Evaluation of Outage Probability in Presence of Interference and Noise with Application to Dual-hop Wireless Systems

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Abstract

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In a wireless network, the main objective of designers is to manage the available resources to maintain good quality of service for all users. As a result, engineers put extra effort in evaluating and improving the performance of wireless systems. Geometric modeling of wireless networks is drawing significant attention with regards to analytically evaluating the performance. In this dissertation, stochastic modeling of users is used to emulate their distribution in the wireless network. The wireless network is analyzed as a single-hop network where there is only one direct link connecting the base station to each user in the cell. Next, probability density function (PDF) of signal to interference ratio (SIR), signal to noise ratio (SNR) and signal to interference plus noise ratio (SINR) are evaluated. Given that outage probability is one of the important factors in studying the performance of a wireless network, outage probability is evaluated based on path loss, SIR, SNR and SINR. Simulation is performed in order to validate the analytical results. To show the application of the work done related to outage probability, a dual-hop wireless system is considered.

Wireless networks face some challenges, such as the presence of users in severe shadowing regions, or obstacles which may diminish the link quality. To overcome these limitations,

dual-hop relaying system is considered. Dual-hop system is used as a cost effective solution in improving the performance of the wireless system. In such a solution, a number of relays is deployed over the cell to help transmission of signals from the base station to mobile stations. In the dual-hop system, a low-quality long-distance link is broken into two better quality links, achieving the required outage probability and enhancing the system performance. The number of deployed relays and their location over the studied area affect the performance of a dual-hop wireless system. Therefore, the required number of relays and their placement over the studied area are investigated, and a desired outage probability is achieved.

Uniform distribution of users cannot describe the real distribution for all cases. Gaussian distribution has malleability, as clustering tendency of users can be controlled in the network using the standard deviation of the distribution. Consequently, Gaussian distribution is considered for users in the studied area to evaluate PDF of SIR and the outage probability.

This dissertation is dedicated

to my parents

and my brother

Acknowledgments

"Our Lord is He who gave everything its creation and then guided it" (Quran 20, 50). Foremost, all praises and thanks are due to Allah for giving me the patience and determination to successfully accomplish my Ph.D. program.

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List of Symbols

d_0	close-in distance
α	free space path loss at the close-in distance
PL	path loss represented by large scale fading
n	path loss exponent which depends on the propagation environment
d	distance between base station and an arbitrary user in the cell
ξ	shadow fading random variable
σ	standard deviation for the shadowing
R	cell radius
σ_G	standard deviation of Gaussian distributed users in a cell
P_m	the transmit power of the main transmission source
L_i	the path loss between the interfering source and the receiver
P_i	the transmit power of the interfering source
L_m	the path loss between the main transmission source and the receiver
l	random variable of path loss between base station and an arbitrary user (in dB)
l_{max}	maximum acceptable level of path loss (in dB)
$f_L(l)$	probability density function for path loss
P_{out}	outage probability
$F_L(l)$	cumulative distribution function for path loss
$Q(x)$	Q function, which is a variation of the error function
ρ	inradius of the hexagonal cell
a	circumradius of the hexagonal cell
L_1	lower boundary of the studied sector

L_2 upper boundary of the studied sector

$F_{SIR}(\gamma)$ cumulative distribution function for signal to interference ratio

γ random variable of signal to interference ratio between base station and any arbitrary user

$f_{SIR}(\gamma)$ probability density function for signal to interference ratio

γ_{min} minimum acceptable level of signal to interference ratio (in dB)

$f_{SNR}(\delta)$ probability density function for signal to noise ratio

δ random variable of signal to noise ratio between base station and any arbitrary user

δ_{min} minimum acceptable level of signal to noise ratio (in dB)

$F_{SNR}(\delta)$ cumulative distribution function for signal to noise ratio

$f_{SINR}(\varphi)$ probability density function for signal to interference plus noise ratio

φ random variable of signal to interference plus noise ratio between base station and any arbitrary user

φ_{min} minimum acceptable level of signal to interference plus noise ratio (in dB)

$F_{SINR}(\varphi)$ cumulative distribution function for signal to interference plus noise ratio

d_m distance between BS and any arbitrary user in presence of an interference source

d_i distance between interfering source and any arbitrary user

R_m radius of the main cell in presence of interference source

R_i radius of the interfering cell

n_m path loss exponent for the main signal

n_i path loss exponent for the interfering signal

ξ_m normal distributed random variable of the main signal with mean $\mu_m = 0\ dB$ and standard deviation σ_m

ξ_i	normal distributed random variable of the interference signal with mean $\mu_i = 0\ dB$ and standard deviation σ_i
μ_H	mean of the ratio of shadowing
σ_H	variance of the ratio of shadowing
G	gain parameter
N	random variable modeling the Gaussian noise with mean equal to zero and standard deviation σ_n
σ_n	standard deviation for the Gaussian noise
$f_D(d)$	probability density function for distance between base station and an arbitrary user
C_n	coefficient of the Gauss quadrature approximation method
t_n	point of the Gauss quadrature approximation method
C	environment-specific constant
d_{BS-RS}	distance between BS and RS
$P_{out,th}$	desired outage probability

List of Acronyms

AF	amplify-and-forward
BS	base station
CCDF	complementary cumulative distribution function
CDF	cumulative distribution function
CDMA	Code Devision Multiple Access
dB	decibels
DF	decode-and-forward
ERF	error function
ERFC	complementary error function
MS	mobile station
PDF	probability density function
QoS	quality of service
RS	relay station
SINR	signal to interference plus noise ratio
SIR	signal to interference ratio
SNR	signal to noise ratio
WiMAX	worldwide interoperability for microwave access
WLAN	wireless local area network

Chapter 1

Introduction

1.1 Motivation

Wireless communication is still growing in terms of number of users. This fact will result in many challenges such as providing and keeping good quality of service. Propagation loss and interference result in unequal distribution of signal, in terms of power over the whole cell [1]. In literature, many solutions are proposed to provide good quality of service over the whole wireless system such as increasing the number of base stations (BSs) with smaller cell size [2, 3]. However, this approach will increase the infrastructure cost. Dual-hop system is considered as a cost effective way to overcome such problem and improve the quality of service of mobile stations (MSs) in a wireless system [3–5]. The performance of a relay-based cellular network is greatly affected by the relay station (RS) location within the cell [5]. Deploying the RS near cell edge will result in a weak BS-RS signal. On the other hand, deploying the RS near the BS will also result in a weak RS-MS signal at the edge of the cell. The number of deployed relays will affect the placement of RS and the performance of the whole system [6]. Therefore, the main limitation in using dual-hop system is finding the required number of relays and their location over the studied area. As a result, this dissertation provides a method to solve this problem and find the required number of relays and their placement in the studied area based on the evaluated outage probability.

In a wireless system, the distance between transmitter and receiver directly affects the quality of service. Thus, there is a strong relation between distribution of wireless terminals and the performance of a wireless system. Based on literature, different methods have been used to evaluate signal to interference ratio [7–9], signal to interference plus noise ratio [10–12] and outage probability [13–16]. However, in these methods the results are not explicitly expressed based on the distribution of users. In this dissertation, uniform distribution of users is considered to evaluate the performance of wireless systems.

In literature, some researchers consider that users are non-uniformly distributed in the studied area, such as:

- When the landscape (i.e. mountains and rivers) and manmade (i.e. buildings) features of an area push users to cluster in specific regions and not in others [17].
- When users tend to be present in parts of the city during the day, and move to other parts during the night. Users moving from commercial to residential area is a clear example of such case [18].

Therefore, Gaussian distribution is applied in this dissertation as an example of a non-uniform distribution of users as well. Then, distribution of users is used to explore the distribution of distances between the BS and each user in the studied area and the performance of wireless system is assessed [19]. Finally, the required number of relays and their location is investigated to reach the desired outage probability.

1.2 Research Objectives and Contributions

In this dissertation, randomness of the location of users with randomness of channel losses are combined together to evaluate the performance of wireless systems. First, the wireless system is emulated based on stochastic modeling. Next, the wireless channel is analyzed to evaluate the distribution of many performance metrics of the studied area. Furthermore, these results are used to analytically evaluate the outage probability. Finally, this approach is applied in a relay-based system to find the required number of relays and their placement to reach the desired outage probability.

Based on the above objectives, the most prominent contributions of this dissertation can be summarized as follows:

- Evaluation of system performance:
  - Evaluation of outage probability based on path loss in a wireless system for uniform and Gaussian distributed users.
  - Evaluation of the probability density function (PDF) of signal to interference ratio (SIR) in a wireless system for uniform and Gaussian distributed users.
  - Evaluation of the PDF of signal to noise ratio (SNR) in a wireless system for uniform distributed users.
  - Evaluation of the PDF of signal to interference plus noise ratio (SINR) in a wireless system for uniform distributed users.

  - Evaluation of outage probability in a wireless system based on the evaluated SIR, SNR and SINR.

- Improving the system performance with relay-based wireless network:
  - Evaluation of the performance of a dual-hop relaying system based on outage probability, taking into consideration the number of the deployed RS and their location.
  - Evaluation of the required number of relays and their location to reach the desired outage probability, which is evaluated based on path loss, SIR, SNR and SINR.

Finally, all the analytical derivations are validated by simulating a wireless system.

1.3 Organization of the Dissertation

The rest of the dissertation is organized as follows:

- In Chapter 2, the literature review is presented by discussing recent works investigated by researchers. Also, background is discussed in order to make the reader aware of some aspects involved in this dissertation.

- In Chapter 3, PDF of SIR, SNR and SINR between a reference and a random point over a wireless network are analytically evaluated in detail. Also, the derived outcomes are verified by simulations.

- In Chapter 4, outage probability is found for a wireless system with uniform distributed users based on path loss, SIR, SNR and SINR. Also, the derived outcomes are simulated for verification.

- In Chapter 5, a dual-hop relaying system is introduced to enhance the performance of a wireless system for different scenarios. Also, an algorithm is developed to find the required number of relays and their locations over the studied area, by enhancing the performance of the system to reach a desired outage probability.

- In Chapter 6, the performance of a wireless system is evaluated with Gaussian distributed users around the BS located at the center of the cell, by assessing the PDF of SIR. Moreover, outage probability is evaluated based on path loss and SIR.

- Finally in Chapter 7, the dissertation is concluded by summarizing the main contributions of this research. Moreover, it is shown in this chapter how this research opens the door for future works in this domain.

Chapter 2

Background and Literature Review

2.1 Introduction

In this chapter, the required background and related literature review are presented. The focus of this dissertation is on evaluating and enhancing the performance of wireless networks. First, the wireless system is modeled by determining the distribution of users over the studied area, and the performance metrics are evaluated in presence of large scale fading. Then, the outage probability is assessed. Finally, a number of relays is deployed to enhance the performance of the studied area.

The rest of this chapter is organized as follows: In Section 2.2, to develop the understanding of the geographic modeling of a wireless network, spatial distribution of users and cell shape are discussed. Then, in Section 2.3, propagation modeling is presented, and different sources that affect the quality of signal from transmitter to receiver are explored.

Also, performance metrics that characterize the quality of a wireless system are presented. In Section 2.4, to expand our insight into dual-hop relaying systems, recent works related to this subject are summarized. Finally, Section 2.5 concludes the chapter.

2.2 Modeling of a Wireless Network

The main objective of this dissertation is to geometrically model wireless networks to explore their characteristics. Spatial distribution of users and cell shape should be taken into consideration to model the geometry of wireless networks.

2.2.1 Spatial Distribution of Users

Different methods are proposed to tackle the spatial distribution of users and emulate the geometry of the studied system in order to analyze the parameters of the wireless system more accurately. Deterministic deployment of nodes is a straightforward approach for the analytical modeling of wireless networks [20,21]. However, this approach is more suitable for fixed sensor networks and it is unfeasible for MSs. Analysis of the human behavior is another approach to predict the distribution of users in a wireless system [22, 23]. Nonetheless, the results found using this method are limited to the studied area. Stochastic modeling of wireless systems, such as planar distribution [24–32] and point process technique [33–36], is impartial and it ensures generalization of the results. In stochastic modeling, selection of a distribution that can best describe users in the studied area is one of the main challenges. In literature, researchers normally use uniform distribution of users [25, 37, 38]. However, uniform distribution of users cannot reflect all real life scenarios. For that reason, other researchers adopt Gaussian distribution for some real life cases, such as:

- Deploying of sensor nodes by dropping them from an airplane according to the central limit theorem [39].

- In rural areas, users tend to be concentrated in the center, with gradually decreasing intensity away from the center [40].

Once the distribution that can best describe how users are distributed in the studied area is decided, the PDF of the distance between the centralized BS and any arbitrary user can be evaluated. In this dissertation, it is assumed that users are either uniform or Gaussian distributed around the BS. PDF of the distances between BS and any user where uniform distribution is used in a circular cell of radius R_m is represented as follows [25]:

$$f_D(d) = \frac{2d}{R_m^2} \quad d \in \ [0, R_m] \tag{2.1}$$

where d is the distance between the BS located at the center of the cell and any arbitrary user.

On the other hand, PDF of distances where users are Gaussian distributed in a circular cell is [41]:

$$f_D(d) = \frac{d.\exp\{(R_m^2 + d_0^2 - 2d^2)/4\sigma_G^2\}}{2\sigma_G^2.\sinh\{(R_m^2 - d_0^2)/4\sigma_G^2\}} \tag{2.2}$$

- d_o is the close-in distance

- R_m is the cell radius

- σ_G is the standard deviation of the Gaussian distributed users

These results are used in the next chapter to evaluate performance metrics based on the most likely distribution of users over the studied area.

2.2.2 Cell Shape

In reality, cells in a wireless communication system have irregular shapes and mainly depend on the geographic characteristics of the studied area. For simplicity in studying wireless networks, researchers tend to consider cells as circular or hexagonal shapes [42]. Circular cells are preferred because of their simplicity. However, circular cells cause overlapping or coverage holes. As a result, complication is added to the theoretical analysis of wireless systems. To overcome this problem, designers use cells with non-intersecting structure. Figure 2.1 shows circular cell shapes that overlap, and different geometric shapes with a tessellating feature where all the studied area is covered without overlaps or gaps [42–45].

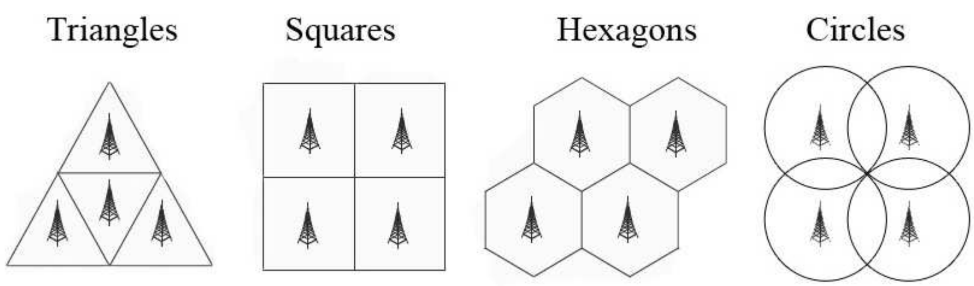

Figure 2.1: Circular cells versus cells with tessellating structures

It is shown in [41], that hexagonal cells are preferred because they have the best possible tessellating option by a comfortable margin compared to square and triangular shapes. Also, in Appendix-A.1, outage probability based on path loss is evaluated for circular and hexagonal cells to show that the difference between them is negligible.

2.3 Evaluation of System Performance

In a wireless system, the received signal is diminished due to many reasons, such as: propagation loss, shadowing, interference and additive white Gaussian noise. The strength of the signal decreases when the signal is emitted from transmitter to receiver [1]. In Appendix-A.2, it is shown how outage probability increases with distance.

Propagation loss is a function of the distance between the transmitter and the receiver. Shadowing is caused by the obstructing sources between the transmitter and the receiver, such as buildings and hills. In wireless systems, shadowing is modeled as a zero-mean log-normal distributed random variable. The path loss (in dB) is:

$$PL = \alpha + \beta \log(X) + \xi \tag{2.3}$$

- α is the free space path loss at the close-in distance d_0
- $\beta = 10n$
- n is the path loss exponent which depends on the propagation environment
- $X = d/d_0$
- d is the distance between BS and any arbitrary user in the cell
- ξ is the shadow fading which is a zero-mean log-normal distributed random variable with standard deviation σ

2.3.1 Performance Metrics

Different performance metrics are used to evaluate the performance of a wireless system. SNR is one of these metrics that can be used in a wireless system where interference is eliminated. On the other hand, in presence of interference, SIR or SINR can be used to characterize the performance of a wireless system. These performance metrics are not only considered as metrics to evaluate the performance of a wireless network, but are also directly involved in computing different essential metrics such as outage probability, coverage, capacity and throughput [33]. Thus, it becomes obligatory to evaluate generic and close form PDFs of these performance metrics taking into consideration the geographic modeling of the studied area.

SIR is the ratio of power of the wanted signal to the total power of the unwanted signals, mainly caused by the interference. For a single interfering source, SIR is expressed as follows:

$$SIR = \frac{P_m \times L_i}{P_i \times L_m} \tag{2.4}$$

- P_m is the transmit power of the main transmission source
- L_i is the path loss between the interfering source and the receiver
- P_i is the transmit power of the interfering source
- L_m is the path loss between the main transmission source and the receiver

In literature, there are many attempts to evaluate SIR. The PDF of the SIR is evaluated in $[8, 46]$ based on propagation loss only, without the use of shadowing. Also, in [7] and [9] PDF of SIR is evaluated by eliminating the large scale fading and just considering the presence of small scale fading. In [47], SINR is evaluated in a cooperative system. However, interference was assumed to impact just relays with negligible interference at the destination. In [10, 11], an upper bound of SINR is evaluated in order to assess the performance of a code division multiple access (CDMA) system.

2.3.2 Outage Probability

Outage probability is the probability of failing to reach satisfactory reception of a signal of interest [16]. Outage probability is considered as a statistical parameter that can reflect the quality of service in a mobile radio system [15]. In literature, outage probability is evaluated using various methods, where each is based on a parameter such as SIR, SINR, etc. In [46], outage probability is found without considering the effect of shadowing. Outage probability is evaluated in [15], by finding the probability that the power of all interfering signals plus noise is greater than the power of the designated user signal. Also, outage probability in [13] has been calculated as the probability of the received SNR failing to reach sufficient signal to noise ratio threshold. In [14], outage probability is calculated based on the same method explained earlier but with using SIR instead of SNR. It is noticed that in most of the referenced methods, the performance is not evaluated based on the geographic modeling of the studied system. However, in our approach, spatial distribution of users and cell shape are taken into consideration in order to model the wireless system and to end up with generic and closed form results. In this dissertation, outage probability is evaluated based on different metrics such as path loss, SIR, SNR and SINR.

2.4 Dual-hop Relaying System

Cellular systems are facing difficulties in providing good quality of service for all users over the cell. It has been shown in [1] that outage probability varies in a circular cell with uniformly distributed users based on the distance between the BS and each user. Result in [1] shows how bad quality of the signal is for users far away from the BS. Thereore, researchers put effort to solve this problem and found many solutions. A well known solution is to use smaller cells and deploy more BSs as in [2, 48]. However, this kind of solution will increase the infrastructure cost of the system. Dual-hop relaying system has less complexity and lower installation cost compared to increasing the number of BSs, to enhance the quality of a wireless system [3–5, 49, 50]. In a dual-hop relaying system, low quality long distance links are broken into two or more high quality links. In such systems, a number of relays is deployed over the area, which play the role of intermediary nodes between the BS and far users, specially near the edge of the cell. The performance of a dual-hop relaying system is greatly affected by the relay location within the cell [3, 5]. In other words, bad placement of relays may lead to decrease in its expected benefits. Deploying relays in a wireless system has many objectives, such as:

- Cell capacity improvement [50]
- Cell coverage extension [3–5]
- Quality of service enhancement [51]

For each of these objectives the location of relays over the cell differ [3, 5]. dual-hop relaying system is used in several papers in literature. In [52], the optimal relay placement to enhance the capacity of a wireless local area network (WLAN) system is found. Also in [6], relays are placed on a circle around the BS to increase the system capacity. Authors in [53] investigate the placement of relays in WiMAX networks. Although useful, only one BS system is considered in these papers. In [54], both user fairness and capacity enhancement is tackled by just considering noise and ignoring interference. In [55] and [56], authors try to minimize the installation cost by minimizing the number of BSs and deploying more relays while maintaining the same system performance. However, authors ignore any interference source in their algorithms. In [3, 5], the optimum placement of relays are found to extend the coverage area. Optimal relay placement regarding the bit rate is found in [4]. The aim of authors in [50] is to locate relays in order to maximize the capacity in a broadband wireless access network. In [51], the optimal relay placement problem is investigated, by minimizing the outage probability which is evaluated based on SNR.

In IEEE 802.16j standard, two modes are described for relay operation: transparent mode and non-transparent mode. The main difference between these two modes of operation is how framing information is transmitted [57].

- Transparent mode: RS does not forward framing information and therefore does not increase the coverage area. The main use of this mode is to enhance the performance of a wireless system within the BS coverage area. This type of relay has lower complexity.

- Non-transparent mode: RS framing information is forwarded by the relay. Hence, forwarding the framing information will result in interference between neighboring RSs. The main use of this mode is to extend the coverage area of a wireless system. This mode of relay is more complex than transparent relay.

