Effects of a Connections Approach on Preservice Teachers' Conceptual Understanding of the Bar Diagram Symbol

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ABSTRACT

Effects of a Connections Approach on Preservice Teachers'

Conceptual Understanding of the Bar Diagram Symbol

Danielle Houstoun

Teachers of mathematics often use diagrams to explain concepts related to quantity. Students of mathematics often have difficulty, however, understanding how the diagrams represent the intended concepts (Uttal, Liu, & DeLoache, 2006). This is consistent with the research on students' difficulties with mathematical symbols and notation (Hiebert, 1992), and previous studies have demonstrated that teachers need to make the connections between symbols and their conceptual referents explicit (Osana & Pitsolantis, 2013). This study examined the impact of instruction that explicitly teaches preservice teachers the conceptual meaning of a mathematical symbol called the "bar diagram." Fifty undergraduate students (N = 50) were assigned to one of three conditions: Bar Diagram with Links (BDL), Bar Diagram with No Links (BDNL), or Comparison. The students in the BDL condition were explicitly shown, with the use of concrete materials, that Bar Diagrams are mathematical symbols used to represent quantity and relations among quantities. The BDNL condition was exposed to the Bar Diagram symbol without any explicit connection to referents. Students in the third condition served as a comparison group. Objectives of this study were to determine whether it is critical to make explicit connections to the conceptual meaning of a mathematical symbol prior to appropriately applying the symbol to (a) solve word

problems presented using text in a story format, (b) view the bar diagrams as pictures that represent quantities for the purpose of solving problems, (c) understand the quantitative meaning of bar diagrams, and (d) use algebraic symbols to solve analogous problems.

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Chapter 1: Statement of the Problem

The current mathematics reform emphasizes teaching mathematics by encouraging students to solve problems by applying meaningful and flexible strategies (National Council of Teachers of Mathematics, 2000). The Quebec Education program (QEP) also emphasizes the importance of problem solving in mathematics with a concentration on word problems (MELS, 2001). Word problems present students with a situational context from which they must extract the necessary information to find a solution. The QEP thus requires teachers to provide instruction to students that focuses on interpreting information in word problems and assisting students to solve problems in flexible and diverse ways (MELS, 2001). Research demonstrates that students often struggle with word problems (Bernardo, 1999; Schleppegrell, 2007; Van Garderen, 2004), however. Therefore, teachers need to provide instruction that is effective in helping them overcome their difficulties.

To this end, teachers are required to possess the necessary knowledge and skills to provide problem solving instruction that meets the learning needs of their students (Ball et al., 2008). Research demonstrates that teachers are quite capable of solving word problems on their own (Van Dooren, Verschaffel, & Onghena, 2003), but are not flexible in the strategies they use and have a difficult time providing conceptual explanations for their solutions (Ball, 1990a; Ball, 1990b). That teachers have difficulties explaining the conceptual nature of mathematical processes is problematic because research shows that students benefit from mathematics instruction that clearly and explicitly explains the meaning behind the conventions (Li, Ding, Caprarpo, & Caprarpo, 2008; Osana & Pitsolantis, 2013; Osana, Przednowek, Cooperman, & Adrien, 2013). Therefore, when teachers do not have in depth knowledge of mathematical concepts nor diverse and flexible strategies for their own problem solving, they tend to not provide instruction that is most favorable for their students. A large body of research shows that preservice teachers, appear to struggle with the conceptual nature of mathematics (Ball, Lubienski, & Mewborn, 2001), which suggests that teacher training programs need to focus on teaching conceptual understanding of problem solving so that they will be able to meet their students' learning needs.

Research demonstrates that diagrams are effective tools for problem solving. Students' self-produced diagrams support them in decoding information in word problems and constructing reasonable and accurate solutions (Koedinger & Terao, 2002; Lewis, 1989; Uesaka, Manalo, & Ichikawa, 2007), but are only effective when students can construct them on their own and understand what they mean (Koedinger & Terao, 2002; Uesaka et al., 2007). Some mathematics curricula emphasize the use of diagrams to visually represent the information presented in word problems. One type of diagram is known as the "bar diagram" and while it has been shown to be an effective problem solving tool for middle school students (Koedinger & Terao, 2002), it has not been adapted in all mathematics classrooms (Singapore Ministry, 1999). Teachers themselves do not exhibit less flexibility when solving word problems and use algebra rather than pictorial representations (Van Dooren et al., 2003). Informal observation suggests that preservice teachers have difficulty understanding the conceptual underpinnings of pictorial representations (H.P. Osana, personal communication, November 7, 2013). This evidence suggests that teachers' difficulties understanding the meaning of the bar diagram will create challenges for them in the classroom because they will be at a disadvantage when providing representations in their lessons.

Ball, Thames, and Phelps (2008) argued that teachers need to understand mathematical representations to provide meaningful instruction to their students. Thus, teacher education programs must focus on exposing preservice teachers to the most effective representational tools for problem solving and providing instruction that makes these representations meaningful. Although the research is not clear on how to assist preservice teachers to understand the conceptual referents of mathematical representations, research conducted on elementary and secondary students suggests that instruction needs to include explicit and meaningful connections between mathematical representations and the concepts they stand for (Li et al., 2008; Osana et al., 2013; Uttal Liu, & DeLoache, 1997). The purpose of this study is to test the impact of explicit explanations of the conceptual representations of bar diagrams with preservice teachers.

The results of this study with have several practical implications. The results will contribute to the literature on the optimal approaches to teaching the meaning of pictorial representations in the context of problem solving. More generally, the results will provide insight on the nature of symbolic understanding in mathematics, particularly in the preservice teacher population. Additionally, the findings will provide practical guidelines on ways to foster teacher understanding and knowledge of problem solving tools in teacher education programs. If the preservice teachers in this study gain conceptual understanding of the bar diagram, they will be better equipped to teach this useful problem solving tool to their students in a meaningful and effective way.

Chapter 2: Literature Review

Symbolic Understanding

Symbols are present everywhere and are essential in many aspects of life. The use of symbols allows for communication and mental manipulation of abstract concepts (Uttal et al., 1997). Symbols have been broadly defined as “something that someone intends to stand for or represent something other than itself” (DeLoache, 2002, p.73). Other scholars have made the connection between symbols and the mental connects that they are made to represent and this connection needs to be distinguished from the physical objects themselves (Potter, 1979). Thus, symbols convey messages that are beyond the superficial features of the physical representations that one sees or touches. To understand the abstract underpinnings of symbols, the interpreter must have a an understanding of the concepts the symbols represent. Otherwise said symbolic understanding occurs when an individual associates the conceptual referent of a symbol with the symbolic representation.

Piaget (1951; as cited in Vasta, Miller, Ellis, Younger, & Gosselin, 2006) explored children’s development of symbolic understanding. Piaget believed young children initially view the world concretely, being unable to think about objects when they are not present or as standing for something else. He claimed that prior to two years of age, children are unable to cognitively understand the abstract nature of concrete objects. More recent research further describes children’s symbolic development. Children under the age of 16-months interact with pictures in picture books as if they are their concrete counterparts, but not symbols for them (DeLoache, Pierroutsakos, Uttal, Rosengren, & Gottlieb, 1998; Murphy, 1978; Perner, 1991), and at 24-months, they are unable to see the symbolic function of pictures as representations of concrete objects (DeLoache & Burns, 1994). Later in development, around three and a half years of age, children begin to show representational skills through their participation in symbolic play, where children use objects to "stand in" as symbols for alternative objects during pretend play (Elder & Pederson, 1978), and start to view pictures and models as representations of their concrete counterparts (DeLoache, 1989; DeLoache & Burns, 1994).

The research discussed above illustrates that very young children do not have the capacity to view symbols both conceptually and concretely at the same time. Research demonstrates that later in development, children experience a cognitive shift that assists them in understanding the abstract nature of symbols. This cognitive shift has been explained by the capacity for "dual representation," which is defined as the understanding that a symbol is both an object in and of itself and a representation of something else (DeLoache, 2000).

Research conducted by DeLoache (1989) demonstrates the developmental differences between young children and their capacity to make the connections between representations and their conceptual referents. The author investigated young children's understanding of a scale model as a representation of a larger room. The author examined two and a half and three year old children's capacity to make the connection between the model of the room and the actual room. Children were shown the model and explicitly told that the scale was a representation of the larger room. Next, children watched as the experimenter hid an object in the scale model. After witnessing this, children were asked to find the hidden object in the larger room. The younger children were unable to find the object in the larger room, but when asked to locate in the scale model, they could do so.

This study illustrates that the younger children were unable to see the scale model as a representation of the larger room, thus failing to make the connection between them.

DeLoache, Miller, and Rosengren (1997) further support the notion that young children do not detect what may seem to be obvious symbol-referent relations. The authors examined two and a half year old children's capacity for dual representation by exposing them to either "symbolic" or "nonsymbolic" conditions. In both conditions, children were shown an object and were asked to locate it after it was hidden. Children in the symbolic condition were shown an object in a scale model and were then asked to find it hidden in a larger room (i.e. DeLoache, 1989). In the nonsymbolic condition, children were presented with a hidden object in a larger room and then shown a machine that could "shrink" toys. Children watched as the experimenters "shrunk" the room and presented a model sized version of the room, making the children believe that the model room was the actual room, only made smaller. Children were then asked to locate the hidden object in the "shrunken" room. Two and a half year old children were much better at finding the hidden object in the nonsymbolic condition because they were not required to consider the model as a representation of the larger room. Thus, this study further shows that dual representation is required for young children to understand the symbolic meaning of the model.

These studies illustrate that children do not possess dual representation at a young age, which interferes with their ability to make connections between symbols and their conceptual referents. Around three years of age, children start to see the abstract underpinnings of objects and can manipulate them both mentally and physically.

Symbols in Mathematics

Mathematics has been described as a type of language that relies on the ability to fluently disconnect symbols from their referents (Pimm, 1987). Broadly speaking, mathematical symbols are elements of a representational system that requires the student to make meaning of primitive visual characteristics that at first do not hold any meaning or interpretation, and to follow rules for combining these characteristics to use them appropriately (Goldin, 1998). Symbols presented in mathematics classrooms are diverse and vary in their visual characteristics. Some examples of representational systems in mathematics include numeration systems, arithmetic algorithms, rational numbers, and algebraic notations (Goldin, 1998; Sherman & Bisanz, 2009). Less conventional mathematical representations include manipulatives (Uttal et al., 2007) and diagrams (Koedinger & Terao, 2002; Uesaka et al., 2007). Although these symbols differ greatly in their visual characteristics, they all broadly encompass a representational system that requires the interpreter to make meaning of them as mathematical symbols.

Mathematics instruction places a great deal of emphasis on the use of mathematical symbols. Because almost everything in mathematics includes symbols, it is essential that students know how to use them with understanding to solve mathematical problems (Hiebert, 1992). Difficulties with symbolic understanding and corresponding concepts have been shown to hinder students' mathematical performance (Koedinger & Nathan, 2004; Sherman & Bisanz, 2009). Therefore, understanding that mathematical symbols are dual in nature does not come automatically.

Research demonstrates that symbolic comprehension of different symbols is a critical component of student success at various points throughout their mathematics education. Sherman and Bisanz (2009), for example, found that second grade children have difficulties solving mathematical problems that involve the equal sign. When students are presented mathematically equivalent problems with manipulatives and no written notation, they perform significantly better than when algebraic symbols are present. Other research shows that older students also demonstrate increased difficulties on mathematical problems when symbols are present. Koedinger and Nathan (2004) compared high school students' performance on problems with and without the presence of mathematical symbols. Specifically, students were presented with word problems, algebraic equations, and word equation problems[1]. Problem difficulty was determined by the location of the unknown value: easy algebra problems were classified as Result-Unknown, whereas more difficult problems were Start-Unknown problems. Results revealed that students performed better on story problems and word equations than on their algebraic symbolic counterparts. In sum, while students may understand many mathematical principles and concepts, the presence of symbols interferes with their ability to interpret the principles being presented, which in turn negatively influences their problem solving abilities.

It has been suggested that students' difficulties with mathematical symbols are the result of their inability to make appropriate connections between the symbols and their meanings (Hiebert & Lefevre, 1986). Some researchers have suggested that dual representation is necessary to understand the conceptual referents of mathematical symbols (Uttal, O'Doherty, Newland, Liu Hand, & DeLoache, 2009). Similar to children's dual representation for scale models, dual representation of mathematical

[1] Koedinger and Nathan (2004) defined word equations as algebraic problems presented in text format rather than algebraic symbols.

symbols requires the understanding of both the physical and representational nature of these symbols. Research suggests that the dual representation of mathematical symbols is not automatic, but requires teachers to make explicit links between the symbol and their conceptual referents (Osana & Pitsolantis, 2013; Osana et al., 2013; Uttal et al., 1997).

Links to Mathematics Teaching and Learning

Instruction plays a critical role in understanding symbols and their conceptual meaning. The mere presence of mathematical symbols in the classroom is not enough for students to understand their meaning (Moyer, 2001; Osana et al., 2013; Sherman & Bisanz, 2009), but teachers who provide students with conceptual explanations for symbols are particularly helpful for students' symbolic understanding. For example, Li et al. (2008) found that when students are provided instruction that highlights the concepts behind the mathematical symbols prior to introducing the symbols themselves, students demonstrated an increased understanding of their conceptual referents, which in turn, supported their mathematical performance. Further, the authors found positive effects when textbooks introduce the concept behind a mathematical symbol (i.e., "same as" for equivalence) prior to introducing the formal mathematical symbol itself (i.e., "="). This research suggests that to understand the conceptual meaning behind a mathematical symbol, students must build their knowledge of the symbol's conceptual referent, and that this knowledge will in turn assist them to interact with symbols in meaningful ways.

Research conducted by Osana et al. (2013) provides additional support for the importance of introducing students to the symbolic meaning of mathematical symbols prior to using them during problem solving. The authors examined first-grade students' perceptions and use of manipulatives based on the definition they were given prior to mathematical instruction. The intervention entailed introducing blue and red plastic chips to students in four different encoding conditions. The four encoding conditions provided students with different explanations of the chips' meaning and use. One condition described the chips as having a quantitative meaning, another condition described the chips as pieces for a checkers game, another condition allowed the children to play freely with the chips, and in a control group, the children were not exposed to the chips at all. Students' perceptions and use of the manipulatives were assessed before and after mathematics instruction that included them. Results showed that students who were explicitly told the quantitative meaning of the chips were more likely to see the chips as having a quantitative representation both before and after instruction. Students in the three other conditions learned to see the objects as mathematical tools, but did not improve in their knowledge of their quantitative meaning. This research suggests that using mathematical symbols during instruction does not necessarily allow students to see the concepts they are meant to convey; instead students can acquire this understanding by having teachers explicitly telling them what the symbols mean prior to using them in a mathematical context.

Other research by Osana and Pitsolantis (2013) further illustrates the importance of mathematics instruction that explicitly explains the conceptual meaning of mathematical symbols and procedures presented symbolically. Fifth-and-sixth grade students were tested to determine the effectiveness of an intervention on their fraction knowledge. Part of the instruction focused on linking concepts to fraction symbols. Results showed that when students received lessons that emphasized the conceptual referents of the symbols and symbolic procedures, their conceptual understanding, and connections between the concepts and symbols, improved. Although students who received instruction that did not link concepts and procedures also improved in their procedural skill, they did not show the same improvement in their conceptual understanding of fractions or their symbols. This study suggests that students do not on their own make the link between the concepts and procedures associated with mathematical symbols and that direct instruction can foster a deeper understanding of the conceptual meaning of these symbols, which could lead to greater flexibility and improved mathematical performance.

