# Multi-Tier Supplier Quality Evaluation in Global Supply Chains

Feras A S Saleh

A Thesis

in

The Department

of

Concordia Institute for Information Systems Engineering (CIISE)

Presented in Partial Fulfillment of the Requirements For the Degree of

Master of Applied Science (Quality Systems Engineering) at

CONCORDIA UNIVERSITY

Montreal, Quebec, Canada

November 2015

CONCORDIA UNIVERSITY

School of Graduate Studies

This is to certify that the thesis prepared

By: Feras A S Saleh

Entitled: Multi-Tier Supplier Quality Evaluation in Global Supply Chains

and submitted in partial fulfillment of the requirements for the degree of

Master of Applied Science (Quality Systems Engineering)

complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

Signed by the final Examining Committee:

| | |
|---|---|
| Dr. Mohammad Mannan | Chair |
| Dr. Govind Gopakumar | Examiner |
| Dr. Jia Yuan Yu | Examiner |
| Dr. Anjali Awasthi | Supervisor |

Approved by: ______________________________

Chair of Department or Graduate Program

______________________________

Dean of Faculty

Date ______________________________

# Abstract

Multi-Tier Supplier Quality Evaluation in Global Supply Chains

Feras A S Saleh

**Concordia University**

Supplier quality evaluation is a critical part of quality management in global supply chains. Poor supplier quality results in not only monetary losses but also negatively impacts the business potential and future growth of buyer organizations. In this thesis, we propose a multi-tier supplier quality evaluation framework based on total cost of ownership and data envelopment analysis for quality management in global supply chains. The proposed approach comprises of three main steps. Firstly, we group the upstream suppliers based on common attributes using hierarchical cluster analysis. Then, we calculate the total cost of ownership for the grouped suppliers and their sub-suppliers using various qualitative and quantitative factors that are vital for quality management in global supply chains. In the third and the last step, we apply data envelopment analysis to compute the efficiencies of various suppliers to identify the best one (s) and recommend for selection. A numerical application is provided.

The proposed approach is very useful to decision makers in benchmarking supplier quality performances and setting improvement targets for supplier quality development in global supply chains.

## Acknowledgments

I would like to express my sincere gratitude to my supervisor, Dr. Anjali Awasthi, for her patience, guidance, and encouragement that she provided during this thesis work. I am truly thankful for her support, time and being there to answer my questions and queries so promptly. Her insightful discussions and suggestions made this work possible.

I would like to thank all the faculty members of Concordia Institute for Information Systems Engineering for the wonderful academic experience and the enormous knowledge they support their students with.

Finally, I would like to thank my wife and the mother of my two daughters, for her understanding and being so supportive during this journey.

Thank you.

# Table of Contents

## List of Figures

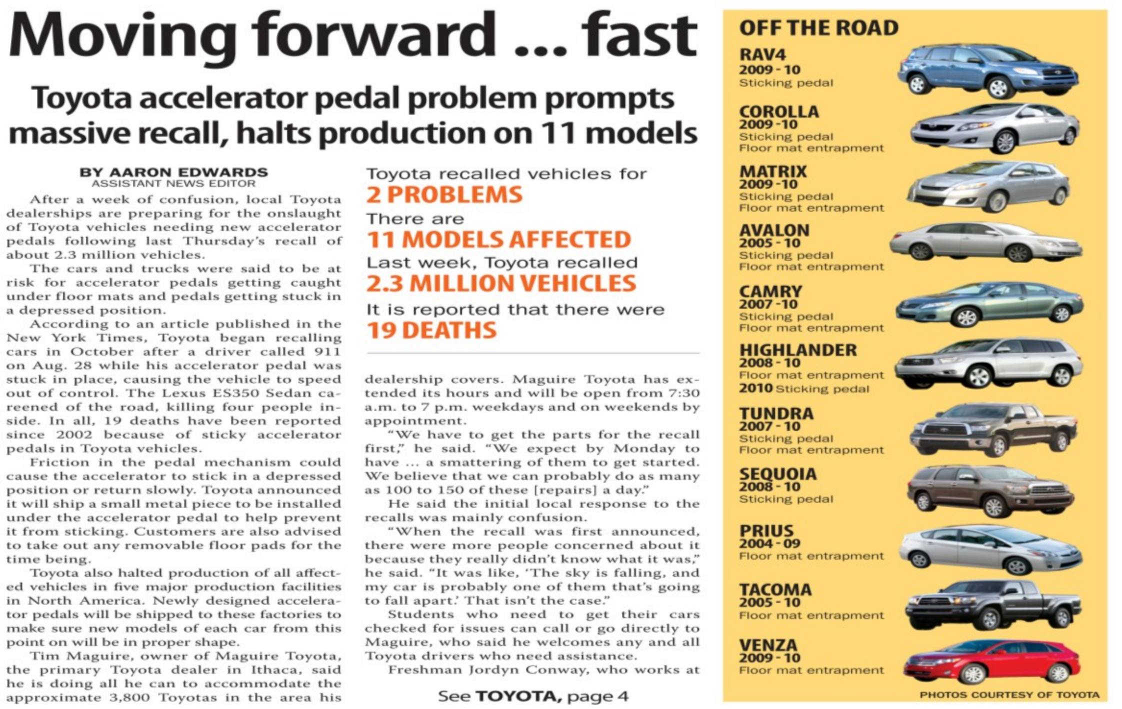
# List of Tables

# Chapter 1

# Introduction

With the complexity of today's supply chains; firms have difficulty keeping track of all the activities happening in the supply chain. Less visibility and control of key processes have become the result of manufacturing, logistics and other roles such as outsourcing (Morehouse and Cardoso, 2011). As a result, the supply chain is now more vulnerable to frauds than before.

Babies got poisoned by contaminated milk in China (Bradley, 2008) because one supplier decided to use melamine instead of protein nitrogen to gain some extra profit. In 2007, Canadian pet food manufacturing and retailer, Menu Food, had a massive recall of one popular pet food product because it caused sickness and death of animals as a result of high melamine level in some ingredients imported from Chinese suppliers (Chen *et al.,* 2014). Moreover, Mattel recalled millions of toys in 2007 because the used paint contained high level of lead (Viswanadham and Samvedi, 2013).



# Moving forward ... fast

## Toyota accelerator pedal problem prompts massive recall, halts production on 11 models

BY AARON EDWARDS
ASSISTANT NEWS EDITOR

After a week of confusion, local Toyota dealerships are preparing for the onslaught of Toyota vehicles needing new accelerator pedals following last Thursday's recall of about 2.3 million vehicles.

The cars and trucks were said to be at risk for accelerator pedals getting caught under floor mats and pedals getting stuck in a depressed position.

According to an article published in the New York Times, Toyota began recalling cars in October after a driver called 911 on Aug. 28 while his accelerator pedal was stuck in place, causing the vehicle to speed out of control. The Lexus ES350 Sedan careened of the road, killing four people inside. In all, 19 deaths have been reported since 2002 because of sticky accelerator pedals in Toyota vehicles.

Friction in the pedal mechanism could cause the accelerator to stick in a depressed position or return slowly. Toyota announced it will ship a small metal piece to be installed under the accelerator pedal to help prevent it from sticking. Customers are also advised to take out any removable floor pads for the time being.

Toyota also halted production of all affected vehicles in five major production facilities in North America. Newly designed accelerator pedals will be shipped to these factories to make sure new models of each car from this point on will be in proper shape.

Tim Maguire, owner of Maguire Toyota, the primary Toyota dealer in Ithaca, said he is doing all he can to accommodate the approximate 3,800 Toyotas in the area his dealership covers. Maguire Toyota has extended its hours and will be open from 7:30 a.m. to 7 p.m. weekdays and on weekends by appointment.

"We have to get the parts for the recall first," he said. "We expect by Monday to have ... a smattering of them to get started. We believe that we can probably do as many as 100 to 150 of these [repairs] a day."

He said the initial local response to the recalls was mainly confusion.

"When the recall was first announced, there were more people concerned about it because they really didn't know what it was," he said. "It was like, 'The sky is falling, and my car is probably one of them that's going to fall apart.' That isn't the case."

Students who need to get their cars checked for issues can call or go directly to Maguire, who said he welcomes any and all Toyota drivers who need assistance.

Freshman Jordyn Conway, who works at

See TOYOTA, page 4

Toyota recalled vehicles for
**2 PROBLEMS**
There are
**11 MODELS AFFECTED**
Last week, Toyota recalled
**2.3 MILLION VEHICLES**
It is reported that there were
**19 DEATHS**

**OFF THE ROAD**

**RAV4**
2009 - 10
Sticking pedal

**COROLLA**
2009 -10
Sticking pedal
Floor mat entrapment

**MATRIX**
2009 -10
Sticking pedal
Floor mat entrapment

**AVALON**
2005 - 10
Sticking pedal
Floor mat entrapment

**CAMRY**
2007 -10
Sticking pedal
Floor mat entrapment

**HIGHLANDER**
2008 - 10
Floor mat entrapment
2010 Sticking pedal

**TUNDRA**
2007 - 10
Sticking pedal
Floor mat entrapment

**SEQUOIA**
2008 - 10
Sticking pedal

**PRIUS**
2004 - 09
Floor mat entrapment

**TACOMA**
2005 - 10
Floor mat entrapment

**VENZA**
2009 - 10
Floor mat entrapment

PHOTOS COURTESY OF TOYOTA

Figure 1.1 Toyota vehicles recall in 2010 (Edwards, 2010, February 4)

As a result, firms have strived to achieve successful supply chain collaboration. Collaboration can improve the traceability and visibility among the supply chain (Sarpong, 2014), which in turn improves the quality of the final product or service. Moreover, collaboration can deliver significant benefits to all parties such as excess inventory reduction, bullwhip avoidance, business synergy enhancement, flexibility and increase joint innovation (Cao and Zhang, 2011). Supply chain performance enhancement (Vereecke and Muylle, 2006) by leveraging the knowledge and resources of suppliers (Cao and Zhang, 2011) are some results of a successful collaboration.

Although product design, warehousing, and distributions centers can all be the subject of quality improvement programs; in this thesis we are focusing on purchasing as it contributes the most to the cost of quality. Both supplier evaluation and selection are essential for the success of the focal firm (Choi and Hartley, 1996; Singh, 2014). We argue that evaluation of the current suppliers is the first step toward a successful collaboration relationship. Evaluation will reveal the weak areas of each supplier and recommend methods for improvement. Moreover, this evaluation will be the basis for supplier development program. We propose an approach based on Total Cost of Ownership (TCO) and Data Envelopment Analysis (DEA) due to their respective advantages.

TCO looks beyond the quoted cost to cover additional true costs related to the entire purchasing cycle. In addition to the quoted price, it may include order placement costs, research costs, transportation costs, receiving costs, inspection costs, holding costs, and disposal costs (Bhutta and Huq, 2002). Consequently, TCO would help to understand the true costs associated with the quality of the purchased items.

DEA is a powerful non-parametric analysis technique that considers both quantitative and qualitative data. DEA does not require the decision maker to assign weights to each indicator but calculates weights from the given data. Additionally, DEA finds the efficient decision making units (suppliers) and computes the amount and source of inefficiency of inefficient suppliers (Cooper *et al*., 2007). It provides improvement targets for inefficient suppliers to become efficient. These targets values can be the basis of a new supplier improvement program.

## 1.1 Problem Definition

Supplier evaluation process is very critical to purchasing management. It is a complex multiple criteria decision making problem that requires careful selection of criteria (Omurca, 2013). Both qualitative and quantitative criteria should be used in evaluating the supplier performance. Additionally, it should speak the language of business, or money, to ensure acceptance among purchasing managers. It must also reflect the network structure of the supply chain. As a result, all the *n*-tier suppliers and the linkages between them should be evaluated for overall quality management in supply chains.

The aim of this thesis is to develop a multi-tier supplier quality evaluation framework for improving the quality of global supply chains. This involves:

1. Grouping of upstream suppliers using hierarchical cluster analysis
2. Evaluation of supplier quality at multiple tiers.
3. Identification of improvement targets for poorly performing suppliers and recommendations generation.

# Chapter 2

# Literature Review

## 2.1 Challenges for quality management in global supply chains

Mentzer *et al*. (2001) define supply chain as a set of firms that have direct upstream and downstream flows of products, services, finances, and information from a source to a customer. According to Bozarth and Handfield (2006), supply chain can be defined as a network of manufacturers and service providers that work together to transform and transport goods from the raw materials stage through the end user. Figure 2.1 illustrates the various stakeholders involved in a supply chain.

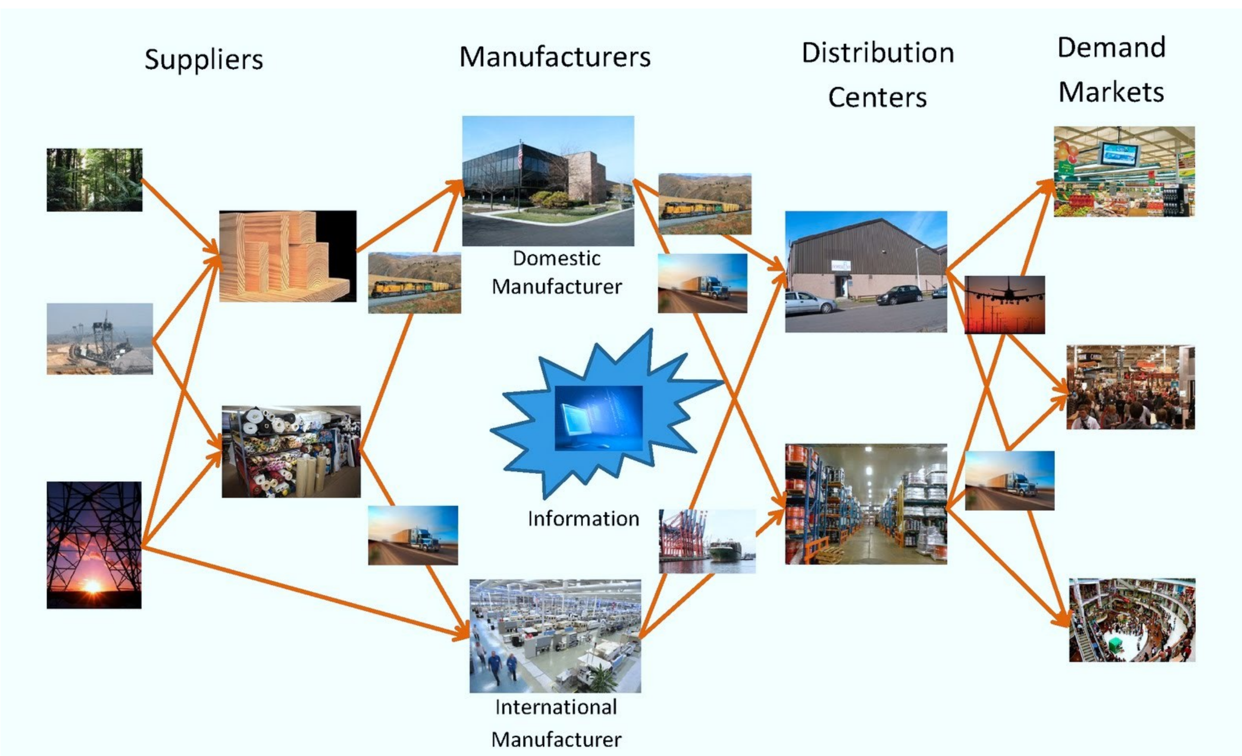



Figure 2.1 Supply chain example (Regan, 2015)

Supply chain can be classified into two main categories: product supply chain and service supply chain. Automobiles, electronics, fresh foods are some examples of product supply chain. Examples of service supply chain are healthcare, education, banking etc. The quality management practices may vary depending on the nature of supply chain. For example, fresh fruit supply chains have a long lead-time and a very high uncertainty level with regards to supply and demand. It requires an efficient management and the use of modern decision technology tools (Soto-Silva *et al.*, 2015). Automobiles industry have very complex supply chains that start from extracting the basic raw materials until the delivery of final product. These supply chains requires innovative approaches to manage different supply chain activities (Vonderembse *et al.*, 2006). In electronics supply chain, demand uncertainty and inventory control challenges are the main side effects due to the short life cycle of these products. Moreover, the complexity of these products requires a wide range of supply materials (Tse *et al.*, 2016). For service supply chains, e.g. healthcare, the supply chain starts from supplying state of the art equipment and medicines from warehouses to clinics and hospitals. Management faces major challenges in managing these supply chains because of their great impacts on public health. It requires precise medical supply according to patient's needs (Jahantigha and Malmir, 2015).

In today's world it is not possible for a single organization to own its entire supply chain like the Ford Company in the first half of the 20$^{th}$ century (Gelderman, 1989). To stay competitive in global market; organizations have to outsource many critical business processes and value chain activities to suppliers thousands of miles away to reduce cost or increase responsiveness (Nieminen and Takala, 2006). Therefore, the supply chain has become more and more complex. Especially when the direct supplier (first tier) outsources part of its business to another supplier (second tier). In this case we have multi-tier suppliers and a multi-tier supply chain. Figure 2.2 presents the diagram of a multi-tier supply chain.

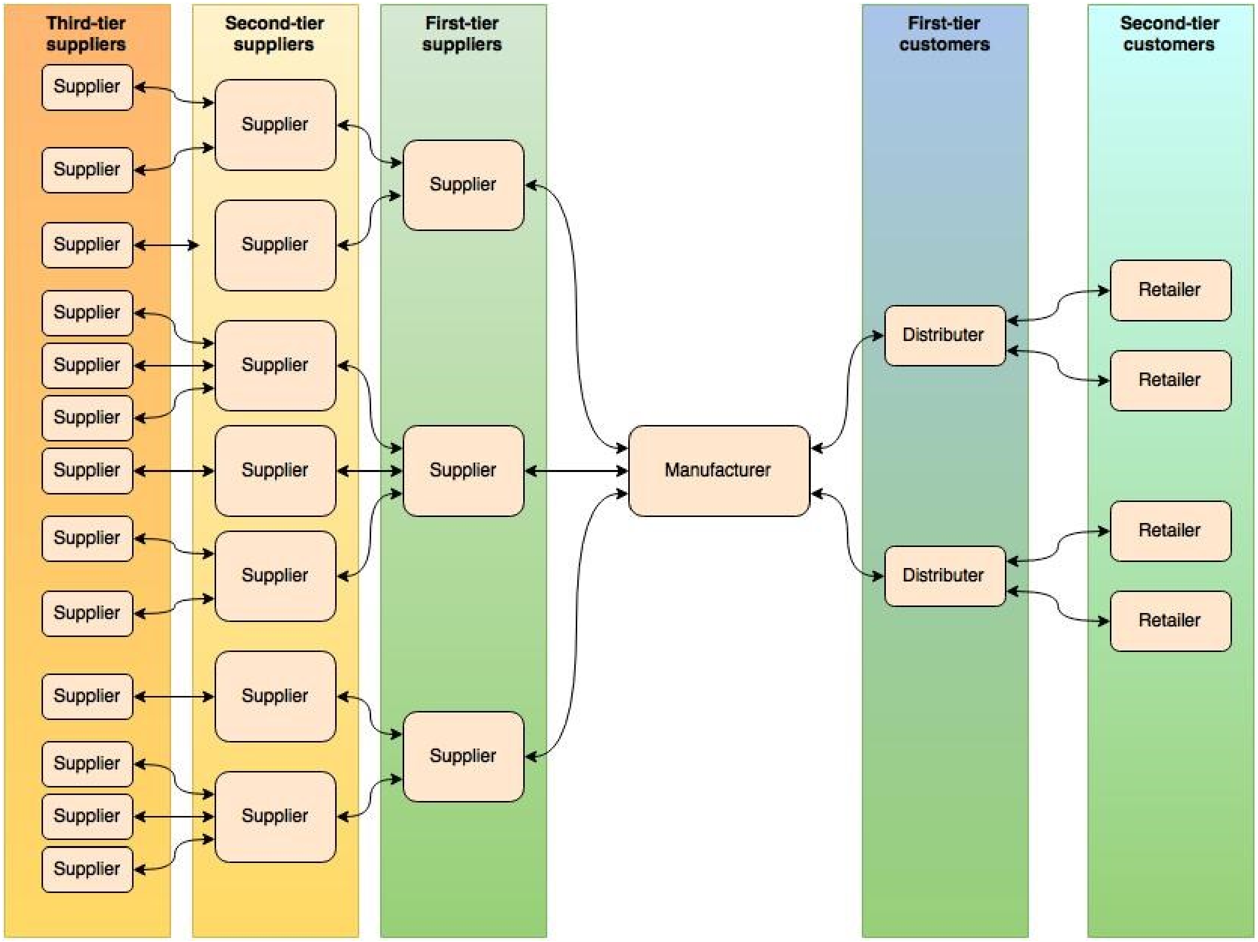



Figure 2.2 Multi-tier supply chain

With globalization, organizations face several challenges in managing supply chain quality (Jain and Benyoucef, 2008; Kuei *et al.*, 2011; Morehouse and Cardoso, 2011; Tse and Tan, 2011; Klassen and Vereecke, 2012; Tachizawa and Wong, 2014). To name a few are:

- Less visibility and control of key processes

  This causes the supply chain to become more vulnerable to frauds than before.

- Protecting the environment

  Considering the environment in order to survive in today's environmentally conscious market.

- Social issues

  Labour practices by global suppliers in developing countries, which includes worker safety, working conditions, workers' rights and child's labour.

- Inventory reduction

  Applying JIT philosophy can lead to significant cost-savings, however, the supply chain may become more vulnerable as they have very little inventory to buffer any interruptions in supply.

- Adopting advanced technologies

  To facilitate effective decision making; firms should identify technology applications that participate effectively in the global progress. Examples of such technology are enterprise resource planning (ERP), customer relationship management (CRM), and product lifecycle management (PLM).

Therefore, firms are striving to achieve successful supply chain collaboration to limit some of these impacts. Collaboration can improve the traceability and visibility among the supply chain members (Sarpong, 2014), which in turn improves the quality of the final product or service.

## 2.2 Collaboration for improving supply chain quality

Anthony (2000) stated that supply chain collaboration occurs "when two or more companies share the responsibility of exchanging common planning, management, execution, and performance measurement information". Furthermore, he suggested "Collaborative relationships transform how information is shared between companies and drive change to the underlying business processes".

In today's business environment; collaboration is more important than ever as technologies are rapidly changing, competition is growing, outsourcing is increasing, and the growth of very specialised companies is taking place (Sarpong, 2014). Several authors have discussed the benefits of successful collaboration. We mentioned some of these benefits in the previous chapter. Other benefits include more sales volume from downstream buyers, lower operational costs, word-of-mouth referrals, and new product and process innovations resulting from a working relationship between trusting partners (Sarpong, 2014). Additionally, collaboration improves and assists the environment and social aspects of sustainability (MacCarthy and Jayarathne 2012).

Many firms have started to realize that it is not enough to collaborate with the first-tier suppliers only. To achieve the full potential benefits of collaboration; firms should collaborate with lower tier suppliers as well. For example, Puma's sustainability report covers up to the fourth tier of suppliers, and Nike is auditing hundreds of second-tier apparel suppliers (Lee *et al.*, 2012). IKEA is working with its suppliers to comply with the code of conduct (Andersen and Skjoett-Larsen, 2009). Similarly, Hewlett-Packard and Migros manage sub-suppliers to ensure they fulfill the requirements of corporate sustainability standards (Grimm *et al.*, 2014).

The first step in collaboration is partner selection which starts with supplier evaluation in buyer-supplier partnerships. In the following section we will explore different approaches for supplier evaluation and reveal their weaknesses and strengths.

## 2.3 Evaluating supplier quality

Supplier evaluation is a multi-criteria decision making problem that involves several qualitative and quantitative factors. Several studies have been conducted to develop decision-making models that can address this problem effectively (Zeydan *et al.*, 2011). Ghodsypour and O'Brien (1998) classify supplier evaluation models into two groups: single objective models and multiple objective models. Single objective models use one criterion, such as the cost, as an objective function and other criteria as constraints. This approach has two weaknesses: it weights all constraints equally which rarely happens in reality, and it faces significant difficulties when considering qualitative factors. Moreover, it is very risky to rely on a single criterion when evaluating a supplier. Therefore, using a multi-criteria approach is preferable over the single criterion approach (Zeydan *et al.*, 2011).

One of the earliest contributions to supplier evaluation and selection criteria was of Dickson (1966). In his study he identified 23 criteria for supplier evaluation. Furthermore, he found that quality, delivery and performance history are the most important criteria (see Table 2.1). Erdem and Gocen (2012) list 60 criteria for supplier evaluation among which price, quality, availability and delivery are the most significant ones. These results confirm the multi-criteria nature of supplier evaluation problem.

| Evaluation Criteria | Rank |
|---|---|
| Quality | 1 |
| Delivery | 2 |
| Performance History | 3 |
| Warranties and claim policies | 4 |
| Production facilities and capacity | 5 |
| Price | 6 |
| Technical capability | 7 |
| Financial position | 8 |
| Procedural compliance | 9 |
| Communication system | 10 |
| Reputation and position in industry | 11 |
| Desire for business | 12 |
| Management and organization | 13 |
| Operating controls | 14 |
| Repair service | 15 |
| Attitude | 16 |
| Impression | 17 |
| Packaging ability | 18 |
| Labor relations record | 19 |
| Geographical location | 20 |
| Amount of past business | 21 |
| Training aids | 22 |
| Reciprocal arrangements | 23 |

Table 2.1 Dickson supplier evaluation criteria

According to Ho *et al.* (2010) and Erdem and Gocen (2012), supplier evaluation and selection approaches can be classified into:

- Linear weighting models such as Analytic hierarchy process, interpretative structure modeling, fuzzy set theory, and total cost of ownership.
- Mathematical programming models such as data envelopment analysis, linear programming, mix integer programming, and goal programming.
- Artificial intelligence models and Statistical/probabilistic models such as case based reasoning, genetic algorithm, neural network, and expert systems.

Supplier evaluation should be extended beyond the first-tier suppliers to cover all suppliers across the supply chain. Many researches consider this as one way to manage the supply chain (Rao, 2002; Zhu and Sarkis, 2004; Andersen and Skjoett-Larsen, 2009; Mueller *et al.*, 2009; Zhu *et al.*, 2012). Ho *et al*. (2010) and Noshad and Awasthi (2015) reviewed several published articles on multi-criteria decision making approaches for supplier evaluation and selection. DEA was the most popular individual approach per Ho *et al*. (2010) while AHP and DEA based approaches were the most popular per Noshad and Awasthi (2015). In the following section we will examine popular approaches found in literature. Additionally, a summary for these approaches is presented in table 2.4.

### 2.3.1 Total Cost of Ownership (TCO)

An IT research and advisory company called Gartner was the original developer of the total cost of ownership (TCO) concept in 1987. They used it to compute the total costs of owning and managing IT infrastructure in a company (Bermen *et al.*, 2007). Later, the concept has been used to calculate total cost of purchased goods in general (Ellram, 1993). TCO is a purchasing tool and a philosophy which is aimed at understanding the real cost of buying a particular good or service from a supplier (Ellram, 1995). It is a complex approach that requires the firm to determine the associated costs and find ways to quantify non-monetary costs if they want to better understand and manage their costs. It may include in addition to the quoted price, order placement costs, research costs, transportation costs, receiving costs, inspection costs, holding costs, and disposal costs (Bhutta and Huq, 2002).

