibrations and settings. In particular, we observe capital charges for Risk and Size Group segments under the *Single Sector* simulation-based implementations of *CreditRisk*$^+$. All models are implemented under an RG calibration, and *CreditRisk*$^+$ settings include the unitary weight setting and fixed normalized standard deviation values of 1, 0.5 and 0.25. Overall EC figures under each implementation correspond to those obtained for the 99.9% percentile, as given in Table 6.1A.

Comparison II *Default correlations under various settings of* CreditRisk$^+$

Tables 6.3A to 6.3C

Table 6.3A provides default correlations under various settings of *CreditRisk*$^+$ alongside those obtained under the *AVM* framework (top panel); see Table 4.2. *CreditRisk*$^+$ default correlations are calculated according to Equation (C.1). We find that the 0.5 and unitary fixed normalized standard deviation setting provide default correlations identical to those obtained in the *AVM*. The unitary weight setting and the 0.25 fixed normalized standard deviation setting provide default correlations that vary from those observed in the *AVM*. Similarities and differences in default correlations are attributed to the “location” of the model specification. For the unitary weight and 0.25 normalized standard deviation settings, we also observe, in Table 6.3A, increasing default correlations with increasing RG (and PD), both overall and when controlling for Size. For these two settings we observe decreasing default correlations with increasing Size.

Given the calibration techniques used on the fixed normalized standard deviation implementations and comparing to the calibrations used for the unitary weight setting of the *CreditRisk*$^+$ model, we suggest that this discrepancy may be attributable to the "location" of our model specification; recall Equations (6.3) and (6.7) which present two alternative methods of specification for the *CreditRisk*$^+$ framework. For the fixed normalized standard deviation setting, our specification of model parameters is concentrated on the segment-specific weights; for the unitary weight setting, our specification is concentrated on the sector risk factor mean and standard deviation. This specification-location aspect of the fixed normalized standard deviation setting is somewhat neutralized in the 0.25 setting as most segment weights are set to one, as can be observed in Table 6.3B. For this setting, we observe that the normalized standard deviation value of 0.25 provides the loss distribution properties closest to those obtained under the *AVM*, while the resultant predominance of segment weight values of 1.0 resemble those under the unitary weight setting. In the 0.5 setting of the fixed normalized standard deviation *CreditRisk*$^+$ implementation a compromise can be found between the loss distribution shape and the consistent calibration of default correlations.

Our results here, as in Chapter 4, show that the data does not, ultimately, reveal an absolute relationship between correlations in an SME credit portfolio and other credit risk characteristics such as PD and Size. For the unitary weight setting, and the 0.25 normalized standard deviation setting, the single sector risk factor parameters – uniform across segments – along with the segment weights, are specified and the remaining component of segment default correlation, PD, is left to determine default correlations; see Equation (C.1) in Appendix C. As such, under those implementations, we observe default correlation patterns reflective of PD patterns: increasing in RG and decreasing in Size. In this respect, we propose that the *AVM* setting – and its equivalence in the 0.5 and unitary normalized standard deviation settings for *CreditRisk*$^+$ - provides the best avenue for the description of the "real" characteristics of the data; put another way, these settings are best suited to give voice to the data and allow it to speak.

Comparison III Misallocation of Capital Charges under Basel II

Figures 6.2 and 6.3; Tables 6.4 to 6.6

In Table 6.4 we compare boosted RG and RG-SG calibrations of the *AVM* to boosted calibrations of the *CreditRisk*$^+$ framework under unitary factor weight settings. As in Tables 6.1, we observe higher capital charges under the *AVM* as compared to *CreditRisk*$^+$ implementations, with *AVM*-derived loss distribution exhibiting significantly higher kurtosis. Figure 6.3 shows generally fatter tails when using the *AVM* under both RG and RG-SG calibrations. Nevertheless, the loss distributions, as depicted in Figures 6.2 and 6.3, show a considerable amount of similarity for the two models under the calibrations and settings described above.

For completeness, Table 6.1B presents loss distribution results for the RG-SG calibrations of the *AVM* and unitary weight setting of the *CreditRisk*$^+$ model. Results show a notable decrease in EC levels under the RG-SG calibration as compared to the RG calibration.

Comparing Economic Capital allocations under the boosted implementations of *CreditRisk*$^{+}$ and the *AVM*, we again find misallocation of capital under Basel II. This result holds such that smaller SME borrowers are undercharged EC under Basel II and larger borrowers are overcharged. For example, we observe capital charges for the smallest and largest borrowers under Basel II (top panel) to be equal to 6.7% and 9.8%, respectively. In contrast, capital charges for the smallest and largest borrowers obtained under the RG calibration of *CreditRisk*$^{+}$ (third panel) are equal to 16.5% and 5.5%, respectively. Comparing boosted RG-SG calibrated capital charges to those obtained under Basel II (Case 2), we observe in Table 6.6 capital charges for the smallest and riskiest borrowers equal to five times those calculated under Basel II, while those for the largest and least risky are one fifth those obtained under Basel II; see the second panel of Table 6.6 for ratios for the ≤ *\$100,000* Size Group – 1-3 Risk Group segment and the >*\$1,000,000* Size Group – 8-9 Risk Group segment, respectively.

Our results indicate EC allocation patterns across Risk and Size Groups in our boosted internally calibrated models contradict those found under Basel II. These results are similar to those of Chapter 4, which are replicated here in boosted *AVM* EC charge allocations. In particular, we once again observe strictly decreasing EC charges with increasing borrower Size segments and decreasing, both at an overall level and when controlling for Risk Group, under RG calibration. Decreasing capital charges are also observed with improving credit quality. Similarly to the *AVM* case, we observe a general amplification of these EC allocation patterns under RG-SG calibrations, with a lower overall portfolio EC charges.

Section 6.3. Discussion of Comparative Results

As shown in Gordy (2000), the underlying driver of loss distributions as derived under *CreditRisk*$^+$ is the specification of the systematic risk factor normalized standard deviation. When detailed SME portfolio data is not available, this specification can be a source of concern to practitioners as an obvious setting may not be necessarily discernible. This critical ambiguity underlines the importance of our work in Chapter 4, in which much time and effort was spent to ensure that our segmentation of the data delivered robust segment-specific estimates of default rate volatilities and averages. Using this data, we are able to proceed with the estimation of both the *AVM* and *CreditRisk*$^+$ from a set of purely internal data.

Our results under a unitary weight setting for *CreditRisk*$^+$ and internally calibrated normalized standard deviation show that under these specifications portfolio loss distributions and capital charges under both models are quite similar. This setting may not, however, provide default correlation parameterizations comparable to the *AVM* due to the exclusive reliance on the overall sector normalized volatility, in this setting, as opposed to values corresponding to the default correlation segments.

Conversely, when calibrating segment-specific weights for a fixed normalized sector volatility, our results show that *CreditRisk*$^+$ may not provide adequately similar SME loss distribution results under popular assumptions for risk factor normalized volatility values of one or greater, as found in Corporate credit portfolio calibrations; see Gordy (2000). Specifically, the unitary normalized standard deviation setting of the *CreditRisk*$^+$ model is shown to display excessively fat tails as compared to the *AVM*, while obtaining identical default correlations.

Our results suggest a *CreditRisk*$^+$ single sector risk factor normalized standard deviation setting of 0.5 for SME portfolios as an acceptable calibration when internal data are not available; see Tables 6.2 and 6.5, and Figures 6.1 and 6.2. This setting, however, has come under some fire in the literature. Wieczerkowski (2003, p. 47) observes that normalized default rate standard deviations of the order of one (or less) correspond to "unrealistically" small asset correlations in *CreditMetrics*, Wieczerkowski (2003, p. 47).

These seemingly contradictory results, that of an inability to generate consistent loss distributions under the two models when the normalized volatility in *CreditRisk*$^+$ has been set to one (or more), and the low resultant asset correlations when the two models are consistently parameterized, may be traced back to a common source. In particular, we suggest that these results may originate from the generally low default rate volatilities observed in SME portfolios, despite their generally higher mean default rates; see Table 4.4. Work in Frye (2008) underlines the significant misestimates of correlations, in a credit portfolio setting, that may arise from the use of market equity data to estimate asset correlations.

As discussed in Chapter 4, Frye (2008, p. 78) highlights some cases in which default and a shortfall of asset values below liability values may not correspond. One of these cases is default due to a liquidity crunch for the borrower resulting in an inability to repay short term debts despite elevated asset values. Another view of the break in the connection between default and shortfall is presented, that in which a shortfall in asset values does not lead to default due, for example, to an extension of further credit by the lending institution. Other assumptions tested include the assumption of known liability values (versus randomly varying liabilities) and the use of unconditional asset correlations (versus conditional asset correlations); see Frye (2008).

Notwithstanding these results on low correlations observed in our data, our implementation of both the *CreditRisk*$^{+}$ and *AVM* frameworks has included a prudential adjustment bringing correlation levels in line with those observed under the Basel II IRB regulatory framework for portfolio credit risk. That our estimated correlations might benefit from such a boost is not uncommon or unexpected; see, for example Dietsch and Petey (2002) and the discussion in Chapter 4. Applying a similar adjustment to their internally estimated asset correlations, Dietsch and Petey (2002) use a normalized standard deviation setting greater than one and a modified to obtain "boosted" capital results comparable to those obtained under the Basel II IRB framework. Boosted internally calibrated capital results are lower in both models as compared to Basel II, these results are similar to those obtained in this Thesis when RG-SG calibrations are used; see Table 4 in Dietsch and Petey (2002, p. 317) and Table 6.5. As expected, the elevated normalized volatility setting in Dietsch and Petey (2002) results in higher capital requirements under the *CreditRisk*$^+$ framework as compared to the *AVM*. Our results suggest that a unitary weight setting of *CreditRisk*$^+$ could reverse this ranking.

Under a simplified setting, Koyluoglu and Hickman (1998) show that the *CreditRisk*$^+$ and *AVM* frameworks similar loss distributions for a wide range of parameter values – wherein the parameters studied included the average default rate, or PD, and the normalized PD standard deviations; see Koyluoglu and Hickman (1998, p. 15). Divergence in model results was observed for very high values of the normalized PD standard deviations, especially in the presence of very high or very low PDs. Additionally, and unsurprisingly, model results were found to differ significantly in the presence of inconsistent parameterizations of the models; see, Koyluoglu and Hickman (1998, p. 15).

Finally, returning to our comparison of boosted EC results we observe under the *CreditRisk*$^+$ framework, as under the *AVM*, a significant break in capital charge allocation as compared to the Basel II IRB framework. Under *CreditRisk*$^+$, as under the *AVM* framework, we observe strictly decreasing capital charges with increasing Size. Under Basel II we note an overestimation of capital charges for the largest borrower segments and an underestimation for the smallest borrower segments.

Our exercise in boosting the parameters of the *CreditRisk*$^+$ model in Chapter 6 reveals significantly lower capital charges than those obtained under Basel II. Further fine-tuning of this boosting methodology could lead to higher capital charges overall, with retain the pattern observed above. Our work in this Chapter, as well as in Chapter 4 reveals that further evidence against Basel II assumptions of correlation relationships with Size and PD. This is especially revealed in our discussion of the various calibrations and settings for *CreditRisk*$^+$. As such, we will have presented the *CreditRisk*$^+$ under three distinct exercises, that of the quantification of single sector and independent multiple sector approximation errors in Chapter 5; that of estimating a parameter-consistent calibration of *CreditRisk*$^+$ to the *AVM*; and that of estimating Economic Capital charges under boosted asset correlations.

Section 6.4. Conclusions

Small and Medium Enterprise loans portfolios present a unique set of challenges to portfolio credit risk modellers and managers. These challenges are rooted, in no small part, in the traditional calibrations of portfolio credit risk models on Corporate credit data, a necessity itself wrought from the general lack of default data available within individual institutions.

As a response to this reliance on external data, the literature has responded by trying to present methods to calibrate portfolio credit risk models to SME data sets, generally constructed over aggregated national data banks. These have presented their own shortcomings, not least of which is the presence of default time series limited in span, and high degrees of diversification within the data sets, leading to a potential dilution of results.

Notwithstanding these data constraints, estimates of SME portfolio credit risk from internal data have faced several challenges. These include the challenge to provide consistent calibrations and parameterizations across varying models of SME portfolio credit risk; the challenge to compare to results obtained under external data sets, and; perhaps as a combination of the previous two points, the challenge to compare to prudential settings for portfolio credit risk, as presented under Basel II.

In presenting prudential guidelines for the treatment of portfolio credit risk, along with specific formulations for the estimation of regulatory capital to meet these risks, Basel II presents specific characterizations of the credit risk among SME borrowers that open themselves up to testing. Specifically, the underlying tool for the estimation of portfolio credit risks lies in an asymptotic single risk factor model calibrated to a 99.9% confidence level, and within which specific dictates on the relationship between a borrower's sensitivity to the systematic risk factor, i.e., their asset correlation, and their size, as well as their probability of default.

For SME borrowers we observe these dictates to include decreasing asset correlations with increasing probabilities of default, and decreasing asset correlations with decreasing borrower size. Furthermore, for the smallest borrowers, with low enough exposures to a given financial institution's portfolio, with loans classified into the Retail-Other asset class, we observe an acknowledgement of a relaxation of these conditions such that asset correlations are no longer related to borrower size, and the relationship between probabilities of default and asset correlations is relaxed. Meanwhile, the underlying theoretical and empirical bases for these settings have been shaky at best, in large part due to the data and modelling restriction constrictions discussed above.

Our work in this Thesis has sought to tackle this broad span of challenges related to the estimation of SME portfolio credit risk and SME credit risk characteristics. The crux of our work has centered on a unique portfolio of Canadian SME borrowers and loans spanning a significant time span and with enough depth to allow for meaningful segmentations of the data. This segmentation allows us to derive robust results on the behaviour of different segments of an SME portfolio, and therefore tackle the question of SME parameter specifications, such as those presented in the Basel II IRB framework. In addition, this data depth and segmentation has allowed us to pursue a consistent calibration of the asset value model (*AVM*) and the *CreditRisk*$^+$ framework for the estimation of portfolio credit risk.

Undertaking an estimation of asset correlations within a setting comparable to the Basel II IRB framework, our results revealed that SME borrowers' asset correlations could not be found to have a strong relationship with either PD or Size. In addition, internally estimated SME asset correlations were found to be considerably lower than those observed in the Basel II IRB framework. This result suggests some similarity with the Retail-Other asset class settings for SME borrowers' asset correlations in the IRB framework, with a strict rejection of the presence of a positive relationship between asset correlations and probabilities of default. An underlying driver in this result, and overall in this overall portfolio credit risk model, was found to be the ratio of the probability of default standard deviation to the average (or unconditional) probability of default, i.e., the normalized PD standard deviation.

The unveiling of the normalized PD standard deviation as being a significant driver of portfolio credit risk has been reviewed in the literature, mostly in theoretical settings. In particular, this driver has been shown to be a critical factor in the calibration of the *CreditRisk*$^+$ model. Given the lack of SME default data available, the calibrations of this factor has relied on Corporate data generally provided by external rating agencies such as Standard & Poor's and Moody's. In addition, traditional calibrations of the *AVM* in this same insufficient default data environment has relied on external market-based equity data, alongside data from external rating agencies to estimate such parameters as the asset correlations.

As such, these external calibrations have presented challenges to the consistent calibrations of the models. When consistent calibrations have been pursued, results have been cast into doubt due to their inconsistent comparison with market-based parameters. Specifically, asset correlations derived from default rate data were found to be very weak when compared to asset correlations derived from market-based equity data.

In an updated review of the literature on the calibration of asset correlations our Thesis presented arguments from the literature on inaccuracies incurred in the use of market-based data to estimate correlations between borrowers in a credit environment. Meanwhile, further evidence from the literature was presented in support correlation levels generally lower to those found using market-based data in an *AVM* framework.

The finding of low correlations has numerous consequences. For one, the use of default data and the low correlations generated by it, allows for a corresponding calibration of the *CreditRisk*$^+$ framework such that the model present similar loss distributions and Economic Capital results. These consistent calibrations are found to be not only legitimate but realistic; suggesting an SME calibration for the *CreditRisk*$^+$ framework single sector normalized risk factor standard deviation in the range of 0.25 to 0.5. This finding is significant in an environment in which suggested calibrations for the *CreditRisk*$^+$ model have been limited to settings derived from Corporate data. Our ability to suggest this calibration is based on the depth of our data and our ability to consistently calibrate the *CreditRisk*$^+$ model to the *AVM* framework using a unitary weight setting of the *Single Sector CreditRisk*$^+$ model.

Our findings of low correlations were also showed to imply a significant result for asymptotic implementations of the single factor *AVM*, such as that calibrated into the Basle II IRB framework. In particular, our results showed a significant increase in granularity effects given the presence of low correlations in an SME portfolio. This effect can be perhaps explained by the predominant role played by idiosyncratic risks in the presence of low correlations. When these idiosyncratic contributions to portfolio risk are assumed away, such as under an asymptotic framework, a significant underestimation of portfolio credit risk may arise. This result was shown to hold even in a very large SME portfolio with several dozen thousand obligors.

To our knowledge, this is the first time the level of asset correlations has been linked to the granularity effect in a portfolio. The granularity has been a focus research since the release of the Basel II guidelines given the asymptotic nature of the framework in which portfolio credit risk is assessed – with links between the granularity effect and PD levels made in the literature. Another aspect of the Basel II framework has attracted significant attention in the literature, that is the single sector aspect which has been shown to provide an overestimation of the portfolio credit risk in contrast to a multiple sector or risk factor setting in which diversity benefits are incorporated into the portfolio credit risk calculation. Our work within the *CreditRisk*$^+$ framework provided a measure of the overestimation that may be incurred when the single risk factor assumption is applied to an SME portfolio. These results were generally in line with those observed in the literature.

Finally, a major consequence of the low level of correlations addressed in this Thesis is the ultimately low capital figures generated, especially when compared to the Basel II framework. To address this issue, and building on the significant default data segmentation available to us through our unique portfolio of SME borrowers and our consistent parameterization of the *CreditRisk*$^+$ and *AVM* frameworks, we are able to propose a prudential adjustment to our model parameters. In particular, we are able to boost asset correlations in the *AVM* to values observed in Basel II, and obtain a consistent parameterization of *CreditRisk*$^+$ through calibrations to the normalized PD standard deviations.

This exercise brings to light significant misallocation of capital in the Basel II framework, due primarily to asset correlation relationships discussed above, such that small SME borrowers obtain discounts to capital charges while larger SME borrowers are shown to have surcharge in capital under Basel II. In addition, through this exercise we are able to confirm results on low asset correlations by comparing them to results for boosted asset correlations. These boosted results show a dissipation of granularity effects observed under internally calibrated asset correlations, given all else constant we are thereby able to reaffirm low correlations as the source of the granularity effect. Taking Size into account in the calibration of PDs and correlations results in an overemphasis in patterns of capital allocation observed under the traditional rating grade calibration of these parameters, while simultaneously lowering overall capital charges.

Our contributions can be summarized as follows:

1. *A Comprehensive Analysis of an SME loans portfolio within a financial institution*
   We provide a very finely detailed description of an SME portfolio, including various breakdowns of risk characteristics, in terms of borrowers and borrower exposures. This level of analysis is difficult to find in the literature as it pertains to one institution, whereas in cases in which SME data has been collected and analyzed it has usually been of an aggregated nature with limited historical time span.

2. *A Detailed Schematic for SME Portfolio Credit Risk Input Data and Structure*
   We are able to segment this portfolio into homogenous groups of borrowers defined along credit risk dimensions such as size, industry, and risk grade. For each borrower segment we generate probabilities of default. Probabilities of default by risk grade, in particular, form the basis of credit risk management frameworks as stipulated under the Basel II prudential guidelines for portfolio credit risk. A financial institution's ability to treat its portfolio credit risk under the most advanced systems within Basel II is thus dependent on its ability to properly estimate the PDs associated with its internal risk grades. Conversely, this data requirement is countered by the need to minimize data requirements and costs at financial institutions. By contrasting the implementation of Basel II frameworks for portfolio credit risk management and internally-calibrated models for portfolio credit risk, we are able to highlight the minimal data requirements stipulated under Basel II. In particular, the significant depth of our unique database allows for a fine segmentation of homogenous segments of our SME portfolio such that a dual-dimension system is defined. As such, we are able to estimate credit risk measures, such as PDs and, later, correlations, for homogenous segments of borrowers defined by risk grade and size. These credit risk measures form the underlying basis on which our work in this Thesis is conducted, both in testing the assumptions and relationships inherent in the Basel II treatment of SME portfolio credit risk, and in establishing internally-calibrated models of our own. The elevated data requirements accompanying the estimation of these models, and our ability to meet them in a robust manner, highlights the unique and important data source on which our results are based.

3. *A View of Conceptual & Pragmatic Implications of Basel II treatments for SMEs*
   We present a comprehensive analysis of the Basel II treatments for SME portfolio credit risk, underlining the assumptions used in the calculation of capital charges under the framework and the impacts these assumptions have on patterns of capital charges across SME borrower segments. In particular, we study the Standardized Approach and the Internal Ratings Based (IRB) Approach to portfolio credit risk, and engage in a Partial Implementation exercise to test the impact of the assumptions within each implementation on capital charges. Our focus is on SME borrower Size segments and our results reveal that the presence of two SME treatment possibilities in Basel II, i.e., the Corporate asset class treatment and the Retail-Other asset class treatment, open the door to dual regimes for SMEs such that the smallest can be treated under a certain set of assumptions and capital rules while the largest are treated under another. Specifically, we find that the Corporate asset class treatment, with its elevated asset correlation values and their programmed positive relationship with Size and negative relationship with PD, provides for elevated capital charges for the largest SME borrowers. The Retail-Other asset class treatment, which maintains the negative relationship with PD but does away with the size-based adjustment for

SME borrowers, maintains lower asset correlation settings for these borrowers and generates capital charge that are decreasing with increasing Size. When applied together across a portfolio of SME borrowers of various Sizes, the result of this dual treatment is a U-shaped capital allocation with increasing borrower size. Even when treating borrowers under one asset class, Corporate, we observe this U-shaped pattern in capital allocations, with our partial implementation exercise revealing the source to be the size adjustment applied to asset correlations.

4. *Empirical findings concerning SME Asset Correlations*
   Working within a single sector asset value model (*AVM*) framework similar to that of the Basel II IRB framework, and using robustly specified segments of our SME portfolio, defined according to Risk and Size Groups (RG and SG, respectively), especially including dual-dimension segmentations, we estimate asset correlations for an SME portfolio. Our results show that SME portfolios typically exhibit low asset correlation values, and we find no empirical evidence of either a positive relationship with size or a negative relationship with PD. This result runs counter to Basel II specifications of a negative relationship between asset correlations and PD, and counters the Corporate asset class assumption of a positive relationship between asset correlation and size. In the finding of low asset correlation values and no relationship between asset correlations and size our results appear to provide some support to specifications under the Retail-Other treatment, however this support remains limited by that treatment's programming of a negative relationship of asset correlations with PD, even if that relationship is weaker than under the Corporate asset class. In addition, our results on the lack of relationships between asset correlations and size and PD, contrast with the literature wherein such relationships have been deduced from generally weak empirical evidence. Our work in defining robust risk-size segments of SME borrowers allows us to present stronger evidence the presence of such relationships, or lack thereof.

5. *Increased Granularity Effects in SME credit portfolios with Low Asset Correlations*
   The presence of low asset correlations increases the approximation error generated by the use of an asymptotic framework in which idiosyncratic risks in a credit portfolio are assumed to be diversified away. The assumption of a fully diversified portfolio, along with the assumption of a single sector or risk factor, forms the underlying basis of the Basel II portfolio credit risk framework. Our findings appear within the context of a very large portfolio and show an approximation error, or granularity effect, of approximately 6%. A figure higher than would be expected for such a large portfolio. This result is the first, to our knowledge, to show empirical evidence of a link between asset correlation values and the granularity effect in a credit portfolio.

6. *Empirical Evidence on the Economic Capital Impact of the Single Sector Assumption*
   Using the *CreditRisk*$^+$ framework we are able to estimate the level of approximation error generated by another underlying assumption in the Basel II framework, the use of a single sector framework for the estimation of portfolio credit risk. Our results show that for our portfolio of SME borrowers the use of a single risk factor can increase EC figures by approximately 40%. The assumption of independence across multiple sectors in our portfolio was shown to underestimate Economic Capital charges by approximately 60%.

7. *A Consistent Calibration of Single Sector* CreditRisk$^+$ *and Asset Value Models for SME Portfolio Credit Risk*
   We calibrate single sector *AVM* and *CreditRisk*$^+$ models and show that the two models provide generally similar results if they are calibrated consistently. Two calibration methodologies are presented for *CreditRisk*$^+$, along with a simulation-based implementation to enhance comparability. Our results show that a calibration of the risk factor weights according to segment-specific ratios of the PD standard deviation to its unconditional mean, in the presence of a fixed sector normalized volatility figure of 0.5, generates segment-specific default correlations consistent with those observed in the *AVM*. In such a setting, the accompanying *CreditRisk*$^+$ loss distribution displays fatter tails that of the *AVM* implementation, and therefore produces higher EC values. Alternatively, we show that a fixed unitary weight setting for the CreditRisk+ model, or a fixed normalized.

8. *An SME portfolio-specific calibration refinement for* CreditRisk$^+$ *models*
   These fixed sector normalized volatility values of 0.5 and 0.25 are not commonly found in the literature, which has tended to focus on calibrations from Corporate borrowers. These calibrations, along with the unitary weight calibration, therefore present SME-specific calibrations of the *Single Sector CreditRisk*$^+$ model.

9. *A thorough Assessment of Basel II approaches to SME credit risk modelling*
   The estimation of low correlation values in the empirical literature is not uncommon, and recent research has highlighted several potential sources for the discrepancy between correlations estimated from default or loss data, and correlations estimated from market-based sources. We present a widely-applied ad hoc boost to our estimated correlations, calibrated from average correlations observed in the application of the Basel II AIRB framework to our portfolio. This prudential adjustment allows us to generate EC figures that are prudentially comparable to the capital charges generated under the AIRB, but also take into account the SME portfolio credit risk characteristics revealed by our study. Our results reveal that Basel II lead to misallocation of capital charges, such that in some cases, smaller and riskier SME borrowers are charged less than larger and safer SME borrowers. These

Basel II capital charges can represent cases of under- or over-charging of capital to borrowers as compared to the capital charges they would incur under internally-calibrated models of portfolio credit risk.

10. *Suggestions for an SME portfolio credit risk management framework*

Taken as a whole our results suggest a choice, not only of internally calibrated portfolio credit risk models for financial institutions and regulators, but also a choice with respect to the calibration and application of regulatory standards and guidelines. In particular, we observe that despite the misallocation capital across SME borrower segments in Basel II, this condition may be alleviated through the removal of size-based adjustments within SME segments. Such a case exists in the Retail-Other treatment of SME borrowers, but is limited in its applicability to all SME segments due to exposure limits and other restrictions on its use. The dropping or easing of these limits may provide avenue through which SME capital allocation within the framework may be corrected. Our work also advises against too quick an attribution of correlation patterns across SME borrower segments. In performing the empirical work we have in this Thesis, and in applying two models to the estimation of SME portfolio credit risk, we have sought to allow the data to speak and have endeavoured to find the calibrations and model specifications that best allow for this. To that end, we found that the simulation-based *AVM* framework provides the freest setting in which to pursue such a study, with limited preconditions or assumptions placed on the overall framework. In addition, this framework presents advantages in its ease of comparability with prudential guidelines. Our adoption of a simulation-based implementation methodology in the *CreditRisk*$^+$ framework, and our successful calibration of the model to our SME portfolio in a manner consistent with that of the *AVM* provides another avenue for SME portfolio credit risk measurement and management, and presents practitioners with a variety of settings to which the model structure can be set without some of the drawbacks usually associated with original model and its suitability to SME portfolios.

References

Akhavein, J., Kocagil, A., and Neugebauer, M. (2005). *A Comparative Empirical Study of Asset Correlations.* Special Report, Fitch Ratings, Quantitative Financial Research.

Akkaya, N., Kurth, A., and Wagner, A. (2004). Incorporating Default Correlations adn Severity Variations. In V. M. Gundlach, and F. Lehrbass, *CreditRisk+ in the Banking Industry* (pp. 129-152). Berlin-Heidelberg: Springer-Verlag.

Allen, L., and Saunders, A. (2002). *Credit Risk Measurement: New Approaches to Value at Risk and Other Paradigms* (Second ed.). New York: John Wiley and Sons, Inc.

Allen, L., DeLong, G. L., and Saunders, A. (2003). *Issues in the Credit Risk Modeling of Retail Markets.* Working Paper, NYU Stern School of Business, New York.

Altman, E. I., and Sabato, G. (2005). Effects of the New Basel Capital Accord on Bank Capital Requirements for SMEs. *Journal of Financial Services Research* (28), 15-42.

Avesani, R., Liu, K., Mirestean, A., and Salvati, J. (2006). *Review and Implementation of Credit Risk Models of the Financial Sector Assessment Program.* IMF.

Basurto, M. A., & Padilla, P. (2006). *Portfolio Credit Risk and Macroeconomic Shocks: Applications to Stress Testing Under Data-Restricted Environments*. Working Paper, International Monetary Fund.

BCBS. (2005). *An Explanatory Note on the Basel II IRB Risk Weight Functions.* Basel: Bank for International Settlements.

BCBS. (2006a, June). *Basel II: Revised international capital framework.* Retrieved March 2009, from Bank for International Settlements: http://www.bis.org/publ/bcbsca.htm

BCBS. (2006b). *Studies on Credit Risk Concentration: An Overview of the Issues and a Synopsis of the Resutls from the Research Task Force Project.* Basel Committee on Banking Supervision Working Paper, Basel Committee on Banking Supervision.

BCBS. (2010). *Basel III: A Global Regulatory Framework for more Resilient Banks and Banking Systems*. Bank for International Settlements.

Bernanke, B., and Gertler, M. (1995). Inside the Black Box: The Credit Channel of Monetary Policy Transmission. *Journal of Economic Perspectives , 9* (4), 27-48.

Bernanke, B., Gertler, M., and Gilchrist, S. (1996). The Financial Accelerator and the Flight to Quality. *The Review of Economics and Statistics , 78* (1), 1-15.

Bluhm, C., Overbeck, L., and Wagner, C. (2003). *An Introduction to Credit Risk Modeling.* U.S.A.: Chapman and Hall.

Burgisser, P., Kurth, A., Wagner, A., and Wolf, M. (1999, July). Integrating Correlations. *Risk Magazine* , pp. 57-64.

Carey, M. (2000). *Dimensions of Credit Risk and their Relationship to Economic Capital Requirements.* Federal Reserve Board.

Cespedes, J. C. (2002). Credit Risk Modelling and Basel II. *Algo Research Quarterly , 5* (1), 57-66.

Chernih, A., Henrard, L., and Vanduffel, S. (2010). Reconciling Credit Correlations. *The Journal of Risk Model Validation , 4* (2), 47-64.

Credit Suisse. (1997). *CreditRisk$^+$: A Credit Risk Management Framework.* London: Credit Suisse Financial Products.

Crouhy, M., Galia, D., and Mark, R. (2000). A comparative analysis of current credit risk models. *Journal of Banking and Finance* , 59-117.

de Servigny, A., and Renault, O. (2002). *Default Correlation: Empirical Evidence.* Working Paper, Standard & Poor's.

Dietsch, M., and Petey, J. (2002). The credit risk in SME loans portfolios: Modeling issues, pricing, and capital requirements. *Journal of Banking and Finance* , 303-322.

Dietsch, M., and Petey, J. (2004). Should SME exposures be treated as retail or corporate exposures? A comparative analysis of default probabilities and asset correlations in French and German SMEs. *Journal of Banking and Finance* , 773-788.

Duellmann, K., and Scheule, H. (2003). *Determinants of the Asset Correlations of German Corporations and Implications for Regulatory Capital.* Working Paper, University of Regensburg.

Duellmann, K., & Masschelein, N. (2006). *Sector Concentration in Loan Portfolios and Economic Capital*. Discussion Paper, Deutsche Bundesbank.

Duellmann, K., Scheicher, M., and Schmieder, C. (2008). Asset Correlations and Credit Portfolio Risk: An Empirical Analysis. *The Journal of Credit Risk , 4* (2), 37-62.

Elizalde, A. (2005). *Credit Risk Models IV: Understanding and Pricing CDOs.* Retrieved from http://www.abelelizalde.com

Emmer, S., & Tasche, D. (2003). Calculating Credit Risk Capital Charges with the One-factor Model. *The Journal of Risk*, 7(2), 85-101.

Frey, R., and McNeil, A. (2003). Dependent Defaults in Models of Portfolio Credit Risk. *The Journal of Risk , 6* (1), 59-92.

Frye, J. (2008). Correlation and Asset Correlation in the Structural Portfolio Model. *Journal of Credit Risk* , 75-96.

Garcia, J., Alderweireld, T., and Leonard, L. (2006). On the Different Approaches for Capital Allocation. *Proceedings of the 4th Actuarial and Financial Mathematics Day.* Royal Flemish Academy of Belgium for Science and Arts.

Gordy, M. B. (2000). A Comparative Anatomy of Credit Risk Models. *Journal of Banking and Finance* (24), 119-149.

Gordy, M. B. (2002). Saddlepoint Approximation of CreditRisk$^+$. *Journal of Banking and Finance* , 1335-1353.

Gordy, M. B. (2003). A Risk-Factor Model Foundation for Ratings-Based Bank Capital Rules. *Journal of Financial Intermediation , 3* (12), 199-232.

Gordy, M. B., and Heitfield, E. (2002). *Estimating Default Correlations from Short Panels of Credit Rating Performance Data.* Working Paper, Federal Reserve Board.

Gordy, M. B., and Howells, B. (2006). Pro-cyclicality in Basel II: Can we treat the Disease without killing the Patient. *Journal of Financial Intermediation* , 395-417.

Gordy, M. B., and Lutkebohmert, E. (2007). Granularity Adjustment for Basel II. *Discussion Paper Series "Banking and Financial Studies" 01/2007* .

Gundlach, M., and Lehrbass, F. (2004). Introduction. In V. M. Gundlach, and F. B. Lehrbass, *CreditRisk$^+$ in the Banking Industry* (pp. 1-6). Springer-Verlag.

Gundlach, V. M. (2004). Basics of CreditRisk$^+$. In V. M. Gundlach, and F. B. Lehrbass, *CreditRisk$^+$ in the Banking Industry* (pp. 7-24). Springer-Verlag.

Gupton, G. M., Finger, C. C., and Bhatia, M. (1997). *Credit Metrics - Technical Document.* New York: Morgan Guaranty Trust Co.

Haaf, H., Reiss, O., and Schoenmakers, J. (2004). Numerically Stable Computation of CreditRisk$^+$. In V. M. Gundlach, and F. B. Lehrbass, *CreditRisk$^+$ in the Banking Industry* (pp. 69-78). Berlin Heidelberg: Springer-Verlag.

Hamerle, A., Liebig, T., and Roesch, D. (2003a). Benchmarking Asset Correlations. *Risk* (16), pp. 77-81.

Hamerle, A., Liebig, T., and Roesch, D. (2003b). *Credit Risk Factor Modeling and the Basel II IRB Approach.* Discussion Paper, Deutsches Bundesbank.

Heitfield, E., Burton, S., and Chomsisengphet, S. (2006). Systematic and Idiosyncratic Risk in Syndicated Loan Portfolios. *Journal of Credit Risk , 2* (3), 3-31.

Hennek, J., and Truck, S. (2006). Asset Correlations and Capital Requirements for SME in the Revised Basel II Framework. *Banks and Bank Systems , 1* (1), 75-92.

Hull, J. C. (2010). *Risk Management and Financial Institutions* (Second Edition ed.). Boston: Prentice Hall.

Industry Canada. (2001). *Supporting Small Business Innovation: Review of the Business Development Bank of Canada.*

Industry Canada. (2009, January). *Small Business Statistics.* Retrieved March 2009, from Industry Canada: http://www.ic.gc.ca/sbstatistics

Jacobson, T., Linde, J., and Roszbach, K. (2005). Credit Risk Versus Capital Requirements under Basel II: Are SME Loans and Retail Credit Really Different? *Journal of Financial Services Reasearch , 28* (1/2/3), 43-75.

Jakubik, P. (2006). *Does Credit Risk Vary with Economic Cycles? The Case of Finland.* Working Paper, Charles University in Prague, Institute of Economic Studies.

Jobst, N., and De Servigny, A. (2005). *An Empirical Analysis of Equity Default Swaps II: Multivariate Insights.* Working Paper, Standard & Poor's.

Jorion, P. (2002). *Value at Risk: The New Benchmark for Managing Financial Risk.* Singapore: McGraw-Hill Education (Asia).

Kluge, D., and Lehrbass, F. B. (2004). Some Remarks on the Analysis of Asset-Backed Securities. In M. Gundlach, and F. Lehrbass (Eds.), *CreditRisk$^+$ in the Banking Industry* (pp. 311-323).

Koyluoglu, U., and Hickman, A. (1998). *A Generalized Framework for Credit Risk Portfolio Models.* Working Paper, Oliver, Wyman & Company and CSFP Capital, Inc.

Lesko, M., Schlottmann, F., and Vorgrimler, S. (2004). Estimation of Sector Weights from Real-World Data. In M. Gundlach, and F. Lehrbass, *CreditRisk$^+$ in the Banking Industry* (pp. 249-258). Springer.

Lopez, J. A. (2002). *The Empirical Relationship between Average Asset Correlation, Firm Probability of Default and Asset Size.* Federal Reserve Bank of San Fransisco.

Lopez, J. A. (2004). The Empirical Relationship between Average Asset Correlation, Firm Probability of Default, and Asset Size. *Journal of Financial Intermediation* , 265-283.

Mausser, H., and Rosen, D. (2008). Economic Credit Capital Allocation and Risk Contributions. In J. Birge, and V. Linetsky (Eds.), *Handbooks in Operations Research and Management Science: Financial Engineering* (Vol. 15, pp. 681-726). Elsevier B.V.

Merino, S., and Nyfeler, M. (2004). Numerical Techniques for Determining Portfolio Credit Risk. In M. Gundlach, and F. Lehrbass (Eds.), *CreditRisk$^+$ in the Banking Industry* (pp. 279-309).

Merton, R. (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance , 29* (2), 449-470.

Nyfeler, M. A. (2000, November 23). Modelling Dependencies in Credit Risk Management. *Diploma Thesis* . Zurich, Switzerland: Swiss Federal Institute of Technology Zurich.

OSFI. (2011). *Capital Adequacy Requirements.* Office of the Superintendent of Financial Institutions.

OSFI. (2012). *Capital Adequacy Requirements*. Guideline, Office of the Superintendent of Financial Institutions Canada.

RMA. (2000). *Credit Risk Capital for Retail Credit Products: A Survey of Sound Practices.*

Roll, R. (1988). R2. *Journal of Finance , 43* (2), 541-566.

Standard & Poor's. (2011). *Default, Transition, and Recovery: 2010 Annual Global Corporate Default Study and Rating Transitions.*

Tarashev, N., and Zhu, H. (2007). Measuring Portfolio Credit Risk: Modelling versus Calibration Errors. *BIS Quarterly Review* , 83-96.

Treacy, W. F., and Carey, M. (2000). Credit risk rating systems at large US banks. *Journal of Banking and Finance* , 167-201.

Vasicek, O. (1987). *Probability of Loss on Loan Portfolio.* San Fransisco: KMV Corporation.

Vasicek, O. (1991). *Limiting Loan Loss Probability Distribution.* San Fransisco: KMV Corporation.

