Mixed-Integer Programming Solution to Zone-Based Air Traffic Management Problem

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A thesis

In

The Department

Of

Mechanical and Industrial Engineering

Presented in Partial Fulfillment of the Requirements for

The Degree of Master of Applied Science (Industrial Engineering) at

Concordia University

Montreal, Quebec, Canada

Winter 2015

School of Graduate studies

Concordia University

This is to certify that the thesis prepared

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Entitled: Mixed-Integer Programming Solution to Zone-Based Air Traffic Management Problem

And submitted in partial fulfillment of the requirements for the degree of Master of

Applied Science (Industrial Engineering)

Complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

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ABSTRACT

The study presented in this research discusses the Air Traffic Flow Management Problem by introducing alternative routing options for aircrafts in a constrained 3-dimensional capacitated airspace. The airspace is divided into a set of capacitated 3-dimensional sectors in order to depict the concept of a free flight situation in which the pilots have more autonomy. This study aims to minimize the total arrival time of all aircrafts to their final destinations while upholding timing and routing constraints and most importantly regarding the capacity constraints through which mid-air collision is avoided and safety is ensured. In order to achieve such a goal, a non-time indexed mixed integer programming model has been developed. Solving the model provides us with a comprehensive flight schedule consisting of the sequence of sectors each flight has to take and the exact departure and arrival times from/to each sector while the capacity constraints defined for all sectors ensure flight safety and collision avoidance at all times.

This model takes multiple airports into consideration and despite the complexity of the problem and its NP-hard nature, is able to be solved for a number of flights on a personal computer using CPLEX. Furthermore, three different solution strategies are introduced in this research in order to tackle real-life size instances. First, we investigated the computational complexity of the problem by considering all flights in the system. Next, a sequential solution methodology is proposed. In the sequential solution method, first the problem is solved for a subset of flights. Next, new set of flights from remaining flight list according to their departure time are added to the airspace by considering the en-route flight plans of previously solved flight sets. The addition of new flights continued until an en-route flight plan for all flights is determined. Obviously the sequential solution method cannot guarantee optimality, yet the problem for large instance can be

solved. Finally, an iterative conflict resolution methodology is proposed. In this method, first we relax some of the constraints so large instances can be solved. Next, flights that conflict with the actual constraint are identified and problem is solved to satisfy only these flights. The iteration is continued until no unresolved conflict is left. Performance of each solution methodology is demonstrated through various case studies.

ACKNOWLEDGEMENTS

Hereby I would like to express my sincere gratitude and appreciation to Dr. Ali Akgunduz for his continuous help and support that made this research possible. His dedication and patience have led me through this journey from the very beginning to the end and his guidance has always shed light to the challenges and hardships of this study.

I would also like to extend my thankfulness to Dr. Ivan Contreras who taught the concepts and technics that made this thesis possible and was there to help throughout many issues.

Special thanks to Majid Bazrafshan and Mohammad Jeyhounian for their help and support whenever problems and impediments were encountered.

Last but not least, I would like to express my deepest appreciation and gratitude to my parents whom made my master studies in Concordia University possible by their unconditional love and support; My father for setting an amazing role model both as a scholar and engineer and my mother for her continuous help and advice in various aspects of my life.

Table of Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Introduction

During 1980s, significant congestion started to affect the United States air traffic network (Octavio Richetta, 1995). Since then, air traffic in Europe and USA has experienced a drastic growth (more than 50 percent in the last 10 years) and moreover a 50 percent traffic increase is anticipated by the year 2018 (Agustin *et al.*, 2010) . These statistics clearly demonstrates that air transportation industry from traffic management perspective, both on the ground and in the air, requires the developments of effective strategies to overcome the challenges that continuously increasing demand for air transportation causes. Recent technological advancements including use of satellites enables authorities to monitor the position and flight specific information of all airplanes in the world in real time. The overall objective of the ATM is to provide guidance to airplanes in such way that complete safety is ensured at all times from departure to arrival with cost effective ways (Fewings, 2010).

Despite all the technological and mathematical advances, the evolution of ATM has been slower to respond to the needs of air traffic growth and the technological evolution. Thus ATM is falling behind technological possibilities and advances. (Pasquini, 2005).

High pressure on the management and planning section of the air traffic management system is due to this continuous expansion of air transportation. Therefore air traffic flow management (ATFM) has become increasingly crucial as a result of the augmenting demand for air traffic transportation and also because congestion problems have become a nettlesome issue in many

European and American airports (Berstimas *et al.*, 2008). Substantial delays which cause significant costs are inflicted upon the air traffic system due to congestion; one example of this cost in the past years is the total yearly delay cost due to congestion for European airlines which was estimated to be about 5.73 billion Euros in 1999. Furthermore, ATC actions which created schedules that are far from optimal schedules cost US airline companies more than 10 billion USD (Dell'Olmo, Lulli, 2003). Another example of these delays in recent years can be found in a report from the Bureau of Transport in the US which reports that 30 per cent of domestic flights arrived more than 15 minutes late in July 2007 (Santos, Robin, 2010). It is also important to note that congestion, in particular, is found mainly in the airspace around airports rather than at the airports themselves (Bertsimas, Patterson, 2000). Leal de Matos and Powell (2003) and a report by Boeing Corp. (2011) predicted that even under the most optimistic airport development cases, capacity shortage will continue to play a crucial role in air traffic congestion both in air and on ground.

These facts and estimations mentioned in the literature of air traffic management all point to this crucial matter that the development of new methods, tools, technics and strategies for controlling the air traffic flow is absolutely vital. Some of these important aspects of this planning and development that can be mentioned are considering the route that aircrafts take, their speed, fuel consumption and the capacity of the airspace. Firstly a general definition of the airspace has to be provided. According to Fewings (2010) the airspace is divided into several segments in which different levels of services are provided under the ATM system. Aside from oceanic areas, the flight information regions (FIRs) are the largest regular segment of airspace over continental areas in use today. This term is employed to describe airspace with specific dimensions in which a flight information service and an alerting service are provided. FIRs are set up according to national borders and the boundaries of territorial waters therefore A small country may have a single FIR whereas several FIRs may exist in a larger country's airspace. Oceanic airspace is divided into oceanic information regions by international agreement through the ICAO (International Civil Aviation Organization) and assigned to a controlling authority bordering that region. In addition, in most countries, a horizontal division of the FIR may also exist. In this case the lower portion of the horizontal division is called the FIR, whereas the airspace above is named upper information region, or UIR (Fewings, 2010).

One of the necessary tools for modernization of air traffic management is the development of advanced air traffic control (ATC) tools for capacity management. The objective of designating such tools is to adjust the flow of air traffic so that it matches the available capacity of the various components of the ATC network such as airports. This matter is known as the ATC flow management problem (FMP). Flow management is particularly important at times when weather conditions significantly decrease the capacity of the ATC system (Richitta, 1995). Statistics gathered by the FAA in 2002 show that flight delays have increased by more than 58 percent since 1995 and cancellations by 68 percent (Nilim, 2004). Delays that are related to weather conditions, which are stochastic in nature, contributed to approximately 80% of the total delays in US since 1995 (Nilim, 2004).

Due to all these complex conditions in ATFM, many scholars have been conducting researches in order to come up with best practical and applicable solutions to deal with problems of the air traffic management industry. Since it was first recognized as a potential problem of future in 1935, industry partners, government agencies and scholars advocated the use of existing air traffic situation to determine the best methods for dealing with en-route air traffic control and mid-air collision hazard (Kraus, 2011) and since then a variety of methods and tools have been designated to confront and handle air sector capacity, congestion, weather conditions and fuel consumption. Of course not all of these researches are capable of implementation. Potts *et al.* (2009) have mentioned a few contributors of this problem; this usually happens due to ignoring some critical operational constraints in the modelling, relaxation of some of the hard constraints in obtaining a solution, or requiring unreasonable computational resources or taking an unreasonable time to solve the problem. One other issue is that more research studies have been conducted on the decision part rather than the control aspects. In other words, coming up with the sequence and schedule for the landing/take-off of aircrafts are more often investigated than determining how to land/takeoff in the assignment slot. Furthermore, effectiveness is one of the essential factors of a solution which can demonstrate the practicality of methods in reality. In real situations, managers are inclined to methods which are able to quickly (in a matter of seconds) find a proper solution (near-optimal) rather than an exact optimal solution achieved after a long computational time. Unfortunately a number of the algorithms that are proposed in literature take far too long to be run by air traffic managers in real time. In addition most studies consider the air traffic problem in a static environment rather than a more realistic dynamic environment. Although, different objective functions such as delay, fuel cost, punctuality, etc. have been considered in researches, a major challenge is to form an integrated model which takes many of these matters into account. Possible types of integration include: integrating runway scheduling, ground movement controlling, and gate assignment; scheduling take-offs and landings at the same time which requires runways at several airports to be scheduled simultaneously (Potts *et al.*, 2009).

Until now, the researches in the realm of ATFM have been mainly focusing on airports' congestion. On this subject, the most widely employed approach by far, has been the assignment of ground delays to departing flights which means postponing their departure time (Bertsimas,2008); Odoni was the first one to formulize this problem(Odoni,1987).

On the other hand, whereas a high number of researches can be found on airport congestion, studies dealing with en-route congestion are not that common (Bertsimas *et al*., 2008). One of the first attempts to include en-route capacity constraints in the ATFM problem was conducted by Helme (1992) who considered managing traffic under normal capacities as a planning problem and the rescheduling of traffic under temporarily reduced capacities as an operational problem and proposed a multi-commodity minimum-cost flow model on a time-space network to assign airborne and ground delays to flights of the network flow model (Helme, 1992). Some work is also done which considers rerouting, at least at a macroscopic level. An example of this type of research is Bertsimas and Stock Patterson who presented a dynamic, multi-commodity, integer network-flow model. Their model addressed routing as well as scheduling decisions, but it did not provide computational performances corresponding with the dimensions of real cases (Bertsimas, 1998). In 2008, Bertsimas, Lulli and Odoni published a paper which also took rerouting into consideration. The scope of their model is to suggest the time of departure, the route, the time required to cross each sector and the time of arrivals taking into account the capacity of all sectors and airports (Bertsimas *et al.*, 2008).

The researches mentioned in this part are just a small instance of the work that has been done in the field of air traffic management. Obviously, in order to reach feasible, practical and applicable results and design models which take critical conditions and constraints into consideration, much more research must be conducted. This study is an effort to reach a solution for the flow of air traffic while regarding the capacity of airspace and assigning the best route to each aircraft among airports.

1.2 Problem definition

The problem tackled in this thesis is a mixed integer programming (MIP) formulation of the aircraft traffic flow management problem (ATFMP) in a constrained airspace with the objective of minimizing the arrival time to the destination for each airplane in a set of airplanes. The airspace is divided into 3D sectors and travelling time and capacity is defined in these sectors. The main objective was to design a mathematical formulation which is able to give us the best route for each aircraft while considering the capacity of each sector and the optimum time to pass each sector. The model is designed to take into account multiple airports and destinations. This model benefits from the usage of a non-time indexed formulation which allows us to know the exact arrival and departure times. Aircrafts enter to or exit from the airspace through multiple given points of entries/exits. The model developed in this research is an MIP NP-hard model and the solution determines a flight plan for aircrafts by identifying the sequence of sectors to be visited in addition to the time it takes to pass each sectors while upholding all constraints at all times.

1.3 Contribution of this research

This research makes a number of contributions to the literature of air traffic management. It develops a model that is non-time indexed which gives much more accurate solutions regarding the arrival and departure times to each sector on the flights path which signifies it from Bertsimas *et al.* (2008) in which the exact time cannot be known for flights. Furthermore, the proposed model differs from Moeini (2012) as it models the aircraft routing between air-segments not on straight arcs. Although straight line travelling constraint does makes sense when a flight is taking off or landing in short altitudes around the airport but in order to depict a free flight situation the flights should have more authority than linear moves on predetermined routes. Another significance of the proposed model from Moeini (2012) is that in the proposed model aircrafts are subject to capacity constraints in air-segments. Finally, the heuristics studied in the thesis, particularly the constraint relaxation algorithm are unique to the research described in this thesis.

1.4 Roadmap of Thesis

The following is organized as follows. First, a categorization of different research subjects in the realm of air traffic management is provided. Second, the model and its distinctive features is discussed and explained, and then the mathematical formulation which represents the problem is discussed. Afterwards, various solution strategies that have been employed to reach a proper solution are presented and compared. Finally this research ends with a summarization of all that has been done and further steps that can be taken.

Chapter 2

Literature Review

2.1 General Theory and Application

There are a number of various subjects that can be mentioned in air traffic management. Some of the researches are based on a specific part of a flight such as take-off, landing or en-route flight planning. Another line of research work focuses on developing models that combine many objectives and sections together. Furthermore, literature is also divided into two areas based on stochastic or deterministic nature. In their work, Wu and Caves (2002) offered a systematic review of research on air traffic management in order to prioritize useful areas of research. In their literature review, research fall into two main categories; the system level and the airport level. The systems level consists of two subgroups; Air traffic flow management (ATFM) and airspace research. The second group which is the airport level is divided to four groups; airport capacity, airport facility utilization, aircraft operations in the airport terminal maneuvering area (TMA) and airport ground operations.

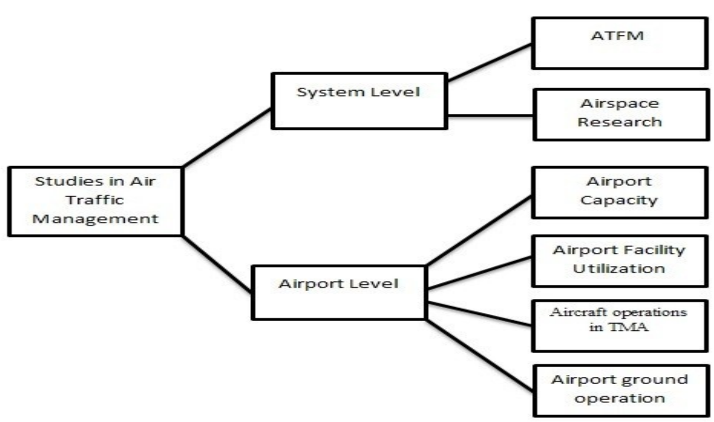

Figure 1-Wu and Caves' classification of Literature in ATFM

In their work Sama *et al*. (2012) also offered another categorization of problems in air traffic management. In their point of view, models are either “basic” or “detailed”. In their basic model, only the runways are included in the terminal control area (TCA), while detailed model schedules aircrafts on other relevant TCA resources.

The following is a general classification of different categories and models in air traffic management research.

2.2 Stochastic vs. Deterministic

Airports, regions of airspace, and specific control points throughout the airspace network can be affected by capacity disruptions which can last for a very long time. Weather conditions are one of the most prominent causes of these disruptions among a number of reasons. Some of the most important parameters through which these disruptions can be characterized include onset (the beginning of the specific condition), the severity of the condition, and its duration. The quantification of such parameters is crucial for modeling yet, they add additional complexity due to their stochastic natures (Churchill, 2011).

As mentioned earlier, extreme weather conditions are the major source of flight delay. Global warming will further contribute to the weather related delays in airline industry. Depending on the severity of the weather conditions, flights might either be canceled or delayed on the ground or aircrafts are redirected to go around these areas to reach safety and comfort, thus due to the cancellation and delay of flights or increased travel times and fuel consumption, extra expenses will occur. In addition to delays, adverse weather also affects safety of flights (Quan *et al.*, 2002).

Due to significant impact of uncontrollable factors to a typical flight, it is logical to formulate air traffic flow management problems as a stochastic model. Typically, literature tackles uncertainty by considering a set of scenarios each corresponding to a time-varying airport capacity (Mukherjee, 2004). According to Ball and Glover (2011), the most comprehensive method used to address this situation is the ground delay programs (GDPs). In their work, they define a GDP as a preemptive measure that holds aircrafts on the ground before they depart their origin airports rather than assigning delays to flights in the air thus the net effect of this measure is that the more costly and more risky airborne delay is reduced and transferred to the ground where it is more easily managed in addition to decreasing the stress on air traffic managers, who have limited options once the aircrafts are airborne (Ball, 2011).

