16-QAM Hierarchical Modulation Optimization in Relay Cooperative Networks

by

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Abstract

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Sara Sallam

Recently, the concept of cooperative networks has attracted special attention in the field of wireless communications. This is due to their ability in achieving diversity with no extra hardware cost. The main drawback that characterizes cooperative networks is that they require extra transmission time slots compared to the traditional non-cooperative networks. Several strategies have been proposed in order to mitigate this disadvantage. One of the most recently adopted techniques is the use of hierarchical modulation. Hierarchical modulation was originally used in Digital Video Broadcast (DVB) applications. Lately, it has been applied in cooperative networks for its ability to transmit relative high data rate with acceptable performance.

In this thesis, the application of a 4/16 QAM hierarchical modulation in cooperative networks is examined. This study focuses on a downlink cellular network scenario, composed of a Base Station, a Relay and two destinations. The Base Station intends to transmit two different streams of data to these two destinations by concatenating the two streams and broadcasting the resulting sequence using a non-uniform 4/16 QAM hierarchical modulation. Unlike previous work, the main contribution in this thesis is the optimization of the 16QAM constellation's parameters according to each user's channel condition. In other words, this method gives each user's data the priority it needs in order to be detected as correctly as possible at the destination. Explicit closed form expressions of Hierarchical modulation Bit Error Rate in relay cooperative networks are derived. These BER expressions are used in order to select the constellation's parameters that will achieve total minimum BER in coded and un-coded schemes. Results prove that the proposed method achieve noticeable improvement in both users performance compared to the use of uniform 16QAM constellation.

To my mother

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List of Symbols and Acronyms

List of Acronyms

Acronym	Explanation
AF	Amplify and Forward
ARQ	Automatic Repeat Request
AWGN	Additive White Gaussian Noise
BCJR	Bahl, Cocke, Jelinek and Raviv
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CRC	Cyclic Redundancy Check
CSI	Channel State Information
DF	Decode and Forward
DNF	De-noise and Forward
DVB	Digital Video Broadcast
EGC	Equal Gain Combining
FEC	Forward Error Correcting codes
FER	Frame Error Rate
HDTV	High Definition Television
HPA	High Power Amplifiers
LLR	Log Likelihood Ratio
MAP	Maximum A Posteriori
MIMO	Multiple Input Multiple Output
ML	Maximum Likelihood
MRC	Maximum Ratio Combining
NC	Network Coding
PCCC	Parallel Concatenated Convolutional Codes

Acronym	Explanation
QAM	Quadrature Amplitude Modulation
RSC	Recursive Systematic Convolutional codes
SC	Switch Combining
SER	Symbol Error Rate
SISO	Soft Input Soft Output
SNR	Signal to Noise Ratio
SPC	Superposition Coding
UEP	Unequal Error Protection
VA	Viterbi Algorithm

List of Symbols

Symbol	Explanation
$\min_{y}\{f(x)\}$	Minimum of the function x in terms of the variable y
$\widehat{d_1}, \widehat{d_2}, \widehat{d'_1}$	Constellation's parameters in the 2nd time slot
$(z)^*$	Complex Conjugate of (z)
L_c	Channel reliability measure
L_e	Extrinsic information
$L^i(\hat{u})$	A posteriori LLR computed at decoder i
N_0	Noise spectral density
P_e	Probability of error
$P_{i,p}$	Punctured version of stream $U_{i,p}$
UE_i	User Equipment i
$U_{i,p}$	Parity bits of encoder# i
$U_{i,s}$	Systematic bits of encoder# i
$d_{x,y}$	Distance between node x and node y
d_1, d_2, d'_1	Constellation's parameters in the 1st time slot
d_{4QAM}	4QAM constellation's parameter
$h_{x,y}$	Fading coefficient for the link from node x to node y
$n_{x,y}$	Noise component in the link from node x to node y
u_i	Input binary stream to user i
u'_i	Decoded binary stream at user i
$\hat{x}$	Received modulated symbol
y_{u_i}	Received signal at user i
$\gamma_{x,y}$	Instantaneous received SNR on the (x, y) link
$\rho^2_{x,y}$	Channel variance on the link from node x to node y
σ^2	Noise variance
DT	Direct Transmission

Symbol	Explanation
E	Symbol Energy
MO	Modulation Optimization
MPO	Modulation and Power Optimization
Q	Q function
$\mathbb{R}$	Real
SR	Source to Relay link
UC	User Cooperation
$\boldsymbol{BS}$	Base Station
$\boldsymbol{G}(.)$	Generator matrix
$\boldsymbol{I}$	Turbo encoder input stream
$\boldsymbol{K}$	Turbo encoder constraint length
$\boldsymbol{erfc}$	Complementary Error Function
$\boldsymbol{l}$	Stream length
$\boldsymbol{s}$	Four bits symbol
$\boldsymbol{x}$	Modulated symbol
$\boldsymbol{\alpha}$	Amplification factor
$\boldsymbol{\mu}$	Path loss exponent
$\boldsymbol{\pi}$	Turbo encoder interleaver

Chapter One

1. Introduction

1.1 Motivation

Recently, the high demand on wireless applications has encouraged extensive research in the wireless communications domain, aiming at analysing and well understanding the wireless environment in order to be able to provide secure, efficient and cost effective wireless services. While these studies have contributed considerably to the development of the present advanced wireless services, more studies are needed to solve some pertaining problems. One of the major concerns is the signal attenuation that wireless networks suffer from. Signal attenuations, also known as fading, occur due to the multipath propagation that the signal experiences in its trajectory from the transmitter to the destination. Several methods were proposed to mitigate channel fading. In recent years, diversity techniques have received considerable attention due to their success in mitigating fading effects. Diversity methods are based on transmitting several copies of the intended signal to the destination. These replicas can be sent on separate intervals of time, which is known as the time diversity. They can also be transmitted over different frequency bands, hence referred to as frequency diversity. Alternatively, they can be sent using multiple transmitting antennas and/or received by multiple receiving antennas as in Multiple Input Multiple Output (MIMO) systems. Such a scheme is known as space diversity. Signals can also be sent through different paths by employing a cooperated multiple nodes transmission scheme that is the cooperative diversity.

Recently, cooperative diversity has been the most practical method and has proven to be the most adaptable to the wireless environments demands with no extra hardware cost. The main disadvantage that characterizes cooperative networks is that they can require extra transmission time slots when compared to the traditional non-cooperative networks. In order to improve the throughput of cooperative networks, coding techniques such as Network Coding (NC) were proposed and extensively studied in the literature [1] [2] [3] [4]. Network Coding performed well in the case of symmetrical channel, in other words, where the channels experienced similar fading. On the other hand, in the case of noticeably dissimilar channels, good channels performance will be dominated by the channel experiencing the worst fading conditions. More recently Superposition Coding (SPC) was proposed to work with, and sometimes as an alternative for, NC. Superposition Coding was first introduced by Cover in [5]. Several comparisons between SPC and NC were presented in the literature [6] [7] [8]. More developed forms of Superposition Coding such as multi-resolution modulation techniques were proposed as a modulation technique to work in parallel with cooperative networks. One of the most employed multi-resolution modulation schemes is the 4/16 QAM Hierarchical modulation. Hierarchical modulation was proposed for use in Digital Video Broadcast (DVB) applications in the early 90's [9]. Recently, Hierarchical modulation was discussed in the literature to mitigate the fading channel effect in the cooperative networks scheme. Explicit closed form expressions of Hierarchical modulation Bit Error Rate (BER) in relay networks were derived in [10]. The BER derivation was expressed in terms of the constellation's distance parameters. The authors proposed a criterion to select the distance parameters that will minimize the refinement bits BER while keeping the error performance of the base bits below a certain threshold. In [11], a downlink multiuser cooperative communication system was developed. It consists of a Base Station, a relay and two mobile users. The motivation behind this paper is to find a method to compensate for the throughput degradation resulting in the use of relay cooperative systems. The authors suggested employing Hierarchical Modulation for simultaneous simulation of the data streams of both mobile users. They mapped one user's data to the base bits location and the other user's data to the refinement bits location. Analysis of the error performance of base bits and refinement bits was carried in a similar manner as in [10]. They simulated their system while employing different constellation parameters (the distance between the constellation points). They compared the base bits and refinement bits performance at the selected distances. The comparison showed that some distance parameters resulted in better performance for the base bits at the expense of degradation of the refinement bits performance and vice versa. They suggested the topic of optimal distance parameters selection as future work.

1.2 Contributions

In this research, a study of the application of a 4/16 QAM hierarchical modulation in cooperative networks is conducted. This study focuses on a downlink cellular network scenario, where a Base Station intends to transmit two different streams of data to two different destinations. These two streams are concatenated together and modulated using a non-uniform 4/16 QAM hierarchical modulation as in [11]. The main contribution in this thesis is the optimization of the 16QAM constellation's distance parameters according to the channel condition of each user. This optimization's goal is to minimize the total Bit Error Rate at the two destinations. In addition, in the case of multiple transmitting nodes, optimization of the total available power distribution on these nodes is examined. Several network topologies are presented in this thesis. First, the concept of optimizing the distance parameters of a hierarchical modulation is explained in the direct transmission scheme. The effect of the optimization is clearly illustrated when compared to the non-optimized scenario (uniform constellation). The performance of the system with the proposed optimization in the case of non-cooperative and cooperative transmissions is investigated. In the case of cooperative transmission, the optimization is conducted in the case of Amplify and Forward (AF) relaying and in the case of Decode and Forward (DF) relaying. Different nodes location scenarios are examined from the point of view of inter-nodes relative distances. Comparison between the Bit Error Rate (BER) in the case of optimized constellation and the case of traditional uniform constellation is presented for each case. Also, the instantaneous probability of error for each stream in the non-cooperative, cooperative AF and cooperative DF cases are derived. Finally, a preliminary study of hierarchical modulation optimization in Turbo coded cooperative networks is presented. In all the above scenarios, optimized constellation always performed better than the traditional uniform 4/16 QAM constellation.

1.3 Thesis Organization

The remainder of the thesis will be organized as follow. In Chapter 2 we present the related literature review along with a background on relay networks, diversity techniques, hierarchical modulation and Turbo codes. In Chapter 3, we present our system model and our proposed constellation optimization method. In Chapter 4, we present the different simulations results in the un-coded and coded environment. And finally in Chapter 5 we conclude our thesis and recommend future work.

Chapter Two

2. Literature Review

In this chapter a detailed literature review and background on the related topics to the proposed research are presented. The discussed topics in this chapter will include diversity techniques, cooperative networks, combining techniques, Network Coding, mutli-resolution modulation and finally Turbo codes.

2.1 Diversity Techniques

Diversity techniques [12], [13] are considered to be the most popular techniques in combating wireless channel fading. The concept of diversity is based on sending one or more redundant copies of the transmitted signal in a way that these copies experience different channel fading conditions. In the case where one of the paths experiences deep fading, there is a chance that other paths will experience better channel conditions and thus the signal can be recovered reliably at the destination. In other words, assuming there exists L branches through which the message can travel and experience independent fading conditions. Then the probability of error on any of these branches is denoted by p, where $p < 1$. The total error probability of the L branches is denoted by p^L where $p^L < p$. Therefore the probability of error is seen to be inversely proportional to the L^{th} power of the average signal to noise ratio. Hence, the diversity techniques are said to improve the performance of the given system. Diversity can be accomplished through several methods. These methods include time diversity, frequency diversity, space diversity and cooperative diversity.

2.1.1 Time Diversity

In time diversity, the signal is repeatedly transmitted over different time intervals. The separating time between any two transmissions need to be greater than the coherence time of the channel. This is to give a chance for variations in the channel conditions to occur and thus insure that any two replicas experience different fading. However, this method might suffer from delays, where the delay depends highly on the coherence time of the channel and on the number of replicas to be sent. In systems where delay is not tolerable, time diversity technique is not a suitable option.

2.1.2 Frequency Diversity

In frequency diversity replicas of the signal are transmitted over different frequency carriers. As in the case of time diversity, the separation between any two frequency carriers needs to be greater than the coherence bandwidth of the channel. This is to assure that each copy of the signal goes through independent fading. However, frequency diversity requires extra bandwidth which is very scarce in wireless systems and in some applications it is even impossible to allocate. Also, frequency diversity entails extra transmitter/receiver for every frequency carrier which adds more to the hardware cost.

2.1.3 Spatial Diversity

Spatial diversity is accomplished by having multiple antennas at the transmitter/receiver. It is more advantageous than time and frequency diversity as it does not require any extra bandwidth and does not suffer from any delays. Multiple Input Multiple Output (MIMO) is one of the spatial diversity applications and it has proven to offer relatively a large capacity increase in comparison to single antenna systems. However, spatial diversity has its own limitations as it requires multiple antennas and physical separating distance between theses antennas to ensure different fading by each signal which is not practical in some of the wireless equipment.

2.1.4 Cooperative Diversity

Cooperative diversity is another form of diversity methods. It uses relays; intermediary nodes, to assist the source in the transmission of the signal. At the destination, the receiver will have one replica from the source; if possible, and a replica from each cooperating relay. Cooperative diversity is mainly used to enhance the coverage when the link between the source and the destination is in a deep fading condition. Also, the concept of using an intermediary node is advantageous as it divides the separating distance between the source and the destination into multiple hops; which reduces significantly the signal attenuation. Finally, the separating physical distance between the source and any intermediary node is large enough so that each replica of the signal is guaranteed to experience independent fading and thus achieves diversity. In the following section, cooperative diversity is discussed in more details, illustrating a simplified cooperative network, presenting the most common relaying protocols and listing the advantages of cooperative networks in the wireless medium.

2.2 Relay Cooperative networks

In cooperative networks, one or more nodes of the network are employed as relay nodes in order to assist the transmitter in delivering its data to the receiver with more reliability. This concept was firstly introduced by van der Meulen in 1971 [14]. He presented a cooperative transmission scheme where the source sends its data to the destination through two different paths. Those are the "source to destination" path and the "source to relay to destination" path. Several studies on capacity bounds of the relay channels were carried out in [15] [16] [17]. More recently, with the high demand on cellular services, more studies such as in [18] [19] were done on the cooperative diversity technique. In both publications, a detailed proposal of the cooperative diversity concept was presented. The strategy for achieving inter nodes cooperation, implementation related matters and the system's performance analysis were discussed. These studies proved that cooperative diversity were entitled to increase the system's capacity, improve the coverage and lead to a more robust system that is less sensitive to channel variations. Advantages of cooperative networks are so many. The authors of [20] summarized the main advantages including the flexibility of the network topology, the ease of forwarding strategy, the modulation and the coding techniques employed. Other advantages included better coverage, increased use capacity and efficiency of power allocation.

To illustrate the basic concept of a cooperative relay network, a simple cooperative network consisting of a Source (S), a destination (D) and a relay (R) is presented in Figure 2.1. A classic cooperative transmission takes place in two phases. In the first phase, the source broadcasts its data to both the relay and the destination. In the second phase, the relay retransmits the source's data in a different time slot which is received by the destination. The destination then combines the two signals using a signal combining technique and recovers the source's data.

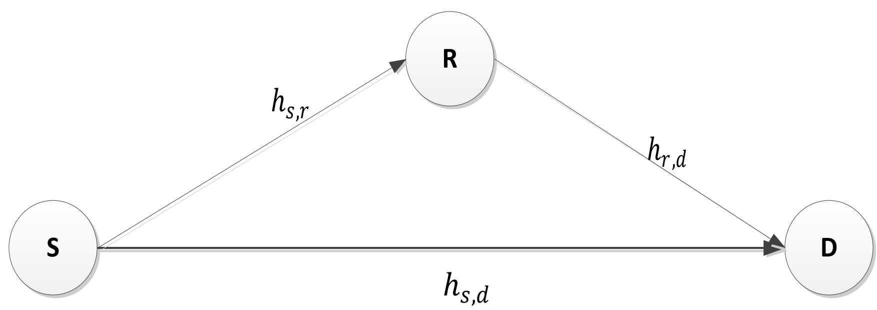

Figure 2.1 A simple cooperative relaying system

Relaying techniques such as Decode and Forward (DF) Amplify and Forward (AF) and others were discussed and compared in the literature as in [16] [21] [22] [23] [24] [25] [26] [27] [28] [29]. Related concepts such as fixed relaying, selective relaying and incremental relaying were discussed in [21] [30] [31] [32] [33]. Finally, the combining techniques at the destination such as Maximal Ratio Combining (MRC), Equal Gain Combining (EGC) and Selective Combining (SC) were proposed in [34]

[35] [36] and explained in details in [37]. In the following sections, the most common forwarding techniques such as DF and AF are explained along with the different combining techniques such as MRC, EGC and SC.

2.2.1 Amplify and Forward

As mentioned in Figure 2.1, the transmitted signal from the source is received by the relay and possibly by the destination. In the case of employing Amplify and Forward technique, also known as the non-regenerative relaying technique, the relay simply amplifies the received signal by an amplification factor α and then re-broadcasts the amplified signal without any further processing. In the case of AF relaying, both the signal and the noise are amplified. While employing AF scheme, it is essential that the destination has full channel information of the Source-Relay link and the Relay-Destination link. At the destination, the receiver combines the two signals coming from the source and the relay using any combining technique. Symbol Error Rate (SER) performance analysis and optimum power allocation for the case of AF were presented in details in [38] and [39].

2.2.2 Decode and Forward

In the case of Decode and Forward (DF), the relay decodes and detects its received signal, then re-encodes a clean version of the signal and re-broadcasts it. The DF relaying, also known as regenerative relaying however might be vulnerable to error propagation in case it erroneously detects the source's signal. At the destination, the signals from the source and the relay are combined together using a combining technique then a decision is made. Performance analysis of the Decode and forward scheme was presented in several articles in the literature including [38], [39], [40].

2.2.3 Compress and Forward

In Compress and Forward [16], the relay does not decode the source's signal, instead it quantizes the signal, and the quantized samples are encoded and broadcasted. At the destination, the signals are not combined; instead, the signal from the relay is used as side information only.

2.2.4 Cooperative network features

Cooperative networks are now becoming more attractive in the wireless communication domain. Due to its direct relation to the work done, the main attributes of cooperative diversity networks are summarized in the paragraphs to follow:

2.2.4.1 Spatial diversity

Cooperative diversity is an attractive diversity method as it profits from the random nature of radio propagation. This is achieved by what is so called, spatial diversity, which is done by exploiting the fact that each signal will go through an independent fade. This occurs under the condition that the separating distance between the signals' sources is big enough for the signals to experience uncorrelated fading.

2.2.4.2 Path loss reduction

The concept of cooperative diversity is based on dividing the distance between the source and the destination into multiple hops with shorter distances. This results in better channel condition as the power of the channel and the distance separating the transmitter and the received are related nonlinearly by

$$\rho_z{}^2 \propto \left(\frac{1}{d_z}\right)^{\mu} \tag{2.1}$$

Where $\rho_z{}^2$ is the variance of the channel z, d_z is the distance separating the transmitter and the receiver, μ is the path loss exponent.

2.2.4.3 Broadcast channel properties

Cooperative networks profit from the broadcast channel properties and make use of the fact that a broadcasted message is not only received by the destination node but can also be overheard by other neighbouring nodes.

2.3 Combining Techniques

In the previous section, the concept of relay networks was discussed. It is based on helping the receivers to detect their received signals more reliably through sending them multiple copies of the transmitted signals. At the receiver, one of the combining techniques [35] that are discussed in this section will be employed to combine the received signals together.

Switch Diversity is a combining technique in which the receiver switches from the current selected branch l to another branch when the instantaneous received SNR on branch l is lower than a predefined threshold. The Switch Diversity is advantageous in the fact that it is easy and cheap to implement. Selection combining (SC) is another technique in which the branch l with the maximum instantaneous received SNR value is selected from the M available branches. SC performs slightly better than Switch Diversity; however its performance degrades with the increasing number of branches. Contrary to the concept of selection is Equal gain combining (EGC) where all the M

available branches are simply co-phased and then summed. Finally, Maximal Ratio combining (MRC) comes as the optimum combining technique in terms of performance. In MRC, each of the M signals is weighted proportionally to its channel quality, co-phased then all signals are summed. This technique maximizes the instantaneous SNR at the combiner's output.

2.4 Drawbacks of cooperative networks

The cooperative networks have proved to be one of the very promising methods in mitigating channel fading [15] [16] [17]. However cooperative networks have two major drawbacks. The first drawback that cooperative networks suffer from is the error propagation resulting from erroneous detection at the relay. Such error propagation results in a degradation of the overall system's performance. Another disadvantage that cooperative networks experiences is its need for extra bandwidth. As can be seen from Figure 2.1, relay networks require more time slots to transmit their data than the traditional communication networks. Due to the operating nature of the relay nodes, they can only operate in half duplex mode, which means that they can either transmit or receive signals at one time. In a uni-directional cooperative network scenario; where the relay assists the source in transmitting its data to the destination; this implies that a whole transmission will require two or more time slots. In a bi-directional scheme (Figure 2.2); where both nodes are exchanging data with the help of the relay; four or more time slots are needed. In the sections to follow, discussions on each of these drawbacks and the work done to overcome each of these disadvantages is presented.

2.4.1 Mitigating error propagation in cooperative networks

As mentioned earlier, forwarding schemes in cooperative networks include Decode and Forward (DF), Amplify and Forward (AF) and others. In case of employing DF scheme, when perfect channel and correct detection at the relay is assumed, the diversity of the system is preserved. However, if erroneous detection at the relay occurs, the relay forwards incorrect messages which results in error propagation and a degradation of the system's performance. Several studies have been proposed to remedy the causes of degradation and control the propagation of the error at the relay. Several papers [20], [33] and [41] have suggested employing a Signal to Noise Ratio (SNR) threshold at the relay. If the SNR of the received signal at the relay is above a predetermined threshold, then the relay can forward the received signal. Otherwise the relay remains silent. Others suggested employing AF forwarding scheme when the relay can not detect the message correctly at the relay and employing DF otherwise [42] [43]. Another employed method to control error propagation is the use of Cyclic Redundancy Check (CRC). In case the CRC check fails, the relay does not forward its decoded message. This method was discussed in [19], [44] and others.

2.4.2 Throughput degradation in cooperative networks

Methods like Network Coding [1], [2], [45], [46] Superposition Coding [15], [47] and multi-level modulations [48], [49] were proposed to solve the dilemma of additional bandwidth resources. Network Coding was initially introduced in the field of computer networks for routing purposes [1], [4], [50]. Recently it has been recommended for the cooperative wireless networks due to its capabilities of compensating for the extra bandwidth that cooperative networks require [51] [52] [53]

[54]. In [52], the authors employed Network Coding at the relay in a bi-directional relaying scheme. Instead of a traditional 4-phase transmission scheme, the Network Coding at the relay reduces the number of needed transmissions to 3-phase transmissions. Thus, the system throughput is enhanced by up to 33%. In a different scenario, a system consisting of two sources transmitting their data to a common destination with the help of a relay was described by Larson *et al*. in [53]. In this context, each source broadcasts its data, which is received by the relay and the destination nodes. The relay combines the data from the two sources using Network Coding and broadcasts the Log Likelihood Ratio (LLR) of the network coded message to the destination. In addition, a comparison between the performance of NC and SPC in an uplink scheme was proposed in [54]. It consisted of multiple users, multiple relays and a Base Station. It was concluded that Network Coding performed better in high rate regions.

The method of Network Coding is based on the addition of two or more signals using a bitwise exclusive OR (XOR). The XOR operation is a logical operation, denoted by $\oplus$, that returns true if exactly one of the inputs is true. Table 2-1 illustrated the XOR operation on the different possibilities of two binary bits x_1 and x_2.

x_1	x_2	$x_1 \oplus x_2$
0	0	0
0	1	1
1	0	1
1	1	0

Table 2-1 XOR Operation

Experimental research has proven that NC was able to increase network throughput in multi-hop wireless networks by conveying the same amount of data in a relatively reduced time that traditional cooperative networks. Such a realization was noticeably found in the bi-directional relay network schemes. Figure 2.2 illustrated the paths of data exchange between source A and source C in a (a) traditional bi-directional relay network and a (b) joint network coded bi-directional relay network.

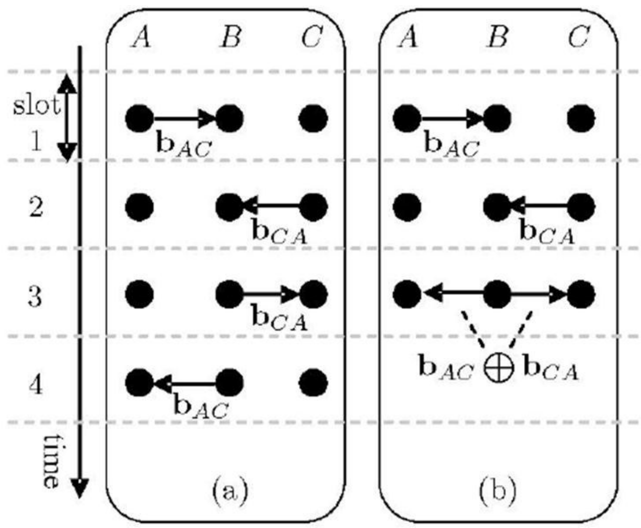

Figure 2.2 Illustration of traditional and coded bi-directional relay network

As illustrated in Figure 2.2, in the network coded scenario, each source sends its data in a different time slot. Source A sends its data b_{AC} in the first time slot and source C sends its data b_{CA} in the second time slot. In the third time slot, the relay B combines the two received symbols and logically adds them using the XOR operator resulting in $r = b_{AC} \oplus b_{CA}$. The resulting symbol r is then broadcasted to the two sources. At each source, the data is extracted by using the XOR operator and adding the source's signal with the signal it has received from the relay B. For example, at source A, the recovered signal is $y_1 = s_1 \oplus r = s_1 \oplus s_1 \oplus s_2 = s_2$. In the same manner, the recovered signal at source 2 is $y_2 = s_2 \oplus r = s_2 \oplus s_1 \oplus s_2 = s_1$.

Several forwarding schemes were studied in the network coded bi-directional cooperative networks schemes. Decode and Forward (DF) was discussed in [52], [46]. Amplify and Forward (AF) was studied in [55], [56]. De-noise and Forward (DNF) was proposed and compared to DF and AF in [57]. In [57], the author concluded that in noiseless channels, DNF and AF can achieve better throughput compared to DF scheme. In addition, at low SNR, DNF has the best throughput of the three schemes.

However, as Network Coding has proven to be very promising in improving the throughput of cooperative networks, it was discovered that such an achievement is only limited to cooperative networks with symmetrical channels. In other words, in relay networks, the rate of the better channel is limited by the rate of the poorer channel when the channels are asymmetric. This has encouraged more research to find other alternatives to be implemented in asymmetrical channels. In [58], a Joint SPC and NC scheme was presented in an asymmetric channels bi-directional relay network scenario. Employing SPC on top of the NC in this scenario enabled the user with the better channel to make use of its advantageous channel by recovering extra information while making sure that the user with the worse channel recovers its basic data. The same concept was presented in [59] but for a multiple user exchange scenario.

Superposition Coding was first introduced by Cover in [5], [15]. Cover examined the achievable joint rates in different broadcast channel schemes. He concluded that “ high joint rates of transmission are best achieved by superimposing high-rate and low-rate information rather than by using time-sharing” [5]. More recently Superposition Coding was adopted in cooperative networks to overcome the need of extra bandwidth. Superposition Coding was thoroughly studied in a three nodes cooperative scheme [60], [61], [62], [63]. In this proposed scheme, two source nodes, node A and node B cooperate together to deliver their information to a destination node D. At the beginning of the transmission, node B broadcasts its data that is received by node A. Node A then superposes node B data on its own data and broadcasts the compound signal. Node B then receives the superimposed signal, where it can extract node A signal since it knows its own signal. On its turn, node B then adds together its own data and node A data using a different Superposition Coding encoder depending on the a-priori information it has. Figure 2.3 illustrates the proposed scheme.

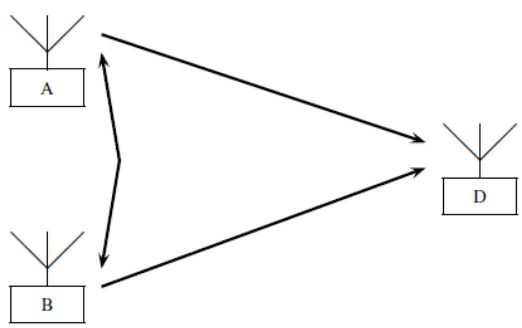

Figure 2.3 Two nodes cooperation to deliver their data to a common destination using SPC

Unidirectional relaying scheme using Superposition Coding at the source node was proposed in [64], [65]. In these two papers, strategies for transmitting the intended message using Superposition Coding are investigated. A proposal is presented regarding dividing the data into two levels and transmitting them employing superposition modulation. Power allocation at the source and the relay is studied to optimize the performance of detection at the destination.

Due to the powerful capabilities of both Network Coding and Superposition Coding on improving the throughput and the performance of cooperative networks, more recent schemes involving joint Network Coding and Superposition Coding have been proposed. Some work considered the two way relaying schemes [58], [6], [66]. In the cited works, the case of asymmetric two way relay channels is considered. The authors in [58] proposed employing Superposition Coding on top of Network Coding at the relay. By employing such a technique, the user with the better channel will benefits from its extra capacity by receiving the extra information embedded in the Superposition Coding. Meanwhile, the user with the worse channel will still receive the data resulting from the Network Coding. The authors have also included channel capacity computations for both users. In [6], a comparison between the use of NC versus the use SPC is presented in a two way wireless communication. It was proven that SC performs better that NC in most of the cases in both scenarios of two and three time slots transmission schemes. On a bigger scale, multiple nodes exchange in cooperative networks using joint network-Superposition Coding was also discussed in [59].

Joint network-Superposition Coding was also discussed in multicast cooperative networks [7], [8], [31]. Signal multicast over asymmetric Rayleigh fading channels is considered in [7], [8]. Superposition Coding is used on top of the Network Coding to send extra data to users with good channel conditions. Results in these two papers showed that additional performance gains were achieved in the proposed scheme when compared to simple Network Coding schemes. In [31] comparison between Superposition Coding and Network Coding with linearly independent global encoding kernels in a multiple source multiple relay scheme is presented. Authors concluded that finite field Network Coding has better performance than Superposition Coding in high rate regions.

A joint Network Coding and Superposition Coding technique was described in [59]. The authors compared this technique with two other techniques; pure time division and pure Network Coding in two different relaying scenarios. They also presented the achievable rate regions for every technique in every scenario and concluded that the joint Network Coding and Superposition Coding technique was not always superior and thus a combination of the three techniques was recommended.

In this section of the literature review, a brief summary of the methods used to mitigate the drawbacks of cooperative networks were presented. The main drawbacks mentioned were the error propagation resulting from erroneous detection at the relay and the need for extra bandwidth compared to traditional transmission schemes. The methods proposed along with the related discussions and the found results were presented. In the coming section, the discussion continues on one of the methods more recently introduced to cooperative networks, which is hierarchical modulation.

2.5 Hierarchical Modulation

Hierarchical modulation is a classification of the multi-level modulations. It was originally introduced in the High Definition Television domain (HDTV) in 1993 [9]. The motive behind employing HM in HDTV systems is that receivers have different channel fading conditions in broadcast channels. The use of multi-resolution modulation allows the receivers located further away from the source or the ones that experience bad channel conditions to receive at least a basic TV image quality. Meanwhile, receivers with good channel quality can receive a HDTV signal quality. Ten years later, hierarchical modulation was standardized in the digital video broadcast (DVB) standards [49].

The concept of hierarchical modulation consists of transmitting multiple streams of data with different protection simultaneously without any additional resources. Hierarchical modulations are capable of prioritizing and dividing the data into different classes of importance. This is useful in poor channel conditions, where the receiver can recover the more protected classes (known as the coarse data) with a satisfactory Bit Error Rate (BER), whereas the less protected classes (known as the enhancement data), are only recovered in better channel conditions [67]. For example, in a 4/16 QAM hierarchical modulation, two 4-QAM are superposed on top of each other as shown in Figure 2.4. The first 4-QAM, in this example referred to as "*constellation A*", is marked by big black dots. Each symbol in the "*constellation A*" is separated by a distance of $2d_1$ from its adjacent symbols. On top of each symbol of the "*constellation A*" there is another 4-QAM constellation, denoted as "*constellation B*". "*Constellation B*" symbols are marked in Figure 2.4 by white small dots. Each symbol in the "*constellation B*" is separated by a distance of $2d_2$ from its adjacent symbols; where $d_2 < d_1$. "*Constellation A*" symbols are known by the term "common bits" or "base bits" as they are common in one quadrant of the constellation. Base bits are assigned to the two Most Significant bits (MSB) in a 4/16 QAM constellation. On the other hand, *Constellation B*" symbols are known by the term "enhancement bits" or "refinement bits". They are more vulnerable to noise as their separating distance is smaller than the separating distance of the base bits. Refinement bits are assigned to the two Least Significant Bits (LSB) of a 4/16 QAM constellation.

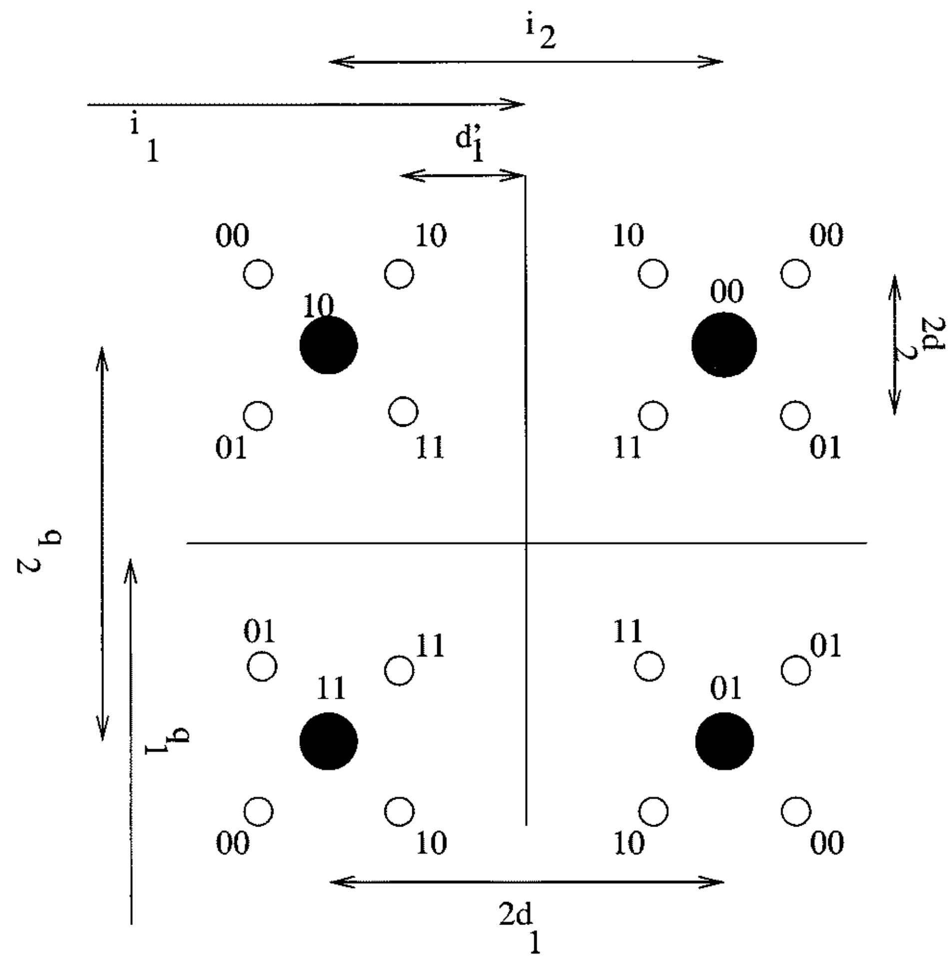

Figure 2.4 Embedded 4/16-QAM constellation with Gray coding

Figure 2.4 illustrates a 4/16 QAM constellation with the locations of the black and white dots and the separating distances. It is to be noted that the black dots are just virtual dots to illustrate the hierarchical protection of the base bits/ refinement bits and to illustrate the overall separating distances between the base bits. It is also to be noted that in this thesis the terms base bits and refinement bits will be used to describe the more protected bits and the less protected bits respectively.

