Half-Duplex Relaying for the Multi-User Channel: Capacity Bounds, Fading Channel Performance and Asymptotical Behaviour

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Abstract

Half-Duplex Relaying for the Multi-user Channel: Capacity Bounds, Fading Channel Performance and Asymptotical Behavior

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Multiple-input multiple-output (MIMO) scheme is able to improve the modern communications system performance in terms of increased throughput or reliability. As a virtually distributed antenna scheme, the relaying can enhance the communication system performance while there is no physical size limitation at the end user. Through decades, many relaying schemes have been extensively investigated for different channels. When the relay is close to the destination in a static channel and perfect channel state information (CSI) is available at the relay in a slow fading channel, the compress-and-forward (CF) scheme is often applied since it performs better than other relaying schemes. However, the CF scheme requires the relay to perform two-step operation (quantization and WZ binning), which increases the cost of implementing such scheme. In addition, having perfect CSI at the relay is not always possible in a wireless channel. To address these problems caused by the nature of the CF scheme, the generalized quantize-and-forward (GQF) scheme is proposed in this dissertation for the half-duplex (HD) multi-user channel.

In this dissertation, the first part focuses on studying the half-duplex (HD) relaying in the Multiple Access Relay Channel (MARC) and the Compound Multiple Access Channel with a Relay (cMACr). A GQF scheme has been proposed to establish the achievable rate regions. Such scheme is developed based on the variation of the Quantize-and-Forward (QF) scheme and single block with two slots coding structure. The achievable rates results obtained can also be considered as a significant extension of the achievable rate region of Half-Duplex Relay Channel (HDRC). Furthermore, the rate regions based on GQF scheme

are extended to the Gaussian channel case. The scheme performance is shown through some numerical examples. In contrast to conventional Full-Duplex (FD) MARC and Interference Relay Channel (IRC) rate achieving schemes which apply the block Markov encoding and decoding in a large number of communication blocks, the GQF developed are based on the single block coding strategies, which are more suitable for the HD channels.

When the relay has no access to Channel State Information (CSI) of the relay-destination link, the GQF is implemented in the slow Rayleigh fading HD-MARC. Based on the achievable rates inequalities, the common outage probability and the expected sum rates are derived. Through numerical examples, we show that in the absence of CSI at the relay, the GQF scheme outperforms other relaying schemes. When the end users have different quality-of-service (QoS) requirements for slow fading channel, it is more precise to use the individual outage related parameters to quantify the scheme performance. The individual outage probability and total throughput are characterized for the HD-MARC. The numerical examples show that the outage probability of the individual users is lower than that of classic Compress-and-Forward (CF) scheme.

The Diversity Multiplexing Tradeoff (DMT) is often applied as a figure of merit for different communication schemes in the asymptotically high SNR slow fading channels. The CF scheme achieves the optimal DMT for high multiplexing gains when the CSI of the relay-destination (R-D) link is available at the relay. However, having the CSI of R-D link at relay is not always possible due to the practical considerations of the wireless system. In this dissertation, the DMT of the GQF scheme is derived without R-D link CSI at the relay. Moreover, the GQF scheme achieves the optimal DMT for the entire range of multiplexing gains.

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Contents

List of Figures

List of Tables

Nomenclature and Abbreviations

Nomenclature

$f()$	Encoding Function
D_i	Destination Node i, $i \in \{1,2\}$
$g()$	Decoding Function
R_i	Information Rate from Source Node i, $i \in \{1,2\}$
R_u	Information Rate from Relay based on Quantization
S_i	Source Node i, $i \in \{1,2\}$
u	Message Index chosen by Relay
W_i	Message sent from Source i, $i \in \{1,2\}$
$\mathcal{W}$	Message set
Y_R	Relay Reception r.v.
$\hat{Y}_R$	Quantized Relay Reception r.v.

Abbreviations

AF	Amplify-and-Forward
AWGN	Additive White Gaussian Noise
CF	Compress-and-Forward
CSI	Channel State Information
CSIR	Channel State Information at Receiver
CSIT	Channel State Information at Transmitter
DDF	Dynamic Decode-and-Forward
DF	Decode-and-Forward
DMT	Diversity-Multiplexing Tradeoff
FD	Full Duplex
GCF	Generalized Compress-and-Forward

GDoF	Generalized Degrees of Freedom
GHF	Generalized Hash-and-Forward
GQF	Generalized Quantize-and-Forward
HD	Half Duplex
IRC	Interference Relay Channel
MAC	Multiple Access Channel
MAF	Multi-access Amplify-and-Forward
MARC	Multiple Access Relay Channel
MIMO	Multiple-Input Multiple-Output
NAF	Nonorthogonal Amplify-and-Forward
NNC	Noise Network Coding
OMARC	Orthogonal Multiple Access Relay Channel
QF	Quantize-and-Forward
QMT	Quantize-Map and Forward
QoS	Quality of Service
RC	Relay Channel
R-D	Relay to Destination
RF	Radio Frequency
r.v.	Random Variable
SIC	Successful Interference Cancellation
SNR	Signal to Noise Ratio
Tx	Trasmitter
WZ	Wyner-Ziv

Chapter 1

Introduction

1.1 Motivation

A conventional point-to-point communication system consists of a sender and a receiver [68, 83] as shown in Fig. 1.1. In a wireless point-to-point system, the information is transmitted through the wireless medium. Modern wireless communication system faces the challenge of the fast growing demand for high data transmission rates and increased reliability. The wireless networks are expected to be capable of supporting as many users as possible while providing the high information exchange rate and the good quality of service. However, since the broadcast nature of the wireless medium, the more users in a network communicate in a wireless environment the more mutual signal interference is generated. Multiple access interference acts as one of the major issues effecting the performance of a wireless system with many users. A simple multi-user channel is often modeled as a two user multiple access channel as shown in Fig. 1.2. On the other hand, the RF frequency spectrum that can be used for practical communication system (cellular, Wi-Fi, etc) is very limited. How to improve the multi-user system performance with limited RF resource became a significant problem in the communication area.

In the late 90s, the Multiple-input Multiple-output (MIMO) was developed to address the

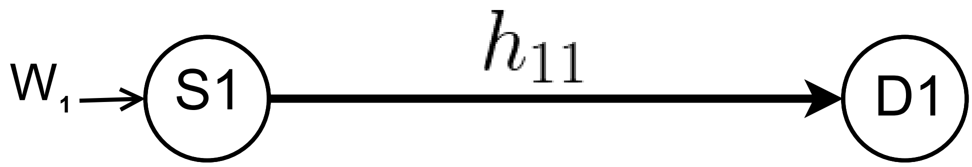


Figure 1.1: Message flow of a point-to-point communication system.

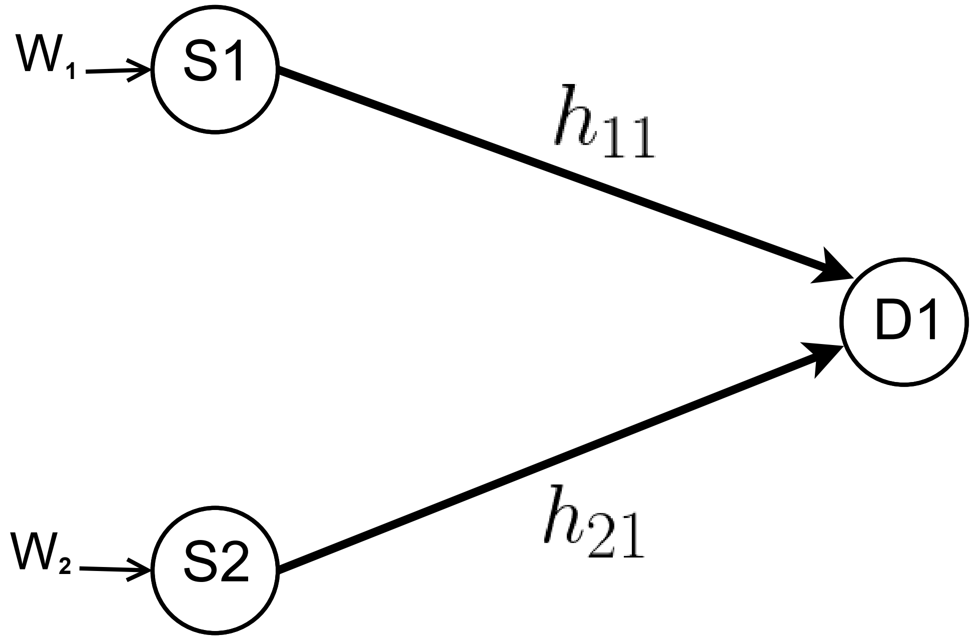


Figure 1.2: A typical two user MAC channel.

above problems. MIMO systems introduce spatial diversity to combat fading and increases spatial multiplexing [4, 24, 86–88]. However, if the end user (say mobile phone users) devices have physical limitations on its size, it is difficult to implement the MIMO. As a practical alternative to the existing MIMO, user cooperation [80, 81] or relaying [54] is often applied where the communication nodes other than the sources serve as virtually distributed antenna.

A typical three nodes relay channel [18, 19] consists of three communication terminals, a source, a destination and a relay as shown in Fig. 1.3. Although the general capacity of a static relay channel is still unknown, it is widely known that relaying can benefit a conventional point-to-point communication channel by cooperating with the transmitter [18, 35]. Furthermore, for the typical two user MAC channel, it is also proved that the relaying

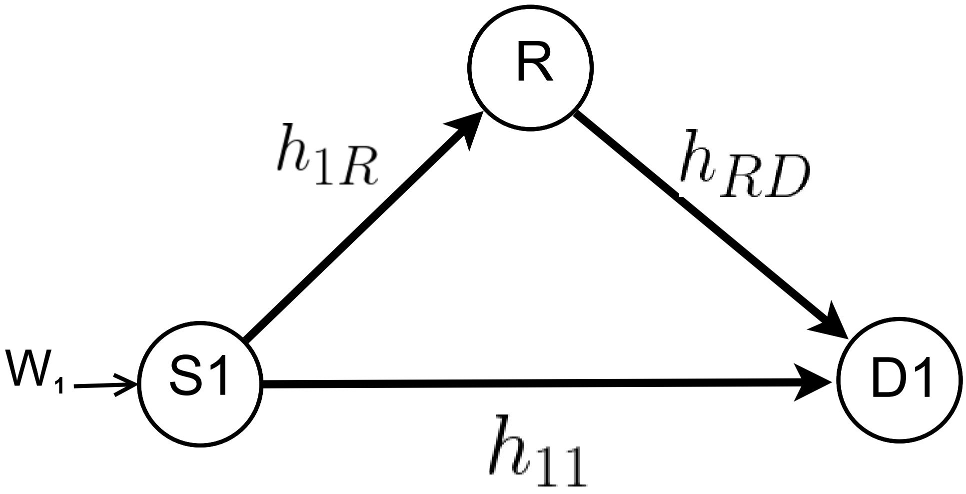
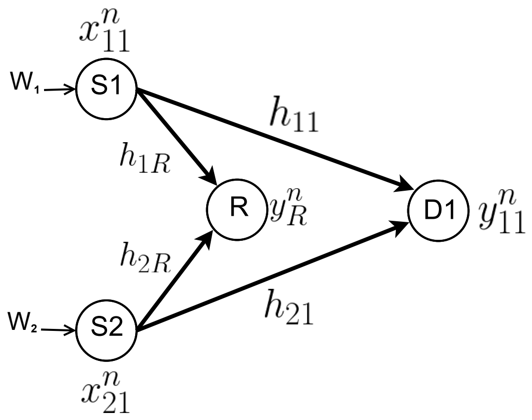
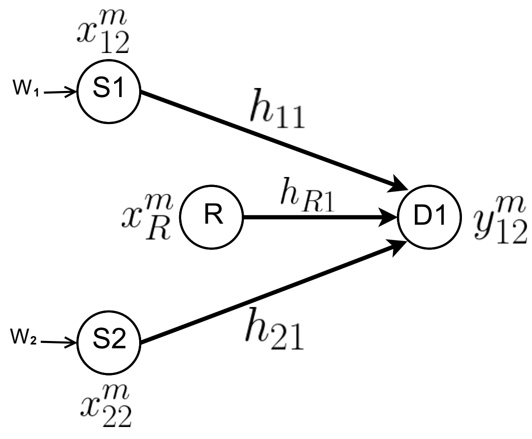


Figure 1.3: A typical three node relaying channel.

can improve the sum achievable rates [31, 48, 71]. From the pioneering work for the relay channel by Cover and El Gamal [18], the fundamental relaying schemes are decode-and-forward (DF) and compress-and-forward (CF) [48, 111]. The CF based schemes are not limited by the decoding capability of the relay, and therefore they can be beneficial in cases where the relay is closer to the destination than to the source. Different variations of the CF based schemes have been investigated in [6, 20, 57, 70, 92, 104].



(a) MARC Slot-1



(b) MARC Slot-2

Figure 1.4: Message flow of the HD-MARC.

In a practical wireless channel, relay usually operates in half-duplex mode [42, 54] where the relay cannot transmit and receive simultaneously. If the relay transmits and receives at the same time, it will have very weak received signal while its transmitting signal is very strong. The HD relay is also called "cheap" relay [42] as it has lower cost from the application perspective. The CF relaying schemes requires two-step operation (quantization and WZ binning) at the relay. For a three node HD-RC, a quantize-and-forward (QF) scheme [111, 113] was proposed which can simplify the relay operation in the sense that only quantization is needed at relay. The QF scheme is a variation of the CF scheme. The QF has the similar achievable rates as the CF while only a low-cost relay is needed. However, for a half-duplex multi-user channel with relay (a two-user HD-MARC shown in Fig. 1.4), the QF scheme cannot directly apply since there are two signals received at relay and three signals received at the destination. Therefore, it is important to develop a relaying scheme that is based from QF to implement the low-cost relay while the scheme can be applied to the multi-user channel, specifically the HD-MARC. This motivates the proposed generalized quantize-and-forward (GQF) scheme in this dissertation.

For wireless communications, fading plays a critical role in the system performance. Fading is a phenomena that signal components received through various propagation paths could add destructively or constructively and cause random fluctuations in the received signal strength due to the scatters in environment and mobile terminals [68, 69, 83, 114]. Fast fading is usually considered positive to the wireless channel as the transmitter can utilize variations of the channel conditions to increase the robustness of the communication. Slow fading is caused by shadowing and multi-path. It is considered as negative for the communication. Relaying can be used to combat fading by its offered spatial diversity. In a slow fading wireless relay channel, the system outage probability can be decreased significantly by the diversity offered from relaying [14, 36, 54]. For the slow fading MARC, the CF based schemes were shown to outperform the DF based schemes when the relay has the complete CSI of the channel [46, 114]. However, the perfect CSI at relay is generally too ideal. When the critical delay constraint exist in the wireless channels, the relay may not be able to obtain the CSI accurately. With only receiver CSI at relay, the QF scheme has outage performance close to the CF scheme with perfect CSI at relay [111]. In a HD-MARC, since there are more signals received at both relay and destination than in the HD-RC, it is essential to investigate the slow fading performance of the GQF scheme.

The outage probability and expected sum rates [39, 44, 85] are widely used as measure of the slow fading system at finite SNR. The diversity-multiplexing tradeoff (DMT) is usually applied to quantify the relationship between reliability and throughput [119] at asymptotically high SNR. The diversity gain describes the slope that the probability of error decreases with increasing SNR. The multiplexing gain shows how fast the system information rate increases with SNR [114]. The DMT of the CF scheme has been investigated in [46, 114]. The CF scheme is DMT optimal in the HD-RC and partially optimal in the HD-MARC with perfect CSI at relay. The DMT of the QF scheme was derived in [111] and is optimal for the HD-RC with only receiver CSI at relay. With more users in the channel, the optimality of the QF scheme cannot directly extend to the HD-MARC. Therefore, the DMT of the GQF scheme needs to be derived and its optimality should be carefully evaluated.

From the engineering perspective, the result from this dissertation can be applied (but not limited to) in the following two systems: 1) a multiuser cellular system with a cheap cost relay. Applying the GQF scheme at relay, no need of the CSI of R-D at relay, the probability of outage is greatly reduced compared to other relaying schemes. 2) a wireless sensor network where the relay is helping the communication between the sensor nodes to the data sink. Adding the cheap cost relay and using GQF scheme can improve the fading performance thus reduce the energy consumed by the sensor nodes.

1.2 Objective and Contributions

1.2.1 Objective of the Dissertation

In this subsection, the objective of this dissertation is presented. The detail for each part is shown in the following:

- The existing relaying schemes can be divided into two categories, which are based on well-known DF and CF [18]. When the relay is close to the destination, the CF scheme is always applied since it is not limited by the decoding capability of the relay. On the other hand, the relay cannot transmit and receive at the same time due to the near-far effect [42, 54] in a wireless relay channel. As a variation of the CF scheme, the QF scheme has been studied for the three node half-duplex relay channel [111]. However, when there are multiple users in the channel, the QF scheme can not be directly applied. A relaying scheme which fits the HD multi-user channel and takes the advantage of CF and QF schemes needs to be investigated. The GQF scheme is proposed to address the problems above. The achievable rate regions of the discrete memoryless and the corresponding AWGN channel need to be derived. The numerical examples shall be provided to compare the performance.

- For slow fading multi-user channels, the common outage probability and expected sum rates are often used as the measurements. Also in practice, having the relay-to-destination (R-D) CSI at relay is too optimistic. The outage related performance of the GQF scheme should be investigated when the relay has only receiver CSI. The fading performance of the GQF scheme needs to be compared with other relaying schemes and verified by numerical examples. Secondly, in practice, each user in a MAC may have different quality-of-service (QoS) requirement [62]. Similarly, for a two-user MARC, the destination failing to decode one of the source messages may not affect the other user's QoS requirement (message decoded successfully by the destination). Therefore, the individual outage related performance of the GQF scheme should be discussed in terms of the individual outage probability and expected sum rate. The numerical examples are also needed to demonstrate the performance.

- When SNR of a slow fading channel is at asymptotically high, the DMT is used as the figure of merit to characterize the performance of different communication schemes. The CF scheme is partially optimal for the HD-MARC with perfect CSI at relay. The QF scheme has been shown to achieve the optimal DMT for the three node relay channel without perfect CSI at relay. Therefore, the GQF scheme is expected to be optimal for the HD-MARC. The DMT of the GQF scheme should be derived first. Then comparison between different relaying schemes needs to be discussed.

1.2.2 Contributions of the Dissertation

This dissertation investigates the performance of the GQF relaying scheme for the half-duplex multi-user channels from achievable rates, outage probability of the finite SNR slow fading and the achievable DMT. The major contributions of the dissertation are summarized in the following:

- The GQF scheme in the static multi-user channel has been characterized first. The achievable rate regions of the discrete memoryless half-duplex MARC, cMACr and the corresponding additive white Gaussian noise (AWGN) channel are established based on the GQF scheme and the classic CF scheme. The achievable rates for the three node relay channel can be treated as special cases from the five node cMACr. The performance comparison between the GQF scheme and the CF scheme is also discussed and shown with numerical example. It is shown that the GQF scheme can provide similar achievable rates while only a simplified relay (no binning necessary) is required.
- Based on the achievable rates, the common outage probability and expected sum rate of the GQF scheme are derived and compared to the classic CF scheme and other common relaying schemes (AF [12,54] and DF [28]). It is shown by the numerical examples that, without relay-to-destination CSI at relay, the GQF scheme outperforms the other schemes and can regain a large portion of the benefit provided by the CF-based schemes (with perfect CSI at relay) over DF-based schemes in the selected topology. Secondly, when each user in a MARC has different QoS requirement, the destination failing to decode one of the source messages may not affect the other user's QoS requirement (message decoded successfully by the destination). The individual outage related performance of the GQF scheme has also been discussed in terms of the individual outage

probability and expected sum rate. The numerical examples are given to demonstrate the differences between the common and individual outage as well as the advantage of the GQF scheme.

- The DMT of the GQF scheme has been derived in the slow fading half-duplex MARC. It is shown that the GQF scheme can achieve the optimal DMT when the relay has no access to the CSI of the relay-destination link while the classic CF scheme can only achieve some part of optimal DMT with complete CSI at relay. The DMT of the GQF scheme is derived without relay-destination link CSI at the relay. It is shown that even without knowledge of relay-destination CSI, the GQF scheme achieves the same DMT, achievable by CF scheme with full knowledge of CSI.

1.3 Organization of the Dissertation

This dissertation is organized in six chapters. The topic of each chapter is briefly introduced as follows.

In Chapter 2, an extensive literature review of the relaying for static and fading multi-user channel and the diversity-multiplexing tradeoff will be presented.

The derivation of the achievable rate region based on the proposed GQF scheme for the Discrete-Memoryless HD-MARC will be given in Chapter 3. The achievable results are extended to the AWGN channel and compared with the classic CF scheme through discussion and numerical examples.

In Chapter 4, the common and individual outage probability will be derived based on the GQF scheme for the slow fading HD-MARC. The numerical examples and discussions will be given to demonstrate the advantage of using GQF scheme when the CSI of the relay-destination link is not available at relay thereafter.

When the receiver SNR increases to infinity, the achievable DMT of the GQF scheme will be derived in Chapter 5. The optimality of the achievable DMT will be provided. The comparison of the achievable DMT from different relaying schemes will be presented.

The conclusion and the contributions of this dissertation will be summarized in Chapter 6 together with the recommendations for the future work.

Chapter 2

Literature Review

In recent decades, developing practical relaying schemes that have low-cost and good performance in both static and fading channels became an interesting topic and great challenge. In this dissertation, the research work will investigate the low-cost relaying schemes for the half-duplex multi-user channels, including both discrete-memoryless channel and additive white gaussian channel, and analyze the performance in the finite slow fading channel and the asymptotical behavior when SNR increases to infinity.

2.1 Relaying

A typical relay channel consists of three communication terminals, source, destination and relay. While a typical communication channel is between one transmitter and one receiver, here a third party, Relay node, is also available to help the message or information propagation from Source to Destination.

Research on Relay channel is originated from Van Der Meulen's work [99–101]. In 1979, an extensive work on relay channel was done by Cover and El Gamal [18]. In their paper, the capacity region was established for the degraded relay channel in which the relay has a better reception compare to the destination. The fundamental approach used to show the achievablity of the degraded relay channel is the famous block Markov encoding scheme.

2.1.1 Relaying schemes

As the most important work in the relaying [18], two fundamental relaying schemes were developed, decode-and-forward (DF) and compress-and-forward (CF). While most of the research in relaying focus on DF and CF, another main relaying scheme, amplify-and-forward (AF), also became popular in the last decade as less delay occurred at relay with this scheme.

Decode-and-Forward

In DF [64, 66], the relay receives the signal from the source, tries to decode the signal to original codeword. If the relay successfully decodes, it re-encodes the message to codeword and transmits to the destination. This scheme ensures that if relay is sending data, it is reliable and error-corrected.

The DF scheme can be divided into three different strategies [48]: 1) irregular encoding/successive decoding [18]; 2) regular encoding/sliding-window decoding [10]; 3) regular encoding/backward decoding [52, 118]. The DF scheme for multiple relays with the above three strategies were also investigated in [32,51,108]. A variation of the DF scheme, dynamic decode-and-forward (DDF), was proposed by Azarian [7]. In the DDF, relay keeps receiving from the source until it is able to successfully decodes. Such scheme offers DMT optimality when the multiplexing gain is low.

Compress-and-Forward

In CF, the relay sends to destination a quantized and compressed version of its reception from the source. The destination performs successive or sequential decoding by combining the signals from both the relay and the source. The relay and destination takes the advantage of the statistical dependence between different receptions where the Wyner-Ziv source coding

[65,106,107] is applied [48]. Since the relay is not limited by decoding of the source message, CF usually offers higher achievable rates than DF when the relay is close to the destination.

The CF scheme was generalized to parallel relay channels [79] and to multiple relays [26,27]. There are also some variations of the CF schemes which were studied in recent years. The hash-and-forward (HF) was proposed by Cover and Kim [20] to find the capacity of the deterministic relay channels, where the relay now hashes its reception. The generalized hash-and-forward (GHF) was introduced by Razaghi to deal with the interference relay channel, where the relay is not constrained to minimize the distortion [70]. A quantize-map-and-forward (QMF) scheme is proposed by Avestimehr and was shown to achieve the approximate capacity of the relay channel [6]. By letting the destination perform joint decoding, the generalized compress-and-forward (GCF) is implemented to derive the achievable rates of the interference relay channel (IRC) [92]. In [57], a Noisy network coding (NNC) scheme was proposed. NNC is able to recover the rate achieved by the classic CF in a three-node relay channel and generally outperforms other CF based schemes in a multiuser channel. However, NNC requires the "long" source messages to be repeated several times which significantly increases the decoding delay. In order to overcome this drawback, a short message noisy network coding (SNNC) was introduced by Wu and Xie [104]. In [37, 49], SNNC was shown to achieve the same rate as NNC in multiple multicast sessions. A unified framework for combining both the CF and DF scheme is also studied recently in [105] for single and multiple relays channels.

Amplify-and-Forward

In AF [53, 54], the relay does not need to decode or quantize its received signals. Instead, it performs an analog operation of amplifying the received signal and send it to the destination or other nodes.

Although AF benefits from low delay and less computing, it also amplifies the noises received from relay to the destinations. To improve the performance of the AF scheme in [54], the source and the relay are allowed to transmit simultaneously in [61] and a scheme nonorthogonal amplify-and-forward (NAF) is developed by Azarian et al [7]. A slotted-AF was investigated in [109] and is shown to outperform the NAF. The multiple-access amplify-and-forward (MAF) scheme is used in deriving the DMT for the MARC [12, 13].

2.1.2 Relaying for point-to-point channel

As there are numerous works on the relaying for the point-to-point channel, this subsection only highlights a few important ones to this dissertation.

Cover and El Gamal [18] have done the pioneering and most important theoretical analysis of the relay channel. In 2007, Chong et al [15] improved Cover and El Gamal's work on the general coding strategy in relay channel with two new schemes that make use of the sequential backward (SeqBack) decoding and simultaneous backward (SimBack) decoding. The new strategies showed higher achievable rate of the Gaussian Relay Channel under some conditions. A deterministic channel model and the QMF scheme were proposed by [6]. It is shown that such scheme achieves the cut-set upper bound to within a gap which is independent of the channel parameters.

Among numerous work through decades, [53, 54] from Laneman et al have made great influences. In their work, the famous AF and DF that uses the cooperation with relay to combat the multi-path fading were developed. It is because of the developed scheme exploited the space diversity that the outage performance of this relay network has been significantly improved. Mitran et al [60] proposed variable time fraction for the space-time relay channel and showed a significant enhancement in terms of the outage probability. Tooher and Soleymani [93–96] further analyzed how to design practical block codes or convolution codes with the variable time-fraction scheme.

2.1.3 Half-duplex relaying

In contrast to the conventional full-duplex (FD) relaying, the half-duplex (HD) relaying plays an important role in practical applications. The relay in this mode does not receive and transmit at the same time or at the same frequency. It has lower cost and hence was named "Cheap" relay channel by Khojastepour [42].