In general, stochastic models are more computationally challenging to solve than deterministic models (Agustin *et al.*, 2010). Consequently, uncertainty or in general stochastic factors have been addressed mainly in the context of the single airport ground holding problem rather than multi airport ground holding problems (Mukherjee, Hansen, 2009). Some papers that have considered the stochastic factor are Glover, Ball (2011) and Andreatta, Dell'Olmo, Lulli (2011). Richetta and Odoni (1993) also used the concept of a scenario tree that represents evolving information about which scenario will be realized.

2.3 Dynamic vs. Static

If the planning horizon is only for an individual period it will fall into the category of static system. Whereas, if the planning horizon includes several periods; dynamic programming approaches are utilized (Agustin *et al*, 2010).

The models studied in literature most commonly deal with static cases where only an individual planning horizon is considered. In static instances, information is assumed to be available for a set of aircrafts. After new information becomes available by the arrival of the aircrafts, the solutions might become modified and updated. However, there exists a number of uncertainties related to matters such as information about aircrafts, operational environment (weather, runway situation, etc.), and taxiways. As time passes, the uncertainty is reduced by the realization of some cases. Thus in actual situations, air traffic management should be discussed in a dynamic environment (Potts, 2009).

Dynamic programming is a method to solve various optimization problems that includes the characteristics of multi-level sub-problems (Farmer, 2008). Some examples of this can be seen in a paper by Leese *et al.* (2001) in which a dynamic programming algorithm is constructed to solve this problem, using a cost function for sequencing take-off aircrafts at one of the simplified holding points at London Heathrow airport.

Another instance of the use of dynamic programming in air traffic management can be seen in Balakrishnan and Chandran (2006) for Aircraft Landing Problem (ALP) where a Dynamic-Programming-based approach that scales linearly in the number of aircrafts is presented; given a set of aircrafts, they attempt to determine the sequence that minimizes the landing time of the last aircraft subject to the operational constraints such as precedence, minimum separation requirement, and possible arrival time windows for aircrafts

2.4 Exact solution vs. Meta-heuristic solution

Alongside conventional and common methods for solving air traffic management models, some researchers have developed heuristic and meta-heuristic methods to solve their mathematical models. According to Oussedik (1998) the most popular stochastic optimization methods are the Simulated Annealing algorithm and Genetic algorithms. The reason of employing heuristic algorithms is to reach solutions in a reasonable time since aircraft sequencing and scheduling at airports is an NP-hard problem (Hancerliogullari, 2013).

Meta-heuristic algorithms offer some advantages for solving air traffic management problems since the exact algorithms may provide the mathematical optimal solution, but usually obtaining the solution requires more time than allowed in real-time cases. In these situations, heuristic algorithms seem more logical to apply (Agustin *et al.*, 2010). Agustin *et al.* (2010) state that a heuristic algorithm does not guarantee the optimality of the solution but usually the solution provided by the different approaches is the optimal one, or it is a quasi-optimal solution, at least. Genetic algorithms and ant colony optimization are examples of meta-heuristic methods used in this field. Genetic algorithms (GA) which use concepts of the evolutionary theory and natural genetics are the set of global and parallel search techniques that are able to progress towards solutions for realistic optimization problems in real-time situations (Potts, 2009). One example of using this method can be seen in Beasley *et al.* (2001). They develop a population of heuristics for solving the aircraft landing scheduling problem based on the order of each aircraft in an optimal landing queue as well as the assigned landing time of each aircraft.

Another example of applying such technics is the development of an ant colony optimization for solving ALP (Potts *et al.*, 2009). Ant colony optimization is a constructive meta-heuristic technic with biological foundations (Agustin *et al.*, 2010). Also Zhan *et al.* (2010) applied ant colony optimization technique to an aircraft arrival sequencing and scheduling problem. Their paper formulates the aircraft arrival sequencing and scheduling problem in the form of a permutation problem and proposes a new solution framework that makes the first attempt at using an ant colony system algorithm based on the receding-horizon-control to solve it. They showed that their receding-horizon-control based ant colony optimization method is robust, effective, and efficient, not only due to the fact that the ant colony optimization algorithm has a strong global search capability and has been proven to be suitable for these kinds of NP-hard problems but also due to the fact that the receding-horizon-control technic can divide the problem with receding time windows to reduce the computational burden and enhance the solution's quality (Zhan *et al.*, 2010).

2.5 Based on subject

In this section a taxonomy of models in air traffic management is offered based on the specific subject which is examined in the research.

2.5.1 Air traffic flow management problem

Agustin *et al.* (2010), state that the basic ATFMP model assumes that the airport capacity for departure and arrivals as well as the air sector capacity are deterministic functions of time, known in advance with certainty, alternative routes and flight speed are not assumed and cancellation is implicitly considered, which means no big decisions can be made about it. Flight continuation is allowed, which means a turnaround time is given for the continued flight. One of the pioneer works on flight planning in real time is presented by Odoni (1987) which minimizes congestion costs. Prior to their works, research mainly focused on the airport's congestion alone (Bertsimas, 2008). The goal of research on ATFMP is to solve actual situations that have a higher degree of complexity than those which can be dealt with using the single airport or multiple airport methodologies because the previous strategies are only applicable in situations that arise frequently in USA where the problems of congestions are confined to airports. In

Europe the situation is also critical in the air sectors and this has motivated some research teams to consider models which provide acceptable solutions (Agustin, 2010).

Recently, Bertsimas, Lulli and Odoni (2009) propose a new Integer Programing approach for the ATFM problem in which they propose a model that aggregates all phases of a flight (i.e., the phase of taking-off, of cruising and of landing; suggesting all the actions to be implemented to achieve the goal of safe, efficient, and expeditious aircraft movement). The prominent feature of their model is that it allows rerouting decisions. Their computations are done in short times (less than 15 minutes) on cases of the size of the entire US air traffic control system which enables their approach to be used as a tool for managing air traffic in the US. They introduce an objective function to minimize the ground holding and air delay by elegantly prioritizing the first one over the second one (Bertsimas, 2009)

2.5.2 Air traffic flow management rerouting problem

The Air traffic flow management rerouting problem is more difficult to solve than the previous problem but it gives a better picture of the real-life problem to solve. This line of research is important because diverting flights is one of the very common daily activities in airports. Due to the impact that such decisions have on the entire airport network, the design of new methodologies are necessary (Agustin, 2010). In this field of study the research conducted by Bertsimas and Stock in 1998 on the Boston Logan, NY La Guardia and Washington National airports is a prominent study. They propose a model for the ATFM problem with en-route capacities and extend their model to account for several variations of the basic problem, most notably, how to reroute flights and how to handle banks in the hub and spoke system (Bertsimas, 1998).

2.5.3 Aircraft Landing Problem (ALP)/Aircraft Take-off Problem (ATP)

In general, ALP and ATP consist of sequencing, scheduling, and runway assignment decision problems. It seems that researchers are more interested in ALP than ATP as ALP has been a frequent subject in literature whereas there are only a few studies on ATP. Some studies on ALP discuss that their model and solution methods can be extended for solving ATP; however these two problems differ from each other significantly. Furthermore, due to the level of uncertainty and type of operational constraints, ATP is a more complicated problem than ALP. Thus the models that have been developed for ALP cannot be easily modified for and applied to ATP (Potts, 2009).

2.5.4 Single Airport Ground-Holding Problem (SAGHP)

Single Airport Ground Holding Problem is the simplest of all methodologies in airport planning and/or aircraft routing. This methodology proposes solutions to the problem of deciding the optimal planning for one single airport, with regard to limitations such as the number of landing and take-off operations that can be handled within a given time periods (Agustin, 2010). The fact underlying ground-holding policies is that airborne delays are much more expensive than ground delays. Thus, it seems logical that one may hold an aircraft on the ground before take-off so, when the aircraft arrives at its destination, it will not have to wait in the air before landing (Vranas, 1994). In the SAGHP, airport arrival capacity forecasts are the base for decision making which includes the allocation of arrival slots to various flights in an optimal or near optimal manner. The objective is to efficiently use the available capacity while absorbing necessary delays by holding the flights on ground (Mukherjee, 2007).

One of the assumptions in this type of strategy is that there is no dependency between the flights taking off and flight arriving. In other words, landing and departures are treated independently.

In this area, many researches focus one introducing methodologies for assigning ground delays to aircrafts since it is obvious that absorbing delays on the ground is safer and less costly yet airborne delays cannot be totally avoided (Agustin, 2010).

The basic models of SAGHP assume that the capacity of an airport where an aircraft is arriving to is a deterministic function of time, known in advance with certainty. The capacity of the departure airports and air sectors are unlimited, no alternative routes are considered, the flight speed is not taken into consideration and arrival advances in the schedule are not allowed. Additionally, no continued flights are permitted (Agustin, 2010). It is clear that such assumptions fall short for handling real life situations since in reality both origin and destination airports have capacity constraints as well as the air sectors that aircraft will fly has limited capacities. Based on the availability of adequate information and the manner in which this information is updated, various versions of the SAGHP are developed to introduce models for real life scenarios (Bertsimas,1997).

2.5.5 Multi-Airport Ground-Holding Problem (MAGHP)

The Multi-Airport Ground-Holding Problem (MAGHP) was the next problem to be developed after the single ground-holding problem. Surprisingly, the best available models in this area are not extension or derivations of models developed for SAGHP (Bertsimas, 1997). The MAGHP decides on making ground-holding allocation for a whole network of airports. The objective of MAGHP is to find a ground delay program which is in consistence with the limitations of the capacity at each airport (Bertsimas, 1998). In this field of work, the relationship between different airports is also taken into consideration (Agustin, 2010).

One of the early works on this subject is by Vranas, Bertsimas and Odoni in 1994; they formulated several integer models for a network of airports and proposed a heuristic algorithm to find a feasible solution to the integer program by rounding the optimal solution of the LP relaxation. Although their main focus is on static MAGHP, they claim that their algorithms can also be applied for dynamic scenarios (Vranas, 1994).

Another instance of research in the MAGHP field was introduced by Brunetta, Guastalla, and Navazio (1998). They use a network consisting of 10 airports in their case study. They also focus on the static scenario to solve their instances using an exact algorithm and two heuristic algorithms based on "priority rules" assigned to flights.

2.6 Shortcomings in the Research of ATFM Literature

There are a number of shortcomings in the realm of air traffic management research. This thesis is an attempt to address some of these issues such as the following. First of all, as stated earlier, the majority of research in air traffic management is focusing on airport congestion (Bertsimas *et al.*, 2008). The work presented in this thesis attempts to address the flow management issue. Moreover, studies in ATFM area are modeled using periodical decision making principles. In time-indexed modeling approaches, planning horizon is divided into equal length time-intervals and decisions are made only at the beginning of a period. Hence, between two consecutive periods, whereabouts of aircrafts is not known. In practice such planning may lead to mid-air conflict. The model proposed in this thesis uses exact arrival and departure times to each segment due to non-time indexed modeling strategy. Finally, reports on air transportation and recent literature suggests that, as well as infrastructure investments, new mathematical tools that are capable of handling dynamic and stochastic nature of air traffic management problems on large instances are urgently needed. The proposed modeling approach in this thesis is unique and has potentials to tackle real life air traffic management problems.

Chapter 3

Formulation of the Air Traffic and Flight Planning Model

In this section the model that is developed in this research will be discussed. In the first part, the MIP formulation of the model is presented and the significance of this model compared to similar studies is discussed.

3.1 Formulation of the air traffic zone-based management model

This paper discusses a mixed integer programming (MIP) formulation of the aircraft routing problem (ATFMP) in a constrained airspace with the objective of minimizing the arrival time to the destination for each airplane in a set of airplanes. As opposed to the airport congestion case, the research literature dealing with en-route congestion is quite sparse (Bertsimas *et al.*, 2008). One of the first attempts to include en-route capacity restrictions in the ATFM problem was by Helme who proposed a multi-commodity minimum-cost flow on a time-space network to assign airborne and ground delay to aggregate flow of flights which are commodities of the network flow model (Helme, 1992).

The model developed in this research is an NP-hard model. The constraints that make this problem hard to solve are the capacity constraints. If the airspace sectors were to be considered with unlimited capacity which is far from reality, we would be able to solve the model for hundreds of flights within a matter of seconds. In Bertsimas and Stock Patterson (1998), the authors prove that the air traffic flow management problem is an NP-hard problem; they do so by proving the theorem that the ATFMP with all capacities equal to one is NP-hard by demonstrating how the job-shop scheduling problem reduces to ATFMP (the limited sector capacity which is equal to one is equivalent to the condition that no two tasks will ever be performed simultaneously on the same processor).

The distinctive features can be summarized as follows.

One of the main differences of this model with the one proposed by Moeini *et al.* (2012) is the definition of the airspace. In fact, the current en-route air traffic control system which covers most part of the air traffic network, allows aircrafts to fly only along certain routes but the increasing demand for air transportation is expected to constantly bring the air traffic system to a saturated and congested state (Pallottino *et al*., 2002). In the model proposed by Moeini *et al*. (2012), it is assumed that all aircrafts enter a 3D mesh network which means the airspace is considered a set of nodes and edges. Aircrafts travel inside the airspace, by visiting nodes through travelling on arcs. Thus the travel time and capacity are defined on each arc or edge. When addressing the subject of safety and collision avoidance many papers take a similar approach since air traffic management (ATM) is currently based on certain routes that pilots have to follow according to a certain flight plan. In order to model such an airspace definition, knowing factors such as the initial coordinates, angle direction, and level flight is a necessity (Alonso-Ayusa *et al.*, 2011). The conflict resolution can also be approached by resolving conflict in the potential conflict regions which are intersection points between aircraft trajectories (Vela *et al.*, 2009). Whereas in our model the airspace is divided into 3D sectors and the travelling time and capacity is defined in these sectors. Here the aim is to extend the airspace considering the concept of free flight, in which pilots and airlines can freely decide on the control of the flight, keeping in touch with air traffic controllers (ATCs) (Alonso-Ayusa *et al.*, 2011). One of the forces behind pushing the air traffic industry towards the concept of free flight is that many airlines in the United States have been complaining about policies such as ground-holding. In the free flight concept, the airlines have a considerable authority and freedom in choosing when to depart, which route to follow, at what speed, etc., for each of their flights, as long as the arrival at the destination airport matches a given time, decided by a central authority which is the FAA in the case of the United States (Andreatta, 2000). The concept of free flight (FF) has been proposed as a way to both manage the consistent increase in air traffic demands, as well as to provide economic benefits to airspace users (Hilburn *et al*., 1997). Concepts such as free flight target the predicted escalation of air traffic by dividing the responsibility for separation among pilots and air traffic controllers (Galdino *et al*., 2007). In the fixed routes situation the aircraft moves along pre-defined and fixed trajectories while the main idea of free flight is user preferred routes and the concept addresses the capability of the aircrafts to self-separate. Therefor upon the entrance to any airspace sector, the aircraft should be able to select whichever trajectory they prefer, in order to shorten the travelled distance. If an aircraft is efficiently and adequately equipped, it will have the freedom to choose its favorable route and speed in real-time which means the responsibility for separation assurance will remain with the aircraft in almost all circumstances excluding emergency situations (Pasquini *et al.*, 2005). The complexity of traffic within a sector is based on the number of flights inside it, and around its borders. Altering the take-off times of flights while ground-held, changing the approached times into the chosen airspace of airborne flights by slowing or speeding up within sectors in the airspace, as well as changes in the altitude are strategies which can resolve the complexity of sectors (Flener *et al*., 2007).

In order to model the airspace by using sectors, an important task would be the definition of arcs. A set of nodes would represent the capacitated sectors and the arcs or routes would depict the relations among the routes/arcs. Therefor there is an arc from node i to node j if i and j are contiguous sectors and sector j can be flown soon after sector i (Bertsimas *et al.*, 2008).

In addition, the capacity constraints that this model proposes are uniquely used in this model thus enabling us to solve the problem with a more realistic approach for larger sets of airplanes. These capacity constraints have been derived from the work of Jaumard, *et al*. (2012).

