An Investigation of the Conditions Under Which Procedural Content Enhances Conceptual Self-Explanations in Mathematics

Annick Lévesque

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Abstract

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The present study was an attempt to understand the nature of young children's self-explanation in the domain of procedural and conceptual knowledge in mathematics. The objective of this study was to examine the relationship between task demands on three outcome variables: (a) quantity of self-explanation, (b) quality of self-explanation (procedural and conceptual), and (c) conceptual knowledge, or an understanding in mathematics of the concepts central to each task. Three different self-explanation tasks were developed for the present study. Each task was based on one of the following theoretical assumptions about the generation of self-explanation: (a) learning through discovery (Self-Explanation Task 1), (b) direct instruction (Self-Explanation Task 2), and (c) effect of surprise (cognitive conflict; Self-Explanation Task 3). Prior knowledge in mathematics was investigated as a possible moderating variable in the analyses.

Thirty second-grade students were interviewed four times. In the first interview, the Number Knowledge Test (NKT) was individually administered to participants to measure their prior knowledge of counting, number, and quantity. In the second, third, and fourth interviews, one of the three self-explanation tasks was administered. A near transfer task was administered at the end of each self-explanation task to measure the knowledge of the concepts central to each.

The data revealed that the task based on direct instruction produced the highest number of self-explanations compared to each of the other two tasks, regardless of prior knowledge. In addition, a significant interaction was found between task type and prior knowledge on the generation of conceptual self-explanations. Specifically, low prior knowledge students generated higher quality self-explanation (i.e., conceptual self-explanations) on the task based on discovery learning, whereas high prior knowledge students produced better quality self-explanations on the task based on surprise. No such interaction was found for procedural self-explanations, but the data revealed that the task based on direct instruction produced a higher number of procedural self-explanations compared to either of the other two tasks. Finally, the results indicated that there was a significant correlation between task type and conceptual knowledge, with evidence suggesting that the task based on surprise was more effective than the task based on direct instruction in enhancing conceptual understanding of key mathematical principles. A number of implications for the classroom are discussed, not least of whieh includes the importance of maintaining a prominent place in the mathematics curriculum for procedural instruction.

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CHAPTER 1: STATEMENT OF THE PROBLEM

Recent research has reported disturbing findings about Canadian and American elementary and high school students. They do not perform at a reasonable level of proficiency in mathematics, probably because they master mostly procedural skills rather than higher order skills (Kilpatrick, Swafford, & Findell, 2001). In Canada, for instance, the results of the 2003 Program for International Student Assessment (PISA) showed that only 6% of Canadian 15- and 16-year-olds were able to conceptualize and apply with understanding the mathematical knowledge they learned (Organization for Economic Co-operation and Development [OECD], 2004). In Québec, high school graduates obtained similar results. In the 2005 final provincial examinations, for example, only 68.2% passed the mathematics test (Québec Ministère de l'Éducation, du Loisir et du Sport, 2007). In sum, Canadian students perform poorly in mathematics; they know how to compute and use formulas, but do not always understand the principles underlying them.

Although mathematics is assumed to be a complex subject by most students and parents (Ashcraft, 2002), research studies showed that the type of instruction, rather than the complexity of the subject, may be a stronger predictor of students' poor performance (Kilpatrick et al., 2001). This finding led many countries to elaborate and implement educational reforms. The province of Québec followed this trend; a reform was notably developed for the primary grades and implemented all over the province in 2001 (Québec Ministère de l'Éducation, 2001). Instead of focusing on the acquisition of mathematical facts and procedures, the new curriculum encouraged instruction that emphasized the understanding of mathematical concepts. This new orientation is reflected in several research studies demonstrating that conceptual instruction, rather than procedural instruction, improves students' mathematical performance (Hiebert & Wearne, 1996; Mack, 1990; Perry, 1991). As a result, teachers were asked to embrace a problem-solving approach that focuses on conceptual instruction to evaluate students' comprehension and reasoning and to foster communication in mathematics.

The effectiveness of the approach promoted in the new mathematics curriculum in Québec is, however, open to debate. So far, the effects of this curriculum on students' learning and development of mathematical competencies have not been studied rigorously, and questions about its effectiveness remain unanswered. Recently, teachers, citizens, and the government have been arguing that the new curriculum diminishes the quality of education because it focuses too much on understanding and competencies at the expense of acquiring factual knowledge (Bédard, 2006; Pierre, 2006; Robitaille, 2008).

In the same ways, scholars have been disputing the issue of whether mathematics instruction should emphasize conceptual understanding or procedural skills (Byrnes, 1992; Byrnes & Wasik, 1991; Hiebert & Wearne, 1996; Kamii & Dominick, 1998; Ohlsson & Rees, 1991). Some of them have compared procedural instruction (i.e., informing learners exactly how to perform sequential steps to solve problems) to conceptual instruction (i.e., why a problem should be solved in a particular way) to find which of the two is more effective in fostering students' mathematical proficiency. Although many studies propose that conceptual instruction seems to be more effective in terms of both conceptual and procedural learning (Mack, 1990; Perry, 1991), some of them suggest, however, that procedural instruction can also be beneficial for conceptual and procedural growth under certain conditions and in certain mathematical domains (see Rittle-Johnson & Siegler, 1998 for a review).

These ambiguous findings about instruction led some researchers to study more closely the nature of these two types of knowledge, conceptual and procedural, involved in mathematical proficiency. Recently, scholars proposed to look at conceptual and procedural knowledge from a different perspective. Basically, they suggested that they might develop iteratively: gains in one type of knowledge lead to gains in the other type (Rittle-Johnson & Alibali, 1999; Rittle-Johnson, Siegler, & Alibali, 2001). In light of these results, conceptual and procedural instruction may both be necessary to develop mathematical competency. If this proves to be the case, this would lead to major modifications in the current mathematics curriculum for primary grades -- that is, a shift toward greater emphasis on standard procedures.

While recent studies suggest that procedural knowledge is important for becoming mathematically competent, this type of knowledge must be taught appropriately. Procedures learned without any links to their underlying principles certainly limit the possibility of conceptual growth (Osana & Pitsolantis, 2009). In addressing this issue, some researchers have attempted to find ways to enhance children's understanding of the mathematical procedure and formulas in science. In particular, they demonstrated that self-explanations might account for the variability in how much students learn from procedural instruction (Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Rittle-Johnson, 2006). Self-explanation is defined by Siegler as "inference concerning how and why events happen" (2002, p. 37) and was found to improve learning and understanding in several domains, notably in mathematics (Neuman, Leibowitz, & Schwarz, 2000; Renkl, 1997, 2003; Siegler, 1995, 1999, 2002; Stigler & Hiebert, 1999). More specifically, the latter research studies found that individuals who use more and better self-explanations while learning mathematical procedures tended to make sense of them, thereby enhancing their understanding of how and why they work.

Although research has explored the benefits of self-explanation on conceptual understanding in mathematics (Renkl, 1997, 2003; Rittle-Johnson, 2006; Tajika, Nakatsu, Nozaki, Neumann, & Maruno, 2007; Wong, Lawson, & Keeves, 2002), little is known about the conditions that prompt children with different mathematical backgrounds to generate the quantity and the type of self-explanations that would lead to growth in understanding while learning a mathematical procedure. This project was an attempt to better understand these conditions in the domain of conceptual and procedural knowledge in mathematics. More specifically, it looked at the impact of task demands on: (a) the quantity of self-explanations, (b) the quality of self-explanations, and (c) conceptual understanding of low and high prior knowledge children. Because most studies on self-explanation were conducted with older children, this research examined the self-explanations of younger children (second grade). As a consequence, this project promises to contribute to the already existing literature on the topic.

The importance of studying self-explanation was vividly highlighted by Siegler (2005), who sees it as an essential tool to promote better mathematics instruction in our schools:

> One reason why requests to explain observations and statements by teachers and textbooks is of such interest is its potential for improving classroom instruction. Encouragement of self-explanations requires no technology or funding to use, can be applied to virtually any subject, and has already shown to improve the learning of persons from kindergarten age through adulthood. Moreover, such questions can be used to supplement almost any curriculum. Thus, self-explanation seems very promising as a means for improving education. (p. 775)

Considering Sielger's position, I conclude that research on the conditions (i.e., types of tasks and prior mathematical knowledge) under which the generation of more and higher quality self-explanations are enhanced and foster students' conceptual understanding of a mathematical procedure is essential for better mathematics instruction and success for all students. Thus, students, educators, and administrators will certainly benefit from the insights offered by this study.

CHAPTER 2: REVIEW OF THE LITERATURE

To help students understand the conceptual basis of mathematical procedures, several scholars have proposed that teachers should encourage students to self-explain during instruction. Self-explanation is generally defined as a metacognitive strategy that leads students to verbalize the task or the information given or beyond (Chi & VanLehn, 1991). Flavell (1976) was the first to define metacognition as "one's knowledge concerning one's own cognitive processes and products or anything related to them. Metacognition refers furthermore to the active monitoring of these processes in relation to the cognitive objects or data on which they bear, usually in service of some concrete goal or objective" (p. 232). Because the growing literature on self-explanation demonstrated its benefits on learning in mathematics (Siegler, 1995, 1999, 2002; Wong et al., 2002), it is reasonable to think that it could help students make sense out of procedures, thereby addressing the actual problem of poor mathematical performance.

Of particular interest, researchers have studied the relationship between individual differences and types of self-explanation. For instance, Chi et al. (1989) examined the relation between academic achievement (i.e., successful and unsuccessful students) and the types of self-explanations the students produced while learning and explaining a procedure. They found that the quantity (i.e., number of statements) and the quality (i.e., self-explanations with links to concepts) of students' self-explanations varied according to their academic performance. More precisely, they demonstrated that successful students generated higher quantity and better quality self-explanations than the unsuccessful students.

Although self-explanation has been studied from different angles, the pedagogical conditions under which the quantity and quality of self-explanations are enhanced and produce growth in conceptual understanding are not sufficiently investigated. In that regard, one underexplored condition is that of task type. Because the design of tasks has been demonstrated to influence students' learning and understanding (Henningsen & Stein, 1997; Lappan, 1993; Osana, Lacroix, Tucker, & Desrosiers, 2006), this variable may indeed influence the effects of self-explanation. A few studies confirm this assumption. For instance, Ainsworth and Loizou (2003) used two different task demands in the domain of biology (text and diagram), and Renkl (2003) presented different formats of worked-out examples (i.e; traditional method of using example-problem versus progressively removing steps in worked-out solutions). Nevertheless, most scholars who investigated the effects of self-explanation in mathematics only used one type of task to address their research questions, more specifically, a worked-out example. Added to this, few studies have examined the effect of task demands as a function of individual characteristics, such as prior knowledge (one notable exception is Osana et al., who studied the relationship between preservice teachers' prior knowledge and their identification of tasks in the elementary mathematics curriculum). Studying these factors in more depth is essential to unlocking the full potential of self-explanation as a tool for understanding the conceptual basis of mathematical procedures.

The Nature of Self-Explanation in Mathematics

Self-explanation has a wide variety of definitions. A review of these justifies my choice of a definition that is best suited to the topic of procedural and conceptual understanding in mathematics, the focus of the present study.

Definition

In a study of middle school students' learning biological texts, Chi, de Leeuw, Chiu, and LaVancher (1994) defined self-explanation as "any utterance that went beyond the information given, namely, an inference of new knowledge" (p. 454). Similarly, Siegler (2002) described self-explanation as inferences about causal connections among objects, events, or how procedures work. Basically, these definitions present self-explanation as a verbal activity that uses oral speech to infer new knowledge on "how" and "why" events happen or the way they are.

Other scholars have broadened these definitions to include specific cognitive processes. Neuman et al. (2000), for instance, offered an operational definition of self-explanation. For them, self-explanations are "utterances that involve not only inference of new knowledge but also clarification of the problem and justification of activities that occur during the problem-solving process" (p. 199).

Whereas previous researchers have focused on the cognitive aspect of self-explanation, others have defined it from an educational point of view, specifically from a learner's perspective. In particular, Roy and Chi (2005) view self-explanation as "a domain general constructive activity that engages students in active learning and insures that learners attend to the material in a meaningful way while effectively monitoring their evolving understanding" (p. 275). Similarly, Crippen and Earl (2007) consider self-explanation as "a form of self-talk where a learner engages in an iterative personal dialog while engaged in problem solving. This dialog aids the learner in identifying problem states and potential solution moves" (p. 811).

Although various definitions exist, it is unquestionably evident that self-explanation is in its broadest sense related to "a family of verbal activities aimed at making something more understandable" (Neuman et al., 2000, p. 3). Considering all of the previous definitions and the purpose of this project, self-explanation is defined here as utterances or verbal mediation that may help the learner to actively make sense out of a mathematical procedure and consequently, to understand it.

The Relationship Between Procedural and Conceptual Knowledge in Mathematics

To become mathematically competent, many scholars confirmed that both procedural and conceptual knowledge are necessary (see Baroody, Feil, & Johnson, 2007; Kilpatrick et al., 2001; Star, 2005). They do not know much, however, about how each form of knowledge influences one another to enhance children's understanding of mathematics. In fact, the mechanism under which both types of knowledge influence one another remains unclear.

This under-researched phenomenon comes after a long history of debate about what conceptual and procedural knowledge are, and how they have been perceived in terms of mathematics learning and instruction. Traditionally, conceptual and procedural knowledge were viewed as distinct entities that were often distinguished by "understanding," for conceptual knowledge, and "skills," for procedural knowledge. Later, Hiebert and Lefevre (1986) expanded these vague definitions. Specifically, they described conceptual knowledge as:

> knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network. (p. 3)

As for procedural knowledge, they defined it as follows:

> One kind of procedural knowledge is a familiarity with the individual symbols of the system and with the syntactic conventions for acceptable configurations of symbols. The second kind of procedural knowledge consists of rules or procedures for solving mathematical problems. Many of the procedures that students possess probably are chains of prescriptions for manipulating symbols. (p. 7)

In response to Hiebert and Lefevre, Byrnes (1992) later offered a simpler definition of these two terms. Precisely, he contended that conceptual knowledge could be defined as "knowing that" and procedural knowledge as a "knowing how."

More recently, Rittle-Johnson and Siegler (1998) have paid attention to the relationship between the two concepts rather than their definitions (see also Rittle-Johnson & Alibali, 1999). This new focus evidently questions the traditional dichotomy in the two forms of knowledge, though some scholars continue to believe that some distinction is still essential in understanding mathematics learning (Hiebert & Wearne, 1996). From this point of view, Rittle-Johnson et al. (2001) reconceptualized these two constructs as follows:

> We define conceptual knowledge as explicit or implicit understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain. We define procedural knowledge as action sequences for solving problems. These two types of knowledge lie on a continuum and cannot always be separated; however, the two ends of the continuum represent two different types of knowledge. These two types of knowledge do not develop independently. (p.175)

Similar to the debate that led to this new conceptualization of procedural and conceptual knowledge, to which this study adheres, the instruction of mathematics in the classroom followed a parallel history of debate and reconsideration in the mathematics education community. In particular, scholars argued about whether mathematics instruction should focus on a conceptual or a procedural understanding of the discipline (Wu, 1999). In an effort to shed light on this issue, Perry (1991) compared procedural instruction to principle-based instruction (i.e., conceptual instruction) on fourth- and fifth-graders' understanding of mathematical equivalence. In her first experiment, she assigned participants to one of two instructional conditions, procedural and principle-based. Her findings showed that students who received the principle-based condition generated better results on a transfer task (i.e., problems different from the ones shown during instruction) than the ones in the procedural condition. Nevertheless, participants in the procedural instruction performed better on comparable problems taught during instruction. In her second experiment, she examined if a combination of both instructional approaches would increase learning. Surprisingly, she found that students receiving a mixture of both instructional conditions performed just like children in the procedural group in her first experiment. Thus, Perry claimed that procedural instruction assists children to correctly use mathematical procedures, but does not lead to understanding beyond competence with those procedures. Consequently, she concluded that principle-based instruction was the best method to foster children's understanding.

While Perry (1991) found that conceptual instruction was more effective than procedural instruction in terms of understanding, recent studies (Rittle-Johnson & Alibali, 1999; Rittle-Johnson et al., 2001) suggest, in contrast, that procedural instruction can lead to a moderate gain in conceptual understanding. These findings put into question the importance of conceptual instruction over procedural instruction. Notably, Rittle-Johnson and Alibali compared procedural and conceptual instruction on fourth- and fifth-graders' knowledge of equivalence. Their study showed that either type of instruction led to increases in children's conceptual and procedural knowledge. In light of these results, Rittle-Johnson and Alibali propose that both forms of knowledge develop iteratively and in tandem, which implies that mathematics instruction should focus on both procedural and conceptual knowledge.

Supporting this speculation, some scholars showed that either forms of knowledge may be acquired first. Rittle-Johnson and Siegler's (1998) review asserted that procedural knowledge may be acquired before conceptual knowledge in certain domains. In particular, they mentioned that preschool children learn to count procedurally (i.e., 1, 2, 3, ...) before fully understanding the concepts that underlie counting, such as cardinality and order-irrelevance (Baroody, 1992; Wynne, 1990). Conversely, they stated that in other mathematical domains, conceptual knowledge may develop before procedural knowledge. They mentioned, for instance, that children usually understand the concepts behind fraction addition before using correct calculation procedures. Taken together, these findings confirm that both procedural and conceptual knowledge should be part of mathematics instruction, at least at the elementary level.

