On Relay Assignment for Cooperative Systems

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Abstract

On Relay Assignment for Cooperative Systems

Xuehua Zhang

In this thesis, we investigate relay assignment for cooperative networks comprising multiple relay nodes and multiple simultaneously transmitting users. The users are grouped in pairs where the nodes comprising each pair are assumed to communicate information between themselves. Such arrangement is normally referred to as a two-way relay channel. The relay nodes are assumed to be able to use network coding when they are selected to help in relaying.

We propose two relay assignment schemes, one that considers all possible permutations of relay assignments and picks the one that results in the best end-to-end (E2E) bit error rate performance. The other one is based on a subset of the available permutations and picks the one that gives the best performance. The latter scheme is devised to make the analysis more tractable and to reduce the computational complexity in finding the best permutation.

In the first part of the thesis, we assume that the relays do not use network coding. We also assume that each pair is helped by one relay. We examine the bit
error rate performance of the proposed relay schemes for both relaying protocols: Amplify-and-forward (AF) relaying and decode-and-forward (DF) relaying. For AF relaying, we derive an upper bound on the E2E bit error rate performance. For DF relaying, we derive closed-form expressions for the E2E bit error rate performance of ideal relaying. It is shown that the full diversity is maintained in all cases, which equals the number of relays. In addition, for DF relaying, we adopt log-likelihood (LLR)-threshold relaying to control error propagation at the relay and give the corresponding performance analysis.

In the second part of the thesis, we consider relay assignment for cooperative systems with multiple two-way relay channels. The relays are assumed to use network coding to simultaneously transmit the signals corresponding to the pairs they are assigned to. We consider the cases when a relay is assigned to a single pair and the case when a relay is assigned (simultaneously) to multiple pairs. To achieve the latter, we use higher order modulation schemes at the relay nodes. We analyze the performance of these schemes over symmetric and asymmetric independent Rayleigh fading channels, and derive closed-form expressions for the end-to-end bit error rate performance. We show that, for all cases, the full diversity is achieved, which equals the number of relays. We present several examples to verify the theoretical results.
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Chapter 1

Introduction

1.1 Centralized versus Distributed Multiple Antenna Systems

Wireless communications suffer from great challenges such as fading, interference and high data rate requirements (but limited bandwidth). Some solutions have been proposed over the past few years to remedy this problem, the most important of which is using the so-called multiple input multiple output (MIMO) technology. In MIMO systems, the transmitter and receiver are normally equipped with colocated multiple antennas. The transmit antennas can be used to enhance the performance without improving the transmission rate such as the case in space-time coding, improve the transmission rates without improving the performance such as the case in layered space-time coding, or striking a balance between performance and transmission rate.
The latter can be achieved by employing multi-layered space-time coding. As for the receive antennas, they are always used to provide receive diversity.

In terms of capacity and information rates achieved by MIMO systems, the capacity increases linearly with the number of transmit or receive antennas (when there are as many transmit antennas as receive antennas). In general, the capacity increases by \( \min\{N_t, N_r\} \) bits per channel use for every 3 dB increase in the signal to noise (SNR), where \( N_r \) and \( N_t \) are the number of transmit and receive antennas, respectively, as compared to the capacity of a single input single output (SISO). The benefit in capacity is achieved without any additional power or bandwidth requirement.

The downside of MIMO technology, however, is the associated complexity. For instance, for every antenna employed, it is required to employ a radio frequency (RF) chain, which is bulky and costly. Also, the power consumption is relatively high due to the complex circuitry. In addition, the overhead required for training can be significant especially when the underlying channel changes relatively fast. In light of these constraints, the MIMO technology is deemed not practical for certain applications where power consumption and/or physical size is an issue. Such applications include cellular networks where it is not practical to mount multiple antennas along with their associated circuitry on a small mobile phone while keeping its size small and its cost affordable. Another example is wireless sensor networks, where the nodes are battery-operated and thus prolonging the battery life as much as possible is a crucial requirement.
As an alternative to using collocated antennas as in MIMO systems, one can achieve the same spatial diversity gain through cooperative diversity [1]-[3]. In cooperative communications, multiple nodes in a wireless network cooperate among themselves to form a virtual antenna array. Using cooperation, it is possible to exploit the spatial diversity of the traditional MIMO techniques without each node necessarily having multiple antennas. The destination receives multiple versions of the message from the source and one or more relays and combines these to obtain a more reliable estimate of the transmitted signal. These cooperative techniques utilize the broadcast nature of wireless signals by observing that a source signal intended for a particular destination can be overheard at neighboring nodes. These nodes, called relays, partners, or helpers process the signals they overhear and transmit towards the destination. At any given time, any node can be a source, relay, or destination. The function of the relay node is to assist in the transmission of the source information to the destination node.

A basic cooperative communication model is illustrated in Fig. 1.1.

![Figure 1.1: Cooperative system model.](image-url)
In the figure, the network has three nodes, one source, one relay and one destination. Due to the broadcast nature of the wireless communication, the relay can overhear the signal from source. It transmits the received signal to the destination according to some relaying protocol, which will be discussed in the next chapter.

The advantage of cooperative communications can be summarized as follows.

1. The flexibility in the network configurations whereby the number of cooperating nodes can be changed according to a specified system performance criterion.

2. The relaying strategy can be adapted to fit various scenarios.

3. Adaptive modulation and coding can be employed to achieve certain performance objectives.

4. The coverage is expected to be better since users will always find relaying nodes close by even if they are at the far end of their cell.

5. A consequence of this is an increased user capacity since the user transmitted power can be better controlled which in turn controls the level of multiple access interference at the access point.

Owing to the advantages of cooperative communications, it has penetrated into the standard of future wireless systems including Long term evolution (LTE), wireless sensor networks (IEEE 802.15.4), fixed broadband wireless systems (WiMax, IEEE 802.16j), Mobile WiMax (IEEE 802.11e), wireless LANs (802.11, a, b, g, n) and cognitive radio/spectrum sharing techniques (IEEE 802.22).
1.2 Problem Statement and Motivation

One of the most active research areas in cooperative communications is selection diversity, which aims at utilizing the system/network resources in a more efficient way. Most of the works consider selecting the best relay, according to a certain criterion, to serve a pair of nodes in a network. A few works have been done for relay assignment for multiple pairs in relay networks. In [4], the author extend the reactive opportunistic relaying proposed in [5] to a relay network with multiple pairs. While for reactive opportunistic relaying, relay selection is performed after the source transmission. That means that all the relays have to try to decode all the information from all sources and tell the destinations whether it has successfully decode which will give burden to the relays and lead to additional overheads. This motivates us to propose relay assignment schemes based on proactive opportunistic relaying. The relay assignment is done prior to the actual transmission from the pairs. Accordingly, once a relay is assigned to a certain pair, that relay needs to decode only the message coming from its respective pair. In addition, no work has been done to solve the following problem: How to allocate the relays in a network environment to the network pairs in conjunction with network coding. To the best of our knowledge, our thesis is the first piece of work to consider this problem.

To elaborate more, consider the system model shown in Fig. 1.2, which represents a cooperative network with $n$ relays and $m$ pairs of nodes. We assume here that the nodes comprising each pair would like to exchange information between each other
through the relay nodes. As such, the channel linking each pair is referred to as a two-way relay channel in this thesis. One of the main objectives in this thesis is to find efficient ways to assign the relay nodes to the network pairs while employing network coding. The relay nodes can operate either in the amplify-and-forward (AF) mode or decode-and-forward (DF) mode. In the AF mode, the relay just amplifies the signal it receives from the nodes and relays it to the destination. On the other hand, in the DF mode, the relay will decode the received signals and relays a clean version of the decoded signal (assuming no decoded errors).

Figure 1.2: Cooperative network with $n$ relays and $m$ pairs.

1.3 Thesis Contributions

The main contributions of the thesis are summarized as follows:
1. We propose four relay assignment schemes for multiple source-destination cooperative networks, two of which take the direct path into consideration while the other two do not. Some of the schemes consider all possible relay assignment permutations and select the best one, whereas others consider only a subset of all permutations, which significantly reduce the computation complexity.

2. We analyze the suboptimal relay assignment scheme for both AF relaying and DF relaying in terms of the end-to-end (E2E) bit error rate performance. For AF, we derive an upper bound on the E2E bit error rate. For DF, we derive closed-form expressions on the E2E bit error rate for genie-aided DF relaying, i.e., when the relays know when errors occur and thus keep silent.

3. In addition, for DF relaying, we adopt a threshold-based relaying scheme to control error propagation. We analyze this scheme and give an expression for the E2E bit error rate performance.

4. We give the diversity proof for our relay assignment schemes. Our relay assignment schemes are shown to achieve full diversity order which is the number of relays plus 1 when there is a direct path.

5. We consider relay assignment schemes for cooperative systems with multiple two-way relay channels cooperating through relays that employ network coding. We analyze the performance of these schemes and derive the closed-form bit error rate expressions for both symmetric and asymmetric Rayleigh fading.
channels. We also analyze the diversity order of our schemes in this case and show full diversity are also achieved.

6. We also consider relaying with $M$-PSK ($M$-ary phase shift keying) in an effort to improve the network throughput. We analyze this system and show that the results are similar, in terms of diversity order, to the case of relaying with BPSK signaling, except for some performance degradation in the former case, as expected.

1.4 Thesis outline

The remainder of this thesis is organized as follows. In Chapter 2, we present some background knowledge including cooperative communications, relay selection and two-way relay channels.

In Chapter 3, we propose our relay assignment schemes for multiple source-destination cooperative networks. The E2E bit error rate expression is derived for both AF relaying and DF relaying. We also adopt LLR threshold-based relaying scheme to control error propagation for DF relaying and analyze the E2E bit error rate performance.

Network coding with higher order modulation schemes is considered in Chapter 4. We analyze the performance of this scheme and derive a closed form expression for the E2E bit error rate.

Finally, some conclusions of this thesis are drawn in Chapter 5. We also discuss in
that chapter some possible future research potential and improvements.
Chapter 2

Background

2.1 Cooperative Communications

In wireless networks, cooperative diversity is an attractive new way to increase throughput, reduce energy requirements and provide resistance to channel fading effects. Due to its substantial gains over non-cooperative communication, cooperative communications has gained a lot of research attention and led to many research publications.

The idea of cooperative communication builds significantly from the work on the classic relay channel model within the information theory community. Cover and El Gamal analyze the capacity of the three-node relay network in AWGN channels [6]. Sendonaris et al extend the idea in [6] to a cooperative mode where the users can act both as information sources and relays. They first present a general information
theoretic model for cooperation between a pair of users, for which the achievable rate regions and outage probabilities are examined [1].

Several cooperative schemes were proposed to achieve cooperative diversity. AF refers to the case when a relay can simply retransmit, or forward, the noisy analog signal received from the source. In [7] Laneman and Wornell first proposed the scheme AF. They also extend the repetition-based cooperative protocols developed in [8] for the multi-user case.

Using DF, the relay try to decode the signal transmitted by the source, and then forward the decoded bits to the destination. The first work proposing a DF protocol for user cooperation was by Sendonaris, Erkip, and Aazhang [1].

Both AF and DF involve a relay repeating the symbols transmitted by the source. In the view of channel coding, repetition coding is not the most efficient use of the available bandwidth. The performance of a cooperative systems can be improved by using channel coding. Coded cooperation is investigated in [9] by Hunter and Nosratinia. In frame 1, the users transmit their own data to each other and to destination. In frame 2, the users transmit each other’ bits given that it successfully decodes the signal of its partner. The overall code can be a convolutional codes and rate compatible convolutional codes. A distributed turbo coded cooperative scheme is proposed in [11].

Compress-and-Forward (CF) was first proposed in Theorem 6 of [6]. The relay compress the received signal without decoding it and forward it to the destination.
Li, Hu [10] proposed using Slepian-Wolf coding in cooperation communication which is the first practical coding scheme for CF.

The idea of distributed space-time coded cooperation was first described in [8]. The source and the assisting relay antennas form a distributed antenna array. The gains of this system are achieved by a proper selection of distributed space-time codes. The space-time coded cooperative is well suitable for multi-hop wireless networks.

### 2.2 Relay Selection

Relay selection for cooperative communication has attracted considerable research interest. Several schemes are proposed and analyzed. The authors in [12] propose a scheme where the relay that contributes the most to the received SNR is selected. It is shown that this scheme achieves full diversity. A selection scheme termed *nearest-neighbour relay selection* is proposed in [13], where the relay that is closest to the source is selected. The diversity order of this scheme with AF relaying is analyzed in [14], and is shown to achieve a diversity of one. The relay selection scheme based on the E2E channel quality for both AF and DF relaying strategies is proposed in [15]. The authors show that using the best relay for cooperation could achieve the same diversity-multiplexing trade-off as that of the space-time coding scheme proposed in [8]. For this selection scheme, there are two main relay selection methods: Proactive
and reactive opportunistic relaying [5].\(^1\) The difference between these two methods are as follow. For proactive opportunistic relaying, the relay selection is based solely on the quality of the subchannels, which takes place before the source actually transmits its signal. Specifically, the relays are ordered according to their respective weakest subchannels, i.e., bottlenecks, and the one exhibiting the best bottleneck is chosen. While for reactive opportunistic relaying, relay selection is performed after the source transmission. That is, the selected relay is the one that has successfully decodes the source’s message and whose relay-destination subchannel is strongest.

Closed-form expressions for the outage and bit error probability of uncoded, threshold-based proactive opportunistic relaying and reactive opportunistic relaying are derived in [16]. It is also shown that the relative performance of the two relaying schemes is highly affected by the threshold. The diversity order of proactive opportunistic AF relaying without direct path is derived in [14], and it is shown that the full diversity is achieved, which equals the number of relays. Closed-form expressions for the outage and bit error probability for proactive opportunistic AF relaying with a direct path between the source and destination can be found in [17], and for proactive opportunistic DF relaying in [18]. It is shown in both cases that the full diversity can be achieved.

\(^1\)Note that the authors in [15], [16] use opportunistic relaying to refer to proactive opportunistic relaying. On the other hand, in [4] and [16], the authors use selection cooperation to refer to reactive opportunistic relaying.
2.3 Network Coding

Cooperative communication has been proven to be an effective way to combat wireless fading by allowing the mobile nodes to share their antennas to achieve spatial diversity [1]-[3]. However, for practical reasons, all communication nodes should operate in the half-duplex mode [3], implying a loss in spectral efficiency. In order to mitigate the spectral efficiency loss and improve the throughput of the cooperative communication network, some efforts have been made to incorporate network coding into cooperative communication.

The concept of network coding was first proposed by Ahlswede, et. al. in [19] as a routing method in lossless wireline networks. Its key idea is that the intermediate node linearly combines the received data instead of sending them directly. By taking advantage of the broadcast nature of the wireless communication, it is natural to apply the idea of network coding to wireless communication. Among all the works, much attention is given to half-duplex two-way relaying as it is a basic building block in most wireless networks.

The two-way relay channel model was first examined by Shannon in [20]. Various relaying protocols have been proposed for the two-way relaying channel. The relay nodes normally operate either in the DF or AF mode [33]. All proposed protocols can be classified into two types: Three-step schemes [21], [22], and two-step schemes [23], [24]. In the former schemes, the relay receives and decodes the bits received from both nodes in the first two steps. In the third step, the relay applies exclusive-or (XOR) to
both bits and broadcasts the resulting bit to both nodes. This results in saving one step in this way as compared to traditional relaying (without network coding). In the two-step schemes, the relay only decodes the sum of the signals received and maps it to a corresponding zero or one. In [25], a two-step AF scheme named analog network coding is presented. A general two-step network coding scheme for AF relaying and DF relaying can be found in [26] and the achievable sum-rates is analyzed. In [27], upper and lower bounds on the average sum rate of two-way AF relaying are derived. The outage performance of AF and DF two-way relaying is analyzed in [28].