Vasicek, O. (2002, December). Loan Portfolio Value. *Risk Magazine* , pp. 160-162.

von Kalckreuth, U. (2001). *Monetary Transmission in Germany: New Perspectives on Financial Constraints and Investment Spending.* Working Paper, European Central Bank

Wahrenburg, M., and Niethen, S. (2000). Vergleichende Analyse alternativer Kreditrisikmodelle. *Kredit und Kapital , 33*, 235-257.

Wieczerkowski, C. (2003). Risk Factor Transformations Relating CreditRisk+ and CreditMetrics. In M. Gundlach, and F. Lehrbass (Eds.), *CreditRisk$^+$ in the Banking Industry* (pp. 45-68). Springer.

Zeng, B., and Zhang, J. (2001). *An Empirical Assessment of Asset Correlation Models.* Research Paper, Moody's KMV.

Appendices

Appendix A – Chapter 4: The Asset Value Model (AVM) Mathematical Framework

The introduction and implementation of the AVM in our Thesis is generally limited to a direct application of the mathematical structures contained within the model for the estimation of asset correlations and in the generation of a loss distribution for our portfolio of SME borrowers; see, for example, Subsection 4.1.1 and Appendix B. In these pages we hope to illuminate the broader conceptual framework that allows for the use of a parsimonious set of empirical data for the estimation of asset correlations and portfolio credit losses using the AVM framework.

Ultimately, our application of the AVM is centered on the use of a factor model to describe the relationship between any two borrowers in the *Financing Company* portfolio. To that end, we introduce two borrowers in our portfolio, firms *(i)* and *(j)*. For these two borrowers we make four principle assumptions.

Assumption 1: A borrower's capital structure is such that his assets are funded by a mixture of equity and debt (with a one year maturity). Should the value of a borrower's assets fall below the value of his debt at its due date, the borrower is in default.

Assumption 2: The value of a borrower's assets can be described by a geometric Brownian motion.

Assumption 3: The asset value dynamics of any two borrowers are correlated through time.

Assumption 4: The correlation structure between borrowers in our portfolio can described through the use of a single factor framework.

Assumption 4 is important in several ways. Firstly, taken with *Assumptions 2* and *3,* it allows us to reduce the relationship between two borrowers to one between two latent stochastic processes, easily described within a Gaussian environment. Secondly, it allows for the parsimonious modeling of relationships across a portfolio of multiple borrowers.

Below we will describe in greater detail the mathematical mechanisms which will allow us to move from a conceptual "balance sheet" framework, to a mathematical "latent variable" one. In doing so, we will eliminate the need for substantial external data in the AVM framework, and limit ourselves to the use of internally observed default rate time series for homogenous segments of borrowers in our credit portfolio. Here homogeneity will signify classes of borrowers with generally similar risk profiles, and identical probabilities of default as well as systematic risk factor weightings. In presenting the AVM mathematical framework below we will rely on conventions presented in Grasselli and Hurd (2010) and Vasicek (2002).

Specifically, we can write the value of a firm's assets as (A^i) and describe its evolution as:

$$dA_t^i = A_t^i \cdot \left[\mu^i dt + \sigma^i dW_t^i\right]. \tag{A.1}$$

For borrowers *(i)* and *(j)*, (W^i) and (W^j) are correlated Brownian motions with a constant correlation parameter of $(\rho^{i,j})$; see, for example, Grasselli and Hurd (2010, p. 93). Choosing a fixed time duration (Δt) – say equal to one year – we can interpret

Equation (A.1) as relating firm *(i)*'s asset return, over the chosen time period, to the standardized normal random variable $\left(X^i\right)$:

$$log\left(A_{\Delta t}^i/A_0^i\right) = \mu^i \Delta t - \frac{1}{2}\left(\sigma^i\right)^2 \Delta t + \sigma^i \sqrt{\Delta t} X^i. \tag{A.2}$$

For our two borrowers we are thus in a bivariate normal setting with a correlation of $\left(\rho^{i,j}\right)$ and standard normal marginal distributions. Under this framework, we are thus able to link, not only, the probability of default for borrower *(i)* to the statistical properties of $\left(X^i\right)$, but also obtain a value for the joint probability of default of borrowers *(i)* and *(j)* to both those borrowers' standard normal latent variables and the correlation parameter $\left(\rho^{i,j}\right)$.

Introducing our single factor framework, we can write, for firm *(i)*:

$$X^i = w_i Z + \sqrt{(1 - w_i^2)} \cdot e_i, \tag{A.3}$$

where *(Z)* is the systematic factor, (e_i) is the idiosyncratic factor and each is an independent standard normal variate. The [asset] correlation between our two borrowers can thus be written as the product of the weights of each borrower's latent variable on the systematic factor:

$$\rho^{i,j} = w_i \cdot w_j. \tag{A.4}$$

In many applications, especially as they relate to publicly traded borrowers, the estimation of the correlation parameter is undertaken through the study of equity returns data – where equity returns act as proxies for asset returns; see, for example Grasselli and Hurd (2010, p. 94). In Subsection 4.1.1 we present methodology introduced in Gordy (2000) for the estimation of asset correlations from historical default data within the AVM framework.

The work in Subsection 4.1.1 elaborates on Equation (A.3) to a setting in which borrowers in a portfolio are placed in homogenous segments for which default rates are observed over time. Using the mathematical framework presented here, in Subsection 4.1.1 and in Appendix B, conditional independence is established between borrowers in the portfolio. For every segment of borrowers in the portfolio, we are thus able to define a non-linear relationship between the segment unconditional probability of default, conditional probability of default variance, and systematic factor weighting. Using a non-parametric methods we use our empirical default rates to estimate the weightings and, by extension, the asset correlation.

Appendix B – Chapter 4: The Vasicek Asymptotic Single Factor Model

Vasicek (2002) defines a portfolio of (*n)* identical borrowers for whom default at the end of a given period (say 1 year) is determined by the comparative level of their assets to their liabilities. Those borrowers whose liabilities exceed their assets at the end of the given period are determined to be in default. For any borrower (i), the probability of default within this one year period is defined as (p). For any two borrowers in this portfolio, the asset value correlation is defined as (ρ).

For any borrower (i) the gross loss given default as a proportion of exposure, is defined as (L_i), such that:

$$L_i = \begin{Bmatrix} 1 & in\ case\ of\ default \\ 0 & otherwise \end{Bmatrix}. \tag{B.1}$$

The proportional portfolio gross loss is then defined as:

$$L = \frac{1}{n}\sum_{i=1}^{n} L_i. \tag{B.2}$$

For each borrower, the dynamics of the asset value are determined by a latent variable X_i, such that the set of latent variables $\{X_1, X_2, \ldots, X_n\}$ is jointly standard normal, and X_i is defined as a function of a systematic factor Y and a borrower-specific idiosyncratic factor Z_i, all of which are defined as mutually independent standard normal variables.

More specifically, we define (X_i) as:

$$X_i = \sqrt{\rho}Y + \sqrt{1-\rho}Z_i. \tag{B.3}$$

Characterising the systematic factor as being representative of the state of the economy, the borrower's dependence on the business cycle can be measured by the weighting $\sqrt{\rho}$ on *Y*. Given the unconditional probability of default (p), a borrower's status at the end of a given time horizon of one year is set to default if:

$$\sqrt{\rho}Y + \sqrt{1-\rho}Z_i < N^{-1}(p), \tag{B.4}$$

where $N^{-1}(\cdot)$ denotes the inverse standard normal cumulative distribution function. Conditional on a fixed realization of (Y) of the state of the economy the conditional probability of default and loss for a borrower (i) is given by:

$$p(Y) = Pr\left[Z_i < \frac{N^{-1}(p) - \sqrt{\rho}Y}{\sqrt{1-\rho}}\middle| Y\right] = N\left[\frac{N^{-1}(p) - \sqrt{\rho}Y}{\sqrt{1-\rho}}\right], \tag{B.5}$$

where the unconditional probability of default (p) is the average of the conditional probabilities over all realizations of (Y). Given (Y), the random variables (X_i) and the borrower defaults, are independent.

Recall, all obligors in the portfolio are identical. This uniformity of borrower characteristics applies not only to probabilities of default and correlations, but also to dollar exposures. Recall, also, (L) in Equation (B.2) giving the portfolio gross proportional loss. Given as a proportion of the total number of borrowers in the portfolio, Equation (B.2) can be considered to give the portfolio default rate at the end of our one year period Elizalde (2005, p. 9). Given the independence of defaults conditional on the realization of (Y) Vasicek (2002) uses the law of large numbers to show that (L) converges to the individual borrower uniform conditional probability of default $p(Y)$. Therefore, the portfolio loss distribution can be defined according to the cumulative distribution function:

$$F(x;p,\rho) = Pr[L \leq x] = Pr[p(Y) \leq x] \qquad \text{(B.6)}$$

$$= Pr\left[N\left[\frac{N^{-1}(p) - \sqrt{\rho}Y}{\sqrt{1-\rho}}\right] \leq x\right]$$

$$= Pr\left[Y \geq \frac{N^{-1}(p) - N^{-1}(x)\sqrt{1-\rho}}{\sqrt{\rho}}\right]$$

$$= 1 - N\left[\frac{N^{-1}(p) - N^{-1}(x)\sqrt{1-\rho}}{\sqrt{\rho}}\right]$$

$$= N\left[\frac{N^{-1}(x)\sqrt{1-\rho} - N^{-1}(p)}{\sqrt{\rho}}\right],$$

with the inverse distribution, or the (α)-percentile value of (L) given by:

$$F(\alpha;1-p,1-\rho) = N\left[\frac{N^{-1}(\alpha)\sqrt{\rho}-N^{-1}(1-p)}{\sqrt{1-\rho}}\right] = N\left[\frac{N^{-1}(\alpha)\sqrt{\rho}+N^{-1}(p)}{\sqrt{1-\rho}}\right]. \qquad \text{(B.7)}$$

The mean and variance of the distribution are respectively given by:

$$E[L] = p \qquad and \qquad Var[L] = BIVNOR(N^{-1}(p), N^{-1}(p), \rho) - p^2. \qquad (B.8)$$

To recap, Vasicek (2002) assumes a portfolio of borrowers with uniform exposures, probabilities of default and asset correlations. For this portfolio, unconditional probabilities of default are given and, without loss of generality, it can also be assumed that recovery (or net loss given default) rates are uniform across borrowers and deterministic Elizalde (2005). Finally, it is assumed that the number of borrowers in the portfolio is sufficiently large, $n \rightarrow \infty$.

Given these assumptions, and building on Vasicek (1987) and Vasicek (1991), Vasicek (2002) provides an asymptotic single factor model for the estimation of portfolio credit losses. This approximating asymptotic portfolio loss distribution is shown to hold even if borrower exposures are not uniform but with a large number of borrowers not one or a few of which are significantly larger than the rest Vasicek (2002).

Appendix C – Chapter 5: CreditRisk$^+$ Default Correlations and Capital Allocation

As a final step, and having generated a portfolio loss distribution, CreditRisk$^+$ provides the framework within which pairwise correlations can be calculated and Economic Capital allocated; see Credit Suisse (1997, pp. 52-57) and Gundlach (2004, p. 13).

In order to calculate pairwise correlation between two obligors, *"h"* and *"i"*, in the same sector *"k"*, the following relation is used:

$$\rho_{h,i} = \frac{\sqrt{(PD_h PD_i)}}{\sqrt{(1 - PD_h)(1 - PD_i)}} w_{h,k} w_{i,k} \left(\frac{\sigma_k}{\mu_k}\right)^2 . \qquad (\mathrm{C}.1)$$

Further details can be found at Credit Suisse (1997, pp. 56-57) and Gundlach (2004, p. 13). In order to determine capital allocation, we first define the portfolio loss distribution variance as derived in Credit Suisse (1997, pp. 54-55):

$$\sigma^2 = \sum_{k=1}^{K} {EL_k}^2 \left(\frac{\sigma_k}{\mu_k}\right)^2 + \sum_{i=1}^{N} EL_i E_i . \qquad (\mathrm{C}.2)$$

Next, we determine each obligor *i*'s capital charge by first calculating the obligor's "Risk Contribution" (RC_i), defined as the contribution of the obligor to the portfolio standard deviation:

$$RC_i = \frac{E_i PD_i}{\sigma}\left(E_i + w_{i,k} EL_k \left(\frac{\sigma_k}{\mu_k}\right)^2\right), \tag{C.3}$$

such that,

$$\sum_{i=1}^{N} RC_i = \sigma. \tag{C.4}$$

To transform RC_i into the "Risk Capital Contribution" (RCK_i), we first define a multiplier to our given loss distribution tail percentile, such that:

$$\xi = \frac{VaR^Q - EL}{\sigma} = \frac{EC^Q}{\sigma}, \tag{C.5}$$

where *Q* denotes the quantile at which the VaR is calculated. Then, RCK_i is defined as:

$$RCK_i = \xi RC_i, \tag{C.6}$$

such that,

$$\sum_{i=1}^{I} RCK_i = EC^Q. \tag{C.7}$$

For further details refer to Credit Suisse (1997, pp. 52-53).

For the *Multiple Correlated Sectors* implementation, Burgisser, Kurth, Wagner, and Wolf (1999, p. 6) show that, for a unitary weight setting, Equaiton (C.2) can be replaced by Equation (5.24) such that the risk contribution for an obligor to the portfolio standard deviation can be written as follows:

$$RC_i^{MC} = \frac{E_i PD_i}{\sigma^{\,(MC)}}\left(E_i + EL_k\left(\frac{\sigma_k}{\mu_k}\right)^2 + \sum_{l:k\neq l} Corr(X_k, X) EL_l \frac{\sigma_k}{\mu_k}\frac{\sigma_l}{\mu_l}\right). \qquad (C.8)$$

For further discussion of the *Multiple Correlated Sectors* implementation, see Subsection 5.1.3.

Figures

Figure 1.1

Organizational Chart – Issues and Results

Chapter 2 – Financing Company Small &Medium Enterprise Loans Portfolio

- Portfolio Segmentation
- SME Portfolio Characteristics
- Input Structure for Portfolio Credit Risk Estimation

Chapter 3 – Basel II & SME Credit Risk

- Credit Risk Assumptions
  - Increasing Correlation with Size
  - Decreasing Correlation with PD
- Input Data Requirements
  - PD by Risk Rating
- Partial Implementation Analysis
- Corporate vs. Retail asset class
- Infinite Granularity Assumption
- Single Risk Factor Assumption
- U-shaped pattern in EC allocations across SME Size segments

Chapter 4 – Asset Value Model Framework

- Robust Data Segmentation
- PD & PD standard deviation by Risk and Size Group
- Empirical normalized PD standard deviation ratios
- Low Asset Correlation Values
- No Evidence of Relationships:
  - ❖ Positive Size-Correlation
  - ❖ Negative PD-Correlation
- Simulation vs. Analytical Model
- Boost to Estimated Correlations
- Granularity Effect observed with Low Correlation Values
- EC Allocation increasing with Size & PD for SME Segments

Chapter 5 – CreditRisk$^+$ Framework

- Single vs. Multiple Risk Factors
- Internally Calibrated Inter-Sector Correlations
- Significant overestimation of overall Economic Capital charges under Single Sector assumption
- Economic Capital Allocation increasing with Size & PD for SME Segments

Chapter 6 – Comparative Analysis of Basel II & SME Models of Portfolio Credit Risk

- Internally calibrated models reveal misallocation of EC across SME segments under Basel II
- Single Sector AVM & CreditRisk$^+$ consistent SME calibration under 0.5 setting for normalized standard deviation
- Unitary weight setting for CreditRisk$^+$ reveals strict restrictions on correlations, positive PD-correlation relation

Figure 1.1 present an organizational flowchart of the structure of this Thesis highlighting the main issues and results tackled in each Chapter.

Figure 2.1A

Borrower Distribution across Risk Ratings and Size Buckets

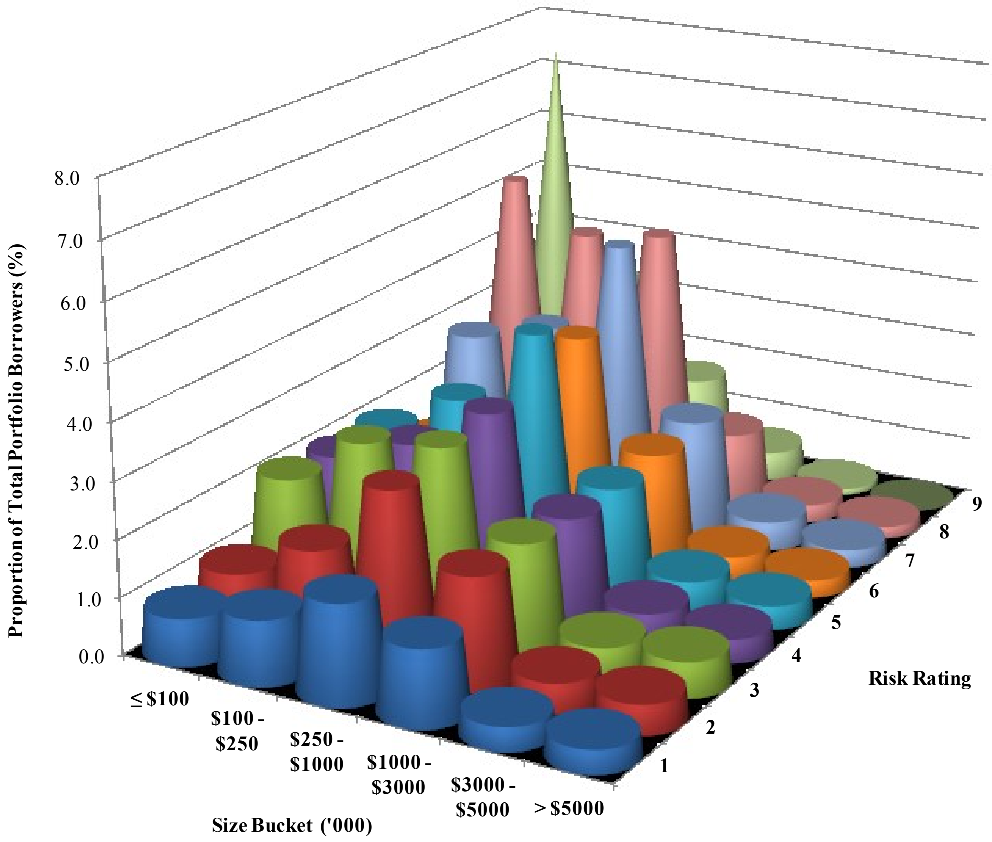


Figure 2.1A gives the distribution of borrowers across Risk Ratings and Size Buckets for the *Financing Company* portfolio as of March 2009. On the z-axis is the proportion of borrowers in the overall portfolio in a given segment, while the x-axis provides the Size Buckets, ranging from ≤$100,000 to >$5,000,000. On the y-axis are the Risk Ratings ranging from 1 (least risky) to 9 (most risky).

Figure 2.1B

$OS Distribution across Risk Ratings and Size Buckets

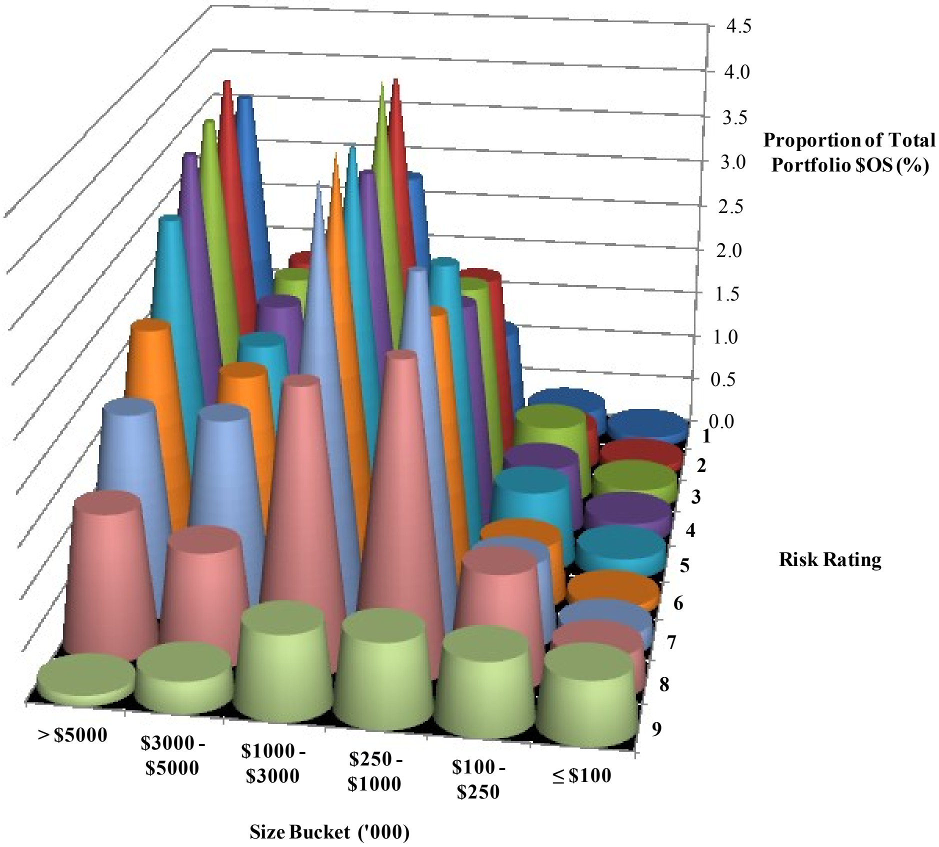


Figure 2.1B shows the distribution of $OS across Risk Ratings and Size Buckets for the *Financing Company* portfolio as of March 2009. On the z-axis is the proportion of overall portfolio $OS in each segment, while the x-axis provides the Size Buckets, ranging from ≤$100,000 to >$5,000,000. On the y-axis are the Risk Ratings ranging from 1 (least risky) to 9 (most risky).

Figure 2.2A

Borrower Distribution across Industries and Size Buckets

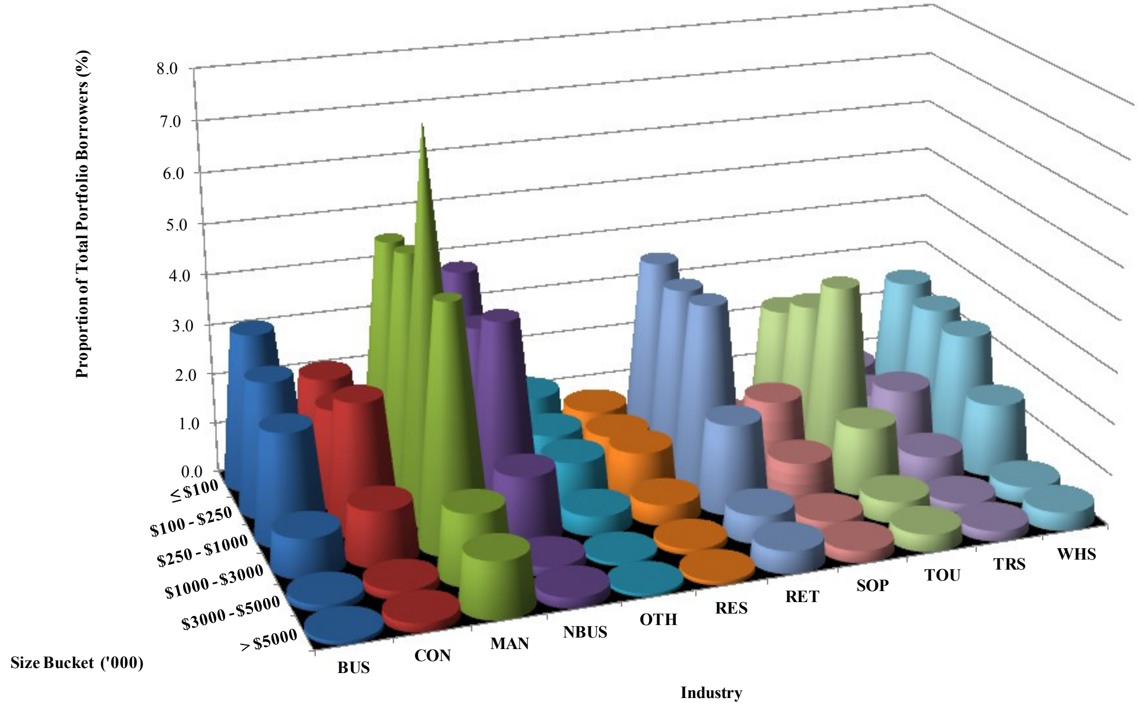


Figure 2.2A shows the distribution of borrowers across Industries and Size Buckets for the *Financing Company* portfolio as of March 2009. On the y-axis is the number of borrowers in each segment, while the z-axis provides the Size Buckets, ranging from ≤$100,000 to >$5,000,000. On the x-axis are the Industries starting, from left to right, with Business Services (BUS); Construction (CON); Manufacturing (MAN); Non-Business Services (NBUS); Resources (RES); Retail (RET); Supplier or Premises (SOP); Tourism (TOU); Transportation and Storage (TRS); Wholesale (WHS); Other (OTH).

Figure 2.2B

$OS Distribution across Industries and Size Buckets

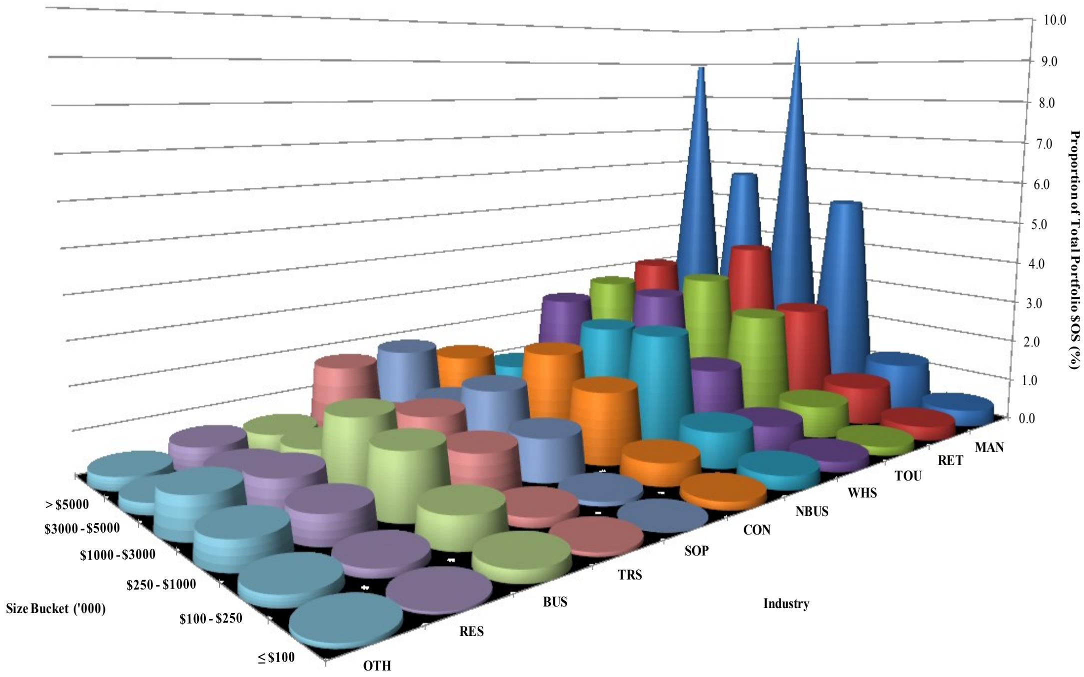


Figure 2.2B gives the distribution of borrower $OS across Industries and Risk Ratings for the *Financing Company* portfolio as of March 2009. On the y-axis is the proportion of overall portfolio $OS in each segment, while the z-axis provides the Size Buckets, ranging from ≤$100,000 to >$5,000,000. On the x-axis are the Industries. Note that for illustrative purpose for this figure, the alphabetical ordering of the industries presented throughout has been altered. The industries, in alphabetical order, are as follows: Business Services (BUS); Construction (CON); Manufacturing (MAN); Non-Business Services (NBUS); Resources (RES); Retail (RET); Supplier or Premises (SOP); Tourism (TOU); Transportation and Storage (TRS); Wholesale (WHS); Other (OTH).

Figure 2.3A

Borrower Distribution across Industries and Risk Ratings

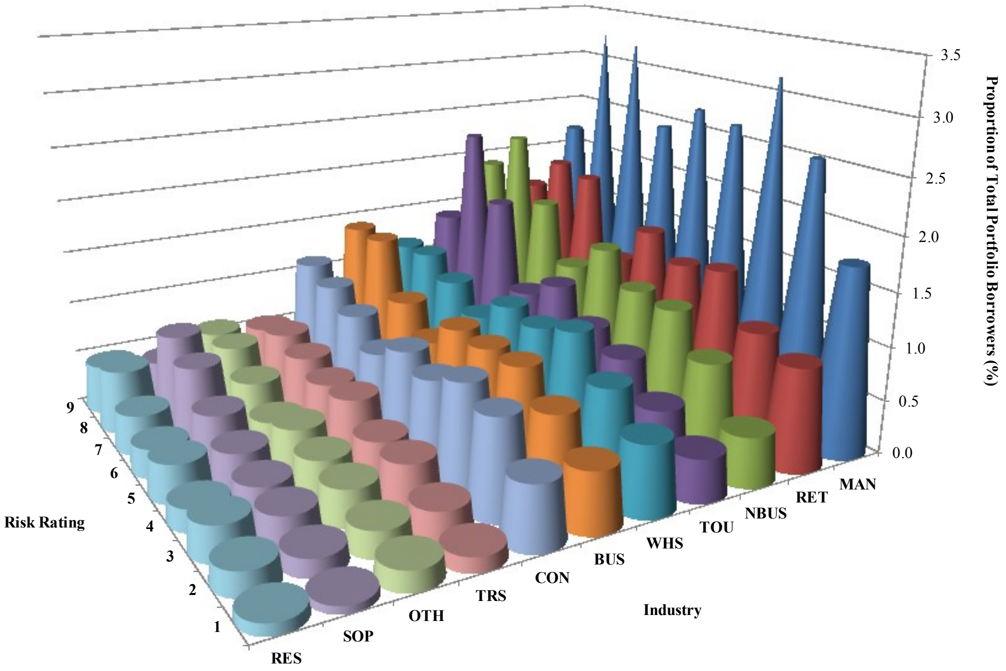


Figure 2.3A shows the distribution of borrowers across Industries and Risk Ratings for the *Financing Company* portfolio as of March 2009. On the y-axis is the proportion of the overall portfolio borrowers in each segment, while the z-axis provides the Risk Ratings, ranging from 1 (least risky) to 9 (most risky). On the x-axis are the Industries. Note that for illustrative purpose for this figure, the alphabetical ordering of the industries presented throughout has been altered. The industries, in alphabetical order, are as follows: Business Services (BUS); Construction (CON); Manufacturing (MAN); Non-Business Services (NBUS); Other (OTH); Resources (RES); Retail (RET); Supplier or Premises (SOP); Tourism (TOU); Transportation and Storage (TRS); Wholesale (WHS).

Figure 2.3B

$OS Distribution across Industries and Risk Ratings

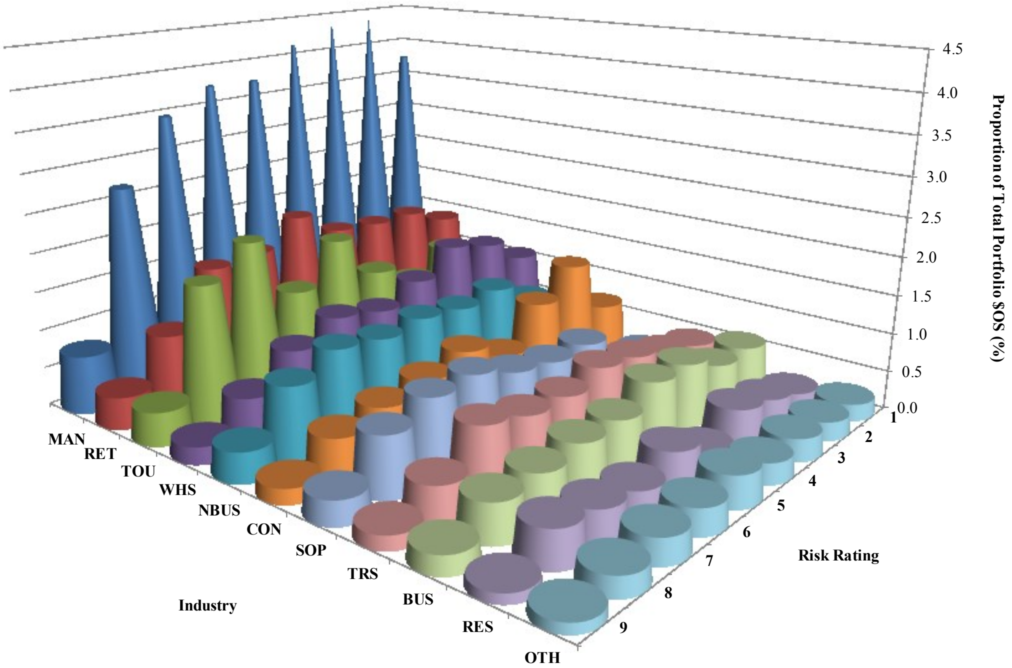


Figure 2.3B gives the distribution of borrower $OS across Industries and Risk Ratings for the *Financing Company* portfolio as of March 2009. On the y-axis is the proportion of overall portfolio $OS in each segment, while the z-axis provides the Risk Ratings, ranging from 1 (least risky) to 9 (most risky). On the x-axis are the Industries. Note that for illustrative purpose for this figure, the alphabetical ordering of the industries presented throughout has been altered. The industries, in alphabetical order, are as follows: Business Services (BUS); Construction (CON); Manufacturing (MAN); Non-Business Services (NBUS); Other (OTH); Resources (RES); Retail (RET); Supplier or Premises (SOP); Tourism (TOU); Transportation and Storage (TRS); Wholesale (WHS).

Figure 2.4A

Borrower Distribution across Geographic Region and Industry

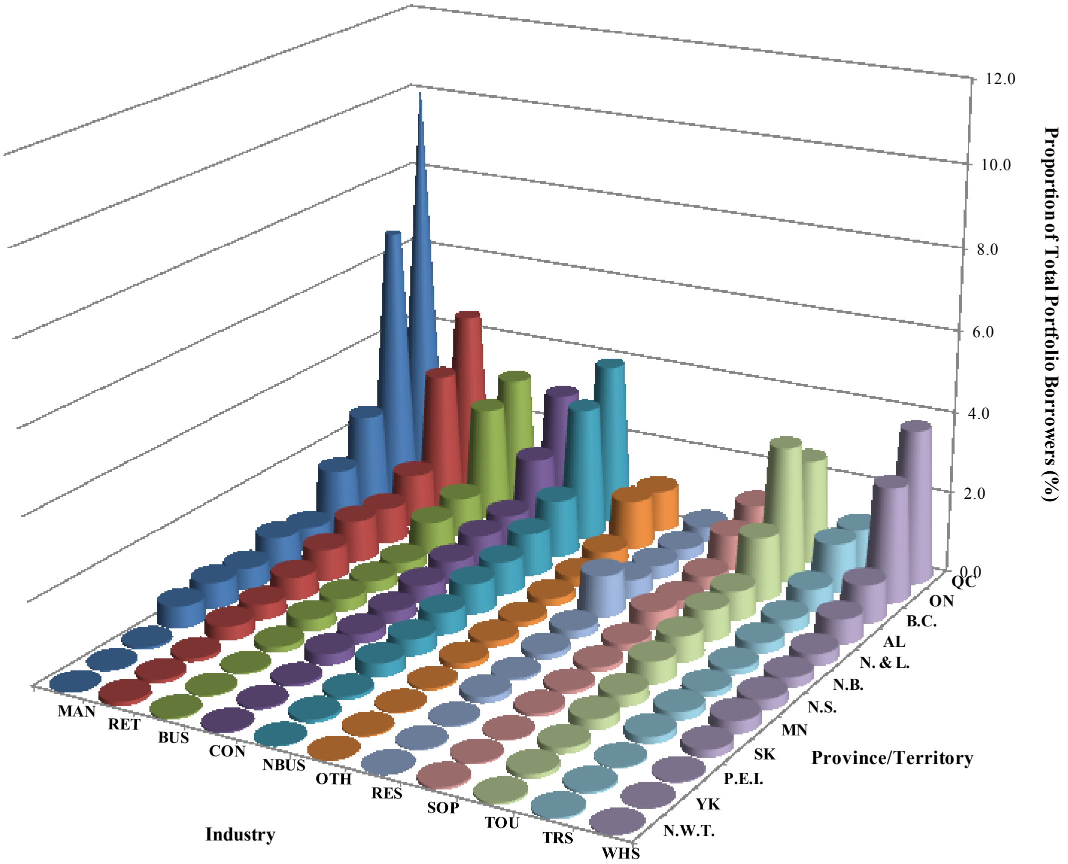


Figure 2.4A shows the distribution of borrowers across Geographic Region and Industry for the *Financing Company* portfolio as of March 2009. On the y-axis is the proportion of overall portfolio borrowers in each segment, while the z-axis provides the Geographic Regions. Note that for illustrative purposes for this figure, the alphabetical ordering of the provinces/territories presented throughout has been altered. The provinces/territories, in alphabetical order, are as follows: Alberta (AL); British Columbia (B.C.); Manitoba (MN); New Brunswick (N.B.); Newfoundland and Labrador (N. & L.); Northwest Territories and Nunavut (N.W.T.); Nova Scotia (N.S.); Ontario (ON); Prince Edward Island (P.E.I.); Quebec (QC); Saskatchewan (SK); the Yukon (YK). On the x-axis are the Industries. Note that for illustrative purpose for this figure, the alphabetical ordering of the industries presented throughout has been altered. The industries, in alphabetical order, are as follows: Business Services (BUS); Construction (CON); Manufacturing (MAN); Non-Business Services (NBUS); Other (OTH); Resources (RES); Retail (RET); Supplier of Premises (SOP); Tourism (TOU); Transportation and Storage (TRS); Wholesale (WHS).

Figure 2.4B

$OS Distribution across Geographic Region and Industry

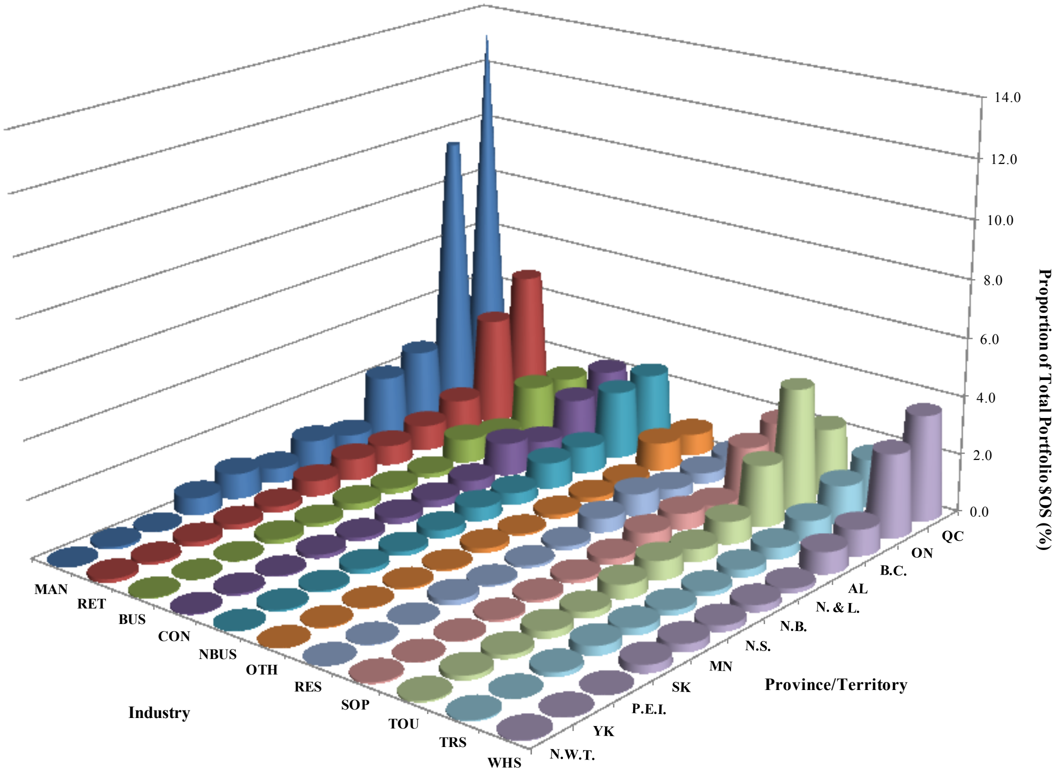


Figure 2.4B gives the distribution of borrower $OS across Geographic Region and Industry for the *Financing Company* portfolio as of March 2009. On the y-axis is the proportion of overall portfolio $OS in each segment, while the z-axis provides the provinces/territories. Note that for illustrative purposes for this figure, the alphabetical ordering of the provinces/territories presented throughout has been altered. The provinces/territories, in alphabetical order, are as follows: Alberta (AL); British Columbia (B.C.); Manitoba (MN); New Brunswick (N.B.); Newfoundland & Labrador (N. & L.); Northwest Territories and Nunavut (N.W.T.); Nova Scotia (N.S.); Ontario (ON); Prince Edward Island (P.E.I.); Quebec (QC); Saskatchewan (SK); the Yukon (YK). On the x-axis are the Industries. Note that for illustrative purpose for this figure, the alphabetical ordering of the industries presented throughout has been altered. The industries, in alphabetical order, are as follows: Business Services (BUS); Construction (CON); Manufacturing (MAN); Non-Business Services (NBUS); Other (OTH); Resources (RES); Retail (RET); Supplier or Premises (SOP); Tourism (TOU); Transportation and Storage (TRS); Wholesale (WHS).