In comparison to the scholarly paper of Bertsimas, Lulli and Odoni (2008), this model also offers some advantages which lies in the usage of a non-time-indexed formulation. In this sense, it is similar to Moeini *et al.* (2012). The majority of literature on this topic formulate aircraft routing problem by using discrete time periods (time-indexed strategies) where t is a period of time and arrival and departure of an aircraft to a node (or sector) can only occur when the time is equal to one of the predefined periods in $t = \{0,1,\ldots,T\}$. Since the aircraft's speed in the air is too high, small miscalculations on flight times may result in mid-air collisions. Non-time indexed formulation eliminates such possibilities since the speed and flight time on each arc is in real-time. The option of rerouting is not considered within this model since all flight schedules are assigned to the set of aircrafts before they start their journey.

In order to come up with a realistic and applicable solution to the air traffic management problem and solve the NP-Hard MIP, it is assumed that aircrafts enter to or exit from the airspace through multiple given points of entries/exits. The problem formulation has been coded in CPLEX optimization studio using OPL programming language and so far is able to solve the problem for a number of aircrafts. Solving the problem will provide us with a solution that determines a flight plan for each aircraft by identifying the sequence of sectors to be visited and the aircrafts' exact arrival and departures times to these sectors in addition to the time it takes to pass each sectors.

Figure 2 is an example of the definition of sectors inside the airspace. This figure shows the 3D sectors covering the US airspace that form the network used for solving a zone based model and the manner in which the airspace is divided to sectors. This figure shows each sector of the US airspace. Each of these sectors is a flight information region (FIR) in which aircrafts move to reach to their final destination.

Figure 2-Airspace division in the US by airspacecoordination.org

The network of sectors can also be depicted as a set of nodes and arcs where the nodes represent the sectors and the arcs represent the existing relationship between the nodes (sectors); obviously a flight can go from one sector to another directly if there is a relation between them. The set of nodes of the digraph represents the set of capacitated elements of the airspace, e.g., sectors and the set of arcs defines the sequence relations. To formally describe the routing conditions, we introduce the following additional notation. For each sector i the subset of sectors which follow i is denoted by S^{+} analogously the subset of sectors that precede i is denoted by S^{-}.

3.1.1 Assumptions

The following assumptions are made in order to model this air traffic management problem:

- An aircraft may only visit a node once during its journey from its origin to destination
- The time each aircraft can spent in each sector in limited between t_{min} and t_{max} minutes.
- A flight has only a single destination point.
- Weather condition is normal with no drastic change during the timeline
- The capacity of all sectors is equal
- The capacity of the entry and exit points are unlimited

The first assumption will prevent circular movements of aircrafts inside the network. Although in real life situations sometimes flights might return to one or more of the previous sectors, in our model in order to avoid introducing other indexes this type of movement is prohibited. The third assumption prevents the flights from entering other exit sectors. The forth assumption is made because the problem this research proposes is a deterministic problem and the last assumption is made for simplicity although in real life the capacity of airports is limited.

3.1.2 Description of model parameters and decision variables

The following is the parameters and decision variables used for the formulation of the problem:

<u>Parameters</u>

F Set of flights, indexed by f

F^+	Set of flights that are already solved in previous iterations
Ext^f	Set of arcs connecting to the entry point of flight *f*
Ent^f	Set of arcs connecting to the exit point of flight *f*
Td^f	Scheduled departure time of flight *f*
S^+	Set of sectors including entry/exit points
S^-	Set of sectors excluding entry/exit points
$N^+(j)$	Set of sectors proceeding sector *j*
$N^-(j)$	Set of sectors before sector *j*
L	Set of links connecting two neighboring sectors, indexed by l. Links $\vec{l}$ and $\overleftarrow{l}$ represent opposite directions between two neighboring sectors.
$L(j)$	Set of links connecting sector j to its neighbors
$L^+(j)$	Set of links entering sector *j*
$L^-(j)$	Set of links leaving sector *j*
CAP_j	Number of flights allowed inside sector *j*
t_{min}^j	Minimum time it takes to cross a sector
t_{max}^j	Maximum time it takes to cross a sector
M	A large real number

Decision Variable

$x_s^f =$ $\begin{cases} 1 & if\ flight\ f\ travels\ in\ sector\ s \\ 0 & otherwise \end{cases}$

$\beta_s^{ff'}$ $\begin{cases} 1 & if\ flight\ f\ departs\ sector\ s\ after\ flight\ f' \\ 0 & otherwise \end{cases}$

$\alpha_s^{ff'}$ $\begin{cases} 1 & if\ flight\ f\ arrives\ sector\ s\ after\ flight\ f' \\ 0 & otherwise \end{cases}$

$\lambda_s^{ff'}$ $\begin{cases} 1 & if\ flights\ f\ and\ f'\ present\ in\ sector\ s\ at\ the\ same\ time \\ 0 & otherwise \end{cases}$

$d_s^f \in R^+$ Departure time for flight *f* from sector s

$a_s^f \in R^+$ Arrival time for flight *f* to sector s

t_s^f Travelling time in sector *s*

3.1.3 Objective Function and Constraints

The objective function of this problem aims at minimizing the total arrival times of all flights to their destination sector. The most common objective functions used in the existing ATFM models in literature, aims to minimize the total delay costs for all flights. One of the downsides of such an approach is that the solution to these models results in an order of flight arrivals that could noticeably differentiate from the initial published flight schedules (Bertsimas, Gupta, 2009). An ideal and comprehensive objective function for this problem could have separated ground delays and airborne delays and emphasized on the minimization of ground delays by assigning less cost to ground holdings and also taking fuel consumption into consideration. Nevertheless the objective function we used in this thesis can be easily adapted to tackle various goals

Objective function

$$MIN \sum_{f \in F} \sum_{l \in L^+(Exit^f)} a_l^f$$

Routing constraints

$$\sum_{l \in L^-(Ent^f)} x_l^f = 1 \qquad \forall f \in F \tag{1}$$

$$\sum_{l \in L^+(Exit^f)} x_l^f = 1 \qquad \forall f \in F \tag{2}$$

$$\sum_{l \in L^+(Ent^f)} x_l^f = 0 \qquad \forall f \in F \tag{3}$$

$$\sum_{l \in L^-(Exit^f)} x_l^f = 0 \qquad \forall f \in F \tag{4}$$

$$\sum_{l \in L(s)} x_l^f \leq 2 \qquad \forall f \in F\,; \forall s \in S^- \tag{5}$$

$$\sum_{l \in L^+(s)} x_l^f \leq 1 \qquad \forall f \in F;\ \forall s \in S^- \tag{6}$$

$$\sum_{l \in L^-(s)} x_l^f \leq 1 \qquad \forall f \in F;\ \forall s \in S^- \tag{7}$$

$$x_{\vec{l}}^{f} + x_{\overleftarrow{l}}^{f} \leq 2 \qquad \forall f \in F, l \in L \qquad (8)$$

$$\sum_{l \in L^{+}(s)} x_{l}^{f} = \sum_{l \in L^{-}(s)} x_{l}^{f} \qquad \forall f \in F;\ \forall s \in S^{-} \qquad (9)$$

The first set of constraints is related to assigning routes to each flight to reach its destination. Constraint (1) ensures that all flights depart from one of the entry points using a single sector adjacent to the entry point. Similarly, constraint (2) ensures that all flights leave the system through one of the sectors adjacent to the available exit points (4 sectors are used as entrance or exit purposes in our experiments). Constraints (3) ensures that no flight goes in the entry point through one of the sectors that only leads to an entry point and constraint (4) makes sure that no flights come out of the exit point and the flights journey ends with the exit point. Constraint (5) ensures that a flight will not go through another exit point other than its designated destination. Constraints (6) and (7) and (8) prevent circular routes for the flights and returns to the same point and make sure each node or sector is visited maximum one time and that flights will not go back to the sectors they have already visited; these constraints are added in order to reduce the complexity of the problem by avoiding the introduction of another index that would signify each arrival and departure to a sector (Moeini *et al.*, 2012). Constraint (6) ensures that only one link is used to enter *sector s* and Constraint (7) ensures that *sector s* is exited through only one link . Constraint (8) states that an airplane can visit a sector only once. Constraint (9) guarantees all aircrafts arriving to a sector will also leave. Figure 3, illustrates constraints 6, 7 and 8 and their difference. According to constraint (6), only one of incomming links ($L^{+}(s)$) can be chosen to fly to *sector s*. Afterwards, sectors 1, 2 and 3 proceed *sector s* and are part of the set $L^{-}(s)$ so only one of them can be chosen to depart from *sector s*.

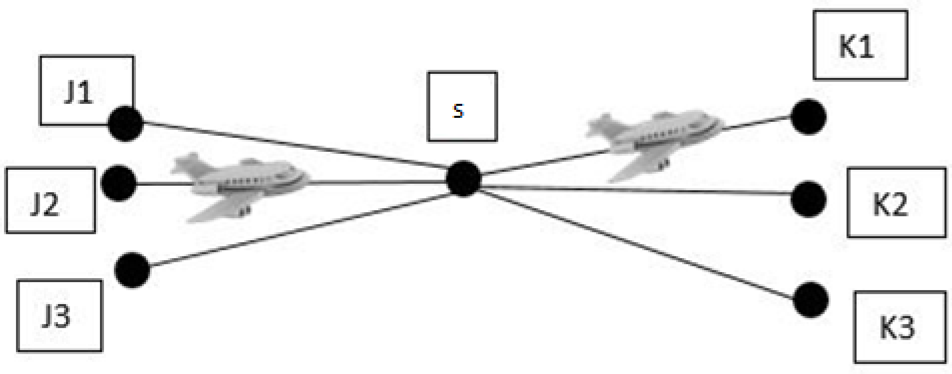

Figure 3-Illustration of constraints 6, 7 and 8

<u>Timing constraints</u>

$$a_l^f \leq Mx_l^f \qquad \forall f \in F\,;\, l \in L \qquad (10)$$

$$d_l^f \leq Mx_l^f \qquad \forall f \in F\,;\, l \in L \qquad (11)$$

$$d_l^f \geq Td^f x_l^f \qquad \forall f \in F; l \in L^{-}(Ent^f) \qquad (12)$$

$$\sum_{\mathrm{l} \in L^{+}(s)} a_{\mathrm{l}}^f + t_s^f + M * \left(\sum_{\mathrm{l} \in L^{+}(s)} x_{\mathrm{l}}^f - 1 \right) \leq \sum_{\mathrm{l}' \in L^{-}(s)} d_{\mathrm{l}'}^f \qquad \forall f \in F; s \in S^{+} \qquad (13)$$

$$t_s^f = \sum_{\mathrm{l} \in L^{-}(s)} d_{\mathrm{l}}^f - \sum_{\mathrm{l}' \in L^{+}(s)} a_{\mathrm{l}'}^f \qquad \forall f \in F;\ s \in S^{-} \qquad (14)$$

$$t_{min}^{s} \sum_{l \in L^{+}(s)} x_{l}^{f} \leq t_{s}^{f} \leq t_{max}^{s} \sum_{l \in L^{+}(s)} x_{l}^{f} \qquad \forall f \in F;\ s \in S^{-} \qquad (15)$$

$$a_{l}^{f} = d_{l}^{f} \qquad \forall f \in F; l \in L \qquad (16)$$

The second set of constraints is introduced to determine traveling time related variables: arrival to a sector and departure from that sector and travelling time on the sector. Constraint (10) and (11) force an arrival or departure time to be set on a particular link if the airplane leaves the given sector from this link. Constraint (12) ensures that aircrafts depart the origin after their scheduled departure time. This constraint can be relaxed in reality if early departure is permissible (not commonly used). Constraint (13) establishes the relationship between arrival time to a sector and departure time from the sector. It also determines the flight time in the sector. Constraint (14) on the other hand removes the impact of inequality used in (13). Since there is an upper and lower bound for the travelling time in a sector, constraint (15) is introduced. Finally in constraint (16), it is ensured that, airplanes do not spend any time on the links; (links are used to follow the navigation of aircrafts). Figure 4 illustrates the usage of sectors for navigation.

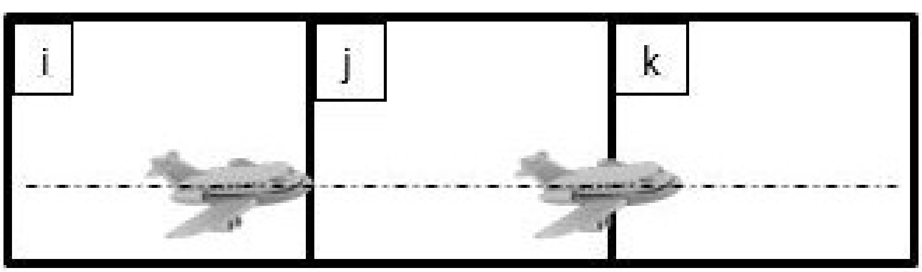

Figure 4-Arrival and Departure to sectors

<u>Capacity constraints</u>

Constraints (17-22) are subject to the following set of flights and sectors.

$$\forall f, f' \in F; f < f'; \forall s \in S^-$$

$$\sum_{l' \in L^-(s)} a_{l'}^{f'} \leq \sum_{l \in L^+(s)} a_l^f + M(1 - \alpha_s^{ff'}) \tag{17}$$

$$\sum_{l \in L^+(s)} a_l^f \leq \sum_{l' \in L^+(s)} a_{l'}^{f'} + M(\alpha_s^{ff'}) \tag{18}$$

$$\sum_{\mathrm{l} \in L^+(s)} a_{\mathrm{l}}^f \leq \sum_{\mathrm{l}' \in L^-(s)} d_{\mathrm{l}'}^{f'} + M(1 - \beta_s^{ff'}) \tag{19}$$

$$\sum_{\mathrm{l}' \in L^-(s)} d_{\mathrm{l}'}^{f'} \leq \sum_{\mathrm{l} \in L^+(s)} a_{\mathrm{l}}^f + M(\beta_s^{ff'}) \tag{20}$$

$$\alpha_s^{ff'} + \alpha_s^{f'f} \geq \sum_{\mathrm{l} \in L^+(s)} x_{\mathrm{l}}^f + \sum_{\mathrm{l}' \in L^+(s)} x_{\mathrm{l}'}^{f'} - 1 \tag{21}$$

$$\beta_s^{ff'} + \beta_s^{f'f} \geq \sum_{\mathrm{l} \in L^+(s)} x_{\mathrm{l}}^f + \sum_{\mathrm{l}' \in L^+(s)} x_{\mathrm{l}'}^{f'} - 1 \tag{22}$$

In order to count the number of flights inside the sectors, three decision variables have been introduced. Constraints (17) and (18) determine if flight f arrives to sector s before flight f' $(\alpha_s^{ff'} = 1)$. Similarly, constraints (19) and (20) identify if flight f arrives to sector s before the departure of flight f' $(\beta_s^{ff'} = 1)$. Finally constraints (21) and (22) are used to associate binary decision variables $\alpha_s^{ff'}$ and $\beta_s^{ff'}$ with the decision variable x_{l}^f as constraints (17-22) should only be considered if both flights f and f' use sector s. Figure 5, illustrates the definition of these three capacity constraints.