2.4 Error Propagation in Cooperative Communications

If the error decoded bits are forward to the destination, these errors cause significant performance degradation at the destination, a problem usually called error propagation. It is one of the challenging problems for cooperative communications. To overcome this problem, different kind of schemes are proposed. One practical approach is that the relay using cyclic redundancy code (CRC) check and only forward the correct frames. However, firstly, the CRC check bring decoding delay and spectral efficiency loss, secondly, a single error in one frame will prevent all other correct bits being forward to the destination which will cause performance loss [31]. We can also set a threshold at the relay, only when the instantaneous source-relay SNR (or
LLR) is larger than the threshold, the relay sends the decoded bit to the destination [29], [30]. Otherwise, it keeps silence. Comparing the two methods as an indication of the reliability of the relay detection, the LLR thresholding scheme takes the instantaneous noise term into consideration, as opposed to SNR thresholding scheme only depend on the instantaneous fading level. It is shown in [29], [30] that LLR thresholding scheme has a better bit error rate performance than SNR thresholding scheme in practical SNR region.

2.5 Conclusions

In this chapter, we have reviewed some of the topics that are related to the work done in this thesis. As mentioned before, in the next two chapters, we will consider relay assignment for cooperative networks with multiple transmitting pairs, we also consider such systems with network coding.
Chapter 3

Relay Assignment Schemes for Multiple Source-Destination Cooperative Networks

3.1 Introduction

From previous chapter, it is shown that most of the above works consider selecting the best relay, according to a certain criterion, to serve a pair of nodes in a network. Relay assignment for multiple pairs in relay networks is considered in [4], [35]-[37]. A relay assignment scheme based on the location of the relays is proposed in [35]. In [36], the authors propose a relay assignment scheme, which is based on maximizing the minimum capacity among all pairs. The authors focus on developing a polynomial
time algorithm which is able to offer a linear complexity for each iteration. The authors in [37] propose a relay assignment scheme based on the distributed relay assignment scheme presented in [4] by considering the user's quality of service (QoS) requirements.

In this chapter, we address the problem of relay assignment to multiple simultaneously transmitting pairs. The relay assignment criteria for two different cases assuming the availability of different sets of instantaneous channel information are studied. Specifically, we propose two relay assignment schemes for both cases in which a relay is assigned to help one pair at a time. The first scheme is optimal since it considers all possible relay assignment permutations and picks the one that achieves the best performance. The second scheme is suboptimal since the search is done over a subset of the possible permutations, leading to tractability in the analysis and less complexity. In both schemes, the assignment is solely based on the channel quality, that is, after estimates of the fading coefficients are obtained at the relays. This implies that the relay assignment is done prior to actual transmission from the pairs. Accordingly, once a relay is assigned to a certain pair, that relay needs to decode only the message coming from its respective pair. We derive an lower bound for the E2E bit error rate of the proposed schemes with AF relaying and exact E2E bit error rate expression of the proposed schemes with DF relaying assuming that relay only forward correct decoded bits to the destination. It is shown that for both AF relaying and DF relaying the maximum diversity is achieved, which equals the num-
ber of relays plus 1. It is assumed here that the number of relays equals or exceeds the number of pairs. However, the assumption that relay only forward the correct decoded bits to the destination is an ideal case.

As LLR thresholding scheme have a better bit error rate performance than SNR thresholding scheme in practical SNR region, we adopt the LLR threshold-relaying scheme proposed in [29] and [30] to our DF scheme to prevent error propagation.

To the best of our knowledge, [4] is the only work that is directly related to what we are proposing here. There are, however, major differences between our proposed schemes and the ones in [4]. For instance, the schemes in [4] are based on reactive opportunistic relaying, that is, the relay is only chosen from the relays that have successfully decoded the received bits. As such, all the relays have to attempt to decode all the messages that they receive before they are assigned. Also relays need to send a message to the destination to indicate that whether or not they have successfully decoded the message. This will consequently incur additional overhead and complexity. Furthermore, only the outage probability performance is considered in [4].

The remainder of the chapter is organized as follows. The system model is presented in Section 3.2. In Section 3.3, we present the proposed relay assignment schemes. Performance analysis is carried out in Section 3.4. In Section 3.5, we analyzed the performance of our relay assignment scheme with LLR-threshold relaying. Several numerical examples and simulation results are given in section 3.6. Section
3.7 concludes this chapter.

3.2 System Model

![Figure 3.1: A cooperative network with m unidirectional communication pairs and n relays.](image)

The system model considered in this chapter is shown in Fig. 3.1. As shown in the figure, the network consists of $m$ pairs and $n$ relays where $n \geq m$. Each of the nodes is equipped with a single antenna and operates in a half-duplex mode. In the first time slot, the source of each pair transmits its signal, i.e., $m$ nodes transmit simultaneously in the first time slot using frequency division multiple access (FDMA) [8]. Depending on the relaying strategy, the assigned relays cooperate in the second time slot (more on this below). Note that only one relay is assigned to each pair, and this assignment is done before actual transmission takes place. As such, each relay will have to decode only the signal coming from the pair it is assigned to.
The network subchannels are assumed to experience independent slow and frequency-flat Rayleigh fading. Let \( h_{S_iR_j} \), \( h_{R_jD_i} \) and \( h_{S_iD_i} \) (for \( i = 1, \ldots, m \), \( j = 1, \ldots, n \)) denote the fading coefficient between the \( i \)th source–\( j \)th relay, \( j \)th relay–\( i \)th destination and \( i \)th source–\( i \)th destination, respectively. The channels gains are modeled as zero mean, unit variance complex Gaussian random variables. All subchannels are assumed to have the same average SNR. Binary phase shift keying (BPSK) modulation is assumed throughout the chapter.

Let \( y_{S_iR_j} \) denote the received signals at the \( j \)th relay from the \( i \)th source, and \( y_{S_iD_i} \) denote the received signals at the \( i \)th destination from the \( i \)th source, which are expressed, respectively, as

\[
y_{S_iR_j} = \sqrt{\rho} h_{S_iR_j} x_{S_i} + n_{S_iR_j}, \quad \text{and} \quad y_{S_iD_i} = \sqrt{\rho} h_{S_iD_i} x_{S_i} + n_{S_iD_i},
\]

where \( \rho \) denotes the average SNR of the subchannel, \( x_{S_i} \) is the signal transmitted from the \( i \)th source, and \( n_{S_iR_j} \) is an additive white complex Gaussian noise (AWGN) sample corresponding to the \( i \)th source–\( j \)th relay subchannel, with zero mean and unit variance. \( n_{S_iD_i} \) are similarly defined.

### 3.2.1 AF Relaying

When AF relaying is used, each relay amplifies the signal it is relaying, and the relayed signal at the \( i \)th destination is expressed as

\[
y_{R_jD_i} = G_j h_{R_jD_i} y_{S_iR_j} + n_{R_jD_i},
\]

where

\[
G_j = \sqrt{\frac{\rho}{|h_{S_iR_j}|^2 \rho + 1}}
\]

is the amplifying coefficient [7]. At the destination, the receiver combines the signals received over the two time slots via maximum ratio combining (MRC) as

\[
\hat{x}_{D_i} = \text{sign} \left( \Re \left\{ y_{S_iD_i} h_{S_iD_i}^* + y_{R_jD_i} h_{R_jD_i}^* G_j \right\} \right) \left( |G_j|^2 + 1 \right) [7].
\]
3.2.2 Genie-aided DF Relaying

Genie-aided relaying implies that a relay cooperates only if the signal is decoded correctly at the relay; otherwise the relay remains silent. We use this scheme as a benchmark for the relaying scheme that we present in the next section.\(^1\) The signal received at the \(i\)th destination is expressed as \(y_{R_jD_i} = \sqrt{\rho} h_{R_jD_i} \hat{x}_{S_iR_j} + n_{R_jD_i}\), where \(\hat{x}_{S_iR_j} = \text{sign} \left( \Re \left\{ y_{S_iR_j} h_{S_iR_j}^* \right\} \right)\). Consequently, if the signal is decoded correctly at the relay, the final decoded bit at the \(i\)th destination using MRC is expressed as \(\hat{x}_{D_i} = \text{sign} \left( \Re \left\{ y_{R_jD_i} h_{R_jD_i}^* + y_{S_iD_i} h_{S_iD_i}^* \right\} \right)\). Otherwise, the decoded bit is \(\hat{x}_{D_i} = \text{sign} \left( \Re \left\{ y_{S_iD_i} h_{S_iD_i}^* \right\} \right)\).

3.2.3 LLR-based Relaying

In genie-aided relaying, the relay is assumed to know exactly when an error occurs, which is idealistic. In addition, when a CRC code is employed, it may not be efficient because in certain cases a whole block of bits is discarded when even a single error occurs, resulting in a degradation in performance [31]. As an alternative, the relay may compute an estimate of the received bit in the form of LLR and subject this LLR value to a pre-determined threshold. If the LLR value exceeds the threshold, a hard decision is made on this bit and is relayed; otherwise, the relay remains silent.

This is the relaying scheme we consider in this section.

\(^1\)However, we remark that, in practice, genie-aided relaying may be achieved by employing a CRC type code for channel coded networks.
The value of the LLR can be computed as $\Lambda_{SR} = 4\sqrt{\rho} \left( |h_{S,R_j}|^2 x + \Re \left\{ n_{S,R_j} h_{S,R_j}^* \right\} \right)$ [29], [32]. The optimal threshold for the LLR-based relaying is derived as $\Lambda_0 = \ln(P_{e,EP} - P_{e,MRC} - P_{e,SD} - 1)$ [29], [30], where $P_{e,MRC}$ represents the bit error rate of ideal relaying, i.e., when the relay forwards all bits correctly. $P_{e,EP}$ represents the bit error rate at the destination given that the relay forwarded a wrong bit to the destination, and $P_{e,SD}$ represents the bit error rate at the destination without relaying.

When $|\Lambda_{SR}| \geq \Lambda_0$, the relay forwards the received bit to the $i$th destination, and the received signal is expressed as $y_{R_jD_i} = \sqrt{\rho} h_{R_jD_i} \hat{x}_{S,R_j} + n_{R_jD_i}$, where $\hat{x}_{S,R_j} = \text{sign} \left( \Re \left\{ y_{S,R_j} h_{S,R_j}^* \right\} \right)$. Consequently, the final decoded bits using MRC at the $i$th destination is expressed as $\hat{x}_{D_i} = \text{sign} \left( \Re \left\{ y_{R_jD_i} h_{R_jD_i}^* + y_{S,D_i} h_{S,D_i}^* \right\} \right)$. When $|\Lambda_{SR}| < \Lambda_0$, the relay remains silent. As such, the corresponding destination has only one copy of the signal, which was sent directly from the corresponding source. In this case, the final decoded bit is $\hat{x}_{D_i} = \text{sign} \left( \Re \left\{ y_{S,D_i} h_{S,D_i}^* \right\} \right)$.

### 3.3 Relay Assignment Criteria

It is intuitive that the optimal assignment scheme will have to depend on the subchannel gains between the relay nodes and the pair nodes, namely, $h_{S,R_j}$ and $h_{R_jD_i}$ (for $i = 1, \ldots, m$, $j = 1, \ldots, n$). The objective here is to optimize the E2E performance. Since there are two hops separating the end nodes in each pair, the weaker link is the one that will dominate the performance. This certainly applies to
all pairs. Therefore, the optimal relay assignment scheme is the one that results in the best subchannel among the weakest ones. Since there is a direct path between the sources and their corresponding destinations, it makes sense to consider such paths in the relay assignment process. In the following subsections, we consider the cases with and without the direct path being considered. We shall start with the case when the direct path is not considered.

### 3.3.1 Full-set Selection Without Direct Path (FSnoDP)

Let $\Phi$ denote the set containing all assignment permutations. Given that there are $m$ pairs and $n$ relays, the size of $\Phi$ is obviously $p_m^n$. Each element of $\Phi$ consists of all the fading coefficients ($2m$ of them) corresponding to that particular relay assignment. To simplify the presentation, let $\phi_k$ denote the $k$th element of $\Phi$ for $k = 1, 2, \ldots, p_m^n$, and let $|h|_{k,\text{min}}^2$ denote the smallest element in $\phi_k$, i.e., the weakest subchannel. Accordingly, the optimal assignment in this scenario, denoted by $\phi_k^*$, has index $k^*$ obtained as

$$k^* = \arg \max_k \{|h|_{k,\text{min}}^2, k = 1, 2, \ldots, p_m^n\}. \quad (3.1)$$

**Example 1** In this example, let $m = 2$ and $n = 3$. Hence, there are six possible ways of assigning one relay to each pair, which are: $(R_1, R_2)$, $(R_2, R_3)$, $(R_3, R_1)$, $(R_1, R_3)$, $(R_2, R_1)$, $(R_3, R_2)$ where the first entry is the relay assigned to the first

---

2We remark that one can also do the relay assignment for AF relaying based on the E2E SNR [14]. In contrast, in our schemes, the assignment is based on the bottleneck link. We provide a performance comparison between these schemes in Section VI where we demonstrate that the E2E SNR-based scheme is marginally better.
pair and the second is the relay assigned to the second pair. In order to simplify the presentation, we use \(|h_{S,R_i,D_i}|^2\) to represent the set \(\min\left\{|h_{S,R_i}|^2, |h_{R_i,D_i}|^2\right\}\) for \(i = 1, 2, j = 1, 2, 3\). Accordingly, we can express \(\Phi\) as

\[
\Phi = \begin{bmatrix}
\phi_1 \\
\phi_2 \\
\phi_3 \\
\phi_4 \\
\phi_5 \\
\phi_6
\end{bmatrix} = \begin{bmatrix}
|h_{S_1,R_1,D_1}|^2, |h_{S_2,R_2,D_2}|^2 \\
|h_{S_1,R_2,D_1}|^2, |h_{S_2,R_3,D_3}|^2 \\
|h_{S_1,R_3,D_1}|^2, |h_{S_2,R_1,D_2}|^2 \\
|h_{S_1,R_1,D_1}|^2, |h_{S_3,R_3,D_2}|^2 \\
|h_{S_1,R_2,D_1}|^2, |h_{S_3,R_2,D_1}|^2 \\
|h_{S_1,R_3,D_1}|^2, |h_{S_3,R_2,D_2}|^2
\end{bmatrix}.
\] (3.2)

Then, the relay assignment to be selected is

\[
k^* = \arg\max_{k} \left\{|h|_{k,\min}^2, \ k = 1, 2, \ldots, 6\right\}.
\] (3.3)

Note that, \(|h|_{k,\min}^2\) represents the minimum of the elements of the \(k\)th row.

### 3.3.2 Subset Selection Without Direct Path (SSnoDP)

Note that there is correlation between certain rows of \(\Phi\) given in (3.2). For instance, rows one and four are correlated since in both cases, \(R_1\) is assigned to the first pair. Such correlation makes it extremely difficult to find a closed form expression for the probability density function (pdf) of the fading coefficients corresponding to the selected relay assignment. This motivates us to consider a suboptimal assignment scheme whereby the permutations considered are those such that there is no
correlation between the fading coefficients. To achieve this, the set $\Phi$ is divided into $N = p_m^n / n$ subsets. Every subset contains all the fading coefficients. However, the subsets that give rise to correlation are dropped. In addition to making the analysis more tractable, the suboptimal scheme reduces the complexity associated with the assignment process since fewer permutations are examined. To make it clearer, consider the following example, which is based on the example given above.