Although there are other selection and evaluation approaches closely related to TCO, such as life-cycle costing, zero-base pricing, cost-based supplier evaluation, and cost ratio method, none of them received significant, widespread support in literature or in practice (Ellram, 1995). Complexity, situation-specific application, over-reliance on some factors and inadequate consideration of others are some of the factors that could be the reasons behind the lack of support (Bhutta and Huq, 2002).

| Author, Year | Incentive to Use TCO | TCO Cost Categories |
|---|---|---|
| Bhutta and Huq (2002) | • Supplier Selection | • Manufacturing<br>• Quality<br>• Technology<br>• After sales services |
| Degraeve and Roodhooft (1999) | • Supplier Selection | Three hierarchic levels:<br>• Supplier level activities<br>• Ordering level activities<br>• Unit level activities |
| Degraeve *et al.* (2005) | • Evaluating organization's strategic procurement options<br>• Purchasing managers performances evaluation<br>• Understanding the costs of purchasing activities | • Cost matrix of supplier, product, order, unit (cash, non cash) versus acquisition, reception, possession, utilization, elimination |
| Dogan and Aydin (2011) | • Supplier Selection | • Product design cost<br>• Downtime cost<br>• Logistics cost<br>• Operation cost<br>• Quality related cost<br>• Administrative cost<br>• Transaction cost |
| Ellram (1993) | • Supplier performance evaluation<br>• Decision making<br>• Supplier selection<br>• Understanding the costs of purchasing activities | • Pre-Transaction costs<br>• Transaction costs<br>• Post-Transaction costs |
| Hurkens e*t al.* (2006) | • Negotiating prices<br>• Identify and prioritize improvement actions<br>• Understand the consequences of changing volume allocation among suppliers. | • Dealer buy<br>• Quality confirmation<br>• Quality check<br>• Adverse buy<br>• Warehousing (Handling, Inventory holding, Storing)<br>• Supplier returns<br>• Supplier monitoring<br>• Cash flow |
| Maltz and Ellram (1997) | • Including non-monetary factors into make/buy decisions | • Management<br>• Quality<br>• Delivery<br>• Service<br>• Communications<br>• Price |

Table 2.2 Total cost of ownership literature review

On the other hand, there has been more focus on TCO among the published articles. Table 2.2 highlights some of these articles along with the incentives of using TCO and the cost categories suggested by different researchers. It can be seen that many researches have considered supplier selection as the main reason for choosing TCO while others considered supplier evaluation as the main motive. But the common reason is to reveal the true cost of purchasing activities (Ellram, 1993). Only (Maltz and Ellram, 1997) considered it from another perspective by incorporating TCO into the make/buy decision.

#### *2.3.1.1 Barriers to TCO*

Despite these efforts, TCO is still a complex approach and many organizations face a lot of difficulties and barriers when trying to implement it. Ellram (1995) considered lack of an accounting and costing system as a major challenge. Many organizations use activity based costing as a way to overcome this barrier (Ellram, 1993; Ellram, 1995; Sohal and Chung, 1998). Culture change is another barrier, a change from price orientation towards understanding total cost. That's why TCO is considered more a philosophy than a tool (Ellram, 1995). Furthermore, there is no standard model for TCO analysis. Using TCO models may vary inside the organization according to the importance of the purchase (Ellram, 1995; Ferrin and Plank, 2002).

#### *2.3.1.2 Benefits of TCO*

TCO delivers many benefits to organizations. Cost transparency is pointed as the basic advantage of applying a TCO approach (Bremen *et al.,* 2007). Other benefits considered by Ellram (1995) are improving the supplier evaluation and selection process, defining the supplier performance expectations for the organization and the supplier, help prioritize areas in which supplier performance would be most beneficial (such as supplier continuous improvement), creating major cost saving opportunities, improving firm's understanding of supplier performance and cost structure, and using cost information as a basis for supplier negotiations. Only Bhutta and Huq (2002) pointed out that TCO helps in building strategic collaboration efforts.

#### *2.3.1.3 TCO Cost Categories*

Table 2.2 illustrates that authors categorized cost according to the functional departments in an organization. Maltz and Ellram (1997) used the categories as management, quality, delivery, service, communications, and price. Ellram (1993) suggested a more general approach based on the order of occurrence: pre-transaction costs, transaction costs, post-transaction costs. Pre-transaction costs involve all costs related to activities that occur prior to order placement such as identifying requirements, searching for a supplier, negotiation, and supplier evaluation. Transaction costs occur at the time of purchasing and receiving the ordered item. Price, delivery charges, and inspection are some examples of transaction costs. Post-transaction costs occur after closing the order. In other words, post- transaction costs include all activities that happen after the organization owns the ordered item such as maintenance costs and quality failures costs (Ellram, 1993).

Hurkens *et al*. (2006) conducted a study on a service sector organization (a vehicle glass repair and replacement company) to show how TCO information can influence strategic decision regarding allocation of volumes. They used the cost categories as buying from dealer, quality confirmation, quality check, adverse buy, supplier returns, supplier monitoring, and cash flow. Bhutta and Huq (2002) categorized production into manufacturing, quality, technology, and after sales services costs to compare between total cost of ownership and analytic hierarchy process approaches. Manufacturing costs include raw material, labor, and machine depreciation. Quality costs are related to activities performed to monitor and control quality such as inspection, cost of rework, and scrap. Designing and engineering costs are some examples of technology costs. Finally after sales costs are related to after sales activities such as warranty and customer claims (Bhutta and Huq, 2002; Garfamy, 2006).

Degraeve and Roodhooft (1999) developed a decision-making model to minimize TCO when selecting a supplier. The model is based on three hierarchical levels. The first level is the supplier's activities that are performed when a specific supplier is used. For example, inspection and quality audit. On the next level we have ordering level activities. Activities on this level should be performed each time an order is assigned to a supplier. Unit level activities are at the last level and performed for a unit in a specific order. For example, production downtown cost related to a defective item purchased from a supplier.

Degraeve *et al*. (2005) presenting a matrix model consisting of supplier level, product level, order level, and unit level activities and their associated life cycle costs. Furthermore, Dogan and Aydin (2011) proposed a model that combines total cost of ownership and Bayesian networks to efficiently select a supplier using product design cost, downtime cost, logistics cost, operation cost, quality related cost, administrative cost, and transaction cost as the cost categories.

Researchers have presented different costs categories to serve different purposes as can be seen from the previous section but no study so far, according to our knowledge, suggested cost categories from collaboration perspective. This thesis is trying to fulfill this gap by incorporating collaboration into our selection of the cost categories.

### 2.3.2 Analytic Hierarchy Process (AHP)

Saaty first developed the Analytic Hierarchy Process (AHP) method in 1980. AHP provides a multiple criteria framework for situations involving intuitive, rational, quantitative, and qualitative aspects. It provides a mathematical model to assign weights to multiple alternatives using a scheme of pairwise comparison (Singh, 2014). AHP allows complex problems to be represented in a hierarchical form that consists of at least three levels, the goal, the criteria, and the alternatives (Bhutta and Huq, 2002). Figure 2.3 shows an example of the hierarchal form of AHP in the supplier evaluation context.

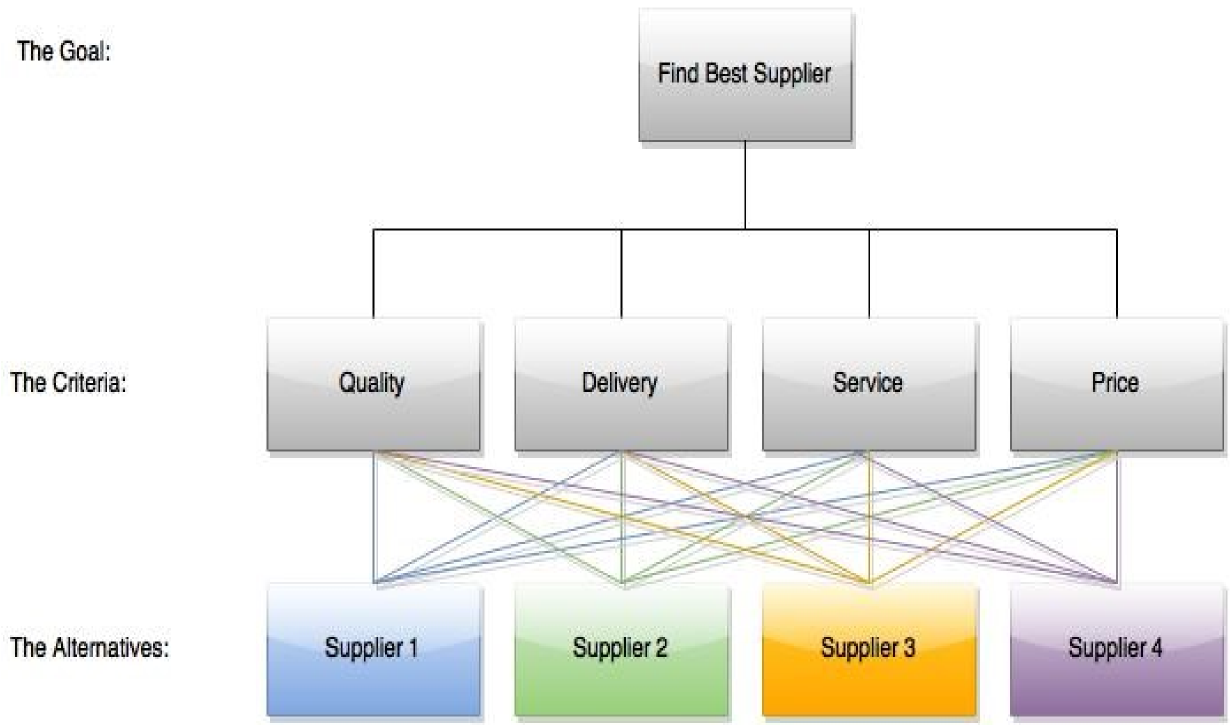



Figure 2.3 Hierarchical Structure

Managerial experience then will drive the computations by assigning weights to each criterion. Thus, quantifying the managerial experience is required at this stage. According to Bhutta and Huq (2002), the scale presented in Table 2.3 is the most common scale used for this analysis. Next step is to construct a matrix that compares each criterion with other using this measurement scale.

| Preference | Numerical Scale |
|---|---|
| Extremely preferred | 9 |
| Very strongly to extremely preferred | 8 |
| Very strongly preferred | 7 |
| Strongly to very strongly preferred | 6 |
| Strongly preferred | 5 |
| Moderately to strongly preferred | 4 |
| Moderately preferred | 3 |
| Equally to moderately to strongly preferred | 2 |
| Equally preferred | 1 |

Table 2.3 AHP Measurement Scale

Once the matrix is ready we can start the calculation process as follows:

1) Compute the sum of each column.
2) Divide each cell by the column total.
3) Calculate row averages.

Similarly, we develop the suppliers' matrix by assigning a score to each supplier with respect to the criteria under consideration. Then, the above steps are followed to calculate the weight of each supplier. Finally, the scores of each supplier are calculated by multiplying the founded scores in the criteria matrix with the weights in the suppliers' matrix, which give us the final scores of each supplier.

Many researchers have used AHP for supplier selection and evaluation (Akarte *et al.*, 2001; Muralidharan *et al.*, 2002; Chan, 2003; Liu and Hai, 2005; Hou and Su, 2007; Bruno *et al.*, 2012; Deng *et al.*, 2014) due to the flexibility, simplicity, and the capability of this approach to incorporate both quantitative and qualitative criteria. On the other hand, AHP has the disadvantage that the number of pairwise comparisons may become very large when having many alternatives and criteria ($n(n-1)/2$). Additionally, another disadvantage is the artificial limitation of the 9-point scale. For instance, if alternative X is 5 times more important than alternative Y, which is 5 times more important than alternative Z. In this case, it is not possible for AHP to handle the fact that alternative X is 25 times more important alternative Z (Macharis e*t al.*, 2004).

### 2.3.3 Goal Programming (GP)

Charnes and Cooper first presented this technique in 1961. Later, Lee and Ignizio improved it in 1972 and 1976 respectively. Goal programming (GP) is a multi-objective decision making technique (Bal *et al.*, 2006). GP involves a set of goals that may often contradict each other. Therefore, it is not possible to satisfy all goals at the same time as achieving one goal may cause another goal to deviate from its own target. Thus, the purpose of GP is to minimize the deviation between achievement of goals and their aspiration levels (Liao and Kao, 2010).

To construct a GP model, we use the following steps (Ignizio, 1976):

i. Define the decision variables.

ii. Determine the goal and system constraints.

iii. Decide the preemptive priority factors and the relative weights.

iv. Define the nonnegative requirement.

The mathematical formulation for GP can be as following (Romero, 2004):

Min $\sum_{i=1}^{q} | f_j(X) - b_i |$

subject to

$X \in F$

where

$F$ is a feasible set

$X$ is an element of $F$

$f_j(X)$ is a linear function of $i^{th}$ goal.

$b_i$ is the aspiration level of the $i^{th}$goal.

$q$ is the total number of goals

GP is useful in the cases of multiple goals that are conflicting and not all achievable. Subsequently, goal programming allows ranking of goals so that the lower priority goals are considered only when higher priority goals are fully satisfied. Also GP is useful when a satisfactory solution is required rather than an optimized solution (Hughes and Grawoig, 1973). As a result, various studies used GP for supplier selection and evaluation either as an individual approach or combined with other approach such as AHP. For instance, Karpak *et al.* (2001) and Osman and Demirli (2010) developed GP model to evaluate suppliers and allocate orders. Çebi and Bayraktar (2003) applied an integrated AHP-GP approach for supplier selection. Weights of suppliers were evaluated using AHP. Then the GP model uses these weights as an input for finding the best supplier. Wang *et al.* (2004), Perçin (2006), Kull and Talluri (2008), Mendoza *et al*. (2008) and Erdem and Gocen (2012) presented different AHP-GP approaches for supplier selection and evaluation.

GP has some problems. One major problem arises from the requirement of setting the goal achieving order, i.e., the weights of each goal. If the model did not produce an acceptable solution then the purchasing managers may alter the priority structure until an acceptable solution is generated. This may be costly and time consuming (Karpak *et al*., 2001). Furthermore, the goal achievement nature of GP may not be appropriate for supplier evaluation for the purpose of collaboration. As stated before, GP will not achieve all goals at the same time and the generated solution is satisfactory rather than an optimum one.

## 2.3.4 Data Envelopment Analysis (DEA)

### *2.3.4.1 Basic Model*

DEA is a popular technique for supplier evaluation and selection (Braglia and Petroni, 2000; Liu *et al.*, 2000; Forker and Mendez, 2001; Narasimhan *et al.*, 2001; Talluri and Baker, 2002; Talluri and Sarkis, 2002; Talluri and Narasimhan, 2004; Garfamy, 2006, Ross *et al.*, 2006; Saen, 2006; Seydel, 2006).

DEA was originally developed by (Charnes *et al.*, 1978) to evaluate non-profit and public sector organizations. It was known as the Charnes, Cooper and Rhoades (CCR) model. Since then DEA has become one of the most effective techniques to measure the performance of organizations such as business firms, government departments, hospitals etc. It does not require the decision maker to define weights for each indicator. It simply calculates the weights from the given data. Moreover, DEA is capable of distinguishing the benchmark entities based on the efficiency score and finding the amount and source of inefficiency of inefficient entities (Cooper *et al.*, 2007).

The objective of DEA is to find the efficiency score of all units under evaluation. These units are called decision-making units or (DMUs). DMU is defined as an entity that consumes inputs to produce outputs and whose performance is to be evaluated (Cooper *et al.,* 2007). DMUs can be the members of a supply chain, firms or simply departments of a single organization.

The first step in evaluating the performance of a DMU is finding a virtual DMU that will be the most efficient unit (called efficient frontier). Then DEA compares all DMUs to the efficient frontier to find their efficiency scores. The efficiency score is defined as weighted sum of outputs divided by weighted sum of inputs. This does not mean we need to assign weights as DEA calculates weights automatically based on the given inputs and outputs. Cooper *et al.* (2007) set the following rules for selecting the inputs and outputs:

- All inputs and outputs should have numerical data which is assumed to be equal or greater than zero.
- The selection of inputs, outputs and DMUs should be relevant to the study.
- Efficiency scores should reflect the following principles:
    i. Smaller input amounts are preferable.
    ii. Larger output amounts are preferable.
- The measurement units across the different inputs and outputs should not be the same.

Suppose we have $n$ different DMUs and each one has $m$ input items and $s$ output items.

$$\text{Efficiency } = \frac{\text{Sum of weighted outputs}}{\text{Sum of weighted inputs}}$$

which can be reformulated per CCR-DEA (Cooper *et al.,* 2007) into the following linear program:

$Min\ \theta_p$

$\sum_j^n x_{ij}\lambda_j \leq \theta_p X_{ip}$ for all $i = 1, \ldots., m$

subject to

$\sum_j^n y_{rj}\lambda_j \geq y_{rp}$ for all $r = 1, \ldots., s$

$\sum_j^n \lambda_j = 1$

where:

$\theta_p$ is the efficiency score of $\text{DMU}_\text{p}$ ( the DMU under evaluation )

$X_{ip}$ is the consumed amount of input $i$ by $\text{DMU}_\text{p}$

$y_{rp}$ is the produced amount of output $r$ by $\mathrm{DMU_p}$

$\lambda_j$ is the computed weights associated with $\mathrm{DMU_j}$ determining whether it is a benchmark for $\mathrm{DMU_p}$

$x_{ij}$ is the consumed amount of input $i$ by $\mathrm{DMU_j}$

$y_{rj}$ is the produced amount of output $r$ by $\mathrm{DMU_j}$

The above equations simply mean that the computed virtual DMU should satisfy two conditions: (i) consume the same or less input amount than $\mathrm{DMU_p}$. (ii) Produce the same or more output than $\mathrm{DMU_p}$.

Moreover, we should note the following:

i. $\mathrm{DMU_p}$ is efficient when $\theta_p = 1$.
ii. $\mathrm{DMU_p}$ is inefficient when $\theta_p < 1$.
iii. Efficiency cannot be greater than 1.

#### *2.3.4.2 Network DEA*

The major drawback of traditional DEA is that it treats DMUs as a "black box" by considering only the initial inputs and the final outputs and omits the actual activities happening inside (Lewis and Sexton, 2004; Kao, 2009; Zhu, 2009; Azbari *et al.,* 2014; Yang *et al.,* 2014). For example, traditional DEA can be used for selecting an efficient supplier among proposed suppliers for a new product but it cannot be used to evaluate the current upstream suppliers, as it will ignore the sub-suppliers and current linkages between them. Therefore, network DEA is preferable in these cases for its advantages over the traditional DEA. Specially, its ability to detect efficiencies missed by the traditional DEA. Network DEA considers the intermediate linkages as the outputs of the previous stages and the inputs to the next stage.

Many approaches for network DEA have been presented over the years. Aoki *et al.* (2010) developed a network DEA model based on the RAM model presented by Cooper *et al.* (1999) to optimize the supply chain. Mirhedayatian *et al.* (2014) proposed a network DEA model to assess green supply chain management in the existence of dual-role factors, undesirable outputs, and fuzzy data. Tajbakhsh and Hassini (2014) developed a multi-stage DEA capable of evaluating the sustainability of the supply chain members. Chen and Yan (2011) created three network DEA models to evaluate the performance of supply chains under the concept of centralized, decentralized, and mixed organizational mechanisms.

Despite its many benefits, DEA has some drawbacks. First, the decision-maker may be confused when defining the input and output criteria. Second drawback comes from the subjective assignment of qualitative criteria. Finally, DEA finds the efficient supplier who generates more output while using less input. Therefore, can an efficient supplier be considered an effective one? (Ho *et al.*, 2010).

This thesis is using the network DEA model presented by Zhu (2009) which embodies the structure of the supply chain to define and evaluate efficiency of the supply chain and its individual members. Moreover, the model yields a list of optimal values for all members that establish an efficient supply chain.

| Technique | Description | Benefits | Drawbacks | Supplier evaluation in literature |
|---|---|---|---|---|
| TCO | A purchasing tool which is aimed at understanding the real cost associated with the entire purchasing cycle | • Cost transparency<br>• Improves supplier evaluation and selection process<br>• Defines the supplier performance expectations<br>• Helps prioritising supplier performance areas<br>• Cost savings opportunities<br>• Better understanding of supplier performance and cost structure<br>• Can be the basis for supplier negotiations<br>• Helps building strategic collaboration efforts. | • Complex approach as it may require methods to quantifying non-monetary costs<br>• Lack of an accounting and costing system<br>• Requires culture change<br>• No standard model | Ellram (1993);Degraeve and Roodhooft (1999);Dogan and Aydin (2011);Bhutta and Huq (2002) |
| AHP | A mathematical model that allows complex problems to be represented in a hierarchal form. | • Flexibility<br>• Simplicity<br>• Can incorporate quantitative and qualitative criteria | • Very complex pairwise comparisons in case of many alternatives and criteria.<br>• Artificial limitation of the 9-point scale. | Akarte *et al.* (2001); Muralidharan *et al.* (2002); Chan (2003); Liu and Hai (2005); Hou and Su (2007); Bruno *et al.* (2012); Deng *et al.* (2014) |
| GP | A multi-objective decision making technique that involves a set of goals that may contradict with each other. | • Helps when we have multiple goals that are conflicted and not all achievable.<br>• Allows ranking of goals so that the lower priority goals are considered only when higher priority goals are fully satisfied<br>• Can be used when satisfied solution is required rather | • Can be costly and time consuming<br>• The generated solution will be a satisfactory solution rather than an optimum one | Karpak *et al*. (2001); Çebi and Bayraktar (2003);Wang *et al*. (2004); Perçin (2006); Kull and Talluri (2008); Mendoza *et al*. (2008); Osman and Demirli (2010); Erdem and Gocen (2012) |

| | | | | |
|---|---|---|---|---|
| | | than an optimized solution | | |
| **Technique** | **Description** | **Benefits** | **Drawbacks** | **Supplier evaluation in literature** |
| DEA | A linear programming based technique to measure the efficiency of a set of entities that have multiple inputs and outputs. | • Non-parametric analysis technique<br>• Considers both quantitative and qualitative data<br>• Finds the efficient supplier as well as the amount and source of inefficiency of inefficient suppliers<br>• Provides improvement targets | • May draw confusion when defining the input and output criteria<br>• Subjective assignment of qualitative criteria<br>• Can mislead the purchase manager, as efficient supplier may not be an effective one. | Braglia and Petroni (2000); Liu *et al.* (2000); Forker and Mendez (2001); Narasimhan *et al.* (2001); Talluri and Baker (2002); Talluri and Sarkis (2002); Talluri and Narasimhan (2004); Garfamy (2006); Ross *et al.* (2006); Saen (2006); Seydel (2006) |

Table 2.4 Summary of popular supplier evaluation techniques

## 2.4 Supplier Quality Evaluation Framework

de Boer *et al.* (2001) developed a decision making framework for supplier selection that expands the purchasing model of Faris et al. (1967) and the model of Kraljic (1983) to include more situations that are not usually found in literature. They differentiate between three types of procurement situations: first time buy, modified rebuy, and straight rebuy of routine and strategic items. Additionally, they propose four phases of the supplier selection process: problem formulation, formulation of criteria, qualification, and supplier selection. Their framework is presented in table 2.5.

First time buy situation usually consists of a brand new product or service with unknown suppliers. In this phase, the level of uncertainty is the highest and no previous historical data about the suppliers is available. As a result, this is the most complex procurement situation and group decision-making is required to solve it.

Modified rebuy is related to buying a new product or service from a known supplier or buying a modified or existing product from a new supplier. The uncertainty level in this case is moderate. On the other hand, in straight rebuy situation we have enough information about the required product or service. Furthermore, an agreement already exists with a known supplier and it is just a matter of placing an order.

| | First Time Buy | Modified Rebuy (Leverage Items) | Straight Rebuy (Routine Items) | Straight Rebuy (Strategic Items) |
|---|---|---|---|---|
| **Problem Definition** | Whether to use a supplier or not | Either to use more, less or other suppliers | Do we need to replace the current supplier? | How to deal with the current supplier? |
| **Formulation of Criteria** | • Various importance levels<br>• Unrepeatable decision<br>• No previously defined criteria or suppliers historical data available | • Moderate to high importance<br>• Repeatable decision<br>• Previously defined criteria and suppliers historical data available | • Low to moderate importance<br>• Repeatable decision<br>• Previously defined criteria and suppliers historical data available | • High importance<br>• Repeatable evaluation<br>• Previously defined criteria and suppliers historical data available |
| **Qualification** | • Various importance levels<br>• Small initial set of suppliers<br>• Sorting rather than ranking<br>• No historical data available | • Large number of initial suppliers<br>• Sorting and ranking<br>• Historical data available | • Large number of initial suppliers<br>• Sorting rather than ranking<br>• Historical data available | • Very small number of suppliers<br>• Sorting rather than ranking<br>• Historical data available |
| **Choice** | • Small initial set of suppliers<br>• Ranking rather than sorting<br>• Several criteria<br>• A lot of interaction<br>• No historical data available<br>• Various importance levels<br>• One time used | • Small to moderate number of initial suppliers<br>• Ranking rather than sorting<br>• How to allocate volume?<br>• Less criteria<br>• Fewer interaction<br>• Historical data available<br>• Model can be used again | • Small to moderate number of initial suppliers<br>• Ranking rather than sorting<br>• Less criteria<br>• Fewer interaction<br>• Historical data available<br>• Model can be used again<br>• Single sourcing rather than multiple sourcing | • Very small number of suppliers (usually one)<br>• Historical data available<br>• It is evaluation rather than selection<br>• Sole sourcing |

Table 2.5 de Boer *et al.* (2001) supplier selection framework

It can be noticed from table 2.5, that although de Boer *et al.* (2001) presented different important levels for the first buy situations but steps of the supplier selection process are the same regardless of the importance. In the rebuy situations, selection steps may vary. In the following paragraphs we will show how de Boer *et al.* (2001) linked these variations to Kraljic (1983) model.

Kraljic (1983) classified the purchase items into leverage, strategic, bottleneck, and routine. In the case of routine item, de Boer *et al.*, (2001) indicated that it is not worthy for an organization to frequently search for new suppliers that could supply the item because of its low value. Therefore, similar routine items are usually purchased from one or two suppliers. As per de Boer *et al.* (2001) any modification related to the specifications of the purchased items are dealt by the current supplier. An evaluation of the supplier is carried out periodically and a new supplier is selected if needed.

Similarly, bottleneck and strategic items have fixed suppliers. The current supplier deals with any changes to the specifications of these items. The reason behind this is different from that of the routine items. These items involve high supply risk because of their unique specifications or the rare material. Therefore, the decision models are only for evaluation and monitoring these suppliers.

On the other hand, modified rebuy situations are usually related to leverage items. In this case, there are many suppliers to select from. A frequent selection of suppliers occurs more often because of high value of these items. An agreed vendor list is the outcome of the first three steps (Problem definition, formulation of criteria and prequalification) and the final supplier is selected from this list (de Boer *et al.*, 2001).

In the section below, we will list the different methods found in literature for different phases of supplier selection process.

### 2.4.1 Problem definition and formulation of criteria

For the problem definition phase, there are not many works on supplier selection. de Boer *et al.* (2001) found two methods that can be used for formulation of criteria. The first technique is called Interpretive Structural Modelling (ISM). The second method is an expert system that covers different phases in the selection process including the supplier selection criteria. This system is based on literature review and knowledge of a senior purchasing manager.

### 2.4.2 Pre-qualification

de Boer *et al*. (2001) defined pre-qualification as the process of selecting an acceptable set of suppliers from the whole set. The methods commonly used in this phase are categorical methods, data envelopment analysis, cluster analysis and case based reasoning systems.

In categorical methods, the supplier is evaluated based on both the buyer's experience and historical data. The supplier is evaluated as positive, neutral or negative. The buyer later gives an overall rating and the suppliers are classified into these three categories.

de Boer *et al*. (2001) suggested DEA and cluster analysis for categorizing suppliers prior to the final selection. Case based reasoning can also be used. It is a software system based on artificial intelligence that aids decision makers with helpful information and experiences from past decisions.

### 2.4.3 Final choice phase

In this phase, linear weighting, total cost of ownership, mathematical programming, statistical and artificial intelligence based models are often used for selecting the appropriate suppliers (de Boer *et al*., 2001).

In linear weighting models, each criterion is given a weight. Then the weights are multiplied by the criteria ratings and summed. AHP is an example of this method.