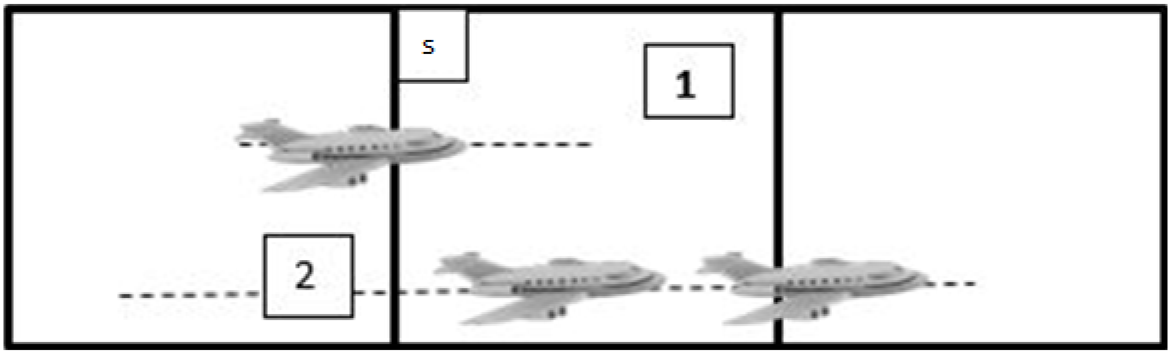

Figure 5-Aircrafts share the same sector for a period of time

For constraint (23- 25): $\forall\, f \,\&\, f' \in F: f \neq f' \,\&\, \forall\, s \in S^-$

$$\alpha_s^{ff'} + \beta_s^{f'f} - 1 \leq \lambda_s^{ff'} \tag{23}$$

$$\lambda_s^{ff'} \leq \alpha_s^{ff'} \tag{24}$$

$$\lambda_s^{ff'} \leq \beta_s^{ff'} \tag{25}$$

$$\sum_{f' \in F: f \neq f'} \lambda_s^{ff'} \leq CAP_s - 1 \qquad \forall\, f \in F \,\&\, \forall\, s \in S^- \tag{26}$$

ATM provides safety by imposing a minimum separating distance between aircrafts through applying the three factors of vertical and horizontal separation standards between each aircraft, the degree of separation depending on radar coverage and the navigation capabilities of individual aircrafts (Fewings, 2010). In this approach, safety distances and standards among flights are embedded in the number of flights which is allowed inside each sector at all times. This number is the definition of sector capacity in this problem. In order to count the number of flights inside the sectors, three decision variables have been introduced. Let us now introduce a set of constraints to tackle the capacity limitation in each sector. Up to this point, using constraints (17-22), we are able to identify if two flights spend any time together in the same sector. Constraints (23-26) are used to count the number of aircrafts already in the sector s at the time flight f arrives to the sector. Consequently, it is guaranteed that at all time, there cannot be more aircrafts in the same sector than the specified capacity (CAP_j).

Figure 6 demonstrates the arrival and departure time of the two flights and shows the time both flights are inside one sector; the time period that falls between a_1 which is the arrival time of the first flight to sector s and d_2 which is the departure time of the second flight from sector *s*, would be the mutual time, the two flights spend in sector *s*.

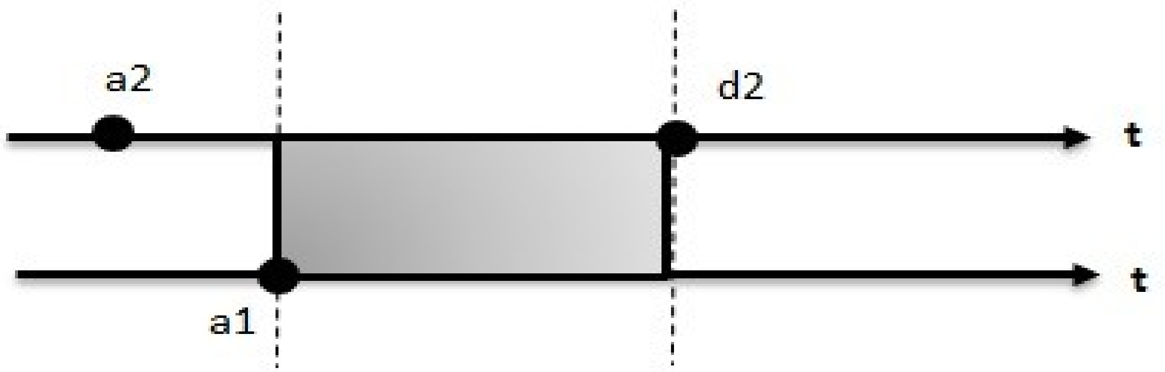

Figure 5-Time two flights spend in the same sector

3.1.4 Extended Formulation

In order to solve the ATFM problem for larger instances within a reasonable time window, a heuristic, namely sequential planning has been proposed. The proposed heuristic has been utilized to solve large scale ATFM instances. The proposed sequential planning method is a two-step iterative approach. First, the problem is solved for a small set of flights(F). Since the departure of aircrafts follows a sequential order, determination of the initial set of flights is according to their departure times from origin airports. After solving the initial set using the original model as described earlier in this thesis, solution set of R that contains flight information for the flight set F is obtained. Accordingly, there exists a unique solution $r \in R$ for all flights in F where $r^f = \left\{x_s^f, d_s^f, a_s^f, \gamma_s^{ff'} : s \epsilon S^-; l \in L(s)\; f, f' \in F\right\}$. Once the problem is solved for the initial set of flights, we add these flights into the flight set with solution(F^C). In the next phase, the new set of flights (F^N) is included in the problem set. Since there already exists a unique solution for $f \in F^C$, the problem is now solved for flights only in F^N in such a way that all constraints for flights in F^C and F^N are satisfied. Once the problem is solved for flights both in F^C and F^N collectively, the solution set R^N is obtained and the flights in F^N are added to the flights in F^C. If there is any flight in F^C with arrival time to the destination airport earlier than the earliest departure time of a flight in the unsolved flight sets, such flights is removed from the F^C set to reduce the computational complexity. In order to apply the sequential solution technique, the following constraints are added to the formulation in regard to the sector capacity.

<u>Capacity constraints for comparing current traffic $\left(f^C \in F^C\right)$ with previous traffic $\left(f^N \in F^N\right)$</u>

All constraints from number (27) to number (35) are subject to the following set of flights and sectors.

$$\forall f^C \in F^C;\ f^N \in F^N : f^s < f^N;\ \forall s \in S^-$$

$$\sum_{l \in L^+(s)} a_l^{f^C} \leq \sum_{l \in L^-(s)} d_l^{f^N} + M(1 - \beta_s^{f^C f^N}) \tag{27}$$

$$\sum_{l \in L^-(s)} d_l^{f^N} \leq \sum_{l \in L^+(s)} a_l^{f^C} + M(\beta_s^{f^C f^N}) \tag{28}$$

$$\alpha_s^{f^C f^N} + \alpha_s^{f^N f^C} \geq \sum_{l \in L^+(s)} x_l^{f^C} + \sum_{l \in L^+(s)} x_l^{f^N} - 1 \tag{29}$$

$$\beta_s^{f^C f^N} + \beta_s^{f^N f^C} \geq \sum_{l \in L^+(s)} x_l^{f^C} + \sum_{l \in L^+(s)} x_l^{f^N} - 1 \tag{30}$$

$$\sum_{l\in L^{+}(s)} a_{l}^{f^{N}} \leq \sum_{l\in L^{+}(s)} a_{l}^{f^{C}} + M(1-\alpha_{s}^{f^{C}f^{N}}) \tag{31}$$

$$\sum_{l\in L^{+}(s)} a_{s}^{f^{c}} \leq \sum_{l\in L^{+}(s)} a_{s}^{f^{N}} + M(\alpha_{s}^{f^{C}f^{N}}) \tag{32}$$

$$\alpha_{s}^{f^{C}f^{N}} + \beta_{s}^{f^{C}f^{N}} - 1 \leq \lambda_{s}^{f^{C}f^{N}} \tag{33}$$

$$\lambda_{s}^{f^{C}f^{N}} \leq \alpha_{s}^{f^{C}f^{N}} \tag{34}$$

$$\lambda_{s}^{f^{C}f^{N}} \leq \beta_{s}^{f^{C}f^{N}} \tag{35}$$

<u>Capacity constraints for New Flights only $\left(f^{N} \in F^{N};\ f^{'N} \in F^{N};\ f^{N} < f^{'N}\right)$</u>

Constraints (36-39) are subject to the following set of flights and sectors.

$\forall f^{N}, f^{'N} \in F^{N}: f^{N} < f^{'N}; \forall s \in S^{-}$

$$\sum_{l\in L^{+}(s)} a_{l}^{f^{N}} \leq \sum_{l\in L^{-}(s)} d_{l}^{f^{'N}} + M(1-\beta_{s}^{f^{N}f^{'N}}) \tag{36}$$

$$\sum_{l\in L^{-}(s)} d_{l}^{f^{'N}} \leq \sum_{l\in L^{+}(s)} d_{l}^{f^{N}} + M(\beta_{s}^{f^{N}f^{'N}}) \tag{37}$$

$$\sum_{l\in L^{+}(s)} a_{l}^{f^{'N}} \leq \sum_{l\in L^{+}(s)} a_{l}^{f^{N}} + M(1-\alpha_{s}^{f^{N}f^{'N}}) \tag{38}$$

$$\sum_{l\in L^{+}(s)} a_{l}^{f^{N}} \leq \sum_{l\in L^{+}(s)} a_{l}^{f^{'N}} + M(\alpha_{s}^{f^{N}f^{'N}}) \tag{39}$$

$$\alpha_{<k>}^{ff'} + \beta_{<k>}^{f'f} - 1 \leq \lambda_{<k>}^{ff'}; \tag{40}$$

$$\lambda_{<k>}^{ff'} \leq \alpha_{<k>}^{ff'}; \tag{41}$$

$$\lambda_{<k>}^{ff'} \leq \beta_{<k>}^{ff'}; \tag{42}$$

Let us now write the capacity constraints for all flights in the system.

$$\sum_{f'^C \in F^C : f^C < f'^C} \lambda_s^{f^C f'^C} + \sum_{f^N \in F^N} \lambda_s^{f^C f^N} \leq \; CAP_s - 1 \quad \forall f^c \in F^C;\; \forall s \in S^- \tag{43}$$

$$\sum_{f^N \in F^N : f^C < f^N} \lambda_s^{f^C f^N} + \sum_{f'^N \in F^N} \lambda_s^{f^N f'^N} \leq \; CAP_s - 1 \quad \forall f^N \in F^{CN};\; \forall s \in S^- \tag{44}$$

Constraint (43) ensures that no aircraft from the current solution shares the airspace with other aircrafts (both $f^C \in F^C$ and $f^N \in F^N$) at the same time that aim to access capacity usage. Similarly, constraint (44) ensures that an aircraft from the new list does not create capacity conflict with other aircrafts (both $f^C \in F^C$ and $f^N \in F^N$) at all times.

Chapter 4

Solution and results

In this section, the designed network used to carry out the experiments is introduced, the proposed air traffic flow planning model is tested on scenarios in which the model is solved for all flights at once and in scenarios in which the model is solved iteratively and each batch of flights is scheduled based on the current existing traffic and also the model is solved by using a third heuristic method developed for this problem discussed in the end of this chapter. In all cases, the objective is to determine conflict free route plans for aircrafts with minimum delay while upholding the limitations of airspace capacity. As mentioned before the non-time indexed formulation allows us to know the exact arrival and departure time for each sector on a flight's path thus safety is ensured through the capacity constraints at all times.

4.1 Network of sectors and data used in solving the problem

Figure 7 introduces the network which is used in experiments to solve the model. As it can be observed from the figure, 4 entry/exit points have been considered for the airspace and including the entry/exit points there is a total of 36 nodes and 98 relations. Two of these entry/exit points are considered more important and bigger airports than the other two so they participate in a larger percentage of air traffic in the network. Figure 8 illustrates the relations among all sectors and shows the arcs (edges) among all sectors. As mentioned before a flight can only go from one sector to another if there is a relation (represented by arc in the figure) between them. For example sectors 25, 5, 6 and 28 can be flown before or after sector 27, so there is an arc from sector 27 to these sectors.

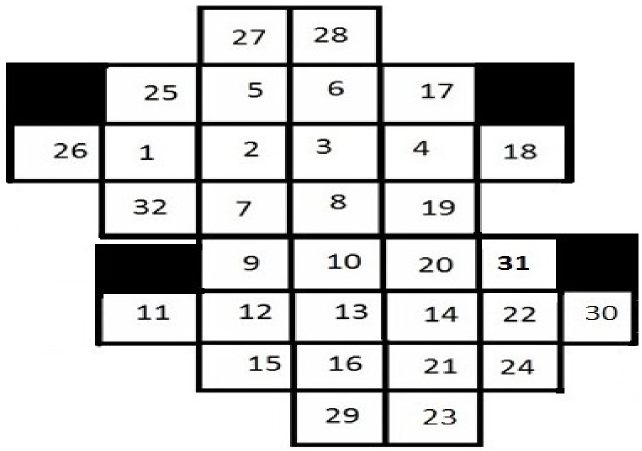

Figure 6-Network Diagram

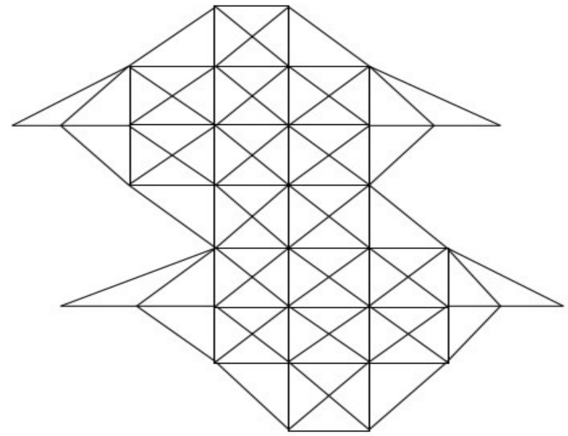

Figure 7-Sector Relations

All scenarios were tested on various traffic conditions. These conditions are discussed and presented for each solution strategy. Corresponding mathematical models were solved in IBM ILOG CPLEX Optimization Studio 12.5.1.0, using Optimization Programming Language (OPL) on a personnel computer with 64 bit operating system, 3.40 GHz Intel Core i7-2600 CPU and 16.0 GB RAM. Airspace around four hypothetical airports is considered. A 3D mesh consisting of 36 sectors including the entry/exit points is considered. Aircrafts enter the airspace through one of the four entry/exit points and exit through another. Approximately each of these four nodes is used 25% of the time. The time Between Arrivals (TBA) is also random following Exponential Distribution with varying average. The volume of all sectors is considered the same and for simplifying purposes the capacity of the entry/exit points are considered unlimited (there is no airport capacity). The travelling time is 15 minutes at minimum for crossing a sector and 30 minutes at maximum. The maximum time is applied because of the fact that air traffic cannot be held en-route for extended periods of time and can only be brought to a halt by the safe landing of the aircraft at a suitable airdrome (Fewings, 2010).

Models were tested for different relative MIP gap values, various sector capacities and different scheduling orders and the impacts on the objective function value is studied in all scenarios. Table 1 provides a sample of the data used to solve this problem.

Table 1-Sample Data

Flight Number	Origin	Destination	Departure Time
1	$Ent_i \ \forall i \in Airports$	$Ext_j \forall i \in Airports$	0
2	$Ent_i \ \forall i \in Airports$	$Ext_j \forall i \in Airports$	0.49
3	$Ent_i \ \forall i \in Airports$	$Ext_j \forall i \in Airports$	0.52
4	$Ent_i \ \forall i \in Airports$	$Ext_j \forall i \in Airports$	0.99
5	$Ent_i \ \forall i \in Airports$	$Ext_j \forall i \in Airports$	3.12

Where $Airports$ is a list of available airports in the network for aircraft to depart or land.

4.2 Simple Solution Strategy

The objective is to solve the model for all aircrafts that are using the airspace during a certain planning horizon. Thus, a non-time indexed network flow problem with a number of F Flights, N nodes and E relations is solved to assign flights on consecutive sectors in order to determine the flight plan in the airspace with a given objective (in our case the objective is to minimize the arrival time to the destination).

The proposed simple model is tested on a number of problems with various sizes. In all experiments in this section, the capacity of the sectors was considered maximum three flights at a time. The impact of relative MIP gap on the average flight time in airspace and total arrival time to destination is investigated. Results provided in table 2 show that, the required computation time increases significantly for the problems with more than 12 aircrafts in the airspace. For all other cases where the problem size is less than 12 aircrafts, the optimal result is obtained in less than 1 minute on a personal computer. The results verified that safety rules which are expressed in the capacity constraints are sustained at all times. Therefore the proposed centralized model is a powerful decision making tool to assist ATC personnel to manage air-traffic in a typical North American airport.

