A GAME THEORETICAL APPROACH FOR REPUTATION PROPAGATION IN ONLINE SOCIAL NETWORKS

Maziar Gomrokchi

A THESIS
IN
THE COMPUTER SCIENCE AND SOFTWARE ENGINEERING

PRESENTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF MASTER OF COMPUTER SCIENCE AT
CONCORDIA UNIVERSITY
MONTRÉAL, QUÉBECE, CANADA

SEPTEMBER 2010
© MAZIAR GOMROKCHI, 2010
NOTICE:

The author has granted a non-exclusive license allowing Library and Archives Canada to reproduce, publish, archive, preserve, conserve, communicate to the public by telecommunication or on the Internet, loan, distribute and sell theses worldwide, for commercial or non-commercial purposes, in microform, paper, electronic and/or any other formats.

The author retains copyright ownership and moral rights in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

In compliance with the Canadian Privacy Act some supporting forms may have been removed from this thesis.

While these forms may be included in the document page count, their removal does not represent any loss of content from the thesis.

AVIS:

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque et Archives Canada de reproduire, publier, archiver, sauvegarder, conserver, transmettre au public par télécommunication ou par l'Internet, prêter, distribuer et vendre des thèses partout dans le monde, à des fins commerciales ou autres, sur support microforme, papier, électronique et/ou autres formats.

L'auteur conserve la propriété du droit d'auteur et des droits moraux qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

Conformément à la loi canadienne sur la protection de la vie privée, quelques formulaires secondaires ont été enlevés de cette thèse.

Bien que ces formulaires aient inclus dans la pagination, il n'y aura aucun contenu manquant.
Abstract

A Game Theoretical Approach for Reputation Propagation in Online Social Networks

Maziar Gomrokchi

Formation of mutual trust between users of an Online Social Network (OSN) is a function of many parameters. One of these parameters that has been widely investigated is the reputation of users. Users interact with each other with different intentions and as a result of their interactions they propagate each other's reputation. In the absence of centralized trusted parties in OSNs, the only way for an agent to estimate others’ reputation is the other agents’ thoughts about that agent. Therefore, intention and behavior of agents in the propagation of each other’s reputation become crucial.

In this thesis, we propose a game theoretic model of reputation propagation among users in OSNs. We use this model to first study the dynamics of propagation and then analyze users’ behavior with respect to their reputation in the network. To do so, we expose the Nash equilibria of the proposed game. Finally, we develop some experiments on the large-scale social network of Epinons and compare our findings in the theoretical part with the observations from the experiments.
Acknowledgments

I would like to extend my sincerest thanks to my supervisor, Dr. Jamal Bentahar, who bared with my unusual problems and guided my research seamlessly and closely over this time. I would like to thank him for all his excellent guidance, precious advice and endless support over the years during my graduate study and research at Concordia University. I would also like to thank my colleague Babak Khosravifar for the priceless help, discussions, support, and experience we have shared.

I would like to thank my lab mates, fellow students and friends who have provided endless inspiration during my stay at Concordia University. Additionally, I would like to thank my relatives in Montreal for their support.

My sincere thank to the relatives in Montreal: my uncles and his lovely family.

Last, but by no mean least, I would like to express my sincerest appreciation and love to my parents, sister, and brothers in Iran and Australia for all their help and support during these past few years.

Finally, I would like to express my sincerest appreciation and love to my parents for all their help and support to fulfill my dreams.
Table of Contents

List of Figures ................................................................. viii

List of Tables ............................................................... ix

1 Introduction, Motivation and Rationale ..................................... 1
   1.1 Context of Research .................................................... 2
   1.2 Motivations ............................................................... 2
   1.3 Summary of Contributions .............................................. 3
   1.4 Thesis Overview ......................................................... 4

2 Literature Review .................................................................. 5
   2.1 Basic Concepts ............................................................ 5
      2.1.1 Trust Definitions in Different Domains ......................... 5
      2.1.2 Trust and Distrust in Social Networks ......................... 8
   2.2 Trust and Distrust Propagation Models in Social Networks ......... 9
      2.2.1 Models based on the Theory of Transitivity .................... 9
      2.2.2 Social Models of Propagation .................................... 10
      2.2.3 Economical Models ............................................... 12
   2.3 Trust Assessment Mechanisms in Social Networks .................. 12
# 2.3 Centralized Mechanisms

- 2.3.1 Centralized Mechanisms .............................................. 13
- 2.3.2 Decentralized Mechanisms ........................................... 14
- 2.4 Summary of Related Work ............................................... 16

# 3 Game Theoretic Analysis of Reputation Propagation in Online Social Networks  

3.1 Technical Background .................................................. 18
3.2 Model Preliminaries ...................................................... 19
   - 3.2.1 Local Reputation .................................................. 20
   - 3.2.2 Reputation Propagation ....................................... 21
3.3 Local Reputation vs. Reputation Propagation ....................... 25
   - 3.3.1 Game Modeling ................................................... 25
   - 3.3.2 Game Analysis .................................................. 29

# 4 Empirical Observations and Analysis  

4.1 Experimental Goals and Scenario ..................................... 35
4.2 Experimental Evaluation ............................................... 36
   - 4.2.1 Characteristic of Social Network Dataset .................... 36
   - 4.2.2 Experiment Setup and Evaluation Criteria ................... 39
   - 4.2.3 Results and Discussion ....................................... 41

# 5 Conclusion and Future Work  

5.1 Conclusion .................................................................. 44
5.2 Future Work ................................................................ 45
   - 5.2.1 The Potential Model ............................................. 46
Bibliography

Appendices
List of Figures

3.1 Visual representation of $\Omega_a^b$ and $\Psi_a^b$ for two sample networks of agents a and b. ............... 23
3.2 One-shot interaction game tree .................................................. 27
4.1 Trust and distrust ratings distribution in epinions network. ................................................... 38
4.2 Distribution of different types of users’ in Epinions network within two periods of observation. .... 40
4.3 MLRD and MELD comparison on sampled users. .......................................................... 43
List of Tables

4.1 Epinions social network statistics. ........................................... 37
4.2 Distribution of different types of users in Epinions network within the period of
6 months observation. ............................................................. 41
4.3 MLRD and MELD performance measurement ............................ 42
Chapter 1

Introduction, Motivation and Rationale

Online Social Networks (OSNs) are considered as the main source of social interactions for Internet users. Formation of social ties can be considered as a result of mutual trust among users. In OSNs, users' relations are extremely sensitive. Peers' relations may be affected not only by their behavior but also by some other external changes in the network. One of the major changes which has a large effect on agents' interaction is the change in the trust relation among users in the network. Typically, OSNs grow very fast and become very large-scale, making repeated interactions between pairs of users infrequent; this inhibits the formation of users' reputation. As the reputation of users propagates among other users, the network map starts changing, which has some effects on social ties. Since the propagation of users' reputation in OSNs is a key factor in the formation of this reputation, we explore this phenomenon in order to have better understanding about patterns of change in user’s reputation, which allows understanding the formation dynamics of trust and distrust relations among agents.

This chapter introduces the context of research in both areas of trust-distrust and reputation propagation. It presents the motivations of this work and describes our contributions. The last
section presents the thesis organization.

1.1 Context of Research

This thesis proposes a model to analyze the reputation propagation in OSNs. Our main focus is on the modeling of users’ interactions with respect to the propagation of their reputation using probabilistic and game theoretic modeling techniques. We present a new approach to model propagation of reputation in OSNs. The main objective of our work is to analyze the behavior of users as rational and selfish agents with respect to their reputation in a social network. This goal is achieved through the design of a game as a model of interactions and its analysis considering both theoretical and experimental perspectives. The contributions of this thesis are discussed in more details in Section 1.3.

1.2 Motivations

Users’ reputation in OSNs can be easily manipulated by their social peers in the network. Many observations [12,17,36,42] made on OSNs show that users (considered as rational agents) ought to be careful about their reputation (i.e., the opinion others have of us). Reputation can be thought of as a source of trust (i.e., firm reliance on the relation between two agents), which means that agents with a good reputation have higher chance to be interpreted as trustworthy agents.

We encapsulate agents’ behavioral perspectives on how to increase their reputation in OSNs and consequently interact with their social peers between two extreme bounds, Presentist and Futurist. We define Presentist agents as these relying on other agents’ current reputation to
make decisions about their interactions, whereas Futurist agents are those that do not rely on the current reputation of other agents; however, they are more interested in the consistency of their future trend of reputation. The objective of futurist agent is to promote their popularity in the network.

Using the pattern of reputation propagation [38, 53] in OSNs, Futurist agents focus on peers with whom they have more chance to build long term and stable relations. Since we are not assuming the existence of any trusted third party to assist agents in their evaluation about each other, the only source for agent a to evaluate agent b's trustworthiness is the public's thought about b. Doing so, two interesting questions arise: 1) Which peers with a specific behavioral perspective (in this thesis we only assume Presentist and Futurist) should agent choose to interact with in order to maximize his reputation in the network?; 2) How do these two behavioral perspectives affect an agent's reputation in his life time in OSN? To answer these two questions, we propose to model the formation of reputation propagation streams in OSNs and analyze the dynamics of agents’ behavior with respect to these patterns of reputation propagation.