Problem Solving and Representation

A central aspect of North American reform in mathematics is about teaching students how to build on their mathematical knowledge through problem solving and to learn a variety of ways to solve mathematics problems (e.g., NCTM, 2000). The reform in Quebec is similar, emphasizing the importance of problem solving in mathematics education and requiring that students acquire the appropriate tools for finding solutions to mathematics problems (MELS, 2001). In the Quebec Education Program (QEP), word problems are presented in a situational context and are at the heart of the mathematics curriculum. The QEP specifically mentions that part of the mathematics curriculum is devoted to helping students decode the information presented within the problem and use representations correctly and flexibly when solving them (MELS, 2001).

Because of the heavy emphasis on word problems in reform-oriented mathematics teaching and learning, researchers have explored students' word problem solving strategies extensively over the last several decades (Carpenter, Fennema, & Franke, 1996; Kintsch & Greeno, 1985; Koedinger, Alibali, & Nathan, 2008; Lewis & Mayer, 1987;

Nathan & Koedinger, 2004; Riley, Greeno, & Heller, 1983). Word problems present students with a situational context in which mathematical information is embedded. Problem solvers are required to create internal representations of the quantities and situation-specific relationships described within the text (Nathan, Kintsch, & Young, 1992; Verschaffel, Torbeyns, Smedt, Luwel, & Van Dooren, 2007). Solving word problems involves interpreting the information that is presented within the text and highlighting key pieces of information that are required to determine a solution.

Researchers have described mathematical problem solving as including two component processes: problem comprehension and problem solution (Lewis, 1989; Lewis & Mayer, 1987). The comprehension phase of problem solving is the part during which the individual translates each sentence of the word problem into an internal representation and forms a coherent mental structure of the problem. During the solution phase, the student plans, monitors, and executes the necessary problem solving procedures (Koedinger & Terao, 2002; Lewis & Mayer, 1987). Some research suggests that one method that appears to help students execute the appropriate problem solving procedures during the solution phase is when they produce diagrams of their internal representations of the problem, regardless of whether the diagrams are their own creations or are demonstrated during instruction (Hembree, 1992; Koedinger & Terao, 2002; Larkin & Simon, 1987). Nunokawa (1994) suggested that when students draw diagrams, they are able to visualize the structure of the problem and change it if necessary. Other research suggests that students' construction of diagrams during mathematical problem solving is particularly helpful because the representations have personal meaning and encourage self-monitoring throughout the problem solving process (Koedinger & Terao, 2002;

Larkin & Simon, 1987). Together, this research suggests that when students draw diagrams of their internal representations, regardless of whether the diagrams are of their own creation or primarily demonstrated by the teacher, they can support dual representation because they provide students the opportunity to create meaning for the diagram itself as well as its mathematical referents.

Research conducted by Koedinger and Terao (2002) illustrates how diagrams can be helpful for students when solving particularly difficult word problems. As part of a middle school mathematics curriculum, 35 sixth-grade students were taught to use “picture algebra” as a tool to solve word problems. Students were taught to translate the information in the word problem into a box diagram by drawing rectangular boxes to represent the quantities described in the problem text. Mathematics curricula in some schools in the United States, Europe, and some Asian countries encourage children to use these representations, sometimes referred to as “bar diagrams” (see Figure 1 for an example of a bar diagram illustration). This “model method” aims to teach students to gain a better understanding of the structure of mathematical problems through the construction of rectangular boxes to represent the known and unknown quantities, and relationships among them (Singapore Ministry, 1999).

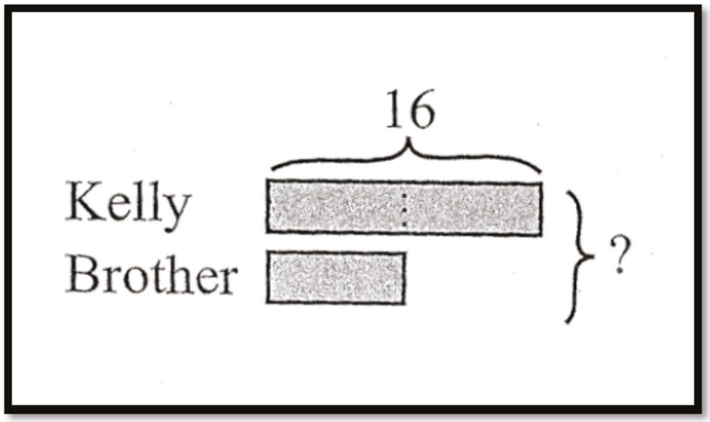


Figure 1. Bar diagram illustration of the following word problem: " Kelly has $16. She has twice as much as her brother. How much money do Kelly and her brother have together?"

After giving instruction to students on "box diagrams" Koedinger and Terao (2002) examined their solutions to three word problems. One word problem was accompanied by a corresponding box diagram, and the other two were not. Results revealed that students' mathematical performance only benefited slightly from the presence of a box diagram. Students' production of their own box diagrams during problem solving also assisted their solution accuracy. The word problem that was identified by the authors to be most difficult was most often solved correctly when students produced box diagrams that accurately depicted the relative size of the quantities presented in the problem.

These results demonstrate that students are capable of representing quantities and the relationships among quantities with the use of box diagrams and this appears to be particularly helpful when solving difficult word problems. Students' production of diagrams is particularly helpful when they accurately represent the information in the text.

Students' ability to appropriately represent information in the text with a picture demonstrates that they are able to understand the conceptual referents of the box diagrams and, most likely, have dual representation for these pictorial symbols. This research suggests that dual representation is required to use diagrams as tools in mathematics problem solving. Students' comprehension of the dual meaning of the box diagram appears to be essential for them to use these symbols as a problem-solving tool.

Teacher Knowledge

As the previous sections of this paper have illustrated, problem solving processes and practices are a concern within mathematics education and research. Under the new mathematics reform, teachers are expected to build their instruction around problem solving and encourage multiple problem-solving strategies in classroom environments that foster students' reasoning and confidence in their own problem solving abilities (MELS, 2001; NCTM, 2000). Researchers argue that teachers must have specific skills and knowledge to provide the kind of instruction demanded by the new mathematics reform. Shulman (1986, 1987) claimed that teachers must have know the subject matter they are teaching as well as be able to translate complex material in ways their students can understand.

Ball et al. (2008) introduced a model describing the knowledge of effective mathematics teachers. The authors proposed that mathematics teachers need, among other types of knowledge, Common Content Knowledge (CCK), which means they have an understanding of the mathematics curriculum in enough detail to solve school mathematics problems on their own. Furthermore, the authors suggested that effective mathematics teachers not only understand the mathematical material they are teaching, but also have Knowledge of Content and Teaching (KCT), which means they have a deep understanding of the appropriate and effective ways to teach concepts to their students in a variety of different contexts. The authors claimed that KCT is essential for high-quality instruction because students are provided with information in ways that makes sense to them and instruction is adapted to meet their learning needs. Additionally, the authors claimed that mathematics teachers need Specialized Content Knowledge (SCK), which is the knowledge and skill required to unpack mathematical concepts so that they are visible and learnable by students. Therefore, teachers require knowledge of both the mathematical content they are teaching as well as teaching strategies and what they can afford for students.

Research suggests that teachers have CCK for word problem solving (Van Dooren et al., 2003), but their solutions do not match the best instructional practices to effectively teach word problem solving to students (Ball, 1990a; Van Dooren et al., 2003). Teachers are required to adjust their teaching methods and provide students with appropriate representations that effectively convey the specific mathematical material they are teaching (Ball et al., 2008). As previously mentioned, one effective way to help students extract the structure of word problems is by using visual tools such as bar diagrams (i.e., Singapore Math, Koedinger & Terao, 2002). Therefore bar diagrams may be a valuable tool for teachers to use in their lessons.

Visual diagrams help students during problem solving, but recent evidence shows that they are helpful when the conceptual meaning of the diagrams is effectively conveyed to students during instruction. Uesaka et al. (2007) suggested that how the teacher uses diagrams during instruction predicts students' use and understanding of diagrams in word problem solving. The authors compared Japanese and New Zealand students' use and perceptions of diagrams as problem solving tools. The emphasis on diagram use in the curricula of these two countries differs. The New Zealand mathematics curriculum stresses the importance of the distinction between understanding the conceptual meaning of a tool and seeing it more generally as a tool for problem solving. In contrast, Japanese mathematics curriculum uses the diagrams within a problem solving context, but without explicitly stressing their role as tools in mathematics.

Uesaka et al. (2007) further found that students' use of diagrams in word problem solving differed between the two countries. Specifically, students from New Zealand were more likely to spontaneously create diagrams to solve word problems compared to Japanese students. Further, when Japanese students did draw their own diagrams during problem solving, the diagrams did not appear to help to solve the problems accurately. Japanese students reported that when they draw diagrams during problem solving, they try to recreate them the way their teachers use them. Thus, the Japanese students did not see the diagrams as problem solving tools, but used them because they thought they were supposed to. This research suggests, therefore, that how teachers use diagrams during instruction is critical to students' perceptions of these diagrams, and in turn, to how helpful these diagrams are to students. It follows that teachers' understanding of how to effectively use diagrams during instruction is related to students' comprehension of these symbols as problem solving tools. In sum, the evidence suggests first, that students need to attach meaning to mathematical symbols, and, second, that teachers' behaviours in the classroom influences how students assign and access this meaning.

Ball et al. (2008) also suggest that teachers need to have a flexible understanding of the material they are teaching so that they can provide instruction that will meet the diverse needs of their students. Preservice teachers demonstrate difficulty, however, with their ability to use diverse methods during problem solving. Specifically, preservice teachers often use problem-solving methods with which they are familiar and demonstrate difficulty providing conceptual justifications of their solutions (Ball, 1990a; Ball, 1990b). Some research also demonstrates that when teachers are presented with word problems, they solve them using algebraic equations (Van Dooren et al., 2008). Together, these findings suggest that teachers find algebraic methods more accessible than other problem solving methods.

This being said, teachers' tendencies to use algebraic methods to solve word problems does not match the problem solving methods preferences of students.. That is, despite teachers' perceptions, students find word problems easier than algebraic equations (Koedinger & Nathan, 2000). Thus, the instruction that teachers may be inclined to use to teach problem-solving may not be the most effective for their students because of the apparent mismatch between their CCK and KCT for word problem solving. As Ball et al. (2008) argued, teachers are required to understand the effective ways to teach mathematical material to their students, which may differ from their own preferred methods.

As previously mentioned, students' production of visual representations is particularly helpful in word problem solving (Koedinger & Terao, 2002; Lewis, 1989; Uesaka et al., 2007). In particular, students find "picture algebra," also known as "box diagrams," helpful in solving difficult word problems (Koedinger & Terao, 2002). Bar diagrams are mathematical symbols that are becoming increasingly popular in some Asian countries (Singapore Ministry, 1999) and are being incorporated in several teacher education programs across the United States (Singapore Math, 2011). Informal observation reveals, however, that preservice teachers have difficulty understanding the quantitative meaning of bar diagram symbols (H. P. Osana, personal communication, November 7th, 2012) and are not inclined to use them in their own word problem solving (Koedinger & Terao, 2002). Guidelines on how to explicitly teach preservice teachers' how to use pictorial representations in mathematical problem solving has not been extensively explored, but there is reason to believe their difficulties are associated with their dual representation of these symbols. Because of the novelty of bar diagrams in North America, many teachers may be unfamiliar with these symbols and thus require explicit explanations of the links between the bar diagram symbol itself and its quantitative referents (Osana & Pitsolantis, 2013; Osana et al., 2013; Uttal et al., 1997).

In sum, children do not automatically understand that symbols are representations of both physical and mental concepts. Throughout their development, under some circumstances, children begin to understand the dual nature of symbols. Dual representation is a critical component in understanding and using mathematical symbols. Understanding mathematical symbols, however, appears to be less intuitive than the symbol-referent relationship examined in DeLoache's work with scale models. Thus, introducing students to mathematical symbols may require explicit instruction on their conceptual referents. Students' problem solving abilities are influenced by their ability to produce diagrams to solve word problems (Koedinger & Terao, 2002; Uesaka et al., 2007) and, in turn their capacity for dual representation. Direct and explicit explanations of the meaning of problem-solving tools influences students' understanding of the mathematical symbols used within the classroom (Osana et al., 2012; Osana et al., 2013; Uesaka et al., 2007), while less is known about adults' dual representation in mathematics, it seems reasonable to expect that teachers themselves must have dual representation for the symbols they are teaching. Such dual representation would seem necessary for mathematics teaching to the extent that it would be central to various aspects of teachers' professional knowledge, including their own problem solving abilities as well as how to teach problem solving to their students.

Present Study

This study explored the effects of instruction on preservice teachers' conceptualizations of the bar diagram. In particular, this research project aimed to examine the effects of two different instructional methods on preservice teachers' understanding and use of the bar diagram symbol. Preservice teachers' problem solving abilities and conceptual understanding of the bar diagram symbol itself were measured before and after an instructional intervention. The goal of this study was to determine if explicit instruction that explains the meaning of the bar diagram symbol assists preservice teachers' ability to attach conceptual meaning to the symbol and in turn, to solve mathematical problems. This topic is important for the reason that teachers' conceptualizations of the topics they teach are critical to their teaching ability, and in turn, to their future students' understanding and academic success (Simon & Shifter, 1993).

My specific research questions are the following. Does explicit instruction on the quantitative meaning of the bar diagram symbol help preservice teachers: (a) accurately solve word problems presented using text in story format, (b) view the bar diagrams as pictures that represent quantities and are used for the purpose of solving problems, (c) understand the quantitative meaning of the bar diagrams, and (d) transfer their knowledge of bar diagrams to solve analogous problems presented using algebraic symbols?

This study was conducted using a three-condition pretest-posttest quasi-experimental design during the fall semester of 2013 with preservice teachers enrolled in an undergraduate mathematics methods course at a large urban university in Canada. Two of the conditions were exposed to instructional sessions that introduced the bar diagram symbol. In one condition, called the Bar Diagram Links (BDL) condition, the participants were taught the meaning of bar diagrams with the use of concrete materials; these materials were used to make explicit connections between the bar diagrams and the quantitative concepts they represented. In the second condition, called the Bar Diagram No Links (BDNL) condition, participants were taught how to use the bar diagrams without explicit instruction on the quantities they were meant to represent (i.e., without the use of concrete materials). The third condition was a comparison condition. The comparison condition was composed of 15 undergraduate students had not taken any mathematics methods courses and selected from an introductory child development course. The participants received no instruction and were not exposed to the bar diagram symbol.

I administered a pretest that evaluated the participants' symbolic understanding of the bar diagram symbol as well as their abilities to solve word problems presented in two different ways, story format and in algebraic symbol format. Three instructional sessions followed the pretest and a structurally similar posttest was administered after the intervention was completed.

Each instructional session consisted of the instructor presenting students with mathematical word equations and giving a detailed solution for each problem presented. Each problem was presented in a word equation format. Word equations are merely the presentation of algebraic equations in word format (e.g., "Starting with 90, if I subtract 66 and then divide by 6, I get a number. What is it?", adapted from Koedinger & Nathan, 2004). In both conditions, the instructor presented the word equation and provided a solution on the chalkboard using bar diagrams to represent the quantities in the problem.