**Example 2** The six possible relay assignments can be divided into two subsets: $\Phi_s_1$ and $\Phi_s_2$, which are defined as

\[
\Phi_s_1 = \begin{bmatrix}
\phi_1 \\
\phi_2 \\
\phi_3
\end{bmatrix} = \begin{bmatrix}
|h_{S_1R_1D_1}|^2, & |h_{S_2R_2D_2}|^2 \\
|h_{S_1R_2D_1}|^2, & |h_{S_2R_3D_2}|^2 \\
|h_{S_1R_3D_1}|^2, & |h_{S_2R_1D_2}|^2
\end{bmatrix},
\]

(3.4)

and

\[
\Phi_s_2 = \begin{bmatrix}
\phi_4 \\
\phi_5 \\
\phi_6
\end{bmatrix} = \begin{bmatrix}
|h_{S_1R_1D_1}|^2, & |h_{S_2R_3D_2}|^2 \\
|h_{S_1R_2D_1}|^2, & |h_{S_2R_1D_2}|^2 \\
|h_{S_1R_3D_1}|^2, & |h_{S_2R_2D_2}|^2
\end{bmatrix}.
\]

(3.5)

Note that all the elements within $\Phi_s_1$ and $\Phi_s_2$ are independent. Hence, we can use either one in the relay assignment process because both are equivalent. If we only choose from $\Phi_s_1$, we have $k^* = \arg\max_k \left\{ |h|_{k,min}^2, \ k = 1, 2, 3 \right\}$, whereas if we use $\Phi_s_2$, then $k^* = \arg\max_k \left\{ |h|_{k,min}^2, \ k = 4, 5, 6 \right\}$.
3.3.3 Full-set Selection with Direct Path (FSDP)

Since there is a direct path, it is expected that the performance improve if the direct path is taken into consideration in the assignment process. This can be accomplished by modifying $|h_{S_iR_jD_i}|^2$ defined above to $\min \left\{ |h_{S_iR_j}|^2, |h_{R_jD_i}|^2 \right\} + |h_{S_iD_i}|^2$. That is, the direct path along with the source-relay-destination link are considered as one unit in forming the different permutations, and the selection is done accordingly.

The rest of the assignment process follows exactly that of the FSnoDP criterion.

3.3.4 Subset Selection with Direct Path (SSDP)

For the subset selection with direct path, it is similar to the SSnoDP criterion except that the terms $|h_{S_iR_jD_i}|^2$ in (3.4) and (3.5) are modified to $\min \left\{ |h_{S_iR_j}|^2, |h_{R_jD_i}|^2 \right\} + |h_{S_iD_i}|^2$. Another difference is that, due to the presence of the direct path in the subsets to be searched over, there is still correlation among the resulting subsets, which again makes it difficult to perform the analysis.

It is clear that among the above four selection criteria, the one that results that the most inferior performance is the SSnoDP. Therefore, the analysis that follows is based on this criterion. We show that the maximum diversity order is achieved with this criterion, suggesting that the same diversity is achieved with the other criteria.

This is also confirmed by simulations.\(^3\)

\(^3\)It should be emphasized here that when $m = 1$, all assignment criteria are equivalent to the opportunistic relaying scheme proposed in [15]. Another special case is that, when $m = 2, n = 2$, FSnoDP is the same as SSnoDP, and FSDP is the same as SSDP.
3.4 Performance Analysis

As mentioned above, due to the correlation between some of the elements of $\Phi$, it is not easy to obtain a closed-form expression for the pdf of the selected permutation. However, when such correlation is not present, obtaining such pdf is straightforward. In this section, we first derive a lower bound on the bit error rate performance for AF relaying. We also derive a closed-form expression for Genie-DF relaying. For both cases, we derive an upper bound to show that the maximum diversity is achieved.

3.4.1 AF Relaying

For notational convenience, we drop the indices since the performance of all pairs is the same. It was shown in [34] that the instantaneous SNR of the weaker link of a two hop channel, denoted by $\gamma_{\min}$ ($\gamma_{\min} = \rho \min \{|h_{SR}|^2, |h_{RD}|^2\}$) is proved to be a tight upper bound on the equivalent one-hop instantaneous SNR. The authors [17] and [18] adopt this idea to derive the bit error rate expression for opportunistic relaying. In this section, we use the same result to derive an expression for the bit error rate expression for our relay assignment scheme. Note that the scheme in [17] and [18] is a special case of the one presented here.

The E2E bit error rate of AF relaying can be lower bounded as

$$P_{e,AF} \geq \int_0^\infty Q\left(\sqrt{2\rho z}\right) f(z) \, dz,$$

where $z = x + y$ and $x = \min \{|h_{SR}|^2, |h_{RD}|^2\}$, $y = |h_{SD}|^2$. For $x$ and $y$ are
independent variables, the pdf of $z$ is the convolution of the pdfs of $x$ and $y$.

$$f(z) = \int_0^\infty f_x(x)f_y(z-x)dx. \quad (3.7)$$

The pdf of $x = \min \{|h_{SR}|^2, |h_{RD}|^2\}$ corresponding to the selected relay is given as [42]

$$f(x) = 2ne^{-2mx}(1 - e^{-2mx})^{n-1} + 2m(2m - 2)e^{-2x} \int_0^x e^{-(2m-2)\theta} (1 - e^{-2m\theta})^{n-1} d\theta, \quad (3.8)$$

and the pdf of $y = |h_{SD}|^2$ corresponding to the source to destination link is given as

$$f(y) = e^{-y}. \quad (3.9)$$

Note that when $m = 1$, the second part of (3.8) is equal to zero. Therefore, we can rewrite (3.8) as

$$f(x) = \begin{cases} 
2ne^{-2x}(1 - e^{-2x})^{n-1}, & m = 1 \\
2ne^{-2mx}(1 - e^{-2mx})^{n-1} + n(4m - 4)e^{-2x} \int_0^x e^{-(2m-2)\theta} (1 - e^{-2m\theta})^{n-1} d\theta, & m \neq 1 
\end{cases} \quad (3.10)$$

Applying binomial expansion to (3.10), we can rewrite $f(x)$ as

$$f(x) = \begin{cases} 
2n \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} e^{-2(j+1)x}, & m = 1 \\
2n \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} e^{-2m(j+1)x} \\
(2m - 2)2ne^{-2x} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \left( \frac{1 - e^{-(2m-2+2mj)x}}{2m+2mj} \right) & m \neq 1 
\end{cases} \quad (3.11)$$
Plugging (3.9) and (3.11) into (3.7) and carrying out the integration, \( f(z) \) can be expressed as

\[
f(z) = \begin{cases} 
2n e^{-z} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{1+2j} [1 - e^{-(1+2j)z}] & m = 1 \\
2n e^{-z} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{2m-1+2mj} [1 - e^{-(2m-1+2mj)z}] \\
+ n(2m-2)e^{-z} (1 - e^{-z}) \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{m-1+mj} & m \neq 1 \\
- n(2m-2)e^{-z} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{(1-e^{-(2m+2mj-1)z})}{(m-1+mj)(2m+2mj-1)} 
\end{cases}
\]

(3.12)

Having obtained a closed-form expression for \( f(z) \), one can now obtain an expression for \( P_e \) by plugging (3.12) into (3.6) and carrying out the integration. The resulting expression is shown below. When \( m = 1 \), we have\(^4\)

\[
P_{e,AF} \geq n \left( 1 - \sqrt{\frac{1}{1 + \rho^{-1}}} \right) \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{1 + 2j} \]

\[
- n \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \left( 1 - \sqrt{\frac{1}{1 + (2 + 2j)\rho^{-1}}} \right) \frac{1}{(2 + 2j)(1 + 2j)},
\]

(3.13)

\(^4\)This result is similar to the one obtained in [17] for opportunistic relaying.
and when \( m \neq 1 \), we have

\[
P_{e,AF} \geq \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{2m + 2mj - 1} \left(1 - \sqrt{\frac{1}{1 + \rho^{-1}}} \right)
\]

\[
- \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{(2m + 2mj - 1)(2m + 2mj)} \left(1 - \sqrt{\frac{1}{1 + \rho^{-1}}} \right)
\]

\[
+ \frac{n(2m - 2)}{2(m - 1 + mj)} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \left(1 - \sqrt{\frac{1}{1 + \rho^{-1}}} \right)
\]

\[
- \frac{n(2m - 2)}{4(m - 1 + mj)} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \left(1 - \sqrt{\frac{1}{1 + 2\rho^{-1}}} \right)
\]

\[
+ \frac{n(2m - 2)}{2} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{(2m - 1 + 2mj)(m - 1 + mj)(2m + 2mj)} \left(1 - \sqrt{\frac{1}{1 + (2m+2mj)\rho^{-1}}} \right)
\]

\[
- \frac{n(2m - 2)}{2} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{(2m - 1 + 2mj)(m - 1 + mj)} \left(1 - \sqrt{\frac{1}{1 + \rho^{-1}}} \right).
\]

(3.14)

While the expressions in (3.13) and (3.14) give a lower bound on the bit error rate performance, they do not yield the diversity order achieved. This motivates us to derive an upper bound on the bit error rate performance, which clearly shows that a diversity of \( n + 1 \) is achieved, i.e., the maximum diversity possible. The derivation is given in the following section.

### 3.4.2 Proof of the Diversity of AF Relaying

In this section, we prove that the expressions in (3.13) and (3.14) have a diversity of \( n + 1 \). In particular, we derive an upper bound on the E2E bit error rate performance and show that the maximum diversity is achieved.
The exact E2E bit error rate of AF relaying can be expressed as

$$P_e = \int_0^\infty \int_0^\infty Q \left( \sqrt{2\rho(u + y)} \right) f(u) f(y) \, du \, dy,$$

(3.15)

where $u = \frac{|h_{SR}|^2 |h_{RD}|^2}{|h_{SR}|^2 + |h_{RD}|^2 + \rho^{-1}}$ and $y = |h_{SD}|^2$. Using Chernoff bound, we can upper bound $P_e$ as [14]

$$P_e \leq P_{e1} + P_{e2}$$

(3.16)

where

$$P_{e1} = \frac{1}{2} \int_{\rho^{-1}}^\infty e^{-\frac{x^2}{2}} f(x) \, dx \int_0^\infty e^{-y^2} f(y) \, dy,$$

and $x = \min \{ |h_{SR}|^2, |h_{RD}|^2 \}$. Plugging (3.9) into the expressions of $P_{e1}$ and $P_{e2}$, after some simply integration, these expressions can be expressed as

$$P_{e1} = \frac{1}{2(\rho+1)} \int_{\rho^{-1}}^\infty e^{-\frac{x^2}{2}} f(x) \, dx$$

and

$$P_{e2} = \frac{1}{2(\rho+1)} \int_0^\rho e^{-\frac{y^2}{4}} \, dy,$$

We now examine the behavior of $P_{e1}$ and $P_{e2}$ in terms of the diversity order. Plugging (3.8) into $P_{e1}$, we can be expressed it as

$$P_{e1} = P_{e11} + P_{e12}$$

(3.17)

where

$$P_{e11} = \frac{1}{2(\rho+1)} \int_{\rho^{-1}}^\infty e^{-\frac{x^2}{4}} \left[ 2n e^{-2m x} (1 - e^{-2mx})^{n-1} \right] \, dx$$

and

$$P_{e12} = \frac{1}{2(\rho+1)} \int_{\rho^{-1}}^\infty e^{-\frac{x^2}{4}} \left[ 2n (2m - 2) e^{-2x} \int_0^x e^{-(2m-2)\theta} (1 - e^{-2m\theta})^{n-1} \, d\theta \right] \, dx.$$
Now we can upper bound $P_{e_{11}}$ as

$$\begin{align*}
P_{e_{11}} &\leq \frac{1}{2(\rho + 1)} \int_{0}^{\infty} e^{-\frac{x^p}{4}} \left[ 2ne^{-2mx}(1 - e^{-2mx})^{n-1} \right] dx \\
&= -\frac{n}{2m(\rho + 1)} \int_{0}^{\infty} e^{-\frac{x^p}{4}} (1 - e^{-2mx})^{n-1} de^{-2mx} \\
&= \frac{n}{2m(\rho + 1)} \int_{0}^{1} t^{\frac{\rho}{8m}} (1 - t)^{n-1} dt \\
&= \frac{n}{2m(\rho + 1)} B(\frac{\rho}{8m} + 1, n) \\
&= \frac{n}{2m(\rho + 1)} \left( \prod_{i=1}^{n} \left( \frac{\rho}{8m} + i \right) \right)^{-1} \\
&= O(\rho^{-(n+1)}). \quad (3.18)
\end{align*}$$

Note that the expression $B(x, y)$ in (3.18) is actually the Beta function given as

[45]

$$B(x, y) = \int_{0}^{1} t^{x-1} (1 - t)^{y-1} dt, \quad [\text{Re } x > 0, \text{Re } y > 0]$$

$$= \frac{\Gamma(x) \Gamma(y)}{\Gamma(x + y)}.$$ 

$P_{e_{12}}$ can be upper bound as

$$\begin{align*}
P_{e_{12}} &\leq \frac{1}{2(\rho + 1)} \int_{0}^{\infty} e^{-\frac{x^p}{4}} \left[ 2n(2m - 2) e^{-2x} \int_{0}^{x} e^{-(2m-2)\theta} (1 - e^{-2m\theta})^{n-1} d\theta \right] dx \\
&= \frac{n(2m - 2)}{4(1 + \rho)^2} \int_{0}^{\infty} e^{-(2m+\frac{\rho}{4})x} (1 - e^{-2mx})^{n-1} dx \\
&= O(\rho^{-(n+2)}). \quad (3.19)
\end{align*}$$

The last two lines of (3.19) follow the similar steps of (3.18). Plugging (3.18) and (3.19) into (3.17), we can conclude that $P_{e1}$ achieve diversity $n + 1$. Plugging (3.8) into
$P_{e_2}, P_{e_2}$ can be expressed as

$$P_{e_2} = P_{e_{21}} + P_{e_{22}}$$

(3.20)

where

$$P_{e_{21}} = \frac{1}{2(\rho + 1)} \int_{0}^{\rho^{-1}} 2ne^{-2mx}(1 - e^{-2mx})^{n-1} \, dx.$$  

(3.21)

and

$$P_{e_{22}} = \frac{1}{2(\rho + 1)} \int_{0}^{\rho^{-1}} 2n(2m - 2)e^{-2x} \int_{0}^{x} e^{-(2m-2)\theta} (1 - e^{-2m\theta})^{n-1} \, d\theta \, dx.$$  

(3.22)

We can upper bound $P_{e_{21}}$ as

$$P_{e_{21}} \leq \frac{n}{(\rho + 1)} \int_{0}^{\rho^{-1}} (1 - e^{-2m\rho^{-1}})^{n-1} \, dx$$

$$= \frac{n}{\rho(\rho + 1)}(1 - e^{-2m\rho^{-1}})^{n-1}$$

$$= O(\rho^{-(n+1)}).$$

(3.23)

In the same way, we can upper bound $P_{e_{22}}$ as

$$P_{e_{22}} = \frac{1}{2(\rho + 1)} \int_{0}^{\rho^{-1}} 2n(2m - 2)e^{-2x} \int_{0}^{x} e^{-(2m-2)\theta} (1 - e^{-2m\theta})^{n-1} \, d\theta \, dx$$

$$\leq \frac{n(m-1)}{(\rho + 1)} \int_{0}^{\rho^{-1}} e^{-2mx} (1 - e^{-2mx})^{n-1} \, dx$$

$$\leq \frac{n(m-1)}{(\rho + 1)} \int_{0}^{\rho^{-1}} (1 - e^{-2m\rho^{-1}})^{n-1} \, dx$$

$$= \frac{n(m-1)}{\rho(\rho + 1)}(1 - e^{-2m\rho^{-1}})^{n-1}$$

$$= O(\rho^{-(n+1)}).$$

(3.24)
Plugging (3.23) and (3.24) into (3.20), we can conclude that $P_{e2}$ achieve diversity $n + 1$.