1.3 Summary of Contributions

The main contribution of this thesis is the proposition of an approach to model the users’ actions in propagation of their reputation in OSNs. Game theory has been used as a formal modeling tool to analyze the propagation of users’ reputation considering their actions. In the first contribution, we encapsulate two major behavioral perspectives among users as Presentist and Futurist. After that, we define two notions of Local Reputation and Reputation Propagation in the context of OSNs. Using these two notions, we deploy on-shot and repeated games which
model agents interactions in the context of OSNs with respect to their number of interactions. We also provide a theoretical analysis of these games.

In the second contribution of this thesis, we analyze the fundamental characteristics of the social network of Epinions [1] and then experimentally verify our findings in the theoretical part by proposing different experimental metrics and scenarios. To do so, we develop an open source software which includes two parts. The first part includes a filtering algorithm that runs on a given dataset and extracts specific agents with some given behavioral perspective. The second part of our software is composed of a behavior analyzer program which first traces the behavior of users (exposed as outputs of filtering program) within a given period of time (in this thesis is six months) and then analyzes their behavior. To this end, we defined Mean Local Reputation Deviation (MLRD), to measure the success of agents in increasing their local reputation and Mean Expansion Chance Deviation (MELD), to measure the success of users with different types in propagating their reputation.

1.4 Thesis Overview

This chapter provides the motivating context and objectives for this work. The remainder of the thesis is organized as follows. Chapter 2 gives an overview of the related work in the areas of trust/distrust and reputation in the context of social networks. Chapter 3 describes our game theoretic model of reputation propagation in OSNs along with a theoretical analysis. Chapter 4 presents some experiments on a given OSN. Chapter 5 concludes this work and identifies some directions for future research opportunities.
Chapter 2

Literature Review

Two main directions of research in the context of trust/distrust between peers in OSNs are: 1) trust/distrust management and 2) trust/distrust propagation modeling and analysis. In both directions the ultimate goal is to predict agents' behaviors with respect to their level of trust/distrust about each other. In this chapter, we review the literature related to models and methods of propagation and management of trust/distrust in social networks. In Section 2.1, the basic concepts and fundamentals are presented. Techniques towards modeling of trust/distrust propagation in social networks are presented in Section 2.2. In Section 2.3, we focus on approaches for managing trust/distrust (providing decentralized/centralized systems to cope with the problem of uncertainty in agents' trustworthiness) in OSNs. Finally, Section 2.4 summarizes the chapter.

2.1 Basic Concepts

2.1.1 Trust Definitions in Different Domains

"...trust is a term with many meanings." (Williamson, 1993: [57]).
“Trust is itself a term for a clustering of perceptions.” (White, 1992).

Before starting to discuss the relation between trust and social networks, we need to discuss definitions of trust in different contexts. In fact, trust is considered as a domain dependent concept and it has various definitions in different domains.

**Sociology:** In sociological literature trust is defined as a concept and tool to obtain a social order. This implies that a higher social order is the outcome of truthful behavior. According to Sztompka’s [55] “trust is a bet on the future contingent actions of others”. In this definition the author mainly focuses on the risk and uncertainty elements of granting trust in mutual relations between peers in a society. Another perspective of trust comes from Coleman’s [11] in this book *Foundation of Social Theory*, which suggests a 4-part definition:

1. “Placement of trust allows actions that otherwise are not possible.”

2. “If the person in whom trust is placed (trustee) is trustworthy, then the trustor will be better off than if he or she had not trusted. Conversely, if the trustee is not trustworthy, then the trustor will be worse off than if he or she had not trusted.”

3. “Trust is an action that involves a voluntary transfer of resources.”

4. “A time lag exists between the extension of trust and the result of the trusting behavior.”

**Cognitive science:** According to the paper written by Castelfranchi and Falcone [14], definitions that arise from cognitive science perspective are generally built based on some assertions. Some examples of these assertions are:

1. Only cognitive agents [25] (an agent with the ability of cognition who endowed with goals, beliefs and reasoning capabilities) can "trust" another agent.
2. Trust basically is a mental state, a complex attitude of an agent \(x\) towards another agent \(y\) about a behaviour/action relevant for the goal.

3. Trust is the mental counter-part of delegation.

_Philosophy:_

"Whatever matters to human beings, trust is the atmosphere in which it thrives.": SISSELA BOK, 1978. [8]. Philosophers mostly discussed the similarities between trust and other notions and rarely gave a precise definition of trust. For example Barbara Misztal in her book [47] describes trust as a notion which does three things in the life of people: 1) it does prediction, 2) creates sense of community, and 3) helps them to have a better life together. Moral philosophers, such as Baier and McLeod had influential contribution in defining trust. One of the most precise definitions that we could extract from philosophical contexts is: “Trust is an attitude that we have towards people whom we hope will be trustworthy, where trustworthiness is a property, not an attitude” [44].

_Economy:_ Trust in this context is defined based on the mutual relation of costumer-provider or supplier-buyer. In the following definition, Hosmer in [24] defined trust by merging two disciplines of organizational theory and philosophy:

"Trust is the expectation by one person, group, or firm of ethically justifiable behavior - that is, morally correct decisions and actions based upon ethical principles of analysis - on the part of the other person, group or firm in a joint endeavor or economic exchange."

After the publication of this definition, some experts disagreed on some parts of it such as the meaning of "ethically justifiable behavior". After Hosmer, Arrow in [6] published his definition of trust between buyer and supplier as follows:
"Trust and similar values, loyalty, or truth telling are examples of what an economist would call "externalities." They are goods; they are commodities; they have real practical value; they increase the efficiency of the system, enable you to produce more goods or more of whatever values you hold in high esteem. But they are not commodities for which trade on the open market is technically possible or even meaningful."

2.1.2 Trust and Distrust in Social Networks

Agents in social networks repeatedly interact with each other and trust among theme emerges along these interactions. Sociological and economical definitions of trust are generally applicable in the context of OSNs. Depending of the type of relation among agents (business or friendship relation they can either explicitly or implicitly reveal their level of trust to each other.

Some researchers have defined trust metrics that are used to measure the trust level of agents in social networks with respect to the context of relation. These metrics in social networks can be classified into two main categories of global and local [60]. Global trust ranks are assigned to an individual based upon complete trust graph information. Numerous global trust metrics have been proposed [21, 32, 35, 52]. Surprisingly, some researchers claim that only local trust metrics are "real" metrics for trust in social networks, since global ones consider the global reputation rather than personalized trust [48]. Most of proposed metrics for trust are applied for scenarios different from the ones encountered in social networks. In fact, research in trust infrastructure and metrics for social networks is still preliminary. Some researchers also have a semantic web approach to the definition of trust in the context of social networks [17, 18, 28].

According to Gans et al. [15], "distrust is regarded as just the other side of the coin, that
is, there is generally a symmetric scale with complete trust on one end and absolute distrust on the other." Guha in [20] and Guha et al. in [21] pointed out the important role of distrust in trust propagation applications, mentioning that "distrust statements are very useful for users to debug their Web of Trust". However, literature in this domain still suffers from lack of precise definitions and models.

From above sections, we conclude that trust is a complex concept that has been defined from different perspectives depending on the context. Because we are dealing social networks in this thesis, we build our model (Chapter 3) based on the sociological perspectives.

2.2 Trust and Distrust Propagation Models in Social Networks

There are different ways for users in a social network to evaluate trustworthiness of other users. One of the most recent approaches is the propagation of trust and distrust. Researchers by analyzing the structure of propagation try to come up with a technique to predict the trustworthiness of others. In this part of this chapter we give an overview on the literature related to different models of analyzing trust and distrust propagation in social networks.

2.2.1 Models based on the Theory of Transitivity

In these models, authors assumed that trust and distrust are transitive concepts and therefore propagate among nodes in social networks. However, they rarely worked on propagation of distrust and mostly focused on trust propagation. In real scenarios, trust is not fully transitive and agents may behave differently in various cases. For example, if agent $a$ trusts agent $b$ and $b$ trusts $c$, we cannot infer that $a$ trusts $c$. This depends on the level of trust that agents have
about each other and on other factors such as different referral paths in the network, meaning that agent $a$ may have been introduced to $c$ from other paths as an untrustworthy agent. Given this issue, researchers proposed different methods of transition and aggregation to resolve this conflict. Guha et al. [21] have provided some new matrix operations in order to aggregate trust values transiting from different paths of relation. They proposed two methods of propagation, atomic propagation and trust-distrust propagation. This approach is somewhat inefficient in terms of both space and time complexity. Hang et al. [22] took the dynamism of environment into consideration and analyzed the propagation of trust in multi-agent environments. They proposed new algebraic methods of trust propagation to analyze the constituting evidence. However, their trust measurement does not take into account the fact that the anarchic behavior of an agent might change his trustworthiness beyond the period of observation.