Figure 2.5A

Borrower Distribution by Geographical Region and Size Bucket

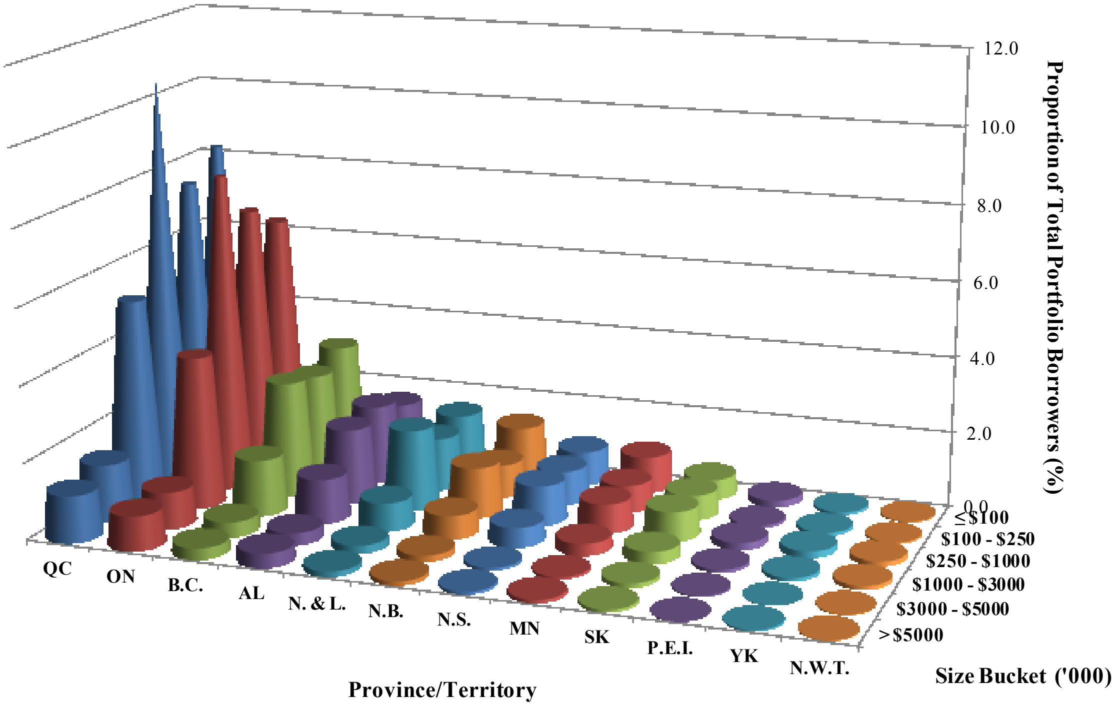


Figure 2.5A shows the distribution of borrowers across Geographic Region and Size Bucket for the *Financing Company* portfolio as of March 2009. On the y-axis is the proportion of overall portfolio borrowers in each segment, while the x-axis provides the Geographic Regions. Note that for illustrative purposes for this figure, the alphabetical ordering of the provinces/territories presented throughout has been altered. The provinces/territories, in alphabetical order, are as follows: Alberta (AL); British Columbia (B.C.); Manitoba (MN); New Brunswick (N.B.); Newfoundland & Labrador (N. & L.); Northwest Territories and Nunavut (N.W.T.); Nova Scotia (N.S.); Ontario (ON); Prince Edward Island (P.E.I.); Quebec (QC); Saskatchewan (SK); the Yukon (YK). On the z-axis are the Size Buckets, ranging from *≤$100,000 to >$5,000,000*.

Figure 2.5B

Distribution of $OS across Geographical Regions and Size Buckets

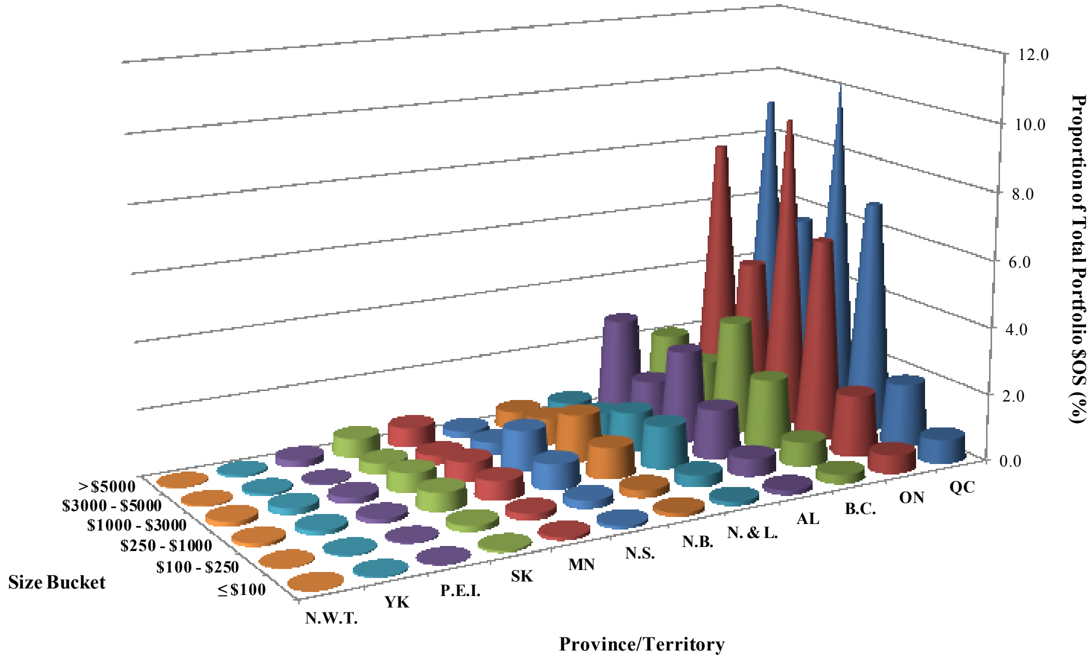


Figure 2.5B gives the distribution of borrower $OS across Geographic Region and Size Bucket for the *Financing Company* portfolio as of March 2009. On the y-axis is the proportion of overall portfolio $OS in each segment, while the x-axis provides the provinces/territories. Note that for illustrative purposes for this figure, the alphabetical ordering of the provinces/territories presented throughout has been altered. The provinces/territories, in alphabetical order, are as follows: Alberta (AL); British Columbia (B.C.); Manitoba (MN); New Brunswick (N.B.); Newfoundland & Labrador (N. & L.); Northwest Territories and Nunavut (N.W.T.); Nova Scotia (N.S.); Ontario (ON); Prince Edward Island (P.E.I.); Quebec (QC); Saskatchewan (SK); the Yukon (YK).). On the z-axis are the Size Buckets, ranging from *≤$100,000* to *>$5,000,000*. See Table 2.6B for more details.

Figure 2.6

Borrower and $OS Concentrations across Industries over Time

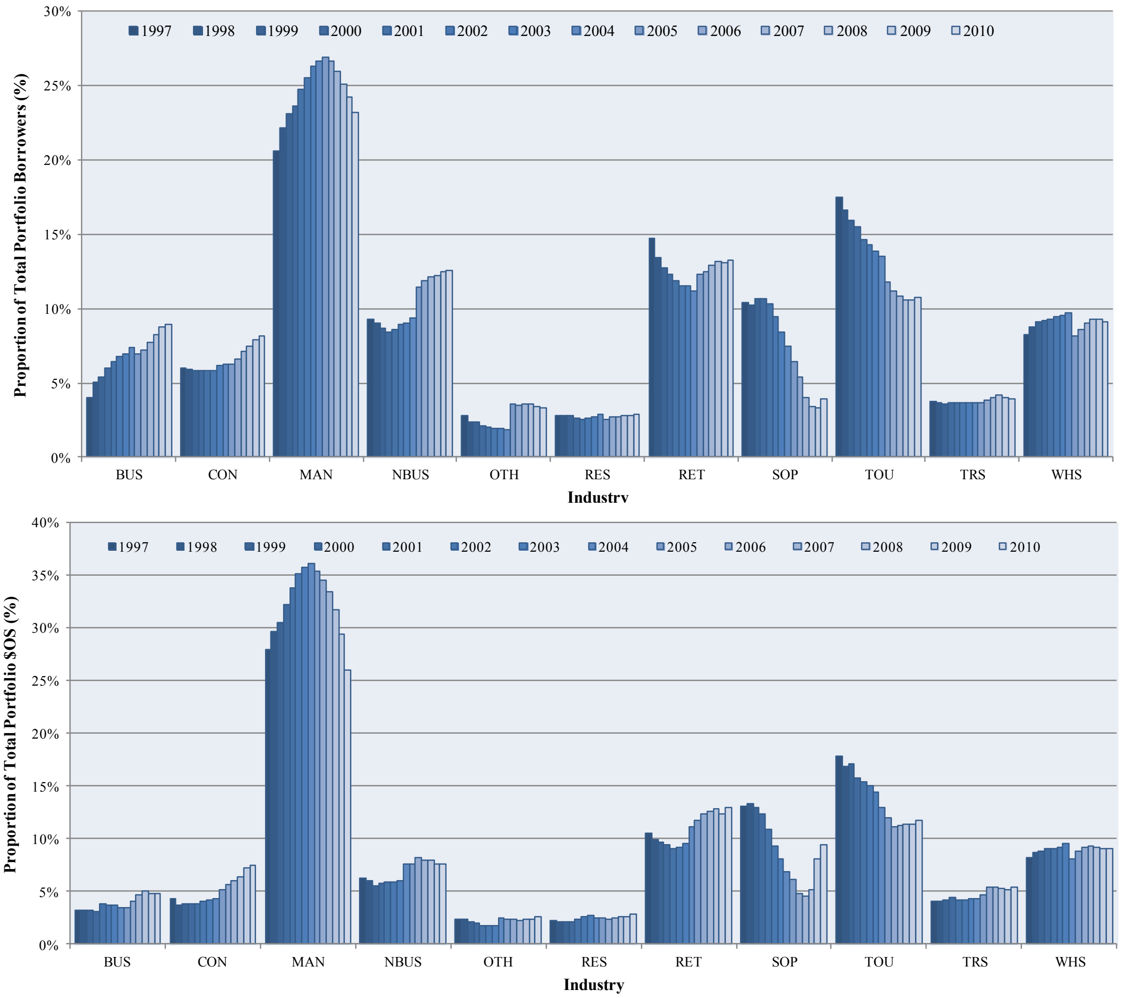
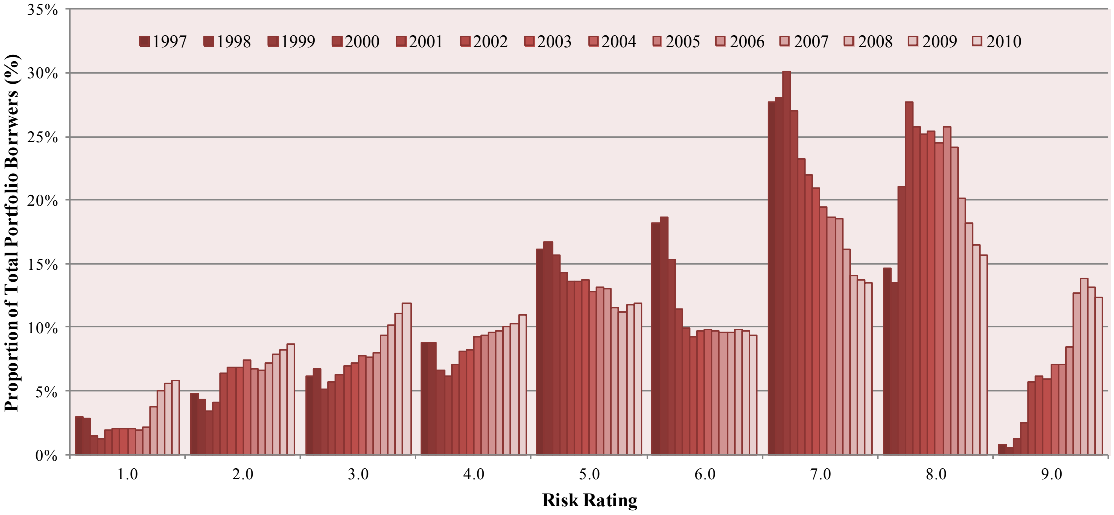


Figure 2.6 displays the concentration of borrowers (top) and $OS (bottom) in a given industry at yearly intervals starting in December 1997 and ending in December 2010. The industries, in alphabetical order, are as follows: Business Services (BUS); Construction (CON); Manufacturing (MAN); Non-Business Services (NBUS); Other (OTH); Resources (RES); Retail (RET); Supplier or Premises (SOP); Tourism (TOU); Transportation and Storage (TRS); Wholesale (WHS). Figure 2.6 indicates MAN to be the predominant industry in the portfolio. Significant increases in borrower concentration, from the start of the evaluation period to the end of it, can be observed for the BUS and NBUS industries, while significant decreases in portfolio concentration are observed for the SOP and TOU industries. Decreases in $OS concentration, from the start of the evaluation period to the end of it, can be observed for the TOU and SOP industries – although for the latter a resurgence is noted in 2009 and 2010. A moderate increase in $OS concentration can be observed for the CON industry. See Table 2.7 for more details.

Figure 2.7

Borrower and $OS Concentrations across Risk Ratings over Time



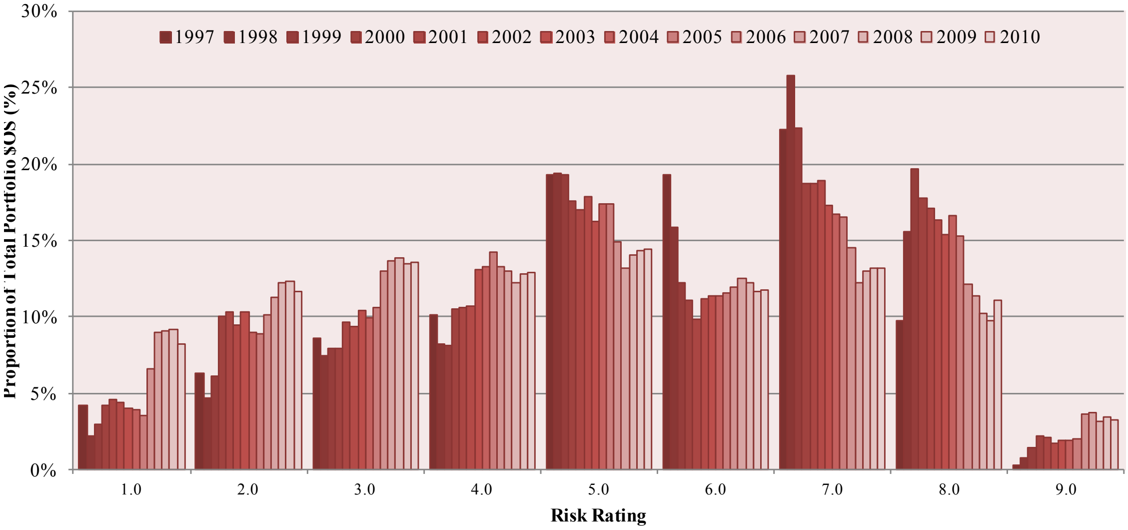


Figure 2.7 displays the concentration of borrowers (top) and $OS (bottom) in a given Risk Rating at yearly intervals starting in December 1997 and ending in December 2010, with Risk Ratings ranging from 1 (least risky) to 9 (most risky). Figure 2.7 indicates a significant but gradual increase in the borrower concentration in the 9 RR, from 1% in 1997 to 12% in 2010, reaching a peak of 14% in 2008. In addition, from 1997 to 2010 we observe a migration in the portfolio towards higher concentrations of borrowers in the 1 to 4 RRs, accompanied by a significant decrease in the concentration of borrowers in the 6 and 7 RRs (along with a moderate decrease in the 5 RR). Comparing 2009 and 2010 to 1997 and 1998, we observe a more uniform distribution of $OS, with the 9 RR accounting for the smallest percentage of $OS in both periods. See Table 2.8 for more details.

Figure 2.8

Borrower and $OS Concentration across Size Buckets over Time

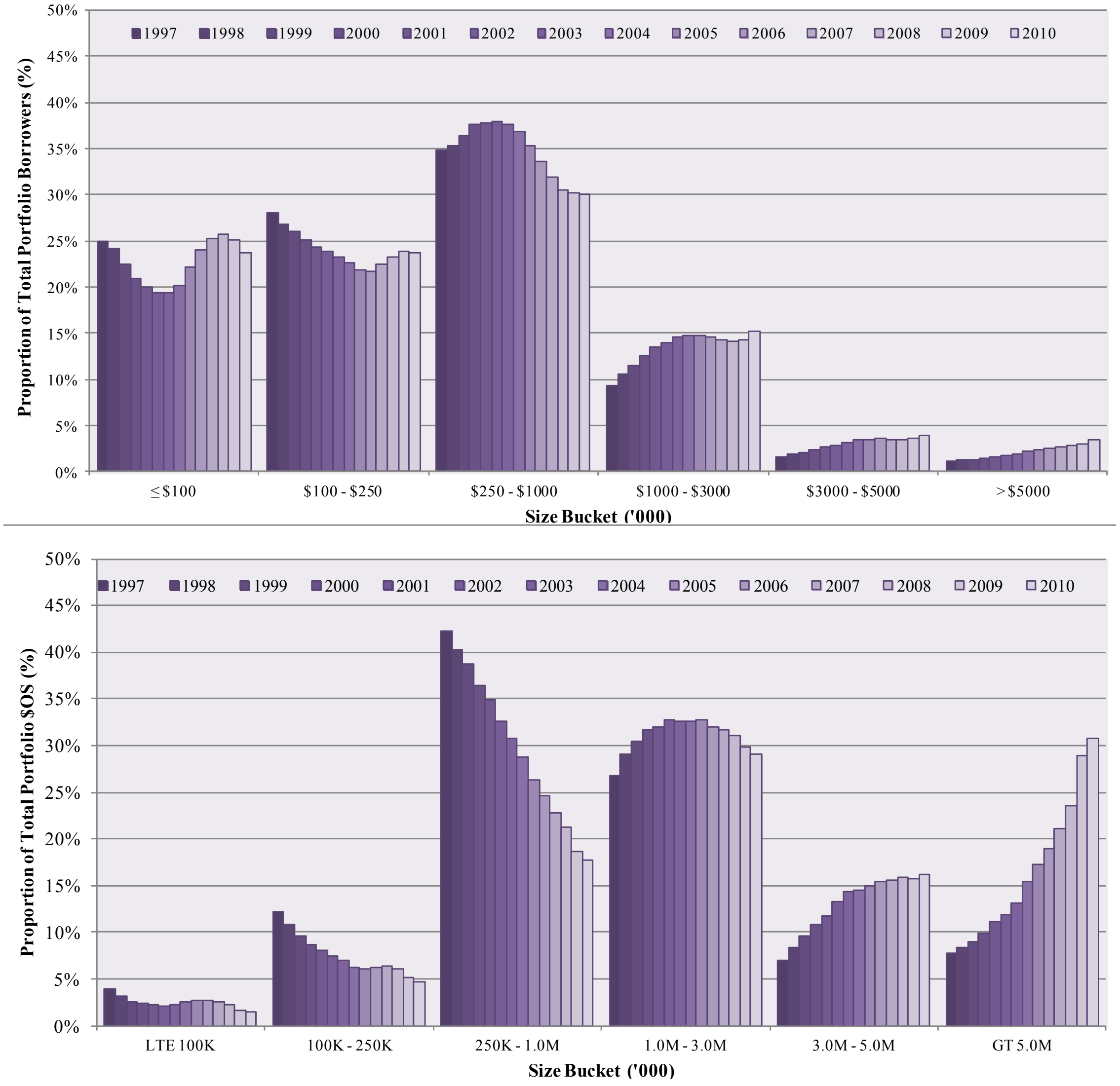


Figure 2.8 displays the concentration of borrowers (top) and $OS (bottom) in a given Size Bucket at yearly intervals starting in December 1997 and ending in December 2010. Size Buckets range from ≤$100,000 to >$5,000,000 and are based on the total commitment to a borrower (including commitment to other "related" borrowers under the same ownership) at last authorization. Results from Table 2.9 indicate a fairly stable distribution of borrowers across the evaluation period, with a slight increase in the concentration of borrowers in Size Buckets of *$3,000,000 or more* going from 1997 to 2010. In terms of $OS concentration, we observe a significant increase (decrease) in the concentration of $OS in Size Buckets of *$3,000,000 or more* (*$250,000 - $1,000,000*) over the evaluation period. See Table 2.9 for more details.

Figure 2.9

Annual Default Rates by Industry

Figure 2.9 provides the annual default rate by Industry for the period starting January 1997 and ending December 2010. The industries, in alphabetical order, are as follows: Business Services (BUS); Construction (CON); Manufacturing (MAN); Non-Business Services (NBUS); Other (OTH); Resources (RES); Retail (RET); Supplier or Premises (SOP); Tourism (TOU); Transportation and Storage (TRS); Wholesale (WHS). To calculate annual default rates, the number of defaulted borrowers over a given calendar year is divided by the number of borrowers at the beginning of that calendar year, so that for the 2008 calendar year, defaults from January 2008 to December 2008 are summed and divided by the number of healthy borrowers as of December 31, 2007. See Table 2.11 and Table 2.14 for more details.

Figure 2.10

Annual Default Rates by Risk Rating

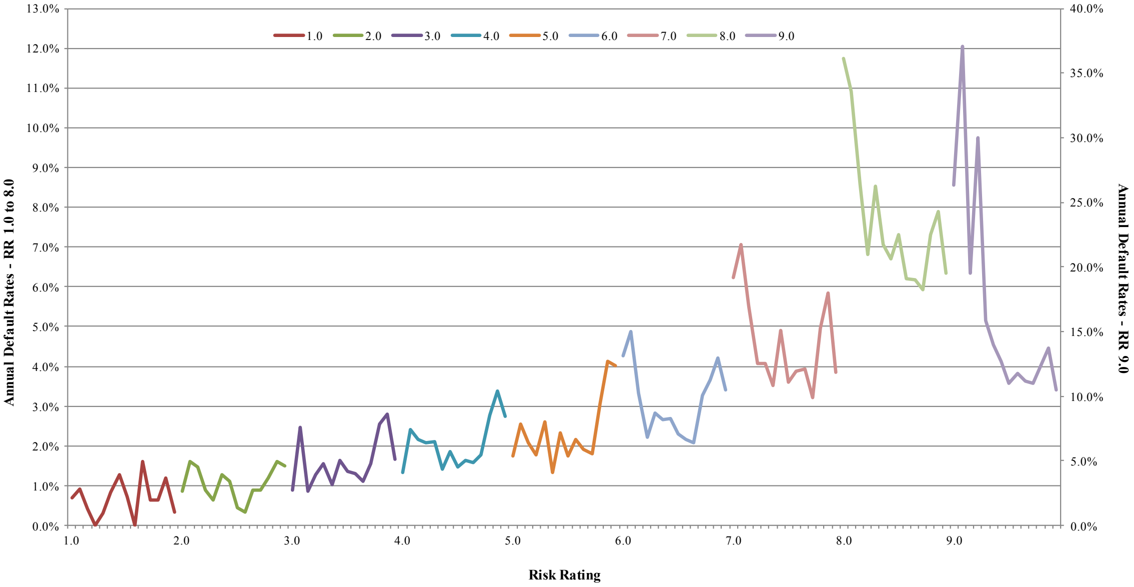


Figure 2.10 shows the annual default rate by Risk Rating for the period starting January 1997 and ending December 2010. Risk Ratings range from 1 (least risky) to 9 (most risky). To calculate annual default rates the number of defaulted borrowers over a given calendar year is divided by the number of borrowers at the beginning of that calendar year, so that for the 2008 calendar year, defaults from January 2008 to December 2008 are summed and divided by the number of healthy borrowers as of December 31, 2007. See Table 2.10 and Table 2.13 for more details.

Figure 2.11

Annual Default Rates by Size Bucket

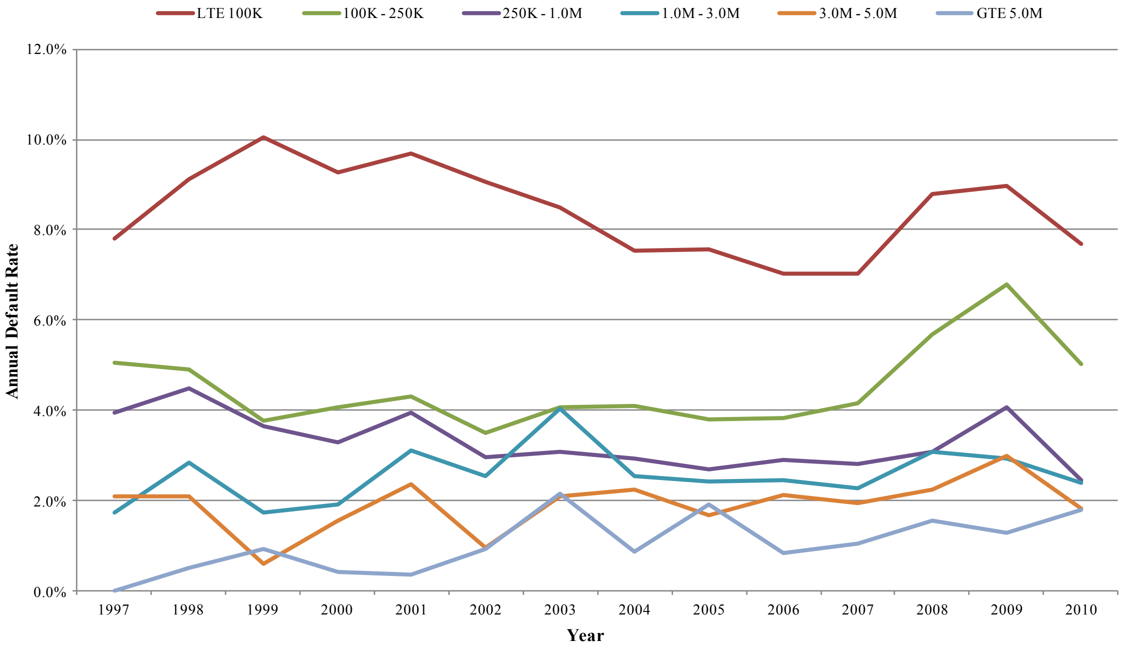


Figure 2.11 provides the annual default rate by Size Bucket for the period starting January 1997 and ending December 2010. Size Buckets, denoted in the graph in (‘000), range from ≤*$100,000* to >*$5,000,000* and are based on the total commitment to a borrower (including commitment to other “related” borrowers under the same ownership) at last authorization. To calculate annual default rates the number of defaulted borrowers over a given calendar year is divided by the number of borrowers at the beginning of that calendar year, so that for the 2008 calendar year, defaults from January 2008 to December 2008 are summed and divided by the number of healthy borrowers as of December 31, 2007. See Table 2.12 for more details.

Figure 3.1

IRB Asset Correlations for Corporate and Retail-Other Asset Classes

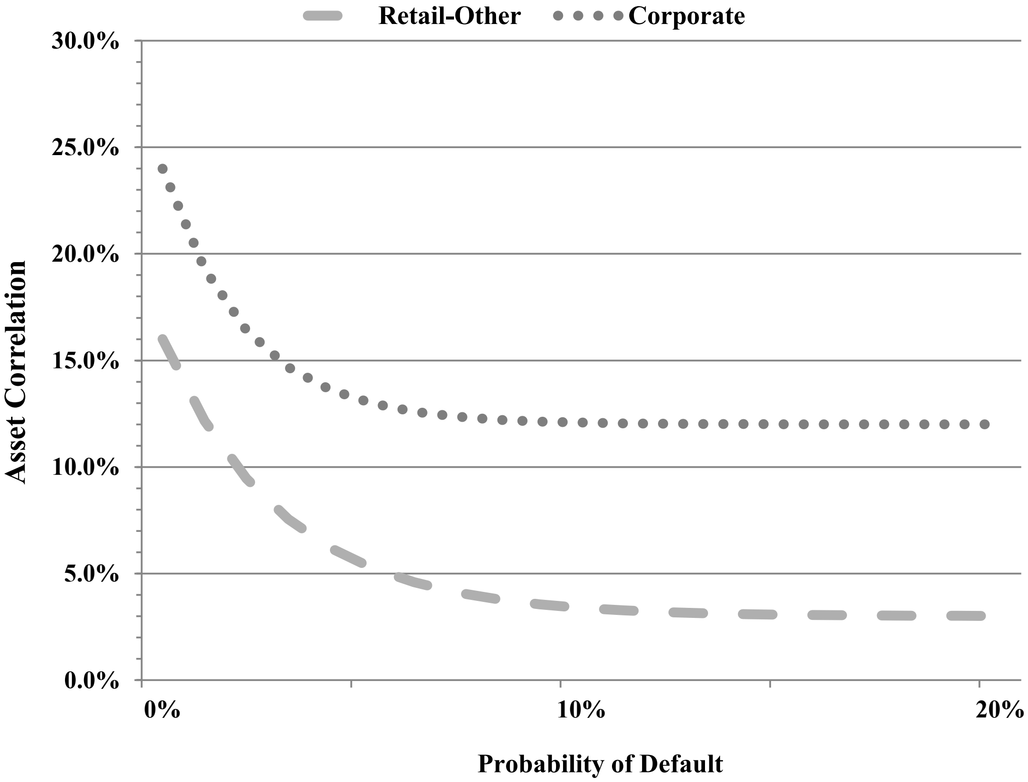


Figure 3.1 plots the Basel II IRB approach asset correlations under the Corporate and Retail-Other asset classes across various values of the Probability of Default and excluding any Size adjustments. See Subsection 3.1.1 for more details.

Figure 3.2

Financing Company PD by Risk Rating and Size Bucket

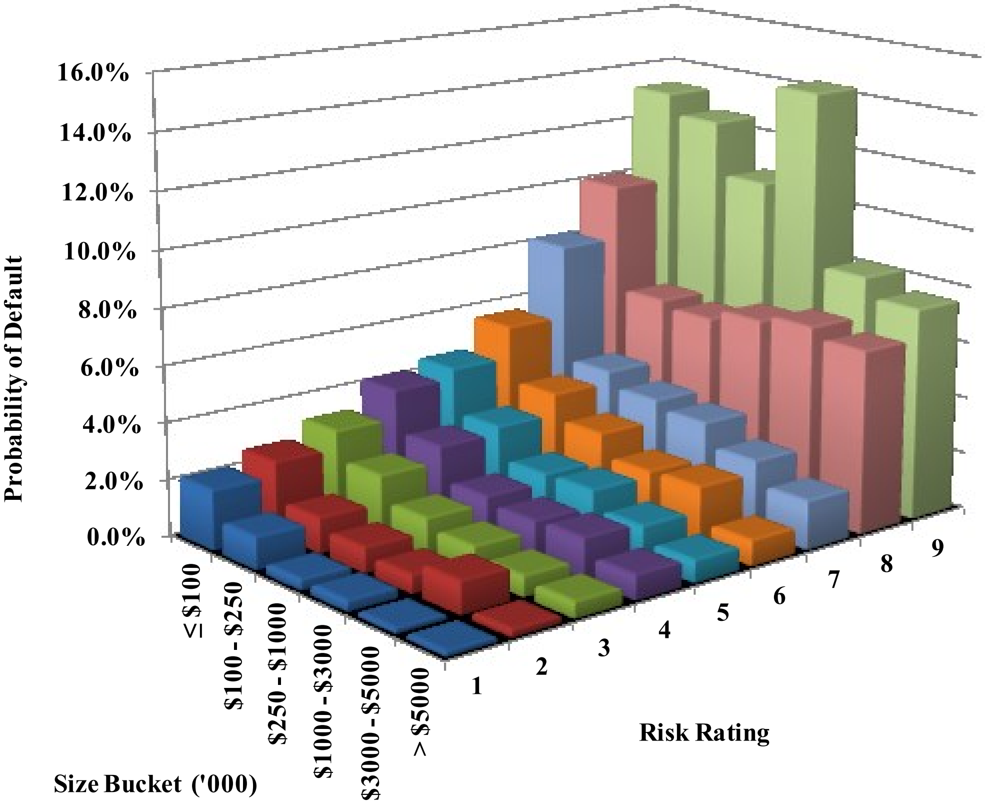


Figure 3.2 – Probability of Default ("PD") figures are calculated by Risk Rating ("RR") – Size Bucket ("Size") segments. PDs are calculated using realized annual default rate data from 1997 to 2010. Results generally indicate decreasing PD with Size and increasing PD with Risk Rating; see Table 3.3 for more details.

Figure 3.3

Probability of Default by Industry and Risk Rating

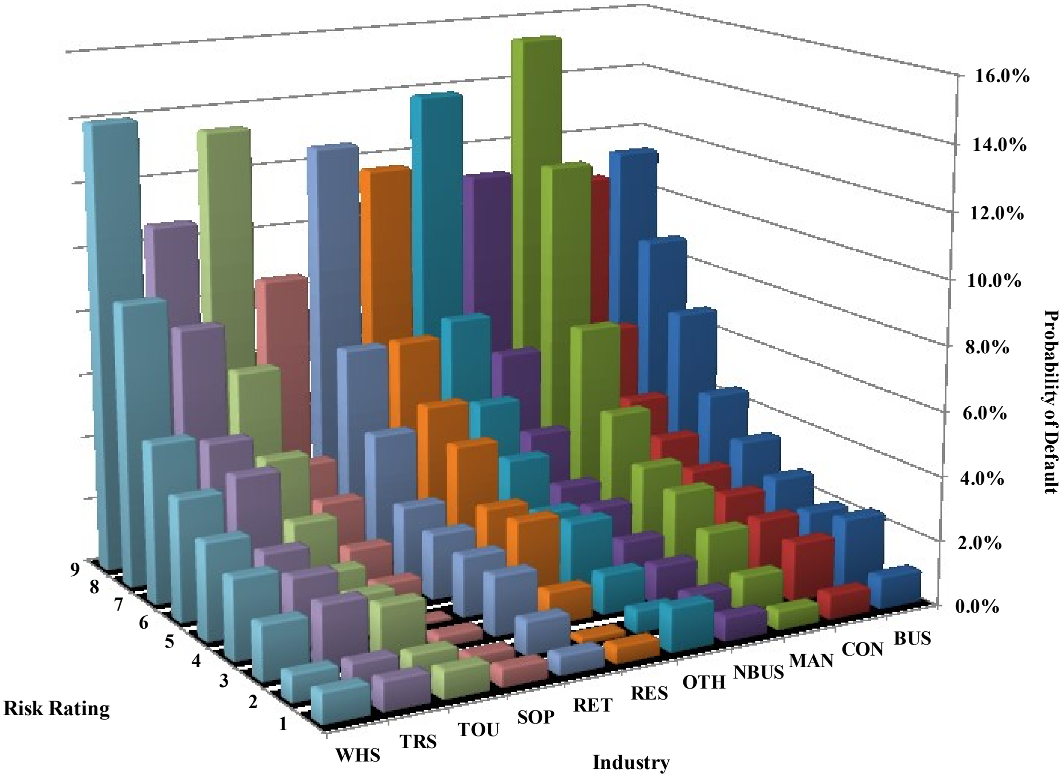


Figure 3.3 depicts the Probability of Default for each Industry-Risk Rating segment, with Risk Ratings ranging from 1 (least risky) to 9 (most risky). Probabilities of Default and their standard of deviations are calculated over a period spanning 14 years starting in January 1997 and ending in December 2010. Defaults were aggregated over a given calendar year and then divided over the count of healthy customers at the start of the year. A weighted average of default rates over the time period then gives the PDs presented above. In addition, defaults were segregated by Risk Rating and Industry as defined in Chapter 2. See Table 3.4 for more details.

Figure 3.4

Probability of Default by Industry and Size Bucket

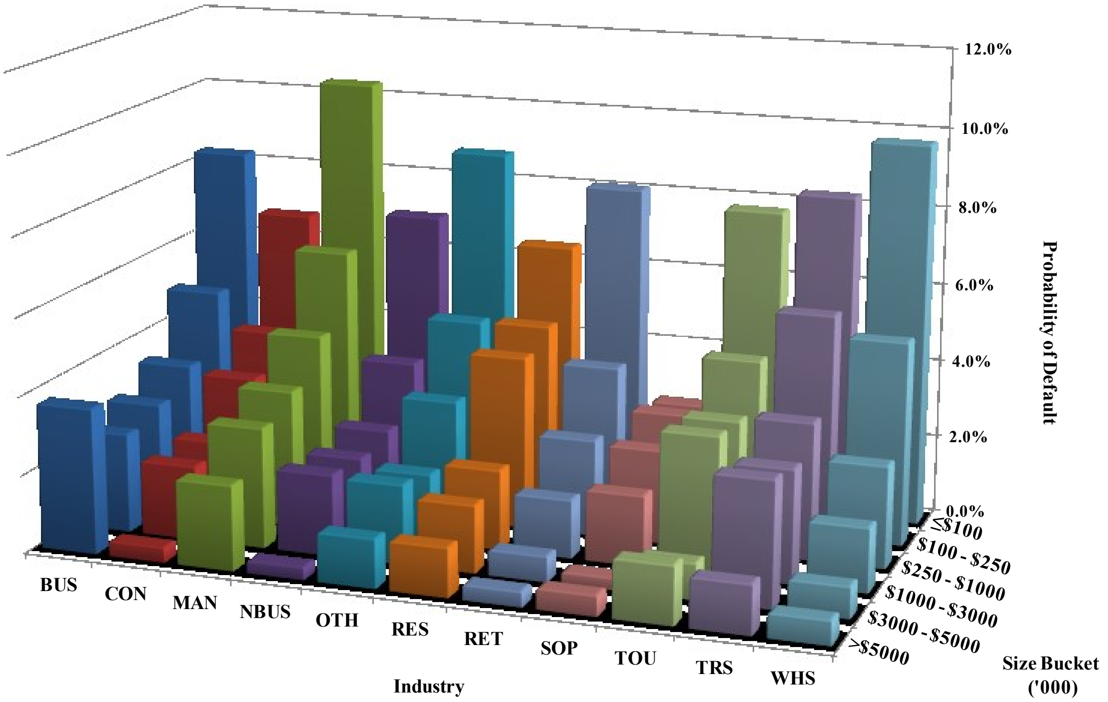


Figure 3.4 depicts the Probability of Default for each Industry-Size Bucket segment. Probabilities of Default and their standard of deviations are calculated over a period spanning 14 years starting in January 1997 and ending in December 2010. Defaults were aggregated over a given calendar year and then divided over the count of healthy customers at the start of the year. A weighted average of default rates over the time period then gives the PDs presented above. In addition, defaults were segregated by Risk Rating and Industry as defined in Chapter 2. See Table 3.5 for more details.