In Laneman's work [53, 54], the AF and DF schemes are implemented with the HD relay. The capacity bounds and the power allocation were presented in [36] for the wireless relay channels. The aforementioned DDF scheme [7] is also built on the HD relaying. Kim et al [43, 46] studied the DF relaying with channel state feedback and CF relaying without WZ coding for the HD-RC.

Besides being applied to conventional three node relay channel, HD relaying is also studied for multi-user channels. For the MIMO shared relay channel with HD constraint, Khojastepour and Wang investigated the cut-set upper bounds and two achievable rates which based on dirty paper [17] and superposition coding [41]. Feng and Sanhu [23] discussed half duplex relaying in the so called diamond relay channel where the channel consists a source, a destination and two relays. The achievable rates for the IRC with and without an out of band relay are derived in [40, 75].

2.1.4 Relaying for multi-user channels

The three node relay channel is the basic model that captures the effect that relaying brings to a communication channel. For the multi-user channels, relaying also provides great benefit in terms of the increased reliability and/or higher throughput.

When the channel consists of a relay, a destination and more than one source, such a channel is named multiple-access relay channel (MARC) [48, 50, 78]. If there are more than one destinations and each destination needs to recover all the messages from the sources, the channel become a compound multiple access channel with a relay (cMACr) [31]. If the destinations are not expected to decode all the messages, the channel is called IRC [58, 72]. In other words, the IRC is an Interference Channel [9, 16, 47] added a relay to help the information prorogation.

The cMACr can be treated as a special case of IRC. The achievable rates based on DF and CF are derived and compared by Gunduz [31]. The IRC has attracted many research interests after Sahin and Erkip's paper which obtained the achievable rates based on the superposition and block Markov coding DF scheme. Based on different ways that relay forwards the messages, signal relaying and interference forwarding are the two strategies used for IRC [76]. In signal relaying, the relay sends to each destination their expected message. The interference forwarding means the relay sends the interfered message to the destination which helps the successful interference cancellation (SIC) at that decoder [21,22]. There are also interesting works on different configurations of the IRC, i.e. 1) the cognitive relay [73, 74, 84, 117] where the relay has complete knowledge of the sources; 2) infrastructure relay [82, 90] where different frequency bands or time slots are available at relay; 3) potent relay [89, 91, 92] where infinity power is assumed at the transmitter of relay.

Relaying for MARC

The Multiple-access Relay Channel (MARC) is a model for networks in which a finite number of sources multicast information to a destination through relay. These networks are widely used in sensor networks and ad hoc networks.

As an extension of the relay channel, the MARC is introduced by Kramer and van Wijngaarden and its upper bound is derived using cut-sets [50]. Later, Kramer et al studied the possible multicast strategies for general relay networks in detail and obtained achievable rates for Gaussian case [51]. The paper [48] summarized the previous work and derived several capacity bounds for a class of MARC.

For the FD-MARC, the achievable rates and upper bounds based on both DF and CF schemes are also obtained by Gunduz [31]. The superposition coding and raptor coding are designed by Gong et al for the HD-MARC based on DF scheme [28, 29]. In HD-MARC, if the sources keep silent while relay transmitting, this channel is referred as orthogonal MARC (OMARC). Hatefi et al studied joint channel network coding for this OMARC in [33, 34]. The AF scheme is modified and improved for the MARC [12,13] . As an extension of the AF in MARC, an analog network coding mappings [112] is also studied by Yao and Skoglund.

For the fading MARC, achievable Rates and opportunistic scheduling is introduced in [77]. Based on an adaptive DF scheme, the outage probability is reduced for the slow fading MARC [102]. The DF scheme is also investigated in over the slow fading MARC in [7]. The CF based schemes were shown to outperform the DF based schemes when the relay has the complete CSI of the channel [46, 114]. However, the perfect CSI at relay is generally too ideal. When the critical delay constraint exists in the wireless channels, the relay may not be able to obtain the CSI accurately. The CF-based schemes are not efficient in a slow fading environment when the relay has no access to the complete CSI [111].

As explained in [62], each user in a MAC may have different QoS requirement such that only use the common outage probability [56] to measure the performance has its limitations. Similarly, for a two user MARC, the destination failing to decode one of the sources messages may not affect the other user's QoS requirement (message decoded successfully by the destination). The individual outage performance needs to be investigated.

2.2 Diversity-Multiplexing Tradeoff

2.2.1 DMT

For MIMO systems and relay channels, the DMT introduced by Zheng and Tse [119] is a fundamental tool between the two measures, reliability and information rate. The reliability depends on the diversity [68] while the information rate relies on the degrees of freedom [24, 88] available for communication. The DMT characterizes the tradeoff between each type of gain (diversity or degrees of freedom) offered by any coding scheme for a given MIMO channel. A scheme is able to have a multiplexing gain r and a diversity gain d if the information rate of such scheme increases as $rlogSNR$ and the average probability of error decays like $1/SNR^d$ [119].

The outage probability and expected sum rate are often used as the measure for the slow fading channels at finite SNR. The DMT quantifies the performance at asymptotically high SNR. It is a powerful tool because it is not only easy enough to derive but also strong enough to offer insightful comparisons for different schemes [114]. Though the general capacity is still unknown for various relay channels, some relaying schemes are already proved to be DMT optimal.

The DMT for the multiple access channel (MAC) is derived in [98]. For the interference channel, the DMTs are derived for the scenarios with and without transmitter CSI (Tx-CSI) in [2, 3]. The DMT of the relay channels are first studied in [7, 54] with the AF and DF schemes. To improve the DMT performance, the NAF and DDF are developed by [7]. The NAF scheme is an AF scheme where the source and relay are not transmitting in the orthogonal channel. The DDF scheme is an adaptive DF scheme where the relay listens to the channel until it decodes the source's message. The optimality of non-dynamic DF relaying over a general three-node Nakagami fading channel is investigated by Kim and Skoglund [45].

Recently, in [38], the DMT of the MIMO half-duplex relay [55] is characterized for arbitrary number of antennas at each node and where the relay is capable of operating dynamically, i.e., it can switch between receive and transmit modes at a channel dependent time. The joint eigenvalue distribution of three mutually correlated random Wishart matrices is the key mathematical tool that is applied. Besides, the QMF scheme [5, 6] is also adopted and shown to achieve the optimal tradeoff for the multiple antenna channel without the global CSI at the relay [67].

THe DMT of the Gaussian MIMO ICR was investigated by Maric and Goldsmith in [59] where all links have same SNR scaling behaviour. A DMT upper bound based on cut-set theorem and an achievable DMT based on CF scheme are studied for the Interference Relay

Channel (IRC). The generalized degrees of freedom(GDoF) of the IRC is derived in [11]. The relay is shown to help the IC achieve a higher GDoF. In [115,116], Zahavi et al derived the achievable DMT of the DF and CF scheme for the full-duplex (FD) IRC in which the channel gain of different links are not scaling with same value.

2.2.2 DMT for MARC

For the MARC, researchers have developed and modified several relaying schemes to improve the DMT performance, i.e. dynamic decode-and-forward (DDF), multiaccess amplify-and-forward (MAF) and compress-and-forward (CF), have been characterized in [7,12,114].

As shown in [114], the CF scheme has its advantages in terms of sustaining to multiple antennas case comparing to the DDF and MAF schemes. Besides, the CF scheme also achieves the optimal DMT upper bound when the multiplexing gain is high enough. To achieve the optimal DMT, the CF scheme needs to have two assumptions: 1) using Wyner-Ziv coding and 2) the relay has perfect channel state information (CSI). Without WZ coding, the DMT of the CF scheme is obtained and compared in [46]. In practical wireless communication systems, having the CSI of relay-destination link at relay is generally too ideal. When the critical delay constraint exists in the wireless channels, the relay may not be able to obtain the CSI accurately. Without perfect CSI, the QF scheme has been shown to achieve the optimal DMT for the three node HD-RC. Though there is multi-user interference at relay in the HD-MARC, it is of great interest to derive the DMT of the GQF scheme and verify its optimality.

Chapter 3

Generalized Quantize-and-Forward Scheme in Static Channels

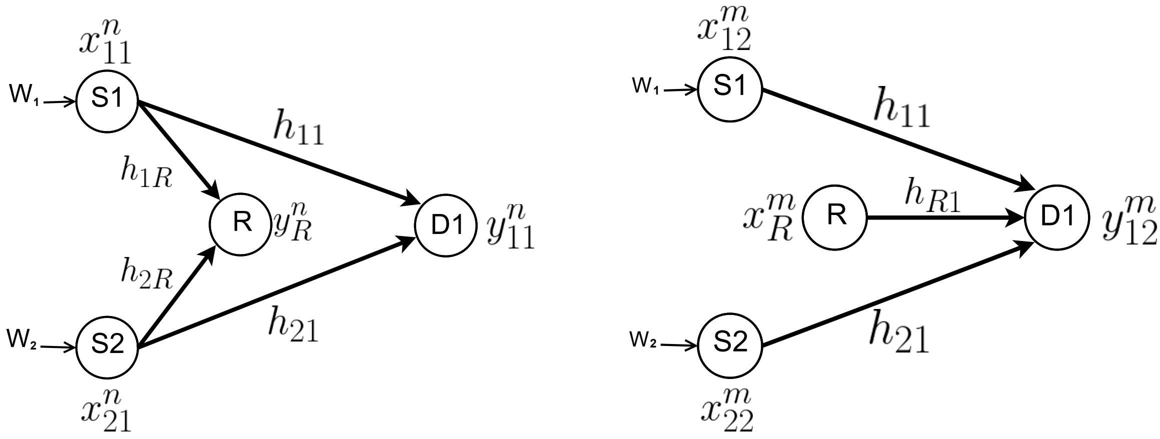
This chapter focuses on studying the half-duplex (HD) relaying in the Multiple Access Relay Channel (MARC) and the Compound Multiple Access Channel with a Relay (cMACr). A generalized Quantize-and-Forward (GQF) has been proposed to establish the achievable rate regions. Such scheme is developed based on the variation of the Quantize-and-Forward (QF) scheme and single block with two slots coding structure. The results in this work can also be considered as a significant extension of the achievable rate region of Half-Duplex Relay Channel (HDRC). Furthermore, the rate regions based on GQF scheme are extended to the Gaussian channel case. The scheme performance is shown through some numerical examples.

3.1 Problem Statement

By cooperating with the transmitter(s), relaying is able to benefit a conventional point-to-point communication channel [18], a multiple access channel [48] and a compound multiple access channel [31]. The fundamental relaying schemes are decode-and-forward (DF) and compress-and-forward (CF). Comparing to the DF based schemes, CF based schemes are not limited by the decoding capability of the relay as studied in [18, 31, 48], etc. Different variations of the CF based schemes have been investigated in [6, 20, 57, 70, 104] for the full-duplex channels.

In [57], a Noisy network coding (NNC) scheme was proposed. NNC is able to recover the rate achieved by the classic CF in a three-node relay channel and generally outperforms other CF based schemes in a multiuser channel. However, NNC requires the "long" source messages to be repeated several times which significantly increases the decoding delay. In order to overcome this drawback, a short message noisy network coding (SNNC) was introduced by Wu and Xie [104]. In [37,49], SNNC was shown to achieve the same rate as NNC in multiple multicast sessions. Usually, the SNNC is applied in the full-duplex relay channel.

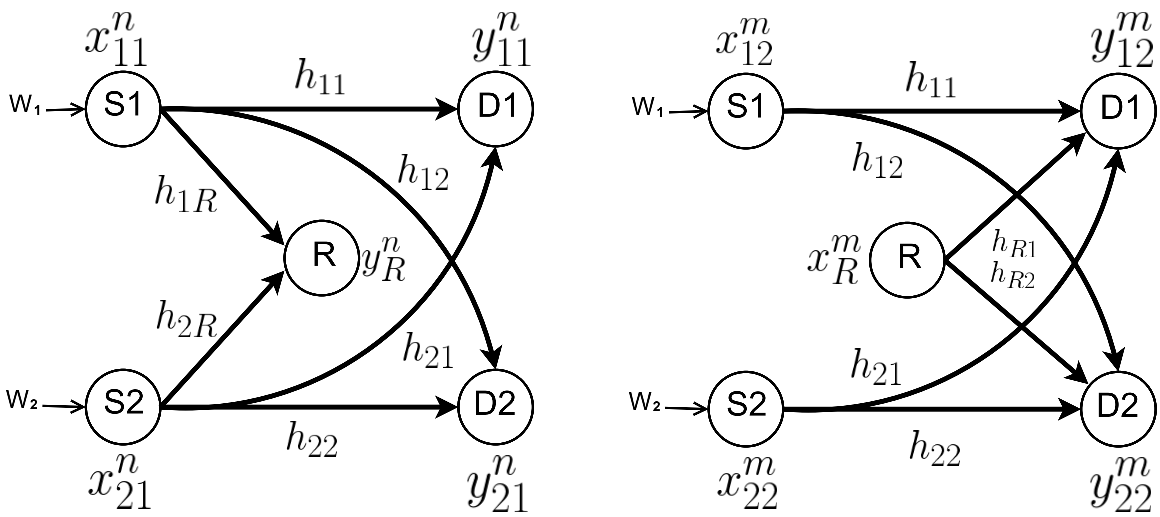
Motivated by the practical constraint that relay cannot transmit and receive simultaneously in wireless communications [54], a quantize-and-forward (QF) scheme, originating from Noisy Network Coding (NNC) [57], has been studied for a fading half-duplex relay channel (HDRC) in [111]. A single block and two slots coding based QF scheme for the H-DRC has been proposed in [111] to derive the achievable rates. In this chapter, the "cheap" half-duplex relay [42] is also considered. Specifically, the achievable rate regions for the half-duplex MARC and cMACr, as shown in Fig.3.1 and Fig.3.2, respectively, are investigated based on a variation of the QF scheme, namely the GQF scheme. Compared with the QF scheme, the proposed GQF scheme not only adopts the single block two slots coding structure but also takes into account the effect of the co-existing interfered message signal at relay. Comparing with the classic CF based schemes, the proposed GQF scheme is able to simplify the operation at relay ("cheaper" relay) while keeping the advantage of the CF based schemes. The relay in the CF scheme usually takes two steps signal processing: 1) compresses (quantizes) its received signal; 2) applies Wyner-Ziv binning of the compressed signal and send the bin index later on. However, the GQF scheme only requires a simplified relay in the sense that no Wyner-Ziv binning is necessary. The relay only needs to send the quantization index. The GQF scheme simplifies the relay encoding and can be implemented in any situation where a low-cost half-duplex relay is needed.



(a) MARC Slot-1

(b) MARC Slot-2

Figure 3.1: Message flow of the Half-Duplex Multiple Access Relay Channel.



(a) Slot-1

(b) Slot-2

Figure 3.2: Message flow of the cMACr.

The performance comparison between the GQF scheme and the CF scheme is also discussed in this chapter. However, in order to fit in the half-duplex MARC and cMACr, the classic CF scheme has been modified. Furthermore, the rate regions based on GQF scheme are extended from the discrete memoryless channel to the Gaussian channel. The scheme performance is shown through some numerical examples.

3.2 System Model

A HD-MARC and a HD-cMACr are considered in this chapter. Since a cMACr naturally reduces to a MARC if the second destination is not present, only the description for the HD-cMACr will be shown in this section.

3.2.1 Half-Duplex cMACr

In Fig. 3.2, a two-user cMACr consists of two sources S_1, S_2 and two destinations D_1 and D_2. Relay R helps the information propagation from sources to destinations by cooperating with the sources. Relay operates in the half-duplex mode, i.e., R is either receiving signals from the source nodes (S_1 and S_2) or transmitting to the destinations (D_1 and D_2).

Assume that each communication block length is l channel uses and divided into two slots. The lengths of the first and the second slot are n and m channel uses, respectively. In the first slot, both S_1 and S_2 broadcast their messages to R , D_1 and D_2. In the second slot, S_1 and S_2 keep transmitting to D_1 and D_2, while R cooperates by transmitting to both destinations as well. Denote x_{i1}^n and x_{i2}^m, as the transmitted sequences by S_i in the first and second slot correspondingly, and x_R^m as the transmitted sequence by R in the second slot, where $x_{ij}^k = [x_{ij,1}, x_{ij,2}, \cdots, x_{ij,k}]$ and $x_R^k = [x_{R,1}, x_{R,2}, \cdots, x_{R,k}]$ for $i, j \in \{1, 2\}$ and $k \in \{n, m\}$. The received sequences at the destination D_i in the first and the second slots are denoted as y_{i1}^n and y_{i2}^m for $i \in \{1, 2\}$, respectively. In the first slot, the received sequence at the relay is y_R^n.

3.2.2 Static HD-cMACr

We consider two channel models. In the discrete memoryless channel case, the source output takes discrete values. In the Gaussian channel case, the source outputs are continuous values generated according to a Gaussian distribution. In both cases, the sources are memoryless, i.e., the value of the source output at any given time is independent of the values at other times. These two static channel models are described as follows:

Discrete Memoryless Channel

In the discrete memoryless HD-MARC, all random variables take values from discrete alphabets. The input random variables are memoryless and are denoted by $X_{11}, X_{12}, X_{22}, X_{22}$, and X_R. Each source S_i, $i \in \{1, 2\}$ chooses a message W_i from a message set $\mathcal{W}_i = \{1, 2, \ldots, 2^{lR_i}\}$, then encodes this message into a length n codeword with an encoding function $f_{i1}(W_i) = X_{i1}^n$ and a length m codeword with an encoding function $f_{i2}(W_i) = X_{i2}^m$, finally sends these two codewords in the corresponding slots. Relay R employs an encoding function based on its reception Y_R^n in the first slot.

Each destination uses a decoding function $g_i(Y_{i1}^n, Y_{i2}^m) = (\hat{W}_1, \hat{W}_2)$ that jointly decodes messages from the receptions in both slots. The channel transition probabilities can be represented by

$$p_{Y_R^n Y_{11}^n Y_{21}^n | X_{11}^n X_{21}^n}(y_R^n, y_{11}^n, y_{21}^n | x_{11}^n, x_{21}^n) = \prod_{i=1}^{n} p_{Y_R Y_{11} Y_{21} | X_{11} X_{21}}(y_{R,i}, y_{11,i}, y_{21,i} | x_{11,i} x_{21,i}) \tag{3.1}$$

and

$$p_{Y_{12}^m Y_{22}^m | X_{12}^m X_{22}^m X_R^m}(y_{12}^m, y_{22}^m | x_{12}^m, x_{22}^m, x_R^m) = \prod_{i=1}^{m} p_{Y_{12} Y_{22} | X_{12} X_{22} X_R}(y_{12,i}, y_{22,i} | x_{12,i} x_{22,i} x_{R,i}). \tag{3.2}$$

for both slots. A rate pair (R_1, R_2) is called achievable if there exists a message set, together with the encoding and decoding functions stated before such that $Pr(\hat{W}_1 \neq W_1 \cup \hat{W}_2 \neq W_2) \to 0$ when $l \to \infty$.

Gaussian Channel

In this case, the input random variable at each transmitter is chosen according to Gaussian distribution. The noise at each receiver is an additive white Gaussian (AWGN) random variable. The signal at each receiver is modeled as the faded transmitted signal corrupted by an AWGN component. For the Gaussian HD-cMACr in Fig. 3.2, the channel transition probabilities are specified in the below:

$$\begin{aligned} y_{i1}^n &= h_{1i}x_{11}^n + h_{2i}x_{21}^n + z_{i1}^n; \\ y_R^n &= h_{1R}x_{11}^n + h_{2R}x_{21}^n + z_R^n; \\ y_{i2}^m &= h_{1i}x_{12}^m + h_{2i}x_{22}^m + h_{Ri}x_R^m + z_{i2}^m, \end{aligned}$$

where h_{ij} denotes the channel coefficient between transmitter i and receiver j for $i \in \{1, 2, R\}$ and $j \in \{1, 2, R\}$, and is a real constant. The noise sequences of z_{i1}^n, z_{i2}^m and z_R^n for $i \in \{1, 2\}$ are generated independently and identically with Gaussian distributions with zero means and unit variances.

In order to clarify the CF based relaying schemes, define the auxiliary random variable $\hat{Y}_R$ as the quantized signal of relay's reception Y_R, i.e., $\hat{Y}_R = Y_R + Z_Q$, where Z_Q is the quantization noise and is an independent Gaussian random variable with zero mean and variance σ_Q^2.

In the first slot, the transmitters have power constraints over the transmitted sequences as

$$\frac{1}{n}\sum_{k=1}^{n} |x_{i1,k}| \leq P_{i1}, \quad \text{for} \ \ i \in \{1, 2\},$$

where $|x|$ shows the absolute value of x. In the second slot, the transmitters have power constraints

$$\frac{1}{m}\sum_{k=1}^{m} |x_{i2,k}| \leq P_{i2}, \quad \text{for} \ \ i \in \{1, 2\}$$

and

$$\frac{1}{m}\sum_{k=1}^{m} |x_R| \leq P_R.$$

3.3 Achievable Rates in Discrete Memoryless Channel

In this section, the achievable rate regions based on the GQF scheme for the discrete memoryless half-duplex MARC and cMACr are presented first. As a reference, the achievable rates based on a modified CF scheme is shown in the second subsection. In the last part of this section, it is shown that the achievable rate regions for the three node HD-RC [111] can be treated as a special case of the result for our five node cMACr.

3.3.1 Achievable Rate Region Based on GQF Scheme

As an essential variation of the classic CF scheme, the QF scheme originates from the NNF [57] and has been investigated in both static and fading relay channel [111]. The GQF scheme generates the QF scheme to multi-user channel and takes into account the effect that relay and destination receives interfered signals from sources. In GQF, relay quantizes its observation Y_R to obtain $\hat{Y}_R$ after the first slot, and then sends the quantization index $u \in \mathcal{U} = \{1, 2, \ldots, 2^{lR_u}\}$ in the second slot with X_R. Unlike the classic CF, no Wyner-Ziv binning is applied at the relay, which simplifies the relay operation. At the destination, decoding is also different in the sense that joint-decoding of the messages from both slots without explicitly decoding the quantization index is performed in GQF scheme.

The achievable rates based on the GQF scheme for the HD-MARC and HD-cMACr will be shown in the following two parts:

Achievable Rate Region for discrete memoryless HD-MARC

In the two-user HD-MARC (shown in Fig. 3.1), there is only one destination D_1. The decoding is finished after D_1 finds both sources messages W_1 and W_2. The following theorem describes the achievable rate region for the discrete memoryless HD-MARC:

Theorem 3.1 *The following rate regions are achievable over discrete memoryless HD-MARC* *based on the GQF scheme:*

$$R_i < min\{a_1(i), b_1(i)\} \tag{3.3}$$

$$R_1 + R_2 < min\{c_1, d_1\} \tag{3.4}$$

where $\beta = n/l$ is fixed,

$$\begin{aligned}
a_k(i) &= \beta I(X_{i1}; X_{j1}, Y_{k1}, \hat{Y}_R) + (1-\beta) I(X_{i2}; X_{j2}, X_R, Y_{k2}) \\
b_k(i) &= \beta[I(X_{i1}; X_{j1}, Y_{k1}) - I(\hat{Y}_R; Y_R | X_{i1}, X_{j1}, Y_{k1})] + (1-\beta) I(X_{i2}, X_R; X_{j2}, Y_{k2}) \\
c_k &= \beta I(X_{11}, X_{21}; Y_{k1}, \hat{Y}_R) + (1-\beta) I(X_{12}, X_{22}; X_R, Y_{k2}) \\
d_k &= \beta[I(X_{11}, X_{21}, \hat{Y}_R; Y_{k1}) + I(X_{11}, X_{21}; \hat{Y}_R) - I(Y_R; \hat{Y}_R)] \\
&\quad +(1-\beta) I(X_{12}, X_{22}, X_R; Y_{k2}),
\end{aligned} \tag{3.5}$$

$i, j, k \in \{1, 2\}$ and $i \neq j$, for all input distributions

$$p(x_{11})p(x_{21})p(x_{12})p(x_{22})p(x_R)p(\hat{y}_R | y_R). \tag{3.6}$$

Proof: Since a two-user HD-MARC can be treated as a reduced case of a two-user HD-cMACr, the above results can be obtained by letting $Y_{21} = Y_{22} = \phi$ at D_2 in the proof for the Theorem 3.2. ■

Achievable Rate Region for discrete memoryless HD-cMACr

In the two-user HD-cMACr (shown in Fig. 3.2), the decoding is done by each of the destinations D_i, $i \in \{1, 2\}$, decodes both messages W_1 and W_2. Therefore, the overall achievable rates takes the minimum of rates achieved by the two destinations. The following theorem describes the achievable rates:

Theorem 3.2 *The following rate regions are achievable over discrete memoryless HD-cMACr based on the GQF scheme:*

$$R_i < min\{a(i), b(i)\} \tag{3.7}$$

$$R_1 + R_2 < min\{c, d\} \tag{3.8}$$

where $\beta = n/l$ is fixed, $a(i) = min\{a_1(i), a_2(i)\}$, $b(i) = min\{b_1(i), b_2(i)\}$, $c = min\{c_1, c_2\}$, $d = min\{d_1, d_2\}$, $a_k(i)$, $b_k(i)$, c_k and d_k were defined in (3.5), $i, j, k \in \{1, 2\}$, $i \neq j$ and $k \in \{1, 2\}$, for all input distributions of (3.6).

Proof: The proof for the achievable rate regions includes the following parts: Codebook generation, Encoding, Decoding and Probability of Error Analysis.

Assume each source message W_i, $i \in \{1, 2\}$ is independent and uniformly distributed in its message set $\mathcal{W}_i = [1 : 2^{lR_i}]$.

Codebook Generation

Assume that the joint pmf factors as in

$$p(x_{11})p(x_{21})p(x_{12})p(x_{22})p(x_R)p(\hat{y}_R|y_R)p(y_{11}, y_{21}, y_R|x_{11}, x_{12})p(y_{21}, y_{22}|x_{12}, x_{22}, x_R).$$

Fix any input distributions from (3.6)

$$p(x_{11})p(x_{21})p(x_{12})p(x_{22})p(x_R)p(\hat{y}_R|y_R).$$

Randomly and independently generate

- 2^{lR_1} codewords $x_{11}^n(w_1)$, $w_1 \in \mathcal{W}_1$, each according to $\prod_{i=1}^n p_{X_{11}}(x_{11,i}(w_1))$;
- 2^{lR_2} codewords $x_{21}^n(w_2)$, $w_2 \in \mathcal{W}_2$, each according to $\prod_{i=1}^n p_{X_{21}}(x_{21,i}(w_2))$;
- 2^{lR_1} codewords $x_{12}^m(w_1)$, $w_1 \in \mathcal{W}_1$, each according to $\prod_{i=1}^m p_{X_{12}}(x_{12,i}(w_1))$;
- 2^{lR_2} codewords $x_{22}^m(w_2)$, $w_2 \in \mathcal{W}_2$, each according to $\prod_{i=1}^m p_{X_{22}}(x_{22,i}(w_2))$;
- 2^{lR_U} codewords $x_R^m(u)$, $u \in \mathcal{U} = \{1, 2, \ldots 2^{lR_U}\}$, each according to $\prod_{i=1}^m p_{X_R}(x_{R,i}(u))$.