2.2.2 Social Models of Propagation

In the social models of propagation models that capture the propagation of users's behavior in a given environment based on sociological theories, researchers applied social science theory in the context of social network in order to predict the pattern of propagation. Authors in [19] described how, within the context of the small world network topology [45], the social concepts of trust can be applied to guarantee the security of ad hoc networks. In [5] authors analyzed dynamics of friendship and enmity in social networks by applying the theory of social balance, proposed in [23]. Social balance theory is a class of theories that explains how the agents tendencies in converging to a state of cognitive harmony and pleasant would influence the structure of a network. In social balance theory the most important component among social agents is assumed
to be "sentiment". These sentiments would lead agents to two groups of *Disliking* and *Liking*. In [23] the authors, after analyzing this dynamics, observed that a friendly link changes to an unfriendly one or vice versa in an imbalanced triad (relationship triangles) to make the triad balanced. They found out that such networks converge to "utopia" (all friendly links). In this research, the obtained results are somehow far from the reality and this is expectable since they did not consider other social theories of propagation such as status theory [41]. In status theory, social status of an agent is defined as the position or rank of an agent within the society or the network that the agent belongs to. Status theory describes agents goal in creating social ties as an intention of improving their social status in the network. For example, if agent $A$ creates a link to agent $B$ this can be interpreted as $B$ has a higher social status in comparison to $A$, since $A$ is the creator of the link. Status theory contradicts the balance theory [23] in some cases. For example, if agent $A$ creates a trust link to agent $B$ (representing that agent $A$ trusts agent $B$) and $B$ creates a trust link to agent $C$, then balance theory suggests that agent $C$ might create a trust link to $A$, but based on status theory, $C$ considers $A$ in a lower status compared to himself and therefore, status theory suggests that agent $C$ might create a distrust link to agent $A$. One of the most recent works in this domain is done by Leskovec *et al.* in [41]. In this work, the authors assumed that the relations between users in online social networks are either friendly or antagonistic. They used two classical theories of friendship (theories concerning about the formation of friendship relations in social communities), status theory and balance theory from phycology and sociology, in order to model the patterns of propagation in real world networks and after extracting some informative perspectives on the link structures. They found that according to balance theory triangles with exactly two positive edges are largely underemphasized in the data, whereas triangles with three positive edges are largely overemphasized. The main result
they obtained is that the theory of status is more effective to present the pattern of trust and distrust propagation, and this can be extended to capture user attitudes and linking tendencies.

### 2.2.3 Economical Models

In these models, agents are assumed to be rational self interested entities, and willing to maximize their utility. The main problem in modeling trust and distrust propagation in social networks using this approach is formalizing the utility with respect to the users' preferences in the network. Users in an OSN do not necessarily have the same interests and preferences and therefore payoff calculation method with respect to their actions should consider all of these preferences, which may conflicting, in some aggregated functions. Because of this difficulty in modeling of payoff functions, researchers have proposed some application-dependent models with some limited and somehow strong assumptions. There are some proposals to analyze and resolve the problem of defining an aggregated utility function \cite{7,26}. In Chapter 3 we propose another probabilistic approach to calculate a piecewise payoff function in order to resolve this problem.

### 2.3 Trust Assessment Mechanisms in Social Networks

A large body of work has been conducted in the field of trust management in various environments \cite{31,32,34,52}. One of these environments that has recently emerged in the area of trust/distrust assessment mechanisms is social networks. Two basic lines of research that are conducted in this area are: decentralized and centralized mechanisms.
2.3.1 Centralized Mechanisms

Researchers have generally studied trust in social networks via some centralized reputation mechanisms [59]. Centralized reputation mechanisms assume the existence of trusted third parties in their prediction of trust rates. The main weakness of these mechanisms is that they cannot be established in large scale networks, where the enormous number of users is interacting with each other and therefore the monitoring of all interactions becomes to some extent impossible [9]. There are different approaches in designing such mechanisms. Two major approaches can be distinguished as follows:

**Offline approaches**

In these approaches, researchers design a central logging mechanism in order to monitor agents' interactions and save them in some log files, in order to be able to trace agents' interactions and therefore keep track of their history [33, 34]. Afterwards, the mechanism employs some offline calculations to measure the trustworthiness of agents. The main problem in applying these mechanisms in practice is related to their computational complexity. These types of mechanisms are hardly adaptable with real time systems.

**Online approaches**

Proposals about the online management of trust in social networks and online communities have normally approached the problem from mechanism design perspective. The intuition behind this is the computationally efficient nature of these techniques. Mechanisms proposed in this literature are mostly incentive compatible [29], meaning that these mechanisms secure the trustfulness of users in their interactions with other users. Miller et al. [46] employed scoring rules [10] in order to elicit truthful reports from agents and after that they proved the incentive
compatibility of designed incentives in the context of signaling reputation mechanisms. The payment schemes are designed based on proper scoring rules (measure of users’ performance while acting under uncertainty). Jurca and Faltings [30] extend the Miller’s work by applying a computational approach to designing an incentive compatible payment method. Instead of scoring rules, they modeled the payment as an optimization problem that minimizes the total budget required.

### 2.3.2 Decentralized Mechanisms

These mechanisms are established with the assumption that there is no central trusted party so that users can refer to in order to assess trust of other users. Each user is equipped with his own trust assessment mechanism. These mechanisms generally have a payment function which calculates the amount of payment that should be provided to the other agents as incentives in order to ensure their truthfulness. The main difference between centralized and decentralized mechanisms is the type of information that they employ in their trust assessment methods. In decentralized mechanisms a user only relies on the available local information which is not necessarily reliable, but in centralized mechanisms there is always a global and reliable source of information available for all users. The decentralized mechanisms are more suitable for social networks, specially online social networks [54], where the monitoring of agent interactions should be adaptable to large-scale settings. These mechanisms are mostly designed based on theory of referral networks [51, 56, 58], where users refer to each other by social ties. Designing and implementing this type of mechanisms for social network environments is a hard task [45]. One of the main problems is the unknown structure of current social networks. The way the network evolves and grows is a factor of many known and unknown parameters [40]. Another major difficulty in
designing such mechanisms is the unpredictable behavior of interacting agents [49]. Two main lines of research to design such mechanisms are discussed in the following sections.

**Probabilistic approaches**

In this line of research, researchers apply different propagability theories in the context of social network analysis in order to come up with some trust prediction algorithms. In the work done by Kuter and Golbeck [39], the authors proposed a new explicit probabilistic interpretation for social networks, which distinguishes between trust and confidence. Therefore, they proposed a new trust inference algorithm called SUNNY that employs probabilistic sampling to separately estimate trust information. Finally, they experimentally compared their work with another trust inference algorithm TIDALTRUST [18]. In another seminal work conducted by Despotovic and Aberer [13], the authors used maximum likelihood estimation and bayesian estimation in order to estimate trust values. Then they examined their work in both peer to peer and social network settings with some available data sets.

**Mechanism design-based approaches**

"Mechanism design is the sub-field of microeconomics and game theory that considers how to implement good system-wide solutions to problems that involve multiple self-interested agents, each with private information about their preferences" [50]. These approaches are built upon design of payment mechanisms to incentivize agents to reveal the truth. Since the use of this notion in the context of OSNs is relatively new, the literature regarding this approach is not mature enough. In [16], Ghosh et al. proposed a decentralized incentive compatible payment (a payment mechanism which ensures that all of the participants are fare best if they truthfully reveal their private information) mechanism in which payments are based on I Owe You (IOU). The IOU is an online decentralized currency, which is based on mutual loans that agents transfer to each
other. The authors conducted a theoretical analysis on the proposed payment infrastructure in a trust network. As a result of their analysis, they proved that under certain circumstances winner determination (finding the best player with respect to the rules defined by the mechanism designer and under the concept of social welfare) is an NP-hard problem. Furthermore, they approximated the solution of winner determination problem with a factor of $1 - \frac{1}{e}$ (meaning that the value of the proposed mechanism is different from the optimal solution with the factor of $1 - \frac{1}{e}$, where $e$ is a very small value).

2.4 Summary of Related Work

In summary, there are a wide variety of approaches being considered to tackle the problem of trust in social networks, but none yet offers complete decentralized model that is adaptable for large-scale settings. Many of the models discussed in this chapter suffer from scalability issue; others rely on central trusted parties and cannot be implemented in large-scale systems, or are very domain specific and dependent on the assumptions or situations under which they can be applied. Convinced of the importance of modeling reputation propagation in OSNs and in general decentralized online communities, we set out to develop a model to implement these issues in the OSN's platform. The resulting model is described in the next chapter.
Chapter 3

Game Theoretic Analysis of Reputation Propagation in Online Social Networks

In Online Social Networks (OSNs), propagation of users' reputation through streams of agents' connections is an important phenomenon that is often disregarded or misunderstood in social network analysis. Modeling the dynamics of this phenomenon and analyzing its effects on users' interactions are the main contributions of this chapter.