While some scholars argue that children's conceptual and procedural knowledge are necessary and develop in tandem, the mechanisms under which these two forms of knowledge influence each other are still unclear. In particular, how procedures can enhance conceptual knowledge is largely unknown and has recently been the focus of some speculation. To address this issue, Rittle-Johnson and Alibali (1999) suggest that "prompting children to explain why procedures are correct may help to increase the impact of procedural knowledge on conceptual knowledge" (p.187). In other words, they propose that self-explanation may be one of the mechanisms that fosters this bidirectional influence of procedural and conceptual knowledge.

The Effects of Self-Explaining in Mathematics

Many studies have shown that self-explanation may help students gain a deeper understanding of the conceptual basis of a procedure or solution in mathematics (Matthews & Rittle-Johnson, 2009; Renkl, 1997, 2003; Rittle-Johnson, 2006; Tajika et al., 2007). This has been called "the self-explanation effect" (Chi et al., 1989). One study by Stigler and Hiebert (1999) illustrates the effect of self-explanation quite vividly. These two scholars showed that Japanese students tend to perform better in mathematics than their American counterparts because they are encouraged to self-explain correct and incorrect mathematical solutions in mathematics class. In fact, Japanese students and teachers spend considerable class time explaining why two different solution procedures generate the same answer, and why seemingly plausible approaches yield incorrect answers. In sum, "encouraging children to explain why procedures work appears to promote deeper understanding of them than simply describing the procedures, providing examples of how they work, and encouraging students to practice them – the typical approach to mathematics instruction in the US" (Siegler, 2002, p. 38).

In line with Stigler and Hiebert (1999), Brown and Kane (1988) looked at the transfer abilities of preschool children when they self-explain. Specifically, they examined children's abilities to apply a rule about mimicry in nature (e.g., a caterpillar, which looks like a snake, is protected from being eaten by birds) to new examples. They found that young children who self-explained the general principle taught, either spontaneously or after the rule was explained by the teacher, applied it better in other contexts compared to the students who did not. Taken together, the previous findings provide evidence that self-explanation might be one possible mechanism that fosters the understanding of a procedure, thereby producing gains in conceptual knowledge. For this reason, it appears to be crucial that educational researchers and practitioners search for instructional interventions that foster self-explanation and as a consequence, lead to desired learning outcomes.

Individual Differences, Types of Self-Explanations, and Learning

Apart from demonstrating the self-explanation effect on understanding, some researchers have shown that individual differences, such as mathematical achievement, can influence the types of self-explanation generated by students. Specifically, some studies compared the types of self-explanations among successful and unsuccessful learners, more precisely in the context of learning new material from worked-out examples in the domain of science and mathematics (Chi et al., 1989; Neuman et al., 2000). In particular, Chi et al.'s seminal study revealed differences in types of self-explanation and in quantity of self-explanation among two groups of students, successful and unsuccessful. In their study, Chi et al. selected ten university students, none of whom had previously taken physics courses before. During the intervention, participants were required to read a text on basic concepts and definitions of Newtonian mechanics. Then, they were asked to think out loud while studying three worked-out examples. Afterwards, they were required to solve two sets of problems independently and think out loud. Finally, a posttest was administered.

According to their performance on the posttest, students were divided into groups of successful and unsuccessful students. Chi et al.'s (1989) findings showed that while solving problems, successful learners generated more and better quality self-explanations. More specifically they: (a) tended to study the worked-out example by giving justifications for each action and relating these actions to the principles and concepts provided in the text, (b) accurately monitored their comprehension failures and successes while studying examples, (c) understood the examples when studying them, (d) used the worked-out examples as a reference when solving a problem, and (e) spent more time studying the worked-out examples, which supported the generation of a large number of self-explanations. In contrast, the unsuccessful learners were more inclined to reread or paraphrase instead of generating inferences, and they monitored their understanding poorly. Chi et al. concluded that successful and unsuccessful learners showed quantitative (i.e., time spent on task and quantity of self-explanations generated) and qualitative (i.e., self-explanations that included inferences and explanations about the underlying concept) differences in their types of self-explanations. Therefore, academic achievement seems to influence the quantity and the types of self-explanations generated by students.

Expanding on these conclusions, Renkl (1997) studied more directly the effects of the qualitative differences in types of self-explanation and learning outcomes. Renkl questioned Chi et al.'s (1989) findings because time on task was not controlled. He argued that time, rather than the quality of self-explanation, may have influenced the results and could explain the higher performance of successful learners. In a follow-up study, Renkl thus replicated Chi et al.'s study, but fixed the time on task for each individual. That way, the pure impact of the self-explanation effect could be isolated.

Renkl (1997) examined the self-explanations of 36 university students who were required to study probability calculation through worked-out examples. Even when controlling for time on task, Renkl found similar results to Chi et al. (1989) in terms of qualitative and quantitative differences in types of self-explanations among successful and unsuccessful students. Furthermore, Renkl found that certain types of self-explanations were significantly related to learning outcomes. Specifically, he found that students who self-explained in a successful style (which he called principle-based and anticipative reasoners) produced higher quality self-explanations and performed better on the posttest. In contrast, the students who self-explained in an unsuccessful style (which he called passive and superficial reasoners) generated low quality self-explanations and did not perform well on the posttest. Similar findings between learning outcomes and quality of self-explanation have been found by Tajika et al. (2007). Taken together, these studies confirm that differences in students' mathematical achievement can lead to quantitatively and qualitatively different types of self-explanations, which in turn appears to impact learning outcomes.

One possible explanation for the discrepant effects of self-explanation for unsuccessful mathematical learners is that these students do not possess as much prior knowledge in mathematics as successful learners. According to Zohar and Aharon-Kravetsky (2005), unsuccessful students often have lower levels of prior knowledge. Because self-explanation requires students to use their previous knowledge to explain the new material to be learned, students with lower prior knowledge (LPK) may not have the necessary mathematical background to generate more and higher quality self-explanation than students with higher prior knowledge (HPK). In that regard, Woloshyn, Pressley, and Schneider (1992), who studied the effect of training in the use of elaborative-interrogation (i.e., formulation of elaborations and inferences about information to be learned), found that prior knowledge was a factor influencing the quality of students' self-explanations.

Nevertheless, research studies that examined the effects of prior knowledge on the quantity and quality of self-explanations generated revealed contradictory findings. For instance, Renkl (1997) found that types of self-explanations did not seem to significantly depend on prior knowledge, which is consistent with the results of Chi and VanLehn (1991). Similarly, Renkl, Stark, Gruber, and Mandl (1998) found that "only the very proximal cognitive prerequisite [topic-specific knowledge] of prior topic knowledge seems to be relevant in this context" (p. 106).

In sum, prior knowledge may be a predictive factor of self-explanation, but likely not the only one. In that regard, Roy and Chi (2005) suggested that future studies on self-explanation should also focus on other factors, such as pedagogical conditions (e.g., task types, learning context, and instructional formats). In this study, I will thus focus on prior knowledge, but also on identifying task demands that could enhance LPK and HPK students' quantity and quality self-explanations and their understanding of mathematical procedures.

Task Effects for Effective Self-Explanation Outcomes

In an effort to shed some light on the pedagogical conditions promoting effective self-explanation, some scholars have investigated the effects of self-explanation on learning within different instructional designs. Specifically, several research studies used spontaneous self-explanation (Bruin, Rikers, & Schmidt, 2007; Chi et al., 1989; Neuman et al., 2000; Renkl, 1997), self-explanation prompts (Chi et al., 1994; Renkl et al., 1998), and self-explanation modeling or training (Bielaczyc et al., 1995; Wong et al., 2002), including feedback (Aleven & Koedinger, 2002), to examine the impact of these instructional features on mathematical achievement. Most of them concluded that the effect of self-explanation on learning is greater when students are trained to self-explain, when they receive a model of it, or when they receive feedback on how to self-explain.

Nevertheless, several other studies demonstrate that even when explicitly trained, prompted, or guided, the quality of students' self-explanation is not always enhanced, indicating the difficulty for some learners to engage in this activity (Roy & Chi, 2005). Thus, much work remains to be done in elucidating the pedagogical conditions under which students of different academic levels tend to produce more and better self-explanations when learning a mathematical procedure.

One interesting pedagogical avenue to explore the effect of self-explanation is that of task demand. Indeed, specific task demands may help unsuccessful and successful students to generate self-explanations that differ in quantity and quality, which in turn may improve growth in understanding. In the self-explanation studies previously described, task demand was not a variable because all participants were requested to perform the same tasks. If it had been considered, the results of their studies would have certainly been different and would have given other insights into the differences in types of self-explanation.

In one notable exception, however, Ainsworth and Loizou (2003) studied the effects of task demands on the quantity and quality of self-explanation and conceptual gains. They presented to the participants the same material to be learned in the domain of biology, but under two formats, text and diagram. For each condition, they asked students to self-explain what was presented in the material. They found that students in the diagram condition generated more (quantity) and better (quality) self-explanations and made greater gains in understanding the human circulatory system, thus showing that task demands can affect self-explanation and learning.

In addition, Matthews and Rittle-Johnson (2009) investigated the relationship between types of self-explanation and the learning of mathematics equivalence problems and two types of instructional formats based on two different task demands: (a) a procedurally-based task demand (i.e., instruction on an add-subtract procedure to use the equal sign properly) and (b) conceptually-based task demand (i.e., instruction about the relational function of the equal sign). Interestingly, they found in their first experiment that when compared to procedurally-based tasks, conceptually-based task demands (a) promoted the generation of high quality self-explanations, (b) produced superior conceptual gains, and (c) allowed students to transfer correct procedures even though they were not explicitly explained. In light of these results, they concluded that task demands may be a variable influencing the generation of high quality self-explanation and conceptual growth.

Also, Rittle-Johnson (2006) evaluated the effectiveness of self-explanation on procedural learning, procedural transfer, and conceptual knowledge in combination with two different instructional formats, direct instruction and discovery learning, that were used to design the tasks. Participants were assigned to one of four conditions based on crossing two factors: (a) direct instruction versus discovery learning and (b) self-explanation versus no self-explanation. Third- through fifth-grade students were asked to perform mathematical equivalence problems on a laptop. Participants in the direct instruction condition were taught a correct add-subtract procedure before solving problems; in the discovery learning condition, participants were prompted to think of a new way to solve the problems and were given feedback on their answers. In the self-explanation condition, students solved a problem and were then shown an additional screen with the correct and incorrect answers of the problem that they had to self-explain. The students in the no self-explanation condition were not shown additional screens. At the end of the intervention, an immediate and a delayed posttest were administered to measure procedural learning, procedural transfer, and conceptual knowledge. Interestingly, Rittle-Johnson found that self-explanation promoted transfer regardless of instructional condition. Nevertheless, neither manipulation (i.e., instruction and self-explanation) promoted greater improvements on conceptual knowledge.

Approaches to Self-Explanation Task Design

In this study, I chose three specific approaches to design three self-explanation tasks, two of which are based on the study of Rittle-Johnson (2006) mentioned above. I will call these approaches: (a) discovery, (b) direct instruction, and (c) surprise. Each of these, which will be discussed below, can be considered respectively as operationalizations of three educational theories that have been used by scholars to explain the teaching and learning of conceptual and procedural knowledge in mathematics: (a) discovery learning, (b) cognitive load theory, and (c) cognitive conflict.

Learning Through Discovery

Some scholars say that children who discover their own procedures (i.e., discovery learning) often have better transfer skills and conceptual knowledge than children who only adopt directly instructed procedures (Hiebert & Wearne, 1996; Kamii & Dominick, 1998), probably because students are constantly trying to find how things work by explaining it to themselves or to others (Goffin & Wilson, 2001). In the domain of mathematics, Goldman, Hasselbring, and the Cognition and Technology Group at Vanderbilt (1997) suggested that instruction for students with or without mathematical difficulties should focus on a discovery approach, which, for these authors, establishes strong connections among students' declarative, procedural, and conceptual knowledge. They argue that the prevalent idea among special educators -- that teaching basic mathematical skills or procedures to students with difficulties is necessary to succeed in mathematics -- may, on the contrary, impede the conceptual understanding of the principles underlying these procedures and thus, prevent these students from becoming mathematically competent. Goldman et al. consider, in fact, that even when procedures are in place, students with learning difficulties often fail to apply that knowledge in meaningful ways when confronted with problem situations. Therefore, they claimed that this kind of isolated skill instruction, even if it fosters fluent retrieval of declarative knowledge and efficient execution of specific procedures, fails to establish relationships among declarative, procedural, and conceptual knowledge, which leads to improvement in conceptual understanding. Based on this premise, therefore, creating conditions such that students are encouraged to discover the conceptual underpinnings of mathematical procedures may positively impact the quality and quantity of their self-explanations and their conceptual knowledge. In this study, I thus used a discovery approach, based on discovery learning, to design the first self-explanation task.

Direct Instruction

Certain information processing theories, such as cognitive load theory (CLT), propose that discovery conditions can overload working-memory capacity because too much information needs to be processed at the same time (Sweller, van Merrienboer, & Paas, 1998). It is indeed well established that humans have limited working-memory capacity (Adams & Hitch, 1998; Baddeley, 1986; Cook, 2006), particularly students with learning difficulties, who have even more restricted memory capacities (Geary, 2004; Gersten & Chard, 1999). An instructional approach that frees up students' working memory, thus giving them the chance to use their cognitive resources to understand the underlying principles of a mathematical procedure, is direct instruction (Kalyuga, 2009). In particular, the initial presentation of the task, in which the students receive prior instruction on the components to be learned, the strategies to solve the task, or the appropriate solution steps, helps consolidate procedures that can then be applied to whole tasks at a later stage (Mayer & Moreno, 2003). According to Mayer, Mathias, and Wetzell (2002), this initial instructional phase on individual to-be-learned components enhances learning.

One of Rittle-Johnson and Alibali's speculations (1999) about how students can understand the conceptual basis of procedures is in line with cognitive load theory: "When children use procedures, they sometimes think about why they work. When procedures are easy to implement, children may not use all their resources in implementing the procedure, and they may have resources available to consider the basis of the procedure" (p. 187). Based on this assumption, tasks that are easy to execute, making available the necessary cognitive resources to understand a procedure, may be conducive to the generation of more and higher quality explanations and gains in conceptual knowledge. In this study, I thus used an approach that reduced students' cognitive load, namely direct instruction, to design the second self-explanation task.

The Effect of Surprise

In the literature, Gredler (1992) claims that cognitive conflict is created when one's expectation or prediction, based on one's current reasoning, does not align with one's past experience or old conception. Although empirical evidence demonstrates the benefit of cognitive conflict as a mean for promoting conceptual change (Druyan, 1997; Hashweh, 1986; Thorley & Treagust, 1987), a review of the literature shows that diverging results were obtained regarding the effectiveness of cognitive conflict as a pedagogical tool. In particular, Zohar and Aharon-Kravetsky (2005) mentioned that students do not always gain conceptual knowledge when engaged in cognitive conflict tasks because they tend to patch their own mental model in a superficial way rather than in a more coherent way. Intrigued by these inconclusive findings, Limon (2001) analyzed a number of studies on cognitive conflict with the intention to propose conditions under which cognitive conflict is effective in the classroom. According to Limon, cognitive conflict implemented in the classroom produced conceptual change only under certain conditions. She postulated that the most important condition is that cognitive conflict must be meaningful to the students. To reach that stage of meaningfulness, students need, among others, a certain amount of prior knowledge. If students have little or no knowledge about the topic, it is difficult to expect any change because their understanding of the new information may be so minimal that the conflict is not meaningful at all. Based on these findings, one can assume that the cognitive conflict may be less effective with low prior knowledge students.

The latter assumption was confirmed by the work of Zohar and Aharon-Kravetsky (2005). In their study on the understanding of photosynthesis in ninth-grade students, they discovered that cognitive conflict may be useful for conceptual change in some circumstances, while in others, it may even be harmful. In particular, they found that cognitive conflict promotes conceptual change with high prior knowledge learners (HPK), but did not have the same effect on low prior knowledge students (LPK). Indeed, HPK students produced more inferences than LPK students about scientific experiments on the photosynthesis phenomenon: they drew conclusions using the essential components of the experiments presented (e.g., the rate of photosynthesis), whereas LPK students were unable to do so. Nevertheless, Zohar and Aharon-Kravetsky clearly stated that these results do not mean that cognitive conflict should never be used with LPK students. Cognitive conflict encourages students to use higher order thinking skills, which is an ability that all students need to practice. Interestingly, they mentioned that metacognitive supports, such as self-explanation, could help LPK students make the cognitive conflict accessible to them.