Calculate the marginal distribution

$$p(\hat{y}_R) = \sum_{x_{11}\in\mathcal{X}, x_{21}\in\mathcal{X}, y_{11}\in\mathcal{Y}, y_{21}\in\mathcal{Y}, y_R\in\mathcal{Y_R}} p(\hat{y}_R|y_R)p(y_R, y_{11}, y_{21}|x_{11}, x_{21})p(x_{11})p(x_{21}).$$

Randomly and independently generate

- 2^{lR_U} codewords $\hat{y}_R^n(u)$, each according to $\prod_{i=1}^n p_{\hat{Y}_R}(\hat{y}_{R,i}(u))$;

Encoding

To send message w_i, the source node S_i transmits $x_{i1}^n(w_i)$ in the first slot and $x_{i2}^m(w_i)$ in the second slot, where $i = 1, 2$. Let $\epsilon' \in (0, 1)$. After receiving y_R^n at the end of the first slot, the relay tries to find a unique $u \in \mathcal{U}$ such that

$$(y_R^n, \hat{y}_R^n(u)) \in \mathcal{T}_{\epsilon'}^n(Y_R, \hat{Y}_R)$$

where $\mathcal{T}_\epsilon^n(Y_R\hat{Y}_R)$ is the ϵ-strongly typical set as defined in [57]. If there are more than one such u, randomly choose one in $\mathcal{U}$. The relay then sends $x_R^m(u)$ in the second slot.

Decoding Destinations, D_1 and D_2, start decoding messages after second slot transmission finishes. Let $\epsilon' < \epsilon < 1$. After receiving in both slots, D_1 and D_2 tries to find a unique pair of the messages $\hat{w}_1 \in \mathcal{W}_1$ and $\hat{w}_2 \in \mathcal{W}_2$ such that

$$(x_{11}^n(\hat{w}_1), x_{21}^n(\hat{w}_2), y_{11}^n, \hat{y}_R^n(u)) \in \mathcal{T}_\epsilon^n(X_{11}X_{21}Y_{11}\hat{Y}_R) \tag{3.9}$$

$$(x_{12}^m(\hat{w}_1), x_{22}^m(\hat{w}_2), x_R^m(u), y_{12}^m) \in \mathcal{T}_\epsilon^m(X_{12}X_{22}X_RY_{12}) \tag{3.10}$$

and

$$(x_{11}^n(\hat{w}_1), x_{21}^n(\hat{w}_2), y_{21}^n, \hat{y}_R^n(u)) \in \mathcal{T}_\epsilon^n(X_{11}X_{21}Y_{21}\hat{Y}_R) \tag{3.11}$$

$$(x_{12}^m(\hat{w}_1), x_{22}^m(\hat{w}_2), x_R^m(u), y_{22}^m) \in \mathcal{T}_\epsilon^m(X_{12}X_{22}X_RY_{22}) \tag{3.12}$$

for some $u \in \mathcal{U}$.

Probability of Error Analysis

Let W_i denote the messages sent from source node S_i for $i = 1, 2$. U represents the index chosen by the relay R. The probability of error averaged over W_1,W_2, U over all possible codebooks is defined as

$$Pr(\epsilon) = Pr(\hat{W}_1 \neq W_1 \cup \hat{W}_2 \neq W_2)$$

$$
\begin{aligned}
&= \sum_{w_1 \in \mathcal{W}_1, w_2 \in \mathcal{W}_2} Pr(\hat{W}_1 \neq w_1 \cup \hat{W}_2 \neq w_2 | W_1 = w_1, W_2 = w_2) p(w_1, w_2) \\
&= Pr(\hat{W}_1 \neq 1 \cup \hat{W}_2 \neq 1 | W_1 = 1, W_2 = 1)
\end{aligned}
$$

This is based on the symmetry of the codebook construction and the fact that the messages W_1 and W_2 are chosen uniformly from $\mathcal{W}_1$ and $\mathcal{W}_2$, the overall probability of error is equal to the probability of error when $W_1 = 1$ and $W_2 = 1$ were selected as the message indices.

Define the following events:

$$
\begin{aligned}
\mathcal{E}_0 &:= \{((Y_R^n, \hat{Y}_R^n(u)) \notin \mathcal{T}_{\epsilon'}^n(Y_R \hat{Y}_R)), \text{for all } u\} \\
\mathcal{E}_{1,(w_1,w_2)} &:= \{(X_{11}^n(w_1), X_{21}^n(w_2), Y_{11}^n, \hat{Y}_R^n(u)) \in \mathcal{T}_\epsilon^n(X_{11} X_{21} Y_{11} \hat{Y}_R) \text{ and} \\
&\qquad (X_{12}^m(w_1), X_{22}^m(w_2), X_R^m(u), Y_{12}^m) \in \mathcal{T}_\epsilon^m(X_{11} X_{21} X_R Y_{12}) \text{ for some } u\} \\
\mathcal{E}_{2,(w_1,w_2)} &:= \{(X_{11}^n(w_1), X_{21}^n(w_2), Y_{21}^n, \hat{Y}_R^n(u)) \in \mathcal{T}_\epsilon^n(X_{11} X_{21} Y_{21} \hat{Y}_R) \text{ and} \\
&\qquad (X_{12}^m(w_1), X_{22}^m(w_2), X_R^m(u), Y_{22}^m) \in \mathcal{T}_\epsilon^m(X_{11} X_{21} X_R Y_{22}) \text{ for some } u\}.
\end{aligned}
$$

Then the probability of error can be rewritten as

$$
\begin{aligned}
Pr(\mathcal{E}) &= Pr((\mathcal{E}_{1,(1,1)} \cap \mathcal{E}_{2,(1,1)})^c \cup_{(w_1,w_2)\in\mathcal{A}} \mathcal{E}_{1,(w_1,w_2)} \cup_{(w_1,w_2)\in\mathcal{A}} \mathcal{E}_{2,(w_1,w_2)} | W_1 = 1, W_2 = 1) \\
&\leq Pr((\mathcal{E}_{1,(1,1)} \cap \mathcal{E}_{2,(1,1)})^c | W_1 = 1, W_2 = 1) + Pr(\cup_{(w_1,w_2)\in\mathcal{A}} \mathcal{E}_{1,(w_1,w_2)} | W_1 = 1, W_2 = 1) \\
&\quad + Pr(\cup_{(w_1,w_2)\in\mathcal{A}} \mathcal{E}_{2,(w_1,w_2)} | W_1 = 1, W_2 = 1) \\
&= Pr((\mathcal{E}_{1,(1,1)} \cap \mathcal{E}_{2,(1,1)})^c \cap \mathcal{E}_0 | W_1 = 1, W_2 = 1) \\
&\quad + Pr((\mathcal{E}_{1,(1,1)} \cap \mathcal{E}_{2,(1,1)})^c \cap \mathcal{E}_0^c | W_1 = 1, W_2 = 1) \\
&\quad + Pr(\cup_{(w_1,w_2)\in\mathcal{A}} \mathcal{E}_{1,(w_1,w_2)} | W_1 = 1, W_2 = 1) + Pr(\cup_{(w_1,w_2)\in\mathcal{A}} \mathcal{E}_{2,(w_1,w_2)} | W_1 = 1, W_2 = 1) \\
&\leq Pr(\mathcal{E}_0 | W_1 = 1, W_2 = 1) + Pr((\mathcal{E}_{1,(1,1)} \cap \mathcal{E}_{2,(1,1)})^c \cap \mathcal{E}_0^c | W_1 = 1, W_2 = 1) \\
&\quad + Pr(\cup_{(w_1,w_2)\in\mathcal{A}} \mathcal{E}_{1,(w_1,w_2)} | W_1 = 1, W_2 = 1) \\
&\quad + Pr(\cup_{(w_1,w_2)\in\mathcal{A}} \mathcal{E}_{2,(w_1,w_2)} | W_1 = 1, W_2 = 1)
\end{aligned}
$$

where $\mathcal{A} := \{(w_1, w_2) \in \mathcal{W}_1 \times \mathcal{W}_2 : (w_1, w_2) \neq (1, 1)\}$. Assume β is fixed, then by covering lemma [1], $Pr(\mathcal{E}_0 | W_1 = 1, W_2 = 1) \to 0$ when $l \to \infty$, if

$$
R_U > \beta I(Y_R, \hat{Y}_R) + \delta(\epsilon'), \tag{3.13}
$$

where $\delta(\epsilon') \to 0$ as $\epsilon' \to 0$. By the conditional typicality lemma [1], $Pr((\mathcal{E}_{1,(1,1)} \cap \mathcal{E}_{2,(1,1)})^c \cap \mathcal{E}_0^c | W_1 = 1, W_2 = 1) \to 0$ as $l \to \infty$.

Note that we only need to show the error analysis for the term $Pr(\cup_{(w_1,w_2)\in\mathcal{A}} \mathcal{E}_{1,(w_1,w_2)} | W_1 = 1, W_2 = 1)$. The result for the term $Pr(\cup_{(w_1,w_2)\in\mathcal{A}} \mathcal{E}_{2,(w_1,w_2)} | W_1 = 1, W_2 = 1)$ can be obtained in a similar fashion.

In order to find the bound of the term $Pr(\cup_{(w_1,w_2)\in\mathcal{A}} \mathcal{E}_{1,(w_1,w_2)} | W_1 = 1, W_2 = 1)$, we define the following events:

$$\mathcal{B}_{11}(w_1, w_2, u) := \{(X_{11}^n(w_1), X_{21}^n(w_2), Y_{11}^n, \hat{Y}_R^n(u)) \in \mathcal{T}_\epsilon^n(X_{11}X_{21}Y_{11}\hat{Y}_R)\}$$
$$\mathcal{B}_{12}(w_1, w_2, u) := \{(X_{12}^m(w_1), X_{22}^m(w_2), X_R^m(u), Y_{12}^m) \in \mathcal{T}_\epsilon^m(X_{11}X_{21}X_RY_{12})\}$$

For simple exposure and convenience, we use $p(u') := p_{U|W_1,W_2}(u'|W_1 = 1, W_2 = 1)$ for the following analysis. The error probability term $Pr(\cup_{(w_1,w_2)\in\mathcal{A}} \mathcal{E}_{1,(w_1,w_2)} | W_1 = 1, W_2 = 1)$ can be written as

$$\begin{aligned}
& Pr(\cup_{(w_1,w_2)\in\mathcal{A}} \mathcal{E}_{1,(w_1,w_2)} | W_1 = 1, W_2 = 1) \\
=& \sum_{u'\in\mathcal{U}} Pr(\cup_{(w_1,w_2)\in\mathcal{A}} \mathcal{E}_{1,(w_1,w_2)} | W_1 = 1, W_2 = 1, U = u') p(u') \\
=& \sum_{u'\in\mathcal{U}} p(u') Pr(\cup_{(w_1,w_2)\in\mathcal{A}} \cup_{u\in\mathcal{U}} (\mathcal{B}_{11}(w_1, w_2, u) \cap \mathcal{B}_{12}(w_1, w_2, u)) | W_1 = 1, W_2 = 1, U = u') \\
\le& \sum_{u'\in\mathcal{U}} p(u') \sum_{(w_1,w_2)\in\mathcal{A}} \sum_{u\in\mathcal{U}} Pr(\mathcal{B}_{11}(w_1, w_2, u) \cap \mathcal{B}_{12}(w_1, w_2, u) | W_1 = 1, W_2 = 1, U = u') \\
=& \sum_{u'\in\mathcal{U}} p(u') \sum_{(w_1,w_2)\in\mathcal{A}} \sum_{u\in\mathcal{U}} Pr(\mathcal{B}_{11}(w_1, w_2, u) | W_1 = 1, W_2 = 1, U = u') \\
& \qquad \times Pr(\mathcal{B}_{12}(w_1, w_2, u) | W_1 = 1, W_2 = 1, U = u')
\end{aligned} \tag{3.14}$$

Notice that the last equation above is based on the fact that $Pr(\mathcal{B}_{11}(w_1, w_2, u) | W_1 = 1, W_2 = 1, U = u')$ is independent of $Pr(\mathcal{B}_{12}(w_1, w_2, u) | W_1 = 1, W_2 = 1, U = u')$. The detail proof of the independence is shown in the appendix (Section 3.6) of this chapter.

Next let us bound $Pr(\mathcal{B}_{12}(w_1, w_2, u) | W_1 = 1, W_2 = 1, U = u')$ for $(w_1, w_2, u) \in \mathcal{W}_1 \times \mathcal{W}_2 \times \mathcal{U}$.

- If $w_1 = 1, w_2 = 1, u = u'$,

$$Pr(\mathcal{B}_{12}(w_1, w_2, u)|W_1 = 1, W_2 = 1, U = u') \leq 1$$

- If $w_1 = 1, w_2 = 1, u \neq u'$, then we have $X_R^m(u) \sim \prod_{i=1}^m p_{X_R}(X_{R,i})$ is independent of $(X_{12}^m(1), X_{22}^m(1), Y_{12}^m) \sim \prod_{i=1}^m p_{X_{12}X_{22}Y_{12}}(x_{12,i}, x_{22,i}, y_{12,i})$

$$Pr(\mathcal{B}_{12}(w_1, w_2, u)|W_1 = 1, W_2 = 1, U = u') \leq 2^{-m(I(X_R;X_{12},X_{22},Y_{12})-\delta(\epsilon))}$$

- Similarly, if $w_1 = 1, w_2 \neq 1, u = u'$, we have

$$Pr(\mathcal{B}_{12}(w_1, w_2, u)|W_1 = 1, W_2 = 1, U = u') \leq 2^{-m(I(X_{22};X_{12},X_R,Y_{12})-\delta(\epsilon))}$$

- if $w_1 = 1, w_2 \neq 1, u \neq u'$,

$$Pr(\mathcal{B}_{12}(w_1, w_2, u)|W_1 = 1, W_2 = 1, U = u') \leq 2^{-m(I(X_{22},X_R;X_{12},Y_{12})-\delta(\epsilon))}$$

- if $w_1 \neq 1, w_2 = 1, u = u'$,

$$Pr(\mathcal{B}_{12}(w_1, w_2, u)|W_1 = 1, W_2 = 1, U = u') \leq 2^{-m(I(X_{12};X_{22},X_R,Y_{12})-\delta(\epsilon))}$$

- if $w_1 \neq 1, w_2 = 1, u \neq u'$,

$$Pr(\mathcal{B}_{12}(w_1, w_2, u)|W_1 = 1, W_2 = 1, U = u') \leq 2^{-m(I(X_{12},X_R;X_{22},Y_{12})-\delta(\epsilon))}$$

- if $w_1 \neq 1, w_2 \neq 1, u = u'$,

$$Pr(\mathcal{B}_{12}(w_1, w_2, u)|W_1 = 1, W_2 = 1, U = u') \leq 2^{-m(I(X_{12},X_{22};X_R,Y_{12})-\delta(\epsilon))}$$

- if $w_1 \neq 1, w_2 \neq 1, u \neq u'$,

$$Pr(\mathcal{B}_{12}(w_1, w_2, u)|W_1 = 1, W_2 = 1, U = u') \leq 2^{-m(I(X_{12},X_{22},X_R;Y_{12})-\delta(\epsilon))}.$$

By using the result above, the error probability can be rewritten as

$$
\begin{aligned}
&Pr(\cup_{(w_1,w_2)\in\mathcal{A}}\mathcal{E}_{1,(w_1,w_2)}|W_1=1,W_2=1)\\
\leq &\sum_{u'\in\mathcal{U}} p(u') \sum_{(w_1,w_2)\in\mathcal{A}} \sum_{u\in\mathcal{U}} Pr(\mathcal{B}_{11}(w_1,w_2,u)|W_1=1,W_2=1,U=u')\\
&\qquad\qquad\qquad\qquad \times Pr(\mathcal{B}_{12}(w_1,w_2,u)|W_1=1,W_2=1,U=u')\\
\leq &\sum_{u'\in\mathcal{U}} p(u') \sum_{w_2\neq 1} Pr(\mathcal{B}_{11}(1,w_2,u')|W_1=1,W_2=1,U=u') \times 2^{-m(I(X_{22};X_{12},X_R,Y_{12})-\delta(\epsilon))}\\
&+\sum_{u'\in\mathcal{U}} p(u') \sum_{w_2\neq 1}\sum_{u\neq u'} Pr(\mathcal{B}_{11}(1,w_2,u)|W_1=1,W_2=1,U=u') \times 2^{-m(I(X_{22},X_R;X_{12},Y_{12})-\delta(\epsilon))}\\
&+\sum_{u'\in\mathcal{U}} p(u') \sum_{w_1\neq 1} Pr(\mathcal{B}_{11}(w_1,1,u')|W_1=1,W_2=1,U=u') \times 2^{-m(I(X_{12};X_{22},X_R,Y_{12})-\delta(\epsilon))}\\
&+\sum_{u'\in\mathcal{U}} p(u') \sum_{w_1\neq 1}\sum_{u\neq u'} Pr(\mathcal{B}_{11}(w_1,1,u)|W_1=1,W_2=1,U=u') \times 2^{-m(I(X_{12},X_R;X_{22},Y_{12})-\delta(\epsilon))}\\
&+\sum_{u'\in\mathcal{U}} p(u') \sum_{w_1\neq 1}\sum_{w_2\neq 1} Pr(\mathcal{B}_{11}(w_1,w_2,u')|W_1=1,W_2=1,U=u') \times 2^{-m(I(X_{12},X_{22};X_R,Y_{12})-\delta(\epsilon))}\\
&+\sum_{u'\in\mathcal{U}} p(u') \sum_{w_1\neq 1}\sum_{w_2\neq 1}\sum_{u\neq u'} Pr(\mathcal{B}_{11}(w_1,w_2,u)|W_1=1,W_2=1,U=u') \times 2^{-m(I(X_{12},X_{22},X_R;Y_{12})-\delta(\epsilon))}\\
\leq &\sum_{w_2\neq 1} Pr(\mathcal{B}_{11}(1,w_2,U)|W_1=1,W_2=1) \times 2^{-m(I(X_{22};X_{12},X_R,Y_{12})-\delta(\epsilon))}\\
&+\sum_{w_2\neq 1}\sum_{u\in\mathcal{U}} Pr(\mathcal{B}_{11}(1,w_2,u)|W_1=1,W_2=1) \times 2^{-m(I(X_{22},X_R;X_{12},Y_{12})-\delta(\epsilon))}\\
&+\sum_{w_1\neq 1} Pr(\mathcal{B}_{11}(w_1,1,U)|W_1=1,W_2=1) \times 2^{-m(I(X_{12};X_{22},X_R,Y_{12})-\delta(\epsilon))}\\
&+\sum_{w_1\neq 1}\sum_{u\in\mathcal{U}} Pr(\mathcal{B}_{11}(w_1,1,u)|W_1=1,W_2=1) \times 2^{-m(I(X_{12},X_R;X_{22},Y_{12})-\delta(\epsilon))}\\
&+\sum_{w_1\neq 1}\sum_{w_2\neq 1} Pr(\mathcal{B}_{11}(w_1,w_2,U)|W_1=1,W_2=1) \times 2^{-m(I(X_{12},X_{22};X_R,Y_{12})-\delta(\epsilon))}\\
&+\sum_{w_1\neq 1}\sum_{w_2\neq 1}\sum_{u\in\mathcal{U}} Pr(\mathcal{B}_{11}(w_1,w_2,u)|W_1=1,W_2=1) \times 2^{-m(I(X_{12},X_{22},X_R;Y_{12})-\delta(\epsilon))}.
\end{aligned}
\tag{3.15}
$$

Next we derive the bound for the term $Pr(\mathcal{B}_{11}(w_1,w_2,u)|W_1=1,W_2=1,U=u')$ using joint typicality lemma. Similarly we have:

- if $w_1=1, w_2\neq 1$, then $X_{21}^n(w_2)\sim\prod_{i=1}^n P_{X_{21}}(x_{21,i})$ is independent of $(X_{11}^n, Y_{11}^n, \hat{Y}_R^n(U))$, therefore

$$Pr(\mathcal{B}_{11}(1, w_2, U)|W_1 = 1, W_2 = 1) \leq 2^{-n(I(X_{21};X_{11},Y_{11},\hat{Y}_R)-\delta(\epsilon))}.$$

Here the bound is good for $(X_{11}^n, Y_{11}^n, \hat{Y}_R^n(U))$ that follows an arbitrary pmf.

- if $w_1 = 1, w_2 \neq 1$, $X_{21}^n(w_2) \sim \prod_{i=1}^n P_{X_{21}}(x_{21,i})$, $\hat{Y}_R^n(u) \sim \prod_{i=1}^n P_{\hat{Y}_R}(\hat{y}_{R,i})$ and $(X_{11}^n(w_1), Y_{11}^n) \sim \prod_{i=1}^n P_{X_{11},Y_{11}}(x_{11,i}, y_{11,i})$ are mutually independent, we have

$$\begin{aligned}
&Pr(B_{11}(1, w_2, u)|W_1 = 1, W_2 = 1) \\
&= \sum_{(x_{11}^n,x_{21}^n,y_{11}^n,\hat{y}_R^n)\in\mathcal{T}_\epsilon^n(X_{11}X_{21}Y_{11}\hat{Y}_R)} p(x_{21}^n)p(\hat{y}_R^n)p(x_{11}^n, y_{11}^n) \\
&\leq 2^{-n(H(X_{21})+H(\hat{Y}_R)+H(X_{11},Y_{11})-H(X_{21},\hat{Y}_R,X_{11},Y_{11})-\delta(\epsilon))} \\
&= 2^{-n(I(X_{21},\hat{Y}_R;X_{11},Y_{11})+I(X_{21};\hat{Y}_R)-\delta(\epsilon))} \qquad (3.16)
\end{aligned}$$

- if $w_1 \neq 1, w_2 = 1$,

$$Pr(\mathcal{B}_{11}(w_1, 1, U)|W_1 = 1, W_2 = 1) \leq 2^{-n(I(X_{11};X_{21},Y_{11},\hat{Y}_R)-\delta(\epsilon))}$$

since $X_{11}^n(w_1) \sim \prod_{i=1}^n P_{X_{11}}(x_{11,i})$ is independent of $(X_{21}^n, Y_{11}^n, \hat{Y}_R^n(U))$. Also notice that, the bound is good for $(X_{21}^n, Y_{11}^n, \hat{Y}_R^n(U))$ that follows an arbitrary pmf.

- if $w_1 \neq 1, w_2 = 1$, $X_{11}^n(w_1) \sim \prod_{i=1}^n P_{X_{11}}(x_{11,i})$, $\hat{Y}_R^n(u) \sim \prod_{i=1}^n P_{\hat{Y}_R}(\hat{y}_{R,i})$ and $(X_{21}^n(w_2), Y_{11}^n) \sim \prod_{i=1}^n P_{X_{21},Y_{11}}(x_{21,i}, y_{11,i})$ are mutually independent, we have

$$\begin{aligned}
&Pr(B_{11}(w_1, 1, u)|W_1 = 1, W_2 = 1) \\
&= \sum_{(x_{11}^n,x_{21}^n,y_{11}^n,\hat{y}_R^n)\in\mathcal{T}_\epsilon^n(X_{11}X_{21}Y_{11}\hat{Y}_R)} p(x_{11}^n)p(\hat{y}_R^n)p(x_{21}^n, y_{11}^n) \\
&\leq 2^{-n(H(X_{11})+H(\hat{Y}_R)+H(X_{21},Y_{11})-H(X_{11},\hat{Y}_R,X_{21},Y_{11})-\delta(\epsilon))} \\
&= 2^{-n(I(X_{11},\hat{Y}_R;X_{21},Y_{11})+I(X_{11};\hat{Y}_R)-\delta(\epsilon))} \qquad (3.17)
\end{aligned}$$

- if $w_1 \neq 1, w_2 \neq 1$, $X_{11}^n(w_1) \sim \prod_{i=1}^n P_{X_{11}}(x_{11,i})$, $X_{21}^n(w_2) \sim \prod_{i=1}^n P_{X_{21}}(x_{21,i})$ and $(Y_{11}^n, \hat{Y}_R^n(U)) \sim \prod_{i=1}^n P_{Y_{11},\hat{Y}_R}(y_{11,i}, \hat{y}_{R,i})$ are mutually independent, we have

$$\begin{aligned}
&Pr(B_{11}(w_1, w_2, U)|W_1 = 1, W_2 = 1) \\
&= \sum_{(x_{11}^n, x_{21}^n, y_{11}^n, \hat{y}_R^n) \in \mathcal{T}_\epsilon^n(X_{11}X_{21}Y_{11}\hat{Y}_R)} p(x_{11}^n)p(x_{21}^n)p(y_{11}^n, \hat{y}_R^n) \\
&\leq 2^{-n(H(X_{11})+H(X_{21})+H(Y_{11},\hat{Y}_R)-H(X_{11},X_{21},Y_{11},\hat{Y}_R)-\delta(\epsilon))} \\
&= 2^{-n(I(X_{11},X_{21};Y_{11},\hat{Y}_R)+I(X_{11};X_{21})-\delta(\epsilon))}
\end{aligned} \tag{3.18}$$

The bound is valid for $Y_{11}^n, \hat{Y}_R^n(U)$that follows an arbitrary pmf.