In the BDL condition, the instructor explicitly wrote the quantity directly above the bar diagram on the chalkboard. Then she made a parallel connection to the quantities presented on the board with concrete materials such as sugar and water, to illustrate the connection between the symbol and its quantitative meaning. In the BDNL condition, numbers were used on the bar diagram, but no explicit conceptual connection was made between the bar diagram and the quantities they represented. After giving the lesson to each condition, the instructor gave the participants time to work in pairs on word equation problems. This time allowed for the participants to ask questions and receive corrective feedback from the instructor and me. I was present during each instructional session and assisted with the preparation of materials and instruction.

I predicted that the BDL condition would outperform the BDNL and Comparison conditions on their ability to accurately solve word problems presented in a story format because their understanding of bar diagrams would allow them to accurately interpret the relationships among the quantities in word problems, which would in turn help them in the solution process. Further, I predicted that students in both the BDL and BDNL conditions would outperform the Comparison condition after instruction on their conceptions of what the bar diagram represented and what it is used for. I predicted that the BDL condition would outperform the Comparison condition on this measure because the instruction provided with the concrete materials and explanations of bar diagram meaning to the BDL condition would allow for the participants to perceive the appropriate meaning and use of the symbol. Further, I predicted that the BDNL condition would outperform the Comparison condition on their conceptions of bar diagrams because they would be exposed to the bar diagram symbol, which might in turn influence their perceptions of it as a mathematical symbol (see Osana et al., 2013). I also predicted that the BDL would outperform both the BDNL and Comparison conditions on their understanding of the bar diagram's quantitative meaning because they received instruction that explicitly connects the meaning of the symbols with the concepts they represent. Finally, I predicted that the BDL condition would outperform the BDNL and Comparison conditions on their ability to solve analogous problems in symbolic form (i.e. algebraic form) because the instruction that emphasizes symbol meaning would elicit symbolic transfer to algebraic contexts.

Chapter 3: Method

Participants and Context

The intervention was conducted over the fall semester of 2013. Fifty undergraduate students ($N = 50$) participated in this study. Forty-one of these undergraduate students ($n = 41$) were enrolled in an elementary teacher education program at an English language university located in a large Canadian metropolitan city. The participants were recruited to participate in this research project through enrollment in the first of three compulsory mathematics methods courses in the program. All the material taught during the intervention was novel to most participants. Because of previous exposure to bar diagrams six participants were removed from the study, resulting in a final sample of 35 participants. The remaining fifteen undergraduate students ($n = 15$) were enrolled in a first year child development course and composed the Comparison condition for this study. Participants in the Comparison condition had not taken any math methods courses previous to being recruited for this study.

The curriculum for the mathematics methods course covers whole number operations, problem solving strategies, the development of children's thinking, and mathematical techniques for effective teacher questioning in mathematics. The instructor of the course is a mathematics teacher educator with 12 years experience working in the field. Because of the relevance of the instructional intervention pertaining to the course curriculum, participation in all project activities, including pretest and posttest measures, were requirements of the course. Consent was acquired for the use of student work as data.

Design

The design of the study was a three condition pretest-posttest quasi-experimental design. Two of the conditions received two different types of instruction that focused on teaching bar diagrams as a mathematical symbol to represent quantity. In one condition, called the Bar Diagram Links (BDL), the instructor used concrete materials to explicitly link the concepts evoked by the concrete materials to the bar diagram symbol. The second condition, called the Bar Diagram No Links condition (BDNL), received instruction on bar diagrams, but without connecting conceptual meaning to bar diagram symbols.

The third condition was a Comparison condition composed of students chosen from another undergraduate course at the university. Participants in the Comparison condition did not receive any instruction on mathematics or problem solving in the course from which they were selected. The Comparison condition was incorporated in this study to control for possible maturation effects, and to test the relative effects of exposure to the bar diagram symbols on the participants' perceptions of these symbols.

The phases of the study are outlined in Figure 2. For the BDL and BDNL conditions, the three phases consisted of a pretest, three instructional sessions, and an isomorphic posttest. For the Comparison condition, there were two phases. The first phase consisted of the same pretest presented to the BDL and BDNL conditions, and the second phase was the same isomorphic posttest.

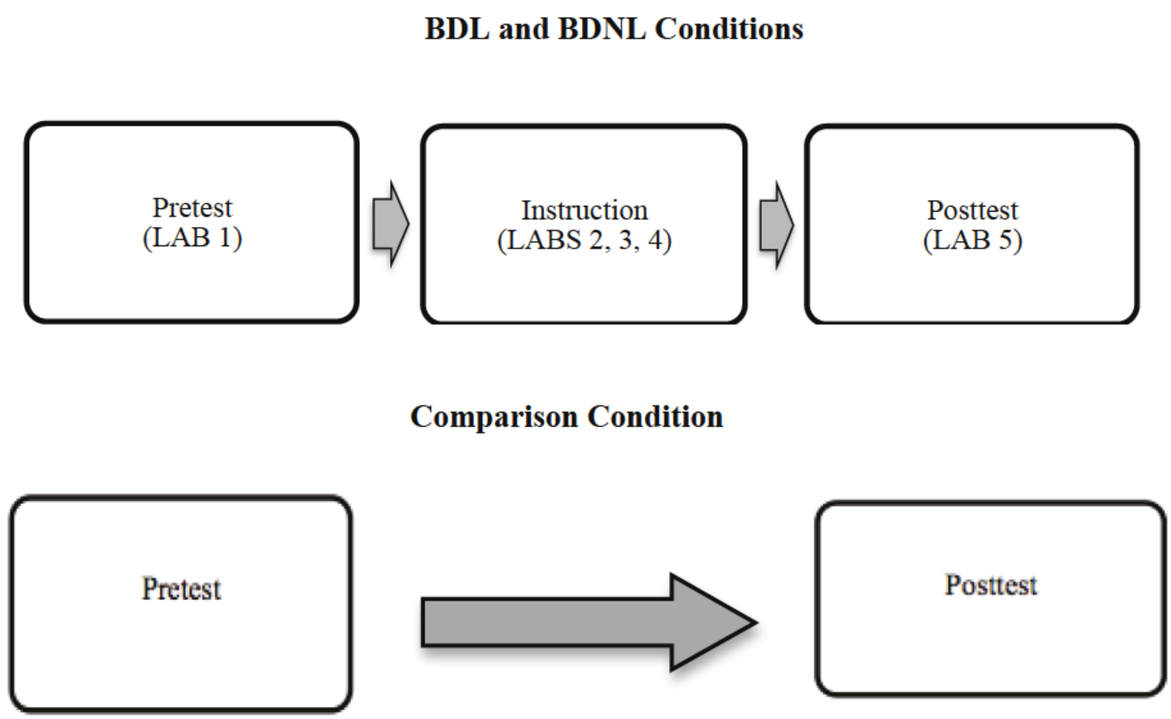


Figure 2. Study Design BDL, BDNL, and Comparison conditions

Outcome measures for this study included: (a) participants' ability to accurately solve word problems presented in a story format; (b) participants' conceptions of what the bar diagram symbol is and what it is used for; (c) participants' understanding of the quantitative meaning of the bar diagram; and (d) participants' ability to accurately solve problems presented using algebraic symbols. The study was designed so that the pretest, posttest, and all instructional sessions for the BDL and BDNL conditions took place during the designated lab portion of the mathematics methods course. Three instructional sessions took place during Labs 3, 4, and 5, and the pretest and posttest in Labs 1 and 5, respectively. The pretest and posttest for the Comparison condition were held in a research lab at the university.

Instructional lab sessions took place once a week for three consecutive weeks and were each two hours in length. Table 1 presents the schedule for each of the conditions over the three weeks. Each two-hour lab session was separated into two instructional sessions. During the first hour, one condition (BDL or BDNL, depending on the week) received instruction, followed by the other condition (BDL or BDNL) receiving instruction in the second hour. The order of the instructional sessions alternated from week to week. Two trained instructors delivered the interventions in both conditions, with the delivery counterbalanced to reduce instructor effects.

Table 1

Schedule of Instructional Lab Sessions Provided to the BDL and BDNL Conditions Over a Three-Week Period

Lab	Hour	Condition	Activity
2	1	BDL	Instructional Session 1
	2	BDNL	Instructional Session 1
3	1	BDNL	Instructional Session 2
	2	BDL	Instructional Session 2
4	1	BDL	Instructional Session 3
	2	BDNL	Instructional Session 3

Measures

Demographics. As part of the pretest, participants in all three conditions were asked to complete a brief demographic questionnaire, presented in Appendix A. The questionnaire consisted of a paper-and-pencil measure taken from Royea (2012). Demographic information collected included gender, age, semester and year participants entered the teacher education program, and questions pertaining to their previous mathematics teaching experience.

Problem Solving and Symbols Test. The Problem Solving and Symbols Test (PSST) is a 19-item paper-and-pencil measure administered to all participants at pretest and 20-item paper-and-pencil measure at posttest (see Appendix B for full version of the tests). I designed the PSST specifically for this study to evaluate the participants' (a) ability to accurately solve word problems, (b) conceptions of what the bar diagram symbol is and what it is used for, (c) understanding of the quantitative meaning of a bar diagram, and (d) ability to accurately solve analogous algebraic equations. Thus, the PSST consists of four subscales: (a) Word Problem subscale, (b) Perceptions of Bar Diagram subscale, (c) Bar Diagram subscale, and (d) Algebraic Problem Solving subscale.

Word Problem subscale. Six items on the PSST were designed to measure participants' ability to accurately solve word problems presented using text in story format. Table 2 contains sample items from the Word Problem Subscale. The problems varied in level of difficulty and were adapted from previous research (Elia, Gagatsis, & Demetriou, 2007; Koedinger et al., 2008; Koedinger & Nathan, 2004). Problem difficulty was manipulated by varying the location of the unknown value. Previous research has shown that students have more difficulty with word problem items that require them to solve for an unknown that is at the start of the problem solving process (Start-Unknown problems), rather than an unknown that is the result of the problem solving process (Result-Unknown problems; Elia, Gagatsis, & Demetriou, 2007; Koedinger & Nathan, 2004). The second manipulation of problem difficulty was based on the number of times the unknown is referenced in the relationships described in the problem. Double referent problems refer to the unknown twice in the word problem and are more difficult than single referent problem types, which only refer to the unknown once (Koedinger et al., 2008). Thus, these two dimensions of problem difficulty resulted in three problem types: Start Unknown-Single Referent, Result Unknown-Single Referent, and Double Referent, with three of each type on the PSST Word Problem subscale.

Two word problem items were presented per page, with ample space to answer each question. Participants were given instructions to "solve the problems in the best way you can" and "show all your work." Participants' responses were scored with 1 point for the correct solution, or 0 points for an incorrect solution.

Table 2

Sample Items of Word Algebra Problems on the Word Problem Subscale of The PSST

Problem Type	Word	Algebra
Single Referent, Result-Unknown	Adam and Emma decided to share their jellybeans. Adam has 14 jellybeans and Emma has 12 jellybeans. If they combine their jellybeans and split them evenly between them, how much does each of them get?	$(14 + 12) / 2 = x$
Single Referent, Start-Unknown	Four friends are renting a hotel room for a weekend trip to Toronto. The entire stay costs them the price of the room plus $43 in incidentals. They split the bill 4 ways and each of them paid $102. How much was the price of the room?	$(x + 43) / 4 = 102$

(table continues)

Problem Type	Word	Algebra
Double Referent	Tom, Sam and Alex are all brothers, and Alex is the youngest in the family. Tom is Alex's age plus 3 and multiplied by 2. Sam is four times Alex's age. If Tom and Sam's age together is 24, how old is Alex?	$2\,(x + 3) + 4x = 24$

Perceptions of Bar Diagrams subscale. Three open-ended items designed to assess participants' conceptions of what the bar diagram symbol is and what it is used for were presented on the Perceptions of Bar Diagrams subscale (see Appendix B for items). Participants were presented with one item that included an image of a bar diagram symbol with the following open-ended questions: "What are these?" and "What are they used for?" An additional open-ended item was included at posttest to assess participants' perceptions of bar diagram use in teaching mathematics. Participants were given ample space to write their responses on the test sheets.

Participants' responses to the item "what are these?" and "what are they used for?" on the Perceptions of Bar Diagrams subscale were grouped together and scored according to a rubric that categorized responses as: (a) Literal, (b) Quantitative, (c) Numerical, and

(d) Symbolic. Participants' responses to these items were grouped together because participants provided responses about both meaning and use in answering both of these questions.

The following rubric was used to code participants' perceptions of bar diagram meaning. Participant responses were classified as Literal when responses described bar diagrams as representations of objects that resemble their visual characteristics, such as patches of grass, tiles, or train cars. Responses were classified as Quantitative when they indicated that bar diagrams are representations of quantities or amounts (e.g., "These are used to visually show quantities.") Responses were classified as Numerical when they referred to bar diagrams as representing numbers or "numerical data" (e.g., "These diagrams allow one to represent numbers in a different way and see how they relate to each other.") Numeric responses differed from Quantitative responses because they would include no mention of the quantities or amounts to which the numbers refer to. Finally, Symbolic responses included descriptions of the bar diagrams as symbols or tools without mentioning their referents (e.g. "They are diagrams used to solve math problems.") Responses were further categorized according to participants' perceptions of bar diagram use. Responses were categorized as either a Problem Solving Math Tool or Other.

Participants' responses to the posttest item "how could you use these in teaching mathematics?" were categorized according to the type of teacher knowledge they reflected. Responses were coded as Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), and Knowledge of Content and Teaching (KCT). Examples of responses that were coded as CCK included descriptions of bar diagrams as tools to assist their own mathematical skills (i.e., "They help me make sense of word problems as I am solving them"). Examples of SCK included descriptions of bar diagrams as tools that assist teachers themselves in their own mathematical learning, which would in turn affect the teaching methods they use (i.e., "Bar diagrams help me make sense of word problems, which will help me to present different types of word problems to my students.") Responses that included descriptions of teaching with bar diagrams to explain mathematical concepts to their students were categorized as KCT (i.e., "They can help explain problem solving to younger children who haven't mastered algebra yet.") Responses were categorized as Other if they did not specify to whom the diagrams were targeted (e.g., "for problem solving…for solving multi-step problems.")

Bar Diagram subscale. The bar diagram subscale consisted of five items: two Word Problem Production items and three Word Problem Selection items (see Appendix B for items). Each Word Problem Production item provided participants with a bar diagram with its corresponding quantities labeled with numbers. Participants were asked to construct a word problem that matched the bar diagram provided. Each Word Problem Selection item provided participants with a bar diagram and a list of three word problems. Participants were asked to select the word problem that best corresponded to the bar diagram given. Participants' responses to each of the Bar Diagram subscale items were awarded 1 point for each correct response and 0 points for each incorrect response.

Algebraic Equations subscale. The Algebraic Equations subscale consisted of six items. Please see Table 2 for sample items and Appendix B for all items. The six items assessed participants' ability to solve problems presented with algebraic symbols. All items were analogous to the items on the Word Problem subscale. The word problem was translated into an algebra equation by using the same relationships between quantities, but expressed with algebraic symbols instead of words (sample analogous problems can be found in Table 2). Two items were Start Unknown-Single Referent, two items were Result-Unknown- Single Referent, and two items were Double Referent.

Participants' responses to the Algebraic Equations subscale items were scored with 1 point for each correct response and 0 points for each incorrect response.