In the domain of mathematics, one of Rittle-Johnson and Alibali's (1999) speculations is in line with cognitive conflict: "in situations in which newly learned procedures result in different solutions than prior procedures, children may find this surprising, and this may lead them to consider the conceptual basis of the new procedure" (p. 187). In other word, students may be surprised when presented with the same problem solved in two different ways. Consequently, this might lead them to think about and understand the underlying principles of the mathematical procedure. This speculation was notably explored by Siegler (2002). While studying mathematical equivalence problems, Siegler found that students who had to explain two different answers for the same problem, a correct and incorrect answer, acquired a deeper understanding of the procedure. Thus, Siegler concluded that this task type encouraged students to reflect more deeply on the meaning of the procedures.

Based on these findings, designing tasks that create an effect of surprise may positively impact the quality and quantity of students' self-explanations and their conceptual knowledge. In this study, I will thus use this approach, which is in line with the cognitive conflict, to design the third self-explanation task.

The Present Study

Given the empirically supported links between the quantity and quality of self-explanation and conceptual knowledge, it appears that self-explanation may be a factor promoting the understanding of the conceptual basis of mathematical procedures. Because differences in types of self-explanations exist among successful and unsuccessful learners, which in turn seems to influence the quality of learning, it is important to look at conditions that promote both the quality and quantity of self-explanations and are related to conceptual understanding for both groups.

In this study, I thus examined the relationship between task type on the nature of students' self-explanations and their conceptual knowledge in mathematics. Three tasks were designed according to three plausible explanations generated in the literature for differences in students' self-explanation and learning: discovery, direct instruction, and surprise. Because this study involves young children (i.e., second-graders), successful and unsuccessful students were identified using their prior knowledge: low prior knowledge students (LPK) and high prior knowledge students (HPK). The model in Figure 1 illustrates the conceptual framework for the present study.

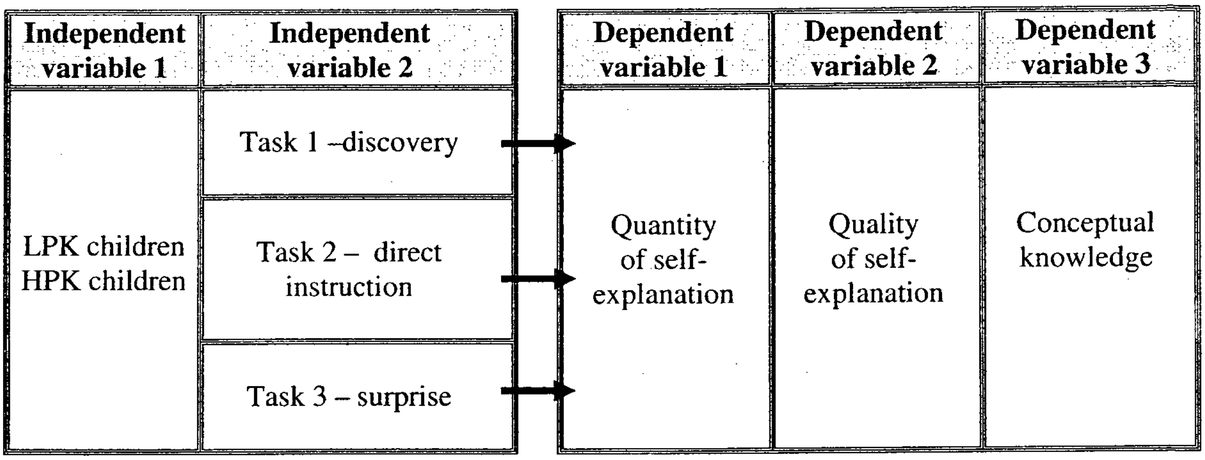

Figure 1. Conceptual framework for proposed investigation.

First, participants' prior knowledge was measured using the Number Knowledge Test (Okamoto & Case, 1996). Then, students' self-explanations were collected during three interviews, each using a different self-explanation task. The quantity of self-explanations was measured by the number of sentences (e.g., at least a subject and a verb) or an answer to a question (e.g., yes, no, or a numerical answer) generated during the interviews. The quality of self-explanations, on the other hand, was measured by coding the students' self explanations using a rubric based on Matthews and Rittle-Johnson's (2009) conceptual-procedural classification scheme. A near transfer task administered at the end of each interview was used to measure conceptual knowledge.

A review of the literature shows that the effects of self-explanations have been well-documented for middle-school and high-school students, university students, and adults (Ainsworth & Loizou, 2003; Chi et al., 1989; Neuman et al., 2000; Renkl, 1997, 2003; Renkl et al., 1998). Only a paucity of studies examined the effect of self-explanation on younger children, probably because their language skills are less developed at this age. Consequently, it is often assumed that the nature of children's self-explanations is similar to that of adults. Young children's self-explanations may, however, be different and produce different outcomes compared to older populations. Therefore, it is important to study younger children's self-explanations to understand them better and to help elementary teachers to use self-explanation as a tool to enhance the conceptual growth of their students.

CHAPTER 3: METHOD

The main goal of the present study was to understand the nature of young children's self-explanation in the domain of procedural and conceptual knowledge in mathematics. The study's objective was to examine the relationship between each of two independent variables, prior knowledge and task demands, on three outcome variables: (a) quantity of self-explanation, (b) quality of self-explanation, and (c) conceptual understanding of specific mathematical procedures selected for this study. Specifically, the research questions were the following:

1. With respect to quantity of self-explanation: (a) Is task type (discovery, direct instruction, and surprise) related to the number of self-explanations produced by the participants? and (b) does student prior knowledge moderate the relationship between task type and the quantity of self-explanations produced by the participants?

2. With respect to the quality of self-explanations: (a) Is task type related to the types of self-explanations produced by the participants (i.e., conceptual and procedural self-explanations)? and (b) does student prior knowledge moderate the relationship between task type and the generation of conceptual and procedural self-explanations produced by the participants?

3. With respect to conceptual knowledge: (a) Is task type related to the participants' conceptual knowledge after engaging in each of the three interview tasks? and (b) does prior knowledge moderate the relationship between task type and the participants' conceptual knowledge?

Research Design

In Figure 2 is a graphical representation of the variables and measures used at the five data collection time points.

Time 1	
Variable	**Measure**
Language Skills	Language Questionnaire

Time 2	
Variable	**Measure**
Prior mathematical knowledge	NKT

Time 3, 4, and 5	
Variables	**Measures**
Quantity of self-explanation	- SET-1[a] - SET-2 - SET-3
Quality of self-explanation	- SET-1 - SET-2 - SET-3
Conceptual knowledge	- Transfer task SET-1 - Transfer task SET-2 - Transfer task SET-3

Legend
SET-1: Self-Explanation Task 1
SET-2: Self-Explanation Task 2
SET-3: Self-Explanation Task 3

Figure 2. Variables and measures used in the present study.

[a] The order of task presentation was counterbalanced across participants.

I randomly selected 30 second-grade students from 4 Cycle I classrooms, whom I individually interviewed four times in March and April 2009 (Times 2 through 5 in Figure 2). At Time 1, the Language Questionnaire was completed by each teacher for each participant in her class. This questionnaire was designed to evaluate the students' general vocabulary and fluency in their mother tongue and subsequently used to control for language ability in the analyses of self explanation.

In the first interview (Time 2), I administered the Number Knowledge Test (Okamoto & Case, 1996) individually to the participants to measure their prior knowledge of counting, number, and quantity. The second interview (Time 3) was conducted approximately one week after the first interview, followed a week later by the third interview (Time 4), and a week after, by the fourth interview (Time 5). In each of the second, third, and fourth interviews, I administered one of the three self-explanation tasks developed for this study using the three approaches previously described: (a) Self-Explanation Task 1 (discovery), (b) Self-Explanation Task 2 (direct instruction), and (c) Self-Explanation Task 3 (surprise). These tasks were used to measure the quantity and quality of the participants' self-explanations. Task presentation was counterbalanced to eliminate any possible effects of order. A transfer task designed to measure the participants' conceptual knowledge was administered immediately following each self-explanation task. All interviews, with the exception of the first one, were videorecorded for subsequent coding and analysis.

Participants and Context

Thirty second-grade children were randomly chosen from four multiage classes (grade 1/2 split), each taught by a different teacher: 9 from class 1, 6 from class 2, 8 from class 3, and 7 from class 4. The age of the children ranged from 7 years and 5 months to 8 years and 4 months.

The participants came from a suburban school in the greater region of Montreal, which serves predominantly French-speaking Canadian, middle-income families.

The school is considered "alternative" because it had a specific educational philosophy based on Freinet's pedagogy, which promotes parent involvement and learning through concrete life experiences (Giroit & Poslaniec, 1985; Lee, 1993; Peyronie, 1999). Teachers in this school do not use textbooks. Instead, they teach writing and reading skills, as well as mathematics, through activities such as pen pals, projects conducted in and out of class, and children's own writing. Because of the innovative nature of many of the school's activities, the students are accustomed to the presence of multimedia equipment, such as video cameras, in their classrooms. In addition, most of the teachers teach in multiage classrooms.

Although textbooks are absent from reading and writing instruction, the teachers use a curriculum called *Challenging Math* (Lyons & Lyons, 2001) for mathematics instruction from grade 1 to grade 6. This curriculum conforms to the underlying principles of the educational reform in Québec, Canada (Québec Ministère de l'Éducation, 2001). Specifically, it emphasizes mathematical reasoning and understanding, problem-solving, and communicating in mathematics. One of the main principles of *Challenging Math* is that understanding a concept is more important than learning mathematical techniques. In other words, comprehension should take place before learning and applying procedures, such as algorithms. For example, addition and subtraction tables are not taught in Cycle I; rather, students are invited to play with numbers and understand the representation of a number (e.g., 6 can be represented by 2 + 4, 6 + 0).

Instruments and Measures

In this section, I will describe the six measures used in this study: (a) the Language Questionnaire, (b) the Number Knowledge Test, (c) Self-Explanation Task 1, (d) Self-

Explanation Task 2, (e) Self-Explanation Task 3, and (f) the measures of conceptual knowledge that followed each of the self-explanation tasks (near transfer tasks). Apart from the Number Knowledge Test, I created the other tests specifically for this study.

Language Skills

Language Questionnaire

Although I know of no study on the relationship between the ability to self-explain and language skills, it is reasonable to think that oral language skills may be related to the generation of self-explanation. In that regard, Aleven and Koedinger (2002) noted that considerable individual differences exist in students' ability to self-explain: some students self-explain more easily than others (Calin-Jageman & Ratner, 2005; Renkl, 1997). A possible reason for this variance is the difference in oral language skill. Therefore, in the present study, a questionaire was developed to control for this factor and was called the Language Questionnaire (see Appendix A).

A review of the literature shows that language comprises different components. In particular, oral fluency and vocabulary seem to be the two most influencial in regard to the ability to express oneself (De Jong & Verhoeven, 1992; Higgs & Clifford, 1982; Iwashita, Brown, McNamara, & O'Hagan, 2008). As a consequence, the Language Questionnaire was designed specifically for this study to measure these two language-related variables. The teachers were asked to rate each of the participants in their classes on vocabulary and fluency using a Likert-like scale from 1 (poor) to 4 (excellent). The Language Questionnaire is presented in Appendix A.

Prior Knowledge Measure

Number Knowledge Test (NKT)

Description of the task. The Number Knowledge Test (Okamoto & Case, 1996) is an instrument developed to evaluate children's understanding of number and quantity. Some scholars modified and normed the original tool for several populations of children in Canada and the United States, and it is now often used as an assessment tool to measure children's acquisition of number, counting, and quantity (Griffin, Case, & Capodilupo, 1995; Griffin, Case, & Siegler, 1994). In this study, a French translation of Griffin and Case's (1997) version of the Number Knowledge Test (NKT) was used (see Appendix B). This version measures students' understanding of magnitude, the concept of "bigger than," and strategies used in counting. Some items require props, such as plastic shapes. More specifically, the NKT comprises a series of structured probes that are categorized into four levels: (a) Level 0, for 4 year-olds; (b) Level 1, for 6 year-olds; (c) Level 2, for 8 year-olds; and (d) Level 3, for 10 year-olds.

Based on a sample of 470 students, Okamoto and Case (1996) calculated that the reliability of the NKT was .93. In addition to these scholars, Gersten, Jordan, and Flojo (2005) found in their study that the Griffin and Case's (1997) version of the NKT was an accurate screening measure to assess number sense. Their results indicated that the NKT was correlated with the Stanford Achievement Test-Ninth Edition (SAT-9), and more specifically, on two of its mathematics subtests: (a) Procedures and (b) Problem Solving. More specifically, the Total Mathematics Score on the SAT-9 was correlated .73 with the NKT, .64 for the Procedures subtest, and .69 for Problem Solving. All correlations were moderately strong and significant ($ps < .01$). This evidence shows that the NKT is a good measure of children's number knowledge.

Administration and scoring. The NKT is administered individually to children. According to the child's age, a particular level is selected, and the interview begins with the items at that level. The administrator reads the items orally to the participant, who responds orally. To pass a level, a child needs to respond correctly to a specific number of items: (a) Level 0 requires a minimum of three correct items to go to Level 1, (b) Levels 1 and 2 each require a minimum of five correct items to go on to the next level, and (c) Level 3 requires four correct items to pass that level. Testing continues until the child does not answer correctly on the required number of items at any given level.

Each item of the NKT is worth 1 point. Some items have two parts. Both parts must be answered correctly to earn a point for that item. The NKT total score, the Child Total Raw Score (CTRS), is calculated by summing all the points earned (the scoring grid is presented in Appendix C). The CTRS ranges from 0 to 32.

Outcome Measures

Measures of Quantity and Quality of Self-Explanation

These self-explanation tasks were previously developed by Lévesque and Osana (2008) in a pilot study because no instrument was found in the literature to measure the quantity and quality of young students' self-explanations while learning new mathematical procedures. Based on previous studies (Calin-Jageman & Ratner, 2005; Goldman et al., 1997; Rittle-Johnson, 2006; Rittle-Johnson & Alibali, 1999; Siegler, 1999, 2002), Lévesque and Osana's self-explanation tasks were developed using the same three theories tested in this study and described earlier: (a) discovery, (b) direct instruction, and (c) surprise. Their results indicated that some modifications to the tasks were necessary to better suit the students' cognitive level, such as using two-digit numbers in the task based on direct instruction and using another format to present the addition equations in the task based on surprise. Thus, the tasks were changed for use in this study to take into account these modifications.

Similar to the study of Calin-Jageman and Ratner (2005), I administered the self-explanation tasks in novice-expert dyad settings where children were required to explain the interviewer's behavior (the expert) or another child's behavior. In addition, the question asked by the expert to measure the quantity and quality of self-explanation was similar to the one used in the study of Calin-Jageman and Ratner because it was shown to be effective with young children. Specifically, these scholars used the following probe: "How did I [the expert] know that [the solution]?" The same question was always used after each self-explanation task to avoid any possible confounds. To prompt students to further self-explain, I also asked other types of questions specific to the task (between one to four additional questions). Lévesque and Osana's (2008) pilot study showed that children at this age sometimes have difficulty expressing their thinking; additional questions are hence necessary to help them articulate their thoughts. These questions will be described in more detail in the following sections.

*Self-explanation task one (SET-*1). This task was developed according to the notion that children who discover their own procedures often have more conceptual knowledge and better abilities to transfer this knowledge to novel problems than children who only adopt directly instructed procedures (Hiebert & Wearne, 1996; Kamii & Dominick, 1998). Based on this premise, I developed a task, the Self-Explanation Task One (SET-1), that required the child to discover concepts of grouping, place value, and the principle of counting in Base 3. Specifically, I played the role of an expert showing a "new" mathematical procedure to the child -- that is, showing how to count in Base 3 and how to represent quantities in Base 3 (Appendix D, part 1). To do so, I counted chips in Base 3 and represented the number on a flip chart. The child, in the role of observer, was asked to explain her understanding of the expert's demonstration. I gave no feedback to the child on any of her statements about the task during the interview.

At the beginning of the task, I said to the child: "I will show you a new way of counting. To do so, I will use chips and a flip chart. I would like you to try to understand the method I use to count them. I will also ask you to think out loud while I count the chips. In other words, tell me everything that is in your head as you watch what I am doing. I will do three examples with you, and you will try to understand my way of counting. After, I will ask you to use my method to count a certain number of chips. Is that OK with you?"

After this introduction, I showed five chips to the participant. I counted three chips with my fingers, one by one, and separated them from the other chips; as a result, a group of three chips was separated from the two chips left over. Then, I put the number "1" in the position of "tens" on the flip chart. After, I counted the chips left over (i.e., 2), and I put the number "2" in the position of "units." I then said that the number is "one two" (i.e., 12). After allowing a few seconds for the child to think, I asked, "How do you think I knew that?" After listening to the child's answer, I also asked, "What does the number 1 on the flip chart represent?" and "What does the number 2 on the flip chart represent?" Then, by pointing with my fingers, I asked: "How come the number 1 is in this position (i.e., "tens")?" and "How come the number 2 is in this position (i.e., units)?"

Subsequently, I showed the second and third examples of Base 3 counting to the child. The second task involved seven chips, and the third involved eight chips. The same actions and questions were used for all examples.