Table 2 depicts the results for various sample sizes. In all cases the relative optimality gap is equal to 1.0E-4.

Table 2-Results of simple solution

Number of flights	Execution time (seconds)	Objective function
3	2.50	196.01
6	2.55	368.33
9	12.77	563.50
12	94.42	772.76
15	1394.90	961.81
18	3726.67	1203.33
21	-	-

Since the problem is NP-hard, the computation time increases exponentially with the number of aircrafts, which makes the approach unattractive for real-time implementation on practical size problems (Potts *et al.*, 2009). Increasing the size to 13 flights will increase the execution time to approximately 7 hours, therefore this methodology of solving the problem, while offering precise arrival/departure times and the shortest path for flying with regard to capacity limitations, is not efficient and possible for larger instances. Obviously, the results offered by this strategy are far from real life situations and large scale cases. One example of addressing large scale cases and real time traffic is in Bertsimas *et al.* (2011), where the authors offer a model that can handle large-scale cases such as 20 airports, 113 sectors and 3000 flights in regional instances and 30 airports, 145 sectors and about 6000 flights in national cases (Bertsimas *et al.*, 2011).

A sample of the solution provided by solving the model is provided in table 3. This table shows the sectors on the route of each flight, the arrival time to and departure time from each sector. It should be mentioned that in all cases the travel time for crossing each sector is equal to 15 minutes since the traffic has not yet reached a point in which it has to increase flight times and delay flights in the air.

Table 3-Solution provided for the first 15 flights

Number of flight	From *node i*	To *node j*	Departure from *node I* = Arrival to *node j*
1	800	31	0
1	31	20	15
1	20	8	30
1	8	2	45
1	2	25	60
1	25	100	75

Number of flight	From *node i*	To *node j*	Departure from *node I* = Arrival to *node j*
2	200	9	0.49
2	9	8	15.49
2	8	4	30.49
2	4	18	45.49
2	18	700	60.49
3	100	25	0.52
3	25	27	15.52
3	27	28	30.52
3	28	17	45.52
3	17	700	60.52
4	800	31	0.99
4	31	14	15.99
4	14	10	30.99
4	10	9	45.99
4	9	200	60.99
5	100	25	3.12
5	25	27	18.12
5	27	6	33.12
5	6	17	48.12
5	17	700	63.12
6	200	9	3.19

Number of flight	From *node i*	To *node j*	Departure from *node I* = Arrival to *node j*
6	9	32	18.19
6	32	26	33.19
6	26	100	48.19
7	700	18	3.46
7	18	19	18.46
7	19	31	33.46
7	31	800	48.46
8	800	31	4.68
8	31	20	19.68
8	20	8	34.68
8	8	2	49.68
8	2	25	64.68
8	25	100	79.68
9	200	9	7.03
9	9	7	22.03
9	7	3	37.03
9	3	17	52.03
9	17	700	67.03
10	100	26	7.09
10	26	32	22.09
10	32	9	37.09

Number of flight	From *node i*	To *node j*	Departure from *node I* = Arrival to *node j*
10	9	200	52.09
11	800	31	15
11	31	20	30
11	20	8	45
11	8	2	60
11	2	25	75
11	25	100	90
12	700	17	7.15
12	17	6	22.15
12	6	5	37.15
12	5	25	52.15
12	25	100	67.15
13	100	26	7.56
13	26	32	22.56
13	32	9	37.56
13	9	200	52.56
14	800	31	15.99
14	31	19	30.99
14	19	18	45.99
14	18	700	60.99
15	200	9	15.49

Number of flight	From *node i*	To *node j*	Departure from *node I* = Arrival to *node j*
15	9	8	30.49
15	8	19	45.49
15	19	18	60.49
15	18	700	75.49

Figure 9 (the time-space diagram) demonstrates the path that each flight has been assigned (in the case with an optimality gap of 1.0E-4), with the time spent in each sectors and proves that the capacity is being upheld since where ever inside the diagram, there are only a maximum of 3 flights inside each sector at all times.

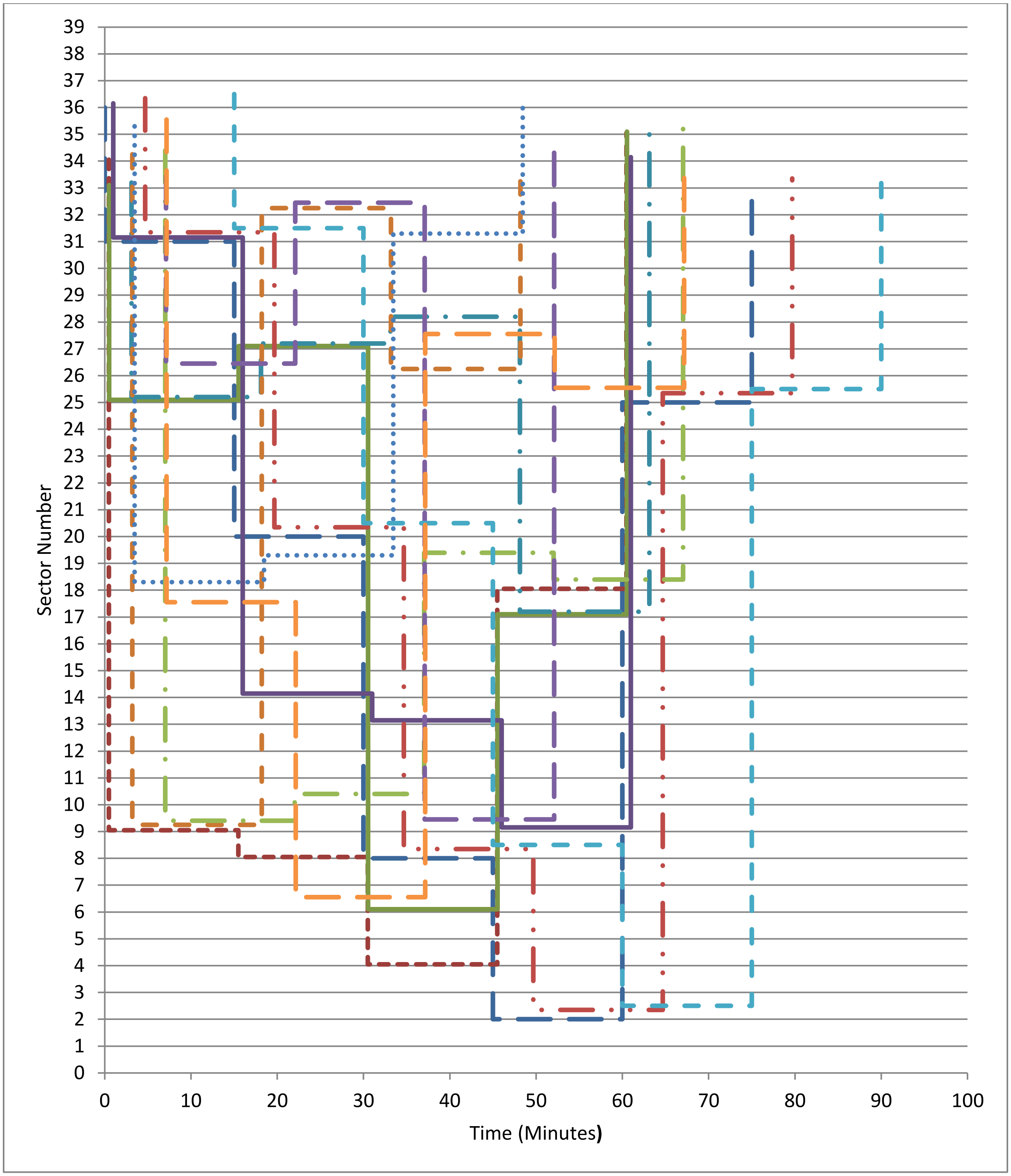

Figure 8-Time-Space Diagram

Table 4 shows the impact of increasing the relative MIP gap tolerance to 0.15 on solving the model.

The relative MIP gap tolerance sets a relative tolerance on the gap between the best integer objective and the objective of the best node remaining.

When the value |best bound-best integer|/ (1e-10+|best integer|) falls below the value of this parameter, the mixed integer optimization is stopped. In this case, to instruct CPLEX to stop as soon as it has found a feasible integer solution proved to be within fifteen percent of optimal, we will set the relative MIP gap tolerance to 0.15. This relative MIP gap may seem far from the optimal solution but we have to consider this important factor that in real life situations, air traffic managers prefer methods which are able to quickly find a proper solution rather than an exact optimal solution obtained after a long computation time

It can be seen that this increase will enable the model to be solved faster and for larger instances while offering good solutions that do not differ that much from the optimal answer.

Table 4-Results for increased tolerance

Number of flights	Execution time (seconds)	Objective function	Average flight time (minutes)
3	1.97	196.01	15
6	2.41	368.33	15
9	11.82	563.50	15
12	66.04	775.23	15
15	221.69	984.28	15.32
18	625.48	1236.72	15.31
20	954.36	1545.50	15.51
21	-	-	-

This increase in the relative MIP gap tolerance will enable us to solve the model for a slightly larger number of flights. It is obvious that further increase of the relative MIP gap will help to expand the set of flights but the answers will grow further from the optimal solution.

In table 5 the effect of increasing the capacity of the sectors has been investigated on the execution time and objective function. In this table the capacity of each sector is increased from 3 to 4. The relative MIP gap tolerance is also set to 0.15 in this case.

Table 5-Results when sector capacity=4 flights

Number of flights	Execution time (seconds)	Objective function	Average flight time(minutes)
3	2.12	196.01	15
6	2.68	368.33	15
9	4.38	563.50	15
12	61.51	764.91	15
15	136.37	950.32	15.01
18	315.38	1264.15	15.31
21	807.54	1477.63	15.20
24	-	-	-

From the table it can be concluded that increase in the capacity of the airspace will generally lead to faster execution time and decrease in the objective function since lesser aircrafts would be forced to take longer paths in order to avoid collision thus the arrival time to the destination would be smaller.

Figure 10 explores the effect of sector capacity on the objective function value which is arrival time to destination for a set of 15 flights. As it can be concluded from the diagram, the larger the capacity, the shorter time it takes to reach the destination thus the objective function (arrival time to destination for all flights) would be smaller.

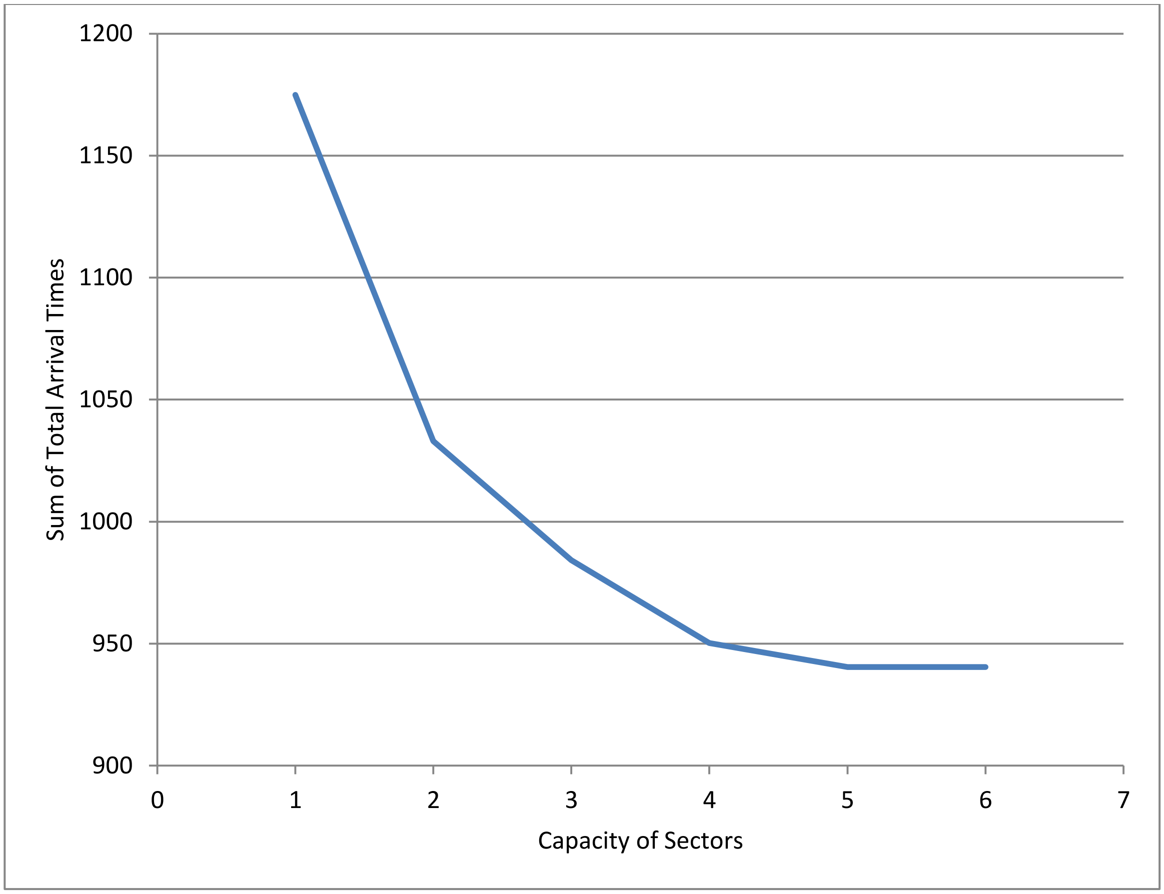

Figure 9-Effect of Capacity on Objective Function Value

Another factor that would help to decrease the computational time is increasing the time between the predetermined departures, needless to say the more time there is between the flights, the less it takes to solve the problem. This effect has been studied in Moeini *et al*. in 2012, where the authors show the effect of increasing the times of departure for each flight. While there is not much difference between the execution times (in some cases lower and in some cases higher for different numbers of flights), the delay cost will increase with increasing the times between departures but it enables them to solve it for a larger number of flights (16 with a 30 second difference and 12 with a 15 second difference).

In our case the time between the scheduled departure of each flight has not been increased so that the data would resemble real life air traffic and an airport's real schedule, but in order to increase the times between each flight, the flights have been put into a random order such that the route planning for them would be different but the release time into the system is still the same. While the previous cases could not be solved for more than 15 flights, in this case the model can be solved for 20 flights but with a larger objective function value. In conclusion, changing the order of flights randomly will enable us to solve the problem for a larger set of flights but with increasing the arrival time to the destination. The relative MIP gap tolerance is also set to 0.15 in this case.

Table 6-Results for flights put into random order

Number of flights	Execution time (seconds)	Objective function	Average flight time(minutes)
3	5.95	250.86	15.00
6	6.15	429.28	15.00
9	10.94	673.30	15.00
12	17.99	871.88	15.00
15	27.59	1362.51	15.00
18	316.30	1362.51	15.30
20	744.22	1594.02	16.10
21	-	-	-

Table 6 summarizes the results of this strategy. The problem can be solved for no more than 20 flights with this strategy.

4.3 Iterative Solution Strategy

In this section, the iterative model for the air traffic flow management problem, is formulated and solved. Again 3 flights at a time are considered for the capacity of the sectors. The method has been adapted to become capable of solving the model for larger sets of flights. Aircrafts depart or approach to an airport independent from each other based on their schedules. The objective is to determine the best route for each aircraft under the given circumstance. Hence the problem is solved for a subset of aircrafts given that F aircrafts are currently in the airspace. When the flight f arrives to the airspace through one of the four entry/exit sectors, the routes (x_s^f), arrival (a_s^f) and departure (d_s^f) times for passing each sector (t_s^f) are transmitted to the approaching set of aircrafts. The objective is to determine x_s^f , a_s^f, d_s^f and t_s^f for the new flights in the current set while all safety rules are ensured. We studied the effectiveness of the model under various traffic conditions. The results show that the iterative model can be solved in real-time on a personal computer. At the beginning of a planning horizon, it is assumed that the airspace among the airports is empty; hence the performance measures (total arrival time for all aircrafts) for earlier flights are significantly better. As the time advances, the traffic conditions updates until the network reaches a steady state level, while the most important criteria which is collision avoidance is guaranteed the whole time.