Relay modes are classified into two categories as follows:

- Amplify-and-Forward relay (AF) is considered as a transparent mode. It is one of the simplest and most popular relaying methods. The signal received by the relay is amplified, frequency translated and retransmitted [57].

- Decode-and-Forward relay (DF) is considered as non-transparent mode. It has the counter protocol to the transparent AF relay. DF relay detects the signal, decodes it and re-encodes it prior to retransmission [57].

Performance enhancement of dual-hop relaying system is a major contribution of this dissertation, which has not been tackled by other researchers in literature to the best of the author's knowledge. Non-transparent DF relays are deployed over the studied are to enhance the system performance. Moreover, the required number of relays and their placement are found in order to reach a desired outage probability.

2.5 Conclusion

In this chapter, the background and literature review of the dissertation were presented. In most reviewed literature, it was found that the evaluation of wireless system performance was not explicitly related to the distribution of users. Therefore, in this dissertation, performance metrics were evaluated in a closed form, by considering the spatial distribution of users and cell shape. Then, a dual-hop relaying system was introduced. One of the objectives of using a dual-hop system was to enhance the outage probability of the studied area. This objective does not seem to be tackled in literature. In the up coming chapters, the performance of a wireless system taking into consideration the geometric modeling of the studied area was derived.

Chapter 3

PDF Evaluation of Performance Metrics for Uniformly Distributed Users

3.1 Introduction

Wireless networks can be viewed as single-hop and multi-hop networks [58]. In single-hop networks, communication is executed through one hop, therefore we have a direct link between BS and MS. Large scale fading, characterized by propagation loss and shadowing, diminishes the link quality from source to destination. In this chapter, the performance of a single-hop wireless system is analytically evaluated, taking into consideration the geographic modeling of the studied area. The results found in this chapter will be used to find the outage probability in a wireless system.

The rest of this chapter is organized as follows: In Section 3.2, the PDF of SIR is evaluated for uniform distributed users in a circular cell around a BS located at the center of the cell. SIR is found in presence of large scale fading characterized by propagation loss and shadowing. Then, numerical and simulation results are performed to validate the results. In Section 3.3, the same approach is used to assess the PDF of the SNR. Numerical and simulation results are compared to validate the results here as well. Afterwards, in Section 2.4, results of Sections 3.2 and 3.3 are used to analytically find the PDF of the SINR, and the results are confirmed by simulation. Finally, Section 3.5 will close the chapter, and a possible direction for using the results is mentioned.

3.2 PDF Evaluation of Signal to Interference Ratio

In this section, PDF of SIR is analytically evaluated. SIR is denoted as the ratio of the wanted signal to the unwanted signals. SIR is a critical parameter that controls capacity and coverage in a wireless system. SIR is evaluated based on the effects of large scale fading described by propagation loss and shadowing which is a random phenomenon. The studied area is considered as a circular cell of radius R_m. Users are uniformly distributed around the BS located at the center of the cell. It is considered that each user has a region of interference. A MS experiences interference only from a source located inside its region of interference of radius R_i, where $R_i > R_m$, as shown in Figure 3.1. Interfering sources are located uniformly within the region of interference of the MS.

It is assumed that there is only one interference signal. It is considered that the transmit powers of the main and interfering sources are equal; therefore, SIR is evaluated as follows

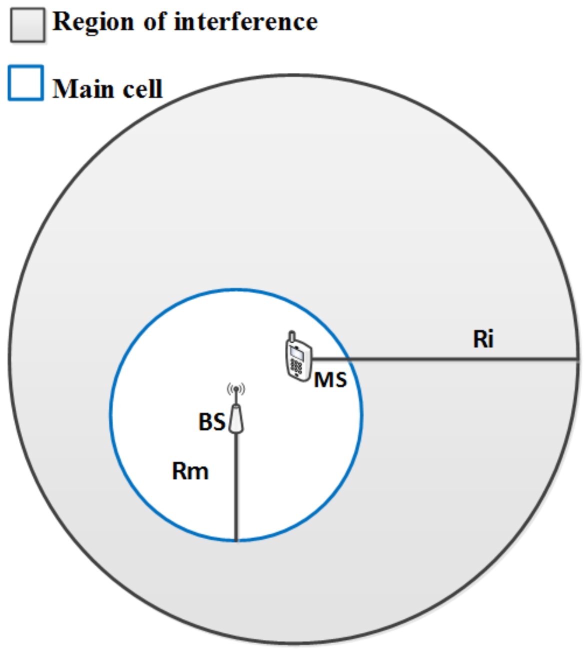

Figure 3.1: Main cell and region of interference for an arbitrary user MS

[59, 60]:

$$SIR = C \times \frac{d_i^{n_i}}{d_m^{n_m}} \times \frac{\exp(\beta \xi_i)}{\exp(\beta \xi_m)} \tag{3.1}$$

- C is an environment specific constant
- d_m is the distance between the uniformly distributed users and the main BS
- d_i is the distance between the uniformly distributed users and the interfering source
- n_m and n_i are path loss exponents for the main and the interference signals, respectively
- $\beta = \ln 10/10$, which is a constant
- ξ_m is a normal distributed random variable of the main signal with mean $\mu_m = 0\ dB$ and standard deviation σ_m

- ξ_i is a normal distributed random variable of the interference signal with mean $\mu_i = 0$ dB and standard deviation σ_i

- $\exp(\beta\xi_m)$ is a log-normal distributed random variable modeling the shadowing of the main signal with mean $\mu_m = 0$ dB and standard deviation σ_m

- $\exp(\beta\xi_i)$ is a log-normal distributed random variable modeling the shadowing of the interference signal with mean $\mu_i = 0$ dB and standard deviation σ_i

To evaluate PDF of SIR, PDFs of the ratio of propagation loss $D = d_i^{n_i}/d_m^{n_m}$ and shadowing $H = \exp(\beta\xi_i)/\exp(\beta\xi_m)$ are found. Then, these two random variables are multiplied together to evaluate PDF of SIR.

First of all, PDF of propagation loss $D = d_i^{n_i}/d_m^{n_m}$ is evaluated. PDFs of d_m and d_i are as shown in (3.2) and (3.3) respectively [61]:

$$f_{D_m}(d_m) = \frac{2d_m}{R_m^2}, \quad 0 < d_m < R_m \tag{3.2}$$

$$f_{D_i}(d_i) = \frac{2d_i}{R_i^2}, \quad 0 < d_i < R_i \tag{3.3}$$

Based on the equation above, PDFs of $X = d_m^{n_m}$ and $Y = d_i^{n_i}$ are directly evaluated as follows:

$$f_X(x) = \frac{2x^{\frac{2}{n_m}-1}}{R_m^2 n_m}, \quad 0 < x < R_m^{n_m} \tag{3.4}$$

$$f_Y(y) = \frac{2y^{\frac{2}{n_i}-1}}{R_i^2 n_i}, \quad 0 < y < R_i^{n_i} \tag{3.5}$$

Knowing that $D = Y/X$, where X and Y are two independent random variables, from [62], PDF of D, $f_D(d)$ becomes:

$$f_D(d) = \int f_X(x) f_Y(zx)|x|dx \tag{3.6}$$

The PDF of D is as follows:

$$f_D(d) = \begin{cases} Ad^{\frac{2}{n_i}-1}, & 0 < d < \mu \\ Bd^{\frac{-2}{n_m}-1}, & \mu < d < \infty \end{cases} \tag{3.7}$$

where $A = \frac{2R_m^{\frac{2n_m}{n_i}}}{R_i^2(n_m+n_i)}$, $B = \frac{2R_i^{\frac{2n_i}{n_m}}}{R_m^2(n_m+n_i)}$, $\mu = \frac{R_i^{n_i}}{R_m^{n_m}}$.

Second, PDF of the ratio of shadowing $H = \exp(\beta\xi_i)/\exp(\beta\xi_m)$ is evaluated. In [62], it is given that the ratio of two independent log-normal random variables is a log-normal variable with mean $\mu_H = \mu_i - \mu_m$ and variance $\sigma_H^2 = \sigma_m^2 + \sigma_i^2$. Therefore, the PDF of $H = \frac{\exp(\beta\xi_i)}{\exp(\beta\xi_m)}$ has a log-normal distribution:

$$f_H(h) = \frac{1}{h\sigma\sqrt{2\pi}} \exp\left(\frac{-(\ln h)^2}{2\sigma_H^2}\right), \quad 0 < h < \infty \tag{3.8}$$

PDF of SIR is evaluated by multiplying the PDF of two independent random variables

D and H. From [62], the PDF of SIR, $f_{SIR}(\gamma)$, is evaluated as follows:

$$f_{SIR}(\gamma) = \int f_D(\frac{\gamma}{h})f_H(h)\frac{1}{h}dh \tag{3.9}$$

where γ is the signal to interference ratio.

Using [63], the integration of (3.9) is:

$$f_{SIR}(\gamma) = \begin{cases} \frac{A\sqrt{C}}{2\sigma_H\sqrt{2}} \exp\left(\frac{C(W+1)^2}{4}\right)\left[1 - erf\left(\frac{CW+C-2\ln(\mu)}{2\sqrt{C}}\right)\right] \times \gamma^{\frac{2}{n_i}-1}, & 0 < \gamma < \mu \\ \frac{B\sqrt{D}}{2\sigma_H\sqrt{2}} \exp\left(\frac{D(W+1)^2}{4}\right)\left[1 + erf\left(\frac{DW+C-2\ln(\mu)}{2\sqrt{D}}\right)\right] \times \gamma^{\frac{-2}{n_m}-1}, & \mu < \gamma < \infty \end{cases} \tag{3.10}$$

- $C = -1 - \frac{2}{n_i}$
- $W = 2\sigma_H^2$
- $D = \frac{2}{n_m} - 1$

In (3.10), a solution is given for PDF of SIR where users are uniform distributed around a BS located at the center of a circular cell.

3.2.1 Numerical and Simulation Results

PDF of SIR derived in (3.10) is numerically evaluated based on the parameters shown in Table 3.1. A wireless system is simulated 100,000 times each for a user located randomly in a circular cell of radius $R_m = 1000m$. In the simulation, each 1 dB is divided into 10 divisions, and the total number of times that it lies in each divisions is counted and averaged to validate the numerical results. High number of simulations is run to get accurate results. Each user is affected by one interfering source located uniformly inside its region of interference, R_i. In Figure 3.2, it is shown that the simulation results closely match the numerical results derived in this chapter.

Table 3.1: System parameters of SIR

Parameters	Values
R_m	1000 m
R_i	2000 m
σ_m	10 dB
σ_i	10 dB
n_m	2
n_i	3

3.3 PDF Evaluation of Signal to Noise Ratio

In this section, PDF of SNR in presence of propagation loss and shadowing is evaluated for uniform distributed users in a circular cell of radius R_m. The SNR is evaluated as follows:

$$SNR = S/N = \frac{P_m \times G}{N} \tag{3.11}$$

- P_m is the transmit power of the main transmission source
- G is the gain parameter
- N is a random variable modeling the Gaussian noise with zero-mean and standard deviation σ_n

The gain parameter G is expressed as:

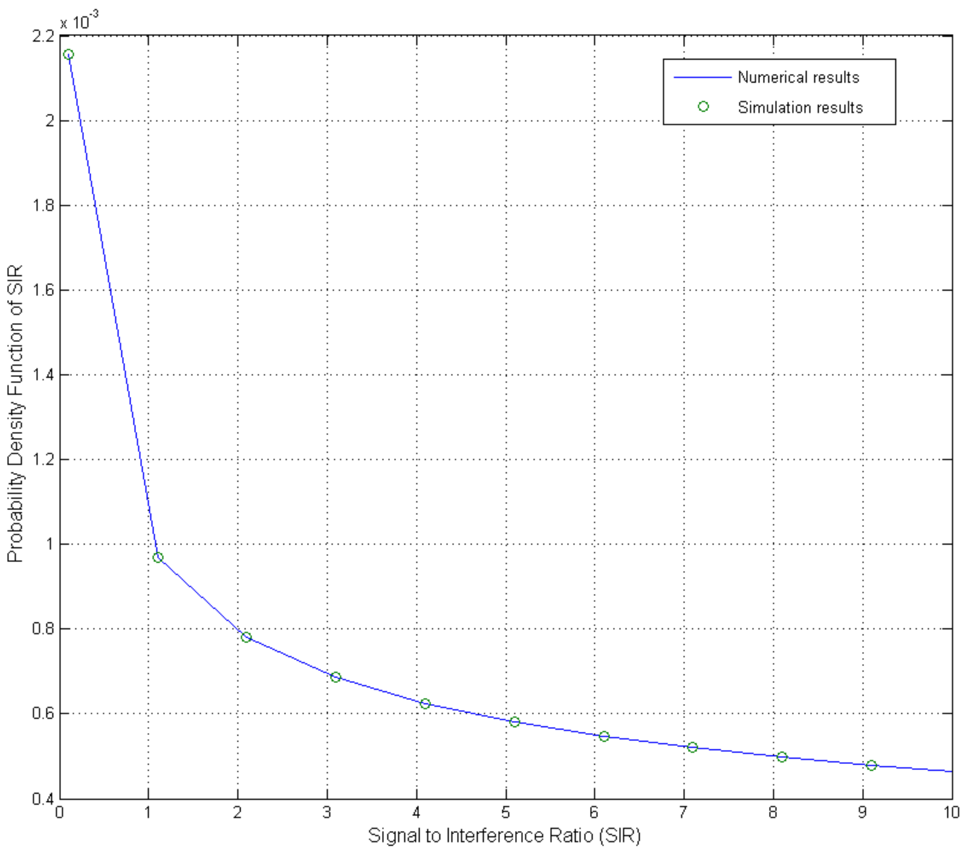

Figure 3.2: PDF of SIR in a circular cell

$$G = \frac{1}{L} \tag{3.12}$$

where L is the path loss between transmitter and receiver.

Like the gain parameter G, the loss parameter L combines effects of propagation loss and shadowing.

$$L = C \times d_m^{n_m} \times \exp(\beta \xi_m) \tag{3.13}$$

- C is an environment-specific constant
- d_m is the distance between the BS and any arbitrary user in the cell
- n_m is the path loss exponent which depends on the propagation environment
- $\beta = \ln 10/10$
- ξ_m is normal distributed random variable representing the main signal with zero-mean and standard deviation σ
- $\exp(\beta\xi_m)$ is a log-normal distributed random variable modeling the shadowing of the main signal with mean $\mu = 0\ dB$ and standard deviation σ

As the result, SNR is evaluated as follows:

$$SNR = \frac{P_m}{N \times d_m^{n_m} \times \exp(\beta \xi_m)} \tag{3.14}$$

To find the distribution of SNR, PDF of $Z = 1/SNR$, $f_Z(z)$, is evaluated. To analytically derive $f_Z(z)$, PDF of the individual components and their products are determined. First, PDF of propagation loss:

$$f_D(d_m) = \frac{2d_m}{R_m^2}, \quad 0 < d < R_m \tag{3.15}$$

Based on the equation above, PDF of $X = d_m^{n_m}$:

$$f_X(x) = \frac{2x^{\frac{2}{n_m}-1}}{R_m^2 n_m}, \quad 0 < x < R_m^{n_m} \tag{3.16}$$

Shadowing is a zero-mean log-normal distributed random variable. Therefore, PDF of shadowing is expressed as follows:

$$f_Y(y) = \frac{1}{y\sigma\sqrt{2\pi}} \exp\left(\frac{-(\ln y)^2}{2\sigma^2}\right), \quad 0 < y < \infty \tag{3.17}$$

with zero-mean and standard deviation σ.

From [62], the multiplication of two independent random variables X and Y is:

$$f_L(l) = \int f_X(\frac{l}{y}) f_Y(y) \frac{1}{y} dy \tag{3.18}$$

Using (3.16), (3.17) and (3.18):

$$f_L(l) = \begin{cases} \frac{\left[1-erf\left(\frac{ab+a-2\ln(R_m^{n_m})}{2\sqrt{a}}\right)\right]}{R_m^2 n_m} \times l^{\frac{2}{n_m}-1}, & 0 < l < R_m^{n_m} \\ \frac{\left[1+erf\left(\frac{ab+a-2\ln(R_m^{n_m})}{2\sqrt{a}}\right)\right]}{R_m^{-2} n_m} \times l^{\frac{-2}{n_m}-1}, & R_m^{n_m} < l < \infty \end{cases} \tag{3.19}$$

- $a = 2\sigma^2$
- $b = -1 - \frac{2}{n_m}$

PDF of the Gaussian noise, N, $f_W(w)$ is:

$$f_W(w) = \frac{1}{\sigma_n \sqrt{2\pi}} exp(-\frac{w^2}{2\sigma_n^2}) \tag{3.20}$$

Consequently, $f_Z(z)$ is evaluated by multiplying two independent random variables found in (3.18) and (3.20), which results in:

$$f_Z(z) = \int f_W(w) f_L(\frac{z}{w}) \frac{1}{w} dw \tag{3.21}$$

As in [62], $SNR = 1/Z$ becomes:

$$f_{SNR}(\delta) = \frac{1}{\delta^2} \times f_Z(1/\delta) \tag{3.22}$$

As a result, PDF of SNR for uniform distributed users around a centralized BS is as follows:

$$f_{SNR}(\delta) = \begin{cases} \frac{\sqrt{d\pi}\left[1 - erf\left(\frac{ab + a - 2\ln(R_m^{n_m})}{2\sqrt{a}}\right)\right]}{R_m^2 n_m \sigma_n \sqrt{2}} \times \delta^{\frac{2}{n_m} - 1}, & 0 < \delta < R_m^{n_m} \\ \frac{\sqrt{d\pi}\left[1 + erf\left(\frac{ab + a - 2\ln(R_m^{n_m})}{2\sqrt{a}}\right)\right]}{R_m^{-2} n_m \sigma_n \sqrt{2}} \times \delta^{\frac{-2}{n_m} - 1}, & R_m^{n_m} < \delta < \infty \end{cases} \tag{3.23}$$

3.3.1 Numerical and Simulation Results

PDF of SNR, derived in (3.23), is numerically evaluated based on the parameters shown in Table 3.2. Also, a wireless system is simulated 100,000 times each for a user located randomly in a circular cell of radius $R_m = 1000m$ to validate the numerical results. In Figure 3.3, it is shown that the simulation results closely match numerical results derived in this chapter.

Table 3.2: System parameters of SNR

Parameters	Values
R_m	1000 m
σ	10 dB
σ_n	10 dB
n_m	2

3.4 PDF Evaluation of Signal to Interference Plus Noise Ratio

In this section, PDF of SINR is analytically evaluated for uniformly distributed users in a circular cell of radius R_m. Noise is considered as additive white Gaussian (AWGN) with mean equal to zero and standard deviation σ_n. SINR is evaluated in presence of propagation loss, as well as shadowing with zero-mean and standard deviation σ:

$$SINR = \frac{S}{I + N} \tag{3.24}$$

First, the PDF of $1/SINR$ is:

$$\frac{1}{SINR} = \frac{I}{S} + \frac{N}{S} \tag{3.25}$$

The PDF of $\frac{I}{S} = \frac{1}{SIR}$, which is the first term of (3.25):

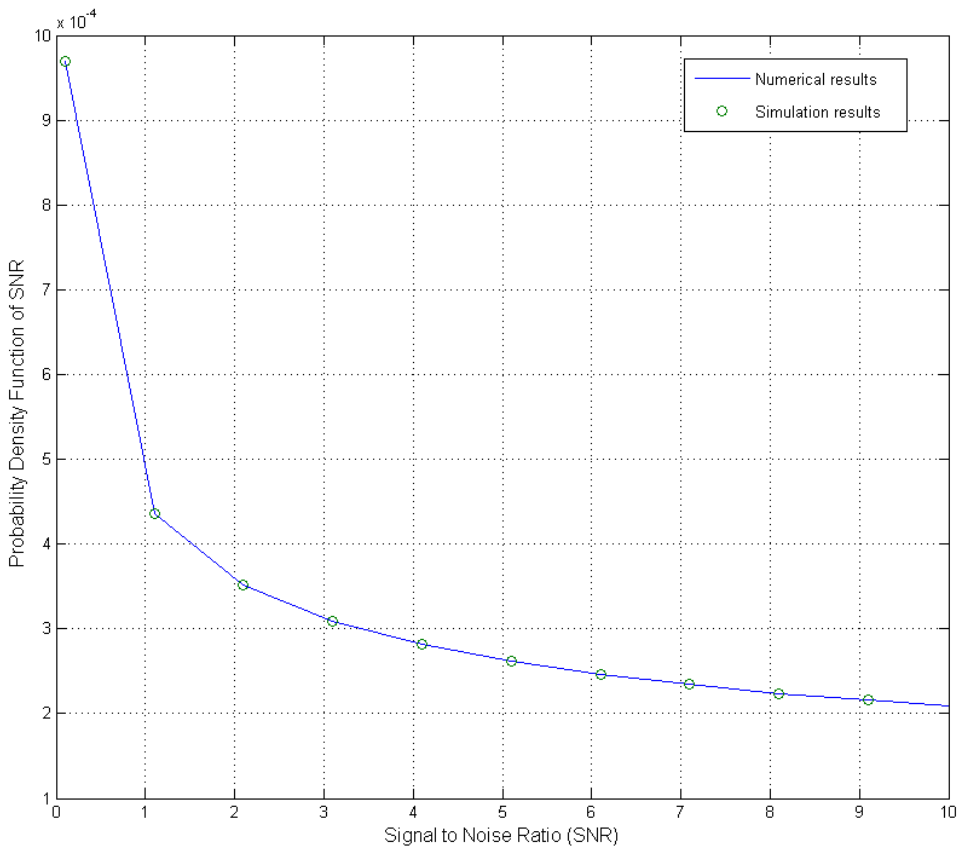

Figure 3.3: PDF of SNR in a circular cell

$$f_{invSIR}(\tau) = \frac{1}{\tau^2} \times f_{SIR}(1/\tau) \tag{3.26}$$

As the result, the PDF of $1/SIR$ is as follows:

$$f_{invSIR}(\tau) = \begin{cases} \frac{B\sqrt{D}}{2\sigma_H\sqrt{2}} \exp\left(\frac{D(W+1)^2}{4}\right)\left[1 + erf\left(\frac{DW+C-2\ln(\mu)}{2\sqrt{D}}\right)\right] \times \tau^{\frac{2}{n_m}-1}, & 0 < \tau < 1/\mu \\ \frac{A\sqrt{C}}{2\sigma_H\sqrt{2}} \exp\left(\frac{C(W+1)^2}{4}\right)\left[1 - erf\left(\frac{CW+C-2\ln(\mu)}{2\sqrt{C}}\right)\right] \times \tau^{\frac{-2}{n_i}-1}, & 1/\mu < \tau < \infty \end{cases} \tag{3.27}$$

Next, the PDF of $\frac{N}{S} = \frac{1}{SNR}$, which is the second term of (3.25) is:

$$f_Z(z) = \begin{cases} \frac{\sqrt{d\pi}\left[1+erf\left(\frac{ab+a-2\ln(R_m^{n_m})}{2\sqrt{a}}\right)\right]}{R_m^{-2} n_m \sigma_n \sqrt{2}} \times z^{\frac{2}{n_m}-1}, & 0 < z < 1/R_m^{n_m} \\ \frac{\sqrt{d\pi}\left[1-erf\left(\frac{ab+a-2\ln(R_m^{n_m})}{2\sqrt{a}}\right)\right]}{R_m^{2} n_m \sigma_n \sqrt{2}} \times z^{\frac{-2}{n_m}-1}, & 1/R_m^{n_m} < z < \infty \end{cases} \tag{3.28}$$

Therefore, to evaluate PDF of SINR, two PDFs in (3.27) and (3.28) should be convolved. The convolution of two functions is the product of their Laplace transform. Thus, the Laplace transforms of these two PDFs are found and multiplied by each other. Later, this result is used to find the convolution of the two functions.