Description of Conditions

The instructor of the course (Instructor 1) delivered half of the instructional sessions to each of the BDL and BDNL conditions. The teaching assistant of the course (Instructor 2) delivered to the remaining instructional sessions. During the BDL and BDNL instructional sessions, the instructor provided direct instruction, which was followed by time for participants to work in pairs on practice problems that reinforced the material covered during the session. All instructional sessions focused exclusively on word equations and their corresponding bar diagram solutions.

BDL.

Instructional session 1. The instructor of the methods course (instructor 1) presented students with four word equation problems (see Appendix C for items in session 1). Each problem was presented to the participants one at a time and solved in specific steps. To illustrate one lesson, consider the following word equation: "I start with some number. I divide it by 2. I add 8 to that number, and get 22. What number did I start with?" The instructor projected the word equation on a projector screen at the front of the class and read it out loud. Then, the instructor drew a bar diagram, with each bar corresponding to the relative size of the quantities in the problem (Koedinger & Terao, 2002). For this problem, the instructor drew a bar and then split the bar in half. She then drew another bar that represented 8 (see Figure 3). The instructor said: “This bar represents an unknown amount,” and she drew a bar on the board. She placed a question mark above the bar. Following, the instructor drew a line down the middle of the bar and said, “I divide this by 2”, and drew a bar half the size of the original bar below it on the chalkboard. Finally, the instructor drew another bar and attached it to the bar that was half the size of the original bar on the chalkboard and said, “This bar represents 8.” Pointing to the bar diagram illustration on the chalkboard the instructor said, “This entire amount equals 22. What number did I start with?” while pointing to the original bar diagram on the chalkboard.

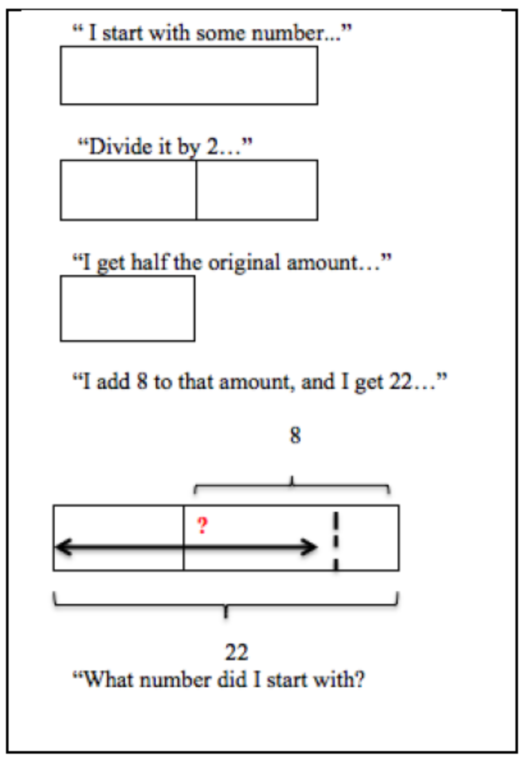


Figure 3. Sample instructional word equation problem as presented on the chalkboard.

As the instructor executed each step with the bar diagrams, she used concrete materials to make the connection between the diagram and its quantitative referent. After drawing the first bar, the instructor said, "This is an unknown amount." Next, the instructor picked up a cylinder filled with an unknown amount of sugar and said; "this is also an unknown amount, which represents the same quantity as this bar." Then the instructor took the cylinder of sugar an emptied half of it into a plastic jug and said, "this is half the original amount, which is the same as that bar," while pointing to the corresponding bar on the chalkboard. The instructor then took a second cylinder of sugar that was labeled 8 and showed it to the class and said, "this amount is the same as this bar," while pointing to the bar labeled 8 on the chalkboard. Finally, the instructor added the cylinder labeled "8" to the other cylinder and said, "this entire amount is 22, which is the same quantity as the bar diagram on the chalkboard." Each of these steps was demonstrated for two Start Unknown-Single Referent and two Double Referent problems (see Appendix C). The instructional portion of the session was approximately 35 minutes.

Following the whole-class instruction on each word equation, participants were given approximately 25 minutes to work in pairs on four word equation problems on a separate handout (see Appendix D) and three additional word problems presented on the board (See Appendix C for materials used). Both instructors and I circulated the classroom and provided corrective feedback to the participants as they worked on the practice problems.

Instructional session 2. During Instructional session 2, Instructor 2, followed the same instructional procedure used in Session 1, with some modifications to the length of instruction and the number of practice problems given to students. The instructor presented four word equation problems to the class and provided the same step-by-step solutions given in Instructional session 1, using bar diagrams and concrete materials in the same way (see Appendix C for materials used). The four word equations presented to the class were two Start-Unknown Single Referent, and two Double Referent, in that order (the problems are presented in Appendix C).

The instructional portion of session 2 was 30 minutes in length. Following the instruction, participants were given 30 minutes to work in pairs on eight word equation problems (presented in Appendix D). Instructor 2 and I circulated the classroom and provided corrective feedback to the participants.

Instructional session 3. In Instructional Session 3, Instructor 1, followed the same instructional sequence used in Session 1 and Session 2, but more time was allocated for participant practice. The instructor presented three word equation problems to the class and provided the same step-by-step solutions given in the two previous instructional sessions (see Appendix C for materials used). The three problems presented to the class were one Result Unknown- Single Referent and one Double Referent, in that order (see Appendix C). The instructional portion of the session was 20 minutes in length, and the participants were given 40 minutes to work in pairs on 10 word equation problems (presented in Appendix D). Instructor 2 and I circulated the classroom and provided corrective feedback to the participants.

BDNL.

Instructional session 1. Instructor 2 used the same script and problems that were used in the BDL condition, but without the use of concrete materials or other tools to explain the conceptual significance of the bar diagram symbols used. The instructor also refrained from using the words “quantity” and “amount” when referring to the bar diagrams.

Participants were presented with the same four word equation problems that were presented to the BDL condition during Instructional session 1. The instructor presented the word equation problems in the same way they were presented to the BDL condition. Specifically, the instructor presented each problem with a step-by-step solution on the chalkboard. The instructor drew bar diagrams in the same “size preserving” manner where all quantities, or parts of quantities, were represented by bars, the relative sizes of which correspond to the relative sizes of the quantities in the word equation (Koedinger & Terao, 2002). Following 35 minutes of instruction, the BDNL condition received 25 minutes to work in pairs on the same seven word equation problems presented to the BDL condition.

Instructional session 2. In Instructional Session 2, Instructor 1 provided the same step-by-step solutions as given in Instructional Session 1. Participants were presented with four word equation problems. The four word equation problems were the same ones provided to the BDL condition, and the instruction lasted 30 minutes in length. Next, the participants were given 30 minutes to work in pairs on eight word equation problems in a separate handout. Both instructors and I circulated the room and provided corrective feedback when needed.

Instructional session 3. During Instructional Session 3, Instructor 2, provided the same step-by-step solutions as given in Instructional Session 1 and Instructional Session 2. Participants were presented with three word equation problems, which were the same as the ones provided to the BDL condition. The instructor provided instruction to the participants for 20 minutes. Next, the participants were given 40 minutes to work in pairs on 10 word equation problems. The instructor and I circulated the room and provided corrective feedback when needed.

Comparison. Participants in the Comparison condition were recruited from an undergraduate child development course at the university. These participants had not taken any university-level mathematics methods courses, previously. I did not delivery any content related to the math methods curriculum or any instruction on bar diagrams to the participants in the Comparison condition. Participants in the Comparison received the same pretest and posttest measures provided to the BDL and BDNL conditions and were given the same amount of time to complete them. There was approximately the same number of weeks between the pretest and posttest measures for all three conditions.

Procedure

Pretest and Posttest

BDL and BDNL. In the first class of the methods course in a designated classroom at the university, participants sat at individual desks. The instructor of the course introduced me and described the study by saying: "Hi, I would like to introduce you to one of my Master's students. This is Danielle Houstoun. She is interested in exploring the teaching of problem solving in preservice teachers. Over a five-week period, she will be in present during lab session activities, during which you will participate in instructional sessions and problem solving tasks. I am going to pass out a consent form for you to read over. All of the activities you will participate in will be compulsory components of the course, but your consent will give me your permission to use your work as data for Danielle's thesis. Please know that none of the work you will complete for this study will affect your grade, but after the study will be completed, you will be asked to write a report about your experience as it pertains to your professional development in mathematics, which will be graded by me, your instructor." Consent forms were then passed out to each student. At this time, the instructor left the room so that the participants could feel free to make a decision about whether or not to participate. During the course, the instructor was not given any information regarding the participation of any students in her class.

After the students had completed the consent form and I collected them all, the instructor returned to the classroom and said, "now you will be asked to think about some word problems on this worksheet. Don't worry if you cannot answer all of the questions. Just try your best. Over the next few weeks of the course, you will learn how to solve these problems. This is important for your development as teachers, given that the Quebec curriculum places a heavy emphasis on problem solving." Next, the instructor and I handed each participant a workbook with all 19 *items* on the PSST. The pretest took between 45 minutes and 60 minutes to complete. After 60 minutes, I collected the workbooks from each participant. I then left the class and stored all consent forms in a locked filing cabinet.

Immediately after the pretest measure, I used a number generator to randomly assign 25 participants to the BDL condition and 21 participants to the BDNL condition. Four participants who had previously taken the course were placed in the BDL condition to ensure the best instruction, but their scores were not used as data. At the start of the second instructional session, a short questionnaire was administered to all participants in the BDL and BDNL conditions asking them, "We have been using diagrams to solve problems. Have you ever seen bar diagrams used in this way before? Six participants had previously seen bar diagrams before and were removed as data for this study. Five days later, the first instructional session took place. Twenty-eight days after the pretest was administered, the posttest was administered to all the participants together in Lab 5. The same procedures used for the pretest were used for the posttest.

Comparison

I attended the last 15 minutes of an undergraduate child development course to recruit 15 participants for the Comparison group in the third week of classes in the fall 2013 semester. I introduced the study and myself by saying: “Hi my name is Danielle Houstoun and I am a Master’s student here at the university in the Department of Education. My supervisor is Dr. Helen Osana, who teaches in the Early Childhood and Elementary Education program in the Department. For my research, I am interested in undergraduate students’ problem solving and their understanding of math symbols. I am hoping that some of you may be interested in this opportunity to be part of a research project. I am looking for about 20 students, who have never previously taken Mathematics Methods I, to complete one worksheet that will take about 30-45 minutes to complete outside of class time two times between now and the end of the semester. The worksheets will ask you to solve some problems and to interpret some pictures in the context of problem solving. The math part isn’t complicated, and I am not interested in right or wrong answers. I am more interested in how you think about math problems.

I would ask that you come to my supervisor’s research lab to fill out the worksheets. You would complete the two worksheets about three or four weeks apart. You will not be graded on this, and your participation is completely independent from this class. Also, your participation will be kept completely confidential and your identity will not be revealed to anyone. If you would like to participate you will receive $20 for the completion of both worksheets. Do you have any questions for me about the project?

Now, I am now going to pass out sheets to fill out if you are interested in participating. If you are interested, could you please write in your name and email and indicate ALL THE TIMES that you are available to come to the research lab to complete the worksheet? The times are listed on this sheet. I will collect the sheets on your way out. If you do not wish to participate, you don't need to hand the sheet back. Once I have all the information from you, I will then organize a few sessions and let you know by email when to come to the research lab. I will contact you within a day or two. Thank you very much for considering this and for your time! "

I contacted the participants from the Comparison condition by email and arranged a time for each participant to complete the pretest. Participants came to the research lab and filled out a consent form. Immediately following this, the participants were given 45 to 60 minutes to complete the pretest. They were given the same procedures and instructions given to the BDL and BDNL conditions. One week following the pretest. I emailed the participants again and set up a time to come back to the research lab to complete the posttest. Participants completed the posttest an average of 28 days after the pretest in the research lab, following the same procedures as the pretest.

Intervention

Instructional sessions for both conditions took place in the classroom designated for the methods course during the second, third, and forth lab portions of the class. At the beginning of each instructional session, I guided the participants into the room and asked them to find a seat at the desks provided. The instructor of the course and her teaching assistant led the instructional sessions in both conditions. Each instructor stood at the front of the class and participants were seated at desks facing the instructor. There was a projector, a laptop computer, a large chalkboard with chalk and an eraser, and the concrete materials used for the BDL condition only, which were placed on a table at the front of the class.

Following the instructional portion of each session, I passed out workbooks with practice problems. After the first hour was completed, the instructor and I took approximately 5 minutes to prepare for the next group to enter the classroom for their instructional session. Between sessions, we collect all workbooks, all props, and erased all previous work from the chalkboard to prevent confounding the conditions. All necessary props were then prepared for the upcoming instructional session. Then, I invited the next group of participants to enter the room, and the same procedures were used to implement the subsequent instructional session.

Chapter 4: Results

The mean age of participants in the BDL group was 25.29 years (*SD* = 6.59) and in the BDNL group, the mean age was 25.35 years (*SD* = 6.29). In the Comparison condition, the mean age was 20.80 years (*SD* = 1.66). Seventy five percent (75%) of the participants in the BDL condition were either in their first or second year of their program, and 70% of the participants in the BDNL condition were either in their second or third year. In the Comparison condition, 71.4% of the participants were in their first year of either the teacher-training program or the BA in Child Studies program.

A large proportion of the participants in all three conditions were female: 92.2% in the BDL condition, 90% in the BDNL condition, and 93.3% in the Comparison group. Similarly, nearly all of the participants in the BDL and BDNL conditions had some previous teaching experience (85.7% and 90%, respectively), and 73.3% of the participants in the Comparison condition reported previous teaching experience.

To address the first research question regarding the effects of the intervention on preservice teachers' word problem solving, an analysis of variance was conducted using the scores on the Word Problem Solving subscale as the dependent measure and time and condition as factors of the independent variable. Separate chi-square tests (at pretest and posttest) were conducted to address the second research question on the effects of the intervention on participants' perceptions of the bar diagrams and what they are used for (i.e., Perceptions of Bar Diagrams subscale). To address the third research question on the effects of the intervention on participants' dual representation of the bar diagram (i.e., a rectangular boxes as well as symbols that represent quantities in mathematical problems), two separate analyses of variance were conducted. Both analysis of variance tests included time and condition as factors, with the Word Problem Selection measure as the dependent measure in one test and Word Problem Production measure as the dependent variable in the other. A final analysis of variance was conducted to address the fourth research question regarding participants' ability to transfer their dual representation of bar diagrams to solve analogous problems presented with algebraic symbols, with time and condition as factors and the Algebra Problem Solving measure as the dependent measure.

Data from the BDL and BDNL conditions were used in all analyses; only two of the analyses included data from the Comparison condition. Because of an administrative error, the Comparison condition received an incorrect version of the posttest, and as a result, the data from the Word Problem Solving, Word Problem Production, and Algebraic Problem Solving subscales could not be used. Therefore, the only analyses that included data for the Comparison condition were those for the Perceptions of Bar Diagrams and Word Problem Selection subscales.

Problem Solving

The means and standard deviations of the participants' percent scores on the Word Problem Solving subscale at both pretest and posttest are presented as function of condition in Table 3. A 2 (condition) x 2 (time) mixed ANOVA was conducted, with condition (BDL, BDNL) as the between-group factor and time (Pretest, Posttest) as the within-group factor. Scores on the Word Problem Solving subscale, in percent, were used as the dependent measure.