2.5.1 Distance parameters

As shown in Figure 2.4, $2d_1$ is the distance between the two fictitious symbols (marked in black circle) while $2d_2$ is the distance between two neighbouring symbols within the same quadrant. $2d'_1$ is another distance parameters which measures the distance between two adjacent symbols in two different quadrants. d'_1 can be related to d_1 and d_2 by

$$d'_1 = d_1 - \left(\frac{\sqrt{M}}{2} - 1\right)d_2 \tag{2.2}$$

where M is the constellation size. Another important parameter is the constellation priority parameter $\lambda = d_2 / d'_1$ which defines the relative message priority, i.e. whether the base bits or the refinement bits are given more priority. In a traditional uniform 4/16 QAM constellation where $d_1 = 2d_2$ the priority parameter $\lambda = 1$. If the base bits are given more priority, then $d_1 > 2d_2$ and the priority parameter is $0 < \lambda < 1$. When $\lambda = 0$, $d_2 = 0$ and the resulting constellation is a QPSK constellation. Otherwise, if the base bits are given less priority, then $d_1 < 2d_2$ and $\lambda > 1$. Thus, by varying these parameters, it is easy to obtain a range of modulation orders starting from a QPSK and ending by a 16QAM with flexible streams prioritization. This makes hierarchical modulation more powerful and more desirable that just simply adaptive modulation technique.

2.5.2 Energies

In a 4/M square QAM constellation, the average energy per symbol E_s is given by [68]

$$E_s = 2d_1^2 + \frac{2}{3}(\frac{M}{4} - 1)d_2^2 \tag{2.3}$$

Which in a 4/16 square QAM constellation reduces to

$$E_s = 2d_1^2 + 2d_2^2 \tag{2.4}$$

If R is defined to be the ratio between the energy of the basic bits and the refinement bits, then

$$R = \frac{2d_1^2}{\frac{2}{3}\left(\frac{M}{4} - 1\right) d_2^2} \tag{2.5}$$

Which in a 4/16 square QAM constellation reduces to

$$R = \frac{d_1^2}{d_2^2} \tag{2.6}$$

2.5.3 Hierarchical modulations in cooperative networks

Recently, hierarchical modulation has been introduced to the context of wireless relay networks [59], [60], [65], [67], [69], [70], [71]. Hierarchical modulation was firstly adopted in the cooperative networks in [69]. An uplink relay communication system that uses channel coding was implemented, where in the first time slot the source sends modulated and encoded data to both the relay and the destination. The data are modulated at the source using hierarchical modulation, where the coarse data are received by both the relay and the destination while the enhancement data is received only by the relay. In the second time slot, the relay then sends additional redundancy bits to the destination. Some other papers proposed hierarchical modulation in order to solve the drawback problem of Network Coding in asymmetric channels [67]. The problem is emphasized in wireless multicast networks, when the data rate of network coded packets must be selected according to the worst channel condition to ensure a reliable multicast. Such a constraint reduces the gain of the Network Coding. In [67] a combined hierarchical modulation and Network Coding algorithm which is known as hierarchically modulated Network Coding, was proposed to accomplish spectral efficiency. Hierarchical modulation was used at user 'A', where its protected data was received by the relay and user 'B'. Meanwhile the relay recovered the less protected data and combined it with user 'B' data using network coding. The results of this scheme were compared to direct transmission, bidirectional Network Coding and coded bidirectional relay. The proposed scheme was proven to have a significantly improved BER.

In another network scenario, where receivers were grouped according to the quality of their receptions, hierarchical modulation was applied in order to classify the transmitted data according to each group reception's quality [71]. In the same context, a multiple relay scheme was presented in [70]. In the presented scheme, the source modulated its signal using a hierarchical 16 QAM, where the data is received by the relays and the destination. According to the CRC check at the relay, the relay re-modulates its received data using the appropriate modulation technique that will improve the system throughput. Non-uniform hierarchical modulation had also been studied in relay networks and it successfully outperformed its uniform counterpart as explained in [72]. Furthermore, an explicit closed form for the BER was derived for cooperative communication systems with hierarchical modulation over Additive White Gaussian Noise (AWGN) channels and Rayleigh fading channels in [10]. This BER derivation was a function of the 16 QAM constellation's distance parameters. In [73] another model implemented a Signal to Noise Ratio (SNR) threshold at the relay in order to decide whether to retransmit the received data or to remain silent while using hierarchical modulation at the source to prioritize the different transmitted data. Performance analyses of hierarchical modulation in several cooperative network schemes were presented in [10], [11], [73].

In conclusion, the concept of hierarchical modulation was introduced in this section. Examples of its important role in HDTV and cooperative network fields were presented. The concept of classes' protection in a hierarchical modulation was explained and the respective energy of each group of protection was computed. A review of the recent related work to hierarchical modulation in cooperative networks was presented. In the coming section, a background along with a brief literature review regarding Turbo codes is presented. A focus on the literature regarding Turbo codes in cooperative networks with the aid of hierarchical modulation is presented.

2.6 Error Correcting Codes: Turbo Codes

Turbo codes are one of the most powerful codes that are widely used in the field of satellite and wireless communications. They were first introduced in 1993 by Berrou, Glavieux and Thitimajshima [74] [75]. Berrou et al. developed a Parallel Concatenated Convolutional Codes (PCCC) using Recursive Systematic Convolutional (RSC) codes with rate $r = {}^{1}\!/_{2}$. Using an interleaver with size N=65,536, they achieved a BER of 10^{-5} at 0.7 dB in an AWGN channel using Binary Phase Shift Keying (BPSK) modulation after 18 iterations. In the following years, several researches, developments, applications and tutorials on Turbo codes have been done. Several papers analyzed the performance of Turbo codes with different settings (such as rate, constraint length, interleaver size…etc) [76] [77] [78]. Tutorials on the principles and applications of Turbo codes were described in details in [79] [80] [81] [82] [83]. Developments of classes of Turbo codes such as Block Turbo codes and Non-Binary Turbo codes took place in [84] [85] [86] [87] [88]. Major applications of Turbo codes such as deep space, satellite communications and cellular networks were presented in [81] [89].

Turbo codes are a high performance class of Forward Error Correcting codes (FEC). The Turbo encoder is primarily composed of two recursive systematic convolutional (RSC) codes concatenated in parallel and separated by an interleaver. On the other end, the Turbo decoder is composed of two iterative Soft Input Soft Output (SISO) decoders that exchange their output data to reach an accurate decision on the received data. Each SISO decoder utilizes the Maximum a posteriori (MAP) algorithm to decode its data. In the following section we will discuss the concept of Forward Error Correcting codes, the composition of Turbo codes encoder and decoder and will briefly discuss the MAP algorithm.

2.6.1 Forward Error Correction Codes

In general, Error Control Coding is divided in two subclasses; Automatic Repeat Request (ARQ) and Forward Error Correction (FEC) codes. An ARQ code detects the presence of errors and asks for a retransmission of the packets in error. An FEC detects and as much as possible correct the messages in error. ARQ requires less bandwidth as it doesn't require any redundancy; however it is not practical in real time application because of its latency which makes FEC more suitable for such applications. FEC is divided in two main subclasses; block codes and convolutional codes. The main difference between block codes and convolutional codes lies in the encoding scheme. Block codes encode their message block by block. They take as input a k symbol block and encode it into an n symbol codeword, where k and n are fixed and relatively long. On the other hand, Convolutional codes keep encoding the data continuously avoiding fixed packet size where k and n are small. We will focus more on the structure of Convolutional Codes as they are the main components of Turbo codes.

2.6.2 On Convolutional encoders

A Convolutional encoder is illustrated in Figure 2.5. As shown in the Figure, a convolutional encoder is composed of a number of shift registers m, that have as input stream u, and as output i branches denoted by v_i. Each v_i is a specific modulo-2 addition of two or more of the data bits in the shift registers. The number of bits in the

u stream is denoted by k while the number of output bits is denoted by n. The rate of such a code is denoted by $= {}^{k}/_{n}$.

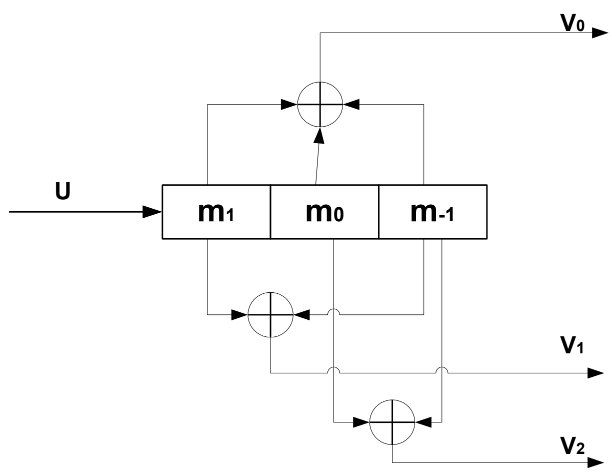

Figure 2.5 Convolutional encoder with rate $r = {}^{1}/_{3}$ and $K = 4$

The number of registers m, shown in Figure 2.5Figure 2.5 Convolutional encoder is related to the constraint length parameter K by the equation $K = m + 1$. In general, a convolutional code is characterized by three parameters: (k, n, m). k is the number of input bits to the encoder at a given time t, while n is the corresponding number of bits out of the encoder. m is the number of shift registers in the convolutional encoder where in binary case, the encoder is a finite state machine with 2^m states that change at every time t. The main disadvantage of convolutional codes is its vulnerability to burst errors. A method to mitigate such a weakness is the use of channel interleaver that scrambles the stream of bits before being transmitted on the channel. At the receiver, after demodulation, a de-interleaver is used to reorder the stream of bits in its original order before being passed to the decoder. By introducing a channel interleaver/ de-interleaver to the system, the burst errors are spread out and appear independent to the decoder [90]. For more details on convolution codes, the reader is referred to [91].

2.6.3 Turbo encoder

A Turbo encoder is the parallel concatenation of two recursive systematic convolutional (RSC) codes separated by an interleaver. The upper encoder receives the stream I while the lower interleaver receives an interleaved version of I denoted by U_{2s}. The interleaver π is a pseudo random interleaver. The interleaver permutates the bits block by block; it takes k bits at a time and interleaves them. Since the two encoders are identical, the systematic bits of the second encoder are omitted. Meanwhile, the parity bits of both encoders are transmitted. The code rate of a Turbo code constructed from two parallel concatenated RSC each of rate one half is of rate one third. For higher code rates, puncturing can be employed. An example of a Turbo encoder is presented in Figure 2.6.

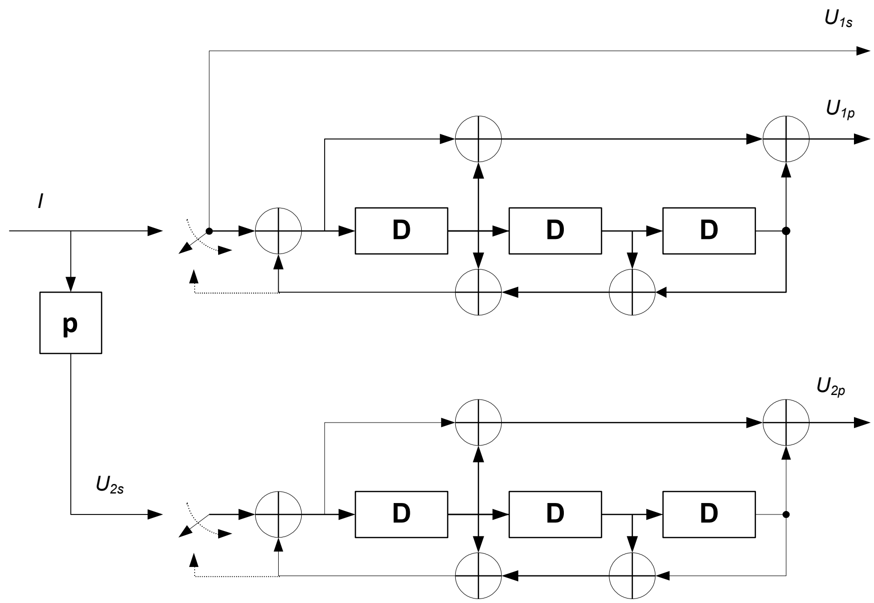

Figure 2.6 A Turbo encoder diagram

The presented Turbo encoder is composed of two RSC, where each can be represented by its generator matrix given by

$$G(D) = \left(1\,, \frac{g_1(D)}{g_2(D)}\right) \tag{2.7}$$

where D is the delay element, $g_1(D)$ is the feed forward polynomial and $g_2(D)$ is the feedback polynomial. For illustration, each RSC forming the Turbo encoder in Figure 2.6 has a generator matrix given by

$$G(D) = \left(1\ , \quad \frac{1 + D + D^2 + D^3}{1 + D + D^3}\right) \tag{2.8}$$

This expression can also be written in octal format using the octal representation of the feedback and feed forward polynomials as in $(g_2, g_1)_{oct}$. The octal representation of the generator polynomials of the RSC in Figure 2.6 is $(15{,}17)_{oct}$.

The corresponding state diagram of the Turbo encoder in Figure 2.6 is presented in Figure 2.7. In the below diagram, a dashed line corresponds to a '1' as input, while the solid line represents an input equal to '0'. Each branch is labeled with its corresponding two bit output.

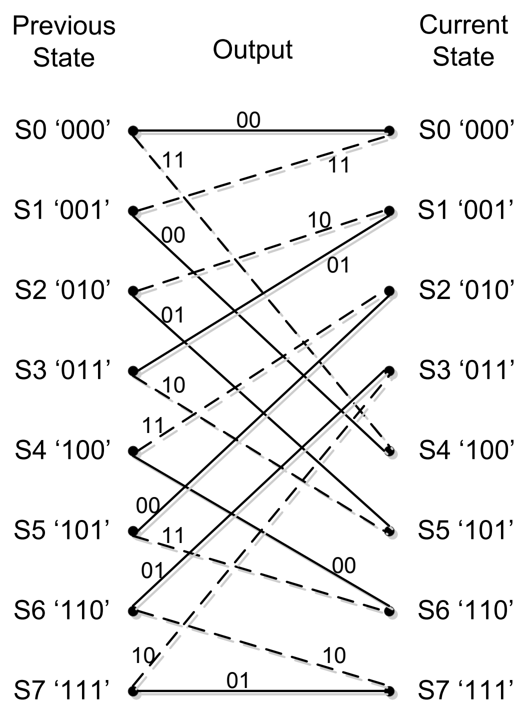

Figure 2.7 A Turbo encoder state diagram

2.6.4 Turbo Decoder

A Turbo decoder diagram is presented in Figure 2.8. A Turbo decoder consists of two soft data input, soft data output (SISO) decoding blocks connected in series via an interleaver that is identical to the one used at the encoder. The Turbo decoder works in an iterative matter, where in each iteration, each SISO decoder passes its output data to the other SISO decoder as extra information to help it with the decoding process. In [81], Valenti summarized the mechanism of the Turbo decoding system by the following system of equations

$$L^1(\hat{u}) = \log \frac{P\left[m_i = 1 | y_{1s}, y_{1p}, L_e^{2\prime}\right]}{P\left[m_i = 0 | y_{1s}, y_{1p}, L_e^{2\prime}\right]} \tag{2.9}$$

$$L^2(\hat{u}) = \log \frac{P\left[\widetilde{m}_i = 1 | y_{1s}', y_{2p}, L_e^{1\prime}\right]}{P\left[\widetilde{m}_i = 0 | y_{1s}', y_{2p}, L_e^{1\prime}\right]} \tag{2.10}$$

where y_{1s} is the received Log likelihood ratio (LLR) of the systematic bits, y_{1p} is the received LLR of the parity bits from the first encoder, y_{1s}' is the interleaved version of the LLR of the systematic bits stream and y_{2p} is the received LLR of the parity bits from the second encoder. $L^i(\hat{u})$ is the a posteriori LLR computed at decoder i, and L_e is the extrinsic value which is related to the LLR by

$$L_e^1 = L^1(\hat{u}) - y_{1s} - L_e^{2\prime} \tag{2.11}$$

$$L_e^2 = L^2(\hat{u}) - y_{1s}' - L_e^{1\prime} \tag{2.12}$$

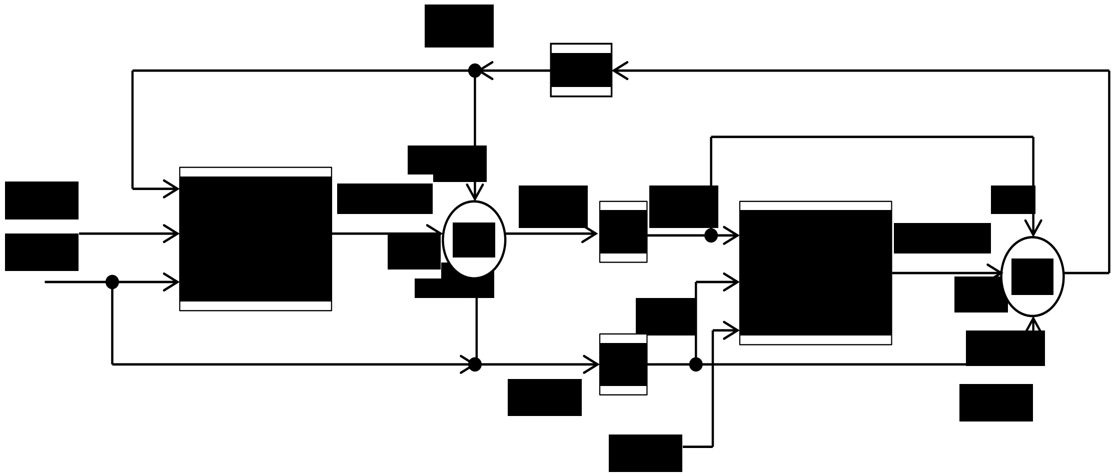

Figure 2.8 Turbo Decoder Diagram

The ensemble of equations (2.9) to (2.12) is solved in an iterative manner as shown in Figure 2.8. The two decoders keep passing their soft information to one another for a predefined number of iterations. This procedure results in a refined estimate of the a posteriori probability of the information bits. At the end of the final iteration, a hard decision is made according to

$$m_i = \begin{cases} 1 \; if \; \tilde{L}^2(\hat{u}) > 0 \\ 0 \; if \; \tilde{L}^2(\hat{u}) < 0 \end{cases} \tag{2.13}$$

where $\tilde{L}^2(\hat{u})$ is the de-interleaved version of $L^2(\hat{u})$

2.6.5 SISO decoder

As mentioned in the previous section, a Turbo decoder consists of two soft data input, soft data output (SISO) decoding blocks ,as shown in Figure 2.9 [92],connected in series via an interleaver that is identical to the one used at the encoder. There are two main inputs to the SISO decoder. The first input is the log likelihood ratio (LLR) of the a priori probability of all information bits $L(u)$.

$$L(u) = \ln \frac{P(u_k = +1)}{P(u_k = -1)} \tag{2.14}$$

At the beginning of the decoding process, equally likely probabilities for the information bits u_k is assumed, where $L(u) = 0$. The second input is a channel reliability measure denoted by L_c. When using the MAP algorithm, the channel Signal to Noise Ratio (SNR) affects the decoding process by augmenting the effect of the channel. So the effect of an error in a systematic bit is also exaggerated. The SNR is measured in the Turbo decoder by the term L_c, also known as the channel reliability measure [81]. The SISO decoder has two outputs. The first output is LLR of a posteriori values

$$L(\hat{u}) = L(u|y) = ln \frac{P(u = +1|y)}{P(u = -1|y)} \tag{2.15}$$

In the case of systematic codes, the soft output of the decoder for the information bit u will be presented as the sum of three terms

$$L(\hat{u}) = L_c \,.\, y + L(u) + L_e(\hat{u}) \tag{2.16}$$

where $L_e(\hat{u})$ is the second output of the SISO decoder. It is the LLR of the extrinsic values for all information bits. $L_e(\hat{u})$ is used as an estimate of the a priori LLR for the other SISO decoder. It computes its soft output information from all the other coded bits in the coded stream. Its value is independent from the values of $L_c \,.\, y$ and $L(u)$.

Figure 2.9 SISO Decoder diagram

2.6.6 The MAP algorithm

There are two known trellis based solution for the problem of estimating the received Markov process sequence. Both methods use the state sequence diagram shown in Figure 2.7 in order to estimate the original sequence. The first method is the Viterbi algorithm (VA) [93]. The second method is the maximum a posteriori (MAP) algorithm [94]. The VA computes the most likely connected path through the trellis diagram leading to the most probable sequence of bits given the received stream y.

$$\hat{s} = \arg\left\{\max_{s} P[s|y]\right\} \tag{2.17}$$

On the other hand, the MAP algorithm aims to compute the most probable state at any given instant i given the received sequence y.

$$\widehat{s_i} = \arg\left\{\max_{s_i} P[s_i|y]\right\} \tag{2.18}$$

Unlike the VA algorithm, the MAP algorithm doesn't require a connected trellis path. The VA algorithm aims to minimize the Frame Error Rate (FER), while the MAP algorithm targets at minimizing the Bit Error Rate (BER). The inputs and outputs of the SISO decoders are Log Likelihood Ratios (LLR) values. In general, the LLR values are the logarithms of the ratio between the probabilities of a given bit $'u'$ being '+1' and the probabilities of the bit $'u'$ being '-1'.

The MAP algorithm, also known as the BCJR algorithm as a reference to Bahl, Cocke, Jelinek and Raviv, was presented by Bahl et al in 1974 as a substitute for Viterbi algorithm for decoding convolutional codes [94]. The algorithm presented was based on forward and backward recursion which makes it more suitable with block oriented processing [90]. Other MAP algorithms that depend on forward recursion only were developed in [95]. Later, the MAP algorithm using the forward backward recursion became known as the *type I* while the one using the forward recursion only became known as the *type II* [96]. MAP *type I* is the one used in Turbo coding as it is more suitable for block codes.

The MAP algorithm provides for each message bit u_k, it's a posteriori probability; the probability that it was sent as a '+1' or '-1' given a noisy observation y. These values are then used to compute the LLR according to (2.15).

$$L(\hat{u}) = L(u|y) = ln\frac{P(u=+1|y)}{P(u=-1|y)} \tag{2.15}$$

$$L(\hat{u}) = \ln\frac{P(u_k=+1|y)}{P(u_k=-1|y)} = \ln\frac{\sum_{u_k=+1}^{(s',s)} P(s',s,y)}{\sum_{u_k=-1}^{(s',s)} P(s',s,y)} \tag{2.19}$$

where we can express the term $P(s',s,y)$ as

$$P(s',s,y) = P\left(s',y_{j<k}\right)\cdot P(s,y_k|s')\cdot P\left(y_{j>k}|s\right) \tag{2.20}$$

$$= \alpha_{k-1}(s') \cdot \gamma_k(s', s) \cdot \beta_k(s) \tag{2.21}$$

Where $y_{j<k}$ represents the received sequence y_j before the receiving the symbol y_k for$(0 < j < k-1)$ and $y_{j>k}$ represents all the sequence of the received symbols following y_k, for $(k+1 < j < K)$. The forward and backward iterations of the MAP algorithms are expressed in [92] as follow

$$\alpha_k(s) = \sum_{(s',s)} \gamma_k(s', s) \cdot \alpha_{k-1}(s') \tag{2.22}$$

$$\beta_{k-1}(s') = \sum_{(s',s)} \gamma_k(s', s) \cdot \beta_k(s) \tag{2.23}$$

$$\gamma_k(s', s) = A_k \cdot B_k \cdot exp\left(\frac{1}{2} L_c \cdot y_{k,1} \cdot u_k + \frac{1}{2}\sum_{v=2}^{n} L_c \cdot y_{k,v} \cdot x_{k,v} + \frac{1}{2} u_k \cdot L(u_k)\right) \tag{2.24}$$

Equations (2.19) and (2.22)-(2.24) yield

$$L(\hat{u}_k) = \ln \frac{\sum_{u_k=+1}^{(s',s)} \gamma_k(s', s) \cdot \alpha_{k-1}(s') \cdot \beta_k(s)}{\sum_{u_k=-1}^{(s',s)} \gamma_k(s', s) \cdot \alpha_{k-1}(s') \cdot \beta_k(s)} \tag{2.25}$$

For more details on the MAP algorithm, the reader is referred to [69] and [92].

2.6.7 Suboptimal algorithms

As mentioned before, the MAP algorithm is optimum for estimating the outputs of a Markov process [92]. However, the MAP algorithm requires a lot of complex computations that include non-linear functions and various additions and multiplications which makes it unpractical to implement. It also suffers round off errors resulting from the presentation of numerical values with finite precision [90]. Other alternatives such as Log Map and Max log Map algorithms were proposed while avoiding the complexity of the MAP algorithm [97].

The main advantage of employing the Log MAP algorithm or the Max Log MAP is that the multiplication in the MAP algorithm becomes and addition in the log domain. According to the *Jacobian algorithm*, the addition in the log domain is performed as follow:

$$\begin{aligned}\ln(e^x + e^y) &= \max(x, y) + \ln(1 + \exp\{-|y - x|\}) \\ &= \max(x, y) + f_c(y - x)\end{aligned} \tag{2.26}$$

Equation (2.26) implies that the addition in the log domain is basically the summation of the maximization operation and a correction function $f_c(.)$. In the case where y and x are different, the correction function $f_c(.)$ tends to be close to zero. In such scenario, it is safe to imply that

$$\ln(e^x + e^y) \approx \max(x, y) \tag{2.27}$$

The Log MAP and the Max Log MAP as mentioned earlier both compute the multiplications as additions in the log domain. The Max Log MAP uses the approximation in equation (2.27) while the Log MAP uses the refined function with the correction function as in equation (2.26).

2.6.8 Turbo Codes and 16-QAM modulation

Even though binary Turbo codes can achieve a significant amount of coding gain, they are not bandwidth efficient unless they are highly punctured. One of the methods adopted to improve the bandwidth efficiency is the use of high order modulation. This has made it interesting to examine the behavior of employing high order modulation with Turbo codes. In this section, a survey on the previous work on Turbo codes and 16 QAM modulation is presented.

The concept of employing high order modulation with Turbo codes was first introduced in [98]. In this paper, Berou et al. proved that this joint scheme achieved a considerable coding gain both on Gaussian and Rayleigh channels. In both channel environments; the Gaussian channel and the Rayleigh fading channel; the described scheme outperformed a 64-state trellis-coded modulation (TCM).

In [99], a joint 16-QAM modulation and binary Turbo codes were employed for phase recovery. The soft bit data was computed at the 16QAM demodulator and used as input to the Turbo decoder. The output of the Turbo decoder was then passed to the phase recovery part to implement iterative Maximum Likelihood (ML) evaluation of the carrier phase error. Meanwhile, the mapping of systematic bits and parity bits in a 16QAM modulation was examined in [100]. The authors used two modulation schemes; 8PSK and 16QAM; and compared them. It was concluded that mapping the systematic bits to be more protected has led to an enhancement in the BER threshold. On the other hand, mapping the parity bits to the more protected bits of the high order modulation resulted in an improved error floor performance on the expense of slight degradation of the BER threshold. In [101], a special non uniform 16QAM modulation was employed to map the systematic and parity bits of a Turbo encoder. The motive behind this mapping was to reduce the gap between the Shannon capacity limit and the performance of Turbo codes at high SNR. The proposed mapping consisted of shaping the constellation's signals in a way that the constellation itself had a "Gaussian like distribution". This 'Gaussian like' shaping had relatively more low energy signals compared to the uniform constellation. This resulted in lower average symbol energy for fixed distance constellations. Such a scheme can be beneficial in Satellite channel environment, as low energy signals are less vulnerable to High Power Amplifiers (HPA) distortion. In addition, a new iterative de-mapping and decoding scheme was proposed. The authors employed an additional soft de-mapper between the two SISO decoders to improve the channel estimation at the second decoder. Performance of the system employing this extra de-mapper resulted in a gain of 0.9 dB at 10^{-5} over linear channels and 1.2dB over non-linear channels.

A study on the design and performance of Unequal Error Protection (UEP) Turbo coded modulation scheme was presented in [102]. The system presented involved multiple streams with different importance to be sent on a wireless channel, using Turbo coding. The study's goal was to achieve UEP through three different scenarios. The first scenario was to achieve Unequal Error Protection by employing non-uniform puncturing after encoding the input stream. The non-uniform puncturing was designed in a way to apply more puncturing to the less important data. The second method was to achieve

UEP by mapping the bits of the most important data to the base bits in a multi-resolution modulation while mapping the least important stream bits to the refinement bits of the same modulation. Finally, the third method was a combination of the first two methods. They concluded that all presented schemes achieved high performance gains for the important streams. Such a gain could not be accomplished using the traditional coded modulation. They also concluded that the performance of the third method has the best performance of the three methods.

Improving the system's BER through adjusting the constellation's symbol energy was employed in [103]. The authors employed different symbol energy and examined the results in different scenarios. However, no information on the channel condition or the kind of Turbo encoder employed was provided. Another scheme was employed in [104] where exchange between the 16QAM demodulator output and the Turbo decoder data takes place in order to improve the accuracy of the decision. They concluded that one iteration between the demodulator and the Turbo decoder is enough to improve the system's performance. However, regular iterations inside the Turbo decoder continue to take place. Different values for the hierarchical 16QAM's priority factor were employed showing the different BER of base bits and refinement bits corresponding to each value.

Chapter Three

3. System Model

3.1 Overview

In this chapter, the proposed system is presented and examined in details. The main constituting nodes of the system are a Base Station (BS) and two User-Ends UE_1 and UE_2. In the presented downlink system, the BS aims at transmitting two different streams of data to these two User-Ends simultaneously. Instead of sending each user's data on a different time slot, the BS concatenates the two streams of data and modulates them by employing a 4/16-QAM hierarchical modulation. In one transmission, the BS sends one 'four bits symbol' where two bits are destined to UE_1 and two other bits are destined to UE_2. Using a high order modulation like a 4/16 QAM results in time saving.

Depending on the surrounding environment of the users and their separating distances to the BS, they experience different fading channel conditions. This results in different message detection accuracy at the destinations. The goal of this system is to minimize the Bit Error Rate (BER) of these two UEs according to their channel conditions.

Such a goal can be achieved by optimizing the energy allocated for modulating each user's data. From the study of the 4/16 QAM hierarchical modulation (Figure 3.1) and from previous applications of Hierarchical modulation in the DVB field, it can be concluded that the 4/16 QAM hierarchical modulation is a suitable method for achieving such an optimization. For example, let's assume that at a specific moment, the instantaneous channel conditions of UE_1 suffer from noticeable degradation. One way to compensate for such degradation is to map the data of UE_1 on the base bits location of the 4/16QAM constellation while mapping the data of UE_2 on the refinement bits location [11]. This solution gives more protection to the data of UE_1 and somehow compensates for the deteriorated performance caused by the degraded channel. Figure 3.1 illustrates a hierarchical 4/16 QAM constellation where the related constellation's distance parameters are explained. As shown in the figure, the distance between two virtual black dots is denoted by $2d_1$ and the distance between two adjacent symbols in the same quadrant is denoted by $2d_2$. The difference in distance between d_1 and d_2 is denoted by d_1' where

$$d_1' = d_1 - d_2 \tag{3.1}$$

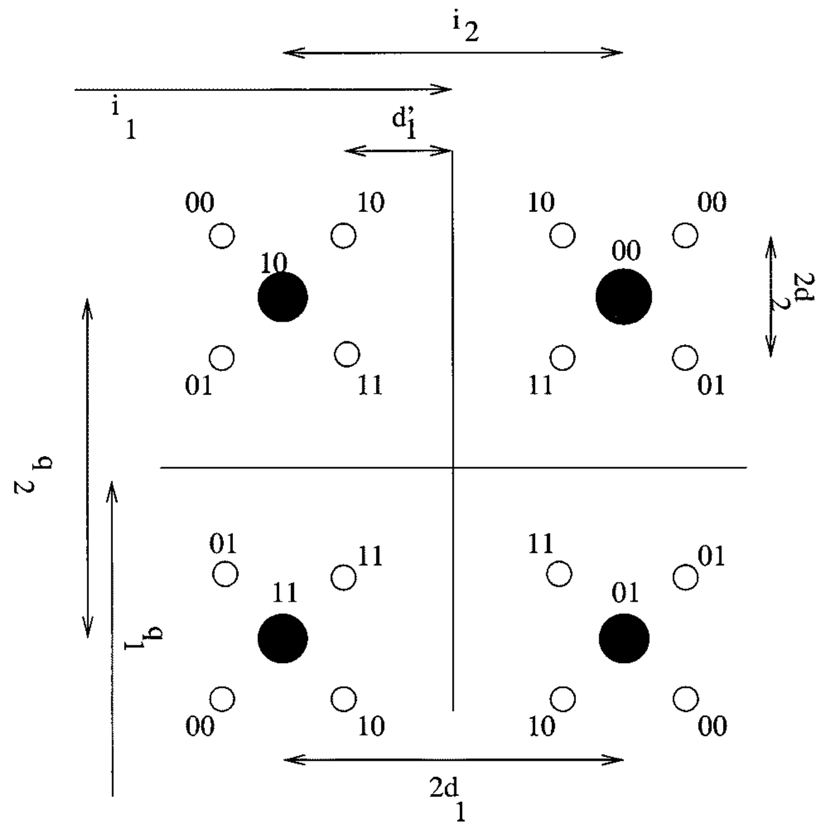

Figure 3.1 Embedded 4/16-QAM constellation with Gray coding

As mentioned in the example given earlier, the 4/16 QAM modulation can be employed to allocate different symbol energy to the different users. In the above mentioned case, if uniform 16QAM modulation is assumed, then the energy allocated to UE_1 is $2d_1^2$ while the energy allocated to UE_2 is $2d_2^2$. In a uniform 16 QAM modulation, the distance parameters d_1 and d_2 are related by $d_1 = 2d_2$. Thus the allocated energy for UE_1 in a uniform 16 QAM constellation is four times the energy allocated to UE_2.

In the presented study, further investigation of this solution is carried by optimizing the energy allocated to each user and not limiting it to the 4:1 ratio mentioned above. An optimization problem is defined where the goal is to minimize the summation of the base bits BER and the refinement bits BER in terms of the distance parameters d_1 and d_2. In other words, the general optimization problem can be expressed as

$$(d_1, d_2) = \arg \min_{(d_1,d_2)} \{Pe_{Base} + Pe_{Ref}\}$$

$$\text{subject to} \begin{cases} 0 < d_2 < d_1 < \sqrt{E/2} \\ E = 2\left({d_1}^2 + {d_2}^2\right) \end{cases} \tag{3.2}$$

where E is the energy per symbol.