Laplace transform for (3.27) is:

$$f(s) = \begin{cases} M_1\Gamma(-N_1)s^{-N_1-1} \\ M_2\Gamma(-N_2)s^{-N_2-1} \end{cases} \tag{3.29}$$

- $M_1 = \frac{B\sqrt{D}}{2\sigma_H\sqrt{2}} \exp\left(\frac{D(W+1)^2}{4}\right)\left[1 + erf\left(\frac{DW+C-2\ln(\mu)}{2\sqrt{D}}\right)\right]$
- $N_1 = \frac{2}{n_m} - 1$

- $M_2 = \frac{A\sqrt{C}}{2\sigma_H\sqrt{2}} \exp\left(\frac{C(W+1)^2}{4}\right)\left[1 - erf\left(\frac{CW+C-2\ln(\mu)}{2\sqrt{C}}\right)\right]$

- $N_2 = \frac{-2}{n_i} - 1$

- $\Gamma(.)$ is Gamma function

Laplace transform for (3.28) becomes:

$$f(s) = \begin{cases} Q_1\Gamma(-W_1)s^{-W_1-1} \\ Q_2\Gamma(-W_2)s^{-W_2-1} \end{cases} \tag{3.30}$$

- $Q_1 = \frac{\sqrt{d\pi}\left[1+erf\left(\frac{ab+a-2\ln(R_m^{n_m})}{2\sqrt{a}}\right)\right]}{R_m^{-2}n_m\sigma_n\sqrt{2}}$

- $W_1 = \frac{2}{n_m} - 1$

- $Q_2 = \frac{\sqrt{d\pi}\left[1-erf\left(\frac{ab+a-2\ln(R_m^{n_m})}{2\sqrt{a}}\right)\right]}{R_m^{2}n_m\sigma_n\sqrt{2}}$

- $W_2 = \frac{-2}{n_m} - 1$

As the result, the Laplace transform for the addition of these two PDFs is:

$$f(s) = \begin{cases} M_1Q_1\Gamma(-W_1)\Gamma(-N_1)s^{-W_1-N_1-2} \\ M_1Q_2\Gamma(-W_2)\Gamma(-N_1)s^{-W_2-N_1-2} \\ M_2Q_2\Gamma(-W_2)\Gamma(-N_2)s^{-W_2-N_2-2} \end{cases} \tag{3.31}$$

The inverse Laplace transform of this function is:

$$f_T(t) = \begin{cases} \frac{M_1Q_1\Gamma(-W_1)\Gamma(-N_1)\times t^{W_1+N_1+1}}{\Gamma(W_1+N_1+1)} \\ \frac{M_1Q_2\Gamma(-W_2)\Gamma(-N_1)\times t^{W_2+N_1+1}}{\Gamma(W_2+N_1+1)} \\ \frac{M_2Q_2\Gamma(-W_2)\Gamma(-N_2)\times t^{W_2+N_2+1}}{\Gamma(W_2+N_2+1)} \end{cases} \tag{3.32}$$

$f_T(t)$ in (3.32) is the PDF of $1/SINR$. Therefore, PDF of SINR is as follows:

$$f_{SINR}(\varphi) = \begin{cases} \frac{M_2 Q_2 \Gamma(-W_2)\Gamma(-N_2)\times\varphi^{-W_2-N_2-3}}{\Gamma(W_2+N_2+1)}, & 0 < \varphi < \mu \\ \frac{M_1 Q_2 \Gamma(-W_2)\Gamma(-N_1)\times\varphi^{-W_2-N_1-3}}{\Gamma(W_2+N_1+1)}, & \mu < \varphi < R_m^{n_m} \\ \frac{M_1 Q_1 \Gamma(-W_1)\Gamma(-N_1)\times\varphi^{-W_1-N_1-3}}{\Gamma(W_1+N_1+1)}, & R_m^{n_m} < \varphi < \infty \end{cases} \tag{3.33}$$

In (3.33), a solution is given for PDF of SINR where users are uniform distributed around a BS located at the center of a circular cell.

3.4.1 Numerical and Simulation Results

In this subsection, numerical and simulation results are performed to confirm the analytical results. First, PDF of SINR derived in (3.33) is plotted based on the parameters shown in Table 3.3. Then, a wireless system is simulated 100,000 times each for a user located randomly in a circular cell of radius $R_m = 1000m$ to validate the numerical results. Each MS is affected by one interfering source uniformly located inside its region of interference R_i. In Figure 3.4, it is shown how the simulation results closely match numerical results derived in this section.

Table 3.3: System parameters of SINR

Parameters	Values
R_m	1000 m
R_i	2000 m
σ_n	10 dB
σ_m	10 dB
σ_i	10 dB
n_m	2
n_i	3

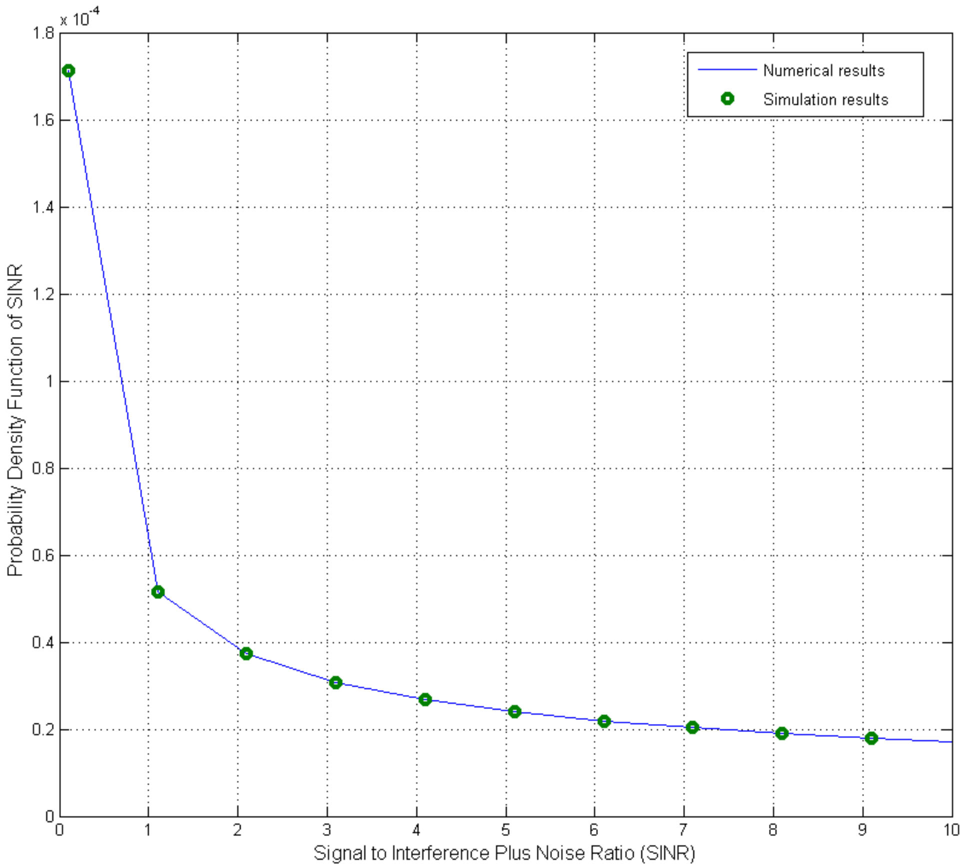

Figure 3.4: PDF of SINR in a circular cell

3.5 Conclusion

The main objective of this chapter was to analytically evaluate the performance of a wireless system using the geometric modeling of the studied area. Therefore, PDF for SIR, SNR and SINR were assessed in presence of large scale fading for randomly distributed users around a reference point. Large scale fading was described by propagation loss, and shadowing which is a random phenomenon. Moreover, numerical and simulation results were performed to validate the results. It was noticed in Figures 3.2, 3.3 and 3.4 that simulation results closely match numerical results. These performance metrics are used in the next chapter to evaluate outage probability.

Chapter 4

Evaluation of Outage Probability

4.1 Introduction

Outage probability is the probability of failing to reach the satisfactory reception of a wanted signal [38]. Outage probability is considered as a statistical parameter that can reflect the quality of service (QoS) in a wireless system [15]. The objective of this chapter is to analytically evaluate outage probability based on different performance metrics found in the previous chapter.

The rest of this chapter is organized as follows: In Section 4.2, outage probability based on path loss is evaluated in presence of large scale fading. In Section 4.3, outage probability based on SIR is assessed. In Section 4.4, outage probability based on SNR is found. In Section 4.5, outage probability based on SINR is analytically evaluated. In addition, numerical and simulation results are compared in each section to confirm the analytical results. Section 3.5 concludes the chapter.

4.2 Outage Probability of Uniformly Distributed Users Based on Path Loss

Outage probability is the probability that the received signal experiences a path loss greater than the acceptable specified value of the path loss l_{max} [38]. Outage probability based on path loss is modeled as the complementary cumulative distribution function (CCDF) of the pass loss at a predefined threshold.

The path loss (in dB) is written as [25]:

$$PL = \alpha + \beta \log(X) + \xi \tag{4.1}$$

- d_o is the close-in distance
- α is the free space path loss at the close-in distance d_0
- $\beta = 10 \times n$
- n is the path loss exponent which depends on the propagation environment
- $X = d/d_0$
- d is the distance between BS and an arbitrary user in the cell
- ξ is the shadow fading which is a zero-mean log-normal distributed random variable with standard deviation σ

Users are uniformly distributed in a cell of radius R_m, where the distance between the BS located at the center of the cell and an arbitrary user is d. Therefore, PDF of the distances from the BS to any user is as follows [25]:

$$f_D(d) = \frac{2d}{R_m^2} \quad d \in \; [0, R_m] \tag{4.2}$$

As shown in [25], PDF of path loss in a circular cell of radius R_m is:

$$f_L(l) = \frac{(\ln 10)}{\beta R_m^2} \exp\left\{ \frac{2\beta(\ln 10)(l - \alpha) + 2\sigma^2(\ln 10)^2}{\beta^2} \right\} \times \{1 - erf(D)\} \tag{4.3}$$

- $D = \frac{l\beta - \alpha\beta - \beta^2 \log(R_m) + 2\sigma^2(\ln 10)}{\sqrt{2}\beta\sigma}$
- erf(x) is the error function as defined: erf$(x) = \frac{2}{\sqrt{\pi}} \int_0^x exp(-t^2)dt$

4.2.1 Evaluation of Outage Probability Based on Path Loss

In this subsection, a solution for outage probability based on path loss for uniformly distributed users in a circular cell is evaluated. To assess the outage probability, PDF of path loss in (4.3) is integrated, and then, its complementary is found. It can be mathematically defined as:

$$P_{out}(l_m ax) = P(l > l_{max}) = F_L(l_{max}) = \int_{l_{max}}^{\infty} f_L(l)dl \tag{4.4}$$

- $F_L(l)$ is the cumulative distribution function (CDF) of path loss
- l is a random variable representing path loss between BS and an arbitrary user
- $f_L(l)$ is the probability density function of path loss

Equation (4.4) can be rewritten as follows:

$$P_{out}(l_max) = \int_{l_{max}}^{\infty} f_L(l)dl = 1 - \int_{0}^{l_{max}} f_L(l)dl \tag{4.5}$$

To evaluate the integral of the right hand side of (4.5), the variables of (4.3) are rearranged as follows:

$$f_L(l) = \overbrace{\frac{(\ln 10)}{\beta R_m^2}\exp\left[\frac{2l(\ln 10)}{\beta} + F\right]}^{I_1(l)} - \overbrace{\frac{(\ln 10)}{\beta R_m^2}\exp\left[\frac{2l(\ln 10)}{\beta} + F\right]\mathrm{erf}\left(\frac{l}{\sqrt{2}\sigma} - E\right)}^{I_2(l)} \tag{4.6}$$

- $E = \frac{\alpha\beta+\beta^2\log(R_m)-2\sigma^2(\ln 10)}{\sqrt{2}\beta\sigma}$
- $F = \frac{2\sigma^2(\ln 10)^2-2\alpha\beta(\ln 10)}{\beta^2}$

Substituting (4.6) in (4.5):

$$P_{out}(l_max) = 1 - \int_{0}^{l_{max}} f_L(l)dl = 1 - \int_{0}^{l_{max}} I_1(l)dl + \int_{0}^{l_{max}} I_2(l)dl \tag{4.7}$$

The results of the first integral is:

$$\int_{0}^{l_{max}} I_1(l)dl = \int_{0}^{l_{max}} \frac{(\ln 10)}{\beta R_m^2}\exp\left[\frac{2l(\ln 10)}{\beta} + F\right]dl = \frac{(\ln 10)}{\beta R_m^2}\exp[F]\frac{\beta}{2(\ln 10)}$$
$$\times\left(exp\left[\frac{2l_{max}(\ln 10)}{\beta}\right] - 1\right) \tag{4.8}$$

The result of the second integral is:

$$\int_0^{l_{max}} I_2(l)dl = \int_0^{l_{max}} \frac{(\ln 10)}{\beta R_m^2} \exp\left[\frac{2l(\ln 10)}{\beta} + F\right] \operatorname{erf}\left(\frac{l}{\sqrt{2}\sigma} - E\right) dl \tag{4.9}$$

The integration of (4.9) is:

$$\int_0^{l_{max}} I_2(l)dl = \frac{(\ln 10)}{\beta R_m^2} \exp[F] \times \int_0^{l_{max}} \exp(Gl) \operatorname{erf}\left(\frac{l}{H} - E\right) dl \tag{4.10}$$

- $G = \frac{2(\ln 10)}{\beta}$
- $H = \sqrt{2}\sigma$

The second component of this integration is:

$$\int_0^{l_{max}} I_2(l)dl = \frac{(\ln 10)}{\beta R_m^2} \exp[F] \times$$

$$\left[e^{GHE}\left(e^{\frac{G^2H^2}{4}} \left(\operatorname{erf}\left(\frac{GH}{2} - \frac{l_{max}}{H} + E \right) - \operatorname{erf}\left(\frac{GH}{2} + E \right) \right) \right) + \right.$$
$$\left. e^{GHE}\left(e^{G(l_{max} - HE)} \operatorname{erf}\left(\frac{l_{max}}{H} - E \right) \right) - \operatorname{erf}(-E) \right] \tag{4.11}$$

Finally, outage probability for uniform distributed users in a circular cell based on path loss is evaluated as follows:

$$P_{out}(l_max) = 1 - \frac{\exp(F)}{2R_m^2} \times \left[\exp\left(\frac{2l_{max}(\ln 10)}{\beta} \right) - 1 + \operatorname{erf}(-E) - \right.$$

$$e^{GHE}\left(\left(e^{\frac{G^2H^2}{4}} \left(\operatorname{erf}\left(\frac{GH}{2} - \frac{l_{max}}{H} + E \right) - \operatorname{erf}\left(\frac{GH}{2} + E \right) \right) \right) + \right.$$

$$e^{G(l_{max}-HE)}\mathrm{erf}\left(\frac{l_{max}}{H}-E\right)\Big)\Big], \qquad 0 < l_{max} < \infty \tag{4.12}$$

4.2.2 Numerical and Simulation Results

In this subsection, numerical and simulation results of outage probability derived in (4.12) are presented based on the parameters shown in Table 4.1. First, we numerically evaluate outage probability based on path loss for $R_m = 100m$ and $R_m = 300m$. Moreover, a

Table 4.1: System parameters of path loss

Channel Environment	Channel parameters
Standard Deviation for Shadowing	10
Path Loss parameters [dB]	$\alpha = 39 \quad \beta = 20$
Supported Distance[m]	$d_0 = 25 \leq d \leq R_m$

wireless system is simulated 100,000 times each for a user located randomly in a circular cell of radius $R_m = 100m$ to validate the numerical results. As shown in Figure 4.1, numerical and simulation results properly match. Also, we conclude from Figure 4.1 that outage probability increases when increasing the cell radius.

4.3 Outage Probability of Uniformly Distributed Users Based on Signal to Interference Ratio

In this section, outage probability based on SIR is evaluated. SIR is assessed based on the effects of large scale fading, described by propagation loss and shadowing which is a random phenomenon for uniform distributed users in a circular cell. Also, simulation and numerical results are performed to confirm the analytical results.