Users in OSNs are considered as autonomous agents, who are rational and self-interested. Diversity of agents behavioral perspectives in terms of level of carefulness about their reputation in the OSN obliged us to limit our theoretical analysis to two extreme cases of Presentist (for agents caring more about their present status in the network) and Futurist (for those caring more about their future status in the network). Although, in Chapter 4 we consider three other types of agents.

In order to analyze the interactions between Futurist and Presentist, we deploy a game theoretical approach. As a result, we expose some Nash behaviors of agents with aforementioned
Presentist and Futurist perspectives in both one shot and repeated interaction games.

3.1 Technical Background

In this section we give a brief overview about some basic concepts and terminology that we use in this thesis. Most of the definitions in this section are extracted from Stanford Encyclopedia of Philosophy or [43].

Game theory: “Game theory is the study of the ways in which strategic interactions among economic agents produce outcomes with respect to the preferences (or utilities) of those agents, where the outcomes in question might have been intended by none of the agents.” [2]

Perfect information game: “A game is said to have perfect information if all players know all moves that have taken place.” [2] Chess is an example of this type of game.

Normal-form game: “In game theory, normal form is a way of describing a game. Unlike extensive form, normal-form representations are not graphical per se, but rather represent the game by way of a matrix.” [2]

Extensive form game: An extensive-form game is a representation of a game in game theory. This form demonstrates the game as a tree. Each node is considered as decision node represents every possible states of the game that is played. A unique player starts the game and the other players sequentially play their actions through the entire tree. The game ends at the terminal nodes and payoffs are assigned to all players. Each player is represented by a non-terminal node and can choose to play an action at non-terminal nodes. Each possible move is an edge leading from that node to another node.

Repeated game: A repeated game (supergame or iterated game) is a given game which is
played repeatedly between the same set of players. A singleton game which is repeated only once is called the base game or stage game or one-shot game. A repeated game which terminates after a finite number of iterations is called a finite repeated game.

**Strategy:** “In game theory, a strategy refers to one of the options that a player can choose.” [2]

**Pure strategy:** “A pure strategy provides a complete definition of how a player will play a game. In particular, it determines the move a player will make for any situation they could face. A player’s strategy set is the set of pure strategies available to that player.” [2]

**Mixed strategy:** A strategy which is a combination of pure strategies with a specific probability assignment to each strategy.

**Solution concept:** “A solution concept is a formal rule for predicting how the game will be played.” [2]

**Nash equilibrium:** Nash equilibrium (named after John Forbes Nash, who proposed it) is a solution concept of a game with two or more players, in which the equilibrium strategies of the players are known to all of them, and no player can increase his payoff by deviating from his strategy.

**Dominant Strategy:** “A strategy is dominant if, regardless of what any other players do, the strategy earns a player a larger payoff than any other. Hence, a strategy is dominant if it is always better than any other strategy, for any profile of other players’ actions.” [2]

### 3.2 Model Preliminaries

In the next section we discuss two notions of *Local Reputation* and *Reparation Propagation*. Before that, we have to define some basic concepts and notations. Many different definitions for
OSNs have been proposed so far [27]. Our definition of OSN is as follows:

**Definition 1.** (OSN): OSN is a tuple \( \Gamma = (\mathcal{Y}, E) \) where \( \mathcal{Y} \) is the set of agents in the network and \( E \in \mathcal{Y} \times \mathcal{Y} \) is the set of agents' connections.

In order to simplify our mathematical formulations we define the following operators:

**Definition 2.** (Ask Operator '⊡'): Let \( a \) and \( b \) be agents in OSNs, \( a \oplus b \) represents \( a \)'s action in asking agent \( b \) for a new friendship relation.

**Definition 3.** (Accept Operator '⊕'): Let \( a \) and \( b \) be agents in OSNs, \( b \oplus a \) represents \( b \)'s action in accepting agent \( a \)'s request for new friendship relation.

**Definition 4.** (Join Operator '▹'): Let \( a \) and \( b \) be agents in OSNs, \( a \triangleright b \) represents the initiation of new friendship relation, means \( a \) joins \( b \)'s friend list after \( a \) initiates the query.

### 3.2.1 Local Reputation

In the absence of a central reputation management system, an agent has to handle his reputation by himself and also has to individually calculate the other's reputation in the network. This means that agent \( a \) may have different measurement about agent \( b \)'s reputation than agent \( c \) does. In this section, we define a notion called Local Reputation (\( LRep(\varphi_a) \)), which agent \( a \) uses to estimate his own reputation in his local friendship network. In other words, agent \( a \) by using this measurement tries to evaluate his reputation from the other agents point of view. Based on *convergence theory* that says "people who wish to act in a certain way come together to form crowds" [4], we can assume that an agent's attitude or behavioral perspective in his friendship network (here
assumed as a crowd) could be the reflection of other agents' behavioral perspectives in that network. Therefore, one possible way of measuring local reputation of agent $a$ in the network is the aggregation of his friends' ($\varphi_a$) reputation. This reputation is assessed by agent $a$ itself.

Agent $a$ uses this notion in order to estimate the contribution of new agents in his Local Reputation in the network. It is important to mention that this notion only estimates the agent's reputation in public scene, since an agent's reputation would have either a direct or indirect impact on his friends' reputation. Rating function $F_a : \varphi_a \rightarrow [0, 1]$ is a personalized function that agent $a$ uses to rate agents in the network, where $\varphi_a$ represents agent $a$'s set of friends. In this thesis we assume that this function exists for each agent $a$ and maps each agent in his friends' set ($\varphi_a$) onto $[0, 1]$. An example of this rating function which we use for our experiment is proposed in Chapter 4. We formulate $LRep(\varphi_a)$ for agent $a$ in the following equation.

$$LRep(\varphi_a) = \frac{\sum_{j \in \varphi_a} F_a(j)}{|\varphi_a|}$$  \hspace{1cm} (1)

To give a better explanation of this notion we provide the following example. A given set of friends for agent $a$ is $\varphi_a = \{c, d, e, f, g\}$ and the set of rankings that $a$ assigns to his friends is: $\{0.6, 0.7, 0.3, 0.9, 0.15\}$, then the $LRep(\varphi_a) = \frac{0.6+0.7+0.3+0.9+0.15}{5} = 0.53$. Therefore agent $a$'s estimation about his reputation in the global scene is: 0.53.

### 3.2.2 Reputation Propagation

Agents with *Futurist* behavioral perspective would like to consider the effect of their actions in propagation of their reputation to friends of friends and so on. In other words, an agent with this behavioral perspective makes friends with the assumption that the new friend would initiate a new
stream of reputation propagation through global network. An example of this action can be found in the network of business firms. Each member of the network would like to expand his business by interacting with larger firm having more opportunity of expansion. Agent with this behavioral perspective establishes a friendship in order to increase his future reputation as well as his friends in the network, instead of only thinking about the level of change on his current reputation with his current set of friends. In order to design a proper notion, we define two probabilities \( P(a \odot b) \) and \( P(b \oplus a | a \odot b) \).

The first probability which we define in equation 2 calculates the probability that agent \( a \) asks agent \( b \) to be his friend.

\[
P(a \odot b) = \frac{\sum_{j \in \varphi_a \cup \{b\}} F_a(j)}{|\varphi_b| + 1}
\]  

(2)

The second probability, \( P(b \oplus a | a \odot b) \), represents the probability of acceptance of agent \( b \) given that agent \( a \) asks him for friendship. This probability, defined in equation 5, considers two cases: 1) an agent does not have any friend; 2) an Agent has at least one friend. For the former case, we assume the probability that agent \( b \) accepts \( a \)'s request is given, \( \beta_{b|a} \), that represents the amount of risk that agent \( b \) is willing to take by accepting agent \( a \) as a friend. In the latter case, first we define two sets of \( \Omega^b_a \) and \( \Psi^b_a \):

\[
\Omega^b_a = \varphi_a - (\varphi_b \cap \varphi_a)
\]

\[
\Psi^b_a = \bigcup_{\forall j \in \varphi_b} (\varphi_j \cap \varphi_a)
\]

(3)

where \( \Omega^b_a \) represents the set of \( a \)'s friends except all mutual friends with \( b \) and \( \Psi^b_a \) represents the set of all agents who are friends of friends of agents in the set \( \Omega^b_a \) and are friends of \( a \) as well.
Figure 3.1: Visual representation of $\Omega^b_a$ and $\Psi^b_a$ for two sample networks of agents a and b.

Figure 3.1 represent an example of couple of friendship networks, where the blue vertical oval represents $\Psi^b_a$ and the red horizontal oval represents $\Omega^b_a$.