- if $w_1 \neq 1, w_2 \neq 1$, $X_{11}^n(w_1) \sim \prod_{i=1}^n P_{X_{11}}(x_{11,i})$, $X_{21}^n(w_2) \sim \prod_{i=1}^n P_{X_{21}}(x_{21,i})$, $\hat{Y}_R^n(u) \sim \prod_{i=1}^n P_{\hat{Y}_R}(\hat{y}_{R,i})$ and $Y_{11}^n \sim \prod_{i=1}^n P_{Y_{11}}(y_{11,i})$ are mutually independent, we have

$$\begin{aligned}
&Pr(B_{11}(w_1, w_2, u)|W_1 = 1, W_2 = 1) \\
&= \sum_{(x_{11}^n, x_{21}^n, y_{11}^n, \hat{y}_R^n) \in \mathcal{T}_\epsilon^n(X_{11}X_{21}Y_{11}\hat{Y}_R)} p(x_{11}^n)p(x_{21}^n)p(y_{11}^n)p(\hat{y}_R^n) \\
&\leq 2^{-n(H(X_{11})+H(X_{21}+H(Y_{11})+H(\hat{Y}_R)-H(X_{11},X_{21},Y_{11},\hat{Y}_R)-\delta(\epsilon))} \\
&= 2^{-n(I(X_{11},X_{21},\hat{Y}_R;Y_{11})+I(X_{11},X_{21};\hat{Y}_R)+I(X_{11},X_{21})-\delta(\epsilon))}
\end{aligned} \tag{3.19}$$

Using the above results for the bound and follow (3.15), we have the following inequalities:

$$\begin{aligned}
&Pr(\cup_{(w_1,w_2)\in\mathcal{A}} \mathcal{E}_{1,(w_1,w_2)}|W_1 = 1, W_2 = 1) \\
&\leq |\mathcal{W}_2| \cdot 2^{-n(I(X_{21};X_{11},Y_{11},\hat{Y}_R)-\delta(\epsilon))} \cdot 2^{-m(I(X_{22};X_{12},X_R,Y_{12})-\delta(\epsilon))} \\
&\quad +|\mathcal{W}_2| \cdot |\mathcal{U}| \cdot 2^{-n(I(X_{21},\hat{Y}_R;X_{11},Y_{11})+I(X_{21};\hat{Y}_R)-\delta(\epsilon))} \cdot 2^{-m(I(X_{22},X_R;X_{12},Y_{12})+I(X_{22};X_R)-\delta(\epsilon))} \\
&\quad |\mathcal{W}_1| \cdot 2^{-n(I(X_{11};X_{21},Y_{11},\hat{Y}_R)-\delta(\epsilon))} \cdot 2^{-m(I(X_{12};X_{22},X_R,Y_{12})-\delta(\epsilon))} \\
&\quad +|\mathcal{W}_1| \cdot |\mathcal{U}| \cdot 2^{-n(I(X_{11},\hat{Y}_R;X_{21},Y_{11})+I(X_{11};\hat{Y}_R)-\delta(\epsilon))} \cdot 2^{-m(I(X_{12},X_R;X_{22},Y_{12})+I(X_{12};X_R)-\delta(\epsilon))} \\
&\quad +|\mathcal{W}_1| \cdot |\mathcal{W}_2| \cdot 2^{-n(I(X_{11},X_{21};Y_{11},\hat{Y}_R)+I(X_{11};X_{21})-\delta(\epsilon))} \cdot 2^{-m(I(X_{12},X_{22};X_R,Y_{12})+I(X_{12};X_{22})-\delta(\epsilon))} \\
&\quad +|\mathcal{W}_1| \cdot |\mathcal{W}_2| \cdot |\mathcal{U}| \cdot 2^{-n(I(X_{11},X_{21},\hat{Y}_R;Y_{11})+I(X_{11},X_{21};\hat{Y}_R)+I(X_{11};X_{21})-\delta(\epsilon))} \\
&\qquad \cdot 2^{-m(I(X_{12},X_{22},X_R;Y_{12})+I(X_{21},X_{22};X_R)+I(X_{12};X_{22})-\delta(\epsilon))}.
\end{aligned}$$

For fixed $\beta = \frac{n}{l}$, $1-\beta = \frac{m}{l}$, if $l \to \infty$, $\epsilon \to 0$ and the following inequalities hold:

$$\begin{aligned}
R_2 &< \beta I(X_{21}; X_{11}, Y_{11}, \hat{Y}_R) + (1-\beta) I(X_{22}; X_{12}, X_R, Y_{12}) \\
R_2 + R_U &< \beta [I(X_{21}, \hat{Y}_R; X_{11}, Y_{11}) + I(X_{21}; \hat{Y}_R)] \\
&\quad +(1-\beta)[I(X_{22}, X_R; X_{12}, Y_{12}) + I(X_{22}; X_R)] \\
R_1 &< \beta I(X_{11}; X_{21}, Y_{11}, \hat{Y}_R) + (1-\beta) I(X_{12}; X_{22}, X_R, Y_{12}) \\
R_1 + R_U &< \beta [I(X_{11}, \hat{Y}_R; X_{21}, Y_{11}) + I(X_{11}; \hat{Y}_R)] \\
&\quad +(1-\beta)[I(X_{12}, X_R; X_{22}, Y_{12}) + I(X_{12}; X_R)] \\
R_1 + R_2 &< \beta [I(X_{11}, X_{21}; Y_{11}, \hat{Y}_R) + I(X_{11}; X_{21})] \\
&\quad +(1-\beta)[I(X_{12}, X_{22}; X_R, Y_{12}) + I(X_{12}; X_{22})] \\
R_1 + R_2 + R_U &< \beta [I(X_{11}, X_{21}, \hat{Y}_R; Y_{11}) + I(X_{11}, X_{21}; \hat{Y}_R) + I(X_{11}; X_{21})] \\
&\quad +(1-\beta)[I(X_{12}, X_{22}, X_R; Y_{12}) + I(X_{12}, X_{22}; X_R) + I(X_{12}; X_{22})], \qquad (3.20)
\end{aligned}$$

the term of probability of error $Pr(\cup_{(w_1,w_2)\in \mathcal{A}} \mathcal{E}_{1,(w_1,w_2)} | W_1 = 1, W_2 = 1) \to 0$. Note that if at least one or more inequalities in (3.20) are not satisfied, the information rates R_1 and R_2 are not achievable. In such case, at least one of the terms in (3.20) has the non-negative power. $Pr(\cup_{(w_1,w_2)\in \mathcal{A}} \mathcal{E}_{1,(w_1,w_2)} | W_1 = 1, W_2 = 1)$ is not tending to 0. Therefore, $Pr(\epsilon)$ is also not tending to 0. By Shannon's definition [19], the chosen R_1 and R_2 are not achievable. Any rate pair that is not within the achievable region will violate at least one of the inequalities in (3.20).

Since the codebook has been independently generated, we can further simplify the above inequalities by substituting $I(X_{22}; X_R) = 0$, $I(X_{12}; X_R) = 0$, $I(X_{11}; X_{21}) = 0$, $I(X_{12}; X_{22}) = 0$ and $I(X_{12}, X_{22}; X_R) = 0$. In the last we can take out R_U by using the inequality (5.9) which is $R_U > \beta I(Y_R, \hat{Y}_R) + \delta(\epsilon')$ and the following are the achievable rates region for D_1:

$$\begin{aligned}
R_2 &< min\{\beta I(X_{21}; X_{11}, Y_{11}, \hat{Y}_R) + (1-\beta) I(X_{22}; X_{12}, X_R, Y_{12}), \\
&\quad \beta[I(X_{21}, \hat{Y}_R; X_{11}, Y_{11}) + I(X_{21}; \hat{Y}_R) - I(Y_R; \hat{Y}_R)] \\
&\quad +(1-\beta) I(X_{22}, X_R; X_{12}, Y_{12})\} \qquad (3.21) \\
R_1 &< min\{\beta I(X_{11}; X_{21}, Y_{11}, \hat{Y}_R) + (1-\beta) I(X_{12}; X_{22}, X_R, Y_{12}),
\end{aligned}$$

$$\beta[I(X_{11},\hat{Y}_R;X_{21},Y_{11})+I(X_{11};\hat{Y}_R)-I(Y_R;\hat{Y}_R)]$$
$$+(1-\beta)I(X_{12},X_R;X_{22},Y_{12})\} \tag{3.22}$$
$$R_1+R_2<min\{\beta I(X_{11},X_{21};Y_{11},\hat{Y}_R)+(1-\beta)I(X_{12},X_{22};X_R,Y_{12}),$$
$$\beta[I(X_{11},X_{21},\hat{Y}_R;Y_{11})+I(X_{11},X_{21};\hat{Y}_R)-I(Y_R;\hat{Y}_R)]$$
$$+(1-\beta)I(X_{12},X_{22},X_R;Y_{12})\}. \tag{3.23}$$

Similarly we can obtain the achievable rate results for D_2. Therefore, the achievable rate region based on the GQF scheme for the HD-cMACr has been shown. ■

The major difference between the GQF scheme and the CF scheme applied in [31] is that relay does not perform binning after quantization of observed sources messages. Moreover, in GQF two destinations perform one-step joint-decoding of both messages instead of sequentially decoding the relay bin index and then the source messages.

3.3.2 Achievable Rate Region Based on modified CF Scheme

The achievable rate regions based on the modified CF scheme are shown for references. The modification includes two parts: First, the relay in the classic CF scheme is now half-duplex; Second, the encoding and decoding are now using a single block two slots structure.

In this CF scheme, relay quantizes its observation at the end of the first slot, implements Wyner-Ziv binning and sends the bin index in the second slot. The destination sequentially decodes the bin index $\hat{s} \in \mathcal{S}$, quantization index $\hat{u} \in B(\hat{s})$ with the side information and finally the source messages $\hat{w}_1 \in \mathcal{W}_1$ and $\hat{w}_2 \in \mathcal{W}_2$ jointly from both slots reception.

The achievable rate regions can be summarized in the following:

Theorem 3.3 *The following rates are achievable over discrete memoryless HD-MARC based on the modified CF scheme:*

$$R_i < a_1(i) \tag{3.24}$$
$$R_1+R_2<c_1 \tag{3.25}$$

subject to

$$\beta[I(Y_R;\hat{Y}_R) - I(Y_{11};\hat{Y}_R)] < (1-\beta)I(X_R;Y_{12}) \tag{3.26}$$

where $i \in \{1,2\}$, $a_1(i), c_1$ *are previously defined as in (3.5), for all the input distributions as in (3.6).*

Proof: Similarly as the GQF scheme, the results can be readily obtained from the proof of Theorem 3.4. ■

Theorem 3.4 *The following rate regions are achievable over discrete memoryless HD-cMACr with the modified CF scheme:*

$$R_i < min\{a_1(i), a_2(i)\} \tag{3.27}$$

$$R_1 + R_2 < min\{c_1, c_2\} \tag{3.28}$$

subject to

$$\beta[I(Y_R;\hat{Y}_R) - min\{I(Y_{11};\hat{Y}_R), I(Y_{21};\hat{Y}_R)\}] < (1-\beta)min\{I(X_R;Y_{12}), I(X_R;Y_{22})\} \tag{3.29}$$

where $i \in \{1,2\}$, $a_1(i), a_2(i), c_1$, c_2 *are previously defined as in (3.5), for all the input distributions as in (3.6).*

Proof: Due to the similarity of CF and GQF schemes, the detailed proof is omitted. Note that the classic CF scheme has been modified to fit the HD MARC channel. Now the relay quantizes the received signal with rate R_U after first slot, applies the Wyner-Ziv binning to further partition the set of alphabets $\mathcal{U}$ into 2^{lR_S} equal size bins and sends the bin index S with $X_R(s)$ in the second slot. The destinations perform successive decoding, i.e. sequentially decode the bin index $\hat{s} \in S$, quantization index $\hat{u} \in B(\hat{s})$ with the side information and finally the source messages $(\hat{w}_1, \hat{w}_2) \in (\mathcal{W}_1, \mathcal{W}_2)$ jointly from both slots' reception. ■

Note that the achievable results (3.24), (3.25), (3.27) and (3.28) should have (3.26) and (3.29) hold, which means the relay-destination link is good enough to support the compression at relay to be recovered at destination(s).

The GQF and the modified CF schemes generally provide different achievable rate regions. However, under some special conditions, both schemes could result in the same achievable rates. Take the achievable rates in the HD-MARC as an example. In Theorem 3.1, the individual rate R_1 is determined by the minimum of $a_1(1)$ and $b_1(1)$. Given (3.26) satisfied in Theorem 3.1, $b_1(1)$ is greater than $a_1(1)$. In such case, R_1 is only determined by $a_1(1)$. The GQF scheme and the modified CF scheme lead to the same individual rate R_1. Similarly, both schemes also result in the same rates R_2 and $R_1 + R_2$. In other words, if the constraint condition ((3.26)) holds, both GQF and modified CF schemes provide the same achievable rates. Therefore, when (3.26) holds and a simplified relay is not required, either the modified CF or the GQF scheme can be applied in the HD-MARC. On the other hand, if a low-cost simplified relay is preferred or (3.26) does not hold, the GQF scheme is a superior choice. This conclusion also applies to the HD-cMACr.

3.3.3 Special Case of The Achievable Rates Result

In this part, we show that the achievable rate regions for the three-node HD-RC [111] can be induced from the aforementioned achievable rate regions for the five-node HD-cMACr. Specifically, by taking $R_2 = 0$ and $X_{21} = X_{22} = Y_{21} = Y_{22} = \phi$, a HD-cMACr reduces to a three-node HDRC which contains S_1, R and D_1.

special case of GQF scheme

For the GQF scheme, since $Y_{21} = Y_{22} = \phi$ and $R_2 = 0$, the individual rate (3.7) become

$$\begin{aligned} R_1 < min\{&\beta I(X_{11}; Y_{11}, \hat{Y}_R) + (1-\beta) I(X_{12}; Y_{12}|X_R), \\ &\beta[I(X_{11}; Y_{11}) - I(Y_R; \hat{Y}_R|X_{11}, Y_{11})] \\ &+(1-\beta) I(X_{12}, X_R; Y_{12})\} \end{aligned} \tag{3.30}$$

where (3.30) is based on the Markov chain $(X_{11}, Y_{11}) \rightarrow Y_R \rightarrow \hat{Y}_R$ that $H(\hat{Y}_R|Y_R) = H(\hat{Y}_R|Y_R, X_{11}, Y_{11})$. Changing the variable names accordingly, we can verify that individual rate of R_1 become the same as in Theorem 1 of [111].

Similarly, the sum rate (3.8) can also be rewritten as

$$\begin{aligned}
R_1 < & min\{\beta[I(X_{11};Y_{11},\hat{Y}_R)+I(X_{21};Y_{11},\hat{Y}_R|X_{11}) \\
& +(1-\beta)[I(X_{12};Y_{12}|X_R)+I(X_{22};X_R,Y_{12}|X_{12})], \\
& \beta[I(X_{11},\hat{Y}_R;Y_{11})+I(X_{21};Y_{11}|X_{11},\hat{Y}_R) \\
& +I(X_{11};\hat{Y}_R)+I(X_{21};\hat{Y}_R|X_{11})-I(Y_R;\hat{Y}_R)] \\
& +(1-\beta)[I(X_{12},X_R;Y_{12})+I(X_{22};Y_{12}|X_{12},X_R)]\} \\
= & min\{\beta I(X_{11};Y_{11},\hat{Y}_R)+(1-\beta)I(X_{12};Y_{12}|X_R), \\
& \beta[I(X_{11},\hat{Y}_R;Y_{11})+I(X_{11};\hat{Y}_R)-I(Y_R;\hat{Y}_R)] \\
& +(1-\beta)I(X_{12},X_R;Y_{12})\} \\
= & min\{\beta I(X_{11};Y_{11},\hat{Y}_R)+(1-\beta)I(X_{12};Y_{12}|X_R), \\
& \beta[I(X_{11};Y_{11})-I(Y_R;\hat{Y}_R|X_{11},Y_{11})] \\
& +(1-\beta)I(X_{12},X_R;Y_{12})\}. \qquad (3.31)
\end{aligned}$$

Observing that (3.31) is the same as (3.30). By changing the variable names accordingly, R_1 from the individual rate and the sum rate become the same as in [111, Th.1]. Therefore the achievable rates based on QF scheme for three-node HD-RC can be treated as a special case of Theorem 3.2 of this work.

special case of modified CF scheme

Follows a similar fashion as the special case of GQF scheme, the individual rate R_1 from (3.27) can be rewritten as:

$$R_1 < \beta I(X_{11};X_{21},Y_{11},\hat{Y}_R)+(1-\beta)I(X_{12};X_{22},Y_{12}|X_R) \qquad (3.32)$$

$$\begin{aligned}
&= \beta[I(X_{11};Y_{11},\hat{Y}_R)+I(X_{11};X_{21}|Y_{11},\hat{Y}_R)] \\
&\quad +(1-\beta)[I(X_{12};Y_{12}|X_R)+I(X_{12};X_{22}|X_R,Y_{12})] \qquad (3.33)
\end{aligned}$$

$$= \beta I(X_{11};Y_{11},\hat{Y}_R)+(1-\beta)I(X_{12};Y_{12}|X_R) \qquad (3.34)$$

where (3.33) is from mutual information identity and (3.34) is from $X_{21} = X_{22} = \phi$.

Similarly using $R_2 = 0$, the sum rate inequality (3.28) can be rewritten as the same as (3.34). Also note that the constraint condition for the achievable rate region (3.29) becomes

$$(1-\beta)I(X_R; Y_{12}) > \beta I(Y_R; \hat{Y}_R) - \beta I(Y_{11}; \hat{Y}_R). \tag{3.35}$$

By changing the variable names respectively, the achievable rate region based on CF scheme of [111] can also be considered as a special case of the result of Theorem 3.4.

3.4 Gaussian Channels and Numerical Examples

The proposed GQF scheme and the modified CF scheme are extended to the Gaussian Channels in this section. Some numerical examples are given to compare the performance of the two schemes in the Half-Duplex Gaussian MARC.

Notice that in a cMACr both destinations need to decode both messages from the sources. The sum achievable rates in a cMACr are always a minimum function of two terms that obtained from the achievability of each destination. Therefore, for clarity of presentation and simplicity of exposition, the extended results of the half duplex Gaussian cMACr are not shown as they have the similar performance and effect in comparing GQF and CF.

For illustration purposes, it is assumed that all the codebooks used are generated according to some zero-mean Gaussian distributions and the optimality of such distribution is not claimed in this work. As pointed by many authors [31, 36, 48], the Gaussian input distributions are not necessarily the optimal distributions which maximize the achievable rate. The optimality of the Gaussian distribution was only proved for the Point-to-Point channel in [19]. Nevertheless, the Gaussian codebooks are still used since they are the most widely used assumption in the literature and make the analysis of characterizing the achievable rates tractable for illustration purpose.

Proposition 3.1 *The following rates are achievable for the Gaussian HD-MARC based on* *the GQF scheme:*

$$
\begin{aligned}
R_i < &\max_{\sigma_Q^2,\beta}\ min\{\frac{\beta}{2}log(1+h_{i1}^2P_{i1}+\frac{h_{iR}^2P_{i1}}{1+\sigma_Q^2})+\frac{1-\beta}{2}log(1+h_{i1}^2P_{i2}),\\
&\frac{\beta}{2}log(\frac{(1+h_{i1}^2P_{i1})\sigma_Q^2}{1+\sigma_Q^2})+\frac{1-\beta}{2}log(1+h_{i1}^2P_{i2}+h_{R1}^2P_R)\} \qquad (3.36)\\
R_1+\ R_2 < &\max_{\sigma_Q^2,\beta}\ min\{\\
&\frac{\beta}{2}log(1+h_{11}^2P_{11}+h_{21}^2P_{21}+\frac{(h_{11}h_{2R}-h_{1R}h_{21})^2P_{11}P_{21}+h_{1R}^2P_{11}+h_{2R}^2P_{21}}{1+\sigma_Q^2})\\
&+\frac{1-\beta}{2}log(1+h_{11}^2P_{12}+h_{21}^2P_{22}),\\
&\frac{\beta}{2}log(\frac{(1+h_{11}^2P_{11}+h_{21}^2P_{21})\sigma_Q^2}{1+\sigma_Q^2})+\frac{1-\beta}{2}log(1+h_{11}^2P_{12}+h_{21}^2P_{22}+h_{R1}^2P_R)\} \qquad (3.37)
\end{aligned}
$$

where $i,j\in\{1,2\}$, P_{ij} is the transmitter i's power in the j slot, P_R is the transmitter power at relay and σ_Q^2 is the variance of the quantization noise.

Proof: Extending the achievable rate result of the GQF scheme from the discrete memoryless case to the AWGN is similar as in [31, 110, 113]. The difference is that the achievable rates is now under the half-duplex multi-user channel. To focus on the discussion of the scheme performance, the detail of the proof is ignored. ■

Assuming the two min terms in (3.37) are two functions of σ_Q^2, i.e., $I_1(\sigma_Q^2)$ and $I_2(\sigma_Q^2)$, respectively. The impact of the different values of the σ_Q^2 on the achievable sum rate is shown in the Fig. 3.3. It can be seen that, for fixed β, $I_1(\sigma_Q^2)$ is a monotonically decreasing function and $I_2(\sigma_Q^2)$ is a monotonically increasing function. Let $I_1(\sigma_Q^2)=I_2(\sigma_Q^2)$, the σ_Q^2 that maximizes the sum rate can be obtained.

As reference, the achievable rates based on the modified CF scheme are shown below:

Proposition 3.2 *The following rates are achievable for the Gaussian HD-MARC with the modified CF scheme:*

$$R_i<\frac{\beta}{2}log(1+h_{i1}^2P_{i1}+\frac{h_{iR}^2P_{i1}}{1+\sigma_Q^2})+\frac{1-\beta}{2}log(1+h_{i1}^2P_{i2}), \qquad (3.38)$$

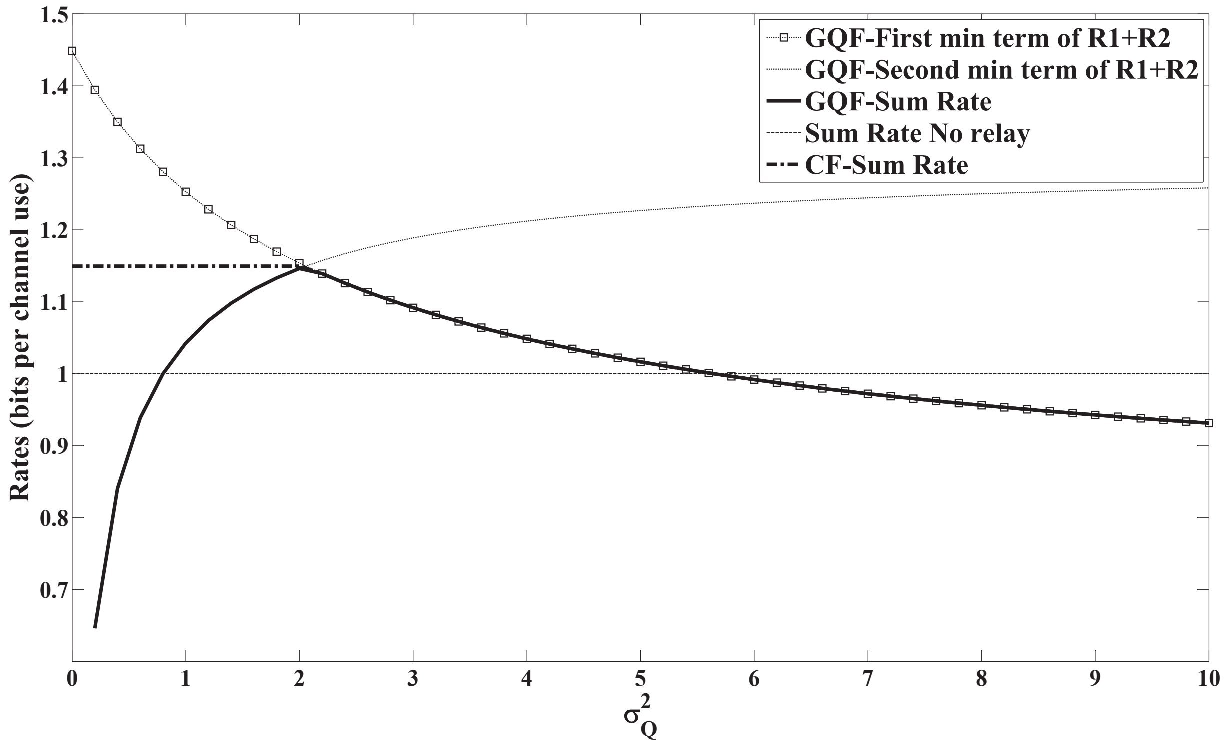


Figure 3.3: Achievable Rates of GQF and CF based schemes with variant $\sigma_q^2, P_{11} = P_{12} = P_{21} = P_{22} = P_R = 1$, and if no relay source powers $P_1 = P_2 = 1.5$, $h_{11} = h_{21} = 1, h_{1R} = 3, h_{2R} = 0.5, h_{R1} = 3, \beta = 0.5$

$$
\begin{aligned}
R_1 + R_2 < & \frac{\beta}{2} log(1 + h_{11}^2 P_{11} + h_{21}^2 P_{21} \\
& + \frac{(h_{11}h_{2R} - h_{1R}h_{21})^2 P_{11}P_{21} + h_{1R}^2 P_{11} + h_{2R}^2 P_{21}}{1+\sigma_Q^2}) \\
& + \frac{1-\beta}{2} log(1 + h_{11}^2 P_{12} + h_{21}^2 P_{22})
\end{aligned} \tag{3.39}
$$

where $i \in \{1,2\}$ *and*

$$
\sigma_Q^2 > \frac{1 + \frac{h_{1R}^2 P_{11} + h_{2R}^2 P_{21} + (h_{11}h_{2R} - h_{1R}h_{21})^2 P_{11}P_{21}}{1 + h_{11}^2 P_{11} + h_{21}^2 P_{21}}}{(1 + \frac{h_{R1}^2 P_R}{1 + h_{11}^2 P_{12} + h_{21}^2 P_{22}})^{\frac{1-\beta}{\beta}} - 1} \tag{3.40}
$$

Proof: Extending the achievable rate result of the modified scheme from the discrete memoryless case to the AWGN is similar to Proposition 3.1. Therefore, the detail of the proof is ignored. ■

The sum rate (3.39) is the same as the first min term of (3.37) when σ_Q^2 in (3.40) is satisfied. Fig. 3.3 also shows (3.39) with different σ_Q^2. (3.40) is the condition that makes modified CF scheme work. A smaller value of σ_Q^2 means $\hat{Y}_R$ is a less compressed observation of Y_R, and hence a higher rate of $\hat{Y}_R$. On the other hand, the channel between relay and destination requires the rate of $\hat{Y}_R$ to be small enough since the compression should be recovered by the destination.

β is the ratio of the first slot taken in a block. It impacts the maximum of the sum rate for both GQF and CF schemes. Fig. 3.4 is shown to demonstrate the impacts where the same settings as Fig. 3.3 except that $\beta = 0.4$. In Fig. 3.4, the maximum sum rates for the both schemes occur when σ_Q^2 approximately equal to 1. As in Fig. 3.4, $\sigma_Q^2 \doteq 2$ leads to the maximum sum rates. If β is changed, the maximum and the σ_Q^2 which gives maximum sum rates are also changed.

The impact of the factor β on the maximum sum achievable rates is shown in Fig. 3.5 where same parameters as Fig. 3.3 and Fig.3.4 are used. It can be seen that in order to maximize the achievable sum rate β should be carefully chosen. Note that if σ_Q^2 was chosen to satisfy (3.40), (3.39) is the same as (3.37). As also shown in the Fig. 3.5, both GQF and CF schemes outperform the case where no relay is available in the channel.