For both $P_{e1}$ and $P_{e2}$ are proved to achieve full diversity. From (3.16), we can conclude that SSnoDP scheme with AF relaying achieve full diversity which is the number of the relays plus 1. In [14], the authors give the diversity order analysis of opportunistic AF relaying without direct path. We extend their results and give the diversity analysis to SSnoDP of AF relaying with direct path. So if we remove the direct path and set $m = 1$, the analysis above is the same as [14]

### 3.4.3 Genie-aided DF Relaying

The bit error probability of Genie-aided DF relaying can be expressed as

$$P_e = P_{e,SR}P_{e,SD} + (1 - P_{e,SR})P_{e,MRC},$$

(3.25)

where $P_{e,SR}$ and $P_{e,SD}$ are the probabilities of making an error over the $S-R$ link and $S-D$ link, respectively. $P_{e,MRC}$ is the bit error rate of ideal relaying, i.e., when the relay forwards all bits correctly. $P_{e,SR}$ can be expressed as

$$P_{e,SR} = \int_0^\infty Q \left( \sqrt{2\rho h} \right) f(h) dh,$$

(3.26)

where $f(h)$ is the pdf of $|h_{SR}|^2$ corresponding to the selected relay, which is given as [42]

$$f(h) = (2m - 1)ne^{-h}\int_0^h e^{-(2m-1)\theta}(1 - e^{-2m\theta})^{n-1}d\theta + ne^{-2mh}(1 - e^{-2mh})^{n-1}.$$

(3.27)
Apply binomial expansion to $f(h)$ in (3.27) and after some simple algebraic manipulations, $f(h)$ can be expressed as

$$f(h) = (2m - 1) ne^{-h} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \left( \frac{1}{2m - 1 + 2mj} - \frac{e^{-(2m-1+2mj)h}}{2m - 1 + 2mj} \right)$$

$$+ n \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} e^{-2m(j+1)h}.$$  \hspace{1cm} (3.28)

Having obtained a close-form expression for $f(h)$, one can now obtain an expression for $P_{e,SR}$ by plugging (3.28) into (3.26) and carrying out the integration. The resulting expression is given as

$$P_{e,SR} = \frac{n}{2} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{2m(j+1)} \left( 1 - \sqrt{\frac{1}{1 + 2m(j+1)\rho^{-1}}} \right)$$

$$+ \frac{n(2m-1)}{2} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{2m - 1 + 2mj} \left( 1 - \sqrt{\frac{1}{1 + \rho^{-1}}} \right)$$

$$- \frac{n(2m-1)}{2} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \left( 1 - \sqrt{1+2m(j+1)\rho^{-1}} \right) \frac{1}{(2m - 1 + 2mj)(2m + 2mj)}. \hspace{1cm} (3.29)$$

This expression is the same as expression (9) derived in [51].

The expression of $P_{e,SD}$ is given in [46] as

$$P_{e,SD} = \frac{1}{2} \left[ 1 - \sqrt{\frac{1}{1 + \rho^{-1}}} \right], \hspace{1cm} (3.30)$$

and $P_{e,MRC}$ can be expressed as

$$P_{e,MRC} = \int_0^\infty Q \left( \sqrt{2\rho z} \right) f(z) \, dz, \hspace{1cm} (3.31)$$
where $z = x + y$ and $x = |h_{RD}|^2$, $y = |h_{SD}|^2$. For $x$ and $y$ are independent variables, the pdf of $z$ is the convolution of the pdfs of $x$ and $y$, given as

$$f(z) = \int_0^\infty f_x(x)f_y(z-x)dx.$$  (3.32)

Since all subchannels are symmetrical, the pdf of $x = |h_{RD}|^2$ and $|h_{SR}|^2$ (corresponding to the selected relay) is the same and is given in (3.28). The pdf of $y = |h_{SD}|^2$ corresponding to the source-to-destination link is given in (3.9). Plugging (3.9) and (3.28) into (3.32) and carrying the integration, $f(z)$ can be expressed as

$$f(z) = 2nev^{-z} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{2m-1+2mj} \left[ 1 - e^{-(2m-1+2mj)z} \right]$$

$$+ n(2m-1)e^{-z} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{(2m-1+2mj)^2} \left( 1 - e^{-(2m-1+2mj)z} \right)$$

$$+ n(2m-1)ze^{-z} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{2m-1+2mj}$$

(3.33)

Having obtained a closed-form expression for $f(z)$, one can now obtain an expression for $P_e$ by plugging (3.33) into (3.31) and carrying out the integration. The resulting
expression is shown below.

\[
P_{e,MRC} = \frac{n}{2} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{2m(j+1)(1-2m-2mj)} \left(1 - \sqrt[1+\rho^{-1}]{\frac{1}{1+2m(j+1)\rho^{-1}}}\right)
\]

\[
+ n \frac{2m-1}{4} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{2 + \sqrt[1+\rho^{-1}]{\frac{1}{1+2m(j+1)\rho^{-1}}}}{2m-1+2mj} \left(1 - \sqrt[1+\rho^{-1}]{\frac{1}{1+\rho^{-1}}}\right)^2
\]

\[
+ n \frac{2m-1}{2} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{(2m-1+2mj)^2(2m+2mj)}
\]

\[
- n \frac{2m-1}{2} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{(2m-1+2mj)^2}.
\]  

(3.34)

An expression for \( P_e \) is obtained by plugging (3.29), (3.30), and (3.34) into (3.25).

This resulting expression, similar to the AF relaying case, does not yield the diversity order achieved. To reveal this diversity, we derive in next section an upper bound on the bit error rate performance and show that a diversity of \( n+1 \) is achieved.

### 3.4.4 Proof of the Diversity of DF Relaying

We can upper bound \( P_e \) as follows

\[
P_e = P_{e,SR}P_{e,SD} + (1 - P_{e,SR})P_{e,MRC}
\]

\[
\leq P_{e,SR}P_{e,SD} + P_{e,MRC}.
\]  

(3.35)

The last line is obtained by assuming that \( 0 \leq P_{e,SR} \ll 1 \), which is true in the practical SNR range. Now we express \( P_{e,SD} \) as
\[ P_{e,SD} = \int_{0}^{\infty} Q \left( \sqrt{2\rho y} \right) f(y) \, dy. \]  
(3.36)

where \( y = |h_{SD}|^2 \).

Plugging (3.27) and (3.9) into (3.26) and (3.36), then using Chernoff bound, we can upper bound \( P_{e,SR} P_{e,SD} \)

\[ P_{e,SR} P_{e,SD} \leq P_1 + P_2, \]  
(3.37)

where

\[ P_1 = \frac{1}{4(\rho + 1)} \int_{0}^{\infty} e^{-\rho h} n e^{-2mh} (1 - e^{-2mh})^{n-1} \, dh. \]  
(3.38)

and

\[ P_2 = \frac{1}{4(\rho + 1)} \int_{0}^{\infty} e^{-\rho h} \left[ (2m - 1) ne^{-h} \int_{0}^{h} e^{-(2m-1)\theta} (1 - e^{-2m\theta})^{n-1} \, d\theta \right] \, dh. \]  
(3.39)

Then using the similar steps of (3.18) and (3.19), we can arrive that the diversity order of \( P_{e,SR} P_{e,SD} \) is \( n + 1 \).

We can express \( P_{e,MRC} \) as

\[ P_{e,MRC} = \int_{0}^{\infty} \int_{0}^{\infty} Q \left( \sqrt{2\rho (x + y)} \right) f(x) f(y) \, dx \, dy, \]  
(3.40)

where \( x = |h_{RD}|^2 \) and \( y = |h_{SD}|^2 \). Then plugging (3.9) and (3.27) into (3.40) and using Chernoff bound, we can upper bound (3.40) as

\[ P_{e,MRC} \leq P_{e,MRC\_1} + P_{e,MRC\_2}, \]  
(3.41)

where

\[ P_{e,MRC\_1} = \frac{1}{2(\rho + 1)} \int_{0}^{\infty} e^{-\rho x} \left[ ne^{-2mx} (1 - e^{-2mx})^{n-1} \right] \, dx. \]  
(3.42)
Then using the similar steps of (3.18) and (3.19), we can arrive that the diversity order of $P_{e,MRC}$ is $n + 1$. So as a whole, we can conclude that Genie-DF relaying achieve full diversity which is $n + 1$.

### 3.5 Threshold-Based Relaying

With LLR-based relaying, a relay computes the LLR values of the received bits and subjects them to a preset threshold. If the LLR value is larger than the threshold, a hard decision on the corresponding bit is made and relayed to the corresponding destination. Otherwise, the relay remains silent. One can also use SNR-based relaying, but the LLR-based relaying has been shown to outperform SNR-based relaying [29], [30].

Let $x$ be the transmitted signal from the source to the relay. The relay then computes the corresponding LLR value as [29], [32] $\Lambda_{SR} = 4\sqrt{\rho} \left( |h_{S,R_j}|^2 x + \Re\{n_{S,R_j}^* h_{S,R_j}^\dagger \} \right)$. Also, let $z = \frac{|A_{SR}|}{4}$, then the bit error probability of the $S - R$ link in terms of $z$ is derived as [32] $Pe_{SR} = \frac{1}{1+e^z}$. The average E2E bit error rate is given by

$$Pe = Pe_{SD}(1 - PT) + Pe_{MRC} (1 - PeT) + Pe_{EP} PeT, \quad (3.44)$$
where $P_{e,MRC}$ represents the bit error rate of ideal relaying, $P_{e,EP}$ represents the bit error rate at the destination given that the relay forwards a wrong bit to the destination and $P_{e,SD}$ represents the bit error rate at the destination without relay. $P_T$ represents the probability that the absolute value of LLR is greater than the threshold and $P_{e|T}$ represents the probability that the absolute value of LLR is greater than the preset threshold $\Lambda_0$ and an error happen at the destination.

All the items in (3.44) have been derived in [29] and [30] for the case that only one pair and one relay. But in our case with relay assignment, all the expressions are need to be re-derived. We first derive the pdf of $z$ which is needed to derive $(1 - P_T)$ and $P_{e|T}$ in (3.44). The expressions for $P_{e,SD}$ and $P_{e,MRC}$ are already given in (3.30) and (3.34) respectively. However, obtaining a closed-form expression for $P_{e,EP}$ is difficult, and is normally obtained numerically [30]. Typically, the value of $P_{e,EP}$ is around 0.5, but in our case, we evaluate it numerically to get accurate values.

### 3.5.1 PDF for $z$

To get the pdf of $z$, we need to derive the pdf of $y = \frac{A_{\Delta h}}{4}$ first. Following the approach of [32], the pdf of $y$ is $f_Y(y) = f_{Y_1}(y) + f_{Y_2}(y)$, where $f_{Y_1}(y) = f_0^\infty \frac{1}{\sqrt{2\pi x}} e^{-\frac{(y-x)^2}{2x}} n \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} e^{-2m(j+1)x} dx$ and $f_{Y_2}(y) = f_0^\infty \frac{1}{\sqrt{2\pi x}} e^{-\frac{(y-x)^2}{2x}} (2m-1) n e^{-x} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \left(\frac{\frac{1}{2m-1+2mj}}{2m-1+2mj} - \frac{\frac{1}{2m-1+2mj}}{2m-1+2mj} e^{-(2m-1+2mj)x}\right) dx$

Correspondingly, we write the pdf of $z$ as

$$f_Z(z) = f_{Z_1}(z) + f_{Z_2}(z),$$

(3.45)
where \( f_{z_1}(z) \) and \( f_{z_2}(z) \) corresponding to \( f_{Y_1}(y) \) and \( f_{Y_2}(y) \), respectively. We now calculate the first part of \( f_{Y_1}(y) \).

Let \( b = \sqrt{x} \) and carry out the integration, we can write \( f_{Y_1}(y) \) as

\[
f_{Y_1}(y) = \frac{1}{\sqrt{\rho [2m(j + 1) + \rho]}} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} e^{-2y \sqrt{\frac{2m(j+1)+\rho}{\rho}}},
\]

where we used the equality \( \int_0^\infty e^{-ax^2} \frac{b}{x^2} dx = \frac{1}{2} \sqrt{\frac{\pi a}{b}} e^{-a/b} \) [45].

The cdf of \( Z_1 \) which is \( F_{z_1}(z) = \int_{-\infty}^{z} f_{Y_1}(y) dy \) can be derived as

\[
F_{z_1}(z) = n \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1 - e^{-2z (\sqrt{1+2m(j+1)\rho^{-1}} + 1)}}{\{2m(j+1) + \rho + \sqrt{\rho [2m(j+1) + \rho]}\}} + n \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1 - e^{-2z (\sqrt{1+2m(j+1)\rho^{-1}} - 1)}}{\{2m(j+1) + \rho - \sqrt{\rho [2m(j+1) + \rho]}\}} \tag{3.46}
\]

Therefore, the pdf of \( Z_1 \) which is \( f_{z_1}(z) = \frac{dF_z(z)}{dz} \) can be derived as

\[
f_{z_1}(z) = n \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1 + e^{4z}}{\sqrt{\rho [2m(j+1) + \rho]}} \times e^{-2z (\sqrt{1+2m(j+1)\rho^{-1}} + 1)}. \tag{3.47}
\]

Using similar steps that lead to \( f_{z_1}(z) \), we can write \( f_{z_2}(z) \) as

\[
f_{z_2}(z) = (2m - 1)n \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{2m - 1 + 2mj} \frac{1 + e^{4z}}{\sqrt{\rho^2 + \rho}} \times e^{-2z (\sqrt{1+\rho^{-1}} + 1)}
\]

\[
- n \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{(2m - 1)n}{2m - 1 + 2mj} \frac{(1 + e^{4z}) e^{-2z (\sqrt{1+2m(j+1)\rho^{-1}} + 1)}}{\sqrt{\rho [2m(j+1) + \rho]}}. \tag{3.48}
\]

Plugging (3.47) and (3.48) into (3.45), we get the pdf of \( z \).