Self-explanation task two (SET-2). This task is based on the notion that when the demand on children's working memory is diminished, it can make cognitive resources available for understanding. In the domain of mathematics, Rittle-Johnson and Alibali (1999) speculated that reducing cognitive load could allow for greater attention paid to the conceptual basis of a procedure. One effective instructional format that has been shown to diminish students' cognitive load is direct instruction (Kalyuga, 2009; Mayer et al., 2002; Mayer & Moreno, 2003). Thus, I developed a task, the Self-Explanation Task 2 (Appendix E), that is based on diminishing the participants' cognitive resources as much as possible. The Self-Explanation Task 2 (SET-2) involves the concept a – a = 0 (otherwise known as "canceling out.") Specifically, I showed the child a simple procedure for solving equations rapidly, which entails cancelling out the identical numbers in the equation. For example, for the equation "15 + 11 – 11 = □," I told the child that by crossing out (i.e., cancelling out) the two identical numbers separated by a minus sign, 11 and 11, the number left gave the answer to the equation, namely 15.

This short instructional phase was intended to show children how to use the procedure without teaching them its conceptual basis. First, I told the child that I would teach her a "trick" (i.e., the procedure) to solve equations. I also told her that she must try to remember the "trick" as much as she can because she would have to use it later to solve other equations. Then, I presented the child with a series of six equations (all of them appear in Appendix E, part 1), and I showed her how to solve them using the cancelling procedure. While doing so, I spoke out loud to show my procedure to the child: "I look at the numbers in the equation, 12 + 15 – 15. I cross out the two identical numbers separated by a minus, 15 and 15 [I cross out the numbers with a pencil], and I know that the remaining number, 12, is the answer to the equation. So I write down 12 after the equal sign." Then, I solved the five remaining equations in the same way to continue the demonstration of the procedure.

At the end of this short instructional phase, the child was required to complete eight equations, using exactly the same procedure that I had just shown her (these equations appear in Appendix E, part 2). These eight equations have the same structure: (a) equations with three whole numbers to the left of the equal sign using addition and subtraction (e.g., four equations of the type a + b – b and four equations of the type b – b + a), (b) two identical numbers on the left side of the equal sign separated by a minus sign, and (c) numbers between 15 and 52. Larger numbers were chosen for each of the problems to minimize the likelihood that students would generate answers to the problems mentally using strategies other than the one demonstrated here.

To solve these eight equations, I asked the child to read the equations out loud (e.g., 33 + 24 – 24 =). In addition, I asked the child to say out loud what she was doing (e.g., "I am crossing the numbers that are the same.") and to tell me the answer while writing it on the sheet. If the child forgot to speak out loud, I reminded her by prompting: "So what are you doing? Please, say it out loud." At the end of the task, I asked the child to explain why she thought the procedure worked. Specifically, I asked the child the following question to examine her reasoning: "How do you think I knew that this trick worked?" I also asked: "Did this trick help you to solve the equations? Tell me how."

Self-explanation task three (SET-3). This task is based on the premise that two different solutions to the same problem can surprise children. According to Rittle-Johnson and Alibali (1999), this surprise may lead the child to reflect upon related concepts and think about why the discrepancy is happening, which in turn may generate conceptual gain.

Based on the possibility that children may reflect on the conceptual basis of two different solutions to the same problem (Siegler, 2002), I developed a task, the Self-Explanation Task 3 (SET-3), that involves the concepts of grouping and place value. Specifically, I showed the child four equations (the equations are presented in Appendix F, part 1), and I told her that another student from another school (Lucas, for the male participants, and Camille, for the female participants) solved these four equations in a different way than the one taught in school (i.e; in a novel way). I also informed her that all of the equations were solved correctly. I then asked the child to look at the first equation, and I left about one minute for her to examine the alternate solution. Then, using the question, "How do you think [Lucas or Camille] knew his or her answer was right?," I asked the child to think out loud about how he or she thought the problem was solved. The same procedure was used for the other three equations on the sheet.

Coding the quantity and quality of self-explanation. To measure the quantity and quality of the participants' self-explanations, children's videotaped interviews were used and were segmented into discrete videoclips, with each videoclip representing one self-explanation chunk (i.e., one sentence, phrase, or answer). All pauses made by the students were not considered in the data set. Because the participants were not only verbalizing, but also writing or counting chips, the video served as a context to code the participants' self-explanations with more fidelity than audiodata alone. In Chi et al.'s (1989) study, self-explanations were segmented at pause boundaries, and a line number was assigned to each chunk on the audio transcript. This methodology was adapted in the present study as a function of the age of the participants and because the data were in video format. The audio portion of the video was segmented into clips at sentence or answer boundaries. More specifically, one videoclip corresponded to one sentence containing: (a) a subject and a verb (e.g., "Cela fait 5."), (b) a subject, a verb, and a subordonant (e.g. "Cela fait 5 parce qu'il a additionne 2 et 3."), or (c) a single word or number to answer a question (e.g., "oui", "3"). Thus, the total number of videoclips were used to compute a measure of self-explanation quantity.

To measure the quality of self-explanations, the segmented videoclips were then coded using a rubric developed for Lévesque and Osana's (2008) pilot study. In the pilot study, young children's self-explanations fell into the two main categories, conceptual and procedural. Matthews and Rittle-Johnson (2009), who investigated the impact of types of instruction and task demands on the generation of self-explanation of young children, found that this type of classification was appropriate for younger students in the context of conceptual and procedural competence in mathematics. They gave each self-explanation a code that represented its quality in terms of conceptual and procedural content. In the present study, however, I created subcategories for each of these two main categories to illustrate more precisely the thinking of the children. Consequently, I coded the children's self-explanations as Conceptual Appropriate, Conceptual Inappropriate, Conceptual Irrelevant, Procedural Correct, and Procedural Incorrect. If not related to any of these types, the self-explanations were coded as "Other," and if they contained both Procedural and Conceptual content, they were assigned both codes. For example, a self-explanation was coded as Conceptual Appropriate when children were able to explain the conceptual basis of a procedure. A comment such as, "The 2 represents two packs of three and the 1, what is left.," shows an appropriate conceptual explanation of Base 3. In contrast, if children only described what they saw or what was done (e.g., "You counted three, then two. You put a 1 and a 2 on the flipchart."), then the self-explanation was placed in the category of Procedural Correct self-explanation. More examples and definitions of each of the types of self-explanations are found in Appendix G.

To calculate inter-rater reliability for coding the children's self-explanations, a random selection of 30% (10/30) of the interviews were coded by two people, an independent rater and the author of the present study. I first met the independent rater to train her on coding SET-2. I began with this task because its structure was familiar to the rater. We then coded two interviews together to make sure our coding was consistent, and that each category was well understood. Next, the rater coded four interviews on her own on which we obtained an agreement of 65%. Because the percentage was below 80%, I met the person again to review the coding of the four interviews and to revise the rubric itself when necessary. Then, the rater coded four new randomly chosen interviews. This time, we obtained an agreement of 80%, and the rest of the interviews were coded by the independent rater. For SET-1 and SET-3, the same reliability procedure was used. For SET-1, we obtained 70% agreement on the first four interviews coded; on the second round (4/30), we obtained 86%. For SET-3, we obtained an agreement of 74% on the first four interviews coded and 89% on the second set (4/30). The rest of the interviews were coded by the independent rater only.

Measures of Conceptual Knowledge

To measure conceptual knowledge, I added a near transfer task at the end of each of the three self-explanation tasks to measure the key concepts in each. The three near transfer tasks required the students to transfer what they had learned during the self-explanation tasks to new situations, success on which provided a measure of the students' conceptual knowledge of key concepts of the self-explanation tasks.

SET-1 Near Transfer Task. At the end of SET-1, children were required to use Base 3 to solve four problems (in Appendix D, part 2). For each problem, they needed to count 9, 10, 13, and 18 chips, respectively, using Base 3. These tasks required children to reproduce by themselves the procedure previously thaught with a higher number of chips, which added additional difficulty and required the application of the concepts of grouping, place value, and the principle of counting in Base 3.

SET-2 Near Transfer Task. At the end of SET-2, children were required to solve four equations of the type b + a – b, which were different from the ones previously solved in the interview (i.e., a + b – b and b – b + a; see Appendix E, part 3). These problems assessed children's understanding of the procedure taught (i.e., the "trick") because children were required to apply their conceptual knowledge, specifically the concept of "cancelling out," to solve the new problems.

SET-3 Near Transfer Task. At the end of SET-3, children were required to solve four equations (see Appendix F, part 2) using the procedure demonstrated by the fictitious student during the interview. These problems assessed children's understanding of the concepts underlying the procedure, specifically the concepts of grouping and place value, because they required children to apply what they had observed and understood before solving the new equations.

Coding for conceptual knowledge. To measure conceptual knowledge, children were asked to explain their solutions for each of the four problems in each set of transfer tasks. I chose to use the children's self-explanations to obtain a measure of conceptual knowledge rather than rely on the answers they provided on each task. This approach was chosen because children can understand a principle and make calculation errors (Sophian, 2007), or, on the contrary, they can provide correct answers because of accurate procedural execution rather than conceptual understanding (Hiebert & Wearne, 1996).

Regardless of the correctness of their answer, I gave a score of 2 when a Conceptual Appropriate Self-Explanation was given to explain the solution of a problem or an equation, a score of 1, when a Conceptual Inappropriate Self-Explanation was given, and a score of 0 for any other type of self-explanation. Therefore, the children's Conceptual Score ranged from 0 to 8 because each task comprised four problems or equations worth a maximum of 2 points each. Higher scores indicated more robust conceptual knowledge.

An important note must be made in regard to the coding of conceptual knowledge of SET-1. Because of the students' lack of knowledge of alternate bases, they did not know that three groups of three chips corresponded to "1" in the position of "hundreds," I considered as conceptually appropriate self-explanations those that illustrated an understanding of grouping by three. For example, in the first problem of SET-1 Near Transfer Task, the students were required to count ten chips using Base 3. If they explained correctly why the three groups of three chips corresponded to a "3" in the position of "tens" and the one chip left to a "1" in the position of "units," I gave a score of 2 because I considered this self-explanation as an understanding of groupings of three.

Procedure

Before starting the project, I approached the four Cycle I teachers to inform them of the study and ask them if they wished to participate. All of them agreed to participate. Using a random number generator, I randomly selected the students from each class to form the sample. I contacted the children's parents by letter and requested their permission to allow their children participate in this study (Appendix H).

After having received the parents' consent, I asked the teachers to complete the Language Questionnaire. Next, I met all of the 30 participants for the first time in a quiet room to introduce them to the study. When meeting the group, I gave details on my project using words children could understand. I also explained to them that my study was confidential, and that only my professor and I would know their names. Finally, I asked children to sign a consent form if they agreed to participate (Appendix H).

A few days after all participants had given consent, the interviews began. Each participant was individually interviewed four times in March and April 2009 and was videotaped three times during that period (during the second, third, and fourth interviews). In the first interview session, I administered the Number Knowledge Test. This test lasted between 10 to 15 minutes. During each of the second, third, and fourth interviews, I administered one of the three self-explanation tasks, which were counterbalanced among participants (see Appendix I for the counterbalancing procedure). These self-explanation task interviews started approximately one week after the administration of the NKT and were spaced one week apart. Each of the three self-explanation tasks took between 10 and 15 minutes each to administer. As a result, each student contributed a maximum of 65 minutes to the project.

Interview 1: Administration of the NKT

Before the administration of the NKT, I selected the appropriate starting testing level according to the age of the participant to be interviewed. Specifically, I used Level 1 for seven-year-old participants and Level 2 was chosen for eight-year-old students. Then, I pulled out the participating child from her classroom for the individual interview. The other children in the class remained with their teacher and continued with the regular classes. I took the child to a quiet room with as few distractions as possible. I welcomed the child and told her what we would be doing: "Today, I will ask you some math questions. Please answer them as best as you can. I will listen to your answers and write them down. I will not tell you if the answer is right or wrong. I just want to find out how you think about numbers." I then read the first item to the child. I waited for her answer and wrote it down on a score sheet for subsequent analysis (Appendix B, second column). Continuation of administration depended on the number of correct answers at any given level, as described earlier.

Interview 2, 3, and 4: Administration of Self-Explanation and Near Transfer Tasks

Once the Number Knowledge Test was administered to all participants, the self-explanation interviews began. Before each interview, I verified which task was to be administered to the selected child following the sequence on the record keeping form (Appendix I). This phase was important because not all participants received the self-explanation tasks in the same order because of the counterbalancing procedure. I also prepared the necessary material for the administration of the chosen task. Then, I pulled out the participating child from her classroom for the self-explanation interview. The other children in the class remained with their teacher and continued regular classes. I took the child to a quiet room with as few distractions as possible. I welcomed the child and told her what we would be doing: "Hi! How are you? Today, we will do some math together. Ready?" Then, the administration of the chosen self-explanation task took place.

CHAPTER 4: RESULTS

To analyze the relationship between task type and prior knowledge on the generation of self-explanations, I calculated a score for the three self-explanation outcome variables: (a) a Quantity Self-Explanation Score, (b) a Conceptual Self-Explanation Score, and (c) a Procedural Self-Explanation Score. In addition to these self-explanation scores, a Conceptual Score was computed to assess the relationship between the task type and prior knowledge variables on the students' ability to acquire the key concepts related to each task.

The Quantity Self-Explanation Score was computed for each task by dividing the total number of videoclips in each interview by the total time (measured in seconds) the participant was engaged in self-explanation in each interview. This ratio controlled for time on task, as the length of each interview varied, particularly across tasks.

The Conceptual Self-Explanation Score was computed for each task by dividing the total number of conceptually appropriate self-explanations in each interview (numerator) by the total number of videoclips in the interview (denominator). This total number of videoclips included all types of self-explanations: (a) Conceptual Appropriate, (b) Conceptual Inappropriate, (c) Conceptual Irrelevant, (d) Procedural Correct, (e) Procedural Incorrect, and (f) Other. Only self explanations coded as Conceptual Appropriate were used in the numerator because the two other conceptual subcategories, Conceptual Inappropriate and Conceptual Irrelevant, were not appropriately related to the key concepts of the tasks. Similarly, the Procedural Self-Explanation Score was computed for each task by dividing the total number of procedurally correct self-explanations in each interview (numerator) by the total number of self-explanations in the interview (denominator). Only self explanations coded as Procedural Correct were used in the numerator because the other procedural category, Procedural Incorrect, did not reflect the appropriate steps or procedures used in the tasks. Both the Conceptual Self-Explanation Score and the Procedural Self-Explanation Score ranged from 0 to 1.00.

Finally, the Conceptual Score was computed for each task by scoring the quality of the self-explanation children produced to explain each of the four problems in the near transfer tasks. Specifically, I gave a score of 2 for a conceptually appropriate self-explanation, a score of 1 for a conceptually inappropriate self-explanation, and a score of 0 for any other type of self-explanation. The Conceptual Score for each participant was calculated by summing these individual scores. Because children gave only one self-explanation to justify their answer to each of the four problems, the Conceptual Score ranged 0 to 8 points.

A median split was used to determine high and low prior knowledge groups. With the Number Knowledge Test (NKT) median score of 20, the low group was formed by putting together scores ranging from 14 to 19 (n = 14), and the high group was created by regrouping scores ranging from 20 to 28 (n = 16). Descriptive statistics (means and standard deviations) are reported for the self-explanation outcome variables (i.e., quantity and quality) for each task in Table 1. Table 2 reports these statistics as a function of prior knowledge. For the Conceptual Score, means and standard deviations are described for each task in Table 3 and as a function of prior knowledge in Table 4.

Table 1

Means and Standard Deviations for the Quantity Self-Explanation Score, the Conceptual Self-Explanation Score, and the Procedural Self-Explanations Score for Each Task

	Quantity Self-Explanation Score		Conceptual Self-Explanation Score		Procedural Self-Explanation Score	
Task types	*M*	*SD*	*M*	*SD*	*M*	*SD*
SET-1	.15	.04	.26	.21	.07	.10
SET-2	.18	.05	.05	.08	.29	.08
SET-3	.14	.06	.29	.26	.14	.16

Table 2

Means and Standard Deviations for the Quantity Self-Explanation Score, the Conceptual Self-Explanation Score, and the Procedural Self-Explanations Score as a Function of Prior Knowledge

	Low Prior Knowledge		High Prior Knowledge	
Task types	*M*	*SD*	*M*	*SD*
	Quantity Self-Explanation Score			
SET-1	.16	.04	.15	.04
SET-2	.17	.05	.19	.05
SET-3	.13	.06	.14	.05
	Conceptual Self-Explanations Score			
SET-1	.37	.23	.17	.14
SET-2	.04	.07	.05	.08
SET-3	.16	.21	.41	.24
	Procedural Self-Explanation Score			
SET-1	.09	.12	.06	.09
SET-2	.30	.08	.29	.08
SET-3	.17	.18	.10	.14

Table 3

Means and Standard Deviations for the Conceptual Score for Each Task

Task types	*M*	*SD*
SET-1	3.20	3.38
SET-2	1.60	3.25
SET-3	4.47	3.85

Table 4

Means and Standard Deviations for the Conceptual Score as a Function of Prior Knowledge

	Low Prior Knowledge		High Prior Knowledge	
Task types	*M*	*SD*	*M*	*SD*
SET-1	2.29	3.50	4.00	3.16
SET-2	1.14	2.91	2.00	3.58
SET-3	2.00	3.42	6.63	2.80

Analyses are reported below and presented in three parts to highlight the relationship between the task type and prior knowledge variables and each outcome measure: (a) quantity of self-explanation, (b) quality of self-explanation, and (c) conceptual knowledge. Before describing these analyses, the correlations between the language measures and the previously mentioned outcome variables are reported. An alpha level of .05 was used for all statistical tests.