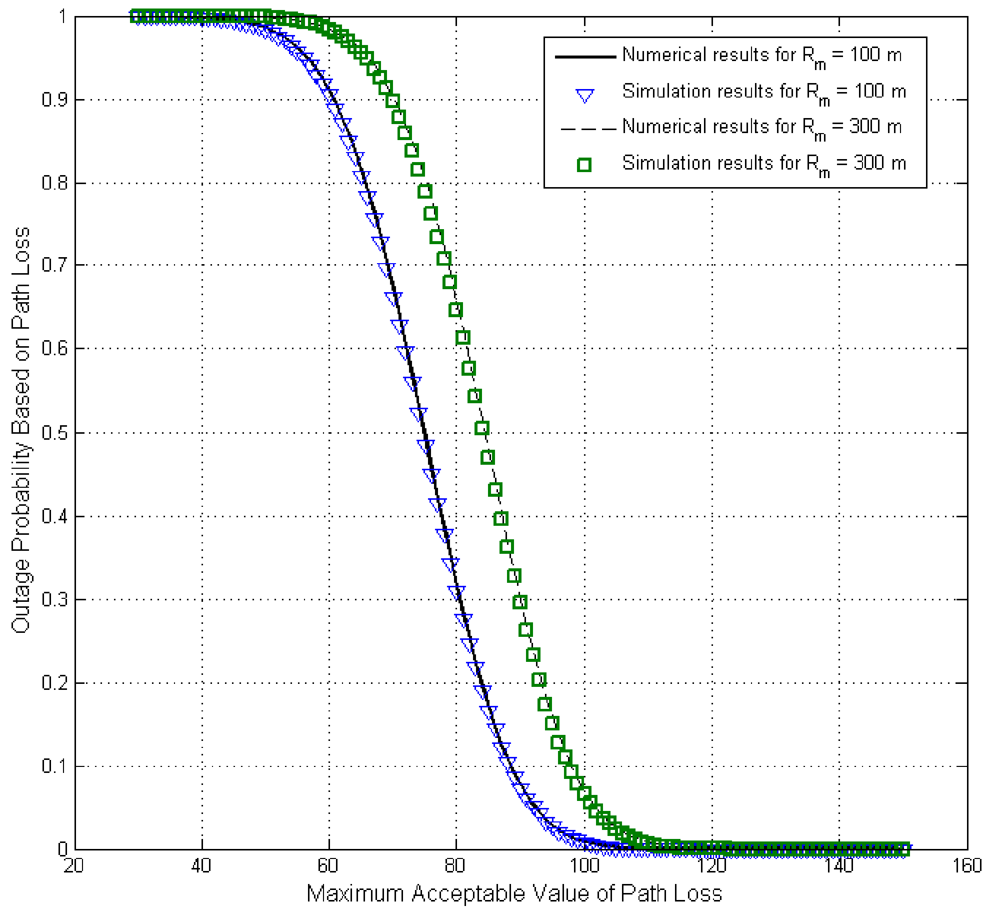

Figure 4.1: Outage probability based on PL threshold in a circular cell for different cell sizes

4.3.1 Evaluation of Outage Probability Based on Signal to Interference Ratio

Outage probability based on SIR is the probability that the received signal does not obtain the required value of SIR. Therefore outage probability based on SIR is mathematically explained as the cumulative distribution function of SIR at a predefined level γ_{min} , which is the minimum acceptable level of SIR when receiving the signal at the receiver:

$$P_{out}(\gamma_{min}) = P(\gamma < \gamma_{min}) = F_{SIR}(\gamma_{min}) = \int_{0}^{\gamma_{min}} f_{SIR}(\gamma)d\gamma \tag{4.13}$$

- $F_{SIR}(\gamma)$ is the cumulative distribution function of SIR
- γ is a random variable representing signal to interference ratio between BS and any arbitrary user
- $f_{SIR}(\gamma)$ is the probability density function of SIR

In the previous chapter, probability density function of signal to interference ratio is evaluated in (3.10). As a result, the outage probability based on SIR is evaluated as follows:

$$P_{out}(\gamma_{min}) = \begin{cases} \frac{An_i\sqrt{C}}{4\sigma_H\sqrt{2}} \exp\left(\frac{C(W+1)^2}{4}\right)\left[1 - erf\left(\frac{CW+C-2\ln(t)}{2\sqrt{C}}\right)\right] \times \gamma_{min}^{\frac{2}{n_i}}, & 0 < \gamma_{min} < \mu \\ 1 - \frac{Bn_m\sqrt{D}}{4\sigma_H\sqrt{2}} \exp\left(\frac{D(W+1)^2}{4}\right)\left[1 + erf\left(\frac{DW+C-2\ln(t)}{2\sqrt{D}}\right)\right] \times \gamma^{\frac{-2}{n_m}}, & \mu < \gamma_{min} < \infty \end{cases} \tag{4.14}$$

4.3.2 Numerical and Simulation Results

Outage probability based on SIR derived in (4.14) is numerically evaluated based on the parameters shown in Table 3.1. On the other hand, a wireless system is simulated 100,000

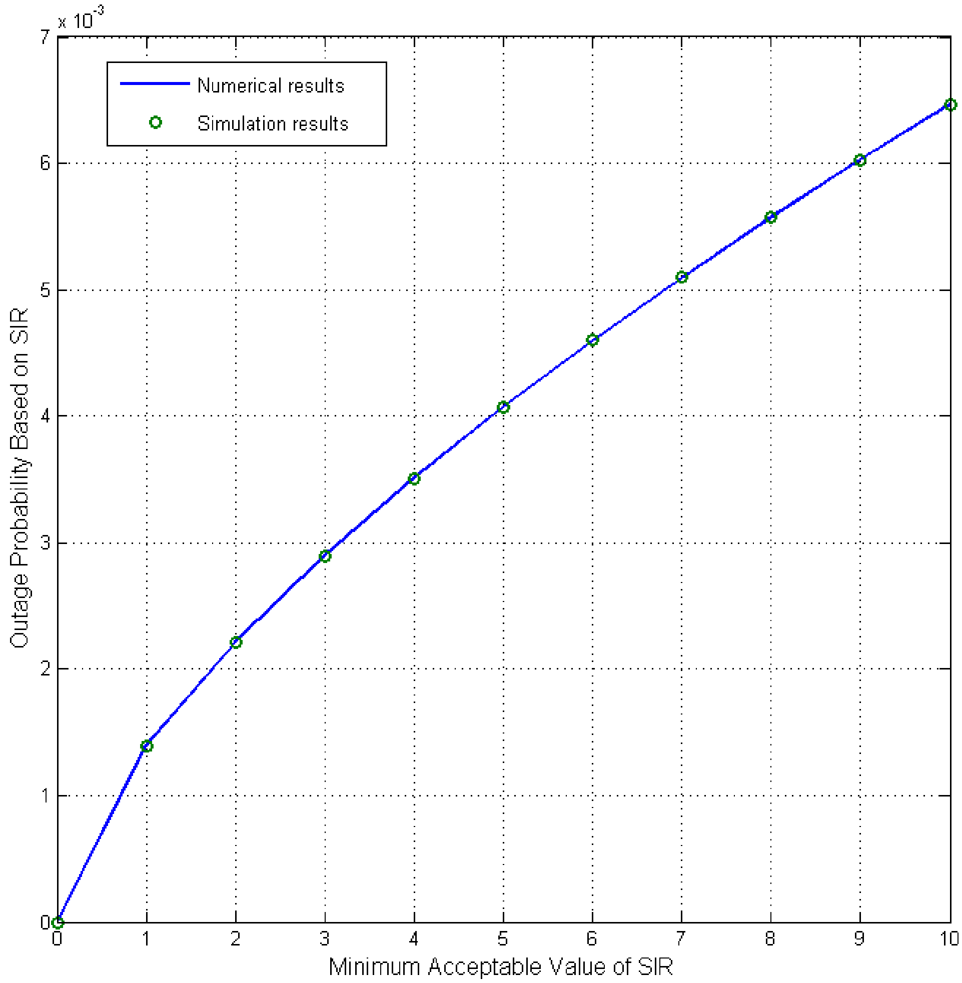

Figure 4.2: Outage probability based on SIR threshold in a circular cell

times each for a user located randomly in a circular cell of radius $R_m = 1000m$ to validate the numerical results. It is shown from the Figure 4.2 that the simulation results match numerical results derived in this section.

Also, it is noticed in Figure 4.2 that increasing the minimum acceptable level of SIR, γ_{min}, results in increasing the outage probability. A user is in outage when he is not reaching the minimum acceptable level of SIR, γ_{min}. Therefore, if the minimum acceptable level of SIR γ_{min} decreases, more users will have good reception and the outage probability will be low. The number of users with good reception starts to decrease when increasing the minimum acceptable level of SIR, γ_{min}.

4.4 Outage Probability of Uniformly Distributed Users Based on Signal to Noise Ratio

In a wireless system where interference is perfectly canceled, SNR becomes a critical parameter to be evaluated. As a result, in this section, outage probability based on SNR is evaluated in presence of large scale fading. It is considered that users are uniformly distributed around a BS located at the center of a circular cell. Also, simulation and numerical results are performed to validate the assessed results.

4.4.1 Evaluation of Outage Probability Based on Signal to Noise Ratio

In this subsection, outage probability is evaluated based on SNR. Outage probability based on SNR is the probability that the received signal does not obtain the required value of SNR. Therefore, outage probability based on SNR is mathematically explained as the cumulative distribution function of SNR at a predefined level δ_{min} which is the minimum acceptable level of SNR to receive the signal at the receiver:

$$P_{out}(\delta_{min}) = P((\delta < \delta_{min})) = F_{SNR}(\delta_{min}) = \int_{0}^{\delta_{min}} f_{SNR}(\gamma) d\delta \tag{4.15}$$

- $F_{SNR}(\gamma)$ is the cumulative distribution function of SNR
- δ is a random variable representing signal to noise ratio between BS and any arbitrary user
- $f_{SNR}(\delta)$ is the probability density function of SNR

In the previous chapter, probability density function of SNR is analytically evaluated in (3.23).

Based on (4.15), outage probability is evaluated as follows:

$$P_{out}(\delta_{min}) = \begin{cases} \frac{n_m\sqrt{d\pi}\left[1-erf\left(\frac{ab+a-2\ln(R_m^{n_m})}{2\sqrt{a}}\right)\right]}{R_m^2 n_m \sigma_n 2\sqrt{2}} \times \delta_{min}^{\frac{2}{n_m}}, & 0 < \delta_{min} < R_m^{n_m} \\ 1 - \frac{n_m\sqrt{d\pi}\left[1+erf\left(\frac{ab+a-2\ln(R_m^{n_m})}{2\sqrt{a}}\right)\right]}{R_m^{-2} n_m \sigma_n 2\sqrt{2}} \times \delta_{min}^{\frac{-2}{n_m}}, & R_m^{n_m} < \delta_{min} < \infty \end{cases} \tag{4.16}$$

In (4.16), outage probability based on the evaluated PDF of SNR is given for uniform distributed users around a BS located at the center of a circular cell.

4.4.2 Numerical and Simulation Results

Outage probability based on SNR derived in (4.16) is analytically evaluated based on the parameters shown in Table 3.2. Also, a wireless system is simulated 100,000 times each for

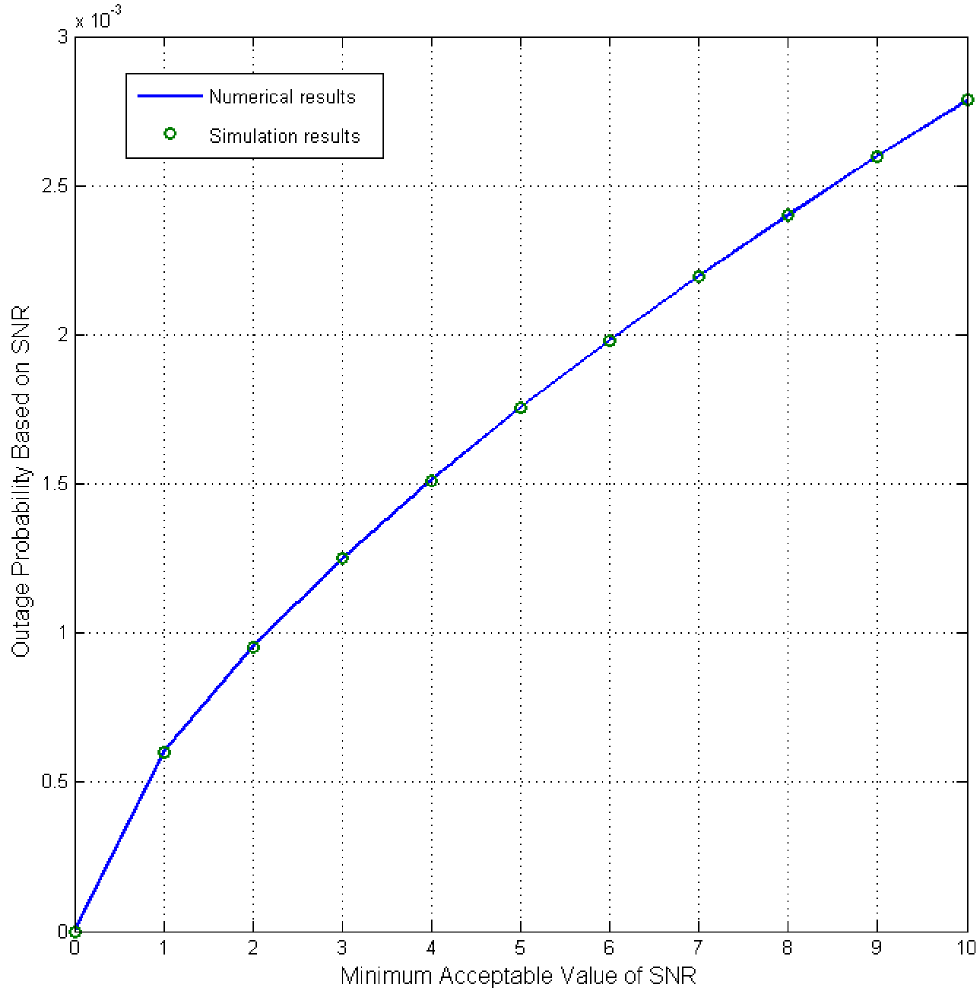

Figure 4.3: Outage probability based on SNR threshold in a circular cell

a user located randomly in a circular cell of radius $R_m = 1000m$ to validate the numerical results. It is shown from Figure 4.3 that the simulation results match numerical results derived in this section.

4.5 Outage Probability for Uniformly Distributed Users Based on Signal to Interference Plus Noise Ratio

In this section, outage probability based on SINR is evaluated. SINR is assessed in presence of propagation loss and shadowing. Moreover, simulation and numerical results are obtained to affirm the analytically derived results.

4.5.1 Evaluation of Outage Probability Based on Signal to Interference plus Noise Ratio

Outage probability based on SINR is the probability that the received signal does not obtain the required value of SINR. Therefore, the outage probability based on SINR is mathematically explained as the cumulative distribution function of SINR at a predefined level φ_{min} which is the minimum acceptable level of SINR to receive the signal at the receiver:

$$P_{out}(\varphi_m in) = P(\varphi < \varphi_{min}) = F_{SINR}(\varphi_{min}) = \int_0^{\varphi_{min}} f_{SINR}(\varphi) d\varphi \tag{4.17}$$

- $F_{SINR}(\varphi_{min})$ is the cumulative distribution function of SINR
- φ is a random variable representing signal to interference plus noise ratio between BS and any arbitrary user

- $f_{SINR}(\varphi)$ is the probability density function of SINR

In the previous chapter, probability density function of SINR was evaluated in (3.33). Therefore, outage probability based on SINR becomes:

$$P_{out}(\varphi_{min}) = \begin{cases} \frac{M_2 Q_2 \Gamma(-W_2)\Gamma(-N_2)}{\Gamma(W_2+N_2+1)} \times \frac{\varphi_{min}^{-W_2-N_2-2}}{-W_2-N_2-2}, & 0 < \varphi_{min} < \mu \\ X + \frac{M_1 Q_2 \Gamma(-W_2)\Gamma(-N_1)}{\Gamma(W_2+N_1+1)} \times \frac{\varphi_{min}^{-W_2-N_1-2}}{-W_2-N_1-2}, & \mu < \varphi_{min} < R_m^{n_m} \\ 1 - \frac{M_1 Q_1 \Gamma(-W_1)\Gamma(-N_1)}{\Gamma(W_1+N_1+1)} \times \frac{\varphi_{min}^{-W_1-N_1-2}}{-W_1-N_1-2}, & R_m^{n_m} < \varphi_{min} < \infty \end{cases} \tag{4.18}$$

$$X = \frac{M_2 Q_2 \Gamma(-W_2)\Gamma(-N_2)}{\Gamma(W_2+N_2+1)} \times \frac{\mu^{-W_2-N_2-2}}{(-W_2-N_2-2)} - \frac{M_1 Q_2 \Gamma(-W_2)\Gamma(-N_1)}{\Gamma(W_2+N_1+1)} \times \frac{\mu^{-W_2-N_1-2}}{(-W_2-N_1-2)}$$

4.5.2 Numerical and Simulation Results

In Figure 4.4, outage probability based on SINR derived in (4.18) is numerically evaluated based on the parameters shown in Table 3.3. Moreover, a wireless system is simulated 100,000 times each for a user located randomly in a circular cell of radius $R_m = 1000m$ to validate the numerical results. It is shown in Figure 4.4 that simulation results match the derived numerical results.

A user is in outage when he is not reaching the minimum acceptable level of SINR, φ_{min}. Therefore, if the minimum acceptable level of SINR, φ_{min}, decreases, more users will have good reception and the outage probability will be low. The number of users with good reception starts to decrease when increasing the minimum acceptable level of SINR, φ_{min}.

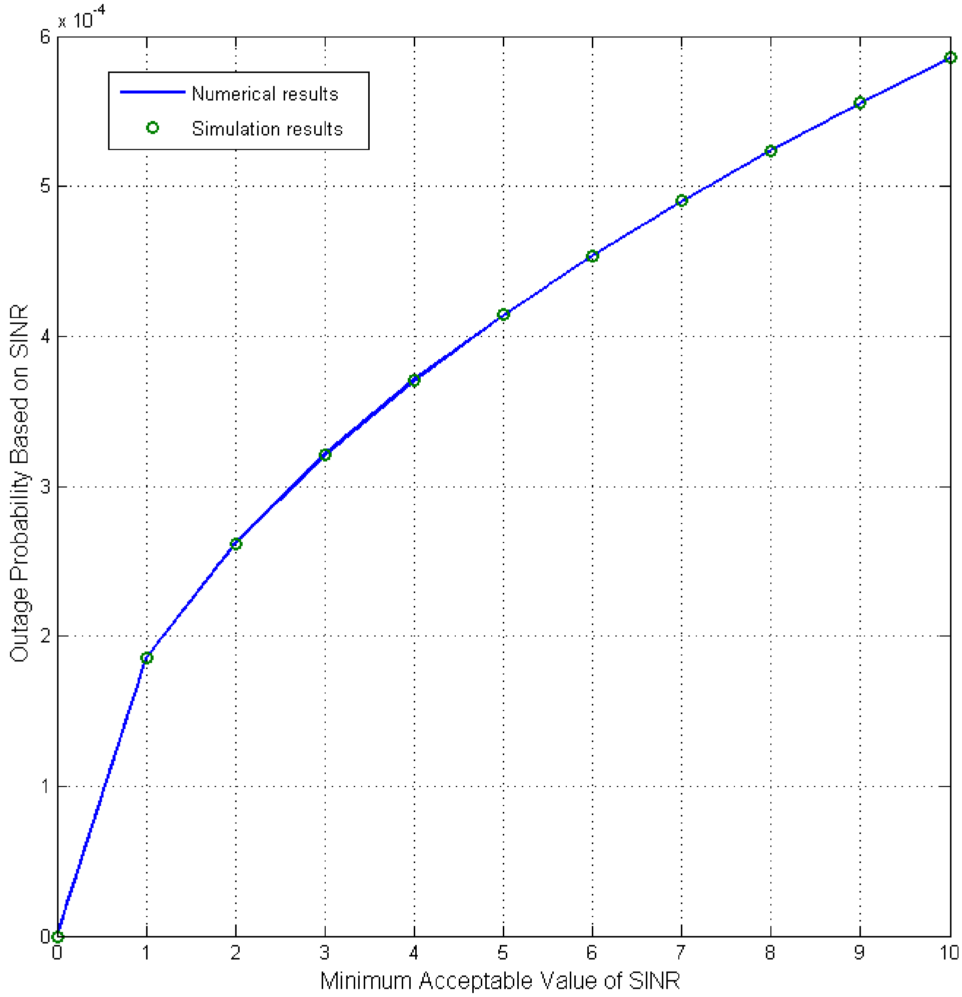

Figure 4.4: Outage probability based on SINR threshold in a circular cell

4.6 Conclusion

The main objective of this chapter was to evaluate outage probability in a wireless system. Consequently, outage probability based on path loss, SIR, SNR and SINR were evaluated. The results of the previous chapter were used to assess outage probability. Uniform distributed users around a centralized BS in a circular cell was geometrically modeled to evaluate the outage probability. Propagation loss and shadowing were considered as the sources that degrade the quality of signal from the BS to the MS. Moreover, comparison of numerical and simulation results were performed in this chapter to affirm the derived results. The results found in this chapter is used in a dual-hop relaying system in the next chapter to enhance the performance of the studied area.

Chapter 5

Performance Improvement of a Wireless System employing Relays

5.1 Introduction

In previous chapters, performance of single-hop wireless networks is evaluated by assessing outage probability of different scenarios. In this chapter, this outage probability is used to improve the system performance deploying dual-hop networks. In dual-hop network, number of relay stations are deployed over the studied area. These relays are used to enhance the quality of the system by helping BSs transmit signals to users located far away. Also, in this chapter, the required number of relays and their placement to achieve a desired performance for the wireless system are found.

The rest of this chapter is organized as follows: In Section 5.2, system model using dual-hop relaying is described. In Section 5.3, performance of dual-hop relaying system is evaluated using analytical results found in previous chapters. In Section 5.4, an approach is presented to find the required number of relays and their placement to reach a specific value of outage probability. Finally, Section 5.4 concludes the chapter.

5.2 System Model

To enhance the performance of a wireless system, a dual-hop relaying system is considered. RS forwards the received signal from BS to MS (downlink) by deploying non-transparent DF mode over the studied area. In this application, it is assumed that there is one interference source randomly located in the region of interference which is a circle centered at the mobile station with a specific radius. Using outage probability of single-hop links, from BS to RS, $P_{out,RS-BS}$, and from RS to MS, $P_{out,RS-MS}$, outage probability for dual-hop DF relaying system, $P_{out,DF}$ is obtained as follows:

$$P_{out,DF} = 1 - (1 - P_{out,BS-RS})(1 - P_{out,RS-MS}) \tag{5.1}$$

Users in the studied area are divided into two different categories based on their performance:

- Users achieving the required performance do not use RS, and receive signal directly from the BS
- Remaining users will receive signal from the nearest RS

Quality of service of the first category of users remains unchanged. On the other hand, quality of service of the second category of users improves. Instead of receiving a weak signal sent directly from BS through a long distance, the users will receive signal through a middle node (RS) located at a shorter distance. Therefore, it is concluded that the location of RSs over the studied area affects the performance of the system. Deploying RS near the cell edge results in a weak BS-RS signal. Deploying the RS near BS also results in a weak RS-MS signal at the edge of the cell. One should keep in mind that the number of deployed relays affects the placement of RS and the performance of the whole system [64].

As a result, in such a system, two factors should be taken into consideration:

- Number of RSs used over the studied area
- Location of RSs over the studied area

5.3 Relay Deployment in a Wireless System

In this section, a number of relays are deployed in a wireless system to improve its performance. Outage probability is compared for single-hop and dual-hop wireless networks. In Figures 5.1, 5.2, 5.3 and 5.4, it is shown how the number of relays affects the performance of the wireless system. As an illustrative example, different number of relays are deployed on a circle with radius equal to half the cell radius and compared with the single-hop system (without relays). In Table 5.1, outage probability is compared for a system without relays, with 3 and 6 relays deployed. It is concluded that increasing the number of relays enhances the quality of the system.