Now we get the pdf of \( z \), then we use it to derive the expressions of \( (1 - P_T) \) and \( P_{e|T} \).
3.5.2 Expressions for \((1 - P_T)\)

\[
1 - P_T = \int_{0}^{\Lambda_2} f(z) dz = \frac{n}{2} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \left[ 1 - e^{-2(\sqrt{1+2m(j+1)p^{-1}+1}) \frac{\Lambda_2}{2}} \right] \left[ 2m(j+1) + \rho + \sqrt{\rho} [2m(j+1) + \rho] \right] + \frac{1 - e^{-2(\sqrt{1+2m(j+1)p^{-1}+1}) \frac{\Lambda_2}{2}}}{2m(j+1) + \rho - \sqrt{\rho} [2m(j+1) + \rho]} \right)
\]

\[
-(2m - 1)n \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{4m - 2 + 4mj} \left[ 1 - e^{-2(\sqrt{1+2m(j+1)p^{-1}+1}) \frac{\Lambda_2}{2}} \right] \left[ 2m(j+1) + \rho + \sqrt{\rho} [2m(j+1) + \rho] \right] + \frac{1 - e^{-2(\sqrt{1+2m(j+1)p^{-1}+1}) \frac{\Lambda_2}{2}}}{2m(j+1) + \rho - \sqrt{\rho} [2m(j+1) + \rho]} \right)
\]

\[
+ \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{4m - 2 + 4mj} \left[ 1 - e^{-2(\sqrt{1+2m(j+1)p^{-1}+1}) \frac{\Lambda_2}{2}} \right] \left[ 2m(j+1) + \rho + \sqrt{\rho} [2m(j+1) + \rho] \right] + \frac{1 - e^{-2(\sqrt{1+2m(j+1)p^{-1}+1}) \frac{\Lambda_2}{2}}}{2m(j+1) + \rho - \sqrt{\rho} [2m(j+1) + \rho]} \right)
\]

3.5.3 Expression for \(P_e|T\)

We can write \(P_e|T\) as

\[
P_e|T = \int_{\Lambda_2}^{\infty} P_e,SRF(z) dz = n \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{e^{-2(\sqrt{1+2m(j+1)p^{-1}+1}) \frac{\Lambda_2}{2}}}{2 \left[ 2m(j+1) + \rho + \sqrt{\rho} [2m(j+1) + \rho] \right]} \left[ 2m(j+1) + \rho + \sqrt{\rho} [2m(j+1) + \rho] \right] + \frac{1 - e^{-2(\sqrt{1+2m(j+1)p^{-1}+1}) \frac{\Lambda_2}{2}}}{2m(j+1) + \rho - \sqrt{\rho} [2m(j+1) + \rho]} \right)
\]

\[
-(2m - 1)n \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{2m - 1 + 2mj} \left[ 2m(j+1) + \rho + \sqrt{\rho} [2m(j+1) + \rho] \right] + \frac{1 - e^{-2(\sqrt{1+2m(j+1)p^{-1}+1}) \frac{\Lambda_2}{2}}}{2m(j+1) + \rho - \sqrt{\rho} [2m(j+1) + \rho]} \right)
\]

\[
+(2m - 1)n \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{1}{2m - 1 + 2mj} \left[ 2m(j+1) + \rho + \sqrt{\rho} [2m(j+1) + \rho] \right] + \frac{1 - e^{-2(\sqrt{1+2m(j+1)p^{-1}+1}) \frac{\Lambda_2}{2}}}{2m(j+1) + \rho - \sqrt{\rho} [2m(j+1) + \rho]} \right) \quad (3.50)
\]

After obtaining the expressions of all the items in (3.44), we get the E2E bit error rate expression of LLR-based relaying for our relay assignment scheme SSnoDP.

Although we only analyze the performance of LLR-based relaying for SSnoDP, this
relaying scheme can apply to FSnoDP, FSDP and SSDP in the same way. In addition, when \( m = 1 \), our scheme reduces to the opportunistic relaying proposed in [15]. Furthermore, when \( m = 1, n = 1 \), our proposed scheme becomes equivalent to the one in [29], including the performance analysis.

3.6 Simulation Results

We present in this section some simulation results to validate the bit error rate expressions derived in the previous sections. In all simulations, we assume that the subchannels are independent and have the same average SNR, i.e., symmetrical. BPSK modulation is used in all simulations.

3.6.1 AF Relaying

In Fig. 3.2, we compare our FSnoDP scheme with three other possible relay assignment schemes, which are random selection, sequential selection and E2E SNR-based selection. In random selection, relay assignment is randomly selected from the set \( \Phi \), which contains all the assignment permutations. It turns out that the performance of this scheme is the same as the case of one pair with one relay. In sequential selection, we first pick out one relay and one pair by finding the largest value of \( |h_{S_iR_jD_k}|^2 \) in \( \Phi \), then remove this pair and this relay from \( \Phi \). The same thing repeats until all the pairs have their corresponding selected relays. As such, all
pairs have equal opportunities to be served by the best relay, the second best relay, etc., leading to equivalent performances among all of them. Furthermore, since the performance in dominated by the case when the pair is assigned last in the process, the overall diversity of this scheme is $n - m + 2$. That is, when the last pair is assigned a relay, only $n - m + 1$ relays are left for assignment, hence the relays contribute only $n - m + 1$ to the diversity, and the additional single diversity comes from the direct path. For example, as shown in the figure, the diversity is two for $m = 3, n = 3$ and three for $m = 2, n = 3$. While it is shown that our scheme achieves full diversity,
which is \( n + 1 \), similar to the diversity of the E2E SNR-based scheme with the latter being slightly superior. An interesting observation is that when \( m = 1 \), sequential selection achieves the same performance as that of our scheme, which is the only case in which sequential selection can achieve full diversity.

Figure 3.3: Bit error rate performance (simulated and theoretical) of SSnoDP with AF relaying

In Fig. 3.3, we compare the simulated bit error rate to the lower bound derived in (3.14) for the SSnoDP scheme. It is shown that the lower bound is quite tight. We also include in the figure results for \( m = 1, n = 2 \) case, which was reported in FIGURE 2 in [17]. This shows that the results in [17] are a special case of our
results for symmetrical subchannels. In addition, it is shown that when $n$ is fixed and $m$ increases, the performance degrades, and when $m$ is fixed and $n$ increases, the performance improves.

Figure 3.4: Bit error rate (simulated) comparison between FSnoDP scheme and SSnoDP scheme with AF relaying.

In Fig. 3.4, we present the bit error rate performance of FSnoDP and SSnoDP for the following cases: $m = 2, n = 2, 3, 4$; and $m = n = 3$. For the $m = n = 2$ case, as shown in the figure, the diversity order is three and the performance of both schemes is the same since both criteria are equivalent, as we have mentioned before. The diversity increases to four in the cases when $n = 3$. In addition, we can see that using
subset selection degrades the performance by about 1 dB at bit error rate $1 \times 10^{-6}$ for both simulated cases. Finally, we observe that the performance of the FSnoDP scheme improves as $m$ decreases from three to two, while the diversity order is the same in both cases. This is attributed to the fact that when $m < n$, the number of ways the relays can be assigned to the $m$ pairs is more.

Figure 3.5: Bit error rate (simulated) comparison between FSDP scheme and SSDP with AF relaying.

In Fig. 3.5, we present the bit error rate performance of FSDP and SSDP for the following cases: $m = n = 2$; $m = 2, n = 3, 4$; and $m = n = 3$. For the case $m = n = 2$, the diversity order is three and the performance for both schemes is the same. It is
observed that the FSDP scheme still outperforms SSDP scheme. However, as shown in the figure, the superiority of FSDP is negligible marginal. In addition, it is shown that both FSDP and SSDP achieve full diversity.

Figure 3.6: Bit error rate (simulated) comparison between FSDP scheme and FSnoDP scheme with AF relaying.

In Fig. 3.6, we compare the bit error rate performance of FSDP and FSnoDP with AF relaying. When $m = 1$, our relay assignment criteria for both cases are equivalent to the opportunistic relaying scheme presented in [15]. It is shown in the figure that, when $m = 2 = 2$, the FSDP scheme outperforms the FSnoDP by less than 1 dB at bit error rate $1 \times 10^{-6}$. While as $n$ increases from two to four, the performance gap
becomes more negligible. This result suggests that the availability of the information of the direct path is not necessary in most of cases. In addition, it also can be observed that when the number of pairs is equal to the number of relays, the performance of the FSnoDP scheme is less than 1 dB worse than the case of $m = 1$ and the performance of the FSDP scheme is very close to the case of $m = 1$. However, when the number of relays is more than the number of pairs, the performance of both FSnoDP and FSDP are very close to the case of $m = 1$. This result suggests that although the pairs share the relays, only little performance degradation is incurred as compared to the case without sharing the relays.

3.6.2 Genie-aided DF Relaying

In Fig. 3.7, we compare the simulated bit error rate to the theoretical one for genie-aided relaying in Section 3.4. We can see the perfect match between the theoretical and simulation results, which corroborates the derived bit error rate expression. As shown in the figures, the performance improves as $n$ increases, as expected. Also the performance improves as $m$ decreases while fixing $n$. In addition, it is shown in the figure that the full diversity of $n + 1$ is achieved, which validates our conclusion in Section 3.4. We also show the performance of ideal relaying in the figure. The performance loss of Genie-DF relaying is about 0.4 dB at bit error rate $1 \times 10^{-6}$ as compared to ideal relaying.
3.6.3 LLR-based DF Relaying

In Fig. 3.8, we compare the simulation results of LLR-threshold relaying with the theoretical one for $m = 2, n = 3$. Here we use the optimum threshold given in [29]. As shown in the figure, the simulation results agree with the theoretical results. In addition, we compare the performance of LLR-based thresholding relaying with other relaying schemes, including DF, CRC, Genie-DF relaying and ideal relaying. For DF relaying, the relay always forwards the decoded bits to the destination. While for CRC relaying, the relay only forwards the correctly decoded frames. We can see that
DF relaying leads to a diversity loss. Both CRC and LLR-threshold relaying achieve significant improvements in the E2E bit error rate, while the latter is superior for all range of SNR shown in the figure.

3.7 Conclusions

In this chapter, we considered relay assignment schemes for relay networks comprising multiple source-destination pairs that communicate with each other simultaneously. In particular, we examined four different assignment schemes, two of which
took the direct path into consideration while the other two did not. In addition, two of these schemes are based on searching over all possible assignment permutations, whereas the other two were based on searching over only a subset on the possible permutations. The latter schemes were devised to reduce the computational complexity and make the analysis tractable. We considered AF and DF relaying, as well as LLR-based relaying. For all cases, we have derived expressions for the bit error rate and showed that all relaying schemes achieved the maximum diversity available, which is $n + 1$. We also compared the proposed assignment schemes with two well studied assignment schemes, namely, random selection and sequential selection and demonstrated the superiority of the proposed schemes.
Chapter 4

Relay Assignment Schemes for Network-Coded Cooperative Systems

4.1 Introduction

In the previous chapter, we have shown our proposed schemes for multiple source-destination pairs. We now direct our interest towards the network-coded cooperative systems.

It is shown that relay selection have been extensively studied for cooperative communications. Recently, some efforts have been made to exploit the selection diversity with network coding in cooperative networks [38]-[40]. Relay selection for
bidirectional DF relaying is first studied in [38]. The authors propose a relay selection criterion that is based on the weighted rate sum of the bidirectional rate pair on the boundary of the achievable rate region. In [39], the authors study the performance of two-way AF relaying with relay selection similar to the opportunistic relaying scheme proposed by Bletsas, et. al. in [15]. A distributed relay selection strategy for selecting the best relay for an AF version of the physical-layer network coding is proposed in [40].

Besides these papers that study the two-way relaying network coding schemes with relay selection for one transmission pair and multiple relays, there are some other papers that study the performance of network coding with relay selection for multiple source-destination pairs. In [41] and [42], the performance of the XOR-based network coding combined with relay selection for multiple source-destination pairs is analyzed. Only the best relay is chosen to help all pairs. The best relay is selected based on the E2E instantaneous channel gains which is similar to the proactive opportunistic relaying proposed in [5].

The authors in [43] propose another network coding scheme for relay selection with multiple source-destination pairs. Also only the best relay is chosen to help all the pairs through network coding. Unlike [41] and [42], the best relay is selected from the relays that have successfully decoded the message from all the sources as the one with the largest mean channel state information (CSI) over the relay-destination links, which is similar to the reactive opportunistic relaying proposed in [5]. However, the
schemes in [41] and [42] suffer impracticality since the destination is assumed to be able to correctly receive the messages sent by all the other sources. The scheme in [43] does not have this assumption where it transmits a random linear combination of the columns in an underlying space time block codes (STBC) [44] matrix as a network coding scheme at relay. But as the number of the pairs increases, the detection complexity of this scheme will be unacceptable. Also we notice that the best relay is chosen from the relays that have successfully decode the message from all the sources which will lead to a nonzero probability that no qualified relays to select from. So although both two schemes are attractive due to their high throughput, it is still not practical in reality.

In light of the above, it is clear that two-way relaying with or without relay selection has been largely studied in many aspects. However, to the best of our knowledge, no work has been done to solve the following problem: How to allocate the relays in a network environment to the network pairs in conjunction with network coding. This is addressed in this chapter. In particular, we propose two relay assignment schemes. One is based on the entire set of relay assignment permutations, and another based on a subset of these permutations. Comparing with the work of [4], there are two main differences. Firstly, the problem that we are trying to solve here is the relay assignment problem for two-way relaying with network coding, while the relay assignment problem in [4] is for one-way relaying without network coding. The schemes in [4] can not be applied directly to solve the problem here. Secondly, although both of
our relay assignment schemes are based on the E2E channel qualities, our schemes are based on proactive opportunistic relaying, while the schemes in [4] are based on reactive opportunistic relaying. Since the relay assignment in our schemes is done before the transmission, only the selected relays need to decode the information of the pairs that they are helping, while for the schemes in [4], all the relays have to try to decode all the information from all sources and tell the destinations whether they have successfully decoded or not, which introduces a computational burden at the relays.

In performing the relay assignment, we consider two cases. In the first case, we assume that a relay is assigned to help only a single pair. As such, the number of relays should be equal or larger than the number of pairs, which is justifiable given that any node in the network can serve as a relay. Consequently, the modulation scheme used at the relays, given that network coding is used, follows the modulation scheme used at the pair nodes being helped. For simplicity, we assume that all nodes in this scenario use BPSK. In the second case, we relax the condition on the number of pairs that can be helped by the same relay. Specifically, we assume that a relay node can help \( k \) pairs at a time by employing a higher order modulation scheme such as \( M \)-PSK where \( k = \log_2 M \) while assuming that the nodes are still using BPSK.

We examine the performance of the proposed assignment schemes on symmetric and asymmetric independent Rayleigh fading channels. In the symmetric case, all subchannels have the same average SNR, whereas in the asymmetric case, the
subchannels have different SNRs. We analyze the performance of these assignment schemes and derive a closed form expression for the E2E bit error rate performance. We show that the maximum diversity order is achieved, which is the number of relays. We also present several examples through which we validate the theoretical results.

The remainder of the chapter is organized as follows. The system model is presented in Section 4.2. In Section 4.3, the proposed relay assignment schemes are presented. We analyze the performance of these schemes assuming BPSK in Section 4.4. Network coding with higher order modulation schemes is considered in Section 4.5. We present several numerical examples in Section 4.6. Section 4.7 concludes this chapter.

4.2 System Model

Figure 4.1: A cooperative network with $m$ bidirectional communication pairs and $n$ relays.
The system model considered in this chapter is shown in Fig. 4.1. As shown in the figure, the network consists of \( m \) pairs (i.e., \( m \) two-way relay channels) and \( n \) relay nodes where \( n \geq m \). Again, the latter assumption is justified because any node in a network can serve as a relay node. The nodes of each pair communicate with each other through one relay node using orthogonal subchannels. For simplicity, we assume that there is no direct path between the source and destination nodes. However, the relay assignments schemes analyzed here can be extended to the case when there is a direct path.

Each of the nodes is equipped with a single antenna and operates in a half-duplex mode. For now, we assume that all nodes use BPSK signaling (higher order modulation schemes will be considered later in this chapter.) In the first time slot, one of the nodes of each pair transmits its signal. That is, \( m \) nodes transmit simultaneously in the first time slot using FDMA [8]. In the second time slot, the remaining \( m \) nodes transmit simultaneously their signals to the relays. The best \( m \) relays are then assigned to the \( m \) pairs where one relay is intended to serve one pair.\(^1\) Each relay node decodes the two received signals (over the two time slots), XORs the decoded signals, and broadcasts the resulting signal to all nodes. Note that the selected \( m \) relays will have to transmit using orthogonal channels (either in time or frequency).