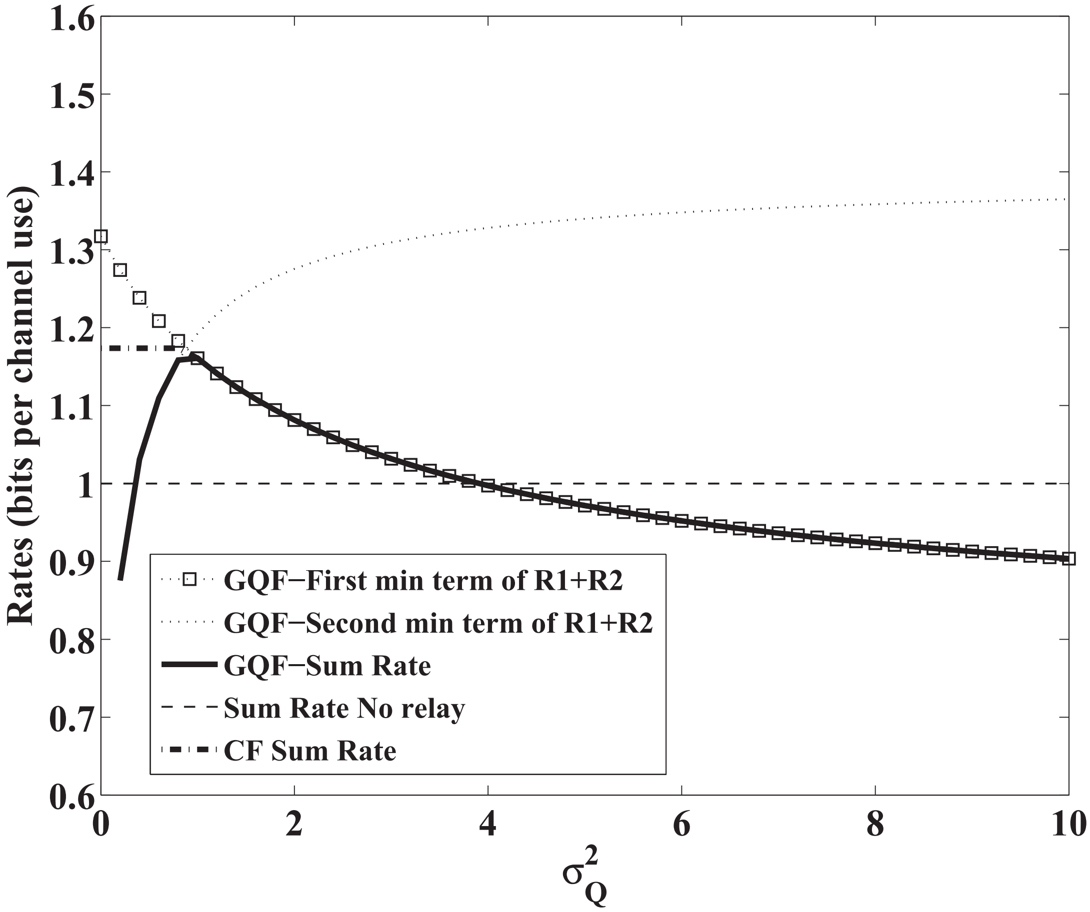


Figure 3.4: Achievable Rates of GQF and CF based schemes with variant $\sigma_q^2, P_{11} = P_{12} = P_{21} = P_{22} = P_R = 1$, and if no relay source powers $P_1 = P_2 = 1.5$, $h_{11} = h_{21} = 1, h_{1R} = 3, h_{2R} = 0.5, h_{R1} = 3, \beta = 0.4$

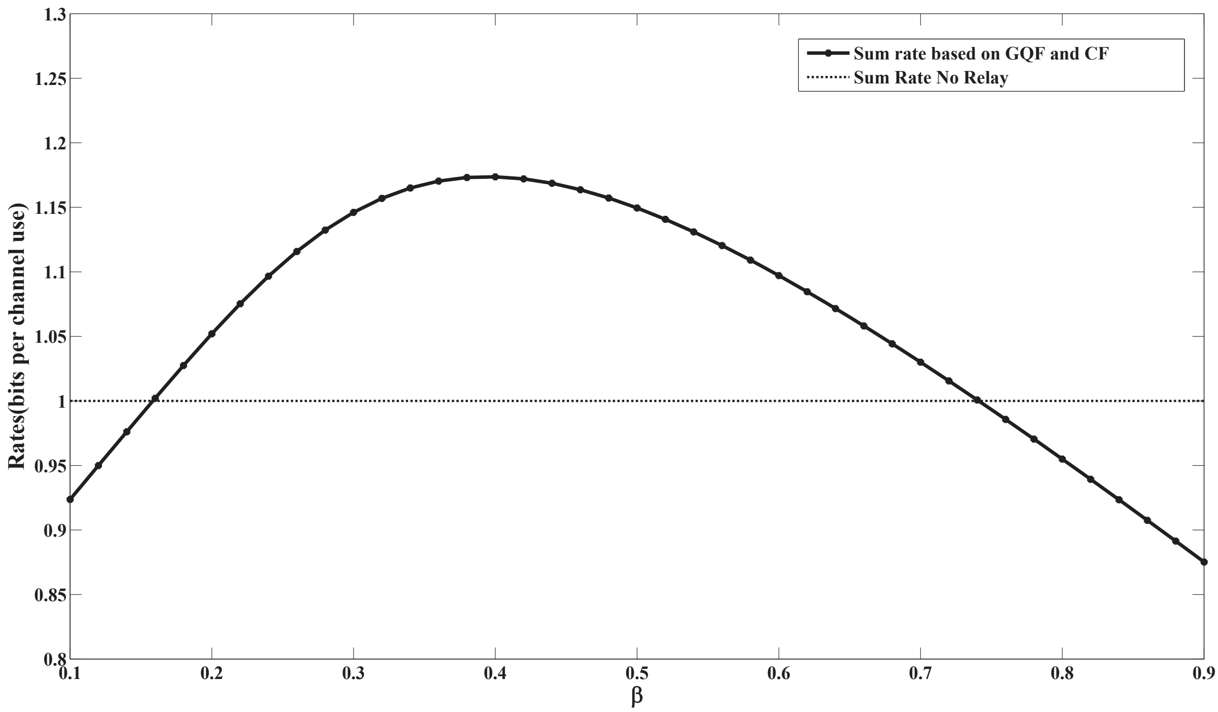


Figure 3.5: Achievable Rates of GQF and CF based scheme with variant β

The achievable sum rates of the GQF scheme with optimized σ_Q^2 are the same as the modified CF scheme over Gaussian HD-MARC. In general, without optimizing σ_Q^2, the CF scheme outperforms the GQF scheme. However, by choosing the optimized value of σ_Q^2, the GQF scheme, in which a low-cost simplified relay is used, is able to provide similar sum rate as the more complicated CF scheme.

3.5 Summary

In this chapter, the Half-Duplex (HD) relaying in the Multiple Access Relay Channel (MARC) and the compound Multiple Access Channel with a relay (cMACr) has been studied. A variation of the QF scheme, the GQF scheme, based on single block coding has been proposed. The GQF scheme employs joint decoding at destinations and uses a low-cost relay. For comparison purpose, a modified CF scheme was also introduced. The achievable rate regions were obtained based on both schemes. It is also shown that the achievable rate regions for the three-node HDRC can be treated as special cases of our results obtained for the five-node channel. As a further development, the achievable rate results were also extended to the Half-Duplex Gaussian MARC. Some numerical examples were provided for performance comparison. The results indicate that the proposed GQF scheme can provide a similar performance as the CF scheme with only a simplified low-cost relay.

3.6 Appendix: Proof of Independence

In this appendix, we will show the independence of two conditional probabilities which are $p(\{x_{11}^n\},\{x_{21}^n\},y_{11}^n,\{\hat{y}_R^n\}|u,w_1,w_2)$ and $p(\{x_{21}^m\},\{x_{22}^m\},\{x_R^m\},y_{12}^m,|u,w_1,w_2)$.

Note that here the random variables inside the brace$\{\}$ denotes the corresponding codebook and other random variables which uses capital font represents the codewords we have chosen. For instance, x_{11}^n means the codebook of x_{11} or all the codewords that can be chosen, while X_{11}^n denotes a specific sequence of codeword that has been selected to transmit according to the message W_1. The following Markov chain exists in our model:

$$\begin{aligned}(W_1,W_2,\{x_{11}^n\},\{x_{21}^n\}) &\to (X_{11}^n,X_{21}^n) \to (Y_{11}^n,Y_R^n,\{\hat{Y}_R^n\})\\ &\to (U,\{X_R^m\}) \to (X_R^m,W_1,W_2,\{x_{12}^m\},\{x_{22}^m\},X_{12}^m,X_{22}^m) \to Y_{12}^m\end{aligned}$$

Then the following probability can be written as:

$$\begin{aligned}
&p(\{x_{11}^n\},\{x_{21}^n\},y_{11}^n,\{\hat{y}_R^n\},\{x_{12}^m\},\{x_{22}^m\},\{x_R^m\},y_{12}^m|u,w_1,w_2)\\
=\ &p(\{x_{12}^m\},\{x_{22}^m\},\{x_R^m\},y_{12}^m|u,w_1,w_2,\{x_{11}^n\},\{x_{21}^n\},y_{11}^n,\{\hat{y}_R^n\})\\
&p(\{x_{11}^n\},\{x_{21}^n\},y_{11}^n,\{\hat{y}_R^n\}|u,w_1,w_2)\\
=\ &p(\{x_{12}^m\},\{x_{22}^m\},y_{12}^m|u,\{x_R^m\},w_1,w_2,\{x_{11}^n\},\{x_{21}^n\},y_{11}^n,\{\hat{y}_R^n\})\\
&p(\{x_R^m\}|u,w_1,w_2,\{x_{11}^n\},\{x_{21}^n\},y_{11}^n,\{\hat{y}_R^n\})\\
&p(\{x_{11}^n\},\{x_{21}^n\},y_{11}^n,\{\hat{y}_R^n\}|u,w_1,w_2)\\
=\ &p(\{x_{12}^m\},\{x_{22}^m\},y_{12}^m|u,\{x_R^m\},w_1,w_2)p(\{x_R^m\}|u,w_1,w_2)
\end{aligned}$$

$$
\begin{aligned}
& p(\{x_{11}^n\}, \{x_{21}^n\}, y_{11}^n, \{\hat{y}_R^n\}|u, w_1, w_2) \\
&= p(\{x_{12}^m\}, \{x_{22}^m\}, y_{12}^m, \{x_R^m\}|u, w_1, w_2) p(\{x_{11}^n\}, \{x_{21}^n\}, y_{11}^n, \{\hat{y}_R^n\}|u, w_1, w_2)
\end{aligned}
$$

Therefore the conditional independence holds for the above mentioned two probabilities.

Chapter 4

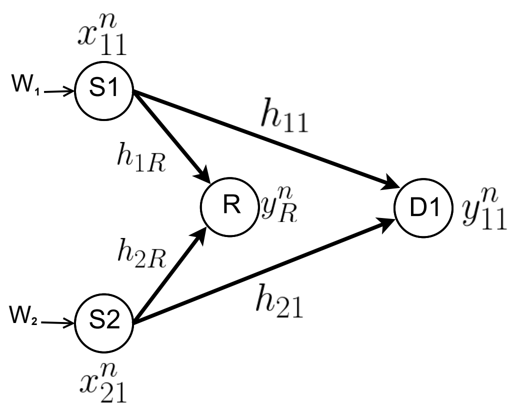
Slow Rayleigh Fading Channel Performance

Since delay sensitive wireless transmission system is generally modeled with the slow fading channel, this chapter focuses on the HD relaying based on the generalized quantize-and-forward (GQF) scheme in the slow rayleigh fading MARC. As perfect CSIT at a low-cost relay is too optimistic in practice, we consider the case that the relay node has no channel state information (CSI) of the relay-to-destination link. The achievable rate regions of the discrete memoryless HD-MARC and the corresponding additive white Gaussian noise (AWGN) channel have been studied in Chapter 3. Based on the achievable rates, the common outage probability and the expected sum rate (total throughput) of the GQF scheme have been characterized. The numerical examples show that when the relay has no access to the CSI of the relay-destination link, the GQF scheme outperforms other relaying schemes, e.g., classic compress-and-forward (CF) [18], decode-and-forward (DF) [28] and amplify-and-forward (AF) [12,54]. In addition, for a MAC channel with heterogeneous user channels and quality-of-service (QoS) requirements, individual outage probability and total throughput of the GQF scheme are also obtained and shown to outperform the classic CF scheme.

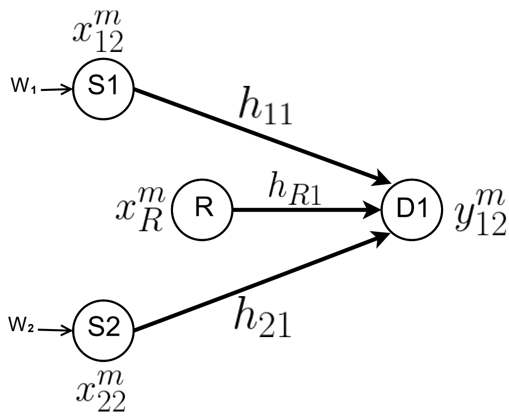
4.1 Background and Problem Statement

With the exact capacity still unknown, static relay channel studies were mainly focused on finding coding schemes that maximizes the achievable rate and techniques that minimizes the upper bound [18]. In the case of relay cooperating with multiple sources, a Multiple Access Channel with a relay (MARC) has been extensively studied in [31, 48, 71]. On the other hand, in a slow fading wireless relay channel, the system outage probability can be reduced significantly by the diversity offered from the relay's cooperation [54].

Motivated by the practical constraint that delay constrains exists in some wireless channels and relay cannot transmit and receive simultaneously in wireless communications [42, 54], a slow fading HD-MARC (shown in Fig. 4.1) is considered in this chapter. In particular, a block fading channel where the channel coefficients stay constant in each block but change independently from block to block is studied. In addition, it is assumed that source and relay have no channel state information of the ongoing channel at transmitter (CSIT). Specifically, the sources have no CSIT, the destination has the receiver CSI (CSIR), and the relay has only the CSI of the source-to-relay link.



(a) MARC Slot-1



(b) MARC Slot-1

Figure 4.1: Message flow of the slow fading HD-MARC.

The fundamental relaying schemes are DF and CF from [18]. The CF based schemes are not limited by the decoding capability of the relay, and therefore were implemented wherever the relay is close to the destination. The DF based schemes have been investigated over the slow fading MARC in [7]. The CF based schemes were shown to outperform the DF based schemes with the relay having the complete CSI of the channel in [46, 114]. However, the perfect CSI at relay is generally too ideal. When the critical delay constraint exists in the wireless channels, the relay may not be able to obtain the CSI accurately. The CF-based schemes are not efficient in a slow fading environment when the relay has no access to the complete CSI [111].

In this chapter, the performance of the GQF scheme, is investigated in the slow Rayleigh fading HD-MARC for both common and individual outage aspects [56, 62]. The achievable rate regions based on the GQF scheme were characterized for the static HD-MARC (both discrete memoryless and AWGN channels) in Chapter 3. Based on the achievable rates, the common outage probability and expected sum rate of the GQF scheme are derived and compared to the classic CF scheme and other common relaying schemes (AF [12, 54] and DF [28]). It is shown by the numerical examples that, without relay-to-destination CSI at relay, the GQF scheme outperform the other schemes and can regain a large portion of the benefit provided by the CF-based schemes (with perfect CSI at relay) over DF-based schemes in the selected topology.

In practice, each user in a MAC may have different QoS requirement [62]. Specifically, for a two user MARC, the destination failing to decode one of the sources messages may not affect the other user's QoS requirement (message decoded successfully by the destination). Therefore, the individual outage related performance of the GQF scheme is also discussed in terms of the individual outage probability and expected sum rate. The numerical examples are given to demonstrate the differences between the common and individual outages as well as the advantage of GQF scheme.

The rest of this chapter is organized as follows: Section 4.2 presents the system model. The common outage related performance of the GQF scheme is derived and discussed in Section 4.3. In Section 4.4, the individual outage performance is studied. Finally the conclusion of this chapter is given in Section 4.5.

4.2 System Model

A slow fading HD-MARC is considered in this chapter. Similarly to Chapter 3, the channel transition probabilities are described by the following:

$$\begin{aligned} y_{11}^n &= h_{11}x_{11}^n + h_{21}x_{21}^n + z_{11}^n \\ y_R^n &= h_{1R}x_{11}^n + h_{2R}x_{21}^n + z_R^n \\ y_{12}^m &= h_{11}x_{12}^m + h_{21}x_{22}^m + h_{R1}x_R^m + z_{12}^m \end{aligned} \tag{4.1}$$

where h_{ij} for $i \in \{1, 2, R\}$ and $j \in \{R, 1\}$ denote the channel coefficients between the transmission node i and the reception node j. For convenience, denote the channel coefficient vector

$$\mathbf{h} := [h_{11}, h_{21}, h_{1R}, h_{2R}, h_{R1}]. \tag{4.2}$$

For the slow fading channel, these coefficients are random variables and stay constant within each block and changes independently over different blocks. In particularly, a Rayleigh fading model is considered in this work. All the channel coefficients of $\mathbf{h}$ are assumed to be mutually independent and circularly symmetric complex Gaussian with zero means and variances σ_{ij}^2 (see [63, 97] for more detailed definition). The elements of the noise sequences of z_{11}^n, z_{12}^m and z_R^n are also circularly symmetric complex Gaussian with zero means and unit variances.

Motivated by the practical applications, the source nodes have no CSI, i.e. no knowledge of $\mathbf{h}$. Hence, each of them can only use a coding scheme with fixed rate $R_i, i \in \{1, 2\}$ to send messages. The relay has only receiver side CSI meaning only h_{1R} and h_{2R} are available. The destination knows $\mathbf{h}$ and therefore has complete CSI.

In Fig. 4.2, the achievable rate regions of a two-user MARC conditioned on the channel state is shown as the region 4 (bounded by two axis and the points C,D,E,F). The instantaneous achievable rate of a certain relaying scheme within a block of transmission is described as $I_{W,j}(\mathbf{h})$, where $j \in \{1, 2, sum\}$ and W denotes the relaying scheme. $I_{W,j}(\mathbf{h})$ is fixed for each block but a random entity determined by $\mathbf{h}$ within the entire transmission. In the

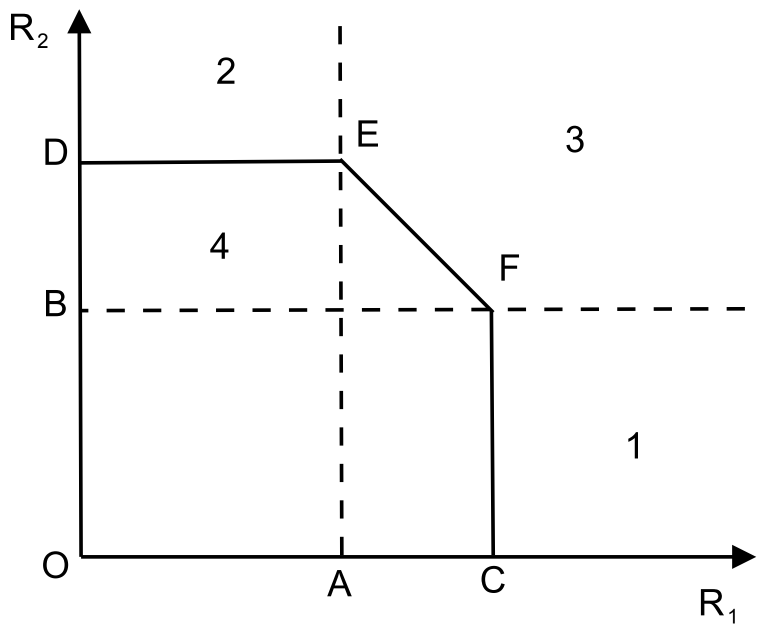


Figure 4.2: Achievable rate region of a 2-user MARC conditioned on channel state

following, the outage and the region of probabilities are taken over the random vector $\mathbf{h}$.

4.2.1 Common outage probability and expected rate

Similarly as in [56, 62], the common outage probability is defined as the probability that the chosen fixed rate pair (R_1, R_2) lies outside the achievable rate region, given $\mathbf{h}$:

$$P_{out,common}(R_1, R_2) = Pr\{R_1 + R_2 > I_{W,sum}(\mathbf{h}) \text{ or } R_1 > I_{W,1}(\mathbf{h}) \text{ or } R_2 > I_{W,2}(\mathbf{h})\}. \quad (4.3)$$

The system throughput or the expected sum rate is defined as in [111] and [62]:

$$\bar{R}_{common}(R_1, R_2) = (R_1 + R_2)(1 - P_{out,common}(R_1, R_2)). \quad (4.4)$$

The common expected rates can be obtained easily once the common outage probability is determined. Hence, the rest of this chapter will focus on deriving the common outage probabilities.

4.2.2 Individual Outage of the MARC

The individual outage event for a given user, say S_1, is defined as the message W_1 can not be decoded correctly at the destination, irrespective of the successful decoding of the message W_2. Define $P_{out,indiv1}(R_1, R_2)$ and $P_{out,indiv2}(R_1, R_2)$ as the individual outage probability of user 1 and 2, respectively. According to Fig. 4.2, the common and individual outage probabilities can be described as the following:

$$P_{out,indiv1}(R_1, R_2) = P_{reg,1} + P_{reg,3} \tag{4.5}$$

$$P_{out,indiv2}(R_1, R_2) = P_{reg,2} + P_{reg,3} \tag{4.6}$$

$$P_{out,common}(R_1, R_2) = 1 - P_{reg,4} = P_{reg,1} + P_{reg,2} + P_{reg,3} \tag{4.7}$$

where $P_{reg,i}$, $i = 1, 2, 3, 4$ denotes the probability that the rate pair (R_1, R_2) lies within the region i. Then the expected sum rate based on the individual outage probability is

$$\bar{R}_{indiv}(R_1, R_2) = R_1(1 - P_{out,indiv1}(R_1, R_2)) + R_2(1 - P_{out,indiv2}(R_1, R_2)). \tag{4.8}$$

4.3 Common Outage Probabilities and Expected Sum Rates of the GQF Scheme

The advantage of the GQF scheme in the slow fading HD-MARC is shown in this section by the following steps. The achievable rates inequalities based on the GQF scheme are rewritten first in order to investigate such scheme in the slow fading channel. Then, based on the achievable rate region result, the common outage events and outage probability of the GQF scheme are characterized. Numerical examples and the discussions are given in the last part to compare the performance of different relaying schemes.

4.3.1 Achievable Rate Region for Fading Channel

In Chapter 3, we have shown the achievable rate region based on the GQF scheme for the static HD-MARC in Theorem 3.1. For simplicity on analyzing the outage performance, we rewrite (3.3), (3.4) and (3.5) as below:

The following rate regions are achievable over discrete memoryless HD-MARC based on the GQF scheme:

$$\begin{aligned} R_1 &< \beta I(X_{11}; Y_{D1}, \hat{Y}_R | X_{21}) \\ &\quad +(1-\beta) I(X_{12}; Y_{D2} | X_{22}, X_R) \quad (4.9) \\ R_1 + R_U &< \beta [I(X_{11}, \hat{Y}_R; X_{21}, Y_{D1}) + I(X_{11}; \hat{Y}_R)] \\ &\quad +(1-\beta) I(X_{12}, X_R; Y_{D2} | X_{22}) \quad (4.10) \\ R_2 &< \beta I(X_{21}; Y_{D1}, \hat{Y}_R | X_{11}) \\ &\quad +(1-\beta) I(X_{22}; Y_{D2} | X_{12}, X_R) \quad (4.11) \\ R_2 + R_U &< \beta [I(X_{21}, \hat{Y}_R; Y_{D1} | X_{11}) + I(X_{21}; \hat{Y}_R)] \\ &\quad +(1-\beta) I(X_{22}, X_R; Y_{D2} | X_{12}) \quad (4.12) \\ R_1 + R_2 &< \beta I(X_{11}, X_{21}; Y_{D1}, \hat{Y}_R) \\ &\quad +(1-\beta) I(X_{12}, X_{22}; Y_{D2} | X_R) \quad (4.13) \\ R_1 + R_2 + R_U &< \beta [I(X_{11}, X_{21}, \hat{Y}_R; Y_{D1}) + I(X_{11}, X_{21}; \hat{Y}_R)] \\ &\quad +(1-\beta) [I(X_{12}, X_{22}, X_R; Y_{D2}), \quad (4.14) \end{aligned}$$

where $\beta = n/l$ is fixed and

$$R_U > \beta I(Y_R, \hat{Y}_R), \quad (4.15)$$

for all input distributions

$$p(x_{11}) p(x_{21}) p(x_{12}) p(x_{22}) p(x_R) p(\hat{y}_R | y_R).$$

When the relay node knows only the CSI of the source to relay link in a AWGN HD-RC, extending the achievable rate results from the discrete memoryless channel to the gaussian channel are not straightforward as noted by [1, 36, 48, 111]. To overcome this problem, a discretization approach based on the ideas in [25,30] has been proposed in [111]. In our work, since the relay node has only the CSI of the two sources to relay link as well, the discretization approach is adopted to AWGN HD-MARC and applied to derive the achievable rates of the GQF scheme. The achievable rates result is the same as Proposition 3.1. However, in order to make some further discussions and distinguish the cases in which relay has the CSI of R-D link or not, we declare a new proposition in the following:

Proposition 4.1 *The following rates are achievable for the Gaussian HD-MARC with the GQF scheme, for which the relay node has only the CSI of S-R link:*

$$\begin{aligned} R_i < \max_{\sigma_Q^2,\beta}\ \min\{ & \frac{\beta}{2} log(1 + h_{i1}^2 P_{i1} + \frac{h_{iR}^2 P_{i1}}{1+\sigma_Q^2}) + \frac{1-\beta}{2} log(1 + h_{i1}^2 P_{i2}), \\ & \frac{\beta}{2} log(\frac{(1+h_{i1}^2 P_{i1})\sigma_Q^2}{1+\sigma_Q^2}) + \frac{1-\beta}{2} log(1 + h_{i1}^2 P_{i2} + h_{R1}^2 P_R)\} \end{aligned} \tag{4.16}$$

$$\begin{aligned} R_1 + R_2 < \max_{\sigma_Q^2,\beta}\ \min\{ & \\ & \frac{\beta}{2} log(1 + h_{11}^2 P_{11} + h_{21}^2 P_{21} + \frac{(h_{11}h_{2R} - h_{1R}h_{21})^2 P_{11}P_{21} + h_{1R}^2 P_{11} + h_{2R}^2 P_{21}}{1+\sigma_Q^2}) \\ & + \frac{1-\beta}{2} log(1 + h_{11}^2 P_{12} + h_{21}^2 P_{22}), \\ & \frac{\beta}{2} log(\frac{(1+h_{11}^2 P_{11} + h_{21}^2 P_{21})\sigma_Q^2}{1+\sigma_Q^2}) + \frac{1-\beta}{2} log(1 + h_{11}^2 P_{12} + h_{21}^2 P_{22} + h_{R1}^2 P_R)\} \end{aligned} \tag{4.17}$$

where $i = 1, 2$ and σ_Q^2 is the variance of the quantization noise Z_Q.

Note that we assume β is fixed, different values of σ_Q^2 can be selected to maximize the sum rate (4.17) or the individual rate (4.16). Those values of σ_Q^2 are shown in the following:

$$\sigma_{Qsum}^2 = \frac{1 + \frac{h_{1R}^2 P_{11} + h_{2R}^2 P_{21} + (h_{11}h_{2R} - h_{1R}h_{21})^2 P_{11}P_{21}}{1 + h_{11}^2 P_{11} + h_{21}^2 P_{21}}}{(1 + \frac{h_{R1}^2 P_R}{1 + h_{11}^2 P_{12} + h_{21}^2 P_{22}})^{\frac{1-\beta}{\beta}} - 1}, \tag{4.18}$$

$$\sigma_{Qindiv,i}^2 = \frac{1 + \frac{h_{iR}^2 P_{i1}}{1 + h_{i1}^2 P_{i1}}}{(1 + \frac{h_{R1}^2 P_R}{1 + h_{i1}^2 P_{i2}})^{\frac{1-\beta}{\beta}} - 1}, \tag{4.19}$$

where $i = 1, 2$.