In this section, the model has been solved for different batch sizes (the number of flights solved altogether at a time); first one aircraft at a time (batch size=1), then 2 by 2 (batch size =2), then 3 by 3 (batch size=3) and so on. The relative MIP gap here is 1.0E-4. With this optimality gap, still the number of flights that can be solved is not that large as the batch size of aircrafts increases.

Table 7 demonstrates the results of the solution. Table 7 compares the total flight time of the iterative model when it is solved for flights one by one and when it is solved for flights 2 by 2 which means the total flight time for flights when they are solved one by one and when they are solved 2 by 2 are compared and so on. It can be concluded that when the model is solved 2 by 2, the steady state of the system is reached after 22 flights and that is when the model with larger batch size starts to give better solutions (in our case, smaller total flight times) and as the number of flights increases the gap between the objective functions of the two models grows larger.

Table 7 summarizes the results of solving the model with batch sizes 1 and 2. The maximum number of flights that could be solved for a batch size of 2 within a reasonable time (less than an hour) were 64 flights while by solving one by one we can obtain a solution for more than a hundred flights in less than 30 minutes approximately.

Table 7-Results for batch sizes 1 and 2 flights

Batch size	Number of flights	Objective function
2	10	630.60
2	20	1368.31
2	30	2361.74
2	40	3472.22
2	50	4738.79
2	60	6205.38
2	64	6844.26
1	10	615.60
1	20	1363.96

Batch size	Number of flights	Objective function
1	30	2366.44
1	40	3489.17
1	50	4790.89
1	60	6298.19
1	64	6952.26

Table 8 shows the results of solving the model iteratively with a relative optimality gap of 1.0E-4 for 15 flights, 3 flights at a time.

Table 8-Sample solution for iterative strategy

Number of flight	From *node i*	To *node j*	Departure from *node i* = Arrival to *node j*	time spent in *sector i*
1	800	31	0	-
1	31	20	15	15
1	20	8	30	15
1	8	2	45	15
1	2	25	60	15
1	25	100	75	15
2	200	9	0.49	-
2	9	7	15.49	15

Number of flight	From *node i*	To *node j*	Departure from *node i* = Arrival to *node j*	time spent in *sector i*
2	7	3	30.49	15
2	3	17	45.49	15
2	17	700	60.49	15
3	100	26	0.52	-
3	26	1	15.52	15
3	1	5	30.52	15
3	5	3	45.52	15
3	3	17	60.52	15
3	17	700	75.52	15
4	800	31	0.99	-
4	31	14	15.99	15
4	14	10	30.99	15
4	10	12	45.99	15
4	12	11	60.99	15
4	11	200	75.99	15
5	100	25	3.12	-
5	25	27	18.12	15
5	27	6	33.12	15
5	6	17	48.12	15
5	17	700	63.12	15

Number of flight	From *node i*	To *node j*	Departure from *node i* = Arrival to *node j*	time spent in *sector i*
6	200	9	3.19	-
6	9	32	18.19	15
6	32	26	33.19	15
6	26	100	48.19	15
7	700	18	3.46	-
7	18	19	18.46	15
7	19	31	33.46	15
7	31	800	48.46	15
8	800	30	4.68	-
8	30	22	19.68	15
8	22	20	34.68	15
8	20	8	49.68	15
8	8	2	64.68	15
8	2	25	79.68	15
8	25	100	94.68	15
9	200	9	7.03	-
9	9	10	22.03	15
9	10	19	37.03	15
9	19	18	52.03	15
9	18	700	67.03	15

Number of flight	From *node i*	To *node j*	Departure from *node i* = Arrival to *node j*	time spent in *sector i*
10	100	26	7.09	-
10	26	32	22.09	15
10	32	9	37.09	15
10	9	200	52.09	15
11	800	31	7.15	-
11	31	19	22.15	15
11	19	3	37.15	15
11	3	2	52.15	15
11	2	25	67.15	15
11	25	100	82.15	15
12	700	17	7.15	-
12	17	28	22.15	15
12	28	5	37.15	15
12	5	1	52.15	15
12	1	26	67.15	15
12	26	100	82.15	15
13	100	26	7.56	-
13	26	32	22.56	15
13	32	9	37.56	15
13	9	200	52.56	15

Number of flight	From *node i*	To *node j*	Departure from *node i* = Arrival to *node j*	time spent in *sector i*
14	800	31	15	-
14	31	19	33.46	18.46
14	19	18	48.46	15
14	18	700	63.46	15
15	200	11	9.41	-
15	11	12	24.41	15
15	12	10	39.41	15
15	10	19	54.41	15
15	19	18	69.41	15
15	18	700	84.41	15

The total arrival time to the destination for all flights (the objective function) this time would be 1025.35 and the average time to pass each sector is 15.05 minutes while if solved the problem for 15 in the simple solution strategy the objective function is 961.81 and the average time for passing each sector is 15 minutes (table 3). But with this strategy we are easily able to solve the model for a number of flights more than twice as the first strategy which was able to solve for 18 flights only.

As the relative optimality gap increases, the problem can be solved for larger number of flights but with this increase, the solution would be better when the number of flight batch is smaller since when this number is small, the problem will reach smaller relative MIP gaps thus the objective function would be better and less further from the optimal solution, but for larger batches of flights, the objective function value would be as far as the gap from the optimal solution, for example, solving the problem with 3 planes at a time would yield to a better solution than solving the problem with 5 airplanes each time when the relative MIP gap is 40%. Thus for this gap, the solution for 3 flights at a time is better than 2 flights at a time but other than that, in all other cases the objective function value which is the total arrival time in our case becomes worse.

Therefor there is not much point in solving the problem with large batches such as 10 flights at a time since with this gap the objective function would be much better with smaller batches of flights and solving the problem one flight at a time would yield to the best solution. It should also be mentioned that putting the flights in random order does not have any significant effect on the ability to solve the model for larger set of flights.

In figure 11, the time-space diagram has been depicted for 20 flights with relative MIP gap of 1.0E-4. The figure shows that the capacity constraints are upheld at all times. Again, in this diagram we can see the sequence of sectors each flight is taking and the time the aircraft spends in those sectors.

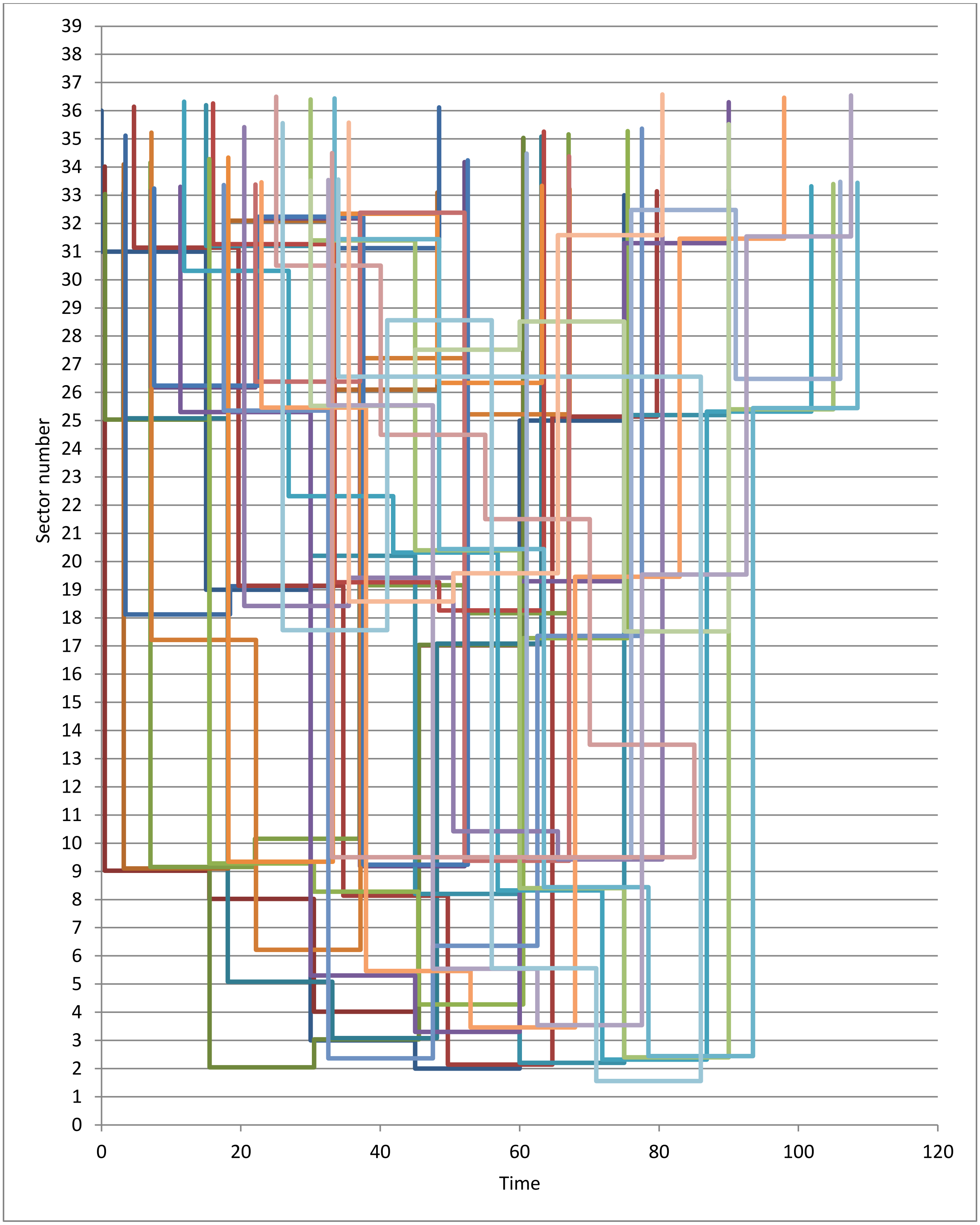

Figure 10-Time-Space diagram when flights solved 1 by 1

The impact of capacity of sectors has been studied in figure 12. The model was solved with 3 flights at a time with a relative MIP gap of 20%. The capacity has been increased from 1 flight to 7 flights in each sector. 27 flights have been solved for each capacity. The reason this relative MIP gap has been chosen is to enable us to solve the problem for this number of flights in a shorter time. Here the objective is to show the impact of capacity on the total arrival time and not to reach solutions near optimality, thus the amount of the gap doesn't hold any importance.

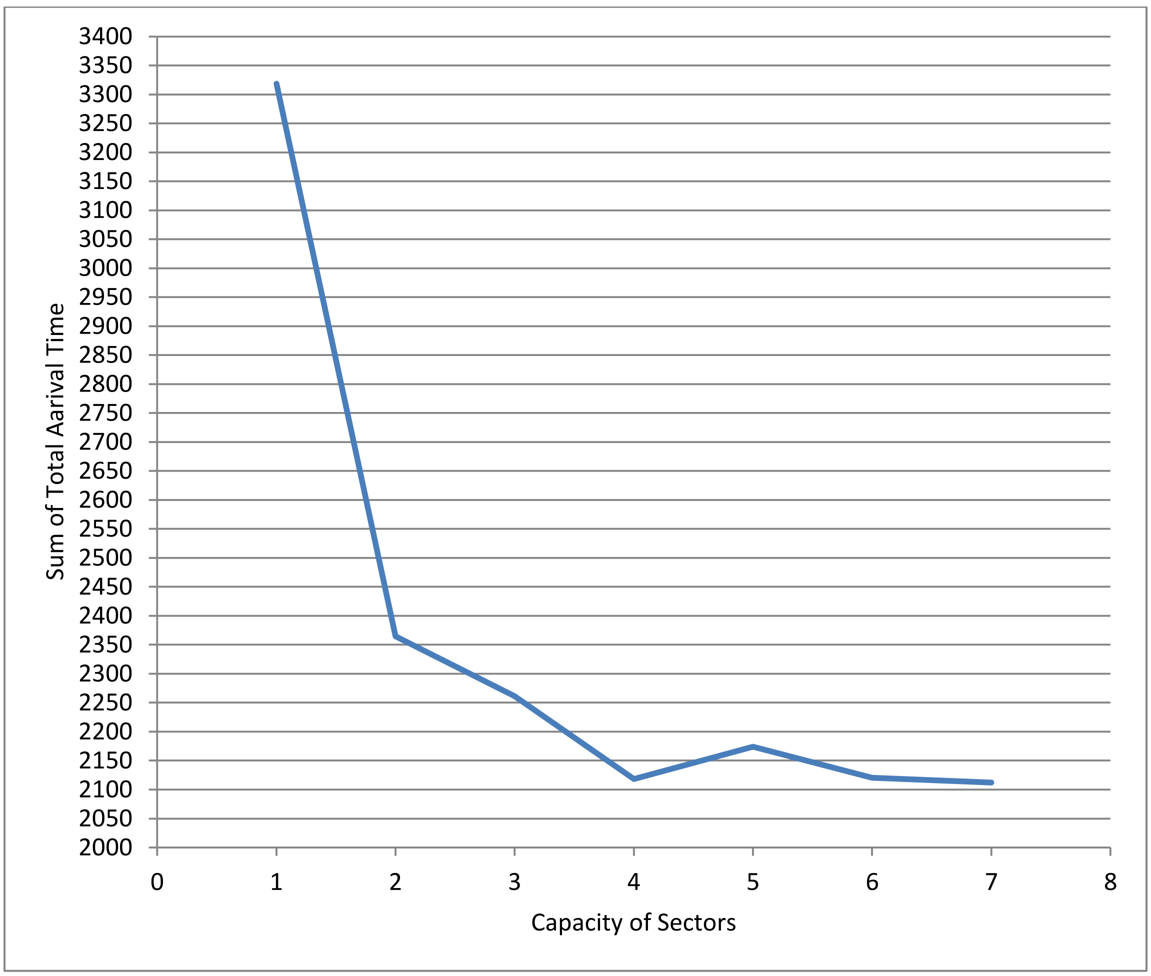

Figure 11-Impact of Capacity on Arrival time of flights

As it can be observed from the graph, In general, decreasing the capacity would increase the objective function which is the arrival time of all flights to their destination because the flights would incur more airborne delays and ground holding due to limited capacity.

In conclusion the first methodology offers better objective values which are closer to the optimal solution, whereas the second methodology allows us to solve the problem for larger sets of flights and in shorter times. In addition, in the second strategy the first flights are given priority to use airspace since the airspace is empty when they enter and other sets of flights would be penalized and the later the flight, the more they are assigned airborne delays and ground holdings while the first strategy penalizes all flights and does not give any priority to the flights.

4.4 Capacity Relaxation strategy and feasibility algorithm

As mentioned before, the main constraints that turn this problem into a complex and hard to solve problem are the capacity constraints. In this section in order to be able to solve the problem for a larger number of flights and in bigger batches of flights, the capacity constraints have been relaxed; not completely ignored but rather increased significantly in order to overcome the complexity and the models inability to schedule more aircrafts within reasonable time limits. It should also be mentioned that the same iterative strategy has also been employed to reach the initial infeasible solution. This solution corresponds to an infeasible flight plan. After solving the problem, the conflicts (flights that make the solution infeasible) are identified and omitted from the flights' schedule. The regular capacity in this section has been considered as four flights per sector as opposed to three flights per sector in the previous section for the sake of simplicity, and for the relaxation, the capacity has been increased to twice as much flights per sector (8 flights). The model has been run for different batches. Another issue in using this strategy is choosing what batch size to start with to reach the infeasible solution. The chosen number of flights (batch size) in this section is 8 flights at a time. This batch size has been chosen for a number of reasons. First of all, for any batch size from 5 to 8, when the system reaches a steady state (around flight 70-75) the bigger batches start resulting in lower total arrival times (better objective functions). For instance, when solved for 80 flights, the total flight time for 5 flights at a time is equal to 9936.78 minutes while when solved for 8 flights at a time the objective function drops to 9257.97 minutes (678.81 minutes difference). As for batch sizes smaller than 5 the objective function indeed yields to smaller amount but the problem is that the number of conflicts is so high that omitting the infeasible flights does not result in a comprehensive flight plan for many flights. Batch sizes larger than 8 are not able to be solved for more than 30 flights. Figure 13 compares the number of conflicts versus time intervals during the time window of flights between the model solved for 2 flights at a time and the model solved for 8 flights at a time. The number 1 on the horizontal axis corresponds to time interval 0-10 minutes, 2 represents 10-20 minutes and so on, As the figure is demonstrating, solving the model for smaller batches of flights at a time, leads to a significantly higher number of conflicts.