The second part of the conditional probability of acceptance is computed in a two-part approach. The first part assumes that the more friends two agents have in common, the higher is the possibility of b accepting a, and the second part takes into account the probability of further connection to agent b’s network. In fact, this part of probability reflects the chance that agent b’s current friends would accept agent a if he joins their community. These two approaches are weighted with $\alpha$ as a factor of proportional relevance and merged in equation 4.

$$\sigma = \alpha \frac{|\varphi_a \cap \varphi_b|}{|\varphi_a \cup \varphi_b|} + (1 - \alpha) \frac{|\Psi^b_a|}{|\Omega^b_a|}$$  \hspace{1cm} (4)

To sum up, the final probability is shown in the following equation:
\[ P(b_\oplus a|a \oplus b) = \begin{cases} \beta_{b \oplus a} & \text{if } \varphi_b = \emptyset; \\ \sigma & \text{otherwise}. \end{cases} \] (5)

Therefore, the probability that agent \( a \) joins agent \( b \) \( (P(a \triangleright b)) \) is calculated as follows:

\[ P(a \triangleright b) = P(b_\oplus a|a \oplus b) \times P(a \oplus b) \] (6)

We define the expansion factor of agent \( a \) on \( b \)'s network upon joining agents \( b \)'s network, denoted by \( C_{a;b} \in [0,1] \), in equation 7. This factor reflects how much agent \( a \) can be successful in expanding his friendship (by propagation of his reputation) upon joining agent \( b \)'s friends list.

\[
C_{a;b} = \frac{\sum_{j \in \varphi_b \cap \varphi_a} \frac{(P_{a(i)} + P_{b(j)})}{2}}{|\varphi_b \cap \varphi_a|} + \frac{\sum_{j \in \Omega_b} P_{b(j)}}{|\varphi_b|} (1 - \frac{|\varphi_a \cap \varphi_b|}{|\varphi_a|}) \] (7)

Note that since \( C_{a;b} \) is a factor that represents agent \( b \)'s potential in expanding of agent \( a \)'s friendship network, these agents have at least one common friend. Therefore \( |\varphi_a \cap \varphi_b| \neq 0 \) is not possible in our case.

For example, if \( \varphi_a \cap \varphi_b = \{c, d, e\} \) and \( a \)'s rankings about these agents are respectively, 0.6, 0.9 and 0.8 and \( b \)'s rankings about them are respectively, 0.7, 0.55 and 0.3. If \( |\varphi_b| = 5 \) and \( |\varphi_a| = 7 \) and \( b \)'s rankings about his friends in \( \Omega_b \) are 0.8 and 0.56 then \( C_{a;b} = 0.657 \). This means that agent \( a \)'s expansion factor upon joining agent \( b \)'s friendship network is 0.56.

Therefore, given \( P(a \triangleright b) \) we can calculate the chance \( (L_{a;b}) \) of agent \( a \) expanding his friendship network upon joining agent \( b \)'s network with the following equation:
In other words, \( L_{ab} \) represents the chance of a friend of \( b \) asks \( a \) to join his network if \( a \) and \( b \) are joined.

In the next section we consider strategic behaviors of agents with different behavioral perspectives inside the OSN. We introduce a game to analyze their strategies to select the best possible set of friends.

3.3 Local Reputation vs. Reputation Propagation

3.3.1 Game Modeling

In this part we focus on the dynamics of agents’ behavior as they establish new friendship relations and on the effects of their actions on each other’s reputation after having \( \tau \) interactions (in the game context we assume each interaction as a game iteration) in OSN. We design a game to model the dynamic behaviors of agents in terms of formation and propagation of reputation in OSNs. We assume agents in OSNs are self-interested, which means they adopt actions that maximize their utility. Agents depending on their behavioral perspective (Presentist or Futurist) interact with an agent who either recommends them to his friends or immediately improves their local reputation. Given this fact, we introduce the Reputation Exchange game as follows:

**Definition 5.** (Reputation Exchange Game): Reputation exchange game is a finite extensive form repeated game with perfect information, represented by a tuple \( (N, A, T, \pi) \), where

- \( N \) is a finite set of players;
• $A = \{A_e, A_t\}$ is the set of action profiles for two types of players: Evaluator (agent who issues a friendship request) and Target (agent who receives the request). $A_e = \{NREC, REC, Ignore, Ask\}$ represents the action profile of evaluator agents in the game. Recommend (REC) represents the action preformed by an agent to propagate an opponent's reputation by an agent as recommending this opponent to the agent friends as a potential choice of friendship and $NREC$ is just a stationary action, which means not recommending this opponent. The ask action represents the evaluator agent's willingness to initiate a friendship, and Ignore represents the opposite. $A_t = \{NREC, REC, Accept, Reject\}$ represents the action profile of target agents in the game. Two possible actions when the Target agent is managing friendship request are Accept and Reject. The Accept action is the action of accepting the request of friendship. The action Reject is the action of rejecting the friendship action. The other actions of $NREC$ and $REC$ are similar to the ones available for Evaluator agents.

• $T = \{(T^1, T^2, T^3, ..., T^n)\}$, is an ordered set of directed trees and each tree $T^i$ represents a one-shot interaction game initiated with agent $i$. $T^i = (V^i, E^i)$, where nodes $V^i$ and edges $E^i$ are elements of directed tree $T^i$. We split $V^i$ into two parts: $TR^i$ (rectangular) and $NTR^i$ (circular), where $TR^i$ represents terminal nodes (payoff nodes) and $NTR^i$ represents non-terminal nodes (decision nodes) in the tree.

• $\pi = \{\pi_1, \pi_2, ..., \pi_{n+1}\}$, where $\pi_1 : A \times A \rightarrow \mathbb{R} \times \mathbb{R}$ represents payoff function of players in the game.

In this game we assume that players can interchange their roles (Evaluator to Target and
Figure 3.2: One shot interaction. Circles represent decision nodes (non-terminals), edge labels represent actions. Payoffs are represented in rectangles (terminals), the top row describes the payoff pair of the evaluator agent, the second row describes payoff pair of the target agent.

vice versa) in the sense that some times they are being evaluated and some times they are evaluating other agents. We assume that agents only play pure strategies. Pure strategy is a strategy from the strategy set covering all possible situations of a player without use of probability distributions over strategies. If target agent b plays REC that means he will propagate agent a’s (Evaluator) reputation to at least one of his friends as a form of recommendation to add a to their friends’ list. We calculate the expected number of agents who are members of agent b’s friends list and might invite agent a to their network in the following formula:

$$[L_{abb} \times |\varphi_b|]$$

If agent b decides to play NREC, then agent a would only expect an increase in his local reputation with the portion of $$\frac{F_a(b)}{\varphi_a}$$.
If the evaluator’s intention in offering friendship to a target agent was to propagate his reputation but that target agent does not do the same thing, that then
evaluator agent might be worse off in this interaction.

Assuming the first agent who starts the game is the agent $a$, therefore agent $a$ will repeatedly play the game with other agents until he decides to play \textit{Ignore} against all other agents and stop playing the game. Furthermore, agent $a$ might stop playing the game when he reaches his threshold of interactions $r$. Figure 3.2 shows the game payoffs upon adopting different actions. Structure of payoff for each player is a 2-part payoff: the first part represents the expected amount of increase on agent’s local reputation and the second part represents the expected number of agents who will add him as a friend and adopt his reputation (believe in his reputation). The top payoff is for the \textit{Evaluator} agent and the bottom one is for the \textit{Target} agent. Figure 3.2 (without Recommendation Pool and New Game Tree parts) represents a one-shot interaction game tree where agent $a$ at the end of the game might receive some other requests of friendship from agents in the \textit{Recommendation Pool} (where all the recommendation from different players are aggregated). Thus, agent $a$’s role may change form \textit{Evaluator} to \textit{Target} agent, who is being evaluated by agents in the recommendation pool and can start playing the game with other agents.

In Figure 3.2, first agent ($a$) preforms his action of \textit{Ignore} or \textit{ask}, if he preforms the \textit{Ignore} action then the corresponding payoff for both agents is $(0,0)$. If $a$ decides to play \textit{ask} then his payoff will depend on the $b$’s action. If $b$ plays \textit{Reject} then $a$ will lose $\varepsilon$ and $b$ will not lose anything, therefore the payoffs would be $(-\varepsilon, 0)$ for agent $a$ and $(0, 0)$ for agent $b$. Agent $a$ loses this amount of local reputation ($\varepsilon$) because his rating about the agent who recommended agent $b$ or was the main source of reaching $b$ is decreased and therefore agent $a$’s local reputation in total is decreased as an effect of this action. If $a$ asks $b$ and $b$ accepts, then both agents may either play \textit{NREC} or \textit{REC}. Their obtained payoffs out of playing actions is represented in Figure 3.2.

In the following subsections we analyze the Nash Equilibrium of the game in two cases:
one-shot ($\tau = 1$) and repeated ($\tau > 1$) interactions.

3.3.2 Game Analysis

One-Shot Game Nash Equilibrium Analysis

Here we assume the case that $\tau = 1$ ($\tau$ represents the number of interactions) and later on we expand our analysis to the case that $\tau > 1$. Figure 3.2 shows the game payoffs for a one-shot game for two players $a$ and $b$. We analyze the Pure Strategy Nash Equilibrium (PSNE) in one-shot game with consideration of two behavioral perspectives of Futurist and Presentist. We have four cases, Presentist – Presentist, Futurist – Presentist, Presentist – Futurist and Futurist – Futurist. For any of these three cases we analyze the nash equilibrium.