Figure 4.1

Internally Calibrated Asset Correlations vs. Basel II Asset Correlations

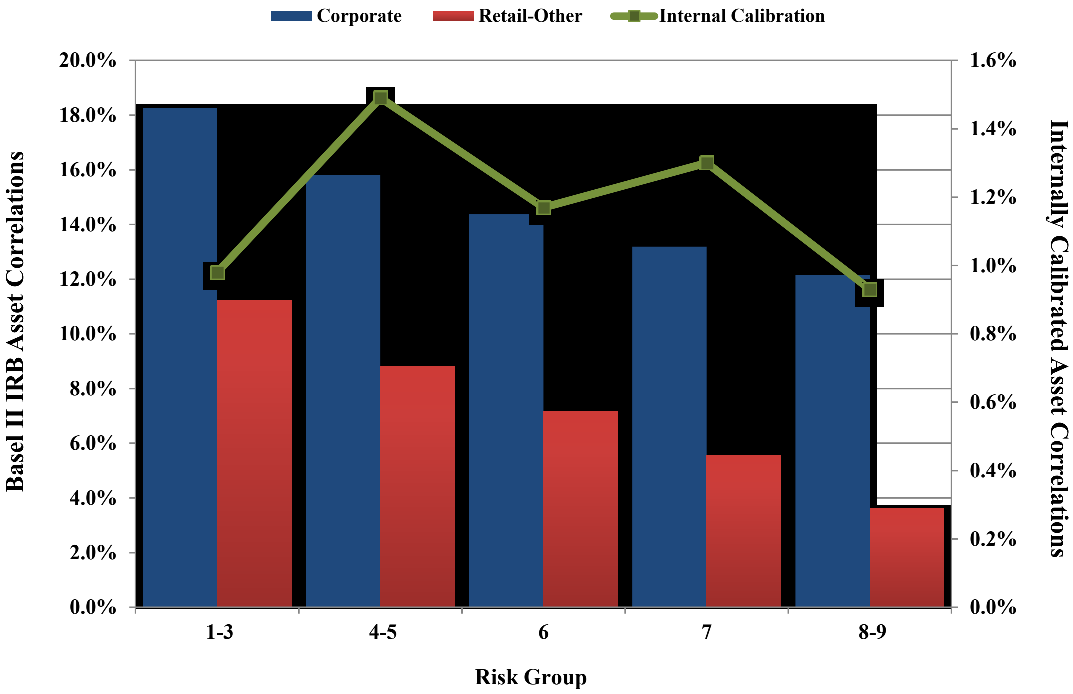


Figure 4.1 plots the Basel II IRB asset correlations under the Corporate and Retail-Other asset classes (excluding Size adjustments) as applied to exposures in the Financing Portfolio in various Risk Groups. These asset correlations are compared to internally calibrated asset correlations as derived according to the model discussed in Subsection 4.1.1 and as presented in Table 4.2.

Figure 4.2

Loss Distributions with Boosted and Non-Boosted Asset Correlations

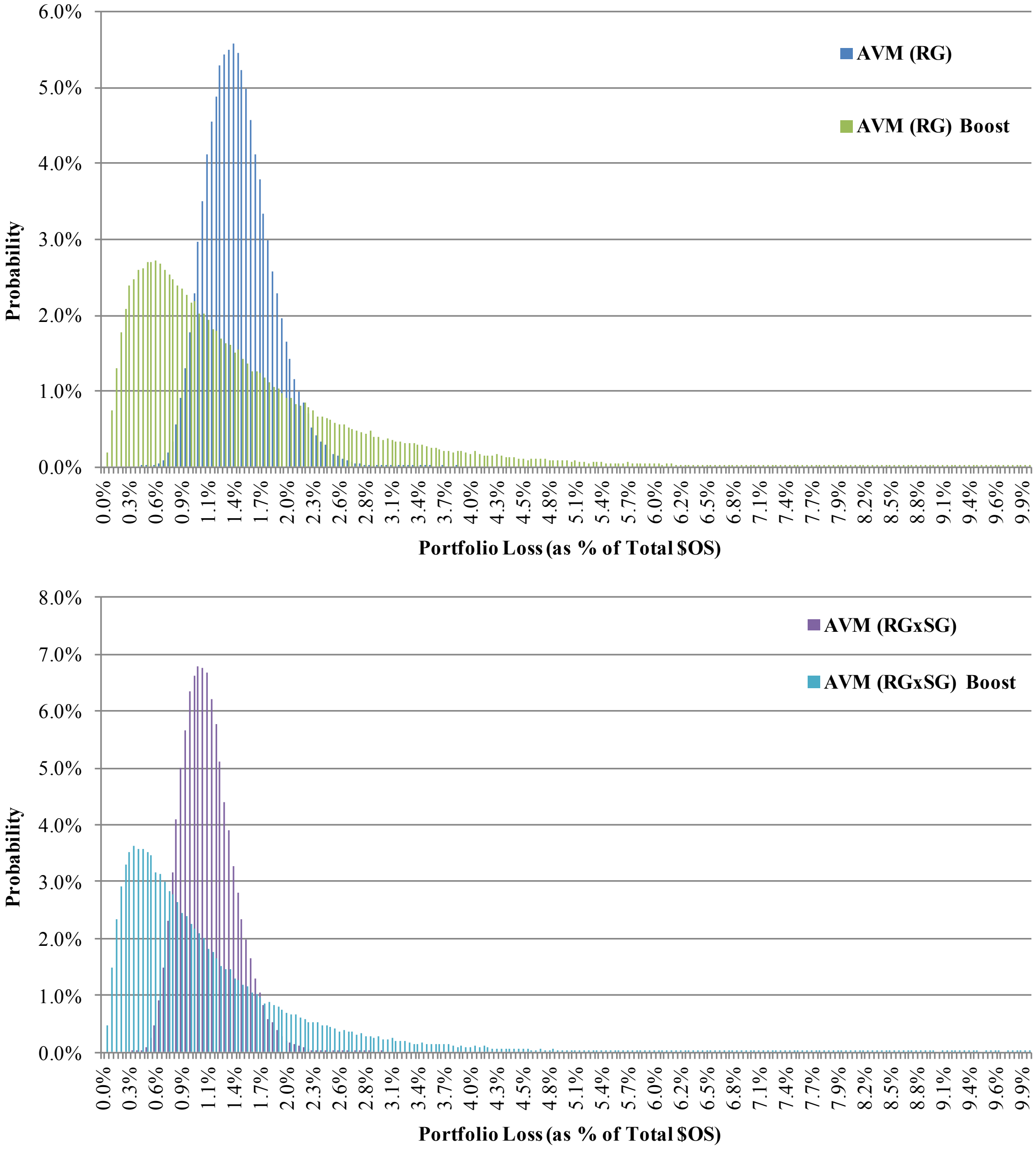


Figure 4.2 depicts simulation-based loss distributions generated using boosted and non-boosted asset correlations. The top panel depicts loss distributions using RG-based asset correlation calibrations; the bottom panel depicts loss distributions calibrated by RG-SG calibrations.

Figure 5.1

CreditRisk$^+$ EC Charges by Industry and Risk Group

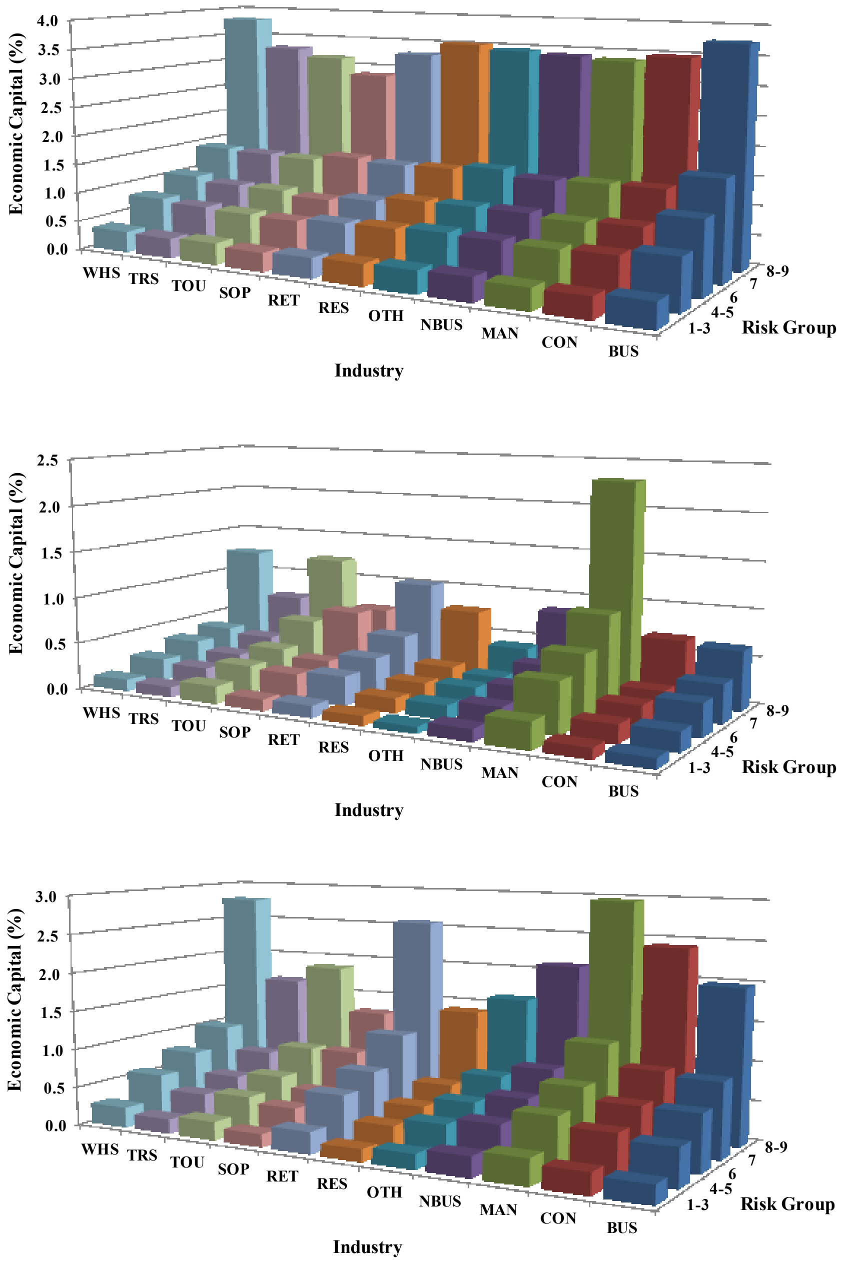


Figure 5.1 – Economic Capital (%) charges are calculated under RG-SG calibrations and various implementations of the CreditRisk$^+$ framework, including (a) *Single Sector* implementation; (b) *Multiple Sector* implementation, and; (c). Industry segregations are used to define sectors – both in the standard *Multiple Sectors* implementation and the *Multiple Correlated Sectors* implementation.

Figure 5.2

CreditRisk$^+$ EC Charges by Industry and Size Group

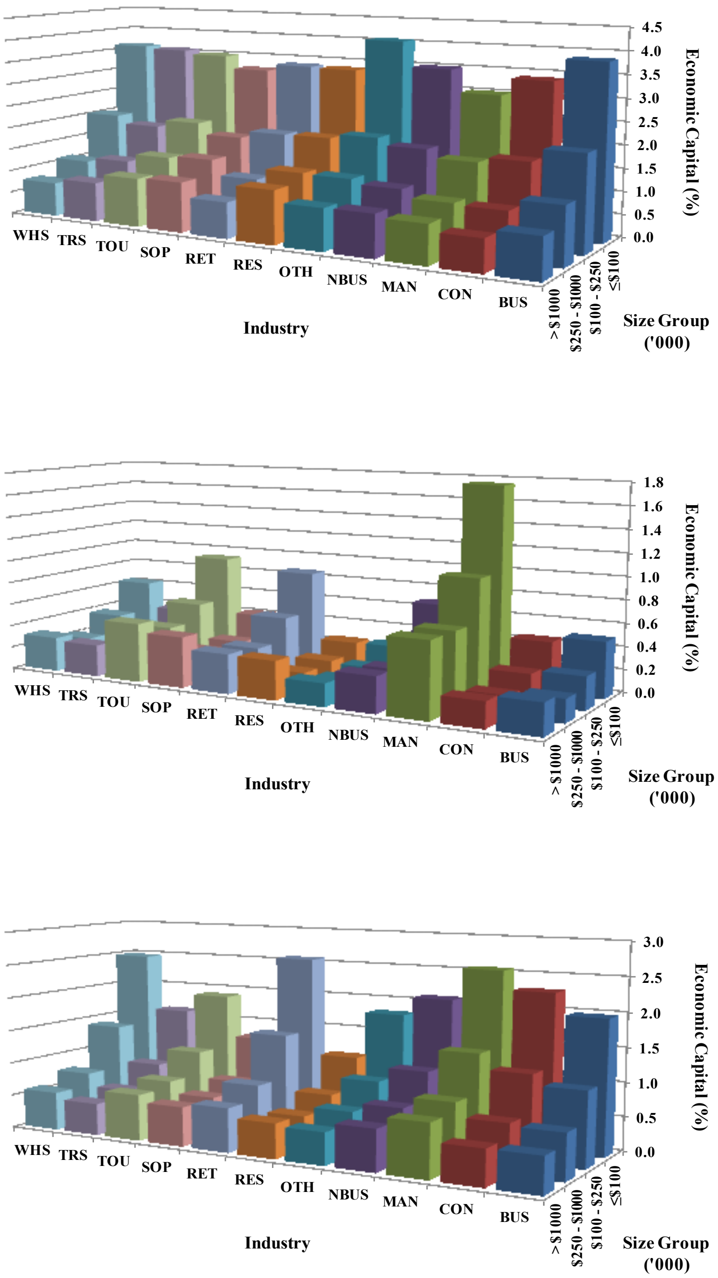


Figure 5.2 – Economic Capital (%) charges are calculated under RG-SG calibrations and various implementations of the CreditRisk$^+$ framework, including (a) *Single Sector* implementation; (b) *Multiple Sector* implementation, and; (c) *Multiple Correlated Sectors* implementation. Industry segregations are used to define sectors – both in the standard *Multiple Sectors* implementation and the *Multiple Correlated Sectors* implementation.

Figure 5.3

Loss Distributions under various CreditRisk$^+$ Implementations

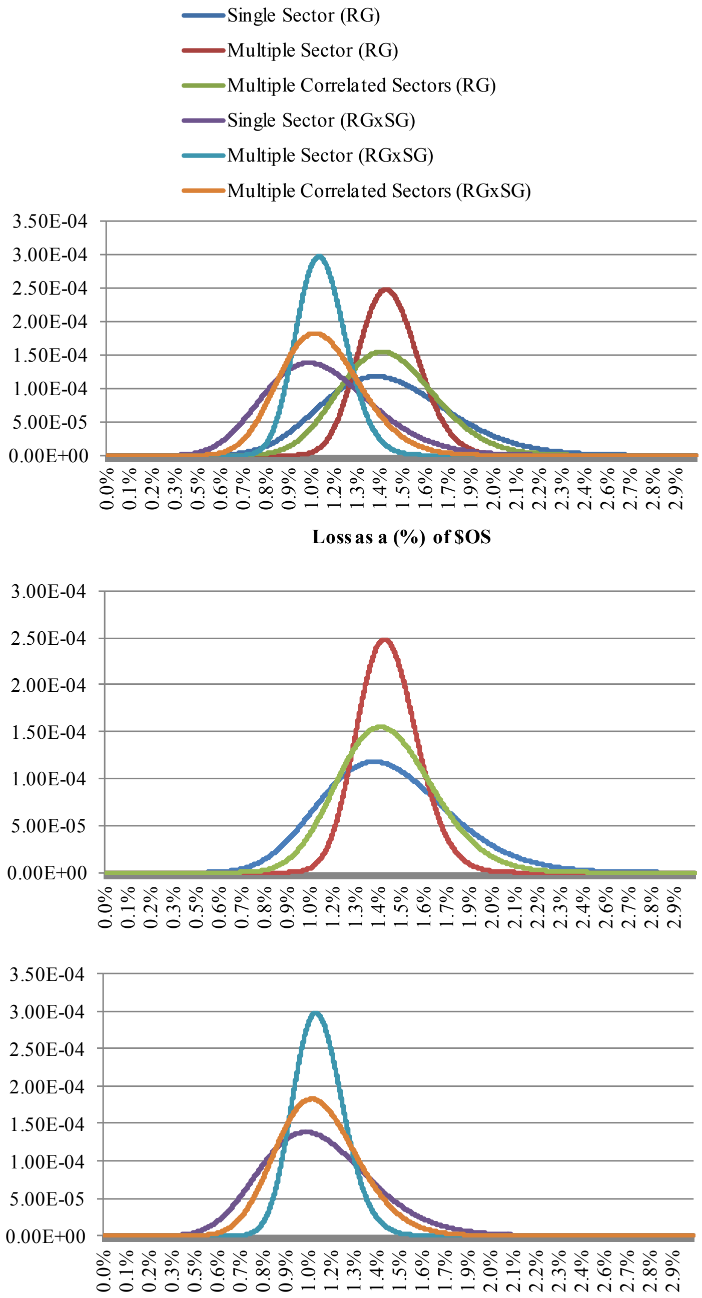


Figure 5.3 depicts the portfolio loss distributions obtained under various implementations of the analytical CreditRisk$^+$ framework. We use both RG- and RG-SG-calibrations of the model according to *Single Sector*, *Multiple Sectors,* and *Multiple Correlated Sectors* implementations. Sectors here are defined by industry.

Figure 6.1

Loss Distribution and Tails under various CreditRisk$^+$ Settings

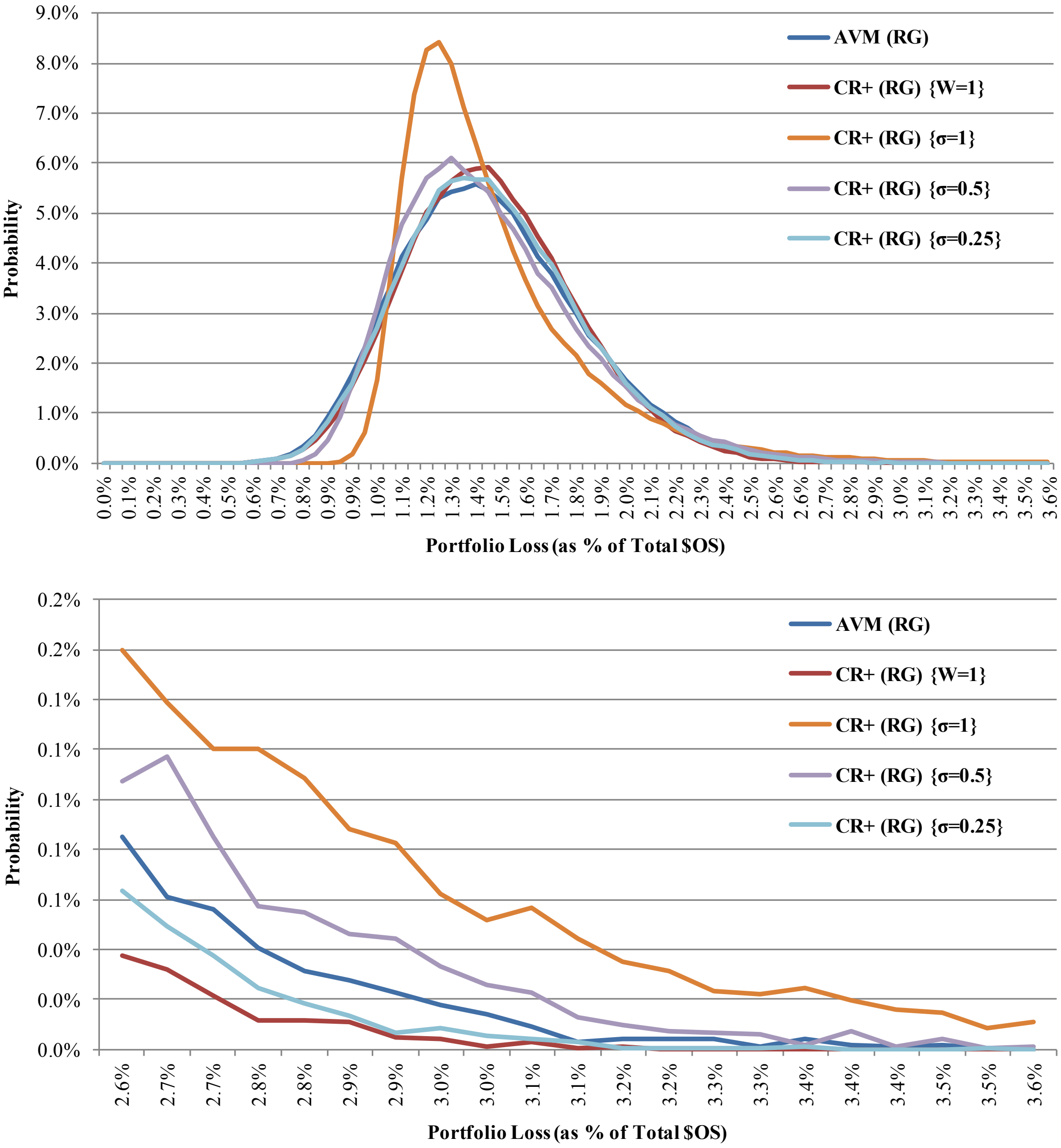


Figure 6.1 presents the loss distributions generated under the single factor simulation-based *AVM* and CreditRisk$^+$ models. The top panel presents results under various implementations using internally-calibrated data, while the bottom panel shows loss distribution tails.

Figure 6.2

Comparing *AVM* and CreditRisk$^+$ Loss Distributions

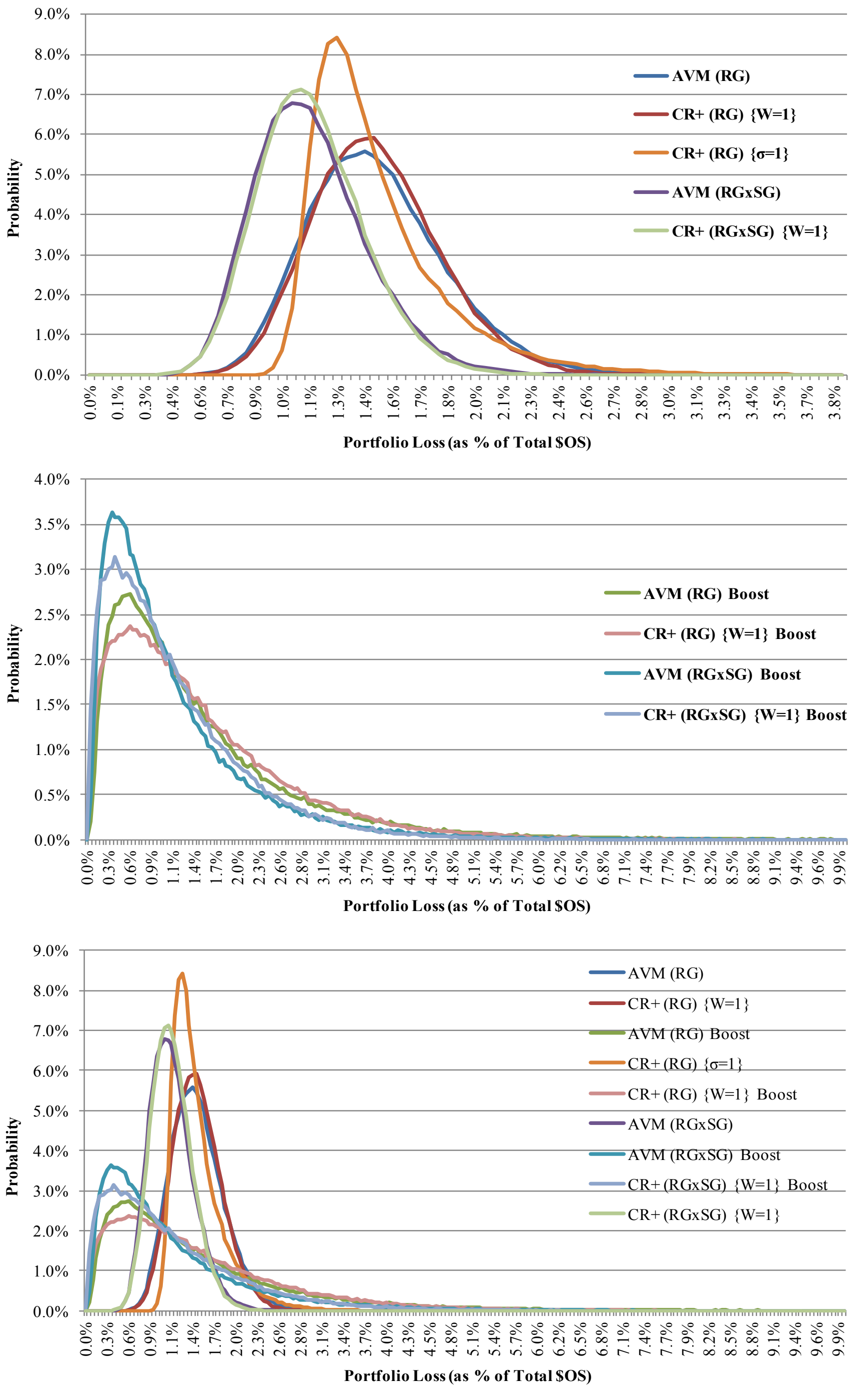


Figure 6.2 presents loss distributions for various calibrations and implementations of the simulation-based *AVM* and CreditRisk$^+$ frameworks. The top panel presents implementations using the RG and RG-SG calibrations of both models, the middle panel presents boosted calibrations of both models, and the third panel presents boosted and non-boosted calibrations together. Distributional statistics can be found in Tables 6.1 and 6.4.

Figure 6.3

Loss Distribution Tails under *AVM* and CreditRisk$^+$

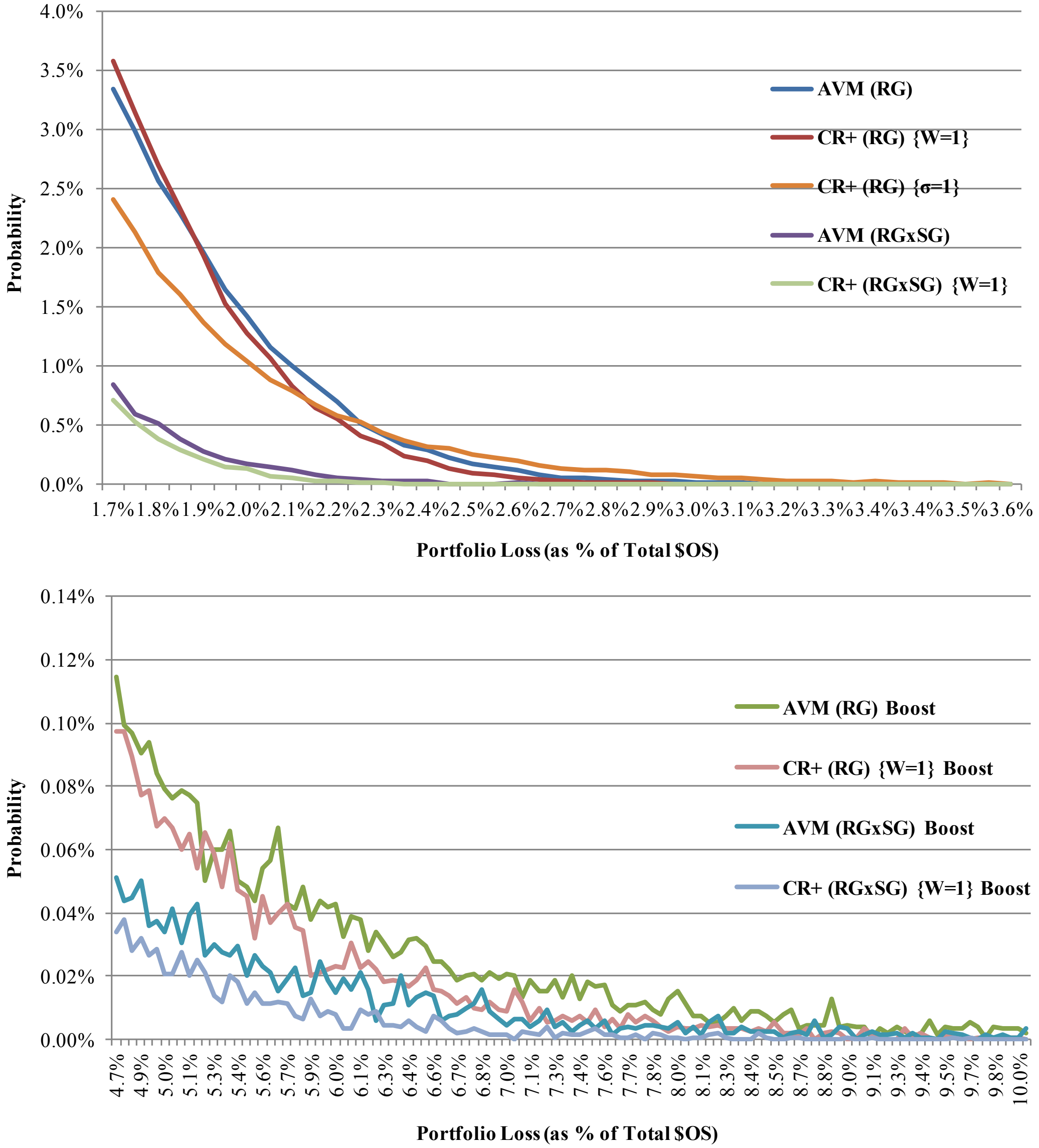


Figure 6.3 presents the tails of the loss distributions shown in Figure 6.1, and includes distributions generated under the single factor simulation-based *AVM* and CreditRisk$^+$ models. The top panel presents results under various implementations using internally-calibrated data, while the bottom panel shows loss distribution tails generated using "boosted" calibrations. Loss distribution statistics are given in Tables 6.1 and 6.2.

Tables

Table 2.1

Cumulative Borrower and $OS Distributions in the *Financing Company* Portfolio

Full Portfolio as of March 2009		
$OS Threshold	**Percentile of Borrowers ≤ Threshold**	**Percentile of Borrower $OS ≤ Threshold**
$10,000	4.7%	0.1%
$25,000	13.6%	0.4%
$50,000	27.4%	1.7%
$100,000	43.0%	4.5%
$150,000	55.5%	8.3%
$250,000	67.3%	13.9%
$500,000	80.3%	25.1%
$1,000,000	89.4%	40.7%
$3,000,000	97. 9%	74.7%
$5,000,000	99.3%	88.0%
$50,000,000	100.0%	100.0%

Table 2.1 provides a description of the *Financing Company* portfolio as of March 2009. The table provides descriptions of the portfolio at various thresholds on the $OS values for a single borrower. As an example, consider the fourth row of the table wherein we examine the $25,000 $OS threshold. From Table 2.1 we observe that approximately 14% of borrowers in the *Financing Company* portfolio have $OS of $25,000 or less and that their cumulative $OS amount to approximately 0.4% of the overall portfolio $OS.

Table 2.2A

Borrowers Segregated into Risk Ratings (RR) and Size Buckets

	For Each RR: Distribution of Borrowers across Size Buckets (%)						
	Size Bucket ('000)						
Risk Rating	**≤ $100**	**$100 - $250**	**$250 - $1000**	**$1000 - $3000**	**$3000 - $5000**	**> $5000**	**Overall**
1	13.9	19.0	28.8	23.4	7.2	7.7	100.0
2	11.3	20.2	34.6	22.5	5.6	5.7	100.0
3	18.8	26.3	28.7	16.4	4.8	4.9	100.0
4	19.2	23.8	31.3	17.0	4.6	4.1	100.0
5	17.0	24.3	35.6	15.6	3.9	3.5	100.0
6	13.1	20.4	38.6	20.0	4.6	3.4	100.0
7	20.7	23.3	35.8	14.6	3.7	1.9	100.0
8	32.4	27.6	28.9	8.1	1.6	1.3	100.0
9	58.1	25.0	12.2	3.7	0.7	0.3	100.0
Overall	24.7	23.9	30.1	14.4	3.7	3.2	100.0
	For Each Size Bucket: Distribution of Borrowers across RR (%)						
1	3.2	4.5	5.5	9.3	11.3	14.0	5.7
2	4.0	7.3	9.9	13.4	13.1	15.6	8.6
3	8.6	12.4	10.8	12.8	14.9	17.5	11.3
4	8.1	10.4	10.8	12.2	13.1	13.5	10.4
5	8.1	11.9	13.8	12.7	12.5	13.1	11.7
6	5.1	8.2	12.3	13.4	12.1	10.2	9.7
7	11.4	13.2	16.1	13.7	13.7	8.2	13.6
8	21.3	18.7	15.5	9.1	7.0	6.9	16.2
9	30.2	13.4	5.2	3.3	2.4	1.1	12.8
Overall	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	Distribution of Borrowers across RR & Size Buckets (%)						
1	0.8	1.1	1.7	1.3	0.4	0.4	5.7
2	1.0	1.7	3.0	1.9	0.5	0.5	8.6
3	2.1	3.0	3.2	1.9	0.5	0.6	11.3
4	2.0	2.5	3.3	1.8	0.5	0.4	10.4
5	2.0	2.8	4.2	1.8	0.5	0.4	11.7
6	1.3	2.0	3.7	1.9	0.4	0.3	9.7
7	2.8	3.2	4.9	2.0	0.5	0.3	13.6
8	5.3	4.5	4.7	1.3	0.3	0.2	16.2
9	7.4	3.2	1.6	0.5	0.1	0.0	12.8
Overall	24.7	23.9	30.1	14.4	3.7	3.2	100.0

Table 2.2A describes the distribution of *Financing Company* borrowers, as of March 2009 across Risk Ratings and Size Buckets. The portfolio consists of borrowers and $OS in Risk Ratings ranging from 1 (least risky) to 9 (riskiest). Size Buckets range from *≤$100,000* to *>$5,000,000* and are based on the total commitment to a borrower at last authorization. Table 2.2A is segregated into three sections: the top section describes the distribution of borrowers across Size Buckets for each Risk Rating; the second section describes the distribution of borrowers across Risk Ratings for each Size Bucket, and; the third section describes the distribution of borrowers in each RR-Size Bucket segment. See also Figure 2.1B.

Table 2.2B

Borrower $OS Segregated into Risk Ratings (RR) and Size Buckets

	For Each RR: Distribution of Borrower $OS across Size Buckets (%)						
	Size Bucket (‘000)						
Risk Rating	**≤ $100**	**$100 - $250**	**$250 - $1000**	**$1000 - $3000**	**$3000 - $5000**	**> $5000**	**Overall**
1	0.8	2.9	12.0	29.1	17.2	38.1	100.0
2	0.6	3.3	15.9	32.4	16.5	31.2	100.0
3	1.5	5.7	16.6	31.7	16.0	28.4	100.0
4	1.5	5.2	17.9	28.8	17.2	29.4	100.0
5	1.4	5.8	22.4	30.6	15.7	24.2	100.0
6	0.8	4.6	22.8	34.8	16.7	20.3	100.0
7	1.8	6.0	27.0	33.8	15.9	15.5	100.0
8	4.2	11.3	30.7	27.7	11.3	14.9	100.0
9	18.5	21.7	24.2	23.3	8.7	3.6	100.0
Overall	2.1	6.0	20.7	31.0	15.7	24.5	100.0
	For Each Size Bucket: Distribution of Borrower $OS across RR (%)						
1	3.7	4.5	5.5	8.9	10.4	14.7	9.5
2	4.0	7.0	9.8	13.2	13.4	16.2	12.7
3	10.0	12.9	10.8	13.8	13.9	15.7	13.5
4	9.0	10.8	10.9	11.7	13.9	15.2	12.6
5	8.9	12.8	14.5	13.2	13.4	13.3	13.4
6	4.8	9.4	13.7	14.0	13.3	10.4	12.5
7	11.1	12.7	16.7	14.0	13.1	8.2	12.8
8	19.6	18.2	14.4	8.7	7.0	5.9	9.7
9	29.0	11.7	3.8	2.4	1.8	0.5	3.3
Overall	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	Distribution of Borrower $OS across RR & Size Buckets (%)						
1	0.1	0.3	1.1	2.8	1.6	3.6	9.5
2	0.1	0.4	2.0	4.1	2.1	4.0	12.7
3	0.2	0.8	2.2	4.3	2.2	3.8	13.5
4	0.2	0.7	2.3	3.6	2.2	3.7	12.6
5	0.2	0.8	3.0	4.1	2.1	3.3	13.4
6	0.1	0.6	2.8	4.3	2.1	2.5	12.5
7	0.2	0.8	3.5	4.3	2.0	2.0	12.8
8	0.4	1.1	3.0	2.7	1.1	1.4	9.7
9	0.6	0.7	0.8	0.8	0.3	0.1	3.3
Overall	2.1	6.0	20.7	31.0	15.7	24.5	100.0

Table 2.2B describes the distribution of *Financing Company* $OS, as of March 2009 across Risk Ratings and Size Buckets. The portfolio consists of borrowers and $OS in Size Buckets ranging from ≤$100,000 to >$5,000,000 and based on the total commitment to a borrower at last authorization. Risk Ratings range from 1 (least risky) to 9 (riskiest). Table 2.2B is segregated into three sections: the top section describes the distribution of borrower $OS across Size Buckets for each Risk Rating; the second section describes the distribution of borrower $OS across Risk Ratings for each Size Bucket, and; the third section describes the distribution of borrower $OS in each RR-Size Bucket segment. See also Figure 2.1A.