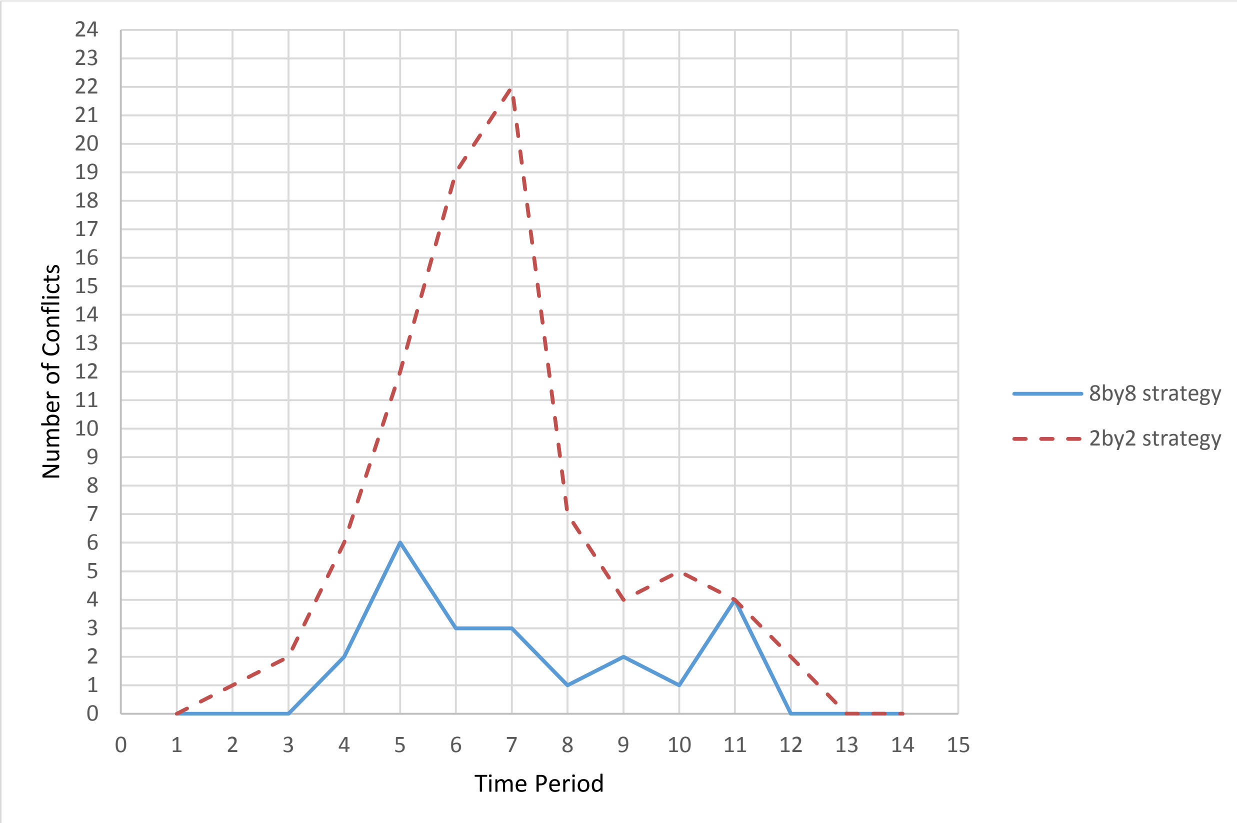

Figure 12-Comparison of number of conflicts

If the regular capacity which is equal to 4 is used to solve the model, it is only able to be solved for 24 flights, while when solved with the sector capacity equal to 8, it can be solved for up to 80 flights at a time. Also the objective function (total arrival time to destination for all flights) would be 2,221.6 for 24 flights with the regular capacity (4 flights per sector) while when solving for the same number of flights with the capacity increased to 8 flights per sector the objective function is equal to 2,237.1 minutes which is almost equal to the previous objective function and there are no flight conflicts when solved for 24 flights in addition to reaching to the solution much faster, thus this method is a suitable method compared to the methods discussed before. After 80 flights, the model again stops being solved for more flights. This is when the infeasible flights are identified and eliminated. After this elimination, again the model has been solved for more flights (100 flights) and then again when it stops working at this point, the solution is examined and turned into a feasible solution after deleting the conflicts. Figure 14 demonstrates the time-space diagram for 8 flights at a time when capacity is equal to 8. The capacity conflicts and the congestion of flights can be easily observed in the diagram. It also should be mentioned that in this section, all models are solved with a relative MIP gap of 15%. The objective function for this number of flights is equal to 9257.97 minutes.

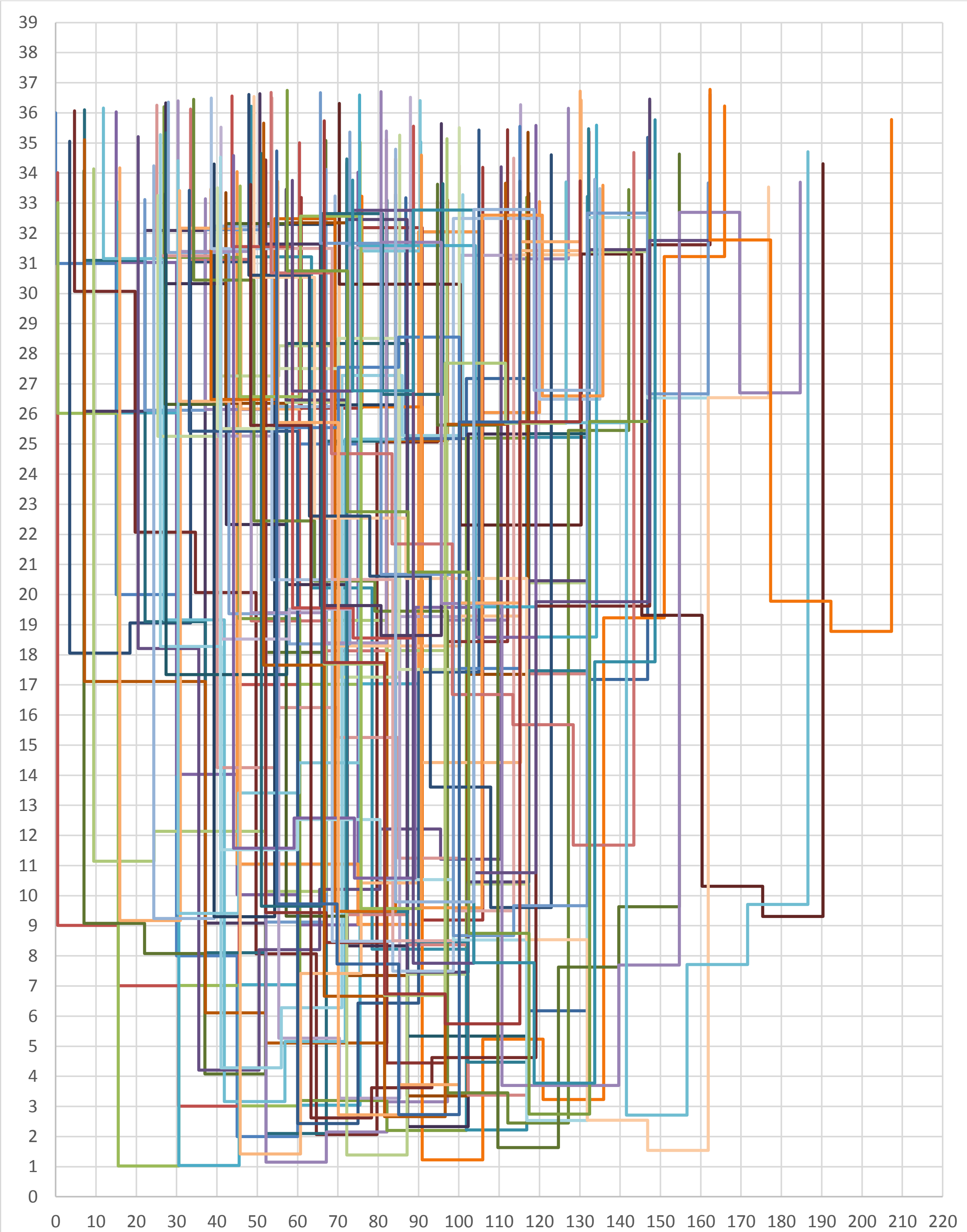

Figure 13-Time-space diagram for batch=8 & Cap=8

The important part of this strategy is designing an algorithm that will omit the smallest possible number of flights in order to reach a feasible solution. First all the sectors that have conflicts inside them are identified, the conflicts inside of each sector can happen in different time periods. In this case, the sectors that have conflicts inside and render the solution infeasible are sectors 2,8,9,18,19, 25,26,31,32. For instance, for sector number 19, the conflict happens at three different periods. The first conflict happens between minutes 50 and 60 with flights 14, 21, 41, 53, 57 and the second conflict happens between minutes 100 and 110 with flights 16,30,60,73 and 71, the third conflict happens with the previous set of flights but with flight 59 instead of flight 60. All other conflicts in different sectors and periods are also identified. After identifying all conflicts, the following algorithm has been developed for omitting flights and has been coded in the programming language JAVA on the same computer all other computations have been carried on:

Input: the conflict matrix that includes the infeasible nodes and all infeasible flights in different periods within those nodes.

1. Find the frequency of each infeasible flight (*F*).
2. Find in how many infeasible sectors these flights can be find (*N*).
3. Choose the flight with the highest *N*. If there is more than one flight with the highest *N*, among them choose the one with the highest *F*. If still there are more than one flight, choose the one which has entered the system the latest. (Bigger flight number).
4. Delete the chosen flight in step 3.
5. Check the number of flights left in all the infeasible sectors that had the deleted flight in step 4 inside of them. If the flights left inside of them are less than or equal to the regular

capacity delete those sectors from the list of infeasible sectors. If not, they remain in the list.

6. Update *F* and *N* for each infeasible flight.
7. Repeat steps 1-6.
8. Continue until the capacity of all sectors inside the infeasible sector list it less than or equal to the regular capacity.

   Output: the infeasible flights

Figure 14 is the input matrix for the first phase of the algorithm (after solving the model for 80 flights). The first number in each row indicates the number of the sector in which the conflict is happening and the rest of the numbers in the row are the numbers of aircrafts that are participating in the conflict at that period of time. For example in sector 2, flights 16, 8, 44, 63 and 73 are in conflict and there is no other conflict in any other time in sector 2, but in sector 26, the conflicts are happening in 5 different times, first there is a conflict with flights 58, 49, 36, 18 and 16 and after this conflict at another time period, another conflict happens with flights 18, 25, 40, 47 and 58 and so on.

$$
\begin{bmatrix}
2 & 16 & 8 & 44 & 63 & 73 & 0 & 0 & 0 \\
8 & 68 & 54 & 38 & 48 & 23 & 0 & 0 & 0 \\
9 & 13 & 33 & 45 & 66 & 74 & 0 & 0 & 0 \\
9 & 13 & 33 & 45 & 66 & 74 & 4 & 0 & 0 \\
9 & 6 & 38 & 48 & 58 & 49 & 0 & 0 & 0 \\
9 & 6 & 38 & 48 & 58 & 80 & 0 & 0 & 0 \\
18 & 14 & 30 & 41 & 57 & 65 & 0 & 0 & 0 \\
19 & 14 & 21 & 41 & 53 & 57 & 0 & 0 & 0 \\
19 & 16 & 30 & 60 & 73 & 71 & 0 & 0 & 0 \\
19 & 16 & 30 & 59 & 73 & 71 & 0 & 0 & 0 \\
25 & 23 & 35 & 46 & 72 & 75 & 0 & 0 & 0 \\
25 & 21 & 34 & 64 & 67 & 74 & 0 & 0 & 0 \\
26 & 58 & 49 & 36 & 18 & 16 & 0 & 0 & 0 \\
26 & 18 & 25 & 40 & 47 & 58 & 0 & 0 & 0 \\
26 & 18 & 25 & 40 & 47 & 58 & 77 & 0 & 0 \\
26 & 20 & 25 & 40 & 47 & 77 & 0 & 0 & 0 \\
26 & 20 & 31 & 40 & 47 & 77 & 0 & 0 & 0 \\
31 & 7 & 14 & 21 & 26 & 37 & 41 & 0 & 0 \\
31 & 7 & 14 & 21 & 26 & 37 & 41 & 50 & 0 \\
31 & 7 & 14 & 21 & 26 & 41 & 50 & 0 & 0 \\
31 & 57 & 50 & 41 & 14 & 7 & 0 & 0 & 0 \\
32 & 31 & 36 & 49 & 58 & 66 & 0 & 0 & 0
\end{bmatrix}
$$

Figure 14-Conflict Matrix

After the algorithm is over, the result is that flights number 58, 41,21,73,23,77,74,50 and 66 are deleted (9 out of 80 flights). The flights that have been deleted will be later programmed and scheduled.

After the first set of infeasible flights has been removed, the model would run again. But this time the batch size of flights have been decreased to 6 flights at a time instead of 8. The algorithm is again implemented on the new flights 'schedule to delete the minimum infeasible flights. In the second execution, the incidents of conflicts (different conflicts during different period in sectors) have increased to 26 as opposed to 22 in the first execution. The sectors that have conflicts inside them are now 2, 3, 9, 19, 20, 25, 26, 31, and 32. After applying the flight eliminating algorithm flights number 105, 94, 83, 71, 100, 93, 86, 106, 101, 88 and 82 (11 flights) are eliminated from the flights' schedule. The result is a flight plan (route and schedule for each flight) for 90 flights. Figure 15 is the time-space diagram for this solution.