In the case of Presentist – Presentist both agents would like to increase their local reputation as opposed to number of their friends.

**Proposition 1.** In case of Presentist – Presentist, if $e \geq \frac{F_a(b)}{|\varphi_a|+1}$ then placing the action Ignore would be the dominant strategy for the Evaluator agent.

**Proof.** In the case that agents are both Presentist and not interested in propagation of their reputation, increase in the second part of payoff is considered as a loss for them. As it is clear from Figure 3.2, the best payoff that agent $a$ as an Evaluator agent can gain out of playing in this game is $-\varepsilon + \frac{F_a(b)}{|\varphi_a|+1}$. If $(-\varepsilon + \frac{F_a(b)}{|\varphi_a|+1}) \geq 0$ then agent initiates the request, otherwise the best action of Evaluator agent is Ignore. Therefore in this case Ignore is a dominant strategy.

$\square$

**Proposition 2.** In case of Presentist – Presentist, if $e < \frac{F_a(b)}{|\varphi_a|+1}$ then (Ask – NREC, Accept – NREC) is PSNE.
Proof. If two players are Presentist, their intention in playing the game and interacting with other users is to increase their local reputation. In this case agents do not like to be recommended and this means that the action REC for both agents is not a favorable action. Therefore, in the case that agents know that they are both Presentist and the Evaluator agent (a) takes the risk of interaction and initiates the request, meaning that \( \varepsilon < \frac{F_a(b)}{||\varphi||+1} \), then the action Ask for Target agent (b) has a better payoff compared to Reject, since b knows that if he plays Reject his payoff is 0 and if he plays accept then there is chance of better payoff. Using backward induction, the Figure 3.2 shows that the best terminal node (rectangular nodes including payoffs) for both agents is the right most node of tree. If b plays NREC and a plays REC then b will be worse off in the second part of his payoff and therefore he would rather to play Reject and if he plays this action, a will worse off as well. But if b plays REC then no matter a plays REC or NREC he will worse off and he will prefer to play Ignore from the beginning. If both agents play REC, then they achieve the best payoff and the only case which agent a might deviate from this action is when \( \varepsilon \geq \frac{F_a(b)}{||\varphi||+1} \), which we assume does not hold since agent a initiates the request. Therefore (Ask – NREC, Accept – NREC) is a PSNE.

Proposition 3. In the case of Futurist – Futurist, (Ask – REC, Accept – REC) is PSNE.

Proof. If both agents adopt the Futurist behavioral perspective then their intention in choosing different strategies is to increase the second part of their payoff and immediate increase in their local reputation (first part of payoff) cannot be considered as the main intention upon which they choose their strategies. Using the backward induction technique, if agent b chooses the action REC then a can choose both REC or NREC and if a chooses NREC then b still prefers to choose REC but b would be worse off, since the second part of his payoff becomes 0. But if a
and b both chooses REC, then b may chooses both Accept or Reject and because $[L_{b\rightarrow a} \times |\varphi_a| > 0$, he will choose Accept. Given b chooses Accept, a may choose Ignore or Ask, and since b adopts the Accept - REC strategy then even if $\varepsilon \geq \frac{F_a(b)}{|\varphi_a|+1}$ because $[L_{ab} \times |\varphi_b| > 0$ he will choose the Ask action. Therefore $(Ask - REC, Accept - REC)$ is PSNE.

\[\square\]

**Proposition 4.** In the case of Futurist – Presentist behavioral perspectives, PSNE is $(Ask - REC, Accept - NREC)$.

*Proof.* In this case the Evaluator agent (a) is assumed to be Futurist and the Target agent (b) is assumed to be Presentist. Therefore the favorable action for agent a is REC and the second part of payoff (which represents the number of recommendation) is in higher priority than the first part (which represents immediate in agent’s local reputation) and for agent b action NREC is a favorable action and the first part of payoff is in higher priority. Knowing this, if agent b plays NREC and a plays REC non of them prefer deviating from their strategies, since both gained the maximum payoff according to the Figure 3.2, so we are done.

\[\square\]

**Proposition 5.** In the case of Presentist – Futurist, if $\varepsilon < \frac{F_a(b)}{|\varphi_a|+1}$ then $(Ask - NREC, Accept - REC)$ is PSNE, otherwise performing the action Ignore would be a dominant strategy.

*Proof.* We start with the fact that agent a’s intention in interaction with other agents is to increase his local reputation. Therefore he always selects an opponent, which maximizes $\frac{F_a(b)}{|\varphi_a|+1} - \varepsilon$. In general, if $(\frac{F_a(b)}{|\varphi_a|+1} - \varepsilon) > 0$ then most likely he starts the game unless he knows that agent b will play the Reject action. But since b is a Futurist agent we know that agent b would prefer to enhance his friendship network and therefore he plays Accept. Again using the backward
induction, if \( b \) chooses the action \( REC \) and \( a \) also chooses \( REC \) then they are both in a stable situation and do not like to deviate from this action; therefore \((Ask - NREC, Accept - REC)\) is \( PSNE \). But if \( \varepsilon \geq \frac{P_b(b)}{|\Phi_b|+1} \) then based on proposition 1 action \( Ignore \) is a dominant strategy, thus the proposition.

\( \square \)

**Repeated Game Analysis**

Here we assume \( \tau > 1 \), meaning agent \( a \) repeatedly plays the game with other agents, belonging to the set of agents in the \textit{Recommendation Pool}, until \( \tau \) is reached, thus each agent has at most \( \tau \) trees. In which agents repeatedly adopt different actions. In this part we analyze the behavior of agents with \textit{Presentist} and \textit{Futurist} behavioral perspectives in the platform of repeated game. We setup our analysis based on evaluator agents' point of view, therefore we assume that agent \( a \) to be an evaluator agent in the rest of this section. We calculate agent \( a \)'s discounted payoff after \( \tau \) interactions in equation 9, where \( \pi_a^t \) represents agent \( a \)'s expected payoff at \( t^{th} \) game.

\[
V_a^\tau = \sum_{j=1}^{\tau} \pi_a^j \theta^{j-1}
\]

where \( \theta \in [0, 1] \) is the discount factor applied to compute the present game value.

Now assuming that agent \( a \)'s opponent(s) in \( \tau_1 \) game(s) play \( NREC \) and in \( \tau_2 \) game(s) play \( REC \) where \( \tau_1 + \tau_2 \leq \tau \), then we have three cases to analyze the behavioral pattern of agent \( a \) during the \( \tau \) interactions. The first case, if \( \tau_1 < \tau_2 \); the second case, if \( \tau_1 > \tau_2 \); and the third case, if \( \tau_1 = \tau_2 \). Considering the aforementioned conditions we analyze the \( PSNE \) of the repeated game for both behavioral perspectives of agent \( a \).
We calculate the upper bound for both agent’s behavioral perspectives, in order to be able to calculate the agent’s payoff in the optimal case and then based on that we find the Nash equilibrium of repeated game. Given the agent $a$ as a Presentist, the upper bound of his expected payoff ($upV^a_{Presentist}$) is calculated in the following equation:

$$upV^a_{Presentist} = -\sum_{j=1}^{\tau_1} \varepsilon_j \theta^{j-1} + \sum_{j=1}^{\tau_1} \sum_{k_1 \in AC^{NREC}} \sum_{i=1}^{\tau_2} F_{a}(k_1) \phi_{k_1} \theta^{j-1} \tag{10}$$

where $AC^{NREC}$ represents the set of agents who played the action $NREC$ in $\tau_1$ interactions of agent $a$.

Notice that agent $a$ obtains the upper bound of his payoff if $\tau = \tau_1$ and $\tau_2 = 0$. Meaning that agent $a$ does not play with any Futurist agent and any Presentist agent whom $a$ plays with does not play $REC$, in the complete period of $a$’s interactions in the network.

If agent $a$ is Futurist, we calculate the upper bound $upV^a_{Futurist}$ for $a$’s expected payoff after $\tau$ interactions. Agent $a$ obtains this upper bound if $\tau = \tau_2$ and $\tau_1 = 0$. This means that $a$ expects to play the game with agents who play the $REC$ action entirely in the who period of his interactions in the network.

$$upV^a_{Futurist} = \sum_{j=1}^{\tau_2} \sum_{k \in AC^{REC}} L_{ab,k} \times |\phi_k| \theta^{j-1} \tag{11}$$

where $AC^{REC}$ represents the set of agents who played the action $NREC$ in $\tau_2$ interactions of agent $a$.