This research examines three different schemes; the direct transmission scheme, the nodes cooperation scheme and the Turbo coded cooperative scheme. In the first two schemes, the instantaneous BER of the base bits and the refinement bits are derived. These derivations are used in computing the values of d_1 and d_2 to achieve optimal total BER. Furthermore, a comparison between the average BER of each user in the optimized modulation case and the uniform modulation case is presented. Later in the results section, it will be shown that this optimization results in a noticeable improvement in BER for both users. Moreover, the presented study is extended to estimate sub-optimal constellation distance parameters for the Turbo coded cooperative scheme from the un-coded cooperative scheme derivations. Simulation results show that these parameters estimates still have a better performance than the parameters of a uniform constellation. In the following subsections, each scheme is presented along with its parameters and necessary derivations. Later in this study, results of each scheme's performance are examined.

3.2 Direct Transmission

In the case of direct transmission, the BS transmits two different streams of data to two destinations without any external assistance from surrounding nodes. It does that by combining these two streams employing a 4/16QAM constellation where two bits of the four bits symbol are assigned to UE_1 and the other two bits are assigned to UE_2. UE_1 and UE_2 are located at distances d_{s,u_1} and d_{s,u_2} respectively from the BS. It is assumed that $d_{s,u_1} \geq d_{s,u_2}$. Thus, the channel condition of UE_2 is generally more favorable than that of UE_1. Hence, to insure acceptable performance at both users, the BS will modulate UE_1's data to be the base bits while UE_2's data to be the refinement bits.

The binary data stream destined to UE_1 is denoted by u_1 and the binary data stream destined to UE_2 is denoted by u_2. Each of these two streams is of length l. At a given time t, where $1 \leq t \leq l$, the Base Station concatenates the two streams in a manner that the four bits symbol at time t will be represented as

$$s(t) = [u_1(2t-1)\ \ u_1(2t)\ \ u_2(2t-1)\ \ u_2(2t)] \tag{3.3}$$

The transmitted modulated symbol is chosen from the set of 16 available symbols, as shown in Figure 3.1 so that the modulated symbol in terms of the constellation's distance parameters is in the form of

$$x(d_1, d_2) = (\pm d_1 \pm d_2) + j(\pm d_1 \pm d_2) \tag{3.4}$$

The Channel State Information (CSI) is assumed to be perfectly known at all nodes before transmission. Such knowledge is important at the source to enable it in determining the constellation's distance parameters values that will achieve minimum BER. On the other hand, such knowledge is mandatory at the destinations in order to enable coherent detection. This knowledge can be achieved via training sequences in the protocol headers resulting in accurate estimates of the link fading coefficients. The transmitted signals experience independent and identically distributed (i.i.d.) slow Rayleigh fading channels. The channel coefficients on the $BS - UE_1$ link and the $BS - UE_2$ link are denoted by h_{s,u_1} and h_{s,u_2} respectively. These channel coefficients are zero mean complex Gaussian random variables with variance ρ^2_{s,u_i} where

$$\rho_z{}^2 = \left(\frac{d_{s,u_1}}{d_z}\right)^{\mu} \tag{3.5}$$

μ is the path loss exponent and is assumed to have a value of 3.5, d_{s,u_1} is the reference distance and d_z is the distance of the link in question.

At the BS, the values of the constellation's distance parameters d_1 and d_2 are selected according to the optimization equation (3.10). The BS then modulates the input stream s into the modulated stream x. It then broadcasts its data that is received by UE_1 and UE_2 as follow:

$$y_{u_1} = \sqrt{E_s}\,.h_{s,u_1}.x + n_{\,s,u_1} \tag{3.6}$$

$$y_{u_2} = \sqrt{E_s}\,.h_{s,u_2}.x + n_{\,s,u_2} \tag{3.7}$$

where E_s is the transmitted symbol energy. n_{s,u_1} and n_{s,u_2} are complex Additive White Gaussian Noise (AWGN) random variables with zero mean and variance σ^2 with spectral density $N_0/2$ per dimension.

At each UE, the received signal y_{u_i} is demodulated and hard decision is applied. However, since each user is interested only in its data, the detection at each user follows the below algorithm:

- At the first User Equipment (UE_1) at time t:

  $If\ Real(y_{u_1}) > 0$

  $then\ u_1'(2t-1) = 0, \qquad else\ u_1'(2t-1) = 1$

  $If\ Imag(y_{u_1}) > 0$

  $then\ u_1'(2t) = 0, \qquad else\ u_1'(2t) = 1$

- At the second User Equipment (UE_2) at time t:

  $If\ Real(y_{u_2}) > \sqrt{E_s} d_1 h_{s,u_2}$

  $then\ u_2'(2t-1) = 0, \qquad else\ u_2'(2t-1) = 1$

  $If\ Imag(y_{u_2}) > \sqrt{E_s} d_1 h_{s,u_2}$

  $then\ u_1'(2t) = 0, \qquad else\ u_1'(2t) = 1$

3.2.1 Distance Parameters selection in Direct Transmission scheme

The constellation's distance parameters are selected based on the instantaneous SNR γ_{s,u_1} and γ_{s,u_2} , where $\gamma_{s,u_i} = \frac{E_s \left|h_{s,u_i}\right|^2}{N_0}$. The goal is to select the parameters (d_1, d_2) that will achieve minimum total instantaneous BER given by $Pe_{Tot}^{DT} =$

$Pe_{Base}^{DT} + Pe_{Ref}^{DT}$. The instantaneous BER for base bits in Direct Transmission (DT) scenario is given by [68] [105]

$$Pe_{Base}^{DT} = \frac{1}{4}\left\{erfc\left((d_1 + d_2)\sqrt{\gamma_{s,u_1}}\right) + erfc\left((d_1 - d_2)\sqrt{\gamma_{s,u_1}}\right)\right\} \tag{3.8}$$

The instantaneous BER for refinement bits in Direct Transmission scenario is given by

$$\begin{aligned} Pe_{Ref}^{DT} = \frac{1}{4}\Big\{2\, erfc\left(d_2\sqrt{\gamma_{s,u_2}}\right) - erfc\left((2d_1 + d_2)\sqrt{\gamma_{s,u_2}}\right) \\ + erfc\left((2d_1 - d_2)\sqrt{\gamma_{s,u_2}}\right)\Big\} \end{aligned} \tag{3.9}$$

Following equation (3.2), the optimization problem can be expressed as

$$\begin{aligned} (d_1, d_2) = \arg\min_{d_1,d_2} \Big\{ & erfc\left((d_1 + d_2)\sqrt{\gamma_{s,u_1}}\right) + erfc\left((d_1 - d_2)\sqrt{\gamma_{s,u_1}}\right) \\ & + 2\, erfc\left(d_2\sqrt{\gamma_{s,u_2}}\right) - erfc\left((2d_1 + d_2)\sqrt{\gamma_{s,u_2}}\right) \\ & + erfc\left((2d_1 - d_2)\sqrt{\gamma_{s,u_2}}\right)\Big\} \end{aligned}$$

$$\text{subject to} \begin{cases} 0 < d_2 < d_1 < \sqrt{E_s/2} \\ E_s = 2\left({d_1}^2 + {d_2}^2\right) \end{cases} \tag{3.10}$$

3.3 Nodes Cooperation

In the case of Direct Transmission, the BS broadcasts its modulated signal that is received by the two destinations without any assistance. However, in the case of nodes cooperation, a surrounding node with favorable reception from the BS assists it in broadcasting another copy of the BS signal. This node can be either a User-End node or an external node. The first case is when one UE has detected the whole four bits symbol correctly and is capable of resending a copy of its data to the other UE using a 4 QAM modulation. The second case is when an external node, a relay node, hears the BS message and can relay this message to the two UEs using a 4/16 QAM constellation. In the following subsections, a presentation of these two scenarios is proposed in details.

3.3.1 User-End Cooperation

In the case of UE Cooperation, the constituting nodes are the same as of the Direct Transmission scheme. However, in this scheme one User-End assists the Base Station in improving the reception at the other User-End. Thus in this scheme, the transmission occurs on two time slots instead of just one. In the first time slot, the BS broadcasts two different streams of data concatenated together using a 4/16QAM constellation to the two User-Ends. Each transmitted signal consists of a four bits symbol, where the first two bits are assigned to UE_1 and the other two bits are assigned to UE_2. The distances separating the BS from UE_1 and UE_2 are denoted by d_{s,u_1} and d_{s,u_2} respectively. The distance separating UE_1 from UE_2 is denoted by d_{u_1,u_2}. It is assumed that $d_{s,u_1} \geq d_{s,u_2}$. In other words UE_2 in general has a more favorable channel condition than that of UE_1. The BS modulates UE_1's data to be the base bits while UE_2's data to be the refinement bits. The channel coefficients on the $BS - UE_1$ link and $BS - UE_2$ link are denoted by h_{s,u_2} and h_{s,u_2} respectively. h_{u_1,u_2} denotes the channel coefficient on the link between the two UEs. The channel coefficients h_{s,u_1}, h_{s,u_2} and h_{u_1,u_2} are zero mean complex Gaussian random variables with variance ρ^2_{s,u_1} , ρ^2_{s,u_2} and $\rho^2_{u_1,u_2}$ respectively

3.3.1.1 Case of correct detection at UE_2

In the case where the channel between the BS and UE_2 is favorable enough for UE_2 to detect its message correctly, the BS decides on the distance parameters according to equation (3.18). The stream s is then modulated into the modulated stream x. The Base Station then broadcasts its data that is received by UE_1 and UE_2 as follow:

$$y'_{u_1} = \sqrt{E_s}\,.\,h_{s,u_1}.x + n_{\,s,u_1} \tag{3.11}$$

$$y_{u_2} = \sqrt{E_s}\,.\,h_{s,u_2}.x + n_{\,s,u_2} \tag{3.12}$$

where E_s is the transmitted symbol energy at the BS. n_{s,u_1}, n_{s,u_2} and n_{u_1,u_2} are complex Additive White Gaussian Noise (AWGN) random variables with zero mean and variance σ^2 with spectral density $N_0/2$ per dimension. UE_2 then demodulates its data, extracts UE_1 data and modulates it using a 4QAM modulation. UE_2 sends the modulated stream $\hat{x}$ to UE_1, where the received signal is expressed as:

$$y''_{u_1} = \sqrt{E_{u_2}}\,.\,h_{u_1,u_2}.\hat{x} + n_{\,u_1,u_2} \tag{3.13}$$

Where $\hat{x} \in \{\pm d_{4QAM} \pm j d_{4QAM}\}$ and d_{4QAM} is the 4QAM constellation's distance parameters. $E_{u_2}$is the transmitting symbol energy at UE_2 where

$$E_s + E_{u_2} = E_{Tot} \tag{3.14}$$

and E_{Tot} is the total available energy for transmission. d_{4QAM} is related to the symbol energy E_{u_2} by

$$E_{u_2} = 2d_{4QAM}^2 \tag{3.15}$$

At UE_1, the two signals y'_{u_1} and y''_{u_1} are combined together using Maximal Ratio Combining (MRC) leading to

$$y_{u_1} = \sqrt{E_s}\left|h_{s,u_1}\right|^2 x + n_{s,u_1}h_{s,u_1}^* + \sqrt{E_{u_2}}\left|h_{u_1,u_2}\right|^2 \hat{x} + n_{u_1,u_2}h_{u_1,u_2}^* \tag{3.16}$$

where z^* represents the conjugate of z.

At each UE, the received signal y_{u_i} is demodulated and hard decision is applied. However, since each user is interested only in its own data, the demodulation process is different at each user. In the case of User-End cooperation it should be taken into consideration that UE_2 needs to detect the whole symbol. The demodulation process follows the subsequent algorithm:

- At the first User Equipment (UE_1) at time t:

  $If\ Real(y_{u_1}) > 0$

  $then\ u'_1(2t-1) = 0, \qquad else\ u'_1(2t-1) = 1$

  $If\ Imag(y_{u_1}) > 0$

  $then\ u'_1(2t) = 0, \qquad else\ u'_1(2t) = 1$

- At the second User Equipment (UE_2) at time t to detect UE_1's data:

  $If\ Real(y_{u_2}) > 0$

  $then\ u''_1(2t-1) = 0, \qquad else\ u''_1(2t-1) = 1$

$If\ Imag(y_{u_2}) > 0$

$then\ u_1''(2t) = 0, \qquad else\ u_1''(2t) = 1$

Where u_1'' is the detected data of UE_1 at UE_2

- At the second User Equipment (UE_2) at time t to detect its own data :

$If\ Real(y_{u_2}) > \sqrt{E_s} d_1 h_{s,u_2}$

$then\ u_2'(2t-1) = 0, \qquad else\ u_2'(2t-1) = 1$

$If\ Imag(y_{u_2}) > \sqrt{E_s} d_1 h_{s,u_2}$

$then\ u_1'(2t) = 0, \qquad else\ u_1'(2t) = 1$

3.3.1.2 Distance Parameters selection in User-End Cooperation scheme in the case of correct detection at UE_2

The constellation's distance parameters are selected based on the instantaneous received SNR on the three links γ_{s,u_1} , γ_{s,u_2}, and γ_{u_1,u_2}. The goal is to minimize the total instantaneous BER given by $Pe_{Tot}^{UC} = Pe_{Base}^{UC} + Pe_{Ref}^{UC}$. In the User-End Cooperation scenario, the BER of the refinement bits Pe_{Ref}^{UC} is the same as in the Direct Transmission case and can be expressed as per (3.9). However, the instantaneous BER for base bits Pe_{Base}^{UC} is computed differently. In the case of User-End Cooperation, if UE_2 is successful in correctly demodulating the entire 16QAM symbol, it will re-modulates UE_1's data using a 4QAM modulation and retransmits it to UE_1. In such a scenario, UE_1 receives two copies of the signal, one from the BS and the other from UE_2. UE_1 combines the two signals using MRC combining then applies hard decision. The resulting signal at UE_1 is given by (3.16)

$$y_{u_1} = \sqrt{E_s}\left|h_{s,u_1}\right|^2 x + \sqrt{E_{u_2}}\left|h_{u_1,u_2}\right|^2 \hat{x} + n_{s,u_1} h_{s,u_1}^* + n_{u_1,u_2} h_{u_1,u_2}^* \tag{3.16}$$

From section A.1.1, the corresponding BER for the Base bits in this scenario can be expressed as

$$Pe_{Base}^{UC} = \frac{1}{4}\left\{erfc\left(\frac{(d_1+d_2)\sqrt{E_s}\left|h_{s,u_1}\right|^2 + d_{4QAM}\sqrt{E_{u_2}}\left|h_{u_1,u_2}\right|^2}{\sqrt{\left|h_{s,u_1}\right|^2 + \left|h_{u_1,u_2}\right|^2}}\right) + erfc\left(\frac{(d_1-d_2)\sqrt{E_s}\left|h_{s,u_1}\right|^2 + d_{4QAM}\sqrt{E_{u_2}}\left|h_{u_1,u_2}\right|^2}{\sqrt{\left|h_{s,u_1}\right|^2 + \left|h_{u_1,u_2}\right|^2}}\right)\right\} \tag{3.17}$$

The optimization problem can be expressed as follow

$$(d_1, d_2) = \arg\min_{d_1,d_2}\left\{erfc\left(\frac{(d_1+d_2)\sqrt{E_s}\left|h_{s,u_1}\right|^2 + d_{4QAM}\sqrt{E_{u_2}}\left|h_{u_1,u_2}\right|^2}{\sqrt{\left|h_{s,u_1}\right|^2 + \left|h_{u_1,u_2}\right|^2}}\right) + erfc\left(\frac{(d_1-d_2)\sqrt{E_s}\left|h_{s,u_1}\right|^2 + d_{4QAM}\sqrt{E_{u_2}}\left|h_{u_1,u_2}\right|^2}{\sqrt{\left|h_{s,u_1}\right|^2 + \left|h_{u_1,u_2}\right|^2}}\right) + 2\,erfc\left(d_2\sqrt{\gamma_{s,u_2}}\right) - erfc\left((2d_1+d_2)\sqrt{\gamma_{s,u_2}}\right) + erfc\left((2d_1-d_2)\sqrt{\gamma_{s,u_2}}\right)\right\}$$

$$\text{subject to} \begin{cases} 0 < d_2 < d_1 < \sqrt{E_s/2} \\ E_s = 2\left({d_1}^2 + {d_2}^2\right) \\ E_{u_2} = 2{d_{4QAM}}^2 \\ E_s + E_{u_2} = E_{Tot} \end{cases} \tag{3.18}$$

3.3.1.3 Case of incorrect detection at UE_2

In the case where UE_2 fails to detect its received signal, it will remain silent and the system will act as a Direct Transmission system. In this case the distance parameters selection will only take place at the BS and the selection will be handled in the same manner as in section (3.2.1).

3.3.2 Relay Cooperation

In the case of Relay Cooperation, the constituting nodes are the same as of the Direct Transmission scheme with the addition of a relaying node. As in the previous mentioned schemes, the BS combines the two streams using a 4/16QAM modulation then broadcasts the modulated signal. The base bits of the 16QAM constellation are destined to UE_1 whereas the refinement bits are assigned to UE_2. In the relay cooperative scheme, the Relay listens to the BS message and forwards it employing either Amplify and Forward (AF) technique or Decode and Forward (DF) technique. After receiving the broadcasted copy from the Relay, each User-End combines the two received signals coming from the BS and the Relay using Maximal Ratio Combining (MRC) then applies hard decision on the resulting signal. Figure 3.2 Illustrates the predicted relay cooperation system.

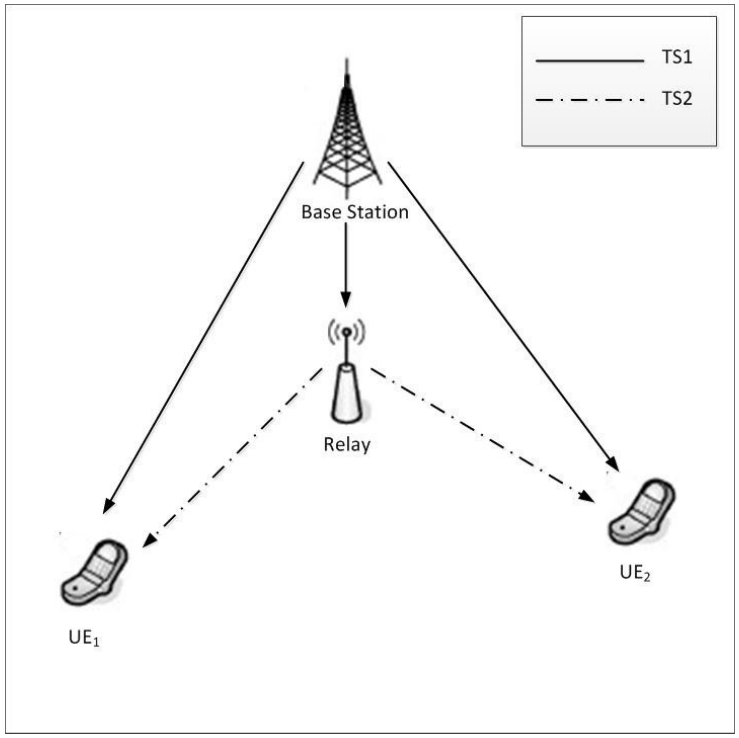

Figure 3.2 Multiuser downlink cooperative system

The distances separating the BS from UE_1, UE_2 and R are denoted by d_{s,u_1} , d_{s,u_2} and $d_{s,r}$ respectively; where $d_{s,u_1} \geq d_{s,u_2} > d_{s,r}$. The distances separating R from UE_1 and UE_2 are d_{r,u_1} and d_{r,u_2} respectively. h_{s,u_1} , h_{s,u_2} and $h_{s,r}$ denote the channel coefficients on the $BS - UE_1$ link, the $BS - UE_2$ link and the $BS - R$ link respectively. h_{r,u_1} and h_{r,u_2} denote the channel coefficients on the $R - UE_1$ link and the $R - UE_2$ respectively. The channel coefficients h_{s,u_1}, h_{s,u_2}, $h_{s,r}$, h_{r,u_1} and h_{r,u_2} are zero mean complex Gaussian random variables with variance ρ^2_{s,u_1} , ρ^2_{s,u_2} , $\rho^2_{s,r}$, $\rho^2_{r,u_1}$and ρ^2_{r,u_2} respectively. In the following section, the two proposed forwarding techniques, Amplify and Forward and Decode and Forward are studied in the relay cooperative scheme. BER of the base bits and the refinement bits are derived for each relaying technique.

3.3.2.1 Decode and Forward scenario

In the case of employing Decode and forward technique, the Relay R detects and decodes the data it receives from the BS. At the relay, a Cyclic Redundancy Check (CRC) is applied in order to check the correctness of the detected data. If the Relay succeeds in correctly decoding the received data, it re-encodes the stream of data and sends it to the two UEs. On the other hand, if the CRC check fails, the Relay node remains silent and the two UEs receive only one copy from the BS.

It is assumed that the channel is constant during the transmission of a whole block of data. Before each block transmission, the BS first collects data on the channel link connecting it to the Relay. From the collected data and from the derived probability of error of the base bits and the refinement bits, the BS can guess whether the Relay will be able to correctly decode the data it receives from the BS or not. Accordingly, the BS modulates its data with the corresponding distance parameters that will ensure minimum bit error at both UEs.

For example, assuming that one block of transmission consists of equal streams of base bits and refinements bits. Each of these two streams is of length l bits. As mentioned earlier, the relay will only forward its received data when this data is detected correctly. Such data can be detected correctly when there exists a set of $[d_1, d_2]$ that satisfies the following conditions

$$Pe_{Base}^{SR} < 1/l \tag{3.19}$$

$$Pe_{Ref}^{DT} < 1/l \tag{3.20}$$

where

$$Pe_{Base}^{SR} = \frac{1}{4}\left\{erfc\left((d_1 + d_2)\sqrt{\gamma_{s,r}}\right) + erfc\left((d_1 - d_2)\sqrt{\gamma_{s,r}}\right)\right\} \tag{3.21}$$

$$Pe_{Ref}^{SR} = \frac{1}{4}\left\{2\, erfc\left(d_2\sqrt{\gamma_{s,r}}\right) - erfc\left((2d_1 + d_2)\sqrt{\gamma_{s,r}}\right) + erfc\left((2d_1 - d_2)\sqrt{\gamma_{s,r}}\right)\right\} \tag{3.22}$$

In the case where there exists several sets of $[d_1, d_2]$ that satisfy the above mentioned conditions then the set of $[d_1, d_2]$ that achieves the minimum total BER is chosen. If such a set does not exist then the relay cannot correctly decode its received information. Hence, the BS will modulate its data using a set of $[d_1, d_2]$ that will achieve optimum performance through the direct transmission only, as per equation (3.10).

3.3.2.1.1 Case of Correct detection at relay

As mentioned in section 3.3.2.1, correct detection at the relay will occur when there exists one or multiple sets of $[d_1, d_2]$ that satisfy conditions (3.19) and (3.20). If there exists only one set of $[d_1, d_2]$, then the BS will use this set for its transmission. However, if there exists multiple sets, then the BS will select the set that will minimize the summation of equations (A.29) and (A.63) accordingly we get

$$\min_{d_1,d_2,d_1',d_2'} \left\{ Q\left(\frac{\left[\sqrt{E_s}(d_1-d_2)\left|h_{s,u_1}\right|^2+\sqrt{E_r}(\widehat{d_1}-\widehat{d_2})\left|h_{r,u_1}\right|^2\right]}{\sigma\sqrt{\left|h_{s,u_1}\right|^2+\left|h_{r,u_1}\right|^2}} \right) \right.$$

$$+ Q\left(\frac{\left[\sqrt{E_s}(d_1+d_2)\left|h_{s,u_1}\right|^2+\sqrt{E_r}(\widehat{d_1}+\widehat{d_2})\left|h_{r,u_1}\right|^2\right]}{\sigma\sqrt{\left|h_{s,u_1}\right|^2+\left|h_{r,u_1}\right|^2}} \right)$$

$$+ 2Q\left(\frac{\sqrt{E_s}d_2\left|h_{s,u_2}\right|^2+\sqrt{E_r}\widehat{d_2}\left|h_{r,u_2}\right|^2}{\sigma\sqrt{\left(\left|h_{s,u_2}\right|^2+\left|h_{r,u_2}\right|^2\right)}} \right)$$

$$- Q\left(\frac{\sqrt{E_s}(2d_1+d_2)\left|h_{s,u_2}\right|^2+\sqrt{E_r}\left(2\widehat{d_1}+\widehat{d_2}\right)\left|h_{r,u_2}\right|^2}{\sigma\sqrt{\left(\left|h_{s,u_2}\right|^2+\left|h_{r,u_2}\right|^2\right)}} \right)$$

$$\left. + Q\left(\frac{\sqrt{E_s}(2d_1-d_2)\left|h_{s,u_2}\right|^2+\sqrt{E_r}\left(2\widehat{d_1}-\widehat{d_2}\right)\left|h_{r,u_2}\right|^2}{\sigma\sqrt{\left(\left|h_{s,u_2}\right|^2+\left|h_{r,u_2}\right|^2\right)}} \right) \right\}$$

$$\text{subject to} \begin{cases} 0 < d_2 < d_1 < \sqrt{\dfrac{E_s}{2}} \\ 0 < \widehat{d_2} < \widehat{d_1} < \sqrt{\dfrac{E_r}{2}} \\ 2(d_1^2+d_2^2) = E_s \\ 2\left(\widehat{d_1}^2+\widehat{d_2}^2\right) = E_r \end{cases} \tag{3.23}$$

$\widehat{d_1}$ and $\widehat{d_2}$ are the modulation's constellation parameters in the second time slot.

After the selection of the constellation's distance parameters, the Base Station modulates its data using 16QAM hierarchical modulation with parameters $d_1$and d_2

then broadcasts the data that it is received respectively by the two user-ends and the relay as

$$y_{s,u_1} = \sqrt{E_s}\, h_{s,u_1} x + n_{s,u_1} \tag{3.24}$$

$$y_{s,u_2} = \sqrt{E_s}\, h_{s,u_2} x + n_{s,u_2} \tag{3.25}$$

$$y_{s,r} = \sqrt{E_s}\, h_{s,r}\, x + n_{s,r} \tag{3.26}$$

where x is the modulated symbol, n_{s,u_1},n_{s,u_2} ,$n_{s,r}$, n_{r,u_1} and n_{r,u_2} are complex Additive White Gaussian Noise (AWGN) random variables with zero mean and variance σ^2 with spectral density $N_0/2$ per dimension. y_{s,u_1}, $y_{s,u_2}$and $y_{s,r}$ are the received signals at UE_1, UE_2 and R respectively. The relay demodulates and detects its data and then re-modulates the data using 16-QAM hierarchical modulation with the optimized distance parameters $\widehat{d_1}$ and $\widehat{d_2}$ as per equation (3.23). The assigned energy for UE_1 data and UE_2 data at the relay is $2\widehat{d_1}^2$ and $2\widehat{d_2}^2$ respectively. The received signals from the relay at the two UEs are given by

$$y_{r,u_1} = \sqrt{E_r}\, h_{r,u_1} \hat{x} + n_{r,u_1} \tag{3.27}$$

$$y_{r,u_2} = \sqrt{E_r}\, h_{r,u_2} \hat{x} + n_{r,u_2} \tag{3.28}$$

where $\hat{x}$ is the re-modulated stream at the relay and E_r is the transmitted energy per symbol at the relay. Each user combines the two received signals from the Base Station and the relay, using MRC. The resultant combined signals at UE_1 and UE_2 are given by

$$y_{u_1} = \sqrt{E_s}\, h_{s,u_1}^2 x + h_{s,u_1}^* n_{s,u_1} + \sqrt{E_r}\, h_{r,u_1}^2 \hat{x} + h_{r,u_1}^* n_{r,u_1} \tag{3.29}$$

$$y_{u_2} = \sqrt{E_s}\, h_{s,u_2}^2 x + h_{s,u_2}^* n_{\,s,u_2} + \sqrt{E_r}\, h_{r,u_2}^2 \hat{x} + h_{r,u_2}^* n_{\,r,u_2} \tag{3.30}$$

where $(z)^*$ represents the conjugate of (z). At each UE, the received signal y_{u_i} is demodulated and hard decision is applied. Since each user is interested only in its data, the detection at each user follows the below algorithm:

- At the first User Equipment (UE_1) at time t:

  $If\ Real(y_{u_1}) > 0$

  $then\ u_1'(2t-1) = 0, \qquad else\ u_1'(2t-1) = 1$

  $If\ Imag(y_{u_1}) > 0$

  $then\ u_1'(2t) = 0, \qquad else\ u_1'(2t) = 1$

- At UE_2:

  $If\ Real(y_{u_2}) > \sqrt{E_s}\,|h_{s,u_2}|^2 d_1 + \sqrt{E_r}|h_{r,u_2}|^2 \widehat{d_1}$

  $then\ u_2'(2t-1) = 0, \qquad else\ u_2'(2t-1) = 1$

  $If\ Imag(y_{u_2}) > \sqrt{E_s}\,|h_{s,u_2}|^2 d_1 + \sqrt{E_r}|h_{r,u_2}|^2 \widehat{d_1}$

  $then\ u_1'(2t) = 0, \qquad else\ u_1'(2t) = 1$

3.3.2.1.2 Case of direct transmission

In the case where the conditions applied in (3.19) and (3.20) resulted in null set, then the relay will not be able to correctly detect its received information. In such a case, the relay remains silent, and the system acts as a direct transmission scheme. The distance parameters are then selected at the BS according to (3.10) and the demodulation procedure follows the algorithm in section 3.2.

3.3.2.2 Amplify and Forward scenario

In the case of employing Amplify and Forward technique, the constellation's distance parameters d_1 and d_2 are the same in both time slots. The amplification factor is given by $\alpha = \sqrt{\frac{E_r}{E_s|h_{s,r}|^2+N_0}}$ and the minimization function can be computed using equation (A.30) and (A.64) yielding

$$(d_1, d_2) = \arg\min_{d_1,d_2}\{Q(\sqrt{E_s}(d_1 - d_2)\delta_1) + Q(\sqrt{E_s}(d_1 + d_2)\delta_1)$$

$$+ 2Q(\sqrt{E_s}d_2\delta_2) + Q(\sqrt{E_s}(2d_1 - d_2)\delta_2)$$

$$- Q(\sqrt{E_s}(2d_1 + d_2)\delta_2)\}$$

$$\text{subject to} \begin{cases} 0 < d_2 < d_1 < \sqrt{\dfrac{E}{2}} \\ 2(d_1^2 + d_2^2) = E \end{cases} \tag{3.31}$$

where

$$\delta_i = \frac{\left(|h_{s,u_i}|^2 + \alpha\, |h_{r,u_i}|^2 |h_{s,r}|^2\right)}{\sigma\sqrt{\left(|h_{s,u_i}|^2 + |h_{r,u_i}|^2 |h_{s,r}|^2\left(\alpha^2 |h_{r,u_i}|^2 + 1\right)\right)}}.$$

and E is the symbol energy at each transmitter. After the optimized distance parameters $d_1$and d_2 have been selected, the Base Station modulates its data then broadcasts it where it is received respectively by the two UEs and the relay as

$$y_{s,u_1} = \sqrt{E_s}\, h_{s,u_1}x + n_{s,u_1} \tag{3.32}$$

$$y_{s,u_2} = \sqrt{E_s}\, h_{s,u_2}x + n_{s,u_2} \tag{3.33}$$

$$y_{s,r} = \sqrt{E_s}\, h_{s,r}\, x + n_{s,r} \tag{3.34}$$

where x is the modulated symbol and n_{s,u_1},n_{s,u_2} and $n_{s,r}$ are complex Additive White Gaussian Noise (AWGN) random variables with zero mean and variance σ^2 with spectral density $N_0/2$ per dimension. y_{s,u_1}, y_{s,u_2} and $y_{s,r}$ are the received signals at UE_1, UE_2 and the relay respectively. The relay simply amplifies the received signal $y_{s,r}$ by an amplification gain $\alpha = \sqrt{\frac{E_r}{E_s|h_{s,r}|^2+N_0}}$ and resends it without any further processing. The received signals from the relay at the User-Ends are given by

$$\begin{aligned} y_{r,u_i} &= \alpha h_{r,u_i}\, y_{s,r} + n_{r,u_i} \quad \text{for } i = 1,2 \\ y_{r,u_i} &= \alpha h_{r,u_i}\sqrt{E_s}\, h_{s,r}\, x + \alpha h_{r,u_i} n_{s,r} \\ &\quad + n_{r,u_i} \end{aligned} \tag{3.35}$$

n_{r,u_1} and n_{r,u_2} are complex Additive White Gaussian Noise (AWGN) random variables with zero mean and variance σ^2 with spectral density $N_0/2$ per dimension. At destination, each User-End combines the two received signals from the Base Station and the Relay using MRC technique. The resultant combined signals at UE_1 and UE_2 respectively are given by

$$\begin{aligned} y_{u_1} &= \sqrt{E_s}\, x\, |h_{s,u_1}|^2 + h^*_{s,u_1} n_{s,u_1} + \alpha\sqrt{E_s}\, x\, |h_{s,r}|^2 |h_{r,u_1}|^2 \\ &\quad + \alpha |h_{r,u_1}|^2 h^*_{s,r} n_{s,r} + h^*_{s,r} h^*_{r,u_1} n_{r,u_1} \end{aligned} \tag{3.36}$$

and

$$y_{u_2} = \sqrt{E_s}\, x \left|h_{s,u_2}\right|^2 + h^*_{s,u_2} n_{s,u_2} + \alpha\sqrt{E_s}\, x \left|h_{s,r}\right|^2 \left|h_{r,u_2}\right|^2 + \alpha\left|h_{r,u_2}\right|^2 h^*_{s,r} n_{s,r} + h^*_{s,r} h^*_{r,u_2} n_{r,u_2} \tag{3.37}$$

The demodulation at each user follows the below algorithm:

- At the first User Equipment (UE_1) at time t:

  $If\ Real\left(y_{u_1}\right) > 0$

  $then\ u_1'(2t-1) = 0, \qquad else\ u_1'(2t-1) = 1$

  $If\ Imag\left(y_{u_1}\right) > 0$

  $then\ u_1'(2t) = 0, \qquad else\ u_1'(2t) = 1$

- At UE_2:

  $If\ Real\left(y_{u_2}\right) > \left(\left|h_{s,u_2}\right|^2 + \alpha\ \left|h_{s,r}\right|^2 \left|h_{r,u_2}\right|^2 \right) d_1\sqrt{E_s}$

  $then\ u_2'(2t-1) = 0, \qquad else\ u_2'(2t-1) = 1$

  $If\ Imag\left(y_{u_2}\right) > \left(\left|h_{s,u_2}\right|^2 + \alpha\ \left|h_{s,r}\right|^2 \left|h_{r,u_2}\right|^2 \right) d_1\sqrt{E_s}$

  $then\ u_1'(2t) = 0, \qquad else\ u_1'(2t) = 1$

3.4 Turbo Coded Relay cooperation

In this section, the analysis is extended to the case of a Turbo coded cooperative network. The system is composed of a BS, Relay and two User-Ends. The Base Station concatenates and encodes the two streams of data. It modulates the resultant stream then broadcasts it. They Relay receives the data along with the two User-Ends. The data is demodulated and decoded at the Relay. A CRC takes place at the Relay. If the CRC results in correct detection, the relay forwards the data using Decode and

Forward technique. The two User-Ends then combine the two signals by summing their resultant LLRs from each path. If the CRC fails, then the Relay remains silent and the destinations receive only one copy of the message from the BS.