Language Measures

Correlations between each of the Vocabulary and Fluency measures and each outcome measure were computed as a function of prior knowledge. Table 5 reports the correlations between Vocabulary and all outcome measures by group, and Table 6 reports the correlations between Fluency and the outcome measures by group. As the data indicate, for each outcome measure, the vocabulary measures were not uniformly correlated across tasks or across groups. Similar findings were revealed for the fluency measure. For this reason, neither of the language measures were used as covariates in the subsequent analyses.

Table 5

Correlations between Vocabulary and Outcome Measures

Outcome Measure	*LPK*	*HPK*
Quantity Measures		
SET-1	.35	-.14
SET-2	.43	.14
SET-3	.11	.04
Quality Measures		
Conceptual self-explanation SET-1	.06	.05
Conceptual self-explanation SET-2	.18	-.01
Conceptual self-explanation SET-3	.10	.03
Procedural self-explanation SET-1	-.07	-.13
Procedural self-explanation SET-2	-.01	.24
Procedural self-explanation SET-3	.19	.42
Conceptual Knowledge Measures		
SET-1	-.32	.33
SET-2	.21	-.07
SET-3	.53	.48

Table 6

Correlations between Fluency and Outcome Measures

Outcome Measure	*LPK*	*HPK*
Quantity Measures		
SET-1	.56*	-.04
SET-2	.52	.29
SET-3	-.03	.15
Quality Measures		
Conceptual self-explanation SET-1	-.20	.39
Conceptual self-explanation SET-2	-.07	-.28
Conceptual self-explanation SET-3	-.23	.14
Procedural self-explanation SET-1	.39	-.19
Procedural self-explanation SET-2	.08	.18
Procedural self-explanation SET-3	.12	.35
Conceptual Knowledge Measures		
SET-1	-.20	.38
SET-2	.34	-.45
SET-3	.18	.39

* $p < .05$

The Relationship Between Task Type and Quantity of Self-Explanation as a Function of Prior Knowledge

To address the first research question of this study, a 3 x 2 mixed design ANOVA was conducted with task as the within-groups factor (SET-1, SET-2, and SET-3), prior knowledge as the between-groups factor (low, high), and quantity of self-explanation as the dependent measure (i.e., the Quantity Self-Explanation Score). The analysis revealed a main effect of task type, $F(2, 56) = 6.43$, $p = .003$; the means are presented in Figure 3. No main effect of group ($p = .659$) or interaction between task type and prior knowledge ($p = .400$) were found.

To determine more precisely the location of the main effect of task type, post-hoc paired-samples *t*-tests were conducted with a Bonferroni correction where an alpha level of 0.017 (.05/3) was used for each test to control for Type I error. A significant difference in the mean quantity of self-explanations was found between SET-1 ($M = .15$, $SD = .04$) and SET-2 ($M = .18$, $SD = .05$), $t(29) = -2.56$, $p = .016$. There was also a significant difference in the mean quantity of self-explanations between SET-2 ($M = .18$, $SD = .05$) and SET-3 ($M = .14$, $SD = .06$), $t(29) = 3.85$, $p = .001$. No significant difference was found between SET-1 and SET-3 ($p = .283$).

Together, these results indicate that task type was related to the quantity of self-explanation generated by the participants regardless of prior knowledge. Specifically, the participants generated more self-explanations in SET-2 (direct instruction) than in the two other tasks.

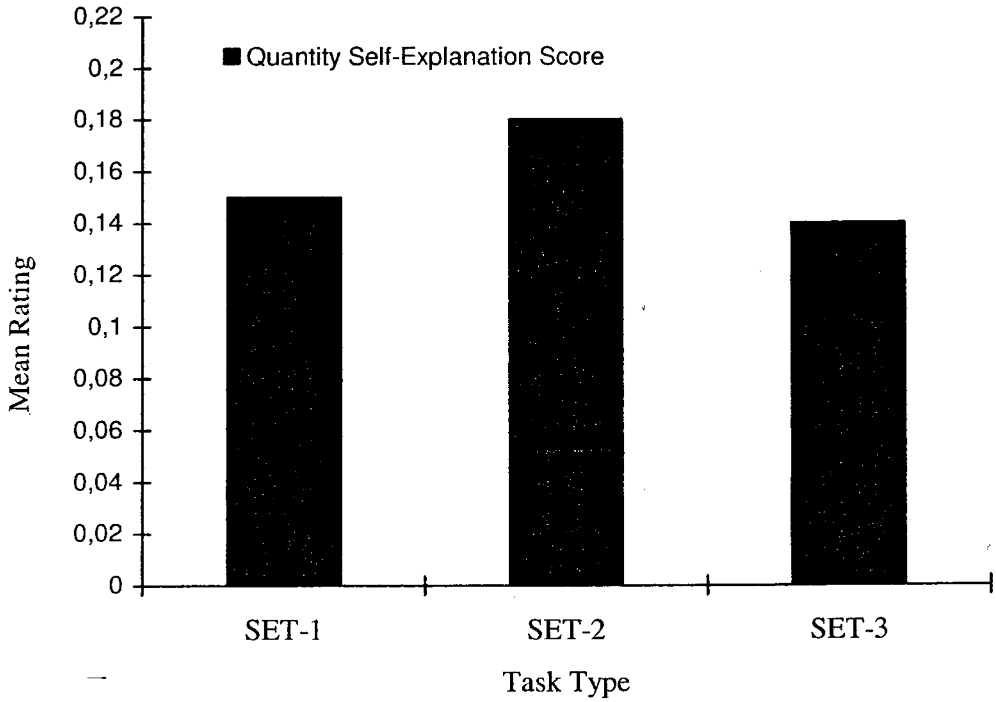

Figure 3. Main effect of task type on the quantity of self-explanation.

The Relationship Between Task Type and Quality of Self-Explanations as a Function of Prior Knowledge

Conceptual Self-Explanation

To address the second research question, a 3 x 2 mixed design ANOVA was conducted with task as the within-groups factor (SET-1, SET-2, and SET-3), prior knowledge as the between-groups factor (low, high), and Conceptual Self-Explanation Score as the dependent measure. The analysis revealed a significant interaction between task type and prior knowledge, $F(2, 56) = 13.04, p < .001$. The means are graphed in Figure 4.

To determine more precisely the location of the differences, post-hoc independent samples *t*-tests were conducted with a Bonferroni correction where an alpha level of 0.017 (.05/3) was used for each test to control for Type I error. A significant difference was found between the high prior knowledge group and the low prior knowledge group for SET-1 (LPK: $M = .37$, $SD = .23$; HPK: $M = .17$, $SD = .14$), $t(28) = 2.78, p = .01$. There was also a significant difference between the two groups for SET-3 (LPK: $M = .16$, $SD = .21$; HPK: $M = .41$, $SD = .24$), $t(28) = -3.01, p = .006$. No significant difference was found between the groups on the frequency of conceptually appropriate self-explanations on SET-2 ($p = .775$).

In addition, a main effect of task type was also found, $F(2, 56) = 18.49, p < .001$, but there was no main effect of group ($p = .584$). Together, these results show that task type and the generation of conceptual self-explanation were related and varied as a function of prior knowledge. Specifically, low prior knowledge students generated more conceptual self-explanations in SET-1 (discovery) and high prior knowledge, in SET-3 (surprise or cognitive conflict).

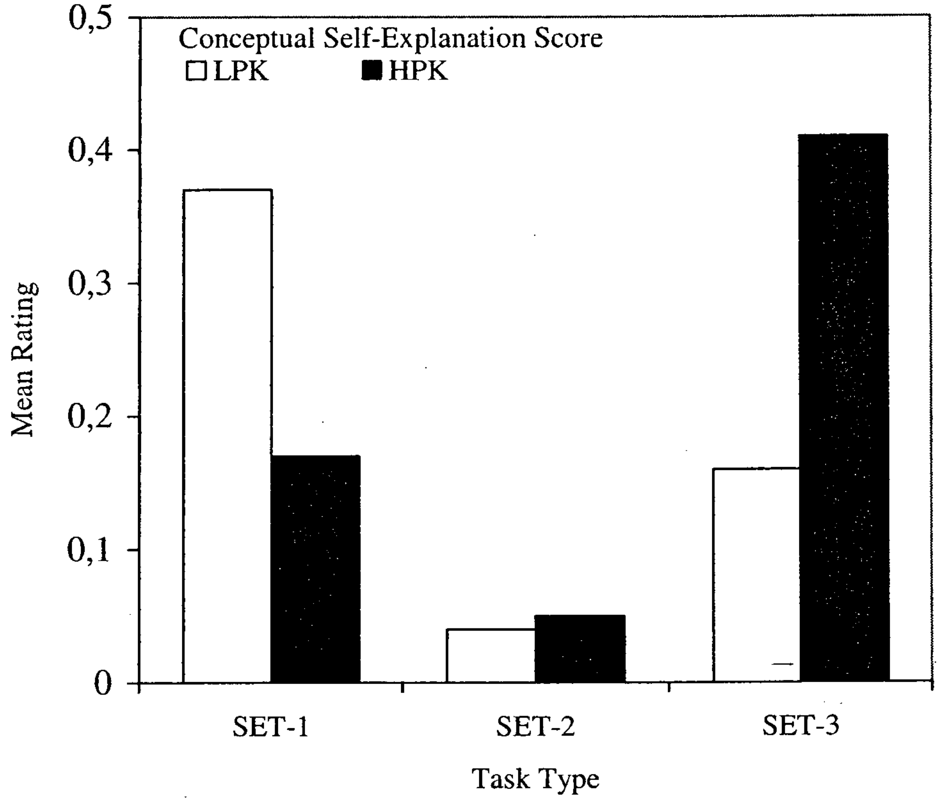

Figure 4. Interaction between task type and prior knowledge on the generation of conceptual self-explanation.

Procedural Self-Explanation

A 3 x 2 mixed design ANOVA was conducted with task as the within-groups factor (SET-1, SET-2, and SET-3), prior knowledge as the between-groups factor (low, high), and Procedural Self-Explanation Score as the dependent measure. The analysis revealed a main effect of task type, $F(2, 56) = 26.05, p < .001$. The means are graphed in Figure 5.

To determine the location of this difference, post-hoc paired-samples *t*-tests were conducted with a Bonferroni correction where an alpha level of 0.017 (.05/3) was used for each test to control for Type I error. A significant difference in the mean number of procedurally correct self-explanations was found between SET-1 ($M = .07, SD = .10$) and SET-2 ($M = .29, SD = .08$), $t(29) = -9.01, p < .001$. There was also a significant difference between SET-2 ($M = .29, SD = .08$) and SET-3 ($M = .14, SD = .16$), $t(29) = 5.01, p < .001$. No significant difference was found in the mean number of procedurally correct self-explanations between SET-1 and SET-3 ($p = .089$).

Also, there was no main effect of group ($p = .099$) and no significant interaction between task type and prior knowledge ($p = .679$). Together, these results indicate that task type was related to the generation of procedural self-explanation regardless of prior knowledge. Specifically, participants produced more procedural self-explanations in SET-2 (direct instruction).

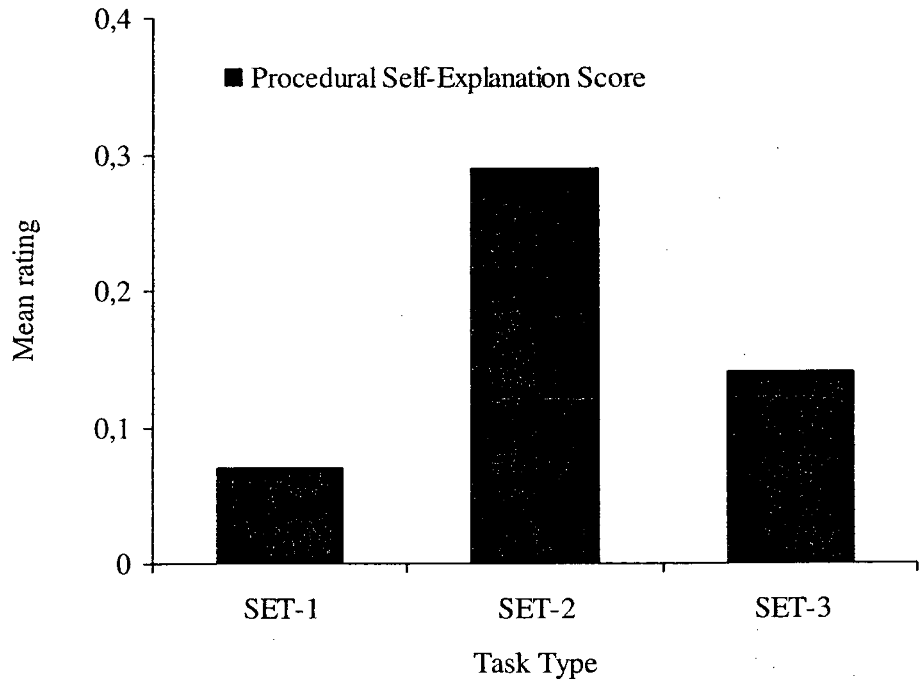

Figure 5. Main effect of task type on the generation of procedural self-explanation.

The Relationship Between Task Type and Conceptual Knowledge as a Function of Prior Knowledge

To address the third research question, a 3 x 2 mixed design ANOVA was conducted with task as the within-groups factor (SET-1, SET-2, and SET-3), prior knowledge as the between-groups factor (low, high), and Conceptual Score as the dependent measure. The analysis revealed a main effect of task type, $F(2, 56) = 5.39$, $p < .007$. The means are graphed in Figure 6.

To determine the location of this difference, post-hoc paired-samples *t*-tests were conducted with a Bonferroni correction where an alpha level of 0.017 (.05/3) was used for each test to control for Type I error. A significant difference in Conceptual Score was found between SET-2 ($M = 1.60$, $SD = 3.25$) and SET-3 ($M = 4.47$, $SD = 3.85$), $t(29) = -3.23$, $p = .003$. No significant difference was found between SET-1 ($M = 3.20$, $SD = 3.38$) and SET-2 ($p = .096$) and SET-1 and SET-3 ($p = .105$).

The ANOVA also revealed a main effect of prior knowledge, $F(1, 28) = 12.32$, $p = .002$. The HPK group performed significantly better on the measure of conceptual knowledge than the LPK group regardless of task type (HPK: $M = 4.2$, $SD = 0.47$; LPK: $M = 1.8$. $SD = 0.50$). No significant interaction between task type and prior knowledge was found ($p = .071$).

The previous results indicate first of all that task type was related to conceptual knowledge regardless of prior knowledge. Specifically, participants' conceptual knowledge was higher on SET-3 (surprise or cognitive conflict) than on SET-2 (direct instruction). Added to this, the findings show that regardless of task type, the high prior knowledge group outperformed the low prior knowledge group.

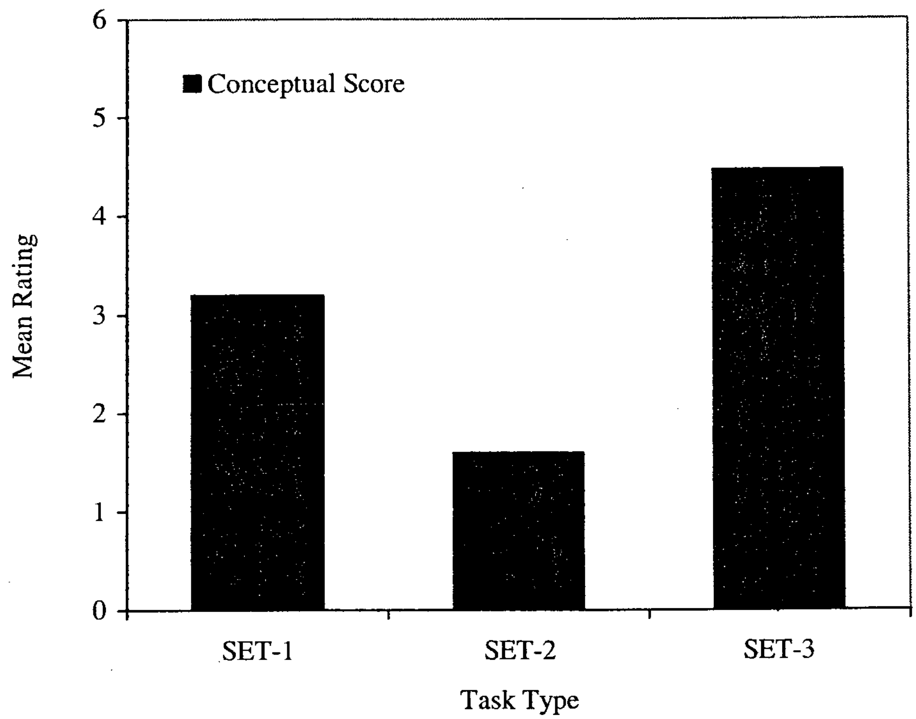

Figure 6. Main effect of task type on conceptual knowledge.