Table 3

Means and (Standard Deviations) of Word Problem Solving Percent Scores at Pretest and Posttest in the BDL and BDNL Conditions (N =35)

	Condition	
	BDL (n =14)	BDNL (n = 21)
Pretest	.79 (.20)	.88 (.16)
Posttest	.77 (.12)	.83 (.17)

Note. BDL = Bar Diagram Links; BDNL = Bar Diagram No Links

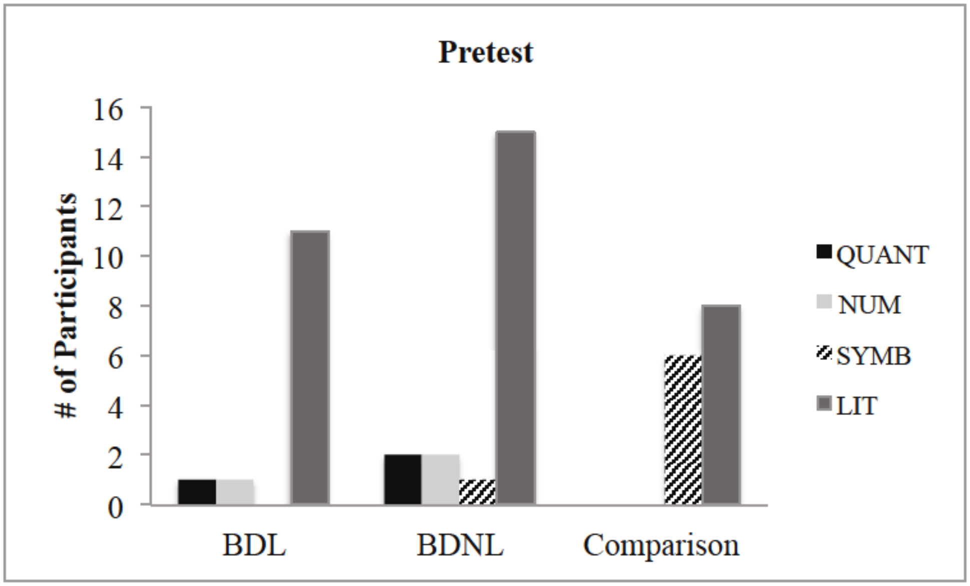
Results revealed no significant main effects for condition, F (2, 33) = 2.04, p =. 16, or time, F (2, 33) = 1.30, p = .25. Results further revealed no significant interaction effect, F (2, 33) = .28, p = .60. The means indicate that no differences were found between conditions on Word Problem Solving performance when averaged across time. Additionally, the means show that there was no difference on participants' performance on Word Problem Solving from pretest to posttest, regardless of condition. Finally, there was no significant difference between conditions in their change from pretest to posttest.

Perceptions of Bar Diagrams

The Perceptions of Bar Diagrams subscale included two items to assess participants' perceptions of bar diagrams by asking two separate questions both at pretest and posttest. The two items, "What are these?" and "What are they used for?," were grouped together, and responses to both questions were used to assess participants' perceptions of (a) bar diagram meaning and (b) bar diagram use. Using these data, two separate analyses were conducted both at pretest and posttest to determine participants' perceptions of bar diagram meaning and use, as a function of condition. Finally, at posttest, the participants were asked how they could use bar diagrams in their own teaching.

Perceptions of Bar Diagram Meaning

Categorization of participants' responses to the items that measured their perceptions of bar diagram meaning were classified as: Literal, Quantitative, Numerical, and Symbolic. Frequency counts and proportions by condition (BDL, BDNL, Comparison) for pretest and posttest are presented in Figure 4.



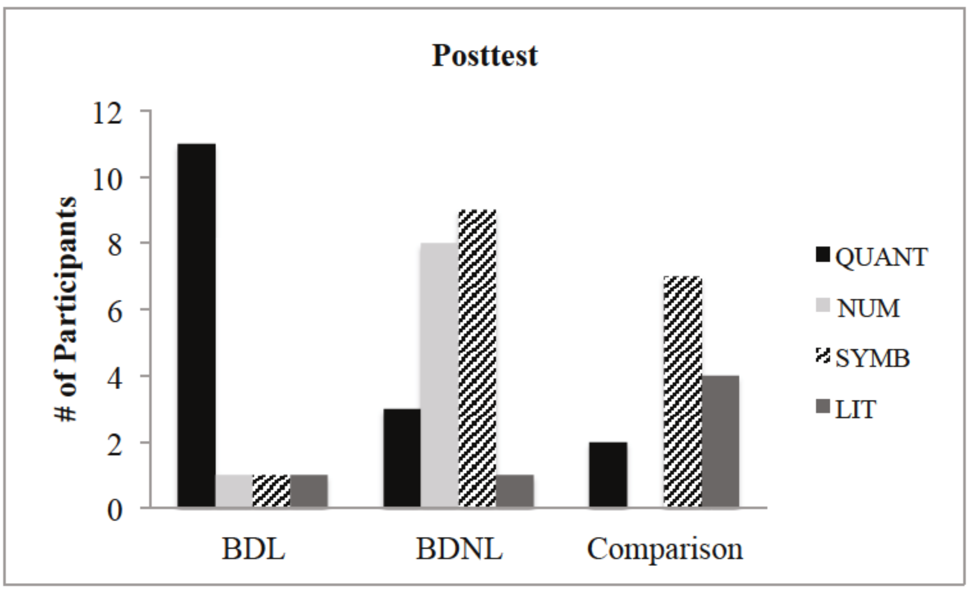


Figure 4. Frequencies of codes assigned to participants' written perceptions of bar diagrams on pretest and posttest.

The frequencies revealed that Literal codes were more frequently assigned to participants' responses than Quantitative, Numerical, or Symbolic codes within each condition at pretest. After instruction, the BDL condition provided more Quantitative responses than the other two conditions. Specifically, 78.6% of the BDL condition described the bar diagrams as representing quantities at posttest, whereas 14.3% of the

BDNL and 20% of the Comparison condition described them as quantities at posttest. Further, posttest frequencies demonstrated that the BDNL condition described the bar diagram as a symbol more often than the other two conditions. Specifically, 42.9% of the BDNL condition described the bar diagram having a symbolic meaning versus 7.1% of the BDL condition and 6.7% of the Comparison condition. Further, the BDNL condition demonstrated more diversity in their responses at posttest than either the BDL and Comparison conditions. For example, 95.3% of the BDNL responses were within three categories (i.e., numerical, symbolic, and quantitative), whereas the majority of the BDL condition's responses were quantitative (78.6%) and 73.4% of the Comparison condition's responses were coded as literal or symbolic.

Two separate 2 x 3 chi-squares, one at pretest and one at posttest, were conducted to test for a relationship between condition and perception of bar diagram as having a quantitative meaning. The frequency counts and proportions at pretest and posttest are presented in Table 4 and Table 5, respectively.

Table 4

Frequencies and Proportions[a] of Quantitative Representations at Pretest for the BDL, BDNL, and Comparison Conditions (N = 44)

	Quantitative Representations		
Condition	No	Yes	Total
BDL	8 (80%)	2 (20%)	10[b] (100%)
BDNL	19 (95%)	1 (5%)	20[b] (100%)
Comparison	14 (100%)	0 (0%)	14[b] (100%)

Note. BDL = Bar Diagram with Links; BDNL = Bar Diagram No Links

[a]Proportions are calculated within condition; [b]Four participants in the BDL and one participant from the BDNL and one participant from the Comparison condition provided no response. .

Pretest results indicated that participants' quantitative responses on pretest were not significantly related to their condition $\chi^2(2) = 3.86$, $p = .15$. Therefore, there was no difference between conditions at pretest and assignment of the quantitative code. Specifically, 80% of the BDL, 95% of the BDNL, and 100% of the Comparison conditions responded non-quantitatively at pretest.

Table 5

Frequencies and Proportions[a] of Quantitative Representations at Posttest for the BDL, BDNL, and Comparison Conditions (N = 49)

	Quantitative Representations		
Condition	No	Yes	Total
BDL	2 (14.3%)	12 (85.7%)	14 (100%)
BDNL	16 (76.2%)	5 (23.8%)	21 (100%)
Comparison	12 (85.7%)	2 (14.3%)	14[b] (100%)

Note. BDL = Bar Diagram with Links; BDNL = Bar Diagram No Links

[a]Proportions are calculated within condition; [b]One Participant from the Comparison condition did not provide a response.

In contrast, posttest results indicated that participant responses were significantly related to their condition $\chi^2(2) = 18.51$, $p < .01$. More specifically, 85.7% of the Quantitative codes were from the BDL condition. In contrast, significantly fewer participants' responses in the BDNL (23.8%) and Comparison (14.3%) conditions were assigned a Quantitative code at posttest.

Perceptions of Bar Diagram Use

Participants' perceptions of bar diagram use were qualitatively analyzed at pretest and posttest. Table 6 and Table 7 present the frequencies and proportions of participants' perceptions of bar diagrams as Non-Math Tools and Math Tools, which was further subcategorizes as Problem Solving Tools, Manipulatives, or Measurement Tools, at pretest and posttest as a function of condition (BDL, BDNL, Comparison).

Examples of Non-Math Tool responses included representations of objects used for non-mathematical purposes (i.e., build, cook, show shape, decoration). Math tool responses were further categorized as Problem Solving Tools (e.g., "To help you solve a problem"), Manipulatives (e.g., "They can be used to separate piles for division or even to determine the total, or for counting"), and Measurement Tools (e.g., "They can be used for measurements.")

Table 6

Frequencies and Proportions[a] of Perceptions of Bar Diagram Use at Pretest for the BDL, BDNL, and Comparison Conditions (N = 48)

	Responses to Bar Diagram Use				
Condition	NMT	PST	MT	MST	Total
BDL	3 (25%)	1 (8.3%)	5 (41.7%)	3 (25%)	12[b] (100%)
BDNL	2 (9.5%)	5 (23.8%)	9 (42.9%)	5 (23.8%)	21 (100%)
Comparison	2 (13.3%)	1 (6.7%)	8 (53.3%)	4 (26.7%)	15 (100%)

Note. BDL = Bar Diagram with Links; BDNL = Bar Diagram No Links; NMT = Non-Math Tool; PST = Problem Solving Tool; MT= Manipulatives; MST = Measurement Tool

[a]Proportions are calculated within condition; [b] Two participants from the BDL condition did not provide a response.

Table 7

Frequencies and Proportions[a] of Perceptions of Bar Diagrams Use at Posttest for the BDL, BDNL, and Comparison Conditions (N = 50)

	Responses to Bar Diagram Use				
Condition	NMT	PST	MT	MST	Total
BDL	0 (0%)	14 (100%)	0 (0%)	0 (0%)	14 (100%)
BDNL	0 (0%)	19 (90.5%)	2 (9.5%)	0 (0%)	21 (100%)
Comparison	1 (6.7%)	3 (20%)	6 (40%)	5 (33.3%)	15 (100%)

Note. BDL = Bar Diagram with Links; BDNL = Bar Diagram No Links; NMT = Non-Math Tool; PST = Problem Solving Tool; MT= Manipulatives; MST = Measurement Tool

[a]Proportions are calculated within condition

The frequencies revealed that participants varied in their perception of bar diagrams as Math Tools at pretest. In particular, participants in all three conditions more often perceived bar diagrams as Non-Math Tools, Manipulatives, or Measurement Tools at pretest. Specifically, 66.7% in each of the BDL and BDNL conditions perceived bar diagrams as either Manipulatives or Measurement Tools at pretest (see Table 6). In contrast, posttest frequencies revealed that participants in the BDL and BDNL conditions responses were more concentrated in one category. Specifically, 100% of the BDL and 90.5% BDNL conditions perceived bar diagrams as problem solving tools, whereas only 20% of the Comparison condition perceived bar diagrams as Problem Solving Tools. Further, the frequencies presented in Table 6 and Table 7 illustrate that the BDL and BDNL conditions' responses were less varied across categories at posttest, whereas the Comparison conditions' responses remained scattered across the various categories at posttest.

Two 2 x 3 chi-squares, one at pretest and one at posttest, were conducted to test for a relationship between participants' perceptions of bar diagrams use (as a Problem Solving Math Tool). The frequency counts and proportions for pretest and posttest are presented in Table 8 and Table 9, respectively.

Table 8

Frequencies and Proportions[a] of Perceptions of Problem Solving Math Tool at Pretest for the BDL, BDNL, and Comparison Conditions (N =47)

	Problem Solving Math Tool		
Condition	No	Yes	Total
BDL	11 (84.6%)	2 (15.4%)	13[b] (100%)
BDNL	15 (75%)	5 (25%)	20[b] (100%)
Comparison	14 (100%)	0 (0%)	14[b] (100%)

Note. BDL = Bar Diagram with Links; BDNL = Bar Diagram No Links[b]

[a]Proportions are calculated within condition; [b]Two participants from the BDL, one participant from the BDNL condition, and one participant from the Comparison condition did not provide a response.

Results indicated that participants' perceptions of bar diagram use as a Problem Solving Math Tool at pretest were not significantly related to condition $\chi^2(2) = 4.06$, $p = .13$. Specifically, 15.4% of the BDL and 25% of the BDNL condition were coded for Problem Solving Math Tools at pretest. These results revealed that there was no significant difference between conditions on their perceptions of bar diagrams as mathematical tools at pretest

Table 9

Frequencies and Proportions[a] of Perceptions of Bar Diagram as a Problem Solving Math Tool at Posttest for the BDL, BDNL, and Comparison Conditions (N =50)

	Problem Solving Math Tool		
Condition	No	Yes	Total
BDL	0 (0%)	14 (100%)	14 (100%)
BDNL	2 (9.5%)	19 (90.5%)	21 (100%)
Comparison	12 (80%)	3 (20%)	15 (100%)

Note. BDL = Bar Diagram with Links; BDNL = Bar Diagram No Links

[a]Proportions are calculated within condition

In contrast, results indicated that perceptions of bar diagram use as a mathematical problem solving tool at posttest was significantly related to condition $\chi^2(2) = 29.11$, $p < .001$. These results revealed that participants' perceptions did depend on the condition they were in. Specifically, 100% of participants from the BDL condition and 90.5% of participants from the BDNL condition perceived bar diagrams as Problem Solving Tools at posttest, whereas 20% of participants from the Comparison condition held these perceptions at posttest.

Perceptions about Bar Diagram Use in Teaching

Participants' perceptions of how to use bar diagrams in their future classrooms were collected at posttest. Participant responses to "How could you use these in teaching mathematics?" were categorized as Common Content Knowledge (CCK), Specialized

Content Knowledge (SCK), or Knowledge of Content and Teaching (KCT). Table 10 presents the frequencies and proportions of the codes assigned to participants' perceptions of the bar diagram as a teaching tool in mathematics classrooms.

Table 10

Frequencies and Proportions[a] of Perceptions of the Use of Bar Diagrams in Mathematics Teaching for the BDL and BDNL Conditions at Posttest (N= 34)

	Types of Teacher Knowledge				
Condition	CCK	SCK	KCT	Other	Total
BDL	0 (0%)	0 (0%)	13 (100%)	0 (0%)	13[b] (100%)
BDNL	0 (0%)	0 (0%)	18 (85.7%)	3 (14.3%)	21 (100%)

Note. BDL = Bar Diagram with Links; BDNL = Bar Diagram No Links; NMT = Non-Math Tool; PST = Problem Solving Tool; MT= Manipulatives; MST = Measurement Tool

[a]Proportions are calculated within condition; [b]One participant from the BDL condition did not provide a response.