In the static MARC (non-fading), sum rate is more important than the individual one since it captures the overall performance of the multi-user channel. Therefore, σ_Q^2 is usually chosen to maximize (4.17). However, in the slow fading MARC where relay has complete CSI [114], sometimes choosing $\sigma^2_{Qindiv,i}$ instead of σ^2_{Qsum} can reduce the common outage probability of the CF based schemes (including classic CF and GQF).

Note that the achievable rates of the GQF scheme with optimized σ_Q^2 are the same as the classic CF scheme over AWGN HD-MARC. However, as shown later in this chapter, the GQF scheme is able to provide significant gain over the CF scheme in the slow fading HD-MARC when the relay node has only receiver CSI.

4.3.2 Outage Probability of the GQF scheme

Based on the achievable rate region result from previous subsection, the common outage events and outage probability of the GQF scheme are characterized.

Outage event and the outage probability of the GQF scheme

Since source nodes have no CSIT, S_1 and S_2 can only use a fixed rate pair of (R_1, R_2) to transmit. The relay node has no CSI of the R-D link, therefore it is not able to adapt to the channel state $\mathbf{h}$ and choose rate R_U accordingly. Instead, the relay can only use a fixed rate of R_U. In order to do so, the relay chooses the auxiliary random variable $\hat{Y}_R$ according to $\hat{Y}_R = Y_R + Z_Q$. The variance of the Z_Q, which is σ_Q^2, is chosen to have

$$R_U = \beta I(Y_R; \hat{Y}_R) = \beta log(1 + \frac{1 + h_{1R}^2 P_{11} + h_{2R}^2 P_{21}}{\sigma_Q^2}). \tag{4.20}$$

As every parameter in (4.20) is known, the relay can choose such σ_Q^2 successfully.

In the GQF scheme, the destination node employs the joint-decoding technique, thus the common outage event happens when either one of the conditions (4.9)-(4.14) is not satisfied. Define the following sets:

$$\mathcal{O}_{R_1} := \{\mathbf{h} : R_1 > \beta I(X_{11}; Y_{D1}, \hat{Y}_R | X_{21})$$

$$+(1-\beta)I(X_{12};Y_{D2}|X_{22},X_R)\} \tag{4.21}$$

$$\mathcal{O}_{R_{1u}} := \{\mathbf{h} : R_1 > \beta[I(X_{11},\hat{Y}_R;X_{21},Y_{D1}) + I(X_{11};\hat{Y}_R)] \\ +(1-\beta)I(X_{12},X_R;Y_{D2}|X_{22}) - R_U\} \tag{4.22}$$

$$\mathcal{O}_{R_2} := \{\mathbf{h} : R_2 > \beta I(X_{21};Y_{D1},\hat{Y}_R|X_{11}) \\ +(1-\beta)I(X_{22};Y_{D2}|X_{12},X_R)\} \tag{4.23}$$

$$\mathcal{O}_{R_{2u}} := \{\mathbf{h} : R_2 > \beta[I(X_{21},\hat{Y}_R;Y_{D1}|X_{11}) + I(X_{21};\hat{Y}_R)] \\ +(1-\beta)I(X_{22},X_R;Y_{D2}|X_{12}) - R_U\} \tag{4.24}$$

$$\mathcal{O}_{R_{12}} := \{\mathbf{h} : R_1 + R_2 > \beta I(X_{11},X_{21};Y_{D1},\hat{Y}_R) \\ +(1-\beta)I(X_{12},X_{22};Y_{D2}|X_R)\} \tag{4.25}$$

$$\mathcal{O}_{R_{12u}} := \{\mathbf{h} : R_1 + R_2 > \beta[I(X_{11},X_{21},\hat{Y}_R;Y_{D1}) \\ +I(X_{11},X_{21};\hat{Y}_R)] + (1-\beta)[I(X_{12},X_{22},X_R;Y_{D2}) - R_U\}. \tag{4.26}$$

In (4.20) R_U has been chosen to satisfy (4.15), the common outage event is determined by the inequalities of the aforementioned six sets. Therefore, the common outage probability of the GQF scheme can be described as

$$P_{out,common}^{GQF}(R_1,R_2,R_U) = Pr\{\mathcal{O}_{R_1} \cup \mathcal{O}_{R_{1u}} \cup \mathcal{O}_{R_2} \cup \mathcal{O}_{R_{2u}} \cup \mathcal{O}_{R_{12}} \cup \mathcal{O}_{R_{12u}}\} \tag{4.27}$$

On the other hand, if the relay node has the access to the complete CSI of all the link, it can adjust its transmission rate of R_U according to that specific channel state $\mathbf{h}$. The outage probabilities of the GQF scheme and the CF scheme are the same in this case. Denote such outage probability as P_{out}^{CSIT}:

$$P_{out}^{CSIT} = Pr\{R_1 + R_2 > I_{GQF,sum}(\mathbf{h}) \text{ or } \\ R_1 > I_{GQF,1}(\mathbf{h}) \text{ or } R_2 > I_{GQF,2}(\mathbf{h})\} \tag{4.28}$$

where $I_{GQF,i}(\mathbf{h})$, $i = 1, 2$ and $I_{GQF,sum}(\mathbf{h})$ are the instantaneous individual and sum rates, i.e., the right hand side of (3.36) and (3.37) given a fixed $\mathbf{h}$, respectively.

4.3.3 Numerical Examples and Discussions

With the outage probabilities from last subsection, some numerical examples of the outage related performance are given in this part to show the benefit of using GQF scheme over HD-MARC.

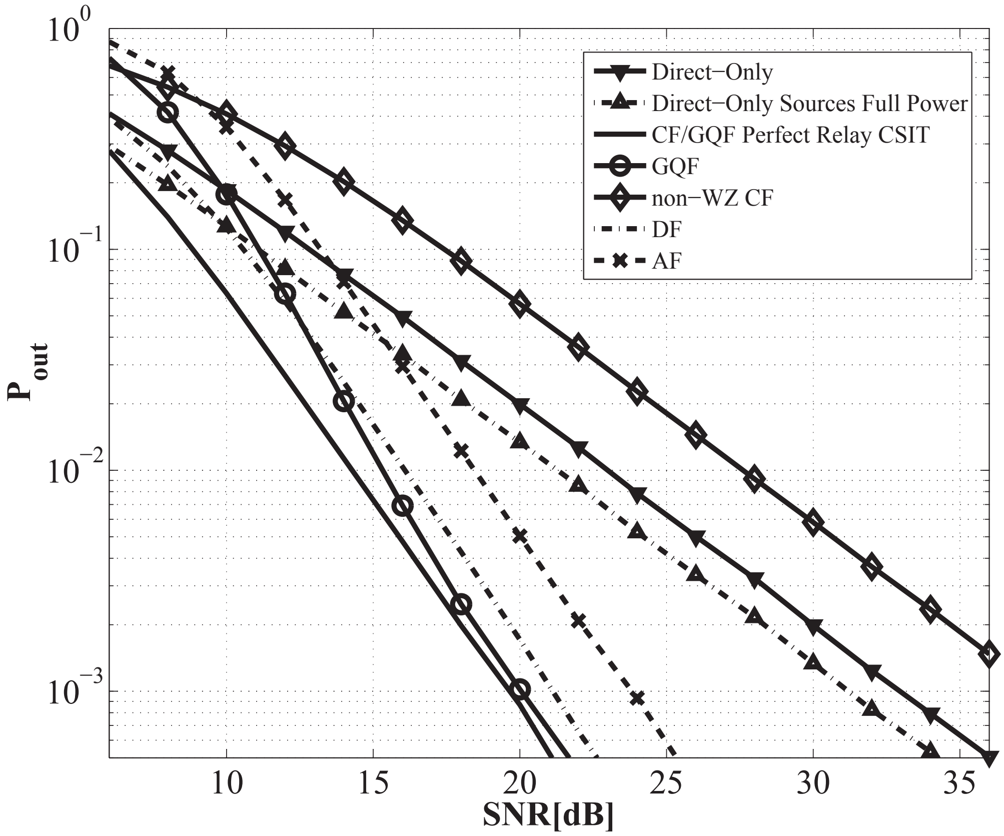


Figure 4.3: Outage Probability of the GQF scheme with $R_U = 3$, $R_1 = R_2 = 1$ and $\beta = 0.5$.

First, a numerical example of the outage probability as functions of SNR is shown in Fig. 4.3. Assuming that $\beta = 0.5$, $R_1 = R_2 = 1$ bit/channel use, $\sigma_i^2 = 1$ for $i \in \{11, 21, 1R, 2R, R1\}$ in (4.2), $P_{11} = P_{21} = P_{12} = P_{22} = SNR$ and $P_R = SNR/(1-\beta)$. In this case, the sources and the relay have the same average power. The outage probabilities of the DF [28], AF [12], non-WZ CF [46] and direct only transmission (no relay) schemes are also shown in the example for comparison.

In the direct transmission scheme the relay is assumed to be silent during the whole block transmission, therefore the system is equivalent to a 2-user half-duplex MAC. The common outage probability of the direct transmission can be described similarly by (4.3). The direct transmission scheme where each of the source node has 1.5 times power is also presented, which considers the case that the relay has kept silent in the whole block and did not consume any power. Hence, the overall power consumed by this direct scheme is the same as other schemes in which the relay transmit in the second block.

The non-WZ CF [46] is a special type of CF scheme that also employs the successful decoding at the destination but simplifies the relay operation without using Wyner-Ziv random binning [103,106,107]. The non-WZ CF scheme has no worse outage probability performance as the classic CF scheme using WZ binning in the slow fading HD-RC as shown in [111], therefore the following numerical examples consider the non-WZ CF for the purpose of comparison.

The outage probabilities of the previously mentioned schemes are shown in Fig. 4.3. It can be seen that without the R-D link CSI at relay, the non-WZ CF scheme performs significantly worse than all the other relaying schemes. The GQF, DF and AF schemes show the diversity advantage of relaying. Even without CSI of R-D link at relay, the GQF scheme with a fixed $R_U = 3$ performs very close to the GQF/CF scheme with complete CSI at relay. Comparing with the AF scheme, the GQF scheme outperforms in all SNR regions considered in the example. The DF scheme has a smaller value of outage probability in low SNR regions. However, in higher SNR regions, the GQF scheme outperforms DF scheme.

Different values of R_U impact the outage probability performance of the GQF scheme. When $R_U = 1$ and $R_U = 2$, the same outage probabilities of those previously mentioned relaying schemes are shown in Fig. 4.4 and Fig. 4.5. The GQF scheme outperforms other schemes in the low to medium SNR. In contrast, for $R_U = 3$, the GQF scheme shows its advantage in the high SNR. Intuitively, if the R_U can be optimized, the GQF scheme is able to show even better outage performance than those with fixed R_U.

In Fig. 4.6, the same outage probabilities of the DF,AF, GQF/CF with complete CSI

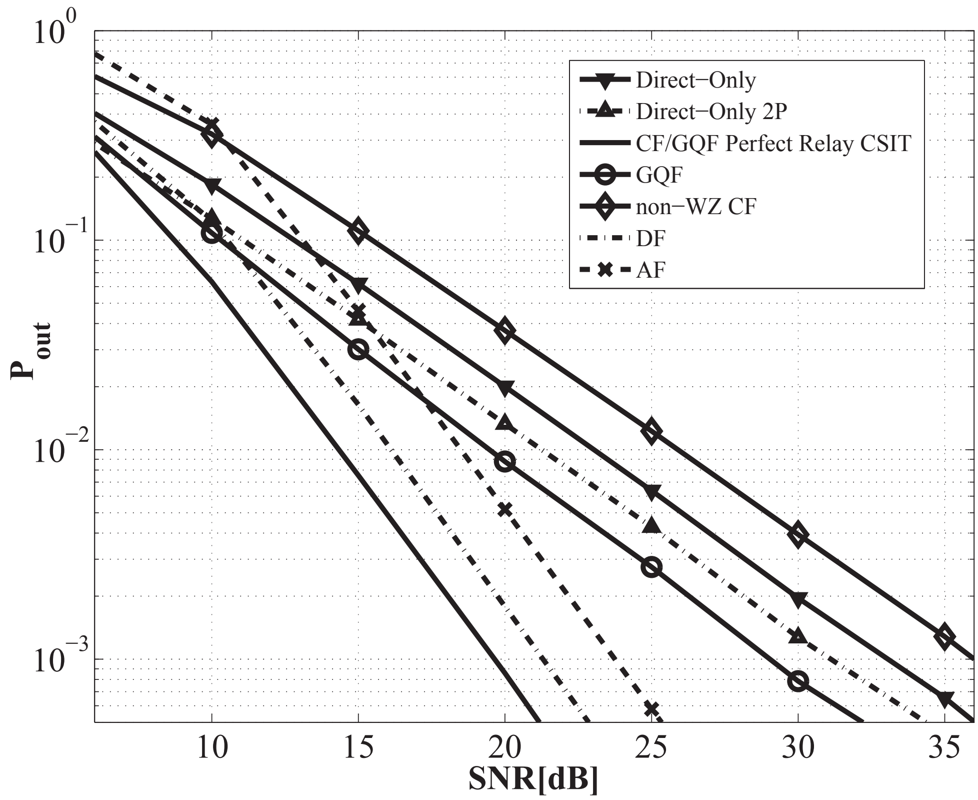


Figure 4.4: Outage Probability of the GQF scheme with $R_U = 1$, $R_1 = R_2 = 1$ and $\beta = 0.5$.

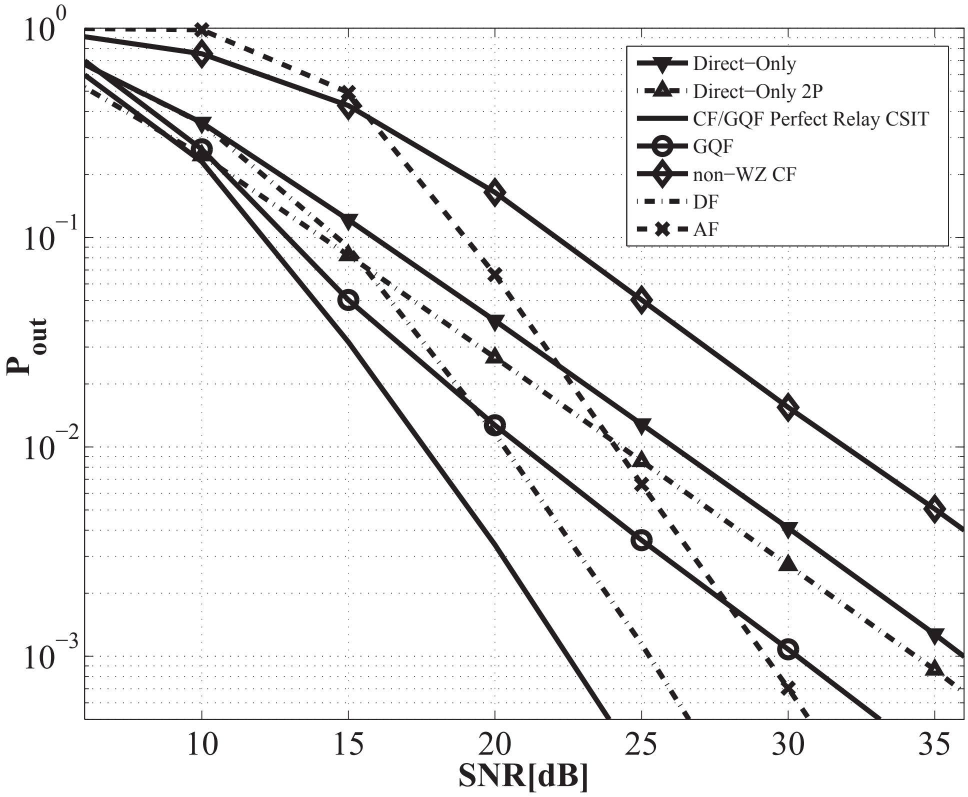


Figure 4.5: Outage Probability of the GQF scheme with $R_U = 2$, $R_1 = R_2 = 1$ and $\beta = 0.5$.

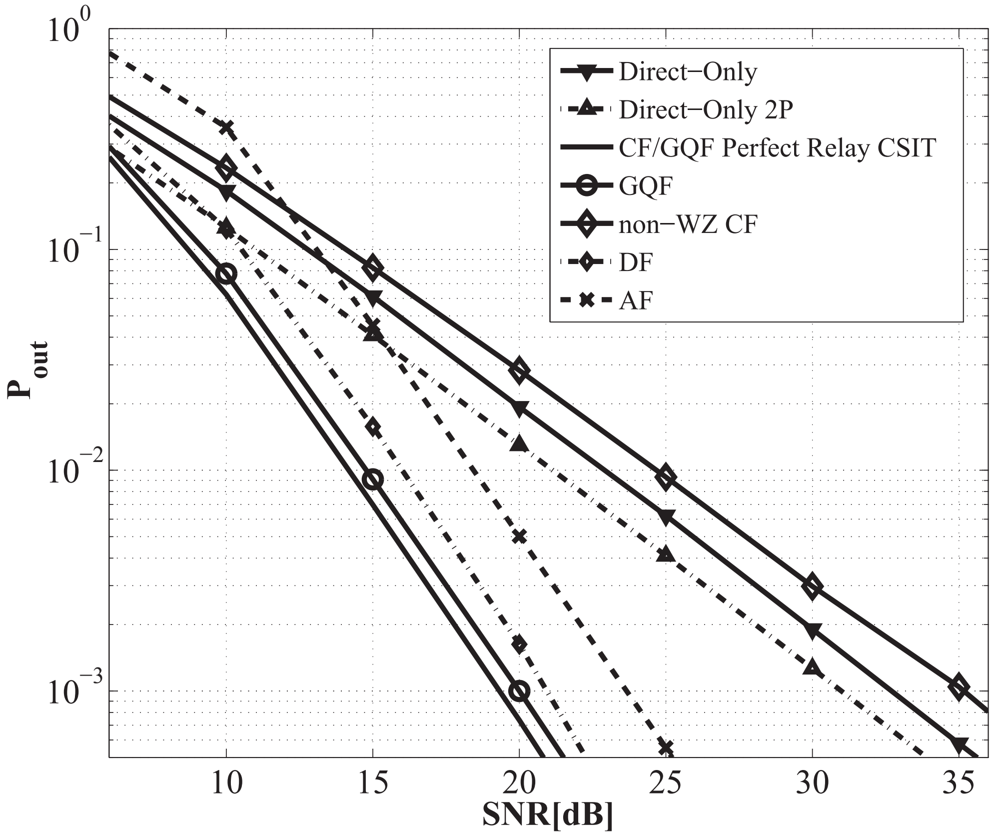


Figure 4.6: Outage Probabilities of the GQF scheme and other schemes, where $R_1 = R_2 = 1$, $\beta = 0.5$ and R_U is optimized for GQF and CF scheme

and the direct transmission are shown together with the optimized R_U for the GQF and CF scheme. With the optimization on the GQF scheme, it outperforms DF scheme in all SNR regions. Without perfect CSI at relay, the CF scheme is inefficient even with the optimized R_U. This is due to successive decoding (Sequential decoding) applied at the destination in the CF scheme. In such a scheme, the destination tries to decode the bin index sent by the relay first, then the compression index with the side information and the sources messages in the last. If the destination is not able to recover the bin index, it treats the signal from the relay X_R as interference and tries to decode the source messages without the help from relay. Hence, in the CF scheme with perfect CSI, the relay can adapt to the channel and chooses the R_U correspondingly. However, without the perfect CSI, using CF scheme become inefficient comparing to the joint-decoding based GQF scheme.

Only taking the common outage probability as the performance measure of the slow fading MARC has some limitations. The result can be affected by the fixed rate pair of (R_1, R_2). In order to gain further insight on the performance of those relaying schemes, the common expected rate is presented in the Fig.4.7. Assuming $\beta = 0.5$, $P_{11} = P_{12} = P_{12} = P_{21} = SNR = 10dB$, $P_R = SNR/(1-\beta)$ and the $\sigma_i^2 = 1$ for $i \in \{11, 21, 1R, 2R\}$, a simulation has been made for the case where the relay is moving towards the destination by increasing the variance of the R-D link σ_{R1}^2. From [48] and [31], if the relay is close to the destination, the CF-based schemes (including GQF, classic Wyner-Ziv CF, non-WZ CF, etc) has better performance than the DF-based schemes. From the Fig. 4.7, it can be seen that in high value of σ_{R1}^2 (relay is close to destination), the GQF and non-WZ CF schemes outperform DF and AF schemes. The GQF scheme outperforms non-WZ CF scheme and perform close to the case GQF/CF with complete CSI. In the region of smaller value of σ_{R1}^2, the GQF and non-WZ CF schemes converge to the line which describes the direct transmission scheme.

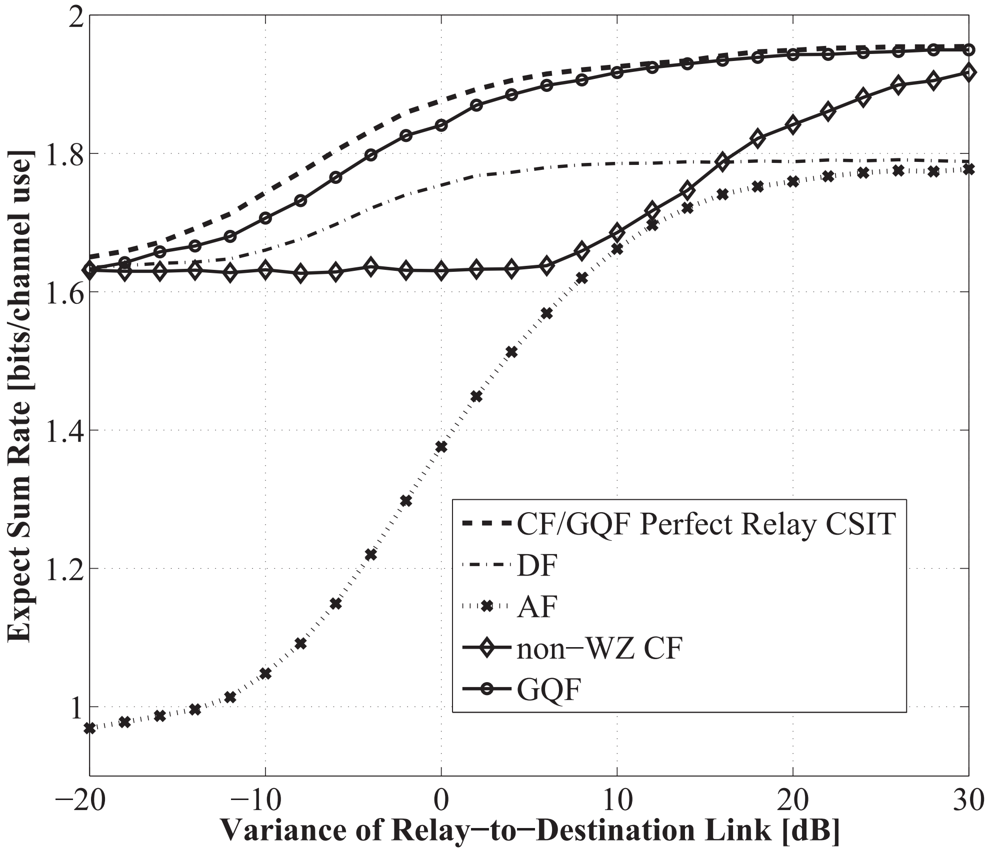


Figure 4.7: Expected Sum Rates of the different schemes based on Common Outage Probabilities

4.4 Individual Outage Probabilities and Expected Sum Rates of the GQF Scheme

In this section, the individual outage probabilities and the expected sum rates of the GQF scheme are characterized. The numerical examples are also provided to show the difference between the common and individual outage-related performance of the GQF scheme and the CF scheme. It is also shown that the GQF scheme still outperforms CF scheme when the relay-to-destination link CSI is not available at relay.

4.4.1 Individual Outage of GQF scheme

Individual outage probability and system total throughput for a slow fading MAC was s-tudied in [56, 62]. We adopt the concept and derivation into the slow-fading HD-MARC. As introduced before, the individual outage probability for a given user, say S_1, is defined as the probability that the message W_1 can not be decoded correctly at the destination D, irrespective of the successful decoding of the message W_2. We characterize the individual outage probability of the GQF scheme in this subsection. Specifically, $P^{GQF}_{out,indiv1}(R_1, R_2, R_U)$ of the source S_1 is derived. The $P^{GQF}_{out,indiv2}(R_1, R_2, R_U)$ for the source S_2 can be obtained similarly.

Note that since relay-to-destination CSI is not available at relay, it chooses a fixed rate R_U to transmit in the second slot. Different choices of R_U will have different impacts on the individual outage probabilities, which is similar to the common outage probability in previous section.

From (4.5) and (4.7), $P^{GQF}_{out,indiv1}(R_1, R_2, R_U)$ can be obtained once $P^{GQF}_{out,common}(R_1, R_2, R_U)$ from (4.27) is known since

$$\begin{aligned} P^{GQF}_{out,indiv1}(R_1, R_2, R_U) &= P_{reg,1} + P_{reg,3} \\ &= P^{GQF}_{out,common}(R_1, R_2, R_U) - P_{reg,2}. \end{aligned} \tag{4.29}$$

Therefore, in the following we focus on $P_{reg,2}$. In Fig. 4.2, given the channel fading state $\mathbf{h}$, any rate pair (R_1, R_2) lies in Region 2 can be explained as follows:

Case 1: the decoder at D can decode the message W_1 successfully while treating the signals of message W_2 as interference, thus S_1 is not in outage;

Case 2: the decoder at D can not decode W_2 even with the successful interference cancelation (SIC) of W_1, hence S_2 is in outage.

Define the following sets:

$$\mathcal{O}_{R_1,indiv1,1} := \{\mathbf{h} : R_1 > \beta I(X_{11}; Y_{D1}, \hat{Y}_R) + (1-\beta) I(X_{12}; Y_{D2}|X_R)\} \quad (4.30)$$

$$\mathcal{O}_{R_1,indiv1,2} := \{\mathbf{h} : R_1 > \beta[I(X_{11}, \hat{Y}_R; Y_{D1}) + I(X_{11}; \hat{Y}_R)] + (1-\beta) I(X_{12}, X_R; Y_{D2}) - R_U\}. \quad (4.31)$$

Then the *Case* 1 corresponds to $\mathcal{O}^c_{R_1,indiv1,1}$ and $\mathcal{O}^c_{R_1,indiv1,2}$, where $\mathcal{O}^c$ defines a complement set of $\mathcal{O}$. The previously defined sets $\mathcal{O}_{R_2}$ in (4.23) and $\mathcal{O}_{R_{2u}}$ in (4.24) describe the *Case* 2. Therefore, $P_{reg,2}$ is calculated as:

$$P_{reg,2} = Pr(\mathcal{O}^c_{R_1,indiv1,1} \cap \mathcal{O}^c_{R_1,indiv1,2} \cap \mathcal{O}_{R_2} \cap \mathcal{O}_{R_{2u}}). \quad (4.32)$$

$P^{GQF}_{out,indiv1}(R_1, R_2, R_U)$ is then obtained by (4.29). The individual outage probability for the source S_2, $P^{GQF}_{out,indiv2}(R_1, R_2, R_U)$, can be derived in a similar fashion. Applying (4.8), the expected sum rate of the GQF scheme based on the individual outage probability can be also characterized.