Table 2.3A

Distribution of Borrowers across Industries and Size Bucket

For Each Industry: Distribution of Borrowers across Size Buckets (%)							
Industry	Size Bucket ('000)						Overall
	≤ $100	$100 - $250	$250 - $1000	$1000 - $3000	$3000 - $5000	> $5000	
BUS	34.8	29.8	24.7	8.5	1.4	0.8	100.0
CON	25.8	24.3	32.3	13.4	2.0	2.1	100.0
MAN	19.0	19.8	31.4	19.6	5.7	4.4	100.0
NBUS	30.7	25.5	30.1	10.6	1.5	1.6	100.0
OTH	33.8	26.2	25.5	11.0	1.7	1.7	100.0
RES	24.7	24.0	30.8	13.8	3.9	2.8	100.0
RET	26.7	25.9	26.9	13.1	4.1	3.2	100.0
SOP	4.0	14.3	44.5	23.8	6.7	6.6	100.0
TOU	21.0	26.1	33.7	12.3	3.8	3.1	100.0
TRS	22.3	20.0	34.3	14.0	4.5	4.9	100.0
WHS	27.0	25.8	24.9	15.2	3.3	3.7	100.0
Overall	24.7	23.9	30.1	14.4	3.7	3.2	100.0
For Each Size Bucket: Distribution of Borrowers across Industries (%)							
BUS	12.5	11.0	7.3	5.2	3.4	2.2	8.9
CON	8.3	8.0	8.5	7.4	4.4	5.2	7.9
MAN	18.5	19.9	25.0	32.6	37.1	33.7	24.0
NBUS	15.5	13.2	12.4	9.1	5.1	6.2	12.4
OTH	4.7	3.7	2.9	2.6	1.6	1.9	3.4
RES	2.8	2.9	2.9	2.7	3.0	2.5	2.8
RET	14.3	14.3	11.8	12.0	14.6	13.5	13.2
SOP	0.5	2.0	5.0	5.6	6.2	7.1	3.4
TOU	9.1	11.6	11.9	9.0	11.1	10.5	10.6
TRS	3.6	3.4	4.6	3.9	5.0	6.2	4.0
WHS	10.2	10.0	7.7	9.8	8.4	11.0	9.3
Overall	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Distribution of Borrowers across Industries & Size Buckets (%)							
BUS	3.1	2.6	2.2	0.8	0.1	0.1	8.9
CON	2.0	1.9	2.6	1.1	0.2	0.2	7.9
MAN	4.6	4.8	7.5	4.7	1.4	1.1	24.0
NBUS	3.8	3.2	3.7	1.3	0.2	0.2	12.4
OTH	1.2	0.9	0.9	0.4	0.1	0.1	3.4
RES	0.7	0.7	0.9	0.4	0.1	0.1	2.8
RET	3.5	3.4	3.6	1.7	0.5	0.4	13.2
SOP	0.1	0.5	1.5	0.8	0.2	0.2	3.4
TOU	2.2	2.8	3.6	1.3	0.4	0.3	10.6
TRS	0.9	0.8	1.4	0.6	0.2	0.2	4.0
WHS	2.5	2.4	2.3	1.4	0.3	0.3	9.3
Overall	24.7	23.9	30.1	14.4	3.7	3.2	100.0

Table 2.3A describes borrowers in the *Financing Company* portfolio, as of March 2009 and following the same specifications as those given for Table 2.2, are segregated into Industries and Size Buckets. The industries, in alphabetical order, are as follows: Business Services (BUS); Construction (CON); Manufacturing (MAN); Non-Business Services (NBUS); Resources (RES); Retail (RET); Supplier or Premises (SOP); Tourism (TOU); Transportation and Storage (TRS); Wholesale (WHS); Other (OTH).

Table 2.3B

Distribution of Borrower $OS across Industries and Size Bucket

For Each Industry: Distribution of Borrower $OS across Size Buckets (%)							
Industry	**Size Bucket ('000)**						**Overall**
	≤ $100	**$100 - $250**	**$250 - $1000**	**$1000 - $3000**	**$3000 - $5000**	**> $5000**	
BUS	4.9	13.2	27.9	32.9	11.5	9.6	100.0
CON	2.7	7.7	25.0	33.0	9.8	21.9	100.0
MAN	1.2	3.8	16.8	31.1	18.5	28.6	100.0
NBUS	3.8	10.2	33.2	31.6	9.6	11.6	100.0
OTH	3.8	9.6	25.4	37.8	10.6	12.8	100.0
RES	1.9	6.6	23.3	30.5	16.6	21.1	100.0
RET	2.4	6.7	19.4	30.3	17.6	23.6	100.0
SOP	0.2	1.8	17.5	29.5	17.1	33.9	100.0
TOU	1.7	6.3	22.7	28.8	16.5	24.0	100.0
TRS	1.5	3.8	19.2	25.9	16.5	33.1	100.0
WHS	2.5	6.5	16.9	33.8	13.6	26.8	100.0
Overall	2.1	6.0	20.7	31.0	15.7	24.5	100.0
For Each Size Bucket: Distribution of Borrower $OS across Industries (%)							
BUS	12.2	11.2	6.9	5.4	3.7	2.0	5.1
CON	8.4	8.3	7.9	6.9	4.1	5.8	6.5
MAN	18.4	19.4	25.2	31.2	36.8	36.4	31.1
NBUS	14.6	13.4	12.7	8.1	4.8	3.8	7.9
OTH	4.3	3.7	2.9	2.9	1.6	1.2	2.4
RES	2.4	2.9	3.0	2.6	2.8	2.3	2.7
RET	15.1	14.5	12.2	12.7	14.6	12.6	13.0
SOP	0.6	1.6	4.6	5.2	5.9	7.5	5.4
TOU	9.4	12.0	12.5	10.6	12.1	11.2	11.4
TRS	3.8	3.3	4.8	4.4	5.5	7.1	5.2
WHS	10.9	9.8	7.4	10.0	8.0	10.0	9.1
Overall	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Distribution of Borrower $OS across Industries & Size Buckets (%)							
BUS	0.3	0.7	1.4	1.7	0.6	0.5	5.1
CON	0.2	0.5	1.6	2.2	0.6	1.4	6.5
MAN	0.4	1.2	5.2	9.7	5.8	8.9	31.1
NBUS	0.3	0.8	2.6	2.5	0.8	0.9	7.9
OTH	0.1	0.2	0.6	0.9	0.3	0.3	2.4
RES	0.0	0.2	0.6	0.8	0.4	0.6	2.7
RET	0.3	0.9	2.5	3.9	2.3	3.1	13.0
SOP	0.0	0.1	1.0	1.6	0.9	1.8	5.4
TOU	0.2	0.7	2.6	3.3	1.9	2.7	11.4
TRS	0.1	0.2	1.0	1.4	0.9	1.7	5.2
WHS	0.2	0.6	1.5	3.1	1.2	2.5	9.1
Overall	2.1	6.0	20.7	31.0	15.7	24.5	100.0

Table 2.3B describes the $OS in the *Financing Company* portfolio, as of March 2009 and following the same specifications as those given for Table 2.2, are segregated into Industries and Size Buckets – as described in Table 2.2B and Table 2.3A.

Table 2.4A

Borrower Distribution by Risk Rating and Industry

For Each Industry: Distribution of Borrowers across Risk Ratings (%)										
Industry	Risk Rating									Overall
	1	2	3	4	5	6	7	8	9	
BUS	6.1	9.0	11.5	11.3	11.3	8.1	10.6	16.1	16.0	100.0
CON	7.2	11.3	12.5	10.3	11.3	8.8	11.3	13.0	14.1	100.0
MAN	7.2	10.6	13.2	11.1	11.3	10.3	13.4	13.7	9.2	100.0
NBUS	3.6	7.3	9.8	10.0	12.1	9.7	13.7	18.4	15.4	100.0
OTH	5.2	6.8	9.7	10.3	12.0	7.9	13.1	17.9	17.1	100.0
RES	3.5	7.5	11.1	7.1	12.4	9.3	12.8	20.6	15.8	100.0
RET	7.2	8.2	11.3	10.7	12.1	9.0	14.4	14.8	12.3	100.0
SOP	1.9	4.7	7.3	7.6	9.4	13.3	20.5	24.1	11.4	100.0
TOU	3.6	5.6	8.2	9.3	11.6	9.5	16.7	22.2	13.2	100.0
TRS	3.2	6.2	10.3	9.7	14.0	12.7	14.5	16.0	13.3	100.0
WHS	6.8	9.8	13.1	11.7	12.1	9.4	11.4	13.0	12.5	100.0
Overall	5.7	8.6	11.3	10.4	11.7	9.7	13.6	16.2	12.8	100.0
For Each Risk Rating: Distribution of Borrowers across Industries (%)										
BUS	9.5	9.3	9.0	9.6	8.6	7.4	6.9	8.8	11.1	8.9
CON	10.0	10.5	8.8	7.9	7.7	7.2	6.6	6.4	8.7	7.9
MAN	30.2	29.6	28.0	25.5	23.2	25.6	23.6	20.2	17.2	24.0
NBUS	7.8	10.6	10.8	12.0	12.8	12.4	12.5	14.1	15.0	12.4
OTH	3.1	2.7	2.9	3.4	3.5	2.8	3.3	3.8	4.6	3.4
RES	1.7	2.5	2.8	1.9	3.0	2.7	2.7	3.6	3.5	2.8
RET	16.5	12.6	13.2	13.5	13.6	12.3	14.0	12.1	12.7	13.2
SOP	1.1	1.8	2.2	2.5	2.7	4.7	5.1	5.0	3.0	3.4
TOU	6.7	7.0	7.8	9.5	10.5	10.5	13.1	14.5	11.0	10.6
TRS	2.3	2.9	3.7	3.8	4.8	5.3	4.3	4.0	4.2	4.0
WHS	11.1	10.6	10.8	10.5	9.6	9.0	7.8	7.5	9.1	9.3
Overall	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Distribution of Borrowers across Industries & Risk Ratings (%)										
BUS	0.5	0.8	1.0	1.0	1.0	0.7	0.9	1.4	1.4	8.9
CON	0.6	0.9	1.0	0.8	0.9	0.7	0.9	1.0	1.1	7.9
MAN	1.7	2.5	3.2	2.7	2.7	2.5	3.2	3.3	2.2	24.0
NBUS	0.4	0.9	1.2	1.2	1.5	1.2	1.7	2.3	1.9	12.4
OTH	0.2	0.2	0.3	0.4	0.4	0.3	0.4	0.6	0.6	3.4
RES	0.1	0.2	0.3	0.2	0.4	0.3	0.4	0.6	0.4	2.8
RET	0.9	1.1	1.5	1.4	1.6	1.2	1.9	2.0	1.6	13.2
SOP	0.1	0.2	0.2	0.3	0.3	0.4	0.7	0.8	0.4	3.4
TOU	0.4	0.6	0.9	1.0	1.2	1.0	1.8	2.4	1.4	10.6
TRS	0.1	0.2	0.4	0.4	0.6	0.5	0.6	0.6	0.5	4.0
WHS	0.6	0.9	1.2	1.1	1.1	0.9	1.1	1.2	1.2	9.3
Overall	5.7	8.6	11.3	10.4	11.7	9.7	13.6	16.2	12.8	100.0

Table 2.4A presents the *Financing Company* portfolio borrowers, as of March 2009 and following the same specifications as those given for Table 2.2, are segregated into Industries (as described in Table 2.3A) and Risk Ratings, ranging from 1 (least risky) to 9 (most risky).

Table 2.4B

Borrower $OS Distribution by Risk Rating and Industry

For Each Industry: Distribution of Borrower $OS across Risk Rating (%)										
Industry	**Risk Rating**									**Overall**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	
BUS	11.2	11.0	15.6	15.8	11.5	11.3	9.9	9.3	4.5	100
CON	9.9	21.4	16.3	8.8	13.0	10.9	9.4	7.6	2.8	100
MAN	12.1	14.1	14.0	13.4	12.1	12.1	11.2	8.6	2.3	100
NBUS	5.8	10.0	13.9	13.0	13.9	13.1	14.2	11.4	4.7	100
OTH	8.9	9.7	11.9	10.9	16.7	13.5	13.1	10.2	5.1	100
RES	5.7	11.7	16.0	5.7	16.4	9.9	13.3	17.0	4.3	100
RET	10.7	12.4	12.5	12.9	15.4	13.0	12.6	7.4	3.1	100
SOP	2.7	3.7	12.7	12.4	14.1	17.8	17.2	13.8	5.5	100
TOU	5.3	10.8	8.2	10.7	15.9	11.5	18.5	15.4	3.6	100
TRS	8.3	10.5	13.5	15.2	12.7	13.0	15.7	7.9	3.2	100
WHS	11.7	15.5	17.0	13.7	11.3	12.5	10.1	6.0	2.3	100
Overal	9.5	12.7	13.5	12.6	13.4	12.5	12.8	9.7	3.3	100
For Each Risk Rating: Distribution of Borrower $OS across Industries (%)										
BUS	6.0	4.4	5.9	6.4	4.4	4.6	4.0	4.9	7.0	5.1
CON	6.8	11.0	7.8	4.6	6.3	5.7	4.8	5.1	5.7	6.5
MAN	39.9	34.6	32.2	33.0	28.0	30.3	27.1	27.7	22.5	31.1
NBUS	4.9	6.2	8.1	8.2	8.2	8.3	8.8	9.3	11.5	7.9
OTH	2.2	1.8	2.1	2.0	2.9	2.6	2.4	2.5	3.7	2.4
RES	1.6	2.4	3.1	1.2	3.2	2.1	2.7	4.6	3.5	2.7
RET	14.7	12.8	12.0	13.3	15.0	13.5	12.8	9.9	12.3	13.0
SOP	1.6	1.6	5.1	5.4	5.7	7.8	7.3	7.7	9.2	5.4
TOU	6.4	9.7	6.9	9.7	13.6	10.5	16.5	18.2	12.8	11.4
TRS	4.6	4.3	5.2	6.3	5.0	5.5	6.4	4.3	5.2	5.2
WHS	11.3	11.1	11.5	9.9	7.7	9.2	7.2	5.7	6.5	9.1
Overal	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Distribution of Borrower $OS across Industries & Risk Rating (%)										
BUS	0.6	0.6	0.8	0.8	0.6	0.6	0.5	0.5	0.2	5.1
CON	0.6	1.4	1.1	0.6	0.8	0.7	0.6	0.5	0.2	6.5
MAN	3.8	4.4	4.4	4.2	3.8	3.8	3.5	2.7	0.7	31.1
NBUS	0.5	0.8	1.1	1.0	1.1	1.0	1.1	0.9	0.4	7.9
OTH	0.2	0.2	0.3	0.3	0.4	0.3	0.3	0.2	0.1	2.4
RES	0.2	0.3	0.4	0.2	0.4	0.3	0.4	0.5	0.1	2.7
RET	1.4	1.6	1.6	1.7	2.0	1.7	1.6	1.0	0.4	13.0
SOP	0.1	0.2	0.7	0.7	0.8	1.0	0.9	0.8	0.3	5.4
TOU	0.6	1.2	0.9	1.2	1.8	1.3	2.1	1.8	0.4	11.4
TRS	0.4	0.5	0.7	0.8	0.7	0.7	0.8	0.4	0.2	5.2
WHS	1.1	1.4	1.6	1.3	1.0	1.1	0.9	0.6	0.2	9.1
Overal	9.5	12.7	13.5	12.6	13.4	12.5	12.8	9.7	3.3	100.0

Table 2.4B presents the *Financing Company* portfolio $OS, as of March 2009 and following the same specifications as those given for Table 2.2, are segregated into Industries (as described in Table 2.3A) and Risk Ratings, ranging from 1 (least risky) to 9 (most risky).

Table 2.5A

Borrower Distribution by Industry and Geographical Region

Region	Industry											Overall
	BUS	CON	MAN	NBUS	OTH	RES	RET	SOP	TOU	TRS	WHS	
For Each Region: Distribution of Borrowers across Industries (%)												
Alberta	10.6	9.6	21.1	13.8	3.2	5.7	11.7	2.9	9.5	4.0	8.0	100.0
British Columbia	8.6	7.0	23.2	12.8	3.4	2.8	11.7	2.8	14.0	5.1	8.5	100.0
Manitoba	10.0	7.7	21.7	13.7	4.4	1.5	12.5	2.4	9.4	6.5	10.1	100.0
New Brunswick	6.0	9.6	18.0	16.8	2.4	4.1	17.1	3.3	14.0	3.4	5.3	100.0
N. & L.	4.1	8.8	9.7	13.1	3.0	16.7	16.8	7.0	12.2	4.0	4.5	100.0
N.W. Territories	10.6	7.6	0.0	3.0	1.5	0.0	33.3	16.7	13.6	12.1	1.5	100.0
Nova Scotia	8.9	8.2	14.5	15.2	4.1	3.6	15.9	3.0	15.5	3.7	7.4	100.0
Ontario	10.0	6.3	24.8	11.8	4.3	0.9	12.3	3.1	11.9	4.3	10.2	100.0
P.E.I.	5.9	11.0	8.5	15.3	3.4	3.4	15.3	2.5	28.0	3.4	3.4	100.0
Quebec	9.1	8.6	29.4	11.4	2.9	1.0	13.1	3.3	7.3	3.1	10.9	100.0
Saskatchewan	5.4	10.6	21.8	13.8	2.0	5.9	13.6	2.9	10.4	6.5	7.2	100.0
Yukon	4.0	8.1	2.0	17.2	5.1	3.0	18.2	6.1	25.3	10.1	1.0	100.0
Overall	8.9	7.9	24.0	12.4	3.4	2.8	13.2	3.4	10.6	4.0	9.3	100.0
For Each Industry: Distribution of Borrowers across Region (%)												
Alberta	8.9	8.9	6.5	8.2	6.9	14.9	6.6	6.3	6.6	7.4	6.3	7.4
British Columbia	10.5	9.6	10.5	11.1	10.8	10.7	9.6	9.1	14.3	13.6	10.0	10.8
Manitoba	3.2	2.7	2.5	3.1	3.6	1.5	2.7	2.0	2.5	4.5	3.1	2.8
New Brunswick	2.9	5.3	3.3	5.9	3.0	6.3	5.6	4.2	5.7	3.6	2.5	4.3
N. & L.	2.8	6.7	2.4	6.3	5.3	35.1	7.6	12.3	6.9	6.0	2.9	6.0
N.W. Territories	0.3	0.2	0.0	0.1	0.1	0.0	0.7	1.3	0.3	0.8	0.0	0.3
Nova Scotia	3.4	3.5	2.0	4.1	4.0	4.3	4.1	3.0	4.9	3.1	2.7	3.4
Ontario	31.1	21.8	28.3	26.0	34.8	8.9	25.6	25.1	30.7	29.1	30.1	27.4
P.E.I.	0.3	0.6	0.2	0.6	0.5	0.6	0.5	0.3	1.2	0.4	0.2	0.5
Quebec	35.1	37.1	42.1	31.5	29.0	12.5	34.1	33.7	23.6	26.7	40.4	34.4
Saskatchewan	1.4	3.1	2.1	2.6	1.4	4.9	2.4	2.0	2.3	3.8	1.8	2.4
Yukon	0.2	0.4	0.0	0.5	0.6	0.4	0.5	0.7	0.9	1.0	0.0	0.4
Overall	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Distribution of Borrowers across Region and Industry (%)												
Alberta	0.8	0.7	1.6	1.0	0.2	0.4	0.9	0.2	0.7	0.3	0.6	7.4
British Columbia	0.9	0.8	2.5	1.4	0.4	0.3	1.3	0.3	1.5	0.5	0.9	10.8
Manitoba	0.3	0.2	0.6	0.4	0.1	0.0	0.4	0.1	0.3	0.2	0.3	2.8
New Brunswick	0.3	0.4	0.8	0.7	0.1	0.2	0.7	0.1	0.6	0.1	0.2	4.3
N. & L.	0.2	0.5	0.6	0.8	0.2	1.0	1.0	0.4	0.7	0.2	0.3	6.0
N.W. Territories	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.3
Nova Scotia	0.3	0.3	0.5	0.5	0.1	0.1	0.5	0.1	0.5	0.1	0.2	3.4
Ontario	2.8	1.7	6.8	3.2	1.2	0.3	3.4	0.9	3.3	1.2	2.8	27.4
P.E.I.	0.0	0.1	0.0	0.1	0.0	0.0	0.1	0.0	0.1	0.0	0.0	0.5
Quebec	3.1	2.9	10.1	3.9	1.0	0.4	4.5	1.1	2.5	1.1	3.7	34.4
Saskatchewan	0.1	0.2	0.5	0.3	0.0	0.1	0.3	0.1	0.2	0.2	0.2	2.4
Yukon	0.0	0.0	0.0	0.1	0.0	0.0	0.1	0.0	0.1	0.0	0.0	0.4
Overall	8.9	7.9	24.0	12.4	3.4	2.8	13.2	3.4	10.6	4.0	9.3	100.0

Table 2.5A describes the *Financing Company* portfolio borrowers, as of March 2009 and following the same specifications as those given for Table 2.2, are segregated into Industries (as described in Table 2.3A) and Geographical Regions.

Table 2.5B

$OS Distribution by Industry and Geographical Region

For Each Region: Distribution of Borrower $OS across Industries (%)

Region	Industry											Overall
	BUS	CON	MAN	NBUS	OTH	RES	RET	SOP	TOU	TRS	WHS	
Alberta	8.8	12.3	23.6	9.7	1.6	8.1	9.4	6.0	8.3	4.2	8.1	100.0
British Columbia	4.8	6.3	25.2	8.7	1.3	3.3	12.1	2.5	20.4	6.8	8.7	100.0
Manitoba	4.7	4.7	38.1	6.2	1.7	1.5	8.7	3.3	9.3	10.1	11.8	100.0
New Brunswick	4.3	6.0	23.7	11.7	1.2	3.1	18.8	5.3	16.3	3.7	5.8	100.0
N. & L.	3.6	8.6	14.8	9.7	3.1	11.6	16.6	11.8	10.6	6.2	3.5	100.0
N.W. Territories	5.3	6.2	0.0	1.3	1.1	0.0	41.1	16.0	15.3	9.7	4.0	100.0
Nova Scotia	7.5	6.8	17.4	9.2	5.0	2.6	18.3	4.0	16.3	5.8	7.2	100.0
Ontario	5.4	5.3	32.2	7.7	3.1	1.1	12.1	5.6	13.8	4.3	9.4	100.0
P.E.I.	0.5	11.5	16.5	10.6	4.8	2.4	17.6	1.1	32.9	0.6	1.5	100.0
Quebec	4.1	6.1	38.5	7.0	2.1	1.1	13.7	5.8	6.5	4.7	10.4	100.0
Saskatchewan	6.1	5.6	26.2	6.6	1.9	7.6	7.3	2.6	10.5	13.7	11.9	100.0
Yukon	1.3	5.5	0.9	8.8	2.2	0.3	28.6	2.1	25.1	25.0	0.1	100.0
Overall	5.1	6.5	31.1	7.9	2.4	2.7	13.0	5.4	11.4	5.2	9.1	100.0
For Each Industry: Distribution of Borrower $OS across Region (%)												
Alberta	15.5	16.9	6.8	11.0	6.1	27.4	6.5	9.9	6.5	7.1	8.0	9.0
British Columbia	9.5	9.8	8.2	11.1	5.6	12.8	9.4	4.6	18.1	13.2	9.7	10.2
Manitoba	2.0	1.6	2.7	1.7	1.6	1.3	1.5	1.3	1.8	4.3	2.9	2.2
New Brunswick	3.2	3.6	2.9	5.7	2.0	4.6	5.6	3.7	5.5	2.7	2.5	3.9
N. & L.	2.8	5.3	1.9	4.9	5.2	17.5	5.1	8.6	3.7	4.7	1.5	4.0
N.W. Territories	0.3	0.2	0.0	0.0	0.1	0.0	0.8	0.8	0.3	0.5	0.1	0.3
Nova Scotia	4.1	2.9	1.6	3.3	5.9	2.8	3.9	2.1	4.0	3.1	2.2	2.8
Ontario	31.8	24.3	31.0	29.0	39.9	13.0	27.9	31.0	36.1	24.8	31.0	30.0
P.E.I.	0.0	0.8	0.2	0.6	0.9	0.4	0.6	0.1	1.2	0.1	0.1	0.4
Quebec	28.0	32.4	42.7	30.3	30.5	14.1	36.4	36.6	19.5	31.2	39.3	34.5
Saskatchewan	2.6	1.9	1.8	1.8	1.8	6.2	1.2	1.0	2.0	5.7	2.8	2.2
Yukon	0.1	0.4	0.0	0.6	0.5	0.1	1.2	0.2	1.2	2.5	0.0	0.5
Overall	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Distribution of Borrowers across Region & Industry (%)												
Alberta	0.8	1.1	2.1	0.9	0.1	0.7	0.8	0.5	0.7	0.4	0.7	9.0
British Columbia	0.5	0.6	2.6	0.9	0.1	0.3	1.2	0.3	2.1	0.7	0.9	10.2
Manitoba	0.1	0.1	0.8	0.1	0.0	0.0	0.2	0.1	0.2	0.2	0.3	2.2
New Brunswick	0.2	0.2	0.9	0.5	0.0	0.1	0.7	0.2	0.6	0.1	0.2	3.9
N. & L.	0.1	0.3	0.6	0.4	0.1	0.5	0.7	0.5	0.4	0.2	0.1	4.0
N.W. Territories	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.3
Nova Scotia	0.2	0.2	0.5	0.3	0.1	0.1	0.5	0.1	0.5	0.2	0.2	2.8
Ontario	1.6	1.6	9.7	2.3	0.9	0.3	3.6	1.7	4.1	1.3	2.8	30.0
P.E.I.	0.0	0.0	0.1	0.0	0.0	0.0	0.1	0.0	0.1	0.0	0.0	0.4
Quebec	1.4	2.1	13.3	2.4	0.7	0.4	4.7	2.0	2.2	1.6	3.6	34.5
Saskatchewan	0.1	0.1	0.6	0.1	0.0	0.2	0.2	0.1	0.2	0.3	0.3	2.2
Yukon	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.1	0.1	0.0	0.5
Overall	5.1	6.5	31.1	7.9	2.4	2.7	13.0	5.4	11.4	5.2	9.1	100.0

Table 2.5B presents the *Financing Company* portfolio $OS, as of March 2009 and following the same specifications as those given for Table 2.2, are segregated into Industries (as described in Table 2.3A) and Geographical Regions.

Table 2.6A

Borrower Distribution by Size Bucket and Region

	For Each Region: Distribution of Borrowers across Size Buckets (%)						
Region	**Size Bucket (‘000)**						**Overall**
	≤ $100	**$100 - $250**	**$250 - $1000**	**$1000 - $3000**	**$3000 - $5000**	**> $5000**	
Alberta	21.9	27.7	26.0	15.4	4.1	4.8	100.0
British Columbia	28.0	25.0	27.5	13.4	3.5	2.6	100.0
Manitoba	33.2	24.9	26.8	10.7	1.5	2.8	100.0
New Brunswick	31.0	19.7	29.8	13.8	3.5	2.2	100.0
N. & L.	24.9	22.9	34.8	12.1	3.3	2.0	100.0
N.W. Territories	4.5	10.6	43.9	34.8	3.0	3.0	100.0
Nova Scotia	26.1	25.8	30.1	14.7	1.8	1.5	100.0
Ontario	23.2	25.5	30.2	14.4	3.5	3.2	100.0
P.E.I.	33.9	11.0	36.4	14.4	3.4	0.8	100.0
Quebec	24.1	22.1	30.8	15.3	4.3	3.5	100.0
Saskatchewan	21.8	27.0	31.9	12.8	4.0	2.5	100.0
Yukon	14.1	18.2	41.4	19.2	1.0	6.1	100.0
Overall	24.7	23.9	30.1	14.4	3.7	3.2	100.0
	For Each Size Bucket: Distribution of Borrowers across Region (%)						
Alberta	6.6	8.6	6.4	7.9	8.3	11.2	7.4
British Columbia	12.3	11.3	9.9	10.0	10.3	9.0	10.8
Manitoba	3.8	2.9	2.5	2.1	1.2	2.5	2.8
New Brunswick	5.5	3.6	4.3	4.2	4.1	3.0	4.3
N. & L.	6.0	5.7	6.9	5.0	5.4	3.9	6.0
N.W. Territories	0.0	0.1	0.4	0.6	0.2	0.2	0.3
Nova Scotia	3.6	3.6	3.4	3.4	1.6	1.6	3.4
Ontario	25.7	29.2	27.5	27.3	25.8	27.9	27.4
P.E.I.	0.6	0.2	0.6	0.5	0.4	0.1	0.5
Quebec	33.6	31.8	35.2	36.4	39.9	37.9	34.4
Saskatchewan	2.1	2.7	2.5	2.1	2.6	1.9	2.4
Yukon	0.2	0.3	0.5	0.5	0.1	0.7	0.4
Overall	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	Distribution of Borrowers across Region & Size Buckets (%)						
Alberta	1.6	2.1	1.9	1.1	0.3	0.4	7.4
British Columbia	5.0	2.7	5.0	1.4	0.4	0.3	10.8
Manitoba	0.9	0.7	0.7	0.3	0.0	0.1	2.8
New Brunswick	1.3	0.9	1.3	0.6	0.1	0.1	4.3
N. & L.	2.0	1.4	2.1	0.7	0.2	0.1	6.0
N.W. Territories	0.0	0.0	0.1	0.1	0.0	0.0	0.3
Nova Scotia	0.9	0.9	1.0	0.5	0.1	0.1	3.4
Ontario	6.3	7.0	8.3	3.9	0.9	0.9	27.4
P.E.I.	0.2	0.1	0.2	0.1	0.0	0.0	0.5
Quebec	8.3	7.6	10.6	5.3	2.0	1.2	34.4
Saskatchewan	0.5	0.6	0.7	0.3	0.1	0.1	2.4
Yukon	0.1	0.1	0.2	0.1	0.0	0.0	0.4
Overall	24.7	23.9	30.1	14.4	3.7	3.2	100.0

Table 2.6A presents the *Financing Company* portfolio borrowers, as of March 2009, are segregated into Size Buckets (as described in Table 2.2A) and Geographical Regions.

Table 2.6B

$OS Distribution by Size Bucket and Region

Region	Size Bucket (‘000)						Overall
	≤ $100	$100 - $250	$250 - $1000	$1000 - $3000	$3000 - $5000	> $5000	
For Each Region: Distribution of Borrower $OS across Size Buckets (%)							
Alberta	1.4	5.8	16.2	30.3	15.1	31.2	100.0
British Columbia	2.6	6.8	20.4	33.0	16.6	20.5	100.0
Manitoba	3.6	7.7	24.8	26.0	11.9	26.0	100.0
New Brunswick	2.7	5.5	23.6	35.3	20.0	12.8	100.0
N. & L.	2.8	8.1	31.4	29.0	16.0	12.7	100.0
N.W. Territories	0.4	3.4	33.0	51.3	7.7	4.1	100.0
Nova Scotia	2.7	8.4	26.5	42.9	12.5	6.9	100.0
Ontario	1.8	6.0	20.0	30.9	15.1	26.2	100.0
P.E.I.	3.3	3.9	27.7	41.9	12.4	10.9	100.0
Quebec	2.0	5.4	19.9	29.8	16.4	26.4	100.0
Saskatchewan	2.2	7.6	24.9	25.8	14.7	24.7	100.0
Yukon	1.3	3.7	22.0	33.2	3.0	36.7	100.0
Overall	2.1	6.0	20.7	31.0	15.7	24.5	100.0
For Each Size Bucket: Distribution of Borrower $OS across Region (%)							
Alberta	6.1	8.6	7.0	8.8	8.7	11.5	9.0
British Columbia	12.7	11.5	10.0	10.8	10.8	8.5	10.2
Manitoba	3.9	2.8	2.7	1.9	1.7	2.4	2.2
New Brunswick	5.0	3.5	4.4	4.4	4.9	2.0	3.9
N. & L.	5.3	5.4	6.0	3.7	4.1	2.1	4.0
N.W. Territories	0.1	0.1	0.4	0.4	0.1	0.0	0.3
Nova Scotia	3.7	3.9	3.6	3.9	2.2	0.8	2.8
Ontario	26.2	29.9	28.9	29.9	28.8	32.2	30.0
P.E.I.	0.7	0.3	0.6	0.6	0.3	0.2	0.4
Quebec	33.6	30.8	33.2	33.2	36.2	37.3	34.5
Saskatchewan	2.3	2.8	2.6	1.8	2.0	2.2	2.2
Yukon	0.3	0.3	0.6	0.6	0.1	0.8	0.5
Overall	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Distribution of Borrower $OS across Region & Size Buckets (%)							
Alberta	0.1	0.5	1.5	2.7	1.4	2.8	9.0
British Columbia	0.3	0.7	2.1	3.4	1.7	2.1	10.2
Manitoba	0.1	0.2	0.6	0.6	0.3	0.6	2.2
New Brunswick	0.1	0.2	0.9	1.4	0.8	0.5	3.9
N. & L.	0.1	0.3	1.3	1.2	0.6	0.5	4.0
N.W. Territories	0.0	0.0	0.1	0.1	0.0	0.0	0.3
Nova Scotia	0.1	0.2	0.7	1.2	0.4	0.2	2.8
Ontario	0.5	1.8	6.0	9.3	4.5	7.9	30.0
P.E.I.	0.0	0.0	0.1	0.2	0.1	0.0	0.4
Quebec	0.7	1.9	6.9	10.3	5.7	9.1	34.5
Saskatchewan	0.0	0.2	0.5	0.6	0.3	0.5	2.2
Yukon	0.0	0.0	0.1	0.2	0.0	0.2	0.5
Overall	2.1	6.0	20.7	31.0	15.7	24.5	100.0

Table 2.6B describes the *Financing Company* portfolio $OS, as of March 2009 and following the same specifications as those given for Table 2.2, are segregated into Size Buckets (as described in Table 2.2A) and Geographical Regions.

Table 2.7

Time Series of Distribution of Borrowers across Industries

	Distribution of Borrowers across Industries											
Calendar Year	**Industry**											**Overall**
	BUS	**CON**	**MAN**	**NBUS**	**OTH**	**RES**	**RET**	**SOP**	**TOU**	**TRS**	**WHS**	
1997	4%	6%	21%	9%	3%	3%	15%	10%	18%	4%	8%	100%
1998	5%	6%	22%	9%	2%	3%	13%	10%	17%	4%	9%	100%
1999	5%	6%	23%	9%	2%	3%	13%	11%	16%	4%	9%	100%
2000	6%	6%	24%	8%	2%	3%	12%	11%	16%	4%	9%	100%
2001	6%	6%	25%	9%	2%	3%	12%	10%	15%	4%	9%	100%
2002	7%	6%	26%	9%	2%	3%	12%	9%	14%	4%	9%	100%
2003	7%	6%	26%	9%	2%	3%	12%	8%	14%	4%	10%	100%
2004	7%	6%	27%	9%	2%	3%	11%	7%	14%	4%	10%	100%
2005	7%	6%	27%	11%	4%	3%	12%	6%	12%	4%	8%	100%
2006	7%	7%	27%	12%	4%	3%	12%	5%	11%	4%	9%	100%
2007	8%	7%	26%	12%	4%	3%	13%	4%	11%	4%	9%	100%
2008	8%	7%	25%	12%	4%	3%	13%	3%	11%	4%	9%	100%
2009	9%	8%	24%	12%	3%	3%	13%	3%	11%	4%	9%	100%
2010	9%	8%	23%	13%	3%	3%	13%	4%	11%	4%	9%	100%
	Distribution of $OS across Industries											
1997	3%	4%	28%	6%	2%	2%	11%	13%	18%	4%	8%	100%
1998	3%	4%	30%	6%	2%	2%	10%	13%	17%	4%	9%	100%
1999	3%	4%	30%	6%	2%	2%	10%	13%	17%	4%	9%	100%
2000	3%	4%	32%	6%	2%	2%	9%	12%	16%	4%	9%	100%
2001	4%	4%	34%	6%	2%	2%	9%	11%	15%	4%	9%	100%
2002	4%	4%	35%	6%	2%	3%	9%	9%	15%	4%	9%	100%
2003	4%	4%	36%	6%	2%	3%	10%	8%	14%	4%	10%	100%
2004	3%	4%	36%	8%	2%	3%	11%	7%	13%	4%	8%	100%
2005	4%	5%	35%	8%	2%	3%	12%	6%	12%	5%	9%	100%
2006	4%	6%	35%	8%	2%	2%	12%	5%	11%	5%	9%	100%
2007	5%	6%	33%	8%	2%	3%	13%	5%	11%	5%	9%	100%
2008	5%	6%	32%	8%	2%	3%	13%	5%	11%	5%	9%	100%
2009	5%	7%	29%	8%	2%	3%	12%	8%	11%	5%	9%	100%
2010	5%	7%	26%	8%	3%	3%	13%	9%	12%	5%	9%	100%

Table 2.7 displays the concentration of borrowers (top) and $OS (bottom) in a given industry at yearly intervals starting in December 1997 and ending in December 2010. The industries, in alphabetical order, are as follows: Business Services (BUS); Construction (CON); Manufacturing (MAN); Non-Business Services (NBUS); Resources (RES); Retail (RET); Supplier or Premises (SOP); Tourism (TOU); Transportation and Storage (TRS); Wholesale (WHS); and Other (OTH). Results from Table 2.7 indicate MAN to be the predominant industry in the portfolio. See Figure 2.6 for accompanying graphics.

Table 2.8

Time Series of Distribution of Borrowers across Risk Ratings

Distribution of Borrowers across Risk Ratings										
Calendar Year	**Risk Rating**									**Overall**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	
1997	3%	5%	6%	9%	16%	18%	28%	15%	1%	100%
1998	3%	4%	7%	9%	17%	19%	28%	13%	1%	100%
1999	1%	3%	5%	7%	16%	15%	30%	21%	1%	100%
2000	1%	4%	6%	6%	14%	11%	27%	28%	3%	100%
2001	2%	6%	6%	7%	14%	10%	23%	26%	6%	100%
2002	2%	7%	7%	8%	14%	9%	22%	25%	6%	100%
2003	2%	7%	7%	8%	14%	10%	21%	25%	6%	100%
2004	2%	7%	8%	9%	13%	10%	19%	24%	7%	100%
2005	2%	7%	8%	9%	13%	10%	19%	26%	7%	100%
2006	2%	7%	8%	10%	13%	10%	19%	24%	8%	100%
2007	4%	7%	9%	10%	12%	10%	16%	20%	13%	100%
2008	5%	8%	10%	10%	11%	10%	14%	18%	14%	100%
2009	6%	8%	11%	10%	12%	10%	14%	16%	13%	100%
2010	6%	9%	12%	11%	12%	9%	13%	16%	12%	100%
Distribution of $OS across Risk Ratings										
1997	4%	6%	9%	10%	19%	19%	22%	10%	0%	100%
1998	2%	5%	7%	8%	19%	16%	26%	16%	1%	100%
1999	3%	6%	8%	8%	19%	12%	22%	20%	1%	100%
2000	4%	10%	8%	10%	18%	11%	19%	18%	2%	100%
2001	5%	10%	10%	11%	17%	10%	19%	17%	2%	100%
2002	4%	9%	9%	11%	18%	11%	19%	16%	2%	100%
2003	4%	10%	10%	13%	16%	11%	17%	15%	2%	100%
2004	4%	9%	10%	13%	17%	11%	17%	17%	2%	100%
2005	4%	9%	11%	14%	17%	12%	17%	15%	2%	100%
2006	7%	10%	13%	13%	15%	12%	14%	12%	4%	100%
2007	9%	11%	14%	13%	13%	13%	12%	11%	4%	100%
2008	9%	12%	14%	12%	14%	12%	13%	10%	3%	100%
2009	9%	12%	13%	13%	14%	12%	13%	10%	3%	100%
2010	8%	12%	14%	13%	14%	12%	13%	11%	3%	100%

Table 2.8 displays the concentration of borrowers (top) and $OS (bottom) in a given Risk Rating at yearly intervals starting in December 1997 and ending in December 2010, with Risk Ratings ranging from 1 (least risky) to 9 (most risky). See Figure 2.7 for accompanying graphics.