The frequencies and proportions revealed that participants' perceptions of bar diagram use in mathematics classrooms were similar for both conditions. Specifically, the responses of all the participants in the BDL and 85.7% in the BDNL condition reflected KCT in describing how bar diagrams could support their practice as mathematics teacher.

Quantitative Use of Bar Diagrams (Dual Representation)

Word Problem Selection

To address the third research question regarding the effects of condition on participants' quantitative understanding of bar diagrams, a 3 (condition) x 2 (time) mixed ANOVA was conducted, with condition (BDL, BDNL, Comparison) as the between group factor and time (pretest, posttest) as the within-group factor. The percent scores on the Word Problem Selection subscale were used as the dependent variable in the analysis. The means and standard deviations of the participants' percent scores on the Word Problem Selection measure at pretest and posttest are presented as a function of condition in Table 11 and graphed in Figure 5.

Table 11

Means and (Standard Deviations) of Word Problem Selection Performance at Pretest and Posttest for the BDL, BDNL, and Comparison Conditions (N = 50)

	Condition		
	BDL (*n* =14)	BDNL (*n* = 21)	Comparison (*n* = 15)
Pretest	.61 (.26)	.61 (.30)	.44 (.21)
Posttest	.81 (.22)	.89 (.16)	.73 (.23)

Note. BDL = Bar Diagram Links; BDNL = Bar Diagram No Links

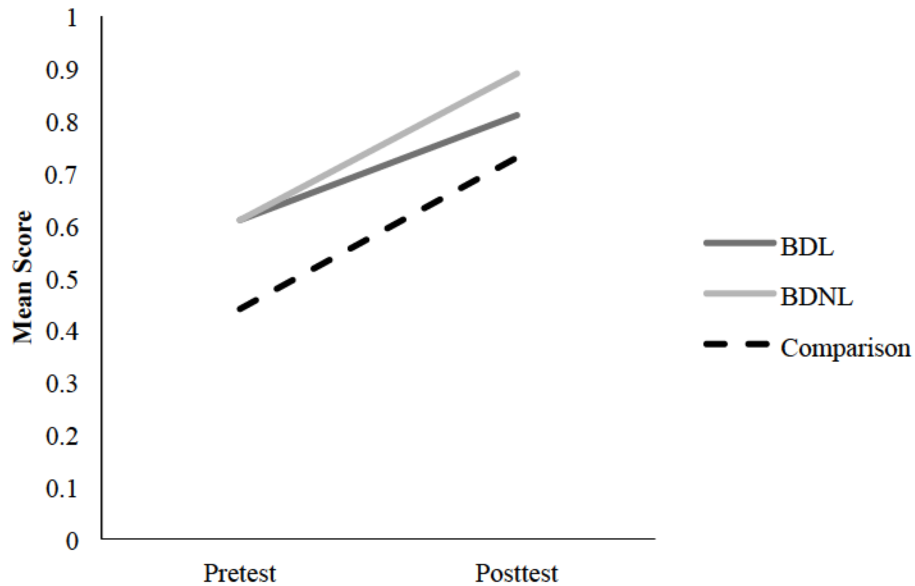


Figure 5. BDL, BDNL, and Comparison conditions' mean scores on the Word Problem Selection measure at pretest and posttest.

The results of the ANOVA revealed both a main effect of time, F (2, 47) = 24.28, $p < .001$ and condition, F (3, 47) = 5.32, $p = .008$. The means revealed a significant difference from pretest to posttest regardless of condition. There was also a significant difference between the means of all three conditions averaged across time. A Bonferroni post hoc test revealed that the significant mean difference was between the BDNL (M = .75, SD = .17) and Comparison (M = .59, SD = .12) conditions, $p = .007$. Finally, results revealed no significant interaction effect, F (3, 47) = .250, $p = .780$. The improved performance was the same for all three conditions from pretest to posttest.

Word Problem Production

To address the third research question regarding participants' quantitative understanding of bar diagrams, a 2 (condition) x 2 (time) mixed ANOVA was conducted, with condition (BDL, BDNL) as the between group factor, and time (pretest, posttest) as the within-group factor. The percent scores on the Word Problem Production subscale were used as the dependent measure in the analysis. The means and standard deviations of the participants' percent scores on the Word Problem Production subscale at pretest and posttest are presented as a function of condition in Table 12 and graphed in Figure 6.

Table 12

Means and (Standard Deviations) of Word Problem Production Percent Scores at Pretest and Posttest for the BDL and BDNL Conditions (N =26)

	Condition	
	BDL (*n* =12[a])	BDNL (*n* = 14[a])
Pretest	.17 (.33)	.04 (.13)
Posttest	.63 (.30)	.86 (.31)

Note. BDL = Bar Diagram Links; BDNL = Bar Diagram No Links

[a]Two participants in the BDL condition and seven participants in the BDNL condition did not provide a response

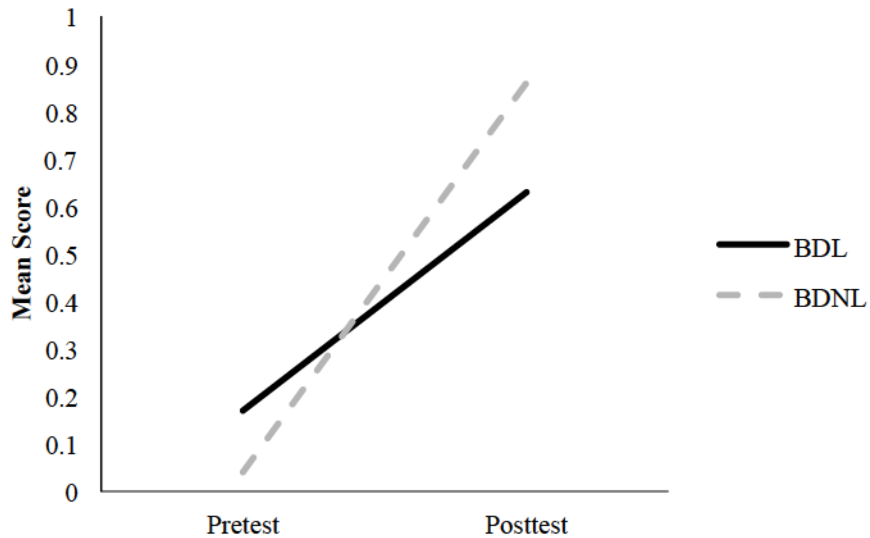


Figure 6. BDL and BDNL pretest and posttest mean scores for the Word Problem Production measure.

Results revealed a main effect of time, F (2, 24) = 83.75, p < .001. No main effect of condition was found F (2, 24) = .251, p = .621. The means indicate that all participants improved from pretest to posttest regardless of condition. The means further reveal that there was no difference between conditions averaged across time (see Figure 6). Results also revealed a significant interaction effect, F (2, 24) = 6.74, p = .016. Specifically, the means showed that change in performance was dependent on condition, with the change in performance from pretest to posttest being greater for the BDNL condition than for the BDL condition.

Algebra Problem Solving

The means and standard deviations of the participants' percent scores on the Algebraic Problem Solving subscale at both pretest and posttest are presented as function of condition in Table 13 and graphed in Figure 7. A 2 (condition) x 2 (time) mixed ANOVA was conducted, with condition (BDL, BDNL) as the between-group factor and time (Pretest, Posttest) as the within-group factor. Scores on the Algebra Problem Solving subscale, in percent, were used as the dependent measure.

Table 13

Means and (Standard Deviations) of Algebra Problem Solving Percent Scores at Pretest and Posttest for the BDL and BDNL Conditions (N =34)

	Condition	
	BDL (n = 13)	BDNL (n = 21)
Pretest	.89 (.17)	.81 (.24)
Posttest	.74 (.36)	.89 (.17)

Note. BDL = Bar Diagram Links; BDNL = Bar Diagram No Links

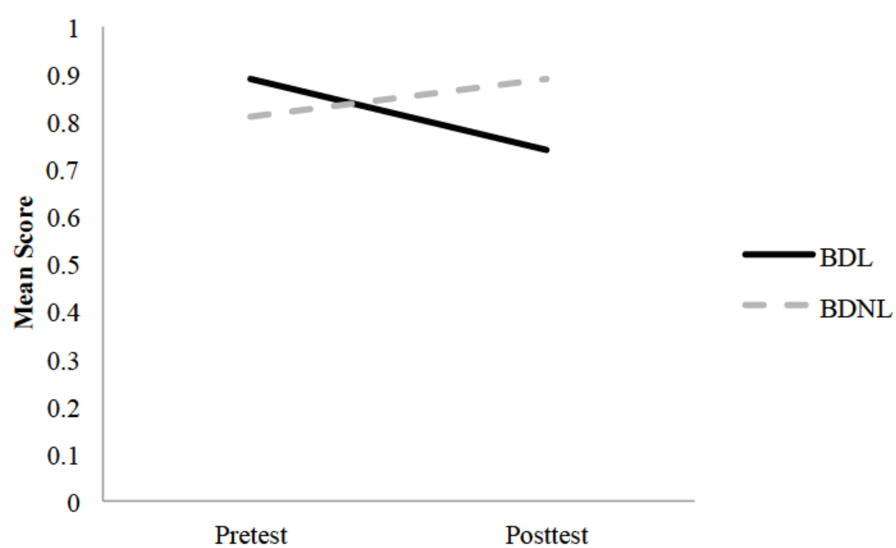


Figure 7. BDL and BDNL pretest and posttest mean scores for the Algebra Problem Solving measure.

Results indicated that there were no main effect of time, *F* (2, 32) = .53, *p* = .472, or condition, *F* (2, 32) = .20, *p* = .655. Results did reveal a significant interaction effect, however, *F* (2, 32) = 5.39, *p* = .027. The interaction effect shows that the change in algebra performance was dependent on condition. Specifically, the BDL group performed less well on the posttest compared to the BDNL condition.

Chapter 5: Conclusions and Discussion

The goal of the present study was to determine whether explicit instruction that explains the meaning of the bar diagram symbol with the use of concrete materials assists preservice teachers to (a) accurately solve word problems, (b) view bar diagrams as images that represent quantities and used to solve mathematical problems, (c) understand the quantitative meaning of bar diagrams, and (d) transfer their knowledge of bar diagrams to solve analogous algebraic mathematical problems. In addition, preservice teachers' views about bar diagrams use in mathematics teaching were explored and analyzed.

The results of the study revealed that there was no change in participants' ability to solve word problems over time, regardless of the intervention they received. These results were contrary to my initial prediction that participants exposed to concrete materials that made explicit connections to bar diagrams would be better able to solve word problems on the posttest compared to participants who were not exposed to concrete materials. These findings indicate that the BDL intervention did not make a difference in participants' abilities to solve word problems, although mean scores on pretest for both conditions were high, indicating a possible ceiling effect. Therefore, the lack of change in word problem solving performance at posttest could be because participants performed particularly well on the word problems at pretest and could thus show little improvement at posttest. Because participants scored high on this measure, at both pretest and posttest, the difficulty level of this measure was perhaps not appropriate for these participants to properly measure their word problem solving performance.

In addition, the results revealed that participants in all three conditions improved in their ability to select word problems that corresponded to the quantities represented as bar diagrams. These findings are contrary to my initial prediction that the BDL condition would outperform the BDNL and Comparison conditions on their ability to select an appropriate word problem for the diagram. One possible theoretical explanation for these findings could be due to what diSessa and Sherin (2000) called "meta-representational awareness." Meta representational awareness is described by diSessa and Sherin as a familiarity with symbols and their purpose, which allows individuals to determine the purpose and meaning of novel symbols (diSessa & Sherin, 2000). Perhaps when adults, maybe even more so undergraduate students specifically, are given diagrams and asked to select the appropriate information that matches them, they are able to tap into their meta-representational competence and use an intuitive understanding of the purpose of the symbol to complete the task. Therefore, improvement in all three conditions on matching the diagram to the appropriate problem could be explained by meta-representational awareness. Further evidence of meta-representational awareness comes from the Comparison conditions improvement on this measure, even after no instruction on the diagrams. Another explanation for the improvement could be methodological: it is possible this measure did not assess something other than dual representation, but further research is needed to identify more precisely the explanation for these findings.

In contrast, results revealed that participants' improvement on creating word problems that accurately reflected the relationships depicted in a bar diagram at posttest depended on the type of instruction they received. Those participants who did not receive instruction with the use of concrete materials were better able to produce word problems that illustrated the quantities in the bar diagram and the relationships among them compared to those who were exposed to the concrete materials. These results are contrary to my initial prediction that the BDL condition would be better able to generate word problems that represented the quantities provided in the diagram than the BDNL condition.

Previous research has shown that the use of concrete materials during instruction can take the participants' attention away from the mathematical concepts being taught (McNeil, Uttal, Jarvin, & Sternberg, 2009; Peterson & McNeil, 2013; Slousky, Kaminski, & Heckler, 2005), which can be used to explain the present results. McNeil et al. (2009) investigated whether the presence of perceptually rich concrete materials would assist students in their performance on word problems. In their experiment, they presented two conditions of fourth-grade students with manipulatives, and assessed their ability to solve problems involving money. In one condition, the manipulatives were perceptually rich, and included bills and coins that resembled real money. The other condition also received manipulatives in the form of bills and coins, but their appearance was less perceptually rich and did not resemble real U.S. currency. Finally, the control did not have access to any concrete materials, but were presented with the same word problems. The results revealed that students in the condition with the perceptually rich materials solved fewer problems correctly than the students in the two other conditions. These results exemplify that providing students with attractive and perceptually salient materials might actually hinder their problem solving performance. The authors suggested that the presence of the bills and coins may have been distracting for the students in their problem solving since they may have already had representations for the money and had difficulty acquiring the representation for it as a tool to assist in their problem solving.

Similar to McNeil et al.'s (2009) findings, participants in the BDNL condition in the present study who did not receive concrete materials during instruction improved more than those in the BDL condition in their ability to create word problems that appropriately represented the given bar diagrams. Perhaps instruction provided to the BDL condition distracted students because they had already made meaning of the materials used (i.e., sugar, water, bread, ribbon) and were thus unable to create alternative representations of the materials. Thus, the concrete materials may have distracted participants in the BDL condition from making connections between the concrete materials and the quantitative referents of the bar diagram.

Another explanation for these unexpected findings could be explained by the divergent focus of instruction provided to the BDL and BDNL conditions. For example, Uesaka et al. (2007) found that instruction that focuses more on providing students with examples of how to *use* diagrams, rather than explaining the *purpose* of the diagram, resulted in students using diagrams more often for problem solving, which in turn improved problem solving performance. Perhaps the instruction in the BDL condition placed too much emphasis on the explanation of diagram meaning, which distracted the students from how the diagram can be used to solve problems. Instruction provided to the BDNL condition placed more emphasis on showing how the diagrams are used as problem solving tools. Perhaps the simpler instruction provided to the BDNL condition enabled participants to make the appropriate connections between the word equations and the bar diagram, without any distracting instruction on the diagrams' meaning.

In addition, the results revealed a significant difference in participants' perceptions of bar diagrams as representations of quantities from pretest to posttest depending on the condition they were in. A larger portion of participants in the BDL, condition perceived bar diagrams as quantitative representations at posttest than in the BDNL, and Comparison conditions. These results were in accordance with my initial prediction that participants exposed to instruction that included the use of concrete materials to explicitly connect the bar diagram symbol to its quantitative meaning would be better able to perceive bar diagrams as representing quantities than participants who had not received such instruction.