4.4.2 Numerical Examples

Two numerical examples of the individual outage performance are shown in this section. The simulation parameters are the same as those in previous section.

It can be seen from Fig. 4.8 that the individual outage probabilities of the GQF and CF scheme are indeed smaller than their common outage probabilities, respectively. In addition,

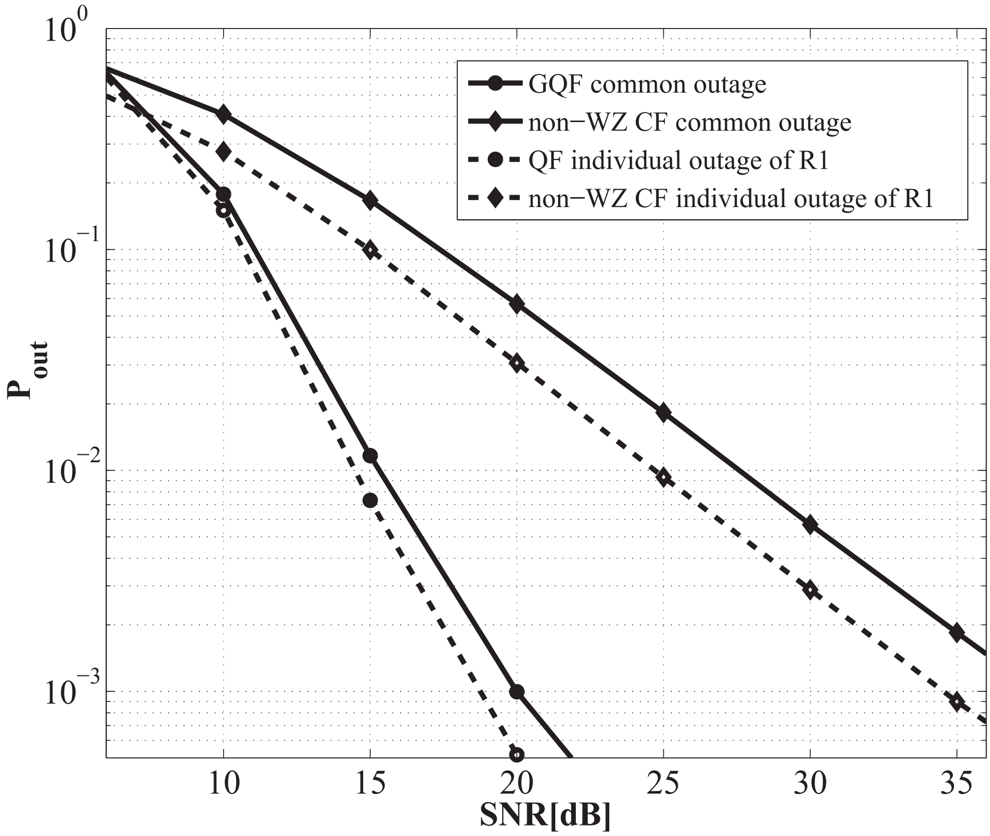


Figure 4.8: Individual outage probabilities

the GQF scheme is preferred than the CF scheme in terms of individual outage probabilities.

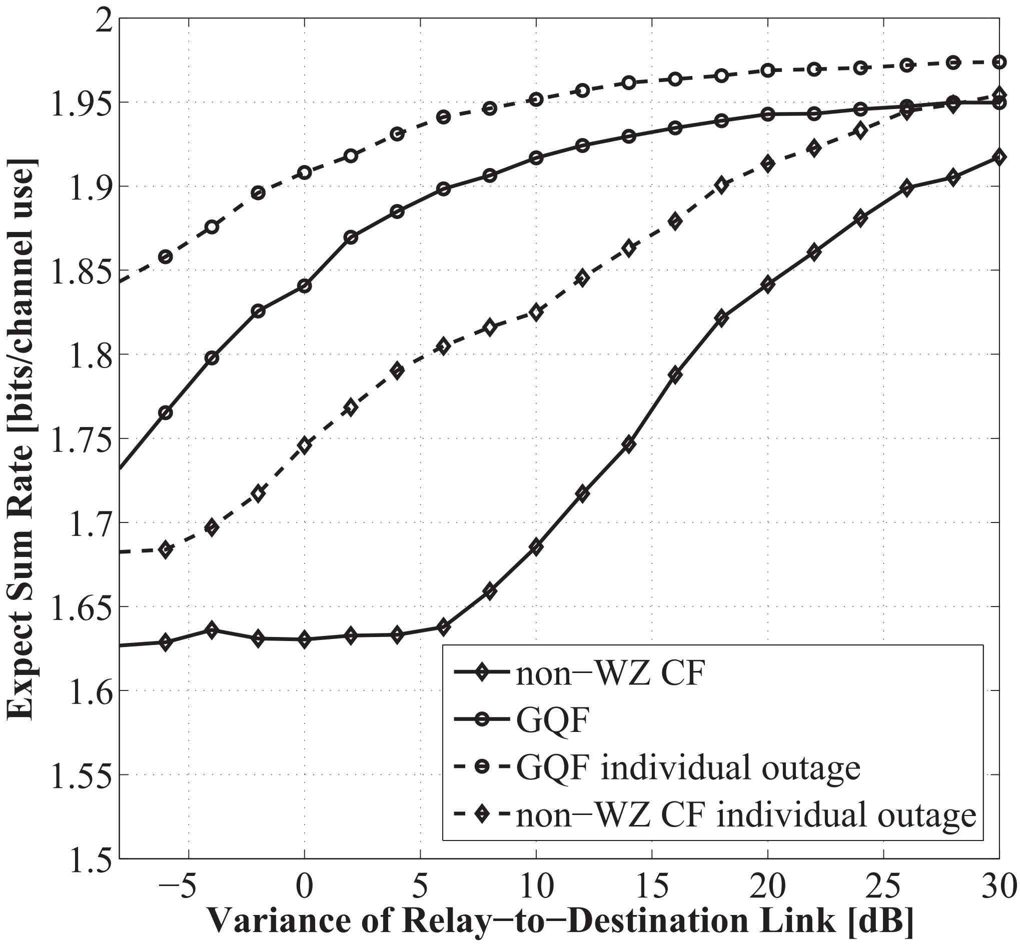


Figure 4.9: Expected Rates of the different schemes individual throughputs

Fig. 4.9 shows the expected sum rates of the GQF and CF scheme in terms of individual outage probabilities are significantly higher than those of common outage probabilities. Moreover, the GQF scheme outperforms the CF scheme in large portion of the σ^2_{RD}. Therefore, the GQF scheme can significantly improve performance of the CF scheme in slow fading HD-MARC whenever the QoS is different or not for both users.

4.5 Summary

In this chapter, the GQF scheme has been studied in the slow fading HD-MARC. Based on the achievable rate region of the GQF scheme, both the common and individual outage probabilities and the expected sum rate were derived. The numerical examples were also presented to show the significant gain obtained by the GQF scheme over the CF scheme when the relay has no access to the CSI of the relay-destination link.

Chapter 5

Diversity-Multiplexing Tradeoff

In Chapter 4, the performance of the GQF scheme over the slow fading HD-MARC is e-valuated at finite SNR. At asymptotically high SNR, the Diversity Multiplexing Tradeoff (DMT) is often used as the figure of merit. Therefore, this chapter investigates the DMT of the GQF relaying scheme. The CF scheme has been shown to achieve the optimal DMT when the CSI of the relay-destination link is available at the relay. However, having the CSI of relay-destination link at relay is not always possible due to the practical considerations of the wireless system. In contrast, in this dissertation, the DMT of the GQF scheme is derived without relay-destination link CSI at the relay. It is shown that even without knowledge of relay-destination CSI, the GQF scheme achieves the same DMT, achievable by CF scheme with full knowledge of CSI.

5.1 Preliminaries

To investigate the DMT of the GQF scheme for the slow fading HD-MARC, some background knowledge, system model and notations are given first.

5.1.1 Background

In the literature, outage-related performance is generally characterized by two different figures of merit. At finite SNR, the outage probability and the expected rates are usually used. When the asymptotically high SNR is present, the diversity-multiplexing tradeoff (DMT), which was introduced by Zheng and Tse in [119], is often served as the performance measurement. The DMT characterizes the multiple-antenna communications in terms of the relationship between system throughput and transmission reliability at asymptotically high SNR.

For the HD-MARC, the DMT of different relaying schemes, i.e. dynamic decode-and-forward (DDF), multi-access amplify-and-forward (MAF) and compress-and-forward (CF), have been characterized in [7, 12, 114]. In [114], it is shown that the CF scheme has its advantages over DDF and MAF scheme in terms of sustaining to multiple antennas case. Besides, the CF scheme is also able to achieve the optimal DMT upper bound when the multiplexing gain is higher than $\frac{4}{5}$. To achieve the optimal DMT, the CF scheme needs to have two assumptions: 1) using Wyner-Ziv coding and 2) the relay has perfect channel state information (CSI). The effect of the Wyner-Ziv coding on the DMT of the CF scheme has been investigated in [46]. In practice, having the CSI of relay-destination link at relay is generally too ideal. When the critical delay constraint exists in the wireless channels, the relay may not be able to obtain the CSI accurately.

In this chapter, we investigate the DMT of the HD-MARC based on the GQF scheme. The CF scheme achieves the optimal DMT of the symmetric HD-MARC when perfect CSI available at relay and $\frac{4}{5} < r < 1$ [114]. To make the DMT result easy to compare, the symmetric rate setup is also applied in this chapter. In [111], it is shown that a QF scheme, which is a variation of the classic CF scheme, achieves the optimal DMT for a half-duplex three-node relay channel (HD-RC) without the R-D link state available at relay. However, receiving more interfering signals at the relay and the destination in the HD-MARC makes the DMT derivation not straightforward from the HD-RC case. By taking into account the multiple access at both relay and destination, the DMT of GQF scheme will be investigated for the multi-user channel. The GQF scheme is expected to be optimal for the entire range of multiplexing gains with only the source-relay CSI at relay.

5.1.2 System Model and Notations

As in Chapter 4, a two-user slow fading half-duplex MARC is considered (Fig. 4.1). The signal transitions are the same as (4.1). In section 4.2, we have defined the notions of outage and outage probability. The common outage probability is used to analyze the DMT performance of different relaying schemes. Therefore, P_{out} means the common outage probability in the rest of this chapter.

As the DMT characterizes the system performance at asymptotically high SNR, we assume each transmitter has the power $P_i = \text{SNR}$ for $i \in \{11, 21, R, 12, 22\}$. Let the information rate $R_i = r_i \log \text{SNR}$ which is increasing with SNR by a fixed ratio r_i, where $0 < r_i < 1$ and $i \in \{1, 2\}$. In order to make our DMT comparable with the previous work [7,12,114], a symmetric MARC is considered. In such case, both S_1 and S_2 operate at the same information rate scaling ratio of r, i.e., $r_1 = r_2 = r$. The outage probability defined in (4.3) can be rewritten as

$$\begin{aligned}
P_{out}(R_1, R_2) &= Pr\{R_1 + R_2 > I_{W,sum}(\mathbf{h}) \text{ or } R_1 > I_{W,1}(\mathbf{h}) \text{ or } R_2 > I_{W,2}(\mathbf{h})\} \\
&= Pr\{r_1 \log \text{SNR} + r_2 \log \text{SNR} > I_{W,sum}(\mathbf{h}) \text{ or} \\
&\qquad r_1 \log \text{SNR} > I_{W,1}(\mathbf{h}) \text{ or } r_2 \log \text{SNR} > I_{W,2}(\mathbf{h})\} \\
&= Pr\{2r \log \text{SNR} > I_{W,sum}(\mathbf{h}) \text{ or} \\
&\qquad r \log \text{SNR} > I_{W,1}(\mathbf{h}) \text{ or } r \log \text{SNR} > I_{W,2}(\mathbf{h})\}
\end{aligned} \tag{5.1}$$

where $I_{W,sum}(\mathbf{h})$, $I_{W,1}(\mathbf{h})$ and $I_{W,2}(\mathbf{h})$ denote the instantaneous sum and individual achievable rate, respectively. The outage probability is a function of both r and SNR, therefore it can be denoted as $P_{out}(r, \text{SNR})$.

At high SNR, the outage exponent (or diversity gain) for each fixed r is defined as

$$d(r) = -\lim_{\mathrm{SNR}\to\infty} \frac{\log P_{out}(r \log \mathrm{SNR})}{\log \mathrm{SNR}},$$

where r is usually referred as the multiplexing gain. This function $d(r)$ describes the fundamental trade-off between the diversity gain and the multiplexing gain.

Following the conventions in [7, 46, 111, 119] for convenience, define the notation of exponential equality as

$$f(\mathrm{SNR}) \doteq \mathrm{SNR}^d \quad \text{if} \quad \lim_{\mathrm{SNR}\to\infty} \frac{\log f(\mathrm{SNR})}{\log \mathrm{SNR}} = d. \tag{5.2}$$

A coding scheme achieves a diversity gain or outage exponent of $d(r)$ for any fixed r when

$$P_{out}(r, \mathrm{SNR}) \doteq \mathrm{SNR}^{-d(r)}.$$

Note that since we have $R_i = r_i \log \mathrm{SNR}$ for $i \in \{1, 2\}$, each source node is transmitting with a rate which increases with the SNR. This means at any fixed SNR, the transmitter at the source i is using a code of rate $r_i \log \mathrm{SNR}$ to send information. Therefore, the DMT is not a performance measure that characterizes a specific code, but a figure of merit of a set (or series) of codes that have the similar structure but different rates according to SNR and the scaling ratio r.

5.2 DMT of the GQF Scheme

To derive the DMT of the GQF scheme in the HD-MARC, the achievable rate region and the corresponding outage event and the outage probability is shown first. In the second part, the DMT upper bound of the symmetric MARC is presented as a reference to measure the performance of different relaying schemes. The achievable DMT of the GQF scheme is derived in the third part. The DMT achieved by other relaying schemes, i.e., DDF, MAF, and classic CF, are described and discussed in the last part.

5.2.1 Achievable Rate Region and Outage Probability

The achievable rate region and outage probability of the GQF scheme for the slow fading HD-MARC have been described in chapter 4. For ease of understanding the derivation of the DMT, the encoding and decoding process, the achievable rates and the outage probability of the GQF scheme are briefly reviewed in this subsection.

In GQF, relay quantizes its observation Y_R to obtain $\hat{Y}_R$ after the first slot, and then sends the quantization index $u \in \mathcal{U} = \{1, 2, \cdots, 2^{lR_U}\}$ in the second slot with X_R. Unlike the conventional CF, no Wyner-Ziv binning is applied. At the destination, decoding is also different in the sense that joint-decoding of the messages from both slots without explicitly decoding the quantization index is performed in GQF scheme.

The following rate regions are achievable over discrete memoryless HD-MARC based on the GQF scheme:

$$\begin{aligned}
R_1 &< \beta I(X_{11}; Y_{D1}, \hat{Y}_R | X_{21}) \\
&\quad + (1-\beta) I(X_{12}; Y_{D2} | X_{22}, X_R) && (5.3)\\
R_1 + R_U &< \beta [I(X_{11}, \hat{Y}_R; X_{21}, Y_{D1}) + I(X_{11}; \hat{Y}_R)] \\
&\quad + (1-\beta) I(X_{12}, X_R; Y_{D2} | X_{22}) && (5.4)\\
R_2 &< \beta I(X_{21}; Y_{D1}, \hat{Y}_R | X_{11}) \\
&\quad + (1-\beta) I(X_{22}; Y_{D2} | X_{12}, X_R) && (5.5)\\
R_2 + R_U &< \beta [I(X_{21}, \hat{Y}_R; Y_{D1} | X_{11}) + I(X_{21}; \hat{Y}_R)] \\
&\quad + (1-\beta) I(X_{22}, X_R; Y_{D2} | X_{12}) && (5.6)\\
R_1 + R_2 &< \beta I(X_{11}, X_{21}; Y_{D1}, \hat{Y}_R) \\
&\quad + (1-\beta) I(X_{12}, X_{22}; Y_{D2} | X_R) && (5.7)\\
R_1 + R_2 + R_U &< \beta [I(X_{11}, X_{21}, \hat{Y}_R; Y_{D1}) + I(X_{11}, X_{21}; \hat{Y}_R)] \\
&\quad + (1-\beta) [I(X_{12}, X_{22}, X_R; Y_{D2}), && (5.8)
\end{aligned}$$

where $\beta = n/l$ is fixed and

$$R_U > \beta I(Y_R, \hat{Y}_R), \tag{5.9}$$

for all input distributions

$$p(x_{11})p(x_{21})p(x_{12})p(x_{22})p(x_R)p(\hat{y}_R|y_R).$$

For convenience of derivation, denote the channel coefficient vector in the slow-fading HD-MARC as $\mathbf{h} := [h_{1D}, h_{2D}, h_{1R}, h_{2D}, h_{RD}]$. Given $\mathbf{h}$, the instantaneous mutual information corresponding to the left hand of (5.3)-(5.8) are denoted as $I_{R_i}(\mathbf{h})$, where $i \in \{1, 1u, 2, 2u, 12, 12u\}$.

As the transmitters have no access to the CSI, S_1 and S_2 can only use a fixed rate pair of (R_1, R_2) to send information. The relay node has no CSI of the relay-destination link, therefore it is not able to adapt to the channel state $\mathbf{h}$ but can only assume a fixed rate of R_U for its transmission. In order to do so, the relay selects the auxiliary random variable $\hat{Y}_R$ and chooses the variance of the Z_Q. Since

$$R_U = \beta I(Y_R, \hat{Y}_R) = \beta \log(1 + \frac{1 + |h_{1R}|^2 P_{11} + |h_{2R}|^2 P_{21}}{\sigma_Q^2}) \tag{5.10}$$

and all the parameters in (5.10) are known at relay, it can choose any fixed R_U successfully.

In the GQF scheme, the destination node employs the joint-decoding technique, thus the outage event happens when either one of the conditions (5.3)-(5.8) not satisfied. The outage event can be defined as the sets of

$$\begin{aligned}
\mathcal{O}_{R_1} &:= \{\mathbf{h} : R_1 > I_{R_1}(\mathbf{h})\} \\
\mathcal{O}_{R_{1u}} &:= \{\mathbf{h} : R_1 + R_U > I_{R_{1u}}(\mathbf{h})\} \\
\mathcal{O}_{R_2} &:= \{\mathbf{h} : R_2 > I_{R_2}(\mathbf{h})\} \\
\mathcal{O}_{R_{2u}} &:= \{\mathbf{h} : R_2 + R_U > I_{R_{2u}}(\mathbf{h})\} \\
\mathcal{O}_{R_{12}} &:= \{\mathbf{h} : R_1 + R_2 > I_{R_{12}}(\mathbf{h})\} \\
\mathcal{O}_{R_{12u}} &:= \{\mathbf{h} : R_1 + R_2 + R_U > I_{R_{12u}}(\mathbf{h})\}.
\end{aligned}$$

As in (5.10), R_U is chosen to satisfy (5.9), the outage probability of the GQF scheme can be described as

$$P_{out}^{GQF}(R_1, R_2, R_U) \quad = \quad Pr\{\mathcal{O}_{R_1} \cup \mathcal{O}_{R_{1u}} \cup \mathcal{O}_{R_2} \cup \mathcal{O}_{R_{2u}} \cup \mathcal{O}_{R_{12}} \cup \mathcal{O}_{R_{12u}}\}. \quad (5.11)$$

5.2.2 DMT upper bound

The DMT upper bound of the symmetric MARC is able to characterize the performance of the different coding/relaying schemes. The closer the DMT achieved by a scheme, the better performance it has. If the achievable DMT equals the upper bound, then such scheme is said to be DMT optimal.

From [7] and [114], the upper bound is

$$d_{upper}(\bar{r}) = \begin{cases} 2 - r, & \text{if } 0 \le r \le \frac{1}{2} \\ 3(1-r), & \text{if } \frac{1}{2} \le r \le 1, \end{cases} \tag{5.12}$$

when both sources taking the same multiplexing gain of $\frac{r}{2}$, $\bar{r} = (\frac{r}{2}, \frac{r}{2})$. Since the cut-set upper bound results in a lower bound on the outage probability, the DMT upper bound of the system can be derived accordingly.

5.2.3 DMT of the GQF scheme

Based on the achievable rates and the outage probability, the DMT of the GQF scheme is derived and discussed in this subsection.

In Section 4.3, we have derived the common outage probability of the GQF scheme when the R-D CSI is not available at the relay. Through numerical examples, the GQF scheme has only a slight degradation compared to the CF/GQF scheme with perfect CSI at the relay. In this subsection, under the condition that relay dose not have the R-D CSI, the DMT of the GQF scheme is derived. The GQF scheme is shown to be optimal for the slow fading HD-MARC in terms of the DMT. The achievable DMT based on the GQF scheme is summarized in the following proposition:

Proposition 5.1 *For the HD-MARC, the GQF scheme achieves the DMT*

$$d_{GQF}(\bar{r}) = \begin{cases} 2-r, & \text{if } 0 \leq r \leq \frac{1}{2} \\ 3(1-r), & \text{if } \frac{1}{2} \leq r \leq 1. \end{cases} \tag{5.13}$$

This $d_{GQF}(\bar{r})$ is optimal as it is equal to the upper bound of the DMT of the HD-MARC.

Proof: The lower bound of the DMT achieved by the GQF scheme will be derived first. Then it is shown that the lower bound meets the upper bound. Hence, the optimal DMT is achieved by the GQF scheme. To find the lower bound on DMT, a lemma needs to be stated and proved first.

Lemma 5.1 *For the case $R_1 = R_2 = \frac{r}{2} log\ SNR$, $R_U = r_U log\ SNR$, and $\beta = r_U = \frac{1}{2}$,*

$$Pr(\mathcal{O}_{R_i}) \doteq SNR^{-(2-r)} \tag{5.14}$$

$$Pr(\mathcal{O}_{R_{12}}) \doteq SNR^{-4(1-r)} \tag{5.15}$$

$$Pr(\mathcal{O}_{R_{12u}}) \doteq SNR^{-3(1-r)} \tag{5.16}$$

where $\mathcal{O}_{R_i}$, $i \in \{1, 1u, 2, 2u\}$, $\mathcal{O}_{R_{12}}$ and $\mathcal{O}_{R_{12u}}$ are the outage events defined previously.

Proof: Following the similar steps as in [7, 46, 111], let $R_1 = R_2 = \frac{r}{2}\log \text{SNR}$, $R_U = r_U \log \text{SNR}$, $\beta = r_U = \frac{1}{2}$, and $\alpha_j = -\log|h_j|^2/\log \text{SNR}$ for $j \in \{11, 21, 1R, 2R, RD\}$. Denote the outage probability

$$Pr(\mathcal{O}_{R_i}) \doteq \text{SNR}^{-d_i}, \tag{5.17}$$

where $i \in \{1, 1u, 2, 2u, 12, 12u\}$. Then the outage exponent or the diversity order can be derived by [46, 111, 119]

$$d_i = \inf_{\mathcal{O}^+_{R_i}} (\alpha_{11} + \alpha_{1R} + \alpha_{21} + \alpha_{2R} + \alpha_{RD}) \tag{5.18}$$

where $\mathcal{O}^+_{R_i}$ is the set

$$\mathcal{O}^+_{R_i} = \{(\alpha_{11}, \alpha_{21}, \alpha_{1R}, \alpha_{2R}, \alpha_{RD}) \in \mathbb{R}^{5+} : \mathcal{O}_{R_i} \text{ occurs}\}. \tag{5.19}$$

outage exponent of d_1

First rewrite $\mathcal{O}_{R_1}$ as

$$\mathcal{O}_{R_1} =: \ \{R_1 > \beta \log(1+|h_{11}|^2 P_{11} + \tfrac{|h_{1R}|^2 P_{11}}{1+\sigma_Q^2}) + (1-\beta)\log(1+|h_{11}|^2 P_{12})\}. \quad (5.20)$$

Perform the change of variables accordingly, $\mathcal{O}_{R_1}^{+}$ can be obtained

$$\mathcal{O}_{R_1}^{+} = \{(\alpha_{11}, \alpha_{21}, \alpha_{1R}, \alpha_{2R}, \alpha_{RD}) \in \mathbb{R}^{5+} : \frac{r}{2} > \frac{1}{2}(1-\alpha_{11}, 1-\alpha_{1R})^{+} + \frac{1}{2}(1-\alpha_{11})^{+}\}. \quad (5.21)$$

Second, in order to solve the optimization problem of d_1, the above set can be partitioned into two cases. d_1 is the minimum of the two solutions.

Case 1: $\alpha_{11} \geq 1$. The inequality in $\mathcal{O}_{R_1}^{+}$ become

$$r > (1-\alpha_{11}, 1-\alpha_{1R})^{+}. \quad (5.22)$$

Based on the relationship between α_{11} and α_{1R}, the above can be further divided into:

Case 1.1: $\alpha_{11} \leq \alpha_{1R}$. We have $\alpha_{1R} \geq 1$ and the optimum values of α's for this case, denoted as a vector $\boldsymbol{\alpha}^*$, are

$$\boldsymbol{\alpha}^* = (\alpha_{11}^*, \alpha_{21}^*, \alpha_{1R}^*, \alpha_{2R}^*, \alpha_{RD}^*) = (1, 0, 1, 0, 0). \quad (5.23)$$

Case 1.2: $\alpha_{11} \geq \alpha_{1R}$. When $\alpha_{1R} \geq 1$, then the optimum $\boldsymbol{\alpha}^*$ are the same as (5.23). However, if $\alpha_{1R} \leq 1$, then (5.22) become

$$r > 1 - \alpha_{1R}. \quad (5.24)$$

The optimum $\boldsymbol{\alpha}^*$ is then

$$\boldsymbol{\alpha}^* = (1, 0, (1-r)^{+}, 0, 0). \quad (5.25)$$

Let d_{1-1} denote the minimum of the outage exponent in Case 1. Case 1.1 and Case 1.2 lead to

$$d_{1-1} = 2 - r \quad (5.26)$$

Case 2: $\alpha_{11} \leq 1$. The inequality in $\mathcal{O}^{+}_{R_1}$ changes to

$$r > (1-\alpha_{11}, 1-\alpha_{1R})^{+} + (1-\alpha_{11}). \tag{5.27}$$

Similarly as Case 1, Case 2 is also divided into two cases.

Case 2.1: $\alpha_{11} \leq \alpha_{1R}$. Then (5.27) becomes

$$r > (1-\alpha_{11}) + (1-\alpha_{11}). \tag{5.28}$$

This leads the optimum $\boldsymbol{\alpha}^{*}$ to be $(1-\frac{r}{2}, 0, 1-\frac{r}{2}, 0, 0)$.