Table 2.9

Time Series of Distribution of Borrowers across Size Buckets

				Distribution of Borrowers across Size Buckets			
				Size Bucket ('000)			
Calendar Year	**≤ $100**	**$100 - $250**	**$250 - $1000**	**$1000 - $3000**	**$3000 - $5000**	**> $5000**	**Overall**
1997	25%	28%	35%	9%	2%	1%	100%
1998	24%	27%	35%	11%	2%	1%	100%
1999	23%	26%	36%	12%	2%	1%	100%
2000	21%	25%	38%	13%	2%	1%	100%
2001	20%	24%	38%	13%	3%	2%	100%
2002	19%	24%	38%	14%	3%	2%	100%
2003	19%	23%	38%	15%	3%	2%	100%
2004	20%	23%	37%	15%	3%	2%	100%
2005	22%	22%	35%	15%	3%	2%	100%
2006	24%	22%	34%	15%	4%	3%	100%
2007	25%	22%	32%	14%	4%	3%	100%
2008	26%	23%	31%	14%	3%	3%	100%
2009	25%	24%	30%	14%	4%	3%	100%
2010	24%	24%	30%	15%	4%	4%	100%
				Distribution of $OS across Size Buckets			
1997	4%	12%	42%	27%	7%	8%	100%
1998	3%	11%	40%	29%	8%	8%	100%
1999	3%	10%	39%	30%	10%	9%	100%
2000	2%	9%	36%	32%	11%	10%	100%
2001	2%	8%	35%	32%	12%	11%	100%
2002	2%	7%	33%	33%	13%	12%	100%
2003	2%	7%	31%	33%	14%	13%	100%
2004	3%	6%	29%	33%	14%	15%	100%
2005	3%	6%	26%	33%	15%	17%	100%
2006	3%	6%	25%	32%	15%	19%	100%
2007	3%	6%	23%	32%	16%	21%	100%
2008	2%	6%	21%	31%	16%	24%	100%
2009	2%	5%	19%	30%	16%	29%	100%
2010	1%	5%	18%	29%	16%	31%	100%

Table 2.9 displays the concentration of borrowers (top) and $OS (bottom) in a given Size Bucket at yearly intervals starting in December 1997 and ending in December 2010. Size Buckets range from ≤$100,000 to >$5,000,000 and are based on the total commitment to a borrower at last authorization. See Figure 2.8 for accompanying graphics.

Table 2.10

Annual Default Rates by Risk Rating

		Risk Rating								
Calendar Year	**Overall**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1997	4.9%	0.7%	0.9%	0.9%	1.3%	1.7%	4.3%	6.2%	11.7%	26.3%
1998	5.5%	0.9%	1.6%	2.5%	2.4%	2.5%	4.9%	7.1%	10.9%	37.0%
1999	4.8%	0.4%	1.5%	0.8%	2.2%	2.1%	3.3%	5.5%	8.6%	19.5%
2000	4.5%	0.0%	0.9%	1.3%	2.1%	1.8%	2.2%	4.1%	6.8%	30.0%
2001	5.0%	0.3%	0.6%	1.6%	2.1%	2.6%	2.8%	4.1%	8.5%	15.8%
2002	4.1%	0.8%	1.3%	1.0%	1.4%	1.3%	2.7%	3.5%	7.1%	14.0%
2003	4.4%	1.3%	1.1%	1.6%	1.9%	2.3%	2.7%	4.9%	6.7%	12.7%
2004	4.0%	0.7%	0.5%	1.4%	1.5%	1.8%	2.3%	3.6%	7.3%	11.0%
2005	4.2%	0.7%	0.6%	1.5%	1.5%	2.5%	2.2%	4.4%	6.8%	11.5%
2006	4.0%	1.0%	1.0%	1.2%	1.5%	1.9%	3.2%	3.3%	5.4%	14.0%
2007	4.6%	0.8%	0.9%	2.0%	2.4%	2.3%	3.3%	4.4%	6.6%	11.7%
2008	5.9%	0.9%	1.7%	2.9%	3.4%	4.4%	4.4%	5.8%	8.4%	14.2%
2009	4.4%	0.5%	1.8%	1.6%	2.8%	4.0%	3.5%	4.5%	6.4%	10.8%
2010	3.7%	0.6%	0.9%	1.9%	2.6%	2.8%	2.9%	3.0%	5.6%	9.3%
Average	4.6%	0.7%	1.1%	1.7%	2.2%	2.5%	3.3%	4.6%	7.3%	12.7%
Std Dev	0.6	0.2	0.4	0.6	0.6	0.9	0.8	1.1	1.5	3.6

Table 2.10 shows the annual default rate by Risk Rating for the period starting January 1997 and ending December 2010. Risk Ratings range from 1 (least risky) to 9 (most risky). To calculate annual default rates, the number of defaulted borrowers over a given calendar year are divided by the number of borrowers at the beginning of that calendar year - so that for the 2008 calendar year, defaults from January 2008 to December 2008 are summed and divided by the number of healthy borrowers as of December 31, 2007. See Figure 2.10 and Table 2.13 for more details.

Table 2.11

Annual Default Rates by Industry

Calendar Year	Industry											
	Overall	**BUS**	**CON**	**MAN**	**NBUS**	**OTH**	**RES**	**RET**	**SOP**	**TOU**	**TRS**	**WHS**
1997	4.9%	9.9%	4.0%	6.1%	3.2%	4.4%	6.1%	4.8%	4.1%	5.1%	3.5%	4.0%
1998	5.5%	10.1%	5.3%	6.4%	4.7%	4.6%	6.8%	4.5%	3.4%	4.8%	4.9%	6.2%
1999	4.8%	7.8%	4.9%	6.1%	4.5%	5.0%	7.7%	3.5%	2.8%	3.7%	3.2%	5.7%
2000	4.5%	7.4%	4.7%	5.4%	3.1%	5.4%	4.5%	3.7%	2.4%	4.9%	4.5%	3.7%
2001	5.0%	7.7%	5.2%	6.5%	3.7%	7.1%	3.1%	4.6%	3.1%	3.9%	4.1%	4.8%
2002	4.1%	6.6%	3.8%	5.3%	4.1%	3.7%	4.1%	2.9%	2.0%	3.3%	6.0%	3.7%
2003	4.4%	6.0%	4.5%	6.0%	3.6%	3.8%	3.0%	3.6%	1.8%	3.9%	5.0%	4.4%
2004	4.0%	5.2%	3.6%	5.1%	3.5%	4.9%	3.8%	3.5%	2.4%	3.5%	4.7%	2.9%
2005	3.9%	5.3%	2.4%	4.9%	3.2%	4.4%	3.5%	3.3%	1.7%	4.0%	4.1%	4.4%
2006	3.9%	3.9%	3.2%	5.0%	3.1%	4.6%	6.0%	3.3%	1.2%	4.3%	3.1%	4.1%
2007	4.0%	3.5%	2.7%	4.8%	3.4%	3.9%	3.4%	3.8%	1.5%	4.7%	3.6%	5.1%
2008	5.1%	4.3%	3.6%	6.1%	4.6%	5.8%	4.5%	5.0%	0.5%	5.1%	6.1%	6.2%
2009	5.7%	5.6%	5.0%	6.5%	4.5%	5.9%	4.1%	5.5%	1.9%	5.6%	7.5%	6.7%
2010	4.2%	4.3%	4.6%	4.8%	3.9%	5.5%	4.2%	4.0%	1.1%	4.1%	3.8%	4.4%
Average	4.6%	5.6%	4.0%	5.6%	3.8%	5.0%	4.5%	4.1%	2.3%	4.4%	4.7%	4.8%
Std Dev	0.1	0.3	0.2	0.1	0.1	0.2	0.3	0.2	0.4	0.2	0.3	0.2

Table 2.11 provides the annual default rate by Industry for the period starting January 1997 and ending December 2010. The industries, in alphabetical order, are as follows: Business Services (BUS); Construction (CON); Manufacturing (MAN); Non-Business Services (NBUS); Resources (RES); Retail (RET); Supplier or Premises (SOP); Tourism (TOU); Transportation and Storage (TRS); Wholesale (WHS); Other (OTH). To calculate annual default rates the number of defaulted borrowers over a given calendar year are divided by the number of borrowers at the beginning of that calendar year, so that for the 2008 calendar year, defaults from January 2008 to December 2008 are summed and divided by the number of healthy borrowers as of December 31, 2007. See Figure 2.9 and Table 2.14 for more details.

Table 2.12

Annual Default Rates by Size Bucket

Calendar Year	Size Buckets ('000)						
	Overall	**≤ $100**	**$100 - $250**	**$250 - $1000**	**$1000 - $3000**	**$3000 - $5000**	**> $5000**
1997	4.9%	7.8%	5.1%	4.0%	1.7%	2.1%	0.0%
1998	5.5%	9.1%	4.9%	4.5%	2.9%	2.1%	0.5%
1999	4.8%	10.0%	3.8%	3.6%	1.7%	0.6%	0.9%
2000	4.5%	9.3%	4.1%	3.3%	1.9%	1.6%	0.4%
2001	5.0%	9.7%	4.3%	4.0%	3.1%	2.4%	0.4%
2002	4.1%	9.0%	3.5%	3.0%	2.5%	0.9%	0.9%
2003	4.4%	8.5%	4.1%	3.1%	4.0%	2.1%	2.1%
2004	4.0%	7.5%	4.1%	2.9%	2.6%	2.2%	0.9%
2005	3.9%	7.6%	3.8%	2.7%	2.4%	1.7%	1.9%
2006	3.9%	7.0%	3.8%	2.9%	2.5%	2.1%	0.8%
2007	4.0%	7.0%	4.1%	2.8%	2.3%	1.9%	1.1%
2008	5.1%	8.8%	5.7%	3.1%	3.1%	2.2%	1.5%
2009	5.7%	9.0%	6.8%	4.1%	2.9%	3.0%	1.3%
2010	4.2%	7.7%	5.0%	2.5%	2.4%	1.8%	1.8%
Average	4.6%	8.3%	4.6%	3.3%	2.6%	2.0%	1.2%
Std Dev	0.1	0.1	0.2	0.2	0.2	0.3	0.4

Table 2.12 provides the annual default rate by Size Bucket for the period starting January 1997 and ending December 2010. Size Buckets range from ≤$100,000 to >$5,000,000 and are based on the total commitment to a borrower (including commitment to other “related” borrowers under the same ownership) at last authorization. To calculate annual default rates the number of defaulted borrowers over a given calendar year are divided by the number of borrowers at the beginning of that calendar year, so that for the 2008 calendar year, defaults from January 2008 to December 2008 are summed and divided by the number of healthy borrowers as of December 31, 2007.

Table 2.13

Annual Default Rate Correlation between Risk Ratings

RR	1	2	3	4	5	6	7	8	9
Overall	0.12	0.57	0.66	0.67	0.52	0.80	0.83	0.67	0.51
1	1.00	0.33	0.23	0.01	0.05	0.16	0.25	0.05	-0.16
2		1.00	0.46	0.65	0.51	0.66	0.54	0.24	0.27
3			1.00	0.81	0.72	0.56	0.41	0.07	0.06
4				1.00	0.90	0.52	0.37	0.00	0.03
5					1.00	0.45	0.27	-0.04	-0.18
6						1.00	0.81	0.72	0.48
7							1.00	0.82	0.67
8								1.00	0.72
9									1.00

Table 2.13 provides the correlation between the annual default rates of borrowers in Risk Ratings 1 (least risky) to 9 (most risky). In addition, we provide the correlation between a given Risk Rating's default rate and that of the overall portfolio ("All") as given in the second row. Correlations are measured over an evaluation period of 1997-2010. See Table 2.10 and Figure 2.10 for more information on the time series of default rates by Risk Rating over the evaluation period.

Table 2.14

Annual Default Rate Correlation between Industries

Industry	BUS	CON	MAN	NBUS	OTH	RES	RET	SOP	TOU	TRS	WHS
Overall	0.55	0.72	0.92	0.67	0.50	0.32	0.86	0.37	0.63	0.50	0.76
BUS	1.00	0.59	0.64	0.16	0.06	0.51	0.23	0.91	0.10	-0.05	0.08
CON		1.00	0.74	0.51	0.51	0.28	0.45	0.45	0.17	0.25	0.36
MAN			1.00	0.57	0.44	0.29	0.68	0.49	0.37	0.40	0.61
NBUS				1.00	0.26	0.31	0.43	-0.06	0.16	0.56	0.76
OTH					1.00	-0.08	0.60	0.04	0.27	0.17	0.33
RES						1.00	0.05	0.39	0.15	-0.32	0.27
RET							1.00	0.16	0.80	0.44	0.66
SOP								1.00	0.01	-0.22	-0.10
TOU									1.00	0.33	0.57
TRS										1.00	0.39

Table 2.14 provides the correlation between the annual default rates across Industries. In addition, we provide the correlation between a given Industry's default rate and that of the overall portfolio as given in the second row. The industries, in alphabetical order, are as follows: Business Services (BUS); Construction (CON); Manufacturing (MAN); Non-Business Services (NBUS); Resources (RES); Retail (RET); Supplier or Premises (SOP); Tourism (TOU); Transportation and Storage (TRS); Wholesale (WHS); Other (OTH). Correlations are measured over an evaluation period of 1997-2010. See Table 2.11 and Figure 2.11 for more information on the time series of default rates by Industry over the evaluation period.

Table 2.15

Annual Default Rate Correlation between Size Buckets

Size Buckets ('000)	≤ $100	$100 - $250	$250 - $1000	$1000 - $3000	$3000 - $5000	> $5000
Overall	0.63	0.75	0.86	0.20	0.39	-0.26
≤ $100	1.00	0.10	0.59	0.10	-0.27	-0.23
$100 - $250		1.00	0.42	0.21	0.69	0.06
$250 - $1000			1.00	0.03	0.26	-0.59
$1000 - $3000				1.00	0.50	0.51
$3000 - $5000					1.00	0.03
> $5000						1.00

Table 2.15 provides the correlation between the annual default rates across Size Buckets. In addition, we provide the correlation between a given Industry's default rate and that of the overall portfolio as given in the second row. Size Buckets range from ≤$100,000 to >$5,000,000 and are based on the total commitment to a borrower (including commitment to other "related" borrowers under the same ownership) at last authorization. Correlations are measured over an evaluation period of 1997-2010. See Table 2.12 and Figure 2.12 for more information on the time series of default rates by Industry over the evaluation period.

Table 3.1

Basel II Standardized Approach Risk Weights

	Retail ("Other") Exposures	Corporate Exposures				
Credit Assessment		AAA to AA-	A+ to A-	BBB+ to BB-	Below BB-	Unrated
Risk Weight	75%	20%	50%	100%	150%	100%

Table 3.1: As seen on page 40 of *OSFI Capital Adequacy Requirement as of January 2011* guidelines.

Table 3.2

Borrower Size Buckets and Annual Sales

Size Bucket ('000)	Loans (%)	$OS (%)	PD	MtM	Sales $ (m)
≤ $100	20%	2%	8.3%	42	$3.6
$100 - $250	23%	6%	4.7%	56	
$250 - $1000	31%	21%	3.2%	74	$4.3 - $5.4
$1000 - $3000	5%	31%	2.6%	97	$12.7
$3000 - $5000	17%	16%	1.9%	116	
> $5000	4%	24%	1.3%	132	$46.1
Overall	100%	100%	4.6%	61	$5.2

Table 3.2 presents summary information on the March 2009 the *Financing Company* portfolio by Size Bucket. In particular, we note in the sixth column average annual sales (in $ millions) by Size Bucket, as well as the Months-to-Maturity in the fifth column for each Size Bucket, and in the fourth column, the weighted average annual default rate – identified as the stationary Probability of Default – for each Size Bucket (as calculated in Chapter 2). In addition, columns two and three show the percentage of the overall March 2009 portfolio accounted for by each Size Bucket, both in terms of number of loans (column two) and $ Outstanding (column three).

Table 3.3

Probabilities of Default by Risk Rating and Size Bucket

Risk Rating	Probability of Default						
	Size Bucket ('000)						Overall
	≤ $100	$100 - $250	$250 - $1000	$1000 - $3000	$3000 - $5000	> $5000	
1	2.1%	1.1%	0.4%	0.4%	0.2%	0.2%	0.7%
2	2.6%	1.2%	0.9%	0.6%	1.1%	0.2%	1.1%
3	3.2%	2.2%	1.3%	1.0%	0.7%	0.6%	1.7%
4	4.2%	2.7%	1.6%	1.3%	1.4%	0.9%	2.1%
5	4.5%	2.9%	1.8%	1.9%	1.3%	0.8%	2.4%
6	5.6%	3.6%	2.8%	2.0%	2.1%	0.9%	3.2%
7	8.2%	4.0%	3.5%	3.3%	2.5%	1.7%	4.6%
8	10.0%	6.3%	6.1%	6.5%	6.8%	6.5%	7.3%
9	13.2%	12.5%	10.6%	14.3%	8.1%	7.5%	12.7%
Overall	8.3%	4.6%	3.3%	2.6%	2.0%	1.2%	4.6%

Table 3.3 presents the Probabilities of Default as calculated over a period spanning 14 years starting in January 1997 and ending in December 2010. Defaults were aggregated over a given calendar year and then divided over the count of healthy customers at the start of the year. A weighted average of default rates over the time period then gives the PDs presented above. In addition, defaults were segregated by Risk Rating and Size as defined in Chapter 2.

Table 3.4

Probabilities of Default by Risk Rating and Industry

	Probability of Default									
Industry	Risk Rating									Overall
	1	2	3	4	5	6	7	8	9	
BUS	1.0%	2.3%	2.0%	2.6%	3.3%	4.4%	6.6%	8.7%	11.3%	5.6%
CON	0.8%	1.8%	2.0%	2.3%	2.6%	3.2%	4.0%	5.9%	10.6%	4.0%
MAN	0.6%	1.1%	1.9%	2.7%	3.0%	4.3%	6.6%	11.5%	15.4%	5.6%
NBUS	0.7%	0.8%	1.2%	1.4%	2.0%	2.2%	3.4%	5.6%	11.1%	3.8%
OTH	1.3%	0.7%	1.2%	2.3%	2.1%	3.3%	4.7%	7.0%	13.8%	5.0%
RES	0.5%	0.2%	1.0%	2.7%	2.5%	4.0%	4.8%	6.5%	11.6%	4.5%
RET	0.6%	1.0%	1.8%	1.9%	2.0%	2.4%	4.2%	6.5%	12.5%	4.1%
SOP	0.5%	0.3%	0.3%	0.1%	0.8%	1.3%	2.3%	3.0%	8.5%	2.3%
TOU	0.8%	0.7%	1.6%	1.3%	1.5%	2.4%	3.9%	6.2%	13.4%	4.4%
TRS	0.9%	0.8%	1.9%	2.2%	2.4%	4.2%	4.7%	7.8%	10.6%	4.7%
WHS	0.9%	0.8%	1.7%	2.5%	3.0%	3.8%	5.0%	8.8%	14.0%	4.8%
Overall	0.7%	1.1%	1.7%	2.1%	2.4%	3.2%	4.6%	7.3%	12.7%	4.6%

Table 3.4 depicts the Probability of Default for each Industry-Risk Rating segment, with Risk Ratings ranging from 1.0 (least risky) to 5.0 (most risky). Probabilities of Default and their standard of deviations are calculated over a period spanning 14 years starting in January 1997 and ending in December 2010. Defaults were aggregated over a given calendar year and then divided over the count of healthy customers at the start of the year. A weighted average of default rates over the time period then gives the PDs presented above. In addition, defaults were segregated by Risk Rating and Industry as defined in Chapter 2.

Table 3.5

Probabilities of Default by Industry and Size Bucket

	Probability of Default						
	Size Bucket ('000)						
Industry	**≤$100**	**$100 - $250**	**$250 - $1000**	**$1000 - $3000**	**$3000 - $5000**	**>$5000**	**Overall**
BUS	8.2%	4.9%	3.3%	2.8%	2.5%	3.7%	5.6%
CON	6.7%	3.9%	3.2%	2.0%	1.9%	0.4%	4.0%
MAN	10.3%	6.2%	4.5%	3.5%	3.1%	2.2%	5.6%
NBUS	6.9%	3.4%	2.1%	1.9%	2.1%	0.3%	3.8%
OTH	8.7%	4.7%	3.1%	1.7%	2.0%	1.2%	5.0%
RES	6.4%	4.8%	4.4%	2.0%	1.7%	1.2%	4.5%
RET	8.1%	3.8%	2.5%	1.5%	0.6%	0.4%	4.1%
SOP	2.5%	2.8%	2.4%	1.8%	0.3%	0.5%	2.3%
TOU	7.8%	4.4%	3.3%	3.5%	0.9%	1.5%	4.4%
TRS	8.3%	5.7%	3.5%	2.9%	3.2%	1.2%	4.7%
WHS	9.8%	5.2%	2.6%	1.6%	0.8%	0.6%	4.8%
Overall	8.3%	4.6%	3.3%	2.6%	2.0%	1.2%	4.6%

Table 3.5 depicts the Probability of Default for each Industry-Size Bucket segment. Probabilities of Default and their standard of deviations are calculated over a period spanning 14 years starting in January 1997 and ending in December 2010. Defaults were aggregated over a given calendar year and then divided over the count of healthy customers at the start of the year. A weighted average of default rates over the time period then gives the PDs presented above. In addition, defaults were segregated by Risk Rating and Industry as defined in Chapter 2.

Table 3.6

Average Asset Correlations under Basel II RR-calibrated Partial Implementations

Average Asset Correlation (%)										
Case 2: AIRB										
Size Bucket ('000)	**Risk Rating**									**Overall**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	
≤ $100	11.8	10.3	8.1	7.1	6.5	5.2	4.0	3.2	3.0	4.9
$100 - $250	11.8	10.3	8.1	7.1	6.5	5.2	4.0	3.2	3.0	5.7
$250 - $1000	11.8	10.3	8.1	7.1	6.5	5.2	4.0	3.2	3.0	6.2
$1000 - $3000	16.8	15.5	13.5	12.6	12.1	10.8	9.6	8.8	8.5	12.3
$3000 - $5000	16.8	15.5	13.5	12.6	12.1	10.8	9.6	8.8	8.5	12.5
> $5000	19.2	17.9	15.9	15.0	14.4	13.2	12.0	11.1	10.9	15.4
Overall	14.2	12.5	9.9	8.9	8.1	7.1	5.4	4.0	3.3	7.5
Case 3: Naïve AIRB										
≤ $100	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	13.9
$100 - $250	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	14.8
$250 - $1000	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	15.2
$1000 - $3000	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	15.8
$3000 - $5000	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	16.0
> $5000	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	16.6
Overall	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	15.0
Case 4: S-AIRB										
≤ $100	16.4	15.1	13.1	12.1	11.6	10.4	9.2	8.3	8.0	9.9
$100 - $250	16.4	15.1	13.1	12.1	11.6	10.4	9.2	8.3	8.0	10.8
$250 - $1000	16.4	15.1	13.1	12.1	11.6	10.4	9.2	8.3	8.0	11.2
$1000 - $3000	16.8	15.5	13.5	12.6	12.1	10.8	9.6	8.8	8.5	12.3
$3000 - $5000	16.8	15.5	13.5	12.6	12.1	10.8	9.6	8.8	8.5	12.5
> $5000	19.2	17.9	15.9	15.0	14.4	13.2	12.0	11.1	10.9	15.4
Overall	16.8	15.4	13.4	12.4	11.8	10.6	9.4	8.4	8.1	11.3
Case 5: R-AIRB										
≤ $100	11.8	10.3	8.1	7.1	6.5	5.2	4.0	3.2	3.0	4.9
$100 - $250	11.8	10.3	8.1	7.1	6.5	5.2	4.0	3.2	3.0	5.7
$250 - $1000	11.8	10.3	8.1	7.1	6.5	5.2	4.0	3.2	3.0	6.2
$1000 - $3000	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	15.8
$3000 - $5000	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	16.0
> $5000	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	16.6
Overall	15.5	13.7	10.9	9.9	9.0	8.1	6.2	4.4	3.5	8.3

Table 3.6 provides a description of average asset correlations over loans by Risk Rating and Size Bucket. Under all implementations presented above Probabilities of Default (PDs) are calibrated by Risk Rating.

Table 3.7

Average Asset Correlations Comparison using PDs calibrated by RR-SB

Average Asset Correlation (%)										
Case 2: AIRB										
Size Bucket ('000)	**Risk Rating**									**Overall**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	
≤ $100	11.8	10.3	8.1	7.1	6.5	5.2	4.0	3.2	3.0	4.9
$100 - $250	11.8	10.3	8.1	7.1	6.5	5.2	4.0	3.2	3.0	5.7
$250 - $1000	11.8	10.3	8.1	7.1	6.5	5.2	4.0	3.2	3.0	6.2
$1000 - $3000	16.8	15.5	13.5	12.6	12.1	10.8	9.6	8.8	8.5	12.3
$3000 - $5000	16.8	15.5	13.5	12.6	12.1	10.8	9.6	8.8	8.5	12.5
> $5000	19.2	17.9	15.9	15.0	14.4	13.2	12.0	11.1	10.9	15.4
Overall	14.2	12.5	9.9	8.9	8.1	7.1	5.4	4.0	3.3	7.5
Case 3: Naïve AIRB (RR)										
≤ $100	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	13.9
$100 - $250	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	14.8
$250 - $1000	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	15.2
$1000 - $3000	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	15.8
$3000 - $5000	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	16.0
> $5000	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	16.6
Overall	20.4	19.1	17.1	16.1	15.6	14.4	13.2	12.3	12.0	15.0
Case 7a: (RR-SB)										
≤ $100	16.2	15.3	14.5	13.5	13.3	12.7	12.2	12.1	12.0	12.8
$100 - $250	18.8	18.5	16.0	15.1	14.8	13.9	13.6	12.5	12.0	14.4
$250 - $1000	22.0	19.8	18.2	17.5	16.8	15.0	14.1	12.6	12.1	16.1
$1000 - $3000	22.1	20.8	19.2	18.1	16.7	16.4	14.4	12.5	12.0	17.3
$3000 - $5000	23.1	18.8	20.4	17.9	18.4	16.2	15.5	12.4	12.2	17.9
> $5000	23.1	22.6	21.1	19.8	20.2	19.7	17.1	12.5	12.3	20.2
Overall	21.0	19.5	17.6	16.6	16.1	15.1	13.8	12.4	12.0	15.5
Case 7d: (RR-SB)										
≤ $100	7.2	6.2	5.3	4.3	4.1	3.6	3.1	3.0	3.0	3.7
$100 - $250	10.1	9.7	7.0	6.0	5.6	4.8	4.4	3.4	3.0	5.3
$250 - $1000	13.7	11.1	9.3	8.6	7.9	5.9	4.9	3.4	3.0	7.1
$1000 - $3000	22.1	20.8	19.2	18.1	16.7	16.4	14.4	12.5	12.0	17.3
$3000 - $5000	23.1	18.8	20.4	17.9	18.4	16.2	15.5	12.4	12.2	17.9
> $5000	23.1	22.6	21.1	19.8	20.2	19.7	17.1	12.5	12.3	20.2
Overall	16.1	14.2	11.3	10.4	9.5	8.9	6.9	4.5	3.5	8.7

Table 3.7 describes average asset correlations calculated over loans in Risk Rating and Size Bucket segments. The top two panels replicate results presented in Table 3.6 for average asset correlations under the Case 2 and Case 3 partial implementations. The bottom two panels present results under two implementations using Probabilities of Default (PDs) calibrated by Risk Rating and Size Bucket.

Table 3.8

Basel IRB Implementations on the *Financing Company* Portfolio

Method	Size Bucket ('000)						
	Portfolio	≤$100	$100 - $250	$250 - $1,000	$1,000 - $3,000	$3,000 - $5,000	>$5000
SA	7.4%	6.0%	6.0%	6.0%	8.0%	8.0%	8.0%
Case 1a: FIRB	8.8%	7.3%	5.7%	4.3%	10.1%	10.0%	11.3%
Case 1b: FIRB (M)	7.4%	7.3%	5.7%	4.3%	8.2%	8.0%	9.0%
Case 2: AIRB	8.1%	7.1%	5.4%	4.0%	9.2%	9.1%	10.3%
Case 3: Naïve AIRB	8.5%	16.6%	12.2%	9.0%	8.1%	7.9%	7.4%
Case 4: S-AIRB	6.9%	12.7%	9.3%	6.8%	6.4%	6.3%	6.9%
Case 5: R-AIRB	6.9%	7.1%	5.4%	4.0%	8.1%	7.9%	7.4%
Case 6a: M-AIRB (M)	12.2%	20.8%	16.4%	12.6%	11.7%	11.5%	11.1%
Case 6b: M-AIRB (Full)	20.2%	21.7%	19.3%	19.0%	20.1%	21.0%	20.9%
Case 7a: PD by RR-SB	7.2%	18.8%	12.4%	8.2%	7.0%	6.5%	4.7%
Case 7d: R-PD by RR-SB	5.6%	7.8%	5.4%	3.8%	7.0%	6.5%	4.7%

Table 3.8 gives the resultant risk capital given various implementations and variations of the Basel II IRB framework. Capital charges are in excess of Expected Loss (EL). EL figures are based on the use of given PD and LGD risk components, respective to the implementation. That is to say, in Cases 1a and 1b (the Foundation IRB cases), pre-set fixed LGDs of 45% for secured loans, and 75% for unsecured loans, in conjunction with the internally estimated "average" PDs (by risk rating) give the EL (%) for each loan. This percentage EL is then multiplied by the $OS (which we use as a proxy for the EAD) and summed for all loans to obtain the values given above. For all other cases, the *Financing Company* downturn LGDs of 73% and 41% were used. For Case 1a, instead of using the standard 2.5 year M as specified in Basel II we follow OSFI convention and use minimum of the actual loan maturity and 5 years. The SA approach classifies borrowers as either Retail or Corporate, with Retail borrowers taking a capital charge of 6%, equal to 75% x 8%, and the Corporate borrowers a capital charge of 100% x 8%; see Table 3.1 for SA risk weights.

Table 3.9

Capital Charges under Case 2 by Risk Rating and Size Bucket

Size Bucket ('000)	Capital Charges (%) under Case 2									
	Risk Rating									Overall
	1	2	3	4	5	6	7	8	9	
≤ $100	4.0	4.7	5.5	5.7	5.9	5.6	6.2	7.2	9.7	7.1
$100 - $250	3.4	4.1	4.8	5.0	4.9	4.9	4.9	5.8	8.6	5.4
$250 - $1000	2.8	3.2	3.7	3.9	3.9	4.0	4.1	4.6	6.4	4.0
$1000 - $3000	6.9	7.6	8.6	9.0	9.3	9.6	10.3	11.7	14.1	9.2
$3000 - $5000	6.8	7.7	8.6	8.8	9.0	9.6	10.7	12.2	14.2	9.1
> $5000	7.7	9.3	10.2	10.5	10.7	11.4	12.2	14.1	20.4	10.3
Overall	6.6	7.3	8.0	8.2	8.1	8.5	8.6	9.1	10.4	8.1

Table 3.9 presents capital charges, as a dollar-weighted percentage of $OS, across portfolio Risk Rating and Size Bucket segments, calculated under the full Basel II implementation (Case 2); see Section 3.3.

Table 4.1

Probability of Default by Risk and Size Group

Probability of Default (%)						
Size Group ('000)	**Risk Group**					**Overall**
	1-3	**4-5**	**6**	**7**	**8-9**	
≤$100	2.81	4.34	5.59	8.15	11.39	8.32
$100 - $250	1.71	2.85	3.63	4.01	7.71	4.58
$250 - $1000	1.00	1.72	2.78	3.50	6.68	3.25
GT $1000	0.68	1.49	1.92	3.02	7.43	2.37
Overall	1.30	2.29	3.24	4.63	8.75	4.56
Variance (%)						
≤$100	0.0056	0.0079	0.0087	0.0306	0.0369	0.0080
$100 - $250	0.0012	0.0084	0.0123	0.0147	0.0418	0.0071
$250 - $1000	0.0010	0.0039	0.0024	0.0105	0.0403	0.0032
GT $1000	0.0005	0.0021	0.0017	0.0023	0.0186	0.0021
Overall	0.0011	0.0045	0.0063	0.0125	0.0237	0.0031
Normalized Standard Deviation						
≤$100	0.27	0.20	0.17	0.21	0.17	0.11
$100 - $250	0.20	0.32	0.31	0.30	0.27	0.18
$250 - $1000	0.31	0.36	0.18	0.29	0.30	0.17
GT $1000	0.35	0.31	0.22	0.16	0.18	0.20
Overall	0.26	0.29	0.24	0.24	0.18	0.12

Table 4.1 presents the Probabilities of Default as calculated over a period spanning 14 years starting in January 1997 and ending in December 2010. Defaults were aggregated over a given calendar year and then divided over the count of healthy customers at the start of the year. A healthy-weighted average of default rates over the time period then gives the PDs presented above. Results are presented according to Risk and Size Groups; see Section 4.2 for definitions and descriptions. PD Variance is calculated according to Equation (4.11). Normalized standard deviations in the third panel are calculated as the ratio of the square root of the variance (second panel) over the PD (first panel).

Table 4.1A

Auxiliary Table A – Probabilities of Default by Risk and Size Group

Size Group ('000)	**Risk Rating**									**Overall**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	
Probability of Default (%)										
≤$100	2.10	2.58	3.18	4.16	4.47	5.59	8.15	10.03	13.19	8.32
$100 - $250	1.13	1.24	2.19	2.72	2.93	3.63	4.01	6.25	12.50	4.58
$250 - $1000	0.36	0.87	1.33	1.56	1.82	2.78	3.50	6.05	10.64	3.25
$1000 - $3000	0.35	0.63	1.02	1.34	1.88	2.02	3.26	6.50	14.25	2.63
$3000 - $5000	0.16	1.12	0.71	1.42	1.27	2.10	2.49	6.79	8.15	2.00
> $5000	0.16	0.24	0.55	0.86	0.77	0.88	1.71	6.46	7.50	1.24
Overall	0.72	1.06	1.72	2.13	2.41	3.24	4.63	7.30	12.68	4.56
Variance (%)										
≤$100	0.0149	0.0177	0.0101	0.0032	0.0110	0.0087	0.0306	0.0457	1.4767	0.0080
$100 - $250	0.0071	0.0021	0.0060	0.0024	0.0128	0.0123	0.0147	0.0370	0.3753	0.0071
$250 - $1000	0.0021	0.0015	0.0034	0.0020	0.0055	0.0024	0.0105	0.0375	1.0265	0.0032
$1000 - $3000	0.0009	0.0025	0.0031	0.0033	0.0013	0.0007	0.0033	0.0189	1.3482	0.0028
$3000 - $5000	0.0007	0.0061	0.0048	0.0167	0.0118	0.0135	0.0119	0.0615	13.1410	0.0033
> $5000	0.0014	0.0019	0.0040	0.0113	0.0006	0.0115	0.0300	0.2388	9.4413	0.0004
Overall	0.0005	0.0008	0.0026	0.0023	0.0062	0.0063	0.0125	0.0293	0.6669	0.0031
Normalized Standard Deviation										
≤$100	0.58	0.52	0.32	0.14	0.23	0.17	0.21	0.21	0.92	0.11
$100 - $250	0.75	0.37	0.35	0.18	0.39	0.31	0.30	0.31	0.49	0.18
$250 - $1000	1.26	0.44	0.44	0.29	0.41	0.18	0.29	0.32	0.95	0.17
$1000 - $3000	0.85	0.79	0.54	0.43	0.19	0.13	0.18	0.21	0.81	0.20
$3000 - $5000	1.67	0.70	0.97	0.91	0.86	0.55	0.44	0.37	4.45	0.29
> $5000	2.36	1.82	1.14	1.24	0.33	1.21	1.01	0.76	4.10	0.16
Overall	0.31	0.26	0.30	0.23	0.33	0.24	0.24	0.23	0.64	0.12

Table 4.1A is analogous to Table 4.1 and presents Probabilities of Default along with PD Variance and normalized standard deviation. Results are presented according to Risk and Size Groups; see Section 4.2 for definitions and descriptions. PD Variance is calculated according to Equation (4.11). Normalized standard deviations in the third panel are calculated as the ratio of the square root of the variance (second panel) over the PD (first panel).

Table 4.1B

Auxiliary Table B – Probabilities of Default by Size and Risk Group

Size Group ('000)	Risk Group				Overall
	1-3	**4-5**	**6-7**	**8-9**	
Probability of Default (%)					
≤$100	2.81	4.34	7.44	11.39	8.32
$100 - $250	1.71	2.85	3.88	7.71	4.58
$250 - $1000	0.85	1.63	3.01	6.86	2.94
Overall	1.30	2.29	4.14	8.75	4.56
Variance (%)					
≤$100	0.0056	0.0079	0.0230	0.0369	0.0080
$100 - $250	0.0012	0.0084	0.0138	0.0418	0.0071
$250 - $1000	0.0007	0.0028	0.0278	0.0278	0.0019
Overall	0.0011	0.0045	0.0088	0.0237	0.0031
Normalized Standard Deviation					
≤$100	0.27	0.20	0.20	0.17	0.11
$100 - $250	0.20	0.32	0.30	0.27	0.18
$250 - $1000	0.32	0.33	0.55	0.24	0.15
Overall	0.26	0.29	0.23	0.18	0.12

Table 4.1B is analogous to Table 4.1 and presents Probabilities of Default along with PD Variance and normalized standard deviation. Results are presented according to Risk and Size Groups; see Section 4.2 for definitions and descriptions. PD Variance is calculated according to Equation (4.11). Normalized standard deviations in the third panel are calculated as the ratio of the square root of the variance (second panel) over the PD (first panel).

Table 4.1C

Auxiliary Table C – Probabilities of Default by Size and Risk Group

Size Group ('000)	Risk Group				Overall
	1-3	4-5	6-7	8-9	
Probability of Default (%)					
≤ $100	2.81	4.34	7.44	11.39	8.32
$100 - $250	1.71	2.85	3.88	7.71	4.58
$250 - $1000	1.00	1.72	3.23	6.68	3.25
> $1000	0.68	1.49	2.53	7.43	2.37
Overall	1.30	2.29	4.14	8.75	4.56
Variance (%)					
≤ $100	0.0056	0.0079	0.0230	0.0369	0.0080
$100 - $250	0.0012	0.0084	0.0138	0.0418	0.0071
$250 - $1000	0.0010	0.0039	0.0054	0.0403	0.0032
> $1000	0.0005	0.0021	0.0013	0.0186	0.0021
Overall	0.0011	0.0045	0.0088	0.0237	0.0031
Normalized Standard Deviation					
≤ $100	0.27	0.20	0.20	0.17	0.11
$100 - $250	0.20	0.32	0.30	0.27	0.18
$250 - $1000	0.31	0.36	0.23	0.30	0.17
> $1000	0.35	0.31	0.14	0.18	0.20
Overall	0.26	0.29	0.23	0.18	0.12

Table 4.1C is analogous to Table 4.1 and presents Probabilities of Default along with PD Variance and normalized standard deviation. Results are presented according to Risk and Size Groups; see Section 4.2 for definitions and descriptions. PD Variance is calculated according to Equation (4.11). Normalized standard deviations in the third panel are calculated as the ratio of the square root of the variance (second panel) over the PD (first panel).

Table 4.2

Internally Calibrated Asset and Default Correlations by Size and Risk Group

Size Group ('000)	**Risk Group**					**Overall**
	1-3	**4-5**	**6**	**7**	**8-9**	
Asset Correlation (%)						
≤ $100	1.32	0.92	0.68	1.33	0.99	0.34
$100 - $250	0.67	1.92	1.88	1.92	1.96	0.77
$250 - $1000	1.31	2.06	0.58	1.70	2.34	0.60
> $1000	1.46	1.45	0.77	0.49	0.93	0.68
Overall	0.98	1.49	1.17	1.30	0.93	0.34
Default Correlation (%)						
≤ $100	0.21	0.19	0.17	0.41	0.37	0.10
$100 - $250	0.07	0.31	0.35	0.38	0.59	0.16
$250 - $1000	0.10	0.23	0.09	0.31	0.65	0.10
> $1000	0.08	0.14	0.09	0.08	0.27	0.09
Overall	0.09	0.20	0.20	0.28	0.30	0.07

Table 4.2 presents asset and default correlations calculated by Risk and Size Group segments. Asset and default correlations correspond to PD and PD variance values given in Table 4.1. For calculation methodology see Subsection 4.1.1; for discussion see Subsection 4.2.2.