Total cost of ownership is a tool and philosophy to achieve a better understanding of the real cost of buying a particular good or service from a supplier (Ellram, 1995). Mathematical programming models aid the decision maker in formulating the purchasing problem into an objective mathematical formula that should be maximized or minimized by altering the values of its variables (de Boer *et al*., 2001).

Statistical models deal with "stochastic uncertainty" related to supplier selection.

Finally, artificial intelligence based models are learning based software systems that can be consulted for selecting an appropriate supplier.

### 2.4.4 The Proposed Framework

de Boer *et al*. (2001) addressed supplier evaluation mainly at one level whereas in this thesis we propose a framework for multi-tier supplier evaluation. Therefore, we are dealing with a different problem as shown below:

- Problem definition

  Evaluate the current *n*-tier suppliers to study the possibility of collaboration.

- Formulation of criteria

  The historical data is available as we are evaluating existing suppliers. Both qualitative and quantitative criteria are used in evaluating the supplier performance.

- Qualification

  Cluster analysis is used to classify suppliers into groups with similar characteristics. This is an essential step when there are a large number of suppliers whose evaluation can be costly and time consuming.

- Final choice

  This phase consists of two steps. First, we find the true cost associated to each supplier using TCO. In the second step, we use the TCO results as input to the DEA model to find the efficient suppliers. TCO and DEA help identify the weak points in supplier performances and suggest improvement targets. These improvement targets cannot be identified if we use another model than DEA in this phase.

# Chapter 3

# Solution Approach

Our solution approach involves three main steps:

1. Grouping of upstream suppliers using hierarchical cluster analysis
2. Multi-tier supplier quality evaluation using TCO and DEA
3. Identifying targets for improving poor quality suppliers performance

## 3.1 Grouping of upstream suppliers using hierarchical cluster analysis

The first step involves classifying suppliers into groups on the basis of similar characteristics. This is an essential step when there are a large number of suppliers that make the evaluation process costly and time consuming. In this step, we will classify only the first-tier suppliers as the lower-tier suppliers will be grouped automatically based on their first-tier suppliers. We base our classification on the following criteria. These criteria were identified based on our previous experience with supply chain projects.

- Cost of Quality

It is the cost related with avoiding poor quality or cost that is encountered as a result of poor quality. It can be categorized into prevention costs, appraisal costs, internal costs, and external failure costs (Evan and Lindsay, 2005).

- Risk

Supply risk is defined as "the probability of an incident associated with inbound supply from individual supplier failures or the supply market occurring, in which its outcomes result in the inability of the purchasing firm to meet customer demand or cause threats to customer life and safety" (Zsidisin, 2003).

- Location

It is a numerical value to represent the supplier geographical location (country name).

- Product type

A numerical value to represent the type of product purchased from the supplier. For instance, raw material, spare parts, and packaging materials.

- Organization profile (size)

It is the number of employees in supplier's firm.

- Cost sustainability

Energy cost, water cost, social policies, fines and penalties.

### 3.1.1 Hierarchical Cluster Analysis

Hierarchical cluster analysis can be classified into: agglomerative approaches and divisive approaches. Agglomerative approach is a bottom-up approach where initially each value is a cluster in its own. Then each pair of clusters is merged together until only one cluster is left. On the other hand, divisive approach considers the whole dataset as one cluster at the beginning, and then splits them into further clusters (Al Salem, 2012).

In order to use these approaches, first distance measure needs to be chosen. Al Salem (2012) listed the following measures to find how two objects are similar or dissimilar:

1. Euclidean distance: It is a popularly used measure. Euclidean distance uses the formula $d_{ij} = \sqrt{\sum_{\mathrm{k=1}}^{\mathrm{p}} \left(x_{ik} - x_{jk}\right)^2}$ to compute the distance between two objects $x_{ik}$ and $x_{jk}$ for suppliers $i$ and $j$ under the $k^{th}$ variable value of the $p$-dimension.

2. Manhattan distance: It calculates total absolute distance between two objects under the $k^{th}$ variable value of the p-dimension using the formula, $d_{ij} = \sum_{\mathrm{k}=1}^{\mathrm{p}} |x_{ik} - x_{jk}|$.
3. Minkowski distance: The previous two measures are a special case of this one.

$$d_{ij} = \left(\sum_{\mathrm{k}=1}^{\mathrm{p}} |x_{ik} - x_{jk}|^r\right)^{1/r} \quad r \geq 1$$

Next step is to link the objects to clusters using one of the below linkage methods as per Al Salem (2012):

1. Single linkage distance: The linkage is determined based on the minimum distance between two objects.
2. Complete linkage: The linkage is determined based on the maximum distance between two objects.
3. Average linkage: The linkage is determined based on the average distance between elements of one cluster and the other clusters.

The linkage of objects to clusters will construct a tree diagram called dendrogram. A dendrogram shows how the elements of each cluster are connected and the distance at which clusters are joined. Figure 3.1 illustrates a sample dendrogram.

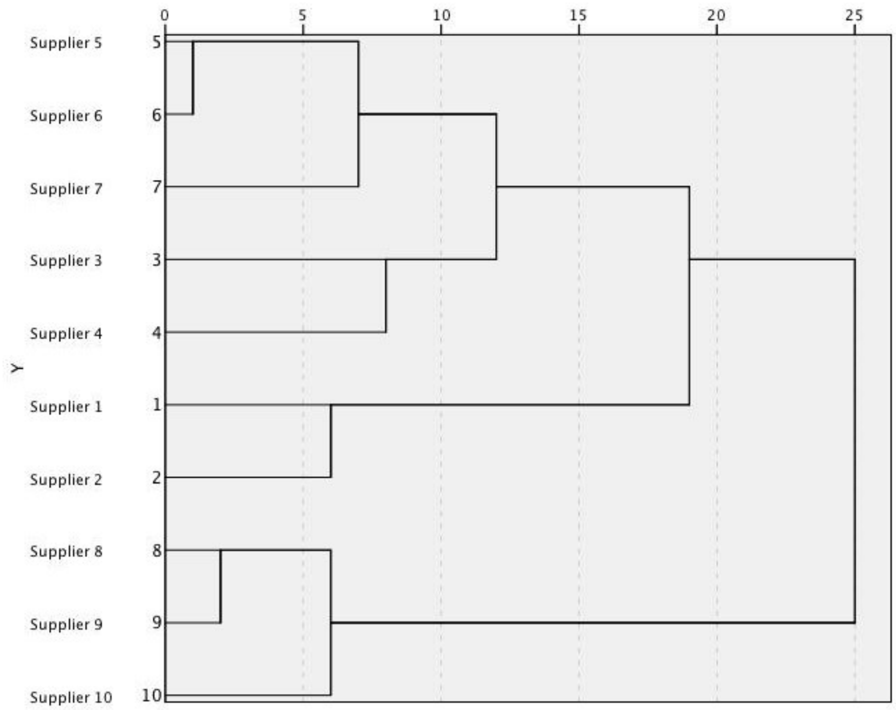


Figure 3.1 A sample dendrogram

In this thesis we are using hierarchical cluster analysis as it is a straightforward clustering technique. Moreover, this technique does not require the number of clusters to be known at the beginning. Once the dendrogram is constructed; the number of clusters can be retrieved from the diagram. We are using the best-cut method to find the number of clusters. Best-cut method is the largest distance width of range between two connected distances. For example in figure 3.2, the best cut lies between distance 19 and 12. This is because the width of range $d_{ij}$ between distance 19 and 12 is equal 7 which is larger than between 12 and 8. Therefore, we have 3 clusters. Cluster 1 contains supplier 5, 6, 7, 3 and 4. Cluster 2 has supplier 1 and 2. Finally the remaining suppliers are in cluster 3.

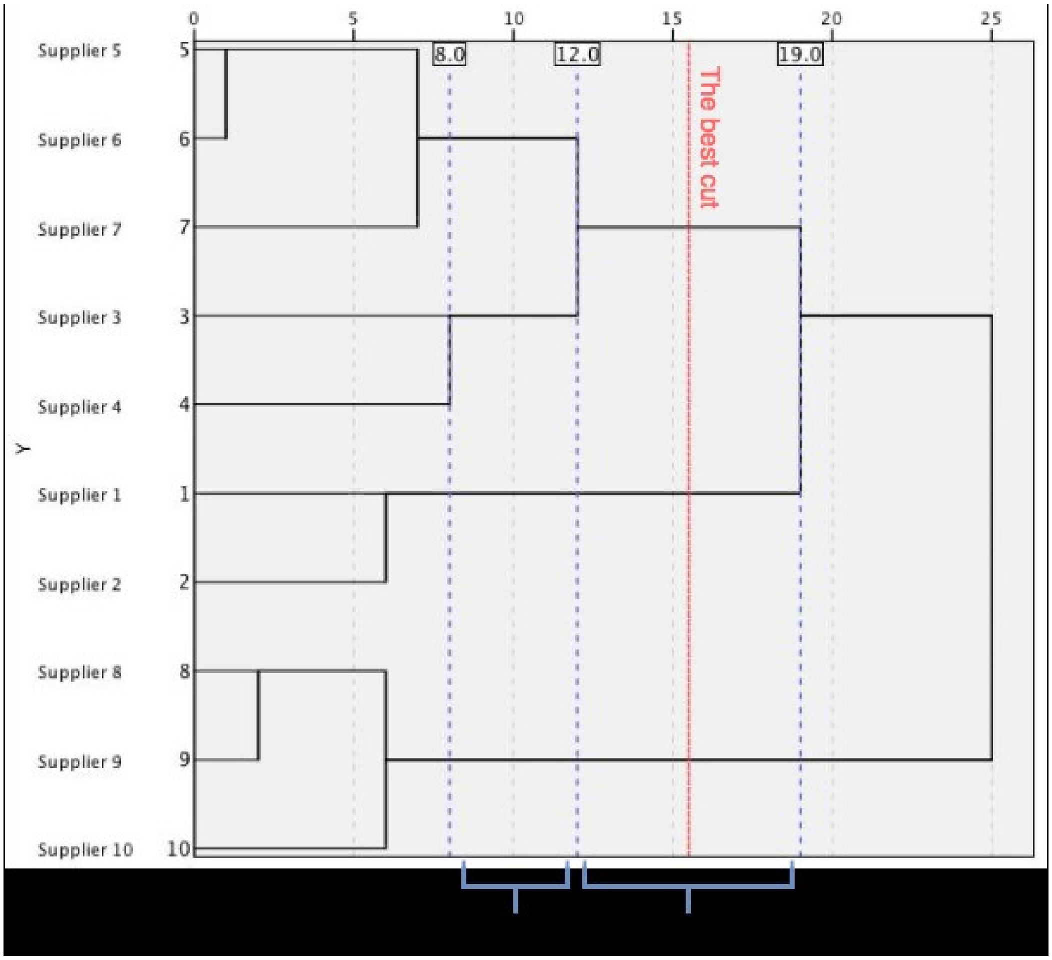



Figure 3.2 Best cut method

Thus, our grouping approach will be as follows:

1. Since our data variables are in different units, normalization is required. We will be using the formula $z = \frac{(x-\bar{x})}{s}$ where $x$ is the data value, $\bar{x}$ is the mean, and $s$ is the standard deviation.
2. Assigning weights to product type values to ensure that suppliers with the same product type stay together.
3. Applying hierarchical clustering using Euclidean distance measures and the complete linkage method.

## 3.2 Multi-tier supplier quality evaluation using TCO and DEA

Our evaluation framework is based on Total Cost of Ownership (TCO) and Data Envelopment Analysis (DEA). The first step involves finding the true cost associated to each supplier using TCO. We will go beyond the quoted price and try to figure out how much it will cost us if we continue dealing with the current suppliers. Furthermore, we will not just evaluate the first tier (direct) suppliers but all the *n*-tier suppliers. This is important if we are aiming towards a successful collaboration.

We will expand the model presented by Garfamy (2006) to include categories related to global supply chain collaboration. Our approach evaluates the whole upstream suppliers whereas Garfamy (2006) approach could be useful to select the best supplier among the suppliers supplying a given part. Therefore, to evaluate our networked suppliers we will need different DEA than the one Garfamy (2006) has used. We will use the Network DEA model presented by Zhu (2009) as stated in the previous chapter.

| TCO Categories | TCO Sub Categories | Author | Global Context | Measurement Method | Product Vs. Service | Input/ Output |
|---|---|---|---|---|---|---|
| Manufacturing Costs | Raw Material | Bhutta and Huq (2002) | Yes | Quantitative costs | P | I |
| | Labour | | Yes | Quantitative costs | P | I |
| | Machine depreciation | | Yes | Quantitative costs | P | I |
| Quality Costs | Quality Audit Cost | Song *et al.* (2007) | Yes | Qualitative cost:<br>Inspection cost and validation cost:<br>Standard cost per hour x time spent | P,S | I |
| | Rework Cost | Song *et al.* (2007) | Yes | Rework cost per unit x number of unit reworked | P | I |
| | Quality Confirmation | Hurkens *et al.* (2006) | Yes | Qualitative cost:<br>Labor cost of quality confirmation:<br>No. of assigned people x Labor cost x Percentage of labor spent on quality confirmation.<br>Quality confirmation cost per supplier:<br>No. of checks required to accept a new item x Labor cost of quality confirmation. | P,S | I |
| Design Costs | Technological capability | Dogan (2011) | Yes | Qualitative cost:<br>He propose an assumption that the design cost will decrease if the supplier is technologically capable, flexible and financially healthy supplier | P | I |
| | Flexibility | | Yes | | P | I |
| | Financial Factors | | Yes | | P | I |
| Logistics Costs | Freight | Dogan (2011) | Yes | Quantitative costs | P | I |
| | Handling and packaging | | Yes | Quantitative costs | P | I |
| | Tariffs, duties and Import fees | | Yes | Quantitative costs | P | I |
| | Customer Service | | Yes | Quantitative costs | P | I |
| | Outbound costs | | Yes | Quantitative costs | P | I |
| | Tariffs warehousing | | Yes | Quantitative costs | P | I |
| After sale service | Service Cost | Bhutta and Huq (2002) | Yes | Quantitative costs | P | I |
| Social / Environmental Costs | Energy Cost | None | Yes | Quantitative costs | P | I |
| | Water Cost | None | Yes | Quantitative costs | P | I |
| | Social Policies | None | Yes | Inspection Cost:<br>No. of assigned people x Labor cost x Percentage of labor spent on inspection. | P,S | I |
| | Fines and Penalties | None | Yes | Quantitative cost | P | I |
| Price | Price | Bhutta and Huq (2002) | Yes | Quantitative costs | P,S | I |

Table 3.1 TCO costs' Categories

### 3.2.1 Total Cost of Ownership

Direct and indirect costs are broken down into 6 categories. These TCO costs can be considered as a guide depending on the industry or service type. Table 3.1 details these cost categories. The total cost of ownership of each supplier is calculated as the quoted price plus the associated costs.

### 3.2.2 Network DEA

Suppose we have $n$ tier suppliers under evaluation ($n$ DMUs). For each supplier we have $J$ observations and a different set of inputs and outputs. If these inputs and outputs are associated with a specific member of the supply chain, we call them "direct" inputs and direct outputs (Zhu, 2009). We refer to them as $\mathrm{DI}^{\Delta}$ and $\mathrm{DO}^{\Delta}$, where, $\mathrm{DI}^{\Delta}$ represents the direct inputs and $\mathrm{DO}^{\Delta}$ represents the direct outputs for a supply chain member Δ.

There are also "intermediate" inputs and outputs between two supply chain members, where usually the outputs from one member become inputs to other member. Calculation of the intermediate measures should be done through coordination among the members of the supply chain (Parlar and Weng, 1997;Thomas and Griffin, 1996). For example, one member would like to maximize the price while the other may prefer to minimize the cost.

In our module we will use the network DEA formula presented by (Zhu, 2009):

$$\Omega^* = min \; \frac{\sum_{i=1}^{n} w_i \Omega_i}{\sum_{i=1}^{n} w_i}$$

subject to

$$( Sub - Sub \; Supplier \; S_1)$$

$$\sum_{j=1}^{J} \lambda_j \, x_{ij}^{S_1} \leq \Omega_1 x_{ij_0}^{S_1} \quad i \in DI^{S_1}$$

$$\sum_{j=1}^{J} \lambda_j \, y_{rj}^{S_1} \geq y_{rj_0}^{S_1} \qquad r \in DO^{S_1}$$

$\sum_{j=1}^{J} \lambda_j \, z_{tj}^{S_1-S_2} \geq z_{tj_0}^{S_1-S_2} \; t = 1, \dots, T$

$\lambda_j \geq 0, j = 1, .., J$

(Sub Supplier $S_2$)

$\sum_{j=1}^{j} \beta_j \, x_{ij}^{S_2} \leq \Omega_2 x_{ij_0}^{S_2} \; i \in DI^{S_2}$

$\sum_{j=1}^{J} \beta_j \, y_{rj}^{S_2} \geq y_{rj_0}^{S_2} \quad r \in DO^{\,S_2}$

$\sum_{j=1}^{J} \beta_j \, z_{tj}^{S_1-S_2} \leq z_{tj_0}^{S_1-S_2} \; t = 1, \dots, T$

$\sum_{j=1}^{J} \beta_j \, z_{mj}^{S_2-S_3} \geq z_{mj_0}^{S_2-S_3} \; m = 1, \dots, M$

$\beta_j \geq 0, j = 1, .., J$

(Supplier $S_3$)

$\sum_{j=1}^{J} \delta_j \, x_{ij}^{S_3} \leq \Omega_3 x_{ij_0}^{S_3} \; i \in DI^{S_3}$

$\sum_{j=1}^{J} \delta_j \, y_{rj}^{S_3} \geq y_{rj_0}^{S_3} \quad r \in DO^{\,S_3}$

$\sum_{j=1}^{J} \delta_j \, z_{mj}^{S_2-S_3} \leq z_{mj_0}^{S_2-S_3} \; m = 1, \dots, M$

$\delta_j \geq 0, j = 1, .., J$

where:

$\Omega_i$ is efficiency score of $\text{supplier}_\text{i}$.

$\Omega^*$ is the optimum efficiency of the supply chain, can be viewed as the supply chain best practice when it's equal to 1

$w_i$ is a user specific weight preference assigned to reflect the preference over each supplier.

$\lambda_j$ is the $j^{th}$ observation computed weights of the sub-sub-supplier.

$\beta_j$ is the $j^{th}$ observation computed weights of the sub-supplier.

$\delta_j$ is the $j^{th}$ observation computed weights of the supplier.

$x_{ij}^{\Delta}$ is the consumed amount of input *i* by supplier $\Delta$ in the $j^{th}$ observation.

$y_{rj}^{\Delta}$ is the produced amount of output *r* by supplier $\Delta$ in the $j^{th}$ observation.

$z_{tj}^{S_1-S_2}$ is the $t^{th}$ intermediate output from sub-sub-supplier $S_1$ to sub-supplier $S_2$ in the $j^{th}$ observation.

$z_{mj}^{S_2-S_3}$ is the $m^{th}$ intermediate output from sub-supplier $S_2$ to supplier $S_3$ in the $j^{th}$ observation.

Additional constraints can be added. For example, if we have intermediate outputs that go from supplier $S_3$ to sub-supplier $S_2$, then we have $\sum_{j=1}^{J} \delta_j \, z_{gj}^{S_3-S_2} \geq z_{gj_0}^{S_3-S_2} \quad g = 1, \ldots, G$.

Basically the module will try to find the efficiency of each supplier by comparing each input or output with different observations of that input or output. Supply chain efficiency $\Omega^*$ is equal to 1 (best practice values of inputs and outputs) when the efficiencies of all suppliers equal 1.

## 3.3 Recommendations for improving supplier quality

The third and final goal of this thesis is to find recommendations to improve the quality of suppliers. We will base our recommendations on the results of cluster analysis as well as TCO and DEA. We will analyze the results of cluster analysis using descriptive statistics. This will give us a better overview of the results. TCO and DEA results will help identify weak points in the supplier performance and suggest improvement targets.

# Chapter 4

# Numerical Application

In this chapter we present two examples to demonstrate the capabilities of our framework. The first example illustrates the hierarchical cluster technique using the criteria presented previously, and the next example shows the application of TCO and DEA approach.

## 4.1 Hierarchical cluster analysis

In a global supply chain, let's assume that we have 20 upstream suppliers. The clustering is done using the following criteria:

- Product type
- Cost of Quality
- Risk
- Location
- Organization profile (size)
- Cost sustainability

The numerical values of these criteria were generated using Excel random number generator routine (see table 4.1). Following steps are used in our clustering technique:

1. Data normalization
2. Assigning weights to product types
3. Applying hierarchical clustering

| Supplier | Product type | Cost of quality | Risk | Location | Organization profile (size) | Cost sustainability |
|---|---|---|---|---|---|---|
| Supplier 1 | 2 | 1098 | 0.54 | 3 | 519 | 606 |
| Supplier 2 | 4 | 1214 | 0.31 | 3 | 126 | 701 |
| Supplier 3 | 5 | 1276 | 0.68 | 2 | 439 | 585 |
| Supplier 4 | 4 | 1302 | 0.31 | 3 | 103 | 747 |
| Supplier 5 | 5 | 1524 | 0.66 | 4 | 167 | 796 |
| Supplier 6 | 1 | 1098 | 0.32 | 4 | 487 | 669 |
| Supplier 7 | 1 | 1399 | 0.67 | 2 | 381 | 734 |
| Supplier 8 | 2 | 1272 | 0.46 | 3 | 258 | 733 |
| Supplier 9 | 3 | 1230 | 0.35 | 2 | 600 | 804 |
| Supplier 10 | 1 | 1017 | 0.33 | 3 | 137 | 605 |
| Supplier 11 | 2 | 1347 | 0.41 | 3 | 239 | 572 |
| Supplier 12 | 2 | 1127 | 0.47 | 3 | 161 | 788 |
| Supplier 13 | 5 | 1050 | 0.33 | 1 | 105 | 782 |
| Supplier 14 | 5 | 1259 | 0.51 | 4 | 485 | 772 |
| Supplier 15 | 4 | 1555 | 0.54 | 1 | 173 | 713 |
| Supplier 16 | 3 | 1513 | 0.6 | 2 | 440 | 603 |
| Supplier 17 | 2 | 1474 | 0.64 | 4 | 553 | 561 |
| Supplier 18 | 5 | 1210 | 0.43 | 3 | 465 | 811 |
| Supplier 19 | 2 | 1020 | 0.53 | 1 | 544 | 755 |
| Supplier 20 | 1 | 1479 | 0.41 | 3 | 137 | 778 |

Table 4.1 Hypothetical upstream suppliers

### 4.1.1 Data Normalization

Since the data is in different units, they need to be normalized first. The data is normally distributed, so we can use the formula presented in the previous chapter. The results of normalization are presented in table 4.2.

| Supplier | Product type | Cost of quality | Risk | Location | Organization profile (size) | Cost sustainability |
|---|---|---|---|---|---|---|
| Supplier 1 | -0.6176 | -1.0058 | 0.5035 | 0.3065 | 1.0785 | -1.1529 |
| Supplier 2 | 0.6827 | -0.3399 | -1.2780 | 0.3065 | -1.1170 | -0.0549 |
| Supplier 3 | 1.3328 | 0.0161 | 1.5878 | -0.7152 | 0.6315 | -1.3956 |
| Supplier 4 | 0.6827 | 0.1653 | -1.2780 | 0.3065 | -1.2455 | 0.4768 |
| Supplier 5 | 1.3328 | 1.4398 | 1.4329 | 1.3283 | -0.8880 | 1.0431 |
| Supplier 6 | -1.2678 | -1.0058 | -1.2006 | 1.3283 | 0.8997 | -0.4247 |
| Supplier 7 | -1.2678 | 0.7222 | 1.5104 | -0.7152 | 0.3075 | 0.3265 |
| Supplier 8 | -0.6176 | -0.0069 | -0.1162 | 0.3065 | -0.3796 | 0.3149 |
| Supplier 9 | 0.0325 | -0.2480 | -0.9682 | -0.7152 | 1.5310 | 1.1355 |
| Supplier 10 | -1.2678 | -1.4708 | -1.1231 | 0.3065 | -1.0556 | -1.1644 |
| Supplier 11 | -0.6176 | 0.4237 | -0.5035 | 0.3065 | -0.4857 | -1.5458 |
| Supplier 12 | -0.6176 | -0.8393 | -0.0387 | 0.3065 | -0.9215 | 0.9506 |
| Supplier 13 | 1.3328 | -1.2814 | -1.1231 | -1.7370 | -1.2343 | 0.8813 |
| Supplier 14 | 1.3328 | -0.0815 | 0.2711 | 1.3283 | 0.8885 | 0.7657 |
| Supplier 15 | 0.6827 | 1.6178 | 0.5035 | -1.7370 | -0.8544 | 0.0838 |
| Supplier 16 | 0.0325 | 1.3767 | 0.9682 | -0.7152 | 0.6371 | -1.1875 |
| Supplier 17 | -0.6176 | 1.1528 | 1.2780 | 1.3283 | 1.2684 | -1.6730 |
| Supplier 18 | 1.3328 | -0.3628 | -0.3486 | 0.3065 | 0.7768 | 1.2164 |
| Supplier 19 | -0.6176 | -1.4536 | 0.4260 | -1.7370 | 1.2181 | 0.5692 |
| Supplier 20 | -1.2678 | 1.1815 | -0.5035 | 0.3065 | -1.0556 | 0.8350 |

Table 4.2 Normalized Data

### 4.1.2 Assigning weights to product types

There are 5 different product types in our example, so we need to assign weights to each product type to ensure that all suppliers of the same product type are grouped together and not divided in different clusters. Therefore, the product types will be multiplied by their corresponding weights. As a result, the clustering technique will group suppliers first based on the product type and then on the remaining criteria. Table 4.3 reports the data after assigning weights.

| Supplier | Product type | Cost of quality | Risk | Location | Organization profile (size) | Cost sustainability |
|---|---|---|---|---|---|---|
| Supplier 1 | -1.2353 | -1.0058 | 0.5035 | 0.3065 | 1.0785 | -1.1529 |
| Supplier 2 | 2.7306 | -0.3399 | -1.2780 | 0.3065 | -1.1170 | -0.0549 |
| Supplier 3 | 6.6640 | 0.0161 | 1.5878 | -0.7152 | 0.6315 | -1.3956 |
| Supplier 4 | 2.7306 | 0.1653 | -1.2780 | 0.3065 | -1.2455 | 0.4768 |
| Supplier 5 | 6.6640 | 1.4398 | 1.4329 | 1.3283 | -0.8880 | 1.0431 |
| Supplier 6 | -1.2678 | -1.0058 | -1.2006 | 1.3283 | 0.8997 | -0.4247 |
| Supplier 7 | -1.2678 | 0.7222 | 1.5104 | -0.7152 | 0.3075 | 0.3265 |
| Supplier 8 | -1.2353 | -0.0069 | -0.1162 | 0.3065 | -0.3796 | 0.3149 |
| Supplier 9 | 0.0975 | -0.2480 | -0.9682 | -0.7152 | 1.5310 | 1.1355 |
| Supplier 10 | -1.2678 | -1.4708 | -1.1231 | 0.3065 | -1.0556 | -1.1644 |
| Supplier 11 | -1.2353 | 0.4237 | -0.5035 | 0.3065 | -0.4857 | -1.5458 |
| Supplier 12 | -1.2353 | -0.8393 | -0.0387 | 0.3065 | -0.9215 | 0.9506 |
| Supplier 13 | 6.6640 | -1.2814 | -1.1231 | -1.7370 | -1.2343 | 0.8813 |
| Supplier 14 | 6.6640 | -0.0815 | 0.2711 | 1.3283 | 0.8885 | 0.7657 |
| Supplier 15 | 2.7306 | 1.6178 | 0.5035 | -1.7370 | -0.8544 | 0.0838 |
| Supplier 16 | 0.0975 | 1.3767 | 0.9682 | -0.7152 | 0.6371 | -1.1875 |
| Supplier 17 | -1.2353 | 1.1528 | 1.2780 | 1.3283 | 1.2684 | -1.6730 |
| Supplier 18 | 6.6640 | -0.3628 | -0.3486 | 0.3065 | 0.7768 | 1.2164 |
| Supplier 19 | -1.2353 | -1.4536 | 0.4260 | -1.7370 | 1.2181 | 0.5692 |
| Supplier 20 | -1.2678 | 1.1815 | -0.5035 | 0.3065 | -1.0556 | 0.8350 |

Table 4.3 Weighted product types

### 4.1.3 Applying hierarchical clustering

IBM SPSS program was used to generate the clustering results (dendrogram). The results are shown in figure 4.1. According to the best cut method, we have 3 clusters.