Case 2.2: $\alpha_{11} \geq \alpha_{1R}$. (5.27) changes to

$$r > (1-\alpha_{1R}) + (1-\alpha_{11}). \tag{5.29}$$

This implies

$$\alpha_{11} + \alpha_{1R} > 2 - r. \tag{5.30}$$

Choosing α_{21}, α_{2R} and α_{RD} equal to zero, the minimum of the outage exponent for Case 2.2 is $2-r$. Combining Case 2.1 and Case 2.2, we have

$$d_{1-2} = 2 - r, \tag{5.31}$$

where d_{1-2} denotes the minimum of the outage exponent in Case 2.

In the last, combing Case 1 and Case 2 and given $d_1 = min(d_{1-1}, d_{1-2})$ we conclude

$$d_1 = 2 - r = 2(1 - \frac{r}{2}). \tag{5.32}$$

outage exponent of d_{1u}, d_2 and d_{2u}

Similarly as deriving d_1, rewrite $\mathcal{O}_{R_{1u}}$ as

$$\begin{aligned}\mathcal{O}_{R_{1u}} =: \{R_1 + R_u > \beta \log[(1+|h_{11}|^2 P_{11})(\frac{1+\sigma_Q^2+|h_{1R}|^2 P_{11}+|h_{2R}|^2 P_{21}}{1+\sigma_Q^2})] \\ +(1-\beta)\log(1+|h_{11}|^2 P_{12}+|h_{RD}|^2 P_R)\}.\end{aligned} \tag{5.33}$$

Table 5.1: Different Cases of Optimization for d_{1u}

Case No.	**Outcomes from**			**Minimum**
	$(1-\alpha_{1R}, 1-\alpha_{2R})^+$	$(1-\alpha_{11})^+$	$(1-\alpha_{11}, 1-\alpha_{RD})^+$	**outage exponent**
Case 1-1-1	$1-\alpha_{1R}$	0	$1-\alpha_{RD}$	$2-r$
Case 1-1-2	$1-\alpha_{1R}$	0	0	$2-r$
Case 1-2-1	$1-\alpha_{1R}$	$1-\alpha_{11}$	$1-\alpha_{11}$	$2-r$
Case 1-2-2	$1-\alpha_{1R}$	$1-\alpha_{11}$	$1-\alpha_{RD}$	$2-r$
Case 2-1-1	$1-\alpha_{2R}$	0	$1-\alpha_{RD}$	$2-r$
Case 2-1-2	$1-\alpha_{2R}$	0	0	2
Case 2-1-1	$1-\alpha_{2R}$	$1-\alpha_{11}$	$1-\alpha_{11}$	$2-r$
Case 2-2-2	$1-\alpha_{2R}$	$1-\alpha_{11}$	$1-\alpha_{RD}$	$2-r$
Case 3-1-1	0	0	$1-\alpha_{RD}$	3
Case 3-1-2	0	0	0	4
Case 3-2-1	0	$1-\alpha_{11}$	$1-\alpha_{RD}$	$3-r$
Case 3-2-2	0	$1-\alpha_{11}$	$1-\alpha_{11}$	$3-r$

Then we have $\mathcal{O}^+_{R_{1u}}$ as

$$\begin{aligned}\mathcal{O}^+_{R_{1u}} = \{&(\alpha_{11}, \alpha_{21}, \alpha_{1R}, \alpha_{2R}, \alpha_{RD}) \in \mathbb{R}^{5+} : \\ &r+1 > (1-\alpha_{11})^+ + (1-\alpha_{1R}, 1-\alpha_{2R})^+ + (1-\alpha_{11}, 1-\alpha_{RD})^+\}. \end{aligned} \tag{5.34}$$

Next, we solve the optimization problem of d_{1u}. Notice that in $\mathcal{O}^+_{R_{1u}}$, $(1-\alpha_{1R}, 1-\alpha_{2R})^+$ has three possible outcomes $1-\alpha_{1R}$, $1-\alpha_{2R}$ and 0. Each of $(1-\alpha_{11})^+$ and $(1-\alpha_{11}, 1-\alpha_{RD})^+$ has two possible outcomes. Based on these outcomes, $\mathcal{O}^+_{R_{1u}}$ can be partitioned into twelve cases. The derivation of the outage exponent for each of these cases is similar to previous subsection. The result of these cases are shown in the Table 5.1.

The eventual outage exponent of d_{1u} takes the smallest value from the last column of the

Table 5.1. Therefore, we have

$$d_{1u} = 2 - r = 2(1 - \frac{r}{2}). \tag{5.35}$$

Similarly as d_1 and d_{1u}, we can find d_2 and d_{2u} as

$$d_2 = d_{2u} = 2 - r = 2(1 - \frac{r}{2}). \tag{5.36}$$

outage exponent of d_{12} and d_{12u}

Following the similar process as previous subsections, we may rewrite $\mathcal{O}_{R_{12}}$ and $\mathcal{O}_{R_{12u}}$ as

$$\begin{aligned}
\mathcal{O}_{R_{12}} =: \{R_1 + R_2 > &\beta \log(1 + |h_{11}|^2 P_{11} + |h_{21}|^2 P_{21} \\
&+ \frac{(h_{11}h_{2R} - h_{1R}h_{21})^2 P_{11} P_{21} + |h_{1R}|^2 P_{11} + |h_{2R}|^2 P_{21}}{1 + \sigma_Q^2}) \\
&+ (1 - \beta)\log(1 + |h_{11}|^2 P_{12} + |h_{21}|^2 P_{22})\}. \qquad (5.37) \\
\mathcal{O}_{R_{12u}} =: \{R_1 + R_2 + R_u > &\beta \log(\frac{(1 + |h_{11}|^2 P_{11} + |h_{21}|^2 P_{21})(1 + \sigma_Q^2 + |h_{1R}|^2 P_{11} + |h_{2R}|^2 P_{21})}{1 + \sigma_Q^2}) \\
&+ (1 - \beta)\log(1 + |h_{11}|^2 P_{12} + |h_{21}|^2 P_{22} + |h_{RD}|^2 P_R)\}. \qquad (5.38)
\end{aligned}$$

Then the corresponding $\mathcal{O}^+_{R_{12}}$ and $\mathcal{O}^+_{R_{12u}}$ are

$$\begin{aligned}
\mathcal{O}^+_{R_{12}} = &\{(\alpha_{11}, \alpha_{21}, \alpha_{1R}, \alpha_{2R}, \alpha_{RD}) \in \mathbb{R}^{5+} : \\
&2r > (1 - \alpha_{11}, 1 - \alpha_{21}, 1 - \alpha_{1R}, 1 - \alpha_{2R})^+ + (1 - \alpha_{11}, 1 - \alpha_{21})^+\}. \qquad (5.39) \\
\mathcal{O}^+_{R_{12u}} = &\{(\alpha_{11}, \alpha_{21}, \alpha_{1R}, \alpha_{2R}, \alpha_{RD}) \in \mathbb{R}^{5+} : \\
&2r + 1 > (1 - \alpha_{11}, 1 - \alpha_{21})^+ + (1 - \alpha_{1R}, 1 - \alpha_{2R})^+ + (1 - \alpha_{11}, 1 - \alpha_{21}, 1 - \alpha_{RD})^+\} \\
& \qquad (5.40)
\end{aligned}$$

Next, we solve the optimization problem of d_{12} and d_{12u}. $\mathcal{O}^+_{R_{12}}$ and $\mathcal{O}^+_{R_{12u}}$ are partitioned into fifteen and thirty six cases respectively. The outage exponent results are shown in Table 5.2 and Table 5.3.

Table 5.2: Different Cases of Optimization for d_{12}

Case No.	**Outcomes from**		**Minimum outage exponent**
	$(1-\alpha_{11}, 1-\alpha_{21})^{+}$	$(1-\alpha_{11}, 1-\alpha_{21}, 1-\alpha_{1R}, 1-\alpha_{2R})^{+}$	
Case 1-1	$1-\alpha_{11}$	$1-\alpha_{11}$	$4(1-r)$
Case 1-2	$1-\alpha_{11}$	$1-\alpha_{21}$	$4(1-r)$
Case 1-3	$1-\alpha_{11}$	$1-\alpha_{1R}$	$4(1-r)$
Case 1-4	$1-\alpha_{11}$	$1-\alpha_{2R}$	$4(1-r)$
Case 1-5	$1-\alpha_{11}$	0	4
Case 2	Similar to Case 1		..
Case 3-1	0	$1-\alpha_{11}$	4
Case 3-2	0	$1-\alpha_{21}$	4
Case 3-3	0	$1-\alpha_{1R}$	$4(1-r)$
Case 3-4	0	$1-\alpha_{2R}$	$4(1-r)$
Case 3-5	0	0	4

Table 5.3: Different Cases of Optimization for d_{12u}

Case No.	**Outcomes from**			**Minimum**
	$(1-\alpha_{11}, 1-\alpha_{21})^+$	$(1-\alpha_{1R}, 1-\alpha_{2R})^+$	$(1-\alpha_{11}, 1-\alpha_{21}, 1-\alpha_{RD})^+$	**outage exponent**
Case 1-1-1	$1-\alpha_{11}$	$1-\alpha_{1R}$	$1-\alpha_{11}$	$3-3r$
Case 1-1-2	$1-\alpha_{11}$	$1-\alpha_{1R}$	$1-\alpha_{21}$	$3-3r$
Case 1-1-3	$1-\alpha_{11}$	$1-\alpha_{1R}$	$1-\alpha_{RD}$	$3-3r$
Case 1-1-4	$1-\alpha_{11}$	$1-\alpha_{1R}$	0	3
Case 1-2	Similar to Case 1-1			..
Case 1-3-1	$1-\alpha_{11}$	0	$1-\alpha_{11}$	$3.5-3r$
Case 1-3-2	$1-\alpha_{11}$	0	$1-\alpha_{21}$	$3.5-3r$
Case 1-3-3	$1-\alpha_{11}$	0	$1-\alpha_{RD}$	$3-2r$
Case 1-3-4	$1-\alpha_{11}$	0	0	5
Case 2	Similar to Case 1			..
Case 3-1-1	0	$1-\alpha_{1R}$	$1-\alpha_{11}$	3
Case 3-1-2	0	$1-\alpha_{1R}$	$1-\alpha_{21}$	3
Case 3-1-3	0	$1-\alpha_{1R}$	$1-\alpha_{RD}$	$3-2r$
Case 3-1-4	0	$1-\alpha_{1R}$	0	3
Case 3-2	Similar to Case 3-1			..
Case 3-3-1	0	0	$1-\alpha_{11}$	5
Case 3-3-2	0	0	$1-\alpha_{21}$	5
Case 3-3-3	0	0	$1-\alpha_{RD}$	4
Case 3-3-4	0	0	0	5

The eventual outage exponent of d_{12} and d_{12u} takes the smallest value from the last column of the Table 5.2 and Table 5.3. Therefore, we have

$$d_{12} = 4 - 4r = 4(1 - r) \tag{5.41}$$

$$d_{12u} = 3 - 3r = 3(1 - r). \tag{5.42}$$

The proof for Lemma 5.1 is finished as we find $d_1 = d_{1u} = d_2 = d_{2u} = 2(1 - \frac{r}{2})$, $d_{12} = 4(1-r)$ and $d_{12u} = 3(1 - r)$.

■

To find a lower bound on the DMT, the union upper bound is applied. The outage probability of the GQF scheme can be upper bounded by

$$\begin{aligned} Pr(\mathcal{O}) &= Pr(\mathcal{O}_{R_1} \cup \mathcal{O}_{R_{1u}} \cup \mathcal{O}_{R_2} \cup \mathcal{O}_{R_{2u}} \cup \mathcal{O}_{R_{12}} \cup \mathcal{O}_{R_{12u}}) \\ &\leq Pr(\mathcal{O}_{R_1}) + Pr(\mathcal{O}_{R_{1u}}) + Pr(\mathcal{O}_{R_2}) + Pr(\mathcal{O}_{R_{2u}}) \\ &+ Pr(\mathcal{O}_{R_{12}}) + Pr(\mathcal{O}_{R_{12u}}). \end{aligned} \tag{5.43}$$

In the symmetric HD-MARC, with any fixed $(\bar{r}) = (\frac{r}{2}, \frac{r}{2})$, β, r_u and $R_1 = R_2 = \frac{r}{2}\log \text{SNR}$, $R_U = r_u \log \text{SNR}$, the outage exponent of the GQF scheme and its lower bound are

$$\begin{aligned} d_{GQF}(\bar{r}, \beta, r_u) &= -\lim_{\text{SNR}\to\infty} \frac{\log \Pr(\mathcal{O})}{\log \text{SNR}} \\ &\geq -\lim_{\text{SNR}\to\infty} \frac{\log \sum_i \Pr(\mathcal{O}_{R_i})}{\log \text{SNR}} \\ &= d^*_{GQF}(\bar{r}, \beta, r_u), \end{aligned} \tag{5.44}$$

where $i \in \{1, 1u, 2, 2u, 12, 12u\}$ and $d^*_{GQF}(\bar{r}, \beta, r_u)$ denotes the lower bound. Let $d_{R_i}(\bar{r}, \beta, r_u)$ represent the outage exponent achieved by the set $\mathcal{O}_{R_i}$, we have

$$d_{R_i}(\bar{r}, \beta, r_u) = -\lim_{\text{SNR}\to\infty} \frac{\log \Pr(\mathcal{O}_{R_i})}{\log \text{SNR}}. \tag{5.45}$$

When SNR $\to \infty$, the union bound outage probability will be dominated by the term with smaller exponent. In other words, the upper bound of the outage probability is mostly determined by the term with smallest diversity order, which is shown as

$$d^*_{GQF}(\bar{r}, \beta, r_u) = \min_{\mathcal{O}_{R_i}} d_{R_i}(\bar{r}, \beta, r_u). \tag{5.46}$$

For each multiplexing exponent r, the outage exponent can be further optimized with the different choices of β and r_u

$$\begin{aligned} d_{GQF}(\bar{r}) &= \max_{\beta, r_u} d_{GQF}(\bar{r}, \beta, r_u) \\ &\geq \max_{\beta, r_u} d^*_{GQF}(\bar{r}, \beta, r_u) \\ &\geq d^*_{GQF}(\bar{r}, \frac{1}{2}, \frac{1}{2}). \end{aligned} \tag{5.47}$$

From *Lemma 5.1*, the outage exponents achieved by each of the outage event are $d_{R_1}(\bar{r}, \frac{1}{2}, \frac{1}{2}) = d_{R_{1u}}(\bar{r}, \frac{1}{2}, \frac{1}{2}) = d_{R_2}(\bar{r}, \frac{1}{2}, \frac{1}{2}) = d_{R_{2u}}(\bar{r}, \frac{1}{2}, \frac{1}{2}) = 2-r$, $d_{R_{12}}(\bar{r}, \frac{1}{2}, \frac{1}{2}) = 4(1-r)$ and $d_{R_{12u}}(\bar{r}, \frac{1}{2}, \frac{1}{2}) = 3(1-r)$. $d^*_{GQF}(\bar{r}, \frac{1}{2}, \frac{1}{2})$ is taking the minimum of the above terms, thus

$$d^*_{GQF}(\bar{r}, \frac{1}{2}, \frac{1}{2}) = \begin{cases} 2-r, & \text{if } 0 \leq r \leq \frac{1}{2} \\ 3(1-r), & \text{if } \frac{1}{2} \leq r \leq 1. \end{cases} \tag{5.48}$$

Notice that when $0 < r < 1$, $3(1-r)$ is always less than $4(1-r)$. Therefore, $d_{R_{12u}}(\bar{r}, \frac{1}{2}, \frac{1}{2})$ is smaller than $d_{R_{12}}(\bar{r}, \frac{1}{2}, \frac{1}{2})$ for all values of r.

Since $d^*_{GQF}(\bar{r}, \frac{1}{2}, \frac{1}{2})$ coincides with the upper bound of the symmetric HD-MARC from [7, 114], the GQF scheme achieves the optimal DMT. This finishes the proof of *Proposition 5.1.* ■

5.2.4 DMT of other relaying schemes and Discussions

As references, the DMT achieved by the DDF scheme [8], MAF scheme [12, 13] and the CF scheme with perfect CSI at the relay [114] are shown in the below:

$$d_{DDF}(\bar{r}) = \begin{cases} 2-r, & \text{if } 0 \leq r \leq \frac{1}{2} \\ 3(1-r), & \text{if } \frac{1}{2} \leq r \leq \frac{2}{3}. \\ \frac{2(1-r)}{r}, & \text{if } \frac{2}{3} \leq r \leq 1. \end{cases} \tag{5.49}$$

$$d_{MAF}(\bar{r}) = \begin{cases} 2-\frac{3}{2}r, & \text{if } 0 \leq r \leq \frac{2}{3} \\ 3(1-r), & \text{if } \frac{4}{5} \leq r \leq 1. \end{cases} \tag{5.50}$$

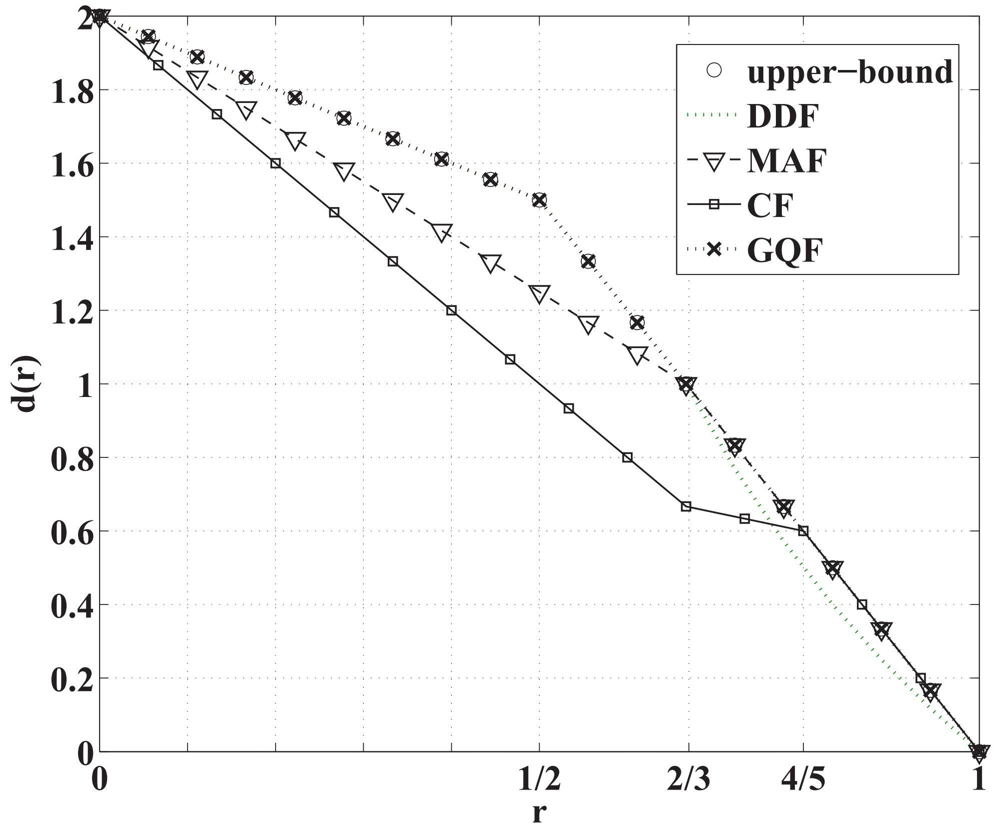


Figure 5.1: DMT of different relaying schemes in symmetric HD-MARC

$$d_{CF}(\bar{r}) = \begin{cases} 2(1-r), & \text{if } 0 \leq r \leq \frac{2}{3} \\ 1-\frac{r}{2}, & \text{if } \frac{2}{3} \leq r \leq \frac{4}{5}. \\ 3(1-r), & \text{if } \frac{4}{5} \leq r \leq 1. \end{cases} \tag{5.51}$$

The achievable DMT of different relaying schemes is shown in Fig. 5.1. Based on the range of r, we compare the DMT in the following two cases.

First, in the low multiplexing gains range, i.e. $r \leq \frac{1}{2}$, the typical outage event happens when only one of the sources is in outage. The MAF scheme is suboptimal in this case. With time sharing at the sources and the relay, the CF scheme achieves the DMT of a 2×1 MISO system. Hence, the CF scheme is optimal for a three-node relay channel but is not optimal for the symmetric HD-MARC. The DDF scheme, which decodes the sources' messages, achieves the DMT upper bound. However, as pointed out in [114], the DDF scheme is not able to sustain its optimality when terminals have multiple antennas. On the other hand, the GQF scheme achieves the optimal DMT and can be extended to the multiple-antenna case.

Second, when $r \geq \frac{1}{2}$, the typical outage event is caused when both of the users are in outage. For $r \geq \frac{2}{3}$, the DDF scheme becomes suboptimal, but the MAF scheme becomes optimal. If $r \geq \frac{4}{5}$, the CF scheme achieves optimal DMT since it compresses both sources together that is more efficient in high data rates. Under the same condition, the GQF scheme is also optimal as it is naturally a variation of the classic CF scheme. For the single antenna case, the MAF, CF and GQF schemes are all optimal. However, as mentioned in [114], it is not easy to extend the MAF scheme to the multiple-antenna case. On the other hand, the CF scheme is still optimal in the multiple-antenna case. The GQF scheme, as the CF scheme's variation, will also be optimal. Therefore, the GQF scheme is not only optimal in the single-antenna case but also in the multiple-antenna case with better extendibility.

The CF scheme mentioned above can be referred as classic (original) CF scheme [111]. Both the classic CF and GQF belong to a type of relaying that is based on compression (CF based schemes). As pointed in [7] compression works better for high multiplexing gains, the classic CF scheme achieves the optimal DMT with the perfect CSI at relay. At the same range of r, the GQF scheme also achieves optimal DMT when relay has only the CSI between source and relay. However, the GQF is also optimal for the rest of range of r since the encoding and decoding processes are different with the classic CF scheme, i.e., no WZ binning is used at the relay and joint-decoding is applied at the destination.

5.3 Summary

In this chapter, the DMT of the GQF scheme is derived for a symmetrical rate setup in an HD-MARC. In GQF, no WZ binning is used at relay and joint-decoding is applied at destination. It is shown that this GQF is optimal for the entire range of multiplexing gains while the classic CF scheme is only optimal for high multiplexing gains. Also while classic CF requires perfect CSI at the relay, the GQF scheme only needs S-R CSI at the relay. In addition, the generalized scheme and its analysis take into account the multi-user interference at relay and destination. This scheme can also be implemented for multi-user scenarios other than the HD-MARC.

Chapter 6

Conclusions and Future Work

6.1 Concluding Remarks

As a virtually distributed antenna scheme, the relaying can improve the communications system performance in terms of increased throughput or higher reliability. When the relay is close to the destination in a static channel and perfect CSI is available at the relay in a slow fading channel, the classic CF scheme is often applied since it performs better than other relaying schemes. However, the CF scheme requires the relay to perform two-step operation (quantization and WZ binning), which increases the cost of implementing such a scheme. In addition, having perfect CSI at the relay is not always possible in a wireless channel. To address these problems caused by the nature of the CF scheme, the GQF scheme is proposed in this work for the multiuser channel.

In this dissertation, the GQF relaying scheme has been extensively investigated for the half-duplex multiuser channels which include the static discrete memoryless, static Gaussian, finite SNR fading and asymptotically high SNR fading channels. The GQF scheme was shown to be able to provide similar achievable rates as the classic CF scheme while having a low-cost relay in the static channels. It is also shown that the GQF scheme can significantly improve the Rayleigh fading performance of the CF scheme when the perfect CSI is not available at the relay. The major contributions of this dissertation research are concluded in the following:

- The HD relaying for the MARC and the cMACr has been studied. A variation of the QF scheme, the GQF scheme, based on single block coding has been proposed. The GQF scheme employs joint decoding at destinations and uses a low-cost relay. For comparison purpose, a modified CF scheme was also introduced. The achievable rate regions were obtained based on both schemes. It is also shown that the achievable rate regions for the three-node HDRC can be treated as special cases of our results obtained for the five-node channel. As a further development, the achievable rate results were also extended to the Half-Duplex Gaussian MARC. Some numerical examples were provided for performance comparison. Those results show that the proposed GQF scheme can provide a similar performance as the CF scheme with only a simplified low-cost relay.

- Based on the achievable rates, the common outage probability and the expected sum rate (total throughput) of the GQF scheme have been characterized. The numerical examples show that when the relay has no access to the CSI of the relay-destination link, the GQF scheme outperforms other relaying schemes, i.e., classic CF, DF and AF. In addition, for a MAC channel with heterogeneous user channels and QoS requirements, individual outage probability and total throughput of the GQF scheme are also obtained and shown to outperform the classic CF scheme.

- At asymptotically high SNR, the DMT is often used as the figure of merit. The CF scheme has been shown to achieve the optimal DMT when the CSI of the relay-destination link is available at the relay. However, having the CSI of relay-destination link at relay is not always possible due to the practical considerations of the wireless system. In contrast, in this dissertation, the DMT of the GQF scheme is derived without relay-destination link CSI at the relay. It is shown that even without knowledge of relay-destination CSI, the GQF scheme not only achieves the same DMT as CF scheme with full knowledge of CSI, but it is also optimal in the low range multiplexing gains.

6.2 Future Works

In future, the studies of this dissertation will be continued in the following related research topics:

- In Chapter 4, the channel coefficients in $\mathbf{h}$ are assumed to have Rayleigh distributions. In order to fully investigate performance of the GQF scheme over slow fading channels, $\mathbf{h}$ with Rician distributions needs to be considered.
- Without the R-D CSI, the CF based relaying schemes show performance degradation in terms of outage probability. If a few bits level feedback from the destination to relay is available, the performance of those CF based relaying schemes (including GQF, non-WZ CF, etc) could be improved.
- The DMT of the symmetric HD-MARC was discussed in Chapter 5. The multiplexing gains of both users S_1 and S_2 under discussion are equal. However, in the practical scenarios, the asymmetric channel, in which there are different multiplexing gains r_1 and r_2, exits sometimes. The achievable DMT based on the GQF scheme needs to be investigated under the asymmetric settings in future.

6.3 Publications

Some papers described the results of this dissertation have been published or submitted in journals and communication field conferences as shown below. The co-author of these papers is Dr. Soleymani who is my supervisor and has given essential guidance on the paper writing, modification and revisions.

- M. Lei and M.R. Soleymani, "Half-duplex relaying for the multiuser channel," *in the Proceedings of the 27's Queen's Biennial Symposium on Communications (QBSC)*, Kingston, pp. 33-37, 2014.

- M. Lei and M.R. Soleymani, "Half-duplex relaying over slow fading multiple access channel," accepted by *IEEE Global Communications Conference (GLOBECOM)*, Austin, USA, 2014.

- M. Lei and M.R. Soleymani, "Performance of the generalized quantize-and-forward scheme over the multiple access relay channel," *IET Communi.*, vol. 8, no. 18, pp.3298-3307, 2014.

- M. Lei and M.R. Soleymani, "Half-duplex relaying over slow fading multiple access channel," accepted by *IEEE Wireless Communications Letters*, 2014

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