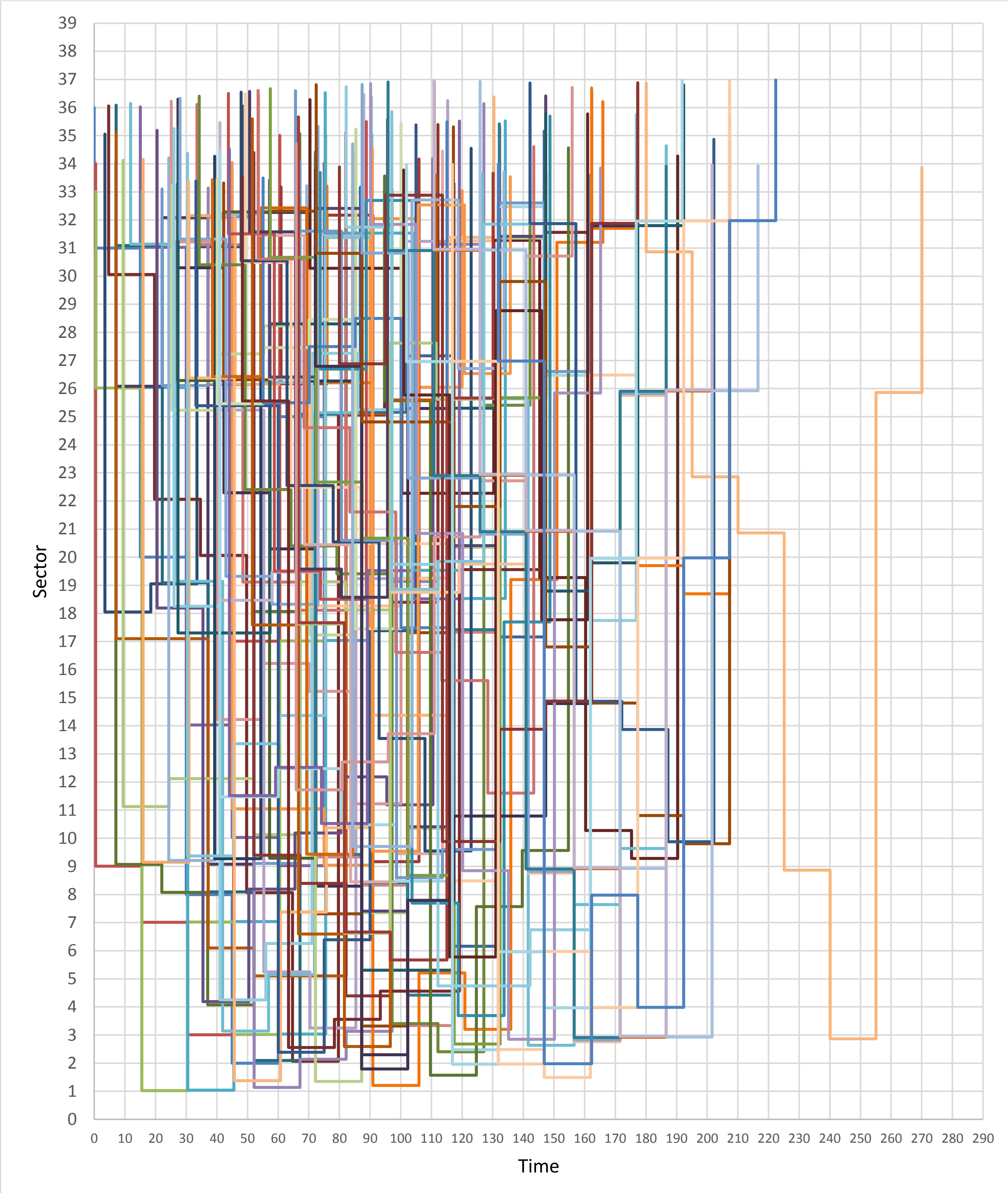

Figure 15-Time-Space diagram for 90 flights

This algorithm can go on for roughly around 200 flights but as the traffic and congestion increases more flights need to be eliminated in order to reach a feasible solution. For demonstrating this algorithm the model has been solved for 110 flights and a total of 20 flights have been eliminated from the flights' schedule in order to reach a feasible solution. Now the flights that have been eliminated must be added again to reach a thorough flight schedule for all flights. In this step, in order to reach a solution, all capacity constraints have been relaxed except for the sectors that have reached full capacity in a period of time. The sectors that are not available for some periods of time are 25, 26, 9, 2, 17, 18, 30, 31, 8, 19 and 20. These sectors have the highest traffic in the network, thus the capacity constraints are not relaxed for these sectors. First the first 10 flights that were eliminated are solved which are flights 21, 23, 41, 50, 58, 66, 71, 73, 74 and 77 and added to the schedule. Then the next 10 flights are solved which are flights 82, 83, 86, 88, 93, 94, 100, 101, 105 and 106 while sectors 10 and 22 are also added to the capacity constraints since there are periods of times in which no capacity is available in these sectors. It should be noted that when adding these previously eliminated flights, the capacity is considered the normal amount which is 4 flights at a time as opposed to 8 to reach a feasible solution, thus all capacity constraints that do not need comparing are relaxed so that the model is able to solve these additional flights. The time-space diagram below in figure 16 demonstrates the route and schedule for all 110 flights. It should be mentioned that one of the advantages of using such a method is that the flights that have the most conflicts are identified and penalized more than other flights by the delay they are assigned and also there is a better use of the sectors in the network since when congested sectors are not available anymore, the flights are sent through other sectors of the network rather than being assigned more excessive delays.

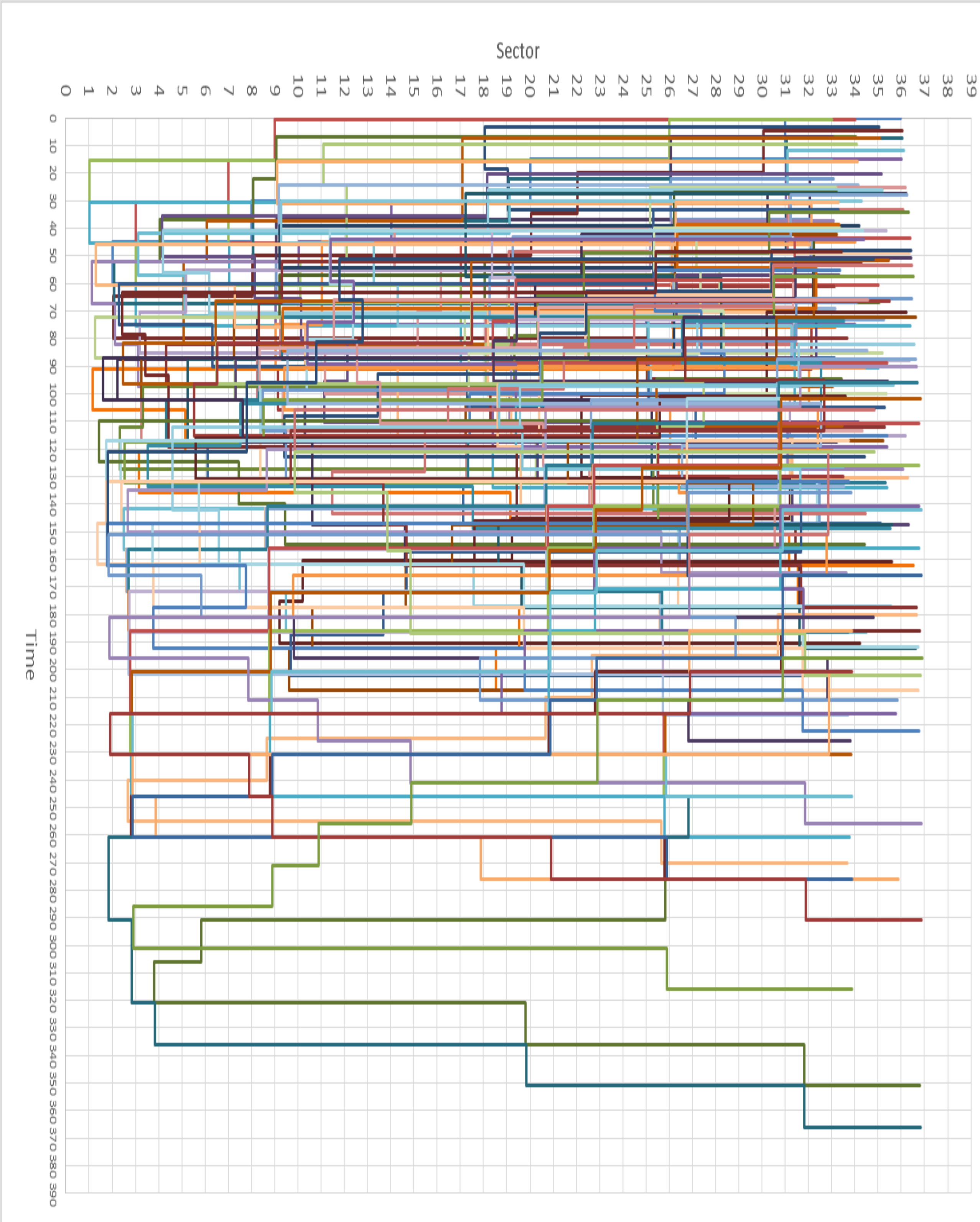

Figure 16-Time-Space diagram for 110 flights

Table 9 summarizes the data and results of this strategy for the 110 flights solved in this section.

Table 9-Summary of results

Total number of flights	110
Total flight time of all flights (airborne) (minutes)	8480.21
Average flight time for each flight (airborne) (minutes)	77.09
Total arrival time of all flights (objective function value) (minutes)	16817.90
Average arrival time for each flight (minutes)	152.89
Total ground holding for all flights (minutes)	8337.69
Average ground holding for each flight (minutes)	75.79
Average time for passing a sector (minutes)	16.19
Relative MIP gap	0.15
Capacity of each sector	4

In this strategy, the smallest flight time is 45 minutes which belongs to various flight numbers such as flights 7, 10, 13, 18 and 20 and the longest flight duration is 134.894 minutes which belongs to flight number 91.

4.5 Comparison of methods

In this section, the result of the heuristic strategy is compared to the results of an iterative solution solved with 2 flights at a time with sector capacity equal to 4 and also the simple strategy with the same capacity and in all cases the relative MIP gap is equal to 0.15.

The heuristic strategy is compared to an iterative strategy solved two flights at a time since this flight batch gives the best answer (objective function-wise). In this case, the problem can be solved for 78 flights, while with the heuristic strategy it can solved for twice as much. For the iterative strategy, the total objective function is equal to 8735.77 while for the heuristic strategy the objective function is 9457.14 which is approximately 722 minutes bigger but the average arrival time for each flight is 152.89 which is only 40 minutes bigger than 111.99 which is the average arrival time for flights solved 2 by 2, thus with the heuristic method, we are able to solve the problem for twice as much as the iterative method with just adding 40 minutes in average to the arrival time of the aircrafts.

In addition there is no advantage in using the simple solution strategy (solving all possible flights at once) over the heuristic method since the problem cannot even be solved for 25 flights.

In conclusion, the iterative solution gives us a better objective function but the heuristic solution enables us to solve the problem for bigger instances without a drastic impact on the average arrival time of flights. Table 10 summarizes the results of comparing these methods.

Table 10-Comparison of strategies

Relaxation vs Iterative with batch size=2	Simple solution strategy	Conclusion
• Iterative solved for 78 flights, Relaxation strategy solved for twice as much. • objective function in iterative strategy= 8735.77. Relaxation strategy, objective function= 9457.14 • Average arrival time for each flight is 152.89 for relaxation (only 40 minutes bigger than 111.99 for iterative)	• There is no advantage in using the simple solution strategy (solving all possible flights at once) over the heuristic method since the problem cannot even be solved for 25 flights.	• Iterative solution gives better objective function but the Relaxation solution enables us to solve the problem for bigger instances without a drastic impact on the average arrival time of flights.

Chapter 5

Conclusions and Future Work

5.1 Conclusions

Currently the air traffic management system which consists of a number of complex subsystems including the human being, infrastructure (both airborne and ground), communications, navigation, and surveillance, with the exception of oceanic areas operates on a sovereign state rather than regional basis. However, gradual measures for changing how the air traffic management system will be operated in the future are being taken due to the pressures for further cost efficiencies, environmental pressures, and the replacement of old technologies by satellite navigation systems (Fewings, 2010) .

This research was an effort to participate in this change toward a better air traffic management system for the future, look over the literature review of Air Traffic Management and construct an optimization model based on non-time indexing which minimizes the total arrival time of all flights to their designated destination. A 3-D mesh network was designed assuming the airspace around a number of airports which consists of a number of sectors in which arcs are used to represent the connection and relations among sectors. The objective function aims to minimize the flight time of each flight thus decreasing delays while giving the flights the best path to fly with all safety constraints. Both airborne delays and ground delays are allowed in the model. Indeed, the model assigns best timing and routes to flights on their journey to their destination. Additionally the air collision is being explicitly avoided within the capacity constraints that just allow a certain number of flights inside each sector at all times. Non-time indexed modeling enables the determination of exact arrival and departure times on each sector. In a controlled airspace, a pilot has the responsibility to uphold all ATC guidance and procedures except in the case of an emergency (Fewings, 2010). The solution provided is practical enough for either cases in which the pilots have more control and authority or for instances in which the air traffic controllers have the majority of responsibility. In conclusion, the following objectives were achieved by solving the model: a) safety constraints and collision avoidance is ensured at all times by using the capacity constraints; b) the exact schedules are derived for arriving and departing to and from each sector.; c) finally, computational time of the model is improved by using different strategies for solving the problem.

In the first section a brief history of studies in this field was provided and why this field has become increasingly important in today's research was discussed. The main reasons for this can be categorized as follows:

- Continuous rise of demand for air transportation
- Growing awareness of the necessity of ATFM due to congestion in the airspace.
- Increase of air traffic delay due to bad weather
- Substantial financial loss due to flight cancelations, long delay durations and airborne delays

Due to these reasons a substantial increase in the number of researches conducted in the field of air traffic management can be observed. These researches take into account various aspects of a flight plan. Some of them just examine a specific part of the flight such as take-off or landing, some others also consider stochastic factors such as weather condition.

This research falls into the category of air traffic flow management problem (ATFMP) and presents a mixed integer program (MIP) to this problem. Thus the main objective of this thesis is to design a mathematical formulation that is able to give the best route for each aircraft while considering the capacity of each sector and other factors.

The distinctive features of this model which has made it prominent from other models can be summarized as follows:

- Dividing the airspace to sectors or zones and defining capacity and distance on zones rather than considering nodes and defining concepts on the arcs between nodes which is an effort to embed the concept of free flight inside the model. Free flight (FF) is a concept in which the pilots are responsible for safety assurance instead of air traffic controllers (Hoekstra *et al.*, 2002). This concept of air traffic management calls for the gradual transfer of separation responsibility from ground air traffic controllers to pilots and aircraft systems (Metzger *et al.*, 2001).
- Using a non-time indexed formulation allowing us to know exact arrival/departure times
- Definition of the capacity constraints for zones/sectors.
- Developing a heuristic method for solving the problem

In order to come up with a realistic and applicable solution to the air traffic management problem and solve the NP-Hard MIP, it is assumed that aircrafts enter to or exit from the airspace through a number of given points of entries thus there are various different directions for the flights. The problem formulation has been coded in CPLEX using OPL language and so far is able to solve the problem for a number of aircrafts. Solving the problem will provide us with a solution that determines a flight plan for each aircraft by identifying the sequence of sectors to be visited and the aircrafts' exact arrival and departures times to these sectors. The problem was solved in three manners, first all the model was solved for all flights at once and flights were assigned routes and schedules at the same time and then the flights were solved batch by batch and finally a heuristic method was developed to solve the problem. The result of each was provided. In conclusion, solving all flights at once yields to a better objective function which means shorter travel time whereas solving the problem iteratively will help to solve the air traffic management problem for larger instances and shorter execution time.

5.2 Future Work

There is a noticeable potential for research to be done in this vast field of science in order to propose an air traffic flow management problem and formulation that would depict the conditions of today's highly demanding air traffic system and to come up with an efficient solution time wise and size wise. Some of the challenges that future research on air traffic management must address is:

- Efficient environmental initiatives
- Employment of automatic systems
- Gradual transfer to free flight
- Joint research opportunities of US and Europe

The desired transformation is aimed to provide scalable technologies for aspects of air traffic management such as communication, navigation, and surveillance infrastructure; one of these examples that can be mentioned is using digital systems in place of analog systems, and using methods of communication that is addressable in comparison to low-bandwidth broadcast systems for communication. These effects will include scalable airborne capabilities for all phases of an aircraft's journey such as separation, sequencing, and precision guidance that enable aircrafts to bring the infrastructure with them as they enter the system. Also, these effects will contain new aircrafts that have qualities such as reduced cost of speed which is made possible through aeronautics technologies and it is only through such innovations that scalable business plans are created for reaching deeply into thinner markets for air traffic and transportation (Holmes, 2004).

As for future work, the following steps can be taken to improve the model:

- Since this model yet needs work to be able to offer a large-scale solution, the model could be expanded to handle more airports with larger sets of flights. The main challenge while solving the model was coming up with a solution methodology that is able to solve the model within a reasonable time frame and to solve it for a large set of flights, therefore the continuation of this research would be exploring different methodologies such as meta-heuristic methods that would offer the advantages of fast execution times and sizeable sets of flights at the same time.
- The objective function of the model can be expanded to consider other factors such as fairness in delay allocation, separation of ground delays and airborne delays, cost of different delays and fuel consumption.

Research in air traffic management includes a vast selection of subjects and some have yet a long way to go to reach to a state of the art method to apply to real life airports and traffic control.

Many scholars and researchers are endeavoring in taking measures to advance the borders of this science. This research was an effort to be part of the ongoing accomplishments.

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