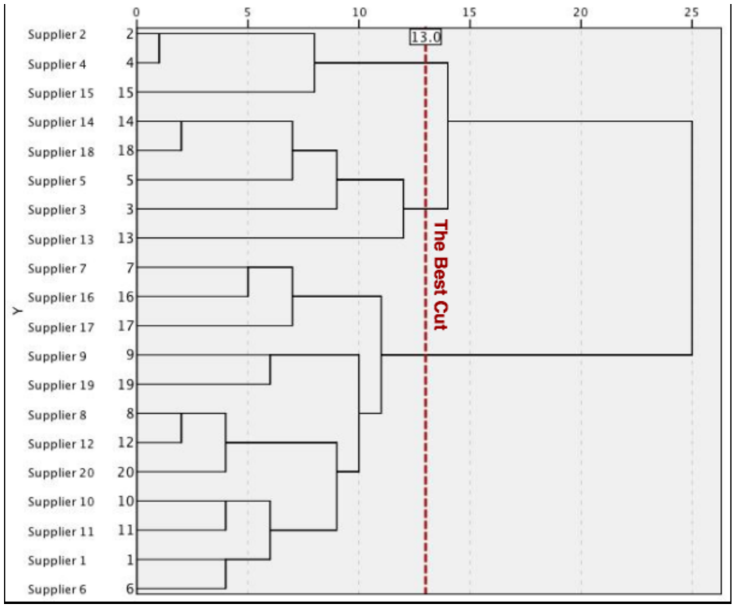


Figure 4.1 Dendrogram with best cut

The final results of cluster analysis are reported in table 4.4.

| Cluster | Supplier | Product type | Cost of quality | Risk | Location | Org. profile (size) | Cost S. |
|---|---|---|---|---|---|---|---|
| 1 | Supplier 1 | 2 | 1098 | 0.54 | 3 | 519 | 606 |
| | Supplier 6 | 1 | 1098 | 0.32 | 4 | 487 | 669 |
| | Supplier 7 | 1 | 1399 | 0.67 | 2 | 381 | 734 |
| | Supplier 8 | 2 | 1272 | 0.46 | 3 | 258 | 733 |
| | Supplier 9 | 3 | 1230 | 0.35 | 2 | 600 | 804 |
| | Supplier 10 | 1 | 1017 | 0.33 | 3 | 137 | 605 |
| | Supplier 11 | 2 | 1347 | 0.41 | 3 | 239 | 572 |
| | Supplier 12 | 2 | 1127 | 0.47 | 3 | 161 | 788 |
| | Supplier 16 | 3 | 1513 | 0.6 | 2 | 440 | 603 |
| | Supplier 17 | 2 | 1474 | 0.64 | 4 | 553 | 561 |
| | Supplier 19 | 2 | 1020 | 0.53 | 1 | 544 | 755 |
| | Supplier 20 | 1 | 1479 | 0.41 | 3 | 137 | 778 |
| 2 | Supplier 2 | 4 | 1214 | 0.31 | 3 | 126 | 701 |
| | Supplier 4 | 4 | 1302 | 0.31 | 3 | 103 | 747 |
| | Supplier 15 | 4 | 1555 | 0.54 | 1 | 173 | 713 |
| 3 | Supplier 3 | 5 | 1276 | 0.68 | 2 | 439 | 585 |
| | Supplier 5 | 5 | 1524 | 0.66 | 4 | 167 | 796 |
| | Supplier 13 | 5 | 1050 | 0.33 | 1 | 105 | 782 |
| | Supplier 14 | 5 | 1259 | 0.51 | 4 | 485 | 772 |
| | Supplier 18 | 5 | 1210 | 0.43 | 3 | 465 | 811 |

Table 4.4 Cluster analysis final results

### 4.1.4 Recommendations

After examining table 4.5, it looks like product number 4 is purchased from 3 small organizations with a high cost of quality and a relatively high cost sustainability. As a result, the supply chain may become more vulnerable with regards to this product. We recommend either engaging these suppliers in a supplier development program or try to deal with new suppliers.

| Complete Linkage | | Minimum | Maximum | Mean | Std. Deviation |
|---|---|---|---|---|---|
| 1 | Product type | 1.00 | 3.00 | - | - |
| | Cost of quality | 1017.00 | 1513.00 | 1256.1667 | 183.98361 |
| | Risk | .32 | .67 | .4775 | .11963 |
| | Organization profile (size) | 137.00 | 600.00 | 371.3333 | 175.68532 |
| | Cost sustainability | 561.00 | 804.00 | 684.0000 | 90.84352 |
| | Number of Suppliers | 12 | | | |
| 2 | Product type | 4.00 | 4.00 | - | - |
| | Cost of quality | 1214.00 | 1555.00 | 1357.0000 | 177.02825 |
| | Risk | .31 | .54 | .3867 | .13279 |
| | Organization profile (size) | 103.00 | 173.00 | 134.0000 | 35.67913 |
| | Cost sustainability | 701.00 | 747.00 | 720.3333 | 23.86071 |
| | Number of Suppliers | 3 | | | |
| 3 | Product type | 5.00 | 5.00 | - | - |
| | Cost of quality | 1050.00 | 1524.00 | 1263.8000 | 170.64642 |
| | Risk | .33 | .68 | .5220 | .14957 |
| | Organization profile (size) | 105.00 | 485.00 | 332.2000 | 181.17726 |
| | Cost sustainability | 585.00 | 811.00 | 749.2000 | 92.96074 |
| | Number of Suppliers | 5 | | | |

Table 4.5 Descriptive statistics

Likewise, product type 5 is purchased from 5 suppliers with a high risk and cost sustainability. Therefore, suppliers can participate in a supplier development program.

To leverage the weak areas and to help plan for an effective supplier development program, it is highly recommended to apply TCO and DEA techniques as per the below example.

## 4.2 TCO and DEA

To illustrate the multi-tier evaluation approach presented in the previous chapter, let's assume that we have 3-tier hypothetical suppliers (see figure 4.2): sub-sub supplier, sub-supplier, and supplier. Each supplier's ultimate output is the part that he supplies to the next tier's supplier.

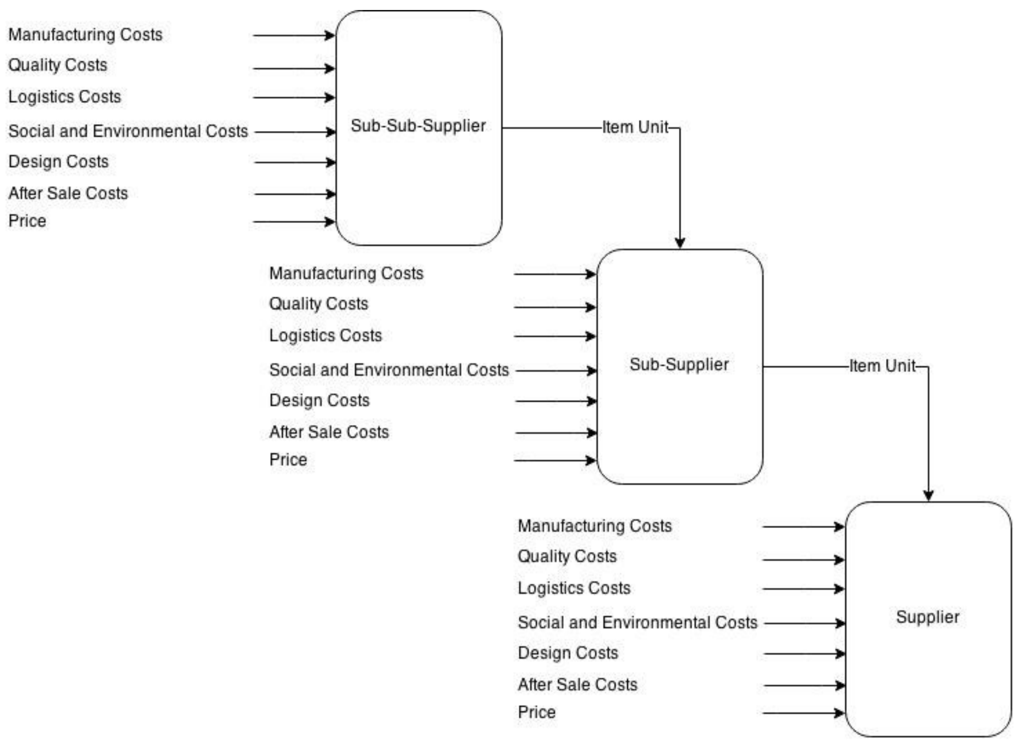



Figure 4.2 Example of 3-tier suppliers

The inputs and outputs of each supplier are based on the following TCO cost categories which were presented in table 3.1:

Inputs:

- Manufacturing Costs, cost amount was generated randomly between these ranges:
    - Sub-sub supplier values ranged between 50 and 130
    - Sub supplier values ranged between 70 and 200
    - Supplier values ranged between 200 and 370
- Quality Costs, cost amount was generated randomly between these ranges:
    - Sub-sub supplier and sub supplier values ranged between 20 and 40
    - Supplier values ranged between 30 and 65
- Logistics Costs, cost amount was generated randomly between these ranges:
    - Sub-sub supplier values ranged between 70 and 170
    - Sub supplier values ranged between 90 and 180
    - Supplier values ranged between 200 and 340
- Social and Environmental, cost amount was generated randomly between these ranges:
    - Sub-sub supplier values ranged between 30 and 80
    - Sub supplier values ranged between 50 and 120
    - Supplier values ranged between 100 and 160
- Design Costs, cost amount was generated randomly between these ranges:
    - Sub-sub supplier values ranged between 50 and 70
    - Sub supplier values ranged between 60 and 99
    - Supplier values ranged between 100 and 160

- After Sale Costs, cost amount was generated randomly between these ranges:
    - Sub-sub supplier and sub supplier values ranged between 20 and 50
    - Supplier values ranged between 30 and 60
- Price, cost amount was generated randomly between these ranges:
    - Sub-sub supplier values ranged between 100 and 220
    - Sub supplier values ranged between 300 and 400
    - Supplier values ranged between 600 and 780

Output:

- Item Unit

The data is presented in table 4.6. Our goal is to evaluate the suppliers and the whole upstream supply chain. For this, we will apply the network DEA presented in the previous chapter to find the efficiency of each supplier starting from observation 1 until 10. The complete linear programming for this example is listed in appendix A. Table 4.7 and figure 4.3 report the obtained efficiency scores of our linear programming.

Please note the following:

$\Omega^*$ is efficiency score of the supply chain

$\Omega_1$ is efficiency score of the sub-sub supplier.

$\Omega_2$ is efficiency score of the sub-supplier.

$\Omega_3$ is efficiency score of the supplier.

| | | TCO Categories | Observation | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
| **Sub Sub Supplier** | Input | Manufacturing Costs | 78 | 70 | 93 | 90 | 73 | 102 | 104 | 123 | 80 | 81 |
| | | Quality | 22 | 26 | 32 | 30 | 28 | 29 | 38 | 38 | 24 | 34 |
| | | Logistics | 116 | 131 | 143 | 101 | 98 | 146 | 105 | 166 | 169 | 154 |
| | | Social and Environmental | 79 | 57 | 50 | 48 | 70 | 62 | 69 | 58 | 58 | 48 |
| | | Design Costs | 50 | 57 | 60 | 60 | 70 | 52 | 67 | 57 | 67 | 59 |
| | | After Sale | 49 | 40 | 46 | 39 | 33 | 38 | 46 | 45 | 34 | 37 |
| | | Price | 107 | 172 | 158 | 150 | 148 | 120 | 128 | 166 | 169 | 126 |
| | Output-Input | Item Unit | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **Sub Supplier** | Input | Manufacturing Costs | 72 | 60 | 104 | 139 | 110 | 106 | 182 | 145 | 143 | 174 |
| | | Quality | 29 | 30 | 24 | 30 | 20 | 28 | 32 | 26 | 30 | 26 |
| | | Logistics | 109 | 170 | 161 | 155 | 142 | 142 | 175 | 172 | 174 | 146 |
| | | Social and Environmental | 95 | 70 | 99 | 75 | 87 | 98 | 103 | 108 | 106 | 97 |
| | | Design Costs | 73 | 61 | 75 | 84 | 81 | 86 | 99 | 85 | 90 | 96 |
| | | After Sale | 39 | 37 | 41 | 42 | 34 | 29 | 41 | 30 | 40 | 40 |
| | | Price | 347 | 355 | 333 | 341 | 348 | 359 | 360 | 368 | 374 | 367 |
| | Output-Input | Item Unit | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **Supplier** | Input | Manufacturing Costs | 288 | 261 | 309 | 230 | 309 | 263 | 284 | 327 | 297 | 313 |
| | | Quality | 45 | 38 | 40 | 42 | 48 | 52 | 52 | 54 | 52 | 63 |
| | | Logistics | 270 | 284 | 276 | 271 | 268 | 299 | 251 | 232 | 301 | 315 |
| | | Social and Environmental | 140 | 126 | 136 | 138 | 144 | 149 | 146 | 142 | 150 | 159 |
| | | Design Costs | 127 | 110 | 138 | 134 | 138 | 132 | 149 | 140 | 141 | 140 |
| | | After Sale | 46 | 35 | 43 | 36 | 40 | 40 | 45 | 45 | 45 | 50 |
| | | Price | 700 | 638 | 658 | 649 | 651 | 668 | 745 | 676 | 734 | 710 |
| | Output | Item Unit | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Table 4.6 Hypothetical data generated using Excel random routine

| Observation | Supply Chain Efficiency | | | |
|---|---|---|---|---|
| | $\Omega^*$ | $\Omega_1$ | $\Omega_2$ | $\Omega_3$ |
| 1 | 0.993 | 1 | 1 | 0.979 |
| 2 | 1 | 1 | 1 | 1 |
| 3 | 0.994 | 0.983 | 1 | 1 |
| 4 | 1 | 1 | 1 | 1 |
| 5 | 0.999 | 1 | 1 | 0.998 |
| 6 | 0.987 | 1 | 1 | 0.960 |
| 7 | 0.978 | 1 | 0.940 | 0.995 |
| 8 | 0.988 | 0.965 | 1 | 1 |
| 9 | 0.935 | 1 | 0.915 | 0.891 |
| 10 | 0.946 | 1 | 0.938 | 0.899 |

Table 4.7 Efficiency scores

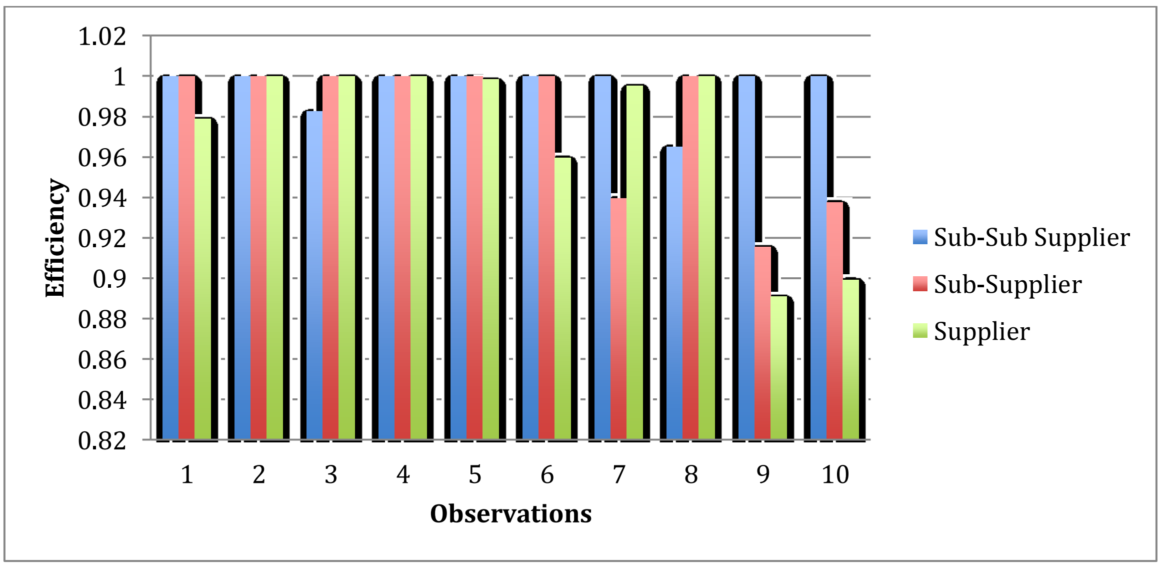



Figure 4.3 Suppliers' efficiency scores

Although a number of observations are efficient but only observations 2 and 4 are indicating a supply chain efficient score, which considers the supply chain best practices that can be used for supplier improvement program. Furthermore, in the first observation only $\Omega_3$ is inefficient with efficiency score of 0.979, which means this supplier should use 97.9% of his resources to become 100% efficient. Similarly, in observations 5 and 6 we notice that $\Omega_3$ is 99.8% and 96% efficient respectively.

Additionally, from figure 4.4 we observe that the sub-sub supplier was efficient in 80% of the observations while the sub-supplier was efficient in 70% of the observations. On the other hand, the supplier was efficient in 40% of the observations only.

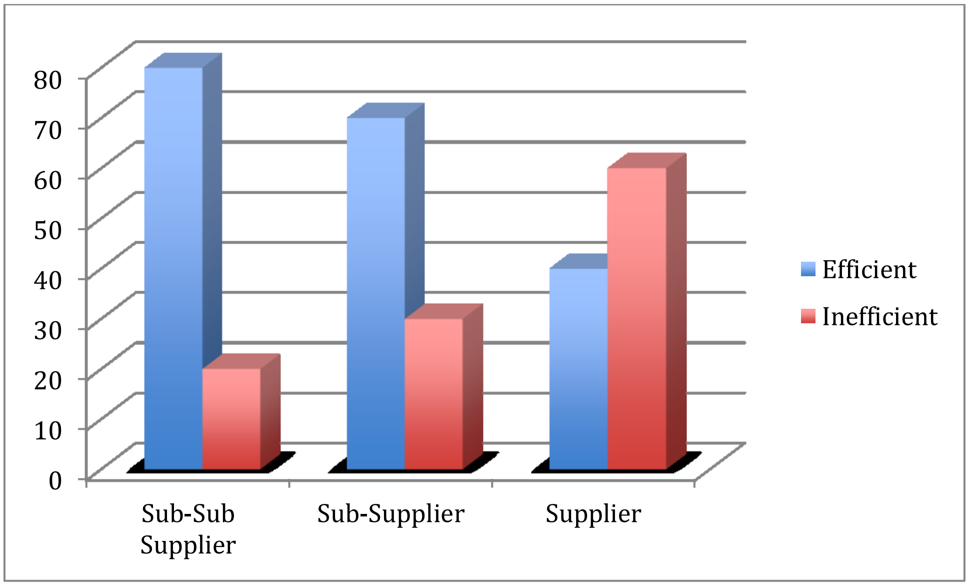



Figure 4.4 Efficiencies of the three suppliers

Table 4.8 reports the input targets that all suppliers should follow to improve their performance. It can be the basis for a supplier development program that aims toward improving the supply chain performance. For example $\Omega_3$ in the first observation should decrease manufacturing cost to be 281.9 from 288, quality from 45 to 44.05, logistics cost from 270 to 264.28, Social and Environmental costs from 140 to 137.04, technology cost from 127 to 124.31, after sales cost from 46 to 45.03, and finally decrease the quoted price to be 685.18. As a result, $\Omega_3$ will be efficient for this observation.

| | TCO Categories | Observation | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **Sub Sub Supplier** | Manufacturing Costs | 78 | 70 | 91.40 | 90 | 73 | 102 | 104 | 118.71 | 80 | 81 |
| | Quality | 22 | 26 | 31.45 | 30 | 28 | 29 | 38 | 36.67 | 24 | 34 |
| | Logistics | 116 | 131 | 140.54 | 101 | 98 | 146 | 105 | 160.21 | 169 | 154 |
| | Social and Environmental | 79 | 57 | 49.14 | 48 | 70 | 62 | 69 | 55.98 | 58 | 48 |
| | Design Costs | 50 | 57 | 58.97 | 60 | 70 | 52 | 67 | 55.01 | 67 | 59 |
| | After Sale | 49 | 40 | 45.21 | 39 | 33 | 38 | 46 | 43.43 | 34 | 37 |
| | Price | 107 | 172 | 155.28 | 150 | 148 | 120 | 128 | 160.21 | 169 | 126 |
| **Sub Supplier** | Manufacturing Costs | 72 | 60 | 104 | 139 | 110 | 106 | 171.03 | 145 | 130.91 | 163.16 |
| | Quality | 29 | 30 | 24 | 30 | 20 | 28 | 30.07 | 26 | 27.46 | 24.38 |
| | Logistics | 109 | 170 | 161 | 155 | 142 | 142 | 164.45 | 172 | 159.29 | 136.91 |
| | Social and Environmental | 95 | 70 | 99 | 75 | 87 | 98 | 96.79 | 108 | 97.04 | 90.96 |
| | Design Costs | 73 | 61 | 75 | 84 | 81 | 86 | 93.03 | 85 | 82.39 | 90.02 |
| | After Sale | 39 | 37 | 41 | 42 | 34 | 29 | 38.53 | 30 | 36.62 | 37.51 |
| | Price | 347 | 355 | 333 | 341 | 348 | 359 | 338.30 | 368 | 342.39 | 344.14 |
| **Supplier** | Manufacturing Costs | 281.90 | 261 | 309 | 230 | 308.52 | 252.39 | 282.67 | 327 | 264.61 | 281.49 |
| | Quality | 44.05 | 38 | 40 | 42 | 47.93 | 49.90 | 51.76 | 54 | 46.33 | 56.66 |
| | Logistics | 264.28 | 284 | 276 | 271 | 267.59 | 286.94 | 249.82 | 232 | 268.18 | 283.29 |
| | Social and Environmental | 137.04 | 126 | 136 | 138 | 143.78 | 142.99 | 145.32 | 142 | 133.64 | 142.99 |
| | Design Costs | 124.31 | 110 | 138 | 134 | 137.79 | 126.68 | 148.30 | 140 | 125.62 | 125.91 |
| | After Sale | 45.03 | 35 | 43 | 36 | 39.94 | 38.39 | 44.79 | 45 | 40.09 | 44.97 |
| | Price | 685.18 | 638 | 658 | 649 | 649.99 | 641.05 | 741.51 | 676 | 653.96 | 638.52 |

Table 4.8 Input targets

### 4.2.1 Recommendations

Based on our findings, following recommendations are generated:

1. The main supplier should join a supplier development program being efficient in only 40% of the observations.
2. Observations 2 and 4 are considered best practices. Therefore, any supplier development program should consider these values.
3. Collaboration is considered an added value only when TCO is between \$1492 to \$1500 for the supplier, \$783 to \$866 for the sub supplier, and \$518 to \$553 for the sub-sub supplier. These values are the results of totalling the TCO categories for each supplier in observations 2 and 4.
4. Table 4.8 is the basis to improve the performance of suppliers in any observation. It can be the basis for negotiation with suppliers.

## 4.3 Sensitivity analysis

To study the sensitivity of our modeling framework to change in input parameters, we will remove one input at a time and then re-evaluate $\Omega_1, \Omega_2$ and $\Omega_3$ to check if they still preserve their efficiency. Then we return that input and remove another one and re-evaluate. We will do that for all inputs until there is no further input left.

### 4.3.1 First Scenario: Evaluate without manufacturing and technology costs

Table 4.9 shows the efficiencies when we delete manufacturing cost. It can be seen that there is no change in efficiencies for the efficient units. Moreover the supply chain best practices are still observations 2 and 4. The same results are observed from table 4.10 when we remove the Technology Cost and re-evaluate.

| Observation | $\boldsymbol{\Omega}^*$ | $\boldsymbol{\Omega_1}$ | $\boldsymbol{\Omega_2}$ | $\boldsymbol{\Omega_3}$ |
|---|---|---|---|---|
| 1 | 0.993 | 1 | 1 | 0.979 |
| 2 | 1 | 1 | 1 | 1 |
| 3 | 0.994 | 0.983 | 1 | 1 |
| 4 | 1 | 1 | 1 | 1 |
| 5 | 0.999 | 1 | 1 | 0.998 |
| 6 | 0.985 | 1 | 1 | 0.955 |
| 7 | 0.970 | 1 | 0.940 | 0.970 |
| 8 | 0.988 | 0.965 | 1 | 1 |
| 9 | 0.934 | 1 | 0.915 | 0.886 |
| 10 | 0.946 | 1 | 0.938 | 0.899 |

Table 4.9 Efficiency scores without manufacturing cost

| Observation | $\boldsymbol{\Omega}^*$ | $\boldsymbol{\Omega_1}$ | $\boldsymbol{\Omega_2}$ | $\boldsymbol{\Omega_3}$ |
|---|---|---|---|---|
| 1 | 0.993 | 1 | 1 | 0.978 |
| 2 | 1 | 1 | 1 | 1 |
| 3 | 0.987 | 0.960 | 1 | 1 |
| 4 | 1 | 1 | 1 | 1 |
| 5 | 0.999 | 1 | 1 | 0.998 |
| 6 | 0.987 | 1 | 1 | 0.960 |
| 7 | 0.978 | 1 | 0.940 | 0.995 |
| 8 | 0.946 | 0.837 | 1 | 1 |
| 9 | 0.935 | 1 | 0.915 | 0.891 |
| 10 | 0.946 | 1 | 0.938 | 0.899 |

Table 4.10 Efficiency scores without technology cost

### 4.3.2 Second Scenario: Evaluate without quality costs and price

Table 4.11-4.12 demonstrate the efficiency scores when we drop quality costs and price respectively. Supply chain best practice is still obtained from observations 2 and 4 for both inputs. However, $\Omega_2$ becomes inefficient in observation 8 when we remove quality cost. Likewise, $\Omega_1$ becomes inefficient in observation 7.

| Observation | $\Omega^*$ | $\Omega_1$ | $\Omega_2$ | $\Omega_3$ |
|---|---|---|---|---|
| 1 | 0.992 | 1 | 1 | 0.976 |
| 2 | 1 | 1 | 1 | 1 |
| 3 | 0.988 | 0.979 | 1 | 0.984 |
| 4 | 1 | 1 | 1 | 1 |
| 5 | 0.999 | 1 | 1 | 0.998 |
| 6 | 0.987 | 1 | 1 | 0.960 |
| 7 | 0.978 | 1 | 0.940 | 0.995 |
| 8 | 0.984 | 0.965 | 0.988 | 1 |
| 9 | 0.935 | 1 | 0.915 | 0.891 |
| 10 | 0.946 | 1 | 0.938 | 0.899 |

Table 4.11 Efficiency scores without quality cost

| Observation | $\Omega^*$ | $\Omega_1$ | $\Omega_2$ | $\Omega_3$ |
|---|---|---|---|---|
| 1 | 0.993 | 1 | 1 | 0.979 |
| 2 | 1 | 1 | 1 | 1 |
| 3 | 0.989 | 0.983 | 0.984 | 1 |
| 4 | 1 | 1 | 1 | 1 |
| 5 | 0.990 | 1 | 1 | 0.969 |
| 6 | 0.977 | 1 | 1 | 0.930 |
| 7 | 0.926 | 0.951 | 0.831 | 0.995 |
| 8 | 0.988 | 0.965 | 1 | 1 |
| 9 | 0.913 | 1 | 0.847 | 0.891 |
| 10 | 0.924 | 1 | 0.911 | 0.860 |

Table 4.12 Efficiency scores without price

### 4.3.3 Third Scenario: Evaluate without logistics and after sale costs

On the other hand, when we calculate efficiency scores without logistics cost we observe that $\Omega_1$ and $\Omega_3$ lose their efficient scores in observations 7 and 8 respectively as seen in table 4.13.