Also, to clarify the importance of the location of relays over the studied area, in Figures 5.5, 5.6, 5.7 and 5.8 six relays are deployed, and then the outage probability is evaluated for different relay locations based on different performance metrics. Also, in Table 5.2, outage

Table 5.1: Effect of the Number of Relays on the performance of a Wireless System

Performance Metric	P_{out} Without Relay	P_{out} With 3 Relays	P_{out} With 6 Relays
$l_{max} = 80\ dB$	33%	20%	16%
$\gamma_{min} = 5\ dB$	0.41%	0.21%	0.13%
$\delta_{min} = 5\ dB$	0.175%	0.078%	0.06%
$\varphi_{min} = 5\ dB$	0.042%	0.023%	0.017%

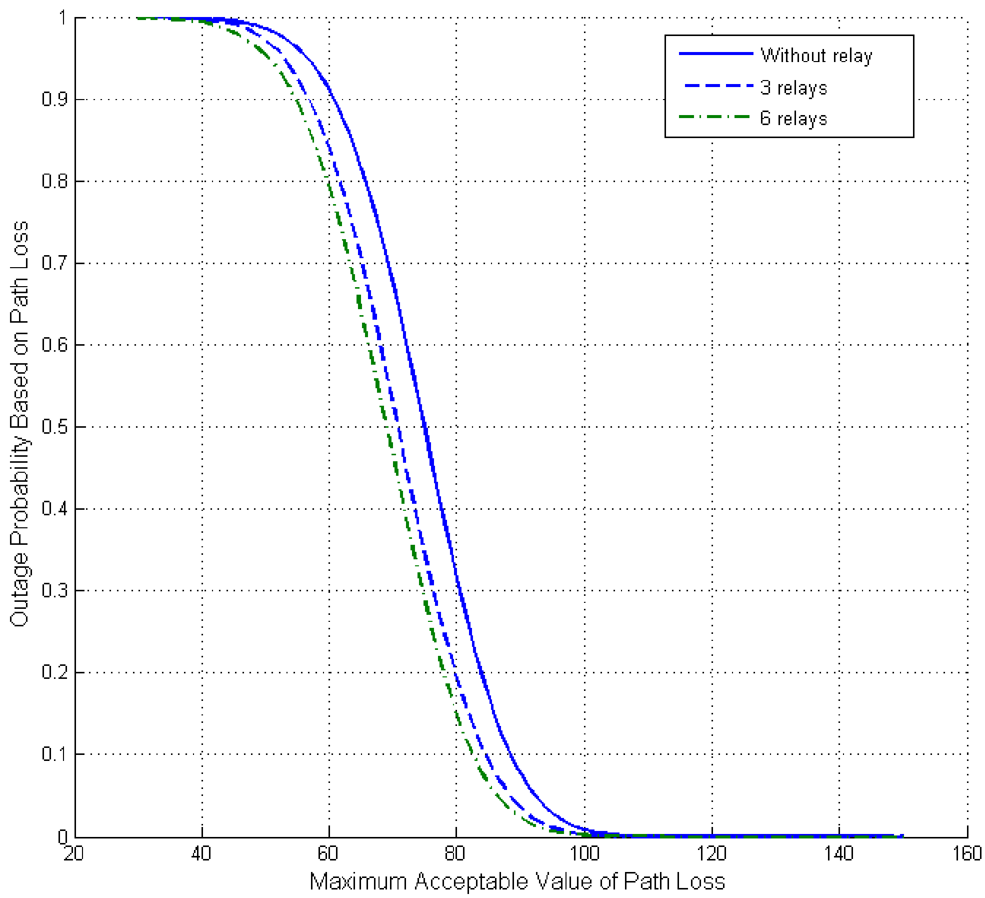

Figure 5.1: Outage probability based on PL threshold for different number of relays located at $0.5 \times R_m$ from the BS

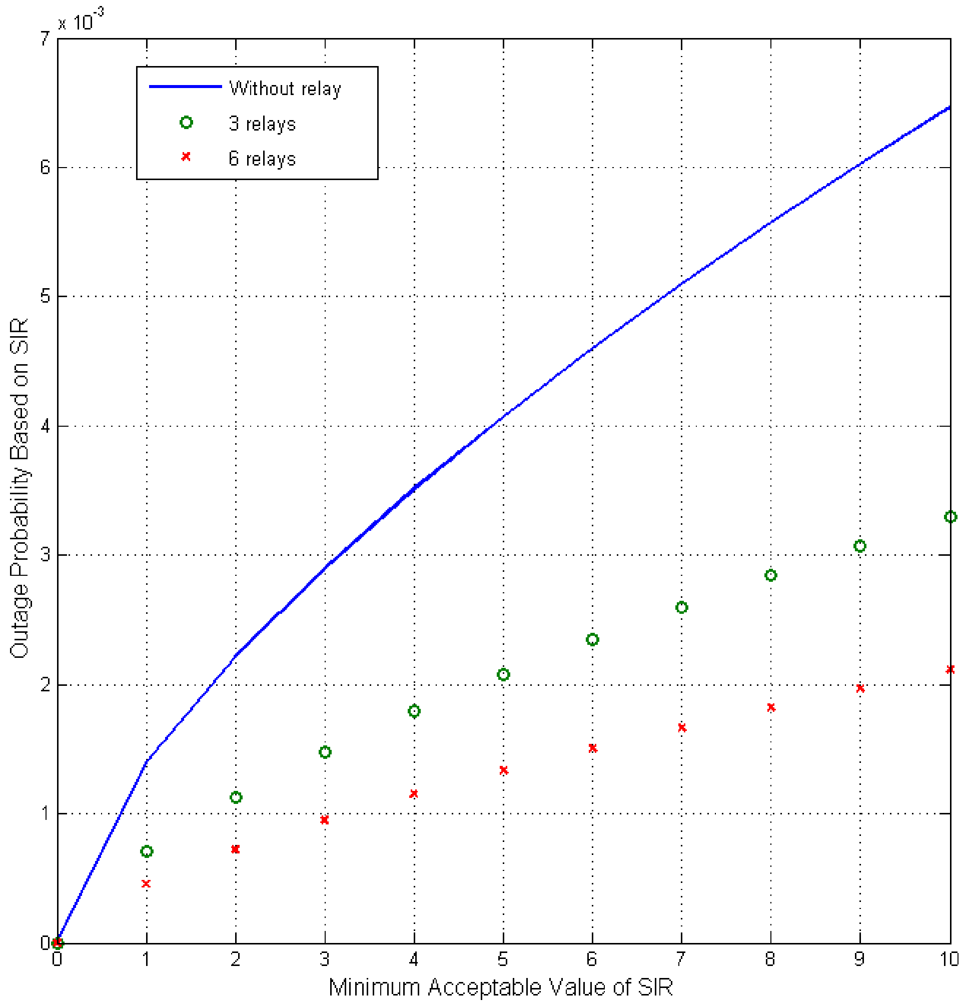

Figure 5.2: Outage probability based on SIR threshold for different number of relays located at $0.5 \times R_m$ from the BS

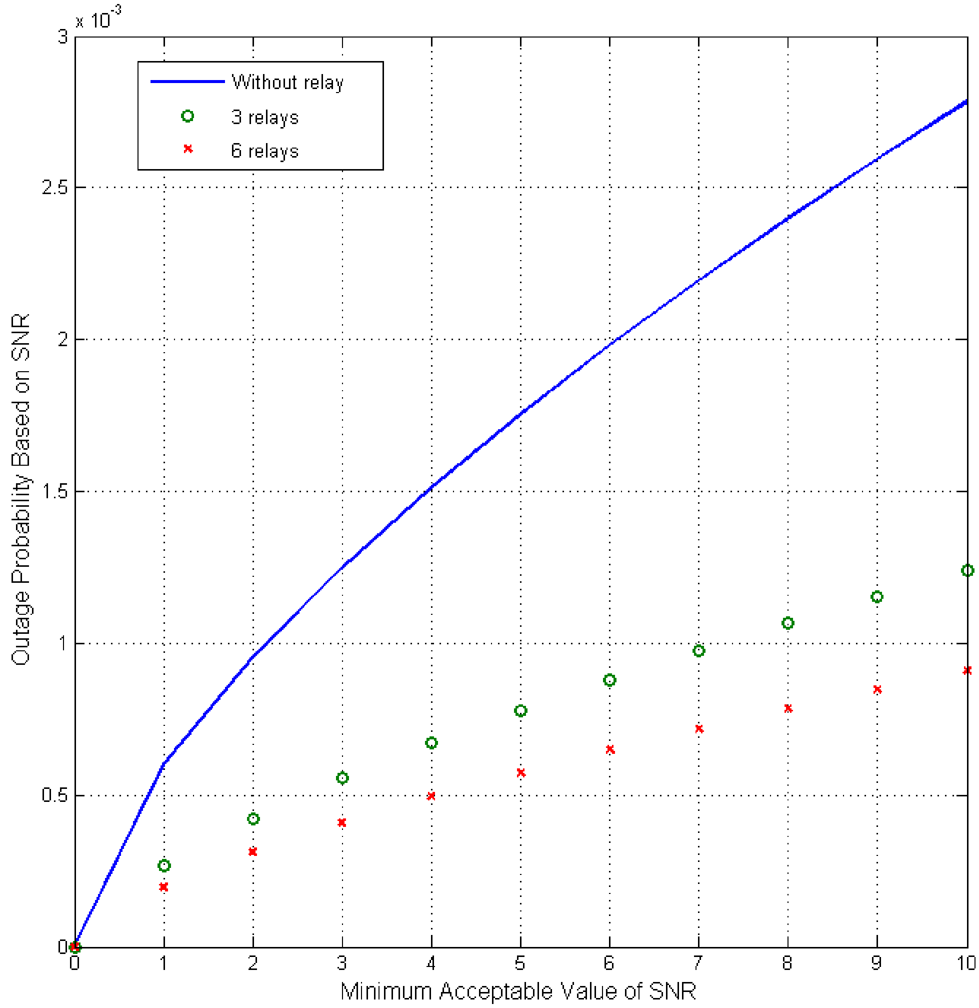

Figure 5.3: Outage probability based on SNR threshold for different number of relays located at $0.5 \times R_m$ from the BS

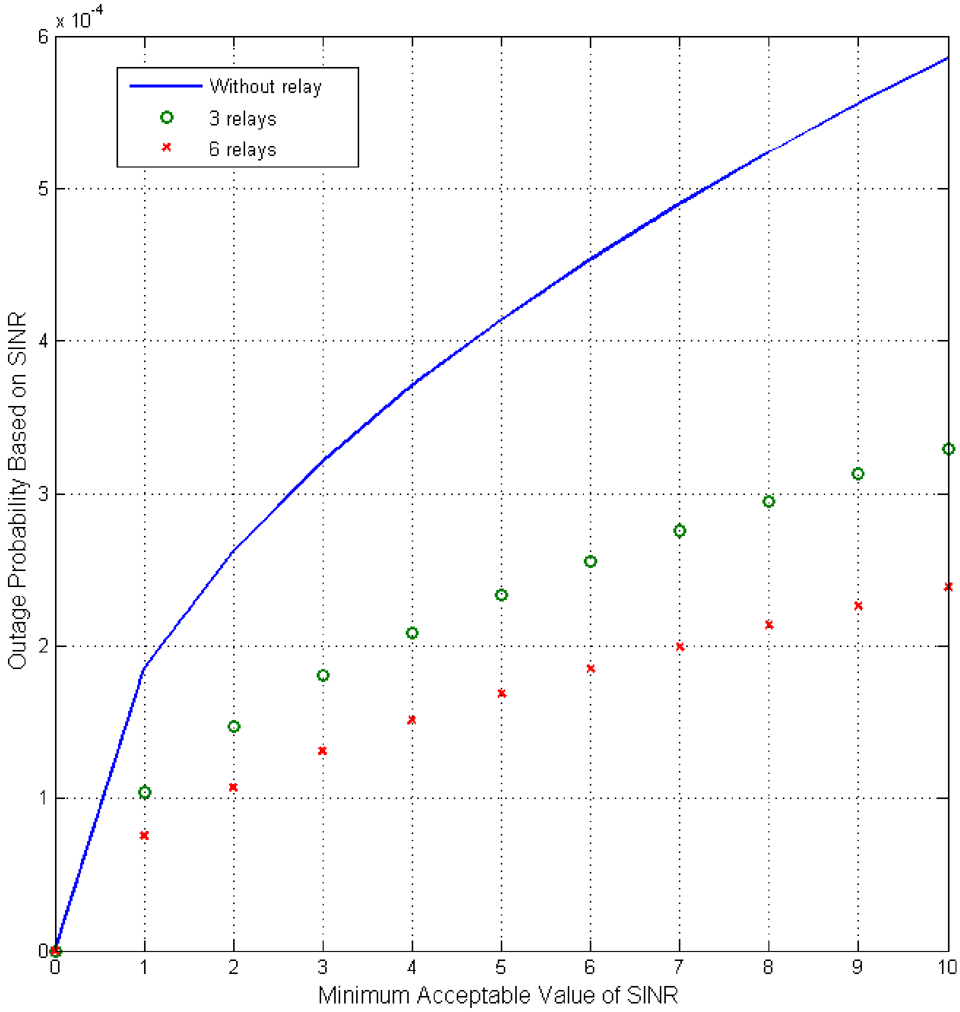

Figure 5.4: Outage probability based on SINR threshold for different number of relays located at $0.5 \times R_m$ from the BS

probability is evaluated for six relays that are deployed based on different d_{BS-RS}. It is noticeable that when BS-RS distance increases, the quality of the system improves until it reaches a turning point, where the outage probability starts increasing.

Table 5.2: Effect of the Location of Relays on the performance of a Wireless System

Performance Metric	P_{out} Without Relay	$0.3 \times R_m$	$0.5 \times R_m$	$0.9 \times R_m$
$l_{max} = 80\ dB$	33%	21%	16%	23%
$\gamma_{min} = 5\ dB$	0.41%	0.18%	0.13%	0.26%
$\delta_{min} = 5\ dB$	0.175%	0.09%	0.06%	0.115%
$\varphi_{min} = 5\ dB$	0.042%	0.028%	0.017%	0.0345%

Finally, it is concluded that there is a relation between the number of relays and their location over the studied area. For example, for $\gamma_{min} = 5$, outage probability based on SIR with three relays deployed at $d_{BS-RS} = 0.5 \times R_m$ is equal to $P_{out}(\gamma) = 0.21\%$. However, outage probability based on SIR with six relays deployed at $d_{BS-RS} = 0.9 \times R_m$ is equal to $P_{out}(\gamma) = 0.26\%$. As a result, an approach to find the required number of relays with their location to achieve a specific outage probability is provided in the next section.

5.4 Required Number of Relays and Their Location in a Wireless System

In this section, a pseudocode is presented in Algorithm 5.4.1 to find the required number of relays and their placement over the studied area. As in input to the algorithm the desired outage probability ($P_{out,th}$) is chosen for a specific threshold of the performance metric. The algorithm starts by evaluation of the outage probability and comparing it with $P_{out,th}$. If $P_{out,th}$ is greater than the desired outage probability, two relays are symmetrically added at a distance $d_{BS-RS} = step$ from the BS, where the *step* is set by the designer based on the

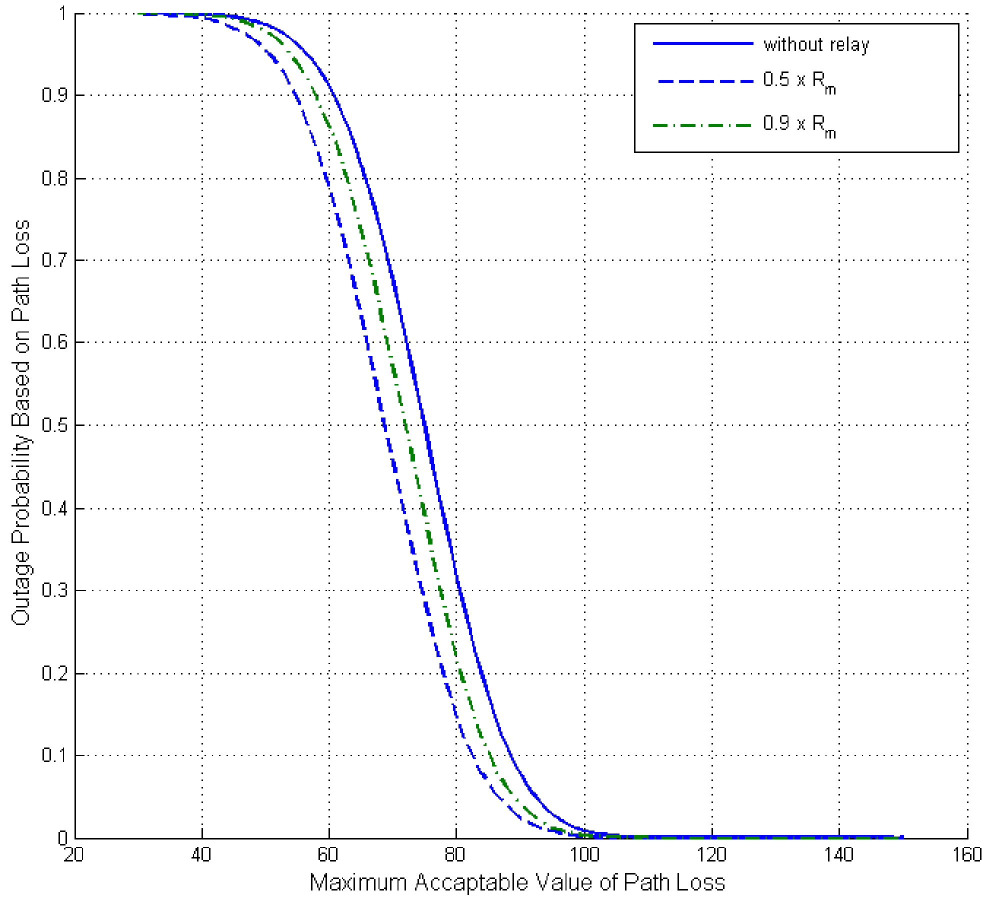

Figure 5.5: Outage probability based on PL threshold for 6 relays located at 0.3 $\times$ R_m, 0.5 $\times$ R_m and 0.9 $\times$ R_m

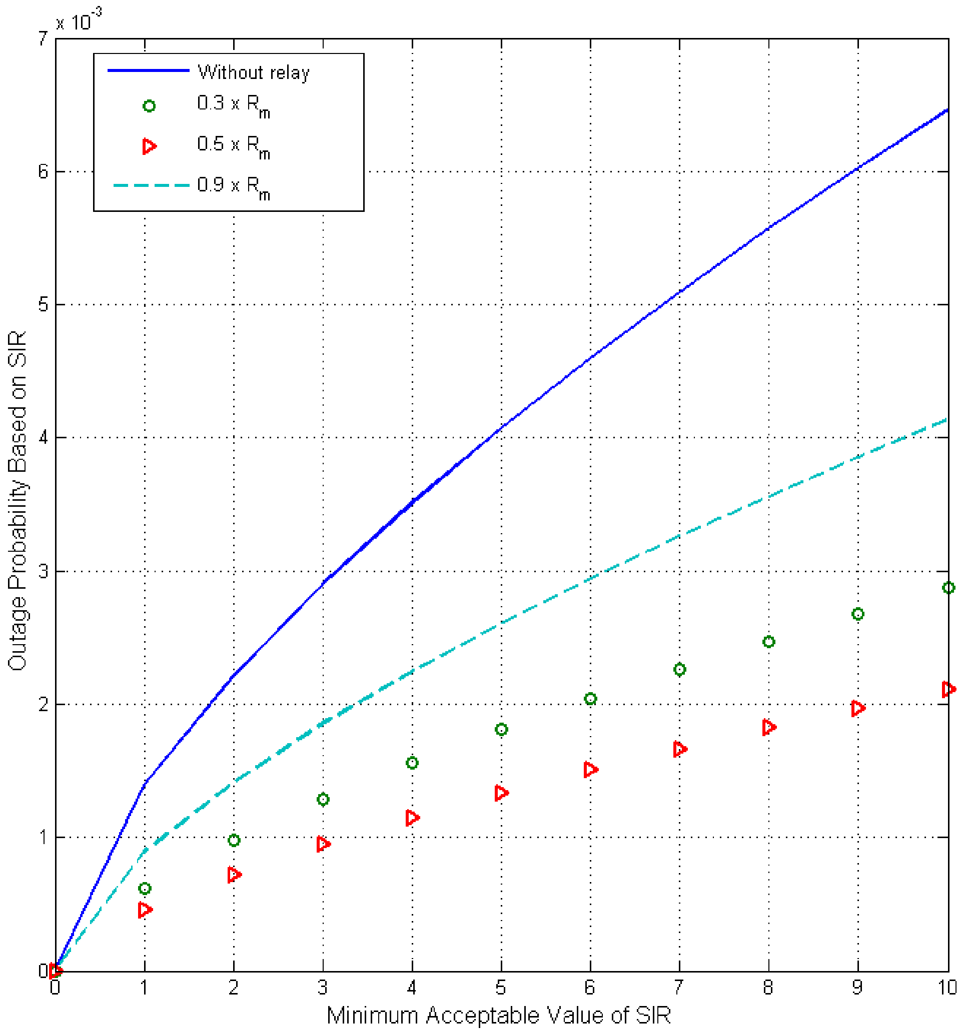

Figure 5.6: Outage probability based on SIR threshold for 6 relays located at $0.3 \times R_m$, $0.5 \times R_m$ and $0.9 \times R_m$