The network subchannels are assumed to experience independent slow and frequency-flat Rayleigh fading. Let \( h_{SiR_j}, h_{DiR_j}, h_{R_jS_i} \) and \( h_{R_jD_i} \) (for \( i = 1, \ldots, m \), \( j = 1, \ldots, n \))

\(^1\)We consider later in this chapter the scenario where a relay is designated to help multiple pairs simultaneously through using higher order modulation schemes.
denote the fading coefficient between the $i$th source—$j$th relay, $i$th destination—$j$th relay, $j$th relay—$i$th source and $j$th relay—$i$th destination, respectively. We consider both independent identically distributed (i.e., symmetric) and independent but non-identically distributed (i.e., asymmetric) Rayleigh fading channels over all transmitting pairs and relays. To avoid ambiguity, we refer to one of the nodes of each pair as source ($S$) and the other node as destination ($D$), although it is a two-way communication between $S$ and $D$.

Let $y_{S,R_j}$ and $y_{D,R_j}$ denote the received signals at the $j$th relay from the $i$th source and $i$th destination (over the two time slots), respectively. These signals can be expressed as $y_{S,R_j} = \sqrt{\rho_{i,j}} h_{S,R_j} x_{S_i} + n_{S,R_j}$ and $y_{D,R_j} = \sqrt{\rho_{j,i}} h_{D,R_j} x_{D_i} + n_{D,R_j}$, respectively, where $\rho_{i,j}$ denotes the average SNR of the $(i,j)$th subchannel, $x_{S_i}$ denotes the transmitted signal from node $S_i$, $x_{D_i}$ denotes the transmitted signal from node $D_i$, and $n_{S,R_j}$ and $n_{D,R_j}$ are AWGN samples with zero mean and unit variance. The relay then uses ML detection to detect the two signals (arriving from the pair nodes over two time slots). That is, $\hat{x}_{S,R_j} = \text{sign}(\Re\{y_{S,R_j}^* h_{S,R_j}^{*}\})$, and $\hat{x}_{D,R_j} = \text{sign}(\Re\{y_{D,R_j}^* h_{D,R_j}^{*}\})$.

After the two signals are detected at the relay nodes, they are XORed as $x_{R_j} = \hat{x}_{S,R_j} \oplus \hat{x}_{D,R_j} = -(\hat{x}_{S,R_j} \hat{x}_{D,R_j})$, and the resulting signal is broadcasted to all nodes in the third time slot. The signals received at the source and destination nodes of each pair are expressed as $y_{R,S_i} = \sqrt{\rho_{j,i}} h_{R,S_i} x_{R_j} + n_{R,S_i}$ and $y_{R,D_i} = \sqrt{\rho_{j,i}} h_{R,D_i} x_{R_j} + n_{R,D_i}$, respectively. Consequently, the final decoded bits at the source and destination of each pair are expressed as $\hat{x}_{S_i} = \text{sign}(\Re\{-x_{S_i} y_{R,S_i}^* h_{R,S_i}^{*}\})$, and $\hat{x}_{D_i} = \text{sign}(\Re\{-x_{D_i} y_{R,D_i}^* h_{R,D_i}^{*}\})$. 
respectively.

4.3 Relay Assignment Criteria

In this section, we discuss two relay assignment criteria. The first criterion considers all possible relay permutations, whereas the second criterion limits the search over a subset of the available permutations. Details of these criteria are in order.

4.3.1 Full Set (optimal) Selection

The optimal assignment scheme depends on the subchannel instantaneous SNR between the relay nodes and the pair nodes, namely, $\gamma_{S_iR_j}$, $\gamma_{R_jD_i}$, $\gamma_{D_iR_j}$, and $\gamma_{R_jS_i}$ (for $i = 1, \ldots, m$, $j = 1, \ldots, n$) where $\gamma_{S_iR_j} = \rho_{i,j} |h_{S_iR_j}|^2$. The other items are similarly defined. The objective here is to optimize the E2E bit error rate performance. Since there are two hops separating the end nodes in each pair, the weaker link is the one that dominates the performance. This certainly applies to all pairs. Therefore, the optimal relay assignment scheme is the one that results in the best subchannel among the weakest ones.

To elaborate, let $\Phi$ be the set containing all assignment permutations. Given that there are $m$ pairs and $n$ relays, the size of $\Phi$ is obviously $p^n_m$. Each element of $\Phi$ consists of all the fading coefficients ($4m$ of them) corresponding to that particular relay assignment. To simplify the presentation, let $\phi_k$ denote the $k$th element of $\Phi$.
for \( k = 1, 2, \ldots, p^n \), and let \( \gamma_{k,\text{min}} \) denote the smallest element in \( \phi_k \), i.e., the weakest subchannel. Accordingly, the optimal assignment, denoted by \( \phi_{k^*} \), has index \( k^* \)

\[
  k^* = \arg \max_k \{ \gamma_{k,\text{min}}, k = 1, 2, \ldots, p^n \}.
\]

**Example 3** In this example, let \( m = 2 \) and \( n = 3 \). Hence, there are six possible ways of assigning one relays to each pair, which are: \((R_1, R_2), (R_2, R_3), (R_3, R_1), (R_1, R_3), (R_2, R_1), (R_3, R_2)\) where the first entry is the relay assigned to the first pair and the second is the relay assigned to the second pair. In order to simply the presentation, we use \( \gamma_{S_i R_j D_i} \) to represent the set \( \{ \gamma_{S_i R_j}, \gamma_{R_j D_i}, \gamma_{D_i R_j}, \gamma_{R_j S_i} \} \), for \( i = 1, 2, j = 1, 2, 3 \). Accordingly, we can express \( \Phi \) as

\[
  \Phi = \begin{bmatrix}
    \phi_1 \\
    \phi_2 \\
    \phi_3 \\
    \phi_4 \\
    \phi_5 \\
    \phi_6
  \end{bmatrix} =
  \begin{bmatrix}
    \gamma_{S_1 R_1 D_1}, \gamma_{S_2 R_2 D_1} \\
    \gamma_{S_1 R_2 D_1}, \gamma_{S_2 R_3 D_2} \\
    \gamma_{S_1 R_3 D_1}, \gamma_{S_2 R_1 D_2} \\
    \gamma_{S_1 R_1 D_1}, \gamma_{S_2 R_3 D_2} \\
    \gamma_{S_1 R_2 D_1}, \gamma_{S_2 R_1 D_2} \\
    \gamma_{S_1 R_3 D_1}, \gamma_{S_2 R_2 D_2}
  \end{bmatrix}.
\]

Then, the optimal relay assignment has index \( k^* = \arg \max_k \{ \gamma_{k,\text{min}}, k = 1, 2, \ldots, 6 \} \).

**4.3.2 Subset (suboptimal) Selection**

For the optimal assignment scheme considers all possible assignments and selects the best one, this scheme endures high computational complexity. In addition, there
is correlation between certain rows of $\Phi$ given in (4.1), which makes the performance analysis difficult. For instance, rows one and four are correlated since in both cases, $R_1$ is assigned to the first pair. Such correlation makes it extremely difficult to find a closed form expression for the pdf of the resulting instantaneous SNRs. These reasons motivate us to consider a suboptimal assignment scheme whereby the permutations considered are not correlated. To achieve this, the set $\Phi$ is divided into $N = p_m^n/n$ subsets. Every subset contains all the fading coefficients. However, the subsets that give rise to correlation are dropped. Consequently, in terms of complexity, the number of permutations to be searched over will be $n$. This is a significant reduction in computational complexity. For instance, when $n = m = 10$, there are $p_m^n = 3628800$ comparisons when using the optimal assignment scheme, whereas the number reduces to 10 when considering the suboptimal one.

**Remark 4** The optimal and subset assignment criteria are equivalent for the following cases: $m = 1$, any $n$; and $m = n = 2$. However, for most other cases, especially as $n$ increases, there will be a performance degradation since the number of permutations to be searched over will be less.

To make it clearer, consider the following example, which is based upon Example 1 above.

**Example 5** The six possible relay assignments can be divided into two subsets: $\Phi_{s_1}$ and
\( \Phi_{s2} \), which are defined as

\[
\Phi_{s1} = \begin{bmatrix}
\phi_1 \\
\phi_2 \\
\phi_3 \\
\end{bmatrix} = \begin{bmatrix}
\gamma_{s_1, r_1, d_1}, & \gamma_{s_2, r_2, d_2} \\
\gamma_{s_1, r_2, d_1}, & \gamma_{s_2, r_2, d_2} \\
\gamma_{s_1, r_3, d_1}, & \gamma_{s_2, r_1, d_2} \\
\end{bmatrix},
\]

(4.2)

and

\[
\Phi_{s2} = \begin{bmatrix}
\phi_4 \\
\phi_5 \\
\phi_6 \\
\end{bmatrix} = \begin{bmatrix}
\gamma_{s_1, r_1, d_1}, & \gamma_{s_2, r_3, d_2} \\
\gamma_{s_1, r_2, d_1}, & \gamma_{s_2, r_1, d_2} \\
\gamma_{s_1, r_3, d_1}, & \gamma_{s_2, r_2, d_2} \\
\end{bmatrix}.
\]

(4.3)

Note that all the elements in \( \Phi_{s1} \) or \( \Phi_{s2} \) are independent. Hence, we can use either one in the relay assignment process because both are equivalent. If we use \( \Phi_{s1} \), then \( k^* \) becomes \( k^* = \arg\max_k \{\gamma_{k,\min}, \ k = 1, 2, 3\} \), and if we use \( \Phi_{s2} \), then \( k^* = \arg\max_k \{\gamma_{k,\min}, \ k = 4, 5, 6\} \).

### 4.4 Performance Analysis

As mentioned above, due to the correlation between some of the elements of \( \Phi \), it is not easy to obtain a closed form expression for the pdf of the selected permutation. However, when such correlation is not present, obtaining such pdf is straightforward. So although we can not give an exact analysis for the optimal scheme, the analysis of the suboptimal scheme gives a performance bound on that of the optimal one. In this section, we analyze the bit error rate performance with the suboptimal assignment scheme for both symmetric and asymmetric Rayleigh fading channels. We first derive
a closed form expression for the bit error rate, and after that we derive an upper bound on the bit error rate to demonstrate that the full diversity order is maintained. Throughout this section, we assume that all nodes use BPSK.

### 4.4.1 The E2E Bit Error Rate Performance

There are two ways of having an error at the relay. When the signal from the source to the relay is detected correctly, but the signal from the destination to the relay is detected incorrectly. The second scenario is the opposite of the first. However, when both signals are in error or both are correct, the relay is not in error. Consequently, the bit error probability at the selected relay $R$ can be obtained as

$$P_{e,R} = P_{e,S_iR} (1 - P_{e,D_iR}) + P_{e,D_iR} (1 - P_{e,S_iR}),$$  \hspace{1cm} (4.4)

where $P_{e,S_iR}$ is the probability of making an error over the $S_i - R$ link. The rest of the variables are similarly defined.

There are also two ways of making an error at either node of each pair. When the relayed bit is in error and received in error, and when the relayed bit is correct but is flipped during transmission. In this case, the bit error rate at node $S_i$ is expressed as

$$P_{e,S_i} = (1 - P_{e,RS_i}) P_{e,R} + (1 - P_{e,R}) P_{e,RS_i}.$$  \hspace{1cm} (4.5)

Plugging (4.4) into (4.5), we have
\[ P_{e,S_i} = (1 - P_{e,RS_i}) [P_{e,SIR} (1 - P_{e,DIR}) + P_{e,DIR} (1 - P_{e,SIR})] \\
+ [1 - P_{e,SIR} (1 - P_{e,DIR}) - P_{e,DIR} (1 - P_{e,SIR})] P_{e,RS_i}. \] (4.6)

Similarly, we can express the error rate expression at node \( D_i \) as

\[ P_{e,D_i} = (1 - P_{e,RDi}) [P_{e,SIR} (1 - P_{e,DIR}) + P_{e,DIR} (1 - P_{e,SIR})] \\
+ [1 - P_{e,SIR} (1 - P_{e,DIR}) - P_{e,DIR} (1 - P_{e,SIR})] P_{e,RDi}. \] (4.7)

In order to get the expression of \( P_{e,S_i} \) and \( P_{e,D_i} \), we need to get expressions for \( P_{e,SIR} \), \( P_{e,DIR} \), \( P_{e,RS_i} \), and \( P_{e,RDi} \).

In the following subsections, we consider the performance of various scenarios. We first start with the case when the subchannels are independent, non-identical Rayleigh fading (i.e., asymmetric). We then consider the semi-symmetric case in which the \( S_i - R \) links are identical, the \( D_i - R \) links are identical, the \( R - S_i \) links are identical, and the \( R - D_i \) links, but these four sets of subchannels are non-identical. We also consider the case when all channels are symmetric, i.e., independent and identical. Clearly, the latter two cases are special cases of the asymmetric case.

### 4.4.2 Asymmetric Channels

We now find expressions for \( P_{e,SIR}, P_{e,DIR}, P_{e,RS_i}, \) and \( P_{e,RDi} \), so that we can get closed-form expressions for \( P_{e,S_i} \) and \( P_{e,D_i} \) defined by (4.6) and (4.7), respectively. To be able to do so, we need to find the pdf of the random variables involved, namely,
Although these variables are independent and different (on average), their pdfs will be similar where we can use one expression to express the four pdfs. As such, to make the presentation simpler, we denote these variables by $\gamma_{ik}$ for $k = 1, 2, 3, 4$ where $\gamma_{i1} = \gamma_{S_1 R_i}$, $\gamma_{i2} = \gamma_{D_i R_i}$, $\gamma_{i3} = \gamma_{R S_i}$, and $\gamma_{i4} = \gamma_{R D_i}$.

The pdf of $\gamma_{ik}$ can then be expressed as [42]

\[
f_{\gamma_{ik}}(\gamma) = \sum_{j=1}^{n} \lambda_{ij,k} e^{-\lambda_{ij,k} \gamma} \left[ (\lambda_j - \lambda_{ij,k}) e^{-(\lambda_j - \lambda_{ij,k})\theta} \prod_{m \neq j} (1 - e^{-\lambda_m \theta}) \right] d\theta 
+ \sum_{j=1}^{n} \left[ \lambda_{ij,k} e^{-\lambda_j \gamma} \prod_{m \neq j} (1 - e^{-\lambda_m \gamma}) \right],
\]

(4.8)

where $\lambda_{ij,1} \triangleq \frac{1}{\bar{\gamma}_{S_i R_j}}$, $\lambda_{ij,2} \triangleq \frac{1}{\bar{\gamma}_{D_i R_j}}$, $\lambda_{ij,3} \triangleq \frac{1}{\bar{\gamma}_{S_i R_j}}$, and $\lambda_{ij,4} \triangleq \frac{1}{\bar{\gamma}_{R_j D_i}}$ (for $i = 1, \ldots, m$, $j = 1, \ldots, n$) and $\lambda_j \triangleq \sum_{i=1}^{m} (\lambda_{ij,1} + \lambda_{ij,2} + \lambda_{ij,3} + \lambda_{ij,4})$. $\bar{\gamma}_{S_i R_j} = \rho_{i,j} E[|h_{S_i R_j}|^2]$ is the average SNR for the $S_i - R_j$ link. The other terms are similarly defined.