Similarly, $\Omega_2$ becomes inefficient in observations 6 and 8 (table 4.14) when we drop after sales cost. Supply chain best practice is still found in observations 2 and 4.

| Observation | $\Omega^*$ | $\Omega_1$ | $\Omega_2$ | $\Omega_3$ |
|---|---|---|---|---|
| 1 | 0.970 | 1 | 1 | 0.911 |
| 2 | 1 | 1 | 1 | 1 |
| 3 | 0.984 | 0.983 | 1 | 0.970 |
| 4 | 1 | 1 | 1 | 1 |
| 5 | 0.993 | 1 | 1 | 0.980 |
| 6 | 0.987 | 1 | 1 | 0.960 |
| 7 | 0.914 | 0.915 | 0.940 | 0.887 |
| 8 | 0.970 | 0.965 | 1 | 0.944 |
| 9 | 0.929 | 1 | 0.915 | 0.870 |
| 10 | 0.943 | 1 | 0.932 | 0.899 |

Table 4.13 Efficiency scores without logistics cost

| Observation | $\Omega^*$ | $\Omega_1$ | $\Omega_2$ | $\Omega_3$ |
|---|---|---|---|---|
| 1 | 0.993 | 1 | 1 | 0.979 |
| 2 | 1 | 1 | 1 | 1 |
| 3 | 0.994 | 0.983 | 1 | 1 |
| 4 | 1 | 1 | 1 | 1 |
| 5 | 0.999 | 1 | 1 | 0.998 |
| 6 | 0.970 | 1 | 0.952 | 0.960 |
| 7 | 0.974 | 1 | 0.928 | 0.995 |
| 8 | 0.959 | 0.965 | 0.911 | 1 |
| 9 | 0.929 | 1 | 0.897 | 0.891 |
| 10 | 0.946 | 1 | 0.937 | 0.899 |

Table 4.14 Efficiency scores without after sale cost

### 4.3.4 Fourth Scenario: Evaluate without social and environmental costs

Finally, we obtain different results when we remove social and environmental costs. Observation 4 is no more the supply chain best practice as $\Omega_2$ becomes inefficient (table 4.15).

| Observation | $\Omega^*$ | $\Omega_1$ | $\Omega_2$ | $\Omega_3$ |
|---|---|---|---|---|
| 1 | 0.993 | 1 | 1 | 0.979 |
| 2 | 1 | 1 | 1 | 1 |
| 3 | 0.967 | 0.901 | 1 | 1 |
| 4 | 0.994 | 1 | 0.983 | 1 |
| 5 | 0.999 | 1 | 1 | 0.998 |
| 6 | 0.987 | 1 | 1 | 0.960 |
| 7 | 0.978 | 1 | 0.940 | 0.995 |
| 8 | 0.968 | 0.904 | 1 | 1 |
| 9 | 0.935 | 1 | 0.915 | 0.890 |
| 10 | 0.945 | 1 | 0.936 | 0.899 |

Table 4.15 Efficiency scores without social and environmental costs

From the above scenarios, we found that removing or omitting some inputs affects the efficiency scores of one or more observations. These results can be summarized below:

1. Removing manufacturing and technology costs does not affect the efficiency scores of any observation.
2. Dropping quality costs and price can only affect the efficiency of sub supplier and sub-sub supplier respectively on one observation only.
3. Evaluating without logistics costs can cause sub-sub supplier and supplier to lose their efficiency score in one observation only.
4. Evaluating without after sales costs can cause sub supplier to become inefficient in two observations, which could be an indication of the importance of this input.
5. Removing the social and environmental costs caused the greatest change in the efficiency score. It caused the sub supplier to become inefficient in the best practice observation. This is an indication of the importance of this input for the DEA model.

## 4.4 Results Validation

Tajbakhsh and Hassini (2014) developed a network DEA model to evaluate the sustainability of the supply chain partners. Their work was based on the envelopment model of CCR-DEA (Cooper *et al.*, 2007). Moreover, to prove the practicality of their model they used the case study from Mirhedayatian *et al.* (2014). The case study consists of data collected from 10 Iranian beverage corporations in a supply chain of four partners: a supplier, a manufacturer, a distributor, and a retailer. Figure 4.5 demonstrates the supply chain network and shows the inputs and outputs of each partner and the intermediate linkages. We are using the same notations of Tajbakhsh and Hassini (2014). Table 4.16 provides the description of each notation.

| Notation | Stage | Description |
|---|---|---|
| $X_1Sup$ | Supplier | Row material cost |
| $X_2Sup$ | Supplier | Transportation cost |
| $Y_1Sup$ | Supplier | Supplier capability factor |
| $Z_1Sup - Man$ | Supplier | Defect-free parts per million |
| $X_1Man$ | Manufacturer | Advertisement cost |
| $X_2Man$ | Manufacturer | Transportation cost |
| $X_3Man$ | Manufacturer | Investment in sustainability design |
| $Y_1Man$ | Manufacturer | CO2 emission |
| $Y_2Man$ | Manufacturer | Average reputation factor |
| $Z_1Man - Dist$ | Manufacturer | Number of green products |
| $X_1Dist$ | Distributer | Transportation cost |
| $X_2Dist$ | Distributer | Personnel cost |
| $Y_1Dist$ | Distributer | Service diversity |
| $Z_1Dist - Ret$ | Distributer | Time deliveries |
| $X_1Ret$ | Retailer | Procurement cost |
| $Y_1Ret$ | Retailer | Average customer satisfaction factor |

Table 4.16 Description of the used notations

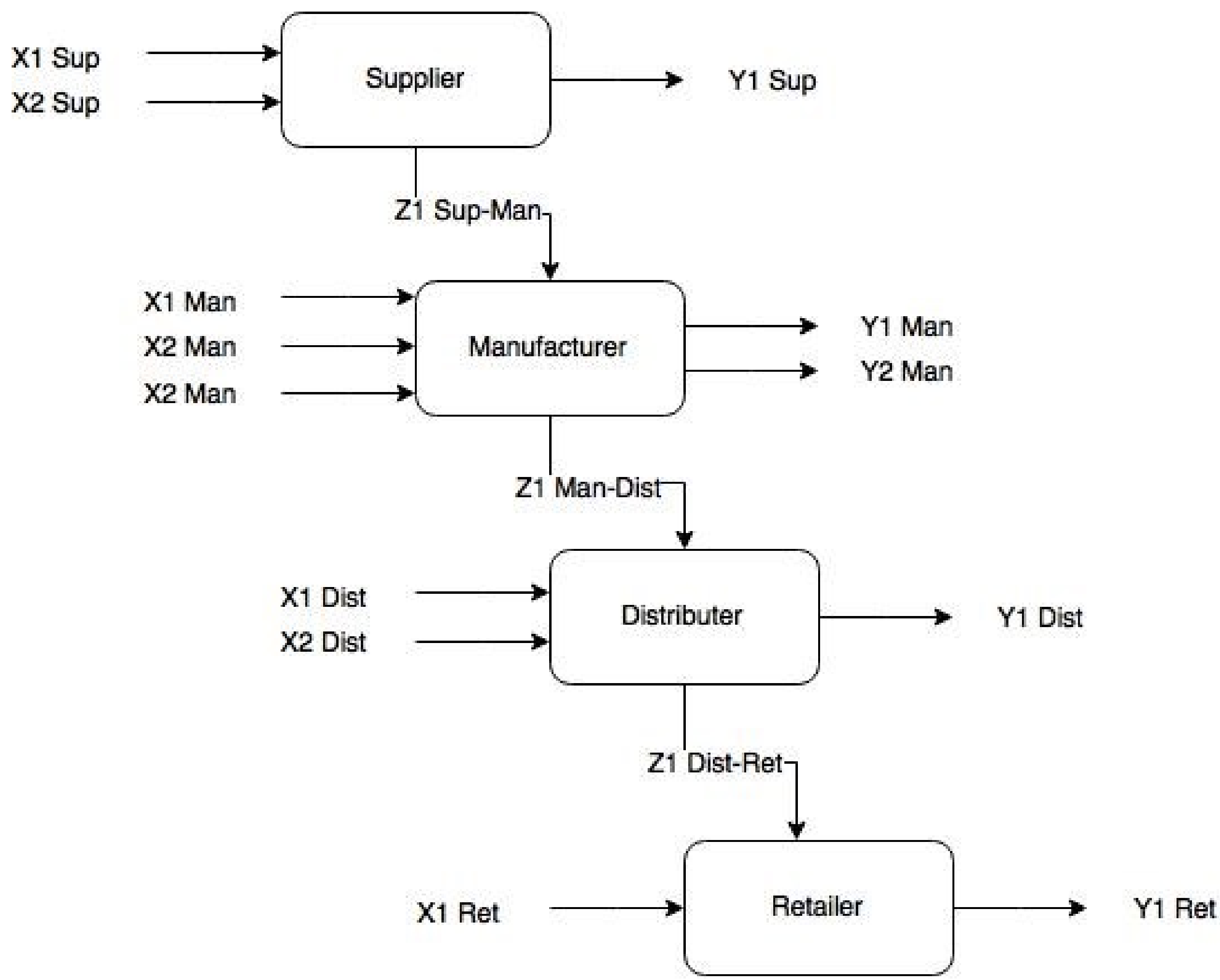



Figure 4.5 The supply chain of the Iranian beverage corporations' case

Tajbakhsh and Hassini (2014) model:

$Min\ \Omega^{*} - \varepsilon\ \times\ Slack$

subject to

( Supplier Sup)

$\sum_{j=1}^{J} \lambda_j\, x_{ij}^{Sup} + s_i^{Sup} = \Omega_{Sup} x_{ij_0}^{Sup} \qquad i \in DI^{Sup}$

$\sum_{j=1}^{J} \lambda_j\, y_{rj}^{Sup} - s_r^{Sup} = y_{rj_0}^{Sup} \qquad r \in DO^{\,Sup}$

$\sum_{j=1}^{J} \lambda_j\, z_{tj}^{Sup-Man} + s_t^{Sup-Man} = z_{tj_0}^{Sup-Man}\ t = 1, \dots, T$

$\lambda_j \geq 0, j = 1, \ldots, J$

(Manufacturer Man)

$\sum_{j=1}^{j} \beta_j \, x_{ij}^{Man} + \; s_i^{Man} = \Omega_{Man} x_{ij_0}^{Man} \, i \in DI^{Man}$

$\sum_{j=1}^{J} \beta_j \, y_{rj}^{Man} - \; s_r^{Man} = y_{rj_0}^{Man} \qquad r \in DO^{\,Man}$

$\sum_{j=1}^{J} \beta_j \, z_{tj}^{Sup-Man} + \; s_t^{Sup-Man} = z_{tj_0}^{Sup-Man} \; t = 1, \ldots, T$

$\sum_{j=1}^{J} \beta_j \, z_{mj}^{Man-Dist} - s_m^{Man-Dist} = z_{mj_0}^{Man-Dist} \; m = 1, \ldots, M$

$\beta_j \geq 0, j = 1, \ldots, J$

(Distributor Dist)

$\sum_{j=1}^{J} \delta_j \, x_{ij}^{Dist} + s_i^{Dist} = \Omega_{Dist} x_{ij_0}^{Dist} \quad i \in DI^{Dist}$

$\sum_{j=1}^{J} \delta_j \, y_{rj}^{Dist} - s_r^{Dist} = y_{rj_0}^{Dist} \qquad r \in DO^{\,Dist}$

$\sum_{j=1}^{J} \delta_j \, z_{tj}^{Man-Dist} + s_t^{Man-Dist} = z_{tj_0}^{Man-Dist} \; t = 1, \ldots, T$

$\sum_{j=1}^{J} \delta_j \, z_{mj}^{Dist-Ret} - s_m^{Dist-Ret} = z_{mj_0}^{Man-Ret} \; m = 1, \ldots, M$

$\delta_j \geq 0, j = 1, \ldots, J$

(Retailer Ret)

$\sum_{j=1}^{J} \gamma_j \, x_{ij}^{Ret} + s_i^{Ret} = \Omega_{Ret} x_{ij_0}^{Ret} \qquad i \in DI^{Ret}$

$\sum_{j=1}^{J} \gamma_j \, y_{rj}^{Ret} - s_r^{Ret} = y_{rj_0}^{Ret} \quad r \in DO^{\,Ret}$

$\sum_{j=1}^{J} \gamma_j \, z_{tj}^{Dist-Ret} + s_t^{Dist-Ret} = z_{tj_0}^{Dist-Ret} \; t = 1, \ldots, T$

$\gamma_j \geq 0, j = 1, \ldots, J$

where:

$\Omega_{Sup}, \Omega_{Man}, \Omega_{Dist}, \Omega_{Ret}$ is efficiency score of the supplier, manufacturer, distributer, and retailer respectively.

$\Omega^*$ is the optimum efficiency of the supply chain

$\lambda_j$ is the $j^{th}$ DMU computed weights of the supplier.

$\beta_j$ is the $j^{th}$ DMU computed weights of the manufacturer.

$\delta_j$ is the $j^{th}$ DMU computed weights of the distributer.

$\gamma_j$ is the $j^{th}$ DMU computed weights of the retailer.

$s_i^{\Delta}$ is the slack amount of input i of supply chain member $\Delta$.

$s_r^{\Delta}$ is the slack amount of output r of supply chain member $\Delta$.

$s_t^{\Delta_1-\Delta_2}$ is the slack amount of input t that produced by supply chain member $\Delta_1$ and consumed by supply chain member $\Delta_2$.

$s_m^{\Delta_1-\Delta_2}$ is the slack amount of output m that produced by supply chain member $\Delta_1$ and consumed by supply chain member $\Delta_2$.

$x_{ij}^{\Delta}$ is the consumed amount of input i by supply chain member $\Delta$ in the $j^{th}$ DMU.

$y_{rj}^{\Delta}$ is the produced amount of output r by supply chain member $\Delta$ in the $j^{th}$ DMU.

$z_{tj}^{\Delta_1-\Delta_2}$ is the $t^{th}$ intermediate input to supply chain member $\Delta_2$ from supply chain member $\Delta_1$ in the $j^{th}$ DMU.

$z_{mj}^{\Delta_1-\Delta_2}$ is the $t^{th}$ intermediate output from supply chain member $\Delta_1$ to supply chain member $\Delta_2$ in the $j^{th}$ DMU.

Slack is the summation of all slacks amounts excluding $s_t^{\Delta_1-\Delta_2}$

Mirhedayatian *et al.* (2014) model:

$$\text{Max}\,\frac{1}{\tau_o^{U^*}} = \sum_{h=1}^{k} w^k \left[ 1 + \frac{1}{R_k^D + R_k^{UD}} \left( \left( \sum_{r_k^D=1}^{R_k^D} \frac{s_r^{k+D}}{y_{r_k^D o}^{k^{DU}}} + \sum_{r_k^{UD}=1}^{R_k^{UD}} \frac{s_r^{k+UD}}{y_{r_k^D o}^{k^{UDU}}} \right) \right) \right]$$

$$s \times t \times \sum_{j=1}^{n} x_{ij}^{k^U} \lambda_j^k \le x_{io}^{k^L}, \quad i = 1,\ldots,m_k, \quad k = 1,\ldots,K,$$

$$\sum_{j=1}^{n} y_{r_k^D j}^{k^{DL}} \lambda_j^k - s_r^{k+D} = y_{r_k^D o}^{k^{DL}}$$

$$\sum_{j=1}^{n} y_{r_k^D j}^{k^{DM}} \lambda_j^k - s_r^{k+D} = y_{r_k^D o}^{k^{DM}}, \quad r_k^D = 1,\ldots,R_k^D, \quad k = 1,\ldots,K,$$

$$\sum_{j=1}^{n} y_{r_k^D j}^{k^{DU}} \lambda_j^k - s_r^{k+D} = y_{r_k^D o}^{k^{DU}}$$

$$\sum_{j=1}^{n} y_{r_k^D j}^{k^{UDL}} \lambda_j^k - s_r^{k+UD} = y_{r_k^D o}^{k^{UDL}}$$

$$\sum_{j=1}^{n} y_{r_k^D j}^{k^{UDM}} \lambda_j^k - s_r^{k+UD} = y_{r_k^D o}^{k^{UDM}}, \quad r_k^{UD} = 1,\ldots,R_k^{UD}, \quad k = 1,\ldots,K,$$

$$\sum_{j=1}^{n} y_{r_k^D j}^{k^{UDU}} \lambda_j^k - s_r^{k+UD} = y_{r_k^D o}^{k^{UDU}}$$

$$\sum_{j=1}^{n} w_{fj}^{k^L} \lambda_j^k = w_{fo}^{k^L}$$

$$\sum_{j=1}^{n} w_{fj}^{k^M} \lambda_j^k = w_{fo}^{k^M}, \quad f = 1,\ldots,F, \quad k = 1,\ldots,K,$$

$$\sum_{j=1}^{n} w_{fj}^{k^U} \lambda_j^k = w_{fo}^{k^U}$$

$$\sum_{j=1}^{n} \sum_{s_{(k,h)}=1}^{S_{(k,h)}} z_{s_{(k,h)}j}^{(k,h)} \lambda_j^k = \sum_{j=1}^{n} \sum_{s_{(k,h)}=1}^{S_{(k,h)}} z_{s_{(k,h)}j}^{(k,h)} \lambda_j^h, \quad \forall (k,h) \ \lambda_j^k, s_r^{k+} \ge 0$$

where:

$m_k$ is number of inputs corresponds to $k^{th}$ supply chain member.

$i$ is a numerical index corresponds to inputs of supply chain members.

$R_k^D$ is number of desirable outputs corresponds to $k^{th}$ supply chain member.

$\mathrm{r}_k^D$ is a numerical index corresponds to corresponds to desirable outputs of supply chain members.

$\mathrm{R}_k^{UD}$ is number of undesirable outputs corresponds to $k^{th}$ supply chain member.

$\mathrm{r}_k^{UD}$ is a numerical index corresponds to corresponds to undesirable outputs of supply chain members.

$K$ is number of supply chain members.

$k$ is a numerical index correspond to supply chain members.

$n$ is the number of DMUs.

$j$ is a numerical index correspond to DMUs.

$x_{ij}^k$ is the $i^{th}$ input of $k^{th}$ supply chain member of $j^{th}$ DMU

$y_{r_k^D j}^{kD}$ is the ${r_k^D}^{th}$ desirable output of $k^{th}$ supply chain member of $j^{th}$ DMU

$y_{r_k^{UD} j}^{kUD}$ is the ${r_k^{UD}}^{th}$ undesirable output of $k^{th}$ supply chain member of $j^{th}$ DMU

$y_{r_k^D 0}^{kD}$ is the ${r_k^D}^{th}$ desirable output of $k^{th}$ supply chain member of the DMU under evaluation $(DMU_0)$

$y_{r_k^{UD} 0}^{kUD}$ is the ${r_k^{UD}}^{th}$ undesirable output of $k^{th}$ supply chain member of the DMU under evaluation $(DMU_0)$

$w_{fj}^k$ is the dual role factor corresponds to $k^{th}$ supply chain member of $j^{th}$ DMU.

$F$ is number of dual role factor corresponds to $k^{th}$ supply chain member of $j^{th}$ DMU.

$f$ is a numerical index of dual role factor corresponds to $k^{th}$ supply chain member of $j^{th}$ DMU.

$Z_{s_{(k,h)} j}^{(k,h)}$ is the Intermediate measures from $k^{th}$ supply chain member to $h^{th}$ supply chain member of $j^{th}$ DMU.

$S_{(k,h)}$ is Number of Intermediate measures from $k^{th}$ supply chain member to $h^{th}$ supply chain member of $j^{th}$ DMU.

$s_{(k,h)}$ is a numerical index of Intermediate measures from $k^{th}$ supply chain member to $h^{th}$ supply chain member of $j^{th}$ DMU.

$s_r^{k+D}$ is the output slack corresponds to the desirable outputs of $k^{th}$ supply chain member.

$s_r^{k+UD}$ is the output slack corresponds to the undesirable outputs of $k^{th}$ supply chain member.

$w^k$ is the supply chain member's assigned weight.

We will use the network model based on Zhu (2009) to calculate the efficiencies of the beverage supply chain partners. We will compare our findings with those of Tajbakhsh and Hassini (2014) and Mirhedayatian *et al*. (2014). The data of the Iranian beverage case study is displayed in table 4.17. Additionally, we assume the same weight preference that Mirhedayatian *et al*. (2014) assigned to each partner: $w_{sup} = 0.3, w_{man} = 0.4, w_{dist} = 0.2, and\ w_{ret} = 0$.

Table 4.18 shows the proposed model results for the Iranian beverage case. We notice that among the three models, $DMU_1$ is the most efficient. Furthermore, we notice that our model results are in agreement with Tajbakhsh and Hassini (2014) for $DMU_1$, $DMU_2$, $DMU_4$, and $DMU_{10}$ which represents 40% of the cases. Moreover, $DMU_1$, $DMU_2$, $DMU_4$, $DMU_5$, $DMU_8$, $DMU_9$, and $DMU_{10}$ have the same rank in both models. The supplier in both models is efficient in four DMUs and the retailer is efficient in two DMUs. On the other hand, we notice that the distributor was efficient in three DMUs of our model while it was efficient in six DMUs of Tajbakhsh and Hassini (2014).

| Stage | Inputs Outputs | DMUs | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
| Supplier | $X_1Sup$ | 290 | 300 | 288 | 320 | 290 | 340 | 325 | 330 | 349 | 295 |
| | $X_2Sup$ | 220 | 345 | 350 | 330 | 275 | 210 | 370 | 250 | 320 | 335 |
| | $Y_1Sup$ | 1250 | 1295 | 1320 | 1259 | 1320 | 1349 | 1329 | 1276 | 1293 | 1302 |
| | $Z_1Sup - Man$ | 999961 | 999966 | 999954 | 999968 | 999947 | 999938 | 999961 | 999955 | 999928 | 999958 |
| Manufacturer | $X_1Man$ | 104 | 125 | 110 | 105 | 135 | 142 | 159 | 130 | 115 | 100 |
| | $X_2Man$ | 139 | 125 | 155 | 132 | 149 | 176 | 125 | 192 | 156 | 145 |
| | $X_3Man$ | 394 | 452 | 329 | 442 | 526 | 349 | 527 | 397 | 309 | 403 |
| | $Y_1Man$ | 0.00645 | 0.00599 | 0.00654 | 0.00556 | 0.00599 | 0.00641 | 0.00562 | 0.00549 | 0.00599 | 0.00575 |
| | $Y_2Man$ | 3 | 2 | 3 | 3 | 2 | 3 | 3 | 2 | 3 | 3 |
| | $Z_1Man - Dist$ | 490 | 523 | 539 | 597 | 479 | 623 | 589 | 532 | 508 | 639 |
| Distributer | $X_1Dist$ | 127 | 147 | 247 | 147 | 184 | 194 | 204 | 215 | 167 | 156 |
| | $X_2Dist$ | 29 | 32 | 28 | 35 | 32 | 35 | 29 | 26 | 37 | 30 |
| | $Y_1Dist$ | 170 | 189 | 172 | 193 | 219 | 189 | 190 | 153 | 189 | 210 |
| | $Z_1Dist - Ret$ | 9590 | 9721 | 10372 | 10333 | 9742 | 11036 | 11553 | 10846 | 10423 | 10467 |
| Retailer | $X_1Ret$ | 102 | 112 | 130 | 100 | 139 | 149 | 147 | 125 | 130 | 104 |
| | $Y_1Ret$ | 4 | 2 | 3 | 4 | 4 | 3 | 4 | 2 | 3 | 2 |

Table 4.17 The data of the Iranian beverage case study

| DMUs | Based on Zhu, 2009 | | | | | | Tajbakhsh and Hassini, 2014 | | | | | | Mirhedayatian, 2014 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\boldsymbol{\Omega^*}$ | $\boldsymbol{\Omega_{sup}}$ | $\boldsymbol{\Omega_{man}}$ | $\boldsymbol{\Omega_{dist}}$ | $\boldsymbol{\Omega_{ret}}$ | **Rank** | $\boldsymbol{\Omega^*}$ | $\boldsymbol{\Omega_{sup}}$ | $\boldsymbol{\Omega_{man}}$ | $\boldsymbol{\Omega_{dist}}$ | $\boldsymbol{\Omega_{ret}}$ | **Rank** | $\boldsymbol{\Omega}$ | **Rank** |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 0.929 | 0.961 | 1 | 0.981 | 0.446 | 6 | 0.929 | 0.961 | 1 | 0.981 | 0.446 | 6 | 0.250 | 8 |
| 3 | 0.933 | 1 | 1 | 0.878 | 0.577 | 5 | 0.945 | 1 | 1 | 0.935 | 0.577 | 3 | 0.467 | 7 |
| 4 | 0.966 | 0.903 | 1 | 0.978 | 1 | 2 | 0.966 | 0.903 | 1 | 0.978 | 1 | 2 | 0.604 | 4 |
| 5 | 0.912 | 1 | 0.849 | 0.998 | 0.729 | 8 | 0.913 | 1 | 0.849 | 1 | 0.731 | 8 | 0.243 | 9 |
| 6 | 0.953 | 1 | 0.943 | 1 | 0.760 | 3 | 0.925 | 1 | 1 | 0.875 | 0.503 | 7 | 0.500 | 5 |
| 7 | 0.926 | 0.894 | 1 | 0.940 | 0.694 | 7 | 0.936 | 0.894 | 1 | 1 | 0.680 | 5 | 0.700 | 3 |
| 8 | 0.761 | 0.898 | 0.708 | 0.843 | 0.400 | 10 | 0.830 | 0.898 | 0.803 | 1 | 0.400 | 10 | 0.500 | 6 |
| 9 | 0.882 | 0.830 | 1 | 0.875 | 0.577 | 9 | 0.907 | 0.830 | 1 | 1 | 0.577 | 9 | 0.800 | 2 |
| 10 | 0.941 | 0.977 | 1 | 1 | 0.481 | 4 | 0.941 | 0.978 | 1 | 1 | 0.481 | 4 | 1 | 1 |

Table 4.18 Calculation results of the Iranian beverage case study

On comparing with Mirhedayatian *et al*. (2014), we observe that our results are more similar to Tajbakhsh and Hassini (2014) than Mirhedayatian *et al*., (2014). This is because our model and Tajbakhsh and Hassini (2014) model are based on the same structure. The only difference is the use of slacks in Tajbakhsh and Hassini (2014) model. On the other hand, Mirhedayatian *et al*. (2014) model is based on a fuzzy structure and non-radial objective function (Tajbakhsh and Hassini, 2014).

We observed from the numerical example and the Iranian beverage case that the based on Zhu (2009) network DEA model has the ability to deal with different supply chain network structures. Additionally, the model can work with large and small number of inputs and outputs.

# Chapter 5

# Conclusions and Future Work

## 5.1 Conclusions

In this thesis we propose a framework for evaluating the quality of multi-tier suppliers in global supply chains. The proposed framework involves three steps. In the first step, suppliers are grouped based on common characteristics using hierarchical cluster analysis. Next for each group, the total cost of ownership of each supplier is calculated using different qualitative and quantitative cost categories. In the third and the final step, data envelopment analysis is used to evaluate the efficiency of each supplier with respect to the computed cost categories.

The proposed framework can help the decision maker answer the following questions:

- Which is the most efficient supplier?
- How can an inefficient supplier become efficient?
- What is the supply chain best practice? In other words, when will the supply chain be efficient?

The strength of the proposed framework is that it takes into account the network structure of the supply chain. It can deal with $n$-tier suppliers.

The limitation is the lack of real data in our study. The numerical examples are based on random data generated by Excel.

## 5.2 Future Work

Future research can involve:

- Other cost categories that are critical to buyer-supplier collaboration.