Although these findings support my initial predictions, they do not correspond with the findings on participants' quantitative understanding of bar diagrams, as measured with the Word Problem Production measure. An explanation for this discrepancy might be because of the language used during instruction. Specifically, the instructors included language such as "quantities" and "amounts" when referring to the bar diagrams during instruction in the BDL condition, but refrained from using such language with the BDNL condition and instead referred to the quantities in bar diagrams as "numbers." These findings correspond to previous research that found that explicitly telling students that a novel symbol had a quantitative referent resulted in students' perceiving the symbol quantitatively after instruction (Osana et al., 2013). In conjunction with the other findings that either demonstrated no condition effects or more improvement from the BDNL than the BDL condition, it is far from certain that the BDL condition's use of the word "quantity" at posttest is an indication of superior understanding of the bar diagram meaning. Rather, these findings might be merely due to the participants' repetition of the language used the by instructor during the intervention.

Results further illustrated a significant difference at posttest between conditions on participants' perceptions of the use of bar diagrams as tools used for problem solving. In line with my initial prediction, both the BDL and BDNL conditions were better able to classify the bar diagram as a mathematical problem solving tool at posttest compared to the Comparison condition. Although the frequencies revealed that all three conditions perceived bar diagrams as a mathematical tools at posttest, the purpose of the mathematical tool differed among the three conditions. Specifically, participants from the Comparison condition more commonly perceived bar diagrams as manipulatives or measurement tools rather than problem solving tools, as articulated in the other two conditions. This is consistent with Osana et al. (2013) who demonstrated that when a novel mathematical symbol is introduced in a mathematical context, students appropriately perceive the purpose of the symbol as a mathematical tool.

The nature of the participants' views of the role of bar diagrams in teaching was similar for both the BDL and BDNL conditions. These results demonstrated that participants in both conditions viewed the diagrams as tools that can help them in teaching problem solving problem solving because they can illustrate the features of the problem. Further, the participants in both groups described the bar diagram as a tool that can help explain specific concepts to their students such as addition, multiplication, division, and representing quantities.

These findings are in line with previous research on preservice teachers' noticing of classroom environments. For example, Star and Strickland (2008) examined preservice mathematics teachers' awareness of classroom features after watching an experienced teacher provide mathematics instruction to eighth grade students. Assessment measures illustrated that preservice teachers mainly focused on what the teacher was doing during the lesson. Specifically, preservice teachers paid attention to the teachers' classroom management techniques and how the teacher presented the lesson (i.e., what materials she used during instruction and how she presented the material). Further, the authors found that preservice teachers rarely noticed aspects of how the teachers critically explained the mathematical content to the students during the lesson. These findings illustrate that when watching an experienced mathematics teacher, preservice teachers mainly focused on teachers' strategy use and are inattentive to mathematical content.

Similarly, the present study's findings illustrate that preservice teachers view the bar diagram as a strategy to present information on problem solving to their students rather than considering how it can assist their own understanding of the structure of the mathematical content. The data illustrated that preservice teachers mainly noticed how the instructor used the bar diagram as a teaching strategy rather than an aid for their own learning of mathematics problem solving. One possible explanation for these findings could be that when preservice teachers watch experienced educators provide instruction, their focus is primarily on how instruction is given because they are seeking practical tools to use in their future classrooms.

Finally, results of the study revealed that participants' ability to solve algebraic problems differed across time depending on condition. Contrary to my initial prediction, the participants who received instruction that made explicit links with concrete materials to the bar diagram symbol did not gain in the ability to transfer their knowledge of bar diagrams to solve analogous algebra problems. Participants who did not receive instruction with concrete materials, in contrast, did improve on their algebra problem solving across time. Sloutsky et al. (2005) examined the role of concrete materials in undergraduate students' transfer mathematical concepts to an isomorphically similar domain. The participants were randomly assigned to one of four conditions, with each condition varying in the degree of perceptual richness of the symbols or objects used during instruction. The authors found that participants in the condition that included perceptually sparse symbols (i.e., basic black symbols) performed significantly better in the transfer domain than the condition that included instruction with perceptually rich symbols (i.e., pictures of real objects). Further, there was no difference between the two perceptually rich conditions (i.e., pictures of real objects and visually rich symbols), indicating that the presence of the perceptually rich symbols and objects hindered participants learning.

Thus, Sloutsky et al.'s (2005) research suggests that the presence of perceptually rich objects during instruction interferes with students' ability to transfer the concepts taught during instruction to similar domains. Their findings are similar to the present study's findings on participants' algebra problem solving, since the condition that did not receive instruction with perceptually rich objects during instruction performed better on the transfer task than the participants who did receive instruction with perceptually rich objects.

Contributions and Implications for Practice

The present study was the first to directly examine preservice teachers' dual representation of bar diagrams. Although researchers have previously investigated the effects of bar diagrams in mathematics instruction on elementary students and adults, there is no literature to my knowledge that has explored this topic with preservice teachers specifically. For example, Koedinger and Terao (2002) investigated 6^{th} grade students' use of bar diagrams to solve word problems after having received instruction on how to use the bar diagrams to break down the structure of the problems. Further, Booth & Koedinger (2012) examined the effectiveness of bar diagrams on the ability of high school and college students' problem solving. The present study's investigation with preservice teachers adds to the literature on bar diagram use in teacher education.

Additionally, the present study differs from other research on diagram use because of the instructional methods used. For example, Koedinger and Terao (2002) provided instruction with bar diagrams to elementary students, but without the use of concrete materials or explanations of bar diagram meaning. Further, Uesaka et al. (2007) examined the differences in instruction with diagrams provided to students in Japan and New Zealand. Specifically, mathematics instruction provided to secondary students in New Zealand emphasizes teaching students how to understand diagrams and use them as a mathematical tools, whereas the mathematics curriculum in Japan emphasizes understanding diagrams without stressing how they are used as tools for problem solving.

The present study adds to the literature on instruction with diagrams in mathematics in that the meaning of the diagram was emphasized in addition to its use in problem solving. Although many of the results were unexpected, these findings contribute to the literature on effective instruction of diagrams in mathematics. In line with Uesaka et al. (2007), the present study's findings suggest that providing multiple examples of how to use diagrams during instruction is an effective method to encourage diagram use and understanding.

Additionally, instruction provided in the present study offers important pedagogical implications. Although unexpected, the findings of the present study suggest that the use of concrete materials to enhance dual representation of the bar diagram symbol may not be the most effective instructional approach. These findings are crucial to inform teachers and teacher educators of the possible interference concrete materials may generate during instruction. More specifically, the findings suggest that if teacher educators present preservice teachers with concrete materials and draw explicit connections to bar diagram meaning during instruction, this approach may interfere with preservice teachers' ability to make the connection between the diagram and the concepts they represent. On the other hand, instruction that provides preservice teachers, with many examples of bar diagrams that correspond to the quantities described in word equations assists their understanding of the quantitative referents associated with the bar diagrams.

Furthermore, the findings of the present study contribute to the literature on symbolic understanding. This study presents empirical evidence to illustrate how preservice teachers interpret mathematical diagrams when they first see them: as objects that correspond to their visual characteristics. Novices to particular visualizations may not interpret their meaning appropriately and may require some assistance. Following mathematics instruction that includes the use of diagrams, researchers often expect that adults' superficial representations will be replaced with the desired ones (Uttal & O'Doherty, 2008). Research with elementary students illustrates that this does not automatically occur (Osana et al., 2013); findings from the present study clearly demonstrate the importance of providing instruction to manage adults' representations of mathematical concepts as well.

Additionally, the findings from the present study contribute to the literature by describing what is salient to preservice teachers in mathematics methods courses. Specifically, the findings demonstrate that preservice teachers primarily focus on how the content in the course will translate to teaching tools in the classroom to the exclusion of focusing on the mathematical content (e.g., CCK and SCK) in the activities. Specifically, preservice teachers appear to focus on how the instruction provides them with tools to use in their own classrooms and they tend not reflect on how the instruction advances their own learning that in turn can support their future teaching practices. The findings provide a clear recommendation for teacher educators to place more emphasis on the importance of content and specialized knowledge in preservice professional development.

Future research can use the present study as a basis for understanding the methods of teaching bar diagrams to preservice teachers can impact their future classroom practices. For example, future research can examine how instruction with the use of concrete materials to explain diagram meaning can either hinder or support teachers' own practices, and in turn, affect student learning. In addition, future research can examine in more detail how participants' perceptions of bar diagram use and meaning actually influences their future teaching practices. Finally, future research could expand on the present study by examining how dual representation for the bar diagram symbol may affect learning of specific mathematical concepts.

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Appendix A

Demographic Questionnaire

Student Number: ______________________________________

Concordia University
Department of Education
EDUC 386/2: Teaching Mathematics I

Participant Demographics

Instructions: Please fill in all the information as accurately as possible. Your information will remain confidential and will only be used for research purposes.

1. Circle your gender: Male/ Female

2. Age: ______

3. When did you begin the ECEE Specialization Program?

   Semester: Fall or Winter

   Year: _____________

4. Do you have any individual or classroom-based teaching experience including substitute teaching, teaching stages, tutoring, working as a classroom aide, etc?

   Circle: **Yes** or **No**

5. If you circled yes, please describe in detail your teaching experienced below.

Type of Teaching Experience	**Details of Responsibilities/Tasks**	**Approximate Duration (in months)**

Please feel free to ask for another sheet if you need more space.

Thank- you!

Appendix B

Problem Solving and Symbols Test (PSST): Pretest

Name: ________________________

Student ID: ____________________

Date: ___________________

Problem Solving Worksheet 1
EDUC 386/2

Instructions:

This worksheet consists of 5 parts. You will be asked to answer the questions in the space provided. You will learn this material over the semester, so don't worry if you are not sure how to answer some of the questions and just try your best. Below are some word problems. Solve the problems in the best way you can. Show all your work.

Part I: Problem Solving

1) Four friends are renting a hotel room for a weekend trip to Toronto. The entire stay costs them the price of the room plus $43 in incidentals. They split the bill 4 ways and each one of them paid $102. How much was the price of the room?

2) Tom, Sam and Alex all brothers, and Alex is the youngest in the family. Tom is Alex's age plus 3 and multiplied by 2. Sam is four times Alex's age. If Tom and Sam's age together is 24, how old is Alex?

3) Adam and Emma decided to share their jellybeans. Adam has 14 jellybeans and Emma has 12 jellybeans. If they combine their jellybeans and split them evenly between them, how much does each of then get?

4) Annie works as a substitute art teacher at a local school. She makes $25 per hour, but $33 of each paycheque is taken out for medical insurance. If her last paycheque was $142, how many hours did she work for that pay period?

5) Sugar can be bought in two bag sizes. The lager bag weighs 20kg, one large bag and one small bag together weighs the same as 5 small bags, how heavy is the smaller bag?

6) Three university classes have been selected to participate in a research study. There are 46 students in each class. Twelve students from each class could not participate. How many students participated in the study?

Part II: Diagrams

Please examine the diagram in the space below:

Please answer the following questions about the diagram in the space provided below:

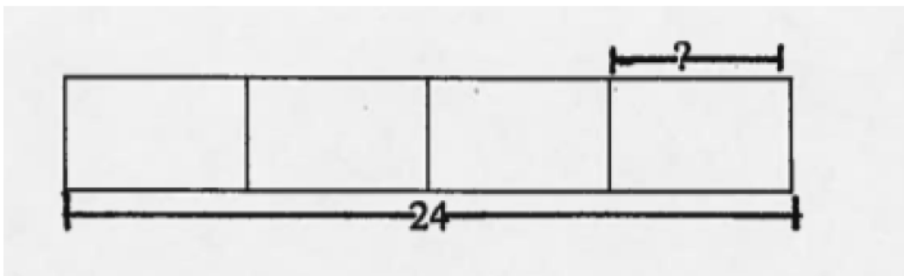
a) What are these?

b) What are these used for?

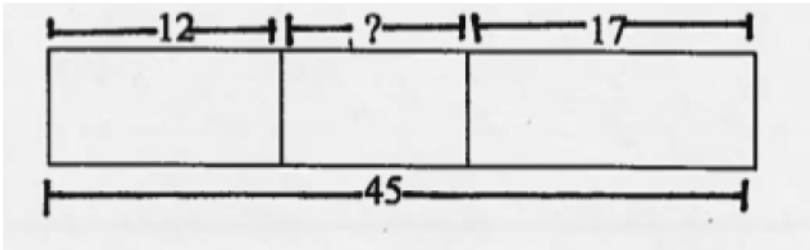
Part III: Writing Word Problems

Instructions:
In each of the following questions you will see a diagram. In the space provided write a word problem that corresponds to the diagram provided.

1)



2)

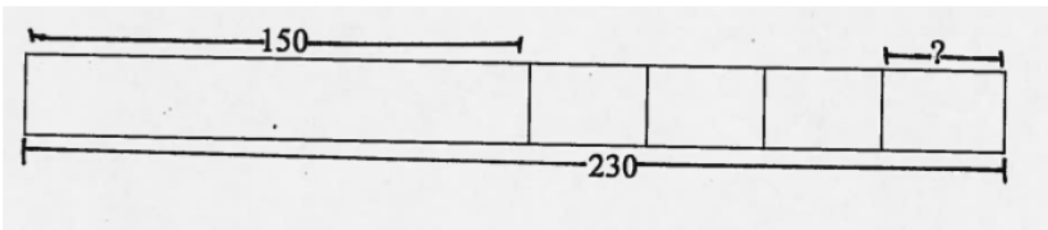


Part IV: Multiple Choice

Instructions:

For each of the following questions, you will be given a diagram and three word problems. Examine the diagram carefully and choose the word problem from the three choices that best corresponds to the diagram. Choose your response by circling the problem.

1)

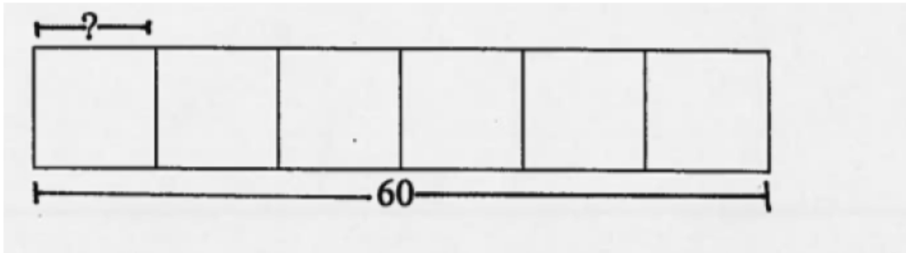


a) Stephanie works part-time at a cell phone store. She makes $150 a week plus $20 commission for every cell phone she sells. If she makes $230 in a week, how many cell phones did she sell?

b) Tim is building a walkway to his front entrance. His walkway will have borders outlining each side of the walkway, which require a specific type of paving stone. The middle of walkway will be composed 4 different types of stones. If Tim needs 150 stones for the borders combined and he buys 230 stones in all, how many stones does Tim have for the middle of the walkway?

c) Diane and Peter are buying patio furniture for their backyard. They want set that is $230 an 4 reclining chairs that $150 each. If they were to buy all of the patio furniture they want, how much would it cost all together?