CHAPTER 5: DISCUSSION

The present study examined the relationship between task type on the quantity and quality of children's self-explanations and on conceptual knowledge in mathematics as a function of prior knowledge. Thirty second-grade students were individually interviewed and were required to self-explain in three different mathematical tasks. Specifically, the students were asked to use self-explanation strategies to explain why specific mathematical procedures worked. In the first task, the students were required to self-explain the behaviour of an expert counting chips in Base 3. This task was designed so that the children would discover the rules behind the interviewer's demonstrated procedure. The second task required the students to self-explain a simple procedure that was directly taught to them, and they were then required to use the procedure repeatedly. The task was designed so that the students' cognitive load would be reduced, thus freeing up space for reflecting on why the procedure works. In the third task, the students were asked to self-explain the novel solutions of another child. This task was designed so that the participants would experience some surprise at, or cognitive conflict over, the unfamiliar solution and thus, be compelled to search from an explanation for how it works. The students' conceptual understanding of all three procedures was measured after each task. Finally, prior mathematical knowledge was examined as a possible moderating variable in the analyses.

First, the data revealed that quantity of self-explanation was related to task type. More specifically, SET-2, a task designed to reduce cognitive load because of its direct instructional format, produced more self-explanations that either of the other two tasks. There was no moderating effect of prior knowledge on the number of self-explanations produced.

Second, the quality of self-explanations was related to both task type and prior knowledge. Specifically, low knowledge students generated more conceptual self-explanations, considered by Matthews and Rittle-Johnson (2009) and Renkl (1997) as high quality self-explanations, in SET-1 (discovery task), whereas high knowledge students produced more of the same high quality self-explanations in SET-3, a task based on the effect of surprise (cognitive conflict). In regard to procedural self-explanations, this low quality self-explanation only differed as a function of task type, not as a function of prior knowledge. Precisely, SET-2, a task based on direct instruction (cognitive load theory), supported the production of procedural self-explanations regardless of participants' prior knowledge.

Finally, the participants' conceptual knowledge was significantly higher in SET-3 than in SET-2 regardless of their prior knowledge. No other differences in conceptual knowledge were found between the tasks.

The Relationship Between Task Type and the Quality and Quanity of Self-Explanation as a Function of Prior Knowledge

The results of the present study put into question the conclusions of some scholars who asserted that unsuccessful students, contrary to successful students, may not have the necessary prior knowledge to generate high quality self-explanations (Zohar & Aharon-Kravetsky, 2005). It also contradicts the findings of Chi and VanLehn (1991), Renkl (1997), and Renkl et al. (1998), who conversely found that prior knowledge did not have an effect on the quality of self-explanation. In contrast, this study showed that not only does prior knowledge influence the quality of self-explanation, but also that low knowledge students are capable of producing high quality self-explanations when engaged in an appropriate task type -- that is, the discovery task for low knowledge students and the surprise task (cognitive conflict) for high knowledge students. Previously, task type was not considered as an influential variable when studying self-explanation; this may partly explain the contradictory results of the present study with previous research, at least with respect to prior knowledge. The current findings are thus more aligned with the recommendation made by Roy and Chi (2005), who suggested that future studies on self-explanation should focus on other factors, such as task demands.

Discovery Task (SET-1)

In this study, the data showed that a task based on discovery learning prompted low knowledge students to generate more conceptual self-explanations, which has been demonstrated to enhance the understanding of a mathematical procedure (Renkl, 1997), than procedural self-explanations. Because discovery conditions are often considered as unsuccessful for low knowledge students, this finding is surprising. In particular, many scholars mentioned that discovery conditions often overload lower achievers' working-memory capacity by requiring too much random-search of the problem space (Kalyuga, 2009; Mayer et al., 2002; Mayer & Moreno, 2003). They thus advocate for more direct instruction that can provide the necessary schemas in a domain that help coordinate information in working memory, which in turn makes available the cognitive resources necessary to engage in cognitive activities. Because discovery conditions seem to overload students' working memory, it is reasonable to think that this approach may reduce low achievers' capacity of producing high quality self-explanations. In the present study, however, the discovery task conversely prompted low knowledge students to produce more conceptual self-explanations.

The design of the task itself may offer some insight into why low knowledge students were responsive to the task based on discovery, at least with respect to providing conceptual self-explanations. One possible reason may be a result of actively engaging low knowledge students in manipulating, linking, and evaluating information, in other words, self-explanation. As Mayer

(2004) mentioned in his article, a key feature of discovery learning is thinking. Therefore, the discovery task in this study may have sufficiently engaged enough low knowledge students in a thinking activity that supported them in the generation of conceptual self-explanation, thereby helping them make sense of the mathematical procedure central to the task. Added to this, Mayer asserted that a discovery task is effective only when some guidance is offered to students, which he called guided discovery. Because the experimenter asked specific questions to guide students during the interview, this was perhaps sufficient to help low knowledge students come into contact with the to-be-learned concepts behind the mathematical procedure.

Apart from actively engaging students, other components of the discovery task may have been beneficial for low knowledge students. In particular, this task provided students with chips. Many studies have demonstrated the benefits of manipulatives, when learning mathematics (see Fuchs & Fuchs, 2001 for a review). Because some evidence suggests that low knowledge students need more concrete supports to learn and construct their understanding, particularly in mathematics (Harris, Miller, & Mercer, 1995; Woodward, Baxter, & Robinson, 1999), this component of the task may have helped them to generate higher quality self-explanations and thus, discover on their own the principles underlying the mathematical procedure.

One could have speculated that the discovery task would have also be beneficial for high knowledge students in addition to the surprise task (cognitive conflict), particularly because a discovery approach stimulates students by requiring random search of the problem, which has been shown to foster conceptual knowledge (Goldman et al., 1997; Hiebert & Wearne, 1996; Kamii & Dominick, 1998). The results of the present study, however, contradict this conclusion: the discovery task was effective for low knowledge students, but not for high knowledge students. One possible reason for this discrepancy is, again, the use of manipulatives, a support that may have been useless or even distracting to high achievers while crucial to low knowledge students' engagement with the underlying principles. In that regard, recent research studies have found that the effectiveness of instructional materials in mathematics is dependent on the expertise of the learner (Clarke, Ayres, & Sweller, 2005; Kalyuga, Ayres, Chandler, & Sweller, 2003). In some circumstances, information, guidance, or visual supports essential for novices may be redundant for those with more expertise. As a result, tools that are effective for novices, such as manipulatives, are less beneficial for more knowledgeable students. These findings may shed some light on the results of the present study with respect to the low number of conceptual self-explanations produced by high knowledge students in the discovery task.

Based on the results of the present study, I conclude that a task based on discovery learning should be used with low knowledge students when teaching a new mathematical procedure because it seems to enhance their conceptual understanding by prompting them to generate high quality self-explanations, namely conceptual self-explanations.

Surprise Task (SET-3)

The result of the present study also showed that a task based on the effect of surprise (cognitive conflict) prompted high knowledge students to generate more conceptual self-explanations, which has been demonstrated to enhance the understanding of a mathematical procedure (Renkl, 1997), than procedural self-explanations. This finding is in line with the work of Zohar and Aharon-Kravetsky (2005), who found that high achievers produced significantly more inferences than low achievers about scientific experiments. Further, the results of the present study provide evidence to support the speculation of Rittle-Johnson and Alibali (1999) about the effect of surprise: "in situations in which newly learned procedures result in different solutions than prior procedures, children may find this surprising, and this may lead them to consider the conceptual basis of the new procedure" (p. 187). The present study supports this speculation by specifying the type of children who benefits from a surprise task, namely high knowledge students.

The benefits of the surprise task for high knowledge students in this study may be due to the high cognitive demand of the task. Indeed, this task appeared to require more abstraction on the part of the students, which may have stimulated high knowledge students to engage themselves more significantly in a cognitive process to acquire new schemas (Ayres, 2006). Because high knowledge students probably possessed the necessary schemas to enhance their understanding from this type of abstraction, they were perhaps challenged by the high demand of the task and therefore, produced more conceptual self-explanations. In the words of Vygotsky (1962), this task was probably in their zone of proximal development.

Although Zohar and Aharon-Kravetsky (2005) mentioned that metacognitive supports, such as self-explanation, could help low knowledge students make the cognitive conflict accessible to them, the present study shows in contrast that self-explanation did not support low knowledge students in producing conceptual self-explanations while engaged in the surprise task. As speculated by Limon (2001), the cognitive conflict is meaningful only if students have a certain amount of prior knowledge. In this study, the low knowledge students may have not possessed this type of knowledge, making it difficult for them to understand the new procedure to be learned.

Based on the results of the present study, therefore, I conclude that a task based on the effect of surprise (cognitive conflict) should be used with high knowledge students when teaching a new mathematical procedure. The evidence from the present study suggests that this task type supports high knowledge students in generating self-explanations that are conceptual in nature, a defining feature of high-quality self-explanation.

Direct Instruction Task (SET-2)

In contrast to the previous tasks, the effect of surprise and the discovery tasks, the results of this study show that the direct instruction task was the only task that was significantly related to the quantity of self-explanations. Specifically, this task generated the highest number of self-explanations, regardless of prior knowledge. These more abundant self-explanations were, however, lower in quality compared to the ones generated in the other two conditions. In other words, the direct instruction task promoted the generation of procedural self-explanations instead of conceptual self-explanations. Because high quality self-explanation, rather than quantity of self-explanation, has been found to be effective in helping students understand a mathematical procedure (Renkl, 1997), it is reasonable to assume that the direct instruction task did not support learning as much as the two other tasks.

The relation between quantity and quality of self-explanation is interesting to juxtapose with respect to the tasks. Although the direct instruction task promoted the generation of a higher number of self-explanation, it did not support the generation of conceptual self-explanation while the discovery and surprise tasks did. This finding suggests that the generation of more self-explanations does not, at least under the conditions set out in this study, foster the understanding of a mathematical procedure. This speculation is in line with the results of Renkl (1997) who found that higher quality of self-explanations, rather than quantity, was a predictor of learning.

As it stands, the direct instruction task in this study, which was designed to reduce the cognitive load of students, did not seem to help students in productively using their free cognitive capacity to generate high quality self-explanations. Instead, they produced a higher number of procedural self-explanations, which has not been found to be beneficial in previous research, at least with respect to conceptual knowledge (Renkl, 1997). These results put into question the conclusions of several scholars who claim that direct instruction is effective for students in learning mathematics, particularly low knowledge students, because of its effect in reducing cognitive load (Kalyuga, 2009; Mayer et al., 2002; Mayer & Moreno, 2003). Also, it contradicts the speculation of Rittle-Johnson and Alibali (1999), who postulated that when procedures are easy to implement, the cognitive load may be sufficiently reduced to help children think about the underlying principles of procedures.

These divergent results may be explained by the redundancy effect, also called the expertise reversal effect, which occurs when the learner is required to process nonessential information or to engage in irrelevant cognitive activities -- that is, any activity not directed to schema acquisition (Chandler & Sweller, 1991; Kalyuga et al., 2003). This effect happens when learners have already mastered the schemas or possess sufficient experience in the domain. Therefore, they may not be sufficiently motivated to be deeply engaged in studying familiar tasks and consequently, provide less mental effort and perform at a lower level (Pass, Tuovinen, van Merriënboer, & Darabi, 2005). In this study, the direct instruction task may have indeed failed to keep the students sufficiently engaged with the task on a conceptual level; it was perhaps too simple. Said differently, this task may have induced a redundancy effect. Added to this, the repetitive nature of the task may have understimulated students who were less engaged in the task and led them to produce more procedural self-explanations (Clarke et al., 2005).

Based on the results of the present study, I conclude that a task based on direct instruction, particularly when the goal of the task is to free up cognitive resources, does not appear to support the students' engagement with the concepts underlying it. Therefore, it should not be used when teaching a new mathematical procedure to students. This conclusion is in line with Ayres (2006), who found that direct instruction on an algebraic task, while reducing cognitive load significantly, did not transfer as well to learning.

The Relationship Between Task Type and Conceptual Knowledge as a Function of Prior Knowledge

Because the results on conceptual knowledge did not reveal any differences in regard to prior knowledge, no comparisons between the two groups can be made here. Nevertheless, the data shows that the participants had a higher conceptual score in the surprise task than in the direct instruction task. This finding is in line with the work of Siegler (1995, 2002) who found that students' understanding of a mathematical procedure improved when they self-explained the solutions of another child, a task design similar to that of the surprise task.

Surprisingly, no difference was found between the discovery and the direct instruction tasks with respect to conceptual knowledge. Because the discovery task in this study supported the generation of conceptual self-explanations for low knowledge students, which in turn appears to foster conceptual knowledge (Renkl, 1997), one could have expected that this condition would have generated higher conceptual scores than in the direct instruction task. This was, however, not the case. Clearly, these contradictory findings raise questions about the coding and the design of the near transfer task because they do not support the previous results of this study regarding the generation of conceptual self-explanations.

In addition, no difference was found between the discovery and the surprise tasks with respect to conceptual knowledge. This non significant difference may, however, be explained.

These two tasks may have promoted the same level of understanding of a mathematical procedure. As mentioned earlier, both tasks were found to support the generation of conceptual self-explanations for low knowledge students, in the case of the discovery task, and high knowledge students, in the case of the surprise task. Therefore, it is reasonable to assume that they are of similar potential in promoting conceptual growth for all participants, regardless of their prior knowledge. Because the results of the present study do not give evidence of such a conclusion, this speculation, therefore, needs further investigation.

In sum, the discovery task and the surprise task seem to be the most promising types of tasks when helping students use self-explanations as a learning tool to make sense out of a mathematical procedure. In particular, the discovery task appears to be more effective with low knowledge students and the surprise task, with high knowledge students. The same cannot be said for the direct instruction task, which generated the lowest scores on the conceptual understanding measure, regardless of level of prior knowledge.

Limitations

First, the study was restricted by the specific mathematical domain and some design features. Specifically, the findings may only be limited to the specific task types and mathematical concepts chosen for this study. In other words, the conclusions may have been different had other task types, with different theoretical bases, been developed. Even if tasks had been based on the same educational theories, but designed differently, the previous results may have been different because components of a task influence the way students interact with them (Osana et al., 2006). For example, a discovery task without manipulatives may not have been as effective for low knowledge students as it was in this study. Second, the results of the present study may not be transferable to classroom settings. Because a one-on-one design was used, which differs greatly from the reality of groups of students in a class, it is clear that the conclusions may not be entirely generalizable to other settings. Specifically, the children may have felt more comfortable expressing themselves in an individual interview setting, thereby generating higher quality self-explanations than had they been in a classroom. Nevertheless, this one-on-one design may be recreated in the classroom when teachers find ways to individually interview children during class time.

Another limitation of this study is the lack of evidence for the validity of the conceptual knowledge measure. A measure with known psychometric properties may have yielded altogether different results. The design of the near transfer tasks may also have had an influence on the conceptual scores; therefore, their design should be revisited to make the necessary modifications that would make the measure of conceptual knowledge more valid. In addition, a pretest and a posttest of conceptual knowledge would have allowed for a better measure of growth in conceptual understanding. An experiment with a control group would also have provided further evidence that the type of task caused, or had an effect, on conceptual growth. This could be addressed in future research.

Finally, the present study was limited to some degree by the small number of participants. The number of students for each self-explanation tasks was relatively small and consequently, the study may have lacked power.

Implications and Future Directions

From a pedagogical point of view, this study provides avenues for changing and enhancing mathematics instruction. I found evidence to suggest that procedures are indeed important – children can, and do, learn from engaging in procedures. Because of the present findings, therefore, there appears to be considerable benefit to including procedural activity in mathematics instruction. Nevertheless, certain conditions make it more optimal than others, such as self-explanation and task type. The findings of the present study give insight to these conditions. First, engagement in the procedures in the discovery and surprise tasks resulted in significantly more conceptual explanations than the task based on direct instruction. Second, the self-explanation of the mathematical procedures required in these two tasks appears to have helped the students understand the underlying concepts behind them, a conclusion shared by Rittle-Johnson (2006) who asserted that self-explanation may be the key to transfer. In fact, the data revealed that the highest level of conceptual knowledge was found after the students had completed the task based on surprise compared to the task based on direct instruction.

Thus, an important finding is that conceptual self-explanations and conceptual knowledge are dependent on the type of task. For teachers, then, this means that they should not only prompt their students to self-explain, but they should also choose the appropriate type of task for their students. As it stands, these simple instructional interventions may offer solutions to the actual problem of Canadian students, many of whom only know how to compute and use formulas, without an understanding of the principles underlying them.