- Other cost categories related to different types of supply chains.
- Results validation with other multi-tier supplier quality evaluation techniques.

Our framework can be used to identify inefficient suppliers and the performance improvement targets which can be the basis of supplier development programs selection at focal organizations. For example, training and implementation of quality tools, committing resources, relationship management etc.

## 5.3 SWOT analysis

Fig 5.1 presents a SWOT (Strengths, Weaknesses, Opportunities, and Threats) analysis of the proposed modeling framework:

| | |
|---|---|
| Strength:<br>• The research topic is crucial for the success of a global supply chain.<br>• Comprehensive literature review covers the key factors related to quality improvement in global supply chain.<br>• The framework uses three techniques: cluster analysis, total cost of ownership and DEA.<br>• The proposed framework takes into account the network structure of the supply chain.<br>• The evaluation process spreads across the *n*-tier suppliers. | Weaknesses:<br>• Lack of real life data.<br>• Not focused on a specific supply chain industry.<br>• Limited number of evaluation criteria. |
| Opportunities:<br>• Finds the efficient supplier as well as the amount and source of inefficiency of inefficient suppliers.<br>• The framework provides improvement targets that can be used as a basis for supplier improvement programs.<br>• Helps building strategic collaboration efforts. | Threats:<br>• Suppliers may reject to share their data with the focal firm.<br>• The evaluation approach could become very complex in long global supply chains.<br>• Other evaluation methods.<br>• Other model classifications. |

Figure 5.1 SWOT analysis

# References


1. Akarte, M.M., Surendra, N.V., Ravi, B., Rangaraj, N. (2001), Web based casting supplier evaluation using analytical hierarchy process, Journal of the Operational Research Society, 52(5), 511–522.
2. Al Salem, A.A. (2012), An integrated model for supplier quality evaluation, (Master's thesis, Concordia University, Montreal, Canada), Retrieved from http://spectrum.library.concordia.ca/974017/
3. Andersen, M., Skjoett-Larsen, T. (2009), Corporate social responsibility in global supply chains, Supply Chain Management: An International Journal, 14, 75‑86.
4. Anthony, T. (2000), Supply chain collaboration: success in the new internet economy, Achieving Supply Chain Excellence Through Technology, 2, 41-44.
5. Azbari, M. E., Olfat, L. , Amiri, M., Soofi, J. B. (2014), A network data envelopment analysis model for supply chain performance evaluation: real case of Iranian pharmaceutical industry, International Journal of Industrial Engineering and Production Research, 25(2), 125-137.
6. Bal H., Örkcü H.H. , Çelebioğlu S. (2006), An experimental comparison of the new goal programming and the linear programming approaches in the two-group discriminant problems, Computers & Industrial Engineering, 50, 296-311.
7. Bhutta, K.S., Huq, F. (2002), Supplier selection problem: a comparison of the total cost of ownership and analytic hierarchy process approaches, Supply Chain Management: An International Journal, 7(3), 126-35.
8. Bozarth, C. C., Handfield, R. B. (2006). Introduction to operations and supply chain management. New Jersey: Pearson Prentice Hall.

9. Bradley, D. (2008), Melamine scandal widens, Sciencebase. Retrieved from http://www.sciencebase.com/science-blog/milky-melamine.html

10. Braglia, M., Petroni, A., (2000), A quality assurance-oriented methodology for handling trade-offs in supplier selection, International Journal of Physical Distribution and Logistics Management, 30(2), 96–111.

11. Bremen, P., Oehmen, J., Alard, R. (2007), Cost-transparent sourcing in china applying total cost of ownership, In: Proceedings of the 2007 IEEE, 262‑266.

12. Bruno, G., Esposito, E., Genovese, A., Passaro, R. (2012), AHP-based approaches for supplier evaluation: problems and perspectives, Journal of Purchasing & Supply Management, 18(3), 159-172.

13. Çebi, F., Bayraktar, D. (2003), An integrated approach for supplier selection, Logistics Information Management, 16(6), 395–400.

14. Cao, M., Zhang, Q. (2011), Supply chain collaboration: Impact on collaborative advantage and firm performance, Journal of Operations Management 29, 163–180.

15. Chan, F.T.S. (2003), Interactive selection model for supplier selection process: An analytical hierarchy process approach, International Journal Production Research 41(15), 3549–3579.

16. Charnes, A., Cooper, W.W., Rhodes, E. (1978), Measuring the efficiency of decision making units, European Journal of Operational Research, 2(6), 429-444.

17. Chen, C., Yan, H. (2011), Network DEA model for supply chain performance evaluation, European Journal of Operational Research, 213, 147–155.

18. Chen, C., Zhang, J., Delaurentis T. (2014), Quality control in food supply chain management: An analytical model and case study of the adulterated milk incident in China, International Journal of Production Economics, 152, 188–199.
19. Choi, T.Y., Hartley, J.L. (1996). An exploration of supplier selection practices across the supply chain. Journal of Operations Management, 14, 333–345.
20. Cooper, W.W., Park, K.S., Paster, J.T. (1999), RAM: a range adjusted measure of inefficiency for use with additive models and relations to other models and measures in DEA, Journal of Productivity Analysis, 11, 5–42.
21. Cooper, W.W., Seiford, L.M., Tone, K. (2007), Data envelopment analysis: a comprehensive text with models, applications, references and DEA-solver software, Springer, USA.
22. de Boer, L., Labro, E., Morlacchi, P. (2001), A review of methods supporting supplier selection, European Journal of Purchasing & Supply Management, 7(2),75–89.
23. Degraeve, Z., Roodhooft, F. (1999), Effectively selecting suppliers using total cost of ownership, The Journal of Supply Chain Management, 35(1), 5‐11.
24. Degraeve, Z., Roodhooft, F., Doveren, B.V. (2005), The use of total cost of ownership for strategic procurement: a company-wide management information system, Journal of the Operational Research Society, 56, 51-59.
25. Deng, X., Hub, Y.,Deng, Y., Mahadevan, S. (2014), Supplier selection using AHP methodology extended by D numbers, Expert Systems with Applications, 41(1), 156–167.
26. Dickson, G. W. (1966), An analysis of vendor selection systems and decisions, Journal of Purchasing, 2, 5–17.

27. Dogan, I., Aydin, N. (2011), Combining Bayesian networks and total cost of ownership, Computers & Industrial Engineering, 61, 1072–1085.

28. Edwards, A. (2010, February 4), Moving forward ... fast Toyota accelerator pedal problem prompts massive recall, halts production on 11 models, The Ithacan, Retrieved from https://mbarriedesigns.wordpress.com/2010/03/

29. Ellram, L.M. (1993), Total cost of ownership: Elements and implementation., International Journal of Purchasing and Materials Management, 29, 2-11.

30. Ellram, L.M. (1995), Total cost of ownership: an analysis approach for purchasing, International Journal of Physical Distribution and Logistics Management, 25(8), 4-23.

31. Erdem, A., Gocen, E. (2012), Development of a decision support system for supplier evaluation and order allocation, Expert Systems with Applications, 39(5), 4927‑4937.

32. Evans J.R., Lindsay W.L. (2005), An introduction to six sigma & process improvement, USA: South-Western Cengage Learning.

33. Ferrin, B. G., Plank, R. E. (2002), Total cost of ownership models: an exploratory study, Journal of Supply Chain Management, 38,18–29.

34. Forker, L.B., Mendez, D., (2001), An analytical method for benchmarking best peer suppliers, International Journal of Operations and Production Management, 21(1–2), 195–209.

35. Garfamy, R.M. (2006), A data envelopment analysis approach based on total cost of ownership for supplier selection, Journal of Enterprise Information Management, 19(6), 662 – 678.

36. Gelderman, C. (1989). Henry Ford: The wayward capitalist. New York: St. Martins Press.

37. Ghodsypour, S. H., O’Brien, C. O. (1998), A decision support system for supplier selection using an integrated analytic hierarchy process and linear programming. International Journal of Production Economics, 56, 199–212.

38. Grimm, J. H., Hofstetter, J. S., Sarkis, J. (2014), Exploring sub-suppliers' compliance with corporate sustainability standards, Journal of Cleaner Production, 1-14.

39. Ho, W., Xu, X., Dey, P.K. (2010), Multi-criteria decision making approaches for supplier evaluation and selection: a literature review, European Journal of Operational Research, 202, 16–24.

40. Hou, J., Su, D. (2007), EJB–MVC oriented supplier selection system for mass customization, Journal of Manufacturing Technology Management 18(1), 54– 71.

41. Hughes, A.J., Grawoig, D.E. (1973), Linear programming: an emphasis on decision making, Don Mills, ON: Addison-Wesley Publishing Company.

42. Hurkens, K., van der Valk, W., Wynstra, F. (2006), Total cost of ownership in the services sector: a case study, The Journal of Supply Chain Management, 42, 27-37.

43. Ignizio, J.P. (1976), Goal programming and extensions. Toronto, ON: Lexington.

44. Jain, V., Benyoucef, L. (2008), Managing long supply chain networks: Some emerging issues and challenges, Journal of Manufacturing Technology Management, 19(4), 469–496.

45. Jahantigh, F.F., Malmir, B. (2015), Development of a supply chain model for healthcare industry, In: Proceedings of the 2015 International Conference on Industrial Engineering and Operations Management.

46. Kao, C. (2009), Efficiency decomposition in network data envelopment analysis: A relational model, European Journal of Operational Research, 192, 949–962.

47. Karpak, B., Kumcu, E., Kasuganti, R.R. (2001), Purchasing materials in the supply chain: Managing a multi-objective task, European Journal of Purchasing and Supply Management, 7(3), 209–216.

48. Klassen, R.D., Vereecke, A. (2012), Social issues in supply chains: Capabilities link responsibility, risk (opportunity), and performance, International Journal of Production Economics, 140(1), 103-115.

49. Kraljic, P., (1983), Purchasing must become supply management, Harvard Business Review 61(5), 109-117.

50. Kuei , C.-H. , Madu , C. N., Lin , C. (2011), Developing global supply chain quality management systems, International Journal of Production Research, 49(15), 4457–4481.

51. Kull, T.J., Talluri, S. (2008), A supply-risk reduction model using integrated multi-criteria decision making, IEEE Transactions on Engineering Management, 55(3), 409–419.

52. Lee, H., Plambeck, E., Yatsko, P. (2012), Embracing green in China [. . .] with an NGO nudge, Supply Chain Management Review, May/June, 38-45.

53. Lewis, H.F., Sexton, T.R. (2004), Network DEA: efficiency analysis of organizations with complex internal structure, Computers and Operations Research, 31, 1365–1410.

54. Liao, C.-N., Kao, H.-P. (2010), Supplier selection model using Taguchi loss function, analytical hierarchy process and multi-choice goal programming, Comp and Industrial Engineering, 58, 571-577.

55. Liu, J., Ding, F.Y., Lall, V., (2000), Using data envelopment analysis to compare suppliers for supplier selection and performance improvement, Supply Chain Management: An International Journal, 5(3), 143–150.

56. Liu, F.H.F., Hai, H.L. (2005), The voting analytic hierarchy process method for selecting supplier, International Journal of Production Economics 97(3), 308– 317.

57. MacCarthy, B.L., Jayarathne, P.G.S.A. (2012), Sustainable collaborative supply networks in the international clothing industry: a comparative analysis of two retailers, Production Planning and Control, 23(4), 252-268.

58. Macharis, C., Springael J., De Brucker, K., Verbeke, A. (2004), Promethee and AHP: the design of operational synergies in multicriteria analysis. Strengthening Promethee with ideas of AHP, European Journal of Operational Research, 153, 307−317.

59. Maltz, A.B., Ellram, L.M. (1997), Total cost of relationship: an analytical framework for the logistics outsourcing decision," Journal of Business Logistics, 18, 45-66.

60. Mendoza, A., Santiago, E., Ravindran, A.R. (2008), A three-phase multicriteria method to the supplier selection problem, International Journal of Industrial Engineering, 15(2), 195–210.

61. Mentzer, J.T., De Witt, W., Keebler, J.S., Min, S., Nix, N.W., Smith, C.D., Zacharia, Z.G. (2001), Defining supply chain management, Journal of Business Logistics, 22(2), 1-25.

62. Mirhedayatian, S.M., Azadi, M., Saen, R.F. (2014), A novel network data envelopment analysis model for evaluating green supply chain management, International Journal of Production Economics, 147, 544–554.

63. Morehouse, J.E., Cardoso, L. (2011), Consumer product fraud – how to stop the fraud now, Quarter 2, Retrieved from http://www.supplychainquarterly.com/topics/Strategy/scq201102fraud

64. Mueller, M., dos Santos, V.G., Seuring, S. (2009), The contribution of environmental and social standards towards ensuring legitimacy in supply chain governance, Journal of Business Ethics, 89(4), 509-523.
65. Muralidharan, C., Anantharaman, N., Deshmukh, S.G. (2002), A multi-criteria group decision-making model for supplier rating, Journal of Supply Chain Management 38(4), 22–33.
66. Narasimhan, R., Talluri, S., Mendez, D. (2001), Supplier evaluation and rationalization via data envelopment analysis: An empirical examination, Journal of Supply Chain Management, 37(3), 28–37.
67. Nieminen, P., Takala, J. (2006), Achieving better on-time delivery performance with the help of internal dependencies in the production, International Journal of Management and Enterprise Development, 3, 181–190.
68. Noshad, K., Awasthi, A. (2015), Supplier quality development: A review of literature and industry practices, International Journal of Production Research, 53(2), 466-487.
69. Omurca, S.I. (2013), An intelligent supplier evaluation, selection and development system, Applied Soft Computing, 13, 690–697.
70. Osman, H., Demirli, K. (2010), A bilinear goal programming model and a modified Benders decomposition algorithm for supply chain reconfiguration and supplier selection, International Journal of Production Economics, 124, 97–105.
71. Parlar, M., Weng, Z.K. (1997), Designing a firm's coordinated manufacturing and supply decisions with short product life cycle, Management Science 43(10), 1329-1344.
72. Perçin, S. (2006), An application of the integrated AHP–PGP model in supplier selection, Measuring Business Excellence, 10 (4), 34–49.

73. Rao, P. (2002), Greening the supply chain: a new initiative in South East Asia, International Journal of Operations and Production Management, 22(5/6), 632-655.
74. Regan, C. (2015), Social supply chain – technology or innovation first, Retrieved from https://smbp.uwaterloo.ca/2015/02/social-supply-chain-technology-or-innovation-first
75. Romero, C. (2004), A general structure of achievement function for a goal programming model, European Journal of Operational Research, 153, 675–686.
76. Ross, A., Buffa, F.P., Dröge, C., Carrington, D. (2006), Supplier evaluation in a dyadic relationship: An action research approach, Journal of Business Logistics, 27(2), 75–102.
77. Saen, R.F. (2006), A decision model for selecting technology suppliers in the presence of nondiscretionary factors, Applied Mathematics and Computation, 181(2), 1609–1615.
78. Sarpong, S. (2014), Traceability and supply chain complexity: confronting the issues and concerns, European Business Review, 26(3), 271 – 284.
79. Seydel, J., (2006), Data envelopment analysis for decision support, Industrial Management and Data Systems, 106(1), 81–95.
80. Simatupang, T.M., Sridharan, R. (2002), The collaborative supply chain, International Journal of Logistics Management, 13(1), 15-30.
81. Simatupang, T.M., Sridharan, R. (2005), An integrative framework for supply chain collaboration. International Journal of Logistics Management 16(2), 257–274.
82. Singh, A. (2014), Supplier evaluation and demand allocation among suppliers in a supply chain, Journal of Purchasing and Supply Management, 20, 167–176.
83. Sohal, A.S., Chung, W.W.C. (1998), Activity based costing in manufacturing: two case studies on implementation, Integrated Manufacturing Systems, 9(3), 137-147.

84. Song, N., Platts, K., Bance, D. (2007), Total acquisition cost of overseas outsourcing/sourcing ‑ a framework and a case study, Journal of Manufacturing Technology Management, 18(7), 858‑875.

85. Soto-Silva, W.E., Nadal-Roig, E., González-Araya, M.C., Pla-Aragones, L.M. (2015), Operational research models applied to the fresh fruit supply chain, European Journal of Operational Research, 000, 1–11.

86. Tachizawa, E.M., Wong, C.Y. (2014), Towards a theory of multi-tier sustainable supply chains: a systematic literature review, Supply Chain Management: An International Journal, 19(5/6), 643-663.

87. Tajbakhsh, A., Hassini, E. (2014), A data envelopment analysis approach to evaluate sustainability in supply chain networks. Journal of Cleaner Production, doi:10.1016/j.jclepro.2014.07.054.

88. Talluri, S., Baker, R.C., (2002), A multi-phase mathematical programming approach for effective supply chain design, European Journal of Operational Research, 141(3), 544–558.

89. Talluri, S., Narasimhan, R., (2004), A methodology for strategic sourcing, European Journal of Operational Research, 154(1), 236–250.

90. Talluri, S., Sarkis, J., (2002), A model for performance monitoring of suppliers, International Journal of Production Research, 40(16), 4257–4269.

91. Thomas, D.J., Griffin, P.M. (1996), Coordinated supply chain management, European Journal of Operational Research, 94(1), 1-15.

92. Tse, Y. K., Tan, K.H. (2011), Managing product quality risk in a multi-tier global supply chain, International Journal of Production Research, 49,139–158.

93. Tse, Y. K., Zhang, M., Akhtar, P., MacBryde, J. (2016), Embracing supply chain agility: an investigation in the electronics industry, 21(1).
94. Vereecke, A., Muylle, S. (2006). Performance improvement through supply chain collaboration in Europe. International Journal of Operations and Production Management, 26(11), 1176-1198.
95. Viswanadham, N., Samvedi, A. (2013). Supplier selection based on supply chain ecosystem, performance and risk criteria. International Journal of Production Research, 51(21), 6484-6498.
96. Vonderembse, M.A., Uppal, M., Huang, S.H., Dismukes, J.P. (2006), Designing supply chains: towards theory development, International Journal of Production Economics, 100(2), 223-238.
97. Wang, G., Huang, S.H., Dismukes, J.P. (2004), Product-driven supply chain selection using integrated multi-criteria decision-making methodology, International Journal of Production Economics, 91(1), 1–15.
98. Yang, Z.F., Shi, Y., Wang, B., Yan, H. (2014), Website quality and profitability evaluation in ecommerce firms using two-stage DEA model, Procedia Computer Science, 30, 4-13.
99. Zeydan, M., Çolpan, C., C. Çobanoğlu, C. (2011), A combined methodology for supplier selection and performance evaluation, Expert Systems with Applications, 38(3), 2741-2751.
100. Zhu, Q. and Sarkis, J. (2004), Relationships between operational practices and performance among early adopters of green supply chain management practices in

Chinese manufacturing enterprises, Journal of Operations Management, 22(3), 265-289.

101. Zhu, Q., Sarkis, J. , Lai, K.H. (2012), Green supply chain management innovation diffusion and its relationship to organizational improvement: an ecological modernization perspective, Journal of Engineering and Technology Management, 29 (1), 168-185.

102. Zhu, J. (2009), Quantitative models for performance evaluation and benchmarking data envelopment analysis with spreadsheets (2nd ed.). Worcester, MA: Springer.

103. Zsidisin, G. A. (2003), A grounded definition of supply risk, Journal of Purchasing and Supply Management, 9, 217–224.

# Appendix A

The complete linear programming of the numerical example in chapter 4:

**Observation 1:**

$Min\ \Omega_1$

$78\lambda_1 + 70\lambda_2 + 93\lambda_3 + 90\lambda_4 + 73\lambda_5 + 102\lambda_6 + 104\lambda_7 + 123\lambda_8 + 80\lambda_9 + 81\lambda_{10} \leq 78\Omega_1$

$22\lambda_1 + 26\lambda_2 + 32\lambda_3 + 30\lambda_4 + 28\lambda_5 + 29\lambda_6 + 38\lambda_7 + 38\lambda_8 + 24\lambda_9 + 34\lambda_{10} \leq 22\Omega_1$

$116\lambda_1 + 131\lambda_2 + 143\lambda_3 + 101\lambda_4 + 98\lambda_5 + 146\lambda_6 + 105\lambda_7 + 166\lambda_8 + 169\lambda_9 + 154\lambda_{10} \leq 116\Omega_1$

$79\lambda_1 + 57\lambda_2 + 50\lambda_3 + 48\lambda_4 + 70\lambda_5 + 62\lambda_6 + 69\lambda_7 + 58\lambda_8 + 58\lambda_9 + 48\lambda_{10} \leq 79\Omega_1$

$50\lambda_1 + 57\lambda_2 + 60\lambda_3 + 60\lambda_4 + 70\lambda_5 + 52\lambda_6 + 67\lambda_7 + 57\lambda_8 + 67\lambda_9 + 59\lambda_{10} \leq 50\Omega_1$

$49\lambda_1 + 40\lambda_2 + 46\lambda_3 + 39\lambda_4 + 33\lambda_5 + 38\lambda_6 + 46\lambda_7 + 45\lambda_8 + 34\lambda_9 + 37\lambda_{10} \leq 49\Omega_1$

$107\lambda_1 + 172\lambda_2 + 158\lambda_3 + 150\lambda_4 + 148\lambda_5 + 120\lambda_6 + 128\lambda_7 + 166\lambda_8 + 169\lambda_9 + 126\lambda_{10} \leq 107\Omega_1$

$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} \geq 1$

$\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, \lambda_4 \geq 0, \lambda_5 \geq 0, \lambda_6 \geq 0, \lambda_7 \geq 0, \lambda_8 \geq 0, \lambda_9 \geq 0, \lambda_{10} \geq 0$

$Min\ \Omega_2$

$72\beta_1 + 60\beta_2 + 104\beta_3 + 139\beta_4 + 110\beta_5 + 106\beta_6 + 182\beta_7 + 145\beta_8 + 143\beta_9 + 174\beta_{10} \leq 72\Omega_2$

$29\beta_1 + 30\beta_2 + 24\beta_3 + 30\beta_4 + 20\beta_5 + 28\beta_6 + 32\beta_7 + 26\beta_8 + 30\beta_9 + 26\beta_{10} \leq 29\Omega_2$

$109\beta_1 + 170\beta_2 + 161\beta_3 + 155\beta_4 + 142\beta_5 + 142\beta_6 + 175\beta_7 + 172\beta_8 + 174\beta_9 + 146\beta_{10} \leq 109\Omega_2$

$95\beta_1 + 70\beta_2 + 99\beta_3 + 75\beta_4 + 87\beta_5 + 98\beta_6 + 103\beta_7 + 108\beta_8 + 106\beta_9 + 97\beta_{10} \leq 95\Omega_2$

$73\beta_1 + 61\beta_2 + 75\beta_3 + 84\beta_4 + 81\beta_5 + 86\beta_6 + 99\beta_7 + 85\beta_8 + 90\beta_9 + 96\beta_{10} \leq 73\Omega_2$

$39\beta_1 + 37\beta_2 + 41\beta_3 + 42\beta_4 + 34\beta_5 + 29\beta_6 + 41\beta_7 + 30\beta_8 + 40\beta_9 + 40\beta_{10} \leq 39\Omega_2$

$347\beta_1 + 355\beta_2 + 333\beta_3 + 341\beta_4 + 348\beta_5 + 359\beta_6 + 360\beta_7 + 368\beta_8 + 374\beta_9 + 367\beta_{10} \leq 347\Omega_2$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \leq 1$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \geq 1$

$\beta_1 \geq 0, \beta_2 \geq 0, \beta_3 \geq 0, \beta_4 \geq 0, \beta_5 \geq 0, \beta_6 \geq 0, \beta_7 \geq 0, \beta_8 \geq 0, \beta_9 \geq 0, \beta_{10} \geq 0$

$Min\ \Omega_3$

$288\delta_1 + 261\delta_2 + 309\delta_3 + 230\delta_4 + 309\delta_5 + 263\delta_6 + 284\delta_7 + 327\delta_8 + 297\delta_9 + 313\delta_{10} \leq 288\Omega_3$

$45\delta_1 + 38\delta_2 + 40\delta_3 + 42\delta_4 + 48\delta_5 + 52\delta_6 + 52\delta_7 + 54\delta_8 + 52\delta_9 + 63\delta_{10} \leq 45\Omega_3$

$270\delta_1 + 284\delta_2 + 276\delta_3 + 271\delta_4 + 268\delta_5 + 299\delta_6 + 251\delta_7 + 232\delta_8 + 301\delta_9 + 315\delta_{10} \leq 270\Omega_3$

$140\delta_1 + 126\delta_2 + 136\delta_3 + 138\delta_4 + 144\delta_5 + 149\delta_6 + 146\delta_7 + 142\delta_8 + 150\delta_9 + 159\delta_{10} \leq 140\Omega_3$

$127\delta_1 + 110\delta_2 + 138\delta_3 + 134\delta_4 + 138\delta_5 + 132\delta_6 + 149\delta_7 + 140\delta_8 + 141\delta_9 + 140\delta_{10} \leq 127\Omega_3$

$46\delta_1 + 35\delta_2 + 43\delta_3 + 36\delta_4 + 40\delta_5 + 40\delta_6 + 45\delta_7 + 45\delta_8 + 45\delta_9 + 50\delta_{10} \leq 46\Omega_3$

$700\delta_1 + 638\delta_2 + 658\delta_3 + 649\delta_4 + 651\delta_5 + 668\delta_6 + 745\delta_7 + 676\delta_8 + 734\delta_9 + 710\delta_{10} \leq 700\Omega_3$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \leq 1$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \geq 1$

$\delta_1 \geq 0, \delta_2 \geq 0, \delta_3 \geq 0, \delta_4 \geq 0, \delta_5 \geq 0, \delta_6 \geq 0, \delta_7 \geq 0, \delta_8 \geq 0, \delta_9 \geq 0, \delta_{10} \geq 0$

**Observation 2:**

$Min\ \Omega_1$

$78\lambda_1 + 70\lambda_2 + 93\lambda_3 + 90\lambda_4 + 73\lambda_5 + 102\lambda_6 + 104\lambda_7 + 123\lambda_8 + 80\lambda_9 + 81\lambda_{10} \leq 70\Omega_1$

$22\lambda_1 + 26\lambda_2 + 32\lambda_3 + 30\lambda_4 + 28\lambda_5 + 29\lambda_6 + 38\lambda_7 + 38\lambda_8 + 24\lambda_9 + 34\lambda_{10} \leq 26\Omega_1$

$116\lambda_1 + 131\lambda_2 + 143\lambda_3 + 101\lambda_4 + 98\lambda_5 + 146\lambda_6 + 105\lambda_7 + 166\lambda_8 + 169\lambda_9 + 154\lambda_{10} \leq 131\Omega_1$

$79\lambda_1 + 57\lambda_2 + 50\lambda_3 + 48\lambda_4 + 70\lambda_5 + 62\lambda_6 + 69\lambda_7 + 58\lambda_8 + 58\lambda_9 + 48\lambda_{10} \leq 57\Omega_1$

$50\lambda_1 + 57\lambda_2 + 60\lambda_3 + 60\lambda_4 + 70\lambda_5 + 52\lambda_6 + 67\lambda_7 + 57\lambda_8 + 67\lambda_9 + 59\lambda_{10} \leq 57\Omega_1$

$49\lambda_1 + 40\lambda_2 + 46\lambda_3 + 39\lambda_4 + 33\lambda_5 + 38\lambda_6 + 46\lambda_7 + 45\lambda_8 + 34\lambda_9 + 37\lambda_{10} \leq 40\Omega_1$

$107\lambda_1 + 172\lambda_2 + 158\lambda_3 + 150\lambda_4 + 148\lambda_5 + 120\lambda_6 + 128\lambda_7 + 166\lambda_8 + 169\lambda_9 + 126\lambda_{10} \leq 172\Omega_1$

$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} \geq 1$

$\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, \lambda_4 \geq 0, \lambda_5 \geq 0, \lambda_6 \geq 0, \lambda_7 \geq 0, \lambda_8 \geq 0, \lambda_9 \geq 0, \lambda_{10} \geq 0$