2)

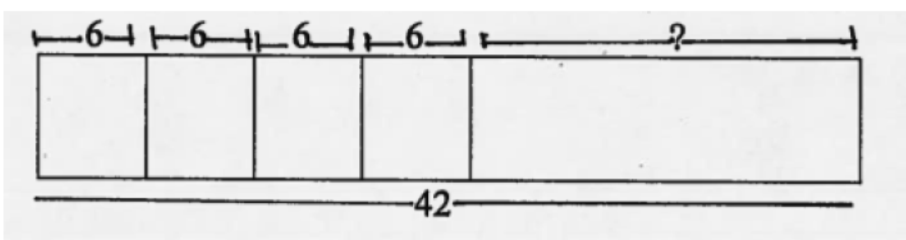


a) Tammy needs to paint the side of a 4-storey building. Each story is painted a different colour. If the building is 60 meters high, what is the height of each storey?

b) Julia is training for a triathlon. On Tuesday's she swims at 60m-length pool. If Julia swims 240m, how many lengths of the pool did she swim?

c) Liam is learning about weight in his grade-2 math class. Liam's interested in comparing his weight to that of his baby brother. Liam weighs his baby brother and finds out that he is a certain amount. Liam then weighs himself and discovers that he is 60lbs. Liam determines that his weight minus the weight of his baby brother is equal to 4 times the weight of his brother. How much does Liam's baby brother weigh?

3)



a) Jodi has 42 sour keys she decides to share them amongst herself and her 5 friends. How many does each person get?

b) Pat is planting his garden. He has 42 tulip bulbs and he wants to put 4 garden beds in the front of his house with 6 tulips bulbs in each bed. How many bulbs does Pat have left to plant?

c) Catering service has 42 tablecloths. They have a big event coming up and need to buy more tablecloths. They buy them in bulk packages of 6. If they buy 4 more packages, how many do they have all together?

Part V: Equations

Solve for the unknown value in each of the following problems. Show your work.

1)

$$\frac{x+43}{4}=102$$

2)

$2\ (x + 3) + 4x = 24$

3)

$$\frac{14+12}{2} = x$$

4)

$25x - 33 = 142$

5)

$x + 20 = 5x$

6)

$(4*6) - 13 = x$

Problem Solving and Symbols Test (PSST): Posttest

Name: ___________________________

Student ID: _______________________

Date: ______________________

Problem Solving Worksheet 2
EDUC 386/2

Instructions:

This worksheet consists of 6 parts. You will be asked to answer the questions in the space provided.

Part I: Problem Solving

Instructions: Below are some word problems. Solve them in any way you wish. Show all your work.

1) Tiffany and her parents go to a restaurant and her parents offer to pay $25 towards the total cost of the meal. The waiter comes to the table with 3 separate bills of $14 each. How much did Tiffany have to pay for her meal?

2) Rena, Emily, and John go tree planting together for the summer. Rena plants a certain number of trees. John plants 5 more than Rena. Emily plants 3 times as many trees as Rena. If the number of trees tat John and Emily plant is 165, then how many trees did Rena plant?

3) Two families go to the park for a picnic. The family of 3 brings 6 sandwiches and the family of 4 brings 15 sandwiches. If they put all the sandwiches together, how many sandwiches does each person get if they get the same number?

4) Four kids in a classroom have a birthday party at the end of the week. Each birthday child brings in the same number of cupcakes. There are 28 children in the class and there are 20 cupcakes left after each child has had one. How many cupcakes did each of the 4 birthday children bring in?

5) Rob walks to his swimming classes every Wednesday. On his way to class one day Rob walks to the store, and then another 6 km to class. The entire distance Rob walks to swimming class is 4 times the distance from his home to the store. What is the distance that Rob walks from home to his swimming class?

6) Heather invites three friends to go see a movie (4 people in total). Heather is feeling generous and decides to pay for everyone to go to the movies. Each movie ticket costs \$8 and she has \$13 on a gift certificate, which she uses towards the price of the movie tickets. How much more does Heather have to pay?

Part II: Diagrams

Please examine the diagram in the space below:

Please answer the following questions about the diagram in the space provided below:

a) What are these?

b) What are these used for?

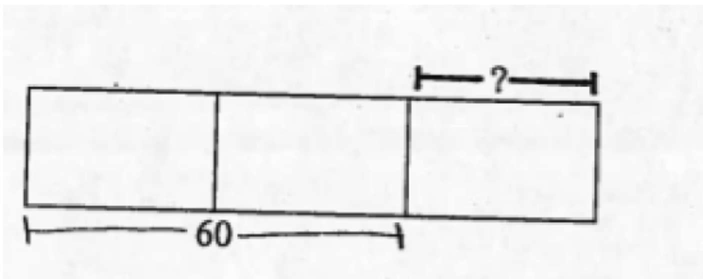
c) How could you use these in teaching mathematics?

Part III: Writing Word Problems

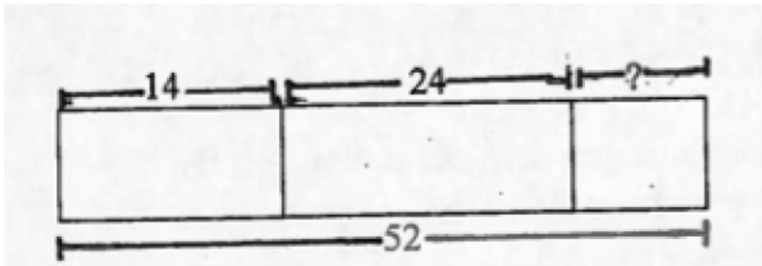
Instructions:

In each of the following questions you will see a diagram. In the space provided write a word problem that corresponds to the diagram provided.

1)



2)



Part IV: Drawing Diagrams

Instructions: For each of the following word problems, please provide a bar diagram that represents the quantities and relationships described in the problem. Makes sure to label all parts of the diagram. You do **NOT** need to solve the problem.

1) Mom won some money. She kept $45 for herself and gave each of her three sons and equal portion for the rest. If each son got $20, how much money did Mom win?

2) Jimmy went apple picking and picked 23 apples. He used 8 apples to make an apple pie. How many apples does Jimmy have left?

3) James makes the same amount of money each day he works at Cup-a-Joe Coffeeshop. Over reading week, he works for 5 days everyday (Monday through Friday). What he earns in 4 days is $360 more than what he earns in one day. How much id James make during reading week?

4) A dress costs twice as much as a skirt. Mrs. Wu bought 2 dresses and 2 skirts. If she paid a total of $150, find the costs of each skirt.

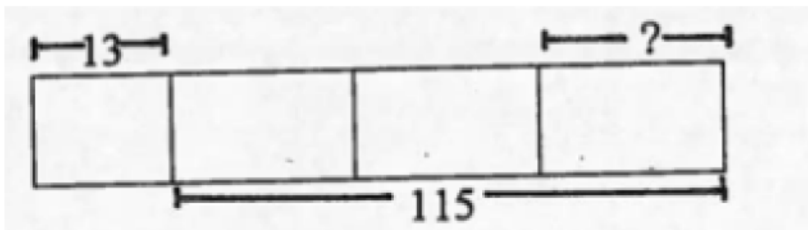
5) Gretchen weights 30kg. Alan is twice as heavy as Gretchen. Jan weighs 18kg less than Alan. What is Jan's weight?

6) There are 3 times as many boys as girls. If there are 24 more boys than girls, how many children are there all together?

Part V: Multiple Choice

Instructions: For each of the following questions, you will be given a diagram and three word problems. Examine the diagram carefully and choose the word problem from the three choices that best corresponds to the diagram. Choose your response by circling the problem.

1)

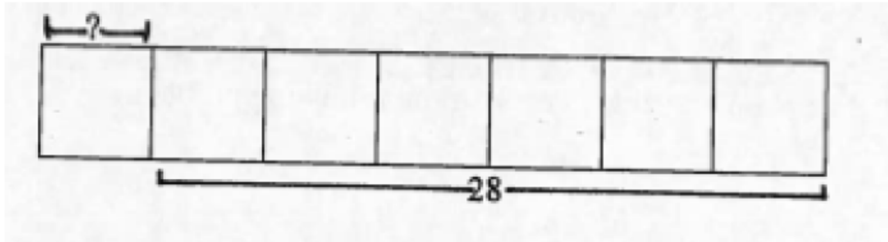


a) A high school tutor charges $13 an hour for tutoring and a certain amount for expenses. If he tutors for 3 hours and makes $115, how much did he charge for travel expenses?

b) There is 13 cm of snow already on the ground and then it snows for 3 days, with an equal amount of snow falling on each day. By the end of the third day there was 115 cm of snow on the ground. How much fell on each day?

c) A musician charges a certain amount for his CD, for every 3 CDs it costs a total of $13 in production costs. If he charges $115 for 3 CDs, how much did each CD cost him?

2)

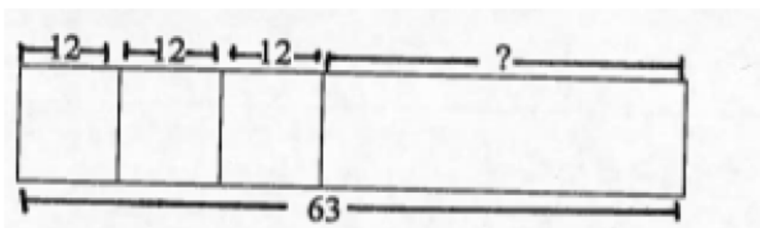


a) Jade and Madison go trick or treating together, Madison collects 28 candies, Jade collects 6 less than Madison. How many candies does Jade collect?

b) Sam has 28 toy cars. He decides to share 1/6th of his toy cars with his brother Max. How many cars does he give to his brother?

c) A store sells two different size boxes of chocolates. The larger box has 28 chocolates in it. The lager box minus the number of chocolates in the smaller box is the same as 6 times the number of chocolates in the smaller box.

3)



a) Jim decides to go golfing on the weekend and finds 63 golf balls in his golf bag. He also finds 3 unopened packages of golf balls in his car. How many golf balls does Jim have?

b) Jenna has $63 dollars and goes to a bookstore and sees a table that has a special on books. Each book is $12 and she decides to buy 3. How much money does Jenna have left?

c) A job opening has 63 people apply, 12 people are selected to come in for 3 different interviews. How many interviews were conducted?

Part VI: Equations

Instructions: Solve for the unknown value in each of the following problems. Show your work.

1)

$4x - 28 = 20$

2)

$(4 * 8) - 13 = x$

3)

$(x + 5) + 3x = 165$

4)

$\frac{x + 25}{3} = 14$

5)

$x + 6 = 4x$

6)

$\frac{6 + 15}{7} = x$

Appendix C

Instructional Materials: Session 1		
	Word Equation	Materials for BDL only
Single Referent, Start Unknown	1) I Start with some number. I divide it by 2. I add 8 to that number, and get 22. What number did I start with?	Two glass cylinders filled with specified amounts of sugar.
	2) I start with some number. I multiply it by 3. I add 20 to that number, and I get 50. What number did I start with?	Bristol board paper in the shape of cylinders, stacked on top of each other to represent the amounts specified.
Double Referent	3) I start with some number. I add 24, and I get 3 times the original number. What number did I start with?	Coloured pieces of ribbon, measured to represent the amounts specified.
	4) I start with some number. I add 15 to it, and I get 4 times the original number. What number did I start with?	Two glass cylinders filled with specified amounts of water.

Instructional Materials: Session 2		
	Word Equation	Materials for BDL only
Single Referent, Start Unknown	1) I start with a number. I divide it by 5. I subtract 3 from that number, and I get 4. What number did I start with?	Coloured pieces of ribbon, measured to represent the amounts specified.
	2) I start with some number. I multiply it by 2. That number is 9 more than half the number I started with. What number did I start with?	Two glass cylinders filled with sugar and labeled with the amounts specified.
	3) I start with some number. I divide it by 3. I then subtract 4, and I get 2. What number did I start with?	Play dough cut into sections.
Double Referent	4) I start with some number. I divide it by 4. That number is 12 less than the number I started with.	Two glass cylinders filled with water and labeled with the amounts specified.

Instructional Materials: Session 3		
	Word Equation	Materials for BDL only
Single Referent, Result Unknown	1) I start with 4 ½. I divide it by 2. Then I add 3. What number do I get?	Three glass cylinders filled with sugar and labeled with the amounts specified.
Double Referent	2) I start with a number. I multiply it by 2 ¼. The number I get is 20 more than the number I started with. What number did I start with?	Coloured ribbon folded into sections of specified amounts.
	3) I start with a number. I divide it by 4. Then I get a number that is 18.75 less than the number I started with. What number did I start with?	A loaf of bread cut into sections of the specified amounts.

Appendix D

Participant Practice Materials: Session 1		
	Word Equation	
Single Referent, Start Unknown	Starting with some number, I divide it by 3. I add 7 to that number and I get 22. What number did I start with?	Starting with a number, I add 15 then I divide that number by 5, I get 9. What number did I start with?
	Starting with some number, I add 64 and I get 4 times the original number. What number did I start with?	Starting with some number, I divide it by 5 and I have 40 less than the number I started with. What number did I start with?
	I start with some number. I add 20 to one tenth of that number, and I get 45. What number did I start with?	I start with a number. I add 12 to twice that number, and I get 32. What number did I start with?
Double Referent	I start with a number and I divide it by 3. The number I started with is 18 more than that number. What number did I start with?	

Participant Practice Materials: Session 2		
	Word Equation	
Single Referent, Start Unknown	I start with a number and multiply it by 6. Then the number is 25 more than the number I started with. What number did I start with?	I start with a number. I divide it by 4. Then I subtract 12 and I get 9. What number did I start with?
	I start with a number and I divide it by 5. Then I subtract 2 from that, and I get 10. What number did I start with?	I start with a number. I multiply it by 3 and I add 35 to that. Then I get 215. What number did I start with?
Single Referent, Result Unknown	Starting with 23, I take away 8 from it and multiply the result by 4. I get a number. What is it?	
Double Referent	I start with a number. I take away 26 from that number, and I have a third of the number I started with. What number did I start with?	I start with a number and divide it by 6. The number I started with is 70 more than that number. What number did I start with?
	I start with a number. I multiply it by three, subtract 15, and I get half of the number I started with. What number did I start with?	

Participant Practice Materials: Session 3		
	Word Equation	
Single Referent, Start Unknown	Starting with a number, I divide it by 4, then I multiply it by 2, I get 37.2. What number did I start with?	Starting with a number, I subtract it by 5.6, then I multiply it by 6, I get 58.2. What number did I start?
	Starting with a number, I multiply it by $2\frac{1}{2}$ and I get 15 more than the number I started with. What number did I start with?	
Single Referent, Result Unknown	Starting with $5\frac{1}{2}$, I divide it by 2 then I add 37. What number do I get?	Starting with 22.03, I multiply it by 4. Then I subtract 18.5 from the result. What number do I get?
Double Reference	Starting with some number, I divide it by 3, and I get a number that is 34.14 less than the number I started with. What number did I start with?	Starting with a number, I divide it by 4 and I get a number that is $63\frac{1}{4}$ less than the number I started with. What number did I start with?

(table continues)

Participant Practice Materials: Session 3		
	Word Equation	
Double Referent	I start with a number and I multiply it by $4\frac{1}{3}$. Then I add 8, and I end up with a number that is 5 times the number I started with. What number did I start with?	I start with a number. I multiply it by 5 and then add 18. I end up with a number that is half of the number I started with. What number did I start with?
	Starting with a number, I multiply it by $2\frac{1}{2}$ and I get 15 more than the number I started with. What number did I start with?	