After some manipulations and carrying out the integration, $f_{\gamma_{ik}}(\gamma)$ can be simplified as

\[
f_{\gamma_{ik}}(\gamma) = \sum_{j=1}^{n} \sum_{\tau \in T_j^n} \text{sign}(\tau) \frac{\lambda_{ij,k}}{\lambda_j - \lambda_{ij,k} + \tau} e^{-\lambda_{ij,k} \gamma} (1 - e^{-(\lambda_j - \lambda_{ij,k} + \tau)\gamma}) 
+ \sum_{j=1}^{n} \sum_{\tau \in T_j^n} \text{sign}(\tau) \lambda_{ij,k} e^{-(\lambda_j + \tau)\gamma},
\]

(4.9)

where the set $T_j^n$ is obtained by expanding the product $\prod_{m \neq j} (1 - e^{-\lambda_m \gamma})$ and then taking the logarithm of each term, and $\text{sign}(\tau)$ corresponds to the sign of each term in the expansion [47]. That is,

\[
T_j^n = \left\{ \tau : \prod_{m=1}^{n} (1 - e^{-\lambda_m \gamma}) = \sum_{\tau \in T_j^n} \text{sign}(\tau) e^{-\tau \gamma} \right\}.
\]

(4.10)
The moment generating function (MGF) of \( \gamma_{ik} \) can be calculated as [48]

\[
M_{\gamma_{ik}}(s) = \int_0^\infty e^{-s\gamma} f_{\gamma_{ik}}(\gamma) \, d\gamma.  \tag{4.11}
\]

Plugging (4.9) into (4.11) and carrying out the integration, we have

\[
M_{\gamma_{ik}}(s) = \sum_{j=1}^n \sum_{\tau \in T_j} \text{sign}(\tau) \frac{\lambda_{ij,k}}{\lambda_j + \tau} \left( \frac{1}{\lambda_{ij,k} + s} - \frac{1}{\lambda_j + \tau + s} \right) + \sum_{j=1}^n \sum_{\tau \in T_j} \text{sign}(\tau) \frac{\lambda_{ij,k}}{\lambda_j + \tau + s}. \tag{4.12}
\]

The average bit error probability (for BPSK) is then given by [48]

\[
P_{e,k} = \frac{1}{\pi} \int_0^{\pi/2} M_{\gamma_{ik}} \left( \frac{1}{\sin^2 \theta} \right) d\theta. \tag{4.13}
\]

Plugging (4.12) into (4.13) and after some manipulations, \( P_{e,ik} \) can be expressed as

\[
P_{e,ik} = \sum_{j=1}^n \sum_{\tau \in T_j} \text{sign}(\tau) \frac{\lambda_{ij,k}}{\lambda_j + \tau} I_1 \left( \frac{1}{\lambda_j + \tau} \right) + \sum_{j=1}^n \sum_{\tau \in T_j} \text{sign}(\tau) \frac{\lambda_{ij,k}(\lambda_j - \lambda_{ij,k})}{\lambda_j - \lambda_{ij,k} + \tau} \cdot \left( \frac{1}{\lambda_{ij,k}} I_1 \left( \frac{1}{\lambda_{ij,k}} \right) - \frac{1}{\lambda_j + \tau} I_1 \left( \frac{1}{\lambda_j + \tau} \right) \right), \tag{4.14}
\]

where \( I_1(c) \) is given as [48]

\[
I_1(c) = \frac{1}{\pi} \int_0^{\pi/2} \frac{\sin^2 \theta}{\sin^2 \theta + c} d\theta = \frac{1}{2} \left( 1 - \sqrt{\frac{c}{1 + c}} \right). \tag{4.15}
\]

Note that when \( k = 1, 2, 3, 4 \), \( P_{e,ik} \) corresponds to \( P_{e,S_iR}, P_{e,D_iR}, P_{e,RSi}, \) and \( P_{e,RDi} \), respectively. Plugging these expressions in (4.6), we obtain the E2E bit error rate expression at node \( S_i \). Similarly, we can get the bit error rate expression at node \( D_i \) using (4.7).
4.4.3 Semi-symmetric Channels

For this case, the bit error rate performance at all $S_i$ for $i = 1, 2, \ldots, m$ is the same because the corresponding subchannels are identical. The same is true for all $D_i$ for $i = 1, 2, \ldots, m$. However, the performance at the nodes of the same pair can be different.

We now define $\lambda_1 \triangleq \frac{1}{\gamma_{SR}}$, $\lambda_2 \triangleq \frac{1}{\gamma_{DR}}$, $\lambda_3 \triangleq \frac{1}{\gamma_{RS}}$ and $\lambda_4 \triangleq \frac{1}{\gamma_{RD}}$. Also, let $\gamma_k$ for $k = 1, 2, 3, 4$ represent $\gamma_{SR}$, $\gamma_{DR}$, $\gamma_{RS}$ and $\gamma_{RD}$, respectively. Note that we dropped the indices from these variables because the performance is the same for all pairs (as explained before). Accordingly, (4.8) can be rewritten as

$$
\begin{align*}
\gamma_k(\gamma) &= n\lambda_k e^{-m\lambda\gamma} (1 - e^{-m\lambda\gamma})^{n-1} \\
&\quad + n\lambda_k e^{-\lambda_k\gamma} \int_0^\gamma [(m\lambda - \lambda_k) e^{-(m\lambda - \lambda_k)\theta} (1 - e^{-m\lambda\theta})^{n-1}] d\theta,
\end{align*}
$$

(4.16)

where $\lambda = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$. Applying binomial expansion to $\gamma_k(\gamma)$ and after some simple algebraic manipulations, (4.16) can be expressed as

$$
\begin{align*}
\gamma_k(\gamma) &= \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} n\lambda_k e^{-\lambda_k\gamma} (m\lambda - \lambda_k) \frac{1}{m\lambda - \lambda_k + m\lambda j} (1 - e^{-(m\lambda - \lambda_k + m\lambda j)\gamma}) \\
&\quad + n\lambda_k \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} e^{-m\lambda(j+1)\gamma}.
\end{align*}
$$

(4.17)

Plugging (4.17) into (4.11) and carrying out the integration, we have

$$
\begin{align*}
\gamma_k(s) &= \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} n\lambda_k (m\lambda - \lambda_k) \frac{1}{m\lambda - \lambda_k + m\lambda j} \left( \frac{1}{\lambda_k + s} - \frac{1}{m\lambda(j+1) + s} \right) \\
&\quad + \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{n\lambda_k}{m\lambda(j+1) + s}.
\end{align*}
$$

(4.18)
Plugging (4.18) into (4.13) and after some manipulations, the bit error rate can be expressed as

\[
P_{e,k} = \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{n\lambda_k(m\lambda - \lambda_k)}{m\lambda - \lambda_k + m\lambda j} \left( \frac{1}{\lambda_k} I_1 \left( \frac{1}{\lambda_k} \right) - \frac{1}{m\lambda(j+1)} I_1 \left( \frac{1}{m\lambda(j+1)} \right) \right) \\
+ \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{n\lambda_k}{m\lambda(j+1)} I_1 \left( \frac{1}{m\lambda(j+1)} \right),
\]

(4.19)

where \( P_{e,k} \) for \( k = 1, 2, 3, 4 \) correspond to \( P_{e,SR}, P_{e,DR}, P_{e,RS}, \) and \( P_{e,RD} \), respectively.

Then plugging the resulting expressions into (4.6) and (4.7), we have the E2E bit error probability at node \( S \) and node \( D \), respectively.

### 4.4.4 Symmetric Channels

For independent, identical Rayleigh fading channels, the average SNRs for all channel is the same, i.e., \( \lambda_k \) defined just after (4.8) are the same, and define \( \beta \triangleq \lambda_k \), implying that \( \lambda = 4\beta \). Consequently, (4.19) simplifies to

\[
P_e = \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{n\beta(4m-1)}{4m-1 + 4mj} \left( \frac{1}{\beta} I_1 \left( \frac{1}{\beta} \right) - \frac{1}{4m\beta(j+1)} I_1 \left( \frac{1}{4m\beta(j+1)} \right) \right) \\
+ \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{n}{4m(j+1)} I_1 \left( \frac{1}{4m\beta(j+1)} \right).
\]

(4.20)

In this case, the bit error rate at all nodes is the same, which can be obtained by plugging (4.20) into (4.6) or (4.7). If we let \( \beta = 1 \), that is, the channel gains are modeled as zero mean, unit variance complex Gaussian random variables, the resulting expression reduces to expression (10) derived in [50].
4.4.5 Achievable Diversity Order

We know if we can show that the full diversity is achieved with the subset assignment criterion, we implicitly show that the same is true for the optimal assignment scheme. While the final expressions derived above are exact, it does not reveal the diversity order. In this section, we derive an upper bound on $P_e$ that proves that the maximum diversity order, which is $n$, is achieved. In order to achieve this goal, we simply assume that all the channels gains are modeled as zero mean, unit variance complex Gaussian random variables, which does not change the diversity order of the system [49].

With this assumption, the average bit error rate of all the links are the same. Then we simply use $P_{e,SR}$ to represent the bit error rate of all links. For $P_{e,S} = P_{e,D}$, we use $P_e$ to represent the E2E bit error rate. To this end, we can upper bound $P_e$ in (4.6) and (4.7) as follows.

$$P_e = 2 (1 - P_{e,SR}) (1 - P_{e,SR}) P_{e,SR} + [1 - 2P_{e,SR} (1 - P_{e,SR})] P_{e,SR}$$

$$\leq 3P_{e,SR}. \quad (4.21)$$

The last line is obtained by assuming that $0 \leq P_{e,SR} << 1$, which is true in the practical SNR range. From (4.21), we know that to prove that $P_e$ achieves full diversity, it is sufficient to prove that $P_{e,SR}$ achieves full diversity.

Note that $P_{e,SR}$ can be expressed as

$$P_{e,SR} = \int_{0}^{\infty} Q \left( \sqrt{2\rho h} \right) f(h) dh, \quad (4.22)$$
where \( f(h) \) is the pdf of \( |h_{SR}|^2 \) corresponding to the selected relay, which is given as [42]

\[
f(h) = n (4m - 1) e^{-h} \int_0^h e^{-(4m-1)\theta} (1 - e^{-4m\theta})^{n-1} d\theta + ne^{-4mh} (1 - e^{-4mh})^{n-1}. \tag{4.23}
\]

Using (4.22) and (4.23), we can express \( P_{e,SR} \) as \( P_{e,SR} = P_{e_1} + P_{e_2} \), where

\[
P_{e_1} = \int_0^\infty Q \left( \sqrt{2\rho h} \right) \left[ ne^{-4mh} (1 - e^{-4mh})^{n-1} \right] dh, \tag{4.24}
\]

and

\[
P_{e_2} = \int_0^\infty Q \left( \sqrt{2\rho h} \right) \left( n (4m - 1) e^{-h} \int_0^h e^{-(4m-1)\theta} (1 - e^{-4m\theta})^{n-1} d\theta \right) dh. \tag{4.25}
\]

We now examine the behavior of the expressions of \( P_{e_1} \) and \( P_{e_2} \) in terms of the diversity order. Using Chernoff bound, we can upper bound \( P_{e_1} \) as

\[
P_{e_1} \leq \int_0^\infty \frac{1}{2} \left[ ne^{-4mh} (1 - e^{-4mh})^{n-1} \right] e^{-\rho h} dh
\]

\[
= -\frac{n}{8m} \int_0^\infty e^{-\rho h} (1 - e^{-4mh})^{n-1} e^{-4mh} = \frac{n}{8m} \int_0^1 t^{\rho} (1 - t)^{n-1} dt. \tag{4.26}
\]

Note that the last expression in (4.26) is actually the Beta function, given as [45]

\[
B(x, y) = \int_0^1 t^{x-1} (1 - t)^{y-1} dt = \frac{\Gamma(x) \Gamma(y)}{\Gamma(x + y)},
\]

for Re \( \{x\} > 0 \) and Re \( \{y\} > 0 \), and \( \Gamma(x) \) is the Gamma function. Armed with the above results, and when \( \rho \) is sufficiently large, we can express the last line of (4.26) as

\[
P_{e_1} \leq \frac{n}{8m} B \left( \frac{\rho}{4m} + 1, n \right) = \frac{n}{8m} \frac{\Gamma \left( \frac{\rho}{4m} + 1 \right) \Gamma(n)}{\Gamma \left( \frac{\rho}{4m} + 1 + n \right)}
\]

\[
= \frac{n}{8m} \left( \prod_{i=1}^n \left( \frac{\rho}{4m} + i \right) \right)^{-1} = O(\rho^{-n}), \tag{4.27}
\]
which suggests that the full diversity order is achieved.

Similarly, using Chernoff bound, we can upper bound $P_{e_2}$ as

\[
P_{e_2} \leq \int_0^\infty \frac{1}{2} e^{-ph} \left( n(4m-1) e^{-h} \int_0^h e^{-(4m-1)\theta} (1 - e^{-4m\theta})^{n-1} d\theta \right) dh
\]

\[
= \frac{1}{2(1 + \rho)} \int_0^\infty e^{-(4m+\rho)h} (1 - e^{-4mh})^{n-1} dh
\]

\[
= \frac{n(4m-1)}{8m(1 + \rho)} \int_0^1 t^{\rho} (1 - t)^{n-1} dt
\]

\[
= \frac{(4m-1) n!}{8m(1 + \rho)} \left( \prod_{i=1}^n \left( \frac{\rho}{4m} + i \right) \right)^{-1} = O \left( \rho^{-(n+1)} \right),
\]

which suggests that the diversity order of this term is $n + 1$, which is one more than the maximum diversity order. Since the performance is dominated by the term with the lower diversity order, we conclude that the diversity order of the bit error rate with the suboptimal assignment criterion is indeed $n$, which is the full diversity. Consequently, the optimal assignment criterion achieves the full diversity as well.

\section{Network Coding with Higher Order Modulation Schemes}

\subsection{Motivation}

So far we assumed that a relay can help up to one pair, suggesting that there should be at least as many relays as pairs. In this section, we relax this condition
where we allow a relay to help more than one pair. Let us assume, without loss of
generality, that a relay can help \( k \) pairs at a time. To achieve this, the relay will have
to employ a \( 2^k \)-ary modulation scheme, while the nodes being helped still employ
BPSK, although other combination of modulation schemes are possible. For this
study, we consider \( M \)-PSK, but other \( M \)-ary modulation scheme can be employed as well.). As such, the throughput will be improved since fewer orthogonal channels (or
time slots) will be used.

To elaborate, suppose that the nodes of \( k \) pairs (2k of them) transmit their signals
using BPSK over 2\( k \) orthogonal channels. The relay selected to help these \( k \) pairs
makes hard decisions on these 2\( k \) bits, and XORs the bits arriving from the nodes
corresponding to the same pair, i.e., there will be \( k \) XORed bits. These \( k \) bits will then
be mapped to one of the \( M \)-PSK symbols. The resulting symbol is then broadcasted
to all nodes using one channel (or time slot). (If multiple relays are helping different
sets of \( k \) pairs, such relays are assumed to use orthogonal channels.) The nodes then
decode the broadcasted symbol and map it back to its corresponding \( k \) bits. The
nodes of each pair know a priori the bit position that corresponds to them. Since
each node has access to its own bit (that was transmitted before), they can recover
their intended signal by XORing the bit extracted form the symbol with their own
bit.

One of the advantages of relaying with higher order modulation schemes is that
\( k - 1 \) time slots or channels are saved compared to relaying with BPSK. However,
this comes at the price of some performance deterioration. To maintain a good performance, more power should be transmitted from the active relays (in this paper we assume that $E_s = kE_b$).