In addition, the results of the present study indicate that the effectiveness of self-explanation in the context of learning a new mathematical procedure is dependent on students' prior knowledge. Specific tasks should be selected according to their level of prior knowledge. In particular, a discovery task for low knowledge students and a task based on the effect of surprise (cognitive conflict) for high knowledge students are likely good choices. This conclusion implies a shift in teaching style for many, urging teachers to differentiate their instruction by planning two different lessons or designing two different tasks for the same mathematical procedure, specifically one for their low achievers and another one for their high achievers. Generally, teachers use the instructional formats that work for low knowledge students, arguing that high knowledge students will learn from any type of instruction (Fuchs & Fuchs, 2001). That may be true, but if their goal is to fully develop the potential of high knowledge students, this study gives evidence that different instructional formats or task types must be designed.

Although this study presents innovative avenues for teaching mathematics differently, it did not, however, offer a practical way to include self-explanation during instruction. As mentioned before, a one-on-one interview design was used in this study, which may have impacted the quantity and quality of self-explanations in ways that may be different in other, more authentic, pedagogical contexts, such as the classroom. Prompting all students to self-explain during instruction and to reflect on the quality of each student's self-explanations is not—a trivial task for teachers. Therefore, future research should focus on the development of instructional programs that facilitate the use of self-explanation in the classroom. Furthermore, these programs should provide teachers with practical and simple tools to differentiate the mathematics instruction for low and high achievers. Finally, these instructional programs should be implemented in different school settings and with teachers with different levels of experience to validate their effectiveness and provide recommendations for successful adaptations to the classroom.

Added to this, the design of the tasks in the present study should be investigated further to define the components that truly make a difference in terms of self-explanation and mathematics learning, particularly the task based on discovery learning and cognitive conflict that was positively related to the outcome measures. In this study, for instance, manipulatives were included in the discovery task. Perhaps other elements would have made this task more amenable to students' conceptual engagement. The study of the essential components of these tasks seems to be crucial in examining ways to improve students' learning. In the same way, other educational theories should be used to design tasks and examine their effect on self-explanation and learning in mathematics.

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Appendix A

Language Questionnaire

Comment jugez-vous la capacité de cet enfant à s'exprimer oralement?

À l'aide de l'échelle d'appréciation ci-dessous, cochez la case correspondant au niveau de l'élève au regard de son vocabulaire et de sa fluidité (rythme à s'exprimer) lorsqu'il s'exprime à l'oral.

Échelle d'appréciation
1 : pauvre
2 : moyen
3 : bien
4 : excellent

		Vocabulaire					Fluidité			
Cote	Nom de l'élève	1	2	3	4		1	2	3	4

Appendix B

Number Knowledge Test

ENGLISH VERSION (NOT USED DURING THE STUDY - SEE THE FOLLOWING PAGES FOR THE FRENCH VERSION THAT WAS ADMINISTERED TO THE PARTICIPANTS)

Number Knowledge Test

(Griffin & Case, 1997)

Preliminary

Let's see if you can count from 1 to 10. Go ahead.

Level 0 (4-year old level): Go to Level 1 if 3 or more correct

1. (Show 5 unordered chips.) Would you count these for me?

2. I'm going to show you some counting chips (show mixed array of 3 red and 4 blue chips). Count just the blue chips and tell me how many there are.

3. Here are some circles and triangles (show mixed array of 7 circles and 8 triangles). Count just the triangles and tell me how many there are.

4. Pretend I'm going to give you 2 pieces of candy and then I'm going to give you 1 more (do so). How many will you have altogether?

Level I (6-year-old level): Go to Level 2 if 5 or more correct

1. If you had 4 chocolates and someone gave you 3 more, how many chocolates would you have altogether?.

2. What number come right after 7?

3. What number comes two numbers after 7? (Accept either 9 or 10)

4a. Which is bigger: 5 or 4?

4b. Which is bigger 7 or 9?

5a. (This time, I'm going to ask you about smaller numbers.) Which is smaller: 8 or 6?

5b. Which is smaller: 5 or 7?

6a. (Show visual array.) Which number is closer to 5:6 or 2?

6b. (Show visual array.) Which number is closer to 7:4 or 9?

7. How much is 2+4? (OK to use fingers for counting)

8. How much is 8 take away 6 (OK to use fingers for counting)

9a. (Show visual array -8 5 2 6- ask child to point to and name each numeral.) When you are counting, which of these numbers do you say first?

9b. When you are counting, which of these numbers do you say last?

Level 2 (8-year-old level): Go to Level 3 if 5 or more correct

1. What number come 5 numbers after 49? (Accept either 54 or 55)

2. What number comes 4 numbers before 60? (Accept either 56 or 55)

3a. Which is bigger: 69 or 71?

3b. Which is bigger: 32 or 28?

4a. (This time I'm going to ask you about smaller numbers.) Which is smaller: 27 or 32?

4b. Which is smaller: 51 or 39?

5a. (Show visual army.) Which number is closer to 21: 25 or 18?

5b. (Show visual army.) Which number is closer to 28:31 or 24?

6. How many numbers are there in between 2 and 6? (Accept either 3 or 4)

7. How many numbers are there in between 7 and 9? (Accept either 1 or 2)

8. (Show card 12 54) How much is 12+54 (No credit if number increased by one with fingers.)

9. (Show card 47 21) How much is 47 take away 21 (No credit if number decreased by one with fingers.)

Level 3 (10-year-old level)

1. What number comes 10 numbers after 99?

2. What number comes 9 numbers after 999?

3a. Which difference is bigger: the difference between 9 and 6 or the difference between 8 and 3?

3b. Which difference is bigger: the difference between 6 and 2 or the difference between 8 and 5?

4a. Which difference is smaller: the difference between 99 and 92 or the difference between 25 and 11 ?

4b. Which difference is smaller: the difference between 48 and 36 or the difference between 84 and 73?

5. (Show card, "13, 39") How much is 13 + 39?

6. (Show card, "36, 18") How much is 36 – 187

7. How much is 301 take away 7?

STUDENT ID [ ]

FRENCH VERSION (USED FOR THIS STUDY)

Number Knowledge Test

Test d'identification des difficultés en mathématiques
(Griffin & Case, 1997)

Matériel : tableau de nombres (dans un cahier, sur une feuille, etc), une trentaine de jetons, formes géométriques en plastique (8 triangles et 7 cercles), petites cartes de nombre (8, 5, 2, 6)

Directives :

- Rencontrer les élèves individuellement pour l'administration de ce test.
- Avant de commencer le test, choisir le niveau approprié à l'âge de l'enfant; s'il ne réussit pas son niveau, choisir le niveau précédent.
- Lire les questions à l'oral.
- Inscrire 1 dans la colonne réponse lorsque l'enfant a répondu correctement à la question; inscrire 0 si sa réponse est incorrecte.
- Le dernier niveau réussi par l'enfant indique le niveau auquel il est rendu au regard des connaissances du nombre et des quantités.

TÂCHE À FAIRE	RÉPONSES	OBSERVATIONS
NIVEAU 0 (maternelle) *Si l'enfant réussit 3 énoncés ou plus, passez au niveau 1.*	**1 : CORRECTE** **0: INCORRECTE**	*Si nécessaire*
PRÉLIMINAIRE : Compte jusqu'à 10		
1. *Montrez 5 jetons.* Peux-tu compter le nombre de jetons pour moi?		
2. *Montrez 3 jetons rouges et 4 jetons bleus mélangés et placés en une seule rangée.* Peux-tu compter le nombre de jetons bleus?		
3. *Montrez 7 cercles et 8 triangles en plastique mélangés et placés en une seule rangée.* Peux-tu seulement compter les triangles et me dire combien il y en a?		
4. Si je te donne 2 bonbons et que par la suite, je t'en donne un autre, combien de bonbons auras-tu? **(rép. 3)**		

NIVEAU 1 ($1^{ère}$ année) *Si l'enfant réussit 5 énoncés ou plus, passez au niveau 2.*	**RÉPONSES** **1 : CORRECTE** **0: INCORRECTE**	**OBSERVATIONS**
1. Si tu as 4 chocolats et que ton papa t'en donne 3 de plus, combien auras-tu de chocolats en tout? **(rép. 7)**		
2. Quel nombre vient tout juste après 7? **(rép. 8)**		
3. Quel nombre vient deux nombres après 7? **(acceptez 9 ou 10)**?		
4a) Quel est le plus grand nombre, 5 ou 4? **(rép. 5)** 4b) Quel est le plus grand nombre, 7 ou 9? **(rép. 9)**		
5a) Quel nombre est le plus petit, 8 ou 6 ? **(rép. 6)** 5b) Quel est le plus petit nombre, 5 ou 7? **(rép. 5)**		
6a) *Montrez un tableau de nombres.* Quel nombre est le plus près de 5 : 6 ou 2 **(rép. 6)** 6b) *Montrez un tableau de nombres.* Quel nombre est le plus près de 7 : 4 ou 9 **(rép. 9)**		
7. Combien font 2 + 4 ? *(l'enfant peut compter avec ses doigts –* **rép. 6***)*		
8. Si j'enlève 6 de 8, combien cela fait-il? *(l'enfant peut compter avec ses doigts –* **rép. 2***)*		
9. *Utiliser les cartes de nombre, 8, 5, 2 ,6. Demandez à l'enfant de pointer un nombre et de le nommer.* a) Quand tu comptes ces nombres, quel nombre dis-tu en premier? **(rép. 2)** b) Quel nombre dis-tu en dernier? **(rép. 8)**		

NIVEAU 2 (2^{e} année) *Si l'enfant réussit 5 énoncés ou plus, passez au niveau 3.*	**RÉPONSES** **1 : CORRECTE** **0: INCORRECTE**	**OBSERVATIONS** *Si nécessaire*
1. Quel nombre vient 5 nombres après 49? **(acceptez soit 54 ou 55)**		
2. Quel nombre vient 4 nombres avant 60? **(acceptez 56 ou 55)**		
3a) Quel nombre est le plus grand 69 ou 71? **(rép. 71)** 3b) Quel nombre est le plus grand, 32 ou 28? **(rép. 32)**		
4a) Quel nombre est le plus petit: 27 ou 32? **(rép. 27)** 4b) Quel nombre est le plus petit: 51 ou 39? **(rép. 39)**		
5a) Quel nombre est le plus près de 21 : 25 ou 18? **(rép. 18)** 5b) Quel nombre est le plus près de 28 : 31 ou 24? **(rép. 31)**		

6. Combien de nombres y a-t-il entre 2 et 6? **(acceptez 3 ou 4)**		
7. Combien de nombres y a-t-il entre 7 et 9? **(acceptez 1 ou 2)**		
8. *Montrez les nombres 12 et 54 sur un tableau des nombres.* Combien font 12 et 54? **(rép. 66)**		
9. *Montrez les nombres 12 et 54 sur un tableau des nombres.* Que devient 47 si je lui enlève 21? **(rép. 26)**		
NIVEAU 3 (3e année) *L'enfant réussit le niveau 3 lorsqu'il a bien répondu à un minimum de 4 questions*	**RÉPONSES** **1 : CORRECTE** **0: INCORRECTE**	**OBSERVATIONS** *Si nécessaire*
1. Quel nombre vient 10 nombres après 99? **(rép. 109)**		
2. Quel nombre vient 9 nombres après 999? **(rép. 1008)**		
3a) Quelle différence est la plus grande? La différence entre 9 et 6 ou la différence entre 8 et 3. **(rép. différence entre 8 et 3)** 3b) Quelle différence est la plus grande : la différence entre 6 et 2 ou la différence entre 8 et 5? **(rép. différence entre 6 et 2)**		
4a) Quelle différence est la plus petite : la différence entre 99 et 92 ou la différence entre 25 et 11. **(rép. différence entre 99 et 92)** 4b) Quelle différence est la plus petite : la différence entre 48 et 36 ou la différence entre 84 et 73. **(rép. différence entre 84 et 73)**		
5. *Montrez les nombres 13 et 39 sur un tableau des nombres.* Combien font 13 + 39? **(rép. 52)**		
6. *Montrez les nombres 36 et 18 sur un tableau des nombres.* Combien font 36 et 18? **(rép. 54)**		
7. Combien fait 301 si je lui enlève 7? **(rép. 294)**		

Appendix C

Number Knowledge Test Scoring

Number Knowledge Test Scoring and Developmental Level Conversion Chart

Raw Score Equivalents	**Developmental Level Score[*]**	**C.A.**
1-3	-0.5	2-3 years
4-6	0.0	3-4 years
7-8	0.5	4-5 years
9-14	1.0	5-6 years
15-19	1.5	6-7 years
20-25	2.0	7-8 years
26-29	2.5	8-9 years

Appendix D

Self-Explanation Task One

Self-Explanation Task (SET-1)

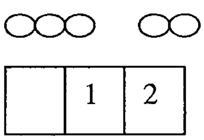
Matériel requis

- 8 pastilles
- 1 compteur de Robinson

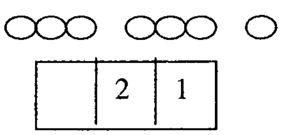
PARTIE 1

1. **Présenter la tâche à l'enfant** :
   Aimes-tu compter toi? Écoute, je vais te présenter une nouvelle façon de compter. Je vais utiliser des pastilles et le compteur de Robinson et tu devras essayer de comprendre comment je compte les pastilles. Lorsque je vais compter, j'aimerais que tu me dises tout ce qui te vient en tête. Je vais faire trois exemples avec toi et tu essaieras pendant ce temps de comprendre comment je compte les pastilles. Après, je vais te demander d'utiliser ma façon de compter avec un certain nombre de pastilles. D'accord?

2. **L'expérimentateur présente sa façon de compter SANS PARLER :**

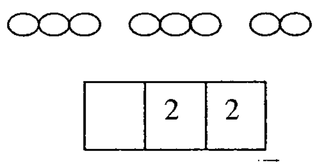


   a. L'expérimentateur aligne cinq pastilles et rappelle à l'enfant de lui dire tout ce qui lui vient en tête pendant qu'il fait la démarche. L'expérimentateur compte trois pastilles une à la fois en les séparant des deux pastilles restantes. Ce faisant, il forme un groupe de trois pastilles séparé des autres pastilles (voir image). Ensuite, il met un « 1 » sur le compteur de Robinson à la position des dizaines. Puis, il compte les deux pastilles restantes et met « 2 » sur le compteur de Robinson à la position des unités. L'expérimentateur laisse quelques secondes à l'enfant pour réfléchir. Par la suite, l'expérimentateur questionne l'enfant.
      - *Comment penses-tu que je savais cela?*
      - *Que représente le nombre « 1» sur le compteur?*
      - *Que représente le nombre « 2 » sur le compteur (en le pointant du doigt)?*
      - *Pourquoi le nombre « 1 » est à cette position? Aurait-il pu être à la place du « 2 »? Et le nombre « 2 », que représente-il? Aurait-il pu être à la place du « 1 »?*



   b. Pour le second exemple, l'expérimenateur aligne sept pastilles et rappelle à l'enfant de lui dire tout ce qui lui vient en tête pendant qu'il fait la démarche. L'expérimentateur compte trois pastilles une à la fois en les séparant des pastilles restantes. Ce faisant, il forme un groupe de trois pastilles séparé des autres pastilles (voir image). Ensuite, il met un « 1 » sur le compteur de Robinson à la position des dizaines. L'expérimentateur compte trois autres pastilles une à la fois en les séparant des pastilles restantes. Ce faisant, il forme un autre groupe de trois pastilles séparé de la pastille restante (voir image; il y a alors deux groupes de trois pastilles de former). Il met alors un « 2 » sur le compteur de Robinson à la position des dizaines. Puis, il compte la pastille restante et inscrit « 1 » sur le compteur de Robinson à la position des unités. L'expérimentateur laisse quelques secondes pour permettre à l'enfant de réfléchir. Par la suite, l'expérimentateur questionne l'enfant.

- *Comment penses-tu que je savais cela?*
- *Que représente le nombre « 2 » sur le compteur?*
- *Que représente le nombre « 1 » sur le compteur (en le pointant du doigt)?*
- *Pourquoi le nombre « 2 » est à cette position? Aurait-il pu être à la place du « 1 »? Et le nombre « 1 », que représente-il? Aurait-il pu être à la place du « 2 »?*



c. Pour le troisième et dernier exemple, l'expérimentateur aligne huit pastilles et rappelle à l'enfant de lui dire tout ce qui lui vient en tête pendant qu'il fait la démarche. L'expérimentateur compte trois pastilles une à la fois en les séparant des pastilles restantes. Ce faisant, il forme un groupe de trois pastilles séparé des autres pastilles (voir image). Ensuite, il met un « 1 » sur le compteur de Robinson à la position des dizaines. L'expérimentateur compte trois autres pastilles une à la fois en les séparant des pastilles restantes. Ce faisant, il forme un autre groupe de trois pastilles séparé des autres pastilles (voir image; il y a alors deux groupes de trois pastilles de former). Il met alors un « 2 » sur le compteur de Robinson à la position des dizaines. Puis, il compte les deux pastilles restantes et inscrit « 2 » sur le compteur de Robinson à la position des unités. L'expérimentateur laisse quelques secondes pour permettre à l'enfant de réfléchir. Par la suite, l'expérimentateur questionne l'enfant.