3.4.1 Encoding Process

In this section, presentation of the structure of the Turbo encoder is illustrated. Related parameters of the Turbo Encoder are given. The process of the Turbo encoding including the input format, the interleaving, the puncturing and the channel interleaving are presented in details in the following subsections.

3.4.1.1 Turbo Encoder Structure

The Turbo encoder used in the proposed system is shown in Figure 3.3. It consists of two recursive systematic convolutional (RSC) encoders, each of constraint length $K = 4$, concatenated in parallel. The feedforward generator is $(15)_{oct}$ and the feedback generator is $(17)_{oct}$. The number of input information bits to the Turbo encoder is denoted by l, $(I_0, I_1, \dots, I_{l-1})$. The two encoders are separated by an interleaver π where U_{2s} is an interleaved version of (U_{1s}).

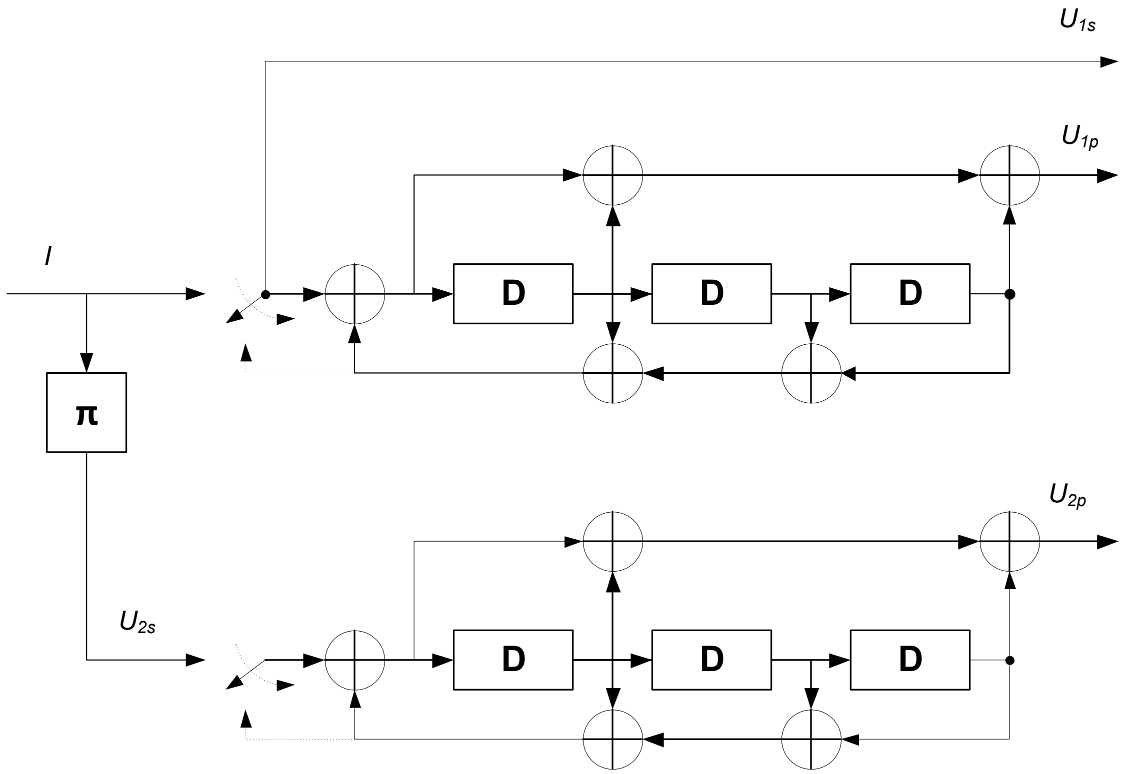

Figure 3.3 System model Turbo encoder with rate 1/3 and generator polynomial $(15{,}17)_{oct}$

At the beginning of the encoding process the two switches are in the upper position. The input stream I of length l is passed to the encoder and the output of the encoder consists of three streams; the systematic bits U_{1s} , the first parity bits U_{1p} , and the second parity bits U_{2p}. The trellis of each encoder is shown in Figure 3.4 The solid black lines represent the '0' input to the encoder while the dashed black lines mark a '1' as input. The output of each encoder is composed of 2 bits marked close to each line where the first bit is the systematic bit U_s and the second bit is the encoded bit U_p.

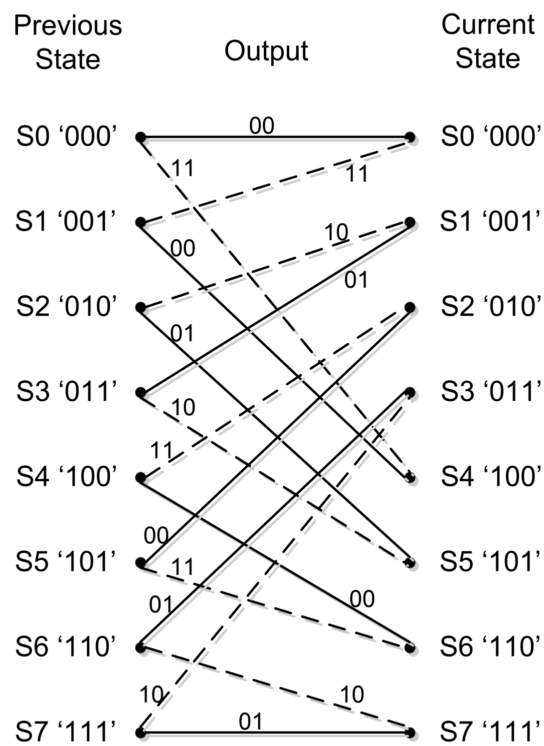

Figure 3.4 Trellis diagram of the system model Turbo encoder

3.4.1.2 Encoder Input

The Base Station has two streams of data I_1 and I_2, destined to two different User Equipments; UE_1 and UE_2 respectively. Each stream is of length $l_j = 1536$ bits, where $j \in \{1,2\}$. The two streams are concatenated together to form one stream I of length $l = 3072$ bits as shown in Figure 3.5.

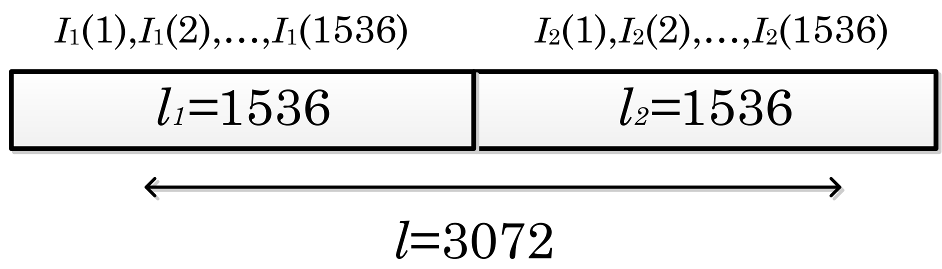

Figure 3.5 Concatenation of Turbo encoder input data stream

Because the Base Station is sending two different streams to two different users, all interleavers used in the encoder will interleave the data in a special manner that keeps the two streams of data separated at all time.

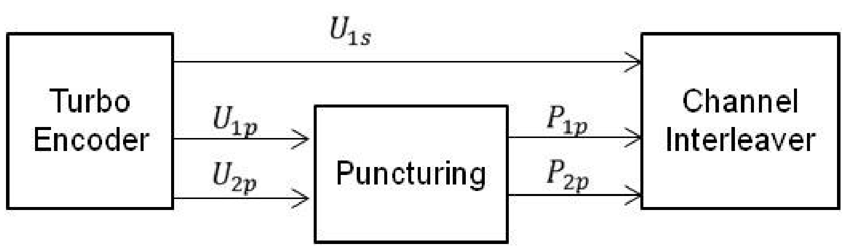

Figure 3.6 Encoding, Puncturing and Channel Interleaving Scheme

The stream $I = \{I_1(1), I_1(2), \ldots, I_1(1536), I_2(1), I_2(2), \ldots, I_2(1536)\}$ is used as the input to the Turbo encoder shown in Figure 3.3. The interleaver π interleaves the input stream on two stages. The first stage uses interleaver π_1 that interleaves the first half of the data (I_1). The second stage uses interleaver π_2 that separately interleaves the second half (I_2). Each interleaver π_i interleaves the corresponding stream I_i resulting in the stream I_i'; where $i = \{1,2\}$. This strategy guarantees that each user's data is interleaved without interfering with the other user's data. The resulting interleaved version of U_{1s} is the stream U_{2s} as shown in Figure 3.7

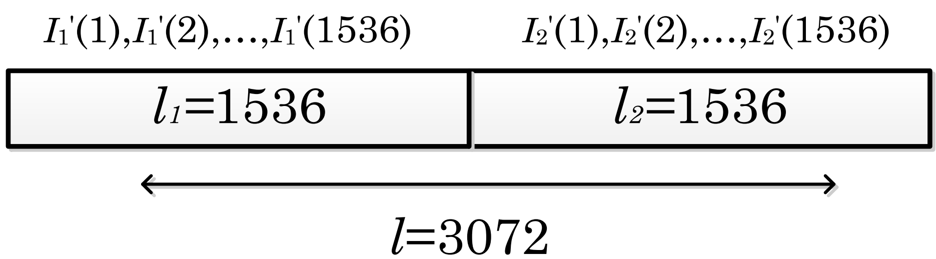

Figure 3.7 Interleaved version of the input stream

The output of the top encoder consists of the systematic bits $U_{1s} = I$ and the parity bits U_{1p}, where $U_{1p}(1{:}\,1536) \in UE_1$ and $U_{1p}(1536{:}\,3072) \in UE_2$. The output of the lower encoder consists of the systematic bits U_{2s} and the parity bits U_{2p}. U_{2s} is the concatenation of the interleaved version of I_1 with the interleaved version of I_2 , where $U_{2s}(1{:}\,1536) = I_1(\pi_1)$ and $U_{2s}(1536{:}\,3072) = I_2(\pi_2)$ as shown in Figure 3.7. The parity bits stream U_{2p} is divided in the same manner as U_{1p} where $U_{2p}(1{:}\,1536) \in UE_1$ and $U_{2p}(1536{:}\,3072) \in UE_2$

After the stream I has been encoded, the trellis of both encoders is forced back to the all zeros state by putting the two switches in the down positions, where the inputs to the two encoders are indicated by the dotted lines. Each encoder will generate $2 * (K - 1)$ tail bits to reach the all zeros state. Since the two encoders are identical, the systematic bits of the second encoder U_{2s} are omitted. Meanwhile, the parity bits of both encoders U_{1p} and U_{2p} are transmitted with the systematic bits of the top encoder U_{1s}.

3.4.1.3 Puncturing

The total number of bits out of the encoder is $l_E = 3 * (l + K - 1) = 3 * (3072 + 3) = 9225$ leading to a code rate $R = \frac{3072}{9225} \cong \frac{1}{3}$. The parity bits U_{1p} and U_{2p} are then punctured in an alternative manner using the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. In other words, all bits in an even number positions will be punctured in stream U_{1p}, while all bits in odd number positions will be punctured in stream U_{2p}. The output of puncturing U_{1p} and U_{2p} is stream $P_{h1} = \{U_{1p}(1), U_{1p}(3), U_{1p}(5), \dots, U_{1p}(l-1)\}$ and stream $P_{h2} = \{U_{2p}(2), U_{2p}(4), U_{2p}(6), \dots, U_{2p}(l)\}$ respectively each of length $l_p = 1536$, where $P_{hj}(1{:}768) \in UE_1$ and $P_{hj}(769{:}1536) \in UE_2$ and $j \in \{1,2\}$ as illustrated in Figure 3.8.

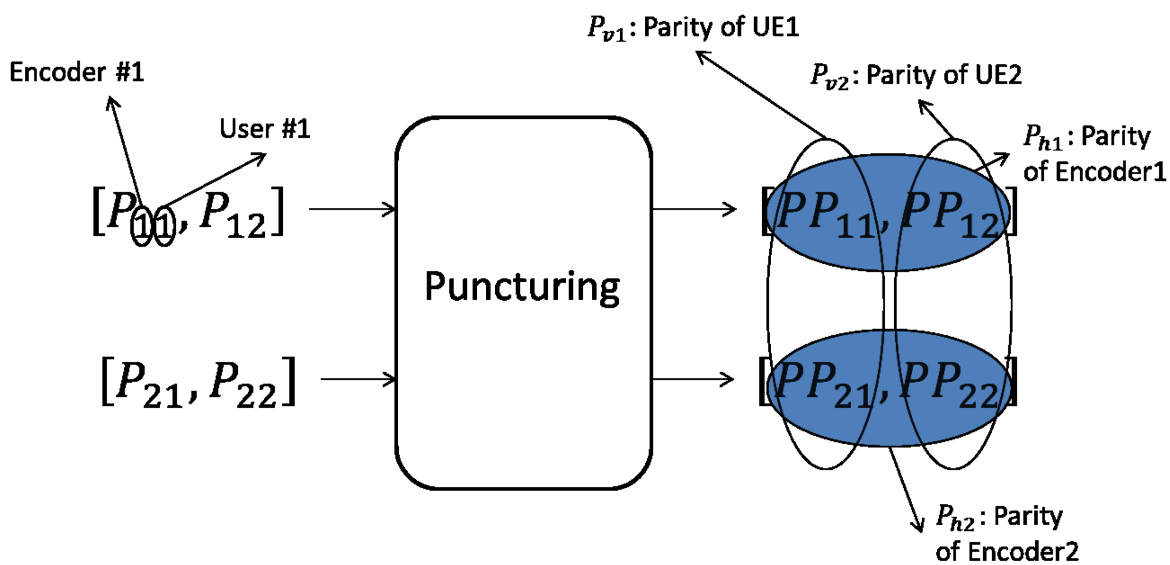

Figure 3.8 Puncturing procedure scheme

Data will be classified into two streams, one stream for UE_1 and another for UE_2. The first stream S_1 will consist of UE_1 data; which is the concatenation of the systematic bits I_1 and the punctured parity bits P_{v1}; $S_1 = [I_1, P_{v1}]$. P_{v1} is the concatenation of the parity bits of UE_1 from the two encoders; $P_{v1} = [PP_{11}, PP_{21}]$; where $PP_{11} = P_{h1}(1{:}768)$ and $PP_{21} = P_{h2}(1{:}768)$. The second stream S_2 is the concatenation of UE_2 data consisting of the systematic bits I_2 and the punctured parity bits defined as P_{v2}; $S_2 = [I_2, P_{v2}]$. P_{v2} is the concatenation of the parity bits of UE_2 coming from the two encoders; $P_{v2} = [PP_{12}, PP_{22}]$, where $PP_{12} = P_{h1}(769{:}1536)$ and $PP_{22} = P_{h2}(769{:}1539)$.

3.4.1.4 Channel Interleaver

In order to simplify the input at the modulation process, inside each stream S_1 and S_2 the systematic bits are interleaved independently from the parity bits. The systematic bits of UE_1, I_1, are interleaved using an interleaver π_{s1} of length $\frac{l}{2} = 1536$ while the parity bits of of UE_1, P_{v1}, are interleaved using an interleaver π_{P1} of the same length as π_{s1}. The same applies for UE_2 where I_2 bits are interleaved using π_{s2} and P_{v2} bits are interleaved using π_{P2}. The result of the channel interleaving process is four separate interleaved streams, each of length $\frac{l}{2} = 1536$. The first stream contains interleaved data of the systematic bits of UE_1 and is denoted by Z_1. The second stream is the interleaved data of the parity bits of UE_1; P_{v1}; It is denoted by Z_2. The third stream is the interleaved version of the systematic bits of UE_2 and is denoted by Z_3. And the final stream contains the interleaved parity bits of UE_2; P_{v2} ; and is denoted by Z_4. The four streams are then passed to the 16 QAM modulator.

The mapping at the input to the 16 QAM modulator consists of 4 streams, each stream will be mapped to one of the 4 bits of the symbol. The first stream Z_1 will be mapped to the first bit b_0. The second stream Z_2 will be mapped to the second position of the 4-bits symbol denoted by b_1. Stream Z_3 will be mapped to the third bit b_2 and stream Z_4 will be mapped to b_3. This mapping will give more protection to the data of UE_1 since it is mapped to the base bits position.

3.4.2 Decoding Process

In this section, presentation of the structure of the Turbo decoder is illustrated. Related parameters of the Turbo decoder are given. The process of the Turbo decoding including the formatting the output of the demodulator, the channel de-interleaving, the de-puncturing and the decoding are presented in details in the following subsections.

3.4.2.1 Input formatting

Since at modulation, the base bits were assigned to the data of UE_1 and the refinement bits were assigned to the data of UE_2, the LLR computed at all b_0 and b_1 positions are those of the systematic and parity bits of UE_1 respectively and the LLRs computed at all b_2 and b_3 positions are those of the systematic bits and parity bits of UE_2 respectively. At the end of the demodulation, a new stream of LLR values Z' of length $2 \cdot l = 6144$ is created. Figure 3.9 illustrates the distribution of data bits in stream Z'

$$if \begin{cases} mod\ (k,4) = 1,2 & then \quad Z'_k \in UE1 \\ mod\ (k,4) = 3,0 & then \quad Z'_k \in UE2 \end{cases}$$

where $k = 1,2,3, \ldots ,6144$.

	Z'_{v1}	Z'_{v2}	Z'_{v3}	Z'_{v4}
# of Symbols	UE1 Systematic	UE1 Parity	UE2 Systematic	UE2 Parity
1	$Z'(1)$	$Z'(2)$	$Z'(3)$	$Z'(4)$
2	$Z'(5)$	$Z'(6)$	$Z'(7)$	$Z'(8)$
… … …	… … …	… … …	… … …	… … …
1536	$Z'(6141)$	$Z'(6142)$	$Z'(6143)$	$Z'(6144)$

Figure 3.9 distribution of data bits at the input of the Turbo decoder

At the decoder, three main steps take place; the de-interleaving step to recover from the channel interleaving taking place at the encoder, then the de-puncturing which consists of filling the previously punctured positions with zeros, then the Turbo decoding to recover the information bits of UE_1 and UE_2.

3.4.2.2 Channel De-interleaving

In order to start the de-interleaving process, the data bits shall be grouped by category as shown in Figure 3.9. Tracing the table from left to right, the systematic bits of UE_1 are in the most left blue column and are denoted by

$$Z'_{v1} = \{Z'(1), Z'(5), Z'(9), \dots, Z'(6141)\} \tag{3.38}$$

The parity bits of UE_1 are denoted by

$$Z'_{v2} = \{Z'(2), Z'(6), Z'(10), \dots, Z'(6142)\} \tag{3.39}$$

The systematic bits of UE_2 are denoted by

$$Z'_{v3} = \{Z'(3), Z'(7), Z'(11), \dots, Z'(6143)\} \tag{3.40}$$

And finally the parity bits of UE_2 are denoted by

$$Z'_{v4} = \{Z'(4), Z'(8), Z'(12), \dots, Z'(6144)\} \tag{3.41}$$

After this grouping, each group is de-interleaved using the corresponding inverse of the interleaver function used at the encoder. The four resulting streams out of the de-interleaving process are denoted as follow

$$I'_1 = Z'_{v1}(\pi_{s1}^{-1}) \tag{3.42}$$

$$P'_{v1} = Z'_{v2}(\pi_{p1}^{-1}) \tag{3.43}$$

and

$$I'_2 = Z'_{v3}(\pi_{s2}^{-1}) \tag{3.44}$$

$$P'_{v2} = Z'_{v4}(\pi_{p2}^{-1}) \tag{3.45}$$

From the two streams $P'_{v1}$and P'_{v2} we recover the two streams $P'_{h1}$and P'_{h2} as illustrated in Figure 3.8 where

$$P'_{h1} = [P'_{v1}(1{:}768), P'_{v2}(1{:}768)] \tag{3.46}$$

and

$$P'_{h2} = [P'_{v1}(769{:}1536), P'_{v2}(769{:}1536)] \tag{3.47}$$

3.4.2.3 De-puncturing

The de-puncturing process takes place in the parity bits of the first encoder, P'_{h1}, and the parity bits of the second encoder, P'_{h2}. It consists of inserting zeros in the previously punctured positions. In other words, a zero is inserted 'after' every bit of the stream P'_{h1} and a zero is inserted 'before' every bit of the stream P'_{h2} giving the two streams U'_{1p} and U'_{2p} where

$$U'_{1p} = \{P'_{h1}(1), 0\,, P'_{h1}(2), 0, P'_{h1}(3), 0\,, \ldots, P'_{h1}(1536), 0\} \tag{3.48}$$

and

$$U'_{2p} = \{0, P'_{h2}(1), 0\,, P'_{h2}(2), 0, P'_{h2}(3)\,, \ldots, 0, P'_{h2}(1536)\} \tag{3.49}$$

3.4.2.4 Turbo Decoder

The Turbo decoder implemented in our system is shown in Figure 3.10. It takes as input the three streams U'_{1s} , U'_{1p} and U'_{2p} where $U'_{1s} = [I'_1, I'_2]$ and U'_{1p} and U'_{2p} are given by equations (3.48) and (3.49). Max Log Map algorithm is implemented in the decoding process for its relative simplicity and fewer computations compared to the Map algorithm as mentioned in section 2.6.6 and 2.6.7

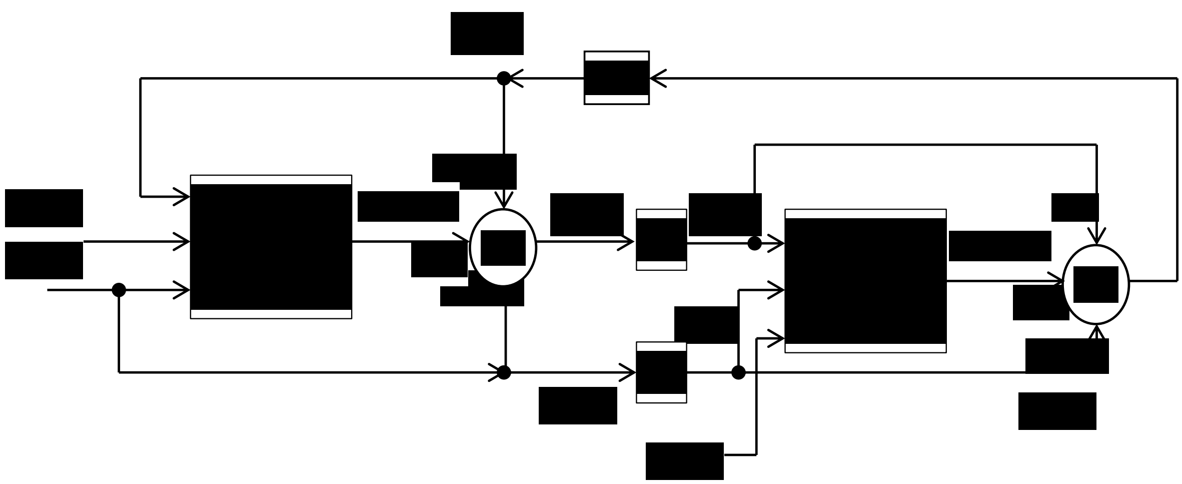

Figure 3.10 Turbo decoder illustration

The two terms U'_{1s} and U'_{1p} computed from previous steps are already in the Log Likelihood Ratio format and they include the channel reliability information so there is no need to compute the L_c term. In the first iteration, the inputs to decoder#1 are U'_{1s},

U'_{1p} and $L_e^{2\prime}$ where $L_e^{2\prime} = L(u)$. $L(u)$ is set to zero at the beginning of the decoding process as equal probability for the input binary bits is assumed.

The group of equations (2.9) to (2.12) are solved at this stage in an iterative manner as shown in Figure 3.10 using Max Log MAP algorithm to obtain an accurate estimate of the *a posteriori probability* (APP) of the information bits. In the presented system, four iterations are employed. At the end of the iterative process a hard decision is made according to

$$m_i = \begin{cases} 1 \; if \; \tilde{L}^2(\hat{u}) > 0 \\ 0 \; if \; \tilde{L}^2(\hat{u}) < 0 \end{cases} \tag{3.50}$$

where $\tilde{L}^2(\hat{u})$ is the de-interleaved version of $L^2(\hat{u})$

3.5 Summary

In this chapter, the proposed system model for this thesis was reviewed. Different schemes for the proposed system including direct transmission, user cooperation, relay node cooperation and finally Turbo coded relay cooperation were presented. Different system parameters related to the modulation/demodulation and encoding/decoding processes were given. In the next chapter, results of all the above mentioned systems will be presented comparing the performance of the traditional uniform constellation with the proposed one.

Chapter Four

4. Results and Discussion

In this chapter, a discussion on the related simulation results is presented. Simulations of direct transmission, user cooperation, relay cooperation and Turbo codes relay cooperation system models are examined. In each of the mentioned systems, comparison between the different users BER in a uniform modulation based system and an optimized hierarchical modulation based system are illustrated. In each of these comparisons, it is proven that the optimized hierarchical modulation improves the performance of both users compared to the uniform modulation.

4.1 Direct Transmission

Although the main focus of this thesis is the study of hierarchical modulation in cooperative networks, the study of hierarchical modulation in direct transmission is mandatory. Its importance lies in the fact that it clearly illustrates how the variation in the distance parameters of a 4/16 QAM constellation can change the performance of both the base and the refinement bits. In this section, it is proven by simulations that optimized selection of the constellation's parameters in fact does improve both the base bits stream and the refinement bits stream BER. As mentioned in Section 3.2, the scheme of the direct transmission consists of a Base Station (BS) sending two different streams of data simultaneously to two different users by employing a hierarchical 16 QAM modulation. The data of UE_1 is mapped to the base bits while the data of UE_2 is mapped to the refinement bits. Simulations in the case of Direct Transmission compare three different inter-nodes distances to examine the effect of the relative physical distances of the nodes on the performance. In order to show such effect, the results are plotted in terms of transmitted SNR, and this transmitted SNR is defined as $SNR_T = {}^{E_s}\!/_{N_o}$, and E_s is the transmitted symbol energy at the BS. In the following simulation results, the distance between the BS and UE_i is denoted by d_{s,u_i} for $i \in \{1,2\}$.

4.1.1 Case $\left(\boldsymbol{d_{s,u_1} = d_{s,u_2}}\right)$

This case studies the scenario where both User-Ends are at the same physical distance from the BS. For simplicity we assume $d_{s,u_1} = d_{s,u_2} = 1$, where the path loss exponent μ is assumed to have a value of 3.5. Comparison between the BER performance of the base bits and the refinement bits using a uniform 16QAM using an optimized hierarchical 16QAM is illustrated in Figure 4.1 and Figure 4.2. In Figure 4.1, the base bits BER using an optimized constellation has a higher performance of 2-4 dB compared to uniform constellation. Meanwhile, Figure 4.2 illustrates the case of refinement bits BER. It is shown in the Figure that the performance using the optimized constellation results in 1-2 dB improved performance than the uniform constellation. Also, as can be predicted beforehand, since UE_1 data is mapped to the base bits, it achieves better performance than the refinement bits user UE_2.

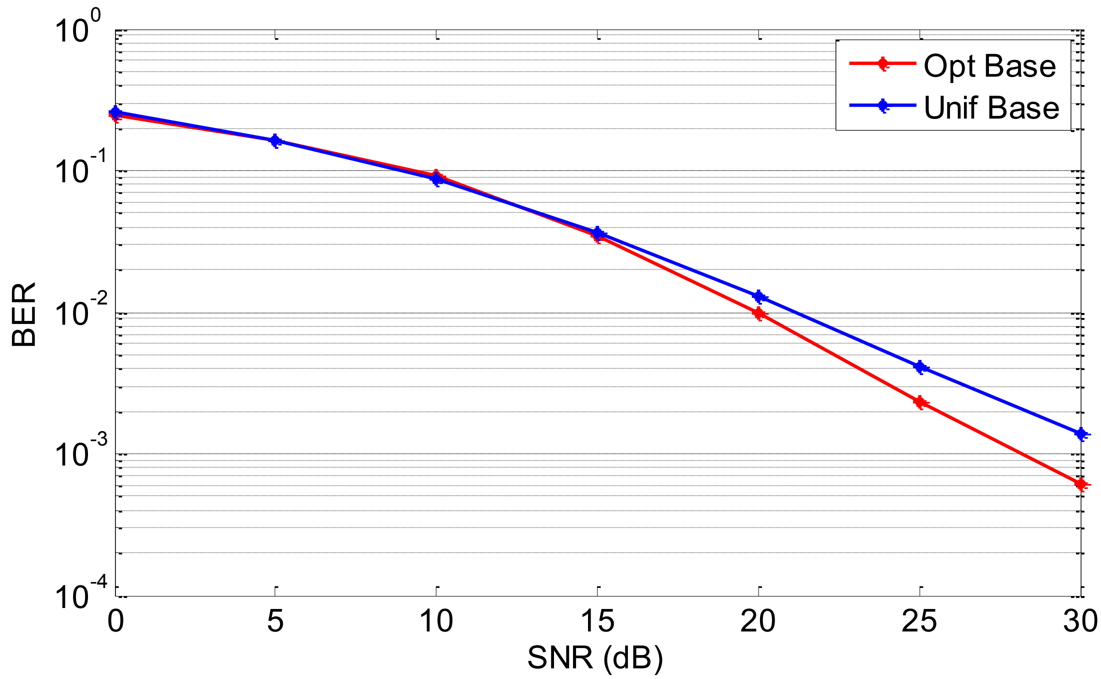

Figure 4.1 Base bits BER, direct transmission, $d_{s,u_1} = d_{s,u_2} = 1$

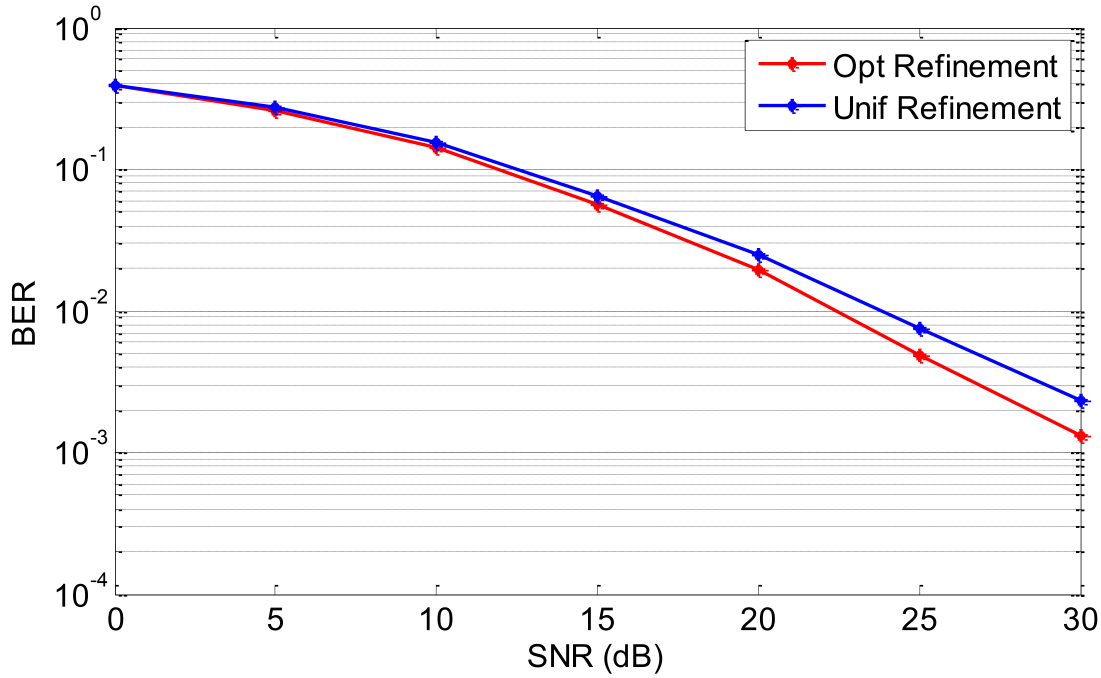

Figure 4.2 Refinement bits BER, direct transmission, $d_{s,u_1} = d_{s,u_2} = 1$

4.1.2 Case $\left(\boldsymbol{d_{s,u_1} = 2d_{s,u_2}}\right)$

In this scenario, one User-End is located closer to the BS than the other. It is assumed that the distance separating that user to the Base Station is half the distance separating the other user from the Base Station. This results in more powerful overall channel condition of UE_2 compared to that of UE_1. Thus to reach almost similar performances at both users, more data protection is applied to UE_1. One of the ways to achieve this unequal protection is by mapping the data of UE_1 to the base bits of the 16-QAM constellation. Comparison between the BER performance of UE_1 and UE_2 in a uniform 16-QAM versus an optimized hierarchical 16-QAM is illustrated in Figure 4.3 and Figure 4.4. In the case of UE_1's data, the improvement in BER performance using the optimized constellation compared to using a uniform constellation is obvious. In low

SNR, (10 dB) the difference in performance is in the range of 2dB. Optimized constellation system outperforms the uniform constellation system by 4 dB of improvement in high SNR region. Meanwhile, in the case of UE_2's data, the optimized constellation results in a slight improved BER performance, 2dB at high SNR, compared to the uniform constellation. In this scheme, the overall performance of the refinement bits user is better than that of the base bits user.