$Min\ \Omega_2$

$72\beta_1 + 60\beta_2 + 104\beta_3 + 139\beta_4 + 110\beta_5 + 106\beta_6 + 182\beta_7 + 145\beta_8 + 143\beta_9 + 174\beta_{10} \leq 60\Omega_2$

$29\beta_1 + 30\beta_2 + 24\beta_3 + 30\beta_4 + 20\beta_5 + 28\beta_6 + 32\beta_7 + 26\beta_8 + 30\beta_9 + 26\beta_{10} \leq 30\Omega_2$

$109\beta_1 + 170\beta_2 + 161\beta_3 + 155\beta_4 + 142\beta_5 + 142\beta_6 + 175\beta_7 + 172\beta_8 + 174\beta_9 + 146\beta_{10} \leq 170\Omega_2$

$95\beta_1 + 70\beta_2 + 99\beta_3 + 75\beta_4 + 87\beta_5 + 98\beta_6 + 103\beta_7 + 108\beta_8 + 106\beta_9 + 97\beta_{10} \leq 70\Omega_2$

$73\beta_1 + 61\beta_2 + 75\beta_3 + 84\beta_4 + 81\beta_5 + 86\beta_6 + 99\beta_7 + 85\beta_8 + 90\beta_9 + 96\beta_{10} \leq 61\Omega_2$

$39\beta_1 + 37\beta_2 + 41\beta_3 + 42\beta_4 + 34\beta_5 + 29\beta_6 + 41\beta_7 + 30\beta_8 + 40\beta_9 + 40\beta_{10} \leq 37\Omega_2$

$347\beta_1 + 355\beta_2 + 333\beta_3 + 341\beta_4 + 348\beta_5 + 359\beta_6 + 360\beta_7 + 368\beta_8 + 374\beta_9 + 367\beta_{10} \leq 355\Omega_2$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \leq 1$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \geq 1$

$\beta_1 \geq 0, \beta_2 \geq 0, \beta_3 \geq 0, \beta_4 \geq 0, \beta_5 \geq 0, \beta_6 \geq 0, \beta_7 \geq 0, \beta_8 \geq 0, \beta_9 \geq 0, \beta_{10} \geq 0$

$Min\ \Omega_3$

$288\delta_1 + 261\delta_2 + 309\delta_3 + 230\delta_4 + 309\delta_5 + 263\delta_6 + 284\delta_7 + 327\delta_8 + 297\delta_9 + 313\delta_{10} \leq 261\Omega_3$

$45\delta_1 + 38\delta_2 + 40\delta_3 + 42\delta_4 + 48\delta_5 + 52\delta_6 + 52\delta_7 + 54\delta_8 + 52\delta_9 + 63\delta_{10} \leq 38\Omega_3$

$270\delta_1 + 284\delta_2 + 276\delta_3 + 271\delta_4 + 268\delta_5 + 299\delta_6 + 251\delta_7 + 232\delta_8 + 301\delta_9 + 315\delta_{10} \leq 284\Omega_3$

$140\delta_1 + 126\delta_2 + 136\delta_3 + 138\delta_4 + 144\delta_5 + 149\delta_6 + 146\delta_7 + 142\delta_8 + 150\delta_9 + 159\delta_{10} \leq 126\Omega_3$

$127\delta_1 + 110\delta_2 + 138\delta_3 + 134\delta_4 + 138\delta_5 + 132\delta_6 + 149\delta_7 + 140\delta_8 + 141\delta_9 + 140\delta_{10} \leq 110\Omega_3$

$46\delta_1 + 35\delta_2 + 43\delta_3 + 36\delta_4 + 40\delta_5 + 40\delta_6 + 45\delta_7 + 45\delta_8 + 45\delta_9 + 50\delta_{10} \leq 35\Omega_3$

$700\delta_1 + 638\delta_2 + 658\delta_3 + 649\delta_4 + 651\delta_5 + 668\delta_6 + 745\delta_7 + 676\delta_8 + 734\delta_9 + 710\delta_{10} \leq 638\Omega_3$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \leq 1$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \geq 1$

$\delta_1 \geq 0, \delta_2 \geq 0, \delta_3 \geq 0, \delta_4 \geq 0, \delta_5 \geq 0, \delta_6 \geq 0, \delta_7 \geq 0, \delta_8 \geq 0, \delta_9 \geq 0, \delta_{10} \geq 0$

**Observation 3:**

$Min\ \Omega_1$

$78\lambda_1 + 70\lambda_2 + 93\lambda_3 + 90\lambda_4 + 73\lambda_5 + 102\lambda_6 + 104\lambda_7 + 123\lambda_8 + 80\lambda_9 + 81\lambda_{10} \leq 93\Omega_1$

$22\lambda_1 + 26\lambda_2 + 32\lambda_3 + 30\lambda_4 + 28\lambda_5 + 29\lambda_6 + 38\lambda_7 + 38\lambda_8 + 24\lambda_9 + 34\lambda_{10} \leq 32\Omega_1$

$116\lambda_1 + 131\lambda_2 + 143\lambda_3 + 101\lambda_4 + 98\lambda_5 + 146\lambda_6 + 105\lambda_7 + 166\lambda_8 + 169\lambda_9 + 154\lambda_{10} \leq 143\Omega_1$

$79\lambda_1 + 57\lambda_2 + 50\lambda_3 + 48\lambda_4 + 70\lambda_5 + 62\lambda_6 + 69\lambda_7 + 58\lambda_8 + 58\lambda_9 + 48\lambda_{10} \leq 50\Omega_1$

$50\lambda_1 + 57\lambda_2 + 60\lambda_3 + 60\lambda_4 + 70\lambda_5 + 52\lambda_6 + 67\lambda_7 + 57\lambda_8 + 67\lambda_9 + 59\lambda_{10} \leq 60\Omega_1$

$49\lambda_1 + 40\lambda_2 + 46\lambda_3 + 39\lambda_4 + 33\lambda_5 + 38\lambda_6 + 46\lambda_7 + 45\lambda_8 + 34\lambda_9 + 37\lambda_{10} \leq 46\Omega_1$

$107\lambda_1 + 172\lambda_2 + 158\lambda_3 + 150\lambda_4 + 148\lambda_5 + 120\lambda_6 + 128\lambda_7 + 166\lambda_8 + 169\lambda_9 + 126\lambda_{10} \leq 158\Omega_1$

$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} \geq 1$

$\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, \lambda_4 \geq 0, \lambda_5 \geq 0, \lambda_6 \geq 0, \lambda_7 \geq 0, \lambda_8 \geq 0, \lambda_9 \geq 0, \lambda_{10} \geq 0$

$Min\ \Omega_2$

$72\beta_1 + 60\beta_2 + 104\beta_3 + 139\beta_4 + 110\beta_5 + 106\beta_6 + 182\beta_7 + 145\beta_8 + 143\beta_9 + 174\beta_{10} \leq 104\Omega_2$

$29\beta_1 + 30\beta_2 + 24\beta_3 + 30\beta_4 + 20\beta_5 + 28\beta_6 + 32\beta_7 + 26\beta_8 + 30\beta_9 + 26\beta_{10} \leq 24\Omega_2$

$109\beta_1 + 170\beta_2 + 161\beta_3 + 155\beta_4 + 142\beta_5 + 142\beta_6 + 175\beta_7 + 172\beta_8 + 174\beta_9 + 146\beta_{10} \leq 161\Omega_2$

$95\beta_1 + 70\beta_2 + 99\beta_3 + 75\beta_4 + 87\beta_5 + 98\beta_6 + 103\beta_7 + 108\beta_8 + 106\beta_9 + 97\beta_{10} \leq 99\Omega_2$

$73\beta_1 + 61\beta_2 + 75\beta_3 + 84\beta_4 + 81\beta_5 + 86\beta_6 + 99\beta_7 + 85\beta_8 + 90\beta_9 + 96\beta_{10} \leq 75\Omega_2$

$39\beta_1 + 37\beta_2 + 41\beta_3 + 42\beta_4 + 34\beta_5 + 29\beta_6 + 41\beta_7 + 30\beta_8 + 40\beta_9 + 40\beta_{10} \leq 41\Omega_2$

$347\beta_1 + 355\beta_2 + 333\beta_3 + 341\beta_4 + 348\beta_5 + 359\beta_6 + 360\beta_7 + 368\beta_8 + 374\beta_9 + 367\beta_{10} \leq 333\Omega_2$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \leq 1$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \geq 1$

$\beta_1 \geq 0, \beta_2 \geq 0, \beta_3 \geq 0, \beta_4 \geq 0, \beta_5 \geq 0, \beta_6 \geq 0, \beta_7 \geq 0, \beta_8 \geq 0, \beta_9 \geq 0, \beta_{10} \geq 0$

$Min\ \Omega_3$

$288\delta_1 + 261\delta_2 + 309\delta_3 + 230\delta_4 + 309\delta_5 + 263\delta_6 + 284\delta_7 + 327\delta_8 + 297\delta_9 + 313\delta_{10} \leq 309\Omega_3$

$45\delta_1 + 38\delta_2 + 40\delta_3 + 42\delta_4 + 48\delta_5 + 52\delta_6 + 52\delta_7 + 54\delta_8 + 52\delta_9 + 63\delta_{10} \leq 40\Omega_3$

$270\delta_1 + 284\delta_2 + 276\delta_3 + 271\delta_4 + 268\delta_5 + 299\delta_6 + 251\delta_7 + 232\delta_8 + 301\delta_9 + 315\delta_{10} \leq 276\Omega_3$

$140\delta_1 + 126\delta_2 + 136\delta_3 + 138\delta_4 + 144\delta_5 + 149\delta_6 + 146\delta_7 + 142\delta_8 + 150\delta_9 + 159\delta_{10} \leq 136\Omega_3$

$127\delta_1 + 110\delta_2 + 138\delta_3 + 134\delta_4 + 138\delta_5 + 132\delta_6 + 149\delta_7 + 140\delta_8 + 141\delta_9 + 140\delta_{10} \leq 138\Omega_3$

$46\delta_1 + 35\delta_2 + 43\delta_3 + 36\delta_4 + 40\delta_5 + 40\delta_6 + 45\delta_7 + 45\delta_8 + 45\delta_9 + 50\delta_{10} \leq 43\Omega_3$

$700\delta_1 + 638\delta_2 + 658\delta_3 + 649\delta_4 + 651\delta_5 + 668\delta_6 + 745\delta_7 + 676\delta_8 + 734\delta_9 + 710\delta_{10} \leq 658\Omega_3$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \leq 1$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \geq 1$

$\delta_1 \geq 0, \delta_2 \geq 0, \delta_3 \geq 0, \delta_4 \geq 0, \delta_5 \geq 0, \delta_6 \geq 0, \delta_7 \geq 0, \delta_8 \geq 0, \delta_9 \geq 0, \delta_{10} \geq 0$

**Observation 4:**

$Min\ \Omega_1$

$78\lambda_1 + 70\lambda_2 + 93\lambda_3 + 90\lambda_4 + 73\lambda_5 + 102\lambda_6 + 104\lambda_7 + 123\lambda_8 + 80\lambda_9 + 81\lambda_{10} \leq 90\Omega_1$

$22\lambda_1 + 26\lambda_2 + 32\lambda_3 + 30\lambda_4 + 28\lambda_5 + 29\lambda_6 + 38\lambda_7 + 38\lambda_8 + 24\lambda_9 + 34\lambda_{10} \leq 30\Omega_1$

$116\lambda_1 + 131\lambda_2 + 143\lambda_3 + 101\lambda_4 + 98\lambda_5 + 146\lambda_6 + 105\lambda_7 + 166\lambda_8 + 169\lambda_9 + 154\lambda_{10} \leq 101\Omega_1$

$79\lambda_1 + 57\lambda_2 + 50\lambda_3 + 48\lambda_4 + 70\lambda_5 + 62\lambda_6 + 69\lambda_7 + 58\lambda_8 + 58\lambda_9 + 48\lambda_{10} \leq 48\Omega_1$

$50\lambda_1 + 57\lambda_2 + 60\lambda_3 + 60\lambda_4 + 70\lambda_5 + 52\lambda_6 + 67\lambda_7 + 57\lambda_8 + 67\lambda_9 + 59\lambda_{10} \leq 60\Omega_1$

$49\lambda_1 + 40\lambda_2 + 46\lambda_3 + 39\lambda_4 + 33\lambda_5 + 38\lambda_6 + 46\lambda_7 + 45\lambda_8 + 34\lambda_9 + 37\lambda_{10} \leq 39\Omega_1$

$107\lambda_1 + 172\lambda_2 + 158\lambda_3 + 150\lambda_4 + 148\lambda_5 + 120\lambda_6 + 128\lambda_7 + 166\lambda_8 + 169\lambda_9 + 126\lambda_{10} \leq 150\Omega_1$

$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} \geq 1$

$\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, \lambda_4 \geq 0, \lambda_5 \geq 0, \lambda_6 \geq 0, \lambda_7 \geq 0, \lambda_8 \geq 0, \lambda_9 \geq 0, \lambda_{10} \geq 0$

$Min\ \Omega_2$

$72\beta_1 + 60\beta_2 + 104\beta_3 + 139\beta_4 + 110\beta_5 + 106\beta_6 + 182\beta_7 + 145\beta_8 + 143\beta_9 + 174\beta_{10} \leq 139\Omega_2$

$29\beta_1 + 30\beta_2 + 24\beta_3 + 30\beta_4 + 20\beta_5 + 28\beta_6 + 32\beta_7 + 26\beta_8 + 30\beta_9 + 26\beta_{10} \leq 30\Omega_2$

$109\beta_1 + 170\beta_2 + 161\beta_3 + 155\beta_4 + 142\beta_5 + 142\beta_6 + 175\beta_7 + 172\beta_8 + 174\beta_9 + 146\beta_{10} \leq 155\Omega_2$

$95\beta_1 + 70\beta_2 + 99\beta_3 + 75\beta_4 + 87\beta_5 + 98\beta_6 + 103\beta_7 + 108\beta_8 + 106\beta_9 + 97\beta_{10} \leq 75\Omega_2$

$73\beta_1 + 61\beta_2 + 75\beta_3 + 84\beta_4 + 81\beta_5 + 86\beta_6 + 99\beta_7 + 85\beta_8 + 90\beta_9 + 96\beta_{10} \leq 84\Omega_2$

$39\beta_1 + 37\beta_2 + 41\beta_3 + 42\beta_4 + 34\beta_5 + 29\beta_6 + 41\beta_7 + 30\beta_8 + 40\beta_9 + 40\beta_{10} \leq 42\Omega_2$

$347\beta_1 + 355\beta_2 + 333\beta_3 + 341\beta_4 + 348\beta_5 + 359\beta_6 + 360\beta_7 + 368\beta_8 + 374\beta_9 + 367\beta_{10} \leq 341\Omega_2$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \leq 1$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \geq 1$

$\beta_1 \geq 0, \beta_2 \geq 0, \beta_3 \geq 0, \beta_4 \geq 0, \beta_5 \geq 0, \beta_6 \geq 0, \beta_7 \geq 0, \beta_8 \geq 0, \beta_9 \geq 0, \beta_{10} \geq 0$

$Min\ \Omega_3$

$288\delta_1 + 261\delta_2 + 309\delta_3 + 230\delta_4 + 309\delta_5 + 263\delta_6 + 284\delta_7 + 327\delta_8 + 297\delta_9 + 313\delta_{10} \leq 230\Omega_3$

$45\delta_1 + 38\delta_2 + 40\delta_3 + 42\delta_4 + 48\delta_5 + 52\delta_6 + 52\delta_7 + 54\delta_8 + 52\delta_9 + 63\delta_{10} \leq 42\Omega_3$

$270\delta_1 + 284\delta_2 + 276\delta_3 + 271\delta_4 + 268\delta_5 + 299\delta_6 + 251\delta_7 + 232\delta_8 + 301\delta_9 + 315\delta_{10} \leq 271\Omega_3$

$140\delta_1 + 126\delta_2 + 136\delta_3 + 138\delta_4 + 144\delta_5 + 149\delta_6 + 146\delta_7 + 142\delta_8 + 150\delta_9 + 159\delta_{10} \leq 138\Omega_3$

$127\delta_1 + 110\delta_2 + 138\delta_3 + 134\delta_4 + 138\delta_5 + 132\delta_6 + 149\delta_7 + 140\delta_8 + 141\delta_9 + 140\delta_{10} \leq 134\Omega_3$

$46\delta_1 + 35\delta_2 + 43\delta_3 + 36\delta_4 + 40\delta_5 + 40\delta_6 + 45\delta_7 + 45\delta_8 + 45\delta_9 + 50\delta_{10} \leq 36\Omega_3$

$700\delta_1 + 638\delta_2 + 658\delta_3 + 649\delta_4 + 651\delta_5 + 668\delta_6 + 745\delta_7 + 676\delta_8 + 734\delta_9 + 710\delta_{10} \leq 649\Omega_3$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \leq 1$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \geq 1$

$\delta_1 \geq 0, \delta_2 \geq 0, \delta_3 \geq 0, \delta_4 \geq 0, \delta_5 \geq 0, \delta_6 \geq 0, \delta_7 \geq 0, \delta_8 \geq 0, \delta_9 \geq 0, \delta_{10} \geq 0$

**Observation 5:**

$Min\ \Omega_1$

$78\lambda_1 + 70\lambda_2 + 93\lambda_3 + 90\lambda_4 + 73\lambda_5 + 102\lambda_6 + 104\lambda_7 + 123\lambda_8 + 80\lambda_9 + 81\lambda_{10} \leq 73\Omega_1$

$22\lambda_1 + 26\lambda_2 + 32\lambda_3 + 30\lambda_4 + 28\lambda_5 + 29\lambda_6 + 38\lambda_7 + 38\lambda_8 + 24\lambda_9 + 34\lambda_{10} \leq 28\Omega_1$

$116\lambda_1 + 131\lambda_2 + 143\lambda_3 + 101\lambda_4 + 98\lambda_5 + 146\lambda_6 + 105\lambda_7 + 166\lambda_8 + 169\lambda_9 + 154\lambda_{10} \leq 98\Omega_1$

$79\lambda_1 + 57\lambda_2 + 50\lambda_3 + 48\lambda_4 + 70\lambda_5 + 62\lambda_6 + 69\lambda_7 + 58\lambda_8 + 58\lambda_9 + 48\lambda_{10} \leq 70\Omega_1$

$50\lambda_1 + 57\lambda_2 + 60\lambda_3 + 60\lambda_4 + 70\lambda_5 + 52\lambda_6 + 67\lambda_7 + 57\lambda_8 + 67\lambda_9 + 59\lambda_{10} \leq 70\Omega_1$

$49\lambda_1 + 40\lambda_2 + 46\lambda_3 + 39\lambda_4 + 33\lambda_5 + 38\lambda_6 + 46\lambda_7 + 45\lambda_8 + 34\lambda_9 + 37\lambda_{10} \leq 33\Omega_1$

$107\lambda_1 + 172\lambda_2 + 158\lambda_3 + 150\lambda_4 + 148\lambda_5 + 120\lambda_6 + 128\lambda_7 + 166\lambda_8 + 169\lambda_9 + 126\lambda_{10} \leq 148\Omega_1$

$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} \geq 1$

$\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, \lambda_4 \geq 0, \lambda_5 \geq 0, \lambda_6 \geq 0, \lambda_7 \geq 0, \lambda_8 \geq 0, \lambda_9 \geq 0, \lambda_{10} \geq 0$

$Min\ \Omega_2$

$72\beta_1 + 60\beta_2 + 104\beta_3 + 139\beta_4 + 110\beta_5 + 106\beta_6 + 182\beta_7 + 145\beta_8 + 143\beta_9 + 174\beta_{10} \leq 110\Omega_2$

$29\beta_1 + 30\beta_2 + 24\beta_3 + 30\beta_4 + 20\beta_5 + 28\beta_6 + 32\beta_7 + 26\beta_8 + 30\beta_9 + 26\beta_{10} \leq 20\Omega_2$

$109\beta_1 + 170\beta_2 + 161\beta_3 + 155\beta_4 + 142\beta_5 + 142\beta_6 + 175\beta_7 + 172\beta_8 + 174\beta_9 + 146\beta_{10} \leq 142\Omega_2$

$95\beta_1 + 70\beta_2 + 99\beta_3 + 75\beta_4 + 87\beta_5 + 98\beta_6 + 103\beta_7 + 108\beta_8 + 106\beta_9 + 97\beta_{10} \leq 87\Omega_2$

$73\beta_1 + 61\beta_2 + 75\beta_3 + 84\beta_4 + 81\beta_5 + 86\beta_6 + 99\beta_7 + 85\beta_8 + 90\beta_9 + 96\beta_{10} \leq 81\Omega_2$

$39\beta_1 + 37\beta_2 + 41\beta_3 + 42\beta_4 + 34\beta_5 + 29\beta_6 + 41\beta_7 + 30\beta_8 + 40\beta_9 + 40\beta_{10} \leq 34\Omega_2$

$347\beta_1 + 355\beta_2 + 333\beta_3 + 341\beta_4 + 348\beta_5 + 359\beta_6 + 360\beta_7 + 368\beta_8 + 374\beta_9 + 367\beta_{10} \leq 348\Omega_2$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \leq 1$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \geq 1$

$\beta_1 \geq 0, \beta_2 \geq 0, \beta_3 \geq 0, \beta_4 \geq 0, \beta_5 \geq 0, \beta_6 \geq 0, \beta_7 \geq 0, \beta_8 \geq 0, \beta_9 \geq 0, \beta_{10} \geq 0$

$Min\ \Omega_3$

$288\delta_1 + 261\delta_2 + 309\delta_3 + 230\delta_4 + 309\delta_5 + 263\delta_6 + 284\delta_7 + 327\delta_8 + 297\delta_9 + 313\delta_{10} \leq 309\Omega_3$

$45\delta_1 + 38\delta_2 + 40\delta_3 + 42\delta_4 + 48\delta_5 + 52\delta_6 + 52\delta_7 + 54\delta_8 + 52\delta_9 + 63\delta_{10} \leq 48\Omega_3$

$270\delta_1 + 284\delta_2 + 276\delta_3 + 271\delta_4 + 268\delta_5 + 299\delta_6 + 251\delta_7 + 232\delta_8 + 301\delta_9 + 315\delta_{10} \leq 268\Omega_3$

$140\delta_1 + 126\delta_2 + 136\delta_3 + 138\delta_4 + 144\delta_5 + 149\delta_6 + 146\delta_7 + 142\delta_8 + 150\delta_9 + 159\delta_{10} \leq 144\Omega_3$

$127\delta_1 + 110\delta_2 + 138\delta_3 + 134\delta_4 + 138\delta_5 + 132\delta_6 + 149\delta_7 + 140\delta_8 + 141\delta_9 + 140\delta_{10} \leq 138\Omega_3$

$46\delta_1 + 35\delta_2 + 43\delta_3 + 36\delta_4 + 40\delta_5 + 40\delta_6 + 45\delta_7 + 45\delta_8 + 45\delta_9 + 50\delta_{10} \leq 40\Omega_3$

$700\delta_1 + 638\delta_2 + 658\delta_3 + 649\delta_4 + 651\delta_5 + 668\delta_6 + 745\delta_7 + 676\delta_8 + 734\delta_9 + 710\delta_{10} \leq 651\Omega_3$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \leq 1$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \geq 1$

$\delta_1 \geq 0, \delta_2 \geq 0, \delta_3 \geq 0, \delta_4 \geq 0, \delta_5 \geq 0, \delta_6 \geq 0, \delta_7 \geq 0, \delta_8 \geq 0, \delta_9 \geq 0, \delta_{10} \geq 0$

**Observation 6:**

$Min\ \Omega_1$

$78\lambda_1 + 70\lambda_2 + 93\lambda_3 + 90\lambda_4 + 73\lambda_5 + 102\lambda_6 + 104\lambda_7 + 123\lambda_8 + 80\lambda_9 + 81\lambda_{10} \leq 102\Omega_1$

$22\lambda_1 + 26\lambda_2 + 32\lambda_3 + 30\lambda_4 + 28\lambda_5 + 29\lambda_6 + 38\lambda_7 + 38\lambda_8 + 24\lambda_9 + 34\lambda_{10} \leq 29\Omega_1$

$116\lambda_1 + 131\lambda_2 + 143\lambda_3 + 101\lambda_4 + 98\lambda_5 + 146\lambda_6 + 105\lambda_7 + 166\lambda_8 + 169\lambda_9 + 154\lambda_{10} \leq 146\Omega_1$

$79\lambda_1 + 57\lambda_2 + 50\lambda_3 + 48\lambda_4 + 70\lambda_5 + 62\lambda_6 + 69\lambda_7 + 58\lambda_8 + 58\lambda_9 + 48\lambda_{10} \leq 62\Omega_1$

$50\lambda_1 + 57\lambda_2 + 60\lambda_3 + 60\lambda_4 + 70\lambda_5 + 52\lambda_6 + 67\lambda_7 + 57\lambda_8 + 67\lambda_9 + 59\lambda_{10} \leq 52\Omega_1$

$49\lambda_1 + 40\lambda_2 + 46\lambda_3 + 39\lambda_4 + 33\lambda_5 + 38\lambda_6 + 46\lambda_7 + 45\lambda_8 + 34\lambda_9 + 37\lambda_{10} \leq 38\Omega_1$

$107\lambda_1 + 172\lambda_2 + 158\lambda_3 + 150\lambda_4 + 148\lambda_5 + 120\lambda_6 + 128\lambda_7 + 166\lambda_8 + 169\lambda_9 + 126\lambda_{10} \leq 120\Omega_1$

$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} \geq 1$

$\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, \lambda_4 \geq 0, \lambda_5 \geq 0, \lambda_6 \geq 0, \lambda_7 \geq 0, \lambda_8 \geq 0, \lambda_9 \geq 0, \lambda_{10} \geq 0$

$Min\ \Omega_2$

$72\beta_1 + 60\beta_2 + 104\beta_3 + 139\beta_4 + 110\beta_5 + 106\beta_6 + 182\beta_7 + 145\beta_8 + 143\beta_9 + 174\beta_{10} \leq 106\Omega_2$

$29\beta_1 + 30\beta_2 + 24\beta_3 + 30\beta_4 + 20\beta_5 + 28\beta_6 + 32\beta_7 + 26\beta_8 + 30\beta_9 + 26\beta_{10} \leq 28\Omega_2$

$109\beta_1 + 170\beta_2 + 161\beta_3 + 155\beta_4 + 142\beta_5 + 142\beta_6 + 175\beta_7 + 172\beta_8 + 174\beta_9 + 146\beta_{10} \leq 142\Omega_2$

$95\beta_1 + 70\beta_2 + 99\beta_3 + 75\beta_4 + 87\beta_5 + 98\beta_6 + 103\beta_7 + 108\beta_8 + 106\beta_9 + 97\beta_{10} \leq 98\Omega_2$

$73\beta_1 + 61\beta_2 + 75\beta_3 + 84\beta_4 + 81\beta_5 + 86\beta_6 + 99\beta_7 + 85\beta_8 + 90\beta_9 + 96\beta_{10} \leq 86\Omega_2$

$39\beta_1 + 37\beta_2 + 41\beta_3 + 42\beta_4 + 34\beta_5 + 29\beta_6 + 41\beta_7 + 30\beta_8 + 40\beta_9 + 40\beta_{10} \leq 29\Omega_2$

$347\beta_1 + 355\beta_2 + 333\beta_3 + 341\beta_4 + 348\beta_5 + 359\beta_6 + 360\beta_7 + 368\beta_8 + 374\beta_9 + 367\beta_{10} \leq 359\Omega_2$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \leq 1$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \geq 1$

$\beta_1 \geq 0, \beta_2 \geq 0, \beta_3 \geq 0, \beta_4 \geq 0, \beta_5 \geq 0, \beta_6 \geq 0, \beta_7 \geq 0, \beta_8 \geq 0, \beta_9 \geq 0, \beta_{10} \geq 0$