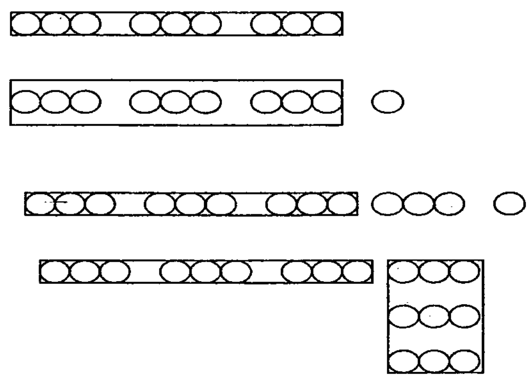
- *Comment penses-tu que je savais cela?*
- *Que représente le nombre « 2 » sur le compteur?*
- *Que représente le nombre « 2 » sur le compteur (en le pointant du doigt)?*
- *Pourquoi le nombre « 2 » est à cette position (en le pointant du doigt – position des dizaines)? Aurait-il pu être à la place du « 2 » (qui est à la position des unités – pointez le nombre avec le doigt)? Et le nombre « 2 », que représente-il (en le pointant du doigt – position des unités)? Aurait-il pu être à la place du « 2 » (qui est à la position des dizaines)?*

<u>PARTIE 2</u>
NEAR TRANSFER TASK

1. Une fois ces questions posées, l'expérimentateur demande à l'enfant de compter de cette façon avec 9 pastilles.

    a. *J'aimerais à présent que tu essaies de compter comme je viens de le faire, mais avec 9 pastilles. Dis-moi à voix haute ce que tu te dis dans ta tête pendant que tu fais la tâche.*

2. Suivant la même démarche, l'expérimentateur demande à l'enfant de compter 10, 13 et 18 pastilles en base 3.



9	1	0	0
10	1	0	1
13	1	1	1
18	2	0	0

Appendix E

Self-Explanation Task Two

STUDENT ID

Self-Explanation Task (SET-2)

PARTIE 1
Exemples résolus par l'expert, c'est-à-dire l'expérimentateur.

$12 + 15 - 15 =$ ______

$34 + 26 - 26 =$ ______

$41 + 37 - 37 =$ ______

$56 + 14 - 14 =$ ______

$28 + 11 - 11 =$ ______

$19 + 34 - 34 =$ ______

PARTIE 2

Résous les équations suivantes.

$$15 + 11 - 11 = \_\_\_\_\_\_\_\_$$
$$33 + 24 - 24 = \_\_\_\_\_\_\_\_$$
$$44 + 35 - 35 = \_\_\_\_\_\_\_\_$$
$$52 + 42 - 42 = \_\_\_\_\_\_\_\_$$
$$18 - 18 + 22 = \_\_\_\_\_\_\_\_$$
$$17 - 17 + 23 = \_\_\_\_\_\_\_\_$$
$$27 + 27 - 27 = \_\_\_\_\_\_\_\_$$
$$18 + 18 - 18 = \_\_\_\_\_\_\_\_$$

- Comment est-ce que j'ai fait pour savoir que ce "truc" fonctionnait?
- Est-ce que ce « truc » t'a aidé à trouver les réponses? Comment ce truc t'a-t-il aidé?

<u>PARTIE 3</u>

Essaie-toi encore!!!

$$15 + 19 - 15 = \_\_\_\_\_\_\_\_\_\_$$

$$32 + 21 - 32 = \_\_\_\_\_\_\_\_\_\_$$

$$42 + 36 - 42 = \_\_\_\_\_\_\_\_\_\_$$

$$51 + 29 - 51 = \_\_\_\_\_\_\_\_\_\_$$

Appendix F

Self-Explanation Task Three

STUDENT ID [ ]

Self-Explanation Task (SET-3)

Pour les garçons

PARTIE 1

Lucas, un garçon du premier cycle provenant d'une autre école, a résolu les équations ci-dessous d'une façon différente de ce que tu as appris en classe. Qu'a-t-il fait? Explique-moi.

$14 + 13 = \boxed{27}$

→ 20+7
27

$26 + 12 = \boxed{38}$

→ 30+8
38

$33 + 15 = \boxed{48}$

→ 40+8
48

$47 + 12 = \boxed{59}$

→ 50+9
59

- Comment Lucas a-t-il su que ses réponses étaient bonnes?

PARTIE 2

C'est à ton tour! Essaie de faire comme Lucas.

$15 + 19 = \square$

$28 + 17 = \square$

$33 + 18 = \square$

$47 + 16 = \square$

STUDENT ID [ ]

Self-Explanation Task (SET-3)
Pour les filles

PARTIE 1

Camille, une élève de premier cycle provenant d'une autre école, a résolu les équations ci-dessous d'une façon différente de ce que tu as appris en classe. Qu'a-t-elle fait? Explique-moi.

$14 + 13 = $ 27
20+7
27

$26 + 12 = $ 38
30+8
38

$33 + 15 = $ 48
40+8
48

$47 + 12 = $ 59
50+9
59

-Comment Camille a-t-elle su que ses réponses étaient bonnes?

PARTIE 2

C'est à ton tour! Essaie de faire comme Camille.

$15 + 19 =$ ☐

$28 + 17 =$ ☐

$33 + 18 =$ ☐

$47 + 16 =$ ☐

Appendix G

Rubric for coding self-explanations

Rubric for coding self-explanations

Coding	*Definition*	*Examples*
Conceptual Appropriate (CA) CODE: 1	**SET - 1** • The student understands that the numbers in the "tens" position on the flip chart represent the groups of three chips and that the numbers in the "units" position represent the remaining chips. • When explaining, the student may describe the action of the experimenter, but do so to explain his or her understanding. The description is always linked to a concept. <u>Keywords</u>: groupe de 3, équipe de 3, paquets de 3	• *« Le 2 représente les deux trois (en pointant les deux groupes de trois pastilles) et le 1 représente un qui reste. »* • *« Tu viens de comme les séparer. Les trois ça fait un groupe et tu mets le 1. Les deux restent, alors tu mets le 2. »* • *Oui ou non*, or similar short answers (when answering correctly a question formulated by the experimenter using the concept of Base 3 or grouping of three chips). • A gesture that shows the understanding. For example, when pointing the two groups of three chips.
	SET - 2 • The student uses correctly the « canceling out » strategy.	• *« Il y a un 11, pis on l'enlève, alors il y en a plus. C'est comme 0. »* • *Oui ou non*, or other similar short answers (when answer correctly a question formulated by the experimenter using the strategy "canceling out")
	SET - 3 • The student uses correctly regrouping "tens" and "units" to explain the problem. • When explaining, the student may describe the action of the experimenter, but do so to explain his or her understanding. The description is always linked to a concept.	• *« Il y a deux dizaines, puis il y a sept unités. »* • *Oui ou non*, or other similar short answers (when answering correctly a question formulated by the experimenter using the concept of regrouping "tens" and "units")
Conceptual Inappropriate (CI) CODE: 2	**SET - 1** • The student has a sense that the numbers being put up on the flip chart represent the groupings of tokens that were made, but does not quite grasp it in terms of Base 3. For example, many	• *« T'as compté les dizaines et après, t'as fait les unités. Les trois ça fait une dizaine, et il t'en reste deux. »* • *« Là, il y a deux groupes (en*

	students said that the second position on the flip chart represented groups of "tens" and the last one, the "units," which they claimed to correspond to the tokens. They said that groups of three made "dizaines" and that groups of one made "unités." • The student clearly understands the concept of the task, but makes an error when explaining it (e.g., using the term "paquet" even for the "units") • The student refers to other concepts than regrouping that are not completely irrelevant or incorrect to explain the task.	*parlant du paquet de trois pastilles et des 2 cubes restants). »* • *Oui ou non*, or other similar short answers (when answering a question formulated by the experimenter with mathematical concepts not related to Base 3 or regrouping, but not completely irrelevant or incorrect to explain the task). • A gesture that shows an understanding, even when not completely correct.
	SET - 2 • The student refers to other concepts than « canceling out, » that are not completely irrelevant or incorrect to explain the task. • The student clearly understands the concept of the task, but makes an error when explaining it (e.g., using the term "ajouter" au lieu de "enlever, soustraire").	• *« C'est comme deux grosses boules qui se touchent, pis elles se détruisent au complet. »* • Oui ou non, or other similar answers (when answering a question formulated by the experimenter with mathematical concepts not related to the strategy "canceling out," but not completely irrelevant or incorrect to explain the task)
	SET - 3 • The student clearly understands the concepts of the task, regrouping and place value, but makes an error when explaining it (e.g., using the word "units" instead of "tens"). • The student refers to other concepts than combining "tens" and "units" that are not completely irrelevant or incorrect to explain the task.	• *« Là, 2 unités (meaning 20 or 2 dizaines) et 7 unités, ça fait 27.»* • *Oui ou non*, or other similar answers (when answering a question formulated by the experimenter with mathematical concepts not related to combining "tens" and "units," but not completely irrelevant or incorrect to explain the task)
Conceptual Irrelevant (CIR) CODE 3	**SET-1; SET-2; SET-3** • The student refers to concepts of the task (see above), but uses or explains them completely incorrectly, showing that the student does not understand them. • The student refers to other concepts that are not related to the task, that do not make sense.	• The experimenter asks what the "1" in the "tens" position represents. The student shows the two cubes left (the good answer it the group of three) • *« On peut faire des groupes de 2 avec cela (en parlant des groupes de trois). »*

Procedural Correct (PC) CODE: 4	**SET-1; SET-2; SET-3** • The student correctly reiterates, describes the actions performed by the experimenter or tells what he or she sees, but not in relation with the key concepts of the task. The student talks in terms of methods or procedures. For example, if he states that two piles of tokens were made and that a "2" and a "0" were put up on the flip chart, without explaining further more the relationships between the numbers and the groups of chips (no links to the concepts), this will be coded as Procedural Correct.	• *"T'as compté 3 et après t'as compté 2. Après ça, t'as mis un 1 et un 2 sur le compteur Robinson. »* • *« Pis là, elle a fait ça. »* • *« Je barre les deux 22.»*
Procedural Incorrect (PI) CODE 5	**SET-1; SET-2; SET-3** • The student incorrectly reiterates or describes the actions performed by the experimenter. • The student uses inappropriate procedures to self-explain or incorrectly tells what he or she sees (not in relation with the concepts). The student talks in terms of methods or a procedure, but incorrectly.	• *« Tu as calculé (en parlant du truc). »* • *« Il a compté sur ses doigts. »*
Other (O) CODE: 6	**SET-1; SET-2; SET-3** • Any statement that does not belong to the five first categories: positive or negative monitoring, reading the equation and saying its answer without explanation, non mathematical comments, etc • When the student says the equation in SET-2. • When the student says the answer in SET-2.	• *« Je ne sais pas…Je comprends pas, c'est difficile. »* • *"15-15+2, cela fait 2."*

Appendix H

Consent forms

Formulaire de consentement PARENT

Recherche au premier cycle -À remettre au plus tard le 18 mars 2009

Par la présente, je déclare consentir à participer à un projet de recherche mené par Mme Annick Lévesque du département en éducation de l'Université Concordia au 1400, boul. de Maisonneuve à Montréal et supervisé par Dr. Helena Osana. Mme Annick Lévesque peut être rejointe par téléphone au 450-474-3040 ou par courriel au anilevesque@cslaval.qc.ca. Dr. Osana peut être rejointe par courriel au osana_h@education.concordia.ca et par téléphone au 514- 848-2424 poste 2543.

A. BUT DE LA RECHERCHE

Cette recherche vise à étudier comment les enfants arrivent à s'expliquer de nouvelles procédures en mathématiques. Cette étude permettra de mieux comprendre ce qui se passe dans la tête des enfants lorsqu'ils essaient de s'expliquer de nouvelles tâches en mathématiques. En examinant cette habileté, je désire améliorer la qualité de l'intervention éducative des enseignants pour permettre aux enfants de mieux comprendre les mathématiques et de faire de meilleurs liens entre les procédures mathématiques qu'ils apprennent et les concepts mathématiques importants à développer au premier cycle.

B. PROCÉDURE

La recherche sera réalisée à l'école alternative l'Envol, au 3661, boul. de la Concorde Est, à Laval, dans quatre classes de premier cycle. Le projet aura lieu au mois de mars et avril 2009. Des enfants de 2e année seront interviewés. Les enfants seront rencontrés individuellement à quatre reprises. À chaque rencontre, l'enfant devra compléter une tâche mathématique et expliquer ce qu'il comprend de cette tâche. Chaque entrevue devrait durer entre 10 à 15 minutes. En tout, la participation de votre enfant à ce projet ne devrait pas dépasser 65 minutes. Comme parent, vous n'aurez rien à faire ou à compléter lors de cette étude. Les données fournies par les enfants resteront confidentielles. Aucun risque n'est associé au projet. Les résultats de cette étude vous seront transmis par courriel.

C. CONDITIONS DE PARTICIPATION

- Je comprends que je puis retirer mon consentement et interrompre à tout moment la participation de mon enfant à ce projet et ce, sans conséquences négatives.
- Je comprends que la participation de mon enfant à cette étude est CONFIDENTIELLE, c'est-à-dire que le chercheur connaît l'identité de mon enfant, mais ne la révélera pas.
- Je comprends que les données de cette étude puissent être publiées, mais que des pseudonymes seront utilisés pour les enfants, l'école et les enseignants, si nécessaire.
- Je comprends le but de la présente étude : je sais qu'elle ne comprend pas de motifs cachés dont je n'aurais pas été informé(e) et qu'elle ne représente aucun risque pour mon enfant.
- Je sais que les résultats de cette recherche me seront transmis via le courriel.

-------------------VEUILLEZ REMPLIR CE COUPON ET LE RETOURNER À L'ÉCOLE POUR LE 18 MARS--------------

J'AI LU ATTENTIVEMENT CE QUI PRÉCÈDE. JE COMPRENDS LA NATURE DE L'ENTENTE ET JE CONSENS LIBREMENT ET VOLONTAIREMENT À CE QUE MON ENFANT PARTICIPE À CETTE ÉTUDE.

NOM du parent (caractères d'imprimerie) ______________________________

NOM de l'enfant (caractères d'imprimerie)______________________________

SIGNATURE du parent ______________________________

Si vous avez des questions concernant vos droits en tant que participants à l'étude, S.V.P. contactez Adela Reid, Agente d'éthique en recherche/conformité, Université Concordia, au 514-848-2424 poste 7481 ou par courriel au adela.reid@concordia.ca

STUDENT ID [ ]

Formulaire de consentement – ENFANT

J'accepte de participer au projet d'Annick. Je comprends qu'elle veut examiner mes habiletés à dire dans mes mots ce que je comprends en mathématiques. Je sais que je devrai faire quatre entrevues de 10 à 15 minutes et que je serai seule avec Annick dans un local à l'extérieur de ma classe. Lors de chacune de ces entrevues, je devrai faire une petite tâche en mathématique et l'expliquer dans mes mots.

Je sais aussi que mes parents sont d'accord que je participe au projet d'Annick et qu'ils pourront savoir ce que j'y ai fait et ce que j'ai répondu. Je comprends aussi que seules Annick et son professeur d'université pourront connaître mon nom, mais qu'elles ne le diront pas à personne.

Je comprends aussi que cette activité n'aura pas de conséquences négatives sur moi. Je sais également que je peux à n'importe quel moment dire que je ne veux plus participer au projet et que je pourrai retourner dans ma classe sans problème.

Je sais pourquoi Annick veut me rencontrer en entrevue individuelle et je me sens à l'aise avec cela. Je sais que le projet ne comporte aucun danger pour moi. Je sais que mon nom ne sera connu que par Annick et son professeur et ce, même si le projet est publié. Je sais aussi que si je ne désire plus faire le projet, je pourrai retourner en classe et qu'Annick comprendra mon souhait.

Pour toutes ces raisons, j'accepte librement et volontairement de participer à ce projet.

NOM :____________________________________

SIGNATURE :______________________________

DATE :___________________________________

Appendix I

Record Keeping Forms

Compilation

Participants	Tâche A (pastilles)	Tâche B (équations)	Tâche C (problèmes)
	Tâche A (pastilles)	Tâche C (problèmes)	Tâche B (équations)
	Tâche B (équations)	Tâche A (pastilles)	Tâche C (problèmes)
	Tâche B (équations)	Tâche C (problèmes)	Tâche A (pastilles)
	Tâche C (problèmes)	Tâche A (pastilles)	Tâche B (équations)
	Tâche C (problèmes)	Tâche B (équations)	Tâche A (pastilles)
	Tâche A (pastilles)	Tâche B (équations)	Tâche C (problèmes)

Compilation

	Tâche A (pastilles)	Tâche C (problèmes)	Tâche B (équations)
	Tâche B (équations)	Tâche A (pastilles)	Tâche C (problèmes)
	Tâche B (équations)	Tâche C (problèmes)	Tâche A (pastilles)
	Tâche C (problèmes)	Tâche A (pastilles)	Tâche B (équations)
	Tâche C (problèmes)	Tâche B (équations)	Tâche A (pastilles)