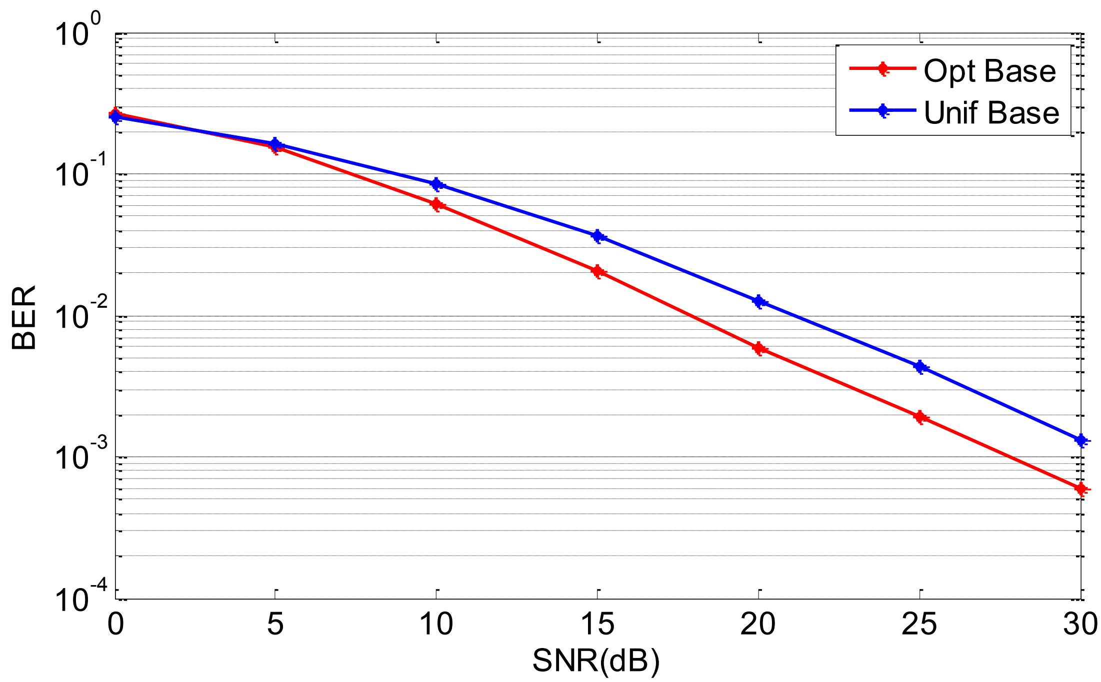

Figure 4.3 Base bits BER, direct transmission, $d_{s,u_1} = 1,\ d_{s,u_2} = 0.5$

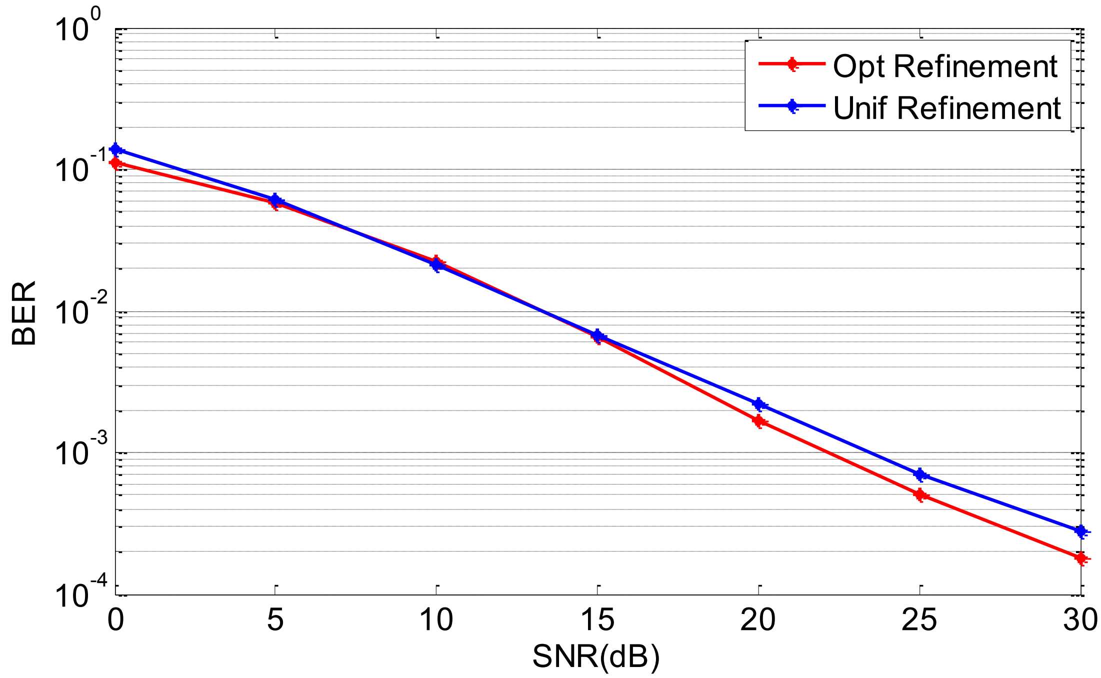

Figure 4.4 Refinement bits BER, direct transmission, $d_{s,u_1} = 1,\ d_{s,u_2} = 0.5$

4.1.3 Case $\left(d_{s,u_1} > 2d_{s,u_2}\right)$

In this scenario, UE_2 is further located closer to the BS than in the previous case. The reason of including this scenario is to show an interesting flip in the performance of the refinement bits. As can be shown in Figure 4.6, the performance of the uniform constellation in the low SNR region for the refinement bits is slightly better than that of the optimized constellation. Observing the performance of the base bits and refinement bits in this scenario, it is noted that at low SNR, the BER of the base bits is in the range of 3×10^{-1}, while the BER of the refinement bits is in the range of 5×10^{-3}. Since the optimization function's goal is to minimize both users BER, the weight of the minimization in this case goes completely to the base bits. This is illustrated in Figure 4.5 and Figure 4.6. In the low SNR region (5-10 dB), the improvement in the performance of the base bits is in the range of 3 dB, costing a degradation of less than 1 dB in the performance of the refinement bits. However, in high SNR region (25-30 dB), greater enhancement in both base bits and refinement bits BER performance is shown in this last scenario; 3-4 dB.

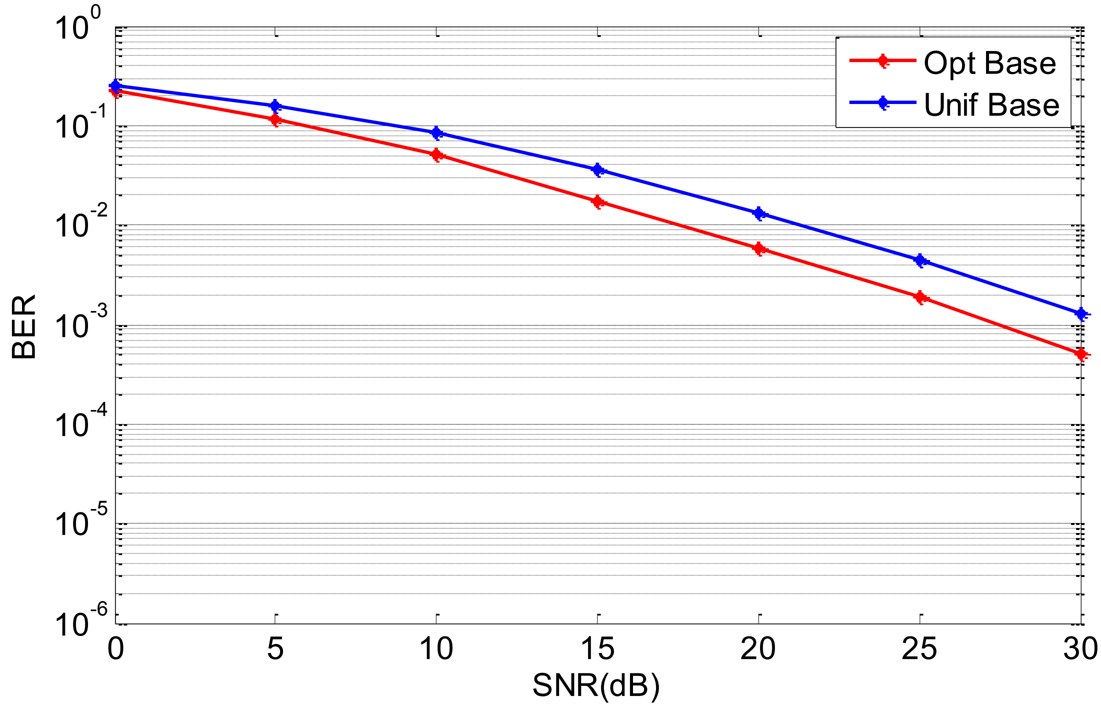

Figure 4.5 Base bits BER, direct transmission, $d_{s,u_1} = 1,\ d_{s,u_2} = 0.2$

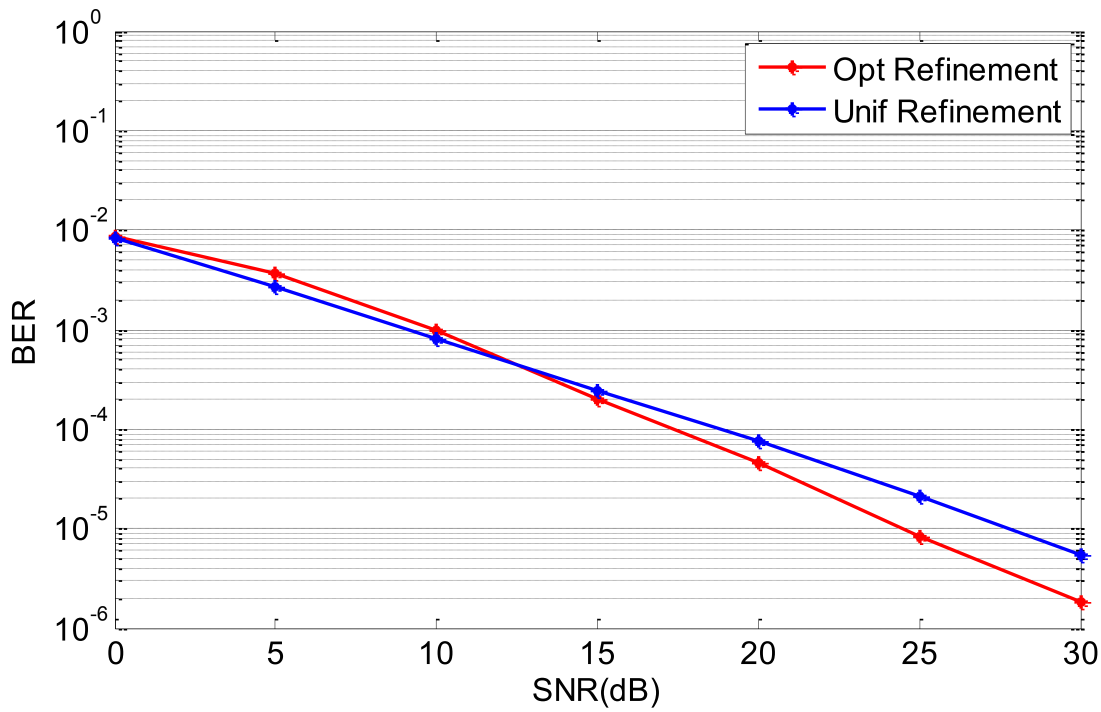

Figure 4.6 Refinement bits BER, direct transmission, $d_{s,u_1} = 1,\ d_{s,u_2} = 0.2$

4.1.4 Summary of results

Finally, summing up the results of the three illustrated cases, two major facts can be concluded. The first conclusion is that the physical distance between the nodes plays an important role in their BER performance. In the second and third case, even though UE_1's data was more protected, UE_2 had a lower BER as it was located at a shorter distance to the BS. The second conclusion is summarized in Figure 4.7. Basically, the Figure illustrates how changing the physical distance of the refinement bits user affects the performance of the base bits user when using an optimized hierarchical modulation. Simulation results of the base bits BER(Figure 4.7) shows that the closer the refinement bits user (UE_2) is to the BS, the better the performance of the base bits user (UE_1) is in the low SNR region. This is due to the fact that the focus of the minimizing process in these cases is dedicated to the base bits user as mentioned in section 4.1.3. However, in high SNR region, the performance of the base bits user (UE_1) in the case of an optimized constellation is the same in the three cases, $Pe \approx 10^{-3}$.

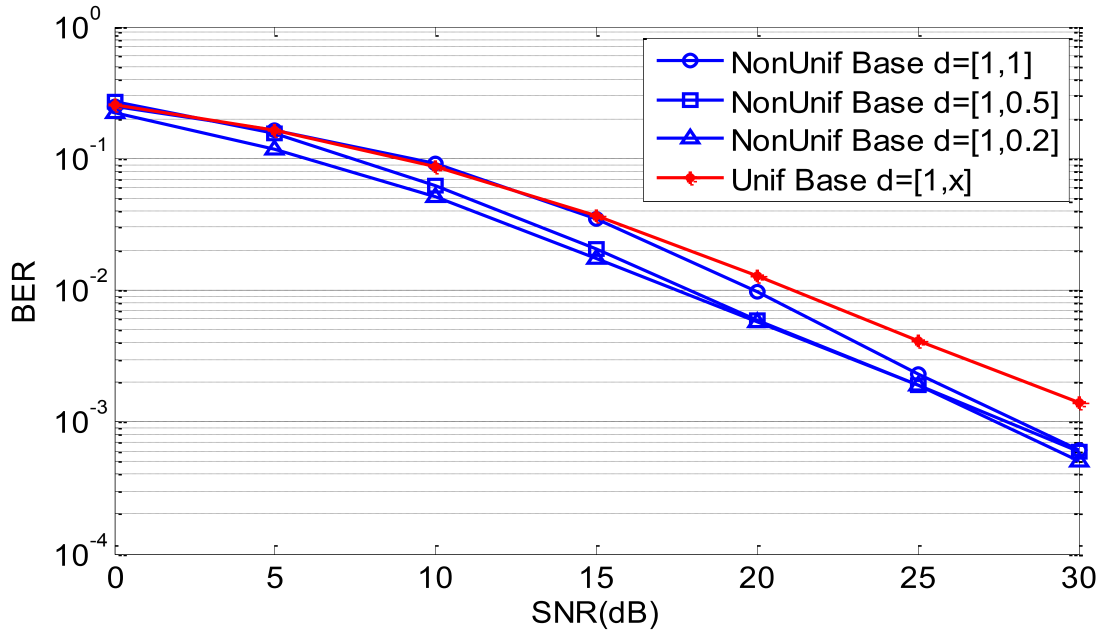

Figure 4.7 Base BER, No cooperation, comparison of different node distance

4.2 User Cooperation Transmission

In this section, one of the two users acts as a relaying node and assists the other node in detecting more accurately its data. The goal of this scheme is to improve the performance of a specific User-End. One reason of prioritizing one user to the other is a relative high importance in correctly detecting this user's data. Another reason is to improve the user's performance in case of weak channel condition. It is assumed that UE_2 is the relaying node in this scenario. UE_2 may or may not retransmit the message it receives according to the link condition between the BS and UE_2. The study in this section includes two specific cases. The first case is when the two nodes are at equal separating distance from the BS. The second case is when one node; UE_2, is closer to BS than UE_1, specifically when $d_{s,u_1} = 2d_{s,u_2}$. In the User Cooperation transmission scheme, both; the 16-QAM constellation's parameters and the total transmitted energy; are optimized in order to achieve minimum total BER. The 16-QAM constellation's parameters control the distribution of the available transmitted energy on the two users at the transmitting node. The optimization of the total transmitted energy is the distribution of energy on the BS and the assisting node in order to achieve minimum BER. Since the goal of this transmission model is to improve the performance of only one user, only the performance of that user (base bits user) is presented in this section. Comparison between the BER performance of UE_1 in user cooperation transmission and in direct transmission is examined.

4.2.1 Case $\left(d_{s,u_1} = d_{s,u_2}\right)$

In this case, the correct reception of UE_1 data is of relative high importance that that of UE_2. It is assumed that both User-Ends are at the same physical distance from the BS, specifically $d_{s,u_1} = d_{s,u_2} = 1$. The path loss exponent μ is assumed to have a value of 3.5. The performance of UE_1 is illustrated in Figure 4.8. A comparison between UE_1's BER when employing a uniform 16QAM modulation and when employing an optimized hierarchical 16QAM modulation is presented. Also a comparison between performance of the Direct Transmission case and the case of user cooperation is illustrated. Results show that the performance of the base bits in the user cooperation model is superior to that of the direct transmission model by 2.5 dB. Meanwhile, optimization of both the transmitted power and the constellation's distance parameters at the BS improves the performance by 2 dB.

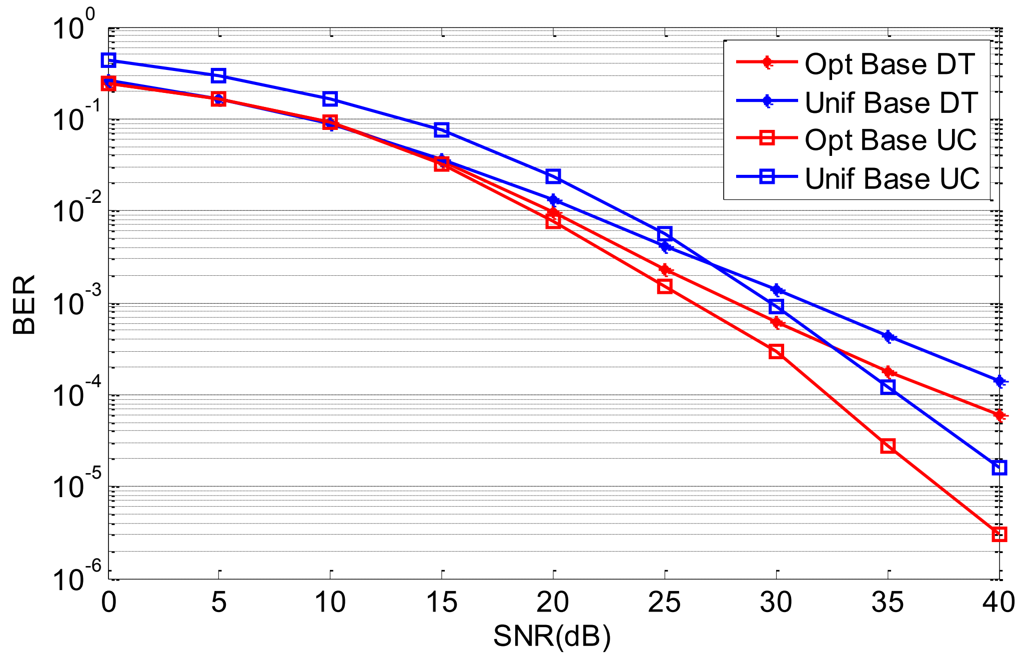

Figure 4.8 Base BER, Direct Transmission vs User Cooperation, $\boldsymbol{d_{u_1} = d_{u_2} = 1}$

4.2.2 Case $\left(\boldsymbol{d_{s,u_1} = 2d_{s,u_2}}\right)$

In this case, the overall channel condition of UE_1 is less favorable than that of UE_2. User cooperation method is employed in this scheme to improve the performance of UE_1.

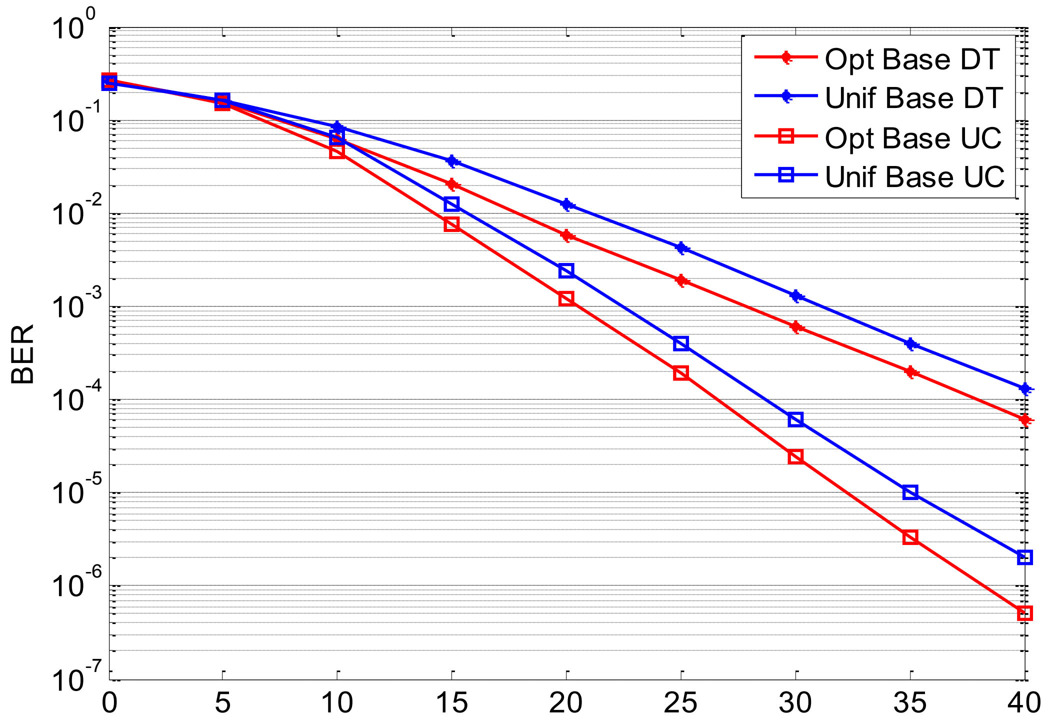

Figure 4.9 illustrates the performance of UE_1 in the Direct Transmission scheme and in the User Cooperation scheme. In the same Figure, a comparison between the BER of UE_1 when employing a uniform 16QAM modulation and when employing an optimized hierarchical 16QAM modulation is illustrated. Results show that the performance of the base bits in the user cooperation model is superior to that of the direct transmission model by 5-8 dB. Meanwhile, optimization of both the transmitted power and the constellation's distance parameters at the BS improves the performance by 2-3 dB.

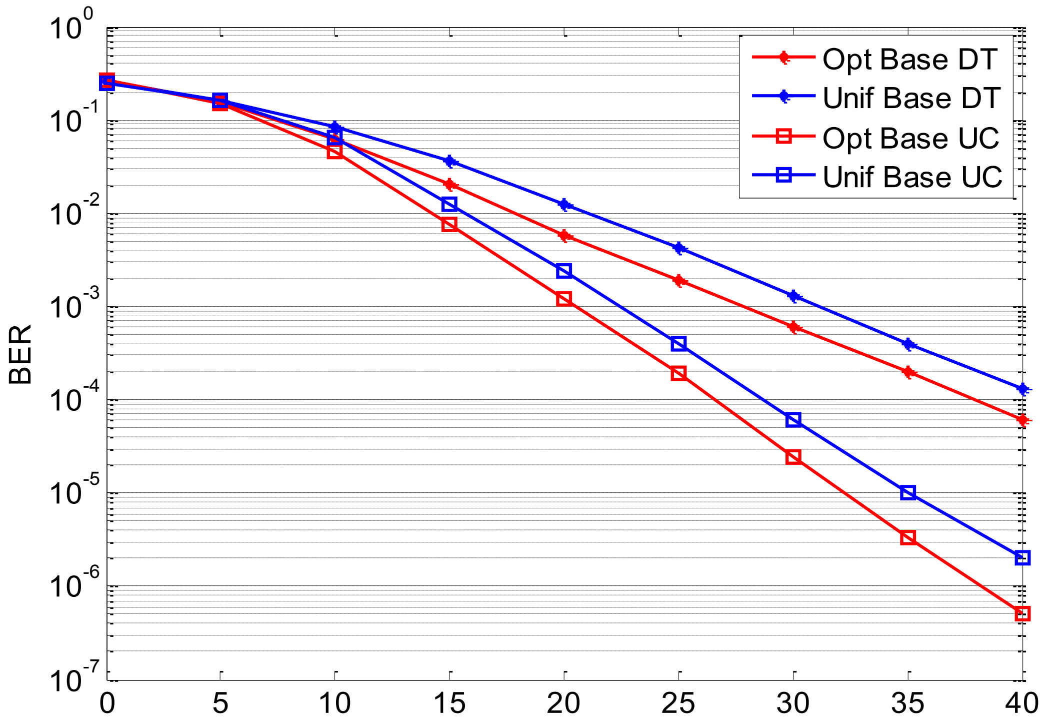

Figure 4.9 Base BER, Direct Transmission vs User Cooperation, $\boldsymbol{d_{u_1} = 1,\ d_{u_2} = 0.5}$

A final note on the user cooperation scheme, when comparing the two illustrated schemes in Section 4.2.1 and 4.2.2, it can be concluded that the performance of UE_1 in the user cooperation scheme is highly affected by the quality of the channel link between the BS and UE_2. Mainly, the better the $BS - UE_2$ link, the higher the probability of UE_2 to assist UE_1 in recovering its data is.

4.3 Relay Cooperative Network Transmission

In this section, optimization of hierarchical modulation in relay cooperative networks is presented. In addition, optimization of total energy distribution on available transmitters (Base Station and Relay) is carried out along with the modulation optimization. Users' performance is studied in both AF and DF relaying techniques. In the following subsections, a study on the effect of changing the relay node location on the performance of the users BER is presented. Later, another study on the effect of changing one destination node location on the BER of both destinations is presented. In each of the presented scenarios, three modulation and energy distribution techniques are examined. The first case employs a uniform constellation with equal energy distribution at the BS and the Relay. The second case employs an optimized hierarchical modulation at both the BS and the Relay while still keeping equal energy distribution at both transmitters. The second case will be referred to as Modulation Optimization (MO). The third case employs an optimized modulation at both the BS and the Relay while optimizing the transmitted energy at these two nodes. The third case will be referred to as Modulation and Power Optimization (MPO). Results in this section are plotted in terms of transmitted SNR, where the transmitted SNR is defined by $SNR_T = E_s/N_0 + E_r/N_0$, where E_s is the transmitted energy at the source and E_r is the transmitted energy at the relay.

4.3.1 Effect of changing the location of the Relay node

In this subsection, analysis of the effect of the relative relay node location to the Base Station is carried out for both AF and DF relaying techniques. This analysis is conducted for both base bits user and refinement bits user UE_2. Three cases of relay node location are examined; the relay is closer to the BS, the relay is half way between the BS and the UEs, the relay is closer to the UEs.

4.3.1.1 Case 1: Relay closer to the Base Station

In the first case, the effect of locating the relay near to the BS is analyzed. Logically, being at a short distance from the BS results in a higher probability of correct message detection at the relay. The distances separating the different nodes of the relay cooperative scheme are represented by a matrix $\boldsymbol{D}$, where $\boldsymbol{D} = [d_{s,u_1}, d_{s,u_2}, d_{s,r}, d_{r,u_1}, d_{r,u_2}]$. In the case where the relay is closer to the BS, the distance matrix is defined as $\boldsymbol{D} = [1,1,0.25,0.9,0.9]$. In Figure 4.10, BER performance of base bits in both AF and DF relaying is examined. As observed in the figure, in the case of DF relaying, both (MO) and (MPO) techniques achieve a better performance than that of the uniform constellation. (MO) outperforms the uniform constellation scheme by 2.5 dB, while (MPO) outperforms the performance of the uniform constellation by 4.5 dB. In the case of AF relaying, also (MO) and (MPO) achieve better BER than the uniform constellation. (MO) achieves a gain of 3 dB while (MPO) achieves a gain of 4 dB compared to the performance of uniform constellation. Overall, the performance of the DF relaying method in this scheme has a better BER for the base bits than when using AF relaying.

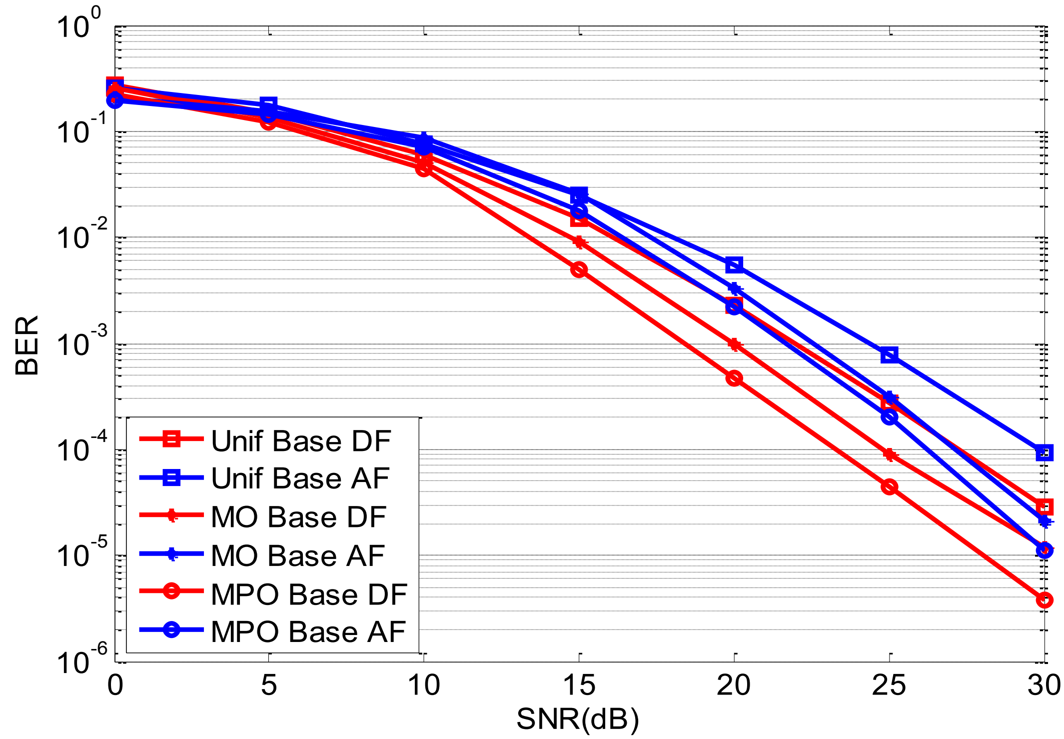

Figure 4.10 Base BER, Relay cooperation, AF and DF schemes, Relay closer to *BS*

In Figure 4.11, in the case of DF relaying, both (MO) and (MPO) techniques achieve a better performance than that of the uniform constellation. (MO) and (MPO) almost have the same performance where they outperform the performance of the uniform constellation by 3 dB. In the case of AF relaying, also (MO) and (MPO) achieve better BER than the uniform constellation. (MO) achieves a gain of 1.5 dB while (MPO) achieves a gain of 4 dB compared to the performance of uniform constellation. Overall, the performance of the DF relaying method in this scheme has a better BER for the refinement bits than when using AF relaying.

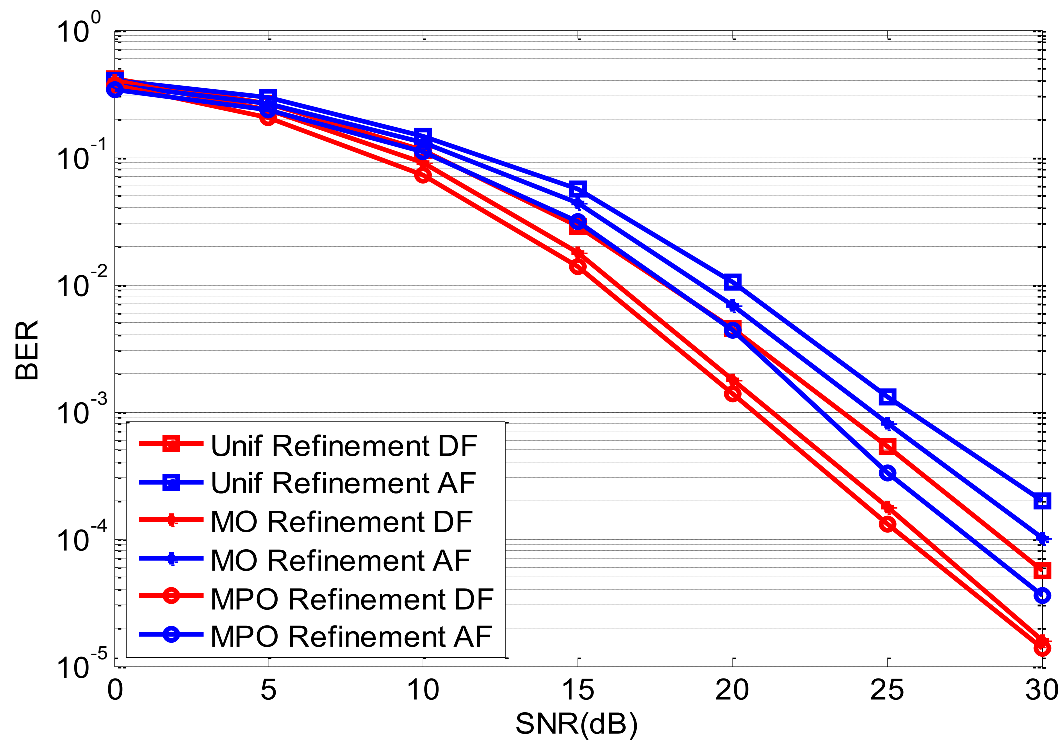

Figure 4.11 Refinement BER, Relay cooperation, AF and DF schemes, Relay closer to BS

4.3.1.2 Case 2: Relay is half way between the Base Station and the User-Ends

Second case examines the model of allocating the relay half way between the BS and the users. In this case, the probability of the relay to correctly detect the BS message is noticeably lower than the previous case. At the same time, the relay is not close enough to the users to guarantee a smooth error free transmission between the relay and the users. The distances matrix in this case is $\boldsymbol{D} = [1,1,0.6,0.6,0.6]$. In Figure 4.12 and Figure 4.13, BER performance of base bits and refinement bits respectively in both AF and DF relaying is examined. In this case, the BER performance in low SNR region is advantageous in the AF relaying regime. This can be interpreted by the fact that when the relay is relatively further away from the BS, its chance to correctly decode the BS message is lower. Thus, in the case of DF relaying, the relay remains silent and the users receives only one copy of the message. However, in the case of AF relaying, where the relay forwards its received message anyway, the probability of achieving diversity is higher. Meanwhile, at high SNR region, when the relay can reliably detect the BS message, DF's performance is superior to AF. Logically, this is because at the user side, the probability of correctly detecting a clean message (DF) is higher than detecting a noisy message (AF).

In Figure 4.12, in the low SNR region (0-10 dB) in DF relaying, the (MO) scheme has the same performance as that of uniform modulation, while the performance of the (MPO) scheme is superior to both of them by an average of 2dB. The performances of (MO) and (MPO) improve at high SNR to outperform the uniform modulation model by 2.5 dB and 5dB respectively. In AF relaying, (MO) has a 1dB gain compared to uniform modulation scenario, while (MPO) outperforms them both in high SNR region (20-30 dB) to reach an average value of 6dB.

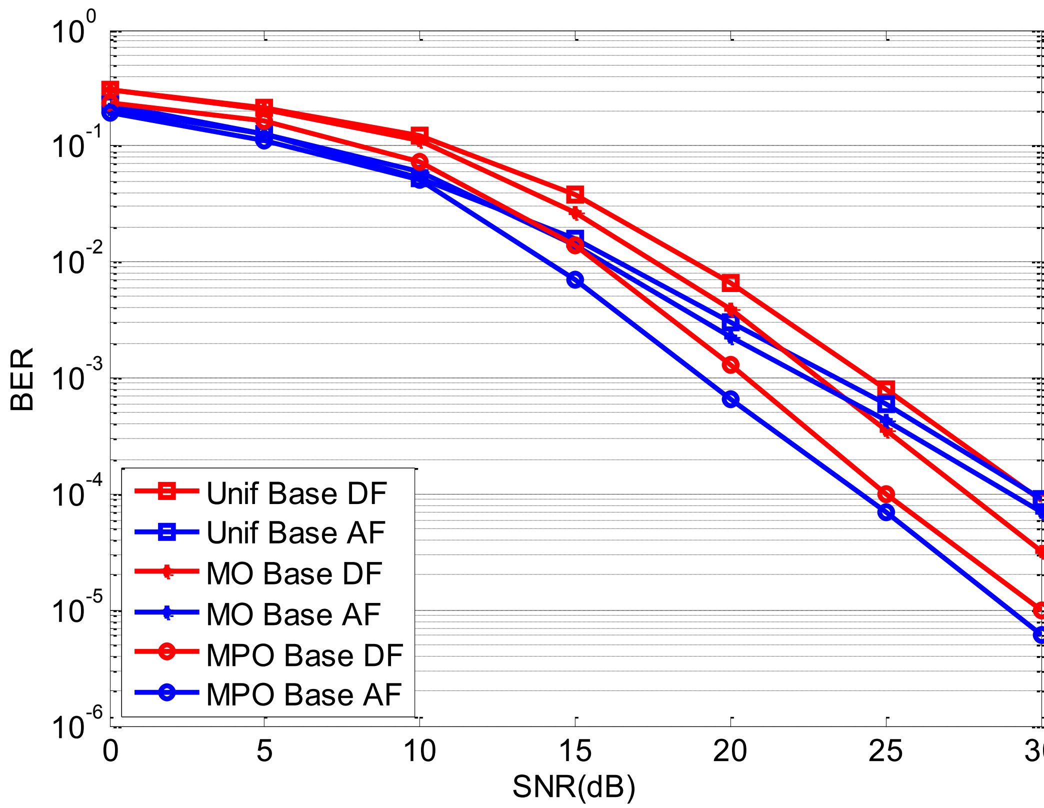

Figure 4.12 Base BER, Relay cooperation, AF and DF schemes, Relay located half way

In Figure 4.13, the (MO) scheme for the refinement bits in the AF regime outperforms the performance of uniform modulation by 2dB. Meanwhile, the performance of the (MPO) scheme is superior to both, the (MO) and the uniform constellation schemes by an average of 4dB and 6 dB respectively at high SNR region (20-30 dB). In the case of DF regime, the performance of the refinement bits in the (MO) scheme outperforms the uniform constellation system by 1.5dB. The performance of the refinement bits using (MPO) in DF relaying is superior to the two other schemes. It outperforms (MO) and uniform modulation by 2.5dB and 4dB respectively.