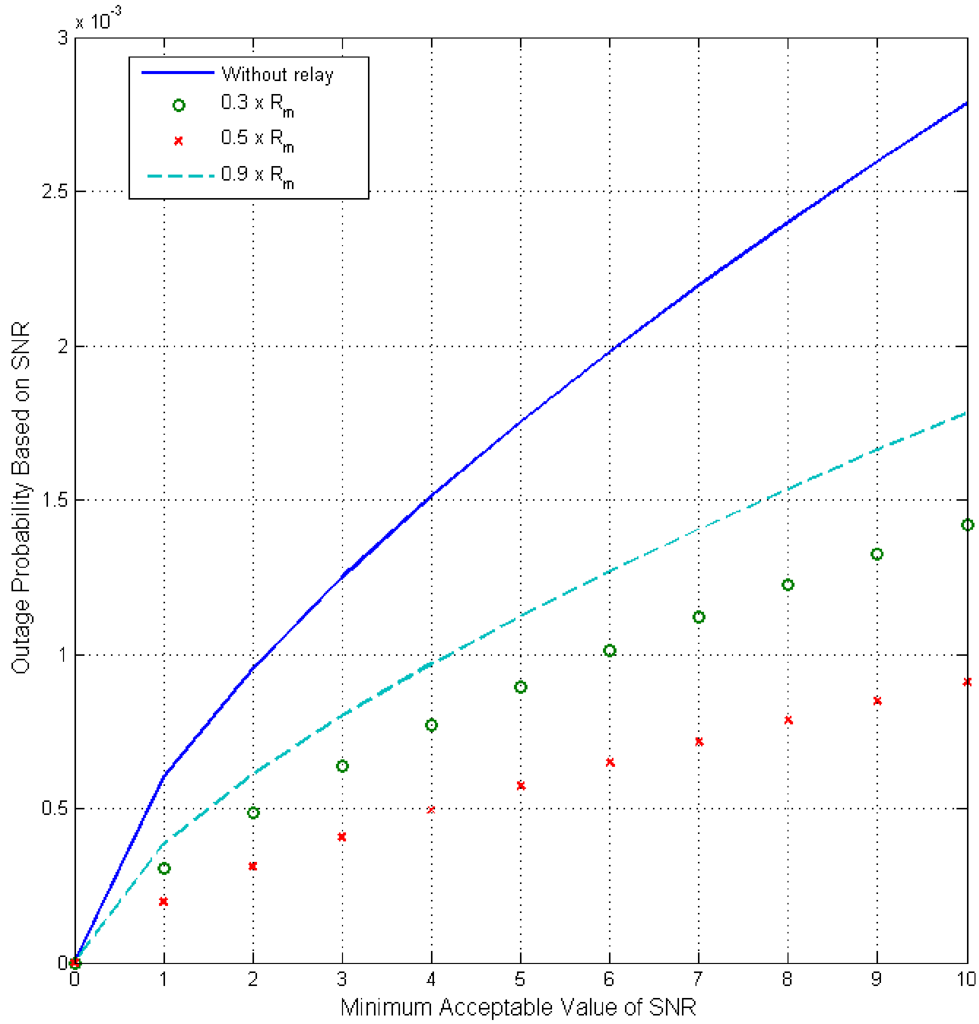

Figure 5.7: Outage probability based on SNR threshold for 6 relays located at $0.3 \times R_m$, $0.5 \times R_m$ and $0.9 \times R_m$

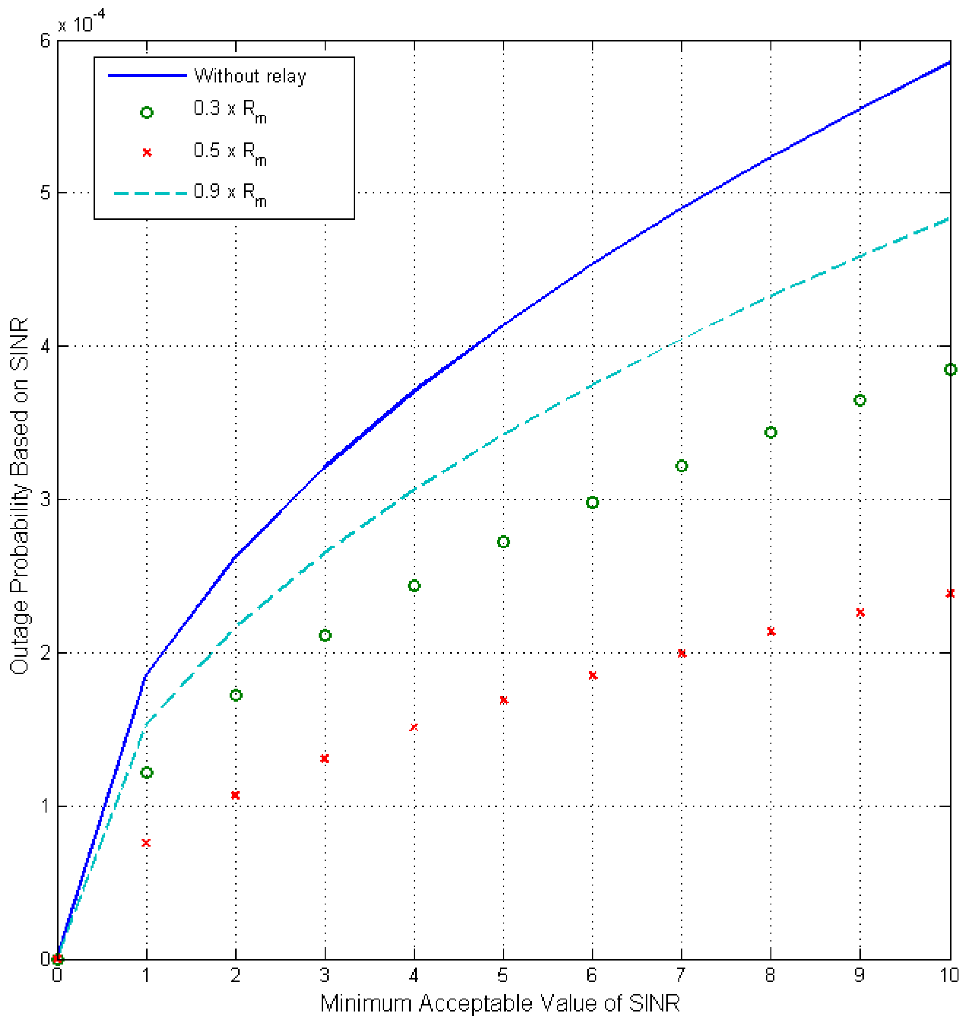

Figure 5.8: Outage probability based on SINR threshold for 6 relays located at $0.3 \times R_m$, $0.5 \times R_m$ and $0.9 \times R_m$

Algorithm 5.4.1: OPTIMUM NUMBER AND PLACEMENT OF RELAYS$(th, d_{out}, step, R_m)$

$numb_{relay} \leftarrow 1$
$d_{BS-RS} \leftarrow R_m - step$
while $P_{out}(th) > d_{out}$
  do
$$\begin{cases} numb_{relay} \leftarrow numb_{relay} + 1 \\ Relay_{angle} \leftarrow 360/numb_{relay} \\ \textbf{while } P_{out}(th) > d_{out} \\ \quad \textbf{do} \\ \begin{cases} d_{BS-RS} \leftarrow d_{BS-RS} + step \\ \textbf{if } d_{BS-RS} > R_m \\ \quad \textbf{then } \{ break \\ ComputeP_{out} \end{cases} \end{cases}$$
output $(numb_{relay}, d_{BS-RS})]$

required accuracy. Then, outage probability is recalculated and compared with $P_{out,th}$. As long as outage probability does not reach $P_{out,th}$, d_{BS-RS} is increased by a *step*. When the present relays reach the cell edge and there is no solution, the number of relays is increased by one. These relays are initially located on a circle with radius equal to *step* and separated by an angle equal to 360 divided by the number of relays, as explained in Algorithm 5.4.1. At the end of this algorithm, the required number of relays and their placement over the studied area are found.

As an illustrative example, simulation is performed taking into consideration that the accuracy is 10%, using parameters given in Table 5.3. Based on Algorithm 5.4.1, it has been shown that four relays are required in a distance $d_{BS-RS} = R/2 + 2 \times step = R/2 + R/5$ to achieve the desired outage probability. As a result, at $\gamma_{min} = 5\ dB$, the outage probability is improved from 0.41% to 0.15% after many iterations of the algorithm 5.4.1.

Table 5.3: Parameters of algorithm 5.4.1

Parameter	value
γ_{min}	$5\ dB$
$P_{out,th}$	0.15%
step	$R/10$
R_m	$1000\ m$

5.5 Conclusion

The main objective of this chapter was to enhance the performance of a wireless system. Therefore, dual-hop relaying system was used to help the BS transmit the signal to the MS. Outage probability found in the previous chapter was used to evaluate the performance of dual-hop relaying systems. However, when relays are deployed in a wireless system, two issues should be taken into consideration: number of deployed relays and their placement. Consequently, an approach was developed to enhance the system performance based on outage probability. Also, the required number of relays and their placement to achieve the desired outage probability in a wireless system was found.

Chapter 6

Performance Evaluation for Gaussian Distributed Users in a Wireless System

6.1 Introduction

As a non-uniform distribution, Gaussian distribution can be used in some cases to model the spreading of users over the studied area. In rural areas for example, it is expected to have a single cell, where users are concentrated in the center of the village, and the number of users gradually decreases with distance. In this chapter, the performance of the wireless system will be evaluated taking into consideration that users are Gaussian distributed.

The rest of this chapter is organized as follows: In Section 6.2 outage probability based on path loss for Gaussian distributed users is evaluated. Also, numerical and simulation results are obtained to affirm the analytical results. Then, in Section 6.3, the PDF of SIR for the case of Gaussian distributed users around centralized BS is evaluated. Then, outage probability based on SIR is analytically found. Moreover, the performance metrics evaluated are confirmed by comparing them with the simulation results. Finally, in Section 6.4, the chapter is concluded.

6.2 Outage Probability for Gaussian Distributed Users Based on Path Loss

In this section, an approximate expression for PDF of outage probability, assuming that users are Gaussian distributed is derived based on path loss stated in [41].

In [41], users are spread based on Gaussian distribution around a BS located at the center of a circular cell with radius R_m. The distribution of the distance between BS and MS is as follows:

$$f_D(d) = \frac{d.\exp\{(R_m^2 + d_0^2 - 2d^2)/4\sigma_G^2\}}{2\sigma_G^2.\sinh\{(R_m^2 - d_0^2)/4\sigma_G^2\}} \qquad d \in [0, R_m] \tag{6.1}$$

where σ_G is the standard deviation of the Gaussian distributed users.

PDF of path loss between a BS and Gaussian distributed users is found in [41]:

$$f_L(l) = \frac{\ln(10) \times 10^{2(l-\alpha)/\beta}}{\sqrt{8\pi}\beta\sigma_G^2\sinh\{(R_m^2 - d_0^2)/4\sigma_G^2\}} \times \exp\{(R_m^2 + d_0^2)/4\sigma_G^2 + (\sqrt{2}\ln(10)\sigma_G/\beta)^2\}\times$$

$$\int_{\mu=\mu_0(l)}^{\mu_L(l)} \exp(\frac{-\mu^2}{2}) \exp(\frac{-10^{2\sigma\mu/\beta} 10^{2\{\beta(l-\alpha)+2\ln(10)\sigma^2\}/\beta^2}}{2\sigma_G^2})\ d\mu \tag{6.2}$$

- $l_0 = \alpha + \beta \log(d_0) - 3\sigma$
- $l_L = \alpha + \beta \log(R) - 3\sigma$
- $0 < l_0 \leq l \leq l_L < \infty$
- $\mu_0(l) \equiv \{\alpha - l + \ln\big(\frac{d_0^{\beta/\ln(10)}}{10^{2\sigma^2/\beta}}\big)\}/\sigma$
- $\mu_L(l) \equiv \{\alpha - l + \ln\big(\frac{R^{\beta/\ln(10)}}{10^{2\sigma^2/\beta}}\big)\}/\sigma$

6.2.1 Evaluation of Outage Probability

Outage probability based on path loss is the probability that the received signal experiences a path loss greater than the acceptable specified value of the path loss l_{max} [38]. To evaluate the outage probability, PDF of path loss in (6.2) is integrated, and its complementary is found. It is mathematically defines as follows:

$$P_{out}(l_{max}) = P(l > l_{max}) = F_L(l_{max}) = \int_{l_{max}}^{\infty} f_L(l)dl \tag{6.3}$$

Equation (6.3) is rewritten as:

$$P_{out}(l_{max}) = \int_{l_{max}}^{\infty} f_L(l)dl = 1 - \int_{0}^{l_{max}} f_L(l)dl \tag{6.4}$$

The integration of the third part of PDF of path loss, shown in (6.5), cannot be analytically evaluated. Therefore, it is evaluated based on the Gauss quadrature approximation method [65].

$$I = \int\limits_{\mu=\mu_0(l)}^{\mu_L(l)} \exp \frac{-\mu^2}{2} \exp \frac{-10^{2\sigma\mu/\beta} 10^{2\{\beta(l-\alpha)+2\ln(10)\sigma^2\}/\beta^2}}{2\sigma_G^2} \; d\mu \tag{6.5}$$

In [65], Gauss quadrature method is divided into two steps. In the first step, the limits of integration are changed from $[\mu_0(l), \mu_L(l)]$ to $[-1, 1]$:

$$I = \int\limits_{\mu=\mu_0(l)}^{\mu_L(l)} f(\mu) \quad d\mu = \int\limits_{-1}^{1} f(t) \quad dt \tag{6.6}$$

- $f(\mu) = \exp \frac{-\mu^2}{2} \exp \frac{-10^{2\sigma\mu/\beta} 10^{2\{\beta(l-\alpha)+2\ln(10)\sigma^2\}/\beta^2}}{2\sigma_G^2}$

Then, variables are changed as follows:

$$\mu = \frac{1}{2} \; [t(\mu_L(l) - \mu_0(l)) + \mu_0(l) + \mu_L(l)] \tag{6.7}$$

$$d\mu = \frac{1}{2} \; (\mu_L(l) - \mu_0(l))dt \tag{6.8}$$

Therefore, the integration of I becomes:

$$I = \int\limits_{-1}^{1} \frac{A}{2} \; f\left(\frac{At + B}{2} \right) dt \tag{6.9}$$

- $A = \mu_L(l) - \mu_0(l) = \frac{\ln Y - \ln X}{\sigma}$

- $X = d_0^{\beta/\ln(10)} / 10^{2\sigma^2/\beta}$

- $Y = R^{\beta/\ln(10)} / 10^{2\sigma^2/\beta}$

- $B = \mu_L(l) + \mu_0(l) = D + \frac{2l}{\sigma}$

- $D = \frac{2\alpha+\ln(X)+\ln(Y)}{\sigma}$

In the second step, an order for the approximation is selected with coefficients and gauss points using the table given in [65]. Then, an approximation of the integration is found as follows:

$$I = \int_{-1}^{1} f(t)dt \approx \sum_{1}^{n} C_n \frac{A}{2}\ f\left(\frac{At_n + B}{2}\right) \tag{6.10}$$

- n is the order of approximation
- C_n are the coefficients
- t_n are Gauss points

After performing these two steps, PDF of path loss is expressed in this approximated form:

$$f_L(l) = \frac{\ln(10) \times 10^{2(l-\alpha)/\beta}}{\sqrt{8\pi}\beta\sigma_G^2 \sinh\{(R_m^2 - d_0^2)/4\sigma_G^2\}} \times$$

$$\exp\{(R_m^2 + d_0^2)/4\sigma_G^2 + (\sqrt{2}\ln(10)\sigma/\beta)^2\}\times$$

$$\sum_{1}^{n} C_n \frac{A}{2}\ f\left(\frac{At_n + B}{2}\right) \tag{6.11}$$

In (6.11), n is the order of approximation. To find n, simulation is performed for $n = 2, 3, 4, 5$, etc.. It is noticed that the approximations of $n = 4$ and $n = 5$ are different, as shown in Figure 6.1. However, for order of more than $n = 4$, approximated signals almost match the main PDF of path loss. Therefore, it is decided to use the fifth order of Gauss quadrature (minimum order) as a solution to approximate the equation for any parameter.

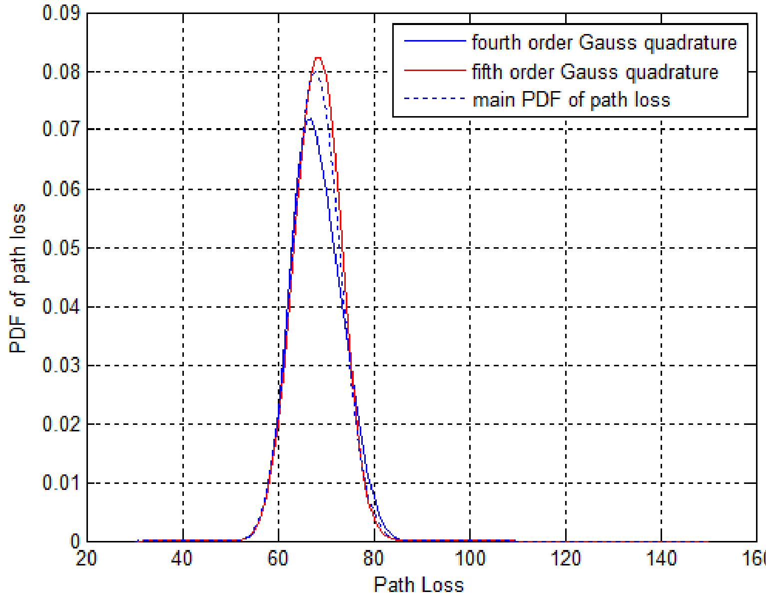

Figure 6.1: Main and approximated functions of different orders

Approximation found in (6.11) is used to evaluate outage probability. First, the variables of (6.11) are rearranged as follows:

$$f_L(l) = \sum_{1}^{n} F \times 10^{\frac{2l}{\beta}} \times G \times C_n \frac{A}{2} \, H_n \exp\left\{ \frac{l(2E_n\sigma - l)}{2\sigma^2} \right\} \tag{6.12}$$

- $F = \frac{\ln(10)10^{\frac{-2\alpha}{\beta}}}{\sqrt{8\pi}\beta\sigma_G^2 \sinh\{(R^2 - d_0^2)/4\sigma_G^2\}}$
- $G = \exp\{(R_m^2 + d_0^2)/4\sigma_G^2 + (\sqrt{2}\ln(10)\sigma/\beta)^2\}$
- $H_n = \exp\{\frac{-E_n^2}{2}\}\exp\left\{ \frac{-10^{\frac{2\sigma\beta E_n - 2\alpha\beta + 4\ln(10)\sigma^2}{\beta^2}}}{2\sigma_G^2} \right\}$
- $E_n = \frac{At_n + D}{2}$

Then, outage probability becomes:

$$\int_0^{l_{max}} f_L(l)dl = \sum_1^n FGC_n \frac{A}{2}\ H_n \times \int_0^{l_{max}} 10^{\frac{2l}{\beta}} \exp\left\{\frac{l(2E_n\sigma - l)}{2\sigma^2}\ \right\}dl \tag{6.13}$$

Knowing that $10^x = e^{x\ln(10)}$, $f_L(l)$ is integrated as follows:

$$\int_0^{l_{max}} f_L(l)dl = \sum_1^n FGC_n \frac{A}{2}\ H_n \times \int_0^{l_{max}} \exp\left\{\frac{l(Z_n - l)}{2\sigma^2}\ \right\}dl \tag{6.14}$$

- $Z_n = 2\sigma_G^2 W \ln(10) + 2E_n\sigma$
- $W = \frac{2}{\beta}$

Equation (6.14) is solved as follows [66]:

$$\int_0^{l_{max}} f_L(l)dl = \sum_1^n FGC_n \frac{A}{2}\ H_n \times \frac{1}{2}\ \sqrt{2\pi}\sigma \times \exp\left\{\frac{Z_n^2}{8\sigma^2}\ \right\}\times$$

$$\left[\mathrm{erf}\left(\frac{2l_{max} - Z_n}{2\sqrt{2}\sigma}\ \right) - \mathrm{erf}\left(\frac{-Z_n}{2\sqrt{2}\sigma}\ \right)\right] \tag{6.15}$$

Finally, outage probability in a circular cell for Gaussian distribution of users around the BS located at the center of the cell is derived as follows:

$$P_{out}(l_{max}) = 1 - \int_0^{l_{max}} f_L(l)dl = 1 - \sum_1^n FGC_n \frac{A}{2}\ H_n \times \frac{1}{2}\ \sqrt{2\pi}\sigma \times \exp\left\{\frac{Z_n^2}{8\sigma^2}\ \right\}\times$$

$$\left[\operatorname{erf}\left(\frac{2l_{max} - Z_n}{2\sqrt{2}\sigma}\right) - \operatorname{erf}\left(\frac{-Z_n}{2\sqrt{2}\sigma}\right)\right], \qquad 0 < l_{max} < \infty \tag{6.16}$$

6.2.2 Numerical and Simulation Results

In this subsection, outage probability derived in (6.16) is numerically evaluated based on the parameters shown in Table 4.1. Additionally, a wireless system of 10,000 users normally dropped around a BS located at the center of circular cell of radius $R_m = 4Km$ is simulated. Results are shown for different network scenarios, where $\sigma_G = R_m/5$ and $\sigma_G = R_m/10$. It is shown in Figure 6.2 that simulation results closely match numerical results. Also, it is observed that when the standard deviation of Gaussian distribution increases, outage probability also increases. When the standard deviation of Gaussian distribution increases the loss in power increases. Therefore, this loss results in increasing the outage probability.