$Min\ \Omega_3$

$288\delta_1 + 261\delta_2 + 309\delta_3 + 230\delta_4 + 309\delta_5 + 263\delta_6 + 284\delta_7 + 327\delta_8 + 297\delta_9 + 313\delta_{10} \leq 263\Omega_3$

$45\delta_1 + 38\delta_2 + 40\delta_3 + 42\delta_4 + 48\delta_5 + 52\delta_6 + 52\delta_7 + 54\delta_8 + 52\delta_9 + 63\delta_{10} \leq 52\Omega_3$

$270\delta_1 + 284\delta_2 + 276\delta_3 + 271\delta_4 + 268\delta_5 + 299\delta_6 + 251\delta_7 + 232\delta_8 + 301\delta_9 + 315\delta_{10} \leq 299\Omega_3$

$140\delta_1 + 126\delta_2 + 136\delta_3 + 138\delta_4 + 144\delta_5 + 149\delta_6 + 146\delta_7 + 142\delta_8 + 150\delta_9 + 159\delta_{10} \leq 149\Omega_3$

$127\delta_1 + 110\delta_2 + 138\delta_3 + 134\delta_4 + 138\delta_5 + 132\delta_6 + 149\delta_7 + 140\delta_8 + 141\delta_9 + 140\delta_{10} \leq 132\Omega_3$

$46\delta_1 + 35\delta_2 + 43\delta_3 + 36\delta_4 + 40\delta_5 + 40\delta_6 + 45\delta_7 + 45\delta_8 + 45\delta_9 + 50\delta_{10} \leq 40\Omega_3$

$700\delta_1 + 638\delta_2 + 658\delta_3 + 649\delta_4 + 651\delta_5 + 668\delta_6 + 745\delta_7 + 676\delta_8 + 734\delta_9 + 710\delta_{10} \leq 668\Omega_3$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \leq 1$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \geq 1$

$\delta_1 \geq 0, \delta_2 \geq 0, \delta_3 \geq 0, \delta_4 \geq 0, \delta_5 \geq 0, \delta_6 \geq 0, \delta_7 \geq 0, \delta_8 \geq 0, \delta_9 \geq 0, \delta_{10} \geq 0$

**Observation 7:**

$Min\ \Omega_1$

$78\lambda_1 + 70\lambda_2 + 93\lambda_3 + 90\lambda_4 + 73\lambda_5 + 102\lambda_6 + 104\lambda_7 + 123\lambda_8 + 80\lambda_9 + 81\lambda_{10} \leq 104\Omega_1$

$22\lambda_1 + 26\lambda_2 + 32\lambda_3 + 30\lambda_4 + 28\lambda_5 + 29\lambda_6 + 38\lambda_7 + 38\lambda_8 + 24\lambda_9 + 34\lambda_{10} \leq 38\Omega_1$

$116\lambda_1 + 131\lambda_2 + 143\lambda_3 + 101\lambda_4 + 98\lambda_5 + 146\lambda_6 + 105\lambda_7 + 166\lambda_8 + 169\lambda_9 + 154\lambda_{10} \leq 105\Omega_1$

$79\lambda_1 + 57\lambda_2 + 50\lambda_3 + 48\lambda_4 + 70\lambda_5 + 62\lambda_6 + 69\lambda_7 + 58\lambda_8 + 58\lambda_9 + 48\lambda_{10} \leq 69\Omega_1$

$50\lambda_1 + 57\lambda_2 + 60\lambda_3 + 60\lambda_4 + 70\lambda_5 + 52\lambda_6 + 67\lambda_7 + 57\lambda_8 + 67\lambda_9 + 59\lambda_{10} \leq 67\Omega_1$

$49\lambda_1 + 40\lambda_2 + 46\lambda_3 + 39\lambda_4 + 33\lambda_5 + 38\lambda_6 + 46\lambda_7 + 45\lambda_8 + 34\lambda_9 + 37\lambda_{10} \leq 46\Omega_1$

$107\lambda_1 + 172\lambda_2 + 158\lambda_3 + 150\lambda_4 + 148\lambda_5 + 120\lambda_6 + 128\lambda_7 + 166\lambda_8 + 169\lambda_9 + 126\lambda_{10} \leq 128\Omega_1$

$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} \geq 1$

$\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, \lambda_4 \geq 0, \lambda_5 \geq 0, \lambda_6 \geq 0, \lambda_7 \geq 0, \lambda_8 \geq 0, \lambda_9 \geq 0, \lambda_{10} \geq 0$

$Min\ \Omega_2$

$72\beta_1 + 60\beta_2 + 104\beta_3 + 139\beta_4 + 110\beta_5 + 106\beta_6 + 182\beta_7 + 145\beta_8 + 143\beta_9 + 174\beta_{10} \leq 182\Omega_2$

$29\beta_1 + 30\beta_2 + 24\beta_3 + 30\beta_4 + 20\beta_5 + 28\beta_6 + 32\beta_7 + 26\beta_8 + 30\beta_9 + 26\beta_{10} \leq 32\Omega_2$

$109\beta_1 + 170\beta_2 + 161\beta_3 + 155\beta_4 + 142\beta_5 + 142\beta_6 + 175\beta_7 + 172\beta_8 + 174\beta_9 + 146\beta_{10} \leq 175\Omega_2$

$95\beta_1 + 70\beta_2 + 99\beta_3 + 75\beta_4 + 87\beta_5 + 98\beta_6 + 103\beta_7 + 108\beta_8 + 106\beta_9 + 97\beta_{10} \leq 103\Omega_2$

$73\beta_1 + 61\beta_2 + 75\beta_3 + 84\beta_4 + 81\beta_5 + 86\beta_6 + 99\beta_7 + 85\beta_8 + 90\beta_9 + 96\beta_{10} \leq 99\Omega_2$

$39\beta_1 + 37\beta_2 + 41\beta_3 + 42\beta_4 + 34\beta_5 + 29\beta_6 + 41\beta_7 + 30\beta_8 + 40\beta_9 + 40\beta_{10} \leq 41\Omega_2$

$347\beta_1 + 355\beta_2 + 333\beta_3 + 341\beta_4 + 348\beta_5 + 359\beta_6 + 360\beta_7 + 368\beta_8 + 374\beta_9 + 367\beta_{10} \leq 360\Omega_2$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \leq 1$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \geq 1$

$\beta_1 \geq 0, \beta_2 \geq 0, \beta_3 \geq 0, \beta_4 \geq 0, \beta_5 \geq 0, \beta_6 \geq 0, \beta_7 \geq 0, \beta_8 \geq 0, \beta_9 \geq 0, \beta_{10} \geq 0$

$Min\ \Omega_3$

$288\delta_1 + 261\delta_2 + 309\delta_3 + 230\delta_4 + 309\delta_5 + 263\delta_6 + 284\delta_7 + 327\delta_8 + 297\delta_9 + 313\delta_{10} \leq 284\Omega_3$

$45\delta_1 + 38\delta_2 + 40\delta_3 + 42\delta_4 + 48\delta_5 + 52\delta_6 + 52\delta_7 + 54\delta_8 + 52\delta_9 + 63\delta_{10} \leq 52\Omega_3$

$270\delta_1 + 284\delta_2 + 276\delta_3 + 271\delta_4 + 268\delta_5 + 299\delta_6 + 251\delta_7 + 232\delta_8 + 301\delta_9 + 315\delta_{10} \leq 251\Omega_3$

$140\delta_1 + 126\delta_2 + 136\delta_3 + 138\delta_4 + 144\delta_5 + 149\delta_6 + 146\delta_7 + 142\delta_8 + 150\delta_9 + 159\delta_{10} \leq 146\Omega_3$

$127\delta_1 + 110\delta_2 + 138\delta_3 + 134\delta_4 + 138\delta_5 + 132\delta_6 + 149\delta_7 + 140\delta_8 + 141\delta_9 + 140\delta_{10} \leq 149\Omega_3$

$46\delta_1 + 35\delta_2 + 43\delta_3 + 36\delta_4 + 40\delta_5 + 40\delta_6 + 45\delta_7 + 45\delta_8 + 45\delta_9 + 50\delta_{10} \leq 45\Omega_3$

$700\delta_1 + 638\delta_2 + 658\delta_3 + 649\delta_4 + 651\delta_5 + 668\delta_6 + 745\delta_7 + 676\delta_8 + 734\delta_9 + 710\delta_{10} \leq 745\Omega_3$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \leq 1$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \geq 1$

$\delta_1 \geq 0, \delta_2 \geq 0, \delta_3 \geq 0, \delta_4 \geq 0, \delta_5 \geq 0, \delta_6 \geq 0, \delta_7 \geq 0, \delta_8 \geq 0, \delta_9 \geq 0, \delta_{10} \geq 0$

**Observation 8:**

$Min\ \Omega_1$

$78\lambda_1 + 70\lambda_2 + 93\lambda_3 + 90\lambda_4 + 73\lambda_5 + 102\lambda_6 + 104\lambda_7 + 123\lambda_8 + 80\lambda_9 + 81\lambda_{10} \leq 123\Omega_1$

$22\lambda_1 + 26\lambda_2 + 32\lambda_3 + 30\lambda_4 + 28\lambda_5 + 29\lambda_6 + 38\lambda_7 + 38\lambda_8 + 24\lambda_9 + 34\lambda_{10} \leq 38\Omega_1$

$116\lambda_1 + 131\lambda_2 + 143\lambda_3 + 101\lambda_4 + 98\lambda_5 + 146\lambda_6 + 105\lambda_7 + 166\lambda_8 + 169\lambda_9 + 154\lambda_{10} \leq 166\Omega_1$

$79\lambda_1 + 57\lambda_2 + 50\lambda_3 + 48\lambda_4 + 70\lambda_5 + 62\lambda_6 + 69\lambda_7 + 58\lambda_8 + 58\lambda_9 + 48\lambda_{10} \leq 58\Omega_1$

$50\lambda_1 + 57\lambda_2 + 60\lambda_3 + 60\lambda_4 + 70\lambda_5 + 52\lambda_6 + 67\lambda_7 + 57\lambda_8 + 67\lambda_9 + 59\lambda_{10} \leq 57\Omega_1$

$49\lambda_1 + 40\lambda_2 + 46\lambda_3 + 39\lambda_4 + 33\lambda_5 + 38\lambda_6 + 46\lambda_7 + 45\lambda_8 + 34\lambda_9 + 37\lambda_{10} \leq 45\Omega_1$

$107\lambda_1 + 172\lambda_2 + 158\lambda_3 + 150\lambda_4 + 148\lambda_5 + 120\lambda_6 + 128\lambda_7 + 166\lambda_8 + 169\lambda_9 + 126\lambda_{10} \leq 166\Omega_1$

$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} \geq 1$

$\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, \lambda_4 \geq 0, \lambda_5 \geq 0, \lambda_6 \geq 0, \lambda_7 \geq 0, \lambda_8 \geq 0, \lambda_9 \geq 0, \lambda_{10} \geq 0$

$Min\ \Omega_2$

$72\beta_1 + 60\beta_2 + 104\beta_3 + 139\beta_4 + 110\beta_5 + 106\beta_6 + 182\beta_7 + 145\beta_8 + 143\beta_9 + 174\beta_{10} \leq 145\Omega_2$

$29\beta_1 + 30\beta_2 + 24\beta_3 + 30\beta_4 + 20\beta_5 + 28\beta_6 + 32\beta_7 + 26\beta_8 + 30\beta_9 + 26\beta_{10} \leq 26\Omega_2$

$109\beta_1 + 170\beta_2 + 161\beta_3 + 155\beta_4 + 142\beta_5 + 142\beta_6 + 175\beta_7 + 172\beta_8 + 174\beta_9 + 146\beta_{10} \leq 172\Omega_2$

$95\beta_1 + 70\beta_2 + 99\beta_3 + 75\beta_4 + 87\beta_5 + 98\beta_6 + 103\beta_7 + 108\beta_8 + 106\beta_9 + 97\beta_{10} \leq 108\Omega_2$

$73\beta_1 + 61\beta_2 + 75\beta_3 + 84\beta_4 + 81\beta_5 + 86\beta_6 + 99\beta_7 + 85\beta_8 + 90\beta_9 + 96\beta_{10} \leq 85\Omega_2$

$39\beta_1 + 37\beta_2 + 41\beta_3 + 42\beta_4 + 34\beta_5 + 29\beta_6 + 41\beta_7 + 30\beta_8 + 40\beta_9 + 40\beta_{10} \leq 30\Omega_2$

$347\beta_1 + 355\beta_2 + 333\beta_3 + 341\beta_4 + 348\beta_5 + 359\beta_6 + 360\beta_7 + 368\beta_8 + 374\beta_9 + 367\beta_{10} \leq 368\Omega_2$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \leq 1$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \geq 1$

$\beta_1 \geq 0, \beta_2 \geq 0, \beta_3 \geq 0, \beta_4 \geq 0, \beta_5 \geq 0, \beta_6 \geq 0, \beta_7 \geq 0, \beta_8 \geq 0, \beta_9 \geq 0, \beta_{10} \geq 0$

$Min\ \Omega_3$

$288\delta_1 + 261\delta_2 + 309\delta_3 + 230\delta_4 + 309\delta_5 + 263\delta_6 + 284\delta_7 + 327\delta_8 + 297\delta_9 + 313\delta_{10} \leq 327\Omega_3$

$45\delta_1 + 38\delta_2 + 40\delta_3 + 42\delta_4 + 48\delta_5 + 52\delta_6 + 52\delta_7 + 54\delta_8 + 52\delta_9 + 63\delta_{10} \leq 54\Omega_3$

$270\delta_1 + 284\delta_2 + 276\delta_3 + 271\delta_4 + 268\delta_5 + 299\delta_6 + 251\delta_7 + 232\delta_8 + 301\delta_9 + 315\delta_{10} \leq 232\Omega_3$

$140\delta_1 + 126\delta_2 + 136\delta_3 + 138\delta_4 + 144\delta_5 + 149\delta_6 + 146\delta_7 + 142\delta_8 + 150\delta_9 + 159\delta_{10} \leq 142\Omega_3$

$127\delta_1 + 110\delta_2 + 138\delta_3 + 134\delta_4 + 138\delta_5 + 132\delta_6 + 149\delta_7 + 140\delta_8 + 141\delta_9 + 140\delta_{10} \leq 140\Omega_3$

$46\delta_1 + 35\delta_2 + 43\delta_3 + 36\delta_4 + 40\delta_5 + 40\delta_6 + 45\delta_7 + 45\delta_8 + 45\delta_9 + 50\delta_{10} \leq 45\Omega_3$

$700\delta_1 + 638\delta_2 + 658\delta_3 + 649\delta_4 + 651\delta_5 + 668\delta_6 + 745\delta_7 + 676\delta_8 + 734\delta_9 + 710\delta_{10} \leq 676\Omega_3$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \leq 1$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \geq 1$

$\delta_1 \geq 0, \delta_2 \geq 0, \delta_3 \geq 0, \delta_4 \geq 0, \delta_5 \geq 0, \delta_6 \geq 0, \delta_7 \geq 0, \delta_8 \geq 0, \delta_9 \geq 0, \delta_{10} \geq 0$

**Observation 9:**

$Min\ \Omega_1$

$78\lambda_1 + 70\lambda_2 + 93\lambda_3 + 90\lambda_4 + 73\lambda_5 + 102\lambda_6 + 104\lambda_7 + 123\lambda_8 + 80\lambda_9 + 81\lambda_{10} \leq 80\Omega_1$

$22\lambda_1 + 26\lambda_2 + 32\lambda_3 + 30\lambda_4 + 28\lambda_5 + 29\lambda_6 + 38\lambda_7 + 38\lambda_8 + 24\lambda_9 + 34\lambda_{10} \leq 24\Omega_1$

$116\lambda_1 + 131\lambda_2 + 143\lambda_3 + 101\lambda_4 + 98\lambda_5 + 146\lambda_6 + 105\lambda_7 + 166\lambda_8 + 169\lambda_9 + 154\lambda_{10} \leq 169\Omega_1$

$79\lambda_1 + 57\lambda_2 + 50\lambda_3 + 48\lambda_4 + 70\lambda_5 + 62\lambda_6 + 69\lambda_7 + 58\lambda_8 + 58\lambda_9 + 48\lambda_{10} \leq 58\Omega_1$

$50\lambda_1 + 57\lambda_2 + 60\lambda_3 + 60\lambda_4 + 70\lambda_5 + 52\lambda_6 + 67\lambda_7 + 57\lambda_8 + 67\lambda_9 + 59\lambda_{10} \leq 67\Omega_1$

$49\lambda_1 + 40\lambda_2 + 46\lambda_3 + 39\lambda_4 + 33\lambda_5 + 38\lambda_6 + 46\lambda_7 + 45\lambda_8 + 34\lambda_9 + 37\lambda_{10} \leq 34\Omega_1$

$107\lambda_1 + 172\lambda_2 + 158\lambda_3 + 150\lambda_4 + 148\lambda_5 + 120\lambda_6 + 128\lambda_7 + 166\lambda_8 + 169\lambda_9 + 126\lambda_{10} \leq 169\Omega_1$

$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} \geq 1$

$\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, \lambda_4 \geq 0, \lambda_5 \geq 0, \lambda_6 \geq 0, \lambda_7 \geq 0, \lambda_8 \geq 0, \lambda_9 \geq 0, \lambda_{10} \geq 0$

$Min\ \Omega_2$

$72\beta_1 + 60\beta_2 + 104\beta_3 + 139\beta_4 + 110\beta_5 + 106\beta_6 + 182\beta_7 + 145\beta_8 + 143\beta_9 + 174\beta_{10} \leq 143\Omega_2$

$29\beta_1 + 30\beta_2 + 24\beta_3 + 30\beta_4 + 20\beta_5 + 28\beta_6 + 32\beta_7 + 26\beta_8 + 30\beta_9 + 26\beta_{10} \leq 30\Omega_2$

$109\beta_1 + 170\beta_2 + 161\beta_3 + 155\beta_4 + 142\beta_5 + 142\beta_6 + 175\beta_7 + 172\beta_8 + 174\beta_9 + 146\beta_{10} \leq 174\Omega_2$

$95\beta_1 + 70\beta_2 + 99\beta_3 + 75\beta_4 + 87\beta_5 + 98\beta_6 + 103\beta_7 + 108\beta_8 + 106\beta_9 + 97\beta_{10} \leq 106\Omega_2$

$73\beta_1 + 61\beta_2 + 75\beta_3 + 84\beta_4 + 81\beta_5 + 86\beta_6 + 99\beta_7 + 85\beta_8 + 90\beta_9 + 96\beta_{10} \leq 90\Omega_2$

$39\beta_1 + 37\beta_2 + 41\beta_3 + 42\beta_4 + 34\beta_5 + 29\beta_6 + 41\beta_7 + 30\beta_8 + 40\beta_9 + 40\beta_{10} \leq 40\Omega_2$

$347\beta_1 + 355\beta_2 + 333\beta_3 + 341\beta_4 + 348\beta_5 + 359\beta_6 + 360\beta_7 + 368\beta_8 + 374\beta_9 + 367\beta_{10} \leq 374\Omega_2$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \leq 1$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \geq 1$

$\beta_1 \geq 0, \beta_2 \geq 0, \beta_3 \geq 0, \beta_4 \geq 0, \beta_5 \geq 0, \beta_6 \geq 0, \beta_7 \geq 0, \beta_8 \geq 0, \beta_9 \geq 0, \beta_{10} \geq 0$

$Min\ \Omega_3$

$288\delta_1 + 261\delta_2 + 309\delta_3 + 230\delta_4 + 309\delta_5 + 263\delta_6 + 284\delta_7 + 327\delta_8 + 297\delta_9 + 313\delta_{10} \leq 297\Omega_3$

$45\delta_1 + 38\delta_2 + 40\delta_3 + 42\delta_4 + 48\delta_5 + 52\delta_6 + 52\delta_7 + 54\delta_8 + 52\delta_9 + 63\delta_{10} \leq 52\Omega_3$

$270\delta_1 + 284\delta_2 + 276\delta_3 + 271\delta_4 + 268\delta_5 + 299\delta_6 + 251\delta_7 + 232\delta_8 + 301\delta_9 + 315\delta_{10} \leq 301\Omega_3$

$140\delta_1 + 126\delta_2 + 136\delta_3 + 138\delta_4 + 144\delta_5 + 149\delta_6 + 146\delta_7 + 142\delta_8 + 150\delta_9 + 159\delta_{10} \leq 150\Omega_3$

$127\delta_1 + 110\delta_2 + 138\delta_3 + 134\delta_4 + 138\delta_5 + 132\delta_6 + 149\delta_7 + 140\delta_8 + 141\delta_9 + 140\delta_{10} \leq 141\Omega_3$

$46\delta_1 + 35\delta_2 + 43\delta_3 + 36\delta_4 + 40\delta_5 + 40\delta_6 + 45\delta_7 + 45\delta_8 + 45\delta_9 + 50\delta_{10} \leq 45\Omega_3$

$700\delta_1 + 638\delta_2 + 658\delta_3 + 649\delta_4 + 651\delta_5 + 668\delta_6 + 745\delta_7 + 676\delta_8 + 734\delta_9 + 710\delta_{10} \leq 734\Omega_3$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \leq 1$

$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \geq 1$

$\delta_1 \geq 0, \delta_2 \geq 0, \delta_3 \geq 0, \delta_4 \geq 0, \delta_5 \geq 0, \delta_6 \geq 0, \delta_7 \geq 0, \delta_8 \geq 0, \delta_9 \geq 0, \delta_{10} \geq 0$

**Observation 10:**

$Min\ \Omega_1$

$78\lambda_1 + 70\lambda_2 + 93\lambda_3 + 90\lambda_4 + 73\lambda_5 + 102\lambda_6 + 104\lambda_7 + 123\lambda_8 + 80\lambda_9 + 81\lambda_{10} \leq 81\Omega_1$

$22\lambda_1 + 26\lambda_2 + 32\lambda_3 + 30\lambda_4 + 28\lambda_5 + 29\lambda_6 + 38\lambda_7 + 38\lambda_8 + 24\lambda_9 + 34\lambda_{10} \leq 34\Omega_1$

$116\lambda_1 + 131\lambda_2 + 143\lambda_3 + 101\lambda_4 + 98\lambda_5 + 146\lambda_6 + 105\lambda_7 + 166\lambda_8 + 169\lambda_9 + 154\lambda_{10} \leq 154\Omega_1$

$79\lambda_1 + 57\lambda_2 + 50\lambda_3 + 48\lambda_4 + 70\lambda_5 + 62\lambda_6 + 69\lambda_7 + 58\lambda_8 + 58\lambda_9 + 48\lambda_{10} \leq 48\Omega_1$

$50\lambda_1 + 57\lambda_2 + 60\lambda_3 + 60\lambda_4 + 70\lambda_5 + 52\lambda_6 + 67\lambda_7 + 57\lambda_8 + 67\lambda_9 + 59\lambda_{10} \leq 59\Omega_1$

$49\lambda_1 + 40\lambda_2 + 46\lambda_3 + 39\lambda_4 + 33\lambda_5 + 38\lambda_6 + 46\lambda_7 + 45\lambda_8 + 34\lambda_9 + 37\lambda_{10} \leq 37\Omega_1$

$107\lambda_1 + 172\lambda_2 + 158\lambda_3 + 150\lambda_4 + 148\lambda_5 + 120\lambda_6 + 128\lambda_7 + 166\lambda_8 + 169\lambda_9 + 126\lambda_{10} \leq 126\Omega_1$

$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} \geq 1$

$\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0, \lambda_4 \geq 0, \lambda_5 \geq 0, \lambda_6 \geq 0, \lambda_7 \geq 0, \lambda_8 \geq 0, \lambda_9 \geq 0, \lambda_{10} \geq 0$

$Min\ \Omega_2$

$72\beta_1 + 60\beta_2 + 104\beta_3 + 139\beta_4 + 110\beta_5 + 106\beta_6 + 182\beta_7 + 145\beta_8 + 143\beta_9 + 174\beta_{10} \leq 174\Omega_2$

$29\beta_1 + 30\beta_2 + 24\beta_3 + 30\beta_4 + 20\beta_5 + 28\beta_6 + 32\beta_7 + 26\beta_8 + 30\beta_9 + 26\beta_{10} \leq 26\Omega_2$

$109\beta_1 + 170\beta_2 + 161\beta_3 + 155\beta_4 + 142\beta_5 + 142\beta_6 + 175\beta_7 + 172\beta_8 + 174\beta_9 + 146\beta_{10} \leq 146\Omega_2$

$95\beta_1 + 70\beta_2 + 99\beta_3 + 75\beta_4 + 87\beta_5 + 98\beta_6 + 103\beta_7 + 108\beta_8 + 106\beta_9 + 97\beta_{10} \leq 97\Omega_2$

$73\beta_1 + 61\beta_2 + 75\beta_3 + 84\beta_4 + 81\beta_5 + 86\beta_6 + 99\beta_7 + 85\beta_8 + 90\beta_9 + 96\beta_{10} \leq 96\Omega_2$

$39\beta_1 + 37\beta_2 + 41\beta_3 + 42\beta_4 + 34\beta_5 + 29\beta_6 + 41\beta_7 + 30\beta_8 + 40\beta_9 + 40\beta_{10} \leq 40\Omega_2$

$347\beta_1 + 355\beta_2 + 333\beta_3 + 341\beta_4 + 348\beta_5 + 359\beta_6 + 360\beta_7 + 368\beta_8 + 374\beta_9 + 367\beta_{10} \leq 367\Omega_2$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \leq 1$

$\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7 + \beta_8 + \beta_9 + \beta_{10} \geq 1$

$\beta_1 \geq 0, \beta_2 \geq 0, \beta_3 \geq 0, \beta_4 \geq 0, \beta_5 \geq 0, \beta_6 \geq 0, \beta_7 \geq 0, \beta_8 \geq 0, \beta_9 \geq 0, \beta_{10} \geq 0$

$Min\ \Omega_3$

$288\delta_1 + 261\delta_2 + 309\delta_3 + 230\delta_4 + 309\delta_5 + 263\delta_6 + 284\delta_7 + 327\delta_8 + 297\delta_9 + 313\delta_{10} \leq 313\Omega_3$

$$45\delta_1 + 38\delta_2 + 40\delta_3 + 42\delta_4 + 48\delta_5 + 52\delta_6 + 52\delta_7 + 54\delta_8 + 52\delta_9 + 63\delta_{10} \leq 63\Omega_3$$

$$270\delta_1 + 284\delta_2 + 276\delta_3 + 271\delta_4 + 268\delta_5 + 299\delta_6 + 251\delta_7 + 232\delta_8 + 301\delta_9 + 315\delta_{10} \leq 315\Omega_3$$

$$140\delta_1 + 126\delta_2 + 136\delta_3 + 138\delta_4 + 144\delta_5 + 149\delta_6 + 146\delta_7 + 142\delta_8 + 150\delta_9 + 159\delta_{10} \leq 159\Omega_3$$

$$127\delta_1 + 110\delta_2 + 138\delta_3 + 134\delta_4 + 138\delta_5 + 132\delta_6 + 149\delta_7 + 140\delta_8 + 141\delta_9 + 140\delta_{10} \leq 140\Omega_3$$

$$46\delta_1 + 35\delta_2 + 43\delta_3 + 36\delta_4 + 40\delta_5 + 40\delta_6 + 45\delta_7 + 45\delta_8 + 45\delta_9 + 50\delta_{10} \leq 50\Omega_3$$

$$700\delta_1 + 638\delta_2 + 658\delta_3 + 649\delta_4 + 651\delta_5 + 668\delta_6 + 745\delta_7 + 676\delta_8 + 734\delta_9 + 710\delta_{10} \leq 710\Omega_3$$

$$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \leq 1$$

$$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} \geq 1$$

$$\delta_1 \geq 0, \delta_2 \geq 0, \delta_3 \geq 0, \delta_4 \geq 0, \delta_5 \geq 0, \delta_6 \geq 0, \delta_7 \geq 0, \delta_8 \geq 0, \delta_9 \geq 0, \delta_{10} \geq 0$$