Based on the abovementioned upper bounds the following conjectures follow:

**Proposition 6.** If the agent $a$ is Presentist and $\tau_1 > \frac{1}{2} \tau$ and $\theta$ is small enough then $NREC$ is a dominant strategy for Evaluator in his game horizon.
Proof. We can proof this by using three cases:

Case 1: If $\tau_1 < \frac{1}{2} \tau$. In this case the number of *Futurist* opponents is greater than *Presentist* ones and therefore the case of *Presentist* – *Futurist* game is the dominant case and based on Proposition 5, *Ask – NREC, Accept – REC* is PNSE. This shows that in this case the *Ask – NREC* might be a dominant strategy but not necessarily (since $\varepsilon \geq \frac{F_0(b)}{|\varphi_a|+1}$ in some cases).

Case 2: If $\tau_1 = \frac{1}{2} \tau$. In this the number of *Futurist* and *Presentist* opponents are the same and there is no dominant case and therefore no conclusion can be derived from this case.

Case 3: If $\tau_1 > \frac{1}{2} \tau$. In this case the number of *Presentist* opponents is greater than *Futurist* ones and therefore the case of *presentist – Presentist* is a dominant case and therefore in based on Propositions 1 and 2 the *NREC* should be a dominant strategy.

Proposition 7. If $\tau_1 = \tau_2$ and $\tau = \tau_1 + \tau_2$ then there is no PSNE in the game regardless of agents' behavioral perspectives.

Proof. This proposition has an straightforward proof, since we cannot identify that which of four options of game that might happen between players is dominant. Therefore, regardless of agent's behavioral perspectives, we cannot find any PNSE in the game.

Proposition 8. If agent a is *Futurist* and $\tau_2 > \frac{1}{2} \tau$ and $\theta$ is small enough, then *Ask – REC* is a dominant strategy for agent a in his game horizon.

Proof. The proof of this proposition is similar to the proof or proposition 6.
Chapter 4

Empirical Observations and Analysis

4.1 Experimental Goals and Scenario

The objective of this chapter is to verify our findings in Chapter 3 through experimentation using a large-scale social network of *Epinions* [1]. In general, to run experiments on social network platforms, researchers face many problems. One of the main problems arises when we want to examine dynamic social behavior of users. Due to some privacy barriers, researchers usually have to run their analysis on static datasets, which are anonymized. Another challenging issue is the consideration of large-scale social networks, where the sampling becomes a necessity. Furthermore, real world networks are normally non-homogenous and therefore the sampling is not an easy task. In this work, in order to tackle these problems, we first employ filtering algorithms to filter five types of users that cover all possible behavioral perspective that an agent may adopt. We deploy then a method to sample a homogenous population of users that includes those five types. In fact, we conduct our experiments to explore the answers of the following questions:
• How do our sampled users (agents) perform?

• Does the behavior of sampled users matches our findings in Chapter 3? (one-shot game)

• How does the history of interaction affect the synthetic (one of five filtered types of agents) agents’ behavior compared to the real agents? (repeated game)

4.2 Experimental Evaluation

In this section we present the experiments we carried out to compare the behavior of real users (with different behavioral perspectives) with our synthetic agent. Synthetic agents represent users that follow the Nash solutions that we found in Chapter 3, in order to interact with other agents. To select synthetic agents from the sampled agents we deployed a filter algorithm that traces agents’ behavior during the period of observation and then selects agents who use the Nash solution concept in over 90% of their social interactions. The filter program that we used, is shared as an open source program for further development [3].

4.2.1 Characteristic of Social Network Dataset

We used data collected from the Epinions [1] social network over a six month period for our experiments. Epinions is an OSN, in which users rate each others reviews on some existing articles. There are approximately 75,879 users in the network with 508,837 user to user rates given, 717,667 trust rates (shown by +1) and 123,705 distrust rates (represented by −1), with 85,000 (64%) users having received at least one rating. Therefore the density of the user-user matrix, which represents the density of ratings as well as sparsity of user connections, is:
Table 4.1: Epinions social network statistics.

<table>
<thead>
<tr>
<th>Dataset statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodes</td>
<td>75879</td>
</tr>
<tr>
<td>Edges</td>
<td>508837</td>
</tr>
<tr>
<td>Number of nodes in the largest weakly connected component, A weakly connected component is a maximal subgraph of a directed graph such that for every pair of vertices u, v in the subgraph, there is an undirected path from u to v and a directed path from v to u.</td>
<td>75877</td>
</tr>
<tr>
<td>Number of edges in the largest weakly connected component</td>
<td>508836</td>
</tr>
<tr>
<td>Number of nodes in the largest strongly connected component, A strongly connected component is a directed graph in which it is possible to reach any node starting from any other node by traversing edges in the direction(s) in which they point.</td>
<td>32223</td>
</tr>
<tr>
<td>Number of edges in the largest strongly connected component</td>
<td>443506</td>
</tr>
<tr>
<td>Average clustering coefficient</td>
<td>0.2283</td>
</tr>
<tr>
<td>Number of triples of connected nodes (considering the network as undirected)</td>
<td>1624841</td>
</tr>
<tr>
<td>Number of connected triples of nodes / number of (undirected) length 2 paths</td>
<td>0.06568</td>
</tr>
<tr>
<td>Maximum undirected shortest path length (sampled over 1,000 random nodes)</td>
<td>13</td>
</tr>
<tr>
<td>90-th percentile of undirected shortest path length distribution (sampled over 1,000 random nodes)</td>
<td>5</td>
</tr>
</tbody>
</table>

\[
\frac{85,000}{75879 \times 75879} \approx 0.000014763
\]

There are also approximately 49,290 users who reviewed 139,738 different items at least once and the total number of reviews are 664,824. Therefore the sparsity of reviews per item and per users are respectively, 13.49 and 4.76. Table 4.2.1 represents the general characteristics related to the given Epinions dataset. Notice that in Epinion social network, users have two main activities: the first one, is the trust/distrust ratings that a user assigns to other users and the second one, is the the ratings that a user assigns to articles. Here, in this experiment, we only analyze the first part of users activity, which considers user to users trust/distrust ratings.

Figure 4.1 represents the distribution of trust and distrust ratings in the OSN. In our experiments we assumed that if a user posts a trust rating for another user, so this means a new friendship relation is established between them and if he posts a distrust rate, so this means that
one of them is rejecting or dishonoring the friendship relation. From Figure 4.1, we can observe that the network is very sparse in terms of trust and distrust ratings. We can interpret the influence of this fact on users’ behavior in the sense that if a user receives a distrust rate, it has considerably high effect on his future interactions in the network. Another natural interpretation of Figure 4.1 is that the rate of distrust action is lower than the rate of trust action (absolute value of slope is less therefore the distribution of distrust rates is less). This reflects the natural tendency of users to favor short-term interactions in comparison to the long-term ones in the Epinions network, since users effort on propagating their negative experience (i.e. Distrust rating of other users) about others is less compared to their positive experience (i.e. Trust rating of other users).
4.2.2 Experiment Setup and Evaluation Criteria

We first define the global rating function we used in our experiments, which we assume common for all users. User \( a \) rates user \( b \) \( (F_a(b)) \) according to their shared reviews (items that they both reviewed) on different items and the deviation of \( b \)'s rate from the average of reviews on that item. Assume user \( b \) has rated \( n \) distinct items and he is sharing \( k \) \( (k \leq n \text{ and } n \neq k) \) reviews with \( a \) and the rest with other users. If \( b \)'s rating on \( n \) items are \( \{r^b_1, \ldots, r^b_n\} \) and \( a \)'s shared reviews with \( b \) are \( \{r^a_1, \ldots, r^a_k\} \) (they shared \( k \) items) then we define \( F_a(b) \) as follows:

\[
F_a(b) = 1 - \left( \frac{k}{n} \sqrt{\frac{\sum_{i=1}^{k} |r^b_i - r^a_i|^2}{k}} + \frac{n-k}{n} \sqrt{\frac{\sum_{i=k+1}^{n} |r^b_i - \bar{r}_i|^2}{n-k}} \right)
\]  

(12)

where \( \bar{r}_i \) is the average of reviews of other users of item \( i \).

In order to observe the effects of users' history on their further interactions, we divide our observation period into two 3-month period and then evaluate users' performance distinctly in each period. As explained earlier in this chapter we developed our experiments on five types of real users strained using our filtering algorithm. 1) Users with 90% to 100% Presentist behavioral perspective, which means more than 90% of the interactions of these users reveal the Presentist behavioral perspective; 2) Moderate users with 40% to 50% Presentist and 40% to 50% Futurist behavioral perspective, which means about half of interactions of these users reveal Futurist and another half interactions reveal Presentist behavioral perspectives; 3) Totally Chaotic users, who are revealing an untraceable perspective during their interaction period; 4) Users with 90% to 100% Futurist behavioral perspective, which means more than 90% of the interactions of these users reveal the Futurist behavioral perspective; and 5) semi - Synthetic users who have behaviors at least 80% similar to the behavior of synthetic agents. As shown
Figure 4.2: Distribution of different types of users' in Epinions network within two periods of observation.