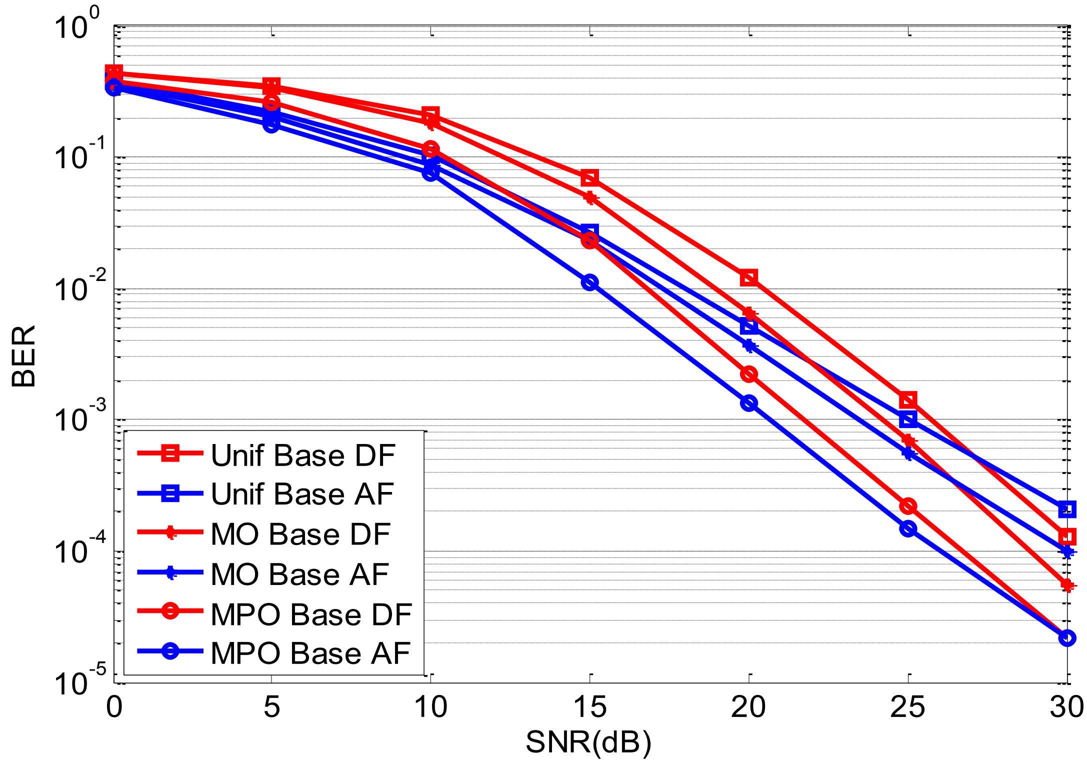

Figure 4.13 Refinement BER, Relay cooperation, AF and DF schemes, Relay located half way

4.3.1.3 Case 3: Relay is closer to the User-Ends

Third case examines the scenario of allocating the relay closer to the users. In this case, the probability of correctly detecting the BS message at the relay is the lowest when compared to the two previous cases. However, the channel link between the relay and the User-Ends is more preferable compared to the previous schemes. The distance matrix in this case is given by $\boldsymbol{D} = [1,1,0.7,0.4,0.4]$. In Figure 4.14 and Figure 4.15, BER performances of base bits and refinement bits respectively in both AF and DF relaying are examined. In this case, the BER performance in low SNR region is advantageous in the AF relaying regime. This can be interpreted, as mentioned in the previous section, by the fact that when the relay is relatively further away from the BS, its chance to correctly decode the BS message is lower. Thus DF relaying's performance is similar to direct transmission while AF can still achieve diversity in some cases.

Figure 4.14 illustrates the performance of the base bits in DF and AF relaying. Examining the case of DF relaying, it is observed that the performance of the base bits user is slightly better in (MO) case than in the uniform modulation scheme. Meanwhile, the performance of the (MPO) scheme is superior to both methods by an average of 4dB. In AF relaying, (MO) has a slight improved performance that that of a uniform modulation scenario. Meanwhile, the performance of (MPO) case outperforms the previous two methods by 5dB in high SNR region.

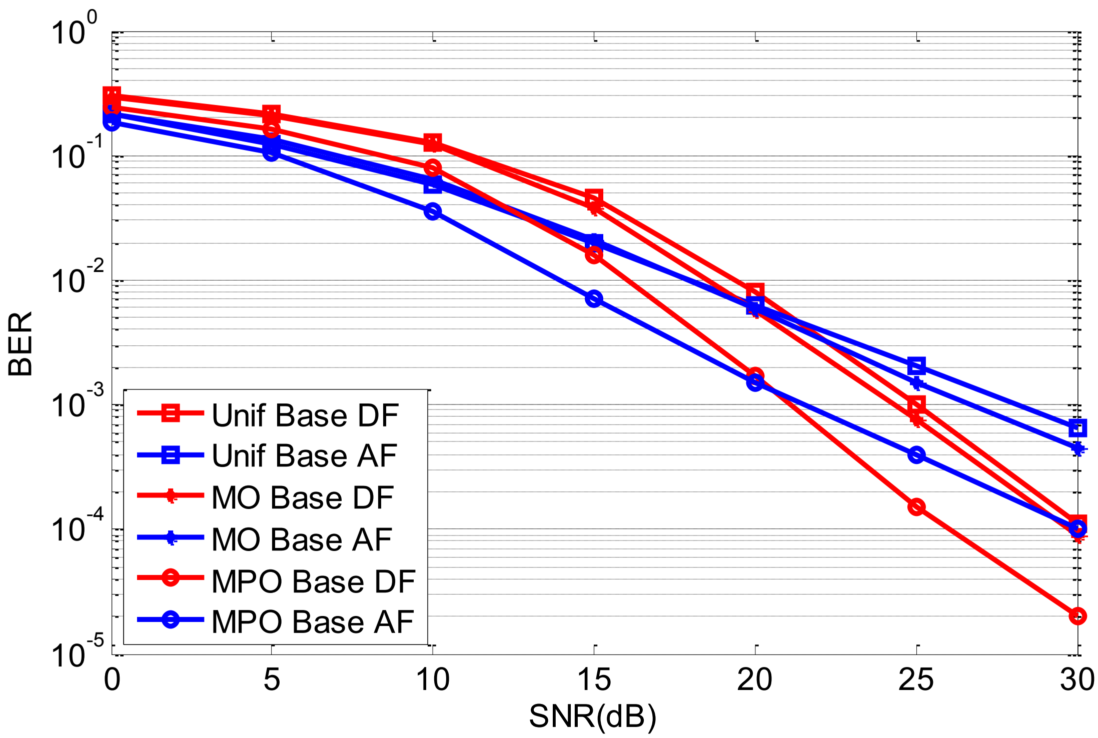

Figure 4.14 Base BER, Relay cooperation, AF and DF schemes, Relay closer to User-Ends

Figure 4.15 presents the performance of the refinement bits user in the case of AF and DF relaying when the relaying node is located closer to the users' side. Examining the performance in the DF relaying scheme, it can be observed that the performance of the (MO) and (MPO) schemes are 1dB and 3 dB superior than that of the uniform modulation case respectively. In the AF relaying technique, (MO) method performs slightly better than the uniform modulation technique while (MPO) outperforms them by almost 5dB.

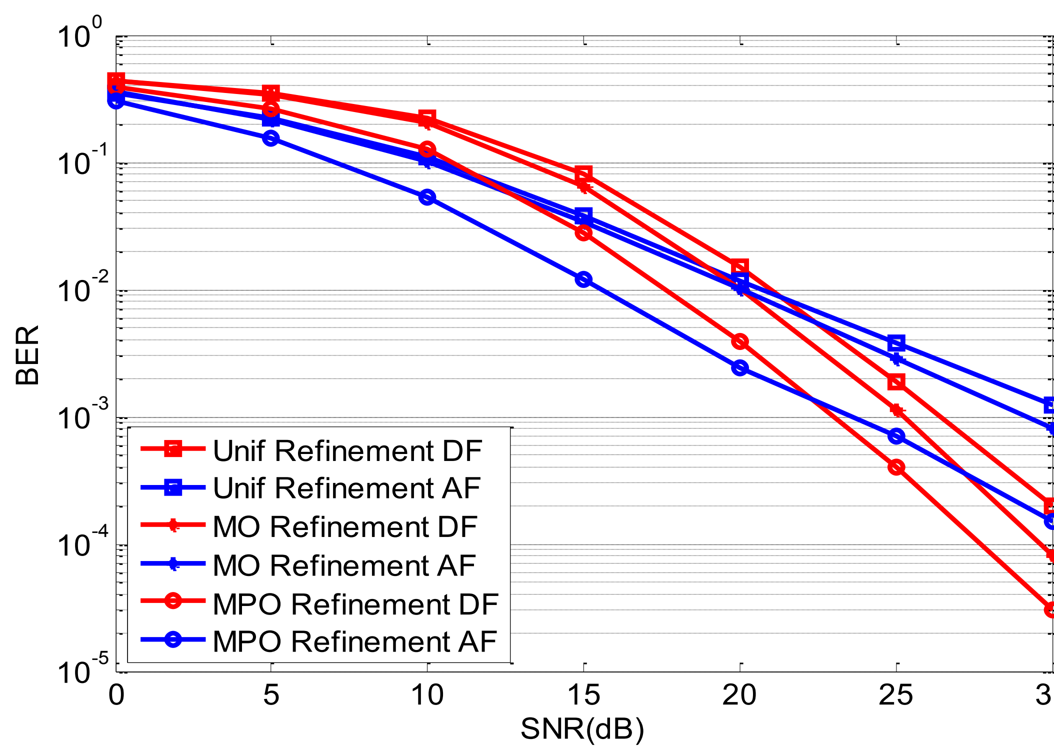

Figure 4.15 Refinement BER, Relay cooperation, AF and DF schemes, Relay closer to User-Ends

Summarizing the three studied physical distances cases, the following can be established. First, the method of optimized hierarchical modulation (MO) would perform best when the relay is closer to the BS. In the case of employing DF relaying, the performance of (MO) scheme for the base bits is not efficient when the relay is closer to the users. Meanwhile, the performance of the refinement bits under the (MO) technique is always advantageous than when employing uniform modulation. However, as the relay is located further from the source, the performance of (MO) slightly degrades. These observations can be interpreted by the base bits and refinement bits characteristics. In general, the performance of the base bits is expected to always outperform the refinement bits performance when the two destinations are at equal distances from the source. When optimizing the hierarchical modulation (MO), the goal is to minimize the total BER. Since the BER of the refinement bits is higher than that of the base bits, most of the time the focus will be to minimize that of the refinement bits to minimize the total BER. Thus, as can be seen from the results, (MO) improves the performance of the refinement bits in all cases, while it becomes less operational for base bits with the increasing separating distance between the relay and the BS.

In the case of employing AF relaying, the performance of both base bits and refinement bits using the (MO) scheme only improves when the relay is closer to the BS. The worst performance of (MO) is when the relay is half way between the BS and the User-Ends. In such a case, its performance resembles the performance of its corresponding uniform modulation. A slight improvement in performance of the (MO) scheme for both base bits and refinement bits compared to the uniform modulation was shown in Figure 4.14 and Figure 4.15. This is interpreted by the fact that when the relay is closer to one end; either the source node or the destination node; the power of the channel link separating the relay from that end is relatively high enough to mitigate the effect of the noise. On the other hand, when the relay is half-way between the source and the destination, both links $h_{s,r}$ and h_{r,u_i} suffer from relatively high path loss effect and the resultant noisy signal is harder to detect.

Finally, optimization of hierarchical modulation along with the optimization of the transmitted energy (MPO) has superior performance compared to the two other methods even in low SNR region. Its performance is superior to that of a uniform modulation by an average of 3.5 dB in DF relaying. In AF relaying, the performance of (MPO) was noticeably superior especially in the last two cases. Thus it can be concluded that when the relay is close to the BS, simple hierarchical modulation optimization is enough to improve the performance. However, the more the relay is further away from the source it is more advisable to employ the (MPO) method.

4.3.2 Effect of changing the location of a User-End

In the previous subsection, we studied the effect of the relative location of the relay to the other nodes on the BER performance at both destinations. In this subsection, analysis of the effect of changing the relative location of one of the users is carried out. BER performance for both users is examined in both AF and DF relaying techniques. As in the previous section, performances of three different modulation and energy distribution cases are examined. The first case employs a uniform constellation with equal energy distribution at the BS and the Relay. The second case employs an optimized modulation at both the BS and the Relay while still keeping equal energy distribution at both transmitters. The second case will be referred to as hierarchical Modulation Optimization (MO). The third case employs an optimized modulation at both the BS and the Relay while optimizing the transmitted energy at these two nodes. The third case will be referred to as Modulation and Power Optimization (MPO). Two cases of User-End location are presented. The first case is when the two users are at an equal distance from the BS; $\left(d_{s,u_1} = d_{s,u_2}\right)$. The second case is when UE_2 is closer to the BS than UE_1; $\left(d_{s,u_1} > d_{s,u_2}\right)$.

4.3.2.1 Case 1: $\boldsymbol{d_{s,u_1} = d_{s,u_2}}$

In the first case, the effect of locating both User-Ends at the same separating distance from the Base Station is examined. The distances separating the different nodes of the relaying cooperative scheme is represented by a distance matrix $\boldsymbol{D}$, where $\boldsymbol{D} = [d_{s,u_1}, d_{s,u_2}, d_{s,r}, d_{r,u_1}, d_{r,u_2}]$. In this scenario, the distance matrix $\boldsymbol{D} = [1,1,0.25,0.9,0.9]$. InFigure 4.16 Base BER, Relay cooperation, AF and DF schemes, case $d_{s,u_1} = d_{s,u_2}$ Figure 4.16, in the case of DF relaying, both (MO) and (MPO) techniques achieve a better performance than that of the uniform constellation. (MO) and (MPO) almost have the same performance where they outperform the performance of the uniform constellation by 3 dB. In the case of AF relaying, also (MO) and (MPO) achieve better BER than the uniform constellation. (MO) achieves a gain of 1.5 dB while (MPO) achieves a gain of 4 dB compared to the performance of uniform constellation. Overall, the performance of the DF relaying method in this scheme has a better BER for the refinement bits than when using AF relaying.

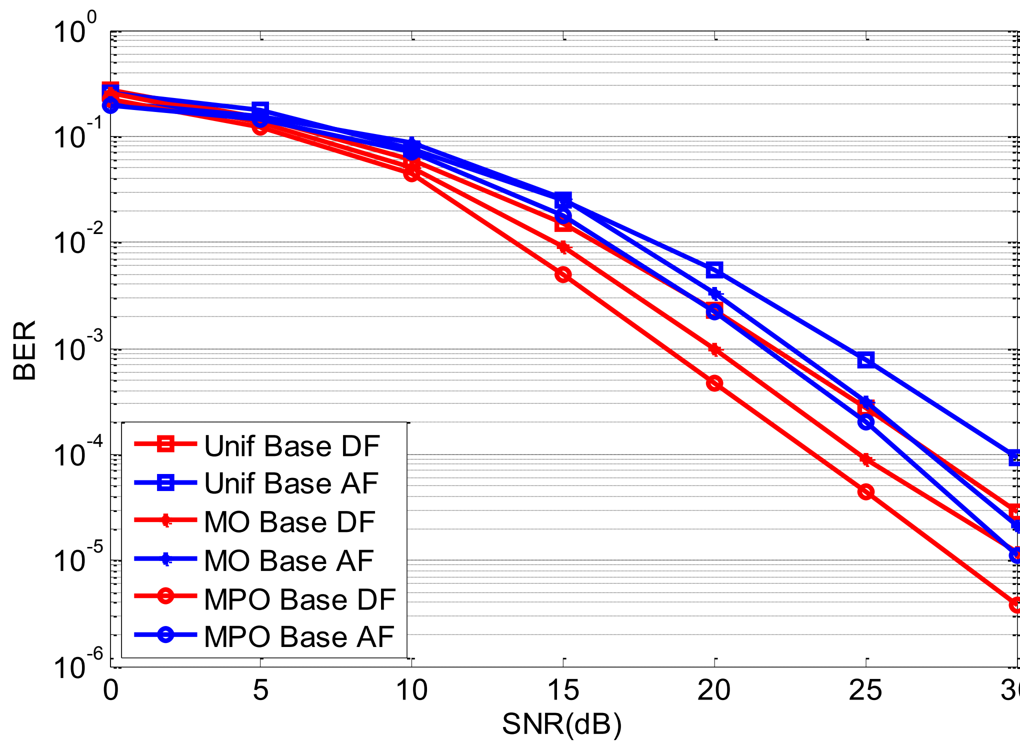

Figure 4.16 Base BER, Relay cooperation, AF and DF schemes, case $d_{s,u_1} = d_{s,u_2}$

Figure 4.17 illustrates BER performance of refinement bits in both AF and DF relaying. In the case of DF relaying, both (MO) and (MPO) techniques achieve a better performance than that of the uniform constellation. (MO) and (MPO) almost have the same performance where they outperform the performance of the uniform constellation by 3 dB. In the case of AF relaying, also (MO) and (MPO) achieve better BER than the uniform constellation. (MO) achieves a gain of 1.5 dB while (MPO) achieves a gain of 4 dB compared to the performance of uniform constellation. Overall, the performance of the DF relaying method in this scheme has a better BER for the refinement bits than when using AF relaying.

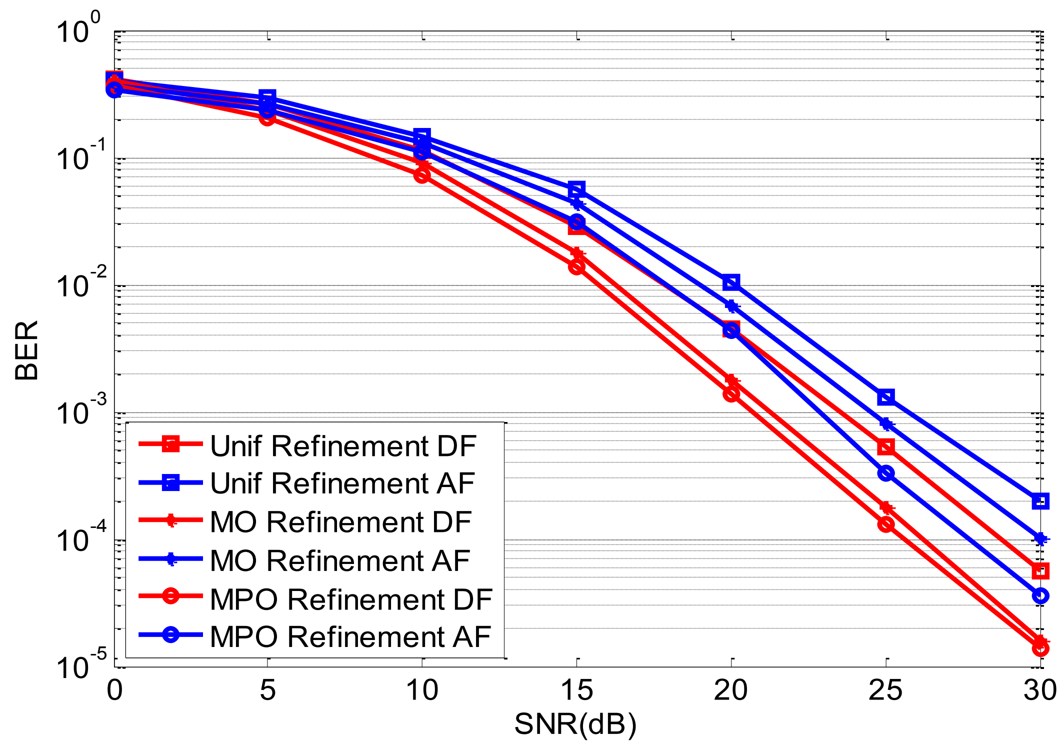

Figure 4.17 Refinement BER, Relay cooperation, AF and DF schemes, case $d_{s,u_1} = d_{s,u_2}$

4.3.2.2 Case 2: $\boldsymbol{d_{s,u_1} > d_{s,u_2}}$

In the second case, we will examine the effect of locating the two users at different distances from the Base Station. More precisely, the refinement bits user UE_2 will be located closer to the BS than the base bits user UE_1. The distance matrix in this scenario is given by $\boldsymbol{D} = [1,0.7,0.25,0.9,0.55]$. In Figure 4.18, BER performance of base bits in both AF and DF relaying is examined. In case of AF relaying, the performance of the (MO) scheme is superior to the uniform constellation by 4 dB in high SNR region. In addition, the performance of the (MPO) scheme is 5.5 dB better than the uniform modulation. Meanwhile, in the case of DF relaying, the performance of the (MO) scheme is superior to the uniform constellation by 4.5 dB in high SNR region. Meanwhile, (MPO) scheme's performance is 5 dB better than the uniform modulation.

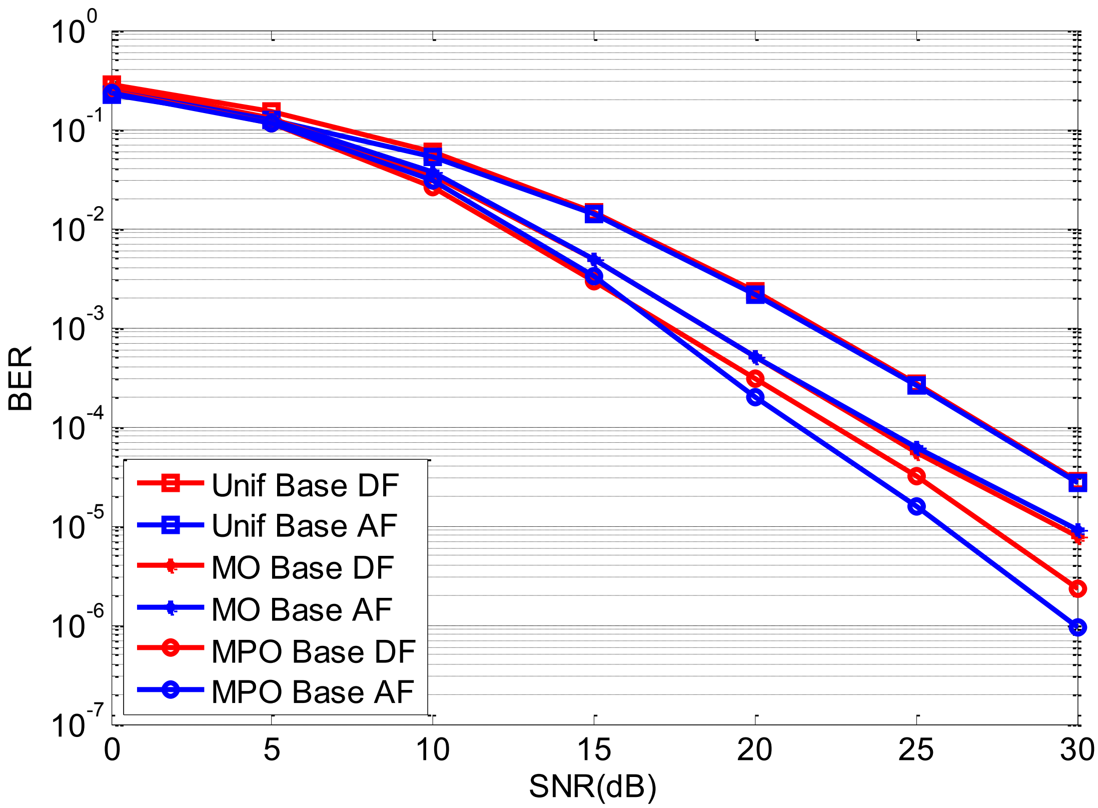

Figure 4.18 Base bits BER, Relay cooperation, AF and DF schemes, case $d_{s,u_1} > d_{s,u_2}$

In Figure 4.19, BER performance of refinement bits in both AF and DF relaying is examined. In case of AF relaying, (MO) scheme's performance is superior to the uniform constellation by 1 dB in high SNR region. On the other hand, the performance of the (MPO) scheme is 3 dB better than the uniform modulation. In the case of DF relaying, the performance of the (MO) is superior to the uniform constellation by 1 dB in high SNR region. Finally, the performance of the (MPO) is 1 dB better than the uniform modulation.

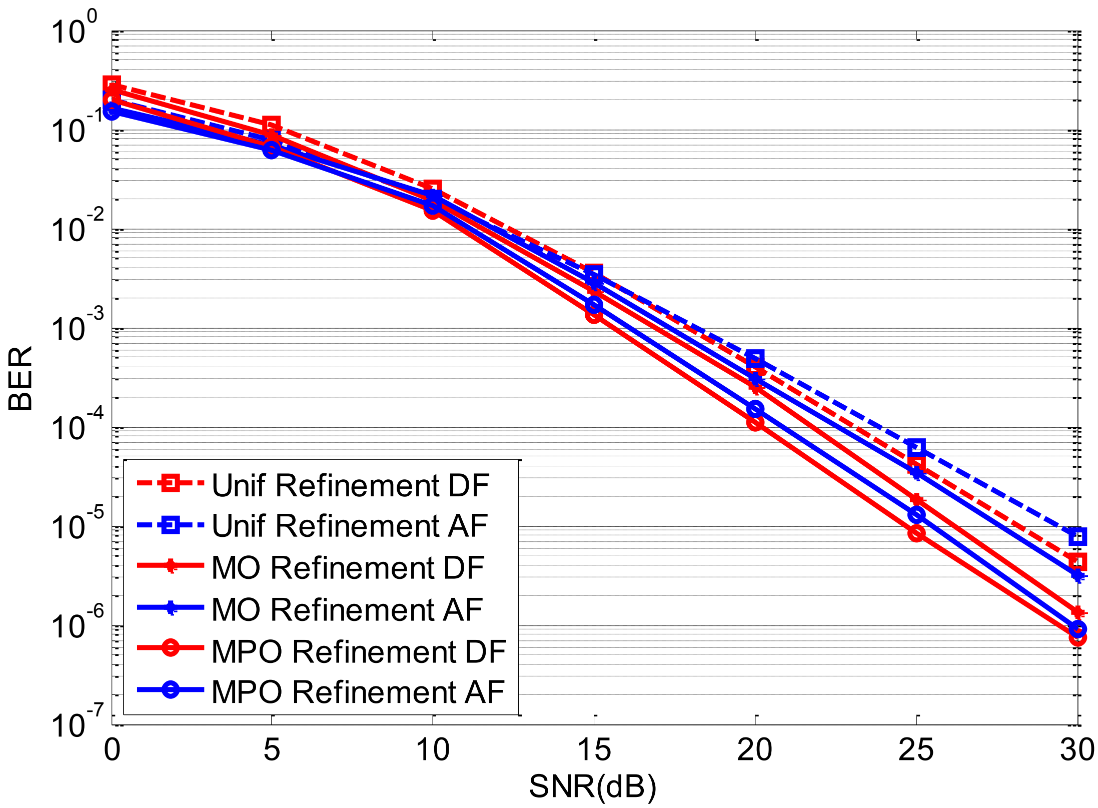

Figure 4.19 Refinement bits BER, Relay cooperation, AF and DF schemes, case $d_{s,u_1} > d_{s,u_2}$

In this section, the effect of changing a destination node's relative location to the Base Station is analyzed. Study of each user's performance in AF and DF relaying is presented. From this study it is concluded that such a change of location affects the performance of both users. As expected, as the refinement bits user moves closer to the BS, its performance improves in both AF and DF relaying. In addition, the performance of base bits also improves with the relative change of distance of the refinement bits user towards the BS. Figure 4.20 and Figure 4.21 illustrates the performance of the base bits in the DF and AF relaying techniques respectively. Each figure compares the performance when $d_{s,u_1} = d_{s,u_2}$, denoted in the figure by $\boldsymbol{D_1}$, to the performance when $d_{s,u_1} > d_{s,u_2}$, denoted in the figure by $\boldsymbol{D_2}$. In the DF relaying technique, the performance of base bits in both distance schemes $\boldsymbol{D_1}$ and $\boldsymbol{D_2}$ using uniform constellation is the same as expected. The performance of the base bits improves in the two optimization methods (MO) and (MPO) by 2-3 dB even in low SNR region. In AF relaying, the same performance occurs under the uniform modulation scheme. Performances of the (MO) and (MPO) schemes outperform the uniform constellation by an average of 3.5 dB in low SNR region.

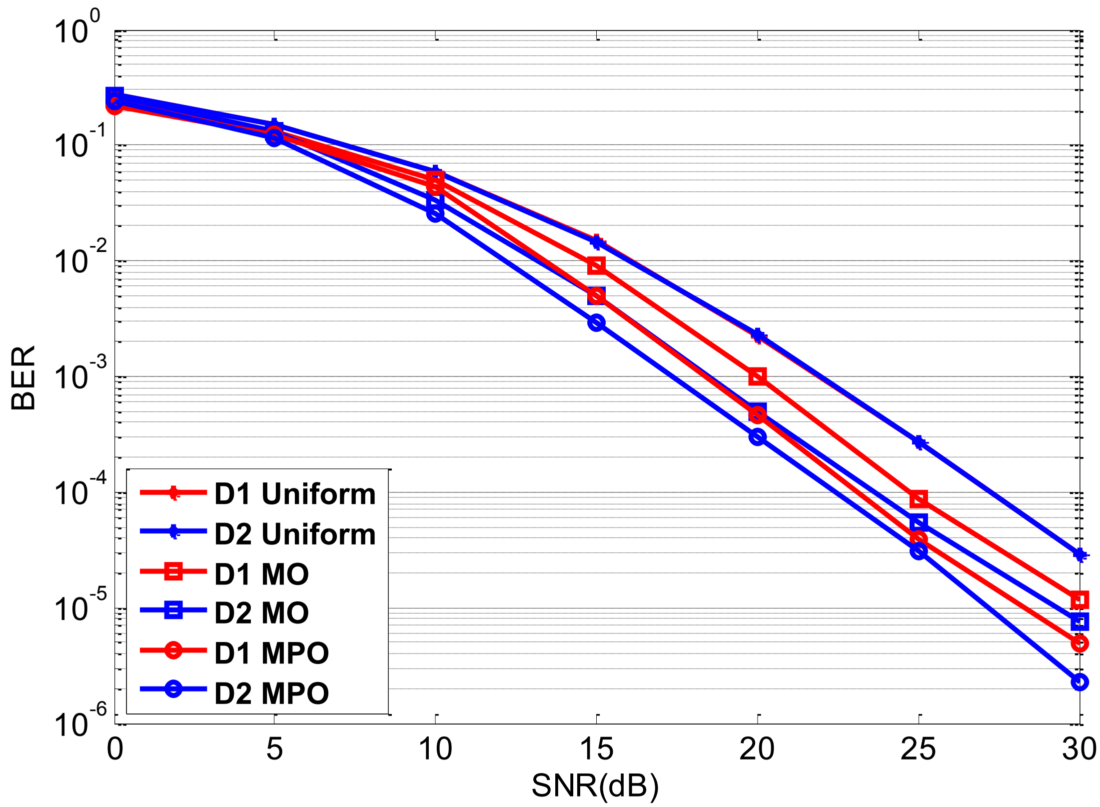

Figure 4.20 Base Bits BER, Relay Cooperation, DF relaying at $\boldsymbol{D_1}$ and $\boldsymbol{D_2}$

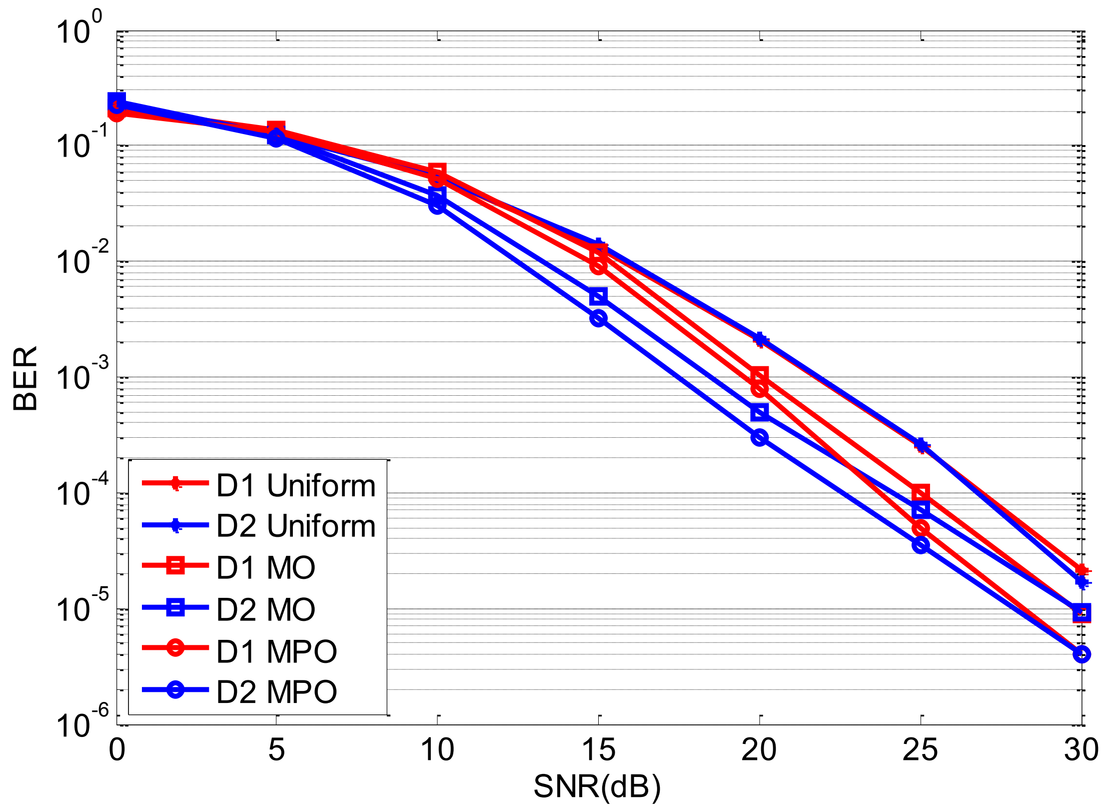

Figure 4.21 Base Bits BER, Relay Cooperation, AF relaying at $\boldsymbol{D_1}$ and $\boldsymbol{D_2}$

4.4 Turbo Coded Cooperative Network Transmission

In this section, results of the Turbo coded cooperative system are discussed. As mentioned earlier, the Turbo coded cooperative system consists of a BS, Relay and two User-Equipment. In simulating the presented system, the distance parameters are chosen based on the selection criteria of an un-coded cooperative network with the same exact channel conditions. This method has proven to be accurate in minimizing the BER of both users in most cases. In this subsection, examples of instantaneous BER resulting from selecting the optimized distance parameters for the corresponding un-coded scenario versus instantaneous BER of a uniform modulation are compared. In few cases, when the selected distance parameters are not optimum, they still achieve BER values very close to the optimum ones. In addition, the overall performance of the system when employing the proposed method is compared with the corresponding case of employing a uniform 16QAM modulation.