As for the relay assignment process, it is similar to the one described in the previous section (i.e., relaying with BPSK). The only difference here is that, once $k$ is determined, these $k$ pairs are treated as a new ‘extended’ pair and they are always assigned together to one relay. For example, when $k = 2$, there are eight subchannels involved, and the relay assigned to help these two pairs follows the same assignment schemes described above for assigning relays to single pairs.

Other issues pertaining to relaying with higher order modulation schemes include the way to select the $k$ pairs to be helped by a certain relay, deciding on the modulation scheme to be used at the relay, the possibility of employing adaptive modulation to reflect the quality of the subchannels, finding efficient ways of informing the pair nodes of the type of modulation used. These issues are not treated in this chapter due to space limitation and can be considered in future work.

4.5.2 Performance Analysis

In this section, we derive exact expressions for the bit error rate over asymmetric, semi-symmetric and symmetric channels. The difference between the analysis here and that given in Section 4.4 is that the $R - S_i$ and $R - D_i$ links employ $M$-PSK signaling, whereas BPSK signaling was assumed over all links in Section 4.4. Assum-
ing Gray code mapping, the bit error rate, $P_b$, and symbol error rate, $P_s$, are related through

$$P_b \approx \frac{1}{\log_2 M} P_s. \quad (4.29)$$

The symbol error rate can be calculated as [48]

$$P_s = \frac{1}{\pi} \int_0^{(M-1)\pi/M} M_\sigma \left( \frac{g_{PSK}}{\sin^2 \theta} \right) d\theta, \quad (4.30)$$

where $g_{PSK} = \sin^2(\pi/M)$.

The pdfs of $\gamma_{RS_t}$ and $\gamma_{RD_t}$ corresponding to the selected relay are given by (4.8) for $k = 3, 4$, respectively. The expression of $M_{\gamma_{ik}}(s)$ is given in (4.12). Plugging $M_{\gamma_{ik}}(s)$ into (4.30) and after some manipulations, the symbol error rate can be expressed as

$$P_{s,ik} = \sum_{j=1}^{M} \sum_{\tau \in T_j^n} \frac{\text{sign}(\tau)\lambda_{ij,k} I_2 \left( \frac{\sin^2(\pi/M)}{\lambda_j + \tau} \right)}{\lambda_j + \tau} \cdot \left( \frac{1}{\lambda_{ij,k}} I_2 \left( \frac{\sin^2(\pi/M)}{\lambda_{ij,k}} \right) - \frac{1}{\lambda_j + \tau} I_2 \left( \frac{\sin^2(\pi/M)}{\lambda_j + \tau} \right) \right), \quad (4.31)$$

where

$$I_2(c) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \frac{\sin^2 \theta}{\sin^2 \theta + c} d\theta$$

$$= \frac{M - 1}{M} \left\{ \left[ \frac{\pi}{2} + \tan^{-1} \left( \frac{\sqrt{\frac{c}{1+c}}}{\frac{M}{(M-1)\pi}} \right) \right] \right\}. \quad (4.32)$$

The remaining variables were defined in the previous section. Then plugging (4.31) into (4.29), we have the expressions for $P_{b,ik}$. Then plugging the resulting expression of $P_{b,ik}$ and (4.14) into (4.6) and (4.7), we have the bit error rate expressions at node $S_i$ and $D_i$, respectively.
For semi-symmetric channels, plugging (4.18) into (4.30) and after some manipulations, the symbol error rate can be expressed as

\[ P_{s,k} = \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{n\lambda_k(m\lambda - \lambda_k)}{m\lambda - \lambda_k + m\lambda_j} \cdot \left( \frac{1}{\lambda_k} I_2 \left( \frac{\sin^2(\pi/M)}{\lambda_k} \right) - \frac{1}{m\lambda(j+1)} I_2 \left( \frac{\sin^2(\pi/M)}{m\lambda(j+1)} \right) \right) \]
\[ + \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{n\lambda_k}{m\lambda(j+1)} I_2 \left( \frac{\sin^2(\pi/M)}{m\lambda(j+1)} \right). \]  \hspace{1cm} (4.33)

where \( P_{s,3} \) represents the symbol error rate of link \( R - S \) and \( P_{s,4} \) represents the symbol error rate for link \( R - D \). We drop the indices for the bit error rate because they are the same for all pairs in this case. Then plugging (4.33) into (4.29), we have an expression for bit error rate, \( P_{b,k} \). By plugging the resulting expressions into (4.6) and (4.7), we have the E2E bit error rate at node \( S \) and node \( D \), respectively.

For the special case of symmetric channels (as defined above \( \beta \triangleq \lambda_k \), implying that \( \lambda = 4\beta \)). Links \( R - S \) and \( R - D \) have the same average bit error rate, which can be easily obtained from (4.33) as

\[ P_s = \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{n(4m - 1)\beta}{4m - 1 + 4mj} \cdot \left( \frac{1}{\beta} I_2 \left( \frac{\sin^2(\pi/M)}{\beta} \right) - \frac{1}{4m\beta(j+1)} I_2 \left( \frac{\sin^2(\pi/M)}{4m\beta(j+1)} \right) \right) \]
\[ + \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \frac{n}{4m(j+1)} I_2 \left( \frac{\sin^2(\pi/M)}{4m\beta(j+1)} \right). \]  \hspace{1cm} (4.34)

Then we can get the expression of the E2E bit error rate similar to the previous cases.
4.6 Simulation Results

We present in this section some simulation results to validate the bit error rate expressions derived in the previous sections. Specifically, we confirm that both assignment criteria achieve the full diversity. We also examine the degradation in performance due to using the subset assignment criterion as opposed to the optimal one.

4.6.1 Relaying with BPSK Modulation

Throughout this section, we assume that all nodes use BPSK. That is, each assigned relay helps only one pair.

In Fig. 4.2, we compare our suboptimal scheme with two other possible relay assignment schemes, which are random selection and sequential selection over symmetric channels. In random selection, relay assignment is randomly selected from the set $\Phi$, which contains all the assignment permutations. It turns out that the performance of this scheme is the same as the case of one pair with one relay. In sequential selection, we first pick one relay and one pair by finding the largest value of $\gamma_{S_iR_jD_i}$ in $\Phi$, then removing this pair and this relay from $\Phi$. The same thing repeats until all pairs have their corresponding selected relays. As such, all pairs have equal opportunities to be served by the best relay, the second best relay, etc., leading to equivalent performances among all of them. Furthermore, since the performance in dominated by the case when the pair is assigned last in the process, the overall diversity of this
scheme is \( n - m + 1 \). That is, when the last pair is assigned a relay, only \( n - m + 1 \) relays are left for assignment, hence the relays contribute only \( n - m + 1 \) to the diversity. For example, as shown in the figure, the diversity is two for \( m = 3, n = 3 \) and three for \( m = 2, n = 3 \). While it is shown that our scheme achieves full diversity, which is \( n \).

In Fig. 4.3, we present the bit error rate performance for the following cases: \( m = n = 2 \); \( m = 2, n = 3 \); and \( m = n = 3 \) over symmetric channels. In all cases, we consider both optimal and suboptimal assignment criteria. For the \( m = n = 2 \) case,
Figure 4.3: Bit error rate performance comparison between the optimal and suboptimal assignment schemes for various $m$ and $n$.

as shown in the figure, the diversity is two and the performance is the same since both criteria are equivalent (as noted above). The diversity increases to three in the cases when $n = 3$. In addition, we can see the degradation in SNR due to the subset assignment scheme, which is a little over 1 dB at bit error rate $1 \times 10^{-5}$. Finally, we observe that the performance of the optimal assignment criterion improves as $m$ decreases from three to two, while the diversity order is the same in both cases. This is attributed to the fact that when $m < n$, the number of ways the relays can be assigned to the $m$ pairs is more, giving rise to the possibility of finding better channel
In Fig. 4.4, we compare the simulated bit error rate to the theoretical expression derived in Section 4.4 for symmetric channels. In our simulations, we set all channel variances to one. It is obvious that all the communication nodes have the same average bit error probability. In particular, we consider the cases: \( m = 2, 3 \) and \( n = 2, 3, 4 \). In all cases, we consider the subset assignment scheme. As shown in the figure, the performance improves as \( n \) increases. Also, the performance improves as \( m \) decreases while fixing \( n \) (similar to the observation in Fig. 4.3.) We can also see
the perfect match between theory and simulations, which validates the derived bit error rate expression for symmetric channels.

Figure 4.5: Bit error rate performance (simulated and theoretical) of the suboptimal assignment scheme over asymmetric channels.

In Fig. 4.5, we show the bit error performance of the suboptimal assignment scheme for asymmetric channels. We consider two cases: \( m = 2 \) and \( n = 2, 3 \). In order to simplify the presentation, we use \( E \left[ \left| h_{S_i R_j D_i} \right|^2 \right] \) to represent the set \( \{ E \left[ \left| h_{S_i R_j} \right|^2 \right], E \left[ \left| h_{R_j S_i} \right|^2 \right], E \left[ \left| h_{D_i R_j} \right|^2 \right], E \left[ \left| h_{R_j D_i} \right|^2 \right] \} \) where \( E \left[ \left| h_{S_i R_j} \right|^2 \right] \) represents the variance of the channel coefficients of the \( S_i - R_j \) link. The other terms are similarly defined. In our simulations, for the case \( m = 2, n = 2 \), we randomly
set $E [ |h_{S_1 R_1 D_1}|^2 ] = (1, \frac{1}{8}, \frac{1}{4}, \frac{1}{2})$, $E [ |h_{S_2 R_2 D_2}|^2 ] = (\frac{1}{12}, 1, \frac{1}{3}, \frac{1}{4})$, $E [ |h_{S_1 R_2 D_1}|^2 ] = (\frac{1}{6}, \frac{1}{14}, \frac{1}{3}, \frac{1}{2})$, and $E [ |h_{S_2 R_1 D_2}|^2 ] = (\frac{1}{9}, \frac{1}{5}, \frac{1}{10}, \frac{1}{6})$. As for the case $m = 2, n = 3$, we set $E [ |h_{S_1 R_1 D_1}|^2 ] = (\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, 1)$. For the case $m = 2, n = 2$, all the nodes have different average bit error rates, which are displayed as four curves in the figure. It is shown from the curves that the full diversity is achieved which is $n = 2$ for all the nodes in the network. While for the case $m = 2, n = 3$, since the channels are semi-symmetric, the average bit error rates are the same for the different pairs, hence there are only two curves for this case. As we expect, the node $S_i \rightarrow D_i$ has a better performance than $D_i \rightarrow S_i$ because the channel from the relay to $D_i$ is better than that from the relay to $S_i$. Also we observe that full diversity is achieved for both nodes which is $n = 3$. In addition, we can also find the perfect match between theory and simulations which validates the bit error rate expression for asymmetric.

4.6.2 Relaying with Higher Order Modulation

In Fig. 4.6, we plot the bit error rate performance for the cases: $m = 6$ and $n = 2, 3$ with 8PSK, and $m = 4, 6$ and $n = 3$ with QPSK over symmetric channels. The suboptimal assignment scheme is used in all cases. As shown in the figure, for 8PSK, when $n$ increases from 2 to 3, the performance improves and the diversity order increases from 2 to 3. Whereas, for QPSK, when $m$ increases from 4 to 6, the performance deteriorates, but the diversity order remains at 3. This suggests that the full diversity is achieved for both cases. We also observe from the figure that the
Figure 4.6: Bit error rate performance (simulated and theoretical) of the suboptimal scheme with 8PSK and QPSK modulation schemes for various $m$ and $n$.

performance of $m = 6$, $n = 3$ with QPSK is better than that of $m = 6$, $n = 3$ with 8PSK. This is attributed to the fact that performance of QPSK is better than 8PSK. We can also see the perfect match between theory and simulations, which validates the derived bit error rate expressions derived in Section 4.5.

4.7 Conclusions

We addressed in this chapter relay assignment schemes for cooperative systems with multiple two-way relay channels cooperating through relays that employ net-
work coding. We presented two assignment criteria: optimal and suboptimal over asymmetric, semi-symmetric and symmetric channels. We derived closed-form expressions for the E2E bit error rate performance and shown that the full diversity order is achieved in all cases. We also considered relaying with $M$-PSK modulation in an effort to improve the network throughput. We have analyzed this system and shown that the results are similar, in terms of diversity order, to the case of relaying with BPSK signaling, except for some performance degradation in the former case, as expected. As mentioned before, the latter subject is far from being done because there are still many issues that remain unaddressed, including deciding when to use higher order modulation schemes and what scheme to use. Furthermore, one may also consider using adaptive modulation to strike a balance between performance and throughput.
Chapter 5

Conclusions

5.1 Concluding Remarks

In this thesis, we have presented two relay assignment schemes for multiple source-destination cooperative networks. Both assignment schemes are based on the quality of the subchannels, implying that estimates of the subchannels should be available before the relays are assigned to the pairs. The first assignment scheme is optimal since it considers all possible relay assignment permutations. The second one is sub-optimal since it considers only a subset of the possible permutations. The advantage of the latter is that it makes the performance analysis more tractable and significantly reduces the complexity of the assignment process.

Firstly, we considered relay assignment schemes for relay networks comprising multiple source-destination pairs that communicate with each other simultaneously.
In particular, we examined the two proposed schemes with and without a direct path between the pair nodes. As such, two of these schemes are based on searching over all possible assignment permutations, whereas the other two were based on searching over only a subset on the possible permutations. The latter schemes were devised to reduce the computational complexity and make the analysis tractable. We considered AF and DF relaying, as well as LLR-based relaying. For all cases, we have derived expressions for the bit error rate and showed that all relaying schemes achieved the maximum diversity available, which is \( n+1 \). We also compared the proposed assignment schemes with two well studied assignment schemes, namely, random selection and sequential selection and demonstrated the superiority of the proposed schemes.

Secondly, we considered relay assignment schemes for cooperative systems with multiple two-way relay channels cooperating through relays that employ network coding. We applied the above mentioned assignment criteria to asymmetric, semi-symmetric and symmetric channels. We derived closed-form expressions for the E2E bit error rate performance and shown that the full diversity order is achieved in all cases. We also considered relaying with \( M \)-PSK modulation in an effort to improve the network throughput. We have analyzed this system and shown that the results are similar, in terms of diversity order, to the case of relaying with BPSK signaling, except for some performance degradation in the former case, as expected.
5.2 Future Work

Our work in this thesis can be expanded in several aspects. They are listed as follows.

1. In our thesis, the performance of suboptimal schemes are analyzed. The performance analysis of the optimal scheme is still unsolved. By further studying the relationship of the optimal scheme and suboptimal scheme, it is promising that a tighter upper bounds on the performance can be obtained.

2. The relay assignment schemes proposed in this thesis are based on the assumption that all the channel state information is available, which means that our schemes are centralized. For a large network, a distributed relay assignment scheme will be more attractive. So one effort can be made to realize our schemes in a distributed way. Another effort can be put on finding more efficient distributed relay assignment schemes.

3. In our analysis, we only derived the E2E bit error rate expressions. The outage probability performance and the diversity multiplexing tradeoff can also be investigated.

4. We considered in this thesis the case when one relay helps one pair. One can study the case when more than one relay help one pair. This might be needed to enhance the performance.
5. Applying our relay assignment schemes to other network coding schemes and investigating the performance.

6. Other issues pertaining to relaying with higher order modulation schemes include the way to select the $k$ pairs to be helped by a certain relay, deciding on the modulation scheme to be used at the relay, the possibility of employing adaptive modulation to reflect the quality of the subchannels, finding efficient ways of informing the pair nodes of the type of modulation used.
Bibliography