In Figure 4.2, we select users in each period based on their distribution in the network within that given time period. Table 4.2 also shows the distribution of users with different types in the network as well as the number of sample users ($N_k$) selected from each type ($k$).

Evaluation Criteria

In order to evaluate the behavior of the users described in the previous section we define two measurements that compare a user's actual local reputation and expansion chance with the best local reputation and expansion chance observed in the network. We are interested in these measurements because the game we have designed assumes that an agent following a pure Presentist strategy will try to optimize his local reputation and a Futurist will try to optimize his expansion chance. We use Mean Local Reputation Deviation ($MLRD$) to measure the success of users with different types in increasing their local reputation, and Mean Expansion Chance Deviation ($MELD$) to measure the success of users with different types in propagating their reputation. We compute these two metrics, defined from time $t_{start}$ (user start making friends in the network)
Table 4.2: Distribution of different types of users in Epinions network within the period of 6 months observation. Sign %, represents the percentage of different type of users in the network and \( N_k \) represents the number of sampled users with type \( k \).

<table>
<thead>
<tr>
<th>Period of Observation</th>
<th>First 3 Months</th>
<th>Second 3 Months</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Users</td>
<td>90% to 100%</td>
<td>Presentist (1)</td>
</tr>
<tr>
<td></td>
<td>Moderate (2)</td>
<td>Chaotic (3)</td>
</tr>
<tr>
<td>Presentist (1)</td>
<td>18.92262</td>
<td>15.75579</td>
</tr>
<tr>
<td>Chaotic (3)</td>
<td>18.92262</td>
<td>15.75579</td>
</tr>
<tr>
<td>Futurist (4)</td>
<td>12.58897</td>
<td>12.58897</td>
</tr>
<tr>
<td>Futurist (4)</td>
<td>12.58897</td>
<td>12.58897</td>
</tr>
<tr>
<td>Semi-Synthetic (5)</td>
<td>47.27096</td>
<td>47.27096</td>
</tr>
<tr>
<td>Semi-Synthetic (5)</td>
<td>47.27096</td>
<td>47.27096</td>
</tr>
</tbody>
</table>


to the time \( t_{stop} \) (user stop making friends in the network), in equations 13. In this equation, \( LRep^i(\varphi_j) \) represents user \( j \)'s local reputation at time unit \( i \) \( \in \{t_{start}, t_{stop}\} \), \( LRep^i(\varphi_{best}) \) represents the best observed local reputation in data set at time unit \( i \), and \( N_k \) is the set of sample users with type \( k \). \( AEC_j^i \) represents the average of user \( j \)'s friends Expansion Chance at time unit \( i \) \( \in \{t_{start}, t_{end}\} \), \( AEC_{best}^i \) represents the highest observed friends community average of expansion chance at time unit \( i \), and \( N_k \) is the set of sample users of type \( k \).

\[
MLRD = \frac{\sum_{j \in N_k} LRep^i(\varphi_{best}) - LRep^i(\varphi_j)}{|N_k|}
\]

\[
MELD = \frac{\sum_{j \in N_k} AEC_{best}^i - AEC_j^i}{|N_k|}
\]

4.2.3 Results and Discussion

Figure 4.2 and Table 4.2 summarize the distributions of user types in the given dataset as well as the number of selection in each user type. The main experimental results are shown in Figure 4.3 and Table 4.3.

Recall that the main objective we target in this thesis is to provide a model of reputation
Table 4.3: MLRD and MELD performance measurement for different types of users in comparison to the best possible performance (lower MLRD and MELD is better).

<table>
<thead>
<tr>
<th>Period of Observation</th>
<th>MELD Metric</th>
<th>MLRD Metric</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>90% to 100% Presentist</td>
<td>Moderate</td>
</tr>
<tr>
<td>Month</td>
<td>Mean</td>
<td>0.361837</td>
</tr>
<tr>
<td></td>
<td>Std.</td>
<td>0.054152</td>
</tr>
<tr>
<td>Month</td>
<td>Mean</td>
<td>0.297293</td>
</tr>
<tr>
<td></td>
<td>Std.</td>
<td>0.078418</td>
</tr>
</tbody>
</table>

propagation in OSNs and to extract some natural solution concepts for users who behave like a self-interested autonomous agent in the network. In Figure 4.3 we observe that the average performance of semi-Synthetic users is better than other users, given the very sparse interaction network of Epinions. In the MELD comparison test, as we have expected from the theoretical part, the Futurist users preformed better than Presentist users and sometimes better than semi-Synthetic users, but since they greedily make friendship with users with high potential expansion rate, they do not preform better than semi-Synthetic users in general. This is due to the fact that they do not act strategically but have a very greedy behavior in making friends, and consequently their MELD trend oscillates. In MLRD experiment, semi-Synthetic users perform much better than the others (Table 4.3) especially in comparison with Presentist users, because semi-Synthetic users adopt different strategies for different users in the network. The performance of Futurist users in this experiment is the worst and it shows the side effects of greedy behavior of this type of agents in the network, given the existence of different types of users with different behavioral perspectives. The observation on the existence of a large number of Presentist users in Epinions network and their lower performance compared to
semi-Synthetic users, especially our MELD comparison test confirms the results obtained in Chapter 3. Similar argument applies to Futurist users in MLRD comparison experiment where again we observe the results obtained in Chapter 3.

Our observations on different types of users over the period of six months illustrate some interesting remarks. 1) Gradual change in users behavioral perspectives by the time passing; 2) Considerable decrease in the percentage of Presentist users which, despite sparsity of users reviews, shows the high effects of greedy and superficial rates on this type of users’ reputation in the network; 3) Considerable decrease in the percentage of Chaotic users and high rate of increase in the number of Moderate and semi-Synthetic users. These points reflect first, high tendency of users to learn from the network, second, rational behavior of these users and finally, effectiveness of the Nash solutions presented in the previous chapter.
Chapter 5

Conclusion and Future Work

The primary goal of the thesis was to model users' reputation propagation in OSNs as outlined in Chapter 1. That goal has been achieved by employing a probabilistic and game theoretic approach in Chapter 3. The model has been implemented and evaluated using a large-scale real world network dataset. An open source software has been provided to filter users' behavior and then trace the filtered users' behavior, which has been explained in Chapter 4. In this chapter we conclude the thesis and propose some potential lines of research as future work. We also provide some possible guidelines to these lines of researches.

5.1 Conclusion

Propagation of reputation as a foundation for modeling trust-distrust propagation in the context of OSNs has received insufficient attention from researchers in the fields of trust and social network. Issues related to reputation propagation are rarely discussed in published work, particularly from game theoretic perspective. Many game theoretic models have been proposed to model trust and
distrust, but only little focus has been shown on reputation propagation.

In this thesis, we have modeled behavioral perspectives of users regarding their reputation in OSNs. We use two concepts of local reputation and reputation propagation to model the main actions of users in order to manipulate their reputation in OSNs. In our analytical approach, to model these two behavioral perspectives, we highlight the Nash solution concepts for users with either short-term (Presentist) or long-term (Futurist) interaction plans. We have filtered users following the behavioral patterns identified in our theoretical analysis in addition to users with different types from the dataset of Epinions. Therefore, the behaviors of these users have been empirically compared and analyzed. We also study the behaviors of users with different types.

As we have expected from the theoretical part, users with a behavior similar to Nash have shown highly adaptive behavior compared to the other types of users. Surprisingly, agents with complete Futurist attitude represent very low rate of adaptation to the short-term interactions and completely Presentist agents also exhibit a very low rate of adaptation in long-term interactions (similar to chaotic agents). This approach is general enough to be applied to a wide variety of online communities, such as online social network advertising, viral marketing and so on.

### 5.2 Future Work

While our work addresses important issues regarding the propagation of reputation in OSNs, it raises other questions and highlights areas that may be fruitful upon further investigation. Here, we briefly address one of the most important research lines among all possible ones. Apart from analyzing the dynamics of reputation propagation in OSNs, this question always arises: how can an agent enforce the propagation of incentives? First step to resolve this problem is to enforce
agents to act truthfully in the domain of OSNs. Therefore, we would like to determine how much incentive is required for agents to maximize their reputation propagation in the network after a certain number of iterations. We believe this can be done by using the idea of Query Incentive Network proposed by Klienberg and Raghavan [37]. A challenging issue is to find out how much incentive is required for agents to maximize their reputation propagation in the network after a certain number of interactions. In the next section we propose a potential model that could be extended to resolve this issue.

5.2.1 The Potential Model

We consider an Online Information Network where there are rational users seeking specific information (IS agents) and there are information providers (IP agent) who are selling information in different categories (e.g. information about car manufacturers or information about cell phone types and brands).

Two main probabilities that affect an IS agents' decision in adopting an actions are: 1) the probability of receiving a query $P'(\cdot)$, at time unit $t$, from other IS agents; and 2) the probability of initiating a query. We assume that the probability $q$ is a constant probability and is known for all agents in the network. However, the probability or receiving a query at time unit $t$ is a dynamic probability, which changes over the time and it depending of several factors.
Bibliography