4.4.1 Example 1

This example examines a Turbo relaying network at SNR = 5dB. The channel coefficients between the components of this model are defined in channel matrix

$$\boldsymbol{C} = \left[\left|h_{s,u_1}\right|^2, \left|h_{s,u_2}\right|^2, \left|h_{s,r}\right|^2, \left|h_{r,u_1}\right|^2, \left|h_{r,u_2}\right|^2\right]$$

In this example, $\boldsymbol{C} = [0.775, 0.129, 0.099, 1.477, 6.846]$. From the $\boldsymbol{C}$ matrix values it can be seen that the $BS - R$ link is not powerful enough for the relay to decode the BS message fully correctly. Thus, the users will only receive one copy of the data coming from the BS. Using equations (3.8) and (3.9), and the values of $\left|h_{s,u_1}\right|^2$and $\left|h_{s,u_2}\right|^2$, d_1 and d_2 can be computed to achieve the minimum total instantaneous BER.

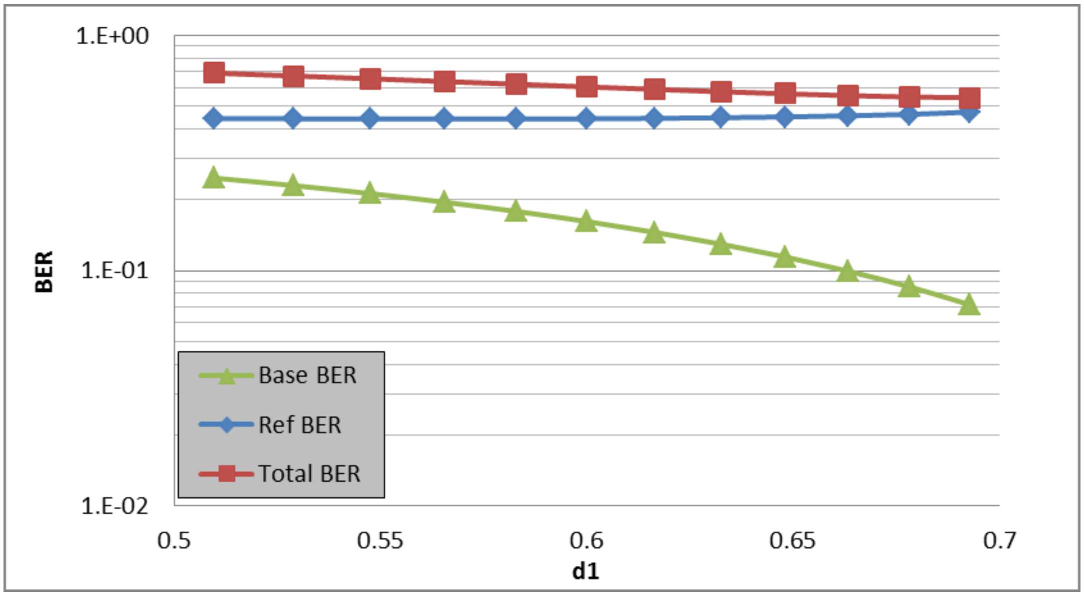

Figure 4.22 BER of example1 in un-coded scenario for different values of d_1

Figure 4.22 illustrates the BER values for the base bits and the refinement bits in case of the un-coded scenario. Even though the BER values are different for the Turbo coded case, the minimum for the total BER curve is the same for both cases. Table 4-1 illustrates the actual total BER achieved in the Turbo code scenario. The red marked BER is the BER achieved when employing a uniform 16QAM constellation, while the green marked BER is the BER achieved when employing the distance parameters computed from equations (3.8)(3.9).

d_1	0.51	0.53	0.55	0.57	0.58	0.60	0.62	0.63	0.65	0.66	0.68	0.69
BER	0.417	0.396	0.388	0.378	0.381	0.372	0.375	0.354	0.343	0.334	0.319	0.315

Table 4-1 Actual BER of example1 in Turbo coded scenario for different values of d_1

4.4.2 Example 2

This example examines a Turbo relaying network at SNR = 10dB. The channel coefficients between the different nodes of this model are defined in channel matrix $\boldsymbol{C}$

where $\boldsymbol{C} = [0.4094, 0.6185, 24.076, 1.0242, 0.112]$. In this example, the $BS - R$ link is good enough for the relay to decode the BS message correctly. Thus, the users will receive two copies of the data; one copy coming from the BS and the other coming from the relay. Using equations (A.29) and (A.63), the values of the channel coefficients in $\boldsymbol{C}$ and the fact that $E_s = E_r = {}^{1}/_{2}\, E_{tot}$, the values of d_1 , d_2 , $\widehat{d_1}$ and $\widehat{d_2}$ can be computed to achieve the minimum total instantaneous BER. Table 4-2 illustrates the total computed instantaneous BER values in the case of the un-coded scenario. The red marked BER is the total instantaneous BER in case of employing a uniform constellation. The green marked BER is the minimum total instantaneous BER. This minimum value is achieved when the constellation's distance parameters set in the first time is $\{d_1 = 0.53, d_2 = 0.47\}$ and when the constellation's distance parameters set in the second time slot is $\{d_1 = 0.69, d_2 = 0.14\}$. Reflecting on the channel values in matrix $\boldsymbol{C}$, such a choice of distance parameters is logical. As can be seen from the channel values, if we consider the two channel links to UE_1; $\left|h_{s,u_1}\right|^2 = 0.4094, \left|h_{r,u_1}\right|^2 = 1.0242$; we will see that the second link is more favorable. Thus, in the second time slot, the constellation's parameters are set to protect UE_1's data by increasing the value of d_1. On the other hand, considering the two channel links to UE_2; $\left|h_{s,u_2}\right|^2 = 0.6185, \left|h_{r,u_2}\right|^2 = 0.112$; we will see that the first link is more favorable. Thus, at the BS, the constellation's parameters are chosen to protect UE_2's data by increasing the value of d_2.

		0.53	**0.57**	**0.60**	**0.63**	**0.66**	**0.69**	**d1**	**TS1**
		0.47	**0.42**	**0.37**	**0.32**	**0.24**	**0.14**	**d2**	
0.53	**0.47**	0.1676	0.1629	0.1630	0.1684	0.1807	0.2074		
0.57	**0.42**	0.1473	0.1439	0.1455	0.1524	0.1666	0.1956		
0.60	**0.37**	0.1286	0.1269	0.1303	0.1390	0.1552	0.1867		
0.63	**0.32**	0.1120	0.1124	0.1177	0.1284	0.1467	0.1809		
0.66	**0.24**	0.0980	0.1007	0.1081	0.1211	0.1416	0.1784		
0.69	**0.14**	0.0873	0.0925	0.1025	0.1179	0.1409	0.1805		
d1	**d2**								
TS2									

Table 4-2 Example2: Theoretical total BER in un-coded scenario for different constellation's parameters values

On the other hand, in the corresponding Turbo coded system, the actual simulated instantaneous BER are shown in Table 4-3. As can be seen from the table, the minimum total instantaneous BER will be achieved when the constellation's distance parameters in the first time slot are $\{d_1 = 0.53, d_2 = 0.47\}$ and in the second time slot $\{d_1 = 0.63, d_2 = 0.32\}$ resulting in a total BER =0.063. Even though these distance parameters are not matching the ones computed in the un-coded system, still the selected parameters in the un-coded case achieve a nearly optimum total BER of value 0.066. Comparing this value to the BER achieved when employing a uniform 16QAM modulation, where the achieved instantaneous total BER =0.118, it can be concluded that our proposed method is more advantageous.

		0.53	0.57	0.60	0.63	0.66	0.69	d1	TS1
		0.47	0.42	0.37	0.32	0.24	0.14	d2	
0.53	0.47	0.145	0.149	0.151	0.145	0.150	0.183		
0.57	0.42	0.084	0.098	0.114	0.125	0.147	0.180		
0.60	0.37	0.064	0.081	0.098	0.117	0.145	0.181		
0.63	0.32	0.063	0.082	0.099	0.118	0.146	0.179		
0.66	0.24	0.066	0.080	0.097	0.117	0.147	0.185		
0.69	0.14	0.066	0.079	0.099	0.118	0.151	0.190		
d1	d2								
TS2									

Table 4-3 Actual BER of example 2 in Turbo coded scenario for different values of d_1

4.4.3 Example 3

In this example, the overall system performance using the proposed method in Turbo coded cooperative relay network is analyzed. The inter-nodes distances in this example are defined by the matrix $\boldsymbol{D}$ as in Section 4.3.1.1 where $\boldsymbol{D} = [1,1,0.25,0.9,0.9]$. Simulations are carried using uniform modulation and optimized hierarchical modulation. Simulation results are illustrated in Figure 4.23 and Figure 4.24. In Figure 4.23, the BER performance of the base bits using uniform modulation is plotted against using optimized hierarchical modulation. Results prove that an improvement in performance is achieved when using the proposed method achieving an average gain of 2 dB. Figure 4.24 illustrates the BER performances of the refinement bits when using uniform modulation and optimized hierarchical modulation. Results prove that an improvement in performance is achieved when using the proposed method achieving an average gain of

1.5 dB.

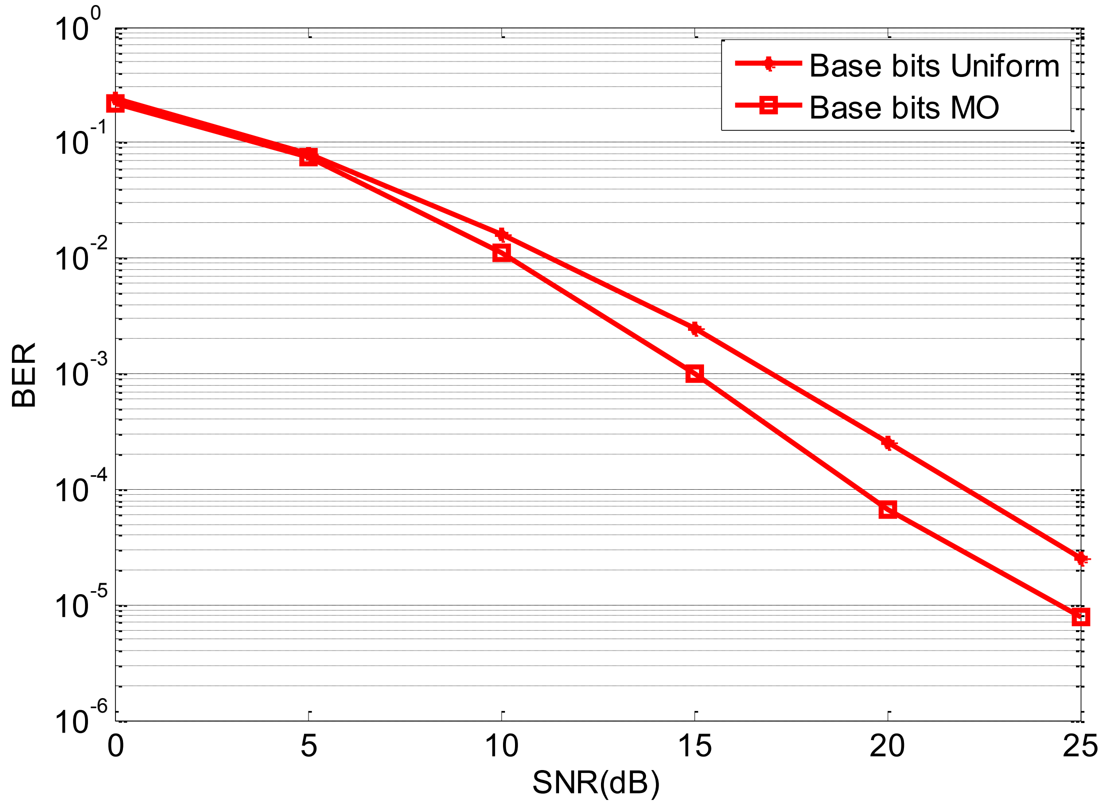

Figure 4.23 Average Base BER in Turbo coded scheme, Relay closer to BS

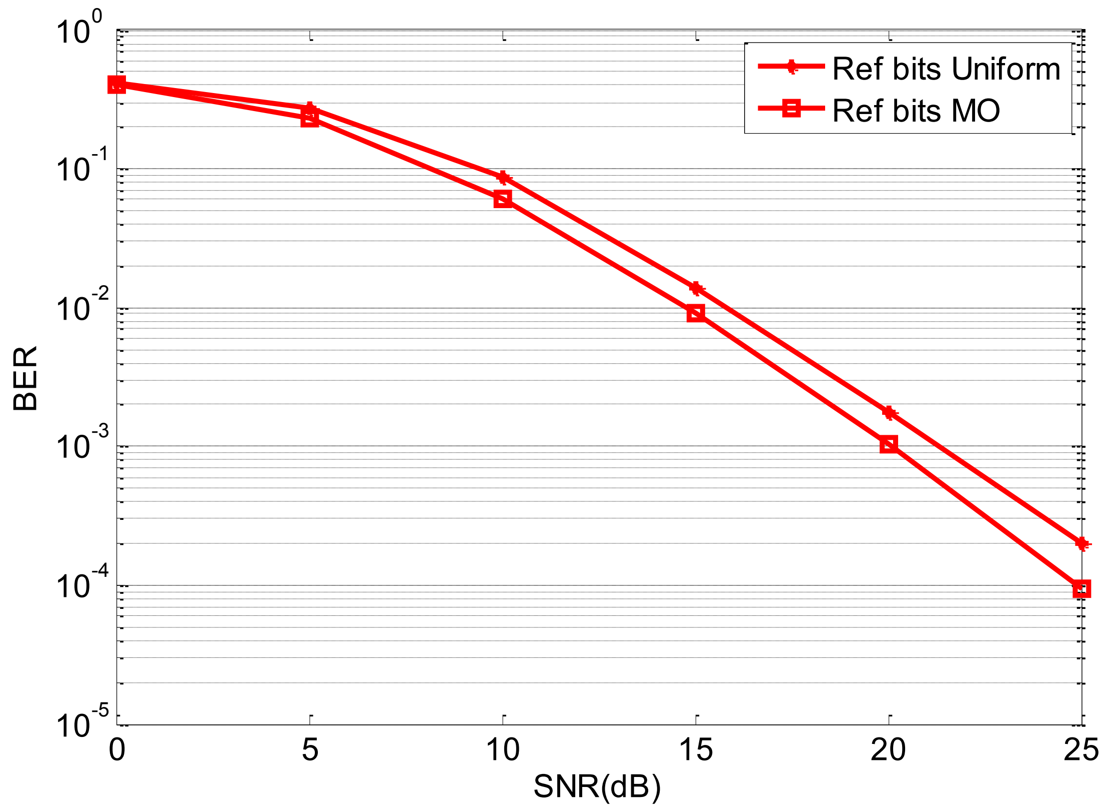

Figure 4.24 Average Refinement BER in Turbo coded scheme, Relay closer to BS

4.5 Summary

In this chapter, simulations analyses of the proposed method are presented. The proposed modulation optimization is applied in several network schemes. These network schemes include Direct Transmission, User Cooperation, Relay Cooperation and Turbo Coded Relay Cooperation. In the Direct Transmission system, three different inter-nodes distance examples are examined. In each example, comparisons between the proposed optimized hierarchical modulation performance and the performance of a regular uniform modulation are presented. It is proven that the proposed method improves the performance of each user; base bits user and refinement bits user; by at least 3.5 dB for base bits user and 1.5 dB for refinement bits user. It is also concluded that under hierarchical modulation application, the change in one user's location affects the performance of both users.

In the case of user cooperation, two examples are presented. The first example evaluates the efficiency of optimized hierarchical modulation when the correct reception of one user's data (UE_1's data) is more important than the other user's data (UE_2's data). In this case, UE_2 relays the data it receives belonging to UE_1 when possible. This solution outperforms the use of simple Direct Transmission employing uniform modulation scheme by 5 dB. Also optimized hierarchical modulation in this scheme outperforms its corresponding uniform modulation by 2dB. In the second example, the user with advantageous channel condition assists the user with less favorable channel condition to better detect its data. In that scheme, the proposed method outperforms its corresponding uniform modulation by at least 2dB.

In relay cooperative network scheme, system evaluations of different relative Relay locations in the network are carried out. It is concluded that the best performance of the suggested scheme is when the relay is located closer to the Base Station. Simulation results vary depending on the relaying protocol used; AF or DF. In DF relaying, the performance of the proposed method is less efficient when the relay is further located from the Base Station. On the other hand in AF relaying, the performance of the proposed system is worst when the relay is located half way between the Base Station and the users. Adjusting the total transmitted energy among the transmitters is also examined in the Relay cooperative scheme. It is shown that such adjustment along with the proposed hierarchical modulation optimization result in even better performances of both users. In the second part of the Relay cooperative system results, a study on varying the location of one of the User Equipment is carried out. In the case where uniform modulation is used, only the user with changing location experiences different BER performances. However, in the case where optimized modulation is used, the performance of both users is affected.

Finally, the presented study is extended to analyse the application of the proposed system in the Turbo coded relay cooperative network environment. In this model, instantaneous selected constellation's parameters are tested to achieve minimum or almost minimum total instantaneous BER. Two examples are presented where it is proven that the proposed method achieves near optimum instantaneous BER values. In the third example, the average total BER for base bits and refinement bits achieved using optimized hierarchical modulation is plotted versus the corresponding achieved BER using uniform modulation. It is shown that for both users improvement in BER performance is achieved when optimized hierarchical modulation is employed.

Chapter Five

5. Conclusion and Future work

Recently, the use of higher order modulation in cooperative networks has acquired special interest in the research field. High order modulations have the capability of transmitting higher data rate than just simple binary modulation. Thus they are more desirable in cooperative network environments to mitigate the need for extra bandwidth. The hierarchical 4/16-QAM modulation is one of the most extensively studied and practically employed in the field of wireless networks due to its simplicity and moderate compromise between performance and data rate.

In this thesis, an adjustable 4/16 QAM hierarchical modulation is developed. It adjusts its distributed energy among the receivers according to the channel condition linking to each user. We apply our proposed protocol to downlink cellular network with a cooperative relay. In this system model, the Base Station aims at delivering two different messages to two different users by concatenating their data together using a 4/16 QAM modulation. The goal of the system is to adjust the modulation parameters in order to minimize the total BER. In order to optimize the hierarchical modulation parameters, theoretical BER for each user in direct transmission and cooperative relay transmission; including AF and DF protocol; are derived in this thesis. Full channel knowledge at all nodes is assumed before transmission. Unlike previous developed systems, the optimization of the hierarchical modulation in this thesis is based on the channel conditions carrying the transmitted data. This resulted in improved performance for all destinations compared to previous work that generally focused on improving the performance only at one User Equipment.

In Chapter 3, the system model studied in this thesis is explained. Different scenarios of transmission have been presented including direct transmission, cooperative transmission and coded cooperative transmission. All related system parameters are explained in details including the modulation/demodulation and encoding/decoding factors. In Chapter 4, simulation results of the different designed schemes are presented. In direct transmission, the use of our proposed system always outperforms the use of uniform modulation. The improvement in performance is even more noticeable when the two destinations are unequally distant from the source. User cooperation simulations improve the performance of users experiencing unfavorable channel conditions. Employment of adjustable hierarchical modulation in this model outperforms its uniform modulation counterpart. In relay cooperative model, several relay locations and user locations are studied, where optimized hierarchical modulation shows to be more efficient in situations than in others. But overall, adjusting the transmitted energy on the transmitters while adjusting the hierarchical modulation energy always show noticeable performance improvement for both destinations. Lastly, our proposed method is extended to Turbo coded cooperative network scenario. The use of theoretically derived optimized modulation parameters for un-coded cooperative environment in Turbo coded cooperative environment is tested. Even though this method may not be optimum all the time, it has proven to improve the performance in most cases and has led to overall improvement in the performance of the User Equipment.

Currently, this specific topic is still under preliminary research. There are a couple of ways in which future work can be conducted for this topic. First, investigation of the performance of optimized high order modulation in other cooperative network topologies such as bidirectional relay networks, or uplink topology can be conducted. Second, computations of actual theoretical BER of base bits and refinement bits in Turbo coded relay cooperative networks can be carried on in future work in order to improve the performance of hierarchical modulation in such an environment. Overall, this is a promising research topic that has proven in this work to achieve a worthwhile performance improvement.

A. Appendix

A.1 Hierarchical modulation BER computation in a Rayleigh fading cooperative network using MRC

In this section, theoretical derivations of the instantaneous BER for the base bits and the refinement bits in a 4/16 QAM constellation in DF and AF relaying using MRC combining technique in Rayleigh fading environment are presented. These derivations are very vital in this work. In the modulation process, these expressions are used to choose the constellation's parameters that will minimize the total BER. In this thesis, the same strategy applied in [68] [105] is followed to derive the required BER expressions.

In this Chapter, the scenario in each relaying technique is briefly explained. In this work, only DF and AF relaying methods are considered. Then the technique for deriving the probability of error of base and refinement bits for each of these relaying methods is explained.

In the Decode and Forward (DF) scenario, the signal is decoded at the relay and a CRC check is applied. If the CRC check fails, the relay remains silent and the users receive their data only from the Base Station. Only in the case of a correct reception of a whole block of transmitted signals, i.e. the CRC check succeeded, these signals are re-modulated and retransmitted with the corresponding constellation distance parameters. At the destination, the signals coming from the Base Station and from the relay are added together using MRC and a hard decision threshold is applied. It is to be noted that in the DF relaying scenario, the constellation's distance parameters chosen during the modulation process at the Base Station can be different from those chosen at the relay.

In the Amplify and Forward (AF) scenario, the signal is amplified at the relay by a certain amplifying gain factor α and is re-broadcasted to the other nodes without any further processing. At each destination, the signals coming from the Base Station and from the relay are added together and a hard decision threshold is applied. In AF relaying scheme, the same constellation's distance parameters are used during modulation at the Base Station and the relay.

A.1.1 Instantaneous BER of Base bits using MRC in a Rayleigh fading environment

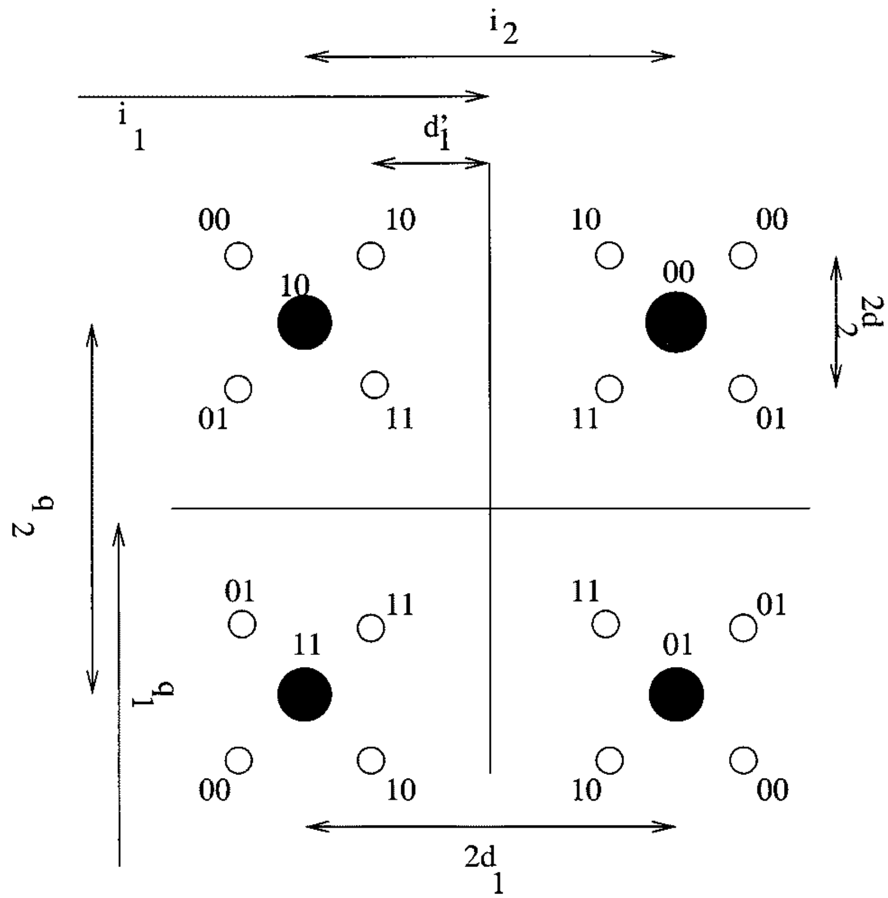

Figure A.1 A hierarchical 4/16 QAM constellation with Gray mapping

Figure A.1 illustrates a hierarchical 4/16QAM constellation and its corresponding 16 symbol binary representation using Grey Coding. As can be seen in the figure, each quadrant shares two common bits represented by a black circle. For illustration purposes, these two bits are denoted by (b_0, b_1). For example, the right upper quadrant symbols share the same base bits $'b_0 b_1 = 00'$. The b_0 bit represents the in-phase common bit while the b_1 bit represents the quadrature-phase common bit.

Let's consider sending a '0' on the bit b_0, the probability of getting an error, i.e. the reception of a '1', means that the resulting signal of combining the two signals coming from the BS and from the relay; denoted by Y; resulted in a reception of signal in the negative side of the y axis. Thus the probability of receiving b_0 in error can be expressed as:

$$Pe(b_0) = P(\mathbb{R}(Y) < 0) \tag{A.1}$$

In the case of receiving b_1; the quadrature phase bit; in error, only the imaginary part of Y will be considered. In this study, only the error probability computations of the in-phase common bit $'b_0'$ are considered. However, error probability computations for the quadrature phase common bit are straight forward. In all the coming computations, all real part notations will be omitted for simplification (i.e. $y_{s,u} = \mathbb{R}(y_{s,u})$).

Let's assume that the symbol"$0x1x$" is sent, which in the 4/16 QAM constellation in Figure A.1 is situated at a distance $(d_1 - d_2)$ from the y axis. Recalling from Chapter3, Equation (3.4), the modulated symbol in this specific case will be $x = d_1 - d_2$. The received signals in the first time slot at the destination and at the relay from the Base Station can be written respectively as

$$y_{s,u} = \sqrt{E_s}(d_1 - d_2)h_{s,u} + n_{s,u} \tag{A.2}$$

$$y_{s,r} = \sqrt{E_s}(d_1 - d_2)h_{s,r} + n_{s,r} \tag{A.3}$$

In the above equations, E_s is the transmitting energy per symbol at the Base Station, $h_{s,u}$ and $h_{s,r}$ are the fading channel coefficients on the source-destination link and the source-relay link respectively. $n_{s,u}$ and $n_{s,r}$ are the AWGN coefficient at the destination and at the relay respectively. d_1, d_2 denote the selected distance parameters in the first time slot.

a) DF Relaying Case

If the CRC check results in successful decoding, the relay re-modulates its signal with new constellation's distance parameters and re-sends its signal. At the destination, the received signal from the relay is expressed as

$$y_{r,u} = \sqrt{E_r}\,(d_1' - d_2')h_{r,u} + n_{r,u} \tag{A.4}$$

E_r is the transmitted energy per symbol at the relay, $h_{r,u}$ is the fading channel coefficient on the relay-destination link and $n_{r,u}$ is the AWGN coefficient at the destination. ${d_1}', {d_2}'$ denote the selected distance parameters in the second time slot.

The two signals $y_{s,u}$ and $y_{r,u}$ are combined at the destination using MRC technique. In this technique, each signal is weighted by its channel coefficient and the two signals are added together. The resultant signal Y is given by:

$$\begin{aligned} Y = \sqrt{E_s}\,(\,d_1 - d_2)\left|h_{s,u}\right|^2 + n_{s,u}\,h_{s,u}^* + \sqrt{E_r}\,(d_1' \\ - d_2')\left|h_{r,u}\right|^2 + n_{r,u}\,h_{r,u}^* \end{aligned} \tag{A.5}$$

Thus, from equations (A.1) and (A.5) the probability of error for the bit b_0 in the symbol "$0x1x$" can be written as follow:

$$Pe(b_0) = P\left(\left[\sqrt{E_s}\,(d_1 - d_2)\left|h_{s,u}\right|^2 + n_{s,u}\,h_{s,u}^* + \sqrt{E_r}\,(d_1' - d_2')\left|h_{r,u}\right|^2 + n_{r,u}\,h_{r,u}^*\right] < 0\right) \tag{A.6}$$

$$= P\left(\left[\sqrt{E_s}\,(d_1 - d_2)\left|h_{s,u}\right|^2 + \sqrt{E_r}\,(d_1' - d_2')\left|h_{r,u}\right|^2\right] < n_{s,u}\,h_{s,u}^* + n_{r,u}\,h_{r,u}^*\right) \tag{A.7}$$

Let's denote $w = n_{s,u}\,h_{s,u}^* + n_{r,u}\,h_{r,u}^*$, where $w \sim \mathcal{N}\left(\mu_w = 0, \sigma_w^2 = \sigma^2\left(\left|h_{s,u}\right|^2 + \left|h_{r,u}\right|^2\right)\right)$. Also, let $z = \left[\sqrt{E_s}\,(d_1 - d_2)\left|h_{s,u}\right|^2 + \sqrt{E_r}\,(d_1' - d_2')\left|h_{r,u}\right|^2\right]$. Using these notations, equation (A.7) can be written as

$$Pe(b_0) = P(z < w)$$

$$= \int_z^\infty \frac{1}{\sqrt{2\pi}\sigma_w} e^{-\frac{1}{2}\left(\frac{x-\mu_w}{\sigma_w}\right)^2} dx$$

let $u = \frac{x-\mu_w}{\sigma_w}$

$$Pe(b_0) = \int_{\frac{z}{\sigma_w}}^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du$$

$$Pe(b_0) = Q\left(\frac{z}{\sigma_w}\right) \tag{A.8}$$

Thus the instantaneous probability of bit error for common bits (base bits) given that the symbol "$0x1x$" was sent using DF relaying over Rayleigh fading channel can be expressed by:

$$Pe(b_0) = Q\left(\frac{\left[\sqrt{E_s}\,(d_1 - d_2)\left|h_{s,u}\right|^2 + \sqrt{E_r}\,(d_1' - d_2')\left|h_{r,u}\right|^2\right]}{\sigma\sqrt{\left|h_{s,u}\right|^2 + \left|h_{r,u}\right|^2}}\right) \tag{A.9}$$

b) AF relaying Case

In the case of relaying the signal using Amplify and Forward, the relay amplifies the received signal $y_{s,r}$ by a gain factor α, where $\alpha = \sqrt{\frac{E_r}{E_s\left|h_{s,r}\right|^2 + N_0}}$ and E_r is the transmitting energy at the relay, E_s is the transmitting energy at the Base Station and N_0 is the noise power spectral density. After the relay amplifies the signal and re-sends it, the received signal at the destination from the relay is given by

$$y_{r,u} = \alpha\, y_{s,r}\, h_{r,u} + n_{r,u} \tag{A.10}$$

$$y_{r,u} = \alpha\, h_{r,u}\, h_{s,r}\sqrt{E_s}\,(d_1 - d_2) + \alpha\, h_{r,u}\, n_{s,r} + n_{r,u} \tag{A.11}$$

where $h_{r,u}$ is the fading channel coefficient on the relay-destination link and $n_{r,u}$ is the AWGN coefficient at the destination. The two signals $y_{s,u}$ and $y_{r,u}$ are combined at the destination using MRC technique. The resultant signal Y is given by:

$$\begin{aligned} Y = {} & \sqrt{E_s}\,(d_1 - d_2)\left|h_{s,u}\right|^2 + n_{s,u}\, h_{s,u}^* \\ & + \sqrt{E_s}\,(d_1 - d_2)\alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2 \\ & + \alpha\left|h_{r,u}\right|^2 h_{s,r}^*\, n_{s,r} + h_{r,u}^* h_{s,r}^*\, n_{r,u} \end{aligned} \tag{A.12}$$

Thus, from equations (A.1) and (A.12) the probability of error for the bit b_0 in the symbol "$0x1x$" using AF relaying can be written as follow:

$$Pe(b_0) = P\Big(\Big[\sqrt{E_s}\,(d_1 - d_2)\left|h_{s,u}\right|^2 + n_{s,u}\,h_{s,u}^* + \sqrt{E_s}\,(d_1 - d_2)\alpha\,\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2 + \alpha\,\left|h_{r,u}\right|^2 h_{s,r}^*\,n_{s,r} + h_{r,u}^* h_{s,r}^*\,n_{r,u}\Big] < 0\Big) \tag{A.13}$$

$$Pe(b_0) = P\Big(\Big[\sqrt{E_s}\,(d_1 - d_2)\left|h_{s,u}\right|^2 + \sqrt{E_s}\,(d_1 - d_2)\alpha\,\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\Big] < n_{s,u}\,h_{s,u}^* + \alpha\,\left|h_{r,u}\right|^2 h_{s,r}^*\,n_{s,r} + h_{r,u}^* h_{s,r}^*\,n_{r,u}\Big) \tag{A.14}$$

Let $w = n_{s,u}\,h_{s,u}^* + \alpha\,\left|h_{r,u}\right|^2 h_{s,r}^*\,n_{s,r} + h_{r,u}^* h_{s,r}^*\,n_{r,u}$,

where $w \sim \mathcal{N}\left(\mu_w = 0, \sigma_w^2 = \sigma^2\left(\left|h_{s,u}\right|^2 + \left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\left(\alpha^2\left|h_{r,u}\right|^2 + 1\right)\right)\right)$. Also, let $z = \left[\sqrt{E_s}\,(d_1 - d_2)\left|h_{s,u}\right|^2 + \sqrt{E_s}\,(d_1 - d_2)\alpha\,\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right]$. Using these notations, and using equation (A.8), the instantaneous probability of bit error for common bits (base bits) given that the symbol "$0x1x$" was sent over Rayleigh fading channel using AF relaying can be expressed by:

$$Pe(b_0) = Q\left(\frac{\sqrt{E_s}\,(d_1 - d_2)\left(\left|h_{s,u}\right|^2 + \alpha\,\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right)}{\sigma\sqrt{\left(\left|h_{s,u}\right|^2 + \left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\left(\alpha^2\left|h_{r,u}\right|^2 + 1\right)\right)}}\right) \tag{A.15}$$

In the previous example, it is assumed that the Base Station wants to send the symbol "$0x1x$" . Now let's assume that the Base Station is sending the symbol "$0x0x$" , which is situated at a distance $(d_1 + d_2)$ from the y axis. The received signals from the Base Station at the destination and at the relay respectively are

$$y_{s,u} = \sqrt{E_s}\,(d_1 + d_2)h_{s,u} + n_{s,u} \tag{A.16}$$

$$y_{s,r} = \sqrt{E_s}\,(d_1 + d_2)h_{s,r} + n_{s,r} \tag{A.17}$$

where d_1 and d_2 denote the selected distance parameters in the first time slot.

a) DF Relaying Case

If the CRC check results in successful decoding, the relay re-modulates its signal with new constellation's distance parameters and re-sends its signal. At the destination, the received signal from the relay is expressed as

$$y_{r,u} = \sqrt{E_r}\,(d_1' + d_2')h_{r,u} + n_{r,u} \tag{A.18}$$

$h_{r,u}$ is the fading channel coefficient on the relay-destination link and $n_{r,u}$ is the AWGN coefficient at the destination. ${d_1}'$, ${d_2}'$ denote the selected distance parameters in the second time slot.