6.3 Outage Probability for Gaussian Distributed Users Based on Signal to Interference Ratio

6.3.1 PDF Evaluation of Signal to Interference Ratio

In this subsection, a solution for PDF of SIR is evaluated. SIR is evaluated based on the effects of large scale fading described by propagation loss and shadowing. The studied area is considered as a circular cell of radius R_m. Users are Gaussian distributed around the BS located at the center of the cell. It is considered that each user has an accessible region of interference. MS experiences interference only from a source located inside its region of interference of radius R_i, where $R_i > R_m$. Interfering sources are located within the accessible region of interference of the user. It is assumed that there is only one interference signal.

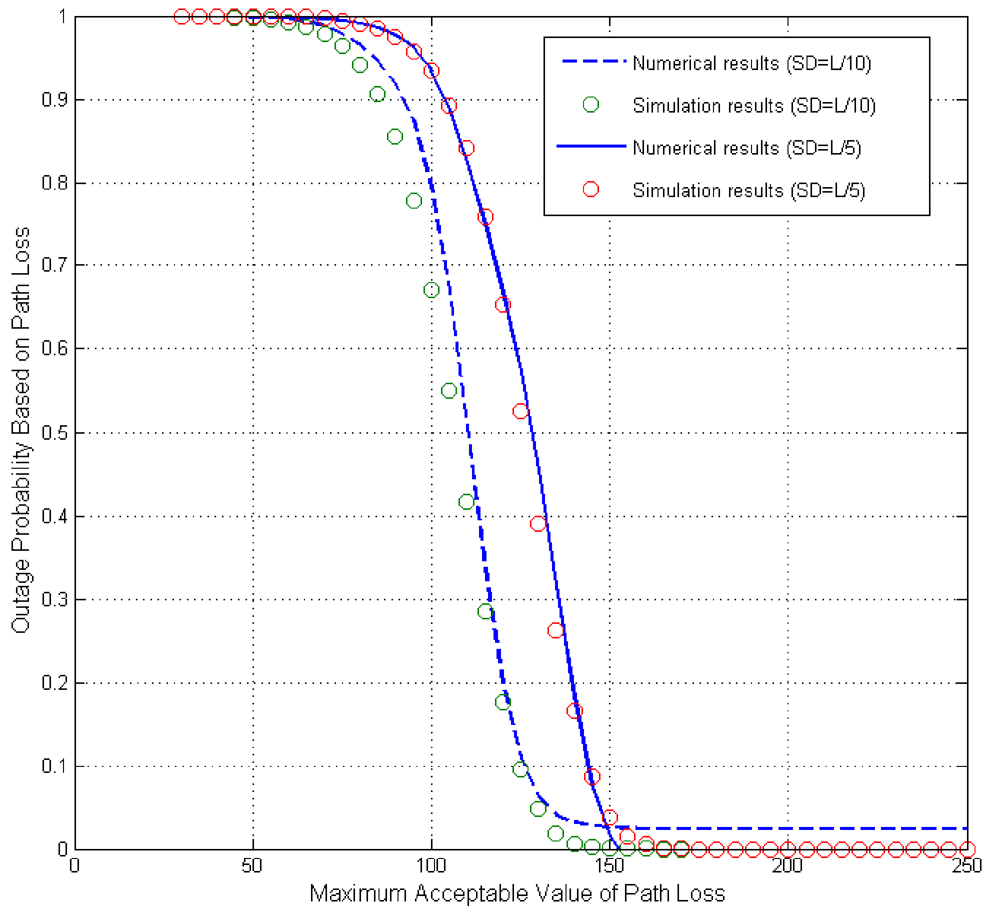

Figure 6.2: Outage probability based on PL threshold for different standard deviation of Gaussian distributed users in a circular cell

First, PDFs of the ratio of propagation loss $D = d_i^{n_i}/d_m^{n_m}$ and shadowing $H = \exp(t\xi_i)/\exp(t\xi_m)$ are found. Then, these two random variables are multiplied together to evaluate PDF of SIR.

PDFs of d_m and d_i are as shown in (6.17) and (6.18) respectively [61]:

$$f_{D_m}(d_m) = \frac{d_m.\exp\{(R_m^2 - 2d_m^2)/4\sigma_G^2\}}{2\sigma_G^2.\sinh\{R_m^2/4\sigma_G^2\}}, \quad 0 < d_m < R_m \tag{6.17}$$

where σ_G is the standard deviation of the Gaussian distribution of users.

$$f_{D_i}(d_i) = \frac{2d_i}{R_i^2}, \quad 0 < d_i < R_i \tag{6.18}$$

Based on the above equations, PDFs of $X = d_m^{n_m}$ and $Y = d_i^{n_i}$ are directly evaluated as follows:

$$f_X(x) = \frac{x^{\frac{2}{n_m}-1}.\exp\left(R_m^2 - 2x^{\frac{2}{n_m}}\right)/4\sigma_G^2}{2n_m\sigma_G^2\sinh\left(R_m^2/4\sigma_G^2\right)}, \quad 0 < x < R_m^{n_m} \tag{6.19}$$

$$f_Y(y) = \frac{2y^{\frac{2}{n_i}-1}}{R_i^2 n_i}, \quad 0 < y < R_i^{n_i} \tag{6.20}$$

Knowing that $D = Y/X$, where X and Y are two independent random variables, from [62], PDF of D, $f_D(d)$ is:

$$f_D(d) = \int f_X(x) f_Y(dx)|x|dx \tag{6.21}$$

Using (6.21), PDF of D is evaluated as follows:

$$f_D(d) = \begin{cases} A \times d^{\frac{2}{n_i}-1} \int\limits_0^{R_m^{n_m}} x^B \exp\left(\frac{-x^{\frac{2}{n_m}}}{2\sigma_G^2}\right)dx, & 0 < d < \mu \\ C \times d^{\frac{2}{n_m}-1} \int\limits_0^{R_i^{n_i}} x^B \exp\left(\frac{-x^{\frac{2}{n_i}}}{2\sigma_G^2}\right)dx, & \mu < d < \infty \end{cases} \tag{6.22}$$

- $A = \frac{\exp(\frac{R_m^2}{4\sigma_G^2})}{n_m n_i R_i^2 \sigma_G^2 \sinh(\frac{R_m^2}{4\sigma_G^2})}$
- $B = \frac{2n_m+2n_i-n_m n_i}{n_m n_i}$
- $C = \frac{\exp(\frac{R_i^2}{4\sigma_G^2})}{n_m n_i R_m^2 \sigma_G^2 \sinh(\frac{R_i^2}{4\sigma_G^2})}$

To solve $f_D(d)$, Gauss quadrature [65] is used. In [65], Gauss quadrature method is divided into two steps. In the first step, the limit of integration is changed from $[0, R_m^{n_m}]$ to $[-1, 1]$ as follows:

$$I = \int\limits_0^{R_m^{n_m}} f(x)dx = \int\limits_{-1}^{1} f(t)dt \tag{6.23}$$

Starting with the first part of $f_D(d)$, let $f(x) = x^B \exp\left(\frac{-x^{\frac{2}{n_m}}}{2\sigma_G^2}\right)$.

Then we change the variables:

$$x = \frac{R_m^{n_m}}{2} \times (1+t) \tag{6.24}$$

$$dx = \frac{1}{2} R_m^{n_m} dt \tag{6.25}$$

As a result, I becomes:

$$I = \int_{-1}^{1} f\left(\frac{R_m^{n_m}}{2}(1+t)\right)\frac{R_m^{n_m}}{2}dt \tag{6.26}$$

In the second step, using the table given in [65], an order is selected for the approximation with coefficients and Gauss points. Then, an approximation for the integration is found as follows:

$$I = \int_{-1}^{1} f(t)dt \approx \frac{R_m^{n_m}}{2}\sum_{n=1}^{k} C_n f\left(\frac{R_m^{n_m}}{2}(1+t_n)\right) \tag{6.27}$$

where k is the order of approximation, C_n are the coefficients and t_n are Gauss points [65]. Applying the same method with the second part $f_D(d)$, as a result $f_D(d)$ is evaluated as follows:

$$f_D(d) \approx \begin{cases} Ad^{\frac{2}{n_i}-1}\frac{R_m^{n_m}}{2} \times \sum_{n=1}^{k} C_n f\left(\frac{R_m^{n_m}}{2}(1+t_n)\right), & 0 < d < \mu \\ Cd^{\frac{2}{n_m}-1}\frac{R_i^{n_i}}{2} \times \sum_{n=1}^{k} C_n f\left(\frac{R_i^{n_i}}{2}(1+t_n)\right), & \mu < d < \infty \end{cases} \tag{6.28}$$

Once PDF of propagation loss is assessed, PDF of the ratio of shadowing $H = \exp(\beta\xi_i)/\exp(\beta\xi_m)$ is evaluated. In [62], it is given that the ratio of two independent log-normal random variables is a log-normal variable with mean $\mu = \mu_i - \mu_m$ and variance $\sigma^2 = \sigma_m^2 + \sigma_i^2$. Therefore, PDF of shadowing has the following log-normal distribution:

$$f_H(h) = \frac{1}{h\sigma\sqrt{2\pi}}\exp\left(\frac{-(\ln h)^2}{2\sigma^2}\right), 0 < h < \infty \tag{6.29}$$

PDF of SIR is evaluated by multiplying PDFs of two independent random Variables D and H. PDF of SIR, $f_{SIR}(\gamma)$, is:

$$f_{SIR}(\gamma) = \int f_D(\frac{s}{h}) f_H(h) \frac{1}{h} dh \tag{6.30}$$

where γ is the signal to interference ratio.

Substituting (6.28) and (6.29) in (6.30):

$$f_{SIR}(\gamma) = \begin{cases} \frac{A\gamma^{\frac{2}{n_i}-1} R_m^{\frac{n_m}{n_i}}}{2\sigma\sqrt{2\pi}} \sum_{n=1}^{k} [C_n f(t_n)] \times \int h^{\frac{-2}{n_i}-1} \exp\left(- \frac{(\ln h)^2}{\sigma^2} \right) dh, & 0 < \gamma < \mu \\ \frac{C\gamma^{\frac{-2}{n_m}-1} R_i^{\frac{n_i}{n_m}}}{2\sigma\sqrt{2\pi}} \sum_{n=1}^{k} [C_n f(t_n)] \times \int h^{\frac{-2}{n_m}-1} \exp\left(- \frac{(\ln h)^2}{\sigma^2} \right) dh, & \mu < \gamma < \infty \end{cases} \tag{6.31}$$

In (6.31), a solution is given for PDF of SIR with Gaussian distribution of users around a BS located at the center of a circular cell.

6.3.2 Evaluation of Outage Probability Based on Signal to Interference Ratio

In this subsection, outage probability based on the evaluated SIR is assessed. As defined previously, outage probability based on SIR is evaluated as follows:

$$P_{out}(\gamma_{min}) = P(\gamma < \gamma_{min}) = F_{SIR}(\gamma_{min}) = \int_0^{\gamma_{min}} f_{SIR}(\gamma) d\gamma \tag{6.32}$$

Therefore, outage probability based on SIR for Gaussian distributed users is evaluated as follows:

$$P_{out}(\gamma_{min}) = \begin{cases} \frac{An_i R_m^{\frac{n_m}{n_i}} \gamma_{min}^{\frac{2}{n_i}}}{4\sigma\sqrt{2\pi}} \sum_{n=1}^{k} [C_n f(t_n)] \times \int h^{\frac{-2}{n_i}-1} \exp\Bigg(-\frac{(\ln h)^2}{\sigma^2}\Bigg) dh, & 0 < \gamma_{min} < \mu \\ 1 - \frac{Cn_m R_i^{\frac{n_m}{n_i}} \gamma_{min}^{\frac{-2}{n_m}}}{4\sigma\sqrt{2\pi}} \sum_{n=1}^{k} [C_n f(t_n)] \times \int h^{\frac{-2}{n_i}-1} \exp\Bigg(-\frac{(\ln h)^2}{\sigma^2}\Bigg) dh, & \mu < \gamma_{min} < \infty \end{cases} \tag{6.33}$$

6.3.3 Numerical and Simulation Results

In this subsection, PDF of SIR found in (6.31) and outage probability found in (6.33) are numerically evaluated based on the parameters shown in Table 3.1, and the results are shown in Figures 6.3 and 6.4, respectively.

Furthermore, a wireless system is simulated 100,000 times each for a user located in a circular cell of radius $R_m = 1000m$ to validate the numerical results. Each user is affected by one interfering source located inside its region of interference R_i. In Figures 6.3 and 6.4, it is shown that the simulation results closely match numerical results derived in this chapter.

In Figure 6.3, numerical and simulation results of PDF of SIR are shown for different sizes of region of interference R_i. In Figure 6.4, numerical and simulation results are presented for Gaussian distributed users with different standard deviations. It is noticed from Figure 6.4 that increasing the standard deviation of Gaussian distribution results in higher outage probability. When standard deviation of Gaussian distribution increases, users became sparse in the cell and the loss increases. Therefore, this loss results in increasing the outage probability.

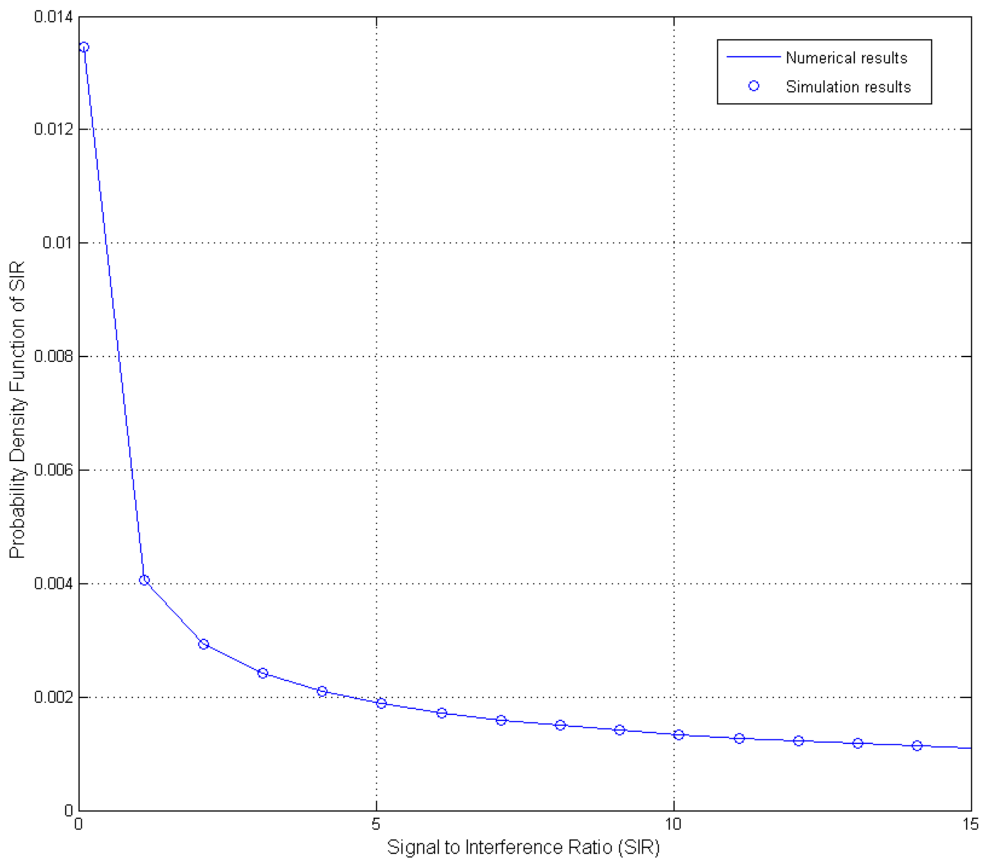

Figure 6.3: PDF of SIR for Gaussian distributed users in a circular cell for different cell sizes

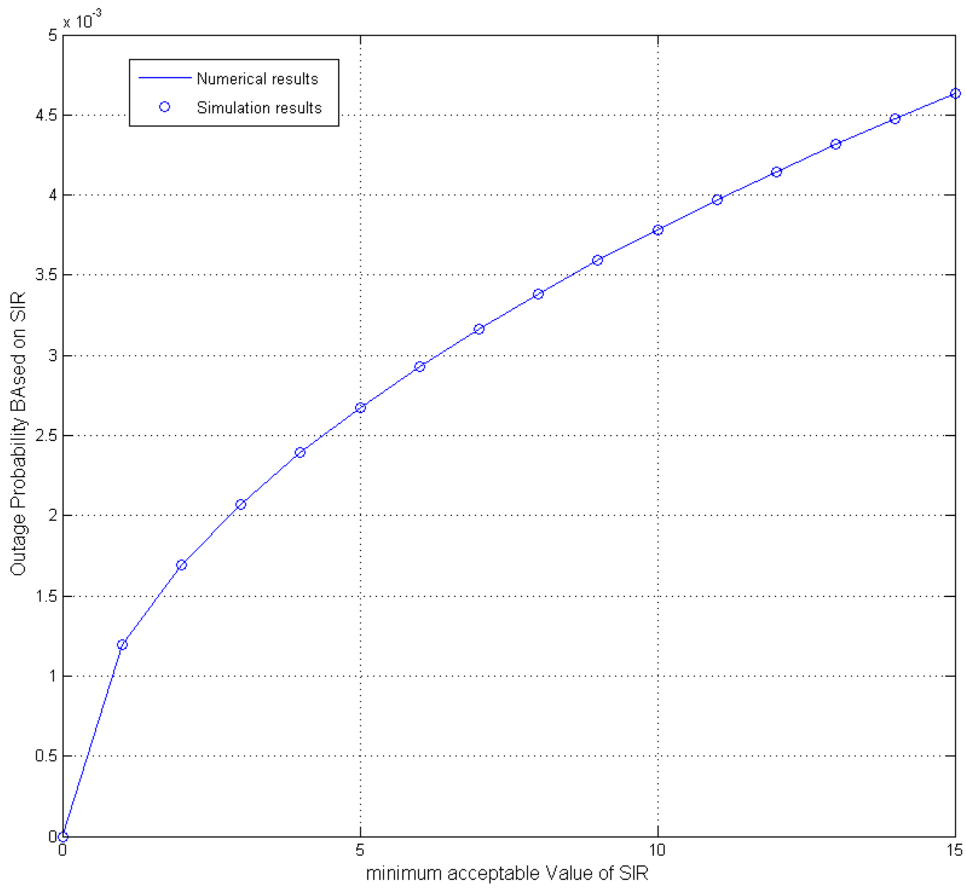

Figure 6.4: Outage probability based on SIR threshold for Gaussian distributed users in a circular cell

6.4 Conclusion

Gaussian distribution is an example of non-uniform distribution of users in a wireless system. In this chapter, it was considered that users follow Gaussian distribution around a BS located at the center of circular cell. Then, outage probability based on path loss was evaluated in presence of large scale fading. Also, PDF of SIR was analytically evaluated. Outage probability was found based on SIR. Finally, following the analytical derivation, simulation was performed to verify the results.

Chapter 7

Conclusion and Future Works

In wireless networks, there is a direct link between the BS and each MS. As a result, the probability density function was analytically determined for different performance metrics: signal to interference ratio (SIR), signal to noise ratio (SNR) and signal to interference plus noise ratio (SINR). To generalize the results, these performance metrics were evaluated by taking into consideration the random distribution of users with respect to a reference point (BS), the cell shape and the random losses affecting the signal. Then, the evaluated results were used to assess outage probability based on each performance metric. Moreover, numerical and simulation results were obtained to validate the analytical outcomes.

As an application of the work done related to outage probability, dual-hop networks were considered. Distance between BS and users is a main factor that diminishes the quality of the signal in a single-hop wireless system. Therefore, in a dual-hop network, relays were deployed over the studied area to play the role of middle nodes by helping the BS transmit to the users located far away. Thus, the bad quality direct link between the transmitter and receiver is replaced by two short and better quality links through an intermediary node.

Outage probability evaluated for single-hop networks was used to evaluate the performance of dual-hop network. It was found that the number of deployed relays and their location are two factors that control the performance of dual-hop networks. Hence, an approach was developed to find the required number of relays needed and their placement over the studied area to reach a desired outage probability.

Although the uniform distribution of users was considered, it was shown that it cannot reflect the real life distribution of users in all the cases. In addition, in some cases, Gaussian distribution describes the position of users over the studied area. Therefore, Gaussian distribution of users was considered and PDF of SIR was evaluated. Moreover, outage probability based on path loss and SIR was analytically assessed.

Finally, the novel contributions of this dissertation can be used in the future to develop our understanding of random wireless networks. Although generic and accurate results were obtained for critical parameters in wireless networks, some enhancements can be added in the future to evaluate the performance of wireless systems more accurately:

- In this dissertation one source of interference was considered. As a result, it will be interesting to evaluate the performance with respect to multiple interfering sources.
- Users were considered to be randomly distributed but stationary over the studied area. As a result, to model a further realistic network, effect of mobility should be taken into consideration.
- It would be interesting if the expressions found in this dissertation are used to evaluate the performance of wireless networks where there are different distributions of users in the same cell.