Table 4.2A

Auxiliary Table A – Internally Calibrated Correlations by Size and Risk Group

Asset Correlation (%)										
Size Group ('000)	**Risk Rating**									**Overall**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	
≤$100	5.27	4.53	1.92	0.40	1.22	0.68	1.33	1.46	27.49	0.34
$100 - $250	6.83	1.94	2.11	0.60	2.74	1.88	1.92	2.38	8.39	0.77
$250 - $1000	11.72	2.50	2.72	1.29	2.59	0.58	1.70	2.53	25.32	0.60
$1000 - $3000	6.42	6.49	3.79	2.60	0.59	0.28	0.61	1.16	23.38	0.73
$3000 - $5000	14.39	6.00	9.30	10.08	8.84	4.80	3.28	3.44	99.99	1.37
> $5000	21.25	17.28	11.10	14.10	1.37	13.73	12.69	12.99	99.99	0.40
Overall	1.24	0.93	1.41	0.88	1.85	1.17	1.30	1.50	14.12	0.34
Default Correlation (%)										
≤$100	0.73	0.71	0.33	0.08	0.26	0.17	0.41	0.51	12.89	0.10
$100 - $250	0.64	0.17	0.28	0.09	0.45	0.35	0.38	0.63	3.43	0.16
$250 - $1000	0.58	0.17	0.26	0.13	0.31	0.09	0.31	0.66	10.79	0.10
$1000 - $3000	0.26	0.40	0.31	0.25	0.07	0.03	0.10	0.31	11.03	0.11
$3000 - $5000	0.46	0.55	0.68	1.19	0.94	0.66	0.49	0.97	98.86	0.17
> $5000	0.88	0.80	0.72	1.33	0.08	1.31	1.78	3.95	98.85	0.03
Overall	0.07	0.07	0.16	0.11	0.26	0.20	0.28	0.43	6.02	0.34

Table 4.2A is analogous to Table 4.2 and presents asset and default correlations calculated by Risk and Size Group segments. Asset and default correlations correspond to PD and PD variance values given in Table 4.1A. For calculation methodology see Subsection 4.1.1; for discussion see Subsection 4.2.2.

Table 4.2B

Auxiliary Table B – Internally Calibrated Correlations by Size and Risk Group

Size Group ('000)	Risk Group				Overall
	1-3	4-5	6-7	8-9	
Asset Correlation (%)					
≤$100	1.32	0.92	1.15	0.99	0.34
$100 - $250	0.67	1.92	1.89	1.96	0.77
$250 - $1000	1.31	2.06	1.02	2.34	0.60
>$1000	1.46	1.45	0.36	0.93	0.68
Overall	0.98	1.49	1.11	0.93	0.34
Default Correlation (%)					
≤$100	0.21	0.19	0.33	0.37	0.10
$100 - $250	0.07	0.31	0.37	0.59	0.16
$250 - $1000	0.10	0.23	0.17	0.65	0.10
>$1000	0.08	0.14	0.05	0.27	0.09
Overall	0.09	0.20	0.22	0.30	0.07

Table 4.2B is analogous to Table 4.2 and presents asset and default correlations calculated by Risk and Size Group segments. Asset and default correlations correspond to PD and PD variance values given in Table 4.1B. For calculation methodology see Subsection 4.1.1; for discussion see Subsection 4.2.2.

Table 4.2C

Auxiliary Table C – Internally Calibrated Correlations by Size and Risk Group

Size Group ('000)	Risk Group				Overall
	1-3	**4-5**	**6-7**	**8-9**	
Asset Correlation (%)					
≤$100	1.32	0.92	1.15	0.99	0.34
$100 - $250	0.67	1.92	1.89	1.96	0.77
$250 - $1000	1.32	1.64	5.44	1.56	0.43
Overall	0.98	1.49	1.11	0.93	0.34
Default Correlation (%)					
≤$100	0.21	0.19	0.33	0.37	0.10
$100 - $250	0.07	0.31	0.37	0.59	0.16
$250 - $1000	0.09	0.18	0.95	0.43	0.07
Overall	0.09	0.20	0.22	0.30	0.07

Table 4.2C is analogous to Table 4.2 and presents asset and default correlations calculated by Risk and Size Group segments. Asset and default correlations correspond to PD and PD variance values given in Table 4.1C. For calculation methodology see Subsection 4.1.1; for discussion see Subsection 4.2.2.

Table 4.3

Asset Correlations Derived from Various Data Sources

Source Study	**Default data source**	**Results (%)**
Gordy (2000, Table 2)	S&P	1.5 - 12.5
Cespedes (2002)	Moody's	10
Hamerle, Liebig, and Roesch (2003a)	Unknown	Max 2.3
Hamerle, Liebig, and Roesch (2003b)	S&P 1982-1999	0.4-6.04
Frey and McNeil (2003, Table 1)	S&P 1981-2000	6.5-6.9-9.1
Dietsch and Petey (2004)	Coface 1994-2001	0.12-10.72
Jobst and De Servigny (2005)	S&P 1981-2003	4.7-14.6
Duellman and Scheule (2003)	DB 1987-2000	0.5-6.4
Jakubik (2006)	BF 1988-2003	5.7
Source Study	**Asset data source**	**Results (%)**
Duellmann, Scheicher, and Schmieder (2008)	MKMV Credit Monitor	10.2
Zeng and Zhang (2001)	MKMV source	9.46 - 19.98
Akhavein, Kocagil, and Neugebauer (2005)	Equity	20.92 - 24.09
Lopez (2002)	MKMV Portfolio Manager	11.25
de Servigny and Renault (2002)	Equity	6

Table 4.3 replicates asset correlation results presented in Chernih, Henrard, and Vanduffel (2010, p. 53) Tables 1 and 2. Results show a large discrepancy between asset correlation results generated from market equity data and those generated from default data sources; see Section 4.2 for further details. S&P: Standard and Poor's; DB: Deutsche Bundesbank; BF: Bank of Finland; MKMV: Moody's KMV.

Table 4.4

Financing Company SME PDs and Ratings as Compared to S&P PDs and Ratings

One Year Default Rates, Average, Standard Deviation, Normalized SD

S&P	Mean	Std Dev	Norm SD	FC RG	Mean	Std Dev	Norm SD
AAA	0.00%	0.00%					
AA+	0.00%	0.00%					
AA	0.01%	0.08%	8.0				
AA-	0.03%	0.10%	3.3				
A+	0.05%	0.15%	3.0				
A	0.07%	0.14%	2.0				
A-	0.07%	0.02%	0.3				
BBB+	0.16%	0.32%	2.0				
BBB	0.26%	0.35%	1.3				
BBB-	0.31%	0.47%	1.5				
BB+	0.67%	0.96%	1.4				
BB	0.88%	0.83%	0.9				
BB-	1.47%	1.79%	1.2	**1-3**	1.30%	0.34%	0.3
B+	2.47%	2.12%	0.9	**4-5**	2.29%	0.67%	0.3
				6	3.24%	0.79%	0.2
				7	4.63%	1.12%	0.2
B	7.17%	4.62%	0.6				
				8-9	8.75%	1.54%	0.2
B-	9.99%	7.95%	0.8				
CCC/C	23.56%	12.69%	0.5				

Table 4.4 provides descriptive statistics for the *Financing Company* SME loans and Standard & Poor's (S&P) rated corporate debt. Statistics are given by rating for the Mean, Standard Deviation (Std Dev) and Normalized Standard Deviation (Norm SD). S&P statistics were measured over the 1921 – 2010 observation period while *Financing Company* (FC) statistics were measured over the 1997 – 2010 period. Results show significantly higher normalized standard deviations for the FC Risk Groups (RGs). Source: Default, Transition, and Recovery: 2010 Annual Global Corporate Default Study and Rating Transitions. Standard & Poor's RatingsDirect on the Global Credit Portal: March 30, 2011

Table 4.5

Average Asset Correlations under Partial Implementations

Method	Size Group ('000)				
	Overall	≤$100	$100 - $250	$250 - $1,000	>$1000
Case 1a: FIRB	7.4%	4.9%	5.7%	6.1%	12.7%
Case 1b: FIRB (M)	7.4%	4.9%	5.7%	6.1%	12.7%
Case 2: AIRB	7.4%	4.9%	5.7%	6.1%	12.7%
Case 3: Naïve AIRB	15.0%	14.0%	14.8%	15.1%	15.8%
Case 4: S-AIRB	11.2%	10.0%	10.8%	11.1%	12.7%
Case 5: R-AIRB	8.2%	4.9%	5.7%	6.1%	15.8%
Case 6a: M-AIRB (M)	15.0%	14.0%	14.8%	15.1%	15.8%
Case 6b: M-AIRB (Full)	15.0%	14.0%	14.8%	15.1%	15.8%
Case 7a: PD by RR-SB	15.4%	12.8%	14.3%	16.0%	17.7%
Case 7d: R-PD by RR-SB	11.6%	8.8%	10.3%	12.0%	17.7%

Table 4.5 restates Table 3.6 in Chapter 3 and uses Size Groups defined in Chapter 4 to present average asset correlations obtained under a Partial Implementation exercise using PD calibrated along Risk Groups instead of the original Risk Ratings as in Chapter 3.

Table 4.6

Restated Partial Implementation Capital Charge Results

Method	Size Group ('000)				
	Overall	≤$100	$100 - $250	$250 - $1,000	>$1000
SA	7.4%	6.0%	6.0%	6.0%	8.0%
Case 1a: FIRB	9.0%	7.0%	5.6%	4.4%	10.6%
Case 1b: FIRB (M)	7.5%	7.0%	5.6%	4.4%	8.6%
Case 2: AIRB	8.2%	6.7%	5.3%	4.1%	9.8%
Case 3: Naïve AIRB	8.6%	15.9%	12.1%	9.1%	8.0%
Case 4: S-AIRB	7.0%	12.1%	9.2%	6.9%	6.7%
Case 5: R-AIRB	7.0%	6.7%	5.3%	4.1%	8.0%
Case 6a: M-AIRB (M)	12.3%	20.2%	16.3%	12.7%	11.6%
Case 6b: M-AIRB (Full)	20.4%	21.1%	19.2%	19.2%	20.8%
Case 7a: PD by RG-SG	7.5%	18.7%	12.2%	8.2%	6.6%
Case 7d: R-PD by RG-SG	6.1%	14.2%	9.2%	6.3%	6.6%

Table 4.6 is analogous to Table 3.8 and restates capital charges obtained in Chapter 3 under the Partial Implementation exercise. In Table 4.6 we use PDs calibrated along Risk Groups, as well as Risk and Size Groups, defined in Chapter 4, and present results by Size Group.

Table 4.7

Boosted Asset Correlations by Risk and Size Group

Size Group ('000)	Asset Correlation (%)					
	Risk Group					Overall
	1-3	4-5	6	7	8-9	
≤ $100	14.7	12.4	10.6	14.8	12.9	7.4
$100 - $250	10.5	17.3	17.2	17.3	17.4	11.3
$250 - $1000	14.7	17.8	9.8	16.5	18.6	9.9
> $1000	15.4	15.4	11.3	8.9	12.5	10.6
Overall	12.8	15.6	13.9	14.6	12.5	7.4
	Default Correlation (%)					
≤ $100	2.90	3.01	2.91	5.15	5.18	2.45
$100 - $250	1.42	3.58	4.04	4.30	6.06	2.80
$250 - $1000	1.55	2.79	1.76	3.77	6.11	1.97
> $1000	1.30	2.11	1.66	1.67	4.05	1.75
Overall	1.52	2.76	2.94	3.80	4.41	1.74

Table 4.7 is analogous to Table 3.6 and presents boosted asset correlation values. A bounded log odds adjustment method is applied to the original internally-calibrated correlations such that the overall (Overall) portfolio asset correlation is boosted from 0.21% to 7.4%, while maintaining existing patterns and relationships by Risk and Size Group. The 7.4% value corresponds to the average correlation across all loans under Case 2 in Section 3.4.

Table 4.8

Internally Calibrated Simulation Based Capital Charges vs. Basel II

Size Group ('000)	Risk Group					Overall
	1-3	4-5	6	7	8-9	
Capital Charges under Case 2 by RG and SG						
≤ $100	5.1%	5.8%	5.6%	6.2%	7.9%	6.7%
$100 - $250	4.4%	4.9%	4.9%	4.9%	6.6%	5.3%
$250 - $1000	3.5%	3.9%	4.0%	4.1%	5.1%	4.1%
> $1000	8.6%	9.7%	10.1%	10.8%	13.2%	9.8%
Overall	7.6%	8.2%	8.5%	8.6%	9.5%	8.2%
Capital Charges under Case 7b Boosted by RG and SG						
≤ $100	12.9%	14.7%	14.1%	23.2%	24.8%	20.2%
$100 - $250	6.0%	13.1%	14.5%	14.5%	21.5%	14.2%
$250 - $1000	4.5%	7.9%	6.3%	10.7%	16.4%	8.8%
> $1000	3.6%	5.9%	5.3%	5.8%	12.7%	5.5%
Overall	3.9%	6.8%	6.0%	8.0%	16.0%	7.0%
Capital Charges under Case 10 Boosted by RG and SG						
≤ $100	6.9%	12.0%	12.7%	16.9%	21.1%	16.1%
$100 - $250	5.9%	10.4%	11.1%	13.4%	17.6%	11.9%
$250 - $1000	4.6%	8.2%	9.1%	11.2%	13.6%	8.9%
> $1000	4.6%	7.8%	8.8%	11.0%	13.4%	7.5%
Overall	4.7%	8.1%	9.0%	11.3%	14.6%	8.2%

Table 4.8 presents Basel II capital charges (Case 2) and those obtained under Cases 7b and Case 10. Case 7b here represents a simulation-based implementation of the asset value model (*AVM*) using Probabilities of Default (PDs) and asset correlations calibrated to RG-SG. Case 10 here represents a simulation-based implementation of the *AVM* using PDs and asset correlations calibrated to RG. Results are presented for Risk and Size Group segments in the portfolio.

Table 4.9

Basel II and Simulation-Based Capital Charges Comparative Ratios

Ratio of Boosted Case 7b to Case 2 Capital Charges						
Size Group ('000)	**Risk Group**					**Overall**
	1-3	**4-5**	**6**	**7**	**8-9**	
≤ $100	2.5	2.5	2.5	3.8	3.1	3.0
$100 - $250	1.4	2.7	2.9	3.0	3.3	2.7
$250 - $1000	1.3	2.0	1.6	2.6	3.2	2.2
> $1000	0.4	0.6	0.5	0.5	1.0	0.6
Overall	0.5	0.8	0.7	0.9	1.7	0.9
Ratio of Boosted Case 10 to Case 2 Capital Charges						
≤ $100	1.4	2.1	2.3	2.7	2.7	2.4
$100 - $250	1.3	2.1	2.3	2.7	2.7	2.2
$250 - $1000	1.3	2.1	2.2	2.8	2.7	2.2
> $1000	0.5	0.8	0.9	1.0	1.0	0.8
Overall	0.6	1.0	1.1	1.3	1.5	1.0
Ratio of Boosted Case 10 to Case 7b Capital Charges						
≤ $100	1.9	1.2	1.1	1.4	1.2	1.3
$100 - $250	1.0	1.3	1.3	1.1	1.2	1.2
$250 - $1000	1.0	1.0	0.7	1.0	1.2	1.0
> $1000	0.8	0.8	0.6	0.5	0.9	0.7
Overall	0.8	0.8	0.7	0.7	1.1	0.9

Table 4.9 presents ratios of capital charges under various three portfolio credit risk models: the Basel II model (Case 2), and; the *AVM*s under Cases 7b and Case 10. Case 7b here represents a simulation-based implementation of the asset value model (*AVM*) using Probabilities of Default (PDs) and asset correlations calibrated to RG-SG. Case 10 here represents a simulation-based implementation of the *AVM* using PDs and asset correlations calibrated to RG. Results are presented for Risk and Size Group segments in the portfolio. Capital charges are described in Table 4.8.

Table 4.10

EC Charge Comparison using Correlations Calibrated by RG

Size Group (‘000)	**Simulation-based Implementation EC (%)**					**Overall**
	Risk Group					
	1-3	**4-5**	**6**	**7**	**8-9**	
≤ $100	0.91	1.92	2.08	2.92	3.78	2.76
$100 - $250	0.77	1.58	1.74	2.29	3.13	1.95
$250 - $1000	0.62	1.28	1.43	1.92	2.42	1.44
> $1000	0.69	1.34	1.53	2.15	2.63	1.32
Overall	0.69	1.35	1.52	2.11	2.73	1.41
	Asymptotic Implementation EC (%)					
≤ $100	0.91	1.93	2.02	2.96	3.83	2.79
$100 - $250	0.79	1.65	1.78	2.34	3.20	2.00
$250 - $1000	0.62	1.31	1.46	1.94	2.47	1.47
> $1000	0.60	1.23	1.38	1.90	2.43	1.18
Overall	0.62	1.28	1.42	1.96	2.66	1.33
	% Change in EC from Asymptotic to Simulation-based Implementation					
≤ $100	-0.2%	-0.7%	2.7%	-1.5%	-1.3%	-1.1%
$100 - $250	-3.1%	-4.6%	-2.6%	-2.1%	-2.1%	-2.7%
$250 - $1000	-0.1%	-2.3%	-1.6%	-1.0%	-2.1%	-1.6%
> $1000	14.9%	8.8%	11.0%	13.2%	8.5%	11.1%
Overall	11.5%	5.3%	7.2%	7.9%	2.7%	6.4%

Table 4.10 presents capital charges generated under a simulation-based implementation of the single factor model presented in Subsection 4.1.2 and using internally-calibrated asset correlations and PDs by RR. The top panel presents the simulation-based capital charges by Risk Rating and Size Bucket. The second panel presents capital charges under an equivalent calibration but using the asymptotic implementation of the single factor model. The bottom panel presents the percentage change in going from the asymptotic implementation to the simulation-based implementation. The figure in the bottom-right corner of the table gives the overall “granularity effect”, on EC, of explicitly modelling idiosyncratic factors.

Table 4.11

EC Charge Comparison using Correlations Calibrated by RG-SG

Size Group (‘000)	Simulation-based Implementation EC (%)					Overall
	Risk Group					
	1-3	4-5	6	7	8-9	
≤ $100	2.1	2.3	2.2	4.4	4.7	3.7
$100 - $250	0.8	2.3	2.7	2.6	4.5	2.6
$250 - $1000	0.6	1.2	0.9	1.8	3.6	1.5
> $1000	0.5	1.0	0.8	0.9	2.5	0.9
Overall	0.6	1.1	0.9	1.3	3.3	1.2
Size	**Asymptotic Implementation EC (%)**					
≤ $100	2.0	2.3	2.2	4.5	4.7	3.7
$100 - $250	0.8	2.3	2.6	2.7	4.6	2.7
$250 - $1000	0.6	1.3	0.8	1.9	3.6	1.6
> $1000	0.5	0.9	0.7	0.8	2.2	0.8
Overall	0.5	1.1	0.8	1.2	3.1	1.1
	% Change in EC from Asymptotic to Simulation-based Implementation					
≤ $100	2.1%	-0.9%	-0.5%	-1.5%	-0.2%	-0.3%
$100 - $250	-3.2%	-1.7%	1.3%	-2.8%	-3.2%	-2.4%
$250 - $1000	-3.5%	-4.5%	2.1%	-2.9%	-1.3%	-2.2%
> $1000	12.9%	10.3%	15.8%	17.7%	14.7%	13.5%
Overall	8.5%	4.8%	10.3%	5.4%	3.9%	5.7%

Table 4.11 presents capital charges generated under a simulation-based implementation of the single factor model presented in Subsection 4.1.2 and using internally-calibrated asset correlations and PDs by RR x Size. The top panel presents the simulation-based capital charges by Risk Rating and Size Bucket. The second panel presents capital charges under an equivalent calibration but using the asymptotic implementation of the single factor model. The bottom panel presents the percentage change in going from the asymptotic implementation to the simulation-based implementation. The figure in the bottom-right corner of the table gives the overall “granularity effect”, on EC, of explicitly modelling idiosyncratic factors.

Table 4.12

EC Charge Comparison using Boosted Correlations Calibrated by RG

Size Group ('000)	Simulation-based Implementation EC (%)					Overall
	Risk Group					
	1-3	4-5	6	7	8-9	
≤ $100	6.9	12.0	12.7	16.9	21.1	16.1
$100 - $250	5.9	10.4	11.1	13.4	17.6	11.9
$250 - $1000	4.6	8.2	9.1	11.2	13.6	8.9
> $1000	4.6	7.8	8.8	11.0	13.4	7.5
Overall	4.7	8.1	9.0	11.3	14.6	8.2
	Asymptotic Implementation EC (%)					
≤ $100	6.7	11.9	12.4	16.6	20.5	15.7
$100 - $250	5.8	10.2	11.0	13.2	17.1	11.6
$250 - $1000	4.6	8.1	9.0	10.9	13.2	8.7
> $1000	4.4	7.6	8.5	10.7	13.0	7.3
Overall	4.5	7.9	8.7	11.0	14.2	8.0
	% Change in EC from Asymptotic to Simulation-based Implementation					
≤ $100	2.8%	0.8%	2.1%	1.5%	3.3%	2.6%
$100 - $250	1.1%	1.9%	1.6%	2.0%	2.9%	2.2%
$250 - $1000	1.2%	1.6%	1.5%	2.4%	3.1%	2.1%
> $1000	2.7%	2.4%	3.3%	3.1%	3.3%	2.9%
Overall	2.4%	2.1%	2.8%	2.8%	3.2%	2.6%

Table 4.12 presents capital charges generated under a simulation-based implementation of the single factor model presented in Subsection 4.1.2 and using boosted internally-calibrated asset correlations and PDs by RR. The top panel presents the simulation-based capital charges by Risk Rating and Size Bucket. The second panel presents capital charges under an equivalent calibration but using the asymptotic implementation of the single factor model. The bottom panel presents the percentage change in going from the asymptotic implementation to the simulation-based implementation. The figure in the bottom-right corner of the table gives the overall "granularity effect", on EC, of explicitly modelling idiosyncratic factors given this boost. Comparing to Table 4.10 we observe a notable decrease of the granularity effect to levels consistent with a well-diversified portfolio.

Table 4.13

EC Charge Comparison using Boosted Correlations Calibrated by RG-SG

Size Group (‘000)	Simulation-based Implementation EC (%)					Overall
	Risk Group					
	1-3	4-5	6	7	8-9	
≤ $100	12.9	14.7	14.1	23.2	24.8	20.2
$100 - $250	6.0	13.1	14.5	14.5	21.5	14.2
$250 - $1000	4.5	7.9	6.3	10.7	16.4	8.8
> $1000	3.6	5.9	5.3	5.8	12.7	5.5
Overall	3.9	6.8	6.0	8.0	16.0	7.0
	Asymptotic Implementation EC (%)					
≤ $100	12.4	14.2	13.6	22.1	23.4	19.2
$100 - $250	5.8	12.8	14.0	14.0	20.7	13.7
$250 - $1000	4.4	7.7	6.0	10.4	15.8	8.5
> $1000	3.4	5.8	5.1	5.5	12.0	5.3
Overall	3.8	6.7	5.8	7.6	15.2	6.8
	% Change in EC from Asymptotic to Simulation-based Implementation					
≤ $100	4.0%	3.5%	4.3%	5.0%	5.8%	5.1%
$100 - $250	2.8%	2.9%	3.7%	3.2%	4.0%	3.5%
$250 - $1000	2.7%	2.4%	4.6%	2.9%	3.9%	3.3%
> $1000	4.0%	2.8%	4.1%	5.2%	5.9%	4.2%
Overall	3.7%	2.8%	4.1%	4.1%	4.9%	3.9%

Table 4.13 presents capital charges generated under a simulation-based implementation of the single factor model presented in Subsection 4.1.2 and using boosted internally-calibrated asset correlations and PDs by RG-SG. The top panel presents the simulation-based capital charges by Risk Rating and Size Bucket. The second panel presents capital charges under an equivalent calibration but using the asymptotic implementation of the single factor model. The bottom panel presents the percentage change in going from the asymptotic implementation to the simulation-based implementation. The figure in the bottom-right corner of the table gives the overall “granularity effect”, on EC, of explicitly modelling idiosyncratic factors given this boost. Comparing to Table 4.11 we observe a notable decrease of the granularity effect to levels consistent with a well-diversified portfolio.

Table 5.1

Default Rate Correlations by Industry for CreditRisk+ Implementation

Industry	**BUS**	**CON**	**MAN**	**NBUS**	**OTH**	**RES**	**RET**	**SOP**	**TOU**	**TRS**	**WHS**
BUS	1	0.59	0.64	0.16	0.06	0.51	0.23	0.91	0.10	0	0.08
CON		1	0.74	0.51	0.51	0.28	0.45	0.45	0.17	0.25	0.36
MAN			1	0.57	0.44	0.29	0.68	0.49	0.37	0.40	0.61
NBUS				1	0.26	0.31	0.43	0	0.16	0.56	0.76
OTH					1	0	0.60	0.04	0.27	0.17	0.33
RES						1	0.05	0.39	0.15	0	0.27
RET							1	0.16	0.80	0.44	0.66
SOP								1	0.01	0	0
TOU									1	0.33	0.57
TRS										1	0.39
WHS											1

Table 5.1 presents annual default rate correlations by Industry, floored at zero, as calculated in Chapter 2.

Table 5.2A

CreditRisk$^+$ EC Charges by Risk and Size Group using RG Calibration

EC (%) under CR$^+$ Single Sector (RG)						
Size Group ('000)	**Risk Group**					**Overall**
	1-3	**4-5**	**6**	**7**	**8-9**	
≤ $100	0.52	0.94	1.26	1.85	3.74	1.93
$100 - $250	0.51	0.92	1.25	1.77	3.64	1.65
$250 - $1000	0.47	0.84	1.18	1.64	3.26	1.30
> $1000	0.51	0.87	1.23	1.73	3.27	1.12
Overall	0.50	0.87	1.22	1.71	3.34	1.21
EC (%) under CR$^+$ Multiple Sector (RG)						
≤ $100	0.16	0.28	0.37	0.55	0.99	0.54
$100 - $250	0.16	0.26	0.39	0.52	1.00	0.47
$250 - $1000	0.16	0.28	0.39	0.53	0.97	0.41
> $1000	0.29	0.45	0.62	0.85	1.53	0.57
Overall	0.26	0.40	0.55	0.74	1.28	0.53
EC (%) under CR$^+$ Multiple Correlated Sectors (RG)						
≤ $100	0.36	0.64	0.85	1.26	2.44	1.28
$100 - $250	0.35	0.62	0.87	1.19	2.40	1.10
$250 - $1000	0.34	0.58	0.81	1.11	2.12	0.88
> $1000	0.40	0.66	0.92	1.27	2.34	0.84
Overall	0.39	0.64	0.89	1.22	2.29	0.87

Table 5.2A – Economic Capital charges are calculated under various implementations of the CreditRisk+ framework. In all cases, default rates by Risk Group are used to define the PD and PD volatility. In addition, industry segregations are used to define sectors – both in the standard *Multiple Sector* implementation and the *Multiple Correlated Sectors* implementation.

Table 5.2B

CreditRisk^{+} EC Charges by Risk and Size Group using RG-SG Calibration

EC (%) under CR^{+} Single Sector (RG-SG)

Size Group ('000)	Risk Group					Overall
	1-3	4-5	6	7	8-9	
≤ $100	1.30	2.06	2.52	3.77	5.64	3.36
$100 - $250	0.77	1.33	1.63	1.77	3.72	1.88
$250 - $1000	0.42	0.73	1.17	1.43	2.88	1.16
> $1000	0.31	0.66	0.84	1.31	3.22	0.88
Overall	0.36	0.74	0.99	1.43	3.33	1.06
EC (%) under CR^{+} Multiple Sector (RG-SG)						
≤ $100	0.39	0.61	0.73	1.11	1.49	0.93
$100 - $250	0.24	0.37	0.50	0.52	1.02	0.54
$250 - $1000	0.14	0.24	0.39	0.46	0.86	0.37
> $1000	0.17	0.35	0.43	0.65	1.52	0.44
Overall	0.18	0.33	0.43	0.61	1.27	0.45
EC (%) under CR^{+} Multiple Correlated Sectors (RG-SG)						
≤ $100	0.88	1.39	1.66	2.51	3.62	2.20
$100 - $250	0.53	0.87	1.10	1.17	2.41	1.24
$250 - $1000	0.29	0.50	0.79	0.96	1.84	0.77
> $1000	0.24	0.49	0.62	0.94	2.27	0.64
Overall	0.27	0.53	0.70	1.00	2.25	0.74

Table 5.2B – Economic Capital charges are calculated under various implementations of the CreditRisk+ framework. In all cases, default rates by RG-SG are used to define the PD and PD volatility. In addition, industry segregations are used to define sectors – both in the standard *Multiple Sector* implementation and the *Multiple Correlated Sectors* implementation.

Table 5.3A

CreditRisk$^+$ EC Ratios under Various Implementations by Risk and Size Group

Size Group ('000)	Risk Group					Overall
	1-3	4-5	6	7	8-9	
Ratio of Single Sector to Multiple Sector Implementations						
≤ $100	3.29	3.36	3.40	3.37	3.77	3.61
$100 - $250	3.21	3.53	3.24	3.38	3.64	3.51
$250 - $1000	2.86	3.05	3.02	3.07	3.37	3.16
> $1000	1.77	1.91	1.98	2.04	2.14	1.98
Overall	1.94	2.15	2.20	2.30	2.61	2.29
Ratio of Single Sector to Multiple Correlated Sectors Implementations						
≤ $100	1.44	1.46	1.48	1.47	1.53	1.51
$100 - $250	1.44	1.49	1.45	1.49	1.52	1.50
$250 - $1000	1.39	1.44	1.46	1.47	1.54	1.48
> $1000	1.27	1.32	1.33	1.37	1.40	1.34
Overall	1.30	1.35	1.37	1.40	1.46	1.39
Ratio of Multiple Correlated Sectors to Multiple Sector Implementations						
≤ $100	2.29	2.30	2.29	2.28	2.46	2.40
$100 - $250	2.24	2.37	2.24	2.27	2.40	2.35
$250 - $1000	2.05	2.11	2.08	2.09	2.19	2.13
> $1000	1.39	1.46	1.49	1.49	1.53	1.48
Overall	1.50	1.59	1.61	1.64	1.80	1.65

Table 5.3A – Economic Capital charges are calculated under various implementations of the CreditRisk+ framework. In all cases, default rates by RG are used to define the PD and PD volatility. In addition, industry segregations are used to define sectors – both in the standard *Multiple Sector* implementation and the *Multiple Correlated Sectors* implementation.

Table 5.3B

CreditRisk$^+$ EC Ratios under Various Implementations by Risk and Size Group

Size Group ('000)	Risk Group					Overall
	1-3	4-5	6	7	8-9	
Ratio of Single Sector to Multiple Sector Implementations						
≤ $100	3.34	3.40	3.44	3.40	3.78	3.60
$100 - $250	3.26	3.56	3.28	3.41	3.64	3.51
$250 - $1000	2.89	3.07	3.04	3.09	3.36	3.16
> $1000	1.76	1.90	1.97	2.02	2.13	1.98
Overall	2.06	2.24	2.30	2.35	2.62	2.38
Ratio of Single Sector to Multiple Correlated Sectors Implementations						
≤ $100	1.47	1.49	1.52	1.50	1.56	1.53
$100 - $250	1.46	1.52	1.48	1.52	1.54	1.52
$250 - $1000	1.42	1.47	1.49	1.50	1.56	1.51
> $1000	1.29	1.34	1.35	1.39	1.42	1.37
Overall	1.34	1.39	1.40	1.43	1.48	1.43
Ratio of Multiple Correlated Sectors to Multiple Sector Implementations						
≤ $100	2.28	2.28	2.27	2.26	2.43	2.35
$100 - $250	2.22	2.34	2.22	2.24	2.36	2.31
$250 - $1000	2.04	2.09	2.04	2.06	2.15	2.10
> $1000	1.36	1.42	1.45	1.45	1.50	1.45
Overall	1.54	1.61	1.64	1.64	1.77	1.67

Table 5.3B – Economic Capital charges are calculated under various implementations of the CreditRisk+ framework. In all cases, default rates by RG-SG are used to define the PD and PD volatility. In addition, industry segregations are used to define sectors – both in the standard *Multiple Sector* implementation and the *Multiple Correlated Sectors* implementation.

Table 5.4A

CreditRisk$^+$ EC Charges by Industry and Risk Group using RG Calibration

EC (%) under CR$^+$ Single Sector (RG)						
Industry	**Risk Group**					**Overall**
	1-3	**4-5**	**6**	**7**	**8-9**	
BUS	0.53	0.94	1.48	1.92	3.68	1.32
CON	0.49	0.86	1.20	1.64	3.42	1.06
MAN	0.51	0.88	1.22	1.72	3.37	1.14
NBUS	0.50	0.85	1.18	1.65	3.38	1.31
OTH	0.50	0.90	1.28	1.75	3.41	1.33
RES	0.50	0.85	1.25	1.70	3.54	1.46
RET	0.47	0.86	1.20	1.68	3.28	1.12
SOP	0.48	0.85	1.19	1.84	2.94	1.41
TOU	0.52	0.83	1.14	1.67	3.18	1.40
TRS	0.50	0.85	1.18	1.71	3.26	1.18
WHS	0.50	0.90	1.28	1.69	3.74	1.09
Overall	0.50	0.87	1.22	1.71	3.34	1.21
EC (%) under CR$^+$ Multiple Sector (RG)						
BUS	0.14	0.23	0.42	0.46	0.58	0.29
CON	0.16	0.22	0.30	0.30	0.65	0.25
MAN	0.41	0.67	0.91	1.23	2.38	0.84
NBUS	0.16	0.22	0.32	0.44	0.78	0.34
OTH	0.09	0.18	0.27	0.23	0.38	0.20
RES	0.14	0.17	0.24	0.29	0.74	0.31
RET	0.18	0.37	0.51	0.63	1.01	0.42
SOP	0.18	0.33	0.40	0.92	0.66	0.48
TOU	0.27	0.34	0.44	0.70	1.17	0.56
TRS	0.17	0.25	0.33	0.44	0.72	0.31
WHS	0.18	0.28	0.45	0.48	1.22	0.35
Overall	0.26	0.40	0.55	0.74	1.28	0.53
EC (%) under CR$^+$ Multiple Correlated Sectors (RG)						
BUS	0.30	0.53	0.86	1.08	1.95	0.73
CON	0.35	0.61	0.84	1.11	2.32	0.74
MAN	0.48	0.82	1.13	1.58	3.09	1.06
NBUS	0.33	0.53	0.75	1.04	2.08	0.82
OTH	0.25	0.46	0.66	0.85	1.62	0.65
RES	0.21	0.33	0.47	0.62	1.36	0.56
RET	0.38	0.71	0.98	1.35	2.57	0.90
SOP	0.24	0.44	0.58	1.05	1.29	0.70
TOU	0.34	0.51	0.70	1.04	1.91	0.86
TRS	0.29	0.48	0.66	0.94	1.75	0.66
WHS	0.37	0.67	0.96	1.23	2.77	0.81
Overall	0.39	0.64	0.89	1.22	2.29	0.87

Table 5.4A – Economic Capital charges are calculated under various implementations of the CreditRisk+ framework. In all cases, default rates by Risk Group are used to define the PD and PD volatility. In addition, industry segregations are used to define sectors – both in the standard *Multiple Sector* implementation and the *Multiple Correlated Sectors* implementation.

Table 5.4B

CreditRisk$^+$ EC Charges by Industry and Risk Group using RG-SG Calibration

EC (%) under CR$^+$ Single Sector (RG-SG)						
Industry	**Risk Group**					**Overall**
	1-3	**4-5**	**6**	**7**	**8-9**	
BUS	0.41	0.86	1.24	1.70	3.73	1.21
CON	0.36	0.76	1.01	1.45	3.46	0.94
MAN	0.36	0.74	0.98	1.44	3.36	0.98
NBUS	0.40	0.76	1.03	1.41	3.40	1.20
OTH	0.38	0.79	1.03	1.52	3.45	1.20
RES	0.36	0.75	1.03	1.44	3.51	1.33
RET	0.35	0.73	0.95	1.42	3.29	0.98
SOP	0.32	0.67	0.87	1.47	2.86	1.20
TOU	0.36	0.69	0.94	1.37	3.14	1.23
TRS	0.34	0.71	0.95	1.39	3.28	1.01
WHS	0.35	0.77	1.03	1.41	3.80	0.94
Overall	0.36	0.74	0.99	1.43	3.33	1.06
EC (%) under CR$^+$ Multiple Sector (RG-SG)						
BUS	0.10	0.20	0.34	0.40	0.63	0.26
CON	0.11	0.19	0.24	0.26	0.67	0.21
MAN	0.27	0.53	0.69	0.98	2.29	0.69
NBUS	0.12	0.21	0.28	0.39	0.84	0.32
OTH	0.07	0.15	0.21	0.20	0.40	0.18
RES	0.10	0.15	0.19	0.24	0.76	0.28
RET	0.13	0.31	0.39	0.52	1.02	0.36
SOP	0.12	0.26	0.29	0.74	0.67	0.40
TOU	0.18	0.29	0.36	0.59	1.21	0.50
TRS	0.11	0.20	0.25	0.35	0.72	0.26
WHS	0.12	0.23	0.34	0.39	1.23	0.29
Overall	0.18	0.33	0.43	0.61	1.27	0.45
EC (%) under CR$^+$ Multiple Correlated Sectors (RG-SG)						
BUS	0.23	0.48	0.71	0.95	1.97	0.66
CON	0.26	0.54	0.72	1.00	2.41	0.67
MAN	0.33	0.66	0.87	1.27	2.95	0.87
NBUS	0.25	0.48	0.64	0.88	2.10	0.75
OTH	0.19	0.39	0.52	0.72	1.61	0.58
RES	0.15	0.29	0.39	0.54	1.40	0.52
RET	0.27	0.59	0.76	1.13	2.56	0.78
SOP	0.16	0.35	0.43	0.84	1.27	0.59
TOU	0.23	0.42	0.56	0.84	1.86	0.74
TRS	0.18	0.37	0.49	0.71	1.64	0.52
WHS	0.26	0.56	0.76	1.01	2.78	0.68
Overall	0.27	0.53	0.70	1.00	2.25	0.74

Table 5.4B – Economic Capital charges are calculated under various implementations of the CreditRisk+ framework. In all cases, default rates by RG-SG are used to define the PD and PD volatility. In addition, industry segregations are used to define sectors – both in the standard *Multiple Sector* implementation and the *Multiple Correlated Sectors* implementation.