The two signals $y_{s,u}$ and $y_{r,u}$ are combined at the destination using Maximal Ratio Combining (MRC) technique. The resultant signal Y is given by:

$$\begin{aligned} Y = \sqrt{E_s}\,(\,d_1 + d_2)\left|h_{s,u}\right|^2 + n_{s,u}\,h_{s,u}^* + \sqrt{E_r}\,(d_1' \\ + d_2')\left|h_{r,u}\right|^2 + n_{r,u}\,h_{r,u}^* \end{aligned} \tag{A.19}$$

Using equations (A.1) and (A.19) the probability of error for the bit b_0 in the symbol "$0x0x$" can be written as follow:

$$Pe(b_0) = P\left(\left[\sqrt{E_s}\,(d_1 + d_2)\left|h_{s,u}\right|^2 + n_{s,u}\,h_{s,u}^* + \sqrt{E_r}\,(d_1' + d_2')\left|h_{r,u}\right|^2 + n_{r,u}\,h_{r,u}^*\right] < 0\right) \tag{A.20}$$

$$= P\left(\left[\sqrt{E_s}\,(d_1 + d_2)\left|h_{s,u}\right|^2 + \sqrt{E_r}\,(d_1' + d_2')\left|h_{r,u}\right|^2\right] < n_{s,u}\,h_{s,u}^* + n_{r,u}\,h_{r,u}^*\right) \tag{A.21}$$

Let's denote $w = n_{s,u}\,h_{s,u}^* + n_{r,u}\,h_{r,u}^*$, where $w \sim \mathcal{N}\left(\mu_w = 0, \sigma_w^2 = \sigma^2\left(\left|h_{s,u}\right|^2 + \left|h_{r,u}\right|^2\right)\right)$. Also, let $z = \left[\sqrt{E_s}\,(d_1 + d_2)\left|h_{s,u}\right|^2 + \sqrt{E_r}\,(d_1' + d_2')\left|h_{r,u}\right|^2\right]$. Using these notations, and using equation (A.8), the instantaneous probability of bit error for common bits (base bits) given that the symbol "$0x0x$" was sent using DF relaying over Rayleigh fading channel can be expressed by:

$$Pe(b_0) = Q\left(\frac{\left[\sqrt{E_s}\,(d_1 + d_2)\left|h_{s,u}\right|^2 + \sqrt{E_r}\,(d_1' + d_2')\left|h_{r,u}\right|^2\right]}{\sigma\sqrt{\left|h_{s,u}\right|^2 + \left|h_{r,u}\right|^2}}\right) \tag{A.22}$$

b) AF relaying Case

In the case of AF relaying, the received signal at the destination from the relay is given by

$$y_{r,u} = \alpha\, y_{s,r}\, h_{r,u} \ + \ n_{r,u} \tag{A.23}$$

$$y_{r,u} = \alpha\, h_{r,u}\, h_{s,r} \sqrt{E_s}\,(d_1 + d_2) + \alpha\, h_{r,u}\, n_{s,r} + n_{r,u} \tag{A.24}$$

where $h_{r,u}$ is the fading channel coefficient on the relay-destination link and $n_{r,u}$ is the AWGN coefficient at the destination. The two signals $y_{s,u}$ and $y_{r,u}$ are combined at the destination using Maximal Ratio Combining technique. The resultant signal Y is given by:

$$\begin{aligned} Y = \sqrt{E_s}\,(d_1 + d_2)\left|h_{s,u}\right|^2 + n_{s,u}\, h_{s,u}^* \\ + \sqrt{E_s}\,(d_1 + d_2)\alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2 \\ + \alpha\left|h_{r,u}\right|^2 h_{s,r}^*\, n_{s,r} + h_{r,u}^* h_{s,r}^*\, n_{r,u} \end{aligned} \tag{A.25}$$

Thus, from equations (A.1) and (A.25) the probability of error for the bit b_0 in the symbol "$0x0x$" using AF relaying can be written as follow:

$$\begin{aligned} Pe(b_0) = P\Big(\Big[\sqrt{E_s}\,(d_1 + d_2)\left|h_{s,u}\right|^2 + n_{s,u}\, h_{s,u}^* \\ + \sqrt{E_s}\,(d_1 + d_2)\alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2 + \alpha\left|h_{r,u}\right|^2 h_{s,r}^*\, n_{s,r} \\ + h_{r,u}^* h_{s,r}^*\, n_{r,u}\Big] < 0\Big) \end{aligned} \tag{A.26}$$

$$\begin{aligned} Pe(b_0) = P\Big(\Big[\sqrt{E_s}\,(d_1 + d_2)\left|h_{s,u}\right|^2 \\ + \sqrt{E_s}\,(d_1 + d_2)\alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\Big] \\ < n_{s,u}\, h_{s,u}^* + \alpha\left|h_{r,u}\right|^2 h_{s,r}^*\, n_{s,r} + h_{r,u}^* h_{s,r}^*\, n_{r,u}\Big) \end{aligned} \tag{A.27}$$

Let $w = n_{s,u}\, h_{s,u}^{*} + \alpha \left|h_{r,u}\right|^2 h_{s,r}^{*}\, n_{s,r} + h_{r,u}^{*} h_{s,r}^{*}\, n_{r,u}$,

where $w \sim \mathcal{N}\left(\mu_w = 0, \sigma_w^2 = \sigma^2\left(\left|h_{s,u}\right|^2 + \left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\left(\alpha^2\left|h_{r,u}\right|^2 + 1\right)\right)\right)$. Also, let $z = \left[\sqrt{E_s}\,(d_1 + d_2)\left|h_{s,u}\right|^2 + \sqrt{E_s}\,(d_1 + d_2)\alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right]$. Using these notations, and using equation (A.8), the instantaneous probability of bit error for common bits (base bits) given that the symbol "$0x0x$" was sent over Rayleigh fading channel using AF relaying can be expressed by:

$$Pe(b_0) = Q\left(\frac{\sqrt{E_s}\,(d_1 + d_2)\left(\left|h_{s,u}\right|^2 + \alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right)}{\sigma\sqrt{\left(\left|h_{s,u}\right|^2 + \left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\left(\alpha^2\left|h_{r,u}\right|^2 + 1\right)\right)}}\right) \qquad \text{(A.28)}$$

Due to symmetry, computing the probability of error of sending the symbol "$1x0x$" and the symbol "$1x1x$" will give the same probability of error of sending the symbol "$0x0x$" and the symbol "$0x1x$" respectively.

Thus, in the case of using DF relaying, the instantaneous probability of error for the common bits (base bits) using MRC in a Rayleigh fading environment can be computed using equation (A.9) and (A.22) yielding

$$Pe_{Base}^{DF} = 0.5\left\{Q\left(\frac{\left[\sqrt{E_s}\,(\,d_1-d_2)\left|h_{s,u}\right|^2+\sqrt{E_r}\,(d_1'-d_2')\left|h_{r,u}\right|^2\right]}{\sigma\sqrt{\left|h_{s,u}\right|^2+\left|h_{r,u}\right|^2}}\right) + Q\left(\frac{\left[\sqrt{E_s}\,(\,d_1+d_2)\left|h_{s,u}\right|^2+\sqrt{E_r}\,(d_1'+d_2')\left|h_{r,u}\right|^2\right]}{\sigma\sqrt{\left|h_{s,u}\right|^2+\left|h_{r,u}\right|^2}}\right)\right\} \tag{A.29}$$

Similarly, in the case of using AF relaying, the instantaneous probability of error for the common bits (base bits) using MRC in a Rayleigh fading environment can be computed using equation (A.15) and (A.28) yielding

$$Pe_{Base}^{AF} = 0.5\left\{Q\left(\frac{\sqrt{E_s}\,(d_1-d_2)\left(\left|h_{s,u}\right|^2+\alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right)}{\sigma\sqrt{\left(\left|h_{s,u}\right|^2+\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\left(\alpha^2\left|h_{r,u}\right|^2+1\right)\right)}}\right) + Q\left(\frac{\sqrt{E_s}\,(d_1+d_2)\left(\left|h_{s,u}\right|^2+\alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right)}{\sigma\sqrt{\left(\left|h_{s,u}\right|^2+\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\left(\alpha^2\left|h_{r,u}\right|^2+1\right)\right)}}\right)\right\} \tag{A.30}$$

A.1.2 Instantaneous BER of Refinement bits using MRC and DF relaying technique in a Rayleigh fading environment

As mentioned earlier, the refinement bits in a 4/16QAM hierarchical constellation are the last 2 bits of any 4 bits symbol. In Figure A.1, the refinement bits are marked by the while circles. They are denoted by (b_2, b_3). Unlike the base bits, refinement bits are not constant in one quadrant. In each quadrant, there exists four different refinement bits; $\{00, 01, 10, 11\}$. Each two adjacent symbol located in the same quadrant are separated by a distance $2d_2$. Like the case in the base bits, bit b_2 represents the in-phase refinement bit, while the bit b_3 represents the quadrature phase refinement bit.

Let's consider sending a '0' on the in-phase common bit b_2, the probability of getting an error, i.e. the reception of a 1, means that the resulting symbol from combining the two signals coming from the BS and from the relay lead to a reception of signal in the $(b_2 = 1)$ region. In other words, if the resultant received signal is denoted by Y the probability of error can be expressed as

$$P_e(b_2) = P\left(-d_1\tilde{h} < \mathbb{R}(Y) < d_1\tilde{h}\right) \tag{A.31}$$

where $\tilde{h}$ is a variable depending on the channel coefficients on the Base Station – destination link and the relay – destination link. In the case of the in-phase refinement bit b_2, only the real part of the signals is considered. In the case of computing the error probability for b_3 , only the imaginary part of the signal Y is considered. In this study, only the error probability computations of the in-phase enhancement (refinement) bit $'b_2'$ are derived. However, error probability computations for the quadrature phase enhancement bit are straight forward. In all the coming computations, all real part notations will be omitted for simplification (i.e. $y_{s,u} = \mathbb{R}(y_{s.u})$).

Let's assume that the symbol"$0x0x$" is sent, which in the 4/16 QAM constellation in Figure A.1 is situated at a distance $(d_1 + d_2)$ from the y axis. The received signals at the destination and the relay from the Base Station can be written respectively as

$$y_{s,u} = \sqrt{E_s}\,(d_1 + d_2)h_{s,u} \ + \ n_{s,u} \tag{A.32}$$

$$y_{s,r} = \sqrt{E_s}\,(d_1 + d_2)h_{s,r} \ + \ n_{s,r} \tag{A.33}$$

E_s is the transmitted symbol energy at the Base Station, $h_{s,u}$ and $h_{s,r}$ are the fading channel coefficients on the source-destination link and the source-relay link respectively. $n_{s,u}$ and $n_{s,r}$ are the AWGN coefficient at the destination and the relay respectively. $d_1,\ d_2$ denote the selected distance parameters in the first time slot.

a) DF Relaying Case

If the CRC check results in successful decoding, the relay re-modulates its signal with new constellation's distance parameters and re-sends its signal. At the destination, the received signal at the destination from the relay is expressed as

$$y_{r,u} = \sqrt{E_r}\,(d_1' + d_2')h_{r,u} \ + \ n_{r,u} \tag{A.34}$$

E_s is the transmitted symbol energy at the Base Station, $h_{r,u}$ is the fading channel coefficient on the relay-destination link and $n_{r,u}$ is the AWGN coefficient at the destination. $d_1{}',\ d_2{}'$ denote the selected distance parameters in the second time slot.

The two signals $y_{s,u}$ and $y_{r,u}$ are combined at the destination using Maximal Ratio Combining technique. In this technique, each signal is weighted by its channel coefficient and the two signals are added together. The resultant signal Y is given by:

$$Y = \sqrt{E_s}\,(d_1 + d_2)\left|h_{s,u}\right|^2 + n_{s,u}\,h_{s,u}^* + \sqrt{E_r}\,(d_1' + d_2')\left|h_{r,u}\right|^2 + n_{r,u}\,h_{r,u}^* \quad \text{(A.35)}$$

Thus, from equations (A.31) and (A.35) we can write the probability of error for the bit b_2 in the symbol "$0x0x$" as follow:

$$Pe(b_2) = P\Big(-\left[\sqrt{E_s}\,d_1\left|h_{s,u}\right|^2 + \sqrt{E_r}\,d_1'\left|h_{r,u}\right|^2\right] < \left[\sqrt{E_s}\,(d_1 + d_2)\left|h_{s,u}\right|^2 + n_{s,u}\,h_{s,u}^* + \sqrt{E_r}\,(d_1' + d_2')\left|h_{r,u}\right|^2 + n_{r,u}\,h_{r,u}^*\right] < \left[\sqrt{E_s}\,d_1\left|h_{s,u}\right|^2 + \sqrt{E_r}\,d_1'\left|h_{r,u}\right|^2\right]\Big) \quad \text{(A.36)}$$

$$e(b_2) = P\Big(-\left[\sqrt{E_s}\,(2d_1 + d_2)\left|h_{s,u}\right|^2 + \sqrt{E_r}\,(2d_1' + d_2')\left|h_{r,u}\right|^2\right] < \left[n_{s,u}\,h_{s,u}^* + n_{r,u}\,h_{r,u}^*\right] < -\left[\sqrt{E_s}\,d_2\left|h_{s,u}\right|^2 + \sqrt{E_r}\,d_2'\left|h_{r,u}\right|^2\right]\Big) \quad \text{(A.37)}$$

Let's denote $w = n_{s,u}\,h_{s,u}^* + n_{r,u}\,h_{r,u}^*$, where $w \sim \mathcal{N}\left(\mu_w = 0, \sigma_w^2 = \sigma^2\left(\left|h_{s,u}\right|^2 + \left|h_{r,u}\right|^2\right)\right)$. Also, let $z_1 = -\left[\sqrt{E_s}\,(2d_1 + d_2)\left|h_{s,u}\right|^2 + \sqrt{E_r}\,(2d_1' + d_2')\left|h_{r,u}\right|^2\right]$ and $z_2 = -\left[\sqrt{E_s}\,d_2\left|h_{s,u}\right|^2 + \sqrt{E_r}\,d_2'\left|h_{r,u}\right|^2\right]$. Using these notations and using equation (A.37)

$$Pe(b_2) = P(z_1 < w < z_2) \tag{A.38}$$

which can be solved using

$$\begin{aligned} Pe(b_2) &= \int_{z_1}^{z_2} \frac{1}{\sqrt{2\pi}\sigma_w} e^{-\frac{1}{2}\left(\frac{x-\mu_w}{\sigma_w}\right)^2} dx \\ &= \int_{-z_2}^{-z_1} \frac{1}{\sqrt{2\pi}\sigma_w} e^{-\frac{1}{2}\left(\frac{x-\mu_w}{\sigma_w}\right)^2} dx \end{aligned} \tag{A.39}$$

$$Pe(b_2) = Q\left(\frac{-z_2}{\sigma_w}\right) - Q\left(\frac{-z_1}{\sigma_w}\right) \tag{A.40}$$

Thus the instantaneous probability of bit error for enhancement bits (refinement bits) given that the symbol "$0x0x$" was sent over Rayleigh fading channel can be expressed by:

$$\begin{aligned} Pe(b_2) = {} & Q\left(\frac{\sqrt{E_s}\, d_2 |h_{s,u}|^2 + \sqrt{E_r}\, d_2' |h_{r,u}|^2}{\sigma\sqrt{\left(|h_{s,u}|^2 + |h_{r,u}|^2\right)}}\right) \\ & - Q\left(\frac{\sqrt{E_s}\,(2d_1 + d_2)|h_{s,u}|^2 + \sqrt{E_r}\,(2d_1' + d_2')|h_{r,u}|^2}{\sigma\sqrt{\left(|h_{s,u}|^2 + |h_{r,u}|^2\right)}}\right) \end{aligned} \tag{A.41}$$

b) AF Relaying Case

In the case of relaying the signal using Amplify and forward, the relay amplifies the received signal $y_{s,r}$ by a gain factor α, where $\alpha = \sqrt{\frac{E_r}{E_s|h_{s,r}|^2 + N_0}}$ and E_r is the transmitting symbol energy at the relay, E_s is the transmitting symbol energy at the Base Station and

N_0 is the noise power spectral density. After the relay amplifies the signal and re-sends it, the received signal at the destination from the relay is given by

$$y_{r,u} = \alpha\, y_{s,r}\, h_{r,u} \;+\; n_{r,u} \tag{A.42}$$

$$\begin{aligned} y_{r,u} = \alpha\, h_{r,u}\; h_{s,r}\, \sqrt{E_s}\,(d_1 + d_2) + \;\alpha\, h_{r,u}\; n_{s,r} \\ +\; n_{r,u} \end{aligned} \tag{A.43}$$

where $h_{r,u}$ is the fading channel coefficient on the relay-destination link and $n_{r,u}$ is the AWGN coefficient at the destination. The two signals $y_{s,u}$ and $y_{r,u}$ are combined at the destination using MRC technique. The resultant signal Y is given by:

$$\begin{aligned} Y = \sqrt{E_s}\,(d_1 + d_2)\left|h_{s,u}\right|^2 + n_{s,u}\, h_{s,u}^* \\ + \sqrt{E_s}\,(d_1 + d_2)\alpha\,\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2 \\ + \alpha\,\left|h_{r,u}\right|^2 h_{s,r}^*\, n_{s,r} \;+ h_{r,u}^* h_{s,r}^*\, n_{r,u} \end{aligned} \tag{A.44}$$

Thus, from equations (A.31) and (A.44) the probability of error for the bit b_2 in the symbol "$0x0x$" using AF relaying can be written as follow:

$$\begin{aligned} Pe(b_2) = P\Big(-\Big[\sqrt{E_s}\, d_1\left|h_{s,u}\right|^2 + \sqrt{E_s}\, d_1\alpha\,\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\Big] \\ < \Big[\sqrt{E_s}\,(d_1 + d_2)\left|h_{s,u}\right|^2 + n_{s,u}\, h_{s,u}^* \\ + \sqrt{E_s}\,(d_1 + d_2)\alpha\,\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2 + \alpha\,\left|h_{r,u}\right|^2 h_{s,r}^*\, n_{s,r} \\ + h_{r,u}^* h_{s,r}^*\, n_{r,u}\Big] \\ < \Big[\sqrt{E_s}\, d_1\left|h_{s,u}\right|^2 + \sqrt{E_s}\, d_1\alpha\,\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\Big]\Big) \end{aligned} \tag{A.45}$$

$$Pe(b_2) = P\Big(-\left[\sqrt{E_s}\,(2d_1 + d_2)\left|h_{s,u}\right|^2 + \sqrt{E_s}\,(2d_1 + d_2)\alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right] < \left[n_{s,u}\,h_{s,u}^* + \alpha\left|h_{r,u}\right|^2 h_{s,r}^*\,n_{s,r} + h_{r,u}^* h_{s,r}^*\,n_{r,u}\right] < -\left[\sqrt{E_s}\,d_2\left|h_{s,u}\right|^2 + \sqrt{E_s}\,d_2\alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right]\Big) \tag{A.46}$$

Let’s denote $w = n_{s,u}\,h_{s,u}^* + \alpha\left|h_{r,u}\right|^2 h_{s,r}^*\,n_{s,r} + h_{r,u}^* h_{s,r}^*\,n_{r,u}$,

where $w \sim \mathcal{N}\left(\mu_w = 0, \sigma_w^2 = \sigma^2\left(\left|h_{s,u}\right|^2 + \alpha^2\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\left(\alpha^2\left|h_{r,u}\right|^2 + 1\right)\right)\right)$.

Also, let $z_1 = -\left[\sqrt{E_s}\,(2d_1 + d_2)\left\{\left|h_{s,u}\right|^2 + \alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right\}\right]$ and $z_2 = -\left[\sqrt{E_s}\,d_2\left\{\left|h_{s,u}\right|^2 + \alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right\}\right]$. Using these notations, and using equation (A.40), the instantaneous probability of bit error for enhancement bits (refinement bits) given that the symbol "$0x0x$" was sent over Rayleigh fading channel using AF relaying can be expressed by:

$$Pe(b_2) = Q\left(\frac{\sqrt{E_s}\,d_2\left\{\left|h_{s,u}\right|^2 + \alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right\}}{\sigma\sqrt{\left(\left|h_{s,u}\right|^2 + \left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\left(\alpha^2\left|h_{r,u}\right|^2 + 1\right)\right)}}\right) - Q\left(\frac{\sqrt{E_s}\,(2d_1 + d_2)\left\{\left|h_{s,u}\right|^2 + \alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right\}}{\sigma\sqrt{\left(\left|h_{s,u}\right|^2 + \left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\left(\alpha^2\left|h_{r,u}\right|^2 + 1\right)\right)}}\right) \tag{A.47}$$

In the previous example, it is assumed that the Base Station wants to send the symbol "$0x0x$", where $b_2 = 0$. Now let’s assume that the Base Station is sending the symbol "$0x1x$", where $b_2 = 1$, which is situated at a distance $(d_1 - d_2)$ from the y axis.

The probability of getting an error in such a case, i.e. the reception of a 0, means that the resulting symbol from combining the two signals coming from the BS and from the relay leads to a reception of signal in the $(b_2 = 0)$ region. In other words, if the resultant received signal is denoted by Y the probability of error can be expressed as

$$P_e(b_2) = P\left(-d_1\tilde{h} > \mathbb{R}(Y)\right) + P\left(\mathbb{R}(Y) > d_1\tilde{h}\right) \tag{A.48}$$

The received signals from the Base Station in the first time slot at the destination and at the relay respectively are

$$y_{s,u} = \sqrt{E_s}(d_1 - d_2)h_{s,u} + n_{s,u} \tag{A.49}$$

$$y_{s,r} = \sqrt{E_s}(d_1 - d_2)h_{s,r} + n_{s,r} \tag{A.50}$$

where d_1 and d_2 denote the selected distance parameters in the first time slot.

a) DF Relaying Case

If the CRC check results in successful decoding, the relay re-modulates its signal with new constellation's distance parameters and re-sends its signal. At the destination, the received signal from the relay is expressed as

$$y_{r,u} = \sqrt{E_r}(d_1' - d_2')h_{r,u} + n_{r,u} \tag{A.51}$$

$h_{r,u}$ is the fading channel coefficient on the relay-destination link and $n_{r,u}$ is the AWGN coefficient at the destination. ${d_1}'$, ${d_2}'$ denote the selected distance parameters in the second time slot.

The two signals $y_{s,u}$ and $y_{r,u}$ are combined at the destination using MRC technique. The resultant signal Y is given by:

$$Y = \sqrt{E_s}(\,d_1 - d_2)\left|h_{s,u}\right|^2 + \,n_{s,u}\,h_{s,u}^* + \sqrt{E_r}(d_1' - d_2')\left|h_{r,u}\right|^2 + \,n_{r,u}\,h_{r,u}^* \tag{A.52}$$

Thus, from equations (A.48) and (A.52) the probability of error for the bit b_2 in the symbol "$0x1x$" can be written as follow:

$$\begin{aligned} Pe(b_2) = P\Big(&-\Big[\sqrt{E_s}d_1\left|h_{s,u}\right|^2 + \sqrt{E_r}d_1'\left|h_{r,u}\right|^2\Big] \\ &> \Big[\sqrt{E_s}(d_1 - d_2)\left|h_{s,u}\right|^2 + \,n_{s,u}\,h_{s,u}^* + \sqrt{E_r}(d_1' \\ &- d_2')\left|h_{r,u}\right|^2 + \,n_{r,u}\,h_{r,u}^*\Big]\Big) \\ &+ P\Big(\Big[\sqrt{E_s}d_1\left|h_{s,u}\right|^2 + \sqrt{E_r}d_1'\left|h_{r,u}\right|^2\Big] \\ &< \Big[\sqrt{E_s}(d_1 - d_2)\left|h_{s,u}\right|^2 + \,n_{s,u}\,h_{s,u}^* + \sqrt{E_s}(d_1' \\ &- d_2')\left|h_{r,u}\right|^2 + \,n_{r,u}\,h_{r,u}^*\Big]\Big) \end{aligned} \tag{A.53}$$

$$\begin{aligned} Pe(b_2) = P\Big(&\Big[\sqrt{E_s}(2d_1 - d_2)\left|h_{s,u}\right|^2 + \sqrt{E_r}(2d_1' - d_2')\left|h_{r,u}\right|^2\Big] \\ &< \left[n_{s,u}\,h_{s,u}^* + n_{r,u}\,h_{r,u}^*\right]\Big) \\ &+ P\Big(\Big[\sqrt{E_s}d_2\left|h_{s,u}\right|^2 + \sqrt{E_r}d_2'\left|h_{r,u}\right|^2\Big] < \left[\,n_{s,u}\,h_{s,u}^* + n_{r,u}\,h_{r,u}^*\right]\Big) \end{aligned} \tag{A.54}$$

Let's denote $w = n_{s,u}\, h_{s,u}^{*} + n_{r,u}\, h_{r,u}^{*}$, where $w \sim \mathcal{N}\left(\mu_w = 0, \sigma_w^2 = \sigma^2\left(\left|h_{s,u}\right|^2 + \left|h_{r,u}\right|^2\right)\right)$. Also, let $z_1 = \left[\sqrt{E_s}(2d_1 - d_2)\left|h_{s,u}\right|^2 + \sqrt{E_r}(2d_1' - d_2')\left|h_{r,u}\right|^2\right]$ and $z_2 = \left[\sqrt{E_s}d_2\left|h_{s,u}\right|^2 + \sqrt{E_r}d_2'\left|h_{r,u}\right|^2\right]$. Using these notations and using equation (A.54)

$$Pe(b_2) = P(z_1 < w) + P(z_2 < w)$$

$$Pe(b_2) = Q\left(\frac{z_1}{\sigma_w}\right) + Q\left(\frac{z_2}{\sigma_w}\right) \tag{A.55}$$

Thus the instantaneous probability of bit error for enhancement bits (refinement bits) given that the symbol "$0x1x$" was sent over Rayleigh fading channel can be expressed by:

$$\begin{aligned} Pe(b_2) = {} & Q\left(\frac{\sqrt{E_s}(2d_1 - d_2)\left|h_{s,u}\right|^2 + \sqrt{E_r}(2d_1' - d_2')\left|h_{r,u}\right|^2}{\sigma\sqrt{\left(\left|h_{s,u}\right|^2 + \left|h_{r,u}\right|^2\right)}}\right) \\ & + Q\left(\frac{\sqrt{E_s}d_2\left|h_{s,u}\right|^2 + \sqrt{E_r}d_2'\left|h_{r,u}\right|^2}{\sigma\sqrt{\left(\left|h_{s,u}\right|^2 + \left|h_{r,u}\right|^2\right)}}\right) \end{aligned} \tag{A.56}$$

b) AF Relaying Case

After the relay amplifies the signal and re-sends it, the received signal at the destination from the relay is given by

$$y_{r,u} = \alpha\, y_{s,r}\, h_{r,u} + n_{r,u} \tag{A.57}$$

$$y_{r,u} = \alpha\, h_{r,u}\, h_{s,r} \sqrt{E_s}(d_1 - d_2) + \alpha\, h_{r,u}\, n_{s,r} + n_{r,u} \tag{A.58}$$

where $h_{r,u}$ is the fading channel coefficient on the relay-destination link and $n_{r,u}$ is the AWGN coefficient at the destination. The two signals $y_{s,u}$ and $y_{r,u}$ are combined at the destination using Maximal Ratio Combining technique. The resultant signal Y is given by:

$$\begin{aligned} Y = {} & \sqrt{E_s}(d_1 - d_2)\left|h_{s,u}\right|^2 + n_{s,u}\, h_{s,u}^* \\ & + \sqrt{E_s}(d_1 - d_2)\alpha \left|h_{r,u}\right|^2 \left|h_{s,r}\right|^2 \\ & + \alpha \left|h_{r,u}\right|^2 h_{s,r}^*\, n_{s,r} + h_{r,u}^* h_{s,r}^*\, n_{r,u} \end{aligned} \tag{A.59}$$

Thus, from equations (A.48) and (A.59) the probability of error for the bit b_2 in the symbol "$0x1x$" using AF relaying can be written as follow:

$$Pe(b_2) = P\Big(-\Big[\sqrt{E_s}d_1|h_{s,u}|^2 + \sqrt{E_s}d_1\alpha\,|h_{r,u}\,|^2|h_{s,r}\,|^2\Big]$$
$$> \Big[\sqrt{E_s}(d_1 - d_2)|h_{s,u}|^2 + n_{s,u}\,h_{s,u}^*$$
$$+ \sqrt{E_s}(d_1 - d_2)\alpha\,|h_{r,u}\,|^2|h_{s,r}\,|^2 + \alpha\,|h_{r,u}\,|^2 h_{s,r}^*\,n_{s,r}$$
$$+ h_{r,u}^* h_{s,r}^*\,n_{r,u}\Big]\Big)$$
$$+ P\Big(\Big[\sqrt{E_s}d_1|h_{s,u}|^2 + \sqrt{E_s}d_1\alpha\,|h_{r,u}\,|^2|h_{s,r}\,|^2\Big] \tag{A.60}$$
$$< \Big[\sqrt{E_s}(d_1 - d_2)|h_{s,u}|^2 + n_{s,u}\,h_{s,u}^*$$
$$+ \sqrt{E_s}(d_1 - d_2)\alpha\,|h_{r,u}\,|^2|h_{s,r}\,|^2 + \alpha\,|h_{r,u}\,|^2 h_{s,r}^*\,n_{s,r}$$
$$+ h_{r,u}^* h_{s,r}^*\,n_{r,u}\Big]\Big)$$

$$Pe(b_2) = P\Big(\Big[\sqrt{E_s}(2d_1 - d_2)|h_{s,u}|^2 + \sqrt{E_s}(2d_1$$
$$- d_2)\alpha\,|h_{r,u}\,|^2|h_{s,r}\,|^2\Big]$$
$$< \Big[n_{s,u}\,h_{s,u}^* + \alpha\,|h_{r,u}\,|^2 h_{s,r}^*\,n_{s,r} + h_{r,u}^* h_{s,r}^*\,n_{r,u}\Big]\Big) \tag{A.61}$$
$$+ P\Big(\Big[\sqrt{E_s}d_2|h_{s,u}|^2 + \sqrt{E_s}d_2\alpha\,|h_{r,u}\,|^2|h_{s,r}\,|^2\Big]$$
$$< \Big[n_{s,u}\,h_{s,u}^* + +\alpha\,|h_{r,u}\,|^2 h_{s,r}^*\,n_{s,r} + h_{r,u}^* h_{s,r}^*\,n_{r,u}\Big]\Big)$$

Let's denote $w = n_{s,u}\,h_{s,u}^* + \alpha\,|h_{r,u}\,|^2 h_{s,r}^*\,n_{s,r} + h_{r,u}^* h_{s,r}^*\,n_{r,u}$,

where $w \sim \mathcal{N}\left(\mu_w = 0, \sigma_w^2 = \sigma^2\left(|h_{s,u}|^2 + |h_{r,u}\,|^2|h_{s,r}\,|^2\left(\alpha^2\,|h_{r,u}\,|^2 + 1\right)\right)\right)$. Also, let $z_1 = \left[\sqrt{E_s}(2d_1 - d_2)\left\{|h_{s,u}|^2 + \alpha\,|h_{r,u}\,|^2|h_{s,r}\,|^2\right\}\right]$ and $z_2 = \left[\sqrt{E_s}d_2\left\{|h_{s,u}|^2 + \alpha\,|h_{r,u}\,|^2|h_{s,r}\,|^2\right\}\right]$. Using these notations, and using equation(A.55), the instantaneous probability of bit error for enhancement bits (refinement bits) given that the symbol "$0x1x$" was sent over Rayleigh fading channel using AF relaying can be expressed by:

$$Pe(b_2) = Q\left(\frac{\sqrt{E_s}d_2\left\{\left|h_{s,u}\right|^2 + \alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right\}}{\sigma\sqrt{\left(\left|h_{s,u}\right|^2 + \left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\left(\alpha^2\left|h_{r,u}\right|^2 + 1\right)\right)}}\right) + Q\left(\frac{\sqrt{E_s}(2d_1 - d_2)\left\{\left|h_{s,u}\right|^2 + \alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right\}}{\sigma\sqrt{\left(\left|h_{s,u}\right|^2 + \left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\left(\alpha^2\left|h_{r,u}\right|^2 + 1\right)\right)}}\right) \tag{A.62}$$

Due to symmetry, computing the probability of error of sending the symbol "$1x0x$" and the symbol "$1x1x$" will give the same probability of error of sending the symbol "$0x0x$" and the symbol "$0x1x$" respectively.

Thus, in the case of employing DF relaying, the instantaneous probability of error for the enhancement bits (refinement bits) using MRC in a Rayleigh fading environment can be computed using equation (A.41) and (A.56) yielding

$$Pe_{REF}^{DF} = 0.5\left\{2Q\left(\frac{\sqrt{E_s}d_2\left|h_{s,u}\right|^2+\sqrt{E_r}d_2'\left|h_{r,u}\right|^2}{\sigma\sqrt{\left(\left|h_{s,u}\right|^2+\left|h_{r,u}\right|^2\right)}}\right) - Q\left(\frac{\sqrt{E_s}(2d_1+d_2)\left|h_{s,u}\right|^2+\sqrt{E_r}(2d_1'+d_2')\left|h_{r,u}\right|^2}{\sigma\sqrt{\left(\left|h_{s,u}\right|^2+\left|h_{r,u}\right|^2\right)}}\right) + Q\left(\frac{\sqrt{E_s}(2d_1-d_2)\left|h_{s,u}\right|^2+\sqrt{E_r}(2d_1'-d_2')\left|h_{r,u}\right|^2}{\sigma\sqrt{\left(\left|h_{s,u}\right|^2+\left|h_{r,u}\right|^2\right)}}\right)\right\} \quad \text{(A.63)}$$

Similarly, in the case of employing AF relaying, the instantaneous probability of error for the enhancement bits (refinement bits) using MRC in a Rayleigh fading environment can be computed using equation (A.47) and (A.62) yielding

$$Pe_{REF}^{AF} = 0.5\left\{2Q\left(\frac{\sqrt{E_s}d_2\left(\left|h_{s,u}\right|^2+\alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right)}{\sigma\sqrt{\left(\left|h_{s,u}\right|^2+\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\left(\alpha^2\left|h_{r,u}\right|^2+1\right)\right)}}\right) + Q\left(\frac{\sqrt{E_s}(2d_1-d_2)\left(\left|h_{s,u}\right|^2+\alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right)}{\sigma\sqrt{\left(\left|h_{s,u}\right|^2+\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\left(\alpha^2\left|h_{r,u}\right|^2+1\right)\right)}}\right) - Q\left(\frac{\sqrt{E_s}(2d_1+d_2)\left(\left|h_{s,u}\right|^2+\alpha\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\right)}{\sigma\sqrt{\left(\left|h_{s,u}\right|^2+\left|h_{r,u}\right|^2\left|h_{s,r}\right|^2\left(\alpha^2\left|h_{r,u}\right|^2+1\right)\right)}}\right)\right\} \quad \text{(A.64)}$$

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