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Appendix A

Appendices

A.1 Outage Probability Based on Path Loss for Hexagonal Cell

In this appendix, the outage probability of a hexagonal cell of inradius (ρ) and circumradius (a) is evaluated based on path loss. As shown in [42], the probability density function of path loss in a hexagonal cell of inradius ρ and circumradius $a = \frac{2\rho}{\sqrt{3}}$, is:

$$
\begin{aligned}
f_L(l) = \frac{\sqrt{3}\ln(10)}{2\beta\rho^2}\Bigg\{\Bigg(100^{(l-\alpha)/\beta}\exp(Q^2)\times \\
\Bigg\{\mathrm{erfc}(\frac{l}{\sqrt{2}\sigma} - M - S - Q) + \frac{\sqrt{3}}{\sqrt{3}-2}\Bigg[\mathrm{erf}(\frac{l}{\sqrt{2}\sigma} - M - S + Q) - \mathrm{erf}(\frac{l}{\sqrt{2}\sigma} - M - T + Q)\Bigg]\Bigg\}\Bigg) \\
-\Bigg(\frac{2\rho\exp(Q^2/4)10^{(l-\alpha)/\beta}}{\sqrt{3}-2}\times\Bigg[\mathrm{erf}(\frac{l}{\sqrt{2}\sigma} - M - S + Q) - \mathrm{erf}(\frac{l}{\sqrt{2}\sigma} - M - T + Q)\Bigg]\Bigg)\Bigg\}
\end{aligned}
\tag{A.1}
$$

- $Q = \sqrt{2}\sigma b^{-1}\ln(10)$

- $M = 2^{-1/2}\sigma^{-1}d_0$
- $S = 2^{-1/2}\sigma^{-1}b\log(\rho)$
- $T = 2^{-1/2}\sigma^{-1}b\log(a)$

After rearranging the components of (A.1), PDF of path loss becomes:

$$f_L(l) = \overbrace{X100^{l/\beta}\text{erfc}\left(\frac{l}{\sqrt{2}\sigma + W_1}\right)}^{I_1(l)} + \overbrace{Y100^{l/\beta}\left(\text{erf}\left(\frac{l}{\sqrt{2}\sigma + W_1}\right) - \text{erf}\left(\frac{l}{\sqrt{2}\sigma} + W_2\right)\right)}^{I_2(l)}$$
$$- \overbrace{Z10^{l/\beta}\left(\text{erf}\left(\frac{l}{\sqrt{2}\sigma + H_1}\right) - \text{erf}\left(\frac{l}{\sqrt{2}\sigma} + H_2\right)\right)}^{I_3(l)} \tag{A.2}$$

- $X = \frac{\sqrt{3}\ln(10)}{2\beta\rho^2}100^{(-\alpha)/\beta}\exp(Q^2)$
- $Y = \frac{3\ln(10)}{(\sqrt{3}-2)2\beta\rho^2}100^{(-\alpha)/\beta}\exp(Q^2)$
- $Z = \frac{2\sqrt{3}\ln(10)}{(\sqrt{3}-2)2\beta\rho^2}100^{(-\alpha)/\beta}\exp(Q^2/4)$
- $W_1 = -M - S + Q$
- $W_2 = -M - T + Q$
- $H_1 = -M - S + \frac{Q}{2}$
- $H_2 = -M - T + \frac{Q}{2}$

PDF of outage probability is as follows:

$$P_{out}(l) = \int_{l_{max}}^{\infty} f_L(l)dl = 1 - \int_{0}^{l_{max}} f_L(l)dl \tag{A.3}$$

Knowing that $10^x = e^{x\ln(10)}$, the results of these three integrations are as follows:

$$\int_0^{l_{max}} I_1(l)dl = \frac{X\beta 100^{\frac{-W_1\sqrt{2}\sigma}{\beta}}}{\ln(100)} \times \Bigg[e^{(\frac{\sqrt{2}\sigma\ln(100)}{2\beta})^2}\operatorname{erf}\left(\frac{l_{max}}{\sqrt{2}\sigma} - \frac{\sigma\ln(100)}{\sqrt{2}\beta} + W_1\right) +$$

$$100^{\left(\frac{W_1\sqrt{2}\sigma + l_{max}}{\beta}\right)}\operatorname{erfc}\left(\frac{l_{max}}{\sqrt{2}\sigma} + W_1\right) - e^{(\frac{\sqrt{2}\sigma\ln(100)}{2\beta})^2}$$

$$\operatorname{erf}\left(-\frac{\sigma\ln(100)}{\sqrt{2}\beta} + W_1\right) - 100^{\frac{W_1\sqrt{2}\sigma}{\beta}}\operatorname{erfc}(W_1)\Bigg] \tag{A.4}$$

$$\begin{aligned}\int_0^{l_{max}} I_2(l)dl = \Bigg\{\frac{Y\beta 100^{\frac{-W\sqrt{2}\sigma}{\beta}}}{\ln(100)} \times \Bigg[100^{\frac{W\sqrt{2}\sigma}{\beta}}\left(100^{\frac{l_{max}}{\beta}}\operatorname{erf}\left(\frac{l_{max}}{\sqrt{2}\sigma} + W\right) - \operatorname{erf}(W)\right) - \\ e^{(\frac{\sigma\ln(100)}{\sqrt{2}\beta})^2}\left(\operatorname{erf}\left(\frac{l_{max}}{\sqrt{2}\sigma} - \frac{\sigma\ln(100)}{\sqrt{2}\beta} + W\right) - \operatorname{erf}\left(-\frac{\sigma\ln(10)}{\sqrt{2}\beta} + W\right)\right)\Bigg]\Bigg\}_{W=W_2}^{W=W_1}\end{aligned} \tag{A.5}$$

$$\begin{aligned}\int_0^{l_{max}} I_3(l)dl = \Bigg\{\frac{Z\beta 10^{\frac{-H\sqrt{2}\sigma}{\beta}}}{\ln(10)} \times \Bigg[10^{\frac{H\sqrt{2}\sigma}{\beta}}\left(10^{\frac{l_{max}}{\beta}}\operatorname{erf}\left(\frac{l_{max}}{\sqrt{2}\sigma} + H\right) - \operatorname{erf}(H)\right) - \\ e^{(\frac{\sigma\ln(10)}{\sqrt{2}\beta})^2}\left(\operatorname{erf}\left(\frac{l_{max}}{\sqrt{2}\sigma} - \frac{\sigma\ln(10)}{\sqrt{2}\beta} + H\right) - \operatorname{erf}\left(-\frac{\sigma\ln(10)}{\sqrt{2}\beta} + H\right)\right)\Bigg]\Bigg\}_{H=H_2}^{H=H_1}\end{aligned} \tag{A.6}$$

So, outage probability of a hexagonal cell is evaluated by using these three integrations:

$$P_{out} = 1 - \frac{X\beta 100^{\frac{-W_1\sqrt{2}\sigma}{\beta}}}{\ln(100)} \times \Bigg[e^{(\frac{\sqrt{2}\sigma\ln(100)}{2\beta})^2}\operatorname{erf}\left(\frac{l_{max}}{\sqrt{2}\sigma} - \frac{\sigma\ln(100)}{\sqrt{2}\beta} + W_1\right)$$

$$100^{\left(\frac{W_1\sqrt{2}\sigma+l_{max}}{\beta}\right)}\text{erfc}\left(\frac{l_{max}}{\sqrt{2}\sigma}+W_1\right)-e^{(\frac{\sqrt{2}\sigma\ln(100)}{2\beta})^2}\text{erf}\left(-\frac{\sigma\ln(100)}{\sqrt{2}\beta}+W_1\right)-100^{\frac{W_1\sqrt{2}\sigma}{\beta}}\text{erfc}(W_1)\Bigg]$$

$$+\Bigg\{\frac{Y\beta 100^{\frac{-W\sqrt{2}\sigma}{\beta}}}{\ln(100)}\times\Bigg[100^{\frac{W\sqrt{2}\sigma}{\beta}}\left(100^{\frac{l_{max}}{\beta}}erf\left(\frac{l_{max}}{\sqrt{2}\sigma}+W\right)-erf(W)\right)-$$

$$e^{(\frac{\sigma\ln(100)}{\sqrt{2}\beta})^2}\left(\text{erf}\left(\frac{l_{max}}{\sqrt{2}\sigma}-\frac{\sigma\ln(100)}{\sqrt{2}\beta}+W\right)-\text{erf}\left(-\frac{\sigma\ln(10)}{\sqrt{2}\beta}+W\right)\right)\Bigg]\Bigg\}_{W=W_2}^{W=W_1}$$

$$-\Bigg\{\frac{Z\beta 10^{\frac{-H\sqrt{2}\sigma}{\beta}}}{\ln(10)}\times\Bigg[10^{\frac{H\sqrt{2}\sigma}{\beta}}\left(10^{\frac{l_{max}}{\beta}}\text{erf}\left(\frac{l_{max}}{\sqrt{2}\sigma}+H\right)-\text{erf}(H)\right)-$$

$$e^{(\frac{\sigma\ln(10)}{\sqrt{2}\beta})^2}\left(\text{erf}\left(\frac{l_{max}}{\sqrt{2}\sigma}-\frac{\sigma\ln(10)}{\sqrt{2}\beta}+H\right)-\text{erf}\left(-\frac{\sigma\ln(10)}{\sqrt{2}\beta}+H\right)\right)\Bigg]\Bigg\}_{H=H_2}^{H=H_1} \quad \text{(A.7)}$$

Simulation and numerical results of outage probability in a hexagonal cell, which is derived in (A.7), is obtained based on the parameters in Table 4.1, as shown in Figure A.1.

Also, comparison between PDF of outage probability for hexagonal and circular cells is presented in Figure A.2.

A.2 Outage Probability Based on Path Loss for a Sector in a Circular Cell

In this appendix, a solution for outage probability of a sector in a circular cell based on path loss is evaluated. Based on [37], PDF of path loss for a specified sector, as shown in Figure A.3, of the cell is:

$$f_L(l)=\overbrace{\frac{E}{2}\times 10^{\frac{2l}{\beta}}\times\text{erf}(\frac{l}{\sqrt{2}\sigma}+D_1)}^{I_1(l)}-\overbrace{\frac{E}{2}\times 10^{\frac{l}{\beta}}\times\text{erf}(\frac{l}{\sqrt{2}\sigma}+D_2)}^{I_2(l)} \quad \text{(A.8)}$$

- $E=\frac{210^c d_0^2\ln 10}{\beta(L_2^2-L_1^2)}$

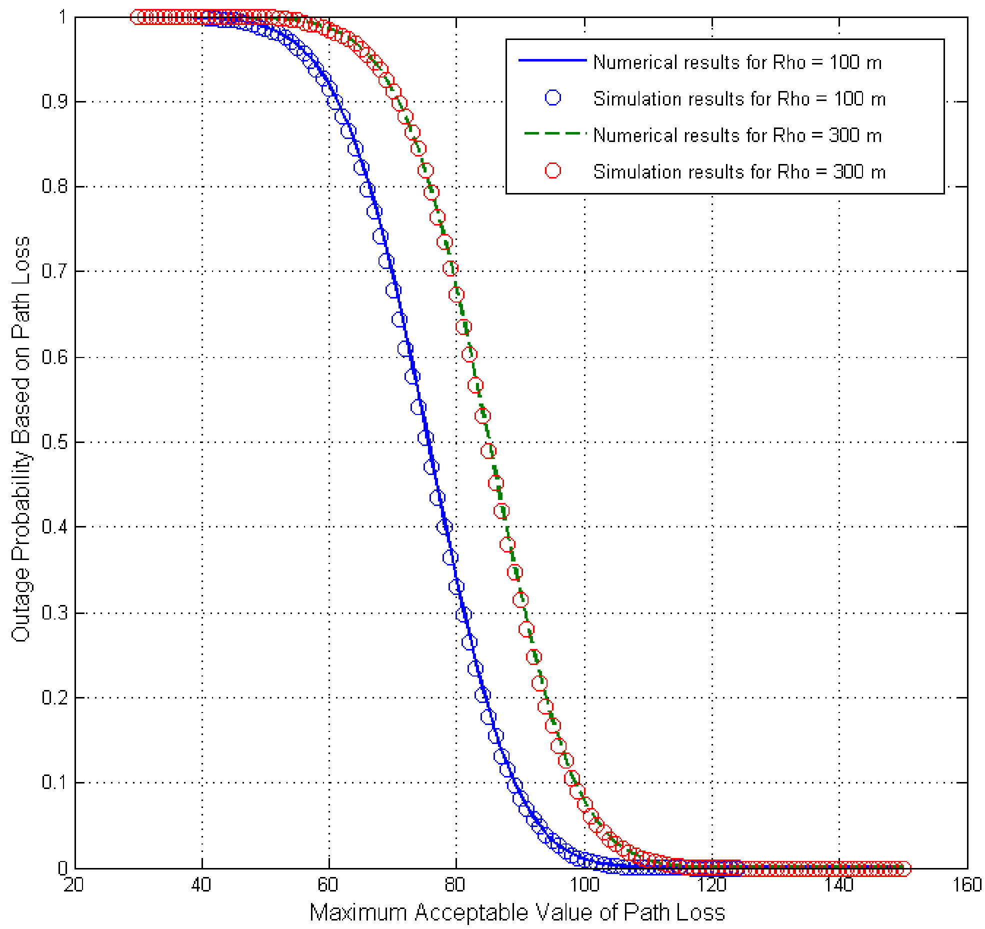

Figure A.1: Outage probability based on PL in a hexagonal cell for different cell sizes

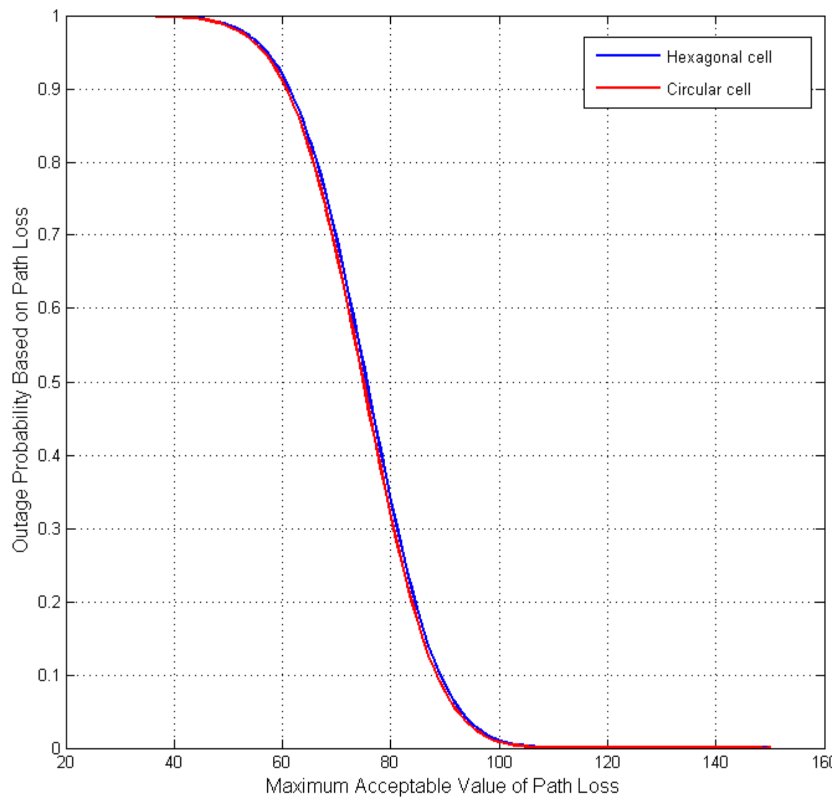

Figure A.2: Outage probability based on path loss for circular versus hexagonal cells

- $C = \frac{2\ln(10)\sigma^2 - 2\alpha\beta}{\beta^2}$

- $D_1 = \frac{-\alpha + \beta\log(d_0) + \frac{2\ln(10)\sigma^2}{\beta} \quad - \beta\log(L_1)}{\sqrt{2}\sigma}$

- $D_2 = \frac{-\alpha + \beta\log(d_0) + \frac{2\ln(10)\sigma^2}{\beta} \quad - \beta\log(L_2)}{\sqrt{2}\sigma}$

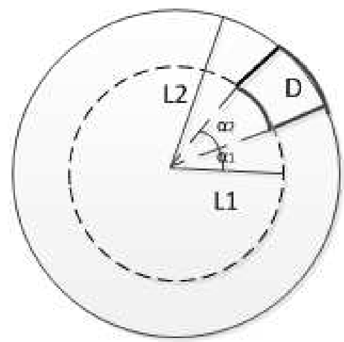

Figure A.3: Circular cell with area of interest "D"

PDF of outage probability is as follows:

$$P_{out} = 1 - \int_{0}^{l_{max}} f_L(l) \quad dl = 1 - \int_{0}^{l_{max}} I_1(l)dl + \int_{0}^{l_{max}} I_2(l)dl \tag{A.9}$$

After many manipulations, outage probability of a specific sector of a circular cell is as follows:

$$P_{out} = 1 - \Bigg[\frac{E\beta 100^{\frac{-D\sqrt{2}\sigma}{\beta}}}{2\ln(100)} \times \Bigg[100^{\frac{-D\sqrt{2}\sigma}{\beta}} \left(100^{\frac{l_{max}}{\beta}} \times erf\left(\frac{l_{max}}{\sqrt{2}\sigma} + D \right) - \mathrm{erf}(D) \right) - \exp\left(\frac{\sigma\ln(100)}{\sqrt{2}\beta} \right)^2 \left(\mathrm{erf}\left(\frac{l_{max}}{\sqrt{2}\sigma} - \frac{\sigma\ln(100)}{\sqrt{2}\beta} + D \right) - \mathrm{erf}\left(- \frac{\sigma\ln(100)}{\sqrt{2}\beta} + D \right) \right) \Bigg] \Bigg]_{D_2}^{D_1} \tag{A.10}$$

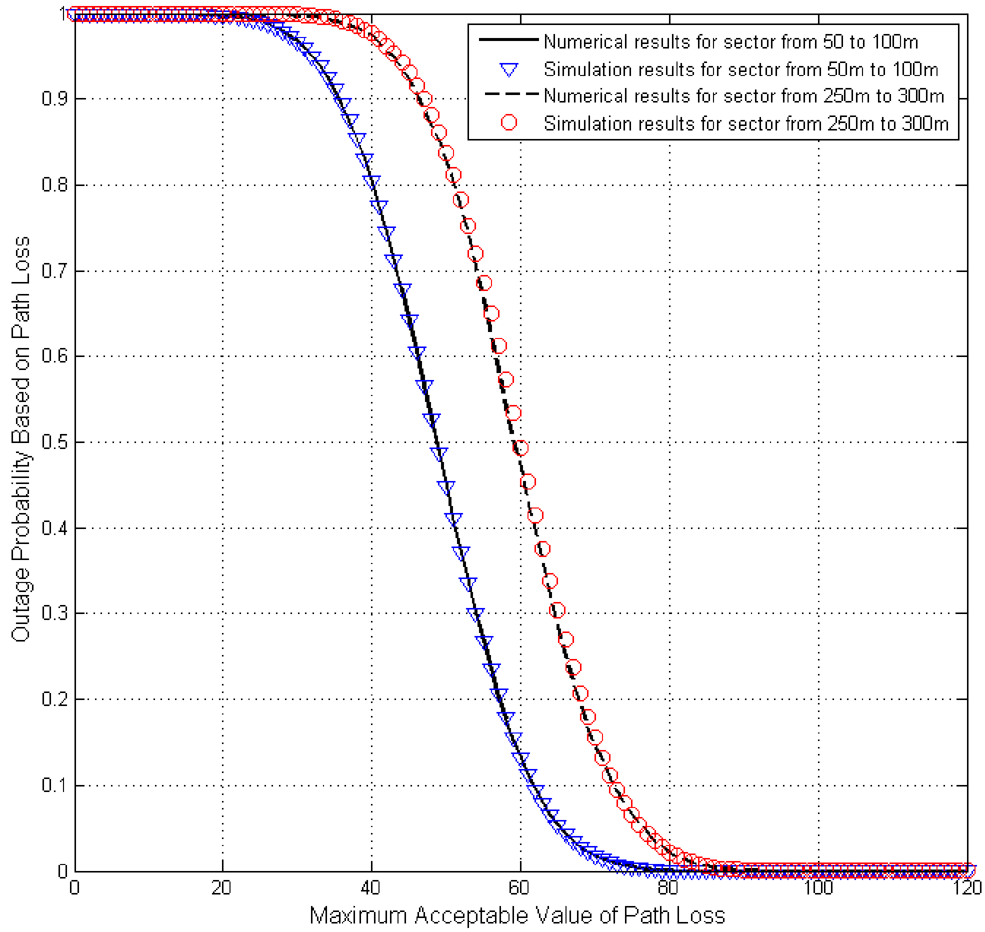

Figure A.4: Outage probability of different sectors of a circular cell based on PL

Finally, numerical and simulation results of outage probability, which is derived in (A.10), are presented based on parameters of Table 4.1, as shown in Figure A.4.

For instance, outage probability based on path loss in a circular cell of radius $R = 300m$ is equal to 0.71, while outage probability based on path loss varies from 0.2 to 0.84 along the radius of the cell.

Table A.1: Outage probability of different sectors

Sector	Outage Probability
0 → 50m	0.2
50m → 100m	0.45
100m → 150m	0.6
150 → 200m	0.7
200 → 250m	0.8
250 → 300m	0.84
0 → 300m	0.71