Table 5.5A

CreditRisk$^+$ Ratios of Various Implementation EC by Industry and Risk Group

Ratio of Single Sector to Multiple Sector Implementations						
Industry	**Risk Group**					**Overall**
	1-3	**4-5**	**6**	**7**	**8-9**	
BUS	3.89	4.15	3.50	4.21	6.35	4.62
CON	3.06	3.91	3.97	5.53	5.28	4.21
MAN	1.25	1.32	1.35	1.40	1.41	1.35
NBUS	3.09	3.78	3.75	3.78	4.31	3.87
OTH	5.32	5.04	4.74	7.49	8.86	6.49
RES	3.52	4.88	5.28	5.82	4.81	4.78
RET	2.64	2.30	2.36	2.68	3.25	2.67
SOP	2.68	2.56	2.99	1.99	4.45	2.95
TOU	1.92	2.43	2.58	2.38	2.71	2.48
TRS	3.01	3.44	3.58	3.92	4.52	3.77
WHS	2.82	3.17	2.88	3.50	3.07	3.07
Overall	1.94	2.15	2.20	2.30	2.61	2.29
Ratio of Single Sector to Multiple Correlated Sectors Implementations						
BUS	1.76	1.78	1.72	1.78	1.89	1.81
CON	1.38	1.43	1.43	1.48	1.47	1.44
MAN	1.06	1.08	1.08	1.09	1.09	1.08
NBUS	1.54	1.59	1.59	1.59	1.62	1.60
OTH	1.97	1.96	1.93	2.07	2.10	2.03
RES	2.36	2.61	2.66	2.72	2.60	2.59
RET	1.24	1.22	1.22	1.24	1.27	1.24
SOP	1.96	1.93	2.03	1.75	2.27	2.02
TOU	1.51	1.61	1.64	1.61	1.66	1.63
TRS	1.72	1.77	1.78	1.82	1.86	1.80
WHS	1.33	1.35	1.34	1.37	1.35	1.35
Overall	1.30	1.35	1.37	1.40	1.46	1.39
Ratio of Multiple Correlated Sectors to Multiple Sector Implementations						
BUS	2.21	2.33	2.03	2.36	3.36	2.55
CON	2.22	2.75	2.78	3.75	3.59	2.93
MAN	1.18	1.23	1.25	1.28	1.29	1.25
NBUS	2.01	2.38	2.36	2.38	2.66	2.43
OTH	2.70	2.58	2.45	3.62	4.21	3.20
RES	1.49	1.87	1.99	2.14	1.85	1.84
RET	2.12	1.89	1.93	2.16	2.55	2.15
SOP	1.37	1.33	1.47	1.14	1.96	1.46
TOU	1.28	1.50	1.57	1.48	1.63	1.53
TRS	1.76	1.95	2.01	2.16	2.42	2.09
WHS	2.12	2.34	2.16	2.55	2.28	2.27
Overall	1.50	1.59	1.61	1.64	1.80	1.65

Table 5.5A – Economic Capital charges are calculated under various implementations of the CreditRisk+ framework. In all cases, default rates by Risk Group are used to define the PD and PD volatility. In addition, industry segregations are used to define sectors – both in the standard *Multiple Sector* implementation and the *Multiple Correlated Sectors* implementation.

Table 5.5B

CreditRisk$^+$ Ratios of Various Implementation EC by Industry and Risk Group

Ratio of Single Sector to Multiple Sector Implementations						
Industry	**Risk Group**					**Overall**
	1-3	**4-5**	**6**	**7**	**8-9**	
BUS	4.12	4.25	3.65	4.25	5.96	4.71
CON	3.28	4.04	4.16	5.51	5.14	4.40
MAN	1.32	1.39	1.42	1.46	1.46	1.42
NBUS	3.20	3.70	3.70	3.65	4.04	3.78
OTH	5.83	5.27	4.96	7.56	8.59	6.77
RES	3.82	5.09	5.54	5.89	4.64	4.80
RET	2.74	2.35	2.43	2.72	3.23	2.74
SOP	2.72	2.52	2.98	1.99	4.24	2.98
TOU	1.97	2.39	2.58	2.34	2.59	2.45
TRS	3.17	3.61	3.85	4.01	4.54	3.96
WHS	2.99	3.31	3.05	3.60	3.09	3.18
Overall	2.06	2.24	2.30	2.35	2.62	2.38
Ratio of Single Sector to Multiple Correlated Sectors Implementations						
BUS	1.79	1.80	1.75	1.80	1.89	1.83
CON	1.37	1.40	1.41	1.45	1.44	1.42
MAN	1.11	1.12	1.13	1.14	1.14	1.13
NBUS	1.56	1.60	1.60	1.60	1.62	1.61
OTH	2.03	2.00	1.98	2.10	2.13	2.07
RES	2.38	2.58	2.63	2.67	2.52	2.54
RET	1.26	1.23	1.24	1.26	1.28	1.26
SOP	1.98	1.93	2.04	1.76	2.25	2.04
TOU	1.55	1.65	1.69	1.64	1.69	1.66
TRS	1.85	1.91	1.94	1.95	2.00	1.95
WHS	1.36	1.38	1.36	1.40	1.37	1.37
Overall	1.34	1.39	1.40	1.43	1.48	1.43
Ratio of Multiple Correlated Sectors to Multiple Sector Implementations						
BUS	2.30	2.36	2.08	2.36	3.15	2.57
CON	2.40	2.88	2.95	3.81	3.58	3.11
MAN	1.19	1.24	1.26	1.29	1.29	1.26
NBUS	2.05	2.32	2.31	2.29	2.49	2.35
OTH	2.87	2.63	2.50	3.59	4.03	3.26
RES	1.60	1.97	2.11	2.21	1.84	1.89
RET	2.18	1.91	1.96	2.17	2.52	2.18
SOP	1.38	1.31	1.46	1.13	1.88	1.46
TOU	1.27	1.45	1.53	1.43	1.53	1.47
TRS	1.71	1.89	1.99	2.05	2.27	2.03
WHS	2.20	2.40	2.24	2.58	2.26	2.32
Overall	1.54	1.61	1.64	1.64	1.77	1.67

Table 5.5B – Economic Capital charges are calculated under various implementations of the CreditRisk+ framework. In all cases, default rates by RG-SG are used to define the PD and PD volatility. In addition, industry segregations are used to define sectors – both in the standard *Multiple Sector* implementation and the *Multiple Correlated Sectors* implementation.

Table 5.6A

CreditRisk$^+$ EC Charges by Industry and Size Group using RG Calibration

EC (%) under CR$^+$ Single Sector (RG)					
	Size Group ('000)				
Industry	**≤ \$100**	**\$100 - \$250**	**\$250 - \$1000**	**> \$1000**	**Overall**
BUS	2.21	1.84	1.41	1.14	1.32
CON	1.93	1.60	1.14	0.93	1.06
MAN	1.68	1.48	1.18	1.09	1.14
NBUS	2.04	1.67	1.37	1.17	1.31
OTH	2.39	1.81	1.48	1.16	1.33
RES	1.94	1.74	1.50	1.39	1.46
RET	1.90	1.70	1.22	1.02	1.12
SOP	1.89	1.62	1.59	1.36	1.41
TOU	2.03	1.81	1.53	1.31	1.40
TRS	2.11	1.59	1.31	1.10	1.18
WHS	2.07	1.74	1.20	0.98	1.09
Overall	1.93	1.65	1.30	1.12	1.21
EC (%) under CR$^+$ Multiple Sector (RG)					
BUS	0.26	0.23	0.20	0.34	0.29
CON	0.24	0.21	0.18	0.29	0.25
MAN	1.03	0.92	0.75	0.86	0.84
NBUS	0.36	0.31	0.28	0.37	0.34
OTH	0.14	0.12	0.13	0.24	0.20
RES	0.13	0.14	0.15	0.38	0.31
RET	0.48	0.44	0.34	0.44	0.42
SOP	0.23	0.21	0.25	0.54	0.48
TOU	0.51	0.47	0.43	0.61	0.56
TRS	0.24	0.19	0.19	0.35	0.31
WHS	0.36	0.32	0.24	0.39	0.35
Overall	0.54	0.47	0.41	0.57	0.53
EC (%) under CR$^+$ Multiple Correlated Sectors (RG)					
BUS	1.13	0.94	0.74	0.67	0.73
CON	1.27	1.06	0.76	0.67	0.74
MAN	1.49	1.32	1.05	1.02	1.06
NBUS	1.21	1.00	0.83	0.76	0.82
OTH	1.08	0.82	0.69	0.60	0.65
RES	0.61	0.56	0.50	0.58	0.56
RET	1.46	1.31	0.95	0.84	0.90
SOP	0.74	0.64	0.65	0.71	0.70
TOU	1.12	1.01	0.86	0.84	0.86
TRS	1.04	0.79	0.66	0.64	0.66
WHS	1.44	1.21	0.84	0.75	0.81
Overall	1.28	1.10	0.88	0.84	0.87

Table 5.6A – Economic Capital charges are calculated under various implementations of the CreditRisk+ framework. In all cases, default rates by Risk Group are used to define the PD and PD volatility. In addition, industry segregations are used to define sectors – both in the standard *Multiple Sector* implementation and the *Multiple Correlated Sectors* implementation.

Table 5.6B

CreditRisk$^+$ EC Charges by Industry and Size Group using RG-SG Calibration

EC (%) under CR$^+$ Single Sector (RG-SG)					
Industry	**Size Group ('000)**				**Overall**
	≤ $100	**$100 - $250**	**$250 - $1000**	**> $1000**	
BUS	3.83	2.11	1.26	0.89	1.21
CON	3.36	1.83	1.02	0.70	0.94
MAN	2.99	1.71	1.05	0.85	0.98
NBUS	3.48	1.89	1.22	0.93	1.20
OTH	4.04	2.04	1.32	0.91	1.20
RES	3.33	1.94	1.34	1.17	1.33
RET	3.35	1.92	1.09	0.79	0.98
SOP	3.20	1.76	1.42	1.10	1.20
TOU	3.48	2.01	1.36	1.07	1.23
TRS	3.56	1.84	1.17	0.86	1.01
WHS	3.61	2.02	1.07	0.74	0.94
Overall	3.36	1.88	1.16	0.88	1.06
EC (%) under CR$^+$ Multiple Sector (RG-SG)					
BUS	0.49	0.28	0.20	0.27	0.26
CON	0.44	0.25	0.17	0.22	0.21
MAN	1.75	1.01	0.64	0.65	0.69
NBUS	0.67	0.38	0.27	0.31	0.32
OTH	0.25	0.14	0.13	0.20	0.18
RES	0.24	0.16	0.14	0.33	0.28
RET	0.85	0.50	0.31	0.34	0.36
SOP	0.41	0.24	0.23	0.44	0.40
TOU	0.92	0.55	0.40	0.52	0.50
TRS	0.39	0.22	0.17	0.28	0.26
WHS	0.62	0.36	0.22	0.30	0.29
Overall	0.93	0.54	0.37	0.44	0.45
EC (%) under CR$^+$ Multiple Correlated Sectors (RG-SG)					
BUS	1.96	1.08	0.66	0.52	0.66
CON	2.27	1.24	0.70	0.51	0.67
MAN	2.54	1.46	0.90	0.77	0.87
NBUS	2.07	1.13	0.74	0.60	0.75
OTH	1.80	0.92	0.61	0.47	0.58
RES	1.09	0.64	0.46	0.51	0.52
RET	2.55	1.47	0.84	0.64	0.78
SOP	1.26	0.70	0.58	0.57	0.59
TOU	1.89	1.10	0.76	0.68	0.74
TRS	1.60	0.84	0.55	0.47	0.52
WHS	2.46	1.38	0.74	0.56	0.68
Overall	2.20	1.24	0.77	0.64	0.74

Table 5.6B – Economic Capital charges are calculated under various implementations of the CreditRisk$^+$ framework. In all cases, default rates by RG-SG are used to define the PD and PD volatility. In addition, industry segregations are used to define sectors – both in the standard *Multiple Sector* implementation and the *Multiple Correlated Sectors* implementation.

Table 5.7A

CreditRisk$^+$ Implementation Ratios by Industry and Size Group

Industry	Size Group ('000)				Overall
	≤ $100	$100 - $250	$250 - $1000	> $1000	
Ratio of Single Sector to Multiple Sector Implementations					
BUS	8.61	8.10	6.93	3.40	4.62
CON	8.00	7.57	6.43	3.23	4.21
MAN	1.63	1.61	1.56	1.28	1.35
NBUS	5.64	5.39	4.88	3.15	3.87
OTH	17.13	14.94	10.99	4.74	6.49
RES	14.51	12.86	9.95	3.66	4.78
RET	3.96	3.84	3.55	2.33	2.67
SOP	8.22	7.69	6.41	2.52	2.95
TOU	3.98	3.86	3.55	2.15	2.48
TRS	8.95	8.34	7.01	3.13	3.77
WHS	5.72	5.48	4.93	2.55	3.07
Overall	3.61	3.51	3.16	1.98	2.29
Ratio of Single Sector to Multiple Correlated Sectors Implementations					
BUS	1.95	1.94	1.91	1.71	1.81
CON	1.52	1.51	1.50	1.39	1.44
MAN	1.12	1.12	1.11	1.07	1.08
NBUS	1.68	1.67	1.65	1.54	1.60
OTH	2.21	2.19	2.15	1.93	2.03
RES	3.16	3.12	3.01	2.39	2.59
RET	1.30	1.30	1.29	1.22	1.24
SOP	2.55	2.53	2.45	1.92	2.02
TOU	1.81	1.80	1.77	1.56	1.63
TRS	2.03	2.02	1.98	1.73	1.80
WHS	1.44	1.44	1.42	1.31	1.35
Overall	1.51	1.50	1.48	1.34	1.39
Ratio of Multiple Correlated Sectors to Multiple Sector Implementations					
BUS	4.41	4.17	3.63	1.99	2.55
CON	5.27	5.00	4.30	2.33	2.93
MAN	1.45	1.44	1.40	1.20	1.25
NBUS	3.36	3.23	2.96	2.04	2.43
OTH	7.74	6.81	5.12	2.45	3.20
RES	4.59	4.12	3.30	1.53	1.84
RET	3.05	2.97	2.76	1.91	2.15
SOP	3.22	3.05	2.62	1.31	1.46
TOU	2.20	2.15	2.01	1.38	1.53
TRS	4.40	4.13	3.53	1.81	2.09
WHS	3.97	3.82	3.46	1.94	2.27
Overall	2.40	2.35	2.13	1.48	1.65

Table 5.7A – Economic Capital charges are calculated under various implementations of the CreditRisk$^+$ framework. In all cases, default rates by Risk Group are used to define the PD and PD volatility. In addition, industry segregations are used to define sectors – both in the standard *Multiple Sector* implementation and the *Multiple Correlated Sectors* implementation.

Table 5.7B

CreditRisk$^+$ Implementation Ratios by Industry and Size Group

Ratio of Single Sector to Multiple Sector Implementations					
	Size Group ('000)				
Industry	**≤ $100**	**$100 - $250**	**$250 - $1000**	**> $1000**	**Overall**
BUS	7.88	7.44	6.41	3.31	4.71
CON	7.70	7.27	6.18	3.20	4.40
MAN	1.71	1.69	1.63	1.31	1.42
NBUS	5.18	4.97	4.51	2.99	3.78
OTH	16.13	14.07	10.41	4.66	6.77
RES	13.66	12.11	9.42	3.51	4.80
RET	3.94	3.82	3.51	2.31	2.74
SOP	7.81	7.31	6.10	2.49	2.98
TOU	3.77	3.65	3.37	2.08	2.45
TRS	9.22	8.54	7.10	3.11	3.96
WHS	5.83	5.57	4.98	2.48	3.18
Overall	3.60	3.51	3.16	1.98	2.38
Ratio of Single Sector to Multiple Correlated Sectors Implementations					
BUS	1.96	1.94	1.91	1.72	1.83
CON	1.48	1.48	1.46	1.36	1.42
MAN	1.17	1.17	1.16	1.11	1.13
NBUS	1.68	1.67	1.65	1.54	1.61
OTH	2.24	2.22	2.17	1.96	2.07
RES	3.05	3.01	2.91	2.32	2.54
RET	1.31	1.31	1.30	1.23	1.26
SOP	2.54	2.51	2.44	1.92	2.04
TOU	1.84	1.83	1.80	1.58	1.66
TRS	2.22	2.20	2.15	1.84	1.95
WHS	1.46	1.46	1.44	1.32	1.37
Overall	1.53	1.52	1.51	1.37	1.43
Ratio of Multiple Correlated Sectors to Multiple Sector Implementations					
BUS	4.03	3.82	3.35	1.93	2.57
CON	5.20	4.92	4.24	2.34	3.11
MAN	1.45	1.44	1.40	1.19	1.26
NBUS	3.09	2.98	2.74	1.94	2.35
OTH	7.19	6.32	4.79	2.38	3.26
RES	4.48	4.03	3.24	1.51	1.89
RET	3.00	2.92	2.71	1.88	2.18
SOP	3.07	2.91	2.50	1.30	1.46
TOU	2.04	1.99	1.87	1.32	1.47
TRS	4.16	3.88	3.30	1.69	2.03
WHS	3.98	3.82	3.45	1.88	2.32
Overall	2.35	2.31	2.10	1.45	1.67

Table 5.7B – Economic Capital charges are calculated under various implementations of the CreditRisk$^+$ framework. In all cases, default rates by RG-SG are used to define the PD and PD volatility. In addition, industry segregations are used to define sectors – both in the standard *Multiple Sector* implementation and the *Multiple Correlated Sectors* implementation.

Table 5.8

Intra-Sector Default Correlations for Single and Multiple Sectors Implementations

Intra-Sector Default Correlations						
Sector	**Squared Volatility Ratio**	**Risk Group**				
		1-3	**4-5**	**6**	**7**	**8-9**
		Probability of Default (%)				
		1.30	2.29	3.24	4.63	8.75
		Default Correlation (%)				
Single	4.62	0.06	0.11	0.15	0.21	0.40
BUS	4.43	0.06	0.10	0.14	0.21	0.39
CON	4.68	0.06	0.11	0.15	0.22	0.41
MAN	4.89	0.06	0.11	0.16	0.23	0.43
NBUS	4.46	0.06	0.10	0.14	0.21	0.39
OTH	4.44	0.06	0.10	0.14	0.21	0.39
RES	4.40	0.06	0.10	0.14	0.20	0.38
RET	4.68	0.06	0.11	0.15	0.22	0.41
SOP	4.33	0.06	0.10	0.14	0.20	0.38
TOU	4.34	0.06	0.10	0.14	0.20	0.38
TRS	4.72	0.06	0.11	0.15	0.22	0.41
WHS	4.75	0.06	0.11	0.15	0.22	0.42

Table 5.8 provides the default correlations obtained in the *Single Sector* and *Multiple Sectors* implementations of the CreditRisk+ framework, along with the components used to calculate them. Specifically, Table 5.8 provides the Probability of Default by RG and the squared volatility ratio by Industry, under the *Multiple Sectors* implementation, and for the portfolio as a whole under the *Single Sector* implementation. Default correlations within a given RG segment are calculated by multiplying the respective PDs and ratios. By and large, default correlations show little variation across industries, and increase with PD, as expected. The highest ratio is obtained in MAN while the lowest is obtained in the SOP industry. This can be attributed to the calibration of the mu and sigma factors for each industry and the mix of borrowers from different RGs in each. Recall from Chapter 2 that SOP has the …

Table 6.1A

AVM and CreditRisk$^+$ Simulation Descriptive Statistics

Loss Distribution Statistics					
	AVM (RG)	CR$^+$ (RG) {W=1}	CR$^+$ (RG) {σ/μ=0.25}	CR$^+$ (RG) {σ/μ=0.5}	CR$^+$ (RG) {σ/μ=1}
Max	3.81%	3.61%	3.50%	3.93%	4.84%
Min	0.42%	0.43%	0.47%	0.64%	0.81%
Skewness	0.586	0.421	0.500	0.922	1.782
Kurtosis	0.584	0.254	0.362	1.267	4.948
Std Dev	0.35%	0.32%	0.34%	0.35%	0.35%
Mean	1.46%	1.45%	1.45%	1.46%	1.45%
Percentile	**Value at Risk**				
75.00%	1.67%	1.66%	1.66%	1.65%	1.60%
90.00%	1.92%	1.88%	1.90%	1.92%	1.90%
95.00%	2.08%	2.02%	2.05%	2.11%	2.14%
99.00%	2.42%	2.30%	2.36%	2.50%	2.69%
99.50%	2.55%	2.41%	2.48%	2.66%	2.92%
99.90%	2.86%	2.65%	2.72%	3.00%	3.43%
99.95%	2.98%	2.73%	2.82%	3.12%	3.68%
99.99%	3.32%	2.93%	3.03%	3.44%	4.14%
Percentile	**Economic Capital**				
75.00%	0.21%	0.20%	0.21%	0.20%	0.14%
90.00%	0.47%	0.42%	0.44%	0.47%	0.45%
95.00%	0.63%	0.57%	0.60%	0.66%	0.68%
99.00%	0.96%	0.85%	0.91%	1.05%	1.24%
99.50%	1.10%	0.95%	1.03%	1.21%	1.47%
99.90%	1.41%	1.19%	1.27%	1.54%	1.98%
99.95%	1.53%	1.27%	1.37%	1.66%	2.22%
99.99%	1.87%	1.48%	1.57%	1.99%	2.68%

Table 6.1A presents loss distribution statistics for losses obtained under the *CreditRisk*$^+$ and *AVM* frameworks. The top panel of Table 6.1A provides maximum and minimum values obtained in 150,000 draws of portfolio losses under each framework, as well as the skewness, kurtosis, mean, and standard deviation of the resultant loss distribution. The middle panel of Table 6.1A provides the Value-at-Risk (VaR), or loss distribution value at various percentiles or critical values. The bottom panel presents Economic Capital values corresponding to VaR values obtained in the middle panel, less portfolio Expected Loss values (approximated by the loss distribution mean values given in the top panel). *CreditRisk*$^+$ and *AVM* values are obtained under Risk Group (RG) calibrations.

Table 6.1B

AVM and CreditRisk$^+$ Simulation Descriptive Statistics

Loss Distribution Statistics				
	***AVM* (RG)**	**CR$^+$ (RG) {W=1}**	***AVM* (RG-SG)**	**CR$^+$ (RG-SG) {W=1}**
Max	3.81%	3.61%	2.99%	2.67%
Min	0.42%	0.43%	0.30%	0.27%
Skewness	0.586	0.421	0.659	0.476
Kurtosis	0.584	0.254	0.750	0.361
Std Dev	0.35%	0.32%	0.29%	0.27%
Mean	1.46%	1.45%	1.12%	1.12%
Percentile		**Value at Risk**		
75.00%	1.67%	1.66%	1.29%	1.29%
90.00%	1.92%	1.88%	1.50%	1.48%
95.00%	2.08%	2.02%	1.64%	1.60%
99.00%	2.42%	2.30%	1.92%	1.85%
99.50%	2.55%	2.41%	2.05%	1.94%
99.90%	2.86%	2.65%	2.31%	2.15%
99.95%	2.98%	2.73%	2.40%	2.24%
99.99%	3.32%	2.93%	2.70%	2.44%
Percentile		**Economic Capital**		
75.00%	0.21%	0.20%	0.17%	0.17%
90.00%	0.47%	0.42%	0.38%	0.36%
95.00%	0.63%	0.57%	0.52%	0.48%
99.00%	0.96%	0.85%	0.81%	0.73%
99.50%	1.10%	0.95%	0.94%	0.83%
99.90%	1.41%	1.19%	1.19%	1.03%
99.95%	1.53%	1.27%	1.29%	1.12%
99.99%	1.87%	1.48%	1.59%	1.33%

Table 6.1B presents loss distribution statistics for losses obtained under the *CreditRisk*$^+$ and *AVM* frameworks. The top panel of Table 6.1B provides maximum and minimum values obtained in 150,000 draws of portfolio losses under each framework, as well as the skewness, kurtosis, mean, and standard deviation of the resultant loss distribution. The middle panel of Table 6.1B provides the Value-at-Risk (VaR), or loss distribution value at various percentiles or critical values. The bottom panel presents Economic Capital values corresponding to VaR values obtained in the middle panel, less portfolio Expected Loss values (approximated by the loss distribution mean values given in the top panel). *CreditRisk*$^+$ and *AVM* values are obtained under Risk Group (RG) and Risk Group – Size Group (RG-SG) calibrations.

Table 6.2

Single Sector CreditRisk$^+$ EC under Various Implementations and Calibrations

EC (%) under Simulation-based AVM (%)

Size Group ('000)	**Risk Group**					**Overall**
	1-3	**4-5**	**6**	**7**	**8-9**	
≤ $100	0.91	1.92	2.08	2.92	3.78	2.76
$100 - $250	0.77	1.58	1.74	2.29	3.13	1.95
$250 - $1000	0.62	1.28	1.43	1.92	2.42	1.44
> $1000	0.69	1.34	1.53	2.15	2.63	1.32
Overall	0.69	1.35	1.52	2.11	2.73	1.41
EC (%) under Simulation-based CR$^+$ {W=1}						
≤ $100	0.65	1.13	1.47	2.22	4.35	2.75
$100 - $250	0.53	1.00	1.35	1.87	3.65	1.82
$250 - $1000	0.43	0.80	1.13	1.52	2.85	1.24
> $1000	0.54	0.86	1.17	1.70	3.14	1.08
Overall	0.53	0.86	1.17	1.67	3.22	1.19
EC (%) under Simulation-based CR$^+$ {σ/μ=0.25}						
≤$100	0.78	1.37	1.80	2.63	3.75	2.59
$100 - $250	0.66	1.16	1.61	2.15	3.14	1.80
$250 - $1000	0.52	0.96	1.30	1.79	2.48	1.31
> $1000	0.63	1.03	1.42	1.96	2.67	1.18
Overall	0.62	1.03	1.41	1.94	2.77	1.27
EC (%) under Simulation-based CR$^+$ {σ/μ=0.5}						
≤ $100	1.00	2.01	2.13	3.28	4.57	3.22
$100 - $250	0.85	1.68	1.89	2.60	3.84	2.26
$250 - $1000	0.67	1.37	1.60	2.16	2.95	1.64
> $1000	0.73	1.39	1.60	2.28	3.06	1.41
Overall	0.73	1.41	1.62	2.29	3.25	1.54
EC (%) under Simulation-based CR$^+$ {σ/μ=1}						
≤ $100	1.26	2.66	2.86	4.26	6.01	4.23
$100 - $250	1.12	2.23	2.53	3.33	5.08	2.98
$250 - $1000	0.89	1.76	2.17	2.73	3.86	2.13
> $1000	0.90	1.73	2.07	2.86	4.04	1.78
Overall	0.91	1.78	2.12	2.88	4.28	1.98

Table 6.2 presents Economic Capital results derived under the *Single Sector* simulation-based implementation of the *CreditRisk*$^+$ framework under various settings and an RG calibration. Results are compared to EC results under the RG-calibrated *AVM*.

Table 6.3A

Intra-Sector Default Correlations Comparison under Single Sector Frameworks

Default Correlations (%)					
Size Group ('000)	**Risk Group**				
	1-3	**4-5**	**6**	**7**	**8-9**
***AVM* Default Correlation (%)**					
≤$100	0.21	0.19	0.17	0.41	0.37
$100 - $250	0.07	0.31	0.35	0.38	0.59
$250 - $1000	0.10	0.23	0.09	0.31	0.65
> $1000	0.08	0.14	0.09	0.08	0.27
Overall	0.09	0.20	0.20	0.28	0.30
Single Sector CreditRisk$^+$ {W=1}					
≤$100	0.17	0.26	0.33	0.48	0.67
$100 - $250	0.10	0.17	0.21	0.24	0.45
$250 - $1000	0.06	0.10	0.16	0.21	0.39
> $1000	0.04	0.09	0.11	0.18	0.44
Overall	0.06	0.11	0.15	0.21	0.40
Single Sector CreditRisk$^+$ {σ/μ=0.25}					
≤$100	0.18	0.19	0.16	0.41	0.37
$100 - $250	0.07	0.18	0.24	0.26	0.52
$250 - $1000	0.06	0.11	0.09	0.23	0.45
> $1000	0.04	0.09	0.09	0.08	0.27
Overall	0.08	0.15	0.20	0.28	0.30
Single Sector CreditRisk$^+$ {σ/μ=0.5}					
≤$100	0.21	0.19	0.16	0.41	0.37
$100 - $250	0.07	0.30	0.35	0.38	0.59
$250 - $1000	0.10	0.23	0.09	0.31	0.65
> $1000	0.08	0.14	0.09	0.08	0.27
Overall	0.09	0.20	0.20	0.28	0.30
Single Sector CreditRisk$^+$ {σ/μ=1}					
≤$100	0.21	0.19	0.16	0.41	0.37
$100 - $250	0.07	0.30	0.35	0.38	0.59
$250 - $1000	0.10	0.23	0.09	0.31	0.65
> $1000	0.08	0.14	0.09	0.08	0.27
Overall	0.09	0.20	0.20	0.28	0.30

Table 6.3A describes the representative default correlations, obtained under the *CreditRisk*$^+$ and *AVM* frameworks, for borrowers within homogenously defined segments of our SME portfolio. Segments here are defined Risk and Size Group.

Table 6.3B

CreditRisk$^+$ Single Sector Risk Factor Weights by Segment and Calibration

Single Sector Risk Factor Weights					
Size Group ('000)	**Risk Group**				
	1-3	**4-5**	**6**	**7**	**8-9**
Probability of Default (%)					
≤$100	2.81	4.34	5.59	8.15	11.39
$100 - $250	1.71	2.85	3.63	4.01	7.71
$250 - $1000	1.00	1.72	2.78	3.50	6.68
> $1000	0.68	1.49	1.92	3.02	7.43
Overall	1.30	2.29	3.24	4.63	8.75
Single Sector CreditRisk$^+$ {σ/μ=0.25}					
≤$100	1.00	0.82	0.67	0.86	0.67
$100 - $250	0.82	1.00	1.00	1.00	1.00
$250 - $1000	1.00	1.00	0.71	1.00	1.00
> $1000	1.00	1.00	0.86	0.64	0.73
Overall	1.00	1.00	0.98	0.96	0.70
Single Sector CreditRisk$^+$ {σ/μ=0.5}					
≤$100	0.53	0.41	0.33	0.43	0.34
$100 - $250	0.41	0.64	0.61	0.61	0.53
$250 - $1000	0.62	0.73	0.35	0.58	0.60
> $1000	0.69	0.62	0.43	0.32	0.37
Overall	0.51	0.59	0.49	0.48	0.35
Single Sector CreditRisk$^+$ {σ/μ=1}					
≤$100	0.27	0.20	0.17	0.21	0.17
$100 - $250	0.20	0.32	0.31	0.30	0.27
$250 - $1000	0.31	0.36	0.18	0.29	0.30
> $1000	0.35	0.31	0.22	0.16	0.18
Overall	0.26	0.29	0.24	0.24	0.18

Table 6.3B describes the risk factor weights obtained under various settings of the *Single Sector* implementation of *CreditRisk*$^+$. Results are presented by various segments, where segments here are defined along Risk and Size Group. The top panel of Table 6.3B presents probabilities of default for the respect segments. Segment risk weights and probabilities of default, along with normalized sector standard deviation settings, are used in the calculation of default correlations in the *CreditRisk*$^+$ framework; see Equation (C.1) and Table 6.3A.

Table 6.3C

Intra-Sector Default Correlations Comparison under Single Sector Frameworks

Ratios of Default Correlations (%)					
Size Group ('000)	**Risk Group**				
	1-3	**4-5**	**6**	**7**	**8-9**
Ratio of *AVM* to Single Sector CreditRisk$^+$ {W=1}					
≤$100	1.21	0.73	0.50	0.85	0.55
$100 - $250	0.73	1.79	1.67	1.59	1.30
$250 - $1000	1.62	2.31	0.55	1.47	1.66
> $1000	2.03	1.61	0.83	0.44	0.61
Overall	1.46	1.84	1.34	1.34	0.74
Ratio of *AVM* to Single Sector CreditRisk$^+$ {σ/μ=0.25}					
≤$100	1.14	1.01	1.00	1.00	1.00
$100 - $250	1.00	1.66	1.49	1.47	1.12
$250 - $1000	1.54	2.11	0.99	1.36	1.44
> $1000	1.90	1.53	1.00	1.00	1.00
Overall	1.07	1.38	1.00	1.00	1.00
Ratio of *AVM* to Single Sector CreditRisk$^+$ {σ/μ=0.5}					
≤$100	1.00	1.01	1.00	1.00	1.00
$100 - $250	1.00	1.00	1.00	1.00	1.00
$250 - $1000	1.00	1.00	0.99	1.00	1.00
> $1000	0.99	1.00	1.00	1.00	1.00
Overall	1.01	1.00	1.00	1.00	1.00
Ratio of *AVM* to Single Sector CreditRisk$^+$ {σ/μ=1}					
≤$100	1.00	1.01	1.00	1.00	1.00
$100 - $250	1.00	1.00	1.00	1.00	1.00
$250 - $1000	1.00	1.00	0.99	1.00	1.00
> $1000	0.99	1.00	1.00	1.00	1.00
Overall	1.01	1.00	1.00	1.00	1.00

Table 6.3C describes ratios of representative default correlations obtained under various settings of the *Single Sector* implementation of *CreditRisk*$^+$ to those obtained under the *AVM* framework. Segments here are defined along Risk and Size Group. See Table 6.3A for corresponding default correlations.

Table 6.4

AVM and CreditRisk^{+} Boosted Implementations Simulation Descriptive Statistics

Loss Distribution Statistics				
	AVM (RG) Boost	CR^{+} (RG) {W=1} Boost	*AVM* (RG-SG) Boost	CR^{+} (RG-SG) {W=1} Boost
Max	16.9%	12.4%	15.2%	9.7%
Min	0.0%	0.0%	0.0%	0.0%
Skewness	2.217	1.614	2.402	1.661
Kurtosis	8.078	3.918	9.668	4.158
Std Dev	1.3%	1.2%	1.0%	0.9%
Mean	1.5%	1.5%	1.1%	1.1%
Percentile		**Value at Risk**		
75.00%	1.9%	2.0%	1.5%	1.5%
90.00%	3.1%	3.0%	2.4%	2.3%
95.00%	4.0%	3.7%	3.1%	2.9%
99.00%	6.2%	5.4%	5.0%	4.3%
99.50%	7.2%	6.2%	5.9%	4.9%
99.90%	9.7%	7.8%	8.1%	6.2%
99.95%	10.7%	8.5%	8.9%	6.8%
99.99%	13.4%	10.2%	11.0%	8.1%
Percentile		**Economic Capital**		
75.00%	0.5%	0.5%	0.3%	0.4%
90.00%	1.6%	1.6%	1.3%	1.2%
95.00%	2.5%	2.3%	2.0%	1.8%
99.00%	4.7%	4.0%	3.9%	3.2%
99.50%	5.8%	4.7%	4.8%	3.8%
99.90%	8.2%	6.4%	7.0%	5.1%
99.95%	9.2%	7.0%	7.8%	5.7%
99.99%	11.9%	8.7%	9.9%	7.0%

Table 6.4 presents loss distribution statistics for losses obtained under the boosted implementations of the *CreditRisk*$^{+}$ and *AVM* frameworks. The top panel of Table 6.4 provides maximum and minimum values obtained in 150,000 draws of portfolio losses under each framework, as well as the skewness, kurtosis, mean, and standard deviation of the resultant loss distribution. The middle panel of Table 6.4 provides the Value-at-Risk (VaR), or loss distribution value at various percentiles or critical values. The bottom panel presents Economic Capital values corresponding to VaR values obtained in the middle panel, less portfolio Expected Loss values (approximated by the loss distribution mean values given in the top panel). *CreditRisk*$^{+}$ and *AVM* values are obtained under Risk Group (RG) and Risk Group – Size Group (RG-SG) calibrations. *CreditRisk*$^{+}$ is boosted under a unitary weight setting.

Table 6.5

Boosted EC Results under Basel II, *AVM*, and *CreditRisk*$^+$

Capital Charges (%) under Basel II (Case 2)

Size Group ('000)	Risk Group					Overall
	1-3	4-5	6	7	8-9	
≤ $100	5.1	5.8	5.6	6.2	7.9	6.7
$100 - $250	4.4	4.9	4.9	4.9	6.6	5.3
$250 - $1000	3.5	3.9	4.0	4.1	5.1	4.1
> $1000	8.6	9.7	10.1	10.8	13.2	9.8
Overall	7.6	8.2	8.5	8.6	9.5	8.2
Boosted Simulation-based *AVM* {RG} EC (%)						
≤ $100	6.9	12.0	12.7	16.9	21.1	16.1
$100 - $250	5.9	10.4	11.1	13.4	17.6	11.9
$250 - $1000	4.6	8.2	9.1	11.2	13.6	8.9
> $1000	4.6	7.8	8.8	11.0	13.4	7.5
Overall	4.7	8.1	9.0	11.3	14.6	8.2
Boosted Simulation-based CR$^+$ (RG) {W=1} EC (%)						
≤ $100	3.8	6.8	9.1	13.7	26.0	16.5
$100 - $250	3.2	5.9	7.9	10.8	21.7	10.8
$250 - $1000	2.5	4.6	6.5	8.9	16.7	7.2
> $1000	2.5	4.4	6.1	8.8	16.5	5.5
Overall	2.6	4.6	6.3	9.0	18.0	6.4
Boosted Simulation-based CR$^+$ (RG-SG) {W=1} EC (%)						
≤ $100	10.3	16.4	19.8	30.5	43.0	30.0
$100 - $250	5.4	9.2	11.4	11.7	24.4	13.3
$250 - $1000	2.5	4.4	7.1	8.6	16.3	7.1
> $1000	1.6	3.7	4.7	7.4	18.0	4.7
Overall	2.0	4.3	5.7	8.4	20.3	6.2
Boosted Simulation-based *AVM* {RG-SG} EC (%)						
≤ $100	12.9	14.7	14.1	23.2	24.8	20.2
$100 - $250	6.0	13.1	14.5	14.5	21.5	14.2
$250 - $1000	4.5	7.9	6.3	10.7	16.4	8.8
> $1000	3.6	5.9	5.3	5.8	12.7	5.5
Overall	3.9	6.8	6.0	8.0	16.0	7.0

Table 6.5 presents capital charges obtained under the boosted simulation-based implementation of the *Single Sector CreditRisk*$^+$ and *AVM* frameworks, as well as those obtained under the Basel II (Case 2) implementation; see Chapter 3 for Basel II partial implementations. *CreditRisk*$^+$ Capital charges are obtained under RG and RG-SG calibrations and a boosted unitary weight setting, and are presented by Risk and Size Group segments, as well as for the overall portfolio.

Table 6.6

Comparative Capital Ratios for Boosted Simulation-based EC Charges

Ratio of Boosted CR^+ (RG) {W=1} Capital Charges to Basel II						
Size Group ('000)	**Risk Group**					**Overall**
	1-3	**4-5**	**6**	**7**	**8-9**	
≤ $100	0.7	1.2	1.6	2.2	3.3	2.5
$100 - $250	0.7	1.2	1.6	2.2	3.3	2.0
$250 - $1000	0.7	1.2	1.6	2.2	3.3	1.8
> $1000	0.3	0.5	0.6	0.8	1.3	0.6
Overall	0.3	0.6	0.7	1.1	1.9	0.8
Ratio of Boosted CR^+ (RG-SG) {W=1} Capital Charges to Basel II						
≤ $100	2.0	2.8	3.5	4.9	5.4	4.5
$100 - $250	1.2	1.9	2.3	2.4	3.7	2.5
$250 - $1000	0.7	1.1	1.8	2.1	3.2	1.8
> $1000	0.2	0.4	0.5	0.7	1.4	0.5
Overall	0.3	0.5	0.7	1.0	2.1	0.8
Ratio of Boosted CR^+ (RG-SG) {W=1} Capital Charges to AVM (RG-SG)						
≤ $100	0.8	1.1	1.4	1.3	1.7	1.5
$100 - $250	0.9	0.7	0.8	0.8	1.1	0.9
$250 - $1000	0.6	0.6	1.1	0.8	1.0	0.8
> $1000	0.5	0.6	0.9	1.3	1.4	0.9
Overall	0.5	0.6	0.9	1.1	1.3	0.9
Ratio of Boosted CR^+ (RG) {W=1} Capital Charges to AVM						
≤ $100	0.5	0.6	0.7	0.8	1.2	1.0
$100 - $250	0.5	0.6	0.7	0.8	1.2	0.9
$250 - $1000	0.5	0.6	0.7	0.8	1.2	0.8
> $1000	0.6	0.6	0.7	0.8	1.2	0.7
Overall	0.6	0.6	0.7	0.8	1.2	0.8

Table 6.6 presents comparative ratios for capital charges obtained under the boosted simulation-based implementation of the *Single Sector CreditRisk*$^+$ frameworks as compared to those obtained under the Basel II (Case 2) implementation and the corresponding *AVM* implementations; see Chapter 3 for Basel II partial implementations. Capital charges are obtained under RG and RG-SG calibrations and a boosted unitary weight setting, and are presented by Risk and Size Group segments, as well as for the overall portfolio. See Table 6.5 for corresponding capital charges.